Strategic Trip Planning: Striking a Balance Between Competition and Cooperation

by

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A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Doctor of Philosophy
in
Electrical and Computer Engineering

Waterloo, Ontario, Canada, 2018

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

In intelligent transportation systems, cooperative mobility planning is considered to be one of the challenging problems. Mobility planning as it stands today is an individual decision-making effort that takes place in an environment governed by the collective actions of various competing travellers. Despite the extensive research on mobility planning, a situation in which multiple behavioural-driven travellers participate in a cooperative endeavour to help each other optimize their objectives has not been investigated. Furthermore, due to the inherent multi-participant nature of the mobility problem, the existing solutions fail to produce ground truth optimal mobility plans in the practical sense - despite their claimed and well proven theoretical optimality.

This thesis proposes a multi-module team mobility planning framework to address the team trip planning problem with a particular emphasis on modelling the interaction between behaviour-driven rational travellers. The framework accommodates the travellers' individual behaviours, preferences, and goals in cooperative and competitive scenarios. The individual behaviours of the travellers and their interaction processes are viewed as a team trip planning game. For this game, a theoretical analysis is presented, which includes an analysis of the existence and the balancedness of the final solution.

The proposed framework is composed of three principal modules: cooperative trip planning, team formation, and traveller-centric trip planning (TCTP). The cooperative trip planning module deploys a bargaining model to manage conflicts between the travellers that could occur in their endeavour to discover a general, satisfactory solution. The number of players and their interaction process is controlled by the team formation module. The TCTP module adopts an alternative perspective to the

individualized trip-planning problem in the sense that it is being behavioural driven problem. This allows for multitudes of traveler centric objectives and constraints, as well as aspects of the environment as they pertain to the traveller's preferences, to be incorporated in the process. Within the scope of the team mobility planning framework, the TCTP is utilized to supply the travellers with personalized strategies that are incorporated in the cooperative game. The concentration problem is used in this thesis to demonstrate the effectiveness of the TCTP module as a behavioural-driven trip planner.

Finally, to validate the theoretical set-up of the team trip planning game, we introduce the territory sharing problem for social taxis. We use the team mobility framework as a basis to solve the problem. Furthermore, we present an argument for the convergence and the efficiency of a coarse correlated equilibrium. In addition to the validation of a variety of theoretical concepts, the territory sharing problem is used to demonstrate the applicability of the proposed framework in dealing with cooperative mobility planning problems.

Acknowledgments

All praise is due to Allah for granting me the strength and knowledge to complete this work. I would like to express my deep and sincere gratitude and appreciation to my supervisor professor Otman Basir for his continuous encouragement, helpful discussion, and insightful guidance throughout the research presented in this thesis.

I would like to express my sincere gratitude to the members of my doctoral committee, Dr. Baher Abdulhai, Dr. Mohamed O. Damen, Dr. Carl Haas, and Dr. Catherine Gebotys for investing time in reading and providing many valuable comments on my thesis.

The Libyan Ministry of Education is highly appreciated for sponsoring me during my study. I am also grateful to Intelligent Mechatronic Systems (IMS) Inc. for allowing me to develop part of my work in an industrial setting.

Last and by far not least, I am indebted to my parents, brothers, and sisters for their continuous support, patience, and prayers. I owe a great deal of gratitude to my wife and my children for their continuous understanding, unlimited encouragement, and unending love during the years of my study.

Dedication

To my mother and father,

my wife and my children,

Al-Hasan, Muhammad, and Omar...

Table of Contents

List of Tables

List of Figures

Nomenclature

$\Delta_{tr_i,t}$ The gain/cost incurred by driver tr_i at instance t

$\hat{P}$ An alternative set of strategies other than the preferred one.

ϑ Retaliation/threat factor.

U The utility cost of the grand coalition.

η The state of the environment and corresponds to a value computed based on trip attributes

Γ_{tr_i} Travellers' doctrine set: a selfish assessment process which assigns a non-negative value to each road segment

κ_r Traffic consistency index

λ, μ Smoothness parameters such that $\lambda > 0$ and $\mu < 1$

ω Concave function that is differentiable over $p_j^{tr_i}$'s such that $p_j^{tr_i} \in P^{tr_i}$

ϕ_r Road comfort index

Π Game referee in the Territory Sharing Game

Σ The trip planning game

Σ^*	Trip planning sub-game.
σ_r	Road safety index
τ_r	Journey time
core	The core of a game representing the solution set of strategic assignments
A^{tr_i}	The set of possible actions within a game
A_r	Set of road attributes
$c_{p_k}^{tr_i}$	Cost value associated with each strategy assigned by a traveller
$D_{R_{p_k}}^{tr_i}$	The mobility planning reward
f^{tr_i}	The destination point of a trip
l_i	Road segment i
P^{tr_i}	Sets of chosen paths/strategies by a traveller
p^{tr_i}	A chosen path by a traveller and a strategic action
$p_{Opt}^{tr_i}$	The optimum path/strategy for a traveller
$r_{Opt}^{tr_i}$	Optimal route
S	A coalition of travellers
S^{GND}	The grand coalition that includes all travellers.
s^{tr_i}	The source point of a trip
tr_i	Traveller in the game

v_σ Characteristic function for a game

ATIS Advanced Traveller Information System

FIS Fuzzy Inference System

R(s^{tr_i},f^{tr_i}) is a region of interest

TCTP Traveller-Centric Trip Planner

TM Transportation modality

TMP Team Mobility Planning

Chapter 1

Introduction

The trip planning problem has become a pivotal consideration of modern daily activities. The process of mobility planning often includes mobility resource selection and cost estimation. Factors such as temporal budgets and anticipated arrival times are examined to determine whether the trip is successful. In this sense, trip planning can be perceived as an individual endeavour. Nevertheless, due to its nature, the trip planning problem is a competitive process in which multiple individuals compete for the same resources. Each decision made by a traveller has an impact on the system and affects other travellers. Hence, there is a need to simultaneously address both facets of this problem.

This chapter presents an overview of the trip planning problems and discusses the motivation and objectives of this research work. The chapter concludes with the outline of this thesis.

1.1 Overview and Motivation

Various solutions have been proposed to approach the problem of mobility planning, ranging from Personal Navigation Devices (PNDs) to the various navigation applications now available for smart devices [1, 2]. These solutions approach the mobility planning problem from a single traveller's perspective. They plan trips independent of the mobility actions and the decisions of other travellers. Thus, they fail to produce an optimal solution in a practical sense.

The main shortcoming of existing mobility planning solutions, with few exceptions such as the work in [3], is that they do not consider the trip planning decisions made by other travellers. To a large extent, this is caused by their perception of mobility planning as an individual behaviour-driven and time-constrained process; for example, this can be observed in the work presented in [4–7]. This view has arisen due to certain technological limitations pertaining to areas such as communications between travellers, data exchange protocols, and the scarcity of the computational power. Although many of these constraints have been reduced or even eliminated, the perception of mobility planning as an individual effort has persisted. Based on this view of planning, it is natural for conflicts to arise among the various trip plans produced by different travellers. These conflicts are best demonstrated by examples of road congestions, intensive road traffic, inefficient parking facilitation, among others. There are numerous situations in which the relationship between travellers is reduced to a competitive process. This competitive process has the potential to be counterproductive to the overall objective of efficient mobility.

Even though the trip planning problem is viewed as an individualized problem, the awareness of its multi-participant nature is practically evident and observed. Regular changes in network conditions often occur due to the decisions made by other travellers. For instance, in dangerous weather conditions, safety becomes a significant concern. Certain routes are closed and others have warnings advising travellers to seek alternatives. As a consequence, many travellers may arrive at the same decision regarding the safest path given their perceived knowledge of the impact of weather on their commuting options. This unawareness of others' decisions may transform what initially would have been considered a safe path into one with increased risk due to congestion. In other words, due to the fact that roads are inherently shared resources, the individualized choice of alternatives has the potential to worsen the situation for all network users.

The above-mentioned scenario of traffic management during weather anomalies signifies the need for a group-centric method in which drivers/travellers coordinate among themselves to obtain a robust optimal decision. Crowd coordination and cooperation is one of the next logical steps of informed trip planning. Modern vehicles are equipped with various means of communication. With applications such as collision avoidance, vehicles can communicate to facilitate cooperation. Furthermore, once a communication channel is established, travellers can share their intentions, desires and future plans in real time.

The technological advancements in fields of communication, localization, and computational processes have allowed the travellers to engage in a cooperative trip planning. For example, social based navigation solutions, such as WAZE, in which travellers communicate among themselves to enhance a multitude of performance factors, have gained adoption due to their better informed trip navigation guidance [8]. These technological improvements and social changes have enabled improvements in trip planning. Crowdsourced data is another example of the transition from traditional trip planning to a more dynamic and personalized approach, as can be seen in the study presented in [4].

Moreover, in the various intelligent transportation systems, there are several examples of team trip planning. The vehicle routing problem for a group of taxis exemplifies a case in which the number of taxis or moving vehicles are optimized to match the required demand and the available routes. Nevertheless, much of the conducted work on this problem has regarded the vehicles as a part of a fleet that is managed ultimately as a single body, and therefore the individual decisions of the drivers are not considered. Another example is that of traditional taxi companies, in which the dispatcher makes the routing decisions for all drivers [9]. Furthermore, in reality, the majority of the taxis are in fact free agents who are executing their own agendas and do not necessarily abide by the commands received from their dispatcher. The best example of a situation in which car drivers are behaving as free agents is that of online transportation network companies. Other applications can be observed in the field of ad-hoc commercial advertisements, election campaign volunteers, and snowplough contractors, among others. The common factor among all these applications is the issue of coordinating between different agents who are aiming to solve different problems such that their independent actions do not negatively impact each other. Therefore, regardless of the application, there is a fundamental need to devise a framework to solve the team trip planning problem.

The primary challenging aspect of solving the team trip planning problem is the lack of a comprehensive platform that captures the needs of the travellers while facilitating a team approach to the problem. Satisfying the needs of the many competing agents in any process is a complex task. Furthermore, for travellers to cooperate, they must be first motivated. For example, the knowledge that their individual gains would be greater through cooperation than if they were to act selfishly can be an effective motivational factor.

The main goal of this thesis is to develop a multi-traveller framework that can facilitate cooperation between travellers so that all travellers can achieve their mobility objectives, subject to individual constraints and strategies. This mobility goal attainment recognizes the potential for conflict between the mobility demands of the different travellers and as such, attempts to strike a balance between these demands to ensure optimal resource utilization. In order to realise this goal, I develop a multi-traveller Team Mobility Planning (TMP) framework that approaches the trip planning problem in the form of a game. A game theoretic approach is developed to manage the problem of cooperation between conflicting travellers. This planning problem is referred to as the team trip planning game.

Game theory is chosen due to its ability to capture the complex dynamics of the team trip planning problem. The individual travellers can be viewed as players, their chosen plans can be formulated as strategies, and the problem can be formulated as a non-cooperative trip planning problem that revolves around the attainment of an equilibrium. The game-theoretic model can be used to improve the personal outcomes for the travellers and the overall state of the system. Furthermore, under appropriate conditions, these travellers are provided with the necessary tools for cooperation. The outcome of such cooperation may be similar to or even better than their non-cooperative outcomes. Hence, to solve the team trip planning problem, it is imperative that it is addressed through both disciplines of game theory: the cooperative and the non-cooperative. In this thesis, travellers are assumed to be able to engage in a collaborative problem solving discourse in which they compete for resources and collaborate to accomplish their individual goals.

1.2 Research Objectives

The research reported in this thesis has three goals: 1) to consider the trip planning problem as a cooperative game for which a solution model must be designed, 2) to consider the trip planning problem as an individualized problem that is approached as a non-cooperative game, and 3) to merge both concepts of cooperation and competition into the solution. These three goals can be achieved by meeting the following objectives:

1. Perform an extensive background survey and literature review on trip planning. The survey should cover the various game theory-based trip planning problems and solutions attempts .
2. Develop a game formulation for the team trip planning problem that encompasses the individual and team aspects of the problem. The mathematical formulation of the problem should facilitate the development of framework for solving the problem.
3. Develop a multi-module team mobility framework to solve the team trip planning problem. To address the aspects of cooperation and competition, this framework should include a traveller-centric trip planning module as well as a cooperative trip planning module.
4. Study the balancedness of the trip planning game and develop an argument regarding the existence of a solution for this game.
5. Develop a bargaining model to solve the game theoretic part of the cooperative trip planning problem. Conduct experimental scenarios that demonstrate the performance of the developed bargaining model and demonstrate its efficiency.
6. Develop a traveller centric-trip planning (TCTP) system that can be used to produce personalized strategies for the team trip planning game.
7. Use the TCTP system as a method to analyze and solve the concentration problem as a non-cooperative trip planning game.
8. Formulate a territory-sharing trip-planning game that can be used to demonstrate the cooperative and competitive aspects of the game.

9. Use the proposed framework to demonstrate that by using the team mobility framework, a solution for the cooperative and the non-cooperative team planning game exists and can be found.

1.3 Thesis Organization

This thesis is composed of the following chapters:

Chapter 1 presents the motivation behind the research work as well as its objectives.

Chapter 2 provides a literature review of various topics related to the research.

Chapter 3 presents a formulation for the team trip planning problem. It presents a model formulation and guidelines for the developed solution. A multi-module framework named Team Mobility Planning (TMP) is introduced in this chapter.

Chapter 4 discusses the balancedness of the trip planning game and develops an argument regarding the existence of a solution for this game. It also presents the bargaining based solution model for the team trip planning game. Experimental scenarios are presented to demonstrate the performance of the proposed solution model.

Chapter 5 presents a traveller-centric trip planning module and discusses the effect of its deployment on the concentration problem and the welfare of the traffic system.

Chapter 6 presents the territory sharing game as a case study to demonstrate the effectiveness of the team mobility planning framework. The notion of regret-models and coarse correlated equilibrium is also discussed in this chapter.

Chapter 7 provides a summary of the contributions of this research work and discusses area for future research.

Chapter 2

Background and Literature Review

2.1 Introduction

Trip planning is a multifaceted research topic and many of its variants found in the literature remain challenging problems. The Travelling Salesman Problem (TSP) and the Vehicle Routing Problem (VRP) are good examples. Moreover, trip planning is inherently concerned with routing and path finding. Routing and path finding are presented in various fields, such as transportation, communication, and networking. Thus, to better understand the trip planning problem, there is a need to study its variants along with the various solutions and algorithms developed over the past decades.

Predominantly, different trip planning problems have been regarded as pure optimization problems, for which various solution algorithms have been developed. Additionally, game theory, as a method of problem modelling, was successfully deployed to solve the trip planning problems. This chapter discusses trip planning in terms of both disciplines of operational research and game theory, with particular emphasis on the latter.

In order to deduce the method of finding a solution, pure optimization approaches develop the (Multi) objective(s) function and problem constraints. In the game theoretic approach, the analysis process is more complicated. In addition to the need to define the objective function and constraints, it is necessary to define the game as cooperative or non-cooperative. Once a game is defined, many issues and challenges, such as the existence and the stability of the solutions, need to be addressed. Furthermore, the different problem formulations, whether pure optimization or game theory, share similarities even if the problem formulations appear to be different. Because of these similarities, the chapter covers many trip planning problems in terms of formulation, solution model development, existence, stability, and convergence analysis. A comprehensive review and understanding of the recent research activities in this area is presented. The following topics are covered in this chapter:

- A brief review of the trip planning problem and some of its variants as combinatorial problems.
- Game theory: a background study on game theory is provided in order to properly understand the theory and introduce relevant terminologies that are particular to this field of study.
- A description of cooperative and non-cooperative trip planning games is presented. Several examples of such games are also discussed.
- Two prominent examples of cooperative and competitive planning games are reviewed: bargaining games and congestion games.
- The challenging issues of trip planning that have motivated the research of this thesis are presented.

The literature work related to the various aspects of this research work can extend beyond the scope of this chapter. Therefore, in situations when the need arises, the appropriate related research will be covered in the other chapters.

2.2 Trip Planning as Optimization Problem

Due to its inherent importance in the daily activity of individuals and organization, trip planning has become an active area of research, particularly in recent years. Trip planning is largely discussed in the literature as an optimization problem in which objective functions and constraints are formulated and utilized in the search for an optimal solution. In other words, trip planning can be viewed as an optimal routing problem.

The most well-known trip planning problem is the shortest route problem. In this problem, a single traveller aims to minimize his/her trip cost as the trip starts from a known pre-determined starting point to a known pre-determined ending point. For a directed graph $G(V, E)$, the shortest path problem can be formulated using linear programming as the following:

$$\text{minimize} \sum_{i,j \in E} C_{i,j} x_{i,j} \tag{2.1}$$

subject to:

$$x_{i,j} > 0 \tag{2.2}$$

$$\sum_{i,j \in E} x_{i,j} - \sum_{i,j \in E} x_{j,i} = \begin{cases} 1, \text{ if i = S;} \\ -1, \text{ if i = D;} \\ 0, \text{ otherwise.} \end{cases} \tag{2.3}$$

where $C_{i,j}$ is the cost of using link $i \to j$, S, D are the source and destination, and $x_{i,j}$ is the decision variable.

One of the most widely-known techniques to solve this problem is Dijkstra's algorithm [10]. Dijkstra's algorithm searches for the shortest path in a graph-modulated map. In general, a cost is associated with each link in the graph, and the route with the minimum cost is chosen. The parameters based on which the cost is calculated differ depending on the nature of the problem.

Another trip planning problem is the vehicle routing problem (VRP). In VRP, the objective is to coordinate a fleet of vehicles in order to determine an optimal route. The VRP can be modified to accommodate a number of constraints, one of which is a time window (VRPTW). Much of the research work on this problem has been directed at developing the best algorithm for solving this problem in a reasonable time [11–14]. The VRPTW is a multi-objective problem that can be used for formulating and solving the trip-planning problem. Many techniques have been proposed in the literature for solving the VRPTW, such as the ant colony technique, in which the use of pheromones enables a fleet of vehicles to cooperate in order to determine the optimal route. In the implementation of the ant colony approach, an ant represents a vehicle. The goal is that through the use of pheromones, the fleet of vehicles will learn which routes effectively minimize the number of required vehicles and the total cost of the trip [15]. Genetic algorithms (GAs) provide a heuristic approach for solving the VRPTW. With GAs the solution space is represented as chromosomes, and at each generation, two parents mate based on specific criteria such as fitness-based selection [16]. For example, in [16] a two-phase GA approach was proposed, in which each chromosome represents a cluster of routes, and the first gene in the chromosome represents the first customer to be served. In the VRPTW, there are multiple participants with multiple objectives.

The Travelling Salesman Problem (TSP) is also considered to be one of the most famous optimization trip planning problems [17]. The most interesting aspect of the TSP as a trip planning problem is that there are numerous problems that can be described similarly to the TSP and can thus be approached in a similar manner. For instance, in [18], Vansteenwegen *et al.* describe a tourist trip planning problem as one in which tourists attempt to travel from a starting point to an end point while crossing specific points of interest within a certain time window. The optimization solution described in [18] can be used to solve this tourist trip planning problem. This problem, which shares similarities with the TSP, is called the Orienteering Problem (OP); however, the traveller is not required to traverse all points of interest.

Another well-known routing problems is the max flow problem, first described in [19]. This problem discusses the issue of maximizing the flows in networks given their capacity. The problem can be stated as the following: given a graph $G(V, E)$, flow f, and link capacity L_c, what is the greatest achievable flow in the network given its capacity?

In the context of vehicle routing, trip planning has received significant attention. A vast proportion of the research focused on routing decisions is concentrated on trip planning based on trip times that are fixed [10] or variant [20]. When other parameters are considered, such as safety, comfortability, or monetary budgets, most of the existing research has been directed towards the consideration of each of these requirements as a separate objective. Hence, for all of these parameters, the routing problem is solved as a multi-objective problem. In [21], Blue *et al.* proposed the use of a bi-objective path search approach for in-vehicle routing. The first objective was to minimize the trip time, while the second objective was to minimize the complexity. Complexity is viewed with regard to lane change, merge and weaving movements such that driving straight ahead has 0 complexity index and performing a U-turn has a complexity index of 0.5. The final decision with regard to the best route is made by performing trade-offs between trip time and trip complexity. Similar work for cyclist routing with a bi-objective function is found in [22], in which monetary and time based objective functions are considered separately for cyclist route choice. In [23], Raith *et al.* viewed trip times and toll costs as separate route choice objectives and aimed to develop heuristic algorithms to obtain a solution for the Multi-Objective Traffic Assignment (MTA) problem. Trip times and toll cost values are considered as cost functions. Iteratively, the shortest and the longest link with positive flow are found for every route and are equilibrated by shifting some of the flow from the longest link to the shortest link until their travel times are equal and the solution is found. Even though the main concept in the aforementioned research was to formulate the routing problem as a multi-objective problem, the actual research only involved the use of a bi-objective problem. Similarly, in [24] Duque *et al.* propose a bi-objective exact algorithm that aggressively prune dominated solutions while at the same time minimizing the trip cost and trip time objectives.

The research studies discussed thus far have the drawback of formulating each cost function, or demand that might arise, as a separate objective function. For a multitude of demands, the optimization problem could become computationally infeasible. A sensible approach is to find a generalized cost function that accommodates many primary cost functions, such as toll cost and trip complexity, and then the optimization function will have to manage single or a limited number of objective function(s). Such an approach is called multi-criteria based vehicle routing. For example, in [25] Chen *et al.* have presented a generalized cost function in which a weighted sum of time and toll cost functions across possible road segments is computed.

An additional problem found in the reviewed literature is that travellers make their decisions to incur a change in the state of the network. This change does not consider the decisions made by others that will also change the state of the environment (i.e., it is entirely possible that the factors on which basis a decision is made are changing as well, which could lead to results that are not optimal). To resolve this problem of conflicting interests and actions, trip planning and its variants have been studied using game theory.

2.3 Game Theoretical Representation of Trip Planning

In the pure optimization approach, multiple-planning decisions are made without consideration for their effects on each other. Game theory can be employed to represent the individual traveller's decision based on pure optimization as well as the consideration of other travellers. In other words, game theory provides mathematical tools to analyze situations in which there are several decision-makers with conflicting interests that lead them to compete, or mutual benefits that causes them to cooperate [26]. If these decision-makers were to compete among themselves to gain access to resources, the non-cooperative game theory assists a decision-maker in establishing the optimal system design in terms of individual planning (i.e., choosing strategies), as well as in terms of infrastructure design.

In terms of infrastructure design, it could be argued that constructing more roads and bridges will improve the overall travelling performance. Nevertheless, an approach that guarantees effective planning is needed. Game theory provides the necessary analytical tools to monitor and assist with the upgrading process [27]. These tools and functionalities not necessarily as available or as powerful when used with traditional pure optimization approaches. Furthermore, in an environment that permits the communication between travellers, cooperation in planning is an intuitive approach that can be observed in the day-to-day practices. The notion of team working is an integral part of the cooperative game theory formulation. For travellers, depending on their geographical location and their previous experiences, knowledge sharing can enhance the overall trip planning process. Game theoretical representation provides the necessary tools to formulate and manifest a cooperative trip planning approach.

Additionally, game theory provides a variety of methods through which trip gains or costs can be distributed among travellers within the same team in a rational and efficient manner. Travellers, through the utilization of game theory, improve their chances of paying less or gaining more by joining a group than by acting alone. The next section provides a background review on game theory.

2.4 Background on Game Theory

Modern game theory studies can be traced back to the early years of the twentieth century in Zermelo's work in [28] (translated in [29]), Von Neumann's work in [30], and most notably in Von Neumann and Morgenstern's seminal work in [31]. A number of definitions have subsequently been proposed on the matter of games and game theory. Osborn and Rubinstein define a game in [32] as "a description of strategic interaction that includes the constraints on the actions that the players can take and the players' interests, but does not specify the actions that the players do take." In [26], Myerson defines game theory as "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers." These definitions serve the purpose of this chapter, as the majority of the work presented hereafter is based on these concepts.

Games are generally categorized into two types: non-cooperative and cooperative [32]. Non-cooperative games are those that involve the analysis of one player's best response given the other players' anticipated actions. The situation in which no player wishes to unilaterally change his/her decision is called a state of equilibrium.

In non-cooperative games, various equilibria can be used to represent the solution. The best-known equilibrium is Nash's equilibrium. Nash's equilibrium exists in a game when each player has a correct expectation of the other players' behaviours and acts rationally. Player are behaving rationally, as indicated in [31], when they make decisions in order to maximize their gains and satisfy their preferences.

A Nash equilibrium always exists whenever there is a game that is convex and has continuous and semi-concave set of utility values [32]. However, in strictly competitive games, such as the zero sum games, a pure Nash equilibrium is non existent. In games with pure strategies, all strategies, and associated utilities, are deterministic, (i.e., a player knows which strategy to use for any particular situation). When players are permitted to have mixed strategies, which are defined through a probabilistic distribution over pure strategies and their associated utilities instead of finding a deterministic payoff in the game, the expected payoff is statistically computed. In general, for n-players in a game, a mixed Nash equilibrium always exists [31]. Nevertheless, even if the existence of an equilibrium is established, the challenge truly lies in how to find this equilibrium.

Non-cooperative games are mostly concerned with choosing the best strategy in a game in order to find an equilibrium state. Although information about the other players' previous actions, preferences, and expected payoffs are largely known, not every equilibrium correspond to the optimum outcome. In non-cooperative games, due to various "bad" strategic choices, players may ultimately arrive at a bad equilibrium. On the other hand, in cooperative games, players can review their possible strategies and outcomes. Once a situation is found in which everybody benefits, an enforced binding agreement between players is established.

The principal challenge in cooperative games is to find a fair allocation for the joint cost or profit. The set of all feasible allocations is called the core. The core was first defined for n-players participating in non zero-sum games by Gillies in [33]. The process of core formulation needs to satisfy aspects of efficiency and rationality for all players. Furthermore, issues such as core balancedness require attention in the process of finding and establishing the final agreement between players. Players, for whom a fair allocation is sought, are grouped in teams named coalitions. Coalitions can be a set of one or more players. The set with all player is called the grand coalition.

2.5 Non-Cooperative Trip Planning

Non-cooperative trip planning games provide a powerful analysis tool, through which an understanding is gained of not only the direct trip planning decision in terms of goal achievement, but also the effect these decisions might have on the surrounding environment as well as the other involved travellers. One of the earliest non-cooperative trip planning theoretic formulations is the Wardrop equilibrium [34]. In this game, no traveller can obtain a better journey time by changing routes. In his work, Wardrop addressed the issue of redistributing the flow of traffic among alternative routes, which led to two criteria. The first criterion, (i.e., the Wardrop equilibrium), states that "The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route." The second criterion states that the average journey time for all journeys is minimum.

The problem addressed by the Wardrop equilibrium is stated as follows: given the flow value Q and the constants b_1, b_2,....,b_D, p_1, p_2,....,p_D for route i, where p_D and b_D are constants related to the road attributes which routes should be taken to obtain the same similar average journey time, t, on all routes and what the value of t.

The journey time, t_i, for the route i is computed as the follows:

$$t_i = \frac{b_i}{1 - \frac{q_i}{p_i}} \tag{2.4}$$

where q_i is a part of the flow Q such that

$$\sum_{1}^{D} q_i = Q \tag{2.5}$$

For $b_1 < b_2 < b_3 .... < b_D$, for route i, b_i is the journey time if the additional flow on that route, q_i, is 0. For j routes in which $t > b_j$ and $t \geq b_{j+1}$, if all these routes are in use, t is always greater than b_i. Thus, for route i

$$q_i = p_i(1 - \frac{b_i}{t}) \text{ where } i = 1, 2, 3, ..., j \tag{2.6}$$

For an *appropriate* t, Q can be computed for j routes as the following:

$$Q = \sum_{i=1}^{j} p_i - \frac{1}{t} \sum_{i=1}^{j} p_i b_i \tag{2.7}$$

In order to solve for several values of t, we can compare the different Q values against the various t values. The value that corresponds to the given Q is the solution of the problem.

The Wardrop equilibrium is based on the assumption that travellers are not co-operating among themselves to find the solution. In this sense, the Wardrop equilibrium can be considered as a special case of Nash equilibrium. Fisk, in [35], noted that Wardrop equilibrium is identical to Nash equilibrium, such that it considers the traveller as a player in a routing game. In [36], Smith altered Wardrop equilibrium by stating that "A traffic distribution is a Wardrop equilibrium when no driver has a less costly alternative route."

2.5.1 Non-Cooperative Networks: Selfish Routing

Road networks in which travellers are sharing resources and selfishly attempting to improve their trip times are referred to as non-cooperative networks. Selfish routing games provide a valuable insight into non-cooperative games. In the next section, non-cooperative networks and selfish routing are discussed in detail.

2.5.1.1 Non-Cooperative Networks

In [37], Feldmann *et al.* studied a non-cooperative network with parallel links connecting one source to one destination. Each traveller has a strategy that, based on certain probabilistic distribution, chooses the route that minimizes their cost. The cost in this network is the maximum latency of the feasible links. The routing problem in [37] is treated as a scheduling problem that does not initially seek an equilibrium. Nonetheless, the authors present an algorithm for Nashification- the process of converting a non-equilibrium solution into a Nash equilibrium. The Nashification algorithm suggests that the user should perform selfish greedy routing in every link until an equilibrium is found. This algorithm comes with the cost of having an exponential execution time. Once the Nashification process is implemented, the research in [38], which discussed the same game setup, can be used to establish the uniqueness and the complexity of the resulted equilibrium. In [38], Fotakis *et al.* provide a proof of existence for pure Nash equilibria for routing games and provide an algorithm to compute the equilibria in polynomial time.

In [39], a measure of performance labelled as the social cost is defined as the expectation of maximum latency caused by the traffic. This measure is considered as a distinguishing factor between the various selfish routing models proposed in the literature. (i.e., the value of this measure indicates the goodness of any non-cooperative trip planning game). In [40], a hybrid model is proposed in which Wardrop equilibrium and the model presented in [39] are used. Their hybrid model is similar to the one in [39] except for the *social cost*, which corresponds to the sum of the expected individual costs. The equilibrium that has the highest social cost is considered as the worst Nash equilibrium in the game. The ratio of the highest and lowest social cost is denoted as the price of anarchy [41, 42].

There are several issues that are usually investigated when analyzing non-cooperative networks. The intuitive approach to reducing the journey times might lie in expanding the infrastructure, such as constructing roads, bridges. Regardless of the financial feasibility of this solution, it is not a guaranteed strategy. Braess paradox describes a situation in which adding resources to a network may in fact cause the travel times to increase. In [43], Steinberg and Zangwill have shown that the Braess paradox is a well likely phenomenon and it might happen under certain conditions. Non-cooperative networks are discussed as methods of analysis of the different possible situations in the attempt to find the system's equilibrium. The decision-maker could exploit this knowledge to produce a policy guaranteeing a successful planning. For instance, by examining the Braess paradox and the price of anarchy, a better network infrastructure design can be made in a form of educated decisions regarding the overall network layout such that the overall traffic is optimized. Furthermore, a dispatcher who is tasked with sending a flow of traffic can make a better decision of how to split the traffic among different paths throughout the understanding of the existing equilibria.

2.5.1.2 Selfish Routing

The research work on selfish routing has been motivated by several practical networking problems such as the Braess paradox. Feldmann *et al.* have noted in [44] that despite the existence of Braess paradox, there are strict sub-networks that have improved performance. Nevertheless, the lack of regularization for large networks in the sense of the availability of an established centralized decision-making system has made it difficult for the improved performance in the sub-network to converge into overall improved performance. For large networks with a dynamic flow of large number of travellers, it is practically impossible to form a central unit for decision making to control the flow of traffic. Hence, the travellers resort to their selfish strategies to maximize their gains and minimize their trip costs. Selfish games are used to characterize and investigate the impact of these selfish strategies.

Selfish routing games were first introduced by Koutsoupias and Papadimitriou in their seminal work on non-cooperative games in [39]. These games were introduced to investigate the consequences of the absence of coordination between the users in a network, even when their information and computational resources are unlimited. In non-cooperative networks, travellers attempt to send their traffic through shared links from a source to a destination while attempting to satisfy personal objective functions. Travellers do not consider the global performance of the network when they make their decisions. They will attempt to selfishly devise their own strategies to minimize their cost by using as many pure strategies as the number of available links [45]. In [46], Roughgarden categorizes selfish routing games into two main categories: 1) non-atomic selfish routing games in which users contribute a negligible amount of traffic to the network, and 2) atomic selfish routing games in which users contribute a non-negligible amount of traffic. The non-atomic equilibrium flow can be described as the following: for pairs of paths between sources and destinations $(S_1, D_1), ..., (S_k, D_k)$, these pairs are called commodities. For commodity i, the set of all paths is ρ_i from S_i to D_i and the feasible path between S_i to D_i is P such that $P \in \rho_i$. There exists an edge cost c_e and edge flow f_e such that $f_e = \sum_{P:e \in P} f_P$ where f_e is the summation of the traffic from all paths that has the edge e. Furthermore, there exists an inelastic traffic T_r such that $\sum_{P\in\rho_i} f_P = T_{r_i}$. The flow f for all feasible paths ρ_i where $i \in \{1, 2, 3, , , , , k\}$ such that P, $\tilde{P} \in \rho_i$. The optimum flow is the flow that satisfy the following:

$$c_P(f) \leq c_{\tilde{P}}(f) \tag{2.8}$$

Non-atomic selfish routing games are concerned with traffic flow according to the network structures rather than the players. However, since players affect the flow in the network in the atomic selfish routing games, the equilibrium flow for these games is presented differently. The commodities in this type of game correspond to the players rather than to the network. For the feasible flow f for the atomic selfish routing game, f is traffic flow in equilibrium if, for every player i and every path $P, \tilde{P} \in \rho_i$ of S_i and D_i paths, and $f_P^{(i)} > 0$:

$$c_P(f) \leq c_{\tilde{P}}(\tilde{f}) \tag{2.9}$$

The term $\tilde{f}$ is the same as f. However, when $\tilde{f}_P^{(i)} = 0$, we have $\tilde{f}_{\tilde{P}}^{(i)} = r_i$. Extensive survey on atomic selfish routing games is found in [47].

2.5.2 Applicability of Non-Cooperative Routing Games

In the process of implementing non-cooperative routing games in real applications, several issues need to be addressed; this includes issues such as who the players are, what are the payoffs, and what are the available actions. According to these issues, a game can be appropriately defined [48]. In this section, a variety of examples describing non-cooperative routing games is reported.

In [49], Levinson introduces a two-player congestion game. In this game, the available strategies consist of the departure times. Each player has the choice to either leave early or on time. Strategies in this game describe the action taken by both players to depart earlier, on time or late. The actions are made independently such that if one player decides to leave earlier, the other player still has the option to leave earlier, late, or on time. For different journey times, different equilibria are reported in [49]. A different type of routing game reported in the literature describes a situation in which a traveller plays against the environment. In such games, the environment is viewed as an evil entity - a demon - whose aim is to cause the traveller to lose. At the same time, the traveller attempts to minimize his/her loss according to the available strategies. An example of these types of game representation of trip planning is the work of Colony in [50]. In this work, a traveller is playing against the nature, represented by traffic flow, which has an effect on the driver's tension. When the flow increases, the tension also increases along with the traveller's losses. The outcome of this game is expected to be in the form of Nash equilibrium. In [51], Bell describes a zero-sum game in which the traveller is playing against an evil entity. The game involves using certain routes as strategies. The travellers attempt at achieving his/her goal by choosing the best routes, while the evil entity is intent on maximizing the trip cost by maximizing the cost of some of these routes. The player and the evil entity can only guess each other's actions and behave accordingly. The goal is to find a mixed Nash equilibrium; a point at which neither the evil entity nor the traveller is able to maximize/minimize the total trip cost. The solution for this game is formulated as the following: for the probability of link i to be chosen, p_i, and the probability of scenario j in which q_j and h_k represent the probability of path k to be chosen, link i has the cost value of c_{ij}, and path k has the cost value of g_{kj}. In addition, we have

$$C = \sum_{ij} p_i * c_{ij} * q_j \tag{2.10}$$

where C is the expected trip cost.

If C^* is the solution for the traveller and D is the solution for the evil entity, they can be found using linear programming as the following:

$$\min_{h_k} C^* \tag{2.11}$$

$$\sum_k g_{kj} * h_k - C \leq 0 \tag{2.12}$$

$$\sum_k h_k = 1 \tag{2.13}$$

$$h_k \geq 0, \forall, k \tag{2.14}$$

and

$$\max_{q_i} D \tag{2.15}$$

$$\sum_j g_{kj} * q_j - D \leq 0 \tag{2.16}$$

$$\sum_j q_j = 1 \tag{2.17}$$

$$q_j \geq 0, \forall, j \tag{2.18}$$

The next section investigates parts of the literature regarding cooperative trip planning.

2.6 Cooperative Trip Planning

In cooperative trip planning games, it is possible for the players to negotiate their policies and strategies and to ultimately establish an enforceable agreement. The key issue is the distribution of the rewards/cost among the players. Other issues include the formulation of the game and definition the players. For example, if the game is represented by a graph, players might be assumed to be situated along the vertices or the edges to denote their ownership of these vertices or edges. The ownership of these resources should limit, the problem of competitiveness between players. Certainly, players may share the ownership of some resources. Through the addressing of these two issues, another problem related to mechanism of creating a group of players can be addressed. These issues are discussed in this section a particular focus on concepts such as the balancedness of a game and its convexity. A game is balanced if its core is a non empty one; and a game is totally balanced if every sub game has a non empty core. The core of a trip planning game is also investigated in this section. Furthermore, several examples of known trip planning games are reviewed in this section in order to stress the aforementioned concepts of balancedness and revenue sharing. The following section commences by reviewing examples of shortest path games.

2.6.1 Shortest Path Games

The shortest path problem as reviewed earlier is a 5-tuple problem $\Sigma = (E,V,L,s,t)$. It has five main elements: 1) the starting point s; 2) the ending point t; 3) the set of nodes V; 4) the set of edges E; and 5) the link cost or simply its length L. The network is represented by a directed graph $G(E,V)$.

In [52], Fragnelli *et al.* presented a class of shortest path games as the following: in this game, edges in the set E are owned by a finite set of players N according to the following mapping: $o : E \to N$ such that $o(e) = i$ (i.e., player i owns the edge e). Now, for a path P, a set of players owning nodes in this path is denoted by $o(P)$. Players can send goods on their owned paths to generate a gain value of g. If there is a coalition, $S \subset N$, that owns a path, such that $o(P) \subset S$ transfers goods along its own paths to generate a gain, g, then the shortest path cooperative situation, σ, can be defined as a 4-tuple game, (E, N, o, g). The gain of a coalition is represented by the following characteristic function:

$$v_\sigma(S) = \begin{cases} g - L_S & L_s < g \\ 0 & \text{otherwise} \end{cases} \tag{2.19}$$

where L_S is the distance, which can be viewed as the cost of the paths owned by the coalition S, and $v_\sigma(S)$ is the characteristic function of the Transferable Utility (TU) game.

The above described game is shown to coincide with a class of monotonic games in which larger coalitions have larger gains. This outcome will lead to a situation in which the core is found to be empty- the game is deemed to be imbalanced. Furthermore, since monotonic games are not guaranteed to be balanced, two restrictions were imposed to prove the balancedness of the shortest path game: 1) For σ and (N, v_σ) to be non-trivial, σ has to have profitable paths, $v_\sigma > 0$. . In other words, for all games, $g \in [0, +\infty)$. 2) Let σ be a non-trivial set in which a shortest-veto (s-veto) player is a player $\in N$ who owns at least a node in every shortest path in Σ. In other words, for player i, $v(N/i) = 0$. V is a set of s-veto players. A game is balanced if V is not empty and every profitable path of σ has a node owned by an s-veto player. These two strong restrictions have to be imposed in order to have a balanced shortest path game. In this game, the gain is not associated with the players but rather with the coalitions.

In [53], Voorneveld and Grahn have presented a shortest path game in which each player has a reward. The shortest path problem can be described as a 5-tuple game $(N, V, (A_i)_{i\in N}, w, (r_i)_{i\in N})$ where for a player, i, $A_i \subseteq V \times V$ is a directed arc in the network, and V is a set representing all nodes in the network including s and t. $(w_i)_{i\in N}$ denotes the cost of using a link in the network by player i. $(r_i)_{i\in N}$ is a reward assigned for a player i where $r_i \in \mathbb{R}^+$. Many differences are found between this definition of the shortest path game and the one in [52]. In [53], players own vertices vs edges in [52]. Furthermore, an edge in [53] can have more than one owner. The shortest path game (N, v) is defined as:

$$\forall S \in 2^N \setminus \{\varnothing\} : v(S) = max\,\{r(S) - c(S), 0\} \tag{2.20}$$

This game is monotonic. Thus, to achieve the non-triviality, the following assumption is made:

$$0 < c(N) < r(N) \tag{2.21}$$

This assumption implies that the reward of the grand coalition is greater than the cost incurred by the coalition's trip, $S \in 2^N \setminus \{\varnothing\}$. The cost of the coalition S that uses path P can be computed as the following:

$$cost(p) = \sum_{k=1}^{m-1} w(i_k, (v_k, v_{k+1})). \tag{2.22}$$

$$c(S) = \begin{cases} \min_{p \in P(S)} cost(P) & if\, P(S) \neq \varnothing \\ \infty & \text{otherwise} \end{cases} \tag{2.23}$$

The core is defined in this work as:

$$C(N, v) = \left\{ x \in \mathbb{R}^N | \sum_{i \in N} x_i(N) = v(N) \text{ and } \sum_{i \in S} x_i(S) \geq v(S),\ \forall S \in 2^N \setminus \varnothing \right\} \tag{2.24}$$

2.6.2 Travelling Salesman Games

Travelling salesman games have received significant attention as a routing game due to its popularity in the operational research studies. In [54], Potters *et al.* describe a fixed route travelling salesman game in which a round trip from s to t is attempted while dividing the total trip cost among the participating players. If this game has a non empty core, then a solution for the traditional travelling salesman problem can be found. The core for this game is defined as

$$C_k(c) = \left\{ x \in \mathbb{R}^N | \sum_{i \in N} x_i(N) = c_k(N) \text{ and } \sum_{i \in S} x_i(S) \leq c_k(S), \ \forall S \in 2^N \setminus \varnothing \right\} \tag{2.25}$$

If c_k is the cyclic trip cost from the home city s and the city k, the core allocation $x = \{x_1, x_2, , , , x_k\}$ has to satisfy the condition that $x_k \leq c_k$ and $\sum_{i \in N} x_i = c_k(N)$. This game has no apparent reward. Sponsors pay at most the cost for the trip from the home to the destination city in a form of a reward. The routes in these games are predefined which means that the players in the coalitions have no choice but to leave their coalition in order to form a better one. In [55], Estevez-Fernandez *et al.* proposed a similar game with a reward sharing scheme that has the core as defined in Equation 2.24. An interesting theorem presented in this paper states that all travelling salesman games with revenues have a non-empty core, given that the conditions of rationality and efficiency exist. However, these conditions do not guarantee the uniqueness of the solution.

Moreover, in nearly all these games, the number of players is an important factor contributing positively or negatively to the balancedness of a game. For example, the game in [54] is shown to be balanced for only 3 players. In [55], the number of players for which the game is guaranteed not to have an empty core is not explicitly indicated. Nevertheless, numerical examples show that a non-empty core exists for 3-player and 4-player games. Furthermore, in [56], Borm *et al.* demonstrated the example of a travelling salesman game in which the game might have an empty core for 6 players.

One of the most important issues in cooperative games with distributed costs/revenues is the question of how to assign the revenue/cost among players in the grand coalition N. Shapley value is an approach that allows for the distribution of the costs among players in certain coalitions while satisfying the following conditions: 1) efficiency, 2) symmetry, 3) additivity, and 4) the irrelevant player property [52]. The Shapley value is computed for player, i, as follows:

$$\Phi_i(N, v) = \frac{1}{N!} \sum_{S \subseteq N \setminus \{i\}} |S|!(|N| - |S| - 1)![v(S \cup \{i\}) - v(S)] \tag{2.26}$$

Shapley value is computed such that when a player i participates in a game, his/her average contribution is $[v(S \cup \{i\}) - v(S)]$. This value is multiplied by the possible ways in which a coalition can be sorted without having the player as a part of it. This value is multiplied by the possible ways in which a coalition can be sorted without having player i being as a member. The result is then multiplied by the possible ways of creating a coalition excluding i and S $((|N| - |S| - 1)!)$. The Shapley value is the average of all collations' contributions. For every convex game, a Shapley value always exists in the core.

2.6.3 Bargaining Games

The discussion thus far has focused on the solutions for cooperative games with side payments (i.e., transferable utilities). However, it remains to be determined how these solutions can be found. An applicable model that will be used in this thesis to solve the team mobility game is the bargaining game model.

The bargaining model is one of the earliest solution models in cooperative games described by Nash in [57] and later extended by extended by the same author in [58]. In this model, rational players will present their sets of anticipations and attempts to reach a satisfactory agreement. Nash described a threat model by which, in 2-person games, each player has an anticipation that he/she wishes to satisfy, while weighing the threat posed by the other player. Both players will exchange their threats in the form of mixed strategies. The players then cease all communications and act independently. If the players find themselves in a situation in which they can retaliate against the opposing party's threat and satisfy their anticipation, then a point of agreement is found. The threat is used to enforce an agreement. Nash's model was extended to an n-person bargaining game by Harsanyi in [59]. In his work, Harsanyi suggested that players could be grouped in syndicates (commonly referred to as coalitions). Each syndicate accommodates a group of players who agree to maximize the syndicate's overall interest. This model uses mixed strategies as threats which the players use against any player who might jeopardize the gain of the syndicate. Various extensions of Nash's bargaining model can be found in the literature in order to resolve the scenario where there is incomplete information [60] or to alter some of Nash's axioms. For instance, In [61], Kali and Smorodinsky replaced the axiom of independence and irrelevant alternatives with the monotonicity axiom.

Recently, in [62], Hart and Mas-Colell proposed a commitment procedure for the bargaining game. They argued that in Non Transferable Utility (NTU) games, coalitions might be difficult to construct due to the possibility that each player cannot gain more rewards or pay less cost in comparison to other players. This argument can be refuted if on the basis that solution in bargaining games "does not exclude cases where, in the end, only one individual could have benefited because the 'fair bargain' might consist of an agreement to use a probability method to decide who is to gain in the end. Any probability combination of available anticipations is an available anticipation." [57]. Furthermore, contrary to the theorem presented by Estevez-Fernandez *et al.* in [55] concerning side payments, the effect of side payments on the final outcomes is another possible action in the bargaining game. Thus, the lack of a side payment in a game should not diminish the possibility of forming a coalition or finding a solution for the game.

2.6.4 Congestion Games

Congestion games, first described by Rosenathal in [63], are potential games in which players' payoffs are affected by the resources they own and the sharing status of these resources. As more players share the same resources, the efficiency of these resources decreases. For instance, routes between two points are shared by the travellers. As more travellers use the same routes, the delay times increase accordingly.

Congestion games in this sense are usually viewed as non-cooperative games and used in the analysis of the price of anarchy and the price of stability for the network. However, recently, a cooperative approach to solve this problem started to gain attention in the literature. In [64], Hayrapetyan described a model in which players who share the resources, i.e., the routes, start forming coalitions. Various coalitions are formed on this basis start to selfishly compete to maximize their objective functions. The aim is that if coalitions were able to maximize the welfare of their participants, the over all gains will outweigh the individual losses. In [65], Fotakis discusses the similarity of cooperative congestion games with their non-cooperative counterparts while demonstrating important issues such as the existence of perfect Nash equilibrium (PNE) and the convergence of the final solution. Cooperative congestion games share similarities with Harsanyi's bargaining models, and the only difference is the method by which a coalition (syndicate) is created.

2.7 Behavioural Driven Trip Planning

The analysis of the human behavioural impact on the process of trip planning can not be extricated from the analysis of team trip planning and selfish planning. Therefore, understanding the various underlying factors affecting the behaviours involved in trip planning and their influence on the entire process is of paramount importance.

A number of researchers have outlined the effect of travellers' experience and behaviour on trip planning and the available planning alternatives. Ben-Akiva *et al.* describe in [66] a model that conceptualizes the dynamic behaviour of the drivers. In this model, travellers identify their trip goals in the form of trip times, trip destination, and budgetary constraints. These goals are processed based on decision rules in order to produce a single suggestion. This model would assist in predicting the traveller's behaviour-driven impact on traffic according to the available information. In [67], Adler *et al.* presented a methodological model based on conflict arousal and motivation to analyze the en-route driver's behaviour, and its subsequent impact on traffic. In [68], Feng and Mingzhe described a traveller-behaviour analysis model that utilizes the Bayesian theory and the decision field theory to predict and explain the traveller's behaviour based on his/her routing preferences. The preferences are limited to one criterion per route such as speed and distance.

Arentaze presented in [69] an adaptive personalized travel information system that models an Advanced Traveller Information System (ATIS) system to respond to the traveller's preference. The model uses Bayesian based method to approximate the individual preferences of each driver based on repetitive sampling. The estimation unit is central and is tasked to deal with multiple inquiries simultaneously. Further efforts were made to model the ATISs according to the behaviour of travellers. For instance, in [70], Jufen and Guiyan presented a navigation model that identifies possible routes based on their expected travel times as well as possible changes in the journey times. User behaviour is defined by their tolerance to the changeability of journey times, with the assumption that travellers are not pre-informed about these changes and can only respond based on their predictions of these changes. However, this study does not consider the fact that modern ATISs have access to online information, and if there is a change in traffic flow, travellers will have knowledge instead of the prediction regarding the changes. Furthermore, by defining the behaviour of the travellers in the context of their tolerance towards possible changes in the trip times, they overlook other travel factors such as monetary budgets, comfortability, the availability of transportation modes, safety, and route familiarity, among others. Ben-Elia and Sheftan maintained in [71] that, in addition to the attitude towards taking risks in trip planning, the amount, nature, and completeness of the provided information to the travellers all play a pivotal role in the trip planning process. Travellers are more inclined to react quickly in a rational way to the changes in the planning factors that might affect their plans, if the correct information is provided. Rationality in [71] is only seen as the compliance of the travellers with the best provided plan. Various simulation studies can be found in [72], which evaluate the travellers' decision making process under various risk assumptions. When travellers are faced with certain risks, their response in terms of aversion is observed. Risk is modelled in their work in terms of information uncertainty such as the uncertainty of the arrival time. Nonetheless, the travellers' personal behaviour regarding the trip planning is not proactively included in the implementation of the trip planning solution. Alternatively, most of the available ATISs process traffic in the same manner for all travellers regardless of their personal preferences. Subsequently, travellers who share the source and destination areas are often presented with the same advice, which generates a number of problems [73].

The term "behaviour" in this research refers to the travellers' personal preferences towards routing. The satisfaction of these preferences can ultimately determine the success of a trip. Furthermore, since travellers' rationality should only be viewed in terms of goal achievement, their preferences (i.e., goals), and travelling behaviours should play a key role in deciding their routing alternatives. This behaviour-driven planning vision is not a substitute for the existing ATISs, but it is complementary.

The behaviour-driven planning can be seen as an additional layer between the ATIS and the travellers, which acts as a proxy for the travellers in terms of processing the available information.

2.8 Outstanding Issues and Motivation

There are many issues that usually arise in trip planning systems. These issues have influenced the proposed trip planning framework. Regardless of the nature of the trip planning approach, whether good or bad, certain issues persist. One of these issues is the over saturation problem. This problem arises from the travellers' mismanagement of the information regarding their trips [66]. The problem is a man-machine interaction problem. The cognition ability and the travelling experience of the user would affect the trip planning decision [74]. Mitigating this issue is difficult since it is related to the amount of control the trip planning system designer needs to give to the user. Furthermore, even with effective trip planning, some problems are deemed to be intrinsically related to the process. Intelligent systems often make similar decisions under similar circumstances. For instance, in a situation where travellers are receiving accurate information about traffic in real-time, the majority of travellers will make the accurate decision of using an alternative route with the least amount of traffic, which will eventually cause traffic oscillation. This problem is called the overreaction problem. There are many solutions that have been devised to resolve this problem, one of which is to utilise the non-cooperative game trip planning model. In this solution model, through their navigation systems, travellers can anticipate the possible action(s) of other travellers and act accordingly [75].

Trip planning is largely dependent on real-time information and individual preferences among the travellers. Thus, it is relatively acceptable to assume that most travellers will choose the best possible paths for their journeys. Departure times are chosen based on their path choices and expected trip times. However, travellers with similar preferences and who have access to the same travelling information will tend to choose similar paths. This situation often results in congestion [66] and is generally referred to in the literature as the concentration problem. While commenting on the Comprehensive Automobile Traffic Control System (CACS) implemented in Japan during 1970s, Kawashima noted in [76] that as the number of vehicles equipped with navigation system increases, the overall traffic management efficiency decreases. The reported navigation systems provided route suggestions based on the shortest route approach. This observation can be extended to the more recent and advanced trip planning systems. Ben-Akiva suggested in [66] that some form of directive planning could be beneficial in reducing this problem. Directive routing can be achieved most effectively through central systems. Nevertheless, this solution has been shown to be impractical and virtually infeasible. In [77], Chang *et al.* suggested that the travelling options should be diversified so that drivers can choose from alternative paths from the same source to the same destination.

The overreaction problem and the concentration problem may seem similar, although this is far from the reality. The overreaction problem is a situation-pertinent event. Once a triggering condition is established, such as congestion or safety hazards, the rationality of the system will create the overreaction problem. On the other hand, the concentration problem is the natural outcome of using a rational intelligent system. Intelligent systems are one of the reasons that congestion exists, where the overreaction is caused by events such as congestion. The resolution of these three problems exemplifies the goal of the research in this thesis. In order to avoid the oversaturation problem, the trip planning system must be able to incorporate cognitional ability and human expertise to influence the trip planning decision. Furthermore, to avoid the problems of both overreaction and concentration, the trip planning system should be capable of correctly anticipating the actions of other travellers or allowing them to coordinate among themselves in order to appropriately direct the traffic and avoid congestion. Furthermore, to deal with the various team mobility problems, any solution has to view the problem as a resource sharing problem. The developed framework in this research is designed to achieve a cooperative trip planning system that is both rational and influenced by the travellers' preferences and demands. Moreover, the framework is designed to address the problem as a resource sharing problem that can be solved through competition between travellers or cooperation between groups of travellers, when applicable.

2.9 Summary

This chapter covered a wide range of topics related to trip planning. Two areas of research were discussed, namely pure optimization-based trip planning and game theoretic-based trip planning. For each area, a background survey was conducted and a variety of trip planning problems were reviewed in terms of problem formulation, solution modelling, as well as the existence and the stability of the final solution.

There are several similarities between some of the reviewed trip planning games and the problem investigated in this thesis. These similarities have assisted in the development of the mathematical formulation of the team trip planning game. However, to the best of the author's knowledge, there is no existing research on cooperative trip planning that suggests a functional framework that addresses the problem of team trip planning from the stage of trip inquiry through to the stage of trip decision making; a framework that handles tasks such as trip information gathering, initial trip planning, and conflict identification and resolution. The work in this thesis aims to construct a fully workable solution that resolves these issues using a unified framework that consists of multiple modules, while also addressing the theoretical issues covered in this literature review.

Chapter 3

Team Mobility Planning: A Game Theoretic Approach[1]

3.1 Introduction

Trip planning games have, to a large extent, been regarded in the literature as non-cooperative games. In nearly all instances of such a representation, the travellers are treated as a passive component of the game: the aim is only to analyze and optimize the environment's resources. This does not mean that using the cooperative approach is a futile endeavour. It is indeed more challenging and exceedingly more complicated to implement. However, with efficient planning, this approach can yield positive results. The rationale is that cooperation is an integral part of the evolution of nearly all aspects of technology and problem solving. The team-trip planning problem is no exception to this rule.

In this chapter, I view the team trip planning game as a cooperative game of competing travellers. I describe a team trip planning game in which travellers initially, selfishly, aim to plan a trip that matches their needs considering the state of

[1]The research work in this chapter has appeared in part in [78]

the existing environment. I demonstrate that the selfish but rational choices of various travellers could be detrimental to the other travellers' optimality. I argue that, through negotiation, each traveller may eventually profit either by improving upon his/her individual optimum gain or, at a minimum, by retaining his/her individual optimum gain. A multi-module framework named Team Mobility Planning (TMP) is described in this chapter.

3.2 Team Trip Planning: Problem and Model Description

3.2.1 Problem Formulation

Consider a group of N travellers, $TR = \{tr_1, tr_2, ., tr_i, ., tr_N\}$, in which a traveler tr_i is contemplating a trip from an initial location s^{tr_i} to a final destination f^{tr_i}. Traveler tr_i can make the trip by following path $p^{tr_i} \in P^{tr_i}$. P^{tr_i} denotes the feasible paths between s^{tr_i} to f^{tr_i} that are available to tr_i. There is a certain cost value c^{tr_i} associated with each path p^{tr_i}. c^{tr_i} is a function of several factors including transportation modals used to complete the trip (e.g, public vs private), environmental conditions along the path, as well as traffic conditions. The notion of cost in this work is multi-aspect, in the sense that it explicitly quantifies monetary costs, temporal costs, safety costs, and comfort costs, to the extent that a multi-criteria cost formulation is employed to facilitate the trip route optimization process. The concept of doctrine is introduced to capture travellers' preferences, demands, and constraints. For each path p^{tr_i}, we use γ^{tr_i} to denote traveler tr_i's doctrine for this path. Γ^{tr_i} denotes the set of all doctrines associated with tr_i's feasible paths P^{tr_i}. The traveler tr_i's desirable route can then be stated as

$$p^{tr_i}_{Opt} = \underset{\forall p^{tr_i}_k \in P^{tr_i}}{Min} \Upsilon(s^{tr_i}, f^{tr_i}, \Gamma^{tr_i}) \tag{3.1}$$

where Υ represents a selfish selection process by which an optimum path is chosen. $p_k^{tr_i}$ represents a possible path, and P^{tr_i} represents the set of all paths. A selfish selection process is a process in which the drivers attempt to maximize their benefits regardless of the impact of their chosen actions on the system- negatively or positively.

For N travellers, their interaction process and their travelling decisions must be formulated such that optimizing traveller tr_i's plan does not negatively impact other travellers', tr_{-i}, chosen routes. To deal with these kinds of interactions, we define a team trip planning game. The team trip planning game, Σ, is a 4-tuple (G, Γ^{tr_i}, s^{tr_i}, f^{tr_i}) such that:

- G is a directed acyclic graph that includes all possible routes, P^{tr_i}. For each traveller tr_i, s^{tr_i} and f^{tr_i} are defined.
- Γ_{tr_i} is a traveller selfish assessment process which assigns a non-negative value to each road segment, $l_j^{p_k}$, denoting the cost of this segment.

A solution set of paths P^{tr_i} is defined for the game Σ such that each path, $p_k^{tr_i}$ connecting s^{tr_i} and f^{tr_i}, is composed of connected links $l_j^{p_k^{tr_i}} \mid l_j^{p_k^{tr_i}} \in L^{p_k^{tr_i}}$. The cost of these paths is $c_{p_k}^{tr_i} = \Sigma_{j=1}^{|L^{p_k^{tr_i}}|} l_j^{p_k^{tr_i}} * \xi_r$, where ξ_r is a weight value that reflects the degree of preference that each traveller has for a certain path. For the mobility planning game Σ, traveller tr_i has an ordered set of strategies: $\left\{P^{tr_i} : P^{tr_i} = \left\{p_1^{tr_i}, p_2^{tr_i}, , , , p_n^{tr_i}\right\}\right\}$ in which $\left\{p_k^{tr_i} \succ p_{-k}^{tr_i} : p_k^{tr_i} = p_{Opt}^{tr_i} \text{ for } tr_i\right\}$. Travellers own road segments, $l_j^{p_k}$s, according to a mapping function $o : L^{p_k} \to TR$ such that $o(l_j^{p_k}) = tr_i$ (i.e., road segment $l_j^{p_k}$ is chosen by traveler tr_i as a possible path). For any path, p_k, $o(L_S^{p_k})$ is a set of connected segments satisfying certain travelling criteria and owned by a group of travellers.

Suppose that for each traveller tr_i there is a travelling cost $c_{p_k}^{tr_i}$ and mobility planning reward $D_{R_{p_k}}^{tr_i}$. Both cost and reward values are non-negative, $\left\{c_{p_k}^{tr_i}, D_{R_{p_k}}^{tr_i} \in \mathbb{R}^+\right\}$, and they belong to the same currency domain. For example, if the reward is monetary, the cost should be also monetary. For $D^{tr_i}_{R_{p_k}} > c^{tr_i}_{p_k}$, traveller tr_i is able to generate profit. Furthermore, assume that a group of travellers, i.e., coalition $S \subset N$, through certain agreements, can generate a profit using only paths P^S owned by the coalition through certain agreements; for example, the set of paths P is owned by a coalition S if $o(P) \subset S$. A situation in which there exists a successful team trip plan is called a team trip planning game (N, v_σ). v_σ is a characteristic function for this game, which is computed as follows:

$$v_\sigma(S) = \begin{cases} \zeta(c^S_P, D_{R_S}) & \text{if S owns } P^S \in \Sigma, \text{ and } \zeta(c^S_P, D_{R_S}) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.2}$$

Where ζ is the mapping function which associates every coalition S with a non-negative value according to c^S_P and D_{R_S}.

The cost of each path, $c^{tr_i}_{p_k}$, for traveller tr_i, is not independent from its reward value, $D^{tr_i}_{R_{p_k}}$. Thus, it is necessary to define a positive function, for the coalition S, that defines the relationship between c^S_P and $D_{P_{R_S}}$. Furthermore, assume that the $c^{tr_i}_P \neq c^{tr_{(-i)}}_P$ and $D^{tr_i}_{R_{p_k}} \neq D^{tr_{(-i)}}_{R_{p_k}}$. This game should be considered a game with Non-Transferable Utility (N-TU). In this cooperative game, the solution, i.e., the core, is described as follows:

$$\text{core} = \left\{ x \in \mathbb{R}^N | \sum_{i \in N} x_i(N) = v(N) \text{ and } \sum_{i \in S} x_i(S) \geq v(S) \ \forall S \in 2^N \setminus \varnothing \right\} \tag{3.3}$$

where $\sum_{i \in N} x_i(N) = v(N)$ guarantees the efficiency of the outcome, and $\sum_{i \in S} x_i(S) \geq v(S) \ \forall S \in 2^N \setminus \varnothing$ guarantees the stability of the game.

Furthermore, each traveller, tr_i, has a set of strategies based on which the decision of the best path, $p^{tr_i}_k$, can be obtained. For each traveller, tr_i, there is a finite non-empty set of actions, paths, P^{tr_i}. For each path, there is a payoff or a utility function

u, such that u: $P^{tr_i} \to \mathbb{R}$. Paths in P^{tr_i} are associated with preference relation $\succsim$ such that $u(p_i^{tr_i}) \geq u(p_j^{tr_i})$ if $p_i^{tr_i} \succsim p_j^{tr_i}$. Cost functions and rewards can correspond to utility values, and the ordering of P^{tr_i} is conducted through the knowledge of $\succsim^{(tr_i)}$. Moreover, P^{tr_i} can be expanded to include a non-route related actions. The chosen departure and the expected arrival times for a trip can be considered as strategic actions, which might change the outcome of the game. For traveller, tr_i, P^{tr_i} can be ordered according to $\succsim^{(tr_i)}$ such that $p_k^{tr_i}$ is better than $p_{k+1}^{tr_i}$ iff $u(p_k^{tr_i}) \geq u(p_{k+1}^{tr_i})$. This game is described as (N, P, u) game.

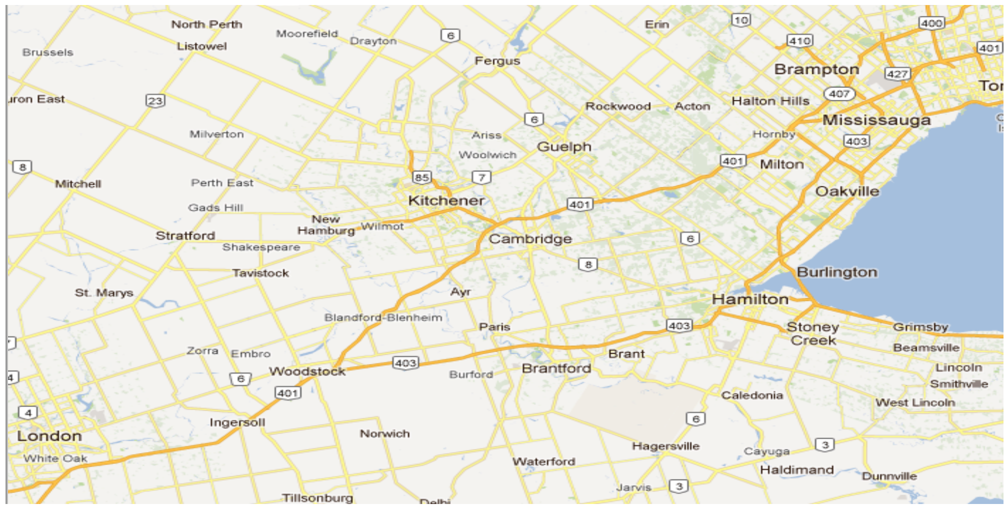
3.2.2 Team Trip Planning Model Formulation

For each traveller, tr_i, the problem of finding the best route can now be stated as follows: Given a graph $G = (V, E)$, where V is the set of all nodes (vertices) in the graph, and E denotes all edges in the graph. The graph represents an area of interest $P(s^{tr_i}, f^{tr_i})$ that includes the starting point s^{tr_i} and the destination point f^{t_i}. This area of interest is defined prior to the commencement of the team trip planning game in order to limit the search space, as shown in Figure 3.1 .

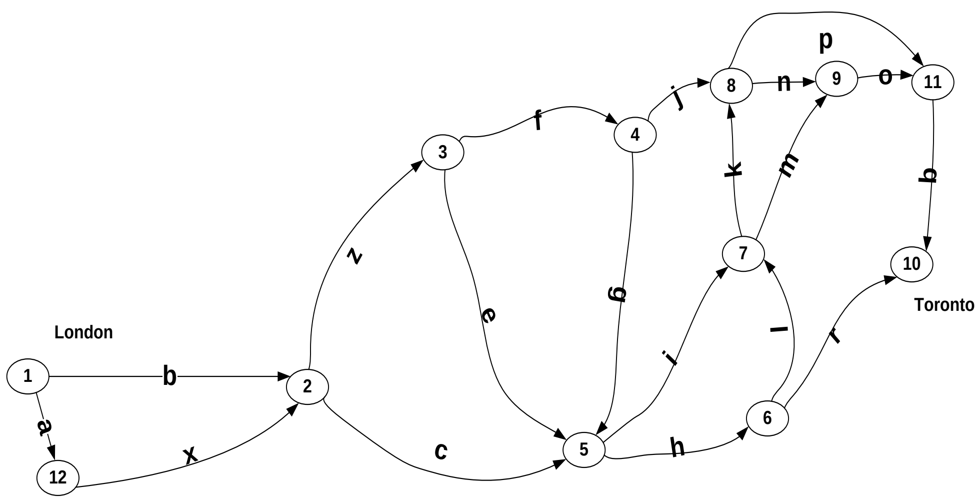
The goal is to find the best path, $p_{Opt}^{tr_x}$ for traveller tr_x with minimum cost value, ξ_r, for n travelling options such that $p_{Opt}^{tr_x} \in \left\{p_1^{tr_x}, p_2^{tr_x}, , , p_n^{tr_x}\right\}$. Based on the preferences-based ordering, $p_{Opt}^{tr_x} = p_1^{tr_x}$. The multi criteria team trip planning problem is formulated to reflect Equation 3.1 such that

$$p_k^{tr_x} = \min \sum_{\forall i,j \in E} \xi_{r_{ij}}\, x_{ij} \tag{3.4}$$

which can be described as a minimum cost optimization problem. The paths with the minimum cost can be ranked in order of the incurred cost. The cost values can be subjective and vary from one traveller to another.



(a) Road network map covers the area of interest taken from Google Maps.



(b) A schematic graph corresponding to the road network of interest.

Figure 3.1: Mapping a satellite map to graph based on the area of interest.

For a team of travellers, $\{tr_1, tr_2, , , , , tr_N\}$, an indication of common *interest* is the utility function $U = \omega(\eta, p_i^{tr_1}, p_j^{tr_2}, , , , p_n^{tr_N})$ where ω is a concave function that is differentiable over $p_j^{tr_i}$s such that $p_j^{tr_i} \in P^{tr_i}$. η is the state of the environment and corresponds to a value computed based on trip attributes such that $\omega(\eta, p_j^{tr_i}) = u^{tr_i}(p_j)$. In the best case scenario, the actions of each traveller are independent from the actions of other travellers and do not affect the state of the world (i.e., $\eta^{tr_1}(tr_2, tr_3, , , tr_N) = \eta^{tr_1}$) such that

$$U = \sum_{i,j} \omega(\eta^{tr_i}, p_j^{tr_i}) \tag{3.5}$$

The formulation in Equation 3.5 corresponds with $v(N) = v(s) : s = S^{GND}$ in Equation 3.3. Where S^{GND} is the grand coalition that includes all travellers.

The optimization problem should be defined as being user-centric with user-specific constraints. Therefore, there is no guarantee that a conflict-free outcome will emerge. Furthermore, conflicts can sometimes occur regardless of the abundance of existing paths. For example, if one path is more attractive than the other paths, most drivers will choose this path, thus resulting in conflicts. Nevertheless, it is important to note that the drivers make their decisions independently from each other, such that the rationality of these decisions is not compromised. Once the drivers arrive at their own personal decisions, the developed cooperative solution offers a procedure by which their conflicts are mitigated. The developed framework is capable of solving each traveller's mobility problem, according to the objective function defined in Equation 3.4, and of the determining of the proper team arrangement such that U is maximized or minimized depending on the application. Additionally, this process, includes an appropriate definition of the relationship between the travellers and the impact of each traveller on the others' expected outcomes.

Example

To illustrate the trip planning problem, consider the following scenario in which only two travellers are driving within the same area and heading to the same destination. In this scenario, according to our problem formulation, these two drivers have different concerns, preferences, and understandings regarding their preferred routes. Thus, the cost of using each road segment is dependent on the driver, as seen in Figure 3.2. Assume that the factors, other than the traffic, contributing to the cost are fixed for this trip and do not change throughout the duration of the trip. Furthermore, if the traffic increases by 1 vehicle, the cost increases by 3 points for both drivers.

For traveller tr_1, $P^{(tr_1)} = \left\{p_1^{(tr_1)}, p_2^{(tr_1)}, , , , p_n^{(tr_1)}\right\}$ such that

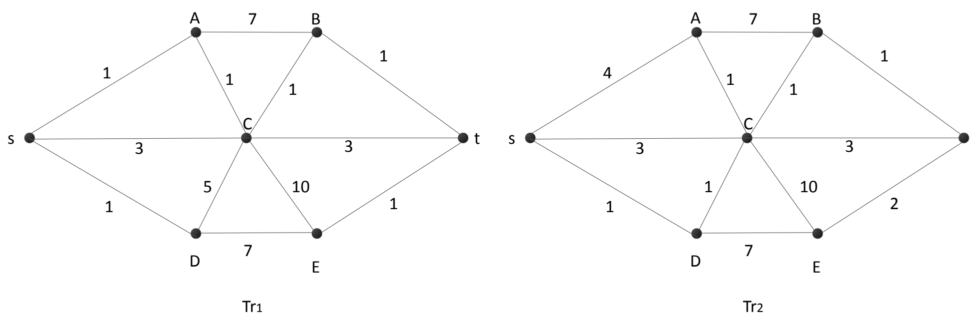


Figure 3.2: Example of two competing travellers.

$$p_1^{(tr_1)} = s \rightarrow A \rightarrow C \rightarrow B \rightarrow t \tag{3.6}$$

$$p_2^{(tr_1)} = s \rightarrow A \rightarrow C \rightarrow t \tag{3.7}$$

.

.

$$p_{n-1}^{(tr_1)} = s \rightarrow C \rightarrow D \rightarrow E \rightarrow t \tag{3.8}$$

$$p_n^{(tr_1)} = s \rightarrow D \rightarrow E \rightarrow t \tag{3.9}$$

and for tr_2, $P^{(tr_2)} = \left\{p_1^{(tr_2)}, p_2^{(tr_2)}, , , , p_n^{(tr_2)}\right\}$ such that:

$$p_1^{(tr_2)} = s \to D \to C \to B \to t \tag{3.10}$$

$$p_2^{(tr_2)} = s \to D \to C \to t \tag{3.11}$$

.

.

$$p_{n-1}^{(tr_2)} = s \to A \to C \to E \to t \tag{3.12}$$

$$p_n^{(tr_2)} = s \to D \to E \to C \to t \tag{3.13}$$

The price model for this example consists of two components: the individual price as per user and the price incurred due to the conflict. Therefore, implicitly for this example, the order of preference is established according to the cost. Rationally, both travellers would choose a preferred path $p_{Opt}^{tr_k} = p_1^{tr_k}$, for $k = 1, 2$. However, if they meet at point C at the same time, the cost of the next segment will increase by 3 points if it is selected by both. If one of them used the segment prior to the other traveller, the cost for the other traveller would increase. Hence, one of them at least will have his/her plan de-optimized. In the worst case scenario, with no agreement or negotiation, both travellers will choose $C \to B \to t$ and the overall cost will increase by 6 points. For both travellers, many of the less desirable paths would be cheaper.

This example is relatively simple, and the cost increase might not be significant. Nonetheless, scaling this example up to N travellers and to many other types of cost factors will result in a significantly more negative scenario. A solution for this example may lead to the necessity to find an agreement such that one traveller will transfer the ownership of p_1 to the other traveller in exchange of some form of compensation. In this case, tr_1 and tr_2 have formed a coalition based on mutual interest. Furthermore, the set A^{tr_k} can include possible starting trip times. A change in one of the traveller's starting times may avert the possibility of both travellers using the same segment at the same time. If this choice is to be made independently by each traveller, this would be a non-cooperative game.

3.3 Team Mobility Planning (TMP) Framework

To address the team trip planning game and its variety of potential classes, a multi-module framework is developed along the lines of the defined team trip planning game, as depicted in Figure 3.3. This framework is a conceptual description of how the team trip planning game will be handled, while addressing various game theoretic issues. Some of these issues are as follows:

- One of the most important conditions of game theory is the assumption that all players are rational. Rational players aim to optimize their objective function(s) based on pre-defined criteria.

- Effective trip planning that is influenced by the traveller's personal preferences is a challenging process to model. In the developed framework, trip planning is a traveller-centric process. Human expertise and understanding is incorporated into the process.

- The developed framework should have a mechanism that can be used for the identification of potential participants in the trip planning game.

To address these challenges, among others, the following description of the developed multi-module framework is presented:

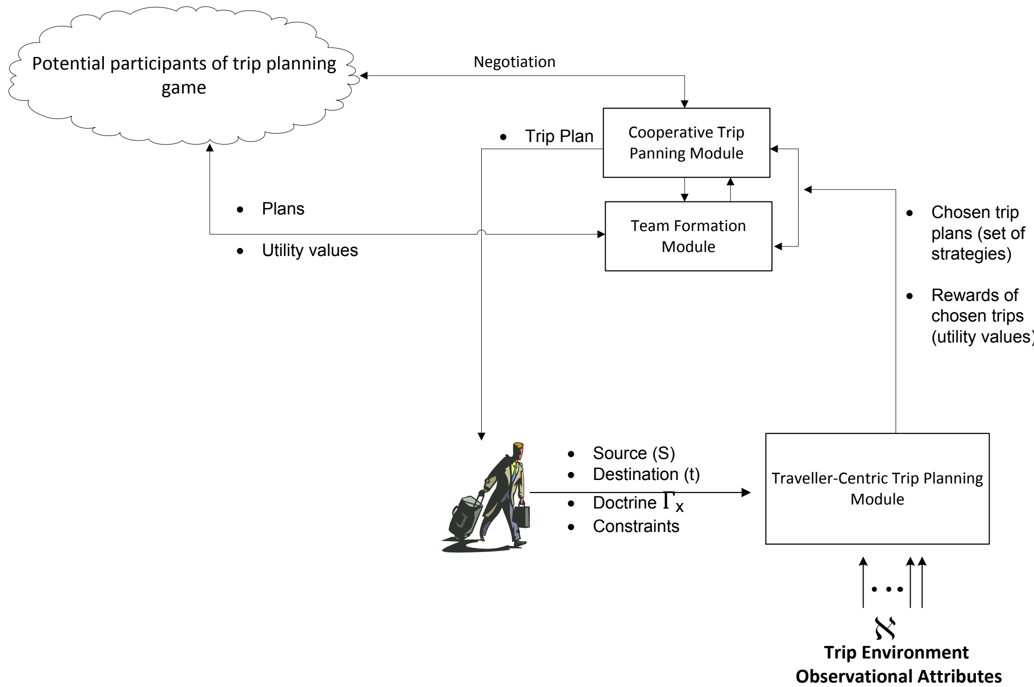


Figure 3.3: Schematic of the Team Mobility Planning (TMP) Framework.

3.3.1 Cooperative Trip Planning Module

In this module, the final trip planning decisions are made. The route assignment, the resource allocation, and the final cooperative operations are also performed. This module deploys the bargaining game as a cooperative approach to resolve the conflicts between the travellers. The cooperation notion is held under the strong assumption that there is always a binding agreement between the travellers and the coalition (i.e., once a final decision is made, all travellers affected by that decision will, with no exception, follow through with their agreed upon actions). Furthermore, in conjunction with the team formation module, the process of coalition creation is implemented. The creation of coalitions is achieved primarily by means of negotiation between the travellers.

3.3.2 Team Formation Module

This module represents the existence of a medium through which the drivers are identified as travellers and as such, can be grouped into coalitions. Their preferences, interests, geographical proximity, and their threats against each other are moderated through the medium. The team consisting of all travellers defined in this module is named the Grand Coalition. The other smaller coalitions are defined in the cooperative trip planning module.

In this developed work, the travellers are using vehicles for the purposes of their commutes. Maps are represented by graphs G in which the optimum route is found. Within the process of finding the optimum route, there may be the necessity to form team(s) to broaden the search space. For instance, in Figure 3.1(b) travellers in the road segments denoted by link j and link k might want to cooperate to discuss their strategies as they could share the same intersection and thus cause congestion.

3.3.3 Traveller-Centric Trip Planning Module

In this module, a Traveller-Centric Trip Planning (TCTP) system is deployed to provide the traveller tr_i with a set of feasible solutions P^{tr_i}. As indicated in the Equation 3.1, each path has its own payoff function u_i. These paths are ordered based on their payoff values. These strategies are formulated and ordered in the TCTP module as doctrines, Γ^{tr_i}. For each chosen path $p(\Gamma^{tr_i})$, there exists a doctrine satisfaction index D_{S_r}, which is used with the cost of the chosen path to define the utility function u_i.

The TCTP module is the first stage of the trip planning process. It is designed such that the travellers are rationally making decision and prioritizing alternatives for existing plans based on the forecast profits or losses. The rationality condition is handled/guaranteed by this module. By adjusting the monetary and temporal constraints, this module will produce multiple paths and multiple utility values which can be used later in the cooperative game.

3.4 Summary

This chapter presented a problem formulation for the team trip planning problem. The problem formulation describes the problem from a single traveller's perspective, as well as the interaction between multiple travellers. The team trip planning problem is described as a cooperative game. A mathematical formulation for the solution model was also developed in this chapter. According to the solution formulation, a Team Mobility Planning (TMP) framework was developed. This framework consists of three modules to cover the aspects of cooperative decision making, team formation, and the traveller's rational planning.

Chapter 4

Cooperative Trip Planning: Bargaining Based Approach[1]

4.1 Introduction

In Chapter 3, the trip planning game is formulated as a cooperative game. In this chapter, I emphasize the notion of cooperation and provide analytical propositions to discuss the balancedness and stability of the described game. The balancedness of a game reflects the existence of a solution for that game. A solution model is presented in this chapter and is further demonstrated using experimental scenarios.

4.2 Balancedness of Team Trip Planning Game

An integral part of developing a solution for any game is the understanding of the characteristics of the game. The most important aspect of any game is its balancedness and stability. In cooperative games, balancedness refers to the existence of at least one coalition of travellers for which the individual outcomes are more than or equal to the gains outside of the coalition. In other words, a game is balanced if it

[1]The research work in this chapter has appeared in part in [78]

has a non-empty core. For non-cooperative games, stability is usually used to refer to the existence of at least one equilibrium. This difference between these two concepts is simple and can facilitate general understanding of the research related to them. Certainly, the term stability can be also extended to the cooperative games [79].

Both terms, the balancedness and the stability of the game, correspond to the possibility of having a solution for the game. Thus, if a game is balanced/stable, there is at least one self-enforcing or externally enforced outcome for the game that conforms to the rules of balancedness and stability. This section addresses the issue of solution convergence at the end of the game. The challenge is to prove the balancedness of the team trip planning game without defining the rules governing the coalition interaction process or conceptualizing the game environment. However, these issues will be intentionally left open at this juncture. Alternatively, to prove the balancedness of the developed game, both the cooperative and non-cooperative sides of this game will be approached.

In general, the non-cooperative game can be described as a general form of cooperative games. Many non-cooperative games have some form of interaction between travellers and indeed, cooperation. In exchanging information prior to the game and revealing their payoffs, travellers are cooperating to a certain extent. In fact, it was proposed that the competitive non-cooperative equilibrium is a part of the cooperative solution- the core [80]. Using this argument, the first proposition of this thesis can be stated.

Proposition 4.2.1. *If the non-cooperative version of a cooperative game has an equilibrium, this equilibrium is a part of the core and the cooperative game is balanced. In other words, the stability of a non-cooperative game is a proof of the balancedness of its cooperative counterpart and* $\sum_{i \in S} x_i(S) \geq v(S)\ \forall S \in 2^N \setminus \varnothing$.

Given that there is a solution for the trip planning optimization function, each traveller will have an equilibrium. Their chosen actions are considered pure strategies, and pure equilibria may exist. This equilibrium refers to an outcome where traveller wishes to unilaterally change his/her optimized solution (A detailed proof is found in [81]). This is true for non-strictly competitive games. As the game progresses, travellers may compete to gain resources to the point that one traveller's gain is another traveller's loss, (i.e., a zero-sum game). This can be overcome if trip re-planning is allowed, which leads to the the following definition and the subsequent proposition.

Definition 4.2.1. *A traveller-centric mobility planning game is a game in which each traveller performs trip re-planning every t time. There is a probability p^k that the traveller will proceed according to the previous plan or $(1-p^k)$ that he/she will move to a different plan (i.e., adopt different strategy). p^k is distributed over the set of all strategies used by each traveller in this game.*

Proposition 4.2.2. *The mobility planning game proposed in Definition 4.2.1 is a mixed game for which at least one mixed equilibrium always exists.*

Proposition 4.2.2 is made with accordance to Nash's definition of mixed strategy equilibria in [82]. Furthemore, based on Proposition 4.2.1, Definition 4.2.1, and Proposition 4.2.2, it can be stated that the cooperative team planning game has at least one solution set in its core, meaning that the game is balanced. That solution set in the core corresponds to the solution of the mixed Nash equilibrium. Although there are other factors that are often studied when analyzing a cooperative game (such as the investigation of all possible sub-games to determine if a game is completely balanced as well as the investigation of the nucleus of a game), for the purpose of the research in this thesis, the important issue is the existence of a solution. This has been established by proving the stability and the balancedness of the described game.

Even if a solution for a game exists, the discovery of that solution could prove to be challenging. The next section describes a bargaining model which is developed as a solution platform for the team trip planning game.

4.3 Bargaining Based Trip Planning Game

Bargaining models are interesting in the sense that some parts of the literature work view them as non-cooperative games [60], while other parts regard them as cooperative games [59, 83, 84]. Thus, the relationship between the cooperative and non-cooperative solution, used for our proof of balancedness/stability, is clearly prominent in this model.

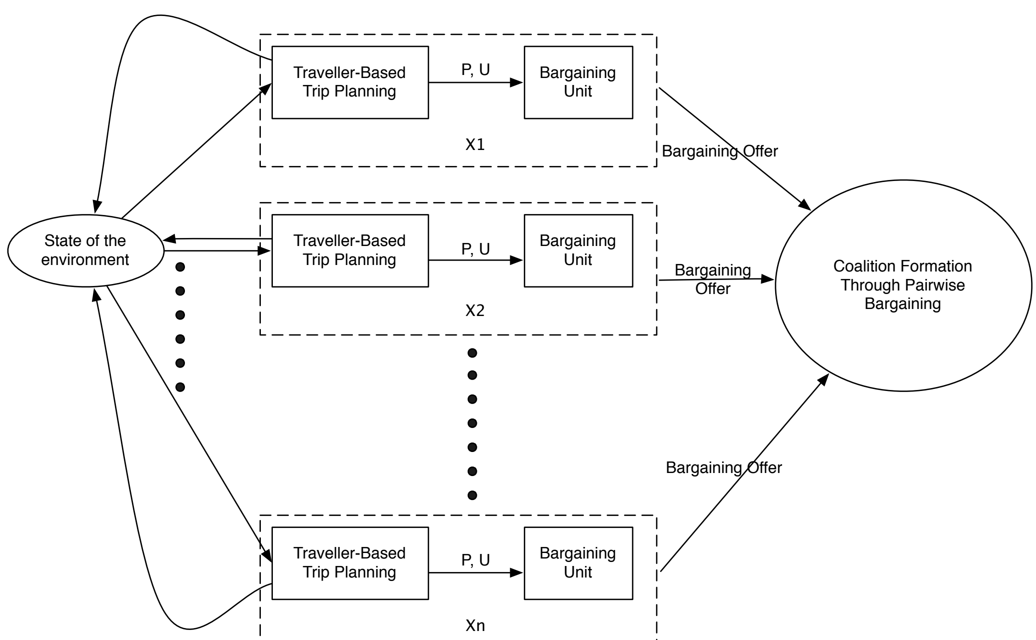


Figure 4.1: Game theoretic framework using the bargaining model.

As aforementioned, a solution for a cooperative game is represented by its core. However, as noted in Chapter 2, there is the problem of finding the elements of the core. In this research work, a bargaining model can be used as a tool for revenue-sharing and cost-allocation (i.e., the bargaining model should provide us with elements of the core). The bargaining model is described in Algorithm 1.

Algorithm 1 The Bargaining Model

procedure COALITION–CREATION
Broadcast P^{tr_i} to other travellers
  for each traveller tr_i **do**
Compute $(|(u(tr^{init}(p)) - u(tr^{init}(\hat{p})))|)|tr_{-i}$
Identify a potential coalition partner
Compute the conflict factor $(|(u(tr^{init}(p)) - (u(tr^{init}(\hat{p})))|tr^{resp}(tr^{resp} = tr_i, i \neq 1))|)$
Compute the threat factor ϑ^{init}
    if p_{resp} responded with an offer **then**
Compute $\vartheta^{respond}$
      if $\vartheta^{init} < \vartheta^{resp}$ **then**
tr^{init} will choose $p^{init} = p_i, i \neq 1$
      else
tr^{resp}will choose $p^{resp} = p_i, i \neq 1$
      end if
Call The Negotiation Procedure
    else
tr^{init} creates a singleton coalition
    end if
Broadcast P^{s^i}
  end for
end procedure

For the proposed bargaining model, it is assumed that the set of strategies P is compact and convex. P is determined through a traveller-centric trip planning module, as seen in Figure 4.1. Furthermore, for each possible point in P, each traveller has a satisfactory payoff. These assumptions are important to guarantee the existence of a solution for the bargaining game. For our game, there is 2-tuple game Σ for N-travellers. Σ has many sub games, Σ^*. These sub games are part of the original game and each of which is a 2-traveller bargaining game.

If the bargaining game is defined such that travellers can accept to pay more than what they would have paid without bargaining, then the rationality condition is violated, which might result in a non-equilibrium solution and hence, an unstable game. To avoid that, a strong rationality assumption is proposed as follows:

Assumption 4.3.1. *For a traveller, tr_i, there exists a set of strategies P^{tr_i} in which each point is less preferred than the solution p^{tr_i} such that $p^{tr_i} \in P^{tr_i}$. During the course of the game, Traveller tr_i is free to choose any strategy in P^{tr_i} which might yield less payoff, or higher cost, than that of p^{tr_i}.*

The previous assumption permits any volunteer choice made by any traveller to give in some of his/her resources to other travellers for the benefit of the group rather than the individual. Groups of travellers engaging in these bargaining games are called coalitions. Hence, the coalition in our game is created based on power play rather than fairness in resource allocation. As noted in [57] and [58], Assumption 4.3.1 does not violate the rationality axiom of cooperative game theory, but it allows for the possibility of creating a beneficial solution for all travellers.

At the beginning of the game, the travellers will list their strategies and associated payoffs. For the conflicting strategies, $\hat{P} : \hat{P} \subset P$, travellers will list the actual payoffs, caused by a conflict of interest. The traveller who has less risk when changing his/her strategy will swerve. In this game, every traveller makes his/her strategies known to other travellers. If there are contradicting strategies, travellers with such contradictions start to contact each other to resolve the conflict of interest. As such, one party will initiate the bargaining game by proposing a *deal*. The initiator, tr_{init}, will suggest that the other traveller, tr_{resp}, should swerve. Furthermore, after making

Algorithm 2 The Negotiation Procedure

procedure NEGOTIATION–ROUNDS
    for each traveller tr_i with new strategy p^{new} **do**
Compute the cost $((u(tr_i(p))|tr_{-i}(p))$
        if $u(tr_i(p^{new}) > u(tr_i(p^{old})$ **then**
            if $u(tr_i(p^j : p^j \neq p^{new}) < u(tr_i(p^{new})$ **then**
                if $\Sigma_k^N P^{tr_k}|(p_i(tr^j)) \leq \Sigma_k^N P^{tr_k}|(tr_i(p^{new}))$ **then**
Change strategy
                end if
            end if
        else
Do nothing
        end if
    end for
end procedure

the strategies, the values of the payoff/cost shown will be recalculated. The deal offered by the first party has a retaliation/threat factor $\vartheta^{tr_{init}}$. $\vartheta^{tr_{init}}$ represents the risk which tr_{resp} has to face when swerving. $\vartheta^{tr_{init}}$ can be any mixed strategy that tr_{init} might use in case the negotiation was not successful. $\vartheta^{tr_{init}}$ is computed as follows:

$$\vartheta^{tr_{init}} = \min_{\hat{P}}(|(u(tr_{init}(p)) - u(tr_{init}(\hat{p}))|, |u(tr_{resp}(p)) - u(tr_{resp}(\hat{p}))|) \tag{4.1}$$

The other party will check the offer and his/her available options for retaliation. If there is another deal in which he/she has a $\vartheta^{tr_{resp}} < \vartheta^{tr_{init}}$, then tr_{init} will be the one swerving. Each party will revisit their sets of available strategies as the negotiation progresses until an agreement is made. If $\vartheta^{tr_{resp}} = \vartheta^{tr_{init}}$, the deal offered by the initiator will be the conclusion of the negotiation. Both parties will have a binding agreement regarding their chosen strategies and they will update their sets of available strategies accordingly. Communications between both parties are maintained at all times. This negotiation process is presented as an iterative code in Algorithm 2.

To further clarify the proposed bargaining model, the bargaining model is used to solve the example described in Chapter 3.

4.3.1 Example 1:

In this example, two drivers working for the same company with different assigned tasks are attempting a trip from the same source to the same destination. According to their preferences, different paths will have different cost values. They need to minimize both of their cost values such that the collateral cost for the company is minimized. Obviously, both drivers need to cooperate to minimized their joint cost.

To simplify the analysis, both drivers will have only two strategies to choose from. They start at the same time with no re-planning (i.e., they follow the path they chose from the beginning).

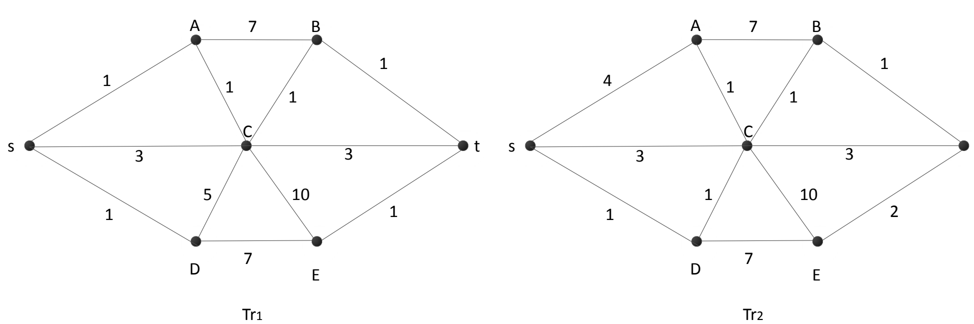


Figure 4.2: Example of two travellers travelling in the same environment.

For traveller tr_1, the chosen strategies are

$$p_1^{(tr_1)} = s \rightarrow A \rightarrow C \rightarrow B \rightarrow t \tag{4.2}$$

$$p_2^{(tr_1)} = s \rightarrow A \rightarrow C \rightarrow t \tag{4.3}$$

For traveller tr_2 the chosen strategies are

$$p_1^{(tr_2)} = s \rightarrow D \rightarrow C \rightarrow B \rightarrow t \tag{4.4}$$

$$p_2^{(tr_2)} = s \rightarrow D \rightarrow C \rightarrow t \tag{4.5}$$

When both drivers choose the same path, the price for each shared route increases by 2 price units. In this case, traveller tr_1 will communicate his/her strategies, $\left\{p_1^{(tr_1)}, p_2^{(tr_1)}\right\}$ and the associated original costs of $\{-4, -5\}$. These prices may increase to $\{-8, -7\}$ if both drivers choose similar routes. tr_1 offers a deal in which tr_2 will choose $p_2^{(tr_2)}$, estimating $\vartheta^{tr_{init}} = -1$. tr_2 has no better offer, and since he/she did not initiate the bargaining game, the *deal* offered by tr_1 is approved. We can verify the optimality of their agreement by analyzing this bargaining game as non-cooperative. If an action and response table is constructed, as shown in 4.1, it can be seen that in the case either tr_1 or tr_2 chooses the best strategy, while the other chooses the second preferred strategy, we will have an outcome of Pareto optimality. This result is in agreement with Nash's axioms for the bargaining game [58].

Table 4.1: Action-response table for tr_1 and tr_2

Traveller	$p_1^{(tr_2)}$	$p_2^{(tr_2)}$
$p_1^{(tr_1)}$	$\{-8, -8\}$	$\{-4, -5\}$
$p_2^{(tr_1)}$	$\{-5, -4\}$	$\{-7, -7\}$

With regards to the increase in the cost value, when analyzing the traffic impact along a path, the path price is usually represented as a function of the traffic. There are several suggestions in the literature with regard to the path cost functions ranging from linear functions to M/G/1-based representations [85]. In this chapter, for simplicity, the increase in the cost value is restricted to a linearly added value. For more details, the research work in [85–87] provide great insights into the topic.

The described bargaining model so far is a 2-travellers model. In the next section, I present a generalization to this model to become an N-traveller mobility planning model.

4.4 N-Traveller Bargaining Game: Creating Coalitions

For a game of N-travellers, the set of available strategies should be in full dimension (i.e., the number of strategies is equal to the number of travellers) [59]. In other words, for N-travellers, each one should have a set of possible strategies in a magnitude of n. Pairwise bargaining with every traveller can be a complicated process, and probably an intractable one. Therefore, for a large number of travellers, it is better to have coalitions of travellers who have already agreed on joint strategies. Forming coalitions in which travellers share certain agreements regarding a set of actions may ease the process of finding a global bargaining agreement. Therefore, instead of having the required strategies for each traveller to be $n = N$, n can be constrained to be equal to $|S_i|$.

In our model, the N-traveller bargaining process is composed of a series of a parallel 2-travellers bargaining sub-games. In each sub-game, only two travellers bargain and find their acceptable joint strategies, P_ϱ. For instance, in Example 1, a plausible set of strategies for a coalition of (tr_1, tr_2) would be:

$$P_\varrho^S = \left\{ \left\{ p_1^{(tr_1)}, p_2^{(tr_2)} \right\}, \left\{ p_2^{(tr_1)}, p_1^{(tr_2)} \right\} \right\} \tag{4.6}$$

In general, for a coalition, S_i, we can define the set of available strategies for the coalition, $P_\varrho^{S_i}$, as the winning strategies in the bargaining process. Moreover, each traveller needs to be a part of a coalition either by joining another traveller and forming a coalition, or by joining an already existing coalition. Travellers join coalitions to increase their gains or minimize their losses. Therefore, if any traveller found that the coalition he/she joined will devalue his/her utility function, he/she can secede from the coalition. Moreover, if there is no coalition that improves a traveller's utility value, then the traveller can form a singleton coalition with his/her preferred strategy. By the end of the game, the travellers who are in single coalitions will have to join the grand coalition, S_{GND}, based on a binding agreement.

The developed team trip planning game is an additive game. Thus, existing coalitions will have a utility value representing their joint gain/loss, which is computed as follows:

$$u_{S_i} = \sum_{j}^{|S|} u_{tr_j}. \tag{4.7}$$

Each coalition will choose a representative to run the negotiation with other coalitions or travellers. The representative is the one with the maximum risk incurred if he/she deviates from the chosen strategies of the coalition. The bargaining is ran according to the changes of the coalition utility value u_{S_i}. The coalition with the least risk of loss will swerve. The coalitions that, when merged, achieve the least deviation risk will swerve and new u_{S_i} will be computed. Furthermore, the bargaining model should be able to accommodate a situation in which travellers do not have an action set of a full dimension. Since our model is eventually reduced to a 2-person bargaining game, each traveller should have at least a 2-D set of actions. The coalition-based bargaining model is further explained in the following example.

4.4.1 Example 2:

In this example, we have four travellers commuting along the routes shown in Figure 4.3. It is decided that no matter which path is chosen, if more than one traveller choose the same subset of road segments, there is an extra cost of 1 price unit for each traveller per each congested road segment. If the travellers were to choose the least preferred path, this change of plan will incur a loss of 1 price unit. The paths available for each traveller are indicated as the following:

$$p_1^{tr_1} = b \rightarrow d \rightarrow f \rightarrow j \rightarrow p \rightarrow q \tag{4.8}$$

$$p_2^{tr_1} = a \rightarrow s \rightarrow c \rightarrow i \rightarrow l \rightarrow r \tag{4.9}$$

$$p_1^{tr_2} = b \rightarrow d \rightarrow f \rightarrow j \rightarrow p \rightarrow q \tag{4.10}$$

$$p_2^{tr_2} = a \rightarrow s \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \rightarrow r \tag{4.11}$$

$$p_1^{tr_3} = b \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \tag{4.12}$$

$$p_2^{tr_3} = a \rightarrow s \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \rightarrow r \tag{4.13}$$

$$p_1^{tr_4} = b \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \tag{4.14}$$

$$p_2^{tr_4} = a \rightarrow s \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \rightarrow r \tag{4.15}$$

The game progresses according to Algorithm 1 and 2. For tr_1, the traveller who poses the highest degree of conflict is tr_2 since both travellers share the same path. tr_2 also identifies tr_1 as the source of the highest conflict, and therefore, both travellers start to communicate. At the same time, tr_3 views tr_4 as the biggest threat and communicates with him/her to initiate the bargaining process.

tr_1 and tr_2 share the same preferred path, which means that both traveller will have to pay extra 6 price units for sharing a path. However, if tr_2 changed his/her strategy, both travellers will have to pay 2 price units plus an extra 1 price unit for tr_2

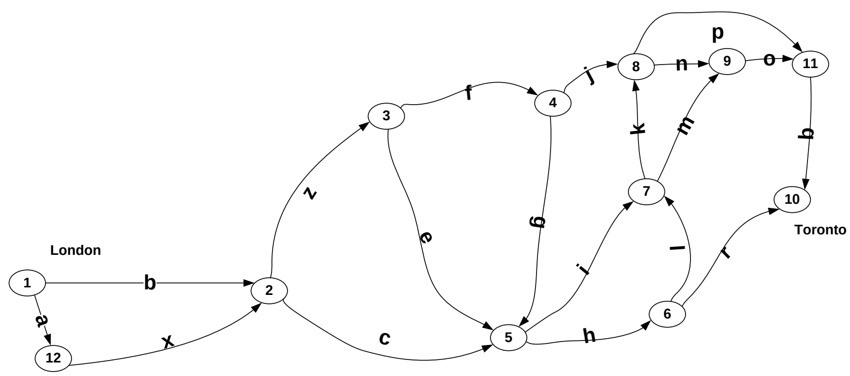


Figure 4.3: Graph representing possible paths.

as a penalty for changing his/her preferred route. Since this is tr_1's initiative, we will consider the 3 price units as tr_2's own utility cost. According to this arrangement, the first coalition is established:

$$S_1 = \{tr_1, tr_2\} \tag{4.16}$$

$$P_{\varrho}^{S_1} = \left\{\left\{p_1^{(tr_1)}, p_2^{(tr_2)}\right\}, \left\{p_2^{(tr_1)}, p_1^{(tr_2)}\right\}\right\} \tag{4.17}$$

For tr_3 and tr_4, the situation is different. Both travellers have the exact same preferred path with conflict cost of 5 price units. If any of them swerved, the swerving party will still have to pay the price of 6 units, and the other party will pay 5 price units. Therefore, in accordance with the approach used for S_1, while assuming that tr_3 is the initiator, the following coalition is established:

$$S_2 = \{tr_3, tr_4\} \tag{4.18}$$

$$P_{\varrho}^{S_2} = \left\{\left\{p_1^{(tr_3)}, p_2^{(tr_4)}\right\}, \left\{p_2^{(tr_4)}, p_1^{(tr_3)}\right\}\right\} \tag{4.19}$$

According to this arrangement, tr_3 is gaining only 1 price unit while tr_4 is gaining nothing. However, by checking the strategy set of S_1, it is clear that for tr_3 and tr_4

to not have a conflict with tr_1 and tr_2, they have to choose $p_1^{(tr_3)}$ and $p_1^{(tr_4)}$. Thus,

$$P_{\varrho}^{S_2} = \left\{ p_1^{(tr_3)}, p_1^{(tr_4)} \right\} \tag{4.20}$$

Furthermore, since forming the second coalition was based on the formation of S_1, the following grand coalition is established.

$$S_{GND} = \{tr_1, tr_2, tr_3, tr_4\} \tag{4.21}$$

$$P_{\varrho}^{S_{GND}} = \left\{ p_1^{(tr_1)}, p_2^{(tr_2)}, p_1^{(tr_3)}, p_1^{(tr_4)} \right\} \tag{4.22}$$

4.5 Simulation Work

In this experimental work, we consider the scenario of N travellers contemplating travel plans from the same source to the same destination, as shown in Figure 4.3. These travellers have limited sets of possible actions. They have to choose, according to their own personal preferences, a set of two actions (i.e., paths). These paths are ordered according to the travellers' preferences. All travellers are departing at the same time. Furthermore, since each option has its own utility cost, and instead of including the actual route costs in the utility cost, it is assumed that each traveller has a sponsor. The sponsor is willing to pay the cost of any available path. Although, there is no additional cost for the travellers if they changed their strategies, they have to pay the plan overlapping cost. In other words, if two travellers choose the same segment, each traveller will pay 1 price unit. The team trip planning problem becomes a cooperative game for which an appropriate cost distribution among all travellers is needed. The final solution assignment is defined according to the following formulation:

$$C(c) = \left\{ x \in \mathbb{R}^N | \sum_{i \in N} x_i(N) = c(N) \text{ and } \sum_{i \in S} x_i(S) \leq c(S) \text{for each } S \in 2^N \setminus \varnothing \right\} \tag{4.23}$$

This core function is used to describe the situation in which the cost value is distributed among the travellers within a game. We observe that $\sum_{i \in S} x_i(S) \leq c(S) \forall S \in 2^N \setminus \varnothing$ to ensure game stability.

4.5.1 Simulation Environment

The simulated road network represents the highways between the city of London-ON and Toronto-ON. Road segments are assigned with cost values reflecting each traveller's personal preferences and demands. These preferences in this simulation work include travelling times, safety threats, and reported traffic speed values. Each traveller has two preferences, and for each preference, there is a generalized cost function. The paths are chosen based on temporal and monetary constraints.

4.5.2 Trip Planning for 10 Travellers

In this simulation scenario, 10-travellers are dispatched according to their personal preferences such that each traveller has only two possible paths from the source to the destination. Their chosen paths are as the following:

$$p_1^{tr_1} = b \to c \to i \to k \to p \to q \tag{4.24}$$

$$p_2^{tr_1} = a \to s \to c \to i \to k \to p \to q \tag{4.25}$$

$$p_1^{tr_2} = b \to d \to f \to j \to p \to q \tag{4.26}$$

$$p_2^{tr_2} = a \to s \to c \to i \to k \to p \to q \tag{4.27}$$

$$p_1^{tr_3} = b \rightarrow d \rightarrow f \rightarrow j \rightarrow p \rightarrow q \tag{4.28}$$

$$p_2^{tr_3} = a \rightarrow s \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \tag{4.29}$$

$$p_1^{tr_4} = b \rightarrow c \rightarrow i \rightarrow l \rightarrow r \tag{4.30}$$

$$p_2^{tr_4} = a \rightarrow s \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \tag{4.31}$$

$$p_1^{tr_5} = b \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \rightarrow r \tag{4.32}$$

$$p_2^{tr_5} = a \rightarrow s \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \tag{4.33}$$

$$p_1^{tr_6} =' b' \rightarrow d \rightarrow f \rightarrow j \rightarrow p \rightarrow q \rightarrow r \tag{4.34}$$

$$p_2^{tr_6} =' a' \rightarrow s \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \tag{4.35}$$

$$p_1^{tr_7} = b \rightarrow c \rightarrow i \rightarrow l \rightarrow r \tag{4.36}$$

$$p_2^{tr_7} = b \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \tag{4.37}$$

$$p_1^{tr_8} = b \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \rightarrow r \tag{4.38}$$

$$p_2^{tr_8} = a \rightarrow s \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \tag{4.39}$$

$$p_1^{tr_9} = b \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \rightarrow r \tag{4.40}$$

$$p_2^{tr_9} = a \rightarrow s \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \tag{4.41}$$

$$p_1^{tr_{10}} = b \rightarrow c \rightarrow h \rightarrow r \tag{4.42}$$

$$p_2^{tr_{10}} = b \rightarrow c \rightarrow i \rightarrow k \rightarrow p \rightarrow q \tag{4.43}$$

If the travellers choose not to cooperate, each traveller will attempt to selfishly use his/her preferred path. The outcome of the game, if each traveller acts selfishly, is indicated in Table 4.2. Also depicted in Table 4.2 is the outcome when the travellers act *irrationally* and chose their second preferred paths. An interesting outcome is observed when one traveller acts differently from the other travellers, in which case the final outcome may prove profitable for the traveller.

Through the use of the bargaining model, the team trip planning problem can be

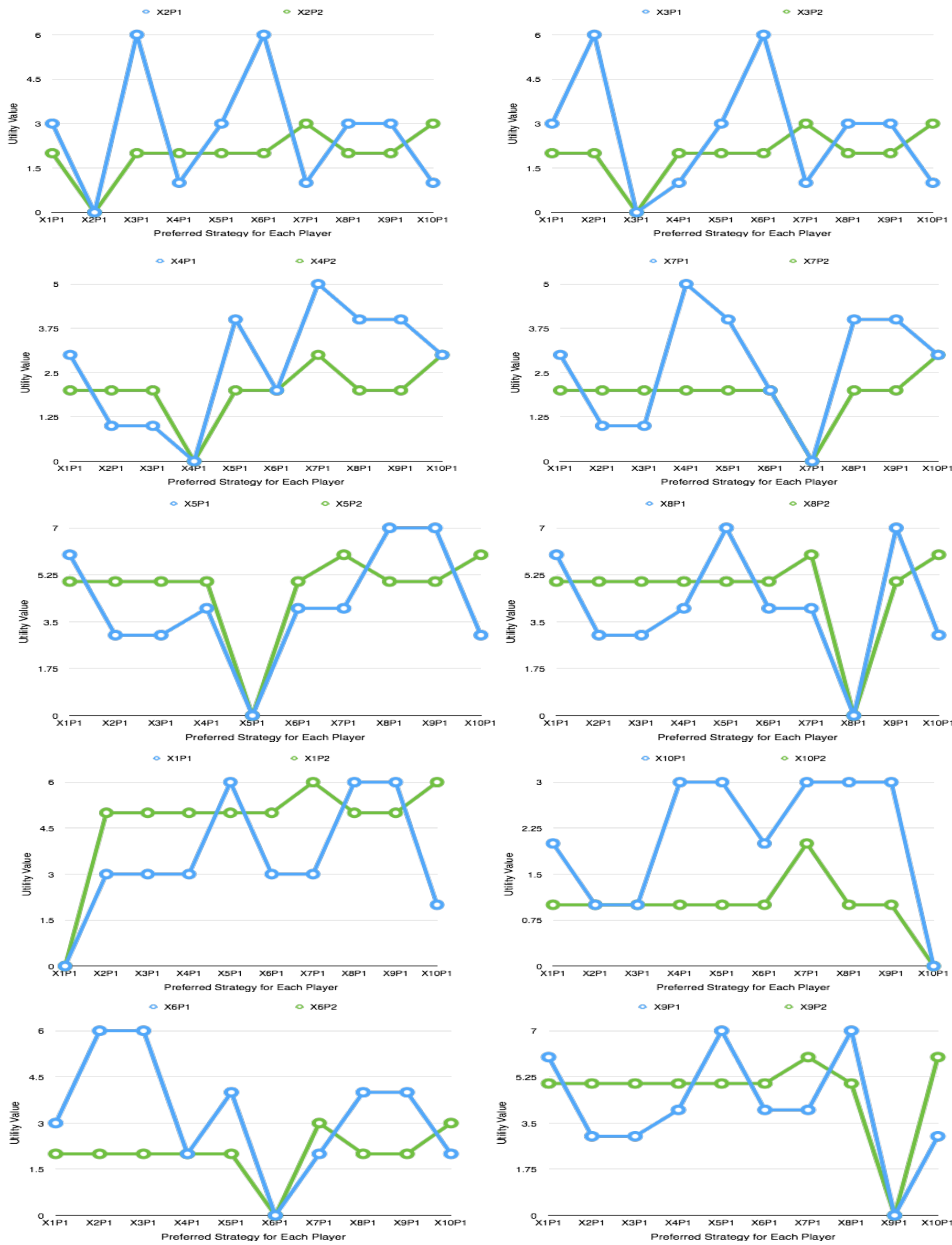


Figure 4.4: Utility values for members of potential coalitions.

Table 4.2: The outcome of selfish mobility planning

Traveller	$P_1^{tr_i}$ vs $P_1^{tr_{-i}}$	$P_2^{tr_i}$ vs $P_1^{tr_{-i}}$	$P_1^{tr_i} vs P_2^{tr_{-i}}$	$P_2^{tr_i}$ vs $P_2^{tr_{-i}}$
tr_1	35	47	47	59
tr_2	27	20	20	59
tr_3	27	20	20	59
tr_4	27	20	20	59
tr_5	41	47	47	59
tr_6	33	20	20	59
tr_7	27	19	19	46
tr_8	41	47	47	59
tr_9	41	47	47	59
tr_{10}	21	10	47	46
$U = \sum_{i,j} \omega(\eta^{tr_i}, p_j^{tr_i})$	320	NA	NA	564

solved, and a fair cost allocation scheme can be found such that each traveller will pay at most the same amount he/she would have paid if the game was non-cooperative. The first step of bargaining is to identify the potential coalitions. According to the utility values, as shown in Figure 4.4, seven coalitions are identified based on the conflict of interest as explained in Algorithm 2. Three coalitions have initially 2-travellers, while the other coalitions are singletons. The utility value of coalition members is important to identify a coalition as an actual coalition. If coalition members are better off playing independently, then they can break away from their coalition. Furthermore, if a traveller in a singleton coalition was negatively impacted due to the agreement among other travellers, he/she should be able to change his/her chosen strategy to improve upon his/her allocated cost value.

According to the chosen actions by the coalitions, as shown in Table 4.3, we have

Table 4.3: Potential coalition and their designated strategies

Coalitions	Members	Strategy
S_1	$\{tr_2, tr_3\}$	$\left\{p_1^{tr_2}, p_2^{tr_3}\right\}$
S_2	$\{tr_4, tr_7\}$	$\left\{p_1^{tr_4}, p_2^{tr_7}\right\}$
S_3	$\{tr_5, tr_8\}$	$\left\{p_2^{tr_5}, p_1^{tr_8}\right\}$
S_4	$\{tr_1\}$	$\left\{p_1^{tr_1}, p_2^{tr_1}\right\}$
S_5	$\{tr_6\}$	$\left\{p_1^{tr_6}, p_2^{tr_6}\right\}$
S_6	$\{tr_9\}$	$\left\{p_1^{tr_9}, p_2^{tr_9}\right\}$
S_7	$\{tr_{10}\}$	$\left\{p_1^{tr_{10}}, p_2^{tr_{10}}\right\}$

the following cost distribution for the grand coalition S^{GND}:

$$\{p_1^{tr_1}, p_1^{tr_2}, p_2^{tr_3} p_1^{tr_4}, p_2^{tr_5}, p_1^{tr_6}, p_2^{tr_7}, p_1^{tr_8}, p_1^{tr_9}, p_1^{tr_{10}}\} = \{30, 24, 27, 26, 36, 30, 25, 48, 36, 26\} \tag{4.44}$$

The total cost of the grand coalition under the new strategy arrangement is

$$U = \sum_{i,j} \omega(\eta^{tr_i}, p_j^{tr_i}) = 308 < 320, \tag{4.45}$$

which indicates that the new arrangement has improved the overall cost value. However, traveller tr_8 and traveller tr_{10} are paying more than what they would have paid if the game was non-cooperative. With regards to traveller tr_8, he/she is a part of coalition S_3 with a binding agreement. Thus, unilateral actions are not permitted until all other singleton coalitions have determined their final strategies. Traveller tr_{10}, on the other hand, is free to change his/her strategy to improve his/her utility value. When tr_{10} changes his/her strategy to $p_2^{tr_{10}}$, we will have the following strategy allocation:

$$\{p_1^{tr_1}, p_1^{tr_2}, p_2^{tr_3} p_1^{tr_4}, p_2^{tr_5}, p_1^{tr_6}, p_2^{tr_7}, p_1^{tr_8}, p_1^{tr_9}, p_2^{tr_{10}}\} = \{29, 28, 25, 24, 34, 29, 24, 47, 38, 22\} \tag{4.46}$$

$$U = \sum_{i,j} \omega(\eta^{tr_i}, p_j^{tr_i}) = 300 < 320. \tag{4.47}$$

This new arrangement has improved tr_{10}'s cost value allocation. However, tr_8's

Table 4.4: The outcome of third round of negotiation

Traveller	$P_1^{tr_i}$vs$P_1^{tr_{-i}}$	$P_2^{tr_i}$vs$P_2^{tr_{-i}}$	1^{st} Round	2^{nd} Round	3^{rd} Round
tr_1	35	59	30	29	31
tr_2	27	59	24	28	27
tr_3	27	59	27	25	20
tr_4	27	59	26	24	23
tr_5	41	59	36	34	36
tr_6	33	59	30	29	28
tr_7	27	46	25	24	21
tr_8	41	59	48	47	38
tr_9	41	59	36	38	40
tr_{10}	21	59	26	22	18
$U = \sum_{i,j} \omega(\eta^{tr_i}, p_j^{tr_i})$	320	564	308	300	282

utility value has not improved, and tr_2 has an added cost. Furthermore, when tr_2 attempts to change strategy, the cost value increases. The next possible action is for tr_5 and tr_8 to withdraw from their potential coalition. The following cost arrangement converges:

$$\{p_1^{tr_1}, p_1^{tr_2}, p_2^{tr_3} p_1^{tr_4}, p_1^{tr_5}, p_1^{tr_6}, p_2^{tr_7}, p_1^{tr_8}, p_1^{tr_9}, p_2^{tr_{10}}\} = \{31, 27, 20, 23, 36, 28, 21, 38, 40, 18\} \tag{4.48}$$

and the overall cost is

$$U = \sum_{i,j} \omega(\eta^{tr_i}, p_j^{tr_i}) = 282 < 320. \tag{4.49}$$

Therefore, the travellers will be paying less than or equal to what they would have paid if the game was non-cooperative. According to these results, the game is concluded, and an enforceable agreement is reached. It is worth noting that this outcome is a result of three rounds of negotiations. Other outcomes are possible if other members of singleton coalitions are allowed to change their strategies. Furthermore, according to Assumption 4.3.1, travellers can choose less favoured strategies within a coalition. Thus, if breaking a coalition did hurt other travellers, tr_5 and tr_8 would have had to re-establish the initial coalition.

As demonstrated in this scenario, and as summarized in Table 4.4, some travellers might have found themselves in a situation in which the choice to join a coalition is not different from the choice to remain alone, if not worse. Furthermore, as the number of travellers increases, the coalition quality as represented by the benefits provided to its members decreases, and the chances of having an empty core increases. This happens partially due to the lack of alternative strategies. Expanding and diversifying the set of strategies will strengthen a traveller's positions during the bargaining process.

4.5.3 Analysis of Team Trip Planning Games Parameters

There are many parameters involved in cooperative trip planning such as the number of travellers (N) and potential coalitions. Next, we investigate the decision sensitivity to these parameters.

Decision Sensitivity to Number of Travellers and Potential Coalitions:

In cooperative trip planning, the size of the team is an important factor that affects the stability of the game. Finding the optimum number of travellers is not an easy task. Ideally, there would be as many travellers as the number of unique strategies. For our case, we have 6 unique paths from the source to the destination. Therefore, to avoid the repeated use of the same path, an optimum number of travellers would be 6, which is not practical. To investigate the issue of the solution stability, the previous

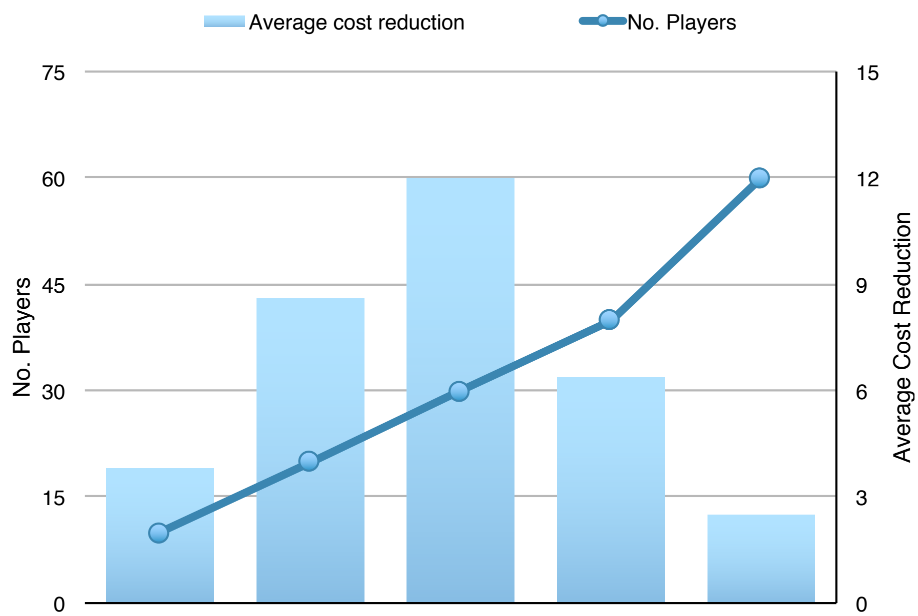


Figure 4.5: Effect of N on the cost allocation process.

experimental scenario is re-examined for different values of N. It is important to observe how the value of N affects the cost allocation process and the potential coalition creation process. Table 4.5 shows that, for various values of N, there are different average cost reduction values per traveller. As seen in Figure 4.5, as the number of travellers increases, the cost reduction value increases until N reaches a peak value (in this case 30); then, the cost reduction value starts to decrease. This can be attributed to the available sets of unique strategies. For 4 unique strategies (paths), we have 6 unique sets of strategies. It is possible that for $N = 30$, these 6 unique sets appear as strategy sets, and thus a better chance to avoid conflict arises. As the value of N keeps increasing, this advantage diminishes due to the wide adaptation of these strategies.

Furthermore, coalitions are created based on the conflict of interest and the added

Table 4.5: Effect of N on the cost allocation process

size of N	Average Cost Reduction
10	3.8
20	8.6
30	12
40	6.375
60	2.5

Table 4.6: Effect of N on the number of potential coalitions

size of N	Potential Coalitions	Formed Coalitions	Average Cost Reduction
10	3	2	3.8
20	3	3	8.6
30	6	3	12
40	6	4	6.375
60	5	4	2.5

profit. As shown in Figure 4.6, as the number of travellers increases, the chances of having more coalitions increase, which can lead to a better cost allocation. Table 4.6 presents the effect of N on potential and existing coalitions and its reflection on the process of cost allocation. The number of created coalitions is different than the number of actual existing coalitions, which had emerged according to the procedure described in Algorithm 2. Without loss of generality, the more coalitions we are able to create and keep, the better cost allocation process we will have. It can be noted that for the highest cost reduction value, the number of formed coalitions is substantially lower than its initial value. This shows that due to the method by which a coalition is chosen and maintained, the solution gets closer to its best cost allocation arrangement.

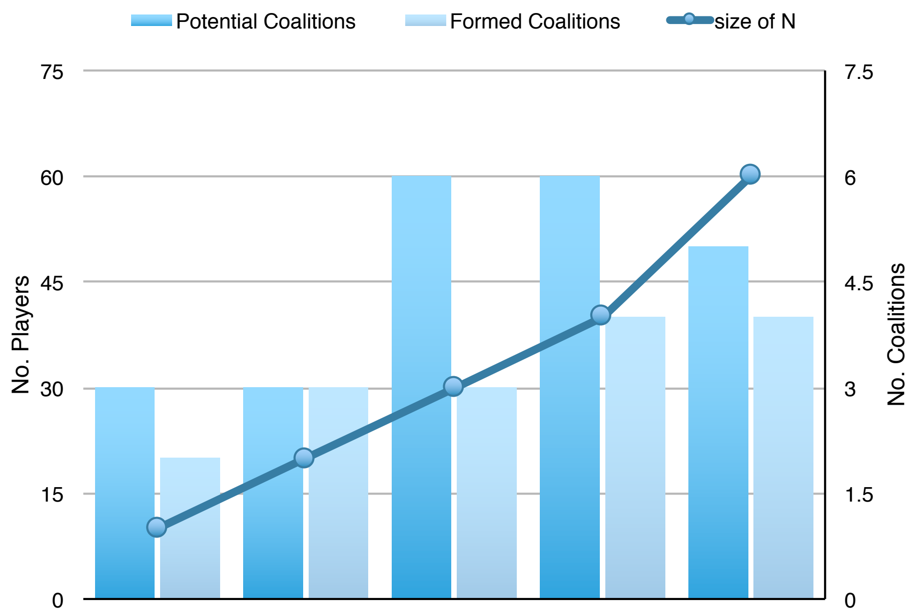


Figure 4.6: Effect of N on the number of potential and established coalitions.

4.5.4 Solution Convergence Analysis

As previously discussed, it can be seen that the final outcome resulting from our game-theoretic model is heuristic and might not be optimal. Therefore, to fully assess the developed model, we need to compare the cost allocation outcomes with the ground truth. In other words, we need to compare the heuristic strategy assignment, and the subsequent cost of the grand coalition, with the optimum deterministic overall cost. However, as the number of travellers increases, the number of possible strategies increases. For example, for N travellers with only 2 possible strategies per traveller, there are 2^N possible solutions. For more than 20 travellers, the real-time centralized-brute-force search for the best solution becomes computationally prohibited. Therefore, validating the heuristic developed solution in comparison with the ground truth is limited to $10 - 20$ travellers. In order to find the optimum strategic assignment, we need to construct more than 1 million possible permutations for 20

Table 4.7: Various study cases with 10 and 20 travellers

Selfish Game Outcome	Bargaining Game Outcome	Optimum outcome	No. Rounds
For 10 Travellers			
330	292	292	4
368	332	324	2
352	328	316	1
368	348	340	2
298	254	238	2
338	280	268	3
316	300	286	3
330	306	296	3
302	272	250	2
354	328	326	2
For 20 Travellers			
2064	1446	1406	2
1544	1456	1456	2
1554	1468	1402	1
1338	1242	1202	2
1354	1230	1180	3
1610	1508	1470	2
1756	1704	1602	1
1628	1542	1482	2
1386	1252	1190	1
1512	1384	1298	2

travellers.

The final solution of the bargaining game is evaluated in terms of its computational complexity and its proximity to the optimum deterministic solution. For the analysis of the computational complexity, it is important to understand how the final decisions are reached. There are two stages needed to find the final solution. In the first stage, travellers determine their preferred strategies through optimizing their objective functions. In the second stage, the travellers will broadcast their strategies so that each traveller can find other travellers with conflicting strategies. These travellers will start the negotiation process in order to form coalitions. The first set of agreed upon strategies is publicized. Then, if a traveller is not satisfied with the final outcome, a second round of negotiation will commence. If we want to implement a centralized solution, the first stage will remain as is. Then, all travellers will send their preferred sets of strategies and their conflict criteria to the central control unit. It is due to the second stage that the exhaustive search approach is becoming infeasible as the number of possibilities grows exponentially ($|S|^N$). Therefore, the superiority of the heuristic developed model in terms of computational complexity is established.

As for the heuristic solution proximity to the exact solution(s), Table 4.7 was established for 20 experimental instances, half of which are constructed with 10 travellers and the other half is constructed with 20 travellers. We can see that regardless of the number of travellers in the game, the solution converges in less than 4 negotiation rounds. At all times the solution was a great improvement over the worst case scenario. However, despite the major improvement and the relative closeness that our solution achieved, only in two instances out of the 20 tested study-cases the solution were equal to the global optimum.

Personalization of Preferences

It is integral to our design that the strategies chosen by the travellers are personalized. Furthermore, to have a balanced cooperative team trip planning game, it is also important for its non-cooperative counterpart to have at least one equilibrium. Therefore, travellers must be able to choose, establish, or modify their strategies based on their personal preferences such that they can engage in a selfish trip planning endeavour.

The traveller-centric trip planning module is responsible for providing each traveller with his/her individualized strategies. The travellers, if left without being enrolled in the bargaining game, should be able to engage in a non-cooperative game that has an equilibrium. The game with re-planning should have a solution that converges to Nash mixed equilibrium. Chapter 5 details the implementation of the traveller-centric trip planning module.

4.6 Summary

This Chapter discussed the existence of a solution for the cooperative trip planning game. A link was made between the cooperative trip planning game and its non-cooperative counterpart to establish the solution's existence and the game's balancedness.

This chapter presented a bargaining model as a solution for the trip planning game. This bargaining model was used for 2-traveller game and for a coalition-based game, for which cases a solution procedure was introduced. According to this solution procedure, any n-person cooperative game can be reduced to a 2-person game.

Experimental scenarios were presented in this Chapter to demonstrate the implementation of the bargaining model. The encouraging results produced by the bargaining model demonstrate the efficiency of the developed cooperative module.

Chapter 5

Traveller-Centric Trip Planning Module [1]

5.1 Introduction

In Chapter 3 and 4, it is indicated that, as a part of the rationality assumption, the players are expected to develop their own strategies based on their own travelling preferences and as such, their sets of strategies have to be ordered based on their defined individualized gains. Additionally, the argument presented for the game's balancedness has been built based on the assumption that there is always an equilibrium for the non-cooperative game. The research in this chapter is concerned with these two issues.

This chapter describes the process of designing and implementing a behavioural-driven individualized trip planning module that enables the travellers to have their own unique strategies. These strategies can be used later for the team trip planning game. The developed module is designed to be a stand-alone behavioural-driven in-vehicle guidance system, which can be used in non-cooperative games. Furthermore,

[1] The research work in this chapter has appeared in part in [88] and [89]

the module, as a part of the TMP framework, is responsible for the rationality of the cooperative planning game, as described in Chapter 3. The research in this chapter focuses on the individualized aspect of the framework and investigates the concentration problem as a non-cooperative game to validate the module's functionality and efficiency.

5.2 Non-cooperative Team Trip Planning Game: Individualized Trip Planning

In the team trip planning game, the travellers are assumed to have personal strategies that have been chosen based on personal/selfish motives. Therefore, if the players were to compete on a non-cooperative/competitive basis, the outcome will correspond to an equilibrium such that Proposition 4.2.1 holds true.

In this thesis, the travellers's strategies are defined as their chosen routing plans for their trips. However, for the team trip planning game to achieve an acceptable stable outcome (i.e., an equilibrium), the travellers should be capable of individualizing their routes according to their preferences. Although the routes suggested by conventional guidance systems can lead to a stable outcome, validating Proposition 4.2.1, the travellers might end with an outcome that contradicts the notion of self-interest. Therefore, in order to enable the travellers to choose the most appropriate strategies, the team trip planning problem is viewed and treated as a non-cooperative game.

5.2.1 Non-Cooperative Team Trip Planning Game

The game Σ is a 3-tuple (S, P, U). In this game, S is the group of travellers involved in the game; P is the set of available strategies for each traveller; and U is the set of expected payoffs. For the game to be realistic, the following conditions are to be met

- The players in S are driven by self-interest. Their ultimate goal is to maximize their personal utility functions regardless of the overall state of the network.
- The set of available strategies for all travellers, $P = p_1 \times p_2 \times p_3 \times ... \times p_N$, is rich and diverse but, ultimately, a limited one. The individual sets, p_ns, are asymmetric. In other words, travellers might have sets with different strategies at their disposal.
- The utility function, U, may vary from one traveller to another according to their best interests. For example, a traveller may consider the cost to be the journey time while another traveller may consider the cost to be the incurred financial cost.

The first condition is intuitive and plays into the notion that the players in any game are assumed to be rational. The second condition might complicate the analysis. For example, if two players, tr_1 and tr_2, have two sets of strategies such that $P_1 \cup P_2 = \phi$, the two players might not be considered to be playing the same game. However, if we consider the case that there is a third player tr_3 who has strategy set P_3 such that $P_1 \cup P_3 \neq \phi$ and $P_2 \cup P_3 \neq \phi$, it is sufficient to say that tr_1 and tr_2 are indirectly playing with each other in relation to tr_3. The third condition requires further discussion. System analyses become difficult when the utilities are not of the same cost domain. If the travellers have different price assessment for the same commodities, it is difficult to formulate an expectation about the final state of the system (i.e., the equilibrium). For instance, travellers on highways are sometimes presented with two choices: toll and toll-free highways. Some travellers choose the toll-free highway according to their financial considerations while other travellers choose the toll highway according to their journey time considerations. In this example, the same commodity has two different utility values: monetary and temporal. Nonetheless, in real life, travellers do have different utilities for the same commodities. While travellers on individual levels can reason their options based on these utility values, it may prove difficult to perform system analysis. To deal with this challenge, I propose the use of two independent utility functions. The first utility function is the traveller's utility function. The traveller may choose to keep his/her function private or share it with others. The second utility function is the system's utility function for each road. The values of these functions are known to all travellers and are used for system analysis to determine the expected best and worst social cost values (i.e., best and worst equilibria).

The user's utility values are of important due to their intrinsic relationship with the personal choices of the traveller. Since the travellers would choose strategies according to their utility-values, these utility-values affect the convergence of an equilibrium, the stability of such an equilibrium, and the quality of the equilibrium. Therefore, there is a need for an individualized trip planner that provides the traveller with rational strategies and cost values associated with these strategies. Hence, the research in this chapter can be divided into two parts:

1. The first part is concerned with the design of an individualized trip planner. The goal is to have a personalized in-vehicle guidance system that allows the travellers to selfishly plan their trips. Although the various game theoretic notions will not be emphasized throughout this part, the travellers will plan their trip according to their best understanding of the state of the environment. In other words, their strategies will be to the best of their expectation, which should allow for Nash equilibrium to converge. If replanning is allowed, a mixed Nash equilibrium should converge.

2. The second part is an analytical one in which the state of the network is inves-

tigated to validate the efficiency of the converging equilibrium. I will use the traffic concentration problem discussed in Chapter 2 as a use-case to demonstrate the (in)efficiency of the developed solution.

Therefore, as a part of the Team Mobility Planning (TMP) framework, I propose a Traveller-Centric Trip Planning (TCTP) module: a novel approach for trip planning that accommodates soft and hard routing criteria. I use the term Traveller-Centric to describe the process through which the traveller's personal planning preferences influence the routing process. The routing choices will be used as strategies in the cooperative game.

5.2.2 Traveller-Centric Trip Planning (TCTP) Module

The individualized trip planning process is executed over two stages. In the first stage, feasible road segments are identified and assessed based on the travellers' routing preferences. The feasible areas are determined through an Advanced Traveller Information System (ATIS). ATIS service providers offer free/subscription-based services to provide a trip planner with updated maps and online/offline traffic information [90, 91]. Few examples of ATISs are TomTom, Google, HERE, INRIX, CoPilot, WorldNavigator, and Ontario HighWays Maps. The road assessment criteria are assumed to be stated by the traveller as linguistic negotiable concepts. For example, a traveller could use a linguistic concept to represent demands such as "high speed" and "congestion free roads". Based on these demands, the system will evaluate the various road segments. A hierarchical fuzzy inference engine is used to compute the cost of each road segment. The cost of the feasible routes are then mapped into linguistic concepts: "*Recommended Route*", "*Marginally Recommended Route*", and "*Not Recommended Route*". The traveller's influence on the system's interpretation of what constitutes an optimum route is represented using the "Traveller's Doctrines". A traveller's doctrine is a set of beliefs that captures the traveller's perception of what is important and significant while planning a trip.

The second stage of the TCTP module is an optimization process through which the traveller's hard demands (constraints) are brought into play in determining the optimum route. There are several possible hard demands that can be used such as the latest accepted arrival time and the temporal and the monetary constraints [92, 93]. Next, I formulate the individualized aspect of the team trip planning game.

5.3 Problem Formulation

We consider a traveller tr_i, contemplating a trip from an initial location s^{tr_i} to a final destination location f^{tr_i}. The trip from s^{tr_i} to f^{tr_i} can be made along one of a set of feasible routes $R(s^{tr_i}, f^{tr_i})$, as shown Figure 5.1. For each route $r \in \mathrm{R}(s^{tr_i},f^{tr_i})$, a set of attributes $A_r(s^{tr_i},f^{tr_i})$ is defined. A_r captures the distance $\delta_r(s^{tr_i},f^{tr_i})$, between s^{tr_i} and f^{tr_i} along route r, Trip-Time $\tau_r(s^{tr_i},f^{tr_i})$ along route r, safety index $\sigma_r(s^{tr_i},f^{tr_i})$, comfort index $\phi_r(s^{tr_i},f^{tr_i})$, and traffic consistency index κ_r. Each route r is constructed as a set of linked road segments $L_r = \{l_1, l_2, ..., l_n\}$, where the first road segment l_1 originates at s^{tr_i}, and the last road segment l_n terminates at f^{tr_i}. For each road segment $l_i \in L_r$, we define $A_r^{l_i}$: a set of attributes similar to that of the route r, vis-a-vis, $\delta_r^{l_i}$, the travel distance along road segment l_i on route r, Trip-Time $\tau_r^{l_i}$, safety index $\sigma_r^{l_i}$, comfort index $\phi_r^{l_i}$, and traffic consistency index $\kappa_r^{l_i}$. The Trip-time $\tau_r^{l_i}$ and safety index $\sigma_r^{l_i}$ are considered to be situation dependent. At traveller tr_i's disposal is a set of transportation modalities $TM = \{\ tm_1, tm_2, ..., tm_m\}$. For each transportation modality tm_i and route $r \in R$, we define a cost function $\xi_r^{tm_i} = f(L_r)$. The cost of the trip from s^{tr_i} to f^{tr_i}, denoted by ξ_r, depends on the route taken,

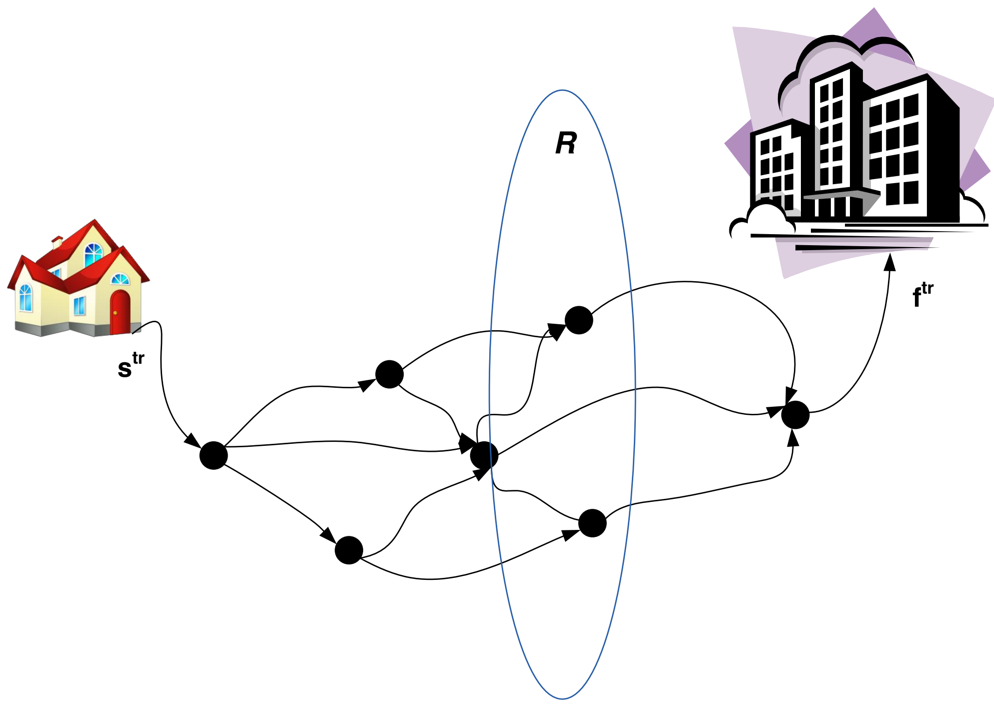


Figure 5.1: Depiction of a map from initial point s^{tr_i} and destination point f^{tr_i}.

the chosen transportation modality, and the contextual information (i.e., traffic, and weather, among others). The notion of cost is multi-aspect, in the sense that it explicitly quantifies monetary costs, temporal costs, and safety costs to the extent that a multi-criteria cost formulation is employed to guide the optimization process. Since the impact and the significance of each aspect of the cost function is traveller dependent, I introduce the traveller's preferences and constraints (i.e., the doctrines). This doctrine is denoted by $\Gamma^{tr_i}(s^{tr_i},f^{tr_i})=\{\gamma_1, \gamma_2, ..., \gamma_m\}$, where γ_i signifies the weight that traveller tr_i assigns to a given attribute. Traveller tr_i's desirable route can be found as the following:

$$r^{tr_i}_{Opt} = \underset{\forall r^{tr_i}_k \in R^{tr_i}}{Min} \Upsilon(s^{tr_i}, f^{tr_i}, \Gamma^{tr_i}), \tag{5.1}$$

where Υ represents the process by which the optimal route $r^{tr_i}_{Opt}$ is found.

The solution to the individualized trip planning problem should be in a form of a guidance system that provides the traveller with individualized routing suggestions.

5.4 Conceptual Architecture: Traveller-Centric Trip Planning (TCTP) Module

The objective of the TCTP is to find optimum routes from s^{tr_i} to f^{tr_i}. However, the optimality of any given route is traveller-centric. The assessment of these routes depends on the travellers' preferences and constraints. Preferences are regarded as the travellers' soft demands or objective(s), while the constraints are considered to be the travellers' hard demands. Figure 5.2 depicts a high level architecture of the TCTP. A variety of sensing devices are deployed to probe traffic and gather the contextual traffic information. This information is gathered, aggregated, and used for future predictions using various ATISs. In the proposed design, it is assumed that a designated ATIS is available for the TCTP which can be used for further routing purposes. The ATIS is responsible for identifying the area of interest R as well as various contextual information pertaining to R. Snow and rain precipitation, black ice, as well as the traffic speed and road occupancy are few examples of the information that can be obtained from an ATIS service provider.

The doctrine based recommendation unit is a key component of the developed TCTP module. As seen in Figure 5.2 and described in Equation 5.1, this module will obtain the source s^{tr_i} the destination f^{tr_i} and the doctrine Γ^{tr_i} from the traveller. It will also receive the traffic attributes, A_r, and the area of interest, R, from the ATIS. This unit will compute the cost value ξ_r. ξ_r is a single value that represents the conclusion of the doctrine assessment unit according to the chosen doctrine, Γ^{tr_i}.

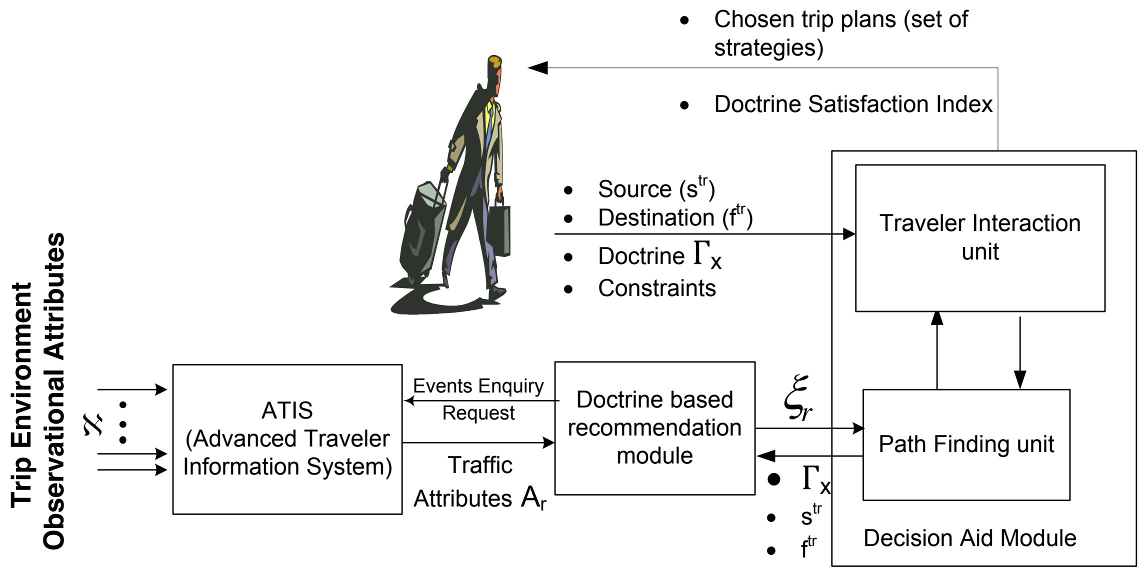


Figure 5.2: Schematic diagram of the TCTP module.

Hence, different doctrines can lead to different ξ_r values.

The traveller interacts with the TCTP module through the traveller decision aid unit. The traveller presents his/her travel preferences and monetary and temporal constraints to the TCTP through this unit. This unit will inform the doctrine-based recommendation unit about the travellers' chosen doctrine(s). Once the ξ_r values are computed, this module will determine and suggest the best route according to the travellers' expressed constraints.

5.5 Road Recommendation Assessment Using Hierarchical Fuzzy Inference Approach

Central to the road's recommendation assessment are the preferences of the travellers. These preferences may span various aspects such as safety, speed, or a weighted combination of both. By integrating the various pieces of contextual information and routing preferences, the TCTP module will perform a traveller-centric assessment to produce a recommendation value for each feasible route. This process can be computationally intractable if traditional crisp computing approaches are employed. Therefore, tools of soft computing are chosen, whereby the inputs to the doctrine-based recommendation unit are represented as linguistic/fuzzy concepts.

The criteria that are used in this research to identify the feasible routes are twofolds: 1) hard constraints such as monetary and temporal constraints, and 2) personal preferences such as road safety, reported traffic speed, and road occupancy. Road safety is defined as a function of weather conditions and road conditions. The presence/absence of snow and black-ice as well as the traffic's speed and congestion are all considered when assessing the safety of any road segment. These factors are chosen in an exemplary context, as more factors influencing trip planning can be employed, as discussed in [94].

5.5.1 Fuzzy Inference Engine in TCTP Module

The fuzzy inference engine in the TCTP module, as depicted in Figure 5.3, receives various traffic information from the ATIS. The received information is then fuzzified so they belong to predefined fuzzy logic subsets. For instance, speed is fuzzified into slow, moderate, or fast. These subsets are represented by generalized bell-shaped membership functions. The membership functions are chosen for all doctrines through a process of trial-and-error. For example, if the road segment has black ice and snow, then we would adjust the membership functions and tune the rules to have the segment assessed as unsafe. The membership functions used in the TCTP module for speed, safety and congestion as well as the road recommendation level are shown in Table 5.1.

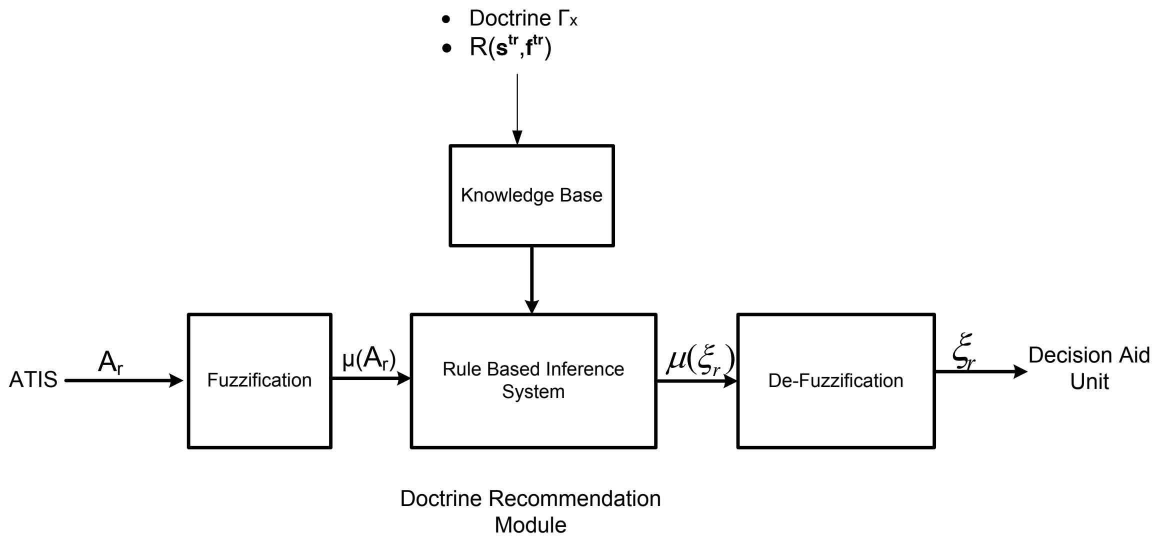


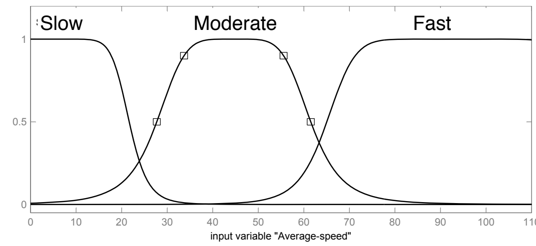
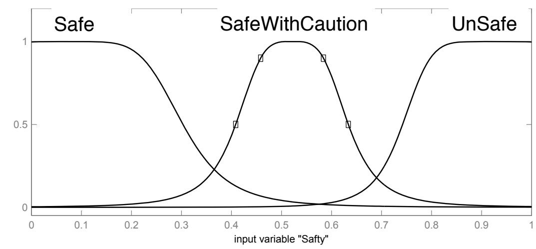
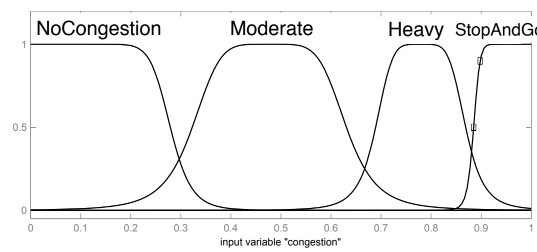
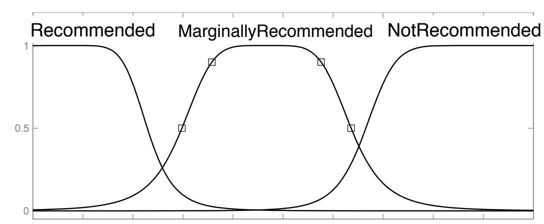
Figure 5.3: Inner schematic of the doctrine based recommendation module.

5.5.2 Concept of Doctrine in TCTP Module

Although most travellers would desire a short journey time, they also have other preferences. For example, traveller may express interest in preferences such as safety, comfortability, and scenery entertainment, among others. The TCTP module addresses these preferences with the concept of doctrine. Traveller's doctrine, Γ^{tr_i}, is a set of beliefs based on which the road recommendation unit perceives the environment. The doctrine determines the way in which the roads are assessed: negatively or positively. Doctrines allow for travellers to prioritize their preferences, which effectively leads to a change in the road recommendation cost value. Based on this change, each doctrine might produce a different preferred path for the same routing problem.

Three doctrines are defined in this chapter as strategic options for the travellers. The first doctrine is the speed doctrine: the roads that have high traffic speed are

Table 5.1: Membership function used in the TCTP system.

<table>
<tr><th>Fuzzy Logic Sets</th><th>Membership Functions</th></tr>
<tr><td>Average speed</td><td>
</td></tr>
<tr><td>Road safety</td><td>
</td></tr>
<tr><td>Congestion</td><td>
</td></tr>
<tr><td>Road recommendation level</td><td>
</td></tr>
</table>

recommended. In the speed doctrine, the highest weight, γ_i, is assigned to the average speed attribute, $S_{average}$, in each road segment. The second doctrine is named the safety doctrine: if the road is safe, then it is recommended. In this doctrine the highest weight, γ_i, is given to the safety index, $\sigma_r(s^{tr_i}, f^{tr_i})$. Nevertheless, safety is a vague concept. For simplicity, safety is defined in terms of the weather conditions that may complicate driving and compromise the travellers' safety. The third doctrine is named the compound doctrines and is concerned equally with safety and speed. If speed and safety demands are relatively satisfied, then the road segment is recommended.

5.5.2.1 The Speed Doctrine

These doctrines are implemented using the fuzzy inference engine. The speed doctrine is represented as follows:

```
IF IsSpeedLow
    Then NotRecommendedRoad
```

The speed index is given a weight that is higher than the weights assigned to the other road attributes $A_r^{l_i}$s. This means that the road congestion and safety index are still being considered in this doctrine. For example, in the case that speed is moderate and there is heavy snow, the road is regarded as not recommended. The following rule shows this relationship:

```
IF IsSpeedModerate & IsUnSafeRoad
& IsHeavyCongestion
    Then NotRecommendedRoad
```

5.5.2.2 The Safety Doctrine

In the safety doctrine, the safety index, $\sigma_r(s^{tr_i}, f^{tr_i})$, is given the highest weight among the other attributes. The following rule shows this restriction:

```
IF IsUnSafe
    Then NotRecommendedRoad
```

In addition, speed and congestion attributes are not ignored in the safety doctrine. For example, the following rule shows that for a road with a moderate safety index, it is assessed as not recommended if speed is low and the congestion is high:

```
IF IsSpeedLow & IsSafeWithCaution
& IsHeavyCongestion
    Then NotRecommendedRoad
```

On the other hand, under the same safety index, but with better speed/congestion conditions, the road is considered to be recommended:

```
IF IsSpeedModerate & IsSafeWithCaution
& IsNoCongestion
    Then RecommendTheRoad
```

5.5.2.3 The Compound Doctrine

For the compound doctrine, the following rules examine the safety index, $\sigma_r(s^{tr_i}, f^{tr_i})$, and the speed index, $S_{average}$, to formulate a road segment assessment:

```
IF IsSafeRoad & IsSpeedSufficient
& IsNoCongestion
    Then RecommendTheRoad
IF IsSpeedLow
    Then NotRecommendedRoad
IF IsUnSafe
    Then NotRecommendedRoad
```

It can be seen that, in this doctrine, equal weights are assigned to the speed and safety attributes, while lower weights are assigned to the other attributes.

The route recommendation assessment in all three doctrines requires a comprehensive understanding of the road safety and the traffic congestion assessment process. For the assessment process to be effective for all doctrines, a hierarchical approach is devised. The safety index is inferred based on two road attributes: black ice and the amount of accumulated snow. The following inference rules show an example of the safety index assessment for inspected road segments:

```
IF IsNoBlackIce & IsNoSnow
    Then IsSafeRoad
IF IsLightSnow & IsNoBlackIce
    Then IsSafeWithCaution
IF &IsHeavySnow&IsBlackIce
    Then IsUnsafe
```

Traffic congestion is another index that is inferred from known attributes: the road segment's width and occupancy. The following rules show the process of estimating traffic congestions for road segments:

```
IF IsMediumWideness & IsLowOccupancy
    Then IsNoCongestion
IF IsWideWideness & IsModerateOccupancy
    Then IsModeratCongestion
IF IsExtrHighOccupancy
    Then IsStpGoCongestion
```

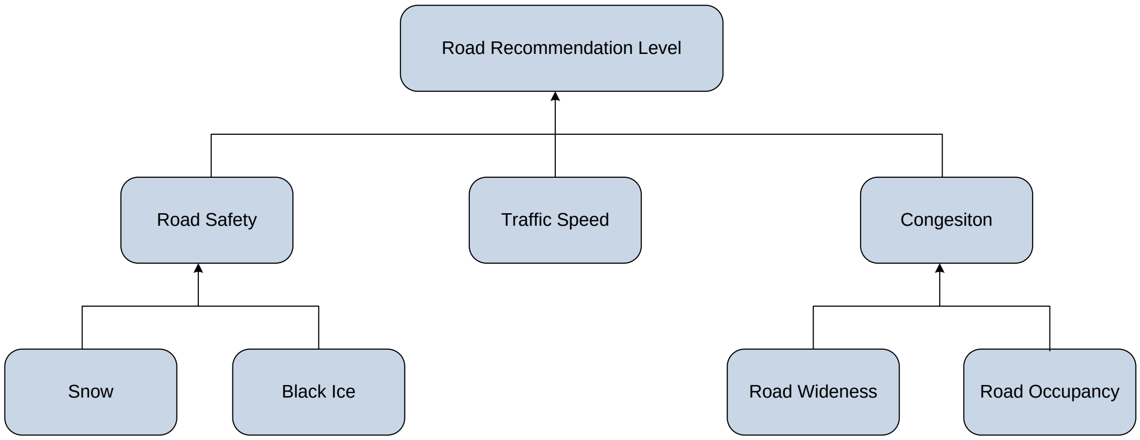


Figure 5.4: The hierarchical order in the situation assessment system.

As shown in Figure 5.4, safety and congestion as basic assessment criteria are assessed based on the data provided by the ATIS. Since more sensory/historical data can be added at the basic level, the process of producing the basic assessment values can be expanded without increasing the complexity of the assessment process. The two levels of assessment are detailed in Table 5.2, highlighting the different ranges of the input values. The output value of the road recommendation level is the cost value, ξ_r, of the assessed road segment. The hierarchical design allows for the integration of more doctrines into the TCTP module.

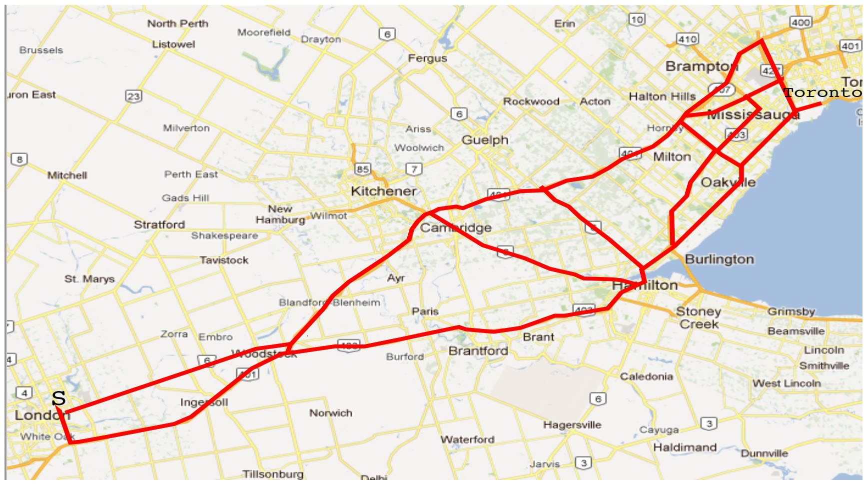
While it is the responsibility of the doctrine-based recommendation unit to assess each road segment, the optimum route is determined by the decision aid unit. In the next section, the decision making procedure is discussed.

Table 5.2: The TCTP Module's Fuzzification Scheme.

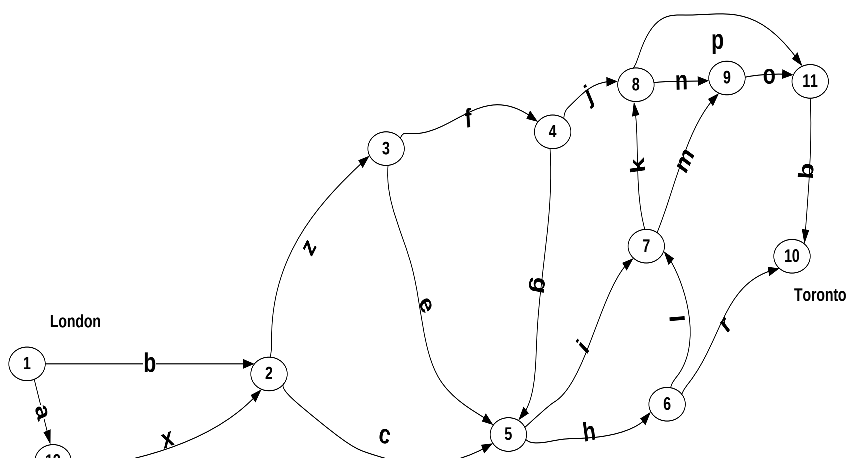
Contextual Information	Range	Membership Function	Hierarchical level
Road Occupancy	0 - 37%	1- Open	First level
	20 - 70%	2- Moderate	
	60 - 92%	3- High load	
	80 - 100%	4- Extremely high	
Road width	0 - 4 m	1- Narrow	First level
	2.5 - 6.8 m	2- Medium	
	5 - 12 m	3- Wide	
Snow degree	0 - 0.4	1- No snow	First level
	0.25 - 0.8	2- Light snow	
	0.6 - 1	3- Heavy snow	
Black ice	0	1- No black ice	First level
	1	2- Black ice	
Congestion	0 - 39%	1- Wide Open Road	Second level
	15 - 80%	2- Moderate Congestion	
	60 - 95%	3- Heavy Congestion	
	95 - 100%	4- Stop and Go	
Safety	0 - 0.55	1- Safe	Second level
	0.17 - 0.87	2- Safe with caution	
	0.6 - 1	3- Unsafe	
Average speed	0 - 35	1- slow	Second level
	15 - 80 Km/hr	2-Medium	
	50 - 120	3- Fast	
Road recommendation	0 - 0.31	1- Recommended	Output level
	0.15 - 0.75	2- Marginally Recommended	
	0.58 - 1	3- Not Recommended	

5.6 Decision-Making Procedure for Route Selection Problem

The final decision about the optimum route is approached as an optimization problem. The optimum path is the one that takes the traveller from his/her initial location to the target destination, subject to the traveller's constraints. To determine the optimum route, two factors are considered. The first is the monetary allowance for the trip, which would include the cash to be spent on gas, toll roads, and parking, among others. The other factor is the traveller's desired journey time. The TCTP module takes advantage of these factors to explore routing options that are optimum in a broad sense as they go beyond the shortest distance and the shortest time in defining optimality. Due to the doctrine's influence, the optimum route might not prove to be the one with the shortest trip time or the shortest distance.



(a) Road network map covering the area of interest taken.



(b) A schematic graph corresponding to the road network of interest.

Figure 5.5: Mapping a satellite map to graph based on the area of interest.

The decision aid unit in the TCTP module has two types of input: 1) The traveller's constraints (i.e., trip-time and trip-monetary constraints) and 2) the cost of each road segment, ξ_i. The decision is to find the optimum route, r_{Opt}, that minimizes the cost, ξ_r, subject to the traveller's temporal, τ_r, and monetary, ψ, constraints. The trip is formulated as a graph-based combinatorial problem. Road segments, L_r, are previously specified by the ATIS. Each $l_i \in L_r$ has a cost value, ξ_i, that is computed by the doctrine-based recommendation unit.

The problem of finding the best route can now be stated as follows: given a graph $G = (V, E)$, where V is the set of nodes (vertices) in the graph, and E denotes the edges in the graph, find the route with minimum cost. The graph represents an area of interest, $R(s^{tr_i}, f^{tr_i})$, that includes the starting point, s^{tr_i}, and the destination point, f^{tr_i}, of the trip. The ATIS defines the area of interest prior to the trip planning to limit the search space, as shown Figure 5.5. The goal is to find the best route, r_{Opt}, with minimum cost value, ξ_r. Travellers are using vehicles for their commute from one point to another. The trip-planning problem with preferences windows is formulated as follows:

$$r_{Opt} = \min \sum_{\forall i,j \in E} \xi_{r_{ij}} \, x_{ij} \tag{5.2}$$

Subject to:

$$\sum_j x_{ij} - \sum_j x_{ji} = \begin{cases} 1 & \text{if } i \text{ is a starting node} \\ -1 & \text{if } i \text{ is a destination node} \\ 0 & \text{otherwise} \end{cases}, \forall i \tag{5.3}$$

$$\sum_{i,j \in A} t_{ij} \, x_{ij} \leqslant \tau^k \tag{5.4}$$

$$\sum_{i,j \in A} m_{ij} \, x_{ij} \leqslant \psi^k \tag{5.5}$$

$$x_{ij} \in \{0, 1\} \tag{5.6}$$

where

ξ_r = Recommendation value,

E = Set of nodes in the net,

τ = Temporal constraint,

ψ = Monetary constraint window,

t_{ij} = Travel time over the segment ij ,

m_{ij} = Cost of travel over the segment ij ,

k = Trip query index,

x_{ij} is the decision variable representing the road segments and is defined as

$$x_{ij} = \begin{cases} 1 & \text{if the road segment is selected} \\ 0 & \text{otherwise} \end{cases} \tag{5.7}$$

The Constraint in Equation 5.3 stipulates that the driver leaves the starting point and eventually arrives at the end point and never uses the same road segment twice. The inequality in Equation 5.4 states that the trip time is never more than τ, as indicated at the query time k. The inequality in Equation 5.5 ensures that the total cost of the road segment does not exceed ψ at query time k. The last constraint in Equation 5.6 is the integrity constraint. Finally, this problem can be solved as a binary integer problem.

The objective function in Equation 5.2 is a cost function that is computed based on the chosen doctrine. Each doctrine can be viewed as an independent soft objective. In terms of hard objectives, the formulation will have to accommodate more constraints corresponding to the desired hard demands. If the optimization function was infeasible, the optimization problem becomes an unconstrained routing problem that can be solved using Dijkstra's algorithm. Next, two measures of assessment that can be used to compare the module's trip suggestions with the preferences of the travellers are defined.

Doctrine Satisfaction Index

The TCTP module provides travellers with individualized routes reflecting their preferences. For the travellers to be able to understand the quality of the suggested route, the TCTP module associate each route with a doctrine satisfaction index. Furthermore, the doctrine satisfaction index can be used within the TMP framework to order the different routes according to their corresponding doctrine satisfaction index.

For traveler tr_i, the minimum trip cost value, ξ_r, for the optimal route, r_{Opt}, is used to compute the doctrine satisfaction index, D_{S_i}. For each road segment l_i, $l_i \in L_r$, there is ξ_i, where $\xi_r = \sum_{\forall l_i \in L_r} \xi_i$. Furthermore, $\forall\ l_i \in L_r$, there is a known trip distance $|l_i|$. D_{S_i} is defined as follows:

$$|L_r| = \sum_{\forall l_i \in L_r} |l_i| \tag{5.8}$$

$$D_{S_i} = \sum_{\forall l_i \in L_r} \frac{\xi_i * |l_i|}{|L_r|}. \tag{5.9}$$

Correspondingly, road doctrine satisfaction index D_{S_r} can be categorized into four levels:

$$D_{S_r} \in \{Highly\ satisfied, Satisfied, Marginally\ satisfied, Unsatisfied\}\,. \tag{5.10}$$

An example of the computation of the route doctrine satisfaction index D_{S_r} is shown in Figure 5.6. The satisfaction levels are mapped to the three membership functions of the road recommendation unit. For instance, if D_{S_i} is computed to be 0.2, then the D_{S_r} is fuzzified as Highly satisfied.

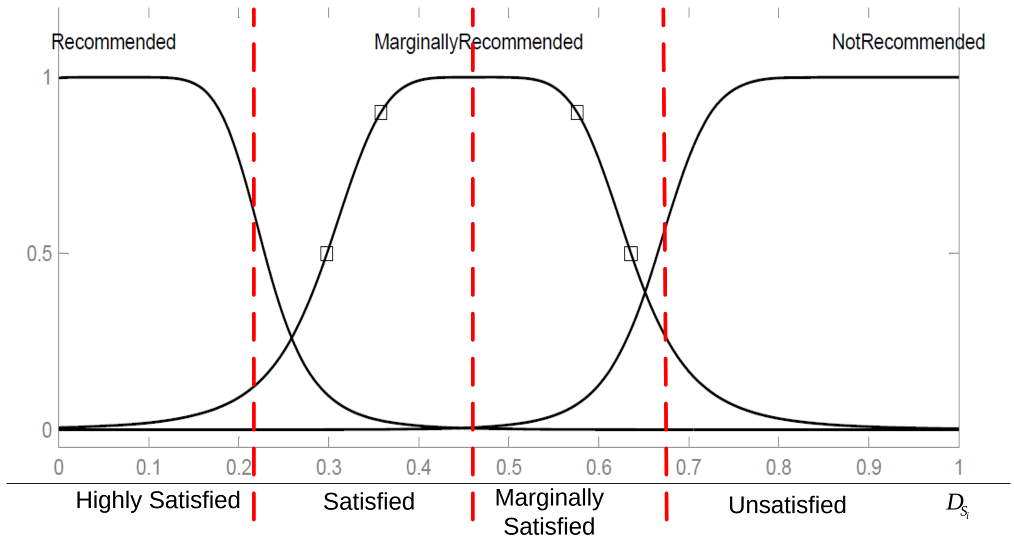


Figure 5.6: Doctrine satisfaction index computation process.

Safety-Risk Exposure Index

Additionally, for the safety doctrine, a trip safety-risk exposure index is provided. Travellers who choose to use this doctrine will be provided with figures and numbers reflecting their safety-risk exposure during the trip. The safety-risk index is computed cumulatively throughout the trip. It is possible that after a repetitive exposure to a low-risk activity, the overall safety risk can be assessed as a medium cumulative risk.

5.7 Individualized Trip Planning Effects on The Concentration Problem

The traffic concentration problem is an unavoidable side-effect of the wide-use of smart routing systems. The problem, as discussed in length in Chapter 2, occurs when multiple-travellers share the same advice regarding their routing plans. This problem is one of the flow control problems affected by two factors: 1) the traffic information; the available information represents the reality and thus can't be altered. 2) The method through which the trip planning system handles the information affects the decision making process, which may subsequently exasperate the problem. In addition, the current ATISs are planning their trips in a non-traveller-centric manner. They aim to find the route with the minimum cost without considering the overall impact of this decision on the other travellers or on the system. Therefore, the overall system performance may degrade [45]. The concentration problem is a result of this form of planning.

On the other hand, even though the TCTP module is a selfish trip planner that aims to minimize the trip cost for the traveller, the TCTP module deals with the provided information differently through the use of doctrines. The rationale is that, if travellers are allowed to affect the trip planning directly through expressing their preferences, their plans will have a better chance of diversifying the routing choices. In other words, each traveller who employs the TCTP module will have an ATIS that is tuned to his/her preferences.

5.8 Simulation Work

The TCTP module, as explained in Section 5.4, depends on various sources of information. For our scenarios, the following assumptions are made:

1. The ATIS is available and can provide the travellers with areas of interest, online traffic attributes A_r, and predictions regarding the traffic information.

2. Communications between the travellers and the ATIS are established and maintained at all times.

For the ATIS, Ontario's 511 interactive map, managed by the ministry of transportation in Ontario and shown in Figure 5.7, is used to provide us with traffic and road conditions. In addition to various weather and traffic Application Program Interfaces

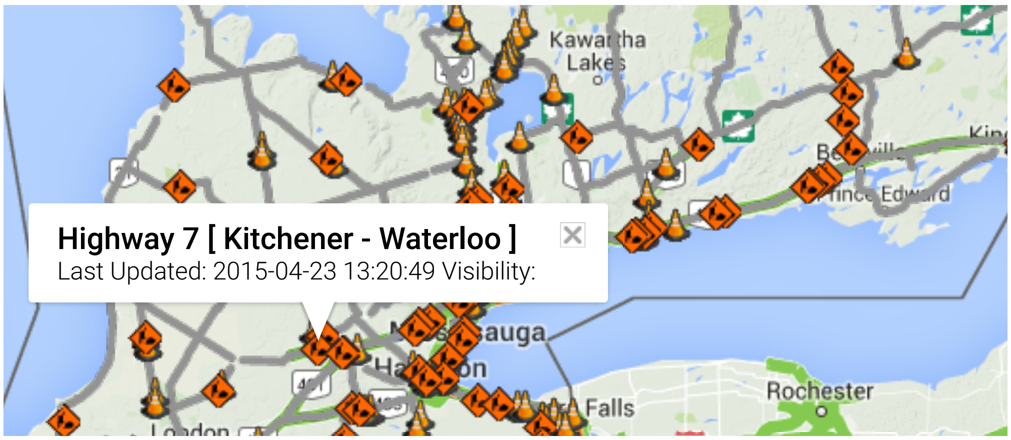


Figure 5.7: Ontario's 511 Interaction TIS map.

(APIs), Google Maps is used to obtain real-time information. The APIs provide us with areas of interest containing various routes connecting the starting point and the end point. Google Maps is known for its popularity as an ATIS, in form of web/mobile API, and therefore it has been chosen in this work for the purpose of comparison [95].

To demonstrate the efficiency of the TCTP module, an experimental work of three parts is carried out. In the first part, the functionality of the doctrine-based unit is discussed. We demonstrate the use of all three doctrines in the recommendation assessment for all road segments within an area of interest R. In the second part, the TCTP module's performance is validated by comparing it with Google Maps' performance. Furthermore, the doctrines' performance is examined under various scenarios. In the third part, the TCTP module's impact on the system's overall efficiency is investigated. For this purpose, through the investigation of various scenarios, in which the travellers are using either the TCTP's doctrines or Google Maps, the concentration problem is discussed.

Table 5.3: The implementation of the FIS for all road segments.

x_{ij}	Road Width	Snow	Black Ice	Occupancy	$S_{average}$ (km/hr)	Safety Assessment	Road Recommendation (Speed Doctrine)	Road Recommendation (Safety Doctrine)	Road Recommendation (Compound Doctrine)
a	6	low	No	med to high	20	0.17	0.73	0.47	0.73
x	9	low	No	med to low	86	0.17	0.14	0.08	0.145
b	6	low	No	low	70	0.17	0.224	0.13	0.224
c	9	low	No	med to high	47	0.17	0.45	0.4	0.45
z	9	high	No	med to low	100	0.86	0.43	0.9	0.819
e	3	medium	Yes	low	71	0.87	0.48	0.90	0.817
f	9	low	No	med to low	80	0.17	0.16	0.105	0.158
g	3	medium	No	med to low	91	0.55	0.15	0.465	0.181
h	9	low	No	high	35	0.17	0.82	0.7	0.787
i	9	low	No	low	95	0.17	0.138	0.07	0.135
j	9	low	No	med to low	100	0.17	0.135	0.07	0.133
k	9	low	No	low	80.1	0.17	0.158	0.07	0.158
l	9	low	No	med to low	82	0.17	0.165	0.12	0.175
m	9	low	No	med to high	30	0.17	0.658	0.313	0.465
n	12	low	No	high	30	0.17	0.665	0.47	0.465
o	12	low	No	high	15	0.17	0.82	0.47	0.791
p	9	low	No	low	110	0.17	0.13	0.07	0.13
q	9	low	No	high	20	0.17	0.829	0.471	0.738
r	9	low	No	high	40	0.17	0.6	0.465	0.47

5.8.1 Part I: Evaluation of The Doctrine Based Recommendation Unit

An important part of the developed system is the Doctrine Based Recommendation unit. As described so far, the unit utilizes a fuzzy inference system to produce road recommendation values in the form of road segments' costs. In this subsection, the functionality of the doctrine based recommendation unit is discussed.

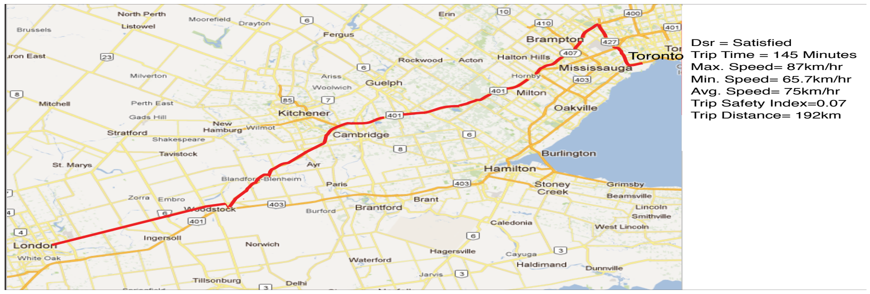
As shown in Table 5.3, for the same values of road's safety index, average speed, and congestion levels, the output varies from recommended to not recommended according to the chosen doctrine. For example, according to the safety doctrine, road segment "a" is recommended to a certain degree. The same road segment has a much higher cost value according to the other two doctrines. This demonstrates that for the safety doctrine, the safety assessment of the road has contributed the most to the final road recommendation value. Conversely, road segment "e" has a high cost according to the safety doctrine as opposed to the speed doctrine. In both cases, the compound doctrine was influenced by either the low reported speed value for “a” of the low safety assessment for “e”. Furthermore, the safety and the speed doctrine consider all attributes contributing to the routing process. For instance, according to the speed doctrine, road segment “x” had an edge cost lower than segment “z” even though the average speed in segment “z” was higher. This is due to the fact that is the safety index is higher in “z” than in “x”. Moreover, even though the safety index was almost the same for all road segments, these road segments had different cost values according to the safety doctrine due to the various values of speed and congestion indices.

5.8.2 Part II: Experimental Implementation and Results

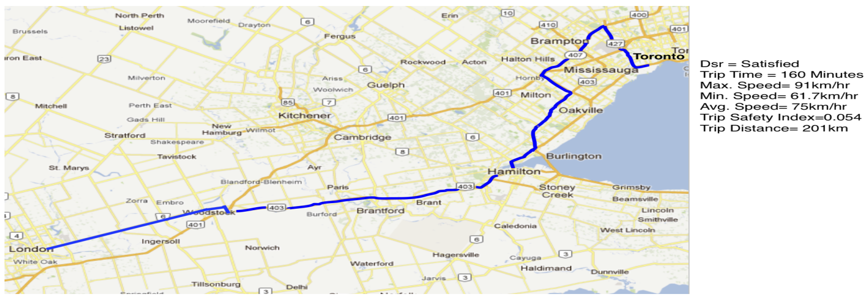
The developed TCTP module is simulated using all three doctrines. The final suggestions are compared with Google Maps’ suggestions for the same trips. Examples of the attributes of the simulated roads, as shown in Figure 5.5(b), are detailed in Table 5.4. Various attributes, A_rs, are used to simulate a dynamic environment. In the following subsections, the doctrines’ effect on the trip planning as well as the doctrine satisfaction index D_{S_r} are investigated for a variety of preferences and constraints.

5.8.2.1 Doctrine Effect on Trip Planning and Doctrine Satisfaction Index

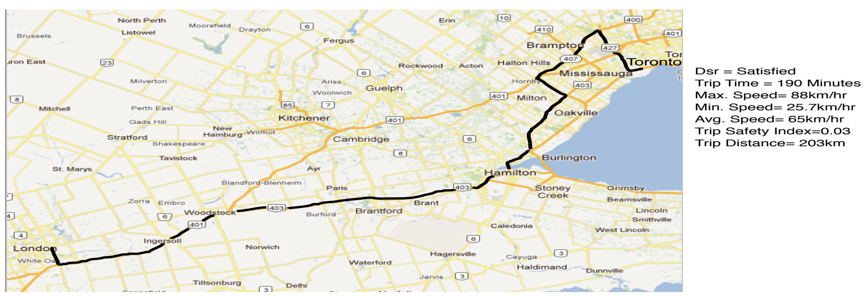
To test the optimality of the routes suggested by the TCTP module with respect to the travellers’ doctrines, three travellers equipped with the TCTP module’s available doctrines are simulated. The final routing suggestions as well as Google Maps’ suggestions are depicted in Figure 5.8. The doctrine satisfaction indices are shown in Table 5.5. It can be seen that the TCTP module provides the travellers with the routes that are influenced by their doctrine as much as possible. The doctrine-based optimum



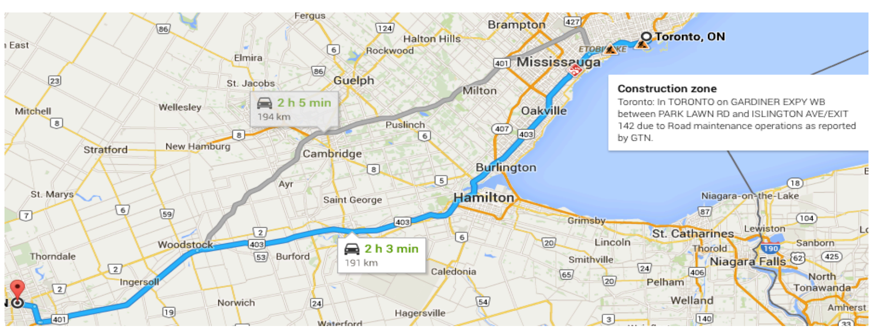
(a) Traveller using TCTP module in the speed doctrine.



(b) Traveller using TCTP module in the compound doctrine.



(c) Traveller using TCTP module in the safety doctrine.



(d) Traveller using Google Maps.

Figure 5.8: Comparison between TCTP module's doctrines and Google Maps.

Table 5.4: Examples of real-time traffic attributes A_r.

x_{ij}	S_{limit} (km/hr)	Price	Snow	Black Ice	Occupancy	$S_{average}$ (km/hr)
a	60	0	low	No	med to high	slow
x	100	0	low	No	med to low	fast
b	80	0	low	No	low	fast
c	100	0	low	No	med to high	medium
z	100	0	high	No	med to low	fast
m	100	0	low	No	med to high	slow
p	100	12	low	No	low	fast
q	100	0	low	No	high	slow
r	100	0	low	No	high	slow

routes are the ones that reflect the traveller's preferences and constraints. Therefore, in this particular scenario, it can be seen that all travellers were able to achieve an acceptable level of satisfaction. Furthermore, the safety index for all travellers can be mapped to the fuzzy set Safe as shown in Table 5.1 and Table 5.2. However, this value is normalized throughout the trip. Hence, using the speed doctrine and the compound doctrine, it is possible that the suggested routes have segments with high safety-risk index as oppose to the suggestions according to the safety doctrine.

Table 5.5: Routing feedback using the TCTP module and Google Maps.

Trip planner	Doctrine/mode	D_{S_r}
The TCTP module	*Speed*	*Satisfied*
The TCTP module	*Compound*	*Satisfied*
The TCTP module	*Safety*	*Satisfied*
Google Maps	Fastest route	NA

Even though the TCTP module is not developed to compete with the existing navigation solutions but rather to complement their shortcomings, it is found that

Google Maps' initial suggestion to be noteworthy. Google Maps suggested two routes with 3 minutes' difference between their estimated trip times. The second path is actually the same path suggested according to the speed doctrine. However, the TCTP module estimated the trip time to take 20 minutes more than Google Maps' estimation. Interestingly, the trip time provided by Google Maps can be achieved if the travellers were able to maintain the maximum speed throughout the trip whether driving through downtown areas or on the highways. This assumption might not be accurate; especially, with the presence of an accident on the highway as well as the construction in and around the destination area, as shown in Figure 5.8(d). In the following scenarios, the different suggestions of each doctrine under various monetary and temporal constraints are investigated.

5.8.2.2 Comparing TCTP Module's Different Doctrines with Open and Limited Resources

In this section, we investigate the TCTP module's performance when presented with different monetary and temporal constraints. Three travellers are dispatched into the

Table 5.6: Doctrine satisfaction levels for the TCTP module's doctrines.

Trip planner	Doctrine/mode	Safety Index	D_{S_r}
The TCTP module	*Speed*	0.4	*Marginally Satisfied*
The TCTP module	*Compound*	0.315	*Satisfied*
The TCTP module	*Safety*	0.17	*Satisfied*

simulation environment. All travellers have unlimited resources with respect to the trip monetary allowance and desired journey time. Each traveller is assigned a unique TCTP doctrine. All three travellers have to start from the same source, s^{tr_i}, and the same destination, f^{tr_i}. To clearly indicate the difference between the three doctrines, speed and safety obstacles are presented in various roads in the form of heavy snow as well as accidental delays. In this scenario, the criterion based on which a plan can

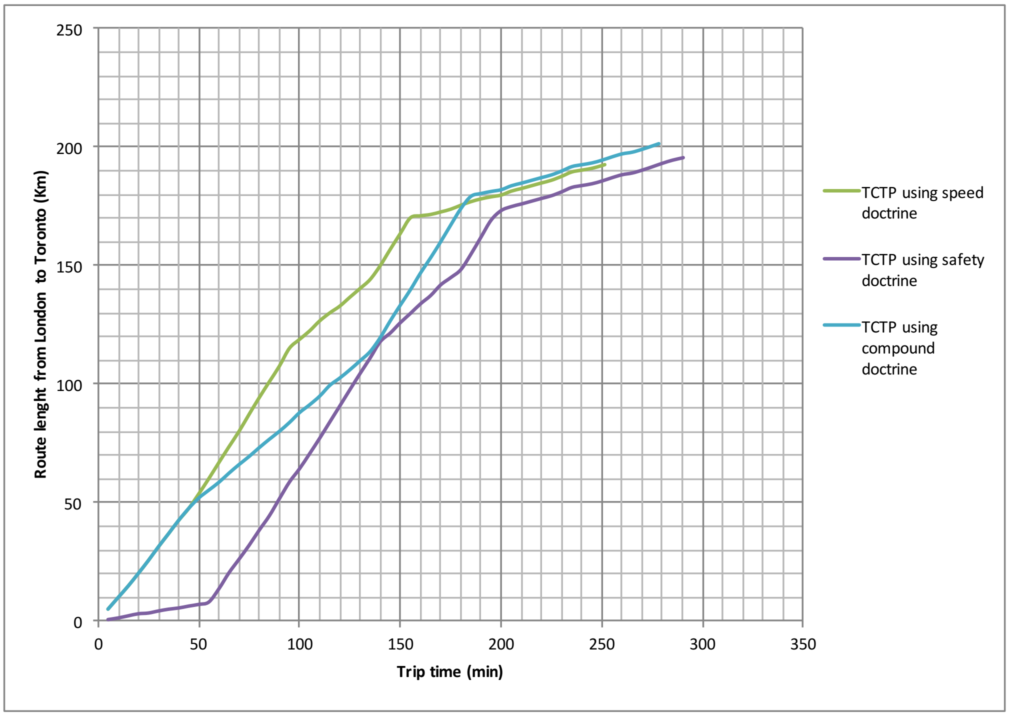


Figure 5.9: Path-planning for preference-based trip.

be viewed as successful is the doctrine satisfaction index.

As can be seen in Figure 5.9 and Table 5.6, due to the variety of obstacles made against the travellers, the trip times were noticeably longer than usual. Furthermore, travellers using the compound and safety doctrines were satisfied with the outcome since safety concerns were addressed. However, since speed is the main requirement for the traveller with the speed doctrine, the result came with doctrine satisfaction assessment of marginally satisfied. In general, the doctrine based navigation system was able to process the information differently for each traveller. Hence, it was able to produced unique suggestions matching the travellers' preferences to a great degree.

In the second use-case, we compare the suggestions provide to three travellers, all of which are using the speed doctrine. However, each traveller has his/her own set of monetary constraints. Our aim is to demonstrate the response of our system according to the hard demands of each traveller.

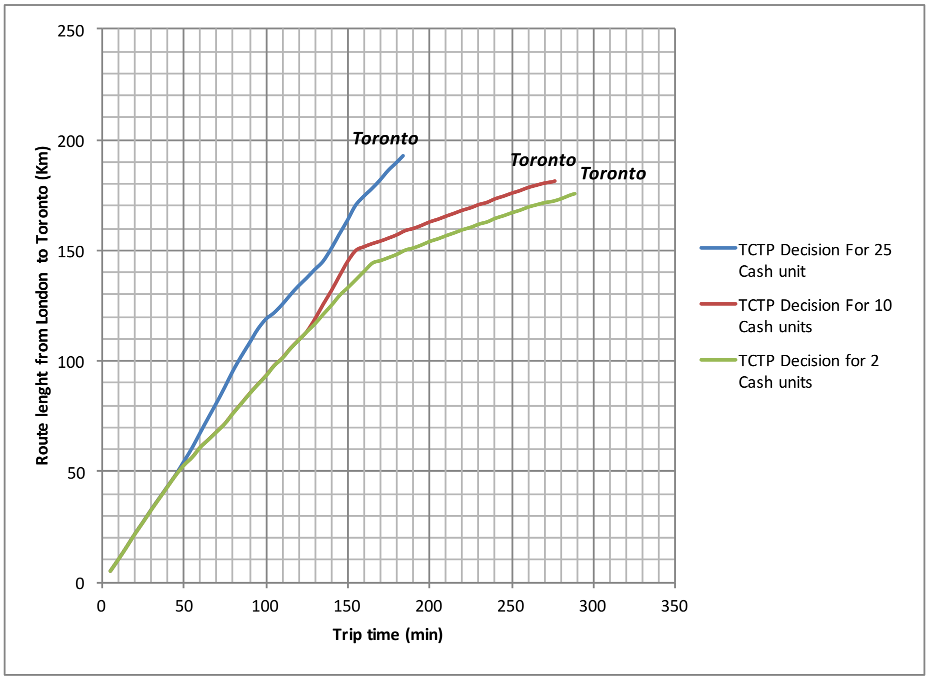


Figure 5.10: Route decision for travellers using TCTP system in speed doctrine with different monetary constraints.

In the simulated environment, certain road segments were, as indicated in Table 5.4, assigned toll cost values: when a traveller crosses these road segment, he/she pays a price for the use of that segment. Other road segments were toll-free, and no monetary cost is incurred as the vehicle traverses them.

As shown in in Figure 5.10, when the monetary allowance is relatively high, the TCTP module has a higher chance of choosing the best possible route. However, when the monetary allowance decreases, the system is forced to choose a route that is relatively slow. Since two travellers have monetary constraints of less than 12 price units, the travellers will have to avoid the toll segment ”p” and use the toll-free segments ”n” → ”o”. The journey time difference between the traveller with an allowance of 25 cash units and the traveller with an allowance of 2 cash units is about 115 minutes.

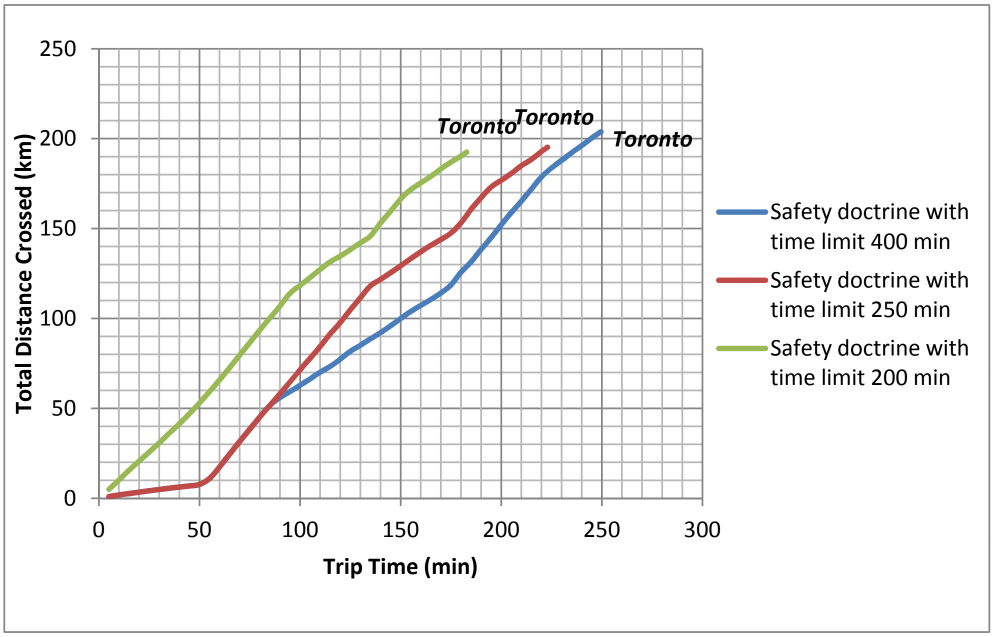


Figure 5.11: Comparison of route decisions for the TCTP module in safety doctrine for different temporal constraints.

5.8.2.3 Comparing The Safety Doctrine for Different Time Windows

The third use-case involves three travellers who are using the safety doctrine under limited constraints. The main constraint that can influence the decision of the TCTP module under the safety doctrine is the desired journey time. As shown in Figure 5.11, the travellers who are using the TCTP module with the safety doctrine were able to meet the journey time constraints. However, as shown in Figure 5.12, the level of safety-risk exposure changes according to the different temporal constraints. It can be seen that when an open journey time is permitted, the safety-risk exposure factor remains stable at a low level for the entire trip. This result shows that using

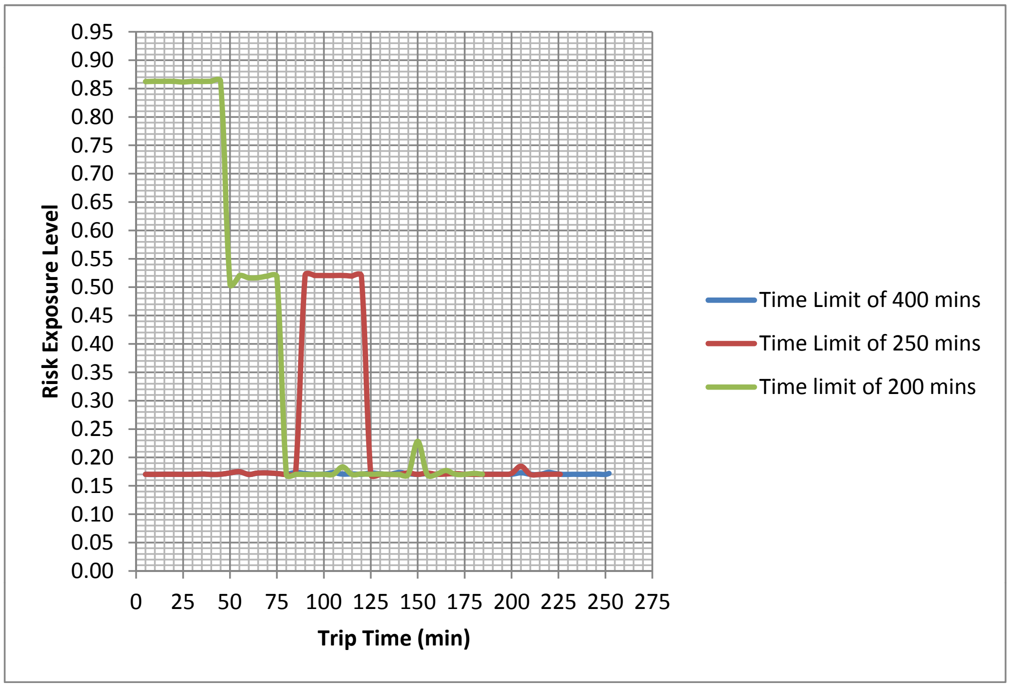


Figure 5.12: The effect of changing the time window on the safety-risk exposure.

the safety doctrine, the TCTP module functions properly by keeping the safety-risk exposure to low levels.

As shown in Figure 5.12, the first traveller with a journey time limit of 400 minutes has sustained a constant low level of safety risk exposure at all times during the trip. However, the other two travellers, who were not given an open time, had different plans for their trips. The traveller with a journey time limit of 250 minutes was exposed to constant medium safety-risk exposure for about 40 minutes. On the other hand, the traveller with a journey time limit of 200 minutes was exposed to high safety-risk exposure for 80 minutes and then low risk for another 10 minutes. The trip times for the three travellers are shown in Figure 5.11.

A comparison between the plans of the speed doctrine and the safety doctrine with a time window of 250 minutes reveals that the plans according to the safety

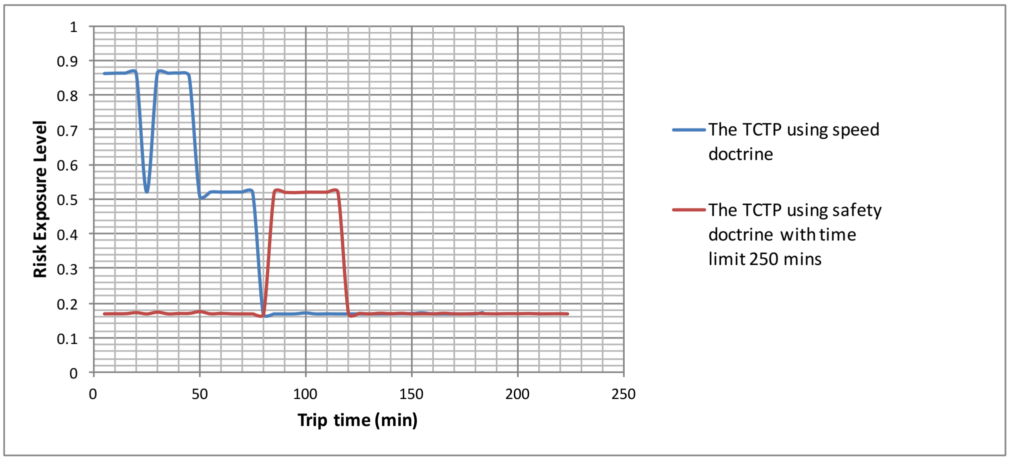


Figure 5.13: Comparison of risk exposure in the safety mode and speed mode.

doctrine have less safety-risk exposure than those of the speed doctrine, as shown in Figure 5.13. In conclusion, as can be seen in Figures 5.10 through 5.13, it can be noted that the decision regarding the best route varies according to the chosen doctrines and imposed constraints.

5.8.3 Part III: Individualized Trip Planning's Effects on Traffic Flow

In this part, we examine the effect of the TCTP module on the traffic flow of a large number of vehicles. In the previous simulation parts, we have established that the TCTP module is at least as good as any other routing system that uses online information. This means that the TCTP module can be also prone to the traffic concentration problem. The traffic concentration problem occurs when a large number of vehicles attempt to travel within the same area at the same time window and receive the same routing suggestions.

To investigate this issue, we have simulated 200 vehicles, equipped with the TCTP module, to plan trips from London, ON to Toronto, ON. For comparison purposes, another 200 vehicles are simulated to use Google Maps as their in-vehicle guidance solution. All vehicles equipped with the TCTP module are equally likely to have any

Table 5.7: System performance using Google Maps vs The TCTP module.

Road Segment	Google Maps	TCTP module's Doctrines
a	0	152
b	200	152
x	0	48
z	0	62
c	200	138
r	200	131
e	0	3
g	0	6
f	0	59
q	200	131
p	0	69

of the three offered doctrines such that each traveller randomly and uniformly chooses one of the three doctrines. As we can see in Table 5.7, when Google Maps is used, during a window of 30 minutes, it consistently offers a single preferred path: "b" $\rightarrow$ "c" $\rightarrow$ "h" $\rightarrow$ "r" (the arc leading to the destination node is "r"). The problem that can be observed is that regardless of the difference in the trip starting times, all vehicles were sent along the road segment "b". Furthermore, all of the vehicles were sent across "c", "r" and "q". This is a clear example of the traffic concentration problem. The guidance system consistently provides the same advice for all travellers within the same area until the system's traffic conditions are degraded. Additionally, there is no self-correction mechanism in deployment thus far to prevent the traffic conditions from worsening. In comparison, when giving more weight to factors such as safety, we see that nearly 25% of the traffic was directed through "a" $\rightarrow$ "x". As shown in Table 5.7, it can be observed that some travellers chose the shorter route, "b" $\rightarrow$ "c" $\rightarrow$ "h" $\rightarrow$ "r", while other travellers chose the route "f" $\rightarrow$ "j"$\rightarrow$ "p"

$\rightarrow$ "q". Next, we discuss and compare the impact of the TCTP module and Google Maps on the system's efficiency.

5.8.3.1 System Efficiency Analysis

To analyze the efficiency of the TCTP module and Google Maps, the Price of Anarchy (PoA) us used. The PoA is the ratio between the optimum system performance and the worst system performance [86]:

$$PoA = \frac{\text{Worst Equilibrium}}{\text{Optimum Outcome}} \tag{5.11}$$

The PoA ratio is referred to as the coordination ratio, which describes the system degradation caused by the travellers' selfish behaviour as well as the effect of the provided information on the decision making process [87]. Therefore, its use as a tool of analysis is deemed appropriate for our application. The analysis is performed on the sub-graph $\hat{G}$. This graph covers the following nodes in Figure 5.5: $\hat{V} = \{2, 3, 4, 5, 6, 8, 9, 10, 11\}$ and $\hat{E} = \{z, f, j, p, c, h, r, q\}$. For $\hat{G}$, we have limited the investigated paths to two routes as shown in Figure 5.14. The 200 vehicles are distributed over these two routes from s^{tr_i} to f^{tr_i}. To simplify the analysis, the transition from one route to another is neglected.

The quadratic function of $c^{l_i} = a^{l_i} \cdot (x^2 + x)$ has been chosen to represent the congestion cost function. There are several suggestions in the literature with regard to the possible cost functions, ranging from linear functions to M/G/1-based representations [85]. In general, the cost functions for all paths in one network can be of the same class or of different classes. Furthermore, the simulated network used to identify the PoA can be a simple one or an elaborately complex one. Nevertheless, with the proper reduction and approximation, the worst PoA can always be found. For better insight into these topics, the subject can be reviewed in [85–87], in which it is covered in great lengths. For simplicity, we have chosen the quadratic function to emphasize the traffic concentration problem and its impact on the system performance. This function represents a cost value that can be interpreted in terms of various factors such as congestion, CO_2 emissions in a given area, and prevalence to accidents, among others.

As shown in Figure 5.14, the two routes have different coefficient values for their quadratic functions. These values represent the length ratio between the two routes.

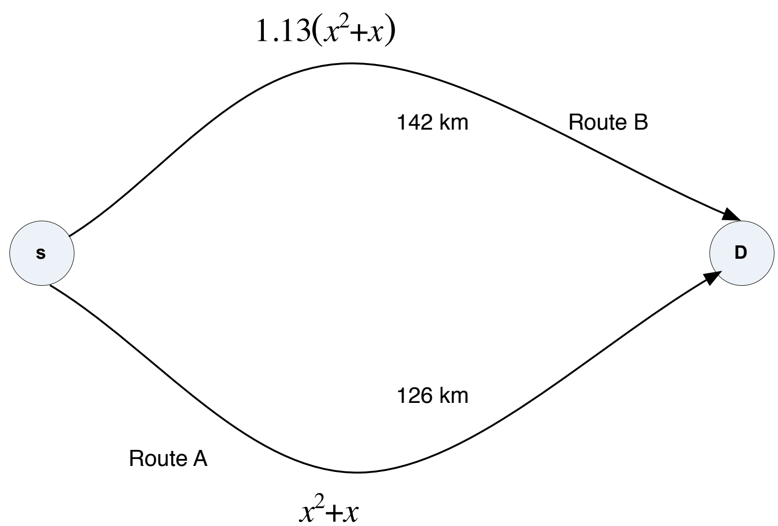


Figure 5.14: Alternative routes with quadratic latency function.

Since both routes have similar attributes except for their total distance, the decision regarding the best route from a selfish perspective is simple: the shortest path. Incidentally, this selfish solution is the same one that was suggested by Google Maps. Route A will be chosen most of the times over route B unless there is a noticeable change in journey time for route A. According to this system, all travellers will have the same advice regarding the route with the least cost, which might lead to the traffic concentration problem. On the other hand, to achieve an ideal traffic distribution,

Table 5.8: System performance analysis.

Chosen Route	Optimum Performance	Worst Performance	Google Maps	The TCTP Module
Route A	105	0	200	131
Route B	95	200	0	69
Overall cost	21,435	45,426	40,200	22,750
PoA	1	2.1	1.87	1.06

the traffic should be split among the alternative routes to minimize the overall cost. However, this will require a central control system, which is virtually impossible to implement. Even if it was possible to have a central control system, it will result in some of the travellers being offered routes against their preferences.

Due to the simplicity of this scenario, we are able to find the cost values of the optimum case scenario and worst case scenario of the traffic distribution. These values are used to set upper and lower limits for Google Maps and the developed TCTP module. As can be seen in Table 5.8, the optimum case scenario would be splitting the traffic in nearly two halves between the two routes. Nevertheless, from the travellers' perspective, some of these plans are not desired since they offer the longer route. The worst case scenario would be when all travellers use Route B, which will result in a maximum overall cost. For Google Maps, it is observed that for a period of more than 30 minutes the advice for all travellers was to use Route A, seen as the optimum route from an individual's point of view. On the other hand, the travellers were split among the two routes when they use the TCTP module. Even though most travellers chose route A, a significant number of them chose route B as their preferred path according to their preferences. This shows the effect that the doctrines might have on the traffic distribution and the overall system performance. Due to this diversity in the routing suggestions, the TCTP shows a superior PoA performance as compared to Google Maps.

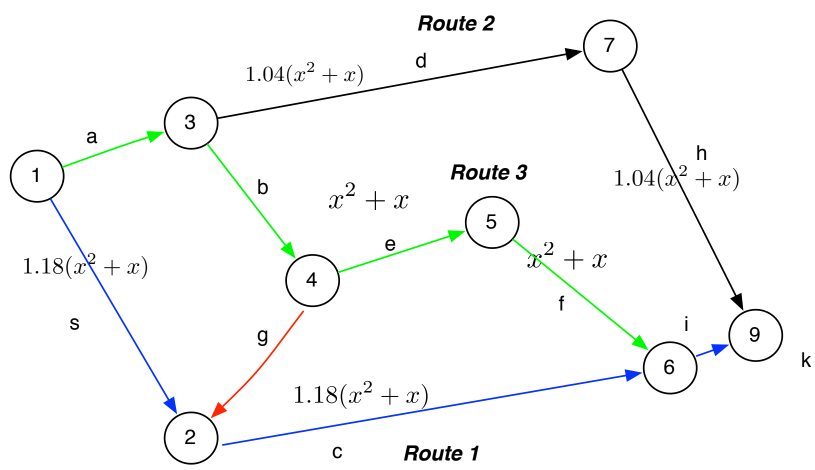


Figure 5.15: Routes with quadratic latency function for downtown areas.

5.8.3.2 The Concentration Problem in Downtown Areas

The concentration problem is often associated with traffic oscillation in downtown areas. In the following scenario, we demonstrate using 4000 vehicles in real-time simulation the differences in suggestions provided by the TCTP module and those provided by Google Maps, Microsoft's Bing Maps, and Nokia's HERE Maps. The routes and the suggested system cost function are shown in Figure 5.15.

Table 5.9: System performance analysis for downtown area.

Chosen Routes	Optimum Performance	Google Maps	Bing Maps	HERE Maps	The TCTP module
Route1	1333	1000	4000	4000	845
Route2	1334	3000	0	0	2180
Route3	1333	0	0	0	975
POA	1	2.04	2.91	2.91	1.28

As shown in Table 5.9, the ultimate strategy would be to split the traffic evenly among all three routes. However, for travellers that are relying on Bing's Maps or HERE's Maps, all traffic is directed along one path regardless of the changes that occur to the network. These changes prompted Google Maps to adjust its plans to suggest faster alternative routes. That is, the traffic is sent along one route until a congestion occurs; then, the traffic is sent to an alternative route until it is also congested. The TCTP module, on the other hand, is less affected by the oscillation problem since speed is a priority for only a portion of the population. Therefore, there is more stability in the network. Furthermore, as shown in Table 5.9, the TCTP-based PoA emerged is superior to the other trip planning solutions. Hence, the results establish the efficiency and the superiority of the converging equilibrium.

5.9 Role of TCTP Module in The TMP Framework

The TCTP Module is discussed thus far as an independent in-vehicle guidance system. For each routing query, the module produces one optimum path per doctrine. Each path has a doctrine satisfaction index associated with it. The travellers have the option of choosing multiple doctrines and arranging the suggested routes according to their D_{S_r} values. The route with the highest index is the preferred route. Therefore, the travellers are to able order their doctrines according to their preferences to preserve Nash's axiom of rationality. In other words, for each doctrine, $\Gamma_j^{tr_i}$, we have $r_{Opt}^{tr_i}(\Gamma_j^{tr_i}) = p_i$. According to the number of available doctrines and their order of priorities, we have $P^{tr_i} = \left\{p_1^{tr_i}, p_2^{tr_i}, .., p_n^{tr_i}\right\}$ where $p_1^{tr_i} = r_{Opt}^{tr_i}(\Gamma_j^{tr_i})$ such that $j = 1, ..., n$, where n is the number of doctrines.

As shown through the experimental scenarios, the equilibrium resulted from the TCTP is efficient in improving the overall social cost of the system. Therefore, the TCTP module and the TMP Framework by extension are poised to deal with the non-cooperative team trip planning game. Furthermore, the strategies created according to the various doctrines can be used in the bargaining game in the cooperative trip planning module. In the next chapter, the TCTP is used to provide the travellers with an ordered set of strategies that can be utilized in the cooperative game.

5.10 Conclusion

In this chapter, a traveller-centric trip planning (TCTP) module was introduced. A full description of the module was presented with emphasis on the doctrine based recommendation unit and the decision-aid unit. To implement the various doctrines, a hierarchical fuzzy system was developed. An optimization based approach was used to find the optimum route. Through the use of the TCTP module, travellers were allowed to be proactive in choosing their routes. The rationale behind this design is to diversify the trip planning process by invoking personal preferences into the routing process at the early stages. Therefore, the concept of doctrines was highlighted as an improvement to the existing trip planning techniques.

The traffic concentration problem was used as a prime example to demonstrate the effectiveness of using a personalized trip planner to diversify and improve the network's performance. Hence, the quality of the emerged equilibrium was established.

Chapter 6

Treatment of The Territory Sharing Problem [1]

6.1 Introduction

In this chapter, I develop a multi-travellers resource sharing game: the Territory Sharing Game. This game is formulated such that the travellers can achieve through cooperation a regret-free outcome that guarantees the welfare of the system. The proposed cooperative model is extended to allow for the deployment of no-regret dynamics and as such, the final outcome will converge to a coarse correlated equilibrium. Additionally, the previously introduced notions of solution existence and stability are revisited. Several simulated scenarios are introduced to demonstrate the effectiveness of the Team Mobility Planning framework.

6.2 Social Taxi Networks

Triggered by the ubiquitous use of information technology, a new economical business model has emerged: the sharing economy. The sharing economy is a platform in which

[1] The research work in this chapter has appeared in part in [96] and [97]

people cooperate using technology to share what can be otherwise an un-utilized inventory on fee/non-fee basis [98]. One application of interest is the social taxi networks [99–102], known as the ride-sourcing platform and free-floating car sharing systems. I coin the term social taxi networks to describe ride-sourcing applications where the service is delivered to riders by utilizing a network of private vehicles. Taxi drivers in these networks communicate with their customers through smartphone applications (Apps). Lyft and Uber are prime examples of such social networks [103]. In this particular sense, the social taxi is similar to a traditional taxi since the provider of the service does not share the destination with the service user, as opposed to ride-sharing applications [104].

The treatment of the territory sharing problem can be observed in literature when examining some of the relocation algorithms for traditional taxis [105] as well as the ride-sharing applications [106]. The main goal of these algorithms is to (re)distribute service providers, such as taxis, in a manner that guarantees that the maximum number of customers is served.

With regards to traditional taxis, the operator has complete control over a fleet of vehicles. Thus, the relocation of vehicles is subject to cost/benefit criteria. This model is similar to the models proposed for some of the social ride-sharing platforms. For example, in [107], Weikl and Bogenberger investigated the possible relocation strategies of the so-called free-floating car sharing systems. They describe two possible strategies: user-based and operator-based. According to the user-based strategy, the users are incentivized by lower costs to use the system despite its long waiting times and intermittent service availability. Nevertheless, not all customers can be influenced by such incentives. According to the operator-based strategy, the number of service-providers is increased to cover all areas. This case assumes that there is complete control over the service providers. The problem with using these two models in social taxi networks is that they do not accommodate the autonomy of the service-providers. For instance, a vehicle owner may prefer to operate in a low density residential area due to its close proximity to their living/work place. On the other hand, most traditional taxi companies might regard that area as a low priority area due to its low population density. If social networks had the same relocation model, the low density areas will be out of coverage.

In the current social taxi networks, service-providers and service-users are independent from the operator. Thus, similar to the user-based strategy, the only manner in which the operator can influence the process is by controlling the fare-prices according to the state of supply and demand. This scheme is problematic since it creates situations in which either the service-users have to pay an expensive fare due to the lack of services, or the service-provider receives lower fare-prices due to high supply of services.

The main concern with the traditional relocation systems is that they operate on the basis of the optimization of the overall system performance, neglecting the welfare of the system users: service-providers and service-users. In this thesis, the territory sharing problem is developed to include the aforementioned concerns as an explicit aspect of the problem.

6.3 Territory Sharing Problem

In social taxi-networks, an increasing number of commuters rely on smartphone apps that allow them to find a transportation service based on their locations. Likewise, service providers (i.e., taxi drivers) are dependent on smartphone apps to connect them with potential customers based on their geographical proximity. Since these apps operate on a location basis, and the drivers choose their own territory, a disturbance in the supply and demand chain is inevitable. The rational thinking of each taxi driver is to choose an area with an elevated chance of having potential customers. For example, downtown areas and shopping districts are areas that have high popularity among customers. Many of these customers are in need for door-to-door transportation services. However, since this is common knowledge, most service providers will target these areas. This will lead to a situation in which we have some areas oversaturated with service providers, and other areas that have low to no existence of service providers. Furthermore, the majority of the smartphone apps governing the social taxi networks have a dynamic fare-rate that changes based on the availability of the service. For peak time periods, the fare-prices are increased and vice versa. This phenomenon is known as the tragedy of the commons [108], and it is pertinent to the situation in which the resources are common and the users are selfish decision makers. I call the problem of managing the routes among drivers based on their preferences the territory sharing game.

There are various issues that can render the territory sharing game a challenging problem. First, there is the issue of formulating the game such that a solution model can be designed. The game consists of three parties: the players, the smartphone app, and the resources. The smart application can be a participant or a game moderator. Depending on its intended role, the game set-up will be different. Furthermore, if the players use the resources as strategies to play among themselves, then the game can be either symmetric or asymmetric. In symmetric games, the players possess the same sets of strategies. For example, if there are only three possible actions and all players have these actions as possible strategies, then the game is symmetric. However, asymmetric games are more general in the sense that the players may have non-identical sets of strategies. Hence, it is important to determine whether the game is symmetric or asymmetric.

Furthermore, once a game's nature is defined, an additional problem arises in determining a solution algorithm that can yield a stable outcome. A stable outcome is a self enforcing outcome that meets certain criteria. This is the second challenge in defining the territory sharing game. In regards to game theory, a stable solution constitutes a stable equilibrium. Identifying the equilibrium sought will assist in designing the solution model. It will also guide the assessment process of the obtained solution, which leads to the third challenge: the evaluation of the effect of the solution model on the overall performance (i.e., the welfare of the system). As discussed thus far, the territory sharing problem suffers from the tragedy of the commons; hence, a stable outcome does not guarantee the efficiency of resource utilization. To assess the (in)efficiency of the proposed solution, the Price of Anarchy (PoA) is used.

In this chapter, a game theoretic formulation for the territory sharing problem is developed. The model describes the problem as a cooperative game in which players with asymmetric sets of strategies cooperate to reach strategic agreements, despite the competition over the resources. An extended version of the bargaining-based solution model introduced in Chapter 4 is developed to allow for the competition procedure to take place, while it facilitates the achievement of the final agreement. The agreement can be enforced using the social taxi networks moderator (i.e., the smartphone app). To ensure that the final solution corresponds to a stable equilibrium, the bargaining framework utilizes the no-regret approach which results in a coarse correlated equilibrium.

6.3.1 Game Description

The ability to enforce the final agreement situates the smartphone app in a position to monitor and moderate, to a certain degree, the territory sharing game. The app will not force drivers to make certain decisions. However, it can incentivize the drivers to participate in the game on the promise that the benefits of this game will outweigh any alternatives. Furthermore, the app can limit the number of participating drivers according to the service demands in certain areas. Thus, the problem of finding the right number of players can be solved.

The desired outcome of the game is for drivers to agree collectively on the use of certain strategies. The drivers will make their agreement based on their best interests. The smartphone app will combine the localization functionality with the decisions committed by the drivers. The app will force the drivers to commit to their announced decisions should they wish to remain in the game. Through this arrangement, a binding agreement is established; and the role of the app ends here. The solution model will balance the process of supply and demand such that the services are available at optimal times for all customers.

6.3.2 Problem Formulation

We consider a group of drivers $TR = \{tr_1, tr_2, , , , , tr_N\}$, each of which is requesting to have ownership of specific territories As. Each driver chooses several areas of interest such that $A^{tr_i} = \left\{A_1^{tr_i}, A_2^{tr_i}, , A_n^{tr_i}\right\}$ with the pre-assigned prices $C^{A^{tr_i}} = \left\{c^{A_1^{tr_i}}, c^{A_2^{tr_i}}, , c^{A_N^{tr_i}}\right\}$. These territories are in the form of paths in the same area such that $A^{tr_i} = P^{tr_i}$, $C^{A^{tr_i}}$ =C$^{P^{tr_i}}$, and $P^{tr_i} \left\{p_1^{tr_i}, p_2^{tr_i}, , p_n^{tr_i}\right\}$. For each path, $p_j^{tr_i}$, a regulatory body Π assigns the usage-price $c^{p_j^{tr_i}}$. This usage-price is equal for all drivers. Let P^{tr_i} be the set of strategies, represented by their actions, and $C^{p_j^{tr_i}}$ be the utility function for driver tr_i over $p_j^{tr_i}$. P^{tr_i} is a compact, differentiable, convex set for which the usage-cost set $C^{P^{tr_i}}$ is computed via a positive non decreasing function. That is, P^{tr_i} is a bounded closed set that contains all of the desired strategies such that each strategy will yield a positive utility value.

The game Σ is a 3-tuple $(TR, P^{tr_i}, C^{P^{tr_i}})$ cooperative territory sharing game. The regulatory body, Π, receives requests to reserve areas from the drivers. The regulatory body computes $C^{P^{tr_i}}$ based on the drivers' concentration in the areas of interest and send this information to the drivers as well as the information about where the drivers are situated. $C^{P^{tr_i}}$ represent the drivers' expected fare price deduction for each territory, i.e., the "loss" for every driver due to the declining fare rate. Once the drivers receive $C^{P^{tr_i}}$, they start the communication to reach an agreement with regards to their chosen strategies (i.e., paths). Furthermore, Π defines a general cost function C. C represents the overall system cost, given the action of the drivers, whereas $C^{P^{tr_i}}$ is a personalized cost function for each driver.

Since the drivers are impacted by their individual decisions, a cooperative scheme is needed to achieve the following:

$$C(s) = \min_{\forall p_j} \sum_{1}^{N} C^{p_j^{tr_i}} \tag{6.1}$$

such that

$$C(s) \leq \sum_{i=1}^{N} C(p_{j^*}^{tr_i}, p_j^{tr_{-i}}) \tag{6.2}$$

Equation 6.1 describes the problem as a game in which the goal is to have a minimum overall cost. Furthermore, in Equation 6.2 the game is expected to arrive at an equilibrium as a competitive game such that if any player unilaterally changed his/her strategy to another strategy, the overall outcome wouldn't improve.

6.3.3 Solution Formulation

Similar to the discussion in Chapter 5, the game described so far has two aspects. The first aspect is the cost that each driver has to pay. Drivers would like to choose strategies that guarantee them the lowest possible conflict cost. The Travellers-Centric Trip Planning (TCTP) module is used to handle the personal strategies such that each driver will have few strategies reflecting their preferences. The second aspect is the overall cost value that results from using the system. An overall cost minimization might require the drivers to cooperate. Drivers who are most interested in increasing their gains or reducing their cost values should form coalitions (Ss). The purpose of these coalitions is to provide their members with a platform through which they can make strategic agreements with regards to the utilization of mutual resources. The resource sharing game is faced usually with the problem of forming cooperating groups that adhere to their agreements. For our targeted application, this is made simple. Customers contact the smartphone app expressing their interest in having a door-to-door transportation service. The drivers contact the app expressing their availability to provide this service. Both parties have no direct interaction, and their communications are managed by the app. Therefore, the smartphone app can force the social taxi drivers to abide by their established agreements.

Nevertheless, the drivers have no interest in joining an agreement that will not benefit them. Subsequently, the solution should guarantee two outcomes. The first outcome is related to the drivers' geographical distribution such that the smartphone app will assign appropriate cost values for various areas based on customer concentration levels. The second outcome is achieved when the drivers are guaranteed to have their gains increase, when they join the cooperative game. These outcomes are best described by the game's core:

$$\text{Core(c)} = \left\{ x \in \mathbb{R}^N | \sum_{i \in N} x_i(N) = c(N) \text{ and } \sum_{i \in S} x_i(s) \leq c(s) \forall s \in 2^N \setminus \varnothing \right\} \quad (6.3)$$

As seen in Equation 6.3, the drivers are interested in joining a cooperative endeavour if they are sure that they will individually benefit from cooperation. The core formulation in Equation 6.3 corresponds to the requirements indicated in Equation 6.1 and Equation 6.2.

6.3.4 Existence and (In)Efficiency of Game Theoretic Solutions: No-regret Models

Before discussing the existence and efficiency of the proposed solution model, it is important to discuss the various equilibria as they relate to the developed cooperative game, as summarized in Figure 6.1. In literature, there is a wide range of problems with a game theoretic presentation. A game theoretic set-up does not guarantee the existence of a solution. A solution exists in a game when an equilibrium outcome converges at the end of the game. Generally, there are three types of equilibria: dominant equilibria, Nash equilibria, and correlated equilibria [109, 110], with each type having its own variants. The dominant equilibria exist if players within a game have strategies that can provide them with the best outcome regardless of the possible strategies of other players. However, for the majority of resource sharing games such a scenario is rare. Nash equilibrium has two main variants: pure equilibria and mixed equilibria. Pure equilibria can be found and analyzed, but they don't exist at all times. On the other hand, the mixed equilibria always exist, but it is difficult to be found. The correlated equilibrium can be seen as a general case of Nash's mixed equilibrium. The correlated equilibrium has two attractive features: first, it always exists, and it can be found; second, the correlated equilibrium analysis is suitable for games with subjective strategies. The correlated equilibrium is primarily described as an outcome of a game in which a random device sends a signal to the players describing/assessing a situation of interest. Therefore, through the signal, the device can affect the players and correlate their choices [110]. In [111], Cigler and Faltings have argued that the existence of a smart device is not needed to produce a correlated equilibrium. Alternatively, for the equilibrium to be produced, it is sufficient to play the game repeatedly and as such, the players will learn from previous rounds such that they incorporate their knowledge in subsequent rounds [112].

According to the team mobility planning framework, drivers view the environment subjectively based on their chosen doctrines. Therefore, they have a fundamentally different understanding and view of the same event. Hence, their negotiation process is governed by their subjective views and subjective utility values. Furthermore, the smartphone app is managing the communication between the drivers; thus, it permits only the drivers within the same area to participate in the game according to certain criteria defined by the app. Therefore, by virtue of having the app, the territory sharing game can be defined as a correlated game with correlated equilibria. However, this definition will pose a major concern of forcing the players to 1) have complete information about other participants in the game, and 2) to have sets of available strategies that are uniformed and symmetrical for all participants. The failure to meet these conditions will limit the successful outcome of the territory sharing game. In most cases, the players are private individuals who would not broadcast their strategies and will not share their expected gains. Furthermore, the symmetry of the game is difficult to achieve since the players have subjective doctrine-based strategies that are personalized to their preferences. This asymmetry of the territory sharing game, and the use of the TCTP module as a mean of strategizing for drivers, would give rise to a problematic situation if the game is viewed as a non-cooperative pure or mixed game. That is, the drivers are incapable of forming expectations of the other drivers' actions. The game's incomplete information and asymmetry are challenging intrinsic aspects of the territory sharing game.

The game's incomplete information might give rise to regrets among players as the game progresses. The need to have no-regret in the game should be addressed as a part of the need to have a stable, self enforcing outcome. The correlated equilibrium might not be sufficient to deal with the regret aspect of the game, where the outcome will result in a poor equilibrium, hence the regret. It is possible to mitigate the regret by “sequentializing” the game such that the repetitive partaking in the game should lead to the vanishing of regret. This outcome corresponds to a coarse correlated equilibrium such that the process of game sequentialization will produce a regret-free outcome through probabilistic strategizing.

The discussion thus far has progressed in a manner in which the different forms of team trip planning will lead to 1) a pure or mixed-equilibrium as argued in Chapter 4 and shown in Chapter 5, 2) a correlated equilibrium as discussed earlier via either the smartphone app or the repetitiveness of the game, or 3) a coarse correlated equilibrium. These equilibria are inter-related such that each equilibrium encompasses the previous ones as shown in Figure 6.1

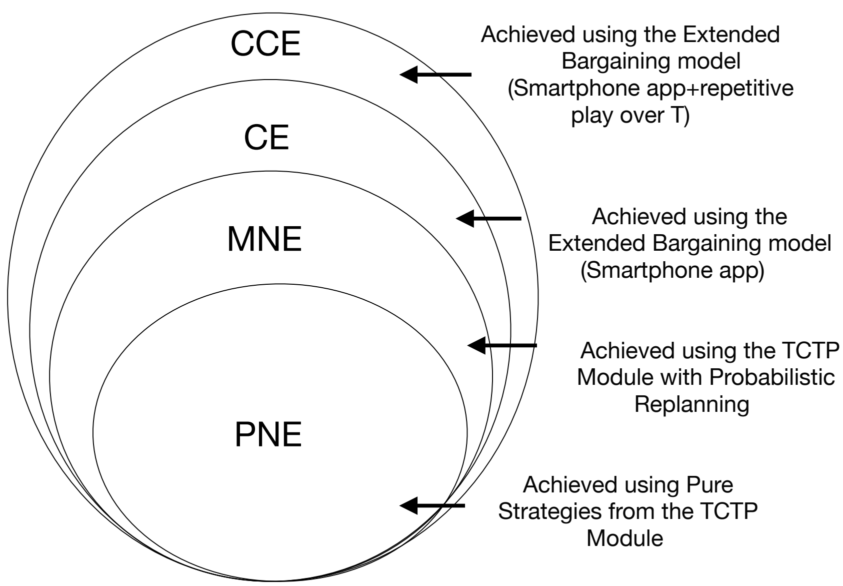


Figure 6.1: The different equilibria in order of simplicity and existence. The original concept of the Figure appears in [113]

The coarse correlated equilibrium is a correlated equilibrium that is *smoothed*. Smooth games admit canonical bounds of the Price of Anarchy (PoA) such that the in(efficiency) of the outcome is mitigated/eliminated [113]. Furthermore, the PoA is the most commonly used criteria in assessing the (in)efficiency of a solution. The PoA, as defined in Chapter 4, is an index that measures the impact of the selfish behaviour of the system's users on the overall system performance by comparing the worst equilibrium with the best possible outcome. For the resource sharing problems, there are several assessments of the PoA values under various assumptions. For example, Johari and Tsitsiklis argued in [114] that for players who have formulated an estimation regarding the impact of their actions on the prices, the lower bound for the aggregated utility is 75% of the best case scenario. The reason for the tight lower bound is that the players are conscious of the consequences of their actions, which subsequently affects their choices. For other scenarios that have less desirable conditions, the PoA's lower bounds decrease. In [115], Bachrach *et al.* proposed that for a wide-range of coalitional games, in which the players expect their utilities to be at least equal to their individual non-cooperative contribution/cost values, the strong PoA is 50% of the optimum scenario. This variation of the lower and upper bounds is attributed to the specific details of the games and the proposed solution methods. Nonetheless, for various games, these bounds provide a general expectation of the gains and losses due to the adoption of the cooperational approaches as opposed to the non-cooperational approaches.

Next, we discuss the integration of the coarse correlated equilibria, the smoothness of the game, and the PoA bounds into the bargaining model, initially described in Chapter 4.

6.4 Extended Bargaining Model

The previous discussion highlighted the fact that the team trip planning game has mixed elements of cooperation and non-cooperation. The bargaining model, as described in Chapter 4, is cooperative in the sense that the final outcome is not self-enforced but enforced through a binding agreement. Furthermore, the cooperative nature of the team trip planning game is a major aspect in the construction of the territory sharing problem as a cooperative game. Nevertheless, the later analysis has heavily deployed tools that have been conventionally used for non-cooperative games. This is hardly a dichotomous employment of these tools and solution methods. The cooperative and non-cooperative aspects of game theory are not viewed as opposing branches. They are rather viewed as two possible methods of approaching team planning problems from two different perspectives and as such, depending on the application, the analyses may overlap. To better highlight the relationship between cooperative and non-cooperative games we quote R. J. Aumann: "*Formally, cooperative games may be considered a special case of non-cooperative games, in the sense that one may build the negotiation and enforcement procedure explicitly into the extensive form of the game. Historically, however, this has not been the mainstream approach. Rather, cooperative theory starts out with a formalization of games (the coalitional form) that abstracts away altogether from procedures and form the question of how each player can best manipulate them for his own benefits; it concentrates, instead, on the possibilities for agreement*" [116]. The solution model in this research work follows the formal definition of cooperative games.

In contrast to the work in many of the recent publications, I have developed a solution model that has a negotiation procedure between the players while the final agreement is enforced through mutual threat or through the game moderator. Hence, similar to Proposition 4.2.1, in the territory sharing game, it is assumed that the drivers will give up on a preferred strategy during the process of negotiation in the pursuit of an agreement. This assumption might contradict Equation 6.2. Therefore, we start by reformulating Equation 6.2 such that for a series of sequential negotiation rounds and agreed upon sets of actions $(P^1, P^2, P^3, , , P^T)$ over T time, Equation 6.2 becomes

$$\frac{1}{T}C(s) \leq \frac{1}{T}\sum_{i=1}^{N} C(p_{j^*}^{tr_i}, p_j^{tr-i}) \tag{6.4}$$

Proof

$$\sum_{1}^{T} C(s^t) = \sum_{1}^{T}\sum_{1}^{N} C(s^{tr_i{}^t}) \tag{6.5}$$

$$\sum_{1}^{T} C(s^t) = \sum_{1}^{T}\sum_{1}^{N}[C(p_{j^*}^{tr_i}, p_j^{tr-i}) + \Delta_{tr_i,t}] \tag{6.6}$$

$$\Delta_{tr_i,t} = C(s^t) - C(p_{j^*}^{tr_i}, p_j^{tr^t_{-i}}) \tag{6.7}$$

The dual use of the sequential playing and the smartphone app as a "referee" can guarantee that we have a correlated equilibrium (CE). However, to guarantee that the drivers will have regret-free outcomes, the smoothness assumption detailed in [46, 109, 117] is used to formulate the following relationship:

$$\sum_{i=1}^{N} C(p_{j^*}^{tr_i}, p_j^{tr-i}) \leq \lambda \cdot C(p_{j^*}^{tr_i}) + \mu \cdot C(p_j^{tr_i}) \tag{6.8}$$

Therefore,

$$\sum_{1}^{T} C(s^t) \leq \sum_{1}^{T} \lambda \cdot C(p_{j^*}^{tr_i}) + \sum_{1}^{T} \mu \cdot C(p_j^{tr_i{}^t}) + \sum_{1}^{T}\sum_{1}^{N} \Delta_{X_i,t} \tag{6.9}$$

For each tr_i, the no-regret model is used to present the following assumption

$$\sum_{1}^{T} \Delta_{tr_i,t} \leq 0 \tag{6.10}$$

Hence

$$\frac{1}{T}C(s^t) \leq \frac{1}{T}\lambda \cdot C(p_{j^*}^{tr_i}) + \frac{1}{T}\mu \cdot C(p_j^{tr_i{}^t}) \tag{6.11}$$

■

For the territory sharing game, Equation 6.9 and Equation 6.10 can be expressed as follows:

$$\sum_1^T C(s^t) \leq \sum_1^T \lambda \cdot C(p_{j^*}^{tr_i}) + \sum_1^T \mu \cdot C(p_j^{tr_i{}^t}) - \sum_1^T \sum_1^N \Delta_{tr_i,t} \tag{6.12}$$

and

$$\sum_1^T \Delta_{tr_i,t} \geq 0, \tag{6.13}$$

where

$p_{j^*}^{tr_i}$ = The optimum strategy of player tr_i,

T = Time window and $t \in T$,

λ, μ = Smoothness parameters such that $\lambda > 0$ and $\mu < 1$,

$\Delta_{tr_i,t}$ = The gain/cost incurred by driver tr_i at instance t.

The game according to Equation 6.8-6.13 allows for the integration of "no regret" dynamics resulting in a coarse correlated equilibrium (CCE).

The bargaining model operates similarly to the model described in Chapter 4. Additionally, the smartphone app Π plays an integral role in assigning a cost function for each area. These cost values ($C^{P^{tr_i}}$s) are communicated to the drivers. Furthermore, the values can be unique to the drivers such that the cost value for $C^{P^{tr_1}}$ can be different than $C^{P^{tr_2}}$. This method is not to centralize the game but to force the drivers to withhold their agreements. Should a driver deviate from an agreement, Π can raise the cost value to prevent the driver from profiting. Furthermore, by controlling the prices, Π can control the number of competing drivers per area. Otherwise, the prices are the same for all players. In all cases, the decisions regarding the strategies are made with correlation to $C^{P^{tr_i}}$.

Once the drivers receive their $C^{P^{tr_i}}$s, they start arranging their areas in order of their preferences. According to their chosen doctrines, and using their TCTP modules, they communicate their strategies to each other via Π. The identity and the number of the drivers involved in the game are decided by Π. Therefore, a case in which the supply and demand are out of balance is avoided. The pairwise negotiation will proceed as discussed in Chapter 4. Once the drivers agree on their chosen strategies, they communicate their agreement to the smartphone app Π. The agreement will then be enforced, and no driver can change their strategy.

The drivers have the right to play the game once and then move on to a different area, or they may join the game again. However, the $C^{P^{tr_i}}$ can be different for each round and not necessarily repeated. Therefore, the choices made by the drivers are strictly correlated with the existence of the smartphone app Π. In the case that Π should relinquish its control over the cost values of various areas, the choices of the players in each round become correlated with their choices in previous rounds. The number of rounds, players, and the cost prices are inferred from the smoothing process of the game such that the outcome converges to a no-regret model.

Next, a simulation work that discusses the territory sharing game and its solutions according to the developed no-regret-based bargaining model is presented.

6.5 Simulation Work

In this simulation work, N drivers who have access to two business areas are simulated. Each area has two paths. Each driver is required to choose at least two non-identical paths. The number of drivers (N) can vary from one experimental set up to another.

Through this simulation, we first demonstrate the territory sharing game with different number of players discussing the expected gains/losses. We will also examine the validity of Equation 6.11 for our model. The simulation work will examine the value of Δ_{tr_i} for all players, if the game was to be played for a period of 30 days. Furthermore, the efficiency of the game model is examined using the Price of Anarchy (PoA). PoA is computed as the following:

$$PoA = \frac{\max_{p_j} \sum_1^N C^{p_j^{tr_i}}}{\min_{p_j} \sum_1^N C^{p_j^{tr_i}}} \tag{6.14}$$

Next, the behaviour of our model for 10 drivers is demonstrated.

6.5.1 Territory Sharing Beween 10 Drivers

In this scenario, 10 drivers randomly choose their sets of strategies. Their profits are penalized by the smartphone app according to their chosen territories. If two drivers chose the same path, the path price will be increased by 2 price units for each driver. Therefore, their collective goal from playing this game is to minimize their overall sharing incidents as well as their individual penalties.

Each player starts the game by broadcasting a set of strategies that has only two alternatives $\{p_1^{tr_i}, p_2^{tr_i}\}$. All drivers announce that their preferred strategy is p_1, and each driver will compute the overall sharing incidents, as shown in Table 6.1. According to Equation 6.12, and as shown in Table 6.2, various coalitions are formed. After the coalitions are established, the first round of negotiation begins. The coalition formation rounds are governed by the two conditions of rationality and efficiency, as indicated in Equation 6.3. As seen in Table 6.1, for the first round, the second condition is met. However, for tr_4 and tr_9, they see their price shares increasing since they joined their respective coalitions. Therefore, over two rounds, the drivers are allowed to examine independently their cost values outside of their coalition.

Table 6.1: Negotiations for cooperative territory sharing game

Players	Selfish Game	Cooperative Game Outcome			Δ_{tr_i}
		1st Round	2nd Round	3rd Round	
tr_1	15	12	9	6	0
tr_2	15	6	3	3	6
tr_3	9	9	9	9	9
tr_4	9	18	9	9	9
tr_5	15	12	9	6	0
tr_6	9	3	6	9	11
tr_7	15	10	7	5	4
tr_8	15	12	9	6	4
tr_9	9	17	20	9	11
tr_{10}	15	5	5	2	7
$\sum_1^{N=10} C^{tr_i}(p_j^{tr_i})$	126	104	86	64	NA

Table 6.2: Coalition formation in the territory sharing game

Coalition No.	Coalition Members	Strategy Agreement
1	tr_1, tr_2	$\left\{p_1^{tr_1}, p_2^{tr_2}\right\}$
2	tr_3, tr_4	$\left\{p_1^{tr_3}, p_2^{tr_4}\right\}$
3	tr_5, tr_7	$\left\{p_1^{tr_5}, p_2^{tr_7}\right\}$
4	tr_6, tr_9	$\left\{p_1^{tr_6}, p_2^{tr_9}\right\}$
5	tr_8, tr_{10}	$\left\{p_1^{tr_8}, p_2^{tr_{10}}\right\}$

After the third round, they confirm that their cost values outside of their coalition are improved, and therefore they secede from their coalitions. In this scenario, the final grand coalition is found as follows:

$$P^{S^{Gnd}} = \left\{p_1^{tr_1}, p_2^{tr_2}, p_1^{tr_3}, p_2^{tr_4}, p_1^{tr_5}, p_1^{tr_6}, p_2^{tr_7}, p_1^{tr_8}, p_2^{tr_9}, p_2^{tr_{10}}\right\} \tag{6.15}$$

6.5.2 Smoothness and Convergence of CCE

The previous scenario included 10 drivers for which a cooperative solution is found. By examining Δ_{tr_i} in the previous scenario, it can be confirmed that the cooperative solution corresponds to a Nash equilibrium for the drivers. However, the possibility of this outcome being consistent for all possible cases can be challenged. If Assumption 4.3.1 is applied and Nash equilibrium did not converge, then the no-regret approach should come into play such that in any case we should have $\sum_1^T \Delta_{tr_i,t} \leq 0$, and the final solution will converge to the CCE. To verify this hypothesis, more than a 1000 scenarios for various values of N were implemented, while simulating situations in which the games are played repeatedly over a period of 30 days ($T \leq 30$).

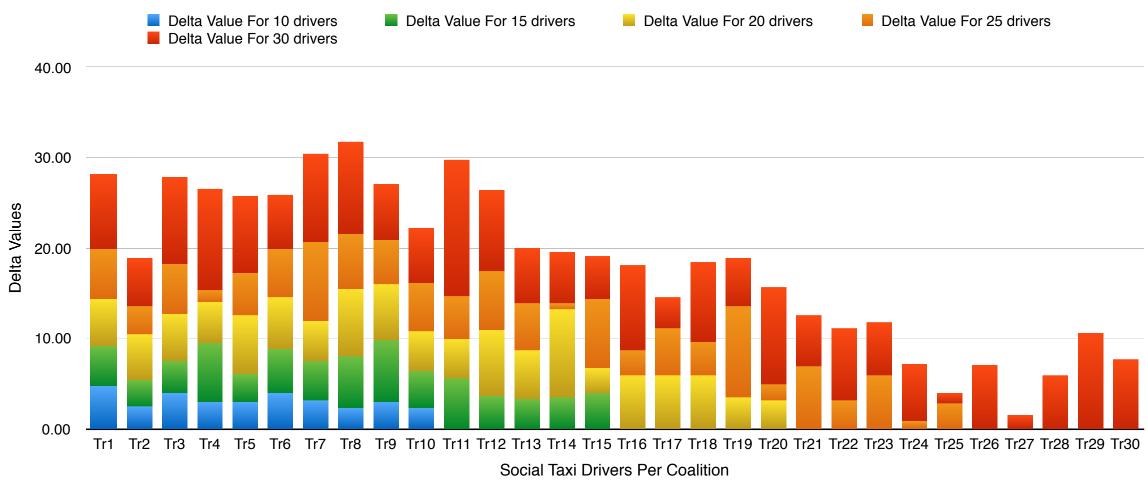


Figure 6.2: Values of $\Delta_{tr_i,t}$ computed over 30 days for $N = 10, 15, 20, 25, 30$.

As shown in Figure 6.2, for up to 30 drivers, the $\sum_1^T \Delta_{tr_i,t}$ has a positive value reflecting the gain achieved by cooperation between drivers. This result demonstrates the ability of our model to provide drivers with an acceptable state of equilibrium. However, it is important to keep in mind that this case is true for when $N \leq 30$, $|P^{tr_i}| = 2$, and $|P^S| = 4$. If the number of players is arbitrarily increased without offering more options, the final result may not converge to a coarse correlated equilibrium (CCE), even if a momentary gain is found. However, an argument can be made that with the existence of the smartphone app, the outcome will converge to a

CE.

As can be seen in Figure 6.3, when more drivers are allowed to compete for customers, the overall results will not conform to Equation 6.11. Therefore, to deal with this issue, there are two options: either to increase the value of T and observe $\sum_{1}^{T} \Delta_{tr_i,t}$ for a longer period of time, or to increase the number of offered strategies. With the former option, a CCE may or may not converge. Whereas, with the latter option, by diversifying and broadening the search space, we are assured to get a CCE. With regards to our application, one of our goals is to distribute the drivers among all possible areas which the customers frequent the most. Therefore, as more drivers express their interest in playing the game, it is logical to increase the coverage areas. The burden of dealing with this issue will fall on Π. For our scenario, we have added two more paths such that $|P^S| = 6$. For each driver, the value of $|P^{tr_i}|$ will remain to be 2. However, each player will have a bigger "pool" of strategies to choose from. As shown in Figure 6.4, we can see that for the same coalition formulation and over the same period of time, a coarse correlated equilibrium has converged.

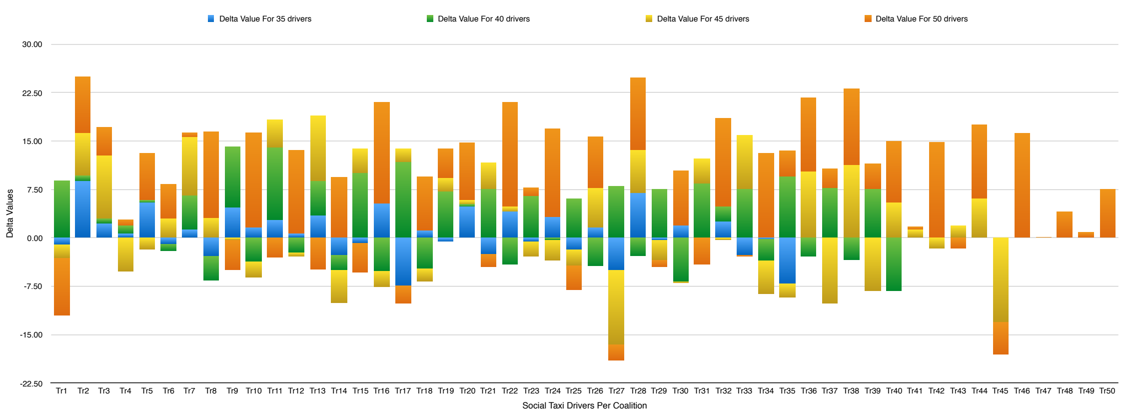


Figure 6.3: Values of $\Delta_{tr_i,t}$ computed over 30 days for $N = (35, 40, 45, 50)$ with $|P^S| = 4$.

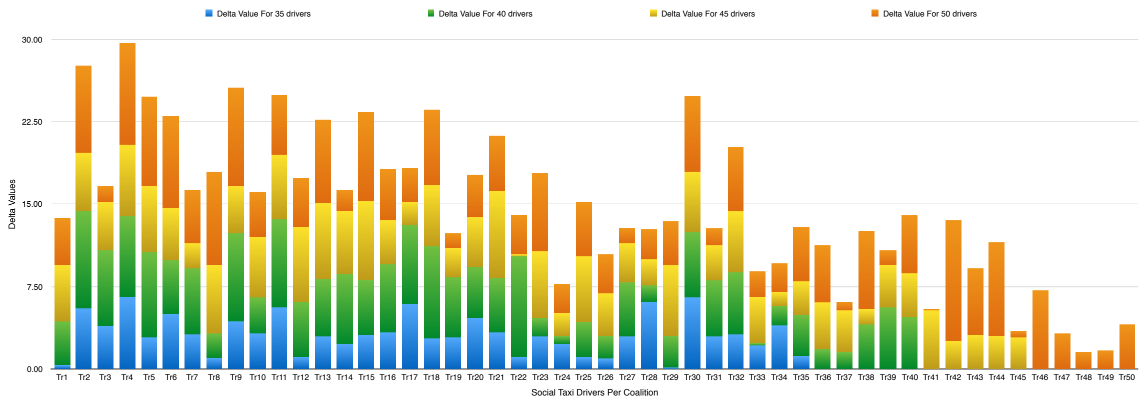


Figure 6.4: Values of $\Delta_{tr_i,t}$ computed over 30 days for $N = (35, 40, 45, 50)$ with $|P^S| = 6$.

6.5.3 Overall System Performance

The simulation work presented has thus far shown the effectiveness of our model in terms of the individual cost distribution and the converged equilibrium. Regardless, since our bargaining model is a heuristic one, a stronger form of validation might be required. For this purpose, we will use the PoA as a tool of assessment. For $N <= 20$ we can compare the converged equilibrium with the best case scenario. However, for $N > 20$ this might be difficult since the search space for $|P^{tr_i}| = 2$ is 2×2^N. Therefore, the assessment will include a comparison with the deterministic best equilibrium as well as a comparison with the estimated "best" equilibrium.

6.5.3.1 Deterministic Equilibrium vs Achieved Equilibrium

We will use the case of $N = 20$ to compare the overall cost of our achieved equilibrium with the deterministic best and worst equilibria which were determined through brute search. As can be seen in Table 6.3, the calculated PoA for 10 days is consistently high. In comparison, the PoA from our model is close to optimum (i.e., close to 1).

Table 6.3: Converged equilibrium for 20 drivers over 10 days

Days	Best Equilibrium	Worst Equilibrium	Achieved Equilibrium	PoA	Achieved PoA
1	370	984	390	2.66	1.05
2	382	1044	392	2.73	1.03
3	564	1120	592	1.99	1.05
4	418	1260	432	3.01	1.03
5	306	816	320	2.67	1.05
6	336	1212	402	3.61	1.20
7	382	1120	392	2.93	1.03
8	364	1022	396	2.81	1.09
9	348	1038	352	2.98	1.01
10	348	1038	388	2.98	1.11

6.5.3.2 Estimated Equilibrium vs Achieved Equilibrium

As mentioned before, as the number of drivers increases, it becomes impossible to find the best and worst equilibria through brute force search. Alternatively, few assumptions are made regarding the estimated best and worst equilibria. First, the worst equilibrium is assumed to be the converging outcome when all drivers choose their least favoured strategy. Therefore, this can be considered as a strong upper bound. For the lower bound (i.e., the best equilibrium), it is rather difficult to make any form of direct estimation of the best equilibrium. Hence, we assume that the system's PoA is $\sqrt{N} = \sqrt{30} = 5.4$, which corresponds to the PoA ratio for a 3^{rd} degree polynomial cost function, and from this value it is possible to compute the best possible outcome. Obviously, these are strong assumptions. However, as seen in Table 6.4, even though the achieved PoA is slightly worse than the case for 10 drivers, our model's efficiency with regards to the overall performance is demonstrated.

Table 6.4: Converged equilibrium for 30 drivers over 10 days

Days	Best Equilibrium	Estimated Worst Equilibrium	Achieved Equilibrium	PoA = $\sqrt{N}$	Achieved PoA
1	502	2712	788	5.40	1.57
2	447	2414	628	5.40	1.40
3	453	2448	795	5.40	1.75
4	468	2526	576	5.40	1.23
5	502	2712	589	5.40	1.17
6	441	2380	556	5.40	1.26
7	488	2634	602	5.40	1.23
8	478	2580	685	5.40	1.43
9	468	2526	816	5.40	1.74
10	459	2480	582	5.40	1.27

6.6 Conclusion

In this chapter, we described a territory sharing game for social taxi networks. In this game, a regulatory body in form of smartphone app allows for a group of drivers to engage in a cooperative endeavour by which they earn the right to operate in certain attraction areas. We have extended the previously developed bargaining model to accommodate the app as a referee for the game. A no regret based approach was developed to ensure that the final outcome of our game will converge to a coarse correlated equilibrium.

To validate the developed model, we conducted an extensive experimental work. Through this work, we demonstrate the implementation of our model, the effectiveness of the no-regret model, and provide an analysis of the overall system efficiency by examining the Price of Anarchy (PoA).

The developed model has been shown to be successful in handling the various examined scenarios. The performance was robust, and the empirical results adhered to the theoretical formulation.

Chapter 7

Conclusions and Future Directions

7.1 Introduction

Team mobility planning has been a point of interest in a multitude of areas of research. As an optimization problem, a variety of solutions were presented to deal with trip planning for both individuals and groups of travellers. However, these solutions are constrained by the nature of their applications. On the other hand, as a game theoretic problem, multiple analyses were presented in the literature categorizing the trip planning game as either cooperative or non-cooperative.

The main objective of the research work in this thesis is to develop a team mobility planning framework. The framework is designed to approach the team trip planning problem as a cooperative game. Furthermore, as a modularized framework, the traveller-centric trip planner module has been developed to independently address the non-cooperative team trip planning. In the following sections I summarize the major contributions of this work, and I provide suggestions for future directions.

7.2 Major Contributions

1. A game formulation of the team trip planning problem that encompasses the individual and the team aspects of the problem has been developed. While the mathematical formulation of the problem sets the foundation for the developed solution work, its formulation has been maintained to be general and usable for other possible solution models.

2. A novel team mobility planning framework has been presented. The framework has been modularized such that each module in it can be considered an independent solution model. For example, the framework in general can be used for cooperative trip planning. On the other hand, the traveller-centric trip planning module can be used as an independent system that deals with non-cooperative team trip planning. Furthermore, the TCTP module has been used as a tool to produce behavioural based strategies that reflect the personal planning beliefs of the travellers.

3. A bargaining model has been formulated to deal with the team trip planning game as a cooperative problem. The model in its original design has produced a stable outcome under an enforced agreement. Furthermore, in the case that an enforced agreement is not reached, the outcome will correspond with mixed Nash equilibria.

4. A novel traveller-centric trip planning module has been presented. The module deploys hard and soft objectives to produce a trip plan that is personalized for each traveller. The use of this module has been shown to produce a better system welfare as compared to existing solutions. Furthermore, the traveller-centric trip planning module has been used to produce the personalized strategies for

each traveller in the team trip planning game.

5. A novel game theoretic formulation of the territory sharing game for social taxi networks has been developed. The formulation has taken into account the usage of smartphone applications (Apps) as the medium of communication between the travellers. Furthermore, the game formulation has situated the smartphone app to be an external referee of the game. The unique position of the app as part of the game, but not as a player, has been used to demonstrate the existence of a correlated equilibrium and a coarse correlated equilibrium.

6. An extension of the bargaining model that deals with the competitive aspect of the game has been presented. In conjunction with the external referee, the bargaining model has produced an outcome that corresponds to a coarse correlated equilibrium. The model has been extended to behave similarly to a no-regret model.

7.3 Future Research Directions

The research work presented in this thesis has addressed the main goals declared herein as research gaps, and has also demonstrated the capabilities of the developed framework in achieving these goals. Additionally, this work has uncovered other issues that deserve further research work.

1. With regards to the team mobility framework, the issue pertaining to the optimum number of players is not addressed in detail. This is an important factor which might lead to the instability of the final outcome as discussed in Chapter 2. Although the smartphone app is used in Chapter 6 to deal with this problem, a more integrated solution might be needed in future. The Price of Anarchy

can be used as a method of upper bounding the number of players based on general assessment of various equilibria. The problem with this approach is that it is localized to each game. Alternatively, dynamic bounding of the different approaches should be investigated.

2. With regards to the traveller-centric trip planner, as shown by the experimental work in this thesis, there are promising results from using the doctrines. However, to understand the exact impact of the doctrines on the traffic requires an experimental work on a large scale under a wide range of scenarios. This should be carried out in future research work.

3. The doctrines chosen in this thesis are limited to three main factors. In future work, it is possible to expand the fuzzy sets to include more doctrines and preferences. The safety doctrine, for example, can be expanded to include other factors such as the rate of traffic accident, visibility, pavement quality, travellers' age and experience, and weather conditions such as fog, rain and hail. Moreover, geographic models could be adopted to consider sand storms instead of snow storms and the existence of sand on roads instead of snow and black ice.

4. For the territory sharing problem, it is possible to add to the set of strategies some form of time stamps. That is, the model will not only operate on location basis, but it will be a locationally and temporally aware model. Furthermore, the time window (T) in the no regret model can be dynamic to serve both short and long term goals.

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Appendices

Appendix A

Publication Related to This Thesis

A.1 Journal Papers

1. Amar, Haitham Masaud, and Otman A. Basir. "A Bargaining-Based Solution to the Team Mobility Planning Game." IEEE Transactions on Intelligent Transportation Systems (2017).

2. Amar, Haitham Masaud, and Otman A. Basir. "A Game Theoretic Solution For The Territory Sharing Problem In Social Taxi Networks." **Accepted in** IEEE Transactions on Intelligent Transportation Systems (2017).

3. Amar, Haitham Masaud, Nabil M. Drawil, and Otman A. Basir. "Traveler Centric Trip Planning: A Behavioral-Driven System." IEEE Transactions on Intelligent Transportation Systems 17.6 (2016): 1521-1537.

4. Drawil, Nabil M., Haitham M. Amar, and Otman A. Basir. "GPS localization accuracy classification: A context-based approach." IEEE Transactions on Intelligent Transportation Systems 14.1 (2013): 262-273.

A.2 Conference Papers

5. Amar, Haitham M., and Otman A. Basir. "A Solution to the Congestion Problem: Profiles Driven Trip Planning." Vehicular Technology Conference (VTC-Fall), 2016 IEEE 84th. IEEE, 2016.

6. Amar, Haitham M., and Otman A. Basir. "A Game Theoretic Approach for the Ride-Sourcing Territory Sharing Problem." Vehicular Technology Conference (VTC Fall), 2015 IEEE 82nd. IEEE, 2015.

7. Amar, Haitham M., Nabil M. Drawil, and Otman A. Basir. "An Evidence-Based Approach for GPS Accuracy Classification." Asian Research Publishing Network (ARPN).VOL. 10, NO. 3, FEBRUARY(2015).

8. Drawil, Nabil, Haitham Amar, and Otman Basir. "Cellular Network Fingerprint Localization Simulation: A Soft Computing Approach." Vehicular Technology Conference (VTC Fall), 2014 IEEE 80th. IEEE, 2014.

9. Drawil, Nabil, Haitham Amar, and Otman Basir. "A solution to the ill-conditioned GPS accuracy classification problem: Context based classifier." GLOBECOM Workshops (GC Wkshps), 2011 IEEE. IEEE, 2011.