Circular Economy: Joint Dynamic Pricing and Recycling Investments

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Abstract

In a circular economy, the use of recycled resources in production is a key performance indicator for management. Yet, academic studies are still unable to inform managers on appropriate recycling and pricing policies. We develop an optimal control model integrating a firm's recycling rate, which can use both virgin and recycled resources in the production process. Our model accounts for recycling influence both at the supply- and demand-sides. The positive effect of a firm's use of recycled resources diminishes over time but may increase through investments. Using general formulations for demand and cost, we analytically examine joint dynamic pricing and recycling investment policies in order to determine their optimal interplay over time. We provide numerical experiments to assess the existence of a steady-state and to calculate sensitivity analyses with respect to various model parameters. The analysis shows how to dynamically adapt jointly optimized controls to reach sustainability in the production process. Our results pave the way to sounder sustainable practices for firms operating within a circular economy.

Keywords: dynamic pricing, recycling investments, optimal control, general demand function, circular economy

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1 Introduction

Stahel (2016) in a paper in *Nature* defines the circular economy as one that "[turns] goods that are at the end of their service life into resources for others, closing loops in industrial ecosystems and minimizing waste." In this paper we address one segment of the entire circular economy, recycling. Before introducing our recycling focused contribution, we will explore the importance of the circular economy. Accenture estimates that by 2030 the circular economy will be valued at $4.5 trillion (Accenture, 2015). "Green products and services are critical to the circular economy" according to the Directorate-General for Environment (European Commission) (2016). Green products, according to the same source are those that last longer, are easier to fix, and are more recyclable. Consumers are actively shaping the circular economy with their purchases and purchase intentions. In a recent consumer survey, Accenture finds that consumers in North America, Europe, and Asia are willing to pay more for products that are designed to be reused or recycled (green products) (Cantwell et al., 2019). Spurred by consumer demand, some industry leaders already focus on making products' environmental benefits clear. For example, the Renault Group has 36% of total mass of new vehicles come from recycled resources, and 85% and 95% of all new vehicles will be recyclable and recoverable, respectively (Renault, 2017). In the food industry, Nestle, PesciCo, and Coca-Coca aim to use 15%, 25%, and 50% of recycled content in packaging in the next decade (Brock, 2020). The growth of the circular economy resulted in Coca-Cola, once opposing the deposit system (Elmore, 2017), piloting them in Scotland (Carrell, 2017). The burgeoning circular economy will drive firms to invest in and innovate greener products, with recycling playing a key role (Maio and Rem, 2015).

However, there is a potential downside to the circular economy. As firms invest in business models, processes, and products in line with the circular economy, the competitive advantage enjoyed by one firm within the circular economy erodes. Practically speaking, the *realized greenness* of a product (Driessen et al., 2013), the amount which this product outperforms competitive products on green attributes, will naturally degrade as other firms innovate newer and greener products. From an innovating firm's perspective, investment into a product's greenness will naturally degrade as other firms also invest and innovate in competing green products.

Given the current setting, firms may find themselves in a Catch 22. On the one hand, a firm will increase sales of green products due to higher consumer demand resulting from the green product. On the other hand, the competitive advantage of the green product will be short-lived due to the relative greenness, how green a product is relative to all other products, of any green product naturally degrading over time (Benchekroun and Long, 2012; Dai and Zhang, 2017; Jørgensen et al., 2010; Saha et al., 2017).

In this paper, we help a firm determine the greenness of a product by determining the proportion of the product to make from recycled resources while controlling for price. However, a firm cannot simply choose any proportion of recycled resources to use, and instead it must invest in manufacturing and recycling processes to attain the desired level of recycled product use in its manufacturing. Using a model inspired by Van Schaik and Reuter (2004) we determine a firm's optimal pricing and recycling investment policies. The key contributions of our work are:

- We propose a dynamic model combining pricing and recycling investments of a profit-maximizing firm. The model incorporates realistic assumptions and settings tied to actual firms' key performance indicators.
- We identify three factors that determine the interplay between recycling investment and product pricing; please see Definition 1 on page 12 for more details.
- We provide a thorough numerical study verifying the analytical results along with steady-state analyses and comparative statics, which allow to infer further managerial insights.

In the remainder of this paper, we first discuss related work in Section 2. In Section 3, we build the optimal control framework used to analyze our model. The analysis of the proposed framework is presented in Sections 4 and 5. For the optimal control framework we use a fixed and finite planning horizon T, and time $t \in [0, T]$ is continuous. Our numerical exploration and experiments are found in Section 6. We discuss the theoretical and managerial contributions in Section 7, by presenting our results and the research questions they address. We conclude the paper in Section 8. For ease of reading the remainder of the paper, Table 1 from Appendix A presents the notations.

2 Related Work

After giving a short overview of the role of recycling within the circular economy we discuss the related work to what we present here. Three streams of literature relate to our work. The first stream examines the role of recycling in retail. The second stream considers recycling in production systems. The third stream explores the interplay and use of optimal control within the circular economy and corporate social responsibility. Prior to turning our attention to each of the streams identified above, we note that the role of operations management within the circular economy is introduced by Agrawal et al. (2019). Partially motivated by the Patagonia example of Agrawal et al. (2019), in our work we consider what fraction of a firm's product should be made from recycled resource.

In a sequence of survey papers, recycling is considered a core component/action of any circular economy strategic framework (Geissdoerfer et al., 2017; Lewandowski, 2016). In a paper by Stahel (2016) the author identifies the circular-economy business models come one of two forms: 1) ones that foster reuse and extended service and 2) that turn discarded goods into resources for new goods via recycling. In this paper, we use the fact that a firm may not use only recycled or only virgin (new) resources, instead, it may use a mixture. We determine the mixture of resources to use so as to maximize profit subject to consumers that are sensitive to green products. With respect to the conceptual frameworks proposed and reviewed by this literature stream, we operationalize recycling decisions in a production process for a firm facing consumers that prefer green products.

We now discuss each stream in turn and discuss how our work relates to each stream. The first stream of literature related to ours focuses on recycling in retail. As early as 1993, Biddle argues that recycling is needed for integration into retailing as it is shown that there is are demand for recycled products (Biddle, 1993). Toyasaki et al. (2011) compare a monopolistic to a competitive setting in a manufacturer-recycler setting. However, the authors do not consider recycling sensitive demand, something we consider in a real-time setting. Esenduran et al. (2019) follow a similar line of research to determine the level of recycling to have by an e-waste firm. The authors consider the flow of e-waste to different recyclers that differentiate on the quality of recycling using a linear term. Unlike these two studies, we consider the demand for products made with recycled resources. Besides, we consider a dynamic setting, whereas the previous studies consider a static setting. The link between consumers interested in purchasing recycled products falls under the larger umbrella of green consumers, and increased demand as recycling increases is well-known (Chen et al., 2019; Pedro Pereira Luzio and Lemke, 2013; Sriram and Forman, 1993). Multiple authors consider the impact of green-sensitive consumers in an operation context (Ghosh and Shah, 2015; Rezaee et al., 2017; Wang et al., 2011). Authors of these studies consider network design problems (Rezaee et al., 2017; Wang et al., 2011) as well as contract design within a supply chain (Ghosh and Shah, 2015). In this paper, we consider a monopolist setting determining the greenness of a product over time with a general (non-linear) demand function for green products (subject to satisfying reasonable structural assumptions). To our knowledge, such analysis is not carried out in this stream of literature and we provide additional insights considering long-term actions a firm may take in the presence of green-sensitive consumers.

The second stream of literature related to our paper focuses on the use of recycled resources in the production process. In a recent paper, Raz and Souza (2018) consider a manufacturing setting where recycled products may be used as a strategic supplier; in the model two competing manufacturers decide how much to recycle. Tian et al. (2019) also show that with market competition producers may recycle their products without any external cooperation. Chang et al. (2019) consider the fact that multiple resources are generated during recycling and suggest ways to use integrated planning to better use all recycled products. Zhu and He (2017) study green product design in supply chains under competition. Guo et al. (2020) study the competition in green product development in fashion markets, cf. relative greenness. In this paper, we do not consider competition, but instead a naturally degrading recycling process. It is known that manufacturing processes naturally degrade over time (Clerc and Muetze, 2012; Yang et al., 2008) either due to machine tolerances or due to a lack of preventative maintenance. In the context of green processes, natural degradation is widely accepted in the literature, as attested by the surveys of Benchekroun and Long (2012); Jørgensen et al. (2010). Green process degradation if no maintenance is made is recently assumed in Chenavaz et al. (2021); Dai and Zhang (2017); Saha et al. (2017). We account for the process natural degradation of the recycling rate and determine how a firm invests in its recycling process in the presence of green-sensitive consumers. Ongoing technology development and natural process quality degradation collectively lead to a decrease in eco-efficiency over time. Consequently, eco-efficiency increases with eco-efficiency investment and decays otherwise.

The third stream of literature linked to our work is that of optimal control applications to the environment, sustainability, and recycling. Optimal control applications to green issues have been regularly compiled. For instance, Jørgensen et al. (2010) survey dynamic games of pollution management and Benchekroun and Long (2012) collaborative environmental management. Recently, Oubraham and Zaccour (2018) present the state of the art in sustainable exploitation of renewable resources. Green consumers are modeled by Saha et al. (2017) in the context of retailer investment in green operations and in preservation technology. Dai and Zhang (2017) consider green process investment with differentiated prices. Zhang et al. (2017) focus on green innovation within a supply chain. Focusing on the circular economy, Chun-Yan (2017) considers inventory management and Sørensen (2018) model the differences between linear and circular economies. Recently, Liu and De Giovanni (2019) look at green process innovation within a supply chain. Corporate social responsibility is also investigated: For instance, Ferrara et al. (2017) and Raza (2018) examine supply chain coordination and Peng et al. (2019) the marketing-mix of price and quality. Highfill and McAsey (2001) propose a pioneering model of optimal control for recycling. Investigating consumer maximization of utility, they show that recycling increases over time. Wanjiru and Xia (2018) look at grey and rainwater recycling. Recycling with manufacturing and remanufacturing is examined by Kiesmüller (2003) for model systems of product recovery with lead times and by Dhaiban et al. (2018) with deteriorating and defective items. The above literature uses linear (parametric) demand functions, constraining consumer behavior. Instead, we model a general (structural) demand function, allowing much freedom on consumer behavior. We look at the linear demand function as a special case.

3 Model Formulation

We now present the model used in our analysis. We start with a general structural approach. That is, we do not assume any particular parametric formulation. As we are considering the impact of recycling on pricing, and recycling investment decisions of a firm we will define the following dynamics:

Recycling Rate: How recycling rate changes over time. In this paper, we also use *greenness* to refer to the recycling rate of a product made by a firm.

Demand: How demand changes with price and recycling rate.

Production Cost: How production cost changes with the recycling rate.

Profit: Combining the previous two dynamics, we determine how profit changes with price and recycling rate.

We write the intertemporal profit of the firm, and then we examine its dynamic optimization problem.

3.1 Recycling

Consumers are increasingly more environmentally conscious, resulting in firms investing in greener processes like a higher recycling rate (Dhaiban et al., 2018; Highfill and McAsey, 2001; Kiesmüller, 2003; Wanjiru and Xia, 2018). In other words, a higher recycling rate (the fraction of recycled resource) used in the production process satisfies green consumers. Investments allow continuous enhancement of recycling over time. Yet, the recycling rate, like any quality process, may degrade slowly over time if not maintained (Dai and Zhang, 2017; Saha et al., 2017). Given that the recycling rate is the fraction of recycled resource used – due to inventions, new developments, technological progress, regulation, etc. – over time the notion of what constitutes a recycled product changes (compared to other firms and underlying standards, cf. relative greenness and greenness competition), leading to a natural degradation of a firm's recycling rate if it stops investing in its greenness. Relative greenness in the literature is common, but not explicit as usually two products are used and the greenness of one product is normalized to zero (Zhou, 2018; Ülkü and Hsuan, 2017). For example, in Denmark, the operating conditions and requirements on the operation of car-dismantling companies, the firms that generate recycled products, fundamentally changed from the mid-1990s to the mid-2000s (Smink, 2007). From a modelling perspective, the degradation of the recycling rate equates to the degradation/depreciation of our model's state variable, an approach commonly used in the green economics literature (Benchekroun and Long, 2012; Jørgensen et al., 2010), though sometimes challenged in the same literature (El Ouardighi et al., 2014, 2016). Thus, the recycling rate increases with investment and decreases otherwise.

With a general function, we model the relationship between recycling rate investment, $u(t) \geqslant 0$, and the corresponding recycling rate, $r(t) \in [0,1]$. Investment expense in recycling, $u(t)$, and recycling rate, $r(t)$, are control and state variables, respectively. The recycling dynamics reads for all t in $(0,T)$ as:

$$\frac{dr(t)}{dt} = R(u(t), r(t)), \text{ with } r(0) = r_0. \tag{1}$$

We assume the recycling dynamics function, (1), $R : \mathbb{R}^+ \times [0,1] \to \mathbb{R}$ to be twice continuously differentiable.

Integrating (1) relates the (cumulative level of) recycling rate to the flow of current investment in greater recylcing $r(t) = r_0 + \int_0^t R(u(s), r(s))ds$. Proportional depreciation of the state variable in green economics is surveyed by Jørgensen et al. (2010) and Benchekroun and Long (2012). Hereafter and when no confusion exists, we omit notational arguments for simplicity. Especially, we often omit the temporal notation in subsequent equations.

Investment, u, increases the recycling rate, r, with diminishing returns. Also, investment loses its effectiveness over time, translating into autonomous decay, $u \geqslant 0$, $r \in [0,1]$:

$$\frac{\partial R}{\partial u} > 0, \ \frac{\partial^2 R}{\partial u^2} < 0, \ \frac{\partial R}{\partial r} \leqslant 0. \tag{2}$$

A parametric example of the structural formulation (1) together with (2) is $\frac{dr}{dt} = \gamma u^{\frac{1}{\gamma}}(1-r) - \delta r$, $r(0) = r_0$ in $[0,1]$, in which the recycling rate increases with the efficiency of the investment, $\gamma > 1$, and depreciates at a constant proportional rate, $\delta > 0$. Its dynamics are similar to the Vidale and Wolfe (1957) model with $\frac{dx}{dt} = \gamma u(1-x) - \delta x$, where the change of the rate of sales depends on the advertising effort u and the fractional market potential x, $x \in [0,1]$. See also Sethi (1973) and Sethi et al. (2008) with $\frac{dx}{dt} = \gamma u(1-x)^{0.5} - \delta x$, where a diminishing impact of the effort u is reflected by quadratic costs.

3.2 Demand

We look now at the demand of consumers. The majority of related literature use a linear demand function of price and product greenness such as, $p \geqslant 0$, $r \in [0,1]$,

$$D = a_0 - a_1 p + a_2 r, \tag{3}$$

with $a_0 > 0$ the market potential, $a_1 > 0$ the sensitivity of demand to price, and $a_2 > 0$ the sensitivity of demand to product greenness (Dai and Zhang, 2017; Saha et al., 2017; Zhang et al., 2017). We now generalize the parametric demand function to a structural demand function.

The price $p \geqslant 0$ is a control variable. The price does not influence a state variable, making price a *static* control variable. The (current) demand function $D : \mathbb{R}^+ \times [0,1] \to \mathbb{R}^+$ is twice continuously differentiable. The demand, D, depends on the price, p, and recycling rate, r, which is a proxy for the greenness of the product. Consumers prefer products with higher recycling rates, meaning consumers value the environment and purchase products that reflect their values. Consequently, investment indirectly affects future demand, via the recycling rate. Formally, we write the demand as, $p \geqslant 0$, $r \in [0,1]$,

$$D = D(p, r). \tag{4}$$

The *direct price effect on demand*, the *direct recycling effect on demand*, and the *cross effect of price and recycling on demand* are given by $\frac{\partial D}{\partial p}$, $\frac{\partial D}{\partial r}$, and $\frac{\partial^2 D}{\partial p \partial r}$, respectively.

Aligned with previous research, demand decreases with the price and increases with product greenness, recycling rate in our case (Eurobarometer, 2013; Laroche et al., 2001; Nielsen, 2014). Further, customers are marginally less sensitive to price with greener products. (See Chenavaz and Jasimuddin (2017) and Masoudi and Zaccour (2018) for a similar interpretation of the cross-derivative assumption.). We account for the three demand assumptions above with, $p \geqslant 0$, $r \in [0,1]$,

$$\frac{\partial D}{\partial p} < 0, \ \frac{\partial D}{\partial r} \geqslant 0, \frac{\partial^2 D}{\partial p \partial r} \leqslant 0. \tag{5}$$

To repeat, in this model, product greenness is proxied by the recycling rate, r. Also, the demand function is assumed not to be "too" convex in the price, $p \geqslant 0$, $r \in [0,1]$,

$$2 - D \frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2} > 0. \tag{6}$$

This assumption is technical and useful for the maximization of profit; it guarantees a unique maximum of the profit function, which will be defined later. Such an assumption of demand convexity is popular in dynamic pricing literature using structural demand functions. It is used for instance in Chenavaz (2012, 2017); Dockner et al. (2000); Jørgensen and Zaccour (2012); Ni and Li (2018); Vörös (2006, 2019). Assumptions (5) and (6) are satisfied with the linear demand function (3) and the Cobb-Douglas (iso-elastic) demand function $D = a_0 p^{-a_1} r^{a_2}$ with $a_0, a_1, a_2 > 0$.

3.3 Production Cost

Unit production cost of virgin resources $c_v \geqslant 0$ and unit production cost of recycled ressources $c_r \geqslant 0$. We assume that virgin resources are more costly than recycled material, $c_v > c_r$. This assumption characterizes, for instance, the case of carbon fiber (Hendriks and Janssen, 2003; Ishak, 2019) and of wood-plastic composites (Keskisaari and Kärki, 2018). That's also the case when virgin resources are taxed by administrative authorities to make it more expensive than recycled material (Söderholm, 2011). Plus, recycling provides an alternative source of material, when few virgin resources are available (Raz and Souza, 2018). For completeness, we explore numerically the alternative case of recycled resources being more expensive than virgin resources, $c_r \geqslant c_v$, in Section 6; numerical experiments show that the results hold.

The firm uses a mixture of recycled and virgin resources to make a product. The fraction of recycled material used is the recycling rate, r, and the corresponding fraction of virgin resources used is $1-r$. The unit cost is therefore the weighted average $rc_r + (1-r)c_v$. As the that virgin resources are more expensive than recycled one, $c_v > c_r$, we normalize $c_r = 0$. Consequently, the unit production cost simplifies to $(1-r)c_v$.

3.4 Profit

The current profit function $\pi : \mathbb{R}^+ \times \mathbb{R}^+ \times [0,1] \to \mathbb{R}$ is assumed twice continuously differentiable. The profit per unit is the retail price minus the cost per unit, $p - (1-r)c_v$. The firm sets the recycling investment, u, before demand is realized, and thus is a fixed cost. Putting the fixed and variable costs together, we obtain the profit function, revenues less costs, $p,\ u \geqslant 0$, $r \in [0,1]$,

$$\pi(p(t), u(t), r(t)) = [p(t) - (1 - r(t))c_v] \cdot D(p(t), r(t)) - u(t). \tag{7}$$

3.5 Firm's Optimization Problem

The firm maximizes the intertemporal profit (or present value of the profit stream) over the planning horizon, by simultaneously choosing the investment in recycling and pricing policies, accounting for the dynamics of the recycling rate. For simplicity, the salvage value of the recycling rate is null. The interest rate is $\alpha \in \mathbb{R}^+$ and the objective function of the firm is:

$$\max_{u(\cdot),\, p(\cdot) \geqslant 0} \int_0^T e^{-\alpha t} \pi(p(t), u(t), r(t)) dt, \tag{8a}$$

$$\text{subject to } \frac{dr(t)}{dt} = R(u(t), r(t)), \text{ with } r(0) = r_0. \tag{8b}$$

4 Model Analysis

We examine here the analytical details of the optimization problem and dynamics proposed in the preceding section. Before our analysis, we present the conditions derived from solving mathematical program (8). Additional details can be found in part 2 of Kamien and Schwartz (1991).[1] In the dynamic setting, with continuous-time, t, there is a potential unique value of the co-state variable $\lambda(t)$ (the counterpart of the Lagrange multipliers in the dynamic setting) for each time t. The Hamiltonian, H, of (8) with the current-value adjoint variable (or shadow price) $\lambda(t)$ for recycling dynamics is:[2]

$$H(p, u, r, \lambda) = [p - (1 - r)c_v]D(p, r) - u + \lambda R(u, r). \tag{9}$$

The Hamiltonian, H, measures the intertemporal profit, summing the current profit, $[p-(1-r)c_v]D(p,r)-u$, and the future profit, λR. We confine our interest to interior solutions for u and p, assuming their existence. The Hamiltonian, H, is assumed strictly concave in investment, u, and price, p. It immediately follows that all optimal decisions must satisfy the first- and second-order conditions of the Hamiltonian, equations (10a)–(10e). Plus, following the maximum principle, we derive equation (10f). Note that all conditions are for $t \in (0, T)$.

[1] Note that static optimization problem is solved via Lagrangian and associated Lagrange multipliers, λ. Similarly, dynamic optimization problems are solved using Hamiltonian and associated co-state variable, $\lambda(t)$.

[2] The Hamiltonian explicitly written with time, t, is:

$$H(p(t), u(t), r(t), \lambda(t)) = [p(t) - (1 - r)c_v]D(p(t), r(t)) - u(t) + \lambda(t)R(u(t), r(t)).$$

Hence, we obtain

$$
\begin{aligned}
\frac{\partial H}{\partial u} = 0 \quad &\Longrightarrow \quad \frac{\partial R}{\partial u} = \frac{1}{\lambda}, && \text{(10a)}\\
\frac{\partial H}{\partial p} = 0 \quad &\Longrightarrow \quad p - (1-r)c_v = -\frac{D}{\frac{\partial D}{\partial p}}, && \text{(10b)}\\
\frac{\partial^2 H}{\partial u^2} < 0 \quad &\Longrightarrow \quad \lambda \frac{\partial^2 R}{\partial u^2} < 0, && \text{(10c)}\\
\frac{\partial^2 H}{\partial p^2} < 0 \quad &\Longrightarrow \quad 2 - D\frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2} > 0, && \text{(10d)}\\
\frac{\partial^2 H}{\partial u^2}\frac{\partial^2 H}{\partial p^2} - \left(\frac{\partial^2 H}{\partial u \partial p}\right)^2 > 0 \quad &\Longrightarrow \quad -\lambda \frac{\partial^2 R}{\partial u^2}\left(2 - D\frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2}\right) > 0, && \text{(10e)}\\
\frac{d\lambda}{dt} = \alpha\lambda - \frac{\partial H}{\partial r} \quad &\Longrightarrow \quad \frac{d\lambda}{dt} = \left(\alpha - \frac{\partial R}{\partial r}\right)\lambda - c_v D - (p - (1-r)c_v)\frac{\partial D}{\partial r}, && \text{(10f)}
\end{aligned}
$$

with the transversality condition $\lambda(T) = 0$.

5 Analytic Results

We are interested in both the recycling investment and pricing policies over time. Before that, we compute the value of λ.

5.1 Value of λ

The shadow price of the recycling rate, λ, represents the impact on future profits of a small increase in the recycling rate. As such, λ captures the intertemporal trade-off between current and future profits and warrants further investigation. Let the recycling rate and price elasticities of demand be $\eta_r = \frac{\partial D}{\partial r}\frac{r}{D}$ and $\eta_p = -\frac{\partial D}{\partial p}\frac{p}{D}$. Substituting η_r, η_p, and (10b) in (10f) implies for all t in $(0, T)$

$$
\frac{d\lambda}{dt} - \left(\alpha - \frac{\partial R}{\partial r}\right)\lambda = -D\left(\frac{\eta_r p}{\eta_p r} + c_v\right), \text{ with } \lambda(T) = 0, \tag{11}
$$

which provides the following lemma:

Lemma 1. *The value of λ over time writes for all t in $[0, T]$*

$$
\lambda(t) = \int_t^T e^{-\left(\alpha - \int_{s-t}^T \frac{\partial R}{\partial r} d\mu\right)(s-t)} D\left(\frac{\eta_r p}{\eta_p r} + c_v\right) ds.
$$

Proof. See Appendix B. □

The shadow price of recycling rate, λ, increases with demand, D, with the markup, $\frac{\eta_r p}{\eta_p r}$, and the product of cost of virgin resources, c_v. In a nutshell, when the recycling rate increases, the intertemporal profit raises because of greater profitable opportunities.

Lemma 2. *The sign of* λ *over time for all* t *in* $(0, T)$ *is given by*

$$\lambda(t) > 0.$$

Proof. Immediate from Lemma 1, noting that D, $\frac{\eta_r p}{\eta_p r}$, $c_v > 0$. □

Consequently, a greater recycling rate always increases the intertemporal profit for three reasons. It pleases consumers, which accepts 1) to buy more (D) and 2) to pay more ($\frac{\eta_r p}{\eta_p r}$); 3) the unit production cost also decreases (c_v).

5.2 Dynamics of Investment in Recycling, $u(t)$

We find that the investment in recycling tends to zero, monotonically, after some time threshold t_1. This result follows from analysis similar to that carried out by Chenavaz (2012, 2017); Ni and Li (2018). We recreate the analysis for completeness in Appendix C. The numerical experiment illustrates the decline of investment, u, at the end of the planning period in Figure 1(b).

Also, investment u may be monotonic or cyclical, independently from the pricing policy, and the formulation of D. The dynamics of u and p are independent. Eventually, starting from some point in time, t_1, investment u decreases over time. Yet, before t_1, all kinds of policies are possible for u. There is much freedom for setting u over time. Yet, there is a strong link between the dynamics of r and p, as we will see in the next subsection.

5.3 Dynamics of Price, $p(t)$, and Rate of Recycling, $r(t)$

In this section we determine the relationship between price and recycling. As demand is a function of both price and recycling the result in this section will allow us to examine demand only concerning recycling. We start with the first-order condition (10b) which holds for all optimal values. We differentiate both sides of the condition with respect to time, t, while accounting for the definitions of cost per product, $c_v(1-r) = c_v(1-r(t))$, and demand, $D = D(p(t), r(t))$:

$$\frac{dp}{dt} = \frac{d}{dt}\left(c_v(1-r) - \frac{D}{\frac{\partial D}{\partial p}}\right) \implies \frac{dp}{dt} = -c_v\frac{dr}{dt} - \frac{(\frac{\partial D}{\partial p}\frac{dp}{dt} + \frac{\partial D}{\partial r}\frac{dr}{dt})\frac{\partial D}{\partial p} - D(\frac{\partial^2 D}{\partial p^2}\frac{dp}{dt} + \frac{\partial^2 D}{\partial p \partial r}\frac{dr}{dt})}{\frac{\partial D}{\partial p}^2}. \tag{12}$$

Note $-\frac{\frac{\partial D}{\partial p}\frac{\partial D}{\partial r}}{\frac{\partial D}{\partial p}^2} = \frac{\eta_r}{\eta_p}\frac{p}{r}$, which we call the markup effect, and we define the sales effect as $D\frac{\frac{\partial^2 D}{\partial p \partial r}}{\frac{\partial D}{\partial p}^2}$.

Proposition 1. *With a general demand function, $D = D(p, r)$, the dynamics of price and recycling are associated as:*

$$\frac{dp}{dt}\left(2 - D\frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}^2}\right) = \frac{dr}{dt}\left(-c_v + \frac{\eta_r}{\eta_p}\frac{p}{r} + D\frac{\frac{\partial^2 D}{\partial p \partial r}}{\frac{\partial D}{\partial p}^2}\right).$$

Proof. Substitute the definition of the markup effect into equation (12) and rearrange. □

Proposition 1 associates price dynamics $\frac{dp}{dt}$ to recycling dynamics $\frac{dr}{dt}$, for a general demand function $D(p, r)$ joint in price and recycling rate. Proposition 1 is based on the sole first-order condition on price (10b). It does not dependent on the first-order condition for investment (10a) and the dynamics of recycling (1). The price, p, may increase or decrease with recycling, r, depending on the relative strength of the cost, markup, and sales effects.[3]

Definition 1. *We define hereafter the three main effects at play in Proposition 1.*

- *The* cost effect, *$-c_v$, measures the impact of higher recycling rate on unit production cost, see Subsection 3.3. A greater recycling rate allows to use less virgin resources, reducing the cost. The cost effect is negative.*
- *The* markup effect, *$\frac{\eta_r}{\eta_p}\frac{p}{r}$, linked to (5), computes the higher consumers' willingness to pay for products with higher recycling rates. The markup effect is positive.*
- *The* sales effect, *$D\frac{\frac{\partial^2 D}{\partial p \partial r}}{\frac{\partial D}{\partial p}^2}$, associated to (5), examines the change in sales after a larger price together with more recycling. Demand increases with recycling, and it increases, even more, when the price is lower. The sales effect is negative.*

The markup effect is the sole cause of any positive relationship between recycling and price. Indeed, the cost and sales effects can only lead to a negative relationship. More precisely, price decreases with greater recycling when the cost and sales effects outweigh the markup effect. On the contrary, price increases with recycling when the markup effect is greater than the combined cost and sales effects.

[3] Similar effects at the demand- and supply-sides are discussed in Chenavaz (2012, 2017); Chenavaz and Jasimuddin (2017); Ni and Li (2018).

If consumers are not aware of the recycling policy of the firm or if they are not sensitive to green products, that is $\frac{\partial D}{\partial r} = 0$, then the demand-side effects do not play a role (markup and sales effects are zero), and price decreases with recycling because of the negative supply-side effect.

Subclasses of Demand Functions

So far, we analyzed a general demand function. We examine here two subclasses of demand functions, namely additively and multiplicatively separable, and present the associated relationship between price and recycling. We suppose that the price effect on demand is modeled with $h = h(p)$ and $h'(p) < 0$; the recycling effect on demand by $l = l(r)$ and $l'(r) \geqslant 0$. These assumptions are aligned with equations (4) and (5).

Corollary 1. *With an additive separable demand function, $D = h(p) + l(r)$, the dynamics of price and recycling are:*

$$\frac{dp}{dt}\left(2 - D\frac{h''}{(h')^2}\right) = \frac{dr}{dt}\left(- c_v + \frac{\eta_r}{\eta_p}\frac{p}{r}\right).$$

Proof. Substitute $D = h(p) + l(r)$ in Proposition 1. □

With a demand function additively separable, the sales effect is zero. Consequently, the price, p, may increase or decrease with recycling, r, depending on the relative strength of the markups and sales effects.

Example 1. *With $D = a_0 - a_1 p + a_2 r$, Corollary 1 yields*

$$\frac{dp}{dt} = \frac{dr}{dt}\frac{1}{2}\left(-c_v + \frac{a_2}{a_1}\right)$$

and according to Lemma 1, λ reads, $0 \leq t \leq T$,

$$\lambda(t) = \int_t^T e^{-(\alpha - a_2)(s-t)} D\left(\frac{a_2}{a_1} + c_v r\right) ds.$$

Price increases with recycling if and only if the ratio of demand sensitivity to recycling over demand sensitivity to price, $\frac{a_2}{a_1}$, is greater than the unit cost of the virgin resource, c_v. Consequently, if consumers are not green, that is $a_2 = 0$, then price decreases with recycling because the markup effect vanishes. Also, when consumers are more recycling- than price-sensitive, the ratio $\frac{a_2}{a_1}$ is positive and arbitrarily large, and the price increases with the recycling rate. In this situation, consumers are willing to pay more for slightly greener products.

Corollary 2. *With a multiplicative separable demand function, $D = h(p)l(r)$, the dynamics of price and recycling are associated as:*

$$\frac{dp}{dt}\left(2 - h\frac{h''}{(h')^2}\right) = \frac{dr}{dt}\left(- c_v\right).$$

Proof. Substitute $D = h(p)l(r)$ in Proposition 1. □

With a multiplicative separable demand function, the markup and sales effects both play a role of same magnitude. Because they exert influence in opposite directions, they cancel each other out. Only remains the cost effect, which is negative. Consequently, price always decreases when the recycling rate increases.

Example 2. *With the Cobb-Douglas (isoelastic) demand function, $D = a_0 p^{-a_1} r^{a_2}$, with $a_0 > 0$, $a_1 > 1$, and $a_2 \geqslant 0$, Corollary 2 yields*

$$\frac{dp}{dt} = \frac{dr}{dt}\left(-\frac{a_2}{a_2+1}c_v\right).$$

The demand sensitivity to price, a_1, plays no role in the relationship between the dynamics of recycling and price. Also, if consumers are not sensitive to product greenness, $a_2 = 0$, then the dynamics of the price are independent of the dynamics of recycling, and the price remains constant over time, as $\frac{dp}{dt} = 0$.

6 Numerical Evaluation

To study how optimally controlled prices and recycling investments evolve, in this section, we provide numerical examples of the model analyzed in the previous sections. In Section 6.1, we describe a discrete-time version of the model, which is used to approximate the solutions of the continuous-time problem discussed in the previous section. Numerical examples are presented in Section 6.2.

6.1 Solution of the Discrete-Time Model

Due to the complexity of the model, an explicit solution is unlikely to be tractable. To numerically approximate the model's solution with time horizon T, T in $\mathbb{N}$, we use a discrete-time version of the model with periods of length h (with $h < 1$ and $1/h$ in $\mathbb{N}$), which in total leads to T/h periods. For each period, the price, p, and the recycling investment, u, have to be chosen from a (discrete) set of admissible prices P and a set of admissible investments U, respectively. For the length of one period (of size one), we use the discount factor $\beta = e^{-\alpha}$. Note, the case of no discounting, i.e., $\alpha = 0$, corresponds to $\beta = 1$ and that the case of discounting, i.e., $\alpha > 0$, refers to $0 \leqslant \beta < 1$.

The current recycling rate r characterizes the state of our Markov decision process and follows the dynamics described in Section 3. The initial recycling rate is r_0, and subsequent rates (states) are $r(t)$ for time t. Just as in the continuous-time setting, price and recycling investment are a function of time, t, and the current recycling rate. For given price and investment decisions (controls) the discounted future profits $G(t)$ from time t on (discounted on time t), $t = 0, h, 2h, ..., T$, are defined by, cp. (7) and (8a),

$$G(t) = \sum_{s=t,t+h,...,T-h} \beta^{h(s-t)} \cdot h \cdot \pi\left(p\left(s, r(s)\right), u\left(s, r(s)\right), r(s)\right).$$

The best possible profits $G(t)$ optimized over all state-dependent Markovian feedback policies (p and u), see (8), are represented by the so-called value function $V(t, r)$, $t = 0, h, 2h, ..., T$, $r \in [0, 1]$, which is determined by the terminal condition $V(T, r) = 0$, $r \in [0, 1]$, and the Bellman equation, $r \in [0, 1]$, $t = 0, h, 2h, ..., T - h$,

$$V(t, r) = \max_{p \in P, u \in U} \left\{ h \cdot \pi(p, u, r) + \beta^h V\left(t + h, r + h \cdot R(u, r)\right) \right\}. \tag{13}$$

As (13) can be recursively computed, we are able to identify (approximated) optimal feedback controls, i.e., prices $p(t, r)$ and recycling investment rates $u(t, r)$, $r \in [0, 1]$, $t = 0, h, 2h, ..., T - h$, which are given by the arg max of (13), i.e., the prices and investments that maximize (13); they depend on time t and the current recycling rate r. In case the arg max of optimal controls are not unique, we choose, e.g., the combination with the largest price and the largest investment, respectively.

6.2 Numerical Examples

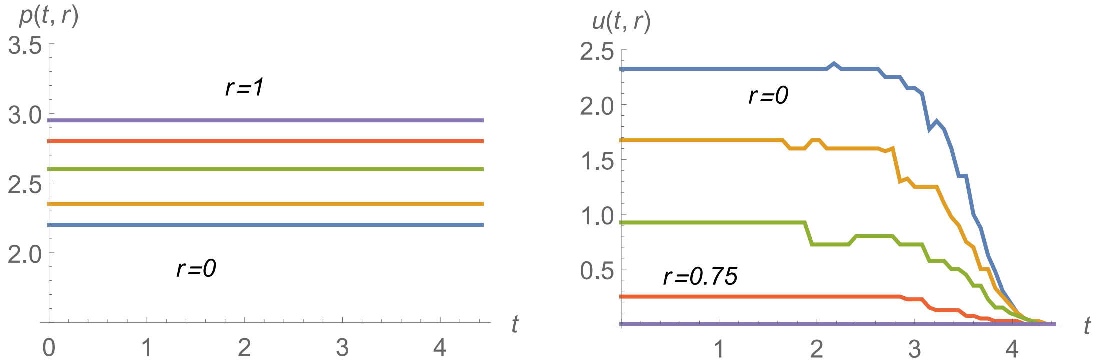
To exemplify the study of the evolution of an optimal solution, we consider the following reproducible numerical example with linear demand, cf. Section 5.3, Example 1.

Example 3. *As a reference case, (if not chosen differently) we let* $T = 4.5$, $h = 0.05$, $\beta = 1$, $r_0 = 0.2$, $c_v = 1$, *and* $c_r = 0$. *We consider the linear demand function* $D(p, r) = max(0, 1 - 0.3p + 0.8r)$, *see Example 1, and the dynamic* $R(u, r) = \gamma u^{\frac{1}{\gamma}}(1 - r) - \delta r$, *where* $\delta = 0.4$, *and* $\gamma = 1.5$. *The profit function* $\pi(p, u, r)$, *cf.* (13), *is defined as in* (7), $p \geqslant 0$, $u \geqslant 0$, $r \in [0, 1]$. *Further, we consider the discretized state space, cf.* $r \in \{0, 0.00025, 0.0005, ..., 1\}$, *and the control spaces, cf.* p *in* $P := \{0, 0.05, 0.1, ..., 5\}$, u *in* $U := \{0, 0.025, 0.05, ..., 5\}$, *to be discretized with arbitrary but suitably chosen granularities.*

In the example, the dynamic R of the recycling rate r is motivated by classical, commonly used functional forms as, for instance, used in the Vidale and Wolfe (1957) model or the Sethi model, see, e.g., Sethi et al. (2008), where the change of the sales rate is modelled through the advertising effort u and the fractional market potential x, $x \in [0, 1]$, via $\frac{dx}{dt} = \gamma u (1 - x)^{0.5} - \delta x$. While in the Sethi model quadratic costs in u are used to reflect a diminishing impact in the (linear) advertising effort, in our model, we use linear costs and a parameter $\gamma > 1$ to model a diminishing effectiveness in the recycling investment effort u. A corresponding variable transformation of u in a similar framework is discussed in Helmes et al. (2013). To model the decay of the recycling rate we used the same linear dynamic.

6.2.1 Feedback Policies

For our reference example, the optimal feedback prices $p(t, r)$ and investment efforts $u(t, r)$ are displayed in Figure 1, which shows that while feedback prices increase with the current recycling rate r for all t, the investment efforts $u(t, r)$ decrease in r. Further, we observe that although the feedback price is time-independent, the investment effort decreases within time t at the end of the planning horizon. Such result conforms the analytic result from Subsection 5.2. As the investment effort is coupled with the evolution of recycling, which affects future profits, the optimal investments to maximize intertemporal profits depend both on time and recycling rate. Instead, the price is decoupled from the evolution of the state and affects neither the recycling rate nor future profits. Thus, the optimal price, the one maximizing intertemporal profits in a given state, is identical to the one maximizing current profits, which in the case of time-homogeneous demand will not depend on time.



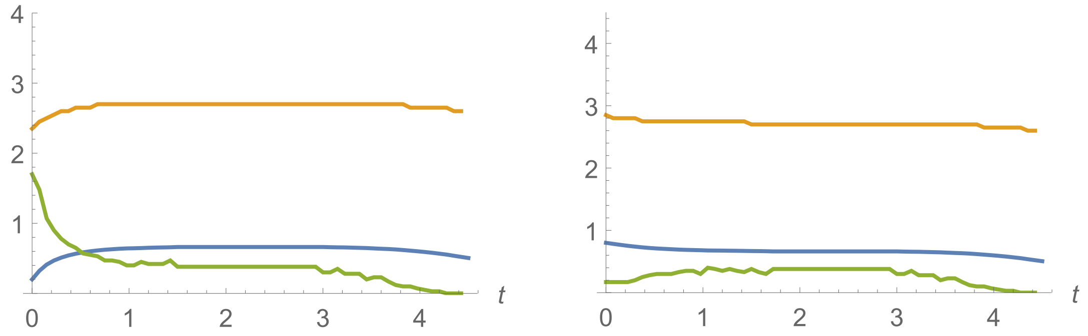
(a) Feedback store price $p(t, r)$

(b) Feedback recycling investment $u(t, r)$

Figure 1: Illustration of feedback prices $p(t, r)$ and recycling investment rates $u(t, r)$ assuming different recycling rates $r = 0$ (blue), $r = 0.25$ (orange), $r = 0.5$ (green), $r = 0.75$ (red), and $r = 1$ (purple) at time t, $0 \leqslant t \leqslant 4.5$; Example 3 for the reference case with $\delta = 0.4$, $c_v = 1$, $c_r = 0$, $\gamma = 1.5$, $\beta = 1$.

6.2.2 Evolution and Sensitivity Analysis

Further, we evaluate the optimal feedback prices and investment efforts of Example 3 to obtain their associated evolution over time (cf. open-loop controls). Figure 2 illustrates prices p, investments u, and the recycling rate r over time for different initial recycling rates r_0 at time $t = 0$, i.e., $r_0 = 0.2$ and $r_0 = 0.8$. We observe that the evolution of both processes only differs in the beginning and then coincide



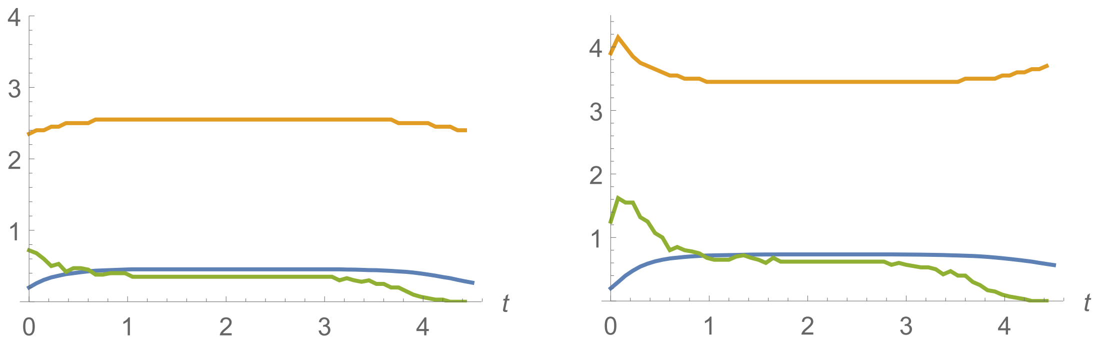
(a) Reference solution with initial rate r_0=0.2, ($\delta = 0.4$, $c_v = 1$, $c_r = 0$, $\gamma = 1.5$, $\beta = 1$) and steady-state: r^*=0.662, p^*=2.7, u^*=0.38

(b) Recycling rate $r_0 = 0.8$ (vs. $r_0 = 0.2$), steady-state: r^*=0.662, p^*=2.7, u^*=0.38

Figure 2: Evaluation over time: price $p(t)$ (orange), investments $u(t)$ (green), and rates $r(t)$ (blue) for the reference case of Example 3 (with $r_0 = 0.2$) vs. a higher initial recycling rate $r_0 = 0.8$, $0 \leqslant t \leqslant 4.5$.

(at a recycling rate level of about 0.662). If the initial recycling rate is comparably small ($r_0 = 0.2$) decreasing investments and increasing prices are used to reach the advantageous recycling level of 0.662. If instead, the initial recycling rate is large ($r_0 = 0.8$) no investments are taken until the recycling level of 0.662 is attained due to the depreciation effect (cf. $\delta > 0$). After a period of stable controls at the end of the time horizon in both cases for r_0 no more investments are taken and the recycling rate decreases.

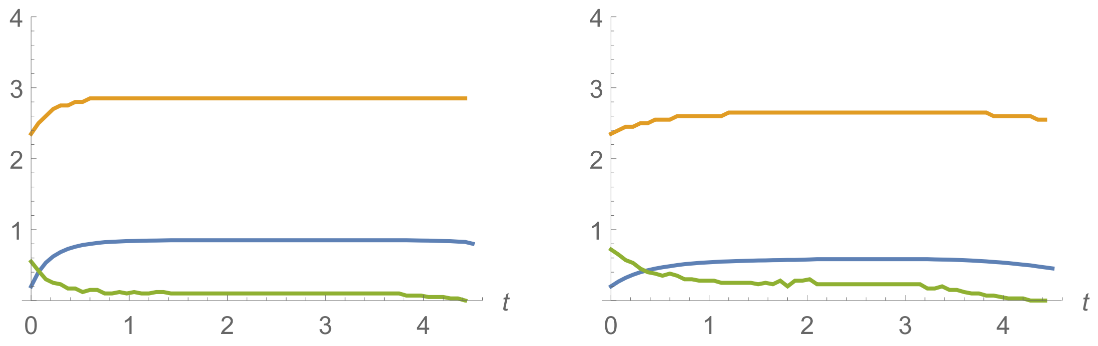
Further, Figures 3 and 4 illustrate how the solution paths of the reference case in Example 3, cf. Figure 2(a), are affected if other model parameters change. Figure 3(a) illustrates the impact of δ. A higher depreciation rate, δ, leads to fewer investment efforts and lower recycling rates in steady-state; prices are smaller but overall hardly affected. Figure 3(b) shows that higher virgin unit cost c_v leads to higher investment efforts and higher recycling rates in steady-state; compared to Figure 3(a) the price path is now of inverse shape (u-shape). Hence, the opportunity to invest in a larger recycling share can even result in decreasing prices. This is the case if the current recycling rate is small and virgin resource



(a) Depreciation factor $\delta = 0.9$ (vs. $\delta = 0.4$), steady-state: r^*=0.454, p^*=2.55, u^*=0.35

(b) Virgin unit costs $c_v = 6$ (vs. $c_v = 1$), steady-state: r^*=0.734, p^*=3.45, u^*=0.62

Figure 3: Impact of depreciation factor δ and virgin unit costs c_v: price $p(t)$ (orange), investments $u(t)$ (green), and recycling rates $r(t)$ (blue) over time, $0 \leqslant t \leqslant 4.5$; Example 3.



(a) Investment effectiveness $\gamma = 4$ (vs. γ=1.5), steady-state: r^*=0.85, p^*=2.85, u^*=0.1

(b) Discount rate $\beta = 0.5$ (vs. $\beta = 1$), steady-state: r^*=0.583, p^*=2.65, u^*=0.23

Figure 4: Impact of investment effectiveness γ and discount rate β: price $p(t)$ (orange), investments $u(t)$ (green), and recycling rates $r(t)$ (blue), over time $0 \leqslant t \leqslant 4.5$; Example 3.

cost, c_v, are sufficiently more expensive than the recycling unit costs c_r.

Figure 4(a) depicts that higher investment effectiveness γ leads to fewer investment efforts and higher recycling rates in steady-state; the price path is hardly affected. Figure 4(b) illustrates that higher discounting (i.e., a lower discount factor β) leads to fewer investment efforts and lower recycling rates in steady-state; prices are slightly smaller. We summarize our observations in the following two remarks.

Remark 1. *The results reveal that the optimal evolution of the model consists of the following three phases:*

- *(i) In the first phase, the controls are used to change the initial state (cf. initial recycling rate r_0) towards a certain steady-state.*
- *(ii) The second phase is characterized by the steady-state, in which all prices and recycling investment rates remain constant.*
- *(iii) The third phase, which starts shortly before the end of the time horizon, describes a decrease in price, investment, and recycling rate. Note that the third phase is expected as analytically determined in Section 5.2.*

Remark 2. *The numerical examples reveal the following sensitivity results (see Figures 2 - 4):*

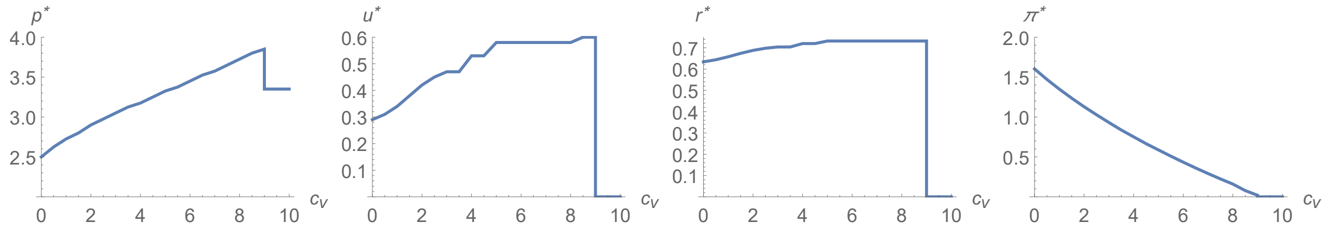
- *The initial recycling rate, r_0, only affects the first phase, cf. Remark 1 (i).*
- *A higher depreciation rate, δ, and a higher discount rate, β, lead to lower recycling rates.*
- *High virgin resource cost, c_v, can lead to prices that are decreasing over time.*
- *With higher investment effectiveness, γ, higher recycling rates can be obtained with lower investments.*

6.2.3 Steady-State

The discrete-time finite horizon model, cf. Section 6.1, allows deriving the solution of the discounted infinite horizon model by approximate dynamic programming techniques (cf. value iteration). Note, the solution of the infinite horizon is characterized by (the transition to) the steady-state, cf. phase (i) and (ii). Hence, the steady-state of the discrete-time model serves as an approximation of the steady-state of the continuous-time model, cf. Sections 3 - 5, with infinite horizon, cf. (8) for $T = \infty$. Because we do not consider time-dependent model parameters, the (feedback) solution of this model will also not depend on time, and the only state is the current recycling rate r.

In this context, let $p^*(r)$ and $u^*(r)$, $r \geq 0$, denote the feedback controls of the continuous-time infinite horizon model. The steady-state is characterized by the (constant) recycling rate r^* that does not change due to time-dynamics, i.e., r^* satisfies $R(u^*(r^*), r^*) = 0$, cf. (8b). Note, the steady-state will not depend on r_0, cf. Figure 2.

The steady-state is, however, affected by δ, γ, (i.e., R), D, β, and most importantly, the unit cost c_v. In the context of Example 3, Figure 5(a)-(b) illustrate the price and the recycling investment rates in steady-state as a function of c_v. We observe that – as long as the unit cost c_v are not too large – the steady-state controls, i.e., price p^* and investment effort u^* increase with c_v. The equilibrium recycling rate r^*, cf. Figure 5(c), is (slightly) increasing with c_v. Further, the demand (i.e., the amount of sales) and the profit rate π^* is decreasing with c_v, cf. Figure 5(d).



(a) price p^* (b) investments u^* (c) rate r^* (d) profit π^*

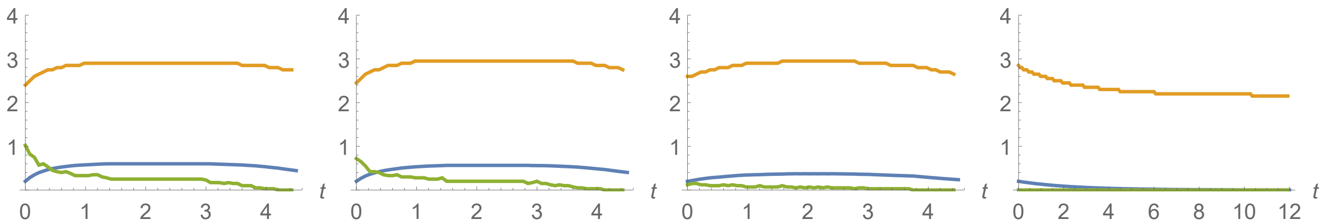
Figure 5: Steady-state solutions for different unit costs c_v: price (a), investments (b), recycling rate (c), and profits (d), $0 \leqslant c_v \leqslant 10$ (vs. $c_r = 0$); Example 3.

Finally, if c_v is sufficiently large the steady-state profits π^* decrease to zero and in turn, the equilibrium collapses, i.e., we obtain $u^* = r^* = 0$ (zero investments) and a sufficiently high price p^* leads to $D^* = 0$ (zero demand) and $\pi^* = 0$ (zero profits). The reason is the following. Although recycled resource is way cheaper, the share of $1 - r^*$ for virgin resource still leads to high unit costs and in turn, to high prices. On the other hand, pushing and keeping the recycling rate r^* close to one is increasingly costly.

The numerical insights support the analytic results and provide a more comprehensive understanding of the solution of the model. In addition, it provides a viable approach to further study the model from a qualitative as well as a quantitative perspective.

6.2.4 Different Unit Cost Structures

As a robustness check, we now evaluate cases with $c_r > 0$ and, in particular, $c_r > c_v$, i.e., when the cost of recycled material exceeds those of virgin resources. For the setting of Example 3, in Figure 6 we illustrate how the optimal solution of the model is affected by the cost parameter c_r (cf. $c_r = 0.8, 1.2, 2.5, 5$, where $c_v = 1$). In the considered setting, we observe that – even if $c_r > c_v$ – the structure of solutions characterized by three phases (cf. Remark 1) remains similar to the one derived for $c_r = 0$, see the reference case of Figure 2(a) with r^*=0.662, p^*=2.7, and u^*=0.38. We find that the impact of c_r is as follows: The higher c_r the lower are the investments and the recycling rate (in steady-state). Prices are higher in steady-state. If c_r is sufficiently large, cf. Figure 6(d) with c_r=5, an investment does not pay-off anymore and the initial recycling rate decreases to zero, which is the steady-state; the price decreases to $p^* = 2.15$ (cf. phase (i)), which optimizes π for $r = u = 0$, cf. (7).



(a) Costs $c_r = 0.8$, with: r^*=0.6, p^*=2.9, u^*=0.25
(b) Costs $c_r = 1.2$, with: r^*=0.564, p^*=2.95, u^*=0.2
(c) Costs $c_r = 2.5$, with: r^*=0.372, p^*=2.95, u^*=0.06
(d) Costs $c_r = 5$, with: r^*=0.0, p^*=2.15, u^*=0.0

Figure 6: Impact of unit costs for recycled material c_r (vs. $c_r = 0$), where virgin costs are $c_v = 1$: price $p(t)$ (orange), investments $u(t)$ (green), and recycling rates $r(t)$ (blue) over time, $0 \leqslant t \leqslant T$; Example 3.

Note, the transition from $c_r < c_v$ to $c_r > c_v$ is not critical. Instead, the structure of the solution (with three phases and positive investments) depends on whether the c_r costs are small enough such that the positive impact of the recycling rate in the demand rate still pays off. Otherwise, we obtain a trivial solution with no recycling investments.

7 Discussion

In this section, we tie the results of Sections 5 and 6. In particular, we explicitly state our theoretical and managerial contributions. By explaining our contributions, we answer the following research questions:

What effects impact recycling policies of a firm? How do the structural properties of the demand function influence the effects at play? What managerial insights can be proposed to the firm? What are the firm's optimal decisions throughout the product life cycle?

7.1 Theoretical Contribution

Theoretically, our work contributes by proposing a structural, non-parametric, model for recycling in the circular economy. A general model allows us to define, structurally, how demand will change with the two decision variables of interest (price and recycling rate). As a consequence, the structural model allowed us to analytically define the three main effects, which accelerate or slow recycling within a firm (see Definition 1). To our knowledge, the analytical definition of these effects is not explicitly stated in the literature. The structural model enables us to identify all situations in which recycling investment is beneficial.

Finally, we provide more specific analytic results for popular subclasses of demand functions, namely additively and multiplicatively separable demand functions, trading-off generality for strength of results. We show, analytically, that simply presupposing a specific demand function, one is eliminating one of the three effects identified in Definition 1. A key contribution to the literature is using a structural model provides insights that cannot be gleaned from parametric models. However, the structural modeling approach does not come with its drawbacks. One main drawback is the complexity of the model, leading us to use a monopoly setting for our analysis.

7.2 Managerial Contribution

The literature echoed in our results, suggests that one of the consequences of the Renault Group listing and touting its business performance metrics (BPMs) related to recyclable products and recoverability of automobiles is that consumer demand and willingness to pay for Renault products increases. More generally, as a firm communicates BPMs (product greenness) to consumers the firm will experience both demand-side and supply-side effects. Namely the three effects listed in Definition 1. The demand-side effects are markup and sales effects. The markup effect captures the consumer's higher willingness to pay for greener products. The sales effect captures the fact demand increases with greenness and the rate of increase decreases with product price. The supply-side effect is captured by the cost effect and captures how unit production cost changes with increased product greenness. Decision makers must consider all three effects when making decisions on improving the greenness of a product. One insight found when considering all three effects is that greener products do not necessarily mean product prices will be higher.

Prices will increase if consumers' willingness to pay increases with product greenness. However, only the markup-effect results in higher prices, in the setting considered in the analytical section of the paper, and other effects may dominate the markup-effect.

In the analytical examination, we assume that the supply-side effect is negative, meaning as greenness increases price decreases. Informally, this means that in the analytical section of our work we assume virgin resources are more expensive than recycled resources ($c_r < c_v$). One of the eventual outcomes of the circular economy is having virgin resources be more expensive than recycled resources. However, that is not currently the case as world economies move from the linear economy to the circular economy. As such, we numerically consider the case when recycled resources are more expensive than virgin resources ($c_r > c_v$). We numerically determine that the results we find in the analytical section, for the case recycled resources are least expensive, hold for the case when recycled resources are most expensive. This, however, is true for the case when the relative magnitudes of virgin and recycled resource costs are close in value. An additional insight from the numerical study is that even if the virgin resource is cheaper than the recycled resource, price may still decrease as product greenness increases, even though in this case the cost-effect suggests higher prices. In particular, this follows from the fact that though both markup and cost effects are negative, i.e., suggesting price should increase, the sales effect may be sufficiently large leading to a decrease in price. However, if virgin resources are sufficiently cheaper than recycled resources, then the firm will not invest in greening the product.

The immediate insights firms and governments can glean from our consideration of different firm behaviors as the relative cost of virgin and recycled resources changes are:

1. A firm is well served to invest in greening a product even when the cost of virgin resources is less than the cost of recycled resources.

2. A firm's rate of investment in greening a product decreases as the cost of recycled resources decreases.

3. A government may not need to incentivize a firm to improve a product's greenness. Instead, a government may want to tax virgin resources, not so they are necessarily more expensive than recycled resources but instead are sufficiently close to in price to recycled resources. Thereafter, the firm will start investing in improving the product's greenness.

Further, considering our numerical results, we may now consider the three phases a firm goes through when making greening decisions. The first phase is the transition phase, from the initial recycling rate. We see that a firm will mutually adjust product price and recycling investments to reach phase 2. There is no clear monotonic relationship between the two, though they must both be adjusted in unison to reach phase 2. Phase 2, in the case of an infinite planning case, is the steady-state of the system, assuming all parameters remain the same. This phase provides the profit-maximizing policy the firm must use. Note that this steady-state will change as the cost of the recycled product decreases and will put the system back in phase 1 before reaching phase 2 again. Phase 3 occurs only with finite time, where the impact of future lost sales is not significant enough to offset the cost of investing in recycling. Naturally, as costs outweigh benefits recycling investments stop. So long as the circular economy is in place and a firm plans on remaining in business, phase 3 will not be observed in practice.

8 Conclusion

In this paper, we consider the interplay between recycling investment, product price, and demand of green consumers within the circular economy. Using a structural model, we identify three effects that determine the level of recycling and price. Namely, the three effects are, cost, markup, and sales effects formally defined in Definition 1. We show that using a non-structural, parametric, approach one may analytically eliminate some effects and only identify a subset of the actual effects. In our analytical results, we show that if the recycled resource used to produce a product is less expensive than the virgin resource, then a firm always has an incentive to invest in recycling. The key insight from our analytical results is the identification and exposition of the interplay between the cost effect, on the supply-side, and the sales and markup effects, on the demand-side, to explain different firm recycling policies. However, it is not clear if the results translated to the case when the recycled resource is more expensive than the virgin resource.

To identify a firm's best course of action in the case the recycled resource is more expensive than the virgin resource we carried out an extensive numerical analysis. Our analysis shows that qualitatively there is no difference in the behavior of the firm between the two cases. An additional benefit of our numerical analysis is we identified how the firm's behavior changes as the relative price of virgin and recycled resource changes. The change of cost between the two resource types is expected within the circular economy. For a detailed discussion of the key takeaways from the numerical analysis please see Sections 6 and 7.2.

Though our work makes substantial contributions to the circular economy and sustainability practices, it leaves many open areas for future work. One future work area is the impact of competition in the considered setting. We show that as the cost of the recycled resource becomes less expensive than the virgin resource, the investment in recycling tends to zero. It is not clear if this behavior is an artifact of the monopoly setting or if this also will happen in a competitive setting. Though we do not explicitly account for strategic consumers, we do consider a general demand function that may subsume some strategic consumer behavior. In the future, it is of interest to understand how/if strategic consumers may impact firms recycling investment and pricing decisions. Finally, we see in the popular literature that governments are using legal and tax frameworks to promote and push industries to operate within the circular economy (Walker et al., 2019). In the future, incorporating a government player is of interest and warrants further consideration.

Appendix

A Notation

Table 1 presents the main notations used in the paper.

Table 1: Main Notations

T	= fixed terminal time of the planning horizon, parameter,
t	= time, continuous,
α	= interest rate, parameter,
$p(t)$	= product price at time t, control variable,
$u(t)$	= investment in recycling at time t, control variable,
$r(t)$	= recycling rate at time t, state variable,
$\frac{dr}{dt}$	$= R(u, r)$ = recycling rate dynamics,
γ	= investment efficiency of the recycling rate, parameter,
δ	= depreciation effect of the recycling rate, parameter,
β	= discount factor (discrete time model), parameter,
$\lambda(t)$	= current-value co-state variable at time t,
$D(p, r)$	= demand,
c_v	= cost of virgin resources,
$c_v(1 - r)$	= unit production cost,
$\pi(p, u, r)$	$= [p - c_v(1 - r)]D(p, r) - u$ = current profit,
$H(p, u, r, \lambda)$	$= \pi + \lambda R$ =current-value Hamiltonian,
$\eta_{D/x}$	$= \left\lvert \frac{\partial D}{\partial x} \frac{x}{D} \right\rvert$ = elasticity of demand with respect to the variable x,
$\eta_{R/u}$	$= \frac{\partial R}{\partial u} \frac{u}{R}$ = elasticity of recycling dynamics with respect to investment.

B Proof of Lemma 1

In this section we present the proofs of some of the results presented in the main text.

Recall the elasticity of recycling rate elasticity of demand $\eta_r = \frac{\partial D}{\partial r} \frac{e}{D}$ and the price elasticity of demand

$\eta_p = -\frac{\partial D}{\partial p}\frac{p}{D}$. Note that $\frac{\frac{\partial D}{\partial r}}{\frac{\partial D}{\partial p}} = -\frac{\eta_r}{\eta_p}\frac{p}{r}$. Substituting η_r, η_p, and (10b) in (10f) implies

$$\frac{d\lambda}{dt} - \left(\alpha - \frac{\partial R}{\partial r}\right)\lambda = -D\left(\frac{\eta_r}{\eta_p}\frac{p}{r} + c_v r\right), \text{ with } \lambda(T) = 0. \tag{14}$$

For simplicity of writing, we abuse the notation using $\int \frac{\partial R}{\partial r} d\mu$ for $\int_{s-t}^{T} \frac{\partial R}{\partial r}(u(\mu), r(\mu))d\mu$. Integrating equation (14) with respect to time provides Lemma 1:

$$\lambda(t) = \int_t^T e^{-(\alpha - \int \frac{\partial R}{\partial r} d\mu)(s-t)} D\left(\frac{\eta_r}{\eta_p}\frac{p}{r} + c_v r\right) ds.$$

Proof. Multiply both sides of (14) by $e^{-(\alpha-\int \frac{\partial R}{\partial r}d\mu)t}$ yields $e^{-(\alpha-\int \frac{\partial R}{\partial r}d\mu)t}\left(\frac{d\lambda}{dt} - (\alpha - \frac{\partial R}{\partial r})\lambda\right) = \frac{d\left(\lambda e^{-(\alpha-\int \frac{\partial R}{\partial r}d\mu)t}\right)}{dt}$
$= e^{-(\alpha-\int \frac{\partial R}{\partial r}d\mu)t}(-D)\left(\frac{\eta_r}{\eta_p}\frac{p}{r} + c_v r\right)$.
Thus, $d\left(\lambda e^{-(\alpha-\int \frac{\partial R}{\partial r}d\mu)t}\right) = e^{-(\alpha-\int \frac{\partial R}{\partial r}d\mu)t}(-D)\left(\frac{\eta_r}{\eta_p}\frac{p}{r} + c_v r\right)dt$.
Therefore, $\int_t^T d\left(\lambda(s)e^{-(\alpha-\int \frac{\partial R}{\partial r}d\mu)s}\right) = \int_t^T e^{-(\alpha-\int \frac{\partial R}{\partial r}d\mu)s}(-D)\left(\frac{\eta_{D/e}}{\eta_p}\frac{p}{e} + c_v r\right)ds$,
and $\lambda(T)e^{-(\alpha-\int \frac{\partial R}{\partial r}d\mu)T} - \lambda(t)e^{-(\alpha-\int \frac{\partial R}{\partial r}d\mu)t} = \int_t^T e^{-(\alpha-\int \frac{\partial R}{\partial r}d\mu)s}(-D)\left(\frac{\eta_r}{\eta_p}\frac{p}{r} + c_v r\right)ds$.
Substituting the transversality condition, $\lambda(T) = 0$, proves the result. □

C Dynamics of Investment in Recycling, $u(t)$, analytical derivation

We find that the firm decreases its investment in recycling as the end of the planning horizon approaches. We find this result by considering the second-order dynamics of demand and recycling investment. We follow here the method and analysis presented by Chenavaz (2012, 2017); Ni and Li (2018). Assume λ is continuously differentiable and the left derivative $\frac{d\lambda(T^-)}{dt} < 0$. Thus, equations (10f) and Lemma 2 ($\lambda(T) = 0$ $\lambda(t) > 0$, for all t in $[0, T)$) hold, then there exists a least one t_1 in $[0, T)$ such that $\frac{d\lambda(t)}{dt} < 0$, for all t in $[t_1, T)$. The result states that λ declines after time t_1.

Moreover, according to (10a), $\frac{d}{dt}(\frac{\partial R}{\partial u}) = \frac{d}{dt}\left(\frac{1}{\lambda}\right) = -\frac{\frac{d\lambda}{dt}}{\lambda^2}$. So, sgn $\left(\frac{d}{dt}(\frac{\partial R}{\partial u})\right) = -$ sgn $\frac{d\lambda}{dt}$, and for all t in $[t_1, T)$, $\frac{d}{dt}(\frac{\partial R}{\partial u}) > 0$. Note that $\frac{d}{dt}(\frac{\partial R}{\partial u}) = \frac{\partial^2 R}{\partial u^2}\frac{du}{dt} + \frac{\partial^2 R}{\partial u \partial r}\frac{dr}{dt}$. On the one hand, according to (2), $\frac{\partial^2 R}{\partial u^2} > 0$. On the other hand, the sign of $\frac{\partial^2 R}{\partial u \partial r}$ is unspecified. Therefore, all possibilities exist for the dynamics of u. Especially, u may be monotonic over time, or cyclical. Assume that R is additively separable, imposing $\frac{\partial^2 R}{\partial u \partial r} = 0$. Thus, for all t in $(0, T)$, we have sgn $\frac{d\lambda}{dt} =$ sgn $\frac{du}{dt}$. Consequently, there exists a least one t_1 in $[0, T)$ such that $\frac{du(t)}{dt} < 0$, for all t in $[t_1, T)$. The result expresses that u falls after time t_1, where t_1 represents the last phase of the period horizon.

D Supplementary Result

We present here an additional result that may be of interest to a reader, but which is not directly tied to pricing and investment policies.

A profit maximizing firm knows the relationship between the optimal price and investment. We now determine this relationship. Let $u^*(p)$ be the investment expense verifying (10a). This investment level maximizes the intertemporal profit for any price. Similarly, let $p^*(u)$ be the price satisfying (10b), which maximizes the intertemporal profit for any investment. The intertemporal profit is maximized with the investment and pricing pair such that $(u^*, p^*) = (u^*(p^*), p^*(u^*))$. In the following, the investment and pricing are called *optimal* in the sense that they maximize the intertemporal profit. For simplicity, we omit now the $*$ superscript notation, when there is no confusion. Denote the elasticity of recycling rate dynamics with respect to investment as $\eta_{R/u} = \frac{\partial R}{\partial u}\frac{u}{R}$, and recall the price elasticity of demand, $\eta_p = -\frac{\partial D}{\partial p}\frac{p}{D}$.

Proposition 2. *The optimal relationship between price and investment writes*

$$\frac{\eta_{R/u}}{\eta_p} = \frac{u}{R}\frac{p - c_v(1-r)}{p}\frac{1}{\lambda}, \tag{15}$$

with λ given in Lemma 1.

Proof. Follows from the definitions of $\eta_{R/u}$, η_p, (10a) and (10b). □

Proposition 2 provides a necessary optimality condition for profit maximization. The left-hand side of (15) is the ratio of supply side elasticity, $\eta_{R/u}$, and demand side elasticity, η_p. The condition states that the ratio of investment elasticity of recycling rate dynamics to price elasticity of demand, $\dfrac{\eta_{R/u}}{\eta_p}$, must equal the level of investment deflated by recycling dynamics, $\frac{u}{R}$, multiplied by the markup rate deflated by the shadow price of the recycling rate, $\frac{p-c_v(1-r)}{C}\frac{1}{\lambda}$.

Note that rearranging (15) yields the Lerner index, $\frac{p-c_v(1-r)}{p} = \frac{\eta_{R/u}}{\eta_p}\frac{R}{u}\lambda$, which measures the market power of the firm. The Lerner index ranges from 0, in a competitive market, to 1, in a monopoly market. Based on Proposition 2, the Lerner index increases with the shadow price of the recycling rate, λ. As the recycling rate decreases over time, so does the Lerner index, meaning that the appeal of the product decreases over time, assuming no additional investment.

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