AT-96-16-2

A Correlation to Quantify Convective Heat Transfer Between Vertical Window Glazings

John L. Wright, Ph.D., P.Eng.
Member ASHRAE

ABSTRACT

Several correlations have been used in recent years for the purpose of calculating convective, center-glass heat transfer rates across gas-filled interpane window cavities. These correlations are examined and their background discussed. The most recent experimental data are gathered and presented. These data were used to develop a new correlation specifically for the purpose of window analysis. The new correlation is not restricted to conditions customarily encountered in window applications (i.e., low Rayleigh number, high aspect ratio). It also is valid with respect to applications involving low aspect ratio (i.e., windows with wide pane spacing or narrow spacing with muntin bars) and/or applications involving high Rayleigh number (e.g., wide pane spacing, krypton fill gas).

INTRODUCTION

Background

Two public domain computer programs are widely used in North America for the solar-optical and thermal analysis of glazing systems. One, called VISION, was written by the author and is distributed through a Canadian university. The other, WINDOW, is offered by a U.S. laboratory. Both include a one-dimensional model for center-glass heat transfer capable of dealing with a wide variety of glazings (e.g., tints, solar control coatings, low-e coatings), substitute fill-gases, pane spacings, and environmental conditions.

The heat transfer models incorporated in VISION and WINDOW are based almost entirely on fundamental principles. This leaves little leeway for uncertainty in the analysis or for deviation between the two programs. However, some phenomena must be dealt with in an empirical manner. Most notably, correlations are used to quantify convective heat transfer within glazing cavities.

The literature contains an abundance of information about rectangular cavities in which a temperature difference between the vertical walls drives a convective flow (Wright and Sullivan 1989). It is customary to quantify the convective heat transfer coefficient between the vertical walls, h_c, as a function of the various independent variables (e.g., temperature difference, ΔT) in a nondimensional format. The Nusselt number, Nu, is used to express the convective heat transfer rate and is proportional to h_c:

$$\mathrm{Nu} = h_c \cdot \frac{l}{k} \text{ or } h_c = \mathrm{Nu} \cdot \frac{k}{l} \tag{1}$$

where

l = cavity width (i.e., pane spacing)
k = thermal conductivity of the fill gas.

Theory shows that Nu is a function of the Rayleigh number (Ra), the Prandtl number (Pr), and the cavity height-to-width aspect ratio (A). Therefore, the functional dependence of the correlation is of the form:

$$\mathrm{Nu} = \mathrm{Nu}(\mathrm{Ra}, \mathrm{Pr}, A). \tag{2}$$

The Rayleigh number is given by

$$\mathrm{Ra} = \frac{\rho^2 l^3 g C_p \Delta T}{\mu k T_m} \tag{3}$$

where

ρ, C_p, and μ = fill-gas density, specific heat, and viscosity;
T_m = mean temperature (absolute) of fill gas;
g = acceleration due to gravity; and

the Prandtl number is

$$\mathrm{Pr} = \frac{\mu C_p}{k}. \tag{4}$$

In the context of a given window application, where the fill gas and pane spacing are fixed, Ra can be interpreted as being proportional to ΔT.

John L. Wright is an adjunct professor in the Department of Mechanical Engineering at the University of Waterloo, Ont., Canada.

The value of Pr for gases never differs appreciably from Pr = 0.71 so Pr can be dropped from Equation 2 for window applications. In addition, many early studies did not deal with, or failed to discern, a dependence of Nu on A so relations used to quantify convective heat transfer in glazing cavities often are called Nusselt vs. Rayleigh or Nusselt/Rayleigh correlations.

The measurements of ElSherbiny (1980) comprise the most suitable single source of data for window analysis because they are based on a well-established procedure carried out over wide ranges of Ra and A with the specific aim of resolving the roles of Ra and A. These data were used to develop a set of correlations (Elsherbiny et al. 1982) that include a "design correlation" plus additional correlations for specific values of A ($A = 5, 10, 20, 40, 80, 110$). It is of particular interest that the design correlation shows Nu to be a function of A only if A is greater than 25. The correlations pertaining to fixed aspect ratios were intended to reproduce the fine-grain structure of the raw Nu/Ra data to a higher accuracy than the design correlation.

Early versions of VISION and WINDOW used Nu/Ra correlations that differ from the correlations used in the current versions—VISION4 and WINDOW 4.1. VISION incorporated the design correlation of ElSherbiny et al. (1982) and WINDOW included a correlation (Rubin 1982) based on DeGraff and Van der Held (1952). WINDOW 2.0 and subsequent versions use the $A = 40$ correlation of ElSherbiny et al. (1982). Although VISION2 and WINDOW 2.0 produced U-factors that were, in general, almost identical, it was discovered that the difference between their correlations was accentuated in the analysis of glazing systems with low-emissivity coatings. Differences in the property values used for argon also were discovered and the resulting discrepancy in U-factors of about 7% was thought to be excessive. Subsequent versions of VISION use a new correlation based on the data of ElSherbiny and a second set of data measured by Shewen (1986) that was more recently compiled. It is the purpose of this paper to present this new correlation.

The DeGraff and Van der Held correlation, two of the ElSherbiny et al. (1982) correlations (the design correlation and the $A = 40$ correlation), and both sets of measured data are shown in Figure 1. Only the high aspect ratio ($A > 25$) portion of the design correlation and the high aspect ratio data are shown. DeGraff and Van der Held did not resolve an aspect ratio dependence in their study. The curves are plotted for Ra < 10,000, which is the range of prime interest for windows (Wright and Sullivan 1989).

Natural Convection in a Vertical Slot

Each Nu/Ra curve starts on the left (at Ra = 0) with Nu = 1 and rises to higher values with increasing values of Ra. If Nu = 1, the total amount of convective heat transfer is equal to the amount that would exist if the gas were stagnant and transferring energy purely by conduction. This is called the *conduction regime*. However, the Nu = 1 condition must not be interpreted as a zero-flow condition. Convection exists for any non-zero ΔT, and local variations in heat flux will exist near the extreme ends of the cavity. At higher values of Ra the amount of convective flow is greater and Nu increases. Information about the local variation of

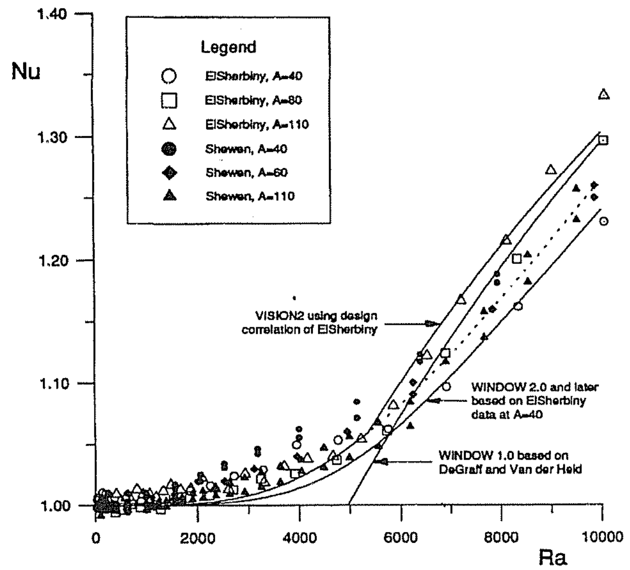

Figure 1 *Nu/Ra correlations used in VISION and WINDOW software plus measured data sets of ElSherbiny and Shewen (A>25).*

Nu can be found in the literature (ElSherbiny et al. 1983; Lee and Korpela 1983; Wright and Sullivan 1994).

Some insights regarding the fluid flow can be gained by examining the approximate method of Raithby et al. (1977) shown in Figure 2. The critical value of Ra at which the flow leaves the conduction regime is a function of A. The convective flow leaves the conduction regime at lower values of Ra in cavities with lower values of A. If Ra is increased sufficiently, instabilities occur that eventually lead to a turbulent boundary layer flow. The transition from laminar to turbulent flow can readily be pinpointed. Turbulent flow is represented by the line that extends upward to the right with a slope of 1/3. The lines inside the knee created by the turbulent boundary layer line and the horizontal axis have a slope of 1/4 and represent laminar boundary layer flow for various values of A. The critical value of Ra for the onset of turbulent flow is a function of A. The flow in enclosures with larger aspect ratios becomes turbulent at smaller values of Ra. The results of this theory suggest that the flow in tall, narrow slots can become turbulent directly from the conduction regime.

The method of Raithby et al. (1977) is shown again in Figure 3 with the experimental data of ElSherbiny superimposed. These data display some trends that are similar to those of the approximate method. However, they do not show the ordered progression (as a function of A) that might be expected inside the knee. The measured Nu/Ra curves are tightly grouped and for $A > 20$ they all depart the conduction regime at Ra $\approx 6 \times 10^3$. It is tempting to conclude, on the basis of the similarity between the shapes of the measured and theoretical curves, that the flow immediately enters the turbulent regime. However, it is not clear whether Nu increases because the flow enters the laminar boundary layer regime, because it becomes turbulent, or because of some other phenomenon. Stability

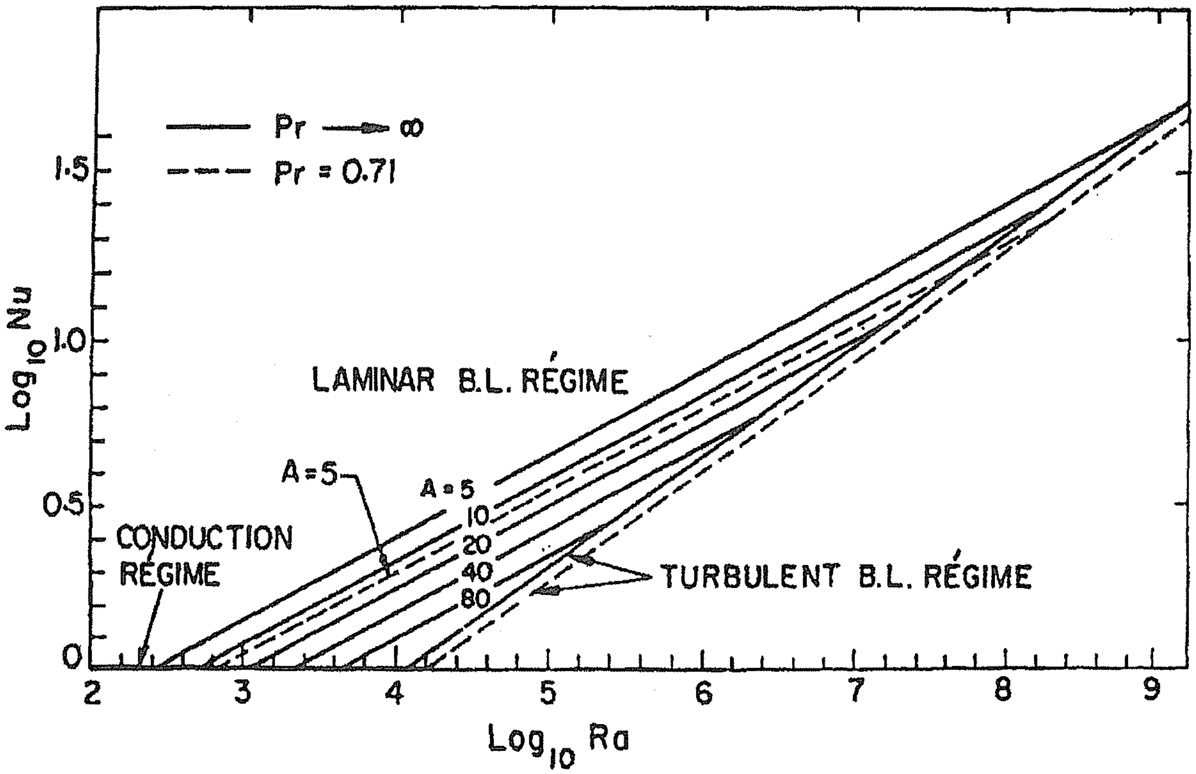

Figure 3 Approximate method of Raithby, Hollands and Unny (1977).

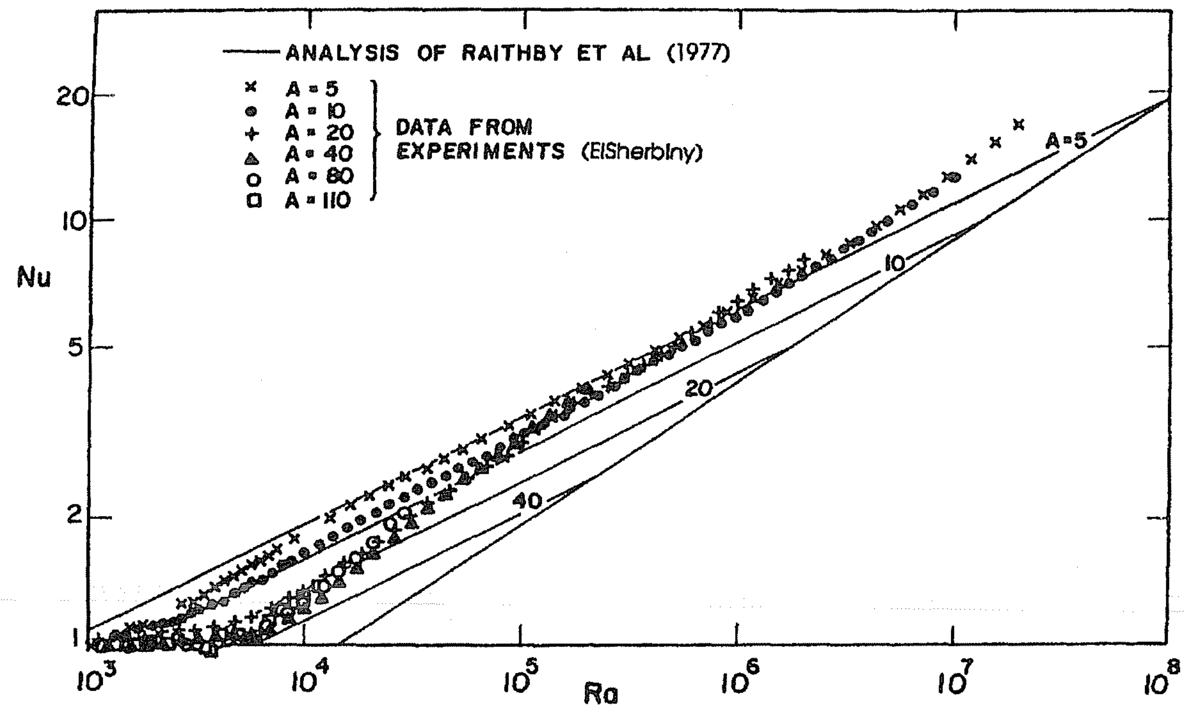

Figure 2 Nu, Ra and A data of ElSherbiny, Raithby and Hollands (1982).

theory predicts (Bergholtz 1978) the onset of a cat's-eye pattern of secondary cells in a tall, gas-filled, vertical slot at Ra $\approx 5.7 \times 10^3$. This matches closely the upswing in the measured data shown in Figures 1 and 3. In fact, the data of ElSherbiny have been closely reproduced up to Ra $\approx 12 \times 10^3$ by computational methods that include the effect of secondary cells (Lee and Korpela 1983; Wright and Sullivan 1994).

ELSHERBINY CORRELATION

Figure 4 shows the design correlation of ElSherbiny et al. (1982), which consists of three functions—Nu_1, Nu_2, and Nu_3:

$$Nu_1 = 0.0605 Ra^{1/3} \tag{5}$$

$$Nu_2 = \left[1 + \left(\frac{0.104 Ra^{0.293}}{1 + \left(\frac{6310}{Ra}\right)^{1.36}}\right)^3\right]^{1/3} \tag{6}$$

$$Nu_3 = 0.242\left(\frac{Ra}{A}\right)^{0.272} \tag{7}$$

$$Nu = Max(Nu_1, Nu_2, Nu_3). \tag{8}$$

Only Nu_3 depends on aspect ratio and Nu_3 only affects Nu if $A < 25$. This is a useful feature because it is convenient, in both the design and rating processes, to calculate center-glass U-factors that depend on the arrangement of various glazings but not on the height of the window.

In this scheme Nu_1, Nu_2, and Nu_3 are each calculated using specified values of Ra and A. Nu is taken as the maximum of the three calculated values. It can be seen in Figure 4 that Nu_1 should be the largest only at large Ra values. However, it has been found that the line for Nu_1, if extended to the left, unintentionally clips the Nu_2 curve. The problem is difficult to see because of the scale used in Figure 4 but it is much more easily seen in Figure 1. The discontinuity shown in the ElSherbiny design correlation near Ra = 5,400 should not exist. The correlation was intended to continue along the dotted line shown in Figure 1 (Hollands 1994). In fact, the design correlation, if applied in a way that avoids this clipping problem, is about 2% above the $A = 40$ correlation and for Ra > 6,000 runs close to the new correlation presented here but still underpredicts the data for Ra < 5,000.

MEASURED DATA

A new correlation was developed on the basis of the two sets of Nu/Ra data available for high aspect ratio cavities. The first set was measured by ElSherbiny (1980), the second by Shewen (1986). Shewen took measurements at $A = 40$, 60, and 110. His apparatus is similar to the apparatus used by ElSherbiny but differs in one significant way. ElSherbiny measured heat transfer over the faces of electrically heated plates embedded in the hot wall of the cavity. Shewen measured heat transfer over the faces of metering plates using the Peltier effect (Shewen et al. 1989).

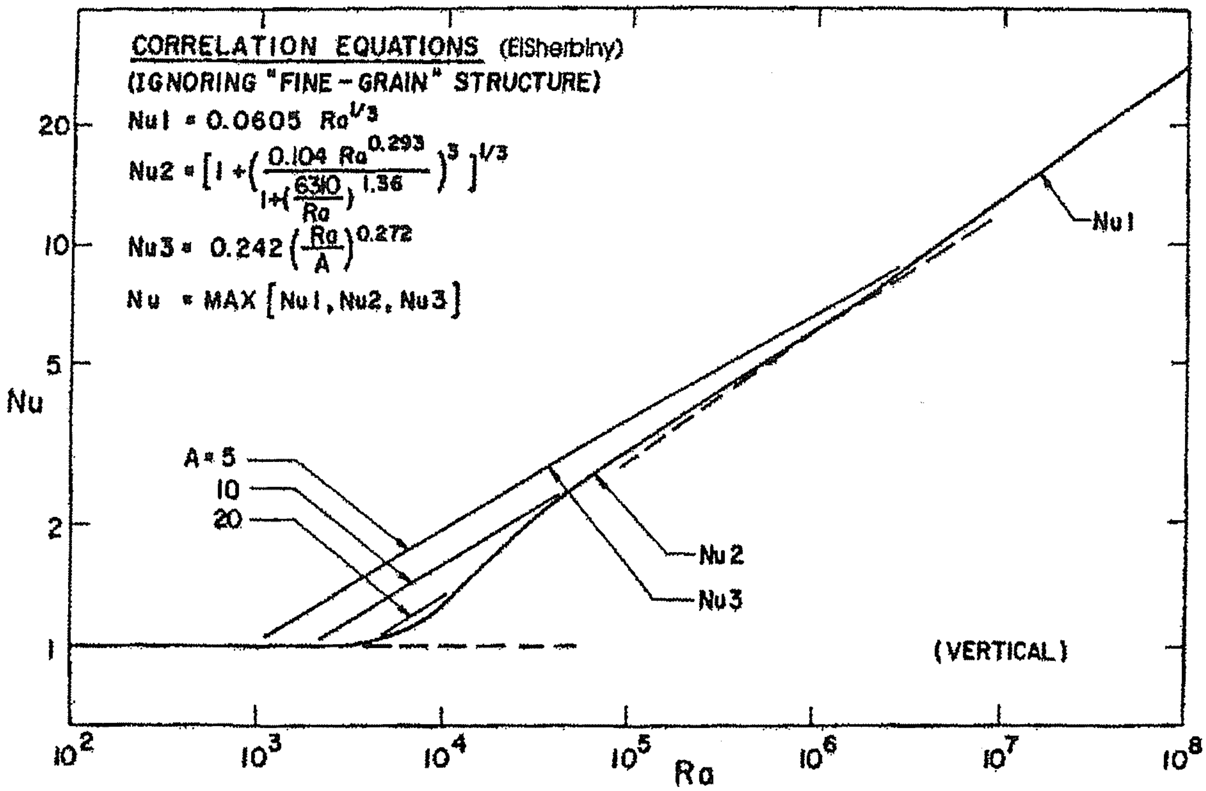

Figure 4 Design correlation of ElSherbiny, Raithby and Hollands (1982).

This technique allowed the metering plates to be either heated or cooled electrically and for each value of Ra, two measurements of Nu were taken—one at the hot wall and the other at the cold wall.

The data sets of ElSherbiny and Shewen are shown in Figures 3 and 5, respectively. Both sets of data ($A \geq 40$ only) are shown in Figure 1 along with the various correlations discussed earlier.

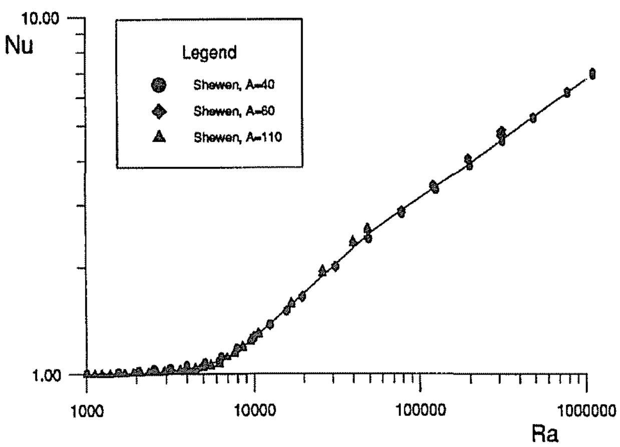

Figure 5 *Nu, Ra, and* A *data of Shewen (1986).*

All correlations clearly underpredict the data, which are tightly grouped, in the range of Ra < 5,000. The ability to predict Nu in this range is important for glazing systems with three or more glazings or for milder weather conditions—including the summer conditions used for calculating solar heat gain and cooling loads.

In the range of Ra between 5,000 and 10,000 several things are worth noting.

- ElSherbiny's data for $A = 80$ and $A = 110$ were close together. This was in accordance with expectation based on theory. It was expected that aspect ratio dependence would disappear at high A and that the lowest curve would correspond to the high values of A. The $A = 40$ data do not follow this pattern because they show Nu lower than the values of Nu measured at $A = 80$ and $A = 110$. This is of little importance because all three data sets are grouped tightly enough that differences between them are of the same order as the experimental error, which is approximately ±3% of Nu. (In fact, the uncertainty in Nu is much less than 3% in this situation. In the high aspect ratio cavities most of the uncertainty arises from uncertainty in Ra (ElSherbiny et al. 1982).
- Shewen's data show the expected progression of Nu with A. Curves drawn through these data are more tightly grouped than those of ElSherbiny. Fortunately, Shewen's group of curves is located within the same band as ElSherbiny's so that in seeking a single correlation, free of aspect ratio, no discrepancy is presented by the two sets of data.
- The width of the band containing all of the high aspect ratio data (ranging up to about ±4%) is reasonable on the basis of two sources of uncertainty. Experimental uncertainties in the measurement of both Nu and Ra contribute several percentage points. In addition, it has been shown that there is more than one fill-gas flow pattern possible. In this range of Ra and A it is certain that secondary cells are present and it also is known that various numbers of cells can exist (e.g., Le Quéré [1990]). Numerical modeling has shown that different numbers of cells may alter the value of Nu by 1% or 2%. The absence of cells at Ra = 10,000 would decrease Nu by about 10% (Wright 1990). Therefore, it is impossible—even using a procedure with zero experimental uncertainty—to repeatedly measure values along a single curve.

On the basis of these observations, no clear pattern can be resolved regarding the influence of A on Nu for $A \geq 40$. Therefore, the approach taken in creating a new correlation is to find a curve that runs directly along the center of the band of data shown in Figure 1.

NEW CORRELATION

A new correlation, independent of A and based on data for $A \geq 40$ and $Ra < 10^6$, has been formulated. The highest priority has been placed on having it trace the center of the data band for Ra < 10,000 and it includes a better representation of the data for Ra < 5,000. The segments of the correlation for Ra > 10,000 have been included to deal with krypton fill-gas in spacings greater than 1/4-in. plus storm windows and other designs that might involve large pane spacings. The correlation is[1]

$$Nu = 0.0673838 \cdot Ra^{1/3} \qquad Ra > 5 \times 10^4 \tag{9a}$$

$$Nu = 0.028154 \cdot Ra^{0.4134} \qquad 10^4 < Ra \leq 5 \times 10^4 \tag{9b}$$

$$Nu = 1 + 1.75967 \times 10^{-10} \cdot Ra^{2.2984755} \qquad Ra \leq 10^4. \tag{9c}$$

The curve defined by Equation 9c is shown in Figure 6 along with the band of measured data. The portions of the curve defined by Equations 9a and 9b (higher Ra) are illustrated by the lines shown in Figure 7.

No upper limit on Ra is stated for Equation 9a. Equation 9a is based on data ranging up to $Ra \approx 10^6$. This limit is unlikely to be exceeded in window calculations—even in the case of wide pane spacings such as those encountered with storm windows or a third glazing added to a double-pane construction. Nonetheless, Equation 9a is expected to be valid well beyond $Ra = 10^6$ because it is of the form $Nu \propto Ra^{1/3}$. The 1/3 exponent results in the 1/3 slope (when log(Nu) is plotted vs. log(Ra)) that has been confirmed with the high-Ra measurements and agrees with the theory discussed in the Introduction. It is interesting to note that Equation 9a can be combined with Equations 3 and 1 to show that h_c is not a function of pane spacing in windows with wide

1. The coefficients used in this correlation include a large number of significant digits. This is done to ensure that the three segments of the correlation are precisely mated and is not meant to suggest that the correlation can offer the corresponding level of accuracy.

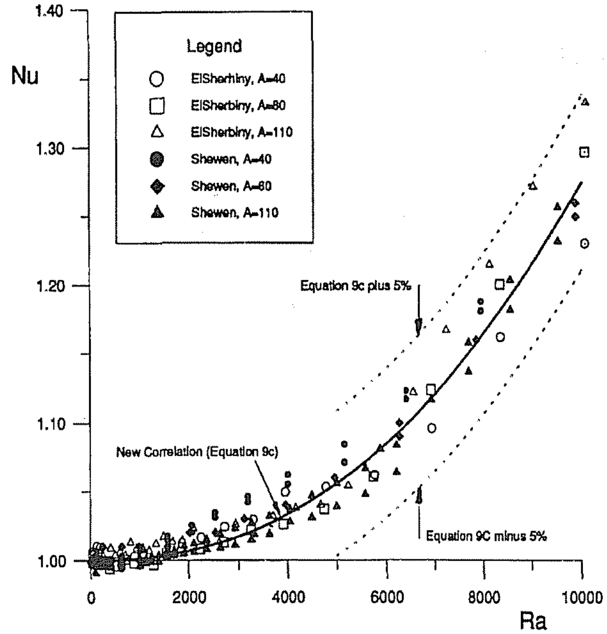

Figure 6 *Nu, Ra, and* A *data of ElSherbiny et al. (1982) and Shewen (1986) plus various correlations.*

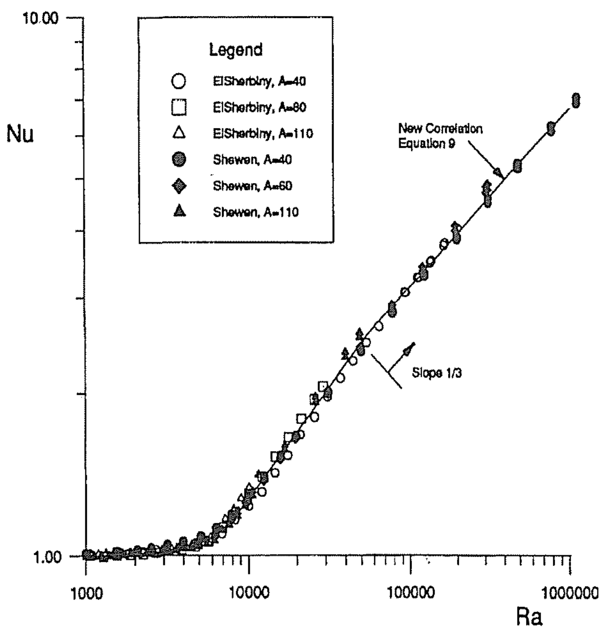

Figure 7 *Nu, Ra, and* A *data of ElSherbiny et al. (1982) and Shewen (1986) plus new correlation.*

pane spacing. This specific effect has been confirmed experimentally (Baker et al. 1989).

LOW ASPECT RATIO

ElSherbiny's data show that the influence of aspect ratio cannot be ignored at low aspect ratio. This is consistent with theory. If it is considered important to calculate Nu correctly at

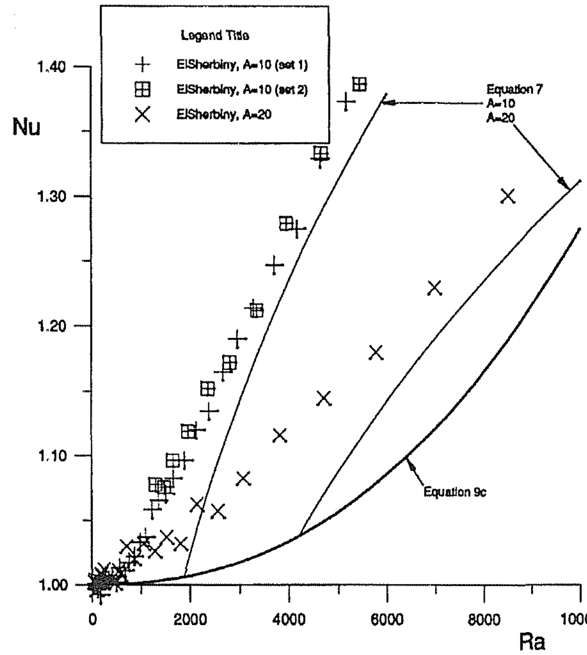

Figure 8 *Nu, Ra, and* A *data of ElSherbiny et al. (1982) with correlation curves—low aspect ratio.*

low A (e.g., short windows, windows with decorative dividers, windows with large pane spacing), then it is possible to use the portion of ElSherbiny's design correlation that deals with aspect ratio. This can be done by taking the maximum of Equation 7 and Equation 9. The correlation given by this combination is shown in Figure 8. The data of ElSherbiny for $A = 20$ and $A = 10$ also are shown.

CONCLUSIONS

A correlation has been developed specifically for quantifying convective heat transfer between vertical window glazings. It is based on the most recent and most reliable sets of measured data. It reproduces the measured results at low Rayleigh numbers more closely than do previous correlations. This allows for more accurate simulation of windows with more than two glazings and windows exposed to moderate weather conditions. The new correlation is not restricted by an upper limit in Rayleigh number and also is valid at low aspect ratio. This feature is useful for simulating short windows, windows with divider bars, or windows with wide pane spacings.

ACKNOWLEDGMENTS

This research was supported by CANMET (Natural Resources Canada). Appreciation also is extended to Dr. E.C. Shewen for permission to publish his measured heat transfer data.

REFERENCES

Baker, J.A., H.F. Sullivan, and J.L. Wright. 1989. A study of pane spacing in glazing systems. *Proc. Solar Energy Society of Canada*, Annual Conference, Penticton.

Bergholtz, R.F. 1978. Instability of steady natural convection in a vertical fluid layer. *Journal of Fluid Mechanics* 84(2): 743-768.

DeGraff, J.G.A., and E.F.M. Van der Held. 1952. The relation between the heat transfer and convection phenomena in enclosed plane air layers. *Applied Scientific Research* A3: 393-409.

ElSherbiny, S.M. 1980. Heat transfer by natural convection across vertical and inclined air layers. Ph.D. thesis, Department of Mechanical Engineering. Waterloo, Ont.: University of Waterloo, Canada.

ElSherbiny, S.M., G.D. Raithby, and K.G.T. Hollands. 1982. Heat transfer by natural convection across vertical and inclined air layers. *Journal of Heat Transfer* 104: 96-102.

ElSherbiny, S.M., K.G.T. Hollands, and G.D. Raithby. 1983. Nusselt number distribution in vertical and inclined air layers. *Transactions of the ASME* 105: 406-408.

Hollands, K.G.T. 1994. Personal communication.

Lee, Y., and S.A. Korpela. 1983. Multicellular natural convection in a vertical slot. *Journal of Fluid Mechanics* 126: 91-121.

Le Quéré, P. 1990. A note on multiple and unsteady solutions in two-dimensional convection in a tall cavity. *Journal of Heat Transfer* 112: 965-974.

Raithby, G.D., K.G.T. Hollands, and T.E. Unny. 1977. Analysis of heat transfer by natural convection across vertical fluid layers. *Journal of Heat Transfer* 99(2).

Rubin, M. 1982. Calculating heat transfer through windows. *Energy Research* 6: 341-349.

Shewen, E.C. 1986. A Peltier effect technique for natural convection heat flux measurement applied to the rectangular open cavity. Ph.D. thesis, Dept. of Mechanical Engineering. Waterloo, Ont.: University of Waterloo, Canada.

Shewen, E.C., K.G.T. Hollands, and G.D. Raithby. 1989. The measurement of surface heat flux using the Peltier effect. *Transactions of the ASME* 111: 798-803.

Wright, J.L. 1990. The measurement and computer simulation of heat transfer in glazing systems. Ph.D. thesis, Dept. of Mechanical Engineering. Waterloo, Ont.: University of Waterloo, Canada.

Wright, J.L., and H.F. Sullivan. 1989. Natural convection in sealed glazing units: A review. *ASHRAE Transactions* 95(1).

Wright, J.L., and H.F. Sullivan. 1994. A 2-D numerical model for natural convection in a vertical, rectangular window cavity. *ASHRAE Transactions* 100(2).