Optics, Loss and Gravity

by

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Author's Declaration

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Abstract

We look at some ideas that can be found from making connections among optics, non-Hermitian dynamics and gravity. Using nonlinear optics, we demonstrate how loss can be used to make brighter sources of thermal light. Also, we simulate accelerating Unruh-DeWitt detectors in 1+1 spacetime using a nonlinear-optical setup engineered to have *variable dispersion*. In our last connection with gravity, we make a connection between a dynamic spacetime metric from linearized gravity and a spacetime-dependent refractive index from a linear-optical setup. From these connections with gravity, we argue that spacetime might be *emergent*, and that one should not quantize the metric to find a quantum gravity theory. Instead, we propose that gravity might have a gauge theory description and that the *problem of quantizing gravity is equivalent to that of quantizing a gauge field*. A gauge theory of gravity will make it possible to include this interaction into the Standard Model, and this inclusion may have implications for physics beyond the Standard Model.

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Dedication

To my two constants: grandmas, Margaret and Georgina.

Life is a game of chess. Checkmate, sometimes. Stalemate, most times. At times, just moving around. Confused. In a daze. -Diwa

Do not follow where the path may lead. Go instead where there is no path and leave a trail. -Ralph Waldo Emerson

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Chapter 1

Introduction

Making connections among different fields of physics has a long history of being a source of new concepts. Ranging from Newton's gravitational law, electromagnetism, special relativity (SR), general relativity (GR) to quantum theory, the benefits of connecting seemingly disjoint ideas about nature leads to much insight about the workings of reality. For Newton, his realization that the laws that govern the motion of objects on earth are the very same ones that celestial bodies obey, thus unifying terrestrial and celestial mechanics, led to the concept of the gravitational force. Maxwell's insight in connecting electricity with magnetism into one propagating electromagnetic wave that always travels at the speed of light in vacuum made him realize that light is an electromagnetic disturbance. SR combines space and time into a single interwoven fabric known as spacetime and this connection explains the constant speed of light in vacuum. After formulating SR, Einstein went on to connect the curvature of the spacetime manifold with gravitational phenomena, giving gravity a neat geometrical formulation and making it consistent with Maxwell's laws i.e., ensuring the gravitational interaction does not propagate at a speed exceeding that of light. Combining particle mechanics and wave theory led to the development of quantum theory which introduced new concepts like uncertainty in physics, a field which hitherto prided itself on determinism (predictive power of mechanics).

In this thesis, we will consider making connections between optics in dielectric materials and other nonlinear optical systems in order to address the problems: what role does the metric play as a degree of freedom for gravity i.e., is it fundamental or not, how can loss in nonlinear optics be useful and how to simulate gravitational phenomena in the lab using nonlinear optics. In these optical systems, we speak of *spatial evolution* rather than *temporal evolution*, with time, *like position in temporal evolution*, becoming an observable.

There is no current consensus on the issue of viewing time as an observable of a system, with famous objections coming from Pauli where it is argued that one cannot have a self-adjoint time observable and also get an energy spectrum that is bounded from below [2]. Pauli's argument against the existence of a time observable is not the final verdict on the matter and there have been counter-claims to it, see [3]. Time is, instead, commonly viewed as external to the system. This view is held both in classical and quantum mechanics [4], where time just labels events without actively participating in any dynamics. This spectator attribute of time conflicts with what general relativity tells us about it, namely, spacetime is dynamic. The resulting dissonance is called the "problem of time". We do not claim to have addressed this problem, however, we propose that to describe spatial evolution in quantum systems, it is *necessary* for time to be an observable of such systems. Besides the time observable, the connections made here tell us something about how loss in optical systems can be useful.

Photon loss in optical systems is generally considered a bad thing, but we argue here that it can have benefits as well. As discussed in [5], loss can have detrimental effects on quantum properties, such as coherence, of systems that are being studied in the lab. Loss also accounts for the attenuation of signal strength that occurs in its transmission through waveguides or optical fibres, and this again has consequences that affect investigations about the quantum properties of light [6]. Here, we will show how loss can be used to make brighter sources of thermal light using nonlinear optics in the context of quantum optics. Our work complements a similar result found in a classical context in [1], so that one may conclude that loss has benefits for making brighter sources of both classical and quantum light. Returning to gravity, we find that the optics-gravity connections made here allow us to draw some conclusions about what a quantum gravity theory is *potentially not*.

The currently accepted theory of gravity is general relativity, where a quantity known as the metric, which is a property of spacetime, is dynamical [7]. Though a successful gravity theory, it breaks down in regions of extreme gravity, such as at the center of blackholes and at the big bang. In these regions, it is believed a quantum gravity theory is needed to replace its classical predecessor, general relativity. Finding such a theory has been challenging and it remains an outstanding problem to this day, even though there have been many attempts, see [8] for a nice survey. Here, we use our connection between a linear-optical system and gravity to deduce that the metric must be viewed as an *emergent* quantity, like the temperature of a thermodynamical system, as opposed to something *fundamental*. This new view suggests that we should not be quantizing the metric to find a quantum gravity theory. The linear-optical-gravity connection requires the notion of a *gravitational refractive index*.

In linear optics, the refractive index characterizes the propagation of light in a dielectric medium. In particular, it describes how light "slows down" as it travels through such media and it does so in different amounts depending on things like frequency, polarization, etc. [9]. Connecting the refractive index with *static* spacetime has been done before, see [10], however, here we connect the refractive index with *dynamical* spacetime and this connection introduces the notion of a gravitational refractive index (GRI). This GRI differs from the optical refractive index by being independent of the properties of light, such as frequency or polarization. In other words, bodies falling *independent of mass*, translates in the optical setting from our connection, as light falling independent of *frequency or polarization*. Some of these connections between optics and other fields rely on a description of spatial evolution in optical systems.

We show here, in Chapter 2, that when systems evolve in space, energy and time become new phase space variables. We do this in two ways: classically and using quantum mechanics. Classically, we write down a new action, which is defined in-terms of momentum and space rather than energy and time. From this new action, we obtain spatial analogues of Euler-Lagrange and Hamilton-Jacobi equations. Using quantum mechanics, we generalize from unitary time (temporal)-evolution to unitary spacetime evolution. This generalization gives a system of four equations, with Schrödinger's equation, describing temporal evolution, being among them and the remaining ones describe spatial evolution. Using standard quantization procedure from classical to quantum mechanics connects our two approaches with momentum viewed as a generator of spatial evolution in a phase space coordinatized using energy and time. As an application of the results in this chapter, we find an optical analogue of the adiabatic theorem and its associated optical analogue of the dynamical phase which play an important role in Chapter 3.

The connection with gravity is done in two ways, in the first instance we connect quantum optics with semi-classical gravity in Chapter 3. Semi-classical gravity is considered an approximation of quantum gravity where we have quantum fields in a classical gravitational background. We simulate a well-known effect in semi-classical gravity known as the Unruh effect. In particular, we show that a certain nonlinear optical process can be used to simulate Unruh-DeWitt detectors (these detectors are used to demonstrate the Unruh effect). The simulation makes it possible to represent accelerating detectors in optical systems using variable dispersion with the further implication that gravitational phenomena can be simulated in optical systems because of the equivalence principle. This has potential applications in testing gravitational phenomena on earth that would otherwise be difficult to do, such as studying event horizon physics without actually making one or needing one in close proximity. We also argued that gravity might have a gauge theory description based on the connection between our optical system and the Unruh-DeWitt system. This last point hints that the problem of quantizing gravity is a problem of quantizing a gauge field. We do not find a gauge theory of gravity here, instead we motivate why it might exist; see here for a survey of attempts to find such a theory [11, 12]. From here, we digress to look at a possible use of non-Hermitian quantum systems.

As a by-product of the results in Chapter 2, we make a connection between optics and non-Hermitian quantum systems in Chapter 4. Here, we show that photon loss, in nonlinear optics, can be used to increase the brightness of pseudothermal light. Light from nonlinear optical sources are at their brightest when certain conditions are met. These conditions are known as phasematching conditions [9]. However, in practice the materials used to produce these sources of light may not make it feasible to implement such phasematching conditions, at least for a desired frequency range, so that for such materials a way of increasing the brightness of the generated light is needed. Here is where our work with loss comes in to help address the problem. Beyond loss, we also look at what the connection between quantum optics and gravity can tell us about gravity at the quantum scale.

The other connection with gravity can be found in Chapter 5, where we make a connection between quantum optics and the weak-field limit of gravity, also known as linearized gravity. This last connection reveals that the spacetime metric can be thought of as a refractive index of some optical material in a way that the dynamics of such a refractive index obeys the Einstein field equations and thus captures the full character of GR spacetime, at least in the weak-field limit. One consequence of making this connection is that it makes clear the point that we should not quantize the metric to find a quantum gravity theory just as we do not quantize the refractive index to do quantum optics. Furthermore, via our connection, we find a relation that interprets the radiation stress-energy tensor as describing photon "mass".

The conclusions drawn here suggest there is much that can be learned about gravity using our understanding of optics. From potentially testing gravitational ideas in the lab using optics to shedding light on what a quantum gravity theory might look like, it is our fervent hope that the ideas presented here will spark some interest into further investigations of the light-gravity connection and maybe help in the search for a *partial* unified theory of everything, where gravity and electromagnetism are not thought of as two separate interactions but, instead, as belonging to some unified interaction, gauge theoretic framework. A summary of our discussions in this direction can be found in Chapter 6.

Chapter 2

Unitary spacetime evolution; its classical origin and application

Since the advent of quantum mechanics, the connection between the momentum operator and the gradient has been derived in different ways: through the commutation relation and, then another way, through arguments from classical mechanics about the generator of translations being the gradient, which is just the momentum. In this chapter we explore alternative motivations for this connection and we also look at some consequences that can be deduced from this connection.

There are two popular ways of finding the momentum operator. One is based on quantum mechanics while the other is classical. From the quantum mechanics' perspective, using a position representation of the Heisenberg algebra, the position-momentum commutation relation, leads to the momentum operator being a gradient in position [4]. In classical mechanics, momentum is the generator of spatial translations. In quantum mechanics, however, the translation operator generates spatial translations and it is a gradient. It is therefore argued that the momentum operator is a translation operator, thus it is connected to the gradient [13].

Here, we explore two new ways to motivate this connection. Firstly, we generalize temporal unitary evolution to spatio-temporal unitary evolution to obtain wave equations that make this connection explicit. Secondly, we derive the associated classical equations from a generalization of the action defined as an integral over a product of quantities with units of energy and time, to a new action that is an integral over a product of quantities with units of momentum and position. This generalization requires a further generalization of the principle of least action from path-variation to a variation in the energy configuration, in order to obtain a spatial analogue of Lagrange's equations of motion and the subsequent spatial analogue of Hamilton-Jacobi's equations. The spatial analogue of the Hamilton-Jacobi's equations will be quantized to obtain the wave equations from the first part. These derived equations will form a starting point of our analysis of photon loss in nonlinear quantum optics in Chapter 4.

These new ways of motivating the momentum-operator-gradient connection are important because they put into perspective the role energy and time play as phase space variables. In particular, having a quantity with units of momentum play a role analogous to that played by the Hamiltonian makes clear the point that the new associated phase space is coordinatized by energy and time, not position and momentum. The generated spatial translations are in this new phase space. The associated Poisson bracket structure is changed to reflect these new phase space variables. Furthermore, we find an interpretation of time as a phase space variable which is tied to the notion of interactions in physical systems.

We then look at two consequences of the spatial evolution wave equations: the spatial analogues of the adiabatic process and dynamical phase. We show how the spatial wave equations for a quantum system undergoing adiabatic-like evolution in space, can be used to find a phase which is analogous to the dynamical phase from the adiabatic theorem. We then use this analogy to set the stage for a discussion of analogues between accelerating bodies and variable dispersion in nonlinear optics, which will be the topic of Chapter 3.

This chapter is organized as follows. We first review the derivation of Schrödinger's equation from the unitarity postulate of quantum mechanics in Section (2.1) and we proceed to generalizing this postulate from unitary temporal evolution to unitary spatio-temporal evolution in Section (2.2). After introducing the spatial evolution equations, in Section (2.3), we use them to find an optical analogue of the dynamical phase from the adiabatic theorem which will be useful in Chapter 3. To better establish the spatial evolution equations, we find their classical origin by introducing new mechanics' concepts in Section (2.4). We look at an example system and discuss the meaning of time, in this context, in Section (2.5). We summarize the results of this chapter in Section (2.6).

2.1 Schrödinger equation from unitary time evolution

We know that one way to derive the Schrödinger equation is to start by assuming that wave functions evolve unitarily in time [14]. If one starts with a wave function $|\psi(t_0)\rangle$ at a particular time, t_0, we can find the wave function $|\psi(t)\rangle$ at a later time, t, when we apply the unitary operator $U(t, t_0)$ on the initial wave function i.e.:

$$|\psi(t)\rangle = U(t, t_0)|\psi(t_0)\rangle. \tag{2.1}$$

It can then be shown that the *temporal* unitarity postulate leads to Schrödinger's equation:

$$\mathrm{i}\frac{d}{dt}|\psi\rangle = H|\psi\rangle, \tag{2.2}$$

with $\hbar = 1$. Given that special relativity treats space and time on the same footing, one may wonder what happens if we generalize the postulate about *temporal* evolution being unitary to say that spacetime evolution is unitary. To the best of our knowledge, this generalisation has not been done before.

2.2 The generalized Schrödinger equation from unitary spacetime evolution

In this section, we present a new way to connect the momentum operator with the gradient. This approach is arguably more straightforward than well-known methods [4]. To make this connection, we generalize the postulate for unitary evolution from just time to *spacetime* so as to find a first-order equation for spatial evolution which is similar to Schrödinger's equation for time. More explicitly, from this generalization, i.e. $U(t, t_0) \to U(x^\mu, x_0^\mu)$, we seek the associated wave equation.

For infinitesimal spacetime displacement, where $x^\mu = x_0^\mu + \delta x^\mu$, the generalized evolution operator about the identity $\mathbb{I}$ can be written as:

$$U(x^\mu, x_0^\mu) = \mathbb{I} - \mathrm{i}P_\mu(x^\mu - x_0^\mu) + O((x^\mu - x_0^\mu)^2). \tag{2.3}$$

With some state vector $|\psi(x^\mu)\rangle$, infinitesimal spacetime evolution gives:

$$|\psi(x^\mu)\rangle = |\psi(x_0^\mu)\rangle - \mathrm{i}P_\mu(x^\mu - x_0^\mu)|\psi(x_0^\mu)\rangle + O((x^\mu - x_0^\mu)^2)|\psi(x_0^\mu)\rangle, \tag{2.4}$$

which means that:

$$|\psi(x^\mu)\rangle - |\psi(x_0^\mu)\rangle = -\mathrm{i}P_\mu(\underbrace{x^\mu - x_0^\mu}_{=\delta x^\mu})|\psi(x_0^\mu)\rangle + O((x^\mu - x_0^\mu)^2)|\psi(x_0^\mu)\rangle, \tag{2.5}$$

which is the same as:

$$\delta|\psi\rangle = -\mathrm{i}P_\mu \delta x^\mu|\psi(x_0^\mu)\rangle + O((\delta x^\mu)^2)|\psi(x_0^\mu)\rangle, \tag{2.6}$$

with $\delta|\psi\rangle = |\psi(x^\mu)\rangle - |\psi(x_0^\mu)\rangle$. Thus,

$$\frac{\delta|\psi\rangle}{\delta x^\mu} = -\mathrm{i}P_\mu|\psi\rangle + O(\delta x^\mu)|\psi(x_0^\mu)\rangle, \tag{2.7}$$

which in the limit that $\delta x^\mu \to 0$ becomes:

$$\mathrm{i}\frac{\partial}{\partial x^\mu}|\psi\rangle = P_\mu|\psi\rangle, \tag{2.8}$$

with:

$$P_\mu = \begin{pmatrix} H \\ -\mathbf{P} \end{pmatrix}, \tag{2.9}$$

and the 3-momentum operator is $\mathbf{P}$. The time-component of Equation (2.8) is the familiar Schrödinger equation. The corresponding spatial components supplement this equation.

The 4-momentum components are not independent of each other. They are related, for example, through the relativistic energy-momentum equation:

$$P^\mu P_\mu = m^2, \tag{2.10}$$

where m is the particle's mass, and $P^\mu P_\mu = H^2 - \mathbf{P}^2$. Thus, the time-component of Equation (2.8) can be written as:

$$\mathrm{i}\frac{\partial}{\partial t}|\psi\rangle = H(\mathbf{P})|\psi\rangle, \tag{2.11}$$

where $H(\mathbf{P})$ is the familiar Hamiltonian operator as a function of the momentum operator, a phase space variable, which is defined by the spatial components of Equation (2.8). What is maybe a little different is that the momentum can also be expressed in-terms of the energy, and the spatial equations can then be used as the wave equation with the Schrödinger equation telling us that the energy operator is a time-derivative.

More explicitly, the spatial components of Equation (2.8) are:

$$-\mathrm{i}\nabla|\psi\rangle = \mathbf{P}(H)|\psi\rangle, \tag{2.12}$$

where now the momentum operator is a function of the energy operator, which is a time-derivative. The spatial wave equations suggest that using the momentum of a particle in a way analogous to the manner in which the Hamiltonian is used i.e., as a generator of time evolution, energy becomes a phase space variable.

The equation of motion for density operators can be obtained as above, with the generalization of unitary evolution in time to that of unitary spacetime evolution. For example, with the density matrix ρ, this generalization means that $\rho \to \rho' = U(x_0^\mu + \delta x^\mu, x_0^\mu)\rho U^\dagger(x_0^\mu + \delta x^\mu, x_0^\mu)$, for the infinitesimal change δx^μ and where $\rho' = \rho(x_0^\mu + \delta x^\mu)$, which all leads to:

$$\rho' = (\mathbb{I} - \mathrm{i}P_\mu \delta x^\mu)\rho(\mathbb{I} + \mathrm{i}P_\mu \delta x^\mu) = \rho - \mathrm{i}[\rho, P_\mu]\delta x^\mu + O((\delta x)^2), \tag{2.13}$$

and gives:

$$\frac{\rho' - \rho}{\delta x^\mu} = -\mathrm{i}[\rho, P_\mu] + O((\delta x)^2). \tag{2.14}$$

In the limit $\delta x^\mu \to 0$, we get the equations of motion:

$$\mathrm{i}\frac{d\rho}{dx^\mu} = [\rho, P_\mu] \tag{2.15}$$

where the time component is the familiar Heisenberg equation of motion. The spatial part of Equation (2.15), together with some nonunitary terms, is used in Chapter (4) to analyze lossy dynamics in a quantum nonlinear optical system.

In the next section we will study the classical origin of the spatial evolution equations derived here. We will find that the idea of the action has to be generalized to include in its definition new quantities analogous to the Lagrangian and Hamiltonian but with units of momentum as opposed to that of energy. The principle of least action has to be extended as well from path variation to energy configuration variation (we can also use time variation instead of energy and find the same results). Furthermore, we will need new phase space variables in going from temporal to spatial evolution. In particular, the position-momentum phase space coordinatization will be replaced with a new coordinatization consisting of energy-time.

2.3 A spatial analogue of both the adiabatic theorem and the dynamical phase

Here we look at one possible application of the spatial evolution equations; in finding the optical analogue of the adiabatic theorem and its, associated optical analogue of the dynamical phase [4]. A system undergoes adiabatic evolution if the state of this system slowly changes in time. For systems undergoing adiabatic evolution, at each point in time they occupy eigenstates that are functionally the same but differ by a *dynamical phase* $\theta_n(t)$ (the subscript n just labels the eigenvalues) which is [4, 15]:

$$\theta_n(t) = \int^t E_n(t')dt', \tag{2.16}$$

where the eigenvalues of the Hamiltonian $E_n(t')$ i.e., the energy is now time-dependent and the lower-limit of the integral can be anything from some value t_0 which can be taken to infinity if need be. We will not explicitly write this lower value from now on. In other words, when a system starts in an *nth*-eigenstate of the Hamiltonian and evolves adiabatically, at each instant in time, this system will occupy the eigenstate of the Hamiltonian at that exact instant and the history of the system is encoded in the phase as it occupies different energy eigenstates at each different point in time hence the integral over an energy as a function of time as the system evolves adiabatically. Put another way, at each instant in time, the system is in the eigenstate $\psi_n(t)$ say, with eigenvalue $E_n(t)$ and at a later time $t + \Delta t$ it is in the state $\psi_n(t + \Delta t)$ with eigenvalue $E_n(t + \Delta t)$, hence the eigenvalues are time-dependent which is why we have the integral.

One of the key things about the adiabatic theorem is that it requires the Schrödinger equation as it applies to quantum systems that evolve slowly in *time*. Thus for a spatial analog of this theorem, we need a spatial analog of the Schrödinger equation which describes the "dynamics" of a quantum system changing in space rather than time. We found such an equation in Equation (2.8) and we will use it to derive a spatial analog of the dynamical phase from the adiabatic theorem.

2.3.1 Review of the adiabatic theorem and the dynamical phase

An adiabatic process is one where a quantum system undergoes gradual change in time. This change can be exponential and as such these processes should not be confused with those of perturbative effects. If the system undergoes adiabatic evolution, the Hamiltonian becomes time-dependent. The state of this system $\psi(t)$ at any time t, is [16, 17]:

$$\psi(t) = Te^{-\mathrm{i}\int_{t_0}^{t} H(t')dt'}\psi(t_0), \tag{2.17}$$

which contains the phase $Te^{-\mathrm{i}\int_{t_0}^{t} H(t')dt'}$, T is the time-ordering operator and the initial state is $\psi(t_0)$, at initial time t_0. The adiabatic theorem tells us, among other things[1], that at each point in time the system will be in the eigenstate of the instantaneous Hamiltonian, which means that if, for example, the energy eigenstates are discrete and labelled by the integer quantum number, n say, then Equation (2.17) implies that:

[1]Under certain strict conditions which we need not concern ourselves with here but which can be found in [18].

$$\psi_n(t) = e^{-\mathrm{i}\int_{t_0}^{t} E_n(t')dt'}\psi_n(t_0), \tag{2.18}$$

that now contains the *dynamical phase* $e^{-\mathrm{i}\int_{t_0}^{t} E_n(t')dt'}$, where there is no more a time-ordering operator because the phase is just a number (or it is proportional to the identity operator). In the next subsection we will look at quantum systems that gradually vary in space in contrast to time and find the spatial analog of the dynamical phase.

2.3.2 Spatial analogue of the dynamical phase

For a one-dimensional quantum system undergoing a gradual spatial evolution (in the z direction, say), a spatial analogue of adiabatic evolution, the dynamics is governed by the spatial component of Equation (2.8):

$$\mathrm{i}\frac{d}{dz}|\psi\rangle = -P(z)|\psi\rangle, \tag{2.19}$$

where the momentum depends on the position z. The general solution of Equation (2.19) is:

$$|\psi(z)\rangle = \mathrm{P}e^{\mathrm{i}\int^{z} P(z')dz'}|\psi_0\rangle, \tag{2.20}$$

where "P" denotes the path-ordered exponential (which is analogous to time-ordering for temporally evolving systems), $|\psi_0\rangle$ is the state at the start position. Path ordering here works as time ordering does for the Hamiltonian. It is required because the momenta at different points do not commute, so as the evolution occurs the order must be preserved and that is why path-ordering is needed. Also, for the spatial analogue of adiabatic evolution, the system is in the eigenstate of the momentum operator at that particular point. Hence Equation (2.20) implies that:

$$|\psi(z)\rangle = \mathrm{P}e^{\mathrm{i}\int^{z} k(z')dz'}|\psi_0\rangle, \tag{2.21}$$

where $k(z)$ is the position-dependent momentum eigenvalue (we used the relation between the momentum eigenvalue p and the wave number eigenvalue k, $p = \hbar k$ where we set $\hbar = 1$) and here $|\psi_0\rangle$ is the state of the system at some initial point in space. The spatial analogue of the dynamical phase is $e^{\mathrm{i}\int^{z} k(z')dz'}$.

This spatial analogue of adiabatic evolution makes space behave like time, forever marching forward. We can see this from how the state of a particle changes under this spatial analogue of adiabatic evolution. To such a particle, space marches forward and this point is captured in the phase which tracks its history as the particle evolves. This time-like behaviour of space is key to simulating accelerating bodies in nonlinear optics, which is discussed in Chapter 3.

2.4 Energy and time as new phase space variables

So far, we have argued that the spatial part of the generalized Schrödinger equation should be treated on the same footing as the temporal part. If we explore a possible classical origin of this spatial part, we are led to an interesting suggestion that perhaps momentum may play a bigger role than just a phase space variable. To begin the exploration, we will introduce a quantity, which is analogous to the Lagrangian, that has units of momentum and is a function of energy, its spatial derivative and position. Through very similar treatments to those used to obtain the Euler-Lagrange and the Hamilton-Jacobi equations, we obtain second-order and first-order analogous equations, respectively. Energy and time become new phase space variables, thus our treatment puts on equal footing the role momentum and energy play in describing motion.

2.4.1 Review of Analytical Mechanics

Newtonian mechanics (or classical mechanics) is wonderful in its ability to account for the motion of all macroscopic non-relativistic objects. At its foundation it works with vectors, examples of which are force, momentum, position, velocities, acceleration, etc. [19]. Newton's philosophy on motion was that objects did not move on their own, or if they were already moving their motion has the same character (constant velocity) unless something acts on it. Newton referred to this acting agent as a force [19]. In the 18th and 19th centuries, Lagrange and Hamilton reformulated classical mechanics into a new field known as analytical mechanics. This new formulation of mechanics did not need the vectors, including forces, used in classical mechanics. It used a different language and we will review it here.

In analytical mechanics, the path followed by all objects is the one that extremizes a quantity known as the action. This observation is known as the principle of least action. The action S is defined as [20]:

$$S = \int dt L, \tag{2.22}$$

where L is known as the Lagrangian. Note that S is a scalar. The Lagrangian is a function of the coordinates (position of the particles), velocity and time i.e. $L = L(q, \dot{q}, t)$ where the coordinates are labeled as q, and $\dot{q}$ is its time derivative (velocity). The principle of least action then implies that the path followed by our object with position $q = q(t)$ (also known as its trajectory) is the one that minimizes the action. Varying the action gives:

$$\delta S = \int dt \delta q \Big(\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} \Big), \tag{2.23}$$

where δq means path variation, which in-turn implies that:

$$\frac{\delta S}{\delta q} = \int dt \Big(\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} \Big). \tag{2.24}$$

The action principle implies that the path taken by this particle is the one for which $\frac{\delta S}{\delta q} = 0$. This condition leads to the equation of motion:

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = 0, \tag{2.25}$$

also known as the Euler-Lagrange equation [20]. Hence we can find the equation of motion of an object without using the concept of forces as used in Newtonian mechanics. The Euler-Lagrange equations are second-order equations. They can be reformulated to become two first-order equations [20]. These first-order equations are known as Hamilton's equations and in this reformulation the Lagrangian is transformed into another quantity that also has units of energy known as the Hamiltonian.

To move from the Euler-Lagrange equation to Hamilton's equations, we must perform a transformation known as the Legendre transform [13]. If we call the Hamiltonian H, then the Legendre transform is:

$$H(q, p, t) = \dot{q}p - L(q, \dot{q}, t), \tag{2.26}$$

where the momentum is $p = \frac{\partial L}{\partial \dot{q}}$. The idea behind this transform is to be able to go back and forth between functions of velocity, such as the Lagrangian, and functions of momentum, such as the Hamiltonian. We will see that defining a function that depends on momentum will split the Euler-Lagrange equation into two first-order equations.

The Hamilton-Jacobi's equations of motion, which are the two first-order equations obtained from the Euler-Lagrange equation, are [13]:

$$\frac{\partial H}{\partial q} = -\dot{p}, \tag{2.27}$$

$$\frac{\partial H}{\partial p} = \dot{q}, \tag{2.28}$$

$$\frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}, \tag{2.29}$$

with the exception of the last part, which applies only to systems where there is an explicit time-dependence in the associated Lagrangian or Hamiltonian.

The position q and the momentum p form a pair which describes the state of the particle. This pair (q, p) is a point in what is called the phase space. In other words any point in phase space describes the state of the particle. This phase space has an additional structure known as the Poisson bracket. The Poisson bracket $\{\ ,\ \}$ acting on two functions of phase space $f(q, p)$ and $g(q, p)$ say, is defined as [19]:

$$\{f, g\} = \frac{\partial f}{\partial q}\frac{\partial g}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial g}{\partial q}. \tag{2.30}$$

Hamilton-Jacobi's equations of motion can be recast in-terms of Poisson brackets as:

$$\{q, H\} = \dot{q}, \tag{2.31}$$

$$\{p, H\} = \dot{p}, \tag{2.32}$$

which can be generalized to any function of the phase space variables, $f(q, p)$ say, to obtain [19]:

$$\{f, H\} = \dot{f}. \tag{2.33}$$

It is in this sense that the Hamiltonian generates time-evolution. From Equation (2.33), we observe that:

$$\{(\ ), H\} = \frac{d}{dt}(\ ), \tag{2.34}$$

which makes it more obvious how the Hamiltonian generates time-evolution since the action of the function in Equation (2.34) on a function of phase space variables, f say, is just the time derivative of that function as written in Equation (2.33). So far we have been exploring a classical theory. To move over to a quantum theory some extensions have to be made to the concepts introduced here.

2.4.2 Quantization of the classical theory

In quantum theory, functions of phase space variables are replaced by operators acting on some Hilbert space [4] (the space of all possible state vectors; a state vector describes the state of the system in quantum theory). In this section, we will represent operators with capital letters so that a phase space function f becomes an operator F in the quantum theory. In the Heisenberg picture, where the dynamics of the quantum system is in the operators as opposed to the state vectors, the time-evolution of the operator is:

$$[F, H] = \mathrm{i}\dot{F}, \tag{2.35}$$

which is known as the Heisenberg equation of motion [19] and [,] is the commutator and it is defined as $[A, B] = AB - BA$ for two operators A and B. The Heisenberg equation of motion is obtained from that of the classical theory by replacing the Poisson bracket with the commutator i.e. $\{\ ,\ \} \to -\mathrm{i}[\ ,\ ]$ which in-turn takes us from Equation (2.33) to Equation (2.35).

In other words, to quantize a classical theory, we promote functions of phase-space variables to operators acting on a space of vectors that describe the state of the particle, the Hilbert space, and the Poisson structure of the classical theory is replaced by the commutator of the quantum theory which allows us to move from dynamics in the classical theory to dynamics in the quantum theory which is the Heisenberg equation of motion (or the vector state equivalent which is the Schrödinger equation).

2.4.3 Classical origin of the spatial part of the generalized Schrödinger equation

After our reviews of analytical, or classical, mechanics and its quantization, we are now ready to extend these concepts to a new description whose quantization will lead to an aspect of the generalized Schrödinger equation presented as the spatial part of Equation (2.8). In this description the “equations of motion” will not have time derivatives appearing. In place of time-derivatives there will be spatial derivatives, or gradients, if you prefer, of quantities such as energy.

We propose a new action:

$$S = \int M(E, E', q)dq, \tag{2.36}$$

with $E = E(q)$ being the generalized energy, E' being the associated spatial derivative or gradient (which is the same as a conservative force) and q being the generalized position which the generalized energy is assumed to be a function of. The quantity M is as yet not defined but it is to be kept in mind that it must have dimensions of momentum so that S has the correct units as an action. From an idea like the principle of least action, which replaces path variation with energy configuration variation (this will be explained shortly), and following steps identical to those used in deriving Equation (2.24), we have that:

$$\frac{\delta S}{\delta E} = \int dq \Big(\frac{\partial M}{\partial E} - \frac{d}{dq}\frac{\partial M}{\partial E'} \Big), \tag{2.37}$$

which from the generalized principle of least action i.e., $\frac{\delta S}{\delta E} = 0$, we obtain the "equation of motion":

$$\frac{\partial M}{\partial E} - \frac{d}{dq}\frac{\partial M}{\partial E'} = 0. \tag{2.38}$$

The second-order equation above i.e., Equation (2.38), can be recast into two first-order equations by introducing the quantity:

$$t = \frac{\partial M}{\partial E'}, \tag{2.39}$$

with t for time since it has units of time, and performing a transformation, like the Legendre transform, from a function of E and E' to that of E and t, which we will call $P = P(E, t, q)$ (the momentum):

$$P(E, t, q) = -E't + M(E, E', q). \tag{2.40}$$

We will now derive the spatial analog of the Hamilton-Jacobi equations.

We begin by first writing down the differential of the momentum dP:

$$dP = \frac{\partial P}{\partial E}dE + \frac{\partial P}{\partial t}dt + \frac{\partial P}{\partial q}dq. \tag{2.41}$$

From Equation (2.40), the differential is:

$$dP = \frac{\partial M}{\partial E}dE - E'dt + (-t + \frac{\partial M}{\partial E'})dE' + \frac{\partial M}{\partial q}dq \tag{2.42}$$

and from Equation (2.39), we know that the mid part of the right-hand-side of Equation (2.42) is zero, thus:

$$dP = \frac{\partial M}{\partial E} dE - E' dt + \frac{\partial M}{\partial q} dq. \tag{2.43}$$

Using Equation (2.38), we can rewrite Equation (2.43) as:

$$dP = t' dE - E' dt + \frac{\partial M}{\partial q} dq. \tag{2.44}$$

Comparing Equation (2.41) to Equation (2.44), we get the spatial analog of the Hamilton-Jacobi equations to be:

$$\frac{\partial P}{\partial t} = -E', \tag{2.45}$$

$$\frac{\partial P}{\partial E} = t', \tag{2.46}$$

$$\frac{\partial P}{\partial q} = \frac{\partial M}{\partial q}, \tag{2.47}$$

where the primes denote spatial derivatives i.e., $t' = \frac{dt}{dq}$ and $v' = \frac{dE}{dq}$.

The quantities E and t define phase space variables and an analogous Poisson bracket can be defined for them i.e., for phase space function f and the momentum P:

$$\{f, P\}_{E,t} = \frac{\partial f}{\partial E}\frac{\partial P}{\partial t} - \frac{\partial f}{\partial t}\frac{\partial P}{\partial E}, \tag{2.48}$$

which becomes, upon using Equation (2.45) and Equation (2.46),

$$\{f, P\}_{E,t} = -\frac{\partial f}{\partial E}\frac{dE}{dq} - \frac{\partial f}{\partial t}\frac{dt}{dq} = -\frac{df}{dq}. \tag{2.49}$$

Hence the momentum generates spatial translations,

$$\{(\ ), P\}_{E,t} = -\frac{d}{dq}(\ ). \tag{2.50}$$

In the quantum theory, this Poisson bracket becomes a commutator and we are left with a spatial analog of Heisenberg's equation:

$$[(\ ), P] = -\mathrm{i}\frac{d}{dq}, \tag{2.51}$$

whose equivalent Schrödinger-like form is just the spatial part of Equation (2.8).

We have shown here that introducing a function of phase space that has units of momentum leads to a spatial analog of the Euler-Lagrange and Hamilton-Jacobi equations. We can perform a Legendre-like transformation to obtain another quantity, analogous to the Hamiltonian, which generates spatial evolution in phase space. The phase space is no longer labeled by position and momentum, but is instead labeled by energy and time and this new function of phase space, with units of momentum, generates translations in this new phase space, in very much the same way the Hamiltonian generates time evolution in the familiar position-momentum phase space. This gives us a new way of thinking of time as an observable in quantum mechanics.

2.5 An example system and the meaning of time

Time as an observable has been considered a problem since the inception of quantum mechanics. The problem stems from concerns about unbounded energies from below (there have been objections to this suggestion, e.g. [21]), deduced from its commutation relation with energy, when it is a self-adjoint operator, to a host of other concerns discussed in the review [22]. But it is also discussed in [22], about the possibility of viewing time as an observable in certain contexts, rather than that of a universal notion or a generally true one for all systems. These objections to a time observable have a common thread; they are based on the assumption that time is just a parameter external to any physical system. As such, it is expected to take on only real values and to be continuous and unbounded from below. These expected features make it difficult to view time as an observable in the context of systems with Hamiltonians that have discrete spectra and which are bounded below (at least for physical systems).

We take a different view of time, here. We will look at some systems that can be analyzed using the concepts developed in the previous section. Namely, we will look at what is meant by time as an observable for these systems, as deduced from the results presented here. We will see that our view of time does not require it to always be continuous or unbounded since it is constructed from properties of the system itself.

Consider an interacting particle with energy H, momentum p, mass m and position q:

$$H(q,p) = \frac{p^2}{2m} + V(q), \tag{2.52}$$

where $V(q)$ is the particle's potential energy. Inverting Equation (2.52) to get momentum as a function of energy, we find:

$$p(E, E') = \sqrt{2m(H - V(q_0) - \nabla V|_{q_0}(q - q_0) + \text{higher orders})}, \tag{2.53}$$

where q_0 is some fixed point, $E = H - V(q_0)$, $\nabla E = E' = \nabla V|_{q_0}(q - q_0)$, and we Taylor expanded the potential. We can go to higher orders in this expansion, however we are interested in the term with a first-order derivative so as to be able to find the time variable.

Performing the derivative of the momentum with respect to E' from Equation (2.53), we obtain the associated time variable from Equation (2.39):

$$t = \frac{\partial p}{\partial E'} = \frac{m}{p}(q - q_0). \tag{2.54}$$

Thus, Equation (2.54) suggests both a local property of time (a dependence on position), in accordance to the first-order equations in Equation (2.46), and a dependence on the mass of the moving, interacting body. Heavier, interacting bodies tend to move slowly and thus take more time to traverse a given distance, in contrast with lighter, interacting ones. For free massive particles, $p = \sqrt{2mE}$, hence p is not a function of the spatial derivative of energy, i.e. E'. As a result, there is no associated notion of time for free particles.

These examples seem to suggest that the notion of time comes from interactions. Imagine, for example, a free particle traveling through space. What does it mean for this particle to be at a certain point in time? This is hard to define. However, if this same particle collides with another particle, this collision signifies an interaction, however brief. This interaction defines an *event*. Such events define moments in time. Collisions happening together define *simultaneous events*, while collisions happening apart define the notion of past and future. Thus, time is intricately linked with the notion of interactions that leave some kind of *stamp* on the particle. Otherwise, it is vague to talk about time for a free particle.

2.6 Summary

We showed that a generalization of unitary *time* evolution to unitary *spacetime* evolution leads to four evolution equations. Three of these equations describe spatial evolution which will be used in Chapter 4, whereas the fourth is the well-known time evolution equation, i.e., the Schrödinger equation. This generalization provides a unifying framework for obtaining the wave equations in quantum mechanics for both temporal and spatial evolution. Furthermore, our generalization makes a straightforward connection between the momentum and gradient.

This connection introduces the new phase variables of energy and momentum and in so doing introduces a notion of a time observable in the quantized theory. This new notion of time as an observable views time as inextricably linked to interactions, to the point where free particles experience no time. Not all interactions give rise to time and this may explain why not all systems admit a time observable. Only those interactions that can be Taylor-expanded to first-order can be used to define the time observable described here.

As an application of the spatial part of the generalized Schrödinger equation, we looked at the spatial analogue of the adiabatic theorem and its associated spatial analogue of the dynamical phase. This spatial analogue can be used to simulate accelerating bodies in optical systems, which has the potential to allow for the testing of gravitational phenomena in the lab setting that are otherwise hard to do, such as the study of the physics of blackhole event horizons. We will further explore this application in Chapter 3. We also found a classical origin of the spatial evolution equations just previously mentioned.

Just as there is a classical origin of Schrödinger's equation i.e., Hamilton-Jacobi's equations, we found here a new set of spatial analogue Hamilton-Jacobi equations, whose quantization yields the spatial evolution equations. This classical description introduced quantities with dimensions of momenta that are analogous to the Lagrangian and Hamiltonian. The quantity with units of momentum, which is analogous to the Hamiltonian, *generates spatial evolution in a phase space with coordinates of energy and time.* Thus this mechanical law, which uses the system's momentum to describe its motion, puts energy and momentum on equal footing, in much the same way as special relativity puts their conjugate: time and space, on equal footing, as well.

Chapter 3

Simulating Unruh-DeWitt detectors using nonlinear optics

Statement of Contributions to Jointly Authored Work Contained in This Chapter of The Thesis

Eugene Adjei, Kevin J. Resch, and Agata M. Brańczyk, *Quantum Simulation of Unruh-DeWitt Detectors with Nonlinear Optics*, Phys. Rev. A **102**, 033506 (2020).

Kevin J. Resch had the original idea and was involved in drafting the article. Agata M. Brańczyk helped develop the formalism and was involved in drafting the article. Eugene Adjei helped develop the formalism and was involved in drafting the article.

The relation between acceleration and gravity is captured in the equivalence principle and it underpins general relativity [7]. This principle posits that one can think of *gravitating bodies as accelerating ones with the gravitational field turned off*: hence the *equivalence* between *acceleration and gravity*. It is therefore not surprising that gravitational phenomena have a recurring theme of accelerating frames. Here we will look at how an optical system engineered to simulate accelerating bodies can be used to simulate the semi-classical gravitational phenomenon known as the Unruh effect.

The Unruh effect is a curious phenomenon, first discovered by W. G. Unruh, in which an accelerating observer sees a thermal bath of particles (whose temperature is proportional to the acceleration) even though an inertial observer sees a vacuum [23, 24]. It is a quantum effect since the observed thermal bath of particles are excitations of an underlying quantum field and it can be considered a gravitational phenomena [25] because falling bodies qualify as accelerating observers, so that since a falling detector is equivalent to an accelerating one they should both see the Unruh effect. These two characteristics of the effect make it an undeniable consequence of QFT in curved spacetime.

There are two main formulations of this effect. One formulation involves finding the average number of quanta of the field as observed by the accelerating observer from the field's quantum description by the inertial one [26, 27]. The other way to obtain this effect is to model the accelerating observer as a detector [28, 29, 30, 24]. This detector is coupled to a field and when the detector accelerates it clicks in response to the presence of radiation when the field is in a vacuum state. This latter way of describing the Unruh effect is called the Unruh-DeWitt model (UDW) and it is what will be considered here.

There have been some proposals for detecting Unruh radiation in the lab. Accelerating electrons were proposed in [31] as a way to test the Unruh effect. In [32], it was discussed that decaying accelerating charged particles can emit radiation which can be conceptually linked with the Unruh effect. In fact, by virtue of the acceleration such a particle could be a proton since the acceleration affects the mean life-time and so it can lower the mean life-time of a proton [32], which is of the order of 10^{25} years, to something more reasonable that can be observed in the lab provided the acceleration is high enough. The idea of using proton decay to observe the Unruh effect is also explored in [33, 34] with emphasis on how it establishes the Unruh effect as a logical consequence of Quantum Field Theory (QFT) so that if one doubts its existence then one must also have issues with QFT. In all these proposals accelerating protons or electrons play the role of an accelerating observer. However, there have also been suggestions of observing the Unruh effect in the lab through the use of ultrabroadband squeezed light pulses as described in [35]. At the core of the issue surrounding the testing of the Unruh effect is the difficulty associated with directly studying the effect due to the large accelerations needed to observe the related Unruh temperature; hence, the Unruh effect is yet to be observed. Quantum simulations offer another way to overcome this challenge.

Quantum simulations have been used to indirectly study the Unruh effect. In [36] they combine techniques of digital quantum simulation of linear and nonlinear optics with boson-qubit mapping to simulate many-body problems in physics which is possible because they can digitally represent the associated Hamiltonians making it possible to study such systems using a computer. Thus, their work paves the way to study the Unruh effect as well. Here [23], they used matter fields based on Bose-Einstein condensates to test the Unruh effect. The Bose-Einstein condensates were set-up to mimic Bogoliubov transformations between frames and the authors observed thermal fluctuations in these matter fields which are consistent with Unruh radiation. Furthermore, in [35], the authors relate spectra from ultrabroadband squeezed pulses interacting with a nonlinear material to that from potential Unruh-Davies experiments, making such a system a viable candidate for testing the Unruh effect.

We propose a new way to simulate the Unruh effect using nonlinear optics. Our proposal differs from other proposals using nonlinear optics because we make a one-to-one correspondence between the nonlinear optical system and the *Unruh-DeWitt model* in a manner that allows for the simulation of accelerating detectors. In particular, we find optical analogues of quantities in the Unruh-DeWitt model such as the acceleration and the detector energy gap. We then connect these with gravitational phenomena using the equivalence principle. In other words, our proposal allows for the simulation of gravitational phenomena in optical systems because it permits the representation of "accelerating photons" which act as our detector.

The nonlinear optical process used here for the simulation is Spontaneous Parametric Down-Conversion (SPDC). In SPDC incident light, called the pump mode, interacting with a nonlinear material leads to the generation of two entangled modes known as the idler and signal modes, as illustrated in Figure (3.1). The nonlinear medium is characterized by a quantity known as the nonlinear susceptibility. *The novel feature in our work is that we simulate accelerating UDW detectors as one of the generated modes of SPDC in a stationary crystal with variable dispersion while the scalar field is simulated by the other mode.*

As we will see, SPDC is a good process to simulate the UDW system. This is so because the Hamiltonian for SPDC, in the classical pump approximation, has the same form as that of the UDW model. Tweaking the SPDC Hamiltonian, one can make a one-to-one correspondence among quantities like the detector, field, coupling between detector and field, acceleration and switching function and their optical analogues.

The material presented here is heavily based on related work we did in [37]. There, our aim was to propose an experiment that can be used to study the Unruh effect, and other closely related phenomena. Here, we are only interested in what can be gained, theoretically, from the connection between our optical system and the UDW system. In other words, we do not propose any experiment here. Instead, we investigate what new insights can be gained about gravity from our optical set-up.

The outline of this chapter is as follows: we introduce the UDW model in Section (3.1), followed by a discussion of the optical system that will be used in this simulation in Section (3.2). It is shown how the two systems are connected in Section (3.3). In Section (3.4), we will discuss how the results of this chapter can be applied to gravity, both *classical* and *quantum*. We summarize the results of this chapter in Section (3.5).

3.1 The Unruh-DeWitt Model

The Unruh-DeWitt model has many applications in different areas of physics, such as semi-classical gravity, which include the investigation of things like entanglement harvesting [38]. It can also be used to model light-matter interactions that do not involve the exchange of angular momentum [28]. We simulate them here as a model for the Unruh effect [39]. To do the simulation, we need to introduce the Hamiltonian describing such a light-matter system.

The Hamiltonian describing this detector-field system is called the Unruh-DeWitt Hamiltonian H_{UDW} and it is [29, 30, 40]:

$$H(\tau)_{UDW} = \lambda\eta(\tau)(\sigma^{\dagger}e^{i\Omega\tau} + \sigma^{-}e^{-i\Omega\tau})\Phi(x(\tau), t(\tau)), \tag{3.1}$$

where Φ is a scalar field (here a function of 1+1 spacetime for simplicity). The ladder operator $\sigma^{\dagger}$ excites the detector while the other ladder operator σ^{-} de-excites it. This 1+1 spacetime has Minkowski coordinates parameterized by the proper time τ and explicitly given, in-terms of this parameter, as:

$$t(\tau) = \frac{c}{a}\sinh\left(\frac{a\tau}{c}\right) \tag{3.2}$$

$$x(\tau) = \frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right). \tag{3.3}$$

The interaction strength between the scalar field and the detector is characterized by λ (and it is assumed to be small). Throughout this chapter, the speed of light in vacuum c is taken to be unity. The detector energy gap is Ω while the scalar field frequency is ω. The accelerating UDW detector has acceleration a and switching function η, which turns the detector on and off. With the mode expansion of the scalar field, H_{UDW} becomes:

$$H(\tau)_{UdW} = \lambda\eta(\tau)(\sigma^{\dagger}e^{i\Omega\tau} + \sigma^{-}e^{-i\Omega\tau})\int\frac{d\omega}{\sqrt{4\pi\omega}}\Big(a(\omega)e^{-i\omega t(\tau)}e^{i\frac{\omega}{c}x(\tau)} + a^{\dagger}(\omega)e^{i\omega t(\tau)}e^{-i\frac{\omega}{c}x(\tau)}\Big). \tag{3.4}$$

In the UDW model of the Unruh effect, the accelerating detector and scalar field are both excited [27]. In other words, by virtue of the detector's acceleration, it experiences an excitation from the excited scalar field, which (the scalar field) would have otherwise been in a vacuum state. Thus, the terms containing the operators a and $\sigma^{\dagger}$ together for the scalar field and detector respectively (and their Hermitian conjugate) do not contribute to the Unruh effect since both scalar field and detector get excited seemingly from the vacuum. Hence, the relevant part of the UDW Hamiltonian that is used to test the Unruh effect is:

$$H(\tau)_{UDW} \sim \lambda\eta(\tau)\int \frac{d\omega}{\sqrt{4\pi\omega}}(\sigma^\dagger a^\dagger(\omega)e^{i(\omega t(\tau)-\frac{\omega}{c}x(\tau))}e^{i\Omega\tau} + \text{h.c.}), \tag{3.5}$$

where h.c. stands for Hermitian conjugate. Using the trajectory of an accelerating particle in 1+1 spacetime from Equation (3.2) and Equation (3.3), $t(\tau) - \frac{1}{c}x(\tau) = \frac{c}{a}\Big(\sinh(\frac{a\tau}{c}) - \cosh(\frac{a\tau}{c})\Big) = -\frac{c}{a}e^{-\frac{a\tau}{c}}$, it can be written as:

$$H(\tau)_{UDW} = \eta(\tau)\lambda\int \frac{d\omega}{\sqrt{4\pi\omega}}(\sigma^\dagger a^\dagger(\omega)e^{-i\omega\frac{c}{a}e^{-\frac{a\tau}{c}}}e^{i\Omega\tau} + \text{h.c.}), \tag{3.6}$$

or put in a form that will aid in connecting the UDW system with SPDC,

$$H(\tau)_{UDW} = \eta(\tau)\lambda\int \frac{d\omega}{\sqrt{4\pi\omega}}(\sigma^\dagger a^\dagger(\omega)e^{i\omega\int^\tau e^{-\frac{a\tau'}{c}}d\tau'}e^{i\Omega\tau} + \text{h.c.}). \tag{3.7}$$

We can construct a quantity called the response function $\mathcal{A}$ from the Hamiltonian in Equation (3.7). The response function characterizing transitions from the joint detector-field state $|g\rangle|0\rangle$, which means the detector is in the ground state $|g\rangle$ and the field is in the vacuum $|0\rangle$, to the state $|e\rangle|1\rangle$, (detector is in the excited state $|e\rangle$ and field is in the excited state $|1\rangle$ as well), is:

$$\begin{aligned}\mathcal{A}(\Omega_s,\Omega) &= \frac{-i}{\hbar}\langle e|\langle 1|a(\Omega_s)\int d\tau H(\tau)_{UDW}|g\rangle|0\rangle,\\ &= \frac{-i\lambda}{\hbar}\int d\tau\eta(\tau)e^{i\Omega\tau}\int \frac{d\omega}{\sqrt{4\pi\omega}}e^{i\omega t(\tau)}e^{-i\frac{\omega}{c}x(\tau)}\delta(\Omega_s-\omega).\end{aligned}$$

Thus, the response function can be written as:

$$\mathcal{A}(\Omega_s,\Omega) = \int d\omega f_a(\omega,\Omega)\delta(\Omega_s-\omega), \tag{3.8}$$

with

$$f_a(\omega,\Omega) = \frac{-i\lambda}{\sqrt{4\pi\omega}\hbar}\int d\tau\eta(\tau)e^{i\Omega\tau}e^{-i\omega\frac{c}{a}e^{-\frac{a\tau}{c}}}, \tag{3.9}$$

or equivalently as:

$$f_a(\omega,\Omega) = \frac{-i\lambda}{\sqrt{4\pi\omega}\hbar}\int d\tau\eta(\tau)e^{i\Omega\tau}e^{i\omega\int^\tau d\tau' e^{-\frac{a}{c}\tau'}}. \tag{3.10}$$

The thermal nature of the observed quantum field is seen through a quantity known as the distribution function $\mathcal{F} = |\mathcal{A}|^2$. This distribution function can be obtained from the UDW Hamiltonian described above. The distribution function for a detector which is on for all time is:

$$\mathcal{F}_a(\Omega, \omega) = \frac{\lambda^2}{2\omega a}\Big(\frac{1}{e^{\frac{2\pi\Omega}{a}} - 1}\Big), \tag{3.11}$$

which is a Planckian distribution with an effective temperature, the Unruh temperature, $T = \frac{a}{2\pi}$. The Unruh temperature is difficult to detect in the lab because it requires high accelerations which, as discussed early on, are difficult to achieve in practice.

3.2 Spontaneous Parametric Down-Conversion system

Spontaneous Parametric Down-Conversion (SPDC) [9] is a nonlinear optical process which involves the generation of an entangled pair of light beams from some incident light interacting with a nonlinear material. The incident light beam is called a pump while the generated entangled pair are called a signal and idler beam, see Figure (3.1).

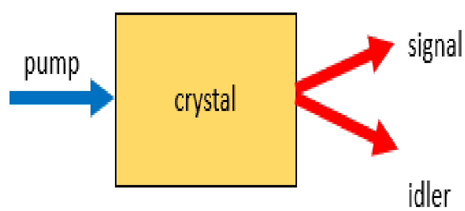

Figure 3.1: This figure describes a nonlinear optical phenomena, where a light beam (labelled as *pump*) generates two entangled beams (labelled as *signal* and *idler*) after its interaction with a nonlinear material, specifically a $\chi^{(2)}$-crystal, whose nonlinearity is characterized by $\chi^{(2)}$.

Here, we will show that the UDW Hamiltonian in Equation (3.4) bears resemblance to the Hamiltonian for SPDC when the pump is classical. So we start with this observation and identify optical analogues of things like the detector, acceleration, coupling between field and detector, and switching function. In our optical analogue system, a detector refers to one of these excited modes of SPDC, here taken to be the idler mode. The role of the scalar field, whose excitations the Unruh-DeWitt detector clicks in response to, is played by the signal field. The optical analogue of acceleration is exponential variable dispersion in the nonlinear-optical material and the role of switching function and coupling between field and detector is played by the nonlinearity, which characterizes the nonlinear material, and the classical pump amplitude respectively.

The SPDC Hamiltonian H_{SPDC} for a nonlinear material with nonlinearity $\chi^{(2)}$, where we have assumed that the tensor nature of the nonlinearity and the vector nature of the electric field can be ignored, and that there is no overlap among modes such as frequency or polarization, is [9]:

$$H(t)_{SPDC} = \int d\mathbf{r}\chi^{(2)} D_p(\mathbf{r},t)D_i(\mathbf{r},t)D_s(\mathbf{r},t), \tag{3.12}$$

where $D(z,t)_{p,s,i}$ are the displacement field amplitudes for the pump mode p, the idler mode i and the signal mode s at transverse position z and time t, $d\mathbf{r}$ is the infinitesimal 3-volume element and $\mathbf{r}$ is the three-position vector (see [41] for a discussion on why we use displacement fields and not electric fields in nonlinear optics). These displacement field amplitudes have the mode decomposition in terms of the creation and annihilation operators $a_l^\dagger$ and a_l respectively:

$$D(\mathbf{r},t)_l \sim \int d\omega_l \Big(\xi_l(x,y)a_l e^{ik_l(\omega_l)z}e^{-i\omega_l t} + \text{h.c.}\Big), \tag{3.13}$$

where $\xi_l(x,y)$ are complex-valued, normalized distribution functions describing the uniform distribution of the transverse mode in the z direction that we have assumed our beams are propagating in, h.c. stands for Hermitian conjugate and l stands for the pump, idler and signal modes. In terms of these creation and annihilation operators and assuming we have a non-depleting, intense, classical pump field with amplitude α, H_{SPDC} becomes:

$$H(t)_{SPDC} \sim \int dz d\omega_i d\omega_s d\omega_p \chi^{(2)} \Big(\xi_p \xi_s^* \xi_i^* \alpha(\omega_p) a_i^\dagger(\omega_i) a_s^\dagger(\omega_s) e^{i(k_p-k_s-k_i)z} e^{-i(\omega_p-\omega_s-\omega_i)t} + \text{h.c.}\Big). \tag{3.14}$$

It will be shown in the next section that for SPDC to simulate UDW detectors, we will need variable dispersion. Variable dispersion plays the role of the detector's acceleration. With variable dispersion, H_{SPDC} gets modified to look like [42]:

$$H(t)_{SPDC} = 2\pi O \int d\omega_p d\omega_s d\omega_i B e^{-it\Delta\omega} \int dz D(z) \Big(e^{i\int_{-\infty}^{z} dz' \Delta k(z')} a_s^\dagger(\omega_s) a_i^\dagger(\omega_i) + \text{h.c.}\Big), \tag{3.15}$$

and

$$B = \frac{\alpha(\omega_p)\chi^{(2)}_{eff}}{2}\sqrt{\frac{\hbar\omega_p\omega_s\omega_i}{\epsilon_0 n_p(\omega_p)n_s(\omega_s)n_l(\omega_l)\pi^3c^3}},$$
$$O = \int dxdy\xi_p(x,y)\xi_s^*(x,y)\xi_i^*(x,y),$$

$n_l(\omega_l)$ is the frequency dependent refractive index for mode l (the modes being signal s and idler i), the position dependent phase mismatch function $\Delta k(z') = k(z')_p - k(z')_s - k(z')_i$, $\Delta\omega = \omega_p - \omega_s - \omega_i$, the nonlinearity is $\chi^{(2)}(z) = \chi^{(2)}_{eff}D(z)$. Performing the time integral enforces energy conservation, which leads to $\mathcal{H}_{SPDC} = \int dt H_{SPDC}(t)$:

$$\mathcal{H}_{SPDC} = \int d\omega_i H(\omega_i)_{SPDC}, \tag{3.16}$$

with ω_p replaced by $\omega_i + \omega_s$ and

$$H(\omega_i)_{SPDC} = 2\pi O\int d\omega_s B\int dz D(z)\Big(e^{i\int_{-\infty}^{z}dz'\Delta k(z')}a_s^\dagger(\omega_s)a_i^\dagger(\omega_i) + \text{h.c.}\Big). \tag{3.17}$$

When we perform the time integral, we get rid of the integral over the pump frequency so as to simplify the appearance of the Hamiltonian for easy comparison with that of the UDW system. In the next section, we will connect the UDW and SPDC systems through their Hamiltonians using Equation (3.7) and Equation (3.17).

3.3 Connecting the two systems

In this section, we show how we can simulate UDW detectors using SPDC. There are optical analogues of the detector, its energy gap and acceleration, coupling between field and detector, and switching function.

To relate the two systems via their Hamiltonians, we Taylor expand the function Δk up to first-order in frequency around central frequencies of the pump Ω_p, the idler Ω_i and the signal $\Omega_p - \Omega_i$. This gives the phasemismatch $\Delta k = \Delta k(\omega_i, \omega_s)$ to be:

$$\Delta k(\omega_i, \omega_s) = \Delta k_1(\omega_i) + \Delta k_2(\omega_s), \tag{3.18}$$

with $\Delta k_1(\omega_i) = k_p(\Omega_p) - k_s(\Omega_p - \Omega_i) - k_i(\Omega_i) - k'_p|_{\Omega_p}\Omega_p + k'_s|_{\Omega_p-\Omega_i}(\Omega_p - \Omega_i) + k'_i|_{\Omega_i}\Omega_i + k'_p|_{\Omega_p}\omega_i + k'_i|_{\Omega_i}\omega_i$ and $\Delta k_2(\omega_s) = \frac{1}{\Delta v}\omega_s$, with $\frac{1}{\Delta v} = k'_p|_{\Omega_p} - k'_s|_{\Omega_p-\Omega_i}$. Thus the Hamiltonian in Equation (3.17) becomes:

$$H(\omega_i)_{SPDC} = 2\pi O \int d\omega_s B \int dz D(z)\Big(e^{i\int^z dz'(\Delta k_1(z'))} e^{i\omega_s \int^z dz' \frac{1}{\Delta v(z')}} a_s^\dagger(\omega_s) a_i^\dagger(\omega_i) + \text{h.c.}\Big). \tag{3.19}$$

The phases appearing in Equation (2.3.2), or variable dispersion, come from an optical analogue of the dynamical phase coming as mentioned in Subsection (2.3.2) of Chapter 3. They give a *time-like feature to space*. More precisely, *variable dispersion* simulates in nonlinear-optical systems the motion of an *accelerating body* in *spacetime*, with the *proper time replaced by the distance travelled by the body within our nonlinear-optical system*. We can find the optical analogue of the detector frequency gap Ω by first setting $\Delta k_1(z', \omega_i) = r(z')C(\omega_i)$ where $r(z)$ is a yet to be determined function of position (or distance travelled) z. From here, using the phase in Equation (3.19), we have that:

$$\int^z dz'(\Delta k_1(z')) = C(\omega_i)\int^z dz'(r(z')). \tag{3.20}$$

Comparing Equation (3.19) to Equation (3.10), Equation (3.20) tells us that the optical analogue of the detector energy gap $\Omega = C(\omega_i)$ provided:

$$r(z) = \frac{d\tau}{dz}. \tag{3.21}$$

To determine an optical analogue of the coupling constant λ and the switching function η in the Unruh-DeWitt system, we make the identification:

$$\frac{-i\lambda}{\sqrt{4\pi\omega}\hbar} \equiv \frac{2\pi O \alpha \chi^{(2)}_{eff}}{2}\sqrt{\frac{\hbar(\Omega_i+\omega_s)\Omega_i\omega_s}{\epsilon_0 n_p(\Omega_i+\omega_s)n_a(\omega_s)n_b(\Omega_i)\pi^3c^3}}, \tag{3.22}$$

which hints that $\alpha\chi^{(2)}_{eff}$, plays the role of the coupling constant λ and it can be shaped in some frequency range much as λ can be tuned. The functional part of the nonlinearity $D(z)$ plays a role very similar to the switching function η.

To further establish the similarity between the two systems, the dispersion relation $\frac{1}{\Delta v}$ must satisfy the equation:

$$\frac{1}{\Delta v(z)} = \frac{1}{r(z)} e^{-\frac{a}{c}z}. \tag{3.23}$$

Solving this equation for the acceleration of the detector gives:

$$a = -\frac{c}{z}\ln\Big(\frac{r}{\Delta v(z)}\Big). \tag{3.24}$$

Thus variable dispersion can be used to model accelerating bodies in optical systems. Given that *accelerating bodies* are *equivalent* to *freely-falling* ones [7], this result suggests a way of testing, in optical systems, the behaviour of bodies in a gravitational field, such as the spectra of radiation near the Event Horizon [43].

There are some conditions that are imposed on the optical parameters in-order to simulate accelerating UDW detectors in our nonlinear-optical setup. For non-zero accelerations, it is required that $\Delta v(z) > r(z)$, for all z. Given the relationship between r and Δk_1 we have another inequality, namely $C\Delta v(z) > \Delta k_1$.

3.4 Relevance to gravity

Our work has potential implications for performing gravitational experiments in the lab using nonlinear optics. Through the equivalence principle, which relates accelerating bodies with freely-falling ones [7], Equation (3.24) tells us that we can simulate gravitational phenomena using variable dispersion. In other words, while nonlinear optical phenomena such as the Kerr effect have been used to simulate gravitational effects such as light spectra near Event Horizons [43] (for example), we propose a different way of using nonlinear optics to simulate gravitational phenomena, i.e. with *variable dispersion*. Beyond the possibility of performing experiments in gravity using nonlinear optics, we can learn something about what a quantum gravity theory will look like from a comparison of the Hamiltonians of the UDW model and that of SPDC.

The Hamiltonians from Equation (3.6) and Equation (3.14) belong to the same family of quadratic Hamiltonians. For SPDC we arrived at its quadratic form in Equation (3.14) from the *classical pump approximation*. Without the classical pump approximation the SPDC Hamiltonian is *trilinear* (details of trilinear Hamiltonians can be found here [44]). This leads us to a tantalizing idea, namely, what happens if we can *think of H_{UDW} as being trilinear as well*. It would imply that the coupling λ in the UDW model may come from a classical approximation of some analogous *gravitational field* just as is the case for the pump amplitude from SPDC considered here. Thus, like the pump field, we expect that gravity has a gauge theory description. This description will make finding a quantum gravity theory a problem of quantizing a gauge theory. While this prospect is intriguing, we do not explore its consequences here, deferring it, instead, to potential future work.

3.5 Summary

We showed that the nonlinear quantum-optical phenomenon known as SPDC can be used to simulate Unruh-DeWitt detectors. In this simulation, one generated mode of SPDC is modeled as the scalar field while the other generated mode is modeled as the detector. The Unruh-DeWitt detector's acceleration is represented in the optical simulation as inverse-group-velocity gradients through variable dispersion. Thus, through the equivalence principle, variable dispersion makes it possible to simulate gravity in our optical system. To see how our optical system can simulate UDW detectors, we started from their Hamiltonians.

In the SPDC Hamiltonian used here in this simulation, the pump mode is assumed to be classical, which we argued may represent *classical gravity*. The pump amplitude together with the constant $\chi^{(2)}_{\text{eff}}$ parameter, that characterizes the strength of the interaction of the different modes in this nonlinear-optical setup, acts as the coupling parameter that appears in the Hamiltonian used to model Unruh-DeWitt detectors. This link between the pump amplitude and the coupling in the UDW system reveals an interesting feature of gravity, namely that gravity might have a gauge theory description just as this pump amplitude. Thus quantizing gravity could be as straight-forward as quantizing a gauge theory, though finding such a gauge theory may not be so simple. Testing the Unruh effect, through experiments for example, can help in finding a quantum gravity theory, because such tests can have potential new insights about gravity at high energy scales. While in [37] we propose an experiment to test the Unruh effect using the connection between SPDC and the UDW model, here we are only interested in the theoretical implications of this connection.

Our work suggests a possible way to investigate, at least, some gravitational phenomena in optical systems via variable dispersion*. Furthermore, our simulation highlights the point that the coupling between the UDW detector and the scalar field comes from a classical approximation of gravity. We can see this from the identification of the pump amplitude, which is assumed to be classical, with this coupling constant. The identification suggests that quantizing gravity may be just like quantizing the electric field or quantizing a gauge theory.*

Chapter 4

Brighter thermal light sources using non-Hermitian optics

Statement of Contributions to Jointly Authored Work Contained in This Chapter of The Thesis

Nicolás Quesada, Eugene Adjei, Ramy El-Ganainy, and Agata M. Brańczyk, *Non-Hermitian Engineering for Brighter Broadband Pseudothermal Light*, Phys. Rev. A **100**, 043805 (2019).

Nicolás Queasda helped develop the formalism and was involved in drafting the article. Eugene Adjei was involved in making plots and in drafting the article. Ramy El-Ganainy had the original idea and was involved in drafting the article. Agata M. Brańczyk helped develop the idea and was involved in drafting the article.

Photon loss in quantum systems is often viewed as something bad. It is linked with the degradation of entanglement [45], making it automatically bad for studying systems which rely on this key feature of quantum mechanics. However, loss may not always be bad even for quantum systems. This intriguing possibility has some benefits in optical systems such as parametric amplifiers [46]. In this chapter, we will explain how loss can be useful in the generation of *brighter* thermal light sources.

Thermal light is an important kind of radiation produced by our sun, which is needed to sustain life here on this planet. Thermal light makes some biological processes such as photosynthesis possible. It has been argued in [47] that to study such biological processes in the lab one needs a reliable source of thermal light. Thermal light refers to light that is incoherent to the point that only one parameter can describe it; for blackbody radiation (from the sun, for example), this parameter is temperature. In general, the parameter is the average of the quantity that has the incoherent, geometric distribution. As such, there are different notions of thermality; there is thermality in the spatial distribution of photons, or in the distribution of their frequency. For single-frequency light, the kind we consider here, thermality is defined in the distribution of the number of photons. Returning back to the use of thermal radiation in the lab, it may be argued that a light-bulb should suffice as a soucre of thermal light. However, using them in the lab for such purposes is impractical since in any given mode, of the myriad of modes produced by a light bulb, there are only few photons. So that a collimated beam from a light bulb is just too weak to be used in the lab for any serious study [48]. The other source of thermal light, where here thermality is in the spatial distribution of photons, in the lab is a rotating glass prism. Light from a laser source is made incident on a rotating glass prism. At different points on the rotating prism the incident radiation acquires a random phase and the resulting different beams are recombined to form spatially incoherent light. This works well for CW lasers but it is debatable as to whether or not a pulsed laser can be used to produce bright thermal light in this way [49]. Radiation from each generated mode of Spontaneous Parametric Down-Conversion (SPDC) is thermal (in the photon-number sense). Hence, SPDC can be another source of thermal light.

Here, we investigate what effect photon loss in the idler mode from SPDC has on the intensity of the signal mode. It is found here that in certain regimes away from what is known as the phasematched regime, the signal mode intensity gets amplified when the idler mode experiences loss. In SPDC, light (known as the pump mode) interacting with a nonlinear material generates two entangled modes historically known as the idler and signal modes and any one of these modes is thermal. Intuitively this can be understood through the observation that without phasematching there is a spontaneous process of *generation* and *recombination*. These two processes are captured in the SPDC Hamiltonian and they are Hermitian conjugates of each other. During the *generation* process, in an approximate sense, photon pairs in the signal and idler modes are created from the pump's interaction with the nonlinear material, however these created photon pairs interact with each other via annihilation and create new photons in the pump mode as a result. The annihilation process is what we call *recombination*. These two competing processes explain the reduced intensity observed in the two modes. Here, we introduce loss in the idler mode to prevent *recombination* from occurring so that there are more photons in the signal mode i.e., not destroyed, making it brighter. Thus, in principle at least, engineered loss can be used to make brighter sources of thermal light.

Quantum systems with loss undergo non-unitary evolution. The equation of motion describing the dynamics of such systems is known as a quantum Lindblad equation. This equation generalizes unitary evolution captured by the Schrödinger equation to include loss in the systems' dynamics due to it's interaction with the environment. In non-unitary evolution, the state of the system is not accurately described by a wave-function, an operator called a density matrix is better suited instead. In describing lossy dynamics in SPDC we will use a system of quantum Lindblad equations with the system states being described by operators. Solving such a system will reveal how loss can increase the brightness of thermal light.

To illustrate this non-Hermitian amplification we begin by reviewing nonlinear sources of thermal light in Section (4.1). We then proceed in Section (4.2) to study the dynamics of a lossy optical system using a quantum Lindblad master equation whose unitary part has a Hamiltonian from the nonlinear optical processes reviewed in the previous the section. Solving these system of equations, we proceed to show the amplification of the intensity in the signal mode as the idler mode experiences loss in Section (4.3). We look at some advantages of studying the effect of loss in quantum nonlinear-optical systems as discussed here in Section (4.4). We summarize and interpret our observations in Section (4.5).

4.1 Nonlinear optical sources of thermal light

In the classical-pump approximation, and assuming phasematching, the output of nonlinear-optical sources such as SPDC can be understood as a source of two-mode squeezed light. The quantum state that describes this light is a two-mode squeezed vacuum (TMSV) state:

$$|TMSV\rangle = S(\zeta)|0\rangle = e^{(-\zeta\hat{a}\hat{b}+\zeta^*\hat{a}^\dagger\hat{b}^\dagger)}|0\rangle = \frac{1}{\cosh\xi}\sum_{n=0}^{\infty}(-1)^n e^{in\phi}(\tanh\xi)^n|n\rangle_a|n\rangle_b \equiv \sum_{n=0}^{\infty}c_n|n\rangle_a|n\rangle_b\,, \tag{4.1}$$

where $S(\zeta) = e^{(-\zeta\hat{a}\hat{b}+\zeta^*\hat{a}^\dagger\hat{b}^\dagger)}$ is known as the two-mode squeezing operator, the single-frequency number-states in modes $j \in \{a,b\}$ are $|n\rangle_j$, and the parameter $\zeta = \xi e^{i\phi}$, where ξ defines the strength of squeezing and ϕ defines the axis of squeezing in phase space[1]. We also defined:

$$c_n = (-1)^n e^{in\phi}\frac{(\tanh\xi)^n}{\cosh\xi}\,. \tag{4.2}$$

[1] The amount of squeezing is proportional to the strength of the nonlinearity and the length of the nonlinear material.

It is well known that tracing out one mode of a single-frequency TMSV leaves the other mode in a single-frequency thermal state [50]:

$$\rho = \mathrm{tr}_b\left[|TMSV\rangle\langle TMSV|\right] = \sum_n |c_n|^2 |n\rangle_a\langle n|_a\,. \tag{4.3}$$

The average number of photons, or the intensity, in mode a is $\sinh^2\xi$ [51]. The goal of this project is to increase the intensity of the thermal state described by the reduced density matrix in Equation (4.3) without increasing the squeezing parameter. To do so, we will need to step outside the phasematching regime.

4.2 Non-Hermitian optics

In this chapter, we study what happens to the intensity of thermal light in one mode of the two-mode squeezed vacuum, when loss occurs in the other mode. In Chapter 4, we saw that we can describe the evolution of a system with respect to space rather than time. The equation that governs this evolution, Equation (2.19), is:

$$\mathrm{i}\frac{d}{dz}|\psi\rangle = -P(z)|\psi\rangle, \tag{4.4}$$

here P is:

$$P = \int d\omega \Big[\Delta k_a(\omega)a^\dagger(\omega)a(\omega) + \Delta k_b(\omega)b^\dagger(\omega)b(\omega) + \xi(a(\omega)b(\Omega-\omega) + \mathrm{h.c.})\Big], \tag{4.5}$$

with mode a photon frequency ω, pump mode frequency Ω, $\Delta k_j = \omega(v_p^{-1} - v_j^{-1})$ where $v_{p/j}$ are the group velocities of the pump and down-converted modes $j \in \{a, b\}$ respectively, and ξ describes the strength of the squeezing.

In the Heisenberg picture, the evolution of observables in a lossy system can be described by the quantum Lindblad master equation for spatial evolving systems which contains a nonunitary part to account for loss. Details of the nonunitary part can be found in [52]. For an observable $\mathcal{O}$, rate of loss γ_b, in mode b, and propagation in the z direction, the equation:

$$\frac{d\langle\mathcal{O}\rangle}{dz} = \frac{i}{\hbar}\langle[P,\mathcal{O}]\rangle + \gamma_b\langle(b^\dagger\mathcal{O}b - \frac{1}{2}\{b^\dagger b, \mathcal{O}\})\rangle, \tag{4.6}$$

describes the evolution of this observable and "{ , }" denote anticommutation [52]. This is what is typically done in open quantum systems where the system interacts with the environment and there is loss [52].

Working with commutators is easier than with anticommutators. We therefore express Eq (4.6) in terms of commutators (see Appendix (A.1)) and work in units for which $\hbar = 1$, which leads to the equation:

$$\frac{d}{dz}\langle\mathcal{O}\rangle = -i\langle[\mathcal{O}, P]\rangle - \frac{\gamma_b}{2}\langle(b^\dagger[b, \mathcal{O}] + [\mathcal{O}, b^\dagger]b)\rangle, \tag{4.7}$$

where P is the momentum defined in Equation (4.5).

4.2.1 The system of equations

We wish to understand the behaviour of the average number of photons $\langle a_\omega^\dagger a_\omega\rangle$ in the frequency mode a_ω (for frequency ω), as a function of loss in mode γ_b.

For the operator $a_\omega^\dagger a_\omega$, we obtain the differential equation from Equation (4.7):

$$\frac{d\langle a_\omega^\dagger a_\omega\rangle}{dz} = \mathrm{i}\xi\langle a_\omega b_{\Omega-\omega}\rangle - \mathrm{i}\xi\langle a_\omega^\dagger b_{\Omega-\omega}^\dagger\rangle\,, \tag{4.8}$$

for the pump frequency Ω. Another important point to note is that ω is an unspecified frequency, however when it is fixed the frequency of the associated mode b photon is also fixed to be $\Omega - \omega$. Within the crystal, therefore, a whole range of frequencies are produced, that are constrained by energy conservation to sum to the frequency Ω of the single-frequency pump. Thus, in a sense, we are tackling a multifrequency problem here as well (not just a single-frequency case).

A couple of points about Equation (4.8). It can be observed that terms containing $\langle a_\omega b_{\Omega-\omega}\rangle$, $\langle a_\omega^\dagger b_{\Omega-\omega}^\dagger\rangle$ appear. Meaning, to understand the evolution of $\langle a_\omega^\dagger a_\omega\rangle$, we need to understand the dynamics of $\langle a_\omega b_{\Omega-\omega}\rangle$, and $\langle a_\omega^\dagger b_{\Omega-\omega}^\dagger\rangle$ as well. The equations of motion of $\langle a_\omega b_{\Omega-\omega}\rangle$, and $\langle a_\omega^\dagger b_{\Omega-\omega}^\dagger\rangle$ follow from Equation (4.7):

$$\frac{d\langle a_\omega b_{\Omega-\omega}\rangle}{dz} = -\mathrm{i}(\Delta k_a + \Delta k_b)\langle a_\omega b_{\Omega-\omega}\rangle - \mathrm{i}\xi\langle a_\omega^\dagger a_\omega\rangle - \mathrm{i}\xi\langle b_{\Omega-\omega}^\dagger b_{\Omega-\omega}\rangle - \mathrm{i}\xi - \langle a_\omega b_{\Omega-\omega}\rangle\frac{\gamma_b}{2} \tag{4.9}$$

$$\frac{d\langle a_\omega^\dagger b_{\Omega-\omega}^\dagger\rangle}{dz} = \mathrm{i}(\Delta k_a + \Delta k_b)\langle a_\omega^\dagger b_{\Omega-\omega}^\dagger\rangle + \mathrm{i}\xi\langle a_\omega^\dagger a_\omega\rangle + \mathrm{i}\xi\langle b_{\Omega-\omega}^\dagger b_{\Omega-\omega}\rangle + \mathrm{i}\xi - \langle a_\omega^\dagger b_{\Omega-\omega}^\dagger\rangle\frac{\gamma_b}{2}. \tag{4.10}$$

Notice that while $\langle a_\omega^\dagger a_\omega \rangle$ doesn't depend on γ_b directly, it depends on γ_b indirectly through $\langle a_\omega b_{\Omega-\omega} \rangle$ and $\langle a_\omega^\dagger b_{\Omega-\omega}^\dagger \rangle$. There is now a new term appearing that contains $\langle b_{\Omega-\omega}^\dagger b_{\Omega-\omega} \rangle$, which comes from the commutator in the unitary part of Equation (4.7). Thus, we must determine the dynamics of the mean of this operator as well. The equation of motion for $\langle b_{\Omega-\omega}^\dagger b_{\Omega-\omega} \rangle$ is:

$$\frac{d\langle b_{\Omega-\omega}^\dagger b_{\Omega-\omega} \rangle}{dz} = \mathrm{i}\xi \langle a_\omega b_{\Omega-\omega} \rangle - \mathrm{i}\xi \langle a_\omega^\dagger b_{\Omega-\omega}^\dagger \rangle - \gamma_b \langle b_{\Omega-\omega}^\dagger b_{\Omega-\omega} \rangle. \tag{4.11}$$

In conclusion, we have a closed system of four coupled differential equations, namely: Equation (4.8), Equation (4.9), Equation (4.10)and Equation (4.11), to solve so as to study the effect of loss in one mode on the intensity of light in the other mode.

4.3 Amplification of thermal radiation from non-Hermitian optics

The system of equations from Section (4.2.1) is solved numerically for nonzero $\gamma_b = \gamma$. We find that loss in mode b can increase the intensity in mode a, as long as the system is sufficiently far away from the phase-matching condition $\Delta k = 0$, where $\Delta k = \Delta k_a + \Delta k_b$. This can be seen in Figure (4.1), where we plot $\langle a^\dagger a \rangle$ (we have dropped the frequency labels on the operators) as a function of Δk, for different values of γ_b. *The black dashed line shows the maximum intensity, optimized over all* γ*.*

We also observe an increase in the intensity in mode b even though loss is occuring here. There is a possibility that this increase intensity in the mode experiencing loss comes from higher-order effects.

In Figure (4.1), we can see that there is a critical point, which we call $\Delta k = \Delta k_{critical}$. Below this, the lossless system yields the maximum intensity. Above this point, the intensity can be increased by introducing loss.

The critical point is shifted close to or further away from phasematching as squeezing ξ and length are adjusted. When ξ is increased (and length kept fixed), the critical point shifts to the right, i.e. further away from phasematching, and it shifts to the left when length is increased (squeezing kept fixed). The squeezing parameter is proportional to the pump amplitude, hence increasing squeezing means increasing the pump power which naturally leads to more photon generation than there would have been away from phasematching thus shifting the critical point, where loss gives gain, further away. For longer

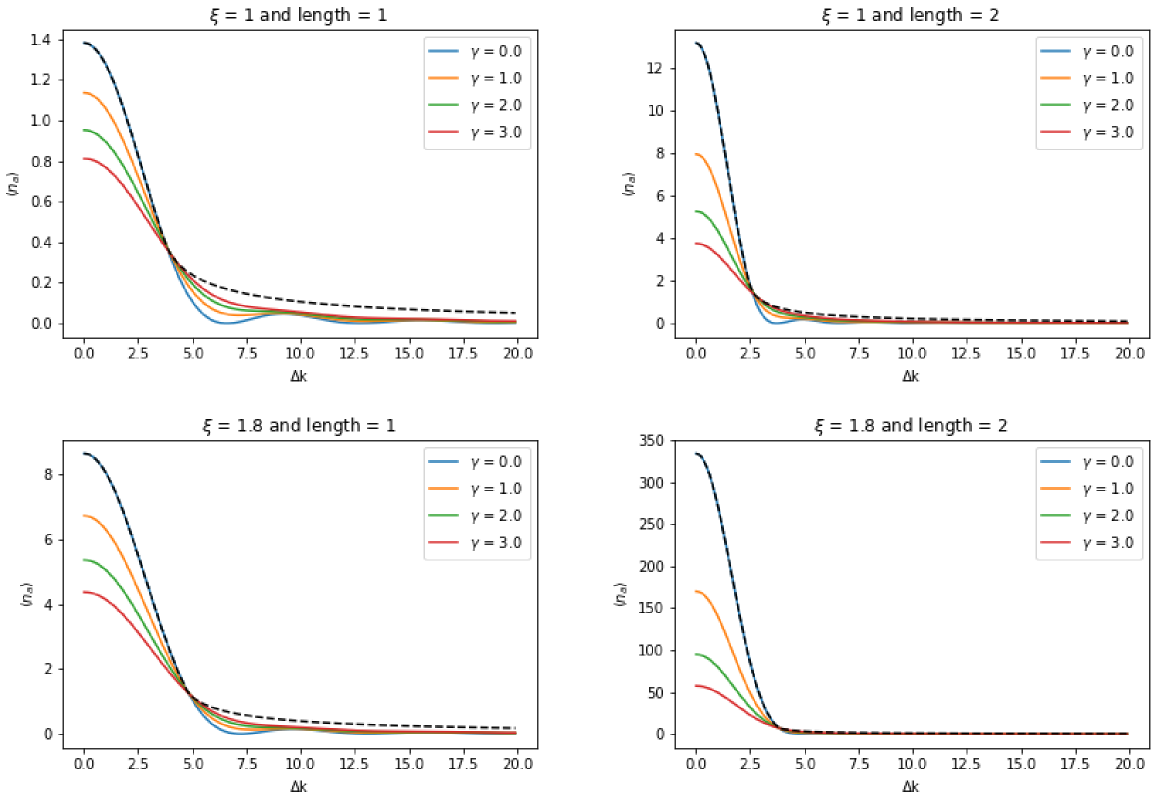

Figure 4.1: Intensity in mode a as a function of Δk, where $\Delta k = \Delta k_a + \Delta k_b$. *Black dashed line is for* $\gamma = \gamma_{\text{opt}}$.

crystals, there is more "recombination" and "photon generation" that can occur and this increased likelihood has the effect of decreasing the intensity so that loss is required for amplification.

This critical point can be identified in Figure (4.2), where we plot the optimal γ corresponding to the choices of ξ and length in Figure (4.1). Identification of the critical point will allow for the determination of the phasemismatch regime required to observe the intensity-increasing effect of optical loss as described in this chapter.

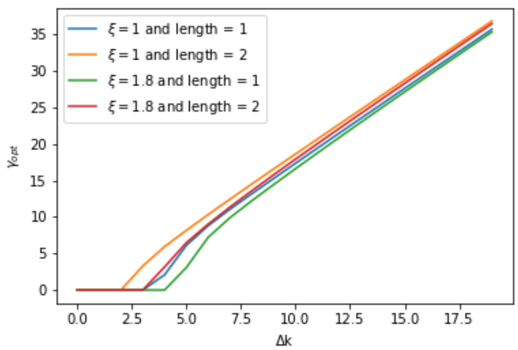

Figure 4.2: The optimum loss against the phasemismatch for different squeezing ξ and length.

4.4 A difference and an advantage with our approach

The behaviour we see here, for quantum nonlinear-optical phenomena like SPDC, has a classical analogue to the case when loss is introduced in classical four-wave mixing, but there is, at least, one difference between the quantum and classical scenario. In Figure 4.3, we see how the intensity changes as a function of position inside the nonlinear medium and we compare it with the classical analogue [1]. This plot did not use optimal γ, but rather a value that gave a similar looking plot to the one from [1]. While a seed is required in the classical case, as indicated in Figure 4.3 b), no such seed is needed in Figure 4.3 a).

It is also worth comparing our approach to Quasi-phase matching (QPM), a common technique in nonlinear optics that also increases the intensity of light generated outside the phasematched regime. QPM enables a positive feedback in the transfer of energy from incoming light modes to generated light modes [9]. We find that QPM works much better, even when compared with optimal γ, see Figure (4.4). Photon loss provides an alternative to QPM for making brighter light sources and future work needs to be done to investigate ways of enhancing its benefits to make it rival those of QPM.

a) b)

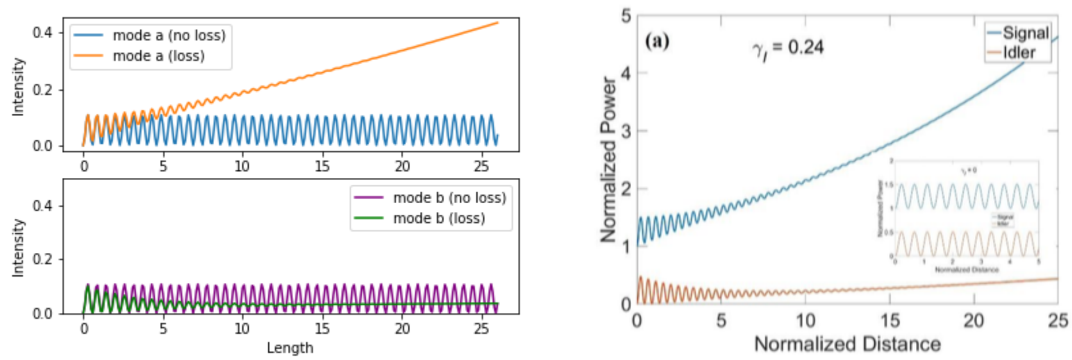

Figure 4.3: a) Intensity against the length for the two modes a and b in both the loss and no loss scenario. b) Analogous plot from Ahmed *et. al* [1]

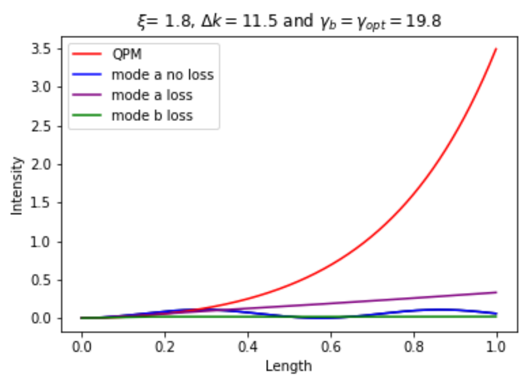

Figure 4.4: A plot of intensity against length which compares the QPM case with the loss and no loss scenario.

4.5 Discussion

Thermal light is characterized as being incoherent light [51], meaning there is randomness in some property of the light. These properties, like frequency or photon number, have a geometric distribution and this distribution is a defining feature of thermality. Artificial sources of thermal light have applications in biological research, for example, into processes like photosynthesis and vision tests. In these areas, it is conceivable that bright thermal light sources will be useful.

In this chapter, we showed that the brightness of thermal light from nonlinear sources can be increased by engineering loss in one of the generated modes. This nonunitary transformation prevents “recombination”, which is one of the competing processes, besides photon generation, that occurs within the nonlinear media away from phasematching. Our work shows that nonlinear materials serve as a source for bright thermal light beams beyond the constraints of phasematching in optical systems. Nonlinear phenomena which generate entangled photon pairs are some examples of sources of thermal light. The setup can be described as having incident radiation on a nonlinear material which leads to a production of other modes of radiation (entangled pairs of photons) and by somehow engineering loss into one of the generated mode, more power gets transferred to the other mode which thus increases in intensity.

The conversion process from the incident radiation to the generated modes through nonlinear interactions initiated by the nonlinear media is most efficient when there is phasematching. The increase in brightness does not occur for all values of phasemismatch. In fact below a certain “critical point” $\Delta k_{critical}$, the lossless scenario has a higher intensity than that of loss. However, beyond $\Delta k_{critical}$ there is gain in the intensity of the generated thermal light. Hence the counter-intuitive statement that “loss brings gain” is shown here to be the case.

The $\Delta k_{critical}$ can be shifted closer and further away from phasematching depending on how the squeezing ξ and length are adjusted. When ξ is increased, $\Delta k_{critical}$ is shifted to higher values of phasemismatch and the opposite effect occurs when ξ is decreased. The “critical point” $\Delta k_{critical}$ is shifted to higher values of phasemismatch for longer crystals and the opposite occurs for shorter crystals.

These results are valid for the case where the pump laser is a single frequency, i.e. continuous wave (CW). The next logical step is to find out what happens when we have a broad-band pump laser. To do this, we must extend our treatment to include correlations between different frequencies. We expect things to get more complicated but we anticipate the principle to remain the same i.e., photon loss improves intensity.

Chapter 5

Gravitational refractive index and light propagation

As we have so far seen in this thesis, analogue systems allow for the cross-pollination of ideas. Furthermore, they allow the study of physical systems that would otherwise be very difficult to recreate in the lab and thus to investigate. For example, it was shown in [43] that event horizons have an optical analogue which consist of fibres and pulses that allow for the study of the behavior of light near event horizons on earth. Beyond the benefit of being able to test concepts using analogue gravity, analogue systems can offer breakthroughs in furthering our understanding of phenomena in the context of what is known once a suitable analogue system has been identified. It is this feature of analogue systems we seek to explore here. In particular, we want to find out what an analogue system of gravity can tell us about quantizing gravity. The theory of gravity we are trying to quantize is Einstein's general relativity.

General relativity (GR) is an established classical theory of gravity that has been verified many times. It is a geometric theory of gravity which views gravitational phenomena as the curvature of spacetime. However, it fails to explain the physics of regions where this curvature is very high, such as at the center of blackholes and at the big bang. This failure of such a hitherto reliable theory has many convinced that a quantum, rather than a classical, theory of gravity is needed to explain phenomena at regions with high spacetime curvature. There have been attempts to find a perturbative quantum gravity theory to address this challenge. However, such a theory is nonrenormalisable, thus it fails to provide any useful predictions about reality in high gravitational regions. This failure has made it necessary to find a nonperturbative theory of quantum gravity. Here, we do not attempt to find a quantum gravity theory, instead we just point out what must *not* be quantized to find such a theory.

The problem of finding a nonperturbative theory of quantum gravity is an outstanding and persistent problem in theoretical physics. Its solution remains elusive despite over half a century worth of effort put into it, a fact evidenced from the many different approaches to tackling it ranging from string theory, to loop quantum gravity, etc. In the absence of a nonperturbative theory of gravity, other indirect approaches are used to study gravitational phenomena not only at the strong but also at the weak regime. One example of an indirect approach to investigating gravitational phenomena is the use of analogue systems.

An example of an analogue system used to test gravitational phenomena are Bos-Einstein Condensates (BEC) . In [53], the authors use dilute gas BEC to make stable sonic blackholes. In fact their system can mimic horizon particle-pair production just as has been proposed to occur for Hawking radiation of blackholes and since Hawking radiation is a consequence of Quantum Field Theory (QFT) applied to curved spacetime (semi-classical gravity), which in-turn is the next best thing for a theory of quantum gravity, this analogue gravity system has potential implications for aiding in finding a theory of quantum gravity. Following the theme of probing semi-classical gravity using BEC, the authors in [54] use the results of an experiment that demonstrated controllable tuning of scattering length in Rubidium 85 to test the predictions of semi-classical gravity, namely particle production from the expanding universe. They had previously shown in [55][56] that a varying scattering length in BEC corresponds to variable speed of light in the effective metric, thus the positive results of the Rubidium 85 experiments encouraged them to push the analogue system further and use it to test other predictions of semi-classical gravity. Another field that has been used to study gravity is optics.

Connecting optics with different fields of physics has been the source of many rich and new ideas and a way of studying phenomena that would be impractical to investigate otherwise. Optics has been used to create event horizons in the lab, see [43]. The authors achieved this by using ultrashort pulses in microstructured fibres to create a medium whose local speed exceeded the effective speed of any wave propagating within it, effectively creating a horizon. They observed blue-shifting of light as predicted by classical optics and they were able to demonstrate, by theoretical calculations, that their system can study semi-classical gravity effects like Hawking radiation. In [57], they use an optical vortex to represent the spacetime in the vicinity of a rotating blackhole. They then proceed further to use the observed resonances to enhance the optical analogue of Hawking radiation and find that this makes it feasible to observe such radiation in the lab. Another connection, discussed in Chapter 3, uses the simulation of *accelerating* Unruh-DeWitt detectors using nonlinear optics [37] to infer a possible gauge theory of gravity, which will in-turn pave the way for a quantum gravity theory. In yet another connection between optics and gravity, a relation between the refractive index and the spacetime metric for static gravitational fields is made, see here [10]. This last connection illustrates how linear optics and gravity are connected.

Here we find a different analogue gravity system using linear optics that seeks to explore how deep the connection is between the refractive index and spacetime. The set-up consist of a linear material with a *spacetime*-dependent linear susceptibility, which contrasts with the spatially-varying refractive index discussed in [10]. This spacetime-dependent linear susceptibility plays the role of an effective spacetime metric for a *dynamical* spacetime. The wave equation describing the propagation of light in this medium is a modified Maxwell's wave equation that can be traced back to a generalisation of Faraday's law for the macroscopic electric (displacement field) and magnetic field. This modified version is meant to mimic general covariance in GR which is its hallmark feature.

For the linear medium we look at two cases. In one case we consider an isotropic medium and in the other case we consider a general non-isotropic medium. The isotropic case is considered because it is simple to work with while the more complicated case is briefly discussed and further investigation is left for future work. For both cases, in the weak-field limit, we observe that the effective spacetime metric is expressed as a perturbation of some fixed background spacetime and the perturbation is our linear susceptibility for both isotropic and anisotropic media. Hence we can apply the machinery of linearized gravity to describe the dynamics of this "optical spacetime." We also find that light propagating in this medium acquires an effective mass which is a function of the material. *This mass is not predicted in the approach which relates the spatially-varying refractive index and the spacetime metric.* Techniques from linearized gravity (it is the weak-field limit of GR) are used here to relate the mass with the radiation stress-energy tensor for the anisotropic case and with the trace of this tensor for the isotropic case. This last part suggests that the effective mass of light propagating in this dynamical isotropic medium depends on the spacetime dimension since the trace of the radiation stress-energy tensor depends on spacetime dimension. This dependence on spacetime dimension means that photons are massless in 4d spacetime and are massive in 3d spacetime. The mass is not defined for spacetime with dimension 2 or 1. This connection between the photon mass and the radiation stress-energy tensor verifies the intuition that the photon is massless due to the vanishing of the trace of the stress-energy tensor. *Another interesting consequence of the work done here is that the connection between a dynamical refractive index and a dynamical spacetime suggests that we should not be quantizing the metric to find a quantum gravity theory since in optics the refractive index is not quantized to find a quantum optics theory.*

This chapter is structured as follows: we first look at Maxwell's equations in the presence of a dynamical medium in Section (5.1). We look at the connection between the 4d spacetime metric and the refractive index in Section (5.2). In Section (5.3) a connection is made between the mass and the stress-energy tensor. From here, we generalize the formalism from isotropic media to anisotropic ones in Section (5.4). We then explain how a modified Faraday's law leads to the modified Maxwell's wave equation used in this chapter in Section (5.5). Finally we summarize what has been done here and interpret the results in Section (5.6).

5.1 Maxwell's wave equation

To explore the connection between an optical medium and the spacetime manifold upon which the metric is supported, we will start from light interacting with a linear material. More precisely, we will start from the Maxwell's wave equation for the electric field components $E_i(\mathbf{r},t)$ of the electric field $\mathbf{E}(\mathbf{r},t)$ interacting with a linear material, resulting in a source term given by the generated polarization field $\mathbf{P}^{(1)}(\mathbf{r},t)$ which is proportional only to the applied field, with the constant of proportionality given by the linear susceptibility, $\chi^{(1)}(\mathbf{r},t)$, assumed to be *time-dependent*:

$$\nabla^2 E_i(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}E_i(\mathbf{r},t) = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 P_i^{(1)}(\mathbf{r},t)}{\partial t^2} - \frac{1}{\epsilon_0}\nabla^2 P_i^{(1)}(\mathbf{r},t), \tag{5.1}$$

where ϵ_0, is the permittivity of free space and $P_i^{(1)}(\mathbf{r},t) = \epsilon_0\chi^{(1)}(\mathbf{r},t)E_i(\mathbf{r},t)$ and c is the speed of light. In addition, we have assumed that the medium is isotropic hence the absence of indices on $\chi^{(1)}(\mathbf{r},t)$. The wave equation, Equation (5.1), is modified to include a term $\frac{1}{\epsilon_0}\nabla^2 P_i^{(1)}(\mathbf{r},t)$. This modification is necessary to ensure that the right-hand-side is relativistically covariant (needed to simulate spacetime) which, we will show, makes it possible to relate the refractive index with the spacetime metric.

We will express the electric field in terms of the gauge potential $\mathbf{A}$ through the relation:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}, \tag{5.2}$$

where we are working in natural units for which $c = 1$. Furthermore, we assumed, without loss of generality, that the Coulomb potential ϕ is zero. [1]

[1] Otherwise the electric field would have been:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} + \nabla\phi. \tag{5.3}$$

While it is perfectly fine to work with the field, we choose to work with the gauge potential here. The equation of motion for the gauge potential can be found from Equation (5.1) to be:

$$n^2(\mathbf{r},t)\nabla^2\tilde{A}^i(\mathbf{r},p_0) + n^2(\mathbf{r},t)p_0^2\tilde{A}^i(\mathbf{r},p_0) = Q, \tag{5.4}$$

where the right-hand-side is:

$$Q = -\mathrm{i}\Big(\frac{\partial}{\partial t}\chi^{(1)}(\mathbf{r},t)\Big)p_0\tilde{A}^i(\mathbf{r},p_0) - \Big(\nabla\chi^{(1)}(\mathbf{r},t)\Big)\cdot\nabla\tilde{A}^i(\mathbf{r},p_0) + \Big(\Big(\frac{\partial^2}{\partial t^2} - \nabla^2\Big)\chi^{(1)}(\mathbf{r},t)\Big)\tilde{A}^i(\mathbf{r},p_0), \tag{5.5}$$

and the temporal Fourier transform of the gauge potential $\tilde{A}^i(\mathbf{r},p_0)$:

$$\tilde{A}^i(\mathbf{r},p_0) = \int dt\; e^{\mathrm{i}p_0 t}A(\mathbf{r},t), \tag{5.6}$$

and $n^2(\mathbf{r},t) = 1 + \chi^{(1)}(\mathbf{r},t)$. In the wave-like approximation, where terms with quadratic differential operators (or momenta for the Fourier transform) acting on a given function dominate those with linear differential operators acting on the same function i.e., $\Big|n^2(\mathbf{r},t)p_0^2\tilde{A}^i(\mathbf{r},p_0)\Big| \gg \Big|-\Big(\frac{\partial}{\partial t}\chi^{(1)}(\mathbf{r},t)\Big)p_0\tilde{A}^i(\mathbf{r},p_0)\Big|$, and $\Big|n^2(\mathbf{r},t)\nabla^2\tilde{A}^i(\mathbf{r},p_0)\Big| \gg \Big|\Big(\nabla\chi^{(1)}(\mathbf{r},t)\Big)\cdot\nabla\tilde{A}^i(\mathbf{r},p_0)\Big|$, with $|\;\;|$ denoting the magnitude, we have the Proca wave equation [2]:

$$\nabla^2\tilde{A}^i(\mathbf{r},p_0) + p_0^2\tilde{A}^i(\mathbf{r},p_0) = -\frac{1}{n^2(\mathbf{r},t)}\Big(\Box\chi^{(1)}(\mathbf{r},t)\Big)\tilde{A}^i(\mathbf{r},p_0), \tag{5.7}$$

with the mass squared $m^2(\mathbf{r},t) = -\frac{1}{n^2(\mathbf{r},t)}\Box\chi^{(1)}(\mathbf{r},t)$ and the d'Alembertian $\Box = \nabla^2 - \frac{\partial^2}{\partial t^2}$. Thus photons traveling in this medium propagate as though they were massive particles and their mass is a function of the medium.

5.2 The 4d spacetime metric and refractive index connection

Such a dynamical linear material must follow some dynamical law. To find this dynamical law, we will look at the propagator. The propagator $G_{\mu\nu}(x - x')$ for spacetime points x

[2]The Proca equation is a relativistic wave equation for massive spin 1 particles.

and x' is defined as the inverse of the differential operator $\mathfrak{D}$ in Equation (5.7) i.e.:

$$\mathfrak{D}\mathrm{i}G_{\mu\nu}(x-x') = g_{\mu\nu}\delta^4(x-x'), \tag{5.8}$$

where the differential operator $\mathfrak{D}$ is:

$$\mathfrak{D} = n^2\nabla^2 + n^2 p_0^2 + \Box\chi^{(1)}(\mathbf{r}, t), \tag{5.9}$$

and $g_{\mu\nu}$ is the 4d spacetime metric.

Using the Fourier transform of the Dirac delta:

$$\delta^{(4)}(x-x') = \int d^4p e^{\mathrm{i}p(x-x')}, \tag{5.10}$$

and the Fourier transform of the the propagator:

$$G_{\mu\nu}(x-x') = \int d^4p e^{\mathrm{i}p(x-x')}\tilde{G}_{\mu\nu}(p), \tag{5.11}$$

we have from Equation (5.8) that:

$$\int d^4p\Big(-n^2\mathbf{p}^2 + n^2p_0^2 - \Box\chi^{(1)}(\mathbf{r}, t)\Big)\mathrm{i}\tilde{G}(p)_{\mu\nu}e^{\mathrm{i}p(x-x')} = \int d^4p g_{\mu\nu}e^{\mathrm{i}p(x-x')}. \tag{5.12}$$

Comparing the left and right-hand-side of Equation (5.12), the Fourier transform of the propagator $\tilde{G}(p)_{\mu\nu}$, with p being the four-momentum, is:

$$\tilde{G}(p)_{\mu\nu} = \mathrm{i}\frac{g_{\mu\nu}}{n^2(\mathbf{r}, t)p_0^2 - n^2(\mathbf{r}, t)\mathbf{p}^2 - \Box\chi^{(1)}(\mathbf{r}, t)}, \tag{5.13}$$

where p_0 is the energy of the photon, and $\mathbf{p}$ is its three-momentum and it is understood that $\mathbf{p}^2$ is the dot product of the three-momentum with itself. Dividing both the numerator and denominator of Equation (5.13) through by the square of the refractive index, it is straightforward to see that the photon propagator, in the presence of the spacetime dependent linear susceptibility, is:

$$\tilde{G}(p)_{\mu\nu} = \mathrm{i}\frac{g_{\mu\nu}}{p^2 - m^2(\mathbf{r}, t) + \mathrm{i}\epsilon}\Big(1 + \chi^{(1)}(\mathbf{r}, t)\Big)^{-1}, \tag{5.14}$$

where $p^2 = p_0^2 - \mathbf{p}^2$, $m^2 = \frac{\Box\chi^{(1)}(\mathbf{r},t)}{n^2(\mathbf{r},t)}$ and the regulator ϵ is included, by hand, to specify the contour used to evaluate the complex integral to find the propagator in spacetime. From

Equation (5.13), we can read off an effective metric $g_{\mu\nu}^{eff}$ experienced by the propagating photon in this dynamical medium to be:

$$g_{\mu\nu}^{eff} \equiv g_{\mu\nu}\Big(1 + \chi^{(1)}(\mathbf{r}, t)\Big)^{-1}. \tag{5.15}$$

Thus, the medium behaves like a 4d spacetime manifold with metric given by Equation (5.15).

5.3 The mass-stress-energy tensor relationship

There are potential interesting implications of Equation (5.15). One could use the effective metric as it appears there in the Einstein field equations and try to solve them to deduce what kind of spacetime they imply. We anticipate this might not be so simple. Here we will consider a simple case where the linear susceptibility is small for all time and space i.e., the so called weak-field limit. We will see that in this limit, the Einstein field equations become linear and thus much simpler to solve. We leave the more complicated case for future work. The effective spacetime metric from Equation (5.15) becomes, in the weak-field limit $\chi^{(1)} \ll 1$:

$$g_{\mu\nu}^{eff}(\mathbf{r}, t) = g_{\mu\nu}\Big(1 - \chi^{(1)}(\mathbf{r}, t)\Big). \tag{5.16}$$

This connection between the refractive index and the effective spacetime metric suggest that the equation of motion for the dynamical linear medium is just the Einstein field equations. As a reminder, the Einstein field equations are:

$$R_{\mu\nu}[g^{eff}] - \frac{1}{2}R[g^{eff}]g_{\mu\nu}^{eff} = 8\pi G T_{\mu\nu}, \tag{5.17}$$

with $R_{\mu\nu}$ being the Ricci tensor, R the Ricci scalar (or the trace of the Ricci tensor), G is Newton's constant and the radiation stress-energy tensor $T_{\mu\nu}$ is given by:

$$T_{\mu\nu} = F^{\mu\alpha}F^{\nu}{}_{\alpha} - \frac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}, \tag{5.18}$$

where the field strength tensor is $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$. Also $\partial^{\mu} = \frac{\partial}{\partial x_{\mu}}$ for spacetime point $x_{\mu} = g_{\mu\nu}x^{\nu}$ and $x^{\nu} = (t, \mathbf{x})$.

We can find what this expression $\Box\chi$ relates to from the Einstein field equations. Given the weak-field limit we are working with, the Einstein field equations simplify from their complicated nonlinear form as they appear in Equation (5.17) to a set of linear equations. This is the origin of the term linearized gravity. Using linearized gravity, we can relate $\Box\chi$ with the radiation stress-energy tensor to obtain the relation:

$$\Box\chi^{(1)}(\mathbf{r},t) = \frac{-16\pi \mathrm{G}}{(d-1)(d-2)c^3} T, \tag{5.19}$$

where d is the spacetime dimension and $T = \frac{1}{4\pi} F^{\mu\nu} F_{\mu\nu} (\frac{4-d}{4})$ is the trace of the radiation stress-energy tensor. The resulting mass-squared is:

$$m^2 = \frac{-(4-d)\pi \mathrm{G}}{(d-1)(d-2)c^3} \frac{F^{\mu\nu} F_{\mu\nu}}{n^2}. \tag{5.20}$$

Thus, photons propagate as massless particles in our optical medium in four spacetime dimensions. However, for spacetime dimensions higher than four, photons propagate as massive particles. It has been argued that the vanishing of the trace of the radiation stress-energy tensor may account for the massless nature of light, and Equation (5.20) seems to verify this intuition.

5.4 Beyond isotropic media

In arriving at a Proca-like wave equation for light from its interaction with some medium, we assumed that the medium was isotropic, which translates to the linear susceptibility having no indices. Here we want to generalize the result of the previous section by relaxing the isotropicity requirement of the linear susceptibility. In this relaxed version, the linear susceptibility changes from $g_{\mu\nu}\chi^{(1)} \to \chi^{(1)}_{\mu\nu}$ and the associated effective metric, in the weak-field limit, is:

$$g^{eff}_{\mu\nu}(\mathbf{r},t) = g_{\mu\nu} - \chi^{(1)}_{\mu\nu}(\mathbf{r},t), \tag{5.21}$$

where $g_{\mu\nu}$ is some fixed metric.

In this new spacetime, the mass-term is just as before but with a different index structure i.e. $\Box\chi^{(1)} \to \Box\chi^{(1)}_{\mu\nu}$. Working in the weak-field limit and using the linearized Einstein field equations, we obtain an expression for the quantity $\Box\chi^{(1)}_{\mu\nu}$ which is:

$$-\Box\chi^{(1)}_{\mu\nu} = \frac{16\pi G}{c^3} T_{\mu\nu} - \partial_\sigma\partial_\mu (\chi^{(1)})^{\sigma}{}_{\nu} - \partial_\sigma\partial_\nu (\chi^{(1)})^{\sigma}{}_{\mu} + \partial_\mu\partial_\nu\chi^{(1)} + g_{\mu\nu}\partial_\rho\partial_\lambda (\chi^{(1)})^{\rho\lambda} - g_{\mu\nu}\Box\chi^{(1)}, \tag{5.22}$$

where the radiation stress-energy tensor $T_{\mu\nu}$ is defined in Equation (5.18) and the trace of the linear susceptibility is $\chi^{(1)} = g^{\mu\nu}\chi^{(1)}_{\mu\nu}$. Hence, we are left with the mass matrix:

$$(m^2)^{\rho}{}_{\nu} = (n^2)^{\rho\mu}(\Box\chi^{(1)}_{\mu\nu}), \tag{5.23}$$

whose eigenvalues correspond to the "mass" of the photon.

On the right-hand-side of Equation (5.22) appears a term known as the gravitational stress-energy pseudotensor. It is the stress-energy tensor associated with the gravitational field itself, which is here represented by the derivatives of the linear susceptibility. If we denote this stress-energy tensor by $T^{\mathrm{Grav}}_{\mu\nu}$ then:

$$T^{\mathrm{Grav}}_{\mu\nu} = \partial_\sigma\partial_\mu(\chi^{(1)})^{\sigma}{}_{\nu} + \partial_\sigma\partial_\nu(\chi^{(1)})^{\sigma}{}_{\mu} - \partial_\mu\partial_\nu\chi^{(1)} - g_{\mu\nu}\partial_\rho\partial_\lambda(\chi^{(1)})^{\rho\lambda} + g_{\mu\nu}\Box\chi^{(1)}. \tag{5.24}$$

The origin of this gravitational energy expressed in-terms of the linear susceptibility further reinforces the idea that the optical linear susceptibility can be viewed as a metric.

5.5 Origin of the modified Maxwell's equation with linear source term

In deriving the connection between the refractive index and gravity we started from a *modified* Maxwell's wave equation for light interacting with a linear medium in Equation (5.1), where the modification is an inclusion of a second-order spatial derivative acting on the polarization field. Here we seek to explain the origin of this modification at the level of Maxwell's equations, with source terms:

$$\nabla \cdot \mathbf{E} = \rho, \tag{5.25}$$
$$\nabla \cdot \mathbf{B} = 0, \tag{5.26}$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{5.27}$$
$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}, \tag{5.28}$$

with the familiar magnetic field $\mathbf{B}$, electric field $\mathbf{E}$, and the sources of these fields being the charge density ρ and the current $\mathbf{j}$.

For light interacting with a linear medium, the electric and magnetic fields become modified to be displacement field $\mathbf{D}$ and the magnetizing field $\mathbf{H}$. These modified fields are believed to be only present in Maxwell's equations with sources, free charge ρ_f and free current $\mathbf{j}_f$ i.e.:

$$\nabla \cdot \mathbf{D} = \rho_f, \tag{5.29}$$
$$\nabla \cdot \mathbf{B} = 0, \tag{5.30}$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{5.31}$$
$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}_f, \tag{5.32}$$

with the argument in support of this being that the bound charges of the linear material is what modifies the fields as such only the fields appearing in Maxwell's equations with sources should get modified.

However, a counter argument against this assertion can be formulated. In this counter-argument, consider the microscopic Maxwell's equations if there were magnetic monopoles [58]:

$$\nabla \cdot \mathbf{E} = \rho, \tag{5.33}$$
$$\nabla \cdot \mathbf{B} = \rho_m, \tag{5.34}$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} - \mathbf{j}_m, \tag{5.35}$$
$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}, \tag{5.36}$$

where ρ_m is the magnetic monopole charge density and the magnetic current is $\mathbf{j}_m$. Clearly, all the equations that make-up Maxwell's equations can be interpreted as all having sources with the sources of the magnetic field just happening to be zero. As such, we propose that rather than modifying just the "source terms" and leaving the fields in Faraday's law untouched, we should modify the fields appearing in all the equations that make-up Maxwell's equations (which includes Faraday's law) to obtain:

$$\nabla \cdot \mathbf{D} = \rho_f, \tag{5.37}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{5.38}$$

$$\nabla \times \mathbf{D} = -\frac{\partial \mathbf{H}}{\partial t}, \tag{5.39}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}_f. \tag{5.40}$$

In the absence of charges and currents, we can obtain Maxwell's wave equation for light interacting within a material from Equations (5.37) - (5.40). This can be done by taking the curl $\nabla\times$ on both sides of Equation (5.39) to obtain:

$$\nabla \times \nabla \times \mathbf{D} = -\frac{\partial}{\partial t}\Big(\nabla \times \mathbf{H}\Big). \tag{5.41}$$

Using the identity true for any vector field $\mathbf{V}$, $\Big(\nabla \times \nabla\Big)\mathbf{V} = \nabla(\nabla \cdot \mathbf{V}) - \nabla^2\mathbf{V}$, Equation (5.41) becomes:

$$-\nabla^2\mathbf{D} = -\frac{\partial}{\partial t}\Big(\nabla \times \mathbf{H}\Big), \tag{5.42}$$

where we have dropped the term $\nabla \cdot \mathbf{D}$ since it is taken be negligible away from charges (see [9] and Equation (5.37)). From Equation (5.40), it implies that Equation (5.42) becomes the wave equation for the displacement field:

$$-\nabla^2\mathbf{D} = -\frac{\partial^2}{\partial t^2}\mathbf{D}. \tag{5.43}$$

Using that $\mathbf{D} = \mathbf{E} + \mathbf{P}$, Equation (5.43) becomes Equation (5.1)[3]:

$$\Box\mathbf{E} = -\Box\mathbf{P}. \tag{5.45}$$

Both sides of Equation (5.44) are relativistic. This twin relativistic nature suggest that the response field propagates at light speed. In the case of a linear material, where the polarization field is proportional to the applied electric field, Equation (5.44) predicts that light travels in this medium at light speed in vacuum.

[3]On the other hand, if we applied steps similar to the ones just used to obtain the wave equation from Maxwell's equations to Equations (5.29) - (5.32), we would obtain:

$$\Box\mathbf{E} = \frac{\partial^2}{\partial t^2}\mathbf{P}. \tag{5.44}$$

5.6 Summary

We showed that light interacting with a linear material propagates as though the photon was massive, with a mass term that is related to the radiation stress-energy tensor (its the trace for isotropic media and with the tensor itself for non-isotropic media). The connection between the photon mass and the trace of the radiation stress-energy tensor came from a wave-like approximation of a modified Maxwell wave equation for light interacting with a linear medium. This linear medium is described by a quantity known as the linear susceptibility (which we assume is spacetime-dependent just like the spacetime metric). The linear susceptibility acts as a conformal factor multiplying the ambient metric so that the product defines a new metric which can be viewed as a perturbation on the ambient metric background in the weak-field limit. In this limit, light traveling in a linear optical material can be viewed as light traveling in a weak gravitational field.

This connection between the mass and the radiation stress-energy tensor suggests that photons must be massless in 4d spacetime and massive in 3d spacetime and spacetime dimension $d > 4$ for isotropic media since the trace of the radiation stress-energy tensor vanishes in 4d and is nonzero for $d > 4$. Furthermore, the photon mass blows-up for spacetime dimensions $d = 1$ and 2. Thus, this analogue gravity system gives the result consistent with the intuition that the photon massless state is due to the vanishing of the radiation stress-energy tensor when spacetime dimension is 4. This analogue system has a strong link with gravity because the wave equation with a source-term governing the dynamics of light in this linear medium was modified from its original form to one that is relativistic.

From this modified Maxwell's wave equation for light interacting with a linear material, we arrived at a Proca wave equation for light. The modification is arrived at by a generalization of Faraday's law from the electric and magnetic fields to the displacement and magnetizing fields. With this modification, we obtain Maxwell's wave equation for the electric field that is driven by a source-term which is identical to the d'Alembertian operator acting on the polarization field as opposed to the commonly used form of a second-order time derivative acting on it. At the level of the gauge potential, we have a Proca-wave equation from an interaction Lagrangian involving the gauge field and the spacetime-dependent linear susceptibility. The modification was necessary in order to ensure that there is general covariance in our analogue gravity system; general covariance is a hallmark feature of GR. The modification also suggests that Faraday's law may be modified in the presence of matter fields contrary to what is presented in the macroscopic Maxwell's equations. This has the interesting outcome that matter fields, used in the macroscopic Maxwell's equations, propagate at the speed of light, even when they are propagating in vacuum.

In conclusion, we showed that light propagating in our analogue gravity system does so with its stress-energy tensor quantifying its mass as though it was a massive object in a gravitational field. The link between mass and radiation stress-energy tensor is intuitive and it has long been suspected that the massless state of light stems from the vanishing of the trace of its stress-energy tensor but no formal proof of this connection has been shown, a problem which is possibly addressed in our system. Also, the wave equation describing propagation of light in our analogue gravity system is a modified version of the standard Maxwell's wave equation with a source term representing the interaction of the propagating light with the material. The modification is used to ensure that everything is covariant so that our system can simulate general covariance as it appears in GR. This modification suggest that Faraday's law gets modified in the presence of matter, which is something that is commonly believed not to be the case. We have also seen here that the spacetime metric can be viewed as a dynamical refractive index of some optical analogue gravity system. The refractive index is a property of a media arising from collective behaviour of some fundamental objects, like atoms, and so *this relation between the metric and refractive index suggests we should not be quantizing the metric to find a quantum gravity theory just as we do not quantize the refractive index to find do quantum optics. There may be atoms of spacetime [59] whose collective behaviour give rise to spacetime itself and the associated metric and so gravity should not be confused with a property of spacetime.*

Chapter 6

Conclusion

In this thesis, we seek to investigate what we can learn from making connections between optics and other fields in physics. The fields we were interested in were non-Hermitian quantum systems (systems with lossy dynamics) and gravity. The goal is to use what we know about optics to infer new ideas about the fields just mentioned.

For lossy dynamics, we see in Chapter 4 that attenuation in one mode of Spontaneous Parametric Down Conversion (SPDC) leads to amplification in the intensity of light in the other mode. We arrived at this observation by solving a system of quantum Lindblad equations for the number operators for the two modes, which capture the intensity in each mode, and two more resulting operators from the non-unitary dynamics. Plots of the intensities for the two generated modes of SPDC against the phasemismatch show that away from phasematching, loss in one mode leads to an increase in the intensity of the other mode.

The use of attenuation to increase the brightness of light has been shown to occur classically for the nonlinear optical process of Spontaneous Four Wave Mixing in [1]. What is novel about our approach is that we demonstrate, quantum mechanically, the same effect loss has on the intensity of light from the nonlinear optical process of SPDC. Hence our work, and what has been done before, shows that loss can have beneficial effects in the lab, namely in making brighter thermal light beyond the constraints of phasematching. From here, we look at what our connections between optics and gravity can tell us about quantum gravity.

In Chapter 5 we look at how a linear-optical system can be used to simulate spacetime, while the associated refractive index simulates a spacetime metric. We do so by studying the propagation of light in this optical material using a modified Maxwell's wave equation.

From this wave equation we find the associated Green's function and through it determine the metric the propagating photons experience. This metric consists of a fixed background with some perturbation given by the refractive index. This form of the metric allows us to use the machinery of linearized gravity to analyze the dynamics of the metric as the photon propagates in our optical material. The photon has an effective mass as it propagates in our linear-optical material and from linearized gravity we find a connection between this mass and the radiation stress-energy tensor. The connection between the spacetime metric and the refractive index suggests that we should not quantize the metric to find a quantum gravity theory just as we do not quantize the refractive index to do quantum optics. We make a suggestion in Chapter 3 on a possible alternative to finding a quantum gravity theory.

In Chapter 3 we look at a quantum simulation of Unruh-DeWitt (UDW) detectors using SPDC. The simulation is achieved by adjusting the SPDC Hamiltonian to match that of the UDW system. Once this is done, we can find optical analogues of quantities in the UDW system such as detector acceleration and frequency gap, and coupling between scalar field and detector. These optical analogues reveal interesting properties about the nature of gravity.

The optical analogue of the coupling between the scalar field and the detector is the pump amplitude. This pump amplitude is treated classically which suggests that the coupling from the UDW system may come from classical gravity. In other words, classical gravity couples the scalar field and detector. This realization further suggests that gravity, like the pump field, may admit a gauge theory description which would make finding a quantum theory of gravity equivalent to quantizing a gauge theory.

Another interesting feature deduced from this simulation is the optical analogue of the detector's acceleration. This optical analogue is variable dispersion. We find that variable dispersion in our optical system allows for the possibility of testing gravitational phenomena in this system. We suspect this to be true because of the equivalence principle; which makes equivalent freely-falling bodies and accelerating ones.

The connections we made here between optics and gravity motivate the idea that gravity may have a gauge theory description. Such a description will make the problem of finding a quantum gravity theory equivalent to the problem of quantizing a gauge theory, which is better understood. Furthermore, such a description will potentially aid in the incorporation of gravity into the Standard Model which includes all known interactions, as gauge theories, with the exception of gravity. This may lead to physics beyond the Standard Model. A unified framework which views all interactions as gauge theories may also lead to a potential Theory of Everything (TOE). We hope our work will help motivate more research into finding a gauge theory of gravity given its potential benefits.

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APPENDICES

Appendix A

Appendix

A.1 Commutators

To get from Equation (4.6) to Equation (4.7) we use the observation that

$$b^\dagger \mathcal{O} b - \frac{1}{2}\{b^\dagger b, \mathcal{O}\} = b^\dagger \mathcal{O} b - \frac{1}{2}(b^\dagger b \mathcal{O} + \mathcal{O} b^\dagger b),$$

which leads to

$$b^\dagger \mathcal{O} b - \frac{1}{2}(b^\dagger b \mathcal{O} + \mathcal{O} b^\dagger b) = b^\dagger [\mathcal{O}, b] + \frac{1}{2}[b^\dagger b, \mathcal{O}], \tag{A.1}$$

which in turn, after using the Leibnitz rule for commutators, simplifies to

$$b^\dagger \mathcal{O} b - \frac{1}{2}(b^\dagger b \mathcal{O} + \mathcal{O} b^\dagger b) = b^\dagger [\mathcal{O}, b] + \frac{1}{2}[b^\dagger, \mathcal{O}] b + \frac{1}{2} b^\dagger [b, \mathcal{O}]. \tag{A.2}$$

Note: the Leibnitz rule for commutators A, B, C states that:

$$[A, BC] = [A, B]C + B[A, C]. \tag{A.3}$$

Finally, we have the result

$$b^\dagger \mathcal{O} b - \frac{1}{2}\{b^\dagger b, \mathcal{O}\} = -\frac{1}{2} b^\dagger [b, \mathcal{O}] - \frac{1}{2}[\mathcal{O}, b^\dagger] b, \tag{A.4}$$

which gives us the part of Equation (4.7) proportional to γ_b.