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1 Highlights of This Work

The contribution of this work can be summarized as follows.

1. This work presents a novel deep-connected architecture of CNN with detailed analytical analysis and extensive experiments on several datasets.
2. A new activation function is presented to approximate arbitrary complex functions with analytical analysis on both forward pass and backward pass.
3. The experiments show the competitive performance of our designed network with less parameters and shallower architecture, compared with other state-of-art models.

Global-Connected Network With Generalized ReLU Activation

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Abstract

Recent Progress has shown that exploitation of hidden layer neurons in convolutional neural networks (CNN) incorporating with a carefully designed activation function can yield better classification results in the field of computer vision. The paper firstly introduces a novel deep learning (DL) architecture aiming to mitigate the gradient-vanishing problem, in which the earlier hidden layer neurons could be directly connected with the last hidden layer and fed into the softmax layer for classification. We then design a generalized linear rectifier function as the activation function that can approximate arbitrary complex functions via training of the parameters. We will show that our design can achieve similar performance in a number of object recognition and video action benchmark tasks, such as MNIST, CIFAR-10/100, SVHN, Fashion-MNIST, STL-10, and UCF YoutTube Action Video datasets, under significantly less number of parameters and shallower network infrastructure, which is not only promising in training in terms of computation burden and memory usage, but is also applicable to low-computation, low-memory mobile scenarios for inference.

Keywords: CNN, computer vision, deep learning, activation

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1. Introduction

Deep convolution neural network (CNN) has achieved great success in ImageNet competition [1]-[2]. In the review papers in [3]-[4], deep learning has been well recognized as powerful tools for the computer vision applications as well as natural language processing in the recent years. This, however, cannot be possible without the availability of massive image/video datasets collected in the Internet-of-things era, as well as the innovation of high-performance parallel computing resources. Exploiting these resources, novel ideas, algorithms as well as modification on the network architecture of deep CNN has been experimented to achieve higher performance in different computer vision tasks. Deep CNNs thus were found to be able to extract rich hierarchical features from raw pixel values and achieved amazing performance for classification and segmentation tasks in computer vision field.

A typical CNN usually consists of several cascaded convolution layers, optional pooling layers (average pooling, max pooling or more advanced pooling), nonlinear activations as well as fully-connected layers, followed by a final softmax layer for classification/detection tasks, where the convolution layer is employed to learn the spatially local-connectivity of input data for feature extraction, pooling layer is for reduction of receptive field and hence prohibits overfitting to some extent, and nonlinear activations for boosting of learned features. This neural network can be learned through the well-known backpropagation algorithm as well as momentum (for running average gradients to avoid fluctuations of stochastic gradient learning) to a very good local optimal point, given good weight initialization and appropriate regularization. Since then, deeper CNN architecture (with more layers) [5] and its variants, e.g., resnet [6] and highway networks [7] have been introduced and experimented on various computer vision datasets, which are shown to achieve state-of-the-art performance. For example, for the ImageNet challenge [1], a computer vision classification task with a massive dataset with 1000 classes of objects, the elegantly designed GoogleNet [5] achieved the best performance on the ILSVRC 2014 competition, which employed 5 million parameters and 22 layers consisting of inception modules. This validates the potential of deep CNN to learn powerful hierarchical information from raw images. However, it is noted that, the advancement of deep CNN largely follows the development of careful weight initialization, advanced regularization methods, and neural arch design. For example, to avoid overfitting

for deep neural networks, some regularization methods are invented, such as dropout [8][9], which turns off the neurons with a certain probability in training. By doing so, the network is forced to learn many variations of itself and generates a natural pseudo-ensemble classifier. In [10], the authors proposed batch normalization algorithm, providing powerful ways for co-adaptation of features learned, which allows for higher learning rates and be less dependable on careful initialization for deep CNN. In [11], the authors considered learning sparse representations of features from neural networks for a multi-task scenario, which in some sense reduces neural network capacity whereas avoiding overfitting to some extent.

In spite of its great success, deep CNN is subject to some open problems. One is that the features learned at an intermediate hidden layer could be lost at classification stage. Another is the gradient vanishing problem, which could cause training difficulty or even infeasibility. They are hence receiving increasing interest in the literature and the industry. In this paper, we are also motivated to mitigate such obstacles by targeting at the tasks of real-time classification on small-scale applications. To this end, the proposed deep CNN system incorporates a globally connected network topology with a generalized activation function. Global average pooling (GAP) on the neurons of some hidden layers as well as the last convolution layers is applied and results in a concatenated vector to be fed into the softmax layer. Henceforth, with only one classifier and one objective loss function for training, we shall enjoy the benefit of retaining rich information fused in the hidden layers while taking minimal parameters so that efficient information flow in both forward and backward stage is guaranteed, and the overfitting risk is avoided. Further, the proposed general activation function is composed of several of piecewise linear functions to approximate complex functions. By doing do, we can not only exploit the hidden layer features via rich interconnections between hidden blocks and final block, but also scales up these learned features with the new designed activation function, leading to a more powerful joint architecture. It will be shown in the conducted experiments that the proposed deep CNN architecture yields similar performance with much less parameters.

The contribution of this work is hence presented as follows.

- We present an architecture which makes full use of features learned at hidden layers, avoiding the gradient-vanishing problem to the most extent. The analytical analysis in closed-form on both forward pass

and backward pass of the proposed architecture is presented.

- We define a generalized multi-piecewise ReLU activation function, which is able to approximate more complex and flexible functions and scales up the learned features. The associated analytical analysis on both the forward pass and the backward pass in closed-form is presented.

- In the conducted experiments, our design is shown to achieve promising performance on several benchmark datasets in computer vision field, but with less parameters and shallower structure.

The rest of the paper is organized as follows. In Section II some the related works are reviewed. In Section III and Section IV, the details of the proposed deep CNN architecture as well as the designed activation function are presented, respectively. Section V evaluates our design on several public datasets, such as MNIST [35], CIFAR-10/100 [36], SVHN [37], Fashion-MNIST [38], STL-10 [39], as well as UCF Youtube Action Video Data Sets [40]. We conclude this paper in Section VI.

2. Related Work

2.1. Activation Design

One key feature of the success of deep CNN architecture is the use of appropriate nonlinear activation functions that define the value transformation from the input to output. It was found that the linear rectifier activation function (ReLU) [12] can greatly boost performance of CNN in achieving higher accuracy and faster convergence speed, in contrast to its saturated counterpart functions, i.e., sigmoid and tanh functions. ReLU only applies identity mapping on the positive side while drops the negative input, allowing efficient gradient propagation in training. Its simple functionality enables training on deep neural networks without the requirement of unsupervised pre-training and paved the way for implementations of very deep neural networks. On the other hand, one of the main drawbacks of ReLU is that the negative part of the input is simply dropped and not updated through backward pass, causing the problem of dead neurons which may never be reactivated again and potentially results in lost feature information through the back-propagation. To alleviate this problem, some new types of activation functions based on ReLU are reported for CNNs. Ng et al [13] introduced the Leaky ReLU assigning a non-zero slope to the negative part, which however

is a fixed parameter and not updated in learning. Kaiming et al pushed it further to allow the slope on the negative side to be a learnable parameter, which is hence named Parameter ReLU (PReLU) in [14]. Further, [15] introduced an S-shaped ReLU function. In [16], the authors introduced the ELU activation function, which assigns an exponential function on the negative side to zeroing the mean activation. The network performance is improved at the cost of the increasing computation burden on the negative side, compared with other variants of ReLU. Nevertheless, all of these functions lack the ability to mimic complex functions in order to extract necessary information relayed to the next level. In addition, Goodfellow et al introduced a maxout function which selects the maximum among k linear functions for each neuron as the output in [17]. While maxout has the potential to mimic complex functions and performs well in practice, it takes much more parameters than necessary for training and thus reduces its popularity in terms of computation and memory usage in real-time and mobile applications.

2.2. Network Architecture Design

The other design aspect of deep CNN is on the network size and the interconnection mechanism of different layers. A natural way to improve performance is to increase its size, by either depth (number of layers) or width (number of units in each layer). This works well suited to the case with a massive number of labelled training data. However, when the amount of training data is small, this potentially leads to overfitting. In addition, a non-necessary large neural net size only ends up with the waste of compute resources, as most learned parameters may finally found to be close to zero and can be simply dropped. Therefore, instead of changing network size, there is an emerging trend in exploiting the interconnection of the network to achieve better performance. The intuition follows that, the learned gradients flowing from the output layer to the input layer, could easily be diluted or even vanished when it reaches the beginning of the network, and vice versa. This hence greatly prohibits performance improvement of neural networks over decades. As discovered in the recent literature, addressing these issues will not only allow the same performance achievable with shallower networks, but also makes deeper networks trainable.

In [5], an inception module concatenating the feature maps produced by filters of different sizes, i.e., a wider network consisting of many parallel convolution networks with filters of different sizes, was proposed to improve the network capacity as well as the performance. In [6], the authors proposed

a residual-network architecture (ResNet) to ease the training of networks, where higher layers only need to learn a residual function with respect to the features in the lower layers. In this way, every node only needs to learn the residual information and thus is expected to achieve better performance with less training time. With the residual module, the authors found that very-deep network (e.g., a 160-layer ResNet was successfully implemented) is trainable and can achieve amazing performance than its counterparts in the literature. In [7], a highway network was proposed by the use of gating units for regulating the flow of information through the network, and achieves the state-of-art performance. In [19], a stochastic depth approach was proposed, which only allows a subset of network modules through and replaced the rest via identity functions in training. By doing so, it was discovered that very-deep networks beyond 1020-layer network is trainable and achieves very high performance. In [20], the authors proposed a neural network macro-architecture based on self-similarity, where the designed network contains interacting sub-paths with different lengths. In [21], a wide residual network is proposed, which decreases depth but increases width of ResNet and also achieves the state-of-art performance in experimentation. In [23], the hypercolumns were designed to extract useful information from intermediate layers for segmentation and fined-grained localization at pixel level, since the final layer information might be too coarse to locate objects. In [24], a multi-scale spatial partition network was proposed to classify text/non-text image classification at patch-level first for all patches from different generated feature maps from hidden layers, and then at the entire image level for classification via voting. In [25], an end-to-end deep Fisher network, which combines both convolution neural networks and Fisher vector encoding, was proposed to outperform standard convolution neural networks and standard Fisher vector methods. In [26], a weak supervision framework was proposed to learn patch features via only image-level supervisions, and integrated multiple stages of weakly supervised object classification and discovery, achieving state-of-art performance on benchmark datasets. In [27], a non-linear mapping with multi-layered deep neural network was employed to extract features with mutilmodal fusion for human pose recovery. In [28], a new multiple instance neural network to learn bag representations was proposed to boost bag classification accuracy and efficiency. In [29], a hierarchical deep neural network with both depth and width architectures into account was proposed for multivariate regression problems. In [30], a deep multi-task learning algorithm was developed to jointly learn features from deep convolutional neural

networks and more discriminative tree classifier for automatic recommendation of privacy settings for image sharing. In [31], a deep multimodal distance metric learning method as well as a structured ranking model were proposed to retrieve images in a precise and efficient way.

Further, in [32], the authors introduced a Network in Network (NIN) architecture that contains several micro multi-layer perceptrons between the convolutional layers to exploit complicated features of the input information, which is shown to reduce the number of parameters while achieve great performance in the public datasets. In [33], the authors introduced a new type of regularization with auxiliary classifiers employed on the hidden layers, to strengthen the features learned by hidden layers. However, the use of the auxiliary classifiers introduces a lot of extra parameters in training, where only the final classifier is employed in the inference stage. In [34], the authors introduced a densely connected architecture within every block to ensure high information flow among layers in the network, achieving state-of-art performance in most public datasets.

3. Global-Connected Net (GC-Net)

In this section, the proposed network architecture, namely Global-Connected Net (GC-Net) is presented, followed by the discussion of the proposed activation function in Sec. 4.

As shown in Fig. 1, the proposed GC-Net, consists of n blocks in total, a fully-connected final hidden layer and a softmax classifier, where a block can have several convolutional layers, each followed by normalization layers and nonlinear activation layers. Max-pooling/Average pooling layers are applied between connected blocks to reduce feature map sizes. The distinguished feature of the proposed GC-Net network architecture from the conventional cascaded structure is that, we provide a direct connection between every block and the last hidden layer. These connections in turn create a relatively larger vector full of rich features captured from all blocks, which is fed as input into the last fully-connected hidden layer and then to the softmax classifier to obtain the classification probabilities in respective of labels. In addition, to reduce the number of parameters in use, we only allow one fully-connected layer to the final softmax classifier, as more dense layers only has minimal performance improvement while requires a lot of extra parameters.

In our GC-Net design, to reduce the amount of parameters as well as computation burden, we shall first apply global average pooling (GAP) to

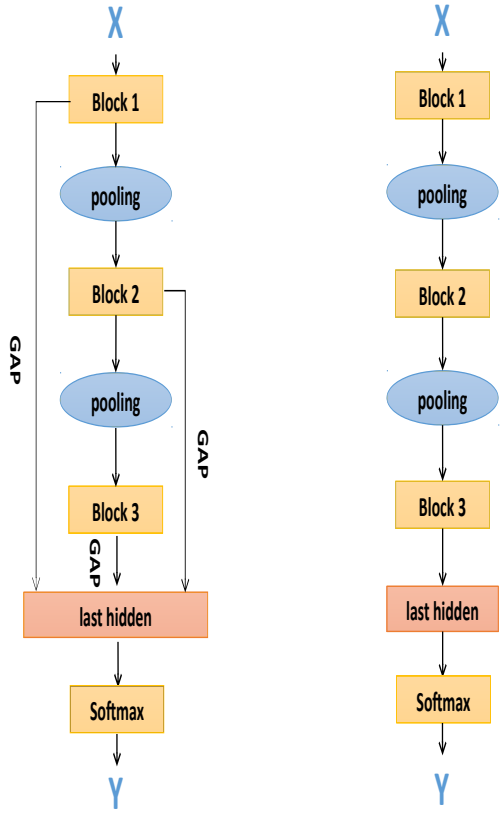

Figure 1: Comparison of our proposed GC-Net (left) with conventional cascaded CNN architecture (right), where GAP stands for global average pooling in GC-Net.

the output feature maps of all blocks and then connects them with the last fully-connected hidden layer. In this sense, we further flatten the neurons obtained from these blocks to obtain the 1-D vector for each blocks, i.e., $\vec{\mathbf{p}}_i$ from block i $(i = 1, \cdots, N)$ of length m_i. We then apply the concatenation operations on all of these 1-D vectors, which hence resulted in a final 1-D vector consisting of neurons from these vectors, i.e., $\vec{\mathbf{p}} = \overrightarrow{(\vec{\mathbf{p}}_1^T, \cdots, \vec{\mathbf{p}}_n^T)}^T$ with its length defined as $m = \sum_{i=1}^{N} m_i$. This resulted vector is then taken as the input to the last fully-connected hidden layer before the softmax classifier for classification. Therefore, to incorporate with this new feature vector, a weight matrix $\mathbf{W}_{m \times s_c} = (\mathbf{W}_{m_1 \times s_c}^T, \cdots, \mathbf{W}_{m_N \times s_c}^T)^T$ for the final fully-connected layer is required, where s_c is the number of classes of the corresponding dataset for

recognition. The final result fed into the softmax function hence is presented as,

$$\vec{\mathbf{c}}^T = \vec{\mathbf{p}}^T\mathbf{W} = \sum_{i=1}^{N} \vec{\mathbf{p}}_i^T\mathbf{W}_i \tag{1}$$

i.e., $\vec{\mathbf{c}} = \mathbf{W}^T\vec{\mathbf{p}}$, where $\mathbf{W}_i = \mathbf{W}_{m_i \times s_c}$ for short. $\vec{\mathbf{c}}$ is the input vector into the softmax classifier, as well as the output of the fully-connected layer with $\vec{\mathbf{p}}$ as input.

Therefore, in the back-propagation stage, defining $dL/d\vec{\mathbf{c}}$ as the gradient of the loss function(denoted by L) with respect to the input fed to the softmax classifier[1], the gradient to the concatenated vector is then given by,

$$\frac{dL}{d\vec{\mathbf{p}}} = \frac{dL}{d\vec{\mathbf{c}}}\frac{d\vec{\mathbf{c}}}{d\vec{\mathbf{p}}} = \mathbf{W}^T\frac{dL}{d\vec{\mathbf{c}}} = ((\frac{dL}{d\vec{\mathbf{p}}_1})^T, \cdots, (\frac{dL}{d\vec{\mathbf{p}}_n})^T)^T \tag{2}$$

Therefore, for the resulted vector $\vec{\mathbf{p}}_i$ after pooling from the output of block i, we obtain its gradient $dL/d\vec{\mathbf{p}}_i$ directly from the softmax classifier.

Further, taking the cascaded back-propagation process into account, except block n, all other blocks will also receive the gradients from its following block in the backward pass. Let us define the output of block i as $\mathbf{B}_i$, and the final gradient of the output of block i from the loss function as $\frac{dL}{d\mathbf{B}_i}$. Then, taking both gradients combing from the final layer and the adjacent block of the cascaded structure into account, the derivation of $\frac{dL}{d\vec{\mathbf{B}}_i}$ is hence summarized in Lemma 1,

Lemma 1. *The full gradient of the output of block i ($i < n$) is given by,*

$$\frac{dL}{d\mathbf{B}_i} = \frac{dL}{d\vec{\mathbf{p}}_i}\frac{d\mathbf{p}_i}{d\vec{\mathbf{B}}_i} + \frac{dL}{d\vec{\mathbf{B}}_{i+1}}\frac{d\mathbf{B}_{i+1}}{d\vec{\mathbf{B}}_i} \tag{3}$$

$$= \frac{dL}{d\vec{\mathbf{p}}_i}\frac{d\mathbf{p}_i}{d\vec{\mathbf{B}}_i} + \sum_{j=i+1}^{n}\left(\frac{dL}{d\vec{\mathbf{p}}_j}\frac{d\vec{\mathbf{p}}_j}{d\vec{\mathbf{B}}_j}\right)\prod_{k=i}^{j-1}\frac{d\mathbf{B}_{k+1}}{d\mathbf{B}_k} \tag{4}$$

where $\frac{d\mathbf{B}_{j+1}}{d\mathbf{B}_j}$ is defined as the gradient for the cascaded structure from block $j+1$ back-propagated to block of j. $\frac{d\mathbf{p}_i}{d\vec{\mathbf{B}}_i}$ is the gradient of the pooled vector of block i, i.e., $\vec{\mathbf{p}}_i$, with respect to the associated output of block i, namely $\vec{\mathbf{B}}_i$.

[1]Note that the gradient $dL/d\vec{\mathbf{c}}$ can be readily derived from the standard back-propagation algorithm and can be found in the textbooks in the literature and hence is omitted.

The proof of Lemma 1 is straightforward from the chain rule in differentiation and hence is omitted. As observed in Lemma 1, each hidden block can receive gradients from its direct connection with the last fully connected layer. Interestingly, the earlier hidden blocks can even receive more gradients, as it not only receive the gradients directly from the last layer, back-propagated from the standard cascaded structure, but also those gradients back-propagated from the following hidden blocks with respect to their direct connection with the final layer. Therefore, the gradient-vanishing problem is expected to be mitigated to some extent. In this sense, the features generated in the hidden layer neurons are well exploited and relayed for classification.

It is noted that our design differs from all the reported research in the literature as it builds connections among blocks, instead of only within blocks, such as ResNet [6] and Dense-connected nets [34]. Our design is also different from the deep-supervised nets in [33] which connects every hidden layer with an independent auxiliary classifier (and not the final layer) for regularization but the parameters with these auxiliary classifiers are not used in the inference stage, hence results in inefficiency of parameters utilization. In our design, in contrast to the deep-supervised net [33], each block is allowed to connect with the last hidden layer that connects with only one final softmax layer for classification, for both the training and inference stages. All of the designed parameters are hence efficiently utilized to the most extent, especially in the inference stage, compared with [33]. It is noted that internal features are also utilized in [23] and [24]. However, in [23], these features are firstly up-sampled and summed up before fed into a tanh activation for final classification. In [24], these intermediate-level feature maps are decomposed into patches for classification at patch level for every patch before the final image-level classification. In our design, however, to reduce computation burden, we do GAP on all of these feature maps and concatenate them into a single vector for classification only at the image level with only one classifier.

Note also that by employing global average pooling (i.e., using a large kernel size for pooling) prior to the global connection in our design, the number of resulted features from all blocks is greatly reduced, which hence significantly simplifies our structure and makes the extra number of parameters brought by this design minimal. Further, this does not affect the depth of the neural network, hence has negligible impact on the overall computation overhead. It is further emphasized that, in back-propagation stage, each block can receive gradients coming from both the cascaded structure and directly from the generated 1-D vector as well, thanks to the newly added connections

between each block and the final hidden layer. Thus, the weights of the hidden layer will be better tuned, leading to higher classification performance.

It is noted that, thanks to parallel computing of GPU cores, as long as the memory space is sufficient, the computation cost of the added features from hidden layers on forward pass of GC-Net in Eqn. (1) is negligible. In addition, computation overhead for the added global average pooling layers on the hidden blocks is also minimal compared with convolution layers. For the backward pass, it is also observed that the computation cost of GC-Net is also minimal as we only need a split operator for gradients of pooled features from different blocks in Eqn. (2) as well as an addition operator to sum the hierarchical gradients and the associated GC-Net connection gradients for specific hidden blocks in Eqn. (3). Therefore, the overall computation overhead brought by GC-Net is minimal and can be safely neglected.

4. Generalized ReLU Activation

To collaborate with GC-Net, a new type of nonlinear activation function is proposed and the details are presented as follows.

4.1. Definition and Forward Phase of GReLU

As shown in Fig. 2, the Generalized Multi-Piecewise ReLU, termed GReLU, is defined as a combination of many piecewise linear functions as presented in (5) as follows.

$$y(x) = \begin{cases} l_1 + \sum_{i=1}^{n-1} k_i(l_{i+1} - l_i) + k_n(x - l_n), & \text{if } x \in [l_n, \infty); \\ \vdots & \\ l_1 + k_1(x - l_1), & \text{if } x \in [l_1, l_2); \\ x & \text{if } x \in [l_{-1}, l_1); \\ l_{-1} + k_{-1}(x - l_{-1}), & \text{if } x \in [l_{-2}, l_{-1}); \\ \vdots & \\ l_{-1} + \sum_{i=1}^{n-1} k_{-i}(l_{-(i+1)} - l_{-i}) + k_{-n}(x - l_{-n}), & \text{if } x \in (-\infty, l_{-n}). \end{cases} \tag{5}$$

As defined in (5), if the inputs fall into the center range of (l_{-1}, l_1), the slope is set to be unity and the bias is set to be zero, i.e., identity mapping is applied. Otherwise, when the inputs are larger than l_1, i.e., they fall into one of the ranges on the positive direction in $\{(l_1, l_2), \cdots, (l_{n-1}, l_n), (l_n, \infty)\}$,

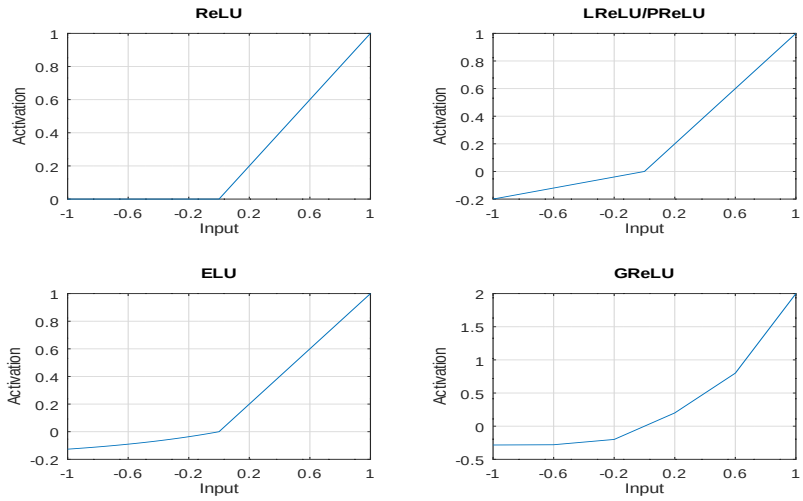

Figure 2: Description of GReLU with other popular nonlinear activation functions. In GReLU, we set the ranges of sections are $(-\infty, -0.6)$, $(-0.6, -0.2)$, $(-0.2, 0.2)$, $(0.2, 0.6)$, $(0.6, \infty)$ and the corresponding slopes for these sections are $(0.01, 0.2, 1, 1.5, 3)$, respectively.

and we assign slopes $(k_1, \cdots, k_n)$ to those ranges, respectively. The bias can then be readily calculated from the multi-piecewise linear structure of the designed function. Similarly, if the inputs fall into one of the ranges on the negative direction in $\{(l_{-1}, l_{-2}), \cdots, (l_{-(n-1)}, l_{-n}), (l_{-n}, -\infty)\}$ we assign $(l_{-1}, \cdots, l_{-(n-1)}, l_{-n})$ to all of those ranges, respectively. By doing so, the use-

ful features learned from linear mappings like convolution and fully-connected operations are hence boosted through the designed GReLU activation function.

To fully exploit the designed multi-piecewise linear activation function, both the endpoints l_i and slopes k_i ($i = -n, \cdots, -1, 1, \cdots, n$) are set to be learnable parameters, and for simplicity and computation efficiency we restrict on channel-shared learning for the designed GReLU activation functions. Further, we do not impose constraints on the leftmost and rightmost points, which are then learned freely while the training goes on.

Therefore, for each activation layer, GReLU only has $4n$ (n is the number of ranges on both directions) learnable parameters, wherein $2n$ accounts for the endpoints and another $2n$ for the slopes of the piecewise linear functions, which is definitely negligible compared with millions of parameters in current popular deep CNN models. For example, GoogleNet has 5 million parameters and 22 layers. It is evident that, with increased n, GReLU can approximate complex functions even better at the cost of extra computation resources consumed, but in practice even a small n ($n = 2$) suffices for image/video classification tasks.

4.2. Relation to Other Activation Functions

It is readily observed that GReLU is a generalization of its prior counterparts. For example, setting the slopes of all sections in the positive range to be unity (shared) and that of the sections in the negative direction another shared slope, it degenerates into leaky ReLU if the update of parameters is not allowed, and PReLU otherwise. Further, by setting the slopes of the negative side to be zero, it is degenerated into ReLU function assuming no update of parameters. In this sense, GReLU is a natural extension to these functions, while has higher potential to learn rich features and thus perform better than its counterparts.

4.3. Backward Phase of GReLU

Regarding the training of GReLU, the gradient descent algorithm for back-propagation is applied. The derivatives of the activation function with respect to the input as well as the learnable parameters are hence given in

(6)-(8) as follows.

$$\frac{\partial y(x)}{\partial x} = \begin{cases} k_n, & \text{if } x \in [l_n, \infty); \\ \vdots & \\ k_1, & \text{if } x \in [l_1, l_2); \\ 1, & \text{if } x \in [l_{-1}, l_1); \\ k_{-1}, & \text{if } x \in [l_{-2}, l_{-1}); \\ \vdots & \\ k_{-n}, & \text{if } x \in (-\infty, l_{-n}). \end{cases} \tag{6}$$

$$\frac{\partial y(x)}{\partial k_i} = \begin{cases} (l_{i+1} - l_i) I\{x > l_{i+1}\} + (x - l_i) I\{l_i < x \leq l_{i+1}\}, \\ \qquad\qquad \text{if } i \in \{1, \cdots, n-1\}; \\ (x - l_i) I\{x > l_i\}, \\ \qquad\qquad \text{if } i = n; \\ (x - l_i) I\{x \leq l_i\}, \\ \qquad\qquad \text{if } i = -n; \\ (l_{i-1} - l_i) I\{x < l_{i-1}\} + (x - l_i) I\{l_{i-1} < x \leq l_i\}, \\ \qquad\qquad \text{if } i \in \{-n+1, \cdots, -1\}. \end{cases} \tag{7}$$

$$\frac{\partial y(x)}{\partial l_i} = \begin{cases} (k_{i-1} - k_i) I\{x > l_i\}, & \text{if } i > 1; \\ (1 - k_1) I\{x > l_1\}, & \text{if } i = 1; \\ (1 - k_{-1}) I\{x <= l_{-1}\}, & \text{if } i = -1; \\ (k_{i+1} - k_i) I\{x <= l_i\}, & \text{if } i < -1. \end{cases} \tag{8}$$

where the derivative to the input is simply the slope of the associated linear mapping when the input falls in its range, and $I\{\cdot\}$ is an indication function returning unity when the event $\{\cdot\}$ happens and zero otherwise.

The back-propagation update rule for the parameters of GReLU activation function can be derived by chain rule as follows,

$$\frac{\partial L}{\partial o_i} = \sum_j \frac{\partial L}{\partial y_j} \frac{\partial y_j}{\partial o_i} \tag{9}$$

where L is the loss function, y_j is the output of the activation function, and $o_i \in \{k_i, l_i\}$ is the learnable parameters of GReLU. Note that the summation is applied in all positions and across all feature maps for the activated output of the current layer, as the parameters are channel-shared. $\frac{\partial L}{\partial y_j}$ is defined as

the derivative of the activated GReLU output back-propagated from the loss function through its upper layers. Therefore, the simple update rule for the learnable parameters of GReLU activation function is

$$o_i \leftarrow o_i - \alpha \frac{\partial L}{\partial o_i} \tag{10}$$

where α is the learning rate. The weight decay (e.g., $L2$ regularization) is not taken into account in updating these parameters.

4.4. Benefits of GReLU

From the discussion above, it is therefore found out that designing GReLU as a multi-piecewise linear function has several benefits, compared to its counterparts. One is that it enables approximation of complex functions whether they are convex functions or not, while most of activation functions however cannot. This demonstrates its stronger capability in feature learning. Further, since it only employs linear mappings in different ranges along the dimension, it inherits the advantage of the non-saturate functions, i.e., the gradient vanishing/exploding effect is mitigated to the most extent. We shall discuss its effect further in the experiment part.

4.5. Computation Complexity Analysis

Due to the fact that we have $2n$ learnable parameters for slopes and $2n$ learnable parameters for sections, it is observed from forward pass in Eqn. (5), at most $\mathcal{O}(\log(2n+1))$ time complexity is required to find the right interval for data points, while the computation cost of the linear operator (e.g., $y = a+b(x-x_0)$) is kind of complexity $\mathcal{O}(1)$ and is minimal. Regarding the backward pass in Eqn. (6)-(8), the computation cost is also bounded up by $\mathcal{O}(\log(2n+1))$ in Eqn. (6) while $\mathcal{O}(2)$ in Eqn. (7) (at most two comparison operators in indication functions) and $\mathcal{O}(1)$ in Eqn. (8) with only one comparison operator. In this sense, the computation cost of one GReLU activation is of the order of $\mathcal{O}(\log(2n+1))$ ReLU activation functions, and is negligible with small n, as the computation cost of ReLU activations in neural networks is minimal compared with other layers in neural models.

5. Experiments and Analysis

5.1. Overall Setting

The following public datasets with different scales, MNIST, CIFAR10, CIFAR100, Fashion-MNIST, SVHN, and UCF YouTube Action Video datasets,

are employed to test the proposed GC-Net and GReLU. Experiments are firstly conducted on small neural nets using the small dataset MNIST and compare the resultant performance with that by the traditional CNN schemes. Then we move to a larger CNN for performance comparison with other state-of-the-art models, such as stochastic pooling, NIN [32] and Maxout [17], for all of these datasets. Due to the complexity of GReLU, we freeze the learning of slopes and endpoints in the first few steps treating it like a Leaky ReLU function, and then starts to learn them thereafter. Our experiments are implemented in PYTORCH with one Nvidia GeForce GTX 1080 and we use ADAM optimizer to update weights for the experiments if not otherwise noted. It is also noted that in all experiments we do not apply data augmentation on these considered datasets[2] Better results are hence expected with extensive search for these parameters..

5.2. MNIST On SmallNet

The MNIST digit dataset [35] contains 70000 28×28 gray-scale images of numerical digits from 0 to 9. The dataset is divided into the training set with 60000 images and the test set with 10000 images.

In this SmallNet experiment, MNIST is used for performance comparison between our model with conventional ones, as well as study on effects of both the GC-Net as well as the new designed activation function. The proposed GReLU activated GC-Net (GC-Net-GReLU) is composed of 3 convolution layers with small 3×3 filters and only 16, 16 and 32 feature maps, respectively. The 2×2 max pooling layer with a stride of 2×2 was applied after both of the first two convolution layers. GAP is applied to the output of each convolution layer and the collected averaged features are fed as input to the softmax layer for classification. The total number of parameters amounts to be only around $8.3K$. For a fair comparison, we also examined the dataset using a 3-convolution-layer CNN with ReLU activation (C-CNN-ReLU), with 16, 16 and 36 feature maps equipped in the three convolutional layers, respectively. Therefore, both tested networks use a similar amount of parameters (if not the same).

In MNIST, neither preprocessing nor data augmentation was performed on this dataset, except we re-scale the pixel values to be within $(-1, 1)$ range.

[2]The hyperparameters used in all the experiments are only manually set due to the computation resources constraint while the slopes and end-points of GReLU are manually initialized.

Table 1: Ablation experiment.

Model	Error Rates
GC-Net + GReLU	0.78%
GC-Net + ReLU	0.90%
C-CNN + GReLU	0.86%
C-CNN + ReLU	1.7%

Table 2: Impact of learning rate.

Learning Rate	Error Rates
1.0	1.3%
0.1	1.03%
0.01	0.78%
0.001	0.85%
0.0001	1.49%

The experiment result in Fig. 3 shows that the proposed GReLU activated GC-Net achieves an error rate no larger than 0.78% compared with 1.7% by the conventional CNN, which is over 50% of improvement in accuracy, after a run of 50 epochs. It is also observed that the proposed architecture tends to converge fast, compared with its conventional counterpart. In fact, for our model, test accuracy exceeds below 1% error rate only starting from epoch 10, while the conventional net reaches similar performance only after epoch 15.

5.2.1. Ablation Study

In this part, we investigate the contribution of GC-Net architecture and GReLU activation separately in the conducted ablation study in Table. 1 with the SmallNet setup, where C-CNN denotes the conventional CNN structure. It is observed that each component of the proposed network independently contributes to the final performance improvement, validating the design of both components. This follows from the fact that, GC-Net exploits the features learned to the most extent, while GReLU boosts the learned features via rich nonlinearities and scales them up for final classification. Since the functions of both components are not quite overlapped, putting them together in the proposed framework should yield higher performance, as demonstrated in the conducted ablation experiments in Table. 1.

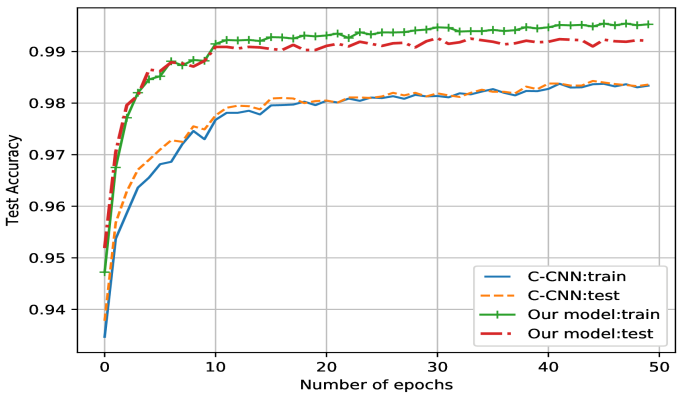

Figure 3: Comparison result of our designed model with conventional CNN (C-CNN) with ReLU activation where x-axis is the number of epochs, and y-axis is the accuracy of MNIST test set.

Table 3: Error rates on MNIST without data augmentation.

Model	No. of Param.(MB)	Error Rates
Stochastic Pooling	0.22M	0.47%
Maxout	$0.42M$	0.47%
DSN+softmax	$0.35M$	0.51%
DSN+SVM	$0.35M$	0.39%
NIN + ReLU	$0.35M$	0.47%
NIN + SReLU	$0.35M + 5.68K$	0.35%
GReLU-GC-Net	$0.078M$	**0.42**%
GReLU-GC-Net	$0.22M$	**0.27**%

5.2.2. Impact of Learning Rate

In this part, we discuss the impact of some optimizer hyperparameter, i.e., specifically the learning rate. We hence conducted experiments on GC-Net with GReLU by applying different learning rates as shown in Table. 2 with the SmallNet setup. As expected, learning rate of 1.0 is kind of too large and cannot reach 1% error rate. Instead, a learning rate of 0.01 is good enough to achieve the highest performance, since MNIST is a simple classification task. With the learning rate as small as 0.0001, the network weights might not be sufficiently updated or probably get stuck at some saddle points, and hence the performance is the worst.

5.2.3. Performance Benchmarking

We have also conducted other experiment on the MNIST dataset to further verify its performance with relatively more complex models. Different from the previous one, we kept all the schemes to achieve similar error rates while observing the required number of trained parameters. Again, we used a network with three convolutional layers by keeping all convolutional layers with 64 feature maps and 3×3 filters. The experiment results are shown in Table 3, where the proposed GC-Net with GReLU yields a similar error rate (i.e., 0.42% versus 0.47%) while taking only 25% of the total trained parameters by its counterparts. The results of the two experiments on MNIST clearly demonstrated the superiority of the proposed GReLU activated GC-Net over the traditional CNN schemes. Further, with roughly 0.20M parameters, a relatively larger network with our framework achieves the state-of-art accuracy performance, i.e., 0.27% error rate, while its benchmark counterparts, DSN, achieves 0.39% error rate with a total of 0.35M parameters.

5.3. CIFAR10

The CIFAR-10 dataset contains 60000 natural color (RGB) images with a size of 32×32 in 10 general object classes. The dataset is divided into 50000 training images and 10000 testing images. All of our experiments are implemented **without data augmentation** and we employ the same preprocessing strategy in [32]. The comparison results of the proposed GReLU activated GC-Net to the reported methods in the literature on this dataset, including stochastic pooling, maxout [17], NIN [32], are given in Table. 4. It is observed that our method achieves comparable performance while taking greatly reduced number of parameters employed in other models. Interestingly, one of our shallow model with only $0.092M$ parameters in 3 convolution layers achieves comparable performance with convolution kernel method. For the experiments with 6 convolution layers and only 0.11M parameters, our model achieves promising performance with error rate around 12.6%. With roughly 0.61M parameters, our model achieves comparable performance in contrast to Maxout with 5M parameters. Actually, compared with NIN consisting of 9 convolution layers and roughly 1M parameters, our model achieves competitive performance, only in a 6-convolution-layer shallow architecture with roughly 60% of parameters of it. These results hence well demonstrate the advantage of our proposed GReLU activated GC-Net method, which accomplishes similar performance with less parameters and a shallower structure (less convolution layers required), and hence is appropriate for memory-efficient and computation-efficient scenarios, such as mobile/embedded applications. In addition, it is observed that, our 9-convolution-layer model is able to achieve around 8.4% error rate with only around 1M parameters without data augmentation.

5.4. CIFAR100

The CIFAR-100 dataset also contain 60000 natural color (RGB) images with a size of 32×32 but in 100 general object classes. The dataset is divided into 50000 training images and 10000 testing images. Our experiments on this dataset are implemented **without data augmentation** and we employ the same preprocessing strategy in [32]. The comparison results of our model to the reported methods in the literature on this dataset, are given in Table. 5. It is observed that our method achieves comparable performance while taking greatly reduced number of parameters employed in other models. As observed in Table. 5, one of our shallow model with only $0.16M$ parameters and 3 convolution layers, achieves comparable performance with deep

Table 4: Error rates on CIFAR-10 without data augmentation.

Model	No. of Param.(MB)	Error Rates
Convolutional kernel	-	17.82%
Stochastic pooling	-	15.13%
ResNet [19] (110 layers)	$1.7M$	13.63%
ResNet [19] (1001 layers)	$10.2M$	10.56%
FractalNet[20] (21 layers)	$38.6M$	10.18%
Maxout	$> 5M$	11.68%
Prob Maxout	$> 5M$	11.35%
DSN (9 conv layers)	$0.97M$	9.78%
NIN (9 conv layers)	$0.97M$	10.41%
Ours (3 conv layers)	$0.092M$	17.23%
Ours (6 conv layers)	**0.11M**	**12.55**%
Ours (6 conv layers)	$0.61M$	10.39%
Ours (8 conv layers)	$0.91M$	**8.43**%

ResNet in [19] of 1.7M parameters. In the experiments with 6 convolution layers, it is observed that, with roughly 10% of parameters in Maxout, our model achieves higher performance. In addition, with roughly 60% of parameters of NIN, our model accomplishes competitive (or even slightly higher) performance with it, which however consists of 9 convolution layers (3 layer deeper than the compared model). This hence validates the powerful feature learning capabilities of our designed GC-Net with GReLU activations. In such way, we can achieve similar performance with shallower structure and less parameters. In addition, one of our deep model with 8 layers achieves performance on par with other state-of-art models.

5.5. Street View House Numbers (SVHN)

The SVHN Data Set contains 630420 RGB images of house numbers, collected by Google Street View. The dataset comes in two formats and we only consider the second format, with all images being of size 32×32 and the task is to classify the digit in the center of the image, however possibly some digits may appear beside it but are considered noise and ignored. This dataset is split into three subsets, i.e., extra set, training set, and test set, and each with 531131, 73257, and 26032 images, respectively, where the extra set is a less difficult set used to be extra training set. Compared with MNIST, it is a much more challenging digit dataset due to its large color and

Table 5: Error rates on CIFAR-100 without data augmentation.

Model	No. of Param.(MB)	Error Rates
ResNet [19]	$1.7M$	44.74%
Stochastic pooling	-	42.51%
Maxout	$> 5M$	38.57%
DSN	$1M$	34.57%
NIN (9 conv layers)	$1M$	35.68%
Ours (3 conv layers)	$0.16M$	44.79%
Ours (6 conv layers)	$0.62M$	35.59%
Ours (8 conv layers)	$0.95M$	**33.74**%

Table 6: Error rates on SVHN without data augmentation.

Model	No. of Param.(MB)	Error Rates
Stochastic pooling	-	2.80%
Maxout	$> 5M$	2.47%
DSN	$1.98M$	1.92%
NIN (9 conv layers)	$1.98M$	2.35%
Ours (6 conv layers)	$0.61M$	2.35%
Ours (8 conv layers)	$0.90M$	2.10%
Ours(10 conv layers)	$1.26M$	**1.96**%

illumination variations.

In SVHN, in data preprocessing, we simply re-scale the pixel values to be within $(-1, 1)$ range, identical to that imposed on MNIST. It is noted that for other methods, local contrast method is employed for data preprocessing. Even so, it is observed in Table. 6 that, our models are quite competitive compared with other benchmark models. For example, one model with only 6 convolution layers and 0.61M parameters, achieves roughly the same performance with NIN, which consists of 9 convolution layers and around 2M parameters. For a deeper model with 8 convolution layers and 0.90M parameters, we achieve competitive performance in contrast to other models, which validates the powerful feature learning capabilities of the designed architecture. Finally, for a model with 10 convolution layers and only 60% parameters of DSN and NIN, our model achieves performance on par with other state-of-art models.

Table 7: Error rates on Fashion-MNIST without data augmentation.

Model	No. of Param.(MB)	Error Rates
AlexNet		10.1%
GoogleNet	-	6.3%
ResNet18 (data augmentation)		5.1%
VGG16	$26M$	6.5%
Google AutoML (24 hour)		6.1%
Capsule Net (data augmentation)	$8M$	6.4%
Ours (8 conv layers)	$0.91M$	5.53%

5.6. Fashion-MNIST Dataset

Fashion-MNIST [38] is a dataset of Zalando's article images, which includes a training set of 60000 examples and a test set of 10000 examples. Every fashion product on Zalando has a set of pictures shot by professional photographers, demonstrating different aspects of the product, i.e. front and back looks, details, looks with model and in an outfit. Similar to MNIST dataset, each image is rescaled to be a 28×28 gray-scale image, with a label from 10 classes. Fashion-MNIST is created to serve as a direct drop-in replacement for the original MNIST dataset to benchmark machine learning algorithms, since it shares the same image size and structure of training and testing splits, however is much more challenging than the naive hand-written digits. It is also noted that our experiment on this dataset is conducted without data augmentation.

From Table. 7, it is observed that the performance of our designed network is competitive with other state-of-art models. For example, compared with capsule network, our network achieves slightly better performance but only with around 12% of parameters in capsule network. In addition, our model achieves competitive performance, even compared with a much deeper resnet18 model with data augmentation. Interestingly, we even outperformed Google AutoML with 24 hour of extensive hyperparameter tuning and neural architecture search. These comparisons hence demonstrate the effectiveness of the designed neural architecture as well as the designed GReLU activation function.

5.7. STL10

The STL-10 dataset [39] is an image recognition dataset which is initially developed to boost unsupervised feature learning, and is inspired by CIFAR-

Table 8: Error rates on STL-10 without data augmentation.

Model	No. of Param.(MB)	Error Rates
Convolutional kernel		37.68%
GoogleNet	-	37.7%
ResNet-50		37.23%
VGG16	$26M$	37.39%
Ours (8 conv layers)	$0.91M$	32.54%

10 dataset. In fact in STL-10 each class has fewer labelled training examples than that of CIFAR-10, however a large number of unlabelled images is provided for unsupervised/semi-supervised learning purposes. In our work, however, we are only interested in learning how to classify these labelled examples and the unlabelled images are neglected in this experiment. In STL-10, the samples are colored images of size 96×96. These images are from 10 classes, i.e., airplane, bird, car, cat, deer, dog, horse, monkey, ship, truck, and is in fact a resized subset of ImageNet. For supervised learning purposes, STL-10 dataset has 500 training images per class and 800 test images per class, and hence is a relatively small dataset compared with datasets from other experiments.

For STL-10, we conduct our experiment by simply training from scratch over the small supervised dataset. The experiment is conducted without data augmentation. It is however noted that, for efficient computation, we down-sample STL-10 images to be compatible with CIFAR-10 images, i.e., of size 32×32 for both experiments (which however has some negative impact on the performance).

The results are reported in Table 8. Note that for GoogleNet, resnet-50 and vgg16, these models are firstly trained on ImageNet and transferred and fine-tuned on STL-10. As is observed, even training from scratch over only the downsampled version of 5000 training images with labels, our designed model is able to achieve better performance than convolutional kernel method and other state-of-art methods.

5.8. UCF YouTube Action Video Dataset

The UCF YouTube Action Video Dataset is a popular video dataset for action recognition. It is consisted of approximately 1168 videos in total and contains 11 action categories, including: basketball shooting, biking/cycling, diving, golf swinging, horse back riding, soccer juggling, swinging, tennis

Table 9: Error rates on UCF Youtube Action Video Dataset without data augmentation.

Model	No. of Param.(MB)	Error Rates
[40]: static features	-	63.1%
[40]: motion features	-	65.4%
[40]: hybrid features	-	71.2%
Ours	-	72.6%

swinging, trampoline jumping, volleyball spiking, and walking with a dog. For each category, the videos are grouped into 25 groups with over 4 action clips in it. The video clips belonging to the same group may share some common characteristics, such as the same actor, similar background, similar viewpoint, and so on. The dataset is split into training set and test set, each with $1,291$ and 306 samples, respectively. It is noted that UCF YouTube Action Video Dataset is quite challenging due to large variations in camera motion, object appearance and pose, object scale, viewpoint, cluttered background, illumination conditions, etc. For each video in this dataset, we select 16 non-overlapping frames clips. However, due to the limitation of GPU memory, we simply resize each frame into size 36×36 and then crop the centered 32×32 for training to guarantee that a batch of 16 samples is fit for 8G memory of GTX1080. Due to the down-sampling implemented on original frames, some performance degradation is expected. Further, we only allow convolution across space within each frame but not across frames over time epochs for this video classification task, This however will definitely have some negative impact on performance, as in fact we do not try to exploit the temporal relationship of frames to the most extent. In another point of view, this implementation only needs a much smaller amount of parameters, and hence can be trained with a small budget and expected to be deployed in real-time video analysis. Even with such simple setup, our designed neural network is capable of achieving higher performance, than the benchmark method using hybrid features in [40].

6. Conclusion

In this work, we have designed an architecture, which makes full use of the hidden layer features, as well as alleviates the gradient-vanishing problem. Further, a generalized linear rectifier activation function was proposed to boost the performance. The combination of the two designs is demon-

strated to achieve state of art performance in several object recognition and video action recognition benchmark tasks, including MNIST, CIFAR-10/100, SVHN, Fashion-MNIST, STL-10 and UCF YouTube Action video datasets, with greatly reduced amount of parameters and even shallower structure. Henceforth, our design can be employed in small-scale real-time application scenarios, as it requires less parameters and shallower network structure whereas achieving matching/close performance with state-of-the-art models.

One can extend our work in several ways. Firstly, one can naturally incorporate our architecture and GReLU with other state-of-art architectures such as resnet and densenet. These architectures are independent of ours as they are more involved in the design of blocks of the deep neural networks, while ours is on the flow of intermediate layers to final classification. In this sense, one can merge these designs together to further improve performance by enjoying benefits of different works. Secondly, due to the fact that different hidden layers provide feature information at different level, diving deeper to utilize them instead of concatenation at the last layer would be of great interest to investigate and expected to boost the performance as well. Further, in this work, we do not tune the hyperparameters to optimize performance, one can fine tune these hyperparameters and observe their impact on the designed network, e.g., optimization on the initialization of GReLU parameters as well as learning rates. In addition, one can apply the designed architecture to deeper neural networks and fine tune it in other interesting computer vision tasks such as autonomous driving.

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