

# New Methods for Channel Allocation Schemes in wireless Cellular Networks

by

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## Abstract

Due to the fast-growing number of cell-phone users, channel allocation scheme for channel assignment plays an important role in cellular networks. Having known that for downlink transmission the base-station aims to transmit signals over a specific channel to different users, broadcasting enhancement gains momentum in cellular networks. Consider a set-up in which the base-station is equipped with  $M$  time/frequency resources and  $M$  clients are being served with their own rate requirements. Optimum solution in a degraded broadcast channel is to send signals to all users over all channels. However, this enlarges the codebook, which leads to an intricate coding/decoding system. Taking this into account, the idea of grouping the clients into smaller subsets has been proposed in this research. The required rate for each client should be satisfied in each subset, which determines the size of the groups. As the highest gain in each subset is obtained by applying broadcasting scenario, one can just focus on finding the best method of grouping; henceforth, each group follows broadcasting scenario.

Similarly, the same scenario applies to uplink transmission in which clients transmit signals to a single base station.

Intuitively, the optimum solution can be achieved using linear programming. However, it turned out linear programming satisfies the zero-one constraint which can be efficiently solved by Hungarian method. It has been shown that this practical method conspicuously outperforms the traditional frequency division method.

## **Acknowledgements**

I would like to express my special appreciation and thanks to my supervisor Prof. Amir Keyvan Khandnai. Your advice on my research and my career has been invaluable.

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A special thanks to my wife. Thanks for encouraging me throughout this experience and your support was what sustained me thus far. A special thanks to my family. I cannot put it into words how grateful I am to my mother and father.

## **Dedication**

This is dedicated to the one I love.

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# Abbreviations

**AWGN** additive white Gaussian noise 3, 6, 8, 34

**FDMA** frequency division multiple access 2, 30, 47

**MAC** multiple access channel 3, 31, 33, 35, 45, 46

**NP** nondeterministic polynomial time 43

# Chapter 1

## Overview and Literature Review

### 1.1 Overview

In the early mobile systems, there was only a single transmitter installed on a very high location. Only one station covers the entire region. The range for the transmitter was up to 50 km. In order to cover this range, the transmitter needs a lot of power. The problem with this approach was that if a frequency was used in the region of coverage, the same frequency cannot be reused by other users. The transmitter needs to be very powerful in this case.

What cellular network suggested is as follows:

Instead of having one transmitter, multiple transmitters with low power can be exploited. Since they have limited power, they can cover a smaller region with respect to the early transmitters. Each station covers a specific area. These transmitters are known as Base Station (BS) and each of them can be used as both transmitter and receiver; that is why they are also called transreceiver. The area that is covered by one BS is known as a cell. Each BS is provided with a portion of entire existent channels (frequencies).

There are some criteria for determining the shape of the cells. For instance, the area of the shapes should not overlap, the shapes should be geometric and should cover the maximum possible area. By considering these benchmarks, the hexagonal shape has been chosen as the shape of the cells, since it covers the maximum area.

Nevertheless, what would be the advantages of cellular networks with respect to the early mobile systems?

If the number of users increases, traffic would also increase and to solve this problem more base station can be installed without using more frequencies. This could be practical by using frequency reuse. In other words, one frequency can be used in another cell if the distance between these two cells is greater than a predefined number.

In order to avoid interference in cellular networks, some methods have been introduced. Under the simplest conditions, a medium can carry only one signal at any moment in time. For multiple signals to share one medium, the medium must somehow be divided, which is known as multiplexing.

Multiple access is nothing but the application of multiplexing. Two well-known ways of multiplexing are time division and frequency division multiplexing. Multiplexing is a technique, in which several messages are combined into a composite signal such that these can be transmitted over a common channel. In [frequency division multiple access \(FDMA\)](#), all clients use the same channel at the same time. Here, the available frequency bandwidth is divided between users accordingly. Each user has its own bandwidth to use and they are allowed to transmit their signals for full time. A different frequency is allocated to each user to avoid interference. In addition, there is a possibility of crosstalk in [FDMA](#), for all the users transmit their signals at the same time.

However, what is the best way of allocating the existent channels to the users?

From the genesis of wireless cellular networks, channel allocation schemes for channel assignment have been the backbone of many studies over cellular networks. Due to the fast-growing number of cell-phone users, how the base station transmits signals to different users and how the clients attempt to transmit their signals to a base station have gained much momentum. This report has been divided into two main parts:

1. Downlink transmission
2. Uplink transmission

In each of the abovementioned parts, the writer attempts to find a decent way of channel allocation for both transmitter (in downlink transmission) and receivers (in uplink transmission).

In the following chapters, it is assumed that the base station is equipped with  $M$  time/frequency recourses, called tones. For downlink transmission, if the base station transmits signals to all clients simultaneously using all tones—which is known as broadcast transmission—the total consumed power would be minimized. In other words, the optimum solution in a broadcast channel is to send signals to all clients, through the entire  $M$  tones by

means of superposition coding and successive decoding. Similarly, for uplink transmission, a [multiple access channel \(MAC\)](#) gives us the highest gain. Nevertheless, when the number of clients increases, these two methods become overly complicated.

On the other hand, if the base station transmits signals to each client through one tone—traditional frequency division—, inter-client interference would disappear. The same scenario happens when in uplink transmission each client transmits signals to a base station through one tone. However, it is obvious that these two methods would not give a reasonable gain and by using them, existed resources have not been exploited properly.

This report aims to find a strategy between mentioned extreme cases for both uplink and downlink transmission, in which one, two, three or four clients are grouped and serviced in one tone based on their rate requirements and channel gains. Careful scrutiny of the proposed method has revealed that corresponding gain is significant in comparison with one-client-per-tone method.

The thesis is organized into two main chapters. Chapter 2 addresses downlink transmission. Section 2.1 describes the linear assignment method, which would be used for optimization purposes in this thesis. In addition, the basic properties of an [additive white Gaussian noise \(AWGN\)](#) channel have been briefly explained in this part. Section 2.2 defines the set-up which is used in this thesis and introduces the idea of grouping the clients. Section 2.3 formulates the idea of grouping the clients such that it could be solvable by using linear optimization. Also, this part analyses three different ways of grouping, which are groups of two, three and four clients. Section 2.4 assesses the advantages of grouping the clients by simulating the cases with MATLAB. Section 2.5, the last part of chapter 2, provides a brief summary of what has been mentioned in chapter 2 and makes a conclusion about what has been obtained based on MATLAB simulations.

Chapter 3 focuses on uplink transmission. Section 3.1 addresses the basic features of a [MAC](#) set-up. In this part, the capacity region of a [MAC](#) system has been plotted and also the way to obtain this region is briefly discussed. Section 3.2 brings up the idea of grouping the clients in uplink transmission and explains why [MAC](#) always leads to the highest gain. Section 3.3 formulates the idea of grouping in uplink transmission and attempts to find a way such that the problem is solvable with linear optimization. This section ends up finding a criterion for optimizing the total rate. This criterion brings about some grouping algorithms which are addressed in section 3.4. Moreover, in section 3.4, the algorithms for different ways of grouping have been assessed by MATLAB. Section 3.5 summarizes chapter 3 and suggests what would be the best way of uplink transmission in the defined set-up.

## 1.2 Literature Review

Many methods have been introduced for channel assignment for mobile communications. Among them, the most related ones have been briefly discussed here.

For downlink transmission, it has been generally accepted as an optimal solution that each subcarrier is allocated to the users with the best channel condition and power is allocated by the water-filling over subcarriers [9], [2]. In [2], in formulating sum-rate maximization problem, a subcarrier could be exploited by multiple users at the same time and the power allocation scheme is obtained by subcarrier assignment for users and power allocation for subcarriers. It has been found that sum-rate of a multiple access channel is maximized when each subcarrier is allocated to one user and total power is distributed over the users based on water-filling. In addition, to simplify the required mathematical calculation to obtain water-filling level, a simple method has been proposed in which users with the best channel gain for each subcarrier are chosen and then the power is evenly distributed among them.

In [11], some sub-carrier allocation algorithms for multiple access scheme in downlink transmission have been proposed. The main goal of this paper is to minimize the total consumed power. This problem has been solved by using improved Hungarian linear assignment algorithm [4]. An optimal subcarrier assignment and power allocation based on utility has been introduced in [9]. In this paper, the optimization of dynamic channel allocation on downlink transmission is analyzed with the purpose of maximizing the aggregate network utility.

For uplink transmission, at first power-frequency allocation is discussed in [14]. It turned out the optimal subcarrier allocation is a simple two-band partition and the power allocation follows the multiuser water-filling; however, the results are valid only when SNR for the two users is the same. In [6], the goal is to maximize the rate-sum capacity. In this paper, by using Karush-KuhnTucker (KKT) condition, a method for subcarrier allocation based on marginal rate function and a method for power allocation based on water-filling have been proposed. Additionally, water-filling method and even power distribution in uplink transmission have the same performance, when they are combined with the proposed method in [6].

# Chapter 2

## Downlink transmission

### 2.1 Linear Assignment

The assignment problem has been studied extensively in optimization. It is defined as the assignment of a number of agents to a number of tasks. It is necessary to perform all tasks by a one-by-one agent-task assignment. Any agent assignment to a task has a defined and distinguished cost.

Linear assignment problem is defined as follows:

If the number of agents and tasks are equal and the total cost for all tasks becomes equal to the sum of the costs for each agent, then the problem is called the linear assignment problem [8].

A schematic view of the assignment problem for n-agents/n-tasks has been illustrated in 2.1.

Mathematically, the linear assignment problem can be defined as follows. Consider a square real-valued weight matrix  $C$ , with one agent and one task sets as  $A$  and  $T$ , respectively. The problem is to find the function  $f : A \rightarrow T$  such that the following cost function is minimized:

$$\sum_{a \in A} C_{a, f(a)} \quad (2.1)$$

One can formulate the problem as a standard linear program with the following objective function:

$$\sum_{i \in A} \sum_{j \in T} C(i, j) \lambda_{ij} \quad (2.2)$$



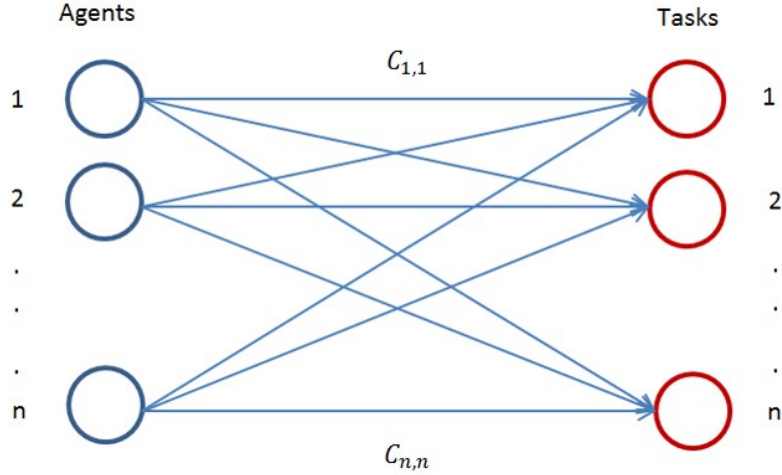


Figure 2.1: Linear assignment problem scheme

and the following constraints:

$$\sum_{j \in T} \lambda_{ij} = 1 \quad \text{for } i \in A, \quad (2.3)$$

$$\sum_{i \in A} \lambda_{ij} = 1 \quad \text{for } j \in T, \quad (2.4)$$

$$\lambda_{ij} \geq 0 \quad \text{for } i, j \in A, T. \quad (2.5)$$

The variable  $\lambda_{ij}$  is equal to 1 if task  $j$  is done by agent  $i$  and otherwise it is equal to 0.

### 2.1.1 Achievable Rate for the Gaussian Channel

Consider an [AWGN](#) channel with output  $Y$ , where  $Y$  is the sum of input  $X$  and noise  $n$ . A schematic view of the set-up has been illustrated in figure 2.2. Meanwhile, the additive noise has a zero-mean normal distribution with variance  $N$ . Hence,

$$Y = X + n, n \sim \mathcal{N}(0, N) \quad (2.6)$$

The channel capacity of an [AWGN](#) channel is

$$C = \max_{EX^2 \leq P} I(X; Y) = \frac{1}{2} \log \left( 1 + \frac{P}{N} \right) \quad (2.7)$$

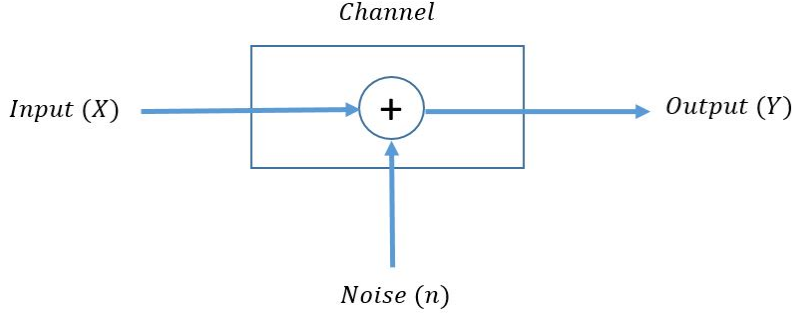


Figure 2.2: Additive white Gaussian channel

where  $I(X; Y)$  is the mutual information between  $X$  and  $Y$ .  $P$  and  $N$  are the power of signal and noise, respectively.

On the other hand, a rate  $R$  is achievable for a Gaussian channel with a power constraint  $P$  if there exists a sequence of codes with code-words satisfying the power constraint. The capacity of the channel is the supremum of the achievable rates [1]. Therefore, we have:

$$R \leq C = \frac{1}{2} \log \left( 1 + \frac{P}{N} \right). \quad (2.8)$$

In addition, given a bandwidth  $w$ , the rate would be

$$R = \frac{w}{2} \log \left( 1 + \frac{P}{N} \right). \quad (2.9)$$

By normalizing  $w = 1$ , the required power to satisfy the known rate  $R_i$  for the  $i^{\text{th}}$  client will be

$$P_i = N_i \times (2^{2R_i} - 1). \quad (2.10)$$

For  $K$  independent Gaussian channels **in parallel** with a common power constraint, it is necessary to distribute the total power among the channels in order to maximize the capacity. For channel  $j$ ,  $Y_j = X_j + n_j, j = 1, 2, \dots, K$  with  $n_j \sim \mathcal{N}(0, N_i)$ , and  $n_1, n_2, \dots, n_K$  are independent. We have

$$C \leq \sum_i \frac{1}{2} \log \left( 1 + \frac{P_i}{N_i} \right), \quad (2.11)$$

where  $\sum P_i = P$ .

Also, the maximum capacity can be obtained by using water-filling method [1].

Henceforth, we consider a Gaussian broadcast channel. As can be seen in figure 2.3, for the sake of simplicity, one transmitter and two receivers are depicted, which can be generalized to M receivers.

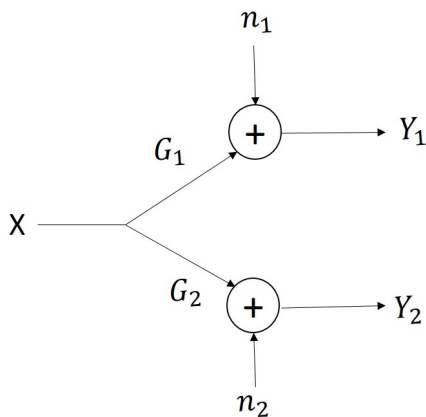


Figure 2.3: Broadcast channel with two receivers

Consider an AWGN channel with output  $Y$ , where  $Y$  is the sum of input  $X$  and noise  $n$ . Meanwhile, the additive noise has a zero-mean normal distribution with variance  $N$ . Thus,

$$Y = X + n, n \sim \mathcal{N}(0, N) \tag{2.12}$$

The transmitter sends signals with power  $P$  to the two receivers with channel gain  $G_1$  and  $G_2$  and independent Gaussian noise distributions. For such a channel, the received signal will be  $Y_1 = G_1.X + n_1$  and  $Y_2 = G_2.X + n_2$ , where  $n_1$  and  $n_2$  are Gaussian random variables with variances  $N_1$  and  $N_2$ . It is necessary to send independent messages at rates  $R_1$  and  $R_2$  to receivers  $Y_1$  and  $Y_2$ , respectively. The channel can be re-characterized as shown in Figure 2.4.

One can find the capacity region for the Gaussian broadcast channel, with signal power constraint  $P$  by time-sharing as follows:

$$R_1 \leq \frac{1}{2} \log \left( 1 + \frac{\alpha P}{N_1} \right), \tag{2.13}$$

$$R_2 \leq \frac{1}{2} \log \left( 1 + \frac{(1 - \alpha)P}{\alpha P + N_2} \right), \quad \text{for } 0 \leq \alpha < 1. \tag{2.14}$$

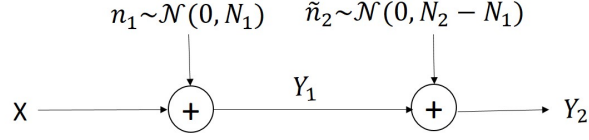


Figure 2.4: Re-characterized broadcast channel

Receiver  $i \in \{1, 2\}$  deals with a constant channel fading gain,  $G_i$ , during the study period. Also, in channel  $k \in \{1, 2\}$ , the signal for one receiver appears as an interference for the other receiver. Therefore, if  $G_1 > G_2$ , we have

$$R_1 \leq \frac{1}{2} \log \left( 1 + \frac{G_1^2 \cdot P_1}{N} \right) \quad (2.15)$$

and

$$R_2 \leq \frac{1}{2} \log \left( 1 + \frac{G_2^2 \cdot P_2}{G_2^2 \cdot P_1 + N} \right), \quad (2.16)$$

in which,  $P = P_1 + P_2$ .

In broadcast set-up, the client with a given channel gain can decode the signals sent to clients with a lower channel gain. Therefore, the rate of the  $i^{\text{th}}$  client in the  $j^{\text{th}}$  tone, denoted as  $R_j^i$  will be

$$R_j^i = \frac{1}{2} \log \left( 1 + \frac{G_i^2 P_j^i}{\sigma^2 + G_i^2 \sum_{m \in A_i} P_j^m} \right), \quad (2.17)$$

in which  $\sigma^2$  is the power of Gaussian noise and  $A_i$  is the set of clients with channel gains higher than the  $i^{\text{th}}$  one. Therefore, we have

$$P_j^i = (2^{2R_j^i} - 1) \times \frac{\sigma^2 + G_i^2 \sum_{m \in A_i} P_j^m}{G_i^2}. \quad (2.18)$$

It is worth mentioning to say that for the broadcast scenario with  $M$  receivers, in order to satisfy the rate requirement for each receiver,  $R_i$ , it is required to have:

$$R_i = \sum_j R_j^i. \quad (2.19)$$

In the broadcast set-up, each client receives  $\frac{1}{M}$  fraction of its required rate in each tone. Therefore,  $R_j^i = \frac{R_i}{M}$ . Now, considering the Equation (2.18), the total power used for the  $i^{th}$  client is

$$P_i = M \times \left(2 \frac{2R_i}{M} - 1\right) \times \frac{\sigma^2 + G_i^2 \sum_{m \in A_i} P_j^m}{G_i^2} \quad (2.20)$$

where

$$P_i = \sum_j P_j^i. \quad (2.21)$$

The above-mentioned equations will be used in the following sections to find the minimum transmitted power such that the required rates are achieved.

## 2.2 System model

In this section, the following set-up for a downlink transmission has been considered:

- There exist one transmitter and  $M$  receivers (clients).
- The transmitter is assumed to have  $M$  frequency bands to service the clients.
- The requested rate for each client is known and fixed over the study period, defined as  $R_i, i \in 1, 2, \dots, M$ .
- There exist  $K = M$  independent Gaussian channels with the variance  $\sigma^2$ . Every channel has a fading gain  $G_i, i \in 1, 2, \dots, M$  which is assumed to be constant and known over the study period.

That being said, we have  $K$  frequency tones available to be assigned for transmissions.

The clients are grouped in  $Q$ . Here, groups of  $Q$  is defined as a grouping of the clients such that each group has at least one and at most  $Q$  clients. Noted that each client can exist in one or more groups at the same time. However, each tone is allocated to one and only one group and applies broadcast scheme to that group which is illustrated in figure 2.5.

In this report, four cases of  $Q = 2$ ,  $Q = 3$  and  $Q = 4$  have been considered and formulated. The examples of a Single-tone and a broadcast scheme for  $k = 5$  have been

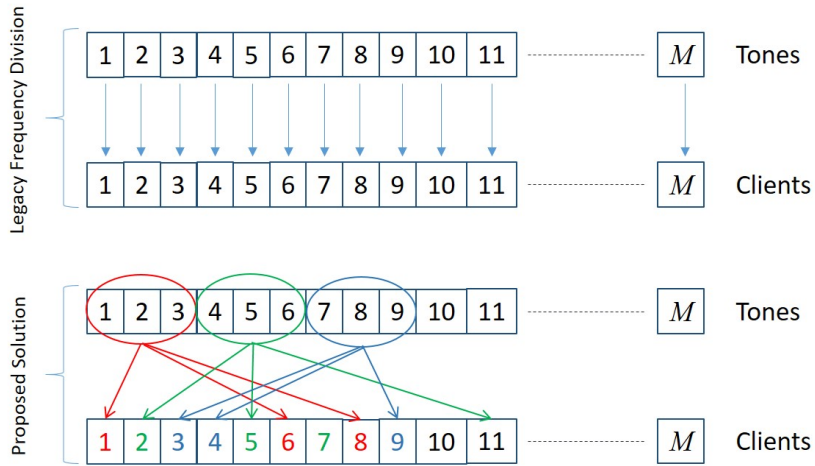


Figure 2.5: Clients grouping solution vs. traditional frequency division

illustrated in Figure 2.6. The examples of grouping of two, three, and four clients have been shown in Figure 2.7. Noted that an arrow from a tone to a client means that the client receives a fraction (or all) of its required rate from that tone. Moreover, in the grouping idea, the broadcast scenario is applied to each group separately.

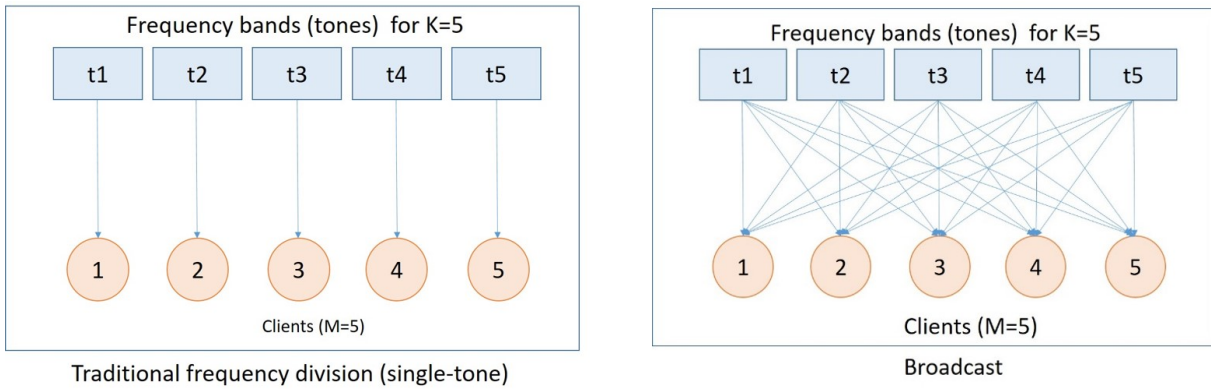


Figure 2.6: Broadcast and traditional frequency division schemes

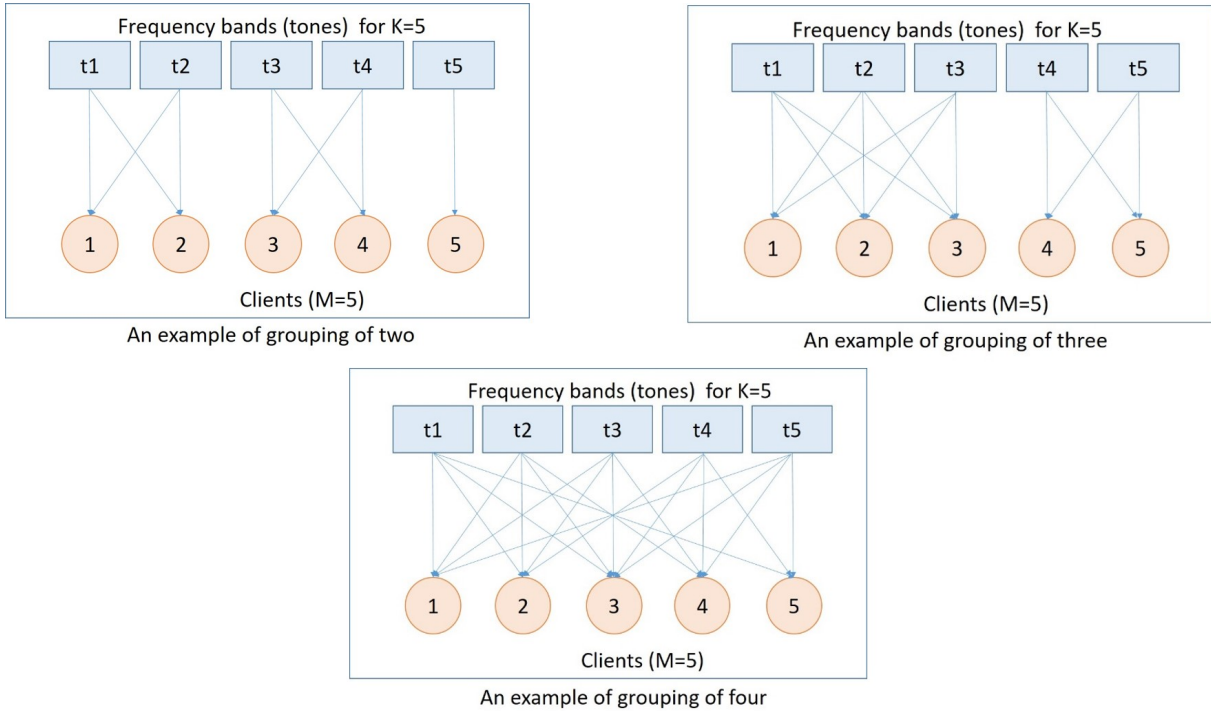


Figure 2.7: Examples of grouping

## 2.3 Problem formulation

In this section, we aim to minimize the total required power to send code-words to the clients, such that all the required rates are achieved. We take advantage of the fact that the total power varies by changing the combination of groups of  $Q$ . Therefore, we have to find the best choice from the possible combinations of groups of  $Q$  which minimizes the total required power. This problem is formulated as the linear assignment problem, described in section 2.1. The agent/task sets are assumed to be a similar set of clients  $A = \{1, 2, \dots, M\}$  for  $Q = 2$ . The latter is to choose the first client (agent) and the second client (task) to be grouped. Moreover, the problem is generalized to  $Q > 2$  by increasing the cost function dimensions.

The required power to transmit code-words to the clients of every possible group is calculated based on the requested rate and the channel gain as calculated in section 2.1.1. These power values generate the weight matrix  $C$ . Then, the general problem can be

formulated as

$$\min_{\lambda} \sum C.\lambda \tag{2.22}$$

in which  $\lambda$  is a matrix with elements showing the grouped clients. If an element has a non-zero value, its index shows the clients that are meant to form a group. The value of that element is corresponding to the number of required tones for that group. There exist some constraints on  $\lambda$  with the following descriptions:

1. Elements of  $\lambda$  matrix can get values between zero and one.
2. The summation over  $\lambda$  elements with any specific index  $i$  should be equal to one, since every client  $i$  needs to be serviced only one time.
3. The summation over all  $\lambda$  elements should be equal to the number of tones.
4. Index order of  $\lambda$  elements has no effect in this problem. Therefore,  $\lambda_{xy}$  and  $\lambda_{yx}$  have the same meaning and get the same values.
5. The summation over  $\lambda$  elements that describe the same group of clients shows the number of tones allocated to that group. For example, the value of  $\lambda_{xy} + \lambda_{yx}$  is the amount of allocated tones to the group of  $x$  and  $y$  in groups of two clients.

In order to clarify the formulation details for groups of two, three and four scenarios, the formulation will be elaborated separately in the following sections.

### 2.3.1 Groups of two clients

For groups of two clients, there exist single element groups and/or groups of size two. The weight matrix  $C$  is a two-dimensional  $M \times M$  matrix. It is generated such that

$$C(i, j) = \begin{cases} P_i & \text{if } i = j, \\ P_{i,j} & \text{if } i \neq j. \end{cases}$$

where,  $P_i$  is the required power to send to only client  $i$  in one tone. Moreover,  $P_{i,j}$  is the summation of the required power to send to clients  $i$  and  $j$  in one tone, such that  $R_i/2$  and  $R_j/2$  are achieved. Achieving half of the requested rate in one tone is due to sharing the channel between two clients. It is obvious that the other half rate is achieved in other tones. Figure 2.8 shows the assignment matrix for group of size 2 with  $M = 4$ .



	1	2	3	4
1	Energy to service client 1 in one tone	Energy to service clients 1 and 2 in one tone (half of their rates)	Energy to service clients 1 and 3 in one tone (half of their rates)	Energy to service clients 1 and 4 in one tone (half of their rates)
2	Energy to service clients 1 and 2 in one tone (half of their rates)	Energy to service client 2 in one tone	Energy to service clients 2 and 3 in one tone (half of their rates)	
3	Energy to service clients 1 and 3 in one tone (half of their rates)	Energy to service clients 2 and 3 in one tone (half of their rates)		
4	Energy to service clients 1 and 4 in one tone (half of their rates)			

Figure 2.8: Weight matrix for grouping of two clients with  $M = 4$

The energy to service two clients is repeated in the matrix two times as shown by the same colors. Therefore, totally we have energy to service two clients in two tones with their rates completely achieved.

Based on equations 2.10, 2.15 and 2.16,  $P_i$  and  $P_{i,j}$  are calculated as

$$P_i = (2^{2R_i} - 1) \cdot (\sigma^2) / G_i^2, \quad (2.23)$$

$$P_{i,j} = \begin{cases} (2^{R_j} - 1)(S_i G_i^2 + \sigma^2) / G_i^2 + S_i, & \text{if } G_i > G_j, \\ (2^{R_i} - 1)(S_j G_j^2 + \sigma^2) / G_j^2 + S_j, & \text{if } G_i \leq G_j, \end{cases}$$

in which  $S_i = (2^{R_i} - 1) \cdot (\sigma^2) / G_i^2$ , and  $S_j = (2^{R_j} - 1) \cdot (\sigma^2) / G_j^2$ .

After generating the weight matrix  $C$  from the possible combinations in groups of two, the cost function, which is meant to be minimized, becomes:

$$\sum_{i \in A} \sum_{j \in A} C(i, j) \cdot \lambda_{ij} \quad (2.24)$$

in which  $\lambda_{ij}$  shows that grouping of  $i$  and  $j$  is chosen if  $\lambda_{ij} \neq 0$ , and not chosen, otherwise. Here, we have  $M^2$  elements of  $\lambda$  as the optimization arguments.

For the constraints, client's rate should be taken into account only one time to be achieved. Therefore, if  $\lambda_{\hat{i}j} = 1$  for some  $\hat{i}$ ,  $\lambda_{ij}$  has to be zero for  $i \neq \hat{i}$ . With the same reason, if  $\lambda_{i\hat{j}} = 1$  for some  $\hat{j}$ ,  $\lambda_{ij}$  has to be zero for  $j \neq \hat{j}$ . Therefore, the constraints are

$$\sum_{j \in A} \lambda_{ij} = 1 \quad \text{for } i \in A, \quad (2.25)$$

$$\sum_{i \in A} \lambda_{ij} = 1 \quad \text{for } j \in A, \quad (2.26)$$

$$\lambda_{ij} \geq 0 \quad \text{for } i, j \in A. \quad (2.27)$$

Here,  $\lambda_{ij} + \lambda_{ji}$  shows how many tones are required for the rate achievement of the group of  $i$  and  $j$ . For example, if  $\lambda_{2,3} = 1$  and  $\lambda_{3,2} = 1$ , two tones should be allocated to clients 2 and 3. The problem is now formulated as a linear programming problem and can be solved easily using Simplex or Hungarian method [7].

For solving the above linear programming problem, we have a  $\lambda$  matrix with  $\lambda_{ij} \geq 0$ . The resulting clients in a group are the  $i$  and  $j$  indexes of  $\lambda_{i,j} \neq 0$ . Moreover, the summation of all possible combinations of a specific  $i$  and  $j$  will be the number of tones that are allocated to these clients. As an example, let's consider  $M = 6$  clients with the requested rates  $R$  and  $K = 6$  tones with the channel gains  $G$ :

$$R = [2.63, \quad 1.55, \quad 2.71, \quad 1.72, \quad 1.12, \quad 2.51].$$

$$G = [0.90, \quad 2.19, \quad 0.51, \quad 0.85, \quad 0.39, \quad 0.65].$$

The minimum power to achieve the requested rates for groups of two clients is the solution of Equation 2.24 with the constraints in Equations 2.25, 2.26 and 2.27. The solution to this problem is as follows:

$$\lambda = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

As it was expected, based on our constraints' definitions, the solution for  $\lambda$  matrix is always symmetric. The above  $\lambda$  results in grouping of clients 1 and 5, clients 2 and 3, and clients 4 and 6.

### 2.3.2 Groups of three clients

We have extended the  $Q = 2$  grouping to  $Q = 3$  grouping such that the weight matrix is changed from a 2-D to a 3-D matrix. In this case, we may have group of size 1, 2 and/or 3. All three dimensions of the weight matrix are from the set  $A = \{1, 2, \dots, M\}$ . Let's define  $C(i, j, k)$  as the total power to send signals to clients  $i, j$  and  $k$  using one tone. Then, we have:

$$C(i, j, k) = \begin{cases} P_i & \text{if } i = j = k, \\ P_{i,j} & \text{if } i = k \text{ or } j = k, \\ P_{i,k} & \text{if } i = j, \\ P_{i,j,k} & \text{if } i \neq j \neq k, \end{cases}$$

in which,

- $P_i$  is the required power to send to only client  $i$  in one tone.
- $P_{i,j}$  and  $P_{i,k}$  are the total power to send code-words to two clients using one tone.
- $P_{i,j,k}$  is the total power to send code-words to three clients using one tone.

Figure 2.9 shows the assignments for the weight matrix in grouping of three clients. Again, we have  $\lambda_{ijk}$  regarding to choose  $i, j$  and  $k$  in a group, if  $\lambda_{ijk} \neq 0$  and do not choose otherwise.

The cost function for the 3-D weight matrix will be:

$$\sum_{i \in A} \sum_{j \in A} \sum_{k \in A} C(i, j, k) \lambda_{ijk} \quad (2.28)$$

The constraints are also extended from the groups of two clients, therefore we have:

$$\sum_{i \in A} \sum_{j \in A} \lambda_{ijk} = 1 \quad \text{for } k \in A, \quad (2.29)$$

$$\sum_{i \in A} \sum_{k \in A} \lambda_{ijk} = 1 \quad \text{for } j \in A, \quad (2.30)$$

$$\sum_{j \in A} \sum_{k \in A} \lambda_{ijk} = 1 \quad \text{for } i \in A, \quad (2.31)$$

$$\lambda_{ijk} \geq 0 \quad \text{for } i, j, k \in A. \quad (2.32)$$

	1	2	3	1	2	3	1	2	3
1	Energy to service client 1 in one tone	Energy to service clients 1 and 2 in one tone (half of their rates)	Energy to service clients 1 and 3 in one tone (half of their rates)	Energy to service clients 1 and 2 in one tone (half of their rates)	Energy to service clients 1 and 2 in one tone (half of their rates)	Energy to service clients 1,2 and 3 in one tone (one third of their rates)	Energy to service clients 1 and 3 in one tone (half of their rates)	Energy to service clients 1,2 and 3 in one tone (one third of their rates)	Energy to service clients 1 and 3 in one tone (half of their rates)
2	Energy to service clients 1 and 2 in one tone (half of their rates)	Energy to service clients 1 and 2 in one tone (half of their rates)	Energy to service clients 1,2 and 3 in one tone (one third of their rates)	Energy to service clients 1 and 2 in one tone (half of their rates)	Energy to service client 2 in one tone	Energy to service clients 2 and 3 in one tone (half of their rates)	Energy to service clients 1,2 and 3 in one tone (one third of their rates)	Energy to service clients 2 and 3 in one tone (half of their rates)	Energy to service clients 2 and 3 in one tone (half of their rates)
3	Energy to service clients 1 and 3 in one tone (half of their rates)	Energy to service clients 1,2 and 3 in one tone (one third of their rates)	Energy to service clients 1 and 3 in one tone (half of their rates)	Energy to service clients 1,2 and 3 in one tone (one third of their rates)	Energy to service clients 2 and 3 in one tone (half of their rates)	Energy to service clients 2 and 3 in one tone (half of their rates)	Energy to service clients 1 and 3 in one tone (half of their rates)	Energy to service clients 2 and 3 in one tone (half of their rates)	Energy to service client 3 in one tone
	1			2			3		

Figure 2.9: Weight matrix for grouping of three clients with  $M = 3$

Similarly, this problem formulation also leads to a linear assignment problem which can be easily solved by Simplex algorithm.

In the obtained  $\lambda$  matrix, we have values other than 0 and 1 for  $Q = 3$  and this should be translated in terms of our set-up. Let us consider the same color elements in the assignment matrix, which shows the same group representatives, in Figure 2.9. As shown, the representatives of each group exist in different places in the assignment matrix. Moreover, based on the above constraints, presenting of a same group with two clients happens three times in a constraint; and presenting of a same group with three clients happens two times in a constraint. Therefore, we have  $\frac{1}{3}$  or  $2 \times \frac{1}{3}$ , or  $3 \times \frac{1}{3}$  values for the  $\lambda$  element for groups of two, and  $\frac{1}{2}$ , or 1 values for  $\lambda$  element for groups of three. As explained before, the summation of the representatives of one group over the whole matrix shows the number of tones that are assigned to that group. Here, each of two-clients and three-clients groups repeated 6 times in the whole matrix. Therefore, at least two and three tones are assigned to two-clients and three-clients groups, respectively. The latter explanation can be extended to any  $M$  clients with their specific values.

To shed more light on what we said, consider an example of groups of three clients in which  $M = 3$  with the following requested rates and channel gains, respectively.

$$R = [1.8834, \quad 1.9844, \quad 2.9572],$$

and

$$G = [1.2876, \quad 0.5803, \quad 0.5679].$$

Solving the above linear assignment problem, we have  $\lambda$  as:

$$\lambda(:, :, 1) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.5 \\ 0 & 0.5 & 0 \end{bmatrix} \quad \lambda(:, :, 2) = \begin{bmatrix} 0 & 0 & 0.5 \\ 0 & 0 & 0 \\ 0.5 & 0 & 0 \end{bmatrix} \quad \lambda(:, :, 3) = \begin{bmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The result is to group 1,2 and 3 in all three tones which is exactly the same as broadcast in this scenario.

$$\lambda_{1,2,3} + \lambda_{1,3,2} + \lambda_{2,1,3} + \lambda_{2,3,1} + \lambda_{3,1,2} + \lambda_{3,2,1} = 3 \quad (2.33)$$

### 2.3.3 Groups of four clients

The groups of four clients is defined as grouping of at most 4 clients for a tone, i.e. we have single element groups, groups of size two, groups of size three, and/or groups of size four. Therefore, for  $Q = 4$ , the weight matrix will be 4-D with every dimension of size  $M$  with the assignment shown in Figure 2.10.

The cost matrix  $C$  has four indexes here in which the matrix element  $C(i, j, k, l)$  represents the total power needed to send code-words to clients  $i, j, k$  and  $l$ . Each matrix element is the total power to send signals to 1,2,3 or 4 clients using one tone. The values of  $\lambda_{ijkl}$  for all  $i, j, k, l \in \{1, 2, \dots, M\}$  is the solution of the following linear assignment problem.

$$\min \sum_{i \in A} \sum_{j \in A} \sum_{k \in A} \sum_{l \in A} C(i, j, k, l) \lambda_{ijkl}, \quad (2.34)$$

$$\sum_{i \in A} \sum_{j \in A} \sum_{k \in A} \lambda_{ijk} = 1 \quad \text{for } l \in A, \quad (2.35)$$

$$\sum_{i \in A} \sum_{j \in A} \sum_{l \in A} \lambda_{ijl} = 1 \quad \text{for } k \in A, \quad (2.36)$$

$$\sum_{i \in A} \sum_{k \in A} \sum_{l \in A} \lambda_{ikl} = 1 \quad \text{for } j \in A, \quad (2.37)$$

Figure 2.10: Assignment matrix for grouping of four clients with  $M = 4$

$$\sum_{j \in A} \sum_{k \in A} \sum_{l \in A} \lambda_{jkl} = 1 \quad \text{for } i \in A, \quad (2.38)$$

$$\lambda_{ijkl} \geq 0 \quad \text{for } i, j, k, l \in A. \quad (2.39)$$

Again, we have the same discussion as Section 2.3.2 for possible values of  $\lambda$  elements. Thus far, the algorithm has determined the best way of grouping the clients and allocates the required tones to the groups. In the following section, the latter is enhanced by focusing on how to satisfy the rate of client  $i$  through specified groups.

### 2.3.4 Augmented solution

Consider the grouping method is performed and the results are available. Having known the result, it is decided that each client, say the  $i^{\text{th}}$  client, would be involved in  $m$  different groups, denoted by the set  $A_i$ . It is possible to further reduce the power if the proportion of the  $i^{\text{th}}$  client's rate to be satisfied in the  $j^{\text{th}}$  group is calculated efficiently. Formulating the problem, we have the following optimization problem:

$$\min \sum_{j \in A_i} P_j^i \quad (2.40)$$

such that

$$R_i = \sum_{j \in A_i} \alpha_j^i R(P_j^i), \quad (2.41)$$

and

$$\sum_{j \in A_i} \alpha_j^i = 1 \quad \text{for } i \in A, \quad (2.42)$$

where  $P_j^i$  is the allocated power to the  $i^{\text{th}}$  receiver through the  $j^{\text{th}}$  group.

### 2.3.5 Uneven tone assignment

Thus far, based on the proposed methods, the number of tones that can be assigned to the groups with the same size has been constant. Now, the question is whether we can get a higher gain if we assign more tones to some groups and less to the others such that the total number of tones remains constant. To clarify the latter statement, consider Figures 2.11 and 2.12. For  $M = 4$  clients and grouping of two clients, four rates needs to be achieved. In the classic grouping, two groups are chosen and two tones are assigned to each group as shown in Figure 2.11. Here, the idea is to have two groups in this way: one group has the clients with the highest rate requirements and the other group has the clients with the lowest rate requirements. Then, three tones are assigned to the first group and one tone is assigned to the second group as shown in Figure 2.12.

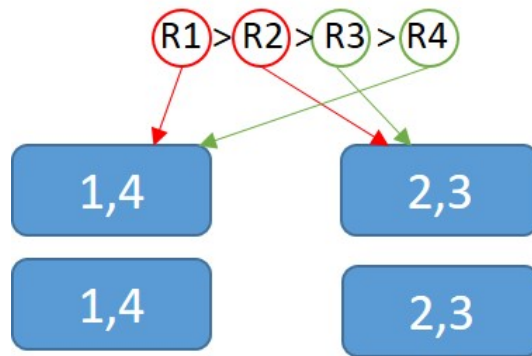


Figure 2.11: grouping in the previous report

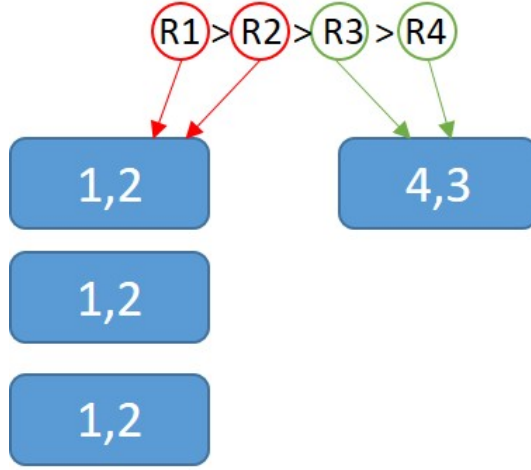


Figure 2.12: uneven tone assignment

Generally speaking, for the idea of uneven tones, the assignment method and the number of tones that are assigned to each group can vary and needs to be studied. For now, we have just considered the discussed scenario with  $M = 4$  and grouping of two clients.

Consider  $M = 4$  clients with requested rates as  $R_1, R_2, R_3, R_4$ , and 4 available tones. The system model and formulation is the same as grouping of two clients discussed in the previous section. What we need to add for uneven tones will be explained here.

First, the requested rates are sorted and the two clients with higher and the two clients with lower rates are chosen as  $max1, max2$  and  $min1, min2$ , respectively. Let us define the tone matrix  $T$  as an  $M \times M$  matrix with all ones except those elements with the following indices:  $(max1, max2), (max2, max1), (min1, min2)$  and  $(min2, min1)$ . The latter elements get  $\frac{3}{2}, \frac{3}{2}, \frac{1}{2}$  and  $\frac{1}{2}$ , respectively, to provide the assigned tones as Figure 2.12.

In order to formulate the problem, the weight matrix is calculated as stated for the grouping method. The only difference is the power for the group of  $(max1, max2)$  and  $(min1, min2)$ . Three tones are assigned to group  $(max1, max2)$ , so the power to satisfy  $\frac{1}{3}$  fraction of the requested rates should be considered. In the same way, the requested rates of  $min1$  and  $min2$  should be considered for the group of  $(min1, min2)$ . Now, the formulation becomes:



$$\sum_{i \in A} \sum_{j \in A} C'(i, j) \cdot \lambda_{ij} \quad (2.43)$$

$$\sum_{j \in A} \frac{\lambda_{ij}}{T_{ij}} = 1 \quad \text{for } i \in A, \quad (2.44)$$

$$\sum_{i \in A} \frac{\lambda_{ij}}{T_{ij}} = 1 \quad \text{for } j \in A, \quad (2.45)$$

$$0 \leq \lambda_{ij} \leq T_{ij} \quad \text{for } i, j \in A. \quad (2.46)$$

$$\sum_{i \in A, j \in A} \lambda_{ij} = M. \quad (2.47)$$

Meanwhile, equation 2.47 needs to be considered in order to keep the total number of tones equal to  $M$ .

In section 2.4, the simulation results for the above groupings and the augmented method would be presented and compared to the broadcast scenario.

## 2.4 Simulation results

For the simulation, the linear assignment problem for groups of two, groups of three and groups of four is considered and solved by Matlab linear programming solver. The fading channels are independent with a Gaussian distribution. Therefore, the channel gains are assumed to have a Rayleigh distribution with average one. In addition, channel noise has a Gaussian distribution with average zero and variance one. Moreover, the requested rates are chosen randomly in a specific range that will be mentioned for each case.

The comparison base for each scenario is the traditional frequency division mode in which the code-words for each client are sent through one and only one tone. The traditional frequency division scheme is called single-tone in all simulations. The grouping results are compared to the broadcast scenario in which the code-words are sent to all  $M$  clients from all  $M$  channels.

Considering 10 clients, i.e.  $M = 10$ , the grouping results for grouping of two, three and four is compared to the traditional frequency division and broadcast. In Figure 2.13, the requested rates are randomly generated such that  $1 \leq R \leq 2$ .

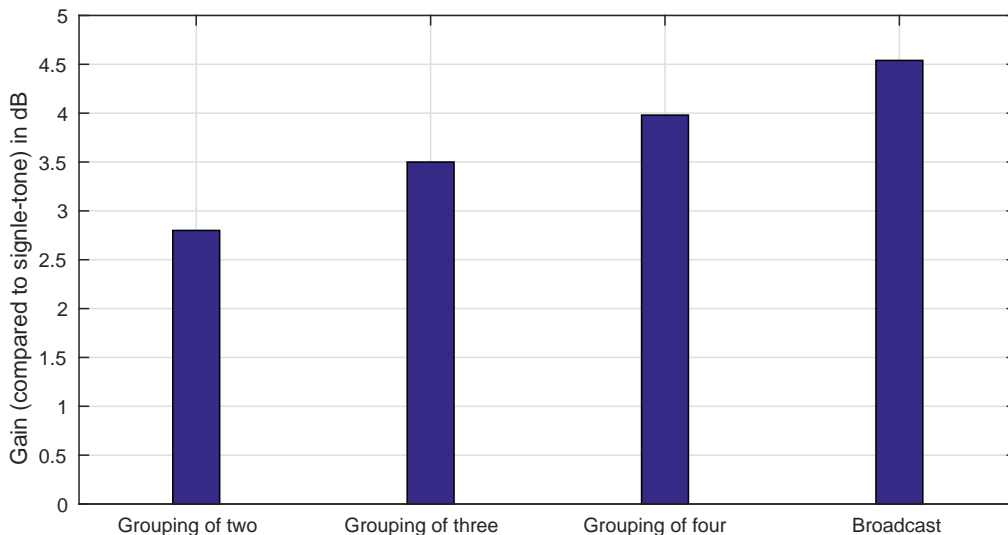


Figure 2.13: Gain comparison based on the traditional frequency division power.  $M = 10$  and  $1 \leq R \leq 2$ .

As shown in Figure 2.13, the required power for groups of two is significantly less than that of traditional frequency division. The improvement continues by increasing  $Q$  from 2 to 4, i.e. from grouping of two to grouping of four. The difference between required power for groups of four and broadcast is less than 1 dB.

The same simulation with  $2 \leq R \leq 3$  leads to the results as shown in figure 2.14. For averagely higher rates, the achieved gain for grouping of two is more than the lower rates and the differences between grouping of four clients and broadcast is still less than 1 dB.

As another setting, for  $1 \leq R \leq 3$ , we have the results as shown in Figure 2.15. Here, the grouping of four has an acceptable gain compared to the broadcast gain like the previous simulations.

For  $3 \leq R \leq 5$ , the result is shown in Figure 2.16. The achieved gain over traditional frequency division scheme is greater than that of lower rates. However, the difference between the gain of grouping of four and broadcast is more than that of lower rates.

Table 2.1 compares different grouping gains for 10 clients and 10 tones. The results are provided in Table 2.2 for 20 clients and 20 tones as well. As can be seen in these tables, the grouping gains are closer to the broadcast gains when the number of clients are closer to the  $Q$  value.

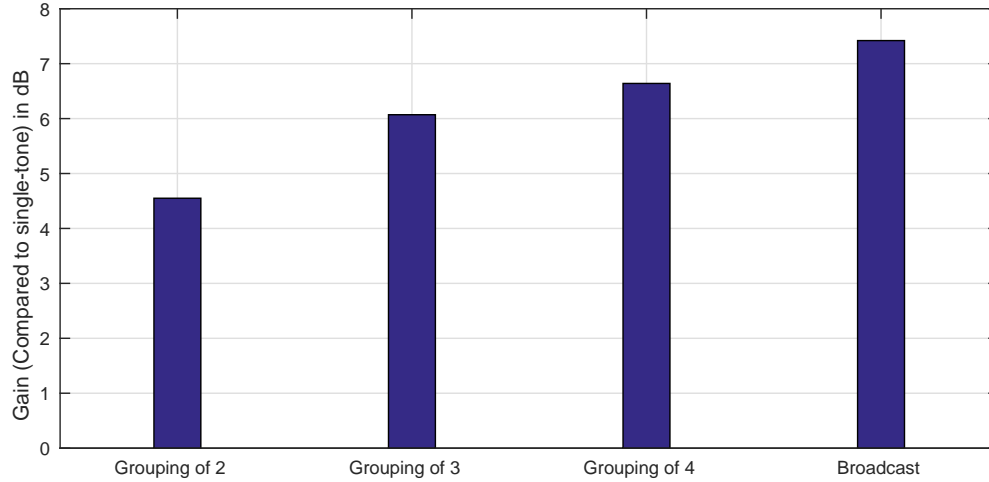


Figure 2.14: Gain comparison based on the single-tone power.  $M = 10$  and  $2 \leq R \leq 3$ .

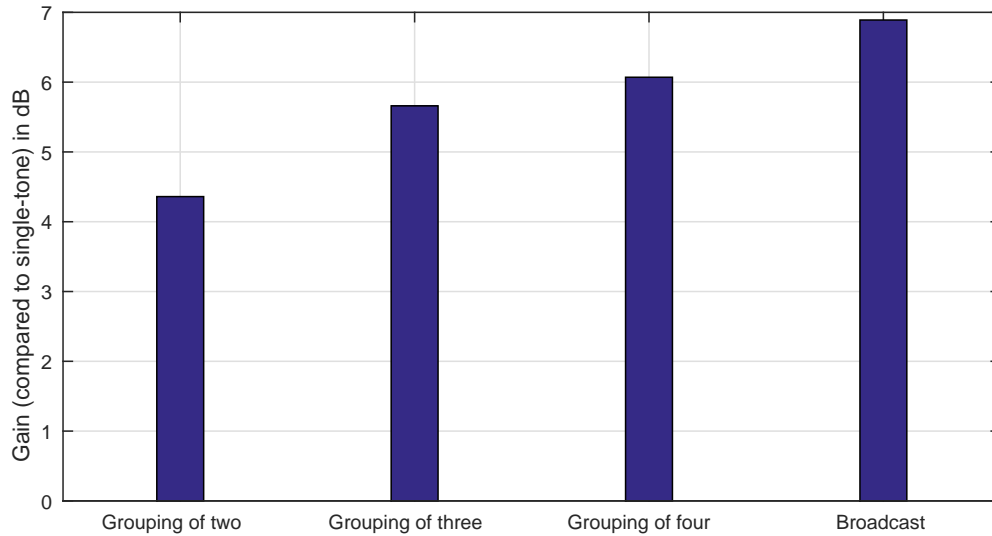


Figure 2.15: Gain comparison based on the single-tone power.  $M = 10$  and  $1 \leq R \leq 3$ .

For the augmented method, the simulation results for  $1 \leq R \leq 2$  have been illustrated in Figure 2.17. The gain achieved by using augmented method depends on the rates.

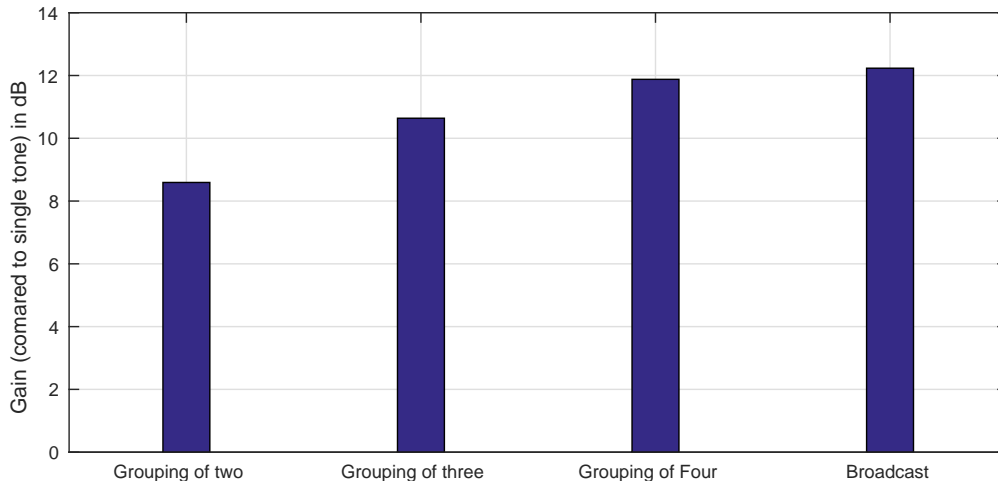


Figure 2.16: Gain comparison based on the single-tone power.  $M = 10$  and  $3 \leq R \leq 5$ .

method/Rate	$R \in [1, 2]$	$R \in [1, 3]$	$R \in [2, 3]$	$R \in \{1, 2, 3\}$
Grouping of 2	2.78	4.03	4.55	4.0
Grouping of 3	3.5	5.66	5.87	5.71
Grouping of 4	3.98	6.07	6.64	6.51
Broadcast	4.54	6.89	7.42	7.01

Table 2.1: Grouping improvements for 10 clients and 10 tones. All values are in dB.

method/Rate	$R \in [1, 2]$	$R \in [1, 3]$	$R \in [2, 3]$	$R \in \{1, 2, 3\}$
Grouping of 2	2.9	4.17	4.88	4.03
Grouping of 3	3.52	5.88	5.97	5.82
Grouping of 4	4.03	6.49	6.89	6.64
Broadcast	4.63	7.45	7.80	7.49

Table 2.2: Grouping improvements for 20 clients and 20 tones. All values are in dB.

However, on average, the improvement is less than 1 dB.

As another setting, for  $1 \leq R \leq 3$ , we have the results for augmented grouping method compared to grouping only as shown in Figure 2.18.

The same simulation with  $2 \leq R \leq 3$  leads to the results as shown in figure 2.19.

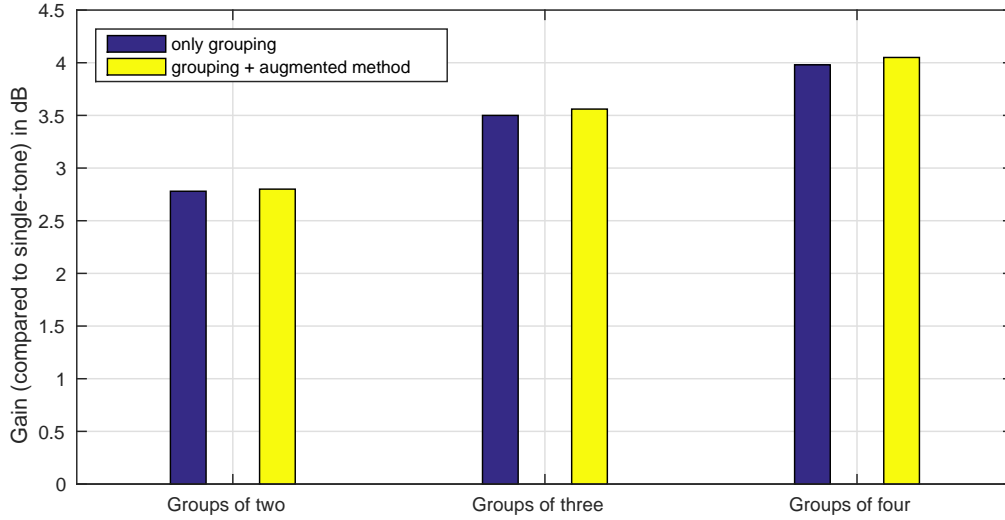


Figure 2.17: Augmented method results.  $M = 10$  and  $1 \leq R \leq 2$ .

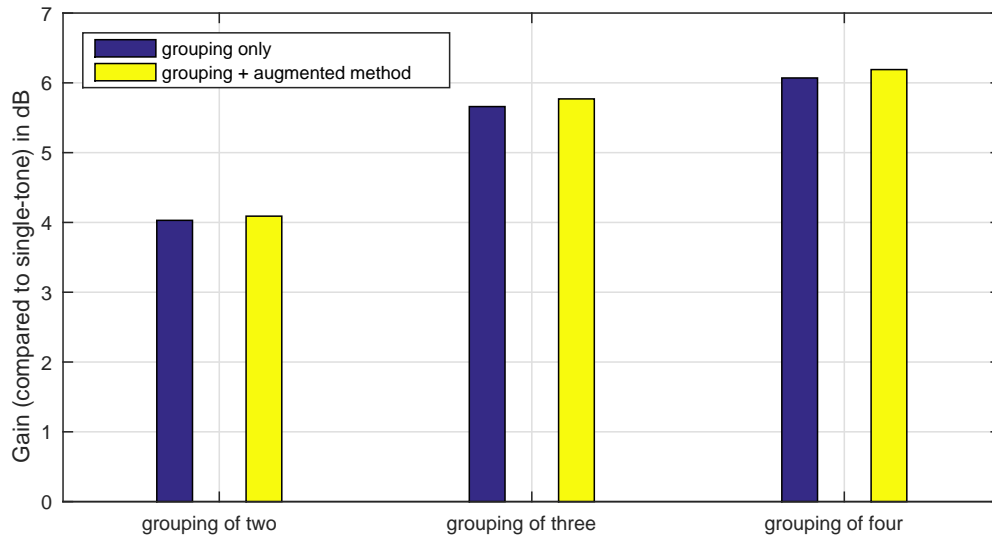


Figure 2.18: Augmented method results.  $M = 10$  and  $1 \leq R \leq 3$ .

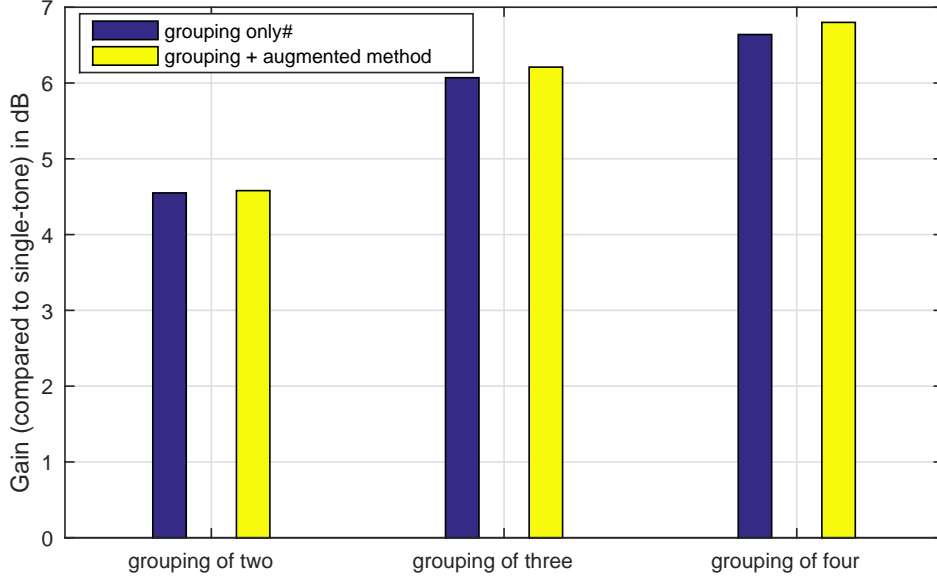


Figure 2.19: Augmented method results.  $M = 10$  and  $2 \leq R \leq 3$ .

Comparing the three augmented method figures, the improvement through the augmented method is more significant in higher rates. The augmented method for grouping of four has the best result among the considered schemes, i.e. its gain is the closest to the broadcast scenario's gain. The difference between the gain of best result and that of broadcast for  $M = 10$  is less than 0.7dB.

Table 2.3 compares different grouping gains with the augmented method for 10 clients and 10 tones.

For the uneven tone assignment, the results for  $M = 4$  and grouping of two are as stated in Table 2.4.

The performance of grouping of three clients for  $2 \leq R \leq 3$ , for different number of clients has been illustrated in figure 2.20. The grouping gain for three clients is exactly the same as the broadcast scenario. As expected, the difference between broadcast and grouping gain increases by growing the number of clients.

Considering different channel models, i.e. shadowing and multi-path with and without line of Sight (LOS), the groups of three clients is compared to the broadcast scenario. Here, we have  $M = 10$  and  $2 \leq R \leq 3$ . The channels are modeled by Log-normal distribution

method/Rate	$R \in [1, 2]$	$R \in [1, 3]$	$R \in [2, 3]$
Grouping of 2	2.78	4.03	4.55
Augmented grouping of two	2.8	4.09	4.58
Grouping of 3	3.5	5.66	5.87
Augmented grouping of 3	3.56	5.77	6.01
Grouping of 4	3.98	6.07	6.64
Augmented grouping of four	4.05	6.19	6.80
Broadcast	4.54	6.89	7.42

Table 2.3: Augmented improvements for 10 clients and 10 tones. All values are in dB.

Method/Rate	$R \in [1, 2]$	$R \in [2, 3]$	$R \in [1, 3]$
Grouping of 2	2.73	4.81	4.26
Uneven tones for grouping of 2	3.08	5.33	4.75
Broadcast	3.75	6.49	5.70

Table 2.4: Uneven tones result for  $M = 4$

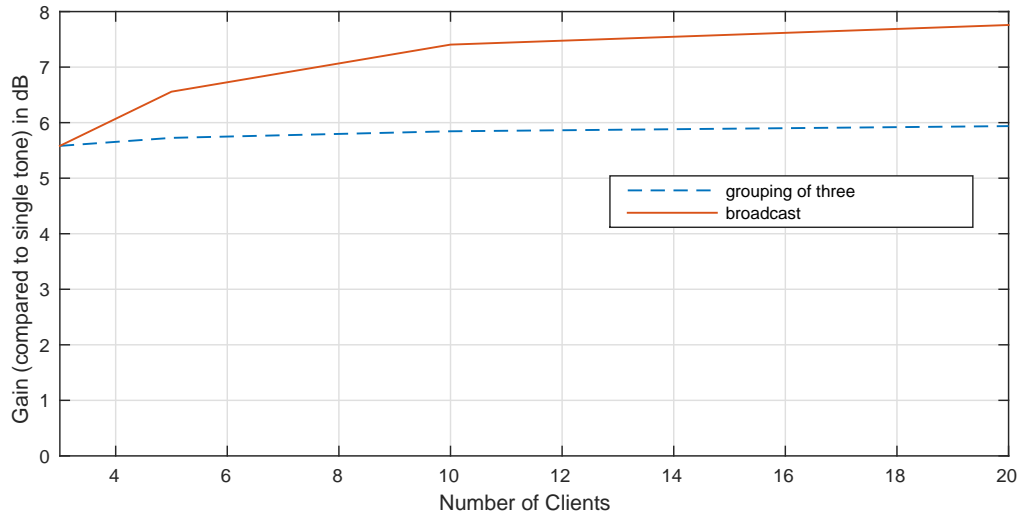


Figure 2.20: Performance for grouping of three clients

with  $\mu = 0$  and  $\sigma = 1$ , Rician distribution with parameters  $s = 1$  and  $\sigma = 1$  and Rayleigh distribution with parameters  $b = \sqrt{\frac{2}{\pi}}$  and  $\mu = 1$ . Figure 2.21 shows that the grouping

idea works well for different kind of channels.

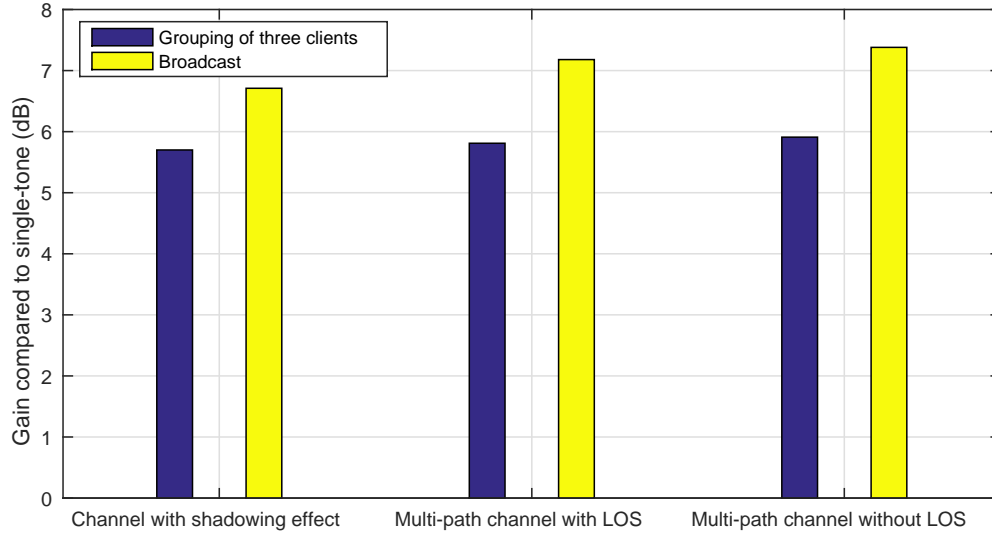


Figure 2.21: Grouping gain over different channels

## 2.5 Summary

### 2.5.1 Conclusions

In this section, the idea of grouping clients was introduced. The channel tones are assigned to the groups of clients with at most  $Q$  members. Optimum (minimum) power is computed to serve all clients at their required rate levels. Different grouping schemes are possible. Among these possible methods, the grouping which minimizes the total power to send code-words to the clients is chosen by formulating and solving the problem as the well-known linear assignment problem. The required power in grouping is significantly less than the single-tone power in which each tone is allocated to one and only one client. Moreover, the grouping gain is not far from the broadcast gain. By increasing the number of clients in the grouping, i.e. increasing  $Q$ , the grouping gain improves. This has been achieved with the cost of increasing the inter-carrier interference and some insignificant computations. However, the grouping assignment gain is significant compared to traditional frequency division method and can be used practically in many applications.



## 2.5.2 List of contributions

In this chapter, a new method for channel allocation in downlink transmission was proposed. The idea of grouping the clients and applying the broadcast set-up in each group has the following merits:

1. It has less complexity with respect to the broadcast set-up, for the size of the code-book used for coding/decoding would be decreased. On the other hand, the gain for the new set-up would remain roughly the same as the broadcast set-up.
2. The gain for the proposed method is significantly higher than the gain for traditional [FDMA](#). This is because in [FDMA](#) the channels are not exploited efficiently.
3. The idea of an uneven tone assignment to each group was raised which outperforms the simple grouping method.

## 2.5.3 Future research

Many different ideas and tests have been left for the future due to lack of time. There is a couple of ideas that I would have liked to try during my research. Future work can possibly concern more about the following topics:

1. Increasing the size of the groups to the numbers greater than four. As the number of existent clients increases, the size of the groups can increase accordingly. However, this needs more work on defining the corresponding optimization problem.
2. The way that the set-up was defined could be also changed. In practice, the number of channels may differ from the number of clients. In our set-up, these numbers are the same.
3. It could be interesting to define a channel which is not Gaussian and after finding the capacity for the new channel, one can test the proposed method over it.

# Chapter 3

## Uplink transmission

### 3.1 Introduction

In uplink transmission, the [MAC](#) set-up gives us the highest gain. However, when the number of clients increases, the size of the codebook used for coding/decoding increases, which leads to a more complex transmitter/receiver system.

The idea of grouping the clients arises out of the following question:

How to decrease the complexity without losing much gain?

It has been shown that grouping the clients would effectively save most of the gain and decrease the complexity. In this method, the clients of one group would not make any interference for the other groups' clients. Meanwhile, in each group, the clients form a [MAC](#) set-up to use the maximum gain of the existent channels.

As an example, consider the grouping of size two. The capacity region for [MAC](#) can be obtained as shown in figure [3.1](#).

Which has been obtained based on the following inequalities:

$$R_1 \leq \frac{1}{2} \log(1 + SNR_1) \tag{3.1}$$

$$R_2 \leq \frac{1}{2} \log(1 + SNR_2) \tag{3.2}$$

$$R_1 + R_2 \leq \frac{1}{2} \log(1 + SNR_1 + SNR_2) \tag{3.3}$$

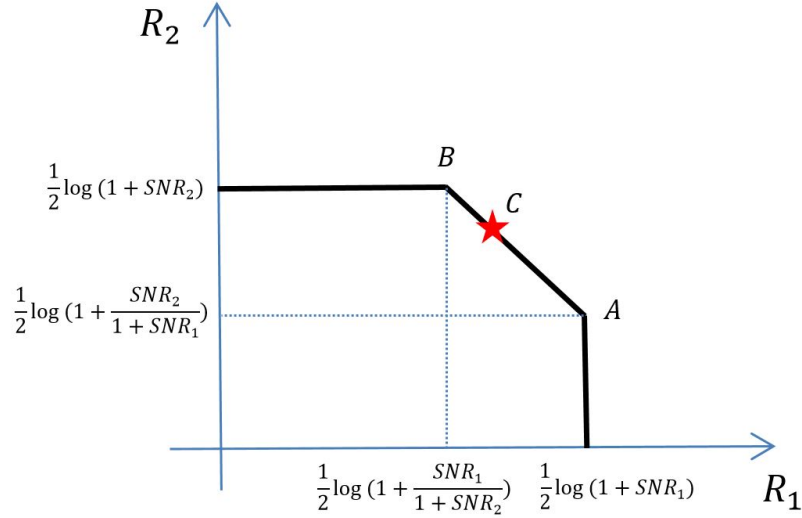


Figure 3.1: Capacity region for *MAC* with 2 clients

In order to achieve point *A* on figure 3.1, first, the decoder decodes the second client's data and treats the first client's data as a noise. Therefore, the achievable rate for the second client would be [13]:

$$R_2 = \frac{1}{2} \log \left( 1 + \frac{SNR_2}{1 + SNR_1} \right) \quad (3.4)$$

Afterward, the receiver subtracts the second client's data from what has been received and then decodes the first client's data. Therefore,

$$R_1 = \frac{1}{2} \log (1 + SNR_1) \quad (3.5)$$

Point *B* can be obtained with the same approach.

In order to obtain a point over the line *AB*, say  $qA + (1 - q)B$ , the system can perform one of the following strategies:

1. Allocate  $(1 - q)$  proportion of time to decode second users data and then  $q$  proportion of time to decode first users data.

2. Allocate  $q$  proportion of time to decode first users data and  $(1 - q)$  proportion of time to decode second users data

That being said, the maximum sum rate in every single group would satisfy inequality 3.3, for the clients form a MAC together.

By generalizing this idea, the capacity region of a group, say  $S$ , with the size less than or equal to  $K$ , can be achieved as follows:

$$\zeta_{uplink} = \cup \{(R_1, \dots, R_K) \geq 0 : \sum_{k \in S} R_k \leq \frac{1}{2} \log(1 + \sum_{k \in S} SNR_k), \forall S \subseteq [1 : K]\} \quad (3.6)$$

Moreover, the sum capacity will be:

$$C_{uplink}^{sum} = \frac{1}{2} \log(1 + \sum_{k=1}^K SNR_k) \quad (3.7)$$

Therefore, the maximum sum rate can be obtained from the following equation:

$$\sum_{i \in S} R_i = \frac{1}{2} \log(1 + \sum_{i \in S} SNR_i) \quad (3.8)$$

Where  $S$  is the set of group elements.

As an example, an uplink channel capacity region for a group of 3 clients is obtained from the following equations:

$$R_k \leq \frac{1}{2} \log(1 + SNR_k), k = 1, 2, 3 \quad (3.9)$$

$$R_1 + R_2 \leq \frac{1}{2} \log(1 + SNR_1 + SNR_2) \quad (3.10)$$

$$R_2 + R_3 \leq \frac{1}{2} \log(1 + SNR_2 + SNR_3) \quad (3.11)$$

$$R_1 + R_3 \leq \frac{1}{2} \log(1 + SNR_1 + SNR_3) \quad (3.12)$$

$$R_1 + R_2 + R_3 \leq \frac{1}{2} \log(1 + SNR_1 + SNR_2 + SNR_3) \quad (3.13)$$

## 3.2 Grouping the clients

Let us consider a set-up in which:

1. The base station is equipped with  $M$  time/frequency resources.
2. Each client's power is bounded by  $P$ .
3. The channel is [AWGN](#) with noise variance  $\sigma^2$  in all transmission channels.
4. The channel gains follow Rayleigh distribution with average 1.

Suppose that we have  $N$  clients to be grouped into  $K$  groups of size  $B$ . For each of these groups, say group  $k$ , the maximum summation rate,  $R_k$ , will be obtained by:

$$R_k = \frac{1}{2} \log \left( 1 + \sum_{i=1}^B \frac{P}{\sigma^2} G_{k_i}^2 \right) \quad (3.14)$$

Considering all  $K$  groups, the problem can be defined as maximizing the total rate over all groups, i.e.

$$\sum_{k=1}^K R_k = \sum_{k=1}^K \frac{1}{2} \log \left( 1 + \sum_{i=1}^B \frac{P}{\sigma^2} G_{k_i}^2 \right) \quad (3.15)$$

The effect of group size on rate improvement has been examined via simulation. In this simulation, the number of clients/channels is equal to 48 and  $\frac{P}{\sigma^2} = 10$ . At this point, the clients have formed the groups randomly and each method's gain has been calculated with respect to the single channel allocation method's gain. The results can be seen in table 3.1.

Random grouping	group size=2	group size=3	group size=4	group size=6	group size=8	group size=12	All- MAC
Gain	0.3662	0.4914	0.5530	0.6155	0.6465	0.6767	0.7208

Table 3.1: Effect of group size on gain. Gains have been obtained with respect to single channel allocation method. All values are in dB.

As it has shown in table 3.1, the rate improves by increasing the group size with the cost of more complexity and delay in decoding the messages. Considering the above maximization problem, two different strategies have been examined. The first one is to define an optimization problem and to solve it efficiently. This would be explained in section 3.3. The second one is to consider the problem as maximizing the product of positive integers. The latter would be elaborated in section 3.4.

### 3.2.1 Why all-MAC set-up always leads to the maximum gain

Suppose that we have  $nk$  channels/clients. Here we have compared two cases:

1. All the channels/clients form a **MAC** set-up.
2.  $nk$  channels/clients are divided into  $n$  groups of size  $k$ . Afterward, each group forms a **MAC** set-up.

By normalizing the bandwidth of each channel to 1, the available bandwidth for the clients in the first case would be  $\frac{1}{nk}$ . Therefore, the maximum sum rate for this case can be obtained from the following formula:

$$C_{MAC}^{sum} = (nk) \times \frac{1}{2} \log \left( 1 + \frac{P}{\sigma^2} \times \frac{\sum_i G_i^2}{nk} \right) \quad (3.16)$$

In the second case, suppose that  $n$  groups of size  $k$  have been arbitrarily chosen such that they form  $n$  disjoint groups. For the sake of simplicity, **channels are denoted by the squared value of their gains**. Therefore, the groups can be shown as follows:

$$S_t = \{G_{t_1}^2, G_{t_2}^2, \dots, G_{t_k}^2\}, 1 \leq t \leq n \quad (3.17)$$

where:

$$\bigcup_t S_t = \bigcup_i S_i, 1 \leq t \leq n, 1 \leq i \leq nk \quad (3.18)$$

$$\bigcap_t S_t = \phi, 1 \leq t \leq n \quad (3.19)$$

Moreover, the sum of each group's elements can be shown by  $s_t$ , which means that:

$$s_t = \sum_{1 \leq i \leq k} G_{t_i}^2 \quad (3.20)$$

In This case, since  $|S_t| = k$ , the available bandwidth for each client would be  $\frac{1}{k}$ . Therefore, the maximum sum rate can be obtained based on the following formula:

$$C_{grouping}^{sum} = \sum_{1 \leq t \leq n} k \times \frac{1}{2} \log \left( 1 + \frac{P}{\sigma^2} \times \frac{s_t}{k} \right) \quad (3.21)$$

Henceforth, we will try to prove that:

$$\sum_{1 \leq t \leq n} k \times \frac{1}{2} \log \left( 1 + \frac{P}{\sigma^2} \times \frac{s_t}{k} \right) \leq (nk) \times \frac{1}{2} \log \left( 1 + \frac{P}{\sigma^2} \times \frac{\sum_i G_i^2}{nk} \right) \quad (3.22)$$

or equivalently:

$$\sum_{1 \leq t \leq n} \log \left( 1 + \frac{P}{\sigma^2} \times \frac{s_t}{k} \right) \leq n \times \log \left( 1 + \frac{P}{\sigma^2} \times \frac{\sum_i G_i^2}{nk} \right) \quad (3.23)$$

By Jensen's inequality [3] for the concave function  $\psi$ :

$$\frac{\sum \psi(x_i)}{n} \leq \psi \left( \frac{\sum x_i}{n} \right) \quad (3.24)$$

Also, it is obvious that  $f(x) = \log(1+x)$  is a concave function, therefore:

$$\frac{\sum f\left(\frac{P}{\sigma^2} \times \frac{s_t}{k}\right)}{n} \leq f\left(\frac{\sum \frac{P}{\sigma^2} \times \frac{s_t}{k}}{n}\right) \quad (3.25)$$

On the other hand,

$$\sum_{1 \leq t \leq n} s_t = \sum_{1 \leq i \leq nk} G_i^2 \quad (3.26)$$

This gives us the desired inequality.

### 3.3 Optimizing the total rates

Let us start with the problem of maximizing the sum rate with  $M$  clients and groups of two clients.

$$\max_{\gamma} \sum_{i,j \in A} \log(1 + C_{ij} \cdot \gamma_{ij}) \quad (3.27)$$

In which  $C_{ij}$  is the summation of SNR of the clients  $i$  and  $j$  in one group. We have some constraints as well:

$$\sum_{i \in A} \gamma_{ij} = 1 \quad (3.28)$$

$$\sum_{j \in A} \gamma_{ij} = 1 \quad (3.29)$$

$$0 \leq \gamma_{ij} \leq 1 \quad (3.30)$$

It seems that we need to add another constraint which satisfies  $R_i \leq \frac{1}{2} \log(1 + SNR_i)$ . Then, based on 3.27, we need to find a solution for this problem:

$$\max_{\gamma} \left( \log \prod_{i,j \in A} (1 + C_{ij} \cdot \gamma_{ij}) \right) \quad (3.31)$$

which is equivalent to:

$$\max_{\gamma} \prod_{i,j \in A} (1 + C_{ij} \cdot \gamma_{ij}) \quad (3.32)$$

This formulation brings up the idea of using related algorithms to find the groups, instead of finding an optimized solution. This idea would be described in the next section. Based on other formulation for optimization, we have:

$$\max \sum_{i \in A} \lambda_i R_i \quad (3.33)$$

$$\sum_{i \in A} \lambda_i P_i \leq P_0 \quad (3.34)$$

$$R_j > R_0, \forall j \in C \quad (3.35)$$

$$\sum_{i \in S_j} \lambda_i > 0, \forall j \in \{1, 2, \dots, K\} \quad (3.36)$$

$$\sum_i \lambda_i = 1 \quad (3.37)$$

where  $A$  is the set of all possible combinations of clients. For example, for two clients and grouping of two,  $A$  includes (1), (2), and (1, 2). In this formulation, the total power is bounded by  $P_0$  and all single rates need to be more than a minimum, say  $R_0$ .

Now, let us define  $R_i^j$  as the maximum rate of client  $j$  in the  $i^{th}$  group. Therefore, the optimization problem can be written as:

$$\max \sum_{i \in A} \lambda_i R_i \quad (3.38)$$

$$= \max \sum_{i \in A} (\lambda_i \sum_j R_i^j) \quad (3.39)$$

$$\max_{\lambda} \sum_i \sum_j \lambda_i R_i^j, \quad (3.40)$$

with the following constraints:

$$\sum_{i \in A} \lambda_i P_i \leq P_0, \quad (3.41)$$

$$\sum_{i \in A, j=J} \lambda_i R_i^j \geq R_0, \forall J \in \{1, 2, \dots, M\}, \quad (3.42)$$



$$\sum_{i \in S_j} \lambda_i \geq 0, \forall j \in \{1, 2, \dots, K\}, \quad (3.43)$$

$$\sum_i \lambda_i = 1. \quad (3.44)$$

We need to deal with both rate and energy parameters in the above inequalities in which their relationship is not linear. As the first step, the problem is reduced to a simpler one with all rate constraints as

$$\max \sum_{i \in A} (\lambda_i \sum_j R_i^j) = \max_{\lambda} \sum_i \sum_j \lambda_i R_i^j = \max_{\lambda} \sum_j \sum_i \lambda_i R_i^j \quad (3.45)$$

$$\sum_{i \in A, j=J} \lambda_i R_i^j \geq R_0, \forall J \in \{1, 2, \dots, M\}, \quad (3.46)$$

$$\lambda_i R_i^j \leq R_{max}^j, \forall i \in A, j \in K \quad (3.47)$$

$$\lambda_i R_i^j + \lambda_i R_i^{\hat{j}} \leq R_{max}^i, \forall j, \hat{j} \in \text{any group} \quad (3.48)$$

$$\sum_i \lambda_i = 1. \quad (3.49)$$

The above problem for grouping of two clients will be as follows:

Case number i	1	2	3
Clients in i	1	2	1,2

Table 3.2: Grouping two clients.

$$\max \sum_{i=1}^3 (\lambda_i \sum_{j=1}^2 R_i^j) = \max \lambda_1 R_1^1 + \lambda_2 R_2^2 + \lambda_3 R_3^1 + \lambda_3 R_3^2 \quad (3.50)$$

$$\lambda_1 R_1^1 + \lambda_3 R_3^1 \geq R_{min}^1 \quad (3.51)$$

$$\lambda_2 R_2^2 + \lambda_3 R_3^2 \geq R_{min}^2 \quad (3.52)$$

$$\lambda_1 R_1^1 \leq R_{max}^1 \quad (3.53)$$

$$\lambda_2 R_2^2 \leq R_{max}^2 \quad (3.54)$$

$$\lambda_3 R_3^2 \leq R_{max}^3 \quad (3.55)$$

$$\lambda_3 R_3^1 \leq R_{max}^4 \quad (3.56)$$

$$\lambda_3 R_3^1 + \lambda_3 R_3^2 \leq R_{max}^5 \quad (3.57)$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1 \quad (3.58)$$

Now, let us divide the problem into two parts. The first part is:

$$\max \lambda_1 R_1^1 + \lambda_2 R_2^2 + \lambda_3 (R_3^1 + R_3^2) \quad (3.59)$$

$$\lambda_1 R_1^1 + \lambda_2 R_2^2 + \lambda_3 (R_3^1 + R_3^2) \geq R_{min}^1 + R_{min}^2 \quad (3.60)$$

$$\lambda_1 R_1^1 \leq R_{max}^1 \quad (3.61)$$

$$\lambda_2 R_2^2 \leq R_{max}^2 \quad (3.62)$$

$$\lambda_3 (R_3^1 + R_3^2) \leq \min \{ R_{max}^3 + R_{max}^4, R_{max}^5 \} \quad (3.63)$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1 \quad (3.64)$$

Then, the second part is:

$$\lambda_1 R_1^1 + \lambda_3 R_3^1 \geq R_{min}^1 \quad (3.65)$$

$$\lambda_2 R_2^2 + \lambda_3 R_3^2 \geq R_{min}^2 \quad (3.66)$$

$$\lambda_3 R_3^2 \leq R_{max}^3 \quad (3.67)$$

$$\lambda_3 R_3^1 \leq R_{max}^4 \quad (3.68)$$

The first problem is a linear optimization problem that can be solved easily.

Always, the result is  $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 1$ . As expected, multiple access channels with both clients is always the best result for this scenario.

The second part can be calculated based on  $\lambda$  values. We have:

$$\frac{R_{min}^1 - \lambda_1 R_1^1}{\lambda_3} \leq R_3^1 \leq \frac{R_{max}^4}{\lambda_3} \quad (3.69)$$

$$\frac{R_{min}^2 - \lambda_2 R_2^2}{\lambda_3} \leq R_3^2 \leq \frac{R_{max}^3}{\lambda_3} \quad (3.70)$$

For a grouping of three clients we have:

$$\max \sum_{i=1}^6 (\lambda_i \sum_{j=1}^2 R_i^j) = \max(\lambda_1 R_1^1 + \lambda_2 R_2^2 + \lambda_3 R_3^3 + \lambda_4 (R_4^1 + R_4^2) + \lambda_5 (R_5^1 + R_5^2) + \lambda_6 (R_6^1 + R_6^2)) \quad (3.71)$$

Case number i	1	2	3	4	5	6
Clients in i	1	2	3	1,2	1,3	2,3

Table 3.3: Grouping three clients.

$$\lambda_1 R_1^1 + \lambda_4 R_4^1 + \lambda_5 R_5^1 \geq R_{min}^1 \quad (3.72)$$

$$\lambda_2 R_2^2 + \lambda_4 R_4^2 + \lambda_6 R_6^2 \geq R_{min}^2 \quad (3.73)$$

$$\lambda_3 R_3^3 + \lambda_5 R_5^3 + \lambda_6 R_6^3 \geq R_{min}^3 \quad (3.74)$$

$$\lambda_1 R_1^1 \leq R_{max}^1 \quad (3.75)$$

$$\lambda_2 R_2^2 \leq R_{max}^2 \quad (3.76)$$

$$\lambda_3 R_3^3 \leq R_{max}^3 \quad (3.77)$$

$$\lambda_4 R_4^1 \leq R_{max}^4 \quad (3.78)$$

$$\lambda_4 R_4^2 \leq R_{max}^5 \quad (3.79)$$

$$\lambda_5 R_5^1 \leq R_{max}^6 \quad (3.80)$$

$$\lambda_5 R_5^3 \leq R_{max}^7 \quad (3.81)$$

$$\lambda_6 R_6^2 \leq R_{max}^8 \quad (3.82)$$

$$\lambda_6 R_6^3 \leq R_{max}^9 \quad (3.83)$$

$$\lambda_4 R_4^1 + \lambda_4 R_4^2 \leq R_{min}^{10} \quad (3.84)$$

$$\lambda_5 R_5^1 + \lambda_5 R_5^3 \leq R_{min}^{11} \quad (3.85)$$

$$\lambda_6 R_6^2 + \lambda_6 R_6^3 \leq R_{min}^{12} \quad (3.86)$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 = 1 \quad (3.87)$$

The problem is separated into two parts. The first part is:

$$\max(\lambda_1 R_1^1 + \lambda_2 R_2^2 + \lambda_3 R_3^3 + \lambda_4(R_4^1 + R_4^2) + \lambda_5(R_5^1 + R_5^3) + \lambda_6(R_6^2 + R_6^3)) \quad (3.88)$$

$$\lambda_1 R_1^1 + \lambda_2 R_2^2 + \lambda_3 R_3^3 + \lambda_4(R_4^1 + R_4^2) + \lambda_5(R_5^1 + R_5^3) + \lambda_6(R_6^2 + R_6^3) \geq R_{min}^1 + R_{min}^2 + R_{min}^3 \quad (3.89)$$

$$\lambda_1 R_1^1 \leq R_{max}^1 \quad (3.90)$$

$$\lambda_2 R_2^2 \leq R_{max}^2 \quad (3.91)$$

$$\lambda_3 R_3^3 \leq R_{max}^3 \quad (3.92)$$

$$\lambda_4(R_4^1 + R_4^2) \leq \min \{R_{max}^4 + R_{max}^5, R_{max}^{10}\} \quad (3.93)$$

$$\lambda_5(R_5^1 + R_5^3) \leq \min \{R_{max}^6 + R_{max}^7, R_{max}^{11}\} \quad (3.94)$$

$$\lambda_6(R_6^2 + R_6^3) \leq \min \{R_{max}^5 + R_{max}^7, R_{max}^{12}\} \quad (3.95)$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 = 1 \quad (3.96)$$

Then, inequalities number [3.72](#),[3.73](#),[3.74](#),[3.75](#),[3.76](#),[3.77](#),[3.78](#),[3.79](#) and [3.80](#) should be checked.

The above problem cannot be solved by separating it into two parts, because the solution for the first part does not have a feasible solution that satisfies the second part.

Considering the above results, it is decided to consider related algorithms in order to group the clients almost efficient.

For the optimization problem, some primary simulations have been done in order to understand the problem and possible results. The uplink grouping is considered for groups of two clients, for total clients of 4 and 6. Five schemes are simulated.

1. Maximizing the minimum rate for grouping.
2. Sorting the channel gains and grouping the biggest with the smallest.
3. Random clients grouping.
4. Multiple access channel with no grouping.
5. Considering uneven tone assignment for random grouping.

(1) and (2) are exactly the same schemes which are better than random grouping but the difference is not significant. The difference between (1), (2), (3) and (4) shows that an optimum or near optimum grouping can improve the grouping gain more than what we have now. The following results are for 6 clients and constant energy which equals one.

method	1	2	3	4	5
Sum rate	0.5954	0.5954	0.5796	0.8639	0.5734

Table 3.4: Sum rate for different schemes considering normalized energy  $\frac{P}{\sigma^2} = 1$

### 3.4 Algorithms for grouping

Suppose that we have  $N$  clients to be grouped into  $K$  groups of size  $B$  and the available power for each client is equal to  $P$ . For each of these groups, say group  $k$ , the maximum summation rate,  $R_k$ , will be obtained by the following formula:

$$R_k = \frac{1}{2} \log(1 + \gamma \times s_k) \quad (3.97)$$

Where  $\gamma = \frac{P}{\sigma^2}$  and  $s_k = \sum_{i=1}^B G_{k_i}^2$  and  $G_{k_i}$  is the channel gain for client  $i$  in group  $k$ . Therefore, we should maximize the following function:

$$\sum_{k=1}^K R_k = \sum_{k=1}^K \frac{1}{2} \log(1 + \gamma \times s_k) = \frac{1}{2} \log \prod_{k=1}^K (1 + \gamma \times s_k) \quad (3.98)$$

Therefore, maximizing  $\sum_{k=1}^K R_k$  is equivalent to maximizing the following term:

$$\prod_{k=1}^K (1 + \gamma \times s_k) \quad (3.99)$$

On the other hand, we have:

$$\sum_{k=1}^K (1 + \gamma \times s_k) = K + \gamma \times \sum_{k=1}^K s_k = K + \gamma \times \sum_{k=1}^N G_i^2 \quad (3.100)$$

In which  $\sum_{k=1}^N G_i^2$  is constant over the study period as each  $g_i$  is assumed to be constant. Consequently, the above summation is constant over the study period.

**Claim:** *The maximum product of positive integers with constant summation happens when all of them are equal.*

This can be easily proven by using  $AM - GM$  inequality. See Appendix A for the proof.

That being said, we should get down to finding  $(1 + \gamma \times s_k)$ 's such that they are roughly equal. In other words, we have to find groups of size  $B$  such that the summation of  $G^2$ 's for the group's elements are roughly equal. Therefore, to optimize the sum rate, the following problem should be taken into account:

Given  $N$  positive numbers, how to distribute them in  $K$  groups with a constant cardinality of  $B$  such that the summation of the elements of each group is roughly equal to the other ones.

Type of grouping	Size=6	Size=12	Size=24	Size=48	Size=72
Random group of size=2	0.3651	0.3643	0.3644	0.3664	0.3658
Grouping around average point of size=2	0.4653	0.3591	0.3078	0.2882	0.2819
Random group of size=3	0.4901	0.4872	0.4904	0.4917	0.4912
Grouping around average point of size=3	0.5398	0.5007	0.4407	0.4091	0.3995
All MAC	0.6140	0.6748	0.7056	0.7208	0.7254

Table 3.5: Grouping algorithm considering the average point. All values are in dB and over single channel allocation.

This problem is an [nondeterministic polynomial time \(NP\)](#) problem that has been studied extensively. Some algorithms have been proposed so far, like KK [5], BLDM [12], LRM [15], Meld [15] etcetera. We have examined different algorithms and provided the results in the simulation section. As it will be shown, some algorithms are not effective and some increase the gain.

### 3.4.1 Grouping around the average point

The first algorithm is known as grouping around the average point. In this algorithm, the average of all channel gains is calculated and denoted by  $\bar{G}$ . Then,  $|G - \bar{G}|$  is calculated and sorted. Grouping starts simply with choosing the first  $K$  clients that are sorted as the smallest ones. The second group is chosen by having the next  $K$  clients and so on. In this way, we try to keep the values in a group that have an average around  $\bar{G}$ .

We have simulated this simple algorithm and rate improvement is provided in 3.5. As shown, the algorithm is only helpful for a small number of clients and is not able to increase the rate for a large number of clients.

### 3.4.2 LRM Algorithm

Suppose that we are meant to group  $N$  numbers to  $K$  groups with a constant cardinality of  $B$  such that the sum of the numbers in each group is equal. "Spread" for each group means the difference of max and min number in each group. This method tries to make the sums roughly equal by decreasing the spread of each group.

The algorithm is defined as follows:

1. Sort out the numbers decreasingly.

2. Put the first  $K$  numbers in one group, the second  $K$  numbers in another group and so on.
3. Find the mean of each group,  $\mu_i$ , and define  $\mu = \sum \mu_i$ .
4. Pick the leftmost number of the group which has the largest spread and put it equal to  $L$ .
5. Pick the rightmost number of the group which has the second largest spread and put it equal to  $R$ .
6. Among the numbers in the group which has the smallest spread, find the number which is closer to  $\mu - L - R$  and define it as  $M$ .
7. If the number of groups is greater than 3, then do the previous step for the other groups as well.
8. Put  $R$ ,  $L$ ,  $M$  (and the other numbers found in the previous step) in one group and eliminate them from the original groups.
9. Return back to the step 4 and do the algorithm until the numbers are over.

### 3.4.3 BLDM Algorithm

Similarly, this method tries to solve the abovementioned problem but the algorithm is slightly different:

1. Sort out the numbers decreasingly.
2. Put the first  $K$  numbers in one group, the second  $K$  numbers in another group and so on.
3. Find two groups which have the largest spread and fold them. By folding, we mean that we have to add the largest number of the first group to the smallest number of the second group and then add the second largest number of the first group to the second smallest number of the second group and so on. This step will decrease the number of groups by one.
4. Return to step 3 until just one group left.
5. Each number in the final group is corresponding to the sum of  $b$  numbers which are meant to form a group.

Type of grouping	Size=6	Size=12	Size=24	Size=48	Size=72
Random group of size=2	0.3651	0.3643	0.3644	0.3664	0.3658
LRM (BLDM) group of size=2	0.5357	0.5891	0.6154	0.6312	0.6357
Random group of size=3	0.4901	0.4872	0.4904	0.4917	0.4912
LRM group of size=3	0.5899	0.6439	0.6704	0.6851	0.6892
BLDM of size=3	0.5899	0.6448	0.6724	0.6887	0.6940
All MAC	0.6140	0.6748	0.7056	0.7208	0.7254

Table 3.6: Effect of size on gain for different algorithms. All values are in dB and over single channel allocation.

### 3.4.4 Simulation results

For the groups of size 2, both LRM and BLDM methods result in the same grouping strategy. In the following, six methods have been compared:

1. When the users randomly form a group of size 2.
2. When the users form a group of size 2 based on LRM (or BLDM) algorithm.
3. When the users randomly form a group of size 3.
4. When the users form a group of size 3 based on LRM algorithm.
5. When the users form a group of size 3 based on BLDM algorithm.
6. When all of the clients use the whole channels simultaneously and make a full **MAC** system.

#### First case, size effect

In this simulation, the effect of size on the rate improvement is compared, while  $\frac{P}{\sigma^2}$  is constant. The gains have been calculated with respect to the single channel allocation set-up. Note that the values are in dB. The comparison has been shown in 3.7.

Here,  $\frac{P}{\sigma^2}$  is fixed and equal to 10 for all types.

It is observed that LRM grouping with size=2 has better performance compared to the random grouping of size=3. In addition, both LRM and BLDM methods result in the same gain for the groups of size 6. Moreover, BLDM for the group size of greater than 6 has a slightly better performance with respect to LRM.



Type of grouping	$\frac{P}{\sigma^2=5}$	$\frac{P}{\sigma^2=10}$	$\frac{P}{\sigma^2=15}$	$\frac{P}{\sigma^2=20}$	$\frac{P}{\sigma^2=50}$
Random group of size=2	0.3773	0.3664	0.3526	0.3423	0.3021
LRM (BLDM) group of size=2	0.6634	0.6312	0.6021	0.5812	0.5043
Random group of size=3	0.5108	0.4917	0.4708	0.4558	0.3991
LRM group of size=3	0.7256	0.6851	0.6513	0.6278	0.5421
BLDM of size=3	0.7293	0.6887	0.6540	0.6298	0.5455
All MAC	0.7667	0.7208	0.6839	0.6589	0.5674

Table 3.7: Effect of power on gain for different algorithms. All values are in dB and over single channel allocation.

### Second case, power effect

In this simulation, the group size is constant and  $\frac{P}{\sigma^2}$  varies. This means that the required rate for each client, which is considered to be the same for all clients, may vary. Here, the number of clients/channels is equal to 48. The results have been shown in 3.7.

## 3.5 Summary

### 3.5.1 Conclusions

In uplink transmission, multiple transmitters can send their messages to the receiver following multiple access channel protocol, which has the best rate achievement but delay and complex decoding at the receiver. Using single channel allocation has the worst rate achievement but less delay and complexity. By grouping the clients into groups of  $K$  clients and having MAC set-up in each group, the total rate improves compared to single channel allocation. Simulation results show that among the considered algorithm, BLDM has the best gain.

### 3.5.2 List of contributions

In this chapter, a new method for channel allocation in uplink transmission was proposed. The idea of grouping the clients and applying the MAC set-up in each group has the following merits:

1. It has less complexity with respect to the All-MAC set-up, for the size of the codebook used for coding/decoding would be decreased. On the other hand, the gain for the new set-up would remain approximately the same as the MAC set-up.
2. The gain for the proposed method is noticeably higher than the gain for [FDMA](#).

### 3.5.3 Future research

The topics which need more work in this chapter could be as follows:

1. Increasing the size of the groups. As the number of clients increases, the size of the groups can increase accordingly. However, this needs more work on defining the corresponding optimization problem and it seems to be time-consuming.
2. In a general set-up, the number of channels may differ from the number of clients. In our set-up, these numbers are the same.
3. As it was discussed in the previous chapter, it could be interesting to define a channel which is not Gaussian and after finding the capacity for the new channel, one can test the proposed method over it.

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# APPENDICES

# Appendix A

## Proof of the claim made in 3.4

**Claim:** *The maximum product of positive integers with constant summation happens when all of the numbers are equal.*

Based on AM-GM inequality we have [10]:

$$\sqrt[n]{x_1 x_2 \dots x_n} \leq \frac{x_1 + x_2 + \dots + x_n}{n} \quad (\text{A.1})$$

Therefore, we have:

$$(x_1 x_2 \dots x_n) \leq \left( \frac{x_1 + x_2 + \dots + x_n}{n} \right)^n \quad (\text{A.2})$$

and equality happens when:  $x_1 = x_2 = \dots = x_n$ .

In the abovementioned claim, the right side of inequality A.2 is constant; therefore the maximum value of  $(x_1 x_2 \dots x_n)$  happens when  $x_1 = x_2 = \dots = x_n$ .