A Profit Sharing Pension Plan

by

Zijing Cui

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Master of Mathematics
in
Actuarial Science

Waterloo, Ontario, Canada, 2020

Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

As Traditional Defined Benefit (DB) plans are declining, more companies are switching to Defined Contribution (DC) plans. However, DC plans have significant disadvantages since employees bear all investment and longevity risk. Hybrid pension plans, lying between DB and DC plans, are designed to meet the needs of both contributors and beneficiaries with better ways of sharing the risks. In this paper, based on the mathematical results from intergenerational risk sharing plans (Hardy *et al.* (2020)), we design a new profit sharing hybrid pension plan. We compare the solvency, contributions, and benefits between the new hybrid plan with the traditional DB plan. We find that the new hybrid design can better manage the volatility of contributions, and it can offer a guaranteed base income that isn't provided in the traditional DB plan if considering the risk of default. The new hybrid plan also offers some flexibility to balance the preference between benefit security and potential for higher income.

Acknowledgements

I would like to express my profound gratitude to my supervisor Professor Mary Hardy. Back in February 2020, I didn't have a special interest to support my research focus. It was Professor Mary Hardy who built my knowledge and shared her insights about the meaning of the Pension field. Based on that, I know that I am not only completing my thesis for graduation, but I hope my research can help the whole society be better off.

I would like to acknowledge Sijia Li, who has worked together with me for the past eight months. I gained more than I expected by listening to each other's ideas and sharing our insights.

Finally, I would like to express my special thanks to my parents and friends for their love, prayers, and caring.

Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

Traditional Defined Benefit (DB) plans are apparently declining all around the world. Many employers switched from DB plans to Defined Contribution (DC) plans, letting employees bear the risk of uncertain pensions. In the 1980s, of all private-sector wage and salary workers participating in a pension plan, 60% were in a DB plan, 17% only in a DC plan, and the remaining 23% were participating in both types of plans. By 2018, of those participating in a plan, merely 2% were solely in a DB plan, 80% were solely in a DC plan, and the remaining 18% were in both, (Rauh *et al.*, 2020).

Fewer and fewer employers choose to provide DB pensions since they prefer not to take the risk of higher costs. Rauh *et al.* (2020) show that, even though theoretically employers can hedge the interest rate risk, mortality risk, and investment risk by asset liability matching, it's hard to achieve in practice since suitable hedging instruments are not generally available.

However, there is an awareness that DC plans do not adequately meet the needs of retirees. From Cooper (2014)'s conclusions, one of the common flaws of DC plans would be excessive volatility of funds, which Cooper deems as "not fit for purpose". Plan members may experience significantly inadequate retirement incomes under unfavorable market conditions. Furthermore, few plan members have the sophisticated financial knowledge to manage their DC assets through retirement.

Since neither DC nor DB plans can fulfill the needs of both employers and employees, it's necessary to design new retirement benefit plans with better structures. Several different

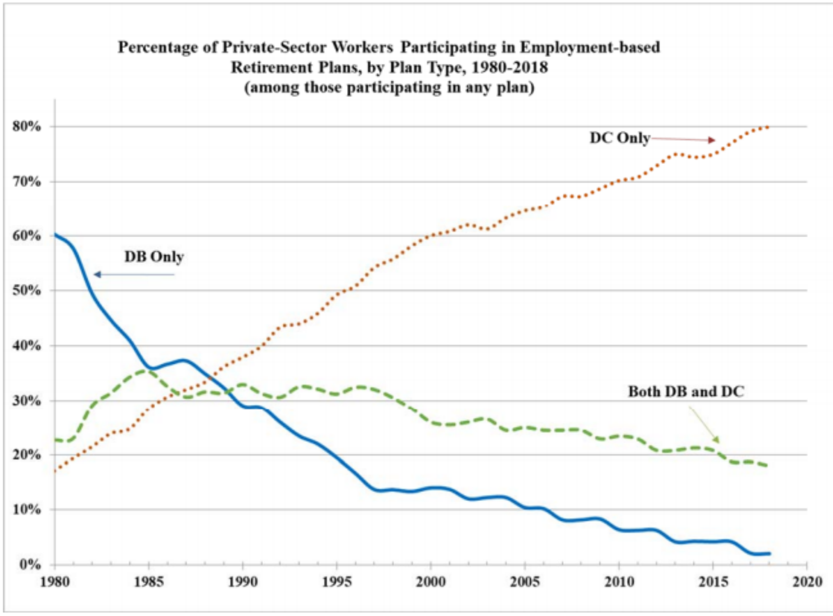


Figure 1.1: Participation in Retirement plans; Bureau of Labor Statistics, Current Population Survey, and U.S. Department of Labor data. From Rauh *et al.* (2020)

types of hybrid pension plans have been studied. One of them would be Hybrid DB-DC plans. Ponds and Riel (2007) studied them from an ALM framework. They concluded that hybrid plans can effectively minimize the risk of under-funding since the solvency risk can be controlled by two steering mechanisms.

Blommestein *et al.* (2009) compared different hybrid pension plans and evaluated their risk-sharing characteristics. Funding ratios and replacement rates were two of their key criteria. By running stochastic simulations, they concluded that the hybrid plans can be efficient and sustainable through risk-sharing. In their paper, hybrid plans are defined as plans lying between traditional DB and individual DC plans, but the plans studied are similar to DC plans. In this paper, we use similar, but more structured, criteria, and we study hybrid plans that are close to the traditional DB plan, since it has a more favorable payment structure from the members' perspective.

Zhu *et al.* (2018) studied the DB Underpin option, which provides a DC plan with a guaranteed minimum benefit based on a traditional DB formula. Zhu *et al.* (2018) considered this plan from sponsor's perspective. The study formulated the problem and presented some theoretical results, followed by some numerical results. In addition to specified contributions into the member's DC account, plan sponsors need to contribute more for the cost of the guarantee. Compared to a pure DB plan, the DB underpin shifts more risk and cost to the employer, so it might not be an attractive option for pension sponsors.

Hardy *et al.* (2020) studied Intergenerational Risk Sharing (IRS) plans, which are risk sharing, target income plans. Unlike a traditional DB plan, both workers and retirees share deficits and surpluses. When the deficit is over a set threshold, both benefits and contributions are adjusted, instead of only contributions in the traditional DB plan. It can offer a more stable income when aggregating both pre- and post- retirement periods. It's a simple and transparent design, with advantages for both contributors and beneficiaries. However, considering the fairness of the pension design, the IRS is more beneficial for younger lives at entry rather than the older.

A new hybrid pension design will be introduced in this paper based on previous research. The new hybrid pension is structured as a guaranteed traditional DB plan with an IRS plan top up. Compared with a pure IRS plan, the new hybrid plan provides a guaranteed pension benefit floor, with additional variable pension payments from the IRS plan, with deficits and surpluses shared by both workers and retirees. We use Monte Carlo simulations to evaluate solvency, contributions, and benefits from both contributor's and beneficiary's perspectives.

We evaluate the pension plan using the key criteria from Hardy *et al.* (2020):

- **Sustainability**: A sustainable plan would have the risk of high costs under a controllable manner even under unfavorable economic conditions. A plan would not be sustainable if the cost of the plan is too volatile.
- **Affordability**: If a plan is affordable, the average total contribution rate would be acceptable for both employers and active members.
- **Adequacy**: A plan provides adequate pensions if an employee can receive sufficient amount of retirement income to make a living through their retirement.
- **Efficiency**: A plan would be inefficient if it either carried too much surplus or paid more than adequate pensions, i.e. collecting too much in contribution.
- **Fairness**: We evaluate the fairness of the plan by considering the costs and benefits for different generations.

The remainder of the thesis is structured as follows. Chapter 2 introduces a model traditional DB pension system, including model set up and analysis of results. Chapter 3 develops the new hybrid pension plan, with some testing results. Chapter 4 compares the new hybrid plan and the traditional DB plan, as well as reporting on sensitivity testings on relevant parameters. Chapter 5 concludes.

Chapter 2

Model Development

A model pension system contains (1) a demographic model (2) a benefit structure to be paid (3) an asset strategy and model and (4) a funding strategy and model. A pension model is developed as an analytical tool so that more integrated decisions can be made by analyzing the relationship between the assets and the liabilities of the plan, the contribution rates of the plan members, and the real income of the members.

The model can be constructed in three sectors:

- Projection of Plan Benefit;
- Projection of Liability and Contributions;
- Projection of Assets and Returns.

A brief description of each sector will be provided below.

2.1 Notation and Assumptions

We use a time horizon of 30 years in our model. We assume that our members enter the plan between age 25 and 55, and retire at age 65. The maximum age is 105. A lump sum, equal to the actuarial value at exit of the deferred pension, is paid if death or withdrawal happens before retirement age.

2.1.1 Input Data

$L(x,0)$:	number of members at age x at time 0, $x = 25, 26, ..., 105$;
$NE(x)$:	number of new entrants at age x, $x \leq 64$;
$ny(x,0)$:	average years of services of members age x at time 0, $x \leq 64$;
$S(x,0)$:	average salary at age x at time 0, $x \leq 64$;
$b(x,0)$:	benefit in payment for $65 \leq x \leq 105$;
$p_x^{(\tau)}$:	probability that (x) is in service at age $x+1$, $x \leq 64$;
p_x:	probability that (x) survives at age $x+1$, $x \geqslant 65$;
α:	accrual rate.

2.1.2 Assumptions

We generated annual time series for equity prices, long term bonds, and inflation from Wilkie (1984), fitted to US 1951-2014 date. We use parameters from Zhang *et al.* (2018). We slightly adjusted the generated time series for the following data and assumptions:

- **Asset returns**: we define $ye(t)$ as return on equities in year $t-1$ to t and $yb(t)$ as return on 20 year long-term bonds based on the Yield to Maturity (YTM) in year t to $t+1$, generated from Wilkie ESG;
- **Inflation rates**: The experienced inflation rates, $js(t)$, are generated from the Wilkie model;
- **Salary growth rate**: We assume the salary growth rates, $gs(t)$, are 50bp higher than the inflation rate $js(t)$.

2.2 Projection of Demographics

To project the demographics, we need to consider plan membership, average salary, and accrued benefit. We define $L(x,t)$ as the number of members age x at time t. We assume all members at age 25 are new entrants. At each year end, the number of members is adjusted by the decrement rate with new entrants added:

$$L(25,t) = NE(25);$$

$$L(x,t) = L(x-1,t-1) \cdot p_{x-1}^{(\tau)} + NE(x) \text{ for } x = 26, 27, .., 65$$
$$\text{for } x \geqslant 66 \text{ , } L(x,t) = L(x-1,t-1) \cdot p_{x-1} \text{ since members retire at age 65.}$$

We then calculate the average years of service of member age x at time t. Every member already in the plan receives one more year of service in each year, and the new entrants at time t will receive a half year of service:

$$ny(x,t) = \frac{(L(x,t) - NE(x))\,(ny(x-1,t-1)+1) + \frac{1}{2}NE(x)}{L(x,t)}$$

Our next step is to project average salaries. Define $S(x,t)$ as the average salary at age x at time t, then

$$S(x,t) = S(x,t-1) \cdot (1 + gs(t)).$$

This implies that $S(x,0)$ acts as a promotional age-based salary scale.

2.2.1 Projection of Accrued Benefits

Benefits are based on a 5-year averaging Final Average Salary:

$$FAS(x,t) = \frac{\sum_{i=1}^{5}(S(\max(25, x-i), \max(0, t-i)) \cdot (1 + gs(0))^{\min(t-i,0)}}{5}$$

We define $b(x,t)$ as the average accrued benefit for age x at time t. Note that for retired members from age 66, the accrued benefit is adjusted annually for cost of living adjustment (COLA):

$$b(x,t) = \alpha \cdot ny(x,t) \cdot FAS(x,t) \text{ for } x \leq 65$$
$$b(x,t) = b(x-1,t-1) \cdot (1 + js(t)) \text{ for } x \geqslant 66$$

The total current accrued benefit for active members age x at time t is:

$$B(x,t) = b(x,t) \cdot L(x,t)$$

The total accrued benefit in payment at time t is:

$$BSUM(t) = \sum_{x \geqslant 65} B(x,t)$$

2.2.2 Projection of Liabilities

This sector performs an actuarial valuation on each future year's plan membership in order to determine the liabilities and annual contributions. We use assumptions and methods that are similar to those used in practice.

- **Actuarial assumptions**: these include decrement assumptions, which are the same as we used in projecting accrued benefits.
- **Economic assumptions**:
  Valuation interest rate $i(t)$: generated from the YTM on long-term risk free bonds, with a 3-year moving average. Typically, actuaries use YTM on corporate bonds for setting the valuation interest rate, so we assume there is a basis point spread between corporate bonds and risk free. 80 basis points are added by setting the average interest rate equal to the average return over time and scenarios, assuming a 60/40 investment split;

  Valuation inflation rate $j(t)$: used when calculating the actuarial liability. We assume $j(t)$ is changed at the same level of valuation interest rate $i(t)$ at each time, so at time t, we have: $j(t) = \min(3\%, \max(0\%, i(t) - 0.0255))$.

  Valuation salary growth rate $g(t)$: 200bp below the interest rate $i(t)$. Our salary growth rate only includes the outside effect of inflation but not inside effect such as promotions. Note that the promotional salary increase has been performed from our input data $S(x, 0)$.
- **Valuation Method**: Traditional Unit Credit (TUC) method;
- **Normal Cost method**: The method used to recognize pension costs for active members to afford service in the current year indicated by TUC methods.

2.2.3 Valuation Projection Method

We define $V(x, t)$ as the actuarial liability for plan members age x at time t, under the TUC valuation method. For active members with age below 65,

$$V(x,t) = B(x,t) \cdot {}_{65-x}\, p_x^{(\tau)} \cdot v_{i(t)}^{65-x} \cdot ar(65,t)$$

where $ar(65,t) = \sum_{k=0}^{40} {}_k p_{65} \cdot v_{i(t)}^k \cdot (1+j(t))^k$, which is the actuarial value of benefit annuity at time t for members retiring at age 65.

For retirees, $V(x,t) = B(x,t) \cdot ar(x,t)$, $x \geqslant 65$;

The Valuation Total In Force at time t is $VTIF(t) = \sum_{x \leq 64} V(x,t)$;

The Valuation Total In Payment at time t is $VTIP(t) = \sum_{x \geqslant 65} V(x,t)$

Our liability projection at time t is $VT(t) = VTIF(t) + VTIP(t)$

The Normal Cost for employees age x at time t:

$$NC(x,t) = ((1+g(t)) \cdot \frac{FAS(x+1,t)}{FAS(x,t)} \cdot \frac{ny(x,t)+1}{ny(x,t)} - 1) \cdot VT(x,t)$$

The total Normal Cost at time t is $NCSUM(t) = \sum_{x \leq 64} NC(x,t)$;

The NC rate is $\dfrac{NCSUM(t)}{\sum S(x,t)}$.

We assume 50% of the costs will be shared by employers 50% and the rest will be paid by active members.

2.3 Projection of Assets and Returns

The plan is assumed to be fully funded on the TUC valuation at the start of the projection. The assets are assumed to be invested in a mix of equities and long-term risk free bonds, rebalanced to maintain the proportions at each year end. For the benchmark results, we assume that the equity weighting is 60%, but in later sections we will allow the parameter to vary.

2.3.1 Projection method

The starting assets $Abf(0)$ would be a proportion k of our current liability. At this moment, we assume that the benefits are fully funded so that $Abf(0) = k \times VT(0)$, $k = 1$. Immediately after the valuation at $t = 0$, we receive the contribution and pay benefits:

$$A(0) = Abf(0) + NCSUM(0) - BSUM(0)$$

Withdrawal Benefit

In our plan, we assume all members can take a lump-sum distribution from the plan without paying a penalty for all kinds of early withdrawal including death. We need to deduct the withdrawal benefits from our asset.

We first determine the number of lives withdrawing at age x in year $t-1$ to t, $LW(x,t)$:

$$LW(x,t) = L(x,t) \times (1 - p_x^{(\tau)}).$$

We then define $WB(x,t)$ as withdrawal benefit of members age x withdrawing at time t, so

$$WB(x,t) = \alpha \cdot (ny(x,t) + \tfrac{1}{2}) \cdot FAS(x,t) \cdot v^{65-(x+\frac{1}{2})} \cdot a(65)$$

Total Cost of withdrawals in year t-1 to t is

$$WSUM(t) = \sum_{x \leq 64} WB(x,t) \cdot LW(x,t)$$

New Entrants Contributions

We assume a new entrant age x who enters the plan at time t pays a half previous year's normal contribution on their entry. The New Entrants' Contribution is:

$$NCNE(t) = \sum_{x} NE(x) \cdot \frac{NC(x-1,t-1)}{L(x-1,t-1)} \cdot 0.5$$

Investment Returns

We assume that we only put our assets in two kinds of investments: equities and long term bonds. The accumulated return rate at time t is

$$R(t) = w \cdot (1 + ye(t)) + (1 - w) \cdot (1 + yb(t))$$

where w is the proportion of assets invested in equities.

Let $Abf(t)$ denote the assets brought forward at time t before paying benefits and receiving contributions:

$$Abf(t) = A(t-1) \cdot R(t) - WSUM(t) \cdot R(t)^{\frac{1}{2}} + NCNE(t) \cdot R(t)^{\frac{1}{2}}.$$

At time t, we receive the Normal Contribution and pay the benefits. Both Normal Contributions $NCSUM(t)$ and Benefits $BSUM(t)$ are assumed to be paid at the start of the year. Then the value of assets at time t, after receiving Normal Contribution and paying benefits, is

$$A(t) = Abf(t) - BSUM(t) + NCSUM(t)$$

2.4 Results

2.4.1 Asset/Liability rate

To analyze the model, we first show the Asset-Liability ratio (A/L): $AL(t) = Abf(t)/VT(t)$ at each time. We construct a matrix of A/L ratios for each projection, for each of the 1000 individual paths, and plot 5%, 25%, 50%, 75%, and 95% quantiles for the metrics at each year end. In addition, we plot 30 random individual paths. These are shown in Figure 2.1.

In Figure 2.1, we can see all the quantiles except 95% are declining as time goes on. The median at all times is below 1.0 and drops to about 0.75 to the end; the worst 5% cases decline to 0 at the end. This indicates that the Normal Contribution is not sufficient to fund the liability.

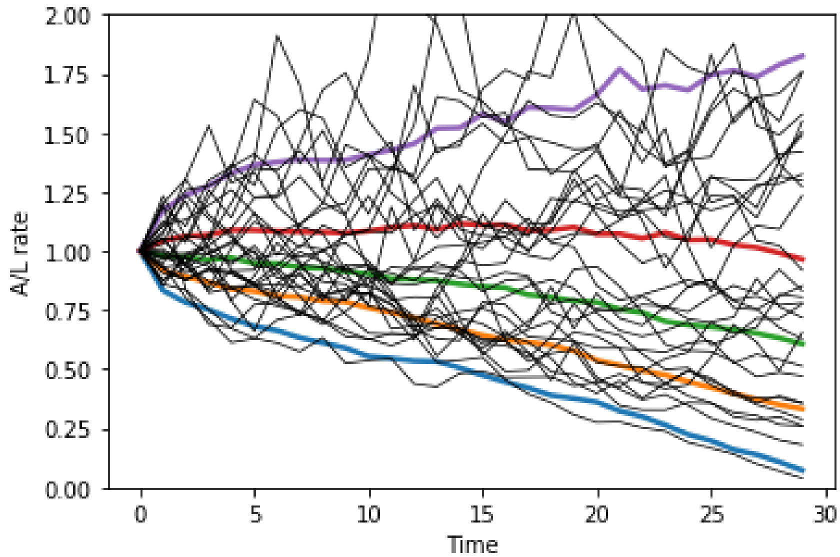


Figure 2.1: Traditional DB Asset/Liability rate; investing 60% in equities. 1000 projections, 5%, 25%, 50%, 75%, 95% quantiles, with 30 sample paths

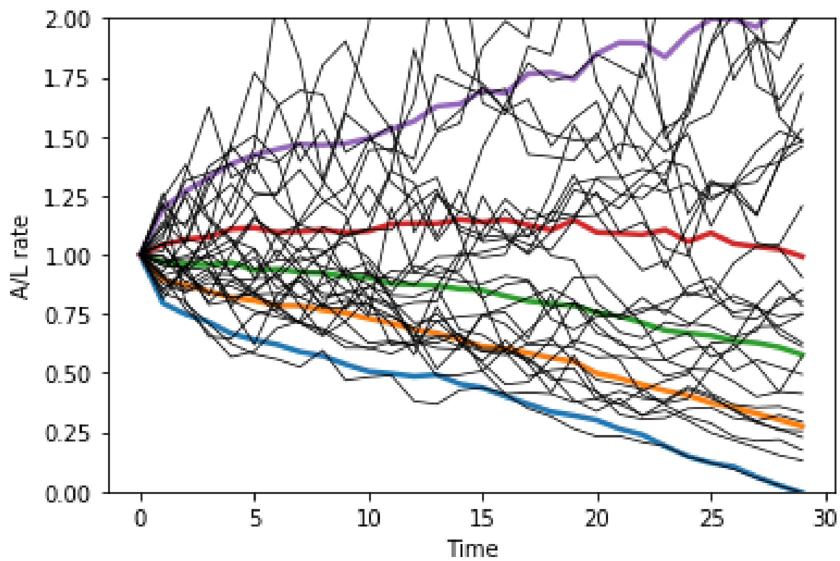


Figure 2.2: Traditional DB Asset/Liability rate; investing 70% in equities. 1000 projections, 5%, 25%, 50%, 75%, 95% quantiles, with 30 sample paths

To fix the problem of deficit, we can either increase the contributions or increase the asset returns. We first try to see if changing the weight on equities solves the issue. Instead of investing 60% of our assets in equities, we invest 70% in equities and 30% in long-term bonds. From Figure 2.2, we can see that investing more in equities stretches out the quantiles plot. The 95% quantile increases to about 1.75. The 5% quantile becomes negative after 25 years, indicating that the plan has run out assets. The individual paths become more volatile. The highest A/L in the paths shown is more than 2.5. However, all the percentiles except 95% are still declining, which means that changing the weights is not able to solve the problem.

In practice additional contributions would make up the deficit. We assume the additional contribution is 10% of any deficit in each year. That represents approximately a 10-year window to recover from deficit. Also if the TUC A/L ratio is greater than 1.2, 20% of the surplus is released as a contribution reduction to avoid gathering too many assets, with a minimum contribution of 0%.

In Figure 2.3, We plot the key features of the A/L ratios again with additional contribution added. We can see that we still have a substantial chance of deficit, but it's not as high as before. The median stays flat at about 1.0 all the time. The 95% percentile remains lower than 1.5 and the 5 percentile doesn't go below 0.75. By definition, we can conclude that the pension plan is reasonably well-controlled.

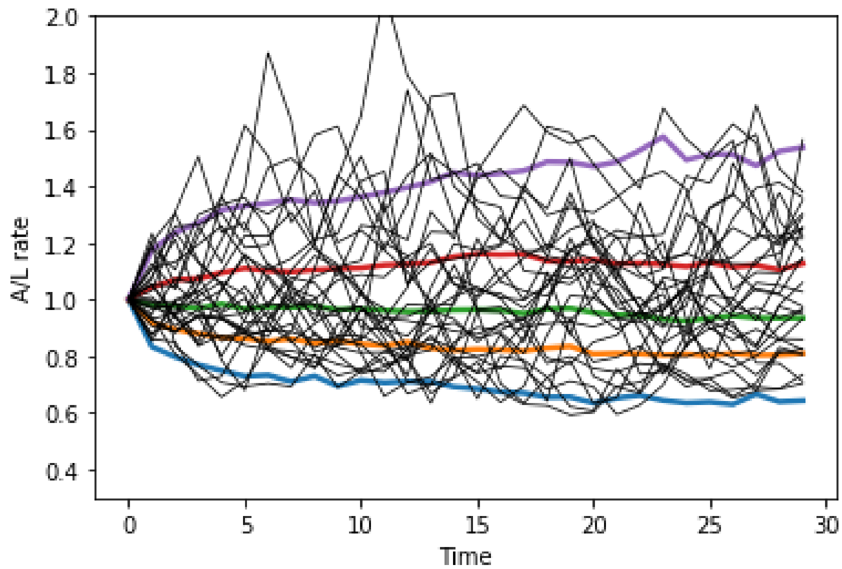


Figure 2.3: Traditional DB plan results. A/L rates including additional contributions. 1000 projections, 5%, 25%, 50%, 75%, 95% quantiles, with the same 30 sample paths

2.4.2 Total Contribution Rates

The Total Contribution rate is the ratio of total contribution to total salaries. We have plotted quantiles and 30 sample paths in Figure 2.4. We see that the 95% quantile of the total contribution is relatively high, at about 40%. From the individual paths we can observe the volatility of total contribution rates is high, which indicates a problem with sustainability, fairness and efficiency:

- The Tradition DB plan may not be sustainable since the sponsors and active workers may not be willing to pay such high contributions;
- The plan is also not fair, since some actives pay 40% contribution, and some pay 0% due to contribution reduction based on market performance, but they are receiving the same benefits;
- The Traditional DB plan has a problem of inefficiency since contributors are over-paying in some years followed by underpaying in other years.

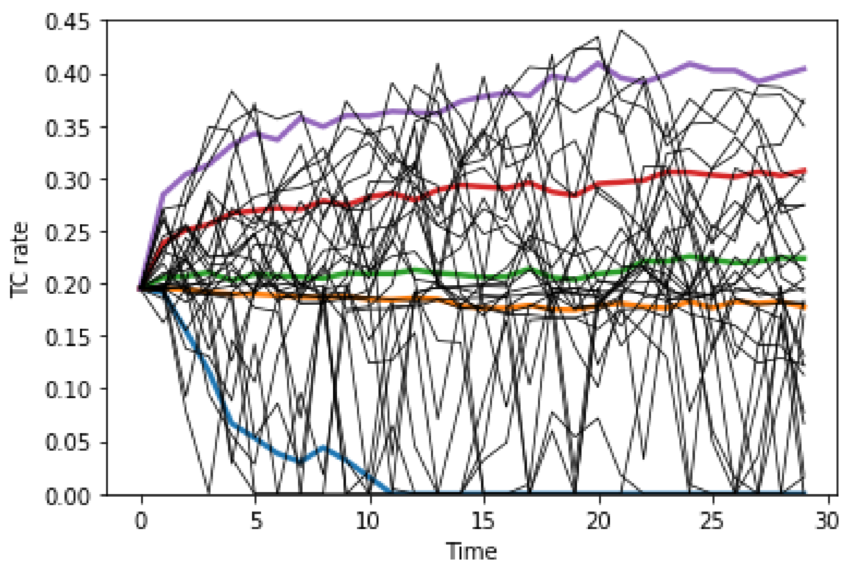


Figure 2.4: Traditional DB plan results. Total Contribution rates including additional contributions(employers and active members each pay 50%). 60% in equities. 1000 projections, 5%, 25%, 50%, 75%, 95% quantiles, with the same 30 sample paths

Chapter 3

A Hybrid Pension Plan

From the previous chapter we can see the existing problems of the Traditional DB plan: the volatility of contribution costs is too high, which results in unaffordable total contribution rates under unfavorable economic conditions. Increasing the equity weighting brings down the average costs, but increases the contribution volatility. Since the existing pension structures are not able to meet the criteria of sustainability and fairness, a new hybrid pension design will be proposed to provide better solutions for existing problems.

We use three key measures for evaluating the characteristics of the pension plan:

- the funding ratio (ratio of assets to liability), indicating the solvency risk which relates to sustainability and adequacy, efficiency of paying pension benefits and the risk of paying additional contributions;
- the total contribution rate, representing the affordability and sustainability of the pension plan;
- the replacement ratio (ratio of benefits to salary), representing the adequacy and efficiency of the benefits.

A better structured pension design will be worth investigating in consideration from different perspectives. Employers will benefit if they have a lower chance of paying substantial additional contributions. Active members would like to see the amount of additional contributions reduced, and they also would like to be confident that their own future benefits are secured. Retirees would like to receive adequate and secure pension benefits.

The new hybrid pension plan includes two separate accounts: a basis account, which will fund an accrual rate of 1.2% as a guaranteed base of income to make a living, and a profit sharing account that tops up the pension income. The following sections will introduce the design of the basis account and the profit sharing account separately. Then we will merge the two accounts together and compare it with the traditional DB plan.

3.1 Basis Account

The Basis Account (BA) is invested using a more conservative investment strategy, so that some proportion of the pension benefit would be secured with little risk. The account is set up in the following way:

- The assets are assumed to be invested 20% in equities and 80% in long-term risk free bonds. Hardy *et al.* (2020) found that the optimal equity weighting for solvency is around 20% to 30%, so our assumption of 20% seems appropriate;
- The Basis Account pays a 1.2% accrual rate, based on the average of the final five years' salary;
- Employers and Employees equally share the Normal Contribution plus 10% of any deficit from the valuation. If $Abf(t) \geqslant 1.2VT(t)$, the 20% of the excess will be released as a contribution reduction, equally shared between employers and employees, with a minimum contribution of 0%;
- The valuation interest rate for the Basis Account is the YTM on long-term bonds with an addition of 15bp, and all other assumptions are the same as the Traditional DB plan in Chapter 2.

In Figure 3.1, we show the quantiles and sample paths for BA's A/L ratios. We note that the BA has the same set up as the traditional DB plan, except for the different asset weights on equities. We can see that BA has a much better control of surplus and deficit, compared with the Traditional DB plan, since it has less investment risk. The 95% quantile stays lower than 1.2, indicating that the BA is paying benefits more efficiently. The 5% quantile is important since the basis account guarantees that the retirees are receiving promised amount of pension benefits, so it should have little risk of default. From the figure we can see that the down side of the asset-liability rato is better controlled since the lowest 5% quantile of the asset-liability ratio is higher than 80%.

Figures 3.2 and 3.3 show the total contribution rates (TCR). Figure 3.2 shows the quantiles and sample paths for the basis account contribution rates, and in Figure 3.3, we overlay the quantile plots of the basis account and a traditional DB plan with the same accrual rate (1.2%) on the same graph. The only difference between the plans is the equity weighting: 20% for the BA and 60% for the traditional DB.

The contribution rates for the BA are much better controlled. Sustainability relates to the 95% quantile for TCR (lower is better). We can observe that the BA 95% TCR quantile is a lot lower than the traditional DB plan's. However, the BA median contribution rate is more expensive since it invests less in equity, which will generate less returns. Fairness relates to the 5% quantile for TCR. The plan is more fair if the 5% quantile TCR is closer to the median and other quantiles. We can observe that the 5% quantile of the contribution rate for Traditional DB plan reaches to 0 at most times, but the BA's 5% TCR quantile is closer to other quantiles. We can conclude that compared to the traditional DB plan, the basis account is more sustainable and more fair.

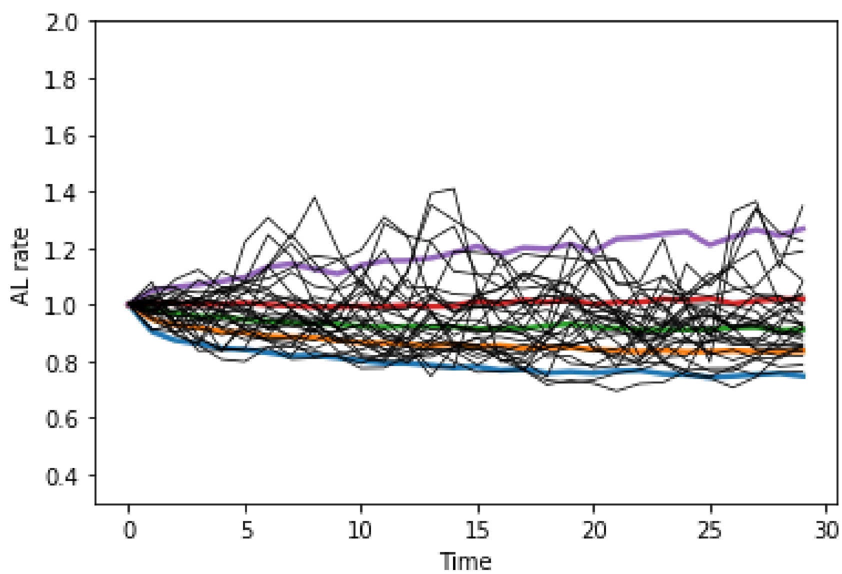


Figure 3.1: Hybrid Plan Basis Account results: Asset/Liability rate. 1000 projections, 5%, 25%, 50%, 75%, 95% quantiles, with the same 30 sample paths as in Figure 2

The Basis Account in the Hybrid plan will provide about 2/3 of the pension payable in the Traditional DB plan in Chapter 2. Since we choose a more conservative investment

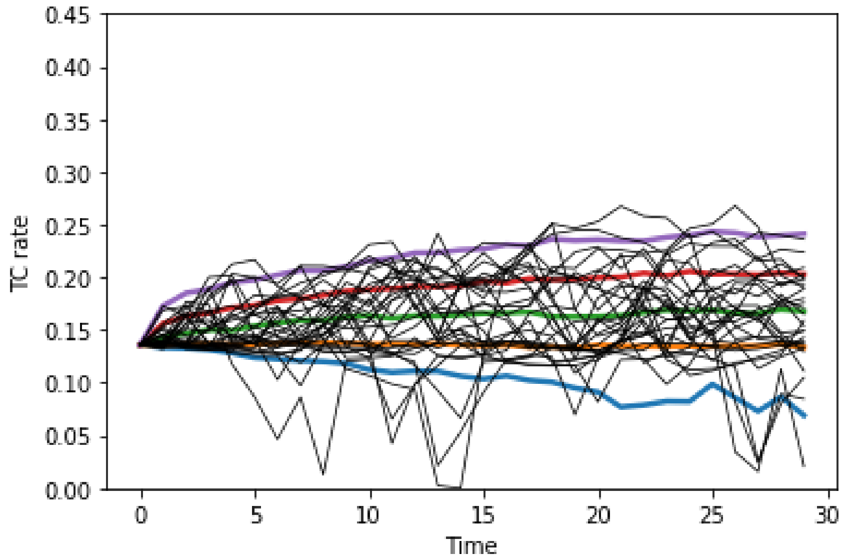


Figure 3.2: Hybrid Plan Basis Account results: Total Contribution rate. 1000 projections, 5%, 25%, 50%, 75%, 95% quantiles, with the same 30 sample paths as in Figure 2

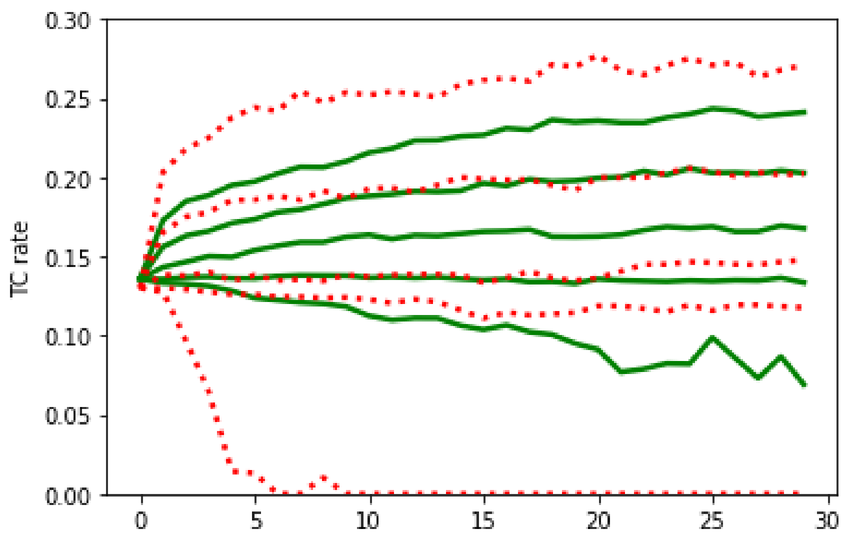


Figure 3.3: Hybrid Plan Basis Account (solid) vs Traditional DB plan (dotted line): Total contribution rates. Same accrual rates. 1000 projections, 5%, 25%, 50%, 75%, 95% quantiles

strategy, the plan has a much lower risk of default so that we can see it as both affordable and sustainable. The build-up of Basis Account serves to secure some benefit for retirees. Next we add a profit sharing account which supplements the guaranteed base pensions from the BA.

3.2 The Profit Sharing Account

With part of the benefit secured by the basis account, we include a profit sharing (PS) account to supplement the guaranteed income. The weight on equities is increased to obtain chances of higher returns. Similar to an IRS plan, both contributions and benefits are adjusted for surpluses and deficits.

Pension paying with Additional profit

The benefit structure of the profit sharing account is similar to the traditional DB plan in Chapter 2. It still pays life annuities after retirement, except the payment amount will be more variable, based on investment returns.

This profit sharing account is built in the following way:

- We invest 80% of assets in equities and 20% in long-term risk free bonds;
- The profit sharing account pays a target of 0.6% accrual rate benefit, based on the average of the final five years' salary. This will bring a total of 1.8% accrual rate when combined with the BA, which equals the accrual rate in the traditional DB plan in Chapter 2;
- If the plan is in deficit, we assume that 10% of the accrued benefit for the PS plan will be deducted. For pensions in payment, this is a real deduction. It does not directly affect the actives in the plan, but does affect the liability value;
- The aggregate total contribution is equal to the Normal Contribution plus 5% of any deficit from the valuation if the asset-liability ratio is lower than 0.8, which will be equally shared between employers and employees;
- If $Abf(t) \geqslant 1.1VT(t)$, 20% of the excess will be shared in proportion to the liabilities for each age group in the plan, including both workers and retirees. We removed the

cap on contribution reduction, which indicates that we allow negative contribution rates in the profit sharing account, but this is adjusted in a later section.

- The values used – 10% benefit deduction rate, 5% additional contribution rate, and 0.8 A/L threshold – are all illustrative and based on numerical exploration.

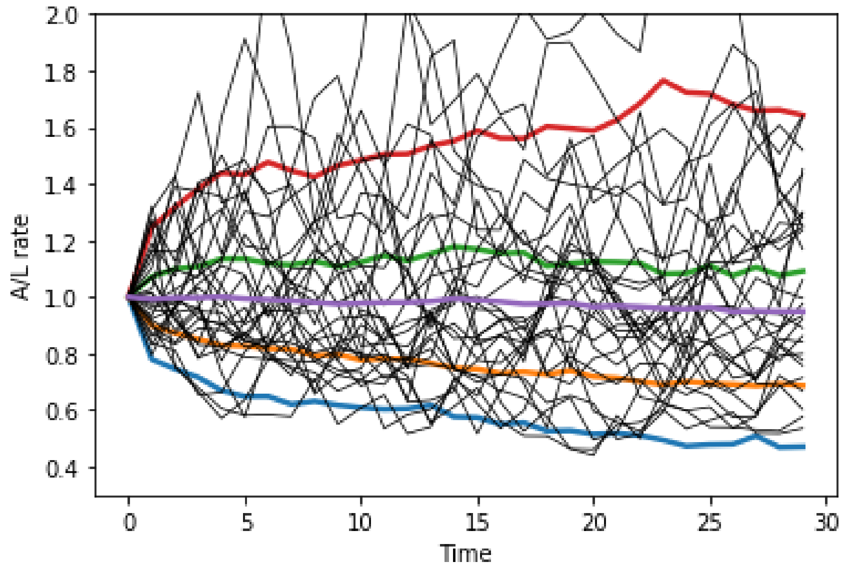


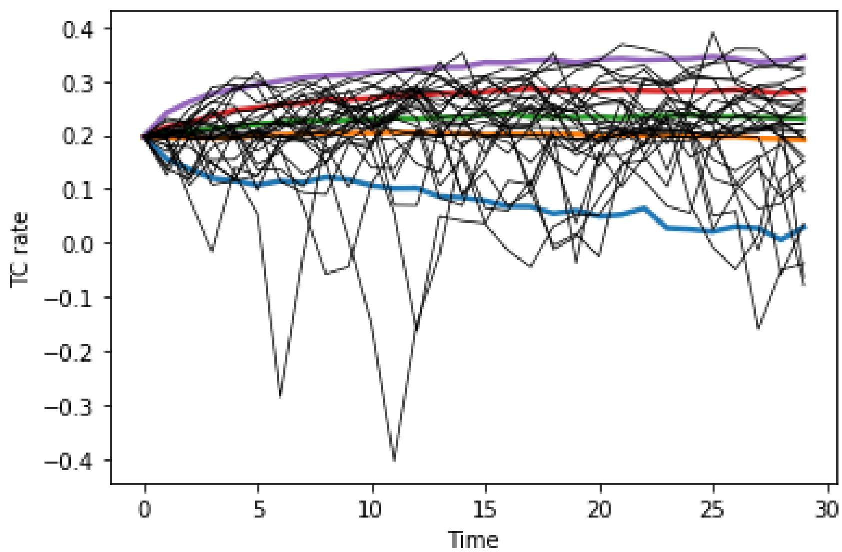
Figure 3.4: Hybrid Plan Profit Sharing Account results: Asset/Liability rates. 1000 projections, 5%, 25%, 50%, 75%, 95% quantiles, with the same 30 sample paths as in Chapter 2

In Figure 3.4, We show the quantiles and the same sample paths for the A/L ratios of the Profit Sharing account. We can see that the account has funding adequacy since the median is steady at 1.0 all the time. Even though we allow negative contribution rate during surplus, the 95% quantile almost reaches 1.4 since we put more weight on asset in equities. There exists several sample projections generating much higher returns than Traditional DB plan, but the lowest 5% quantile almost reaches 0.6, which indicates that the plan is experiencing higher risk at the same time.

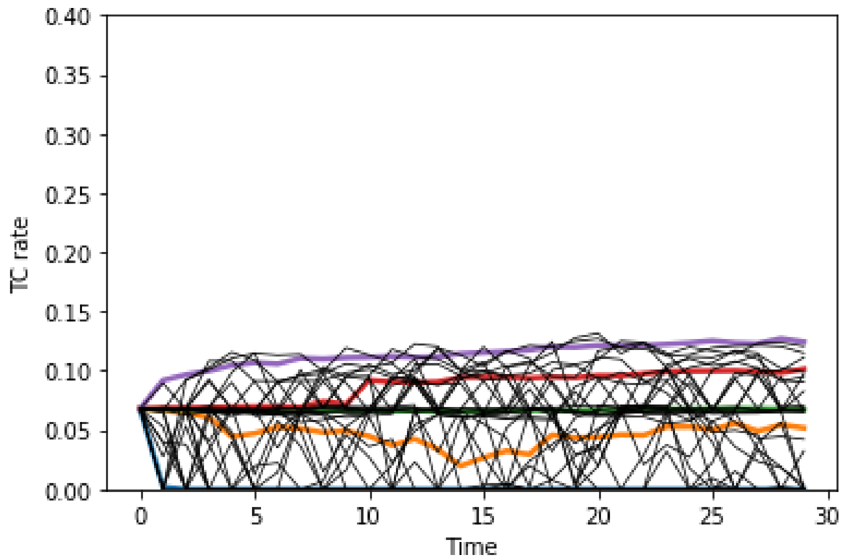
In Figure 3.5(a), we can observe that the total contribution rate plot is more beneficial for the contributors. The 95% quantile stays lower than 10% since we both cut the pension payments and add the contributions at a smaller amount when the plan is in deficit, so the risk of the contribution rate is well-controlled as well. However, since we allow negative contribution rates when sharing the 20% of surplus, the lowest 5% quantile goes lower than -5%, and we do have as low as -40% total contribution rate of sample projections when the assets perform really well. The windfall investment return makes the pension plan not fair, for example, for the plan members who retire before participating. The negative contribution rate is also not practical in reality due to the tax implications.

As a result, we add back the constraint of 0% minimum contribution rates. In Figure 3.5(b), we can see that the 95% quantile is still about 10%, and the lowest 5% are floored at 0%. This avoids the circumstances of paying large amounts of profits when returns are extremely high.

The profit sharing account serves to supplement the guaranteed income. In next section, we will merge the profit sharing account with the Basis account as a new Hybrid Pension plan and compare it with the Traditional DB plan in Section 2.



(a) TC rate: no constraint



(b) TC rate: 0% minimum constraint

Figure 3.5: Hybrid Plan Profit Sharing Account results: Total Contribution rates (a) no constraint, and (b) with constraint of 0% minimum contribution rates. 1000 projections, 5%, 25%, 50%, 75%, 95% quantiles, with the same 30 sample paths as in Chapter 2

Chapter 4

Hybrid Plan Results

In this section, we merge the Basis Account and the Profit Sharing Account together as a Hybrid Pension Plan, and we compare the results with the Traditional DB plan from Chapter 2.

4.1 Results

4.1.1 Asset-Liability ratio

The asset-liability ratio is important since it represents the solvency and funding adequacy of the pension plan. In real life, the DB benefits are not secured by current pension regulation since underfunding is allowed (Davis, 2011). We would like to see the A/L ratio stay close to 1.0. If the ratio is really low, the plan might default. On the other hand, having a high A/L ratio means we have collected more assets than we need, which shows that we are not delivering benefits efficiently.

In Figure 4.1, we show the the quantiles and sample paths for the Hybrid Pension A/L ratio. We can see that overall the A/L ratio is well-controlled. The median is at about 1.0 as time goes on, and the 95% quantile and 5% quantile stay between 1.4 and 0.8. There are some individual paths generating really high returns, but the individual paths at most times fit within the key features, which indicates that the volatility of A/L ratio is not high.

Comparing the quantile plots of Hybrid plan and the Traditional DB plan A/L's in Figures 4.1 and 4.2, we find that the A/L quantiles of the Hybrid Plan are better controlled than the Traditional DB plan. The 95% quantile of the Hybrid Plan is a lot lower than the Traditional DB plan. It represents that the hybrid plan is much more efficient in paying pension benefits. The median of both the hybrid pension plan and the Traditional DB plan are close to 1.0, which means that both plans are adequately by funded. The 5% quantile of the Hybrid pension plan is slightly higher than Traditional DB plan, which indicates that the hybrid plan is less risky.

We note that the quantile plot might not be the best way to display the results, since we merge the A/L of the hybrid plan, as

$$\frac{Abf_{Basis}(t) + Abf_{Prof}(t)}{VT_{Basis}(t) + VT_{Prof}(t)}.$$

However, the liability in the Basis Account is guaranteed, where the Profit Sharing Account, we only have a target liability.

To better illustrate this, in Figure 4.3 we show 6 individual paths of A/L rates in the Basis Account, Profit Sharing Account, hybrid plan, and Traditional DB plan. We observe that the A/L rates in the BA, displayed by the red solid line, stay close to 1.0 in all 6 graphs. After year 15 in the third graph and the last 5 years of the fourth graph, we can see that the economic scenarios are unfavorable, so the other two lines go down to 0.4 and 0.6. However, the A/L rates in the BA don't fall below 0.8. We can conclude that the volatility of the BA A/L rate is low, which meets the needs of providing guaranteed benefits. On the other hand, A/L rates in PS account (orange solid lines) are very volatile due to the large weight on equities. The PS can generate lots of returns in the first and fifth graphs under favorable economic scenarios, but in the third or the fourth graph, it will pay no benefits for a while but still collects contributions.

We can observe that the A/L rates of the Traditional DB plan (green slashed lines) and the Hybrid plan (blue solid lines) have similar shapes, but the hybrid plan's A/L rates are less volatile in all cases. Even though the Profit sharing account in Hybrid plan is more risky than traditional DB plan, we invest only about $\frac{1}{3}$ of our assets in the PS account, so that in total, only around 40% asset are invested in equities in the Hybrid plan, compared with 60% in the Traditional DB plan. We discuss it further in Section 4.2.1.

By comparing the A/L rates in the Hybrid and traditional DB plans, we conclude that the splitting between BA and PS in hybrid plan provides a stable guaranteed base payment that is more secure than the pension benefits in the traditional DB plan.

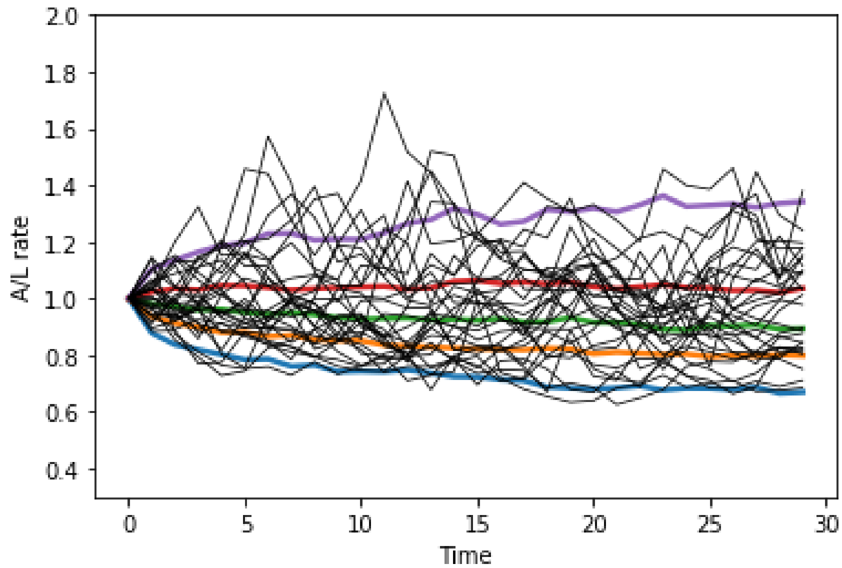


Figure 4.1: Hybrid Plan: Asset-Liability rate results. 1000 projections, 5%, 25%, 50%, 75%, 95% quantiles, with the same 30 sample paths

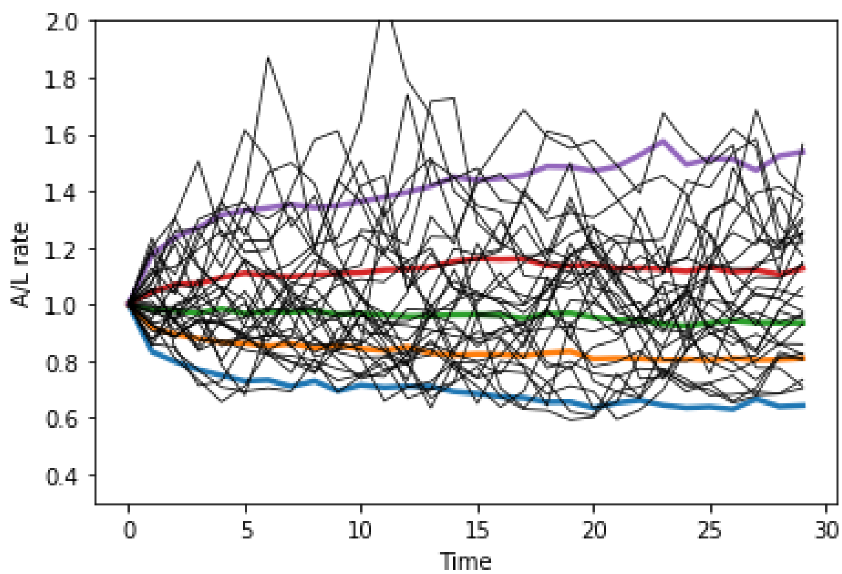


Figure 4.2: Traditional DB plan: Asset-Liability rate results. 1000 projections, 5%, 25%, 50%, 75%, 95% quantiles, with the same 30 sample paths

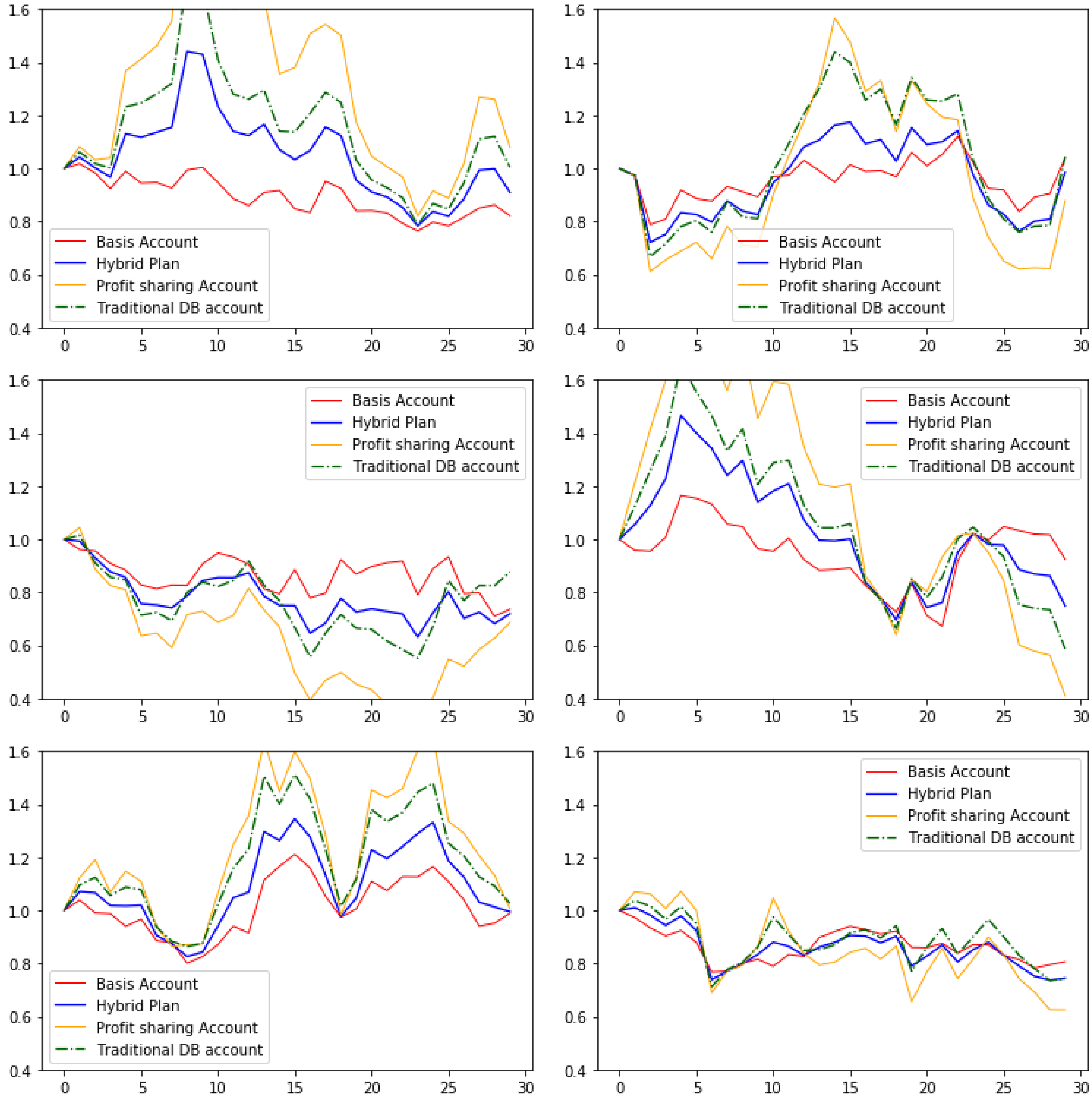


Figure 4.3: A/L rates: Traditional DB plan, Basis Account, Profit Sharing Account, and hybrid plan. Same Economic Scenarios in each graph

4.1.2 Total Contribution Rate

The Total Contribution rate is another important measure since it reflects the sustainability and affordability of the pension plan. If the 95% quantile of the total contribution rate is high, it means that the plan has risk of higher costs. For many companies, the risks of high contribution costs may dominate those of their own business (Clark and Monk, 2006). It is one of the main reasons that fewer companies are willing to sponsor traditional DB plan.

In Figure 4.4, we plot the contribution rate quantiles and sample paths of hybrid plan. We observe that the median of the total contribution rates for the hybrid plan slightly increase to about 22% at the end. The 95% quantile stays about 30%. The 5% quantile stays at about 15% for the first 15 years and then decreases.

In Figure 4.5, we compare the total contribution rates of the Hybrid pension plan and the Traditional DB plan in the same graph. We can observe that both 95% and 75% quantiles of the Hybrid pension plan are lower than the Traditional DB plan, and more obviously for the 95%. We can see that the risk of high costs is significantly reduced for the Hybrid pension plan. However, the median contribution rate of the Hybrid Plan is higher. This is understandable, since we invest less in equity for the Hybrid plan, which means that our assets are not able to generate as high return as in the Traditional DB plan. Comparing the 25% and 5% quantiles, the Hybrid plan shows fewer chances of 0% contribution rates, which illustrates that the Hybrid pension is more fair than the Traditional DB plan. Overall, both the Hybrid plan and the Traditional DB plan are affordable, but the Hybrid pension plan is more expensive, which can be seen as a trade-off of guaranteeing some proportion of benefits. The risk of contribution cost is better controlled in the Hybrid pension plan.

In Figure 4.6, we generate individual paths of the contribution rates for both the Hybrid plan and the Traditional DB plan under the same 6 economic scenarios as the A/L rates in Figure 4.3. We observe that in each scenario, the waves for the Hybrid and the Traditional DB plans are at the same level and trending in the same direction, but the contribution rates of the Traditional DB plan are much more volatile than the Hybrid plan. For example, if we consider we find from Figure 4.3 that the Traditional DB A/L rates reach to 1.6 at time 5 but fall down below 0.6 at time 30, driving a contribution rate 0% at time 5 and about 50% at time 30 correspondingly, which is highly inefficient and unfair. However, since the A/L rate in hybrid plan in the same scenario is much less volatile, the contribution rate in hybrid plan stays between 10% and 30% all the time.

The comparison between hybrid plan and traditional DB plan total contribution rates demonstrates that the hybrid plan is more sustainable and more fair than the traditional DB plan.

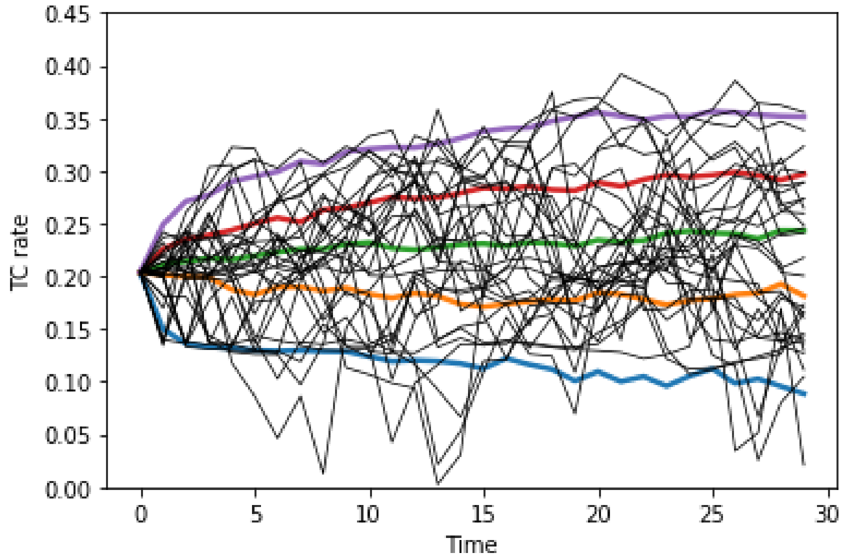


Figure 4.4: Total Contribution rate - Hybrid plan results. 1000 projections, 5%, 25%, 50%, 75%, 95% quantiles, with the same 30 sample paths

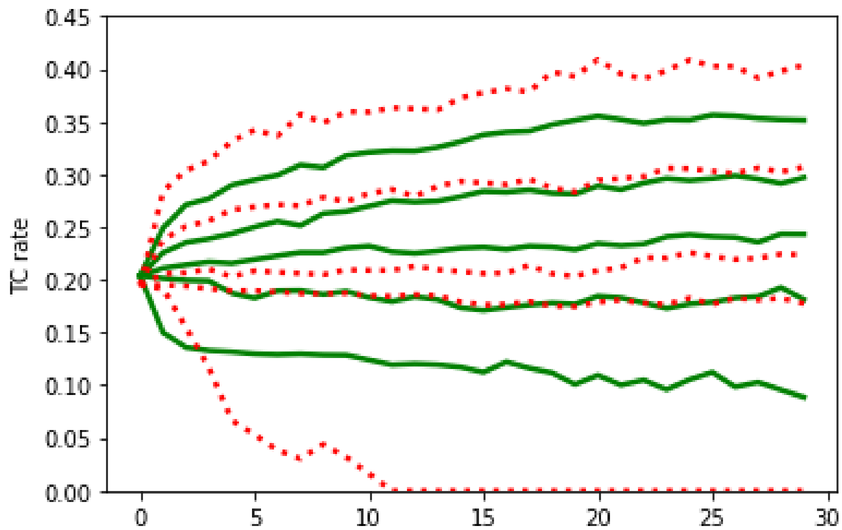


Figure 4.5: Total Contribution rate comparison Between Hybrid Plan and Traditional DB Plan. 1000 projections, 5%, 25%, 50%, 75%, 95% quantiles

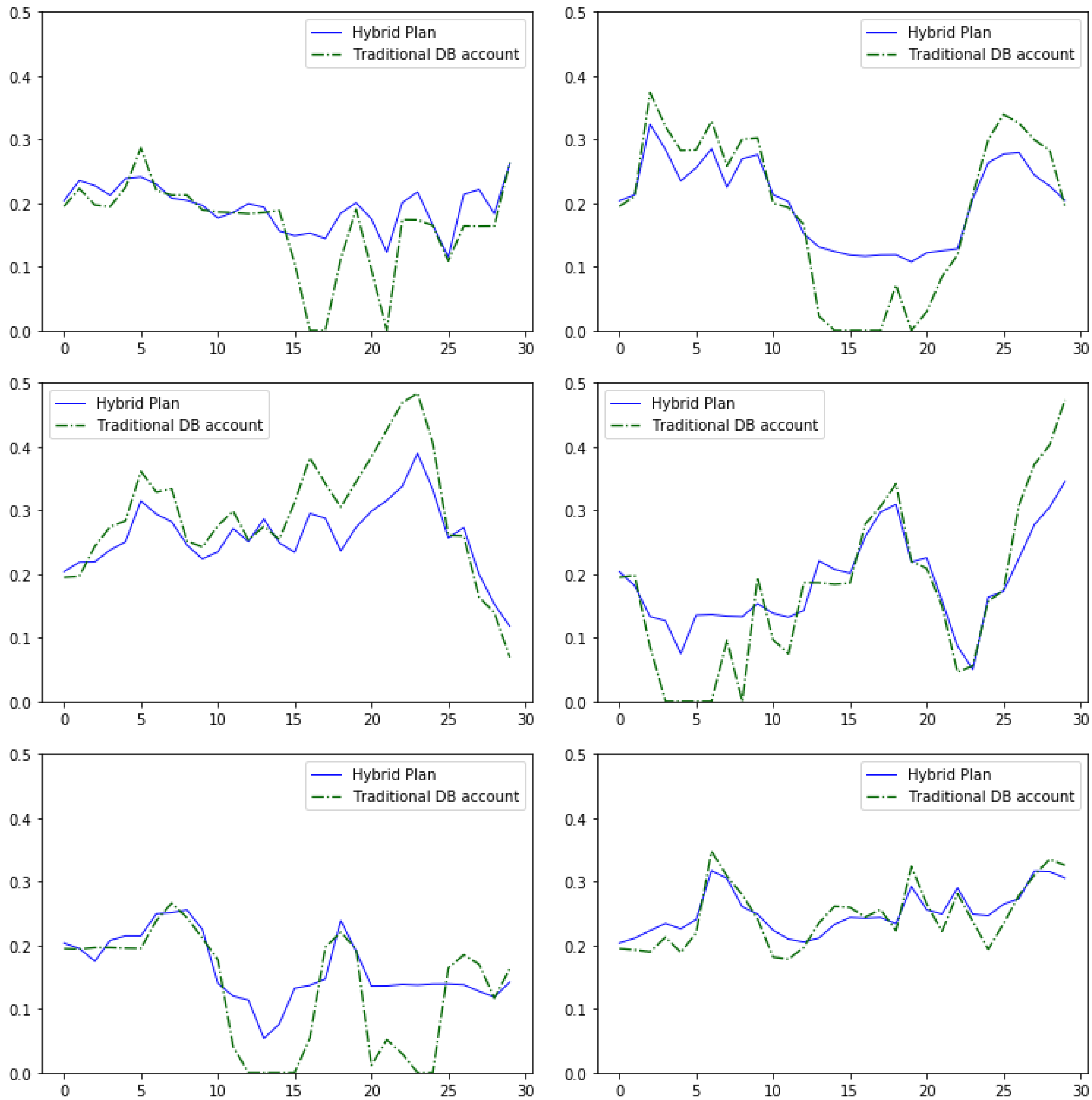
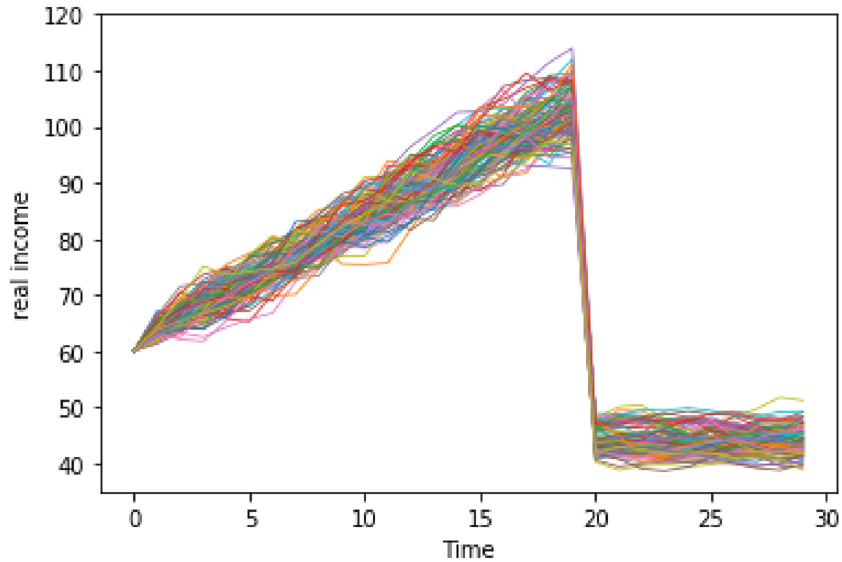


Figure 4.6: Total Contribution rates: Traditional DB plan, Hybrid plan. Same Economic Scenarios in Figure 4.3

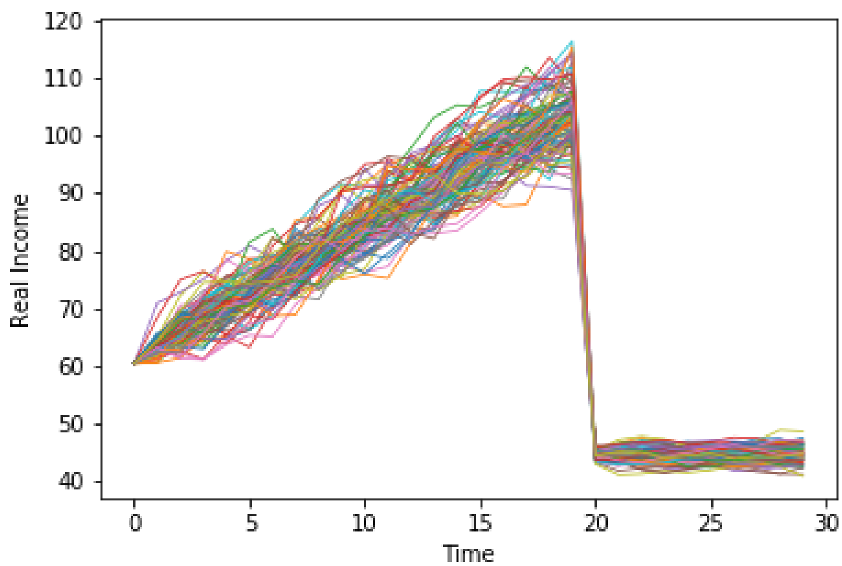
4.1.3 Real income

Considering the pension plan from each individual plan members' level, we care about the real income, which represents how much plan members will receive in hand. In Figure 4.7 we show 100 paths of the inflation-adjusted income, for a life who is age 45 at the start of the projection and stays in the plan till age 75. The real income will be net of 50% of the contribution rate before retirement; after retirement, the income is the pension benefit. In Figure 4.8 we show the histograms of the replacement rates, $RR = \frac{B(65, 20)}{FAS(64, 19)}$, in the hybrid plan, for an active member age 45 at the start of the projections. The following comments can be made:

- The salary trends upwards due to the use of promotional salary scale, which indicates that the salaries increase more than inflation each year.
- The variation in salary values between age 46 and 64 come from the differences of the total contribution rates. The Traditional DB plan's real income has a larger range of variation from time 0 to 20 since the total contribution rate is from about 0% to 35%, where the hybrid pension plan's is from 15% to 30%.
- After retirement, the pension benefit for the Traditional DB plan is highly predictable if assuming that there is no default, but there are a broad range of variation for hybrid pension's benefits. The downside of the benefits are under unfavorable economic scenarios, where there will be 10% benefit reduction. However, when the investments perform well, retirees can receive benefits with a replacement rate up to 48% annually.
- With the same accrual rate, there are some chances for the hybrid pension to have a replacement rate higher than traditional DB plan, but about half of the simulations generate replacement ratios lower than traditional DB plan.
- In the Hybrid plan, 29.75K of the benefits will be guaranteed annually, which brings a replacement ratio of 29%, based on 25 years of services; all benefits are guaranteed in traditional DB plan, unless the plan defaults.



(a) Real income-Hybrid plan



(b) Real income-Traditional DB plan

Figure 4.7: Projected real income for an active member age 45 at the start of the projections; Comparison between 4.7(a)Hybrid plan and 4.7(b)Traditional DB plan; same 100 paths

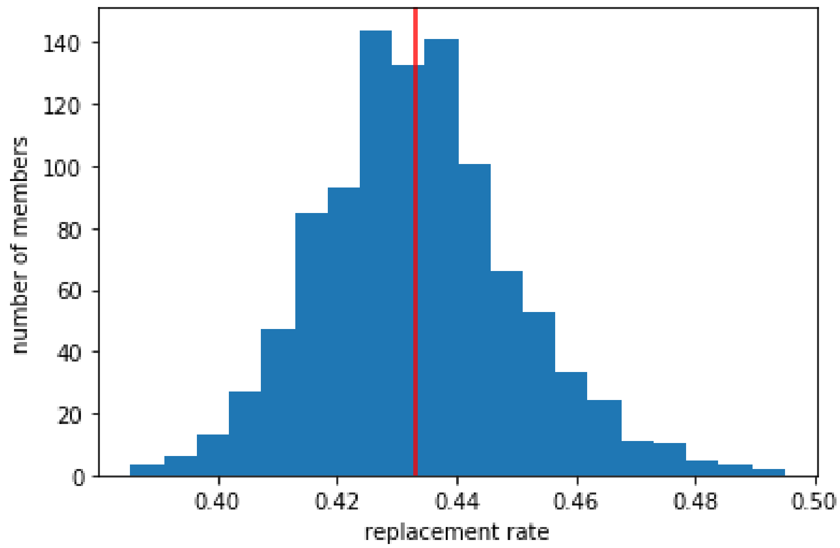


Figure 4.8: Hybrid Plan: Replacement Ratio at age 65 over 5-year Final Average Salary at age 64. 1000 simulation projections. The red line indicates the replacement rate of Traditional DB plan

4.2 Sensitivity Testing

In this section, we adjust some of the illustrative and arbitrary parameters and see how the changes impact our results. For each test, we perform 1000 simulations and observe the following measures:

- **Probability of A/L lower than 0.65**: indicating the plans' risk of default;
- **Median of A/L**: indicating the funding adequacy of the plan;
- **Probability of Total Contribution rate higher than 35%**: representing the sustainability and fairness of the plan;
- **Median of Total Contribution rate**: representing the affordability of the plan; For an active member age 45 at the start of the projection, we present:
- **Guaranteed benefit after retirement**: representing the amount of money provided by Basis Account;

- **Average real income at age 50**: showing the salary amount net of the contributions for an active worker;
- **Average benefit at age 70**: showing the pension benefit amount after retirement.
- **Probability of benefits higher than Traditional DB**: showing the chances of receiving higher benefits in hybrid plan

4.2.1 Changing the Accrual Rate and the Equity Weighting

From the previous section, we observe that the median contribution rate in the Hybrid pension plan is slightly higher than the Traditional DB plan. This is understandable since we invest, approximately,

$$\frac{\alpha_{Basis}}{\alpha_{Basis} + \alpha_{Prof}} \times 20\% + \frac{\alpha_{Prof}}{\alpha_{Basis} + \alpha_{Prof}} \times 80\% = 40\%$$

in equities, which is less than the 60% used in the Traditional DB plan. This makes the hybrid pension plan more expensive, on average. In this section, with all other assumptions stay the same, we test two adjustments:

1. The weight on equities for the basis account is increased to 35%;
2. The weight on equities for the basis account is increased to 25.6%; the accrual rates are changed to 0.01 in the basis account and 0.008 in the profit sharing account.

so that in total we have a 1.8% accrual rate and 50% weight on equities. We compare the results with a Traditional DB plan with a 50% weight on equities

		Traditional DB plan	Hybrid Plan 1 (BA/PS)	Hybrid Plan 2 (BA/PS)
	α	1.80%	1.2%/0.6%	1.0%/0.8%
	Weight on Equities	50%	35%/80%	26%/80%
A/L	$P(A/L < 0.65)$	1.48% [0.38%]	0.02%/7.39% (1.54% joint)	0.0%/7.39% (1.98% joint)
	Median	0.9571 [0.0302]	0.95 [0.03] (0.9579/0.9186)	0.9427 [0.0298] (0.943/0.9186)
TC rate	$P(TCR > 35\%)$	5.830% [0.74%]	1.540% [0.39%]	2.430% [0.487%]
	Median	21.420% [0.68%]	21.200% [0.67%]	21.490%[0.68%]
Income	Guaranteed income	44.6	29.75	24.79
	E[income at age 50]	71.79	71.97	72.19
	E[income at age 70]	44.6	44.29	44.06
	P(age 70's pension>47)	0	29.50%	24.4%

Table 4.1: Sensitivity Test: impact on solvency, contributions and benefits of Traditional DB plan, Hybrid plan 1: increasing BA's equity weight, and Hybrid plan 2: adjusting α and increasing BA's equity weight. 1000 projections

We summarize the results in the Table 4.1 and make the following comments:

- The probability of the A/L falling below 0.65 demonstrates that the risk of default in the Traditional DB plan is much higher than the BA, but lower than the PS. In Hybrid plan 1, increasing the weight on equities in the BA will slightly increase the risk of default, but it's relatively low compared with the Traditional DB plan.
- Compared to the Traditional DB plan, the Hybrid plan is more sustainable and better at controlling the risk of high costs in all cases since the probability of high contributions is much smaller than for the Traditional DB plan.
- With the same weight on equities, the affordability of the traditional DB plan and the Hybrid plan are similar. By comparing Hybrid plan 1 and 2, we observe that increasing the BA's accrual rate will increase the total contribution.

- In Hybrid plan 2, the guaranteed amount is reduced, due to the decrease in the accrual rate for the basis account. Theoretically all pensions are guaranteed in the Traditional DB plan, but we don't take the risk of default into account.
- The average real income increases for active members before retirements since the contribution rate in the basis account is lower, and more surplus can be shared from profit sharing account. However, the average pension benefits decreases due to pension adjustment.
- The average pension benefits in the Hybrid plan after retirement is slightly lower than in the Traditional DB plan, but there are more chances of receiving a higher pension than traditional DB plan, and less risk of default.

By comparing Hybrid Plans 1 and 2, with increasing proportion in the profit sharing account, the real income of active workers increases, but the guaranteed pension decreases, as does the average pension at age 70. It shows that the profit sharing account appears more beneficial for younger generations. This will balance out for workers who enter the plan in their early ages and stay in the plan long enough, but it might not be fair for those entering the plan late or withdrawing early. In the next section, we will adjust the illustrative parameters in profit sharing account and observe the results.

4.2.2 Changing parameters in profit sharing accounts

From the sensitivity tests in the previous section, we find that the profit sharing account favors younger generation, and it may not meet our criteria of fairness. We don't have any problem in surplus since it's shared in proportion to liabilities to each age. However, in deficit, we deduct 10% of benefits when A/L is lower than 1.0, and we add additional contributions of 5% of deficits if A/L rate goes lower than 0.8. There are multiple ways to improve the fairness, including

1. Increasing the amount of additional contribution to 6.6% and decreasing the pension reduction to 5%;
2. Changing the A/L threshold for both contribution and pension adjustment to 0.9.

		Hybrid Plan 1	Hybrid Plan 2	Hybrid Plan 3
PS Pension Adjustment	Amount	10.0%	5.0%	10.0%
	A/L Threshold	1.00	1.00	0.90
AC in PS	Amount	5.0%	6.67%	5.0%
	A/L Threshold	0.8	0.8	0.9
A/L rate in PS	$P(A/L < 0.6)$	7.46% [0.83%]	6.41% [0.77%]	7.00% [0.8%]
	Median	90.90%	91.58%	92.24%
TC rate (Hybrid)	$P(TCR > 35\%)$	3.080% [0.54%]	4.48% [0.65%]	1.08% [0.32%]
	Median	21.450%	21.452%	21.72%
Income (Hybrid)	Guaranteed income	29.8	29.8	29.8
	E[income at age 50]	72.13	72.03	72.01
	E[income at age 70]	43.97	44.45	44.14
	P(age 70's pension>45)	23.00% [1.33%]	26.40% [1.39%]	26.40% [1.39%]

Table 4.2: Sensitivity Testing: Impact on solvency, contributions, and benefits by changing amount of pension deduction and contribution rate in the PS and A/L rate threshold of paying. 1000 projections

We summarize the results in Table 4.2 and make the following comments:

- We note that this sensitivity testing only changes the PS account and so does not impact the A/L rate or the guaranteed income in the BA.
- Both adjustments improve the solvency and funding adequacy. The risk of default decreases in both cases, and the median A/L rate increases.
- Both increasing the amount of contribution and adjusting the threshold will increase the total contribution costs. Increasing the amount contributed has larger impact on sustainability and affordability.
- Adjusting the amount of pension deduction and its additional contribution impacts

the severity of the changes in income; adjusting the thresholds impact the frequency. We can observe that both changes can improve fairness, since in both cases, more contributions are made by active workers and less pensions are deducted for retirees. However, decreasing the reduction amount will provide more chances of receiving higher benefits than traditional DB plans, which may be inefficient if benefits become excessive, or unfair, if the amount of benefits varies significantly depending on the retirement date.

Chapter 5

Conclusion

In this paper, we first modeled a traditional DB pension plan and analyzed the results from both contributor's and beneficiary's perspectives, and then we introduced a new hybrid pension plan based with similar set up of both traditional DB plan and IRS plan. We compared the traditional DB plan to the hybrid pension plan and summarize its difference based the five loose criteria from Hardy *et al.* (2020): affordability, sustainability, adequacy, efficiency, and fairness:

- **Affordability**: We find that the traditional DB plan is affordable based on a 20.3% median level of total contribution rate. The new hybrid pension plan's median contribution is about 21.42%, which is 1.12% higher. It's understandable that the new hybrid pension plan is more expensive since in the basis account we invest more in long-term bonds which will generate less return on average, and that will lead to a higher contribution rate compared to the traditional DB plan. In Chapter 4, sensitivity tests show that the traditional DB plan and Hybrid plan, with identical accrual rates and equity weighting, bring a similar level of contribution costs. As a result, both the traditional DB plan and hybrid pension plan can be identified as affordable.

- **Sustainability**: One disadvantage of the traditional DB plan is that it's not seen as sustainable. We can see that the volatility of contribution rate is really high from Figure 2.4. The hybrid pension plan improves sustainability with a much smaller volatility of contribution rates as seen in Figure 3.2. The 95% quantile of the total contribution rate is reduced by about 3% at all times for the new hybrid pension plan, which is beneficial for both employers and active members.

- **Adequacy**: The main advantage of the traditional DB plan is the adequacy and predictability of pension incomes. However, we did not take into account of the possibility of default. The new hybrid pension plan pension payments are not as predictable as traditional DB, however, about 2/3 of the pension is better secured, since the basis account is much less likely to default. The supplemental payment from the profit sharing account uses intergenerational risk sharing, which might cause pension deductions in deficit, but offers a chance of receiving higher income, since we invest more in equities.

- **Efficiency**: The investment strategy and payment structure largely improve the efficiency of the pension plan as well. Since 2/3 of assets are invested in the basis account, with overall less equity weighting, excessive surpluses can be avoided. The profit sharing account is structured as risk-sharing. It reduces the chances of paying too many contributions for excessive pension benefits.

- **Fairness**: The Hybrid plan improves the fairness by better controlling the volatility of contributions, but, the downside is the benefits are more volatile. A deficit in basis account is shared by the contributors, but in the profit-sharing account, the retirees share the deficit, whereas only contributors share the risk in traditional DB plan. On the other hand, in the profit sharing account, both contributors and retirees share the surpluses, which improves fairness. However, considering the perspective of different age groups, the profit sharing accounts are more favorable younger generations based on the initiative parameters, but the problem can be reduced, as shown in Chapter 4.

In summary, the hybrid plan can provide a better payment structure than DC plans, and it can be more efficient and sustainable, through risk-sharing, than both Traditional DB and DC plans. The hybrid plan design is simple and not costly, and it takes both contributors' and beneficiaries' welfare into account. It provides some flexibility to meet different needs. For example, the accrual rates and weight on equities can be adjusted for the both basis account and the profit sharing account to fit the desired balance of guaranteed pension income, risk tolerance, and cost.

References

Blommestein HJ, Janssen P, Kortleve N, Yermo J (2009). "Moving Beyond the 'DB Vs. DC' Debate: The Appeal of Hybrid Pension Plans." *Rotman International Journal of Pension Management*, **2**. ISSN 0149-3333.

Clark GL, Monk AHB (2006). "The 'crisis' in defined benefit corporate pension liabilities Part I: Scope of the problem." *Pensions: An International Journal*, **22**. ISSN 43-54.

Cooper J (2014). "Are Defined Contribution Pension Plans Fit for Purpose in Retirement?" *Seattle University Law Review*, **37**.

Davis RB (2011). "Is your Defined-Benefit Pension Guaranteed?: Funding Rules, Insolvency Law and Pension Insurance." *IRPP study*.

Hardy M, Saunders D, Zhu X (2020). *Risk Sharing Pension Plans: Sustainability, Affordability, Adequacy, and Fairness.*

Ponds EH, Riel Bv (2007). "The Recent Evolution of Pension Funds in the Netherlands: The Trend to Hybrid DB-DC Plans and Beyond." *SSRN*.

Rauh JD, Stefanescu I, Zeldes SP (2020). "Cost Saving and the Freezing of Corporate Pension Plans." *NBER working paper*. ISSN 0270-6448.

Wilkie AD (1984). "A stochastic investment model for actuarial use." *Transactions of the Faculty of Actuaries*, **37**.

Zhang S, Hardy M, Saunders D (2018). "Updating Wilkie's Economic Scenario Generator for U.S. Applications." *North American actuarial journal*, **22**. ISSN 1092-0277.

Zhu X, Hardy M, Saunders D (2018). "Valuation of a Early Exercise Defined Benefit (DB) Underpin Hybrid Pension Benefit." *Scandinavian Actuarial Journal*, **2018(9)**.

Appendix A

Appendix

Table A.1: Membership information

x	In force	Ave. Salary	Ave. service		x	In force	Ave. Salary	Ave. service
25	17	32.0	0.5		45	89	66.5	10.04
26	32	33.0	0.97		46	93	68.5	10.36
27	45	35.3	1.45		47	97	70.5	10.69
28	55	36.7	1.92		48	100	72.3	11.01
29	64	38.2	2.39		49	104	74.1	11.35
30	70	39.7	2.89		50	107	76.0	11.80
31	74	41.3	3.39		51	110	77.9	12.24
32	77	42.9	3.88		52	111	79.8	12.78
33	78	44.6	4.38		53	113	81.8	13.31
34	78	46.4	4.94		54	112	83.8	14.06
35	75	48.1	5.58		55	111	85.5	14.93
36	76	49.7	6.18		56	108	87.2	15.93
37	78	51.5	6.75		57	106	89.0	16.93
38	79	53.3	7.29		58	103	90.7	17.93
39	80	55.1	7.80		59	101	92.6	18.93
40	81	57.1	8.29		60	98	94.4	19.93
41	82	59.1	8.75		61	98	96.3	20.93
42	83	60.8	9.09		62	98	97.3	21.93
43	85	62.7	9.41		63	97	98.2	22.93
44	87	64.5	9.73		64	97	99.2	23.93

Table A.2: Survival rates

x	$q_x^{(\tau)}$		x	$q_x^{(\tau)}$		x	$q_x^{(\tau)}$		x	$q_x^{(\tau)}$
25	0.100246		45	0.03075		65	0.005915		85	0.057665
26	0.100252		46	0.02082		66	0.006619		86	0.064554
27	0.100258		47	0.02090		67	0.007409		87	0.072237
28	0.100266		48	0.02098		68	0.008297		88	0.080798
29	0.100274		49	0.02108		69	0.009294		89	0.090326
30	0.100284		50	0.02118		70	0.010413		90	0.100917
31	0.100295		51	0.02130		71	0.01167		91	0.112675
32	0.100307		52	0.02144		72	0.013081		92	0.125708
33	0.090324		53	0.02159		73	0.014664		93	0.140128
34	0.080343		54	0.02176		74	0.01644		94	0.156052
35	0.070364		55	0.02195		75	0.018433		95	0.173599
36	0.060388		56	0.02217		76	0.020668		96	0.192887
37	0.050415		57	0.02241		77	0.023175		97	0.21403
38	0.05044		58	0.02268		78	0.025984		98	0.237134
39	0.050469		59	0.02299		79	0.029132		99	0.262294
40	0.050501		60	0.01336		80	0.032658		100	0.289584
41	0.050537		61	0.00379		81	0.036607		101	0.319055
42	0.050578		62	0.00423		82	0.041025		102	0.350723
43	0.050623		63	0.00473		83	0.045968		103	0.384564
44	0.040682		64	0.00529		84	0.051493		104	0.4205

For active members, the survival rates are all decrements including death. All active lives who reach age 65 are assumed to retire immediately. For retirees, death would be the only decrement.