Hessian Estimation Based Adaptive and Cooperative Extremum Localization

by

Huseyin Demircioglu

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

The thesis is on Hessian estimation based adaptive and cooperative extremum localization via a single mobile sensory agent as well as a network of multiple such agents.

First, we develop a continuous time adaptive extremum localization of an arbitrary quadratic function $F(\cdot)$ based on Hessian estimation, using the measured signal intensity via a single mobile sensory agent. A gradient based adaptive Hessian parameter estimation and extremum localization scheme is developed considering a linear parametric model of field variations.

Next, we extend the proposed single agent based Hessian estimation and extremum localization scheme to consensus based cooperative distributed scheme to be implemented by a network of such sensory agents.For the networked multi-agent case, a consensus term is added to the base adaptive laws to obtain enhanced estimation cooperatively. Stability and convergence analysis of the proposed scheme is studied, establishing asymptotic convergence of the Hessian parameters and location estimates to their true values robustly, provided that the motion of agent(s) satisfies certain persistence of excitation(PE) conditions. Furthermore, we show that for a network of connected agents, the PE requirements can be distributed to the agents so that the requirement on each agent is more relaxed and feasible.

Later, we design an adaptive motion control scheme for steering a mobile sensory agent in 2D toward the source of a signal field $F(\cdot)$ using the signal intensity the agent continuously measures at its current location. The proposed adaptive control design is based on the Hessian estimation based adaptive extremum localization. Results are displayed to verify that the proposed scheme is stable, provides asymptotic convergence of the Hessian parameter and extremum location estimates to their true values and the agent location to the source location, robustly to signal measurement noises.

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I would like to express my gratitude to my supervisor, Dr. Baris Fidan, for his continuous support and guidance. I would like to thank my readers, Dr. Hyock Ju Kwon and Dr. Soo Jeon, for their feedback.

Thank you to my colleague and friend Cem Berke Cetin for making my time at Waterloo all the better. Also, I would like to thank my family for supporting me through this time of my life.

Dedication

This is dedicated to my family.

Table of Contents

List of Figures

Chapter 1

Introduction

1.1 Scope and Motivation

Extremum localization is a fundamental problem in nature that is relevant to many complex applications, such as environmental monitoring [1], search and rescue operations [2], odor source detection [3], and pollution sensing [4]. The problem of localizing the source of a signal has been approached in the manner of deploying a fixed network of sensors that collect measurements to cooperatively estimate the source location.

In the existing literature, extremum localization is achieved by computing the sensory measurements of a group of mobile robots. The gradient of these signals gathered by the agents are estimated by different methods such as extremum seeking techniques, least square methods, etc. Existing approaches fail to do error analysis by considering the quality of the estimation algorithm and do not consider the effect of measurement noise and the performance is analyzed only in simulation. Also, only few sources explore the problem of multirobot source seeking in three dimensional space.

The aim of this thesis is to overcome the main drawbacks of related works following a new approach based on estimation of the gradient and the Hessian matrix of unknown signal fields from the signal strength measurements by sensory agents.

1.2 Objectives and Contributions

A set of novel solutions to the extremum seeking problem where an autonomous vehicle (or group of vehicles) is required to locate the source of a certain signal based on measurements of the signal's strength at different positions will be studied in the thesis research. As the first step of this research, an adaptive Hessian estimation based source localization algorithm has been developed for a mobile sensory agent with guaranteed accuracy. As the second step, utilizing a consensus protocol, the single agent localization scheme design is extended to a multi-sensory-agent adaptive localization scheme in order to have a faster and more accurate estimate of the source location. Later the effects of signal field modeling uncertainties and measurement noises will be studied.

1.3 Thesis Outline

This thesis is organized as follows: Chapter 2 provides background information and literature review on extremum localization. The chapter explains the objectives of extremum localization, its application areas and the related problems, and surveys the proposed solutions in the literature. Chapter 3 presents the proposed adaptive Hessian estimation and extremum localization scheme for a single agent case. In Chapter 4, extension of this scheme to multi agent networks is introduced. Single-agent and multi-agent cases are compared based on their accuracy and convergence rate of the estimation of the extremum location. In Chapter 5, an adaptive steering control scheme of agents toward the source is introduced. The conclusions are presented in Chapter 6

Chapter 2

Background and Literature Review

For more than a decade, source localization has been extensively studied, motivated by its promising applications, see e.g., [5, 6, 7, 8]. The general undertaking in these problems is that at least one sensory agent finds the location of a signal source with the assistance of sensory measurement. To pinpoint the source, various types of estimation schemes have been proposed relying upon the setting and limitations of the specific localization task.

2.1 Target Localization Problems and Measurement Technologies

Generally, localization is achieved utilizing some information about the relative position of a single agent or a network of multiple agents to a source such as time of arrival (TOA) [9, 10], bearing / angle of arrival (AOA) [11, 12], time difference of arrival (TDOA) [13, 14], received signal strength(RSS) [15, 16].

The TOA principle is a method for measuring the distance between a sensor and an object, based on the time difference between the emission of a signal and its return to the sensor, after being reflected by an object. In the TOA approach [17, 18], the local clock at the target and those at the sensors are assumed to be synchronized so that the TOA information can be locally obtained at each sensor using time stamps. Various types of signals (also called carriers) can be used with the TOA principle. A common application is for positioning system in industrial environment [19].

The AOA measurements are also known as the bearing measurements or the direction of arrival measurements. In the AOA approach, each sensor node is equipped with an antenna array which can be used to estimate the angle of arrival of the target signal. With multiple AOA estimates from different sensor locations, it is possible to determine the location of the target. In [20], authors propose a localization and orientation scheme based on angle of arrival information between neighbor nodes. In [21], a technique based on antenna arrays and AOA measurements is carefully discussed.

TDOA approach refers to determining the difference in arrival times of a signal received at two spatially separated sensors. In contrast to TOA, the TDOA approach does not rely on the sensor clocks to be synchronized with that of the target. It has applications in direction finding and source localization [22, 23, 24, 25]. For example, in the [25], multiplying the TDOA by the signal propagation speed gives the range difference between the source and two receivers. Each range difference defines a hyperbola on which the target must lie in the two-dimensional space, and thus the source position can be obtained from the intersection of at least two hyperbolas.

Localization task can be conveniently accomplished using a small number of measurements without noise. But, noise in signals is inevitable in practice, hence, agents need to possess an estimator to compensate the uncertainties due to measurement noise as well as the source's motion, which can be studied under adaptive target localization. A source position estimation algorithm is proposed in [26, 27, 28], where the agent is able to measure its distance to the source location. It is shown that this estimation algorithm is exponentially stable under a persistent excitation (PE) condition and robust to drifts in the source location. It is also demonstrated in the simulation results of [26, 27, 28] that the algorithm operates well in presence of sensor noise as well.

In [29], to compensate uncertainties in signal permittivity and path loss coefficients for the electromagnetic signal based distance measurement, the authors propose a geometric cooperative technique with RSS and TOF based range sen- sors. The introduced technique is combined with a recursive least squares (RLS)-based adaptive localization scheme and an adaptive motion control law, in order to perform adaptive target localization robust to uncertainties in environmental signal propagation coefficients. This approach is applied to the problem of tracking biomedical robotic capsules for gastro-intestinal endoscopy and medication applications in [6].

The above approaches all utilize sensor units providing geometric measurements, such as distance, bearing, distance difference, directly related to relative position of the target or the signal source. In many applications, as opposed to distance/direction measurement, RSS is used to estimate the gradient of the unknown signal field of interest and locate the extremum point where the gradient of the field is zero. RSS approach can be defined as measuring the signal strength emitted by the source. The source could be a point of chemical contamination and the signal would be the chemical's concentration in the environment, for instance. Alternatively, the source could be a radio transmitter and the signal would be a radio frequency transmission. In [1], control of a mobile sensor network is studied to perform a gradient climbing task in an environment with an unknown potential field using noisy measurements of the field partially decoupling the formation stabilization problem from the gradient climbing mission. In [30] control laws are proposed to distributively self-steer a circular vehicle formation towards the source of a signal field using only direct measurements of that signal at individual locations of the vehicles. In [31], similar to [30], the authors propose a combination of a cooperative control law to stabilize the agents to a circular formation and a distributed consensus-based source-seeking algorithm and examine the performance of algorithm for different cases such as asynchronous communication between agents, noisy measurements, multiple or time-varying sources. In [32], the authors study a combined formation acquisition and cooperative extremum seeking control scheme for a team of three robots moving on a plane in order to find the extremum point of an unknown signal field by on-board signal measurement.

2.2 Extremum Localization

Localization task can be conveniently accomplished using a small number of measurements without noise. But, noise in signals is inevitable in practice, hence, agents need to possess an estimator to compensate the uncertainties due to measurement noise as well as the source's motion, which can be studied under adaptive target localization. A source position estimation algorithm is proposed in [26, 27, 28], where the agent is able to measure its distance to the source location. It is shown that this estimation algorithm is exponentially stable under a persistent excitation (PE) condition and robust to drifts in the source location. It is also demonstrated in the simulation results of [26, 27, 28] that the algorithm operates well in presence of sensor noise as well.

In [29], to compensate uncertainties in signal permittivity and path loss coefficients for the electromagnetic signal based distance measurement, the authors propose a geometric cooperative technique with RSS and TOF based range sensors. The introduced technique is combined with a recursive least squares (RLS)-based adaptive localization scheme and an adaptive motion control law, in order to perform adaptive target localization robust to uncertainties in environmental signal propagation coefficients. This approach is applied to the problem of tracking biomedical robotic capsules for gastro-intestinal endoscopy and medication applications in [6].

The majority of the extremum localization strategies proposed in the current literature exploit the measurements collected by a group of mobile robots to estimate properties of the signal, such as the model parameters of the scalar field of interest or its gradient. There are several approaches to estimate the gradient of a signal, such as extremum seeking techniques [33], least-squares methods [34, 35], consensus-based parametric algorithms [36], and cooperative Kalman filters [37]. However, all these strategies do not explicitly quantify the quality of the estimation algorithm nor the impact of measurement noise and at best, the performance is analyzed only in simulation. Such error analysis is of paramount importance in practical applications to properly design formation shapes or exploration strategies [38]. Such analysis is often avoided in the literature since no specific formation patterns of the robots are enforced, thus resulting in algorithms that are highly nonlinear in the relative distance measures with high communication and computational complexity. Only few works addressed the problem of multirobot source seeking in three-dimensional (3-D) scenarios [39, 40] or proposed strategies to estimate the Hessian of the unknown signal from noisy measurements [34, 41, 42].

In this master thesis, we first develop an adaptive Hessian estimation based extremum localization scheme for a single sensory agent. Then, we extend the design for networks of multiple sensory agents. The goal is adaptive Hessian estimation and extremum localization of a (signal) field F using (i) a single sensory agent that consistently measures the intensity of F at its current location while moving and (ii) a network of multiple such sensory agents. Beyond the existing literature, including [1, 30, 31, 32] , the aimed contribution of these designs is three-folds: (1) On-line identification of more detailed information about the signal field F than just the extremum of it. (2) More accurate and faster localization of the extremum utilizing this extra information. (3) Using multiple agents, to distribute the motions to satisfy PE condition and to reduce the convergence time. A more realistic scenario that includes the effects of modelling uncertainties in the signal field and measurement noises will also be studied. Indoor experiments using a fleet of ground mobile robots and an indoor localization system will be conducted to evaluate the performance of the proposed adaptive localization algorithms.

Chapter 3

Single Agent Adaptive Source Localization

3.1 The Extremum Localization Problem

The main objective of the adaptive estimator design in this thesis is to produce an accurate estimate of the location of the extremum(maximum) of a quadratic (signal field) function $F(\cdot) : \mathbb{D} \longrightarrow \mathbb{R}$, for a compact state (or location) domain $\mathbb{D} \subset \mathbb{R}^m$, formulated by

$$F(y) = c_1 - \frac{1}{2}(y-x)^T H (y-x) \tag{3.1}$$

where c_1 is an unknown positive constant and H is an unknown $m \times m$ positive definite matrix. For $m \in \{2,3\}$, (3.1) typically represents the strength of a signal emitted by a source at location(state) $x \in \mathbb{R}^m$ measured by a sensory node at location (state) $y \in \mathbb{R}^m$ [43, 44, 45]. The idea for using a quadratic function as a profile of the signal field is rooted in the fact that any smooth function can be approximated locally by its Taylor expansion near each extremum point. For a general nonlinear smooth function $F_g(\cdot)$, the gradient $\nabla F_g(y)$ will vanish at the extremum point $y = x$, we can write [46] :

$$F_g(x + y_r) = F_g(x) + \frac{1}{2} y_r^T \nabla^2 F_g(x) y_r + h.o.t \tag{3.2}$$

where $y_r = y - x$. The approximation (3.2) enables us to extract the gradient of the field using averaging methods [47] and find the location of the extremum point. Assuming that

$F_g(\cdot)$ is a positive concave signal field function, $\nabla^2 F_g(x)$ is negative definite and c_1 and H in (3.1) matches, respectively, with $F_g(x)$ and $-\nabla^2 F_g(x)$ in (3.2). For brevity, neglecting the higher order terms ($h.o.t.$) in (3.2), we focus on the representation (3.1) in this thesis, and formally define the extremum localization problem for this representation.

In order to devise an adaptive localization algorithm, we use the adaptive parameter identification based framework proposed in [26, 27, 28]. We use the notation in [26] for derivative operation and asymptotically equal signals: s denotes the derivative operator, i.e., given a function f of time t, $sf := \dot{f} = df/dt$. $\frac{1}{s+a}f(t) := \int_0^t e^{-a\tau} f\tau d\tau$. For two vector functions f, g of the same dimension, $f(\cdot) \approx g(\cdot)$ if there exist λ, M such that $\|f(t) - g(t)\| \leq Me^{-\lambda t}$ for all $t \geq 0$. Moreover, we use the following assumption for the estimation of Hessian matrix parameters;

Assumption 3.1 *The Hermitian matrix H satisfies the following:*

1. $H_{ii} > 0$ *for all* $i = 1, \cdots, m$.
2. H *is strictly diagonally dominant which means* $|H_{ii}| > \sum_{i \neq j} |H_{ij}|$ *for all* $i, j = 1, \cdots, m$.

Lemma 3.1 *The set S_H of $m \times m$ symmetric real matrices P satisfying the two conditions of Assumption 3.1 is convex. Furthermore, any $P \in S_H$ is positive definite.*

Proof: For any arbitrary matrix pair $P_1, P_2 \in S_H$, $P = P_1 + P_2$ satisfies both conditions of Assumption 3.1, hence, $P \in S_H$, establishing convexity of S_H. For any given arbitrary $P \in S_H$, Theorem 6.1.10 of [48] implies that P is positive definite.

For the drift analysis, we utilize the following assumption;

Assumption 3.2 *The agent trajectory $y : \mathbb{R} \to \mathbb{R}^m$ is twice differentiable, the source trajectory $x : \mathbb{R} \to \mathbb{R}^m$ is differentiable and there exist $M_1, M_2, M_3, M_4, \epsilon > 0$ such that for all $t \in \mathbb{R}$*

$$||y(t)|| \leq M_1, \; ||\dot{y}(t)|| \leq M_2, \; ||\ddot{y}(t)|| \leq M_3, \tag{3.3}$$

$$||x(t)|| \leq M_4, \; ||\dot{x}(t)|| \leq \epsilon. \tag{3.4}$$

Problem 3.1 *Consider the quadratic signal field function in (3.1). Suppose that a sensory agent has access to the field measurement $F(y)$ at its current location y. Design an adaptive identification scheme to estimate the target location x at which F takes its maximum value, and derive the conditions under which the estimate $\hat{x}(t)$ converges to x asymptotically.*

3.2 The Adaptive Hessian Estimation and Localization Scheme

In order to devise an adaptive localization algorithm, we use the adaptive parameter identification based framework proposed in [26, 27, 28]. We use the notation in [26] for the derivative operation and asymptotically equal signals: s denotes the derivative operator, i.e., given a function f of time t, $sf := \dot{f} = df/dt$. $\frac{1}{s+a}f(t) := \int_0^t e^{-a\tau} f\tau d\tau$. For two vector functions f, g of the same dimension, $f(\cdot) \approx g(\cdot)$ if there exist λ, M such that $\|f(t) - g(t)\| \leq Me^{-\lambda t}$ for all $t \geq 0$. We derive a parametric model that is linear in unknown parameters of the system, i.e., the elements of Hessian matrix H and the location(state) x of the extremum. Noting that (3.1) is not linear parametric, we take time derivative of (3.1) for this purpose. Assuming that x is constant, i.e., $\dot{x} = 0$, we obtain

$$\begin{aligned}
\dot{F}(y) &= -\dot{y}^T H(y-x) \\
&= -\dot{y}^T Hy + \dot{y}^T Hx \\
&= -\frac{1}{2}\frac{d}{dt}\left(y^T Hy\right) + \frac{d}{dt}\left(y^T\right) Hx \\
&= -\frac{1}{2}\frac{d}{dt}\left(H_{11}y_1^2 + 2H_{12}y_1y_2 + \cdots + H_{22}y_2^2 + 2H_{23}y_2y_3 + \cdots + H_{mm}y_m^2\right) + \frac{d}{dt}\left(y^T\right) Hx
\end{aligned} \tag{3.5}$$

which can be written as

$$\dot{F}(y) = \theta^{*T}\frac{d\Psi}{dt}, \tag{3.6}$$

$$\theta^* = \Big[H_{11}, H_{12}, \cdots, H_{1m}, H_{22}, \cdots, H_{mm}, \underbrace{x^T H_1, \cdots, x^T H_m}_{x^T H}\Big]^T \in \mathbb{R}^{\frac{m(m+3)}{2}}, \tag{3.7}$$

$$\Psi = \Big[\tfrac{-1}{2}y_1^2, -y_1y_2, \cdots, -y_1y_m, \tfrac{-1}{2}y_2^2, \cdots, \tfrac{-1}{2}y_m^2, y^T\Big]^T \in \mathbb{R}^{\frac{m(m+3)}{2}}, \tag{3.8}$$

where H_i denotes the ith column (= transpose of the ith row) of H. In order to eliminate need for explicit differentiation of the available signals, $z(\cdot)$ and $\phi(\cdot)$ are introduced as the state variable filtered versions of $F(\cdot)$ and $\Psi(\cdot)$, respectively, as follows;

$$\dot{\xi}_1(t) = -a\xi_1(t) + F(y(t)), \tag{3.9}$$

$$\xi_1(0) = 0, \tag{3.10}$$

$$z(t) = -a\xi_1(t) + F(y(t)), \tag{3.11}$$

$$\dot{\xi}_2(t) = -a\xi_2(t) + \Psi(t), \tag{3.12}$$

$$\xi_2(0) = [0, \ldots, 0]^T \in \mathbb{R}^{\frac{m(m+3)}{2}}, \tag{3.13}$$

$$\phi(t) = -a\xi_2(t) + \Psi(t), \tag{3.14}$$

for some $a > 0$. It can be seen in (3.9)–(3.14) that the measurements of the location (state) $y(t)$ of the sensory agent and the field intensity $F(y(t))$ at that location are sufficient to generate the signals $z(t)$ and $\phi(t)$.

Lemma 3.2 *Suppose $\theta^* \in \mathbb{R}^{\frac{m(m+3)}{2}}$ is a constant, and $z(t), \phi(t)$ are defined by (3.9)–(3.14) with $a > 0$. Then there holds:*

$$z(\cdot) \approx \theta^{*T}\phi(\cdot). \tag{3.15}$$

Proof: Using (3.9)–(3.11), we obtain;

$$\dot{z}(t) + az(t) = \frac{d}{dt}\left\{F\left(y\left(t\right)\right)\right\}, \tag{3.16}$$

where $a > 0$. With operator notation, i.e., using s to denote the differentiator operator,

$$z(\cdot) \approx \frac{s}{s+a}\left\{F(\cdot)\right\}. \tag{3.17}$$

Similarly,

$$\phi(\cdot) \approx \frac{s}{s+a}\left\{\Psi(\cdot)\right\}. \tag{3.18}$$

Therefore,

$$z(\cdot) \approx \frac{s}{s+a}\left\{F(\cdot)\right\} \approx \frac{1}{s+a}\left\{\theta^{*T}\dot{\Psi}(\cdot)\right\} \approx \theta^{*T}\frac{s}{s+a}\left\{\Psi(\cdot)\right\} \approx \theta^{*T}\phi(\cdot). \tag{3.19}$$

Using (3.15) as the linear parametric model, and (3.9)–(3.14) to generate the regressor signals in this model, we design the following gradient based adaptive estimation algorithm [49, 50] to identify θ^*:

$$\dot{\hat{\theta}} = \gamma\phi(z - \hat{\theta}^T\phi), \tag{3.20}$$

where $\hat{\theta}$ denotes the estimate of θ^* and $\gamma > 0$ is a scalar design constant. To be able to extract the information of the elements of H and the location(state) of the source (x) from the estimation of θ^*, we consider the following partitioning of θ^* and $\hat{\theta}$:

$$\theta^* = \begin{bmatrix} \theta_H^* \\ \theta_x^* \end{bmatrix}, \quad \hat{\theta} = \begin{bmatrix} \hat{\theta}_H \\ \hat{\theta}_x \end{bmatrix} \tag{3.21}$$

where $\theta_H^* \in \mathbb{R}^{\frac{m(m+1)}{2}}$ is composed of the entries of θ^* that are independent of x, $\theta_x^* = Hx \in \mathbb{R}^m$, $\hat{\theta}_H$ and $\hat{\theta}_x$ are the estimates of θ_H^* and θ_x^*, respectively. Since all the elements of H exist in θ_H^*, we can form $\hat{H}$ (the estimate of H) from $\hat{\theta}_H$. In order to obtain the estimate $\hat{x}$ of the source location (state) x, we utilize the definition of θ_x^* to form the adaptive law

$$\hat{x} = \hat{H}^{-1}\hat{\theta}_x. \tag{3.22}$$

In order to implement (3.22), $\hat{H}$ needs to be guaranteed to be invertible. We guarantee invertibility of $\hat{H}$ applying parameter projection to (3.20), utilizing Assumption 3.1.

Lemma 3.3 *The set S_H of $m \times m$ symmetric real matrices H satisfying the two conditions of Assumption 3.1 is convex. Furthermore, any $H \in S_H$ is positive definite.*

Proof: For any arbitrary matrix pair $H^1, H^2 \in S_H$, $H = H^1 + H^2$ satisfies both conditions of Assumption 3.1, hence, $H \in S_H$, establishing convexity of S_H. For any given arbitrary $H \in S_H$, Theorem 6.1.10 of [48] implies that it is positive definite. ■

To assure $\hat{H}$ is non-singular, we apply parameter projection on the elements of $\hat{\theta}_H$ in consideration of Assumption 3.1 and (3.20) with the parameter projection is re-designed as;

$$\dot{\hat{\theta}} = \underset{\hat{\theta}_H \in S_H}{\text{Proj}} \{\gamma\phi(z - \hat{\theta}^T\phi)\}, \tag{3.23}$$

where the convex set S_H is defined as the set of all vectors $\hat{\theta}_H$ such that the corresponding $m \times m$ matrix $\hat{H}$ satisfies Assumption 3.1, and $\underset{\hat{\theta}_H \in S_H}{\text{Proj}} \{\cdot\}$ is the parameter projection operator [49, 50] defined to maintain $\hat{\theta}_H$ in S_H.

Remark 3.1 *If H is a diagonal matrix, the vectors θ^* and Ψ in (3.7)–(3.8) can be redefined in reduced form as follows:*

$$\theta^* = \Big[H_{11}, \cdots, H_{mm}, x^T H\Big]^T \in \mathbb{R}^{2m} \tag{3.24}$$

$$\Psi = \Big[\frac{-1}{2} y_1^2, \cdots, \frac{-1}{2} y_m^2, y^T\Big]^T \in \mathbb{R}^{2m} \tag{3.25}$$

For a general case, since H is a symmetric matrix with real elements, we can deduce that by choosing appropriate coordinates, we can diagonalize the matrix H and hence, design the identification algorithm based on the reduced order model (3.15),(3.24),(3.25).

In the next section, we analyze the stability of the proposed adaptive estimation and localization scheme.

3.3 Stability and Convergence

3.3.1 Stationary Extremum Localization

Note that the base adaptive law (3.20) and the adaptive law (3.23) with parameter projections can be rewritten, respectively, as

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} = -\gamma \phi \phi^T \tilde{\theta}, \tag{3.26}$$

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} = \underset{\hat{\theta}_H \in S_H}{\text{Proj}} \{-\gamma \phi \phi^T \tilde{\theta}\}, \tag{3.27}$$

where $\tilde{\theta} = \hat{\theta} - \theta^*$. Hence, the aimed convergence of the estimate $\hat{\theta}$ to actual θ^* is equivalent to the convergence of $\tilde{\theta}$ to zero.

Theorem 3.1 *Suppose $\theta^* \in \mathbb{R}^{\frac{m(m+3)}{2}}$ is a constant. Consider $z(t)$ and $\phi(t)$ defined in (3.9)–(3.14), with $a > 0$. Then for each of the base adaptive law (3.26) and the adaptive law (3.27) with parameter projection , there exist $\rho_1, \rho_2, \lambda > 0$ such that for all $t \geq 0$ and $||\theta^*(0)||$*

$$||\tilde{\theta}(t)|| \leq (\rho_1 ||\theta^*(0)|| + \rho_2) e^{-\lambda t} \tag{3.28}$$

if and only if there exist $\alpha_1 > 0$, $\alpha_2 > 0$, $T > 0$ such that for all $t \geq 0$

$$\alpha_1 I \leq \int_t^{t+T} \phi(\tau) \phi(\tau)^T d\tau \leq \alpha_2 I. \tag{3.29}$$

Proof: It is established in the literature (see, e.g., [51]) that (3.26) is exponentially asymptotically stable if and only if (3.29) holds. Moreover, it is proven in [50] that the parameter projection does not affect the properties of the gradient adaptive laws deducted on the Lyapunov analysis and it can only make the time derivative of Lyapunov function more negative. Hence, (3.27) is also exponentially asymptotically stable if and only if (3.29) holds. ■

3.3.2 Drift in Extremum Location

The drift analysis in [26] can be applied here as well, without requiring significant modifications. Making the Assumption 3.2, we can introduce the following lemma.

Lemma 3.4 *Under Assumption 3.2, for $z(t)$ and $\phi(t)$ defined in (3.9)–(3.14), there exists $M_5 : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that for a suitable K_1 depending only on M_1, M_2, M_4 and a,*

$$|z(t) - \theta^{*T}\phi(t)| \leq M_5(t), \quad \forall t \geq 0 \tag{3.30}$$

and

$$M_5(\cdot) \approx K_1\epsilon. \tag{3.31}$$

Proof: Using the operator notation in the proof of Lemma 3.2, it is achieved that

$$\begin{aligned} z(\cdot) \approx& \frac{s}{s+a}\left\{F(\cdot)\right\} \\ \approx& \frac{1}{s+a}\left\{-\left(\dot{y}(\cdot)-\dot{x}(\cdot)\right)^T H\left(y(\cdot)-x(\cdot)\right)\right\} \\ \approx& -\frac{s}{s+a}\left\{\frac{1}{2}y^T(\cdot)Hy(\cdot)\right\} + \frac{1}{s+a}\left\{\frac{1}{2}\dot{y}^T(\cdot)Hx(\cdot)\right\} + f(\cdot) \end{aligned} \tag{3.32}$$

where

$$f(\cdot) = \frac{1}{s+a}\left\{\frac{1}{2}\dot{x}^T(\cdot)H\left(y(\cdot)-x(\cdot)\right)\right\}. \tag{3.33}$$

In consideration of Assumption 3.2, there exists a $F; \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, such that for all $t \geq 0$,

$$|f(t)| \leq F(t) \tag{3.34}$$

and

$$F(\cdot) \approx \frac{M_1 + M_4}{a}\epsilon. \tag{3.35}$$

Now, consider the second term in (3.32)

$$\frac{1}{s+a}\left\{\frac{1}{2}\dot{y}^T(\cdot)Hx(\cdot)\right\} \approx Q(\cdot) \tag{3.36}$$

where with $C \in \mathbb{R}^m$,

$$\begin{aligned}
Q(t) =& e^{-at}\int_0^t e^{a\tau}\dot{y}^T(\tau)Hx(\tau)d\tau \\
=& e^{-at}\left[\left(\int_0^\tau e^{as}\dot{y}(s)ds + C\right)^T Hx(\tau)\right]_0^t - e^{-at}\int_0^t\left(\int_0^\tau e^{as}\dot{y}(s)ds + C\right)^T H\dot{x}(\tau)d\tau \\
=& \left[\left(\int_0^\tau e^{-a(t-s)}\dot{y}(s)ds + Ce^{-at}\right)^T Hx(\tau)\right]_0^t - G(t), \qquad (3.37) \\
G(t) =& e^{-at}\int_0^t\left(\int_0^\tau e^{as}\dot{y}(s)ds + C\right)^T H\dot{x}(\tau)d\tau. \qquad (3.38)
\end{aligned}$$

Thus, as $a > 0$, and adding the first term in (3.32), we obtain

$$-\frac{s}{s+a}\left\{\frac{1}{2}y^T(\cdot)Hy(\cdot)\right\} + Q(\cdot) \approx \theta^{*T}\phi(\cdot) - G(\cdot). \tag{3.39}$$

Moreover, from (3.38), it is obtained that

$$|G(t)| \leq e^{-at}M_2\lambda_{max}(H)\epsilon\left[\frac{e^{at}-1}{a^2} + t\left(||C|| - \frac{1}{a}\right)\right] \tag{3.40}$$

Then the result follows from (3.30)–(3.40). ■

Then in the view of Theorem 3.1, we have the following result.

Theorem 3.2 *Suppose Assumption 3.2 holds, and there exist $\alpha_1, \alpha_2, T > 0$ such that $\forall t \geq 0$. Consider $z(t)$ and $\phi(t)$ defined in (3.9)–(3.14). Then $\hat{\theta}(t)$ in (3.23) obeys for some K obtained from $M_1, M_2, M_4, \gamma, a, T, \alpha_1$ and α_2, lim $sup_{t\to\infty}|\hat{\theta}(t) - \theta^*(t)| = K\epsilon$.*

Proof: Due to (3.23) there holds

$$\begin{aligned}\dot{\tilde{\theta}}(t) =&\dot{\hat{\theta}}(t) - \dot{\theta}^*(t)\\ =&\gamma\phi(t)(z(t) - \hat{\theta}^T(t)\phi(t)) - \dot{\theta}^*(t)\\ =&-\gamma\phi(t)\phi^T(t)\tilde{\theta}(t) + \gamma\phi(t)(z(t) - \theta^{*T}(t)\phi(t)) - \dot{\theta}^*(t)\\ =&-\gamma\phi(t)\phi^T(t)\tilde{\theta}(t) + G_2(t)\end{aligned} \tag{3.41}$$

where

$$G_2(t) = \gamma\phi(t)(z(t) - \theta^{*T}(t)\phi(t)) - \dot{\theta}^*(t). \tag{3.42}$$

Then because of Lemma 3.4, (3.4) and the fact that $\hat{\phi}(\cdot)$ is bounded, there exists a $K_5 > 0$ obtained from M_1, M_2M_4, γ and a, and an $M_6 : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, obeying $M_6(\cdot) \approx K_5\epsilon$ such that $|G_2(t)| \leq M_6(t)\forall t \geq 0$. Hence the result follows from the exponential asymptotic stability of (3.23). ■

3.4 Simulation Results

In this section, we provide simulation results to exhibit the performance of the proposed scheme in Section 3.2. For all examples, the state number, the adaptation gain and the filter pole are selected as $m = 2$(considering the localization of extremum in 2-D plane.), $\gamma = 1$ and $a = 0.5$, respectively and the signal field is formed as $F(y) = 3-(y-x)H(y-x)$ where the Hessian matrix is $H = \begin{bmatrix} 1 & 0.2 \\ 0.2 & 2 \end{bmatrix}$.

Scenario 3.1 *Assume the extremum location is at* $x = \begin{bmatrix} 1 & 2 \end{bmatrix}^T$ *and the sensory agent's trajectory is given by* $y = \begin{bmatrix} \sin(4t) + \sin(5t) & \sin(2t) + \sin(3t) \end{bmatrix}^T$. *Using the adaptive estimation algorithm* (3.23)*, the Hessian matrix and the source location estimates converge to their actual values exponentially as seen in Figure 3.1.*

Scenario 3.2 *Consider the same conditions in Scenario 3.1, but with white noise with variance(0.05) on* $F(t)$ *measurement of the sensory agent. Figure 3.2 displays that the localization is accomplished with some errors scaled with the noise magnitude.*

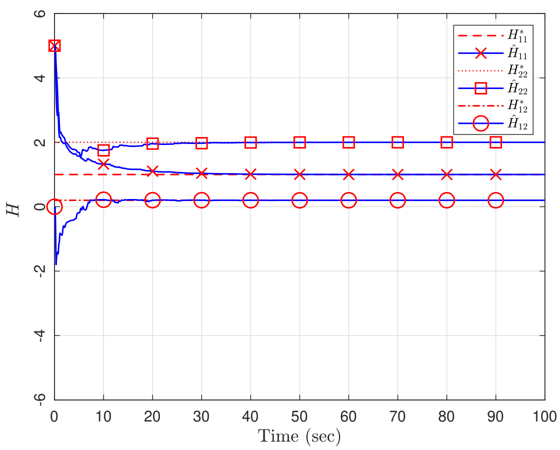
Scenario 3.3 *There is a slow drift movement in the location of extremum as* $x(t) = [1 + 0.5 \sin \frac{\pi}{750}t,\ 2 + 0.5 \sin \frac{\pi}{750}t]^T$*. As expected from Subsection 3.3.2, the simulation results in Figure 3.3 show that the adaptive estimation algorithm in* (3.23) *is applicable for the drift case.*

Scenario 3.4 *Combine the two circumstances in Scenarios 3.2 and 3.3. There is* $F(t)$ *measurement noise with variance(0.05) and drift in the location of extremum point as* $x(t) = [1 + 0.5 \sin \frac{\pi}{750}t,\ 2 + 0.5 \sin \frac{\pi}{750}t]^T$*. The simulation results in Figure 3.4 demonstrate the adaptive estimation algorithm in* (3.23) *works well despite the extremum location drift and noise in sensing.*

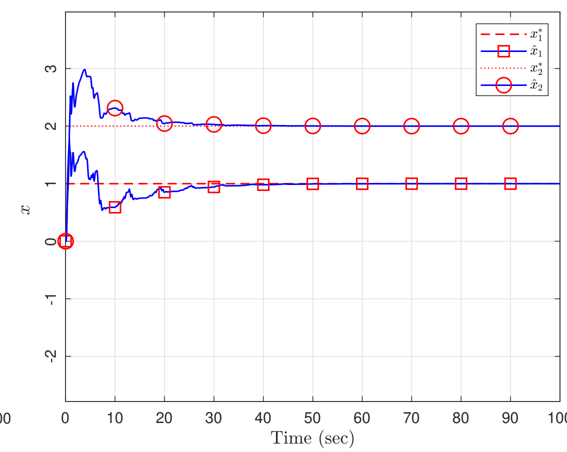
3.5 Summary and Concluding Remarks

In Chapter 3, we have designed an adaptive scheme for Hessian estimation and extremum localization of quadratic signal field functions by a sensory agent measuring the signal intensity. The proposed scheme is effective in extracting more detailed information about such signal fields and utilizes this information for more accurate and faster localization of the extremum. The stability of the proposed adaptive estimation and localization scheme has been proven for both stationary and slowly drifting extremum cases. Simulation results are presented in the presence of realistic measurement noise and drift in extremum location that exhibit the performance of the proposed scheme.

Implementation of the proposed scheme on cooperative systems of multiple mobile sensory agents is studied in the next chapter.



(a) H estimate of agent



(b) x estimate of agent

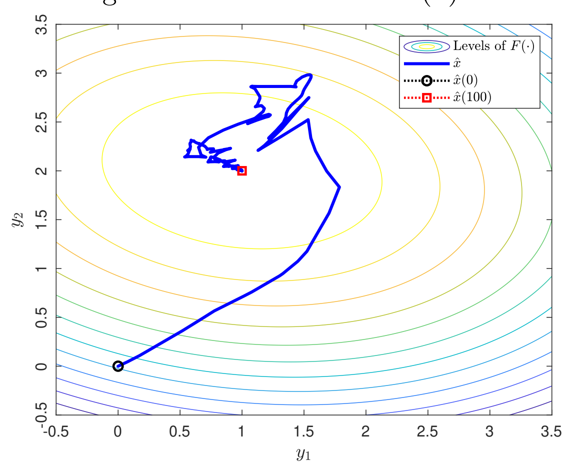
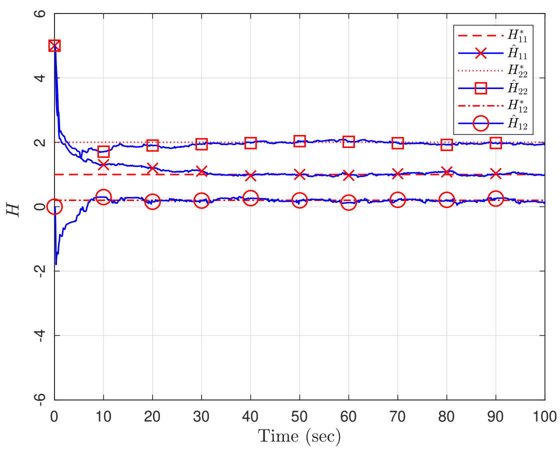
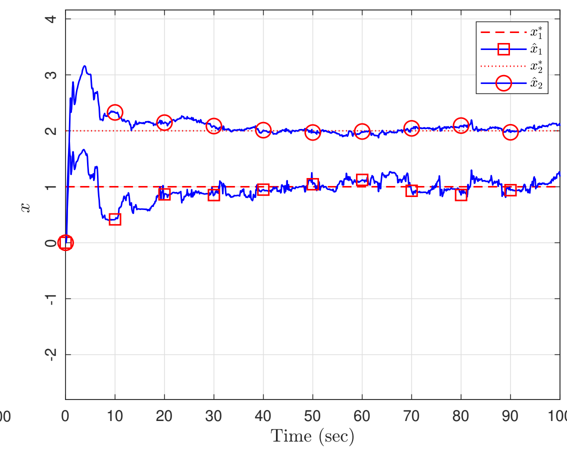


Figure 3.1: Location estimation for $x(t) = [1,\ 2]^T$, $y(t) = [\sin(4t) + \sin(5t),\ \sin(2t) + \sin(3t)]^T$, $a = 0.5$.



(a) H estimate of agent



(b) x estimate of agent

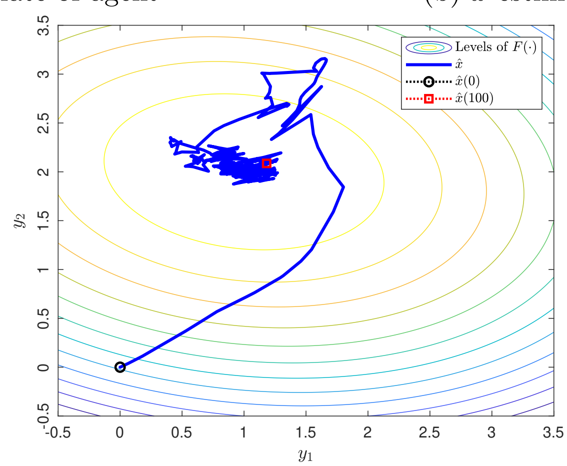


Figure 3.2: Location estimation for $x(t) = [1, 2]^T$, $y(t) = [\sin(4t)+\sin(5t),\ \sin(2t)+\sin(3t)]$. Noise in sensing the signal intensity with variance(0.05).

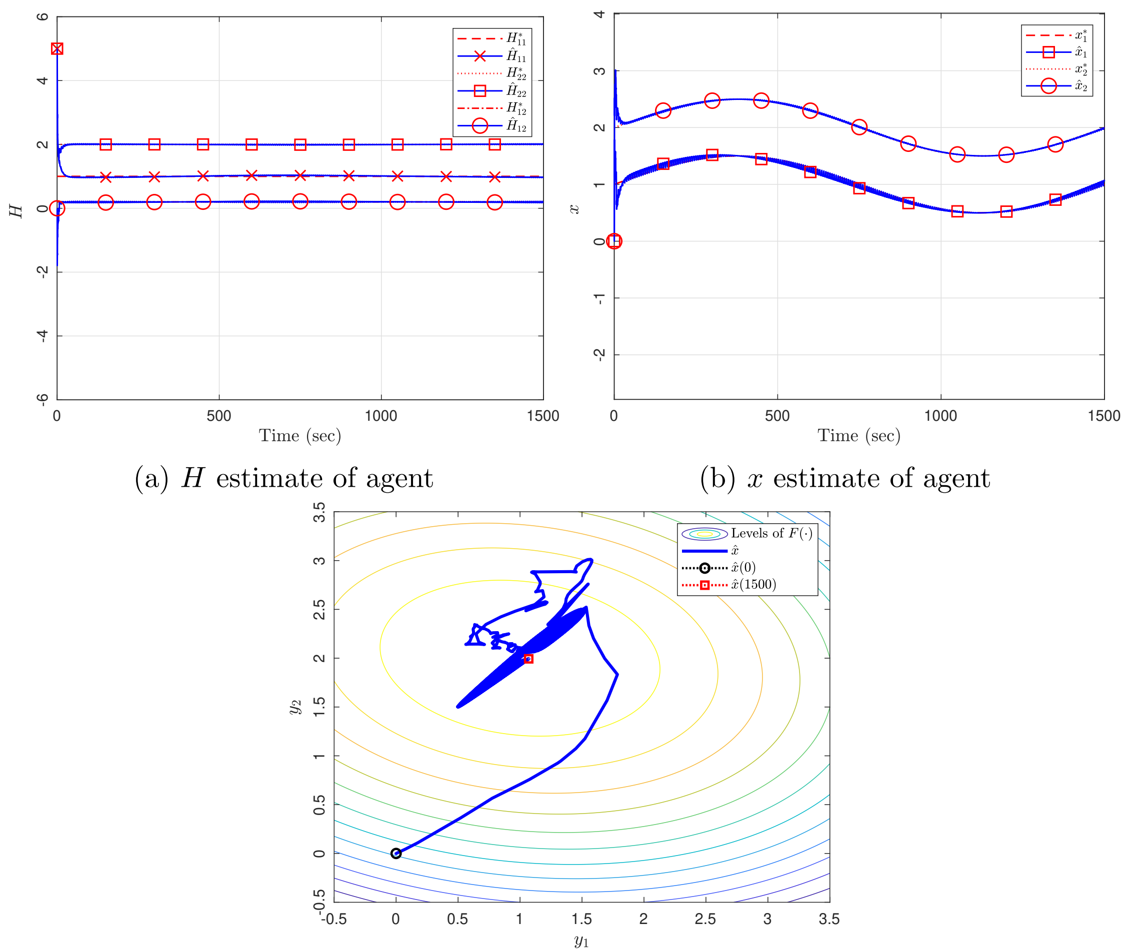
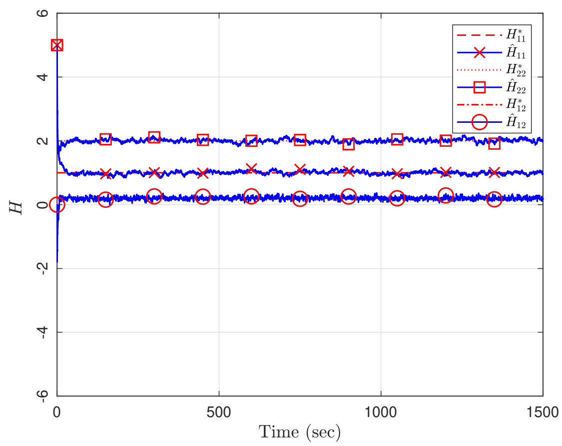
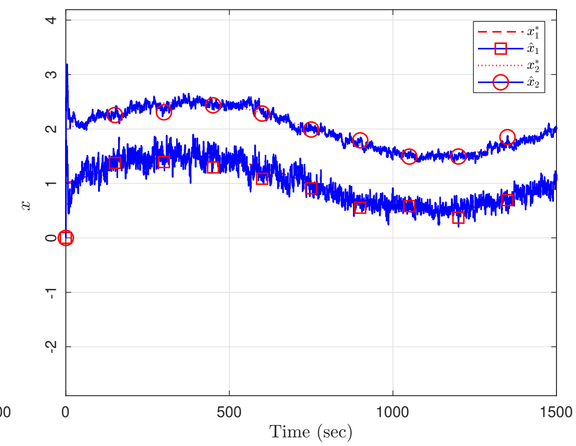


Figure 3.3: Location estimation for $x(t) = [1+0.5\sin\frac{\pi}{750}t,\ 2+0.5\sin\frac{\pi}{750}t]^T$, $y(t) = [\sin(4t)+\sin(5t),\ \sin(2t)+\sin(3t)]^T$, $a = 0.5$.



(a) H estimate of agent



(b) x estimate of agent

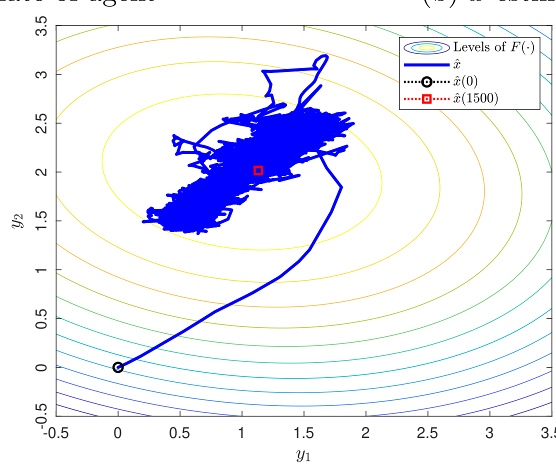


Figure 3.4: Location estimation for $x(t) = [1{+}0.5\sin\frac{\pi}{750}t,\ 2{+}0.5\sin\frac{\pi}{750}t]^T$, $y(t) = [\sin(4t){+}\sin(5t),\ \sin(2t){+}\sin(3t)]^T$, $a = 0.5$. Noise in sensing the signal intensity with variance(0.05).

Chapter 4

Multi Agent Adaptive Source Localization

Before presenting the algorithm for the multi sensory agents, we introduce the graph theoretical notions, define the localization problem of interest, and review the PE property in parameter identification. Here, we state the theorems and definitions in abridged form and refer to [52] for full versions of these theorems. We represent a network of n agents by a directed graph $G = (V, \varepsilon)$ of order n, which consists of a vertex set V of n elements and another set of edges, $\varepsilon \subset V \times V$. We index the elements of the vertex set with $\{1, ..., n\}$. A graph is called undirected if $(j, i) \in \varepsilon$ whenever $(i, j) \in \varepsilon$. A weighted graph is a triplet $D = (V, \varepsilon, A)$ where A stands for weighted adjacency matrix which has the following properties: for each i, j in the index set the entry $a_{ij} = a_{ji} = 1$ if $(i, j) \in \varepsilon$, and $a_{ij} = 0$ otherwise. Furthermore, we assume that $a_{ii} = 0$ for all vertices. d_i denotes the number of edges which is connected to the i^{th} vertex. We introduce the following lemma to use in the stability analysis.

Lemma 4.1 *The Laplacian Matrix*

$$L = diag(d_1, \cdots, d_n) - A \tag{4.1}$$

has the following properties

1. *$L1_n = 0_n$, i.e. 1_n is an eigenvector of matrix L which is associated with the zero eigenvalue.*

2. *For a connected graph G, the eigenvalues λ_i, $i \in \{2, \cdots, n\}$ are strictly positive.*

Problem 4.1 *Consider the quadratic signal field function in (3.1) and a network of N sensory agents which communicate through an undirected network graph $G = (\mathbb{V}, E)$ with vertex set $\mathbb{V}$ and edge set E. Suppose that each agent $i = \{1, \cdots, N\}$ has access to the measurement $F(y_i)$ of the target field at its current location y_i. Design a distributed identification scheme to estimate the target location x at which F takes its maximum value, and derive the conditions under which the estimates $\hat{x}_i(t)$ converges to x asymptotically.*

4.1 The Adaptive Hessian Estimation and Localization Scheme

The proposed distributed estimation algorithm is composed of n individual estimators, each in the form of Section 3.2, and a distributed consensus algorithm to produce a single common estimate $\hat{x}$ of the target location x, based on the indivudual estimates $\hat{x}_i$, $i = 1, \ldots, N$, of the agents.

To generate the individual estimates, the parametric modeling in Section 3.2 is followed. For each agent $i \in \{1, \ldots, N\}$ similarly to (3.9)-(3.14), the regressor signals are generated as follows:

$$\dot{\xi}_{1i}(t) = -a\xi_{1i}(t) + F(y_i(t)), \tag{4.2}$$

$$\xi_{1i}(0) = 0, \tag{4.3}$$

$$z_i(t) = -a\xi_{1i}(t) + F(y_i(t)), \tag{4.4}$$

$$\dot{\xi}_{2i}(t) = -a\xi_{2i}(t) + \Psi_i(t), \tag{4.5}$$

$$\xi_{2i}(0) = [0, \ldots, 0]^T \in \mathbb{R}^{\frac{m(m+3)}{2}}, \tag{4.6}$$

$$\phi_i(t) = -a\xi_{2i}(t) + \Psi_i(t). \tag{4.7}$$

The parametric models are obtained similarly to (3.15) as

$$z_i(\cdot) \approx \theta^{*T}\phi_i(\cdot). \tag{4.8}$$

Without introducing consensus,the individual estimator of agent with and without projection would be, respectively

$$\dot{\hat{\theta}}_i = \gamma\phi_i(z_i - \hat{\theta}_i^T\phi_i), \tag{4.9}$$

$$\dot{\hat{\theta}} = \underset{\hat{\theta}_H \in S_H}{\text{Proj}} \{\gamma\phi_i(z_i - \hat{\theta}_i^T\phi_i)\}, \tag{4.10}$$

In order to produce a common reliable estimate for all the agent, consensus is introduced to (4.9) and (4.10), leading to the distributed estimator without parameter projection and the distributed estimator with parameter projection

$$\dot{\hat{\theta}}_i = \gamma\phi_i\left(z_i - \hat{\theta}_i^\top\phi_i\right) + \sum_{j\in N_i}(\hat{\theta}_j - \hat{\theta}_i) \tag{4.11}$$

where N_i denotes the set of neighbors of Agent i.To extract information of the elements of H and the extremum location x, we partition $\hat{\theta}_i = \begin{bmatrix}\hat{\theta}_{Hi} & \hat{\theta}_{xi}\end{bmatrix}^T$ same as in (3.21), hence, we obtain the estimate of the extremum location as;

$$\hat{x}_i = \hat{H}_i^{-1}\hat{\theta}_{xi}, \tag{4.12}$$

where $\hat{H}_i$ is built using the entries of $\hat{\theta}_{Hi}$ Similar to the single agent case, a parameter projection is used to guarantee that $\hat{H}_i$ is always invertible and (4.11) is modified as follows:

$$\dot{\hat{\theta}}_i = \underset{\hat{\theta}_{Hi}\in S_H}{\operatorname{Proj}}\left\{\gamma\phi_i\left(z_i - \hat{\theta}_i^\top\phi_i\right) + \sum_{j\in N_i}(\hat{\theta}_j - \hat{\theta}_i)\right\}. \tag{4.13}$$

In the next section, we analyze the convergence properties of (4.13).

4.2 Stability and Convergence Analysis

(4.11), (4.13), respectively, imply

$$\dot{\tilde{\theta}}_i = -\gamma\phi_i\phi_i^\top\tilde{\theta}_i + \sum_{j\in N_i}(\tilde{\theta}_j - \tilde{\theta}_i), \tag{4.14}$$

$$\dot{\tilde{\theta}}_i = \underset{\hat{\theta}_{Hi}\in S_H}{\operatorname{Proj}}\left\{-\gamma\phi_i\phi_i^\top\tilde{\theta}_i + \sum_{j\in N_i}(\tilde{\theta}_j - \tilde{\theta}_i)\right\}, \tag{4.15}$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta^*$. Next, using the stack vector $\tilde{\Theta} = [\tilde{\theta}_1^\top, \cdots, \tilde{\theta}_n^\top]^\top$ and the definition of the Laplacian matrix, (4.14), (4.15), respectively, can be rewritten as

$$\frac{d}{dt}\tilde{\Theta} = -(L \otimes \mathbf{I}_n)\tilde{\Theta} - \gamma\Omega\tilde{\Theta}, \tag{4.16}$$

$$\frac{d}{dt}\tilde{\Theta} = \underset{\hat{\theta}_H \in S_H}{\text{Proj}} \left\{-(L \otimes \mathbf{I}_n)\tilde{\Theta} - \gamma\Omega\tilde{\Theta}\right\}, \tag{4.17}$$

where

$$\Omega = \begin{bmatrix} \phi_1\phi_1^\top & \cdots & 0 \\ \vdots & \cdots & \vdots \\ 0 & \cdots & \phi_n\phi_n^\top \end{bmatrix}. \tag{4.18}$$

The following lemma summarizes the main results we establish for (4.16):

Lemma 4.2 *Suppose that the directed graph G is connected and Assumption 3.2 holds. Then the adaptive law (4.11) guarantees the following:*

1. *The signals $\hat{\theta}_i$ are bounded for all $i = 1, \ldots, n$ and for all $t \geq 0$.*
2. *For all i, j, $\hat{\theta}_j(t) - \hat{\theta}_i(t)$ converges to 0 as $t \to \infty$.*
3. *If there exist positive real numbers $\alpha_1, \alpha_2 > 0$, $T > 0$ such that for all $t \geq 0$*

$$\alpha_1 I \leq \int_t^{t+T} \sum_{i=1}^{n} \phi_i(\tau)\phi_i^\top(\tau)d\tau \leq \alpha_2 I \tag{4.19}$$

holds, then $\tilde{\theta}_i(t) = \hat{\theta}_i(t) - \theta^*$ asymptotically converges to 0 as $t \to \infty$.

Proof The result is a direct corollary of Theorem 1 of [53].

Conjecture 4.1 *The estimation algorithm (4.13) with the defined projection possesses all the properties of (4.11) that are established in Lemma 4.2 and in addition guarantee that $\hat{\theta}_{Hi}(t) \in S_H$ for all $t \geq 0$, provided $\hat{\theta}_{Hi}(0) \in S_H$ and $\theta_H^* \in S_H$*

Proof The proof will be along the relevant analyses in [49, 50]. The effect of consensus however needs to be studied yet.

Remark 4.1 (4.19) *forms an idea of collective PE. For the network with a single agent,* (4.19) *reduces to* (3.29) *which is adequate to achieve θ^* convergence as in* (3.23) *where the extra consensus term in* (4.13) *has no role.*

Remark 4.2 *Even if not all ϕ_i are persistently exciting, a few of them are sufficient for* (4.19) *to hold due to its linearity of the integral. Moreover, persistence of excitation for* (4.19) *can be achieved when sum of each agent's motion satisfy the case for a single agent.*

4.3 Simulation Results

In this section, we present the simulation results of the proposed adaptive Hessian estimation algorithm for both single and multi sensory agents. We first set up the simulation using a field function with elliptical level sets of the form in plane ($m = 2$);

$$F(y) = F^* - (y - x)^\top H (y - x). \tag{4.20}$$

where $F^* = 3$, $x = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $H = \begin{bmatrix} 1 & 0.2 \\ 0.2 & 2 \end{bmatrix}$.The control gains values used for all simulations are $a = 0.5$, $\Gamma = 5I$. There are different scenarios we simulate to compare the performances of the single agent and the multi-agent cases.

Scenario 4.1 (single-agent): Assume the sensory agent's trajectory is as following

$$y_1 = \begin{bmatrix} \sin 2t + 1 & \sin 3t + 1 \end{bmatrix}^\top.$$

Using the adaptive estimation algorithm (3.23), the Hessian matrix and the source location estimates converge to their actual values exponentially as seen in Figure 4.1.

Scenario 4.2 (multi-agent): Assume a sensory agent moves in a trajectory as in Scenario 4.1. Now, we add 2 sensory in the network of agents where all the agents are connected each other. The trajectories of those agents are as $y_2 = \begin{bmatrix} -1 & \sin 4t + 1 \end{bmatrix}^\top$, $y_3 = \begin{bmatrix} \sin t - 1 & 1 \end{bmatrix}^\top$. Figure 4.2 displays that the localization is accomplished. When the results of the simulations for Scenarios 4.1 and 4.2 are compared, it can be seen that using multi-agent algorithm helps to localize the extremum faster.

Scenario 4.3 (single-agent): Assume the sensory agent's trajectory is as following

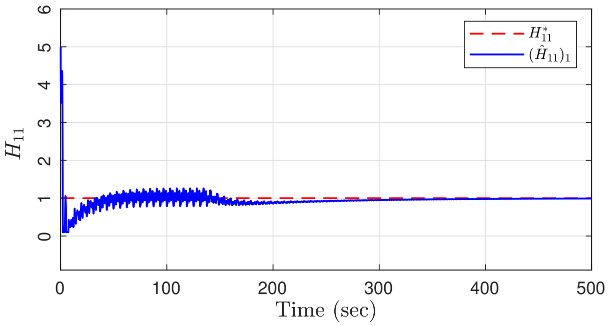
$$y_1 = \begin{bmatrix} \sin 2t + 1 & 1 \end{bmatrix}^\top.$$

Since the P.E. condition is not satisfied for the algorithm (3.23), the Hessian matrix and the source location estimates do not converge to their actual values as seen in Figure 4.3.

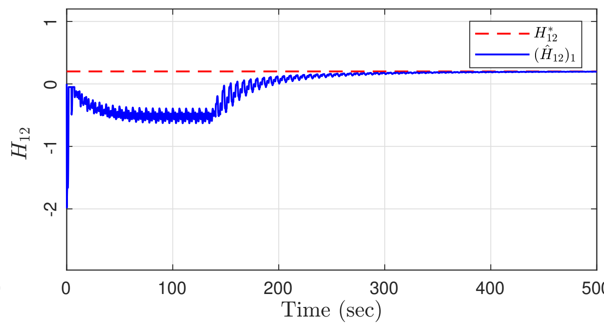
Scenario 4.4 (multi-agent): Assume a sensory agent moves in a trajectory as in Scenario 4.3. Now, we add 2 sensory in the network of agents where all the agents are connected each other. The trajectories of those agents are as $y_2 = \begin{bmatrix} -1 & \sin 4t + 1 \end{bmatrix}^\top$, $y_3 = \begin{bmatrix} \sin t - 1 & 1 \end{bmatrix}^\top$. Figure 4.4 displays the results of the simulation. Contrary to Scenario 4.3, the localization task is accomplished due to the fact that the P.E. condition is distributed among the agents.

Scenario 4.5 (single-agent): Assume the same case in Scenario 4.1.Now, there is a slow drift movement in the location of extremum as $x(t) = \begin{bmatrix} 3 + 0.5 \sin \frac{\pi}{1000} t, & 2 + 0.5 \sin \frac{\pi}{1000} t \end{bmatrix}^\top$. As expected from Subsection 3.3.2, the simulation results in Figure 4.5 show that the adaptive estimation algorithm in (3.23) is applicable for the drift case.

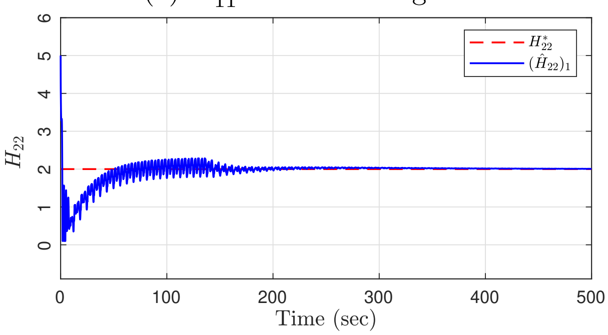
Scenario 4.6 (multi-agent): Assume the same case in Scenario 4.2.Now, there is a slow drift movement in the location of extremum as $x(t) = \begin{bmatrix} 3 + 0.5 \sin \frac{\pi}{1000} t, & 2 + 0.5 \sin \frac{\pi}{1000} t \end{bmatrix}^\top$. The simulation results in Figure 4.6 show that the adaptive estimation algorithm in (4.13) is applicable for the drift case.



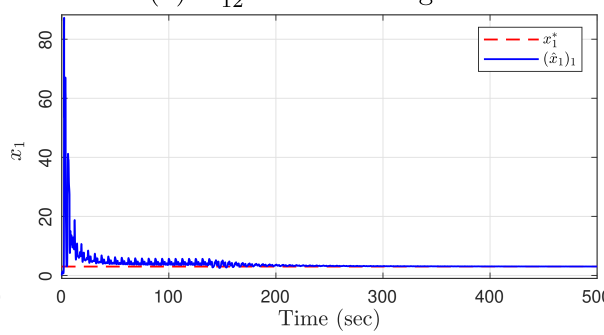
(a) H^*_{11} estimate of agent



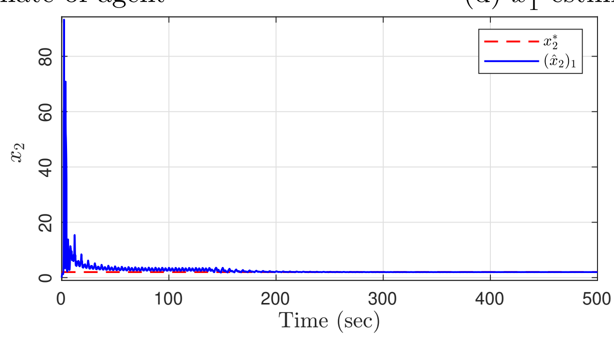
(b) H^*_{12} estimate of agent



(c) H^*_{22} estimate of agent

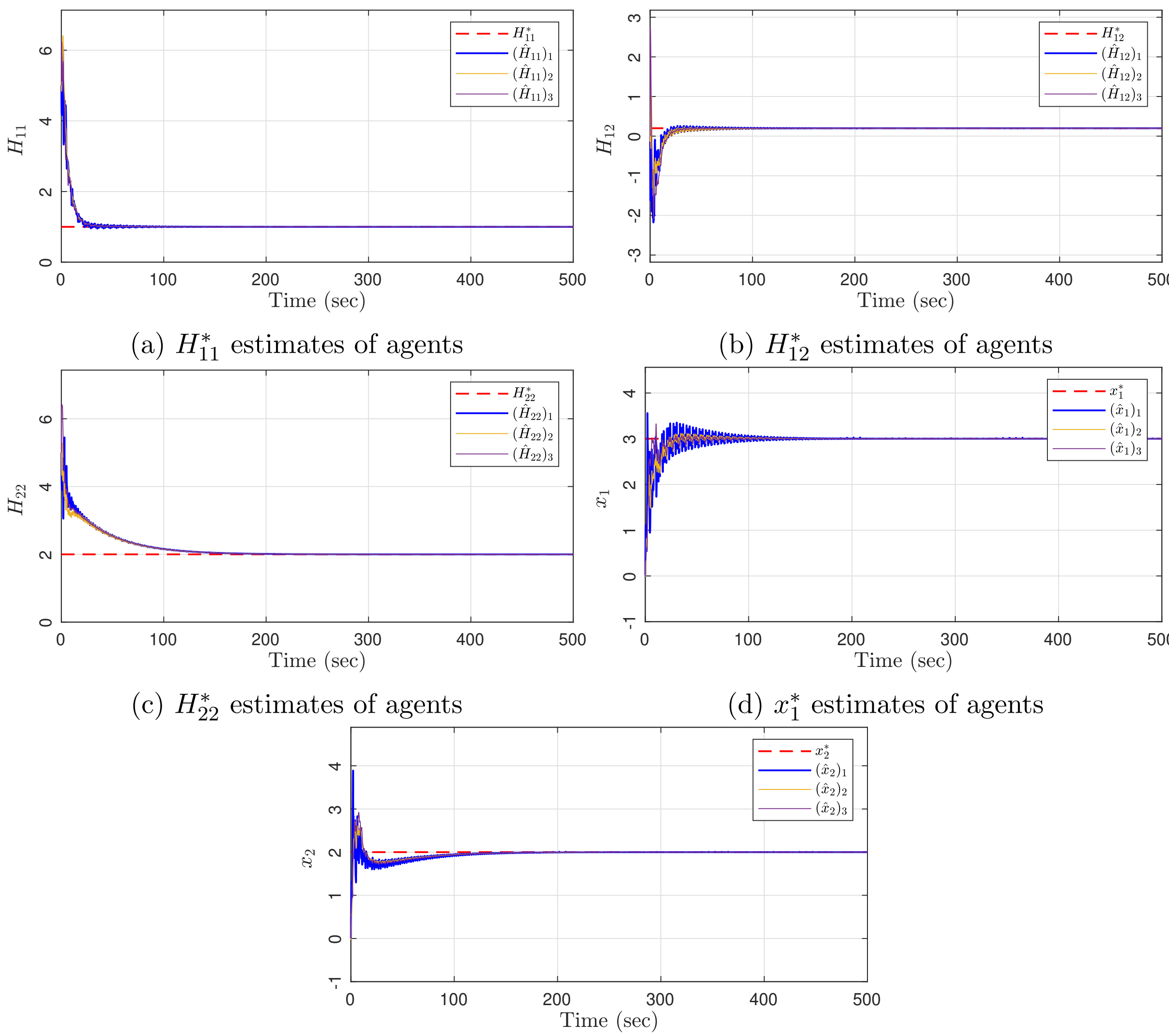


(d) x^*_1 estimate of agent



(e) x^*_2 estimate of agent

Figure 4.1: Simulation Results of Scenario 4.1



(a) H_{11}^* estimates of agents

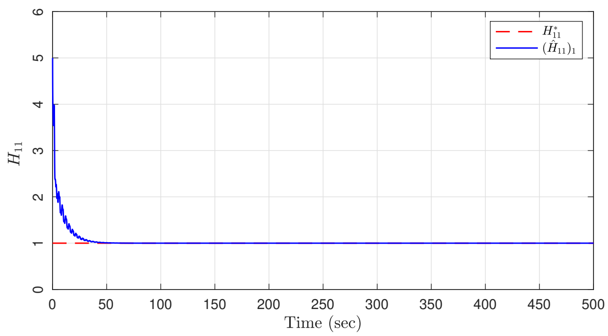
(b) H_{12}^* estimates of agents

(c) H_{22}^* estimates of agents

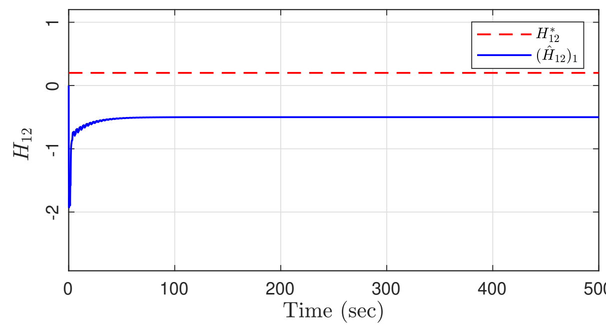
(d) x_1^* estimates of agents

(e) x_2^* estimates of agents

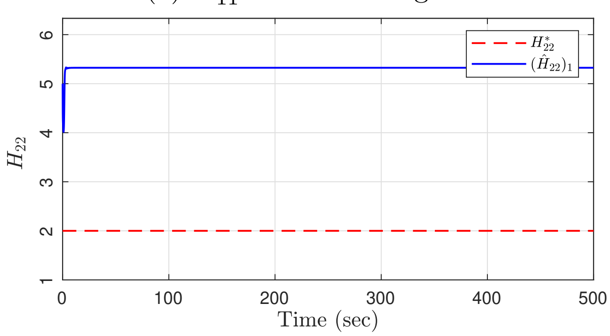
Figure 4.2: Simulation Results of Scenario 4.2



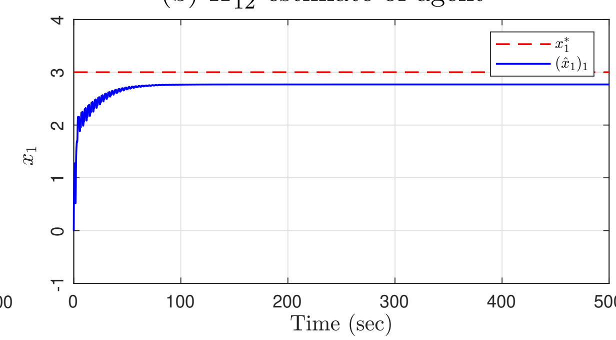
(a) H_{11}^* estimate of agent



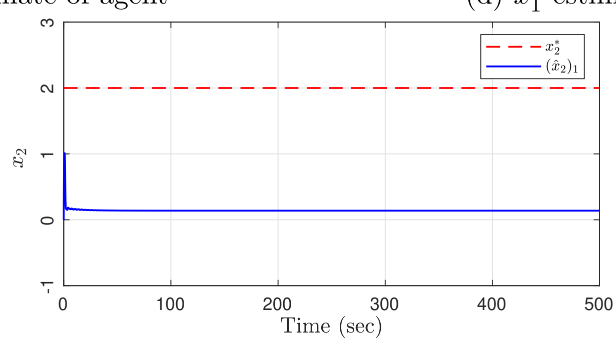
(b) H_{12}^* estimate of agent



(c) H_{22}^* estimate of agent

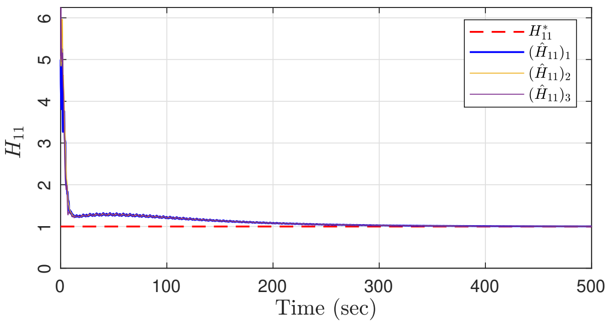


(d) x_1^* estimate of agent

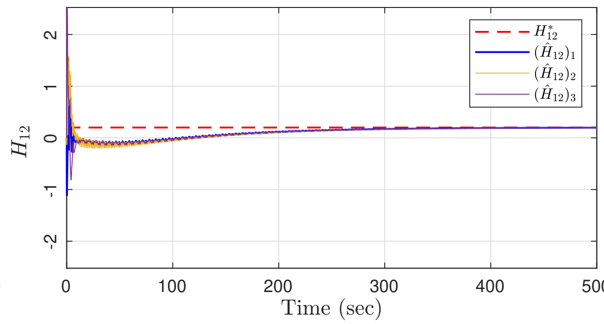


(e) x_2^* estimate of agent

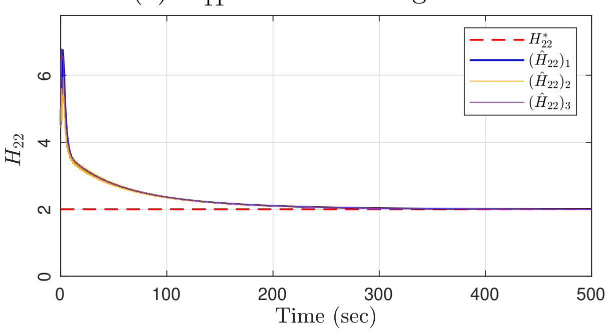
Figure 4.3: Simulation Results of Scenario 4.3



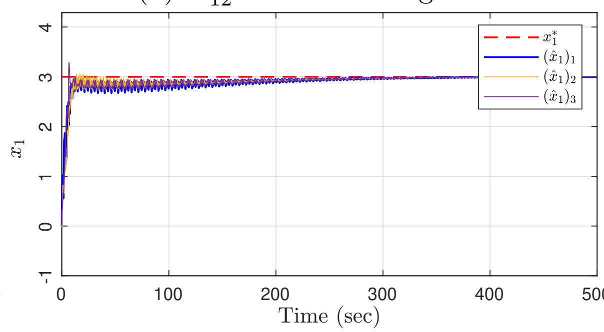
(a) H_{11}^* estimates of agents



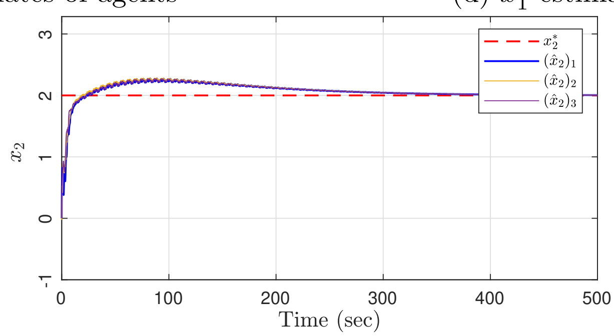
(b) H_{12}^* estimates of agents



(c) H_{22}^* estimates of agents

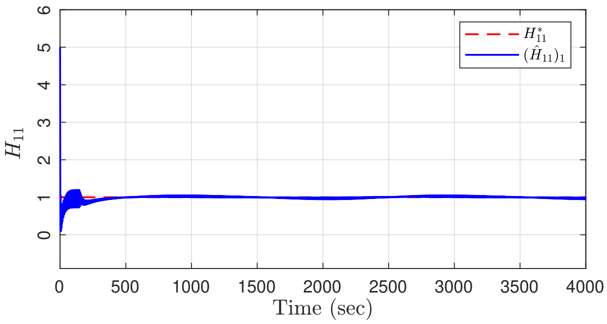


(d) x_1^* estimates of agents

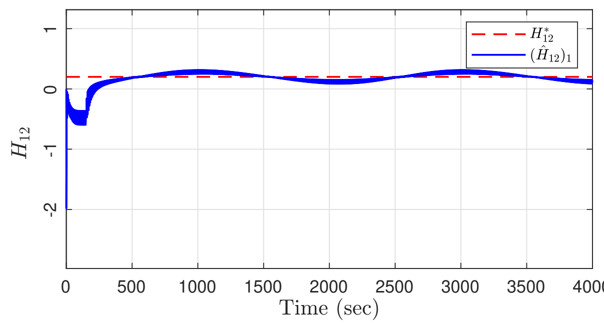


(e) x_2^* estimates of agents

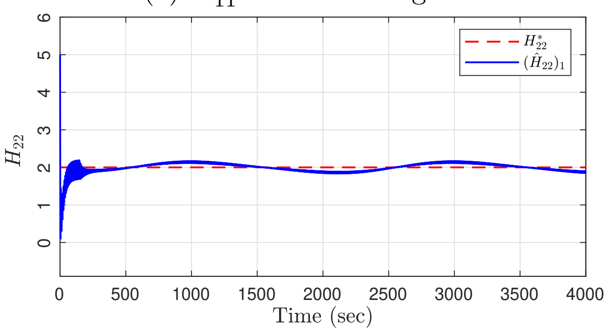
Figure 4.4: Simulation Results of Scenario 4.4



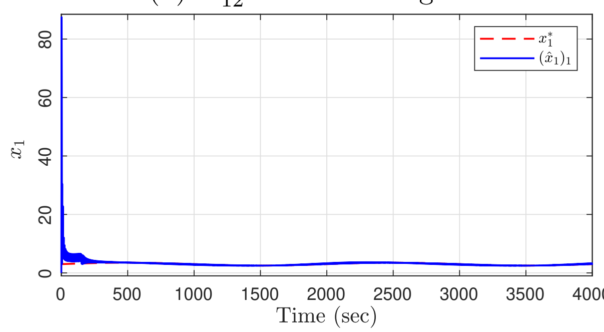
(a) H_{11}^* estimate of agent



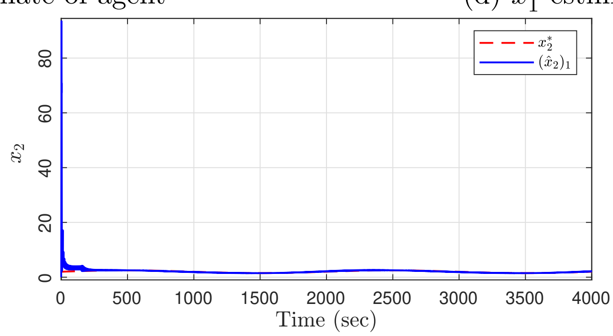
(b) H_{12}^* estimate of agent



(c) H_{22}^* estimate of agent

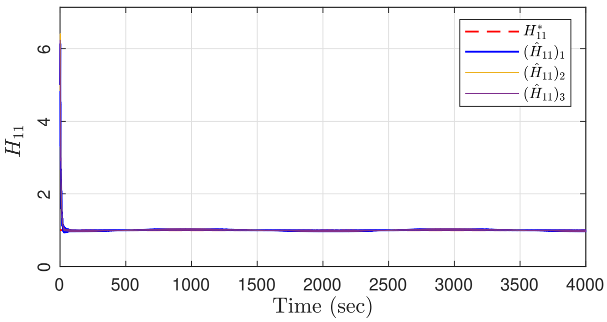


(d) x_1^* estimate of agent

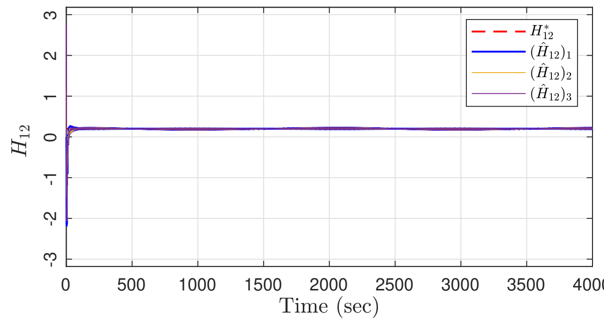


(e) x_2^* estimate of agent

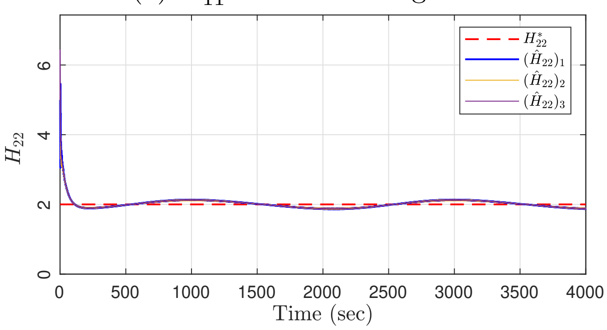
Figure 4.5: Simulation Results of Scenario 4.5



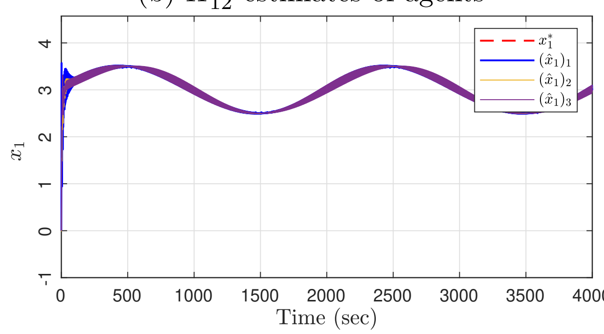
(a) H_{11}^* estimates of agents



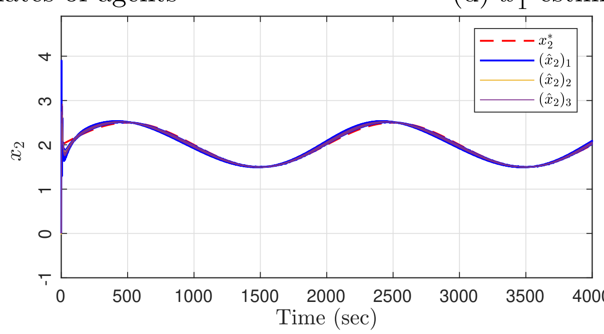
(b) H_{12}^* estimates of agents



(c) H_{22}^* estimates of agents



(d) x_1^* estimates of agents



(e) x_2^* estimates of agents

Figure 4.6: Simulation Results of Scenario 4.6

4.4 Summary and Concluding Remarks

In Chapter 4, we have studies continuous time distributed adaptive extremum localization of an arbitrary quadratic function $F(\cdot)$ via a network of multiple mobile sensory agents based on Hessian estimation, as an extension of Chapter 3. A consensus term is added to the base adaptive laws, distributed implementation of the estimation scheme introduced in Chapter 3, to obtain enhanced estimation cooperatively. Stability and convergence analysis of the proposed scheme is discussed, focusing on the effects of use of the cooperative multi-agent setting on persistence of excitation of the regressor signals and robust convergence of the Hessian parameter and location estimates to their true values. In particular, we have shown that for a network of connected agents, the persistence of excitation requirements can be distributed to the agents so that the requirement on each agent is more relaxed and feasible.

Ongoing and future related research directions include experimentation of the proposed scheme on indoor mobile robots, particularly QBot-2 ground robots [54].

Chapter 5

Adaptive Steering Control of Agents Towards the Source

In this chapter, we design an adaptive motion control scheme for steering a mobile sensory agent moving in 2D towards the source of a signal field $F(.)$ using the signal intensity the agent continuously measures at its current location. The signal field $F(.)$ is modeled to be a quadratic function of location, and has its extremum (maximum) at the signal source location x. The proposed adaptive control design is based on on-line estimation of the Hessian parameters of $F(.)$ and the extremum location in Chapter 3. Some simulation test results are displayed to verify the established properties of the proposed scheme as well as robustness to signal measurement noise.

5.1 Problem Definition

The main task of this chapter is to have a sensory agent A search for and move on the plane towards a signal source located at an unknown position $x \in \mathbb{R}^2$. For the sake of the extremum seeking algorithm, we consider a velocity integrator kinematic model of the agent motion for the purpose of a control design to produce the required velocity v, as

$$\dot{y}(t) = v(t) \tag{5.1}$$

where $y(t), v(t) \in \mathbb{R}^2$ are the position and velocity vectors of the agent, respectively.

The signal strength at any point $y(t)$ due to the signal source at x is denoted by $F(y)$, where $F(\cdot) : \mathbb{R} \to [0, \infty)$ is an unknown function which satisfies the following assumptions, similar to [32].

Assumption 5.1 *i The function F, its gradient $\nabla F = \begin{bmatrix} \partial_1 F & \partial_2 F \end{bmatrix}^T = \begin{bmatrix} \frac{\partial F}{\partial y_x} & \frac{\partial F}{\partial y_y} \end{bmatrix}^T$, and its Hessian $\nabla^2 F = \begin{bmatrix} \partial_{xx} F & \partial_{xy} F \\ \partial_{yx} F & \partial_{yy} F \end{bmatrix}^T = \begin{bmatrix} \frac{\partial^2 F}{\partial y_x^2} & \frac{\partial^2 F}{\partial y_x \partial y_y} \\ \frac{\partial^2 F}{\partial y_y \partial y_x} & \frac{\partial^2 F}{\partial y_y^2} \end{bmatrix}^T$ are continuous and the entries of $\nabla^2 F$ are continuously differentiable.*

ii The function F has a single maximum at point $x \in \mathbb{R}$.

iii There exists a scalar $M_H > 0$ s.t. $\forall y \in \mathbb{R}^2$, $||\nabla^2 F(y)|| \leq M_H$.

iv At x, $\nabla^2 F(x)$ is negative definite.

v There exists a scalar $L_H > 0$ s.t. $\forall y_1, y_2 \in \mathbb{R}^2$, $||\nabla^2 F(y_1) - \nabla^2 F(y_2)|| \leq L_H ||y_1 - y_2||$.

vi There exists a scalar $G_H > 0$ s.t. $\forall y_1, y_2 \in \mathbb{R}^2$, $||\nabla^2 F(y_1) - \nabla^2 F(y_2)|| \leq G_H$.

Next, we formulate the adaptive steering control problem explained above.

Problem 5.1 *Consider a signal field $F(\cdot)$ which satisfies Assumption 5.1. Suppose that a sensory agent A with motion kinematics (5.1) has continuous access to the field measurement $F(y)$ at its current location y. Design an adaptive identification scheme to estimate the target location x at which F takes its maximum value, and an adaptive motion control scheme to steer the agent A to the target location x, i.e., to have $y(t)$ asymptotically converge to x.*

5.2 Proposed Adaptive Extremum Seeking Control Scheme

We approach Problem 5.1 using a parameter identifier based adaptive control framework [50], formulating the localization of the source by the mobile agent A as a parameter identification problem and designing gradient and RLS based adaptive localization algorithms to produce the estimate $\hat{x}$ of the source location of x, as in [55]. This estimate is fed to the adaptive motion control law of the agent A, which is designed in a way to move the mobile agent towards $\hat{x}$.

The structure of the proposed extremum seeking control scheme is shown in Figure 5.1. Designs of the two key components of the proposed scheme, the adaptive extremum localization algorithm and the motion control law, are explained in detail in Section 5.3 and Section 5.5, respectively.

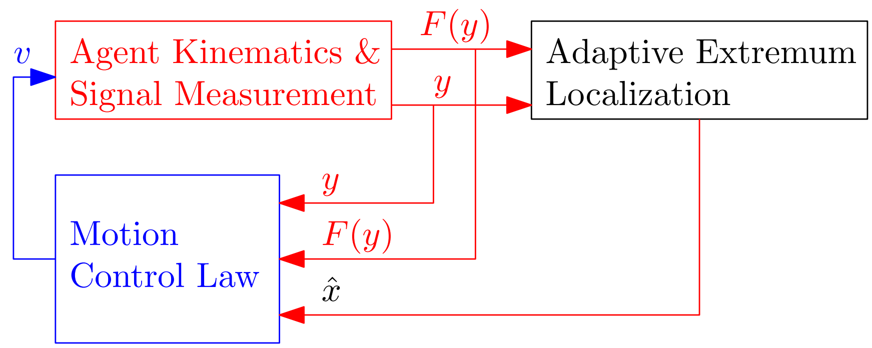


Figure 5.1: Structure of the proposed extremum seeking control scheme.

5.3 Adaptive Extremum Localization

For adaptive extremum localization, we use the adaptive Hessian estimation and localization scheme designed in Chapter 3. For a general nonlinear smooth function $F(\cdot)$, its gradient $\nabla F(y)$ will vanish at the extremum point, we can write [46]:

$$F(y) = F(x) + \frac{1}{2}(y-x)^T \nabla^2 F(x)(y-x) + \text{h.o.t.} \tag{5.2}$$

where y and x are the locations of the agent and the source, respectively. Using (5.2) and neglecting the h.o.t. , we derive a parametric model that is linear in unknown parameters of the system, i.e., the elements of Hessian matrix H and the location(state) x of the extremum as in [55]. Approximating $F(y)$ as a quadratic function such that

$$F(y) \approx c_1 - \frac{1}{2}(y-x)^T H (y-x) \tag{5.3}$$

and taking time derivative of (5.3) with the assumption of constant x, we get

$$\begin{aligned} \dot{F}(y) \approx & -\dot{y}^T H(y-x) = -\dot{y}^T H y + \dot{y}^T H x \\ = & -\frac{1}{2}\frac{d}{dt}\left(y^T H y\right) + \frac{d}{dt}\left(y^T\right) Hx \\ = & -\frac{1}{2}\frac{d}{dt}\left(H_{11}y_1^2 + 2H_{12}y_1y_2 + H_{22}y_2^2\right) \\ & + \frac{d}{dt}\left(y^T\right) Hx \end{aligned} \tag{5.4}$$

which can be written as

$$\dot{F}(y) \approx \bar{z}(t) = \theta^{*T}\frac{d\Psi}{dt}, \tag{5.5}$$

$$\theta^* = \Big[H_{11}, H_{12}, H_{22}, \underbrace{x^T H_1, x^T H_2}_{x^T H}\Big]^T \in \mathbb{R}^5, \tag{5.6}$$

$$\Psi = \Big[\ \ \tfrac{-1}{2} y_1^2, -y_1 y_2, \tfrac{-1}{2} y_2^2, y^T\Big]^T \in \mathbb{R}^5, \tag{5.7}$$

where H_i denotes the ith column (= transpose of the ith row) of H. We propose the following base gradient algorithm to generate the estimate $\hat{\theta}$ of θ^*:

$$\begin{aligned}\dot{\hat{\theta}}(t) &= \gamma\left(\bar{z}(t) - \hat{\bar{z}}(t)\right)\bar{\phi}(t)\\ \hat{\bar{z}}(t) &= \hat{\theta}^\top(t)\bar{\phi}(t)\end{aligned} \tag{5.8}$$

It is well known that in (5.8), $\hat{\theta}(t)$ converges exponentially to θ^*, provided $\bar{\phi}(t)$ is sufficiently persistently exciting. In order to eliminate need for explicit differentiation of available signals, $z(\cdot)$ and $\phi(\cdot)$ are introduced as the state variable filtered versions of $F(\cdot)$ and $\Psi(\cdot)$, as given in (3.9)–(3.14), but constrained to the specific case for "m=2".

Using (3.15) as linear parametric model, and (3.9)–(3.14) to generate the regressor signals in this model, we design the following gradient based adaptive estimation algorithm [50, 49] to identify θ^*:

$$\dot{\hat{\theta}} = \gamma\phi(z - \hat{\theta}^T\phi), \tag{5.9}$$

where $\hat{\theta}$ denotes the estimate of θ^* and $\gamma > 0$ is a scalar design constant. To extract the information of the elements of H and the location(state) of the source (x) from the estimate of θ^*, we consider the partitioning of θ^* and $\hat{\theta}$ in (3.21) ;

We generate the estimate $\hat{x}$ of the source location x, utilizing the equality $\theta_x^* = Hx$, as

$$\hat{x} = \hat{H}^{-1}\hat{\theta}_x. \tag{5.10}$$

In order to implement $\hat{H}$ in (5.10), $\hat{H}$ is required to be non-singular. This requirement is met applying parameter projection, assuming that H satisfies Assumption 3.1 for $m = 2$.

To assure that $\hat{H}$ is non-singular, we apply parameter projection on the elements of $\hat{\theta}_H$ based on Assumption 3.1, and (5.9) is modified to apply parameter projection as;

$$\dot{\hat{\theta}} = \underset{\hat{\theta}_H \in S_H}{\text{Proj}}\,\{\gamma\phi(z - \hat{\theta}^T\phi)\}, \tag{5.11}$$

where the convex compact set S_H is defined as the set of all vectors $\hat{\theta}_H = [\hat{H}_{11}, \hat{H}_{12}, \hat{H}_{22}]^T$ such that the corresponding 2×2 matrix $\hat{H}$ satisfies Assumption 3.1 for $m = 2$, and $\underset{\hat{\theta}_H \in S_H}{\text{Proj}} \{\cdot\}$ is the parameter projection operator [49, 50] defined to maintain $\hat{\theta}_H$ in S_H.

In the next section, we introduce an alternative adaptive extremum seeking scheme which uses Least Square(LS).

5.4 LS-based Extremum Seeking

In the alternative to the gradient based adaptive estimation algorithm (5.9) and (5.11), we propose the one based on LS as follow;

$$\dot{\hat{\theta}}(t) = P(t)\phi(z(t) - \hat{\theta}^T\phi(t)), \tag{5.12}$$

$$\dot{P}(t) = \Gamma P(t) - P(t)\phi(t)\phi(t)^\top P(t) \tag{5.13}$$

where $\Gamma > 0$ is the fixed forgetting factor and $P(0) > 0$ is the design matrix defining the scake of the pnalty on deviation from the initial estimate.

The LS based algorithm has the same stability and convergence properties as a gradient algorithm in Section 5.3. Its main advantage is to be capable of being fine-tuned for faster settling and or being less sensitive to measurement noises which make it more useful for the motion control law defined in the next section.

5.5 The Motion Control Law

The motion control law of the proposed extremum seeking control scheme depicted in Figure 5.1 is designed similarly to [27], with one main component aiming to drive $y(t)$ to the estimate $\hat{x}(t)$ of the target location x and one auxiliary component used to provide the PE level required in Theorem 3.1, as follows:

$$\dot{y}(t) = \dot{\hat{x}}(t) - \beta\left(y(t) - \hat{x}(t)\right) + \epsilon\left(F(y)\right) y_a(t) \tag{5.14}$$

where $\hat{x}$ is the position estimate of the source generated by (5.10) in Section 5.3 together with (5.9) or (3.23), or (5.12) $\beta > 0$ is a control gain constant. The auxiliary control signal is formed as

$$\epsilon\left(F(y)\right) y_a(t) \in \mathbb{R}^2, \tag{5.15}$$
$$y_a(t) = \dot{\sigma}_1(t) + \dot{\sigma}_2(t) + \dot{\sigma}_3(t), \tag{5.16}$$

with

$$\dot{\sigma}_i(t) = A_i \sigma_i(t) \text{ for } i = 1, 2, 3,$$
$$A_i = c_i \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \tag{5.17}$$

where c_1, c_2, c_3 are distinct positive numbers defining 3 distinct frequencies of $y_a(t)$, making it sufficiently rich of order 6 [50] which is one more than the size of the unknown constant parameter vector θ^*, so that the PE condition (3.29) of Theorem 3.1 is satisfied and

$$\epsilon(F(y)) = \bar{F} - F(y) \tag{5.18}$$

is used to attenuate the effect of the auxiliary signal as the agent comes closer to the source, where $\bar{F}$ is a known upper bound of $F(y(t))$ satisfying $\bar{F} \geq F^* = F(x)$. with the assumption that such upper bound is known.

The design idea of (5.14), (5.16) and (5.18) are inspired from Eqs. (3.1), (3.2) and (3.3) of [27], the difference comes from the kind of measurement, such that [27] uses the measurement of relative distance $\|y - x\|$ to the source, while we use the signal strength $F(y)$ measured at agent location y.

The selection of the auxiliary signal $y_a(t)$ in the form of (5.16) is to provide PE for estimation of θ^*. This is guaranteed along the following properties of (5.16), (5.17).

Lemma 5.1 *The auxiliary signal components (5.16), (5.17) satisfy the following:*

1. *A_i is skew-symmetric.*

2. *$\dot{\sigma}_i$ in (5.17) is PE for any arbitrary nonzero initial value $\sigma_i(0)$ such that there exist positive $T, \alpha_1, \alpha_2 > 0$ for all $t \geq 0$ to hold*

$$\alpha_1 ||\sigma(0)||^2 I \leq \int_t^{t+T} \dot{\sigma}(\tau)\dot{\sigma}(\tau)^\top d\tau \leq \alpha_2 ||\sigma(0)||^2 I. \tag{5.19}$$

3. *For every $\theta \in \mathbb{R}^n$, and every $t \in \mathbb{R}^+$, there exists $t_1(t, \theta) \in [t, t + T_1]$, dependent on θ and t, such that $\theta^\top \dot{\sigma}(t_1(t, \theta)) = 0$.*

4. *Along Lemma 3.1 of [27], considering A_i is skew-symmetric, $||\sigma(t)|| = ||\sigma(0)||$ for all $t \geq 0$.*

5.6 Simulation Results

In this section, we provide simulation results to exhibit the performance of the proposed scheme in Section 5.2. For all examples, $\gamma = 1$ and $a = 0.5$, respectively and the signal field is formed as $F(y) = 3 - (y-x)H(y-x)$ where the Hessian matrix is $H = \begin{bmatrix} 1 & 0.2 \\ 0.2 & 2 \end{bmatrix}$.

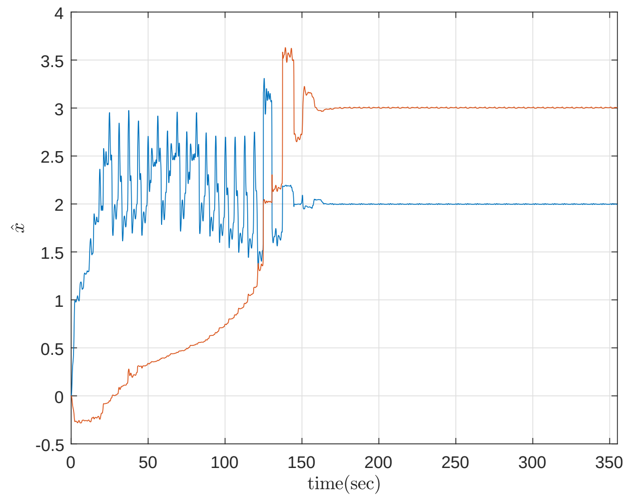
Scenario 1: Assume the extremum location is at $x = \begin{bmatrix} 1 & 2 \end{bmatrix}^T$. Using the adaptive estimation algorithm (5.11), and the motion control law in (5.14), the sensory agent can be steered to the source's location as seen in Figure 5.2.

Scenario 2: Consider the same conditions in Scenario 1, but with white noise with variance(0.05) on $F(t)$ measurement of the sensory age. Again, using the adaptive estimation algorithm (5.11), and the motion control law in (5.14), the sensory agent can be steered to the source's location as seen in Figure 5.3.

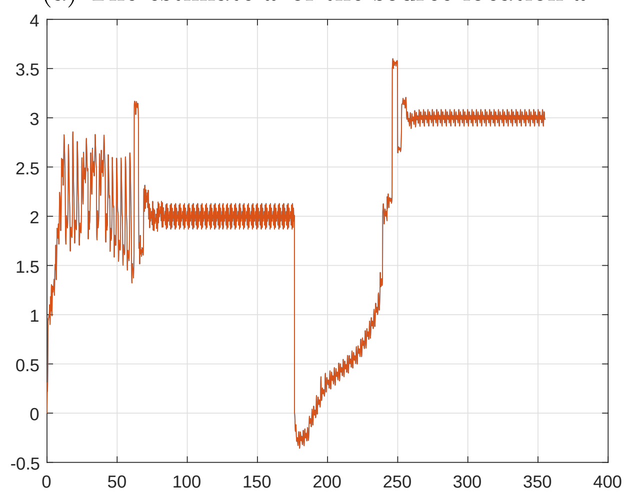
5.7 Summary and Concluding Remarks

In Chapter 5, we have designed an adaptive motion control scheme for steering a mobile sensory agent in 2D toward the source of a signal field $F(\cdot)$ using the signal intensity the agent continuously measures at its current location. The use of the localization scheme designed in Chapter 3 and its least-squares version is effective in extracting more detailed information about such signal fields and utilizing this information in more accurate and faster steering of the agent to the extremum. The stability of the proposed adaptive estimation and localization scheme and motion control law has been justified by the presented simulation results, as well as the robustness to measurement noises and the performance of the proposed scheme.

Ongoing and future related research directions include implementing the proposed scheme on autonomous vehicle and cooperative extensions of the design where more than one sensory agent are utilized.

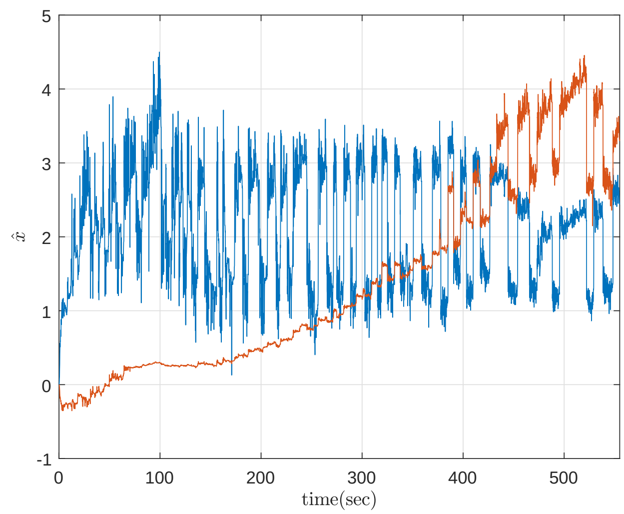


(a) The estimate $\hat{x}$ of the source location x

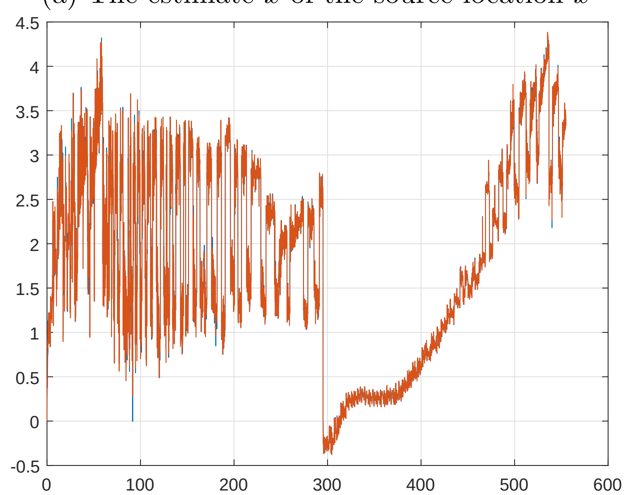


(b) The position y of the agent A

Figure 5.2: Location estimation for $x(t) = [3,\ 2]^T$, $a = 0.5$.



(a) The estimate $\hat{x}$ of the source location x



(b) The position y of the agent A

Figure 5.3: Location estimation for $x(t) = [3,\ 2]^T$, $a = 0.5$. Noise in sensing the signal intensity with variance(0.05).

Chapter 6

Conclusion

In this thesis, robust and practical adaptive extremum localization algorithms have been proposed for extremum localization of an arbitrary quadratic function $F(\cdot)$ based on Hessian estimation, using the measured signal intensity via a single mobile sensory agent as well as a network of multiple such agents. The function $F(\cdot)$, in practice, represents a signal field with a source located at the maximum point of $F(\cdot)$. Gradient and least-squares based adaptive Hessian parameter estimation and extremum localization schemes have been developed considering a linear parametric model of field variations. For the networked multi-agent case, a consensus term is added to the base adaptive laws to obtain enhanced estimation cooperatively. Stability and convergence analysis of the proposed scheme is provided for the single agent case, establishing asymptotic convergence of the Hessian parameters and the location estimate to their true values robustly, provided that the motion of the agent satisfies certain persistence of excitation (PE) conditions. For the case with a network of connected agents, similar results are partially established formally and justified via numerical solutions. Further for the multi-agent setting, it is demonstrated that the PE requirements can be distributed to the agents so that the requirement on each agent is more relaxed and feasible.

The potential future research directions include formal proof of the conjectural analysis results for multi-agent settings, further study of the optimality and optimization of the proposed adaptive algorithms and schemes, analysis of the effects of non-ideal characteristics of real-life signal fields and uncertainties and deviations from the assumed models. Another particular future research topic is real-time experimentation with mobile ground vehicles, particularly QBot-2 ground robots [54].

Later, an adaptive motion control scheme has been designed for steering a mobile sensory agent in 2D toward the source of the signal field modeled by $F(\cdot)$, using the signal intensity the agent continuously measures at its current location. The Hessian based localization scheme designed in the earlier part of the thesis is utilized to effectively extract more detailed information about such signal fields and to provide accurate and faster steering of the agent to the extremum. Simulation results are presented in the presence of realistic measurement noise that exhibit the performance of the proposed scheme.

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