When Does Eco-Efficiency Rebound or Backfire? An Analytical Model

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Abstract

It is known that an eco-efficiency strategy, which saves resources in the production process, may be offset by a rebound effect; it may even backfire. Less known are the exact conditions under which eco-efficiency rebounds or backfires. This article fills the gap by providing an analytical model of the rebound and backfire effects. We propose an optimal control framework of dynamic pricing and eco-efficiency investment, for which eco-efficiency reduces the unit production cost and boosts the demand of environmentally concerned consumers. Results, which hold with a general demand formulation, examine the analytic conditions for the rebound and backfire effects. They also highlight the possibility of a reverse rebound effect. Such results pave the way to sounder sustainability strategies.

Key words: Pricing, rebound effect, eco-efficiency, dynamic pricing, sustainability, optimal control

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1 Introduction

Eco-efficiency, a term coined and championed by Stephan Schmidheiny, a founding member of the World Business Council for Sustainable Development, is the act of a business using natural resources more efficiently (Bank, 2002). By producing more eco-efficiently companies can lower their environmental impact, thus contributing to Sustainable Development. Eco-efficiency has also been linked to the business case for sustainability (Dyllick and Hockerts, 2002): Eco-efficient firms may at the same time experience higher profits (WSJ, 2016; Bank, 2002) than less eco-efficient firms. However, eco-efficiency does not always reduce resource use. For example, increased automobile fuel efficiency leads to more driven miles resulting in net fossil fuel consumption greater than initially expected (Jenkins Jr., 2018). Smaller than expected decrease in resource use even though product eco-efficiency increases is referred to as the *rebound effect* (Khazzoom, 1980). In the United States of America, the government assumes the rebound effect cancels out at least 10% of CO_2 savings from increased car fuel-efficiency (Van Benthem and Reynaert, 2015). In addition, 15% of fuel savings are lost due to older cars being used longer to account for the increased cost of newer, more fuel-efficient, vehicles (Van Benthem and Reynaert, 2015). Similar results are found in the Japanese automobile market in which fuel usage and cost lead to consumption rebounds, and fuel economy incentives and standards are positively correlated with consumption rebounds (Yoo et al., 2019). The rebound effect is observed not only in the automotive industry, but also in-home energy use with eco-efficient appliances such as LED lights (McGinty, 2014; Zink and Geyer, 2016). An additional source of the rebound effect may stem from *backstop technologies*, technologies that help to replace non-renewable by renewable resources (Dasgupta and Heal, 1979). For example, as society uses up non-renewable resources, it invests in technologies such as photovoltaics to eventually replace non-renewable with renewable resources. Consequently, the price for renewable resources comes down, stimulating demand, creating economies of scale and inciting further investments in these technologies. Ultimately renewable resources replace non-renewable resources. This cycle of investments, leading to higher demand, lower prices and even higher investments and demand results in a rebound effect. In fact, this trend is already being observed. In a recent market analysis, it is observed that the marginal cost of production of electricity from renewable sources is below that of non-renewable sources (Lazard, 2018). As production cost of electricity decreases, no matter the source, energy consumption will increase (Billette de Villemeur and Pineau, 2016; Pineau et al., 2011; Pineau and Zaccour, 2007; Tillman, 2018), potentially offsetting any benefits brought on by the use of renewable electricity production methods (Aghion et al., 2016; Fowlie et al., 2018). Along the same lines, Yi et al. (2020) find no evidence showing energy efficiency leads to a reduction in haze pollution; such relationship is explained by a strong rebound effect in energy consumption.

In this paper, we interpret resource use widely; a further discussion on how we consider resource use and its relationship to the sustainability literature is found in Section 3.3.

In extreme cases rather than decreasing resource use, higher eco-efficiency might not only result in a less than expected reduction of resource use, but may increase resource use. This is referred to as the *backfire effect*, also known as Jevons paradox (Jevons, 1866). The size and importance of the rebound effect are subject of both empirical and theoretical research (Fowlie et al., 2018; Greening et al., 2000; Saunders, 2008). Improving eco-efficiency, though the rebound effect may occur, can still result in a net reduction of the impact of the firm on the environment. We argue in this paper that current research on the rebound effect has oversimplified the relationship between eco-efficiency and resource use; most papers in the literature use parametric (specific) functions of demand and cost. Parametric functions imply the existence or non-existence of certain types of rebound Saunders (2008). Several parametric model formulations directly eliminate, for example, *the reverse rebound effect*, also referred to as *super-conservation*, where the realized environmental savings are greater than expected (Saunders, 2008). By using a structural (general) approach, we only posit loose structural properties of the functions. The structural approach imposes little constraint on the relationship between the variables. This allows all possible relationships between changes in eco-efficiency and *total resource use*, the amount of resource required to meet demand. Total resource use is determined by the product of demand and the *unit resource use*, the amount of resource required to make a single product. There are five different types of rebound that may occur as eco-efficiency increases:

1. Total resource use increases (the backfire effect); the decrease of unit resource use due to an increase of eco-efficiency is more than offset by increased demand.

2. Total resource use is unchanged (*the total rebound effect*); the decrease of unit resource use is exactly offset by increased demand.

3. Total resource use is partially offset (*the partial rebound effect*); the decrease of unit resource use is partially offset by increased demand.

4. Total resource use decreases as much as predicted by the increase in eco-efficiency (*the zero rebound effect*); demand does not change with eco-efficiency.

5. Total resource use decreases more than expected (the reverse rebound effect); demand decreases with eco-efficiency, discussed in greater detail in Section 5.

By using a general approach, we can now account for non-linearities and dynamics in the relationship between eco-efficiency and resource use. Our structural results replicate the rebound effects documented in the literature. Moreover, we analytically explain the drivers of these rebound effects. The contributions of this work are:

- Propose a model of a profit-maximizing firm engaging in dynamic policies of pricing and investment in eco-efficiency.
- Consider jointly both supply- and demand-side in characterizing a firm's dynamic behavior with a general demand and cost formulations.
- Describe the dynamics between pricing and eco-innovation that lead to the reverse rebound, the partial rebound, and the backfire effects.

In the remainder of the paper, we discuss in Section 2 the related literature, and we formulate our model in Section 3. Our analytical results are presented in Section 4, and we discuss the implications and managerial takeaways of our results in Section 5. Section 6 concludes the paper with future research directions and a general overview.

2 Literature Review

Proposed by Jevons (1866), the rebound effect regained traction in the 1980s when reintroduced by Khazzoom (1980) to explain why energy eco-efficient appliances do not lead to their anticipated energy savings. Since 1980 many authors show that the rebound effect does indeed occur. The surveys of Greening et al. (2000) and Sorrell et al. (2009) both concentrate on the particular case of household appliances and examine how higher efficiency of these appliances leads to higher demand. This is referred to as the direct rebound effect. Research usually distinguishes between two further effects: an indirect/secondary and an economy-wide rebound effect (Greening et al., 2000; Sorrell and Dimitropoulos, 2008). Definitions of each type of rebound effect and descriptions of their fundamental economic principles appear in Table 1.

Greening et al. (2000) survey empirical studies that support the existence of the direct rebound effect. As this effect is potentially the easiest to identify, it has the largest volume of related studies. Only recently a number of studies look at indirect and economy-wide effects (e.g., Ai et al. (2020); Chitnis et al. (2013); Freire-González (2011); Gava et al. (2020); Thomas and Azevedo (2013)). However, regardless of the type of rebound effect, Greening et al. (2000) note that "the real controversy [in the rebound effect] lies in the identification of sources and size of rebound."

As reflected by the descriptions of the three effects in Table 1, most research on rebound focuses on the link between higher efficiency, lower prices and increased demand. Changes in demand are a possible source of rebound. Other causes of rebound are possible. For example, recently Corbett (2018), using the work of Asensio and Delmas (2016), argues that big-data may be used to identify the cause of rebound. Still, to our knowledge,

	Actual Energy savings	
‘Engineering’ estimate of energy savings	Direct effect	Improved energy efficiency for a particular energy service will decrease the effective price of that service and should therefore lead to an increase in consumption of that service. This will tend to offset the reduction in energy consumption provided by the efficiency improvement
	Indirect effect	The lower effective price of the energy service may lead to changes in the demand for other goods, services and factors of production that also require energy for their provision. For example, the cost savings obtained from a more efficient central heating system may be put towards an overseas holiday
	Economy-wide effect	A fall in the real price of energy services may reduce the price of intermediate and final goods throughout the economy, leading to a series of price and quantity adjustments, with energy-intensive goods and sectors likely to gain at the expense of less energy-intensive ones.

Table 1: Three different types of rebound effects; Source: Own figure based on Greening et al. (2000). Explanations from Sorrell and Dimitropoulos (2008)

no published papers using the proposed methods exist in the literature. Saunders (1992) uses neoclassical growth theory to evaluate how energy efficiency impacts energy use. He argues that higher energy efficiency makes energy less scarce, compared to other production factors, and can result in a substitution of capital or labor with energy. In addition, higher energy efficiency can fuel economic growth, which also results in higher energy consumption. What is interesting about this macroeconomic approach is that it opens up the analysis of rebound to the supply side. However, there are two weaknesses to this approach. First, this approach does not explicitly consider the demand side simultaneously with the supply side. Second, as Saunders (2008) notes, depending on the parametric function used to describe a rebound effect, only certain types of rebounds can be observed.

As such, to determine the source of rebound effects and to mitigate potential bias by only allowing certain types of rebound, in this paper we consider a general (that is structural, rather than parametric) approach, and we analytically determine under what conditions rebound may occur. Contrary to most research, assuming a static setting like Greening (2000), we consider a dynamic setting.

We are not the first to consider analytical models of the rebound effect. Agrawal and Bellos (2017) consider an analytical model of servicizing and note, that under some conditions, the partial rebound effect may occur. Wirl (1996) builds an incentive compatible mechanism for energy consumption while accounting for the partial and total rebound effects. Benjaafar et al. (2019); Jacobs and Subramanian (2012) show that the partial rebound effect may occur in the sharing economy. Chen et al. (2019) show that even when emissions are penalized in a decentralized supply chain, rebound and backfire may occur. Raz et al. (2013) show that the partial rebound effect is more likely to occur when there is high future demand uncertainty. All of the preceding papers show that rebound may occur, though they do not explicitly consider the relationship between both supply and demand in a dynamic setting. Benchekroun and Chaudhuri (2014, 2015) represent exceptions, in that they use dynamic analysis, but focus on the partial rebound effect of pollution and not on the partial rebound effect of resource consumption. In particular, the authors look at the partial rebound effect in the context of transboundary pollution and international agreements, and not in our context of resource use. In their research, rebound occurs as a by product; rebound is not the main interest of their analysis. As such, Benchekroun and Chaudhuri (2014, 2015) give neither the conditions nor the rationale for each kind of rebound. In addition, all of the papers discussed in this paragraph consider specific functional forms of consumer demand and profit, something we do not in our work.

Using a structural approach, we incorporate both demand and supply side impacts of improved eco-efficiency and find that each contribute to either the total or partial rebound effects or the reverse rebound effect. To our knowledge, the explicit link using a structural approach with general functions between these two sides, supply and demand, of rebound have not been considered simultaneously in analytical models. Like the majority of research on rebound we limit ourselves to the direct rebound effect, detailed in Table 1. Unlike preceding research, we integrate supply and demand, which allows us to identify further effects beyond the effect that increased eco-efficiency can have on lowering costs and stimulating demand. The markup and sales effect are two new effects that we introduce to the research on rebound.

3 Model Formulation

The model developed in our paper considers not only the supply side of rebound but also the demand side. Classically, rebound considers the supply side, i.e., production costs decrease, resulting in lower prices and in turn higher demand, leading to higher resource use than anticipated (Khazzoom, 1980). However, as noted in the literature (De Pelsmacker et al., 2005; HaBrookshire and Norum, 2011; Kang et al., 2012), consumers are interested in environmentally friendly products. Similar findings are found by consumer research firms (Nielsen, 2014). This can lead to a new form of rebound. A profit-maximizing firm, aware of the interest of consumers in environmentally friendly products, may charge more for eco-efficient and, thus, resource-saving products. Upward pressure on prices may lead to the reverse rebound effect (also know as *super-conservation* (Saunders, 2008)). The resource saving effect of lower demands adds to the resource-saving effect of higher eco-efficiency, resulting in resource consumption that is lower than originally projected.

In this paper we develop an optimal control model with a monopolist firm setting (product) price, $p(t)$, and (eco-efficiency) investment, $u(t)$, over time, t. The firm's eco-efficiency investment impacts the eco-efficiency, $e(t)$, of the product. Consumers prefer eco-efficient products, stimulating product demand. Similarly, consumers also prefer lower priced products. Increasing eco-efficiency leads to reduced resource use per unit, leading to lower resource cost per unit, which is the only per-unit cost we consider in our model. If no additional eco-efficiency investment is made, then the eco-efficient processes decay over time, leading to a loss of eco-efficiency (Benchekroun and Long, 2012; Dai and Zhang, 2017; Saha et al., 2017; Zhang et al., 2017). The decay may come from degradation of machines, or by the natural improvement of the entire industry leading to a relative degradation. The firm's profit is a function of its decision on the product price, $p(t)$, and the eco-efficiency investment, $u(t)$, as well as the level of eco-efficiency, $e(t)$. Price and eco-efficiency investment are the control variables, and eco-efficiency is the state variable.

In the remainder of this section we build the optimal control framework used to analyze our model. For the optimal control framework, we use a fixed and finite planning horizon T, and time $t \in [0, T]$ is continuous. For ease of reading the remainder of the paper, Table 2 presents the notations.

Table 2: Main Notations

T = fixed terminal time of the planning horizon,
r = interest rate,
$p(t)$ = product price at time t (control variable),
$u(t)$ = investment in eco-efficiency at time t (control variable),
$e(t)$ = eco-efficiency level at time t (state variable),
$\frac{de}{dt}$ $= E(u, e) =$ eco-efficiency dynamics,
$\lambda(t)$ = current-value co-state variable at time t,
$D(p, e)$ = demand,
$R(e)$ = resource per product,
$R(e)D(p, e)$ = total resource,
$C(R(e))$ = unit production cost,
$\pi(p, u, e)$ $= [p - C(R(e))]D(p, e) - u =$ current profit,
$H(p, u, e, \lambda)$ $= \pi + \lambda E =$current-value Hamiltonian,
$\eta_{D/x}$ $= \left|\frac{\partial D}{\partial x}\frac{x}{D}\right| =$ elasticity of demand with respect to the variable x,
$\eta_{E/u}$ $= \frac{\partial E}{\partial u}\frac{u}{E} =$ elasticity of efficiency dynamics with respect to investment.

3.1 Eco-Efficiency

Consumers have an increasing level of environmental consciousness and the firm invests in more environmentally friendly and eco-efficient production processes for this reason (Herring and Sorrell, 2009). On the one hand, it is well known in the literature that eco-efficiency investment results in eco-efficiency gains (Herring and Sorrell, 2009). Technological improvements drive continuous enhancement of eco-efficiency over time. On the other hand, ongoing economy-wide technology development degrades what was a comparatively high eco-efficient process in the past to a low eco-efficient process in the present. Also, the quality of any process erodes if it is not maintained (Dai and Zhang, 2017; Saha et al., 2017). Ongoing technology development and natural process quality degradation collectively lead to a decrease in eco-efficiency over time. Consequently, eco-efficiency increases with eco-efficiency investment and decays otherwise.

Using a general functional form, we model the relationship between eco-efficiency investment, $u(t) \in \mathbb{R}^+$, and the associated improvement of eco-efficiency, $e(t) \in \mathbb{R}^+$. Investment expense, $u(t)$, and eco-efficiency, $e(t)$, are control and state variables in our model, respectively. The eco-efficiency dynamics write as

$$\frac{de(t)}{dt} = E(u(t), e(t)), \text{ with } e(0) = e_0, \tag{1}$$

where the eco-efficiency dynamics function $E : \mathbb{R}^{2+} \to \mathbb{R}$ is twice continuously differentiable. The structural formulation (1) generalizes a natural parametric formulations, such as $\frac{de(t)}{dt} = u(t) - \delta e(t)$ used in Dai and Zhang (2017); Saha et al. (2017); Zhang et al. (2017), in which the stock of eco-efficiency depreciates at the constant proportional rate $\delta > 0$. Integrating (1) relates the stock (or cumulative level) of eco-efficiency to the flow of current investment $e(t) = e_0 + \int_0^t E(u(s), e(s))ds$. Proportional depreciation of the state variable in green economics is surveyed by Benchekroun and Long (2012); Jørgensen et al. (2010) and challenged by El Ouardighi et al. (2014, 2016). Hereafter and when no confusion exists, we omit notational arguments for simplicity. Especially, we often omit the temporal notation in subsequent equations.

Investment u increases eco-efficiency e, with diminishing returns. Also, investment loses its effectiveness over time, translating into autonomous decay:

$$\frac{\partial E}{\partial u} > 0, \ \frac{\partial^2 E}{\partial u^2} < 0, \ \frac{\partial E}{\partial e} < 0. \tag{2}$$

The structural formulation (1) together with (2) generalizes the parametric formulation $\frac{de}{dt} = \sqrt{u} - \delta e$ (Karray and Martín-Herrán, 2009). We do not assume any cross effect, that is, the sign of $\frac{\partial^2 E}{\partial u \partial e}$ is left undetermined (for a discussion of the sign of the cross derivative, see Pineau et al. 2011).

Regardless of specific effects on the demand function, the structural formulation (1) with (2) allows a general formulation for the two most salient features of eco-efficiency dynamics. It accounts for the lasting effect of eco-efficiency investment in changing preferences of consumers, together with the tendency of eco-efficiency to erode.

3.2 Demand

We now turn our attention to consumer demand. Most related literature considers a linear demand function of price and product greenness such as

$$D = a_0 - a_1 p + a_2 e, \tag{3}$$

where $a_0 > 0$ represents the market potential, $a_1 > 0$ is the price sensitivity of demand, and $a_2 > 0$ measures the demand expansion effect of eco-efficiency (Dai and Zhang, 2017; Saha et al., 2017; Zhang et al., 2017). To ensure demand is positive when price equals the minimum production cost, c_0, and the eco-efficiency is null, it is assumed that $a_0 > a_1 c_0$ (Martín-Herrán and Taboubi, 2015; Zaccour, 2008).

The price $p \in \mathbb{R}^+$ is a control variable. The price does not influence a state variable, making price a *static* control variable. The (current) demand function $D : \mathbb{R}^{2+} \to \mathbb{R}^+$ is twice continuously differentiable. The demand, D, depends on the price, p, and eco-efficiency, e, which is a proxy for the greenness of the product. Though the demand function is known, the demand will change as product price and eco-efficiency changes, thus allowing us to endogenize demand changing with price and level of eco-efficiency. Consumers prefer products that are more eco-efficient in the sense that they require fewer resources. Consequently, investment indirectly affects future demand via eco-efficiency. Formally, we write the demand as:

$$D = D(p, e). \tag{4}$$

The *direct price effect on demand*, the *direct eco-efficiency effect on demand*, and the *cross effect of price and eco-efficiency on demand* are given by $\frac{\partial D}{\partial p}$, $\frac{\partial D}{\partial e}$, and $\frac{\partial^2 D}{\partial p \partial e}$, respectively.

In line with the literature, demand falls with price and rises with product greenness, eco-efficiency in our case (Eurobarometer, 2013; Laroche et al., 2001; Nielsen, 2014). Further, customers are marginally less price sensitive for greener products. (See Chenavaz and Jasimuddin (2017), Masoudi and Zaccour (2018), and Chenavaz et al. (2020) for a similar interpretation of the cross-derivative assumption.). The three assumptions of demand discussed above are formally captured by:

$$\frac{\partial D}{\partial p} < 0, \frac{\partial D}{\partial e} \geqslant 0, \frac{\partial^2 D}{\partial p \partial e} \leqslant 0. \tag{5}$$

The case $\frac{\partial D}{\partial e} = 0$ represents the situation of consumers not knowing or not caring about the eco-efficiency of a product. Also, the demand function is assumed not to be "too" convex in the price:

$$2 - D\frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2} > 0, \tag{6}$$

a technical assumption, ensuring that the firm chooses the profit-maximizing price, as opposed to a profit-minimizing price. Equation (6), tied to the strict concavity of the Hamiltonian function later defined, implies that the demand function is not "too" convex in the price. A similar assumption of demand convexity is widely used by papers in the dynamic pricing literature that use structural demand functions, for instance in Chenavaz (2012, 2017); Dockner et al. (2000); Jørgensen and Zaccour (2012); Ni and Li (2018); Vörös (2006, 2019). Assumptions (5) and (6) are verified for example by both a linear demand function (3) and a Cobb-Douglas demand function $D = a_0 p^{-a_1} e^{a_2}$ with $a_0, a_1, a_2 > 0$.

3.3 Resource and Production Cost

In this subsection we formally define the link between eco-efficiency, resource use, and unit production cost. It is common in the sustainability literature to distinguish between economic and environmental capital (Costanza and Daly, 1992; Harte, 1995). In this context, capital is defined as "a stock that yields a flow of valuable goods or services into the future" (Costanza and Daly, 1992, p. 38). Natural capital provides a flow of goods, for example energy, and services, for example the recycling of waste material. Following other research in the field, we refer to these flows as resources. Production, be it for consumption or to build up further economic capital, is unthinkable without natural capital. Analogously, exploiting natural capital requires economic capital. Economic and environmental capital are therefore in a complementary relationship (Costanza and Daly, 1992). As both of them are scarce they need to be used efficiently.

A major challenge in the assessment of resource use is the scope of the assessment. Production, use and ultimately disposal of goods typically cover many different steps and every step is linked to the use of environmental capital. Changes in one step can lead to higher or lower resource use in another step. Assessments, therefore, aim to take a life-cycle approach where resources use is covered cradle-to-grave (Rebitzer et al., 2004) or even cradle-to-cradle (McDonough and Braungart, 2010). Theoretically, for all but the most simple products, there is a quasi-infinite number of steps that would need to be considered to cover all resource flows; in practice, the number of steps that can be considered due to time and resource-constraints is limited (Raynolds et al., 2000).

We acknowledge that higher resource use in one step can lead to lower resource use in another step. Using aluminium in car production is for example linked to higher energy use during production but results in lighter and more fuel-efficient cars during the use phase (Miller et al., 2000). This is distinct from our argument in this article. Rather than looking at the effect that a change in eco-efficiency has on resource use in another step, we look at the impact it has on resource use in the same step. We, therefore, do not distinguish between different steps in the following.

As just mentioned, resource is considered in a general way, as an environmental input that disappears or degrades in the production process. Resources can be raw material (cotton used to produce jeans), water (as for producing beverages such as Coca-Cola), or energy. In the case for energy, energy-efficient products are referred to as eco-efficient (Sheffi, 2018). As a firm becomes more eco-efficient, the amount of resource required to produce a single good decreases. The resource required for one product as a function of the firm's eco-efficiency, e, is given by the function $R : \mathbb{R}^+ \to \mathbb{R}^+$, which is once continuously differentiable. That is, $R = R(e)$. The more eco-efficient the firm, the lower resource consumption per product (Herring and Sorrell, 2009), formally:

$$\frac{dR}{de} \leqslant 0. \tag{7}$$

The (unit) production cost is a function of the resource required for one product, R. Formally, we have $C : \mathbb{R}^+ \to \mathbb{R}^+$, which is once continuously differentiable; that is, $C = C(R)$. Because resource is costly to buy, greater resource requirements increase production cost:

$$\frac{dC}{dR} \geqslant 0. \tag{8}$$

By definition, as eco-efficiency increases the amount of resource required per product and the cost per product decreases. Similarly to Benchekroun and Chaudhuri (2014, 2015), the relationship reads:

$$\frac{dC}{de} = \frac{dC}{dR}\frac{dR}{de} \leqslant 0. \tag{9}$$

The implication of equation (9) is that cost decreases with eco-efficiency. Mathematically, this relationship follows from the intuitive relationships captured in equations (7) and (8). Managerially, the relationship is needed to ensure the markup effect, defined in Definition 4.1, is due to the firm manipulating price to maximize profit, and not due to an increase in production costs.

The *total resource* is the net amount of raw resource use to satisfy all demand. We capture this relationship by RD. Total resource is used to identify the type of rebound in Section 4.

3.4 Profit

The current profit function $\pi : \mathbb{R}^{3+} \to \mathbb{R}$ is assumed twice continuously differentiable. The profit per unit is the retail price minus the cost per unit, $p - C$. As discussed previously, the cost per unit is a function of the resource use per unit, R, which is, in turn, a function of the level of eco-efficiency, e. Thus the per-unit production cost is variable, $C(R(e))$. The firm sets the eco-efficiency investment, u, prior to demand being realized, and thus is a fixed cost. Putting the fixed and variable costs together, we derive the following profit function, revenues less costs:

$$\pi(p, u, e) = [p - C(R(e))]D(p, e) - u. \tag{10}$$

Larger eco-efficiency allows for greater profits to be made ($\frac{\partial \pi(p,u,e)}{\partial e} > 0$ follows from (5) and (9)).

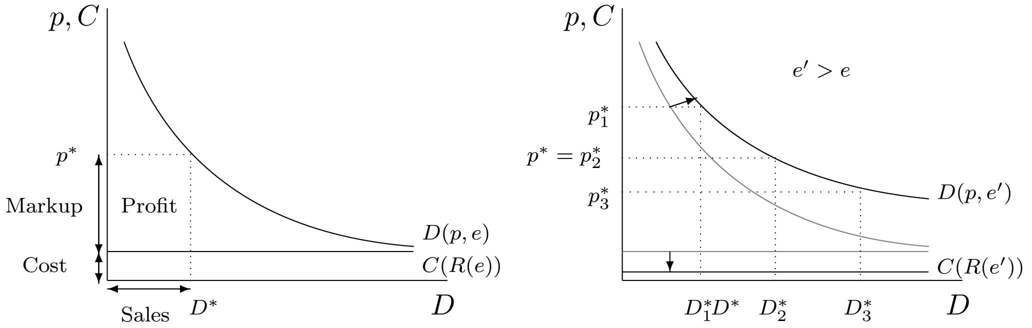
3.5 Firm's Optimization Problem

The firm maximizes the intertemporal profit (or present value of the profit stream) over the planning horizon, by simultaneously choosing the investment and pricing policies, accounting for the dynamics of eco-efficiency. For simplicity, the salvage value of eco-efficiency is zero. The interest rate is $r \geqslant 0$ and the objective function of the firm is:

$$\max_{u(s),\, p(s) \geqslant 0,\, \forall s \in [0,T]} \int_0^T e^{-rt} \pi(p(t), u(t), e(t)) dt, \tag{11a}$$

$$\text{subject to } \frac{de(t)}{dt} = E(u(t), e(t)), \text{ with } e(0) = e_0. \tag{11b}$$

3.6 Model Discussion



(a) Optimal Price and Demand for an Eco-Efficiency Level

(b) Optimal Price and Demand with better Eco-Efficiency

Figure 1: The Effect of Eco-Efficiency on Price and Demand

Prior to presenting our analytical results, we first discuss the intuition of our model and our expected results.

In Figure 1 we present two panels, the first panel, Figure 1(a), shows the base case of a firm prior to a change in eco-efficiency, e. The second panel, Figure 1(b) exhibits the same firm after an increase in eco-efficiency to $e' > e$. Figure 1(b) depicts "two" demand curves, one is the demand curve from Figure 1(a), with the eco-efficiency level e, and the other is the new demand curve, with the eco-efficiency level e'. The influence of eco-efficiency on demand is captured by the shift in the demand curve up and to the right. An additional consequence of increased eco-efficiency is the decrease in the unit cost, captured by the lower cost line $(C(R(e')))$, relative to the original cost line $(C(R(e)))$. Lower unit cost and larger demand collectively impact the potentially new product price. If demand increases with eco-efficiency, then potentially the firm may charge a higher price while maximizing profit, relative to the base case (this is captured by p_1^* in Figure 1(b), relative to p^*). Alternatively, if cost per product decreases, then the firm may decrease price, increasing even further demand, again maximizing profit, relative to the base case (this is captured by p_3^* in Figure 1(b), relative to p^*).

Section 4 derives and presents the analytical factors that determine the magnitude, and direction of the two arrows in Figure 1(b) capturing the change in demand and unit cost as eco-efficiency increases. The two arrows, in the second panel are manifestations of the three effects discussed in Definition 4.1, namely cost, markup, and sales. The combination of the three effects will determine the price after the eco-efficiency increase $(p^*|e')$, relative to the price before the eco-efficiency increase price $(p^*|e)$. The three effects are separated into two categories. The first category, consisting of the markup and sales effects, impacts the demand. The second category, comprised of the cost effect, affects the supply. The collective impact of the markup and sales effects on demand is ambiguous and may lead to an increase or decrease in demand; in the example in Figure 1(b), the markup and sales effects are captured by the diagonal arrow in the panel. Due to the cost effect, profit increases if price remains unchanged, and is exhibited by the downward arrow in Figure 1(b). Because of the different effects at play, Figure 1 shows that the reverse, zero, partial, and total rebound effects, and also the backfire effect, are possible. Note that the zero rebound effect case, formally given by $D^*|e' = D^*|e$, appears as an exception. Figure 1 provides insights, by qualitatively capturing the three effects in Definition 4.1. We quantifying the three effects in the next section.

4 Model Analysis and Results

In this section, we fill the analytical details of the intuition provided in the preceding section. Before our analysis, we present the conditions derived from solving mathematical program (11), for additional details, we refer interested readers to part 2 of Kamien and Schwartz (1991). [1] [2] In the dynamic setting, with time, t, being

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[1] Recall that static optimization problems are solved via Lagrangian and associated Lagrange multipliers, λ. Whereas dynamic optimization problems are solved using Hamiltonian and associated co-state variable, $\lambda(t)$.

[2] We solve the problem via the Maximum Principle. Alternatively, it is also solvable with the Principle of Optimality from dynamic optimization (Kamien and Schwartz, 1991, Section 21).

continuous, we have a potentially unique value of the co-state variable $\lambda(t)$ (the counterpart of the Lagrange multipliers in the dynamic setting) for each instance of t. The Hamiltonian, H, of (11) with the current-value adjoint variable (or shadow price) $\lambda(t)$ for eco-efficiency dynamics is: [3]

$$H(p,u,e,\lambda) = [p - C(R(e))]D(p,e) - u + \lambda E(u,e). \tag{12}$$

The Hamiltonian, H, measures the intertemporal profit, summing the current profit, $[p - C(R(e))]D(p,e) - u$, and the future profit, λE. We confine our interest to interior solutions for u and p, assuming their existence. The Hamiltonian, H, is assumed strictly concave in investment, u, and price, p. It immediately follows that all optimal decisions must satisfy the first- and second-order conditions of the Hamiltonian, equations (13a)–(13e). In addition, following the maximum principle, we derive equation (13f). Note that all conditions are for $t \in (0,T)$.

$$\begin{aligned}
\frac{\partial H}{\partial u} = 0 \quad &\Longrightarrow \quad \frac{\partial E}{\partial u} = \frac{1}{\lambda}, && \text{(13a)}\\
\frac{\partial H}{\partial p} = 0 \quad &\Longrightarrow \quad p - C = -\frac{D}{\frac{\partial D}{\partial p}}, && \text{(13b)}\\
\frac{\partial^2 H}{\partial u^2} < 0 \quad &\Longrightarrow \quad \lambda\frac{\partial^2 E}{\partial u^2} < 0, && \text{(13c)}\\
\frac{\partial^2 H}{\partial p^2} < 0 \quad &\Longrightarrow \quad 2 - D\frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2} > 0, && \text{(13d)}\\
\frac{\partial^2 H}{\partial u^2}\frac{\partial^2 H}{\partial p^2} - \left(\frac{\partial^2 H}{\partial u \partial p}\right)^2 > 0 \quad &\Longrightarrow \quad \lambda\frac{\partial^2 E}{\partial u^2}\left(-2 + D\frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2}\right) > 0, && \text{(13e)}\\
\frac{d\lambda}{dt} = r\lambda - \frac{\partial H}{\partial e} \quad &\Longrightarrow \quad \frac{d\lambda}{dt} = \left(r - \frac{\partial E}{\partial e}\right)\lambda + \frac{dC}{dR}\frac{dR}{de}D - (p - C)\frac{\partial D}{\partial e}, && \text{(13f)}
\end{aligned}$$

with the transversality condition $\lambda(T) = 0$.[4]

The second-order condition (13c) together with the diminishing returns of investment, equation (2), impose: [5]

$$\lambda(t) > 0, \ \forall t \in [0,T), \tag{14}$$

according to which greater eco-efficiency always increases the intertemporal profit. Integrating (13f) with respect

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[3] Note that the Hamiltonian written while explicitly taking into account time, t, is:

$$H(p(t),u(t),e(t),\lambda(t)) = [p(t) - C(R(e(t)))]D(p(t),e(t)) - u(t) + \lambda(t)E(u(t),e(t)).$$

[4] Note that for an infinite time horizon, the transversality condition for a free terminal state and infinite terminal time is $\lim_{t\to\infty} e^{-rt}\lambda(t) = 0$. Consequently, the optimality conditions (13a)-(13f) remain the same with either finite or infinite time horizon.

[5] Condition $\dfrac{\partial E}{\partial u} > 0$ from (5) together with (13a) give the same conclusion.

to time yields the value of the co-state variable, $\lambda(t)$:

$$\lambda(t) = \int_t^T e^{-\left(r - \int \frac{\partial E}{\partial e} d\mu\right)(s-t)} D \left(\frac{\eta_{D/e}}{\eta_{D/p}} \frac{p}{e} - \frac{dC}{dR} \frac{dR}{de} \right) ds. \tag{15}$$

Where $\eta_{D/e}$ and $\eta_{D/p}$ are the demand elasticities to eco-efficiency and price, respectively. The analytical details of finding (15) are provided in Appendix A. We note that the co-state variable is directly linked to the markup and cost effects, defined in Definition 4.1. The markup effect, $\frac{\eta_{D/e}}{\eta_{D/p}} \frac{p}{e}$, captures the price increase that consumers accept to pay following greater eco-efficiency. The markup effect increases with the relative demand sensitivity to eco-efficiency and to price, $\frac{\eta_{D/e}}{\eta_{D/p}}$, and the eco-efficiency-deflated price, $\frac{p}{e}$. The cost effect, $\frac{dC}{dR} \frac{dR}{de}$, measures the unit production cost reduction after larger eco-efficiency. For completeness, we present the dynamics of investment in eco-innovation in Appendix B.1, which are in line with the consensus in the literature. Similarly, we consider the optimal relationship between investment in eco-efficiency and price in Appendix B.2.

In the remainder of this section, we present our main analytical findings. We initially establish the relationship between price and eco-efficiency in Proposition 1. Recall that rebound is defined with respect to eco-efficiency and total resource use. Total resource use is defined as the product of demand, a function of price and eco-efficiency, and resource per product, a function of eco-efficiency. Using Proposition 1, we determine the relationship between demand and eco-efficiency in Proposition 2. Using Proposition 2, the relationship between total resource use and eco-efficiency is established in Proposition 3.

4.1 Dynamics of Eco-Efficiency and Price

In this section, we determine the relationship between price and eco-efficiency. As demand is a function of both price and eco-efficiency the result in this section will allow us to examine demand only with respect to eco-efficiency. We start with the first-order condition (13b) which holds for all optimal values. We differentiate both sides of the condition with respect to time, t, while accounting for the definitions of cost per product, $C = C(R(e(t)))$, and demand, $D = D(p(t), e(t))$:

$$\frac{dp}{dt} = \frac{d}{dt} \left(C - \frac{D}{\frac{\partial D}{\partial p}} \right) \implies \frac{dp}{dt} = \frac{dC}{dR} \frac{dR}{de} \frac{de}{dt} - \frac{\left(\frac{\partial D}{\partial p} \frac{dp}{dt} + \frac{\partial D}{\partial e} \frac{de}{dt}\right) \frac{\partial D}{\partial p} - D\left(\frac{\partial^2 D}{\partial p^2} \frac{dp}{dt} + \frac{\partial^2 D}{\partial p \partial e} \frac{de}{dt}\right)}{\frac{\partial D}{\partial p}^2}. \tag{16}$$

Note $-\frac{\frac{\partial D}{\partial p} \frac{\partial D}{\partial e}}{\frac{\partial D}{\partial p}^2} = \frac{\eta_{D/e}}{\eta_{D/p}} \frac{p}{e}$, which we call the markup effect, and we define the sales effect as $D \frac{\frac{\partial^2 D}{\partial p \partial e}}{\frac{\partial D}{\partial p}^2}$, please see

Definition 4.1. Substituting the definition of the markup effect into equation (16) and rearranging we have:

$$\frac{dp}{dt}\left(2 - D\frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2}\right) = \frac{de}{dt}\left(\frac{dC}{dR}\frac{dR}{de} + \frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e} + D\frac{\frac{\partial^2 D}{\partial p \partial e}}{\frac{\partial D}{\partial p}^2}\right). \tag{17}$$

Equation (17) links the dynamics of price $\frac{dp}{dt}$ to the dynamics of eco-efficiency $\frac{de}{dt}$, for a joint price and eco-efficiency demand function $D(p, e)$. Equation (17) is derived solely from the first-order condition on price (13b) and is independent from the first-order condition for investment (13a) and the dynamics of eco-efficiency (1). Let $\frac{dp}{de}$ be the *total effect of eco-efficiency on price*, we measure this effect in Proposition 1.

Proposition 1. With $D = D(p, e)$, the total effect of eco-efficiency on price writes as:

$$\frac{dp}{de} = \frac{\overbrace{\frac{dC}{dR}\frac{dR}{de}}^{-} + \overbrace{\frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e}}^{+} + \overbrace{D\frac{\frac{\partial^2 D}{\partial p \partial e}}{\frac{\partial D}{\partial p}^2}}^{-}}{\underbrace{2 - D\frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2}}_{+}}. \tag{18}$$

Proof. Assuming that control p depends on state e, we apply the time elimination method (Mulligan and Sala-i Martin, 1991). More precisely, p is supposed to be a once continuously differentiable function $p : \mathbb{R}^+ \to \mathbb{R}^+$ of e, that is $p = p(e)$. Suppose $\frac{de}{dt} \neq 0$, we obtain $\frac{\frac{dp}{dt}}{\frac{de}{dt}} = \frac{dp}{de}$. Note that the signs of $\frac{dp}{dt}$ and $\frac{de}{dt}$ are unknown, thus the sign of $\frac{dp}{de}$ is unknown. Rearranging (17) gives the result. ☐

Equation (18) quantifies the effect of eco-efficiency on price. On the right-hand side of (18), the denominator $2 - D\frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2}$ is strictly positive due to the second-order condition (13d), similar condition is assumed by Chenavaz (2012, 2017); Dockner et al. (2000); Jørgensen and Zaccour (2012); Ni and Li (2018); Vörös (2006, 2019). The numerator is the sum of a supply-side effect, namely the cost effect, and two demand-side effects, namely the markup and sales effects. [6]

Definition 4.1. We now formally define and discuss the three effects.

- The *cost effect*, $\frac{dC}{dR}\frac{dR}{de}$, tied to (9), measures the decrease in the unit production cost associated with greater eco-efficiency. The cost effect is negative.
- The *markup effect*, $\frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e}$, associated with (5), determines the increase in consumers' willingness to pay following an increase in eco-efficiency. The markup effect is positive.

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[6] Similar effects of sales and markup appear in Chenavaz (2012, 2017) and of cost in Chenavaz (2017), Chenavaz and Jasimuddin (2017), and Ni and Li (2018).

- The *sales effect*, $D\frac{\frac{\partial^2 D}{\partial p \partial e}}{\frac{\partial D}{\partial p}^2}$, linked to (5), quantifies the change in sales after higher price together with larger eco-efficiency. Demand increases with eco-efficiency, and it would increase even more with a lower price. The sales effect is negative.

Note that the markup effect is the sole cause of any positive relationship between eco-efficiency and price. Indeed, the cost and sales effects can only lead to a negative relationship. These observations lead to the following remarks:

Remark 1. Price decreases with eco-efficiency if the cost and sales effects outweigh the markup effect. Conversely, price increases with eco-efficiency if the markup effect is greater than the combined cost and sales effects. Putting the above observations together it follows that: If the cost and sales effects are greater than the markup effect, then price decrease with eco-efficiency. Conversely, if the markup effect is greater than both the cost and sales effects, then price increases with eco-efficiency. The described relationship is captured in Figure 2.

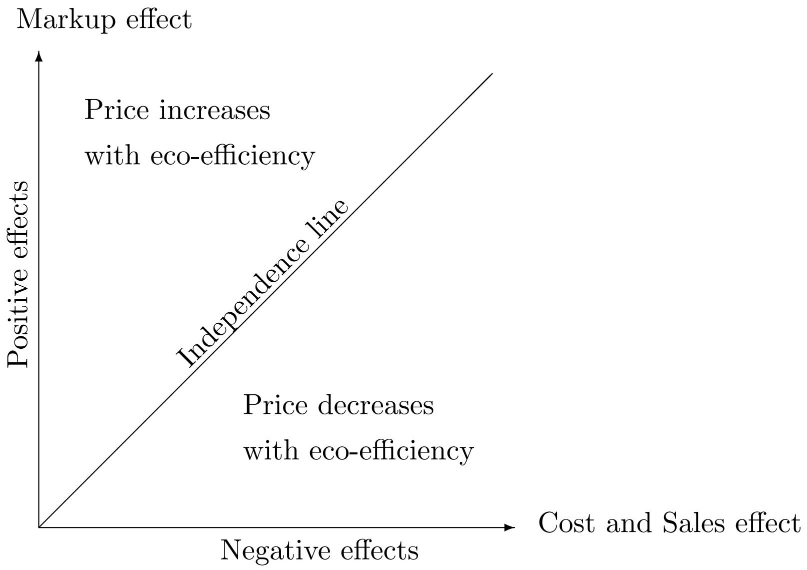

Figure 2: Price and eco-efficiency relationship

Remark 2. If consumers are not aware of the eco-efficiency policy of the firm or if they are not interested in green products, that is $\frac{\partial D}{\partial e} = 0$, then the demand-side effects do not play a role (markup and sales effects are zero), and price decreases with eco-efficiency because of the negative supply-side effect. Also, if eco-efficiency does not impact the resource use, that is $\frac{dR}{de} = 0$, then the supply-side effect does not exert influence (cost effect is zero), and price may increase or decrease with eco-efficiency because of the opposing demand-side effects.

Remark 3. The markup effect, $\frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e}$, is made of the first order derivatives of the demand with respect to price and to eco-efficiency, namely $\frac{\partial D}{\partial p} < 0$ and $\frac{\partial D}{\partial e} \geqslant 0$. The sales effect, $D\frac{\frac{\partial^2 D}{\partial p \partial e}}{\frac{\partial D}{\partial p}^2}$, is made of the cross derivative of demand with respect to price and eco-efficiency, namely $\frac{\partial^2 D}{\partial p \partial e} \leqslant 0$. Note that $\frac{\partial^2 D}{\partial p \partial e} < 0$ implies $\frac{\partial D}{\partial p} < 0$ and $\frac{\partial D}{\partial e} > 0$, but $\frac{\partial D}{\partial p} < 0$ and $\frac{\partial D}{\partial e} > 0$ does not imply $\frac{\partial^2 D}{\partial p \partial e} < 0$, for instance if the demand is additively separable in the price and eco-efficiency ($D(p, e) = h(p) + l(e)$). Consequently, the existence of a sales effect implies the existence of a markup effect, but the existence of a markup effect does not imply the existence of a sales effect. In other words, when the sales effect plays a role, then the markup effect also exhibits influence, but the reciprocal does not hold.

Subclasses of Demand Function

So far we considered a general demand function. In this section, we consider two subclasses of demand functions, namely additively and multiplicatively separable, and present the associated relationship between price and eco-efficiency. We first consider additively separable demand functions.

Corollary 1. If $D = h(p) + l(e)$, that is D is additively separable, the total effect of eco-efficiency on price writes as:

$$\frac{dp}{de} = \frac{\frac{dC}{dR}\frac{dR}{de} + \frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e}}{2 - D\frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2}}. \tag{19}$$

Proof. With $D = h(p) + l(e)$, the cross derivative $\frac{\partial^2 D}{\partial p \partial e}$ is null and the sales effect in (18) disappears. □

In the additive separable demand function case, the total effect of eco-efficiency on price depends on the (negative) cost effect and (positive) markup effect. Following an increase in eco-efficiency, the price may either rise or fall, depending on the relative strength of the cost and markup effects. An example of an additively separable function is the linear demand function $D = a_0 - a_1 p + a_2 e$ with $a_0, a_1, a_2 > 0$.

Next, we move to a multiplicatively separable demand function.

Corollary 2. If $D = h(p)l(e)$, that is D is multiplicatively separable, the total effect of eco-efficiency on price writes as:

$$\frac{dp}{de} = \frac{\frac{dC}{dR}\frac{dR}{de}}{2 - D\frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2}} \leqslant 0. \tag{20}$$

Proof. With $D = h(p)l(e)$, the markup and sales effect are of same magnitude, but opposite signs. They cancel each other out. □

In the multiplicative separable demand function case, the total effect of eco-efficiency on price depends on the sole (negative) cost effect. When eco-efficiency rises, then the price falls. An instance of multiplicative separability is the Cobb-Douglas demand function, $D = a_0 p^{a_1} e^{a_2}$ with $a_0, a_1, a_2 > 0$.

4.2 Dynamics of Eco-Efficiency and Demand: Rebound Effect

So far, the dynamics of price and eco-efficiency are discussed. However, rebound is defined with respect to total resource use, the product of demand and resource per product, $RD = R(e(t))D(p(t), e(t))$. Before turning our attention to total resource use and eco-efficiency, we first determine the dynamics of demand and eco-efficiency. To determine the dynamics of eco-efficiency and demand, we differentiate demand, $D(p(t), e(t))$, with respect to time, t, to find:

$$\frac{dD}{dt} = \frac{\partial D}{\partial p}\frac{dp}{dt} + \frac{\partial D}{\partial e}\frac{de}{dt}. \tag{21}$$

Proposition 2. The total effect of eco-efficiency on demand is

$$\frac{dD}{de} = \underbrace{\frac{\partial D}{\partial p}}_{-} \frac{\overbrace{\frac{dC}{dR}\frac{dR}{de}}^{-} + \overbrace{\frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e}}^{+} + \overbrace{D\frac{\frac{\partial^2 D}{\partial p \partial e}}{\frac{\partial D}{\partial p}^2}}^{-}}{\underbrace{2 - D\frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2}}_{+}} + \underbrace{\frac{\partial D}{\partial e}}_{+}. \tag{22}$$

Proof. Similarly to the proof of Proposition 1, we eliminate time in equation (21). With $p = p(e)$, we have $D = D(p(e), e)$. Substitute $\frac{dp}{de}$ from (18) into (21), after eliminating time. Rearranging gives the result. ☐

The total effect of eco-efficiency, e, on demand, D, may be either positive or negative, because of opposing effects. Below we identify under which conditions the effect is positive and under which conditions the effect is negative. Indeed, the direct price effect, $\frac{\partial D}{\partial p}$, is negative, whereas the direct eco-efficiency effect, $\frac{\partial D}{\partial e}$, is positive. The first term of the right hand side of (22) captures the indirect effect of eco-efficiency on demand, which exists because of the intermediary role of price. Informally, eco-efficiency impacts the price, which in turn impacts demand. Such impact may be positive or negative. In the following remarks, we present some insights from the derived relationship on the dynamics of eco-efficiency and demand.

Remark 4. Understanding rebound requires to distinguish between the direct eco-efficiency effect on demand, $\frac{\partial D}{\partial e}$, and the total eco-efficiency effect on demand, $\frac{dD}{de}$. The direct effect is always positive by assumption (5), meaning that consumers favor sustainable products. The total effect is a result and may be positive or negative, capturing both the direct and the indirect effects of eco-efficiency on demand.

We acknowledge that the backfire effect is defined with respect to total resource use. However, to determine the existence of a rebound effect, reverse, zero, or positive, we only need to consider the sign of the total effect of eco-efficiency on demand, $\frac{dD}{de}$. After eco-efficiency changes, the new resource per product is determined and fixed. As such, it is sufficient to only consider the change in demand with change in eco-efficiency. Note that we refer to positive rebound to capture the partial and total rebound effects along with the backfire effect.

Remark 5. If the total effect of eco-efficiency is to increase demand, $\frac{dD}{de} > 0$, then a positive rebound is observed. The positive rebound is more likely with strong direct price, cost, and sales effects and low direct eco-efficiency and markup effects.

Remark 6. If the total effect of eco-efficiency is to decrease demand, $\frac{dD}{de} < 0$, then a reverse-rebound effect is observed. A strong enough markup effect is required for the reverse-rebound effect, meaning that consumers must be eco-sensitive, that is $\frac{\partial D}{\partial e} > 0$.

In this section, we established the dynamics between eco-efficiency and demand. We also found that understanding how demand changes with eco-efficiency is sufficient to identify the existence of reverse, zero, or positive rebound. We have yet to disentangle positive rebound into its components: the partial rebound, total rebound and backfire effects. We carry out the disentanglement next when we determine the dynamics of eco-efficiency and total resource use.

4.3 Dynamics of Eco-Efficiency and Total Resource: Partial Rebound, Total Rebound, and Backfire Effects

So far, we only considered the relationship between demand and eco-efficiency, and gained some insights into the existence of rebound. We now establish the relationship between total resource use and eco-efficiency, which is required to disentangle the different types of positive rebound: the partial rebound, total rebound and backfire effects. Formally, total resource use corresponds to the resource per product multiplied by the demand, that is $RD = R(e(t))D(p(t), e(t))$. In order to determine the dynamics between eco-efficiency and total resource, we take a similar approach as in the previous two sections and differentiate total resource use with respect to time and then use the time elimination method. We now differentiate total resource use with respect to time:

$$\frac{d(RD)}{dt} = \frac{dR}{de}\frac{de}{dt}D + R\left(\frac{\partial D}{\partial p}\frac{dp}{dt} + \frac{\partial D}{\partial e}\frac{de}{dt}\right). \tag{23}$$

Eliminating time from equation (23) offers:

Proposition 3. The total effect of eco-efficiency on total resource reads

$$\frac{d(RD)}{de} = \underbrace{\frac{dR}{de}}_{-} \underbrace{D}_{+} + \underbrace{R}_{+} \left(\underbrace{\frac{\partial D}{\partial p}}_{-} \frac{\overbrace{\frac{dC}{dR}\frac{dR}{de}}^{-} + \overbrace{\frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e}}^{+} + \overbrace{D\frac{\frac{\partial^2 D}{\partial p \partial e}}{\frac{\partial D}{\partial p}^2}}^{-}}{\underbrace{2 - D\frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2}}_{+}} + \underbrace{\frac{\partial D}{\partial e}}_{+} \right), \tag{24}$$

with $\frac{dp}{de}$ given by (18).

Proof. Similarly to the proof of Proposition 1, we assume a function D depending on e, which yields $RD = R(e)D(e)$. Substitute $\frac{dp}{de}$ from (18) and rearranging (23), after eliminating time, gives the result. ☐

The total effect of eco-efficiency, e, on total resource, RD, may be either positive or negative, due to opposing effects, please see the signs associated with the under and over braces associated with each term in (24). Next we identify under which conditions the effect is positive and under which conditions the effect is negative. Compacting (24), we have $\frac{d(RD)}{de} = \frac{dR}{de}D + R\frac{dD}{de}$. Informally, we say $\frac{dR}{de}D$ is the *resource-side effect* of eco-efficiency, i.e., it informs how resource use changes with eco-efficiency, assuming demand is unchanged. Similarly, $R\frac{dD}{de}$ is the *consumer-side effect* of eco-efficiency, i.e., it informs how consumer demand changes with eco-efficiency, assuming resource use is unchanged. From (22) we know $\frac{dD}{de}$ may be either positive or negative. In the case $\frac{dD}{de}$ is negative, then total resource use decreases with eco-efficiency. However, if $\frac{dD}{de}$ is positive, then the relative magnitude of the consumer-side and resource-side effects matter. Recall that the backfire effect occurs when the total resource use after eco-efficiency improvements exceeds total resource use before the improvements. Using equation (24), we say backfire occurs if $\frac{d(RD)}{de} > 0$. Similarly, by definition of the total rebound effect, total resource use remains unchanged after an increase in eco-efficiency, leading to the following condition: $\frac{d(RD)}{de} = 0$. The partial rebound effect may be identified only when using both Propositions 2 and 3. In particular, the partial rebound effect occurs when the total resource use decreases, $\frac{d(RD)}{de} < 0$, but demand increases, $\frac{dD}{de} > 0$, as eco-efficiency increases. We now make a few remarks regarding the appearance of the partial rebound, total rebound, and the backfire effects.

Remark 7. If the consumer-side effect is of the same magnitude and opposite sign of the resource-side effect ($R\frac{dD}{de} = -\frac{dR}{de}D$), then the total rebound effect occurs.

Remark 8. If the resource-side effect ($\frac{dR}{de}D$) is strong "enough," then regardless of the consumer-side effect ($R\frac{dD}{de}$), no backfire effect appears $\left(\frac{d(RD)}{de} < 0\right)$.

Remark 9. If the consumer-side effect is positive ($R\frac{dD}{de} > 0$), as dictated by Remark 5, and strong "enough," then the backfire effect appears $\left(\frac{d(RD)}{de} > 0\right)$.

Remark 10. If the consumer-side effect is negative ($R\frac{dD}{de} < 0$), as dictated by Remark 6, then no backfire effect appears $\left(\frac{d(RD)}{de} < 0\right)$. Only if demand increases with eco-efficiency ($\frac{dD}{de} > 0$), see Remark 5, will the partial rebound effect occur.

Proposition 3, equation (24), may look like a paradox. Consumers purchase eco-efficient products to use less resources. However, if consumers' affinity for eco-efficient products is high, in the sense that consumers' willingness to buy more eco-efficient products is large ("large" $\frac{\partial D}{\partial e}$), then the backfire is more likely to appear. Meaning that consumer drive for more eco-efficient products may lead to an increase in total resource use. Alternatively, a firm may capitalize on consumers' willingness to pay more eco-efficient products by charging a higher price, then the reverse rebound effect is more likely to occur. These and other insights are further discussed in the next section.

5 Discussion

In the section, we summarize our contributions and discuss the theoretical and managerial implications.

5.1 Contributions Summary

Price plays a key role in the literature on the rebound effect. The usual assumption in the rebound literature is that efficiency increases lead to (relative) price decreases, because of the cost effect (cf. Definition 4.1). The literature furthermore assumes that demand decreases with price, which results in lower efficiency-induced prices causing higher demand for goods and thus higher than expected resource use (see, for example, Khazzoom (1980)). If we assume that more eco-efficient products are more desirable, then lower prices as a result of a more eco-efficient production are only one possibility. There is a wealth of literature that shows both empirically as well as conceptually that consumers are willing to pay a price premium for products that are more environmentally-friendly. In this context, the empirical literature looks at consumers willingness to pay a price premium for environmentally-friendly products (Galarraga Gallastegui, 2002). The conceptual literature argues that environmental aspects can serve as a differentiation factor that allows to charge higher prices (Vlosky et al., 1999). For profit-maximizing firms, it is not necessarily wise to pass on the cost-savings to its customers, but a combination of higher prices and lower output can result in higher profits, the resource-saving effect of which has to date not been considered in the literature on the rebound effect.

One contribution of our work is to consider the demand side impact of improving eco-efficiency via the markup effect. We see that the markup effect can mitigate the appearance of the partial rebound, total rebound, and backfire effects. In fact, if price increases sufficiently with eco-efficiency, as previously discussed, then the total resource use will decrease, explaining the reverse rebound effect. From an empirical perspective, identifying the impact of the markup effect on the rebound effect may be difficult, as the positive rebound effect will be mitigated or not observed at all when the markup effect is strong. Prior work uses parametric functions to model rebound implicitly limiting the types of rebound observed, something we do not in our structural equation approach, allowing us to identify the markup effect. In fact, the only work we are aware of that identifies the reverse rebound effect, does not explain why the reverse rebound effect occurred (Saunders, 2008). With our work, we provide insight not only as to the existence of the reverse rebound effect, but also as to why and when the reverse rebound effect occurs.

Another contribution of our work is to distinguish the direct eco-efficiency effect on demand, $\frac{\partial D}{\partial e}$, and the total eco-efficiency effect on demand, $\frac{dD}{de}$, see Remark 4. In the literature, consumers favor sustainable products. However, we identify that the total eco-efficiency effect on demand is made up of two effects: direct and indirect. The direct effect captures the relationship between eco-efficiency and demand, everything else being equal; the indirect effect accounts for the intermediary role of price in the eco-efficiency and demand relationship, as eco-efficiency affects the price, which in turn impacts the demand. The intermediary relationship between eco-efficiency and price is captured by the three effects discussed in Definition 4.1, namely, cost, markup, sales effects. We find that though the direct eco-efficiency effect is always positive, the total eco-efficiency effect may be either positive or negative, depending on the strength of the cost, markup, and sales effects. It is, therefore, a fallacy of the existing rebound literature to conclude that demand always increases with eco-efficiency.

5.2 Theoretical Discussion

We begin our discussion by reviewing the factors that lead to rebound, captured in Table 3.

Table 3 summarizes the results. The first column defines the kind of rebound; columns 2, 3, and 4 link the definition of the rebound effect to the different results of our model given by Propositions 1, 2, and 3. We present our results as necessary and sufficient condition(s) and implication(s). For each kind of rebound, the condition(s) listed is sufficient and necessary for the kind of rebound effect to appear. The implications are properties and must hold whenever the condition(s) is(are) observed. Eco-efficiency backfires if, after an eco-efficiency increase, the index is greater than 100 (Column 1). The condition for the backfire effect is that total resource uses increases with eco-efficiency (Column 4). The implications are that price increases with eco-efficiency (Column 2), and demand decreases with eco-efficiency (Column 3). With the total rebound effect, the index remains at 100 (Column 1). The condition is that total resource remains unchanged (Column 4), implying that the price increases and the demand decreases with eco-efficiency. With the partial rebound effect, the index falls between

Table 3: Curation of Analytical Results

(1) Kind of Rebound	(2) Proposition 2	(3) Proposition 3	(4) Proposition 4
Backfire	$\frac{dp}{de} < 0$	$\frac{dD}{de} > 0$	$\frac{d(RD)}{de} > 0$
$i > 100$	Implication	Implication	Condition
Total Rebound	$\frac{dp}{de} < 0$	$\frac{dD}{de} > 0$	$\frac{d(RD)}{de} = 0$
$i = 100$	Implication	Implication	Condition
Partial Rebound	$\frac{dp}{de} \in \mathbb{R}$	$\frac{dD}{de} > 0$	$\frac{d(RD)}{de} < 0$
$0 < i < 100$	Implication	Condition	Condition
Zero Rebound	$\frac{dp}{de} > 0$	$\frac{dD}{de} = 0$	$\frac{d(RD)}{de} < 0$
$i = 0$	Implication	Condition	Implication
Reverse Rebound	$\frac{dp}{de} > 0$	$\frac{dD}{de} < 0$	$\frac{d(RD)}{de} < 0$
(Super-Con.) $i < 0$	Implication	Condition	Implication

Index of total resource use $RD_i = 100$. Conditions are necessary and sufficient.

0 and 100 (Column 1). The two joint conditions are that demand increases and total resource decreases with eco-efficiency (Columns 3 and 4). There is no implication with respect to the effect of eco-efficiency on price, which can be positive or negative (Column 2). The zero rebound effect imposes an index of 0. The condition is that demand is independent from eco-efficiency (Column 3). The implications are that price increases and total resource decreases with eco-efficiency. With the reverse rebound effect, the index is negative (Column 1). The condition is that demand decreases with eco-efficiency (Column 3). It implies that price increases and total resource decreases with eco-efficiency (Column 4).

To conclude this section, we combine Corollary 2 with the rows in Table 3 describing the zero and reverse rebound effects. As noted in Corollary 2, multiplicatively separable demand leads to price always decreasing as eco-efficiency increases. However, whenever the zero or reverse rebound effects are present price must increase with eco-efficiency as found in our analysis, please see Table 3. Putting the two findings together, we note that whenever a multiplicatively separable demand function is used, then neither the zero rebound effect nor the reverse rebound effect will be observed. This result echos the finds of Saunders (2008) in which the existence of various rebounds are predicated on the production function used, please see the author's discussion of the Cobb-Douglas production function as one example of a multiplicatively separable function. Our results generalize those of Saunders (2008) to all multiplicatively separable demand functions.

5.3 Managerial Discussion

In the management literature markets are characterised in different ways. The arguably most influential characterization is by Porter (1985). Porter distinguishes between two dimensions. The first dimension is the source of competitive advantage for the companies in the market. He distinguishes between lower costs and differentiation as source for competitive advantage. The second dimension is the competitive scope. The scope can be narrow or broad and defines the scope of activities of the companies. Companies that address one or few specific segments of a market address a narrow scope, while companies that address all or most of a market have a broad scope.

Porter (1985) uses the four combinations depicted in Table 4 to characterize how companies gain a competitive advantage. We can interpret our analytical results analogously and distinguish between a high and low mark-up effect on the one hand and a cost and sales effect on the other hand. A high mark-up effect points to companies that aim to differentiate and a low mark-up effect to companies that seek cost-leadership. When companies target specific segments of a market, their strategy to gain competitive advantage is focused. It is usually argued that differentiation allows companies to achieve higher prices (Porter, 1985) and, while they might be offset to some degree by higher costs (Rappaport, 1987), also higher margins and therefore higher profits. Cost-leadership, conversely, allows companies to grab more market share, which can compensate for lower margins, resulting in turn in higher profits. In the case of niche markets, the information and specialized knowledge that companies have on their market protect their margins from eroding (Porter, 1980). Mass markets, on the other hand, are likely to be far more competitive.

Mark-up	High	Differentiation focus $\frac{dp}{de} > 0$ Partial, zero, and reverse rebounds	Differentiation $\frac{dp}{de} \in \mathbb{R}$ All rebounds
	Low	Cost focus $\frac{dp}{de} \in \mathbb{R}$ All rebounds	Cost leadership $\frac{dp}{de} < 0$ Partial and total rebounds and backfire
		Niche	Mass market
		Cost and Sales (Market size)	

Table 4: Relationship between price and eco-efficiency in a Porter matrix (Porter, 1985)

Our analytical results match the dominant view in the management literature, when we combine the effects of mark-up and market size with an increase in eco-efficiency, depicted in Table 4, we find: 1) Companies with high mark-ups and in a niche market are doubly protected from market pressures on their prices; for such firms, we expect a positive impact on their prices with an increase in eco-efficiency, leading to the partial, zero, or reverse rebound effects. This setting is captured in the upper-left-hand corner of the Porter matrix. 2) Companies operating in a low mark-up, mass market setting face double price pressures; for such firms, we expect prices to decrease and demand to increase with an increase in eco-efficiency, leading to the partial rebound, total rebound, or backfire effects to occur. This setting is captured in the lower-right-hand corner of the Porter matrix. 3) For firms operating in either low mark-up and niche market setting or high mark-up and mass market setting there is only a single price pressure; for such firms, it is not clear how the market will react to an increase in eco-efficiency, all forms of rebound and backfire effects may occur, and must be handled on a case-by-case basis. These two cases are captured in the lower-left-hand and upper-right-hand corners of the Porter matrix.

6 Conclusion

The starting point of existing research on rebound is that increases in eco-efficiency lead to increases in demand, compensating partially, fully or even overcompensating efficiency-induced reductions of resource use. We argue that this view is due to the restrictive design of existing research that focuses on identifying rebound, while we analytically determine when and why eco-efficiency rebounds. The work we present in this paper is the first to use a structural (opposing parametric) approach to formulate general (non-linear) demand and cost functions. Such a general approach offers a more complete understanding of the role of eco-efficiency improvement on the existence of rebound. That is, our structural approach allowed us to move beyond the simple relationship that links any increase in eco-efficiency to increases in demand. In fact, we find that the relationship is far richer than posited by others; the three effects identified in this paper, cost, markup, and sales effects, may collectively even lead to a decrease in demand and thus an even stronger reduction of resource use when eco-efficiency improves, generating the reverse rebound effect. Of the three effects discussed, the markup effect in isolation, if sufficiently strong, will always lead to a decrease in demand when eco-efficiency increases. Practically speaking, our findings suggest that increasing eco-efficiency does not necessarily lead to positive rebound; positive rebound is not inevitable. Calling any form of rebound effect a certainty (Alcott, 2008) is as premature as calling rebound effects, in general, not significant (Lovins, 1988). There may very well be instances where increased eco-efficiency leads to higher prices, in turn, lower consumption, and lower overall total resource use. Taking a practical lens to our results, we show how the markup effect can mitigate the effect lower marginal costs can have on the rebound effect. As mentioned above the marginal cost of production of electricity is lower for renewable than for non-renewable energy sources Lazard (2018). Renewable resources emit less CO_2-emissions when producing electricity. The literature suggests that in a highly competitive marketplace lower marginal costs result in lower prices. Lower prices risk at the same time to stimulate demand and are linked to a positive rebound. In a highly competitive marketplace the lower marginal costs of renewable energies might therefore result in a reduction of CO_2-emissions that is below expectations. The markup effect that we identify in our paper suggests that this link between lower costs and higher than expected CO_2-emissions is not inevitable. The markup effect can act as a countervailing force that drives up prices and, in this way, reduces CO_2-emissions. Our dynamic setting identifies the interplay between the different forces that link an increase of the CO_2-efficiency of electricity production to changes of total CO_2-emissions.

The insights from our work suggest that existing literature on the rebound effect needs to widen its lens to capture the full spectrum of possible relations between eco-efficiency and resource use. We think that our work is a necessary first step in this direction. Specifically, the three effects we identify are not fully understood empirically. Identifying each effect is an open question, and answering this question will help understand rebound further. The use of general functions for demand and cost to model rebound is new, and we think other, related fields in green economics and sustainability may also benefit from using this general approach. Theoretically, the work presented here is deterministic in nature. However, it is difficult for anyone to know what the future will bring, and as such a stochastic model may be more appropriate to use. Notwithstanding philosophical questions regarding knowing future distributions, a stochastic model will provide insights not available in the model presented here. A stochastic variant of firm objective (11) may use either the Maximum Principle (optimal control) or the Principle of Optimality (Hamilton-Jacoi-Bellman formulation). Interested readers may find both approaches used in the literature (Caulkins et al. 2011; Chenavaz et al. 2020 use the Maximum Principle; Schlosser 2017 uses the Principle of Optimality; comparison of the two approaches is carried out by Sethi 1983). Taking the lens of rational expectations, we expect the properties gleaned from a stochastic model to mirror those found in this work. However, a stochastic model will further inform a firm manager and policy makers on issues that cannot be addressed with the model used in the paper. For example, variance of returns and costs may be analyzed; so can the probability of ruin/bankruptcy for a firm be determined as it invests in eco-efficiency while demand and eco-efficiency decay change in a stochastic manner. Insights provided by stochastic models will provide further insights on the financial aspects of the rebound effect, something not explicitly covered by our work. Finally, our work suggests an additional research question in terms of public policy. Knowing that rebound is not a certainty and that higher efficiency can even lead to additional unexpected reductions of resource use, opens up the scope of possible regulations. Put colloquially, not all increases in eco-efficiency are equal. The policy value of some increases in eco-efficiencies might very well be in the unexpected chain of causalities they trigger. This poses the challenge to policy makers to put into place the correct incentives for eco-efficiency based on their rebound potential. To better understand how eco-efficiency increases link to higher or lower resource use is a necessary first step that research needs to take.

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A Proofs

In this section, we present the proofs of some of the results presented in the main text.

A.1 Proof of equation (15)

Recall the elasticity of eco-efficiency elasticity of demand $\eta_{D/e} = \frac{\partial D}{\partial e}\frac{e}{D}$ and the price elasticity of demand $\eta_{D/p} = -\frac{\partial D}{\partial p}\frac{p}{D}$. Note that $\frac{\frac{\partial D}{\partial e}}{\frac{\partial D}{\partial p}} = -\frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e}$. Substituting $\eta_{D/E}$, $\eta_{D/p}$, and (13b) in (13f) implies

$$\frac{d\lambda}{dt} - \left(r - \frac{\partial E}{\partial e}\right)\lambda = D\left(\frac{dC}{dR}\frac{dR}{de} - \frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e}\right), \text{ with } \lambda(T) = 0. \tag{25}$$

We abuse the notation using $\int \frac{\partial E}{\partial e} d\mu$ for $\int_{s-t}^{T} \frac{\partial E}{\partial e}(u(\mu), e(\mu))d\mu$. Integrating equation (25) with respect to time provides equation (15):

$$\lambda(t) = \int_t^T e^{-\left(r - \int \frac{\partial E}{\partial e} d\mu\right)(s-t)} D\left(\frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e} - \frac{dC}{dR}\frac{dR}{de}\right) ds.$$

Proof. Multiply both sides of (25) by $e^{-\left(r-\int \frac{\partial E}{\partial e} d\mu\right)t}$ yields $e^{-\left(r-\int \frac{\partial E}{\partial e} d\mu\right)t}\left(\frac{d\lambda}{dt} - (r - \frac{\partial E}{\partial e})\lambda\right) = \frac{d\left(\lambda e^{-\left(r-\int \frac{\partial E}{\partial e} d\mu\right)t}\right)}{dt} = e^{-\left(r-\int \frac{\partial E}{\partial e} d\mu\right)t} D\left(\frac{dC}{dR}\frac{dR}{de} - \frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e}\right)$. Thus, $d\left(\lambda e^{-\left(r-\int \frac{\partial E}{\partial e} d\mu\right)t}\right) = e^{-\left(r-\int \frac{\partial E}{\partial e} d\mu\right)t} D\left(\frac{dC}{dR}\frac{dR}{de} - \frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e}\right) dt$. Therefore, $\int_t^T d\left(\lambda(s) e^{-\left(r-\int \frac{\partial E}{\partial e} d\mu\right)s}\right) = \int_t^T e^{-\left(r-\int \frac{\partial E}{\partial e} d\mu\right)s} D\left(\frac{dC}{dR}\frac{dR}{de} - \frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e}\right) ds$, and $\lambda(T)e^{-\left(r-\int \frac{\partial E}{\partial e} d\mu\right)T} - \lambda(t)e^{-\left(r-\int \frac{\partial E}{\partial e} d\mu\right)t}$ $= \int_t^T e^{-\left(r-\int \frac{\partial E}{\partial e} d\mu\right)s} D\left(\frac{dC}{dR}\frac{dR}{de} - \frac{\eta_{D/e}}{\eta_{D/p}}\frac{p}{e}\right) ds$. Substituting the transversality condition, $\lambda(T) = 0$, proves the result. $\square$

B Supplementary Results

In this section, we present some results that may be of interest to a reader but are not directly tied to determining rebound.

B.1 $u(t)$ Dynamics

We follow here Chenavaz (2012, 2017); Ni and Li (2018). Recall equations (13f) ($\lambda(T) = 0$) and (14) ($\lambda(t) > 0$, for all $t \in [0, T)$). Assuming that λ is continuously differentiable and the left derivative $\frac{d\lambda(T^-)}{dt} < 0$ implies that it exists a least one $t_1 \in [0, T)$ such that $\frac{d\lambda(t)}{dt} < 0$, for all $t \in [t_1, T)$. The result states that λ declines after time t_1. Moreover, according to (13a), $\frac{d}{dt}(\frac{\partial E}{\partial u}) = \frac{d}{dt}\left(\frac{1}{\lambda}\right) = -\frac{\frac{d\lambda}{dt}}{\lambda^2}$. So, $\operatorname{sgn}\left(\frac{d}{dt}(\frac{\partial E}{\partial u})\right) = -\operatorname{sgn}\frac{d\lambda}{dt}$, and for all $t \in [t_1, T)$, $\frac{d}{dt}(\frac{\partial E}{\partial u}) > 0$. Note that $\frac{d}{dt}(\frac{\partial E}{\partial u}) = \frac{\partial^2 E}{\partial u^2}\frac{du}{dt} + \frac{\partial^2 E}{\partial u \partial e}\frac{de}{dt}$. In the case of additive separability of E in u and e (which implies $\frac{\partial^2 E}{\partial u \partial e} = 0$), we have $\frac{d}{dt}(\frac{\partial E}{\partial u}) = \frac{\partial^2 E}{\partial u^2}\frac{du}{dt}$. According to (2), $\frac{\partial^2 E}{\partial u^2} < 0$, and thus $\operatorname{sgn}\frac{du}{dt} = \operatorname{sgn}\frac{d\lambda}{dt}$.

Therefore, investment falls after time t_1. Mathematically, we have:

$$\exists\, t_1 \in [0, T) \;\bigg|\; \frac{du(t)}{dt} < 0, \;\; \forall t \in [t_1, T). \tag{26}$$

The dynamic innovation (26) depends solely on the first-order condition for innovation, (13a); investment is independent from the first-order condition for price, (13b). Investment may increase ($\frac{du}{dt} > 0$) at the product launch, from $t = 0$ to t_1. Though investment rate falls ($\frac{du}{dt} < 0$) for the remaining planning period, from t_1 to T. However, the firm will always have positive investment, as indicated by investment rule (13a). The implication of this result is aligned with Vörös (2006, 2013), in which quality may first increase and then decrease. In the special case $t_1 = 0$, it is also compatible with Li and Rajagopalan (1998), according to which quality always decreases over time.

B.2 Optimal Relationship between Price and Investment

A profit maximizing firm knows the relationship between the optimal price and investment. We now determine this relationship. Let $u^*(p)$ be the investment expense verifying (13a). This investment level maximizes the intertemporal profit for any price. Similarly, let $p^*(u)$ be the price satisfying (13b), which maximizes the intertemporal profit for any investment. The intertemporal profit is maximized with the investment and pricing pair such that $(u^*, p^*) = (u^*(p^*), p^*(u^*))$. In the following, the investment and pricing are called *optimal* in the sense that they maximize the intertemporal profit. For simplicity, we omit now the $*$ superscript notation, when there is no confusion. Denote the elasticity of eco-efficiency dynamics with respect to investment as $\eta_{E/u} = \frac{\partial E}{\partial u}\frac{u}{E}$, and recall the price elasticity of demand, $\eta_{D/p} = -\frac{\partial D}{\partial p}\frac{p}{D}$.

Proposition 4. The optimal relationship between price and investment writes

$$\frac{\eta_{E/u}}{\eta_{D/p}} = \frac{u}{E}\frac{p-C}{p}\frac{1}{\lambda}, \tag{27}$$

with λ given in equation (15).

Proof. Follows from the definitions of $\eta_{E/u}$, $\eta_{D/p}$, (13a) and (13b). ☐

Proposition 4 provides a necessary optimality condition for profit maximization. The left-hand side of (27) is the ratio of supply side elasticity, $\eta_{E/u}$, and demand side elasticity, $\eta_{D/p}$. The condition states that the ratio of investment elasticity of eco-efficiency dynamics to price elasticity of demand, $\dfrac{\eta_{E/u}}{\eta_{D/p}}$, must equal the level of investment deflated by eco-efficiency dynamics, $\frac{u}{E}$, multiplied by the markup rate deflated by the shadow price of eco-efficiency, $\frac{p-C}{C}\frac{1}{\lambda}$.

Note that rearranging (27) yields the Lerner index, $\frac{p-C}{p} = \frac{\eta_{E/u}}{\eta_{D/p}}\frac{E}{u}\lambda$, which measures the market power of the firm. The Lerner index ranges from 0, in a competitive market, to 1, in a monopoly market. Based on Proposition 4, the Lerner index increases with the shadow price of eco-efficiency, λ. As eco-efficiency decreases over time, so does the Lerner index, meaning that the appeal of the product decreases over time, assuming no additional investment.