# Cooperative Autonomous Vehicle Speed 

 Optimization Near Signalized Intersectionsby

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## Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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#### Abstract

Road congestion in urban environments, especially near signalized intersections, has been a major cause of significant fuel and time waste. Various solutions have been proposed to solve the problem of increasing idling times and number of stops of vehicles at signalized intersections, ranging from infrastructure-based techniques, such as dynamic traffic light control systems, to vehicle-based techniques that rely on optimal speed computation. However, all of the vehiclebased solutions introduced to solve the problem have approached the problem from a single vehicle point of view. Speed optimization for vehicles approaching a traffic light is an individual decision-making process governed by the actions/decisions of the other vehicles sharing the same traffic light. Since the optimization of other vehicles' speed decisions is not taken into consideration, vehicles selfishly compete over the available green light; as a result, some of them experience unnecessary delay which may lead to increasing congestion. In addition, the integration of dynamic traffic light control system with vehicle speed optimization such that coordination and cooperation between the traffic light and vehicles themselves has not yet been addressed.

As a step toward technological solutions to popularize the use of autonomous vehicles, this thesis introduces a game-theoretic-based cooperative speed optimization framework to minimize the idling times and number of stops of vehicles at signalized intersections. This framework consists of three modules to cover issues of autonomous vehicle individual speed optimization, information acquisition and conflict recognition, and cooperative speed decision making. It relies on a linear programming optimization technique and game theory to allow autonomous vehicles heading toward a traffic light cooperate and agree on certain speed actions such that the average idling times and number of stops are minimized. In addition, the concept of bargaining in game theory is introduced to allow autonomous vehicles trade their right of passing the traffic light with less or without any stops. Furthermore, a dynamic traffic light control system is introduced to allow the cooperative autonomous vehicles cooperate and coordinate with the traffic light


to further minimize their idling times and number of stops. Simulation has been conducted in MATLAB to test and validate the proposed framework under various traffic conditions and results are reported showing significant reductions of average idling times and number of stops for vehicles using the proposed framework as compared to a non-cooperative speed optimization algorithm. Moreover, a platoon-based autonomous vehicle speed optimization scheme is posed to minimize the average idling times and number of stops for autonomous vehicles connected in platoons. This platoon-based scheme consists of a linear programming optimization technique and intelligent vehicle decision-making algorithm to allow vehicles connected in a platoon and approaching a signalized intersection decide in a decentralized manner whether it is efficient to be part of the platoon or not. Simulation has been conducted in MATLAB to investigate the performance of this platoon-based scheme under various traffic conditions and results are reported, showing that vehicles using the proposed scheme achieve lower average values of idling times and number of stops as compared to two other platoon scenarios.

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## Dedication

This thesis is dedicated to the memory of my father. He was a very dear friend, and he will always be missed. I dedicate this work to his memory. I also dedicate this thesis to my mother. She is more than a mother. Her strength, endurance, character, friendliness, and love mean a lot to me. Thank you Mom for your sacrifices.

Finally, I wish to acknowledge the gratitude I owe to my family, who offered me unconditional love and support during the course of this thesis.

## Table of Contents

List of Figures ..... xii
List of Tables ..... xiv
1 Introduction ..... 1
1.1 Autonomous Vehicles ..... 2
1.2 Challenges ..... 3
1.3 Motivation ..... 4
1.4 Thesis Objectives ..... 6
1.5 Thesis Organization ..... 7
2 Background and Literature Review ..... 9
2.1 Safety, Environmental and Economical Impacts ..... 10
2.2 Challenges and Related Issues ..... 12
2.3 Traffic Management near Intersections ..... 13
2.4 Vehicle Speed Optimization ..... 14
2.5 Adaptive Cruise Control ..... 17
2.5.1 Cooperative Adaptive Cruise Control ..... 18
2.5.2 Platoon Formation in Urban Areas ..... 19
2.6 Fuzzy Logic Control ..... 20
2.7 Traffic Light Control Systems ..... 21
2.8 Game Theory ..... 23
2.8.1 Shortest Path and Congestion Games ..... 24
2.8.2 Dynamic Traffic Light Control Games ..... 25
2.9 Summary ..... 26
3 A Cooperative Framework for Autonomous Vehicle Speed Optimization ..... 28
3.1 Problem Statement ..... 29
3.1.1 Signalized Roadway Intersection Setting ..... 29
3.1.2 Problem Formulation ..... 31
3.2 Stability of the Speed Optimization Game ..... 32
3.3 The Cooperative Speed Optimization Framework ..... 33
3.3.1 Autonomous Vehicle Speed Optimization Module ..... 34
3.3.2 Information and Conflict Recognition Module ..... 42
3.3.3 Cooperative Decision Making Module ..... 42
3.4 Cooperative Credit-Point Bargaining ..... 49
3.4.1 The Marginal Contribution ..... 51
3.4.2 The Core ..... 52
3.5 Simulation Tests and Results ..... 55
3.6 Summary ..... 62
4 Cooperative Autonomous Vehicle-Dynamic Traffic Light Control System ..... 64
4.1 Problem Statement ..... 65
4.2 Cooperative Autonomous Vehicle-Dynamic Traffic Light Control ..... 67
4.2.1 Decision Making Unit ..... 68
4.2.2 Control Unit ..... 71
4.3 Simulation Tests and Results ..... 75
4.4 Summary ..... 79
5 Platoon-based Autonomous Vehicle Speed Optimization near Signalized Intersec- tions ..... 80
5.1 Problem Statement ..... 81
5.2 Platoon-based Autonomous Vehicle Speed Optimization Scheme ..... 82
5.2.1 Platoon Formation and Control ..... 83
5.2.2 Leading-Autonomous-Vehicle Speed Optimization ..... 84
5.2.3 Intelligent Vehicle Decision-Making Algorithm ..... 87
5.3 Simulation Tests and Results ..... 88
5.4 Summary ..... 91
6 Conclusion and Future Research ..... 93
6.1 Conclusion ..... 93
6.2 Future Research ..... 96
Bibliography ..... 98

## List of Figures

1.1 The autonomous car of Google Corporation [3]. ..... 3
3.1 A simple example of a traffic light scenario. ..... 30
3.2 Schematic depiction of the cooperative speed optimization framework. ..... 34
3.3 Cooperative speed optimization logic ..... 46
3.4 A sub-network with three signalized intersections ..... 56
3.5 Total average idling time at SI1, SI2, and SI3 ..... 58
3.6 Total average number of stops at SI1, SI2, and SI3. ..... 58
3.7 Total average energy consumption of vehicles approaching SI1, SI2, and SI3. ..... 60
3.8 Speed trajectories of six vehicles approaching intersection 1, (a) vehicles using the CSOF, (b) vehicles using the NCSO. ..... 61
3.9 Activation distance analysis for intersection 2, (a) total average idling time, (b) total average number of stops. ..... 62
4.1 An example scenario of autonomous vehicles approaching a static-signal-timing traffic light ..... 66
4.2 Schematic depiction of the cooperative autonomous vehicle-dynamic traffic light control system. ..... 67
4.3 Block diagram of the fuzzy logic control system. ..... 70
4.4 Membership functions of extra vehicles in the green-light phase. ..... 72
4.5 Membership functions of extra vehicles in the red-light phase. ..... 72
4.6 Membership functions of green light extension. ..... 73
4.7 Total average idling time at SI1, SI2, and SI3. ..... 76
4.8 Total average number of stops at SI1, SI2, and SI3 ..... 77
4.9 Input-output of fuzzy inference system. ..... 78
5.1 An example scenario of a platoon approaching a signalized intersection ..... 81
5.2 Logic of platoon-based autonomous vehicle speed optimization scheme ..... 83
5.3 A signalized intersection of single-lane four-roadways ..... 89
5.4 Total average idling time in three hours ..... 90
5.5 Total average number of stops in three hours ..... 91

## List of Tables

3.1 Action and response table of a two-player game ..... 47
3.2 Action and response table of a two-player game ..... 48
3.3 Reduction in average idling time at signalized intersection 1. ..... 59
3.4 Reduction in average idling time at signalized intersection 2. ..... 59
3.5 Reduction in average idling time at signalized intersection 3. ..... 59
4.1 Decision-making action table. ..... 69
4.2 Reduction in average idling time at signalized intersection 1. ..... 77
4.3 Reduction in average idling time at signalized intersection 2. ..... 77
4.4 Reduction in average idling time at signalized intersection 3. ..... 78
5.1 Reduction in average idling time at the signalized intersection. ..... 91

## Nomenclature

| Abbreviations |  |
| :--- | :--- |
| ACC | Adaptive Cruise Control |
| AI | Artificial Intelligence |
| AV | Autonomous Vehicle |
| CACC | Cooperative Adaptive Cruise Control |
| CAV-DTLC | Cooperative Autonomous Vehicle-Dynamic Traffic Light Control |
| CCC | Conventional Cruise Control |
| CM | Centroid Method |
| CSOF | Cooperative Speed Optimization Framework |
| CSP | Constraint Satisfaction Problem |
| CU | Control Unit |
| DC-GLOSA | Driver Centric-Green Light Optimal Speed Advisory |
| DMU | Decision Making Unit |


| DSRC | Dedicated Short Range Communication |
| :---: | :---: |
| DTLCS | Dynamic Traffic Light Control System |
| EAV | Electric Autonomous Vehicle |
| EV | Electric Vehicle |
| FIS | Fuzzy Inference System |
| FL | Fuzzy Logic |
| FLC | Fuzzy Logic Control |
| FLCr | Fuzzy Logic Controller |
| GLOSA | Green Light Optimal Speed Advisory |
| IoT | Internet-of-Things |
| IR-LED | Infr Red-Light Emitting Diode |
| IVDMA | Intelligent Vehicle Decision-Making Algorithm |
| IVSOP | Intelligent Vehicle Speed Optimization Platoon |
| LP | Linear Programming |
| LSOP | Leader only Speed Optimization Platoon |
| MCP | Marginal Contribution Principle |
| MFM | Mamdani Fuzzy Model |
| MPC | Model Predictive Control |
| NB | Nash Bargaining |


| NCSO | Non-Cooperative Speed Optimization |
| :---: | :---: |
| NE | Nash Equilibrium |
| NSOP | No Speed Optimization Platoon |
| OPP | Object Oriented Programming |
| PAVSOS | Platoon-based Autonomous Vehicle Speed Optimization Scheme |
| PD | Poisson Distribution |
| PNs | Petri Nets |
| RB | Rule Base |
| RFID | Radio Frequency Identification |
| ROI | Region of Interest |
| SDV | Self-Driving Vehicle |
| SI | Signalized Intersection |
| STL | Smart Traffic Light |
| TL | Traffic Light |
| TTAA | Time Token Allocation Algorithm |
| USI | Un-Signalized Intersection |
| V2I | Vehicle to Infrastructure |
| V2V | Vehicle to Vehicle |
| WSNs | Wireless Sensor Networks |

## Symbols

$\Delta \quad$ Amount of energy consumed or gained at time step $t$
$\eta \quad$ Efficiency of the autonomous vehicle's motor
$\lambda \quad$ Traffic light arrival rate
$\mu \quad$ Traffic light departure rate
$\mu_{E}(E) \quad$ Membership function of the fuzzy set of the green-light time extension
$\mu_{g}\left(E X_{g}\right) \quad$ Membership function of the fuzzy set of extra vehicles in the green-light phase
$\mu_{r}\left(E X_{r}\right) \quad$ Membership function of the fuzzy set of extra vehicles in the red-light phase
$\rho \quad$ Air density coefficient
$\tau_{i} \quad$ Index of time token $i$ within the green light time of the traffic light
$a_{i} \quad$ Lower boundary of time token $\tau_{i}$
$a_{r} \quad$ Air drag coefficient
$A_{s} \quad$ Cross sectional area of the autonomous vehicle
$A V_{i} \quad$ Autonomous vehicle $i$
$b_{i} \quad$ Upper boundary of time token $\tau_{i}$
$C_{i t}^{P} \quad$ Average idling time cost of a platoon of vehicles
$C_{\max } \quad$ Maximum capacity of the autonomous vehicle battery
$C_{s v}(i) \quad$ Idling time cost of vehicle $A V_{i}$ incurred by the sequence of velocities $s v$
$C P\left(A V_{i}\right) \quad$ Credit point balance of vehicle $A V_{i}$

| $D\left(L_{j}\right)$ | Traffic density on road segment $L_{j}$ |
| :--- | :--- |
| $d_{i j}(t)$ | Headway distance between vehicle $i$ and vehicle $j$ at time step $t$ |
| $d_{i}(t)$ | Distance of vehicle $A V_{i}$ to the stop line of the traffic light at time step $t$ |
| $E$ | Extension game |
| $E C_{i}^{a c}$ | Acceleration energy |
| $E C_{i}^{d c}$ | Deceleration energy |
| $E C_{i}^{e d}$ | Energy consumed by electric devices |
| $E C_{i}^{l o s s}$ | Loss of energy at time step $t$ |
| $E C_{i}^{P C}$ | Potential consumed energy at time step $t$ |
| $E C_{i}^{P G}$ | Potential gained energy at time step $t$ |
| $E C_{i}^{T}$ | Total energy consumed by autonomous vehicle $A V_{i}$ |
| $E W$ | East-West roadway phase |
| $E X_{g}$ | Extra vehicles in the green-light phase |
| $E X_{r}$ | Extra vehicles in the red-light phase |
| $f\left(\left\{A V_{i}\right\}\right)$ | Value of player $A V_{i}$ prior to playing a game |
| $f(\{S\})$ | Value of a subset of players $S$ prior to playing a game |
| $f_{r}$ | Friction coefficient |
| $E$ | Cooperative speed optimization game |
| $E$ |  |


| $G L P$ | Green-light phase |
| :---: | :---: |
| $l_{d}(t)$ | Distance of vehicle $A V_{i}$ between current and previous locations at time step $t$ |
| $L_{j}$ | Road segment $j$ |
| $m$ | Mass of the autonomous vehicle |
| $M\left(A V_{i}\right)$ | Mode value of vehicle $A V_{i}$ |
| $M C\left(A V_{i}\right)$ | Marginal contribution of player $A V_{i}$ |
| $N$ | A set of players |
| $N_{a r r}$ | Number of vehicles arriving at the traffic light during the whole cycle |
| $N_{\text {dep }}$ | Number of vehicles departing the traffic light during green light time |
| $N_{E X_{g}}$ | Number of extra vehicles in the green-light phase |
| $N_{E X_{r}}$ | Number of extra vehicles in the red-light phase |
| $N_{G L P}$ | Number of vehicles in the green-light phase |
| $N_{p}$ | Number of vehicles connected in a platoon |
| $N_{q}$ | Number of vehicles currently in the queue |
| $N_{R L P}$ | Number of vehicles in the red-light phase |
| $N_{t k n}$ | Number of time tokens |
| $N_{T L}$ | Number of traffic lights on a path $P$ |
| $N E$ | Not extension game |
| $N E X{ }_{g}$ | No extra vehicles in the green-light phase |

$N E X_{r} \quad$ No extra vehicles in the red-light phase
$P^{e d}(j) \quad$ Power consumed by electric device $j$
$p_{0 i} \quad$ Initial location on a path
$p_{f i} \quad$ Final location on a path
$P_{i}^{w r} \quad$ Power of the autonomous vehicle's electric motor
$R_{g} \quad$ Remaining green light time
$R_{r} \quad$ Remaining red light time
$R L P \quad$ Red-light phase
$R N\left(A V_{i}\right) \quad$ Random number value of vehicle $A V_{i}$
$R N(T L) \quad$ Random number value of the traffic light

A subset of players
$S_{E} \quad$ Fuzzy set of the extension of the green-light time
$S_{g} \quad$ Fuzzy set of the extra vehicles in the green-light phase
$s_{i} \quad$ The optimal speed of vehicle $A V_{i}$ at time step $t+1$
$S_{r} \quad$ Fuzzy set of the extra vehicles in the red-light phase
SN South-North roadway phase
Sequence of velocities
$t^{e d}(j) \quad$ Time that electric device $j$ takes in use
$t_{a c} \quad$ Acceleration time

| $T_{C}$ | Traffic light cycle duration |
| :---: | :---: |
| $t_{d c}$ | Deceleration time |
| $T_{e x}$ | Defuzzified extension time |
| $T_{g}$ | Green light time duration |
| $T_{q}$ | Time needed to clear all the vehicles in the queue |
| $T_{r}$ | Red light time duration |
| $T_{s d}$ | Time token duration |
| $T_{\text {slot }}$ | Time token location in the traffic light memory |
| $T L_{S}$ | Traffic light system |
| $T T I_{i}$ | Time to intersection of vehicle $A V_{i}$ |
| $u(t)$ | Elevation of the road segment at time step $t$ |
| $v_{i}(t)$ | Speed of vehicle $A V_{i}$ at time step $t$ |
| $v_{\text {max }}$ | Maximum speed limit |
| $v_{\text {min }}$ | Minimum speed limit |
| VIN | Autonomous vehicle identification number |
| $x_{a v_{i}}$ | Value received by player $A V_{i}$ after playing a game |
| $x_{i}(t)$ | Location of vehicle $i$ at time step $t$ |
| $x_{S}$ | Value received by the subset of players $S$ after playing a game |
| $Y_{D M U}$ | Output of the decision making unit |

$Y_{E} \quad$ Output of the extension game
$Y_{N E} \quad$ Output of the no extension game

## Chapter 1

## Introduction

In the USA alone, there were about six million vehicle crashes in 2010. Human errors caused approximately $93 \%$ of these crashes [1]. Over the past few years, technology and automobile industries have concentrated on the human driving process in their efforts to automate the transportation system. Recently, various car models have started to include automatic characteristics and features, such as parking assistant systems that automatically steer vehicles into available parking spaces. Some automobile companies have made leaps toward manufacturing Autonomous Vehicles (AVs), which can implement different levels of automatic functions as a means of achieving a safer and more efficient transportation system. AVs are believed by many industrial companies and stakeholders to have the potential to dramatically improve the current transportation system. The next section provides a brief introduction to AVs.

### 1.1 Autonomous Vehicles

AVs, also called Self-Driving Vehicles (SDVs), have different levels of intelligence and capability to perform partial or full automatic driving tasks. AVs can be categorized, based on their level of autonomy, into four major levels [2-4]. In Level-one AVs, the driver is in full control of the vehicle, and the automation is limited to very specific control functions, such as cruise control, anti-lock brakes and an electronic stability control system. The automation of Level-two AVs is extended to multiple and more integral control functions, such as Adaptive Cruise Control (ACC), Cooperative Adaptive Cruise Control (CACC), lane centering and lane changing, and autonomous parking capability. Other than when these automatic systems are functioning, the driver is responsible for monitoring the roadway at all times. Level-three AVs are able to monitor the surrounding environment and navigate autonomously, so the drivers are not expected to monitor the roadway at all times. However, under certain critical conditions the drivers must be attentive and ready to take back control of the vehicle for a short period of time. The Google car [3] is an instance of this level of self-driving (Figure 1.1). Level-four AVs are those capable of monitoring the surrounding environment and performing all driving tasks autonomously for an entire trip. The driver may input a destination to the vehicle, but is not expected to intervene during operation. Therefore, these AVs may operate without passengers or with passengers who cannot drive (e.g., disabled people, non-drivers or elderly people, etc).

Reportedly, AVs have multiple major positive impacts on the transportation system, such as reduced driver stress and public transportation cost; improved mobility of the non-driving, disabled and elderly people; reduced accident risks; increased road capacity (e.g., by forming platoons of vehicles travelling in a row with safe headway distances, resulting in less speed variations); and reduced parking costs [1-4]. However, many issues need to be resolved before AVs can be practically operated on the roads. The next section addresses a few challenges associated with the widespread development and use of AVs.


Figure 1.1: The autonomous car of Google Corporation [3].

### 1.2 Challenges

Security and privacy risks are one category among the many important issues to be resolved before AVs are widely marketed. In scenarios of comprehensive AV adoption, Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communication systems may be vulnerable to data abuse [2]. Hence, the communication systems must be robust and controllable in cases such as when there is cyber attack risk. In addition, in order to enhance road safety and driver convenience, AVs require high-technology sensors, special navigation systems, and software upgrades; as a result, purchase prices will be increased by thousands of dollars. Also, these technological components must meet high installation, testing, and maintenance standards, which add more cost to the ownership of AVs [3]. Consequently, low and middle income people will not be able to afford AVs. Moreover, laws for the liability, standardization, and certification have to be legislated. There are concerns about who owns the risk when highly autonomous vehicles crash; therefore, solutions to legalize and legislate the use of AVs have to be found [1].

Most importantly, the impact of the wide use of AVs on road congestion, especially in urban environments, is uncertain and has to be investigated. On one hand, AVs will have many positive impacts on the transportation system, such as reducing public transportation cost due to fewer
human-driving activities, reducing public transportation demand through increased rates of carsharing and better mobility of the non-driving, disabled, and elderly people. Moreover, a high adoption of AVs will reduce parking costs as many people can drive to work or school then program their AVs to return home while they are working or studying. On the other hand, all of these positive impacts may increase the number of vehicles on roads; consequently, traffic congestion in urban areas might rise. As forming platoons of vehicles reduces the inter-vehicle spacing and speed variations on roads, this could improve the road capacity on highways but only when the penetration rate is high [3]. Therefore, the impacts of AVs on road congestion in urban areas cannot be predicted and need to be addressed through research. It is reported in [2] that in order to obtain positive impacts on road congestion, cooperation between AVs and infrastructure is necessary. Cooperative AV systems could improve traffic management, and therefore reduce congestion in urban environments.

### 1.3 Motivation

Road congestion in urban environments, especially near Signalized Intersections (SIs), has been a major cause of significant fuel waste and time delay. It is reported in [1] that in the USA every year, 4.8 billion hours is wasted by traffic congestion. This significant delay costs around $\$ 100$ billion of fuel waste and $\$ 23$ billion due to congestion impacts on trucking operations. These facts present a strong need to devise technological congestion-reduction techniques for AVs . Long and cumulative idling times of vehicles at SIs are considered a major contributor to traffic congestion within urban environments. Therefore, various solutions have been proposed to solve the problem of the increasing idling times at Traffic Lights (TLs), ranging from infrastructurebased techniques such as dynamic TL control systems [5-7] to vehicle-based techniques that rely on vehicle optimal speed computation [8] [9]. These techniques have one common objective, that is, to minimize the vehicles' idling times and number of stops at TLs.

However, all of the vehicle-based solutions introduced to solve the idling-time-minimization problem have approached the problem from a single-vehicle point of view. In fact, traffic mobility is a function of all the vehicles sharing a travel resource and is dependent on the constraints imposed by the environment and infrastructure. Therefore, it is natural for conflicts to arise between vehicles sharing the same travel resource. Speed optimization for vehicles approaching a TL is an individual decision-making process governed by the actions/decisions of the other vehicles sharing the same TL. Hence, interest conflicts are expected to occur. The major drawback of existing vehicle speed optimization solutions is that the optimization of other vehicles' speed decisions is not taken into consideration. As a result, vehicles selfishly compete over the available green light, often causing to each other unnecessary delays. In addition, even though there has been interesting research conducted on dynamic TL control, the existing work does not integrate dynamic TL control with vehicle speed optimization such that coordination and cooperation between the TL and vehicles are implemented to minimize idling times and number of stops.

As a step forward in devising technological solutions to popularize the use of AVs, the main motivating objective of this thesis is to introduce a game-theoretic AV speed optimization solution to minimize the idling times and number of stops of AVs at TLs. The main motivating objective for the rest of the research is to integrate the dynamic TL control with the game-theoretic AV speed optimization such that a dynamic-TL-AV-based cooperative technological system is devised to further minimize vehicles' idling times and number of stops. The task of satisfying the needs of many participants in a process is extremely complex. Game theory is recognized as a solid mathematical platform to capture the complexity of team-aware optimization problems. For instance, by cooperating with each other, vehicles can exchange strategic actions (e.g., speeds) in some form of compensation to minimize their idling times at TLs. Both categories of game theory, cooperative and non-cooperative, have been well developed and can be used to model the AV minimization problem of idling times and number of stops as a cooperative strategic game.

### 1.4 Thesis Objectives

Existing research on vehicle optimal speed computation to minimize the idling times and number of stops at SIs does not consider the interaction and cooperation between vehicles. Thereby, the aim of this thesis has been to allow AVs, travelling toward a TL, to interact and cooperate with each other as well as with the TL control system such that the average idling times and number of stops are minimized. Thus, to model the AV speed optimization as a cooperative process, the following research objectives are addressed:

- Developing a problem formulation using game theory to address the interaction and cooperation between AVs approaching an SI.
- Investigating the feasibility and stability of the game-theoretic formulation.
- Developing a cooperative AV speed optimization solution framework to model the interaction and cooperation between AVs approaching an SI.
- Developing a bargaining model to allow the AVs heading toward a TL trade their rights of passing through without delay.
- Developing a dynamic TL control system to allow the cooperative AVs interact and cooperate with the TL to achieve further minimization of idling times and number of stops.
- Investigating the impacts of speed optimization on connected-AV platoons when approaching SIs.
- Finally, conducting extensive simulation experiments to test and validate the performance of the introduced techniques.


### 1.5 Thesis Organization

This thesis is divided into six chapters as follows: Chapter 2 provides extensive background information and a literature review addressing the environmental, economical, and safety impacts of AVs. In addition, challenges creating barriers to the widespread use of AVs are also discussed. Then, research on vehicle-centric speed optimization techniques as well as dynamic TL control systems is addressed. Following that, research conducted on vehicle speed control and platooning systems such as ACC and CACC as well as fuzzy logic control is summarized. Background information is presented on game theory as a team-based optimization platform with special attention given to the shortest-path and congestion games.

Chapter 3 states the problem of increasing idling times and stop numbers at a TL as the number of vehicles utilizing the TL increases. The idling time and stop number minimization problem for AVs approaching a TL is formulated as a strategic game, and the stability of the AV speed optimization game is discussed as well. Furthermore, this chapter introduces a gametheoretic Cooperative Speed optimization Framework (CSOF) for solving the problem of idling time and stop number minimization and achieving better traffic efficiency at SIs. Moreover, a bargaining model using cooperative game theory is introduced to allow the AVs to trade their rights of passing TLs without delay.

Chapter 4 presents a dynamic TL control system to allow coordination and cooperation between the cooperative AVs and the TL system as a pace forward for further minimization of idling times and number of stops of AVs at TLs. This cooperative AV-TL control system consists of decision making and control. The decision making is based a theoretic game structured as a three-player game including the roads with green-light, roads with red-light, and the TL system. The control part employs fuzzy logic to adjust the signal timings according the traffic volume on the roadways.

Chapter 6 addresses a platoon-based autonomous vehicle speed optimization scheme to minimize the idling times and number of stops of connected-AV platoons when approaching TLs. This cooperative scheme consists of a speed optimization procedure conducted by the leader of the platoon and intelligent algorithm for decision making to be run by every follower in the platoon such that the average idling times and stop numbers of the AV-platoons are minimized.

Finally, Chapter 7 summarizes concluding remarks of what has been addressed in the thesis and presents a discussion on the simulation results obtained within each chapter. In addition, a future research plan is outlined, addressing the future directions of the thesis.

## Chapter 2

## Background and Literature Review

This chapter provides background and a literature review on a broad range of AV development, challenges and problems. First, it addresses a number of research studies conducted with regards to the road safety, environmental, and economical impacts of adopting AVs as an alternative to conventional vehicles that are fully driven by humans. Second, issues and challenges associated with the development of AVs are discussed with special attention given to AV speed optimization and control near SIs. Third, it addresses research conducted on dynamic TL control systems and their contribution to the problem of TL efficiency improvement. Finally, background information on game theory is presented, focusing largely on shortest-path and congestion games as well as the applications of game theory to the dynamic TL control.

### 2.1 Safety, Environmental and Economical Impacts

According to [1], approximately 15 deaths per 100,000 people were caused by about six million vehicle crashes in the USA in 2010. Of these crashes, $93 \%$ were caused by human error. In addition, around 2.3 million people were treated in hospitals due to car crash injuries throughout the USA in 2009. AVs can significantly reduce the accident rate and have positive impacts on road safety since most of fatal accidents are caused by alcohol, distraction, drugs or fatigue. Even if the accident is caused by a vehicle failure or the environment, additional human factors such as inattention or speeding usually maximize the severity of the crash and thus, injury consequences. A recent study in [2] states that only 5\% of road crashes are due to vehicle failure. Accidents resulting from human-driving mistakes constitute approximately $90 \%$ of the overall accident rate; therefore, widespread AV adoption could reduce the overall accident rate by $90 \%$ [10]. The positive safety impacts of AVs are further extended to having vehicles that are much lighter in weight and more energy efficient than conventional vehicles, as some safety features (e.g., reinforced steel bodies and airbags) can be neglected; according to one authority, $20 \%$ reduction in weight corresponds to $20 \%$ increase in efficiency [1].

Green-house-gas emissions can also be reduced when AVs are widely used [2]. Since AVs are equipped with high-technology capabilities, they will be able to navigate more efficiently and smoothly than human drivers throughout any journey. When there is high market penetration of AVs , major reductions can be reached with respect to parking problems, energy consumption and $\mathrm{CO}_{2}$ emission [3]. Road vehicle automation could help increase road capacity by facilitating the formation of platoons of vehicles, which reduce the road space required per vehicle [2]. Forming platoons of vehicles alone would reduce fuel consumption on highways by about 20\% [1]. Cited by [4], the SARTRE project reports that with a platoon of vehicles, the fuel savings are approximately $8 \%$ for the lead vehicle and $14 \%$ for the following vehicles when the average speed of the platoon is $85 \mathrm{~km} /$ hour and the headway between every two vehicles is 6 m . The signifi-
cant benefit of AV emission reduction will be mainly in urban areas and achieved by minimizing speed variations and stop-and-go driving. $50 \%$ congestion reduction results in $8 \%$ fewer traffic accidents and $5 \% \mathrm{CO}_{2}$ emission reduction [4]. Hence, AVs will certainly have a positive impact on the environment.

In [1], KPMG LLP and the Center for Automotive Research (CAR) cooperatively conducted a research study in which leading technologists, automotive industry leaders, academicians, and stakeholders were interviewed on the convergence of AV adoption and implications for investment in AVs. They concluded that due to the benefits expected from using them, AVs will be widely adopted in the coming few years. In addition, the Victoria Transport Policy Institute has conducted a study in [3] about the impacts of the four levels of AVs on transportation planning. This research pays attention to the benefits and costs of AVs, exploring the impacts on the parking supply and public transit demand. The results illustrate that the adoption of AVs by wealthy people will perhaps start in the 2020s or 2030s. The impacts of increasing safety, reducing road congestion, reducing emissions, and reducing parking congestion will be significant only when AVs become common, adopted by low-income people, most likely in the period from 2040s to 2060s. The widespread use of Level-four AVs can also reduce vehicle ownership by making people rely more on self-driving taxis and car-sharing [11]. Moreover, in [10], a research study sponsored by the Eno Center for Transportation presented the benefits of generally adopting AVs in the USA. It is reported that AVs have the potential to reduce crashes, congestion, fuel consumption, and parking needs; and to ease the mobility of elderly and disabled people. When the market penetration rate of AVs is only $10 \%$, the annual savings of the USA economy could reach $\$ 25$ billion. An anonymous survey with over 450 responses was conducted in [12] to explore the feelings, beliefs and expectations of participants toward the technology of AVs. It is shown that the productivity, efficiency and environmental impacts are accepted by the participants. However, the cost and legal structures were not acceptable; therefore, it was concluded that the promotion of AVs must be further developed so that the technology becomes accepted by the public.

### 2.2 Challenges and Related Issues

Even though some levels of AVs will be convenient for long-distance travelers, non-drivers, commercial drivers and disabled people, they require high-technology sensors, special navigation and mapping systems, and software upgrades; therefore, several thousands of dollars will likely be added to the vehicle purchase and ownership prices. Thus, the cost of purchase and ownership is expected to be prohibitively high. Furthermore, in order to enhance road safety and passenger convenience, these technological components have to meet high manufacturing, installation, testing, and maintenance standards, which add more costs on top of the vehicle purchase and ownership prices [3]. For instance, $\$ 70,000$ is the cost of the Light Detection and Ranging (LIDAR) technology used by Google's AV [1] [10]. Additional costs occur with other sensors; consequently, such high costs will not be affordable by low-income people, constituting a barrier to the large-scale market penetration of AVs. It is expected that when the purchase cost is reduced, the adoption of AVs will likely be widened; hence, further developments to reduce the total cost of purchase and ownership are greatly needed before stakeholders and investors invest in this emerging technology [1].

AVs may also introduce some privacy and security concerns, such as cyber terrorism as they are equipped with computer technology and V2V communications [2]. Communication systems must be robust and safe from corruption and cyber attacks. It is expected that several security issues will emerge when AVs become popular. For instance, hackers or unauthorized parties may capture data and compromise the vehicle owner by tracking the vehicle or identifying the owner's residence. Furthermore, hackers could provide fake information to drivers or attack the whole transportation system, causing collisions and traffic disruptions [1] [10]. Another example would be a computer virus that may be programmed and spread across vehicles along a period of time; consequently, all the targeted vehicles simultaneously increase their speeds. It may not be possible to completely secure the whole system since each AV represents an access point in the
targeted group [10]. In addition, AVs may use data generated by other vehicles (e.g., the Google car), so "there are privacy concerns over who would have access to this data and how it might be used" [2]. Therefore, further developments are needed with regards to AV security and privacy. The protection against such security and privacy threats may include the removal of identifying information, the aggregation of data within the vehicle rather than broadcasting a large amount of raw data over a fleet of vehicles, encryption, and the use of security functions at each layer of software and hardware [1].

Legislation is expected to impact the fast adoption of AVs [1]. For example, in Europe, the legal framework imposes that vehicles must be controlled by a driver all time of operation, which illegalizes at least level-three and level-four AVs. Some issues must be resolved with respect to liability law, regulatory law, standardization, and certification [4]. An important question would explore the case of crashes involving AVs designed such that the driver is no longer in control. Hence, the ambiguity behind risk responsibility must be resolved before high market penetration of AVs occurs [1].

### 2.3 Traffic Management near Intersections

Traffic management especially near intersections has been an important area of research. An autonomous intersection management system is proposed in [15], where incoming vehicles are assigned priorities by a controller to pass through USIs safely based on a defined brake-safe state. A queue control algorithm is addressed in [16] to control the average queue size at SIs and thereby alleviate congestion in road networks. A traffic management framework is introduced in [17] to reduce traffic congestion at SIs by incorporating a dynamic vehicle rerouting strategy and a TL control system. To provide emergency safety responses at SIs, a TL control system based on deterministic and stochastic Petri Nets (PNs) is designed in [18] where TL control
strategies are implemented to ensure the safety of traffic in cases of accidents. An Internet-ofThings (IoT)-based platform containing three main units is introduced in [19] for emergency vehicle priority and self-organized traffic control management at SIs.

Based on the self-organizing biological concept, a design of a self-organizing network with a set of local rules to manage traffic priority is presented in [20] where priority is given to emergency vehicles to expedite their response times. A heuristic approach is proposed in [21] to resolve space-time conflicts such that AVs are able to pass through USIs safely with minimum delay. A cooperative framework is introduced in [22] to allow adjacent TL control systems to coordinate their signal-timing settings so as to globally optimize the traffic efficiency and travel time at multiple intersections. To manage traffic at USIs, a distributed traffic management protocol is addressed in [23] to allow vehicles to cross safely and with high level of driver's comfort. A study is presented in [24] to analyze the difference based on the passing order of vehicles at USIs between the cooperative driving of the ad hoc negotiation strategy and the planning strategy. A cooperative mechanism is developed in [25] to autonomously manage the crossing of AVs through USIs by adjusting the entry times and speeds of AVs in a certain core area.

### 2.4 Vehicle Speed Optimization

This section considers research conducted to address the impacts of vehicle speed optimization on traffic efficiency near SIs. Traffic efficiency at TLs can be achieved by minimizing the idling times of vehicles as well as stop-and-go driving. As presented next, limited research has been conducted on centric-speed optimization (i.e., the vehicles individually and independently compute their optimal speeds to meet the green-light time) to minimize the idling times and number of stops at TLs.

An algorithm called Green Light Optimal Speed Advisory (GLOSA) has been proposed in [8]
to minimize the number of stop times at SIs through a journey. The impacts of this algorithm on traffic efficiency and average trip time were reported. The performance analysis of the same algorithm is investigated again in [9] using the performance metrics of average fuel consumption and average stop times at SIs. Vehicles equipped with this algorithm are assumed to communicate among each other through V2V communication and with the TL through V2I communication to gather information about the TL characteristics. Based on the time that the vehicle would take to get to the TL plus the time remaining in the current phase, depending on the current phase light, an optimal speed is recommended to the driver to follow.

A real-world testing and evaluation of GLOSA is presented in [26] where important factors that are simplified in simulations but affect the results are taken into consideration. Investigating the ride comfort for buses approaching SIs, a genetic algorithm-based method is proposed in [27], showing that the installation and use of GLOSA on buses results in significantly smoother speeds, better ride comfort and arrival during green-light time. A modified version of GLOSA called Driver-Centric GLOSA (DC-GLOSA) is introduced in [28]. DC-GLOSA takes into consideration the acceleration and braking of vehicles to achieve better fuel saving and driving comfort. A multi-segment GLOSA system called R-GLOSA is proposed in [29] to provide an AV with the optimal speed when approaching an SI assuming that the AV can have access to the TL schedules along the whole path of travel. In large-scale simulation, the potentials and limitations of GLOSA are investigated in [30]. A performance comparison between the single-segment and multi-segment GLOSA approaches is addressed in [31], concluding that during free-flow conditions, the multi-segment GLOSA achieves better results in terms of travel time and fuel efficiency. To easily enable the application of GLOSA, a windshield mounted smart-phone prototype is proposed in [32] for detecting and predicting the schedules of TLs.

An approach for a single-vehicle optimal speed computation is introduced in [33] to reduce fuel consumption and $\mathrm{CO}_{2}$ emissions by reducing stop-and-go driving in urban areas particularly at TLs. In this approach though, the case of a driver cruising the vehicle speed to pass a TL
is not considered. The vehicles are assumed to communicate through V2V communication and with the TL through V2I communication within a certain distance called the Region of Interest (ROI). The TLs are assumed to be static with fixed cycle/phase duration. The approach proposed in [33] is used in [34] for a single-vehicle optimal speed computation to reduce $\mathrm{CO}_{2}$ emissions. Since [34] pays attention to the reduction of $\mathrm{CO}_{2}$ by reducing stop-and-go driving, the case where the driver may cruise the vehicle speed to pass the TL is taken into account. The TLs are assumed to be dynamic, and vehicles can communicate at a high reliability with the TLs through V2I communication within a distance of 100 m .

To minimize the overall emissions of vehicles, an approach is presented in [35] to provide the vehicle with optimal speeds toward an SI to avoid unnecessary acceleration and deceleration. By analyzing the driving behaviour as well as the signal phasing and timing information, an eco-driving model is introduced in [36] to provide the drivers of Electric Vehicles (EVs) with the optimal speed profiles when approaching an SI. To minimize the energy consumed along a certain path, an analytical model is proposed in [37] to compute a time-dependent optimal speed profile for EVs approaching an SI.

The impacts of V2I communication, namely vehicle to TL communication, on fuel and emission reductions have been investigated in [38]. In this research, the focus is on the key factors that influence fuel and emission reductions. These key factors are the gear choice and distance at which the highest fuel and emission savings are achieved. It is concluded that the maximum fuel and emission reductions can be achieved by integrating optimal speed computation with the gear choice factor such that stops at TLs are to some extent avoided. A vehicle speed advisory method along with a fuel consumption model to reduce fuel consumption when passing an SI are introduced in [39]. Using the time computed to the TL and vehicle acceleration and deceleration capabilities, the major contribution of this research is the computation of a fuel-optimal speed profile during deceleration and acceleration phases based on the deceleration rate (i.e., when approaching the TL) and acceleration rate (i.e., after passing the TL).

Noticeably, the approaches and techniques proposed for optimal speed computation of vehicles approaching a TL are incomplete, failing to efficiently minimize the vehicles' idling times and number of stops. All of these studies have focused only on single-vehicle optimal speed computation, and therefore, cooperation between the vehicles and TLs to optimize vehicles' speeds and reduce stop-and-go driving at TLs has not been addressed at all yet.

### 2.5 Adaptive Cruise Control

The underlying idea of Adaptive Cruise Control (ACC) is based on the theory of the Conventional Cruise Control (CCC) system. The CCC system can maintain the speed of a vehicle at a desired value set by the driver. Since in congested traffic conditions the CCC system becomes less useful, a radar system was added to it, creating ACC. The addition of the sensory radar technology has enabled the system to adjust the headway distance of a vehicle from another vehicle travelling ahead. The ACC system decreases the need for the driver to continuously adjust speed to match that of a vehicle ahead, and so the system is designed to reduce driving workload and increase driver comfort and convenience [40].

There are two modes of operation for ACC (i.e., set-speed control and follow control). When the system is in use and no vehicle is present within the predefined headway distance of the ACC vehicle, the system functions in the set-speed control mode. The objective of this mode is to control the speed of the vehicle to that set by the driver. If a forward vehicle is detected by the radar, the ACC system functions in the time-gap mode (i.e., follow mode). During this mode, the system adjusts the vehicle speed to maintain a time gap between the two vehicles. Therefore, the ACC system reduces the driver workload by: (1) maintaining a desired preset speed; (2) applying limited deceleration when approaching a slower vehicle ahead on the same lane; (3) keeping a safe headway distance between the two vehicles; and (4) accelerating back to the desired preset
speed when no slower vehicle is detected ahead [40]. However, when forming a platoon of vehicles on the road, the ACC system has several drawbacks. String stability, which is considered as a major barrier to ACC improving road capacity, is defined as the oscillations or perturbations resulting from the speed variations of the leading vehicle in a platoon of vehicles [41-44]. These speed shock-waves are amplified by the following vehicles' drivers when pressing the braking pedals harder and harder one after another to avoid collision when the leading vehicle brakes or decelerates.

### 2.5.1 Cooperative Adaptive Cruise Control

To improve the performance of ACC, V2V communication capability was added to the system [40]. The updated version of ACC is called Cooperative Adapted Cruise Control (CACC). CACC, which relies on V 2 V communication, has improved the performance of a platoon of vehicles and thus, reduced the impacts of string stability [40,41,45]. Including V2V communication has improved the performance of the system because the leading vehicle can communicate with the rest of the platoon, transmitting its expected acceleration/deceleration rates before it performs its actions. CACC has proven to help increase the capacity of highways only when the penetration rate of the vehicles equipped with CACC is high. There are two positive impacts of CACC on highways. First, it reduces the inter-vehicle spacing, which improves highway capacity. V2V communication allows vehicles equipped with CACC to travel at closer headways, resulting in extremely small time gaps between vehicles. Second, it improves the homogeneity of traffic flow by reducing the effects of string stability to the lowest level [4, 40, 41]. As reported in [40], "a cause of congestion is the occurrence of shockwaves in traffic platoons".

### 2.5.2 Platoon Formation in Urban Areas

There have been three streams of research being conducted in the field of platoon formation in urban areas. The first one focuses on optimizing the traffic flow of platoons using dynamic traffic signal control. A self-scheduling control model is proposed in [46] where a view of incoming traffic is used to determine whether to extend the TL timing or not. A similar platoon-based TL control algorithm has been introduced in [47] to optimize the traffic flow of platoons.

The second research stream focuses on platoon formation and management near Un-Signalized Intersections (USIs). A platoon-based traffic control strategy has been proposed in [48] to improve traffic efficiency at USIs, where a Petri Nets model is introduced to describe the traffic behaviour. Similarly, a model has been introduced in [49] to manage the movement of platoons at USIs. Other platoon management techniques to improve traffic efficiency as compared to a TL system have been introduced in [50-53].

The last stream focuses on platoon management near SIs, where the platoon has no control over TLs but one way communication from the TL to vehicles is available. Investigating the effects of platoon formation near SIs has been addressed in [54], concluding that platoons on major roadways should have the right of way even if this causes longer waiting times for vehicles on minor roads. Experimental results for platoon modelling and management when approaching SIs have been reported in [55], while [56] has studied the fuel-optimal strategies of platoons departing SIs. An analytical model to predict delays of platoons at SIs has been developed in [57].

As per the literature review conducted with respect to platoon formation in urban areas, very few papers address platoon management near SIs [54-57]. These papers do not address the issue of minimizing platoon-based idling times and number of stops at SIs.

### 2.6 Fuzzy Logic Control

Fuzzy logic control (FLC) has proven to be a promising technological tool for vehicle automatic speed control. A Fuzzy Logic Controller (FLCr) has been proposed in [58] to automatically control the throttle and brake of an AV , where the output of the FLCr is the pressure on the throttle and brake pedals. Since the objective of this study is to achieve human-like driving at low speeds, the FLCr has been shown to perform well for smooth stop-and-go driving. However, the effect of string stability was not reduced as V2V communication is not considered in the whole process. A lateral and longitudinal FLCr for AVs has been introduced in [59]. The longitudinal FLCr is used for AV speed control. The AV speed control in this research consists of two main applications. The first is a single AV speed control, where the target speed is set by the driver beforehand. Then, another FLCr is embedded to automatically control the vehicle's throttle and brake to maintain the target speed. However, there is no experimental validation of this application. The second application is a stop-and-go driving application, where two real AVs were used to validate the stop-and-go FLCr at low speeds (i.e., a maximum of $30 \mathrm{~km} / \mathrm{hour}$ ). An algorithm using an FLCr has been proposed in [60] to automatically control the speed of an AV . The objective of this research is to make the abrupt changes in the FLCr output smoother. This is achieved by integrating another loop in the controller structure. This proposed algorithm is used to automatically control the throttle pressure and thus, the acceleration of the vehicle at high speeds (e.g., from $80 \mathrm{~km} / \mathrm{hour}$ to $100 \mathrm{~km} / \mathrm{hour}$ ); however, the control of the brake is not included in the process.

FLC has shown to be successful for automatically controlling the speed of a single vehicle or a platoon of vehicles mainly to keep safe headway distances between vehicles, so it has the same fundamental idea as the CACC but with the assumption that the whole process is fully automated (i.e., no human intervention). Therefore, speed optimization of AVs approaching a TL for improving traffic efficiency has not been considered in the applications of FLC.

### 2.7 Traffic Light Control Systems

This section provides a review of TL control systems and their impact on managing traffic. There are mainly two TL control systems: static TL control and dynamic TL control. The mechanism of the former depends on using a static predetermined schedule of the phase and/or cycle duration for each direction of the intersection. The controller of this TL system is an electro mechanical one with a dial timer to allow repetitions of fixed phase duration intervals. Since phase duration cannot be adapted to longer/shorter intervals, this type of TL control can achieve a limited management of traffic under normal conditions. The Dynamic TL Control System (DTLCS) consists of two main units: a detector unit to detect the volume of traffic on roadways and a control unit to control the signal timings by giving more congested roads longer green-light times so that congestion at SIs is alleviated [5-7]. Some research has been conducted with respect to the development and applications of the DTLCSs.

A design of a DTLCS that relies on the use of Infra Red-Light Emitting Diode (IR-LED) transmitters and receivers is introduced in [61] to measure the traffic volume on junction roads. A DTLCS based on intelligent Radio-Frequency-Identification (RFID) is developed in [62]. Integrated with a certain algorithm and data base approach, this developed DTLCS can handle multi-vehicle, multi-lane, and multi-road-junction scenarios to manage traffic. The modelling of a DTLCS as a stochastic hybrid system is introduced in [63] where online gradient estimators of a cost metric are derived with respect to the controllable light cycles. The estimators are then used to iteratively adjust the controllable cycles to efficiently manage traffic under various traffic conditions. A DTLCS is proposed in [64] based on a traffic flow prediction model and a Model Predictive Control (MPC) optimization method. Using sonars for dynamic TL control, a DTLCS is addressed in [65], performing the calculation of the green and red-light times for different lanes based on predictive machine learning algorithm. This predictive machine learning algorithm uses historic traffic data to reinforce the dynamic behaviour of the system.

FLC has been shown to have the capability to apply real-life rules similar to the way humans would control traffic flow near SIs. A FLC method applied to dynamically control the signal timings of a TL system using a microscopic simulator is presented in [66]. A TL control system is proposed based on FLC in [67] to model unusual traffic conditions where the duration of the green-light phase is extended or terminated depending on the number of vehicles approaching the green-light phase and that of vehicles waiting in the queue during the red-light phase. A FLC-based TL system is introduced in [68] to dynamically control the traffic and pedestrians at SIs. The number of vehicles and pedestrians are estimated using cameras mounted at the SI. A centralized FLC-based TL system is proposed in [69] to manage traffic flow at SIs with machine vision algorithms used to report the traffic volume on the roadways. The moving traffic to and from nearby intersections is considered in the computations of the FLC system. A FLC system is presented in [70] with the objective of achieving smooth traffic flow by minimizing the waiting time of vehicles at SIs. A FLC method is applied in [71] to isolated four-roadway SIs where the decision of extending or terminating the light signal of a certain roadway is produced using a three-level fuzzy controller model.

To model the input-output membership functions of a FLC method for dynamic TL control, the Mamdani fuzzy logic system is posed in [72], while the Sugeno fuzzy logic system is introduced in [73]. A FLC approach for dynamic TL control is developed in [74], generating the fuzzy logic rules based on evolutionary algorithms and using real statistical traffic data. The design and implementation of a Smart Traffic Light (STL) based on FLC is addressed in [75] where Wireless Sensor Networks (WSNs) are used for traffic data collection. The input of the FLC is the traffic quantity and waiting time of each lane, while the output is the priority degree that determines the order of the green light for the different lanes. To overcome traffic problems such as congestion, accidents, and irregularity, a TL control system based on FLC is proposed in [76] where traffic volume and queue length information is obtained using WSNs and image processing techniques. An intelligent TL control system using a statistical multiplexing method
algorithm is proposed in [77]. The functionality of this statistical multiplexing algorithm is based on statistical information obtained by using the conservation law of vehicles and traffic density on the four roadways. A hierarchical WSN is installed at the intended intersection to provide the necessary information on each roadway.

### 2.8 Game Theory

Game theory was introduced in the early years of the twentieth century in [78] and [79]. Several definitions of game theory have been introduced. According to [80], game theory is "the study of mathematical models of conflict and cooperation between intelligent rational decision makers". A theoretic game is defined in [81] as a "description of strategic interaction that includes the constraints on the actions that the players can take and the players' interests but does not specify the actions that the players do take". Game theory categorizes two types of games: 1) noncooperative games and 2) cooperative games.

Non-cooperative games provide a detailed model of strategic actions that players can take, so these games provide the analysis of one player's action or response given other players' anticipated actions. The case where no player wishes to unilaterally change the current strategy, provided that the other players do not change their current strategies, is called the state of equilibrium. These games model the best strategy or response of every player to the strategies of other players such that an equilibrium state is found. The most known equilibrium used to represent the solution to these games is Nash Equilibrium (NE) [81]. However, in strictly competitive games, such as zero-sum games, a pure NE does not exist. In addition, in pure strategy games where the associated utilities are deterministic, an NE may not exist. When the players can have mixed (i.e., stochastic/probabilistic) strategies, the expected payoffs are statistically computed. In such cases, a mixed NE always exists, even for n-player games [79] [81].

On the other hand, cooperative games concentrate on the modelling of outcomes that result when the players form groups or teams. Once the players come to a situation in which everyone gets at least as much payoff as they do without interacting or cooperating with the others, a binding agreement between the players is made by which they are committed to implement certain strategies. Finding a fair allocation for the joint cost or payoff is the main challenge in cooperative games. The set of all feasible allocations is called the core. The core was first mathematically defined for n-player non-zero-sum games in [82]. Many issues need to be resolved, such as determining how the cost/revenue is distributed between the players. Another issue with cooperative games is the necessity to prove whether the game is balanced or not. If there is at least one fair allocation to the joint cost/revenue, then the core is not empty. In this case the game is said to be balanced [83].

### 2.8.1 Shortest Path and Congestion Games

Shortest-path and congestion problems have been mathematically modeled as games using game theory. A shortest-path game with transferable utility (TU) is introduced in [84]. The focus of this research is on the allocation of profits generated by the coalitions of players that own nodes in a road network. The objective of players owning nodes on a path is to transport goods through the path with a minimum cost. A shortest-path game is presented in [85] where players can own road segments in a road network. Each player in the game receives a non-negative reward if he/she transports a good from the source to the destination. Congestion games, first described in [86], are viewed usually as non-cooperative resource allocation games. Congestion games are considered to be potential games in which players' payoffs are influenced by the resources that players use and are dependant on the number of players that share these resources. For instance, in symmetric-network congestion games (i.e., all the players have the same source and destination), paths between the source and destination are considered resources that are shared by
the travelers using them. A model is presented in [87] showing that players who share resources (i.e., routes) can form coalitions to selfishly compete against each other to maximize their values. If the coalitions are able to maximize the payoffs of their participants, the gain will outweigh the losses they might cause to each other. A discussion is conducted in [88] about the similarities between cooperative congestion games and their non-cooperative counterparts, where important issues are demonstrated, such as the existence of and the convergence to a pure strategy NE.

### 2.8.2 Dynamic Traffic Light Control Games

Game theory has also been applied to the dynamic TL control problem. A model is introduced in [89] for TL system control based on a Markov Chain game with the objective of minimizing the queue lengths at multiple SIs. A two-player cooperation game is proposed in [90] for TL signal timing control applied to a two-phase SI. Similar research to [89] is presented in [91] where a non-cooperative game to model the TL signal timing control problem is introduced based on game theory and modeled as a finite controlled Markov Chain. However, the TL model in [91] is applied to a single SI. Based on Cournot's Oligopoly game, a game theory model is presented [92]. A novel game theory optimization algorithm is proposed in [93] for TL signal timing control, where the Nash Bargaining (NB) [94] is used to find the optimal strategy of the TL signal timing control problem.

Noticeably, the development of game theory and its applicability to the transportation problems have been limited to shortest-path, congestion and dynamic TL control games. Therefore, game theory has not yet been developed and applied for vehicle speed optimization to allow vehicles to cooperate with each other and with TLs, coordinating their speed actions when approaching an SI to minimize idling times and stop numbers.

### 2.9 Summary

This chapter has presented a broad background on AVs, including their environmental, economical, and road safety impacts, as well as the comfort provided when using them in the transportation system. It also briefly discussed challenges and related issues associated with the growth of AV technology and the automation of the transportation system, such as purchase and ownership costs, privacy and security concerns, and legislation concerns. A review of the existing research on optimal speed computation for vehicles approaching a TL has been addressed. A background has been presented to explore speed control models, such as ACC and CACC for vehicle platoons with a research review focusing on vehicle platooning formation near SIs. In addition, research that relies on the use of FLC to control the speeds of AVs by automatically controlling the pressure on the throttle and brake was reviewed. Existing research conducted on DTLCSs addressing a variety of techniques and approaches including FLC has been addressed. Finally, background on game theory being applied to analyze the conflicting interests of multiagent systems has been conducted. Special attention has been given to the applicability of game theory to transportation problems, and the research conducted on shortest-path, congestion, and dynamic TL control games.

It has been shown that despite the diversity of research in the area of vehicle optimal-speed computation and dynamic TL control, existing research does not fully solve the problem of minimizing the vehicles' idling times and number of stops at SIs. Interestingly, the approaches and models proposed for optimal speed computation to minimize the idling times and number of stops of vehicles at TLs have been limited to considering a single-vehicle optimal speed computation. Therefore, cooperation between multiple vehicles to minimize the idling times and number of stops at TLs has not been addressed yet. In addition, integration of cooperative-vehicle speed optimization with dynamic TL control such that both could cooperate and coordinate their actions to improve traffic efficiency and thus minimize the average idling times and number of
stops has not been addressed yet. In this thesis, it is argued that a cooperative speed optimization scenario, developed and modelled using game theory, in which AVs approaching a TL could cooperate with one another and with the TL would be an effective mechanism to minimize the vehicles' idling times and number of stops, and thus improve traffic efficiency near SIs.

## Chapter 3

## A Cooperative Framework for Autonomous Vehicle Speed Optimization

In this chapter, the idling time and stop number minimization problem for AVs approaching a TL is stated to illustrate the complexity associated with it. Then, the AV speed optimization process is viewed as a theoretic game between players (i.e., AVs). The stability of the formulated AV speed optimization game is also discussed. Following that, a Cooperative Speed Optimization Framework (CSOF) is proposed to model the AV speed optimization as a cooperative process. This framework relies on Linear Programming (LP) and game theory, consisting of three modules to address issues of AV individual rational speed optimization, information and conflict recognition, and cooperative speed optimization decision making. Furthermore, a cooperative bargaining model, based on the Marginal Contribution Principle (MCP), is introduced to allow AVs to trade their rights of passing SIs with less delay. Finally, simulation is conducted to test and validate the performance of the CSOF under various traffic conditions investigating the average idling times, average number of stops, and average energy consumption as AVs approach SIs. The results obtained from the simulation tests are reported and discussed.

### 3.1 Problem Statement

### 3.1.1 Signalized Roadway Intersection Setting

Consider, as an illustrative example, the two-lane four-roadway SI in Figure 3.1. For simplicity, assume that the TL control system has a two-phase static cycle where East-West roadways are one phase and North-South roadways are the other phase. Each phase has a signal design of Green-Yellow-Red; however, for simplicity, the yellow-light time is assumed to be part of the green-light-time duration. The key TL parameters are the green-light-time duration $T_{g}$, the red-lighttime duration $T_{r}$, and the TL cycle duration $T_{c}=T_{g}+T_{r}$. These parameters are assumed to be constant, e.g., $T_{g}=24 \mathrm{sec}, T_{r}=36 \mathrm{sec}$, and $T_{c}=60 \mathrm{sec}$. Assume that there is V2I communication such that the vehicles heading toward the TL can receive signal timing information and that every AV is conducting speed optimization re-planning to have a chance of meeting the green-light time.

Definition 1. Speed optimization re-planning is a game in which each AV performs speed optimization every time step $t$. There is a probability $p$ that the $A V$ will proceed according to the previous strategy at time step $t-1$ and a probability $(1-p)$ that it will move to a different strategy (i.e., adopt a different speed).

To illustrate the complexity of the problem, consider a scenario where for a certain cycle, the arrival and maximum departure rates of a certain roadway at the TL are $\lambda=0.25 \mathrm{veh} / \mathrm{sec}$ and $\mu=0.333 \mathrm{veh} /$ sec, respectively [95]. Therefore, on this particular roadway, the number of AVs arriving during the red time is $N_{\text {arr }}=\lambda T_{r}=(0.25)(36)=9$ veh, while the maximum number of AVs that can depart the TL during the green time is $N_{\text {dep }}=\mu T_{g}=(0.333)(24)=8$ veh. Making this setting, assume that the TL has just turned green for the East-West directions. Consider the case of two AVs travelling on the West roadway performing speed optimization


Figure 3.1: A simple example of a traffic light scenario.
re-planning. Taking the queue size into account, according to the computations by these AVs , each of them can pass within the current green light. Since only one AV can pass through, the other will experience unexpected delay, waiting for the next green light. Hence, AVs negatively impact the objectives of each other.

This example is a simple scenario that consists of a two-player competitive game with one TL, so the delay-time cost may not be significant. However, scaling up this example to $N$ players, multiple TLs along a certain path, and many other types of cost factors will result in a much more complicated scenario. A solution to this example may require finding a cooperative agreement between the players such that one player re-optimizes its own speed to meet the next green light in exchange of some form of compensation, allowing the other player to pass through smoothly.

### 3.1.2 Problem Formulation

Consider a group of AVs travelling within a locality with $m$ TLs. Each AV with index $A V_{i}$ contemplates speed optimization to minimize its idling times at TLs from an initial location $p_{0 i}$ to a final destination $p_{f i}$. The trip from $p_{0 i}$ to $p_{f i}$ is made on a path, $P\left(p_{0 i}, p_{f i}\right)$, constructed from a set of road segments ending with TLs, $L=\left\{L_{1}, L_{2}, \ldots, L_{m}\right\}$. The speed $v_{i}(t)$ of each $A V_{i}$ belongs to a set of feasible speeds, $V=\left\{\bar{v}_{1}, \bar{v}_{2}, \ldots, \bar{v}_{f}\right\}$. The cost of the trip for $A V_{i}$ on a road segment $L_{j}$, where $j=1,2, \ldots, m$, denoted by $C_{s v}^{L_{j}}(i)$, explicitly models its idling time. For $A V_{i}$, the cost of travelling over road segment $L_{j}$ incurred by choosing a time indexed sequence of velocities, $s v$, is defined as follows:

$$
C_{s v}^{L_{j}}(i)=\left\{\begin{array}{rll}
t_{i} & \text { if } & \text { stop }  \tag{3.1}\\
0 & \text { if } & \text { no stop }
\end{array}\right.
$$

where $t_{i}$ is the idling time of $A V_{i}$ at the TL positioned at the end of road segment $L_{j}$. The total cost for $A V_{i}$, incurred over a path $P\left(p_{0 i}, p_{f i}\right)$ that is composed of the road segments $L_{1}^{P}, \ldots, L_{N_{T L}}^{P} \in L$ (sequentially), is the summation of idling times at all TLs along the path, i.e.,

$$
\begin{equation*}
C_{s v}^{P}(i)=\sum_{j=1}^{N_{T L}} C_{s v}^{L_{j}^{P}}(i) . \tag{3.2}
\end{equation*}
$$

where $N_{T L}$ is the number of TLs on the path $P$, and $s v$ denotes the sequence of velocities for road segment $L_{j}^{P}$. The overall aim is to minimize $C_{s v}^{P}$ for each $A V_{i}$. To provide a sub-optimal solution to the above overall task, we follow a decentralized approach and consider each road segment $L_{j} \in L$ of the locality separately. We propose a Time Token Allocation Algorithm (TTAA) for the TL and a cooperative distributed conflict resolution scheme for the vehicles in each such $L_{j}$.

For player $A V_{i}$, the optimal speed value in the set of possible speeds may lead to a time token
$\tau_{i}$ within the green light. The time token $\tau_{i}$ is the index of a time window assigned by the TL using the TTAA. For player $A V_{i}$, the cost associated with $\tau_{i}$ is the minimum (e.g., player $A V_{i}$ will pass through the TL without stopping, $\left.C_{s v}^{L_{j}}(i)=0\right)$.

We define a cooperative speed optimization game, $G$. In this game, AVs with conflicting allocated time tokens agree to take certain speed actions to resolve the conflict. For each player $A V_{i}$, there is a finite non-empty set of speed actions $V$. There is an idling time cost, $C_{s v}^{L_{j}}(i)$, associated with each sequence of actions $s v$. Action sequences are associated with a preference relationship such that $C_{s v^{*}}^{L_{j}}(i)<C_{s v}^{L_{j}}(i)$ means $s v^{*} \succ s v_{j}$ (i.e., the sequence of actions $s v^{*}$ is preferred over that $s v$ as it incurs less cost).

### 3.2 Stability of the Speed Optimization Game

An important aspect in developing a solution to any game is understanding the game properties. For non-cooperative games, stability is the most important property, while for cooperative games, balancedness is the most important one. In non-cooperative games, stability reflects the existence of a solution to the game. In other words, if an NE exists, then the game is stable. In cooperative games, though, the existence of the solution is reflected by the balancedness. If the core (i.e., the set of feasible payoff/cost allocations other than which no subset of players can achieve better outcome) is not empty, the game is balanced. In general, non-cooperative games can be considered as part of cooperative games. Many non-cooperative games have some form of interaction between players. For instance, when players exchange information and reveal their payoffs prior to the game, players are cooperating to some extent. In addition, the theory of repeated games in non-cooperative games studies and analyzes the possibility of cooperation in perpetual relationships [96]. In such games, players may agree to swerve (i.e., lose) a game but in return gain a reward such as a promise to allow them to win next time they play the game.

Since there is a solution to the optimization function each AV is attempting to solve, each AV will have an optimal action. The chosen actions of AVs are considered pure strategies, and therefore, there always exists pure equilibrium. A pure equilibrium means that no AV wishes to unilaterally change its optimized solution. However, this is true only for non-strictly competitive games. As the game progresses, AVs compete to gain resources such that one AV's gain is another AV's loss. Therefore, by considering the speed optimization re-planning game (Definition 1), it can be proven that a solution exists.

A mixed strategy game, which always has a mixed equilibrium, is a game in which the strategies available to the players are not deterministic but are regulated by probabilistic rules [96] [94]. Thereby, from Definition 1, it is concluded that there is a probability distribution over all the strategies available to every AV in the game. Hence, the speed optimization replanning game is a mixed strategy game for which a mixed equilibrium always exists. Thus, it has been established that there is a solution to the formulated speed optimization game, proving that equilibrium exists and the game is stable.

### 3.3 The Cooperative Speed Optimization Framework

Schematics of the CSOF we propose to solve the speed optimization re-planning game are provided in Figure 3.2. To formulate and conduct the AV speed optimization as a cooperative process, this framework relies on LP and game theory. It consists of three modules to address (i) AV individual rational speed optimization, (ii) information and conflict recognition, and (iii) cooperative speed optimization decision making. The CSOF is designed to function on multiple-lane roadways with two essential rules. First, AVs using the CSOF in free motion can smoothly overtake each other on the roadway to comply to certain speed actions resulting from their interaction and cooperation. Second, under certain traffic conditions such as when overtaking is not possible,
a safe following distance between consecutive AVs is maintained. Consecutive AVs are modeled to maintain a minimum time gap of two seconds in order to avoid collision [98] [99]. All the AVs are identical in length and have an average length of 5 m .


Figure 3.2: Schematic depiction of the cooperative speed optimization framework.

### 3.3.1 Autonomous Vehicle Speed Optimization Module

The objective of this module is to provide each player (i.e., vehicle), $A V_{i}$, with the optimal speed at every time step $t$. Based on the Time to Intersection $T T I_{i}$ and using the TTAA, the TL may allocate $\tau_{i}$ to $A V_{i}$, an integer value indicating the index of a time window during which $A V_{i}$ can pass the intersection smoothly. The speed $v_{i}(t)$ of $A V_{i}$ at time step $t$ is a function of the traffic density $D\left(L_{j}\right)$ on road segment $L_{j}$. The linearity of the relation between traffic density and speed under
mild generic assumptions is justified in [100], concluding that as traffic concentration/density increases, speed decreases. The maximum speed $A V_{i}$ can travel at, $v_{\max }$, will only occur when there are no other vehicles on the roadway. In general, the speed of $A V_{i}$ goes to zero as the road reaches the maximum density, $v_{i}(t)$ converges to 0 as $D\left(L_{j}\right)$ converges to $D_{\max }\left(L_{j}\right)$. Therefore, considering a linear relation between the traffic density and speed, the speed $v_{i}(t)$ of $A V_{i}$ with respect to the traffic density $D\left(L_{j}\right)$ on road segment $L_{j}$ is

$$
\begin{equation*}
v_{i}(t)=v_{\max }\left(1-\frac{D\left(L_{j}\right)}{D_{\max }\left(L_{j}\right)}\right) \tag{3.3}
\end{equation*}
$$

$A V_{i}$ is allocated a token $\tau_{i}$ only if $T T I_{i}$ falls within the upcoming green-light time, i.e., $T T I_{i} \leq$ $R_{g}$ or $R_{r}<T T I_{i} \leq R_{r}+T_{g}$ where $R_{g}$ and $R_{r}$ are the remaining green-light and red-light times respectively. For $A V_{i}$ approaching a TL, the speed that minimizes the idling-time cost is found as follows:

## Light is Green

As $A V_{i}$ receives upcoming signal information from the TL, indicating that the current light is green, there are three possible cases in terms of $T T I_{i}$ and $R_{g}$ :

- Case 1: $T T I_{i} \leq R_{g}$. Using the current speed, $A V_{i}$ will be able to pass through within the remaining green-light time. The TL allocates a time token $\tau_{i}$ to $A V_{i}$. Thereby, $A V_{i}$ maintains its speed to pass during the assigned time token.

$$
\begin{aligned}
& s_{i}=v_{i}(t) \\
& \text { subject to }: \\
& v_{i}(t) \leq d_{i}(t) / a_{i} \\
& v_{i}(t) \geq d_{i}(t) / b_{i} \\
& v_{i}(t) \leq v_{\max } \\
& v_{i}(t) \geq v_{\min }
\end{aligned}
$$

where $T T I_{i}=d_{i}(t) / v_{i}(t)$ sec, $d_{i}(t)$ is the distance of $A V_{i}$ to the stop line of the TL at time step $t, s_{i}$ is the optimal speed of $A V_{i}$ at time step $t+1$, and $v_{\max }$ and $v_{\text {min }}$ are the maximum and minimum speed limits on the road segment respectively, while $a_{i}$ and $b_{i}$ represent the lower and upper boundaries of the allocated time token respectively.

$$
\begin{gather*}
a_{i}=\left(\tau_{i}-1\right) \frac{1}{\mu}  \tag{3.4}\\
b_{i}=\tau_{i} \frac{1}{\mu} \tag{3.5}
\end{gather*}
$$

where $\mu$ is the departure rate in $v e h / s e c$.

- Case 2: $R_{g}+T_{r} \geq T T I_{i}>R_{g}$. The vehicle is not allocated a time token, and the speed of the vehicle is optimized so that $T T I_{i}$ is sufficient to meet the next green light.

$$
\begin{aligned}
& s_{i}=\min \quad v_{i}(t) \\
& \text { subject to }: \\
& v_{i}(t) \leq d_{i}(t) /\left(R_{g}+T_{r}+T_{q}\right) \\
& v_{i}(t) \geq d_{i}(t) /\left(R_{g}+T_{r}+T_{g}\right) \\
& v_{i}(t) \leq v_{\max } \\
& v_{i}(t) \geq v_{\min }
\end{aligned}
$$

where $T_{q}$ is the time needed to clear all the vehicles in the queue, and it is found as follows:

$$
\begin{equation*}
T_{q}=\frac{n(t)}{\mu} \tag{3.6}
\end{equation*}
$$

where $n(t)$ denotes the number of vehicles currently in the queue. If the current speed does not allow $A V_{i}$ to be part of the green-light time but the maximum speed of the roadway does, the speed optimization system will accelerate the speed of $A V_{i}$ such that it is allocated a token.

$$
\begin{aligned}
& s_{i}=\max \quad v_{i}(t) \\
& \text { subject to }: \\
& v_{i}(t) \leq d_{i}(t) / a_{i} \\
& v_{i}(t) \geq d_{i}(t) / b_{i} \\
& v_{i}(t) \leq v_{\max } \\
& v_{i}(t) \geq v_{\min }
\end{aligned}
$$

- Case 3: $R_{g}+T_{r}+T_{g} \geq T T I_{i}>R_{g}+T_{r}$. AV $V_{i}$ will maintain its current speed as $T T I_{i}$ leads
$A V_{i}$ to be part of the green-light of the next cycle; However, $A V_{i}$ will not yet be allocated a time token.

$$
\begin{aligned}
& s_{i}=v_{i}(t) \\
& \text { subject to }: \\
& v_{i}(t) \leq v_{\text {max }} \\
& v_{i}(t) \geq v_{\text {min }}
\end{aligned}
$$

## Light is Red

If the information received by the vehicle from the TL indicates that the current light is red, there are three possible cases in terms of $T T I_{i}$ and $R_{r}$ :

- Case 1: $T T I_{i}<R_{r}$. $A V_{i}$ will not be allocated a time token, and its speed is optimized such that it will meet the next green light.

$$
\begin{aligned}
& s_{i}=\min \quad v_{i}(t) \\
& \text { subject to }: \\
& v_{i}(t) \leq d_{i}(t) /\left(R_{r}+T_{q}\right) \\
& v_{i}(t) \geq d_{i}(t) /\left(R_{r}+T_{g}\right) \\
& v_{i}(t) \leq v_{\max } \\
& v_{i}(t) \geq v_{\min }
\end{aligned}
$$

- Case 2: $R_{r}<T T I_{i} \leq R_{r}+T_{g} . A V_{i}$ is allocated a time token within the upcoming green-light time.

$$
\begin{aligned}
& s_{i}=v_{i}(t) \\
& \text { subject to }: \\
& v_{i}(t) \leq d_{i}(t) /\left(R_{r}+a_{i}\right) \\
& v_{i}(t) \geq d_{i}(t) /\left(R_{r}+b_{i}\right) \\
& v_{i}(t) \leq v_{\max } \\
& v_{i}(t) \geq v_{\min }
\end{aligned}
$$

- Case 3: $T T I_{i}>R_{r}+T_{g} . A V_{i}$ is not allocated a time token and its speed is optimized to meet the green-light of the next cycle.

$$
\begin{aligned}
& s_{i}=\min \quad v_{i}(t) \\
& \text { subject to }: \\
& v_{i}(t) \leq d_{i}(t) /\left(R_{r}+T_{g}+T_{r}+T_{q}\right) \\
& v_{i}(t) \geq d_{i}(t) /\left(R_{r}+T_{r}+2 T_{g}\right) \\
& v_{i}(t) \leq v_{\max } \\
& v_{i}(t) \geq v_{\text {min }}
\end{aligned}
$$

## Energy Consumption Model

In this thesis, all the AVs involved in the cooperative process are assumed to be Electric Autonomous Vehicles (EAVs); therefore, the energy consumption model presented in [101] has been modified to compute the instant energy consumed by every AV at time step $t$. The problem of battery constraint (i.e., it is not possible to recuperate energy into the battery, regenerate energy from downhill edges and during deceleration phases into the battery, if the battery is already
fully charged) is solved by dynamically modifying and adjusting the energy cost. The energy regenerated from downhill edges and deceleration phases is stored into the available free capacity of the battery until the battery is full. The rest of any regenerative energy is lost. The total energy cost consumed by $A V_{i}$ at time step $t$ consists of multiple sub-costs as follows:

- Potential Consumed/Gained Energy: the potential energy $E C_{i}^{P}(t)$ at time step $t$ is consumed from the battery during the uphill travel and is gained into the battery during the downhill travel. The potential consumed and gained energies are

$$
\begin{align*}
E C_{i}^{P C}(t) & =\frac{1}{\eta}[m g u(t)]  \tag{3.7}\\
E C_{i}^{P G}(t) & =-\eta[m g u(t)] \tag{3.8}
\end{align*}
$$

where $\eta$ is the efficiency of $A V_{i}, m$ is the mass of $A V_{i}, g$ is the gravity factor, and $u(t)$ is the elevation of the road segment at time step $t$. In addition, we define the following parameters: $C_{\max }$ is the battery maximum capacity; $J$ is the battery charge level, where $J \leq C_{\max }$; $U$ is the remaining free capacity of the battery, where $U=C_{\max }-J$; and $\Delta$ is the amount of energy consumed or gained at time step $t$. Therefore, this potential consumed/gained energy takes the following value:

$$
\begin{equation*}
E C_{i}^{P}(t)=\max \{\Delta,-U\} \tag{3.9}
\end{equation*}
$$

- Loss of Energy: the loss of energy at time step $t$, which is always consumed from the battery, occurs due to aerodynamic and rolling resistances.

$$
\begin{equation*}
E C_{i}^{l o s s}(t)=\frac{1}{\eta}\left[f_{r} m g l_{d}(t)+\frac{1}{2} \rho A_{s} a_{r} v_{i}^{2}(t) l_{d}(t)\right] \tag{3.10}
\end{equation*}
$$

where $f_{r}$ is the friction coefficient, $l_{d}(t)$ is the distance between the current and previous locations, $\rho$ is the air density coefficient, $A_{s}$ is the cross sectional area of $A V_{i}$, and $a_{r}$ is the air drag coefficient. This loss of energy takes the following value:

$$
\begin{equation*}
E C_{i}^{l o s s}(t)=\Delta \tag{3.11}
\end{equation*}
$$

- Acceleration/Deceleration Energy: the acceleration energy $E C_{i}^{a c}(t)$ at time step $t$ is consumed from the battery as $A V_{i}$ accelerates to a higher speed while the deceleration energy $E C_{i}^{d c}(t)$ at time step $t$ is recuperated and stored into the battery as $A V_{i}$ comes to a lower speed.

$$
\begin{gather*}
E C_{i}^{a c}(t)=\frac{1}{\eta} P_{i}^{w r} t_{a c}  \tag{3.12}\\
E C_{i}^{d c}(t)=-\eta P_{i}^{w r} t_{d c} \tag{3.13}
\end{gather*}
$$

where $P_{i}^{w r}$ is the power of the electric motor of $A V_{i}$, while $t_{a c}$ and $t_{d c}$ are the times taken by $A V_{i}$ during acceleration and deceleration respectively. During acceleration/deceleration phases, this type of energy takes the following value:

$$
\begin{equation*}
E C_{i}^{a d}(t)=\max \{\Delta,-U\} \tag{3.14}
\end{equation*}
$$

- Energy Consumed by On-Board Electric Devices: this energy is not path related and is consumed directly from the battery at time step $t$ by the on-board electric devices such as air conditioner, windshield wipers, etc.

$$
\begin{equation*}
E C_{i}^{e d}(t)=\sum_{j=1}^{n} P_{i}^{e d}(j) t_{i}^{e d}(j) \tag{3.15}
\end{equation*}
$$

where $P_{i}^{e d}(j)$ is the power withdrawn at time step $t$ by the electric device $j$ and $t_{i}^{e d}(j)$ is the time that device $j$ takes in use.

Therefore, taking into consideration the driving style factor $D S$ to model different driving modes, the total energy cost consumed by $A V_{i}$ at time step $t$ is computed as

$$
\begin{equation*}
E C_{i}^{T}(t)=\left[E C_{i}^{P}(t)+E C_{i}^{\text {loss }}(t)+E C_{i}^{a d}(t)\right] * D S \tag{3.16}
\end{equation*}
$$

### 3.3.2 Information and Conflict Recognition Module

In this module, AVs are recognized as rational players based on their interests, preferences and threat to other players. If two or more players are allocated the same time token, the TL informs them that they have a conflict. Players with conflicting time tokens communicate to share their strategies and associated costs. Consequently, they start to negotiate to find a binding agreement based on which they can cooperate and agree on certain speed actions. Once an agreement is reached, all the players abide by the rules to apply those actions.

### 3.3.3 Cooperative Decision Making Module

In this module, the final speed optimization decisions are made. Speed assignments, resource allocation, and cooperative speed optimization decisions are finalized. The input to this module is in the form of various strategic speeds and associated costs. The cooperative game notion in this module is based on the assumption that players, representing the AVs, can reach a binding agreement with which they apply certain strategic actions. Once a decision is made, all the players involved in the game, with no exception, follow the decision. The speed optimization game played in this module is conducted in conjunction with the information and conflict recognition
module described earlier. The idling-time cost to a player is guaranteed not to be greater than what it would be without cooperation, asserting the following rationality axiom.

Axiom 1. At time step $t$, there exists an optimal strategy $v_{k}$ for player $A V_{i}$ such that $C_{v_{k}}^{L_{j}}(i) \leq$ $C_{v_{j}}^{L_{j}}(i), \forall k, j \in V$ (i.e., the cost associated with this optimal strategy is less than or equal to that associated with any other strategy player $A V_{i}$ can take). However, player $A V_{i}$ is free to choose any other strategy that might yield a higher cost, but only in exchange for a reward.

The above-stated axiom permits that a player may give some of its resources to other players for the benefit of the group rather than the individual but only in exchange for a reward. This proposed cooperative module also includes the TL rather than only the vehicles. Once an AV is within range of the communications radio (e.g., DSRC), it will communicate with the TL to inquire about the current light signal and queue information, and also request a time token within the upcoming green-light time. The TL allocates time tokens to the players using Algorithm 1. In Algorithm 1, VIN is the AV identification number, $N_{q}$ is the number of vehicles currently in the queue, $T_{s d}$ is the slot (i.e., time token) duration, and $T_{s l o t}$ is the time token location in the TL memory. According to this algorithm, the TL gives priority in allocating tokens to the queued AVs. The rest of the green-light time is segmented as tokens and offered to the approaching AVs.

When two players are allocated the same time token, one of them will initiate the negotiation to start the game. Players with conflicting tokens will share their available strategies and associated costs, listing the costs caused by the conflict rather than the expected ones. It is assumed that each AV has a mode property, which may take one of three values at a time: Rush Mode, Normal Mode, or Relaxed Mode.

- Rush Mode: is used for urgent and emergency situations (e.g., the AV must be at the hospital shortly).
- Normal Mode: is used when there is no emergency; the AV may yield the road to others.
- Relaxed Mode: is used when there is plenty of time. The AV would yield the road to other vehicles comfortably.

```
Algorithm 1 : Single Roadway Time Token Allocation Algorithm
Input: VIN \(_{i}, T T I_{i} ;\) Output: \(\tau_{i}\)
    if Light is Green then
        if \(\left(T T I_{i} \leq R_{g}\right)\) then
        for \(j=N_{q}+1: N_{d e p}\) do
            \(a=T s d *(j-1) ;\)
            \(b=T s d * j ;\)
            if \(\left(T T I_{i} \geq a \& T T I_{i} \leq b\right)\) then
                \(\operatorname{Tslot}(1, j)=V I N_{i} ;\)
                \(\tau_{i}=j ;\)
                break;
            end if
        end for
        else
        \(\tau_{i}=0 ;\)
    end if
    else
        if \(\left(T T I_{i}<R_{r}\right)\) then
        \(\tau_{i}=0 ;\)
        else if \(\left(T T I_{i}>R_{r}+\left(T s d * N_{q}\right) \& T T I_{i} \leq R_{r}+T_{g}\right)\) then
        for \(j=N_{q}+1: N_{\text {dep }}\) do
            \(a=R_{r}+T s d *(j-1) ;\)
            \(b=R_{r}+T s d * j ;\)
            if \(\left(T T I_{i} \geq a\right.\) \&\& \(\left.T T I_{i} \leq b\right)\) then
                \(\operatorname{Tslot}(1, j)=V I N_{i} ;\)
                \(\tau_{i}=j ;\)
                break;
            end if
        end for
    else
        \(\tau_{i}=0 ;\)
    end if
    end if
```

Different integers, e.g., 0, 1, and 2, are used to represent the modes Relaxed, Normal, and Rush
respectively. Players first play the game based on the mode type. The vehicle $A V_{i}$ using the highest mode value, $M\left(A V_{i}\right)$, wins (i.e., uses the current time token), while the one using the mode with the smaller value loses (i.e., slows down and requests a different time token). However, the TL grants the loser a credit point and deducts a credit point from the winner. The winner of the mode-based game is determined as

$$
\begin{equation*}
A V_{\text {winner }}=\max \left(M\left(A V_{1}\right), M\left(A V_{2}\right)\right) \tag{3.17}
\end{equation*}
$$

If any two players have the same mode value, they will decide the winner based on the credit points, $C P\left(A V_{i}\right)$, they have. The one with the most will eventually win the game. Again, a credit point is deducted from the winner, and a credit point is granted to the loser. In this case, the winner is determined as

$$
\begin{equation*}
A V_{\text {winner }}=\max \left(C P\left(A V_{1}\right), C P\left(A V_{2}\right)\right) \tag{3.18}
\end{equation*}
$$

If both players have the same mode value and credit point, a random number-generation procedure between the TL and the players is conducted to resolve the conflict. Each of the players as well as the TL generates a random number. The one whose generated number, $R N\left(A V_{i}\right)$, is closer to that of the TL, $R N(T L)$, wins the current time token but loses a credit point. The other gains a credit point and requests a different token. The winner of the game is determined as follows:

$$
\begin{equation*}
R N_{1}=\left|R N(T L)-R N\left(A V_{1}\right)\right| \tag{3.19}
\end{equation*}
$$

$$
\begin{equation*}
R N_{2}=\left|R N(T L)-R N\left(A V_{2}\right)\right| \tag{3.20}
\end{equation*}
$$

$$
A V_{\text {winner }}=\left\{\begin{array}{lcl}
A V_{1} & \text { if } & R N_{1}<R N_{2}  \tag{3.21}\\
A V_{2} & \text { otherwise }
\end{array}\right.
$$



Figure 3.3: Cooperative speed optimization logic

Figure 3.3 illustrates the cooperative decision-making logic used in this module. To further clarify the cooperative speed optimization game, an example of two AVs approaching a TL is presented next.

Example 1. Consider the problem setting of Section 3.1.1 and assume that the TL has just turned green for the East-West directions. After communicating with the TL, the two AVs, approaching the TL from the West, have been allocated, at time step $t$, the same and only-remaining time token. Both vehicles will have only two strategies to choose from; $v_{1}(t)^{A V_{i}}$ corresponding to using the
current time token and $v_{2}(t)^{A V} V_{i}$ corresponding to minimizing speed and requesting a token within the next green light, as summarized in Table 3.1.

Table 3.1: Action and response table of a two-player game

| Players/Strategies | $v_{1}(t)^{A V_{2}}$ | $v_{2}(t)^{A V_{2}}$ |
| :---: | :---: | :---: |
| $v_{1}(t)^{A V_{1}}$ | $\left(C_{v_{1}(t)}^{L_{j}}(1), C_{v_{1}(t)}^{L_{j}}(2)\right)$ | $\left(C_{v_{1}(t)}^{L_{j}}(1), C_{v_{2}(t)}^{L_{j}}(2)\right)$ |
| $v_{2}(t)^{A V_{1}}$ | $\left(C_{v_{2}(t)}^{L_{j}}(1), C_{v_{1}(t)}^{L_{j}}(2)\right)$ | $\left(C_{v_{2}(t)}^{L_{j}}(1), C_{v_{2}(t)}^{L_{j}}(2)\right)$ |

When the players choose the same strategy resulting in the use of strategy profile $\left(C_{v_{1}(t)}^{L_{j}}(1), C_{v_{1}(t)}^{L_{j}}(2)\right)$ $\operatorname{or}\left(C_{v_{2}(t)}^{L_{j}}(1), C_{v_{2}(t)}^{L_{j}}(2)\right)$, the cost is high for both of them. When either of them chooses the optimal strategy while the other chooses the second preferred strategy resulting in the use of strategy profile $\left(C_{v_{2}(t)}^{L_{j}}(1), C_{v_{1}(t)}^{L_{j}}(2)\right)$ or $\left(C_{v_{1}(t)}^{L_{j}}(1), C_{v_{2}(t)}^{L_{j}}(2)\right)$, the game is stable. In this case, the strategy profile has an NE. The NE is defined based on the NE concept [81] [94] as follows:

Definition 2. A Nash Equilibrium of a strategic game $G$ is a strategy profile in which every player $A V_{i} \in G$ replies to the other players' actions, using the action that incurs the minimum cost/maximum profit.

Hence, the binding agreement between the players would enforce them to choose different strategies as doing so leads to the optimal conflict-resolution solution. The player who wins the current time token is decided as described earlier, which is based on the mode and credit point values. The player with the poorer mode or credit point value will agree to swerve but in return get a credit point and so have a greater chance to win next time. A numerical representation of such a game is shown in Table 3.2.

Table 3.2: Action and response table of a two-player game

| Players/Strategies | $v_{1}(t)^{A V_{2}}$ | $v_{2}(t)^{A V_{2}}$ |
| :---: | :---: | :---: |
| $v_{1}(t)^{A V_{1}}$ | $(4,4)$ | $(0,2)$ |
| $v_{2}(t)^{A V_{1}}$ | $(2,0)$ | $(3,3)$ |

## Multi-Phase Cooperative Speed Optimization Game

When more than two vehicles are allocated the same time token, a multi-phase cooperative procedure is implemented to resolve the conflict. For each player, cooperating with every other player makes the process extremely complex. Therefore, the multi-phase game is composed of twoplayer sub-games. In each sub-game, only two players cooperate to find their acceptable joint strategies. Then, the winners of the two-player sub-games will play another sub-game to determine the winner of the only available time token. To further clarify the multi-phase cooperative speed optimization game, an example of four AVs is presented next.

Example 2. Let us recall Example 1 but consider a case in which four AVs are approaching the TL from the West direction. In the first phase of this multi-phase game, two two-player sub-games are played as described earlier; as a result, a winner is nominated from each sub-game, $G_{1}$ and $G_{2}$. For instance, if $A V_{1}$ and $A V_{3}$ are the winners of the two-player sub-games, then the optimum solutions resulting from these sub-games would state the strategies as follows:

$$
\begin{equation*}
V^{G_{1}}=\left\{v_{1}(t)^{A V_{1}}, v_{2}(t)^{A V_{2}}\right\} \tag{3.22}
\end{equation*}
$$

$$
\begin{equation*}
V^{G_{2}}=\left\{v_{1}(t)^{A V_{3}}, v_{2}(t)^{A V_{4}}\right\} \tag{3.23}
\end{equation*}
$$

After playing the second phase of the game, if $A V_{1}$ wins over $A V_{3}$, the final strategy assignment will take the following form:

$$
\begin{equation*}
V^{G}=\left\{v_{1}(t+1)^{A V_{1}}, v_{2}(t+1)^{A V_{2}}, v_{2}(t+1)^{A V_{3}}, v_{2}(t+1)^{A V_{4}}\right\} \tag{3.24}
\end{equation*}
$$

Therefore, a credit point will be deducted from $A V_{1}$, and each of the losing players, $A V_{2}, A V_{3}$ and $A V_{4}$, will receive a credit point from the $T L$ and minimize its own speed to meet the next green light.

### 3.4 Cooperative Credit-Point Bargaining

This section introduces the concept of bargaining in cooperative games to allow AVs, sharing common interest, to come into groups and trade credit points. AVs, heading toward a TL, may trade credit points to help players, with low credit point balance, increase their balance and thus, maximize their chance of winning time tokens. If an $A V$ has extra credit points or if it is not truly in rush to pass without delay, it may offer to sell credit points to others that are in need for those points. Hence, AV s that are in rush, but have no enough credit point balance to pass through the TL without any delay, have a chance to buy credit points on the way and thereby, maximize their chance of winning tokens and passing through without any delay.

The typical team-based cooperative game consists of a set of players, willing to get together in a group, and a characteristic function, which specifies the values created by different subsets of players. Within the context of this research, the set of all players interested to trade and perform transactions is called the grand coalition, while the different subsets of players within the grand coalition are called sub-coalitions. A cooperative game is a tuple of two elements $(N, f)$, where $N=\left\{A V_{1}, \ldots, A V_{n}\right\}$, is a finite set of AVs willing to trade credit points, and $f$ is a function that
maps subsets $S \subseteq N$ to their total value induced when the members of $S$ come together to trade credit points.

If a player is selling a credit point and receiving equal offers from multiple buyers, a random number generation procedure is conducted to determine who buys the point. In this case, the player whose generated number is closer to that of the seller will buy the point. The other case is when the seller is receiving non-equal offers from multiple buyers. To clarify the conceptual formulation of the $A V s$ ' characteristic function of the bargaining game for the latter case, an example is presented next.

Example 3. Consider three $A V s, N=\left\{A V_{1}, A V_{2}, A V_{3}\right\}$, heading toward a $T L$, where $A V_{1}$ is a seller of a credit point, while $A V_{2}$ and $A V_{3}$ are two buyers. Consider the case that $A V_{1}$ has only one credit point to sell at $\$ 3$ and each of the buyers contemplates to buy at most one credit point. $A V_{2}$ is willing to pay $\$ 5$, while $A V_{3}$ is willing to pay $\$ 8$. The characteristic function, $f$, of this game is defined as follows:

$$
\begin{aligned}
& f\left(\left\{A V_{1}\right\}\right)=f\left(\left\{A V_{2}\right\}\right)=f\left(\left\{A V_{3}\right\}\right)=0 \\
& f\left(\left\{A V_{1}, A V_{2}\right\}\right)=5-3=2 \\
& f\left(\left\{A V_{1}, A V_{3}\right\}\right)=8-3=5 \\
& f\left(\left\{A V_{2}, A V_{3}\right\}\right)=0 \\
& f\left(\left\{A V_{1}, A V_{2}, A V_{3}\right\}\right)=8-3=5
\end{aligned}
$$

In the above formulation, it is noted that no player can create any value on its own because no transaction can take place. When players $A V_{1}$ and $A V_{2}$ come together and transact, the total value is the difference between the credit point cost and the buyer's offer, which in this case is $\$ 2$. Players $A V_{2}$ and $A V_{3}$ cannot create any value when getting together since each of them is a buyer (i.e., no transaction can take place). Finally, the value created by the grand coalition, including
$A V_{1}, A V_{2}$ and $A V_{3}$, is $\$ 5$. Although there are two buyers in the game, $A V_{1}$ can transact only with one of them; consequently, the rational player would transact with the buyer that is offering a higher price. In this case, it is $A V_{3}$.

The important point now is how to fairly split or divide the overall value, $\$ 5$, created by the grand coalition, $f(\{N\})$. Bargaining in cooperative games has been recognized as a feasible method to fairly divide the overall value created by the players [97] [102]. The solution concept in coalitional cooperative games is the core, and bargaining can be used to find at least one allocation or division of values, which satisfies the core conditions. In order to address how the bargaining concept is applied to the credit point trade problem, the marginal contribution concept is addressed.

### 3.4.1 The Marginal Contribution

The marginal contribution concept [103] provides the analytical reasoning of bargaining. Let $N \backslash A V_{i}$ be the subset of $N$ that contains all the AVs except $A V_{i}$. The marginal contribution of $A V_{i}$ is $f(\{N\})-f\left(\left\{N \backslash A V_{i}\right\}\right)$ and denoted by $M C_{A V_{i}}$. For example, the marginal contributions of the previously defined game are

$$
\begin{aligned}
& M C_{A V_{1}}=f(\{N\})-f\left(\left\{N \backslash A V_{1}\right\}\right)=5-0=5 \\
& M C_{A V_{2}}=f(\{N\})-f\left(\left\{N \backslash A V_{2}\right\}\right)=5-5=0 \\
& M C_{A V_{3}}=f(\{N\})-f\left(\left\{N \backslash A V_{3}\right\}\right)=5-2=3
\end{aligned}
$$

Definition 3. An allocation, $\left(x_{a v_{1}}, x_{a v_{2}}, \ldots, x_{a v_{n}}\right)$, which is a collection of numbers representing the division of the overall value, where $x_{a v}$ indicates the value received by $A V_{i}$, is individually rational if $x_{a v_{i}} \geq f\left(\left\{A V_{i}\right\}\right), \forall i \in\{1,2, \ldots, n\}$.

Definition 4. An allocation, $\left(x_{a v_{1}}, x_{a v_{2}}, \ldots, x_{a v_{n}}\right)$, is efficient if $\sum_{i=1}^{n} x_{a v_{i}}=f(\{N\})$.

Definition 5. An individually rational and efficient allocation, $\left(x_{a v_{1}}, x_{a v_{2}}, \ldots, x_{a v_{n}}\right)$, satisfies the Marginal Contribution Principle if $x_{a v_{i}} \leq M C_{A V_{i}}, \forall i \in\{1,2, \ldots, n\}$.

To see how the overall value can be fairly divided among the players, we recall Example 3. Since player $A V_{2}$ has zero marginal contribution, Definition 5 poses that this player will not receive any value. Player $A V_{3}$ has a marginal contribution of $\$ 3$,so it cannot receive more than this value and player $A V_{1}$ can receive at most $\$ 5$ of value. Since it is assumed that the players are perfectly rational, player $A V_{1}$ will sell the credit point to $A V_{3}$ because it has offered a higher price. Besides, $A V_{3}$ has to pay at least $\$ 5$ in order to secure the credit point; However, the price at which both players agree to finalize the transaction may be any price between $\$ 5$ and $\$ 8$. Therefore, one possible allocation of the core is $(\$ 5, \$ 0, \$ 0)$. As can be noted, bargaining does not specify all the divisions of the overall value. Hence, all the possible allocations, which represent the core elements, must be found.

### 3.4.2 The Core

The core is the solution concept of coalitional cooperative games, containing the set of all feasible payoff/cost allocations other than which no subset or coalition of players can achieve better outcome. The most common approach in the literature that deals with finding the core elements is the Shaply Value [104]. However, within this research, an Artificial Intelligence (AI) technique is used to find the core elements. Let $x(S)$ be the sum of the values received by the AVs in the subset $S$, such that

$$
\begin{equation*}
x(S)=\sum_{i \in S} x_{a v_{i}} \tag{3.25}
\end{equation*}
$$

According to [103], the core has two main properties, summarized in the following theorems.

Theorem 1. An allocation, $\left(x_{a v_{1}}, x_{a v_{2}}, \ldots, x_{a v_{n}}\right)$, is part of the core if it is efficient and every subset $S$ of $N$ is individually rational such that $x(S) \geq f(\{S\})$ is satisfied.

Proof. let $S$ include only $A V_{i}$ such that $S=\left\{A V_{i}\right\}$ for $i=1,2, \ldots, n$

Noticeably, $x\left\{A V_{i}\right\}=x_{a v_{i}}$ both represent the values received by $A V_{i}$.
Therefore, the condition $x(S) \geq f(\{S\})$ is in fact the individual rationality condition $x_{a v_{i}} \geq$ $f\left(\left\{A V_{i}\right)\right\}$ 。

In addition, let the marginal contribution of a subset $S$ of $N$ be $M C_{S}=f(\{N\})-f(\{N \backslash S\})$.

Theorem 2. An allocation, $\left(x_{a v_{1}}, x_{a v_{2}}, \ldots, x_{a v_{n}}\right)$, is part of the core if it is efficient and every subset $S$ of $N$ satisfies the Marginal Contribution Principle $x(S) \leq M C_{S}$.

Proof. Using the individual rationality condition, consider $N \backslash S$

$$
\begin{gather*}
x(N \backslash S) \geq f(\{N \backslash S\})  \tag{3.26}\\
x(N \backslash S)=x(N)-x(S) \tag{3.27}
\end{gather*}
$$

By efficiency, we have

$$
\begin{equation*}
x(N)=f(\{N\}) \tag{3.28}
\end{equation*}
$$

Substituting (3.27) into (3.26)

$$
\begin{equation*}
x(N)-x(S) \geq f(\{N \backslash S\}) \tag{3.29}
\end{equation*}
$$

Substituting (3.28) into (3.29)

$$
\begin{equation*}
x(S) \leq f(\{N\})-f(\{N \backslash S\})=M C_{S} \tag{3.30}
\end{equation*}
$$

Therefore, the core of the cooperative credit point bargaining game is defined as follows:

$$
\begin{equation*}
\left\{\left(x_{a v_{1}}, x_{a v_{2}}, \ldots, x_{a v_{n}}\right): \sum_{i \in N} x_{a v_{i}}=f(\{N\}), \text { and } x(S) \geq f(\{S\}), \forall S \in N\right\} \tag{3.31}
\end{equation*}
$$

where $f(\{S\})$ is the sum of the values of the members of $S$ prior to playing the game, and $x(S)$ is the sum of the values received by each of the members of $S$. To find the core elements, we propose that the problem is formulated as a Constraint Satisfaction Problem (CSP). The most common CSP solving techniques are Backtracking Search and Local Search [105]. For instance, the feasible allocations in Example 3 are the points $\left(x_{a v_{1}}, x_{a v_{2}}, x_{a v_{3}}\right)$, such that

$$
\begin{aligned}
& x_{a v_{1}}+x_{a v_{2}}+x_{a v_{3}}=5 \\
& \text { subject to }: \\
& x_{a v_{1}}+x_{a v_{2}} \geq 2 \\
& x_{a v_{1}}+x_{a v_{3}} \geq 5 \\
& x_{a v_{2}}+x_{a v_{3}} \geq 0 \\
& x_{a v_{1}} \geq 0, x_{a v_{2}} \geq 0, x_{a v_{3}} \geq 0
\end{aligned}
$$

The domains of $x_{a v_{1}}, x_{a v_{2}}$ and $x_{a v_{3}}$ are

$$
\begin{aligned}
& \operatorname{dom}\left(x_{a v_{1}}\right)=\{\text { any value between } 0 \text { and } 5\} \\
& \operatorname{dom}\left(x_{a v_{2}}\right)=\{\text { any value between } 0 \text { and } 5\} \\
& \operatorname{dom}\left(x_{a v_{3}}\right)=\{\text { any value between } 0 \text { and } 5\}
\end{aligned}
$$

By solving this problem as a CSP, the core of the game is

$$
\begin{gathered}
\text { Core }=\left\{\left(x_{a v_{1}}, x_{a v_{2}}, x_{a v_{3}}\right): \sum_{i=1}^{i=3} x_{a v_{i}}=f(\{N\}), \text { and } x(S) \geq f(\{S\}), \forall S \in N\right\} \\
\text { Core }=\{(\$ 2, \$ 0, \$ 3)(\$ 3, \$ 0, \$ 2)(\$ 4, \$ 0, \$ 1)(\$ 5, \$ 0, \$ 0)\} .
\end{gathered}
$$

### 3.5 Simulation Tests and Results

Simulation was conducted to test and validate the performance of the proposed CSOF in a detailed MATLAB environment using the concept of Object Oriented Programming (OOP). A twolane roadway sub-network containing three SIs was chosen in Waterloo, ON, Canada, to conduct the simulation (Figure 3.4). The SIs are as follows: SI1, Westmount Road North with Columbia Street West; SI2, Westmount Road North with Bearinger Road; and SI3, Northfield Drive West with Weber Street North. Every SI has a static TL system such that TL1, TL2, and TL3 for SI1, SI2, and SI3 respectively. Each TL control system has a two-phase cycle where the East-West roadways are one phase and South-North roadways are the other phase. Each phase has a signal design of Green-Yellow-Red; however, for simplicity, the yellow-light time is assumed to be part of the green-light-time duration. To enhance safety, one second of red-light time is given to all the roadways between every two consecutive phases.

In order to overcome randomization and capture the real behaviour of traffic, the simulation was run for more than three hours. The maximum and minimum speed limits on any roadway in the network are $v_{\max }=60 \mathrm{~km} / \mathrm{hour}$ and $v_{\min }=10 \mathrm{~km} / \mathrm{hour}$ respectively where the maximum number of vehicles a road segment may have is assumed to be $85 \%$ of the maximum density, $D_{\max }\left(L_{j}\right)$.


Figure 3.4: A sub-network with three signalized intersections.

AVs are generated randomly into the network based on Poisson Distribution (PD) from ten generation points determined in advance. The amount of traffic to make left/right turns or move straightforward at every SI was specified in percentage prior to the AV generation process such
that traffic is equally distributed throughout the network and every SI receives equal random arrivals through its roadways. The generated AVs travel at an average speed of $50 \mathrm{~km} / \mathrm{hour}$ until they get within the activation distance (i.e., the distance at which the vehicles get within the V2I communication range and start cooperating). The activation distance was fixed at 500 m . The performance of the CSOF is compared to a Non-Cooperative Speed Optimization algorithm (NCSO) (i.e., the vehicles individually and independently compute their optimal speeds). Once they are within range, AVs start speed optimization, based on CSOF or NCSO, to catch the green light when they arrive at the TL. The NCSO algorithm is a complete speed optimization procedure that includes all the possible scenarios based on road and signal timing constraints. Besides, it takes into consideration the queue lengths at every roadway when computing the optimal speeds. Therefore, the NCSO algorithm represents a higher benchmark than the stat-ofart in the literature, where queue lengths are not taken into account most of the time.

Figures 3.5, 3.6, and 3.7 show the total average idling time, total average number of stops, and total average energy consumption at SI1, SI2, and SI3, comparing the CSOF to the NCSO algorithm. As can be seen, in under-saturation traffic conditions (i.e., the number of arriving AVs is within the TL capacity), the two techniques on average achieve nearly the same average values of idling times and number of stops. As the traffic volume increases to reach over-saturation traffic conditions (i.e., the number of arriving AVs is greater than the TL capacity), CSOF outperforms NCSO by achieving lower average idling times and average number of stops. This is because the conflicting passing times of vehicles through the intersections are resolved by CSOF. All the AVs using CSOF, meant to arrive during the green-light time, are allocated time tokens before reaching the intersections. As such, when they arrive, they are able to pass through smoothly during their allocated times. In addition, due to the road and signal timing constraints, the AVs that could only arrive at the intersections during the red-light times were not allocated time tokens before reaching the intersections. These AVs joined the queues with less waiting times. As mentioned previously, the time needed to clear the queue is excluded from that available as
time tokens to the approaching AVs. The figures show that as the number of AVs approaching the intersections increases, the average idling times and number of stops become greater. The reductions in average idling times that have been achieved by CSOF when compared to NCSO for SI1, SI2, and SI3 are summarized in Tables 3.3, 3.4, and 3.5 respectively.


Figure 3.5: Total average idling time at SI1, SI2, and SI3.


Figure 3.6: Total average number of stops at SI1, SI2, and SI3.

Table 3.3: Reduction in average idling time at signalized intersection 1.

| AVs (veh/hour) | 300 | 600 | 900 | 1200 | 1500 | 1800 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NCSO (sec) | 0.0344 | 0.6417 | 4.6417 | 5.9479 | 6.1433 | 7.5194 |
| CSOF (sec) | 0.0344 | 0.4188 | 1.2729 | 1.8547 | 2.3975 | 2.9791 |
| Reduction (\%) | 0 | 35 | 73 | 69 | 61 | 60 |

Table 3.4: Reduction in average idling time at signalized intersection 2.

| AVs (veh/hour) | 300 | 600 | 900 | 1200 | 1500 | 1800 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NCSO (sec) | 0.0196 | 0.1533 | 4.2899 | 4.9364 | 5.5783 | 6.5815 |
| CSOF (sec) | 0.0174 | 0.0565 | 0.3732 | 0.8435 | 1.3683 | 2.4757 |
| Reduction (\%) | 11 | 63 | 91 | 83 | 75 | 62 |

Table 3.5: Reduction in average idling time at signalized intersection 3.

| AVs (veh/hour) | 300 | 600 | 900 | 1200 | 1500 | 1800 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NCSO (sec) | 0.1174 | 0.1348 | 4.0920 | 4.6674 | 5.9448 | 6.1098 |
| CSOF (sec) | 0.0478 | 0.0978 | 0.4094 | 0.7739 | 1.6835 | 2.5674 |
| Reduction (\%) | 59 | 27 | 90 | 83 | 72 | 58 |

In addition, CSOF has achieved lower average energy consumption. With CSOF, as soon as a vehicle is allocated a time token, it maintains its speed to pass the intersection smoothly during its allocated token. Hence, in general there are less speed variations using CSOF. To justify
this, Figure 3.8 captures the speed trajectories of six AVs approaching SI1 from the Westmount Road North direction from the moment they joined the activation distance until they passed the stop line of the intersection. On average, the reductions in speed variations using CSOF are not significant as compared to NCSO. As a result, the energy savings of AVs using CSOF have not been significant as reported in Figure 3.7. It can be noticed in Figure 3.7 that in general, the energy consumption of AVs does not increase as the number of AVs approaching SIs increases. This is because it is assumed that the electric motor an AV operates is able to regenerate energy during downhill and deceleration phases. In addition, the energy consumption model, presented in Section 3.3.1, depends on many road and environment factors when computing the total energy consumption at every time step; hence, the energy consumption may not have a linear relation with the increasing number of AVs.


Figure 3.7: Total average energy consumption of vehicles approaching SI1, SI2, and SI3.

Furthermore, the total average idling time and number of stops achieved by CSOF were investigated with respect to the V2I activation distance for SI2. It is assumed that the V2I communication radio is available in a range of up to 800 m away from the intersection, so it was varied from 200 m to 800 m in steps of 100 m . Figure 3.9 depicts the total values of average idling times and number of stops achieved by CSOF.


Figure 3.8: Speed trajectories of six vehicles approaching intersection 1, (a) vehicles using the CSOF, (b) vehicles using the NCSO.

It is seen that the optimal point of activation is near 500 m . At shorter distances, some AVs arrive during the red-light time due to insufficiency of the time available to get allocated tokens, or reallocated tokens after playing a game, and to adjust speeds accordingly for low average values of idling times and stop numbers. At further activation distances, the average idling times and stop numbers slightly increase. This is due to the speed limit and signal timing constraints the AVs have to deal with as soon as they join the activation distance and start the token allocation process. Hence, the best allocation of tokens apparently occurs at an activation distance near 500 $m$.


Figure 3.9: Activation distance analysis for intersection 2, (a) total average idling time, (b) total average number of stops.

### 3.6 Summary

This chapter has presented the idling time and stop number minimization problem of AVs approaching an SI. A simple example has been addressed to demonstrate how AVs could negatively impact each other's plans. Then, the speed optimization process for AVs approaching a TL has been formulated as a cooperative theoretic game. The game defined the players, their available actions, and the costs associated with their actions. In addition, it has been proven that equilibrium in the formulated game exists and the game is stable. Furthermore, a cooperative speed
optimization framework, CSOF, has been presented to formulate the AV speed optimization as a cooperative process. This framework consists of three modules addressing issues of AV rational speed optimization, information and conflict recognition, and cooperative speed decisionmaking. Thus, the proposed framework describes the problem from a single and multiple AVs' point of view. It is argued that when AVs approaching a TL interact and cooperate with each other, the average idling times and number of stops at the TL can be minimized, improving traffic efficiency. Moreover, this chapter introduced an AV bargaining approach to allow AVs, sharing common interest, to get together and trade credit points so that AVs could maximize their chances of passing through the TL without delay. Simulation tests have been conducted to test and validate the performance of CSOF and obtained results are reported. The CSOF was compared under various traffic conditions to a non-cooperative speed optimization, NCSO, algorithm where AVs individually conduct speed optimization. It is concluded that the CSOF outperformed the NCSO by achieving lower values of total average idling times, average number of stops and average energy consumption.

## Chapter 4

## Cooperative Autonomous Vehicle-Dynamic Traffic Light Control System

In this chapter, a Cooperative Autonomous Vehicle-Dynamic Traffic Light Control (CAV-DTLC) system is introduced. The hypothetical concept behind this system is that incorporating the dynamic TL functionality with the AVs, cooperating based on the CSOF, would achieve further minimization of average idling times and number of stops. The proposed CAV-DTLC has the capability to model over-saturation traffic conditions, consisting of a decision-making unit based on a theoretic game and a control unit that relies on FLC to control the signal timings of the TL. Simulation tests are conducted to validate the performance of the CAV-DTLC under various traffic conditions as compared to the CSOF and results obtained from the simulation tests are reported.

### 4.1 Problem Statement

Consider a two-lane four roadway SI, as illustrated in Figure 4.1. For simplicity, it is assumed that the TL control system has a two-phase static cycle $T L_{S}=\{E W, S N\}$, i.e., the East-West roadways are one phase and North-South roadways are the other phase. Each phase has a signal design of Green-Yellow-Red; however, for simplicity, the yellow-light time is assumed to be part of the green-light time duration. The parameters of the TL are the green-light time duration $T_{g}$, red-light time duration $T_{r}$, and TL cycle duration $T_{c}=T_{g}+T_{r}$. These parameters are assumed to be constant, e.g., $T_{g}=22 \mathrm{sec}, T_{r}=28 \mathrm{sec}$, and $T_{c}=50 \mathrm{sec}$. We assume that the TL has fixed maximum arrival and departure rates, $\lambda$ and $\mu$ in $v e h / s e c$, respectively. Therefore, the maximum number of AVs that can arrive at the TL from each roadway during the red time is $N_{a r r}=\lambda T_{r}$ vehs, and the maximum number of AVs that can depart the TL from each roadway during the green-light time is $N_{d e p}=\mu T_{g}$ vehs.

Consider a set of AVs, $N=\left\{A V_{1}, A V_{2}, \ldots, A V_{n}\right\}$, travelling toward the SI from each direction and receiving signal-timing information from the TL through V2I communication. Further assume that the AV in $N$ have the capability to conduct the following cooperative speed optimization game:

Definition 6. The cooperative speed optimization game is a game in which a set of AVs, $N=$ $\left\{A V_{1}, A V_{2}, \ldots, A V_{n}\right\}$, heading toward a $T L$, cooperate and reach a binding agreement to implement certain speed actions. In this game, an $A V_{i} \in N$, travelling toward a $T L$ from any direction, either is allocated a time token $\tau_{i}$, an integer value indicating a time window within the green light, during which it passes the TL smoothly or agrees to slow down to meet the green light of the next cycle in return of a compensation.


Figure 4.1: An example scenario of autonomous vehicles approaching a static-signal-timing traffic light.

Given the above setting, the problem of interest is to minimize $C_{s v}^{L_{j}}(i)$, i.e., the idling-time cost of every $A V_{i} \in N$ at $T L_{S}$ that is located at the end of road segment $L_{j}$, when $N<N_{d e p}$ for one roadway phase, e.g., $S N \in T L_{S}$, but $N>N_{\text {dep }}$ for the other phase $E W \in T L_{S}$ such that less AVs on $E W \in T L_{S}$ experience unexpected delay, waiting for the next green light to pass through.

To clarify the extent of the problem, consider a certain cycle where $\lambda=0.393 \mathrm{veh} / \mathrm{sec}$ and $\mu=0.455 \mathrm{veh} / \mathrm{sec}$. Thus, for this particular cycle, $N_{a r r}=\lambda T_{r}=(0.393)(28)=11 \mathrm{veh}$, and $N_{\text {dep }}=\mu T_{g}=(0.455)(22)=10 \mathrm{veh}$. Now, let us consider the moment at which the TL has just turned green for $E W \in T L_{S}$. Let two AVs travelling on the East roadway play a cooperative speed optimization game. Since there is only one token $\tau_{i}$ available at the TL, one of these AVs must come to a complete stop and wait for the next green light to pass through. This simple two-AV
example shows that cooperation between the AVs alone is not fully effective for the minimization of idling times and stop numbers, requiring the incorporation of a dynamic TL system into the AV-cooperative process.

### 4.2 Cooperative Autonomous Vehicle-Dynamic Traffic Light Control

The objective of introducing the CAV-DTLC system is to achieve further minimization of idling times and number of stops for AVs approaching a SI. The fundamental concept of CAV-DTLC is to incorporate the dynamic TL functionality into the cooperative process the AVs conduct based on the CSOF. Generally, the dynamic TL system has the ability to adjust the signal timings according to the traffic volumes on the roadways, which has shown better results than the static TL system.


Figure 4.2: Schematic depiction of the cooperative autonomous vehicle-dynamic traffic light control system.

As depicted in Figure 4.2, the CAV-DTLC system we propose consists of a theoretic-game Decision-Making Unit (DMU) that decides whether the green-light time for a certain phase
should be extended or not, and an FL-based Control Unit (CU) to generate the signal-timing commands. It is assumed that there is reliable V2I communication capability; therefore, the TL periodically receives information about the traffic volumes of each phase (i.e., the TL receives reliable information about the number of vehicles, within the distance of communication, willing to pass through). The decision of green-light time extension and control of the signal timings are performed only once for every green-light phase after a certain predefined time interval has elapsed (e.g., $\Delta_{t}=5 s e c$ ) from the realization of over-saturation traffic conditions.

### 4.2.1 Decision Making Unit

Through the DMU, the TL decides, based on the traffic volume of a certain signal phase, whether the green-light time should be extended or not. The decision to extend or not extend the greenlight time is made based on a game formulated as a two-player game including the Green-Light Phase (GLP) and the Red-Light Phase (RLP). Thus, the set of players in the game is

$$
\begin{equation*}
N=\{G L P, R L P\} \tag{4.1}
\end{equation*}
$$

Generally, for each player $i \in N$, we define a non-empty set of $m$ actions $A_{i}=\left\{a_{1}, \ldots, a_{m}\right\}$ (i.e., the set of strategies available to player $i$ ), and a preference relation $\succsim_{i}$ on $A_{i}$. In addition, $A_{i}$ is associated with a non-empty set of $m$ costs $C_{i}=\left\{c_{1}, \ldots, c_{m}\right\}$ such that $c_{j} \in C_{i}$ is incurred by taking action $a_{j} \in A_{i}$, where $j=1, \ldots, m$. If player $i$ prefers $a_{j}$ over $a_{k}, \forall j, k \in A_{i}$, i.e., $a_{j} \succsim_{i} a_{k}$, then $c_{j} \leq c_{k}, \forall j, k \in C_{i}$. Making this setting, each player $i \in N$, in the proposed game, can have an action of either Extra or Not Extra AVs (i.e., Extra AVs means that there are more AVs willing to pass through than the TL capacity). Therefore, the actions possible for each player in the game are

$$
\begin{align*}
& G L P=\left\{\begin{array}{lcc}
E X_{g} & \text { if } & N_{G L P}>N_{d e p} \\
N E X_{g} & \text { otherwise }
\end{array}\right.  \tag{4.2}\\
& R L P=\left\{\begin{array}{lll}
E X_{r} & \text { if } & N_{R L P}>N_{d e p} \\
N E X_{r} & \text { otherwise }
\end{array}\right. \tag{4.3}
\end{align*}
$$

where, $N_{G L P}$ is the number of AVs in the GLP, $N_{R L P}$ is the number of AVs in the RLP, $E X_{g}$ means extra AVs in the GLP, $N E X_{g}$ means no extra AVs in the GLP, $E X_{r}$ means extra AVs in the RLP, and $N E X_{r}$ means no extra AVs in the RLP.

Table 4.1: Decision-making action table.

|  | $Y_{E} \Rightarrow E$ |  |
| :---: | :---: | :---: |
| Players and Strategies | $E X_{r}$ | $N E X_{r}$ |
| $E X_{g}$ | $(0,1)$ | $(0,1)$ |
| $N E X_{g}$ | $(1,2)$ | $(2,3)$ |
| $Y_{N E} \Rightarrow N E$ |  |  |
| Players and Strategies | $E X_{r}$ | $N E X_{r}$ |
| $E X_{g}$ | $(2,0)$ | $(3,0)$ |
| $N E X_{g}$ | $(0,0)$ | $(0,0)$ |

The proposed game, described in Table 4.1, consists of two sub-games, namely, Extension game $E$ and No Extension game $N E$. In Table 4.1, the numerical values, $0,1,2$ and 3, represent the costs incurred by the DMU when taking the actions of $E$ or $N E$ based on $N_{G L P}$ and $N_{R L P}$. The DMU produces the final output $Y_{D M U}$ after conducting a two-stage decision-making process. First, each sub-game produces the strategy profile cost that represents the given input values.

Then, the DMU produces $Y_{D M U}$ based on the concept of Nash Equilibrium (NE) [81], comparing the two strategy profiles resulting from the first stage, i.e., the DMU produces a strategy profile $Y_{D M U}$ in which each player $i \in N$ replies to the action cost of the other player, $c_{-i}$, using the action that incurs the minimum cost. Thus, the final output of the DMU is

$$
Y_{D M U}= \begin{cases}\left(c_{-i}, c_{i}\right) & \text { if } \quad \sum\left(c_{-i}, c_{i}\right) \leq \sum\left(c_{-j}, c_{j}\right)  \tag{4.4}\\ \left(c_{-j}, c_{j}\right) & \text { otherwise }, \forall c_{i} \in C_{E} \& c_{j} \in C_{N E}\end{cases}
$$



Figure 4.3: Block diagram of the fuzzy logic control system.

### 4.2.2 Control Unit

FLC has been shown to have the capability to apply real-life rules similar to the way humans would control traffic flow near SIs as traffic is mostly not deterministic and tends to be subjective, approximate and qualitative. Thus, it allows inexact traffic data to be manipulated to reflect varying degrees of truth such that more realistic signal timings can be applied. The block diagram of the FLC system, introduced to implement the signal timings of the CAV-DTLC system, is depicted in Figure 4.3, where the input variables to the FLC system are the number of extra AVs in the GLP, $N_{E X_{g}}$, and the number of extra AVs in the RLP, $N_{E X_{r}}$. The output of the FLC system is the amount by which the green-light time is extended, $E$.

## Fuzzification

Fuzzification is used to establish the membership degrees of the FLC input/output variables, where the crisp values are represented by membership functions. The membership function maps the elements of the input/output variable onto numerical values in the interval $[0,1]$. We define a fuzzy set including the membership function grades for each input variable. The fuzzy sets of all possible values of the input variables, $N_{E X_{g}}$ and $N_{E X_{r}}$, respectively, are defined as follows:

$$
\begin{gather*}
S_{g}=\frac{\mu_{g}\left(E X_{g_{1}}\right)}{N_{E g_{g_{1}}}}+\frac{\mu_{g}\left(E X_{g_{2}}\right)}{N_{E X_{g_{2}}}}+\cdots+\frac{\mu_{g}\left(E X_{g_{f}}\right)}{N_{E X_{g_{f}}}}  \tag{4.5}\\
S_{r}=\frac{\mu_{r}\left(E X_{r_{1}}\right)}{N_{E X_{r_{1}}}}+\frac{\mu_{r}\left(E X_{r_{2}}\right)}{N_{E X_{r_{2}}}}+\cdots+\frac{\mu_{r}\left(E X_{r_{f}}\right)}{N_{E X_{r_{f}}}} \tag{4.6}
\end{gather*}
$$

where $\mu_{g}\left(E X_{g_{i}}\right) \in[0,1]$ represents the membership-function grade of the element $N_{E X_{g_{i}}}$ of the fuzzy set $S_{g}$, and $\mu_{r}\left(E X_{r_{i}}\right) \in[0,1]$ is the membership-function grade of the element $N_{E X_{r_{i}}}$ of
the fuzzy set $S_{r}$. Each input variable is represented by four linguistic descriptive memberships, Zero, Few, Medium, and Many, to model the numbers of extra AVs in the GLP and RLP. The membership functions of the fuzzy sets $S_{g}$ and $S_{r}$, which are designed to be trapezoidal, are depicted in Figures 4.4 and 4.5 respectively.


Figure 4.4: Membership functions of extra vehicles in the green-light phase.


Figure 4.5: Membership functions of extra vehicles in the red-light phase.

The output of the FLC system determines by how much the green-light time is extended. We define a fuzzy set, $S_{E}$, to model the output variable $E$ as follows:

$$
\begin{equation*}
S_{E}=\frac{\mu_{E}\left(E_{1}\right)}{E_{1}}+\frac{\mu_{E}\left(E_{2}\right)}{E_{2}}+\cdots+\frac{\mu_{E}\left(E_{f}\right)}{E_{f}} \tag{4.7}
\end{equation*}
$$

where $\mu_{E}\left(E_{i}\right) \in[0,1]$ represents the membership-function grade of the element $E_{i}$ of the fuzzy set $S_{E}$. The output variable is represented by five linguistic descriptive memberships, Zero, Very Short, Short, Medium, and Long, to model the extension time of the green light. The membership functions of the fuzzy set $S_{E}$, designed to be trapezoidal, are depicted in Figure 4.6.


Figure 4.6: Membership functions of green light extension.

## Fuzzy Inference System

We define a set of if-then fuzzy inference rules to be the essential operation of the Fuzzy Inference System (FIS). The fuzzy inference rules connect the input-output fuzzy variables, providing a Rule Base (RB) based on which the decision of green-light time extension is made. The rule base of the FIS is as follows:

- Rule Base

If $N_{E X_{g}}$ is Zero and $N_{E X_{r}}$ is Zero, then $E$ is Zero
If $N_{E X_{g}}$ is Zero and $N_{E X_{r}}$ is Few or Medium or Many, then $E$ is Zero

If $N_{E X_{g}}$ is Few and $N_{E X_{r}}$ is Zero, then $E$ is Short
If $N_{E X_{g}}$ is Medium or Many and $N_{E X_{r}}$ is Zero, then $E$ is Long
If $N_{E X_{g}}$ is Few and $N_{E X_{r}}$ is Few, then $E$ is Short
If $N_{E X_{g}}$ is Few and $N_{E X_{r}}$ is Medium or Many, then $E$ is Very Short
If $N_{E X_{g}}$ is Medium and $N_{E X_{r}}$ is Few, then $E$ is Medium
If $N_{E X_{g}}$ is Medium and $N_{E X_{r}}$ is Medium or Many, then $E$ is Short
If $N_{E X_{g}}$ is Many and $N_{E X_{r}}$ is Few, then $E$ is Medium
If $N_{E X_{g}}$ is Many and $N_{E X_{r}}$ is Medium or Many, then $E$ is Short

To calculate the membership grades of possible values of the fuzzy output variable $E$, the Mamdani Fuzzy Model (MFM) is used in the FIS. The MFM is designed to process two input membership grades for $N_{E X_{g}}$ and $N_{E X_{r}}$, and produce one output membership grade for $E$. In the MFM, the composition/aggregation of the fuzzy variables is conducted based on the max-min operator. The output of the MFM is produced as the fuzzy membership function

$$
\begin{equation*}
\mu_{F}(E)=\max _{k=1 \rightarrow r b_{t}}\left[\min \left(\mu_{g}\left(E X_{g}\right), \mu_{r}\left(E X_{r}\right)\right)\right] \tag{4.8}
\end{equation*}
$$

where $k$ denotes the case number of the RB, and $r b_{t}$ denotes the total number of cases of the RB.

## Defuzzification

The output of the MFM in the FIS is a fuzzy value that is represented by a membership function. This value is defuzzified to provide the crisp value for the extension of the green-light time, $E$. The defuzzification method used in this thesis is the Centroid Method (CM) [106]. The CM computes the center of gravity of the membership function resulting from the MFM. If the FLC
system decides to extend the green-light time for a certain phase, the defuzzified extension time $T_{e x}$ is segmented and offered as time tokens to be available to the approaching AVs. The AVs can be allocated tokens based on the TTAA. The number of tokens $N_{t k n}$ that can be added to the current tokens of a roadway is found as follows:

$$
\begin{equation*}
N_{t k n}=\frac{T_{e x}}{T_{t k d}} \tag{4.9}
\end{equation*}
$$

where $T_{t k d}=\frac{1}{\mu}$ is the token duration in sec/veh.

### 4.3 Simulation Tests and Results

Simulation was conducted to investigate the performance of the proposed CAV-DTLC system. The simulation was performed in MATLAB using the concept of OOP. The road network used to conduct the simulation tests is the same one presented in Section 3.5 (Figure 3.4), containing three SIs in Waterloo, ON, Canada. The SIs are SI1, SI2, and SI3 with TL systems of TL1, TL2, and TL3 respectively. To overcome randomization in traffic behavior, the simulation was run for more than two hours.

The performance of the CAV-DTLC was compared to those of the CSOF and NCSO. With NCSO, there is no cooperation as the AVs individually and independently perform speed optimization to meet the green-light time. With CSOF, cooperation is performed only among the AVs, while with CAV-DTLC, the AVs cooperate among themselves and with the dynamic TL system. A PD model was used to generate the AVs randomly into the road network. In order to show the difference between the three techniques in terms of performance, some roadways were given more traffic than others during the vehicle random generation process. The activation distance of the communication radio was fixed at 500 m . The metrics used to validate the per-
formance of the three techniques are the total average idling times and total average number of stops at the defined SIs.

Figures 4.7 and 4.8 illustrate the total average idling times and total average number of stops with respect to the increasing traffic volume at SI1, SI2, and SI3, comparing CAV-DTLC with CSOF and NCSO. It can be seen from the figures that the CAV-DTLC, where the dynamic TL interacts and cooperates with the cooperative AVs, has outperformed the CSOF, where cooperation is performed only between the AVs, and the NCSO, where there is no cooperation between the AVs. This is because the unnecessary stops and delays are reduced in the CAV-DTLC scenario via the dynamic functionality of the TLs.


Figure 4.7: Total average idling time at SI1, SI2, and SI3.

A remarkable observation is that in under-saturation traffic conditions, there is not much difference in the total average idling times and stop numbers achieved by the three techniques. However, as the traffic volume increases until it reaches the over-saturation traffic conditions, the difference between the three techniques in terms of total average idling times and stop numbers becomes clear. Thus, the CAV-DTLC has proven to be more efficient in terms of traffic management, achieving lower average values of idling times and stop numbers.


Figure 4.8: Total average number of stops at SI1, SI2, and SI3.

Tables 4.2, 4.3, and 4.4 illustrate the reductions in average idling times that have been achieved by the CAV-DTLC at SI1, SI2, and SI3 respectively as compared to the CSOF.

Table 4.2: Reduction in average idling time at signalized intersection 1.

| AVs (veh/hour) | 300 | 600 | 900 | 1200 | 1500 | 1800 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CSOF (sec) | 0.0194 | 0.3798 | 0.9465 | 1.0663 | 1.4232 | 1.6202 |
| CAV-DTLC (sec) | 0.0145 | 0.2698 | 0.4177 | 0.1901 | 0.2669 | 0.5421 |
| Reduction (\%) | 25.00 | 28.98 | 55.86 | 82.17 | 81.24 | 66.54 |

Table 4.3: Reduction in average idling time at signalized intersection 2.

| AVs (veh/hour) | 300 | 600 | 900 | 1200 | 1500 | 1800 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CSOF (sec) | 0.0081 | 0.1613 | 0.3863 | 1.2516 | 1.6795 | 2.7243 |
| CAV-DTLC (sec) | 0.0065 | 0.1161 | 0.1153 | 0.3149 | 0.5619 | 0.7477 |
| Reduction (\%) | 20 | 28 | 70 | 75 | 67 | 73 |

Table 4.4: Reduction in average idling time at signalized intersection 3.

| AVs (veh/hour) | 300 | 600 | 900 | 1200 | 1500 | 1800 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CSOF (sec) | 0.0129 | 0.1379 | 0.4685 | 1.0489 | 1.5929 | 2.1633 |
| CAV-DTLC (sec) | 0.0032 | 0.1153 | 0.1438 | 0.1 | 0.1769 | 0.3013 |
| Reduction (\%) | 75 | 16 | 69 | 90 | 89 | 86 |

Furthermore, Figure 4.9 depicts a snapshot of the input-output of the FIS. As can be seen in the figure, in this case, a roadway with green-light has got 10 extra AVs, while another roadway with red-light has got only 1 extra AV. Therefore, the green-light phase was extended by 10.5 sec . This extension portion was segmented as tokens and made available to the AVs in the green-light phase. Hence, the minimization of the total average idling times and number of stops has been more efficient with the CAV-DTLC scenario.


Figure 4.9: Input-output of fuzzy inference system.

### 4.4 Summary

This chapter has introduced a cooperative AV-TL control system, CAV-DTLC, consisting of decision-making and control units. The proposed system has the ability to adjust the signal timings under different traffic conditions. The decision-making unit is a theoretic game formulated by the TL to decide whether the green-light time for a certain phase/roadway should be extended, while a fuzzy logic controller is embedded in the control unit to determine by how much the green-light time should be extended. The purpose of proposing the cooperative AV-TL system is to allow the TL to be dynamic such that it interacts and cooperates with the cooperative AVs , while the objective is to achieve further minimization of the total average idling times and number of stops at SIs. Simulation tests were conducted to investigate the performance of the CAV-DTLC as compared to the CSOF and NCSO. It has been demonstrated that the CAV-DTLC outperforms the CSOF and NCSO by achieving lower average values of idling times and number of stops under various traffic conditions.

## Chapter 5

## Platoon-based Autonomous Vehicle Speed Optimization near Signalized Intersections

This chapter addresses the AV cooperation in urban areas through platoon formation, in particular the impacts of AV platoon formation on average idling times and number of stops at SIs. A Platoon-based Autonomous Vehicle Speed Optimization Scheme (PAVSOS) is proposed to allow AVs, approaching an SI, to decide in a decentralized manner whether to be part of the platoon or not such that the average idling times and number of stops are minimized. The PAVSOS relies on V2V and V2I communication and consists of an LP speed optimization procedure to be conducted by the leading AV and an Intelligent Vehicle Decision Making Algorithm (IVDMA). Simulation tests are conducted and analysis results are reported, investigating the performance of the PAVSOS in terms of average idling times and average number of stops at an SI in comparison with two other AV platoon scenarios.

### 5.1 Problem Statement

Consider the intersection depicted in Figure 5.1 as a single-lane, four-roadway SI. For simplicity, assume the TL control system has a two-phase static cycle $T L_{S}=\{E W, S N\}$, where East-West roadways are one phase and North-South roadways are the other phase. Assume that a number of AVs connected in a platoon, $N_{p}=\left\{A V_{1}, A V_{2}, \ldots, A V_{n}\right\}$, are travelling toward the intersection from the West roadway direction. The TL parameters are defined as follows: green-light-time duration, $T_{g}$; red-light-time duration, $T_{r}$; and TL cycle duration, $C$, where $C=T_{g}+T_{r}$.


Figure 5.1: An example scenario of a platoon approaching a signalized intersection

As the objective of the platoon is to be connected, the problem of interest is to minimize $C_{i t}^{P}$, i.e., the average idling-time cost of the platoon at an SI when $N_{p}>N_{d e p}$ and $N_{p} \leq N_{d e p}$. First,
assuming that there is no V2I communication available, if the number of AV s in the platoon is greater than the number of AVs that can pass through the intersection within the green-light time of a single cycle (i.e., $N_{p}>N_{\text {dep }}$ ), then some of the platoon AVs will certainly stop and experience delay. If the number of AVs in the platoon is less than or equal to those that can pass through the intersection within the green-light time of a single cycle (i.e., $N_{p} \leq N_{d e p}$ ), then some of the platoon AVs still may experience delay. This delay occurs because at least the leading AV is not optimizing its speed based on the TL signal-timing information.

Second, assuming that there is V2I communication and that only the leading AV is able to optimize its own speed based on the TL signal timing and its own Time-to-Intersection, $T T I_{L}$, such that it has a great chance to meet the green-light time when arriving at the intersection. In this case, the following AVs are controlling their speeds based on the leader's speed. Considering, $N_{p} \leq N_{d e p}$, if the leading AV takes less time to reach the intersection than the remaining greenlight time, $T T I_{L} \leq R_{g}$, not necessarily the last vehicle in the platoon does, (e.g.,TTI $I_{n}>R_{g}$ ). As a result, some of the platoon AVs may still stop and experience delay. Therefore, the minimization of idling time and stop-and-go driving of a platoon of AVs is a complex problem that requires intelligent algorithmic tools to allow the platoon-AV members to connect/disconnect based on individual interests.

### 5.2 Platoon-based Autonomous Vehicle Speed Optimization Scheme

In this Section, we propose the Platoon-based AV Speed Optimization Scheme (PAVSOS) to minimize the average idling times of platoons of AVs approaching SIs. PAVSOS consists of an LP speed optimization procedure to be conducted by the leading AV and an intelligent decisionmaking algorithm, IVDMA, to be run by every following AV such that the platoon can have a
greater chance of meeting the green-light time when arriving at the intersection. The objective of the IVDMA is to enable the following AVs to reason and decide whether it is efficient for them to be part of the platoon or not. The logic of the PAVSOS is depicted in Figure 5.2.


Figure 5.2: Logic of platoon-based autonomous vehicle speed optimization scheme

### 5.2.1 Platoon Formation and Control

It is assumed that the AV platoons are formed and being controlled using a unidirectional stringstable CACC [107] [108]. Constant headway time is maintained between consecutive AVs where
the relative distance measured from $A V_{i}$ to $A V_{j}$ is defined as

$$
\begin{equation*}
d_{i j}(t)=x_{i}(t)-x_{j}(t) \tag{5.1}
\end{equation*}
$$

### 5.2.2 Leading-Autonomous-Vehicle Speed Optimization

Computing the optimal speed for the platoon leading-AV, $A V_{L}$, when approaching an SI is subject to some constraints such as time to intersection, $T T I_{L}$, maximum and minimum speed limits on the roadway, $v_{\max }$ and $v_{\text {min }}$, current light signal, and queue size. The optimal speed of $A V_{L}$ at time step $t$ travelling toward a TL (i.e., the speed that minimizes the idling-time cost of the leader at time step $t$ ) can be found through linear programming as follows:

## Light is Green

As $A V_{L}$ receives upcoming signal information from the TL, indicating that the current light is green, there are three possible cases in terms of $T T I_{L}$ and $R_{g}$ :

- Case 1: $T T I_{L} \leq R_{g}$. In this case, using the current speed, $A V_{L}$ will be able to pass through within the remaining green-light time.

$$
\begin{aligned}
& s_{L}=v_{L}(t) \\
& \text { subject to }: \\
& v_{L}(t) \leq d_{L}(t) / T_{q} \\
& v_{L}(t) \geq d_{L}(t) / T_{g} \\
& v_{L}(t) \geq v_{\min } \\
& v_{L}(t) \leq v_{\max }
\end{aligned}
$$

where $T T I_{L}=d_{L}(t) / v_{L}(t) \sec , v_{L}(t)$ is the speed of $A V_{L}$ at time step $t, d_{L}(t)$ is the distance of $A V_{L}$ to the stop line of the TL at time step $t, s_{L}$ is the optimal speed of $A V_{L}$ at time step $t+1$.

- Case 2: $R_{g}+T_{r} \geq T T I_{L}>R_{g}$. In this case, the speed of the vehicle is optimized over the distance to the TL so that $T T I_{L}$ is sufficient to meet the next green light.

$$
\begin{aligned}
& s_{L}=\min \quad v_{L}(t) \\
& \text { subject to }: \\
& v_{L}(t) \leq d_{L}(t) /\left(R_{g}+T_{r}+T_{q}\right) \\
& v_{L}(t) \geq d_{L}(t) /\left(R_{g}+T_{r}+T_{g}\right) \\
& v_{L}(t) \geq v_{\min } \\
& v_{L}(t) \leq v_{\max }
\end{aligned}
$$

In addition, if the current speed does not allow $A V_{L}$ to be part of the current green-light time but the maximum speed of the roadway does, the speed optimization system will accelerate the speed of $A V_{L}$ to pass cautiously during the current green-light time.

$$
\begin{gathered}
s_{L}=\max \quad v_{L}(t) \\
\text { subject to }: \\
v_{L}(t) \leq d_{L}(t) / T_{q} \\
v_{L}(t) \geq d_{L}(t) / T_{g} \\
v_{L}(t) \geq v_{\min } \\
v_{L}(t) \leq v_{\max }
\end{gathered}
$$

- Case 3: $R_{g}+T_{r}+T_{g} \geq T T I_{L}>R_{g}+T_{r}$. In this case, $A V_{L}$ will maintain its current speed as
$T T I_{L}$ leads it to be part of the next green-light time.

$$
\begin{aligned}
& s_{L}=v_{L}(t) \\
& \quad \text { subject to: } \\
& \quad v_{L}(t) \leq d_{L}(t) /\left(R_{g}+T_{r}+T_{q}\right) \\
& v_{L}(t) \geq d_{L}(t) /\left(R_{g}+T_{r}+T_{g}\right) \\
& v_{L}(t) \geq v_{\min } \\
& v_{L}(t) \leq v_{\max }
\end{aligned}
$$

## Light is Red

If the information received by $A V_{L}$ from the TL indicates that the current light is red, there are three possible cases in terms of $T T I_{L}$ and $R_{r}$ :

- Case 1: $T T I_{L} \leq R_{r}$. In this case, the speed of $A V_{L}$ is optimized such that $A V_{L}$ will meet the upcoming green-light time.

$$
\begin{aligned}
& s_{L}=\min \quad v_{L}(t) \\
& \text { subject to }: \\
& v_{L}(t) \leq d_{L}(t) /\left(R_{r}+T_{q}\right) \\
& v_{L}(t) \geq d_{L}(t) /\left(R_{r}+T_{g}\right) \\
& v_{L}(t) \geq v_{\min } \\
& v_{L}(t) \leq v_{\max }
\end{aligned}
$$

- Case 2: $R_{r}<T T I_{L} \leq R_{r}+T_{g}$. In this case, $A V_{L}$ will maintain its current speed to smoothly
pass within the upcoming green-light time.

$$
\begin{aligned}
& s_{L}=v_{L}(t) \\
& \quad \text { subject to }: \\
& v_{L}(t) \leq d_{L}(t) /\left(R_{r}+T_{q}\right) \\
& v_{L}(t) \geq d_{L}(t) /\left(R_{r}+T_{g}\right) \\
& v_{L}(t) \geq v_{\min } \\
& v_{L}(t) \leq v_{\max }
\end{aligned}
$$

- Case 3: $T T I_{i}>R_{r}+T_{g}$. In this case, the speed of $A V_{L}$ is optimized to meet the green-light time of the next cycle.

$$
\begin{aligned}
& s_{L}=\min \quad v_{L}(t) \\
& \text { subject to: } \\
& v_{L}(t) \leq d_{L}(t) /\left(R_{r}+T_{g}+T_{r}+T_{q}\right) \\
& v_{L}(t) \geq d_{L}(t) /\left(R_{r}+T_{r}+2 T_{g}\right) \\
& v_{L}(t) \geq v_{\min } \\
& v_{L}(t) \leq v_{\max }
\end{aligned}
$$

### 5.2.3 Intelligent Vehicle Decision-Making Algorithm

In this section, the IVDMA is introduced (Algorithm 2) to allow the platoon-following AVs to check their status as they are connected to the platoon. As such, the objective of minimizing the average idling time for the platoon as a whole can be achieved. Through V2V communication, AVs can share their TTIs. The IVDMA is run by every follower in the platoon in order to be able to decide whether being part of the platoon would minimize its own idling time or not. If not,
then the follower can disconnect from the platoon and become a leader for its followers, forming a new platoon. The new leader then is able to communicate with the TL and run the speed optimization linear program and thus, have a better chance of meeting the green-light time.

```
Algorithm 2 : Intelligent Vehicle Decision-Making Algorithm for Vehicle \(j\)
    Receive information from TL
    if Light is Green then
        if \(T T I_{j-1} \leq R_{g}\) and \(T T I_{j}>R_{g}\) then
            Disconnect from platoon
            optimize speed accordingly
        else
            Keep headway time
        end if
    else
        if \(T T I_{j-1} \leq R_{r}+T_{g}\) and \(T T I_{j}>R_{r}+T_{g}\) then
            Disconnect from platoon
            optimize speed accordingly
        else
            Keep headway time
        end if
    end if
```


### 5.3 Simulation Tests and Results

In order to test and validate the performance of PAVSOS, simulation was conducted. The simulation was performed in MATLAB using the concept of OOP. An SI of single-lane four-roadways was built (Figure 5.3). The TL control system is a two-phase system with East-West roadways as one phase and South-North roadways as another phase. The maximum and minimum speeds allowed on the roadways are $V_{\max }=60 \mathrm{~km} / \mathrm{hour}$ and $V_{\min }=10 \mathrm{~km} / \mathrm{hour}$ respectively. The TL parameters are as follows: Green-light time, $T_{g}=24 \mathrm{sec}$; Red-light time, $T_{r}=36 \mathrm{sec}$; Cycle time, $C=T_{g}+T_{r}=60 \mathrm{sec}$. For simplicity, the yellow-light time is neglected and assumed to be part of the green-light time. AVs are generated randomly through each roadway direction according to

PD. AVs travel through the intersection in a straightforward movement (i.e., no left and/or right turns allowed). In order to capture the real behaviour of traffic, the simulation was run for more than three hours. The headway time between every two consecutive vehicles in the platoon is set to 2 sec.


Figure 5.3: A signalized intersection of single-lane four-roadways

The performance metrics used to investigate the performance of PAVSOS are average idling time and average number of stops. The platoon that uses PAVSOS is compared to two other platoons. The first is a regular platoon of AVs assumed to be connected through a CACC with no V2I communication. In this platoon, there is no speed optimization being implemented, so it is named No Speed Optimization Platoon (NSOP). The average speed of this platoon is set to the maximum speed allowed on the roadway, $V_{\max }$. The second is a regular platoon of AVs
assumed to be connected through a CACC with only the leading-AV communicating with the TL and implementing speed optimization. This platoon is named Leader only Speed Optimization Platoon (LSOP). The platoon that functions based on PAVSOS is named Intelligent Vehicle Speed Optimization Platoon (IVSOP).

The results are depicted in two graphs and one table. Comparing the three platoon scenarios, Figures 5.4 and 5.5 illustrate the average idling times and number of stops for the whole intersection with respect to the number of platooned AVs. The number of vehicles is varied from a platoon of two vehicles up to twelve vehicles. As can be seen in the graphs, LSOP has outperformed NSOP, while IVSOP has outperformed both NSOP and LSOP, achieving lower total average idling times and number of stops. Based on the performance validation, the reductions being achieved in terms of average idling times between IVSOP with the NSOP and LSOP are illustrated in Table 5.1.


Figure 5.4: Total average idling time in three hours


Figure 5.5: Total average number of stops in three hours

Table 5.1: Reduction in average idling time at the signalized intersection.

| Number of Vehicles | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IVSOP vs. NSOP (\%) | 100 | 98 | 96 | 96 | 94 | 93 | 92 | 91 | 91 | 88 | 81 |
| IVSOP vs. LSOP (\%) | 99 | 60 | 50 | 50 | 41 | 41 | 37 | 45 | 60 | 59 | 51 |

### 5.4 Summary

This chapter has introduced a speed optimization scheme, PAVSOS, to minimize delay and stop-and-go driving for connected AV platoons at SIs. The scheme relies on V2V and V2I communication capabilities so that vehicles could receive signal timing and queue information from TLs. It includes a speed optimization LP technique and intelligent decision-making algorithm, IVDMA, to allow the platoon-AV members to decide in a decentralized manner whether it is efficient to part of the platoon or not. The platoon functioning based on PAVSOS was compared to two other platoons to validate its performance. The first is a platoon that does not perform any speed optimization, NSOP. The second is a platoon that has only the leading AV communicat-
ing with the TL through V2I communication and performing speed optimization, LOSP. It has been reported that the platoon using the PAVSOS outperformed the two other platoon scenarios, achieving lower average idling times and number of stops.

## Chapter 6

## Conclusion and Future Research

### 6.1 Conclusion

AVs are expected to eventually have major positive impacts on the transportation systems by reducing driver stress and public transportation cost; improving the mobility of non-drivers, the disabled and elderly people; reducing accident risk; increasing road capacity; and reducing parking costs. However, in general, these positive impacts are expected to raise the number of vehicles on the roads. Long idling times of AVs at SIs, which may lead to congestion, will be a major cause of significant fuel waste and time delay. This thesis has analyzed and modelled the problem of idling time and stop number minimization of AVs at SIs as a cooperative speed optimization process.

Extensive background information and a comprehensive literature review on AVs in general and vehicle speed optimization techniques in particular were addressed, showing the challenges facing the wide adoption of AVs as well as the outstanding issues to be dealt with to achieve
further idling time and stop number minimization and thus improve traffic efficiency at TLs. In this regard, vehicle-centric speed optimization scenarios, explaining how AVs would act selfishly and negatively impact each other's goals, were demonstrated. In addition, the problem was viewed and formulated as a cooperative game between the AVs , and game theory was proven to have the potential to model and analyze the interaction and coordination of AVs' strategic speed actions.

Following the game theoretic formulation, a Cooperative Speed Optimization Framework (CSOF) was proposed to perform the cooperative speed optimization process between AVs approaching a TL. The proposed CSOF consists of three main modules to address issues of AVcentric speed optimization, information gathering and conflict recognition, and cooperative decision making. In the AV-centric speed optimization module, a Linear Programming (LP) speed optimization procedure is performed to provide every $A V$ with the optimal speed such that the idling time and number of stops at the TL are minimized. In addition, within the process of finding optimal speeds, AVs can request and own time tokens within the green light during which they can smoothly pass through. Therefore, via this module, AVs satisfy the rationality condition in the sense that each would know the optimal speed choice among all possible speeds. In the information and conflict recognition module, AVs are recognized as rational players. AVs with conflicting time tokens are contacted by the TL to start a negotiation process to resolve the conflict. Thus, AVs with conflicting tokens communicate with each other to share their speed strategies and associated costs. In the cooperative decision-making module, the final token allocations are made and speed assignments are finalized. AVs are assumed to abide by the rules and can reach binding agreements, based on which, the time token conflicts are resolved. AVs involved in a conflict would accept the need to swerve and decelerate their speeds, requesting different time tokens, only in exchange for rewards. The TL can reward AVs that accept losing the game, which maximizes their chances to win the next time they have a conflict. Thus, the theory of the cooperative decision-making module was conceptualized and formulated as a
repeated game, representing cooperation between the AVs in ongoing token allocation conflicts.

Furthermore, a cooperative credit point bargaining model was introduced to allow AVs to trade credit points as they are travelling toward an SI. AVs heading toward an SI can come into groups to sell and buy credit points. The theoretical concept of this model is to allow AVs that are in rush to pass through without delay but have no enough credit points to buy credit points and use them to pass through smoothly without delay. AVs with extra credit points and/or are not in rush to pass through can sell credit points to those in need. Simulation tests were conducted to test and validate the performance of the CSOF under various traffic conditions. A sub-network consisting of three SIs in the neighbourhood of the University of Waterloo, ON, Canada was chosen to conduct the simulation tests. The performance of the CSOF was compared to a Non-Cooperative Speed Optimization (NCSO) technique with which AVs independently and individually perform speed optimization to meet the green light of a TL. The results reported in this thesis demonstrate the effectiveness of the CSOF by achieving lower average values of idling times and number of stops under various traffic conditions. Due to the less speed variations resulting from AVs using the CSOF, the CSOF has also shown less energy consumption as compared to the NCSO technique.

Moreover, this thesis has proposed a Cooperative Autonomous Vehicle-Dynamic Traffic Light Control (CAV-DTLC) system based on game theory and fuzzy logic control. The CAVDTLC system consists of a decision making and control units. It has the ability to adjust the signal timings of the TL under different traffic conditions. The decision-making unit is a theoretic game formulated by the TL to decide whether the green-light time for a certain phase/roadway should be extended, while a fuzzy logic controller is embedded in the control unit to determine by how much the green-light time should be extended. The purpose of proposing the CAV-DTLC is to allow the TL to be dynamic such that it interacts and cooperates with the cooperative AVs, while the objective is to achieve further minimization of the average idling times and number of stops at SIs. Simulation tests were conducted to investigate the performance of the CAV-DTLC
as compared to the CSOF and NCSO. It has been demonstrated that the CAV-DTLC has outperformed the CSOF and NCSO by achieving lower average values of idling times and number of stops under various traffic conditions.

Besides, this thesis has proposed a Platoon-based Autonomous Vehicle Speed Optimization Scheme (PAVSOS) with the aim to minimize idling times and number of stops for connectedAV platoons at SIs. The scheme relies on V2V and V2I communication capabilities so that AVs could receive signal timing and queue information from TLs. It includes a speed optimization technique and intelligent decision-making algorithm to minimize the platoon average idling times and number of stops at SIs. Simulation tests were conducted to validate the performance of the PAVSOS. The AV platoon functioning based on PAVSOS was compared to two other platoon scenarios. The first is No Speed Optimization platoon (NSOP), which has no V2I communication and thus, no speed optimization being implemented. The second is a Leader only Speed Optimization Platoon (LSOP) with which only the platoon leading AV implements speed optimization when approaching an SI to meet the green-light time. It has been reported in the results that the platoon using the PAVSOS outperformed the two other platoon scenarios, achieving lower average idling times and number of stops.

### 6.2 Future Research

One important point to be investigated in the near future is the implementation and testing of the proposed techniques under various traffic conditions using a larger road network with different geometrical designs of SIs and various cycle/phase signal-timing settings. Since the current geometrical design of SIs is limited to two-lane roadways with no independent left and right lanes, SIs with independent left and right lanes will be considered. Besides, the design of the cycle/phase timings will be developed to more complicated scenarios such that TL cycles with
more than two phases are implemented. For instance, the traffic making left turns at an SI might have its own phase and thereby have its own time token schedule. Having completed the experimentation design and settings, real-life traffic data will be used to test and validate the comparative performance between the CAV-DTLC with CSOF and NCSO.

Memory and time complexities of the proposed techniques are another important point to investigate. It is important to verify how much memory space is needed and how much time is taken by the TL system to manage the token allocation process. Another question to raise is how costly it is in terms of memory space to keep a record of Vehicle Identification Numbers (VINs) for AVs passing an SI as this might be beneficial for security and safety purposes. In addition, what ways are possible to keep the token allocation process efficient and reasonably inexpensive in terms of memory and time usage. It is also important to investigate the computational cost of the speed optimization, conflict resolution, and token allocation processes and find out strategies and means to overcome any expensive computational costs.

Signal-timing coordination between neighbouring TL systems over a large area is significantly important research point to be addressed so as to help minimize the idling times and stop numbers of AVs at SIs. According to the novel development of techniques introduced in this thesis, the TL signal-timing system has been limited to a dynamic phase-based timing adjustment. With signal-timing coordination between neighbouring TL systems, the dynamic timing adjustment is performed to the cycles such that, based on traffic volume conditions, certain phases may be expanded beyond the cycle timing limit. Therefore, a cycle-based dynamic-TL control system will be proposed to allow TL systems over a large area to coordinate their dynamic cyclebased signal-timing settings such that global minimization of idling times and stop numbers is achieved. The performance analysis of such a cycle-based dynamic-TL system will be compared with that of the techniques introduced in this thesis (i.e., CAV-DTLC and CSOF).

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