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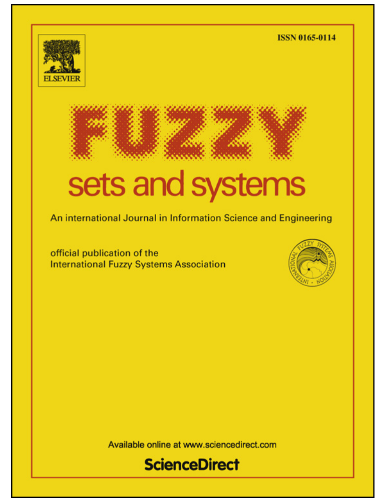

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Highlights

- A new preference structure is designed for modeling complex preferences.
- This preference structure combines fuzzy preference with levels of preference strength.
- The preference structure is incorporated into the graph model for conflict resolution.
- New definitions for graph model stability concepts of human behavior are presented.
- The flexible methodology is demonstrated using a water allocation conflict in China.

Fuzzy levels of preference strength in a graph model with multiple decision makers

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Abstract: A new hybrid preference structure combining multiple-level strength of preference and fuzzy preference is proposed to facilitate the modeling and analysis of strategic conflicts involving multiple decision makers (DMs) with complex preferences using the Graph Model for Conflict Resolution (GMCR). The new preference structure, named fuzzy strength of preference, provides a more comprehensive and flexible representation of DMs' relative preferences among states. A key contribution of this paper is to redefine four graph model stability definitions for fuzzy preferences, fuzzy Nash stability (FNash), fuzzy general metarationality (FGMR), fuzzy symmetric metarationality (FSMR), and fuzzy sequential stability (FSEQ), as general stabilities, strong stabilities, and weak stabilities at each level, permitting deeper analysis of graph models with the new preference structure. The resulting methodology can be utilized to model and analyze complex multiple-DM conflicts, thereby enhancing the capability of the graph model to provide strategic insights. A graph model of the Zhanghe River water allocation conflict in China demonstrates that the system can find the evolution path of a conflict and give new strategic insights for both practitioners and researchers. The fuzzy strength of preference framework makes GMCR more capable for addressing a wider range of practical conflicts.

Keywords: Decision analysis; Graph model for conflict resolution; Multiple levels of preference; Fuzzy preference; Stability definitions

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1. Introduction

The Graph Model for Conflict Resolution (GMCR) methodology, which was first proposed by Kilgour et al. [1] and Fang et al. [2,3], extends metagame analysis [4] and conflict analysis [5,6], and has been improved and broadened by many researchers and utilized to investigate various kinds of conflicts, such as by Yu et al. [7], He et al. [8], and Xu et al. [9], to name but a few. As a useful and flexible method to model and analyze real-world strategic conflicts, the GMCR methodology only requires decision makers' (DMs') relative preferences, which is much easier to be obtained than cardinal utility values. Moreover, the possible evolution paths and resolution of a conflict can be effectively determined for furnishing decision advice for both practitioners and researchers through conducting stability analyses and other related analyses according to DMs' relative preference information under the framework of GMCR.

As one of the key elements in both conflict modeling and analysis processes in GMCR, DMs' relative preference information can have significant influence on the stability analysis findings. Therefore, to model real-world preferences better, different types of individual preference structure have been introduced into the basic framework of GMCR. Specifically, crisp preference [3], three-level strength of preference [10,11], and multi-level strength of preference [12] are used to represent a DM's certain preferences, while unknown preference [13], fuzzy preference [14], Grey-based preference [15], hybrid preference combining three-level strength of preference and unknown preference [16], and hybrid preference combining three-level strength of preference and fuzzy preference [17] are the preference structures which can express both certain and uncertain preferences.

In some real-world conflicts, however, DMs may possess multi-level strength of preference regarding some pairs of feasible states and hold fuzzy preferences with respect to other feasible states. In such cases, the separate multi-level strength of preference structure, fuzzy preference structure, and three-level fuzzy strength of preference structure are no longer applicable. A hybrid preference structure which can represent both multi-level strength of preference and fuzzy preference should thus be developed such that a graph model under this hybrid preference structure can be established. Actually, Yu et al. [17] proposed a preference structure which can represent both three-level strength of preference and fuzzy preference, and defined a graph model under this kind of preference structure with two DMs. However, the three-level fuzzy strength of preference structure is limited in its ability to represent the intensity of relative preference. Accordingly, the main objective of this paper is to extend the three-level fuzzy strength of preference framework to a multi-level fuzzy strength of preference framework in GMCR having two or more DMs so that general conflict problems can be modeled and analyzed.

In Section 2, the existing frameworks of GMCR under crisp, strength of, and fuzzy preferences are separately reviewed. In Section 3, under the framework of GMCR with multiple DMs, the fuzzy strength of preference structure, which combines both multi-level strength of preference and fuzzy preference, is proposed. Moreover, four graph model stability definitions for fuzzy preferences which consist of fuzzy Nash stability (FR), fuzzy general metarationality (FGMR), fuzzy symmetric metarationality (FSMR), and fuzzy sequential stability (FSEQ), are extended and defined for each level as fuzzy general stability definitions, fuzzy strong stability definitions, and fuzzy weak stability definitions, such that the new hybrid preference structure can be utilized in the analysis of a conflict. In Section 4, the applicability and flexibility of the new fuzzy strength of preference framework within GMCR to investigate strategic conflicts are illustrated through modeling and analyzing a water allocation conflict in the Zhanghe River basin in China. Finally, conclusions and directions for future research are drawn in Section 5.

2. The graph model for conflict resolution (GMCR) under different preference structures

In general, a graph model for conflicts with multiple DMs can be represented by $G = \langle N, S, \{A_i\}_{i\in N}, P\rangle$, where $N = \{1, 2, \dots, i, \dots, n-1, n\}$ is the set of DMs; $S = \{s_1, s_2, \cdots, s_k, \cdots, s_t, \cdots, s_v, s_w\}$ is the set of feasible states; $D = \langle S, \{A_i\}_{i\in N}\rangle$ is the set of directed graphs, where S is the set of nodes; $A_i \subseteq S \times S$ denotes the directed arcs, representing possible state transitions by DM i; and $P = \{P_i, i \in N\}$ is the set of relative preference information on S for all of the DMs. Crisp preferences in GMCR are expressed by the binary relations "is indifferent to," $\sim_i$, and "is (strictly) preferred to," $\succ_i$. Note that $\{\succ_i, \sim_i\}$ is complete on S, which means that precisely one of $s_k \sim_i s_t$, $s_k \succ_i s_t$, or $s_t \succ_i s_k$ is true when a DM possesses crisp preference between the two states.

2.1. GMCR under strength of preference

In some real-world situations, a DM may feel strongly about differences between certain pairs of states. In such cases, more information concerning DM's preferences is available than is required for relative preferences, but not as much as is needed for cardinal preferences. Therefore, the strength of preference structure was proposed by Hamouda et al. [10,11] and Xu et al. [12] to depict the intensity of relative preference and thus can be employed in stability and sensitivity analyses under the framework of GMCR to provide highly informative strategic results.

Specifically, the binary relation $\succ_i$ was further divided by Hamouda et al. [10,11] into two binary relations $>_i$ and $\gg_i$, with the explanations that $s_k >_i s_t$ means that DM i mildly prefers state s_k to s_t, while $s_k \gg_i s_t$ means that DM i strongly prefers state s_k to s_t. Hence, the three-level strength of preference is represented by $\{\gg_i, >_i, \sim_i\}$, which was further extended by Xu et al. [12] to multi-level strength of preference $\left\{\overbrace{> \ \cdots \ >}^{r}{}_i, \ \cdots, \gg_i, >_i, \sim_i\right\}$, where $r > 0$.

The four basic solution concepts, Nash stability (Nash) [18,19], general metarationality (GMR) [4], symmetric metarationality (SMR) [4], and sequential stability (SEQ) [5,6], are usually employed to calculate stabilities within the crisp preference framework of GMCR. Under the three-level strength of preference framework of GMCR, the solution concepts of GMR, SMR, and SEQ, were further divided into strong and weak stabilities according to the strength of the possible sanctions [10,11]. Therefore, if a particular state s_k is general stable, then s_k is either strongly stable or weakly stable. Under the multi-level strength of preference structure [12], the general stabilities are constituted by stabilities at each level of preference, including the Nash solution concept.

2.2. GMCR under fuzzy preference

The concept of fuzzy sets was invented by Zadeh [20,21,22]. Subsequently, this idea and associated logic were further developed by other researchers [23-26], to name but a few, see [27] and the references contained therein for overviews. In application, fuzzy sets were widely used to express uncertain information in various decision making problems, such as the consensus reaching model involving a group of decision makers [28-30], which is focused on finding a consensus or a collective preference among different experts even there are non-cooperative behaviors. The consensus reaching model is usually used when an academic conference committee wants to select a best paper or a science foundation committee hopes to find outstanding projects to support [29], to name just a few. Note that the objective and manner to find a collective preference in the consensus reaching model is totally different from the stability analysis in the graph model. Stability in a graph model is based on the idea of moves and countermoves by the DMs. This idea of stability is strategic. In a graph model, a DM may choose to move or stay at any initial state; the order of moves is immaterial. GMCR investigates strategic conflicts in the form of strategic interactions modeled on directed graphs.

There is uncertainty or vagueness in DMs' preferences in many real-world contexts. Due to the cultural and educational backgrounds, lack of information, personal habits, or the inherent vagueness of human judgment, DMs may be unclear or uncertain about preferences between two states, and may wish to or may be able to express their preference as a fuzzy preference degree which indicates the grade or extent to which the preference for one state over the other is certain. Thus, the fuzzy preference structure was developed by Bashar et al. [14] under the framework of GMCR so as to gain strategic insights into a conflict having uncertain preference information. Note that the analysis module of the fuzzy preference framework under GMCR does not need to assume that this fuzzy preference relation satisfies any consistency properties such as those in [31-34]. This is because it only considers relative fuzzy preference information in which states are compared with respect to preference in a pairwise fashion. As a result of defining any type of preference in terms of pairwise comparisons as well as having key stability definitions based on pairwise preference comparisons, the overall GMCR methodology is deigned to work under both transitive and intransitive preference.

A fuzzy preference relation is a pairwise preference degree, which is expressed using numerical values between 0 and 1, and is interpreted as the level or degree of certainty of preferring one state to the other. Note that a fuzzy preference structure can represent both certain and uncertain preference relationships. Under the framework of GMCR, the fuzzy preference relation is formally defined as below.

Definition 1 (Fuzzy Preference): DM i's fuzzy preference information on S is a fuzzy relation over S, denoted by a matrix $\tilde{\mathcal{R}}_i = (r_{kt})_{w \times w}$, where the membership function is $\mu_{\mathcal{R}}: S \times S \longrightarrow [0, 1]$, and $\mu_{\mathcal{R}}(s_k, s_t) = r_{kt}$ represents the preference degree that state s_k is to be preferred to s_t, with the conditions that $r_{kk} = 0.5$ and $r_{kt} + r_{tk} = 1$.

The interpretations of preference degree r_{kt} are as follows:

1) $r_{kt} = 1.0$ means that state s_k is definitely preferred to s_t, which is equivalent to $s_k \succ_i s_t$;
2) $r_{kt} > 0.5$ indicates the degree of certainty that state s_k is preferred to s_t; the bigger the r_{kt}, the more certain it is that s_k is preferred to s_t;
3) $r_{kt} = 0.5$ means that it is equally likely to prefer state s_k to s_t or the other way around, which is similar to $s_k \sim_i s_t$;
4) $r_{kt} < 0.5$ indicates the degree of certainty that state s_t is preferred to s_k; the smaller the r_{kt}, the more certain it is that s_t is preferred to s_k;
5) $r_{kt} = 0$ means that state s_t is definitely preferred to s_k, which is equivalent to $s_t \succ_i s_k$.

Based on the concepts of Fuzzy Relative Strength of Preference (FRSP) and Fuzzy Satisficing Threshold (FST), Bashar et al. [14] defined the four fuzzy stability definitions comprised of FNash, FGMR, FSMR, and FSEQ. See Bashar et al. [14] for details.

3. GMCR under fuzzy strength of preference

In some real-world conflicts, DMs may have multi-level strength of preferences with respect to some pairs of feasible states, and may possess fuzzy preferences regarding other feasible states. In such cases, the separate multi-level strength of preference framework of GMCR and fuzzy preference framework of GMCR are no longer suitable. A hybrid preference structure, which is capable of dealing with both multi-level of preference strength and fuzzy preference, should be developed and then a graph model under this hybrid preference structure can be established.

3.1. Fuzzy strength of preference

Here, the fuzzy preference relations and the multi-levels of preference strength are combined. The structure of the new hybrid preferences, fuzzy strength of preferences (or fuzzy levels of preferences), is now explained.

Definition 2 (Fuzzy Strength of Preference): DM i's fuzzy strength of preference information on S is a fuzzy relation over S, denoted by a matrix $\tilde{\mathcal{R}}_i^S = (r_{kt})_{w\times w}$, where the membership function is $\mu_{\mathcal{R}}: S \times S \longrightarrow f(l) = [0,1] \cup \{2^0, 2^1, \dots, 2^l, \dots\} \cup \{1-2^0, 1-2^1, \dots, 1-2^l, \dots\}$, $l \geq 0$, and $\mu_{\mathcal{R}}(s_k, s_t) = r_{kt}$ represents the preference degree that state s_k is to be preferred to s_t, with the conditions that $r_{kk} = 0.5$ and $r_{kt} + r_{tk} = 1$.

The interpretations of preference degree r_{kt} are as follows:

1) $r_{kt} = 2^l$ $(l \geq 0)$ indicates that state s_k is preferred to s_t at fuzzy strength level $l+1$, which is equivalent to $s_k \overbrace{> \cdots >}^{l+1}_i s_t$. In particular, one could have the following cases:

   1.1) $r_{kt} = 4$ $(l \geq 2)$ means that state s_k is very strongly preferred to s_t, which is equivalent to $s_k \ggg_i s_t$;

   1.2) $r_{kt} = 2$ $(l \geq 1)$ indicates that state s_k is strongly preferred to s_t, which is the same as $s_k \gg_i s_t$;

   1.3) $r_{kt} = 1$ $(l \geq 0)$ means that state s_k is mildly preferred to s_t, which is equivalent to $s_k >_i s_t$.

2) $r_{kt} \in (0.5, 1)$ $(l \geq 0)$ indicates the degree of certainty that state s_k is preferred to s_t; the bigger the r_{kt}, the more certain it is that s_k is preferred to s_t.

3) $r_{kt} = 0.5$ $(l \geq 0)$ means that it is equally likely to prefer state s_k to s_t or the other way around, which is similar to $s_k \sim_i s_t$.

4) $r_{kt} \in (0, 0.5)$ $(l \geq 0)$ indicates the degree of certainty that state s_t is preferred to s_k; the smaller the r_{kt}, the more certain it is that s_t is preferred to s_k.

5) $r_{kt} = 1 - 2^l$ $(l \geq 0)$ indicates that state s_t is preferred to s_k at fuzzy strength level $l+1$, which is equivalent to $s_t \overbrace{> \cdots >}^{l+1}_i s_k$. Specifically, one could have the following cases:

5.1) $r_{kt} = -3$ $(l \geq 2)$ means that state s_t is very strongly preferred to s_k, which is equivalent to $s_t \ggg_i s_k$;

5.2) $r_{kt} = -1$ $(l \geq 1)$ indicates that state s_t is strongly preferred to s_k, which is the same as $s_t \gg_i s_k$;

5.3) $r_{kt} = 0$ $(l \geq 0)$ means that state s_t is mildly preferred to s_k, which is equivalent to $s_t >_i s_k$.

Taking $S = \{s_1, s_2, s_3, s_4\}$ for example, the matrix $\tilde{\mathcal{R}}_i^S$ in Table 1 describes DM i's fuzzy strength of preferences. For instance, the maximum number "$2^l = 2^3$" in the first row and second column of $\tilde{\mathcal{R}}_i^S$ means that the fuzzy strength of preference of state s_1 over s_2 for DM i is 8, which also indicates that the maximum l in this case is 3, and therefore one can say that DM i holds four levels of fuzzy strength of preferences.

Table 1 Matrix $\tilde{\mathcal{R}}_i^S$: DM i's fuzzy strength of preferences.

$$\tilde{\mathcal{R}}_i^S = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{pmatrix} 0.5 & 8 & 1 & 0.9 \\ -7 & 0.5 & 0.6 & 0.7 \\ 0 & 0.4 & 0.5 & 0.8 \\ 0.1 & 0.3 & 0.2 & 0.5 \end{pmatrix} \end{array}$$

When $l = 0$, the fuzzy strength of preference structure is actually the fuzzy preference structure proposed by Bashar et al. [14], whereas in this paper this situation is described as one-level fuzzy strength of preference. When $l = 1$, the fuzzy strength of preference structure includes both the fuzzy preference structure as well as the three-level strength of preference structure proposed by Hamouda et al. [10,11], whereas in this paper this situation is regarded as two-level fuzzy strength of preference. Analogously, for $l \geq 0$, one can say there are $l+1$ levels of fuzzy strength of preference, which means that the fuzzy strength of preference structure contains the fuzzy preference structure as well as the multi-level strength of preference structure proposed by Xu et al. [12].

Table 2 provides a clear comparison of different preference structures under GMCR mentioned above. Note that the assumption of transitivity is not required for the crisp preference, strength of preference, and fuzzy preference frameworks of the graph model. To be consistent with this novel feature of the graph model, the fuzzy strength of preference structure of GMCR is also developed such that transitivity of preference is not assumed in any stability calculations. This is because when calculating stability of a given state utilizing most of the commonly used stability definitions in GMCR states are compared only two at a time with respect to preference. From Table 2, one can see that Yu et al. [17] proposed the three-level fuzzy strength of preference structure, which is extended to the multi-level fuzzy strength of preference structure in this paper.

Table 2 Different preference structures under GMCR.

Preferences	Representations	References
Crisp preference	$\{\succ_i, \sim_i\}$	Fang et al. [3]
Strength of preference	Three levels: $\{\gg_i, \succ_i, \sim_i\}$	Hamouda et al. [10,11]
	Multiple levels: $\left\{\overbrace{\succ \cdots \succ}^{l}{}_i, \cdots, \gg_i, \succ_i, \sim_i\right\}$	Xu et al. [12]
Fuzzy preference	$\mu_{\mathcal{R}}: S \times S \longrightarrow [0, 1]$	Bashar et al. [14]
Fuzzy strength of preference	Three levels: $\mu_{\mathcal{R}}: S \times S \longrightarrow [0, 1] \cup \{-1, 2\}$	Yu et al. [17]
	Multiple levels: $\mu_{\mathcal{R}}: S \times S \longrightarrow f(l) = [0,1] \cup \{2^0, 2^1, \dots, 2^l, \dots\} \cup \{1-2^0, 1-2^1, \dots, 1-2^l, \dots\}, l \geq 0$	Proposed in this paper

3.2. Stability definitions

The concept of fuzzy relative strength of preference is proposed by Bashar et al. [14] to assess how strongly or certainly a DM prefers one state to another. For the hybrid preference structure in this paper, the fuzzy relative strength of preference is re-defined as follows:

Definition 3 (Fuzzy Relative Strength of Preference (FRSP)): For $i \in N$, and $l \geq 0$, let r_{kt}^{i} be DM i's preference degree of state s_k over s_t. Then DM i's FRSP of state s_k over s_t is $\alpha^{i}(s_k, s_t) = r_{kt}^{i} - r_{tk}^{i}$, where $\alpha^{i}(s_k, s_t) \in g(l) = \begin{cases} [-1, 1] \cup \{1 - 2^{l+1}, 2^{l+1} - 1\}, & l = 0 \\ g(l-1) \cup \{1 - 2^{l+1}, 2^{l+1} - 1\}, & l \geq 1 \end{cases}$. Denoting $\alpha^{i}(s_k, s_t) = \alpha_{kt}^{i}$, DM i's FRSP over S can be described by matrix $(\alpha_{kt}^{i})_{w \times w}$, in which $\alpha_{kt}^{i} = -\alpha_{tk}^{i}$ and $\alpha_{kk}^{i} = 0$.

In the analysis step of GMCR, one vital task is to justify whether it is better to move from the current state to some other state for a DM. Every DM with fuzzy preferences in the framework of GMCR may choose a level or several levels of FRSP to determine whether a move from one state to another is worthwhile or not. This level of FRSP is regarded as the fuzzy satisficing threshold of the DM, which is defined as below.

Definition 4 (Fuzzy Satisficing Threshold (FST)): For $i \in N$, and $l \geq 0$, DM i's FST is represented by γ_i. Then, $\alpha^{i}(s_t, s_k) \geq \gamma_i$ $(0 < \gamma_i \leq 1)$ indicates that DM i would be willing to transfer from state s_k to s_t.

Note that different DMs in a conflict may hold different FSTs to determine their own fuzzy stable states; one specific DM, for instance, may have different FSTs at different times or under different circumstances. The fuzzy levels of preference framework of GMCR are capable of handling different FSTs of DMs to predict different possible resolutions to the conflict.

Before furnishing the definitions of the solution concepts, it is necessary to introduce the unilateral movement (UM) lists and the fuzzy unilateral improvement (FUI) lists for an individual DM as well as a group of DMs. For $i \in N$, and $H \subseteq N$, let $R_i(s_k)$ and $R_H(s_k)$ denote the UM list from state s_k for DM i and the coalition H, respectively. Specifically, DM i's UM list $R_i(s_k)$ is the set of states that DM i can transfer to from state s_k in one step. Mathematically, $R_i(s_k) = \{s_t \in S: (s_k, s_t) \in A_i\}$. The coalition H's UM list is the set of states that DMs in H can transfer to from state s_k by a legal sequence of DMs in H, where a legal sequence means that a DM cannot move in succession. $R_H(s_k)$ can be constructively defined as follows in Definition 5.

Definition 5 (UM for a Coalition): For $i \in H$, and $H \subseteq N$, let $H = \{1, 2, \cdots, m\}$, define coalition H's UM set $R_H(s_k)$ inductively as follows: 1) if $s_t \in R_i(s_k)$, then $s_t \in R_H(s_k)$ and $i \in \Omega_H(s_k, s_t)$, where $\Omega_H(s_k, s_t)$ denotes all last DMs who are capable of moving from s_k to s_t in one step; and 2) if $s_t \in R_H(s_k)$, $s_v \in R_i(s_t)$, and $\Omega_H(s_k, s_t) \neq \{i\}$, then $s_v \in R_H(s_k)$ and $i \in \Omega_H(s_k, s_v)$.

The fuzzy unilateral improvement lists for an individual DM and for coalition H from state s_k, $\tilde{R}^+_{i,\gamma_i}(s_k)$ and $\tilde{R}^+_{H,\gamma_H}(s_k)$, are defined below in Definitions 6 and 7, respectively.

Definition 6 (Fuzzy Unilateral Improvement (FUI)): For $i \in N$, and $l \geq 0$, let γ_i denote DM i's FST, and $R_i(s_k)$ represent DM i's set of reachable states from state s_k. Then state $s_t \in R_i(s_k)$ is called a FUI for DM i from s_k iff $\alpha^i(s_t, s_k) \geq \gamma_i$. The FUI list for DM i from s_k, denoted by set $\tilde{R}^+_{i,\gamma_i}(s_k)$, is defined as $\tilde{R}^+_{i,\gamma_i}(s_k) = \{s_t \in R_i(s_k) : \alpha^i(s_t, s_k) \geq \gamma_i\}$.

Note that in this paper, the meaning of $\tilde{R}^+_{i,\gamma_i}(s_k)$ differs from that of Bashar et al. [14]; there, it denotes all one-level unilateral improvements from s_k by DM i, i.e., $\tilde{R}^+_{i,\gamma_i}(s_k) = \{s_t \in R_i(s_k) : \alpha^i(s_t, s_k) \geq \gamma_i, l = 0\}$, whereas here, it includes all unilateral improvements, no matter how many levels there are, i.e., $\tilde{R}^+_{i,\gamma_i}(s_k) = \{s_t \in R_i(s_k) : \alpha^i(s_t, s_k) \geq \gamma_i, l \geq 0\}$.

Definition 7 (FUI for a Coalition): For $i \in H$, and $H \subseteq N$, let $H = \{1, 2, \cdots, m\}$, and $\gamma_H = \{\gamma_1, \gamma_2, \cdots, \gamma_m\}$. Define the set $\tilde{R}^+_{H,\gamma_H}(s_k)$ inductively as follows: 1) if $s_t \in \tilde{R}^+_{i,\gamma_i}(s_k)$, then $s_t \in \tilde{R}^+_{H,\gamma_H}(s_k)$ and $i \in \tilde{\Omega}^+_{H,\gamma_H}(s_k, s_t)$, where $\tilde{\Omega}^+_{H,\gamma_H}(s_k, s_t)$ denotes all last DMs who are capable of making a fuzzy unilateral improvement from s_k to s_t in one step; and 2) if $s_t \in \tilde{R}^+_{H,\gamma_H}(s_k)$, $s_v \in \tilde{R}^+_{i,\gamma_i}(s_t)$, and $\tilde{\Omega}^+_{H,\gamma_H}(s_k, s_t) \neq \{i\}$, then $s_v \in \tilde{R}^+_{H,\gamma_H}(s_k)$ and $i \in \tilde{\Omega}^+_{H,\gamma_H}(s_k, s_v)$.

The set $(N - i)$ is one example of the coalition set H, which represents all DMs in N except for DM i. In Table 3, for instance, $R_{N-i}(s_t)$ and $\tilde{R}^+_{(N-i),\gamma_{(N-i)}}(s_t)$ denote the UM list and the FUI list for coalition $(N - i)$ from state s_t, respectively.

With the above definitions, definitions of solution concepts called as fuzzy general stabilities, fuzzy strong stabilities, and fuzzy weak stabilities are given in Table 3. For the three-level strength of preference structure, if a state is stable, then it is either strongly stable or weakly stable based on sanctioning strength. The general stabilities include stabilities at each level of preference within the multiple levels of preference structure. Under the fuzzy strength of preference structure, the fuzzy general stabilities contain stabilities at each fuzzy strength level of preference, while the fuzzy strong stabilities of Nash, GMR, SMR, and SEQ at different levels can be distinguished according to the strength level of the sanctions. Actually, the Nash, GMR, SMR, and SEQ stabilities at fuzzy strength level two or higher belong to the fuzzy strong stabilities, while others are part of the fuzzy weak stabilities.

From the definitions given in Table 3, one can see that a state is FNash stable for a DM if the DM cannot move to a preferred state; a state is FGMR for a DM if all of the DM's unilateral improvements are sanctioned by subsequent unilateral moves of others; a state is FSMR for a DM if all of the DM's unilateral improvements are still sanctioned by others, even after a possible response by the original DM; and a state is FSEQ for a DM if all of the DM's unilateral improvements are blocked by subsequent unilateral improvements of others. Moreover, if a state satisfies FNash stability, then the state must also satisfy FGMR, FSMR, and FSEQ; if a state satisfies FSMR or FSEQ, then the state must also satisfy FGMR.

Table 3 Stability definitions under fuzzy strength of preferences.

Fuzzy general stability concepts	Definitions or conditions
$FGNash$	$s_k \in S_i^{FGNash}$, iff $\tilde{R}_{i,\gamma_i}^{+}(s_k) = \phi$
$FGGMR$	$s_k \in S_i^{FGGMR}$, iff for each $s_t \in \tilde{R}_{i,\gamma_i}^{+}(s_k)$, there is at least one $s_v \in R_{N-i}(s_t)$ to make $\alpha^i(s_v, s_k) < \gamma_i$
$FGSMR$	$s_k \in S_i^{FGSMR}$, iff for each $s_t \in \tilde{R}_{i,\gamma_i}^{+}(s_k)$, there is at least one $s_v \in R_{N-i}(s_t)$ to make $\alpha^i(s_v, s_k) < \gamma_i$ and $\alpha^i(s_w, s_k) < \gamma_i$ for all $s_w \in R_i(s_v)$
$FGSEQ$	$s_k \in S_i^{FGSEQ}$, iff for each $s_t \in \tilde{R}_{i,\gamma_i}^{+}(s_k)$, there is at least one $s_v \in \tilde{R}_{(N-i),\gamma_{(N-i)}}^{+}(s_t)$ to make $\alpha^i(s_v, s_k) < \gamma_i$
Fuzzy strong stability concepts	**Definitions or conditions**
$FSNash$	$s_k \in S_i^{FSNash}$, iff $\tilde{R}_{i,\gamma_i}^{+}(s_k) = \phi$, and there is at least one $s_t \in R_i(s_k)$ to make $\alpha^i(s_t, s_k) = 1 - 2^{l+1}$, $l \geq 1$
$FSGMR$	$s_k \in S_i^{FSGMR}$, iff for each $s_t \in \tilde{R}_{i,\gamma_i}^{+}(s_k)$, there is at least one $s_v \in R_{N-i}(s_t)$ to

	make $\alpha^i(s_v, s_k) = 1 - 2^{l+1}$, $l \geq 1$
$FSSMR$	$s_k \in S_i^{FSSMR}$, iff for each $s_t \in \tilde{R}_{i,\gamma_i}^{+}(s_k)$, there is at least one $s_v \in R_{N-i}(s_t)$ to make $\alpha^i(s_v, s_k) = 1 - 2^{l+1}$ and $\alpha^i(s_w, s_k) = 1 - 2^{l+1}$ for all $s_w \in R_i(s_v)$, $l \geq 1$
$FSSEQ$	$s_k \in S_i^{FSSEQ}$, iff for each $s_t \in \tilde{R}_{i,\gamma_i}^{+}(s_k)$, there is at least one $s_v \in \tilde{R}_{(N-i),\gamma_{(N-i)}}^{+}(s_t)$ to make $\alpha^i(s_v, s_k) = 1 - 2^{l+1}$, $l \geq 1$
Fuzzy weak stability concepts	Definitions or conditions
$FWNash$	$s_k \in S_i^{FWNash}$, iff $s_k \in S_i^{FGNash}$ and $s_k \notin S_i^{FSNash}$
$FWGMR$	$s_k \in S_i^{FWGMR}$, iff $s_k \in S_i^{FGGMR}$ and $s_k \notin S_i^{FSGMR}$
$FWSMR$	$s_k \in S_i^{FWSMR}$, iff $s_k \in S_i^{FGSMR}$ and $s_k \notin S_i^{FSSMR}$
$FWSEQ$	$s_k \in S_i^{FWSEQ}$, iff $s_k \in S_i^{FGSEQ}$ and $s_k \notin S_i^{FSSEQ}$

Notice that different DMs may hold different FSTs in determining their own fuzzy stable states. For $l \geq 0$, if every DM's FST in a conflict graph model is 1, the fuzzy strength of preference GMCR framework coincides with the multiple levels of preference GMCR framework [12]. If $l = 0$, the fuzzy strength of preference GMCR framework coincides with the fuzzy preference GMCR framework [14]. If $l = 0$, and every DM's FST in the conflict graph model is 1, the fuzzy strength of preference GMCR framework coincides with the crisp preference GMCR framework [3]. If $l = 1$, and every DM's FST in the conflict graph model is 1, the fuzzy strength of preference GMCR framework coincides with the three-level strength of preference GMCR framework [10,11].

4. Real-world conflict application

Chu et al. [35] investigated a water allocation dispute in the Zhanghe River basin between one upstream and two downstream provinces in China utilizing the crisp preference framework of GMCR. The Zhanghe River is located in the southern Haihe basin, which originates in the upstream province of Shanxi and flows across the downstream provinces of Henan and Hebei. There are reservoirs in the upstream river region, and a number of canals for the irrigation purpose in the downstream of the river. Additionally, small hydropower stations were constructed along the river. Water demand in the Zhanghe River basin exceeds supply. Since the upstream province of Shanxi requires water for electricity generation and industrial development, it does not want to release more water to the downstream provinces of Henan and Hebei without compensation. Because the two downstream provinces require large volumes of water for agriculture irrigation and also a certain amount of water for industrial development, Henan and Hebei often attempted to obtain more water through taking illegal actions such as blowing up water facilities, destroying infrastructure, and physical assault. Sometimes, they jointly purchased water from Shanxi.

Water resources conflicts in the border area of the three provinces, particularly in the 108.44 km section along the Zhanghe River, arise continually. The Zhanghe River Upstream Management Bureau (ZRUMB) governs the 108.44 km section and acts as a mediator to resolve the conflicts between the upstream and the downstream provinces. Some water allocation agreements were signed among the three provinces with the involvement of ZRUMB. However, these agreements were mainly aimed at addressing a particular conflict and did not have long-term applicability. This is evidenced by the Agreement on Solving Water Disputes that was reached to resolve the conflict of building water projects in 1991; the Agreement on Abolishing Illegal and Criminal Activities and Solving Water Disputes that was reached to resolve the conflict of the destruction of water projects between Henan and Hebei in 1998; and the agreements on the particular water transfers organized by ZRUMB in 2001, 2002, 2004 and 2005 [35]. Consequently, water conflict arose again whenever a new water shortage occurred during the dry irrigation season. The Zhanghe River water conflicts follow a pattern: conflict; new agreement; conflict; new agreement; conflict; and so on.

In practice, water transfer at an appropriate price from Shanxi to Henan and Hebei was an effective method to solve the water demand–supply imbalances. As a win-win situation, water transfer in the Zhanghe River basin was successfully organized on five occasions with the facilitation of ZRUMB. Therefore, a long-term reliable water transfer system having an effective price mechanism between the upstream and downstream provinces is suggested to be constructed to solve the problem fundamentally. The above-mentioned Zhanghe River water allocation conflict is remodeled and analyzed in this article employing the fuzzy strength of preference framework of GMCR. In the conflict model established in this paper, one of the conflict patterns: conflict; new agreement, is chosen.

4.1. Conflict modeling

4.1.1. DMs and options

The southern and northern parts of the downstream Zhanghe River belong to the provinces of Henan and Hebei, respectively. Since the aims and behaviors of Henan and Hebei are similar, they are regarded as one DM in this paper. The downstream DM, together with the upstream province Shanxi and the mediator ZRUMB, constitute the three DMs or parties in the conflict model. Table 4 shows the DMs and details about each DM's options.

Table 4 DMs and options.

DMs	Options	Descriptions
DM 1 Shanxi	A_1: Retain	Use water without considering for the needs of downstream provinces
	A_2: Transfer	Transfer water to the downstream with compensation
DM 2 Henan and Hebei	B_1: Retain	Follow existing agreements
	B_2: Resist	Take illegal actions to obtain water
	B_3: Purchase	Purchase water from Shanxi
DM 3 ZRUMB	C_1: Enforce	Enforce existing agreements
	C_2: Encourage	Encourage to reach a new water transfer agreement

4.1.2. Feasible states

Theoretically, three DMs with seven options will produce 2^7 = 128 states. Practically, however, only 12 states are feasible as shown in Table 5, where "Y" and "N" imply that an option is selected by the corresponding DM or not, respectively.

Table 5 Feasible states.

DMs	Options	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}
DM 1 Shanxi	A_1: Retain	Y	Y	Y	N	N	N	Y	Y	Y	N	N	N
	A_2: Transfer	N	N	N	Y	Y	Y	N	N	N	Y	Y	Y
DM 2 Henan and Hebei	B_1: Retain	Y	N	N	Y	N	N	Y	N	N	Y	N	N
	B_2: Resist	N	Y	N	N	Y	N	N	Y	N	N	Y	N
	B_3: Purchase	N	N	Y	N	N	Y	N	N	Y	N	N	Y
DM 3 ZRUMB	C_1: Enforce	Y	Y	Y	Y	Y	Y	N	N	N	N	N	N
	C_2: Encourage	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y

4.1.3. Graph model

The circles and the directed arcs in the graph model in Fig. 1 display the feasible states and all the possible movements or state transfers of each DM in the conflict model, respectively. For instance, the dotted line with both arrows between states s_1 and s_2 indicates that DM 2 is able to transfer between the two states through changing its option selections.

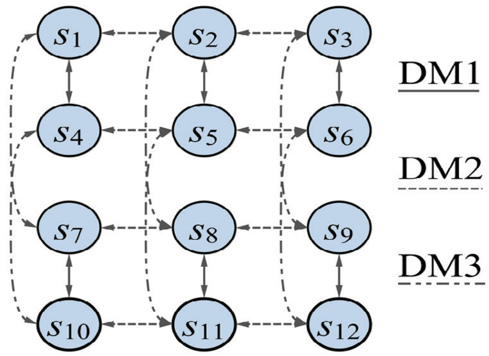

Fig. 1. Graph model.

4.1.4. Preferences

In the Zhanghe River water allocation conflict, Shanxi province (DM 1) is the key to resolving the problem. Shanxi's preference is quite complicated and is as shown in the matrix in Table 6. From Table 6, one can see that Shanxi holds fuzzy strength of preferences. Specifically, the least preferred situation for Shanxi is that in which Henan and Hebei (DM 2) chooses option B_2 (Resist). Shanxi prefers to select option A_1 and prefers that ZRUMB (DM 3) selects option C_1 when DM 2 chooses option B_1 (Retain), while Shanxi prefers to select option A_2 (Transfer) and prefers that DM 3 selects option C_2 (Encourage) when DM 2 chooses option B_2 (Resist) or B_3 (Purchase). Additionally, DM 1's preferences over states s_3, s_4, s_9, and s_{10}, and over states s_7 and s_{12}, are fuzzy; while DM 1 strongly prefers state s_1 to s_7, strongly prefers state s_{12} to states s_3, s_4, s_6, s_9, and s_{10}, strongly prefers states s_6 and s_7, to states s_3, s_4, s_9, and s_{10}, strongly prefers states s_3, s_4, s_9, and s_{10}, to states s_2, s_5, s_8, and s_{11}, very strongly prefers state s_1 to states s_3, s_4, s_9, and s_{10}, very strongly prefers states s_6, s_7, and s_{12}, to states s_2, s_5, s_8, and s_{11}, and extremely strongly prefers state s_1 to states s_2, s_5, s_8, and s_{11}. See Table 6 for details about the preference degrees of DM 1.

Compared to DM 1, the preferences of DM 2 and DM 3 are quite clear and simple. DM 2 prefers that DM 1 chooses option A_2 and DM 3 selects option C_2, while the priorities of its own options are B_3, B_1, and B_2. DM 3 prefers that DM 2 does not choose option B_2 and DM 1 selects option A_2. DM 3 also hopes that DM 2 can choose option B_3 and itself selects option C_2 to successfully transfer the water. Therefore, the crisp preferences of DM 2 and DM 3 are as shown in Table 7.

Table 6 Matrix $\tilde{\mathcal{R}}_1^S$: DM 1's fuzzy strength of preferences.

$$
\tilde{\mathcal{R}}_1^S = \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \\ s_9 \\ s_{10} \\ s_{11} \\ s_{12} \end{matrix}
\overset{\begin{matrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} & s_{11} & s_{12} \end{matrix}}{
\begin{pmatrix}
0.5 & 8 & 4 & 4 & 8 & 1 & 2 & 8 & 4 & 4 & 8 & 1 \\
-7 & 0.5 & -1 & -1 & 0 & -3 & -3 & 1 & -1 & -1 & 0 & -3 \\
-3 & 2 & 0.5 & 1 & 2 & -1 & -1 & 2 & 0.9 & 1 & 2 & -1 \\
-3 & 2 & 0 & 0.5 & 2 & -1 & -1 & 2 & 0 & 0.1 & 2 & -1 \\
-7 & 1 & -1 & -1 & 0.5 & -3 & -3 & 1 & -1 & -1 & 0 & -3 \\
0 & 4 & 2 & 2 & 4 & 0.5 & 0 & 4 & 2 & 2 & 4 & -1 \\
-1 & 4 & 2 & 2 & 4 & 1 & 0.5 & 4 & 2 & 2 & 4 & 0.8 \\
-7 & 0 & -1 & -1 & 0 & -3 & -3 & 0.5 & -1 & -1 & 0 & -3 \\
-3 & 2 & 0.1 & 1 & 2 & -1 & -1 & 2 & 0.5 & 0.6 & 2 & -1 \\
-3 & 2 & 0 & 0.9 & 2 & -1 & -1 & 2 & 0.4 & 0.5 & 2 & -1 \\
-7 & 1 & -1 & -1 & 1 & -3 & -3 & 1 & -1 & -1 & 0.5 & -3 \\
0 & 4 & 2 & 2 & 4 & 2 & 0.2 & 4 & 2 & 2 & 4 & 0.5
\end{pmatrix}}
$$

Table 7 Crisp preferences of DM 2 and DM 3.

DMs	Preferences
DM 2	$s_{12} \succ s_{10} \succ s_{11} \succ s_6 \succ s_4 \succ s_5 \succ s_9 \succ s_7 \succ s_8 \succ s_3 \succ s_1 \succ s_2$
DM 3	$s_{12} \succ s_6 \succ s_{10} \succ s_4 \succ s_9 \succ s_3 \succ s_7 \succ s_1 \succ s_{11} \succ s_5 \succ s_8 \succ s_2$

4.2. Conflict analysis

4.2.1. Stability analysis

The stability analysis of the Zhanghe River water allocation conflict is conducted through employing the stability definitions given in Table 3, with the results as shown in Table 8. In Table 8, "E" is the abbreviation of equilibrium, which indicates that the states are stable under the given fuzzy stability definition for all DMs. Moreover, "✓" indicates that under the given fuzzy general stability definition, the state in the particular row is stable for the corresponding DM or for all DMs. In particular, "✓" with superscript, such as "$✓^2$" and "$✓^3$" in Table 8, represents that under the given fuzzy strong stability definition, the state in the particular row is strongly stable for the corresponding DM at fuzzy strength level 2 and level 3, respectively. Note that from the fuzzy weak stability definitions given in Table 3, one can see that all of the stabilities indicated by "✓" without any superscript in Table 8 are the fuzzy weak stabilities. Actually, the fuzzy general stabilities are constituted by the fuzzy strong stabilities and the fuzzy weak stabilities. Since DM 1 holds fuzzy strength of preferences, and DM 1 may have different FSTs to determine the stable states, two different FSTs of DM 1 are taken into account in the analysis in this paper, and they are: 1) $\gamma_1 = 0.3$; and 2) $\gamma_1 = 0.6$. As DM 2 and DM 3 hold crisp preferences, they possess an equal FST of 1, i.e., $\gamma_2 = \gamma_3 = 1.0$.

However, under the two different groups of FSTs, the stability analysis results are indifferent, which illustrates that although DM 1's preferences over some feasible states are fuzzy, the fuzziness is weak, i.e., the fuzzy preference degrees of DM 1 are quite close to 0 or 1.

From Table 8, one can see that state s_{12} is the FGNash stable state for all DMs in the conflict, which means that it is the most likely solution for the conflict since no DM wants to move away from it unilaterally. States s_3, s_4, s_5, s_6, s_7, s_9, s_{10}, s_{11}, and s_{12} are the FGGMR and FGSMR stable states for all DMs in the conflict, which indicates that these states might be solutions for the conflict under certain circumstances. One can also notice that for DM 1, state s_1 and states s_6, s_7, and s_{12} are the FSNash stable states at fuzzy strength level 3 and level 2, respectively, while states s_3, s_4, s_6, s_7, s_9, s_{10} and s_{12} are the FSGMR and FSSMR stable states at fuzzy strength level 2, which reflect DM 1's fuzzy strength of preferences clearly. Note that although state s_1 is the most prefered or stable state for DM 1, it is not likely to be the solution for the conflict since it is not prefered by both DM 2 and DM 3.

Table 8 Stability analysis results.

FSTs	States	FGNash: FSNash, FWNash				FGGMR: FSGMR, FWGMR				FGSMR: FSSMR, FWSMR				FGSEQ: FSSEQ, FWSEQ			
		DM1	DM2	DM3	E	DM1	DM2	DM3	E	DM1	DM2	DM3	E	DM1	DM2	DM3	E
$\gamma_1 = 0.3$ $\gamma_2 = 1.0$ $\gamma_3 = 1.0$	s_1	$\checkmark^{3}$				$\checkmark^{3}$		✓		$\checkmark^{3}$		✓		$\checkmark^{3}$			
	s_2																
	s_3		✓			$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓		✓		
	s_4					$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓			✓	
	s_5	✓				✓	✓	✓	✓	✓	✓	✓	✓	✓			
	s_6	$\checkmark^{2}$	✓			$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓		
	s_7	$\checkmark^{2}$		✓		$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$		✓	
	s_8			✓			✓	✓			✓	✓				✓	
	s_9		✓	✓		$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓		✓	✓	
	s_{10}			✓		$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓			✓	
	s_{11}	✓		✓		✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	
	s_{12}	$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓
$\gamma_1 = 0.6$ $\gamma_2 = 1.0$ $\gamma_3 = 1.0$	s_1	$\checkmark^{3}$				$\checkmark^{3}$		✓		$\checkmark^{3}$		✓		$\checkmark^{3}$			
	s_2																
	s_3		✓			$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓		✓		
	s_4					$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓			✓	
	s_5	✓				✓	✓	✓	✓	✓	✓	✓	✓	✓			
	s_6	$\checkmark^{2}$	✓			$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓		
	s_7	$\checkmark^{2}$		✓		$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$		✓	
	s_8			✓			✓	✓			✓	✓				✓	
	s_9		✓	✓		$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓		✓	✓	
	s_{10}			✓		$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓			✓	
	s_{11}	✓		✓		✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	
	s_{12}	$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓	$\checkmark^{2}$	✓	✓	✓

Fig. 2 shows how the conflict could evolve from the status quo, state s_1, to the final equilibrium, state s_{12}. In the dry irrigating season, the shortage of water is so serious for the Zhanghe River basin that Shanxi is not willing to transfer more water to the downstream at this stage. As a result, the downstream provinces sometimes choose to take illegal actions to obtain more water, which makes the conflict move from state s_1 to state s_2. In such situations, as the management institution of the Zhanghe River basin, the ZRUMB prefers to solve the conflict through encouraging water transfer from the upstream to the downstream because enforcing the existing agreements is not effective anymore, as shown in the transition from state s_2 to state s_8 in Fig. 2. The downstream's behavior of taking illegal actions may have negative consequences, which are not preferred by both ZRUMB and Shanxi. Under the pressure of both the downstream and the ZRUMB, and considering that it can benefit from selling water to the downstream, Shanxi is very likely to transfer water for a reasonable price, thus, the conflict may move further from state s_8 to state s_{11}. Finally, the downstream prefers to stop taking illegal actions and start purchasing water from Shanxi for an appropriate price and the mediation of ZRUMB, and the conflict moves from state s_{11} to the equilibrium state s_{12}.

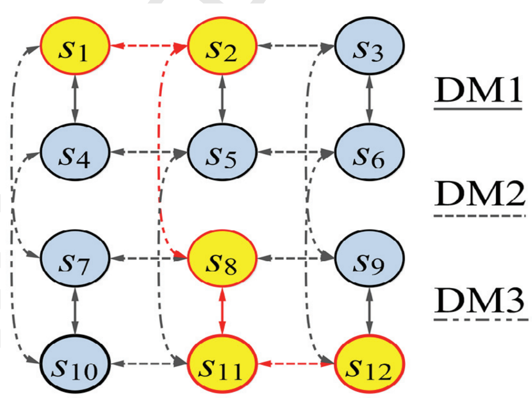

Fig. 2. The possible evolution path.

4.2.2. Discussion of results

From the above analysis of the Zhanghe River water allocation conflict, one can find that firstly, Shanxi's fuzzy strength of preference is quite clear and is well reflected in the stability analysis results. Specifically, although Shanxi very strongly prefers state s_1, Shanxi also strongly prefers state s_{12}, which means that Shanxi would like to transfer water to the downstream if it can obtain appropriate compensation. Both states s_6 and s_7 are the FSNash stable states for DM 1 means that DM 1's behavior is more influenced by DM 2 than DM 3. Additionally, there is not a very big difference for DM 1 between retaining the status quo and transferring water with compensation, because DM 1 can benefit from both situations. Therefore, if DM 2 and DM 3 can accommodate DM 1's preferences, they can facilitate the water transfer without any illegal actions taken by DM 2.

Secondly, it is difficult to reach an agreement on water transfer if the upstream and the downstream attempt to do so on their own. As the mediator with certain power to manage the Zhanghe River water resources, the ZRUMB can effectively facilitate the water transfer between the upstream and the downstream in the conflict. For instance, a reasonable price for water has a direct effect on whether Shanxi is willing to transfer water to the downstream provinces or not. However, the upstream desires a higher price but the downstream prefers to pay a lower price. An appropriate water price acceptable to both the upstream and the downstream is difficult to be set without the fair and just analysis and effort of the third-party ZRUMB. In fact, several water transfers have been implemented successfully in the past with the assistance of ZRUMB. An effective price mechanism for cross-regional water transfer in the Zhanghe River region should be established with the efforts of all parties involved.

Note that the modeling and analysis of the Zhanghe River water allocation conflict in this paper are different from those provided by Chu et al. [35]. In the conflict model established by Chu et al. [35], there are a total of four DMs as the two downstream provinces, Henan and Hebei, are regarded as two DMs though the options of the two DMs are the same and the preferences of the two DMs are very similar; four DMs with ten options result in 14 feasible states kept in the model; all DMs' preferences are simple crisp; and the stability analysis results are that the states where Shanxi chooses to transfer water, both Henan and Hebei cooperate together to purchase water from Shanxi, and the ZRUMB select to either enforce the existing agreements or facilitate the three provinces to reach a water transfer agreement, are the equilibrium states or possible solutions.

However, in this paper, there are a total of three DMs since the two downstream provinces with almost the same options and preferences are regarded as one DM in order to simplify the model; the feasible states are also further simplified to 12; Shanxi holds fuzzy strength of preference which is more consistent with the real DM in practice; the stability analysis finds out that state s_{12} (the upstream chooses "transfer", the downstream selects "purchase", while the ZRUMB take the option "encourage") is the most possible solution for the conflict, which is in accordance with the reality; Shanxi's fuzzy strength of preference is well reflected in the stability analysis results, which gives more strategic understanding of the conflict; and the simulation evolution path of the conflict accurately indicates the importance of the third party, the ZRUMB, in the mediation and solution of the conflict.

5. Conclusions

In this paper, a new fuzzy strength of preference structure containing both multiple levels of preference strength and fuzzy preference is proposed, and a new graph model under this fuzzy strength of preference is presented, which can be utilized to study real-world conflicts to make the analytical findings to be more consistent with reality, as well as to obtain strategic insights even when both multi-level strength of preference and fuzzy preference exist. Actually, the fuzzy strength of preference framework of GMCR is capable of consistently dealing with crisp preference, multi-level strength of preference, and/or fuzzy preference, which is an important function for both researchers and practitioners to find strategic insights. Specifically, the graph model under this new preference structure is capable of employing DMs' different FSTs and allowing DMs' multi-level strength of preferences over some feasible states to calculate different possible resolutions of a conflict. One key contribution of this work is that the four fuzzy stability definitions are extended for employment with stability analyses when fuzzy strength of preference is considered. Specifically, the four fuzzy stability definitions are redefined into fuzzy general stability definitions, fuzzy strong stability definitions at each level, and fuzzy weak stability definitions. The analysis of the Zhanghe River water allocation conflict demonstrates the applicability and feasibility of the fuzzy strength of preference framework of GMCR.

Note that the fuzzy strength of preference framework of GMCR does not require assuming transitivity of DMs' preferences. Generally, preference structures introduced in this paper can be used to express intransitive preferences. Based on the list of preference statements, the crisp preference [36], the fuzzy preference [37], three-level strength of preference [38], and the unknown preference [39] can be calculated, while developing an option prioritization technique that can represent the fuzzy strength of preference is one direction of future research. Additionally, the fuzzy strength of preference framework of GMCR might also be integrated with other developments under the GMCR framework, such as coalition analysis [40] and status quo analysis [41].

Declarations of interest: None.

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