Skill mismatch in the labour market

by

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Declaration

This thesis consists of material all of which I authored or co-authored: see Statement of Contributions included in the thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Statement of Contribution

While the main ideas in Chapter 1 were developed jointly by my supervisors, Francisco M. Gonzalez and Matthew Doyle, and myself, I have made the major contribution to the work involved in proving the results of Chapter 1.

Abstract

This thesis contains three chapters on skill mismatch in the labour market.

Chapter 1 provides a theory of *ex ante* skill mismatch, which we define as a situation where firms create jobs that workers search for and accept, even though they do not make the most productive use of their skills. The core idea is that, in the presence of asymmetric information about workers' outside options, the value of on the job search is higher for workers employed in such jobs. The theory provides new insights into the returns to education as well as the impact of on the job search on labour market mismatch. It also provides an explanation for the declining fortunes of educated American workers in recent decades.

Chapter 2 studies a competitive search equilibrium with exogenous skill mismatch, where educated workers apply to routine jobs only if they face a high cost searching for cognitive jobs. The purpose is to examine whether a simple model with exogenous mismatch can explain the adverse labour market outcomes of educated workers. Under a negative shock to routine jobs, the model fails to generate a fall in the employment rate together with a decline in the job-to-job transition rate. Compared to endogenous mismatch equilibrium, an equilibrium with exogenous mismatch does not incorporate the trade-off between job finding rates and wages when unemployed workers choose to search in different job sectors. The comparison suggests that understanding the mechanism of skill mismatch is essential to understanding the labour market outcomes of educated workers.

Chapter 3 shows that displacement of high-school workers from routine jobs can be understood as the labour-market response to an adverse selection problem. The adverse selection problem arises because employment contracts do not systematically discriminate against education, even though over-qualified workers are relatively more likely to quit routine jobs. The labour market equilibrium distorts the labour market outcomes of high school graduates by inefficiently increasing their wage at the expense of higher unemployment rate, in order to separate them from overqualified college graduates. In addition, the labour market response to the adverse selection problem creates a demand for post-secondary vocational education, which is valuable because it acts as an entry barrier that prevents college graduates from using routine jobs as stepping-stones towards better jobs.

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Dedication

to Shengkai

Contents

Contents vii

List of Figures viii

List of Tables ix

1 Skill mismatch in competitive search equilibrium 3

- 1.1 Introduction . . . 3
- 1.2 The Model . . . 7
  - 1.2.1 Environment . . . 7
  - 1.2.2 Competitive search equilibrium . . . 10
- 1.3 Equilibrium with one type of job . . . 14
  - 1.3.1 Revealing equilibrium . . . 19
  - 1.3.2 Non-revealing equilibrium . . . 21
- 1.4 Equilibrium Mismatch . . . 26
- 1.5 Conclusion and Implications . . . 29
  - 1.5.1 Incomplete information in the labour market . . . 30
  - 1.5.2 On-the-Job Search . . . 30
  - 1.5.3 labour Force Participation . . . 32
  - 1.5.4 Mismatch . . . 33
  - 1.5.5 The Returns to Education . . . 36

2 Endogenous and exogenous mismatch in the labour market 37

- 2.1 Introduction . . . 37
- 2.2 Model and Equilibrium . . . 40
  - 2.2.1 The model . . . 40
  - 2.2.2 Characterization of Equilibrium . . . 43
- 2.3 Numerical Simulation . . . 46
  - 2.3.1 Baseline Economy . . . 46

2.3.2 Employment . . . 47
2.3.3 Wage . . . 50
2.3.4 Job flows . . . 52
2.4 Compared to the Endogenous Mismatch Equilibrium . . . 55
2.5 Conclusion . . . 58

3 Adverse selection in the labour market and the demand for vocational education 60
3.1 Introduction . . . 60
3.2 Model . . . 64
3.2.1 Agents and Markets . . . 64
3.2.2 Worker's Problem . . . 67
3.2.3 Employer's Problem . . . 68
3.2.4 Equilibrium Refinement . . . 69
3.2.5 Equilibrium . . . 70
3.3 Equilibrium with Crowding-out . . . 71
3.4 Vocational Education and Training . . . 79
3.4.1 VET as Entry Barrier for College Workers . . . 80
3.4.2 A labour market with VET . . . 81
3.4.3 Post-Secondary Education and the Skill-Biased Technological Changes 84
3.5 Conclusion . . . 87

Bibliography 88

A Appendices 91
A.1 Appendices of Chapter 1 . . . 91
A.1.1 Proof of Proposition 1 . . . 91
A.1.2 Proof of Proposition 2 . . . 91
A.1.3 Proof of Proposition 3 . . . 100
A.1.4 Proof of Proposition 4 . . . 104
A.1.5 Proof of Proposition 5 . . . 105
A.2 Appendices of Chapter 2 . . . 106
A.2.1 Problem of high school workers . . . 106
A.2.2 Comparative Statics . . . 107
A.3 Appendices of Chapter 3 . . . 109
A.3.1 Proof of Proposition 1 . . . 109
A.3.2 Proof of Proposition 2 . . . 112

A.3.3 Proof of Proposition 3 116
A.3.4 Proof of Proposition 4 118
A.3.5 Proof of Proposition 5 118

List of Figures

1.1 Fraction of routine jobs employment filled by college graduates 34
1.2 Fraction college graduates employed in routine jobs 35

3.1 Labour Market Flow 72

A.1 Comparative Statics-Scenario 1 107
A.2 Comparative Statics-Scenario 2 108
A.3 Comparative Statics-Scenario 4 109
A.4 Non-College Job Markets 110

List of Tables

1.1 Comparative statics . . . 31

2.1 Parameter Values . . . 47
2.2 Baseline Economy . . . 47
2.3 Employment . . . 49
2.4 Wage . . . 50
2.5 Job to Job Flows . . . 53
2.6 Baseline Economy- Endogenous mismatch . . . 56
2.7 Mismatch Equilibrium with a lower α_2 . . . 57
2.8 Comparative Statics - Endogenous Mismatch . . . 57

3.1 Match Productivity . . . 64
3.2 Comparative Statics . . . 85

Introduction

This three-chapter thesis focuses on puzzles connected to skill mismatch and the role of education in the labour market. The increasing employment of college graduates in jobs that do not typically require a college degree has raised widespread concerns about skill mismatch, the labour market misallocation between worker skills and the skill requirements of jobs. The analysis in this thesis helps understand skill mismatch and its implications for the adverse labour market outcomes, including the decline of employment and participation rates, for both educated and less educated workers.

Chapter 1 of this thesis provides a theory of why unemployed, college educated workers choose to search for jobs in which they are relatively likely to end up mismatched. In the presence of asymmetric information about workers' outside options, the value of searching for non-college jobs is higher because the value of on-the-job search is higher for workers employed in such jobs. It offers a novel perspective on the returns to education, which is the sum of the value of searching for non-routine jobs (college jobs) plus the option value of searching for routine jobs (non-college jobs).

Using this insight, Chapter 1 argues that the destruction of routine jobs, driven by skill biased technological change, automation and off-shoring, also explains the adverse labour market outcomes of educated workers. A decline in the value of routine jobs reduces the value of labour force participation to college educated workers. Furthermore, a larger fraction of unemployed college educated workers are forced to direct their search towards non-routine jobs, which causes a decline in their employment rates as well as a fall in job-to-job transitions.

While Chapter 1 explains the labour market trends of college graduates through the mechanism of skill mismatch, the idea that mismatched jobs have option values seems to apply intuitively to any economy with skill mismatch, whether it arises under asymmetric information or caused by random shocks. For this reason, Chapter 2 studies a simpler model with exogenous skill mismatch, where educated workers apply to routine jobs only if they face high costs searching for cognitive jobs. Under a negative shock to routine jobs, the model fails to generate a fall in the employment rate together with a simultaneous decline in the job-to-job transition rate. Compared to the model in Chapter 1, an equilibrium with exogenous mismatch does not incorporate the trade-off between job finding rates and wages when unemployed workers choice to search in different job sectors. The comparison suggests that understanding the mechanism of skill mismatch is essential to understand the labour market outcomes of educated workers.

In the third chapter, I study the impact of skill mismatch on the labour market opportunities of less educated workers. The analysis in Chapter 3 shows that displacement of high-school graduates from employment can be understood as the labour-market response to adverse selection affecting the creation of jobs involving mainly routine tasks. This is because employment contracts do not discriminate against education, even though overqualified workers are relatively more likely to quit routine jobs. Consequently, a separating equilibrium in the labour market penalizes high school workers by increasing their unemployment rate above its efficient level.

This mechanism of displacement in Chapter 3 helps understanding the value of vocational education, which is post-secondary education that focuses primarily on providing occupationally specific preparation, and the striking increase of post-secondary vocational education in the United States over the past few decades. Vocational education has a higher market value than it is commonly thought, because it acts as an entry barrier to prevent college graduates from using some routine jobs as stepping-stones towards better jobs.

This thesis contributes to our understand of skill mismatch in the labour market. It addresses important research questions regarding the mechanism of skill mismatch, the impacts of skill mismatch on the labour market outcomes of workers with different educational backgrounds. It also provides useful implications in understanding the role and return to education in the labour market.

Chapter 1

Skill mismatch in competitive search equilibrium

1.1 Introduction

A substantial minority of U.S. college educated workers are employed in jobs that are routine task based or otherwise do not require a college education.[1] The observation that a large number of workers are mismatched, in the sense that they are employed in jobs that do not make the most productive use of their skills, requires an explanation. One hypothesis is that workers sort themselves into jobs efficiently, either according to assortative matching in frictionless competitive markets[2] or by non-assortative but efficient matching.[3] This explanation denies the existence of mismatch as it implies that college workers employed in routine jobs are, in fact, well suited to those jobs. The implication is that apparent labour market mismatch is, in fact, an artifact of incomplete data (on unobserved ability, for example). A second explanation draws from random matching models in which workers search for jobs in which they expect to be well-matched, but some times experience bad luck in the matching process.[4] This approach is consistent with the notion that some workers are employed in jobs that do not make the most productive use of their skills, but views mismatch as an entirely *ex post* phenomenon.

By contrast, in this chapter, we develop a competitive search model in which, in equilibrium, workers choose to search for and accept jobs in which they know they are less productive. That is, we develop a theory of *ex-ante* equilibrium mismatch. Our theory, ap-

[0] This chapter is co-authored with Francisco M. Gonzalez and Matthew Doyle.

[1] Beaudry et al. (2016), Abel et al. (2014).

[2] Acemoglu and Autor (2011), Beaudry et al. (2016).

[3] Eeckhout and Kircher (2011).

[4] Lise et al. (2016).

plied to the labour market for college educated workers, generates important insights about the returns to education and the role of on-the-job search in labour markets. The theory also implies that several adverse trends that have impacted college educated workers in recent years can be understood as the result of the interaction between *ex-ante* labour market mismatch and the well documented destruction of routine jobs, which is thought to have been caused by some combination of automation and offshoring.

The ability of workers to search on the job is at the heart of our theory. This is because the option value of on-the-job search, which is an important component of the value of a job, depends on whether a worker is well matched in a job. College educated workers, who are most productive in jobs comprised primarily of non-routine tasks, can be induced to search for routine jobs if the value of on-the-job search from a routine job is sufficiently greater than from a non-routine job.

Informational asymmetries naturally give rise to a situation where the value of on-the-job search in routine jobs exceeds that in non-routine jobs. A worker's willingness to switch jobs depends on her ability to elicit a retention offer from her current employer, which is private information to the parties in the match. The fact that potential poachers do not observe the workers' outside option when they make a job offer creates an adverse selection problem: while mismatched workers search on the job in order to transition from a poor match to a better one, well-matched workers have an incentive to mimic the on-the-job search behavior of these mismatched workers. Since well-matched workers seek retention offers rather than new jobs, the poaching of employed workers creates more surplus when the pool of applicants contains a higher fraction of mismatched workers.

This creates a coordination problem. First, workers' incentive to search for jobs where they expect to be less productive, that is, jobs where they are more likely to be mismatched *ex post*, is greater when the future returns to search on the job are expected to be higher. Second, firms are more likely to attempt to poach workers in markets where they believe the fraction of mismatched workers to be high, implying that the return to search on the job for a worker is higher in markets where workers are more likely to be mismatched. These two effects reinforce each other and they can lead to a labour market coordination failure.

We characterize a unique equilibrium with *ex ante* mismatch, in which a positive fraction of unemployed, college educated workers apply for routine jobs, despite the fact that they produce relatively little in those jobs. Routine jobs, while paying lower wages in equilibrium, are appealing to unemployed, college-educated workers because they are easier to find and because they offer attractive opportunities for on-the-job search. Routine jobs being easier to find is an equilibrium result, and is not due to any assumed advantage in either the cost of creating routine jobs or the ease of matching. Despite the fact that, from a purely technological perspective, routine jobs are unambiguously worse than non-routine jobs, in equilibrium these jobs are more valuable to unemployed college workers than are non-routine jobs because the value of on-the-job search is higher in routine jobs. Indeed, in the equilibrium with *ex ante* mismatch, workers only apply to the non-routine jobs when applying to routine jobs is relatively costly.

Not surprisingly, the mismatch equilibrium is inefficient. We show that the constrained efficient outcome, which can also be supported as an equilibrium, fully reveals private information and exhibits positive assortative matching, in the sense that college educated workers only ever search for non-routine jobs. Furthermore, in the efficient case the option of searching for routine jobs has no value for college educated workers. In the mismatch equilibrium, relative to the efficient outcome, the fraction of college educated workers who search for routine jobs is inefficiently high and the private returns to education are inefficiently low.

The mismatch equilibrium offers an interesting perspective on several facets of the labour market for college educated workers: First our analysis offers a novel perspective on the return to education, which comprises wages, job finding rates, and the value of on-the-job search. In our framework, the value of labour market participation to college graduates, which closely corresponds to the return to education, is the sum of the value of searching for non-routine jobs plus the option value of searching for routine jobs, implying that the option to search for a routine jobs represents an important component of the value of a college education. This differs from the conventional wisdom, which views the observation of college workers in routine jobs as a failure of education rather than as an important part of the return to education.

Our perspective on mismatch also has strong implications concerning the role of on-the-job search in labour markets. The traditional perspective, which views mismatch as an *ex post* phenomenon, suggests that on-the-job search is efficiency enhancing, as it represents a corrective force that pushes the labour market towards a more efficient allocation. In contrast, when mismatch is seen as an *ex ante* phenomenon originating in the optimal search decisions of workers, it becomes clear that on-the-job search can also be an important *cause* of mismatch, as it gives workers an incentive to search for jobs for which they are otherwise not well suited.

Finally, we argue that several trends affecting college educated American workers can be understood as a consequence of the reduction in the option value of searching for routine jobs, which has been driven by factors such as skill biased technological change, automation, and offshoring. Ample statistical evidence suggests that the value of routine jobs, whether cognitive or manual, has fallen in recent decades.[5] For example, it is clear that many jobs in

[5] See Jaimovich and Siu (2015), for example.

the legal profession are under pressure from legal startups which use machine learning and text recognition software to perform routine tasks previously requiring hours of labour[6]. We show that, in a mismatch equilibrium, the effects of this shock closely mirrors a number of recent changes in the labour market for college educated workers.

In an equilibrium with *ex ante* mismatch, where the option value of routine jobs is positive, a decline in the value of these jobs reduces the value of labour force participation to college educated workers. Furthermore, as routine jobs decline in value, a greater fraction of unemployed college educated workers are forced to direct their search towards non-routine jobs, which causes a decline in their employment rates as well as a fall in job-to-job transitions. We show that this reduction in the value of labour market participation to college educated workers is consistent with a rising wage premium, both for college workers relative to high school workers and for workers in non-routine jobs relative to routine jobs. Furthermore, according to this view and in stark contrast to conventional perspectives, the worsening labour market fortunes of educated workers corresponds to a *fall* in the degree of observed skill mismatch in the economy. Thus our theory provides a parsimonious explanation of a number of, otherwise disparate, outcomes in the market for college educated workers.

While our theory is not the only potential explanation of why college educated workers search for routine jobs, existing alternatives rely on partial equilibrium settings in which the supply of jobs is exogenous. Consider the following alternatives: i) young workers learn by experimentation, ii) the supply of non-routine, abstract jobs lags the supply of college educated workers, iii) routine jobs are stepping stone jobs that facilitate promotion, and iv) routine jobs are easier to get.All of these explanations, while plausible on the surface, cannot be trivially reconciled with the idea that the vector of job types created by the economy is an endogenous variable, which suggests that the economy could create the sorts of jobs that suit the skills and education levels of its workforce. It is possible that each or any of these alternate explanations could be made consistent with endogenous job creation through the use of plausible technological assumptions about the relative ease of creating different kinds of jobs but, to our knowledge, no such theory has been worked out, and it is not obvious that any such model could be made to be consistent with other facets of the labour market for educated workers.

Our analysis builds on previous work on competitive search equilibrium with adverse selection by Guerrieri et al. (2010) and Chang (2014) to address the interaction between skill

[6]In 2010, for example, software of the e-discovery firm Clearwell was used by the law firm DLA Piper to search through a half-million documents under a court-imposed deadline of one week (New York Times, *Armies of Expensive Lawyers, Replaced by Cheaper Software*, March 4, 2011).

mismatch and search on the job. Our specification of search on the job combines elements of directed search that are standard in competitive search models (Shi, 2009) and elements of bargaining that are standard in random matching models (Postel-Vinay and Robin, 2002). The resulting model retains the block-recursive structure of competitive search equilibria under complete information while allowing employers to counter outside offers, which is the most natural assumption and it renders our framework remarkably tractable by limiting the scope for job quits, thereby eliminating the sorts of wage ladders found in Delacroix and Shi (2006). Our model is sufficiently tractable that we are able to work with the decentralized equilibrium directly, rather than by way of a planning problem, which in turn enables us to study equilibria that are not constrained efficient.

The chapter proceeds as follows: in Section 2 we lay out the economic environment and define a competitive search equilibrium. In Section 3 we consider a simplified version of the model, with only one type of job, in order to facilitate a more intuitive presentation of some of the key mechanisms of the model. In Section 4 we characterize a mismatch equilibrium. In section 5 we discuss the main implications of our analysis. Technical proofs are in the Appendix.

1.2 The Model

1.2.1 Environment

Time is discrete. All agents are risk neutral and discount the future at a rate $r > 0$. There is a unit measure of workers who are either employed or unemployed. An unemployed worker searches for a job and receives a flow benefit from unemployment equal to $b \geq 0$. An employed worker produces. Subsequently, a separation shock makes her unemployed with probability $\delta > 0$. Otherwise, the worker can search for a different job while employed.

There are two observable types of jobs and the measure of type-i jobs will be determined endogenously by free entry, for $i = 1, 2$. Workers are *ex ante* identical and they are relatively more productive in type-1 jobs. A worker-job match produces y_h units of output with probability α_i and y_l units of output with probability $1 - \alpha_i$, for $i = 1, 2$, where $b < y_l < y_h$ and $0 < \alpha_2 < \alpha_1 < 1$. *Ex post*, a worker is mismatched when he is in a low-productivity match. The symmetry in the realizations of labour productivity across jobs simplifies the analysis by helping to limit potential job-to-job transitions. The relevant asymmetry between the two jobs is simply that a worker is less likely to be mismatched in type-1 jobs.

As a matter of interpretation, we think of workers as college graduates, type-1 jobs as jobs involving cognitive, non-routine tasks and type-2 jobs as jobs involving cognitive, but routine tasks. Below we consider the case where there is a second type of worker who is relatively more productive in type-2 jobs (e.g., a high school graduate).

Employers incur an entry cost $k > 0$ in order to post a vacancy. We assume that $(r+\delta)\,k < \alpha_2\,(y_h - y_l)$ to allow for positive job-to-job transitions from and to either type of job. Unemployed workers face a positive opportunity cost of searching for jobs where they are more likely to be mismatched. Specifically, an unemployed worker incurs a cost c if she searches for type-2 jobs in a given period, where c is the realization of a random variable that is independent across workers and over time. For simplicity, we assume that c is a draw from an exponential distribution: $F\,(c) = 1 - \exp\{-\theta c\}$, for $c \geq 0$,with $\theta > 0$.

The structure of the labour market is as follows. Each period a continuum of markets may open up. A market is characterized by an employment contract x, which is specified below, and a market queue length q, which is equal to the ratio of workers to employers in that particular market. Each employer can post any feasible contract and each worker can direct her search to any market.

Let $Q : X \to \mathbb{R}_+$, where X is the set of feasible contracts and $Q\,(x)$ denotes the queue length associated with a contract x, which is defined as the measure of workers searching for x divided by the measure of employers posting x. Matching is bilateral, so each employer meets at most one worker and vice versa. Workers who search in a market where $Q\,(x) = q$ meet an employer with probability $f\,(q)$ and employers in the same market meet a worker with probability $qf\,(q)$. We assume that $f\,(q)$ is twice differentiable, strictly decreasing and convex, with $f\,(0) = 1$ and $f\,(\infty) = 0$. We also assume that $qf\,(q)$ is strictly increasing and concave, approaching 1 as q converges to ∞. These assumptions ensure that the elasticity of job creation, given by $\eta\,(q) = -qf'\,(q)\,/f\,(q)$, is such that $0 = \eta\,(0) < \eta\,(1) \leq 1$, with $\eta'\,(q) > 0$. For simplicity, we also assume that $\eta\,(q)$ is concave, with $\eta\,(\infty) = 1$.[7]

When a worker and a potential employer meet, both the worker's labour market status and her wage, if currently employed, are observed by the potential employer. Then, the productivity of the potential match is drawn randomly and observed by both parties. However, if the worker is already employed, the productivity of her current match is not observed by the potential employer. Subsequently, employers decide whether or not to make formal offers. We assume that employers make take-it-or-leave-it offers, they can counter outside offers and wages can only be renegotiated by mutual agreement. Workers then decide whether to accept any offers. New matches start producing next period.

Since workers are unable to commit not to search on the job, and employers are unable to commit not to counter outside offers, it will facilitate presentation to specify contracts in

[7] An example of a matching technology that satisfies these assumptions is $M\,(u,v) = uv/\,(u+v)$, where u is the measure of unemployed workers while v is the measure of vacancies.

terms of fixed entry wages, taking into account that wages can be renegotiated by mutual agreement, rather than including retention policies as part of the contract. Then, a worker that gets a credible outside offer can choose to terminate the current fixed wage contract and agree to a "new" contract with a different wage, which lasts until a new outside offer arrives. If the outside offer is credible and if a better counteroffer is feasible, the employer then commits to a new fixed wage contract, and so on. Retention policies will depend on history only through the worker's current wage, which is a sufficient statistic for the payoff-relevant history of the current contract.

A contract $x = \{j, g_w^u, g_w^e\}$ specifies a type of job $j \in \{1, 2\}$ and wage policies $g_w^u : \{y_l, y_h\} \to [0, y_h]$ and $g_w^e : \{1, 2\} \times [0, y_h] \times \{y_l, y_h\} \to [0, y_h]$, where $g_w^u(y')$ denotes the entry wage to be offered to unemployed workers when y' is the realization of match productivity and $g_w^e(j, w, y')$ denotes the entry wage to be offered to a worker currently employed in a type-j match earning a wage w when y' is the realization of match productivity in the new match. Note that g_w^e is a function of a worker's observable characteristics before she enters the contractual relationship $\{j, w\}$ and the realization of match productivity y', but it cannot be made contingent on the worker's unobservable type y.

Our assumptions imply that employed and unemployed workers in effect do not compete for the same jobs. Moreover, neither workers nor employers can be forced to participate in a match. That is, employers cannot commit to make a formal job offer and workers cannot commit to accept such an offer before observing the realized match productivity. In this sense, matches are pure inspection goods, rather than experience goods. These assumptions are made to highlight the role of incomplete information about workers' outside options. Below, we argue that our main results continue to apply more generally, as long as match productivity in type-2 jobs is sufficiently easier to observe upon inspection. We think of type-2 jobs as those involving routine tasks and type-1 jobs as those involving non-routine tasks, so this asymmetry is the natural case.

Our specification of search on the job combines elements of directed search that are standard in competitive search models (Shi, 2009) and elements of bargaining that are standard in random matching models (Postel-Vinay and Robin, 2002). The resulting model retains the block-recursive structure of competitive search equilibria under complete information while allowing employers to counter outside offers, which is both the most natural assumption and it renders our framework remarkably tractable by limiting the scope for job quits, in particular, by ruling out wage ladders, which significantly complicate previous analyses of competitive search on the job (Delacroix and Shi, 2006).

We focus on the adverse selection problem that arises from the combination of limited commitment and asymmetric information. Since match productivity is unobserved by third parties, a worker's current labour productivity is private information to the worker *vis-a-vis* potential new employers. Consequently, poaching offers cannot discriminate between workers with different outside options, unless (equilibrium) wages reveal match productivity. Since workers are unable to commit not to search on the job and employers are unable to commit not to counter outside offers, workers in high-productivity matches have an incentive to seek outside offers solely in order to elicit retention offers from their current employer.

Finally, we assume that employers face a small costs of making offers, so they will never make offers that they know will be rejected with certainty. This assumption rules out potential equilibria where mismatched workers are able to elicit retention offers. For simplicity, we assume these costs are negligible and so we are not explicit about them.

1.2.2 Competitive search equilibrium

Let $s \equiv \{i, w, y\} \in S$ denote a worker's payoff-relevant state, where a worker can be unemployed ($i = 0$), employed in a type-1 job ($i = 1$) or employed in a type-2 job ($i = 2$); $w \in [0, y_h]$ denotes her current wage and $y \in \{y_l, y_h\}$ denotes current match productivity. Unemployed workers are associated with the state $s_u = \{\emptyset, b, b\}$, by convention, and the feasible state space is given by $S = \{s_u\} \cup S_e$, where $S_e = X \times [0, y_h] \times \{y_l, y_h\}$.

We focus on stationary equilibria. A competitive search equilibrium Moen (1997) specifies a mapping Q from feasible contracts to market queues. Workers direct their search across all feasible contracts, taking as given the market queue length $Q(x)$ for all $x \in X$. Workers' decisions must be optimal at any information set, which includes their own state $s \in S$ and the distribution of workers across states, that is, the aggregate state of the economy $\psi : S \to [0, 1]$. However, it will become clear that stationary competitive search equilibria are *block recursive* in the sense that the agents' value functions and therefore their equilibrium strategies are not a function of the aggregate state. Thus, for simplicity, we are not explicit about the potential dependence of the agents' value functions on the aggregate state ψ.

Accordingly, let $V(s)$ denote the value function of a worker evaluated in state s. Let $U(s, c, x, Q(x))$ denote the expected surplus to a worker with current state s from searching for x, with associated queue length $Q(x)$, when c is the realized cost of searching for type-2 jobs. The worker meets an employer with probability $f(Q(x))$, in which case a draw y' of match productivity is realized and a wage offer w_o is made. Workers reject any offer w_o from a type-j employer such that $V(s) > V(s_o)$, where $s_o = \{j, w_o, y'\}$. Instead, if $V(s) < V(s_o)$, the decision of a worker with current state $s = \{i, w, y\}$ amounts to choosing whether to accept the offer, in which case her state becomes $s_o = \{j, w_o, y'\}$, or reject the offer, in which case her state remains unchanged, if the worker was unemployed (if $s = s_u$), or it becomes

$s_c = \{i, w_c, y\}$ if the worker was employed (if $s \neq s_u$) and she got a wage counteroffer w_c. Employed workers only renegotiate contracts if they have a credible outside option; so $V(s_c) > V(s)$ if and only if $V(s_o) > V(s)$.

Thus, we have

$$V(s) = w + \frac{\delta V(s_u)}{1+r} + (1-\delta)\left\{\frac{V(s)}{1+r} + \mathbb{E}_c\left\{\max_{x\in X} U(s,c,x,Q(x))\,|s\right\}\right\}, \tag{1.1}$$

for all s, where

$$U(s,c,x,Q(x)) = \begin{cases} (1-j)\,c + f(Q(x))\,\mathbb{E}_{y'}\left\{\max\left\{0, \frac{V(s_o)}{1+r} - \frac{V(s)}{1+r}\right\}\right\} & \text{if} \quad s = s_u \\ \\ f(Q(x))\,\mathbb{E}_{y'}\left\{\max\left\{0, \frac{V(s_o)}{1+r} - \frac{V(s)}{1+r}, \frac{V(s_c)}{1+r} - \frac{V(s)}{1+r}\right\}\right\} & \text{if} \quad s \neq s_u, \end{cases}$$

where $\mathbb{E}_c$ and $\mathbb{E}_{y'}$ denote expectations taken with respect to the exogenous variables c and y', respectively; $s = \{i, w, y\}$, $s_o = \{j, w_o, y'\}$ and $s_c = \{i, w_c, y\}$, where the worker anticipates that $w_o = g^u_w(y')$ if $s = s_u$ and $w_o = g^e_w(i, w, y')$ if $s \neq s_u$, taking as given that $w_c = g_r(s, j, w_o)$, where g_r is her current employer's retention policy.

Since $c \geq 0$ is a random draw from F if $s = s_u$, but $c = 0$ if $s \neq s_u$, we let

$$g_x(s,c) \in \begin{cases} \arg\max_{x\in X} U(s,c,x,Q(x)) & \text{if} \quad s = s_u \\ \\ \arg\max_{x\in X} U(s,0,x,Q(x)) & \text{if} \quad s \neq s_u, \end{cases} \tag{1.2}$$

where it should be noted that a contract $x = \{j, g^u_w, g^e_w\}$ specifies the type of job and the wage policies for the employed and the unemployed.

Moreover, we let

$$g_a(s, s_o, w_c) \in \begin{cases} \arg\max_{a\in[0,1]} \{aV(s_o) + (1-a)V(s)\} & \text{if} \quad s = s_u \\ \\ \arg\max_{a\in[0,1]} \{aV(s_o) + (1-a)\max\{V(s), V(s_c)\}\} & \text{if} \quad s \neq s_u \end{cases} \tag{1.3}$$

for all $s \in S$, $s_o \in S_e$ and $w_c \in [0, y_h]$, where $g_a(s, s_o, w_c) = 1$ if a worker in state s accepts an offer to work for a type-j employer at the wage w' when w_c is her current employer's counteroffer, with $s \equiv \{i, w, y\}$, $s_o \equiv \{j, w_o, y'\}$ and $s_c = \{i, w_c, y\}$ We let $w_c = b$ if $s = s_u$, by convention.

Next, the present value of an ongoing match to the employer, denoted by $J_f(s)$, solves

$$\begin{aligned} \frac{J_f(s)}{1+r} \quad &= \quad \frac{y-w}{r+\delta+(1-\delta)f(Q(g_x(s,0)))} + \left(1 - \frac{r+\delta}{r+\delta+(1-\delta)f(Q(g_x(s,0)))}\right) \\ &\quad \times \mathbb{E}_{y'}\left\{\max_{w_c}\left\{(1-g_a(s,s_o,w_c))\frac{J_f(s_c)}{1+r}\right\}\right\} \end{aligned}$$

$$\text{subject to } w_c \geq w, \tag{1.4}$$

for all $s = \{i, w, y\} \neq s_u$, where $s_o = \{j, w_o, y'\}$ and $s_c = \{i, w_c, y\}$. Let $g_r(s, j, w_o)$ denote a solution to this problem (1.4). Note that employers in an ongoing match need to anticipate the worker's search policy (g_x) and her acceptance policy (g_a), which are common knowledge in equilibrium. Note that the retention policy g_r is contingent on the worker's state $s = \{i, w, y\}$, which includes her type y, since it becomes contractible once the match is formed. Moreover, retention offers cannot be made contingent on the realized match productivity associated with an outside offer (i.e., the worker's potential future type), since it is unobserved by the incumbent employer. Of course, the observed wage offer may reveal match productivity in equilibrium.

Given $Q(x)$, workers searching for x do not need to account for the composition of workers in that market. By contrast, employers posting x need to anticipate not only the likelihood of meeting a worker, given by $Q(x) f(Q(x))$, but also the composition of the pool of workers searching for that contract. The problem is that potential employers have incomplete information about the quality of ongoing matches and therefore, about the outside option of workers who may search on the job. Accordingly, we let $\mu(\cdot\,|x)$ denote a probability distribution on S, for each $x \in X$. An employer posting x incurs a flow cost k and meets a worker with probability $Q(x) f(Q(x))$, in which case the expected surplus to the employer is given by $\mathbb{E}_s\{J(s,x)\,|x\}$, where $J(s,x)$ is the expected value of the employer's surplus conditional on meeting a state-s applicant and $\mathbb{E}_s\{\cdot\,|x\}$ is taken with respect to $\mu(\cdot\,|x)$. Thus, the value of posting x to an employer is given by

$$-k + Q(x) f(Q(x))\, \mathbb{E}_s\{J(s,x)\,|x\},$$

where

$$J(s,x) = \mathbb{E}_{y'}\left\{\max\left\{0, g_a(s, s_o, g_r(s, j, w_o))\, \frac{J_f(s_o)}{1+r}\right\}|s\right\}, \tag{1.5}$$

with $s = \{i, w, y\}$, $s_o = \{j, w_o, y'\}$, where $J_f(s_o)$ satisfies equation (1.4) and where by convention, we set $g_r(s_u, j, w_o) = b$ for all (j, w_o), with $s_u = \{0, b, b\}$.

The maximum in equation (1.5) reflects the fact that employers cannot commit to participate in the match. Rather than including an explicit hiring policy, we have embedded the employer's hiring decision into the wage policy. Our specification of payoffs implies that the employer can always make a low enough offer, conditional on realized match productivity, that the worker will reject with certainty. Equation (1.5) also reflects the fact that poachers anticipate the current acceptance policies of the workers they attract (g_a) and the retention policies of their current employers (g_r). In order to minimize clutter, we do not include these explicitly as arguments in the value function J.

Definition 1 *A stationary equilibrium consists of a set of posted contracts $X^* \in X$, value* *functions* $V : S \to \mathbb{R}_+$ *and* $J : S \times X \to \mathbb{R}_+$*, policy functions* $g_x : S \times \mathbb{R}_+ \to X \cup \emptyset$*,* $g_a : S \times S_e \times [0, y_h] \to [0,1]$*, and* $g_r : S_e \times \{1,2\} \times [0, y_h] \to [0, y_h]$*, a function* $Q : X \to \mathbb{R}_+$*, a distribution* $\psi : S \to [0,1]$ with support on $S^* \subseteq S$, and a conditional distribution $\mu : S \times X \to [0,1]$ with support on $S^* \times X$ *such that:*

(A) Workers' optimal search and acceptance: (A1) V *satisfies (1.1);* g_x satisfies *(1.2);* g_a *satisfies (1.3).* (A2) $g_x(s,c) \in X^* \cup \emptyset$, for all $s \in S$. (A3) For any $s \in S$ and any $x \in X$, $\mu(s|x) = 0$ if $U(s,c,x,Q(x)) < U(s,c,g_x(s,c),Q(g_x(s,c)))$, for all $c \in \mathbb{R}_+$, where U is given by *(1.1).* (A4) For any $x \notin X^*$, $Q(x) = 0$ if $\mu(s|x) = 0$ for all $s \in S$.

(B) Optimal contract posting and retention with free entry: (B1) g_r and J *solve (1.4) and (1.5). Moreover, for any* $x \in X^*$*,* $Q(x) f(Q(x)) \int_S J(s,x)\, d\mu(s|x) = k$*.* (B2) For any $x' \notin X^*$ there does not exist any queue $q \in \mathbb{R}_+$ and any beliefs $\mu'(\cdot|x')$ on S with support on S^* such that $qf(q) \int J(s,x')\, d\mu'(s|x') \geq k$, where, for any feasible (s,c), $\mu'(s|x') > 0$ if and only if $U(s,c,x',q) > U(s,c,g_x(s,c),Q(g_x(s,c)))$.

(C) Market clearing: *For any* $x \in X^*$, $\int_S \mathbb{E}_c\{\mathbb{I}_x(g_x(s,c))|s\}\, d\psi(s) > 0$ and

$$\mu(s|x) = \frac{\psi(s)\,\mathbb{E}_c\{\mathbb{I}_x(g_x(s,c))|s\}}{\int_S \mathbb{E}_c\{\mathbb{I}_x(g_x(s,c))|s\}\, d\psi(s)},$$

for all $s \in S$, where $\mathbb{I}_x(g_x(s,c)) = 1$ *if* $g_x(s,c) = x$ *and* $\mathbb{I}_x(g_x(s,c)) = 0$ *if* $g_x(s,c) \neq x$*.*

(D) Aggregate consistency: $S^* = \{s \in S : \psi(s) > 0\}$, where for all $s \in S$,

$$\int_{S^*} \Pr(s_{t+1} = \widetilde{s}|s_t = s)\, d\psi(\widetilde{s}) = \int_{S^*} \Pr(s_{t+1} = s|s_t = \widetilde{s})\, d\psi(\widetilde{s}),$$

where $\Pr(s_{t+1}|s_t)$ *is the unique distribution associated with* g_x*,* g_a *and* g_r*.*

Part (A1) ensures that workers' search and acceptance policies are optimal for all states, taking as given the market queue length for all contracts. Part (B1) ensures that retention policies are optimal, and employers posting equilibrium contracts make zero profits. Parts (A2) and (B2) ensure that the mapping from contracts to market queues and the beliefs of employers are such that there are no profitable deviations from the equilibrium path. Part (C) ensures that employers' beliefs are consistent with the workers' search strategies through Bayes' rule along the equilibrium path. It ensures that any contract that is posted in equilibrium, that is, any $x \in X^*$, attracts a positive mass of workers and that the distribution of workers searching for any equilibrium contract is exactly what the employers posting those contracts expect. Free entry of employers then ensures the correct market clearing queue.

Part (D) ensures that employers' and workers' equilibrium strategies generate a stationary distribution of worker and jobs and characterizes the set of equilibrium states S^*, which consists of the unemployed plus the support of the equilibrium wage distribution for workers in each type of job. This condition requires that the aggregate flows in and out of any state in S^* must be equal to each other at all times. A formal statement of the transition probability $\Pr(s_{t+1} = s' \,|s_t = s)$ is straightforward, but cumbersome. We provide one in the specific context of the equilibria we characterize below.

Part (A4) rules out a continuum of equilibria that we think are supported by implausible beliefs, where employers do not post some contracts because they believe they will not attract workers while workers do not search for those contracts because they believe too many other workers would be searching for them as well.

Parts (A2)-A3) and (B2) in our equilibrium definition build on the concept proposed by Guerrieri et al. (2010) and extended by Chang (2014). Parts (A2) and (A3) together require that employers posting an off–equilibrium contract must believe that the only workers the contract would ever attract must be indifferent between the off-equilibrium contract and their preferred equilibrium contract. Part (B2) then requires an equilibrium to be robust to deviations where some worker could strictly profit from searching for an off-equilibrium contract x', provided that the employer posting x' has off-equilibrium beliefs that assign probability zero to workers that would not actually profit from searching for x'. It is critical that this requirement applies only to workers that participate in the labour market in equilibrium, that is, only if $s \in S^*$. Intuitively, if a state s is not in the support of the aggregate state ψ, then employers should assign probability zero to the event that such a worker would ever search for any contract.

Part (B2) rules out equilibria where off-equilibrium poaching contracts are not posted because potential employers believe they would only attract unemployed workers, while employed workers do not search on the job because they believe that there are no off-equilibrium poaching contracts. Part (B2) is important in our setting because beliefs need to be specified over observable worker characteristics as well as unobservable worker types and, because of the possibility of search on the job, workers in different equilibrium states $s \in S^*$ are not indifferent between all equilibrium contracts $x \in X^*$.

1.3 Equilibrium with one type of job

In this section we examine a model of an economy with one type of job. Such an economy cannot, by definition, exhibit the kind of *ex ante* mismatch that we are interested in understanding. However, both the informational externality that affects firm wage setting and the adverse selection are present in the one job economy. Presenting this, simplified, version of the model allows us to introduce these key elements without the complexity of the full model. Furthermore, by first presenting a one-type-of-job version of the model, we can highlight an important technical problem that has impeded research in this area to date as well as present our resolution of this problem. Throughout this section we suppose that the single type of job created by the economy is of type 1.

Our assumptions about the nature of counteroffers impose a lot of structure on the problem. First, we assume that employers cannot commit to not counter outside offers. In the absence of commitment, incumbent firms will match any offer up to the worker's current productivity. Second, we assume that poaching firms never make offers that will be rejected with certainty. Collectively, these assumptions imply that workers who are known to be in high productivity matches cannot profit from on-the-job search. The reason is that workers in high productivity matches who receive outside offers will elicit retention offers from their current employer, who will be willing to pay up to y_h to retain the worker. Consequently, there are no gains from trade between workers in high productivity matches and potential poaching firms. This implies, for example, that workers in high productivity matches cannot search in separate markets, as no poaching firm would enter a market populated solely by such job seekers.

In principle, workers known to be in high productivity matches could pool with mismatched workers, which would crowd out mismatched workers. However, high productivity matched workers seek outside options only to elicit a retention offer, and since poaching firms do not make offers that will be rejected with certainty, such workers will not get outside offers. However, this possibility is ruled out by our equilibrium refinement (B2), since there would be alternative contracts that could be posted where employers would make non-negative profits and mismatched workers searching for them would be strictly better off, without making well-matched workers worse off. Note that this means that workers in high productivity matches can only profit from on-the-job search if their match productivity is not revealed in equilibrium, in which case they may be able to pool with mismatched workers.

We further assume that poaching firms cannot commit to offers before observing match productivity, which implies that the maximum wage a poaching wage can offer a worker with whom it has a low productivity match is y_l. Note that, since incumbent firms are willing to make retention offers up to the total amount of worker productivity, poaching firms never make offers to workers with whom they would form a low productivity match.

This sharply defines the set of possible job and wage transitions as follows: all job switches occur when a worker in a low productivity match meets a firm with which she has a high productivity match. A worker in a high productivity match (who can search on the job by pooling) who meets a firm with which she would also form a high productivity match will elicit both a job offer from the poaching firm and a retention offer from the incumbent firm. Therefore, job switches and wage changes reveal that a worker is now employed in a high productivity match. An implication of this is that the job ladder has at most one rung. A worker who moves reveals that she has moved into a high productivity match and will no longer be the target of poaching firms while a worker who accepts a retention offer reveals that she is currently employed in a high-productivity match and will also no longer be the target of poaching firms.

Our model shares the well known property that the allocation supported by a competitive search equilibrium can be characterized as the solution of a certain dynamic programming problem. However, in our model there exist two equilibria: in one wages reveal match productivity, while in the other wages do not reveal match productivity. While each equilibrium outcome corresponds to the solution of a distinct dynamic programming problem, we will present these two problems using one set of Bellman equations.

To that end, let $\rho \in \{1-\alpha_i, 1\}$ denote the fraction of currently mismatched workers among all on-the-job searchers, where we can restrict attention to two types of situations: one where wages are revealing and, consequently, only currently mismatched workers search on the job ($\rho = 1$), and another where wages are non-revealing and, consequently, both currently well-matched and mismatched workers in type-i jobs search on the job ($\rho = 1-\alpha_i$).

We begin with the search problem of an employed worker. It will be convenient to index job types by i and j, and to assume that unemployed workers search for type-i jobs while employed workers search for type-j jobs, even though in this section we are assuming that $i = j = 1$, so our analysis in this section will extend readily to the case with multiple type of jobs.

For a given value of $\rho \in \{1-\alpha_i, 1\}$, the value of employment to a worker in state $s = \{i, w, y\} \neq s_u$, for $w \in [0, y_h]$ and $y \in \{y_l, y_h\}$, who searches for type-j jobs is given by:

$$\overline{V}(s,\rho) = w + \frac{\delta \overline{V}(s_u,\rho)}{1+r} + (1-\delta)\left\{\frac{\overline{V}(s,\rho)}{1+r} + U_j(s,\rho)\right\}, \tag{P1}$$

where

$$U_j(s,\rho) = \max_{w',q'}\left\{f(q')\,\alpha_j\left[\max\left\{0, \frac{w'}{r+\delta} + \left(\frac{\delta}{r+\delta}\right)\frac{\overline{V}(s_u,\rho)}{1+r} - \frac{\overline{V}(s,\rho)}{1+r}\right\}\right]\right\}$$

subject to

$$\begin{aligned} k &\leq q' f(q')\,\alpha_j\left(\frac{y_h - w'}{r+\delta}\right)\rho, \\ w' &\geq y_l, \\ U_j(\{i, w, y_h\}, 1) &= 0. \end{aligned}$$

Denote a solution to Problem (P1) by $\{w_e(s,\rho), q_e(s,\rho)\}$, with $q_e(\{i, w, y_h\}, 1) = \infty$ and $w_e(\{i, w, y_h\}, 1) = 0$. This normalization simply captures the fact that well-matched workers do not crowd out mismatched workers in a revealing equilibrium.

$U_j(s,\rho)$ represents the option value of on-the-job search for type-j jobs to an employed worker. As discussed above, the structure of our counteroffer game implies that employed workers whom wages reveal to be in a high productivity match cannot profit from on-the-job search. Workers who are indistinguishable from workers in low productivity matches can search on the job, and the option value of this search is given by the constrained optimization problem above.

Workers who can profit from on-the-job search face a relatively straightforward competitive search problem. The first constraint imposes that poaching firms must make non-negative expected profits. In this constraint, $\rho \in \{1-\alpha_i, 1\}$ is used to index the two problems. When $\rho = 1$, the non-negative profit constraint is written as if all poaching offers are accepted by workers. This version of the problem corresponds to the equilibrium where wages reveal match productivity, in which case workers in high productivity matches cannot profit from on-the-job search. When $\rho = 1 - \alpha_i$, the non-negative profit constraint is written as if poaching offers are accepted by workers with probability $1 - \alpha_i$. This version of the problem corresponds to the equilibrium in which wages do not reveal productivity, high productivity workers search on the job, and a fraction $1 - \alpha_i$ of applicants to poaching firms reject job offers in favour of retention offers. The other constraint, $w' \geq y_l$, reflects the assumption that poachers recognize the fact that employed workers can only be recruited if the poaching offer exceeds the worker's current productivity. It is easy to verify that a solution to Problem (P1) is such that

$$q_e\left(\{i, w, y_h\}, 1 - \alpha_i\right) = q_e\left(\{i, w, y_l\}, 1 - \alpha_i\right)$$

and

$$w_e\left(\{i, w, y_h\}, 1 - \alpha_i\right) = w_e\left(\{i, w, y_l\}, 1 - \alpha_i\right),$$

which reflects the fact that well-matched workers and mismatched workers have identical incentives in a non-revealing equilibrium. Both types of workers compete for the same outside offers, where subsequent retention offers elicited by currently well-matched workers will just match the outside offers that will be accepted by currently mismatched workers.

The Bellman equation for an unemployed worker who searches for type-i jobs is:

$$\overline{V}\left(s_u, \rho\right) = b + V_i\left(\rho\right), \tag{P2}$$

where

$$V_i\left(\rho\right) = \frac{\overline{V}\left(s_u, \rho\right)}{1+r} + \max_{w_l, w_h, q}\left\{f\left(q\right)\left[\alpha_i \frac{\overline{V}\left(\left\{i, w_h, y_h\right\}, \rho\right)}{1+r} + \left(1-\alpha_i\right)\frac{\overline{V}\left(\left\{i, w_l, y_l\right\}, \rho\right)}{1+r} - \frac{\overline{V}\left(s_u, \rho\right)}{1+r}\right]\right\}$$

subject to

$$
\begin{aligned}
k &\leq qf(q)\left[\alpha_i\left(\frac{y_h}{r+\delta}-\frac{w_e(\{i,w_h,y_h\},\rho)}{r+\delta}+\frac{w_e(\{i,w_h,y_h\},\rho)-w_h}{r+\delta+(1-\delta)\alpha_j f(q_e(\{j,w_h,y_h\},\rho))}\right)\right.\\
&+\left.(1-\alpha_i)\left(\frac{y_l-w_l}{r+\delta+(1-\delta)\,\alpha_j f\left(q_e\left(\{j,w_l,y_l\},\rho\right)\right)}\right)\right],\\
w_l &\leq y_l,\, w_h\leq y_h \text{ and } w_h\begin{cases}=w_l & \text{if } \rho=1-\alpha_i\\ \neq w_l & \text{if } \rho=1.\end{cases}
\end{aligned}
$$

Denote a solution to Problem (P2) by $\left\{w_u^l(i,\rho), w_u^h(i,\rho), q_u(i,\rho)\right\}$.

The last constraints in Problem (P2) reflect the facts that employers cannot commit to pay wages that exceed the worker's marginal product and wages reveal a worker's current match productivity if and only if entry wages vary across realizations of match productivity.

The first constraint is the non-negative profits constraint, incorporating all possibilities for on-the-job search allowed under our assumptions about counteroffers. The two terms within the parentheses in the first line reflect the profits an employer enjoys when it forms a high productivity match with an unemployed job seeker. The first term is the expected discounted value of the profits received if the employer were to pay the future retention offer $w_e(\{i,w_h,y_h\},\rho)$. The second term reflects the temporary extra profits due to the fact that the entry wage w_h of a high productivity worker is lower than the retention offer the worker will elicit as soon as she receives an outside offer. The denominator reflects the three sources of discounting: the discount rate (r), the exogenous probability of job destruction (δ), and the probability that such a worker receives an outside offer from a poaching firm ($(1-\delta)\alpha_j f(q_e(\{j,w_h,y_h\},\rho))$), in which case the incumbent firm will match and the worker's wage will change. The term within the parentheses in the second line represents the profits a firm enjoys when if forms a low productivity match with an unemployed job seeker. The structure of our counteroffer game implies that such workers are always able to search on the job, never elicit retention offers, and quit whenever they meet a poaching firm with which they form a high productivity match.

The following proposition implies that any allocation supported by an equilibrium of the model must solve a version of the above problems.

Proposition 1. *In a revealing equilibrium with positive quits, the equilibrium allocation solves problems (P1) and (P2) with $\rho=1$. In a non-revealing equilibrium with positive quits, the equilibrium allocation solves problems (P1) and (P2) with $\rho=1-\alpha_i$.*

The two possible equilibrium types correspond to the cases where wages either reveal or do not revealing the productivity realization of workers in jobs found from unemployment. Whether wages do or do not reveal this information is critical because it determines whether workers with a high productivity realization can profit from on-the-job search. Revealing equilibria correspond to typical competitive search equilibria. The contribution of this chapter is to provide a characterization of non-revealing equilibria and to illustrate the implications of such equilibria for labour market mismatch.

Using problems (P1) and (P2) to characterize equilibrium allocations is non-trivial due to the fact that the objective function in problem (P2) is not generally concave in $\{w_l, w_h, q\}$. The main complication arises because poachers do not take workers' future quit rates as given, but rather they understand that workers' future quit rates are a function of their current wages. To see why, consider how a worker' current wage affects her trade-off between quit rates and future wages. For a given current wage, a worker is willing to quit at a relatively slower rate only in exchange for relatively higher future wages. The higher her current wage, the lower the *ex post* surplus she can obtain from a given wage and thus, the lower the worker's quit rate. Since a given (future) wage represents a smaller proportional share of the wage gain in the worker expected surplus for workers with higher current wages, a worker's quit rate declines with her current wage at a decreasing rate. While this property is as one would expect, it implies that the worker's value function $\overline{V}(\{i, w, y\}, \rho)$ may not be a concave function of w, which is problematic. In general, it is unclear whether or not the properties of $q_e(\{i, w, y\}, \rho)$ ensure that both the worker's surplus and the employer's surplus are well-behaved with respect to w.

The above problem complicates significantly the analysis of competitive search on the job (e.g., Delacroix and Shi (2006)). In the appendix, we show that this problem can be easily addressed by viewing the solution to (P1) as a mapping from the workers' quit rates to their current wages, rather than the reverse. This approach is crucial as it allows us to solve directly for the equilibrium, as opposed to characterizing a constrained efficient outcome that corresponds to the equilibrium allocation. This approach allows us to examine both efficient and inefficient equilibria.

1.3.1 Revealing equilibrium

It is instructive to begin with the revealing equilibrium in order to highlight our constructive approach to characterizing equilibrium allocations in the familiar setting of standard competitive search equilibria with complete information.

Proposition 2. *Assume that $(y_h - b) / (y_h - y_l) \geq (r + \delta + \alpha_1) / (r + \delta + (1 - \delta)\alpha_1)$. There is a number $k_0 > 0$ such that there is a revealing equilibrium for all $k \leq k_0$. The corresponding equilibrium allocation is uniquely characterized by equations (1.6)-(1.9) and (1.11) below, and it maximizes the present value of aggregate production net of search costs.*

In a revealing equilibrium the wage distribution has three mass points: one wage for each productivity realization for workers who find jobs out of unemployment, and one wage for workers who find jobs via on-the-job search. Equilibrium transitions are as follows: All job offers made to unemployed workers are accepted. Unemployed workers who meet a firm with which they form a low productivity match conduct on-the-job search. These workers change jobs upon meeting another firm with which they form a high productivity match. Workers in high productivity matches do not profit from search on the job. Accordingly, as discussed above, our equilibrium refinement (B2) implies that they will not crowd out mismatched workers in a revealing equilibrium. Jobs are destroyed both exogenously (at rate δ) and, for the case of low productivity matches, endogenously by quits.

The key feature of the revealing equilibrium is that low productivity workers are paid their marginal product:

$$w_u^l(i,1) = y_l. \tag{1.6}$$

This is an important feature of separating equilibria, and is at the core of the constrained efficiency of the revealing equilibrium. Intuitively, the *ex ante* match surplus is maximized when the employer assigns all of the match surplus to mismatched workers *ex post*, in which case they quit exactly when it is efficient to do so. Such a surplus division is optimal from the viewpoint of employers, because they are able to maximize surplus extraction when workers are well-matched *ex post*.

Otherwise, the revealing equilibrium satisfies the usual zero profit and matching efficiency conditions of competitive search models. In particular, $\{w_e(s,1), q_e(s,1)\}$ is the unique pair $\{w', q'\}$ that solves

$$q' f(q') \alpha_j \left(\frac{y_h - w'}{r+\delta}\right) = k \tag{1.7}$$

and

$$\frac{w' - y_l}{r + \delta + (1-\delta)\alpha_j f(q')} = \left(\frac{1-\eta(q')}{\eta(q')}\right)\left(\frac{y_h - w'}{r+\delta}\right). \tag{1.8}$$

Equation (1.7) requires that the expected value of a vacancy to potential poachers equals the cost of posting the vacancy. It implies that employers are willing to offer higher wages and suffer reductions in the net present value of their profits only if they expect to fill their vacancies at a faster rate.

Equation (1.8) is the familiar condition of matching efficiency from standard competitive search equilibrium models. The left side of the equation is the present value of forgone wages while a currently mismatched worker searches on the job. It is easy to verify that the Bellman equation in problem (P1) implies that it is equal to the worker surplus in the new match. Recalling that $\eta(q)$ is the elasticity of job creation and $1-\eta(q)$ is the elasticity of job finding, this matching-efficiency condition implies that the ratio of the worker's surplus to the firm's surplus in new matches equals the ratio of their matching elasticities.

Similarly, $\{w_u^h(i,\rho), q_u(i,\rho)\}$ is the unique pair $\{w, q\}$ that satisfies

$$qf(q)\alpha_i\left(\frac{y_h - w}{r+\delta}\right) = k \tag{1.9}$$

and

$$\alpha_i\frac{\overline{V}\left(\{i,w,y_h\},1\right)}{1+r}+(1-\alpha_i)\frac{\overline{V}\left(\{i,y_l,y_l\},1\right)}{1+r}-\frac{\overline{V}\left(s_u,1\right)}{1+r}=\left(\frac{1-\eta\left(q\right)}{\eta\left(q\right)}\right)\alpha_i\left(\frac{y_h-w}{r+\delta}\right) \tag{1.10}$$

together with the Bellman equation in Problem (P2).

Note that the condition for matching efficiency in the market for unemployed workers (i.e. equation (1.10)) is completely standard. This is because of the result that, in equilibrium, firms earn no profit from low productivity workers. Consequently, the firm's match surplus is entirely a function of the profits it makes when employing high productivity workers. Since these workers cannot profit from on-the-job search in a revealing equilibrium, employers have no incentive to set wages in order manipulate their quit rates.

One can verify that equations (1.9) and (1.10), together with the Bellman equation in Problem (P2) imply that $q_u(i,\rho)$ is the unique value of q that solves

$$\frac{y_h-b}{r+\delta}-\frac{(1-\alpha_i)\,k}{\eta\left(q_e(s,1)\right)q_e(s,1)f\left(q_e(s,1)\right)\alpha_j}=\frac{k}{\eta\left(q\right)qf\left(q\right)}+\left(\frac{1-\eta\left(q\right)}{\eta\left(q\right)}\right)\frac{k}{\left(r+\delta\right)q}. \tag{1.11}$$

This completes the characterization of the unique equilibrium allocation associated with a revealing equilibrium.

The assumption that $(y_h - b)/(y_h - y_l) \geq (r+\delta+\alpha_1)/(r+\delta+(1-\delta)\alpha_1)$ made in Proposition 2 is sufficient to ensure that unemployed workers are willing to accept job offers when match productivity is low. Otherwise, a revealing equilibrium with positive job creation may not exist if k is sufficiently low. The assumption requires that the difference between y_l and b be sufficiently large. The smaller the job destruction rate the less restrictive the assumption is.

On the equilibrium path, beliefs about the composition of the pool of applicants must be correct, both in the market for unemployed workers as well as the separate market for workers searching on the job. Moreover, in the latter employers have the most optimistic beliefs, as they believe that their job offers will be accepted with certainty. Therefore, it is straightforward to support the above equilibrium allocation. Clearly, neither the equilibrium mapping Q nor off-equilibrium beliefs that support the equilibrium allocation are unique.

1.3.2 Non-revealing equilibrium

We now turn to non-revealing equilibria. In the Appendix we prove the following proposition.

Proposition 3. *Assume that* $(1-\alpha_1)(1-\delta) > (r+\delta)$ *There is a number* $k_1 > 0$ *such that there is a non-revealing equilibrium for all* $k \leq k_1$. *The allocation supported by a non-revealing equilibrium is uniquely characterized in the Appendix.*

In a non-revealing equilibrium the wage distribution has two mass points. Since wages do not differ across productivity realizations, all jobs found out of unemployment pay an identical wage. In principle, there could be two wages in the on-the-job search market, as *ex post* mismatched workers accept poaching offers whereas *ex post* well-matched workers transition to retention wages. However, since both types of workers have the same current wage, their incentives to search on the job are identical, so the equilibrium poaching and retention wages are identical.

Equilibrium transitions are as follows: All job offers made to unemployed workers are accepted, and all workers employed in jobs found out of unemployment search on the job, with workers who are well-matched *ex post* mimicking the on-the-job search behavior of workers who are mismatched *ex post*. As a result of pooling, all workers searching on the job face the same matching probabilities. Workers with low productivity realizations in their first jobs change jobs upon meeting another employer firm with which they form a high productivity match. Workers with high productivity realizations in their first jobs receive retention offers upon meeting another employer with which they form a high productivity match. Jobs are destroyed both exogenously (at rate δ) and, in the case of workers in low productivity matches, endogenously by quits.

Consider the search problem of a worker that is currently employed in a type-i job earning a wage w and searching for a type-j job. Once again, it will be convenient to distinguish between different types of jobs even though we are assuming that $i = j = 1$. throughout this section. It is easy to verify that an interior solution of Problem (P1), $\{w\prime, q'\}$, satisfies the familiar matching efficiency condition

$$\frac{w'-w}{r+\delta+(1-\delta)\,\alpha_j f(q)} = \left(\frac{1-\eta(q')}{\eta(q')}\right)\left(\frac{y_h-w'}{r+\delta}\right),$$

according to which the ratio of the worker's surplus to the employer's surplus equals the ratio of their matching elasticities. With respect to this, the only difference with the revealing equilibrium allocation is that entry wages (w) may not be equal to the low realization of match productivity (y_l). It also satisfies the usual zero-profit condition

$$q'f(q')(1-\alpha_i)\,\alpha_j\left(\frac{y_h-w'}{r+\delta}\right) = k. \tag{1.12}$$

Note that, in a non-revealing equilibrium, potential poachers need to anticipate that a fraction $(1-\alpha_i)$ of their pool of applicants are currently mismatched, and so a fraction α_i will turn down their job offers because they are only searching to elicit a retention offer from their current employer.

Even if one takes as given that wages are non-revealing, that is, even if one takes as given that $w \equiv w_l = w_h$ in Problem (P2), solving this problem is non-trivial because the objective function is not generally concave in $\{w, q\}$. Fortunately, one can address this problem by viewing the solution to Problem (P1) as a mapping from the workers' quit rates to their entry wages, rather than the reverse, and then treat current and future quit rates as the relevant choice variables in Problem (P2). We follow this approach in the proof of Proposition 2 to characterize the equilibrium allocation in the constrained efficient equilibrium. In the Appendix, we show that this approach can be followed more generally to characterize the allocation in the non-revealing equilibrium and prove Proposition 3.

To understand the properties of the equilibrium allocation in a non-revealing equilibrium, it is useful to consider the transformed problem in some detail. To that end, use the above first-order conditions to express the worker's entry wage as a function of (the future) q':

$$\widetilde{W}(q') \equiv y_h - \left(\frac{k}{q' f(q')(1-\alpha_i)\alpha_j}\right)\left(r + \delta + \left(\frac{1-\eta(q')}{\eta(q')}\right)(r + \delta + (1-\delta)\alpha_j f(q'))\right). \tag{1.13}$$

Observe that employers understand i) that all workers search on the job, and ii) that job finding probabilities in the on-the-job search market depend on the wages earned by workers in their current jobs. Employers take this effect into account and set current wages, in part, in order to influence future quit rates. Let $\widetilde{V}_0(i, q')$ denote the value of a type-i job to an employed worker expressed as a function of q':

$$\begin{aligned} \widetilde{V}_0(i, q') &\equiv \alpha_i \overline{V}\left(\left\{i, \widetilde{W}(q'), y_h\right\}\right) + (1-\alpha_i)\overline{V}\left(\left\{i, \widetilde{W}(q'), y_l\right\}\right) \\ &= \overline{V}\left(\left\{i, \widetilde{W}(q'), y_h\right\}\right) = \overline{V}\left(\left\{i, \widetilde{W}(q'), y_l\right\}\right) \end{aligned}$$

and let $\widetilde{M}_0(i, q')$ denote the *ex ante* surplus associated with a type-i match:

$$\widetilde{M}_0(i, q') \equiv \alpha_i \widetilde{M}\left(\left\{i, \widetilde{W}(q'), y_h\right\}\right) + (1-\alpha_i)\widetilde{M}\left(\left\{i, \widetilde{W}(q'), y_l\right\}\right),$$

where $\widetilde{M}\left(\left\{i, \widetilde{W}(q'), y_h\right\}\right)$ is the *ex post* surplus associated with a high-productivity match and $\widetilde{M}\left(\left\{i, \widetilde{W}(q'), y_l\right\}\right)$ is the *ex post* surplus associated with a low-productivity match.

It is useful to understand the connection between the total surplus of a match and its allocation between a worker and her employer. To that end, note first that the surplus in low-productivity matches is given by

$$\begin{aligned} \frac{\widetilde{M}\left(\left\{i, \widetilde{W}(q'), y_l\right\}\right)}{1+r} &= \frac{\overline{V}\left(\left\{i, \widetilde{W}(q'), y_l\right\}\right)}{1+r} - \frac{\overline{V}(s_u)}{1+r} \\ &\quad + \left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q')}\right)\frac{y_l - \widetilde{W}(q')}{r+\delta} \end{aligned}$$

where the first line on the right side is the part of the match surplus that goes to the worker and the second line is the part that goes to the employer, which consists of a flow of profits equal to $y_l - \widetilde{W}(q')$ for as long as the worker stays with the employer, where the term in parentheses is the probability that the worker will find an outside offer, which the employer anticipates she will accept with probability one.

The surplus in high-productivity matches is more interesting. In particular,

$$\begin{aligned}
\frac{\widetilde{M}\left(\left\{i,\widetilde{W}(q'),y_h\right\}\right)}{1+r} &= \frac{\overline{V}\left(\left\{i,\widetilde{W}(q'),y_h\right\}\right)}{1+r} - \frac{\overline{V}(s_u)}{1+r} \\
&\quad + \left(1 - \frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q')}\right)\frac{y_h - w_e\left(\left\{i,\widetilde{W}(q'),y_h\right\}\right)}{r+\delta} \\
&\quad + \left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q')}\right)\frac{y_h - \widetilde{W}(q')}{r+\delta}.
\end{aligned}$$

The first and the third lines in the right side are the obvious counterparts of those in low-productivity matches. The second line reflects the fact that *ex post* well-matched workers will search for outside offers solely in order to elicit a retention offer from their current employer.

It is easy to verify that an interior solution for the current and future labour market queues $\{q,q'\}$ must satisfy the following conditions:

$$\frac{\widetilde{V}_0(i,q')}{1+r} - \frac{\overline{V}(s_u)}{1+r} = \lambda_i q\left(\frac{1-\eta(q)}{\eta(q)}\right)\frac{k}{qf(q)}, \tag{1.14}$$

$$\lambda_i = \frac{f(q)\,\partial\widetilde{V}_0/\partial q'}{qf(q)\left(\partial\widetilde{V}_0/\partial q' - \partial\widetilde{M}_0/\partial q'\right)} \tag{1.15}$$

and

$$qf(q)\left(\frac{\widetilde{M}_0(i,q')}{1+r} - \frac{\widetilde{V}_0(i,q')}{1+r} + \frac{\overline{V}(s_u)}{1+r}\right) = k, \tag{1.16}$$

where λ_i is the multiplier associated with the employer's zero-profit constraint, given by equation (1.16). Equation (1.14) coincides with the standard matching efficiency condition if and only if the multiplier equals $1/q$, which is the usual case in standard competitive search equilibria analyzed in the literature. Consider equation (1.15). The multiplier is the expected value of surplus to the worker associated with a higher labour market queue at the margin $(f(q)\,\partial\widetilde{V}_0/\partial q')$ evaluated in terms of the employer's surplus $\left(qf(q)\left(\partial\widetilde{V}_0/\partial q' - \partial\widetilde{M}_0/\partial q'\right)\right)$. The expected surplus of a match is maximized at $\partial\widetilde{M}_0/\partial q'$, which implies that $\lambda_i = 1/q$. In the Appendix, we show that this happens exactly at the corner when $\widetilde{W}(q') = y_l$.

At an interior non-revealing equilibrium $\lambda_i \geq 1/q$ and the match surplus is not maximized, except in the special case where the first-order conditions hold at the corner and $\lambda_i = 1/q$.

The problem is that, while employers can lower the workers' future quit rates, by raising the entry wages they offer in the first place, they also have an impact on the outside offers the workers will get, because workers with higher wages have an incentive to elicit higher outside offers. Since they cannot prevent well-matched workers to seek outside offers, the allocation of surplus at the margin is allocated disproportionately to the worker and so employers do not typically have an incentive to raise entry wages all the way to y_l.

Observe that the corner allocation is still inefficient because it induces too little entry of employers in the market for unemployed workers. This is because, relative to a revealing equilibrium, employers are forced to share too much surplus with the worker, as high productivity workers who receive outside offers are staying with their current employer, but they are able to extract some of the surplus. In the Appendix, we show that the allocation in a non-revealing equilibrium is uniquely characterized, although in general we cannot guarantee whether or not the allocation is interior.

To understand why non-revealing wages can be supported in equilibrium note the existence of an informational externality, whereby firms in the market for unemployed workers do not take into account the informational value of wages to poachers. The consequence of this externality is that employers have no direct incentive to post revealing wages. This means that non-revealing wages can be equilibrium wages as long as unemployed workers choose to search for non-revealing contracts when revealing contracts are feasible. This occurs because the option to search on the job constitutes an important component of the value of a job, but the value of this option depends on the beliefs of both workers and potential poaching firms.

Consider a potential equilibrium with non-revealing wages. Now suppose an unemployed worker is considers searching for a contract with revealing wages. The value of on-the-job search for such a job, however, depends on the off equilibrium beliefs of poaching firms about the current match quality of their applicant pool. These beliefs are, in general, unrestricted. In particular, condition (B2) of our equilibrium definition has no bite off the equilibrium path. Consequently, if potential poachers are sufficiently pessimistic about the composition of the applicant pool in the on-the-job search market associated with a revealing contract (i.e. they believe it will contain a high percentage of well-matched workers looking for retention offers) then the returns to on-the-job search associated with deviations to a revealing contract are sufficiently low that such deviations are unprofitable to workers. Of course, neither the equilibrium mapping Q nor off-equilibrium beliefs that support the equilibrium allocation are unique.

Formally, if $s \notin S^*$, beliefs are arbitrary and we may assume that employers believe that $\mu(s\,|x) = 0$ for all $s \notin S^*$. In the Appendix, we show that the assumption that

$(1-\alpha_1)(1-\delta) > (r+\delta)$ made in Proposition 3 then ensures that no type-1 employer can profit from offering a deviating contract where wage offers to unemployed workers are made conditional on match productivity. The assumption requires that the probability of mismatch in type-1 jobs is sufficiently high, that the jobs are sufficiently durable and that workers value future payoffs sufficiently.

Finally, observe that wages that do not reveal match productivity create adverse selection in the on-the-job search market since, under non-revealing wages, workers in matches with high productivity cannot be identified and, therefore, have an incentive to search on the job in order to elicit retention offers from their current employers. Non-revealing wages, therefore, increase the value of on-the-job search to workers with a high productivity realization relative to the case where wages reveal match quality, thereby preventing well-matched workers from searching on the job. The overall effect of this adverse selection problem, however, is to depress the returns to poaching firms, which reduces the entry of poachers and therefore, depresses the returns to on-the-job search overall. This reduction is concentrated on poorly matched workers. Note that this adverse selection problem is worse the higher is α_1, because a large value of α_1 implies that many workers are well matched to begin with, and therefore only searching on the job to elicit retention offers.

1.4 Equilibrium Mismatch

In this section we extend our analysis to the economy with both types of jobs. Recall that type-2 jobs are less likely to result in high productivity matches than type 1 jobs while being identical to type-1 jobs in terms of creation costs and the matching function. The main result, of both the section and the chapter, is that despite the unambiguous technological inferiority of type-2 jobs, there exist equilibria in which type-2 jobs are created alongside type 1 jobs, and some workers choose to search for type-2 jobs. We refer to equilibria in which workers purposefully search for inferior jobs, despite the availability of better jobs, as exhibiting *ex ante* mismatch.

It is easy to verify that an equilibrium allocation must solve the obvious analogue of Problems (P1) and (P2). To that end, note that our assumptions about counteroffers continue to restrict the possible job and wage transitions as explained in section 1.3. Consider the value of employment to a worker in the two-job economy. It is straightforward to verify that any allocation supported by an equilibrium with positive quits must be such that employed workers only ever search for type-1 jobs. Intuitively, workers are expected to be more productive in type-1 jobs and, consequently, type-1 employers always drive type-2 employers out of any market where employed workers search.

Taking this into account, Problem (P1), with $j = 1$, can be used to characterize equilibrium allocations, except that now it ought to be recognized that the value functions and the corresponding policy functions are functions of $\rho_1, \rho_2 \in \{1 - \alpha_i, 1\}$, rather than simply ρ, where ρ_i denotes the fraction of currently mismatched workers employed in type-i jobs among all those searching for type-1 jobs, for $i = 1, 2$. With a slight abuse of notation we will continue to denote those functions as before.

One can verify that equilibrium allocations in the two-job economy must satisfy the analogue of Problem (P2), with

$$\overline{V}(s_u, \cdot) = b + \max\{V_1, V_2\}, \tag{1.17}$$

where V_i is given by Problem (P2) with $\rho = \rho_i$, for $i = 1, 2$, and where it should now be understood that $\overline{V}(s_u, \cdot) \equiv \overline{V}(s_u, \rho_1, \rho_2)$.

In order to characterize an equilibrium allocation, first note that, due to the technological inferiority of type-2 jobs, if wages in type-1 matches reveal productivity the equilibrium is constrained efficient. In this case, no type-2 jobs are created, the equilibrium allocation solves P1 and P2, with $\rho = 1$, and it is as given by Proposition 2, with $j = 1$. Note that, if wages in both types of jobs are revealing, then neither type of job suffers from the adverse selection problem. In this case, the higher productivity of type-1 jobs makes them more attractive to all searchers. If wages in type-1 jobs are revealing, but wages in type-2 jobs are non-revealing then type-2 jobs, in addition to being less productive, also suffer from the adverse selection problem. Clearly, the allocation in a revealing equilibrium is unique within the class of revealing equilibria, but it can be supported in a continuum of different ways.

Thus, the existence of a mismatch equilibrium requires that wages in type-1 jobs are non-revealing, in which case on-the-job search from these jobs suffers from the adverse selection problem created by *ex post* well-matched workers searching for retention offers. The key to understanding mismatch equilibria is to note that the potential adverse selection problem is more severe in markets for type-1 jobs, because workers in these jobs are relatively less likely to be *ex post* mismatched, and therefore more likely to be searching on the job in order to elicit retention offers than are workers in type-2 jobs. Potential poaching firms understand this and, consequently, are less willing to enter markets where workers in type-1 jobs search on the job. This lowers the value of on-the-job search for workers in type-1 jobs. When this effect is sufficiently strong, type-2 jobs have an equilibrium advantage over type-1 jobs despite being inferior along traditional technical dimensions.

There are only two cases to consider: the case where wages do not reveal productivity in either type of job, and the case where wages reveal productivity in type-2 jobs but not type-1 jobs. In the former case, however, the degree of asymmetry may not be powerful enough to support the creation of both types of jobs. Consequently, in the remainder of this section we examine the case where type-1 entry wages are non-revealing and type-2 entry wages are fully revealing. Proposition 4 provides sufficient conditions for existence of an equilibrium where both types of jobs are created, for this case. Below, we argue that this situation can also be interpreted as an extreme case of labour markets where match productivity is imperfectly observable and it is relatively easier to observe in type-2 jobs, so they are subject to a less severe adverse selection problem.

Proposition 4. *Assume that* $(1-\alpha_1)(1-\delta) > (r+\delta)$*. There are numbers* $\widehat{\alpha} \in (0, \alpha_1)$ *and* $\widehat{k} > 0$ *such that there is an equilibrium with positive job quits where both types of jobs are created for all* $\alpha_2 \in (\widehat{\alpha}, \alpha_1)$ *and all* $k \in \left(0, \widehat{k}\right)$*.*

The condition in the proposition, $(1-\alpha_1)(1-\delta) > (r+\delta)$ ensures that no type-1 employer can profit from offering a deviating contract where wage offers to unemployed workers are made conditional on match productivity. The assumption requires that the probability of mismatch in type-1 jobs is sufficiently high, that the jobs are sufficiently durable and that workers value future payoffs sufficiently. Under this assumption, Proposition 4 says that an equilibrium with both types of jobs can be constructed whenever the probability of *ex post* mismatch in type-2 matches is sufficiently close to that in type-1 matches and the employers' search costs are sufficiently low.

It is easy to see that the unemployed workers' optimal search policy is characterized by a cutoff c_0 such that they search for type-2 jobs if and only if their current realization of the search cost c is smaller than the cutoff. Noting that unemployed workers will search for type-2 jobs if and only if their idiosyncratic search cost c is smaller than the utility gain $V_2 - V_1$, we have that

$$V(s_u, \cdot) - b = V_1 + F(c_0)\left(V_2 - \mathbb{E}(c \,|c \leq c_0) - V_1\right). \tag{1.18}$$

That is, the net value of unemployment $(V(s_u, \cdot) - b)$ to a worker is equal to the value of searching for jobs where she is more productive (V_1) plus the option value of searching for jobs where she is less productive, which consists of the expected utility gain $F(c_0)(V_2 - V_1)$ minus the expected search costs $F(c_0)\,\mathbb{E}(c\,|c \leq c_0)$.

Since searching for type-1 jobs is costless, an equilibrium allocation must have $V_2 - V_1 \leq c_0$, with equality if and only if $V_2 \geq V_1$. Thus, either the option value of searching for type-2 jobs is non-negative, or else only type-1 jobs are created in equilibrium. This is the sense in which constructing an equilibrium with *ex ante* mismatch is non-trivial: it requires that the value of search for jobs where the unemployed is relatively less productive to be relatively higher in equilibrium. That is, it requires that $c_0 > 0$. The assumption that $\alpha_2 \in (\widehat{\alpha}, \alpha_1)$ in the proposition ensures that the two types of jobs are sufficiently similar that unemployed workers would strictly prefer to search for type-2 jobs if it were costless to do so. It is clear that there is a number $\widehat{\alpha} \in (0, \alpha_1)$ such that this is the case.

In the economy with two jobs workers in type-1 and type-2 jobs conduct on-the-job search on separate markets, because job type is an observable component of a worker's labour market state. It is then easy to see that the allocation supported by an equilibrium with ex ante mismatch is such that V_i is given by Problem (P2) subject to (1.18), for $i = 1, 2$, with $\rho = 1 - \alpha_1$ for $i = 1$, and $\rho = 1$ for $i = 2$.

The next proposition provides further insight on the properties of mismatch equilibria.

Proposition 5. *Consider an interior equilibrium where wages in type-*1 *jobs are non-revealing and wages in type-*2 *jobs are revealing. (i)* $b + V_2 > \overline{V}(s_u, \cdot) > b + V_1$*. (ii)* $f(q_u(2, \cdot)) > f(q_u(1, \cdot))$

Part (i) of the proposition implies that type-2 jobs have higher value than type-1 jobs. This reflects the fact that adverse selection in the on-the-job search market associated with type-1 jobs reduces the values of those jobs. Since wages in type-2 jobs are revealing, the adverse selection does not affect on-the-job search for workers in type-2 jobs.

Part (ii) says that type-2 jobs are also easier to get. Since adverse selection reduces the value of on-the-job search from type-1 jobs, the wage received in those jobs constitutes a larger portion of the value of the job. In type-2 jobs, in contrast, the current wage is relatively less important because the option value of search is higher. Therefore, it is relatively cheaper for firms to compensate workers with job finding probability, rather than with wages, in the market for type-2 jobs.

Unsurprisingly, given the adverse selection problem, labour market outcomes in the mismatch equilibrium are inefficient. In particular, the degree of equilibrium mismatch, defined as the fraction of workers searching for type-2 jobs, is inefficiently high relative to the efficient benchmark, under which all workers search for type-1 jobs. Furthermore, the value of unemployment in a mismatch equilibrium is inefficiently low, due to the fact that the equilibrium fails to maximize the present value of aggregate production net of search costs.

1.5 Conclusion and Implications

This chapter has developed an original theory of *ex-ante* labour market mismatch. We begin this section by arguing that the coordination failure we have introduced above is not necessary, and that *ex-ante* mismatch identical ours can be generated from informational frictions alone. We go on to show that mismatch equilibria like that of our model provide several novel insights about labour markets. We focus on the experiences of college educated

American workers and we think of the type-1 jobs of the previous sections as corresponding to non-routine jobs, which make full use of a college education, while type-2 jobs correspond to routine jobs, which do not.

1.5.1 Incomplete information in the labour market

Consider the model from the previous section, except that the productivity of type-i entry matches is observed with some probability $\nu_i \in [0, 1]$, for $i = 1, 2$, and suppose that $\nu_2 \geq \nu_1$. For simplicity, suppose that poachers always observe the realization of match productivity before hiring. Our model is the special case where $\nu_2 = \nu_1 = 1$. If $\nu_1 < 1$, the revealing equilibrium fails to exist and moreover, all equilibria are inefficient. If $\nu_2 = \nu_1 = 0$, there is an equilibrium where wages are non-revealing in both markets. This equilibrium also exists in our model. Moreover, in such an equilibrium the adverse selection problem is necessarily more severe in markets where workers employed in type-1 jobs search, precisely because the proportion of mismatched workers in those markets is relatively low. So everything else equal, the returns to search on the job are higher for workers currently employed in type-2 jobs. We have not been able to establish analytically that this equilibrium can exhibit mismatch and our numerical simulations of the model suggest that it does not. It is clear, however, that all that is needed to support mismatch is that the adverse selection problem is sufficiently more severe in markets where workers employed in type-1 jobs search. Our analysis can be understood as an extreme version of this problem. Whether mismatch results from coordination failure or from the fact that non-routine match productivity is imperfectly observable, the structure of the equilibrium is the same and so are the relevant externalities.

1.5.2 On-the-Job Search

The conventional view, derived from viewing mismatch as an *ex post* phenomenon, suggests that on-the-job search is an important channel by which labour market mismatch is corrected, as it enables workers in unsuitable jobs to transition to better matches without experiencing unemployment. According to this view, high job-to-job transitions are generally viewed as a positive signal about the ability of labour markets to allocate workers to jobs. This view underlies much of the academic literature (Moscarini and Thomsson (2007), Kambourov and Manovskii (2008)) as well as the analysis of other labour market observers, such as Goldman Sachs who claim that "Businesses adding and losing workers and people quitting and taking other jobs – what economists call "churn"— are generally good measures of economic confidence."[8]

[8] Goldman Sachs 2014.

Table 1.1: Comparative statics

Variable	$\alpha_1 \uparrow$	$\alpha_2 \downarrow$
Employment rate	$\uparrow$	$\downarrow$
Fraction of routine jobs	$\downarrow$	$\downarrow$
Value of unemployment	$\uparrow$	$\downarrow$
Non-routine/routine wage premium	$\uparrow$	$\uparrow$
Job-to-job transition rate	$\downarrow$	$\downarrow$

An *ex ante* perspective on mismatch, however, calls for a radically different view. Our analysis implies that , in a mismatch equilibrium, on-the-job search is also the *cause* of mismatch. This suggests that there is no simple relationship between the rate of job-to-job transitions, the extent of labour market mismatch, and the efficiency of labour markets.

To illustrate this point further, we perform a pair of comparative static exercises. Given the richness of the setting, it is not possible to generate analytical results. In lieu of these, we solve the model numerically.[9] Our numerical results should not be mistaken for a serious calibration of the model, which is beyond the scope of this chapter. This is because the primary objective of this chapter is to introduce a rigorous theory of *ex ante* mismatch. As a consequence of this focus, we employ some fairly extreme symmetry assumptions (for example, that the productivity of good and bad match realizations is identical across job types) that, while greatly simplifying the analytical proofs of existence of equilibrium, render our model quantitatively unrealistic.

The first comparative static, which we report in the first column of Table 1.1, is the equilibrium response to an increase in α_1, which is the probability that a college worker is highly productive in a non-routine job. The rise in α_1 induces unemployed workers to reallocate their search towards non-routine jobs (though this effect is mitigated by the fact that the value of on-the-job search from routine jobs also increases). This reallocation of search effort causes mismatch in the economy to fall, as a smaller fraction of college educated workers choose to search for routine jobs. This further implies that job-to-job transitions also fall, because well-matched workers in non-routine jobs do not search on-the-job. Furthermore, when α_1 rises, the gap between expected productivity in a non-routine and routine job also rises, which is reflected in an increase in the wage premium associated with non-routine work, and therefore a rise in overall wage inequality within the group of

[9]For instance, an interior mismatch equilibrium exists under the following parameterization: $y_l = 1$, $y_h = 1.1$, $\alpha_1 = 0.4$, $\alpha_2 = 0.35$, $k = 0.1$, $\delta = 0.1$, $b = 0.1$, $r = 0.02$, $\theta = 1$.

college workers. Since the increase in α_1 also raises the expected productivity of non-routine jobs, firms create more such jobs, both in markets where unemployed workers search, and in markets where workers search on-the-job. The result of is an increase in overall employment, average wages, and the value of unemployment.

These results stand in sharp contrast to the conventional wisdom, as a decline in job-to-job transitions does not indicate either increased mismatch or fall in labour market efficacy. Instead, a decline in job-to-job flows in a mismatch equilibrium can indicate precisely the opposite. The fall in transitions reflects the fact that workers do not search on-the-job precisely because they are well matched. Furthermore, the labour market outcomes are better in the sense that the value of unemployment is higher.

Our second comparative static concerns the equilibrium response to a fall in α_2, which we report in the second column of Table 1.1. As was the case for an increase in α_1, the increase in the gap between the expected productivity of non-routine and routine jobs following a fall in α_2 causes an increase in the wage premium paid to workers in non-routine jobs. Also as before, as workers reallocate their search towards non-routine jobs, both mismatch and job-to-job transitions fall. In this case, however, the decline in α_2 results in a fall in the surplus associated with routine jobs. Consequently, firms are willing to create fewer such jobs in equilibrium. Since routine jobs have value to college workers in a mismatch equilibrium, their destruction causes unemployment, average wages, and the value of unemployment to fall. In this case, while the decline in job-to-job transitions is associated with a fall in equilibrium mismatch, this reduction in mismatch corresponds to a fall in the welfare of labour market participants, as the routine jobs they would prefer to search for become less profitable and less available.

1.5.3 labour Force Participation

It is straightforward to extend our model to incorporate a labour force participation decision. Suppose that workers have heterogeneous opportunity costs of participating in the labour force. Formally, there is an exogenous and constant flow benefit l from staying out of the labour force, which is worker-specific and is discontinued when a worker enters the labour force. For simplicity, we assume that l is a one-time draw from a distribution H.

Clearly, labour force participation decisions are determined solely by the exogenous opportunity costs of participation and the returns to labour market search. The latter are independent of participation decisions and so optimal participation decisions are characterized by a single cutoff l_0, where

$$\overline{V}(s_u, \cdot) = \left(\frac{1+r}{r}\right) l_0.$$

Workers participate in the labour force if and only if their opportunity cost l is smaller than the cutoff l_0. In this setting, $\overline{V}(s_u, \cdot)$ can be understood as the equilibrium value of labour market participation to college educated workers. The important point is that the participation rate will track the value of participation. We address some important implications of this below.

1.5.4 Mismatch

While the model presented in this chapter only contains one kind of worker, it is conceptually simple to incorporate differing educational levels into the framework. We assume that high school educated workers cannot be employed in non-routine jobs. As long as educational status is observable a separating competitive search equilibrium exists, under which the model of Section 3 describes the labour market facing college educated workers.[10] In this section we investigate the ability of our model, extended to allow for both high school and college educated workers as well as a labour force participation decision, to understand labour market outcomes over the past two decades.

We begin with the widely accepted observation that automation and offshoring have resulted in a fall in the productivity of routine jobs over recent decades.[11] While the impact of these changes on less educated workers has been widely studied, much less has been written about the impact of these changes on college educated workers. In a mismatch equilibrium, however, it is clear that changes in the productivity of routine jobs will also impact college educated workers. We model the impact of automation on routine jobs as a decline of α_2, as this corresponds to a fall in the expected productivity of routine jobs and, in particular, a fall in the fraction of routine jobs that are high productivity.

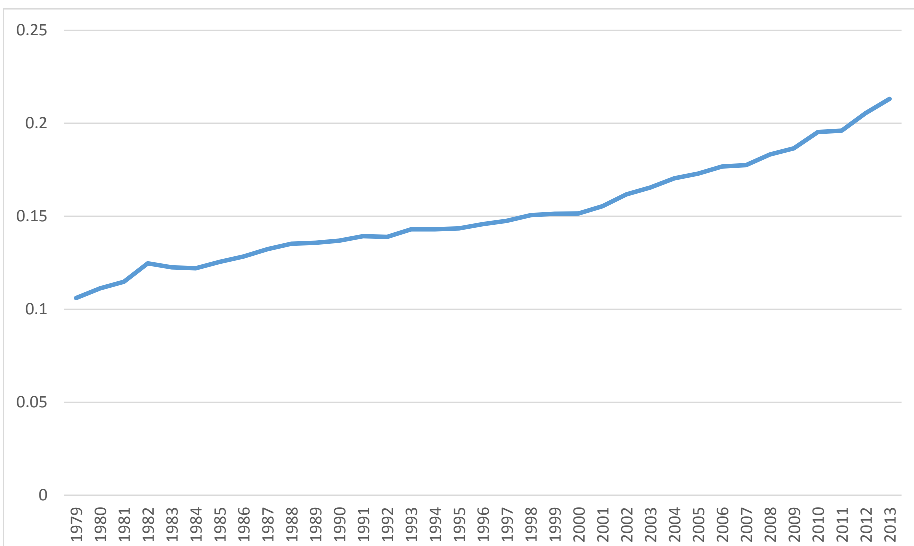
It is clear from the results presented in Table 1.1 that a fall in α_2 generates results corresponding to the well documented fall in the employment rates of college graduates, rising within-group inequality for the highly educated workers (Lemieux (2006), Autor et al. (2008)), and the decline in job-to-job transition rates (Davis et al., 2012). The discussion of the previous sub-section implies that the labour force participation rates of college educated workers will fall, as observed in the data, because this rate mirrors the behaviour of the value of unemployment. The extended model further implies that the fall in the productivity of

[10]Note that in such an economy, college and non-college educated workers do not compete for the same jobs. For an example of such competition see Chapter 3.

[11]It is apparent that automation, for example, could be viewed both as reducing the productivity of routine jobs, for which it is a substitute, while simultaneously increasing the productivity of non-routine jobs, with which it is a complement. It seems clear, however, that in the past two decades the dominant technological shock has been the decline in the productivity of routine jobs (Acemoglu and Autor (2011), Autor (2015), Jaimovich and Siu (2015)).

routine jobs can also explain the fall in employment and participation rates of high school graduates as well as a rising college/high school wage premium (Autor, 2015). Because high school educated workers are more concentrated in routine jobs, the decline in the productivity of these jobs will generally have a more severe impact on high school educated workers relative to college educated workers. Overall, these results suggest that a number of recent trends in the market for college educated workers can be understood as responses to a fall in the productivity of routine jobs in a mismatch equilibrium.

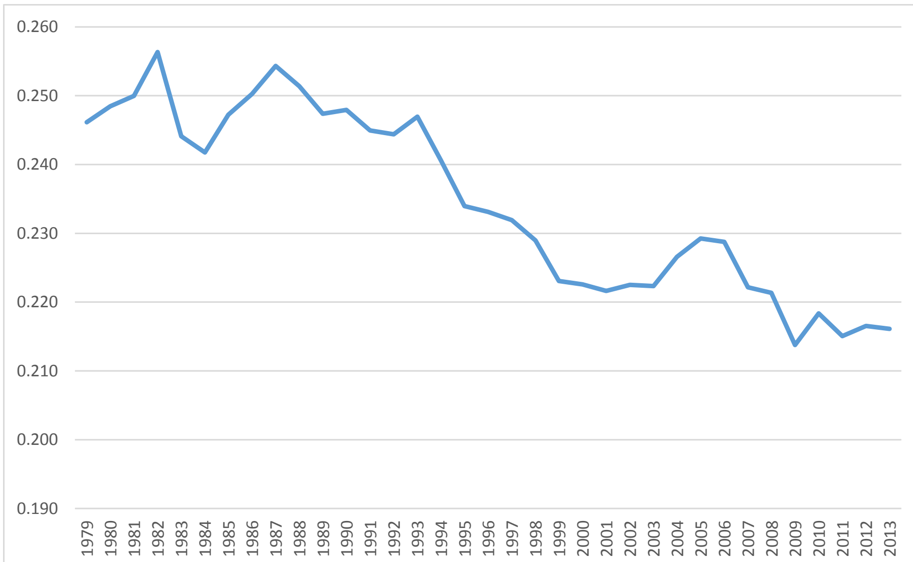
Figure 1.1: Fraction of routine jobs employment filled by college graduates



Perhaps the most apparently counterfactual implication of our explanation is the suggestion that observed mismatch should have fallen, along with the value of routine jobs. Figure 1, for example, documents the fact that the fraction of routine jobs filled by workers with a college degree has risen over recent decades.[12] This sort of data could be interpreted as a rise in mismatch related to college educated workers moving down the occupational

[12]Our data is taken from Beaudry et al. (2016), and we thank Ben Sand for making it available to us. The data covers men and women between the ages of 25 and 54, and is drawn from the U.S. Current Population Survey (CPS) from 1980-2013. The division of jobs into routine and non-routine is based on the types of tasks predominantly performed within them employing the categorization of Acemoglu and Autor (2011) and Autor and Dorn (2013).

Figure 1.2: Fraction college graduates employed in routine jobs



ladder and into less skilled intensive jobs, perhaps in response to a fall in the creation of non-routine jobs (for example, Beaudry et al. (2016)). We argue that this interpretation is problematic, and that the figure may be more simply explained as the result of a decline in the population of non-college educated relative to college educated workers over the same time period. In Figure 2 we take a different look at the data by plotting the fraction of college workers employed in routine jobs, which we view as corresponding to the faction of mismatched college educated workers. It is clear from the figure that, according to this measure at least, mismatch in the market for college educated workers has fallen steadily over time. This in spite of large growth in of the population of college educated workers.

The data in Figure 2 is consistent with the conclusions of other observers who have attempted to analyze the link between adverse labour market outcomes and labour market mismatch. On attempts to understand recent rises in unemployment as a consequence of increases in mismatch, Krugman writes "structural stories come in two variants: geography and skills. The geography story says that workers are in the wrong places; the skill story that they lack the right know how. At this point both stories have been thoroughly debunked."[13].

[13]Krugman, New York Times, June 8, 2012.

Altig reaches the same conclusion: "we have yet to find much evidence that problems with skill-mismatch are more important post-recession than they were pre-recession."[14]

1.5.5 The Returns to Education

Ex ante mismatch also has implications for the returns to education which, in the extended model, corresponds to the difference between the value of unemployment to college versus high school educated workers. Under the conventional, *ex post* mismatch, the observation of college workers employed in routine jobs appears to represent a failure of education. That is, such workers would appear to have ended up in jobs that they could have found without a college education.

In a mismatch equilibrium, however, the observation that college educated workers search for and are employed in routine jobs does not imply a failure of college education, as the option value of searching for routine jobs is positive and constitutes an important component of the value of labour force participation to these workers. In the absence of a college education, workers would have the set of choices that high school educated workers face, which is of substantially lower value than the value of the set of choices facing a college educated worker. Furthermore, the value of routine jobs to college workers differs from the value of such jobs to high school educated workers as it embeds the option of on-the-job search, which is of greater value to college educated workers. Finally, in a mismatch equilibrium, the adverse selection problem associated with on-the-job search from non-routine jobs means that the equilibrium exhibits an inefficiently high degree of equilibrium mismatch and, consequently, an inefficiently low return to education.

[14] Dave Altig, research director, Atlanta Fed, June 2012.

Chapter 2

Endogenous and exogenous mismatch in the labour market

2.1 Introduction

Chapter one has demonstrated a novel perspective that the return to education is the sum of the value of searching for jobs that make full use of a college education, plus the option value of searching for jobs with lower skill requirements. It also implies that the destruction of routine jobs, as observed in the past few decades, worsens the option value of routine jobs which helps explaining the adverse labour market outcomes of educated workers. However, the idea that routine jobs have value seems to apply intuitively to any economy as long as educated workers sometimes search for routine jobs. It is not clear whether the source of mismatch matters in terms of understanding the observed labour market trends. In Chapter 1, mismatch arises in the presence of both asymmetric information and limited commitment while the equilibrium with endogenous mismatch relies heavily on the adverse selection problem. A natural question then arises: could models with simpler explanations, for example some college graduates are low in ability to find non-routine jobs, be enough to explain the adverse labour market outcomes of educated workers?

In this chapter, I consider an economy where mismatch occurs exogenously. Educated workers search for routine jobs because some college graduates face high costs to find a well-matched job. I then investigate how a decline in the value of routine jobs would affect the labour market outcomes of educated workers under such an economy. While the model implies that the return to education falls under a negative shock to routine jobs, it fails to generate the adverse trends of falling employment rate with declining job-to-job transition rate simultaneously. Compared to Chapter 1, the two models respond distinctly in employment and job mobility and the reason lies in the incentives of workers to search for routine jobs in the first place. While workers trade-off for higher wage and lower job finding rates when searching for non-routine jobs instead of routine jobs when unemployed in Chapter 1, such incentives does not apply to models with exogenous mismatch. The comparison suggests that understanding the mechanism of mismatch is essential to study the labour market outcomes of educated workers.

The model follows Chapter 1 closely. College workers can choose to search for non-routine jobs or routine jobs. *Ex post* they are more likely to form a high productive match with a non-routine job than with a routine job. Both employed and unemployed workers could search for jobs. When employed workers receive an outside offer, their current employer could choose to make a counter offer.

Unlike the previous chapter, information about productivity is perfectly observable. Under perfect information, the value of searching for non-routine jobs is strictly higher than the value of searching for routine jobs. I assume that unemployed workers search for routine jobs because they face a non-negative cost when applying for non-routine jobs. The cost is a random draw for each unemployed worker, which could vary from family constraints to geographical limitations or simply that some workers are low in employable abilities even after completing college education. All workers can continue to search while employed.

I present the comparative static results of the model using numerical simulations. The aim of the simulation exercise is to examine whether a simpler model that assumes mismatch could also deliver the adverse labour market outcome while being consistent with other observed labour market trends. The previous chapter has argued that the destruction of routine jobs, driven by factors such as skill biased technological change, automation and off-shoring, reduces the labour market return to college workers. It also explains the adverse labour market outcomes including the fall in employment rate and labour force participation rate of educated workers, as well as the decline in job-to-job transitions. I consider the same negative shock of routine jobs in the simulation and check whether the model with exogenous mismatch could deliver the same results.

Under a negative shock in the routine job sector, the model generates a decline in the return to search for educated workers, suggesting that the incentive of workers to participate in the labour market is lower. However, the model fails to generate a decline in employment and a fall in job-to-job transitions simultaneously. In the scenario where the employment rate falls, the job-to-job transition rate increases, while in the case where the job-to-job transition rate falls, the employment rate rises. The negative shock of routine jobs affects the equilibrium allocation mainly through two effects. When the direct negative effect on the routine-job sector dominates, employment falls because employers post fewer vacancies while job-to-job transitions increase because a higher fraction of employed workers search on job. The shock also causes an equilibrium effect which leads to a higher fraction of workers to search for non-routine jobs when unemployed. When this effect is large, employment increases while the change in job-to-job transitions is ambiguous and depends on the level of uncertainty in matching quality. When workers direct their search to non-routine jobs instead of routine jobs, their likelihood to be employed increases. Overall, the two main effects together could not explain the fall in both employment and job mobility for educated workers.

Previous literature has studied the impact of skill-biased technological changes on models with exogenous mismatch. For example, Gautier (2002) and Albrecht and Vroman (2002) present random matching models in which workers search for jobs with heterogeneous skill requirements. While both of the papers suggest that wage inequality rises and employment falls under negative shocks to low skill jobs, these models do not incorporate on-the-job search and therefore cannot study the impact of the shocks on the job-to-job transitions. Dolado et al. (2009) studies a random matching model with heterogeneous jobs and on-the-job search. In their model, however, the job finding rate for unemployed workers and employed workers are assumed to be equal. Besides, the paper does not consider the comparative static under the negative shock of low skilled jobs. The literature of random matching does not provide an answer of how models with exogenous mismatch would respond to the declining trend of routine jobs.

Compared to the model in Chapter 1, one important difference between the two types of mismatch equilibria is how workers trade off job search for routine versus non-routine jobs. In Chapter 1, routine jobs have lower wages but are easier to find and have higher value of on-the-job search than non-routine jobs. In this Chapter, routine jobs are strictly worse than non-routine jobs in both wage and the job finding rate. When a negative shock of routine jobs directs workers to search more for non-routine jobs, the trade-off between the two types of jobs leads to a lower employment rate and lower job-to-job flow in a model with endogenous mismatch. On the contrary, when mismatch is assumed endogenously, switching to non-routine jobs makes workers better-off in both the extensive margin and intensive margin of their labour market outcomes. The fundamental difference in the reasons behind mismatch causes the difference in the trade-offs of applying for different jobs, which turn out to be important in understanding the adverse labour market outcomes of college workers.

The rest of the chapter is organized as follows: Section 2 lays out the model and characterizes the equilibrium. Section 3 carries out the simulation exercises. Section 4 compares the simulation results with the mismatch equilibrium in Chapter 1 (hereafter referred to as the endogenous mismatch equilibrium). Section 5 concludes.

2.2 Model and Equilibrium

2.2.1 The model

The model of this chapter follows closely the framework in the previous chapter. Time is discrete. Risk neutral agents discount future at a rate r. Consider a unit measure of college graduates for now. [1] A worker, either employed or unemployed, can apply for one of the two types of jobs with job type $i \in \{1, 2\}$. Following chapter 1, I consider type-1 jobs as non-routine jobs while type-2 jobs as routine jobs. The productivity of a worker-job match is realized only after a worker meets with an employer with a job offer. With probability α_i, the match is of high quality and the output per period is y_h; with probability $1 - \alpha_i$, the output is $y_l < y_h$. College workers are more productive in type-1 jobs given that $\alpha_1 > \alpha_2$.

The main departure from the previous chapter is that the productivity of a match is observable to all agents. Taking away asymmetric information eliminates the adverse selection problem, therefore the endogenous mismatch equilibrium. I assume that when workers are unemployed, they face a positive cost of searching for non-routine jobs. The cost is a random draw from an exponential distribution $F(c) = 1 - exp\{-\theta c\}$ i.i.d. across unemployed workers and over time. One can think of this cost as any exogenous force that makes searching for jobs that fully use their education costly. Examples include geographical constraints, constraints that prevent people from working full-time, lack of access to networks, etc. The cost could also reflect the ability of workers in the sense that some college graduates cannot fulfil the requirements of non-routine jobs after graduating from post-secondary education. Note that this assumption is the opposite of the previous chapter, where workers pay a non-negative cost to search for routine jobs. The assumption here is necessary to have some workers apply to routine jobs under perfect information. Without asymmetric information, the value of searching for routine jobs is unambiguously lower than searching for non-routine jobs for an

[1] The second type of worker, the high school graduates, is introduced in section 3.

unemployed worker. In fact, without any search cost, the model with perfect information would generate the identical equilibrium allocation as the wage revealing equilibrium under asymmetric information, as characterized in chapter 1.

The rest of the model follows chapter 1. A worker's state is denoted as $s \equiv \{i, \omega, y\}$, which contains the labour market information of a worker such as whether the worker is employed; if employed, the type of employer, the wage and the matching productivity. The state of an unemployed worker is $s_u = \{0, b, b\}$, where b is the flow income for unemployed workers. The labour market is structured with a continuum of markets. Each market is characterized by an employment contract x and a market queue length q. In each period, employers can post any feasible contract with cost k per period and workers (both unemployed and employed) direct their search across all feasible contracts. A contract $x = \{j, g^u_\omega, g^e_\omega\}$ specifies the job type j and wage policy $g^u_\omega(y')$ for unemployed workers and $g^u_\omega(y, y')$ for employed workers. Wages are offered contingent on the productivity realization of the new match y' and the current matching productivity of a worker y which is observable. A mapping $Q : X \to \mathbb{R}_+$ specifies the queue length, which equals the ratio of workers to job offers, associated with each contract. Matching is bilateral and the matching technology follows the assumptions as listed in Chapter 1. Let $f(q)$ denote the probability that a worker meets an employer when searching for contract x with market queue length $q = Q(x)$, and $qf(q)$ is then the probability that an employer meets a worker searching in the same market.

Once a worker and an employer meet, the worker decides whether to accept the contract. If a contract is accepted, an unemployed worker becomes employed and an employed worker switches to a new employer. Employed workers face an exogenous separation shock and become unemployed with probability δ. Once an employed worker receives an outside offer, her current employer choose to make a take-it-or-leave it counter offer. The value function of a worker in state s is as follows:

$$V(s) = \omega + \frac{\delta V(s_u)}{1+r} + (1-\delta)\left\{\frac{V(s)}{1+r} + \mathbb{E}_c\left\{\max_{x \in X} U(s, c, x, Q(x))|s\right\}\right\} \tag{2.1}$$

where

$$U(s, c, x, Q(x)) = \begin{cases} (2-j)c + f(Q(x))\mathbb{E}_{y'}\left\{\max\left\{0, \frac{V(s_o)}{1+r} - \frac{V(s)}{1+r}\right\}\right\} & \text{if } s = s_u \\ f(Q(x))\mathbb{E}_{y'}\left\{\max\left\{0, \frac{V(s_o)}{1+r} - \frac{V(s)}{1+r}, \frac{V(s_c)}{1+r} - \frac{V(s)}{1+r}\right\}\right\} & \text{if } s \neq s_u \end{cases}$$

where $\mathbb{E}_c$ and $\mathbb{E}_{y'}$ denote expectations with respect to c and y', respectively. The initial state is $s = \{i, \omega, y\}$, while $s_o = \{j, \omega_o, y'\}$ is the state of workers after searched offer.

The state $s_c = \{i, \omega_c, y\}$ is the state of the worker after accepting a counter offer. The worker anticipates the wage offer to be $\omega_o = g^u_\omega(y')$ when unemployed, $\omega_c = g^e(\omega, y, y')$ when employed, and $\omega_c = g^r(s, y', \omega_o)$ when anticipating a counter offer to ω_o. Notice that since match productivity is perfectly observable, wages offered to employed workers are conditional on the worker's current match productivity. Let g_x be the search policy of a worker, then

$$g_x(s,c) = \begin{cases} \arg\max_{x \in X} U(s, c, x, Q(x)) & \text{if } s = s_u \\ \arg\max_{x \in X} U(s, 0, x, Q(x)) & \text{if } s \neq s_u \end{cases} \tag{2.2}$$

A worker's acceptance policy g_a satisfies

$$g_a(s, s_o, \omega_c) = \begin{cases} \arg\max_{\alpha \in [0,1]} \{\alpha V(s_o) + (1-\alpha) V(s)\} & \text{if } s = s_u \\ \arg\max_{\alpha \in [0,1]} \{\alpha V(s_o) + (1-\alpha) \max\{V(s), V(s_c)\}\} & \text{if } s \neq s_u \end{cases} \tag{2.3}$$

where $g_a(s, s_0, \omega_c) = 1$ if the worker accepts the poaching offer and switches to a new employer. Now consider the present value of an employer matched with a worker in state s. The expected return for the employer $J(s)$ must satisfy

$$\begin{aligned} \frac{J(s)}{1+r} =& \frac{y-\omega}{r+\delta+(1-\delta)f(Q(g_x(s,0)))} + \left(1 - \frac{r+\delta}{r+\delta+(1-\delta)f(Q(g_x(s,0)))}\right) \\ & \times \mathbb{E}_{y'}\left\{\max_{\omega_c}\left\{(1-g_a(s,s_o,\omega_c))\frac{J(s)}{1+r}\right\}\right\} \end{aligned} \tag{2.4}$$

Finally, the expected return of posting a contract x to an employer is:

$$-k + Q(x) f(Q(x)) \mathbb{E}_{y'}\left\{\max\left\{0, g_a(s, s_o, g_r(s, j, \omega_o))\frac{J(s_o)}{1+r}\right\}\right\} \tag{2.5}$$

where k is the cost to maintain a vacancy on the market for each period. Let g_r be the retention policy of an employer, then

$$g_r(s, y', \omega_o) = \arg\max_{\omega_c \in [\omega, y_h]} \left\{(1 - g_a(s, s_o, \omega_c)\frac{J(s_c)}{1+r}\right\} \tag{2.6}$$

With no asymmetric information, the definition of a stationary competitive search equilibrium is standard following the literature.

Definition 6. *A stationary competitive search equilibrium consists of a set of posted contracts $X^* \in X$, value functions V, J and policy functions g_x, g_a, g_r, a mapping Q and a distribution ϕ of workers in different states, such that:*

(A) Optimality for workers: V satisfies (2.1); g_a satisfies (3.7); g_x satisfies (2.2) and $g_x(s,c) \in X^*$ for all $s \in S$.

(B) Optimality for employers: J satisfies (2.4) and g_r satisfies (2.6); for any $x \in X^*$

$$Q(x)f(Q(x))\mathbb{E}_{y'}\left\{\max\left\{0, g_a(s, s_o, g_r(s, j, \omega_o))\frac{J(s_o)}{1+r}\right\}\right\} = k \tag{2.7}$$

(C) Aggregate consistency conditions are satisfied.

The aggregate consistency condition guarantees that for each state, the inflow of workers equals the outflow. Instead of providing a general condition, I list the aggregate consistency conditions for the equilibrium I characterize in the next subsection.

2.2.2 Characterization of Equilibrium

As demonstrated in Chapter 1, the allocation of a competitive search equilibrium can be characterized by solving a set of optimization problems. Since employers can make counter offers and poachers observe perfectly the match productivity of an employed worker, no employers poach employed workers with productivity y_h. In equilibrium only workers with matching realization y_l search on job. Following Chapter 1, the on-the-job search problem of an employed worker with state $s = \{i, \omega, y_l\}$ is:

$$V(s) = \omega + \frac{\delta V(s_u)}{1+r} + (1-\delta)\left\{\frac{V(s)}{1+r} + U(s)\right\} \tag{2.8}$$

where

$$U(s) = \max_{\omega', q'}\left\{f(q')\alpha_1\left[\max\left\{0, \frac{\omega'}{1+r} + \frac{\delta}{r+\delta}\frac{V(s_u)}{1+r} - \frac{V(s)}{1+r}\right\}\right]\right\} \tag{2.9}$$

subject to

$$k \leq q' f(q')\alpha_1\left(\frac{y_h - \omega'}{r+\delta}\right) \tag{2.10}$$

$$\omega' \leq y_l \tag{2.11}$$

Denote $\{q^e(s), \omega^e(s)\}$ the set of solutions to the above question. Let $V(s)$ be the value of employment for workers with productivity y_h and state $s = \{i, \omega, y_h\}$ is

$$V(s) = \omega + \frac{\delta V(s_u)}{1+r} + (1-\delta)\frac{V(s)}{1+r} \tag{2.12}$$

The value of an unemployed worker satisfies:

$$V(s_u) = b + \mathbb{E}_c \{\max\{V_1 - c, V_2\}\} \tag{2.13}$$

where

$$V_i = \frac{V(s_u)}{1+r} + \max_{\omega_l,\omega_h,q} \left\{ f(q) \left[\alpha_i \frac{V(\{i,\omega_h,y_h\})}{1+r} + (1-\alpha_i) \frac{V(\{i,\omega_l,y_l\})}{1+r} - \frac{V(s_u)}{1+r} \right] \right\} \tag{2.14}$$

subject to

$$k \le q f(q) \left[\alpha_i \left(\frac{y_h - \omega_h}{r+\delta} \right) + (1-\alpha) \frac{y_l - \omega_l}{r + \delta + \alpha_1 f(q^e(\{i,\omega_l,y_l\}))} \right] \tag{2.15}$$

$$\omega_l \le y_l; \;\; \omega_h \le y_h \tag{2.16}$$

Let $\{\omega_1(y_h), \omega_1(y_l), \omega_2(y_h), \omega_2(y_l), \omega^e, q_1, q_2, q^e, c_0\}$ be the equilibrium allocation, characterized by the following equations:

$$\omega_1(y_l) = \omega_2(y_l) = y_l \tag{2.17}$$

$$q^e f(q^e) \alpha_1 \left(\frac{y_h - \omega^e}{r+\delta} \right) = k \tag{2.18}$$

$$\frac{y_h - y_l}{r+\delta} = \left(\frac{k}{q^e f(q^e)} \alpha_1 \right) \left[1 + \left(\frac{1-\eta(q^e))}{\eta(q^e)} \right) \left(\frac{r + \delta + (1-\delta)\alpha_1 f(q^e)}{r+\delta} \right) \right] \tag{2.19}$$

$$c_0 = \frac{1-\eta(q_1)}{\eta(q_1)} \frac{k}{q_1} - \frac{1-\eta(q_2)}{\eta(q_2)} \frac{k}{q_2} \tag{2.20}$$

where $v(c_0) \equiv F(c_0)[c_0 - \mathbb{E}(c|c \le c_0)] = c_0 - \frac{1}{\lambda}F(c_0)$. For $j = \{1,2\}$

$$\omega_j(y_h) = y_h - \frac{k(r+\delta)}{\alpha_j q_j f(q_j)} \tag{2.21}$$

$$\frac{y_h - b}{r+\delta} - \frac{(1-\alpha_j)k}{\eta(q_b) q_b f(q_b) \alpha_1} = \frac{v(c_0)}{r+\delta} + \frac{k}{\eta(q_j) q_j f(q_j)} + \frac{1-\eta(q_2)}{\eta(q_2)} \frac{k}{(r+\delta) q_2} \tag{2.22}$$

The equilibrium allocation has properties similar to the efficient equilibrium discussed in Chapter 1, where wages reveal the productivity of workers. In the equilibrium, contracts offered to an unemployed worker pay the marginal product if the productivity realization is low, regardless of the job type. These workers change jobs with the identical job finding rate and receive the same poaching wage. The value of on-the-job search is identical for them.

Workers with high productivity realization earn different wages when matched with different types of employers. Unemployed workers also have different job finding rates when applying for routine jobs and non-routine jobs.

Without asymmetric information, non-routine jobs have higher value than routine jobs for unemployed workers. To see this: the option value of on-the-job search is identical regardless of a worker's current employer type. Non-routine jobs have higher ex ante match surplus than routine jobs. Given free entry of firms, workers search for routine jobs only if their realization of search costs is high such that it exceeds the gain from searching for non-routine jobs.

Aggregate Consistency condition

The equilibrium transitions are as follows: unemployed workers search for type-1 jobs if the realized cost is smaller than the cut-off cost c_0, which is determined endogenously. Otherwise they search for type-2 jobs. Once an unemployed worker receives an offer, she accepts the offer and searches on-the-job only if the productivity realization is y_l. When searching on-the-job, workers search for type-1 jobs only and switch jobs only if the productivity realization with the poacher is high. All employed workers might become unemployed in any period with probability δ. It is now straight forward to characterize the aggregate consistency conditions for different states of workers in equilibrium. In the steady state, the inflow and outflow of each state of workers are equal. The aggregate state of economy ψ is characterized as follows:

$$\psi(s_u) = \frac{\delta\pi}{\delta + [1 - F(c_0)]f(q_2) + F(c_0)f(q_1)} \tag{2.23}$$

$$\psi(\{1, y_l, y_l\}) = \frac{F(c_0)(1-\alpha_1)f(q_1)}{\delta + (1-\delta)\alpha_1 f(q^e)}\psi(s_u) \tag{2.24}$$

$$\psi(\{1, \omega_1(y_h), y_h\}) = \frac{F(c_0)\alpha_1 f(q_1)}{\delta}\psi(s_u) \tag{2.25}$$

$$\psi(\{2, y_l, y_l\}) = \frac{[1 - F(c_0)](1-\alpha_2)f(q_2)}{\delta + (1-\delta)\alpha_1 f(q^e)}\psi(s_u) \tag{2.26}$$

$$\psi(\{2, \omega_2(y_h), y_h\}) = \frac{[1 - F(c_0)]\alpha_2 f(q_2)}{\delta}\psi(s_u) \tag{2.27}$$

$$\psi(\{1,\omega^e,y_h\}) = \frac{(1-\delta)\alpha_1 f(q^e)}{\delta}(\psi(\{2,y_l,y_l\}) + \psi(\{2,y_l,y_l\})) \tag{2.28}$$

Given the fixed rate of exogenous job destruction, the level of employment is determined by the fraction of unemployed workers who search for routine jobs, as well as the job finding rates of searching for both types of jobs when unemployed. The flow of job-to-job transitions is affected by the equilibrium variables that determine the measure of low productive workers, as well as the equilibrium quit rate of these workers.

2.3 Numerical Simulation

In this section, I present the results of some simulations. The purpose of these exercises is to investigate whether a simple model of mismatch with no adverse selection can explain the observed labour market trends in the U.S., which include the declines in employment and participation rates of educated workers, the rise in wage inequality and the fall in job-to-job transitions.

I focus on how the equilibrium with exogenous mismatch change under a negative shock on routine job sector. The intuition from Chapter 1 suggests that a decline in the value of routine jobs, driven by the skill-biased technological change like automation and off-shoring, reduces the option value of routine jobs and therefore lowers the value of labour force participation to college educated workers. Moreover, workers are more likely to search for non-routine jobs when unemployed which causes a decline in their employment rates as well as a fall in job-to-job transitions. While none of the above intuitions seem to require the mechanism of adverse selection, I show in this section that the model with exogenous mismatch fails to capture some of the trends mentioned above. I compare the difference between the two types of models in the next section.

2.3.1 Baseline Economy

The exogenous variables of the baseline economy are listed in Table 2.1. Time is measured in quarters. The productivity of an ex-post low match y_l is normalized to 1. The flow cost of posting vacancies is set at 0.1, while flow income of unemployment is also set at 0.1. The job destruction rate is 0.1 per quarter, which means on average a match lasts for two and a half years. The discount factor r is set at 0.02, which is equivalent to a discount rate of 0.98. The exogenous cost for searching for type-1 jobs follows an Exponential distribution with parameter λ, where λ is set to 1. The matching function is assumed to be $f(u, v) = \dfrac{uv}{u+v}$, which satisfies all the assumptions on the match technology indicated in Chapter 1.

Table 2.1: Parameter Values

Variables	y_l	α_1	α_2	k	δ	b	r	λ
Value	1	0.4	0.35	0.1	0.1	0.1	0.02	1

Table 2.2 presents the value of the equilibrium allocation variables under the baseline economy. As discussed earlier, non-routine jobs dominate routine jobs in terms of expected return to search. In particular, non-routine jobs offer a higher wage and have a high job finding rate for unemployed workers, while the values of on-the-job search are equal given that the job turnover rate and the wages received after job switches are identical. Workers apply to routine jobs only when searching for non-routine jobs are costly.

Table 2.2: Baseline Economy

Variables	$f(q_1)$	$f(q_2)$	$f(q^e)$	$\omega_1(y_h)$	$\omega_2(y_h)$	ω_1^e	$\omega_1(y_l) = \omega_2(y_l)$	c_0
Value	0.739	0.736	0.356	0.985	0.970	1.053	1	0.002

Now consider a decline in α_2 as the negative change to routine jobs. The decline in α_2 reduces the expected productivity in type 2 jobs, without complicating the job-to-job transition problem by adding more potential job ladders. I consider three scenarios with different levels of y_h ($y_h \in \{1.1, 2, 4\}$). The reason for different productivity levels is because a negative shock on routine jobs affects the equilibrium variables not only through a direct effect on the return of routine jobs, but also through an equilibrium effect that changes the search behavior of workers. Non-monotonic results arises when the two effects affect the aggregate variables in the opposite directions. A higher level of y_h amplifies the equilibrium effects while with a lower level of y_h, the direct effects dominate. As I will discuss later, in this chapter that some labour market outcomes of the model behave quite differently at different levels of y_h.

The following subsections present separately the results of the simulation for employment, wages inequalities and job flows.

2.3.2 Employment

The employment rate and labour force participation rate of college graduates have declined over the last 15 years. Meanwhile, the share of routine employment among all employment of college workers has also dropped.[2] Table 2.3 presents the simulation results of the model on the employment rate, the share of routine employment among employed workers and the value of participating in the labour market under a negative productivity change of routine jobs.

Consider first the employment rate. Since the labour force participation decision is not modelled explicitly, all workers search actively when unemployed. Therefore, the employment rate in the model, defined as the ratio of employment to population, captures also the employment changes with respect to the labour force. As α_2 declines, the employment rate of workers declines under scenario one, where y_h equals to 1.1. The employment rate increases in scenarios two and three, where the productivity difference between the two types of jobs is larger ($y_h = 2, 4$).

The employment rate of workers is determined jointly by the job finding rates of unemployed workers searching for type-1 jobs and type-2 jobs, as well as the fraction of workers searching for type-2 jobs when unemployed. A negative shock on the routine-job sector lowers the job finding rate in the sector directly. It also directs unemployed workers more likely to search for non-routine jobs, where the job finding rate is higher. The negative shock also generates an equilibrium effect which increases the job finding rate in the non-routine-job sector. This is because when the value of being unemployed falls as the result of the negative shock, workers adjust their search and move out of unemployment faster by looking for jobs that offer lower wages. In scenario one, the match surplus of the two types of jobs are not very different, the first effect dominates and the employment rate declines as type 2 jobs become less productive, which reduces the incentives for employers to create these jobs. In scenarios two and three, however, a decline of α_2 raises the productivity gap between the two types of jobs more effectively. As a result, workers are more likely to search for non-routine jobs. Given that the job finding rate is higher for non-routine jobs and a decline in α_2 increases the job finding rate in non-routine jobs, overall employment increases as more workers searching for non-routine jobs when unemployed.

With a negative shock of the routine-job sector, the share of routine employment over

[2]See Figure 1.2.

Table 2.3: Employment

Variables		$\alpha_2 = 0.35$	$\alpha_2 = 0.25$	$\alpha_2 = 0.15$	Sign of change
Employment rate	$y_h = 1.1$	88.04%	88.04%	88.03%	decline
	$y_h = 2$	88.74%	88.75%	88.77%	increase
	$y_h = 4$	89.38%	89.42%	89.45%	increase
Routine employment share	$y_h = 1.1$	62.07%	54.07 %	46.62 %	decline
	$y_h = 2$	46.60%	30.86 %	19.84%	decline
	$y_h = 4$	36.23 %	15.26%	6.25%	decline
Return to unemployment	$y_h = 1.1$	44.90	44.59	44.30	decline
	$y_h = 2$	75.53	74.20	73.18	decline
	$y_h = 4$	147.55	145.32	144.23	decline

total employment decreases in all scenarios and the magnitude of the decline (as percentage) increases when y_h is higher. On the one hand, routine jobs become more difficult to get since the reduced surplus lowers employers' incentives to create routine jobs. On the other hand, a smaller share of unemployed workers search for routine jobs when unemployed. As a result, the share of routine employment amongst the total employment always declines, regardless of how total employment changes. The fall in the share of routine employment is quite significant, as compared to the magnitude of changes in employment. In scenario three, for example, the share of routine employment declines from 36 percent to 6 percent, and the fraction of unemployed workers searching for non-routine jobs increases from 29 percent to 82 percent.

The value of unemployment, which is the sum of the discounted value of all future incomes for an unemployed worker, taking into account both wages and job finding rates. The return to unemployment declines as α_2 decreases in all scenarios. The negative shock of routine jobs reduces the value of searching for routine jobs, which is part of the value for workers to participate in the labour market. As modelled in Chapter 1, a fall in the return to unemployment could transfer easily into a fall in the labour force participate rate of workers. In an economy where workers have heterogeneous reservation values to participate in the labour market. A decline in the expected return to search for jobs would lead to fewer people entering the labour market and lower the labour force participation rate.

The fall in the return to unemployment helps understand the effect of the shock on the non-routine job sector. While the expected productivity of non-routine jobs remains unchanged, workers' optimal search behaviour changes. A fall in the expected return to unemployment lowers the option value of search again in the next period if a worker fails to find an offer. Consequently, workers have less incentive to remain unemployed and search towards job posts that offer lower wages but have higher probabilities to meet with an employer. Although the expected return to participating in the labour market declines, the job finding rate of searching for non-routine jobs increases.

2.3.3 Wage

The next objective is to examine how the model responds to the negative shock in term of wage changes. Table 2.4 below lists the comparative statics of a negative change in α_2 for wages and wage inequalities. The first variable is the mean wage of college workers. The second variable is the wage premium of non-routine jobs, measured by the relative ratio between the mean wage of college workers employed in non-routine jobs versus the mean wage of college workers employed in routine jobs. I also examine whether the model captures the increase in skill-wage premium over the last two decades. For this purpose, I introduce one more type of worker: the high school graduate. The third variable in Table 2.4 is the skill wage premium, which is measured by the ratio between the mean wage earned by college graduates and the mean wage earned by high school graduates.

Table 2.4: Wage

Variables		$\alpha_2 = 0.35$	$\alpha_2 = 0.25$	$\alpha_2 = 0.15$	Sign of change
Mean wage of college graduates	$y_h = 1.1$	1.009	1.003	0.997	decline
	$y_h = 2$	1.714	1.694	1.683	decline
	$y_h = 4$	3.363	3.346	3.352	non-monotonic
non-routine over routine wage gap for college graduates	$y_h = 1.1$	1.07	1.09	1.11	increase
	$y_h = 2$	1.19	1.27	1.43	increase
	$y_h = 4$	1.25	1.37	1.67	increase
Skill wage premium	$y_h = 1.1$	1.00	1.01	1.03	increase
	$y_h = 2$	1.01	1.07	1.20	increase
	$y_h = 4$	1.02	1.11	1.32	increase

When α_2 declines, the mean wage of college graduates declines in the first two scenarios. In the third scenario, mean wage first decreases then increases while α_2 decreases. The magnitude of changes are small in all cases (smaller than 2%). In all scenarios, both the mean wage for workers employed in routine jobs and the mean wage for workers employed in non-routine jobs decline. The reasons, however, are different. A negative shock of routine jobs lowers the mean wage of workers employed in routine sectors directly because the match surplus of these jobs are lower and employers offer lower wages to compensate. The shock also reduces the expected entry wage in the non-routine-job sector indirectly through effect of a lower expected return of unemployment. However, as α_2 fall, more workers apply to non-routine jobs when unemployed since the expected wage is higher than searching for routine jobs. This effect increases the mean wage. In the first two scenarios, the direct effect on routine jobs dominates and the average wage increases for college workers. In scenario three, the effect of switching job search of unemployed workers dominates as α_2 becomes smaller, leads to the ambiguous total effect on the changes of mean wage.

Although the mean wages of both job sectors decline following the shock on α_2, the magnitude is larger in the routine-job sector than in the non-routine-job sector. The second variable in Table 2.4 presents the relative ratio of average wage received by college graduates in non-routine job-sector versus in routine-job sector. In all scenarios, this within group wage difference increases as routine jobs become less productive. It is worth noticing that the equilibrium wage distribution in this model is special. Workers with ex-post productivity realization equal to y_l always receive their marginal productivity. The equilibrium poaching wages, determined by the current wage of employed workers and the matching surplus of new jobs, are not affected by the changes of α_2. Changes of mean wage in each job sector are caused only by the changes in the entry wage of unemployed workers who's productivity realization is y_h. It is intuitive to see why the wage gap between the two sectors increases because decline in α_2 reduces the entry wage in the routine-job sector directly, while it only affects the entry wage in the non-routine-job sector through the general equilibrium effect on the return to unemployment.

Skill-wage premium: Introducing high school graduates

To see if the model captures the increase of college/high school wage premium, I now introduce another type of worker: high school graduates. Assume for simplicity that high school graduates differ from college graduates only in that they cannot perform type-1 jobs. [3] High school workers are equally productive in routine jobs as college graduates: a high school worker produces y_h with probability α_2 and produces y_l with probability $1 - \alpha_2$. An employed high school worker can also search on the job, but they search for routine jobs only. The problem of high school workers are listed in the appendix.

Notice that the problems of high school workers is studied separately from the problems of college workers. One reason it that employers could make offers conditional a worker's

[3] The fraction of high school graduates in non-routine cognitive jobs is small in the data.

observable type, so that the two types of workers do not compete for the same jobs. Even if employers make wage offers unconditional on the observable type, high school and college graduates have incentives to search in different submarkets of routine jobs and their optimal job-search decisions are not affected by each other. This is because search is directed, meaning that all agents take into account the trade-off between wages and job finding rates. This implies workers with different outside options (including different on-the-job search options) would prefer to apply for different contracts. While high school graduates and some college graduates search for type-2 jobs when unemployed, they do not compete for the same contract. For each contract posted with a routine job, employers anticipate only one type of workers to apply. The following equations characterize the equilibrium allocation $\{\omega(y_h), \omega(y_l), \omega_h^e, q, q_h^e\}$ of high school workers:

$$\omega(y_l) = y_l \tag{2.29}$$

$$q_h^e f(q_h^e)\alpha_2 \left(\frac{y_h - \omega_h^e}{r+\delta}\right) = k \tag{2.30}$$

$$\frac{y_h - y_l}{r+\delta} = \left(\frac{k}{q_h^e f\left(q_h^e\right)\alpha_2}\right)\left(1 + \left(\frac{1-\eta\left(q_h^e\right)}{\eta\left(q_h^e\right)}\right)\left(\frac{r+\delta+(1-\delta)\,\alpha_1 f\left(q_h^e\right)}{r+\delta}\right)\right) \tag{2.31}$$

$$\frac{y_h - b}{r+\delta} - \frac{(1-\alpha_2)\,k}{\eta\left(q_h^e\right) q_h^e f\left(q_h^e\right)\alpha_2} = \frac{k}{\eta(q)\, q f(q)} + \left(\frac{1-\eta(q)}{\eta(q)}\right)\frac{k}{(r+\delta)\, q} \tag{2.32}$$

Since high school workers can search for routine jobs only, a decline in α_2 lowers the surplus of matches when workers search out of unemployment, it also lowers the return to on-the-job search. Unlike college graduates, high school workers cannot adjust to the shock by searching for non-routine jobs. Therefore, a negative shock of routine jobs affects high school graduates more steeply than college graduates.

The third variable in Table 2.4 is the skill-wage premium. It is measured by the ratio of the mean wage of college graduates over the mean wage of high school graduates. The skill-wage premium rises in all scenarios as the shock on routine jobs hurts high school graduates deeper than college graduates. When there is a negative shock to the productivity of routine jobs, both the expected entry wage and the poaching wage of high school workers fall unambiguously. Although the mean wage of college graduates also declines under some scenarios, the impact of the shock is smaller than on high school workers because the share of routine employment declines once these jobs become less profitable.

2.3.4 Job flows

The last part of the simulation exercises focuses on the labour market dynamics, in particularly the job mobility of college graduates. Many have documented a fall in job-to-job mobility since the mid 90s.[4] Table 2.5 lists the simulated results on the changes of job-to-job transitions of college graduates following a negative shock on routine jobs.

Table 2.5: Job to Job Flows

Variables		$\alpha_2 = 0.35$	$\alpha_2 = 0.25$	$\alpha_2 = 0.15$	Sign of change
Job-to-job flow	$y_h = 1.1$	0.0096	0.0110	0.0122	increase
	$y_h = 2$	0.0124	0.0135	0.0140	increase
	$y_h = 4$	0.0127	0.0131	0.0129	non-monotonic
Job-to-job flow rate	$y_h = 1.1$	3.65%	4.16 %	4.63 %	increase
	$y_h = 2$	4.65%	5.06 %	5.26%	increase
	$y_h = 4$	4.75%	4.88 %	4.81 %	non-monotonic

Job-to-job transitions in the model are "efficient" in the sense that matching productivities always increase when workers switch employers, within or across job sectors. Another feature is that changes in α_2 do not alter the quit rate of employed workers. In equilibrium, only workers whose current matching productivity is y_l search on the job. These workers receive wage y_l and they search for non-routine jobs only. A change in α_2 does not affect either the current wage of these workers nor the match surpluses of their future jobs, therefore has no impact on the optimal on-the-job search decisions. Nevertheless, a negative shock on routine jobs affects the job-to-job flow of college workers by changing the pool of job switchers through the inflows from unemployment. A fall in α_2 changes the measure of inflow from unemployment to employment, as well as the share among the inflow of workers whose productivity realizations are low and will therefore continue to search on job.

As shown in Table 2.5, both the measure of workers who switch jobs (job-to-job flow) and the job-to-job flow rate, defined as the measure of job-to-job flow over employment, follow similar patterns. In scenarios one and two, job-to-job transitions increases. In scenario three, as α_2 decreases, the job-to-job flow first increases then decreases. To understand this pattern, I look separately the stock of ex-post low productive workers in the non-routine-job sector($\psi(\{1, y_l, y_l\})$ and in the routine-job sector($\psi(\{2, y_l, y_l\})$. The measure of the first

[4]See (Davis et al., 2012) and Moscarini and Vella (2002)

group always increases as α_2 decreases. Since not only a larger fraction of workers search for non-routine jobs, the likelihood to find these jobs also increases, while the probability of be ex-post low productive remains unchanged. However, the measure of workers employed in routine jobs with productivity y_l changes non-monotonically. As α_2 declines, workers are more likely to have low matching productivity conditional on meeting with a routine job employer, even though the total inflow into routine employment declines. The second effect is larger with a high level of y_h, leads to a non-monotonic change in the overall job-to-job transition rate in scenario three.

Simulation summary

Under a negative change in the routine-job sector, the model generates the increased wage inequalities, both within college workers and between educated and less educated groups. The model also suggests that the return to search declines, implying a reduced incentive for workers to participate in the labour market. However, the model fails to replicate the changes in employment and job mobility at the same time. With a negative shock of routine jobs, the model is able to generate a decline in employment rate when the productivity difference between routine and non-routine jobs is small. However, the job-to-job transition rate increases rather than decreases as the observed trend. When the productivity difference between the two jobs is relatively large, as α_2 goes down, the model response of employment is to the opposite direction of the data observation. It implies that the employment rate increases for college graduates, together with an ambiguous effect on the job-to-job transition rate. Figures that summarize the changes of employment and job-to-job flows are presented in the appendix.

Two effects determine the changes of employment and the changes of job-to-job transition under the negative shock. One is the direct impact of the shock on routine jobs, including the decreases in wage and job finding rate in the routine-job sector and the increase in the fraction of mismatched workers among workers who are hired out of unemployment. The second effect is the equilibrium effect that directs unemployed workers to search towards non-routine jobs instead of routine jobs. This effect increases the job finding rate while reduce the share of mismatched workers. The shock also affects the equilibrium allocations in the non-routine-job sector. But this effect is quantitatively dominated by the other two effects mentioned above, in determining the aggregate employment and job-to-job transition.

In scenario one where the productivity difference between y_h and y_l is small, the first effect dominates. Even though the inflow of workers into employment decreases, job-to-job transition rate increases because the share of mismatched workers also increases. In the other two scenarios, the second effect dominates and the employment rate increases since more workers are searching for jobs that are easier to find. The job-to-job flow is determined jointly by both the size of the inflow into employment, and the fraction of mismatched workers. On the one hand the increase of employment enlarges the potential job switchers, on the other hand more workers searching for non-routine jobs means fewer workers end up mismatched. In either case, the model cannot generate declines in both employment and job-to-job flows.

In general, a simple model with exogenous mismatch fails to capture the adverse labour market outcomes of educated workers: it cannot generate the expected changes for both the stock (employment rate) and the flow (job-to-job transition) of employment at the same time.

2.4 Compared to the Endogenous Mismatch Equilibrium

In this section, I present a similar numerical simulation on an equilibrium with endogenous mismatch. Comparing the results of both endogenous and exogenous mismatch equilibria, I show that the reason why workers search for inferior jobs is the key to understanding the adverse labour market outcomes of educated workers.

I construct an equilibrium with endogenous mismatch from Chapter 1 as follows: college workers search for both routine and non-routine jobs when unemployment. In the routine-job markets, equilibrium wages offered to unemployed workers reveal their productivity realizations. An unemployed worker continues to search after receiving an offer from a routine job employer only if the productivity realization is low. In the non-routine-job market, wages do not differ across productivity realizations and all workers who are employed out of employment search on the job. The equilibrium has a "separating" feature (revealing equilibrium) in the routine-job markets and a "pooling" feature (non-revealing equilibrium) in the non-routine-job sector, meaning that poachers face adverse selection problem only when searching for workers currently employed in non-routine jobs. As a result, the value of on-the-job search is higher for workers employed in routine jobs than in non-routine jobs.

Simulation

I use the same benchmark parameters as in Table 2.1. The only difference is that the cost of search applies to non-routine jobs instead of routine jobs. Table 2.6 presents the equilibrium allocations for the baseline economy. Compared with non-routine jobs, routine jobs are easier to find when workers search out of unemployment. They also pay a lower expected wage and have a higher endogenous job turnover rate through job-to-job transitions. Moreover, the poaching wage for workers employed in routine jobs is higher than the poaching/retention wage of workers who search out of employment with non-routine jobs. The return to on-the-job search is higher for workers employed in routine jobs since the job switching rates are higher and workers receive higher wages after job transitions.

Comparing the equilibrium allocations, one can see the fundamental differences between the two types of equilibria. In terms of matching surplus, routine jobs are unambiguously worse than non-routine jobs. However, routine jobs have higher value for unemployed workers in the endogenous mismatch equilibrium because job mobility is higher. For unemployed workers, routine jobs are easier to find than non-routine jobs. Once employed, workers are more likely to obtain a wage increase. The high mobility compensates the lower wages offered with routine jobs and leads to high value in routine jobs unemployed workers. In the equilibrium with exogenous mismatch, however, routine jobs have lower job finding rates while paying lower wages. Workers strictly prefer to search for non-routine jobs if the search cost is not taking into account.

Table 2.6: Baseline Economy- Endogenous mismatch

Variables	$f(q_1)$	$f(q_2)$	$f(q_1^e)$	$f(q_2^e)$	ω_1	$\omega_2(y_h)$	$\omega_2(y_l)$	ω_1^e	ω_2^e	c_0
Value	0.732	0.74	0.28	0.36	0.98	0.97	1	1.03	1.05	0.028

Constructing an endogenous mismatch equilibrium is restricted, in the sense that it does not exist under a broad range of parameter values. It is unfortunate because I cannot present the full simulation exercises under three scenario as in the last section. Instead, I present the numerical exercise under $y_h = 1.1$, the case where the difference between productivity realizations is the smallest, with drop of α_2 from 0.35 to 0.3. Because the change of α_2 is smaller relative to the previous exercise, the changes of the equilibrium variables are small in magnitude. [5] However, I focus on the sign of changes and find that equilibrium with endoge-

[5] In general, the effect of changing α on the overall economy is small when the productivity realizations are similar. See Chapter 1 for a simulation with relative larger effects.

nous mismatch responses quite differently to the shock, comparing with the previous section .

Table 2.7 lists the equilibrium allocation of the mismatch economy with a lower α_2. Compare with the baseline economy, job finding rates for unemployed workers decline in the routine-job sector (direct effect through the negative shock) and increase slightly in the non-routine-job sector (indirect effect through the decline of return to unemployment). Wages offered to unemployed workers decline in both job sectors, while the magnitude is much smaller in the non-routine-job sector. The job finding rates increase slightly for employed workers with non-routine jobs, and remain constant for workers employed in routine jobs.

Table 2.7: Mismatch Equilibrium with a lower α_2

Variables	$f(q_1)$	$f(q_2)$	$f(q_1^e)$	$f(q_2^e)$	ω_1	$\omega_2(y_h)$	$\omega_2(y_l)$	ω_1^e	ω_2^e	c_0
Value	0.732	0.737	0.280	0.356	0.981	0.948	1	1.031	1.053	0.006

Table 2.8 presents the comparative statics of the model under a decline in α_2. With a negative shock of routine jobs, the model shows a decline in the share of routine employment for college workers as well as a fall in the value of return to search for unemployed workers. As discussed in the previous section, a fall in the productivity of routine jobs lowers the option value of these jobs and directs workers to search for non-routine jobs instead. The shock affects high school workers and college workers asymmetrically and raises the skill wage premium. It also increases wage inequality between college workers employed in different job sectors. All these effects are the same as in the model with exogenous mismatched presented in the previous section.

The differences between the simulation results of the two types of models lie in the joint movement of employment rates and job-to-job transition rates. As shown in Table 2.8, when α_2 decreases, employment rate of college workers falls, while the job-to-job flow, measure both as the ratio of employment and the size of the flow decrease. In the endogenous mismatch equilibrium, workers trade off higher wages with lower job finding rates as well as low job-to-job transition rates, if they search for non-routine jobs instead of routine jobs when unemployed. A negative change in the routine-job sector not only lowers the job arrival rate within the sector but also direct workers searching for jobs that are harder to find, therefore reduce the overall flow from unemployment to employment. Both the equilibrium effect on directing job search towards non-routine jobs and the direct negative effect on routine jobs reduce employment. When workers search for non-routine jobs instead of routine jobs, the

Table 2.8: Comparative Statics - Endogenous Mismatch

Variables	$\alpha_2 = 0.35$	$\alpha_2 = 0.30$	Sign of change
Employment rate	26.40%	26.39%	decline
Routine employment share	1.78%	0.36%	decline
Return to unemployment	44.70	44.69	decline
Mean wage of college graduates	1.006	1.006	decline
non-routine over routine wage gap (college)	1.024	1.032	increase
Skill wage premium	1.000	1.008	increase
Job-to-job flow	0.008	0.008	decline
Job-to-job flow rate	3.03 %	3.02 %	decline

job-to-job transition rate also falls because the return to on-the-job search is lower in non-routine-job sector.

In contrast, in the economy where mismatch was caused by exogenous cost, workers face no such trade off. Workers are strictly better off, with higher wage and higher job finding rate, when searching for non-routine jobs instead of routine jobs. The direct effect and the equilibrium effect have opposite impacts on the changes of employment as well as on the changes of job-to-job transition. When the direct effect dominates, the job-to-job flow increases because a higher fraction of employed workers search on job. When the effect of switching jobs dominates, the employment rate responds in the wrong direction since workers gain from a higher job finding rate. The exogenous mismatch model cannot generate a decline in both employment and job-to-job transition.

How workers trade-off wages with job finding probabilities when deciding which type of jobs to search, is important in determining the level of employment and job mobility. It is the main difference between the two types of mismatch models, which links to the fundamental research question asked in Chapter 1, "why do workers search for mismatched jobs in the first place? "

2.5 Conclusion

This chapter follows the implications from Chapter 1 and examines whether a simple model with exogenous mismatch can also explain the adverse labour market outcomes of educated workers. Under a negative shock to routine jobs, the model generates a fall in the return to education. However, it fails to generate a fall in employment rate together with a decline in job-to-job transition rate. Compare with Chapter 1, equilibrium with exogenous mismatch does not have the trade-off between job finding rates and wages when workers search for different jobs. The comparison suggests that whether mismatch arises endogenously or exogenously, is essential to understand the labour market outcomes of educated workers.

Chapter 3

Adverse selection in the labour market and the demand for vocational education

3.1 Introduction

There is a common perception that high school graduates are being displaced from jobs involving mainly routine tasks. These jobs are increasingly automated, offshored, or performed by over-qualified workers, that is, workers whose qualifications exceed job requirements. The roles of skill-biased technological change and globalization in the determination of labour market conditions faced by less educated workers are widely recognized,[1] but the role of over-qualification remains unclear. Many studies assume that educated and uneducated workers compete for the same scarce jobs.[2] Yet, firm-level data does not support the view that over-qualified workers directly crowd out less educated workers.[3] Moreover, over-qualified workers are likely to use their current job as a stepping stone towards better jobs. In fact, more than 30 percent of overeducated workers switch into matched jobs within a year.[4] But if over-qualified workers have a relatively higher quit rate, it is unclear why they should be the ones displacing other workers, especially from routine jobs, where differences in productivity are not likely to be very large.

[1] See Autor et al. (2003) on routine jobs, Goos et al. (2014) on skill biased technological change versus globalization and Autor et al. (2014) on exposure to international trade.

[2] See Gautier (2002), Albrecht and Vroman (2002), Dolado et al. (2009), Acemoglu and Autor (2011), Beaudry et al. (2016) and Barnichon and Zylberberg (2014).

[3] Gautier et al. (2002) find no evidence at the firm-level that firms upgrade their work force during low employment years.

[4] See Clark et al. (2014). Also see Van den Berg et al. (2002) for an example of stepping stone jobs.

In this chapter, I show that displacement of high-school graduates from employment can be understood as the labour-market response to an adverse selection problem. The adverse selection problem arises because employment contracts do not systematically discriminate against education, even though over-qualified workers are relatively more likely to quit those routine jobs. Comparing with high school graduates, overqualified college graduates are less attracted to routine jobs that offer a higher wage but is associated with a lower matching rate. Consequently, the labour market equilibrium generates inefficient unemployment of high school graduates. Moreover, I argue that this mechanism explains why vocational education has a higher market value than it is commonly thought.

Most studies of the role of education in the labour market focus on high school and college workers. Yet the increasing role of vocational education, which is the post-secondary education that focuses primarily on providing occupationally specific preparation, is striking. In 2010, for example, about 1,650,000 bachelor degrees and about 1,442,000 vocational education and training (VET) credentials were awarded in the U.S. VET credentials awarded per year increased by 60 percent from 2000 to 2010, growing much faster than the 34 percent increase of bachelor degrees. In this chapter I argue that an important part of the labour-market value of vocational education stems from the fact that it acts as an entry barrier to exclude overqualified college graduates from some routine jobs. In this context, skill-biased technological change can explain the rise of vocational education over the past decades.

I begin by analyzing a dynamic frictional labour market with two types of workers and two types of jobs. I have in mind jobs that involve mainly cognitive tasks, but some of those jobs involve mainly routine tasks and some involve mainly non-routine tasks. Workers are assumed to have different educational backgrounds and are referred to as high school and college workers. For simplicity, I assume that all workers can perform routine tasks, and that high school and college workers are equally productive in these tasks. However, only college workers can perform non-routine tasks. College workers employed in routine jobs can search for non-routine jobs while on-the-job.

In this context, routine jobs have value for college workers as a stepping stone toward non-routine jobs. Employers offering these jobs understand that college workers are relatively less profitable than high school workers, because their quit rate is higher. However, employment contracts cannot discriminate effectively against education — either because such a form of discrimination is illegal or because college graduates cannot be compelled to disclose their college degree. This creates an adverse selection problem.

I show that any competitive search equilibrium of the model separates college and high school graduates searching for routine jobs into different markets.[5] The separation, however, is at the cost of generating inefficient unemployment for high school graduates. This is because college workers are less willing to wait for a routine job than high school workers are. College workers search for routine jobs to transition into better jobs, so they are willing to accept a relatively lower wage as long as the job-finding rate is sufficiently high. In contrast, high school graduates are more likely to wait longer for a relatively higher wage, since they have a longer expected job tenure. In equilibrium, jobs that attract high school graduates offer a relatively higher wage,[6] and a corresponding inefficiently low job-arrival rate to discourage educated workers from applying. As a result, high school workers are displaced from employment by over-qualified college workers.

In this context, I argue that post-secondary vocational education has a higher market value than is commonly thought. The displacement effect through adverse selection is the key to understanding the value of vocational education as an entry barrier. Besides the obvious benefit of skill development, vocational education acts as an entry barrier because jobs requiring vocational education can exclude over-qualified workers from seeking stepping-stone opportunities. By extending the baseline model to include educational choices, I show that the adverse selection problem creates a demand for vocational education. Since employment contracts cannot discriminate against workers with more education, but can condition on required training, vocational education successfully excludes college workers from seeking stepping-stone opportunities. As a result, employers are able to offer contracts that do not suffer from the distortion of the adverse selection problem. I show that introducing costly vocational education into the labour market makes workers ex-ante better off. Vocational education helps screen workers who are more committed to the occupation and improves labour market efficiency.

Occupational licensing is an extreme example of entry barriers as it is often viewed to have no impact on productivity. According to Kleiner and Krueger (2013), in 2008 nearly 30 percent of the workers with more than high school education, but not a bachelor degree, were required to hold a license. Many popular fields of vocational education such as health

[5] I extend the work of Guerrieri et al. (2010) on competitive search equilibrium with adverse selection to study a dynamic labour market equilibrium that includes on-the-job search.

[6] Wage here represents the total labour income from the job that includes for example benefits and pensions. Jobs with high turnover rate are often less generous in these benefits.

care and “trades” in areas such as manufacturing, construction, repair, and transportation, also prepare students to obtain occupational licensing. One concern is that, as an entry barrier, occupational licensing may cause job losses by increasing employment costs. While this concern is widespread in policy papers (see Kleiner (2005)) and in the media, my results imply that occupational licensing also has an important benefit as an entry barrier. Imposing restrictions on entry provides protection for low to middle-skilled workers from the competition of college graduates seeking for stepping-stone jobs. Entry barriers such as vocational education and occupational licensing can mitigate the distortion caused by adverse selection by screening out workers who are not pursuing a career in the particular occupation. This is in sharp contrast to standard competitive market models, where a barrier to entry creates monopoly power and is unambiguously welfare reducing.

Finally, I relate the outcomes of this model to the increase in post-secondary educational attainment over the past few decades. Autor et al. (2010) point out that one puzzle in the U.S. labour market is that the relative supply of college-educated workers is not growing fast enough, given the steep rise in the college-versus-high-school earnings ratio. This chapter suggests that much of the increase in post-secondary education may take the form of non-bachelor post-secondary vocational education. The reason for this is that skill-biased technological change has not only increased the return to college, relative to high school, but also the return to vocational education. I simulate the model to examine the effect of a skill-biased technological change that is complementary to non-routine jobs but substitute to routine jobs, combined with an increase in university tuition fees to replicate the education of 2010. I find that while the increasing productivity of non-routine jobs leads to a direct increase in the educational attainment at university level, it also lowers the return to high school by further displacing high school workers from employment. Therefore such a change also increases the incentive for high school workers to attain both college and vocational education. In addition, the reduction in the productivity of some routine jobs has also worsened the labour market outcome of high school workers, leading to an increased popularity of VET. Overall, together with an increase in university tuition, skill-biased technological change can explain the increase of both college degrees and post-secondary VET certificates over the past thirty years.

The theory I propose is in contrast with previous work that emphasizes the displacement or “crowding-out” of less-educated workers by high skilled-workers. In the frictionless competitive models of Acemoglu and Autor (2011) and Beaudry et al. (2016), no externality is generated by employing high-skilled workers in less-skilled jobs, since replacing less-educated workers by high-skilled ones is an efficient market adjustment. In models with random matching, such as Gautier (2002), Albrecht and Vroman (2002) and Dolado et al. (2009), search externalities rely crucially on the assumption that high-skilled workers and less-educated workers are matched randomly with the same routine employers. What remains unexplained is whether this displacement exists when workers have incentives to apply to different employers. I show that over-qualified workers create a negative spillover effect on low-skilled workers even when they search for different routine jobs. Allowing workers to direct their search is crucial for understanding the mechanism that underlies the displacement of high school graduates and its policy implications.

The rest of the chapter is organized as follows. Section 2 lays out the model. Section 3 characterizes the equilibrium with the displacement effect. Section 4 extends the model with educational choices and discusses the implication on vocational education. I also present a numerical simulation to discuss the implications of skill biased technological changes. Section 5 concludes.

3.2 Model

3.2.1 Agents and Markets

Time is discrete and continues forever. All agents are risk neutral and discount the future at a rate $r > 0$. There are two types of workers, distinguished by their observable educational background. Let $i \in \{1, 2\}$ indicate the types of workers, where type-2 workers can be interpreted as workers with a college/university degree while type-1 workers represent workers with a high school diploma only. The measure of workers is normalized to 1, with a fixed fraction π of college workers. There are also two types of jobs and $j \in \{1, 2\}$ indicates the job types. Type-2 jobs represent jobs that involve mostly non-routine cognitive tasks and require workers to have a college/bachelor degree. Type-1 jobs involve mostly routine tasks and the skill requirements are met by all workers. The measure of jobs is determined endogenously by the free entry of employers. Each job requires only one worker to operate. Non-routine jobs produces output y_2 while routine jobs produces $y_1 < y_2$. Match productivity is summarized in Table 1. For simplicity, I assume that college and high school workers are equally productive in preforming a routine job. The main results of the chapter still hold if college workers are less or more productive than high school workers in a routine job.

There are four stages within each period: production, separation, search and matching.

Table 3.1: Match Productivity

Worker	Job	
	Non-college	College
High School	y_1	0
College	y_1	$y_2 > y_1$

In the beginning of a period, all workers are either employed or unemployed. During the production stage, employed workers produce and collect wages while unemployed workers receive benefit b. During the separation stage, an exogenous separation shock occurs with probability $\delta > 0$. Once a match is hit by the shock, the worker becomes unemployed and starts job-searching in the next period while the job is destroyed. During the search stage, employed workers and workers who were unemployed at the beginning of the period can search. Meanwhile, firms create job vacancies and post wage contracts for each vacancy. During the last stage, workers and employers are brought into contact to form new matches. When an employed worker receives an outside offer, her current employer can choose to make a take-it-or-leave-it counter-offer. The worker then chooses whether to accept the poaching offer and switch to new employer or stay with the incumbent employer.

To understand the impact of over-education on displacing high school workers, it is important to have some college workers applying to routine jobs in equilibrium. I assume that when searching for college jobs, an unemployed college worker faces a cost $c > 0$.[7] The cost is a random draw from a continuous distribution with cumulative distribution function $F(\cdot)$ and is i.i.d. across workers and over time. It reflects the relative difficulty between applying for a college versus a non-college job for a college worker. The cost is realized before the search stage, so that in equilibrium some college workers apply for a type-1 job while others apply for a type-2 job based on the realization of the cost. For simplicity, I assume no cost is incurred when workers search on job. Given that on-the-job search has values for mismatched workers to improve matching quality, none of my results rely on this assumption.

A worker's labour market state includes whether the worker is employed or unemployed, as well as the wage and the job type if the worker is employed. Let ℓ denote the labour market state of a worker. Given a wage ω and a job type j, the labour market state of an employed worker is a pair $\ell = (\omega, j)$. For an unemployed worker, her the labour market state is denoted by $(b, 0)$. The state of a worker also includes her type, which is the education

[7] Over-education is treated in a reduced-form way, since the focus of this chapter is on less educated workers. See Chapter 1 for a model with endogenous mismatch of college worker.

attainment of the worker. The state of a worker is perfectly observable.

I focus on a labour market where each employer post a single contract conditional on workers' states. In general, firms can post menus that offer different contracts with different conditions on the state of a worker. With free entry of firms, however, the equilibrium where firms post menus of contracts is equivalent to the equilibrium where each firm posts a single conditional contract in separated markets. This is because in an equilibrium with menus, each contract on the menu must generate zero profit for the firms. If firms can make positive profits with some contracts that specify certain states of workers, there is a profitable deviation for other firms to post menus that attract only these workers. Since all firms make zero expected profits with all contracts in an equilibrium with menus, one can think equivalently as a market where employers posts contracts instead of menus. The contracting environment is that employers commit to a posted wage, which remains constant until an outside offer is received by the employee. Once an outside offer is received, employers can respond by offering a counter wage. Any renegotiation of the wage is based on mutual agreement.

The key assumption of the model is that employers with non-college job vacancies cannot post contracts that are conditional on the education attainment of a worker. If employers could make distinct offers to college and high school workers, given that college workers are more likely to quit a routine job for a non-routine job, they would pay a lower wage to college workers for the same job. Restricting employers from offering conditional wage for a routine job eliminates discrimination against workers with more education. In practise, paying equally productive workers different wages might subject to legal risks and job posts that exclude workers with more education from applying are rare. One reason that such exclusion is difficult to implement is because committing not to hire over-qualified workers ex-ante is not credible ex-post. With search frictions and costs of maintaining a vacancy, employers might prefer to hire an overqualified worker once they meet instead of leaving the vacant job unfilled. Another reason is that if employers try to exclude over-qualified workers through restricting job posts, workers might respond by omitting credentials that are not related to the job. Because employers cannot make contracts conditional on education for a routine job, they might face both college and non-college workers applying for the same vacancy. This particular contractual limitation causes an adverse selection problem that works as if education is not observable: being the less desired type for routine job employers, college workers might mimic the search behaviour of high school workers when applying for a routine job.

The structure of the labour market is as follows. Each period a continuum of markets may open. A market is characterized by an employment contract x and a market queue length q, which equal the measure of job applicants divided by the measure of vacancies in that particular market. Markets are indexed by contracts. A contract $x = \{(\omega, j), \ell\} \in X$ includes a job offer of a pair (ω, j) which specifies the wage ω and the job type j on the employer, as well as the labour market state of workers ℓ that the offer is made to. Notice that the contract is not conditional on the type of a worker because only college workers are qualified to non-routine jobs and routine job employers face non-discrimination constraint. Let $Q : X \to \mathbb{R}^+$ be the queue mapping function and $Q(x)$ is the queue length associated with with a contract x. Search is directed as opposed to random matching. Each employer can post one contract with a flow cost $k_j > 0$ for job type j and workers observe all posted wage and decide where to search. Both employers and workers take into account the trade-off that a wage is associate with higher market queue length for any given job.

Workers and firms in the same market are brought into contact by a matching technology. Matching is bilateral, so that each worker meets at most one employer and vice versa. Workers who search in a market with market queue length q meet an employer with probability $f(q)$, and employers in the same market meet a worker with probability $qf(q)$. Following the standard search literature, I assume that $f(q)$ is twice differentiable, strictly decreasing and convex, with $f(0) = 1$ and $f(\infty) = 0$. I also assume that $qf(q)$ is strictly increasing and concave, approaching 1 as q converges to ∞. In addition, I assume that the elasticity of the matching function with respect to job creation $\eta(q)$, defined as $\dfrac{qf'(q)}{f(q)}$ is concave, with $\eta(\infty) = 1$. This assumption is not necessary but simplifies the existence proof of an equilibrium.

I assume that $(r+\delta)k_2 < y_2 - y_1$. This assumption guarantees enough profits for employers to poach an employed worker and ensures that job-to-job transition is possible. On-the-job search plays an important role because the difference between a high school worker and a college worker, once employed in a routine job, is their on-the-job search opportunities. Without on-the-job search, college and high school workers are naturally separated when searching for a routine job given their different reservation values of unemployment.

3.2.2 Worker's Problem

Let $s = \{\ell, i\} \in S$ denote the state of a worker of type i with labour force status ℓ, and S be the set of all possible states of a worker. Let $V(s)$ denote the discounted lifetime income of a worker in state s. Notice that employed and unemployed workers search in different markets as specified by the required labour market states of contracts.

When unemployed, a worker receives benefit b and decides where to search after the search cost is realized. When an unemployed worker i searches for a job offer (ω, j) in market $x = \{(\omega, j), (b, 0)\}$, the probability of meeting an employer is $f(Q(x))$ and the state of the worker becomes $s_o = \{(\omega, j), i\}$ once matched. The value function $V(s)$ for a unemployed worker in state $s = \{(b, 0), i\}$ satisfies:

$$V(s) = b + \mathbb{E}_c \max_{(\omega,j)} \left\{ (1-i)(1-j)c + f(Q(x)) \max \left\{ \frac{V(s)}{1+r}, \frac{V(s_o)}{1+r} \right\} + [1 - f(Q(x))] \frac{V(s)}{1+r} \right\} \tag{3.1}$$

Similarly, employed workers collect wages and decide where to search on-the-job if survive the exogenous separation shock. Consider a type i worker who is currently employed in a type j job and receives wage ω. Given the worker's current labour market state $\ell = (\omega, j)$, if she searches for an offer (ω', j') in market $x = \{(\omega', j'), \ell\}$, the probability that she meets a poaching employer is $f(Q(x))$. Once the worker receives an offer (ω', j'), her current employer can respond by offering a counter-wage ω_c, which equals zero when no counter-offer is made. If the worker accepts the outside offer, she switches to the new employer j' and forms a new match starting the next period with state $s_o = \{(\omega', j'), i\}$. Otherwise, the worker remains matched with her current employer. Her state becomes $s_c = \{(\omega_c, j'), i\}$ with new wage ω_c if the counter-offer is accepted, otherwise the worker remains in state $s = \{\ell, i\}$. The value function $V(s)$ of an employed worker in states $s \neq \{(b, 0), i\}$ satisfies:

$$V(s) = \omega + \frac{\delta V(s_u)}{1+r} + (1-\delta) \left\{ \max_{(\omega', j')} \left\{ f(Q(x)) \max \left\{ \frac{V(s)}{1+r}, \frac{V(s_o)}{1+r}, \frac{V(s_c)}{1+r} \right\} + [1 - f(Q(x))] \frac{V(s)}{1+r} \right\} \right\} \tag{3.2}$$

Denote by $g(s, c)$ the search policy of a worker in state s with cost realization c ($c = 0$ if the worker is employed). Also denote by $g^a(s, s_o, s_c)$ the acceptance policy such that $g^a(s, s_o, s_c)$ is the probability that a worker in state s accepts the outside offer. In equilibrium, employers take the worker's optimal search and acceptance policies as given.

3.2.3 Employer's Problem

Consider a type-j employer matched with a worker of type i receiving wage ω. Denote by $H(s)$ the present value of an on-going match to the employer, where $s = \{(\omega, j), i\}$ is the state of the employee. $H(s)$ equals the discounted sum of profits for the employer matched with the worker in state s. Employers takes as given that if the match is not hit by the exogenous shock, the worker searches on-the-job according to the search policy g: the worker searches for outside offer $g(s,0)$ in market $x' = \{g(s,0), s\}$ and receives an outside offer with probability $f(Q(x'))$. Once an outside offer is received, the employer chooses whether to make a counter-offer ω_c, taking as given the the acceptance rule g^a. If the worker accepts the outside offer, her state becomes $s^o = \{g(s,0), i\}$ and the current match is destroyed with zero value left for the employer. Otherwise, the worker stays with the incumbent employer. The state of the worker remains at s or changes to $s_c = \{(\omega_c, j), i\}$, if she accepts the counter offer. Since workers always apply for a higher wage when searching on-the-job, optimal retention wage ω_c must be higher than the current wage. The present value of the match for the employer $H(s)$ satisfies the following:

$$H(s) = y_j - \omega + (1-\delta)\left\{ f(Q(x')) \max_{\omega_c} \left\{ [1 - g^a(s, s_o, s_c)] \frac{H(s_c)}{1+r} \right\} + [1 - f(Q(x'))] \frac{H(s)}{1+r} \right\} \tag{3.3}$$

Denote the optimal retention policy of an employer by $g^r(s, s_o)$. The retention policy is contingent on the employee's current state.

Now consider firms with empty vacancies. Employers choose how many vacancies to create and where to locate them. Because contracts are conditional on workers' labour force status but not on their types, employers need to form expectations about the distribution of the applicants attracted to each posted contract. Let $\mu(\cdot|x)$ be the conditional distribution function of workers who are attracted by contract $x = \{(\omega, j), \ell\}$ across types. Let $J(x)$ be the ex-ante return of posting contract x to an employer. $J(x)$ is given by

$$J(x) = -k_j + Q(x) f(Q(x)) \sum_{i=1}^{2} \mu(i|x) g^a(s^i, s^i_o, s^i_c) \frac{H(s^i_o)}{1+r} \tag{3.4}$$

where j is the job type posted. The states s^i, s^i_o, s^i_c of a worker of type i satisfy $s^i = \{\ell, i\}$, $s^i_o = \{\{\omega, j\}, i\}$ and $s^i_c = \{g^r(s^i, s^i_o), i\}$ respectively for $i \in \{1, 2\}$.

3.2.4 Equilibrium Refinement

Without specifying the belief off the equilibrium path, the model could have numerous equilibria supported by different off-equilibrium beliefs. I now introduce a refinement that follows closely the equilibrium concepts in Chang (2014) and Guerrieri et al. (2010). The idea of the refinement is that a deviating contract must attract workers who have the strongest incentive to apply. Formally, denote $\tilde{V}(x, q, s)$ the expected return to a worker in state

$s = \{\ell, i\}$ searching in market x with queue length q. The incentive of a worker i applying to deviating contract x is determined by the longest queue that a worker is willing to accept:

$$q(x, i) \equiv \sup\{\tilde{q} \geq 0 : \tilde{V}(x, \tilde{q}, s) \geq V(s)\} \tag{3.5}$$

Note that $\tilde{q} = 0$ if $\tilde{V}(\hat{x}, \tilde{q}, s) \geq V(s)$ has no solution. For contract x, $q(x, i)$ is the longest queue that a worker i is willing to accept in order to deviate. If $q(x, i)$ is positive, then the workers is indifferent between her equilibrium allocation and applying to contract x with queue length equal to $q(x, i)$. If x could attract some positive measure of workers, the type of the worker who has the strongest incentive to apply–the type with the longest $q(x, i)$–must have been attracted. Then the expectation function $\mu(\cdot|x)$ must satisfy that only workers who have the strongest incentive to apply are attracted. That is, belief must satisfy

$$\mu(i|x) = 0 \text{ if } i \notin \arg\sup_{i \in I}\{q(x, i)\} \text{ for any } x \in X. \tag{3.6}$$

This refinement arises naturally as a single employer does not serve the entire market under bilateral matching. It implies that when more that one type of worker is attracted by an off-equilibrium contract, their longest acceptable queue must be identical. It also implies that the queue mapping must satisfy:

$$Q(x) \equiv \sup_{i \in I} q(x, i)$$

The queue length associated with off-equilibrium contracts is determined by the longest acceptable queue of the attracted worker type. To understand this, consider the following adjustment process: if an employer posts an off-path contract x, some workers might find it profitable to apply if the market queue is sufficiently small. As workers flow in, the worker-over-employer ratio, i.e. the queue length increases. The flow-in of type i workers stops when they are indifferent between their equilibrium market and market x. They prefer not to apply to the market if the worker-over-employer ratio continues to increase if other types of workers with stronger incentives still flow in. This process stops when the queue length of the market reaches the longest acceptable queue of the worker type with the strongest incentive to apply. Therefore, the longest accepted queue of workers attracted to the contract determines $Q(x)$ for off-equilibrium contracts.

3.2.5 Equilibrium

Definition 7. *A stationary competitive search equilibrium consists of a set of posted contracts $X^* \subseteq X$, a market queue mapping Q, a value function for workers V, a value function for* *employers H, a search policy g, an acceptance rule g^a, a retention policy g^r, a conditional distribution function μ :, a distribution ϕ such that:*

(i) Workers optimize: $V(s)$ satisfies (3.1) when $s \in S^u$ and satisfies (3.2) for $s \in S \setminus S^u$ with $g(s,c)$ is the associated policy function and $g(s,c) \in X^*$ for all $c \in \mathbb{R}$, $s \in S$; and $g^a(s, s_o, s_c)$ is the acceptance rule such that

$$g^a(s, s_o, s_c) = \arg\max_{a \in [0,1]} \{aV(s_o) + (1-a)\max\{V(s), V(s_c)\}\} \tag{3.7}$$

(ii) Employers optimize: $H(s)$ satisfies (3.3) for all $s \in S^e$ with $g^r(s, s_o)$ the corresponding policy function;

(iii) For any $x \in X^*$, firms make zero profits: $J(x) = 0$

(iv) Beliefs are consistent: for any $x \in X^*$ and any state $\{\ell, i\} \in S$,

$$\mu(i|x) = \frac{\phi(\{\ell, i\})}{\sum_{i \in \{1,2\}} \phi(\{\ell, i\})}$$

(v) Steady-State conditions are satisfied.

(vi) Beliefs satisfy equilibrium refinement: $\mu(i|x)$ satisfies (3.6)

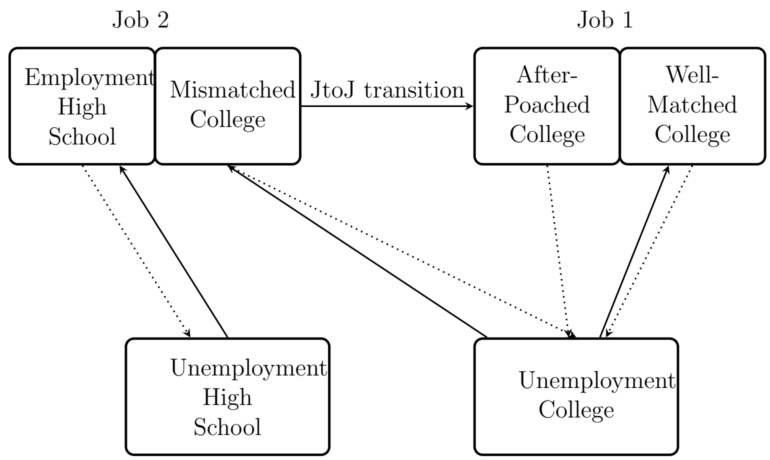
The general statement of the steady-state conditions is cumbersome and therefore not listed here, I provide the steady-state condition in the context of the equilibrium characterized below.

Condition (i) ensures that workers' search and acceptance policies are optimal for all states, taking as given the market queue length of all contracts and employers' optimal counter-offer strategy. Condition (ii) and (iii) ensures that, given the optimal search strategy of the worker, retention policies are optimal, and equilibrium contracts generates zero profits for employers. Condition (iii) ensures that for equilibrium contracts, employers beliefs are consistent with the workers search strategies through Bayes rule. Condition (v) ensures the law of motion for the aggregate state of the economy is stationary. Finally, condition (vi) restricts the off-equilibrium beliefs such that for any contract, only workers with the strongest incentives are attracted.

3.3 Equilibrium with Crowding-out

In this section, I study an equilibrium where the labour outcome of high school workers are distorted with inefficiently high unemployment rate. The equilibrium can be understood by

Figure 3.1: Labour Market Flow



studying a set of constrained optimization problems. [8] The constrained optimization problems have an intuitive structure where workers maximize expected return to search subject to firms make non-negative profits and the incentive compatibility constraints of all types hold. After stating the problems, I show that a separating equilibrium can be supported with the solution of the problems and the equilibrium always exists. Under mild conditions, the incentive compatibility constraint of a college worker is binding and the equilibrium suffers from an adverse selection problem so that the unemployment rate of high school workers is inefficiently high.

Figure 1 presents the equilibrium flow of workers. High school workers search for non-college jobs only. Once employed, they do not search on-the-job. College workers can search for both college and non-college jobs when unemployed. An employed college worker search on-the-job only if when matched with a non-college job employer. Job-to-job transitions occur when a college worker switches from a non-college job employer to a college job employer. All employed workers face an exogenous separation shock. Once hit by the shock, a worker becomes unemployed.

In equilibrium, job-to-job transitions occur only when switching employers generates a productivity gain. This is because the search cost is a sink cost for an employer who has already matched with a worker and it is optimal for the incumbent employer to counter offer up to the productivity of the match, once an outside offer is received by the employee. When there exists a negligible small cost of making offers, an employer find it not profitable

[8] See a full discussion in Guerrieri et al. (2010).

to poach a worker who is equally productive with the current employer and with the poaching employer. Similarity, an incumbent employer do not make counter-offers if the workers is more productive with the poaching employer than in the current match. As a result, only college workers who are matched with non-college job employers search on-the-job. The search problems that need to be considered in the equilibrium are: the on-the-job search problem of an employed college worker, the search problem of an unemployed college worker for both college and non-college jobs and the search problem of an unemployed high school worker.

Consider first the on-the-job search problem of an employed college worker. Denote the value of a college worker who is currently employed in a non-college job with wage ω by $V(\{(\omega,1),2\})$. The worker searches for a college job and faces the following search problem. The worker chooses across all pairs of (ω',q) to maximize the expected return to search, subjects to the constraint that the poaching employer makes non-negative profits and the poaching wage is no less than her current matching productivity. For any given pair (ω',q), the worker meets an employer with probability $f(q)$ and forms a new match with value $V(\{(\omega',2),2\})$ to the worker with the new employer. With probability $1-f(q)$, the worker stays with the incumbent employer and her state remains unchanged. $V(\{(\omega,1),2\})$ satisfies:

$$V(\{(\omega,1),2\}) = \omega + \max_{\omega',q}\left\{ f(q)\frac{V(\{(\omega',2),2\})}{1+r} + [1-f(q)]\frac{V(\{(\omega,1),2\})}{1+r} \right\} \qquad (\mathbf{P-1})$$

Subject to

$$-k_2 + qf(q)\left(\frac{y_2-\omega'}{r+\delta}\right) \geq 0$$

$$\omega' \geq y_1$$

The value of a match with a college job for a college worker satisfies

$$\frac{V(\{(\omega,2),2\})}{1+r} = \frac{\omega}{r+\delta} + \frac{\delta}{r+\delta}\frac{V(\{(b,0),2\})}{(1+r)}$$

which equals to the discount sum of wages with the value of becoming unemployment $(V(\{(b,0),2\})$ with probability δ.

Denote the problem by $(\mathbf{P-1})$. The first constraint is the non-negative profits constraint for a college job employer searching for a worker in the market with wage offer ω' and queue length q. The employer pays cost k_2 to create the job and with probability $qf(q)$ she meets a worker. The expected profits of a match for the employers equals to $\frac{y_2 - \omega'}{r + \delta}$, which is the flow profits $y_2 - \omega'$ of the match discounted by the common discount rate r and the job separation risk δ. The second constraint restricts the equilibrium poaching wage to be no less than the worker's current productivity. Because the incumbent employer is able to make counter offers up to y_1, no poaching employer in equilibrium posts wages lower than y_1 to attract a mismatched college worker.

Denote the solution to the above problem by $\{q^e(\omega), \omega^e(\omega)\}$. Notice that the solution to the on-the-job problem depends on the worker's current wage. The probability that a college worker switches to a college job employer depends on the wage ω that she receives from the non-college job employer.

Next, I show the problem of an unemployed college worker searching for a non-college job employer, denoted by $(\mathbf{P-2})$. Denote the return to search for a non-college job to an unemployed college worker by V_2^1, such that

$$V_2^1 = \max_{\omega,q} \left\{ f(q) \frac{V(\{(\omega,1),2\})}{1+r} + [1-f(q)] \frac{V(\{(b,0),2\})}{1+r} \right\} \qquad (\mathbf{P-2})$$

s.t.

$$-k_1 + qf(q) \frac{y_1 - \omega}{r + \delta + (1-\delta) f(q^e(\omega))} \leq 0$$

An unemployed college worker searches across all pairs of (ω, q) subject to the constraint that the non-college job employer makes non-negative profits. With probability $f(q)$, the worker meets an employer and is matched with a non-college job and starts search on-the-job in the next period. With probability $1 - f(q)$, the worker does not find an employer and remain unemployed with value $V(\{(b,0),2\})$. Notice that the employer takes into consideration that the quit rate of the worker depends on the wage ω and discounts the expected profit of a match with $r + \delta + f(q^e(\omega))$, where $f(q^e(\omega))$ is the endogenous separation rate for the match.

Now consider the search problem of an unemployed college worker looking for a college job, denoted by $(\mathbf{P-3})$. As in the problem $(\mathbf{P-2})$, an unemployed college worker searches across all pairs of (ω, q) subject to the constraint that the college job employer makes non-negative profits. Different from searching for a non-college job, a college worker do not search on-the-job once matched with a college job. Denote the return to search for a college job to an unemployed college worker by V_2^2, such that

$$V_2^2 = \max_{(\omega,q)} \left\{ f(q)\frac{V(\{(\omega,2),2\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),2\})}{1+r} \right\} \qquad (\mathbf{P-3})$$

s.t.

$$-k_2 + qf(q)\frac{y_2-\omega}{r+\delta} \geq 0$$

The search problem is similar as the on-the-job search problem $(\mathbf{P-1})$, except that the workers remains unemployed with value $V(\{(b,0),2\})$ if she does not meet an employer within a period.

What remains in the search problems of a college worker, is to determine whether to search for a college job or a non-college job when unemployed. It is easy to see that there exists a cut-off level cost $\bar{c}$, such that a worker searches for a non-college job if and only if the realization of the search cost is larger than $\bar{c}$. Since the search cost c follows distribution with cumulative density function $F(c)$, an unemployed college worker searches en-ante for a college job with probability $F(\bar{c})$ with return to search V_2^2 and pays the cost. With probability $1-F(\bar{c})$, the worker searches optimally for a non-college job with return to search V_2^1 and does not pay the search cost. The value of unemployment to a college worker $V(\{(b,0),2\})$ equals:

$$V(\{(b,0),2\}) = b + [1-F(\bar{c})]V_2^1 + F(\bar{c})[V_2^2 - \mathbb{E}(c|c<\bar{c})] \qquad (3.8)$$

The cut-off cost $\bar{c}$ must satisfy that an unemployed college worker with cost realization $\bar{c}$ is indifferent between searching for a college job and pay cost $\bar{c}$ or searching for a non-college job without paying the cost. The cut-off $\bar{c}$ is defined by the following equation:

$$V_2^2 - \bar{c} = V_2^1 \qquad (3.9)$$

Now consider the search problem of an unemployed high school worker. Like college workers, an unemployed high school worker maximizes the expected return to search by choosing across all pairs (ω,q) subjects to the constraint that the employer makes non-negative profits. Unlike college workers, the optimization problem of an unemployed high school workers is also subject to an incentive compatibility (IC) constraint. For any given pair (ω,q), the worker meets an employer with probability $f(q)$ and forms a match with value $V(\{(\omega,1),1\})$. Otherwise, the worker remain unemployed with the value denoted by $V(\{(b,0),1\})$. Denote the search problem of an unemployed high school worker by $(\mathbf{P-4})$, such that

$$V(\{(b,0),1\}) = b + \max_{\omega,q}\left\{f(q)\frac{V(\{(\omega,1),1\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),1\})}{1+r}\right\} \qquad (\mathbf{P-4})$$

s.t.

$$-k_1 + qf(q)\frac{y_1-\omega}{r+\delta} \leq 0 \qquad \text{(Non-zero profits constraint)}$$

$$f(q)\frac{V(\{(\omega,1),2\})}{1+r} + [1-f(q)]\,\frac{V(\{(b,0),2\})}{1+r} \leq V_2^1 \qquad \text{(IC constraint)}$$

the value of a match for a high school worker satisfies:

$$\frac{V(\{(\omega,1),1\})}{1+r} = \frac{\omega}{r+\delta} + \frac{\delta}{r+\delta}\frac{V(\{(b,0),1\})}{(1+r)}$$

The IC constraint reflects the adverse selection problem and is the key to the displacement mechanism. Because of the non-discrimination against education restriction, contracts offered to high school workers can not exclude college workers from applying. At any given (ω, q), an employer expects less profits from matching with a college worker since the worker uses the job as a stepping-stone. The pair (ω, q) that makes an employer earning zero-profits by hiring a high school worker would generate negative profits if hiring a college worker. In equilibrium, no employer posts contracts to high school workers that would also attract college workers. The equilibrium contracts offered to high school workers must satisfy that a college worker weakly prefers search in her own market with value V_2^1 than searching for the same (ω, q) as high school workers, the value of which equals the left-hand-side of the IC constraint. One can see from problem $(\mathbf{P-1})$, when the IC constraint is binding, the solution to an unemployed high school worker's search problem is no longer constrained optimal.

I now show that the properties of a competitive search equilibrium can be understood by studying the above problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$. Proposition 1 below establishes the linkage between the problems and the equilibrium.

Proposition 1. *i) Any equilibrium allocation must solve problems* $(\mathbf{P-1})$ *to* $(\mathbf{P-4})$. *ii)A solution to problems* $(\mathbf{P-1})$ *to* $(\mathbf{P-4})$ *can be supported as an equilibrium.*

The proof of this proposition follows closely after Guerrieri et al. (2010) and can be found in the appendix. Statement i) says that any solution to the set of problems generates an equilibrium. It is easy to see that the search problems in markets with college jobs are the standard competitive search problems without adverse selection. In markets with non-college jobs, however, the equilibrium expectation function is not degenerated because of the adverse selection problem. To show that workers maximize the expected return to search given the expectation function and the corresponding queue mapping function, I consider a truncated expectation function. For any wage post that is lower than the wage of an "indifference allocation", with which both the non-negative constraint and IC constraint in problem $(\mathbf{P-4})$ are binding, the employers expect only college workers to apply; otherwise the employers expect only high school workers to apply. With this expectation function and other corresponding equilibrium objects, the solution to problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$ can be supported as a separating equilibrium, where college and high school workers search in different markets for a non-college employer.

The second statement of Proposition 1 says that one can find any equilibrium by solving problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$. Notice that problems $(\mathbf{P-2})$ and $(\mathbf{P-4})$ are constructed under the assumption that an employer with a non-college job expects to meet either a college worker or a high school worker but not a mixed distribution of them. Therefore, statement ii) implies that any equilibrium must be separating. The explanation of no pooling equilibrium is as follows. If there were pooling in equilibrium, employers could attract the high school workers by offering a higher wage and break the proposed equilibrium. This is because unlike college workers who search for non-college jobs because they are a stepping stone to better jobs, high school workers are more willing to remain unemployed and search for jobs offering a relatively higher wage. The rational refinements on the off-path expectation function ensures that a deviating wage attracts the type of workers who have the strongest incentives to apply. Therefore, offering a higher wage attracts the desired type of workers, in which case are the high school workers, and is a profitable deviation.

Given that any solution to the set of problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$ generates an equilibrium and any equilibrium must solve the problems, the existence of a separating equilibrium follows immediately.

Proposition 2. *There is a separating equilibrium.*

The sorting mechanism of the separating equilibrium is as follows. The market sorts unemployed workers by education according to their tolerance to remaining unemployed. College workers are less willing to remain unemployed, since they search for non-college jobs because they are a stepping stone to better jobs, so they are willing to search for jobs offering a relatively lower wage as long as the job finding rate is sufficiently high. In a separating equilibrium, the market makes the jobs that attract high school workers sufficiently hard to get so as to discourage educated workers from applying.

The next proposition states the main result of the chapter. Under mild condition, the separating equilibrium suffers from an adverse selection problem and the labour market outcomes of high school workers are distorted.

Proposition 3. *There is a number $\bar{k} > 0$ such that for any $k_2 \in (0, \bar{k})$, an equilibrium suffer from adverse selection, and the unemployment rate of high school workers is inefficiently high.*

I show that when the cost of posting a college job is sufficiently small, the IC constraint of a college worker is binding. In this case, the constraint efficient allocation to a high school worker, which is the solution to the optimizing problem where high school workers maximizing expected return to search subject to employers making zero-profits, also attracts college workers. This means when the IC constraint is binding, no employer offers the wage that a high school worker finds optimal to apply. Instead, the market makes the jobs that attract high school workers sufficiently hard to get in order to exclude college workers from applying. The reason is that when searching for a non-college job, a college workers have lower tolerance to remaining unemployed than a high school worker. As a market solution to the adverse selection problem, the market sorts workers with different career paths by distorting the matching probabilities of high school workers.

In order for the market to distort the equilibrium matching rate and separate workers, the wage offered to high school workers is higher than its efficient level, even though the return to search, which is the discounted total labour income of a high school worker over time, is inefficiently low. In fact, the model implies that the skill premium, measured as the wage differential between high school and college workers, underestimates the true returns to education. This is because the true returns to education take into account not only the wage when a worker is employed, but also the job finding probability when a worker is unemployed, as well as the discounted value of future opportunities when a worker is employed. In this model, even though the skill premium is inefficiently low, as the wage of a high school workers is above its efficient level, the true returns to education are inefficiently high.

In the literature, many have discussed the crowding-out effect of over-qualified high-skilled workers on the employment of less-educated workers. These studies have taken as given that educated and uneducated workers do compete for the same scarce jobs. For example, in competitive framework with no search frictions, less-educated workers are replaced by more productive high-skilled workers in jobs that are less skill intensive and in routine tasks (see Acemoglu and Autor (2011) and Beaudry et al. (2016)). Yet firm level study has found no evidence supporting that firms upgrade their work force for low-skilled jobs in economic downturns (Gautier et al., 2002). In these models, no externality is generated by high-skilled workers employing in less-skilled jobs since replacing less-educated workers by high skilled ones is an efficient market adjustment. In search literature with random matching models, high-skilled workers create search externalities by applying to low-skilled jobs, which affects the creation of these jobs (see Gautier (2002), Albrecht and Vroman (2002) Dolado et al. (2009)). These externalities rely crucially on the assumption that high-skilled workers and less-educated workers are matched randomly with employers offering low-skilled jobs. However, the observation that the proportions of college workers vary across non-college occupations (Vedder, 2012) suggests that college workers do not search randomly for low-skilled jobs. Without imposing the assumption of direct competitions, the crowding-out effect of the above discussed models are not valid.

In this chapter, the search behaviour of college workers generates a negative spillover effect on the unemployment rate of high school workers. The effect is similar to the crowding-out effect in the literature, but the mechanism is quite different. The high unemployment rate of high school workers is a market response to an adverse selection problem, where employment contracts cannot exclude college workers seeking non-college jobs as a stepping-stone. The displacement of high school workers is inefficient and is not caused by productivity driven adjustments. Even if college workers are less productive in non-college jobs, the distortions on high school workers still exists. The mechanism does not rely on the direct competition assumption between the workers neither. This means even without observing college and high school workers competing for the same jobs, the negative spillover effects caused by over-qualified college workers could still harm high school workers. Difference in mechanism presumably generates different implications. In the next section, I proceed by discussing the implication of this model on the post-second vocational education and training.

3.4 Vocational Education and Training

In the previous section, I show that the labour market responses to the adverse selection problem by distorting the labour market outcomes of high school workers. In the this section, I discuss the implication of the model. I argue that post-secondary vocational education and training (VET) has value as an entry barrier to college workers and is an institutional solution to the adverse selection problem.

VET here refers to the nonbaccalaureate post-secondary level education that focuses primarily on providing occupationally specific preparation. In the United State, VET has been growing fast in the past few decades. As showed in the beginning of the chapter, the total number of VET certificates awarded per year has increased over 60 percent from 2000 to 2010, growing much faster than the 34 percent increase of bachelor degrees.[9] VET credentials take the form of either a post-secondary certificate or an associates degree and are provided mostly by community colleges in the U.S.

In this section, I first argue that the fast-growing VET can be understood as a response to the adverse selection problem introduced in the previous sections. By acting as an entry barrier to college workers, VET helps improve the labour market outcomes of high school workers and increases labour market efficiency. I formalize this intuition by extending the baseline model to include educational choices. I show that VET is an useful labour market institution that mitigates the adverse selection problem. I also show that in contrast to the competitive models where barriers to entry are viewed as rent-seeking distortions to the market, introducing costly VET as an entry barrier to labour market is welfare improving. Finally, I preform a numerical simulation and examine the effects of skill-biased technological changes as well as an increase in college tuitions on the changes of educational choices.

3.4.1 VET as Entry Barrier for College Workers

With the insight of the mechanism presented in this chapter, I argue that the displacement of high school workers caused by the adverse selection problem creates a demand for VET. The intuition behind such a demand is that VET has the value as an entry barrier to college workers. By requiring VET, a non-college job employer can exclude college workers from seeking stepping-stone opportunities and can therefore offer non-distorted wages to high school workers. By obtaining VET, high school workers gain access to markets that do not suffer from the adverse selection problem and improve their labour market performance. As long as the cost of VET does not excess the gain from receiving non-distorted labour market allocations, high school workers have incentives to pursue VET.

The adverse selection problem discussed in this chapter and the role of VET as an entry barrier are similar to the standard Spence's signalling model of education (Spence, 1973). In the signalling model, worker's unobserved productivity creates an adverse selection problem

[9] National Center for Education Statistics (NCES).

and educational credentials can be used to signal worker's abilities. In this chapter, the on-the-job search and the incompleteness of contracting to exclude over-educated applicants are the cause of the adverse selection problem. Notice that in this environment, workers with more education are the less desired type to the employers, because they are more likely to quit a lower-skilled job in favour of high-skilled jobs. In this sense, on-the-job search changes the direction of the adverse selection problem in terms of educational levels. Using VET, employers can condition job offers on required training and exclude college workers from seeking stepping-stones. Therefore, VET helps mitigate the adverse selection, like the role of educational credential in Spence's model, by resolving the contracting limitation of non-discrimination against education. Next, I formally present this intuition by extending the baseline model to include educational choices and VET.

3.4.2 A labour market with VET

Consider now a third type of jobs: the VET jobs (denoted by v). The difference between non-college jobs and VET jobs is that VET jobs require VET credentials while non-college jobs do not require any specific educational credential and can be performed by all workers. To separate the value of VET as an entry barrier from productivity, I assume in the model that the VET jobs and non-college jobs are equally productive such that $y_2 > y_1 = y_v$.

The distribution of workers' educational attainment is determined endogenously. All workers are ex-ante identical. They make a one-time educational choice at period zero before entering the labour market. The choice is made based on the cost and the expected return of education. Let e denote the cost of post-secondary education to a worker, where e follows a distribution with cumulative function $F_e(\cdot)$, and is i.i.d. across all workers. A worker chooses whether to pursue post-secondary education and within the post-secondary educational system whether to pursue a VET credential or a bachelor degree. A workers with cost e needs to pays αe for a bachelor degree and βe for VET with $\alpha > \beta$. In this model, all workers have high school diplomas.

The search problem of high school workers and college workers are identical as in the baseline model. A worker with a VET credential can apply for both non-college jobs and VET jobs. However, the worker in equilibrium applies to VET jobs only and do not search on-the-jobs once matched.[10] Unlike high school workers, however, the search problem for

[10] For a worker with a VET credential, the cost of VET is a sunk cost and the optimal return to search for VET jobs weakly dominates the return to search for non-college jobs. Once matched with a VET job, there is no option value from on-the-job search, therefore the worker does not search on-the-job.

workers with VET is not subject to an adverse selection problem. Therefore workers with VET credentials receive efficient allocations by maximizing expected return to search subject to employers making zero-profits. Denote by U_h, U_v, U_c the equilibrium return to search for a high school worker, a worker with a VET credential and a worker with a bachelor degree respectively. Within an educational group, all workers are identical.

To determine the distribution of workers, consider the following two cutoffs. Let $\bar{e}$ be the cutoff cost to post-secondary education so that a worker with educational cost $\bar{e}$ is indifferent between pursuing VET or obtaining a high school diploma only. The cutoff to post-secondary education $\bar{e}$ must satisfy

$$U_h = U_v - \beta\bar{e} \tag{3.10}$$

Similarly, let $\underline{e}$ be the cutoff so that a worker with educational cost $\underline{e}$ is indifferent between obtain a VET credential or a bachelor degree. The cutoff $\underline{e}$ must satisfy:

$$U_v - \beta\underline{e} = U_c - \alpha\underline{e} \tag{3.11}$$

If $\bar{e} \leq \underline{e}$, a worker chooses either to attain post-secondary education at university level or obtain only a high school diploma with no demand for VET. In the following proposition, I show that when the labour market suffers from the adverse selection problem and the cost of VET is not too large, the demand of VET is positive.

Proposition 4. *There is a level $\bar{\beta} \in (0, \alpha)$ such that for any $\beta \in (0, \bar{\beta})$, there exists an equilibrium with positive vocational education, for any $k_1 \in (0, \bar{k})$, with $\bar{k}$ defined in Proposition 3.*

When the cost of VET is not too large relative to the cost of university education, $\bar{e} > \underline{e}$. In this case, some workers find it optimal to pursue VET. In equilibrium, workers with an educational cost e smaller than $\underline{e}$ go to universities and can search for both non-college jobs and college jobs after graduation. Workers with a cost above $\bar{e}$ do not pursue post-secondary education and can only apply to non college jobs. Workers with a educational cost in between $\underline{e}$ and $\bar{e}$ choose VET. Workers with VET credentials can apply to both non-college and VET jobs, but would choose only VET in equilibrium. In this model, while the productivity gains of university education drives some workers to pursue post-secondary education, the entry barrier role of VET also encourages workers with only high school diploma to pursue post-secondary education. Next I discuss the welfare implications of VET.

At period 0, all workers are identical. Consider the situation when there is a demand for VET in the market. A worker makes educational choices based on the realization of her educational cost. With probability $F_e(\underline{e})$, the worker goes to universities for a college degree and pays the educational cost αe. With probability $1 - F_e(\bar{e})$, the worker holds a high school diploma only. With probability $F_e(\bar{e}) - F_e(\underline{e})$, the worker attains post-secondary education for VET and pays the educational cost βe. The ex-ante utility of a worker denoted by U_0 follows:

$$U_0 = F_e(\underline{e})U_c + [F_e(\bar{e}) - F_e(\underline{e})]U_v + [1 - F_e(\bar{e})]U_h - \beta\mathbb{E}[e|\underline{e} \leq e < \bar{e}] - \alpha\mathbb{E}[e|e \leq \underline{e}] \quad (3.12)$$

Since all employers in equilibrium make zero ex-ante profits, I can compare the total welfare of different economies by comparing the levels of U_0. Consider two labour markets with different educational institutions: one does not have VET and the other one offers VET which is costly. The follow proposition states the role of VET on improving welfare:

Proposition 5. *The optional to undertake VET makes workers better off ex-ante, for* $k_1 \in (0, \bar{k})$ *and* $\beta \in (0, \bar{\beta})$.

Introducing costly VET acting as an entry barrier for college workers can improve welfare by solving the adverse selection problem. VET attracts high school workers and workers who would choose university education when VET is absent. The gain from VET for high school workers is that workers with VET credentials gain access to jobs that exclude college workers and receive labour market outcomes that are not distorted by the adverse selection problem. Some workers who would pursue university education in the absence of VET also benefit from VET. These workers have relatively low educational costs amongst all workers so that they choose university level education. However, they have relatively high costs amongst college students so that their gain from the university education net the cost is small. With the option of VET, these workers benefit from applying to VET because even though the return to VET is less than the return to a university credential, the cost they could save from pursuing VET instead of university credential surpasses the difference in returns. So long as the cost to VET is not too large relative to gaining a bachelor degree, introducing VET to the labour market is strictly welfare improving.

One commonly debated form of entry barriers is occupational licensing. According to Kleiner and Krueger (2013), in 2008 nearly 30 percent of the workers with more than high school education, but not a bachelor degree, were required to hold a license. Many popular fields of VET such as health care and collective "trades" fields of manufacturing, construction, repair, and transportation, also prepare students for obtaining occupational licensing or certifications. One concern of the occupational licensing is that as an entry barrier, occupational licensing may cause job losses by increasing employment costs (see Kleiner (2005)). This is because in a market with perfect competitive, an entry barrier is a rent-seeking distortion to the market. However, using the insight of these chapter, I argue that an entry barrier also has an important benefit to employment. In particular, imposing entry barriers on certain occupations provides protections for low to middle-skilled workers from the competition of over-qualified college workers seeking for stepping-stones.

Notice that the mechanism of the displacement of high school workers through adverse selection is the key to the welfare implication on VET. If displacement of high school workers happens in efficient labour market (see Acemoglu and Autor (2011), Beaudry et al. (2016)), a barrier to entry creates monopoly power and is a distortion to the market. Introducing new tasks or jobs that are equally productive as the existing ones but requiring additional training cost is not welfare improving. Even though low skilled workers could benefit from gaining access to jobs that exclude competition of college workers, the economy losses from incurring training cost without increasing productivity.

3.4.3 Post-Secondary Education and the Skill-Biased Technological Changes

In this section, I use some simulations of the model to help understand the effects of important labour market trends on the increase of post-secondary education in the past thirty years. In particular, I consider the effects of skill-biased technological changes (SBTC), which complement skilled cognitive jobs but substitute less-skilled routine jobs, [11] on educational choices. I also discuss the impact of an increase in college tuition on VET.

Benchmark Parameter Values

Assume the matching technology is giving by $M(u,v) = \dfrac{uv}{u+r}$. Condisder the following parameter values.[12] The productivity of non-college jobs y_1 is normalized to 1 and the productivity of college jobs y_2 equals 2.5. I relax the assumption that the productivity of VET jobs being equal to the productivity of non-college jobs and set $y_v = 1.5$. The flow income of unemployment b is set to be 0.1. Time is measured in quarters. Let $r = 0.01$, which is equivalent to an annual discount rate of 0.96. Given that job-to-job transition is endogenous

[11] See Autor et al. (2003) on routine jobs, Goos et al. (2014) on skill biased technological change versus globalization and Autor et al. (2014) on exposure to international trade.

[12] I found the IC constraint is always binding in all simulation examples with different parameters.

in this model, the exogenous separation rate is set to be $\delta = 0.05$, consistent with the finding of Sahin et al. (2010). I assume the search cost c follows exponential distribution with parameter $\lambda = 1$ and the educational cost e follows beta distribution with parameter $(4, 4)$. The cost of posting college vacancies k equal to 0.2. The educational cost rate α and β are set to be 450 and 135 respectively, so that the educational attainment of the benchmark economy has 10 percent college graduates, 10 percent of VET workers and 80 percent high school workers, matching roughly the distribution of educational attainment in the 1970s. [13]

Column 2 in Table 3.2 presents the distribution of educational attainment and the key labour market outcomes of different workers in the benchmark economy. The economy has 80 percent high workers and 10 percent each for college and VET workers. Workers with more education have higher wage and lower unemployment rate. The college-versus-high-school wage ratio is 2.47, matching the college-versus-high-school earnings ratio in the early 80s (Acemoglu and Autor, 2011).

Next, I turn to simulations. I consider a positive shock to the productivity of college jobs, an negative shock to the productivity of non-college jobs and a scenario that combines both shocks with an increase in college tuition. The increase in productivity of VET jobs has straight forward effects and is not examined here.

Table 3.2: Comparative Statics

	Benchmark	Scenario 1	Scenario 2	Scenario 3
Education Attainment				
College	10%	98%	10%	33%
VET	10%	0	52%	30%
High School	80%	2%	38%	37%
Unemployment Rate				
College	6.0%	5.6%	6.0%	5.6%
VET	6.5%	6.5%	6.5%	6.5%
High School	9.6%	9.7%	10.3%	10.7%
Mean Wage				
College	2.42	4.92	2.42	4.92
VET	1.46	1.46	1.46	1.46
High School	0.98	0.98	0.68	0.68

[13] U.S. Bureau of labour Statistics. I use here the group of workers with some college as well as workers with associate degrees to proximate VET workers.

Increased productivity of college jobs

Consider an increase in the productivity of college jobs with $y_2 = 5$. The results are presented in the second column of table 3.2 under Scenario 1. With a 100 percent increase in the productivity of college jobs while keeping the educational cost to university constant, 98 percent of workers would choose to become a college graduate and no worker prefers VET. The positive shock to college jobs has two effects on a worker's ex-ante educational choice. Increasing the productivity of college jobs raises the return to university education directly with a large increase in the wage and a mild decrease in the unemployment rate. It also lowers the return to search for a high school worker through the adverse selection effect. When ex-ante return to search for an unemployed college worker is higher, the worker becomes more "picky" about jobs even when applying for a stepping-stone job. As a result, the workers is more patient to remain unemployed and is less willing to accept a job with a lower wage, making it more difficult for the market to separate workers. The further distortion on high school workers are reflected in the increase of the unemployment rate and a negligible wage rise. The second effect, however, is quantitatively dominated by the first one.

Decreased productivity of non-college jobs

Now consider a fall on the productivity of non-college jobs by 30 percent so that $y_1 = 0.7$. This fall can be thought as automation and offshoring processes that substitute routine-based less-skilled workers. The results are presented in the third column of table 3.2 under Scenario 2. The negative shock reduces the return to search to an unemployed high school worker by decreasing the wage and increasing the unemployment rate. It increases the incentive for a high school graduate to obtain post-secondary education. However, the increase in post-secondary education is captured entirely by a 32 percent increase in VET without changing the fraction of university graduates. The negative shock on non-college jobs has little effect on the return to search for an unemployed college worker. This is because when a non-college jobs is less productive, fewer unemployed college workers choose to apply for a non-college job.

Increased university tuition

Finally, I consider a scenario with the two shocks discussed previously and a 100 percent increase in university cost α. As shown in the last column of Table 3.2 under Scenario 3, the economy has 33 percent college workers, 30 percent VET workers and 37 percent high school workers, similar to the education attainment in the U.S. in 2010.[14] The relative wage

[14] U.S. Bureau of labour Statistics.

ratios between college workers versus VET workers and high school workers are 3.4 and 7.2 respectively. While the increased productivity on college jobs attracts workers to enrol in universities, the negative shock on non-college jobs and the increase in the cost to colleges relative to the cost to VET encourage workers to attain VET instead.

I relate these results to the argument of Autor et al. (2010) who points out that one puzzle in the U.S. labour market is that given the steep rise in the college-versus-high-school earnings ratio, the relative supply of college-educated workers is not growing fast enough. This chapter shows that much of the increase in post-secondary education has taken the form of non-bachelor level post-secondary VET. Because skill-biased technological changes has increased not only the return to college relative to high school but also the return to non-university VET. Together with an increase in college tuition and costs, VET becomes an another popular educational investment amongst students.

3.5 Conclusion

In this chapter, I argue that the labour market generates inefficient unemployment of high school graduates as a mechanism to separate high school graduates from overqualified college graduates searching for some types of routine jobs. This is how the labour market resolves the adverse selection problem arising from the fact that employment contracts in those routine jobs do not discriminate between high school and college graduates. In this context, the demand for vocational education arises because it allows employers to exclude applicants who treat the job as a stepping-stone. As a result, a vocational credential provides high school workers with access to markets that do not suffer from the distortion of over-educated workers and improve their labour market outcomes.

The model helps understand the effects of skill-biased technological change on the increase of post-secondary education in the past thirty years. Using numerical simulations, I show that skill biased technological changes increase the return to post-secondary education at both university level and non-university level in the form of vocational education. Together with an increase in college tuitions, skill biased technological changes can explain both the increase in college education and post-secondary vocational education over the past thirty years.

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Appendix A

Appendices

A.1 Appendices of Chapter 1

A.1.1 Proof of Proposition 1

It is easy to see that an allocation that does not solve problems (P1) and (P2), for a given value of $\rho \in (0, 1]$, cannot be supported by a competitive search equilibrium with positive quits, for the proposed equilibrium would necessarily violate condition (B2) in the equilibrium definition. The discussion leading to Proposition 1 implies that a revealing equilibrium must have $\rho = 1$ whereas a non-revealing equilibrium must have $\rho = 1 - \alpha_i$, as required. **QED**

A.1.2 Proof of Proposition 2

Throughout this proof we maintain the assumption that $\rho = 1$ and we drop the argument ρ from all functions. We keep track of job types under the assumption that unemployed workers search for type-i jobs and employed workers search for type-j jobs, where it is understood that $i = j = 1$ throughout this proof. We first prove existence and uniqueness of the candidate equilibrium allocation. It will become clear that it can be supported by a revealing equilibrium. Then, we show that the revealing equilibrium is constrained efficient.

We begin by characterizing the solution to Problem (P1) as a function of a worker's current wage.

Lemma 1. *Let $s = \{i, w, y_l\}$. For any $w \in [0, y_l]$, $\{w_e(s), q_e(s)\}$ is given by the unique pair (w', q') with $y_l \leq w' < y_h$ and $0 < q_a \leq q \leq q_b < \infty$ that solves the following conditions:*

$$q' f(q') \alpha_j \left(\frac{y_h - w'}{r + \delta} \right) = k,$$

$$\frac{w' - w}{r + \delta + (1-\delta)\alpha_j f(q')} \geq \left(\frac{1-\eta(q')}{\eta(q')}\right)\left(\frac{y_h - w'}{r+\delta}\right)$$

and $q' \geq q_a$ *with complementary slackness, where* q_a *is given by*

$$q_a f(q_a)\alpha_j \left(\frac{y_h - y_l}{r+\delta}\right) = k$$

and $q_b > q_a$ *is given by*

$$\frac{y_h - y_l}{r+\delta} = \left(\frac{k}{q_b f(q_b)\alpha_j}\right)\left(1 + \left(\frac{1-\eta(q_b)}{\eta(q_b)}\right)\left(\frac{r+\delta+(1-\delta)\alpha_j f(q_b)}{r+\delta}\right)\right). \tag{A.1}$$

Proof: The relevant first-order conditions for an interior solution of problem (P1) with $\rho = 1$ are given by:

$$\lambda q' = 1,$$

where λ is the relevant Lagrange multiplier, and

$$\frac{w'}{r+\delta} + \left(\frac{\delta}{r+\delta}\right)\frac{\overline{V}(s_u)}{1+r} - \frac{\overline{V}(\{i, w, y_l\})}{1+r} = \lambda q' \left(\frac{1-\eta(q')}{\eta(q')}\right)\left(\frac{y_h - w'}{r+\delta}\right),$$

together with the zero-profit constraint

$$q' f(q')\alpha_j \left(\frac{y_h - w'}{r+\delta}\right) = k.$$

This is the first condition stated in the lemma. The second condition follows from combining the first two first-order conditions above and the fact that the Bellman equation implies that a solution to the problem must be such that

$$\frac{w'}{r+\delta} + \left(\frac{\delta}{r+\delta}\right)\frac{\overline{V}(s_u)}{1+r} - \frac{\overline{V}(\{i, w, y_l\})}{1+r} = \frac{w' - w}{r+\delta+(1-\delta)\alpha_j f(q')}.$$

Clearly, $w_e(\{i, w, y_l\}) \geq y_l$ if and only if $q_e(\{i, w, y_l\}) \geq q_a$. Our assumption that $(r+\delta)k < \alpha_2(y_h - y_l)$ ensures that $0 < q_a < \infty$.

Combining the two conditions stated in the proposition implies that an interior solution $q_e(\{i, w, y_l\})$ is the unique value of q' that solves

$$\frac{y_h - w}{r+\delta} = \left(\frac{k}{q' f(q')\alpha_j}\right)\left(1 + \left(\frac{1-\eta(q')}{\eta(q')}\right)\left(\frac{r+\delta+(1-\delta)\alpha_j f(q')}{r+\delta}\right)\right). \tag{A.2}$$

It follows that $w \leq y_l$ implies that $q_e(\{i, w, y_l\}) \leq q_b$. Clearly, $\infty > q_b > q_a > 0$. **QED**

Invert (A.2) to express the worker's current wage as a function of q':

$$W(q') \equiv y_h - \left(\frac{k}{q' f(q')\alpha_j}\right)\left(r+\delta+\left(\frac{1-\eta(q')}{\eta(q')}\right)(r+\delta+(1-\delta)\alpha_j f(q'))\right), \tag{A.3}$$

for all $q' \in [q_a, q_b]$.

Lemma 2. $W(q)$ *and* $\overline{V}(\{i, W(q), y_l\})$ *are strictly increasing and concave functions of* q *on* $[q_a, q_b]$.

Proof: It is easy to verify that the Bellman equation for $\overline{V}(\{i, w, y_l\})$ implies that

$$\begin{aligned}\frac{\overline{V}(\{i, W(q), y_l\})}{1+r} &= \left(\frac{\delta}{r+\delta}\right)\frac{\overline{V}(s_u)}{1+r} + \left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q)}\right)\frac{W(q)}{r+\delta} \\ &+ \left(1 - \frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q)}\right)\frac{w_e(\{i, W(q), y_l\})}{r+\delta} \end{aligned} \tag{A.4}$$

and, using the first-order conditions stated in Lemma 1, one can write

$$\frac{\overline{V}(\{i, W(q), y_l\})}{1+r} = \frac{y_h}{r+\delta} + \left(\frac{\delta}{r+\delta}\right)\frac{\overline{V}(s_u)}{1+r} - \frac{k}{\eta(q)\, q f(q)\, \alpha_j}. \tag{A.5}$$

One can verify that

$$\frac{\partial}{\partial q}\left(\frac{\overline{V}(\{i, W(q), y_l\})}{1+r}\right) = \frac{k}{q f(q)\, \alpha_j}\left(\frac{\eta'(q)}{(\eta(q))^2} + \frac{1}{q}\left(\frac{1-\eta(q)}{\eta(q)}\right)\right),$$

which is positive *on* $[q_a, q_b]$. A sufficient condition for it to be strictly decreasing on $[q_a, q_b]$ is that $\eta'(q)/(\eta(q))^2$ is a decreasing function, which follows from the concavity of η. Hence $\overline{V}(\{i, W(q), y_l\})$ is strictly concave *on* $[q_a, q_b]$, as required.

Next, differentiating equation (A.2) with respect to w and q one can verify that

$$\frac{\partial W(q)}{\partial q} = (r+\delta+(1-\delta)\alpha_j f(q))\frac{\partial}{\partial q}\left(\frac{\overline{V}(\{i, W(q), y_l\})}{1+r}\right),$$

which is positive and strictly decreasing on $[q_a, q_b]$, because both f and $\partial\overline{V}/\partial q$ are positive and strictly decreasing on $[q_a, q_b]$. Hence, $W(q)$ is strictly increasing and concave *on* $[q_a, q_b]$, as required. **QED**

Let $M(s)$ denote the match surplus as a function of the worker's state and note that

$$\frac{M(\{i, w, y_h\})}{1+r} = \frac{\overline{V}(\{i, w, y_h\})}{1+r} - \frac{\overline{V}(s_u)}{1+r} + \frac{y_h - w}{r+\delta} \tag{A.6}$$

and

$$\frac{M(\{i, W(q), y_l\})}{1+r} = \frac{\overline{V}(\{i, W(q), y_l\})}{1+r} - \frac{\overline{V}(s_u)}{1+r} + \frac{y_l - W(q)}{r+\delta+(1-\delta)\alpha_j f(q)}. \tag{A.7}$$

Lemma 3. *$M(\{i, w, y_h\})$ is independent of w; $M(\{i, W(q), y_l\})$ is a strictly concave function of q on $[q_a, q_b]$ and it is maximized at $q = q_b$; $M(\{i, W(q), y_l\}) - \overline{V}(\{i, W(q), y_l\})$ is a strictly decreasing and convex function of q on $[q_a, q_b]$.*

Proof: Fix $\overline{V}(s_u)$. Noting that

$$\frac{\overline{V}(\{i, w, y_h\})}{1+r} = \frac{w}{r+\delta} + \frac{\delta}{r+\delta}\frac{\overline{V}(s_u)}{1+r}$$

one can write

$$\frac{M(\{i, w, y_h\})}{1+r} = \frac{y_h}{r+\delta} - \frac{r\overline{V}(s_u)}{1+r},$$

which is independent of q. Using (A.5), together with (A.3) and (A.7), one can write

$$\begin{aligned}\frac{M\left(\{i,W\left(q\right),y_l\}\right)}{1+r} &= \frac{y_h}{r+\delta}-\frac{r\overline{V}\left(s_u\right)}{1+r}-\frac{r+\delta}{r+\delta+\left(1-\delta\right)\alpha_j f\left(q\right)}\left(\frac{y_h-y_l}{r+\delta}\right)\\ &\quad -\left(1-\frac{r+\delta}{r+\delta+\left(1-\delta\right)\alpha_j f\left(q\right)}\right)\left(\frac{k}{qf\left(q\right)\alpha_j}\right). \qquad \text{(A.8)}\end{aligned}$$

where $M\left(\{i,w,y_h\}\right) > M\left(\{i,W\left(q\right),y_l\}\right)$ whenever $y_h > y_l$. Differentiating equation (A.8) one can verify that

$$\begin{aligned}\frac{\partial}{\partial q}\left(\frac{M\left(s\right)}{1+r}\right) &= \left(\frac{1-\delta}{q^2\left[r+\delta+\left(1-\delta\right)\alpha_j f\left(q\right)\right]}\right)\\ &\quad\times\left(\left(1-\eta\left(q\right)\right)k-\left(\frac{\left(r+\delta\right)\eta\left(q\right)}{r+\delta+\left(1-\delta\right)\alpha_j f\left(q\right)}\right)\left(qf\left(q\right)\alpha_j\left(\frac{y_h-y_l}{r+\delta}\right)-k\right)\right).\end{aligned}$$

for $s=\{i,W\left(q\right),y_l\}$. The term in the first line is decreasing in q since both $qf\left(q\right)$ are strictly increasing on $[q_a,q_b]$. The terms in the second line are also decreasing in q since $f\left(q\right)$ is decreasing and $\eta\left(q\right)$ and $qf\left(q\right)$ are increasing on $[q_a,q_b]$, and $qf\left(q\right)\alpha_j\left(y_h-y_l\right)\geq\left(r+\delta\right)k$ for $q\geq q_a$. Hence $M\left(\{i,W\left(q\right),y_l\}\right)$ is strictly concave on $[q_a,q_b]$. It is now easy to verify that equation (A.1) is a necessary and sufficient condition for $\partial M\left(\{i,W\left(q\right),y_l\}\right)/\partial q=0$. Hence $M\left(\{i,W\left(q\right),y_l\}\right)$ is maximized at $q=q_b$.

Using equations (A.5) and (A.8) one can write

$$\begin{aligned}\frac{M\left(s\right)-\left(\overline{V}\left(s\right)-\overline{V}\left(s_u\right)\right)}{1+r} &= \left(\frac{k}{qf\left(q\right)\alpha_j}\right)\left(\frac{r+\delta}{r+\delta+\left(1-\delta\right)\alpha_j f\left(q\right)}+\frac{1-\eta\left(q\right)}{\eta\left(q\right)}\right)\\ &\quad-\left(\frac{y_h-y_l}{r+\delta+\left(1-\delta\right)\alpha_j f\left(q\right)}\right),\end{aligned}$$

for $s=\{i,W\left(q\right),y_l\}$, and differentiating this equation one can verify that

$$\begin{aligned}\frac{\partial}{\partial q}\left(\frac{M\left(s\right)-\left(\overline{V}\left(s\right)-\overline{V}\left(s_u\right)\right)}{1+r}\right) &= \left(\frac{\left(1-\delta\right)\alpha_1 f'\left(q\right)}{\left[r+\delta+\left(1-\delta\right)\alpha_j f\left(q\right)\right]^2}\right)\left(y_h-y_l-\frac{\left(r+\delta\right)k}{qf\left(q\right)\alpha_j}\right)\\ &\quad-\left(\frac{k}{qf\left(q\right)\alpha_j}\right)\left(\left(\frac{1-\eta\left(q\right)}{q}\right)\left(\frac{r+\delta}{r+\delta+\left(1-\delta\right)\alpha_j f\left(q\right)}+\frac{1-\eta\left(q\right)}{\eta\left(q\right)}\right)+\frac{\eta'\left(q\right)}{\left(\eta\left(q\right)\right)^2}\right),\end{aligned}$$

for $s=\{i,W\left(q\right),y_l\}$. The term in the first line of the right side is negative since $f'\left(q\right)<0$ and $qf\left(q\right)\alpha_j\left(y_h-y_l\right)\geq\left(r+\delta\right)k$ for $q\geq q_a$. The term subtracted in the second line is positive since $\eta\left(q\right)<1$ and $\eta'\left(q\right)>0$. Hence $M\left(s\right)-\left(\overline{V}\left(s\right)-\overline{V}\left(s_u\right)\right)$, for $s=\{i,W\left(q\right),y_l\}$, is a strictly decreasing function of q on $[q_a,q_b]$. Moreover, the term in the first line of the right side is an increasing function of q, because $f'\left(q\right)$ and $qf\left(q\right)$ are increasing and $f\left(q\right)$ is decreasing. The term subtracted in the second line is a decreasing function of q, since $qf\left(q\right)$ and $\eta\left(q\right)$ are increasing and $f\left(q\right)$ and $\eta'\left(q\right)/\left(\eta\left(q\right)\right)^2$ are decreasing. Hence, $M\left(s\right)-\left(\overline{V}\left(s\right)-\overline{V}\left(s_u\right)\right)$, for $s=\{i,W\left(q\right),y_l\}$, is a strictly convex function of q. **QED**

Next, note that Problem (P2) can be formulated as

$$\overline{V}(s_u) = b + V_i, \tag{P3}$$

where

$$V_i = \frac{\overline{V}(s_u)}{1+r} + \max_{w,q,q'} \left\{ f(q) \left(\frac{V_0(i,w,q')}{1+r} - \frac{\overline{V}(s_u)}{1+r} \right) \right\}$$

subject to

$$k \leq qf(q) \left(\frac{M_0(i,w,q')}{1+r} - \frac{V_0(i,w,q')}{1+r} + \frac{\overline{V}(s_u)}{1+r} \right),$$

$$q' \in [q_a, q_b],\ w \leq y_h, w \neq W(q')$$

where

$$V_0(i,w,q') = \alpha_i \overline{V}(\{i,w,y_h\}) + (1-\alpha_i)\overline{V}(\{i,W(q'),y_l\}),$$

and

$$M_0(i,w,q') = \alpha_i M(\{i,w,y_h\}) + (1-\alpha_i) M(\{i,W(q'),y_l\}).$$

Let $\left\{w_u^h(i), q_u(i), q_e^l(i)\right\}$ denote a solution to Problem (P3) while disregarding the constraint $w \neq W(q')$. Even though the objective is not concave in $\{w,q,q'\}$, we prove below that the solution is unique (and it is such that $w_u^h(i) \neq W\left(q_e^l(i)\right)$). It is then easy to see that $\left\{w_u(i), W\left(q_e^l(i)\right), q_u(i)\right\}$ solves problem (P2), since $q_e^l(i) = q_e\left(\left\{i, W\left(q_e^l(i)\right), y_l\right\}\right)$.

One can readily verify that an *interior* solution to Problem (P3) is such that the total surplus of the match is maximized. Specifically, it must be that $\partial M_0(i,w,q')/\partial q' = 0$, which requires that $\partial M(\{i,W(q'),y_l\})/\partial q' - 0$. Hence, Lemma 3 implies that $q_e^l(i) = q_b$, where q_b is given by equation (A.1). Comparing (A.1) and (A.2), it follows that $W\left(q_e^l(i)\right) = y_l$. Hence, $w_u^l(i,1) = y_l$ as indicated in (1.6).

Next, note that

$$\begin{aligned} V_i &= (1-f(q_u(i)))\frac{\overline{V}(s_u)}{1+r} + f(q_u(i))\frac{V_0\left(i, w_u^h(i), q_e^l(i)\right)}{1+r} \\ &= (1-f(q_u(i)))\frac{\overline{V}(s_u)}{1+r} + f(q_u(i))\left(\frac{\overline{V}(s_u)}{1+r} + \left(\frac{1-\eta(q_u(i))}{\eta(q_u(i))}\right)\frac{k}{q_u(i)f(q_u(i))}\right), \end{aligned}$$

where the first equality comes from the Bellman equation in Problem (P3) and the second equality follows from the matching-efficiency condition (1.10) and the zero-profit condition (1.9). It follows that

$$V_i = \frac{\overline{V}(s_u)}{1+r} + \left(\frac{1-\eta(q_u(i))}{\eta(q_u(i))}\right)\frac{k}{q_u(i)},$$

which, together with the fact that $\overline{V}(s_u) - b = V_i$, implies that

$$\frac{r\overline{V}(s_u)}{1+r} = b + \left(\frac{1-\eta(q_u(i))}{\eta(q_u(i))}\right)\frac{k}{q_u(i)}.$$

Using this equation, together with equations (1.9) and (1.10) and the fact that

$$\begin{aligned}\frac{V_0\left(i, w_u^h(i), q_e^l(i)\right)}{1+r}-\frac{\overline{V}(s_u)}{1+r} &= \alpha_i \frac{w_u^h(i)}{r+\delta}+(1-\alpha_i)\left(\frac{y_h}{r+\delta}-\frac{k}{\eta\left(q_e^l(i)\right) q_e^l(i) f\left(q_e^l(i)\right) \alpha_j}\right) \\ &\quad -\left(\frac{r}{r+\delta}\right)\frac{\overline{V}(s_u)}{1+r},\end{aligned}$$

it follows that $q_u(i)$ satisfies equation (1.11) in the text.

The right side of (1.11) is strictly decreasing in $q_u(i)$, it converges to ∞ as $q_u(i)$ approaches 0 and it converges to k as $q_u(i)$ approaches ∞. Hence, there is a unique solution $q_u(i) \in (0, \infty)$ that solves the equation if and only if

$$\frac{y_h-b}{r+\delta}-\frac{(1-\alpha_i)k}{\eta(q_b) q_b f(q_b) \alpha_j} > k.$$

There is a number $k_a > 0$ such that this inequality holds for all $k \in (0, k_a)$. To prove this, differentiate (A.1) to verify that

$$\partial q_b/\partial k > 0 \text{ and } \frac{\partial}{\partial k}\left(\frac{k}{\eta(q_b) q_b f(q_b)}\right) > 0,$$

with

$$\lim_{k\to 0} q_b = 0 \text{ and } \lim_{k\to 0}\left\{\frac{k}{\eta(q_b) q_b f(q_b) \alpha_j}\right\} = \frac{y_h-y_l}{r+\delta+(1-\delta)\alpha_j} < \frac{y_h-y_l}{r+\delta} < \frac{y_h-b}{r+\delta}.$$

Next, we verify that $\overline{V}(s_u) \le \min\left\{\overline{V}\left(\left\{i, w_u^h(i), y_h\right\}\right), \overline{V}(\{i, y_l, y_l\})\right\}$. To that end, note that

$$\begin{aligned}\overline{V}(\{i, y_l, y_l\})-\overline{V}(s_u) &= V_0\left(i, w_u^h(i), q_e^l(i)\right)-\overline{V}(s_u) \\ &\quad -\alpha_i\left[\overline{V}\left(\left\{i, w_u^h(i), y_h\right\}\right)-\overline{V}(\{i, y_l, y_l\})\right]\end{aligned}$$

and

$$\begin{aligned}\overline{V}\left(\left\{i, w_u^h(i), y_h\right\}\right)-\overline{V}(s_u) &= V_0\left(j, w_u^h(i), q_e^l(i)\right)-\overline{V}(s_u) \\ &\quad +(1-\alpha_i)\left[\overline{V}\left(\left\{i, w_u^h(i), y_h\right\}\right)-\overline{V}(\{i, y_l, y_l\})\right],\end{aligned}$$

where

$$V_0\left(i, w_u^h(i), q_e^l(i)\right) = \alpha_i \overline{V}\left(\left\{i, w_u^h(i), y_h\right\}\right)+(1-\alpha_j)\overline{V}\left(\left\{i, w_l\left(j, q_e^l(i)\right), y_l\right\}\right),$$

and use the fact that

$$\frac{V_0\left(i, w_u^h(i), q_e^l(i)\right)}{1+r}-\frac{\overline{V}(s_u)}{1+r} = \left(\frac{1-\eta(q_u(i))}{\eta(q_u(i))}\right)\frac{k}{q_u(i) f(q_u(i))}$$

and the fact that

$$\frac{\overline{V}\left(\left\{i, w_u^h(i), y_h\right\}\right)}{1+r}-\frac{\overline{V}(\{i, y_l, y_l\})}{1+r} = \frac{k}{\eta(q_b) q_b f(q_b) \alpha_j}-\left(\frac{k}{\alpha_i q_u(i) f(q_u(i))}\right)$$

to write

$$\frac{\overline{V}(\{i, y_l, y_l\})-\overline{V}(s_u)}{1+r} = \frac{k}{\eta(q_u(i)) q_u(i) f(q_u(i))}-\alpha_i\left(\frac{k}{\eta(q_b) q_b f(q_b) \alpha_j}\right), \tag{A.9}$$

where

$$\frac{\overline{V}\left(\left\{i, w_u^h\left(i\right), y_h\right\}\right) - \overline{V}\left(s_u\right)}{1+r} = \left(\frac{1}{\eta\left(q_u\left(i\right)\right)} - \frac{1}{\alpha_i}\right)\frac{k}{q_u\left(i\right)f\left(q_u\left(i\right)\right)} + \left(1-\alpha_i\right)\left(\frac{k}{\eta\left(q_b\right)q_b f\left(q_b\right)\alpha_j}\right).$$

Differentiating equation (1.11), one can verify that $\partial q_u\left(i\right)/\partial k > 0$, with

$$\lim_{k\to 0} q_u\left(i\right) = \lim_{k\to 0}\left\{\frac{k}{q_u\left(i\right)f\left(q_u\left(i\right)\right)}\right\} = 0$$

and

$$\lim_{k\to 0}\left\{\frac{k}{\eta\left(q_u\left(i\right)\right)q_u\left(i\right)f\left(q_u\left(i\right)\right)}\right\} = \left(\frac{r+\delta}{1+r+\delta}\right)\left(\frac{y_h-b}{r+\delta} - \left(1-\alpha_j\right)\frac{y_h-y_l}{r+\delta+\left(1-\delta\right)\alpha_j}\right).$$

It follows that there is a number $k_b > 0$ such that $\overline{V}\left(\left\{i, w_u^h\left(i\right), y_h\right\}\right) > \overline{V}\left(s_u\right)$ for all $k \in (0, k_b)$. Moreover,

$$\lim_{k\to 0}\left\{\frac{\overline{V}\left(\left\{i, y_l, y_l\right\}\right) - \overline{V}\left(s_u\right)}{1+r}\right\} = \frac{y_h-b}{1+r+\delta} - \left(\frac{\alpha_j+r+\delta}{1+r+\delta}\right)\left(\frac{y_h-y_l}{r+\delta+\left(1-\delta\right)\alpha_j}\right).$$

This limit is positive if and only if $\left(y_h-b\right)/\left(y_h-y_l\right) \geq \left(r+\delta+\alpha_j\right)/\left(r+\delta+\left(1-\delta\right)\alpha_j\right)$, which is ensured by the assumption in Proposition 2. It follows that there is a number $k_c > 0$ such that $\overline{V}\left(\left\{i, y_l, y_l\right\}\right) > \overline{V}\left(s_u\right)$ for all $k \in (0, k_c)$.

Furthermore, note that

$$\lim_{k\to 0}\left\{\frac{\overline{V}\left(\left\{i, w_u^h\left(i\right), y_h\right\}\right)}{1+r} - \frac{\overline{V}\left(\left\{i, y_l, y_l\right\}\right)}{1+r}\right\} = \frac{y_h-y_l}{r+\delta+\left(1-\delta\right)\alpha_j} > 0,$$

which implies that $\lim_{k\to 0} w_u^h\left(i\right) > y_l$.

The above arguments together imply that there is a number $k_0 > 0$ such that $k \in (0, k_0)$ is sufficient for $q_u\left(i\right) \in \left(0, \infty\right)$ and $\overline{V}\left(s_u\right) \leq \min\left\{\overline{V}\left(\left\{i, w_u^h\left(i\right), y_h\right\}\right), \overline{V}\left(\left\{i, y_l, y_l\right\}\right)\right\}$, and $w_u^h\left(i\right) > y_l$. Since $w_u^h\left(i\right) \neq y_l$, equilibrium wages reveal the current productivity of employed workers, as required.

It is straightforward to characterize ψ. The unemployment rate is given by

$$\psi\left(s_u\right) = \frac{\delta}{\delta+f\left(q_u\right)},$$

where $q_u = q_u(i, 1)$. The wage distribution has three mass points: two wages for workers who find jobs out of unemployment — a wage w_u^h for those who are well matched and a wage w_u^l for those who are mismatched — and a wage w_e for those previously mismatched workers who find jobs out of employment. The mass of workers earning the wage w_u^l is

$$\psi\left(\left\{i, w_u^l, y_l\right\}\right) = \left(\frac{\left(1-\alpha_i\right)f\left(q_u\right)}{\delta+\left(1-\delta\right)\alpha_j f\left(q_e\right)}\right)\psi\left(s_u\right),$$

where $w_u^l = w_u(i, y_l, 1)$ and $q_e = q_e\left(\{j, w_u^l, y_l\}, 1\right)$. The mass of workers earning the wage w_u^h is

$$\psi\left(\left\{w_u^h, y_h\right\}\right) = \left(\frac{\alpha_i f\left(q_u\right)}{\delta}\right)\psi\left(s_u\right)$$

and the mass of workers earning the wage w_e is

$$\psi\left(\{j, w_e, y_h\}\right) = \left(\frac{(1-\delta)\,\alpha_j f\left(q_e\right)}{\delta}\right)\psi\left(\left\{i, w_u^l, y_l\right\}\right).$$

One can verify that $f\left(q_u\right)$ is an increasing function of y_l, y_h and α_1, and $f\left(q_e\right)$ is an increasing function of $(y_h - y_l)$ and α_1 in the revealing equilibrium.

It is straightforward to verify that there are functions Q and μ that support the allocation characterized by equations (1.6)-(1.9) and (1.11). If $s \in S^*$, beliefs must be correct and the construction of Q is standard. If $s \notin S^*$, beliefs are arbitrary and we may assume that employers believe that $\mu\left(s\,|x\right) = 0$ for all $s \notin S^*$.

Lemma 4. *The revealing equilibrium allocation maximizes the present value of aggregate production net of search costs.*

Proof: First, note that the state of the economy at the beginning of each period can be summarized by $\{u, m\}$, where $u \in [0,1]$ is the measure of unemployed workers, and $m : \{y_l, y_h\} \to [0,1]$, where $m(y)$ denotes the measure of employed workers with match productivity y. Let $p(y)$ denote the probability with which a match has productivity realization y. Let $x_u(y)$ denote the probability with which a meeting between an unemployed worker and a job is turned into a match given the productivity realization y, and $x_e(y'|y)$ denote the probability with which a meeting between a worker and a job with productivity realization y' is turned into a match given that the worker is currently employed in a job with match productivity y. Finally, let q_u denote the labour market queue where unemployed workers search for jobs, and $q_e(y)$ denote the labour market queue where employed workers search given that they are currently employed in jobs with productivity y.

Aggregate output can be written as:

$$Y(u,m) = bu + \sum_y ym(y) - k_u\frac{u}{q_u} - (1-\delta)k_e\sum_y\frac{m(y)}{q_e(y)}. \tag{A.10}$$

Denote by $\hat{u}$ the measure of unemployed workers one period ahead, and by $\hat{m}(y)$ the measure of employed workers with match productivity y one period ahead. Then,

$$\hat{u} = \left(1 - \sum_y f(q_u)x_u(y)\right)u + \delta\sum_y m(y), \tag{A.11}$$

and,

$$\begin{aligned}\hat{m}(y) &= p(y)f(q_u)x_u(y)u + (1-\delta)m(y)\left[1 - p(y)f(q_e(y))x_e(y'|y)\right] \\ &+ (1-\delta)\sum_{y'} m(y')p(y)f(q_e(y'))x_e(y|y').\end{aligned} \tag{A.12}$$

The allocation that maximizes aggregate output net of search costs can be characterized as the solution to the planning problem:

$$J(u,m) = \max_{q_u,x_u,q_e,x_e} \left\{ Y(u,m) + \frac{J(\hat{u},\hat{m})}{1+r} \right\}, \tag{A.13}$$

subject to equations (A.10)-(A.12). $J(u,m)$ is the unique solution to the planner's problem and can be written as:

$$J(u,m) = J_u u + \sum_y m(y) J_e(y),$$

where

$$J_u = \max_{q_u,x_u} b - \frac{k_u}{q_u} + \sum_y p(y) f(q_u) x_u(y) \frac{J_e(y)}{1+r} + \left(1 - \sum_y p(y) f(q_u) x_u(y)\right) \frac{J_u}{1+r}. \tag{A.14}$$

and

$$\begin{aligned} J_e(y) \quad &= \quad \max_{x_e,q_e} y - (1-\delta)\frac{k_e}{q_e(y)} + \delta \frac{J_u}{1+r} + (1-\delta)\left[1 - \sum_{y'} p(y') f(q_e(y)) x_e(y'|y)\right] \frac{J_e(y)}{1+r} \\ &+ \quad (1-\delta) \sum_{y'} p(y') f(q_e(y)) x_e(y'|y) \frac{J_e(y')}{1+r}. \end{aligned} \tag{A.15}$$

It is easy to verify that at the optimum $q_e(y_h) = \infty$. This implies:

$$J_e(y_h) = y_h + \delta \frac{J_u}{1+r} + (1-\delta)\frac{J_e(y_h)}{1+r} > J_e(y_l), \tag{A.16}$$

It is also easy to verify that $x_e(y_h|y_l) = 1$ and $x_e(y_l|y_l) \in [0,1]$ at the optimum. This means that the planner's problem has multiple solutions, all of which yield the same optimal value. The multiplicity concerns the probability with which the planner instructs workers to accept or reject lateral job moves. We characterize the solution when $x_e(y_l|y_l) = 0$.

The necessary condition of (A.15) with respect to $q_e(y_l)$ can be written:

$$\left[\frac{J_e(y_h)}{1+r} - \frac{J_e(y_l)}{1+r}\right] = \frac{k_e}{\alpha_1 q_e(y_l) f(q_e(y_l)) \eta(q_e(y_l))} \tag{A.17}$$

and the Bellman equation for $J_e(y_l)$ gives:

$$\frac{J_e(y_l)}{1+r} = \frac{1}{r+\delta+(1-\delta)f(q_e(y_l))\alpha_1}\left(y_l - y_h - (1-\delta)\frac{k_e}{q_e(y_l)}\right) + \frac{J_e(y_h)}{1+r}$$

The above equations, along with the expression for $J_e(y_h)$, yield equation (A.1), which defines the equilibrium value of $q_e(y_l)$.

Conjecture that $x_u(y) = 1$ for $y = \{y_l, y_h\}$. The necessary condition of (A.14) with respect to q_u can be written:

$$\frac{r}{r+\delta}\frac{J_u}{1+r} = \frac{y_h}{r+\delta} - \frac{k_u}{q_u f(q_u)\eta(q_u)} - \frac{(1-\alpha_1)k_e}{\eta(q_b) q_b f(q_b)\alpha_1}. \tag{A.18}$$

From the Bellman equation for J_u:

$$\frac{rJ_u}{1+r} = b + \frac{k_u}{q_u}\left(\frac{1-\eta(q_u)}{\eta(q_u)}\right). \tag{A.19}$$

Combining these two equations yields equation (1.11) from the text, where $i = j = 1$, and $q_e(s, 1) = q_b$. This defines the equilibrium value of q_u.

To show that $x_u(y) = 1$ for $y = \{y_l, y_h\}$, combine equations (A.17) and (A.18) to obtain:

$$\frac{J_e(y_l)}{1+r} - \frac{J_u}{1+r} = \frac{k_u}{q_u f(q_u)\eta(q_u)} - \alpha_1 \frac{k_e}{q_b f(q_b)\eta(q_b)\alpha_1}.$$

The right side is identical to the right side of equation (A.9) and is therefore positive under the same conditions. Since $J_e(y_h) > J(y_l)$, if follows that when all low productivity matches are accepted, all high productivity matches are accepted. **QED**

This concludes the proof of Proposition 2. **QED**

A.1.3 Proof of Proposition 3

This proof parallels that of the first part of Proposition 2. Throughout the proof we maintain the assumption that $\rho = 1 - \alpha_i$ and we drop the argument ρ from all functions. As before, we keep track of job types under the assumption that unemployed workers search for type-i jobs and employed workers search for type-j jobs, where it is understood that $i = j = 1$ throughout this proof. We first prove existence and uniqueness of the candidate equilibrium allocation. Then we show how it can be supported by a non-revealing equilibrium.

The first-order conditions for an interior solution of Problem (P1) are given in the main text. We now have that $q_e(s) = q_e(\{i, w, y_l\}) = q_e(\{i, w, y_h\})$ and it is easy to verify that $q_e(s) \in [\widehat{q}_a, \widehat{q}_b]$, where $w_e(s) \geq y_l$ if and only if $q_e(s) \geq \widehat{q}_a$ and $\widetilde{W}(q_e(s)) \leq y_l$ if and only if $q_e(s) \leq \widehat{q}_b$ and where $\widehat{q}_a$ and $\widehat{q}_b$ are given by

$$\widehat{q}_a f(\widehat{q}_a)(1-\alpha_i)\alpha_j \left(\frac{y_h - y_l}{r+\delta}\right) = k \tag{A.20}$$

and

$$\frac{y_h - y_l}{r+\delta} = \left(\frac{k}{\widehat{q}_b f(\widehat{q}_b)(1-\alpha_i)\alpha_j}\right)\left(1 + \left(\frac{1-\eta(\widehat{q}_b)}{\eta(\widehat{q}_b)}\right)\left(\frac{r+\delta+(1-\delta)\alpha_j f(\widehat{q}_b)}{r+\delta}\right)\right) \tag{A.21}$$

respectively. Clearly, $\infty > \widehat{q}_b > \widehat{q}_a > 0$.

Proceeding as before, Problem (P2) can be formulated in the present case as

$$\overline{V}(s_u) = b + V_i, \tag{$P4$}$$

where

$$V_i = \frac{\overline{V}(s_u)}{1+r} + \max_{q,q'} \left\{ f(q) \left(\frac{\widetilde{V}_0(i, q')}{1+r} - \frac{\overline{V}(s_u)}{1+r}\right)\right\}$$

subject to

$$k \leq qf(q)\left(\frac{\widetilde{M}_0(i,q')}{1+r} - \frac{\widetilde{V}_0(i,q')}{1+r} + \frac{\overline{V}(s_u)}{1+r}\right),$$

$$q' \in [q_a, q_b],\ w \leq y_h,$$

where $\widetilde{V}_0(i,q')$ and $\widetilde{M}_0(i,q')$ are defined in the main text. Let $\{q_u(i), q_e^l(i)\}$ denote a solution to Problem ($P4$).

Noting that

$$\frac{\widetilde{V}_0(i,q')}{1+r} - \frac{\overline{V}(s_u)}{1+r} = \frac{y_h}{r+\delta} - \left(\frac{r}{r+\delta}\right)\frac{\overline{V}(s_u)}{1+r} - \frac{k}{\eta(q')q'f(q')(1-\alpha_i)\alpha_j},$$

and using (1.12)-(1.13) and the definition of $\widetilde{M}\left(\left\{i, \widetilde{W}(q'), y_l\right\}\right)$ given in the main text, one can verify that

$$\begin{aligned}\frac{\widetilde{M}_0(i,q')}{1+r} &= \frac{y_h}{r+\delta} - \left(\frac{r}{r+\delta}\right)\frac{\overline{V}(s_u)}{1+r} \\ &\quad - (1-\alpha_i)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q')}\right)\left(\frac{y_h-y_l}{r+\delta}\right) \\ &\quad - (1-\alpha_i)\left(1 - \frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q')}\right)\left(\frac{k}{q'f(q')(1-\alpha_i)\alpha_j}\right).\end{aligned}$$

Lemma 5. *(i) $\widetilde{W}(q)$ and $\widetilde{V}_0(i,q)$ are strictly increasing and concave functions of q on $[\widehat{q}_a, \widehat{q}_b]$. (ii) $\widetilde{M}_0(i,q)$ is a strictly concave function of q on $[\widehat{q}_a, \widehat{q}_b] \subset (0,\infty)$ and it is maximized at $q = \widehat{q}_b$; $\widetilde{M}_0(i,q) - \widetilde{V}_0(i,q)$ is a strictly decreasing and convex function of q on $[\widehat{q}_a, \widehat{q}_b]$.*

Proof: It replicates the arguments in Proposition 2 with minor changes. **QED**

The first-order conditions for an interior solution of Problem ($P4$) are given by equations (1.14)-(1.16) in the main text.

Following similar steps as in the proof of Proposition 2 one can verify that an interior solution to Problem ($P4$) satisfies

$$\frac{y_h-b}{r+\delta} - \frac{k}{\eta(q')q'f(q')(1-\alpha_i)\alpha_j} = \lambda_i q\left(\frac{1-\eta(q)}{\eta(q)}\right)\frac{k}{q}\left(\frac{1}{f(q)} + \frac{1}{r+\delta}\right), \tag{A.22}$$

where λ_i is given by (1.15), and

$$\begin{aligned}\frac{k}{qf(q)} &= -(1-\alpha_i)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(q')}\right)\left(\frac{y_h-y_l}{r+\delta}\right) \\ &+ \left(\frac{k}{q'f(q')(1-\alpha_i)\alpha_j}\right)\left(\frac{(1-\alpha_i)(r+\delta)}{r+\delta+(1-\delta)\alpha_j f(q')} + \frac{1-\eta(q')}{\eta(q')} + \alpha_i\right).\end{aligned} \tag{A.23}$$

Lemma 6. *Assume that $(r+\delta)k < (1-\alpha_1)\alpha_1(y_h-y_l)$. Equations (1.15), (A.22) and (A.23) have a unique solution (λ_i, q, q'), with $q \in (0,\infty)$, $q' \in (q_c, q_d)$, and $\lambda_i q \geq 1$, where*

$$\frac{y_h-b}{r+\delta} - \frac{k}{\eta(q_c)q_c f(q_c)(1-\alpha_i)\alpha_j} = 0,$$

$$\frac{\widetilde{M}_0(i, q_d)}{1+r} - \frac{\widetilde{V}_0(i, q_d)}{1+r} + \frac{\overline{V}(s_u)}{1+r} = k$$

and where $q_c < \widehat{q}_b < q_d$.

Proof: Differentiating equation (1.15) one can verify that the following inequality is necessary and sufficient for $\partial \lambda_i q / \partial q' < 0$:

$$\frac{-\partial^2 \widetilde{M}_0 / \partial q'^2}{-\partial^2 \widetilde{V}_0 / \partial q'^2} > \frac{\partial \widetilde{M}_0 / \partial q'}{\partial \widetilde{V}_0 / \partial q'}.$$

The left side of the inequality is greater than one, since $\widetilde{M}_0 - \widetilde{V}_0$ *is a strictly convex function of* q'. The right side is smaller than one, since $\widetilde{M}_0 - \widetilde{V}_0$ *is a strictly decreasing function of* q'. Hence, $\partial \lambda_i q / \partial q' < 0$. Moreover, note that $\lambda_i q \geq 1$ if and only if $\partial \widetilde{M}_0 / \partial q' \geq 0$. Accordingly, (A.22) characterizes q as a strictly decreasing function of q', where the right side converges to 0 as q approaches ∞ and it converges to ∞ as q approaches 0. Thus, $\infty > q > 0$ if and only if $q' > q_c$. Similarly, (A.23) characterizes q as a strictly increasing function of q', where the left side converges to ∞ as q approaches 0 and it converges to k as q approaches ∞. Thus, $\infty > q > 0$ if and only if $q' < q_d$. Together, (A.22)-(A.23) imply that $q' \in (q_c, q_d)$ and, hence, $\infty > q > 0$.

To verify that $\widehat{q}_b < q_d$, write (A.21) as

$$\begin{aligned} \frac{\alpha_i k}{\eta(\widehat{q}_b)\, \widehat{q}_b f(\widehat{q}_b)(1-\alpha_i)\alpha_j} \quad &= \quad -(1-\alpha_j)\left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j f(\widehat{q}_b)}\right)\left(\frac{y_h - y_l}{r+\delta}\right) \\ &+ \left(\frac{k}{\widehat{q}_b f(\widehat{q}_b)(1-\alpha_i)\alpha_j}\right)\left(\frac{(1-\alpha_i)(r+\delta)}{r+\delta+(1-\delta)\alpha_j f(\widehat{q}_b)} + \frac{1-\eta(\widehat{q}_b)}{\eta(\widehat{q}_b)} + \alpha_i\right). \end{aligned}$$

Comparing this with (A.23), it follows that $\widehat{q}_b < q_d$ if and only if

$$\frac{\alpha_i}{\eta(\widehat{q}_b)\, \widehat{q}_b f(\widehat{q}_b)(1-\alpha_i)\alpha_j} > 1.$$

A sufficient condition for this is $\alpha_i = \alpha_j$, which is the case here. Hence, $\widehat{q}_b < q_d$.

To verify that $\widehat{q}_b > q_c$, note that (A.21) implies that

$$\frac{k}{\eta(\widehat{q}_b)\, \widehat{q}_b f(\widehat{q}_b)(1-\alpha_i)\alpha_j} < \frac{y_h - y_l}{r+\delta},$$

which, together with the fact that $y_l > b$, implies that $\widehat{q}_b > q_c$. **QED**

If an interior non-revealing equilibrium exists, it is uniquely characterized by equations (1.12), (1.13), (1.15), (A.22) and (A.23). Recall that $w_e(s) \geq y_l$ if and only if $q_e(s) \geq \widehat{q}_a$ and $\widetilde{W}(q_e(s)) \leq y_l$ if and only if $q_e(s) \leq \widehat{q}_b$, but we know only that $q' \in (q_c, q_d)$. Hence, we need to verify that the candidate interior solution for q' is such that $q' \in [\widehat{q}_a, \widehat{q}_b]$.

There are only three possible solutions to problems (P1) and (P2) with $\rho = 1 - \alpha_i$. One is the interior allocation characterized above, provided that it is such that $q' \in [\widehat{q}_a, \widehat{q}_b]$. Another is the corner allocation that solves $q' \in \widehat{q}_b$, $\widetilde{W}(q') = y_l$, together with (1.12) and (A.23). In principle, a third possibility is the corner allocation such that $q' \in \widehat{q}_a$ and $w_e(s) = y_l$. However, it is easy to verify that there is a number $k_e > 0$ such that this case will never arise whenever $k \in (0, k_e)$, which is the relevant case below.

Therefore, in order to construct an equilibrium, consider the other two possible allocations and select the one that provides unemployed workers with the higher welfare. It is straightforward to characterize ψ. The unemployment rate is given by

$$\psi(s_u) = \frac{\delta}{\delta + f(q_u)},$$

where $q_u = q_u(i, 1-\alpha_i)$. The wage distribution has two mass points: one wage w_u for workers who find jobs out of unemployment, and one wage w_e for workers who find jobs via on-the-job search. The mass of workers earning the wage w_u is

$$\psi(\{i, w_u, y_l\}) + \psi(\{i, w_u, y_h\}) = \left(\frac{f(q_u)}{\delta + (1-\delta)\alpha_j f(q_e)}\right)\psi(s_u),$$

where $w_u = w_u(i, y_l, 1-\alpha_i) = w_u(i, y_h, 1-\alpha_i)$ and $q_e = q_e(\{j, w_u, y_l\}, 1-\alpha_i) = q_e(\{j, w_u, y_h\}, 1-\alpha_i)$. The mass of workers earning the wage w_e is

$$\psi(\{j, w_e, y_l\}) + \psi(\{j, w_e, y_h\}) = \left(\frac{(1-\delta)\alpha_j f(q_e)}{\delta}\right)[\psi(\{i, w_u, y_l\}) + \psi(\{i, w_u, y_h\})],$$

where $w_e = w_e(\{j, w_u(i, 1-\alpha_i), y_h\}, 1-\alpha_i) = w_e(\{j, w_u(i, 1-\alpha_i), y_l\}, 1-\alpha_i)$.

Furthermore, one can verify that $f(q_u)$ is an increasing function of y_l, y_h and α_1, and $f(q_e)$ is an increasing function of $(y_h - y_l)$ and α_1.

It remains to prove that there are mappings Q and μ that support the candidate equilibrium allocation.The construction of Q is standard. If $s \in S^*$, beliefs must be correct. If $s \notin S^*$, beliefs are arbitrary and we may assume that employers believe that $\mu(s|x) = 0$ for all $s \notin S^*$. It only remains to prove that a type-i employer posting a contract offering revealing wages will not attract any unemployed workers while making non-negative profits. We show that there is a number $\widehat{k}_d > 0$ such that this is the case for all $k \in \left(0, \widehat{k}_d\right)$. To see why, it is sufficient to consider the case where potential poachers will never hire workers with $s \notin S^*$. In this case, one can verify that the value of searching for a revealing contract to an unemployed worker, denoted by V_i^d is continuous in k with

$$\lim_{k\to 0} \frac{V_i^d}{1+r} = \alpha_i \frac{y_h}{r+\delta} + (1-\alpha_i)\frac{y_l}{r+\delta} + \left(\frac{r}{r+\delta}\right)\frac{\overline{V}(s_u)}{1+r},$$

whereas the candidate equilibrium allocation has

$$\lim_{k\to 0} \frac{\widehat{V}_i}{1+r} = \left(1 - \frac{r+\delta}{r+\delta+(1-\delta)\alpha_j}\right)\frac{y_h}{r+\delta} + \left(\frac{r+\delta}{r+\delta+(1-\delta)\alpha_j}\right)\frac{y_l}{r+\delta} + \left(\frac{r}{r+\delta}\right)\frac{\overline{V}(s_u)}{1+r}.$$

Hence, $\lim_{k\to 0} \widehat{V}_i > \lim_{k\to 0} V_i^d$ if and only if $(1-\alpha_i)\alpha_j/\alpha_i > (r+\delta)/(1-\delta)$. Since $\alpha_i = \alpha_j$, all that is needed is $(1-\alpha_i)(1-\delta) > (r+\delta)$ as assumed in the proposition. **QED**

A.1.4 Proof of Proposition 4

Replicating the approach we followed in the proofs of propositions 2 and 3 for the two-job economy, one can verify that an interior equilibrium allocation where entry wages are revealing in type-2 matches, but not in type-1 matches, provided that it exists, can be constructed as follows.

Step 1. For a given value of $\widehat{c}$, find values of q_1, q_1', q_2 and q_2' such that q_2 solves

$$\frac{y_h - b}{r+\delta} - \frac{(1-\alpha_2)\,k}{\eta(q_b)\,q_b f(q_b)\,\alpha_1} = \frac{F(\widehat{c})\,(\widehat{c} - \mathbb{E}(c\,|c \leq \widehat{c})) - \widehat{c}}{r+\delta} + \frac{k}{\eta(q_2)\,q_2 f(q_2)} + \left(\frac{1-\eta(q_2)}{\eta(q_2)}\right)\frac{k}{(r+\delta)\,q_2}, \tag{A.24}$$

with $q_2' = q_b$, and (q_1, q_1') solve

$$\frac{y_h - b}{r+\delta} - \frac{k}{\eta(q_1')\,q_1' f(q_1')\,(1-\alpha_1)\,\alpha_1} = \frac{F(\widehat{c})\,(\widehat{c} - \mathbb{E}(c\,|c \leq \widehat{c}))}{r+\delta} + \lambda_1 q_1 \left(\frac{1-\eta(q_1)}{\eta(q_1)}\right)\frac{k}{q_1}\left(\frac{1}{f(q_1)} + \frac{1}{r+\delta}\right), \tag{A.25}$$

where λ_1 is given by the analogue of (1.15), for $i = 1$, and

$$\frac{k}{q_1 f(q_1)} = -(1-\alpha_1)\left(\frac{r+\delta}{r+\delta+(1-\delta)\,\alpha_1 f(q_1')}\right)\left(\frac{y_h - y_l}{r+\delta}\right) + \left(\frac{k}{q_1' f(q_1')\,(1-\alpha_1)\,\alpha_1}\right)\left(\frac{(1-\alpha_1)(r+\delta)}{r+\delta+(1-\delta)\,\alpha_1 f(q_1')} + \frac{1-\eta(q_1')}{\eta(q_1')} + \alpha_1\right). \tag{A.26}$$

Step 2. Use those values of q_1, q_1', q_2 and q_2' to calculate the implied values of V_1 and V_2 as a function of $\widehat{c}$ and let

$$D(\widehat{c}) = V_2 - V_1. \tag{A.27}$$

Step 3. We are seeking to establish existence of a fixed point

$$D(c_0) = c_0 > 0. \tag{A.28}$$

Similarly, one can verify that a corner equilibrium allocation where entry wages are revealing in type-2 matches, but not in type-1 matches, with $\widetilde{W}(q_1') = y_l$, provided that it exists, can be constructed following the same steps, except that equation (A.25) is replaced with $q_1' = \widehat{q}_b$. As before, it is easy to verify that there is a number $k_f > 0$ such that these two cases exhaust all feasible cases whenever $k \in (0, k_f)$, which is the relevant case below.

Suppose that $\widehat{c} = 0$, in which case all arguments in propositions 2 and 3 hold. First, suppose the solution at $\widehat{c} = 0$ is interior. Clearly there is a number $\alpha_a \in (0, \alpha_1)$ such that $D(0) > 0$ for all $\alpha_2 \in (\alpha_a, \alpha_1)$. Now start increasing the value of $\widehat{c}$. Following the same arguments we used in the proof of Proposition 3, one can verify that there is a number $c_a \in (0, \infty)$ such that there is a solution to the equations above with $q' \in (\widetilde{q}_c(\widehat{c}), \widetilde{q}_d)$ where $\widetilde{q}_c(\widehat{c}) < \widehat{q}_b < \widetilde{q}_d$, for all $\widehat{c} \in [0, c_a)$. There are only two possibilities. If there exists an interior equilibrium, then there is some value $c_0 \in (0, c_a)$ such that $D(c_0) = c_0$.

Otherwise, it must be that $q_1' = \widehat{q}_b$. Replicating the arguments in the proof of Proposition 2, one can verify that, for given $\widehat{c}$, there is a number $\widehat{k}_g > 0$ such that there is a unique value of $q_2 \in (0, \infty)$ that solves equation (A.24), with $q_2' = q_b$, for all $k \in \left(0, \widehat{k}_g\right)$. Moreover, $q_2 > 0$, and thus, V_2 is bounded, for all $\widehat{c} \geq 0$ since $\lim_{\widehat{c} \to \infty} [\widehat{c} - F(\widehat{c})(\widehat{c} - \mathbb{E}(c \,|c \leq \widehat{c}))] = 1/\theta < \infty$. Similarly, the arguments in the proof of Proposition 3 imply that a non-revealing equilibrium allocation in the market for type-2 jobs can be supported for sufficiently small values of $k > 0$, provided that $(1 - \alpha_1)(1 - \delta) > (r + \delta)$, as assumed in the proposition. Since V_2 remains bounded as we increase $\widehat{c}$, there must exist a fixed point $D(c_0) = c_0$.

It is now straightforward to characterize ψ. The unemployment rate is given by

$$\psi(s_u) = \frac{\delta}{\delta + (1 - F(c_0)) f(q_u(1, \cdot)) + F(c_0) f(q_u(2, \cdot))}.$$

The wage distribution across type-2 jobs consists of two mass points. The mass of workers earning the wage $w_u^l(2, \cdot) = y_l$ is

$$\psi(\{2, y_l, y_l\}) = \left(\frac{F(c_0)(1 - \alpha_2) f(q_u(2, \cdot))}{\delta + (1 - \delta)\alpha_1 f(q_e^l(2, \cdot))}\right) \psi(s_u);$$

the mass of workers earning the wage $w_u^h(2, \cdot)$ is

$$\psi\left(\left\{2, w_u^h(2, \cdot), y_h\right\}\right) = \left(\frac{F(c_0)\alpha_2 f(q_u(2, \cdot))}{\delta}\right) \psi(s_u).$$

The wage distribution across type-1 jobs consists of three mass points. The mass of workers earning the wage $w_e(\{2, y_l, y_l\}, \cdot)$ is

$$\psi(\{1, w_e(\{2, y_l, y_l\}, \cdot), y_h\}) = \left(\frac{(1 - \delta)\alpha_1 f\left(q_e^l(2, \cdot)\right)}{\delta}\right) \psi(\{2, y_l, y_l\});$$

the mass of workers earning the wage $w_u^l(1, \cdot) = w_u^h(1, \cdot)$ is

$$\psi\left(\left\{1, w_u^h(1, \cdot), y_h\right\}\right) + \psi\left(\left\{1, w_u^l(1, \cdot), y_l\right\}\right) = \left(\frac{(1 - F(c_0)) f(q_u(1, \cdot))}{\delta + (1 - \delta)\alpha_1 f(q_e^l(1, \cdot))}\right) \psi(s_u),$$

where $(1 - \alpha_1)\psi\left(\left\{1, w_u^h(1, \cdot), y_h\right\}\right) = \alpha_1 \psi\left(\left\{1, w_u^l(1, \cdot), y_l\right\}\right)$; the mass of workers earning the wage $w_e\left(\left\{1, w_u^l(1, \cdot), y_l\right\}\right) = w_e\left(\left\{1, w_u^h(1, \cdot), y_h\right\}\right)$ is

$$\psi\left(\left\{1, w_e\left(\left\{1, w_u^l(1, \cdot), y_l\right\}\right), y_h\right\}\right) = \frac{(1 - \delta)\alpha_1 f\left(q_e^l(1, \cdot)\right)}{\delta} \left(\frac{(1 - F(c_0)) f(q_u(1, \cdot))}{\delta + (1 - \delta)\alpha_1 f(q_e^l(1, \cdot))}\right) \psi(s_u).$$

This concludes the proof of Proposition 4. **QED**

A.1.5 Proof of Proposition 5

An equilibrium where both types of jobs are created must be such that $V_2 - V_1 = c_0 > 0$. Part (i) then follows from the fact that $V(s_u) - b = V_1 + F(c_0)(V_2 - \mathbb{E}(c \,|c \leq c_0) - V_1)$. Part (i) follows from the fact that an interior equilibrium where wages in type-1 jobs are non-revealing and wages in type-2 jobs are must be such that

$$V_2 - V_1 = \left(\frac{1-\eta\left(q_u\left(2,\cdot\right)\right)}{\eta\left(q_u\left(2,\cdot\right)\right)}\right)\frac{k}{q_u\left(2,\cdot\right)} - \lambda_1 q_u\left(1,\cdot\right)\left(\frac{1-\eta\left(q_u\left(1,\cdot\right)\right)}{\eta\left(q_u\left(1,\cdot\right)\right)}\right)\frac{k}{q_u\left(1,\cdot\right)},$$

with $V_2 - V_1 > 0$ and $\lambda_1 q_u\left(1,\cdot\right) \geq 1$. **QED**

A.2 Appendices of Chapter 2

A.2.1 Problem of high school workers

Since high school workers only search foe one type of jobs, let $s_h = \{\omega, y\}$ be state of high school workers. The problem of an employed high school worker with state $s_h = \{\omega_l, y_l\}$ follows:

$$V(s_h) = \omega + \frac{\delta V(s_h^u)}{1+r} + (1-\delta)\left\{\frac{V(s_h)}{1+r} + U(s_h)\right\} \tag{A.29}$$

where

$$U(s_h) = \max_{\omega',q'}\left\{f(q')\alpha_2\left[\max\left\{0, \frac{\omega'}{1+r} + \frac{\delta}{r+\delta}\frac{V(s_h^u)}{1+r} - \frac{V(s_h)}{1+r}\right\}\right]\right\} \tag{A.30}$$

subject to

$$k \leq q' f(q')\alpha_2\left(\frac{y_h - \omega'}{r+\delta}\right) \tag{A.31}$$

$$\omega' \leq y_l \tag{A.32}$$

Denote the solutions $\{q_h^e(\omega_l, y_l), \omega_h^e(\omega_l, y_l)\}$. For employed workers with productivity realization y_h, the value function for workers in state $s_h = \{\omega_h, y_h\}$ satisfies

$$V(s_h) = \omega + \frac{\delta V(s_h^u)}{1+r} + (1-\delta)\frac{V(s_h)}{1+r} \tag{A.33}$$

The expected return of an unemployed high school worker is

$$V(s_h^u) = b + V \tag{A.34}$$

where

$$V = \frac{V(s_h^u)}{1+r} + \max_{\omega_l,\omega_h,q}\left\{f(q)\left[\alpha_2\frac{V(\{\omega_h, y_h\})}{1+r} + (1-\alpha_2)\frac{V(\{\omega_l, y_l\})}{1+r} - \frac{V(s_h^u)}{1+r}\right]\right\} \tag{A.35}$$

subject to

$$k \leq qf(q)\left[\alpha_2\left(\frac{y_h-\omega_h}{r+\delta}\right)+(1-\alpha)\frac{y_l-\omega_l}{r+\delta+\alpha_2 f(q_h^e(\{\omega_l,y_l\}))}\right] \tag{A.36}$$

$$\omega_l \leq y_l;\;\; \omega_h \leq y_h \tag{A.37}$$

A.2.2 Comparative Statics

The figure below are the comparative statics in all scenarios for a decline in α_2. Variables included are the job finding rate for unemployed workers searching for routine jobs ($f(q_2)$), the job finding rate for unemployed workers searching for cognitive jobs ($f(q_2)$), the employment rate and job-to-job flow rate of college workers.

Figure A.1: Comparative Statics-Scenario 1

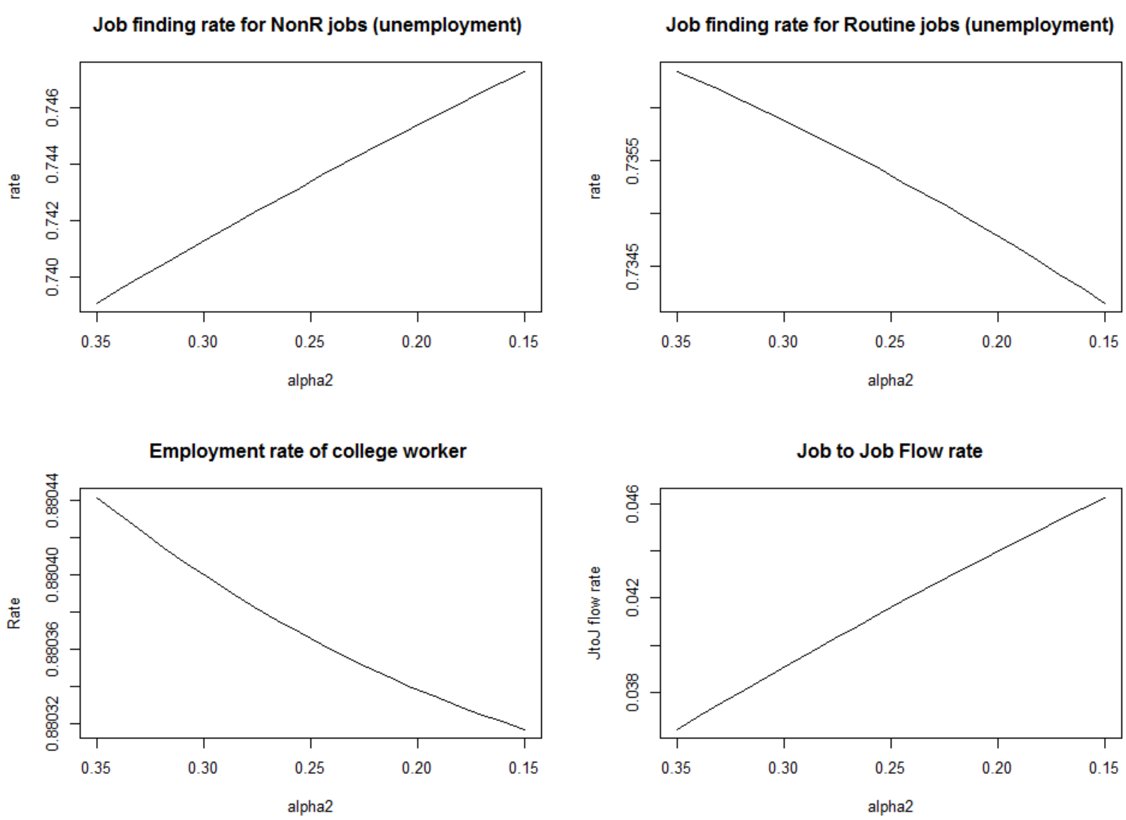


Figure A.2: Comparative Statics-Scenario 2

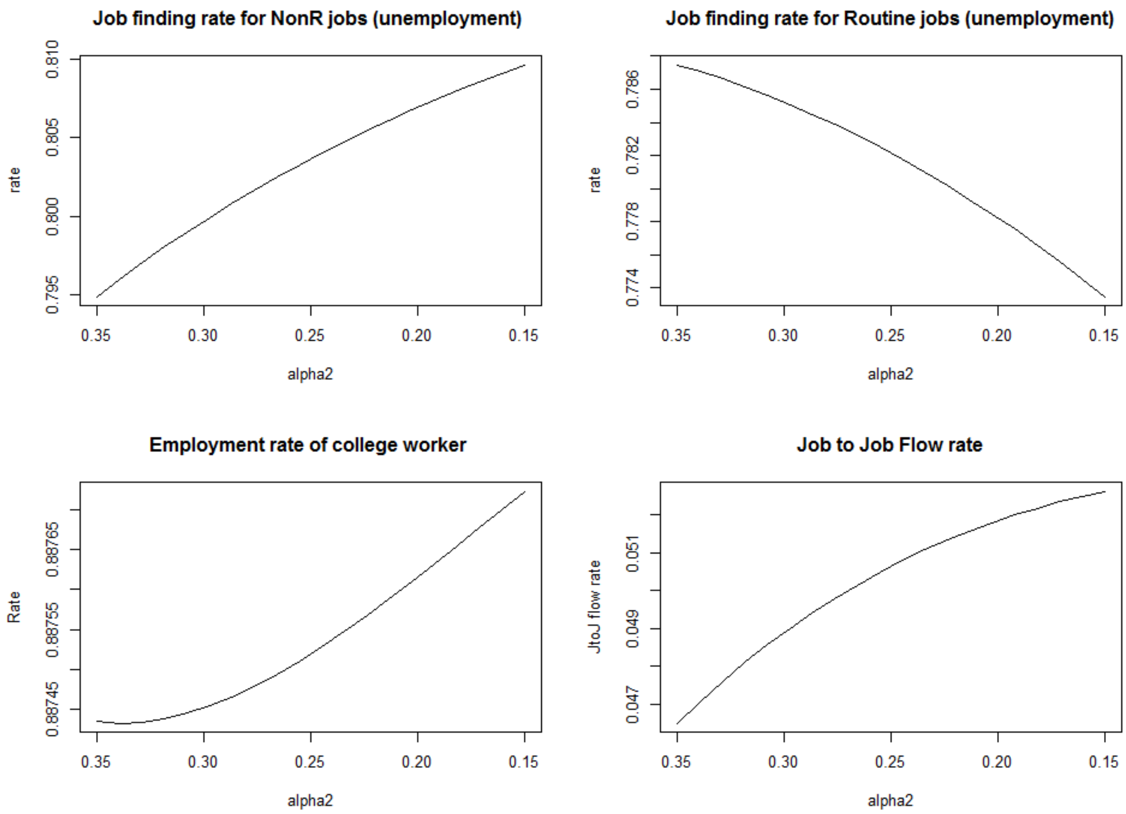
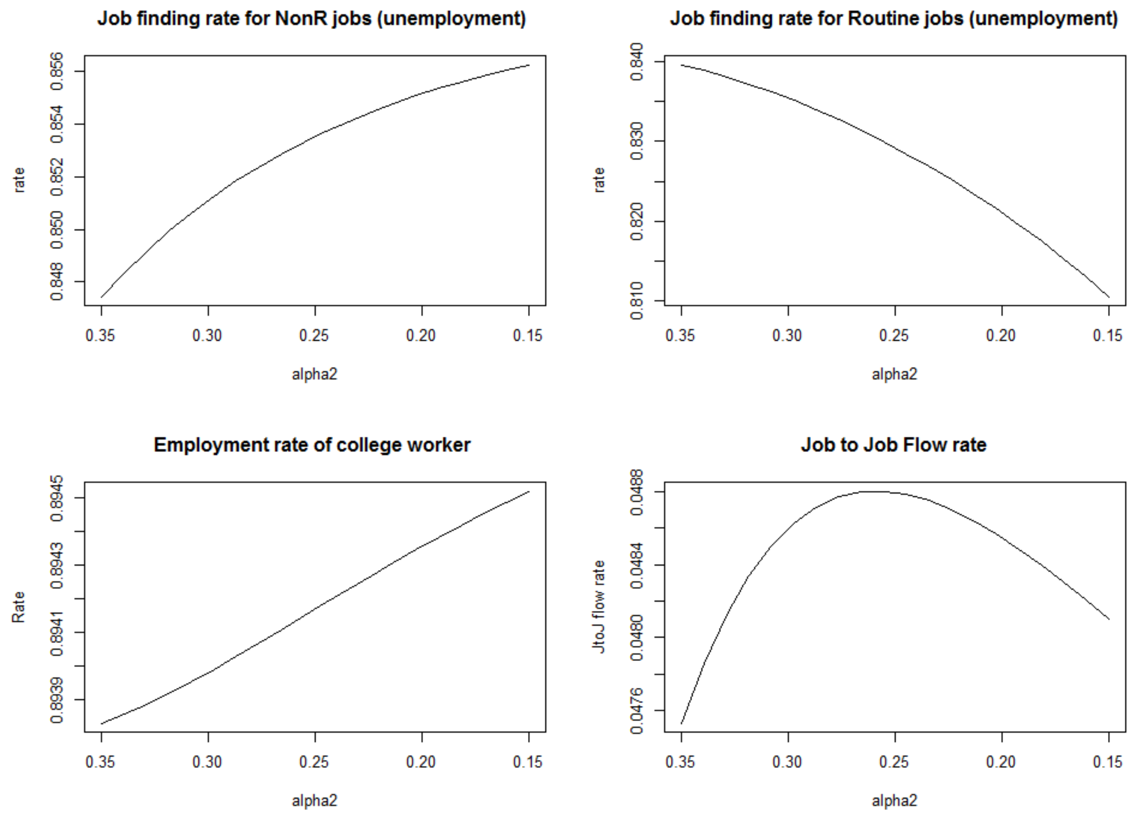


Figure A.3: Comparative Statics-Scenario 4



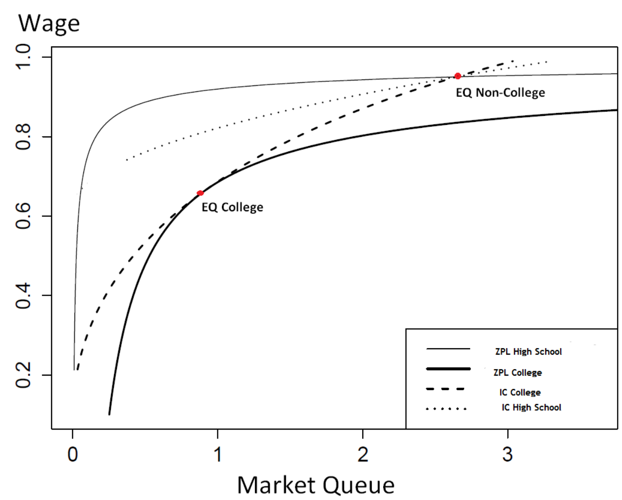
A.3 Appendices of Chapter 3

A.3.1 Proof of Proposition 1

Now I prove the first statement of Proposition 1. I first state that any equilibrium must be separating. The reason why no pooling equilibrium is as follows. Consider a contract with wage ω in non-college job market such that both types of workers apply in equilibrium. Contract x with the corresponding queue q must lines between the zero-profit line of hiring high school workers alone and the zero-profit line of hiring college workers alone, in the space of wages and market queue as shown in Figure 2. However, at (q, ω), the slope of indifference curves of college workers is stepper than that of high school workers. This is because unlike college workers who search for non-college jobs because they are a stepping stone to better jobs, high school workers are more willing to remain unemployed and search for jobs offering a relatively higher wage. This means for a higher wage, high school workers are willing to accept a larger queue to be indifferent. Following the equilibrium refinement, there always exists a profitable deviation for a routine employer to post a wage higher than ω such that only high school workers are attracted. Therefore, pooling equilibrium does not exists.

Given that any equilibrium must be separating, the rest of the proof of statement 1 follows the equilibrium definition. By definition, an equilibrium must be that workers choose where to apply to maximize expected return to search and employer make zero profits, as is characterized in problems ($\mathbf{P}-1$) to ($\mathbf{P}-4$). Any equilibrium must be separating implies that the incentive compatibility constraint of a college worker must hold. This is guaranteed by the solution of problem ($\mathbf{P}-4$). Therefore, any equilibrium allocation must solve problems ($\mathbf{P}-\mathbf{1}$) to ($\mathbf{P}-\mathbf{4}$).

Figure A.4: Non-College Job Markets



Finally, I prove the second statement of Proposition 1 hold by constructing the construction. Denote the solution to problems ($\mathbf{P}-1$) to ($\mathbf{P}-4$) as the follow four pairs (ω_2^2, q_2^2), (ω^e, q^e), (ω_2^1, q_2^1),(ω_1, q_1). I first construct the objectives of a potential equilibrium and show that all the equilibrium conditions as defined in Definition 1 could be satisfied with these objectives.

- The set of contracts X^* includes: $\{(\omega_2^2, 2), (b, 0)\}$, $\{(\omega^e, 2), (\omega_2^1, 1)\}$, $\{(\omega_1, 1), (b, 0)\}$ and $\{(\omega_2^1, 1), (b, 0)\}$,

- The retention policy g^r such that for any $s = \{(\omega, j), i\}, s_o = \{(\omega', j'), i\}$:

$$g^r(s, s_o) = \begin{cases} 0, & \text{if } j < j' \\ \omega' & \text{if } j \geq j' \end{cases} \tag{A.38}$$

- The market queue mapping Q satisfies:

  i any wage ω posted from a college employer attracting unemployed college workers and employed college workers, Q is such
$$k = qf(q)\frac{y_2 - \omega}{r + \delta}$$

  ii any wage ω posted from a non-college job employer attracting unemployed workers Q is such that

$$\begin{cases} k_1 = qf(q)\dfrac{y_1 - \omega}{r + \delta + (1-\delta)f(q^e(\omega))}, & \text{if } \omega < \omega_2^1 \\ V_2^1 = f(q)\frac{V(\{(\omega,1),2\})}{1+r} + [1 - f(q)]\dfrac{V_2}{1+r}, & \text{if } \omega_2^1 \leq \omega \leq \omega_1 \\ k_1 = qf(q)\dfrac{y_1 - \omega}{r + \delta}, & \text{if } \omega > \omega_1 \end{cases} \tag{A.39}$$

- The conditional distribution function μ such

  i any contract x posted from an college job employer: $\mu[2|x] = 1$

  ii any contract $x = \{(\omega, 1), (b, 0)\}$ posted from an non-college job employer

$$\mu[2|x] = \begin{cases} 1, & \text{if } \omega < \omega_1 \\ 0, & \text{if } \omega \geq \omega_1 \end{cases} \tag{A.40}$$

- The distribution of workers $\phi = \{u_1, u_2, e_1, e_2^1, e_2^2(\omega_2^2), e_2^2(\omega^e)\}$ such that the inflow equals outflow at each state

$$\begin{aligned}
&u_1 f(q_1) = (1 - \pi - u_1)\delta \\
&e_1 = 1 - \pi - u_1 \\
&u_2[F(\bar{c})f(q_2^2) + [1 - F(\bar{c})]f(q_2^1)] = (\pi - u_2)\delta \\
&e_2^1[\delta + (1-\delta)f(q^e)] = u_2[1 - F(\bar{c})]f(q_2^1) \\
&e_2^1(1-\delta)f(q^e) = \delta e_2^2(\omega^e) \\
&\pi - u_2 - e_2^1 - e_2^2(\omega^e) = e_2^2(\omega_2^2)
\end{aligned}$$

I now show that all of equilibrium conditions from Definition 1 hold. Condition (i) In all college job market, since Q traces zero profit condition, workers maximize expected return to search given Q in $(\mathbf{P-1})$ and $(\mathbf{P-2})$. In non-college job market, Q ensures that college worker maximized given employer makes zero profits while high school worker maximized given the incentive compatibility constraint, as in $(\mathbf{P-4})$. The search policy g is given by the solution of each optimization problem.

Condition (ii) The retention policy g^r maximize the continuation value of a match for employers and the zero-profit conditions satisfies (3.3). Condition (iii) has been shown in Lemma 1 that the zero profit condition binds for all equilibrium contracts. Condition (iv) and (v) are satisfied by the construction of μ and ϕ. Finally, the constructed expectation function μ also ensures that no pooling deviation is profitable because any deviation wage attracts at most one type of worker. *QED*

A.3.2 Proof of Proposition 2

I show that a solution exists to problem $(\mathbf{P-1})$ to $(\mathbf{P-4})$.

Consider first the problem $(\mathbf{P-3})$ of college workers. The first order conditions with respect to q and ω jointly gives

$$\frac{V(\{(\omega,2),2\})}{1+r}-\frac{V_2}{1+r}=\frac{1-\eta(q)}{\eta(q)}\frac{k_2}{qf(q)} \tag{A.41}$$

Substituting $\frac{V(\{(\omega,2),2\})}{1+r}$ and the zero-profit condition and denote the solution to this market (ω_2^2, q_2^2) which solves the following equations for any given V_2

$$\frac{y_2}{r+\delta}-\frac{r}{r+\delta}\frac{V_2}{1+r}=\frac{k_2}{\eta(q_2^2)q_2^2f(q_2^2)} \tag{A.42}$$

$$\omega_2^2=y_2-\frac{(r+\delta)k_2}{q_2^2f(q_2^2)} \tag{A.43}$$

The expected return to search for a college workers V_2^2 can be expressed as

$$V_2^2=\frac{1}{r}\left[y_2-\frac{k_2(r+\delta)}{\eta(q_2^2)q_2^2f(q_2^2)}\right]+\frac{1-\eta(q_2^2)}{\eta(q_2^2)}\frac{k_2}{q_2^2} \tag{A.44}$$

Next consider the on-the-job search problem $(\mathbf{P}-1)$. Ignore the last constraint, the first order condition for ω' and q jointly gives

$$\frac{\omega'}{r+\delta}+\frac{\delta}{r+\delta}\frac{V_2}{1+r}-\frac{V(\{(\omega,1),2\})}{1+r}=\left[\frac{1-\eta(q)}{\eta(q)}\right]\left(\frac{y_2-\omega'}{r+\delta}\right) \tag{A.45}$$

Substitute the stationary $V(\{(\omega,1),2\})$ from (A.58), an interior solution $q^e(\omega)$ and $\omega^e(\omega)$ satisfies

$$\frac{\omega^e(\omega)-\omega}{r+\delta+(1-\delta)f(q^e(\omega))} = \left[\frac{1-\eta(q)}{\eta(q)}\right]\left(\frac{y_2-\omega'}{r+\delta}\right) \tag{A.46}$$

together with the zero profit condition

$$\omega^e(\omega) = y_2 - \frac{k_2(r+\delta)}{q^e(\omega)f(q^e(\omega))} \tag{A.47}$$

Denote q_a such that

$$q_a f(q_a)\left(\frac{y_2-y_1}{r+\delta}\right) = k_2 \tag{A.48}$$

$q^e(\omega) \geq q_a$ if and only if $\omega^e(\omega) \geq y_1$. Also let q_b be such that

$$\frac{y_2-y_1}{r+\delta} = \frac{k_2}{q_b f(q_b)}\left[1+\frac{1-\eta(q_b)}{\eta(q_b)}\frac{r+\delta+(1-\delta)f(q_b)}{r+\delta}\right] \tag{A.49}$$

then $q^e(\omega) \leq q_b$ if and only if $\omega \leq y_1$. For any $\omega \in [0, y_1]$, the pair $(\omega^e(\omega), q^e(\omega))$ is uniquely given by (A.47) and (A.50) with $y_1 \leq \omega^e(\omega) < y_2$ and $0 \leq q_a \leq q \leq q_b < \infty$, where (A.50) is given by

$$\frac{\omega^e(\omega)-\omega}{r+\delta+(1-\delta)f(q^e(\omega))} \geq \left[\frac{1-\eta(q)}{\eta(q)}\right]\left(\frac{y_2-\omega'}{r+\delta}\right) \tag{A.50}$$

with equality if $\omega^e(\omega) > y_1$. Rewrite equations (A.46) so that the wage from the off-job search problem is a function of the quit rate

$$\omega(q^e) = y_2 - \frac{k_2}{q^e f(q^e)}\left(r+\delta+\frac{1-\eta(q^e)}{\eta(q^e)}[r+\delta+(1-\delta)f(q^e)]\right) \tag{A.51}$$

Substituting ω and transform the surplus to the worker as a function of the quit rate. Denote $\hat{V}(q^e) \equiv \dfrac{V(\{(\omega,1),2\})}{1+r}$

$$\hat{V}(q^e) = \frac{y_2}{r+\delta} - \frac{k_2}{q^e f(q^e)\eta(q^e)} + \frac{\delta}{r+\delta}\frac{V_2}{1+r} \tag{A.52}$$

$$\frac{d\hat{V}(q^e)}{dq^e} = \frac{k_2}{q^e f(q^e)}\left(\frac{1}{q^e}\frac{1-\eta(q^e)}{\eta(q^e)} + \frac{\eta'(q^e)}{\eta^2(q^e)}\right) \tag{A.53}$$

Next consider the following transformed problem from $(\mathbf{P}-2)$. Using (A.52) the off-job search problem for a college worker looking for a non-college job can be reformed as

$$\max_{q',q}\left\{f(q)\hat{V}(q')+[1-f(q)]\frac{V_2}{1+r}\right\} \tag{$\mathbf{P-2'}$}$$

s.t.

$$-k_1+qf(q)\left[\hat{S}(q')-\hat{V}(q')-\frac{V_2}{1+r}\right]\leq 0$$

Where $\hat{S}(q')$ is the total surplus of the match and $\hat{S}(q')-\hat{V}(q')-\dfrac{V_2}{1+r}=\dfrac{y_1-\omega(q')}{r+\delta+(1-\delta)f(q')}$, with

$$\begin{aligned}\frac{d\left(\hat{S}(q^e)-d\hat{V}(q^e)+\frac{V_2}{1+r}\right)}{dq^e}&=\frac{(1-\delta)f'(q^e)}{[r+\delta+(1-\delta)f(q^e)]^2}\left(y_2-y_1-\frac{k_2(r+\delta)}{q^ef(q^e)}\right)\\&-\frac{k_2}{q^ef(q^e)}\left[\frac{1-\eta(q^e)}{q^e}\left(\frac{1-\eta(q^e)}{\eta(q^e)}+\frac{r+\delta}{r+\delta+(1-\delta)f(q^e)}\right)+\frac{\eta'(q^e)}{[\eta(q^e)]^2}\right]\end{aligned} \tag{A.54}$$

I now solve the problem ($\mathbf{P-2'}$) with the first order conditions with respect to q' and q given by

$$\lambda q=-\frac{\dfrac{d\hat{V}(q')}{dq'}}{\dfrac{d}{dq'}\left(\hat{S}(q')-\hat{V}(q')-\dfrac{V_2}{1+r}\right)} \tag{A.55}$$

$$\hat{V}(q')-\frac{V_2}{1+r}=\lambda q\left[\frac{1-\eta(q)}{\eta(q)}\right]\frac{k_1}{qf(q)} \tag{A.56}$$

Denote the solution to problem $\mathbf{P-2'}$ as (q^e, q_2^1), which satisfies

$$\frac{y_2}{r+\delta}-\frac{k_2}{\eta(q^e)q^ef(q^e)}-\frac{r}{r+\delta}\frac{V_2}{1+r}=\lambda q_1^2\left[\frac{1-\eta(q_1^2)}{\eta(q_1^2)}\right]\frac{k_1}{q_1^2f(q_1^2)} \tag{A.57}$$

The return to search for a non-college job V_2^1 for college worker satisfies:

$$V_2^1=\frac{1}{r}\left(y_2-\frac{k_2(r+\delta)}{\eta(q^e)q^ef(q^e)}-\lambda q_2^1\left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right]\frac{k_1(r+\delta)}{q_1^2f(q_1^2)}\right)+\lambda q_2^1\left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right]\frac{k_1}{q_2^1} \tag{A.58}$$

Substituting V_2^1 and V_2^2 from (A.58) and (A.44) into equation (3.9), the cut-off $\bar{c}$ solves

$$\bar{c} = \left[\frac{1-\eta(q_2^2)}{\eta(q_2^2)}\right]\frac{k_2}{q_2^2} - \lambda q_2^1 \left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right]\frac{k_1}{q_2^1} \tag{A.59}$$

From equation (3.8), the expected return to search for a college worker V_2 follows

$$V_2 - b = V_2^2 - \bar{c} + \nu(\bar{c}) = V_2^1 + \nu(\bar{c}) \tag{A.60}$$

where $\nu(\bar{c}) = F(\bar{c})[\bar{c} - \mathbb{E}(c|c \le \bar{c})]$. The three equations listed below together equation (A.59) solves the search problem of a college worker with unknowns $\bar{c}$, q_2^1, q_2^2 and q^e.

$$\frac{y_2 - b + \bar{c} - \nu(\bar{c})}{r+\delta} = \frac{k_2}{\eta(q_2^2)q_2^2 f(q_2^2)} + \left[\frac{1-\eta(q_2^2)}{\eta(q_2^2)}\right]\frac{k_2}{(r+\delta)q_2^2} \tag{A.61}$$

$$\frac{y_2 - b - \nu(\bar{c})}{r+\delta} - \frac{k_2}{\eta(q^e)q^e f(q^e)} = \lambda q_2^1 \left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right]\frac{k_1}{f(q_2^1)q_2^1} + \lambda q_2^1 \left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right]\frac{k_1}{(r+\delta)q_2^1} \tag{A.62}$$

$$y_2 - \frac{k_2}{q^e f(q^e)}\left[r+\delta+\left[\frac{1-\eta(q^e)}{\eta(q^e)}\right](r+\delta+(1-\delta)f(q^e))\right] = y_1 - \frac{[r+\delta+(1-\delta)f(q^e)]k_1}{q_2^1 f(q_2^1)} \tag{A.63}$$

Next, I am going to show that there exists a solution for equations (A.61), (A.62) and (A.63) for a given $\bar{c} > 0$. I then follow by showing that a fix point exists for a function $D(\bar{c}) = \bar{c}) > 0$.

Fix a $\bar{c} > 0$. For equation (A.61), the right hand side converges to infinity when q_2^2 converges to 0 and converges to $k_2 + \dfrac{\nu(\bar{c}) - \bar{c}}{r+\delta}$. Therefore, there always exists a solution $q_2^2(\bar{c})$ such that equation (A.61) holds. For equation (A.62), since λq_2^1 is a function of q^e, notice first that $\dfrac{d(\lambda q_2^1)}{dq^e} < 0$. To see this, differentiate $\lambda q_2^1)$ with respect to q^e and notice that

$$\frac{\partial^2 \hat{S}/\partial(q^e)^2}{\partial^2 \hat{V}/\partial(q^e)^2} > \frac{\partial \hat{S}/\partial q^e}{\partial \hat{V}/\partial q^e}$$

The left side of the inequality is greater than one, since $\hat{S} - \hat{V}$ is a strictly convex function of q. The right side is smaller than one, since $\hat{S} - \hat{V}$ is a strictly decreasing function of q. Hence, $\dfrac{d(\lambda q_2^1)}{dq^e} < 0$, which suggests q_2^1 is a decreasing function of q^e in equation (A.62). The right hand side of equation (A.62) converges to infinity when q_2^1 converges to 0 and converges to $\dfrac{\nu(c)}{r+\delta}$ when q_2^1 converges to infinity. Therefore, there exists a $q_c(\bar{c}) > 0$ such that

$$\frac{y_2 - b}{r+\delta} - \frac{k_2}{\eta(q_c)q_c f(q_c)} = \frac{\nu(\bar{c})}{r+\delta}$$

and $q_c'(\bar{c}) > 0$. Then $q_2^1 < \infty$ if and only if $q^e > q_c(\bar{c})$. For equation (A.63), the left hand side converges to infinity when q_2^1 converges to 0 and to k_1 when q_2^1 converges to infinity. Define q_d such that

$$\frac{1}{r+\delta+(1-\delta)f(q_d)}\left[\frac{k_2(r+\delta)}{q_d f(q_d)} - (y_2 - y_1)\right] + \frac{k_2}{q_d f(q_d)} = k_1$$

and $q_2^1 < \infty$ if and only if $q^e < q_d > q_b$. Given that $q_a \leq q_b$, then for any $\bar{c}$, there is a solution to the equations (A.61), (A.62) and (A.63).

Now I construct a function $D(\bar{c})$ equal to the right hand side of (A.59). $D(0)$ satisfies

$$D(0) = \frac{k_1 \lambda q_2^1}{\eta(q_2^1)q_2^1 f(q_2^1)} - \frac{k_2}{\eta(q_2^2)q_2^2 f(q_2^2)} + \frac{y_2 - y_1}{r+\delta+(1-\delta)f(q^e)} + \frac{k_2(1-\delta)}{q^e}\frac{1}{r+\delta+(1-\delta)f(q^e)}$$

and $D(0) > 0$ with $q_2^2 > q_2^1$. From equation (A.61), when $\bar{c}$ increase, q_2^2 decrease, therefore the first item in $D(\bar{c})$ increases. From equation (A.62) and (A.63), when $\bar{c}$ increases, q_2^1 increases and q^e increases, therefore, the second term of $D(\bar{c})$ decreases and $D(\bar{c})$ is an increasing function of $\bar{c}$. Given that $D(0) > 0$ and $D(\bar{c})$ is finite, there exists a fixed point to $D(\bar{c}) - \bar{c}$ with $\bar{c} > 0$. *QED*

A.3.3 Proof of Proposition 3

I show that when the cost of posting college job k_2 converges to zero, the IC constraint is binding. When $k_2 \to 0$, $f(q^e) = 1$ and $\omega^e = y_2$. A non-college job employer receives one period profit from a college worker such. The optimal search problem for a college worker is :

$$V_2^1 = \max_{\omega,q}\left\{f(q)\frac{V(\{(\omega,1),2\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),2\})}{1+r}\right\} \tag{A.64}$$

s.t.

$$-k_1 + qf(q)\frac{y_1 - \omega}{1+r} \leq 0$$

where

$$V(\{(\omega,1),2\}) = \omega + \frac{\delta V(\{(b,0),2\})}{1+r} + \frac{(1-\delta)V(\{(y_2,2),2\})}{1+r}$$

The F.O.C. satisfies:

$$\frac{V(\{(\omega,1),2\})}{1+r}-\frac{V(\{(b,0),2\})}{1+r}=\frac{1-\eta(q)}{\eta(q)}\frac{k_1}{qf(q)} \tag{A.65}$$

Substituting $\dfrac{V(\{(\omega,1),2\})}{1+r}$ and the zero profit condition and denote q_0 the solution to the problem (A.64), then equation (A.65) becomes

$$\frac{y_1}{r+\delta}-\frac{(1-\delta)y_2}{(1+r)(r+\delta)}-\frac{rV(\{(b,0),2\})}{(1+r)(r+\delta)}=\frac{k_1}{\eta(q_0)q_0f(q_0)} \tag{A.66}$$

Compare to the FOC of the high school worker's search problem where the IC constraint is absent, where q_1 denote the equilibrium queue length of a high school worker's allocation

$$\frac{y_1}{r+\delta}-\frac{rV(\{(b,0),1\})}{(1+r)(r+\delta)}=\frac{k_1}{\eta(q_1)q_1f(q_1)} \tag{A.67}$$

Compare the two equations. Given than $V(\{(b,0),2\}) > V(\{(b,0),1\})$, and $\eta(q)qf(q)$ is an increasing function in q, $q_0 > q_1$. This implies that when k_2 converges to 0, a college worker strictly prefer the undistorted allocation of a high school worker with queue length q_1 to her own optimal allocation with queue length q_0.

To see this, consider the following three allocations: allocation (q_0, ω_0) is the optimal allocation of a college worker, (q_1, ω_1) is the undistorted optimal allocation of a high school worker as well as (q_0, ω_0') where ω_0' is the wage that generates zero-profits for an employer if the corresponding queue is q_0 and the worker does not search on-the-job. A college workers strictly prefer allocation (q_0, ω_0') to (q_0, ω_0) because for the same matching rate the wage ω_0' is larger than ω_0 when workers are expected to search on-the-job. A college workers also prefers allocation (q_1, ω_1) to (q_0, ω_0'). This is because at q_0, employers is welling to give up more wage for an increase in matching rate when they do not expect workers to search on the job.

Finally, since a college worker prefers (q_0, ω_0') to (q_0, ω_0), the equilibrium allocation (q_1^*, ω_1^*) of a high school worker that makes a college worker indifferent must have $q_1^* > q_0 > q_1$. The matching rate of a high school worker is lower than its efficient level and the unemployment rate of high school workers is inefficiently high.

A.3.4 Proof of Proposition 4

The cutoffs of post-secondary education $\bar{e} = \dfrac{U_v - U_h}{\beta}$ and the cutoff of pursuing bachelor degree $\underline{e} = \dfrac{U_c - U_v}{\alpha - \beta}$. Denote $\bar{\beta}$ such that $\bar{e} = \underline{e}$. At $\bar{\beta}$, a worker with cost $\bar{e}$ is indifferent between obtaining a bachelor degree or a VET credential. $\bar{\beta}$ solves:

$$\bar{\beta} = \alpha \frac{U_v - U_h}{U_c - U_h} \tag{A.68}$$

Given that $U_v > U_h$ and $U_c > U_h$, $\bar{\beta} > 0$ for any $\alpha > 0$. QED

A.3.5 Proof of Proposition 5

Fix α. Consider $\bar{e}(\beta) = \dfrac{U_v - U_h}{\beta}$ with $\dfrac{d\bar{e}}{d\beta} < 0$. Similarly, $\underline{e}(\beta) = \dfrac{U_c - U_v}{\alpha - \beta}$ with $\dfrac{d\bar{e}}{d\beta} > 0$. Consider the ex-ante utility of a worker U_0 as the function of β such that

$$U_0(\beta) = F_e(\underline{e}(\beta))U_c + [F_e(\bar{e}(\beta)) - F_e(\underline{e}(\beta))]U_v + [1 - F_e(\bar{e}(\beta))]U_h - \beta \mathbb{E}[e|\underline{e}(\beta) \leq e < \bar{e}(\beta)] - \alpha \mathbb{E}[e|e \leq \underline{e}(\beta)] \tag{A.69}$$

$$\frac{dU_0(\beta)}{d\beta} = -\int_{\underline{e}(\beta)}^{\bar{e}(\beta)} e f_e(e) de \tag{A.70}$$

The ex-ante utility of a worker in a economy without VET equals $U_0(\bar{\beta})$ with $\beta = \bar{\beta}$ defined in Proposition 4. At $\beta = \bar{\beta}$, $\dfrac{dU_0(\beta)}{d\beta} = 0$. The ex-ante utility of a worker in a economy with VET equals $U_0(\beta)$ with $\beta < \bar{\beta}$, in which case $\dfrac{dU_0(\beta)}{d\beta} < 0$ and the ex-ante utility of a worker is strictly increasing as β decreases. By continuity, $U_0(\bar{\beta}) < U_0(\beta)$ for any β such that $0 < \beta < \bar{\beta}$. The ex-ante utility of a worker is higher in economy with VET. Since all employers in equilibrium makes zero expected profits, the total ex-ante welfare of an economy increases by introducing VET. *QED*