Bi-objective p-hub Location Problems

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

In this thesis, we introduce, model, and solve bi-objective hub location problems. The two well-known hub location problems from the literature, the p-hub median and p-hub center problems, are unified under a bi-objective setting considering the single, multiple, and r-allocation strategies. We developed a 3-index and a 4-index mixed-integer programming formulation for each of the allocation strategies. All the formulations are tested on the CAB dataset from the literature using a commercial optimization software. We observe the effect of different priorities given to the objectives on the locations of hub nodes, allocations, and the CPU time requirements with different allocation strategies under different values of problem parameters.

Keywords: p-hub median, p-hub center, hub location, mixed-integer programming formulations.

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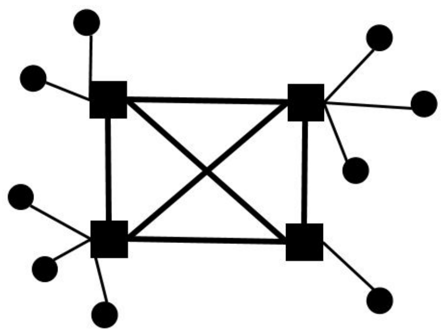
Chapter 1

Introduction

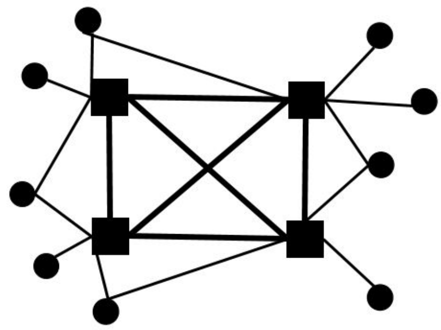
Hubs are facilities, which can be used as transshipment consolidation, or sorting points to connect origin-destination (O-D) pairs in networks. Their main feature is to connect high number of O-D pairs with less number of links. Instead of serving each O-D pair with a direct link, flows with the same origin but different destinations are merged at hubs, and then these flows are combined with flows with the same destinations but different origins. Thus hub networks reduce network setup costs by eliminating direct links between O-D pairs. Moreover, the transportation costs decrease by means of economies of scale on the inter-hub links.

Hubs are commonly used in telecommunication, computer, logistics, and transportation networks. Hubs are employed in passenger and freight airline networks, truckload and less-than-truckload networks, postal and parcel delivery networks, and public transportation networks. There is usually a large traffic flow between O-D pairs in such networks, and economies of scale is achieved by the use of hub facilities.

Hub location problems are location problems that include decisions on locating hub facilities and/or designing hub networks, and mostly aim to optimize a cost or a service based objective function. There are two main decisions in hub location problems: where to locate hubs and how to allocate non-hub nodes to those hubs. Hub location problems are difficult problems in nature because they require determining the locations of hub facilities as well as the routes of flows through the network. Hub location problems are mostly classified based on the allocation strategies of non-hub nodes to hub nodes. There are two basic allocation strategies: single allocation and multiple allocation (Figure 1.1). In single allocation, each demand node must be allocated to a single hub, whereas, in multiple allocation, demand nodes can be allocated to any number of hubs.



(a) Single allocation hub network.



(b) Multiple allocation hub network.

Figure 1.1: Single and multiple allocation hub networks.

Figure 1.1 illustrates two different examples of hub networks. In this figure, squares denote hub facilities, bold lines inter-hub links, and thin lines allocation links. Figure 1.1a shows a single allocation hub network. Note that all nodes are assigned to at most one hub, while Figure 1.1b illustrates a multiple allocation hub network when nodes can be allocated to more than one hub facility. More recently, a new allocation strategy referred as r-allocation is introduced (Yaman, 2011). In r-allocation, demand nodes can be allocated to at most r hubs.

Hub location problems have a lot of similarities with the classical location problems, yet hub location problems have three main key features, which classical location problems do not include. First, in hub location problems, the demand is specified between pairs of O-D nodes, but in classical location problems, demand is specified for each node. Second, hub facilities are consolidation or collection points, which sort and separate the flow, yet facilities in classical location problem do not have this function. Third, economies of scale exist on inter-hub links, which reduce the transportation cost between hubs, while there is usually no transportation of flow between facilities in the classical location problems.

In this study, we model and solve bi-objective hub location problems. We consider two common hub location problems from the literature, the p-hub median and p-hub center problems, and unify them under a single bi-objective problem. We model all possible allocation strategies for this new problem: single, multiple, and r-allocation. The p-hub median and p-hub center problems have not been combined under a single *bi*-criteria problem in the literature before. Our aim is to analyze the resulting hub networks when these two different objectives are considered at the same time.

The outline of this thesis is as follows. In Chapter 2, we define the p-hub median and p-hub center problems and provide a brief review of the relevant literature. Chapter 3 presents different mathematical models that we developed for the single, multiple, and r-allocation p-hub location problems. Detailed computational experiments with these models are provided in Chapter 4. We finalize the study with a conclusion presented in Chapter 5.

Chapter 2

Literature Review

In this chapter, we analyze the related studies on hub location problems. Since we are considering a bi-objective p-hub location problem with median and center objectives, we particularly focus on the literature on the p-hub median and p-hub center problems. Moreover, we also review the multi-objective studies on hub location problems.

The literature on hub location problems started with the pioneering works of O'Kelly (1986a,b, 1987). O'Kelly (1987) presented a quadratic integer programming formulation of the problem, and a heuristic solution methodology. The original problem introduced by O'Kelly (1987) is later referred as the single allocation p-hub median problem. In this problem, the aim is to locate p hub facilities so as to minimize total transportation costs. O'Kelly (1987) defined a constant economies of scale factor (α) to reflect the discounted transportation costs on the inter-hub links.

Campbell (1994a) presented linear integer programming formulations for the single and multiple allocation p-hub median problems. Moreover, hub center and hub covering problems have been introduced for the first time in this study. The p-hub center problem aims to locate p hub facilities while minimizing the maximum transportation or service cost between O-D pairs. Hub covering problems, on the other hand, locate hub facilities where demand points are within the service radius. Campbell (1994a) classified hub covering problems in three categories. The first one is where the maximum cost from the origin to its destination does not exceed an upper bound. The second one is when the cost for each link between an O-D pair do not exceed an upper value. The last one where each origin-hub transportation cost and hub-destination cost do not exceed a specified value.

Several reviews on hub location problems are presented in the literature. Campbell (1994b), O'Kelly and Miller (1994) presented the first surveys related to hub location problems. Klincewicz (1998) reviewed hub location applications on telecommunication networks. Bryan and O'Kelly (1999) study air transportation networks related to hub location problems. Alumur and Kara (2008), Farahani et al. (2013) classified the papers published in the area of hub location. One can refer to additional reviews by Campbell et al. (2002), Campbell and O'Kelly (2012), and Contreras (2015).

A few data sets are introduced in the literature for testing hub location problems. The first one is the CAB dataset introduced by O'Kelly (1987). This dataset is evaluated by the Civil Aeronautics Board (CAB) based on airline passenger interactions between 25 U.S. cities in 1970. The second data set is based on Australia Post (AP) data introduced by Ernst and Krishnamoorthy (1996). This dataset includes 200 postal districts for the postal delivery network of Sydney. The third one is based on the Turkish postal delivery system data proposed by Kara and Tansel (2000). This dataset consists of 81 nodes representing the cities in Turkey. All of these datasets are readily available in OR Library (Beasley, 1990).

In the next section, we review studies on the p-hub median problem. The p-hub center literature is reviewed in Section 2.2. In Section 2.3, we review the studies on multi-objective hub location problems. We also present bi-objective solution strategies at the end of this section.

2.1 The p-hub median problem

As defined in the previous section, p-hub median problems aim to locate p hubs to minimize the total transportation cost. In this section, we review the studies on p-hub median problems.

The p-hub median problem is NP-hard. For the single allocation version, it is proved that even though the hub locations are fixed, the allocation part of the problem still remains NP-hard (Kara, 1999).

Campbell (1994a) proposed the first linear integer programming formulation for the single allocation p-hub median problem. This formulation uses decision variables with four indices. Skorin-Kapov et al. (1996) proposed a new mixed-integer formulation again with four-indexed decision variables. The LP relaxation of this new formulation yields good lower bounds with less than 1% below the optimal objective function value.

O'Kelly et al. (1996) proposed a formulation for the single allocation problem with symmetrical flow data to reduce the problem size. The study focused on sensitivity of the solutions to inter-hub discount factor and exact solution methodologies for hub location problems. Sohn and Park (1998) presented a reduced size formulation, and mixed-integer formulation again for the single allocation problem with symmetrical flows.

Ernst and Krishnamoorthy (1996) presented a new formulation using 3-index decision variables. This formulation has fewer variables and constraints and it yields quite efficient solution times for optimality when compared with other formulations in the literature using commercial solvers.

Ebery (2001) proposed a different formulation with 2-index decision variables for the single allocation problem. This formulation has the least number of decision variables in the literature, yet the solution times required for the optimality were slower than the formulation presented by Ernst and Krishnamoorthy (1996).

Campbell (1992) proposed the first linear integer programming formulation for the multiple allocation p-hub median problem. The formulation contains 4-index decision variables. This paper also presented models for the problems with flow thresholds and fixed costs. Skorin-Kapov et al. (1996) presented a new mixed-integer formulation. This formulation contains less constraints and variables than the Campbell (1992) formulation and provides tighter LP relaxation results.

Ernst and Krishnamoorthy (1998) presented a 3-index formulation for the multiple allocation p-hub median problem. This formulation utilizes the similar idea with the single allocation formulation presented in Ernst and Krishnamoorthy (1996). This formulation provided faster solution times for optimality than other multiple allocation formulations.

There are a number of studies proposing heuristic algorithms to solve hub location problems in the literature. Ilić et al. (2010) proposed one of the best heuristics in terms of solution quality and CPU time required for the solutions of the single allocation p-hub median problem. This algorithm uses variable neighbourhood search (VNS) and solves very large problems with up to 1,000 nodes and 20 hubs in reasonable CPU times. Marić et al. (2013) presented a memetic algorithm (MA) for solving the uncapacitated single allocation hub location problem. This heuristic provides very good results especially with large-scale problems.

Contreras et al. (2011a) presented a Bender's decomposition algorithm for the multiple allocation uncapacitated hub location problem. The algorithm is able to solve instances with up to 500 nodes within resonable CPU times. Contreras et al. (2011b), on the other hand, proposed a branch-and-price algorithm for the capacitated single allocation hub location problem.

2.2 The p-hub center problem

The p-hub center problem locates p hubs with the aim of minimizing the maximum transportation cost on the hub network. In this section, the studies on the p-hub center problem are provided.

Campbell (1994a) introduced p-hub center problems to the literature and proposed the first linear integer programming formulations for the single and multiple allocation versions of the problem. Additionally, the author discussed and classified the p-hub center problem in three categories. The first one is that the maximum cost for any origin-destination pair is minimized. This type of hub center problems are important for perishable or time sensitive items in hub networks. The second one minimizes the maximum cost for any single link. This type is important when preserving or processing at hub locations are effective. The last one minimizes the cost only between a hub and origin. This type has similar features with the second type under special conditions. Even though, there are different types of p-hub center problems, literature focused only on the first type as we defined.

The literature on the p-hub center problems is scarce compared with the p-hub median problem. Kara and Tansel (2000) presented several basic model linearizations and a new formulation for the single allocation p-hub center problem, which provide faster solution times than other formulations shown by computational experiments in the study. Ernst et al. (2009) proposed new mixed-integer programming formulations for both single and multiple allocation problems that are superior to the previous p-hub center formulations. Meyer et al. (2009) proposed an ant colony optimization algorithm for the single allocation p-hub center problem. This algorithm provides high quality solutions for large-scale problems with up to 400 demand nodes.

2.3 Multi-objective hub location studies

Multi-objective optimization has been used for simultaneous optimization of more than two different objective functions for mathematical programming problems. This method is commonly applied when optimality factor is based on more than one conflicted trade-off. We consider median and center type objectives in this thesis because they are the most employed objectives in the hub location literature. These objectives are minimizing total transportation cost, and minimizing the maximum transportation cost in the network. These are two conflicting objectives, thus bi-objective optimization is used in our models.

There are not many studies considering multiple-objectives in hub location problems. Costa et al. (2008) presented two bi-criteria single allocation hub location problems. In the first model the objectives are minimizing the summation of cost and minimizing the total time to process the flow entering hubs. In the second model, the objectives are minimizing the summation of cost and minimizing the maximum service time for the hubs.

Köksalan and Soylu (2010) studied two bi-criteria multiple allocation p-hub location problems. The former problem in this study minimizes the total transportation costs and total travelling costs between hubs and origin-destination points, where the problem turns into classical facility location problem. The latter problem focuses on the service delay at the hubs. The objectives in this problem are minimizing the total transportation cost and minimizing the maximum delay at each hub. Moreover, an evolutionary algorithm is proposed.

Mohammadi et al. (2011) proposed a multi-objective imperial algorithm to find near-optimal solutions for capacitated hub covering problems. The first objective is minimizing the total transportation cost while the second objectice is minimizing service times in the hubs. Mohammadi et al. (2013) used a novel stochastic multi-objective model for hub covering problems under uncertainty. Their objectives are minimization of total current investment costs and total maximum transportation time between each origin-destination pair.

As can be seen, there are not any hub location studies in the literature combining median and center objectives together in a bi-objective problem.

In this thesis, we study bi-objective hub location problems. The two conflicting objective functions can be combined with several different strategies. The first strategy we suggest is the weighting method from the literature (Fishburn, 1967). The strategy is as follows:

$$Min \qquad \beta_1 Obj_1 + \beta_2 Obj_2 \tag{2.1}$$

where β_1, $\beta_2 \geq 0$ and $\sum_{i=1}^{2} \beta_i = 1$. This bi-objective function provides pareto optimal solutions for different weights given to the objectives. We use this weighting method in our study as both of our objectives are in the same units.

The second strategy which could be used is the ϵ-constraint method (Haimes et al., 1971). In this method, one objective is selected to be optimized, while the other objective is added as a constraint with its upper or/and lower bound. For illustration:

$$Min \qquad Obj_1 \tag{2.2}$$

$$s.t. \quad Obj_2 \leq \epsilon_{Obj_2} \tag{2.3}$$

Where ϵ_{Obj_2} is an upper bound for the second objective. The model minimizes the first objective where the second objective is less than its upper bound.

The third strategy is no-preference method (Proos et al., 2001). This method is used on the scaling of the objective functions to make them dimensionless. For this reason, using ratio of the objective functions with their ideal solutions, or finding gaps between ideal solutions and current objective functions can be used. For instance:

$$Min \qquad Obj_1/Obj_1^{ideal} + Obj_2/Obj_2^{ideal} \tag{2.4}$$

Obj_1^{ideal} and Obj_2^{ideal} stand for individual optimum solutions for the two objectives. Moreover, this formulation takes the ratio of them the make the bi-objective function dimensionless. Another non-preference method can be as follows (Proos et al., 2001):

$$Min \qquad \|Obj_1 - Obj_1^{ideal}\| + \|Obj_2 - Obj_2^{ideal}\| \tag{2.5}$$

This formulation finds the gaps between optimum solutions and the current objectives, and minimizes the total gap.

In addition to the stated methods, the hybrid methods, combining two or more different methodologies, can be used. Moreover, there are other strategies that can be used for the problem. One may refer for further strategies that as presented in Marler and Arora (2004), and Proos et al. (2001).

Chapter 3

Mathematical Models

In this chapter, we introduce mathematical models for bi-objective p-hub location problems. We study single, multiple, and r-allocation versions of the problem and propose two different mathematical formulations for each version. The parameters used in all of the formulations are defined as follows:

N	Set of nodes.
w_{ij}	Amount of demand originated at node $i \in N$ destined to node $j \in N$.
c_{ij}	Unit transportation cost from node $i \in N$ to node $j \in N$.
α	Transportation cost discount factor ($0 \leq \alpha \leq 1$).
p	Number of nodes to be selected as hubs.
β_1	Weight of the total transportation cost median objective.
β_2	Weight of the maximum transportation cost center objective.

3.1 Single allocation formulations

In single allocation, each demand node is allocated to exactly one hub node, which means that every demand point can receive and send flow through at most one hub. We introduce two different mixed-integer programming models for the single allocation version of the problem in the subsequent sections.

3.1.1 The 4-index formulation

We firstly introduce a formulation with decision variables consisting of four indices, referred as the 4-index formulation. To formulate this model we used ideas from the p-hub median formulation presented by Skorin-Kapov et al. (1996) and the p-hub center formulation presented by Campbell (1994a). The decision variables that are used for the 4-index model are given below:

$$x_{ik} = \begin{cases} 1, & \text{if node } i \in N \text{ is allocated to hub } k \in N, \\ 0, & \text{otherwise.} \end{cases}$$

y_{ijkm}	Fraction of demand originated at node $i \in N$ destined to node $j \in N$ which is transferred from hub $k \in N$ to hub $m \in N$.
obj_1	Total transportation cost.
obj_2	Maximum transportation cost.

The 4-index formulation for the single allocation bi-objective p-hub location problem with the given variables and parameters is as follows:

$$Min \quad \beta_1 obj_1 + \beta_2 obj_2 \tag{3.1}$$

$$s.t. \quad obj_1 = \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{m \in N} w_{ij} y_{ijkm} (c_{ik} + \alpha c_{km} + c_{mj}) \tag{3.2}$$

$$obj_2 \geq \sum_{k \in N} \sum_{m \in N} y_{ijkm} (c_{ik} + \alpha c_{km} + c_{mj}) \qquad \forall i, j \in N \tag{3.3}$$

$$\sum_{k \in N} x_{kk} = p \tag{3.4}$$

$$\sum_{k \in N} x_{ik} = 1 \qquad \forall i \in N \tag{3.5}$$

$$x_{ik} \leq x_{kk} \qquad \forall i, k \in N \quad (3.6)$$

$$\sum_{k \in N} \sum_{m \in N} y_{ijkm} = 1 \qquad \forall i, j \in N \quad (3.7)$$

$$\sum_{k \in N} y_{ijkm} \leq x_{jm} \qquad \forall i, j, m \in N \quad (3.8)$$

$$\sum_{m \in N} y_{ijkm} \leq x_{ik} \qquad \forall i, j, k \in N \quad (3.9)$$

$$x_{ik} \in \{0, 1\} \qquad \forall i, k \in N \quad (3.10)$$

$$y_{ijkm} \geq 0 \qquad \forall i, j, k, m \in N \quad (3.11)$$

Objective function (3.1) is the weighted sum of two objectives. It minimizes the weighted sum of median and center objectives. Equation (3.2) computes the total transportation cost, while constraints (3.3) compute the maximum transportation cost in the network. Constraint (3.4) ensures that the model locates exactly p hub facilities. Constraints (3.5) allocates a node to a single hub. Constraints (3.6) make allocations to be made only for located hubs. Constraints (3.7) states that all demands are to be satisfied. Constraints (3.8) and (3.9) assure that the flow is routed only through feasible allocations. Constraints (3.10) define the binary variables and constraints (3.11) are the non-negativity constraints.

3.1.2 The 3-index formulation

In addition to the formulation introduced in the previous section, we introduce another formulation using decision variables with three indices rather than four. We formulated this model based on the median formulation of Ernst and Krishnamoorthy (1998) and the center formulation of Ernst et al. (2009). The decision variables used in the 3-index formulation are defined as follows:

$$x_{ik} = \begin{cases} 1, & \text{if node } i \in N \text{ is allocated to a hub at node } k \in N, \\ 0, & \text{otherwise.} \end{cases}$$

y_{ikm}	Amount of demand originated at node $i \in N$ which is transferred from hub $k \in N$ to hub $m \in N$.
r_k	Maximum distance between hub $k \in N$ and the nodes which are allocated to it (radius of hub $k \in N$).
obj_1	Total transportation cost.
obj_2	Maximum transportation cost.

The 3-index formulation of the single allocation problem is as follows:

$$Min \quad \beta_1 obj_1 + \beta_2 obj_2 \tag{3.1}$$

$$s.t. \quad obj_1 = \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} w_{ij} c_{ik} x_{ik} + \sum_{i \in N} \sum_{k \in N} \sum_{m \in N} \alpha c_{km} y_{ikm} + \sum_{i \in N} \sum_{j \in N} \sum_{m \in N} w_{ij} c_{mj} x_{jm} \tag{3.12}$$

$$obj_2 \geq r_k + \alpha c_{km} + r_m \qquad \forall k, m \in N \tag{3.13}$$

$$\sum_{k \in N} x_{kk} = p \tag{3.4}$$

$$\sum_{k \in N} x_{ik} = 1 \qquad \forall i \in N \tag{3.5}$$

$$x_{ik} \leq x_{kk} \qquad \forall i, k \in N \tag{3.6}$$

$$\sum_{m \in N} y_{ikm} - \sum_{m \in N} y_{imk} = \sum_{j \in N} w_{ij} x_{ik} - \sum_{j \in N} w_{ij} x_{jk} \qquad \forall i, k \in N \tag{3.14}$$

$$\sum_{m \in N} y_{ikm} \leq \sum_{j \in N} w_{ij} x_{ik} \qquad \forall i, k \in N \tag{3.15}$$

$$r_k \geq c_{ik} x_{ik} \qquad \forall i, k \in N \tag{3.16}$$

$$x_{ik} \in \{0, 1\} \qquad \forall i, k \in N \tag{3.10}$$

$$y_{ikm} \geq 0 \qquad \forall i, k, m \in N \tag{3.11}$$

In this formulation, constraint (3.12) calculates the total transportation cost, and constraints (3.13) calculates the maximum transportation cost between pair of nodes in the network. Constraints (3.14) are flow balance constraints. Constraints (3.15) force the flow to be correctly routed on the network. Finally, constraints (3.16) calculates the maximum cost radius of every hub in the network.

3.2 Multiple allocation formulations

Multiple allocation problems allow nodes to receive and send flow through more than one hub, which means non-hub nodes can be allocated to more than one hub node. In the following two subsections, we introduce two different mixed-integer programming formulations for the multiple allocation bi-objective p-hub location problem.

3.2.1 The 4-index formulation

Similar to the single allocation version, we fist introduce a 4-index formulation of the problem. We adopt the formulation presented by Skorin-Kapov et al. (1996) for the median part and the formulation presented by Ernst et al. (2009) for the center part of the model. The decision variables required for this formulation are provided below:

$$x_k = \begin{cases} 1, & \text{if node } k \in N \text{ is a hub,} \\ 0, & \text{otherwise.} \end{cases}$$

y_{ijkm}	Fraction of demand originated at node $i \in N$ destined to node $j \in N$ which is transferred from hub $k \in N$ to hub $m \in N$.
obj_1	Total transportation cost.
obj_2	Maximum transportation cost.

Note that, we require one less index in defining the x variables in the multiple allocation version of the problem. We can now present the 4-index formulation of the problem.

$$Min \qquad \beta_1 obj_1 + \beta_2 obj_2 \tag{3.1}$$

$$s.t. \quad obj_1 = \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{m \in N} w_{ij} y_{ijkm} (c_{ik} + \alpha c_{km} + c_{mj}) \tag{3.2}$$

$$obj_2 \geq \sum_{k \in N} \sum_{m \in N} y_{ijkm} (c_{ik} + \alpha c_{km} + c_{mj}) \qquad \forall i, j \in N \tag{3.3}$$

$$\sum_{k \in N} \sum_{m \in N} y_{ijkm} = 1 \qquad \forall i, j \in N \tag{3.7}$$

$$\sum_{k \in N} x_k = p \tag{3.17}$$

$$\sum_{k \in N} y_{ijkm} \leq x_m \qquad \forall i, j, m \in N \tag{3.18}$$

$$\sum_{m \in N} y_{ijkm} \leq x_k \qquad \forall i, j, k \in N \tag{3.19}$$

$$x_k \in \{0, 1\} \qquad \forall k \in N \tag{3.10}$$

$$y_{ijkm} \geq 0 \qquad \forall i, j, k, m \in N \tag{3.11}$$

Constraint (3.17) ensures that the model locates exactly p hubs. Constraints (3.18) and (3.19) represent that flow is routed only through established hubs. The rest of the constraints of the model are as defined as before.

3.2.2 The 3-index formulation

We additionally introduce a 3-index formulation as well. To model this formulation, we adapted the formulations of Ernst and Krishnamoorthy (1998) and Ernst et al. (2009). The decision variables defined for multiple allocation 3-index formulation are given below:

$$x_k = \begin{cases} 1, & \text{if node } k \in N \text{ is a hub,} \\ 0, & \text{otherwise.} \end{cases}$$

$$u_{ikj} = \begin{cases} 1, & \text{If node } i \in N \text{ is allocated to hub } k \in N \text{ to send flow to node } j \in N. \\ 0, & \text{otherwise.} \end{cases}$$

$$v_{imj} = \begin{cases} 1 & \text{If node } j \in N \text{ is allocated to hub } m \in N \text{ to receive flow originated} \\ & \text{from node } i \in N. \\ 0, & \text{otherwise.} \end{cases}$$

y_{ikm}	Amount of demand originated at node $i \in N$ which is transferred from hub $k \in N$ to hub $m \in N$.
h_{imj}	Amount of demand originated at node $i \in N$ flowing from hub $m \in N$ to node $j \in N$.
z_{ik}	Amount of flow from node $i \in N$ to hub $k \in N$.
obj_1	Total transportation cost.
obj_2	Maximum transportation cost.

Note that the radius variable (r_k) that we used in the single allocation 3-index formulation is not used in this model. Since a demand node can use a different hub for sending flow to each destination, we cannot use the radius idea to model the multiple allocation version of the problem. The mixed-integer programming formulation for the 3-index multiple allocation problem is as follows:

$$Min \quad \beta_1 obj_1 + \beta_2 obj_2 \tag{3.1}$$

$$s.t. \quad obj_1 = \sum_{i \in N} \sum_{k \in N} c_{ik} z_{ik} + \sum_{i \in N} \sum_{k \in N} \sum_{m \in N} \alpha c_{km} y_{ikm} + \sum_{i \in N} \sum_{j \in N} \sum_{m \in N} c_{mj} h_{imj} \tag{3.20}$$

$$obj_2 \geq \sum_{k \in N} (c_{ik} + \alpha c_{km}) u_{ikj} + \sum_{n \in N} c_{nj} v_{inj} -$$

$$\alpha(1 - v_{imj}) c_{max} \qquad \forall i, j, m \in N \qquad (3.21)$$

$$\sum_{k \in N} x_k = p \qquad (3.17)$$

$$\sum_{k \in N} z_{ik} = \sum_{j \in N} w_{ij} \qquad \forall i \in N \qquad (3.22)$$

$$\sum_{m \in N} h_{imj} = w_{ij} \qquad \forall i, j \in N \qquad (3.23)$$

$$\sum_{m \in N} y_{ikm} - \sum_{m \in N} y_{imk} = z_{ik} - \sum_{j \in N} h_{ikj} \qquad \forall i, k \in N \qquad (3.24)$$

$$z_{ik} \leq \sum_{j \in N} w_{ij} x_k \qquad \forall i, k \in N \qquad (3.25)$$

$$h_{imj} \leq w_{ij} x_m \qquad \forall i, j, m \in N \qquad (3.26)$$

$$u_{ikj} \leq x_k \qquad \forall i, j, k \in N \qquad (3.27)$$

$$v_{imj} \leq x_m \qquad \forall i, j, m \in N \qquad (3.28)$$

$$\sum_{k \in N} u_{ikj} = 1 \qquad \forall i, j \in N \qquad (3.29)$$

$$\sum_{m \in N} v_{imj} = 1 \qquad \forall i, j \in N \qquad (3.30)$$

$$u_{ikj}, v_{imj}, x_k \in \{0, 1\} \qquad \forall i, j, k, m \in N \qquad (3.31)$$

$$y_{ikm}, h_{imj}, z_{ik} \geq 0 \qquad \forall i, j, k, m \in N \qquad (3.32)$$

Constraint (3.20) presents the total transportation cost. Constraints (3.21) calculate the maximum transportation cost in the network, where c_{max} is the maximum unit transportation cost. Constraints (3.22)-(3.24) are flow balance constraints. Constraints (3.25) and (3.26) assure that the flows between nodes and hubs exist when hubs are located. Similarly, constraints (3.27) and (3.28) make sure that nodes are allocated only to established hubs. Constraints (3.29) and (3.30), on the other hand state that the demand between an origin destination pair is routed using at most one origin and one destination hub.

3.3 r-allocation formulations

In r-allocation problems, each node can be assigned to at most r hubs. If r is 1, the problem reduces to the single allocation problem; on the other hand, if r is equal to the maximum number of nodes, the problem becomes the multiple allocation problem. Thus, r-allocation is a generalization of both of the allocation rules (Yaman, 2011). Similar to other allocation rules, we present two formulations for the r-allocation version of the problem.

3.3.1 The 4-index formulation

We present a mixed-integer formulation for the problem with decision variables of four indices in this section. We used ideas from the r-allocation p-hub median formulation presented by Yaman (2011) to model this problem. However, there is not any study in the literature presenting a model for the p-hub center version of the r-allocation problem. The decision variables used for r-allocation 4-index formulation are given below:

$$x_{ik} = \begin{cases} 1, & \text{if node } i \in N \text{ is allocated to hub } k \in N, \\ 0, & \text{otherwise.} \end{cases}$$

y_{ijkm}	Fraction of demand originated at node $i \in N$ destined to node $j \in N$ which is transferred from hub $k \in N$ to hub $m \in N$.
obj_1	Total transportation cost.
obj_2	Maximum transportation cost.

The formulation is as follows:

$$Min \quad \beta_1 obj_1 + \beta_2 obj_2 \tag{3.1}$$

$$s.t. \quad obj_1 = \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{m \in N} w_{ij} y_{ijkm} (c_{ik} + \alpha c_{km} + c_{mj}) \tag{3.2}$$

$$obj_2 = \sum_{k \in N} \sum_{m \in N} y_{ijkm} (c_{ik} + \alpha c_{km} + c_{mj}) \qquad \forall i, j \in N \tag{3.3}$$

$$\sum_{k \in N} x_{kk} = p \tag{3.4}$$

$$x_{ik} \leq x_{kk} \qquad \forall i, k \in N, \tag{3.6}$$

$$\sum_{k \in N} y_{ijkm} \leq x_{jm} \qquad \forall i, j, m \in N \tag{3.8}$$

$$\sum_{m \in N} y_{ijkm} \leq x_{ik} \qquad \forall i, j, k \in N \tag{3.9}$$

$$\sum_{k \in N} \sum_{m \in N} y_{ijkm} = 1 \qquad \forall i, j \in N \tag{3.7}$$

$$\sum_{k \in N} x_{ik} \leq r \qquad \forall i \in N \tag{3.33}$$

$$x_{ik} \in \{0, 1\} \qquad \forall i, k \in N \tag{3.10}$$

$$y_{ijkm} \geq 0 \qquad \forall i, j, k, m \in N \tag{3.11}$$

This formulation uses the same set of constraints introduced in the previous formulations, except for constraints (3.33). These constraints make sure that the nodes are allocated to at most r hubs.

3.3.2 The 3-index formulation

In this section, an the r-allocation formulation containing three indices is given. We used ideas from Ernst et al. (2009) and Yaman (2011) to formulate this model. The decision variables used for the 3-index r-allocation formulation are defined as follows:

$$x_{ik} = \begin{cases} 1, & \text{if node } i \in N \text{ is alocated to hub } k \in N, \\ 0, & \text{otherwise.} \end{cases}$$

$$u_{ikj} = \begin{cases} 1, & \text{If node } i \in N \text{ is allocated to hub } k \in N \text{ to send flow to node } j \in N. \\ 0, & \text{otherwise.} \end{cases}$$

$$v_{imj} = \begin{cases} 1 & \text{If node } j \in N \text{ is allocated to hub } m \in N \text{ to recieve flow originated} \\ & \text{from node } i \in N. \\ 0, & \text{otherwise.} \end{cases}$$

y_{ikm}	Amount of demand originated at node $i \in N$ which is transferred from hub $k \in N$ to hub $m \in N$.
h_{imj}	Amount of demand originated at node $i \in N$ flowing from hub $m \in N$ to node $j \in N$.
z_{ik}	Amount of flow from node $i \in N$ to hub $k \in N$.
obj_1	Total transportation cost.
obj_2	Maximum transportation cost.

The r-allocation 3-index mixed-integer programming formulation we proposed is as follows:

$$Min \quad \beta_1 obj_1 + \beta_2 obj_2 \tag{3.1}$$

$$s.t. \quad obj_1 = \sum_{i \in N} \sum_{k \in N} c_{ik} z_{ik} + \sum_{i \in N} \sum_{k \in N} \sum_{m \in N} \alpha c_{km} y_{ikm} + \sum_{i \in N} \sum_{m \in N} \sum_{j \in N} c_{mj} h_{imj} \tag{3.20}$$

$$obj2 \geq \sum_{k \in N} (c_{ik} + \alpha c_{km}) u_{ikj} + \sum_{n \in N} c_{nj} v_{inj} - \alpha(1 - v_{imj}) c_{max} \qquad \forall i, j, m \in N \tag{3.21}$$

$$\sum_{k\in N} x_{kk} = p \tag{3.4}$$

$$x_{ik} \leq x_{kk} \qquad \forall i,k \in N \tag{3.5}$$

$$\sum_{k\in N} z_{ik} = \sum_{j\in N} w_{ij} \qquad \forall i \in N \tag{3.22}$$

$$\sum_{m\in N} h_{imj} = w_{ij} \qquad \forall i,j \in N \tag{3.23}$$

$$\sum_{m\in N} y_{ikm} - \sum_{m\in N} y_{imk} = z_{ik} - \sum_{j\in N} h_{ikj} \qquad \forall i,k \in N \tag{3.24}$$

$$\sum_{k} u_{ikj} = 1 \qquad \forall i,j \in N \tag{3.29}$$

$$\sum_{m} v_{imj} = 1 \qquad \forall i,j \in N \tag{3.30}$$

$$\sum_{k\in N} x_{ik} \leq r \qquad \forall i \in N, \tag{3.33}$$

$$z_{ik} \leq \sum_{j\in N} w_{ij}x_{ik} \qquad \forall i,k \in N \tag{3.34}$$

$$h_{imj} \leq w_{ij}x_{jm} \qquad \forall i,j,m \in N \tag{3.35}$$

$$u_{ikj} \leq x_{ik} \qquad \forall i,j,k \in N \tag{3.36}$$

$$v_{imj} \leq x_{jm} \qquad \forall i,j,m \in N \tag{3.37}$$

$$u_{ikj}, v_{imj}, x_{ik} \in \{0,1\} \qquad \forall i,j,k,m \in N \tag{3.10}$$

$$y_{ikm}, h_{imj}, z_{ik} \geq 0 \qquad \forall i,j,k,m \in N \tag{3.11}$$

The constraints of this model are almost the same with the multiple allocation 3-index formulation's. However, the location decision variable has an extra index to account for allocations in the r-allocation formulation. Hence, constraints (3.25)-(3.28) are modified accordingly as (3.34)-(3.37). Moreover, constraints (3.33) limiting the number of allocations for r is introduced.

3.4 Comparison of the formulations

In this section, we demonstrate the number of variables and constraints in each of the formulations. Let $|N| = n$, then from Table 3.1, single allocation 4-index mixed-integer formulation has $O(n^2)$ binary, $O(n^4)$ continuous variables, and $O(n^3)$ constraints, while single allocation 3-index formulation has $O(n^2)$ binary, $O(n^3)$ continuous variables, and $O(n^2)$ constraints. As can be seen, the single allocation 3-index formulation has less number of continuous variables and constraints than 4-index formulation has. On the other hand, note that there are less number of binary variables and constraints in the 4-index formulations of the multiple and r-allocation problems. However, there are more continuous variables in these formulations compared with the 3-index versions.

Table 3.1: Number of variables and constraints in the formulations.

	Single Allocation		Multiple Allocation		r-allocation	
	4-index	3-index	4-index	3-index	4-index	3-index
Number of binary variables	n^2	n^2	n	$2n^3 + n$	n^2	$2n^3 + n^2$
Number of continuous variables	$n^4 + 2$	$n^3 + n + 2$	$n^4 + 2$	$2n^3 + n^2 + 2$	$n^4 + 2$	$2n^2 + n^2 + 2$
Number of constraints	$2n^3 + 3n^2 + n + 2$	$5n^2 + n + 2$	$2n^3 + 2n^2 + 2$	$4n^3 + 5n^2 + n + 2$	$2n^3 + 3n^2 + n + 2$	$4n^3 + 6n^2 + 2n + 2$

Chapter 4

Computational Experiments

In this chapter, we test our formulations and analyze the resulting locations of hubs on the Civil Aeronautics Board (CAB) dataset O'Kelly (1987). This dataset is formed based on airline passenger interactions between 25 U.S. cities (Figure 4.1). Each city is a potential hub location. The data on the demand between each pair of cities (w_{ij}) and costs (c_{ij}) are presented in OR Library Beasley (1990). We scaled the demand values so that the total demand adds up to one as customarily done in the literature. Due to this scaling, it is possible that the center objective (the maximum cost) can be greater than the median objective (the total cost) in our results. We test different values for the number of hubs (p) the transportation cost discount factor between hubs (α), and the weights of the objectives (β_1, and β_2). We take α values to be 0.2 (the highest discount), 0.4, 0.6, and 0.8 (the lowest discount). Number of hubs are varied from 2 to 8. We tested different values for β_1 and β_2 between 0 and 1. As mentioned before, we used the weighting method as both of the objectives are in same units (cost). However, we would like to note that the choice of weights may effect the bi-objective model as one objective function can be larger than the other one.

All the numerical experiments were performed on a computer with AMD A-10-7300

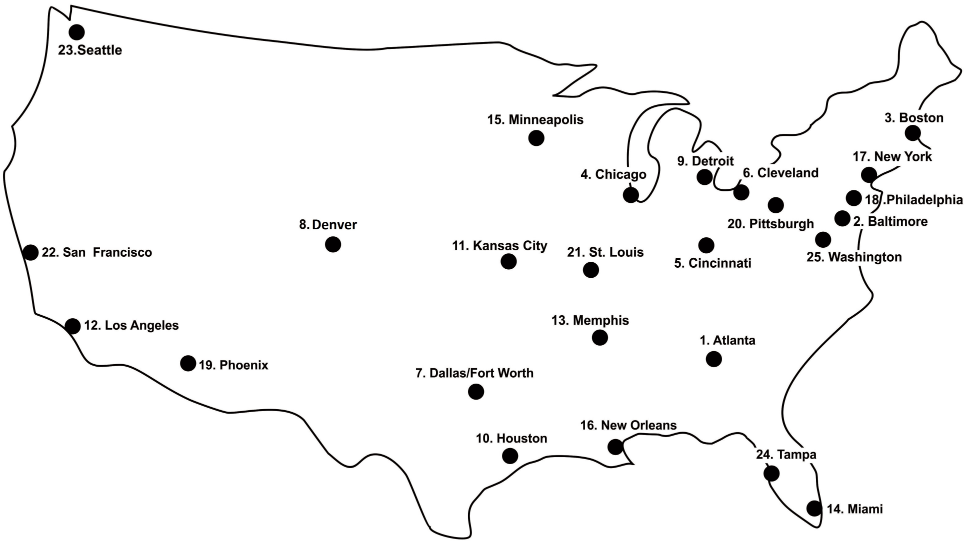


Figure 4.1: CAB dataset.

4-core 1.90 GHz processor and 12.0 GB of RAM. The formulations were coded and solved using *IBM ILOG CPLEX Optimization Studio 12.7*. We changed absolute and relative mixed-integer programming gap tolerances as 10^{-9} for all the experiments.

In the next section, we compare the performances of the 4- and 3-index formulations using instances from the CAB dataset. In section 4.2, the computational results for single allocation, multiple allocation, and r-allocation problems are provided using selected formulations. We demonstrate the trade-off between the two objectives in the last section.

4.1 Comparison with the formulations

In this section, we compare each of the formulations that we proposed in Chapter 3 using the CAB dataset. Initially, we compare the number of variables and constraints in all the formulations in Table 4.1. Table 4.1 has three parts for single allocation, multiple allocation, and r-allocation formulations. For each allocation rule, we calculated the number of binary and continuous variables and the number of constraints in the proposed 4- and

3-index formulations with the CAB dataset containing 25 nodes.

Table 4.1: Number of variables and constraints in the formulations ($n = 25$).

	Single Allocation		Multiple Allocation		r-allocation	
	4-index	3-index	4-index	3-index	4-index	3-index
Number of binary variables	625	625	25	$31,275$	625	$31,875$
Number of continuous variables	$390,627$	$15,627$	$390,627$	$31,877$	$390,627$	$31,877$
Number of constraints	$33,152$	$3,152$	$32,502$	$65,652$	$33,152$	$66,302$

Observe from Table 4.1 that both of the single allocation formulations has exactly the same number of binary variables. However, the 3-index single allocation formulation has less number of continuous variables and constraints than the 4-index multiple allocation formulation. The 4-index multiple allocation formulation has the least number of binary variables among all the given formulations, and it has less number of constraints than the 3-index multiple allocation formulation. However, the 4-index formulation has much more number of continuous variables than the 3-index multiple allocation formulation. When we look at the r-allocation formulations, they have quite similar features with the multiple allocation formulations. Again, while the 4-index formulation has less number of binary variables and constraints than the 3-index formulation, the 3-index formulation has less number of continuous variables.

Table 4.2 presents and compares the CPU time requirements to obtain optimal solutions with the proposed formulations. For this analysis, we set a 12-hour (43,200 seconds) maximum time limit to *CPLEX*. The results are observed with $p = 3$, and $p = 4$, additionally, with $r = 2$ for the r-allocation formulations. We tested all formulations with four different α values. β_1 and β_2 values are both taken as 0.5 for this analysis.

From Table 4.2 it can be observed that the 3-index formulation for the single allocation problem works faster than the 4-index formulation. Both of the formulations have ex-

Table 4.2: CPU time requirements in seconds with the formulations when $p = 3$ and 4.

	α	Single allocation 4-index	Single allocation 3-index	Multiple allocation 4-index	Multiple allocation 3-index	r-allocation 4-index	r-allocation 3-index
$p = 3$	0.2	199.8	26.1	170.4	3,046.4	158.4	11,137.7
	0.4	353.1	46.9	161.0	30,804.9	181.5	42,816.6
	0.6	273.8	48.3	96.6	43,200	128.2	43,200
	0.8	153.6	40.8	95.7	43,200	198.5	43,200
$p = 4$	0.2	212.8	55.0	247.4	25,169.6	238.6	26,184.2
	0.4	250.2	53.0	177.5	43,200	235.0	43,200
	0.6	304.0	81.1	146.9	43,200	230.0	43,200
	0.8	274.2	96.1	100.1	43,200	216.2	43,200
Average CPU time (s)		253.4	56.4	149.2	34,377.6	198.1	37,017.3

actly the same number of binary variables; however, note from Table 4.1 that the 3-index formulation has less number of continuous variables and constraints.

When we analyse the solution times with the multiple allocation formulations from Table 4.2, it is obvious that the 4-index formulation is more consistent and has faster solution times than the 3-index formulation. It seems that more number of binary variables and constraints makes the 3-index formulation slower and less efficient.

We can interpret similar conclusions from Table 4.2 for the r-allocation formulations (as with the multiple allocation formulations). The 4-index r-allocation formulation is clearly more efficient than the 3-index r-allocation formulation on these instances.

Observe from the average CPU times reported in the last row of Table 4.2 that the 3-index single allocation formulation is significantly faster than all the other formulations, while the 3-index r-allocation formulation is the slowest. The 3-index multiple allocation formulation has slightly lower CPU time requirement than the 3-index r-allocation formulation.

Given the results that we obtained in this section, we decided to use the 3-index single, the 4-index multiple, and the 4-index r-allocation formulations for the remaining computational experiments.

4.2 Computational results

In this section, we present results with the selected single, multiple, and r-allocation formulations on the CAB dataset. The experiments are done with p values ranging from 1 to 8, four α values (0.2, 0.4, 0.6, and 0.8) and three different combinations for (β_1, β_2): (1, 0), (0.5, 0.5), and (0, 1).

Table 4.3 demonstrates solutions of the single allocation problem. The first column indicates eight different p values tested for the problem, and the second column lists the corresponding α value for each instance (Note that there is no economies of scale between hubs when only a single hub is located, so the solutions are independent from the α value when $p = 1$). The table is divided into three main parts. The first part represents solutions of the problem when $\beta_1 = 1$ and $\beta_2 = 0$ (p-hub median solutions). The second part is for $\beta_1 = 0.5$ and $\beta_2 = 0.5$, while the third part is for $\beta_1 = 0$ and $\beta_2 = 1$ (p-hub center solutions). Furthermore, the first columns in each part show the median objective function value (Obj_1), and the second columns report the center objective function value (Obj_2) at optimality. The third columns list optimal hub locations for each instance, and, lastly, the fourth columns indicate the CPU times in seconds required to obtain the optimal solutions.

Observe from Table 4.3 that different solutions are obtained under different β values given to the two objectives - median and center. When the weights of the objectives changes, objective function values and optimum hub locations change. Note that when $p = 1$, equally weighted problem and the center objective weighted problem result in exactly

Table 4.3: Solutions of the single allocation problem with the 3-index formulation.

		$\beta_1 = 1, \beta_2 = 0$				$\beta_1 = 0.5, \beta_2 = 0.5$				$\beta_1 = 0, \beta_2 = 1$			
p	α	Obj_1	Obj_2	Hub locations	CPU time(s)	Obj_1	Obj_2	Hub locations	CPU time(s)	Obj_1	Obj_2	Hub locations	CPU time(s)
1		1,491	4,074	5	1.2	1,781	3,013	11	2.7	1,781	3,013	11	1.5
2	0.2	1,001	2,539	12,20	0.7	1,074	2,183	5,22	6.0	1,266	2,132	21,22	16.5
	0.4	1,102	2,964	12,20	1.2	1,213	2,404	5,8	16.3	1,442	2,403	8,21	9.4
	0.6	1,201	3,565	12,20	2.7	1,264	2,620	5,8	22.4	1,505	2,559	8,21	7.6
	0.8	1,294	3,990	12,20	7.8	1,315	2,836	5,8	15.5	1,762	2,715	8,21	7.1
3	0.2	767	2,577	4,12,17	1.4	821	1,937	13,17,22	24.9	1,062	1,923	5,19,23	12.3
	0.4	902	2,968	4,12,18	3.7	1,084	2,101	1,8,20	43.2	1,195	2,101	1,6,8	15.5
	0.6	1,034	3,328	2,4,12	7.8	1,186	2,342	1,8,20	42.1	1,469	2,340	8,9,16	11.7
	0.8	1,159	3,792	2,4,12	16.6	1,241	2,587	8,13,20	36.2	1,518	2,554	6,8,16	8.0
4	0.2	630	2,244	4,12,17,24	1.3	716	1,672	11,18,22,24	52.0	945	1,620	9,16,19,23	10.3
	0.4	788	2,592	1,4,12,17	2.5	922	1,885	12,13,18,23	51.5	1,127	1,885	12,13,20,23	13.4
	0.6	939	2,941	1,4,12,17	9.7	1,049	2,216	8,17,21,24	80.0	1,353	2,183	19,21,22,23	14.0
	0.8	1,088	3,289	1,4,12,18	23.6	1,148	2,559	4,8,16,18	100.1	1,423	2,454	19,21,22,23	7.9
5	0.2	538	1,974	4,7,12,14,17	1.2	655	1,351	11,12,17,23,24	32.5	791	1,346	6,11,12,23,24	9.1
	0.4	708	2,298	4,7,12,14,17	2.0	871	1,600	11,12,14,18,23	74.0	877	1,600	11,12,18,23,24	14.8
	0.6	877	2,789	4,7,12,14,17	11.8	948	2,092	12,14,18,21,23	89.0	1,182	2,004	13,18,19,22,23	14.1
	0.8	1,034	3,180	1,4,7,12,18	30.0	1,191	2,378	13,19,20,22,23	116.7	1,337	2,371	6,8,16,22,23	11.0
6	0.2	491	1,974	4,6,7,12,14,17	1.9	507	1,328	4,7,12,17,23,24	34.7	647	1,225	11,12,13,14,23,25	16.0
	0.4	660	2,366	4,6,7,12,14,17	3.8	697	1,571	4,7,12,14,17,23	52.7	876	1,473	11,18,19,22,23,24	13.8
	0.6	828	2,776	4,6,7,12,14,17	15.4	943	1,898	14,18,19,21,22,23	93.6	1,063	1,860	11,14,17,19,22,23	10.6
	0.8	991	3,166	1,4,6,7,12,17	30.9	1,098	2,299	18,19,21,22,23,24	96.2	1,374	2,254	11,12,14,17,22,23	6.7
7	0.2	448	1,688	4,6,7,12,14,17,22	1.3	484	1,120	4,8,12,14,16,17,23	29.2	499	1,120	4,8,12,14,16,18,23	13.1
	0.4	622	2,121	4,6,7,12,14,17,22	3.4	696	1,470	4,7,14,17,19,22,23	75.0	836	1,390	11,12,18,19,22,23,24	12.6
	0.6	795	2,554	4,6,7,12,14,17,22	18.6	913	1,785	8,12,17,21,22,23,24	78.7	1,035	1,755	3,11,19,22,23,24,25	8.1
	0.8	960	3,141	1,4,6,7,12,17,25	71.1	1,018	2,275	4,7,12,17,22,23,24	83.5	1,223	2,221	3,5,19,21,22,23,24	8.3
8	0.2	415	1,658	1,4,6,7,12,14,17,22	1.5	1,054	486	4,8,14,16,17,19,22,23	39.0	501	1,054	4,8,14,16,18,19,22,23	15.4
	0.4	589	1,979	1,4,6,7,12,14,17,22	5.4	689	1,349	5,11,12,17,19,22,23,24	57.8	773	1,336	11,12,14,18,19,21,22,23	8.3
	0.6	763	2,456	1,4,6,7,12,14,17,22	16.8	822	1,760	4,7,12,17,19,22,23,24	62.8	1,020	1,715	3,11,12,18,19,22,23,24	4.7
	0.8	929	2,585	1,4,6,7,8,12,17,25	46.7	968	2,250	4,7,12,17,20,22,23,24	38.3	1,150	2,181	5,6,8,12,13,14,22,23	2.7

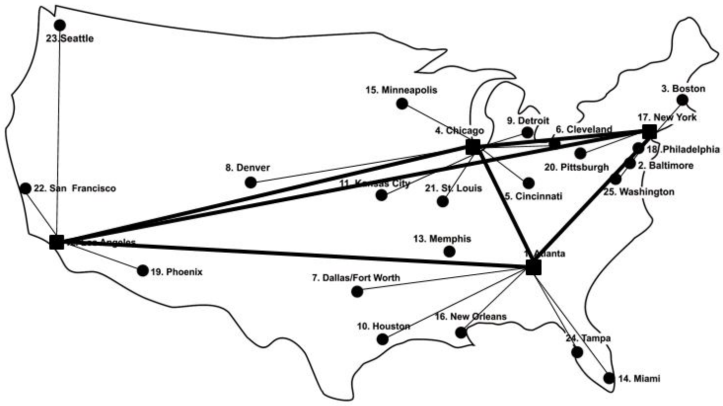
the same solution. In all the other instances, on the other hand, all resulting solutions are different under different β values. In most of the instances, the objective function values obtained under the equally weighted problem fall between the values obtained under the two extreme weighted problems.

Different hub locations are selected under different p values. Note that locations obtained with smaller p values are not a subset of the locations obtained with larger p values. This shows that the decision maker needs to determine the locations of all the hubs at once rather than locating the hubs incrementally. For a given p value, hub locations tend to change with different values of α. When the discount factor value is close to 1, the distances between hubs become smaller. Hence optimum locations are sensitive to the economies of scale discount factor.

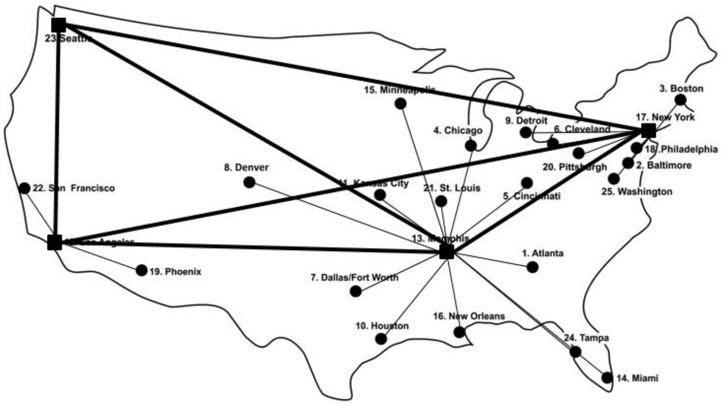
Lastly, observe from Table 4.3 that when $\beta_1 = 1$ and $\beta_2 = 0$, CPU times required for the optimal solutions increase when α increases. On the other hand, note that the CPU times decrease with the increase in α values when $\beta_1 = 0$ and $\beta_2 = 1$. With β_1,$\beta_2 = 0.5$, we cannot deduce a conclusion on the effect of α on the CPU times. When we observe the effect of p values on the CPU time again we cannot derive a generic conclusion. One would expect the CPU times to increase with increasing values of p; however, note from Table 4.3 that this is not the case. When we calculated the average CPU times of the instances under different β values, we observed that $\beta_1 = 0.5$, $\beta_2 = 0.5$ provided the slowest CPU times, while $\beta_1 = 1$, $\beta_2 = 0$ is the fastest.

Figure 4.2 depicts solutions of the single allocation problem with $p = 4$ and $\alpha = 0.4$ under different weights given to the objectives. In this figure, squares denote hubs, bold lines the inter-hub connections, and the thin lines the allocation connections.

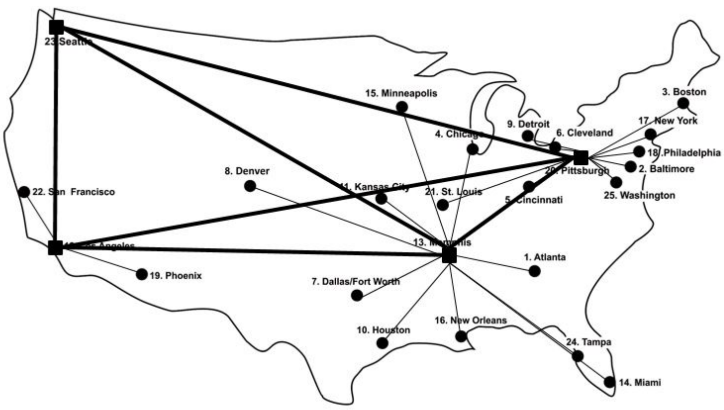
It can be observed from the figure that the distances between hubs are closer with the median objective problem compared with the other two solutions. In the center objective weighted solution, on the other hand, the distances between hubs tend to be far. In the



(a) $\beta_1 = 1$ and $\beta_2 = 0$.



(b) $\beta_1 = 0.5$ and $\beta_2 = 0.5$.



(c) $\beta_1 = 0$ and $\beta_2 = 1$.

Figure 4.2: Single allocation solutions with different β values when $p = 4$ and $\alpha = 0.4$.

equally weighted objective solution the locations of the three hubs are the same with the center weighted objective solution; however, one hub is located in Pittsburgh (20) rather then Philadelphia (18). Since Pittsburgh (20) is more centrally located than Philadelphia (18) the total transportation cost is 922 with the equally weighted solution whereas it is 1127 with the center weighted solution although both solutions yield the same maximum transportation cost (Obj_2) of 1885.

Table 4.4 shows solutions for the multiple allocation problem. As it is in the previous table, the first column is for eight different p values tested for the problem, and the second column presents α value for each instance. The table is separated into three main parts for three different β_1 and β_2 combinations. The first column of every part indicates the optimal hub median objective function value (Obj_1), the second column lists the optimal hub center objective function (Obj_2), the third column reports optimal hub locations, and lastly, the fourth column indicates CPU time requirements in seconds.

Observe from Table 4.4 that in general the model resulted in different solutions under different β values. Hub locations are exactly the same in a few solutions when (β_1, β_2)=(0.5, 0.5) and (β_1, β_2)=(0, 1); for instance, when $p = 2$ and $\alpha = 0.2$, or $p = 3$ and $\alpha = 0.8$. Center objective function (Obj_2) values are quite close for the equally weighted problem and the center weighted problem, even though selected hub locations are different. On the other hand, the differences between the objective function values and the selected hub locations of the median weighted and the equally weighted problem are greater and these two solutions have less similarities. As expected, objective function values under the equally weighted problem is always between the two extreme weighted problem solutions.

Similar to the single allocation solutions, solutions with smaller p values are not subsets of the solutions with larger p values in multiple allocation. On the other hand, observe from Table 4.4 that for a given p value, α value has less effect on the locations of hubs in multiple allocation than single allocation problem. More comparable solutions are obtained with different α values.

Table 4.4: Solutions of the multiple allocation problem with the 4-index formulation.

		$\beta_1 = 1, \beta_2 = 0$				$\beta_1 = 0.5, \beta_2 = 0.5$				$\beta_1 = 0, \beta_2 = 1$			
p	α	Obj_1	Obj_2	Hub locations	CPU time(s)	Obj_1	Obj_2	Hub locations	CPU time(s)	Obj_1	Obj_2	Hub locations	CPU time(s)
1		1,491	4,073	5	20.3	1,781	3,013	11	21.4	1,781	3,013	11	31.9
2	0.2	996	3,777	12, 20	13.7	1,066	2,050	5,22	57.0	1,622	2,050	5,22	544.1
	0.4	1,073	4,202	12, 20	12.7	1,191	2,404	5,8	72.7	1,769	2,403	8,21	283.2
	0.6	1,137	4,627	12, 20	12.9	1,229	2,620	5,8	78.1	1,958	2,559	8,21	398.6
	0.8	1,180	5,052	12, 20	12.0	1,190	2,739	8,20	21.6	2,248	2,715	8,21	435.8
3	0.2	753	3,925	12,17,21	14.1	814	1,915	13,17,22	158.3	1,352	1,912	5,16,22	1,229.5
	0.4	870	4,416	4,12,17	12.3	1,005	2,108	1,8,18	151.8	1,719	2,065	5,12,23	798.2
	0.6	949	3,708	4,12,17	12.0	1,098	2,244	8,18,24	90.8	1,790	2,244	8,18,24	437.9
	0.8	1,020	5,397	4,12,17	17.2	1,154	2,516	8,17,24	79.3	1,585	2,516	8,17,24	378.9
4	0.2	618	4,463	4,12,17,24	12.2	704	1,671	11,17,22,24	255.8	1,427	1,620	9,16,19,23	1,393.2
	0.4	754	4,652	4,12,17,24	11.4	870	1,863	12,13,18,23	186.1	1,575	1,774	9,12,16,23	1,345.1
	0.6	866	4,860	1,4,12,17	11.4	928	2,190	4,8,17,24	156.7	1,816	2,127	1,12,17,23	1,087.6
	0.8	952	5,397	1,4,12,17	11.2	1,004	2,500	4,8,17,24	104.0	1,694	2,438	3,8,18,24	501.4
5	0.2	530	4,375	4,7,12,14,17	10.5	649	1,292	11,12,17,23,24	135.6	1,192	1,292	11,12,18,23,24	1,694.4
	0.4	676	4,934	4,7,12,14,17	10.7	783	1,600	11,12,14,18,23	157.2	1,372	1,600	11,12,18,23,24	1,034.8
	0.6	805	4,860	4,7,12,14,17	11.0	846	2,024	4,12,17,23,24	88.7	1,564	1,916	1,18,19,22,23	807.8
	0.8	910	5,193	4,7,12,17,24	11.0	1,015	2,356	12,13,18,22,23	105.1	1,739	2,289	17,19,22,23,24	484.2
6	0.2	485	4,974	4,6,7,12,14,17	13.0	527	1,292	4,11,12,17,23,24	231.7	1,225	1,053	11,12,14,16,23,25	2,230.0
	0.4	632	4,202	4,7,12,14,17,20	10.2	729	1,466	14,17,19,21,22,23	108.5	1,191	1,466	17,19,21,22,23,24	754.0
	0.6	765	4,627	4,7,12,14,17,20	10.3	850	1,835	14,17,19,21,22,23	107.6	1,701	1,823	8,12,18,22,23,24	829.0
	0.8	881	5,052	4,7,12,14,17,20	10.6	924	2,250	4,12,17,22,23,24	45.0	1,687	2,221	3,6,12,22,23,24	500.0
7	0.2	442	4,330	4,6,7,12,14,22	9.8	475	1,120	4,8,12,14,16,17,23	148.5	864	1,120	4,8,12,16,17,23,24	1,825.0
	0.4	595	4,202	4,7,12,14,17,20,22	10.1	637	1,431	4,7,14,17,19,22,23	177.3	1,118	1,337	11,12,18,19,22,23,24	736.0
	0.6	736	4,627	4,7,12,14,17,20,22	10.5	785	1,759	4,8,12,17,22,23,24	99.2	1,569	1,738	2,3,19,21,22,23,24	999.0
	0.8	857	5,052	4,7,12,14,17,20,22	10.3	874	2,250	4,7,12,17,22,23,24	47.9	1,612	2,181	1,3,9,12,14,22,23	355.0
8	0.2	410	4,330	1,4,6,7,12,14,17,22	10.5	474	1,052	4,8,14,16,17,19,22,23	135.8	860	1,049	4,8,16,18,19,22,23,24	1,338.0
	0.4	566	4,202	4,7,8,12,14,17,20,22	10.2	630	1,297	4,11,12,17,19,22,23,24	138.2	1,022	1,284	11,12,17,19,21,22,23,24	1,228.0
	0.6	709	4,627	4,7,8,12,14,17,20,22	10.5	746	1,735	4,7,12,17,19,22,23,24	46.1	1,410	1,715	2,3,5,8,12,22,23,24	1,261.0
	0.8	838	5,052	4,7,8,12,14,17,20,22	11.1	875	2,181	3,4,12,13,14,18,22,23	38.2	1,711	2,181	3,12,14,21,22,23,24,25	376.0

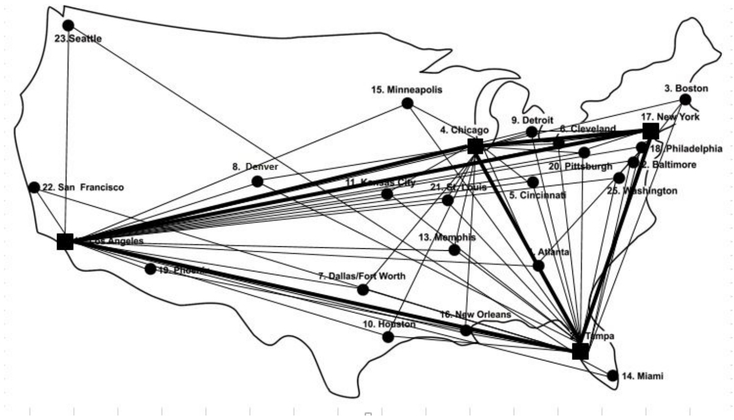
CPU time requirements are mostly steady with different p and α values for $\beta_1 = 1$, $\beta_2 = 0$. For $\beta_1 = 0.5$, $\beta_2 = 0.5$ and $\beta_1 = 0.5$, $\beta_2 = 1$, CPU times tend to decrease when α increases. Similar to the results obtained with the single allocation problem, the multiple allocation model has the slowest solution times when $\beta_1 = 0$, $\beta_2 = 1$, whereas $\beta_1 = 1$, $\beta_2 = 0$ provides the fastest results.

Figure 4.3 demonstrates solutions of the multiple allocation problem with $p = 4$ and $\alpha = 0.4$ under different weights given to the objectives. Squares presents hub locations, bold lines inter-hub connections, and thin lines node allocations, just as in the previous figure.

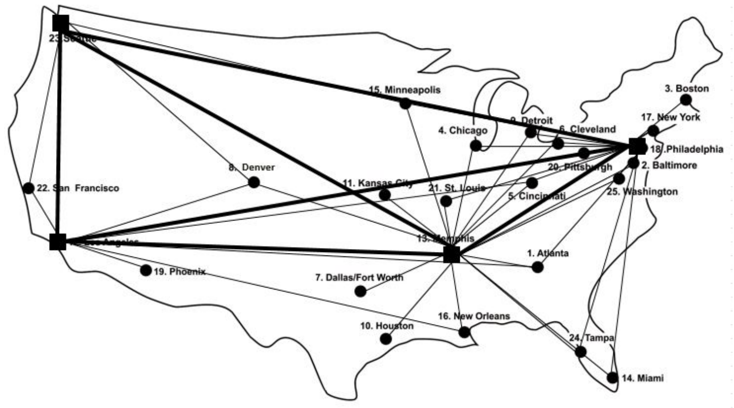
Note from Figure 4.3 that distances between the hubs are the smallest in the center weighted solution, while this variation is the largest in the median weighted solution. Moreover, the nodes are allocated to, generally two, but sometimes at most three hubs in the equally weighted solution; however, the nodes are allocated, mostly, to four hubs in the center weighted solution, whereas there are a few nodes allocated to four hubs in the median weighted solution.

Table 4.5 shows solutions for the r-allocation problem. We observed from the multiple allocation solutions that a node is generally allocated to at most three hubs. Thus we tested two different r values; $r = 2$, and $r = 3$. These values are presented in the first column of Table 4.5. We tested five different p values for this analysis, which are listed in the second column. The reminder of Table 4.5 is organized as previous tables.

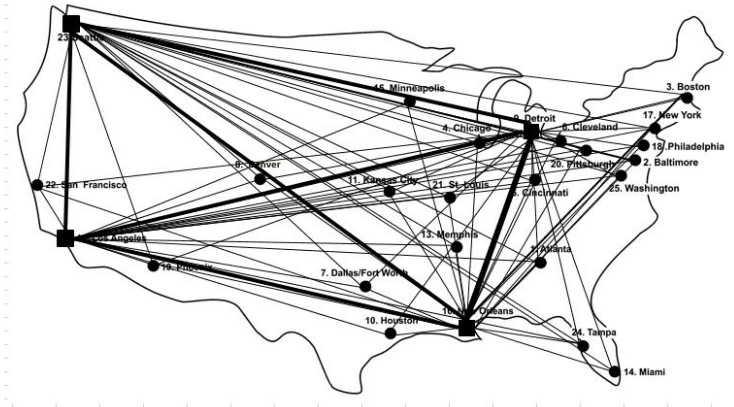
Observe from Table 4.5 that similar to the results obtained with the other two models the median and center solutions result in different hub locations and objective function values under different β values. However, the problem gives the same or close objective function values when $\beta_1 = 0.5$, $\beta_2 = 0.5$ and $\beta_1 = 0$, $\beta_2 = 1$. For example, hub locations and the center objective function (Obj_2) values are exactly the same when $p = 3$ and $\alpha = 0.6$ with both of the r values.



(a) $\beta_1 = 1$ and $\beta_2 = 0$.



(b) $\beta_1 = 0.5$ and $\beta_2 = 0.5$.



(c) $\beta_1 = 0$ and $\beta_2 = 1$.

Figure 4.3: Multiple allocation solutions with different β values when $p = 4$ and $\alpha = 0.4$.

Table 4.5: Solutions of the r-allocation problem with the 4-index formulation.

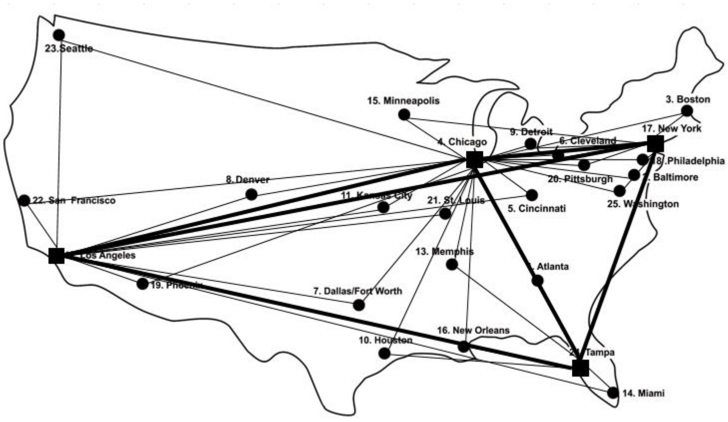
			$\beta_1 = 1, \beta_2 = 0$				$\beta_1 = 0.5, \beta_2 = 0.5$				$\beta_1 = 0, \beta_2 = 1$			
	p	α	Obj_1	Obj_2	Hub locations	CPU time(s)	Obj_1	Obj_2	Hub locations	CPU time(s)	Obj_1	Obj_2	Hub locations	CPU time(s)
$r = 2$	1		1,491	4,073	5	40	1,781	3,013	11	52	1,781	3,013	11	59
	2	0.2	996	3,541	12,20	17	1,066	2,050	5,22	84	1,396	2,050	5,22	5,619
		0.4	1,073	4,202	12,20	13	1,191	2,404	5,8	86	2,029	2,403	8,21	3,193
		0.6	1,137	4,391	12,20	13	1,229	2,620	5,8	69	2,023	2,559	8,21	5,657
		0.8	1,180	5,052	12,20	12	1,189	2,739	8,20	41	2,057	2,715	8,21	13,813
	3	0.2	753	3,893	12,17,21	13	814	1,915	13,17,22	158	1,409	1,912	1,5,22	9,107
		0.4	860	3,435	4,12,17	12	1,014	2,108	1,8,18	182	1,564	2,065	5,12,23	21,456
		0.6	950	4,263	4,12,17	12	1,123	2,244	8,18,24	128	1,798	2,244	8,18,24	10,162
		0.8	1,024	3,965	12,18,21	21	1,110	2,587	8,13,18	198	1,931	2,516	8,17,24	6,825
	4	0.2	618	3,947	4,12,17,24	11	704	1,671	11,17,22,24	261	1,233	1,620	9,16,19,23	13,462
		0.4	759	3,435	4,12,17,24	17	870	1,863	12,13,18,23	240	1,300	1,863	12,13,18,23	17,345
		0.6	878	3,766	1,4,12,17	19	944	2,190	4,8,17,24	233	1,448	2,127	1,12,17,23	17,598
		0.8	968	4,114	4,12,13,17	28	1,034	2,500	4,8,17,24	222	1,602	2,438	3,6,8,24	10,304
	5	0.2	530	1,974	4,7,12,14,17	9	649	1,292	11,12,17,23,24	128	859	1,292	11,12,18,23,24	15,533
		0.4	682	3,486	4,7,12,14,17	12	788	1,600	11,12,14,18,23	218	1,234	1,600	11,12,18,23,24	14,186
		0.6	819	3,766	4,7,12,17,24	13	862	2,024	4,12,14,17,23	179	1,563	1,958	1,19,22,23,25	16,643
		0.8	935	4,114	1,4,7,12,17	41	1,025	2,356	12,13,18,22,23	231	1,581	2,305	19,20,22,23,24	18,624
$r = 3$	1		1,491	4,073	5	40	1,781	3,013	11	51	1,781	3,013	11	59
	2	0.2	996	3,958	12,20	14	1,066	2,050	5,22	79	1,384	2,050	5,22	6,465
		0.4	1,073	4,202	12,20	12	1,190	2,404	5,8	116	2,025	2,403	8,21	5,516
		0.6	1,137	3,946	12,20	12	1,229	2,620	5,8	98	1,965	2,559	8,21	6,806
		0.8	1,180	5,052	12,20	11	1,189	2,739	8,20	21	2,154	2,715	8,21	11,783
	3	0.2	753	3,718	12,17,21	12	814	1,915	13,17,22	237	1,318	1,912	1,5,22	9,425
		0.4	860	3,772	4,12,17	12	1,005	2,108	1,8,18	146	1,608	2,065	5,12,23	10,854
		0.6	949	4,313	4,12,17	12	1,098	2,244	8,18,24	75	1,676	2,244	8,18,24	9,604
		0.8	1,020	4,917	4,12,17	17	1,154	2,516	8,17,24	69	1,984	2,516	8,17,24	4,819
	4	0.2	618	2,995	4,12,17,24	11	704	1,671	11,17,22,24	261	1,372	1,620	9,16,19,23	10,984
		0.4	754	4,416	4,12,17,24	11	870	1,863	12,13,18,23	229	1,401	1,774	9,12,16,23	11,031
		0.6	866	3,766	1,4,12,17	11	928	2,190	4,8,17,24	175	1,523	2,127	1,12,18,23	12,945
		0.8	952	4,721	1,4,12,17	11	1,006	2,500	4,8,17,24	109	1,897	2,438	3,6,8,24	11,401
	5	0.2	530	2,098	4,7,12,14,17	9	649	1,292	11,12,17,23,24	199	783	1,292	11,12,17,23,24	11,176
		0.4	678	2,282	4,7,12,14,17	10	783	1,600	11,12,14,18,23	205	1,189	1,600	11,12,14,18,23	10,036
		0.6	807	4,637	4,7,12,17,24	11	846	2,024	4,12,17,23,24	134	1,431	1,916	1,17,19,22,23	12,961
		0.8	912	3,674	1,4,7,12,17	19	1,019	2,356	12,13,18,22,23	127	1,788	2,289	17,19,22,23,24	9,555

A comparison of Tables 4.4 and 4.5 reveals that the results obtained with the r-allocation model are very similar to the results obtained with the multiple allocation problem. Note that the multiple allocation problem provides a lower bound for the r-allocation problem. Hence, the objective function values obtained with the multiple allocation model will always be less than or equal to the values obtained with r-allocation.

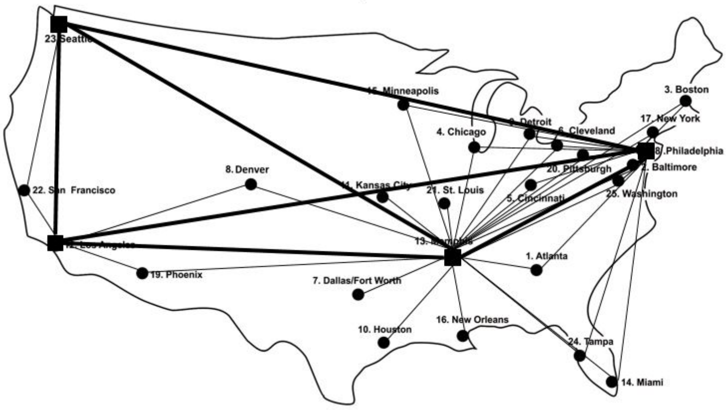
Observe form Table 4.5 that CPU times increase considerably when $\beta_1 = 0$, $\beta_2 = 1$ with the center weighted objective function. The median weighted instances, on the other hand, are solved within a few seconds. Equally weighted objective problem is slower than the median objective weighted problem but faster than the center objective weighted problem.

Figure 4.4 illustrates the solutions of the r-allocation problem when $p = 4$, $\alpha = 0.4$, and $r = 2$ with different weights given to the objectives. Observe from the figure that hubs are located on the same nodes as the multiple allocation problem when $\beta_1 = 1$, $\beta_2 = 0$ and $\beta_1 = 0.5$, $\beta_2 = 0.5$. The objective function values, on the other hand, are different because of the allocations. The multiple allocation problem yields an optimum median objective of 754 whereas 2-allocation problem yields an optimum median objective of 759. This is because a non-hub node is allocated to more than two hub nodes in the optimum multiple allocation solution. In the center objective weighted 2-allocation problem, two of the four hubs are located at different nodes than the multiple allocation version of the problem.

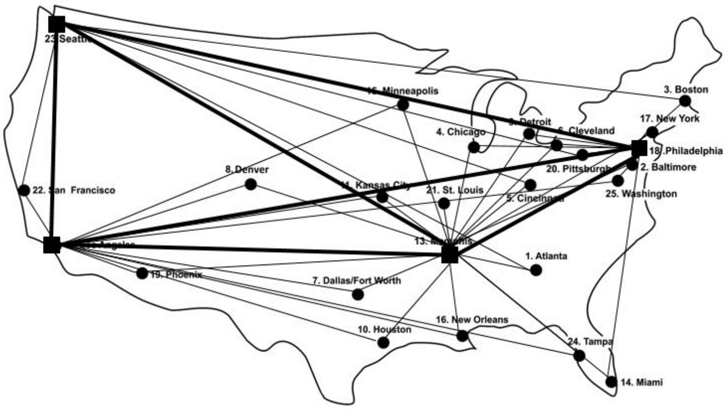
Lastly, we compared single, multiple, and r-allocation solutions. We observed from Tables 4.3, 4.4, and 4.5 that resulting optimal hub locations are similar under all the three problems. In terms of objective function values, as noted before, multiple allocation solutions provide a lower bound both to r-allocation and single allocation problems. Similarly, r-allocation problem provides a lower bound for the single allocation problem. Hence, the highest objective function values are obtained with single allocation. Regarding the CPU time requirements, single allocation formulation is the fastest among all the formulations. The slowest formulation, on the other hand, is the r-allocation model.



(a) $\beta_1 = 1$ and $\beta_2 = 0$.



(b) $\beta_1 = 0.5$ and $\beta_2 = 0.5$.



(c) $\beta_1 = 0$ and $\beta_2 = 1$.

Figure 4.4: 2-allocation solutions with different β values when $p = 4$, $\alpha = 0.4$.

4.3 Trade-off between two objectives

In this section, we provide trade-off curves to observe the effects of weights given to the objectives on the solutions. For this analysis, we use $p = 4$ and $\alpha = 0.4$. We tested 11 different weight combinations by incrementing the β values by 0.1 between 0 and 1.

We first analyze the single allocation problem. Table 4.6 presents results with eleven different β combinations. The first two columns list the β values. The median objective function values are shown in the third column, and the center objective function values are given in the fourth column. The last column provides the optimal hub locations in the solutions.

Table 4.6: Single allocation solutions ($p = 4$, $\alpha = 0.4$).

β_1	β_2	Obj_1	Obj_2	Hub locations
1	0	788	2,592	1,4,12,17
0.9	0.1	807	2,327	4,12,16,17
0.8	0.2	834	2,170	14,17,21,22
0.7	0.3	922	1,885	12,13,18,23
0.6	0.4	922	1,885	12,13,18,23
0.5	0.5	922	1,885	12,13,18,23
0.4	0.6	922	1,885	12,13,18,23
0.3	0.7	922	1,885	12,13,18,23
0.2	0.8	922	1,885	12,13,18,23
0.1	0.9	922	1,885	12,13,18,23
0	1	1,127	1,885	12,13,20,23

Note from Table 4.6 that the seven out of eleven strategies result in the same hub locations. The optimal solution and the objective function values remain the same when $\beta_1 \in [0.1, 0.7]$ and $\beta_2 \in [0.3, 0.9]$. Figure 4.5 displays the trade-off curve for the values presented in Table 4.6.

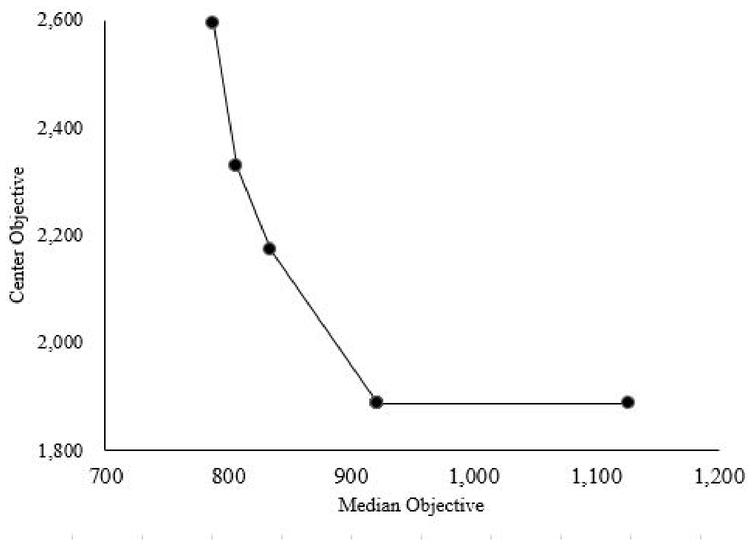


Figure 4.5: Trade-off curve for the single allocation problem when $p = 4$, $\alpha = 0.4$.

Figure 4.5 clearly shows that the center weighted objective should not be adopted as a solution. Note that this solution is dominated by the $\beta_1 = 0.1$, $\beta_2 = 0.9$ solution that has a better value of the median objective and exactly the same value of the center objective.

Secondly, we analyze the multiple allocation problem. Table 4.7 presents solutions with eleven different β combinations. Similar to the previous table, the first two columns present β values, while the third and fourth columns provide the objective function values, and the last column shows the hub locations.

In Table 4.7, there are five different hub-location groups shown as optimal out of the eleven different strategies. Five of the solutions indicate the hubs should be located at nodes 9, 12, 16, and 23, while three of the solutions designate hub nodes at 12, 13, 18, and 23. Additionally, hub locations are selected at nodes 4, 12, 17, and 24 at two strategies. There is quite a few variations in these solutions.

Figure 4.6 provides the trade-off curve with the multiple allocation problem. Observe from the figure that as a decision maker the extreme solutions (median and center) do not look very promising by means of the objective function values. A small compromise in one

Table 4.7: Multiple allocation solutions ($p = 4$, $\alpha = 0.4$).

β_1	β_2	Obj_1	Obj_2	Hub locations
1	0	754	4,652	4,12,17,24
0.9	0.1	754	2,362	4,12,17,24
0.8	0.2	797	2,066	14,17,21,22
0.7	0.3	870	1,863	12,13,18,23
0.6	0.4	870	1,863	12,13,18,23
0.5	0.5	870	1,863	12,13,18,23
0.4	0.6	981	1,774	9,12,16,23
0.3	0.7	981	1,774	9,12,16,23
0.2	0.8	981	1,774	9,12,16,23
0.1	0.9	981	1,774	9,12,16,23
0	1	1,575	1,774	9,12,16,23

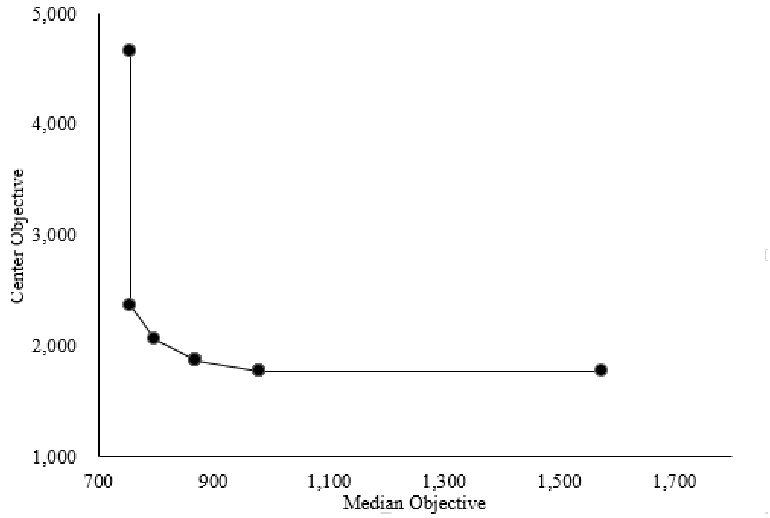


Figure 4.6: Trade-off curve for the multiple allocation problem when $p = 4$, $\alpha = 0.4$.

objective may result in a huge gain in the other objective.

Table 4.8: 2-allocation solutions ($p = 4$, $\alpha = 0.4$).

β_1	β_2	Obj_1	Obj_2	Hub locations
1	0	759	3,435	4,12,17,24
0.9	0.1	761	2,362	1,4,12,17
0.8	0.2	799	2,066	14,17,21,22
0.7	0.3	870	1,863	12,13,18,23
0.6	0.4	870	1,863	12,13,18,23
0.5	0.5	870	1,863	12,13,18,23
0.4	0.6	870	1,863	12,13,18,23
0.3	0.7	870	1,863	12,13,18,23
0.2	0.8	870	1,863	12,13,18,23
0.1	0.9	870	1,863	12,13,18,23
0	1	1,300	1,863	12,13,18,23

Lastly, we analyze the trade-off between the objectives for the 2-allocation problem. Table 4.8 demonstrates the results in the same way with the two previous tables. Note from Table 4.8 that the optimal median and center objective function values are slightly higher than the multiple allocation solutions because at most two hub allocations are allowed in this problem. Eight of the solutions indicate that the hubs should be located at nodes 12, 13, 18, and 23.

Figure 4.7 shows the trade-off curve with the given solutions for the 2-allocation problem. Note that it has a similar shape with the trade of curve of the multiple allocation solution. The decision maker must avoid adopting the extreme solutions because as can be observed from this curve a small change in weights has a great influence on the other objective function value.

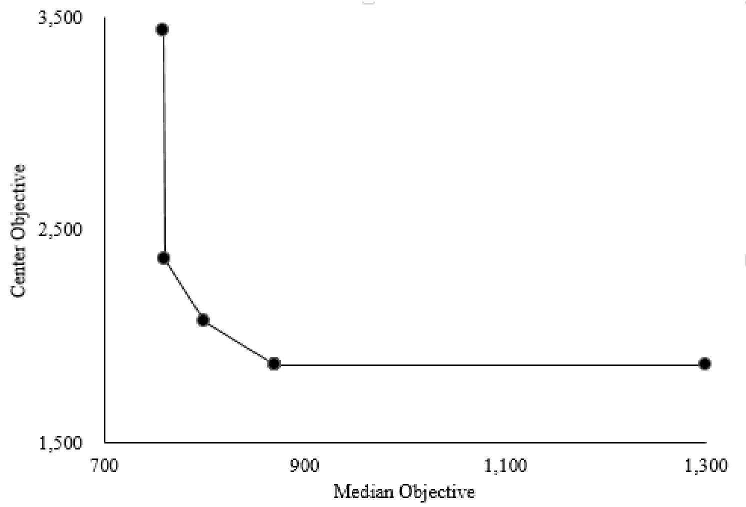


Figure 4.7: Trade-off curve for the 2-allocation problem when $p = 4$, $\alpha = 0.4$.

Chapter 5

Conclusion

In this thesis, we defined and studied bi-objective p-hub location problems. We unified p-hub median and p-hub center problems under a bi-objective problem considering the single, multiple and r-allocation strategies. Our aim is to find the optimal locations of hubs and the optimal allocations of the demand nodes to these hubs.

First, we developed different mathematical models combining p-hub median and p-hub center problems for each possible allocation strategy. We used a weighted sum objective. A 3-index and a 4-index mixed-integer programming formulation have been developed for each of the single, multiple, and r-allocation strategies.

All of the developed formulations were compared based on their CPU time requirements to obtain optimal solutions using the commercial solver IBM CPLEX. Then, the formulation, which provided the fastest solution time, was selected for each allocation strategy for further computational experiments. It was shown that the 3-index single allocation formulation yields the fastest solution times, and the 3-index r-allocation formulation yields the slowest ones. The 3-index multiple allocation formulation gives slightly faster solution times than the 3-index r-allocation formulation. 4-index formulations provide better solution times than 3-index formulations for the multiple and r-allocation strategies.

We have performed numerical experiments on selected formulations using the CAB dataset from the literature. We varied the values of the problem parameters while testing our formulations. More specifically, we varied the number of hubs to be established, the economies of scale parameter, and the weights given to the median and center objectives. For each instance, we analyzed the values of the median and center objectives, the locations of the hub nodes, the allocations, and the CPU time requirements. We observed the effect of changes in the problem parameters on the results.

The multiple and the r-allocation models with $r = 2$ and 3 yielded very similar results. Moreover, the multiple allocation formulation provides a lower bound for the single and r-allocation formulations. The single allocation formulation yields the highest objective function values.

With all the models, hubs were located at different nodes under different p values. It has been observed that the locations obtained with smaller p values are not a subset of the locations obtained with larger p values. Hence, rather than selecting the locations of the hubs incrementally, the decision maker needs to determine the locations of all hubs at once.

In general, the models resulted in different optimal hub locations under different weights given to the median and center objectives. We further analysed the trade-off between the two objective functions for each allocation strategy. The results showed that the decision maker must avoid implementing neither the optimal median nor the optimal center solution, as better solutions can be obtained with a small compromise in one objective.

In this study, we used a constant economies of scale factor as commonly done in the literature. As a future research direction, one may use and model a flow-dependent economies of scale factor. This will definitely yield more realistic results. Furthermore, all the data that we used were deterministic, we have not considered any uncertainty in the data. However, real life certainly involve several uncertain conditions. Therefore, the models can further be extended considering uncertainty. Lastly, we assumed complete inter-hub networks. Formulations for the bi-objective p-hub location problems on incomplete inter-hub networks can be another direction for future research.

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