

Strategic Network Design for Delivery by Drones under Service-based Competition

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

In today's world, E-commerce is a fast growing industry and e-retailers are looking for innovative ways to deliver customer orders within short delivery times at a low cost. Currently, the use of drone technology for last-mile delivery is being developed by such companies as Amazon, FedEx, and UPS. Drones are relatively cheaper and faster than trucks but are limited in range and may be restricted in landing and takeoff. Most of the work in the Operations Research literature focusses on the operational challenges of integrating drones with truck delivery. The more strategic questions of whether it is economically feasible to use drones and the effects on distribution network design are rarely addressed. These questions are the focus of this work. We consider an e-retailer offering multiple same day services using both existing vehicles and drones, and develop a facility location problem under service-based competition where the services offered by the e-retailer not only compete with the stores (convenience, grocery, etc.), but also with each other. The competition in the market is incorporated using the *Multinomial Logit* (MNL) market share model. To solve the resulting nonlinear mathematical formulation we develop a novel logic-based Benders decomposition approach. We also show that the nonlinear model can be transformed into a linear mixed integer formulation. Computational experiments show that our algorithm outperforms direct solution of the linear formulation. We carry out extensive numerical testing of the model and perform sensitivity analyses over pricing, delivery time, government regulations, technological limitations, customer behavior, and market size. The results show that government regulations play a vital role in determining the future of drone delivery.

Acknowledgements

I would like to thank my supervisor, Professor Fatma Gzara for her continuous support and motivation.

Dedication

This work is dedicated to my hometown, Dera Ghazi Khan.

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Chapter 1

Introduction

Use of unmanned aerial vehicles (UAVs) or drones for medical purposes in remote areas of South Africa and Germany, and the recent announcement of Amazon's Prime Air service to deliver packages within 30 minutes, have put drones in the spotlight. Companies like Google, Walmart, DHL, and Zookal have also joined the race of drone delivery [37, 42]. A drone would take a parcel from the warehouse, vertically take-off from its base station, and fly to customer's doorstep. At arrival, the parcel is dropped and the drone returns to the warehouse to begin next delivery. Since drones can fly over congested road networks, drone delivery greatly reduces delivery time. In today's world, online ordering is increasing rapidly. Companies are looking for new technologies to delivery goods to customers within short time windows to maximize market share. Several companies including FedEx, UPS, and Amazon are offering same-day delivery services[41]. Unlike traditional truck delivery, drones fly without a human pilot and are battery operated. Some studies suggest delivery using drone reduce overall cost[11, 53]. This makes drones more attractive for last mile delivery, in which customers want their orders to be delivered quickly with low delivery



(a) Amazon Prime Air



(b) DHL Parcelcopter

Figure 1.1: Parcel delivery using Drones

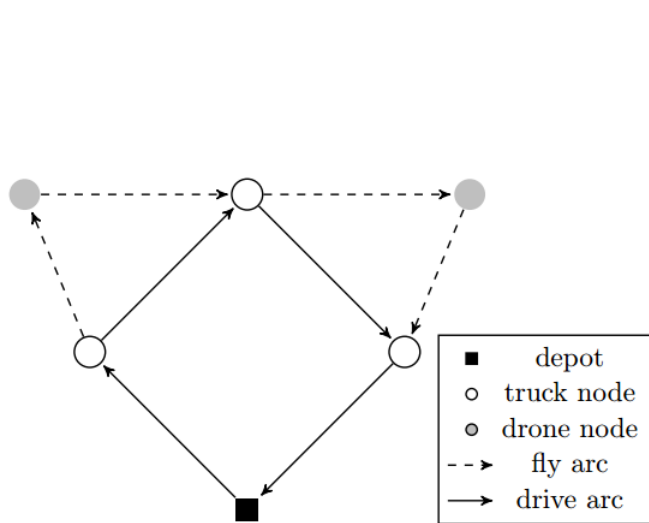
charges.

Challenges

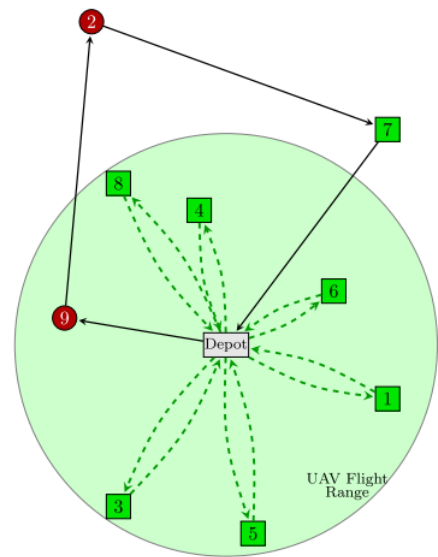
There are some limitations that need to be addressed to make delivery by drone possible. Figure 1.1(a) shows the drone prototype presented by Amazon to make 30-minute deliveries. The drone has a range of only 20 km which means that it can only deliver a parcel within 10 km of the warehouse. The drone can carry a parcel weighing up to 2.5 kg. Fortunately, 70% of Americans live within 8 km of a Walmart store and 86% of the parcels delivered by Amazon weigh less than 2.5 kg [11]. Another drawback for package delivery using drones is congested airspace. Currently, drones operate outside the restrictive airspace but a drone logistics network would surely cause congestion of airspace and hence would require their integration in the air traffic system. Although the aviation authorities do believe that drones can be operated safely in airspace, it is going to be a challenging task. The future of drones critically depends on government regulations which are currently quite restrictive in the United States. Experts believe that once regulatory issues are addressed, technological limitations will not hinder the use of drones for last-mile delivery.

Truck-drone delivery to address limited range

Due to range limitations, a drone network would require many facilities to open in close proximity to the customer locations. Opening additional facilities would increase the inventory holding and facility costs, however this may be compensated for the reduced outbound transportation costs and increased demand due to reduced delivery times. There have been few studies to address operational challenges with drone delivery. To address limited range of drones, one possible solution proposed by Agatz et al. [16] is shown in Figure 1.2(a), in which a drone collaborates with a delivery truck. The truck would take a main route avoiding the congested areas while the drone flies to congested areas, reducing the transportation time and distance. In this way, the truck would cover several customer locations, and the drones would simultaneously cover other customer locations, one by one, returning to the truck after each delivery. The following transportation problem consists of both assignment and routing problems where the assignment decision determines which vehicle to use, drone or truck, and the routing decision determines the sequence in which customers would be served by each vehicle type. However, the model proposed by Agatz et al. [16] tries to minimize delivery time without considering costs associated with vehicle type. Murray and Chu [55] propose two mathematical programming models where trucks deliver parcels in collaboration with drones, as shown in Figure 1.2(b). The paper focuses on a new variant of the traditional Traveling Salesman Problem (TSP) to determine the optimal assignment of the customers to drones flying along with trucks to make deliveries. They call the problem the Flying Sidekick Traveling Salesman Problem (FSTSP). The paper also presents a mathematical model for the case where customers are located in close proximity to the distribution center (DC). This problem is called the Parallel Drone Scheduling TSP (PDSTSP), finding optimal truck and drone usage to make all deliveries.



(a) Agatz et al. [16]



(b) Murray and Chu [55]

Figure 1.2: Truck-drone delivery

These studies mainly focus on the operational challenges posed by delivery drone without taking into account strategic issues related to the impact on the distribution network and overall profits.

Cost analysis

There have been some studies to estimate the costs associated with drone delivery. Hickey [42], for instance, presents a report on the successful delivery of crucial medical products in the landlocked area of South Africa, Lesotho, by a start-up company, *Matternet*. Matternet delivered medical products and collected blood samples to be brought back to the hospital for tests. In this particular case, drones are a perfect solution as blood samples are time-sensitive, small, and light-weight. Similarly, the area did not have air traffic and hence

the whole process was automated with specified landing areas where drones are recharged automatically. Drones in this case would take 15 minutes to take a 2 kg of cargo to locations as far as 11 km. Surprisingly, it costed only \$0.24 per delivery [42]. This practical example shows that drones are cost effective. Similarly, a guest editorial published in IEEE by D'Andrea [25] suggested that fuel cost directly associated with drone delivery is \$0.10 for a parcel of 2 kg within a 10 km range. However, these figures do not incorporate other costs such as operator salary, facility costs, amortization of the drone, and maintenance. Assuming a vehicle costs \$1000, adding 20% for yearly maintenance yields an annual cost of \$400 which equals \$1/day. If a drone makes 10 runs per day, each parcel delivery would cost approximately an additional \$0.10 [11]. The total cost per delivery would equal \$0.20, excluding other operational costs. The above calculations assume that the drone network would be automated, however government regulations may require operators to monitor drones. In the traditional truck delivery, driver salary is one of the main components of delivery costs. For a driver-less truck, delivery costs may be significantly lower. Drone technology available for the last-mile delivery part of the supply chain can only take a single parcel to a single destination. A traditional truck, on the other hand, can make multiple deliveries per route. Hence, it is important to consider all costs associated with drone delivery including facility and operator's costs to study the impact of the drone network on the last mile delivery. In a report by ARK Invest, Keeney [52] takes into account these factors to estimate unit delivery cost using drones for Amazon. In the study, facility up-gradation costs, operators salary, fuel cost, and internet Bandwidth costs are considered to estimate total unit delivery cost. Keeney [52] estimates unit cost to be close to \$1. In the analysis, it is assumed that government regulations would allow an operator to monitor 5-10 drones simultaneously. However, other industry reports suggest 1-2 drones would be allowed per operator [9, 11]. Assuming this case, it could cost Amazon between

\$15 to \$20 per package .

Our contribution

The drone technology could bring about a revolution in logistics due to decreased delivery costs, which is usually one of the main factors that parcels are delivered in days rather than in hours. From an Operations Research perspective, there have been some studies addressing operational challenges associated with drone technology, we are not aware of a study addressing the effect of delivery by drone on the distribution network. This paper studies the effect of drones on distribution network design while taking into account three important measures of a network design: (1) facility location, (2) services offered, and (3) response time. These measures not only affect customer demand, but also affect network costs. Consider Amazons Prime Air service, where delivery would be made to the customer location from a warehouse using drones within 30 minutes. Since the current drone technology limits drone range to 10 km, it would require more technologically advanced facilities to be built in close proximity to customer locations resulting in increased facility costs. Increasing the number of facilities also increases the holding costs due to the disaggregation of demand. On the other hand, outbound transportation costs are significantly reduced due to the close proximity to the customer locations. Similarly, with 30-minute delivery time, Amazon would be able to capture a higher market share, leading to higher revenues, compensating for the increased inventory and facility costs.

Since drone delivery is currently in the testing phase, it is hard to predict its effect on customer demand. To capture customer behavior, we adopt the *multinomial market share model*, also referred to as the *brand choice*, and the *attraction model*. In such models, the customer demand captured by a service is probabilistic and depends on the utility



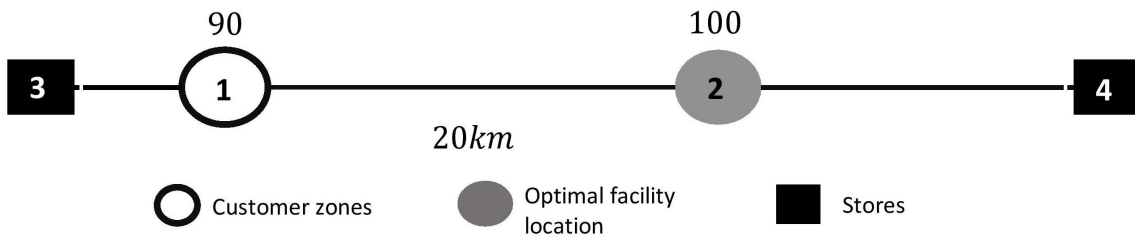
Figure 1.3: Top selling *Prime Now* products in each city in 2015 [22]

(or attraction) derived from that service relative to the utility derived from other services available in the market. Currently, Amazon offers Prime Now service to its prime members in which delivery is made within 1-2 hours. Figure 5.1 summarizes the top selling products in cities where Prime Now service is available. Studying the products being ordered for 1 to 2-hour delivery, it is observed that customers usually order products that are readily available at departmental and convenience stores, and are bought when required, such as bottled water, towels, diapers, etc. As such, drone delivery will not only compete with services already offered by the e-retailer itself but also with stores nearby.

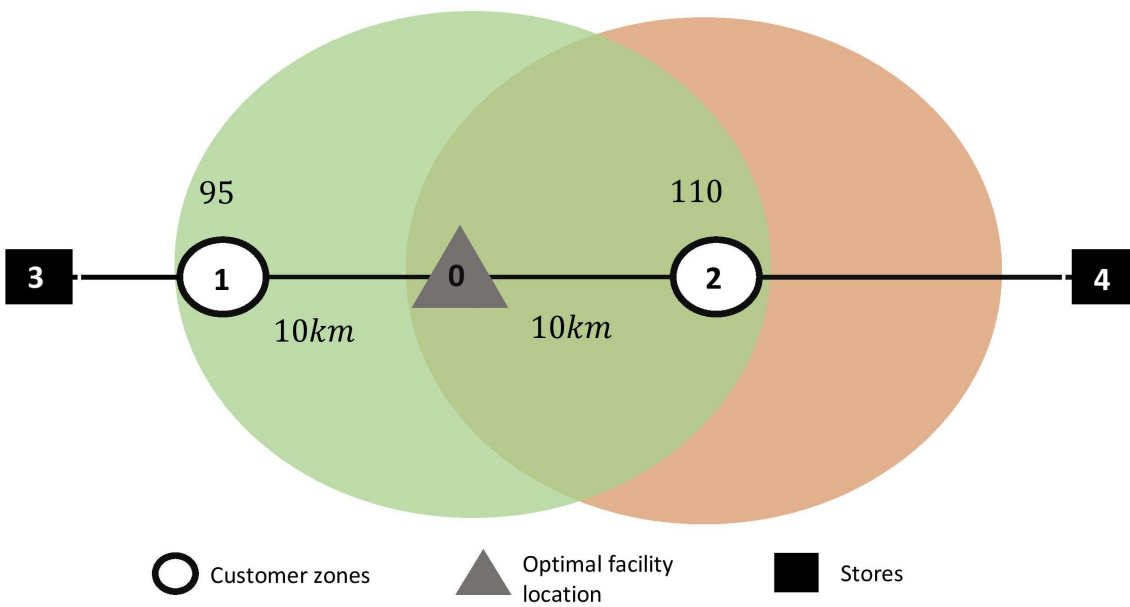
Figure 1.4 illustrates a two customer zone network, each with similar demand characteristics. In Figure 1.4(a), the e-retailer offers only one service using existing vehicles. The demand captured by the e-retailer from customer zone 2 is higher than 1 due to competition

with stores. Since zone 2 is farther away from store 4, the demand captured by the store is lower due to high travel time and costs. In this scenario, the optimal facility location to minimize total delivery costs is at node 2. In Figure 1.4(b), the e-retailer introduces drone delivery along with existing service. Note that node 2 is not the optimal facility location when drone service is available. Since the drones have a limited range of 10 km, the optimal facility location is node 0, in order to offer both existing and drone service in all customer zones. This illustrates the impact of drone delivery on network design. We therefore develop a facility location problem with multiple services where the last-mile delivery is being carried out using both traditional vehicles and drones in a competitive market.

The outline for thesis is as follows. Chapter 2 presents a detailed literature review on drone delivery, competitive network design, market share models, and logic-based Benders decomposition. In Chapter 3, we formally define the problem. A mixed integer non-linear program is presented to model a facility location problem under service-based competition. Since nonlinear formulations are very hard to solve using commercial optimization software, Chapter 4 presents a novel logic-based Benders decomposition approach to solve the model to optimality within reasonable time. In Chapter 5, the numerical testing results of an actual network over New York City are analyzed. In Chapter 6, we present computational results of the solution methodology used. Finally, in Chapter 7, concluding remarks are presented.



(a) Without drones



(b) With drones

Figure 1.4: Optimal network configuration

Chapter 2

Literature review

Our work relates to three bodies of literature including delivery by drones, competitive facility location problem, and logic-based Benders decomposition. In this section, we review the literature in these areas and position our work accordingly.

Delivery by drones

The amount of literature for drone delivery has significantly increased in the last few years. Murray and Chu [55] are the first to address operational challenges associated with drone delivery. The authors suggest a new variant of the Traveling Salesman Problem (TSP), in which a drone is operated from a vehicle to make deliveries. They assume that drones can only depart and land at a customer location. Ha et al. [39], Agatz et al. [16], Ha et al. [40], and Ponza [58] consider a very similar problem of truck-drone delivery. Ferrandez et al. [35] use k-means clustering to find the optimal location of drone launch sites from the truck and present a genetic algorithm to solve the TSP. Dorling et al. [27] develop two

multi-vehicle routing problems (mVRPs) based on the drone energy consumption model. In the model, multiple drones are used to make deliveries from a depot. An extension of the TSP with drones is presented by Wang et al. [67] where a vehicle routing problem with a fleet of trucks and drones is considered. Tavana et al. [63] propose a cross-docking truck-scheduling problem where only direct shipments are allowed using drones while other deliveries are made using a fleet of trucks. In contrast, Ulmer and Thomas [66] consider a dynamic vehicle routing problem using a fleet of trucks and drones for same day delivery, where customer orders need to be fulfilled in a given time interval. Finally, Campbell et al. [21] use continuous approximation modeling technique to model a hybrid system where drones make simultaneous deliveries along with trucks. They present strategic analysis of the truck-drone delivery and their findings suggest substantial cost savings in suburban areas.

Other than Hong et al. [44], researchers are more focused on the operational challenges associated with drones without addressing key network design questions. Hong et al. [44] develop a maximal coverage location model with a given number of warehouses and charging stations. The objective is to maximize drone coverage while minimizing the average network distance between the warehouse and charging stations. However, the location of the warehouses is fixed and the model decides on locating charging stations. Drone delivery would require reconfiguration of the distribution network which would effect networks costs and demand.

Strategic network design questions must be addressed before dealing with operational challenges. There are many factors that must be taken into account while designing a distribution network. One of the most important factors is the presence of competitors offering the same product or service. In a network with drones, to fulfill an order within 30 minutes, an e-retailer would enjoy having a monopoly for providing a unique service.

However, as discussed earlier, the e-retailer competes with existing stores and should also consider the competition between its own services. Stores that are located in close proximity to customer locations have zero waiting time to buy a products, there customers may prefer to do shopping at a store to avoid waiting for the product to arrive by shopping online. Nevertheless, a customer incurs travel cost and travel time to buy a product from a store. For this reason, it is important to examine customer behavior when they have an alternative choice to an online purchase that will be delivered to their doorstep in just 30 minutes.

Since our work relates to facility location problem in a competitive setting, we now present different market share models and review the literature for competitive facility location problems.

Market share or demand models

A variety of different market share models are used in the literature. In this chapter, we discuss five commonly used models as presented by Cooper et al. [24]. Linear, multiplicative and exponential models are the basic market share models used. In the linear model, the market share captured by a player (e.g. facility, firm, or service) is a linear function of the attributes of all competing players. Attributes are the factors against which the players are competing in the market, e.g. price, delivery time, distance from the facility, etc. In the multiplicative model, market share is given by the product of the attributes (raised to a power depending on the relative importance of the attribute) of all competing players. Similarly, exponential models estimate market share as the exponential of the attributes of all competing players. Note that these models are closely related to one another. Both multiplicative and exponential models can be transformed into log-linear models.

To meet *logical-consistency requirements*, market share captured by a player must be nonnegative and the sum of the market share of all players must be less than or equal to 1. Surely, none of the above mentioned models meet these requirements. There are two commonly used models that meet the *logical-consistency requirements*: the Multiplicative Competitive Interaction (MCI) model, and the Multinomial logit (MNL) model. In this chapter, we follow the notations used by Cooper et al. [24]. Consider a set of players M in a market competing against K attributes or factors. In these models, market share captured by each player $m \in M$ is defined as $\frac{A_m}{\sum_{m \in M} A_m}$ where A_m is attractiveness or utility

of the player m based on its attributes. In MCI model, $A_m = \exp(\alpha_m) \prod_{k=1}^{k=K} X_{km}^{\beta_k}$, and in

MNL model, $A_m = \exp(\alpha_m + \sum_{k=1}^{k=K} \beta_k X_{km})$, where α_m is the constant attraction factor or inherent attractiveness of player m , X_{km} is the value of k^{th} attribute for player m , and β_k denotes the sensitivity of the market to the k^{th} attribute. To ensure *logical-consistency requirements* are met, we use the MNL model to predict market shares of the services in the market. The model formulation and methodology in this thesis is also applicable to the MCI model as well.

In facility location literature, *gravity models*, also referred to as *spatial interaction models*, are widely used. Gravity models are a special case of MNL and MCI market share models originating from the work of Huff [48]. In these models, facilities compete against each other based on the distance from customer location and other attraction factors. In next section, we review the literature for competitive facility location problems in which such models are used.

Competitive facility location problem

There are three main types of competition addressed in the literature. Static competition is when the new entrant assumes that the attributes of the existing competitors do not change following its entrance into the market. [57]. Dynamic competition is when the new entrant makes decisions assuming that competitive characteristics of the existing rivals may change, following its entrance into the market (see [38, 20, 73, 54]). Competition with foresight is when the rivals (follower) soon join the market once the new entrant (leader) enters the market [31].

There is a great deal of literature addressing competitive facility location problems. Meng et al. [54] study a dynamic competitive facility location problem where a manufacturer intends to enter the existing decentralized supply chain with three tiers: manufacturer, retailer, and customers. The study focuses on both operational (pricing) and strategic decisions (facility location) in which the market share or demand is a linear function of the price charged to customers by the two competing supply chains. Drezner and Drezner [28] consider a facility location problem with foresight, in which a future competitor would enter the market and locate its facility at the best site given the location of the existing firm. In such situation, the facility location decision should be made taking future competition into account. The paper formulates a stackelberg equilibrium problem using the gravity model where the objective is to maximize one's market share. The gravity model used in the problem only considers the distance from the customer as competing factor while assuming all other attractiveness factors are constant. Rezapour and Farahani [60] develop a bi-level model for a new SC in a competitive network structure. In their problem, strategic decisions are made once for all, with the exception to operational decisions; price and service level are adjustable in the future. The objective is to maximize

future profits of the SC where the demand is a linear function of all prices and service levels in the market. Berman and Krass [18] develop a nonlinear static competitive facility location problem by incorporating both spatial and flow interaction market share models, in which facilities derive their demand from both dedicated-trip customers and intercepting customers passing by a facility. Wu and Lin [71] solve a similar problem to Berman and Krass [18] called the *flow-capturing location allocation problem*. Aboolian et al. [15] presents a location-allocation problem for a web services provider in a duopolistic competition. The paper investigates the optimal facility location, customer allocation, and number of servers at each facility to maximize the provider's profit, while taking into account the facility location of the existing rival providing the same service. In this study, a customer patronizes the provider offering, minimum expected waiting time with the demand follows a Poisson distribution. Fernández et al. [34] consider a facility location problem along with competitive pricing, with the assumption that a customer buys the product from the facility offering the lowest price. They then provide three different choice models to break ties in the case each facility offers same price. Aboolian et al. [13] develop a competitive facility location problem using the spatial interaction model to optimize both location and design decisions. In the study, demand is elastic, i.e. as the total utility of the service increases, the market expands.

In our problem, we assume static competition where the characteristics of stores and services already being offered by the e-retailer would not change following the entrance of the unique service with drones. Our work consider a unique competition in which e-retailer's own facilities do not compete against each other but rather the services are competing against each other. These services also compete with the existing stores that are in close proximity to the customer location. Hence, we consider strategic factors along with time-based and price-based competition to make optimal location and service decision. We

use a *multinomial logit model* (MNL) to estimate market share captured by each service in a competitive environment (refer to [24] for additional market share models).

Logic-based Benders decomposition

The resulting formulation is a mixed integer non-linear program. We therefore develop a logic-based Benders decomposition approach to solve the facility location problem under competing services. Logic-based Benders decomposition generalizes the classical Benders decomposition approach by relaxing the linear subproblem requirement [45, 46]. In this approach, the original problem is divided into a master problem and subproblem(s). In the master problem, some decision variables and constraints are fixed/removed. The optimal solution of the master problem is directed to the subproblem(s) to find Benders cuts that are then added back to the master problem. This iterative process continues until the optimal solution is found. Logic-based Benders decomposition approach has been used in a variety of optimization problems including scheduling problems [49], network design [36], and location problems [32, 69].

Chapter 3

Problem Definition

In this work, we address the problem of simultaneous optimization of strategic and operational decisions for an e-retailer that is opening new facilities to offer a set of services S . The services are 30-minute delivery using drones ($s = 0$), 2-hour delivery ($s = 1$), and 12 hour delivery ($s = 2$). In the market, it competes with existing independent stores (retail, convenience, and department). The existing stores offer a single service (3), which is in-store shopping. The e-retailer wants to decide on the optimal network configuration by opening facilities from a set of discrete candidate locations J offering services in $S = \{0, 1, 2\}$. Each facility $j \in J$ has unlimited capacity and incurs a fixed opening cost L_j and a fixed cost F_s of offering a service $s \in S$.

The customer demand originates from a set of finite customer zones I with two types of packages $P = \{0, 1\}$ where $p = 0$ denotes packages that are not deliverable by drone and $p = 1$ refers to packages that may be delivered by drones. A package is defined as a bundle of products a customer buys in one order. A package cannot be delivered by a drone due to two reasons: (1) it weighs more than 5 lbs, or (2) landing at the customer location is

not possible. A binary parameter a_{sp} is calculated apriori which indicates whether service $s \in S$ can deliver package $p \in P$. Another binary parameter, r_{ijs} , indicates whether customer zone $i \in I$ can be served by facility $j \in J$ using service $s \in S$. $r_{ijs} = 1$ if the distance between i and j is less than the maximum range of the service. The e-retailer earns a fixed profit $\alpha\pi_p$ per package $p \in P$ where α is the percentage margin and π_p is the package value. To deliver a package to customer zone $i \in I$ from facility $j \in J$ using service $s \in S$, the e-retailer incurs a delivery cost c_{ijs} and earns a service charge q_s . The maximum demand at customer zone $i \in I$ for package type $p \in P$ is N_{ip} . Proportion of the maximum demand captured by service $s \in S$ D_{isp} , is a function of the utility of customer zone $i \in I$ for package $p \in P$ derived from all services $S_o = S \cup \{3\}$. We propose detailed discussion of the market model and the utility and demand expressions in Section 3.2, and focus on deriving the mathematical model first in Section 3.1.

Our primary goal is to develop a model that addresses strategic issues for an e-retailer offering multiple services using both trucks and drones. We focus on studying the economic feasibility of drones and its effect on network design taking into account service pricing, technological limitations, government regulations, customer behavior, and competition. Our model addresses the following key questions: (1) the location of the facilities and the services offered at open facilities, and (2) the services be made available to each customer zone $i \in I$. These questions are interrelated. For example, the number of facilities to be located is dependent on the type of services being offered. market size, and customer utility for each service. For 30-minute delivery using drones, more facilities are expected to open compared to same-day delivery with the hope of capturing higher market share. The goal is to make these decisions optimally such that the overall profits of the e-retailer are maximized.

3.1. Drone network design under competition (DNDC)

Three sets of binary decision variables and two sets of continuous nonnegative decision variables are defined as:

$$\begin{aligned}
w_j &= \begin{cases} 1, & \text{if candidate facility } j \in J \text{ is opened,} \\ 0, & \text{otherwise.} \end{cases} \\
x_{js} &= \begin{cases} 1, & \text{if service } s \in S \text{ is offered at facility, } j \in J \\ 0, & \text{otherwise.} \end{cases} \\
y_{ijs} &= \begin{cases} 1, & \text{if facility } j \in J \text{ serves customer zone } i \in I \text{ using service } s \in S, \\ 0, & \text{otherwise.} \end{cases}
\end{aligned}$$

D_{isp} = demand derived by service $s \in S$ for package $p \in P$ in customer zone $i \in I$.

d_{ijsp} = demand captured by facility $j \in J$ using service $s \in S$ for package $p \in P$ in customer zone $i \in I$.

DNDC formulation is then developed as the nonlinear mixed integer program [NP].

$$[\text{NP}]: \quad \max \quad \sum_{i \in I} \sum_{j \in J} \sum_{s \in S} \sum_{p \in P} (\alpha\pi_p + q_s - c_{ijs})d_{ijsp} - \sum_{j \in J} \sum_{s \in S} F_s x_{js} - \sum_{j \in J} L_j w_j \quad (3.1)$$

$$\text{s.t.} \quad \sum_{j \in J} y_{ijs} \leq 1 \quad \forall i \in I, s \in S, \quad (3.2)$$

$$y_{ijs} \leq r_{ijs} x_{js} \quad \forall i \in I, j \in J, s \in S, \quad (3.3)$$

$$x_{js} \leq w_j \quad \forall j \in J, s \in S, \quad (3.4)$$

$$d_{ijsp} \leq M y_{ijs} \quad \forall i \in I, j \in J, s \in S, p \in P, \quad (3.5)$$

$$\sum_{j \in J} d_{ijsp} \leq D_{isp} \quad \forall i \in I, s \in S, p \in P, \quad (3.6)$$

$$(3.12), (3.13), (3.14), (3.17),$$

$$y_{ijs} \in \{0, 1\} \quad \forall i \in I, j \in J, s \in S, \quad (3.7)$$

$$x_{js} \in \{0, 1\} \quad \forall j \in J, s \in S, \quad (3.8)$$

$$w_j \in \{0, 1\} \quad \forall j \in J, \quad (3.9)$$

$$d_{ijsp} \geq 0 \quad \forall i \in I, j \in J, s \in S, p \in P \quad (3.10)$$

The objective function (3.1) maximizes the overall profitability of the e-retailer expressed as the difference between revenues and delivery costs and fixed facility and service costs. and fixed costs. Constraint (3.2) ensures that customer zone $i \in I$ is served using service $s \in S$ by only one facility. For customer zone $i \in I$, service $s \in S$ may be offered by facility $j \in J$ if it is available and the distance between zone i and facility j is less than the maximum range of service s as indicated by constraint (3.3). Additionally, constraint (3.4) ensures that a facility $j \in J$ can offer services only if it is open. Constraint (3.5) ensures that demand d_{ijsp} for package $p \in P$ using service $s \in S$ can only be satisfied by facility $j \in J$ if the customer zone $i \in I$ is assigned to it. Constraint (3.6) restricts the total demand $\sum_{j \in J} d_{ijsp}$ of customer zone $i \in I$ for package $p \in P$ using service $s \in S$ to the demand captured by that service D_{isp} . The latter depends on the utility that the customer zone $i \in I$ derives from that service relative to the utility derived from other services. Note that constraints (3.2), (3.5), and (3.6) limit the demand satisfied (d_{ijsp}) to either 0 or D_{isp} . Constraints (3.7), (3.9), and (3.8) are binary requirements for variables w_j , x_{js} , and y_{ijs} respectively. Constraint (3.10) is the nonnegative requirement for variable d_{ijsp} . Model [NP] is only completely defined when variable D_{isp} is well defined. Constraints (3.12), (3.13), (3.14), and (3.17) refer to the demand model equations detailed next.

3.2. Demand model

As discussed earlier in Section 2, five main market share models are used in the literature. Linear, exponential, and multiplicative models do not meet logical consistency requirements and standard MCI models are unrealistic when any negative attribute of the demand model takes a value close to zero. For instance, consider delivery price. If MCI model is used, utility $\rightarrow \infty$ as service charges $\rightarrow 0$. We therefore use a *multinomial logit model* (MNL)

to predict the demand captured by each service in a competitive environment. The overall profitability of the e-retailer depends not only on competition with other stores, but also on competition between its own services. A customer zone $i \in I$ has a maximum demand N_{ip} for package $p \in P$ which is distributed between the services in S_o that are offered to the market. The proportion of the maximum demand captured by the market is a function of the total utility of customer zone $i \in I$ for package $p \in P$, denoted by $g(U_{ip})$. Note that $g(U_{ip})$ allows for the consideration of elastic demand with respect to total utility. The demand of customer zone $i \in I$ for package $p \in P$ captured by service $s \in S$ is D_{isp} and is dependent on the utility the customer derives from that service, denoted by u_{isp} . The total utility of customer zone $i \in I$ for package $p \in P$ is $U_{ip} = \sum_{s \in S_o} u_{isp}$. Services in S_o compete on five distinct factors: (1) inherent attractiveness, β_{0s} (2) travel time, TT_{is} , (3) travel cost, TC_{is} , (4) additional service charges, q_s , and (5) delivery/waiting time, WT_s . We assume that there is no competition between the stores and customers visit their nearest store. It is further assumed that there is a static competition between services offered by the e-retailer and stores, i.e., the characteristics of the services offered by the stores will not change once delivery by drone service is made available.

The utility derived by customer zone $i \in I$ for package $p \in P$ using service $s \in S_o$ as:

$$u_{isp} = \exp(\beta_{0s} - \beta_{tt}TT_{is} - \beta_{tc}TC_{is} - \beta_{wt}WT_s - \beta_{dp}q_s) \quad (3.11)$$

where β_{0s} is a parameter capturing the inherent attractiveness of service $s \in S_o$, and β_{tt} , β_{tc} , β_{wt} , and β_{dp} are sensitivity parameters. Parameters $\beta_{tt} \geq 0$ and $\beta_{tc} \geq 0$ relate to the sensitivity of the customer to travel time TT_{is} and travel cost TC_{is} to a store, i.e. $s = 3$. Parameters $\beta_{wt} \geq 0$ and $\beta_{dp} \geq 0$ indicate the sensitivity of the customer to the service charge q_s and service time WT_s when making online purchases using services $s \in S$.

The utility derived by the customers is inversely proportional to travel time, travel cost, waiting time, and service charge as given by equation (3.2). When a customer orders online, $TT_{is} = 0$ and $TC_{is} = 0$ since the package is delivered to his/her doorstep. Moreover, online orders from customer zone i for a particular package p may be fulfilled from an open facility j that is able to provide service s to the customer, i.e. $a_{sp}Y_{ijs} = 1$. Note that for packages that are not deliverable by drone service $a_{sp} = 0$, which ensures that the utility derived by customer zone $i \in I$ using drone service is zero for package type $p = 0$. Consequently, the utility derived by customer zone $i \in I$ when package $p \in P$ is ordered online using service $s \in S$, UR_{isp} , is a function of the e-retailer's decision for which facilities to open and what services to offer:

$$UR_{isp} = \left(\sum_{j \in J} a_{sp} y_{ijs} \right) \exp(\beta_{0s} - \beta_{wt} WT_s - \beta_{dp} q_s) \quad (3.12)$$

On the other hand, when buying from a store, the customer does not incur additional service charge over the retail price, and does not wait for the purchase to be delivered. Since customer utility is inversely proportional to the travel time to the store, a customer is expected to use the nearest store. Hence the utility derived from other stores is equal to zero, and US_{ip} , the utility derived by customer $i \in I$ to buy package $p \in P$ from the nearest store is

$$US_{ip} = \exp(\beta_{03} - \beta_{tt} TT_{ip} - \beta_{tc} TC_{ip}). \quad (3.13)$$

The index s is dropped from the expression of the utility derived from store since stores offer only one type of service. The total utility function of customer zone $i \in I$ for package

$p \in P$ is

$$U_{ip} = \sum_{s \in S} UR_{isp} + US_{ip} \quad (3.14)$$

and the proportion of the maximum demand of customer $i \in I$ for package $p \in P$ captured by service $s \in S$ is:

$$MS_{isp} = \frac{UR_{isp}}{U_{ip}}. \quad (3.15)$$

The standard market share model assumes that demand is perfectly inelastic which limits the applicability of the model to capture market expansion or shrinkage. When more services are available to the customers the probability of lost sales decreases. As a result, the overall demand increases. Similarly, it is nearly impossible to incorporate all competing firms and services in a mathematical model. As a result, the maximum demand can never be captured by the services included in the model. In reality, a proportion of demand is always lost. Consider the assumption that a customer will always prefer nearest store, though it is possible that some customers may not prefer the nearest store. Similarly, a part of the demand may also be lost to other e-retailers. Hence, we use the exponential expenditure function also (Berman and Krass [19]) to determine the proportion of the maximum demand N_{ip} that is captured by all services offered by the e-retailer and the store. The expenditure function is

$$g(U_{ip}) = 1 - \exp(-\lambda U_{ip}) \quad (3.16)$$

and the total demand captured is $N_{ip} \times g(U_{ip})$.

Parameter λ represents the elasticity of demand with respect to total utility U_{ip} . When

the demand is perfectly inelastic, $\lambda \rightarrow \infty$, $g(U_{ip}) \rightarrow 1$, and the maximum demand is fully captured. Parameter λ can also be used as a measure of the market size. When market size is small, λ is low. The demand of customer i for package p captured by service $s \in S$ is $D_{isp} = N_{ip} \times g(U_{ip}) \times MS_{isp}$, or

$$D_{isp} = \frac{N_{ip} (1 - \exp(1 - \lambda U_{ip})) \left(\sum_{j \in J} a_{sp} y_{ijs} \right) \exp(\beta_{0s} - \beta_{wt} WT_s - \beta_{dp} q_s)}{\sum_{s \in S} \left(\left(\sum_{j \in J} a_{sp} y_{ijs} \right) \exp(\beta_{0s} - \beta_{wt} WT_s - \beta_{dp} q_s) \right) + \exp(\beta_{03} - \beta_{tt} TT_{ip} - \beta_{tc} TC_{ip})} \quad (3.17)$$

The expression of the demand D_{isp} is nonlinear and is function of the decision variable Y_{ijs} . In fact, D_{isp} is a decision variable, and Constraints (3.12), (3.13), (3.14), and (3.17) must be included as constraints in model [NP] to obtain a complete formulation of the problem.

[NP] is highly nonlinear due to the demand constraints. Although model NP can be linearized (see Appendix A for the linear formulation), it is computationally expensive. We therefore develop a novel logic-based Benders decomposition approach to solve the problem to optimality. This is detailed in next section.

Chapter 4

A Logic-Based Benders Decomposition Approach

Since model [NP] is nonlinear, we develop a logic-based Benders decomposition approach to solve the problem. When the location decisions are fixed, the only remaining decisions are assignment variables that maximize profit. Once the opened facilities and the services they provide are known, the assignment decisions reduce to allocating each customer zone to the nearest open facility offering the service. We exploit this feature to decompose [NP] into a location-service master problem (LSMP) that makes locational decisions, and a set of customer service-assignment subproblems (CSASPs) where the assignment decisions choose which services will be provided to each customer zone $i \in I$. LSMP determines the facilities to be opened and the services made available at each opened facility. Given that the locational decisions are known, [NP] is divided into $|I|$ subproblems. To offer service $s \in S$ to a customer zone, it is assigned to the nearest opened facility offering that service. Since in practice the set of services offered by an e-retailer are limited, and possible

combinations of services offered to a customer equals $2^{|S|}$, it is computationally possible to use enumeration to find the optimal combination of the services offered.

In Sections 4.1 and 4.2, models LSMP and CSASP are developed respectively. Section 4.3 details Benders optimality cuts and in Section 4.5 cut coefficients are calculated.

4.1. Location-Service Master Problem

The LSMP is given by:

$$[\text{LSMP}]: \quad \max \sum_{i \in I} Z_i - \sum_{j \in J} \sum_{s \in S} F_s x_{js} - \sum_{j \in J} L_j w_j \quad (4.1)$$

$$\text{s.t.} \quad \sum_{j \in J} \sum_{s \in S} \sum_{p \in P} (\alpha \pi_p + q_s - c_{ijs}) d_{ijsp} - Z_i = 0 \quad \forall i \in I, \quad (4.2)$$

$$\sum_{j \in J} d_{ijsp} \leq D_{isp}^{max} \quad \forall i \in I, s \in S, p \in P, \quad (4.3)$$

$$d_{ijsp} \leq r_{ijs} D_{isp}^{max} x_{js} \quad \forall i \in I, j \in J, s \in S, p \in P, \quad (4.4)$$

$$\sum_{j \in J} \sum_{s \in S} d_{ijsp} \leq TD_{ip}^{max} \quad \forall i \in I, p \in P, \quad (4.5)$$

$$\text{cuts}, \quad (4.6)$$

$$(3.4), (3.8), (3.9), (3.10)$$

$$Z_i \geq 0 \quad \forall i \in I, \quad (4.7)$$

where decision variable Z_i captures the total revenue minus the delivery cost associated with serving customer zone $i \in I$, as defined by constraint (4.2). The objective function maximizes the total profit, which is the same as (3.1). Constraints (4.3), (4.4) and (4.5) are valid constraints, and are added to tighten the relaxation. Constraint (4.3) ensures

that the total demand satisfied, $\sum_{j \in J} d_{ijsp}$, does not exceed the maximum possible demand of the customer zone $i \in I$ for package $p \in P$ using service $s \in S$, D_{isp}^{max} . D_{isp}^{max} is only achieved that service is made available to the customer zone $i \in I$ for package $p \in P$. This result is proven in Theorem 1.

Theorem 1 *The maximum demand D_{isp}^{max} , captured by service $s \in S$ for the package $p \in P$ from customer zone $i \in I$ is achieved when no other service is made available.*

Proof of Theorem 1. Recall equation (3.14) and (3.17). Equation (3.15) says that the total utility U_{ip} increases as the number of services offered increases. Taking the derivative of D_{isp} with respect to U_{ip} :

$$\frac{\partial D_{isp}}{\partial U_{ip}} = -\frac{N_{ip}UR_{isp}(1 - (\exp(-\lambda U_{ip}) \times (\lambda U_{ip} + 1)))}{U_{ip}^2} < 0 \quad (4.8)$$

as $\lambda > 0$, $U_{ip} > 0$, and $\exp(-\lambda U_{ip}) \times (\lambda U_{ip} + 1) > 1$. Hence, as total utility U_{ip} increases, D_{isp} decreases. Therefore, the maximum demand that may be captured by service $s \in S$, D_{isp} is achieved when only service $s \in S$ is available i.e., $y_{ijs} = 1$ and $y_{ijs'} = 0 \forall s' \neq s$.

■

On the other hand, the e-retailer captures maximum demand, TD_{ip}^{max} relative to competition, when it provides all services to the customer $i \in I$ for package $p \in P$ as proven in Theorem 2. Constraint (4.5) limits the total demand captured from customer zone $i \in I$ for package $p \in P$ to TD_{ip}^{max} .

Theorem 2 *The maximum total demand TD_{ip}^{max} for package $p \in P$ from customer zone $i \in I$ is captured by an e-retailer when all services in S are made available.*

Proof of Theorem 2.

Using equations (3.12) - (3.17), total demand captured TD_{ip} , by the e-retailer is expressed as:

$$TD_{ip} = \frac{N_{ip}(1 - \exp(-\lambda U_{ip}))(U_{ip} - US_{ip})}{U_{ip}}, \quad (4.9)$$

where $(U_{ip} - US_{ip}) \geq 0$ is the total utility derived by customer zone $i \in I$ for package $p \in P$ given the services offered by the e-retailer. Taking the derivative of TD_{ip} with respect to total utility U_{ip} :

$$\frac{\partial TD_{ip}}{\partial U_{ip}} = \frac{N_{ip}e^{-\lambda U_{ip}}(US_{ip}(e^{\lambda U_{ip}} - 1) + \lambda U_{ip}(U_{ip} - US_{ip}))}{U_{ip}^2} > 0 \quad (4.10)$$

as $\lambda > 0$, $U_{ip} > 0$, $(e^{\lambda U_{ip}} - 1) > 0$, and $\lambda U_{ip}(U_{ip} - US_{ip}) \geq 0$. Hence, as U_{ip} increases, TD_{ip} increases. Therefore, maximum demand TD_{ip}^{max} , is achieved when the e-retailer offers all services in S to the customer $i \in I$ for package $p \in P$. ■

Finally, constraint (4.6) are Benders optimality cuts that are added to the [LSMP] each time the subproblem is solved until an optimal solution is found. Deriving optimality cuts from the subproblem solution is explained in detail in Section 4.3.

4.2. Customer Service-Assignment Subproblems

When location and service decisions (w, x) are known, [NP] reduces to $|I|$ customer-service assignment subproblems $[SP_i]$. We denote known location and service decisions as \bar{w} and

\bar{x} respectively. $[SP_i]$ is modeled as:

$$[SP_i] : \max \sum_{j \in J} \sum_{s \in S} (\alpha \pi_p + q_s - c_{ijs}) d_{ijsp} \quad (4.11)$$

$$y_{ijs} \leq r_{ijs} \bar{x}_{js} \quad j \in J \quad s \in S, \quad (4.12)$$

$$(3.3) - (3.5), (3.7), (3.10), (3.12) - (3.14), \&(3.17).$$

Subproblem $[SP_i]$ finds an optimal assignment of customer zone $i \in I$ to offered services at open facilities. Such an assignment depends on the demand values determined by constraints (3.12) - (3.14), and (3.17). Since the latter are highly nonlinear, $[SP_i]$ remains challenging to solve. We explain two main characteristics of $[SP_i]$ and develop an enumeration-based algorithm to solve it. When \bar{x} and \bar{w} decisions are known, to offer service $s \in S$, the customer zone $i \in I$ is assigned to the nearest facility offering that service to minimize delivery costs c_{ijs} . The customer zone $i \in I$ can only be served using service $s \in S$ from facility $j \in J$ if $r_{ijs} \bar{x}_{js} = 1$. This not only ensures that the facility offers the service but is also within its maximum range to offer that service. To serve the customer zone $i \in I$ from facility $j \in J$ using service $s \in S$, the delivery cost CM_{ijs} is

$$CM_{ijs} = \begin{cases} c_{ijs}, & \text{if } r_{ijs} \bar{x}_{js} = 1, \\ M, & \text{otherwise.} \end{cases} \quad (4.13)$$

To offer service $s \in S$ to customer zone $i \in I$, minimum delivery cost $MC_{is} = \min_{j \in J} (CM_{ijs})$. This reduces the decision to whether service $s \in S$ is offered to customer zone $i \in I$. Note that when no facility can offer service $s \in S$ to customer zone $i \in I$, the minimum delivery cost $MC_{is} = M$, i.e., there is a large penalty which ensures that service s will not be assigned to customer zone i . The utility derived UR_{isp} by customer zone $i \in I$ for package

$p \in P$ using service $s \in S$ is then a constant and is denoted by:

$$\overline{UR}_{isp} = a_{sp} \exp(\beta_{0s} - \beta_{wt} WT_s - \beta_{dp} q_s) \quad i \in I, \quad s \in S, \quad p \in P. \quad (4.14)$$

Consider a set E containing all possible subsets of the services in S . Let $A_{se} = 1$, if service $s \in S$ is in set $e \in E$. As such, the demand of customer zone $i \in I$ for package $p \in P$ using service $s \in S$ when the set $e \in E$ is selected can be precalculated as:

$$\overline{U}_{ipe} = \sum_{s \in S} \overline{UR}_{isp} A_{se} + US_{ip} \quad i \in I, \quad p \in P, \quad e \in E, \quad (4.15)$$

$$\overline{MS}_{ispe} = \frac{\overline{UR}_{isp} A_{se}}{\overline{U}_{ipe}} \quad i \in I, \quad s \in S, \quad p \in P \quad e \in E, \quad (4.16)$$

$$\overline{D}_{ispe} = N_{ip} (1 - \exp(-\lambda_p \overline{U}_{ipe})) \overline{MS}_{ispe} \quad i \in I, \quad s \in S \quad p \in P \quad e \in E, \quad (4.17)$$

where \overline{U}_{ipe} is the total utility derived by customer zone $i \in I$ for package $p \in P$ when the set of services $e \in E$ is selected. \overline{MS}_{ispe} calculates the market share of package $p \in P$ from customer $i \in I$ captured by service $s \in S$ when the set of services available is $e \in E$ as computed in Equation (4.16). Finally, Equation (4.17) computes the total demand captured by the service $s \in S$ when set $e \in E$ of the services is offered to customer zone $i \in I$ for product $p \in P$. The subproblem $[SP_i]$ can now be:

$$\overline{SP}_i = \max_{e \in E} \left\{ \sum_{s \in S} \sum_{p \in P} (\alpha \pi_p + q_s - MC_{is}) \overline{D}_{ispe} \right\} \quad \forall \quad i \in I. \quad (4.18)$$

Algorithm 1: Customer-service assignment subproblem Algorithm (CSASPA)

Data: obtain solution \bar{x} from [LSMP]

```
1 for customer zone  $i \in I$  do
2   for service  $s \in S$  do
3     for facility  $j \in J$  do
4       compute delivery cost:
5        $CM_{ijs} = \begin{cases} c_{ijs}, & \text{if } r_{ijs}\bar{x}_{js} = 1 \\ M & \text{otherwise} \end{cases}$ 
6     end
7     compute minimum delivery cost:
8      $MC_{is} = \min_{j \in J}(CM_{ijs})$ 
9   end
10  obtain optimal solution:
11   $\overline{SP}_i = \max_{e \in E} \{ \sum_{s \in S} \sum_{p \in P} (\alpha\pi_p + q_s - MC_{is}) \overline{D}_{ispe} \}$ 
12 end
```

4.3. Logic-based Benders cuts

Consider a solution $(\bar{x}, \bar{w}, \bar{x})$ obtained from [LSMP] at iteration k . The master problem [LSMP] provides an upper bound to the original problem, and a lower bound is calculated using master problem and subproblems as $\sum_{i \in I} \overline{SP}_i - \sum_{j \in J} \sum_{s \in S} F_s \bar{x}_{js} - \sum_{j \in J} L_j \bar{w}_j$. Since the demand model is not included in the master problem [LSMP] and an upper bound is used for the demand captured D_{isp} , at any given iteration, $\overline{Z}_i \geq \overline{SP}_i$. When the operating profits \overline{Z}_i , in [LSMP] equals the operating profits calculated in subproblem $\overline{SP}_i \forall i \in I$,

the [LSMP] solution is optimal. At a given iteration k , let $O_k = \{j \in J, s \in S : \bar{x}_{js} = 1\}$. If $\bar{Z}_i > \overline{SP}_i$ for $i \in I$, we add following Benders optimality cuts to the master problem [LSMP]:

$$Z_i \leq \overline{SP}_i + M \sum_{j \in O_k} \sum_{s \in O_k} (1 - x_{js}) + M \sum_{j \notin O_k} \sum_{s \notin O_k} x_{js} \quad i \in I \quad (4.19)$$

A valid Benders cut is defined by Chu and Xia [23] as any logical expression that eliminates the current master solution $(\bar{X}, \bar{w}, \bar{Z})$ if its not feasible to the original problem [NP], and it must not eliminate any solution that is feasible to the original problem [NP]. For the current solution \bar{x} at iteration k , $O_k = \{j \in J, s \in S : \bar{x}_{js} = 1\}$. If the same set of the services are offered in subsequent iterations, $\sum_{j \in O_k} \sum_{s \in O_k} (1 - x_{js})$ and $\sum_{j \notin O_k} \sum_{s \notin O_k} x_{js}$ equal 0, reducing the cut to

$$Z_i \leq \overline{SP}_i \quad (4.20)$$

which is violated if Z_i takes a value greater than \overline{SP}_i . As such, the cut (4.19) ensures that either the solution \bar{x} is changed or the operating profit is reduced to \overline{SP}_i for all customers and products i.e. it eliminates the current solution $(\bar{x}, \bar{w}, \bar{Z})$ if it is infeasible. This proves that the cut satisfies the first condition. If the set of services offered changes, the right-hand-side of (4.19) increases by at least M which is a significantly large number and thus the equation holds trivially. Now Z can take any value as the cut (4.19) is non-binding for this new set of \bar{x} . This proves that cut only removes a solution that is infeasible to the original problem NP.

The effectiveness of cut (4.19) depends on the value M . A large M is likely to result in total enumeration. Therefore, it is important to calculate smaller coefficients in (4.19)

without eliminating any solution that is feasible to the original problem. For customer zone $i \in I$, let γ_{ijs} be the maximum change in operating profit Z_i when service $s \in S$ offered at facility $j \in J$ (i.e., $\bar{x}_{js} = 1$) is closed. Also, let δ_{ijs} be the maximum change in operating profit Z_i when the service $s \in S$ that is not offered at facility $j \in J$ (i.e., $\bar{x}_{js} = 0$) is opened. Note that $\gamma_{ijs} \leq 0$ since closing an opened service $s \in S$ at a facility $j \in J$ can never improve the operating profits. Similarly, $\delta_{ijs} \geq 0$ because offering a new service at a facility can never reduce the operating profits. A tighter valid optimality cut is as follows:

$$Z_i \leq \overline{SP}_i + \sum_{j \in O_k} \sum_{s \in O_k} \gamma_{ijs}(1 - x_{js}) + \sum_{j \notin O_k} \sum_{s \notin O_k} \delta_{ijs}x_{js} \quad i \in I \quad (4.21)$$

Since the maximum change is considered in computing γ_{ijs} and δ_{ijs} , cut (4.21) is still a valid Benders cut. At iteration k , consider service $s \in S$ that is available at the facility $j \in J$ (i.e., $\bar{x}_{js} = 1$). The minimum possible decrease in \overline{SP}_i equals γ_{ijs} if that service s at facility j is closed. Similarly, at k^{th} iteration, consider a service $s \in S$ that is not available at facility $j \in J$ (i.e., $\bar{x}_{js} = 0$). The maximum possible increase in SP_i then equals δ_{ijs} if that service s is made available at facility j .

4.4. Calculating cut coefficients

The cut coefficients are designed to capture the change in operating profits where γ_{ijs} captures the minimum possible decrease in \overline{SP}_i when an available service $s \in S$ at facility $j \in J$ is closed and δ_{ijs} captures the maximum possible increase in \overline{SP}_i when a service $s \in S$ at facility $j \in J$ is made available. To calculate γ_{ijs} , note that we want to compute minimum decrease (maximum change) in Z_i value when service s at facility j is closed. Note further that operating profits are maximized when all services are made available at

all facilities because the delivery costs are minimized by serving a customer zone $i \in I$ from the nearest facility. For a given service $s \in S$ at facility $j \in J$, γ_{ijs} can be computed as:

$$\gamma_{ijs} = M_{ijs}^{max} - M_i^{max} \quad (4.22)$$

where M_{ijs}^{max} denotes operating profits earned from customer zone $i \in I$ when only service s at facility j is not available and M_i^{max} denotes optimal operating profits earned when all services are made available at all facilities. $M_{ijs}^{max} = \overline{SP}_i$ when $\bar{x}_{js} = 0$ and $\bar{x}_{j's'} = 1 \forall \{j' \in J, s' \in S\} - \{j' = j, s' = s\}$. Similarly, $M_i^{max} = \overline{SP}_i$ when $\bar{x}_{js} = 1 \forall j \in J, s \in S$. Similarly, to calculate δ_{ijs} , we need to calculate the maximum possible contribution by service $s \in S$ at facility $j \in J$ to operating profit earned from customer zone $i \in I$. As proved by Theorem 1, total demand captured by a service s from customer zone $i \in I$ for package $p \in P$ is maximized when only that service is made available. Hence, maximum profit contribution by service $s \in S$ at facility $j \in J$ is achieved when it is the only available service to customer zone $i \in I$. For each customer zone $i \in I$, maximum increase δ_{ijs} is then computed as:

$$\delta_{ijs} = M_{ijs} - M_i^{min} \quad (4.23)$$

where $M_{ijs} = \overline{SP}_i$ when $\bar{x}_{js} = 1$ and $\bar{x}_{j's'} = 0 \forall \{j' \in J, s' \in S\} - \{j' = j, s' = s\}$, and M_i^{min} equals zero when all facilities are closed i.e., $\bar{x}_{js} = 0 \forall j \in J, s \in S$.

4.5. Location - Assignment Benders Algorithm (LAB)

Logic-based location-assignment Benders algorithm (referred to as LAB) is an iterative process that alternates between [LSMP] and [SP]. At a given iteration k , an optimal solution (\bar{x}) to [LSMP] is used to solve subproblems [SP]. If $\bar{Z}_i = \overline{SP}_i \forall i \in I$, the solution $(\bar{w}, \bar{x}, \bar{Z})$ is optimal. This rarely happens in early iterations since the demand D_{isp} in [LSMP] is overestimated. If $\bar{Z}_i \neq \overline{SP}_i$, optimality cuts (4.21) are calculated and added to [LSMP]. The algorithm stops when for each customer zone $i \in I$, $\bar{Z}_i = \overline{SP}_i$. The overall iterative algorithm (LAB) is given in Algorithm 2.

Algorithm 2: Location-Assignment Benders Algorithm(LAB)

Data: Compute benders cut coefficients γ_{ijs} , δ_{ijs} and demand values \bar{D}_{ispe}

```
1 k=1;
2 for customer zone  $i \in I$  do
3    $\bar{Z}_i = \infty$ ;
4    $\bar{SP}_i = 0$ ;
5 end
6 //Main loop
7 while  $\sum_{i \in I} \bar{Z}_i \neq \sum_{i \in I} \bar{SP}_i$  do
8   Solve [LSMP], obtain solution  $(\bar{x}, \bar{Z})$  and update  $\bar{Z}_i$ ;
9   Solve [SP] using CSASPA algorithm, and update  $\bar{SP}_i$ ;
10  if  $\sum_{i \in I} \bar{Z}_i = \sum_{i \in I} \bar{SP}_i$  then
11    stop, optimal solution for [NP] is found;
12  else
13    for customer zone  $i \in I$  do
14      Derive the optimality cut and add to [LSMP];
15      k=k+1;
16    end
17  end
18 end
19 //Optimal solution
20 Location decisions  $(x, w)$  are calculated using latest [LSMP] solution ;
21 Assignment decisions,  $y$  are calculated from latest [SP] solution;
```

Chapter 5

The case of NYC

In this chapter we study a facility location problem for an e-retailer offering drone service along with other same day delivery services. The model is solved over an actual network in New York City (NYC). Given that Amazon.com is the leading e-retailing company in the US and has announced its plan to deliver packages using drones within 30 minutes, we generate data based on the online information available regarding its demand characteristics and services offered for same day delivery.

The data used for the numerical testing is presented Section [5.1](#). In Section [5.2](#), comparative analysis of the network with and without drones is presented. We also carry out detailed sensitivity analysis to study the effect of customer behavior, government regulations, technological limitations, and the attributes of drone service on optimal network configuration.

5.1. Data used

In this section, we present the data used in the computational experiments. An actual network is constructed for NYC which is the most populated city in the US with a population of around 8.5 million, over an area of 789 km². NYC is divided into five boroughs with Manhattan being the most densely populated area. Each borough consists of community districts (CDs) that are further divided into Neighborhood Tabulation Areas (NTAs) based on the population density. Figure 5.1 illustrates NTAs based division of NYC. Each NTA is considered as a customer zone where the centroid of each zone is used as demand point. Areas with zero population (shaded in grey), like Airport, parks, and cemeteries are excluded. The location of stores in NYC are determined using Google Earth and each store is considered as a competitor. To determine candidate facility locations, 20 out of 67 stores are assumed to be candidate facility locations. The competitive stores and candidate facility locations are illustrated in Figure 5.1. For the base case scenario, we consider three potential delivery services to be offered by the e-retailer: (1) 30-minute delivery using drones, (2) 2-hour, and (3) same-day (12-hour). Note that same-day and 2-hour delivery are currently offered by Amazon in NYC.

As discussed earlier, for the same day delivery, customers usually order products that are readily available at convenience and retail stores. We therefore use grocery market data to estimate demand values. In 2015, US grocery store sales amounted to \$606 billion and online grocery shopping captures \$7 billion [17]. The grocery market in each customer zone is calculated using population [2]. We assume that each package $p \in P$ has an average value of \$20. The American Community Survey data [6] presents housing characteristics in different community districts. For the base case, we assume that drones can only deliver packages to buildings where the number of housing units is less than or equal to 9, i.e.

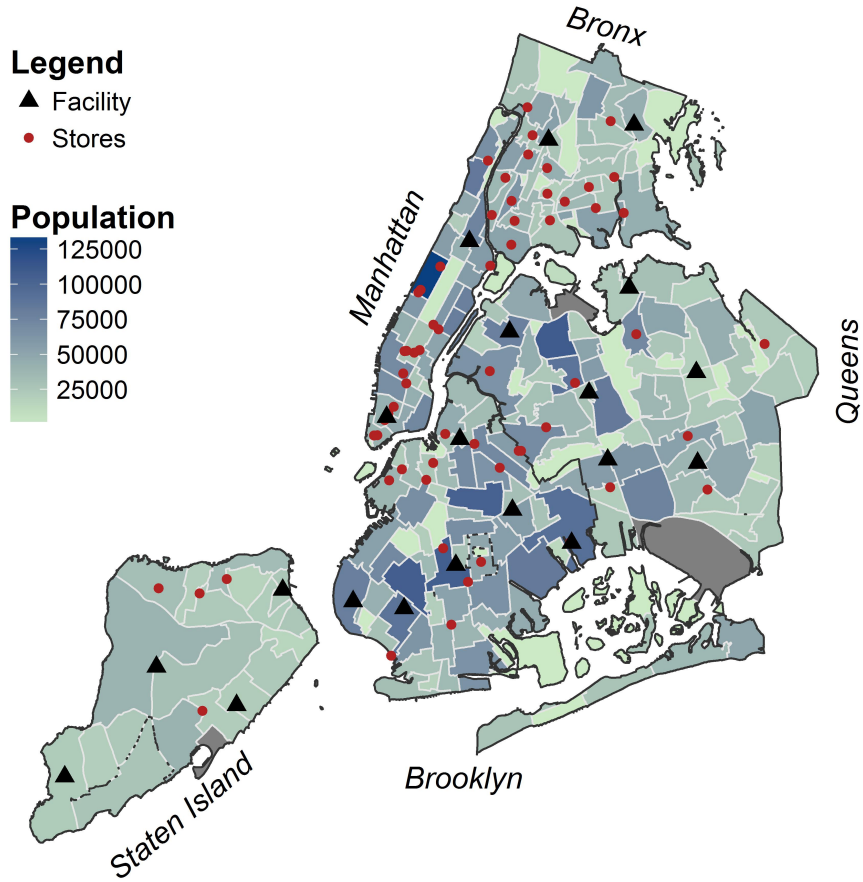


Figure 5.1: New York City Network

delivery is not possible to customers living in apartment buildings. We also assume that 86% of the packages weigh less than 2.5 kg. The demand N_{ip} for each package $p \in \{0, 1\}$ in customer zone $i \in I$ is then calculated by distribution demand based on the percentage of packages that are deliverable by drones

$$N_{i1} = \frac{606 \times 10^8}{\pi_1} \times \frac{Population_i}{US\ population} \times pr(\text{weight} \leq 5) \times pr_i(\text{Building size} \leq n)$$

$$N_{i0} = \frac{606 \times 10^8}{\pi_0} \times \frac{Population_i}{US\ population} \times (1 - pr(\text{weight} \leq 5)) \times (1 - pr_i(\text{Building size} \leq n))$$

where $pr(\text{weight (lbs)} \leq 5)$ denotes the percentage of the packages that weigh less than 2.5 kg while $pr_i(\text{Building size} \leq n)$ denotes the percentage of buildings in customer zone $i \in I$ having less than or equal to n units. As such, N_{i1} equates to the demand in customer zone $i \in I$ that is deliverable by drones and N_{i0} is the demand that is not deliverable by drones.

To estimate the facility location costs, we use leasing rates in NYC presented by Jll [50] in a monthly report for 2015 and the online leasing website *LoopNet* [7]. The minimum (min_b), and maximum (max_b), lease rates per sq.ft in different boroughs (B) is estimated. The results are summarized in Table B.1. We further assume that each candidate facility covers an area of 50,000 sq.ft. The intuition behind our assumption is the fact that Amazon’s Distribution Center in Manhattan is 50,000 sq.ft. Labor and miscellaneous costs are estimated based on the study conducted by Boyd Company [8]. Since in [8], the analysis is based on 500,000 sq.ft distribution center, therefore we scaled labor and other costs to a 50,000 sq.ft distribution center. On average, 50,000 sq.ft facility will incur other costs of \$1.5 million. As such, facility costs at $j \in J$ is $L_j = 50,000 * U(min_b, max_b) + 1,500,000$. Yearly facility costs at each candidate location are presented in Table B.2. To offer same-day or 2-hour delivery service at a facility, an additional cost $F_s = \$250,000$ will be incurred. To compute yearly costs of offering 30-minute delivery using drones, we use the analysis provided by Keeney [52]. Since the analysis presented in [52] assumes up-gradation of all Amazon’s distribution centers, we scaled down the costs based on the proportion of NYC’ population to the entire population of the US. We estimate an additional yearly cost of \$1,000,000 is incurred to offer drone service at a facility.

To estimate the delivery costs c_{ijs} , we assume that transportation costs are fixed per unit and independent of distance. As shown in Table 5.1, same-day delivery costs \$6, 2-hour delivery costs \$10, and drone delivery costs \$2.5 per unit. For same-day delivery, the unit cost of \$6 is used based on the analysis presented by Wohlsen [70]. For 2-hour

delivery, Amazon launched its Flex Program where independent drivers are paid \$18-\$25 per hour to use their own cars to deliver Amazon Prime Now packages [64]. Assuming that a driver is paid \$20 per hour and makes 4 deliveries in two hour time window, unit delivery cost = $\frac{20 \times 2}{4} = \10 . These values are well aligned with Amazon’s delivery charges for each type of service. We further assume that the delivery charges (q_s) for each service equals the delivery cost. The assumption can be justified by the fact that e-retailers do not earn profits from delivery charges. Amazon, for instance, reports revenue earned from delivery to be even less than delivery costs incurred. We also assume that a facility can serve all customers within 40 km radius for same-day delivery. Similarly, for 2-hour delivery, the radius is assumed to be 20 km and for drones, we consider its range to be restricted to 10 km. We use the *geosphere* package in *R* to compute distances between nodes[43]. The package calculates shortest distance between two points on surface of the sphere using *Haversine* formula as shown in Table 5.1.

For drone delivery, there have been few studies to estimate unit delivery costs [25, 52]. The values used in this paper are based on a study conducted by Keeney [52]. We made some changes to the assumption in [52]. Table B.3 illustrates detailed calculations to estimate delivery cost per unit for drones. Let DD be the yearly demand for drone service. Assuming that the demand is uniformly distributed, we calculate the average hourly demand by dividing the yearly demand over 365×24 . However, in real life demand may fluctuate significantly due to seasonality factors. To make sure we carry enough drones to meet peak time demand as well, the average hourly demand is multiplied by a safety factor of 2. Since a drone can fly at a speed of up to 40 km/h, it will take at most 30 minutes for a drone to fly from the distribution center to a customer location and back to the distribution center. As such, a single drone can make up to two deliveries in one hour. Hence, the number of drones required equals half of the maximum hourly demand.

| | |
|--|---|
| $L_j : 50000 \times U(\min_b, \max_b) + 1,500,000$ | yearly facility costs |
| $\pi_0 = \pi_1 = 20, \alpha = 0.30$ | package price and profit margin |
| $F_0 = \$1,000,000$ | 30-minute delivery-by-drone - yearly facility costs |
| $F_1 = \$250,000$ | 2-hour delivery - yearly facility costs |
| $F_2 = \$250,000$ | same-day delivery - yearly facility costs |
| $c_{ij0} = q_0 = \$2.6$ | delivery cost and charges per package using drone service |
| $c_{ij1} = q_1 = \$10$ | delivery cost and charges per package using 2-hour service |
| $c_{ij2} = q_2 = \$6$ | delivery cost and charges per package using same-day service |
| $r_0 : 10 \text{ km}$ | Range of delivery-by-drone |
| $r_1 : 20 \text{ km}$ | Range of two-hour delivery at a facility |
| $r_2 : 40 \text{ km}$ | Range of same day delivery at a facility |
| $d : 2r \times \arcsin\left(\sqrt{\sin^2\left(\frac{\varphi_1 - \varphi_2}{2}\right) + \cos(\varphi_1)\cos(\varphi_2)\sin^2\left(\frac{\lambda_1 - \lambda_2}{2}\right)}\right)$ | distance between two points with coordinates (λ_1, φ_1) & (λ_2, φ_2) where λ is the longitude and φ is the latitude, in radians. [44] |
| $\beta_{03} = 0.00, \beta_{00} = -2.22, \beta_{01} = -2.00, \beta_{02} = -2.00,$ | Inherent attractiveness of the services available. |
| $\beta_{tt} = 1.4$ | Travel time (in hours) sensitivity parameter. |
| $\beta_{tc} = 0.035$ | Travel cost sensitivity parameter. |
| $\beta_{wt} = 0.092$ | Delivery time (in hours) sensitivity parameter. |
| $\beta_{dp} = 0.34$ | Delivery charges (in dollars) sensitivity parameter. |
| $\lambda = 0.5$ | Demand elasticity parameter, elastic. |

Table 5.1: NYC example input parameters

The e-retailer would be required to buy extra batteries so that there is no waiting time to recharge drones. Many reports suggest that the FAA new regulations for drone delivery would require certified operators to monitor the drone activity [11, 52]. However, the number of drones an operator can manage simultaneously are highly speculated. Keeney [52] assumes that an operator would be permitted to manage 5-10 drones simultaneously. Other studies, on the other hand, suggest that an individual operator will be required for each drone flight [9]. For the base case scenario, we assume that an operator will be allowed to monitor 10 drones at the same time. In the subsequent section, we present sensitivity analysis to study the affect of the number of drones per operator. To estimate the number of operators required, we use the peak demand. However, it should be noted that in our calculations, we assume a linear relationship between the number of operators required and hourly demand. Given that hourly demand in a metropolitan city like NYC is high, and assuming a linear relationship between operators required and hourly demand does

not impact the delivery charges significantly. Note further that the number of operators required has the largest impact on unit delivery cost as it constitutes 92% ($\$2.397/\2.61) of the total unit cost. Once the number of drones, batteries, and operators required are calculated, cost figures are estimated as shown in Table B.3.

To estimate the sensitivity parameters for the demand model, we use the results presented by Schmid et al. [62] who study the consumer choice behavior for online and in-store shopping. The analysis is presented for groceries which is well aligned with the scope of this work. The inherent attractiveness for online shopping is estimated relative to that of in-store shopping. As such, $\beta_{03} = 0.0$ and $\beta_{0s} = -2.00$. A negative inherent attractiveness implies that customers have negative attraction towards online shopping relative to in-store shopping. For the 2-hour delivery and same-day, $\beta_{01} = -2.00$ and $\beta_{02} = -2.00$ respectively. For the drone service $s = 1$, we assume that its inherent attractiveness $\beta_{00} = -2.2$. Inherent attractiveness for the drones is selected be more negative than other online services due to other reasons such as ease of use etc. Schmid et al. [62] estimate the average value of travel time $VOTT = \frac{\beta_{tt}}{\beta_{tc}} = \$40.0/\text{hr}$ and travel cost sensitivity $\beta_{tc} = 0.035$. As such, we estimate travel time sensitivity $\beta_{tt} = VOTT \times \beta_{tc} = 40.0 \times 0.035 = 1.4$. Similarly, the study estimates the value of delivery time $VODT = \frac{\beta_{wt}}{\beta_{dp}} = \$6.5/\text{day} = \$0.27/\text{hr}$. We therefore estimate delivery time (in hours) sensitivity $\beta_{wt} = 0.27 \times \beta_{dp}$. Note that in [62] different delivery times are used, the sensitivity to delivery price and delivery time cannot be used as reported in the study. To calculate these parameters, note that in the utility function (3.11), the only unknown parameter is sensitivity with respect to service charge β_{dp} . Based on the grocery market, and assuming that currently only 2-hour and same-day service are available to the customers, we select β_{dp} such that the market share captured by the e-retailer is equal to online grocery market in the US. Based on our calculations, $\beta_{dp} = 0.34$ and $\beta_{wt} = 0.092$. We assume demand elasticity $\lambda = 0.5$.

We believe a survey or e-retailer’s transaction data is required to reduce errors in estimating demand model parameters. We refer the readers to [24] for other survey-based and POS data-based approaches to estimate parameters in the market share models. Nonetheless, our work is mainly focused on studying the effect of drones under different settings, the calculated parameters provide a reasonable benchmark for sensitivity analyses.

5.2. Analysis of the results

For the base case scenario, we analyze the optimal network configuration with and without drones. The optimal solution is illustrated in Figure 5.2. In the absence of drone service, only one facility is opened offering 2-hour (denoted by 1) and same-day (denoted by 2) delivery service. The same-day service is available in all customer zones while the shaded region in Figure 5.2(a) shows the region where 2-hour service is available. Figure 5.2(b) illustrates the optimal network configuration when the drone service is made available. Three facilities are opened each offering drone service where the blue shaded region depicts coverage of the drones. The optimal facility location without drones is no more optimal when drones are included. This shows that offering drone service not only requires e-retailers to open more facilities but it would also require the relocation of existing facilities. The 2-hour service is now available at two facilities and even though some customers are not being served using drones but now 2-hour service is available to them. This is depicted by red circles in Figure 5.2(b). Without drones, 2-hour service is available to 95% of the customer zones. However when the drone service is included, 2-hour service covers 98% of the customer zones.

In the presence of drones, more facilities need to be opened resulting in higher facility costs. However, the drones are able to cover their high facility costs through increased

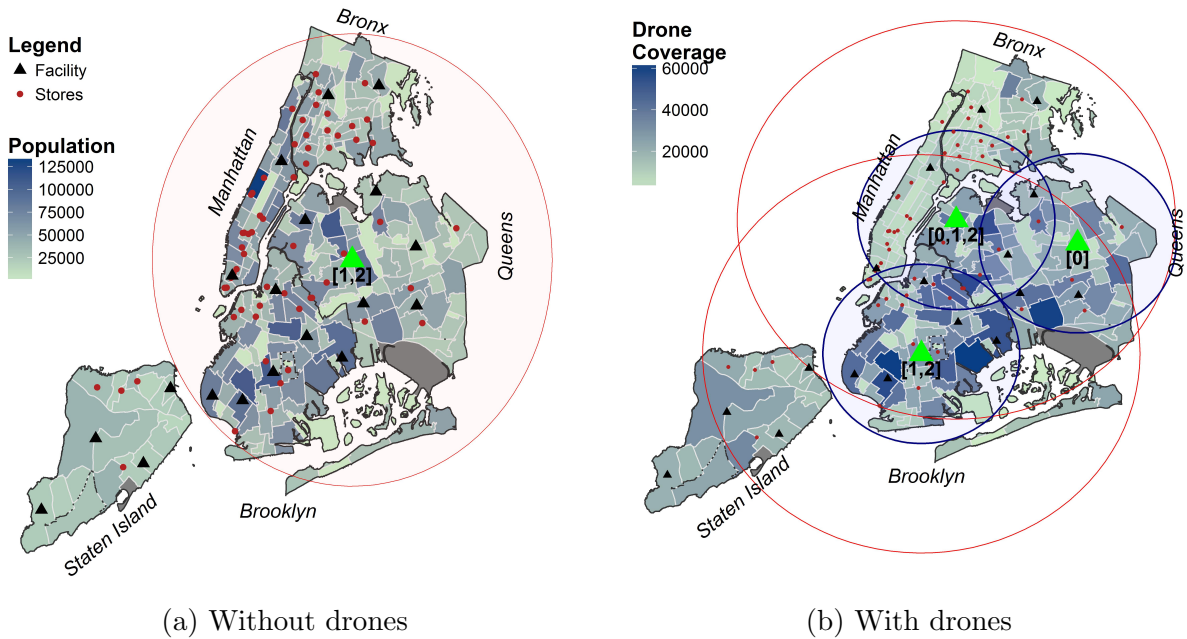
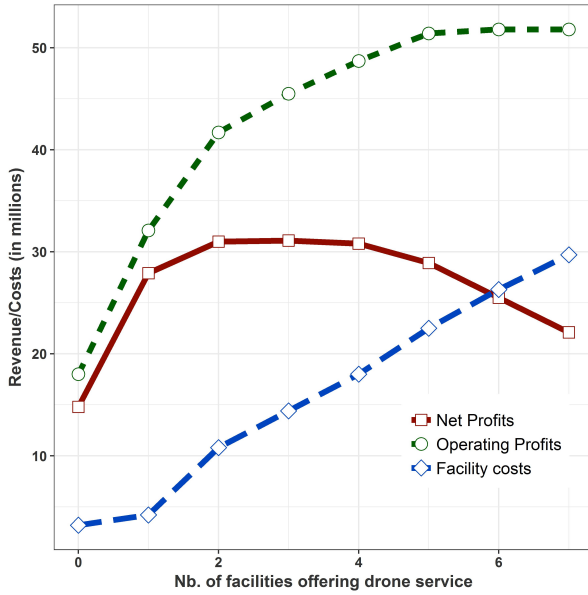


Figure 5.2: NYC optimal network configuration

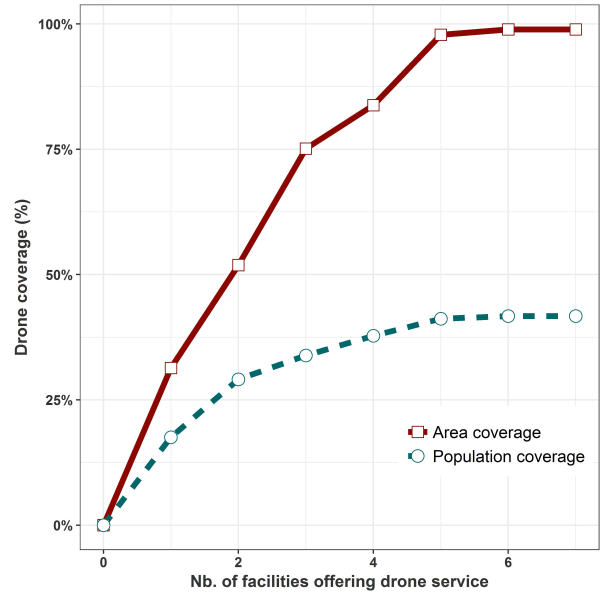
demand as shown in Figure 5.3(a). Similarly, Figure 5.3(b) shows that as the number of facilities increases, drone coverage increases but at a decreasing rate. Opening more facilities may increase drone coverage in terms of land area, but population coverage can not exceed 41% due to technological limitations like building size and package weight. Figure 5.3(b) shows that in terms of area coverage, five facilities are enough to cover 99% of NYC. In the next section, we present detailed sensitivity analysis to study the effect of different parameters on facility locations.

5.2.1 Effects of competition

In this section, we analyze the effect of drones under different demand characteristics. We study the effect of customer sensitivity travel time and travel cost on the added value by



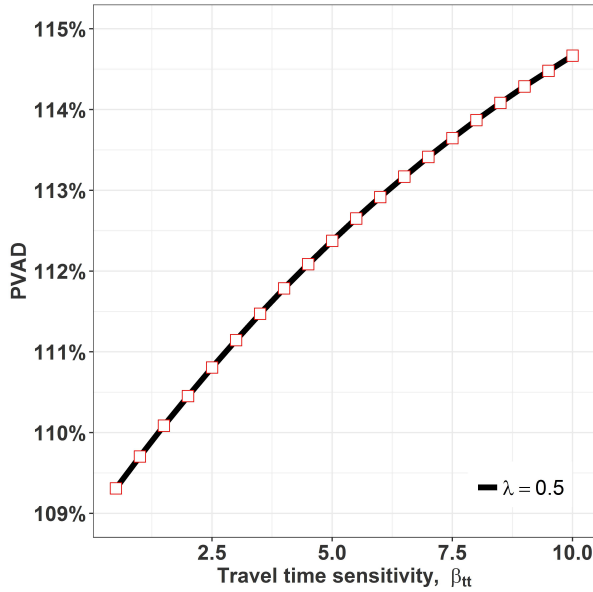
(a) Operating Profits vs facility costs



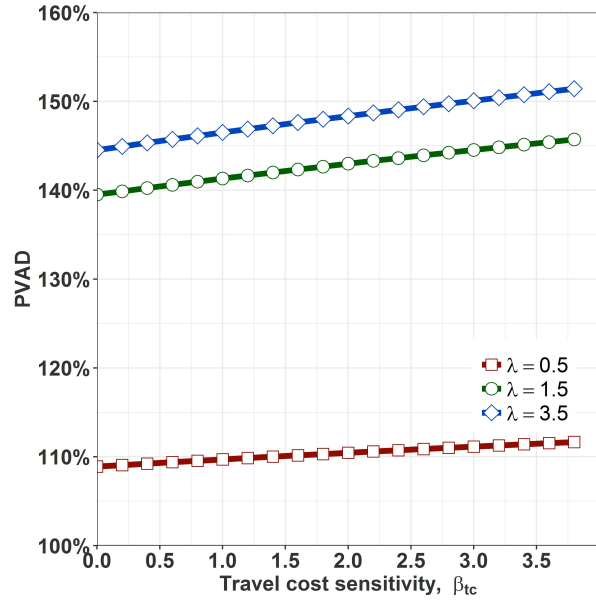
(b) Drone coverage

Figure 5.3: Trade-offs between net revenue, costs and coverage

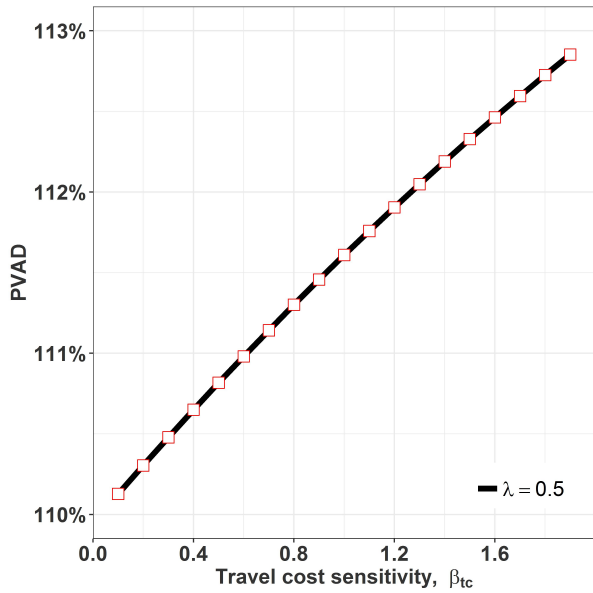
the drones as shown in Figure 5.4. We calculate the percentage value-added by drones (PVAD) as the percentage increase in e-retailer’s profits when drone service is offered. When customers are more sensitive to travel time and travel cost, the PVAD increases. This is shown in Figure 5.4(a) and 5.4(c). When customer sensitivity to travel time and travel cost increases, the utility derived by in-store shopping decreases. As such, customers are more willing to shop online. In the presence of the drone delivery, the total utility derived by customers increases and e-retailer is able to capture higher demand. This leads to capturing demand which could have been lost if the drone service is not available. This implies that the drone service is more favorable when the stores are farther away from customer locations. Figures 5.4(b) and 5.4(d) show the effect of demand elasticity on PVAD. When the demand is inelastic, the market is able to capture a greater proportion of the maximum demand which leads to increased PVAD. We also observe that PVAD is more



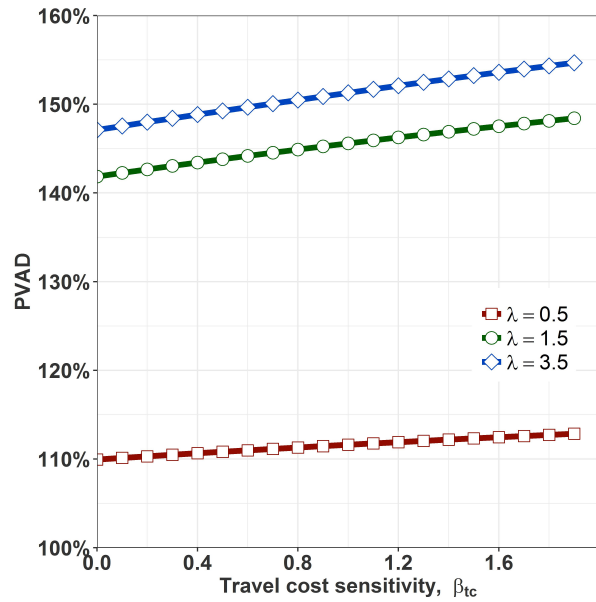
(a) Travel time sensitivity β_{tt} at $\lambda = 0.5$



(b) Effect of β_{tt} under inelastic demand



(c) Travel time cost β_{tc} at $\lambda = 0.5$



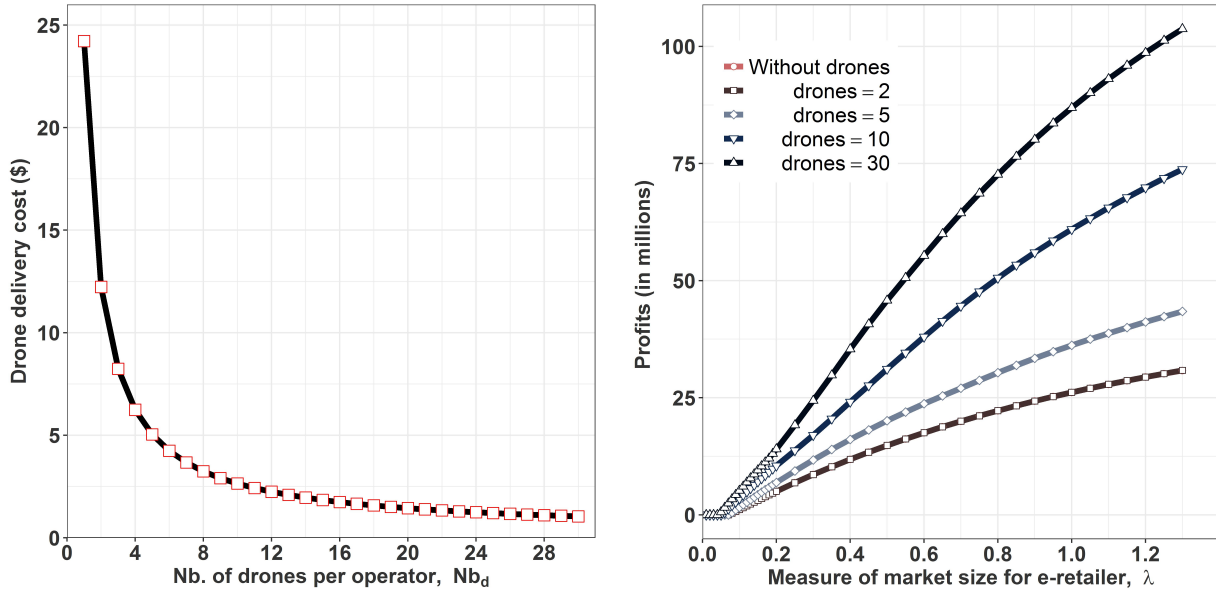
(d) Effect of β_{tc} under inelastic demand

Figure 5.4: Effect of the competition on percentage value-added by drones

sensitive to store competition for inelastic demand. For instance, when β_{tt} changes from 0.0 to 3.8, the change in PVAD $\Delta PVAD = 3\%$ at $\lambda = 0.5$. For $\lambda = 3.5$, $\Delta PVAD = 7\%$. This is because when the demand is inelastic and the customer sensitivity to in-store shopping increases, the demand is not lost but is rather captured by the e-retailer.

5.2.2 Effect of government regulations

Government regulations are expected to play a vital role in determining the future of drones in last-mile delivery. To determine the effect of the regulations on drones, we present sensitivity analysis over different scenarios. As discussed earlier, government regulations may require firms to hire certified operators to monitor drone activity during its flight. However, the number of drones an operator will be allowed to monitor simultaneously is not yet clear. Some reports suggest only 1 to 2 drones per operator will be allowed while others believe this value may be as high as 30. In this section, we study the effect of the number of drones per operator Nb_d on e-retailer's optimal network. Based on the calculations in Table B.3, we estimate drone delivery costs using the number of drones per operator. The effect of Nb_d on drone delivery cost is illustrated in Figure 5.5(a). Note that the regulations can significantly impact the drone delivery costs varying from \$24 to only \$1.



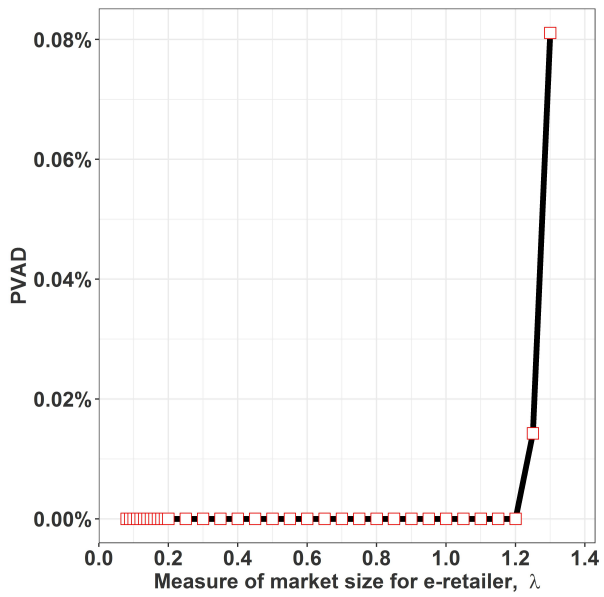
(a) drone delivery cost as a function of Nb. of drones per operator (b) Drones per operator effect under different market sizes

Figure 5.5: Effect of regulations for drones on same day delivery market

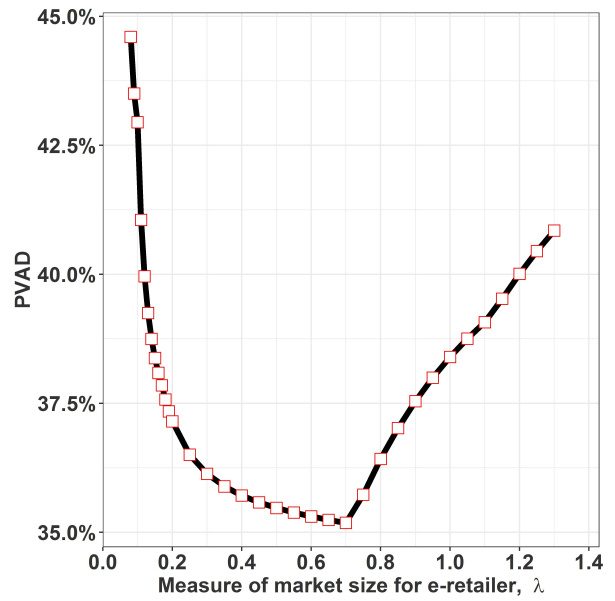
We study the effect of government regulations on an e-retailer’s profits and percentage value-added by drones (PVAD) under different market sizes as shown in Figures 5.5(b) and 5.6. When $Nb_d = 1$, drone delivery costs around \$24, and the drone service is not offered even when λ is significantly high. For $Nb_d = 2$, drone service is only offered when the market size is very large. Such a large value of λ is only possible when the e-retailer has monopoly and the market size is also large. However, the value-added by the drones is still quite low as shown in Figure 5.6(a) where the maximum $PVAD = 0.08\%$. For $Nb_d = 5$, drone service is feasible in all markets where the same-day delivery is already available as shown in 5.7(b). When the government regulations are more restrictive, the drones are feasible for the big e-retailers in urban areas. When the government regulations are less restrictive, the value-added by the drones is significant and the e-retailer opens

more facilities as shown in Figure 5.7(c) and 5.7(d). If the government regulations are less restrictive, drones will allow e-retailers to enter the new small markets as shown in Figures 5.7(c) and 5.7(d). This allows big e-retailers to enter new markets where same day service is not yet available and small e-retailers may start offering same day service in urban areas. It is interesting to note that PVAD is maximum in lower markets when the regulations are less restrictive as shown in Figures 5.6(b), 5.6(c) and 5.6(d). This is due to the fact that the drones allow e-retailer to capture higher demand which enables it to cover their facility costs. As such, drone service allows e-retailer to enter the markets where previously, same day delivery is not feasible due to low demand.

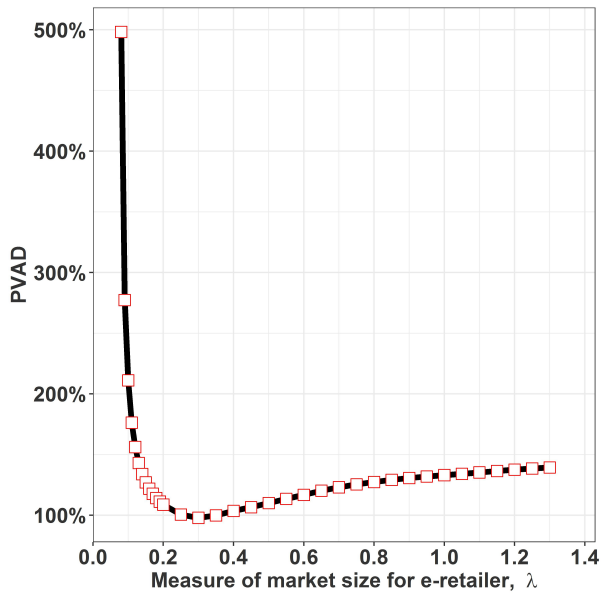
Based on the above analysis, it is observed that government regulations will play a vital role in determining the future of drones. The same day service may extend to the regions with low demand given that the regulations are less restrictive. However, if these regulations are more restrictive, drone service would be restrictive to densely populated areas where the technological limitations would further hinder the value-added by the drones.



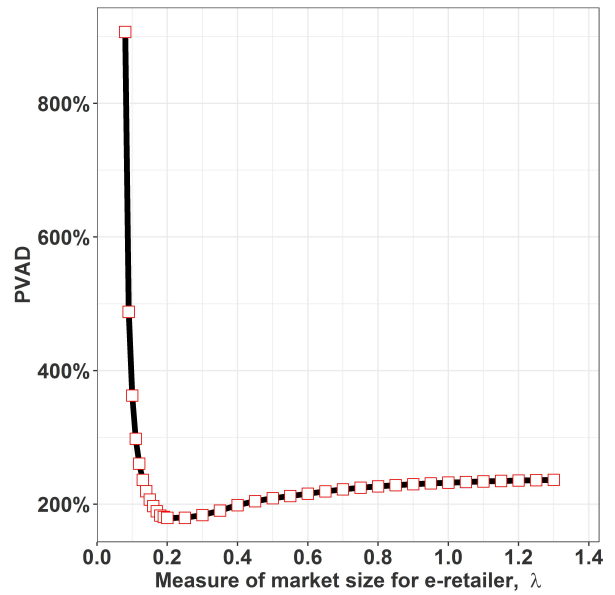
(a) Nb. of drones per operator = 2



(b) Nb. of drones per operator = 5

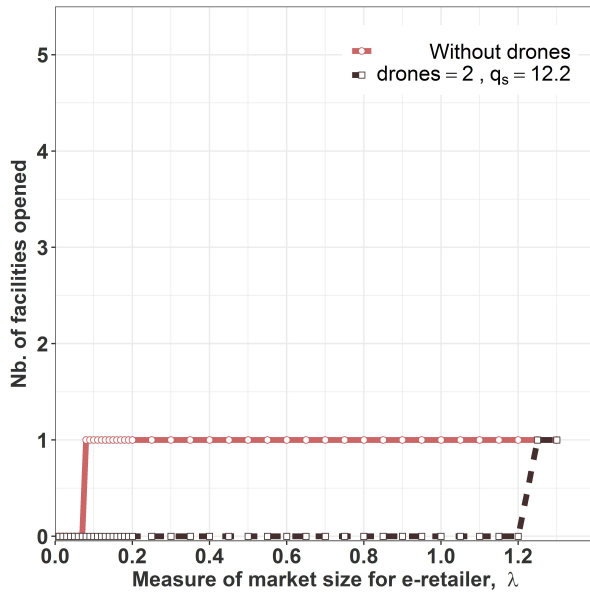


(c) Nb. of drones per operator = 10

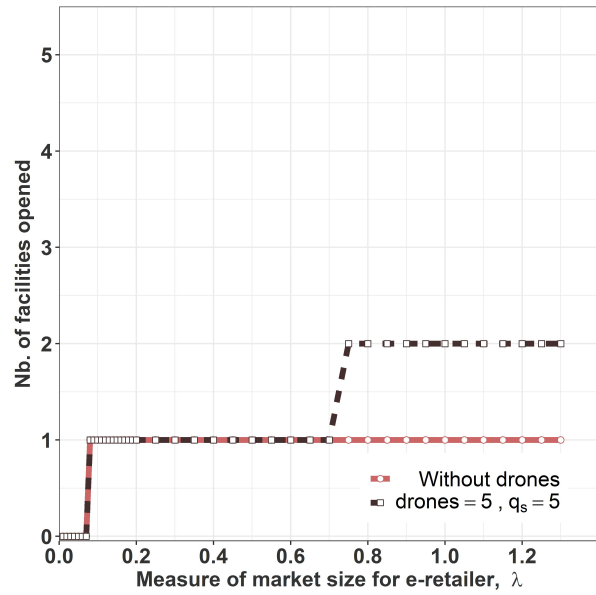


(d) Nb. of drones per operator = 30

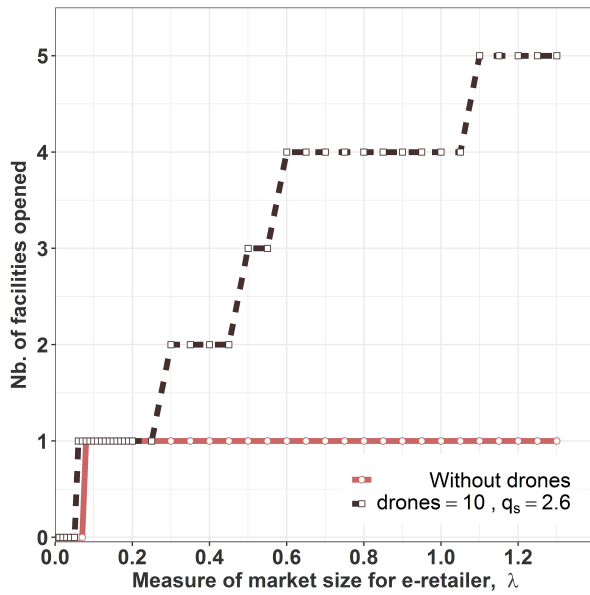
Figure 5.6: Effect of government regulations on PVAD



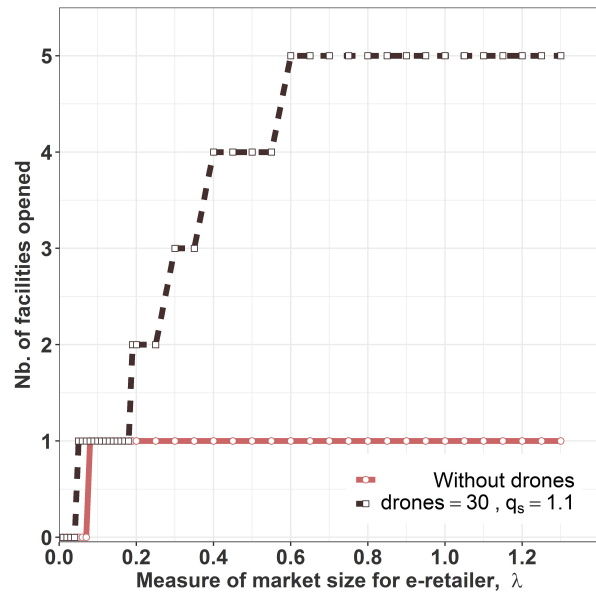
(a) Nb. of drones per operator = 2



(b) Nb. of drones per operator = 5



(c) Nb. of drones per operator = 10



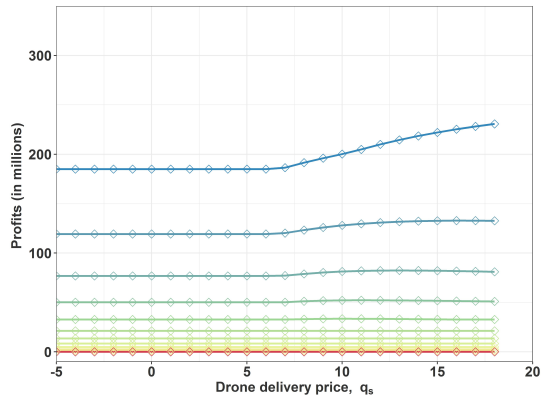
(d) Nb. of drones per operator = 30

Figure 5.7: Effect of government regulations on the number of drones facilities

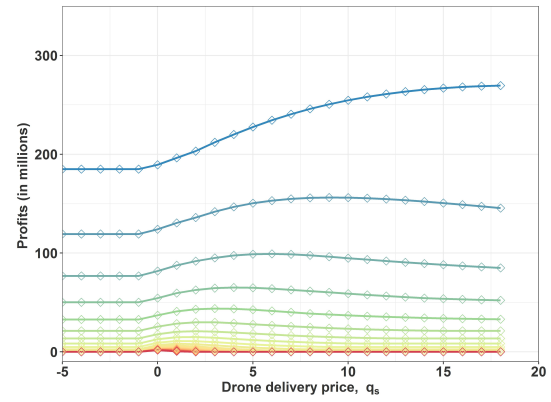
5.2.3 Effects of drone delivery service charge

Government regulations on the number of drones per operator, Nb_d has a significant impact on the drone delivery costs. As such, these regulations play a key role in the pricing decisions of drone service. In this section, we study the effect of pricing or service charge under different values of Nb_d and customer price sensitivity β_p . The service charge of drone service is varied from $-\$5$ to $\$18$ and for the price sensitivity parameter β_{dp} , we use values ranging from 0.05 to 1.00. The negative service charge can be interpreted as discounts offered to customers for using drone service. When Nb_d is high, delivery costs are low which allows the e-retailer to charge lower prices to maximize its demand which in turns leads to higher profits. This is illustrated in Figure 5.8. The optimal pricing decision greatly depends on price sensitivity β_{dp} . When customers are less sensitive to price, the optimal delivery price is high as the customers are more willing to pay. The effect of the pricing decision on e-retailer's profits reduces when the delivery costs are high and price sensitivity is low as shown in Figure 5.8. This is because at higher delivery costs, the e-retailer does not have much margin to vary the price and the customer will have to pay higher service charges. As a result, the demand captured is not significantly improved. In such a scenario, drones will be used to offer a premium service to the customers who are willing to pay high service charge. When the delivery costs are low enough that the e-retailer can offer discounts, added value by drones is maximized. When $Nb_d = 30$, the delivery cost equals $\$1$. The profit margin per delivery in our example equals $\$6$. This allows the e-retailer to offer discounts to capture higher demand leading to higher profits as shown in 5.8(c) and 5.8(d).

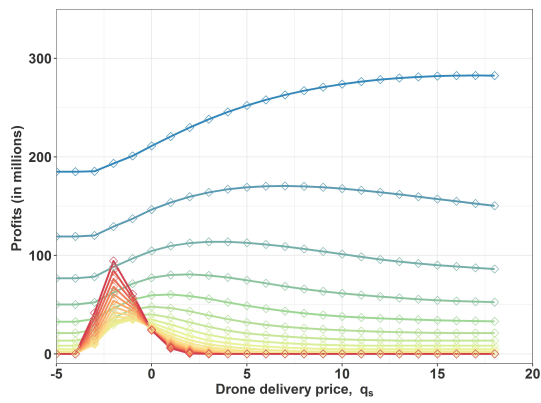
It is interesting to note that value-added by drones is maximized when the customers are very sensitive to price and the government regulations allow 30 drones per operator as



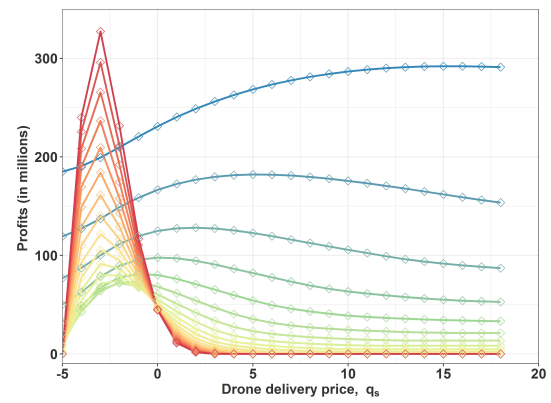
(a) $Nb_d = 2$



(b) $Nb_d = 5$

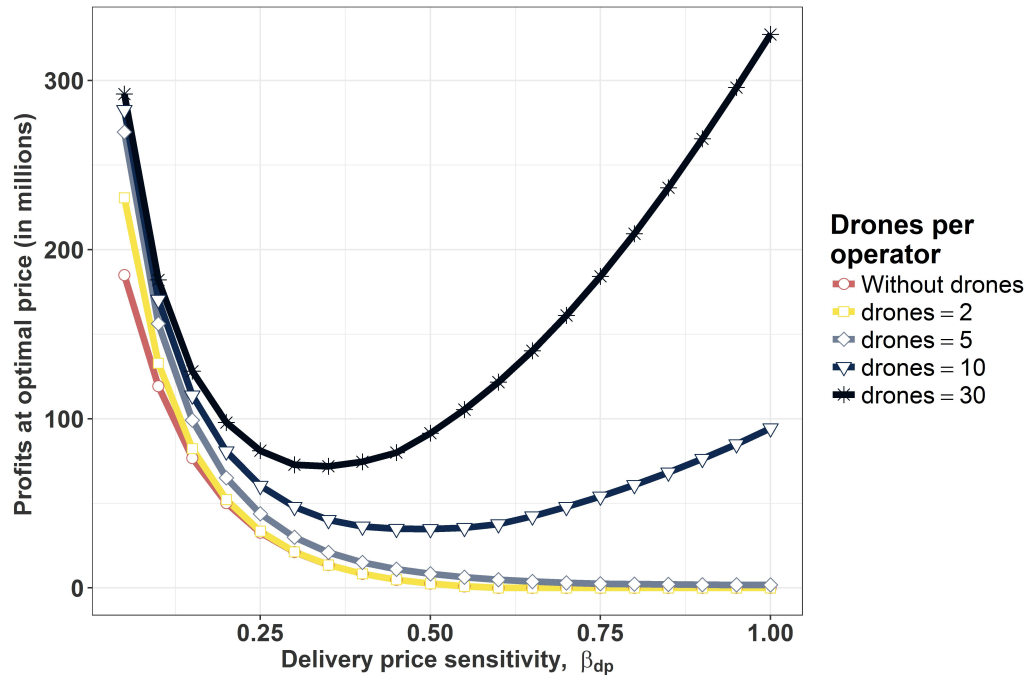


(c) $Nb_d = 10$

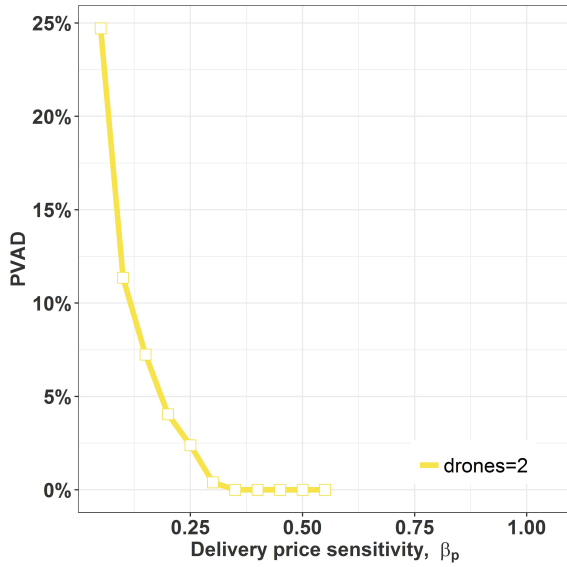


(d) $Nb_d = 30$

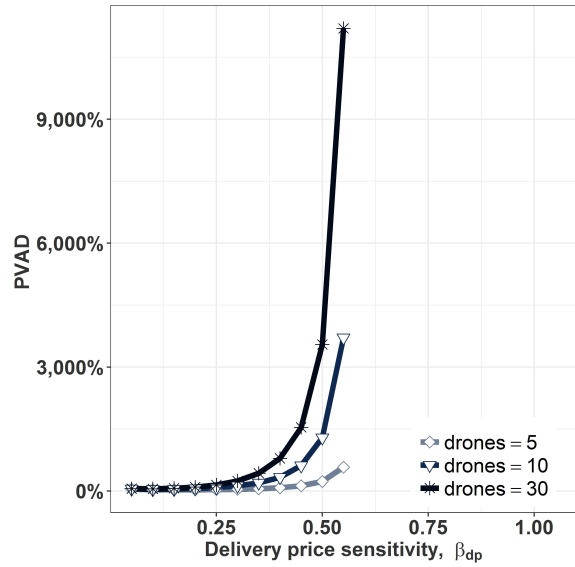
Figure 5.8: Pricing analysis under government regulations and customer price sensitivity



(a) Effect of customer price sensitivity



(b) PVAD for $Nb_d = 2$



(c) PVAD for $Nb_d = 2, 5, 10, 30$

Figure 5.9: Profits under optimal pricing

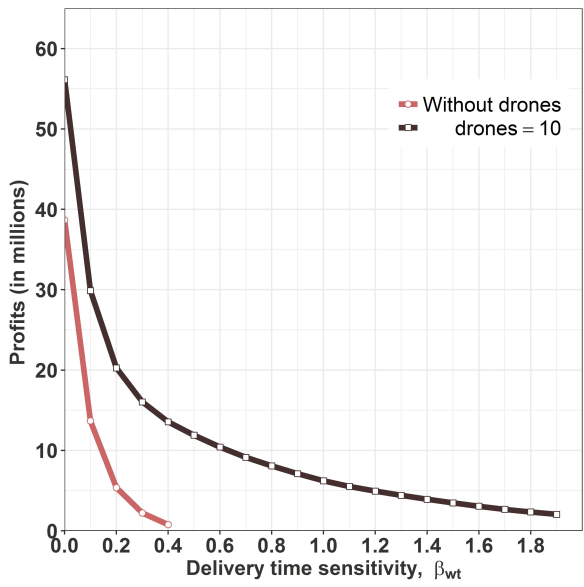
shown in 5.9(a). When customers are more sensitive to price, offering discounts over the retail price significantly increase the demand captured by the e-retailer as the customers now prefer online shopping over in-store shopping. When $Nb_d = 2$, the percentage value-added(PVAD) is maximized if the customers are less sensitive to the price as shown in Figure 5.9(b). However, when the delivery costs for the drones are lower than other services offered by e-retailer, PVAD is maximized when customers are very sensitive to price as shown in Figure 5.9(c).

When government regulations are less restrictive, the e-retailer enjoys low delivery costs which enables it charge lower price to maximize its profits. Increased demand due to low delivery charges would allow the e-retailer to enter new markets with low demand. However, if these regulations result in high delivery costs, drones are used for a premium service available to the customers who are willing to pay higher service charges for a 30-minute delivery.

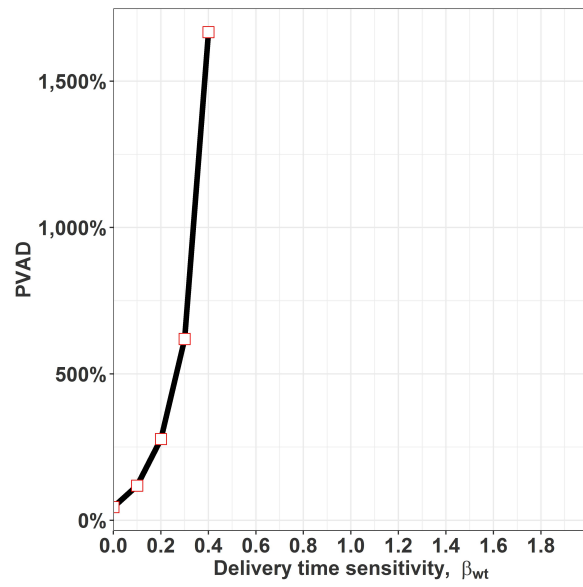
5.2.4 Effect of customer’s delivery time sensitivity

Since drones can fly over congested road networks and only one delivery is made per trip, reduced delivery time is one of the most attractive features of drones. We study customer sensitivity to delivery time on the value-added by the drones. As delivery time sensitivity β_{wt} increases, the demand captured by e-retailer decreases leading to reduced profits as shown in Figure 5.10(c). In the absence of drones, when $\beta_{wt} > 0.4$, the e-retailer does not offer same day service due to low demand. However, when drone service is available, even when $\beta_{wt} = 1.8$, the e-retailer still offers same day service. Due to very low delivery time using drones, the e-retailer is able to capture demand from the time sensitive customers. As shown in Figure 5.10(b), PVAD is maximized when the customers are more sensitive

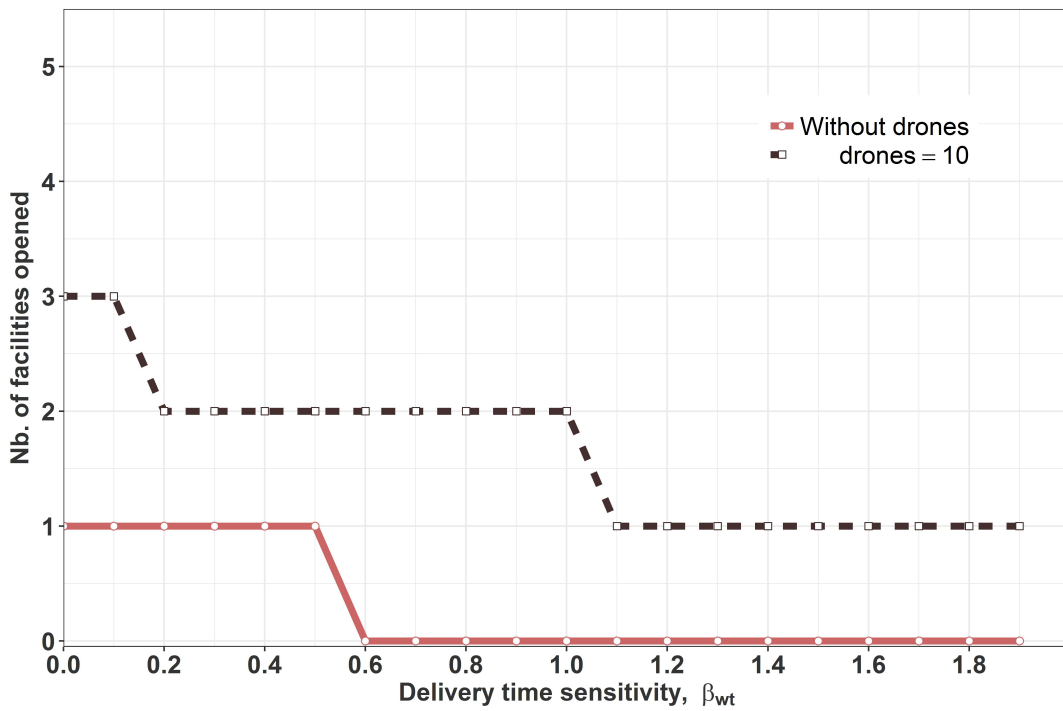
to delivery time. When drones are available, not only is the e-retailer able to capture high demand in the regions where same day service is already available but it would also allow the e-retailer to start offering same day in new markets where the customers are more sensitive to delivery time. Note also that e-retailer's optimal network configuration is more sensitive to customer behavior in the presence of drones as illustrated in [5.10\(c\)](#). Without drones, e-retailer's decision is restricted to either offer the same day service in a given city or not. However, when the drones are introduced, the number of facilities opened is more sensitive to customer behavior. When customers are less sensitive to delivery time, more drone facilities are opened. Fewer facilities are opened when the customer sensitivity is high.



(a) E-retailer's Profits



(b) PVAD



(c) Facilities opened

Figure 5.10: Effect of delivery time sensitivity on drone network

5.2.5 Effect of technological limitations

Currently, drone technology is in the development phase and it can have a significant impact on the distribution network of an e-retailer. In this section, we study the effect of the drone technology on optimal network design. As discussed earlier, we use American community survey data to estimate the percentage of the population living in different types of buildings. Figure 5.11 shows the percentage of the population that has access to drone delivery based on building size and weight of the package being ordered.

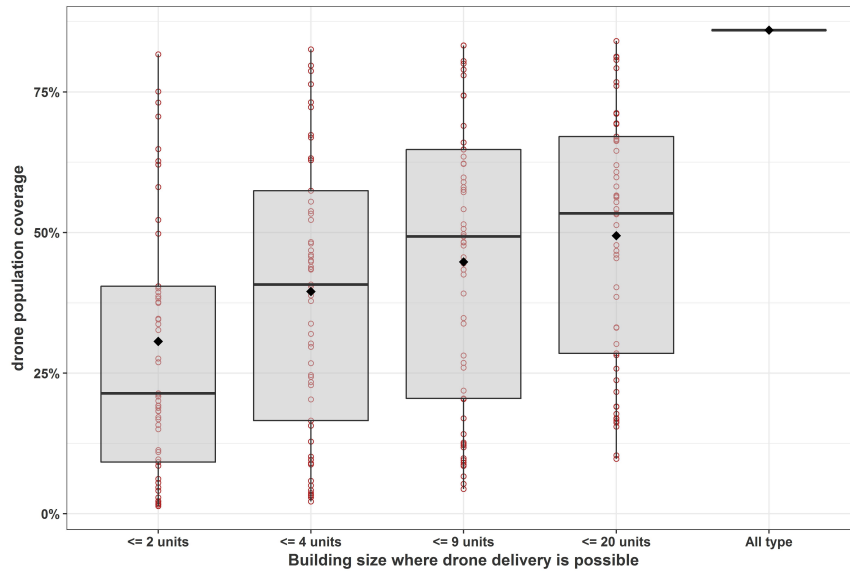
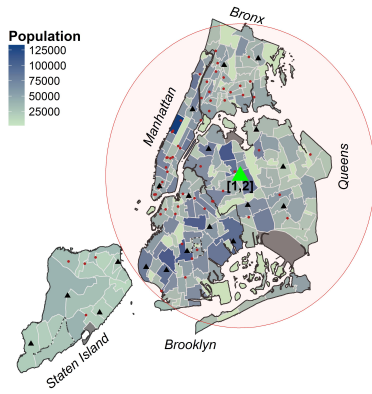


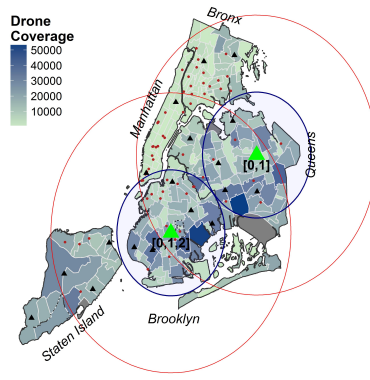
Figure 5.11: Population deliverable by drones vs building size where drone is deliverable

Figure 5.12 shows how the drone technology can effect the optimal network configuration. If the drone technology restricts delivery to only homes (building size ≤ 4), only two facilities are opened. Although, Manhattan region has the highest population density, the drone service is not available when the technology restricts delivery to only homes (size ≤ 4). This is due to the fact that only 5% of the population in Manhattan region

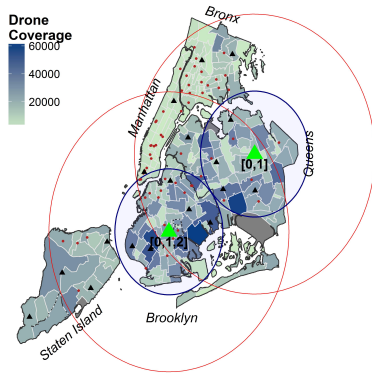
lives in buildings with 4 units or less. This is shown by the light shaded region depicting lower drone coverage [5.12\(b\)](#). In Staten Island, 100% of the population lives in buildings with size 4 units or less. The population density in this region, however, is significantly low and hence it is not served. This shows that under technological limitations, drones are more favorable in sub-urban areas. Urban areas usually have tall buildings limiting drone coverage while rural areas do not have enough demand to cover facility costs associated with drone delivery. If the drone can serve to buildings with higher number of units, Manhattan region becomes favorable for the drones. When drones can deliver to all types of buildings, 86% of the population can receive drone delivery. As such, the effect of drones is significant as it can capture high demand due to greater coverage. As shown in [Figure 5.12\(f\)](#), four facilities are opened to serve the customers using drones. This shows as advance technology is available, urban areas are more profitable. Rural areas, on the other hand, are not profitable due to low demand which may not cover the facility costs. Our analysis also shows that Brooklyn and Queens are the most favorable boroughs for the drones due to high population and small building sizes. Technologically advance drone system would not only increase the drone's population coverage but it also impact the location and coverage of other services offered by the e-retailer. This analysis shows that the technology has a significant effect on the added value by drones and optimal network configuration with drones.



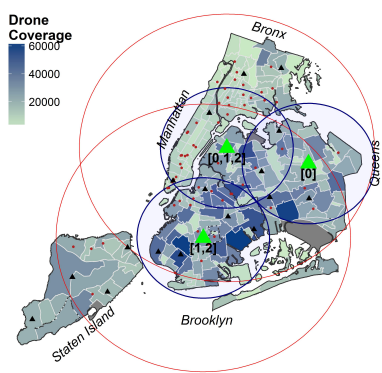
(a) Without drones



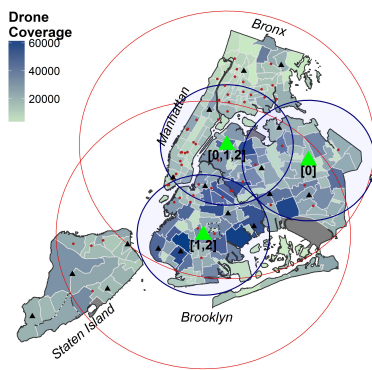
(b) building size ≤ 2



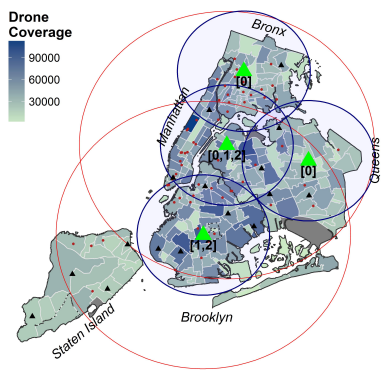
(c) building size ≤ 4



(d) building size ≤ 9



(e) building size ≤ 20



(f) All type buildings

Figure 5.12: Effect of the type of buildings where drone can deliver

Chapter 6

Computational results

We conducted several experiments to study the efficiency of the LAB algorithm by randomly generating nodes over a 1600 km² square region. The number of customer nodes or demand points $|I|$ are varied between 50 and 150 (in steps of 50). For the candidate facility locations, we varied it between 10 and 40 (in steps of 10). To compute the maximum demand (N_{ip}), each customer zone $i \in I$ and the package $p \in P$ is assigned a weight ω_{ip} from a uniform distribution $[0,1]$. The demand is then calculated as:
$$N_{ip} = \frac{\omega_{ip}}{\sum_{i \in I} \sum_{p \in P} \omega_{ip}} \times 16 \text{ million.}$$
 The 16 million is the total maximum demand for all customers as calculated in Section 5.1. We assume $\lambda = 10.0$, which is high enough to challenge the algorithm at higher demand values. The facility costs are randomly generated at three levels: low ($[1\text{M}, 2\text{M}]$), medium ($[2\text{M}, 3\text{M}]$), high ($[3\text{M}, 4\text{M}]$). To study the effect of demand, each instance is solved for two different values for the total stores: (10, 100). All other parameters are the same as used for the base case in Section 5.1, if not explicitly stated otherwise. For a given $|I|$, $|J|$, facility costs level, and number of stores, 10 random instances are generated resulting in a total of 720 instances.

To validate the effectiveness of the LAB algorithm, we compare its CPU time with the mixed integer linear formulation of the model NP. The IP formulation is given in Appendix A. The LAB algorithm and IP model are coded in C++ Visual Studio 2013 and solved using CPLEX version 12.6.1 on a 64-bit Windows 10 with Intel(R) core i7-4790 3.60GHz processors and 8.00 GB RAM. Each instance is executed to an optimality gap of 1e-09 or up to 3600 seconds in CPU time.

The results are summarized in Tables 6.1 and 6.2, for which the values are reported as the average of the random instances for each combination of $|I|$ and $|J|$. The number of facilities opened is denoted by $\sum_{j \in J} W_j$. *Iter* denotes the number of iterations carried out by the LAB algorithm. *Gap* refers to the optimality gap: $(UB - LB)/LB$. CPU times are reported in seconds and the ratio of CPU time of IP model and LAB algorithm is denoted by *Time ratio*.

Our algorithm performs exceptionally well compare to the IP formulation. On average, the LAB algorithm is 300 times faster than solving the IP model using B&C. The IP model fails to converge in 25% of the instances while the LAB algorithm is able solve all instances to optimality where maximum CPU time reported is 26 seconds. “n/a” denotes the instance where CPLEX fails to find a feasible solution in 3600s. The IP model fails at three instances to find a feasible solution. This further supports the need for an efficient algorithm to solve large scale problems to optimality. As expected, increasing the number of stores reduces the computational difficulty of the problem due to lower demand captured by e-retailer. As seen in the tables, the problem gets harder when the number of facilities opened increases. This happens when either facility costs are low or the demand captured is higher. The LAB algorithm, however, performs very well under such scenarios. When facility costs are low or demand is high, the number of iterations of the algorithm increases. This is because in such instances, the master problem [LSMP] has incentive to switch to

other solutions. The LAB algorithm may iterate multiple times between master problem and subproblems, but it is still able to outperform the IP model in which all decisions are made simultaneously.

| $ J $ | $ I $ | $\sum_{j \in J} W_j$ | LAB algorithm | | | IP | | Time ratio |
|--|-------|----------------------|---------------|------|----------|------|----------|------------|
| | | | Iter | Gap | CPU time | Gap | CPU time | |
| 10 | 50 | 2 | 3 | 0.00 | 0.31 | 0.00 | 40.61 | 130 |
| 10 | 100 | 3 | 2 | 0.00 | 1.16 | 0.00 | 322.31 | 279 |
| 10 | 150 | 3 | 11 | 0.00 | 3.47 | 0.00 | 552.02 | 159 |
| 20 | 50 | 4 | 3 | 0.00 | 0.97 | 0.00 | 218.23 | 225 |
| 20 | 100 | 5 | 6 | 0.00 | 6.95 | 0.00 | 2256.13 | 324 |
| 20 | 150 | 4 | 3 | 0.00 | 8.47 | 0.00 | 2836.53 | 335 |
| 30 | 50 | 3 | 3 | 0.00 | 1.78 | 0.00 | 486.63 | 273 |
| 30 | 100 | 5 | 14 | 0.00 | 11.39 | 0.25 | 3601.20 | 316 |
| 30 | 150 | 6 | 5 | 0.00 | 12.91 | 0.30 | 3601.23 | 279 |
| 40 | 50 | 6 | 11 | 0.00 | 7.56 | 0.00 | 2676.30 | 354 |
| 40 | 100 | 4 | 3 | 0.00 | 14.23 | 0.16 | 3601.31 | 253 |
| 40 | 150 | 6 | 7 | 0.00 | 22.43 | n/a | 3600.11 | 161 |
| <i>Average F_j : low</i> | | 4.3 | 5.9 | 0.00 | 7.64 | n/a | 1982.72 | 257 |
| 10 | 50 | 3 | 3 | 0.00 | 0.28 | 0.00 | 54.17 | 193 |
| 10 | 100 | 2 | 4 | 0.00 | 1.03 | 0.00 | 161.78 | 157 |
| 10 | 150 | 2 | 2 | 0.00 | 2.06 | 0.00 | 517.80 | 251 |
| 20 | 50 | 2 | 3 | 0.00 | 1.19 | 0.00 | 344.16 | 290 |
| 20 | 100 | 2 | 3 | 0.00 | 2.39 | 0.00 | 539.69 | 226 |
| 20 | 150 | 2 | 5 | 0.00 | 5.95 | 0.00 | 2097.67 | 352 |
| 30 | 50 | 2 | 3 | 0.00 | 1.47 | 0.00 | 829.84 | 565 |
| 30 | 100 | 1 | 2 | 0.00 | 9.70 | 0.00 | 2819.36 | 291 |
| 30 | 150 | 1 | 2 | 0.00 | 10.98 | 0.89 | 3600.92 | 328 |
| 40 | 50 | 2 | 5 | 0.00 | 4.58 | 0.00 | 1666.11 | 364 |
| 40 | 100 | 2 | 3 | 0.00 | 8.95 | 0.56 | 3600.86 | 402 |
| 40 | 150 | 2 | 2 | 0.00 | 23.92 | 0.9 | 3601.89 | 151 |
| <i>Average F_j : Medium</i> | | 1.9 | 3.1 | 0.00 | 6.04 | 0.20 | 1652.85 | 274 |
| 10 | 50 | 1 | 2 | 0.00 | 0.19 | 0.00 | 68.97 | 368 |
| 10 | 100 | 1 | 2 | 0.00 | 0.63 | 0.00 | 280.08 | 448 |
| 10 | 150 | 1 | 2 | 0.00 | 1.84 | 0.00 | 695.19 | 377 |
| 20 | 50 | 1 | 2 | 0.00 | 0.69 | 0.00 | 225.30 | 328 |
| 20 | 100 | 1 | 2 | 0.00 | 1.00 | 0.00 | 413.16 | 413 |
| 20 | 150 | 1 | 2 | 0.00 | 7.20 | 0.00 | 945.52 | 131 |
| 30 | 50 | 2 | 2 | 0.00 | 2.44 | 0.00 | 696.38 | 286 |
| 30 | 100 | 2 | 2 | 0.00 | 9.13 | 0.51 | 3601.08 | 395 |
| 30 | 150 | 1 | 4 | 0.00 | 9.95 | 0.90 | 3602.30 | 362 |
| 40 | 50 | 2 | 3 | 0.00 | 1.80 | 0.00 | 1014.14 | 564 |
| 40 | 100 | 1 | 2 | 0.00 | 12.34 | 0.86 | 3600.66 | 292 |
| 40 | 150 | 1 | 2 | 0.00 | 14.16 | n/a | 3601.36 | 254 |
| <i>Average F_j : High</i> | | 1.3 | 2.3 | 0.00 | 5.1 | n/a | 1562.0 | 351 |

Table 6.1: Results for Number of stores = 10

| J | I | $\sum_{j \in J} W_j$ | LAB algorithm | | | IP | | Time ratio |
|--|-----|----------------------|---------------|------|----------|------|----------|------------|
| | | | Iter | Gap | CPU time | Gap | CPU time | |
| 10 | 50 | 3 | 3 | 0.00 | 0.36 | 0.00 | 57.73 | 161 |
| 10 | 100 | 2 | 3 | 0.00 | 1.16 | 0.00 | 361.39 | 313 |
| 10 | 150 | 3 | 4 | 0.00 | 2.83 | 0.00 | 685.00 | 242 |
| 20 | 50 | 3 | 3 | 0.00 | 1.02 | 0.00 | 229.11 | 226 |
| 20 | 100 | 4 | 2 | 0.00 | 2.70 | 0.00 | 647.44 | 240 |
| 20 | 150 | 3 | 3 | 0.00 | 7.22 | 0.00 | 1560.64 | 216 |
| 30 | 50 | 4 | 5 | 0.00 | 3.50 | 0.00 | 1098.03 | 314 |
| 30 | 100 | 3 | 4 | 0.00 | 7.33 | 0.50 | 3600.59 | 491 |
| 30 | 150 | 2 | 2 | 0.00 | 15.28 | 0.50 | 3600.91 | 236 |
| 40 | 50 | 2 | 3 | 0.00 | 3.13 | 0.00 | 794.44 | 254 |
| 40 | 100 | 3 | 2 | 0.00 | 9.69 | 0.36 | 3606.33 | 372 |
| 40 | 150 | 3 | 6 | 0.00 | 26.41 | n/a | 3600.48 | 136 |
| <i>Average F_j : low</i> | | 2.9 | 3.3 | 0.00 | 6.72 | n/a | 1653.51 | 267 |
| 10 | 50 | 1 | 2 | 0.00 | 0.20 | 0.00 | 54.31 | 267 |
| 10 | 100 | 1 | 2 | 0.00 | 0.72 | 0.00 | 112.31 | 156 |
| 10 | 150 | 1 | 2 | 0.00 | 1.28 | 0.00 | 494.52 | 386 |
| 20 | 50 | 1 | 2 | 0.00 | 1.23 | 0.00 | 226.00 | 183 |
| 20 | 100 | 1 | 2 | 0.00 | 1.89 | 0.00 | 312.03 | 165 |
| 20 | 150 | 2 | 6 | 0.00 | 9.38 | 0.00 | 1750.52 | 187 |
| 30 | 50 | 1 | 2 | 0.00 | 3.81 | 0.00 | 790.50 | 207 |
| 30 | 100 | 2 | 3 | 0.00 | 5.22 | 0.00 | 2017.86 | 387 |
| 30 | 150 | 1 | 2 | 0.00 | 7.73 | 1.03 | 3600.59 | 466 |
| 40 | 50 | 2 | 3 | 0.00 | 3.33 | 0.00 | 1044.17 | 314 |
| 40 | 100 | 1 | 2 | 0.00 | 4.91 | 0.86 | 3601.22 | 734 |
| 40 | 150 | 2 | 2 | 0.00 | 15.88 | 0.80 | 3600.27 | 227 |
| <i>Average F_j : Medium</i> | | 1.3 | 2.5 | 0.00 | 4.63 | 0.22 | 1467.03 | 307 |
| 10 | 50 | 1 | 2 | 0.00 | 0.20 | 0.00 | 47.69 | 235 |
| 10 | 100 | 1 | 2 | 0.00 | 0.52 | 0.00 | 130.39 | 253 |
| 10 | 150 | 1 | 2 | 0.00 | 0.67 | 0.00 | 615.58 | 916 |
| 20 | 50 | 1 | 2 | 0.00 | 1.03 | 0.00 | 170.33 | 165 |
| 20 | 100 | 1 | 2 | 0.00 | 2.75 | 0.00 | 353.31 | 128 |
| 20 | 150 | 1 | 2 | 0.00 | 2.66 | 0.00 | 585.50 | 220 |
| 30 | 50 | 1 | 2 | 0.00 | 1.44 | 0.00 | 476.66 | 332 |
| 30 | 100 | 1 | 2 | 0.00 | 5.31 | 0.00 | 1043.66 | 196 |
| 30 | 150 | 1 | 2 | 0.00 | 8.36 | 0.00 | 3392.33 | 406 |
| 40 | 50 | 1 | 2 | 0.00 | 1.95 | 0.00 | 695.81 | 356 |
| 40 | 100 | 1 | 2 | 0.00 | 7.13 | 0.00 | 2671.27 | 375 |
| 40 | 150 | 1 | 2 | 0.00 | 14.89 | 1.09 | 3600.25 | 242 |
| <i>Average F_j : High</i> | | 1.0 | 2.0 | 0.00 | 3.91 | 0.09 | 1148.56 | 319 |

Table 6.2: Results for Number of stores = 100

Chapter 7

Conclusion & Future Research

In this thesis, we have studied the effect of drones on optimal network configuration. We considered competition between services offered by an e-retailer and stores. We present a nonlinear formulation that incorporates consumer behavior to predict demand for all services offered using the *Multinomial logit (MNL)* market share model. We also presented a novel logic-based Benders decomposition approach to solve the problem to optimality within seconds. The proposed LAB algorithm performs very well over the linear formulation of the original model NP. The efficient LAB algorithm allows us to carry out detailed sensitivity analysis over operational decisions.

The analysis showed that facility location decisions are more sensitive to customer behavior in the presence of drones. In the absence of drones, the e-retailers can either open a single facility or not offer same day delivery service at all. However, in the presence of drones, the overall network's profits greatly depend on the number and location of facilities opened. This is due to the limited range of drones and hence each facility should be opened only if the demand captured covers facility costs. Our analysis also shows

that government regulations play a vital role in determining the target market for drone delivery. When government regulations are highly restrictive, drone delivery costs are high and in such scenario, drone service is only available to the customers who are willing to pay a premium price for 30-minute delivery. However, if government relaxes the number of drones per operator criteria, drones are very cost effective. In such scenario, the e-retailer can maximize its profits by offering discounts over the retail price to capture higher demand and drone service can be used by a greater proportion of the population. It would also allow e-retailers to enter new small markets where same day delivery services are not yet available due to lower demand. Technological limitations also impact the drone network. In densely populated areas such as Manhattan, added value by drones is not significant due to lower population coverage as majority of the population lives in apartments. Areas with low population density (e.g., Staten Island) have housing units that are more favorable for drone service. However, such areas do not have enough demand to cover fixed costs associated with drones.

The work presented in this thesis has some limitations that need to be addressed in future. We believe that a good market survey is required to estimate sensitivity parameters used in demand model. In the future, we plan to develop a solution methodology where pricing and delivery times are considered as decision variables for all types of services. A possible extension of our work might be to incorporate charging stations to extend the range of drones at the expense of higher delivery times. Another possible extension of the model could be to consider capacitated facility location problem and incorporate inventory holding costs. Nevertheless, the study presents managerial insights for optimal network design in the presence of delivery-by-drones. This study also provides insights for an e-retailer making strategic decisions such as facility location and service selection, along with operational decisions such as pricing and delivery time.

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APPENDICES

Appendix A

Linear formulation of model NP

Three sets of binary decision variables and one set of continuous nonnegative decision variables are defined as:

$$t_{ie} = \begin{cases} 1, & \text{if customer zone } i \in I \text{ is offered set of services } e \in E \\ 0, & \text{otherwise} \end{cases}$$

$$w_j = \begin{cases} 1, & \text{if candidate facility } j \in J \text{ is opened} \\ 0, & \text{otherwise} \end{cases}$$

$$x_{js} = \begin{cases} 1, & \text{if service } s \in S \text{ is offered at facility } j \in J \\ 0, & \text{otherwise} \end{cases}$$

$$d_{ijspe} = \begin{cases} \text{demand captured by facility } j \in J \text{ using service } s \in S \text{ for package } p \in P \text{ in} \\ \text{customer zone } i \in I \text{ when set of services offered is } e \in E \end{cases}$$

The model NP is transformed into mixed integer program [IP] as:

$$[\text{IP}]: \max \sum_{i \in I} \sum_{j \in J} \sum_{s \in S} \sum_{p \in P} \sum_{e \in E} (\alpha \pi_p + q_s - c_{ijs}) d_{ijspe} - \sum_{j \in J} \sum_{s \in S} F_s x_{js} - \sum_{j \in J} L_j w_j \quad (\text{A.1})$$

$$\text{s.t.} \quad \sum_{e \in E} t_{ie} = 1 \quad \forall i \in I, \quad (\text{A.2})$$

$$x_{js} \leq w_j \quad \forall j \in J, s \in S, \quad (\text{A.3})$$

$$d_{ijspe} \leq M t_{ie} \quad \forall i \in I, j \in J, s \in S, p \in P, e \in E, \quad (\text{A.4})$$

$$d_{ijspe} \leq M r_{ijs} x_{ijs} \quad \forall i \in I, j \in J, s \in S, p \in P, e \in E, \quad (\text{A.5})$$

$$\sum_{j \in J} d_{ijspe} \leq \bar{D}_{ispe} \quad \forall i \in I, s \in S, p \in P, e \in E, \quad (\text{A.6})$$

$$t_{ie} \in \{0, 1\} \quad \forall i \in I, e \in E, \quad (\text{A.7})$$

$$x_{js} \in \{0, 1\} \quad \forall j \in J, s \in S, \quad (\text{A.8})$$

$$w_j \in \{0, 1\} \quad \forall j \in J, \quad (\text{A.9})$$

$$d_{ijspe} \geq 0 \quad \forall i \in I, j \in J, s \in S, p \in P, e \in E. \quad (\text{A.10})$$

Appendix B

Data used

| Borough | Minimum | Maximum |
|---------------|---------|---------|
| Manhattan | 70 | 100 |
| Staten Island | 14 | 21 |
| Brooklyn | 65 | 75 |
| Queens | 14 | 25 |
| Bronx | 30 | 40 |

Table B.1: Yearly Lease Rate (\$) per sq. ft in Boroughs

| ID | Borough Name | NTA code | Yearly Cost (\$) |
|----------------|---------------|----------|------------------|
| 1 | Brooklyn | BK82 | 5,000,000 |
| 2 | Queens | QN49 | 2,450,000 |
| 3 | Queens | QN01 | 3,000,000 |
| 4 | Queens | QN18 | 2,700,000 |
| 5 | Staten Island | SI11 | 2,750,000 |
| 6 | Staten Island | SI24 | 2,600,000 |
| 7 | Brooklyn | BK28 | 5,300,000 |
| 8 | Bronx | BX13 | 3,600,000 |
| 9 | Staten Island | SI45 | 2,650,000 |
| 10 | Bronx | BX06 | 3,550,000 |
| 11 | Staten Island | SI37 | 2,450,000 |
| 12 | Manhattan | MN24 | 5,550,000 |
| 13 | Queens | QN41 | 2,700,000 |
| 14 | Brooklyn | BK31 | 5,000,000 |
| 15 | Queens | QN53 | 3,000,000 |
| 16 | Manhattan | MN11 | 6,100,000 |
| 17 | Brooklyn | BK81 | 5,300,000 |
| 18 | Brooklyn | BK42 | 5,250,000 |
| 19 | Brooklyn | BK72 | 5,150,000 |
| 20 | Queens | QN70 | 2,650,000 |
| Average | | | 3,837,500 |

Table B.2: Yearly facility Costs

| Parameters | Estimated Value |
|---|--|
| Yearly demand of delivery-by-drones | DD |
| Avg hourly demand, H | $\frac{DD}{365 \times 24}$ |
| maximum hourly demand, $H^{max} = H \times 2.0$ | $\frac{DD}{365 \times 24} \times 2.0$ |
| Nb. Of drones required, $NbDrones = \frac{H^{max}}{2}$ | $\frac{DD}{365 \times 24} \times 2 \times \frac{1}{2}$ |
| Nb.of batteries required, $NbBatteries = NbDrones$ | $\frac{DD}{365 \times 24} \times 2 \times \frac{1}{2}$ |
| Nb. Of drones per operator, $DronesPerOperator$ | 10 |
| Nb.of operators required per hour $NbOperators = \frac{H^{max}}{DronesPerOperator}$ | $\frac{DD}{365 \times 24} \times 2 \times \frac{1}{10}$ |
| Nb.of hours an operator works | 8 |
| Total Nb. of operators required, $TotalNbOperators = NbOperators \times \frac{24}{8}$ | $\frac{DD}{365 \times 24} \times 2 \times \frac{1}{10} \times \frac{24}{8}$ |
| Costs calculations | |
| Cost per drone | 3000 |
| Cost per battery | 200 |
| Total battery cost, $BC = 200 \times NbBatteries$ | $\frac{DD}{365 \times 24} \times 2 \times \frac{1}{2} \times 200$ |
| Total drone cost, $DC = 3000 \times NbDrones$ | $\frac{DD}{365 \times 24} \times 2 \times \frac{1}{2} \times 3000$ |
| Total purchasing cost, $PC = BC + DC$ | $(3000 + 200) \times \frac{DD}{365 \times 24} \times 2 \times \frac{1}{2}$ |
| Amortization rate, r | 20% |
| Yearly amortized cost of drones & Batteries, $AC = r \times PC$ | $(3000 + 200) \times \frac{DD}{365 \times 24} \times 2 \times \frac{1}{2} \times 0.20$ |
| Yearly operators salary | 70,000 |
| Total yearly operators salary cost, $OC = 70000 \times NbOperators$ | $\frac{DD}{365 \times 24} \times 2 \times \frac{1}{10} \times \frac{24}{8} \times 70000$ |
| Total yearly costs, $TC = AC + OC$ | $(3000 + 200) \times \frac{DD}{365 \times 24} \times 2 \times \frac{1}{2} \times 0.20 + \frac{DD}{365 \times 24} \times 2 \times \frac{1}{10} \times \frac{24}{8} \times 70000$ |
| Battery charging cost per delivery (in dollars) | 0.10 |
| Cost per delivery, $\frac{TC}{DD}$ | $\left(\frac{(3000+200) \times \frac{DD}{365 \times 24} \times 2 \times \frac{1}{2} \times 0.20 + \frac{DD}{365 \times 24} \times 2 \times \frac{1}{10} \times \frac{24}{8} \times 70000}{DD} \right) + 0.10$ |
| | $= 0.117 + 2.397 + 0.10 \approx \mathbf{2.61}$ |

Table B.3: Estimation of delivery by drone cost per package