A Quantitative Model of Perception of Randomness in Structured Space

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

The literature on the perception and generation of randomness suggests that people's conception of randomness deviates from true randomness in predictable and consistent ways. In general, people see patterns or repetitions as evidence of non-randomness (Nickerson, 2002). In the 2D domain (e.g., grids) in particular, people believe that random chance occurrences do not fall in clusters, in similar locations (e.g., same row or column), or on the corners and edges of the space (Falk, Falk & Ayton, 2009). A common explanation given is that spread-out and patternless occurrences in the interior of the grid are locally representative of what people believe random chance occurrences to look like. But why do people make this association in the first place? Given that random sequences are minimally compressible (Nickerson, 2002), Falk and Konold (1997) suggest that people judge the randomness of a sequence based on their tacit ability to encode it. Cells that are clustered or located on the edges of a grid are easier to encode (i.e., describe, memorize) and are thus judged as less random. This explanation, however, fails to account for people's strong preference for the center of the grid, which is in itself a location that is easy to encode. Additional explanations based on positional biases for the center (Christenfeld, 1995) and variety seeking tendencies, including diversification bias (Read & Lowenstein, 1995), choice bracketing (Read, Lowenstein & Rabin, 1999), partition dependence (Fox, Ratner, & Lieb, 2005), and distinctiveness (Ayal & Zakay, 2009), only add to a patchwork of theories that cannot in themselves provide a complete explanation of the observed behaviour. Therefore, the main research objective of this thesis is to formally characterize and explain people's choices when they generate random selections in structured two-dimensional space.

In Study 1, people's choices were formally observed in a controlled experiment. Participants searched for an item (prize) that was hidden in a 9x9 grid by a random process. Trying to 'match' that random process, they generated selections that avoided the edge of the grid, and were spread out such that they were rarely near each other or in the same row or column. Based on analysis of data from Study 1 as well as data from Falk et al. (2009), we observed that people group cells in a 2D grid by proximity - e.g., cells in the immediate vicinity of a selected cell, or by similarity - e.g., cells in the same row or column. Cells pertaining to a group are judged as having similar attributes, including similar probability assessments. Given a selected cell, we defined its 'coverage' to be the perceptually-formed grouping to which it belongs: fundamentally, a cell 'covers' similar or nearby cells. We then proposed that people judge the randomness of selected cells by their perceived coverage: the higher the coverage, the more random are the cells perceived. The effect of the grouping size on a group element's judged probability was confirmed in Study 2. Based on the above, we designed a quantitative model that evaluates coverage in a 2D grid, taking into account two factors. First, the size of an individual cell's coverage is directly affected by the size of the grid. Second, the aggregate coverage of multiple cells is not equal to the sum of coverages of the individual cells; rather, the calculation takes into account the amount of overlap between those individual cells' coverages. It was shown that individual cells that fall on the edges and sets of cells that are clustered together all have low coverage values. We tested the validity of the model in two experiments (Studies 3A and 3B) conducted using different 2D 6-cell arrangements. Study 3A showed that a cell's calculated coverage predicts participants' judgements of its randomness. Study 3B showed that the pairs of cells that participants judge to be most random have the highest aggregate coverage, as predicted, but a low sum of their individual cells' coverages. In conclusion, our model of coverage provides a self-contained explanation and prediction of people's perception of randomness in structured 2D spaces. Notably, the model provides a single explanation for both the edge avoidance and spreading aspects of choice.

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The opportunity to pursue extended studies at the University of Waterloo is in itself a testament to the selfless sacrifices of my parents Mira and Perparim Zacaj. They were smart and hardworking engineers whose talents were underutilized in communist and later post-communist Albania. They later sacrificed their personal successes and ambitions to give me and my sister Linda a life of dignity and opportunity in Canada. I am humbled and thankful.

And finally, I want to thank my husband Adam who has supported the completion of this thesis not only emotionally – as is often the case with spouses – but also technically. He was instrumental in the programming of the computer interfaces for Study 1 and Study 2 , often working on the computer code for hours in the evenings. Adam, you are an amazing husband, friend, collaborator, and dad.

Dedication

To the memory of my dear uncle Valentin Prifti, who is sorely missed.

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1. Introduction

Consider the following scenario: You and a friend are playing a game of cards. You shuffle the deck well, spread the cards on a table and give your friend three guesses to find the ace of hearts. Your friend will likely not pick three back-to-back cards; rather, she will try to select three ‘random’ cards, spreading her three choices and thus selecting cards from the beginning, middle, and end ‘regions’ of the deck, in a manner similar to that shown in Figure 1. She will likely also avoid selecting cards at both ends of the deck.

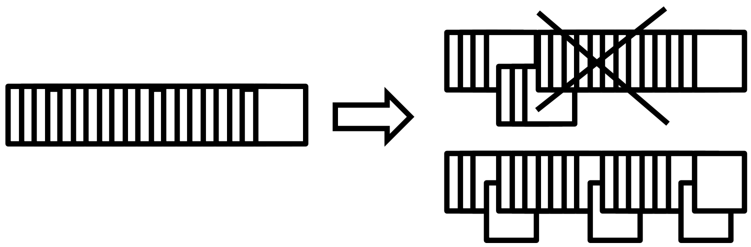


Figure 1 An imaginary game: Find a specific card in a well-shuffled deck

Due to two particular characteristics of the stimulus - the low probability that any of the cards is the ace of hearts and the small number of guesses available to find the desired card - the overall probability of success is just 3/52. Given that the deck was well-shuffled, any of the cards in the deck had exactly the same probability of being the ace of hearts. One could argue that given the equal odds, the least effortful choice would be to simply select the first 3 cards. Instead, it is easily imagined that people are more likely to try and produce what they consider a ‘random’ set of selections. At the moment, without further consideration of whether one choice is more random than the other, at least two questions arise: Why do people not choose three consecutive cards? And why do they avoid the very beginning and end cards in the deck?

The main research objective of this thesis is to formally characterize and explain the combined spreading and edge avoidance phenomena described above. In this thesis we propose that the simple example described above illustrates some particular tendencies: First, the structure of the problem space (i.e., the cards on the table) is subject to perceptual processes on the part of the observer, who will perceive it as broken down into separate groupings (e.g., beginning, middle, and end). Second, elements within the same groupings are assumed to have some level of dependence on each other, such that the probability of one element affects the assumed probability of other elements in its grouping. In the cards example, once a card is selected, the player is less likely to select other cards immediately next to it. Third, the attributes (e.g., location, size) of these subjectively formed groupings affect the subjective assessments of probability of the grouping's elements (e.g., cards in the middle are more likely to be selected than cards at the ends of the deck).

This thesis is structured in the following way: The remainder of this chapter provides further evidence for the observed spreading phenomenon in existing literature as well as in a new experimental study (Study 1). In Chapter 2 we search for an explanation for the phenomenon in existing literature, evaluate potential existing explanations, and after finding them lacking, lay the groundwork for a new theoretical explanation. According to the proposed theory, spreading behaviour results from two drives. First, people avoid searching close to a location where search has been previously unsuccessful (discussed in Chapter 3). Second, people seek to explore a new, large region (Chapter 4, Study 2). We then formalize this theory and test it by predicting and observing people's selections in small 6-item search spaces (Chapter, 5, Study 3). We conclude with a summary discussion of the contribution and its limitations and ponder future directions for research (Chapter 6).

1.1 Spreading behaviour in the literature

Starting with the assumption that, at least at some level, people try to generate a random series of selections when they draw the three cards from the shuffled deck, we look to find other examples of spreading behaviour in the literature on people's perception and generation of randomness.

Multiple studies and anecdotes confirm that our judgement of whether a chance event is random is often flawed. One of the often quoted stories that confirm biases in the perception of randomness is from the bombing of the city of London during WWII. The bombing did not seem random to its residents because certain areas of the city were bombed more than others. To confirm whether this was true, the city was divided into equal squares and the number of bombs in each square counted. It was found that the bombing had been indeed random (i.e., each area had been hit approximately the same number of times), but the perception of clusters made it seem like it possessed a certain pattern (Feller (1957) as cited by Bar-Hillel and Wagenaar (1991)).

Another example comes from a recent study, in which participants were asked to choose 3 papers from 7 numbered arbitrarily 1-7 (Rubinestein & Salant, 2006). While 39% of participants chose to select papers 1-3 (likely an outcome of a minimal effort 'strategy'), 44% chose one of the [1,4,7], [2, 4, 6], [2,5,7], [1, 4, 6], [1,3,7], and [2,4,7] combinations, which represent just about 17% of the potential choices. Thus, selections that avoided neighbouring papers (i.e., 'spread' selections) were overrepresented.

People's difficulty in producing truly random sequences has been a much debated topic, with the earliest attributions to Reichenbach (1949) and many contributions since (as summarized by Bar-Hillel and

Wagenaar (1991) and Oskarsson, Van Boven, McClelland and Hastie (2009)). In a review of this literature, Nickerson (2002) states that:

> *"People tend to avoid events they presumably believe would be considered evidence of nonindependence – repetitions, long runs, various types of regularities – and consequently produce them less frequently than would a process like that of tossing a coin or rolling a die…Binary sequences that people produce when they are trying to produce random ones have been reported typically to have more alternations (fewer repetitions) than would be expected by chance."* (p. 339)

The bias towards more alternations is of particular interest. In truly random processes, when the outcome is a binary event with equal probability, the probability of alternations (calculated as (r-1)/(n-1), where *r* is the number of runs and *n* is the length of the sequence) should be close to 0.5 (Falk & Konold, 1997). Yet, when people try to mimic the products of such random processes, they tend to create sequences that have a probability of alternation closer to 0.6 (Falk & Konold, 1997; Oskarsson, et al., 2009). In other words, people judge over-alternating sequences as more random. When a sequence has 'clusters' of repeating elements it is judged as less random. This tendency has been shown in both one-dimensional (1D) sequences and two-dimensional (2D) grids (Falk & Konold, 1997).

An early example of over-alternations in the 2D domain comes from Falk (1975), as summarized by Falk, Falk, and Ayton (2009). In one study, participants were shown 10x10 grids of squares and asked to colour 10 of the squares in a random way. In general, participants avoided colouring neighbouring cells and cells on the edge of the grid. In another study, one set of participants coloured 50 of the 100 squares, while another set judged the randomness of the produced grids. Grids that were rated to be most random had a more 'spread' appearance to them – 'clumps' of colored squares were minimized. In both studies, participants avoided marking neighbouring cells as much as possible, thus resulting in high alternation rates.

While the studies reviewed above as well as other examples reviewed by Falk and Konold (1997) are informative, the stimuli they use have significant differences from the problem context we are after. In most of those examples, if the objective had been finding a particular square, the resulting probability of success would have been ½. For example, in the 10x10 matrix, 50 of the 100 squares are coloured. In head/tail-type sequences, the probability of each entity is also ½. These constitute interesting, however special cases. Our particular concern is spreading behaviour in the more general search context, where it is implied that the probability of a singular location to hold the desired object and the number of opportunities one has to guess its location may both be low. Below we summarize a limited number of prior studies that fit these criteria.

In a rare experiment with a 2D stimulus, Lisanby and Lockhead (1991) asked participants to mark where a raindrop might fall in a rectangle drawn on a piece of paper, and then again, to mark a dot in the rectangle in a non-random location. Participants placed their guesses of where the rain-drop might fall (i.e., what locations in the square were random[1]) along the diagonals of the square and avoided the edges and corners of the structure. In the 'non-random' condition, they guessed non-random locations to be dead on the centre, in the corners, and along the edge. The authors' explanation for the observed patterns was that participants tried to place the random selections (i.e., raindrops) in 'regions of balance' away from non-random referents (corners, centre, etc.,) but not exactly at the balance points; rather, they were located slightly away from them, so that they may appear more unpredictable.

In a more recent study by Ayal and Zakay (2009), participants were shown a computer program that randomly generated 3 integers from the 1-100 set and – in the case of the gain condition - were tasked with producing 3 numbers from 1 to 100 in the hopes that at least one of the numbers they came up with would coincide with one of the randomly generated ones. Participants were not shown visually how the integers 1 to 100 might be structured; it is, however, plausible that without any other visual cues participants would simply imagine them arranged in a 1D sequence. The spreading (or, as the authors called it, the perceived diversity) of the numbers selected by participants was measured by their range and standard deviation. When compared to control and loss conditions, the numbers that participants guessed in the gain condition had a large range and standard deviation.

Also in 2009, Falk et al. conducted a series of studies in which they asked participants to mark 3 cells in a 5x5 grid in different conditions: (1) cooperative (where they were told to mark 3 cells so that someone else would be able to easily guess *or* guess the 3 cells that someone else marked hoping for the participant to find them), (2) competitive (where they were told to mark 3 cells so that someone else would not be able to easily guess *or* guess the 3 cells that someone else marked hoping for the participant to not find them), (3) random (the participant was asked to select 3 cells at random), (4) indefinite (the participant was given no direction), and (5) aesthetic (the participant was told to mark the cells so that the 3 selections were pleasing to the eye). It was found that under all conditions certain cells of the grid were more desirable than others; in no condition were all cells equally selected. In the competitive and random conditions in particular – participants preferred to select cells that were internal to the grid compared to cells that were located on the edges. They also avoided selecting adjacent cells.

Finally, in a study of commercially produced (yet human-generated) advent calendars[2], Sanderson (2014) found that in trying to randomly spread out the numbers 1 to 24 in the calendar space, people take into

[1]The results where the same as when the participants were asked to produce a random location, mark the first place they could think of, and predict where the raindrop might fall (Lisanby & Lockhead, 1991).

[2] "*An advent calendar is a collection of 24 containers, labelled 1, …, 24 and each containing some surprise. Every day from December 1 to 24, one opens the container labelled with that day's date to obtain the surprise*."(Sanderson, 2014, p.2)

account the location of previous numbers, such that the next location is chosen far from the previous locations.

Based on the evidence from the studies reviewed above, there is reason to believe that when people attempt to generate random selections in 1D or 2D spaces, predictable patterns arise such as the 'spread' of selections and the differentiation between the edge and the internal space. In this thesis, we are interested in the generation of these patterns in the context of searching for a randomly 'hidden' item in a structured space, generally assuming a low overall probability of success. Given a structured 2D search space – i.e., one in which the individual locations are ordered in rows and columns - it is believed that perception has an important role in subjective assessments of probability. People 'see' distinct groupings; for example, the edge is perceived as distinct from the more internal region of the space and individual cells are perceived in groupings such as quadrants, rows, and columns. People also avoid selections in locations that are close or similar to prior selections. Based on scattered findings from the small number of related studies (reviewed above), and given a context in which people are searching for a randomly located item in a structured space, we formulate the following predictions:

Prediction 1: *People will avoid selecting locations at the edge of the search space.*

Prediction 2: *People will avoid selecting cells in the vicinity of prior (unsuccessfully guessed) locations*

Prediction 3: *People will avoid selecting locations that are in the same row or column as prior (unsuccessfully guessed) selections*

1.2 Study 1 – Controlled observation of choices in structured 2D space

Method. Participants were brought into an office and asked to sit in front a computer monitor. The monitor showed 81 small squares arranged in a 9x9 grid. The task was framed as a game the objective of which was finding a randomly placed item. Participants were told that a random (computerized) process had assigned a $10 gift certificate to one of the cells in the grid. They were then asked to make three selections; each selection represented an opportunity to uncover the square that contained the prize. After each selection, participants received feedback on whether it had been successful. A screen capture of the computer interface used to administer the study is shown in Appendix A.1. At the conclusion of the task, participants were interviewed by the experimenter and asked to answer open-ended questions, such as "Tell me, how did you go about making your selections?", "You seem to have avoided the edges – was that on purpose?", and "You seem to have spread your choices – why is that?" All interviews were audio recorded and later transcribed.

Participants. The study was completed by 37 participants. All were students in two undergraduate courses in Management Sciences at the University of Waterloo, who completed the study for course credit. No demographic statistics were formally recorded.

Results. The selections of all participants were collected and their coordinates analyzed. Specific properties of the selections were identified as summarized in Table 1. (More detail about how each property is calculated for each participant is provided in Appendix A.3.)

Table 1: Search behaviour property descriptions

Property	Description
Distance from Edge	Smallest distance from the edge of the search space to the three selections of each participant, averaged amongst all participants
Minimum Distance	Minimum (Euclidian)[1] distance between selections for each participant, averaged amongst all participants
Area	Area of triangle created by the three selections of each participant, averaged amongst all participants
Same XY	Number of pairs of selections that fall in the same row or column, averaged amongst all participants

A series of 90000[2] selections (of 3 locations each) were randomly generated to approximate the theoretical attributes of a truly random distribution of choices. Given this assumption, participant selections were compared to the theoretical attributes using a 1-sample student's t-test. The results of the comparison are summarized in Table 2.

Table 2 Comparison of participant selections to randomly generated sequences of selections

Property	**Theoretical mean**	**Study participants (N=37)**		**t-statistic (36 df)**	**p-value**
		Mean	Standard Deviation		
Distance from Edge (units)	0.24	0.46	0.51	2.60	0.02
Minimum Distance (units)	2.83	3.85	1.39	4.46	0.00
Area (square units)	6.33	8.62	6.29	2.22	0.04
Same XY (# of occurrences)	0.60	0.16	0.37	7.10	0.00

[1] Using city-block (Manhattan) distance yields a similar effect.
[2] 90000 instances of randomly generated sequences of three selections was a number large enough to allow the identified measures to converge to two decimal places of accuracy

When compared to randomly generated ones, participant selections are on average more distant from the grid's edge; as predicted participants showed a tendency to avoid making selections on or near the edge. In addition, again when compared to randomly generated ones, participant selections have on average a high minimum distance between them. In a similar vein, the area of the triangle created by the selections of each participant is also larger. These two findings demonstrate that, as predicted, participants avoided clustering their choices and instead chose to spread them in the grid. Finally, participants avoided making selections that fell in the same row or column. Overall Study 1 strongly supported our three predictions, providing a baseline characterization of human behaviour in the context of searching for a randomly located item in a structured space.

Of particular interest are the opposing tendencies described by the first and second predictions. The first suggests that people avoid the edges, while the latter suggests that they try to spread their guessed locations. Of course maximum spreading would be achieved if all selections were on the edge. This is certainly not the case with the observed selections. Taking a closer look, we observe how far the participants place their second selection in relation to the first selection and the edge. This distance is presented as a ratio to the maximum possible distance (the latter calculated as distance of the first selection from the edge). Interestingly, over 67% of the participants placed their second selection at 0.5 - 0.7 of the maximum distance. They chose to make their second selection far from the first, but not too far; the desire to spread was counteracted by the desire to stay away from the edge.

We also analyzed the qualitative data collected through participant interviews, which were transcribed and analyzed using QSR NVivo 8 software. The complete transcribed text is presented in Appendix A.6, while Table 3 below summarizes the main types of comments that were gathered by the participants when describing their 'strategy' in searching for the square containing the prize.

Table 3 Summary of properties of the search as described in the participants' interviews

Property	Description
Spread 26 references from 22 (59.5%) respondents	Participants try to space out their selections as much as possible, often with the goal of 'covering' the most area. Sometimes, the search area is divided in regions, and then each selection 'samples' one region. *See Appendix A.4.1 for instances of descriptions of this behaviour*
Random selections 26 references from 22 (59.5%) respondents	Participants describe their 3 selections as random. *See Appendix A.4.2 for instances of descriptions of this behaviour*
Distinction between 'middle' and 'edge, corners' of the grid 35 references from 25 (67.6%) respondents	Participants differentiate between the middle/centre and the edge/corners. The prize is perceived to be in the middle, and not likely at all to be in the edges or corners. *See Appendix A.4.3 for instances of descriptions of this behaviour*

Changing of region after negative feedback 18 references from 16 (43.2%) respondents	Participants do not like to place a selection too close to a previous selection. Once a selection is revealed to be bad, the next selection will be far away from the first, in a different area. *See Appendix A.4.4 for instances of descriptions of this behaviour*

A majority of the respondents commented on their strategy to spread throughout the grid, in order to 'cover' as much of the search space as possible.

> *"I don't know I just have a feeling that if I cover a lot more space I'd have a greater chance."*

The spreading/covering strategy was realized by sampling from different regions or areas of the grid:

> *"So I just picked three spaces. I guess I spaced them out. I thought about picking 3 in a row, but I divided I guess in my head…maybe it will be in this quadrant, or this quadrant, or this quadrant, so I guess I picked one from each"*

However, even though most described a very specific strategy that spread their selections throughout the grid by sampling from different areas, most participants insisted that their search was 'random' and that they had simply selected three random cells:

> *"I didn't really use any strategies; I just kinda randomly picked boxes"*

The formation of regions was constrained by the perception of 'specialty' cells in the grid. Participants did not select cells that were on the edges of the grid or in the corners. Instead, they believed that the square containing the prize was likely to be 'non-distinct':

> *"Umm, I don't know it's hard to explain. If you think you got a grid, and [the prize]'s hidden there, odds are that it's not going to be, you know, in the corners. Odds are just as much it being there as not, but you only have 3 choices. And I'm trying to make odds as good as possible. [I'm] trying to rule out a couple of places."*

Whenever participants clicked on a square, feedback was provided to let the participant know that that square did not contain the prize, encouraging them to try again. The responses indicated that knowing that they had clicked on a square that did not contain the prize drove them to make their next selection somewhere far from the original one. It appeared that the spreading behaviour was driven by two principles: the desire to cover as much of the grid as possible and the desire to place a subsequent selection at a distance from an original 'failed' selection:

> *"For some reason I didn't want to pick anything close to it, even though I know that doesn't make any sense. But I picked this one after because it was totally in a different area, so I tended to pick something that was further away."*

1.3 Discussion

The designed experiment was intended to provide a controlled environment in which to observe choice in the longshot search context. All cells of the grid were equally likely to contain the prize. Moreover, the cells were completely independent; information about one square did not reduce the uncertainty in any of the other cells. As such, there was no strategy that participants could have employed to secure higher chances of finding the prize. Thus, the rational expected behaviour was the random selection of three locations in the grid. Participants certainly seemed to believe that they were doing precisely this. In the post-experiment interviews, a majority of participants insisted that their selections were ‘just random’. Yet, a comparison to objectively random selections suggested that clear patterns could be observed in participant choices. Given the considerable size of the grid and limits to visual attention (Klemmer & Frick, 1953), participants could not discern all individual cells simultaneously; the configuration highlighted the potential perceptual groupings of cells, depending on their location in the grid.

First, as predicted, the edge of the grid emerged as a distinct region that was avoided. This was also confirmed by the interview responses where a majority of the participants admitted to having differentiated between the edge/corners and the rest of the grid. The experimental results suggest that compared to the edge, the interior of the grid was perceived as more likely to contain the randomly located prize.

Second, participants avoided making selections in locations that were near or similar to prior unsuccessful selections. Rather, they chose to spread their selections, placing them at a distance from earlier selections and in different rows/columns. This was made evident by the large minimum distance between selections, the large average area of the triangle formed by the three selections, and the low frequency of pairs of selections that were placed in the same row or column. The behaviour was also self-reported by the participants in the interviews. A majority admitted trying to space out their selections as much as possible and avoiding the placement of a selection too close to a previous one.

Overall, the results indicate that we are dealing with two interrelated phenomena: First, the structure of the search space (or more generally, the presentation of the problem) is subject to the observer’s perception, who will perceive the space as broken down into separate groupings, which are created by proximity and similarity (e.g., edges, rows, columns). Second, the search behaviour is strongly affected by these perceived groupings – individual cells within the same grouping lose their independence and are believed to give the same or similar results. Seeking to find a theoretical explanation for our findings, the next chapter provides a review of the literature and evaluates the effectiveness of existing theories in explaining the observed behaviour.

2. In search of an explanation

Study 1 revealed that when it came to the task of guessing the location of a randomly assigned prize, most participants believed that their strategy was to produce a series of random guesses themselves. Yet, their selections differed from random in consistent and predictable ways. In particular, participants did not assign equal probabilities to all cells (and as a result avoided the edge of the grid in favour of the more internal cells) and tried to keep their choices spread out such that they were not in the same row, column, or other perceptually-formed sub-region of the grid. It appears that the resulting behaviour may have been influenced by positional biases, perceptual groupings of the search space, and beliefs about what randomness is and what it ought to look like.

2.1 Positional attributes and preference for the middle

In Study 1 we found that participants avoided selecting cells located at the edge of the grid, preferring instead to make their selections further from the edge than would have occurred should their selections have been truly random. The avoidance of the edge has also been reported by the very few studies that use similar 2D stimuli (Lisanby & Lockhead, 1991; Falk et al., 2009). So why is the edge avoided?

Numerous studies in different domains have shown that the position of an item in a list of otherwise similar or even identical items can greatly affect the likelihood of that item being chosen. Examples include choosing paths from a set of same-length paths (Christenfeld, 1995), meals from a list of menu items (Dayan & Bar-Hillel, 2011), pictures arranged in a line (Rodway, Schepman, & Lambert, 2012), hotels from a list in a travel website (Eyal & Fleischer, 2014), and candidates from a list in a ballot card (Kim, Krosnick, & Casasanto, 2014).

Preference for the position is shown to be dependent on context: in contexts that are similar to Study 1 – identical and equal probability options, no other contextual distractors – preference is consistently for centrally located items. For example, Christenfeld (1995) found that when having to choose from identical options (e.g., choosing from up to 9 rows of stocked cans or from a row of 4 bathroom stalls), people gravitated towards the middle. In another experiment, people had to put an 'x' in a row of 3 circles and circle an 'x' in a row of 4 'x's. In both cases people preferred the central locations, with, especially in the case of the 4 'x's, the distribution of choices leaning towards the left side. The preference for centrally located items has also been replicated in a number of other studies, e.g., Colman and Stirk (1999), Shaw, Bergen, Brown, and Gallagher (2000), and Falk et al. (2009). An early explanation for this tendency was that central locations are the 'least thought' (or minimal effort) options (Christenfeld, 1995). Shaw et al. (2000) wondered if the preference for centrality was tied to a preference for symmetry but their experiment did not support their hypothesis. In a similar vein, Falk et al. (2009) believed that the selection of the central cell when participants were instructed to select one randomly was an unintentional carry-over from other conditions in which that choice made sense (e.g., the center cell was a preferred choice in the 'aesthetic' condition due to its symmetry). According to a more recent explanation, central locations are chosen more often because visual attention is given to the central location in the moments before making a choice (Atalay, Bodur, & Rasolofoarison, 2012).

The explanations above are limited in their ability to explain the behaviour of Study 1. While Study 1 is framed as low probability search, in the studies reviewed above participants casually selected an item among many, with any of the items being equally and fully rewarding. In Study 1, although the cells had equal probability of offering the prize (and so, objectively, any of them would have been a good selection), only 1 of the 81 cells had a reward attached to it. Participants were actively looking for the prize. In their interviews they suggested that while their intention was to make random selections, they had vague strategies in mind for how to do this – exerting a 'minimal effort' in making their choice did *not* seem to be at all a common strategy. Thus, it seems unlikely that participants were avoiding the edge as a result of a casual tendency to 'going for the middle'.

2.2 Perceptual groupings in the search space

One of the decisions made in the design of Study 1 was the presentation of the structured search space, which ultimately came in the form of 81 individual cells structured in a 9x9 grid. Theories of visual attention inform us that people are generally limited in their ability to see multiple objects at once (Duncan, 1984). When it comes to two-dimensional stimuli, people are only able to discern about 24 unique positions, or roughly a 5x5 matrix (Klemmer & Frick, 1953). The chosen matrix size (9x9) was conducive to some level of grouping/simplification of the stimulus, or at the very least inability on the part of the subjects to 'keep track' of all individual 81 locations. Participants had a tendency to group subsets of individual cells and accordingly adjust their judgement of probability for those cells.

How was the grouping/simplification of the stimulus achieved? Much of our understanding on how perceptual grouping occurs is based on Gestalt theory (see Wagemans et al. (2012) for a recent review). The meaning of Gestalt comes from German and can be roughly translated to "shape" or "form". In Gestalt theory a *gestalt* refers to any "segregated whole". An object (in space or time) is said to possess the Gestalt quality when it can be described as "regular", "simple", "harmonious", and even "symmetrical" (Kohler, 1935, pp. 191-193). Wholes are perceived as such as a result of a specific organization of the stimuli. The organization is effortless and immediate, without any deliberation on the part of the perceiver. The spontaneous arrangement of the individual components occurs according to some well-established principles, commonly referred to as the laws of perceptual organization. Wertheimer (1950) describes many such principles and factors, most notably the factors of proximity and similarity. Items that are perceived to be in proximity of each other are somehow 'united' to create a combined whole; the perceiver would in fact have difficulty perceiving the objects themselves individually, unless as part of, or in relationship to the whole. Similarly, objects that are similar in nature will be likely grouped together, even if they are not in proximity to each other. Tversky (1977) refined the definition of similarity by describing it as a feature-matching process; the more features two objects have in common (and the fewer features they don't have in common), the more similar they are. When an arrangement of various shapes – some close to each other, and some further away – is given, the factors of similarity and proximity will determine how the perceiver will experience the arrangement. Depending on the configuration, one factor may be predominant over the other (Wertheimer, 1950).

A regular structure such as the 9x9 grid used in Study 1 lends itself easily to groupings based on both proximity and similarity. In the participant interviews it became clear that they were well-aware of their selections' membership to various groupings. First, the grid is effortlessly divided into rows and columns. Second, the edge is also a particularly strong perceptual grouping (Biederman & Ju, 1988). Cells belonging to the edge are in contact with the empty space surrounding the grid, in stark contrast to the internal cells of the grid which are all surrounded by other cells. Finally, more 'fuzzy' perceptual groupings are also created according to more general location properties (e.g., cells in the top-left quadrant of the grid).

Overall, Gestalt theory provides a good theoretical framework for explaining the observed perceptual groupings of subsets of grid cells by participants. Therefore, the next questions concern the observed behaviour of participants' selections with respect to those groupings. Why did they avoid making selections nearby prior selections? Why did they avoid making multiple selections in the same row or column? And finally, why did participants appear to avoid making selections on or near the edge of the grid?

2.3 The production and perception of randomness

In general, people make use of simple mathematics with ease. Simple judgements of probability are not difficult for us. We understand probability as a ratio of chances for and against us; the larger that ratio the better our chances. However, as they increase in complexity, problems involving judgements of probability can quickly become difficult. In a review of the literature on subjective probability (defined as "*our judgements of the likelihood of uncertain events*" (Kahneman & Tversky, 1972, p. 430)), Einhorn & Hogarth (1981) observe that "*the picture of human judgement and choice that emerges…is characterized by extensive biases and violation of normative models*…" (p. 55). Complex problems involving judgements of probability reveal significant departures of subjective probability assessments from objective probability.

Likely one of the most relevant series of experiments in so far as resemblance to our Study 1 is one of the studies conducted by Falk et al. (2009), as described in Section 1.1. Participants coloured 3 of the 25 cells (arranged in a 5x5 grid) in a variety of conditions: competitive, co-operative, random, aesthetic, and 'no instructions'. Of particular relevance to us are the competitive and (especially) the random condition as they most resemble the setup in our Study 1. In those conditions, participants avoided edges and over-alternated (i.e., avoided marking adjacent, or neighbouring cells). Because they found strong correlations among conditions (cooperative, competitive, random), the authors suggested that people have 'default tendencies' with regards to how they approach these problems and only make slight adjustments according to the condition; these default tendencies are symptomatic of a shared understanding of what random choice might look like. Similarly, participants in our Study 1 insisted that since the process that had assigned the prize to one of the cells was random, all they had to do was also randomly select cells to guess the prize's location; to them, the locations they chose in the grid appeared to be random.

Yet, randomness, as a concept, is very elusive. People's conception of what is random (in both perception and production) is different from what is objectively defined as such (Lisanby & Lockhead, 1991). Nickerson (2002) explains that the *objective* concept of randomness typically involves the properties of equiprobable outcomes, independence and unpredictability of sequential outcomes, and minimal compressibility (i.e., the inability to describe a sequence of outcomes by a rule, or procedure). However, when it comes to people's *subjective* concept of randomness, it typically manifests as an avoidance of any evidence of non-randomness or regularities, such as repetitions. That is why, when people try to produce random sequences they end up having too few runs and too many alternations, compared to what would have been produced by a random process. While our ultimate interest is in the production and perception of randomness in 2D search spaces, much of the relevant literature concerns 1D sequences of usually equiprobable entities (e.g., a series of coin tosses).

2.3.1 Local representativeness

According to the representativeness heuristic, people judge the probability of a sample based on its perceived similarity to its parent population or to the process that generated it (Kahneman & Tversky, 1972). In the case when the population is generated by a random process, the sample must also appear to be generated by such a process. This can be determined by factors such as the perceived irregularity (or absence of patterns) in the sample outcomes and the extent to which the sample is *locally representative* of the parent population (i.e., the sample exhibits the properties of the population not just as a whole, but also in all of its subparts). Local representativeness is thought to be directly related to small sample bias (Tversky & Kahneman, 1971), because people expect even very small samples (or parts of the sample) to be representative of the population (Kahneman & Tversky, 1972).

One way to look at participants' avoidance of selecting nearby cells and spreading them throughout the grid is that these selections were locally representative of randomness. In other words, participants were trying to make three selections as randomly as a computer would have. While this explanation could be attractive, it is far from satisfactory. The computer had assigned a prize to just *one* cell. If the participants believed that they had to match that random process, why not make *one* 'random' selection and then, upon receiving negative feedback, select nearby cells? Even if one accepts that spreading and the avoidance of edges is representative of chance, the question remains as to why that is.

2.3.2 Over-alternations

The avoidance of selecting cells near prior unsuccessful locations might be looked at as a form of over-alternation. Alternations refer to the number of times the symbol types change in a sequence of 2D set. A binary sequence (i.e., one where two symbol types occur with identical frequency) of length n, can be characterized by its probability of alternations P(A)=(r-1)/(n-1), where r is the number of runs in the sequence, and n is the sequence length (Falk & Konold, 1997). In other words, P(A) is equal to the number of observed transitions divided by the total number of possible transitions. For example, the sequence *OXXO* has P(A) = (3-1)/(4-1) = 0.67. In 2D grids, P(A) is calculated in a similar manner, by counting the transitions along both the vertical and horizontal directions (Falk & Konold, 1997). For example, in the case illustrated in Figure 2, the number of transitions in the horizontal and vertical directions is 6 each, whereas the total (possible) number of transitions in each direction is 8 transitions per row(column) multiplied by the total number of rows (columns). Therefore the P(A) of the configuration of selections in Figure 2 is (6+6)/(8*9+8*9) = 1/12 = 0.083. While, generally, when generating random sequences people tend to over-alternate, maximum alternation (P(A)= 1, e.g., *OXOX*) results in a perfect pattern that is considered too regular and thus not random. In a review of the literature on the subjective generation and perception of random (mostly binary) 1D and 2D sets, Falk and Konold (1997) observed a fairly stable probability of alternation P(A) of about 0.6 across many studies.

The concept of alternations could be well suited to characterizing the participant selections in the 9x9 grid of Study 1. The 9x9 grid offers a maximum of 144 transitions between cells along horizontal and vertical directions. When only 3 selections are made, the number of transitions (i.e., r-1) can vary from a minimum of 4 (e.g., when all selections are grouped in the corner) to a maximum of 12 (when all selections are sufficiently spread out, such that there no two selections in cells adjacent or immediately diagonal to each other), with the resulting P(A) varying from 0.028 to 0.083. On the surface, the over-alternation model could easily explain the results of Study 1: selections that are judged more random, are more spread, and as expected, have a higher P(A). Similarly, selections that are on the edge or corners have lower total alternations and thus judged less random. However, it is possible to achieve the maximum P(A) with selections that we know participants would find not at all random, such as the example shown in Figure 2. Thus, in Study 1 over-alternation displays itself quite differently from the typical examples provided for the negative recency effect. Strictly speaking, participants could have over-alternated simply by avoiding neighbouring cells – still, they could have easily ended up with all three selections localized in one area of the grid (or all in one row as shown), rather than having them all spread out in the entire space.

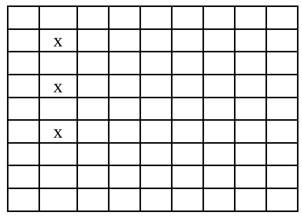


Figure 2 Example of 3 selections in the 9x9 grid - high P(A), yet low probability of selection

2.3.3 Positive and negative recency effects

In the literature on the production of binary sequences in particular, the number of alternations has been described as being affected by two opposing tendencies: the negative and positive recency effects. The former is directly related to the number of alternations, whereas the latter to the run length (see Scholl and Greifeneder (2011) for a discussion of both) .

When it comes to populations with a specific proportion value (e.g., coin tosses with an outcome proportion value of ½), people believe that each sequence will maintain the population's proportion, and that each deviation from that proportion will be quickly counteracted by an opposite deviation. For example, coin tosses that result in a sequence of 3 Heads will be followed by a sequence of Tails (even though each coin toss is an independent event), in order to preserve the 'fairness' of the toss. This belief is called the negative recency effect (or gambler's fallacy). Some explanations for this effect are tied to memory limitations (Tune, 1964): participants can only create random sequences over a small number of elements and end up creating sequences that have too many alternations and are not overall random (Kareev, 1995). People try to ensure that even small portions of the sequence are locally representative of the population by retaining its Heads/Tails proportion (Kahneman & Tversky, 1972).

Over-alternating is opposed by another tendency, termed the positive recency (or hot hand) effect. There are some domains (e.g., basketball shots (Gilovich, Vallone, & Tversky, 1985) and roulette winnings (Wagenaar, 1988)) in which people believe in streaks, and more commonly, that a positive streak will continue. According to a representativeness-based explanation, very small streaks (e.g., two good hits in a row) are representative of the generating process (in this case the skill of the player on that particular day), and thus, longer positive streaks will be expected (Gilovich, Vallone, & Tversky, 1985)[1]. Another suggested explanation is that the hot hand phenomenon is a rational and adaptive approach that is sometimes applied inappropriately to situations in which outcomes are independent. Since in our daily experience resources are often clustered together, then it is normal to expect a streak, e.g., if a prey has been encountered then a similar prey can be found nearby (Wilke & Barrett, 2009). In a similar vein, Hsu, Griffiths, and Schreiber (2010) suggest that when trying to make 'random' selections, people avoid the kinds of regularities that they would encounter in the natural world. In a study using pictures of natural scenes, the authors found that neighbouring regions have similar intensity values – and since nature scenes are not considered random, having neighbouring cells with similar values would be evidence of regularity and non-randomness.

Participants in Study 1 could potentially be using (a poor adoption) of the positive recency effect. Perhaps people view the absence of resources (i.e., prizes) as also existing in clumps. According to this explanation, after making an unsuccessful guess, people assume that locations that do not contain the prize are clumped together in the vicinity of that earlier guess and so they must make the second guess away from the first. It seems plausible that the characteristic of the earlier guess (i.e., its failure to contain the prize) is being generalized to cells surrounding it. This explanation is further pursued in Chapter 3.

2.3.4 Implicit encoding

In trying to explain why binary sequences that were judged most random had a 'sweet spot' of P(A) = 0.6 (i.e., lots of alternations, but not so many that they make a pattern), Falk & Konold (1997) proposed that people judge the randomness of a sequence based on their (tacit) ability to encode the sequence, the ease (or more commonly, difficulty) of which can be judged or measured (e.g., through ease of memorization). Good patterns are easy to encode and can thus be easily compressed into a computer instruction that can reproduce them. In their studies, the authors provided participants with long binary sequences and asked them to judge whether the sequences were random, to memorize them, and to assess the difficulty of memorizing them. They found a correlation between these three measures: the sequences that were judged more random were also more difficult to memorize (as evidenced by the time it took participants to memorize them, as well as their own assessment of this difficulty).

The encoding explanation is used to explain some of the behaviour observed in the experiments of Falk et al. (2009) with 5x5 grids, which as previously noted, are highly relevant to our research question due to the similarity of their stimulus to the one used in Study 1. Of all their conditions (co-operative,

[1] Here representativeness is used to explain both positive and negative recency tendencies, which are quite opposite in nature. Ayton and Fischer (2004) suggested that people employ the negative recency bias when they believe that the source is a natural event (e.g., a coin toss), and the positive recency bias when the source is human performance (e.g., throwing of darts). Another explanation is that some sporting events (e.g., golf putting (Gilden & Wilson, 1995)) have a natural positive recency and people are just assuming this is also the case in basketball.

competitive, random, aesthetic, and no instructions), the random condition is the most conceptually similar. It was found that in that condition participants avoided edges and over-alternated (i.e., avoided marking adjacent, or neighbouring cells). There were also some strong effects with regards to the selections of particular locations in the grid. For example, cell B4 – the cell diagonal to the top-left corner – was very popular. The authors suggested that its popularity might be explained by its lack of saliency (i.e., it has no distinguishing features, just like the number 7 in a string from 1 to 10). Another way of saying this is that describing the location of B4 would take longer than describing the location of, say, corner cells. The location of B4 is not easily encoded, and thus more random-seeming (Falk & Konold, 1997). Another explanation offered relates to B4 having medium complexity, where complexity is calculated based on the number of cells that are similar (e.g., by rotation)[1]. Apparently items of medium complexity are judged as the most random (Garner, 1970; Falk & Konold, 1997). A similar explanation provided is that B4 belongs to the largest equivalency set (i.e., it belongs to the 16 cells that have complexity 4), and thus is perceived more random (Teigen, 1984).

The concepts of ease of encoding, complexity, and equivalency sets can be used selectively to explain some of the behaviour observed, but they fail to explain the contradictions in the participant selections. For example, it is suggested that B4 is popular due to its medium complexity and difficulty to encode, yet the dead-middle of the grid (low complexity and easy to encode) is also very popular. Similarly, it is suggested that B4 is popular because it is in the largest equivalency set, but so are the mid-edge and the corner cells, yet they are not very popular. Although certainly strong, these concepts fail to describe participants' choices in their entirety and can only be used piece-wise to explain some but not all aspects of choice.

2.4 Variety, diversification, and distinctiveness

One way of thinking about spreading is as a tendency to make selections from a variety of regions, rows, and columns. Existing literature suggests that when faced with making multiple choices at once, people are generally variety-seeking. A typical illustration is people's purchase of a variety of candy, or a variety of yogurt flavours. Simonson (1990) found that people seek a greater variety when making decisions about multiple purchases simultaneously (e.g., buying yogurt for the week) compared to when making those choices sequentially (e.g., buying yogurt every day). Read and Lowenstein (1995) suggested that higher variety-seeking in simultaneous choice (as compared to sequential choice) is not normative, but rather a bias, which they called diversification bias. People choose more variety than they actually end up wanting. This bias arises as a result of the way sets of choices are subjectively partitioned. When choices

[1] B4 has a complexity of 4 (i.e., there are 3 other locations like it). In the 5x5 grid, complexity ranges from 1 (in the case of the cell right in the centre of the grid) to 8 (in the case of the cell above B4 (B5)).

are bracketed together, a diversification heuristic is employed; whereas, when choices are partitioned into smaller brackets, the preferred choice is made each time – i.e., a utility maximization heuristic is employed. To illustrate, in an experiment with children trick-or-treating on Halloween, Read and Lowenstein (1995) found that when children were asked to choose two candy bars from two piles of two different candies, they diversified - all children chose two different candies. In contrast, when the children were asked to choose one candy from two different candy piles, twice (i.e., at different houses), they chose whichever candy they liked the best, leading to many ending up with two of the same kind of candy. Read, Loewenstein and Rabin (1999) called this phenomenon choice bracketing; the most variety is sought under conditions of broad bracketing (e.g., buying yogurt for the week). Broad and narrow brackets imply large and small sets of choices, respectively.

Why is variety sought in broad bracketing? If people choose variety, what is that variety with respect to? According to Fox, Ratner and Lieb (2005), people seek to spread their choices across different groupings (or what they refer to as partitions) using naïve diversification or uniform distribution (e.g., the 1/n rule (Benartzi & Thaler, 2001)). Partition dependence thus arises when different – subjective - partitions will lead to different choices. The less informed the user is about attributes and differences between partitions (or the more equal they seem), the more likely she is to use the diversification heuristic (Fox et al., 2005).

A similar framework comes from Ayal and Zakay (2009), who have proposed that people use diversity to minimize risk. They suggest that a richer diversity of options is perceived to be less risky; thus, in line with prospect theory (Kahneman & Tversky, 1979), they find that people prefer diversity under gains (i.e., risk avoidance under gains), but avoid it under losses (risk seeking under losses). One of the components of diversity is the distinctiveness of options. To illustrate, in an experiment they presented participants with a computerized program that randomly selected three numbers from the 1-100 sequence. Participants were then asked to write down three numbers under conditions of gain (win money for each hit), loss (lose money for each hit), and neutrality ("guess what the computer will pick"). It was found that the distinctiveness of the selections – measured using range and standard deviation - was highest for the condition of gain and lowest for the condition of loss.

Diversification and distinctiveness can be useful conceptualizations of the spreading behaviour observed in Study 1. In particular, the finding that the diversification heuristic finds its most use when the options are 'hidden' and equal in probability is an important one. The grid experiment of Study 1 is an extreme version of lack of information. While participants might feel that the edge is less likely to contain the prize and thus be less likely to 'diversify' to the edge, within the grid there is no further preference between the individual cells, so the diversification heuristic – i.e., diversification between (the remaining) areas of the grid - prevails. Yet, how is this diversification achieved? With respect to what partitions do people diversify? Do people create the partitions first, and then choose one cell from each partition, or do they re-create partitions after each choice?

Similar difficulties are encountered when using the distinctiveness concept: although it is generally understood that being in different rows or columns could justify calling two cells distinct, distinctiveness in other cases can be established only *after* the participants have made all three choices. One could potentially contour fuzzy distinct regions surrounding each selection, but it is not clear that participants are actually creating these regions before they make their selections.

2.5 Summary and conclusion

Based on the literature summarized in the sections above, there are a few theories and conceptualizations that can, to some extent, explain various aspects of the patterns observed in Study 1. Those patterns are the avoidance of the cells located on the edge of the grid and of cells that are close or similar to previously unsuccessful cells.

We attempted to explain the avoidance of the edge using theories of positional bias, which suggest that people are naturally drawn to choose items from the center of a set of equal or similar items. Central locations afford symmetry, are minimally effortful, and draw our visual attention. These explanations, as well as the context they imply do not map well to the stimulus used in Study 1 and the observed behaviour of participants. Rather than casually selecting their 3 cells from the middle of the grid, participants were careful to spread their choices throughout the grid, each subsequent selection informed by the knowledge of the failure to find the prize in prior selections.

Participants in Study 1 insisted that they were just ‘selecting cells randomly’, so it appeared attractive to describe the participants’ selections as displaying the same characteristics – over-alternations and minimal compressibility – as the ones reported in the literature on the production and perception of randomness. The explanation that participants were over-alternating in order to maintain local representativeness of what they expected randomness to look like was found to be lacking; over-alternation could have been achieved without requiring spreading of the selections throughout the grid. Similarly, the explanation that selections are deemed most random when they are least easy to encode was also limited in its applicability: easy to encode locations such as the dead-centre were found to be popular.

Finally, some relevant insights came from the literature on diversification and distinctiveness, which suggest that people choose variety, especially under conditions of gain and simultaneous choice/broad bracketing. In particular, when people have little information about what distinguishes partitions in the set, they are more likely to seek variety with respect to the various partitions, attempting a near-uniform distribution of their choices through each partition. While these theories could describe very well participants' behaviour of 'seeking variety' with regards to perceptual groupings (be it more easily defined rows/columns or more fuzzy sub-regions of the grid), they are intended to describe behaviour where the partitions are already pre-formed before the participant makes their choices. This was not the case in Study 1, where the grid is homogenous and few obvious partitions (e.g., the edge) exist. So while the labels appear helpful (diversification, variety, spreading), the explanations are not.

In conclusion, the review of the literature has uncovered a host of somewhat relevant theories and concepts; however, to our knowledge, no theory alone is able to explain participants' selections in Study 1 in their entirety. One of the major difficulties has been that context of Study 1, a search context with no information and very low probability of success, is different from all the contexts that are examined by the reviewed literature. Thus, an opportunity exists to develop a new theory that can both explain Study1 and also successfully predict people's choices when faced with search task in structured spaces of different sizes and shapes.

This new theory is built in three steps. In Chapter 3, we suggest that people avoid neighbouring cells after an unsuccessful guess because they generalize its properties (e.g., not containing the prize) to cells that are deemed to be like it, whether it be as a result of proximity or similarity. We name this concept the selection's coverage and find strong evidence of it in both the 9x9 grid from Study 1 as well as the 5x5 grid of Falk et al. (2009). In Chapter 4, we propose that participants are drawn to larger 'unexplored' areas of the grid and test the hypothesis that items belonging to larger sets are judged to be of higher probability, even if the sets are only perceptual and otherwise have no bearing on the objective probability of their elements. Finally, building on these first two steps, in Chapter 5 we propose a new theory that explains the participants' selections as maximizing coverage. We also design and test a simple model to calculate coverage mathematically in some small pre-defined 2D search spaces.

3. The concept of 'coverage'

When participants interacted with the grid in Study 1, grouping/simplification of the stimulus occurred at two different times. First, when people looked at the 9x9 grid, they were not able to discern between all 81 cells (Klemmer & Frick, 1953; Duncan, 1984). The large number of cells was grouped or simplified to some level, with cells within a grouping becoming fairly indistinguishable from each other. Such groupings might have been the edges, the grid quadrants, etc. Second, once participants selected a cell in a chosen grouping, that (unsuccessful) cell served as a point of reference for new groupings of cells, which could have occurred by proximity (e.g., cells that are in the immediate vicinity of the selected cell), or by similarity (e.g., other cells in the same row or column).

The focus of this chapter is on the second grouping process, that is, what happens once participants select a cell. Study 1 results as well as other supporting literature (Falk et al., 2009) suggest that people avoid selecting cells that are *near* or *like* cells that have resulted unsuccessful in prior selections. In other words, participants believe that these neighbouring or otherwise similar cells have a low probability of containing the prize. More generally, the location of cells relative to a 'known' (or reference) cell affects their (subjective) probability. We thus propose that participants assign similar attributes (in this case the odds of being assigned a prize) to cells that are perceptually grouped together, whether the grouping principle is proximity or similarity. We define a cell's ***coverage*** to be the set of other cells whose probability assessments are affected by knowledge about that cell's attributes.

The main objective of this chapter is to understand how coverage is created as a result of grouping by proximity and similarity. Of interest are the subjective probability assessments of cells inside and outside the coverage area and the effect of the size of the search space on the size of a cell's coverage.

3.1 Coverage by proximity

We first analyze the effect of grouping by proximity on people's assessment of probabilities. We refer to the group of cells whose probability assessments are affected by knowledge (of lack of success) of a nearby cell as the latter cell's *coverage by proximity* (or CP). Based on our understanding that people avoid making selections near a prior selection, the effect of a cell's outcome on probability assessments of neighbouring cells is assumed to decrease the further you are from it. Thus in our chosen framework, CP is assumed to decrease radially (but not necessarily linearly) the further one is from the cell of reference.

Taking the context of a structured NxN grid, let x_{ij} be any cell x located in row i and column j . It can be argued that cells immediately adjacent to x_{ij} ($x_{i-1,j}$, $x_{i,j-1}$, $x_{i+1,j}$, and $x_{i,j+1}$) are perceptually closer than cells immediately diagonal to it ($x_{i-1,j-1}$, $x_{i-1,j+1}$, $x_{i+1,j-1}$, and $x_{i+1,j+1}$); however, for simplification, we assume that all have the same distance of 1 unit from x_{ij}, with that distance increasing in increments of 1 the further you are from it. Given that equidistant cells form (roughly) concentric circles around the reference cell, we refer to this measure as radial distance. Given this framework, we can calculate the radial distance between any two cells in the grid. To illustrate, in the 9x9 grid of Study 1, given a cell of reference x_{11} in the top-left corner of the grid, other cells in the grid can be as close as 1 unit and as far as 8 units of radial distance from x_{11} (Figure 3). Given data from two different grid sizes – 9x9 in Study 1 and 5x5 in Falk et al. (2009), in the next three sections we analyze how probability assessments compare at different radial distances and deduce the size of CP in each grid.

x	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	2	3	4	5	6	7	8
3	3	3	3	4	5	6	7	8
4	4	4	4	4	5	6	7	8
5	5	5	5	5	5	6	7	8
6	6	6	6	6	6	6	7	8
7	7	7	7	7	7	7	7	8
8	8	8	8	8	8	8	8	8

Figure 3 Illustration of radial distances from a corner reference cell in the 9x9 grid

3.1.1 Probability assessments and radial distances in the 9x9 grid

To better understand participant choices in the 9x9 grid, we first determine the expected theoretical distribution of radial distances between any two cells. Given that in the 9x9 grid radial distance between cells may range from 1 to 8 units, and given any selected (or reference) cell x, we calculate the likelihood that the next selected cell y will be at distance d_{xy} equal to 1, 2, 3, …, 8. For example, consider the reference cell x_{11} in the top-left corner of the grid. Only 3 of the remaining 80 cells (x_{21}, x_{22}, and x_{12}) are located at a radial distance of 1 unit; thus, assuming that all cells are equally likely to be selected, the probability that the next selected cell will be at a distance of 1 unit is 3/80. We then aggregate these probabilities for all 81 cells and calculate the *expected* likelihood of occurrence for each of the 8 radial distances d_{xy}. We perform a similar analysis on all pairs of selections from Study 1. For each participant with selections 1, 2, and 3, we calculate d_{12}, d_{23}, and d_{13}, which, when combining the selections of 37 participants results in 111 radial distance values, all ranging from 1 to 8. Given this data, we calculate the *observed* probability of occurrence of each radial distance. [1]

[1] Note that the calculation of observed probabilities is not perfectly matched to the method used for the calculation of expected probabilities. For simplification, pairs of selections in each sequence of three selections are taken independently. For example, the observed distance between the second and the third selection is compared to the expected distance between two selections; however, this is not a perfect comparison – the existence of a prior (first selection) further limits the availability of locations for the third location. This is not captured in the calculation of expected probabilities. The same simplification is made in Sections 3.1.2, 3.2.1, and 3.2.2. It is believed that this simplification does not significantly affect the results of the comparison.

Figure 4 overlays the observed and expected radial distance distributions in the 9x9 grid. Both are curvilinear, with clear differences in the middle and in the extremes (Kolmogorov-Smirnov one-sample test $D = 0.17$, $p < 0.01$). The observed probabilities of occurrence for the 'low' (1 or 2 units) and 'high' (6 units and up) radial distances are visibly lower than expected. This difference is in line with our earlier finding in Chapter 1 that participants avoided making selections too close to a previous selection. In addition, a selection that is 6, 7, or 8 units away from a prior selection is likely to fall on, or near the edge, and, as we already know, participants avoided making these types of selections. An opposite trend is found with regards to 'medium' (3-5 units) radial distances where the observed probabilities are visibly higher than expected. As already observed in Chapter 1, most participants chose to make their selections about halfway between a prior selection and the edge.

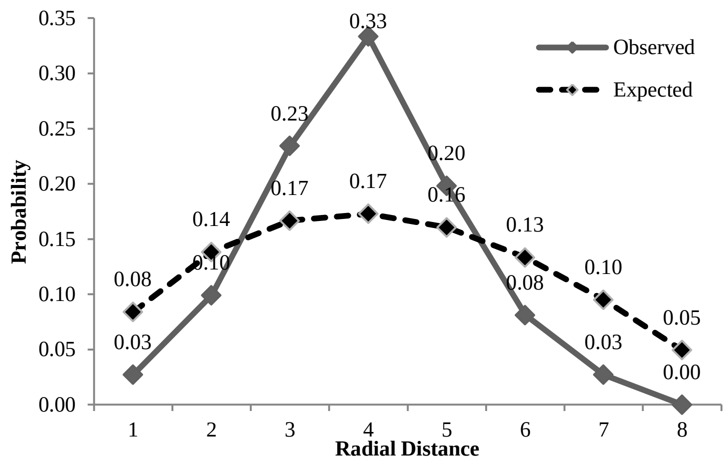


Figure 4 Observed and expected probabilities of radial distances in 9x9 grid

3.1.2 Probability assessments and radial distances in the 5x5 grid

To better understand participant choices in the 5x5 grid, we first determine the expected theoretical distribution of radial distances between any two cells. Given that in the 5x5 grid radial distance between cells may range from 1 to 4 units, and given any selected (or reference) cell x, we calculate the likelihood that the next selected cell y will have a radiance distance of d_{xy} equal to 1, 2, 3, or 4. For example, consider the reference cell x_{11} in the top-left corner of the grid. Only 3 of the remaining 24 cells are at a radial distance of 1 unit; thus, assuming again that all cells are equally likely to be selected, the probability that the next selected cell will be at a distance of 1 unit is 3/24. We then aggregate these probabilities for all 25 cells and calculate the *expected* likelihood of occurrence for each of the 4 radial distances d_{xy}. We perform a similar analysis on all pairs of selections from the random condition of Falk et al. (2009). For each of the 250 participants with selections 1, 2, and 3, we calculate d_{12}, d_{23}, and d_{13}, resulting in 750 radial distance values, all ranging from 1 to 4. Given this data, we calculate the *observed* probability of occurrence for each radial distance.

Figure 5 overlays the observed and expected radial distance distributions in the 5x5 grid. As in the case of the 9x9 grid, both distributions are curvilinear, with clear differences in the middle and in the extremes (Kolmogorov-Smirnov one-sample test D = 0.12, $p < 0.01$). The observed probabilities of occurrence for the 'low' (1 unit) and 'high' (4 unit) radial distances are visibly lower than expected. This difference is in line with the observation by Falk et al. (2009) that participants avoided making selections too close to a previous selection or on the edges. An opposite trend is found with regards to the 'medium' (2-3 units) radial distances where the observed probabilities are visibly higher than expected. Overall, findings of the 5x5 grid follow closely those of the 9x9 grid.

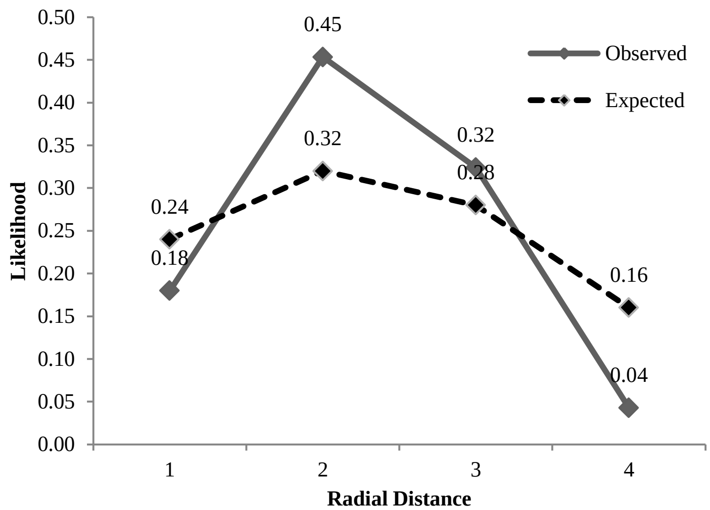


Figure 5 Observed and expected probabilities of radial distances in 5x5 grid

3.1.3 The size of CP

To determine the size of a cell's CP, we focus on the smaller radial distances (1-2 units). When comparing Figures 4 and 5, in the lower radial distances the observed and theoretical lines cross at different points. In the case of Study 1, the likelihood of cells being at a distance of 1 or 2 units from each other was smaller than expected, with that trend reversing itself at a distance of 3. In the case of Falk et al. (2009), that reversal occurs at a distance of 2 units. Put another way, in the 9x9 grid, participants are demonstrating a reluctance to place their selections within 1 or 2 units of a prior (unsuccessful) selection, signalling that their assessment of the probability of those cells having the prize was low. In contrast, in the 5x5 grid, participants are demonstrating a reluctance to place their selections within just 1 unit of the prior selection, implicitly showing they are attributing low probability only to the cells immediately adjacent or diagonal from the prior selected cell.

It can thus be suggested that the size of the grid, or more generally the size of the search space, influences the size of a cell's CP. Roughly speaking, in the 9x9 grid of 81 cells, a cell covers all cells that are 1 or 2 radial units away from it (24 cells in total, or 29.6% of the set). Similarly, in the 5x5 grid of 25 cells, a cell covers all cells that are 1 radial unit away from (8 cells in total or 33.3% of the set). Regardless of the grid size, a cell's CP seems to be sized at about 1/3 of the grid.

3.2 Coverage by (row/column) similarity

In addition to grouping by proximity, we also analyze the effect of grouping by similarity on people's assessment of probabilities. We refer to the group of cells whose probability assessments are affected by knowledge about a similar cell as the latter's cell *coverage by similarity* (or CS). While it can be argued that cells can be grouped by similarity based on different attributes such as the row or column they belong to, symmetry, type (e.g., corners), etc., we limit the scope of our analysis to just groupings by rows and columns. Based on our understanding that people avoid making selections in the same row/column (r/c) as a prior selection, the effect of a selected cell's outcome on probability assessments is expected to be strongest in that cell's r/c and decreasing the further an r/c is from the cell of reference. We thus refer to it as the r/c distance between two cells. Thus in our chosen framework, coverage by similarity is assumed to decrease the further one is from the r/c of the cell of reference.

We refer to the distance between two cell's rows or columns as their r/c distance. Using the same notation as in the section on coverage by proximity, we declare that all cells in row *i* and column *j* have an r/c distance of 1 from cell x_{ij}, with that distance increasing in increments of 1 the further a cell's r/c is from row *i* and column *j*. Given this framework, we can calculate the r/c distances between any two cells in the grid. To illustrate, in the 9x9 grid of Study 1, given a cell of reference x_{11} in the top-left corner of the grid, other cells in the grid can be as close as 1 unit and as far as 9 units of row distance from x_{11} (Figure 6). It should be noted that due to the symmetry of the grid, similarity calculations with respect to row groupings are identical to those of column groupings.

x	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9

Figure 6 Illustration of row distances from a corner reference cell in the 9x9 grid

As in the case of proximity, given data from two different sizes of grids – 9x9 in Study 1 and 5x5 in Falk et al. (2009), in the next three sections we analyze how probability assessments compare at different r/c distances and deduce the size of coverage by r/c similarity in each grid.

3.2.1 Probability assessments and r/c distances in the 9x9 grid

To better understand participant choices in the 9x9 grid, we first determine the expected theoretical distribution of r/c distances between any two cells. Given that in the 9x9 grid r/c distance between two cells may range from 1 to 9 units, and given any selected (or reference) cell x, we calculate the likelihood that the next selected cell y will be at a r/c distance d_{xy} equal to 1, 2, 3, …, 9. For example, consider the reference cell x_{11} in the top-left corner. Of the remaining 80 cells in the grid, 8 are located at a row distance of 1 unit, with another 8 located at a column distance of also 1 unit. Assuming that all cells are equally likely to be selected, the probability that the next selected cell will be at an r/c distance of 1 from x is 8/80. We then aggregate these probabilities for all 81 cells and calculate the *expected* likelihood of occurrence for each of the 9 r/c distances d_{xy}. We perform a similar analysis on all pairs of selections from Study 1. For each participant with selections 1, 2, and 3, we calculate r/c distances d_{12}, d_{23}, and d_{13}, which, when combining the selections of 37 participants result in 111 distance values, all ranging from 1 to 9. Given this data, we calculate the *observed* probability of occurrence for each r/c distance.

Figure 7 overlays the observed and expected r/c distance probability distributions in the 9x9 grid. While they have similar shapes, the distributions have clear differences in the middle and in the extremes (Kolmogorov-Smirnov one-sample test for rows $D = 0.13$, $p<0.05$; for columns $D = 0.11$, not-significant at alpha = 0.05). The rationale for the differences is similar to the discussion of proximity in the previous section. Overall, it appears that the likelihood that selections were made in the same r/c as a prior selection or in an adjacent r/c was less than what was expected theoretically. Similar observations can be made on the other end of the spectrum, where the observed likelihood of r/c distances greater than 7 units was less than expected. These differences are in line with our earlier finding in Chapter 1. As we already know, participants avoided selections in the same row or column as a prior selection. Additionally, if two cells are 7 or more rows/columns part, it is likely that one, if not both cells are on or near the edge, which is also something participants avoid. An opposite trend is found with regards to cells at 3-5 r/c units of distance. For example, 61% of selected pairs of cells in Study 1 had cells at a distance of 3-5 rows from each other, while the expected likelihood was 39%.

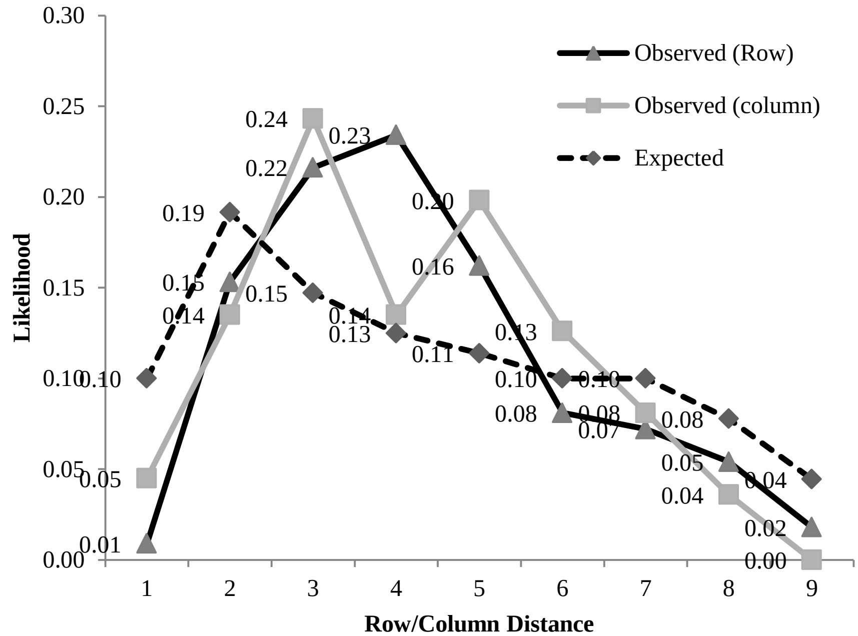


Figure 7 Observed and expected probabilities of r/c distances in the 9x9 grid

Of note in Figure 7 is the jaggedness of the observed column curve compared to the observed row and expected curves. We are not aware of any reasons that would explain the dip in the observed column curve at 4 units. In addition, no such irregularity is observed in the analysis of the much larger dataset from the 5x5 grid of Falk et al. (2009), as is shown in the next section in Figure 8. We thus attribute this irregularity to the limited data from Study 1 and predict that a larger data set would smooth out the observed column curve to more closely resemble the others.

3.2.2 Probability assessments and r/c distances in the 5x5 grid

To better understand participant choices in the 5x5 grid, we first determine the expected theoretical distribution of r/c distances between any two cells. Given that in the 5x5 grid r/c distance between cells may range from 1 to 5 units, and given any selected (or reference) cell *x*, we calculate the likelihood that the next selected cell y will have an r/c distance d_{xy} equal to 1, 2, 3, 4, or 5. For example, consider the reference cell x_{11} in the top-left corner of the grid. Of the remaining 24 cells in the grid, 4 are located at a row distance of 1 unit, with another 4 located at a column distance of also 1 unit. Assuming that all cells are equally likely to be selected, the probability that the next selected cell will be at an r/c distance of 1 from x is 4/24. We then aggregate these probabilities for all 25 possible points of reference and calculate the *expected* likelihood of occurrence for all of the 5 r/c distances d_{xy}. We perform a similar analysis on all pairs of selections from the random condition of Falk et al. (2009). For each of the 250 participants with selections 1, 2, and 3, we calculate r/c distances d_{12}, d_{23}, and d_{13}, resulting in 750 values. Given this data, we calculate the *observed* likelihood of occurrence for each r/c distance. The results are very similar to those that emerged from the analysis of data from Study 1.

Figure 8 overlays the observed and expected r/c distance probability distributions in the 5x5 grid. Both the row and column curves are significantly different from the theoretical distribution (Kolmogorov-Smirnov one-sample test for rows D = 0.17, $p<0.01$; for columns D = 0.13, $p<0.01$). The rationale for the differences is similar to the discussion of radial distances in the previous section. The likelihood that selections were made in the same r/c as a prior selection (distance of 1) was less than what was expected theoretically. Similar observations can be made on the other end of the spectrum, where participants were less likely than expected to select cells that were 4 or 5 r/c units away from a prior selected cell. Again the explanation is tied to the avoidance of selecting cells in the same r/c as a previous selection and also avoiding selections on the edge of the grid. Additionally, as was also found from the 9x9 grid data, the likelihood that participants placed their cells at a distance in the medium range (in this case 2-3 units) was higher than what was expected theoretically. Overall, findings from the 5x5 grid follow closely those of the 9x9 grid.

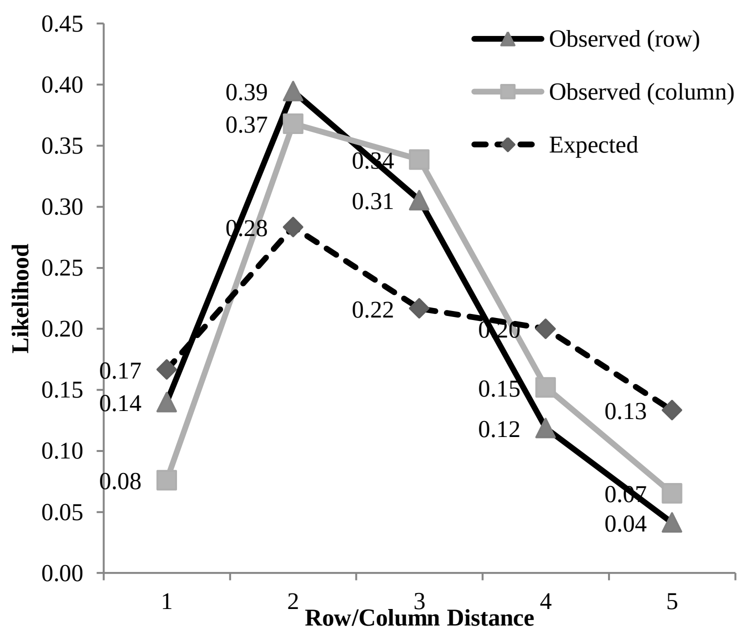


Figure 8 Observed and expected probabilities of r/c distances in the 5x5 grid

3.2.3 The size of CS

To determine the size of a cell's CS, we focus on the smaller r/c distances (1-2 units). When comparing Figures 7 and 8, we find a similar trend to what we found when we looked at CP: when we zero in on the lower r/c distances, the observed and expected probability graphs cross at different points for the 9x9 and

5x5 grids. In the case of Study 1, the likelihood of cells being at an r/c distance of 1 or 2 units from each other was much smaller than expected, with that trend reversing itself at a distance of 3. In the case of Falk et al. (2009), that reversal occurs at an r/c distance of 2 units. Again, as previously discussed, this is an important difference that is an assumed result of the difference in size between the grids. In the 9x9 grid, participants are demonstrating a reluctance to place their selections in the same r/c, or in an r/c adjacent to the r/c of a prior selection, signalling that their assessment of the probability of those cells having the prize was low. By contrast, in the 5x5 grid, participants demonstrated a reluctance to place their selections within just the same r/c of the prior selection, implicitly showing that they are attributing low probabilities only to that set.

It can thus be suggested, in the case of grouping by r/c similarity, as was the case in grouping by proximity, that the size of the grid, or more generally the size of the search space, influences the size of a cell's r/c similarity coverage. In the 9x9 grid of 81 cells, a cell covers 3 rows and columns (i.e., 44 cells not including the selected cell), or 54% of the set. In the 5x5 grid of 25 cells, a cell covers just other cells in the same row or column (i.e., 8 cells not including the selected cell), or 32% of the set in each case. Although the ratios are not as similar as the ones observed in the case of coverage by proximity, there is nevertheless a strong sense that CS, like CP, is directly affected by the size of the search space – the larger the space, the larger the coverage.

3.3 Discussion

The analysis of participant choices in the 9x9 and 5x5 grids demonstrated that probability distributions of different distances between two selections (whether they were calculated radially or by row or column) deviated from theoretically derived probability distributions consistently and predictably. It was shown (once again) that once a cell has been selected, the probability of selecting a second cell near or like it was less than would be expected had the selections been made truly randomly. In this chapter we introduced a new concept – coverage – as the set of cells whose probability judgements are affected by knowledge of the outcome of a prior selection. We defined CP and CS as those sets formed by proximity and similarity, respectively.

The size and attributes of CP and CS were investigated in two separate contexts: the 9x9 grid of Study 1, where participants were making three selections in the hopes of finding a randomly located prize, and the random condition in Falk et al. (2009), where participants were simply making 3 random selections in a 5x5 grid. Overall, it was found that the size of both CP and CS increased as the size of the grid increased.

There are some limitations to the analysis presented so far. First, while we have conducted a similar analysis in the 9x9 and 5x5 grids, there are some differences in those experiments. There may be a psychological difference between searching for a randomly located prize and simply randomly selecting three cells in the grid. In particular, in Study 1, participants received feedback after each selection, whereas in Falk et al. (2009) there was no wrong answer (and no feedback was necessary). At this time, we are inclined to believe that this difference is not particularly important. When asked about what prompted to make their selections the way they did, participants in Study 1 expressed that they were just selecting cells randomly; in trying to find a prize that was placed by a random process, participants were attempting to recreate a random process of their own. In addition, Falk et al. (2009) reported correlations between participant selections in the competitive and random conditions, evidence that participants were defaulting to 'selecting randomly' in both cases.

A second limitation relates to the interaction of CP and CS. While we discovered the parameters of CP and CS in the 5x5 and 9x9 grids, we did not investigate how 'much' a cell is covered when under both the CP and CS of one or more selections. Rather, the two were discovered and analyzed separately and independently from each other. It is unclear at this point what the relative strength of these two types of coverage is, how they compare, and how this comparison might be affected by factors such as the size and shape of the grid and the number of selections.

Finally, the parameters of CP and CS were determined in the specific circumstance where 3 selections were made in both the 5x5 and 9x9 grids. For example, it is reasonable to assume that the size of CP that was discovered - 1/3 of the grid – was directly related to the number of selections: to cover as much of the grid as possible given 3 selections, each selection must cover roughly 1/3 of the grid. It is possible that CP and CS depend on the ratio of the size of the grid and the number of selections. Currently, our analysis is not sufficiently developed to describe the sensitivity of CP and CS to these parameters.

An interesting theoretical discussion can be had by tying the concept of coverage to the theory of generalization, which, Shepard (1987) argues, is psychology's first general law. The theory of generalization is as such:

> "*Whenever a response has been learned in one stimulus situation, similar stimulus situations will also tend to elicit that response in proportion to their similarity, and that stimulus situation will also tend to elicit similar responses in proportion to their similarity….It should be recognized that this is an exceedingly adaptive principle. As a rule, similar situations require similar responses*." (Logan, 1970, pp. 128-129)

Multiple studies with animals have helped establish orderly gradients of generalization; an animal's response to a stimulus can be strictly predicted by the gradient of similarity of that stimulus to the original (training) stimulus (Shepard, 1987). It is generally accepted that the generalization function – the probability that the animal will generalize their response given the psychological distance between the stimulus and the reference – decays exponentially (or in some cases in Gaussian form (Nosofsky, 1992)) (Shepard, 1987).

The observed participant behaviour of assigning similar probability assessments to cells that are perceptually grouped together is evocative of the theory of generalization – one could think of the properties of a prior selected cell that had resulted unsuccessful as being generalized to cells that are like it, whether this likeness is due to being close or being in the same row or column. In addition, the larger the psychological distance between a cell and a prior selection (whether one measures this psychological distance with regards to Euclidian distance, similarity, or a combination of the two), the less likely one is to generalize the selection's properties to that cell, and the more likely to choose that cell as the next selection. This interpretation is also supported by the data from the 9x9 grid: when plotting the probability that a participant will not be selecting a cell (i.e., the probability that the cell is being judged as too similar to a prior selection and that the selection's attributes are generalized to the cell) versus the (psychological) distance between the cell and the prior selection, the function that is achieved is Gaussian.

There is thus undoubtedly an interesting resemblance of the concept of coverage to the concept of (stimulus) generalization; however, this idea is beyond the scope of this thesis and will not be further explored.

Looking ahead, in Chapter 4, we investigate an important attribute of coverage: its size. Through an experiment (Study 2), we demonstrate that the size of a grouping affects the subjective probability of its elements. This finding is used to further develop the concept of coverage, resulting in a coverage maximization theory that is tested in Chapter 5.

4. The size of coverage

As participants interacted with the grid of Study 1, their choice context changed with every selection. Initially, before a participant had made any selections, the search space was blank. As previously mentioned, due to people's inability to discern between all 81 cells, grouping/simplification of the stimulus occurred (e.g., edges, grid quadrants, etc.) Once the participants selected a cell in a chosen grouping, that cell enforced its own CP and CS (as discussed in Chapter 3). Thus, when making the second, and later, the third selection, participants were left to choose from the remaining cells and faced a new choice context - new groupings could be formed under the added constraint of avoiding areas covered by their previous selection(s). The thesis of this chapter is that, regardless of the choice context, participants try to choose a location in the grid that affords them the greatest coverage possible.

4.1 Evidence from Study 1

We begin by providing evidence of people's preference for locations that maximize coverage in the 9x9 grid used in Study 1. Participants made two types of decisions: choosing a first cell in the unexplored grid, and choosing a second (or third) cell after an unsuccessful first (or second) selection.

Of interest in the first type of decision is participants' established avoidance of cells on the edge and corners of the grid. A defining feature of those cells compared to internal cells in the grid is their low CP and CS. For example, consider Figure 9. Based on the analysis in Chapter 3, a typical cell (*x*) inside the grid has a CP of 24 cells. In contrast the largest CP achieved by cells on the edge (*e*) and corners (*c*) of the grid is 14 and 8 cells respectively. Participants' avoidance of edge and corner cells could be construed as a tendency to seek cell locations that maximize CP (thus avoiding low-coverage cells).

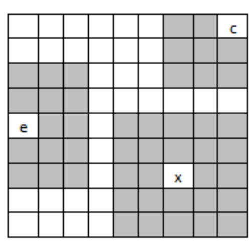


Figure 9 CP of typical internal, edge, and corner cells in 9x9 grid

A similar conclusion can be reached with respect to CS (see Figure 10 for illustration). Based on the analysis in Chapter 3, a typical cell inside the grid (e.g., *x*) has a total CS of 44 cells (when summing up both similarity by column and row, whereas edge (e.g., *e*) and corner (e.g., *c*) cells would be limited to 38 and 31 cells respectively.

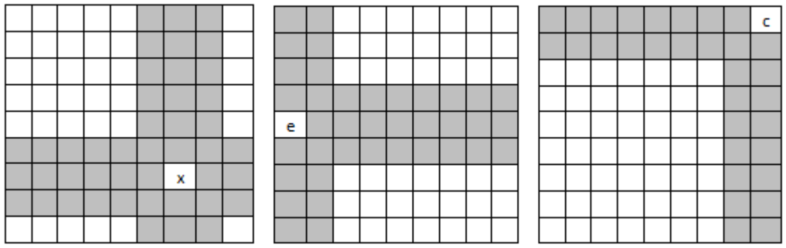


Figure 10 CS of typical internal, edge, and corner cells in 9x9 grid

The second type of decision occurs once one or more prior selections have been made in the grid and resulted unsuccessful. Consider the case when a selection (*x*) has been made and its coverage established (see Figure 11 for illustration). Cells *a* and *b* are both feasible alternatives for the location of the second selection. However, evidence from Section 3.1.1 suggests that location *b* would be preferred to location *a*. As was observed in Figure 4, the largest probability of a second selection (and at the same time largest deviation from expected probability) is at a radial distance of 4 units. Thus, rather than making their second selection immediately outside the coverage area at a radial distance of 3 units from *x* (i.e., cell *a*), participants are more likely to choose instead cells that are further (i.e., cell *b* which is 4 units away). The preference for cell location *b* supports the 'maximizing-coverage' prediction; cell *b* adds 23 'new' cells to the total CP of the two selections, which is larger than the added CP brought by the selection of cell *a* (20 cells).[1]

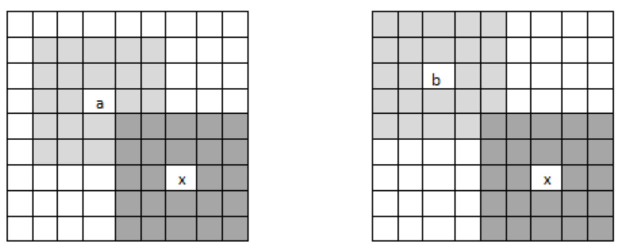


Figure 11 CP of cells at a radial distance of 3 and 4 units from a prior selection in the 9x9 grid

In that same type of decision, a similar analysis can also be performed with respect to CS. For example, consider the CS of a first selection *x*, and two alternative second selections *a* and *b*, at a row distance of 3 and 4 units respectively (see Figure 12 for illustration). While *a* and *b* are both feasible alternatives (being outside of the coverage area of *x*), evidence from Section 3.2.1 would suggest that *b* would be preferred to *a*. As was observed in Figure 7, the largest probability of a second selection (and at the same time largest deviation from expected probability) is at a row distance of 4 units. Therefore, the predicted preference for cell *b* also supports our prediction that participants seek to maximize coverage; cell *b* brings an added CS of 26 cells, which is larger than that brought by cell *a* (17 cells). Of note is that in this

[1] The exact calculation of the total CP of two selections is discussed in detail in Chapter 5.

example we chose to discuss row similarity, rather than column similarity. The reason for this is the identified irregularity in the observed coverage by column similarity curve in Figure 7, as discussed in Section 3.2.1.

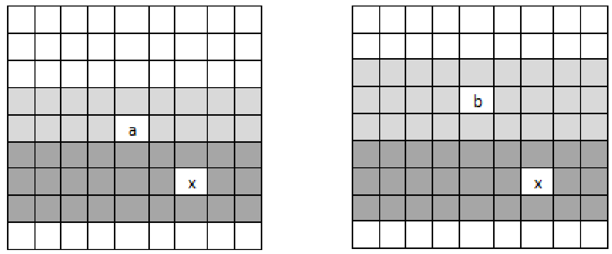


Figure 12 CS of cells at a row distance of 3 and 4 units from a prior selection in the 9x9 grid

In summary, known patterns of choice from Study 1 are consistent with the prediction that participants seek to select cells in the grid that afford the highest possible coverage. We further suggest that the reason for this is because while participants rightly assume that the prized cell is more likely to be in a larger grouping compared to a smaller one, they wrongly believe that *individual* cells within a larger grouping have a higher probability of containing the prize than *individual* cells in smaller groupings. We further explore this explanation in the next section.

4.2 Evidence from the literature: category size bias and multi-stage lotteries

People's (erroneous) belief that individual chance events have an increased likelihood of occurrence when they belong to a larger category (compared to a smaller one) has been recently demonstrated in a series of experiments by Isaac and Brough (2014). For example, in one experiment participants were presented with an urn of 15 numbered balls of different colours – blue, grey, and white. While ball number 8 was always grey, the different conditions varied the number of balls belonging to each of the three colour categories. It was found that participants judged the likelihood of randomly selecting ball number 8 from the urn as being larger when the number of grey balls in the urn was larger, thus demonstrating what the authors termed 'category size bias'. Categorization has thus the effect of distorting how participants process the information and of altering their choices. The mechanism provided by the authors to explain this bias was categorical inheritance: people expect an item to possess the attributes of a category it belongs to. Since larger categories are more likely to occur (e.g., a grey ball is more likely to be selected when there are more grey balls), people believe (erroneously) that individual items in those categories (e.g., grey ball number 8) are also more likely to occur.

Objectively, the judgement of the category size and its effect on the individual outcome's probability ought to be just the first calculation step in calculating the *overall* probability of that outcome. A second, often forgotten step, is calculating the likelihood of selecting the desired outcome from that chosen category. Previously, two-step probabilities (also known as multi-stage lotteries) have been explored by Ronen (1973). Compound probabilities were explicitly broken down into two sub-events, each with their own well-defined probability outcomes. In one experiment, participants were asked to choose between different sequences of two-probability events. For example, a sequence was composed of two chance events: picking a blue marble from a bag of blue and white marbles, and then picking a red marble from a bag of red and green marbles. While the total (compound) probability of picking a blue marble from the first bag followed by a red marble in the second bag was constant, participants systematically preferred those sequences in which the first event was higher in probability (and as a consequence, those that had a second event with lower probability), demonstrating a tendency towards early success. The preference for multi-stage lotteries with decreasing probabilities was also more recently confirmed by Budescu and Fischer (2001). Multiple explanations have been provided, with the most promising being that early success provides participants with more ‘hope’ and a higher likelihood of ‘staying in the game’, which is of subjective value (as also suggested by Ronen, 1973). As one participant stated, “*The progress from one state to the other means something, it’s better to lose at a later stage”* (Budescu & Fischer, 2001, p. 200) It is to be noted that while the preference for higher first-stage probabilities has been demonstrated in lottery-type domains, it may be supressed in other scenarios (Chung, von Winterfeldt, & Luce, 1994).

In light of the above, we suggest that people’s selection of an individual cell from the 9x9 grid of Study 1 can be modelled as a two-stage lottery. Consider an individual cell *x* that is grouped with a number of other cells in grouping *X* (e.g., the CP of *x*) that is contained in the search space *N* (i.e., the 81 cells of the grid). For cell *x* to contain the prize, a sequence of two separate events must have occurred: the prize must have been placed in the grouping *X and* the participant must have chosen the cell *x* in *X* that actually contained the prize. Thus, the probability that *x* in *X* contains the prize can be decomposed into a sequence of two probabilities: (1) the probability that *X* contains the prize, which is equal to X/N, and (2) the probability that *x* in *X* contains the prize, which is equal to 1/X. A multiplication of the two probabilities yields a total probability of 1/N, which is equal to the objective probability. Yet, given participant choices and prior research on the preference for early success in multi-stage lotteries, it would appear that participants do not assign equal probabilities to all cells *x* in the grid. Instead, the size of the category/grouping *X* affects their subjective probability and, given that participants seem to prefer choosing from the larger subsets, it is implied that of the two probabilities, X/N holds a higher weight in participants’ probability calculations than 1/X.

4.3 Study 2 – Effect of the group’s size on an element’s probability

The groupings created in the grid are perceptual and only loosely inferred from participants’ choices and post-experiment interviews. To test the prediction and explanation offered above, we designed an experiment that retained the search aspect of the grid while removing the ambiguity of the perceptual groupings that were formed based on the grid structure. Rather than presenting the search space as a number of cells in a structured grid, the game was instead framed as a quest to select a red marble from a jar that contained a mix of blue and red marbles. The objective likelihood of this chance event was 1/10. Perceptual groupings of the marbles were enforced by effectively making a separation of the marbles in the original jar and segregating the marbles in two additional jars. Thus, marbles were grouped according to the common region principle, recently proposed by Palmer (1992). According to this principle, items will be grouped together if they are "*located within a common region of space, i.e., if they lie within a connected, homogeneously colored or textured region or within an enclosing contour*" (p. 438). Palmer argued that this principle differs from Wertheimer's principles in that it was an extrinsic principle (the organization occurs because of something other than the items to be grouped), whereas all of Wertheimer's principles are based on intrinsic factors – the items themselves dictated the organization.

Method. Two conditions were designed. In an online survey participants were shown a short video developed in Microsoft Powerpoint. In the narrated (and subtitled) video an opaque jar was shown to contain a mix of 10(100 marbles): 9 (90) blue marbles and 1(10) red marble(s). A hand was shown to mix the marbles well and then the contents of the jar were poured into two additional jars. 8(80) of the marbles were shown to be poured into one jar and 2(20) in the other. Two hands were then shown to randomly draw one marble from each of the jars. Without revealing the colours of the selected marbles, participants were asked to predict which of the two marbles most people would judge to be more likely to be red. They were also asked to provide an explanation and probability calculation to support their choice. An example of the stimulus is provided in Appendix B.1.

Participants. A total of 185 (55% male, mean age = 34.5 years) participants were recruited using Amazon Mechanical Turk. All participants completed both Condition 1 and Condition 2, in randomized order. A few participants declined completing both conditions, and instead completed just one. They were not excluded from the analysis.

Results. No positional effects were found with regards to the left-right presentation of jars ($\chi^2 < 0.4$, $p > 0.5$). In addition, no order effects were found with regards to the presentation of the experimental conditions ($\chi^2 < 1.8$, $p > 0.18$). Results were thus aggregated.

In both conditions, a majority of participants judged the marble selected from the larger jar to be more likely to be red (Condition 1: 81%, N=185, $p<0.001$; Condition 2: 62%, N=176, $p<0.001$). However, the proportion of participants that made that judgment significantly decreased in Condition 2 compared to

Condition 1(χ^2= 15.38, p<0.001)[1]. The full dataset of explanations provided by participants for their choices is given in Appendix B.2. Typical answers for each condition are provided in Table 4 below:

Table 4 Study 2 - Typical participant explanations for their choices

	Condition 1	**Condition 2**
Explanation for choice of marble from <u>big</u> jar	*"There are more marbles in [the bigger] bowl, so most people would guess it has a higher chance of having the red marble."*	*"[The bigger bowl] should have more red balls total even though the proportion is likely to be the same as [the smaller bowl]."*
Explanation for choice of marble from <u>small</u> jar	*"Because if the red marble is in [the smaller] bowl, there is a 50 percent chance of getting it"*	*"There are less balls in total in [the smaller] bowl, which most people might think would give them a better chance"*

4.4 Discussion

In Study 2, as in Study 1, participants were presented with a problem in which the prized element was hidden in a set of identical items, with a low overall probability of it being found. While in Study 1 participants created their own perceptual groupings of the cells in ways that came natural given the grid's structured nature, in Study 2 the marbles were forced by the experimenter into two distinct groups (the larger and smaller jar). In both cases, the groupings should have had no bearing on participant choices. In the case of Study 1, all cells in the grid had exactly the same probability of containing the prize, regardless of their location and the size of their coverage. Similarly, in Study 2 every individual marble had the same exact probability of being red – 1/10; the placement in jars was an artificial separation and it should have had no effect on participants' judgments. Yet, Study 2 demonstrated that participants believed that when directly comparing a marble that had been grouped in the larger category (larger jar) with one that had been grouped in the smaller category (smaller jar), the former had a higher probability of being red than the latter. In this case, the effect of the grouping's size on the individual elements' probability judgments was demonstrated unambiguously.

Participants' reasoning (based on their written explanations) was that the larger jar was more likely to contain the prized red marble (or, in the case of Condition 2, likely to contain more red marbles). While this is true, participants make no mention of the second stage of the probability calculation – while the

[1] An interesting effect can also be observed in the difference between male and female participants. Overall, it is observed that female participants, regardless of condition, are more likely to judge the marble selected from the bigger jar as more probable to be red (χ^2= 3.93, p<0.05). A suggested explanation is based on prior research, which has shown small, but significant differences in men and women's self-reported thinking styles; while men score themselves higher on 'rational ability', women score themselves higher on 'experiential ability and engagement' (Pacini & Epstein, 1999). Thus, according to this explanation women exhibit a stronger category size effect because they can more easily arrive at intuitive answers.

larger jar likely has more red marbles, it also has a larger number of blue marbles, which implies a smaller probability of actually selecting a red marble. In contrast, the minority participants who chose the smaller jar argued that, had one (or more) red marbles ended up in the smaller jar, the probability of selecting a red marble from there was high. These participants, unlike the other group, chose to focus their attention on the second stage of the probability calculation. Overall, the results of Study 2 indicate that most participants focus on the first stage of the probability calculation and choose alternatives that have higher first-stage probabilities, in line with prior research (Ronen, 1973; Budescu & Fischer, 2001).

A second interesting finding of Study 2 is the decrease in Condition 2 from Condition 1 of the portion of participants that believed the marble from the larger jar was more likely to be red. The explanation for this trend is subtly inferred in the answers given by participants in Condition 2 that chose the marble from the small jar – they point out that given that 20 marbles ended up in the small jar, it is very likely that at least some of them were red. In fact, assuming a proportional distribution of the red marbles between the two jars, the probability of selecting a red marble from the small jar in Condition 2 is 2/20. In contrast, in Condition 1, it is much more difficult intuitively to proportionally 'split' the sole red marble between the two jars. Objectively, the probability of selecting a red marble from the small jar in Condition 1 is 0.2/2. In reality, however, most participants just guessed that the marble had ended up in the larger jar, making the probability of finding it in the smaller jar effectively 0. An alternative explanation for the difference between the two conditions can be argued in the form of the ratio bias effect (Miller, Turnbull, & McFarland, 1989) – 2/20 might seem more attractive than 0.2/2; however, the same explanation could also be used to predict an increased preference for the larger jar in the 100 marble condition – a 8/80 probability might seem more attractive than 0.8/8.

In Chapter 3 we discovered that people do not process individual outcomes independently; rather, due to perceptual grouping influenced by the context in which the outcomes are presented, probability judgments of any outcome in a group are strongly affected by knowledge about other outcomes in that group. In general, outcomes in a group are assumed to have similar probability judgments, even if the grouping process should have no relevant effect on their objective probability. In this chapter we have shown that there is a second factor that affects the probability judgment of an outcome in a group – the group's size. Generally, the larger the group's size, the higher the subjective probability of any individual outcome that is contained in the group. We now have evidence to suggest that when people select a cell in the grid, the size of the coverage that that cell can afford (which is dependent on the location of that cell as well as the location of prior selections) affects the judged probability that the cell contains the prize: the larger the available coverage, the higher the judged probability. In the next chapter, we construct and test a generalized coverage-based model of people's choices in the multi-selection search context.

5. The coverage maximization theory

In the previous chapters we have concluded that in the context of searching for a randomly hidden prize in structured space, people attempt to make 'random' selections in the space. The generation of these 'random' selections is such that: (1) a selection has coverage, or set of cells that are judged to have similar probabilities, and (2) people seek to select cells that afford the highest coverage. Thus, summing up the predictions and evidence presented in the previous chapters, we can now suggest that people's choices in the long-shot search context can be modelled as a tendency to maximize coverage. The objective of this chapter is to test coverage maximization as a model of choice in the multi-selection search context.

In Chapter 3 we defined and found evidence for coverage based on both radial proximity (CP) and row/column similarity (CS). However, we did not delve into the interaction of the proximity and similarity grouping principles and thus did not construct a 'total coverage' model that incorporated the two. Taking into consideration the complexity of modelling and testing choice when both CP and CS might be at play, in this chapter we choose to model and test the 'coverage maximization' model of choice focusing on CP alone, taking CS into consideration only as a tie-breaker in cases of identical CP.

Having derived the coverage principles from the 9x9 and 5x5 grids, in this chapter we define and test the coverage maximization model in the context of the smallest search space that we believe maintains the main attributes of the multi-selection search paradigm – 6-cell structures. Moreover, to minimize the influence of CS, we choose, as much as possible, simple structures that lack clear row/column organization. Despite its small size, we believe the 6-cell search context maintains the main psychological attributes of the multi-selection search paradigm. In addition, the choice presents several advantages, including the simplification of the computation of coverage, people's general familiarity with similarly sized odds (e.g., throwing the dice), and – important for testing purposes – it offers a minimal number of possible choices to study participants.

5.1 The case of the single selection

We first define and test coverage maximization in the context of a single selection. Having previously derived the coverage principles from the 9x9 and 5x5 grids, in this chapter we begin by modelling coverage in a simple 6-cell structure presented as a 2x3 grid. The 2x3 grid affords a simple, yet 2D configuration of the 6 cells.

5.1.1 Defining single-selection CP in the 2x3 grid

In Chapter 3, we found that in the 9x9 and 5x5 grids, the covered cells were all immediately adjacent to or just one cell removed from the selected cell. A cell's CP, depending on its location, was comprised of at most 29.6% and 33.3% of the total cells in the 9x9 and 5x5 grids respectively, indicating that the size of a cell's CP was proportional to the size of the grid. We propose that CP in the 2x3 grid can be modelled as shown in Figure 13: a cell's coverage is extended to just ½ unit of radial distance, covering half of each immediately adjacent cell and a quarter of each cell that is immediately diagonal to it. For example, the two types of cells in the grid - *x* in the middle and *c* in the corner - have a total CP of 2 and 1.25 cells respectively. This ensures that a selected cell's CP extends to at most 33% of the total grid area, in similar proportion to the CP in the larger grids.

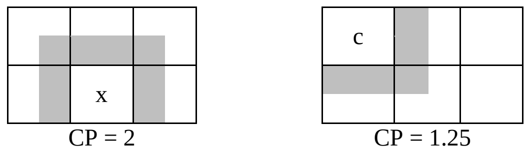


Figure 13 Coverage by proximity of a single selection in the 2x3 grid

Given a selection's calculated CP, coverage maximization as a model of choice can be tested against real participant selections. In particular, given a context in which people are asked to judge the likelihood that a particular location in the search space contains a randomly placed prize, we can test whether these judgments correlate with our predictions that people will prefer to select cell locations that have the highest coverage. We thus begin by first focusing on the case when a participant makes a single selection in the search space and put forward the following hypothesis:

Hypothesis 1: In the case of a single selection, cells that have higher coverage are judged to be more likely to contain a randomly placed prize.

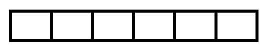
Recall that one of the most relevant alternative models of choice reviewed in Chapter 2 was over-alternations. The premise of the model was that the higher the probability of alternations P(A), the more random a series of selections will appear to people. If, however, P(A) becomes too large, the alternations become to appear as a pattern in themselves. Thus, in binary sequences/grids, choices that are judged the most random have P(A) around 0.6 (Falk & Konold, 1997). Study 3A below sought to not only test Hypothesis 1, but to also assess the predictive power of the over-alternations model in comparison to the coverage maximization model.

5.1.2 Study 3A - Single selection in 6-cell structure

Method. The survey was administered as a 3 page handout. The first page provided a description of the task, as shown below. In the following two pages, participants were shown 7 different 6-cell structures. In all structures, every cell was adjacent to at least another cell (i.e., no cells were 'floating').

For each structure, they were also shown 3[1] different shaded cells and asked to rank the cells based on their judgement of frequency of selection by the fictitious game participants. To avoid rank-order and orientation biases, efforts were taken to randomize the order of presentation of the structures, the order of the options to be ranked, and the left-right orientation of the structures as much as possible, within the constraint of the paper-based delivery. A version of the complete questionnaire is provided in Appendix C.1.

"A while back the following game was played with a large number of people. People were shown various 6-cell shapes. An example of such a shape is shown below:



In each structure, a computer had randomly assigned a prize to one of the cells. The game players had to select the cell where they felt the prize might be.

As you can imagine, players selected some cells more frequently than others.

In the next page, you will see a variety of 6-cell structures. Examples of selected cells by people are shaded. Your task is to rank the selected cells in order from the one that you expect was the most frequently selected to the one that you expect was the least frequently selected:"

Participants. The questionnaire was administered as an ungraded activity to students in an undergraduate Management Sciences course, an elective taken by students from a variety of disciplines.

Results. The questionnaire was completed by 57 of the 60 students in attendance. Of interest was the level of agreement of participant rankings with the predicted coverage-based rankings. The metric chosen was the correlation between several judges and a criterion ranking (T_C), which is the average of the Kendall rank-order correlations between the participants and the predicted ranking (as described in Siegel and Castellan Jr, 1988, pp. 281-284). The higher is T_C, the more in agreement judges are with the criterion ranking.

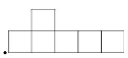
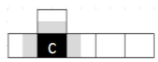
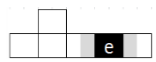
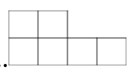
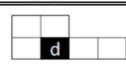
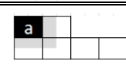
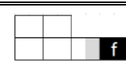
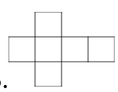
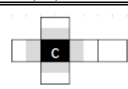
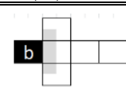
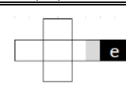
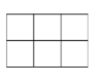
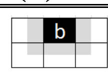
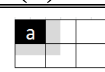
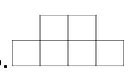
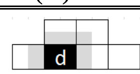
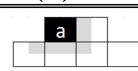
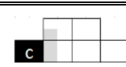
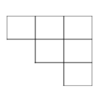
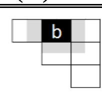
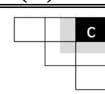
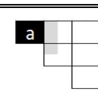
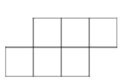
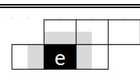
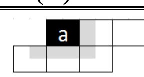
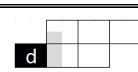
Table 5 shows all the structures presented to participants, as well as the options of shaded cells to be ranked and their calculated CP and P(A) values[2]. For example, in the case of structure 1, cells *c*, *e*, and *f*

[1] In the case of structure #4, participants were only shown two shaded cells. The reason was that there were only two unique CP values (i.e., multiple cells had identical CPs)
[2] In all cases, N = 57, except in the case of structure #6 (N = 56)

have a calculated CP of 1.5, 1, and 0.5, respectively. The shaded cells are ranked and presented in order of descending CP. The table also shows the correlation value T_C and its associated statistical significance.

The results of study 3A support Hypothesis 1. In all structures, there is an observed agreement among participant rankings and expected rankings. In all but structure #7, that agreement is significant at a level of 0.01.

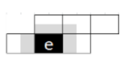
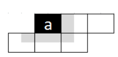
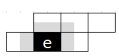
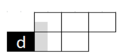
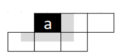
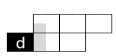
Table 5 Results of Study 3A

6-cell shape	Cells selected by fictitious game players, from highest P(A) to lowest			Value and significance of T_c
1. 	 CP = 1.5 P(A) = 3/5	 CP = 1 P(A) = 2/5	 CP = 0.5 P(A) = 1/5	**0.44 (p < 0.000)**
2. 	 CP = 1.75 P(A) = 3/6	 CP = 1.25 P(A) = 2/6	 CP = 0.5 P(A) = 1/6	**0.24 (p = 0.003)**
3. 	 CP = 2 P(A) = 4/5	 CP = 1 P(A) = 1/5	 CP = 0.5 P(A) = 1/5	**0.37 (p < 0.000)**
4. 	 CP =2 P(A) = 3/7	 CP =1.25 P(A) = 2/7	**N/A**	**0.44 (p < 0.000)**
5. 	 CP =1.75 P(A) = 3/6	 CP =1.5 P(A) = 2/6	 CP = 0.75 P(A) = 1/6	**0.31 (p < 0.000)**
6. 	 CP =1.75 P(A) = 3/6	 CP =1.25 P(A) = 2/6	 CP =0.75 P(A) = 1/6	**0.30 (p < 0.000)**
7. 	 CP =1.75 P(A) = 3/6	 CP =1.5 P(A) = 2/6	 CP =0.75 P(A) = 1/6	**0.08 (p = 0.182)**

Of particular interest is the case of structure #7, where the correlation coefficient T_C was not found to be significant. Further decomposing the pair-wise rankings as summarized in Table 6, we find that, as expected, the selections with the higher CP - *e* and *a* - are generally ranked higher than the low-CP cell *d*.

Unexpectedly, however, when comparing selections *e* and *a* to each other participant rankings go against our CP-based predictions. One possible explanation for the observed pattern of choice could be that cell *a* is located closer to the top-left region of the structure. In Study 1 as well as the studies of Falk et al. (2009) it was noted that people exhibited a preference for selections in the top-left of a structure, a tendency possibly borrowed from the learned behaviour of reading a page from top-left to right-bottom. When paired with the almost identical values of CP between *a* and *e* (difference of only 0.25), this preference may explain participants' higher-than-expected ranking of *a*.

Table 6 Pairwise comparisons of selection rankings for structure #7

Selected cells & their CP		**Results**	**Value & significance of T_c**
 CP = 1.75	 CP = 1.5	# of participants that ranked 'e' higher than 'a' = 20 # of participants that ranked 'a' higher than 'e' = 36	-0.29 (p = 0.12)
 CP = 1.75	 CP = 0.75	# of participants that ranked 'e' higher than 'd' = 33 # of participants that ranked 'd' higher than 'e' = 20	0.18 (p = 0.12)
 CP = 1.5	 CP = 0.75	# of participants that ranked 'a' higher than 'd' = 38 # of participants that ranked 'd' higher than 'a' = 18	0.36 (p = 0.006)

With regards to how coverage maximization performs compared to a competing model – over-alternations, in the case of cells chosen in the study, their CP correlated with their P(A) values, thus assuming that the higher the P(A), the more random the selection, P(A) made the same (good) predictions about participant choices. The only exception was in the case of Figure 3, where cells *b* and *e* had identical P(A) values but different CP values. In this case, participant rankings were in agreement with predictions made by CP: *b* was ranked higher than *e* (T_c = 0.23, p = 0.056). In contrast, over-alternations theory would have predicted that there would be no difference between the two cells. Nevertheless, given that for the most part, P(A) and CP values made the same predictions with regards to participant choices, Study 3A could not adequately compare the two models and could not categorically reject over-alternations theory as a suitable alternative to coverage maximization.

5.2 The case of two selections

In this section, we test the coverage maximization model in the context of two selections. As such, the first step is determining CP is calculated in the case of multiple selections.

5.2.1 Defining double-selection coverage in the 2x3 grid

According to the framework described in the previous section, a cell in the vicinity of a selection can be covered by proximity at a level of 0, ¼, or ½. A question arises with regards to what happens when more than one selection is made. In this case, two complications arise: (1) a selection may be made within a prior selection's CP, and (2) the CPs of 2 or more selections may overlap.

One approach, termed the *additive* approach, could be to simply add up the CPs of each individual selection, as if they had been made alone and independently. For example, according to this approach, if both cells *x* and *c* had been selected in the 2x3 grid (Figure 13), their total coverage would be the sum of those selections' individual CPs, i.e., 2 + 1.25 = 3.25.

Another approach, termed the *holistic* approach, takes a more 'gestalt' view of coverage, as illustrated in Figure 14. First, we assume that if a cell is selected, its selected status becomes its only attribute, and its inclusion in any prior or future selections' CPs is discarded. Thus, since cells *c* and *x* are selected, the inclusion in each other's CPs is not counted. Second, to determine the 'coverage level' of other cells in the grid, we take into consideration whether they are under the CP of 1 or 2 selected cells. Consider the following case: A first selection *x* has been made, and its coverage shaded in light grey, as shown in Figure 14. We next consider what happens when the second selection *c* is made. The new selection brings its own CP, shaded in dark grey. The top right corner of the cell below *c*, and the bottom left corner of the cell immediately to the right of *c* are under the overlapping CP of both *x* and *c*, and thus shaded in black. Several options could be imagined for calculating the total coverage incurred by such a cell that is under the CP of multiple selections (i.e., in an area of overlapping CPs).

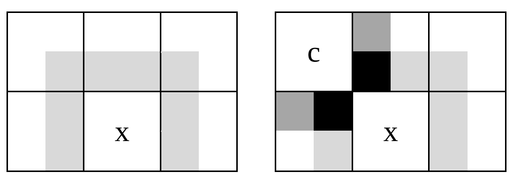


Figure 14 Illustration of the CP of two selections in 2x3 grid

Option 1. According to this option, we assume that once an area of a cell has been covered, the added coverage brought on by an overlapping CP does not affect its perceived coverage. The area (A) of the cell that is under overlapping CP is counted just once, therefore total CP in the grid is calculated as:

$$CP_{Option\ 1} = A_{light\ grey} + A_{dark\ grey} + A_{black} = 1.25 + 0.5 + 0.5 = 2.25$$

Option 1 has the advantage of simplicity, but one might question whether added (overlapping) CP onto a cell does not in fact increase its perceived coverage. Recall that the definition of CP is the set of cells whose probability judgements are affected due to their proximity to a selected cell. Once selection *x* has been made, because of its CP, the likelihood that the next selection will be located in the cell immediately to its left becomes lower. One could imagine that once selection *c* is made, the likelihood of that cell being selected in a potential third try is even lower than before. A lower judgment of probability would imply a higher coverage. Option 2 below presents an alternative that takes the added coverage into account while also adjusting its influence given that it is added to prior coverage.

Option 2. With each new overlap, the CP added is discounted by 50%. Therefore, total CP in the grid is calculated as:

$$CP_{Option\ 2} = A_{light\ grey} + A_{dark\ grey} + 1.5 * A_{black} = 1.25 + 0.5 + 1.5 * 0.5 = 2$$

Option 2 is chosen for modelling (holistic) CP in double-selection choices. Its suitability compared to Option 1 is further discussed later in this chapter.

As in the case of a single selection, given a selection pair's calculated CP in a small grid, coverage maximization as a model of choice can be tested against real participant selections. In particular, given a context in which people are asked to judge the likelihood that the selection of two particular locations in the search space is going to result in finding a randomly placed prize, we can test whether these judgments correlate with our predictions based on the model described above. Based on our understanding of coverage and its effect on choice we would predict that people will prefer to select cell locations that have the highest coverage.

Hypothesis 2: In the case of two selections, pairs of selections that have higher coverage are judged to be more likely to contain a randomly placed prize.

A second objective relates to validating the holistic approach taken for calculating coverage in the double-selection case.

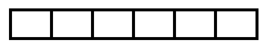
Hypothesis 3: In the case of two selections, coverage is perceived holistically (as calculated by Option 2), rather than as the sum of coverages of selections taken independently (as calculated by Option 1).

As in the case of the single-selection case, in Study 3B (described below) we sought to not only test Hypotheses 2 and 3, but to also assess the predictive power of the over-alternations model (Falk & Konold, 1997) in comparison to the coverage maximization model.

5.2.2 Study 3B - Two selections in 6-cell structures

Method. The context used in Study 3B was very similar to the one used in Study 3A, but rather than being administered as a paper handout, the survey was conducted online. The first page provided a description of the task, as shown below. In the following pages, each participant was shown a random selection of 4 of the 7 structures used in Study 3A. For each structure, participants were shown 3 different pairs of shaded cells and asked to rank them based on their judgement of frequency of selection by the fictitious game participants. In general, for each structure, selection pairs were chosen such that the holistic and additive CP calculations would result in different, if not opposing predictions. To avoid rank-order and left-right orientation biases, the order of presentation of the structures, the order of the ranking options, and the orientation of the structures were all randomized. A version of the complete questionnaire is provided in Appendix C.2.

A while back the following game was played by a large number of people. People were shown various 6-cell shapes. An example of such a shape is shown below:



In each structure, a computer had randomly assigned a prize to one of the cells.

As part of the game, people could select two cells, in the hopes that one of the selected cells could be where the prize might be.

As you can imagine, people selected some pairs of cells more frequently than others.

In the next pages, you will see four 6-cell structures. Examples of pairs of cells selected by people are shown shaded in blue. Your task is to rank the selected pairs of cells in order from the pair that you expect was the most frequently selected to the pair that you expect was the least frequently selected.

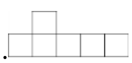
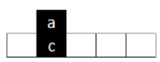
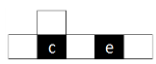
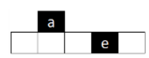
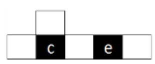
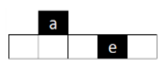
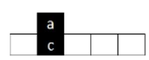
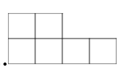
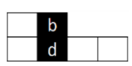
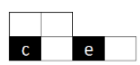
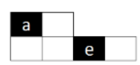
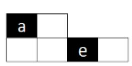
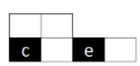
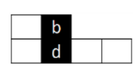
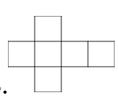
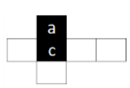
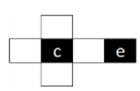
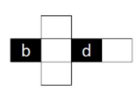
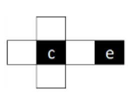
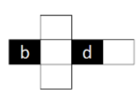
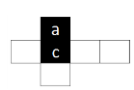
Participants. A total of 113 participants were recruited using Amazon Mechanical Turk.

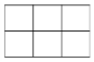
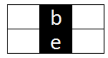
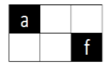
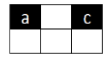
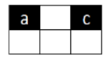
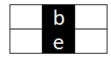
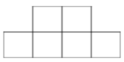
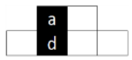
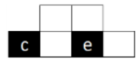
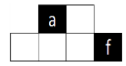
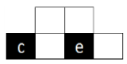
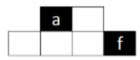
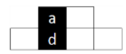
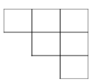
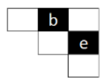
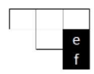
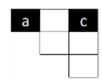
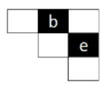
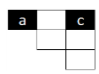
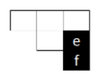
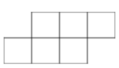
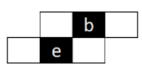
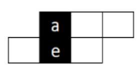
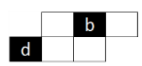
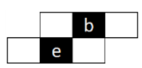
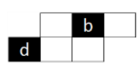
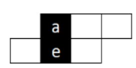
Results. As in Study 3A, of interest was the level of agreement of participant rankings with the hypothesized ranking based on CP values. Therefore, the correlation between several judges and the criterion ranking (T_C) was once again chosen as the measurement metric.

Table 7 shows all the structures presented to the participants, as well as the options of selection pairs to be ranked and their total CP values, calculated both holistically and additively. For example, in the case of structure 1, pairs *ce, ac,* and *ae* have a calculated additive CP of 2.5, 2.5, and 2, respectively. The selection pairs are ranked and presented in order of descending CP. In the case when a calculated CP (whether holistically or additively) was identical for two pairs of selections, the ranking was further decided based on an aspect of coverage that was deliberately chosen not to model in this chapter – coverage across rows and columns (CS). In Chapter 3 we found evidence that a selection covered all other cells in the same row (in the 5x5 grid) and even cells in adjacent rows and columns (in the 9x9 grid). In the case of the simple 6-cell structures used in Study 3B, it can be conceived that a selection covers all other cells in the same row, especially when the structure may lend itself more easily to the perception of rows and columns. For example, in the case of structure #1, pairs *ac* and *ce* have identical additive CP values. In this case, the *ac* pair was predicted to rank higher than *ce* because since it has a selection in each of the two rows, it performs better in ‘covering’ all rows and columns of the structure. In contrast, *ce* has both selections in the bottom row. By design, expected (predicted) rankings differed depending on whether the holistic or additive CP calculation was used. Therefore, the correlation value T_C and its associated statistical significance is shown for both predicted rankings.

Finally, for each pair of selection, the calculated P(A) value is provided below the holistic CP value (for convenience). While in most cases, P(A) values are in the same order as holistic CP values, there are cases (holistic) CP values can distinguish between pairs where P(A) values cannot (e.g., *bd* and *ac* in structure #3).

Table 7 Results of Study 3B

6-cell shape:	**Selected pairs by (fictitious) game players, from highest total CP to lowest.**				**Value& significance of Tc**
1.  N = 56	Additive	 CP= 2.5	 CP = 2.5	 CP = 2	0.02 (p = 0.417)
	Holistic	 CP = 2.5 P(A) = 5/5	 CP = 2 P(A) = 3/5	 CP = 1.25 P(A) = 2/5	**0.21 (p = 0.007)**
2.  N = 61	Additive	 CP = 3.25	 CP = 2.5	 CP = 2.5	-0.07 (p = 0.175)
	Holistic	 CP = 2.5 P(A) = 4/6	 CP = 2.5 P(A) = 4/6	 CP = 1.875 P(A) = 3/6	**0.19 (p = 0.012)**
3.  N = 57	Additive	 CP = 3	 CP = 2.5	 CP = 2.5	-0.25 (p = 0.001)
	Holistic	 CP = 2.5 P(A) = 5/5	 CP = 2.5 P(A) = 3/5	 CP = 1.75 P(A) = 3/5	**0.24 (p = 0.003)**

Structure	Model				Correlation
4.  N = 56	Additive	 CP = 4	 CP = 2.5	 CP = 2.5	-0.14 (p = 0.04)
	Holistic	 CP = 2.5 P(A) = 4/7	 CP = 2.5 P(A) = 4/7	 CP = 2.5 P(A) = 4/7	**-0.07** **(p = 0.18)**
5.  N = 59	Additive	 CP = 3.25	 CP = 2.5	 CP = 2.5	-0.01 (p = 0.45)
	Holistic	 CP = 2.5 P(A) = 4/6	 CP = 2.25 P(A) = 3/6	 CP = 1.875 P(A) = 3/6	*0.05* *(p = 0.34)*
6.  N = 60	Additive	 CP = 3.5	 CP = 2.5	 CP = 2	0.08 (p = 0.19)
	Holistic	 CP = 2.75 P(A) = 6/6	 CP = 2 P(A) = 3/6	 CP = 1.375 P(A) = 2/6	**0.24** **(p = 0.002)**
7.  N = 60	Additive	 CP = 3.5	 CP = 3.25	 CP = 2.5	0.06 (p = 0.27)
	Holistic	 CP = 2.75 P(A) = 6/6	 CP = 2.5 P(A) = 4/6	 CP = 1.875 P(A) = 3/6	**0.20** **(p = 0.009)**

Overall, Hypothesis 2 is supported in the results of structures #1, 2, 3, 6 and 7. In all these cases, there was a statistically significant correlation between the participant rankings and the expected rankings that were based on the holistic CP calculation. For those structures, the null hypothesis that rankings are made randomly is rejected at a level of significance of 0.01.

Of particular interest is the case of structure #4: all pairs had the same holistic CP, so we had attempted to predict ranking based on the differences in CS. This proved to be a difficult task. While, the pair *af* was ranked the highest as it covered 2 rows and 2 columns, it was more difficult to discern between *ac* and *be*, which covered 1 row/2 columns and 2 rows/1 columns respectively. An arbitrary decision was made to rank *ac* higher than *be*. Yet, it was found that there was no general agreement among participants and no correlation with predicted rankings. *As it would have been predicted by the CP values alone, no ranking order emerged from the participants' choices*. One explanation for this is that structure #4, unlike other tested structures, has a highly regular and symmetric shape. It is thus possible that there are more factors at play than we have been able to capture (e.g., similarity according to symmetry) and that those factors are affecting coverage in more ways than we are able to take into account at this time.

In the case of structure #5, it is found that there is a non-significant correlation between participant rankings and the rankings predicted through the holistic CP calculation. An explanation for this can be found again in CS. In particular, while the *ce* pair has a calculated CP that is just 0.25 higher than the *af* pair, the second pair has better CS, having a selection in each of the structure's two rows. It is possible that given the low difference in CP, the large difference in CS has a significant impact on the perceived total coverage achieved by each pair, resulting in a fairly even ranking of the pairs.

The additive CP calculation was not successful in predicting participant rankings in any of the structures. When considering the significant predicting power of the holistic CP calculation, it can be concluded that coverage is perceived holistically rather than additively, as proposed by Hypothesis 3.

A final point is with regards to the competing model – over-alternations. Overall, in the case of the double selection (Study 3B), the predictive power of P(A) is not as good as in the case of the single selection (Study 3A). First, in some cases (e.g., *ae* and *ce* of structure #2), pairs of selections have the same P(A), but different (holistic) CP values. Thus CP is able to distinguish between and predict participants' rankings of selection pairs when P(A) cannot. Second, according to the over-alternations model, maximal alternations appears too regular and thus are not judged as random; yet, in some selection pairs that have high CP (and as predicted are also highly ranked by participants) have the maximum possible number of alternations P(A) = 1 (e.g., *ce* of structure #3 and *be* of structure #7). This would contradict the prediction that the selections that are judged as most random have a higher than expected, but not maximal P(A).

5.3 General discussion

Studies 3A and 3B tested the coverage maximization model of choice in a controlled context of search in small 6-cell structures. Whether one or two selections were made in the structure, participants believed that selections or pairs of selections that had the most coverage were also the ones more likely to be chosen to find a randomly located prize.

When it comes to the case when more than one selection is made, Study 3B strongly supported the 'gestalt' view of coverage: participant choices were in agreement with the rankings predicted by the holistic CP calculation and either uncorrelated, or negatively correlated with the rankings predicted by the additive CP calculation. What this means in practice is that a selection pair containing a cell that would have had a high CP had it been a single selection, might end up not being chosen if the second cell it is paired with results in a low overall coverage for the pair. With regards to how holistic CP was calculated, in Section 5.2 two options were presented for calculating the value of overlapping CP, with the second ultimately chosen in making the ranking predictions in Study 3B. In all 7 structures, both options would have predicted the same rank order for all selection pairs – with deviations only in the magnitude of difference of calculated CP between the pairs. Therefore, without further studies it is impossible to confirm whether Option 2 is indeed superior to Option 1.

As discussed at the beginning of this chapter, in studies 3A and 3B we chose to not use CS in determining the predicted rankings. The models of CP and CS in the 9x9 and 5x5 grids arrived at in Chapter 3 have not been yet been developed to the extent of the interaction of the two principles of perceptual organization. We assumed that given the small size of the chosen structures and their irregularities, CP would be dominant and the influence of CS would be minimized. Generally, choosing small simple 6-cell structures and basing rankings mostly on the calculated values of CP and only using a CS-type judgement to break ties proved sufficient in predicting participant choices. A future study could more carefully investigate the interplay of CP and CS and whether one is more dominant, especially as the grid size changes. Yet, modelling CS and assessing its influence and interplay with CP can prove difficult, especially in the smaller structures such as the ones used in studies 3A and 3B. In Chapter 3 coverage by similarity was only analyzed and defined based on groupings by rows and columns; however, one could easily imagine cell groupings that are formed based on other similarity principles, e.g., symmetry. In the smaller structures, in particular, where the rows and columns groupings are not as salient, it would be difficult to ascertain if a cell is covered by row/column similarity, by symmetry, or both.

An important question that was asked was whether the over-alternations model could have made the same predictions with respect to participant choices. While this model made good predictions in Study 3A (but no better than the coverage maximization model), its predictive power was weak in Study 3B, where it was not always able to distinguish between selection pairs.

Another alternative theory – ease of encoding (Falk & Konold, 1997) – was not formally tested in this chapter. That is because we were not able to develop a suitable measure for the 'encodability' of the single selections and the selections pairs, due to the small size of the structures and the limited number of selections. Nevertheless, in Section 6.2 of the next chapter we further discuss the merits of this theory and assess its potential ability to predict participant choices in Studies 3A and 3B.

In summary, Study 3A and 3B showed that coverage maximization is a useful model for predicting participant choices in the case of searching for a randomly located item in a structured space. In the next chapter we summarize the main findings of this thesis, outline its overall contribution, and discuss the model's limitations and future research directions.

6. General discussion and opportunities for future research directions

6.1 Summary of findings

This thesis began with the example of the simple task of 'randomly' choosing three cards from a deck of well-shuffled cards. Experience told us that most people would avoid the very first and the very last cards and spread their choices such that one card was selected from each of the beginning, middle, and end portions of the deck. A review of the literature found other examples of spreading-like behaviour from contexts as varied as dropping bombs (Feller, 1957), selecting papers (Rubinestein & Salant, 2006), creating advent calendars (Sanderson, 2014) and more generally the perception and generation of 'random' 1D sequences (Falk & Konold, 1997; Ayal & Zakay, 2009) and 2D patterns (Falk, 1975; Lisanby & Lockhead, 1991; Falk et al., 2009; Hsu et al., 2010). They confirmed that when people try to generate random patterns, whether in 1D or 2D contexts, the selections are predictably 'spread'. In addition, people differentiate between the internal region of space and the edge, generally avoiding the latter.

Of particular interest in this thesis was the special case of the longshot (i.e., overall low probability of success) search in the 2D domain. Thus, the first study aimed to formally capture the spreading tendency in a 9x9 grid of cells. Participants were asked to select 3 cells from the 81-cell grid, in efforts to uncover a prize that was randomly assigned to one of the cells. All cells had exactly the same probability of containing the prize and were completely independent of each other. Yet, as predicted, participants avoided making selections at the edge of the grid and spread their three selections, such that a selection was rarely in a location too close or too similar (i.e., in the same row or column) to a prior selection.

The question that arose was thus "Why do people spread?" Study 1 results as well as other supporting literature suggested that people avoid selecting cells that are *near* or *like* cells that have resulted unsuccessful in prior selections. We thus proposed that participants assign similar attributes (in this case the odds of being assigned a prize) to cells that are perceptually grouped together, whether the grouping principle is proximity or similarity. A selection's coverage by proximity (CP) was defined as the group of nearby cells whose probability assessments are influenced by the outcome of that selection. Similarly, a selection's coverage by row/column similarity (CS) in the grid was defined as the set of cells in the same or adjacent row/columns whose probability assessment are affected by the outcome of that selection. If a selection has resulted unsuccessful, the probability that other cells in that selections' CP or CS will be selected next is lower than we would expect if the selections were made randomly. Both CP and CS were investigated in depth using data from the 9x9 grid of Study 1 as well as the 5x5 grid of Falk et al. (2009).

Another important prediction with respect to coverage was that in the context of search, participants try to choose a location in the grid that affords them the greatest coverage possible. This prediction was tested in Study 2, using a new context where the size of the grouping (i.e., coverage) was enforced rather than left to (individuals') perceptual groupings. As in Study 1, participants were presented with a problem in which the prized element (a red marble) was hidden in a set of identical items (blue marbles), with a low overall probability of it being found. The marbles were forced by the experimenter into two distinct groups (the larger and smaller jar). In both cases, the groupings should have had no bearing on participant choices. Yet, participants believed that when directly comparing a marble that had been grouped in the larger category (larger jar) with one that had been grouped in the smaller category (smaller jar), the former had a higher probability of being red than the latter. The effect of the grouping's size on the individual elements' probability judgments, as also previously found by Isaac and Brough (2014), was demonstrated unambiguously.

The findings of Study 1 and Study 2, together with our understanding of the concept of coverage were synthesized into the thesis of this dissertation: people's choices in the context of searching for a randomly located item can be modelled as a tendency to maximize coverage. The model was successfully tested in Studies 3A and 3B, using simple 6-cell structures. Participants were asked to judge the likelihood of certain cells being selected when trying to guess the location of a randomly assigned prize. Both cases of single and double selections were tested and in each, coverage maximization as a model of choice reliably predicted participant preferences. The higher the coverage achieved by the selection or pair of selections, the higher they were ranked by participants as a suitable choice for finding the prize. *In summary, the coverage maximization model has provided a plausible answer to the questions posed at the beginning of this thesis: avoidance of the edges and spreading of choices are both outcomes of coverage maximization*.

6.2 Alternative explanations and contribution

Various existing theories were explored in Chapter 2 in order to find an early explanation for spreading and the avoidance of the edge. On a first look, it seemed that a simple explanation for people's choices would be representativeness (Kahneman & Tversky, 1972) – people were making selections that they thought were random, thus, the attributes of these selections (avoidance of edges, spreading) were representative of what participants thought selections made by a random process ought to look like. It was unclear, however, why those attributes were in fact representative of chance.

One potentially good model for predicting participants' choices came in the form of over-alternations as a form of negative recency. In binary 1D and 2D structured spaces, Falk and Konold (1997) confirmed that sequences that were judged the most random had a probability of alterations P(A) at a "sweet spot" of roughly 0.6. In other words, while in general, the sequences that had a higher number of alternations were judged to be more random, when the number of alternation approached its maximum, patterns of alternations emerged that made the sequence appear less random. This framework can model choice in binary contexts very well, but its predictive power is lost in more general contexts. As illustrated in Chapter 2, in the 3-selection context of the 9x9 grid of Study 1, one can achieve a high P(A) with three selections spread out in just one row and still result in a pattern of selections that most participants would judge non-random. The alternations framework comes closest to our coverage theory in the context of the 6-cell structures used in Study 3. Those contexts were not quite binary, but with the probability of success being 1/6 and 1/3 in Study 3A and 3B respectively, the parameters are at least comparable. In the case of the single selection (Study 3A), there was an almost perfect correlation between P(A) and CP; thus P(A) (as did CP) made the same (good) predictions about participant choices. The predictive power of P(A) is not as good in the case of the double selection (Study 3B). First, in some cases, pairs of selections would have the same P(A), but different CPs. Thus CP was able to distinguish between and predict participants' rankings of selection pairs better than P(A) could. Second, in some cases, the selection pairs that had high CP (and as predicted were also highly ranked by participants) had the maximum possible number of alternations (P(A) = 1). This would contradict the prediction that the selections that are judged as most random have a higher than expected, but not maximal P(A).

From a calculation point of view, the measurement of alternations is very similar to the measurement of coverage. In particular, the differences between the two are very small in smaller structures (e.g., 6-cell structures of Study 3), with just one point of departure: in addition to cells adjacent to a selection, which would also be counted by alternation, CP also takes into consideration cells diagonal to it. Conceptually, however, coverage is well-defined and substantially different from the alternations model. These differences become obvious in the cases of larger grids (where coverage increases to beyond just adjacent cells) and multiple selections (where we have a more subtle understanding of how cells that are close or similar to more than one selection are influenced). Finally, alternations can only resemble one aspect of coverage – CP – whereas we know that probably assessments are also influenced by the principle of organization of similarity, as we have defined through CS.

Falk & Konold (1997) suggested that over-alternating sequences may be more difficult to encode, thus proposing that people judge the randomness of a sequence based on their (tacit) ability to encode the sequence. In their studies, they measured this through the ease (or more commonly, difficulty) of memorizing the sequence. There are cases where coverage maximization and ease of encoding would make similar predictions. For example, one could argue that cells at the edge of a grid are both easier to encode and also have lower coverage – both resulting in being perceived as less random. Another example is three adjacent cells compared to three cells spread out in the grid. Again, one could argue that the three adjacent cells would be easier to encode/memorize/copy compared to the spread-out cells, with the latter, in both cases being thus judged as more random. Thus, the question that arises is "Could encoding be used to explain the results of Study 3"? Our answer is "not likely". For example, consider the case of the first 6-cell structure of Study 3A (Table 5). Cell *c* has a considerably larger CP than cell *f*, which also correlated with participant perceptions who ranked *c* higher than *f*. The ease of encoding theory would suggest that the selection that is considered more random would also be harder to memorize. However, one would have difficulty justifying *c* as more difficult to memorize than *f*. While *f* can be easy described (and therefore remembered) as the cell at the very right, c also has a very special location at the intersection of the two segments. It can thus be concluded that while the model of over-alternations and theory of encoding can often make good predictions, such that in some cases they overlap with the predictions made by the coverage maximization model, the latter has overall better predictive power and is more generalizable to non-binary 1D and 2D settings.

Overall, the literature review conducted in chapter 2 did not provide a satisfactory account for the spreading and edge avoidance behaviour. In contrast, the coverage maximization model that was developed and tested in this thesis threads together the concepts of proximity, similarity, and size, and provides a cohesive explanation that alone can explain people's choices in the context of searching for a randomly located item in a variety of set sizes and selection opportunities.

When spreading behaviour was described and confirmed in Chapter 1, there was a temporary temptation to label the behaviour as a bias; 'spreading bias' would join a long existing list of cognitive biases[1]. An important contribution of this thesis, however, is the delivery of a systematic theory and rationale for this bias, and a detailed description of the processes in place that sum up to the observed spreading behaviour. Much of the current discussion on subjective probability is framed around dual-process theories. They may use different labels – e.g., *cognitive-experiential self-theory* (Epstein, 2003) and *System 1 vs System 2* (Stanovich & West, 2000; Kahneman & Frederick, 2002) – however, fundamentally these theories describe human cognition as subject to two kinds of processes: the first type (System1) is fast, effortless, and intuitive; the second (System 2) is slow, effortful, and rational. Some of the most well-known subjective probability biases – e.g., the ratio bias effect (Miller et al., 1989) - have been interpreted or reinterpreted as due to shortcomings or inappropriate use of System 1 or conflict between the two systems (Denes-Raj & Seymour, 1994; Pacini & Epstein, 1999) . In that vein, one could argue a similar explanation for participant choices in the grid: people use their intuition and experience of what random selections ought to look like (i.e., System 1) when making their selections. However, the account that we have proposed in this thesis shows that there exists a 'calculus' for the observed pattern of choices. People

[1] See for example https://en.wikipedia.org/wiki/List_of_cognitive_biases

make mistakes in their judgements, but these mistakes occur because of our perceptual limits. Mathematically, all cells in the grid are independent and their configuration is of no relevance to their probabilities. Objectively, under the right framing, groupings of the cells could impact judgements of probability. For example in the 9x9 grid, it is objectively correct to state that the probability that the prize is in one of the 4 corners of the grid is smaller than the probability that the prize is in one of the 49 internal cells of the grid. The size of the grouping does matter objectively, just as it does subjectively. People's failing however, is in taking this a step further and expecting a singular corner cell to be less likely of containing the prize than a singular internal cell. Because of human perception, the cells within a grouping (whether by proximity or similarity) lose their independence. The selection of a cell becomes psychologically equivalent to the selection of all other cells in its proximity. Thus, the larger the grouping size, the larger the number of cells people feel they have already sampled through that one selection.

6.3 Limitations and opportunities for future work

Despite its success in predicting behaviour in the controlled experiment of Study 3, the coverage maximization model is in its infancy and thus has important limitations, providing ample opportunity for future research.

First, in this thesis we have considered the process of generating random selections and the process of searching for a randomly assigned prize as interchangeable. The reason for this was because in Study 1, when asked about what prompted them to make their selections the way they did, most participants expressed that they were just selecting cells randomly. It was argued that in trying to find a prize that was placed by a random process, participants were attempting to recreate a random process of their own. This assumption was crucial in that it allowed us to use data from the 'random condition' of Falk et al. (2009) in order to extract the attributes of CP and CS in the 5x5 grid – a stepping stone in building our coverage maximization model. The assumption was also supported by the findings of Falk et al. (2009) themselves, who reported correlations between participant selections in the random and competitive conditions. Future studies could further investigate whether the behaviour of randomly selecting cells in the grid would differ (whether in process or outcome) from the behaviour of selecting cells with the intention of finding a randomly located item.

A second limitation of the model is in the development of CS. In the larger grids (5x5 and 9x9), the groupings of cells according to rows and columns was salient enough that we were able to determine the parameters of coverage by row/column similarity. However, grouping by similarity can also be based on other factors, such as symmetry. This kind of grouping would be particularly salient in smaller grids; however, we have so far not accounted for this type of similarity in our CS calculation. Thus, an opportunity exists for future research that investigates coverage by 'symmetric' similarity.

Even with respect to CP, which we believe has been developed more robustly that CS, the model is still only developed in a very contrived context of a regular grid of cells. One may wonder whether CP can be easily used in other contexts. For example consider this prior study: Shaw et al. (2000) compared participant preferences when choosing to sit in one of three chairs where one of the end chairs was occupied by a backpack. In both the left-occupied and right-occupied conditions 68% of participants chose the center chair and 32% chose the unoccupied end chair. One could take this example as evidence that rejects the coverage theory; after all the backpack should have covered more than just the chair it was occupying and more participants should have avoided the centre chair. There is, however, an important detail: the chairs were 3 feet apart. It is plausible that the chairs were distant enough that the backpack's CP was limited to the chair it was occupying. This example may provide a clue that even in the context of the grid of cells, CP might be affected not only by the size of the set and the number of selections, but also by the physical distance of the cells from each other.

Another limitation relates to the interaction of CP and CS. While we discovered the parameters of CP and CS in the 5x5 and 9x9 grids, we did not investigate how 'much' a cell is covered when under both the CP and CS of one or more selections. In Study 3A and Study 3B, we used a search space context that purposefully minimized the influence of CS, allowing us to test the coverage maximization model using just CP and only referring to CS when needed as a tiebreaker. Future research could help test the relative strength of CP and CS, model their interaction more precisely, and discover their sensitivity to contextual factors such as the size and shape of the search space.

Finally, the parameters of CP and CS were determined in the specific circumstance where 3 selections were made in both the 5x5 and 9x9 grids. For example, it is reasonable to assume that the size of CP that was discovered **-** 1/3 of the grid – was directly related to the number of selections: To cover as much of the grid as possible given 3 selections, each must cover roughly 1/3 of the grid. There exists thus an opportunity for future research that more carefully investigates the effect of the number of selections on the size and other attributes of coverage.

6.4 Conclusion

In conclusion, this thesis has described people's choices when looking for an item that they believe could be anywhere in the structured search space (i.e., its location is randomly assigned). A newly developed model – coverage maximization – explains and predicts important aspects of choice including the avoidance of edges, avoidance of neighbouring selections, and the spreading of choices in the search space.

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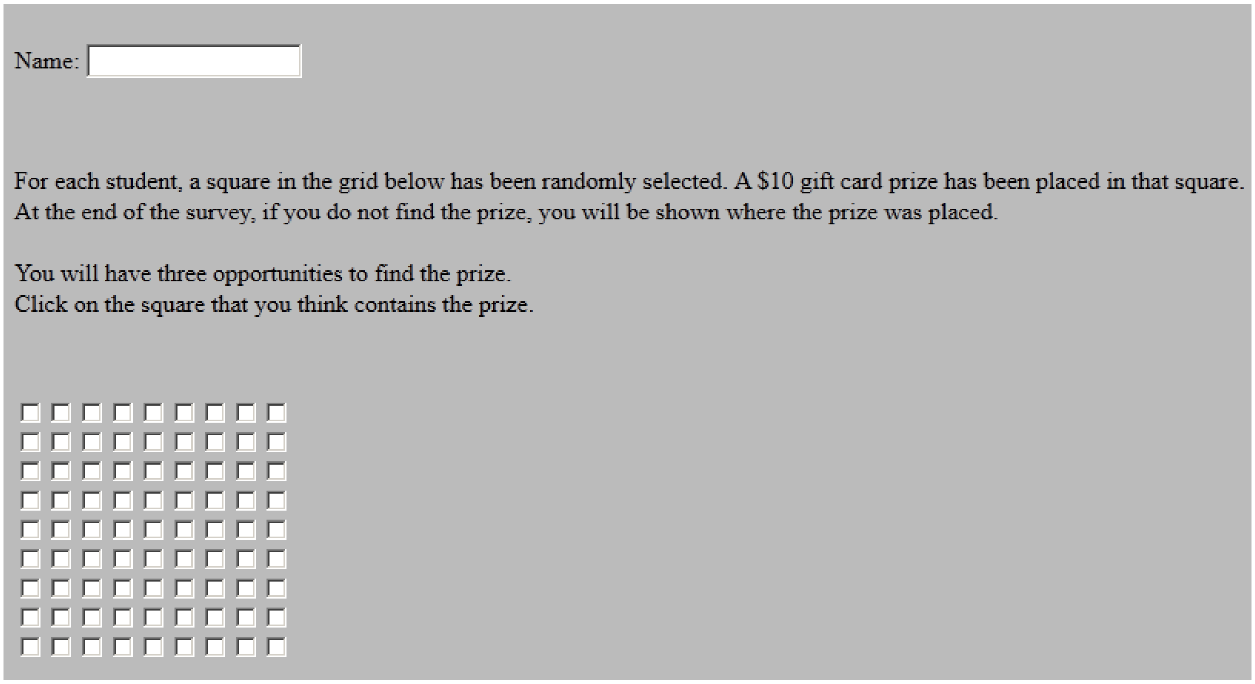
Appendix A: Study 1 supporting materials, data and methods

A.1 Study user interface



A.2 Data

Mapping of choices can be seen below. For example, a choice in the top-left corner is labelled as '11'.

11	12	13	14	15	16	17	18	19
21	22	23	24	25	26	27	28	29
31	32	33	34	35	36	37	38	39
41	42	43	44	45	46	47	48	49
51	52	53	54	55	56	57	58	59
61	62	63	64	65	66	67	68	69
71	72	73	74	75	76	77	78	79
81	82	83	84	85	86	87	88	89
91	92	93	94	95	96	97	98	99

Below are the selections of each of the 37 participants. The choices are given using the mapping in the table above. For example, choice 1 (C1) of Participant 1 is cell 45, which is the cell that is located in the fourth row from the top and the fifth column from the left.

Participant	C1	C2	C3
1	45	79	83
2	78	52	29
3	38	56	24
4	44	37	25
5	87	61	36
6	38	11	37
7	22	96	37
8	22	37	74
9	55	23	87
10	65	27	33
11	13	56	83
12	45	91	15
13	57	32	84
14	25	65	19
15	48	64	24
16	37	64	78
17	22	88	19
18	33	74	58
19	59	68	38
20	55	37	93
21	45	63	87
22	46	63	97
23	52	27	88
24	55	33	68
25	44	87	28
26	26	53	99
27	43	17	75
28	35	77	52
29	38	53	27
30	55	83	19
31	63	26	78
32	24	56	61
33	14	99	82
34	37	62	98
35	43	17	55
36	55	33	11
37	33	55	88

A.3 Equations for calculating properties of choices

The distance between each pair of selections (x, y) was calculated using the Euclidian distance formula:

$$Distance\ between\ any\ two\ selections_{Participant} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

The minimum distance between selections for each participant was found by calculating the pair distances between the first, second, and third selections and finding the smallest distance.

$$Minimum\ Distance\ between\ Selections_{Participant}$$
$$= Min\left[\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}, \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}, \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2)}\right]$$

The overall **average minimum distance** between selections for all participants was found by averaging the above across all participants:

$$Average\ Minimum\ Distance\ Between\ Selections_{All\ participants}$$
$$= \frac{\sum_i^N Minimum\ Distance\ between\ Selections_i}{N}$$

Another measure that was calculated for each participant was the **area of the triangle** created by the three selections:

$$Area_{Participant} = \left|\frac{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)}{2}\right|$$

The overall average area was found by averaging the above across all participants:

$$Average\ Area_{All\ participants} = \frac{\sum_i^N Area_i}{N}$$

It was important to know where the selections were falling relative to the edge of the search space. Hence, another measure was the minimum **distance from the edge**, the smallest distance of the selections from the edge. The equations below demonstrate the calculation made for each selection:

$$Minimum\ distance\ from\ left\ or\ right\ edge_{selection} = Min[|x - 1|, |x - 9|]$$

$$Minimum\ distance\ from\ top\ or\ bottom\ edge_{selection} = Min[|y - 1|, |y - 9|]$$

$$\therefore \quad Minimum\ distance\ from\ any\ edge_{selection}$$
$$= Min[Minimum\ distance\ from\ left\ or\ right\ edge_{selection}, Minimum\ distance\ from\ top\ or\ bottom\ edge_{selection}]$$

Accordingly, the minimum distance from the edge is found for each participant, and then, for all participants:

$$Minimum\ Distance\ from\ Edge_{Participant}$$
$$= Min[Min.dist.from\ any\ edge_{selection\ 1}, Min.dist.from\ any\ edge_{selection\ 2}, Min.dist.from\ any\ edge_{selection\ 3}]$$

$$Minimum\ Distance\ from\ Edge_{All\ participants} = \frac{\sum_i^N Minimum\ Distance\ from\ Edge_{Participant\ i}}{N}$$

Finally, it was also important to measure how often participants were placing 2 or more selections in the same or column. For each participant this was calculated as the sum of all instances when pairs of selections had the **same x or y** coordinate. This value was then averaged across all participants.

A.4 Transcripts of interviews

A.4.1 Instances of descriptions of 'spread' behaviour

1	"To me it's random...I did try to do a bit of a spread, trying to go into 3 different locations"
2	"I don't know why I didn't bunch them all together, i just figured, kinda like playing battleship, before you have anything, just throw them all over the board."
3	"I guess you look at it…I went not entirely as far away as possible, and then the third one I put as far away as possible from the first two... but you spread them out as far as possible.
4	"…and I didn't have any reasons for the next two, I just happened to choose them in different regions of the map...so I didn't choose all three in a row right next to each other.. I choose them in different spots to maximize my chances of finding the prize"
5	"…so you think of different areas.."
6	*"Why not all together"* "I thought about it, but I thought I'd have a better chance if I put them all in different areas."
7	"Technically it would make more sense to be all in one area..it was just something I thought...I don't know why exactly I did this, to be honest with you. I honestly I do not have a reason why I did that. I tried to try different areas"
8	*"You spread out your choices...why?"* "I thought the chances of getting the prize is a lot higher if I just spread them" *"Would you say that you tried to cover the whole space"* "Yes, yes. I don't know I just have a feeling that if I cover a lot more space I'd have a greater chance."
9	*"Experiential? You picked one, why wouldn't your picks be all in one area?"* "Umm...maybe just, like personal habits or something. I think that if I just focus on one point the chance will be lower. I think I don't know why, but I just did that
10	*Once you made your first pick, why did you go over here?"* "Just to increase the spread.." *"And why is that?"* "To possibly increase my chances. I don't know how the program is written. I mean I just increased the spread. There's no difference between me clicking there and me clicking there...so, I just thought to be random, but I guess by nature you just increase the spread."
11	"*Which one was your first pick?"* "This one...top left." *"Second"* "This one, because I guess it was the farthest away, I guess to check out the other half of the map. I don't know. "The third was a corner"
12	"So I just picked three spaces. I guess I spaced them out. I thought about picking 3 in a row, but I divided I guess in my head…maybe it will be in this quadrant, or this quadrant, or this quadrant, so I guess I picked one from each"
13	" First I went for the middle one, I guess, and then the remaining two picks I took ones from opposite quadrants."
14	*"Strategy?"* "Kinda I guess, to cover the most area." *"Okay, once you made that pick and it was bad, were there any particular boxes that you wouldn't have selected?"* "it would have been like the middle one from one of the other four corners. I tried not to cluster, I wanted to spread out more" *"Any other thoughts?"* "Not really, I just tried to diversify where I pick my points"
15	*Your first choice was here…once you picked there, did it constrain where your next one was going to be?"* "Yeah I guess I didn't want to pick the second one right next to it. Maybe a few rows and columns apart" *Why?* "I don't know...when you see lottery numbers being picked, they're usually far apart from each other so, it was just like"
16	*"What went through your mind?"* "I just have been focusing on one square, the first one, since I started. I saw a pattern there, in that corner. It's mostly this square pattern that I see. When I picked this, I saw another square that was about this side. So corner doesn't, work, so let me try another edge..." *"So you created 4x4 areas, and then tried to look at different spots in these areas? Once you picked your first one, were there any areas you wouldn't have picked?"*

	"I wouldn't have picked anything in this square." *"So you wouldn't have picked anything in the square you had already explored"* "I wouldn't have picked one, two , and three next to each other. It was not even an option for me. I didn't even think about it." *"Why?"* "It's just the variety. Saying it could be anywhere. I think my brain works that way. I would like to have a more expanded region of the three checks, compared to three checks that take away small area
17	*"Was there any thoughts that went through your mind?"* "I just kinda tried to pick three different areas that were far from each other, I guess I could cover most of the space."
18	*Why not close by?* "I don't know, I just put them farther apart."
19	“Spread them out I guess. I know it doesn't really matter whether it's right next door or really far, but I've been playing battleship...no particular reason. For some reason I had a tendency to pick them in a square so that they were equidistant, but then I thought that would be unlikely. More guesswork"
20	*"Why not all together"* "Chances are lower if they're all together. They're not, but intuitively I guess..
21	“I figured since it was kinda random, it didn't really matter where I picked, so I just tried to spread them out a little. I don't know, i just thought they should be spread out. I figured I could cover more area, I don't know why I did that! I just figured that if I covered more area, I would reveal more. I figured I should try to explore every part of the square. Cause you only get three choices, so I figured I’d do a triangle.
22	*"Avoidances?"* "Kinda far away, I didn't want to make it near the first one. 9x9 grid, so I didn't want to select all three in one small area. I felt like if I had more range of selection...I know it doesn't matter...but I just did...
23	“I always think that it's never going to be close to where you think it's going to be, it's always going to be far.. so that's why I always try to pick the biggest area, try to spread them out...I don't know I just...cause I feel like the odds of getting it are better if you spread it rather than putting all your eggs in one basket."
24	"I thought about doing that (putting them together), but for some reason , it felt like it was slimming the chance of hitting it, because they were all so close, and it felt like it was just one, instead of 3"
25	*"why not all together?"* "I think I personally would have spread them out, but I don't know if that would statistically help me all, but I think that I would just like to take out a larger portion of the boxes.
26	“I guess it wasn't random, I don't know I tried to cover as many regions...I guess if you could cut it into thirds"

A.4.2 Instances of descriptions of ‘selecting randomly’

1	They're just like random
2	but then i really picked it random, where i was looking, i picked there.."
3	Umm.. I'm trying to think of ways that I could have gotten the answer, but in the end it just is random so I clicked random squares
4	Uhhh...similar to Battleship, but there was no feedback, so it was randomly selecting a location, but I wouldn't pick 3 in a row, I didn't think it would be adjacent to where I had picked already, so I thought it would be best to sort of random my move..
5	"Uhh...not really for strategy I guess, the first one was completely random,
6	It's supposed to be random so it doesn't matter where I pick."
7	"It was pretty random...I don't know.
8	First one it wouldn't matter where it is...I just picked any random one
9	But then, i just randomized them in no particular pattern.
10	The first one was really randomly...that's the thing
11	no I just thought this was totally random because it's produced by a random number generator, so it didn't really matter. So yeah, just by intuition."
12	.so, I just thought to be random, but I guess by nature you just increase the spread."
13	Okay, each box has equal chances to win the prize, then 3/81 is a pretty small number. I guess, if I'm not really lucky today, I probably don't get the prize, so I just randomly picked three boxes, pretty much." Uhh...well I guess I chose it randomly and doesn't really have any reason that why I picked the second one, it just happened to be that one."
14	"When you did this, did you have any strategy?" "I didn't really use any strategies, I just kinda randomly picked boxes"
15	Basically I figured it was all random, so it didn't matter where I picked. So I just picked three spaces
16	"Was there something prohibiting you from putting them all in one row?" "No, initially I was going to...when you first told me it was random I thought it doesn't matter, so I'd put them all in one row, but once I started clicking... No I wouldn't say that I looked at one point and I clicked it. i just kinda moved the mouse around and clicked."
17	It was just luck, so the chances aren't too good, it's just 1/27. So since it's randomly generated, I just randomly clicked"

18	But then when you think about it...because it was randomly selected didn't think too much about it, it was just like click, click, click."
19	"I didn't have a strategy, I just picked 3 random boxes..."
20	"There wasn't really a particular strategy, since all the boxes are build the same. So really i just randomly picked a place." "There wasn't until the third one where I picked the corner, whereas the first two were kind of random."
21	I thought it was randomly selected, so I didn't have a strategy or anything, I was just randomly clicking.."
22	Basically, I just didn't know where to start, so I just started clicking randomly
23	"I did them all random. I just looked and then I picked the one that shone at that moment."
24	Uhh, I don't know, just, I was just trying my intuition, if i get a feeling, so the last one I was tryingto hit the centre, not reason for that all, just random selection"
25	"Randomly, I don't know, it wasn't much of a process

A.4.3 Instances of descriptions of distinctions between 'centre' and 'edge'

1	"I was going to go for the centre"
2	guess I tended to pick in the middle and not in the outside...I don't know why.. I didn't pick the edges. "The reason that I didn't choose the edges, I suppose is, I kinda separate into an edge probability and then everything except the edge probability, so I thought, well, it's probably not going to be on the edge, it's probably going to be inside the edge…I don't know why..
3	Why no edges? "Not unconsciously, umm, sort of to me it's very little chance of actually succeeding in selecting the right square so I didn't..."
4	"I wasn't really looking...I don't know...I didn't think putting them in the middle...even though it was random...I doubted it...and then I was thinking of going at the very first square…but I guess the chance it's the same but it didn't seem like it would be there"
5	" I don't know ..it just.. there's as much chance of it being there as anywhere else...so...I guess that's why the prizes are hard to come by...they're hidden in the edge and not many people go there"
6	"I was originally going to select the first 3 on the edge, one, two, and three, but I thought there was no reason to do that."
7	I don't know...the way I tried to think of it, middle and then sides.. that's the approach i usually take "Umm.. normally I probably wouldn't have considered the corners, just because from experience it's never usually in the corners"
8	"Well, at first, I though, kinda looked at the whole box, the column, and the rows, and thought, well, it might be at the centre. "Were there any places you avoided?" "Bottom right for some reason, just didn't feel like..." What about the edges? "Umm, no, didn't cross my mind at all. I just I don't I focused on the centre"
9	"Were there places you avoided?" "Corners
10	I don't know…psychologically those people who created this square...obviously the corners and the centre the prize cannot be under those
11	"Uhh, I think that it's… I think that that would be a common behaviour, in that situation, people would not click the box on edge or on the angle, or like very at the edge ofthe picture...cause I think the experience tells me that I have a bigger chance if I click on the centre of the picture."
12	"Why not the edges?" "It's just from habit. You typically stay away from edges…I don't know"
13	"Avoidances?" "Logically, I want to say that it's from past experiences, but it seems like it wouldn't be in the very corner, or it's usually in the middle. There's no basis for that, inherently I thought about it being around there" "What do you mean by past experience?" "Umm, I don't know it's hard to explain. If you think you got a grid, and it's hidden there, odds are that it's not going to be you know in the corners. Odds are just as much it being there as not, but you only have 3 choices. And I'm trying to make odds as good as possible. Trying to rule out a couple of places.." "Why rule out the corners?" "No reason, it's just something that I do."
14	" First I went for the middle one, I guess, and then the remaining two picks I took ones from opposite quadrants." "Third one is at the edge, why?" "I just tried to try a fringe case I guess."
15	"What about the fact that the edges didn't really.." "You just think that maybe it's not there. chances are it's in the middle. When you first look at it makes you focus right in the middle, so that why I picked in the middle. "
16	Why that particular box for your first pick?"

	"I guess nothing in particular, but mostly because it was in the middle. I guess it was what my eye looked at first." "Why no edges?" "No reason, I don't think there is a reason. I was going to pick this one and then i decided to change. I had a feeling it was that one, but clearly my feeling was wrong." "Any general thoughts?" "When I first was going to pick I was going to pick something on the edge, but I decided not to and picked something in the middle. I don't know why, it doesn't make any difference."
17	I don't know, like, in my first pick I just chose the middle, because I think it's just unintended. I think the probability of having it in the middle is higher, even though I know that it's the same, but it just looks better so I picked the middle "Any avoidances?" "The corners..uhh, i guess, cause I play battleships, and when I play I don't play corners cause I think the chance is low, even though I know it's the same. In my mind it's telling me it can't be in the corners
18	"Why no edges?" "I assumed it be somewhere closer to the middle, but not exact middle.
19	Why corner? "I kinda thought that maybe there was some trick to it, maybe they wanted to hide it well, so they went to the corner, people wouldn't know to look there".
20	Umm, the first one I chose this one first, I guess it just felt right. It's sort of central but off to the side a little bit. Second one, I wanted to go with the extremities, and the third one I wanted to get back to the middle, but not too close to the one I had already chosen "Why on the edge?" "I thought to pick it on the edge, the edges get often neglected, i didn't want to miss out. In general, I feel like people tend to think that they'd be more in the middle of the maze, as opposed to in the edges.
21	I picked extremities, all four corners, close to the centre, as best as I could" "Why neglect the edges?" "Just intuitively it's the border, so it's not going to be there. But chances are the same I guess"
22	"I didn't even consider the edges at all. In my mind, I just thought they wouldn't be there. I just thought it would be somewhere in the centre-ish "
23	First I thought maybe well, I wasn't really sure how this was going to go, so I just picked one to see how it was going to tell you whether it was right or wrong. That was just a totally random one, like a test one. Second one I picked the corner, just because it was a corner. I always pick one corner in any situation. "
24	Uhh, I don't know, just, I was just trying my intuition, if i get a feeling, so the last one I was trying to hit the centre, not reason for that all, just random selection"
25	Well I started out in the middle, just because it seems like it intersects all of the quadrants Why corner "it was a long shot, like a shot in the dark, they're evenly spaced, so it looked nice

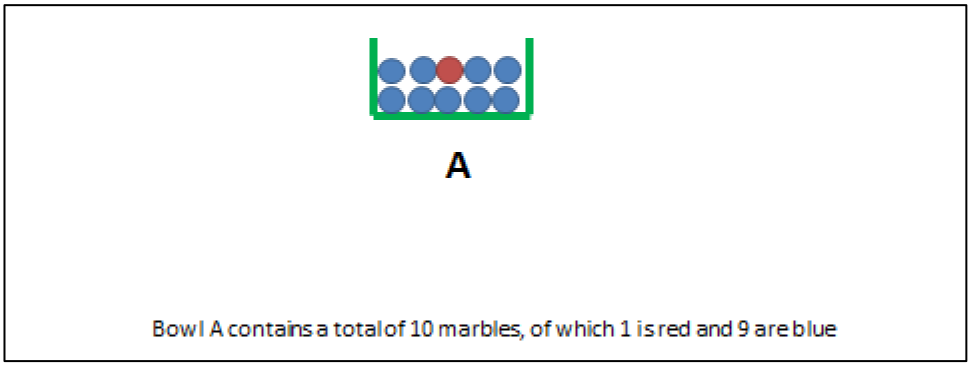
A.4.4 Instances of descriptions of changing region after negative feedback

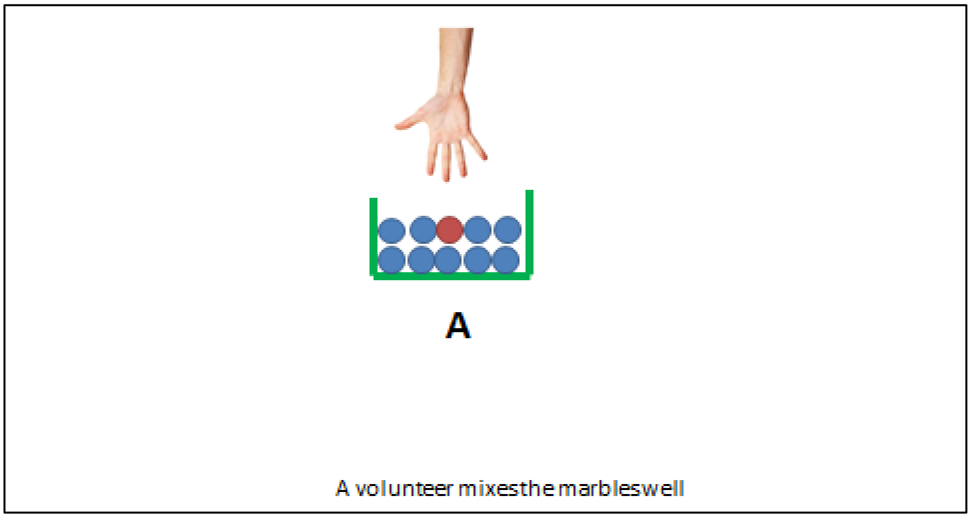
1	I thought if I couldn't find it here, then I should go somewhere else
2	I picked one that like say if I put it in the top left, the other one would be in the bottom right, somewhere where it wouldn't be close to the one I already did"
3	"The only strategy...I wouldn't pick the second one too close to the first one...so it's really, but then i really picked it random, where i was looking, i picked there.." "there's just small probability if I pick two really close to one another. There's probably more probability if I go somewhere else, different area there might be more probability" "Why? I guess if I pick the one that's over here on the spot, i feel that if i take a chance at a different position, then there's still a whole lot of other boxes on the other side that I could check, so I wouldn't waste all my three clicks on one little area…because there's so many choices, I wouldn't just stay in these four boxes here.
4	"I guess proven wrong, if I click there, it's kinda like playing minesweeper, you push a check mark and if it's not there... all of the surrounding squares are disqualified, so I thought of changing it up a little bit...but it still ends up being relatively consistent...
5	"Uhhh...similar to Battleship, but there was no feedback, so it was randomly selecting a location, but I wouldn't pick 3 in a row, I didn't think it would be adjacent to where I had picked already, so I thought it would be best to sort of random my move.. "Why not 3 in a row?" "Just seems unlikely that if you were to select...now in this situation it would have been to my advantage...but to me it seems unlikely that it would happen to be adjacent to the location that I already chose. There's no real empirical rational about that...it's just intuition.."
6	"I guess it doesn't matter, it could have been anywhere, but it seems like if I go back to the battleship example, if you miss one place, chances are it's not going to be right next to it..it seems like even though there's just as much chance…in my head I thought it must be farther...on the other side"

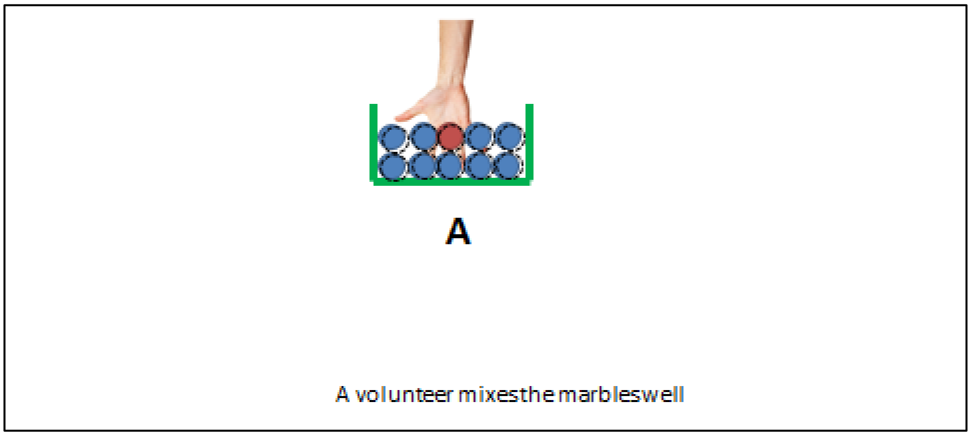
7	"Because I divided the whole map into 4 quadrants and then basically chose one in each quadrant cause I'm thinking if it's not in that one fourth of the map, if I use all of three of chances in that one place then it's worse than if I put all three in different quadrants."
8	"Were there are any areas you wouldn't have gone?" "Uhh, I guess. Uhh, well cause the prize can be anywhere, I first chose the top half, and then it's not there, so I guess unconsciously i picked the bottom half."
9	"Were there any areas youwouldn't have considered" "I wouldn't have picked boxes right around it, for now reason, that's just how my mind works I guess." "You spread them out quite a bit.." "I guess just to there was on the far sides of the boxes?" "Why?" "I'm not sure why, because it could be any box really, but I just figure that it most likely not be right beside it."
10	Your first choice was here...once you picked there, did it constrain where your next one was going to be?" "Yeah I guess I didn't want to pick the second one right next to it. Maybe a few rows and columns apart" Why? "I don't know…when you see lottery numbers being picked, they're usually far apart from each other so, it was just like" It just didn't...it's not...when you look at it...two things right next to each other doesn't have a high chance of being picked...that's how you think about it. First thing that crosses your mind
11	"What went through your mind?" "I just have been focusing on one square, the first one, since I started. I saw a pattern there, in that corner. It's mostly this square pattern that I see. When I picked this, I saw another square that was about this side. So corner doesn't, work, so let me try another edge.." "So you created 4x4 areas, and then tried to look at different spots in these areas? Once you picked your first one, were there any areas you wouldn't have picked?" "I wouldn't have picked anything in this square." "So you wouldn't have picked anything in the square you had already explored" "I wouldn't have picked one, two , and three next to each other. It was not even an option for me. I didn't even think about it." "Why?" "It's just the variety. saying it could be anywhere. I think my brain works that way. I would like to have a more expanded region of the three checks, compared to three checks that take away small area.
12	"Constraints on your next pick" "For some reason I didn't want to pick anything close to it, even though I know that doesn't make any sense. But I picked this one after because it was totally in a different area, so I tended to pick something that was further away."
13	For the second one, since I failed in the middle, maybe something upper left, and then that was wrong so I went down right.
14	Usually don't click the one right next to it, cause it just feels that I'll have a greater chance somewhere else. Intuition. Logically speaking every box has the same chance. so I don't know why I went somewhere else instead of the box right next to it." "why do people do it like that?" "Just seems like the first one when i chose it, i wasn't close at all, cause there were so many boxes, so when I chose it , it didn't feel like it would be right next to it." "So you felt completely wrong.." "I wouldn't have guessed somewhere next to it"
15	Suddenly it just felt like I shouldn't be in the top and so I picked somewhere in the middle, and to the left. Because my first one was in the top, and the prize wasn't there, so it gave me the feeling that maybe it shouldn't be there on the top. And then the third one...total chance! I just waved the mouse around and it sort of landed close there"
16	"Nothing it was just random. I was thinking to go a little bit far from the first one. I don't know, I didn't think about it, it was just involuntary.

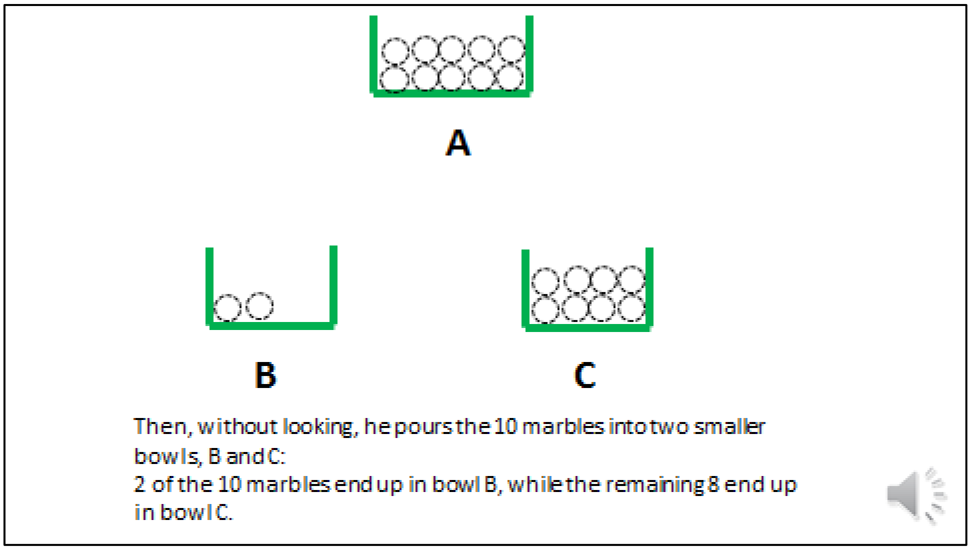
Appendix B - Study 2 supporting materials and data

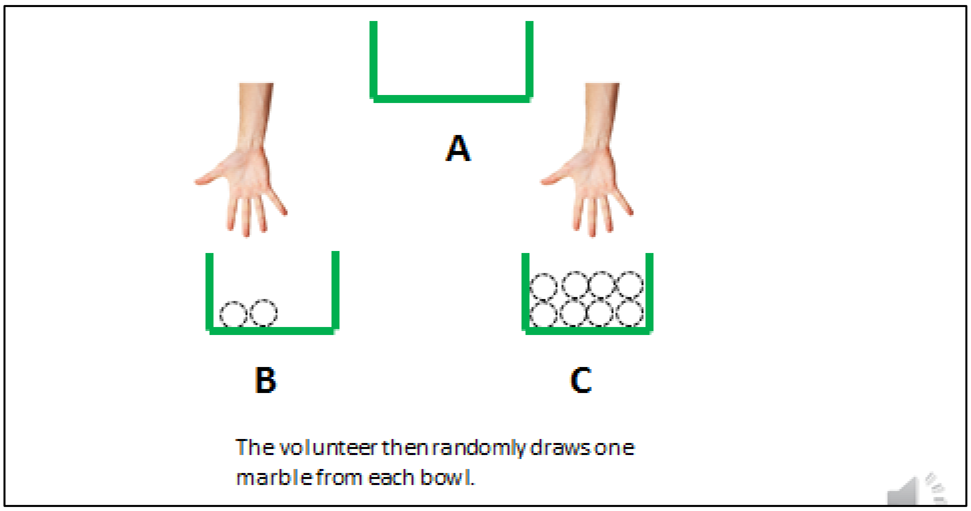
B.1 Example of graphics used in the video materials provided to participants

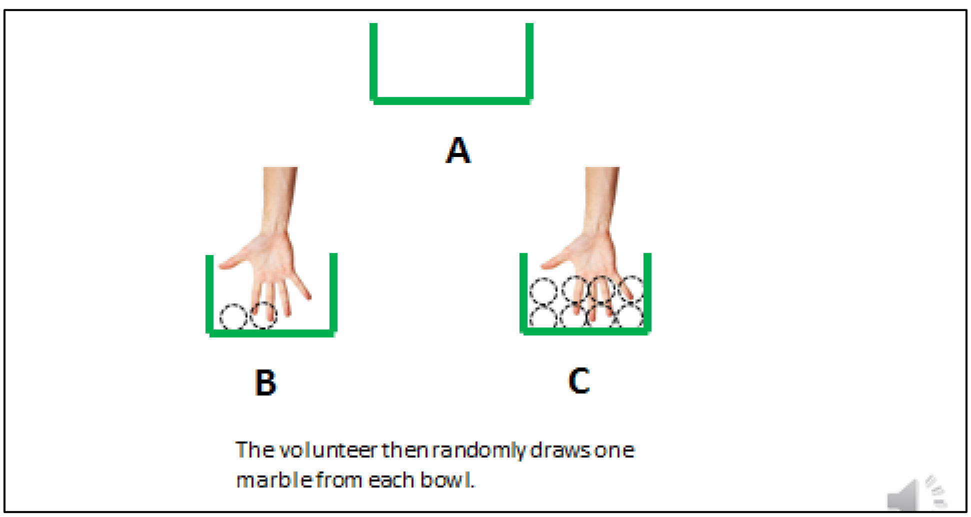







A
B
C
Then, without looking, he pours the 10 marbles into two smaller bowls, B and C:
2 of the 10 marbles end up in bowl B, while the remaining 8 end up in bowl C.


A
B
C
The volunteer then randomly draws one marble from each bowl.


A
B
C
The volunteer then randomly draws one marble from each bowl.

B.2 Complete transcripts of reasons provided by participants for their choices

B.2.1 Condition 1 – Choice of marble from big jar

It is more likely the marbles went into the large bowl because more marbles went into the large bowl.
more marbles
With a larger sample size there may be a higher likelihood that the red marble is in this bowl
The probability of red marble falling in bowl C is more.
It may appear more likely that the red marble would be in the bowl with more.
There is the possibility of having more red marbles in that bowl, while the other bowl may not have any red.
More marbles
The larger variety makes it seem like you have a higher chance
More options
The likelihood of the one red marble going into bowl C is far more likely.
Again, people may think the larger number of marbles give them more chances to win.
With more marbles there is more of a chance to find a red one
because there are more marbles and a better chance of getting the red marble.
better odds
There is a higher chance the red marble was drawn into bowl c then bowl b.
there are 8 chances for the red marble to be in bowl C, while only 2 chances for it to be in bowl B
The marble has a higher chance of being in bowl C.
more balls, higher odds
Bowl C is more likely to have the red ball.
There is a very good chance that B will not have a red marble
has more marbles
better chance went into bowl c
Because there are more marbles and a greater chance of drawing red.
there are more to choose from, not just two
There are more chances to pick a red marble in this bowl.
they would assume that it gives you a better chance since there are more marbles
there are more balls in the bowl
It seems that with more marbles the chance would be higher
Because there are more marbles in the bowl
Because there are more marbles, so the probability is greater to draw the red marble.
More marbles could mean more chance of finding the red.
more chance the red ended up in it
more marbles in the bowl means more of a chance to get the red one
Bowl C had 8 marbles in it, so the odds are higher of getting the red marble.
Since there is only one red ball, bowl C would be more attractive as there are more balls. Which gives it a better chance.
More marbles so people would think there is a higher chance that the red marble is in there.
there\'s a bigger chance that the red is in C
Chances are low that a red marble was placed in bowl B

The red ball can only be in one bowl and more than likely it will be that bowl
Probability of the red marble ending up in the bowl C is higher than bowl b.
Bowl B only holds two marbles, and therefore is unlikely to have a red marble at all.
more marbles available
it just seems like you would have a better shot since there are more balls in that bowl
More marbles in Bowl C
There is only 1 red marble, so it is more likely that the red marble is in the bigger bowl. Bowl B could possibly only have blue marbles.
a larger group of marbles gives a better chance of picking a red one
more marbles may mean better chance of getting the red marble
You have better odds because there are 8 marbles in C opposed to just 2 in B.
I think Bowl C appears better because it\'s got more choices but not too many.
There is a better chance the red marble ended up in this bowl.
There are more marbles to choose from in bowl C and chances are the red marble is in that bowl.
There is a higher chance of being successful because the amount of balls in the bowl C probably has the red ball.
To me, nothing, but I suppose people would somehow think that they have a better shot with a bigger bowl. It\'s a one in ten shot no matter what bowl you draw from.
There are so few marbles in bowl b, that it could be likely the red marble isn\'t in bowl B
There is a higher chance of picking the red marble out of a bigger set of data
There are more balls in Bowl C so there is more of a chance of getting the right ball.
It is more likely that the red marble is drawn from it.
In this case the chances that one of the two marbles in bowl B is red, is low.
There\'s a 50/50 chance of winning
Bowl C would bring a higher winning ratio
It has more marbles so more likely of a chance that it landed in that bowl.
Maybe better odds. There may be no red marble in the bowl with 2 marbles.
more marbles drawn from would seem to give a greater chance of a red one
There are more marbles in Bowl C, so more chances.
greater number of balls
there is more of a chance the red ball is in the larger bowl
More options.
There are most likely more red marbles.
more marbles inside
higher chance the red marble is in bowl c because of numbers.
Because it\'s more likely the red marble wound up in that bowl, even though the overall odds of getting it are the same either way
More marbles are in there
There\'s a good chance the small bowl wouldn\'t have any red marbles.
Bowl C has a higher chance for the Red Marble to be in it.
Higher chance of choosing a red marble.
Bowl C because people might have a greater chance of drawing the red marble. It depends on how many red marbles are in the small bowl. There might be a 0% chance if there are no red marbles in the bowl.
Most of the marbles went into bowl C. Since there was only one red marble, the probability was that it went into that bowl.
There is a higher chance that the red marble ended up in bowl c.
It doesn\'t matter which bowl because the probability is still the same

It is easier to draw one out and there are more to chose from.
More marbles means higher probability.
There are likely to be more red marbles in the bowl with more marbles in general
There is a better chance that the red marble is in that bowl.
There is a much better chance that the red ball ended up in Bowl B.
the randomness of order.
More marbles, therefore greater chance of getting the one
More marbles were placed into Bowl B so there is a higher chance that the red marble went into this bowl.
There are more chances to get a red marble
There are more marbles total in this bowl, which would lead people to believe it has a higher chance of having the red marble.
more marbles increases your odds
Just more options on marbles, the chances of getting the red one goes way up.
It\'s more likely that the red marble will be in the bowl with more marbles in it.
There\'s a chance there might not be a single red marble in bowl c. thats why
because there are more marbles in that bowl and it gives you a better chance of getting the red one
more marbles in that bowl
because there is a larger chance of getting a red marble from bowl b
More chances to get the right marble?
Because there are more opportunities for a marble to be red
Since there are more marbles to pick from, it seems like there is more chance that the red one is in there.
there are more marbles in bowl B, so most people would guess it has a higher chance of having the red marble.
There are more marbles to choose from.
there is more marbles in there
there is a better chance of the red marble actually being in the bowl for Bowl B because it has a larger sample size.
More marbles are in B so it is more likely the red marble is there
There are more marbles in bowl B. People will deduce that it is more likely the red marble is in bowl B. Therefore it is most likely.
more chance of the red one being in there
Theres a better chance of a red in Bowl B.
There are more marbles in Bowl B to choose from
there are more balls
There might be a better chance because bowl b has more marbles.
Because there is more chance of the red marble being in bowl B
there are more balls in bowl b, so the chance of drawing the red marble is greater
You only more marbles to choose from. lesser chance of pulling a red marble.
there are more marbles in bowl b so there is a better chance the red one went in there
more options
only 2 marbles are in c
There is a higher chance that the red marble is in bowl B because there are more marbles in there.
you have more chances to pick the red marble
More balls in the bowl, higher chance that red is one of the eight
There is more of a chance that the red marble ended up there.
Bigger chance of getting red marble since there are more marbles.

There is a bigger likelihood than only blue were taken out of the larger bowl because there is only one red. So it is likely still in the larger bowl.
It has more balls so more chances.
They have more options and there is more probability that they\'ll end up with red ball.
There\'s a greater chance of the red marble being in the larger bowl.
It looks like there is only a one in five chance of the marble going into the little bowl
the more marbles in the bowl the greater chance you will get the red one
it has more marbles in it
It would be extremely unlikely that the red marble would land into bowl C
There are more marbles, so more chances to be correct, but actually it wouldn\'t make a difference.
There is a better chance of the red marble being in the bigger bowl
It seems more likely that the red ball would be mixed in with the bowl with more balls. It seems less likely that out of 10 balls the red one would be in the bowl of 2.
More marbles to choose from
There are more marbles in bowl B, so a higher chance of there being, and picking a red marble.
More marbles equals more chances.
More opportunities for a red marble
More balls in bowl b give better odds of having the red ball than the bowl C
Because bowl B have more marbles changes of red marble will be find in B is higher than C
There are more marbles
More balls in my mind may equate to a better chance that the red ball is there. I actually would go for the 2 balls myself.
There is a greater chance of the red marble landing into bowl B
The likelyhood that the red ball is in bowl B is higher 8:10.
with only 2 marbles in bowl c - there might not even be a red ball in there
Bowl B is more likely to contain the red marnle
Only one marble in the bowls is red, so it seems more likely that it would be in the one with the most marbles to choose from.
Bowl B is more attractive because it at least offers the illusion that there are more chances of getting a red marble.
It contains more marbles.
more marbles in bowl B
Bowl B will have more chances of having the red ball
with only 2 marbles the likelyhood that the 1 red marble is 1 of the 2 is much greater-seemingly-than being 1 red out of 8

B.2.2 Condition 1 – Choice of marble from small jar

Because if the red marble is in Bowl B there is a 50 percent chance of getting it
Appears simpler
I think they\'d think they\'d have a greater chance drawing from the small bowl.
Because there are less marbles to choose from in bowl b
More of a chance to select what you want.
if indeed the red one ended up here then you will have a 50 percent chance of getting it, still 1 in 10 or a 10 percent chance, odds are either 50 percent but still ultimately only 10 percent to get the red ball
I have a 1 in 2 chance of getting a red marble assuming that the red marble was dropped in bowl B.
There are fewer marbles, even though there is a smaller chance of the red one being present.
I think people would look at the bowl with 2 marbles and think \"hey so it\'s 50/50 right?\"

there is an equal chance of the ball being red or blue
Greater odds of winning
Less effort to draw marble.
in my mind better odds
is less i think is red
There are only two choices, so it appears that the probability of finding the red marble is 50%.
Because if the ball is there they have a 50/50 chance to win.
What makes this choice appear more attractive is that a person seems to have a better chance of drawing the proper marble from one with less blue marbles then one with red marbles. In otherwords the probability is higher.
If one of the marbles happens to be red, then there is a 50% chance of getting the marble
Because there are less options in the bowl.
They think they have a 50/50 chance of getting the red marble
less marbles in the probability.
Less risk at grabbing the red marble.
Since there are only two marbles in Bowl C, there is a 50-50 chance of getting red marble.
Once again, it\'s attractive because it seems like a higher chance with a smaller total.
there are fewer marbles
I think it\'s easier for people to understand and visualize.
only two marbles to choose from
If the red one fell in there, 50% chance of picking it
There are fewer blue balls in bowl C.
I think Bowl C is more attractive because it\'s a 50/50 chance that the ball is red.
It has a smaller sample size
Because most people will think the odds are 50/50, forgetting that their a smaller chance the red marble will be in bowl C
I guess because it appears to be a 50/50 chance, even though it\'s really not
Nothing, there is no statistically higher probability of selecting the marble in either side.
less marbles one had to be red

B.2.3 Condition 2 – Choice of marble from big jar

More marbles would offer a greater chance in most people minds.
It is more likely that there are multiple red marbles in the larger batch
The probability of finding a red marble is more in bowl C.
Probably most people would prefer it since it has more marbles.
More marbles may mean more chances to select the winning marble.
Since it is a large bowl
Because bowl C has more marbles in general
There is a greater chance of their being a red marble in this bowl.
because it has more sample.
It has the most choices
Bowl C has more marbles
It appears that there would be a higher likelihood of getting a red marble
There are more red marbles total in that bowl so people would believe they had a higher chance of winning.

more of a chance to get a red one
there are more marbles.
because there are more marbles at it appears you have a better chance of drawing one
More chance for red balls being drawn into bowl C
The likelihood is higher that there would be more red marbles in bowl C.
more marbles
There are more chances the red balls would be in the larger amount.
There are more balls in C
There are more marbles
there are more marbles in C
It should have more red balls total even though the proportion is likely to be the same.
more chance to get red marble
more balls in this bowl
because there are more probability to have red marbels.
More marbles seems more likely to produce a red one.
higher % of drawing a red marble
I think people would look at the larger number and think they had a better chance there.
Same reason as before. Bowl C would have a better chance of getting a red ball because there are more balls.
Better probability of choosing a particular color
people think they have a better chance when they see more
more marbles
In Bowl B
visually
More marbles in the bowl
There are more marbles and therefor more chances that a red one will be in there.
there are less chances for the balls to be blue
a larger chance of picking a red marble based on a larger group of marbles
more marbles = better chance the red one is in there
Bowl C contains more marbles so that odds of a red being in there are better.
although perhaps untrue
There is a bigger chance of it being in the bowl with more marbles.
There is more chance of a red being in the larger bowl because there is a bigger likelihood that more blue were taken out of the bowl than red. This is because there are more blue to begin with. So more red are still in the larger bowl than the smaller.
It has more balls to chose from.
There are more options to choose from
In bowl B
More marbles would have ended up in C.
the more marbles you have a better chance of getting the red
There are more marbles and therefore a higher chance that there are red ones presents.
Because there are more marbles in bowl c
There will be more marbles in larger bowl and a higher likelihood that one red marble would be drawn.
More marbles to choose from
picking from a larger quantity of marbles so there would seem to be a greater chance of a red marble

There should be more total red balls in bowl C
There is more likely to be more red marbles in bowl C.
If more marbles are in Bowl C
Higher chance of the red marble being in the bowl c
Bowl C has more marbles so that at aleast gives the appearance that there is a greater chance of getting a red marble from bowl C.
Because on average
There are more overall marbles in there
Bowl C has higher odds of containing more red marbles.
The more marbles the higher the chance there will be more red ones.
It might seem that the concentration of red balls is higher because the bowl is fuller
People would assume they have a larger chance due to the larger amount of marbles
It appears attractive because there are likely 8 red marbles in Bowl B
There are more marbles
I think there are more chances of getting a red marble
It contains more marbles
I would like to draw from the bowl b because there are more marbles and it makes the chances of getting a red marble go way up.
greater chance there is a red marble
There are a greater number of marbles to choose from in bowl B.
More chances of getting the red marble
Because there are more marbles in B so most people figure there is a better chance of getting a red one in B since there are more red marbles in B than in C
There\'s a higher chance of it being in bowl B.
larger number of balls
There are more marbles and a great chance of drawing red.
there are more to choose from
There are more marbles in B
They think the more marbles the better the choice.
because there are more marbles in this bowl
It seems like you have a better chance of picking out a red marble because there are more.
Because there are more marbles which would yield a high er probability of not selecting a red marble.
Bowl B had 80 marbles in it
It seems that there are a lot of marbles but a bigger chance to have a red one
There are more marbles and a better chance of drawing a red one.
there are more marbles in b
Because most of the red balls would probably end up in bowl b since it has the most marbles making the probability of drawing a red marble higher.
more marbles in that bowl
more balls more chances
That there are more red marbles in there
More options to choose from.
More red marbles in it
There will probably be more red marbles in that bowl to choose from.
I could see people thinking that because more total marbles went into bowl B that the chances of the red marbles also being in there are increased.

Because most people would assume that more of the red marbles would go into this bowl.
More marbles to choose from
There are more marbles in that bowl, so a larger opportunity to pick a red marble.
They\'ll assume that since there\'s a small amount of red marbles, the bowl with the most marbles will have a higher chance of holding that red marble.
There are more options to get a red marble
More marbles to choose from.
There\'s more marbles
more balls
it is more likely that the red balls are in bowl B
More options.
There should be more red balls in 80 marbles
Because there is more marbles.
more marbles inside
B has more marbles.
Bowl B contains more marbles thus has a higher chance on containing a red marble.
Higher chance of drawing a red ball.
Having more balls in bowl B has better odds of drawing a red ball.
Because it looks like people will have a greater chance of picking a red marble.
There are more marbles, so people might think that there is more of a chance of a red marble being drawn.
more options to choose from probably more red marbles

B.2.4 Condition 2 – Choice of marble from small jar

Less marbles to choose from in the bowl.
less marbles
Because there are fewer marbles, it makes people feel like they are more likely to draw the red marble.
people would think less balls in the bowl would make it more likely to pick a red one
I think there is a large enough grouping in B to draw a red but the smaller size would make people think they would be more likely to draw it
Fewer marbles means a greater chance for getting a red marble
there are less marbles in bowl b so that would lead people to believe there was a better chance
Less chances of getting a blue marble, and easier to pick one.
There are less marbles so people may think their is a better chance of picking the red marble.
There may be more of the reds in the bowl with lesser marbles.
more marbles
There are less marbles to choose from.
I think it feels like there is a higher chance of drawning a small number from a smaller selection rather than a small number from a large selection.
fewer marbles
Less marbles would me a supposed better chance of picking red.
It makes the odds appear to be more favorable.
They might believe they can get a better chance with B.
Only 20 marbles went into Bowl B. If 90 of the 100 marbles are red, it seems like there would be more red marbles would be in Bowl B.

Even though it\'s not true, it appears you have better odds with Bowl B since there are only 20 marbles. You feel like your odds have increased.
There are less marbles to choose from.
There is a higher chance of pulling out the marble I want because there is less marbles to choose from.
seems to me as the best selection, it\'s easier to imagine less marbles?
I don\'t really know
The bowl contains fewer marbles, so if there were to be one more red marble than there statistically should have been, then it would have a larger affect on the probability of pick the red marble assuming that the red marbles werent even distrubuted
it is smaller it gives you a better chance
Less marbles
Better odds
It seems that if the red marble was in this bowl you would have a greater chance of pulling one out since it has less.
Maybe in this case there might be more chance with less marbles, (more chance of red) but more marbles overall.
lest marble annd red
I think Bowl B would be more attractive because there are less bowls to chose from increasing the chance.
There are a lot more blue marbles, so bowl C has many more blue marbles than red ones
Neither of them seems more attractive to me. I wouldn\'t have a preference.
Because there are 20 marbles in bowl B
There\'s smaller variables in this one.
There are less marbles that could be blue in bowl B then bowl C.
Fewer marbles
There are less balls in total in bowl c, which most people might think would give them a better chance
There aren\'t as many blue marbles to contend with in
less marbles. Easier to draw a red marble
There\'s fewer chances of getting a wrong choice.
There is a better chance of choosing a red marble from a small pool of marbles
a chance that more red marbles may be in the 20
You have a smaller sample, and a higher chance of making multiple red marbles in the smaller bowl.
Seems like the likely choice.
I think you narrow your chances to a better one
More of a chance
has less marbles to pick from
Since there are more marbles in bowl C, it appears to me that there would be a better chance of selecting two red marbles.
less marbles in bowl c increases the probability
less marbles more chance
Fewer total marbles
Theres a higher chance of choosing a red ball from bowl c
There are fewer marbles in that bowl
It just seems like there would be more of a chance to pick red.
It appears there is a greater chance of the red marble being pulled.
Greater chance
There are fewer marbles in bowl C, yet enough to have a higher chance of pulling a red marble.
Bowl C has less marbles so if they is a red one inside the bowl, you have a better chance of finding it.
There are less blue marbles in the bowl

There are fewer blue balls in bowl C
Less balls, I guess better chance of drawing a red ball.
Because marbles (bowl C) is less marbles compared to B
I just scared of Bowl B It has so many blue marbles, i feel like my chances are better with a smaller stock to choose from
Smaller sample size
The bowl with fewer marbles gives a better chance to choose red.
theer is a decent chance that one or more of the red balls went into c
There are fewer balls to pick from which may contain a higher number of red balls. They may have a higher chance of winning.
Fewer to pick from seems to imply and I infer that my chances are better choosing from fewer marbles.Perhaps deceptively-fewer seems that my red marble is in the fewer bowl so when I reach to select from fewer-fewer means fewer BLUE marbles
The ratio would stay about the same while splitting the marbels, the small bowl of 20 would have less blue marbels, increasing the odds of picking a red one.

Appendix C – Study 3 supporting materials

C.1 Study 3A - Example of paper-based questionnaire supplied to study participants

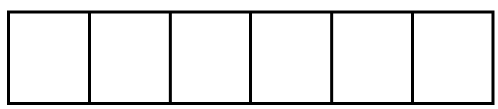
MSCI 311

MODULE 8 – DECISION MAKING

ACTIVITY

Version A

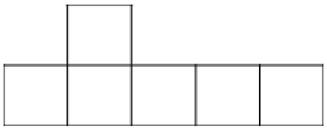
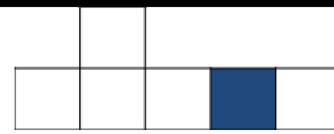
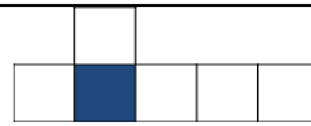
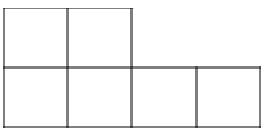
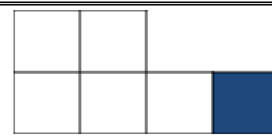
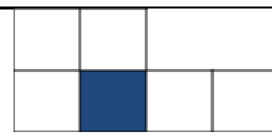
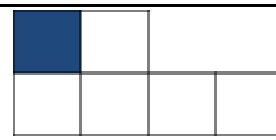
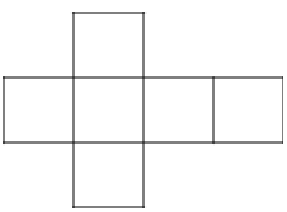
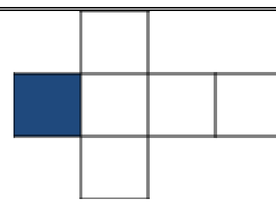
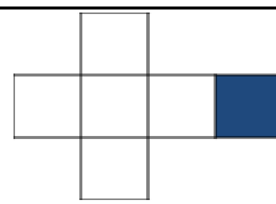
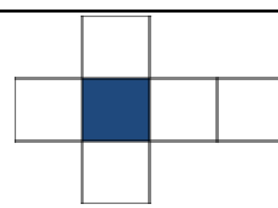
A while back the following game was played with a large number of people. People were shown various 6-cell shapes. An example of such a shape is shown below:

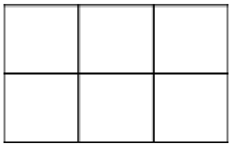


In each structure, a computer had randomly assigned a prize to one of the cells. The game players had to select the cell where they felt the prize might be.

As you can imagine, players selected some cells more frequently than others.

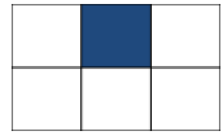
In the next page, you will see a variety of 6-cell structures. Examples of selected cells by people are shaded. Your task is to rank the selected cells in order from the one that you expect was the **most frequently** selected to the one that you expect was the **least frequently** selected:

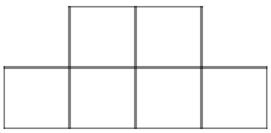
6-cell shape shown to participants:	Rank	Selected Cell
 Rank the selected cells in order from the one that you expect to be the <u>most frequently</u> selected (1) to the one that you expect to be the <u>least frequently</u> selected (3):	(___)	
	(___)	
	(___)	
 Rank the selected cells in order from the one that you expect to be the <u>most frequently</u> selected (1) to the one that you expect to be the <u>least frequently</u> selected (3):	(___)	
	(___)	
	(___)	
 Rank the selected cells in order from the one that you expect to be the <u>most frequently</u> selected (1) to the one that you expect to be the <u>least frequently</u> selected (3):	(___)	
	(___)	
	(___)	



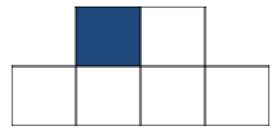
Rank the selected cells in order from the one that you expect to be the **most frequently** selected (1) to the one that you expect to be the **least frequently** selected (2):

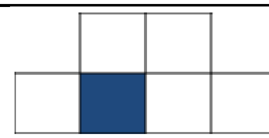
(___) 

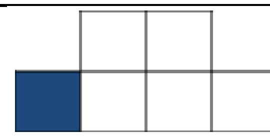
(___) 

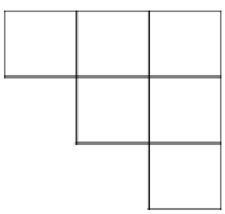


Rank the selected cells in order from the one that you expect to be the **most frequently** selected (1) to the one that you expect to be the **least frequently** selected (3):

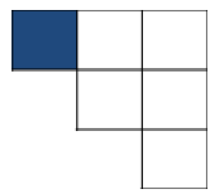
(___) 

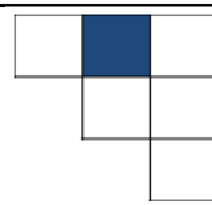
(___) 

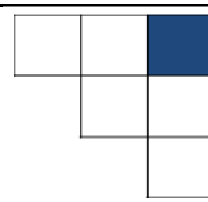
(___) 

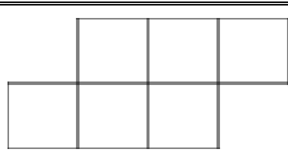


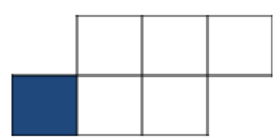
Rank the selected cells in order from the one that you expect to be the **most frequently** selected (1) to the one that you expect to be the **least frequently** selected (3):

(___) 

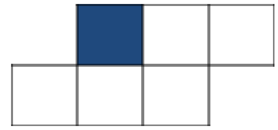
(___) 

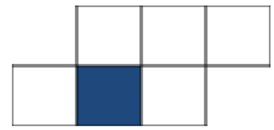
(___) 



(___) 

Rank the selected cells in order from the one that you expect to be the <u>most frequently</u> selected (1) to the one that you expect to be the <u>least frequently</u> selected (3):

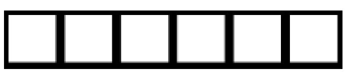
(___) 

(___) 

C.2 Study 3B - Example of web-based questionnaire supplied to study participants

Welcome to our survey!

A while back the following game was played by a large number of people. People were shown various 6-cell shapes. An example of such a shape is shown below:



In each structure, a computer had randomly assigned a prize to **one** of the cells.

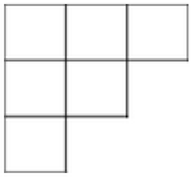
As part of the game, people could select **two** cells, in the hopes that one of the selected cells could be where the prize might be.

As you can imagine, people selected some pairs of cells more frequently than others.

In the next pages, you will see four 6-cell structures. Examples of pairs of cells selected by people are shown shaded in blue. Your task is to rank the selected pairs of cells in order from the pair that you expect was the **most frequently** selected to the pair that you expect was the **least frequently** selected.

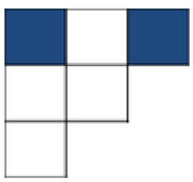
>>

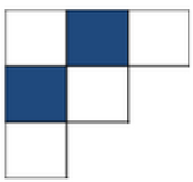
Below is the 6-cell structure shown to participants:

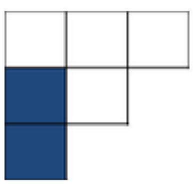


Rank the selected pairs of cells from the pair you expect was the most frequently selected (1) to the pair you expect was the least frequently selected (3).

Drag and drop the images to choose your ranking.

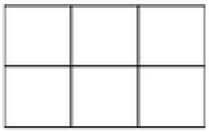






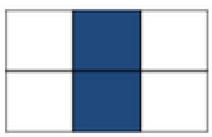
>>

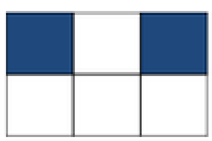
Below is the 6-cell structure shown to participants:

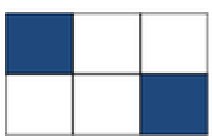


Rank the selected pairs of cells from the pair you expect was the most frequently selected (1) to the pair you expect was the least frequently selected (3).

Drag and drop the images to choose your ranking.

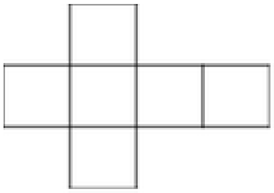






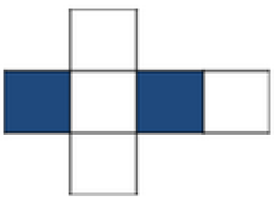
>>

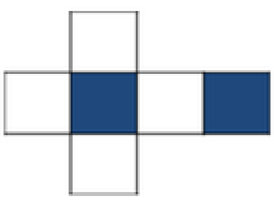
Below is the 6-cell structure shown to participants:

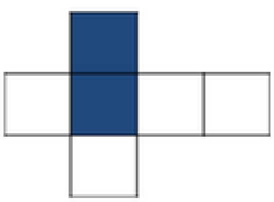


Rank the selected pairs of cells from the pair you expect was the <u>most frequently</u> selected (1) to the pair you expect was the <u>least frequently</u> selected (3).

Drag and drop the images to choose your ranking.

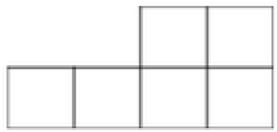






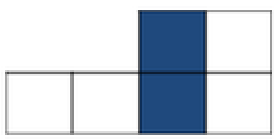
>>

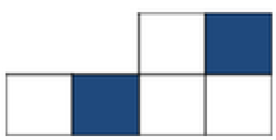
Below is the 6-cell structure shown to participants:

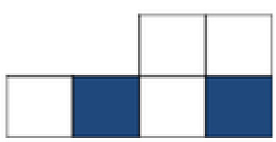


Rank the selected pairs of cells from the pair you expect was the most frequently selected (1) to the pair you expect was the least frequently selected (3).

Drag and drop the images to choose your ranking.







>>

Thank you for participating.

Your validation code for MTurk is snGDByerTrkBzqCfFelF

Please copy/paste this code into the MTurk box