

Commodity Prices, Stock Prices and Economic Activity in a Small Open Economy

by

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Declaration

This thesis consists of material all of which I authored or co-authored: see the Statement of Contributions enclosed in the Thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Statement of Contribution

Chapters 1 and 2 were written solely by myself. Chapter 3 is co-authored with my supervisor, Professor Jean-Paul Lam. While the original idea for the paper was Professor Lam's, I contributed to its extension and development. The version of the paper which appears in this thesis was written jointly by myself and Professor Lam.

Abstract

This thesis is comprised of three papers which jointly examine the role of commodity prices as well as other asset prices in influencing the evolution of economic activity in a small-open economy (SOE). Using Canada as the quintessential small-open economy, each chapter adopts a particular approach to investigating this dynamic relationship. It is hoped that the contribution made in this thesis to understanding the relationship will aid policy-makers as they attempt to address the associated policy questions which are often fraught with difficulties and uncertainty.

In chapter 1 the use of a recursively identified Vector Auto-Regression (VAR) is employed to study the impact of commodity price shocks on Canada's macro-economy. While similar analysis has been carried out before, this has tended to focus solely on the impact of oil prices. Additionally, the analysis has tended to focus on aggregate output, while neglecting the specific sectoral impact. Given that each sectors' exposure to commodity price movements will be different, one would also expect varying sectoral responses to these shocks. Chapter 1 attempts to focus on this and thus offers a level of insight into the operation of the Canadian macro-economy which has not been extensively addressed in the literature.

The results suggest that indeed there is divergent sectoral responses to commodity price shocks, using a broad measure of commodity prices. The commodity producing sectors of the economy respond favourably to an unexpected rise in commodity prices, whilst the manufacturing sector is negatively impacted by such movements. We also found evidence that policy-makers may attempt to contain any inflationary pressures emanating from rising commodity prices by raising interest rates.

Chapter 2 delves even further into the dynamics of this relationship by employing a

Dynamic Stochastic General Equilibrium (DSGE) model. In this chapter we extend the analysis undertaken in chapter 1, where we are again attempting to ascertain the sectoral responses to a commodity price shock. The use of this modelling framework however allows us to analyse that relationship in a manner which is internally consistent and also in-line with our beliefs about the behaviour of economic agents. Additionally, the DSGE model allows us to conduct counter-factual policy experiments which were not possible using the VAR framework.

The results of the model are generally in-line with those found in chapter 1, as the commodity price shock has differing impacts on the various sectors of the economy. The results suggest that just examining the aggregate effects of commodity price shocks could overshadow important sectoral differences which are subsumed in these aggregate figures. Additionally, the counterfactual policy exercises indicate that actions taken by the Central Bank during the Global Financial Crisis positively impacted Canada's economic performance during the crisis and the period immediately after.

In the final chapter, co-authored with Jean-Paul Lam, we seek to quantify the interdependence between stock prices and monetary policy using an underidentified Structural VAR (SVAR) for Canada and the United States. We find that employing a recursive identification leads to counterfactual responses for the stock market following a monetary policy shock. In the underidentified VAR, the stock market and monetary policy are allowed to simultaneously react to each other's shock through a combination of short-run, long-run and sign restrictions. Unlike many studies in this literature, we impose a minimal number of restrictions on the short-run and long-run matrix, allowing the data to uncover the relationship between the variables in the SVAR. We find that an increase of 25 basis points (*b.p.*) in the policy rate of the central bank leads to a fall of about 1.75% in stock prices in Canada and to a fall of about 1.25% in stock prices in the U.S. This effect of

monetary policy on stock prices is larger in Canada compared to the U.S. mainly because sectors that are interest rate sensitive, such as financials and energy account for a much larger share of the stock index in Canada compared to the U.S. Following a stock market shock, the short-term interest, industrial production, inflation and commodity prices rise both in Canada and in the U.S. A 1% increase in the stock market leads to an increase of about 27 *b.p.* in the overnight rate in Canada while it leads to an increase of about 10 *b.p.* in the Federal funds rate.

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Finally, I would like to thank my family for their patience and encouragement. To my kids who are always asking "Mommy are you still working?", I say "I'm finally done." And to my husband Andrew, thank you for your continued love, support and understanding throughout the process of completing my doctoral studies.

Dedication

To Andrew, Arianna, Alcaldo, Salea and Sakeri. Only made possible through your patience and support.

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Introduction

Commodity prices have experienced significant volatility over the past twenty years. In fact, relative to the previous decade, commodity prices have on average more than doubled, with some subgroups, such as energy related products experiencing above average volatility and growth. Commodity-exporting economies such as Canada have therefore benefited from favourable terms of trade (ToT) movements. However, the recent down-swing in many of these commodity prices has resulted in a reversal of fortunes and renewed policy discussions focused on the effects of commodity price fluctuations on output, inflation and other macroeconomic variables, as well as appropriate policy responses to deal with such volatility.

Given the above, the objective of the first two chapters of this thesis is to analyse the effects of commodity price shocks in a small open economy commodity exporter, using Canada as the prototypical example. To carry out this exercise, two different methodologies are employed, firstly a structural VAR and secondly a DSGE model. The VAR allows us to examine and uncover the patterns and dynamics present in the data. The second method, the DSGE model complements this approach and enriches the analysis, while getting around the problems often involved in identifying the structural shocks in a VAR setting. It also allows us to build our model of the economy with a structural micro

foundation and also analyse the propagation mechanism of commodity price shocks.

Chapter 1 considers a recursively identified VAR with commodity prices, aggregate and sector-specific value-added output, as well as other macroeconomic indicators as variables. Following Zha (1999) the model is estimated by maximum likelihood, with the optimal lag length chosen by minimizing the Akaike Information Criterion (AIC). Impulse responses are obtained to a one standard deviation commodity price shock and the corresponding 68 percent confidence bands calculated through the Monte Carlo procedure proposed by Zha. The primary advantage of using VAR models is their ability to provide assessments of the dynamic responses of key macroeconomic variables to shocks without requiring a complete structural model of the economy. Their usefulness in uncovering the transmission mechanism of macroeconomic shocks has contributed significantly to their popularity in the literature. For this reason they are employed in this thesis to examine the relationship between the Canadian macro-economy and commodity prices.

In chapter 2, we examine this topic even further, by examining the general equilibrium linkages throughout the economy. While reduced form analysis is useful for examining the relationship among a subset of variables, the actual economy is much more complicated and consist of numerous links among multiple variables. Due to the problem of over-parametrisation which we would quickly encounter in a VAR setting, this framework does not offer us the flexibility to fully explore these linkages. For this reason, an alternative modelling framework is needed, as is provided by a dynamic stochastic general equilibrium model - the subject of focus for chapter 2. We will seek to answer a similar question to that posed in chapter 1, but this alternative framework should allow us to explore the more complex relationships which exist within the Canadian economy. The model employed is a calibrated version of the DSGE model developed in Dib (2008), to which we make some minor modifications largely to improve tractability. In our context, the model

is also applied to examine a different question than that which was addressed by Dib. More specifically, while Dib examined welfare effects, we will be seeking to assess what the macroeconomic and sectoral impacts are to a commodity price shock. Additionally, we conduct counter-factual policy exercises to examine how the economy would have evolved in response to alternative policy actions adopted by the monetary authority when faced with drastically declining commodity prices following the onset of the global financial crisis.

Chapter 3 is a joint paper between my supervisor, Professor Jean-Paul Lam and myself. In this chapter, we investigate the interactions between stock prices and monetary policy using an underidentified SVAR for Canada and the United States. In the model, the stock market and monetary policy are allowed to simultaneously react to each other's shock through a combination of short-run, long-run and sign restrictions. Unlike many studies in this literature, we impose a minimal number of restrictions on the short-run and long-run matrix, thereby allowing the data as much as possible to uncover the relationship between the variables in the SVAR. Allowing for this simultaneity is particularly useful as asset prices, such as stocks, generally contain information which is relevant to the conduct of monetary policy. This is so as they are considered to be leading indicators, either of inflation or economic activity and both of these variables are of relevance to an inflation-targeting central bank. Our results suggest that stock prices and monetary policy respond to each other's shock on impact. This response is generally significant and consequently, studies which impose a zero restriction on impact may be understating the interrelationship between stock prices and monetary policy.

Chapter 1

Sectoral Impact of Commodity Price Shocks on the Canadian Economy

1.1 Introduction

In the media and among official sources, movements in commodity prices have often been cited as significantly affecting economic activity in Canada.¹ See Poloz (2014), Faruquee et al. (2015), Rubin (2015) and Kirby (2014) among others. To illustrate, in a presentation by the Governor of the Central Bank in 2015, to Calgary Economic Development he noted that “Natural resources have been a big economic story for this country since the time of European contact.” He further noted that “Any economy that relies on natural resources will naturally be challenged by large movements in their prices. These shocks are more than

¹As used in this paper, commodities refer to raw materials or primary agricultural products that can be bought and sold on international market, and are interchangeable with other products of the same type. More specifically, the definition includes, agriculture, forestry, fishing, mining, quarrying, oil and gas extraction.

just swings in national income; they also force businesses to make decisions about the way resources such as capital and labour are allocated. These decisions often lead to difficult adjustments, but they are necessary for maximizing our economy’s potential.” Given the often repeated notion of the importance of commodity prices, an analytical assessment of their impact is relevant.

This study is further motivated by the observation that although the Canadian economy shares some similarities with other industrial nations, such as the US (see Backus et al. (1995), Schmitt-Grohe (1998)), it does exhibit significant differences which are largely related to the influence of commodity prices. In particular, it is observed that Canada’s trade balance is positively correlated with commodity prices, and the real exchange rate tends to appreciate in response to higher commodity prices as well. Such effects are absent for other non-commodity exporting industrial nations. Thus answers are sought to the question, what are the aggregate and sector-specific impact of commodity price shocks in the Canadian context? In particular, we will seek to assess the impact of movements in an aggregate commodity price index on both overall economic activity as well as sector specific output. As a significant exporter of a wide range of commodities, the Canadian economy is sensitive to changes in global commodity prices which can affect economic activity both through the terms of trade, as well as via changing demand for other Canadian exports, such as manufactured goods.

For Canada, a significant proportion of exports is comprised of commodities. Consequently, rising commodity prices which began in 2002, due to higher prices for oil, gold and agricultural commodities, led to an appreciating exchange rate, rising Gross National Income and higher standards of living as measured by the Index of Economic Well Being, (see Osberg et al. (2016)). Further contributing to the commodity boom, was the increased demand for commodities from rapidly expanding Asian economies such as China.

The combination of these factors, meant that Canadians benefited from higher purchasing power during that period, and this was translated into above average economic activity, such as higher real incomes, consumption and investment.

These favourable conditions have persisted until quite recently. The slowdown in global economic activity precipitated by the financial crisis in the US, resulted in declining commodity prices and a reversal of fortunes for major commodity exporters like Canada. Given the relatively large reliance on commodity exports and the great degree of volatility to which their prices are subject, quantifying the effects of commodity price shocks is undoubtedly an important issue.

Previous papers have generally agreed on the importance of commodity price movements and terms of trade shocks for the Canadian economy, see Vasishtha and Maier (2013) for example. What has been lacking in the literature however, is an assessment of the sectoral impact of these shocks. Intuitively, one expects commodity price shocks to affect the productive sectors of the economy differently, since each sector has a different level of exposure (both directly and indirectly) to movements in commodity prices. Furthermore, economic theory suggests that these relative price changes should induce a reallocation of resources and as such, contribute to the differential sectoral impact.² The objective of this paper is to assess the impact of a broad-based measure of commodity prices on sectoral GDP and thereby offer some useful insights about the particular structure of the Canadian economy.³

To accomplish this objective, a structural vector autoregressive (SVAR) model is es-

²Industries which produce commodities, like the energy sector are expected to be positively affected by an increase in commodity prices, whereas those who use commodities as a significant input and rely heavily on exports, such as manufacturing, are expected to be negatively impacted.

³The measure of commodity prices employed is the Bank of Canada's Total Commodity Price Index. According to the Bank, this is a price index of the spot or transaction US dollar prices of 24 commodities produced in Canada and sold in world markets.

timated, over the period February 1997 to April 2016. We build on and extend previous works which have examined the importance of international shocks in the Canadian context, see Cushman and Zha (1997), Zha (1999), Bhuiyan (2012), among others. The SVAR framework is particularly useful in assessing the empirical relationship between commodity prices and macroeconomic variables in Canada, as it provides a summary of the joint movements within the data. By imposing a set of identifying restrictions on the estimated residuals we can analyse the dynamic impact of the structural shocks on the system of variables. In the current setting, such impulse-response analysis, allows for the empirical assessment of the effects of unexpected commodity price shocks.⁴

The major finding of the study is that commodity price shocks have a significant positive impact on the output of the commodity producing sectors of the economy. Manufacturing however, is negatively impacted by increases in commodity prices over the long term. Domestic inflation is also found to increase in response to rising commodity prices. However, the impact is short lived, and quite likely reflects the response of policy makers, as the interest rate also rises significantly to counteract these inflationary pressures. As has been noted previously, this study also finds a strong positive relationship between commodity prices and the exchange rate.

The remainder of the paper is organized as follows. Section 1.2, briefly discusses literature relevant to the topic under consideration. Section 1.3 examines the relationship between commodities and Canadian economic activity, further highlighting the importance of commodity prices. Section 1.4 discusses the methodology employed, including the data and identifying assumptions. The results of the model are presented in Section 1.5, where there is a discussion of the baseline model as well as some robustness checks. Finally,

⁴Fernandez-Villaverde et al. (2007) also illustrate that if the variables chosen for the model are relevant and if appropriate identifying restrictions are imposed, then the impulse responses of a structural VAR adequately portray the dynamic behaviour of the macroeconomic variables due to the structural shocks.

Section 1.6 concludes.

1.2 Literature Review

Of all the commodities, oil has undoubtedly received the most attention and since the pioneering work of Darby (1982) and Hamilton (1983) in the U.S. context, several studies have sought to assess the relationship between oil prices and the macro-economy in both developed and developing countries. Previous papers which have examined the relationship between resource prices and economic activity in Canada, have also tended to focus primarily on the energy sector and more specifically on oil prices. These include articles by Stuber (2001), Kremmidas (2011), and Schaufele (2016). The general conclusion of these papers is that Canada's economic fortunes are closely tied to that of the energy sector.

A subset of these papers specifically examine the uncertainty associated with energy prices and how economic activity is affected by this uncertainty. Jo (2012) for example, examines the time-varying volatility associated with oil prices in order to assess how oil price uncertainty could affect real economic activity. She finds that uncertainty relating to oil prices has a significantly negative impact on both global and Canadian economic activity. Similarly, Bashar et al. (2013) examine the link between oil price uncertainty and macroeconomic activity in the context of Canada. Their results suggest that while shocks to oil price level do not affect the level of aggregate output, oil price uncertainty does exert a considerable influence on the Canadian economy. Huang et al. (2005) using a multivariate threshold autoregressive model for the U.S, Canada, and Japan also finds that oil-price volatility has an effect on these economies if the change was below the threshold level. However, if the change exceeds the threshold, then movements in the level of oil prices better explains macroeconomic variables rather than oil-price volatility.

Focusing more on changes to commodity prices itself, rather than its volatility, Charnavoki and Dolado (2014) use a combination of structural VAR methodology, structural dynamic factor model and factor augmented VAR⁵, to categorise the main world commodity price shocks and assess the impact of these shocks on a small commodity exporting economy, such as Canada. They find that a rise in commodity prices generates a positive effect on the trade balance and results in an appreciation of the real exchange rate. These responses have been coined the “commodity currency effect” and was pioneered in the work of Chen and Rogoff (2003) and expanded in other studies such as Cashin et al. (2004).

Vasishtha and Maier (2013) also use a large scale factor-augmented VAR (FAVAR) model to analyse how global developments, including movements in commodity prices, impact the Canadian economy. They conclude that Canada benefits from higher commodity prices through a positive terms of trade shock. This benefit however, is tempered by the extent to which higher commodity prices lower global economic activity, as this would negatively impact Canada’s non-commodity exports. Similar results are also reported by Korhonen and Ledyeva (2010), in their study of net oil-producing and oil-consuming countries. They find that for net oil-producers, such as Russia and Canada, a positive oil price shock significantly increases GDP. However, oil-producers are also negatively impacted by the indirect effects of an oil price increase. Namely, higher oil prices result in lower economic activity in oil-importing countries, such that their volume of oil-imports may decline.

In the stream of literature that examines industry impact, Lee and Ni (2002) employ a VAR model to examine the effects of shocks to oil prices across several different industries in the United States. Their results suggest that for most industries, positive oil price shocks

⁵Structural dynamic factor models were largely developed in the work of Stock and Watson (2005) and Forni et al. (2009) and for factor augmented VARs see Bernanke et al. (2005), Boivin and Giannoni (2008) and Mumtaz and Surico (2009).

result in a significant reduction of output. Similarly, Fukunaga et al. (2010) conclude that the impact of oil price changes on industrial output in the U.S. and Japan is significant and is influenced by industry characteristics. Kilian and Baumeister also demonstrate the macroeconomic implications of oil price shocks on several industrial nations, including the US and Canada in an extensive series of publications.⁶

Jimenez-Rodriguez (2008) focuses specifically on the manufacturing industry in six OECD countries⁷ and report that a positive oil price shock decreases the level of manufacturing output in all countries.⁸ However, this shock produces different responses across sub-industries within the manufacturing sector. Guidi (2010) examines the responsiveness of the UK manufacturing and services sector to oil price shocks, and finds that while the manufacturing sector contracts significantly in response to a positive shock, the services sector is relatively unaffected.

Among developed nations, numerous other papers find that positive oil price shocks have a negative impact on industrial output, (see Burbidge and Harrison (1984), Gisser and Goodwin (1986), Hooker (1996), Rotemberg and Woodford (1996) and Schmidt and Zimmermann (2007)). The negative impact of a positive oil price shock on manufacturing and industrial output, is not unexpected, as industries in these sectors tend to have a high energy content in their production processes. Economic theory would suggest that the energy-intensity of the industry under consideration would affect the magnitude of its response, and this expectation has been borne out in the above cited studies.

Additionally, there are several studies which examine the effects of commodity price

⁶See Kilian and Barsky (2004), Kilian (2008a), Kilian (2008b), Kilian (2009), Lewis and Kilian (2011), Kilian (2014), Baumeister and Kilian (2014), Baumeister and Kilian (2016), Baumeister et al. (2010), and Baumeister and Peersman (2013).

⁷These countries included France, Germany, Italy, Spain, the US and UK

⁸See also Jimenez-Rodriguez and Sanchez (2005) which examine the relationship between oil price shocks and real GDP growth in a subset of OECD countries.

movements on economic activity within the context of developing countries. Farzanegan and Markwardt (2009) examine the relationship between oil price shocks and key macroeconomic variables in Iran by applying a VAR approach. They find that positive oil price shocks significantly affect inflation as well as the growth of industrial output. In contrast, Olomola and Adejumo (2006) and Iwayemi and Fowowe (2011) find that for Nigeria, oil price shocks do not have a major impact on most macroeconomic variables. Eltony and Al-Awadi (2001) find evidence that oil price shocks are important in explaining macroeconomic variables, and in particular government spending, in Kuwait. For the Philippines, Raguindin and Reyes (2005) report that positive shocks to oil prices result in a sustained reduction in domestic output and also that negative shocks have a larger impact than positive ones. For China, Du et al. (2010) and Tang et al. (2010) report that oil price increases negatively affect output and investment, but positively affect the inflation and interest rates, with the real effects being much more persistent than the nominal ones. Silva (2011) report that for a subset of Latin American countries, domestic output increase after a positive commodity price shock. These results are generally in line with a priori expectations.

While the above cited studies have focused on the impact of commodity price shocks at the aggregate level and a few focus on the industry impact of oil price shocks, to date, studies which focus on the differential sectoral impact of a broad-based commodity price index, specifically in the Canadian context, are limited. Knop and Vespignani (2014), is among the few which have examined this sectoral issue in the context of Australia. They find that commodity price shocks largely affect the mining, construction and manufacturing industries, whereas the financial and insurance sectors were relatively unresponsive.

This paper follows in the spirit of Knop and Vespignani (2014), however, in contrast to them, we take commodity prices to be an international variable and it is therefore exogenous

to the domestic economy. They however, assumed that domestic variables could affect commodity prices with a lag, and so Australia was not strictly modelled as a small-open economy, since it could impact the international price of its commodity exports (albeit with a lag).

This paper maintains the assumption that Canada is a small open economy (SOE) and net exporter of commodities. That is, while Canada participates in the international trade of commodities, it is small enough compared to its trading partners that it is not able to alter world commodity prices, and hence, is a price-taker. Additionally, unlike many of the studies which have examined the impact of commodity prices on Canada, this paper employs the use of the Bank of Canada's Commodity Price index, rather than just a sole focus on oil prices. This index includes a wide range of industrial and agricultural commodities, and is therefore relatively more representative of the commodity price shocks that Canada faces rather than an index which solely focuses on oil prices. This is important because oil prices can exhibit significant volatility which are specific to the global political economy in which oil prices are determined and as such, are unrelated to the conditions influencing other commodity prices.

Table 1.1 presents a survey of studies which have examined the impact of commodity price shocks on economic activity, both in the Canadian and international context.

**Table 1.1: Literature Survey -
Commodity Price Shocks and Economic Activity (VARs)**

Authors	Data	Sample Period	Methodology	Main Findings
Soojin Jo (2012)	world crude oil production real price of crude oil world industrial production	1958Q2 - 2008Q3	VAR with stochastic volatility of oil prices	Oil price uncertainty af- fects world real economic activity in a significantly negative way
Charnavoki et al. (2014)	world: real economic activity, inflation, real commodity prices domestic: real activity indicators, inflation series, exchange rates, financial variables	1975Q1 - 2010Q4	VAR Dynamic factor model FAVAR	Positive commodity price shocks, positively impact the external balance and appreciates the exchange rate
Vasishtha et al. (2013)	real activity: output growth, industrial production, exports, employment inflation: CPI, producer price index, core inflation, wage growth, import prices others: monetary aggregates, various interest rates	1985M1 to 2008M5	FAVAR	Canada benefits from a positive commodity price shock, but only to the ex- tent that this shock does not significantly dampen global economic activity
Bashar et al. (2013)	industrial production, CPI, money stock, overnight interest rate, exchange rate, federal funds rate, oil price, oil price uncertainty	198M1 - 2011M12	SVAR	Shocks to the level of oil prices do not play a ma- jor role in shaping out- put variations in Canada, however output is signifi- cantly affected by shocks to the volatility of oil prices
Knop et al. (2014)	world: real GDP, inflation, interest rate domestic: commodity prices, non-farm real GDP, sector-specific real GVA, industry profits before income tax, nominal GVA, inflation rate, interest rate, exchange rate	1993Q1 - 2013Q1	SVAR	Mining and construction were largely impacted by commodity price shocks, whereas financial and in- surance sectors were rel- atively unaffected. The manufacturing sector was also impacted, however re- sponses varied among the sub-indices.

**Table 1.1: Literature Survey -
Commodity Price Shocks and Economic Activity (VARs) Cont'd**

Authors	Data	Sample Period	Methodology	Main Findings
Lee et al. (2002)	macro variables: M2, 3-month T-bill rate, CPI, 10-year treasury rate, industrial production, oil price industry variables: industry-level output and price	1959M1 - 1997M9	Identified VAR	Both the demand and supply of industries are affected by oil price shocks. Oil price shocks primarily reduce the supply of oil-intensive industries while they mainly reduce the demand of other industries.
Jimenez-Rodriguez (2008)	real oil price in domestic currency; sector-specific industrial output	1975M1 - 1998M12	Recursively identified VAR	A positive oil price shock lowers the level of aggregate manufacturing output in all countries under study.
Guidi (2010)	real oil price; sector-specific industrial output; sector-specific real wage; real effective exchange rate; inflation; short-term interest rate; long-term interest rates	1970Q1 - 2005Q4	Recursive VAR	Positive oil price shocks negatively affect the UK manufacturing sector however, the services sector is relatively unaffected.
Ichiro et al. (2010)	world: crude oil output; industrial output; spot crude oil price domestic: aggregate industrial production; industry production; industry producer prices	1973M1 - 2008M12	Block-Recursive Identified VAR	Unanticipated oil price increases negatively impact the US economy both at the aggregate and industry levels, however the impact on Japans economy is ambiguous at the aggregate level, but positive for oil-intensive industries.
Farzanegan et al. (2009)	Industrial GDP per capita, real public consumption expenditures, real imports, real effective exchange rate, inflation, real oil prices changes	1975Q2 - 2006Q4	VAR	Positive oil price shocks increase the real effective exchange rate and the growth of industrial output in Iran.

**Table 1.1: Literature Survey -
Commodity Price Shocks and Economic Activity (VARs) Cont'd**

Authors	Data	Sample Period	Methodology	Main Findings
Iwayemi et al. (2010)	real GDP, government expenditures, inflation, real exchange rate, net exports	1985Q1 - 2007Q4	unrestricted VAR	Positive oil price shocks do not have a significant impact on macroeconomic variables in Nigeria, but there does appear to be asymmetric effects, as negative oil price shocks do significantly affect output and the real exchange rate.
Du et al. (2010)	real GDP, inflation, real oil price, short-term interest rate, money supply	1995M1 - 2008M12	VAR	Inflation and economic growth in China are significantly impacted by world oil shocks, however China's economic activity fails to significantly affect world oil prices.
da Silva (2011)	foreign: real GDP, inflation, fed funds rate, world commodity price index domestic: real GDP, inflation, interest rate, exchange rate	2000M1 - 2011M1	SVAR	Real GDP, domestic inflation and interest rates in a subset of Latin American countries increase after a positive commodity price shock.

1.3 A Closer Look at Commodities and the Canadian Economy

To place this study in context it is useful to examine the role played by commodities in the Canadian economy. In looking at the sectoral composition of the Canadian economy some salient features are observed, the more prominent of which are highlighted in Table 1.2.

From this table we can see that a significant proportion of production in the commodities sector is exported, and that relative to the manufacturing and services sector, commodities contribute significantly more to net exports. This observation is further collaborated by Figure 1.1 and Figure 1.2, which emphasise the high degree of export dependence on the commodity sector.⁹ These figures reveal that commodity exports consistently account for on average, 48 percent of total exports and 14 percent of real GDP. These are relatively high figures for any economy. Even during the period of the global financial crisis, there was no significant departures from these averages.

Table 1.2: Sectoral Composition of the Canadian Economy

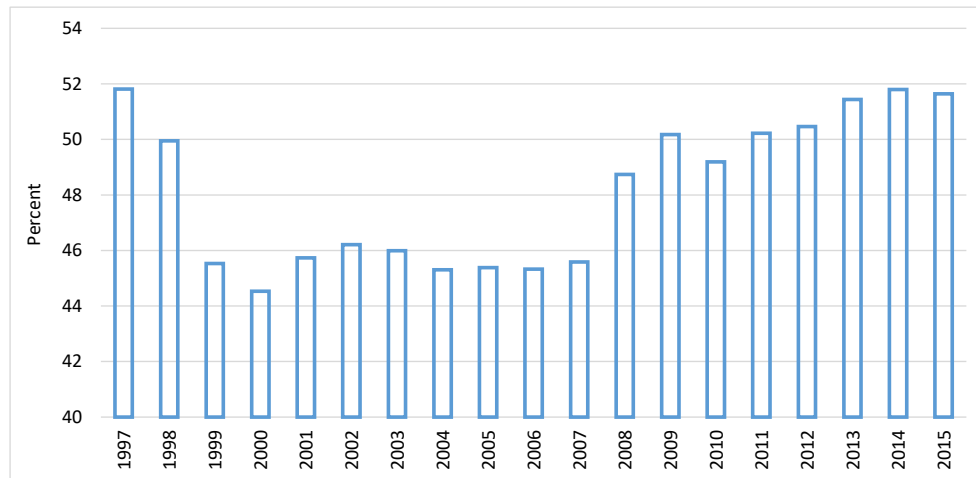
	Gross Value Added (% of total)	Employment (% of total)	Export (% of industry production)	Net export (% of total GDP)
Primary Commodity Sector	8.4	5.5	32.7	2.3
<i>Agriculture, forestry and fishing</i>	3.0	4.0	17.3	0.6
<i>Mining and quarrying</i>	5.4	1.4	45.1	1.6
Tradable Sector	16.4	15.4	42.3	-2.4
<i>Manufacturing</i>	16.4	15.4	42.3	-2.4
Non-tradable Sector	75.2	79.2	5.7	0.8
<i>Utilities</i>	3.1	0.9	6.0	0.2
<i>Construction</i>	6.1	6.0	-	-
<i>Services</i>	66.0	72.2	6.9	0.6

Source: Charnavoki and Dolado (2014); Data: CANSIM, average over 1975-2010

Additionally, according to Natural Resources Canada estimates, in 2015 natural resources (which includes primary commodities such as forestry, mining and energy) directly and indirectly accounted for 1.75 million jobs in Canada (see Figure A.1a), represented 17 percent of nominal GDP (see Figure A.1b), contributed over 40% of total non-residential

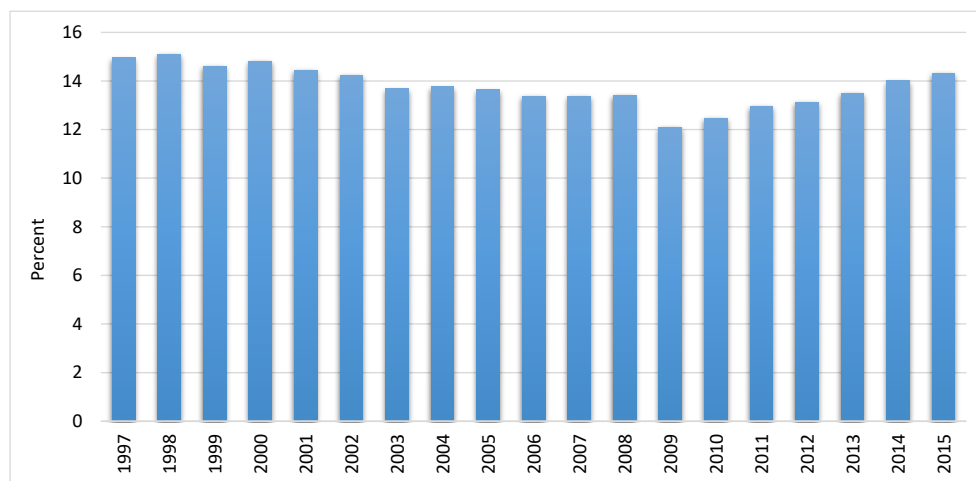
⁹Throughout this thesis, commodity sector, resource sector and natural resource sector are all used interchangeably.

Figure 1.1: Commodity Exports as a Percentage of Total Goods Exports



Source: CANSIM, author's calculations

Figure 1.2: Commodity Exports as a Percentage of Real GDP



Source: CANSIM, author's calculations

capital investment (see Figure [A.1c](#)) and over the period 2010 - 2014, contributed \$27 billion on average, to government revenues (see Figure [A.1d](#)).

Agriculture and Agri-Food Canada also reported that in 2014 the agricultural and agri-food industry provided one in eight jobs in Canada, which translated into employment of over 2.3 million people. They also noted that this industry contributed over \$100 billion annually to Canada’s gross domestic product (GDP), representing 6.6 percent of Canada’s GDP in 2014.¹⁰ Given this highlighted importance, it is necessary to not only understand, but also to quantify how shocks to the price of commodities is likely to impact domestic economic activity. In the next section, the model and data that will be used to do this is presented.

1.4 Empirical Strategy

1.4.1 Methodology

The analysis is carried out, with a structural vector auto-regressive (SVAR) model in order to assess the relationship between commodity prices and macroeconomic activity in Canada. This system may be expressed as in equation [1.1](#), where the constant term and any exogenous variables have been omitted for ease of exposition:

$$A_0 y_t = \sum_{l=1}^p A_l y_{t-l} + \varepsilon_t \tag{1.1}$$

where A_0 is a $n \times n$ contemporary parameter matrix, y_t , a $n \times 1$ column vector of endogenous variables at time t , A_l a $n \times n$ parameter matrix of lagged coefficients, and ε_t , a $n \times 1$

¹⁰Information taken from [An Overview of the Canadian Agriculture and Agri-Food System 2016](#).

column vector of structural disturbances, $t = 1, \dots, T$ is the sample size and p , the lag length of the VAR.¹¹ The structural disturbances are assumed to be independently and identically distributed with mean zero and variance-covariance matrix Ω .

We may further write the model more compactly in matrix form as follows:

$$Ay_t = Gz_t + \varepsilon_t \tag{1.2}$$

where $A = A_0$, $G = [A_1 \dots A_p]$ and $z_t = [y_{t-1} \dots y_{t-p}]'$. In equation 1.2, G is the $n \times np$ matrix of lagged coefficients and z_t is the $np \times 1$ column vector of lagged endogenous variables. Thus, the parameters of the structural model are contained in A and G . Since A captures the contemporaneous relationship of the variables, direct estimation of equation 1.2 can prove difficult. However, given that A is invertible, the reduced-form of the model may be obtained by pre-multiplying equation 1.2 by A^{-1} to obtain:

$$y_t = Bz_t + u_t \tag{1.3}$$

where $B = A^{-1}G$ and $u_t = A^{-1}\varepsilon_t$, with variance-covariance matrix Σ .¹² It can further be shown that Σ and the structural variance-covariance matrix Ω , are related according to $\Sigma = A^{-1}\Omega A^{-1'}$. As the regressors of equation 1.3 are all predetermined and not contemporaneous, it can be estimated efficiently and consistently using conventional techniques, such as maximum likelihood, and the structural shocks of interest, recovered by imposing appropriate identifying restrictions.

In the identification process, it is important to correctly characterise the relationship between the foreign and domestic variables in the model. Canada's economy is approxi-

¹¹The parameters of the individual equations in the VAR are contained in the rows of A_0 and A_t

¹² u_t is the reduced form matrix of residuals.

mately one-tenth as large as the economy of the United States,¹³ with about 75 percent of Canada’s exports destined for the US, and about 20 percent of US exports for Canada. Thus, it seems unreasonable to conclude that economic activity in Canada could significantly affect US or world variables. In fact, Zha (1999) has shown that implausible conclusions can be reached when the small-open economy (SOE) structure of the Canadian economy is ignored within a VAR setting.¹⁴ Consequently, in this study, such a structure is imposed via a block exogeneity assumption.

In doing so, we note that y_t is comprised of two blocks. A domestic block y_{dt} , which contains the Canadian variables and a foreign block y_{ft} , which contains the US and world variables. Thus the structural model in equation 1.2 can be written as:

$$\begin{pmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} y_{ft} \\ y_{dt} \end{pmatrix} = \begin{pmatrix} G_{11} & 0 \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} z_{ft} \\ z_{dt} \end{pmatrix} + \begin{pmatrix} \varepsilon_{ft} \\ \varepsilon_{dt} \end{pmatrix} \quad (1.4)$$

where we have restricted the parameters of A and G so that economic developments in Canada do not affect foreign variables either contemporaneously, $A_{12} = 0$, or with a lag, $G_{12} = 0$.

In the reduced-form model, the SOE assumption is represented as follows:

$$\begin{pmatrix} y_{ft} \\ y_{dt} \end{pmatrix} = \begin{pmatrix} B_{11} & 0 \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} z_{ft} \\ z_{dt} \end{pmatrix} + \begin{pmatrix} u_{ft} \\ u_{dt} \end{pmatrix} \quad (1.5)$$

¹³As measure by real GDP in 2015.

¹⁴Zha (1999) found for example, that when the block exogenous structure of the US-Canada economic relationship is ignored, then shocks to Canadian variables contributed 67 percent to fluctuations in US interest rates, 62 percent to fluctuations in US price level and 45 percent to fluctuations in US output. Such results are anomalous to the actual economic relationship which exists between the US and Canada. Johnson and Schembri (1990) and Souki (2008) also found that relative to domestic shocks, U.S. based shocks are more important in explaining Canadian macroeconomic variability.

with the contemporaneous restrictions imposed in the A matrix, which as stated above, is related to the estimated reduced-form variance-covariance matrix, Σ .

The structural innovations in equation 1.4 can be recovered from equation 1.5 if an appropriate identification method can be found for the system. The identification method requires placing additional zero restrictions on the A matrix based on theoretical beliefs about the contemporaneous interactions among the variables. In, the baseline analysis, the structural shocks are identified using a recursive ordering approach. No additional restrictions are imposed on the lagged coefficients except the block-exogeneity restrictions on the foreign variables in the domestic equations, as shown above in the matrix representations of the model.

The foreign block is kept in the lower-triangular order of output, inflation, interest rate and finally commodity prices. Commodity prices are placed in the foreign block since it is believed that Canada is a price-taker and is too small to significantly affect the world prices of the commodities which it exports. This assumption is fairly standard in the literature which examines the impact of external shocks on small open economies, (see Silva (2011), Knop and Vespignani (2014), among others). It is also placed last because it is a relatively fast moving variable and its value generally reflects all available relevant information rather quickly.¹⁵ For the domestic block, the variables are also ordered lower triangular in the form output, inflation, interest rate and exchange rate. Thus, output is the most exogenous of the domestic variables and exchange rate, the most endogenous such that it is impacted by the shocks of all the other domestic variables.

Following Cushman and Zha (1997) and Zha (1999), the foreign variables are treated

¹⁵Commodities are often traded on futures markets using financial contracts, and so their prices tend to fluctuate daily and incorporate all publicly available information. Consequently, placing them last in the external block of the VAR reflects their usage as financial assets which can adjust almost instantaneously to news concerning the other foreign variables, such as foreign interest rates.

as exogenous from Canada's point of view in order to strictly maintain the small-open-economy assumption. In his paper, Zha (1999) developed an algorithm to estimate block recursive VARs. This algorithm uses a generalized block Monte Carlo method to obtain maximum likelihood (ML) estimates of the parameters and to calculate error bands. Zha applied this methodology to examine monetary policy shocks in the Canadian context. However, the methodology, which is also able to handle highly parameterized models, will be employed here to examine the impact of commodity price shocks. Thus, to study the question posed in this chapter, the methodology formulated by Zha (1999) is utilised such that the model is estimated via maximum likelihood and the block exogenous structure of the model is accounted for.

While VAR models are an established tool in macroeconomics for describing the data generating process of time-series variables and have the advantage that statistical inference can be undertaken using standard methods, direct interpretation of the VAR parameters is often difficult. Consequently, analysing the interaction among variables via direct examination of the coefficients is also rendered difficult. Therefore, to examine the interaction among the variables, impulse response functions (IRFs) are generally employed. Since these are determined from the coefficients of the estimated parameters, they too are estimates and their estimation uncertainty is often indicated by plotting error bands along with the estimated IRF coefficients. The error bands reported in this study are based on the Monte Carlo simulation method outlined in Zha (1999).¹⁶

In much of the applied VAR literature, either 68 percent or 95 percent error bands are normally reported. However, Sims (1987) presents arguments against the use of 95 percent error bands in VAR studies, on the basis that "there is no scientific justification for testing

¹⁶The method involves draws made directly from the posterior distribution of the VAR coefficients, where the error bands are calculated using Monte Carlo integration over several replications. The .16 and .84 fractiles are then graphed to delineate the .68 probability band.

hypotheses at the 5 percent significance level in every application.” He instead, advocates that the significance of the impulse responses derived from the VAR coefficient estimates, be treated differently from coefficient estimates obtained in standard econometric models. It is an inherent feature in VAR models that a large number of the estimated parameter coefficients are not statistically different from zero, and consequently are insignificant when evaluated at the 5 percent level. This automatically translates into extremely large error bands for the impulse response functions. Despite this, it has generally been found by both empirical and theoretical researchers, that the estimates from reduced-form VARs provide a useful and valid summary of the data generating process. Allowing for this, Sims and Zha (1999) recommend the use of 68 percent error bands for the estimated impulse response functions. Thus this paper, adopts this probability level when evaluating the IRFs.¹⁷

1.4.2 Data

The model is estimated using seasonally adjusted monthly data for the period 1997:M2 to 2016:M4. Based on the Akaike Information Criterion (AIC), a lag length of 2 was also selected.¹⁸ The model includes four foreign and five domestic variables. The foreign variables are US industrial production as a measure of output, US CPI inflation, the federal funds rate and the Bank of Canada commodity price index.¹⁹ The US is used to represent the foreign economy as it is Canada’s closest, and main trading partner. The domestic

¹⁷Consequently, the constructed error bands contain the true impulse response functions with 68 percent probability. This is generally equivalent to the one standard deviation confidence intervals that are usually computed in standard econometric models, under the assumption of normality (or more specifically, of normally distributed error terms).

¹⁸Other lag-length selection criteria, such as Schwarz Information Criterion (SIC) or Hannan-Quinn Criterion (HQC) for example, suggested alternative lag-lengths of 1, 3 and 4. Two was selected to maintain a balance between omitted variables and over-parametrisation. As discussed in our robustness section however, the relevant results are not unduly affected by changes in lag length.

¹⁹Here again we reiterate that commodity prices are viewed as a foreign variable, because Canada is a price taker in its export market.

variables are total real gross value added (GVA), a measure of sectoral production as given by industry real gross value added, CPI inflation, the overnight rate and the US/Canada bilateral exchange rate. All US variables are taken from the FRED database, and all domestic variables from CANSIM. Further information about the data may be found in Table A.1 in Appendix A. For each sector of the domestic economy, a separate VAR is estimated.²⁰ An aggregate model that excludes sectoral production measures is also estimated to examine the impact of commodity price shocks on the macro variables.

These choice of variables are fairly standard in models of this type and are meant to capture the main drivers of economic activity in the domestic economy while maintaining model parsimony. US output, inflation and policy rate are intended to capture the impact of economic conditions in advanced economies on Canada²¹ and commodity prices are included as our main variable of interest, but also because it captures the fact that Canada is a significant commodity exporter.

GVA and its components, enter the model as log deviations from a quadratic trend.²² Specifying output as a deviation from trend bears consistency with the way this variable often enters the reaction function of policy makers. That is, output is usually thought of in terms of deviation from potential (or its long-run trend) and it is for this reason that we prefer a trend specification for these variables over first differences. The consumer price index (CPI) enters the model in monthly changes (that is, inflation) and so does

²⁰For the sectoral models, the measure of total GVA included is net of the value-added of the particular industry under review, such that total output is equal to the sum of total GVA in the model plus industry GVA.

²¹Several papers (see Cushman and Zha (1997) and Bhuiyan (2012)) have also postulated that the Canadian monetary policy function responds to changes in the US policy rate. Thus suggesting that these US variables can affect Canada's macro variables.

²²The use of a quadratic trend accounts for the reduction in the general pace of economic growth in the US since the 1900's. The results remain virtually unchanged if a linear trend is used instead for the output variables. Similarly, using growth rates did not yield significantly different results.

commodity prices. The interest rate variables are in levels and the exchange rate in (log) levels as well. The Augmented Dickey Fuller (ADF) test indicates stationarity of the transformed variables at conventional levels of significance. A dummy variable for the period October 2008 to September 2009, was also included in the regression equations to account for the unusual decline in economic activity which took place during the global financial crisis. All equations were also estimated with a constant.

1.5 Results

1.5.1 Baseline Model

Aggregate Model

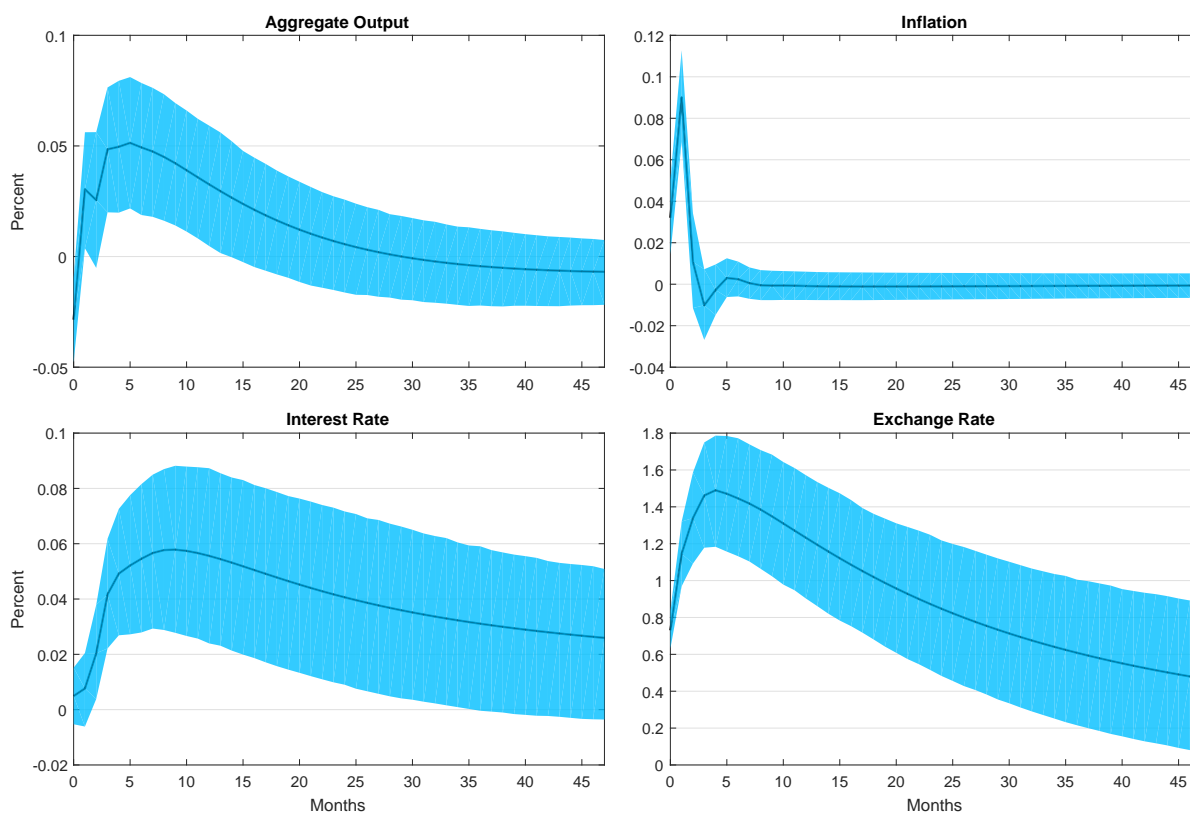
While we aim to examine the impact of commodity price shocks on sectoral production in Canada, we begin our analysis by assessing the impact of these shocks on key Canadian macroeconomic variables for the economy as a whole. Figure 1.3 shows the response of aggregate output, inflation, interest rate, and the exchange rate to a one standard deviation shock to commodity prices.

On impact, aggregate output falls in response to the commodity price shock, but begins to rise immediately thereafter and continues to do so until it peaks in the ninth month. This fall in production may be due to an initial decline in economic activity in the non-commodity producing sectors of the economy, which is not fully offset by the higher production levels of the commodity producing sectors.

Inflation increases contemporaneously in response to higher commodity prices, but declines in subsequent periods, and by about the tenth month, has returned to baseline. The

relatively rapid return of inflation to pre-shock levels quite likely reflects two complementary factors. Firstly, the appreciation of the exchange rate which results in lower prices for imported goods and thus lower inflation levels, and secondly, rising interest rates which serves to reduce demand and dampen the associated inflationary pressures. Figure 1.3 also shows that interest rates initially increase in response to the commodity price shock, as the monetary authority attempts to anchor inflationary expectations.

Figure 1.3: Aggregate Responses to a Commodity Price Shock



Notes: The vertical axis units are deviations from the unshocked path, which is the baseline. Blue lines are the impulse response estimates. Blue areas are 68 percent probability intervals about the impulse response point estimates.

The exchange rate initially appreciates in response to the commodity price shock and continues to do so before peaking at month five. This response is statistically significant over the entire impulse horizon, and reinforces the result of the strong relationship between commodity prices and exchange rate movements which have been found for Canada in previous studies, (see for example Chen and Rogoff (2003)).

Production Sectors

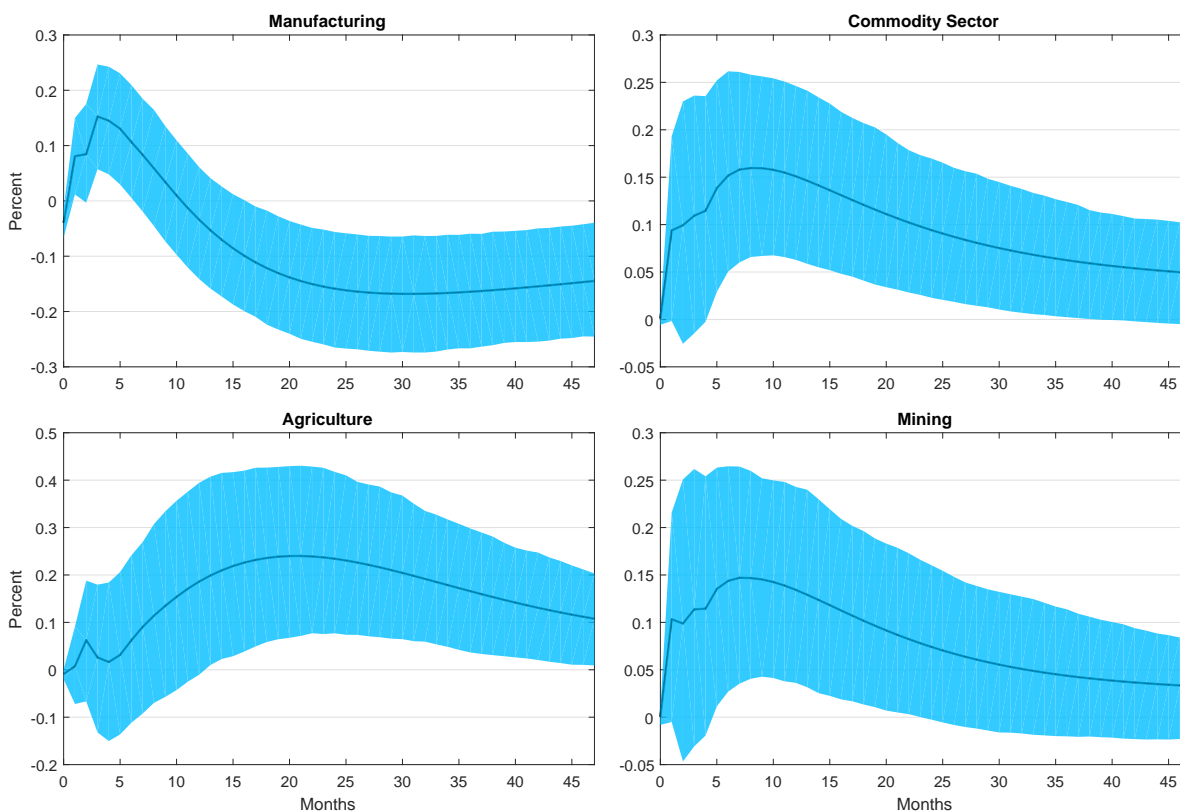
Figures 1.4 and 1.5 show how different industries respond to the commodity price shock. Consistent with the decline in aggregate output on impact, manufacturing output also declined on impact. It then begins to rise, until reaching its peak approximately five months after the initial shock. Manufacturing output declines from this point onwards, and falls below its baseline level twelve months after the shock, remaining below this level even after four years. These responses are statistically significant over the impulse horizon and offers evidence that the manufacturing industry in Canada is negatively affected by rising commodity prices. Similar results for the UK manufacturing sector were found by Guidi (2010).

Rising commodity prices may negatively impact the manufacturing sector in two respects. Firstly, if commodities are an important input into the production process of manufacturing firms, then higher costs for these inputs, either have to be absorbed by the firm,²³ thereby reducing profits, or passed on to the consumer, which is likely to reduce demand and therefore sales. Secondly, and perhaps more importantly in the Canadian context, rising commodity prices lead to an appreciating exchange rate (see Figure 1.3).

²³In which case, rising commodity prices is expected to have a smaller absolute effect on manufacturing output.

Since a significant portion of Canada’s manufactured goods are exported,²⁴ this raises the price of Canada’s manufactured exports and thus reduces foreign demand,²⁵ resulting in a contraction of output.

Figure 1.4: Production Sector Responses to a Commodity Price Shock



Notes: The vertical axis units are deviations from the unshocked path, which is the baseline. Blue lines are the impulse response estimates. Blue areas are 68 percent probability intervals about the impulse response point estimates.

²⁴Approximately 42 percent of total manufacturing production is exported (see Table 1.2).

²⁵Domestic demand may also fall since imported manufactured items, which are a substitute for the domestically produced ones, will also be cheaper to purchase, and thus domestic consumers may engage in expenditure switching towards the foreign item.

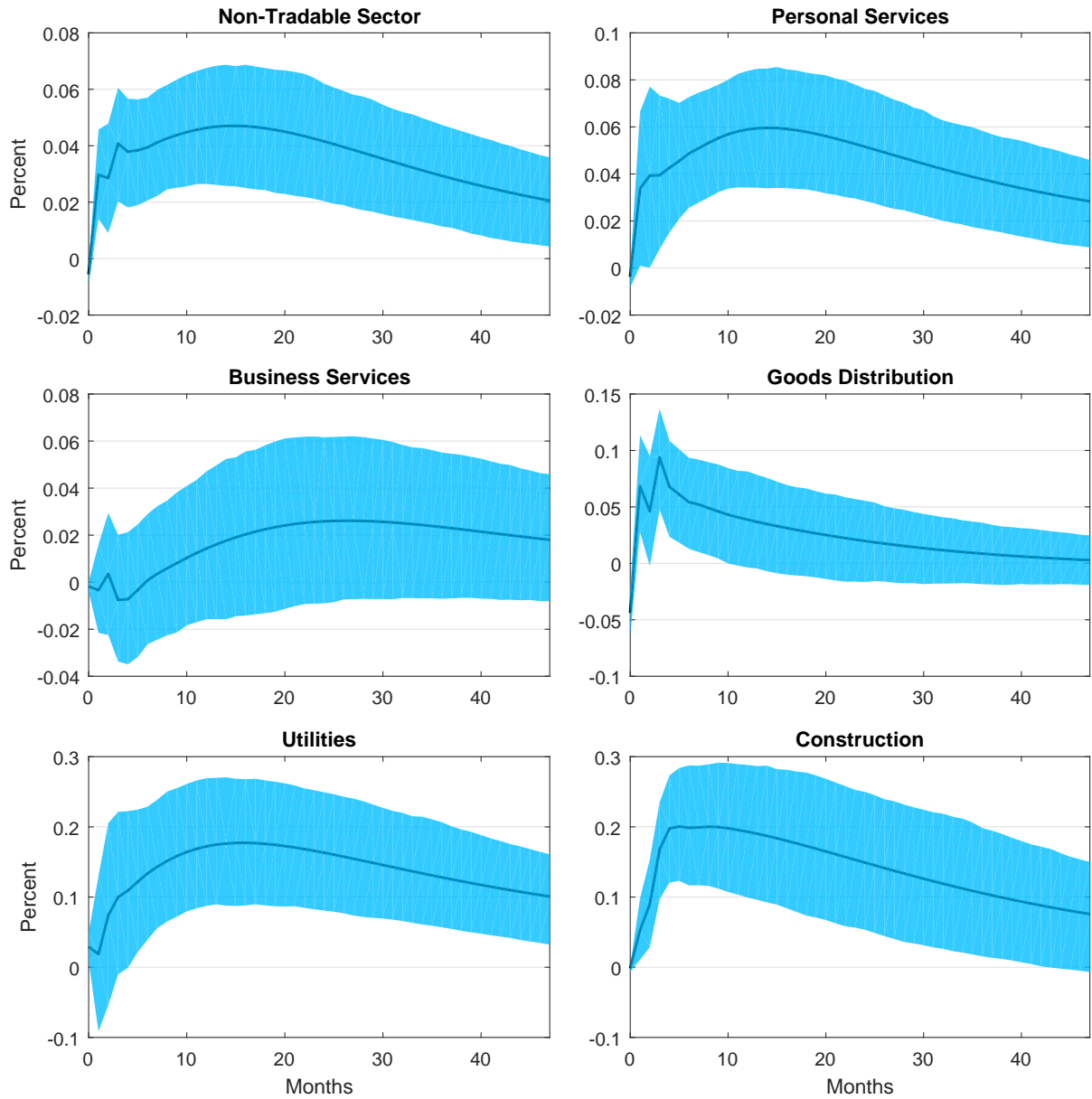
In contrast to the manufacturing sector, Figure 1.4 shows that the commodity producing sectors of the economy rise unequivocally in response to the positive commodity price shock, with the greatest impact occurring about ten months after the shock. The lower panel of Figure 1.4 disaggregates this response into the two component comprising the commodity sector - *Agriculture, forestry and fishing* and *Mining and quarrying*.

Unlike the mining sector, which rises immediately following the shock, agriculture begins to rise in earnest only after the fifth month, and becomes statistically significant from month twelve onwards. Output in the mining sector, however, immediately displays a sharp rise and continues to do so, though at a slower pace, eventually peaking at 0.15 percent higher than it otherwise would have been in the absence of the shock. Also, whereas output in the mining sector peaks at around month ten, output in the agricultural sector, peaks almost two years after the initial shock. This reflects differences in the underlying nature of the two industries. While mining operations can increase output fairly quickly in response to positive price signals, by bringing into operation previously idle mines, or increasing the intensity of operation at currently open mines (given that they were not operating close to or at full capacity), an increase in agricultural output will necessitate waiting on the appropriate growing season and growing period.

Figure 1.5 illustrates the response of the non-tradable sector to the positive commodity price shock. As with manufacturing, output in this sector, declines on impact, but rises thereafter. However, unlike manufacturing, this sector does not experience a decline below its baseline level. The figure also shows the response of the three major sub-categories of this sector - *Utilities, Construction* and *Services*, which for ease of analyses has been further broken down into Personal Services, Business Services and Goods Distribution.

With respect to *Services*, there was a decline on impact for all three sub-sectors and this contributed significantly to the decline which was observed for the non-tradable sector as

Figure 1.5: Non-Tradable Sector Responses to a Commodity Price Shock



Notes: The vertical axis units are deviations from the unshocked path, which is the baseline. Blue lines are the impulse response estimates. Blue areas are 68 percent probability intervals about the impulse response point estimates.

whole on impact of the shock. The response of Personal Services, which includes education and accommodation, was significant over the entire forecast horizon, suggesting that higher commodity prices positively impact the balance sheet of households, which contributes to higher demand for these services and consequently, higher real sectoral output. This has often been cited as the wealth effect of higher commodity prices on Canadian households, as the higher commodity price, essentially represents a positive income shock, such that households feel more secure, thereby spending more and stimulating demand for these services. Business services²⁶ also rose in response to the commodity price shock, however this result was not statistically significant.

Goods distribution, which includes the sectors of wholesale, retail and transportation, increased as well in response to the commodity price shock. The significance of the response however, dissipated after the tenth month, as it returned to its baseline level. The initial decline for this sub-sector may be attributable to the fact that commodities are a significant intermediate input in the production processes of firms in this sector. At the same time, however, the output of this sub-sector is utilised by commodity producing firms, thus the initial negative effect of higher input prices is offset by higher demand for their output.

A similar situation also prevails in the construction sector, although the response here is much more pronounced and significant, relative to the goods distribution sector. Output in the construction sector rises significantly in response to the commodity price shock, being approximately 0.2 percent higher up to ten months after the shock. The output of this sector is utilised heavily by companies in the mining sector, and so as output in the mining sector goes up, there may also be increased demand for the output of this sector, as the construction of new mining sites take place. Robust economic activity, resulting in greater

²⁶Business services includes the sectors of Information, Finance, Real Estate, Professional Services, Management and Administration

business and personal wealth, may also lead to greater demand for houses and corporate structures, thereby bolstering demand for the construction sector.

The increase in utilities in response to the commodity price shock may appear puzzling at first glance, since a significant input into the production of utilities is energy commodities.²⁷ However, although energy commodities are an input into the production of utilities, utilities are also used by firms engaged in the production of both agricultural and industrial commodities. Thus, as these primary commodity producing firms increase output in response to the positive price signal, they also demand a greater amount of utilities, thereby resulting in higher output in the utility sector as well. All industries of the economy need electricity to operate, either directly or indirectly. Thus, when these sectors respond positively to the commodity price shock, consumption of utilities will also increase.

Variance Decompositions

The previous two sections showed that commodity price shocks can have a meaningful impact on the Canadian economy. To further assess this relationship, the forecast error variance decomposition can be investigated.²⁸ While the impulse response functions capture the dynamic response of the endogenous variables to a commodity price shock and consequently show the effects of that shock on the adjustment path of the variables, the

²⁷According to Statistics Canada, this sector comprises establishments primarily engaged in operating electric, gas and water utilities. These establishments generate, transmit, control and distribute electric power; distribute natural gas; treat and distribute water; operate sewer systems and sewage treatment facilities; and provide related services, generally through a permanent infrastructure of lines, pipes and treatment and processing facilities.

²⁸Forecast error variance decompositions specify the proportion of the forecasting error of a variable, at a given horizon, that is due to a particular shock. Decomposition results are derived from the coefficients of the moving average representation of the VAR system in conjunction with the variance of the shock under consideration.

variance decomposition divides the variation in the endogenous variables into that which is attributable to the various structural shocks of the model. As a result, we are able to obtain an estimate of the quantitative importance of each structural shock in driving the dynamics of the particular endogenous variable. Both the IRFs and variance decomposition are useful in assessing how shocks to the endogenous variables propagate throughout the system, and together they allow us to better assess the pass-through of external shocks to the economic variables. Examining the variance decomposition in particular, allows us to determine if commodity price shocks are an important source of Canadian macroeconomic volatility. Tables 1.3 and 1.4 report the variance decompositions for the aggregate and industry models, respectively.

Table 1.3 shows that commodity price shocks contribute significantly to the variability of Canadian macro-variables, most notably to inflation and the exchange rate. For inflation, commodity price shocks explain, on average, 12 percent of the variability and is the third most important factor contributing to inflation volatility after inflation's own shock and shocks to trading partner inflation. For the exchange rate, the impact of a commodity price shock is even more pronounced, as this shock explains approximately 30 percent of exchange rate variability at all horizons.²⁹ The impact of commodity price shocks on aggregate output and interest rates, however, are less marked, contributing on average 3 percent and 4 percent respectively.

Table 1.4 illustrate that commodity price shocks explain a reasonable amount of the variability of industry variables, particularly at longer horizons. This is especially evident for the non-tradable sectors of the economy, where this shock contributes more than 10 percent of the variability for Utilities, Construction and Personal Services at the four year

²⁹The only other significant factor in explaining exchange rate volatility, are shocks to the exchange rate itself.

Table 1.3: Variance Decomposition - Aggregate Variables

Variables	Horizons (months)				
	6	12	24	32	48
Aggregate Output	2.7324	3.5273	3.1915	2.9943	2.8868
Inflation	12.2421	12.2375	12.2420	12.2471	12.2539
Interest Rate	3.3555	4.6018	4.5407	4.3197	3.9952
Exchange Rate	31.3208	33.9414	33.4324	32.1835	29.5524

Each cell in the table shows the percentage of variance explained by the commodity price shock, at the indicated horizon.

horizon. Almost 6 percent of the variability of the manufacturing industry is also explained by shocks to commodity prices at this horizon. For the commodity producing sectors, commodity price shocks explain an increasing amount of the forecast error variance, reaching a modest proportion of about 4 percent at the longest horizon. These sectors, however, are primarily influenced by their own shocks, shocks to foreign output and shocks to the interest rate. This result is not unreasonable, given that much of their output is exported to the foreign economy (see Table 1.2) and firms in these industries are often engaged in financial contracts, which are sensitive to interest rate movements.³⁰ Commodity price shocks explain little of the variance of the other industries in the economy.

³⁰These firms often have significant debt, due to the high cost of capital investment, which are tied to interest rates prevailing in the economy.

Table 1.4: Variance Decomposition - Industry Variables

Industry	Horizons (months)				
	6	12	24	32	48
Agriculture	0.0559	0.5338	2.6459	3.7859	4.6956
Mining	0.8262	2.0595	3.3395	3.6303	3.8663
Manufacturing	1.7388	1.3900	2.4616	3.8937	5.9481
Utilities	0.8366	2.8140	6.7220	8.3639	10.0538
Construction	4.3390	6.8424	8.7480	9.4504	10.2301
Business Services	0.0436	0.0856	0.5719	0.9110	1.3154
Personal Services	1.8925	4.7103	8.8454	10.1662	11.3023
Goods Distribution	2.7530	2.7303	2.4513	2.3871	2.3349

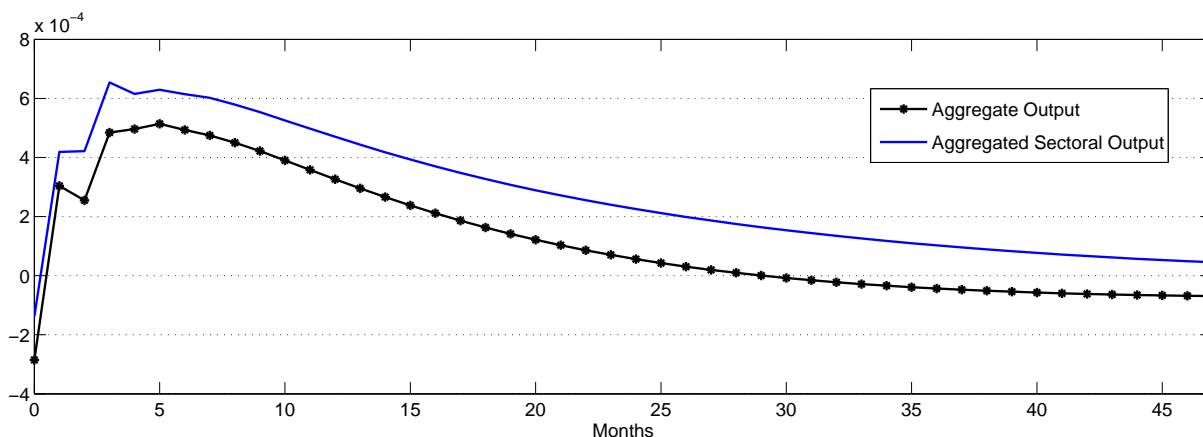
Each cell in the table shows the percentage of variance explained by the commodity price shock, at the indicated horizon.

1.5.2 Robustness Checks and Sensitivity Analysis

Output: Aggregate versus Sectoral Models

Since a separate VAR is estimated for each sector of the economy, one may be interested to check the relationship of our sectoral models with the aggregate model. To examine whether the results from our sectoral models are consistent with those obtained from the aggregate model, we compare these responses in Figure 1.6. In this figure, the output responses of the various sectoral models have been aggregated, with each sector's output weighted by its mean share of total output over the sample period. As can be seen, the results are fairly consistent. The commodity price shock triggers a decline of output on impact in both cases. Responses over the impulse horizon is also similar, although the response from the aggregated sectoral model does remain above that for the aggregate model. This difference however is not statistically significant, as it still lies within the confidence bands of the aggregate model.

Figure 1.6: Output - Aggregate vs. Sectoral Models



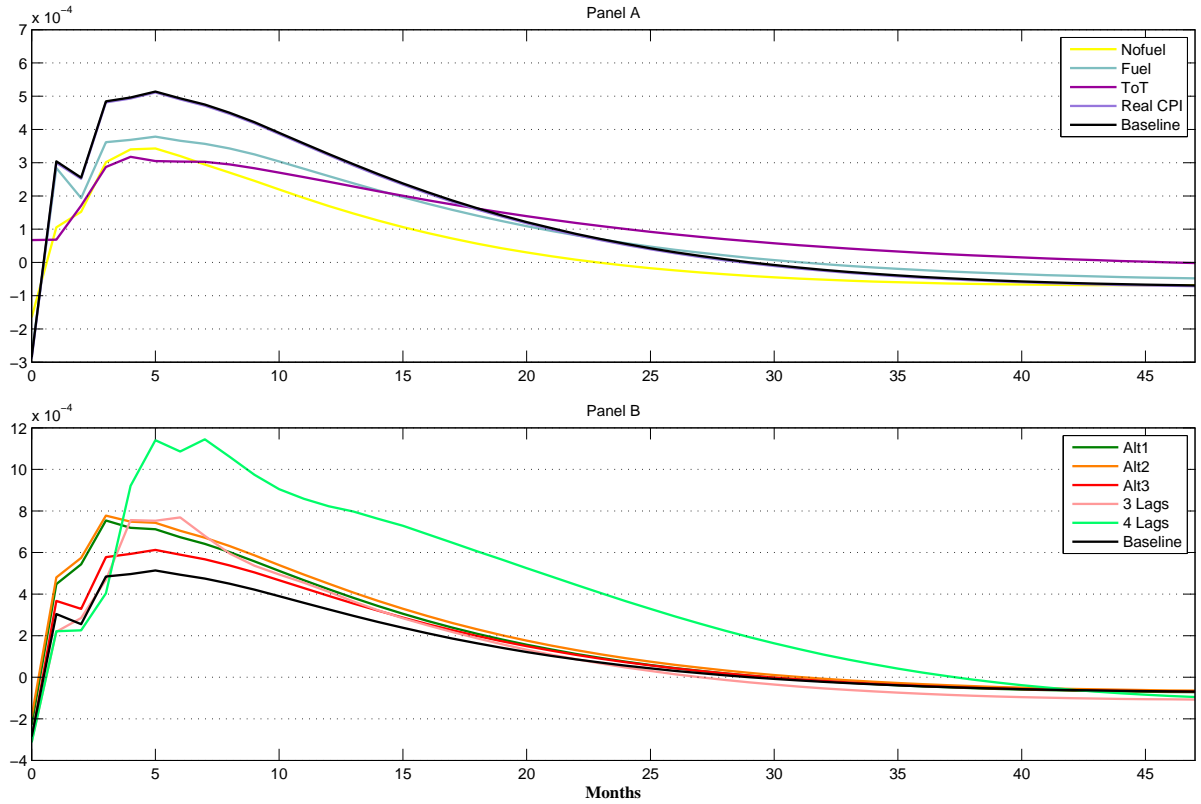
Robustness Checks: Aggregate Model

In this section we examine the robustness of our results to various changes in the specification of the model. The alternative model specifications examined include, changing the index used to represent commodity prices. These are reported in Panel A of Figure 1.7. Here, both the Non-Fuel and Fuel sub-index of the total commodity price index are used. A measure of the terms-of-trade as well as the real commodity price index is also used.³¹ In Panel B of Figure 1.7 we report the results of changing the ordering of commodity prices in the foreign block of the model as well as that of increasing the number of lags in the model. While lag selection tests suggested the use of two lags, it was also possible to use alternative lag lengths. In alternative ordering 1, 2 and 3, commodity prices is ordered before foreign output, inflation, and interest rate respectively, in the identification scheme,

³¹It is often argued that Canada's economic fortunes are significantly impacted by changes to its terms of trade. As used in the paper, the terms-of-trade is calculated as the ratio of total merchandise exports to total merchandise imports. This measure therefore, captures more than just changes to the price of commodity exports. The real commodity price index is obtained by dividing the nominal index by the US consumer price index.

whereas in the baseline, commodity prices is ordered last.

Figure 1.7: Responses of Aggregate Output to a Commodity Price Shock



Notes: **Nofuel** - Commodity price index which excludes fuel prices; **Fuel** - Commodity price index with only fuel prices; **ToT** - Trade of Terms index; **Real CPI** - Real Commodity Price Index; **Alt 1,2,3** - Alternative Identification Scheme 1,2,3; **3(4) Lags** - Model with 3 (4) lags

The response of aggregate output to the commodity price shock under the various scenarios is fairly similar. The difference between the baseline and the real CPI are virtually indistinguishable, suggesting that the baseline results are not unduly affected by nominal concerns. The response using the Fuel and Non-Fuel sub-indices are also similar to that

obtained under the baseline, though the peak responses are lower. In the case of aggregate output therefore, this would suggest that studies which focus exclusively on fuel prices may provide estimates which are only a lower bound for the response of output to a commodity price shock. Studies which employ a broader measure of commodity prices however, may capture more dynamic and complete responses. On impact, the terms-of-trade measure shows a positive response to the price shock, relative to the baseline which reflects a decline on impact. The terms-of-trade response is also somewhat more persistent, even-though its peak response is lower.

With respect to the alternative identification schemes, all three offer a similar response to the commodity price shock. Relative to the baseline, they imply a somewhat larger response of aggregate output to the price shock, however, by two years the differences in responses are virtually indistinguishable. Changing the lag order also results in higher peak responses compared to the baseline. This is particularly evident when 4 lags are used. However, even in this case, all responses have converged to the baseline by year four.

Robustness Checks: Sectoral Models

The response of output in each sector to the commodity price shock under the various scenarios outlined in the previous section, are presented in Figure A.2 of Appendix A. Generally, the response of the various sectors are similar to that observed for aggregate output, and as such we will just examine a few areas in which they differ.

The sectoral response to both the *Fuel* and *Non-Fuel* sub-indices are similar to those observed for aggregate output. However, in the case of the *Commodity* sector, represented in the right panel of sub-graph (a), we see that contrary to the baseline, when only the *Non-Fuel sub-index* is used, commodity output actually declines the first few months after

the price shock, before starting to increase from month three onwards. This reflects a decline in both the *Agricultural* and *Mining* sectors. These initial declines however, are not statistically significant and may reflect the use of non-energy inputs in the production of output for these sectors.³² Higher prices for these inputs may result in the initial declines observed. Similar initial declines are observed for *Personal Services* and *Utilities*, when the Non-Fuel sub-index is used, but again these declines are not statistically significant.³³

As with aggregate output, the use of the Real Commodity Price Index produced virtually indistinguishable results from the baseline case in the sectoral models. In general, the use of the Terms-of-Trade index, produced results that imply a somewhat smaller output response over the impulse horizon relative to the baseline. This suggests that direct price shocks to commodities exported by Canada might have a greater impact on sectoral output, than does changes in the terms of trade, which also factors in import price changes.

With respect to employing alternative identifying assumptions, the various sectoral responses are generally in-line with the baseline case. Of all the sectors considered however, *Utilities* was the most sensitive to these ordering assumptions. For *Utilities*, all three alternative orderings, resulted in a larger increase in output on impact, than that observed in the baseline case. The peak response also occurred much sooner than the baseline, at approximately four months. By month thirty however, the responses are virtually identical to the baseline.

Increasing the lag lengths did not significantly alter the response of sectoral outputs to the commodity price shock. The model with three lags was generally consistent with

³²The effect in the mining sector is not significant over the entire impulse horizon, whereas the effect on agricultural output does not become significant until after the 27th month, at which point agricultural output is increasing.

³³The effect for *Personal Services* becomes significant only after month nine and for *Utilities*, after one year.

the baseline result. For most sectors, the model with four lags produced a higher peak response, but generally returned to the baseline level by the eight month. These larger number of lags could be allowing the model to capture more interactions between sectoral output and the commodity price shock, thus accounting for the higher peak responses. The baseline therefore, with two lags, may represent a lower bound on the effect of commodity price shocks on output.

1.6 Conclusion

This chapter evaluates the impact of commodity price shocks on Canadian economic activity. In addition to examining the aggregate impact, we also sought to examine how these shocks affected sectoral production. In order to do so, we employed a structural vector autoregressive model, identified using a recursive ordering approach and utilising a block exogeneity assumption. The sectoral impact of commodity price shocks has not been extensively discussed in the literature and so represents an area in which we contribute to the current literature. This is important as we find that the effect of the commodity price shock on aggregate output masks significant differences in responses across industries. Thus a failure to account for differential sectoral responses could result in policy actions that if not appropriately targeted, may be beneficial to some industries, while causing harm to others.

Our analysis reveals that commodity price shocks play a significant role in positively impacting the output of *Commodity* producing sectors within the economy. In contrast, over the medium to long term, the *Manufacturing* sector is negatively affected by increases in commodity prices. Output in the *Non-Tradable* sector, though declining on impact, rises thereafter and does not experience a decline below its baseline level, as was observed

for *Manufacturing*.

With respect to other aggregate variables of interest, we find that domestic inflation rises in response to positive commodity price shocks. This impact however, does not last very long, and the relatively quick return to baseline probably reflects the actions of policy makers, as interest rates also increase significantly to moderate inflationary pressures. The strong positive correlation between commodity prices and the exchange rate, which has been reported in other studies, was also found in the current study.

Our findings are generally invariant to a range of robustness checks, including the use of alternative ordering assumptions, changing the lag lengths and employing different measures for the commodity price shock. These findings suggest that overall, commodity prices are an important source of disturbances that affect Canadian economic performance. While the analysis undertaken thus far represents a useful first step to understanding the sectoral impact of these changes within Canada, it is possible to extend this work. For example, alternative identification mechanisms, such as long-run or sign restrictions, may also be employed to identify the commodity price shock. Additionally, one may also attempt to further disentangle the relationship between specific commodity prices and sectoral output. Such extensions would serve to further elaborate upon the results reported here and may be worth exploring in future research.

Chapter 2

Commodity Prices in a Multi-Sector DSGE Model of the Canadian Economy

2.1 Introduction

In this chapter, we analyse in more detail the transmission mechanisms that can explain the dynamics observed in the VAR examined in chapter one. This analysis is based on a micro-founded approach, where a Dynamic Stochastic General Equilibrium (DSGE) model is employed. The model described in this chapter is a slightly modified version of the DSGE model developed by Dib (2008). Dib's model is a small open economy New Keynesian model with several features which are standard in the literature. It also encapsulates some elements which are specific to the Canadian economy, such as a commodity-producing and exporting sector. It also contains a tradable manufacturing sector and a non-tradable

sector, which is comprised primarily of services.

While our study differs in terms of topic of interest and question of focus, the methodology we employ is similar to Dib. The modifications we make to the model include (i) removing monopolistic competition in the labour market, (ii) adding a scaling factor to hours worked, (iii) adding a permanent productivity process, (iv) incorporating price-stickiness via Rotemberg, rather than the Calvo-Yun mechanism, (v) incorporating local currency pricing rather than producer currency pricing, (vi) relaxing the law of one price in the commodity sector, to allow for short-run deviations and (vii) modelling the foreign economy as a three equation system, rather than as AR(1) processes. These changes were made either to increase the tractability of the model and make it easier to solve, given its fairly complex nature or to incorporate some real-world features into the model, without again making it overly complicated.

As was illustrated in chapter one, commodity prices can have a significant impact on the Canadian economy and it is useful for policy to know the mechanisms behind the impact. Work in this area is therefore motivated by Canada's experience with respect to commodity price movements in recent history. More specifically, since the beginning of 2002, commodity prices have been rising and this has been accompanied by an appreciation of the Canadian dollar and an increasing share of commodities in total exports, but since 2008 there has been a notable fall in these prices, with the consequent knock-on effects. The decline in commodity prices following the global financial crisis also brought with it significant questions about the ability of the Canadian economy to withstand these shocks and has spurred renewed interest in Canada's economic resilience.

This paper quantitatively highlights the role of commodity price shocks in affecting Canada's macro variables with special emphasis on the sectoral impact of these shocks. This focus on the sectoral impact is generally absent from the literature, with the working

DSGE models employed by the Central Bank, being the notable exception. The Central Bank's models, of course, are necessarily larger and more complicated than the one examined in this thesis. However the present paper fills an important gap in the literature, as a tractable model is employed, which though simpler, still captures the salient features of a multi-sectoral commodity exporting small open economy and allows for meaningful analysis of relevant questions, such as, what are the differential sectoral impacts of rising commodity prices? The multi-sectoral structure of the model is also important, especially for an economy such as Canada, that is susceptible to international shocks - for example, commodity price shocks - which are largely sectoral in nature. This framework therefore offers a more realistic economic environment for Canada and thus can be used to assess if commodity price shocks are an important source of economic fluctuations in the Canadian context.

The DSGE modelling framework is also quite useful for conducting scenario analysis. This functionality has been highlighted by Smets and Wouters (2003), who note that these models provide a coherent theoretical framework within which to assess the likely impact of the realisation of certain risks as well as the wide range of policy responses available. When employed in this way, this class of model possess an advantage over simpler reduced-form time series models, such as vector auto-regressions, in that they make transparent the economic mechanisms at work within the model. In employing the model to study the policy response of the Central Bank when confronted with declining commodity prices during the Global Financial Crisis (GFC), this study is among the few to evaluate this response using a DSGE analysis.

Though several studies have looked at unconventional monetary policy during the period of the GFC (see Gambacorta et al. (2014), Neely (2015), Fratzscher et al. (2016), Hamilton and Wu (2012)), very few have examined the impact of reducing interest rate to

its effective lower bound in the Canadian context¹ and sought to quantify the macroeconomic effect of this specific policy action. Additionally, some of the previous analysis was largely qualitative in nature, (see BIS (2008), Zorn et al. (2009) for example) and much of it focused on the U.S. or European countries (see Stock and Watson (2012), Baumeister and Benati (2013), Wu and Xia (2016), Krishnamurthy and Vissing-Jorgensen (2011)). The Canadian response in terms of lower interest rates has been largely unaddressed in the literature, and our paper is the first to assess the sectoral impact of this policy action within a DSGE framework as previous DSGE models of Canada have not been applied to address this issue.² The present paper thus represents a useful contribution to the literature.

Our results suggest that just examining the aggregate effects of commodity price shocks on Canadian macroeconomic variables, may not present a very clear picture as very important sectoral differences are concealed by the aggregate figures. A rise in commodity prices has an uneven impact on sectoral output, even-though aggregate output increases. As expected, there is a rise of output in the commodity producing sector. Output in the non-tradable sector also rises initially, but then experiences a slight decline, whereas output in the manufacturing sector declines on impact and remains below its steady state level, even after five years. The results of the counter-factual policy exercises also reveal that actions taken by the Central Bank were able to prevent a more protracted decline in economic activity following the drop in commodity prices during the GFC, however had these policies been adopted sooner, the results could have been even better.

¹The effective lower bound is considered to be 0.25% by the Bank of Canada, see Zorn et al. (2009).

²Dib (2003) for example, developed a DSGE model for Canada which lacked the sectoral nature, and also never examined the response of monetary policy in the context of a real-world commodity price shock. Dib et al. (2008) also employed a DSGE framework to undertake welfare analysis. Other papers focused on estimating the equilibrium interest rates for Canada in the DSGE framework (Lam and Tkacz 2004) and Murchison et al. (2004) developed a DSGE model to understand the dynamic relationships in Canadian macroeconomic data, but again, did not assess the monetary policy response to an actual commodity price shock.

The remainder of the chapter is organised as follows. Section 2.2 briefly examines the literature relevant to the current area of research. The description of the model is presented in section 2.3. Section 2.4 outlines the model parametrization process and section 2.5 examines the results. Section 2.6 discusses some policy implications of these results and section 2.7 offers some concluding remarks.

2.2 Review of the Literature

As highlighted in chapter one, a number of papers have examined the relationship between commodity prices and the Canadian economy, with emphasis largely focused on energy prices. Unlike chapter one, however, the current chapter is related to previous studies that employ New Keynesian open economy models to examine the impact of shocks on the evolution of the macro-economy. Numerous such studies have been done in the international context, however, with specific reference to Canada, the literature is much more limited.

Mendoza (1995) is one of the earliest studies to examine the impact of terms of trade shocks. These shocks, in the Canadian context are often attributed to movements in commodity prices. Mendoza's study incorporated both developing and developed countries, including Canada. He found that terms of trade shocks were an important source for driving business cycles and argued that almost 50 percent of the variability in output could be attributed to these shocks. DePratto et al. (2009) also estimated a New Keynesian general-equilibrium open economy model for Canada, the United States and the United Kingdom, to examine how changes in oil prices affected the macro-economy.³ They found that in general, energy prices affected the economy primarily through the supply side, as

³The authors did not develop a full DSGE model, but rather employed a structure similar to Gali (2008) as the basis for their empirical model.

the demand-side effects were not substantial. They also found that higher oil prices had a temporary negative effect on both the output gap and on trend growth, which translated into a permanently lower level of potential and actual output. No evidence was found to support the notion that oil price changes had an asymmetric effect on the economy.

Dissou (2007) utilised a forward-looking dynamic general equilibrium model to assess the impact of a sustained increase in energy prices on the Canadian economy.⁴ He concluded that a permanent 20 percent increase in world oil prices resulted in a 0.4 percent increase in aggregate output above its baseline level. In a more recent paper by McKenzie and Carbone (2016), the authors employed a similar computable general equilibrium (CGE) framework, and found that in steady-state, a 10 percent reduction in oil prices resulted in a 1 percent decline in national output for Canada. While both these models employed a general equilibrium framework, they did not utilise the dynamic stochastic New Keynesian modelling techniques which have become the standard in the literature. Such a framework is exploited in the current paper.

The most complete and fully-specified DSGE model currently in use to capture the dynamics of the Canadian economy, is that employed by the Bank of Canada. The terms of trade economic model (ToTEM), is a state-of-the-art open-economy DSGE model, which is more richly specified than the one considered here.⁵ The main features of the model are outlined in Murchison and Rennison (2006). ToTEM was later updated and revised to

⁴The model was calibrated based on the structure of the Canadian economy as illustrated in the social accounting matrix (SAM) for 2002.

⁵ToTEM is the end result of several rounds of model developments which have been undertaken at the Bank of Canada over several decades. This started with the small annual model (SAM), which was a small scale theoretical model of about 25 equations, that was really more geared towards answering specific policy questions, rather than providing a coherent structure of the economy. SAM was eventually replaced by the quarterly projection model (QPR) over the period 1993 - 2005. This model built on SAM, but also featured model-determined asset prices among other significant improvements. ToTEM, the most recent evolution of the Bank's forecasting and policy models, extends the capabilities of QPM but also incorporates optimizing behaviour for the model's agents, both in and out of steady state.

include financial linkages to the real economy and this updated version is referred to as ToTEM II (see Dorich et al. (2013)).⁶ The onset of the global financial crisis in particular, highlighted the need for the inclusion of these financial frictions, so as to capture the additional channels through which shocks may affect the real economy.

Despite the existence of ToTEM, a model such as the one examined here, still fills an important gap in the literature. This is so because it presents a tractable, easily understandable and readily available model (the Central Bank model is really only accessible to staff at the Bank) which would be useful to the business sector in general, and financial firms in particular. These are entities that would be interested in carrying out scenario analysis and producing forecasts for the macro-economy that are based on a set of internally consistent assumptions about the underlying structure of the Canadian economy and is premised on a clearly articulated and easily understood micro foundation.

In the international context, several models have been developed for the US and Euro area. The Australian Central Bank, also relatively recently began the utilisation of DSGE models in their forecasting and policy exercises. Among the papers published for Australia, these include studies by Jaaskela and Nimark (2011) and Rees et al. (2016). Jaaskela and Nimark (2011) found that both foreign and domestic shocks were important drivers of the Australian business cycle, while Rees et al. (2016) highlight the importance of resource price shocks. They found that higher resource prices increased domestic income, resulting in a sustained expansion in the domestic economy and an appreciation of the real exchange rate. The model also suggested that resource price shocks did not play a great role in explaining the volatility of most Australian macroeconomic variables, with the exception of export growth, interest rates and the nominal exchange rate.

An important concern is that commodity price shocks may affect developing countries

⁶This updated version, replaced ToTEM in June 2011.

differently. For Chile, Medina and Soto (2007) estimated a DSGE model with nominal and real rigidities, to describe the sources of business cycle fluctuations in Chile. This model was later extended by Fornero and Kirchner (2014) and Fornero et al. (2014) and used to specifically analyse the impact of commodity price shocks in Chile. Their results suggested that the effect of commodity price increases was expansionary, and that this expansion was largely driven by the positive responses of investment in the commodity sector which then spilled over to the non-commodity producing sectors of the economy. Similarly, for Russia, Malakhovskaya and Minabutdinov (2014) developed a New Keynesian DSGE model which they used to assess the importance of commodity price shocks for the Russian economy. They found that despite a strong impact on GDP from the commodity price shock, business cycles in Russia were primarily driven by domestic factors.

This paper builds on and complements the existing literature which investigates the macroeconomic effects of commodity price shocks. This is achieved by employing the DSGE modelling framework, which has become standard in the literature, but includes sectoral disaggregation, which is absent from many of the papers cited. Table 2.1 briefly summarises some of the studies which have employed a General Equilibrium (GE) framework to examine the relationship between macroeconomic activity and commodity price shocks, both in the Canadian and international context.

**Table 2.1: Literature Survey -
Commodity Price Shocks and Economic Activity (GE Models)**

Authors	Data	Sample Period	Methodology	Main Findings
Canada				
Murchison et al. (2006)	Not stated	1980 - 2004	fully calibrated DSGE model	There is a significant and sustained increase in consumption in response to a commodity price shock. Additionally, the exchange rate appreciates, inflation rises and there is an increase in aggregate output.
Dissou (2007)	data from the Canadian input-output table, national accounts, trade statistics and government accounts	calibrated to 2002 data	calibrated CGE model	A permanent 20 percent increase in world oil prices resulted in a 0.4 percent increase in aggregate output above its baseline level.
Dorich et al. (2013)	consumption (personal expenditures), residential investment (residential construction), business investment, inventory investment (total business inventories), government spending (government expenditure), exports and imports, core CPI, short-term interest rates, fiscal variables, foreign variables, NFA, CAB, commodity prices, exchange rates	1980Q1 - 2012Q2	DSGE model estimated using a Covariance Matrix Adaptation Evolution Strategy (CMA-ES) algorithm to maximise the likelihood	Results were qualitatively similar to those obtained in Murchison and Rennison (2006), thought they tended to be smaller in magnitude. "Overall, our simulation suggests that, regardless of the source of the commodity price shock, the effects are, on net, positive, since gross domestic income, wealth and GDP all rise. In all cases, the Canadian dollar appreciates, but its adverse impact on manufacturing exports is partially offset by the reduced costs of imported production inputs."

**Table 2.1: Literature Survey -
Commodity Price Shocks and Economic Activity (GE Models) Cont'd**

Authors	Data	Sample Period	Methodology	Main Findings
McKenzie et al. (2016)	Input-output tables for Canada	calibrated to 2007 data	Static multi-sector, multi-region CGE model (calibrated)	A 10 percent reduction in oil prices results in a 1 percent reduction in national output.
Other Developed Countries				
Jaaskela et al. (2011)	Inflation, consumption, exchange rate, employment, nonfarm GDP, imports, exports, world GDP, commodity price inflation, commodity demand	1993Q2 - 2007Q3	estimated DSGE (Bayesian)	Both foreign and domestic shocks are important contributors to the Australian business cycle. Commodity demand shocks had the largest impact on export growth, explaining approximately 25 percent of its variance.
Rees et al. (2016)	Australian GDP, consumption, investment, public demand, resource exports, non-resource exports, sectoral output, headline inflation, sectoral inflation, commodity prices, domestic interest rate, foreign inflation, interest rate and GDP	1992Q1 - 2013Q4	estimated DSGE (Bayesian)	Higher commodity prices had an expansionary effect on the domestic economy and resulted in an appreciation of the real exchange rate. Commodity price shocks, however, did not play a large role in explaining the volatility of most Australian macroeconomic variables.
Developing Countries				
Fornero et al. (2014)	GDP, commodity production, consumption, investment, government consumption, headline inflation, core inflation, interest rate, exchange rate, current account balance, foreign output, interest rate, inflation. Emerging market bond index	2001Q3-2013Q4	calibrated DSGE	A large proportion of the above-average growth of investment in Chile between the period 2004 to 2010 was explained by movements in commodity prices. Thus, these price fluctuations have been a significant driving force of the observed investment cycle in Chile. price shocks

**Table 2.1: Literature Survey -
Commodity Price Shocks and Economic Activity (GE Models) Cont'd**

Authors	Data	Sample Period	Methodology	Main Findings
Malakhovskaya et al. (2014)	private final consumption expenditures per capita, producer price index, interest rate, wage, real exchange rate, per capita revenues from the export of oil based commodities, foreign inflation, interest rate and output	1999Q3 - 2011Q3	estimated DSGE (Bayesian)	Despite a strong impact on GDP from the commodity price shock, business cycles in Russia were primarily driven by domestic factors.
Multi-Country Studies				
Mendoza (1995)	GDP, consumption, investment, trade balance, terms of trade, real exchange rate	1955 - 1990 for G-7 countries 1960 - 1990 for Developing Countries	calibrated RBC	Terms of trade shocks account for nearly half of actual GDP variability. Terms of trade shocks cause real appreciations and positive interest rate differentials.
DePratto et al. (2009)	Output growth, domestic interest rate, exchange rates, domestic inflation, foreign interest rate, output and inflation	1971Q1 - 2008Q1	semi-structural New Keynesian model estimated by maximum likelihood	Higher oil prices had a temporary negative effect on both the output gap and on trend growth, which translated into a permanently lower level of potential and actual output.

2.3 Model Description

The benchmark set-up of the model closely follows the multi-sector open economy model developed in Dib (2008) with a few modifications. The general overview of the model is presented below, with the detailed model equations outlined in the accompanying technical appendix B for the chapter. The model is composed of three types of economic units.

These are households, firms and policy-makers. Households are perfectly competitive in the labour market and derive utility from consumption.⁷ Savings and investment in the economy is also undertaken by the households, who may purchase either domestic or foreign bonds and invest in sector-specific capital stock.

Table 2.2: Firms in the Model

Firm	Market Structure	Inputs Used	Uses of Output
Commodity	Perfect Competition	Labour, Capital, Land	Manufacturing production Non-tradable production Exports
Non-Tradable	Monopolistic Competition	Labour, Capital, Commodities	Final Good production
Manufacturing	Monopolistic Competition	Labour, Capital, Commodities	Final Good production Exports
Import	Monopolistic Competition	Foreign Im- ported Good	Final Good production
Final Good	Perfect Competition	Non-tradable good, Manu- factured good, Imported good	Consumption (HH & Gov't) Investment

The firms in the economy are divided into five sectors as illustrated in Table 2.2 - (i) a perfectly competitive commodity-producing sector, (ii) a non-tradable goods producing sector, (iii) a manufactured goods producing sector, (iv) an imports sector and (v) a final domestic good producing sector. Manufacturing and non-tradable goods producing firms employ the use of labour, capital and commodities in their production process. They also possess some market power and thus can set prices for their output. Commodities are produced by a perfectly competitive firm using labour, capital and land, which has

⁷Dib (2008) originally assumed monopolistic competition in the labour market. However, to increase the tractability of the model, this assumption was dropped.

a fixed exogenous supply.⁸ This firm has no pricing power, and thus takes the price of commodities, which is determined in foreign markets, as a given. The imports sector purchases foreign goods and sells them on the domestic market, with each firm selling a differentiated product. The perfectly competitive domestic final good firm combines the output of the non-tradable sector, the import sector and a fraction of the output of the manufacturing sector to produce the final good. This final good is then used for consumption or investment by households or for government expenditure. The fraction of output produced by the commodities sector and the manufacturing sector, which is not utilized domestically, is exported to foreign markets.

The monetary authority in the model adjusts nominal interest rate with the target of stabilising the inflation rate and aggregate output. Whilst fiscal policy is modelled as an exogenously determined spending process which is financed by the proceeds of lump-sum taxation imposed on the households. A schematic overview of the model may be viewed in section B.1 of technical appendix B.

2.3.1 Households

There is a continuum of households, indexed by $h \in (0, 1)$, which obtain utility from consumption, and disutility from labour effort.⁹ The representative household thus maximises lifetime utility which is given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \xi_{c,t} \ln (C_{h,t} - dC_{h,t-1}) - A_L \frac{H_{h,t}^{1+\chi}}{1+\chi} \right\} \quad (2.1)$$

⁸The presence of land in the production function of commodities, allows for the existence of more than one tradable sector in equilibrium (see Bouakez et al. (2009)).

⁹Since in equilibrium all households will make the same choices and thus their aggregate(average) optimal choices will coincide, we can drop the h subscript to increase the ease of exposition.

where β is the discount factor, d is the parameter that controls habit persistence and determines the weight that households place on previous consumption, χ is the inverse of the Frisch labour supply elasticity and controls the responsiveness of hours worked to changes in the real wage. Though not present in Dibs' model, in the spirit of Adolfson et al. (2007), A_L is included as a scaling factor to ensure the compatibility of average hours worked in the model with that observed in the data. $\xi_{c,t}$ is a consumption preference shock and follows a stationary autoregressive process.

The households labour bundle, $H_{h,t}$ is a constant elasticity of substitution (CES) composite of labour supplied to each sector and is given by:

$$H_{h,t} = \left[H_{n,ht}^{1+\varsigma} + H_{m,ht}^{1+\varsigma} + H_{x,ht}^{1+\varsigma} \right]^{\frac{1}{1+\varsigma}} \quad (2.2)$$

where $H_{n,ht}$, $H_{m,ht}$, $H_{x,ht}$ signifies hours allocated by household h to the non-tradable (n), manufacturing (m) and commodities sectors (x), respectively and ς denotes the willingness of households to substitute labour across sectors.

Households are able to purchase domestic treasury bonds, B_{ht} and also have access to incomplete international financial markets in which they can trade bonds denominated in foreign currency, B_{ht}^* . In addition to labour, households also supply capital to firms in the productive sectors.¹⁰ The profits of firms with market power, Γ_{ht} , are remitted to households, since it is assumed that households own equity in these firms. Households are also assumed to own an endowment of land, L , which provides them with rental income and to pay lump-sum taxes, T_{ht} , to the government. The household may use its income received to purchase new bonds (foreign or domestic), or to purchase some of the final

¹⁰The productive sectors in the model refer to the non-tradable, manufacturing and commodity producing sectors.

good which it then allocates between consumption goods or investment goods.

The nominal budget constraint of the household is therefore given by:

$$P_t C_{ht} + P_t I_{ht} + \frac{B_{ht}}{R_t} + \frac{s_t B_{ht}^*}{\kappa_t R_t^*} \leq \sum_{j=n,m,x} (R_{j,ht} K_{j,ht} + W_{j,ht} H_{j,ht}) \quad (2.3)$$

$$+ B_{ht-1} + s_t B_{ht-1}^* + R_{L,t} L + \Gamma_{ht} - T_{ht}$$

where P_t is the price of the final good in the economy, $I_{ht} = I_{n,ht} + I_{m,ht} + I_{x,ht}$ is total investment by household h in the productive sectors, $R_{j,ht}$ and $W_{j,ht}$ are the rental rate of capital and the wage rate in sector $j = n, m, x$, $R_{L,t}$ is the rate of return on land, R_t and R_t^* are the gross nominal interest rates on domestic and foreign bonds respectively, Γ_{ht} is total profits remitted to household h and T_{ht} are lump-sum transfers to the government. s_t is the nominal exchange rate and κ_t is a country specific risk premium, which is positively related to the amount of outstanding foreign debt and a risk shock, as outlined in equation 2.4. The risk shock, Ψ_t follows a stationary autoregressive process.

$$\kappa_t = \exp \left[-\varkappa \left(\frac{s_t B_t^*}{P_t Y_t} \right) + \Psi_t \right] \quad (2.4)$$

The sector-specific capital stock, evolves according to law of motion:

$$K_{j,ht+1} = (1 - \delta) K_{j,ht} + \Upsilon_t \left[1 - F_t \left(\frac{I_{j,ht}}{I_{j,ht-1}} \right) \right] I_{j,ht} \quad (2.5)$$

where δ is the depreciation rate which is common to all sectors. F_t represents the cost of adjusting investment in sector j , that is, it represents the cost associated with turning

investment into physical capital and takes the form $F(\cdot) = \frac{\Phi^k}{2} \left(\frac{I_{j,ht}}{I_{j,ht-1}} - \mu \right)^2$.¹¹ γ_t is a stationary investment-specific shock which alters the rate at which investment is transformed into productive capital (see Greenwood et al. (2000)).

The representative household, h , maximises utility, equation 2.1, subject to equations 2.3 and 2.5 to obtain the first-order conditions which are highlighted in appendix B.3.

2.3.2 Non-Tradable Sector

The non-tradable sector, indexed by n , consists of a continuum of intermediate-good producing firms indexed by $i \in [0, 1]$. The output of firm i is produced using capital, labour and commodity inputs. Each firm then sells their output to a representative competitive firm, who we refer to as the retailer. The retailer transforms the differentiated intermediate products into a homogeneous non-traded good which it then sells to the final good producing firm. The composite non-traded good is produced by the retailer employing the following constant elasticity of substitution technology:

$$Y_{n,t} = \left(\int_0^1 (Y_{n,it})^{\frac{\theta_n-1}{\theta_n}} di \right)^{\frac{\theta_n}{\theta_n-1}} \quad (2.6)$$

where $Y_{n,it}$ is the output of firm i and θ_n controls the elasticity of substitution among the output of the different non-traded firms. Optimization by the retailer yields the following demand function for each firm's output:

¹¹The function $F_t(\cdot)$ satisfies the usual assumptions such that in steady state, the marginal cost of a small adjustment to the rate of investment growth is zero, but that these costs rise substantially as the desired change in investment becomes larger. This is formally represented as $F(\mu) = F'(\mu) = 0$ and $F''(\mu) > 0$, where μ is the steady-state rate of productivity growth.

$$Y_{n,it} = \left(\frac{P_{n,it}}{P_{n,t}} \right)^{-\theta_n} Y_{n,t} \quad (2.7)$$

where $P_{n,it}$ is the price of firm i 's output and $P_{n,t}$ is the price of the composite non-traded good. The production function of non-tradable firm i is given by:

$$Y_{n,it} \leq A_{n,t} (\mu_t H_{n,it})^{\alpha_n} (K_{n,it})^{\gamma_n} (Y_{x,it}^n)^{1-\alpha_n-\gamma_n} \quad (2.8)$$

where $H_{n,it}$, $K_{n,it}$ and $Y_{x,it}^n$ are inputs of labour, capital, and commodities used by firm i in its production process and α_n , γ_n and $1 - \alpha_n - \gamma_n$, the respective input shares. $A_{n,t}$ is a stationary, sector-specific technology shock, whilst μ_t is a stochastic trend in labour productivity which evolves according to a random walk with drift.¹²

Unlike Dib (2008), who uses a Calvo - Yun mechanism (see Calvo (1983) and Yun (1996)), price stickiness is incorporated in our model by assuming that firms face a quadratic adjustment cost when changing prices, as initially conceptualized by Rotemberg (1982).¹³ Given this cost, firms choose prices and factor inputs to maximise real profits, where the profit function is given by:

$$\Gamma_{n,it} = \frac{P_{n,it} Y_{n,it}}{P_t} - \frac{MC_{n,it} Y_{n,it}}{P_t} - \frac{\tau_n}{2} \left[\frac{P_{n,it}}{\Pi_{n,t-1}^\eta \Pi^{1-\eta} P_{n,it-1}} - 1 \right]^2 \frac{P_{n,t} Y_{n,t}}{P_t} \quad (2.9)$$

The quadratic adjustment cost term in squared brackets tells us that in changing its price between period $t-1$ and t , the firm is able to account for the policymaker's inflation target,

¹²This permanent productivity process was not included in Dib's model, however this modification was included here as the real economy does grow over time, and this allows us to capture that aspect of the data.

¹³This change made it less complicated to model price stickiness. Although to a first order approximation, both methods yield similar results, the Rotemberg method was easier for us to implement.

Π , as well as sector-specific inflation, $\Pi_{n,t-1}$, before incurring a cost. $MC_{n,it}$ is the nominal marginal cost faced by firm i , and is given by:

$$MC_{n,it} = \frac{\varepsilon_{n,t}}{A_{n,t}} \left[\frac{W_{n,t}}{\alpha_n \mu_t} \right]^{\alpha_n} \left[\frac{R_{n,t}}{\gamma_n} \right]^{\gamma_n} \left[\frac{P_{x,t}}{1 - \alpha_n - \gamma_n} \right]^{1 - \alpha_n - \gamma_n} \quad (2.10)$$

where $P_{x,t}$ is the domestic currency price for commodities and $\varepsilon_{n,t}$ is a stationary mark-up shock that alters marginal costs in the non-tradable sector for reasons unrelated to changes in factor costs.

2.3.3 Manufacturing Sector

The manufacturing sector, indexed by m , consists of a continuum of intermediate-good producing firms indexed by $i \in [0, 1]$. The output of firm i , which may be used domestically or exported, is produced using capital, labour and commodity inputs. In the domestic market, each firm sells their output to a competitive retailer, who transforms the differentiated intermediate products into a homogeneous manufactured good which it then sells to the final good producer. In the export market, manufacturing firms sell their output to an exporter, who transforms the differentiated intermediate products into a homogeneous manufactured good which is exported. Firms are able to price discriminate between the domestic and foreign market, and thus can set different prices for each. The composite manufactured good is produced employing the CES production technology:

$$Y_{m,t}^l = \left(\int_0^1 (Y_{m,it})^{\frac{\theta_m - 1}{\theta_m}} di \right)^{\frac{\theta_m}{\theta_m - 1}} \quad \text{for } l \in \{d, ex\} \quad (2.11)$$

where d indicates goods sold in the domestic market and ex , those which are exported. $Y_{m,it}$ is the output of firm i and θ_m controls the elasticity of substitution among the output

of the different manufacturing firms. Optimization by the retailer and exporter yields the following demand functions for each firms output, for the relevant market:

$$Y_{m,it}^d = \left(\frac{P_{m,it}}{P_{m,t}} \right)^{-\theta_m} Y_{m,t}^d \quad (2.12)$$

$$Y_{m,it}^{ex} = \left(\frac{P_{m,it}^*}{P_{m,t}^*} \right)^{-\theta_m} Y_{m,t}^{ex} \quad (2.13)$$

where $P_{m,it}$ and $P_{m,it}^*$ is the price of firm i 's output in the domestic and foreign markets, respectively and $P_{m,t}$ ($P_{m,t}^*$) is the price of the domestic (foreign) composite manufactured good. The production function of manufacturing firm i is given by:

$$Y_{m,it} \leq A_{m,t} (\mu_t H_{m,it})^{\alpha_m} (K_{m,it})^{\gamma_m} (Y_{x,it}^m)^{1-\alpha_m-\gamma_m} \quad (2.14)$$

where $H_{m,it}$, $K_{m,it}$ and $Y_{x,it}^m$ are inputs of labour, capital, and commodities used by firm i in its production process and α_m , γ_m and $1 - \alpha_m - \gamma_m$, the respective input shares. $A_{m,t}$ is a stationary, manufacturing-specific technology shock which follows a first-order autoregressive process (see equation 2.42).

As in the non-tradable sector, manufacturing firms face a quadratic adjustment cost when changing prices. In particular, these costs are incurred in the currency of the market in which the firm's output is sold, so that prices are sticky in local currency terms. In this regard, we modify Dib's assumption of producer currency pricing (PCP) which allows for a complete pass-through of the exchange rate to prices, and instead assume that firms engage in local currency pricing (LCP), which may lead to lower pass-through of exchange rate to import prices and thus temper the effects of exchange rate movements. ¹⁴ Given

¹⁴Under PCP, the law of one price always holds, however there have been some empirical studies which

this cost, firms choose prices and factor inputs to maximise real profits, where the profit function is given by:

$$\begin{aligned}
\Gamma_{m,it} = & \frac{P_{m,it}Y_{m,it}^d}{P_t} + \frac{s_t P_{m,it}^* Y_{m,it}^{ex}}{P_t} - \frac{MC_{m,it}^d Y_{m,it}^d}{P_t} \\
& - \frac{MC_{m,it}^{ex} Y_{m,it}^{ex}}{P_t} - \frac{\tau_m}{2} \left[\frac{P_{m,it}}{\Pi_{m,t-1}^\eta \Pi_{m,it-1}^{1-\eta}} - 1 \right]^2 \frac{P_{m,t} Y_{m,t}^d}{P_t} \\
& - \frac{\tau_m^*}{2} \left[\frac{P_{m,it}^*}{(\Pi_{m,t-1}^*)^\eta \Pi_{m,it-1}^{1-\eta}} - 1 \right]^2 \frac{s_t P_{m,t}^* Y_{m,t}^{ex}}{P_t}
\end{aligned} \tag{2.15}$$

$MC_{m,it}^l$ is the nominal marginal cost faced by firm i in market $l \in \{d, ex\}$, and is given by:

$$MC_{m,it}^l = \frac{\varepsilon_{m,t}^l}{A_{m,t}} \left[\frac{W_{m,t}}{\alpha_m \mu_t} \right]^{\alpha_m} \left[\frac{R_{m,t}}{\gamma_m} \right]^{\gamma_m} \left[\frac{P_{x,t}}{1 - \alpha_m - \gamma_m} \right]^{1 - \alpha_m - \gamma_m} \tag{2.16}$$

where $\varepsilon_{m,t}^l$ is a stationary mark-up shock.

While the domestic demand for manufactured goods, $Y_{m,t}^d$, is the result of agent's optimising behaviour in the model, we assume that foreign demand for manufactured exports, $Y_{m,t}^{ex}$, is given by:

$$Y_{m,t}^{ex} = \omega^* \left(\frac{P_{m,t}^*}{P_t^*} \right)^{-\nu^*} Y_t^* \tag{2.17}$$

where Y_t^* and P_t^* is the foreign output and price level, respectively. The elasticity of demand by foreigners for domestic manufactured output is given by $-\nu^*$ and the parameter ω^* governs the share of manufactured exports in total foreign expenditure.

show that this is not always the case, particularly in the short-run (see Meese and Rogoff (1988) and Edison and Fisher (1991) for example). Thus, utilizing LCP allows us to account for this temporary deviation from the law of one price in the short-run.

2.3.4 Commodity Sector

The commodity sector, indexed by x , produces a homogeneous output under perfect competition. Given these conditions, the sector can be viewed as consisting of a single firm that produces output according to the following Cobb-Douglas technology:

$$Y_{x,t} \leq A_{x,t} (\mu_t H_{x,t})^{\alpha_x} (K_{x,t})^{\gamma_x} (\mu_t L)^{1-\alpha_x-\gamma_x} \quad (2.18)$$

where $H_{x,t}$, $K_{x,t}$ and L are inputs of labour, capital, and land used to produce the commodity output and α_x , γ_x and $1 - \alpha_x - \gamma_x$, the respective input shares. $A_{x,t}$ is a stationary, sector-specific technology shock which follows a first-order autoregressive process.

The commodity producing firm is a price-taker and thus selects the amount of capital and labour input, each period so as to maximise profits which is given by:

$$\Gamma_{x,t} = P_{x,t} Y_{x,t} - W_{x,t} H_{x,t} - R_{x,t} K_{x,t} - R_{L,t} L \quad (2.19)$$

where $P_{x,t}$ is the domestic currency price of commodities.

The foreign currency price of commodities, $P_{x,t}^*$, is entirely determined in world markets and remains unaffected by domestic economic developments. For this sector, the law of one price (LOP) is assumed to hold in the long run. However, in contrast to Dib, we allow for deviations from LOP in the short-run. In particular, we allow for a delay in the pass-through of foreign currency commodity price changes to the domestic currency price which the commodity producing firm receives. This assumption coincides with empirical literature which have found that the law of one price does not seem to hold in the short-run.¹⁵ It also allows us to account for real world frictions, such as the fact that sale

¹⁵See Rogoff (1996), Sarno and Taylor (2002) and Taylor and Taylor (2004) for an extensive review of

contracts for commodities are generally written up with pre-specified prices, such that changes in international commodity prices may take some time to filter through to the price that the domestic firm receives. Taking the above into consideration, we select the following functional specification, for the price of resources in the domestic currency: $P_{x,t} = (s_t P_{x,t}^*)^{\frac{1}{2}} (P_{x,t-1})^{\frac{1}{2}}$, where $P_{x,t}^*$ is the price of commodities in the foreign currency. Even though the pass-through of the foreign currency price is not immediate, it does occur rather rapidly, with half of any change feeding through immediately and almost all of the change within one year.

2.3.5 Import Sector

The import sector, indexed by f , consists of a continuum of domestic importers indexed by $i \in [0, 1]$, that import a homogeneous foreign intermediate product, for the foreign price P_t^* . Each firm, via a one to one process, transforms the imported good into a differentiated variety, $Y_{f,it}$.¹⁶ Each firm sells its variety in a domestic, monopolistically-competitive market to produce the imported composite good, $Y_{f,t}$, which is produced according to the following CES production technology:

$$Y_{f,t} = \left(\int_0^1 (Y_{f,it})^{\frac{\theta_f - 1}{\theta_f}} di \right)^{\frac{\theta_f}{\theta_f - 1}} \quad (2.20)$$

where θ_f controls the elasticity of substitution among the various import firms' varieties. Optimization yields the following demand function for each variety:

the Law of One Price and Purchasing Power Parity literature.

¹⁶We may conceptualise this as each firm just applying its brand name to the homogeneous import, so as to create product differentiation.

$$Y_{f,it} = \left(\frac{P_{f,it}}{P_{f,t}} \right)^{-\theta_f} Y_{f,t} \quad (2.21)$$

where $P_{f,it}$ is the price for firm i 's variety and $P_{f,t}$ is the price of the composite imported good.

Importing firms also face quadratic price adjustment costs, and thus choose prices to optimise profits, which is given by:

$$\Gamma_{f,it} = \frac{P_{f,it}Y_{f,it}}{P_t} - \frac{MC_{f,it}Y_{f,it}}{P_t} - \frac{\tau_f}{2} \left[\frac{P_{f,it}}{\Pi_{f,t-1}^\eta \Pi^{1-\eta} P_{f,it-1}} - 1 \right]^2 \frac{P_{f,t}Y_{f,t}}{P_t} \quad (2.22)$$

where the importers' nominal marginal cost, $MC_{f,it}$, is given by:

$$MC_{f,it} = \varepsilon_{f,t} s_t P_t^* \quad (2.23)$$

with $\varepsilon_{f,t}$ representing a stationary mark-up shock.

2.3.6 Final Good Sector

The final good sector consist of a perfectly competitive representative firm who uses the composite non-trade good, $Y_{n,t}$, the composite domestic manufactured good, $Y_{m,t}^d$, and the composite import good, $Y_{f,t}$, to produce a final good, FG_t , according to the CES technology:

$$FG_t = \left[\omega_n^{\frac{1}{\nu}} Y_{n,t}^{\frac{\nu-1}{\nu}} + \omega_m^{\frac{1}{\nu}} Y_{m,t}^{d \frac{\nu-1}{\nu}} + \omega_f^{\frac{1}{\nu}} Y_{f,t}^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}} \quad (2.24)$$

where $\omega_n + \omega_m + \omega_f = 1$ and ω_n , ω_m and ω_f controls the share of non-tradable, manufactured and imported goods respectively in the final good. ν is the elasticity of substitution among those three inputs. Profit maximisation, along with the zero profit condition¹⁷, implies the following functional form for the aggregate price index in the economy:

$$P_t = [\omega_n P_{n,t}^{1-\nu} + \omega_m P_{m,t}^{1-\nu} + \omega_f P_{f,t}^{1-\nu}] \quad (2.25)$$

2.3.7 Government

The government is assumed to raise revenues by imposing lump-sum taxes on households, T_t , and by issuing domestic bonds, B_t . These revenues are used to finance government expenditure, G_t , and to repay outstanding debt, B_{t-1} . Thus, the government budget constraint is given by:

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t} \quad (2.26)$$

Government spending is modelled as an exogenous process, which evolves according to:

$$\ln \left[\frac{G_t}{\mu_t} \right] = (1 - \rho_g) \ln(g) + \rho_g \ln \left[\frac{G_{t-1}}{\mu_{t-1}} \right] + \varepsilon_g \quad (2.27)$$

where g is the steady state level of government spending, ρ_g a stationary autoregressive coefficient and ε_g a normally distributed white noise term. Domestic debt is assumed to be in zero net supply in equilibrium, such that $B_t = 0$ for all t .

¹⁷We can impose the zero profit condition, since the final good firm is perfectly competitive.

2.3.8 Monetary Authority

The monetary authority is assumed to pursue a Taylor-type rule (see Taylor (1993) and Taylor (1999)) in adjusting the nominal interest rate, R_t .¹⁸ This rule is given by:

$$\ln\left(\frac{R_t}{R}\right) = \rho_r \ln\left(\frac{R_{t-1}}{R}\right) + (1 - \rho_r) \left[\varrho_\pi \ln\left(\frac{\Pi_t}{\Pi}\right) + \varrho_y \ln\left(\frac{Y_t}{Y}\right) \right] + \varepsilon_{r,t} \quad (2.28)$$

where Π_t is the CPI inflation rate and Y_t is the relevant measure of aggregate output in the economy, (see equation 2.36). R , Π and Y are the steady-state values of R_t , Π_t and Y_t , respectively. $\varepsilon_{r,t}$ is a normally distributed white-noise error term, which is interpreted as the monetary policy shock. The rule indicates that the central bank adjusts current interest rates in response to past rates as well as in response to deviations of inflation from target and output from its long-run equilibrium level.

2.3.9 Market Clearing Conditions

Equilibrium in the goods market require that the following three conditions be satisfied:

$$Y_{m,t} = Y_{m,t}^{ex} + Y_{m,t}^d \quad (2.29)$$

$$Y_{x,t} = Y_{x,t}^{ex} + Y_{x,t}^n + Y_{x,t}^m \quad (2.30)$$

$$FG_t = C_t + I_t + G_t \quad (2.31)$$

¹⁸Taylor rules are simply monetary policy rules that delineate how a central bank should systematically adjust its interest rate policy instrument in response to changes in inflation and other macroeconomic variables.

Equation 2.29 indicates that all output of the manufacturing sector must either be exported or used domestically. Similarly, equation 2.30 indicates that output of the commodity sector must either be exported abroad, or be used in the production process of the non-tradable and manufacturing firms. Equation 2.31 outlines the uses of the domestic final good, which must either be consumed, used for investment, where $I_t = I_{n,t} + I_{m,t} + I_{x,t}$, or used for government expenditure.

The trade balance or nominal net exports, NX_t , is given by the sum of commodity exports and manufacturing exports net of import values:

$$NX_t = P_{x,t}Y_{x,t}^{ex} + s_t P_{m,t}^* Y_{m,t}^{ex} - s_t P_t^* Y_{f,t} \quad (2.32)$$

The current account equation, which governs the evolution of net foreign assets, B_t^* , in the economy is given by the sum of the previous net foreign asset position and the trade balance:

$$\frac{s_t B_t^*}{\kappa_t R_t^*} = s_t B_{t-1}^* + NX_t \quad (2.33)$$

Commodities are used as an input in the production processes of the the non-tradable and manufacturing sectors. Consequently, value added in these sectors will be lower than total production, and in order to calculate the relevant measure for aggregate output in the economy, this needs to be taken into consideration. We do this by subtracting commodity inputs and constructing a measure of value added output in these sectors as follows:

$$Y_{n,t}^{va} = Y_{n,t} - \frac{P_{x,t}}{P_{n,t}} Y_{x,t}^n \quad (2.34)$$

$$Y_{m,t}^{va} = Y_{m,t} - \frac{P_{x,t}}{P_{m,t}} Y_{x,t}^m \quad (2.35)$$

where $Y_{n,t}^{va}$ and $Y_{m,t}^{va}$ are valued added output in the non-tradable and manufacturing sectors, respectively. The relevant measure for real GDP, Y_t , is therefore obtained by summing the value-added in each productive sector:

$$Y_t = \left[\frac{P_{n,t}}{P_t} \right] Y_{n,t}^{va} + \left[\frac{P_{m,t}}{P_t} \right] Y_{m,t}^{va} + \left[\frac{P_{x,t}}{P_t} \right] Y_{x,t} \quad (2.36)$$

2.3.10 Foreign Economy

The foreign economy is modelled as a simple three equation system, consisting of an IS curve, a New Keynesian Phillips curve (NKPC) and a Taylor rule for the monetary authority. The IS curve governs foreign output, the Phillips curve, foreign inflation and the Taylor rule, foreign interest rate. These were originally modelled as exogenous AR(1) processes, however this change allowed for more transparency and economic rationale in the modelling of the foreign economy.

The equations are presented below in log-linearised form, where a hat over a variable indicates log-deviation from its steady state value.

The foreign IS curve:

$$\hat{y}_t^* = \mathbb{E}_t \hat{y}_{t+1}^* - (\hat{r}_t^* - \mathbb{E}_t \hat{\pi}_{t+1}^*) - \hat{\xi}_{y^*,t} \quad (2.37)$$

The foreign Phillips curve:

$$\hat{\pi}_t^* = \beta \mathbb{E}_t \hat{\pi}_{t+1}^* + \kappa^* \hat{y}_t^* + \varepsilon_{\pi^*,t} \quad (2.38)$$

The foreign Taylor rule:

$$\hat{r}_t^* = \rho_{r^*} \hat{r}_{t-1}^* + (1 - \rho_{r^*}) \left(\varrho_{\pi^*} \hat{\pi}_t^* + \varrho_{y^*} \hat{y}_t^* \right) + \varepsilon_{r^*,t} \quad (2.39)$$

where y_t^* is foreign output, r_t^* is foreign interest rate, π_t^* is foreign inflation, $\xi_{y^*,t}$ is a foreign demand shock that follows a AR(1) process, κ^* is a parameter which governs the slope of the foreign NKPC, $\varepsilon_{\pi^*,t}$ is a white noise cost push shock and $\varepsilon_{r^*,t}$ is a shock to foreign monetary policy.

We also include in this section, the equation which governs the evolution of the foreign price for commodities. In particular, we maintain the small open-economy assumption, such that the price for commodities is entirely determined in world markets but is subject to commodity-specific price shocks. Thus relative commodity prices in terms of the foreign currency, evolves according to:

$$\hat{p}_{x,t}^* = \rho_{p_x^*} \hat{p}_{x,t-1}^* + \varepsilon_{p_x^*,t} \quad (2.40)$$

where $p_{x,t}^* = \frac{P_{x,t}^*}{P_t^*}$ is the relative price of commodities in the foreign currency, and $\hat{p}_{x,t}^*$ is the log deviation of $p_{x,t}^*$ from its steady state value, p_x^* . $\varepsilon_{p_x^*,t}$ is the commodity price shock and $\rho_{p_x^*}$, a stationary autoregressive coefficient.

2.3.11 Exogenous Processes

The growth rate of the labour productivity process, $\Delta\mu_t = \ln(\mu_t/\mu_{t-1})$, is specified as follows:

$$\Delta\mu_t = \ln(\mu) + \varepsilon_{\mu,t} \quad (2.41)$$

where $\ln(\mu)$ is the trend rate of growth and $\varepsilon_{\mu,t}$ is an uncorrelated and normally distributed innovation, with zero mean and constant variance, σ_μ^2 .

The structural shocks that evolve according to a first-order autoregressive process are specified as follows:

$$\hat{\varphi}_t = \rho_\varphi \hat{\varphi}_{t-1} + \varepsilon_{\varphi,t} \quad \varepsilon_{\varphi,t} \sim N(0, \sigma_\varphi^2) \quad (2.42)$$

for $\varphi = \{\xi_c, \Psi, \Upsilon, A_n, A_m, A_x, \xi_{y^*}\}$. The remaining shocks, $\varepsilon_n, \varepsilon_m^d, \varepsilon_m^{ex}, \varepsilon_f, \varepsilon_g, \varepsilon_r, \varepsilon_{\pi^*}, \varepsilon_{r^*}$ and $\varepsilon_{p_x^*}$, are specified as serially uncorrelated and normally distributed white-noise processes.

2.4 Calibration

The model is calibrated at a quarterly frequency to capture the prominent features of the Canadian economy.¹⁹ Table 2.3 reports some relevant calibrated parameters. Steady state productivity growth, μ , is set to 0.58 percent and inflation, to an annualised rate of 2.84 percent. These figures are consistent with the averages observed over the period 1981:Q1 to 2016:Q1, for which data was available.

¹⁹Some parameters of the model are assigned values prior to the calibration because they are non-identified or the data used contain only limited information about them. The remaining parameters are calibrated to capture the salient features of the Canadian economy.

Table 2.3: Key Calibrated Parameters

Parameter	Description	Value
Technology and Policy		
μ	Steady-state productivity growth rate	1.0058
π	Steady-state inflation rate	1.0071
δ	Capital depreciation rate	0.025
\varkappa	Risk premium coefficient	0.001
b^*	Governs steady-state net foreign assets	-0.008
g	Governs steady-state government spending	0.772
Households		
β	Discount factor	0.9983
χ	Intertemporal labour supply elasticity	1
ς	Inter-sectoral labour supply elasticity	1
Final Goods Sector		
ν	Inter-sectoral elasticity of substitution in domestic final good	0.8
ω_n	Controls the share of non-tradables in final good production	0.59
ω_m	Controls the share of manufactured goods in final good production	0.10
ω_f	Controls the share of imports in final good production	0.31
Non-Tradable Sector		
α_n	Labour's share in production	0.66
γ_n	Capital's share in production	0.28
θ_n	Elasticity of substitution in non-tradable sector	5
Manufacturing Sector		
α_m	Labour's share in production	0.63
γ_m	Capital's share in production	0.26
θ_m	Elasticity of substitution in manufacturing sector	6
ν^*	Elasticity of substitution between domestic and foreign goods	0.8
ω^*	Governs the share of manufactured goods that are exported	1.9
Commodity Sector		
α_x	Labour's share in production	0.39
γ_x	Capital's share in production	0.41
Imports Sector		
θ_f	Elasticity of substitution in imports sector	6

The household's discount factor, β , is parametrised to 0.9983, which along with the previous two parameters imply a steady state real interest rate of about 6 percent. This is close to the average value of 5.7 percent observed in the data.²⁰ We calibrate both χ , which is the inverse of the Frisch labour supply elasticity and ς , which governs the elasticity of labour supply across sectors, to one. This is the generally accepted value in the literature, (see Bouakez et al. (2009) and Justiniano and Preston (2010)) and allows labour across sectors to be imperfect substitutes. Again, consistent with the literature, we set δ , the capital depreciation rate, to 2.5 percent.

Based on the study of Macklem et al. (2000), we calibrate the shares of capital (γ_x), labour (α_x) and land ($1 - \gamma_x - \alpha_x$) in the production function of the commodities sector to be 0.41, 0.39 and 0.2 respectively. Macklem et al. calculated these shares from the Canadian 1996 medium-level of aggregation input-output tables. The similar parameters for the non-tradable and manufacturing sectors are also calibrated according to this study. For the non-tradable sector capital (γ_n), labour (α_n) and commodity input shares ($1 - \gamma_n - \alpha_n$) are 0.28, 0.66, and 0.06, respectively. For the manufacturing sector, the associated input shares are 0.26 for capital (γ_m), 0.63 for labour (α_m) and 0.11 for commodity inputs ($1 - \gamma_m - \alpha_m$).

The parameter, θ_l for $l \in \{n, m, f\}$, governing the elasticity of substitution within the non-tradable, manufacturing and import sector is set equal to 6 for the manufacturing and import sectors and 5 for the non-tradable sector. This implies an average price mark-up of about 20 percent in the manufacturing and import sectors in steady state and 25 percent in the non-tradable sector.²¹ The inter-sectoral elasticity of final demand, ν , and the elasticity

²⁰Interest rates have since declined dramatically, particularly as a consequence of the global financial crisis. However, we sought to match the average in the data, as this represented a much more stable period and for steady-state analysis we generally do not want to focus on periods of significant volatility.

²¹Empirically, price mark-ups are generally found to be higher in the non-tradable sectors of the economy, since there is generally less direct competition between this sector of the domestic economy and the rest of the world. Consequently, less substitutes are available and non-tradable firms are able to exert

of demand for domestic manufactured goods in the foreign economy, ν^* , are also set equal to 0.8. These values are consistent with the range of estimates in previous literature that have examined small open economy models for Canada (see Ambler et al. (2004), Ortega and Rebei (2006) and Dib (2011)).

The parameter ω^* which controls the share of domestic manufactured goods that gets exported is calibrated to 1.9 so that the ratio of manufactured exports to GDP in the model, matches that in the data. The parameters ω_n , ω_m , and ω_f that govern the share of non-tradable, manufactured and imported goods in the production of the domestic final good are set to 0.59, 0.10, and 0.31, respectively, to match the share of these sectors in Canadian GDP. Finally, the parameters g and b^* which govern the steady state level of government spending and net foreign assets, respectively are set to match their relevant counterparts in the data.²²

2.4.1 Solution Process

The solution process for the system of equations involves (log) linearising the equilibrium conditions of the model around their deterministic steady state values using a first-order Taylor-series approximation. These (log) linearised equations are then entered into Dynare,²³ and the program solves for the coefficients of the policy function, g . The policy function is a set of equations that relate the current values of the variables to their past values and to the current shocks. Thus we can trace how the endogenous variables in the

greater market power. The price mark-up is calculated as $\left(\frac{\theta_i}{\theta_i-1} - 1\right) \times 100$.

²² g is set to match the government spending to GDP ratio and b^* is set to match the net-foreign asset to GDP ratio.

²³Dynare is a software platform for solving and performing policy exercises with a wide class of economic models, in particular dynamic stochastic general equilibrium (DSGE). It employs a collection of MATLAB subroutines to accomplish these tasks.

system would respond to exogenous shocks. Greater details of the solution process may be found in Appendix [B.2](#).

2.5 Model Results

We now use the model from the previous section to assess the impact of commodity price shocks on the Canadian economy. In particular, we highlight the important asymmetric sectoral effects that these shocks can have. Table [2.4](#) compares some relevant steady-state ratios of the model to their averages in the data over the period 1981Q1 to 2016Q1.

Table 2.4: Steady-State Ratios of the Model

Ratio	Data Average	Model
Expenditure (per cent of GDP)		
Household Consumption	52.1	51.3
Private Investment	16.8	24.8
Government Spending	24.8	23.9
Exports	27.7	21.7
Imports	25.2	21.7
Production* (per cent of GVA)		
Non-Tradable	76.9	66.3
Manufacturing	12.9	19.1
Commodities	10.1	14.6
External Trade (per cent of exports)		
Commodity Exports	43.8	43.1
Manufacturing Exports	56.2	56.9
Investment Demand (per cent of private investment)		
Non-Tradable	49.1	59.7
Manufacturing	18.2	17.4
Commodities	32.7	22.9

Notes: * The data averages for production span the period 1997Q1 to 2016Q1 due to availability.

The model was generally able to capture many of the key features of the data. In

particular, the steady state behaviour of household consumption, government spending, commodity and manufacturing exports were captured fairly well. The model however, gives a higher result for the behaviour of private investment, while yielding somewhat lower results for the production of non-tradables and investment demand for commodities.

2.5.1 Volatility and Correlations

To further assess the model we examine the standard deviations, σ_z , and autocorrelations, $E(z_t, z_{t-1})$, implied by the model relative to similar statistics for the data over the period 1997Q1 to 2016Q1. These results are reported in Table 2.5 for some variables of interest.

Table 2.5: Moments of the Data and Model

Variable	Symbol	Data		Model	
		σ_z	$E(z_t, z_{t-1})$	σ_z	$E(z_t, z_{t-1})$
Real Exchange Rate	q_t	4.63	0.76	4.73	0.82
Commodity Prices	$p_{x,t}^*$	14.03	0.76	14.63	0.77
Output	Y_t	1.19	0.89	2.43	0.81
Non-Tradable Value-Added	$Y_{n,t}^{va}$	0.79	0.89	3.08	0.80
Manufacturing Value-Added	$Y_{m,t}^{va}$	3.37	0.87	2.86	0.86
Commodity Output	$Y_{x,t}$	2.97	0.77	6.95	0.93
Inflation	π_t	0.47	0.10	0.26	0.39
Interest Rate	r_t	0.48	0.88	0.30	0.85

Notes: The HP filter with $\lambda = 1600$ was applied to the data prior to calculation. This transformation, resulted in data series which were consistent with their model counterparts, thus allowing for meaningful comparisons.

For several variables, the model is able to replicate the volatility. However, output in the non-tradable and commodity producing sectors are the two notable exceptions, where the model yields larger standard deviations for these variables relative to the data. The model

also produces somewhat lower volatility for inflation and the interest rate relative to the data, but captures it quite closely for the real exchange rate and commodity prices.

The autocorrelations of the model are generally in line with those found in the data. The model does however produce greater persistence for commodity output and inflation than the data. The autocorrelation of 0.77 generated for commodity prices in the model is very close to the 0.76 observed in the data, highlighting the fact that the model is able to accurately capture movements of commodity prices.

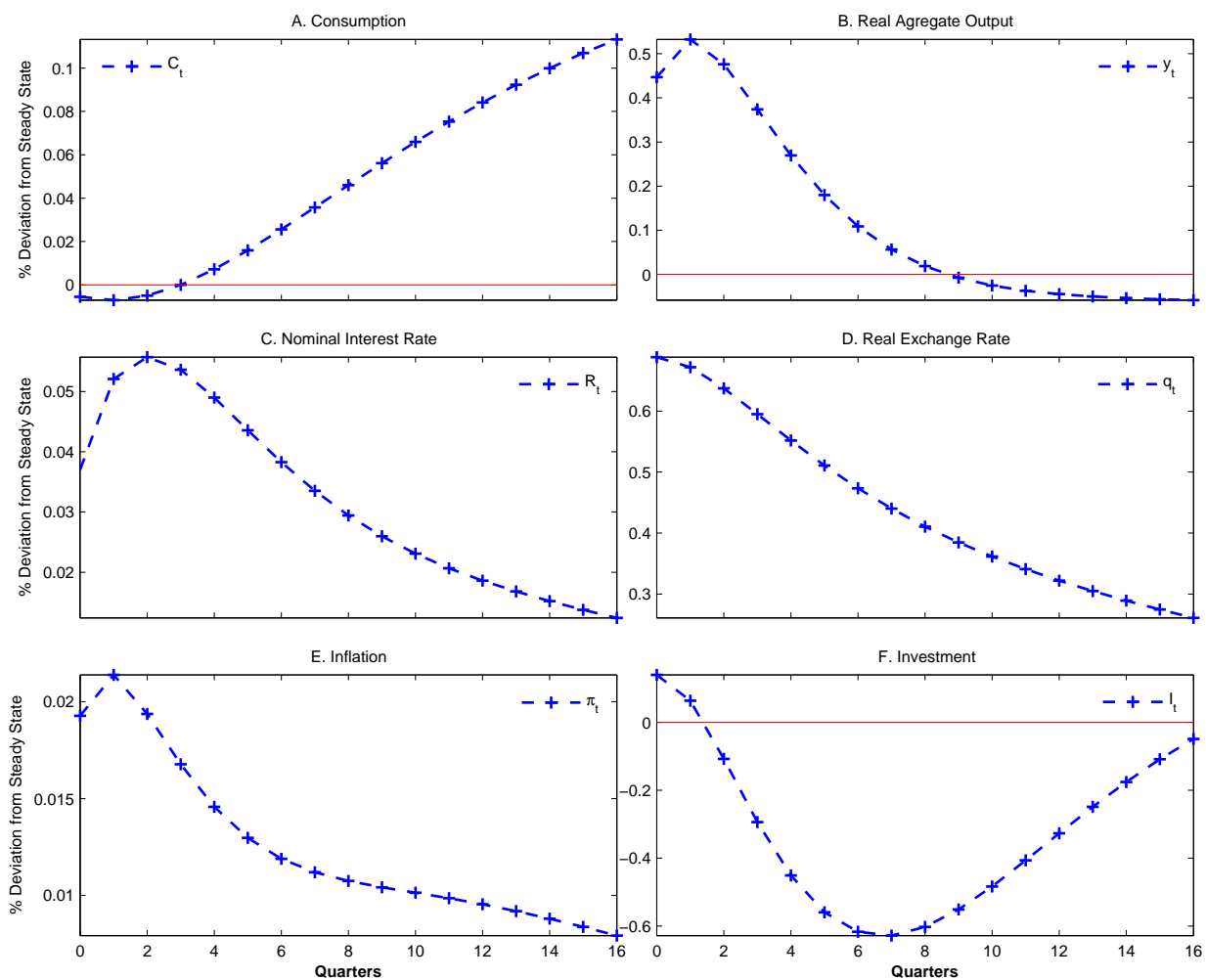
2.5.2 Impulse Responses

Figures 2.1 and 2.2 plot the impulse responses to a positive 10 percent innovation in commodity prices. This one-time shock represents an exogenous increase of commodity prices in world markets.

Higher commodity prices result in a notable rise in aggregate output (relative to the baseline in which commodity prices did not change). This is largely due to the rise in domestic income stemming from the increase in commodity prices and greater commodity production. Initially, there is also an increase in aggregate investment. By the second quarter however, aggregate investment falls below baseline before returning to steady-state and this largely reflects the decline in investment which occurs in the manufacturing and non-tradable sectors, whereas the initial increase reflected higher investment in the commodity producing sector.

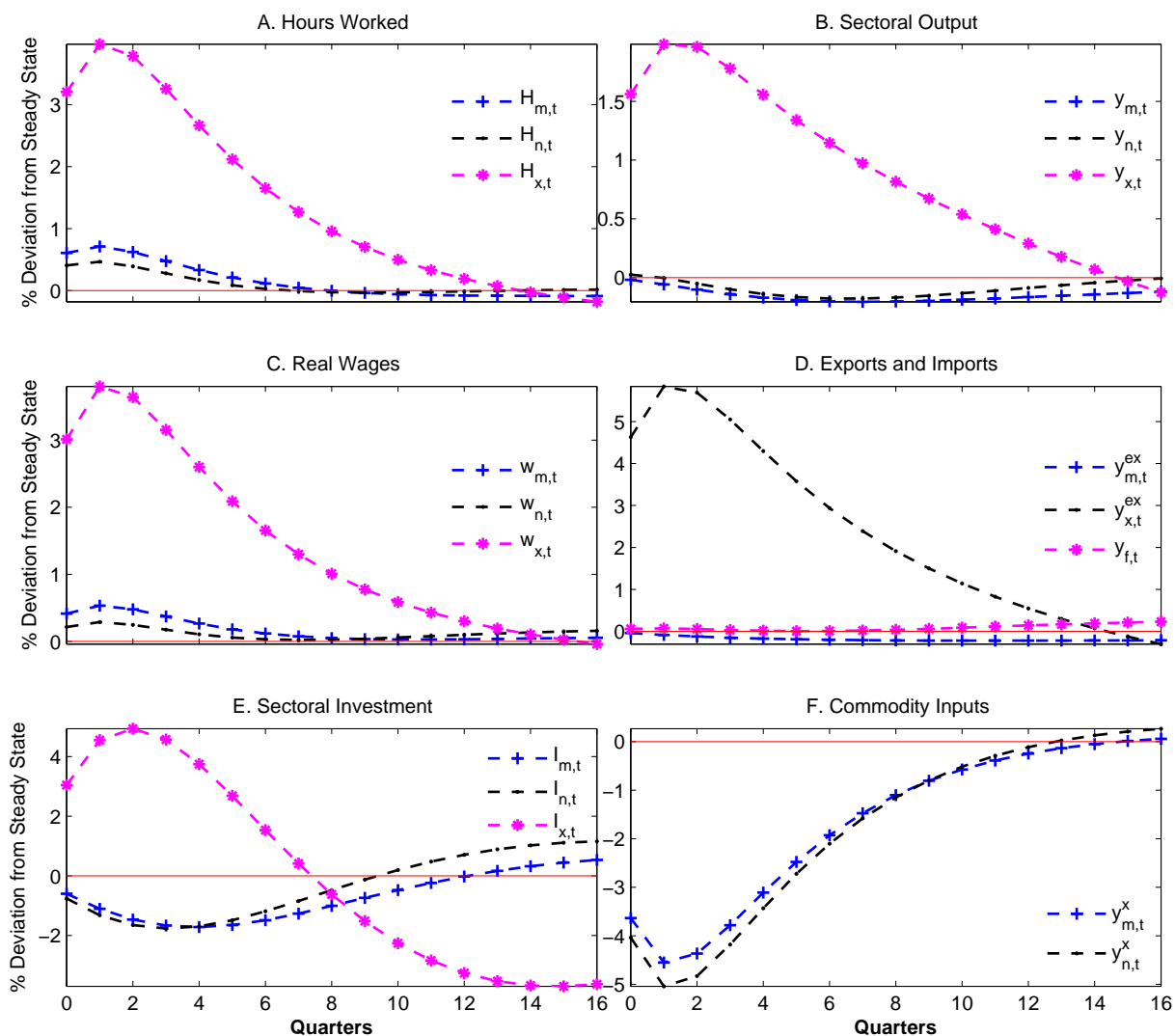
Overall, consumption also increases in response to the shock, however, this is first preceded by an initial decline. This short run decline in consumption is partly due to the lower income (wages) in the manufacturing and non-tradable sectors and also to habit persistence in the consumption patterns of households. However, this is quickly superseded

Figure 2.1: Aggregate Responses to a Positive 10% Commodity Price Shock



(after two quarters) by the rise in household wealth emanating from the commodity sector. Similar results have been found in the consumption pattern of households in other models for Canada. Murchison and Rennison (2006), for example found a sustained increase in consumption (of about 0.4 percent for the first five years), which lasted for about twenty years in response to a commodity price increase.

Figure 2.2: Sectoral Responses to a Positive 10% Commodity Price Shock



The real exchange rate appreciates on impact due to the rise in commodity prices before gradually returning to its steady state level. In spite of the appreciation, there is an increase in commodity exports as commodity producing firms positively respond (that is,

increase production in response) to exogenously higher commodity prices, which is denominated in the foreign currency.²⁴ Manufacturing exports, on the other hand, experience a decline, as the higher exchange rate makes manufactured exports less competitive internationally. Even after four years, manufactured exports has not returned to its steady state level. There is also a marginal increase in import volumes, highlighting the expenditure switching effect of the currency appreciation, as the higher domestic demand is facilitated by consuming more imports.

While output in the commodity producing sector increases in response to the shock, the output of firms in the manufacturing and non-tradable sectors that use commodities as an input, decline as the rise in commodity prices results in a shifting of resources across sectors. Manufacturing output declines virtually on impact, while the decline in the non-tradable sector occurs after the first quarter. This decline is less pronounced than that observed for manufacturing and recovery occurs much sooner.²⁵ As can be seen in panel F of figure 2.2 there is a marked decline in demand for commodity inputs from both sectors. These sectors also experience a decline in investment, whereas the opposite is true for the commodity sector. The commodity sector, is relatively more productive and thus there is a notable rise in the real wage in this sector relative to the marginal increases observed for the other two sectors.

Inflation rises on impact, reflecting rising aggregate demand in the economy in response to the higher commodity prices. These inflationary pressures are largely evident in the non-tradable and manufacturing sectors as higher input costs, primarily for commodities, drive up their relative prices.²⁶ To combat the rising level of inflation, the monetary authority

²⁴This is further reinforced by the fact that domestic demand for commodities, in the production of manufactured and non-tradable goods decline.

²⁵This is consistent with the results obtained in chapter one, utilising the VAR framework.

²⁶Inflation in the import sector, actually declines reflecting the effect of the currency appreciation, which causes imports to be cheaper.

proceeds to increase the interest rate, which rises above its steady state level in order to contain inflation.

2.5.3 Variance Decomposition

We now examine the unconditional variance decomposition of some relevant macroeconomic variables to determine which shocks the model suggests are important for the development of economic activity in Canada. These results are presented in Table 2.6.

Table 2.6: Unconditional Variance Decomposition

Variable	Shock					
	Productivity	Demand	Supply	Commodity	Monetary	World
Output (Y_t)	69.2	2.9	5.2	13.6	7.8	1.4
Consumption (C_t)	58.4	25.2	1.5	9.3	0.7	4.8
Investment (I_t)	85.3	1.2	4.0	1.1	6.4	2.0
Manuf. Exp. ($Y_{m,t}^{ex}$)	2.9	0.1	77.0	6.3	0.0	13.8
Cmdty. Exp. ($Y_{x,t}^{ex}$)	29.6	1.0	2.0	54.1	1.7	11.7
Imports ($Y_{f,t}$)	57.2	3.1	11.7	12.2	6.0	9.9
Inflation (π_t)	17.9	0.3	68.0	4.2	1.0	8.6
Interest Rate (r_t)	39.2	2.8	8.3	18.7	16.4	14.6
Cmdty. VA ($Y_{z,t}$)	36.5	1.3	2.8	44.4	2.6	12.3
NonTrad. VA ($Y_{n,t}^{va}$)	77.5	3.7	5.2	2.7	7.9	2.9
Manuf. VA ($Y_{m,t}^{va}$)	41.7	0.7	42.5	7.3	1.5	6.4
Real FX Rate (q_t)	3.1	0.4	1.0	17.7	1.2	76.5

Notes: Manuf. Exp.- Manufacturing Exports; Cmdty. Exp. - Commodity Exports; VA - Value-Added; NonTrad. VA - Non-Tradable Value-Added

For ease of analysis and greater clarity, the shocks of the model have been grouped into six broad categories. The first includes the ‘Productivity’ shocks: these are the labour-augmenting technology shock (ε_μ), the investment shock (ε_r), and the sector-specific technology shocks ($\varepsilon_{A_n}, \varepsilon_{A_m}, \varepsilon_{A_x}$). The ‘Demand’ category includes shocks to consumption

preferences (ε_{ξ_c}) and the government spending shock (ε_g). The ‘Supply’ group includes the mark-up shocks in the non-tradable (ε_n), manufacturing ($\varepsilon_m^d, \varepsilon_m^{ex}$) and import (ε_f) sectors. Shocks to commodity prices ($\varepsilon_{p_x^*}$) are contained in the ‘Commodity’ group and shocks to domestic interest rates (ε_r) in the ‘Monetary’ group. The final group consists of the ‘World’ shocks which include the risk premium shock (ε_ψ) as well as shocks to foreign output ($\varepsilon_{\xi_{y^*}}$), inflation (ε_{π^*}), and interest rates (ε_{r^*}).

The model indicates that much of the variation in economic activity in Canada can be explained by the productivity shock. Approximately 69% of the variation in output is explained by the productivity shock, with shocks to commodity prices contributing approximately 14% to this variation. 58% of the volatility in consumption is also explained by the productivity shock although a quarter of this volatility is also attributable to demand shocks. Except for consumption, demand shocks do not appear to be a significant source of volatility for Canadian macroeconomic variables.

Volatility of manufacture exports is largely due to supply shocks, with world shocks contributing 14%, particularly the risk premium shock. Commodity price shocks are estimated to explain over half the variance of commodity exports, with productivity and world shocks jointly explaining an additional 41%. Supply shocks are found to be largely responsible for the variation in inflation. This is primarily due to mark-up (cost-push) shocks in the non-tradable sector. Productivity and world shocks explain an additional 18% and 9%, respectively. However, the impact of monetary policy shock is relatively negligible.

Although the productivity shock on its own explains 39% of the variability in interest rates, commodity shocks, monetary policy shocks and world shocks jointly explain almost half of the variance of this variable. Variation in commodity output is primarily due to commodity, productivity and world shocks, respectively. While variability in the output of the non-tradable sector is largely due to productivity shocks, output variability

in the manufacturing sector is approximately equally affected by productivity and supply shocks. Variation in the real exchange rate is largely explained by world shocks, followed by commodity price shocks.

Overall, the model indicates that much of the variation in economic activity is due to productivity shocks, followed by commodity price shocks and then supply and world shocks. Thus, the predictions of the model are in line with the empirical results from chapter 1 and adds to the literature by providing further insights on the importance and contribution of various shocks to Canadian output. In addition, a useful advantage of employing a DSGE framework is the possibility of decomposing or disaggregating the dynamics of macroeconomic variables, such as GDP growth, according to the different structural shocks of the model. This exercise will give us further information on the importance of the various shocks for driving economic dynamics and we carry out this exercise in the next section.

2.5.4 Historical Decomposition of GDP Growth

As noted, we now turn to an examination of the structural shocks which the model indicates were important in driving the Canadian business cycle over the period 1997Q1 to 2016Q1. We achieve this by computing the historical decomposition of Canada's demeaned GDP growth rate over this period. The historical decomposition essentially attributes all deviations from steady-state in the variable under consideration to the structural shocks of the model, so that we can assess the contribution of each shock to the evolution of the variable.²⁷

²⁷To compute the historical decomposition, we used as observable variables real GDP, household final consumption expenditure, business gross fixed capital formation, the sum of general government final consumption expenditure and general government gross fixed capital formation (i.e. government consump-

The historical decomposition therefore allows us to visualize both the direction and magnitude of contribution of each exogenous shock to the movement of the endogenous variable. Additionally, unlike the variance decomposition, which tells us how each exogenous shock contributes to the second moment (that is, the variance) of the endogenous variables, the historical decomposition enables us to assess how each has contributed to the first moment of the endogenous variables. The results of this exercise are presented in Figure 2.3 below.²⁸

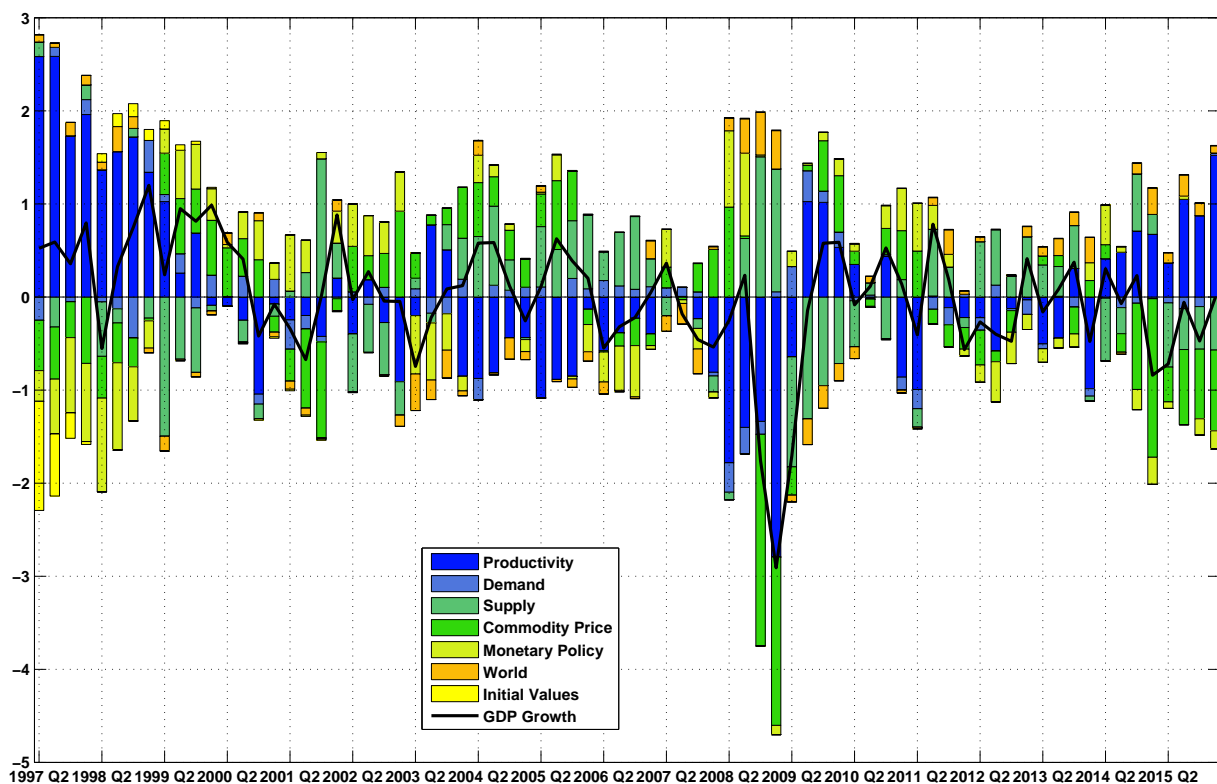
As with the variance decomposition, the shocks have been grouped into the same six broad categories to simplify the visual representation and improve clarity. According to the results, over the entire period, productivity shocks have played an important role in explaining GDP growth. This shock contributed positively to growth both in the early and latter parts of the sample, but its contribution during the mid-period (2000Q2 to 2009Q2 and 2011Q1 to 2014Q1) has generally been negative. This was particularly evident during the period of the global financial crisis, when at its height, large negative productivity shocks deducted almost 3 percentage points off of GDP growth.

Supply shocks have also been a significant contributor to GDP growth over the period. While its influence was relatively muted in the early parts of the sample, post-1999Q1 its impact has grown consistently. In fact, during the period 2004Q1 to 2007Q1, positive supply shocks counteracted the effect of a series of negative productivity shocks, which

tion), commodity exports, manufacturing exports, Gross Value-Added in the non-tradable, manufacturing and commodity producing sectors, CPI inflation, services inflation as a proxy for inflation in the non-tradable sector, the overnight rate, the nominal exchange rate, the BoC commodity price index, US real GDP, CPI inflation and the federal funds rate. All variables are measured as quarterly growth rates, with the exception of inflation and interest rates, which are measured as quarterly averages in levels. Finally, the variables are demeaned using their sample averages.

²⁸In the graph, the actual data is represented by the solid line while the contribution of each shock to the historical movement of (demeaned) GDP growth is shown in the form of a stacked bar chart. Thus, the contributions indicated by each bar sums to the value given by the solid line in each period.

Figure 2.3: Historical Decomposition of GDP Growth



otherwise would have resulted in below average GDP growth for that period. Its positive contribution during the period of the GFC also helped to mitigate the impact of that event, however, in the period immediately following the crisis, the contribution of this shock turned negative as the economy started to recover.

According to the model, commodity price shocks played just as large a role as supply shocks in accounting for the movements in GDP growth, thus implying that this shock is particularly important in the Canadian context. It contributed to above average growth in the period 1999Q2 to 2000Q2 and along with supply shocks, helped to offset the negative

productivity shocks during the period 2004Q1 to 2005Q4. As with productivity shocks, the commodity price shock contributed significantly to the decline in growth which occurred during the GFC. In fact, close to the trough of the recession, this shock detracted 2.3 percentage points from GDP growth. During the recovery phase immediately following the crisis however, commodity price shocks have generally been positive. In contrast, towards the end of the sample (2014Q3 onwards) the contribution of this shock has been negative, likely reflecting the decline in energy prices which has been counteracting the pace of economic recovery in Canada.

The role played by the remaining shocks in the model, are not as significant as the three discussed. The monetary policy shock, for example, has generally been contractionary over the period, with the most notable exception being at the start of the crisis and in its immediate aftermath where this shock contributed positively to GDP growth. Overall, the results of the historical decomposition are in-line with those obtained from the variance decomposition, in terms of the relative importance of the various shocks in driving Canadian economic activity. The results indicate that economic activity is largely driven by productivity, commodity price and supply shocks. Given these findings, we now examine some possible implications of the model, using it as a tool to carry out some policy counterfactuals.

2.5.5 Counterfactual Policy Exercises

We now employ the model to assess interesting counterfactual policy questions, which shed light on the possible implication of alternative policy actions in response to external shocks which have affected the Canadian economy. With the onset of the global financial crisis, and the ensuing economic downturn in many countries, policy-makers undertook various

actions with the goal of mitigating the effect on this crisis on their domestic economy. The Canadian experience was no different. Canadian policy-makers however did not express significant concerns about the crisis until after the collapse of Lehman Brothers in September 2008.²⁹ This was evident from the Bank of Canada's (BoC) 3rd September 2008 press release, where they noted that eventhough domestic demand in Canada had slowed, it had remained strong and decided to keep the target for the overnight rate fixed at 3 percent. Following the collapse however, the Bank undertook several rounds of aggressively reducing interest rates, so as to stimulate consumer demand and promote economic activity.³⁰ These actions could lead one to ask the questions of what would have happened had the Central Bank not reduced interest rates when it did, or alternatively what if the BoC had started to aggressively reduce rates sooner, prior to the collapse of Lehman Brothers and the intensification of the crisis which was accompanied by precipitously declining commodity prices?

In this section, we attempt to answer these questions with the design of two policy experiments. The first of these examine the impact on some relevant macro-variables of the Central Bank not lowering interest rates when it did, but rather keeping it fixed at its relatively high level of 3 percent as at September 2008 for an additional two quarters. In the second experiment we analyse what would have been the economic consequences if the Central Bank had begun to aggressively cut interest rates sooner. These simulation experiments are labelled scenarios 1 and 2 respectively, in the discussion which follows.

In scenario 1, we examine how the economy would have behaved in the aftermath of the

²⁹Lehman Brothers was a global financial intermediary and the insolvency of this firm is recognised by many as the key event which triggered the intensification of significant upheavals in global financial markets.

³⁰This was particularly important in the Canadian context, as commodity prices, which had undergone an extended period of growth was beginning to decline rather quickly over a relatively short period of time. This reflected the deterioration of economic activity in U.S., Japan and Europe and resulted in an uncertain outlook for Canada's economic future.

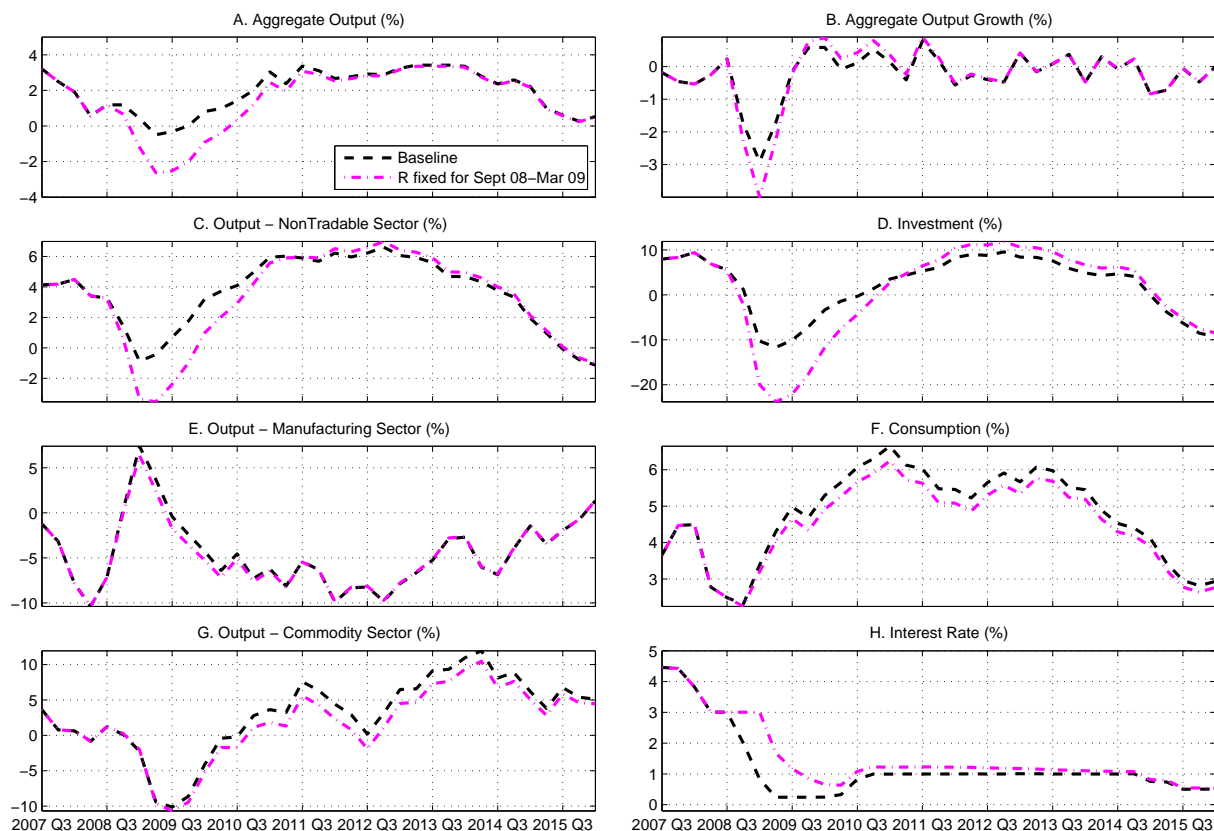
GFC and declining commodity prices if the Central Bank had not intervened by lowering rates. Thus we are able to assess how the economy would have evolved in the context of lower commodity prices, but with relatively higher interest rates, than that which actually prevailed. To eliminate the effects of the easing of monetary policy³¹ in response to the crisis and falling commodity prices, we keep the interest rate constant at its relatively high level of 3 per cent as at September 2008 for the quarters of September 2008 through to March 2009. Thus, in Scenario 1 we feed all the generated shocks into the system but hold the interest rate constant from September 2008 to March 2009. In order to obtain the constant interest rate, this required a change in the path of the monetary policy shock while keeping the other shocks at their original values. Essentially, a sequence of precisely determined negative monetary policy shocks were feed into the system, such that equation 2.28 endogenously gave rise to a fixed interest rate for the period September 2008 to March 2009. The results of scenario 1 are presented in Figure 2.4 which shows the time path of relevant macro variables from this experiment.

Note that in Figure 2.4 the baseline and scenario paths coincide until the September 2008 quarter, when we start to hold interest rates constant. Interest rate is held constant at 3 percent from September 2008 to March 2009, and then allowed to evolve endogenously thereafter, where it declined to 1.7 percent in the June 2009 quarter. In the data (the black dotted line in panel H of Figure 2.4), the interest rate was gradually reduced from 3.0 percent to 0.25 percent over the same period, resulting from the monetary easing actions of the Central Bank.

Under Scenario 1, without this policy intervention, the simulated time path of aggregate output and aggregate output growth differ significantly from their data path in the post-

³¹The BoC reduced the overnight target rate from 3 per cent to 0.5 per cent over the period September 2008 to March 2009.

Figure 2.4: Time-Path of Variables Under a Fixed Interest Rate - Scenario 1



Notes: Baseline represents the time path of the variables had interest rates not been held constant for the Sept. 2008, Dec. 2008 and Mar. 2009 quarters. Aggregate GDP growth is reported as quarter-over-quarter percentage change, interest rates as the quarterly average of the daily values and all other variables as percentage deviations from steady-state.

crisis period. Without the policy intervention, aggregate output would have fallen below its steady state level beginning in the March 2009 quarter and remained so until the September 2010 quarter. In contrast, the baseline path, which accounts for the monetary intervention of the BoC, would have only been lower than steady state for two quarters (June 2009 and September 2009).

These marked differences are also reflected in the rate of aggregate output growth (see panel B). Output growth declined to 2.9 percent and 4.0 percent in the December 2008 and March 2009 quarters, when interest rates are held constant, as opposed to declines of only 1.8 percent and 2.9 percent in the data. Thus we can see that without the intervention of the Central Bank, the recession would have been much more severe in the aftermath of the crisis. Recovery however, was stronger under scenario 1, than that which was actually observed in the data. Thus it seems there may be some trade-off between the severity of the recession versus the strength of the recovery, which may necessitate a choice as to which path is preferable - a more severe recession with a stronger recovery or a less severe recession with a more muted and protracted recovery.

Similar to aggregate output, we also see notable differences in the time paths of investment and consumption under scenario 1 versus the baseline. Consumption remained consistently below the baseline value, reflecting the negative impact of the GFC on consumer confidence and hence offers some justification for the Central Bank seeking to support consumer demand, by lowering rates. At its trough, investment was 23.8 percent lower than its steady state value when interest rates are held at 3 percent versus a decline of only 11.7 percent in the baseline. The higher interest rate, combined with weak consumer demand and declining aggregate output, contributed greatly to this precipitous decline in investment. As with aggregate output, the recovery in investment was stronger under scenario 1. Which also confirms that there is a trade-off present in the policy choices available to the monetary authorities.

With respect to sectoral output, we see a similar pattern. Output in all three sectors is lower in scenario 1 than in the baseline. This is most notable for the non-tradable sector, which constitutes the largest sector of the economy. The impact on the manufacturing sector is less noticeable, while the higher rates lead to a moderate, but protracted decline

in the commodity producing sector.

In scenario 2, we examine the likely macroeconomic conditions that would have prevailed had the Central Bank aggressively began lowering rates sooner, that is, prior to September 2008. The design of this policy experiment necessitated identifying a sequence of three monetary policy shocks in equation 2.28 for the quarters March 2008 to September 2008, such that the endogenous path for interest rates followed a predefined counterfactual trajectory, while the remaining shocks in the model assumed their previously computed values. The predefined trajectory for interest rates which we sought to match was the reduction in interest rates which occurred over the period December 2008 to June 2009.³² This experiment is particularly useful as it allows us to assess whether the impact of the recession and falling commodity prices would have been milder had the Central Bank pursued a policy of expansionary monetary policy sooner. The results of this experiment are displayed in Figure 2.5.

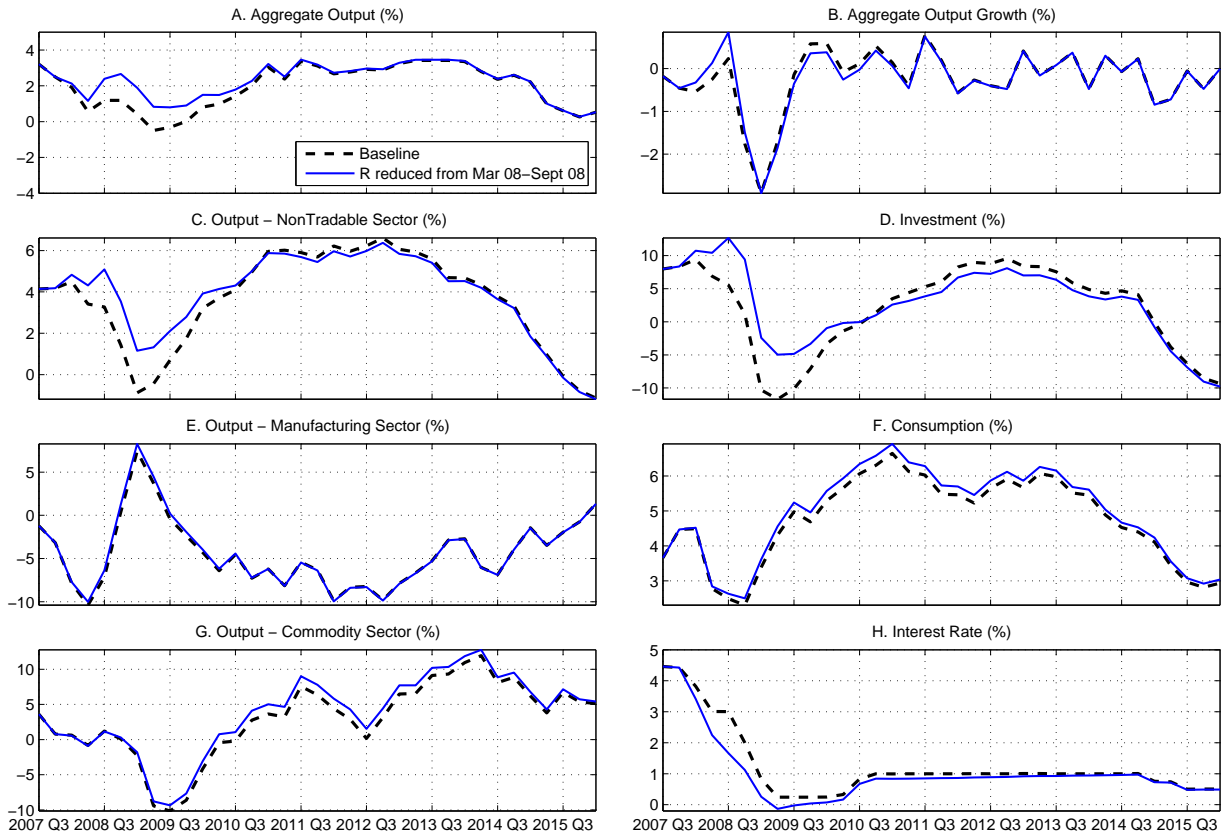
As can be seen from Figure 2.5 interest rates declined much earlier under scenario 2 than in the baseline case, from 3.4 percent to 1.7 percent over the quarters March 2008 to September 2008 versus a much more muted decline of 3.8 percent to 3.0 percent over the same period in the data.

The earlier decline in interest rates lead to higher growth in aggregate output over the period, however, the effect of the crisis on output growth was still significant, and in fact, aggregate output growth post-crisis was lower than the baseline case. Despite this, the level of aggregate output would have been distinctly higher had the course of policy action examined in scenario 2 been undertaken by the monetary authority.

The lower interest rate also positively impacted investment and consumer demand.

³²We essentially brought these interest rate reductions forward by three periods, and then allowed rates to evolve endogenously thereafter.

Figure 2.5: Time-Path of Variables when Interest Rates are Lowered Earlier - Scenario 2



Notes: Baseline represents the time path of the variables when interest rates were not reduced aggressively in the Mar. 2008, June 2008 and Sept 2008 quarters. Aggregate GDP growth is reported as quarter-over-quarter percentage change, interest rates as the quarterly average of the daily values and all other variables as percentage deviations from steady-state.

Whereas output was 11.7 percent below trend in the June 2009 quarter in the absence of the policy experiment, it was only 5 percent below trend in the case of scenario 2. The impact on consumption, though moderate, was persistent, as consumption remained above the baseline case for a significant period.

Sectoral output was also higher under scenario 2, relative to the baseline. Output in the non-tradable sector was particularly higher in the earlier periods of the sample, though it went marginally below baseline in the post-June 2010 period. For the commodity sector, the impact was largely felt in the longer horizons, as opposed to the shorter horizons. The differential time response of these two sectors likely reflect their differing underlying structures. This is so, as projects which take advantage of the lower interest rates in the commodity-producing sector (agriculture, mining, oil extraction etc.) are expected to see increased production only after a significant lag, whereas the impact of lower interest rates on output in the non-tradable sector (made up largely of services industries) is expected to be more immediate. The impact on the manufacturing sector was relatively muted, although it too was marginally higher.

Thus, it appears that pursuing a policy of reducing interest rates sooner would have had an overall beneficial effect on the economy. We do note however, that the results obtained here should be viewed as a lower bound on the possible effect of policy actions on the economy since the Central Bank, not only engaged in monetary easing by lowering the interest rate (which is the only policy action examined here), but also employed non-conventional monetary policy, such as quantitative easing (where funds were made directly available to the financial sector). Incorporating quantitative easing, we would expect to see an even larger effect of policy action on the economy.

2.6 Policy Implications

From a policy perspective, our results from this and the previous chapter, support the view that decision makers should take commodity price movements into account and in particular how these movements impact each sector of the economy. While positive commodity

price shocks may be beneficial to commodity producing sectors of the economy, these same shocks are likely to negatively impact Canada's manufacturing and non-tradable sectors. Consequently, policy makers should consider implementing programmes which offset these possible negative effects by channelling some of the windfall from the commodity producing sectors into those sectors which are negatively impacted by rising commodity prices.

These actions may include policies such as taxing the consumption of luxury goods and services which are fuelled by higher personal incomes from employment in the commodity producing sector,³³ (also by collecting royalties and taxes from the commodity producing firms themselves) and using those additional revenues to create a stabilisation fund which may be accessed for capital investment projects by firms in the manufacturing and non-tradable sectors (or be used for other projects and activities which boost productivity and competitiveness in these sectors). This same fund, may also be useful to support activities in the commodity producing sector itself when it is faced with a significant negative shock which results in drastically declining prices and output, such as that which materialised during the GFC.

2.7 Concluding Remarks

In this chapter we have sought to examine the impact of commodity price shocks on a small commodity exporting open economy, using Canada as the stylized example. This is directly related to what we attempted to achieve in chapter 1, however, here we employ a different methodological framework, more specifically, a Dynamic Stochastic General Equilibrium model. This extended framework allows us to examine linkages in the economy that were

³³Our model showed a significant rise in consumption, in response to the positive commodity price shock.

not possible with the method employed in the previous chapter and thus represents an extension of the work undertaken there. Additionally, the DSGE model allowed us to conduct scenario analysis, so that we could examine the possible economic impact of certain counterfactual policy actions in response to the large negative commodity price shock which accompanied the global financial crisis.

The model employed was that developed by Dib (2008), with some minor modifications. Dib's focus, in his paper, was specifically on how commodity price shocks affected the exchange rate and the associated welfare effects. He did not exploit the sectoral nature of the model to assess how each sector responded to relevant shocks, such as shocks to commodity prices. In this way, the focus of our paper is different from Dib, and reflects our contribution to the literature in an area which has been relatively unaddressed. Additionally, using the model to examine the policy response to a real-world commodity price shock, represents a unique contribution of our paper. While the policy response to the GFC in the Canadian context has been examined in many ways (see BIS (2008), Zorn et al. (2009)), this paper is the first to assess the policy response within a DSGE framework and examine how this action influenced the sectoral outcome for Canada.

The model was generally able to capture key features of the data and showed that the Canadian economy was highly responsive to changes in the prices of commodity exports. The evolution and volatility of aggregate GDP growth in particular, was significantly influenced by changes in commodity prices, as illustrated by its behaviour during the GFC. These results suggest that commodity price shocks, along with productivity and supply shocks, have been important in driving the Canadian business cycle.

The disaggregated nature of the model also allowed us to analyse the differing sectoral responses in the economy. We found that although a positive commodity price shock, resulted in an unambiguous increase in aggregate output, this largely reflected increased

output in the commodity producing sectors, as output in both the manufacturing and non-tradable sectors declined. Similar diverging responses were noted with respect to sectoral investment. Thus the disaggregated nature of the model is of particular importance in analysing and understanding the transmission mechanism of such external shocks throughout the economy, as otherwise, important sectoral differences may be overlooked when only an aggregated model is examined.

With respect to policy actions undertaken in response to significant negative commodity price shocks, we can conclude from our counterfactual policy exercises that the actions taken by the monetary authority may necessarily involve some trade-off between attenuating the severity of a recession versus heightening the strength of recovery. The action of the Central Bank during the GFC, also appeared to have positively impacted the economic outcome for Canada, while at the same time, earlier actions by the Bank may have proved to be even more beneficial.

Despite what has been accomplished with the model thus far, there exists room for improvement and these remain areas for further extension and future research. In particular, the current model was calibrated based on available data, however, it would be useful and informative to estimate some parameters of the model to extract even more information from the available data in a consistent and transparent manner. Additionally, the GFC has shown that financial linkages within the economy may be important in assessing how the economy responds to certain shocks. Incorporating such linkages and a well specified financial sector is thus an important area for future research.

Chapter 3

Monetary Policy and the Stock Market in Canada and the U.S.

3.1 Introduction

Among asset prices, stock prices are typically the most closely monitored by the general public and decision makers. Changes in asset prices affect the economy through various channels. In particular, fluctuations in stock prices have a direct impact on the balance-sheet, wealth, collateral value and liquidity of households and firms. Moreover, since stock prices can be volatile and at times can deviate from fundamental values for prolonged periods, abrupt changes in their prices can have disastrous implications for the economy (Bordo and Jeanne 2002). On the other hand, changes in the policy instrument of central banks and short-term rates have a direct and immediate effect on stock prices. Since stock prices play a significant role in the transmission of monetary policy and monetary policy in turn can have an important impact on stock prices, it is critically important to understand

how precisely they are interconnected.

Many papers have studied how monetary policy and stock prices are connected in the U.S (for example, Neri (2004), Bernanke and Kuttner (2005), Gilchrist et al. (2009), Nisticó (2012)) using either a Structural Vector Autoregression (SVAR) or a small-scale DSGE model. Studies using Canadian data are not as prevalent. To identify monetary policy and stock price shocks, most papers that employ a SVAR usually assume a recursive or short-run identification where a particular causal relationship between stock prices and monetary policy is imposed. These papers either assume that monetary policy respond with a lag to a shock in stock prices or the reverse.

There are two reasons why this type of identification is problematic. First, as we show in the paper, assuming a recursive identification can lead to the wrong and counterfactual response for stock prices, interest rates and other variables following a monetary policy shock. For example, if we assume that stock prices react with a lag to a monetary policy shock, a positive monetary policy shock leads to an increase in stock prices in Canada. Second, there is ample empirical evidence that monetary policy and stock prices react to each other simultaneously and without any delay. Rigobon and Sack (2004) for example find that the probability of a 25 basis point (*b.p.*) tightening at the next FOMC meeting immediately increases to just over 50 per cent if there is an unexpected 5 per cent increase in the Standard & Poor's 500 index. Bernanke and Kuttner (2005) find that a 25 basis point unexpected easing in the Federal funds rate instantly leads to a 1.25 per cent increase in the stock market, that is a stock market multiplier of 5. Coincidentally, this is by about the same magnitude that the S&P/TSX index rose when on January 21, 2015, the Bank of Canada surprised markets by decreasing the overnight rate by 0.25 per cent.¹

¹Data from BAX futures or the 1-month Treasury bill indicated that market participants were not pricing in any reduction in interest rates.

The literature has attempted to resolve this simultaneity and identification problem in two ways. First, as in Rigobon and Sack (2004), high frequency data are employed to formally test whether there is no feedback from interest rates to stock prices. The idea here is to measure the responsiveness of stock prices to monetary policy shocks that are identified based on changes in variance of the policy rate on days of FOMC announcements. However, if one wants to also study the reaction of other variables that are only available on a lower frequency basis, this approach would be difficult to implement. The second approach is to use lower frequency data and solve the simultaneity problem in a different way. Bjørnland and Leitemo (2009) in their paper overcome this simultaneity problem by assuming that monetary policy shocks have no long-run impact on real stock prices. The additional zero restriction gained by assuming this long-run constraint removes the need to impose a zero coefficient between stock prices and monetary policy in the short-run impact matrix. By doing so, they have an exactly identified SVAR while allowing stock prices and monetary policy to react to each other's shocks contemporaneously.

The identifying restrictions used by Bjørnland and Leitemo (2009) are thus very appealing and are a big improvement on the previous literature as the simultaneity problem is solved while at the same time, the SVAR is exactly identified. However, to achieve identification, they nevertheless have to impose fairly rigid and possibly ad-hoc assumptions on the causal relationship between the other variables in the model instead of letting the data reveal these causal relationships. As a result, their findings may not be robust to different recursive orderings.

In this paper, we start with the same premise as in Bjørnland and Leitemo (2009) by allowing interest rates and stock prices to react contemporaneously to each other's shock but the novelty here is our agnostic view regarding the identification of all the shocks in the model and about the short-run and long-run relationship among all the variables. We

employ a six-variable VAR and then impose a minimal set of identifying restrictions that are used in the literature on some equations while leaving the other equations unrestricted. First, in the baseline model, we do not impose any short-run restrictions on the model. We allow all variables to react contemporaneously to each other, including stock prices and interest rates.² Second, we assume that monetary policy and demand shocks are neutral in the long-run, a common assumption in the VAR literature.³ Since our SVAR remains underidentified, we use sign restrictions to identify all the impulse responses that are consistent with the reduced form VAR. We derive our sign restrictions from a small scale DSGE model which include stock prices, monetary policy and confidence shocks. We believe that our approach is more flexible as it solves the simultaneity problem while at the same time, it does not rely on a number of short-run identifying assumptions that may be ad-hoc or not entirely consistent with the data. The methodology that we use to combine short-run, long-run and sign restrictions in the underidentified SVAR is adapted from Rubio-Ramírez et al. (2010) and Binning (2013).

Our results suggest that employing recursive identification in monetary VARs of this type can lead to counterfactual responses for the stock market following a monetary policy shock and for the interest rate following a stock market shock. They therefore highlight the dangers of using this particular identification strategy to study the relationship between stock prices and monetary policy and this is another contribution of our paper. Using Canadian data, we find that a positive monetary policy shock leads to an increase in stock prices, if we assume that stock prices react with a lag to monetary policy shocks. In contrast, if we assume that stock prices react immediately to monetary policy shocks,

²We also impose as a robustness test the usual assumption that monetary policy has no contemporaneous impact on output and inflation, a common assumption in the monetary policy literature.

³Bjørnland and Leitimo (2009) assume that monetary policy has no impact in the long-run on real stock prices.

we obtain the expected response, however the response of interest rates to a stock market shock is contrary to expectations. Moreover, employing a recursive identification also leads to counterfactual responses for some macro variables following a monetary policy shock. Industrial production and inflation both increase following an increase in interest rates in the Canadian and U.S. VAR.

When we employ the underidentified VAR, we do not obtain any counterfactual responses and we find that the stock market and monetary policy have an important impact on each other in Canada and in the U.S. We find that an increase of 25 *b.p* in the policy rate of the central bank leads to a fall of about 1.75% in stock prices in Canada and to a fall of about 1.25% in stock prices in the U.S. The stock market, industrial production, inflation and commodity prices react negatively to a positive monetary policy shock in Canada and the U.S. Our estimates for the U.S are in line with the empirical literature. Estimates in the empirical literature range from 0.5 to 2.5 per cent for the U.S. Our estimates for Canada are larger compared to the few papers that estimate this relationship. We argue that the effect of the monetary policy shock in Canada is larger compared to the U.S. mainly because the sectors that are interest rate sensitive such as materials, financials, industrials and energy account for a much larger share of the stock index in Canada compared to the U.S. These sectors account for around 75% of the TSX index and only 33% of the S&P 500 index. On the other hand, sectors that are less sensitive to interest rate changes such as telecoms, utilities, health care and consumer staples account for a much larger share of the U.S. stock market (28%) compared to the Canadian stock market (13%).

Following a positive stock market shock, the short-term interest rate, industrial production, inflation and commodity prices rise both in Canada and in the U.S. In Canada, a 1% increase in the stock market leads to an increase of about 27 basis points in the

overnight rate, while it leads to an increase of about 10 basis points in the Federal funds rate. Our estimates for the U.S. are in line with the empirical literature. Typical estimates range from 2 to 12 basis points for the U.S.

3.2 Literature Review

The literature studying the interaction between stock market and monetary policy can be divided into two types of approaches: SVAR and other types including event-studies and DSGE models. Most of the studies have used U.S. data and there is little work for Canada. Most VAR studies including those by Thorbecke (1997), Patelis (1997) and Neri (2004) analyse the effect of monetary policy on stock prices using a recursive VAR and assume that either stock prices react to monetary policy shocks with a lag or the reverse. Single-equation studies such as that by Ioannidis and Kontonikas (2006) also follow in the same vein. All of these studies find that an increase in the short-term interest rate decreases stock prices, at least in the short-run. Neri (2004) for example, finds that a 1 per cent increase in the short-term interest rate leads to a 3.6 per cent decrease in stock prices in the U.S. after four months. Li et al. (2010) analyse the impact of monetary policy shocks on stock prices in Canada and the U.S. using short-run restrictions. They use a non-recursive system and impose many strong assumptions to exactly identify their SVAR. They find that the response of stock prices in Canada to an unanticipated monetary policy shock is much smaller than in the U.S. A 25 basis point change in the policy rate leads to a 0.8 per cent change in stock prices in Canada and to a 4 per cent change in stock prices in the U.S. This peak response is achieved after four months in Canada and after 17 months in the U.S. They argue that these differences are caused mostly by differences in the degree of openness of the asset markets in Canada and the U.S. They do not study how changes

in stock prices affect the policy rate of the central bank.

Other studies such as Bjørnland and Leitemo (2009) and Rigobon and Sack (2003; 2004) assume a non-recursive structure and allow stock prices and monetary policy to react to each other's shock contemporaneously. Bjørnland and Leitemo (2009) find great interdependence between the federal funds rate and real stock prices in the U.S. They show that a 25 basis points increase in the federal funds rate, decreases real stock prices by 1.25 to 1.75 per cent. On the other hand, a one per cent increase in stock prices lead to an increase in the federal funds rate by around 4 basis points. Rigobon and Sack (2004) use a simultaneous identification approach and high frequency data to formally test whether there is no feedback from interest rates to stock prices. Their strategy relies on identifying changes in the variance of the short-term rates on FOMC and Humphrey-Hawkins testimony meetings and examining the response of several stock price indices on these days. They find that a 25 basis points increase in the short-term interest rate leads to a fall of 2.4% in the Nasdaq index and to a fall of 1.7% in the S&P 500 index.

Bernanke and Kuttner (2005) use an event-study approach to analyse the effects of unanticipated changes in the federal funds rate on stock prices. They use data from the futures market to estimate the expected value of the federal funds rate and then compare this expected value with the actual decision of the Federal Reserve. By doing so, they get a measure of the unanticipated changes in the federal funds rate target. They find that there is a five-fold increase in stock prices following an unanticipated cut in the federal funds rate target. Gurkaynak et al. (2005) also use an event-study approach similar to Bernanke and Kuttner (2005) and find that unanticipated changes in the federal funds rate target lead to approximately a five-fold change in stock prices. They also study the impact on stock prices from unanticipated changes in the future path of the federal funds rate arising from changes in the statement that accompanies FOMC announcements. They find that

stock prices are also severely impacted by these revisions in expectations.

Studies by Castelnuovo and Nisticò (2010), Airaudo et al. (2007), and Challe and Giannitsarou (2014) use small-scale DSGE models to examine the effects of policy shocks on stock prices. These papers also find that an increase in the policy rate of the central bank leads to a decrease in stock prices. The estimates from the DSGE models are in line with the empirical literature as they find that a 25 basis points change in the policy rate typically leads to a 1% change in stock prices. Table 3.1 summarizes some of the literature which examines the relationship between monetary policy and stock prices.

Table 3.1: Literature Survey - Stock Prices and Monetary Policy

Author(s)	Data	Sample Period	Methodology	Main Findings
VAR Studies				
Patelis (1997)	excess stock returns, federal funds rate, various measures of interest rate spread, nonborrowed reserves, total reserves, dividend yield, T-bill rates	1962M3 - 1994M11	VAR with short-run identification	A contractionary monetary policy shock is generally associated with lower expected stock returns initially, and higher returns in the longer-term.
Rapach (2001)	implicit GDP deflator (price level variable), 3-month T-bill rate, real GDP, S&P 500 stock price index	1959Q3 - 1999Q1	VAR identified with only long-run restrictions	A 1% positive interest rate shock, leads to an approximate 6% decline in stock prices. A similar 1% positive stock prices shock, leads to a 0.05 percentage point rise in interest rates.
Neri (2004)	world commodity price index, nominal exchange rate, industrial production, CPI, short-term interest rate, monetary aggregate, stock market index	1985M1 - 2000M12	VAR identified with short-run non-recursive restrictions	A 1% exogenous increase in short-term interest rates, result in a 4% (1%) decline in stock market indices for the U.S. (Canada).

Table 3.1: Literature Survey - Stock Prices and Monetary Policy Cont'd

Author(s)	Data	Sample Period	Methodology	Main Findings
Bjørnland and Leitemo (2009)	CPI, commodity price index, industrial production, federal funds rate, S&P 500 stock price index	1983M1 - 2002M12	VAR with short-run and long-run identification	A 1% increase in the federal funds rate (stock prices) leads to a 9% decline (0.04 percentage point increase) in real stock prices (interest rates).
Li et al. (2010)	industrial production index, consumer price index, S&P 500 index, TSE 300 index, CAD/USD exchange rate, O/N interest rate, Fed funds rate, M2, oil prices	1988M1 - 2003M12	VAR identified with short-run non-recursive restrictions	On impact, an unanticipated 25 bp increase in the federal funds rate resulted in a 0.55% decline in stock prices. A similar increase in the Canadian overnight rate lead to a negligible 0.003% decrease in Canadian stock prices.
Other Methodologies				
Thorbecke (1997)	industrial production, inflation rate, a commodity price index, federal funds rate, non-borrowed reserves, total reserves, stock returns (CRSP database)	1967M1-1990M12	Multiple methods: VAR (short-run recursive identification), GMM, Event-study	a one standard deviation increase in the fed funds rate, on impact, leads to a 0.8% decline in stock prices.
Rigobon and Sack (2003)	3 month T-bill rate, S&P 500 stock price index	Mar1985 - Dec1999 (daily data)	Heteroskedasticity-based approach	A 5 percent rise (fall) in the S&P 500 index increases the probability of a 25 b.p. tightening (easing) of the interest rate by about a half.
Rigobon and Sack (2004)	Dow Jones Industrial Average, S&P 500, Nasdaq, Wilshire 5000, Treasury yields, eurodollar futures rates	January 3, 1994 - November 26, 2001 (daily data with 78 policy dates included)	Heteroskedasticity-based approach	an unanticipated 25 b.p. increase in short-term interest rate leads to a 1.7% fall in the S&P 500 index and a 2.4% fall in the Nasdaq index.

Table 3.1: Literature Survey - Stock Prices and Monetary Policy Cont'd

Author(s)	Data	Sample Period	Methodology	Main Findings
Bernanke and Kuttner (2005)	federal funds rate, the Center for Research in Security Prices (CRSP) value-weighted index	Jun 1989 - Dec 2002 (131 “events” in total)	Event-study	A hypothetical unexpected 25 basis-point reduction in the Federal funds rate target is, on average, associated with an approximate 1% rise in broad stock indices.
Ioannidis and Kontonikas (2006)	nominal stock return, short-term interest rate	1972M1 - 2002M7	single equation estimation (OLS)	stock returns are significantly affected by changes in monetary policy.
Airaudo <i>et al</i> (2007)	not applicable	-	calibrated DSGE	The optimal interest rate rule for the policy-maker is one which responds to a stock price index.
Castelnuovo and Nisticò (2010)	real per capita GDP, real per capita consumption, real S&P 500 index, real wages, per capita hours worked, inflation, fed funds rate	1954Q3 - 2007Q2	estimated DSGE	A 25 bp unexpected rise in the federal funds rate causes stock-prices to decline by about 20 bp. In contrast, a 1% shock to stock-prices results in a 12 bp interest rate hike on-impact.
Challe and Giannitsarou (2014)	not applicable	-	calibrated DSGE	The model predicts that a positive shock to the policy rate causes a decline in stock prices on impact, and this is in-line with empirical estimates.

3.3 Exact and Partial Identification

As discussed in Kilian (2013), in the VAR literature, there are a number of potential sources for identifying structural shocks from the reduced-form VAR. Economic theory is

often used to identify structural shocks and theory can inform us about the long-run and short-run responses of some variables to shocks. For example, firms may not change their investment plans immediately following a technology shock as physical constraints and lags in decision-making may prevent them from increasing physical investment immediately. As a result, investment may not respond immediately to certain shocks, but rather, respond with a lag. Similarly, we can use economic theory to impose restrictions on the long-run response of some variables to some shocks. This idea is used in many papers, for example in Galí (1999) who assumes that technology shocks are the only shocks that have a permanent effect on labour productivity. He derives this restriction from a neoclassical production function.

Identifying shocks using short-run and long-run restrictions however are not without any drawbacks. The limitations of both of these approaches are well discussed in the literature (Kilian (2013)). In recent years, VARs that identify structural shocks by sign restrictions have become more popular. The sign restriction approach was pioneered by Faust (1998) in the context of identifying monetary policy shocks. One of the advantages of sign restriction is that there is no need to impose linear restrictions between the reduced form and structural errors. In this way, sign restrictions are more flexible than imposing restrictions on the impact matrix or on the long-run matrix. In sign identified structural VARs, restrictions are imposed on the sign of the responses of some variables to some structural shocks. The source of restrictions usually takes the form of the impulse response functions from a DSGE model or in some cases, a partial equilibrium model. Unlike short-run and long-run restrictions, sign restrictions do not imply a unique point estimate but rather a set of models that are consistent with the imposed restrictions. These structural responses are admissible as long as they satisfy the set of identifying restrictions imposed by the researcher.

A VAR that combines short and long-run restrictions as in Bjørnland and Leitemo (2009) cannot be solved using a lower triangular Cholesky decomposition of the long-run covariance matrix since the solution involves solving a non-linear problem. In their paper, Rubio-Ramírez et al. (2010) propose an alternative approach to transform the non-linear problem into a linear one using a QR decomposition.⁴ They introduce an efficient algorithm by imposing short and long-run restrictions in exactly identified models. Their main idea is to use the QR decomposition to find an appropriate rotation matrix that satisfies the zero short-run and long-run restrictions imposed by the researcher. Binning (2013) extends the methodology of Rubio-Ramírez et al. (2010) to models that do not have enough zero restrictions and thus are not globally defined. In the case where there are not enough zero restrictions, the model will produce a set of impulse response functions that may or may not be consistent with theory. Sign restrictions based on a model can then be used to discard all the impulse response functions that are not consistent with theory. This will produce a band of impulse response functions that are consistent with the zero, as well as with the theory based sign restrictions imposed by the researcher.

3.3.1 Exactly Identified Models with Short and Long-run Restrictions

We explain the methodology of Rubio-Ramírez et al. (2010) for exactly identified models and Binning (2013) methodology for underidentified models using a stylized VAR model. Once the methodology is explained, we describe the specific VAR we use to analyse the interconnectedness between stock prices and monetary policy.

⁴The QR decomposition of a matrix is a procedure in linear algebra which decomposes a matrix A into a product Q and R (that is, $A = QR$) such that Q is an orthogonal matrix and R is an upper triangular matrix.

Consider the following structural representation of a VAR(q) model

$$B_0 z_t = B_1 z_{t-1} + B_2 z_{t-2} + \cdots + B_q z_{t-q} + \epsilon_t = B Z_{t-1} + \epsilon_t \quad (3.1)$$

where z_t is a vector of endogenous variables, $Z'_{t-1} = [z'_{t-1}, z'_{t-2} \cdots z'_{t-q}]$, $B = [B_1, B_2 \cdots, B_q]$ and ϵ_t is a vector of structural shocks. The reduced-form representation of the VAR(q) is given by:

$$z_t = A_1 z_{t-1} + A_2 z_{t-2} + \cdots + A_q z_{t-q} + u_t = A Z_{t-1} + u_t \quad (3.2)$$

where $A = [A_1, A_2 \cdots, A_q] = B_0^{-1} B$ and u_t is a vector of reduced-form shocks. The reduced-form VAR can be estimated by OLS. We can see that the structural shocks are linear combinations of the reduced form shocks since $u_t = B_0^{-1} \epsilon_t \Leftrightarrow \epsilon_t = B_0 u_t$. If we assume that the structural shocks are uncorrelated and have unit variance, such that $E(\epsilon_t \epsilon_t^T) = I$, using the reduced form covariance matrix, we can show that:

$$E(u_t u_t^T) = (B_0^{-1}) E(\epsilon_t \epsilon_t^T) (B_0^{-1})^T = (B_0^{-1}) (B_0^{-1})^T = \Sigma \quad (3.3)$$

Knowledge of the reduced form parameters from OLS estimation does not allow us to pin down a unique B_0^{-1} without any identifying restrictions, as there are many B_0^{-1} matrices that satisfy $(B_0^{-1})(B_0^{-1})^T = \Sigma$. In the case of a recursive identification or Cholesky decomposition, the matrix B_0^{-1} is lower triangular. If we assume that matrix V is a Cholesky lower triangular matrix that satisfies $VV^T = \Sigma$ where $V = B_0^{-1}$, then there exists an orthogonal matrix K such that $(B_0^{-1})(B_0^{-1})^T = VKK^T V^T = VV^T$.

To illustrate the methodology of Rubio-Ramírez et al. (2010) for exactly identified models with short and long-run restrictions, we use a stylized VAR model with three variables ($m = 3$). The VAR is specified as $[\Delta Y \ \pi \ R]$ where ΔY represents the growth

rate in output, π the growth rate in the CPI and R is the short-term nominal interest rate. All variables used in the VAR are assumed to be $I(0)$, that is, stationary. We assume that there are three shocks in the VAR: monetary policy, demand and supply. Since we have three variables, to exactly identify the VAR, we need to impose three zero restrictions. As an illustration, we assume that a monetary policy shock does not have a contemporaneous effect on output. Moreover, we assume that demand and monetary policy shocks have no effect on output in the long-run. These three restrictions exactly identify the model. In this case, the impact matrix B_0^{-1} is given by:

$$B_0^{-1} = \begin{array}{c} \Delta Y \\ \pi \\ R \end{array} \begin{array}{ccc} MP & D & S \\ \left[\begin{array}{ccc} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{array} \right] \end{array} \quad (3.4)$$

The long-run restrictions imply that the long-run matrix is given by:

$$\Theta_1 = A(1)^{-1}B_0^{-1} = \begin{array}{c} \Delta Y \\ \pi \\ R \end{array} \begin{array}{ccc} MP & D & S \\ \left[\begin{array}{ccc} 0 & 0 & \times \\ \times & \times & \times \\ \times & \times & \times \end{array} \right] \end{array} \quad (3.5)$$

where $A(1) = (I_3 - A) = (I_3 - A_1 - A_2 - A_3)$

The algorithm of Rubio-Ramírez et al. (2010) consists of constructing a draw for Θ_1 using matrix A and an initial draw from the short-run impact multiplier matrix $C = chol(\hat{\Sigma})$.⁵ They label the short-run impact matrix $L_0 = B_0^{-1}$ and the long-run matrix

⁵Our explanation of the algorithm follows closely the description contained in Rubio-Ramírez et al.

$L_\infty = \Theta_1 = A(1)^{-1}B_0^{-1}$. L_0 and L_∞ are both $m \times m$. Hence, we first have $L_0^* = C$ and $L_\infty^* = [I - A]^{-1} \times C$. The initial impulse response function consistent with the matrix C is then stacked into a single matrix denoted by $f(B, A)$ where

$$f(B, A) = L = \begin{bmatrix} L_0 \\ L_\infty \end{bmatrix} = \begin{array}{c|ccc} & MP & D & S \\ \hline \Delta Y & 0 & \times & \times \\ \pi & \times & \times & \times \\ R & \times & \times & \times \\ \hline \Delta Y & 0 & 0 & \times \\ \pi & \times & \times & \times \\ R & \times & \times & \times \end{array} \quad (3.6)$$

The L matrix therefore contains m structural shocks and $2m$ variables. Rubio-Ramírez et al. (2010) show that zero restrictions imposed on each structural shock can be represented in terms of matrix Q_j where the number of columns in Q_j is equal to the number of rows in L , that is $2m$. If the rank of Q_j is q_j , then q_j is also the number of zero restrictions associated with the j^{th} shock. They show that the zero restrictions imposed on each shock are satisfied if and only if

$$Q_j L e_j = 0 \quad (3.7)$$

for $1 \leq j \leq m$ and where e_j is the j^{th} column of I_m . Rubio-Ramírez et al. (2010) re-order the columns in L from highest rank to lowest rank for the corresponding Q_j matrix. Using our stylized VAR, the restrictions for the three shocks can be written as

(2010) and Binning (2013).

$$Q_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_3 = 0_{3 \times 6}$$

Q_1 contains the restriction that monetary policy shocks affect output with a lag in the short-run and have no effect on output in the long-run. Q_2 contains the restriction that demand shocks do not have any permanent effect on output. Q_3 contains no restrictions and is thus a matrix of zeros. It follows that the SVAR is exactly identified if and only if $q_j = m - j$ for $1 \leq j \leq m$. Once this is achieved, each $Q_j L$ for $j = m$ is calculated and a QR decomposition is performed on each. The last column of the orthogonal matrix obtained by applying the QR decomposition for each j is then used to construct a rotation matrix. The rotation matrix is multiplied by L_0 to find the structural impact multiplier matrix. In our example, for all three shocks to be identified, the rank condition for the first, second and third shock must respectively be $q_1 = 2$, $q_2 = 1$ and $q_3 = 0$. It is clear that our model is exactly identified.

3.3.2 Underidentified Models with Short-run, Long-run and Sign Restrictions

We use the same stylized VAR to extend the methodology to models that are underidentified. We assume this time the following restrictions

$$f(B, A) = L = \begin{bmatrix} L_0 \\ L_\infty \end{bmatrix} = \frac{\begin{array}{c} \Delta Y \\ \pi \\ R \\ \Delta Y \\ \pi \\ R \end{array} \begin{array}{c} MP \\ U \\ U \\ MP \\ U \\ U \end{array} \begin{bmatrix} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \\ 0 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix}}{\quad} \quad (3.8)$$

The only shock we identify is a monetary policy shock. The other two shocks are left undefined and we assume that monetary policy affects output with a lag and has no effect on output in the long-run. In this case the Q matrices will be given by:

$$Q_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_2 = Q_3 = 0_{3 \times 6}$$

where Q_1 includes the restriction that monetary policy does not affect output contemporaneously and monetary policy is neutral in the long-run. For the model to be exactly identified, the rank conditions for the first, second and third shock must respectively be $q_1 = 2$, $q_2 = 1$ and $q_3 = 0$. With the restrictions we imposed, the rank conditions for the first, second and third shocks are respectively 2,0,0. Hence the first shock is uniquely identified but the other two shocks are not, as for $j = 2, 3$, $q_j < m - j$. As a result, we have an under-identified model. The under-identified SVAR will generate a band of impulse response functions that are consistent with the reduced-form VAR. Binning (2013) applies sign restrictions with the zero restrictions to discard all the impulse response functions that are inconsistent with the restrictions imposed by the researcher.

In terms of algorithm, in the under-identified model, Binning (2013) shows that one of the key differences is that the initial short-run impact matrix used in L is randomized by post-multiplying the Cholesky matrix C by a rotation matrix calculated from Q . This is done as the model is under-identified and by doing so, L is now constructed from a set of models consistent with the reduced form VAR and the imposed zero restrictions. Once this is done, a second rotation matrix is constructed using the same procedure as in the fully identified VAR and an impulse response function is generated. If the impulse response function satisfies the sign restrictions imposed, the draw is kept, otherwise it is discarded. In many cases, the sign restrictions will come from a model. In our example, we could use a simple New Keynesian three equation model to generate the sign restrictions on the impulse responses of each of the variables. Since the VAR model would match exactly the simple New Keynesian model, this could be executed fairly easily.

3.4 VAR Model with Stock Prices

The baseline VAR that we employ in this paper is a six variable monthly VAR consisting of industrial production (IP), inflation, (π) commodity prices (Pcom), short-term interest rates (i), stock market index (sm) and an index measuring consumer confidence (conf). We use a stationary VAR and the yearly growth rate of each variable is taken when there is evidence of non-stationarity. We test for the optimal number of lags and several of the tests we employ for lag-selection suggest four lags. We identify only five shocks in the VAR: a demand (D), supply (S), monetary policy (MP), stock market (SM) and a confidence shock (C). The sixth shock is left unidentified (U). Since we have 6 variables, to exactly identify the VAR, we would require 15 zero restrictions. If we employ a recursive identification as many papers employ in this literature, this would involve making strong

assumptions regarding the timing of the response of each variable to the six shocks. In particular, we would be forced to make an assumption about the ordering of stock prices and interest rates and in doing so, explicitly assume whether or not they react with a lag or contemporaneously to each other. Many studies order stock prices after the interest rate variable, thereby assuming that stock prices react contemporaneously to a monetary policy shock but interest rate reacts with a lag to a shock in stock prices. We show later that this type of ordering leads to the right response for stock prices following a monetary policy shock but to the wrong response for interest rates following a stock market shock.

As we have argued in this paper, we take an agnostic view about how certain variables respond to shocks in the short-run and we use the following minimal restrictions in the baseline model:⁶

- (i) Monetary policy and demand shocks have no long-run effect on industrial production

With our set-up, we have the following L matrix:

⁶We are aware that being agnostic is no “free lunch” as it involves a larger set of models (possibly problematic) to be retained in the admissible set. This remains an unresolved question in the literature. For this reason, we provide some robustness tests on our identification strategy and we find that our results are robust.

$$L = \begin{bmatrix} L_0 \\ L_\infty \end{bmatrix} = \begin{array}{c} \Delta IP \\ \pi \\ \Delta Pcom \\ i \\ \Delta Stock\ market \\ Confidence \\ \Delta IP \\ \pi \\ \Delta Pcom \\ i \\ \Delta Stock\ market \\ Confidence \end{array} \begin{array}{c} D \quad MP \quad S \quad U \quad SM \quad C \\ \left[\begin{array}{cccccc} x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \\ 0 & 0 & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \end{array} \right] \end{array} \quad (3.9)$$

where the rows describe the variables and the columns the associated shocks. In this case the Q matrices will be given by:

$$Q_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Q_3 = Q_4 = Q_5 = Q_6 = 0_{6 \times 12}$$

where the matrices Q_1 and Q_2 respectively include the restrictions that demand and monetary policy shocks are neutral in the long-run. Clearly, with the imposed restrictions, the rank conditions are not satisfied and the model is underidentified. As we discussed earlier, for the model to be exactly identified, the rank condition for the six shocks must respectively be $q_1 = 5$, $q_2 = 4$, $q_3 = 3$, $q_4 = 2$, $q_5 = 1$ and $q_6 = 0$. With the restrictions we imposed, the rank condition for the six shocks are respectively 1,1,0,0,0,0. Consequently, the model is underidentified.

3.5 DSGE Model with Stock Prices

We use a simple DSGE model with monetary policy, stock prices and confidence shocks to impose restrictions on the sign of the IRF on impact in the SVAR. As the model is similar to Nisticó (2012) and other papers, except for the confidence shock, we do not provide a detailed account of the model. Nisticó (2012) extends the Blanchard-Yaari perpetual youth model with the inclusion of equities (see Yaari (1965) and Blanchard (1985)). The model shares many common features with the simple New Keynesian model and the linearized version is given by the following equations:

$$x_t = \frac{1}{1+\psi} E_t x_{t+1} + \frac{\psi}{1+\psi} s_t - \frac{1}{\sigma} [i_r - E_t \pi_{t+1} - r_t^n] \quad (3.10)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \epsilon_t^\pi \quad (3.11)$$

$$s_t = \beta E_t s_{t+1} + \lambda E_t x_{t+1} - [i_r - E_t \pi_{t+1} - r_t^n] + \epsilon_t^s \quad (3.12)$$

$$i_t = \rho i_{t-1} + (1-\rho) [r_t^n + \phi_\pi \pi_t + \phi_x x_t + \phi_s s_t] + \epsilon_t^{mp} \quad (3.13)$$

$$\epsilon_t^\pi = \gamma_\pi \epsilon_{t-1}^\pi + \eta_t^\pi \quad (3.14)$$

$$\epsilon_t^{mp} = \gamma_{mp} \epsilon_{t-1}^{mp} + \eta_t^{mp} \quad (3.15)$$

$$\epsilon_t^s = \gamma_s \epsilon_{t-1}^s + \text{conf}_{t-4} + \eta_t^s \quad (3.16)$$

Equation 3.10 is a forward-looking IS-type relation that relates the output-gap x_t to its expected future value, real stock prices s_t and the real short-term interest rate ($i_t - E_t \pi_{t+1}$). In the model, agents can allocate their portfolio between a riskless asset and equities issued by monopolistically competitive firms. Real stock prices affect the output-gap in this model through wealth-effects on private consumption. Equation 3.11 is the New Keynesian Phillips curve. Equation 3.12 describes the dynamics of real stock prices. Real stock prices are driven by its expected future value, the expected value of output in the future, the difference between the real rate and the neutral (or natural) interest rate and a stochastic shock. The stochastic shock driving stock prices is described in Equation 3.16. We assume that stock prices are driven by its own shock given by ϵ_t^s and another fundamental shock which we interpret as a confidence shock. Our confidence shock takes a similar form as news shocks in the literature (see Matsumoto et al. (2011), Schmitt-Grohe and Uribe (2012) and Fratzscher and Straub (2013) for example). These confidence shocks represent information about expectations of consumers and firms regarding the future fundamentals of the economy. Surveys of firm and consumer sentiment can provide information about

how optimistic or pessimistic consumers and firms are regarding future uncertainty which in turn can have implications for stock prices and real economic activity. Equation 3.13 depicts the reaction function of the central bank. The central bank is assumed to react to stock prices directly in addition to the output-gap and inflation. The rest of the model describes the behaviour of the shocks in the model.

The model is calibrated using similar values as Nisticó (2012). Figure C.1 in Appendix C presents the impulse response of the model following a shock to the short-term interest rate. As expected, the output-gap, inflation and stock market all fall following an increase in the short-term interest rate. Figures C.2 and C.3 show the impulse response function from the model following a stock market shock and a confidence shock respectively. A positive shock in the stock market leads to an increase in the output-gap, interest rates and inflation. In the model, the shock to the stock market creates a positive wealth effect that leads to an increase in output and firms who are able to change their prices take advantage of the increase in the demand for their goods by increasing prices. The nominal interest rate increases since the central bank in its Taylor rule reacts to the increase in these variables. Figure C.3 shows that an expected boost in confidence increases the output-gap, inflation and interest rates. The effect on the output-gap and the other variables is fairly large. The confidence shock as modelled is anticipated expectations about future fundamentals. The shock has an immediate effect on stock prices which in turn has a positive wealth effect on consumers and hence output. The increase in demand leads to an increase in prices as firms who are able to change prices in any given period do so to take advantage of the change. The shock from confidence is more persistent than the other shocks in the model.

We use the responses from our stylized model to impose the sign restrictions in our VAR. We should point out that our small DSGE model does not map directly into our VAR as we are employing a VAR model that contains more variables such as commodity

prices.⁷ In the VAR, we treat confidence as an endogenous variable rather than driven by completely exogenous factors. Moreover, the confidence shock in the VAR is also assumed to be unanticipated whereas in the theoretical model the confidence shock is anticipated. Nevertheless, the stylized model is still informative regarding the correlation between these variables and the signs they take on impact.

3.6 Underidentified VAR with Stock Prices

The L stacked matrix with our zero and sign restrictions is given by matrix 3.17. In addition to the zero restrictions in the long-run, we assume that monetary policy shocks have a positive impact on interest rates and a negative impact on the stock market, industrial production and inflation. All of these restrictions come from our small stylized model. We assume that a demand shock has a positive effect on industrial production and inflation while a supply shock has a positive effect on industrial production but a negative effect on inflation. Using the responses from the model, we assume that a positive shock from the stock market has a positive impact on industrial production, inflation, interest rates and the confidence index. All of these responses are consistent with the DSGE model. Finally, the sixth shock is left completely unidentified.

⁷In our robustness test, we employ a smaller VAR that maps more directly into the small theoretical model.

$$\begin{bmatrix} L_0 \\ L_\infty \end{bmatrix} = \begin{array}{c|cccccc} & D & MP & S & U & SM & C \\ \hline \Delta IP & + & - & + & x & + & + \\ \pi & + & - & - & x & + & x \\ \Delta Pcom & x & x & x & x & x & x \\ i & x & + & x & x & + & x \\ \Delta Stock\ market & x & - & x & x & + & + \\ Confidence & x & x & x & x & + & + \\ \hline \Delta IP & 0 & 0 & x & x & x & x \\ \pi & x & x & x & x & x & x \\ \Delta Pcom & x & x & x & x & x & x \\ i & x & x & x & x & x & x \\ \Delta Stock\ market & x & x & x & x & x & x \\ Confidence & x & x & x & x & x & x \end{array} \quad (3.17)$$

3.6.1 Data

We use monthly data from January 1980 to December 2017. For Canada, we use data on industrial production, total CPI, the overnight rate, the TSX/S&P stock index, the Bank of Canada non-energy commodity price index, and the OECD consumer confidence index. The OECD consumer confidence index for Canada surveys households about their current and expectations of future economic situation and their current and expected major purchases. For the U.S, we use the industrial production, total CPI, the federal funds rate, the S&P 500 stock index, the world bank index of non-energy commodity prices and the index of consumer sentiment from the University of Michigan Survey of Consumers. This survey contains approximately 50 questions that track consumer attitudes

and expectations about the economy. Consumer confidence has historically been a good predictor of consumer spending for the next three to six months.⁸ Each series is tested for stationarity and the year over year growth rate is taken when we needed to transform the data. In our VAR, only the interest rate and the confidence index are in levels whereas all other variables are employed as their rate of change.⁹ Figures C.4 and C.5 in Appendix C show the data utilised for both countries.

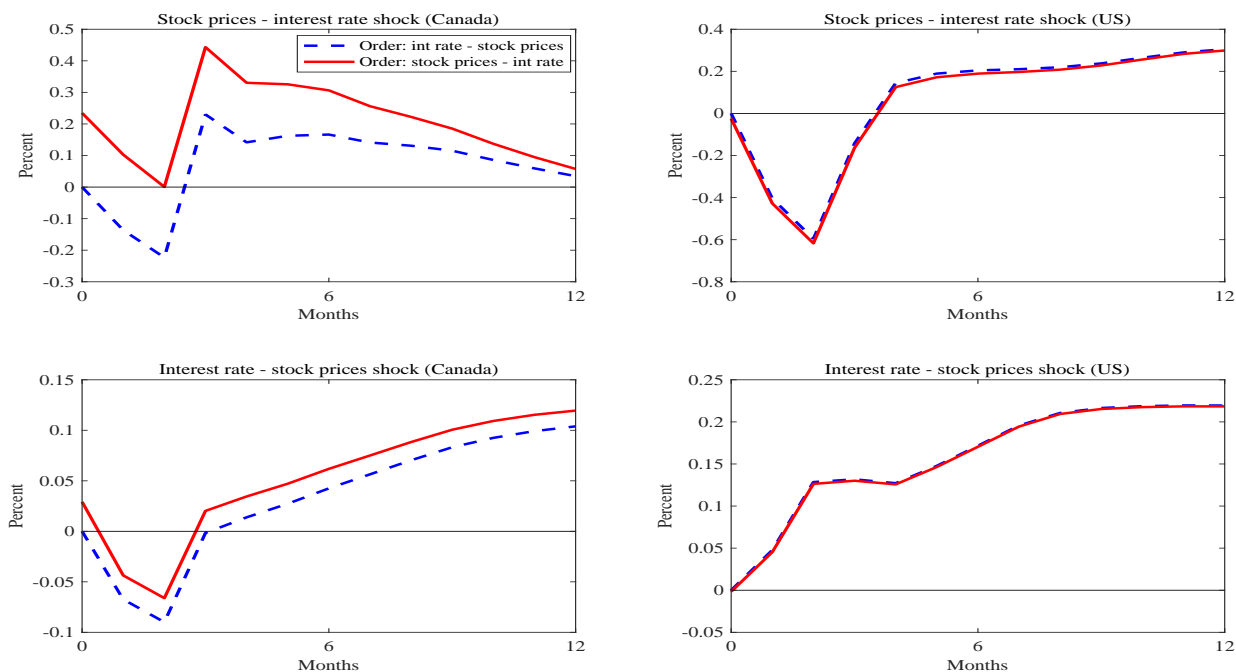
3.7 Results with Short-Run Identification

We first report the results for the VAR where we employ short-run restrictions to identify the monetary and stock market shocks. The results are shown in Figure 3.1 below and Figures C.6 — C.9 in Appendix C. As the first panel of Figure 3.1 shows, if we employ a short-run recursive identification scheme and we assume that stock prices react with a lag to a monetary policy shock, then a positive monetary policy shock leads to an increase in stock prices for Canada (the red solid line). The response of stock prices shows how the recursive assumption with this particular ordering can lead to a counterfactual response for stock prices. We obtain the expected response if the ordering is reversed. That is, stock prices fall following a positive monetary policy shock if we assume that monetary policy reacts with a lag to stock prices. This is the ordering used in the vast majority of papers that employ this type of identification scheme. However doing this leads to a counterfactual response for interest rate following a stock market shock. As shown in the third panel of Figure 3.1, we find that the policy rate in Canada falls following a shock to

⁸The University of Michigan Survey of Consumers is the most commonly used measure in the literature to assess confidence for the U.S. We also run the model with the OECD consumer confidence index for the U.S. and obtained fairly similar results to those reported.

⁹We also test our data for structural breaks. Our results were very similar whether a dummy was used to account for possible breaks at the beginning of the 1980s and for the 2007-2008 recession.

Figure 3.1: Response of Stock Prices and Interest Rates to Shocks in Canada and the U.S. using a Short-run Identification



Notes: Response of stock prices (to a positive interest rate shock) and interest rates (to a positive stock price shock) in Canada and the U.S. using a short-run identification.

stock prices (the blue dashed line). This result is at odds with many empirical findings and is also not supported by the DSGE model we employ in the paper. As shown in Figure C.6, the response of some macro variables is also incorrect following a monetary policy shock. Industrial production increases following a positive monetary policy shock when we assume that stock prices react with a lag to a monetary policy shock.

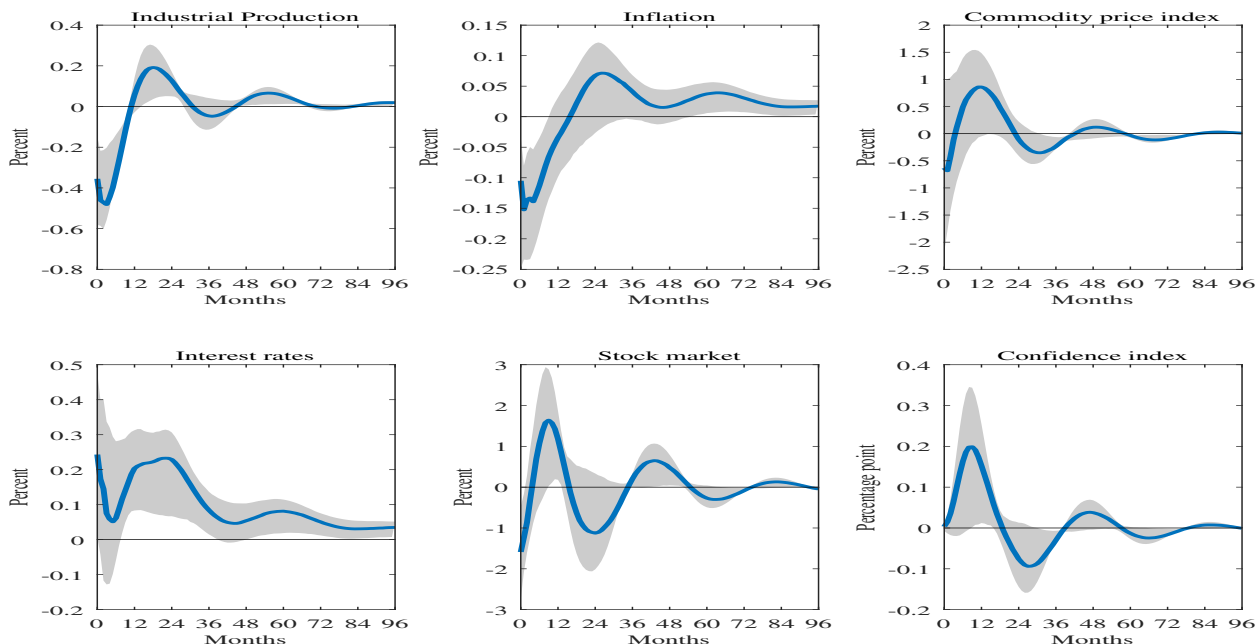
As shown in Figure 3.1, when U.S. data is employed, we find that stock prices fall following a monetary policy shock, while interest rates rise following a positive shock to stock prices. The results are robust to the ordering of stock prices and interest rates. However,

we obtain some counterfactual responses for industrial production and inflation following a positive monetary policy shock as shown in Figure C.8. They both increase following a rise in the Federal Funds rate. This counterfactual response vanishes if inflation and industrial production are allowed to react contemporaneously to a monetary policy shock as in our underidentified VAR. Thus monetary VARs that employ a recursive identification strategy in this context, can lead to counterfactual responses. For this reason and the fact that empirical evidence suggests that monetary policy and stock prices react to each other simultaneously, we argue that this type of identification procedure can be problematic when we specifically aim to study the interaction between stock prices and monetary policy.

3.8 Results with the Underidentified VAR

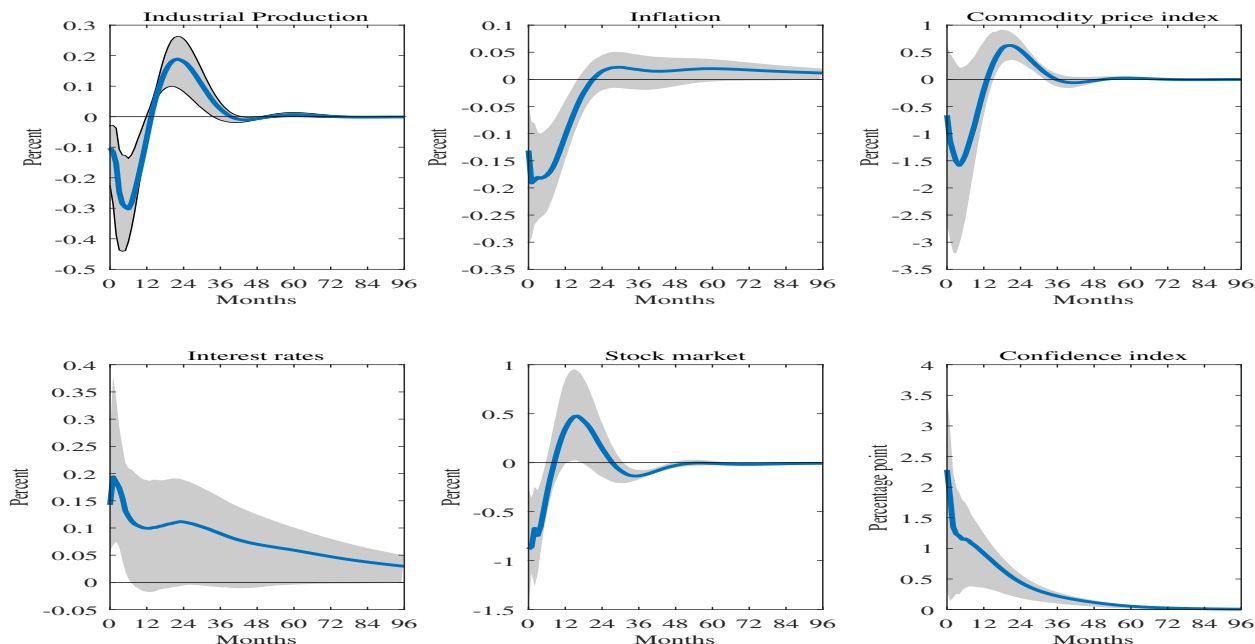
Results from the underidentified VAR are shown in Figures 3.2 - 3.6. We simulate the model 1000 times and report the vector of pointwise posterior median responses of the impulse response function from the VAR as well as the pointwise 68% posterior confidence bands. Results from the monetary policy shock for Canada and the U.S. are shown respectively in Figures 3.2 and 3.3. An increase in the interest rate leads to an immediate fall in industrial production, inflation, commodity prices and the stock market in both countries. The fall in stock prices following the increase in interest rates is consistent with the decrease in industrial production, as the higher cost of borrowing and the increase in the discount rate when combined, depresses stock prices. It takes approximately 10 months in the U.S. and 5 months in Canada for the stock market to increase above its initial value. As interest rate decline back to its initial level and industrial production starts to recover from its trough, these contribute to expectations of higher future profits and dividends for firms and hence, result in higher stock prices.

Figure 3.2: Response of SVAR Model to a Monetary Policy Shock in Canada



From Figures 3.2 and 3.3, we find that an increase of 25 basis points in the overnight rate leads to a fall of about 1.75% in stock prices in Canada, whereas a similar increase in the U.S. federal funds rate leads to a fall of about 1.25% in stock prices. Our results are in line with Bjørnland and Leitemo (2009), Rigobon and Sack (2004) and Bernanke and Kuttner (2005) who also find similar estimates for the U.S. Their estimates range between 0.75% and 1.25%. Our results however, are different from Li et al. (2010) for Canada. They find that stock prices fall by around 0.8% following a 25 basis points increase in the overnight rate. They also report a much smaller impact of monetary policy on stock prices in Canada compared to the U.S. Our results are likely different due to differences in identification strategies. Whereas they identified all the shocks in a 7 variable VAR, we decided not to impose such rigid assumptions on the causal relationship among the

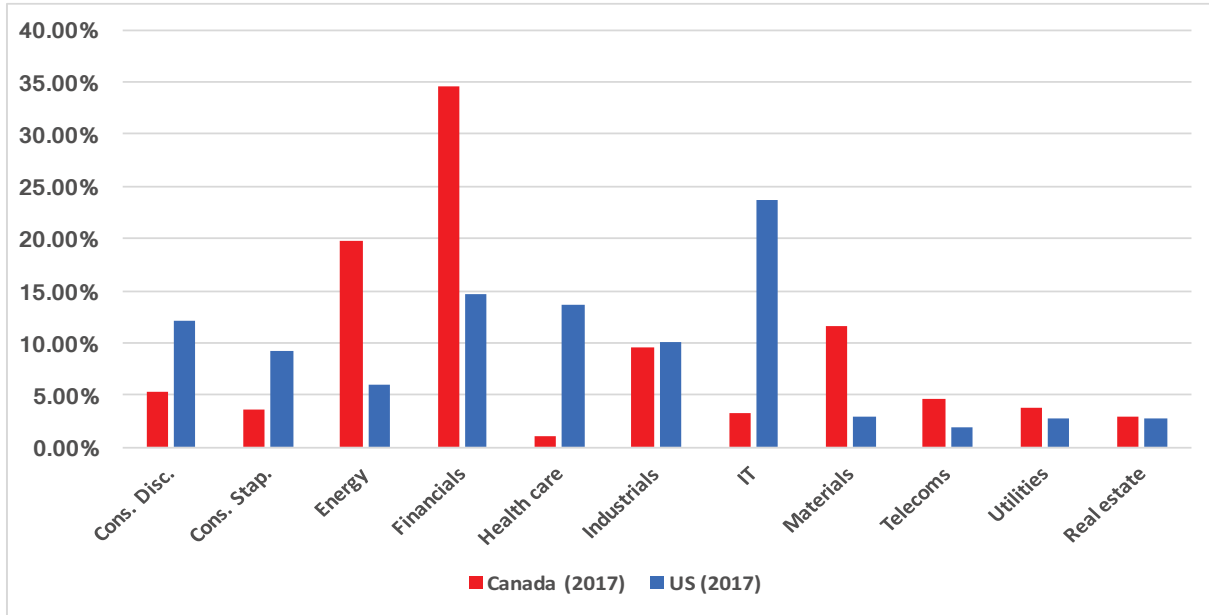
Figure 3.3: Response of SVAR model to a Monetary Policy Shock in the U.S.



variables in the model but instead let the data reveal these causal relationships. We believe our approach is more robust and less sensitive to ordering assumptions.

The impact of monetary policy shocks is larger in Canada compared to the U.S. for two reasons. Firstly, the TSX index has a heavy concentration of companies in the financial, energy, industrials and materials sectors compared to the U.S. stock index. In 2017, taken together, these sectors accounted for over 75% of the TSX index and only about 33% of the S&P index for the U.S. On the other hand, sectors that are less sensitive to interest rate changes such as telecoms, utilities, health care and consumer staples account for a much larger share of the U.S stock market (28%) compared to the Canadian stock market (13%). Figure 3.4 shows the share of each of the 11 sectors in Canada and in the U.S. in 2017.

Figure 3.4: TSX and S&P 500 Sector Weightings in 2017

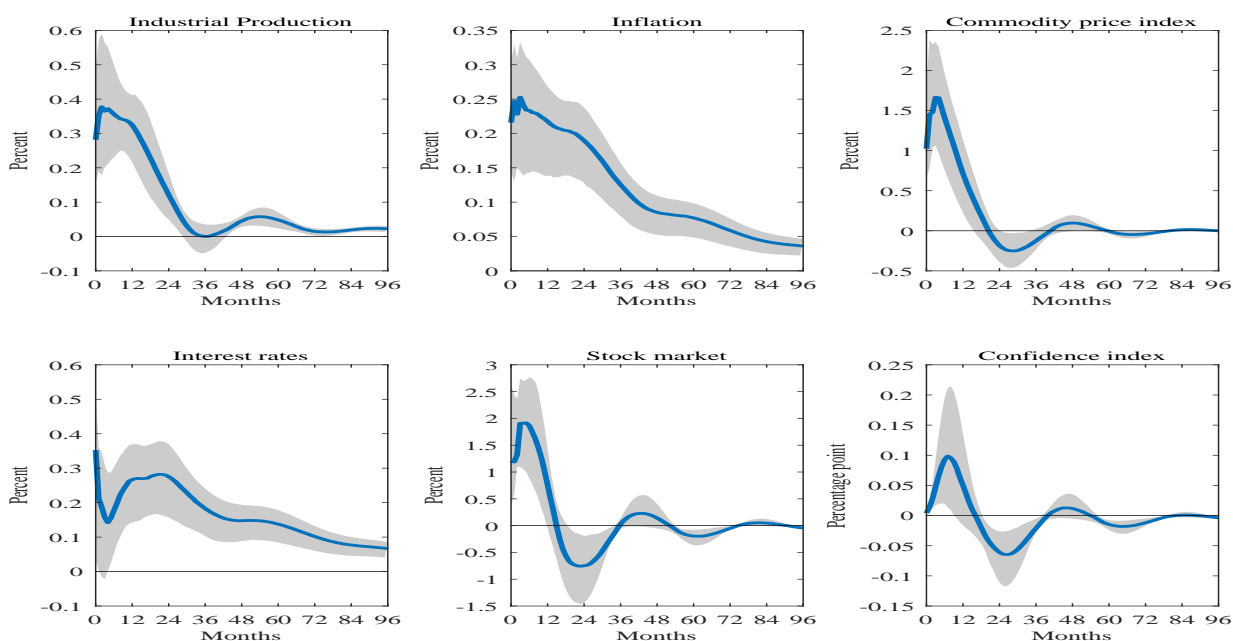


It is evident from the graph that the share of sectors that are more sensitive to interest rate changes such as financial, energy, materials and industrials are much larger in Canada than in the U.S. Since these sectors are more sensitive to changes in interest rates compared to other sectors such as consumer staples, utilities and telecoms which account for a much larger share of the index in the U.S, it is not surprising that monetary policy has a larger impact on stock prices in Canada compared to the U.S. Secondly, the stock market in Canada responds more to domestic credit conditions than in the U.S. This occurs as the percentage of sales to foreign countries is higher for companies in the U.S. compared to Canadian companies. As a result, Canadian companies are more dependent on demand and credit conditions at home than U.S. companies. According to Goldman Sachs, foreign sales accounted for 44% of total revenue for U.S. companies that were part of the S&P 500 in 2016.¹⁰

¹⁰We do not have a comparable figure for Canada, but since financial companies account for approx-

Figures 3.5 and 3.6 show the response of short-term interest rates and other variables to a positive stock market shock in Canada and the U.S. respectively.

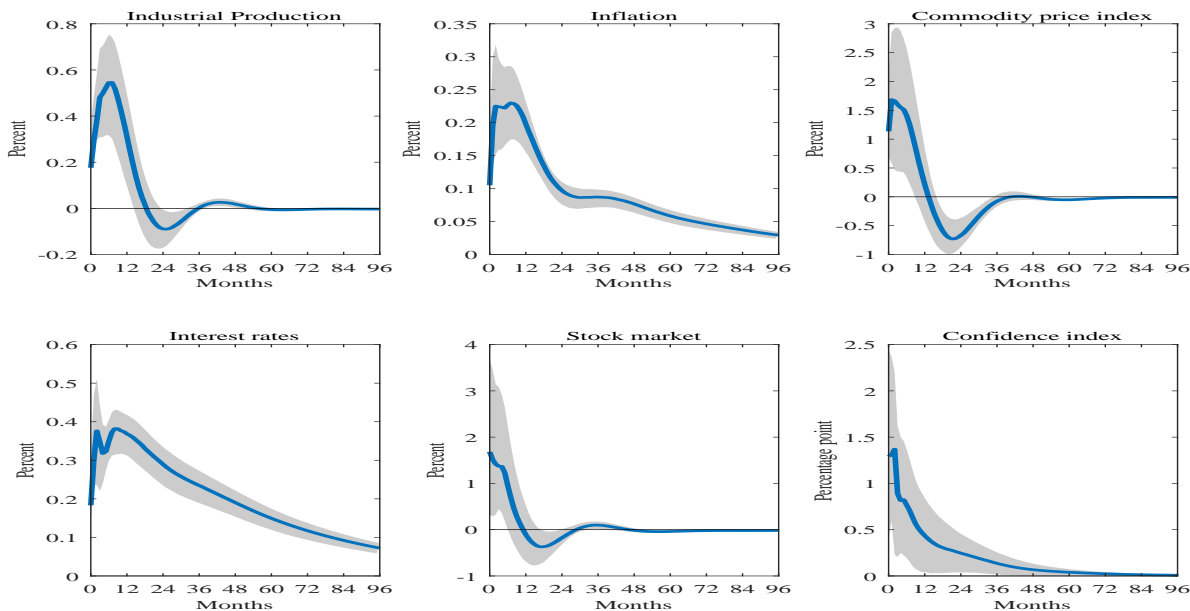
Figure 3.5: Response of SVAR Model to a Stock Market Shock in Canada



A positive stock market shock leads to an increase in industrial production, inflation, interest rates and commodity prices in both countries. As in the DSGE model, the increase in the stock market has a positive wealth effect on consumers and increases investment for firms. The consumer confidence index increases following the stock market shock in Canada and the U.S. We find that in Canada, a 1% increase in the stock market leads to an increase of approximately 27 basis points in the overnight rate whereas a similar increase leads to an approximately 10 basis points increase in the federal funds rate. Our results for the

imately 35% of the TSX index and as they derive a large share of their revenue domestically, it is very likely that foreign sales account for less of the total revenues of all of the companies listed on the TSX compared to the US.

Figure 3.6: Response of SVAR Model to a Stock Market Shock in the U.S.



U.S. are in the range of results found by other papers. For example, many of the papers cited previously have an estimated range from 2 to 10 basis points for the U.S. The Bank of Canada reacts more strongly to the stock market shock as the initial impact of the shock on inflation and industrial production is much more pronounced in Canada than in the U.S. In Canada, a 1% increase in stock prices lead to a 0.17% rise in inflation and to an increase of 0.24% in industrial production. In the U.S, for a similar increase in stock prices, inflation rose by only 0.05% and industrial production by 0.09%.

3.9 Robustness

To test the robustness of our results, we estimate the same VAR this time imposing the common assumption that industrial output and inflation react with a lag to a monetary

policy shock. All remaining identifying assumptions are the same as before. Our results are depicted in Figures C.10 – C.13 of Appendix C. When we impose these two additional short-run restrictions, we find that a 25 basis point increase in the policy rate leads to a fall of 1.93% in Canadian stock prices and to a fall of 1.45% for the U.S. These results are very similar to the ones we obtain when we employed the baseline VAR, and in fact are within the 68% confidence interval of the baseline model. Note that with these minimal number of short-run restrictions, as opposed to employing a fully identified recursive scheme, we obtain responses for industrial production and inflation for the U.S. which are in line with real world observations and economic theory. They both fall following an increase in the policy rate. On the other hand, when Canadian data is employed, although industrial production falls following a policy rate increase, we obtain a price puzzle, which we did not have in the baseline model. For the stock market shock, we find that a 1% increase in the stock markets leads to an approximate 35 basis point change in the policy rate in Canada and to an approximate 8 basis point change in the policy rate in the U.S. These results are again very similar to the results we obtain in the baseline VAR.

We also perform another robustness check by using a smaller VAR with only 4 variables – industrial production, inflation, the interest rate and a measure of stock prices. We use the same identification as in the baseline case, that is we impose only the restrictions that demand and monetary policy shocks are neutral in the long-run. Our results are shown in Figures C.14 – C.17 of Appendix C. Using the 4-variable VAR, we find that an increase of 25 basis points in the overnight rate leads to a fall of about 0.6% in stock prices in Canada, whereas a similar increase in the U.S. federal funds rate leads to a fall of about 1% in stock prices. The response of the stock market in Canada is smaller than in the U.S. when this model is employed and the median responses for both countries are smaller than those obtained in the baseline model. Regarding the shock to stock prices, using the

4-variable VAR, we find that in Canada, a 1% increase in the stock market leads to an increase of about 15 basis points in the overnight rate, whereas a similar increase leads to about a 7 basis points increase in the federal funds rate. Again these results though similar in direction to those obtained in the baseline VAR are all lower in magnitude.¹¹

3.10 Conclusion

We investigate the interconnectedness between stock prices and monetary policy using an underidentified SVAR for Canada and the U.S. We allow stock market and monetary policy to simultaneously react to each other through a combination of short-run, long-run and sign restrictions. We also impose a minimal set of restrictions on the rest of the model, and thereby allow the data to reveal the relationship among the different variables in the VAR. Our results for the U.S are in line with many previous studies. Our study provides new insights on the relationship between stock prices and monetary policy in Canada. We find that stock prices react more strongly to monetary policy shocks in Canada compared to the U.S. and argue that this is mainly due to the composition of the TSX index compared to the S&P 500. Sectors such as financial and energy are fairly sensitive to movements in interest rates and these sectors account for approximately 60% of the index in Canada whereas they account for a much smaller share in the U.S. (see Figure 3.4). We also show that using a fully identified recursive strategy in the context of a monetary VAR, can lead to counterfactual results for some macro variables. We do not obtain any such counterfactual results when we employ our underidentified VAR.

¹¹We also employ the Dow Jones Industrial Average and the S&P/TSX 60 for Canada to test for the robustness of our results. We find very similar results when these stock indices are employed.

References

- Adolfson, M., Laseen, S., Linde, J., and Villani, M. (2007). “Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through”. *Journal of International Economics* 72.2, pp. 481–511.
- Airaudo, M., Nisticó, S., and Zanna, L. F. (2007). *Learning, Monetary Policy and Asset Prices*. LLEE Working Paper No. 48. LUISS Guido Carli.
- Ambler, S., Dib, A., and Rebei, N. (2004). *Optimal Taylor Rules in an Estimated Model of a Small Open Economy*. Working Paper No. 36. Bank of Canada.
- Backus, D. K., Kehoe, P. J., and Kydland, F. E. (1995). *International Business Cycles: Theory and Evidence*. Ed. by e. Cooley Thomas F. Princeton: Princeton University Press, pp. 26–356.
- Bashar, O., Wadud, I, and Ali Ahmed, H. (2013). “Oil Price Uncertainty, Monetary Policy and the Macroeconomy: The Canadian Perspective”. *Economic Modelling* 35, pp. 249–259.
- Baumeister, C. and Benati, L. (2013). “Unconventional Monetary Policy and the Great Recession: Estimating the Macroeconomic Effects of a Spread Compression at the Zero Lower Bound”. *International Journal of Central Banking* 9.2, pp. 165–212.
- Baumeister, C. and Kilian, L. (2014). “Do oil price increases cause higher food prices?” *Economic Policy* 29.80, pp. 691–747.

- Baumeister, C. and Kilian, L. (2016). “Forty Years of Oil Price Fluctuations: Why the Price of Oil May Still Surprise Us”. *Journal of Economic Perspectives* 30.1, pp. 139–60.
- Baumeister, C. and Peersman, G. (2013). “Time-Varying Effects of Oil Supply Shocks on the US Economy”. *American Economic Journal: Macroeconomics* 5.4, pp. 1–28.
- Baumeister, C., Peersman, G., and Robays, I. V. (2010). “The Economic Consequences of Oil Shocks: Differences across Countries and Time”. In: *Inflation in an Era of Relative Price Shocks*. Ed. by R. Fry, C. Jones, and C. Kent. RBA Annual Conference Volume. Reserve Bank of Australia.
- Bernanke, B. S., Boivin, J., and Elias, P. (2005). “Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach”. *Quarterly Journal of Economics* 120.1, pp. 387–422.
- Bernanke, B. and Kuttner, K. (2005). “What Explains the Stock Market’s Reaction to Federal Reserve Policy?” *Journal of Finance* 60.3, pp. 1221–1257.
- Bhuiyan, R. (2012). “Monetary Transmission Mechanisms in a Small Open Economy: A Bayesian Structural VAR Approach”. *Canadian Journal of Economics* 45.3, pp. 1037–1061.
- Binning, A. (2013). *Underidentified SVAR models: A Framework for Combining Short and Long-Run Restrictions With Sign-Restrictions*. Working Paper No. 2013-14. Norges Bank.
- BIS (2008). *Central Bank Operations in Response to the Financial Turmoil*. CGFS Papers, No. 31. Bank for International Settlements.
- Bjørnland, H. C. and Leitimo, K. (2009). “Identifying the Interdependence between US Monetary Policy and the Stock Market”. *Journal of Monetary Economics* 56.2, pp. 275–282.

- Blanchard, O. (1985). “Debt, Deficits, and Finite Horizons”. *Journal of Political Economy* 93.2, pp. 223–47.
- Boivin, J. and Giannoni, M. (2008). *Global Forces and Monetary Policy Effectiveness*. NBER Working Paper Series:13736. Cambridge, MA: National Bureau of Economic Research.
- Bordo, M. D. and Jeanne, O. (2002). “Monetary Policy and Asset Prices: Does ‘Benign Neglect’ Make Sense?” *International Finance* 5.2, pp. 139–164.
- Bouakez, H., Cardia, E., and Ruge-Murcia, F. J. (2009). “The Transmission Of Monetary Policy In A Multisector Economy”. *International Economic Review* 50.4, pp. 1243–1266.
- Burbidge, J. and Harrison, A. (1984). “Testing for the Effects of Oil-Price Rises Using Vector Autoregressions”. *International Economic Review* 25.2, pp. 459–84.
- Calvo, G. A. (1983). “Staggered Prices in a Utility-Maximizing Framework”. *Journal of Monetary Economics* 12.3, pp. 383–398.
- Cashin, P., Cespedes, L. F., and Sahay, R. (2004). “Commodity Currencies and the Real Exchange Rate”. *Journal of Development Economics* 75.1, pp. 239–268.
- Castelnuovo, E. and Nisticò, S. (2010). “Stock Market Conditions and Monetary Policy in a DSGE Model for the U.S.” *Journal of Economic Dynamics and Control* 34.9, pp. 1700–1731.
- Challe, E. and Giannitsarou, C. (2014). “Stock Prices and Monetary Policy Shocks: A General Equilibrium Approach”. *Journal of Economic Dynamics and Control* 40, pp. 46–66.
- Charnavoki, V. and Dolado, J. (2014). “The Effects of Global Shocks on Small Commodity-Exporting Economies: Lessons from Canada”. *American Economic Journal: Macroeconomics* 6.2, pp. 207–237.

- Chen, Y. and Rogoff, K. (2003). “Commodity Currencies”. *Journal of International Economics* 60.1, pp. 133–160.
- Cushman, D. O. and Zha, T. (1997). “Identifying Monetary Policy in a Small Open Economy under Flexible Exchange Rates”. *Journal of Monetary Economics* 39.3, pp. 433–448.
- Darby, M. R. (1982). “The Price of Oil and World Inflation and Recession”. *American Economic Review* 72.4, pp. 738–751.
- DePratto, B., Resende, C. de, and Maier, P. (2009). “How Changes in Oil Prices Affect the Macroeconomy”. *Bank of Canada Working Paper, No. 33*.
- Devereux, M. B. and Engel, C. (2003). “Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility”. *Review of Economic Studies* 70.4, pp. 765–783.
- Dib, A. (2003). “An Estimated Canadian DSGE Model with Nominal and Real Rigidities”. *Canadian Journal of Economics* 36.4, pp. 949–972.
- Dib, A. (2008). *Welfare Effects of Commodity Price and Exchange Rate Volatilities in a Multi-Sector Small Open Economy Model*. Working Paper No. 8. Ottawa, Canada: Bank of Canada.
- Dib, A. (2011). “Monetary Policy in Estimated Models of Small Open and Closed Economies”. *Open Economies Review* 22.5, pp. 769–796.
- Dib, A., Mendicino, C., and Zhang, Y. (2008). *Price Level Targeting in a Small Open Economy with Financial Frictions: Welfare Analysis*.
- Dissou, Y. (2007). *Energy Prices and Real Economic Activity in Canada: A Multi-Sector Dynamic General Equilibrium Analysis*. Working Papers, No. 0707E. University of Ottawa, Department of Economics.

- Dorich, J., Johnston, M., Mendes, R., Murchison, S., and Zhang, Y. (2013). *ToTEM II: An Updated Version of the Bank of Canada's Quarterly Projection Model*. Technical Report, No. 100. Bank of Canada.
- Du, L., Yanan, H., and Wei, C. (2010). "The Relationship between Oil Price Shocks and China's Macro-economy: An Empirical Analysis". *Energy Policy* 38.8, pp. 4142–4151.
- Edison, H. J. and Fisher, E. O. (1991). "A Long-run View of the European Monetary System". *Journal of International Money and Finance* 10.1, pp. 53–70.
- Eltony, M. N. and Al-Awadi, M. (2001). "Oil Price Fluctuations and their Impact on the Macroeconomic Variables of Kuwait: A Case Study using a VAR Model". *International Journal of Energy Research* 25.11, pp. 939–959.
- Engel, C. (2000). "Local-Currency Pricing and the Choice of Exchange-Rate Regime". *European Economic Review* 44.8, pp. 1449–1472.
- Faruqee, H., Lusinyan, L., and Pescatori, A. (2015). *Commodity Slump Could Finally Bring Balance to Canadas Economy*. Online publication. The Globe and Mail.
- Farzanegan, M. R. and Markwardt, G. (2009). "The Effects of Oil Price Shocks on the Iranian Economy". *Energy Economics* 31.1, pp. 134–151.
- Faust, J. (1998). "The robustness of identified VAR conclusions about money". *Carnegie-Rochester Conference Series on Public Policy* 49.1, pp. 207–244.
- Fernandez-Villaverde, J., Rubio-Ramirez, J., Sargent, T. J., and Watson, M. W. (2007). "ABCs (and Ds) of Understanding VARs". *American Economic Review* 97.3, pp. 1021–1026.
- Fornero, J. and Kirchner, M. (2014). *Learning About Commodity Cycles and Saving- Investment Dynamics in a Commodity-Exporting Economy*. Working Papers Central Bank of Chile, No. 727.

- Fornero, J., Kirchner, M., and Yany, A. (2014). *Terms of Trade Shocks and Investment in Commodity-Exporting Economies*. Working Papers Central Bank of Chile. 18th Annual Conference of the Central Bank of Chile, Commodity Prices and Macroeconomic Policy.
- Forni, M., Giannone, D., Lippi, M., and Reichlin, L. (2009). “Opening the Black Box: Structural Factor Models with Large Cross Sections”. *Econometric Theory* 25.5, pp. 1319–1347.
- Fratzscher, M. and Straub, R. (2013). “Asset Prices, News Shocks, and the Trade Balance”. *Journal of Money, Credit, and Banking* 45.7, pp. 1211–1251.
- Fratzscher, M., Duca, M. L., and Straub, R. (2016). “ECB Unconventional Monetary Policy: Market Impact and International Spillovers”. *IMF Economic Review* 64.1, pp. 36–74.
- Fukunaga, I., Hirakata, N., and Sudo, N. (2010). *The Effects of Oil Price Changes on the Industry-Level Production and Prices in the U.S. and Japan*. NBER Working Paper Series. Cambridge, MA: National Bureau of Economic Research.
- Gali, J. (1999). “Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?” *American Economic Review* 89.1, pp. 249–271.
- Gali, J. (2008). *Monetary Policy, Inflation, and the Business Cycle :An Introduction to the New Keynesian Framework and Its Applications*. Princeton, N.J.: Princeton University Press.
- Gambacorta, L., Hofmann, B., and Peersman, G. (2014). “The Effectiveness of Unconventional Monetary Policy at the Zero Lower Bound: A Cross-Country Analysis”. *Journal of Money, Credit, and Banking* 46.4, pp. 615–642.
- Gilchrist, S., Yankov, V., and Zakrajsek, E. (2009). “Credit Market Shocks and Economic Fluctuations: Evidence from Corporate Bond and Stock Markets”. *Journal of Monetary Economics* 56.4, pp. 471–493.

- Gisser, M. and Goodwin, T. H. (1986). “Crude Oil and the Macroeconomy: Tests of Some Popular Notions: A Note”. *Journal of Money, Credit and Banking* 18.1, pp. 95–103.
- Greenwood, J., Hercowitz, Z., and Krusell, P. (2000). “The role of investment-specific technological change in the business cycle”. *European Economic Review* 44.1, pp. 91–115.
- Guidi, F. (2010). “The Economic Effects of Oil Prices Shocks on the UK Manufacturing and Services Sectors”. *The IUP Journal of Applied Economics* 0.4, pp. 5–34.
- Gurkaynak, R. S., Sack, B., and Swanson, E. T. (2005). “Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements”. *International Journal of Central Banking* 1.1, pp. 55–93.
- Hamilton, J. D. and Wu, J. C. (2012). “The Effectiveness of Alternative Monetary Policy Tools in a Zero Lower Bound Environment”. *Journal of Money, Credit, and Banking* 44, p. 3.
- Hamilton, J. (1983). “Oil and the Macroeconomy since World War II”. *The Journal of Political Economy* 91, 228248.
- Hooker, M. A. (1996). “What happened to the oil price-macroeconomy relationship?” *Journal of Monetary Economics* 38.2, pp. 195–213.
- Huang, B.-N., Hwang, M. J., and Peng, H.-P. (2005). “The Asymmetry of the Impact of Oil Price Shocks on Economic Activities: An Application of the Multivariate Threshold Model”. *Energy Economics* 27.3, pp. 455–476.
- Ioannidis, C. and Kontonikas, A. (2006). *Monetary Policy and the Stock Market: Some International Evidence*. Departmental Discussion Papers. University of Glasgow.
- Iwayemi, A. and Fowowe, B. (2011). “Impact of Oil Price Shocks on Selected Macroeconomic Variables in Nigeria”. *Energy Policy* 39.2, pp. 603–612.

- Jaaskela, J. P. and Nimark, K. (2011). “A Medium-Scale New Keynesian Open Economy Model of Australia”. *Economic Record* 87.276, pp. 11–36.
- Jimenez-Rodriguez, R. (2008). “The Impact of Oil Price Shocks: Evidence from the Industries of Six OECD Countries”. *Energy Economics* 30.6, pp. 3095–3108.
- Jimenez-Rodriguez, R. and Sanchez, M. (2005). “Oil Price Shocks and Real GDP Growth: Empirical Evidence for Some OECD Countries”. *Applied Economics* 37.2, pp. 201–228.
- Jo, S. (2012). “The Effects of Oil Price Uncertainty on the Macroeconomy”. *Bank of Canada Working Paper*.
- Johnson, D. R. and Schembri, L. L. (1990). “Atheoretical and Structural Approaches to Modelling Canadian-American Macroeconomic Interaction”. *Empirical Economics* 15.4, pp. 367–382.
- Justiniano, A. and Preston, B. (2010). “Monetary Policy and Uncertainty in an Empirical Small Open-Economy Model”. *Journal of Applied Econometrics* 25.1, pp. 93–128.
- Kilian, L. (2008a). “A Comparison of the Effects of Exogenous Oil Supply Shocks on Output and Inflation in the G7 Countries”. *Journal of the European Economic Association* 6.1, pp. 78–121.
- Kilian, L. (2008b). “The Economic Effects of Energy Price Shocks”. *Journal of Economic Literature* 46.4, pp. 871–909.
- Kilian, L. (2009). “Not All Oil Price Shocks Are Alike: Disentangling Demand and Supply Shocks in the Crude Oil Market”. *American Economic Review* 99.3, pp. 1053–69.
- Kilian, L. (2013). “Structural vector autoregressions”. In: *Handbook of Research Methods and Applications in Empirical Macroeconomics*. Ed. by N. Hashimzade and M. A. Thornton. Cheltenham, UK: Edward Elgar Publishing, Inc. Chap. chapter 22.
- Kilian, L. (2014). “Oil Price Shocks: Causes and Consequences”. *Annual Review of Resource Economics* 6.1, pp. 133–154.

- Kilian, L. and Barsky, R. (2004). "Oil and the Macroeconomy Since the 1970s". *Journal of Economic Perspectives* 18.4, pp. 115–134.
- Kirby, J. (2014). *What a Commodity Bust Would Mean for Canada's Economy*. Online publication. Macleans. (Visited on 08/22/2016).
- Knetter, M. M. (1995). "Pricing to Market in Response to Unobservable and Observable Shocks". *International Economic Journal* 9.2, pp. 1–25.
- Knop, S. and Vespignani, J. (2014). *The Sectorial Impact of Commodity Price Shocks in Australia*. Discussion Paper Series. Australia: University of Tasmania, Tasmanian School of Business and Economics.
- Korhonen, I. and Ledyeva, S. (2010). "Trade Linkages and Macroeconomic Effects of the Price of Oil". *Energy Economics* 32.4, pp. 848–856.
- Kremmidas, T. (2011). *The Changing Effects of Energy Price Shocks on Economic Activity and Inflation*. Economic Policy Series. The Canadian Chamber of Commerce.
- Krishnamurthy, A. and Vissing-Jorgensen, A. (2011). "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy". *Brookings Papers on Economic Activity*, pp. 215–287.
- Lam, J.-P. and Tkacz, G. (2004). "Estimating Policy-Neutral Interest Rates for Canada Using a Dynamic Stochastic General Equilibrium Framework". *Swiss Journal of Economics and Statistics* 140.1, pp. 89–126.
- Lee, K. and Ni, S. (2002). "On the Dynamic Effects of Oil Price Shocks: A Study Using Industry Level Data". *Journal of Monetary Economics* 49.4, pp. 823–852.
- Lewis, L. T. and Kilian, L. (2011). "Does the Fed Respond to Oil Price Shocks?" *Economic Journal* 121.555, pp. 1047–1072.

- Li, Y. D., Iscan, T. B., and Xu, K. (2010). “The Impact of Monetary Policy Shocks on Stock Prices: Evidence from Canada and the United States”. *Journal of International Money and Finance* 29.5, pp. 876–896.
- Macklem, T., Osakwe, P., Pioro, H., and Schembri, L. (2000). *The Economic Consequences of Alternative Exchange Rate and Monetary Policy Regimes in Canada*. in Proceedings of a conference held by the Bank of Canada. Bank of Canada, November 2000.
- Malakhovskaya, O. and Minabutdinov, A. (2014). “Are Commodity Price Shocks Important? A Bayesian Estimation of a DSGE Model for Russia”. *International Journal of Computational Economics and Econometrics* 4.1-2, pp. 148–180.
- Marston, R. C. (1990). “Pricing to Market in Japanese Manufacturing”. *Journal of International Economics* 29.3-4, pp. 217–236.
- Matsumoto, A., Cova, P., Pisani, M., and Rebucci, A. (2011). “News Shocks and Asset Price Volatility in General Equilibrium”. *Journal of Economic Dynamics and Control* 35.12, pp. 2132–2149.
- McKenzie, K. and Carbone, J. (2016). “Going Dutch? The Impact of Falling Oil Prices on the Canadian Economy”. *Canadian Public Policy* 42.2, 168180.
- Medina, J. P. and Soto, C. (2007). *The Chilean Business Cycles Through the Lens of a Stochastic General Equilibrium Model*. Working Papers Central Bank of Chile. Central Bank of Chile.
- Meese, R. A. and Rogoff, K. (1988). “Was It Real? The Exchange Rate-Interest Differential Relation over the Modern Floating-Rate Period”. *Journal of Finance* 43.4, pp. 933–948.
- Mendoza, E. G. (1995). “The Terms of Trade, the Real Exchange Rate, and Economic Fluctuations”. *International Economic Review* 36.1, pp. 101–137.
- Mumtaz, H. and Surico, P. (2009). “The Transmission of International Shocks: A Factor-Augmented VAR Approach”. *Journal of Money, Credit, and Banking* 41, pp. 71–100.

- Murchison, S. and Rennison, A. (2006). *ToTEM: The Bank of Canada's New Quarterly Projection Model*. Technical Report, No. 97. Bank of Canada.
- Murchison, S., Rennison, A., and Zhu, Z. (2004). *A Structural Small Open-Economy Model for Canada*.
- Neely, C. J. (2015). "Unconventional Monetary Policy Had Large International Effects". *Journal of Banking and Finance* 52, pp. 101–111.
- Neri, S. (2004). *Monetary Policy and Stock Prices: Theory and Evidence*. Economic Working Papers No. 513. Banca Italia.
- Nisticó, S. (2012). "Monetary Policy and Stock-Price Dynamics in a DSGE Framework". *Journal of Macroeconomics* 34.1, pp. 126–146.
- Olomola, P. and Adejumo, A. (2006). "Oil Price Shock and Macroeconomic Activity in Nigeria". *International Research Journal of Finance and Economics* 3, pp. 28–34.
- Ortega, E. and Rebei, N. (2006). *The Welfare Implications of Inflation versus Price-Level Targeting in a Two-Sector, Small Open Economy*. Working Paper No. 12. Bank of Canada.
- Osberg, L., Sharpe, A., and Thomas, J. (2016). "How Much do Changing Terms of Trade matter for Economic Well-Being in Canada?"
- Patelis, A. D. (1997). "Stock Return Predictability and the Role of Monetary Policy". *Journal of Finance* 52.5, pp. 1951–1972.
- Poloz, S. (2014). *Canadas Hot - and Not - Economy*. Speech. Saskatoon, Saskatchewan: Saskatchewan Trade and Export Partnership. URL: <http://www.bankofcanada.ca/2014/04/canadas-hot-not-economy/> (visited on 08/22/2016).
- Poloz, S. (2015). *Riding the Commodity Cycle: Resources and the Canadian Economy*. Speech. Calgary: Calgary Economic Development. (Visited on 08/22/2016).

- Raguindin, C and Reyes, R (2005). *The Effects of Oil Price Shocks on the Philippine Economy: A VAR Approach*. Working Paper. Philippines: University of the Philippines School of Economics.
- Rapach, D. E. (2001). “Macro Shocks and Real Stock Prices”. *Journal of Economics and Business* 53.1, pp. 5–26.
- Rees, D. M., Smith, P., and Hall, J. (2016). “A Multi-sector Model of the Australian Economy”. *Economic Record* 92.298, pp. 374–408.
- Rigobon, R. and Sack, B. (2003). “Measuring The Reaction of Monetary Policy to the Stock Market”. *The Quarterly Journal of Economics* 118.2, pp. 639–669.
- Rigobon, R. and Sack, B. (2004). “The Impact of Monetary Policy on Asset Prices”. *Journal of Monetary Economics* 51.8, pp. 1553–1575.
- Rogoff, K. (1996). “The Purchasing Power Parity Puzzle”. *Journal of Economic Literature* 34.2, pp. 647–668.
- Rotemberg, J. J. (1982). “Monopolistic Price Adjustment and Aggregate Output”. *Review of Economic Studies* 49.4, pp. 517–531.
- Rotemberg, J. J. and Woodford, M. (1996). “Imperfect Competition and the Effects of Energy Price Increases on Economic Activity”. *Journal of Money, Credit, and Banking* 28.4, pp. 550–577.
- Rubin, J. (2015). *Will a Smaller Global Commodities Appetite Benefit Canada?* Online publication. The Globe and Mail.
- Rubio-Ramírez, J. F., Waggoner, D. F., and Zha, T. (2010). “Structural Vector Autoregressions: Theory of Identification and Algorithms for Inference”. *The Review of Economic Studies* 77.2, pp. 665–696.
- Sarno, L. and Taylor, M. P. (2002). “Purchasing Power Parity and the Real Exchange Rate”. *IMF Staff Papers* 49.1, pp. 65–105.

- Schaufele, B. (2016). *The Impact of Oil Price Shocks on Canadian National and Provincial GDP*. Policy Brief. Ivey Energy, Policy, and Management Centre: Western University.
- Schmidt, T. and Zimmermann, T. (2007). *Why are the Effects of Recent Oil Price Shocks so Small?* Ruhr Economic Papers.
- Schmitt-Grohe, S. (1998). “The International Transmission of Economic Fluctuations: Effects of U.S. Business Cycles on the Canadian Economy”. *Journal of International Economics* 44.2, pp. 257–287.
- Schmitt-Grohe, S. and Uribe, M. (2012). “What’s News in Business Cycles”. *Econometrica* 80.6, pp. 2733–2764.
- Silva, M. E. A. da (2011). “Commodity Price Shocks and Business Cycles in Emerging Economies”. Universidade Federal de Pernambuco, Brazil.
- Sims, C. (1987). “Vector Autoregressions and Reality: Comment”. *Journal of Business and Economic Statistics* 5.4, pp. 443–449.
- Sims, C. A. and Zha, T. (1999). “Error Bands for Impulse Responses”. *Econometrica* 67.5, pp. 1113–1155.
- Smets, F. and Wouters, R. (2003). “An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area”. *Journal of the European Economic Association* 1.5, pp. 1123–1175.
- Souki, K. (2008). “Assessing the Effects of U.S. Shocks on the Canadian Economy Using Alternative Identification Methods”. *North American Journal of Economics and Finance* 19.2, pp. 193–213.
- Stock, J. and Watson, M. (2005). “Implications of Dynamic Factor Models for VAR Analysis”. *NBER Working Paper Series* w11467.
- Stock, J. H. and Watson, M. W. (2012). “Disentangling the Channels of the 2007-09 Recession”. *Brookings Papers on Economic Activity*, pp. 81–156.

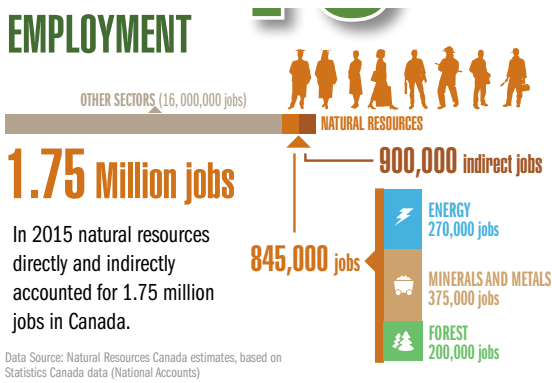
- Stuber, G. (2001). *The Changing Effects of Energy Price Shocks on Economic Activity and Inflation*. Bank of Canada Review. Bank of Canada.
- Tang, W., Wu, L., and Zhang, Z. (2010). “Oil Price Shocks and Their Short and Long-Term Effects on the Chinese Economy”. *Energy Economics* 32, S3–14.
- Taylor, A. M. and Taylor, M. P. (2004). “The Purchasing Power Parity Debate”. *Journal of Economic Perspectives* 18.4, pp. 135–158.
- Taylor, J. B. (1993). “Discretion versus Policy Rules in Practice”. *Carnegie-Rochester Conference Series on Public Policy* 39, pp. 195–214.
- Taylor, J. B., ed. (1999). *Monetary Policy Rules*. Chicago: University of Chicago Press.
- Thorbecke, W. (1997). “On Stock Market Returns and Monetary Policy”. *Journal of Finance* 52.2, pp. 635–654.
- Vasishtha, G. and Maier, P. (2013). “The Impact of the Global Business Cycle on Small Open Economies: A FAVAR Approach for Canada”. *North American Journal of Economics and Finance* 24, pp. 191–207.
- Wu, J. C. and Xia, F. D. (2016). “Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound”. *Journal of Money, Credit, and Banking* 48.2-3, pp. 253–291.
- Yaari, M. E. (1965). “Uncertain Lifetime, Life Insurance, and the Theory of the Consumer”. *Review of Economic Studies* 32.2, pp. 137–150.
- Yun, T. (1996). “Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles”. *Journal of Monetary Economics* 37.2, pp. 345–370.
- Zha, T. (1999). “Block Recursion and Structural Vector Autoregressions”. *Journal of Econometrics* 90.2, pp. 291–316.
- Zorn, L., Wilkins, C., and Engert, W. (2009). “Bank of Canada Liquidity Actions in Response to the Financial Market Turmoil”. *Bank of Canada Review/Revue de la Banque de Canada*, pp. 3–22.

APPENDICES

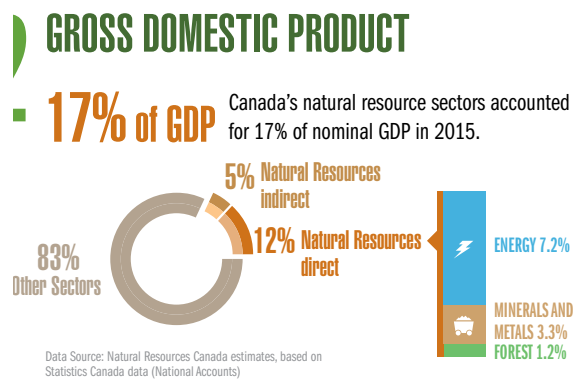
Appendix A

Appendix to Chapter [1](#)

Figure A.1: Contributions of Natural Resources to the Canadian Economy



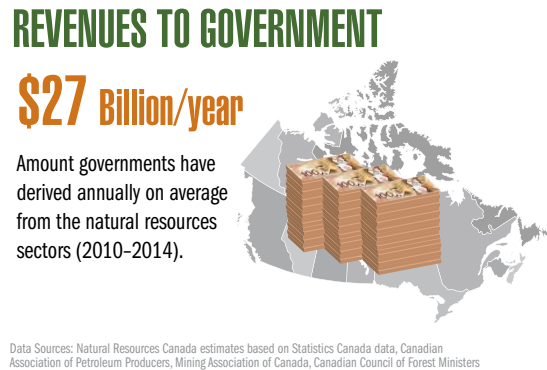
(a) Contributions to Employment



(b) Contributions to GDP



(c) Contributions to Capital Expenditure



(d) Contributions to Government Revenues

Table A.1: Data Description Table

Data	Data Source	Data Description
US CPI	FRED	Consumer Price Index for All Urban Consumers: All Items, Index 1982-1984=100, Monthly, Seasonally Adjusted; series identifier - CPIAUCSL
US Industrial Production	FRED	Industrial Production Index, Index 2012=100, Monthly, Seasonally Adjusted; series identifier - INDPRO
Federal Funds Rate	FRED	Effective Federal Funds Rate, Percent, Monthly, Not Seasonally Adjusted; series identifier - FEDFUNDS
Commodity Price Index	CANSIM	Bank of Canada Commodity Price Index - a chain Fisher price index of the spot or transaction prices in U.S. dollars of 24 commodities produced in Canada and sold in world markets. Commodity weights are updated on an quarterly basis using recent commodity production and price data, Index 1972=100. series identifier - v52673496
CAN/US Exchange Rate	CANSIM	CDN\$ per US\$, noon spot rate, average; series identifier - v37426
Domestic Interest Rate	CANSIM	Canada Overnight Money Market Rate; series identifier - v122514
Domestic CPI	CANSIM	Consumer Price Index: All Items, Index 2002=100, Monthly, Seasonally Adjusted; series identifier - v41690914
Total GVA	CANSIM	GDP at basic prices, by North American Industry Classification System (NAICS), monthly, Chained (2007) dollars; All industries series identifier - v65201210
Industry GVA	CANSIM	GDP at basic prices, by NAICS, monthly, Chained (2007) dollars; various industries series identifier - multiple series identifiers taken from Table 379-0031

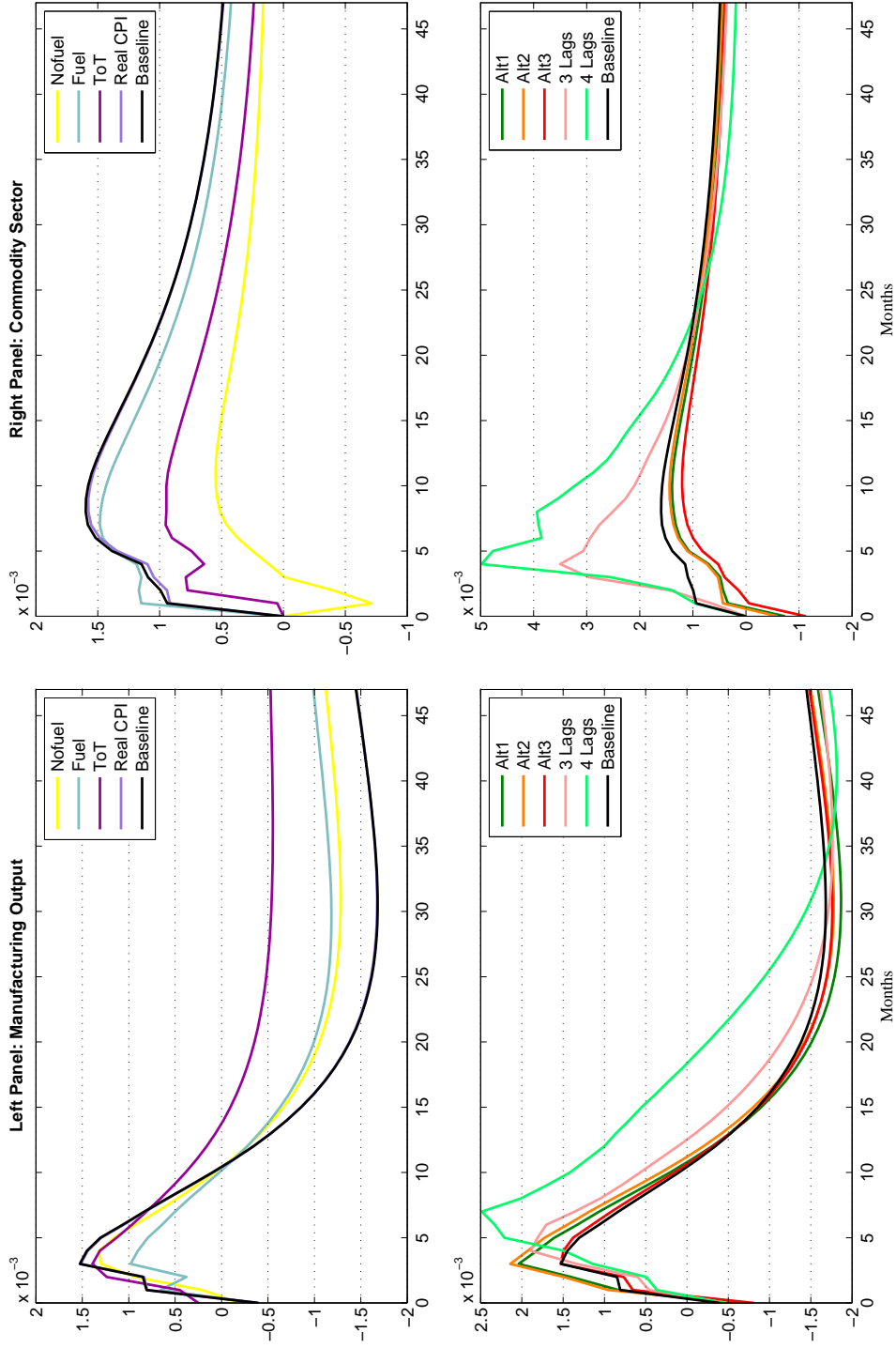
Federal Reserve Economic Data (FRED) - a financial and economic database maintained by the Research Division of the Federal Reserve Bank of St. Louis.

CANSIM - Canadian Socioeconomic Database, provided and maintained by Statistics Canada

GVA - Gross Value Added. This is total industry output net of inputs.

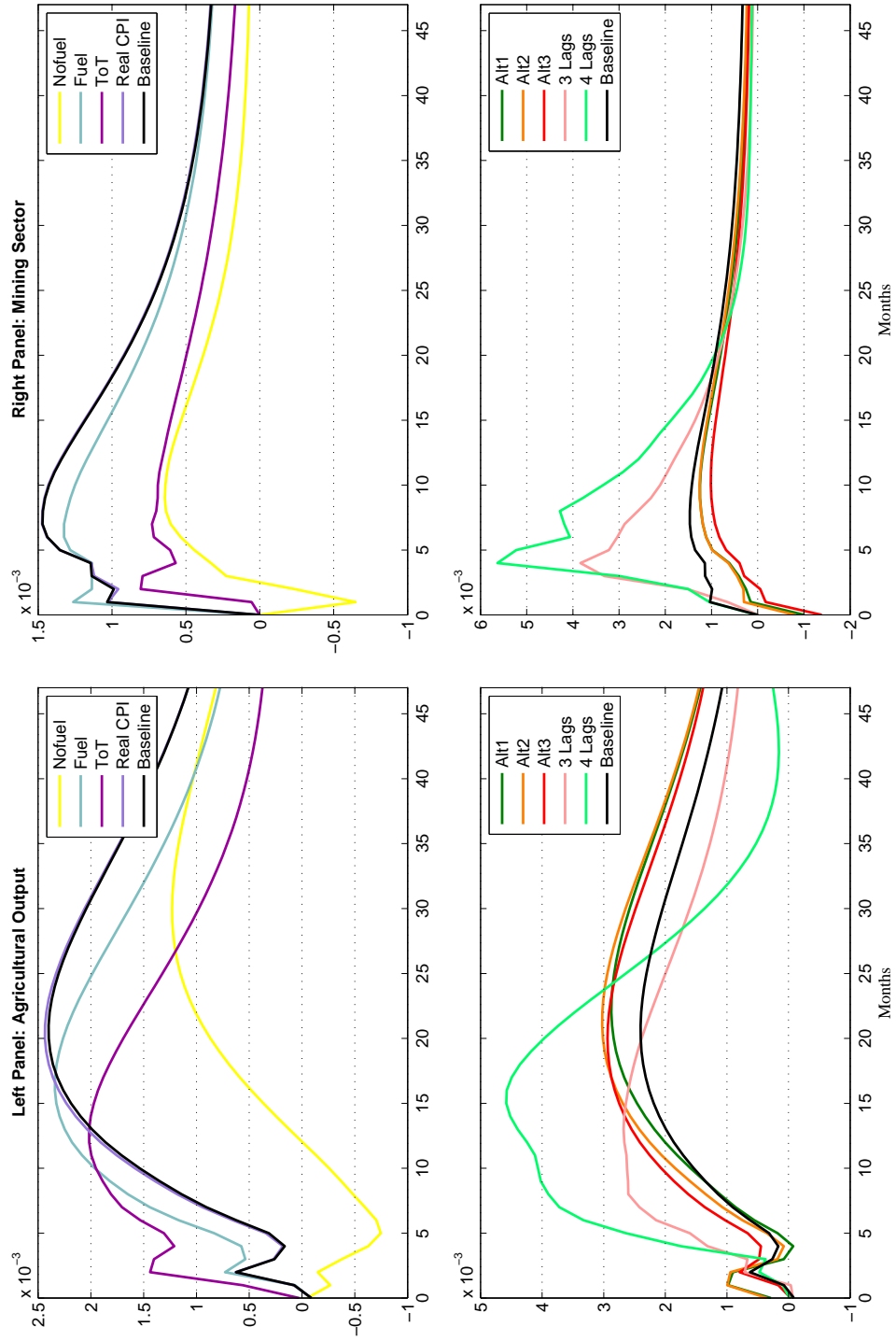
Figure A.2: Sectoral Responses to a Commodity Price Shock

(a) Manufacturing and Commodity Sector

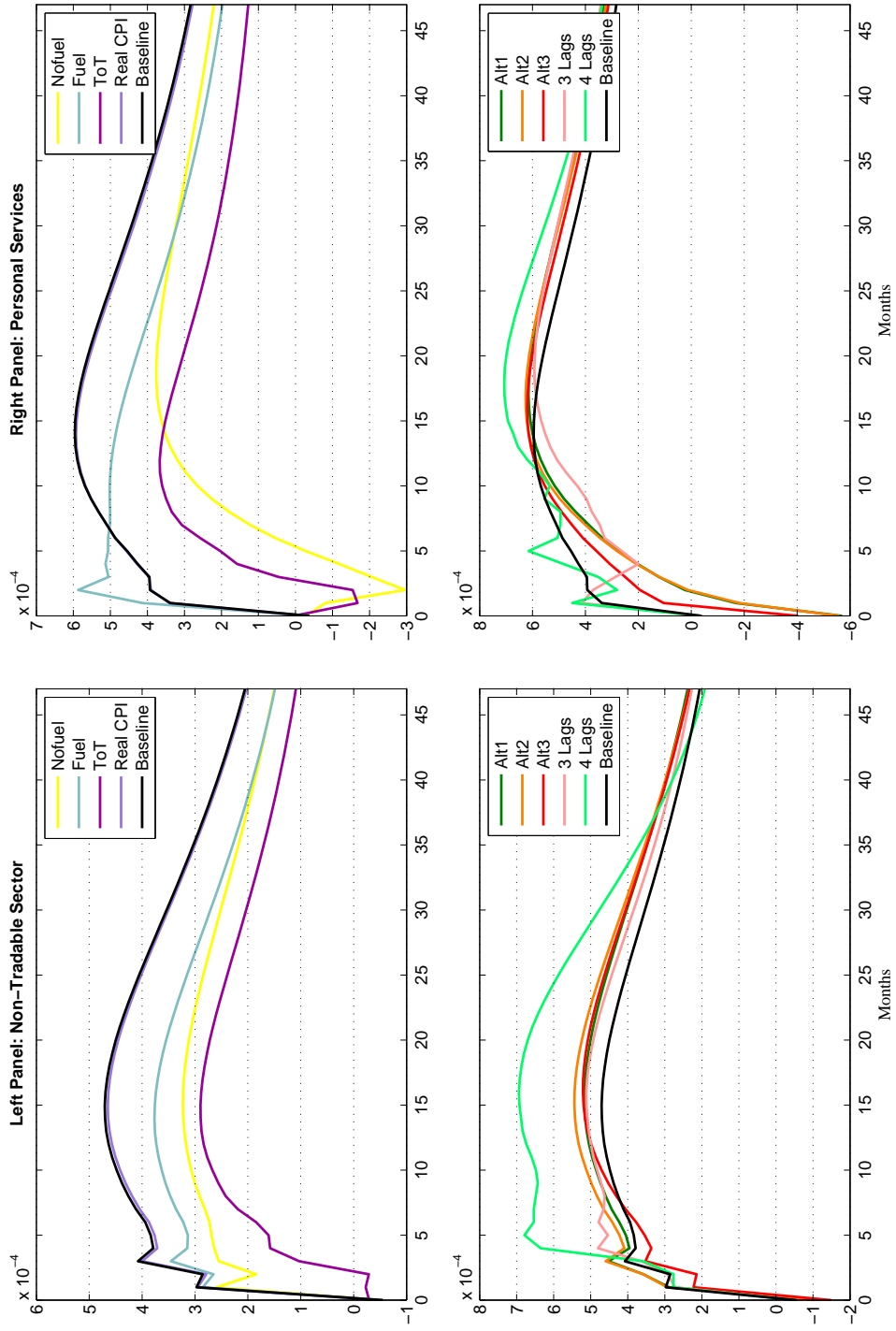


Notes: **Nofuel** - Commodity price index which excludes fuel prices; **Fuel** - Commodity price index with only fuel prices; **ToT** - Trade of Terms index; **Real CPI** - Real Commodity Price Index; **Alt 1,2,3** - Alternative Identification Scheme 1,2,3; **3(4)** - Lags - Model with 3 (4) lags

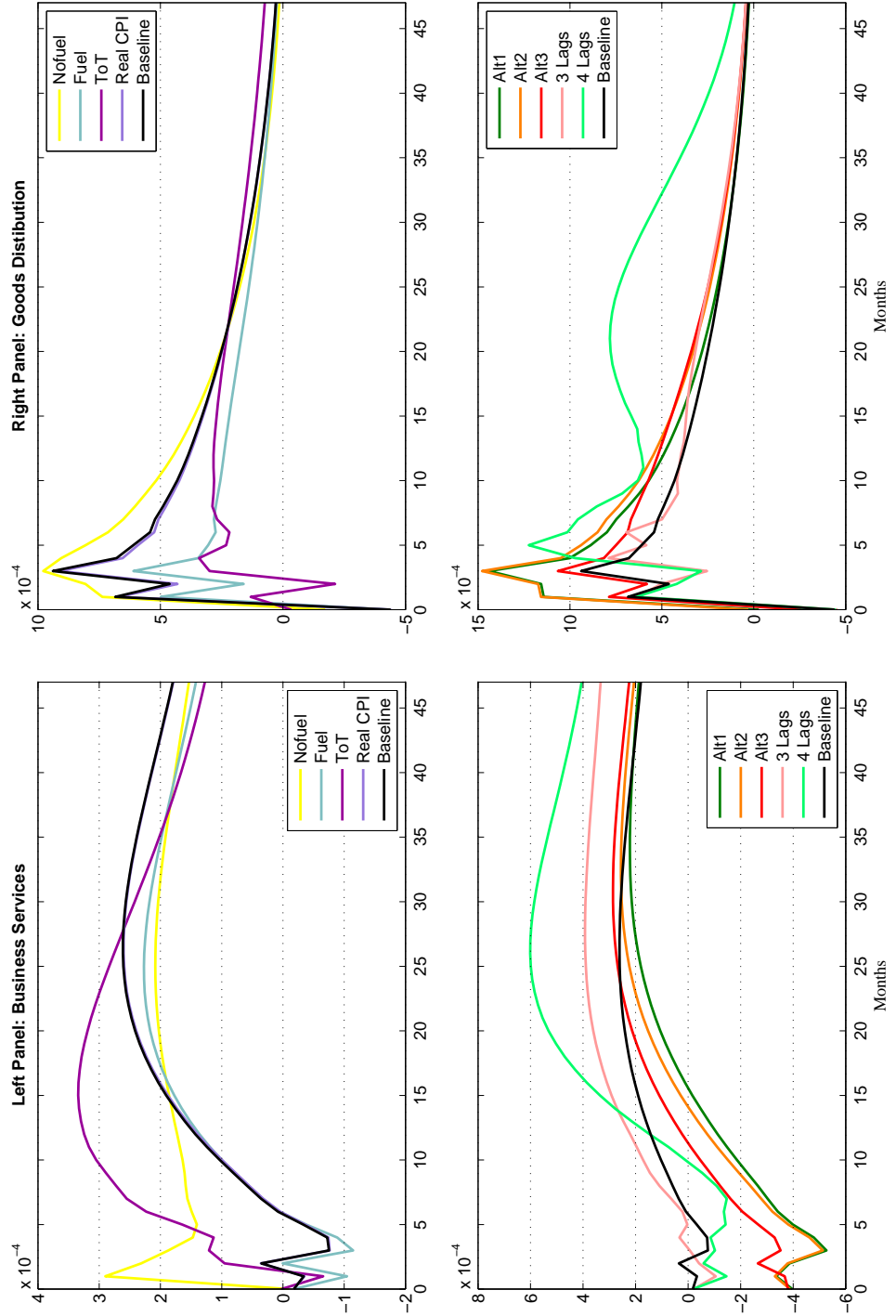
(b) Agriculture and Mining



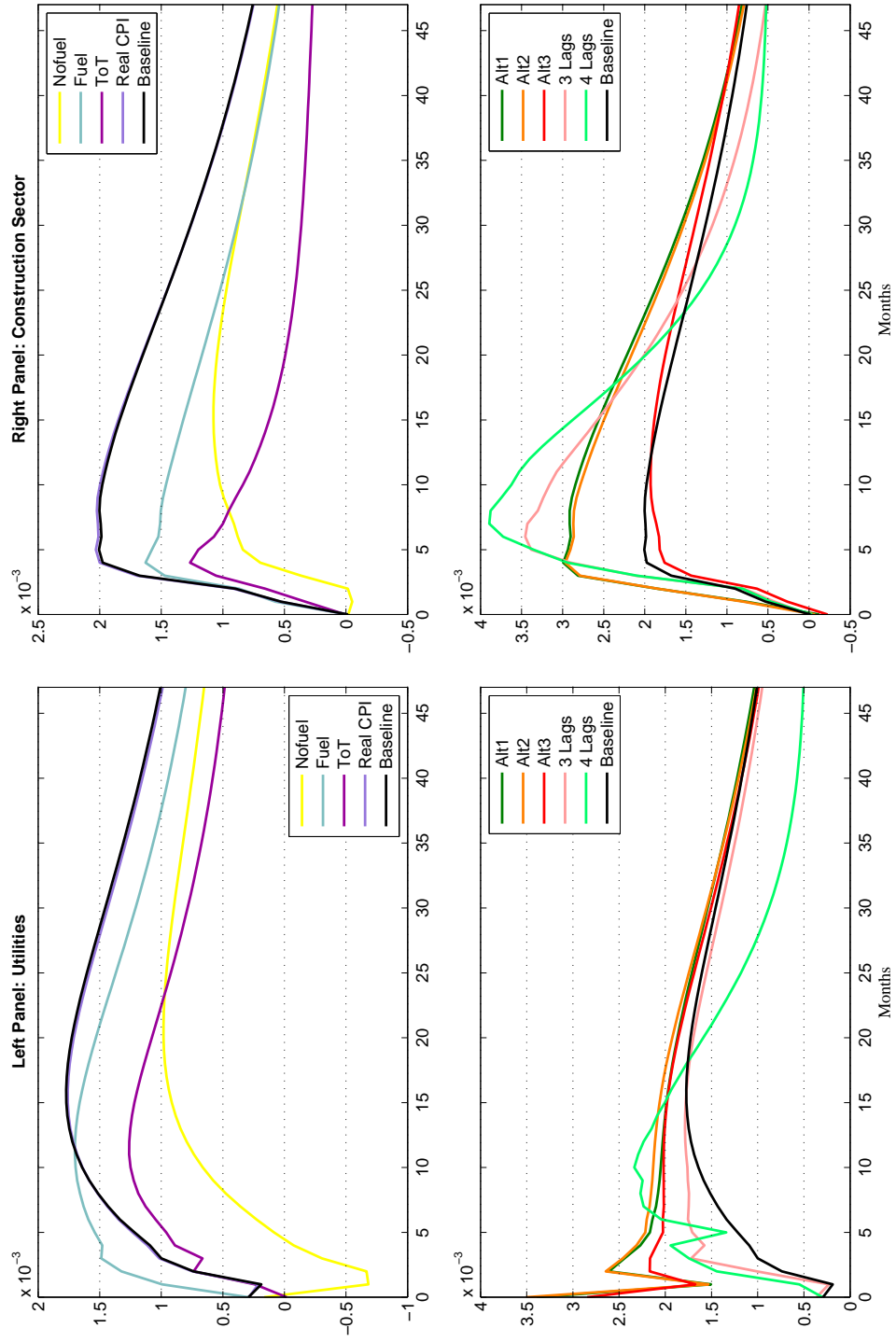
(c) Non-Tradable Sector and Personal Services



(d) Business Services and Goods Distribution



(e) Utilities and Construction Sector

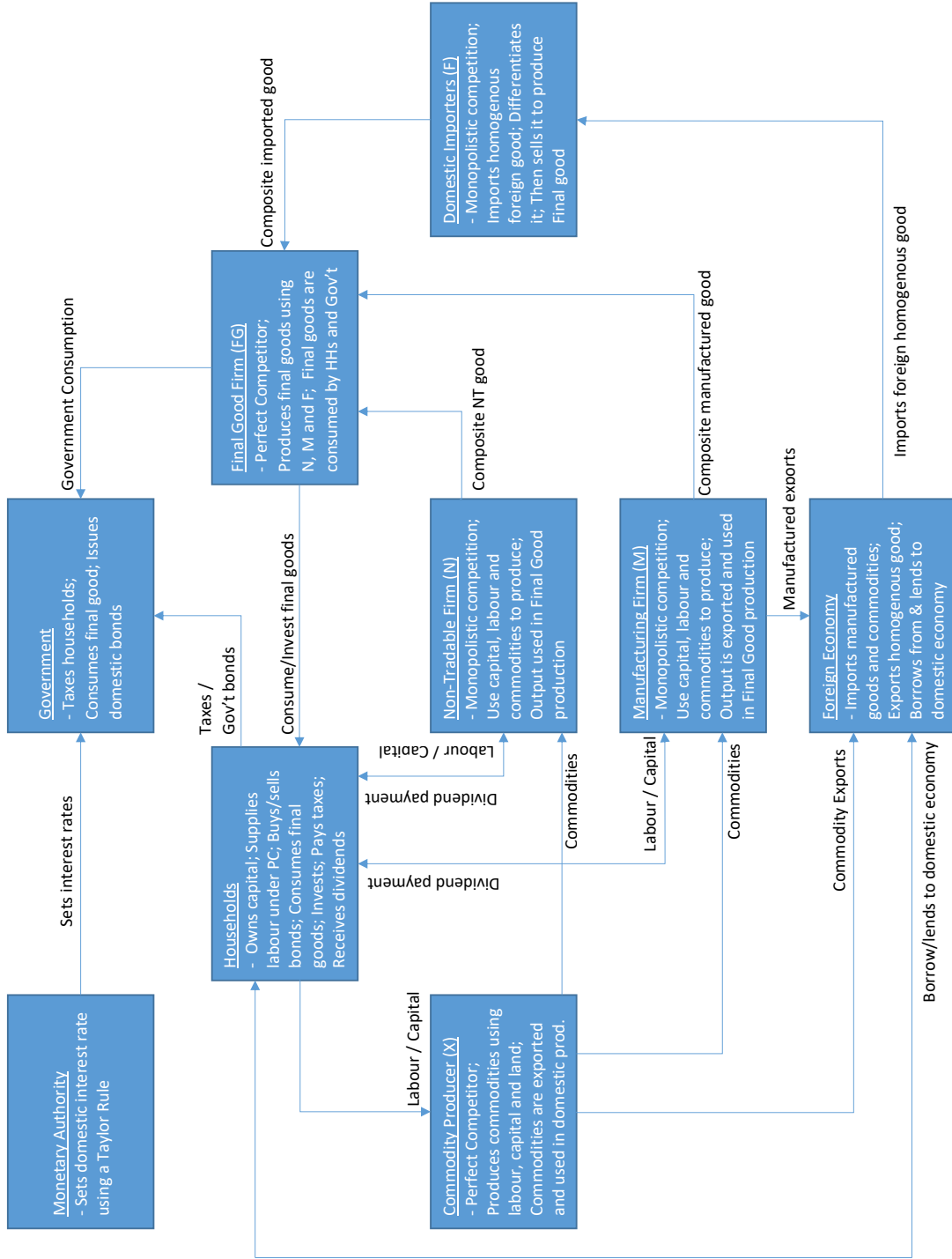


Appendix B

Technical Appendix to Chapter [2](#)

B.1 Schematic Overview of the Model

Figure B.1: Overview of the Model



B.2 How the Model is Solved

A DSGE model may be characterised as a collection of first-order and equilibrium conditions that assume the general form:¹

$$\mathbb{E}_t\{f(x_{t+1}, x_t, x_{t-1}, u_t; \theta)\} = 0 \tag{B.1}$$

$$\begin{aligned} \mathbb{E}_t(u_t) &= 0 \\ \mathbb{E}_t(u_t u_t') &= \Sigma_u \end{aligned} \tag{B.2}$$

where

$f(\cdot)$ are functional forms

x is a vector of endogenous variables

u is a vector of exogenous stochastic shocks and

θ is a vector of parameters

The solution to this system, the DSGE model, is a set of policy functions which are defined as a set of equations that relate the current values of the variables to their past values and to the current shocks while satisfying the set of equations given in [B.1](#). This policy function we denote as:

$$x_t = g(x_{t-1}, u_t) \tag{B.3}$$

To derive the policy function we begin as follows: we first note that equation [B.3](#) may be iterated forward to give us x_{t+1} :

¹This section closely follows the exposition given in the Dynare User Guide

$$\begin{aligned}
x_{t+1} &= g(x_t, u_{t+1}) \\
&= g(g(x_{t-1}, u_t), u_{t+1})
\end{aligned}
\tag{B.4}$$

A new function F may then be defined, such that:

$$F(x_{t-1}, u_t, u_{t+1}) = f(\underset{x_{t+1}}{g(g(x_{t-1}, u_t), u_{t+1})}, \underset{x_t}{g(x_{t-1}, u_t)}, \underset{x_{t-1}}{x_{t-1}}, \underset{u_t}{u_t})
\tag{B.5}$$

Equation B.5 allows us to rewrite our system in B.1, in terms of past variables, current shocks and future shocks:

$$\mathbb{E}_t[F(x_{t-1}, u_t, u_{t+1}); \theta] = 0
\tag{B.6}$$

The non-stochastic steady state of the DSGE model given in equation B.1 is defined as:

$$f(\bar{x}, \bar{x}, \bar{x}, 0) = 0
\tag{B.7}$$

having the property that:

$$\bar{x} = g(\bar{x}, 0)
\tag{B.8}$$

where \bar{x} is the steady-state value of x .

We can therefore linearise the re-written system given in equation B.6 by taking the First-order Taylor series approximation of it around this steady state:

$$\begin{aligned}
\mathbb{E}_t \left\{ F^{(1)}(x_{t-1}, u_t, u_{t+1}) \right\} &= \mathbb{E}_t \left\{ f(\bar{x}, \bar{x}, \bar{x}) + f_{x_{t+1}}(g_{x_{t-1}}(g_{x_{t-1}}\hat{x}_{t-1} + g_{u_t}u_t) + g_{u_t}u_{t+1}) \right. \\
&\quad \left. + f_{x_t}(g_{x_{t-1}}\hat{x}_{t-1} + g_{u_t}u_t) + f_{x_{t-1}}\hat{x}_{t-1} + f_{u_t}u_t \right\} \\
&= 0
\end{aligned} \tag{B.9}$$

where $\hat{x}_{t-1} = x_{t-1} - \bar{x}$, $f_{x_{t+1}} = \frac{\partial f}{\partial x_{t+1}}$ and $g_{x_{t-1}} = \frac{\partial g}{\partial x_{t-1}}$

Taking expectations we get:

$$\begin{aligned}
\mathbb{E}_t \left\{ F^{(1)}(x_{t-1}, u_t, u_{t+1}) \right\} &= f(\bar{x}, \bar{x}, \bar{x}) + f_{x_{t+1}}(g_{x_{t-1}}(g_{x_{t-1}}\hat{x}_{t-1} + g_{u_t}u_t)) \\
&\quad + f_{x_t}(g_{x_{t-1}}\hat{x}_{t-1} + g_{u_t}u_t) + f_{x_{t-1}}\hat{x}_{t-1} + f_{u_t}u_t \\
&= (f_{x_{t+1}}g_{x_{t-1}}g_{x_{t-1}} + f_{x_t}g_{x_{t-1}} + f_{x_{t-1}})\hat{x}_{t-1} + (f_{x_{t+1}}g_{x_{t-1}}g_{u_t} + f_{x_t}g_{u_t} + f_{u_t})u_t \\
&= 0
\end{aligned} \tag{B.10}$$

Since future shocks, u_{t+1} , only enter equation B.9 with their first moments, they drop out after taking expectations of the linearised equations.² We have two unknown variables in equation B.10. These are $g_{x_{t-1}}$ and g_{u_t} , each of which will help us recover the policy function g .

Since equation B.10 holds for any \hat{x}_{t-1} and any u_t , the value of each parenthesis must necessarily be zero. Thus, we can solve each bracketed expression sequentially. Working with the first, gives us a quadratic expression which can be solved for $g_{x_{t-1}}$. Having recovered $g_{x_{t-1}}$, we then proceed in a relatively straightforward manner to recover g_{u_t} from

²If we recall from equation B.2, the expected value of these error terms are assumed to be zero.

the second parenthesis.

Finally, notice that a first-order linearisation of equation B.3, which is the policy function g , yields:

$$x_t = \bar{x} + g_{x_{t-1}}\hat{x}_{t-1} + g_{u_t}u_t \quad (\text{B.11})$$

And now that we have $g_{x_{t-1}}$ and g_{u_t} , we have derived the approximate policy function and have succeeded in solving our DSGE model. To assess the impact of shocks on the model via impulse response functions, we would simply iterate the policy function, B.11, starting from an initial value given by the steady state.

B.3 Household Optimisation Problem

The representative household chooses C_t , $H_{j,t}$, $K_{j,t+1}$, $I_{j,t}$, b_t and b_t^* to maximise utility, equation 2.1, subject to the budget constraint, equation 2.3, and the capital accumulation equation 2.5:

$$\max \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \xi_{c,t} \ln(C_t - dC_{t-1}) - A_L \frac{[H_{n,t}^{1+\varsigma} + H_{m,t}^{1+\varsigma} + H_{x,t}^{1+\varsigma}]^{\frac{1+\chi}{1+\varsigma}}}{1+\chi} \right\} \quad (\text{B.12})$$

subject to

$$C_t + I_t + \frac{b_t}{R_t} + \frac{q_t b_t^*}{\kappa_t R_t^*} = \sum_{j=n,m,x} (r_{j,t} K_{j,t} + w_{j,t} H_{j,t}) + \frac{b_{t-1}}{\pi_t} + \frac{q_t b_{t-1}^*}{\pi_t^*} + r_{L,t} L + \Gamma_t - \tau_t \quad (\text{B.13})$$

and

$$K_{j,t+1} = (1 - \delta)K_{j,t} + \Upsilon_t \left[1 - \frac{\Phi_K}{2} \left(\frac{I_{j,t}}{I_{j,t-1}} - \mu \right)^2 \right] I_{j,t} \quad (\text{B.14})$$

The Lagrangian for this problem is:

$$\begin{aligned} \mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t & \left[\left\{ \xi_{c,t} \ln(C_t - dC_{t-1}) - A_L \frac{[H_{n,t}^{1+\varsigma} + H_{m,t}^{1+\varsigma} + H_{x,t}^{1+\varsigma}]^{\frac{1+\chi}{1+\varsigma}}}{1+\chi} \right\} - \right. \\ & \lambda_t \left\{ C_t + I_t + \frac{b_t}{R_t} + \frac{q_t b_t^*}{\kappa_t R_t^*} - \sum_{j=n,m,x} (r_{j,t} K_{j,t} + w_{j,t} H_{j,t}) - \frac{b_{t-1}}{\pi_t} - \frac{q_t b_{t-1}^*}{\pi_t^*} - r_{l,t} L - \Gamma_t + \tau_t \right\} - \\ & \left. \lambda_{j,t}^K \left\{ K_{j,t+1} - (1 - \delta)K_{j,t} - \Upsilon_t \left[1 - \frac{\Phi_K}{2} \left(\frac{I_{j,t}}{I_{j,t-1}} - \mu \right)^2 \right] I_{j,t} \right\} \right] \end{aligned} \quad (\text{B.15})$$

FOC w.r.t. C_t :

$$\frac{\partial \mathcal{L}}{\partial C_t} : \quad \beta^t \xi_{c,t} \frac{1}{C_t - dC_{t-1}} - \beta^t \lambda_t - \beta^{t+1} \xi_{c,t+1} \frac{d}{C_{t+1} - dC_t} = 0 \quad (\text{B.16})$$

which implies:

$$\lambda_t = \xi_{c,t} \frac{1}{C_t - dC_{t-1}} - \beta \xi_{c,t+1} \frac{d}{C_{t+1} - dC_t} \quad (\text{B.17})$$

$$\lambda_t = \xi_{c,t} (C_t - dC_{t-1})^{-1} - d\beta \xi_{c,t+1} (C_{t+1} - dC_t)^{-1} \quad (\text{B.18})$$

FOC w.r.t. $H_{j,t}$: (for $j = n, m, x$)

$$\frac{\partial \mathcal{L}}{\partial H_{j,t}} : \quad -\beta^t A_L \frac{1}{1+\varsigma} [H_{n,t}^{1+\varsigma} + H_{m,t}^{1+\varsigma} + H_{x,t}^{1+\varsigma}]^{\frac{\chi-\varsigma}{1+\varsigma}} (1+\varsigma) H_{j,t}^{\varsigma} + \beta^t \lambda_t w_{j,t} = 0 \quad (\text{B.19})$$

which implies:

$$\lambda_t w_{j,t} = A_L [H_{n,t}^{1+\varsigma} + H_{m,t}^{1+\varsigma} + H_{x,t}^{1+\varsigma}]^{\frac{\kappa-\varsigma}{1+\varsigma}} H_{j,t}^\varsigma \quad (\text{B.20})$$

$$\lambda_t w_{j,t} = A_L H_t^{\kappa-\varsigma} H_{j,t}^\varsigma \quad (\text{B.21})$$

FOC w.r.t. $K_{j,t+1}$: (for $j = n, m, x$)

$$\frac{\partial \mathcal{L}}{\partial K_{j,t+1}} : \quad -\beta^t \lambda_{j,t}^\kappa + \beta^{t+1} \lambda_{j,t+1}^\kappa (1 - \delta) + \beta^{t+1} \lambda_{t+1} r_{j,t+1} = 0 \quad (\text{B.22})$$

which implies:

$$\lambda_{j,t}^\kappa = \beta \lambda_{j,t+1}^\kappa (1 - \delta) + \beta \lambda_{t+1} r_{j,t+1} \quad (\text{B.23})$$

FOC w.r.t. $I_{j,t}$: (for $j = n, m, x$)

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial I_{j,t}} : \quad & -\beta^t \lambda_t + \beta^t \lambda_{j,t}^\kappa \Upsilon_t \left[1 - \frac{\Phi_\kappa}{2} \left(\frac{I_{j,t}}{I_{j,t-1}} - \mu \right)^2 \right] - \beta^t \lambda_{j,t}^\kappa \Upsilon_t \Phi_\kappa \left(\frac{I_{j,t}}{I_{j,t-1}} - \mu \right) \frac{I_{j,t}}{I_{j,t-1}} \\ & + \beta^{t+1} \lambda_{j,t+1}^\kappa \Upsilon_{t+1} \Phi_\kappa \left(\frac{I_{j,t+1}}{I_{j,t}} - \mu \right) \left(\frac{I_{j,t+1}}{I_{j,t}} \right)^2 = 0 \end{aligned} \quad (\text{B.24})$$

FOC w.r.t. b_t :

$$\frac{\partial \mathcal{L}}{\partial b_t} : \quad -\beta^t \frac{\lambda_t}{R_t} + \beta^{t+1} \frac{\lambda_{t+1}}{\pi_{t+1}} = 0 \quad (\text{B.25})$$

which implies:

$$\frac{\lambda_t}{R_t} = \beta \frac{\lambda_{t+1}}{\pi_{t+1}} \quad (\text{B.26})$$

FOC w.r.t. b_t^* :

$$\frac{\partial \mathcal{L}}{\partial b_t^*} : \quad -\beta^t \frac{\lambda_t q_t}{\kappa_t R_t^*} + \beta^{t+1} \frac{\lambda_{t+1} q_{t+1}}{\pi_{t+1}^*} = 0 \quad (\text{B.27})$$

which implies:

$$\frac{q_t \lambda_t}{\kappa_t R_t^*} = \beta \frac{\lambda_{t+1} q_{t+1}}{\pi_{t+1}^*} \quad (\text{B.28})$$

B.4 Non-Tradable Firm Optimisation Problem

B.4.1 Cost-minimisation Problem

Non-tradable good producers choose the most efficient level of input subject to their available level of technology. That is, they minimize input costs given factor prices and thus firm i faces the following cost minimisation problem:

$$\min \quad W_{n,t} H_{n,it} + R_{n,t} K_{n,it} + P_{x,t} Y_{x,it}^n \quad (\text{B.29})$$

subject to

$$Y_{n,it} = A_{n,t} (\mu_t H_{n,it})^{\alpha_n} (K_{n,it})^{\gamma_n} (Y_{x,it}^n)^{1-\alpha_n-\gamma_n} \quad (\text{B.30})$$

The Lagrangian for this problem is:

$$\mathcal{L} = W_{n,t} H_{n,it} + R_{n,t} K_{n,it} + P_{x,t} Y_{x,it}^n + MC_{n,it} \left[Y_{n,it} - A_{n,t} (\mu_t H_{n,it})^{\alpha_n} (K_{n,it})^{\gamma_n} (Y_{x,it}^n)^{1-\alpha_n-\gamma_n} \right] \quad (\text{B.31})$$

FOC w.r.t. $H_{n,it}$:

$$\frac{\partial \mathcal{L}}{\partial H_{n,it}} : \quad W_{n,t} - MC_{n,it} \alpha_n A_{n,t} (\mu_t H_{n,it})^{\alpha_n - 1} (K_{n,it})^{\gamma_n} (Y_{x,it}^n)^{1-\alpha_n-\gamma_n} = 0 \quad (\text{B.32})$$

$$W_{n,t} = MC_{n,it} \alpha_n \frac{Y_{n,it}}{H_{n,it}} \quad (\text{B.33})$$

FOC w.r.t. $K_{n,it}$:

$$\frac{\partial \mathcal{L}}{\partial K_{n,it}} : \quad R_{n,t} - MC_{n,it} \gamma_n A_{n,t} (\mu_t H_{n,it})^{\alpha_n} (K_{n,it})^{\gamma_n - 1} (Y_{x,it}^n)^{1 - \alpha_n - \gamma_n} = 0 \quad (\text{B.34})$$

$$R_{n,t} = MC_{n,it} \gamma_n \frac{Y_{n,it}}{K_{n,it}} \quad (\text{B.35})$$

FOC w.r.t. $Y_{x,it}^n$:

$$\frac{\partial \mathcal{L}}{\partial Y_{x,it}^n} : \quad P_{x,t} - MC_{n,it} (1 - \alpha_n - \gamma_n) A_{n,t} (\mu_t H_{n,it})^{\alpha_n} (K_{n,it})^{\gamma_n} (Y_{x,it}^n)^{-\alpha_n - \gamma_n} = 0 \quad (\text{B.36})$$

$$P_{x,t} = MC_{n,it} (1 - \alpha - \gamma_n) \frac{Y_{n,it}}{Y_{x,it}^n} \quad (\text{B.37})$$

Marginal Cost

Using equations [B.33](#), [B.35](#) and [B.37](#) we can derive the expression for the firm's marginal cost.

Using [\(B.33\)](#) and [\(B.35\)](#) we get:

$$\begin{aligned}
\frac{W_{n,t}}{R_{n,t}} &= \frac{\alpha_n}{\gamma_n} \frac{Y_{n,it}}{H_{n,it}} \frac{K_{n,it}}{Y_{n,it}} \\
&= \frac{\alpha_n}{\gamma_n} \frac{K_{n,it}}{H_{n,it}} \\
K_{n,it} &= \frac{\gamma_n}{\alpha_n} \frac{W_{n,t}}{R_{n,t}} H_{n,it}
\end{aligned} \tag{B.38}$$

Using (B.33) and (B.37) we get:

$$\begin{aligned}
\frac{W_{n,t}}{P_{x,t}} &= \frac{\alpha_n}{1 - \alpha - \gamma_n} \frac{Y_{n,it}}{H_{n,it}} \frac{Y_{x,it}^n}{Y_{n,it}} \\
&= \frac{\alpha_n}{1 - \alpha - \gamma_n} \frac{Y_{x,it}^n}{H_{n,it}} \\
Y_{x,it}^n &= \frac{1 - \alpha - \gamma_n}{\alpha_n} \frac{W_{n,t}}{P_{x,t}} H_{n,it}
\end{aligned} \tag{B.39}$$

Expressions B.38 and B.39 can be substituted into the first-order condition for labour, equation B.33, to derive the final expression for the firm's marginal cost. From equation B.33, we get:

$$\begin{aligned}
MC_{n,it} &= W_{n,t} \frac{1}{\alpha_n} \frac{H_{n,it}}{Y_{n,it}} \\
&= W_{n,t} \frac{1}{\alpha_n} \frac{H_{n,it}}{A_{n,t} (\mu_t H_{n,it})^{\alpha_n} (K_{n,it})^{\gamma_n} (Y_{x,it}^n)^{1 - \alpha_n - \gamma_n}}
\end{aligned} \tag{B.40}$$

Using expressions B.38 and B.39, we get:

$$\begin{aligned}
MC_{n,it} &= W_{n,t} \frac{1}{\alpha_n} \frac{H_{n,it}}{A_{n,t} (\mu_t H_{n,it})^{\alpha_n}} \left(\frac{\alpha_n}{\gamma_n} \right)^{\gamma_n} \left(\frac{R_{n,t}}{W_{n,t}} \right)^{\gamma_n} \left(\frac{1}{H_{n,it}} \right)^{\gamma_n} \left(\frac{\alpha_n}{1 - \alpha_n - \gamma_n} \frac{P_{x,t}}{W_{n,t} H_{n,it}} \right)^{1 - \alpha_n - \gamma_n} \\
&= \frac{1}{A_n} (W_{n,t})^{\alpha_n} (\alpha_n)^{-\alpha_n} (\mu_t)^{-\alpha_n} \left(\frac{R_{n,t}}{\gamma_n} \right)^{\gamma_n} \left(\frac{P_{x,t}}{1 - \alpha_n - \gamma_n} \right)^{1 - \alpha_n - \gamma_n} \\
&= \frac{1}{A_n} \left(\frac{W_{n,t}}{\alpha_n \mu_t} \right)^{\alpha_n} \left(\frac{R_{n,t}}{\gamma_{n,t}} \right)^{\gamma_n} \left(\frac{P_{x,t}}{1 - \alpha_n - \gamma_n} \right)^{1 - \alpha_n - \gamma_n}
\end{aligned} \tag{B.41}$$

All firms face the same marginal costs, so we can drop the i subscript. A shock term, $\varepsilon_{n,t}$, is also included to account for changes in marginal costs that are unrelated to factor input costs. Thus we get the expression for marginal costs, given in the text:

$$MC_{n,t} = \frac{\varepsilon_{n,t}}{A_n} \left(\frac{W_{n,t}}{\alpha_n \mu_t} \right)^{\alpha_n} \left(\frac{R_{n,t}}{\gamma_{n,t}} \right)^{\gamma_n} \left(\frac{P_{x,t}}{1 - \alpha_n - \gamma_n} \right)^{1 - \alpha_n - \gamma_n} \tag{B.42}$$

A similar procedure to the one above, yields the relevant equations for the manufacturing sector.

B.4.2 Profit Maximisation Problem

Since these intermediate good producers have some monopoly power, they set their prices by maximising profits subject to the relevant demand constraint.

$$\max_{\{P_{n,it+l}\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{l=0}^{\infty} D_{t,t+l} \left\{ \frac{P_{n,it+l} Y_{n,it+l}}{P_{t+l}} - \frac{MC_{n,it+l} Y_{n,it+l}}{P_{t+l}} - \frac{\tau_n}{2} \left(\frac{P_{n,it+l}}{\Pi_{n,t+l-1}^\eta \Pi^{1-\eta} P_{n,it+l-1}} - 1 \right)^2 \frac{P_{n,t+l} Y_{n,t+l}}{P_{t+l}} \right\} \tag{B.43}$$

subject to

$$Y_{n,it+l} = \left(\frac{P_{n,it+l}}{P_{n,t+l}} \right)^{-\theta_n} Y_{n,t+l} \quad (\text{B.44})$$

where $D_{t,t+l} = \beta^l \frac{U_{c,t+l}}{U_{c,t}} = \beta^l \frac{\lambda_{t+l}}{\lambda_t}$ is the relevant stochastic discount factor. Substituting the demand function into the profit function yields:

$$\begin{aligned} \mathbb{E}_t \sum_{l=0}^{\infty} \beta^l \frac{\lambda_{t+l}}{\lambda_t} \left\{ \frac{P_{n,it+l}}{P_{t+l}} \left(\frac{P_{n,it+l}}{P_{n,t+l}} \right)^{-\theta_n} Y_{n,t+l} - MC_{n,it+l}^r \left(\frac{P_{n,it+l}}{P_{n,t+l}} \right)^{-\theta_n} Y_{n,t+l} \right. \\ \left. - \frac{\tau_n}{2} \left(\frac{P_{n,it+l}}{\Pi_{n,t+l-1}^\eta \Pi^{1-\eta} P_{n,it+l-1}} - 1 \right)^2 \frac{P_{n,t+l} Y_{n,t+l}}{P_{t+l}} \right\} \end{aligned} \quad (\text{B.45})$$

which may be re-written as:³

$$\begin{aligned} \mathbb{E}_t \sum_{l=0}^{\infty} \beta^l \frac{\lambda_{t+l}}{\lambda_t} \left\{ \left(\frac{P_{n,it+l}}{P_{n,t+l}} \right)^{1-\theta_n} \frac{P_{n,t+l}}{P_{t+l}} Y_{n,t+l} - MC_{n,it+l}^r \left(\frac{P_{n,it+l}}{P_{n,t+l}} \right)^{-\theta_n} Y_{n,t+l} \right. \\ \left. - \frac{\tau_n}{2} \left(\frac{P_{n,it+l}}{\Pi_{n,t+l-1}^\eta \Pi^{1-\eta} P_{n,it+l-1}} - 1 \right)^2 \frac{P_{n,t+l} Y_{n,t+l}}{P_{t+l}} \right\} \end{aligned} \quad (\text{B.46})$$

³where $MC_{n,it+l}^r = \frac{MC_{n,it+l}}{P_{t+l}}$ is the real marginal costs.

Taking the derivative of B.46 with respect to $P_{n,it+l}$ yields:

$$\begin{aligned} \beta^l \frac{\lambda_{t+l}}{\lambda_t} & \left\{ (1 - \theta_n) P_{n,it+l}^{-\theta_n} \left(\frac{1}{P_{n,t+l}} \right)^{1-\theta_n} \frac{P_{n,t+l}}{P_{t+l}} Y_{n,t+l} + \theta_n MC_{n,it+l}^r (P_{n,it+l})^{-\theta_n-1} \left(\frac{1}{P_{n,t+l}} \right)^{-\theta_n} Y_{n,t+l} \right. \\ & \left. - \tau_n \left(\frac{P_{n,it+l}}{\Pi_{n,t+l-1}^\eta \Pi^{1-\eta} P_{n,it+l-1}} - 1 \right) \frac{1}{\Pi_{n,t+l-1}^\eta \Pi^{1-\eta} P_{n,it+l-1}} \frac{P_{n,t+l} Y_{n,t+l}}{P_{t+l}} \right\} + \beta^{l+1} \frac{\lambda_{t+l+1}}{\lambda_t} \\ & \left\{ \tau_n \left(\frac{P_{n,it+l+1}}{\Pi_{n,t+l}^\eta \Pi^{1-\eta} P_{n,it+l}} - 1 \right) \frac{P_{n,it+l+1}}{\Pi_{n,t+l}^\eta \Pi^{1-\eta} P_{n,it+l}^2} \frac{P_{n,t+l+1} Y_{n,t+l+1}}{P_{t+l+1}} \right\} = 0 \end{aligned} \quad (\text{B.47})$$

In a symmetric equilibrium, all firms face the same problem and will thus charge the same price. Imposing this symmetric equilibrium and evaluating expression B.47 at the current period, we get the first-order condition for price setting:

$$\begin{aligned} (1 - MC_{n,t}^r)^{\theta_n} = & 1 - \tau_n \left(\frac{\Pi_{n,t}}{\Pi_{n,t-1}^\eta \Pi^{1-\eta}} - 1 \right) \frac{\Pi_{n,t}}{\Pi_{n,t-1}^\eta \Pi^{1-\eta}} + \\ & \tau_n \beta \frac{\lambda_{t+1}}{\lambda_t} \left\{ \left(\frac{\Pi_{n,t+1}}{\Pi_{n,t}^\eta \Pi^{1-\eta}} - 1 \right) \frac{\Pi_{n,t+1}}{\Pi_{n,t}^\eta \Pi^{1-\eta}} \frac{Y_{n,t+1}}{Y_{n,t}} \right\} \end{aligned} \quad (\text{B.48})$$

Log-linearizing equation B.48 about the steady-state of the variables, yields the New Keynesian Phillips curve.

B.5 The Log-linearized System of Equations

The log-linearized equations of the model are presented in this section. Since the model features a permanent productivity shock, captured by μ_t , this implies that real variables

also grow at the same rate in the long-run. The optimality and market-clearing conditions of the model must therefore first be normalised by the technology factor so as to be made stationary. It is these stationary conditions that are then log-linearized and used to solve the model. Lower-case letters indicate that variables have been made stationary with the trend level of technology, that is $x_t = \frac{X_t}{\mu_t}$. A hat over a variable indicates it is in terms of log deviation from its steady state value, $\hat{x}_t = \ln x_t - \ln x$. Where variables without a time subscript refer to steady state values.

The household's inter-temporal consumption equation is:

$$\begin{aligned}
(\mu - d)(\mu - \beta d)\hat{\lambda}_t &= (\mu^2 - \mu d)\hat{\xi}_{c,t} - (d\beta\mu - d^2\beta)\hat{\xi}_{c,t+1} - (\mu^2 + d^2\beta)\hat{c}_t \\
&\quad + \mu d(\hat{c}_t + \beta\hat{c}_{t+1} - \hat{\mu}_t + \beta\hat{\mu}_{t+1})
\end{aligned} \tag{B.49}$$

Household labour supply decision for each sector is:

$$\hat{w}_{j,t} = (\chi - \varsigma)\hat{h}_t + \varsigma\hat{h}_{j,t} - \hat{\lambda}_t \quad \text{for } j \in \{n, m, x\} \tag{B.50}$$

Household investment decision for each sector is:

$$\hat{\lambda}_{j,t}^k = \Phi^k \mu^2 \left[(1 + \beta)\hat{i}_{j,t} - \hat{i}_{j,t-1} - \beta\hat{i}_{j,t+1} - \beta\hat{\mu}_{t+1} + \hat{\mu}_t \right] + \hat{\lambda}_t - \hat{Y}_t \quad \text{for } j \in \{n, m, x\} \tag{B.51}$$

where $\lambda_{j,t}^k$ is the shadow price of productive capital.

The equilibrium condition for capital in each sector is given by:

$$\hat{\lambda}_{j,t}^k = \hat{\lambda}_{t+1} + \hat{r}_{j,t+1} + \hat{\mu}_t - \hat{\mu}_{t+1} - \frac{\beta(1 - \delta)}{\mu} \left(\hat{\lambda}_{t+1} + \hat{r}_{j,t+1} - \hat{\lambda}_{j,t+1}^k \right) \tag{B.52}$$

The law of motion for capital is:

$$\hat{k}_{j,t+l} = \frac{1-\delta}{\mu}(\hat{k}_{j,t} - \hat{\mu}_t) + \frac{\mu-1+\delta}{\mu}(\hat{\Upsilon}_{j,t} + \hat{i}_{j,t}) \quad \text{for } j \in \{n, m, x\} \quad (\text{B.53})$$

The household's CES composite labour supply bundle is defined as follows:

$$\hat{H}_t = \left(\frac{H_n}{H}\right)^{1+\varsigma} \hat{H}_{n,t} + \left(\frac{H_m}{H}\right)^{1+\varsigma} \hat{H}_{m,t} + \left(\frac{H_x}{H}\right)^{1+\varsigma} \hat{H}_{x,t} \quad (\text{B.54})$$

The sectoral production functions are given by:

$$\hat{y}_{j,t} = \hat{A}_{j,t} + \alpha_j \hat{H}_{j,t} + \gamma_j \hat{k}_{j,t} + (1 - \alpha_j - \gamma_j) \hat{y}_{x,t}^j \quad \text{for } j \in \{n, m, x\} \quad (\text{B.55})$$

Phillips curves for the non-tradable and manufacturing sectors are:

$$\hat{\pi}_{j,t} = \frac{\beta}{1+\eta\beta} \hat{\pi}_{j,t+1} + \frac{\kappa_j}{1+\eta\beta} \hat{m}c_{j,t} + \frac{\eta}{1+\eta\beta} \hat{\pi}_{j,t-1} + \varepsilon_{j,t} \quad \text{for } j \in \{n, m\} \quad (\text{B.56})$$

where $\kappa_j = \frac{\theta_j-1}{\tau_j}$.

Real marginal costs in the non-tradable and manufacturing sectors are:

$$\hat{m}c_{j,t} = \alpha_j \hat{w}_{j,t} + \gamma_j \hat{r}_{j,t} + (1 - \alpha_j - \gamma_j) \hat{p}_{x,t} - \hat{p}_{j,t} - \hat{A}_{j,t} \quad \text{for } j \in \{n, m\} \quad (\text{B.57})$$

where $p_{j,t}$ denotes the relative price of good j , such that $p_{j,t} = \ln\left(\frac{P_{j,t}}{P_t}\right)$.

The capital to labour ratio in the non-tradable and manufacturing sectors are:

$$\hat{h}_{j,t} = \hat{k}_{j,t} + \hat{r}_{j,t} - \hat{w}_{j,t} - \hat{\mu}_t \quad \text{for } j \in \{n, m\} \quad (\text{B.58})$$

The capital to commodities ratio in the non-tradable and manufacturing sectors are:

$$\hat{y}_{x,t}^j = \hat{k}_{j,t} + \hat{r}_{j,t} - \hat{p}_{x,t} - \hat{\mu}_t \quad \text{for } j \in \{n, m\} \quad (\text{B.59})$$

The Phillips curve for the import sector:

$$\hat{\pi}_{f,t} = \frac{\beta}{1 + \eta\beta} \hat{\pi}_{f,t+1} + \frac{\kappa_f}{1 + \eta\beta} \hat{m}c_{f,t} + \frac{\eta}{1 + \eta\beta} \hat{\pi}_{f,t-1} + \varepsilon_{f,t} \quad (\text{B.60})$$

where $\kappa_f = \frac{\theta_f - 1}{\tau_f}$.

Marginal cost for the import sector (modified to accommodate possible deviations from the Law of one price):

$$\hat{m}c_{f,t} = \hat{q}_t - \hat{p}_{f,t} \quad (\text{B.61})$$

Labour demand in the commodities sector:

$$\hat{h}_{x,t} = \hat{p}_{x,t} + \hat{y}_{x,t} - \hat{w}_{x,t} \quad (\text{B.62})$$

Capital demand in the commodities sector:

$$\hat{k}_{x,t} = \hat{p}_{x,t} + \hat{y}_{x,t} - \hat{r}_{x,t} \quad (\text{B.63})$$

The definition of the domestic-currency price of commodities:

$$\hat{p}_{x,t} = \frac{1}{2}(\hat{q}_t + \hat{p}_{x,t}^*) + \frac{1}{2}\hat{p}_{x,t-1} \quad (\text{B.64})$$

Foreign currency price of commodities:

$$\hat{p}_{x,t}^* = \rho_{p_x^*} \hat{p}_{x,t-1}^* + \varepsilon_{p_x^*,t} \quad (\text{B.65})$$

Foreign demand for manufacture exports:

$$\hat{y}_m^{ex} = -\nu^*(\hat{p}_{m,t}^*) + \hat{y}_t^* \quad (\text{B.66})$$

Domestic demand for non-tradable, manufactured and imported goods:

$$\hat{y}_{j,t} = -\nu \hat{p}_{j,t} + \hat{f} g_t \quad \text{for } j \in \{n, m, f\} \quad (\text{B.67})$$

Market clearing in the manufacturing sector:

$$y_m \hat{y}_{m,t} = y_m^{ex} \hat{y}_{m,t}^{ex} + y_m^d \hat{y}_{m,t}^d \quad (\text{B.68})$$

Market clearing in the commodities sector:

$$y_x \hat{y}_{x,t} = y_x^{ex} \hat{y}_{x,t}^{ex} + y_x^n \hat{y}_{x,t}^n + y_x^m \hat{y}_{x,t}^m \quad (\text{B.69})$$

Uses of the domestic final good:

$$fg(\hat{f} g_t) = c \hat{c}_t + \hat{i}_t + g \hat{g}_t \quad (\text{B.70})$$

Definition of aggregate investment:

$$\hat{i}_t = i_n \hat{i}_{n,t} + i_m \hat{i}_{m,t} + i_x \hat{i}_{x,t} \quad (\text{B.71})$$

Real value-added in the non-tradable and manufacturing sector:

$$y_j^{va} \hat{y}_{j,t}^{va} = y_j \hat{y}_{j,t} - \frac{p_x}{p_j} y_x^j \hat{y}_{x,t}^j \quad j \in \{n, m\} \quad (\text{B.72})$$

Total real valued-added:

$$y^{va} \hat{y}_t^{va} = p_n y_n^{va} \hat{y}_{n,t}^{va} + p_m y_m^{va} \hat{y}_{m,t}^{va} + p_x y_x \hat{y}_{x,t} \quad (\text{B.73})$$

The current account equation:

$$\begin{aligned} \frac{b_t^*}{\kappa r^*} &= \frac{b_{t-1}^*}{\pi^* \mu} + \frac{p_x y_x^{ex}}{y^{va}} (\hat{p}_{x,t} + \hat{y}_{x,t}^{ex} - \hat{y}_t^{va}) + \frac{p_m^* y_m^{ex}}{y^{va}} (\hat{p}_{m,t}^* + \hat{y}_{m,t}^{ex} - \hat{y}_t^{va}) \\ &\quad - \frac{q y_f}{y^{va}} (\hat{q}_t + \hat{y}_{f,t} - \hat{y}_t^{va}) \end{aligned} \quad (\text{B.74})$$

Uncovered interest rate parity condition:

$$\hat{q}_{t+1} - \hat{q}_t = \hat{\pi}_{t+1}^* - \hat{\pi}_{t+1} + \hat{r}_t - \hat{r}_t^* - \hat{\kappa}_t \quad (\text{B.75})$$

Country-specific risk premium:

$$\hat{\kappa}_t = -\varkappa(b_t^*) + \hat{\Psi}_t \quad (\text{B.76})$$

By definition of the real exchange rate:

$$\hat{q}_t - \hat{q}_{t-1} + \hat{\pi}_t - \hat{\pi}_t^* = \Delta \hat{s}_t \quad (\text{B.77})$$

Definition of the aggregate price index implies:

$$0 = \omega_j p_j^{1-\nu} (1 - \nu) \hat{p}_{j,t} \quad j \in \{n, m, f\} \quad (\text{B.78})$$

where $\hat{p}_{j,t}$ is the relative price in sector j , which evolves according to:

$$\hat{p}_{j,t} = \hat{p}_{j,t-1} + \hat{\pi}_{j,t} - \hat{\pi}_t \quad (\text{B.79})$$

The Taylor rule of the monetary authority is:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r) (\varrho_\pi \hat{\pi}_t + \varrho_y \hat{y}_t^{va}) + \varepsilon_{r,t} \quad (\text{B.80})$$

The foreign economy:

$$\text{IS curve:} \quad \hat{y}_t^* = \hat{y}_{t+1}^* - (\hat{r}_t^* - \hat{\pi}_{t+1}) - \hat{\xi}_{y^*,t} \quad (\text{B.81})$$

$$\text{Phillips curve:} \quad \hat{\pi}_t = \beta \hat{\pi}_{t+1} + \kappa^* \hat{y}_t^* + \varepsilon_{\pi^*,t} \quad (\text{B.82})$$

$$\text{Taylor rule:} \quad \hat{r}_t^* = \rho_{r^*} \hat{r}_{t-1}^* + (1 - \rho_{r^*}) \left(\varrho_{\pi^*} \hat{\pi}_t^* + \varrho_{y^*} \hat{y}_t^* \right) + \varepsilon_{r^*,t} \quad (\text{B.83})$$

Appendix C

Appendix to Chapter 3

C.1 Response of the Small-scale DSGE Model to Shocks

Figure C.1: Response of DSGE Model to a Monetary Policy Shock

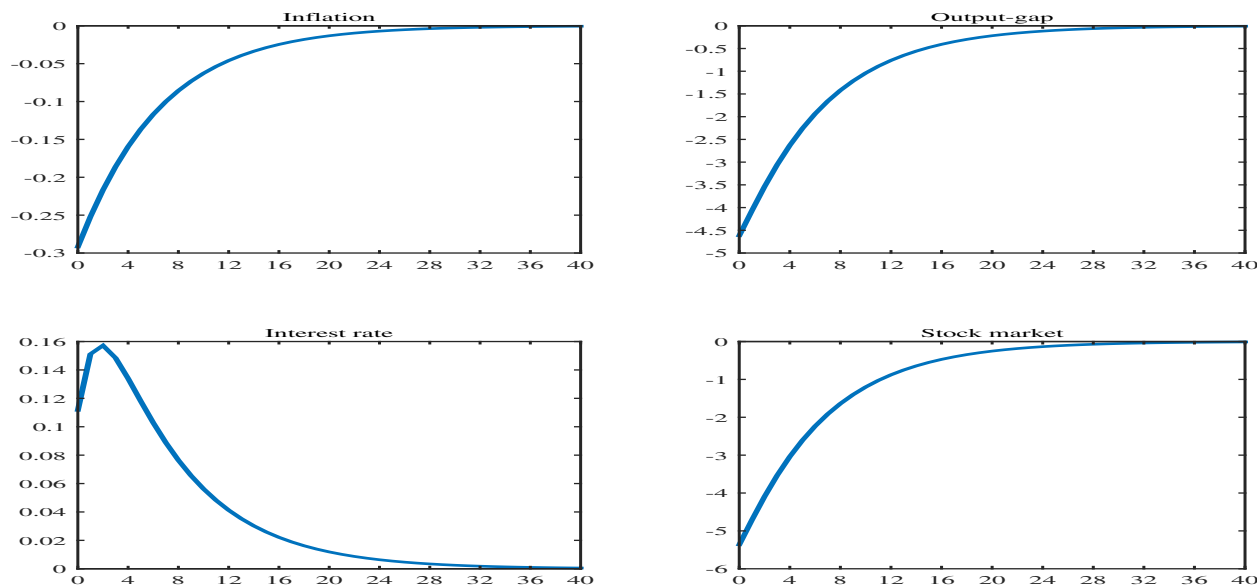


Figure C.2: Response of DSGE model to a Stock Market Shock

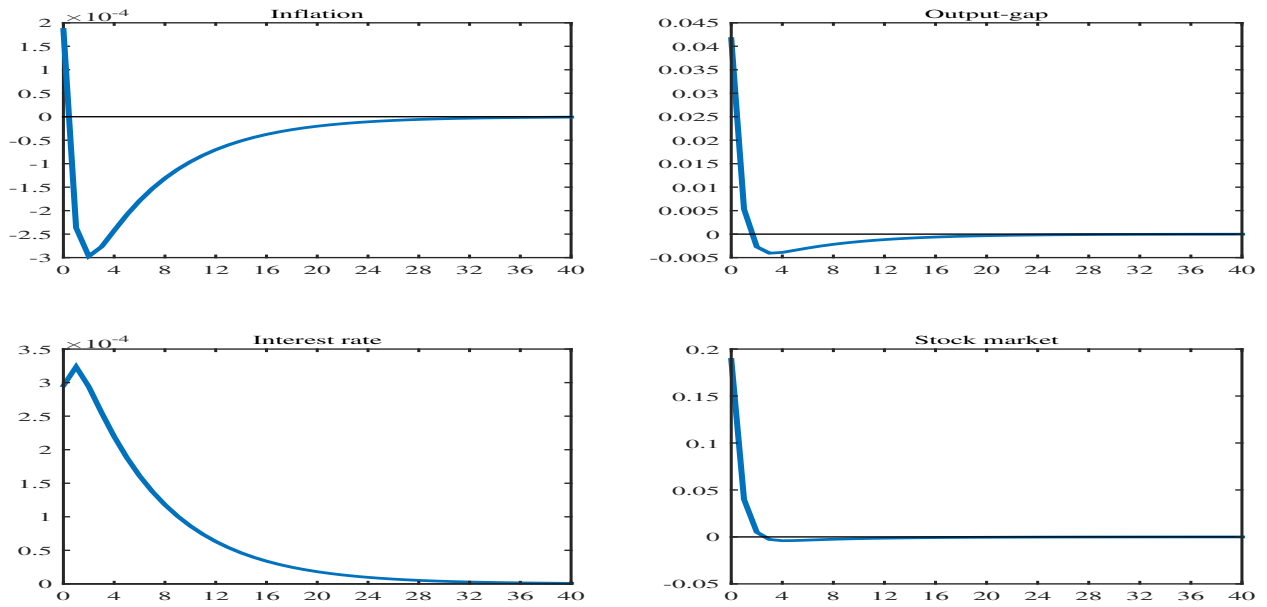
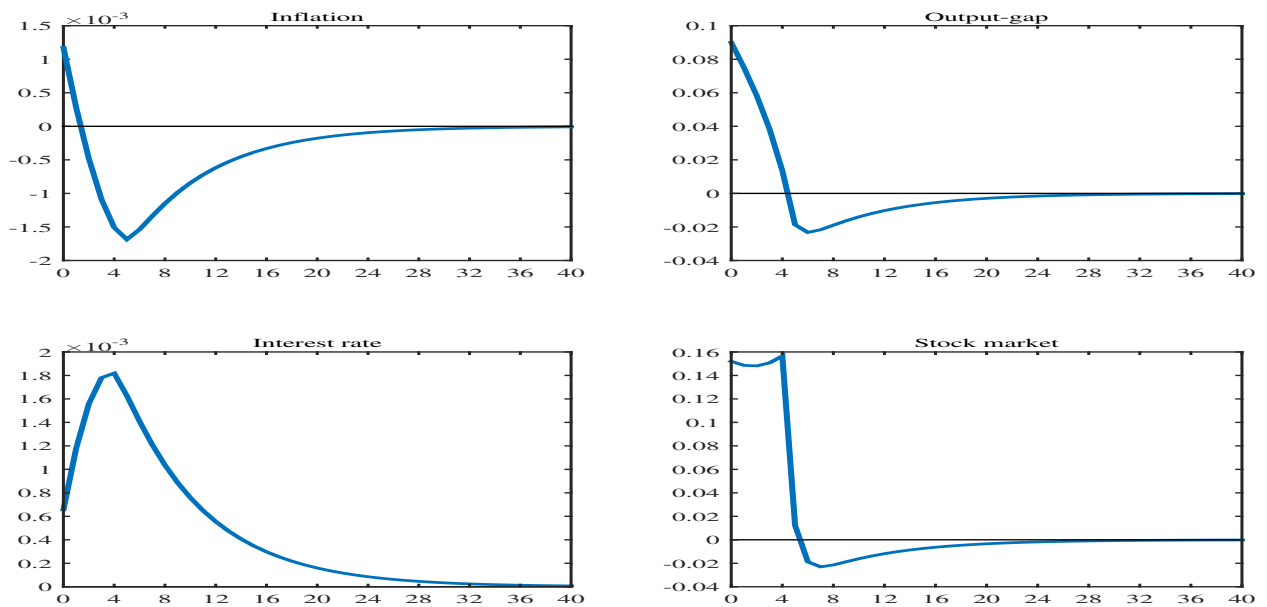


Figure C.3: Response of DSGE Model to a News Shock



C.2 Data Series

Figure C.4: Canadian Data

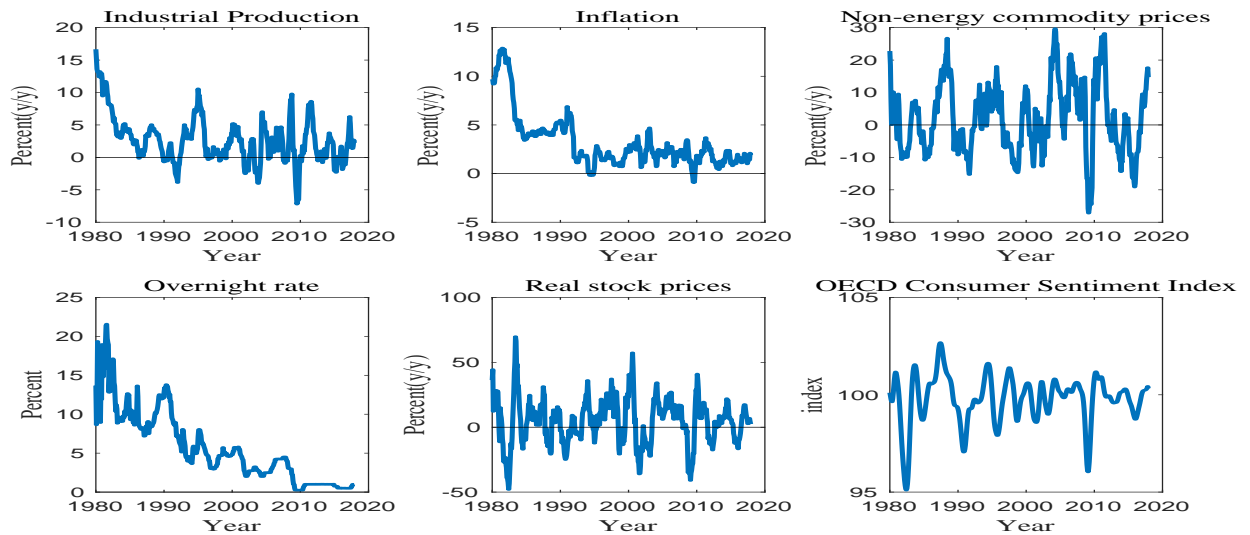
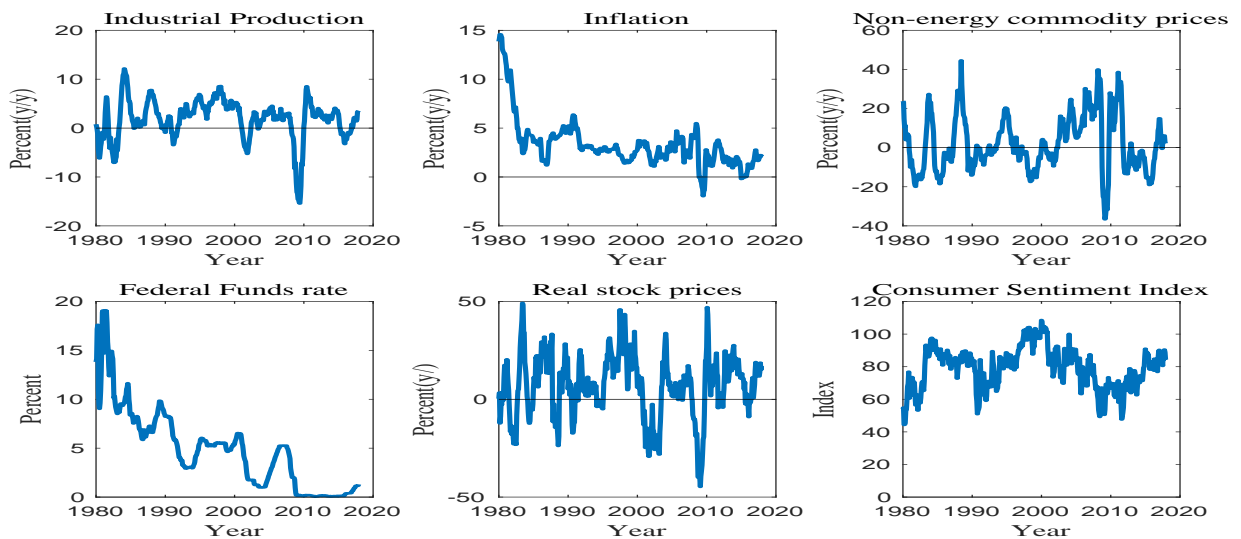


Figure C.5: US Data



C.3 Impulse Responses: Short-run Identification

Figure C.6: Response of SVAR Model to a Monetary Policy shock in Canada- Short-run Identification

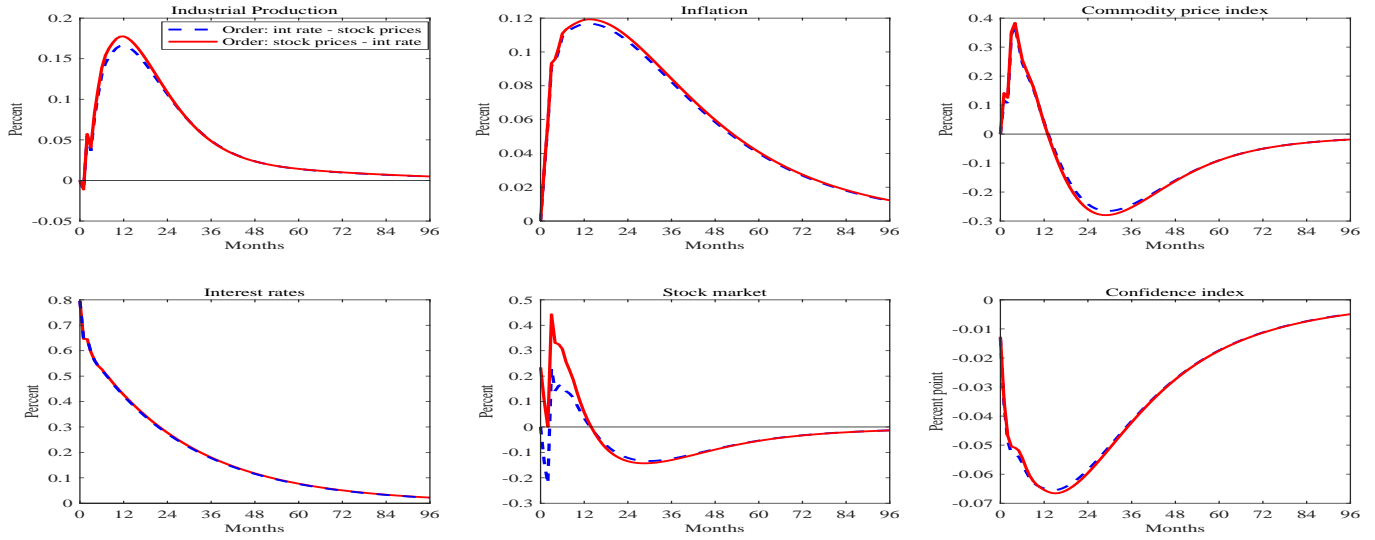


Figure C.7: Response of SVAR Model to a Stock Market Shock in Canada- Short-run Identification

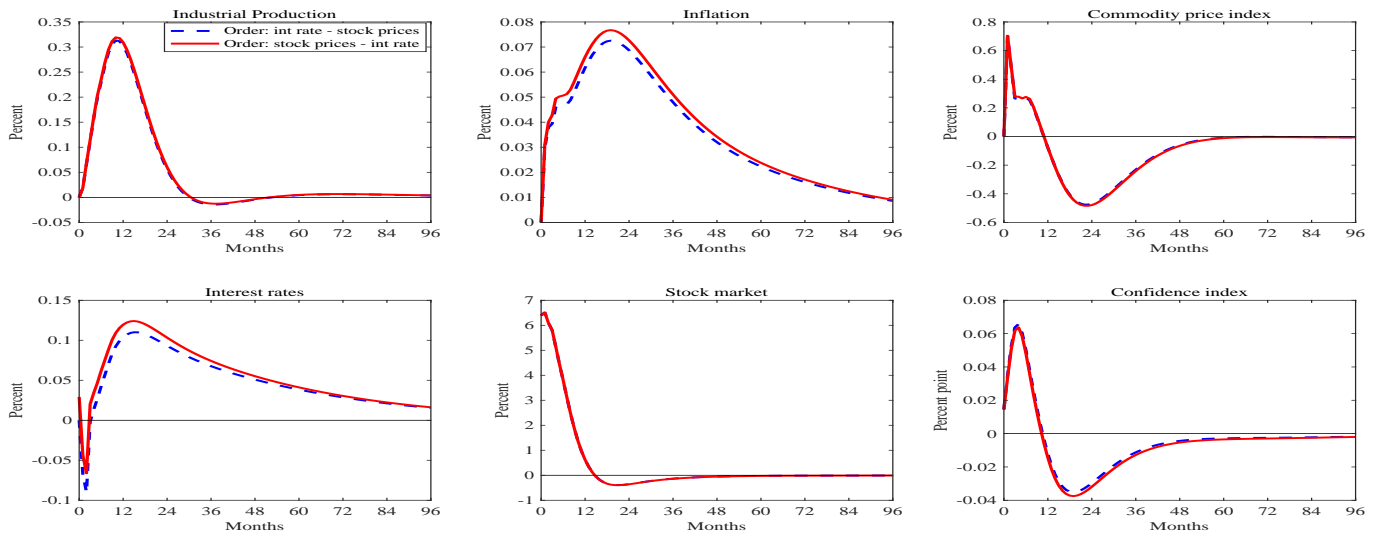


Figure C.8: Response of SVAR model to a monetary policy shock in the U.S.-
Short-run Identification

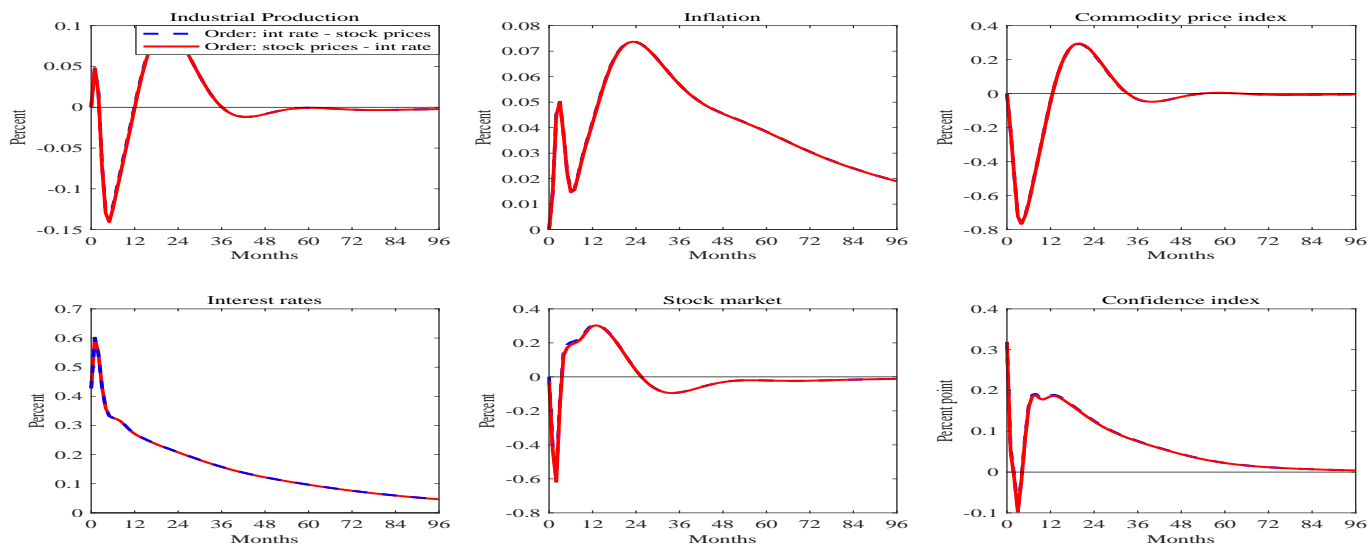
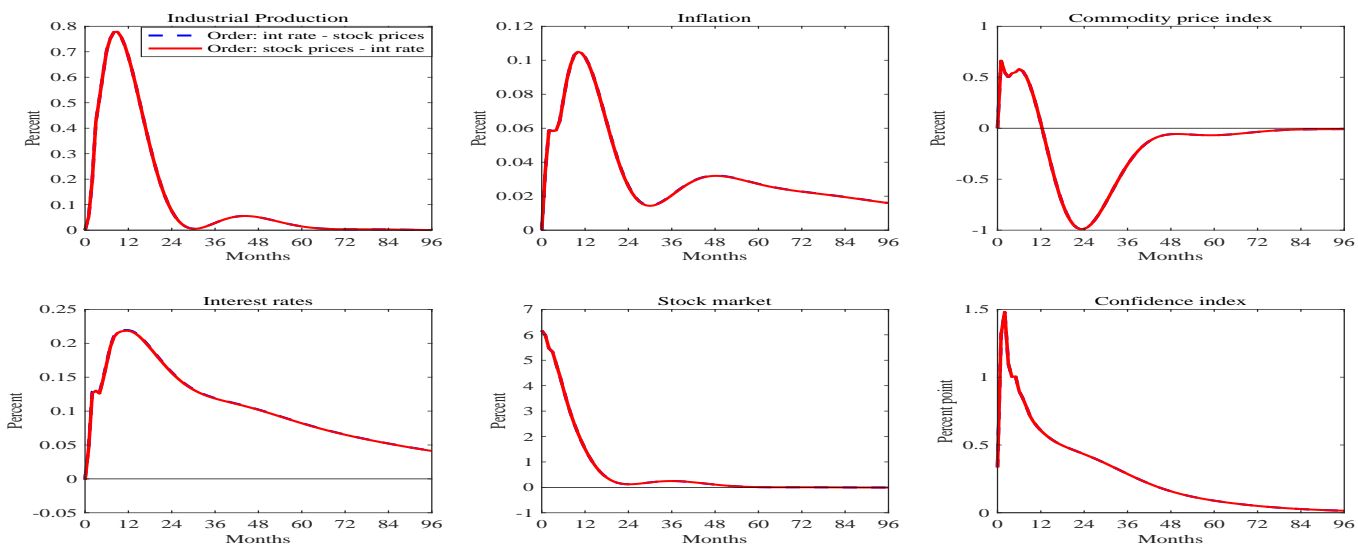


Figure C.9: Response of SVAR model to a stock market shock in the U.S -
Short-run Identification



C.4 Robustness Graphs: Short-run Restrictions

Figure C.10: Response of SVAR model to a policy rate shock in Canada with some short-run restrictions

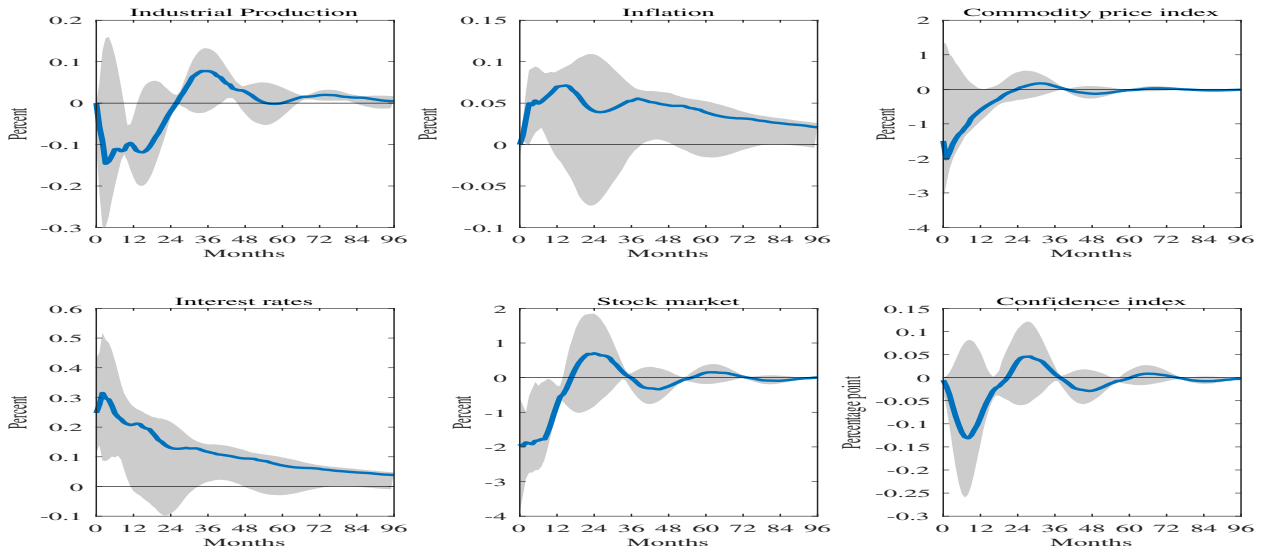


Figure C.11: Response of SVAR model to a policy rate shock in the U.S. with some short-run restrictions

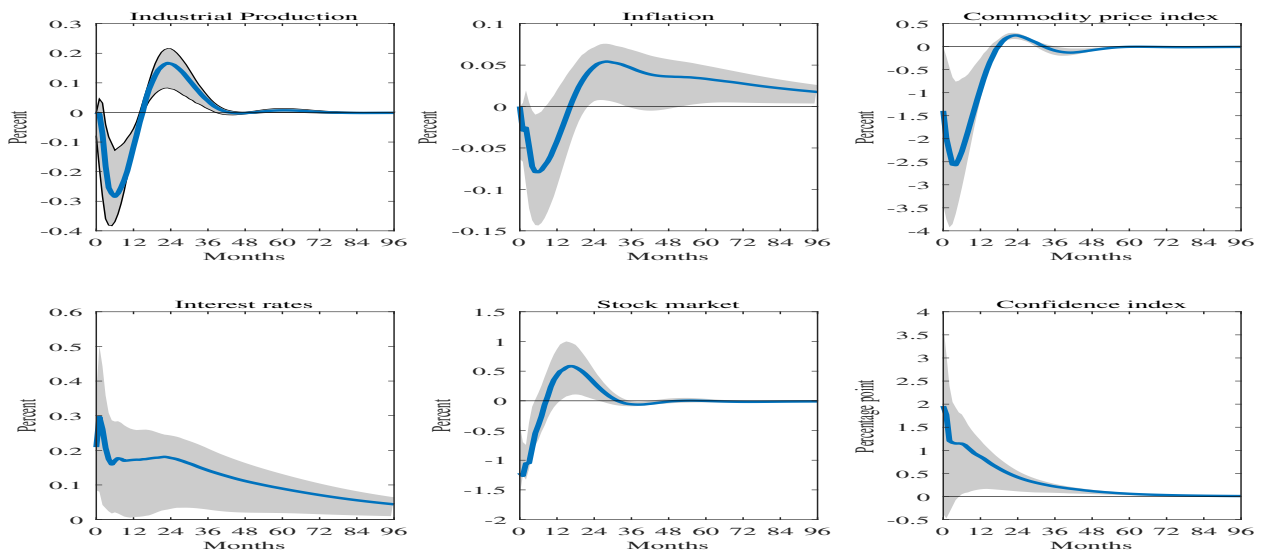


Figure C.12: Response of SVAR model to a stock market shock in Canada with some short-run restrictions

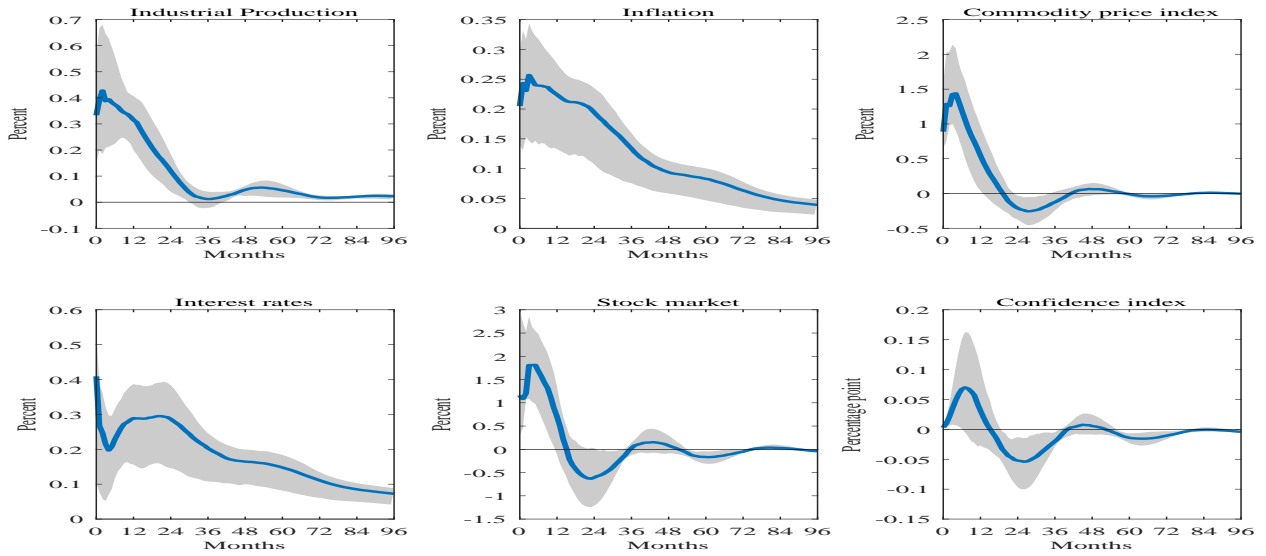
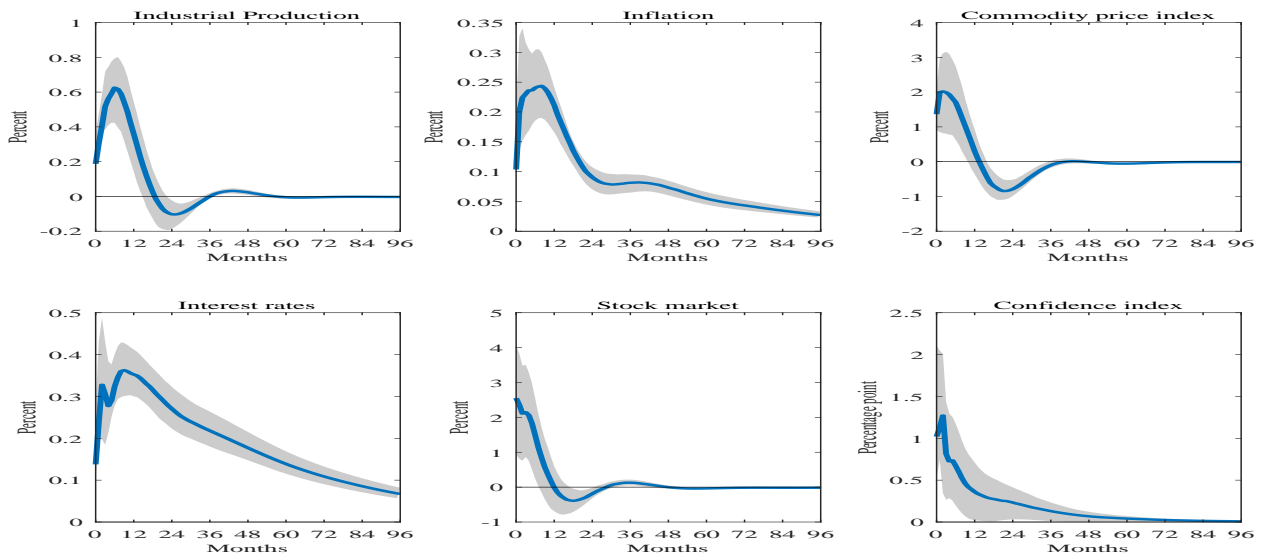


Figure C.13: Response of SVAR model to a stock market shock in the U.S. with some short-run restrictions



C.5 Robustness Graphs: 4-variable VAR

Figure C.14: Response of 4-variable SVAR model to a policy rate shock in Canada

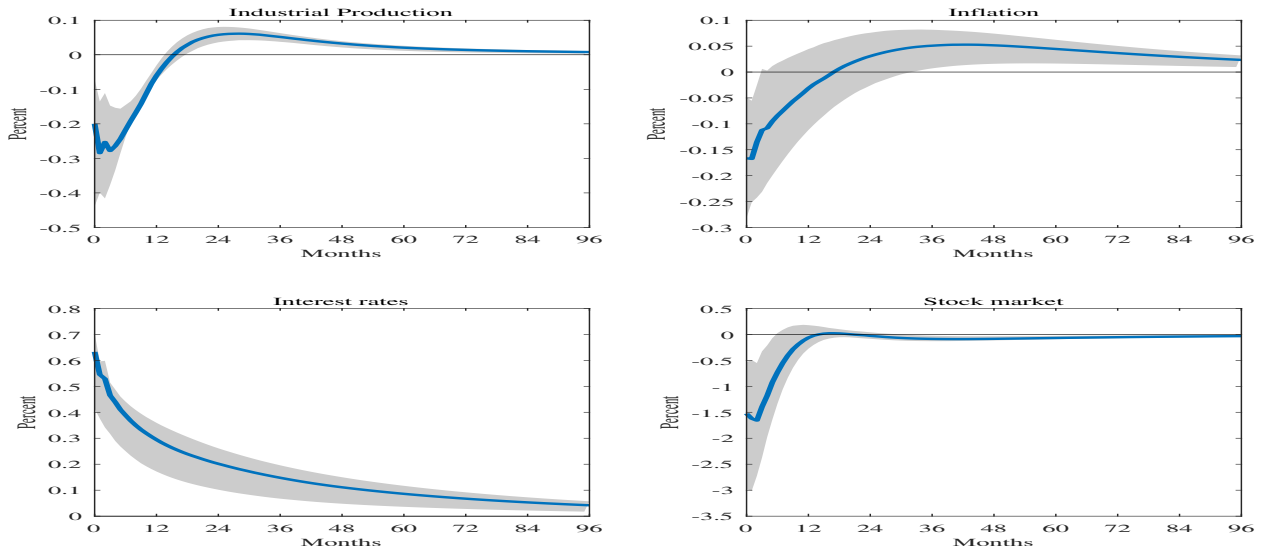


Figure C.15: Response of 4-variable SVAR model to a policy rate shock in the U.S.

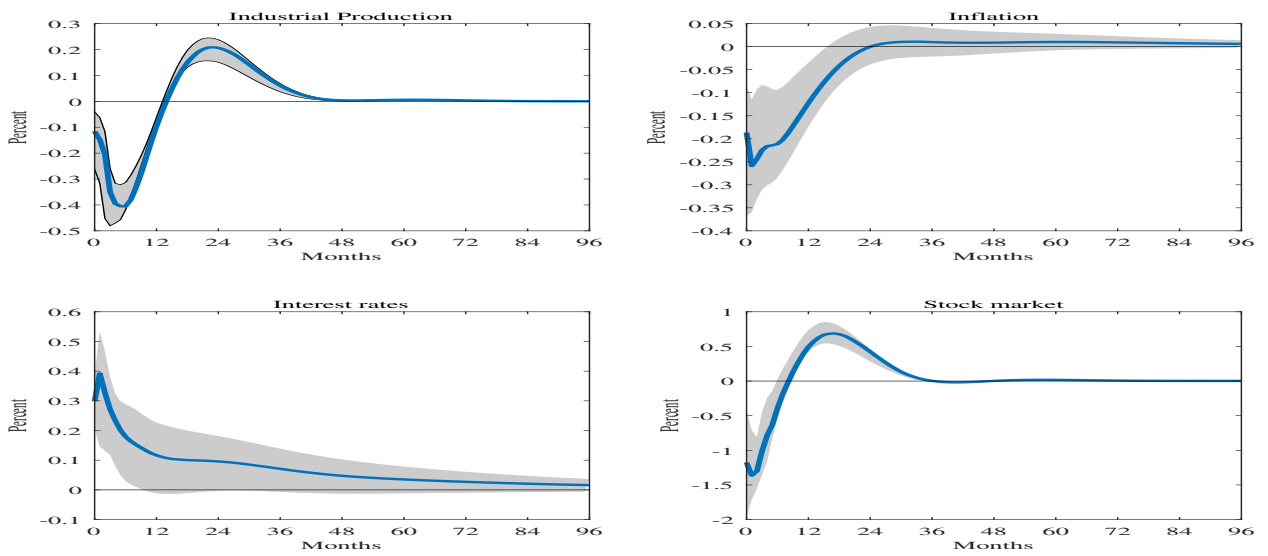


Figure C.16: Response of 4-variable SVAR model to a stock market shock in Canada

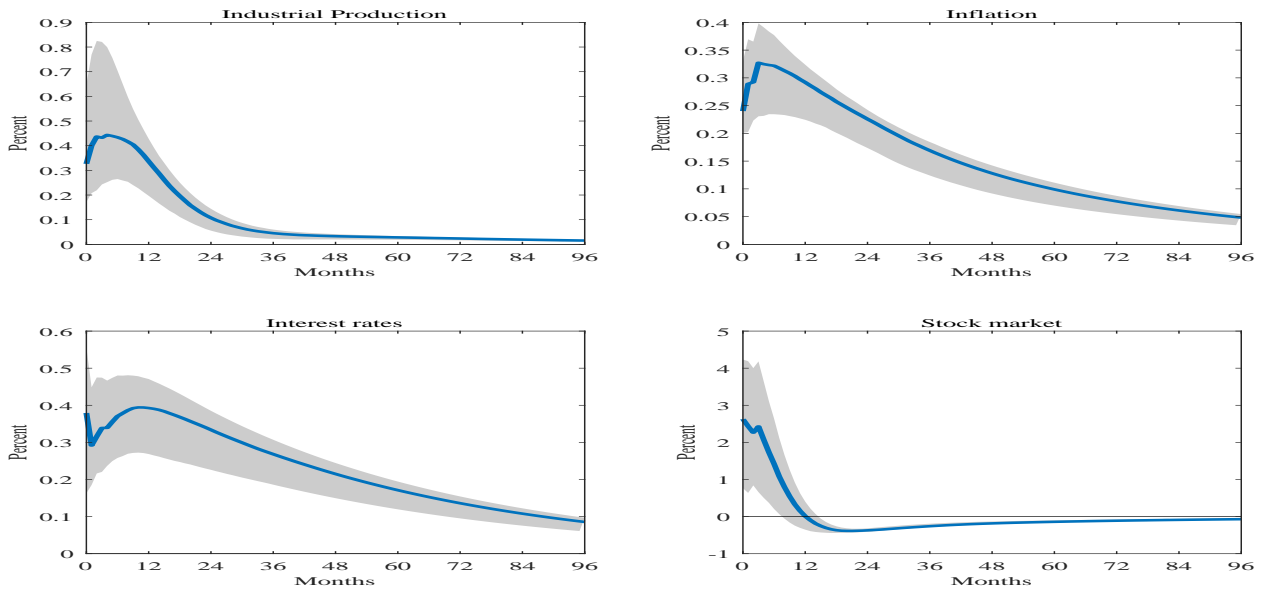


Figure C.17: Response of 4-variable SVAR model to a stock market shock in the U.S.

