DYNAMIC ROUTE PLANNING FOR LAST-MILE DELIVERY

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

There has never been a time with more demand than now for e-retailing and as a consequence last-mile services. The growth in demand is also bringing significant challenges. With the abundance of options, customers are ever more demanding and expecting more control. With the existing strategies, matching customers' foregoing expectations causes significant economic burdens and ecological disturbances. As a result, e-retailers need to define efficient routing strategies for their last-mile services. This thesis is motivated by identifying efficient routing strategies, in terms of environmental impacts, service time and cost, for last-mile delivery services. We investigate different routing strategies for the last-mile delivery problems, with a focus on same-day services. The corresponding problem is known as the last-mile same-day delivery problem and is dynamic due to the nature of service requests.

In the first part, we investigate vehicle and drone integrated delivery systems. We consider an alternative way to integrate drones into conventional vehicle delivery systems, such that drones resupply vehicles with the future orders of customers while vehicles deliver available orders to customers. We evaluate the impact of the drone resupply system based on a case of the problem in which a single vehicle and a single drone are dedicated to the service area. We introduce a mixed-integer programming model for the delivery problem with known requests. For the dynamic problem in which the requests reveal dynamically throughout the horizon, we propose a periodic reoptimization algorithm as a solution approach. We compare the performance of the drone resupply system to the conventional vehicle only delivery systems over several practical instances that differ in terms of customer preferences and system settings. Through computational experiments, we showed that the drone resupply system outperforms the conventional system with respect to operational time, cost and carbon emissions levels.

In the second part of the thesis, we evaluate the impact of outsourcing strategy in a multi-period delivery problem. Given the relevance of the problem in practice, we suggest that exploitable stochastic information might be gathered for the dynamically revealed information. To the best of our knowledge, we are the first to introduce outsourcing in the literature of dynamic multi-period vehicle routing problems with probabilistic information. We model the corresponding problem as a Markov decision process. We propose a multi-stage programming model and a progressive hedging algorithm to solve the decision problems. We evaluate several planning strategies to evaluate the impact of postponement and outsourcing decisions. Based on the computational experiments, we determined the best delivery strategy in terms of cost over different practical settings.

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Dedication

"I have been freaking out for a long time, thinking I am never going to finish this special and be working on it forever. And recently, I have been feeling like, "Oh, man... Maybe I am getting close to done with this. Maybe I am going to finish it after all." And that has made me completely freak out. Because if I finish this special, that means that I have to, um, not work on it anymore. And that means, I have to just live my life. Ad so, I am not going to do that and I am not going to finish this special. I am going to work on it forever, I think. So yes, I am going to work on this forever and I am never going to release it. I am not talking to anybody now. I am just talking to myself. ... And goodbye, and let's keep going. "

- Bo Burnham: Inside

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Chapter 1

Introduction

The World Economic Forum (2020) (Deloison et al. (2020)) announced an estimate of 78% growth globally in the demand for the last-mile delivery services by 2030. One of the main reasons for the growth in last-mile delivery services is the rising trend for e-retailing. In the past decade, demand for e-retailing services almost tripled globally and the share of e-retailing in the market is expected to grow to reach 20% by 2023 (Deloison et al. (2020)). However, as encouraging as this news is for the future of e-retailing, the growth also causes significant challenges. With the growth in e-retailing, the customers' expectations are getting more and more important in defining the structure of e-logistics. Therefore, e-retailing customers have ever more power to dictate the features of the last-mile services. The e-retailing customers are highly price-sensitive, expecting fast delivery services, with individually scheduled delivery time windows, options to choose alternative delivery locations and shipment traceability (Allen et al. (2016)). Furthermore, to match customers' foregoing expectations, the number of delivery vehicles in the top 100 cities globally is expected to increase by 36% until 2030, which will cause an increase by 32% in emissions from delivery traffic (Deloison et al. (2020)). However, as the world is getting more and more aware of the ecological issues, the global goal is to design conscious alternatives that help towards reducing the negative externalities, such as carbon emissions. Hence, the last-mile delivery services are often considered to be the most critical and expensive operation of e-retailing.

To have the best of both worlds, the e-retailers must design efficient routing strategies. Thus, the resulting routing problem is of importance both in practice and transportation science and logistics literature. On the one hand, the e-retailers, and so last-mile delivery services, might benefit from optimization techniques to reduce undesirable outcomes, such as cost and negative ecological impacts, and offer better service, which is ever faster. On the other hand, the resulting problem corresponds to a case of the vehicle routing problem, and so NP-Hard (Garey and Johnson (1979)).

In the following thesis, we investigate different routing strategies for the problems arising in the last-mile services. We focus on same-day delivery services, which require requests to be served on the same day that the request is placed. An important challenge in the corresponding problem is to plan the routes while customers simultaneously place new orders to be delivered during the day. By the definition, the last-mile same-day delivery problems are dynamic problems. That is, the subset of the problem information is not available at the time of planning and reveals dynamically throughout the execution of the plan (Pillac et al. (2013)). The objective of the research is to propose efficient routing strategies for the dynamic last-mile delivery services, such as same-day last-mile delivery services.

As the first strategy, we analyze the impact of the vehicle and drone integrated delivery systems in last-mile same-day delivery problems. The idea to use drones in last-mile delivery is initially proposed by the e-retailing giant Amazon in 2013 (Prime). Ever since, it has been gaining attention among several other e-retailers and third-party delivery companies, such as UPS (Staff (2020)) and DHL (DHL (2019)). Using drones for the last-mile delivery might be advantageous in terms of both speed and cost (Wolhsen (2014)), besides being an important step towards designing eco-conscious systems (Goodchild and Toy (2018)). However, there are significant challenges regarding drone delivery: regulations (Baloch and Gzara (2020)) and concerns ensuring a safe release to the ground during the drop-off and deciding on a safe location for unattended packages following the drop-off (Wolhsen

(2014)). With the motivation of eliminating most of the prior challenges and concerns regarding drone delivery services in practice, we investigate an alternative way to use drones in the delivery systems, such that drones are integrated to resupply vehicles with dynamically revealing orders while vehicles deliver orders to customers (Dayarian et al. (2020)). The corresponding problem is known as the vehicle routing problem with drone resupply (VRPDR) and is a dynamic routing problem due to the nature of the service requests. The purpose is to evaluate the performance of the alternative drone delivery system in same-day last-mile delivery services both in terms of feasibility and efficiency. We evaluate the impact of the drone resupply system based on a special case of the problem over a service area dedicated to a single vehicle and a single drone. We compare the performance of the drone resupply system to the conventional vehicle only delivery systems over several different practical settings. We provide detailed observations regarding the impact of the drone resupply system on the operational time, cost and carbon emissions levels. Our research contributes important findings to the literature by analyzing and identifying several settings in which the drone resupply system outperforms conventional systems in same-day last-mile delivery.

Then, we investigate the impact of outsourcing on last-mile same-day delivery problems. Outsourcing is an important resource and a common practice to improve the resilience of the distribution network in times of demand fluctuations, such as in peak seasons like Christmas, events like Black Friday and Cyber Monday in the United States (Allen et al. (2016)) or in times of crisis like COVID-19 pandemic (OECD (2020)). Besides ensuring the feasibility of delivery when the demand fluctuates, outsourcing might also be an economically convenient option. For example, outsourcing the requests in isolated locations might introduce opportunities for reducing distribution costs. We consider a problem in which requests might either be served with a private vehicle from the fleet or outsourced to a common carrier. Furthermore, the problem is a multi-period problem, since the requests placed towards the day might be postponed to the following day. Then, the decisions to consider in a day are, whether to serve a request with a private vehicle either at the current or the next day or to outsource to a common carrier, and the route for each vehicle. Given the relevance of the same-day delivery services in practice, we suggest that exploitable stochastic information might be gathered for the dynamically revealed information. Hence, the corresponding problem is categorized as a dynamic and stochastic routing problem. To the best of our knowledge, we are the first to introduce outsourcing in the literature of dynamic multi-period vehicle routing problems with probabilistic information (DVRPP). We refer to the corresponding research problem as the dynamic and stochastic multi-period vehicle routing problem with partial outsourcing (DS-MVRPPO). Here, the purpose is to evaluate the cost-efficiency of outsourcing in same-day last-mile delivery services. We evaluate four different planning strategies and conduct detailed computational experiments to observe the impact of these strategies on postponement and outsourcing decisions. First and foremost, our research shows that outsourcing helps reduce the service cost in same-day last-mile delivery. Moreover, our research contributes a detailed understanding of the impact of different outsourcing settings on the outsource percentage and service cost.

The remaining of this thesis is organized as follows. In Chapter 2, we discuss the motivation for the system under consideration and present the formal definition of the corresponding problem, the VRPDR. Then, in Chapter 3, we present a solutions approach and detailed observations on the impact of the drone resupply system in the last-mile same-day delivery problems. Following, in Chapter 4, we discuss the motivation for the strategy under consideration and present the formal definition of the corresponding problem, the DS-MVRPPO. Then, in Chapter 5, we present two different solutions approaches and detailed observations on the impact of postponing and outsourcing requests in the last-mile same-day delivery problems. Finally, in Chapter 6, we present concluding remarks and discuss future research directions.

Chapter 2

Dynamic Last-Mile Delivery with Drone Resupply

In this chapter, we introduce the motivation for the dynamic last-mile delivery problem with drone resupply. We review and discuss the relevant research in the literature. We then present the formal definition of the problem and propose a mathematical programming model for a special case of the problem.

2.1 Introduction

There has been significant growth in e-retailing, especially in online shopping followed by delivery to the customer. However, instant gratification is still an important advantage of brick-and-mortar retailers against e-retailers. In fact, due to the significant growth in e-retailer options, customers are even more demanding nowadays with regards to the speed of the delivery service. A recent analysis on customer expectations from Joerss et al. (2016) shows that up to 27% of the customers make a choice based on the service times. As a consequence, e-retailers are working towards offering ever-faster delivery services that bring near-instant gratification to online shoppers (Howard (2014)), such as same-day delivery services. On the other hand, while fast delivery is a key factor of success for e-retailers, the dynamic nature of the same-day delivery services leads to significant challenges with respect to both cost and logistics of the operation. To meet customers' expectations both in terms of demand and service, the number of delivery vehicles in the top 100 cities globally is expected to increase by 36% until 2030, which will cause an increase by 32% in emissions from delivery traffic (Deloison et al. (2020)). However, customers are aware of the ecological issues now more than ever and so tend towards options that are also conscious and working towards the reduction of the negative externalities, such as carbon emissions. Hence, e-retailers are searching for efficient routing strategies.

One of the most innovative ideas is proposed in 2013 by the e-retailing giant Amazon, with the reveal of the "Amazon Prime Air" project, which aims to deliver orders to the customer with drones in 30 minutes (Prime). Amazon has been conducting pilot tests with drones in the United Kingdom, Canada and the Netherlands as of 2016 (Prime). Meanwhile, Google's parent company, Alphabet Wing has been doing pilot tests with drones in Australia, Finland and the United States since 2018 (Wing). In 2016, UPS started tests with drones to perform commercial deliveries to remote or difficult-to-access locations. In 2017, Walmart started tests with drones in Central New York and at Griffiss International Airport in Rome (Doran (2019)).

Using drones for the last-mile delivery services might be advantageous in terms of both speed and cost; drones avoid congestion by using shortcuts that are not available in conventional networks and have low transportation costs per kilometre compared to conventional vehicles (Wolhsen (2014)). Furthermore, using drones in delivery systems is an important step towards designing eco-conscious systems, since drones decrease fuel usage and result in the reduction of carbon emissions compared to conventional vehicles (Goodchild and Toy (2018)). However, a significant challenge of using drones is to make sure that the orders are delivered properly to the customers (Kobie (2016)). There are several issues regarding the drop-off of the packages, such as ensuring a safe release to the ground during the drop-off and deciding on a safe location for unattended packages once dropped off. The safety of the delivery system is an important drawback against willingness to adapt the drone technology (Wilke). However, the most important challenge remains to be the regulations. Current Federal Aviation Administration (FAA) regulations severely limit the effective flight range of drones (Baloch and Gzara (2020)). Even without these restricting regulations, drones are limited in terms of both maximum flight range and capacity. Nonetheless, it is possible to reduce these challenges and concerns by integrating drones into vehicles in the delivery systems.

Research in the transportation literature started to address the use of vehicle and drone integrated delivery systems for last-mile delivery problems following the research of Murray and Chu (2015). Most research considers settings in which both the vehicle and drone are responsible for delivering orders to the customers. However, these settings still do not address the challenges and concerns regarding the safety of the drone delivery systems. To the best of our knowledge, the research of Dayarian et al. (2020) is the first to investigate an alternative way to integrate drones into vehicle delivery systems, such that drones resupply vehicles with dynamically revealing orders for same-day delivery. One advantage of the drone resupply delivery system is that it is likely to be much faster and less costly than the conventional vehicle delivery system, in which the vehicle has to return to a distribution center to pick up the orders that are placed dynamically and have to be delivered to the customer the same day. Further, the drone resupply delivery system eliminates most of the prior challenges and concerns regarding drone delivery services in practice. Though the idea is quite recent, US Army already utilized a similar system and resupplied a submarine using drones in October 2019 (Sutton (2019)). As the drone resupply delivery system addresses almost all of the foregoing expectations without posing an additional challenge, the research on the system requires further attention, to evaluate the appropriateness and impact of the system in practical applications of same-day delivery problems.

In this research, we investigate the performance of the drone resupply delivery system in the last-mile same-day delivery problems. The delivery problem under consideration addresses the dynamism of the last-mile same-day delivery problems for which the drone integrated delivery systems are proposed in practice. We evaluate the delivery problem for an area dedicated to a single vehicle and a single drone. The purpose of our research is to provide both strategic and operational observations on the drone resupply system. On the operational side, we formulate the optimization problem to decide on vehicle and drone routes, given the set of requests. To the best of our knowledge, we are first to introduce a mathematical programming model for the corresponding static routing problem. For the reoptimization approach proposed to address future requests, we investigate different routing strategies, such as different short-term strategies based on different objectives and different solution strategies to solve the re-routing problems. On the strategic side, we analyze the impact of the drone resupply system in practical applications compared to vehicle only systems. We examine the performance of the drone resupply system for different delivery service strategies that differ with respect to service time guarantee and the earliest possible time that the request can be delivered. To the best of our knowledge, this research is the first in the literature to consider time windows in drone integrated dynamic delivery problems. The findings of our research are important to evaluate whether the drone resupply system is applicable to practical problems and the impact of the problem settings on the solution performance.

The remaining of this chapter is organized as follows. In Section 2.2, we review the research relevant to the problem under consideration and discuss our contribution to the literature. In Section 2.3, we present the delivery system and the formal description of the corresponding delivery problem. Then, in the next chapter, we discuss the proposed solution approach and the conclusion of the computational experiments.

2.2 Literature Review

In this section, we review the literature related to the travelling salesman problem with drone resupply (TSPDR) introduced in Section 2.1. We first review the literature on integrated last-mile delivery systems in Section 2.2.1, such as the drone resupply system. Then, in Section 2.2.2, we review the literature that considers the use of drones to meet dynamic requests, as the same-day request in the problem under consideration.

2.2.1 Synchronization in Vehicle Routing

The vehicle routing problem with multiple synchronization constraints (VRPM) is a vehicle routing problem (VRP) with additional synchronization constraints with regards to spatial, temporal, and load aspects (Drexl (2018)). The routing problems for the delivery systems that integrate conventional vehicles to unmanned aerial vehicles, such as drones, are instances of the VRPM literature. Singhal et al. (2018) categorize the literature on drone integrated problems according to the application areas as civilian, environment and defence. The use of drone integrated systems in last-mile delivery problems is a civilian application. In the following, we review the research on the VRPMs, emerging from the civilian application of the drone integrated delivery systems, focusing especially on last-mile delivery problems.

Murray and Chu (2015) are the first in the literature to introduce delivery problems that integrate vehicle and drone. Their research discusses two delivery problems, which differ with respect to the integration between the vehicle and drone in the delivery system. The first problem, the flying sidekick travelling salesman problem (FSTSP), is an instance of the VRPM and requires additional synchronization constraints between the vehicle and drone. However, in the second problem, the parallel drone scheduling travelling salesman problem (PDSTSP), the vehicle and drone are performing deliveries without synchronization. Here, we focus on the research on the VRPMs and so the review is based on the FSTSP.

The FSTSP is a generalization of the well-known travelling salesman problem (TSP). The problem considers a set of customers, each of which must be served exactly once, by either a vehicle or a drone. The drone operates in coordination with the vehicle, as a sidekick. The objective of the FSTSP is to minimize the time required to serve all customers and return to the depot, for both the vehicle and drone. Murray and Chu (2015) formulate the corresponding problem with a mixed-integer linear programming (MILP) model and propose a heuristic solution approach. Ponza (2016) extends the research of Murray and Chu (2015) and proposes a simulated annealing heuristic for the FSTSP. Following, Es Yurek and Ozmutlu (2018) propose an optimization-based heuristic, which is an iterative algorithm based on a decomposition approach. Further, Jeong et al. (2019) suggest a further extension for the FSTSP, by taking the drone energy consumption and flying zones into consideration. The authors present a mathematical model and a heuristic solution approach for their extension.

Following the assumptions of the FSTSP of Murray and Chu (2015), Ha et al. (2018) consider the problem to minimize operational costs, which include the total transportation costs of vehicles and the cost resulting from the time that a vehicle wastes while waiting for the other. The authors propose a MILP model by extending the model of Murray and Chu (2015), with additional constraints that capture the cost of waiting. Ha et al. (2018) also propose a heuristic solution approach for the corresponding problem. Then, Phan et al. (2018) extend the research of Ha et al. (2018) to integrate multiple drones into a single vehicle. In their extension, the objective does not penalize the wait time, unlike in Ha et al. (2018). The authors refer to the corresponding problem as the travelling salesman problem with multiple drones (TSP-mD). Phan et al. (2018) adopt the heuristic approach in Ha et al. (2018) to solve the TSP-mD.

Furthermore, Agatz et al. (2018) consider a drone integrated delivery problem, quite similar to the FSTSP of Murray and Chu (2015). The authors refer to the corresponding problem as the travelling salesman problem with drone (TSP-D). According to the TSP-D, in contrast to the FSTSP, the drone might meet with the vehicle at the node where both split. The TSP-D also considers different speeds for the vehicle and drone, unlike in the FSTSP. Agatz et al. (2018) formulate the corresponding problem with a mixed-integer linear programming (MILP) model and propose a heuristic solution approach, in which the construction of the drone route is based on either local search or dynamic programming. Bouman et al. (2018) extend the research of Agatz et al. (2018), by proposing an exact solution method based on dynamic programming for the TSP-D. de Freitas and Penna (2020) propose a metaheuristic to solve both the FSTSP and the TSP-D and show that the proposed heuristic outperforms the existing methods.

Ferrandez and Sturges (2016) extend the TSP-D to equip vehicles with multiple drones.

Following, Wang et al. (2017) further extend the research of Ferrandez and Sturges (2016) to consider multiple vehicles which are equipped with multiple drones. The authors referred to this problem as the vehicle routing problem with drones (VRP-D) and analyzed several worst-case scenarios. Schermer et al. Schermer et al. (2018) introduce valid inequalities to the VRP-D as presented by Wang et al. Wang et al. (2017) and propose a metaheuristic algorithm based on variable neighbourhood search.

2.2.2 Drones in Dynamic Vehicle Routing

The problem under consideration is dynamic, due to the nature of the same-day delivery problems. In the same-day delivery problems, new requests are dynamically revealed throughout the day, to be served during the day. That is, a subset of requests is available from the beginning while the remaining are revealed throughout the planning horizon. No further information is available on the subset of dynamically revealing requests. According to the categorization of the dynamic routing problems in Pillac et al. (2013), the problem under consideration is dynamic and deterministic.

In the dynamic and deterministic routing problems, part or all of the information is not available and unknown. The critical information about the problem reveals dynamically, without neither deterministic nor stochastic information available, while en route. That is, the complete set of information is only available by the end of the planning. As a consequence, the exact solution approaches can provide an optimal solution for the current state of the problem, but cannot guarantee the solution will remain optimal once new information reveals. A common approach is to progressively construct the solution for the complete problem based on the available information. The corresponding approach is known as the reoptimization approach. Examples of successful implementations of the reoptimization approach to the dynamic delivery problems as in here are presented in Kilby et al. (1998) and Chen and Xu (2006). In the following, we review the limited research on the dynamic delivery problems, especially with dynamic requests or same-day requests, that integrates vehicles and drones.

Ulmer and Thomas (2018) are the first in the literature to consider a vehicle and drone integrated system in the same-day delivery problem. In their research, the delivery system consists of heterogeneous vehicles and drones and vehicles and drones require no synchronization. In a sense, the PDSTSP of Murray and Chu (2015) is a special version of the corresponding problem, in which all customer orders are available at the time of planning, i.e., static. The authors propose approximate dynamic programming (ADP) as a solution approach.

Although Ulmer and Thomas (2018) are the first in the literature to suggest using drone integrated delivery systems in the same-day delivery problems, Dayarian et al. (2020) are the first to consider the delivery system, with additional synchronization constraints. The authors introduce a routing problem in which dynamically revealing requests have to be served with respect to a common service time guarantee by vehicles, while drones resupply vehicles with the new requests. The objective of the corresponding problem is to maximize the number of requests served. Dayarian et al. (2020) refer to the problem as the vehicle routing problem with drone resupply. The authors focus on the single vehicle and single drone case and propose a heuristic solution approach. For the computational experiments, Dayarian et al. (2020) consider two different strategies for the drone resupply system: (1) restricted-resupply, in which all the requests loaded to the vehicle must be served before the next drone resupply, (2) flexible-resupply, in which drone resupply might take place at any time during the delivery route. Their computational experiments illustrate that the flexible-resupply strategy outperforms the restricted-resupply.

2.2.3 Contribution

The interest in drone integrated delivery systems in the context of last-mile delivery problems has been growing since the research of Murray and Chu (2015). However, most of this research assumes that all of the critical information about the problem is available and known at the time of planning, i.e., static problems. Considering that the idea of using drones is initially proposed to meet ever-growing service expectations of customers, such as same-day delivery services, it is crucial to consider the dynamic aspect of such delivery problems. Furthermore, in all but one of the abovementioned researches in the review, both the vehicle and drone are responsible for making deliveries. However, using drones for delivery is rather challenging in practice, due to regulations and difficulties in ensuring proper delivery. To the best of our knowledge, the VRPDR as presented in Dayarian et al. (2020) is the only research in the literature that considers all these aspects; a dynamic same-day delivery problem, which integrates vehicles and drones in a delivery system, in a way that drone is not responsible for the deliveries. Table 2.1 summarizes the problems in the literature that considers drone integrated delivery systems for last-mile delivery problems.

Table 2.1: Review of the literature on drone integrated routing problems

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Problem	Objective	# Vehicles	#Drones	System	Sync	Demand	Time Window
Flying Sidekick travelling Salesman Problem (FSTSP)	Minimize completion time	Single	Single	Vehicle - Drone: Delivery	Yes	Static	No
travelling Salesman Problem with Drone (TSP-D)	Minimize operation cost	Single	Single	Vehicle - Drone: Delivery	Yes	Static	No
Vehicle Routing Problem with Drones (VRP-D)	Minimize completion time	Multiple	Multiple	Vehicle - Drone: Delivery	Yes	Static	No
Same-Day Delivery Routing Problems with Heterogeneous Fleets of Drones and Vehicles (SDDPHF)	Maximize requests served	Multiple	Multiple	Vehicle - Drone: Delivery	No	Dynamic	No
Vehicle Routing Problem with Drone Resupply (VRPDR)	Maximize requests served	Multiple	Multiple	Vehicle: Delivery Drone: Resupply	Yes	Dynamic	No

For the research under consideration, we further investigate the drone resupply delivery system introduced in Dayarian et al. (2020) to address a last-mile same-day delivery problem. As the drone resupply strategy, we consider the flexible-resupply strategy, since it is shown to be the better performing strategy by Dayarian et al. (2020). Note that, in our research, we evaluate a different objective function based on the sustainability initiative of the e-retailer such as Amazon (Prime,Wilke). That is, we consider the minimization of environmental impact instead of maximization of the number of requests served. Following, we further extend the research of Dayarian et al. (2020) to evaluate the performance of the drone resupply system under different problem settings, such as different time windows strategies based on the practical applications of the same-day delivery problems. We investigate both different and common service times, in addition to the common service time assumption of Dayarian et al. (2020). Furthermore, instead of assuming that a request is ready for delivery once placed as in Dayarian et al. (2020), we investigate different strategies for the earliest possible delivery times of the requests. Table 2.1 illustrates that the problem under consideration in this research is the first research to consider these characteristics together. We propose a periodic reoptimization to solve the problem. We introduce a mathematical programming model and propose a simulated annealing based metaheuristic to solve the static problem corresponding to the current state. We provide extensive computational analyses to evaluate the performance of the drone resupply system over the conventional vehicle only delivery systems in same-day delivery problems.

2.3 Problem Definition and Formulation

In this section, we first present the delivery system and introduce the corresponding delivery problem (TSPDR) in Section 2.3.1. Then, in Section 2.3.2, we introduce the mathematical model for the static case of the problem.

2.3.1 Problem Definition

We consider a last-mile same-day delivery problem in which orders of the customers arrive dynamically throughout a planning horizon T. Each order of each customer defines a service request that must be fulfilled with respect to a service time window. Due to the nature of the same-day requests, the corresponding delivery system and the problem are dynamic. That is, a subset of requests is available from the beginning while the remaining are revealed throughout the planning horizon. In the following drone resupply delivery system, the vehicles and drones are integrated to perform different and complementary operations of the last-mile delivery system. At the beginning of the planning, vehicles pick up orders that are available and start delivering to customers from the distribution center. Subsequently, drones resupply vehicles with the new requests that are placed while vehicles are en route. We assume that drones are responsible for resupplying vehicles with the new requests, while vehicles perform the delivery of orders to the customers. Note that, both the initially available and dynamically revealed requests might need to be fulfilled during the horizon, as the research under consideration addresses the same-day requests. The vehicle routes should start and end at the distribution center and could include multiple stops in between. On the other hand, the drone routes should also start and end at the distribution center, but could include just one stop, which is the meeting location for resupply. Resupply might take place at any customer location along the routes of vehicles and vehicles have to be stationary at the meeting location until the transfer of the order is complete. The vehicle has to be present at the meeting location when the drone reaches the location. That is, the vehicle can wait for the drone at the meeting location but the drone cannot, since we consider settings in which there is no safe ground for the drone to land at the meeting location.

At a given time t in the planning horizon, let the set of available requests be denoted as $N \in \{0, 1, \cdots, n-1\}$ and the distribution center as n. Let the time when request $i \in N$ is placed be denoted as θ_i . Following, request $i \in C$ must be fulfilled with regards to a service time window. Let the time window for request $i \in N$ be denoted as $[a_i, b_i]$, such that a_i and b_i represent the earliest and latest time request $i \in N$ must be served respectively. The earliest service time a_i of request i might either be the same as θ_i or later. The latest time window b_i is defined as $a_i + S_i$ in which S_i represents the service time for request i. The service time S_i might either be the same or change for each request. Note that, different settings for the service time window define different settings for the delivery systems. Also, let the weight of request $i \in N$ be denoted as w_i. Request $i \in N$ is characterized with the following: customer location, order placement time (θ_i), service time window ($[a_i, b_i]$) and weight (w_i).

The capacity of the vehicles is infinite, as the weights of the parcels are insignificant compared to the capacity of the vehicles. However, the capacity of the drones is finite and expressed in terms of weight ρ. The vehicles and drones also differ with respect to service network and speed. The vehicles follow the road network, and so the distances might be asymmetric. The drones use the Euclidean distance, and so the distances satisfy the triangle inequality. Let the average speed of the vehicles and drones be denoted as v^k and v^d respectively. Here, $v^d \geq v^k$, based on the fact that the drone neither makes multiple stops on a single trip nor gets stuck in congestion at the road network, unlike the vehicle.

The aforementioned describes a delivery system that consists of multiple vehicles and multiple drones, in which drones resupply vehicles with dynamic orders while vehicles deliver the orders to the customers. The drone resupply proposes a rather new approach to integrating drones into delivery systems, and so the research on the subject is rather limited. For the research under consideration, we analyze the impact of the drone resupply in the last-mile delivery problems over different settings. For that, we focus on the delivery problem of a single vehicle that is resupplied by a single drone, rather than addressing the whole delivery problem of the system. In other words, we consider the delivery problem of the drone resupply system for an area dedicated to a single vehicle and a single drone. Note that, dividing the service area into zones and assigning vehicles to each zone prior to routing, is also a common approach in practice to handle high volumes of requests. The corresponding problem is known as the zoning problem and is not part of the routing problem considered in this research. Here, the problem is to find a delivery plan, for a predetermined zone, which minimizes environmental impact through fuel usage. With regards to the delivery system under consideration here, minimizing the fuel usage of the system also corresponds to minimizing the distance (time) travelled by the delivery vehicle. We refer to the corresponding research problem as the travelling salesman problem with drone resupply (TSPDR).

2.3.2 Mathematical Formulation

The TSPDR is a dynamic problem by definition, in which there is no further information available on the subset of requests that reveals throughout the horizon. As a consequence, it is not possible to capture the entire problem with a single mathematical model and a re-optimization procedure is proposed as a solution approach based on the static case of the TSPDR.

To better understand the re-optimization approach, and also to highlight the dynamics of the drone resupply delivery system, assume that there exists a known moment θ in the planning horizon at which the values of all previously unknown parameters are now available. That is, all of the dynamic requests are revealed and the corresponding problem at θ is static. Let $N \in \{0, 1, \cdots, n-1\}$ defined as the set of available requests at θ as per the definition in Section 2.3.1. Then, let the requests in N be further categorized as requests that are already loaded to the vehicle at the time of the planning (N^v) and that are resupplied by the drone following the plan (N^d). Note that, $N = N^v \cup N^d$ and $N^v \cap N^d = \emptyset$. Furthermore, let the starting location of the vehicle be denoted as s. At $\theta = 0$, s corresponds to the distribution center n as per the problem definition. For $\theta > 0$, s corresponds to either the location of customer $i \in N$ that the vehicle is at, if the vehicle is at a stationary position, or the next location according to the route, otherwise. Finally, let the decisions regarding the routes of the vehicle and drone be denoted with variables based on the vehicle's position on the route. That is, the drone is integrated with the vehicle with respect to the route of the vehicle. Then, the corresponding problem at θ is formulated with a mathematical model as follows.

Sets

N^v : Set of requests that are already loaded to the vehicle

N^d : Set of requests that are to be resupplied by the drone

$\bar{N} = N^v \cup N^d \cup \{n\}$: Set of requests and the distribution center

Parameters

$[a_i, b_i]$: Service time window for request i

w_i : Weight of request i

d^v_{ij}, d^d_{ij} : Travel distance from request i to j with the vehicle, drone

t^v_{ij}, t^d_{ij} : Travel time from request i to j with the vehicle, drone

v^v, v^d : Speed of the vehicle, drone on average

ρ : Weight capacity of the drone

Decision Variables

$$y_{ik} = \begin{cases} 1, & \text{if the vehicle delivers request } i \text{ on the } k\text{-th position on the vehicle route,} \\ 0, & \text{otherwise} \end{cases}$$

$$z_{ik} = \begin{cases} 1, & \text{if the vehicle picks up (drone resupplies) request } i \text{ at the } k\text{-th} \\ & \text{position on the vehicle route,} \\ 0, & \text{otherwise} \end{cases}$$

$$x_{kl} = \begin{cases} 1, & \text{if the drone meets the vehicle at the } l\text{-th position after meeting at } k\text{-th} \\ & \text{position on the vehicle route,} \\ 0, & \text{otherwise} \end{cases}$$

p^v_k, p^d_k = Arrival time of the vehicle, drone to the k-th position on the vehicle route

q^v_k = Departure time of the vehicle from the k-th position on the vehicle route

Mathematical Model

$$\min \sum_{i \in \bar{N}} \sum_{j \in \bar{N}} \sum_{k=1}^{n} d_{ij}^{v} y_{i(k-1)} y_{jk} \tag{2.1}$$

$$\text{s.t.} \quad y_{s0} = y_{nn} = 1 \tag{2.2}$$

$$\sum_{i \in \bar{N}} y_{ik} = 1 \qquad \forall k \in \{0, 1, ..., n\} \tag{2.3}$$

$$\sum_{k=0}^{n} y_{ik} = 1 \qquad \forall i \in \bar{N} \tag{2.4}$$

$$\sum_{k=0}^{n} x_{nk} = 1 \tag{2.5}$$

$$\sum_{l=0:l \neq n}^{n-1} x_{nl} - \sum_{k=0:k \neq n}^{n-1} x_{kn} = 0 \tag{2.6}$$

$$\sum_{k=0:k \neq l}^{n} x_{kl} - \sum_{m=1:l \neq m}^{n} x_{lm} = 0 \qquad \forall l \in \{0, 1, ..., n-1\} \tag{2.7}$$

$$p_l^d \geq [p_k^d + (t_{in}^d + t_{nj}^d) y_{ik} y_{jl}] x_{kl} \qquad \forall i, j \in \bar{N}, \forall k, l \in \{0, 1, ..., n\} \tag{2.8}$$

$$p_k^v \geq q_{k-1}^v + t_{ij}^v y_{i(k-1)} y_{jk} \qquad \forall i, j \in \bar{N}, \quad \forall k \in \{1, 2, ..., n\} \tag{2.9}$$

$$p_l^d \geq p_l^v x_{kl} \qquad \forall k, l \in \{0, 1, ..., n-1\} \tag{2.10}$$

$$q_k^v \geq p_k^v \qquad \forall k \in \{0, 1, ..., n-1\} \tag{2.11}$$

$$y_{il} \leq \sum_{k=0:k \leq l}^{n-1} z_{ik} \qquad \forall i \in N^d, \forall l \in \{0, 1, ..., n\} \tag{2.12}$$

$$\sum_{k=0}^{n-1} z_{ik} = 1 \qquad \forall i \in N^d \tag{2.13}$$

$$z_{il} \leq \sum_{k=0}^{n} x_{kl} \qquad \forall i \in N^d, l \in \{0, 1, ..., n-1\} \quad (2.14)$$

$$p_k^v \geq \sum_{i \in N} a_i y_{ik} \qquad \forall k \in \{0, 1, ..., n\} \quad (2.15)$$

$$p_k^v \leq \sum_{i \in N} b_i y_{ik} \qquad \forall k \in \{0, 1, ..., n\} \quad (2.16)$$

$$\sum_{i \in N} w_i z_{ik} \leq \rho \qquad \forall k \in \{0, 1, ..., n\} \quad (2.17)$$

$$y_{ik},\ z_{ik},\ x_{kl} \in \{0, 1\} \qquad \forall i \in \bar{N}, \forall k, l \in \{0, 1, ..., n\} \quad (2.18)$$

$$p_k^v,\ q_k^v,\ p_k^d \in \mathbb{R}^+ \qquad \forall k \in \{0, 1, ..., n\} \quad (2.19)$$

Objective (2.1) of the problem is to minimize the fuel usage of the delivery system, i.e. total distance travelled by the vehicle. Constraint (2.2) ensures that the first and last positions on the route of the vehicle are the starting location s, which is the distribution center for $\theta = 0$ and a customer $i \in N$ for otherwise, and the distribution center respectively. Constraint (2.3) and (2.4) respectively state that the vehicle must be at exactly one customer on each position on the route and each customer must be served exactly once. Note that, there is no such requirement for the drone and the drone might resupply the vehicle with as many orders as possible at once under the capacity constraints. Throughout the planning horizon, the drone might make multiple trips to the distribution center while the vehicle makes a single trip starting and ending at the distribution center. However, here we formulate the routing decision of the drone as though the drone is also making a single trip. According to this, Constraint (2.5) and (2.6) together state that the drone starts and ends its route at the distribution center, as in the Constraint (2.2) for the vehicle. Constraint (2.7) states the flow balance constraints for the route of the drone, i.e., the drone must leave every meeting location that it arrives at. Then, Constraint (2.8) ensures that there is enough time for the drone to make a trip to the distribution center in between meetings with the vehicle. That is, Constraint (2.8) sets the travel time from i to

j for the drone to the total travel times from i to the distribution center (t^d_{in}) and from the distribution center to node j (t^d_{nj}). Constraint (2.9) states the arrival time of the vehicle. Constraint (2.10) states that the arrival time of the drone must not be prior to that of the vehicle, since the drone cannot wait for the vehicle at the meeting location according to the problem definition. Furthermore, Constraint (2.11) ensures that the departure time of the vehicle is not before the arrival time of the drone to that location, since the vehicle has to remain stationary at the meeting location until the resupply is complete. If the drone does not meet with the vehicle at that location, then the constraint is redundant, as the arrival time for the drone is going to be zero at that location. Constraint (2.12) states that an order that is not loaded to the vehicle cannot be delivered to the customer. Following, Constraint (2.13) states that the orders that are not loaded to the vehicle are resupplied by the drone along the route. Constraint (2.14) defines the relationship between x_{kl} and z_{ik} variables. That is, if the drone resupplies the vehicle on the k-th position on the route, then both must meet at the corresponding location. Constraint (2.15) and (2.16) ensure time window constraints for the vehicle and Constraint (2.17) ensures the capacity constraints for the drone. Finally, Constraint (2.18) and (2.19) define the domains of the decision variables.

The aforementioned mathematical model includes non-linear terms both in the objective function and constraints such as Constraint (2.8) and (2.10). We first linearize the term $y_{i(k-1)}y_{jk}$ and then redefine Objective (2.1) and Constraint (2.9) with σ_{ijk} as follows.

$$\sigma_{ijk} \leq y_{i(k-1)} \qquad \forall i, j \in \bar{N} : j \neq i, \forall k = 1, 2, ..., n \tag{2.20}$$

$$\sigma_{ijk} \leq y_{jk} \qquad \forall i, j \in \bar{N} : j \neq i, \forall k \in \{1, 2, ..., n\} \tag{2.21}$$

$$\sigma_{ijk} \geq (y_{i(k-1)} + y_{jk}) - 1 \qquad \forall i, j \in \bar{N} : j \neq i, \forall k \in \{1, 2, ..., n\} \tag{2.22}$$

Likewise, for Constraint (2.8), we linearize $(y_{ik}y_{jl})x_{kl}$ with γ_{ikjl} and redefine as follows.

$$\beta_{ikjl} \leq y_{ik} \qquad \forall i, j \in \bar{N} : j \neq i, \forall k \in \{0, 1, ..., n+1\} \tag{2.23}$$

$$\beta_{ikjl} \leq y_{jl} \qquad \forall i, j \in \bar{N} : j \neq i, \forall k \in \{0, 1, ..., n+1\} \tag{2.24}$$

$$\beta_{ikjl} \geq (y_{ik} + y_{jl}) - 1 \qquad \forall i, j \in \bar{N} : j \neq i, \forall k, l \in \{0, 1, ..., n+1\} \tag{2.25}$$

$$\gamma_{ikjl} \leq \beta_{ikjl} \qquad \forall i, j \in \bar{N} : j \neq i, \forall k, l \in \{0, 1, ..., n+1\} \tag{2.26}$$

$$\gamma_{ikjl} \leq x_{kl} \qquad \forall i, j \in \bar{N} : j \neq i, \forall k, l \in \{0, 1, ..., n+1\} \tag{2.27}$$

$$\gamma_{ikjl} \geq (\beta_{ikjl} + x_{kl}) - 1 \qquad \forall i, j \in \bar{N} : j \neq i, \forall k, l \in \{0, 1, ..., n+1\} \tag{2.28}$$

Again, for Constraint (2.10), we linearize $p_l^v x_{kl}$ with α_{kl} as follows.

$$\alpha_{kl} \leq M x_{kl} \qquad \forall k, l \in \{0, 1, ..., n+1\} : k \neq l, k \neq (n+1) \tag{2.29}$$

$$\alpha_{kl} \leq p_l^v \qquad \forall k, l \in \{0, 1, ..., n+1\} : k \neq l, k \neq (n+1) \tag{2.30}$$

$$\alpha_{kl} \geq p_l^v - M(1 - x_{kl}) \qquad \forall k, l \in \{0, 1, ..., n+1\} : k \neq l, k \neq (n+1) \tag{2.31}$$

Chapter 3

Solution of the Last-Mile Delivery Problem with Drone Resupply

In this chapter, we present a periodic reoptimization approach for the travelling salesman problem with drone resupply (TSPDR) from Section 2.3. We also present a simulated annealing based metaheuristic approach, in addition to the mathematical programming approach, to solve the static problems in reoptimization. We evaluate the impact of the drone resupply delivery system over the conventional vehicle only delivery systems. We discuss the conclusions of the research and future directions.

3.1 Solution Approach

In this section, we first present a periodic reoptimization approach to solve the dynamic problem from Section 2.3.1 in Section 3.1.1. Then, in Section 3.1.2, we present a heuristic approach, in addition to the mathematical programming approach from Section 2.3.2, to solve the static problems addressed at each period of the reoptimization approach.

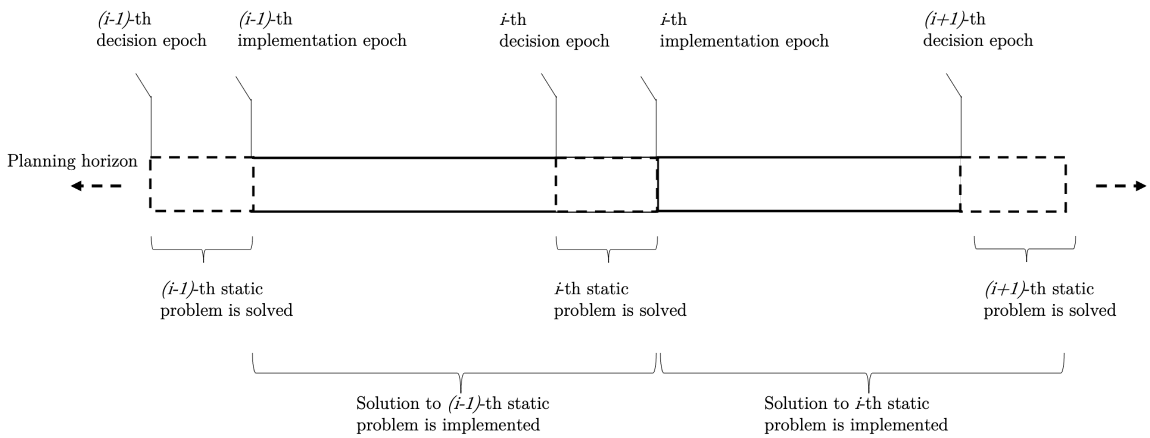
In Section 2.3, we discuss that the drone resupply delivery system is proposed for the dynamic same-day delivery problem and so, the problem under consideration (TSPDR) is dynamic. Furthermore, since the application considered here is rather new, we assume that there is no further exploitable information available on the dynamic requests. This further categorizes the TSPDR as a dynamic and deterministic problem (Pillac et al. (2013)). Note that, a possible extension to this problem is to examine the dynamic and stochastic case, in which exploitable stochastic information is available on the dynamic requests.

3.1.1 Periodic Reoptimization

The reoptimization approaches iteratively construct a solution to the dynamic problem based on the previous solutions. Each iteration corresponds to a new problem state, in which the information from the previous states (iterations) is updated based on the solutions and then the new information that is revealed since the previous state (iteration) is introduced to the available information. Each problem state corresponds to a new delivery problem as in Section 2.3.2. Note that, here the terms iteration and state are used interchangeably.

Pillac et al. (2013) categorize the reoptimization approaches into two as periodic and continuous reoptimization. Periodic reoptimization approaches start with an optimization procedure that solves the problem for the initial information that is available at the beginning of the planning horizon. Following, the procedure periodically solves the static problem corresponding to the current state, either whenever the available information changes or at a given interval of time. Since the problem at each iteration is static, the periodic reoptimization benefits from extensive research on the static counterpart of the problem. Figure 3.1 illustrates the generic procedure in periodic reoptimization approaches. On the other hand, continuous reoptimization approaches maintain information on good solutions from the previous states and perform optimization throughout the planning horizon. Whenever the available information changes, a decision procedure updates the solution based on the information on previous good solutions. The continuous reoptimization approach does not require the decisions to be optimized, unlike the periodic approach, but at the expense of more elaborate implementation. In this research, we propose a periodic reoptimization to benefit from the research on static problems. Furthermore, to overcome the computational setback of the approach, we use a simulated annealing based metaheuristic to solve the static problems.

Figure 3.1: A flow of periodic reoptimization approach.



The periodic reoptimization approach presented in Algorithm 1 reoptimizes the routes at given intervals of time, i.e., decision epochs. Since we consider same-day delivery problems, the planning horizon (T) corresponds to a day. Then, let the duration of each decision epoch be denoted as δ, such that δ is a divisor of T. The reoptimization period should be chosen considering the trade-off between the solution performance and computational requirements of the solution algorithm. That is, δ should be short enough to incorporate new information into the decision as soon as possible but also long enough to perform the route without requiring constant interruption. Let the time of the decision be denoted as $t \in T$, such that t is updated with δ increments of time until $T - \delta$. At $t = 0$, the first optimization problem is defined based on the requests available at the beginning of the day. The requests that are available at $t = 0$ (N_0), are the ones that were not delivered on the previous day and the ones that are placed in between the previous and current day.

Since the vehicle starts the route from the distribution center at $t = 0$, the first problem reduces to a well-known static problem, i.e., the travelling salesman problem with time windows (TSPTW). The mathematical model of the TSPTW is provided in Appendix **??**. Following, at $t = \delta$, the second problem, which includes the requests that were not served until $t = \delta$ and the requests that are placed in the time between $t = 0$ and $t = \delta$ (N_1^+) is defined. As the vehicle is already en route at $t = \delta$, the drone must resupply the requests from N_1^+ to the vehicle, the second problem corresponds to the static problem in Section 2.3.2, such that $\theta = \delta$ and $N^d = N_1^+$. Note that, we assume that the vehicle cannot be interrupted en route. As a consequence, if the vehicle is en route at $t = \delta$, then the route starts at (s) the next location that the vehicle is headed to at t. The algorithm repeats the foregoing steps for $t >= \delta$ as follows.

Algorithm 1: Periodic reoptimization approach.

$t \leftarrow 0$
$N_0 \leftarrow$: set of requests that are available at t=0
$\delta \leftarrow$: time between decision epochs
Solve TSPTW in **??** with N_0
for all $s \in \{1, 2, ..., (T/delta) - 1\}$ **do**
    $N_s^- \leftarrow$: Set of requests that the vehicle either served or en route to serve until $t+\delta$
    $N_s^+ \leftarrow$: Set of new requests that are placed in between t and $t + \delta$
    $N^v \leftarrow (N_s \setminus N_s^-)$
    $N^d \leftarrow N_s^+$
    Solve the static TSPDR in Section 2.3.2 with N^v and N^d
    Update $t \rightarrow t + \delta$
    Update $i \rightarrow i + 1$
end for

Note that, we analyze the performance of drone resupply in the last-mile delivery problems by comparing it to conventional vehicle only delivery systems. The static case of the conventional vehicle only delivery problem corresponds to the TSPTW, as the static

TSPDR at $t = O$. For the dynamic TSPTW, we ensure that the vehicle resupplies new requests to be served from the distribution center by adding Constraint (3.1) to Appendix **??** and again using the aforementioned periodic reoptimization approach.

$$y_{il} \leq \sum_{k=0:k\leq l}^{n+1} y_{ik} \quad \forall i \in N^d, \forall l \in \{0, 1, ..., n+1\} \tag{3.1}$$

3.1.2 A Simulated Annealing based Metaheuristic

It is possible to solve the static cases of the TSPTW and TSPDR in Algorithm 1 with the mathematical programming model presented in Appendix **??** and Section 2.3.2. In addition to the mathematical programming approach, we also present a simulated annealing based metaheuristic as a solution approach. The main motivation behind proposing a heuristic approach is twofold. First, since the complete set of requests is only available by the end of the planning horizon, exact solution approaches provide an optimal solution just for the current state of the problem. That is, exact solution approaches cannot guarantee an optimal solution for the complete problem. Then, heuristic approaches, that produce good enough solutions, can reduce the computational effort required for the periodic reoptimization approach, without compromising the performance of the solution.

The simulated annealing (SA) algorithm has a significant impact among other heuristic algorithms for its simplicity and efficiency in solving combinatorial optimization problems (Talbi). The SA algorithm starts with an initial solution and iterates as follows.

Phase 1: Generating Initial Solution

We generate an initial delivery plan in two stages: (1) we first generate a feasible delivery route for the vehicle based on service time windows and then, (2) we define a feasible resupply route for the drone based on the corresponding vehicle delivery route. We generate an initial delivery route with respect to the earliest due date (EDD) rule. That is, if the service due time of request i is earlier than that of j ($b_i < b_j$), then the vehicle must serve request i before j ($y_{ik} = y_{jm} = 1 : k < m$). In case of a tie ($b_i = b_j$), the algorithm follows shortest processing time (SPT) rule. That is, if the travel distance (time) from request h to i is shorter than that to j ($d_{hi} < d_{hj}$), then the vehicle must serve request i after h ($y_{i(k+1)} = 1 : y_{hk} = 1$). Once we generate the delivery route, we define a feasible resupply route subject to the corresponding delivery route in the mathematical programming model in Section 2.3.2. We also refer to the corresponding solution as the parent solution.

Phase 2: Generating Neighbor Solutions

We generate neighbour solutions by randomly exchanging the positions of the requests on the parent delivery route. We again use the mathematical programming model to define a feasible resupply route for the neighbour delivery route. Note that, if random exchange results in an infeasible delivery plan, because either the delivery route is not feasible or there exists no feasible resupply route, the corresponding neighbour solution is not considered in the evaluation phase. We generate multiple neighbour solutions from the initial solution and evaluate neighbour solutions according to the objective function (2.1). That is, we choose the delivery plan with minimum environmental impact among neighbour plans. We refer to the corresponding neighbour solution as the candidate solution.

Phase 3: Selecting Solutions

The candidate solutions with better performance according to (2.1) are always accepted as the parent solution for the next iteration while the worsening solutions are accepted with a probability. The acceptance probability depends on the amount of degradation of the objective function and the control parameter of the algorithm, known as temperature.

At each iteration, starting from an initial temperature, the temperature and as a result, the acceptance probability decreases, according to a cooling procedure. We use the geometric schedule, in which the temperature at the current iteration (T_i) is updated based on a cooling ratio (α) as $T_i = \alpha T_{i-1}$. The algorithm terminates once it reaches a predetermined number of iterations. Here, the initial temperature (T_0), cooling ratio (α) and the number of iterations are set following a parameter tuning procedure.

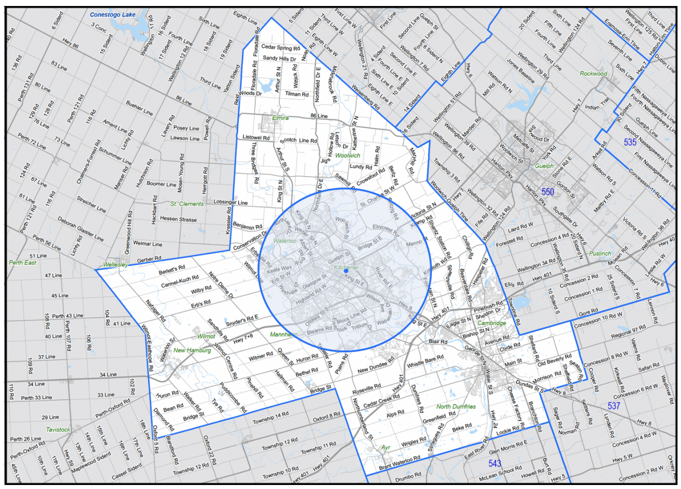
3.2 Computational Experiments

In this section, we first present the design of the computational experiments and then discuss the computational results. In Sections 3.2.1 and 3.2.2, we respectively provide the details of the instance generation process and propose different delivery service strategies for the last-mile delivery service. In Sections 3.2.3 and 3.2.4, we discuss strategies for the reoptimization approach, such as the impact of different short-term strategies on the long-term objective and different strategies to solve the static problems to the computational effort. In Section 3.2.4, we conduct detailed experiments to evaluate the performance of the drone resupply delivery system over the conventional vehicle only delivery system.

3.2.1 Instance Generation

For computational experiments, we evaluate a service area in the Kitchener-Waterloo-Cambridge area in Ontario, Canada. Online shopping in Canada is surging by 15% in spending each year and is expected to surpass 33 billion USD by 2024 Post; Coppola. Kitchener-Waterloo-Cambridge area covers a total of 121.16 square miles with a density of 3,864 residents per square mile in Ontario and offers a convenient environment for drone resupply with the tallest building in the region being just over 270 feet Canada. As illustrated in Figure 3.2, we set the distribution center at the center of the area's network and consider the area in 5 miles range from the distribution center to be the service zone. Note that, we dedicate a single vehicle and a single drone to serve random requests in the zone, as discussed in Section 2.3.1. We consider random order weights according to Uniform $[3, 5]$ (Prime). We represent the locations of the customers in terms of coordinates. We consider the shortest network distances for the vehicle using GraphHopper Routing API (GraphHopper) and the Euclidean distances for the drone using the estimation of the great-circle (haversine) formula (Wright).

Figure 3.2: Geographical representation of the service zone and distribution center



To evaluate the impact of the system parameters on the performance, we consider $v^v = 35, v^d \in \{50, 70\}$ (Ponza (2016), Wilke) and $\rho \in \{10, 15\}$ (Prime) for the drone. For each setting, we generate 5 different instances. That is, we investigate total of 20 instances for each of the delivery service strategies described in the following section.

3.2.2 Service Strategies

We consider three delivery service strategies, two hourly and one daily delivery strategies, that differ in terms of the characteristics of delivery time windows. For all of the strategies, we consider that the delivery service operates from 8:00 AM to 8:00 PM, which doubles the horizon considered in Dayarian et al. (2020). The strategies differ with respect to the service time guarantee (S_i), as a consequence the duration of the decision epochs in the reoptimization approach, and the earliest time that the request can be delivered (a_i).

According to hourly delivery services, all requests have a common service time guarantee, i.e., four hours. We update the set of available requests every two hours, i.e., $\delta = 2$ hours. and so re-route five times during the planning horizon. The parameter δ is chosen to ensure the feasibility of the solution and the practicality of the solution approach. We consider two different hourly delivery strategies: due-time (DT) and time-window (TW). In the due-time strategy, the earliest delivery time is the same as the order time, i.e., $a_i = \theta_i$, and so the time window is $[\theta_i, \theta_i + 4]$. In the time-window strategy, customers choose the earliest delivery time, which might be later than the order placement time, i.e., $a_i \geq \theta_i$. The order must be delivered to the customer in the time window $[a_i, a_i + 4]$. Even though the research in the literature, including Dayarian et al. (2020), rather focuses on daily delivery services, Joerss et al. (2016) points out that a critical percentage of customers will be expecting delivery services with time windows, and so 69% of e-retailers indicate an interest in investing to delivery service options with time windows Buhler (2016).

Daily delivery (DD) service corresponds to the well-known same-day (SD) delivery service, in which the online requests placed before 2:00 PM must be delivered during the same day and the ones placed after the cutoff time must be delivered before 2:00 PM on the following day Amazon. Service time guarantee differs for each request, depending on the time the request is placed. Joerss et al. (2016) reveals that 20 to 25 percent of customers are willing to pay significant premiums for their orders to be delivered on the same day, and so most e-retailers like Amazon are now offering same-day delivery services. We consider service time guarantees as hard constraints of the problem, and so the routes that do not meet the service time as infeasible. However, the approach presented here can simply be adapted to prevent infeasibility, by incurring a penalty in the objective function for every request that is not served during the guaranteed service time. In order to ensure feasibility, we set $\delta = 6$ hours. That is, we update the delivery route once the information regarding the set of remaining requests to be fulfilled during the day is revealed.

For all service strategies, we generate a random number of requests according to Uniform $[l, u]$. Note that, the following demands correspond to the demand in a predetermined service zone, that is dedicated to a single vehicle and drone. For hourly delivery strategies, we consider $u \in \{15, 20, 25\}$ and $l = 0$. We set the lower bound to zero in order to capture low demand periods, such as work hours. For daily delivery strategy, we consider $u \in \{10, 14, 20\}$ and $l = 2$. Each request is associated with a random location and weight as described in the foregoing section.

3.2.3 Routing Strategies

Prior to the evaluation of the drone resupply delivery system, we evaluate the performance of the solution performance for different settings. Reoptimization approaches as in Section 3.1.1, that is based on the repeated optimization of the static problems at given intervals, are the commonly accepted solution approaches in the literature to solve dynamic problems as in here (Angelelli et al. (2007)). To guarantee that the solutions of the static problems lead to efficient solutions for the dynamic problem, we discuss the impact of solution strategies for the reoptimization approach, such as the impact of different short term strategies on the long term objective and different strategies to solve the static problems to the computational effort.

We first evaluate the impact of the objective function on the solution performance. That is, whether we should consider the same objective as in the dynamic problem also for the static problems and if not, then what is the better performing objective to consider. In the TSPDR, the objective is to find the delivery plan with minimum negative externalities to the environment, i.e., the plan with the minimum fuel usage. We evaluate two different objective functions: (1) minimization of vehicle travel time (distance) and (2) minimization of service completion time. Note that, since the drones operate with electricity, minimizing environmental impact translates into minimizing the travel time (distance). Let z_V and z_S denote the value of the performance indicator that we compare for the first (1) and second (2) objective in order. Then, we calculate the percent change (Δ) as in Equation (3.2).

$$\Delta = \frac{z_S - z_V}{z_S} \times 100 \tag{3.2}$$

Tables 3.1 and 3.2 illustrate the performance of different objective functions for different performance indicators.

Table 3.1: Percent change (Δ) in hourly strategies for different objective functions.

Number of Requests	(v^d, c^d)	Δ Travel Time				Δ Wait Time		Δ Service Time	
		Vehicle		Drone					
		DT	TW	DT	TW	DT	TW	DT	TW
U[0, 15]	(50, 10)	-3.27	-2.37	-1.7	1.29	0.71	1.79	-0.49	0.64
	(70, 10)	-0.92	0.62	0	1.86	0	0.74	-0.2	0.72
	(50, 15)	0	-1.04	0	0.57	-0.06	0.35	-0.04	-0.04
	(70, 15)	-1.64	3.79	0	2.24	0.2	1.73	-0.2	2.15
U[0, 20]	(50, 10)	-1.34	-4.88	11.54	9.14	4.81	2.29	2.22	-0.65
	(70, 10)	-4.25	0.23	-0.93	4.2	-0.16	-1.8	-1.43	-1.19
	(50, 15)	0.5	1	12.8	11.73	2.11	-3.06	1.43	-1.33
	(70, 15)	1.67	0.48	-1.56	10.39	-0.93	-2.59	-0.12	-1.66
U[0, 25]	(50, 10)	0.08	2.9	0	-1.18	0.68	2.62	0.41	2.75
	(70, 10)	-0.65	2	0.66	2.82	3.72	-2.03	2.3	-0.63
	(50, 15)	-3.35	-1.49	3.69	6.1	4.4	-0.14	0.79	-0.77
	(70, 15)	-3.44	1.4	1.95	7.68	-3.32	-1.07	-3.36	-0.23
U[5, 20]	(50, 10)	-1.92	1.2	6.92	5.6	0.48	0.06	0.18	0.2
	(70, 10)	-3.08	0.18	5.46	2.35	0.3	0.01	0	0.03
	(50, 15)	-1.69	1.25	0.75	4.62	0.45	0.02	0.18	0.18
	(70, 15)	-6.82	-1.4	4.87	3.13	0.67	0.23	0	0.09
Average		-1.88	0.24	2.78	4.53	0.88	-0.05	0.1	0.02

Table 3.2: Percent change (Δ) in daily strategy for different objective functions.

Number of Requests	(v^d, c^d)	Δ Travel Time Vehicle	Δ Travel Time Drone	Δ Wait Time	Δ Service Time
U[2, 10]	(50, 10)	1.51	-31.26	0.9	1.06
	(70, 10)	-2.6	-9.4	-1.41	-1.64
	(50, 15)	1.31	-12.38	-0.52	-0.04
	(70, 15)	-0.11	-16.08	-0.41	-0.36
U[2, 14]	(50, 10)	-2.53	-13.29	3.53	1.57
	(70, 10)	1.64	4.18	0.36	0.66
	(50, 15)	3.5	-14.78	-2.49	-0.41
	(70, 15)	-0.83	1.55	-0.27	-0.41
U[2, 20]	(50, 10)	0.65	-9.51	-0.06	0.04
	(70, 10)	1.11	-13.18	0.12	0.23
	(50, 15)	1.18	9.86	0.02	0.18
	(70, 15)	-1.27	4.09	-0.39	-0.48
Average		0.3	-6.14	-0.05	0.03

The foregoing results highlight the importance of evaluating the impact of different short-term strategies over the long-term objective. Results show that, even though the second (2) objective minimizes the service completion time for the information available in each static problem, better service completion time values are achieved in the overall dynamic problem with the first (1) objective. Overall, results show that considering the first (1) objective in the short term, results in better solutions in terms of vehicle travel time (distance), i.e., fuel usage, and service time in the long term. Note that, even though the second (2) objective results in better solutions in terms of wait time overall, the difference is not significant. Hence, we consider the first (1) objective for the short-term objective in the static problems, which is the same as the long-term objective of the dynamic problem.

Following, we evaluate the impact of the approach used to solve the static problems on the computational performance. We consider two different solution approaches: (1) mathematical programming model (MM) (2) simulated annealing (SA) based metaheuristic approach. Furthermore, we consider two different static problems for the comparison: (1) the first optimization problem from Section 3.1.1 (TSPTW) (2) the static case of the

TSPDR from Section 2.3.2.

Table 3.3: Computational performances for the SA metaheuristic and MM.

TSPTW				TSPDR			
MM		SA		MM		SA	
Gap (%)	Run Time (sec)	Ratio	Run Time (sec)	Gap (%)	Run Time (sec)	Ratio	Run Time (sec)
26.03	TL	1	0.54	85.42	TL	1.23	1.36
0	1746.94	1.02	0.81	87.92	TL	0.96	1
0	1836.53	1.07	0.74	90.37	TL	1	1.01
0	5133.31	1.01	0.53	87.91	TL	1.13	0.83
0	1106.59	1	0.36	83.77	TL	1.44	0.79
12.75	TL	1	0.51	88.71	TL	1.38	0.89
13.54	TL	1	0.61	92.15	TL	1.31	0.64
14.32	TL	1	0.88	88.99	TL	1.16	0.47
0	3461.28	1	0.6	90.76	TL	1.24	0.7
7.41	3876.17	1.01	0.62	88.45	TL	1.2	0.85

Table 3.3 illustrates the advantages of using the SA metaheuristic approach over the MM in terms of the solution performance and computational time. The percentage gap and ratio respectively denote the optimality gap between the lower and upper bounds on the optimal value of MM and the ratio of best objective function value from the SA metaheuristic to that from the MM, in the time limit (TL) of 90 minutes. For the first problem (TSPTW), the SA metaheuristic approach reaches the optimal solution in 40% of the instances with an average ratio of 1.01 overall instances. For the 50% of the instances, for which the optimal solution cannot be obtained in the time limit, the SA metaheuristic approach finds the same best feasible solution as the MM with an average of 0.64 seconds. The advantage of the SA metaheuristic is even more obvious in the static case TSPDR. For the second problem, the MM cannot prove the optimality of the solution in the time limit for any of the instances. However, the SA metaheuristic results with an average ratio of 1.20 in an average run time of 0.85 seconds. Further, in one of these instances, the solution from the SA metaheuristic is better than the best solution from the MM in the time limit. Hence, we conclude that the SA metaheuristic achieves good solutions with

less computational effort, compared to the MM.

3.2.4 Evaluation of Drone Resupply Delivery System

We evaluate the performance of the drone resupply delivery system over the vehicle only delivery systems for different service strategies and settings. For both delivery systems, we use the following solution approach to solve the resulting routing problems: SA based metaheuristic from Section 3.1.2 to solve the static problem and reoptimization approach from Section 3.1.1 to solve the dynamic problem. Let z_C and z_P denote the value of the performance indicator that we compare for the conventional vehicle only and drone resupply delivery systems in order. Then, we calculate the percent change (Δ) relative to the conventional system as in Equation (3.3). Note that, $\Delta > 0$ indicates that the drone resupply outperforms the conventional system, whereas $\Delta < 0$ indicates otherwise, for the corresponding performance indicator. We evaluate performance indicators such as vehicle travel time, service time and cost. The service time represents the total time required to serve all the requests, i.e., the time between the vehicle's departure from the distribution center to the vehicle's arrival to the final location en route. The corresponding time includes the waiting time for the drone and the travelling time to the distribution center to resupply in the drone resupply and vehicle only delivery systems respectively. Further, to estimate the service cost, travel costs for the vehicle and drone are set to 50 and 2 units respectively in each delivery system (Wolhsen (2014)). According to the problem definition, the service cost includes only the travel costs, which are the fuel usage and charging costs for the vehicle and drone respectively.

$$\Delta = \frac{z_C - z_P}{z_C} \times 100 \tag{3.3}$$

We also analyze the strategic planning decisions impacting the operational decisions, by addressing the capacity utilization of both the vehicle and drone. The utilization of the vehicle refers to the ratio of the total time that the vehicle is en route to the total service time, whereas the utilization of the drone is calculated as in Equation (3.4). Further, we examine the endurance of the drone based on the minimum, maximum and average flight duration (in hours), to evaluate the settings that the drone resupply system is applicable to.

$$\text{Utilization} = \frac{\text{Total weight}}{\text{Number of trips} \times \text{Capacity}} \times 100 \tag{3.4}$$

Hourly Delivery Service Strategies

Table 3.4 summarizes the performance of the drone resupply system for the due-time (DT) and time-window (TW) service strategies. As expected, the drone resupply delivery system results in less negative environmental externalities, i.e., results in less fuel usage, compared to vehicle only delivery systems. For both the DT and TW service strategies, the drone resupply system decreases the fuel usage by around 20% according to Figure 3.5. Even though the drone resupply system outperforms the vehicle delivery system also in terms of service time according to Figure 3.6, the improvement is not as significant as in the vehicle travel time. This is explained by the capacity constraints of the system. That is, in order to resupply the same amount, the drone might need to do more trips to the distribution center in the drone resupply system compared to the vehicle in the vehicle only system, due to the capacity constraints, which increases the wait time. Note that, in the vehicle only system, the number of trips to the distribution center does not depend on the number (weight) of the requests but the number of re-routes. As a consequence, the improvement in service time is less than that of in the vehicle travel time. This interpretation is supported by the decrease in service time as the drone speed increases. An importation observation is that considering that the vehicle spends idle time waiting for the drone in the current system setting, it is possible to serve more customers in the meantime.

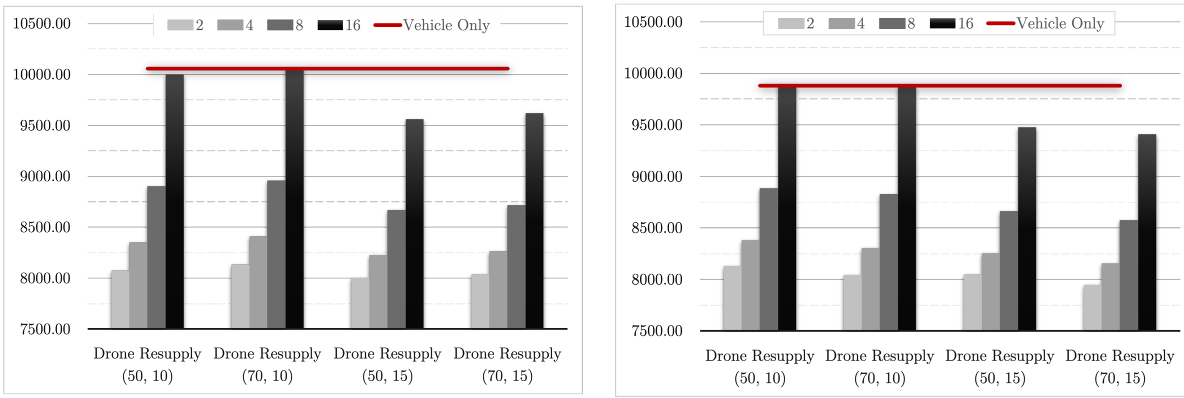
Table 3.4: Percent change (Δ) for hourly service strategies.

# Requests	(v^d, c^d)	Δ Time Vehicle DT		Service DT		Δ Service Cost DT	
		DT	TW	DT	TW	DT	TW
U[0, 15]	(50, 10)	19.58	17.04	0.16	0.28	17.09	14.73
	(70, 10)	20.91	19.74	0.57	0.64	18.41	17.32
	(50, 15)	21.87	15.77	0.29	0.12	19.92	13.83
	(70, 15)	20.47	19.27	0.53	0.63	18.61	17.34
U[0, 20]	(50, 10)	14.77	14.81	0.45	0.35	12.12	12.22
	(70, 10)	12.15	17.51	0.54	0.85	9.34	14.83
	(50, 15)	16.50	17.30	0.76	1.00	14.37	15.46
	(70, 15)	18.24	18.22	1.14	1.24	16.02	16.34
U[0, 25]	(50, 10)	32.25	29.54	2.97	2.16	29.05	26.73
	(70, 10)	31.68	26.95	3.42	1.88	28.73	24.04
	(50, 15)	29.95	27.54	3.45	2.01	27.51	25.17
	(70, 15)	30.30	28.53	3.29	2.39	27.72	26.08
U[5, 20]	(50, 10)	15.98	15.97	0.07	0.10	14.09	13.99
	(70, 10)	15.75	15.70	0.14	0.16	13.97	13.73
	(50, 15)	15.98	15.54	0.07	0.09	14.32	134.13
	(70, 15)	12.46	14.99	0.08	0.16	10.82	13.57
Average		20.55	19.65	1.12	0.88	18.26	17.47

Besides, the results show that the drone resupply is the more cost-effective delivery system, with around 18% savings in cost for both of the service strategies. To have a better understanding of the cost-effectiveness of using drone resupply, we further investigate the impact of the cost of the drone, by incrementally increasing the charging cost. Figure 3.3 illustrates the influence of the cost of the drone on the cost performance of the drone resupply system, which outperforms vehicle only system still when the charging cost increases to the fourth power of the initial cost.

Table 3.5 presents the utilization of the vehicle and drone. As in the service time results,

Figure 3.3: Total service cost in hourly service strategies for different charging costs.



(a) Due-time service strategy

(b) Time-window service strategy

the results indicate that the vehicle can serve more requests in the current setting of the drone resupply system. It is possible to do so: (1) either by considering a same size service zone with a higher request rate (2) or by increasing the size of the service zone with the same request rate, since even the maximum duration is in the practical endurance range of the drone (Murray and Chu (2015)) (3) or by changing the system setting, such that multiple drones resupply single vehicle. Such changes could further highlight the benefits of the drone resupply system. Following, a possible future research direction might be deciding the service zones and the zones, i.e., vehicles, that the drone resupplies. Note that in a setting where the service zone is considered as a variable, the flight duration of the drone must be constrained according to the practical endurance range in the problem definition. Nonetheless, the solution approaches presented here are still applicable to such problem settings.

Table 3.5: Capacity utilization and flight duration for hourly service strategies.

# Requests	$(\hat{vd}, \hat{cd})$	Utilization				Flight Duration (min, max, avg)	
		Vehicle		Drone			
		DT	TW	DT	TW	DT	TW
U[0, 15]	(50, 10)	30.61	28.17	50.57	54.88	(0.10, 0.66, 0.34)	(0.09, 0.60, 0.34)
	(70, 10)	30.39	27.75	50.57	54.88	(0.10, 0.48, 0.27)	(0.08, 0.45, 0.25)
	(50, 15)	29.8	27.99	39.23	44.97	(0.13, 0.67, 0.37)	(0.15, 0.63, 0.35)
	(70, 15)	30.77	27.72	41.51	44.97	(0.10, 0.48, 0.26)	(0.07, 0.48, 0.24)
U[0, 20]	(50, 10)	42.77	41.31	60.05	58.58	(0.15, 0.80, 0.43)	(0.08, 0.80, 0.38)
	(70, 10)	43.75	40.27	61.44	60.05	(0.12, 0.58, 0.34)	(0.07, 0.56, 0.30)
	(50, 15)	41.84	40.7	45.12	53.54	(0.18, 0.81, 0.46)	(0.12, 0.80, 0.43)
	(70, 15)	41.73	40.66	49.25	54.81	(0.13, 0.53, 0.29)	(0.06, 0.51, 0.30)
U[0, 25]	(50, 10)	46.04	46.61	94.08	63.73	(0.20, 1.00, 0.62)	(0.22, 1.00, 0.62)
	(70, 10)	47.36	47.26	61.86	63.73	(0.17, 0.71, 0.47)	(0.12, 0.72, 0.43)
	(50, 15)	48.79	46.54	50.41	60.21	(0.18, 0.97, 0.62)	(0.22, 0.92, 0.58)
	(70, 15)	47.18	45.97	49.06	46.78	(0.19, 0.69, 0.44)	(0.16, 0.69, 0.44)
U[5, 20]	(50, 10)	12.86	12.54	56.09	52.47	(0.03, 0.19, 0.12)	(0.01, 0.20, 0.11)
	(70, 10)	12.93	12.64	56.09	52.47	(0.03, 0.14, 0.09)	(0.01, 0.14, 0.08)
	(50, 15)	12.86	12.65	45.4	45.4	(0.02, 0.19, 0.13)	(0.02, 0.19, 0.1)
	(70, 15)	13.42	12.8	45.4	45.4	(0.02, 0.14, 0.10)	(0.02, 0.14, 0.07)
Average		33.32	31.97	53.51	53.55	(0.11, 0.57, 0.34)	(0.09, 0.55, 0.32)

The results indicate that the drone resupply system outperforms vehicle only system, in terms of service time and cost, besides the environmental advantages of the system, in both of the hourly service strategies. We further discuss for which of the two hourly strategies the improvement is more significant and our understanding of the reason. We observe marginally less improvement, in terms of both time and cost, in TW strategy to that of DT strategy. This difference is explained by the unbalanced number of requests over the interval in the TW strategy. That is, even though the number of requests placed in an interval is bounded by the same values in both strategies, the number of requests that must be served in an interval is different, since the TW service allows to postpone service time window later into the future unlike DT. As a consequence, there might be intervals such that, even though there are new requests to be served and the vehicle is idle, none of the new requests can be served in the TW strategy.

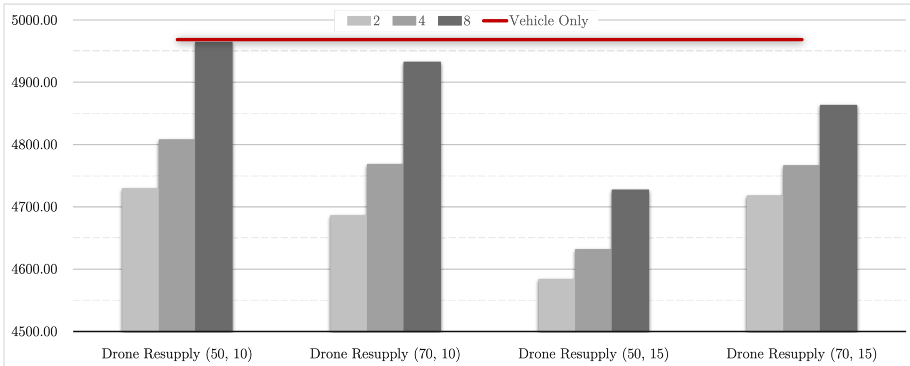
Daily Delivery Service Strategy

Tables 3.6 and 3.7 summarize the performance for the daily delivery (DD) service strategy. As in the hourly service strategies in Section 3.2.4, also in here, the drone resupply system outperforms vehicle only system both in terms of service time and cost, besides resulting in less negative environmental impact. Further, Figure 3.4 illustrates that the drone resupply system is the more profitable delivery system, even when the charging cost of the drone is cubed.

Table 3.6: Percent change (Δ) for daily service strategy.

# Requests	(v^d, c^d)	Δ Time		Δ Service Cost
		Vehicle	Service	
U[2,10]	(50, 10)	5.89	0.02	4.23
	(70, 10)	3.2	-0.04	1.64
	(50, 15)	7.68	0.04	6.75
	(70, 15)	6.17	0.05	5.22
U[2,14]	(50, 10)	5.13	0.01	3.86
	(70, 10)	5.07	0.03	3.74
	(50, 15)	6.08	0.01	5.28
	(70, 15)	4.07	0	3.18
U[2,20]	(50, 10)	6.83	-0.09	5.43
	(70, 10)	10.69	0.08	9.13
	(50, 15)	9.02	0.26	7.95
	(70, 15)	7.07	0.19	6.05
Average		6.41	0.05	5.2

Figure 3.4: Total service cost in daily service strategy for different charging costs.



However, the improvement here is as not as significant as in the hourly service strategies in Section 3.2.4 according to Figures 3.5 and 3.6. In DD, the average cost improvement is on average almost 12.5% less than that of in DT and TW strategies. This difference is explained by the number of times the vehicle is resupplied, because of the capacity constraints, in the DD strategy. In DT and TW strategies, the vehicle is resupplied every 2 hours, i.e., the time between decision epochs, to be able to meet 4 hours service time guarantee. On the other hand, in DD, there exists a single cutoff time at 2:00 PM, at which vehicle is resupplied. In the vehicle only system, the vehicle returns to the distribution center only once during the cut-off time for resupply, since the vehicle has no capacity. However, in the drone resupply system, the drone might need to do more than a single trip, due to the capacity constraints of the drone. That is, as the service time guarantee increases, the number of deadlines decreases and as a result number of trips to the distribution center decreases in the vehicle only system; however, the change in service time guarantee has no impact on the number of trips to the distribution center in the drone resupply system.

Figure 3.5: Percentage change (Δ) in carbon fuel usage (vehicle distance)

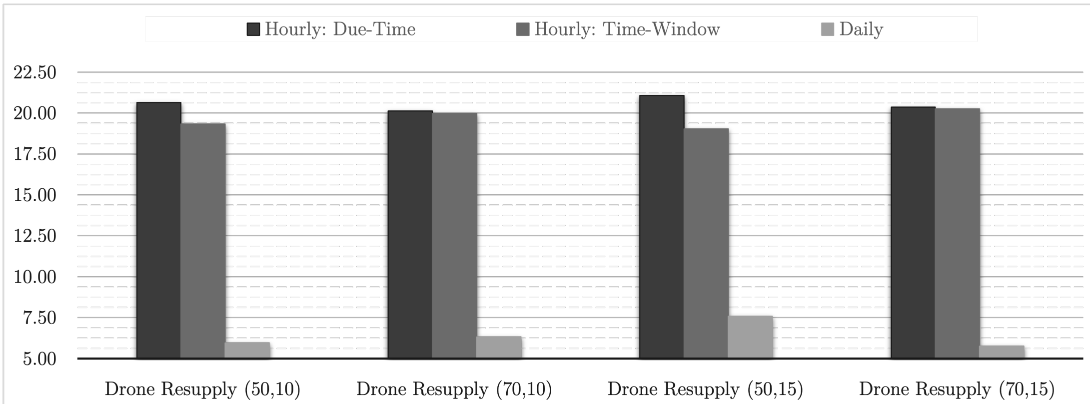


Figure 3.6: Percentage change (Δ) in service time (vehicle travel and wait time)

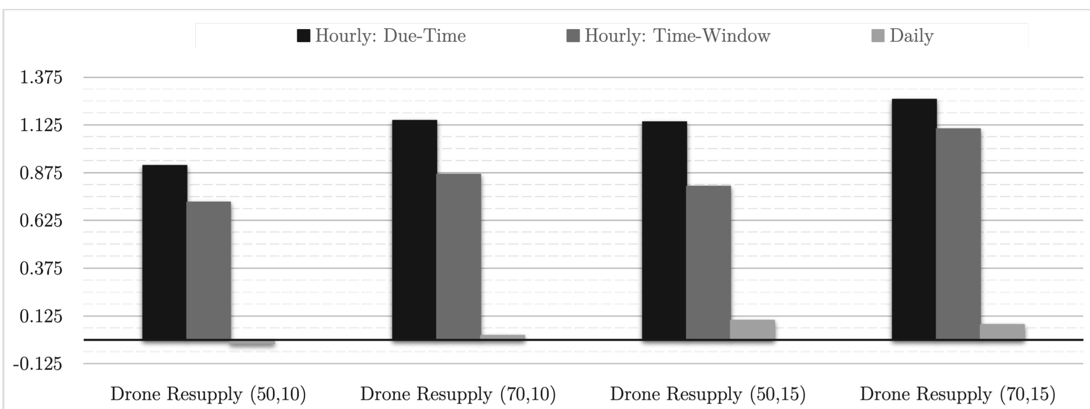


Table 3.7 present the utilization of the vehicle and drone in DD, which supports the observations from DT and TW strategies. That is, it is possible to serve more requests in the current setting of the drone resupply system. Possible future research directions based on these observations are further discussed in the conclusion.

Table 3.7: Capacity utilization and flight duration for daily service strategy.

# Requests	(v^d, c^d)	Utilization		Flight Duration (min, max, avg)
		Vehicle	Drone	
U[2,10]	(50, 10)	25.43	40.33	(0.26, 0.48, 0.88)
	(70, 10)	25.92	40.33	(0.21, 0.35, 0.27)
	(50, 15)	25.11	36.44	(0.31, 0.36, 0.33)
	(70, 15)	25.46	36.44	(0.25, 0.26, 0.25)
U[2,14]	(50, 10)	32.7	46.83	(0.44, 0.66, 0.55)
	(70, 10)	31.86	49.3	(0.30, 0.44, 0.37)
	(50, 15)	32.64	38.33	(0.39, 0.55, 0.47)
	(70, 15)	33.83	38.33	(0.23, 0.33, 0.28)
U[2,20]	(50, 10)	14.47	47.67	(0.12, 0.16, 0.14)
	(70, 10)	14.48	47.67	(0.10, 0.13, 0.11)
	(50, 15)	14.27	37.33	(0.10, 0.17, 0.13)
	(70, 15)	14.56	37.33	(0.07, 0.12, 0.09)
Average		24.23	41.36	(0.23, 0.33, 0.28)

3.3 Conclusion and Future Research

In this research, we evaluated the performance of an alternative drone integrated delivery system compared to conventional vehicle only delivery systems for several practical last-mile delivery settings. Unlike most of the research on drone integrated delivery systems in the literature, we considered a delivery system based on Dayarian et al. (2020), in which drones are not responsible for delivering orders to the customer, but responsible for resupplying vehicles with dynamically revealing requests. The delivery problem under consideration (TSPDR) accomplished to address the dynamism of the last-mile same-day delivery problems for which the drone integrated delivery systems are proposed in practice. We considered the delivery problem for an area dedicated to a single vehicle and a single drone. We proposed a reoptimization approach for the dynamic problem. Further, we investigated different routing strategies for the reoptimization approach, such as different short-term strategies based on different objectives and different solution strategies to solve the re-routing problems. We introduced an exact solution method based on mathematical programming, besides a simulated annealing based metaheuristic. We also investigated different delivery service strategies that differ with respect to service time guarantee and the earliest possible time that the request can be delivered.

The findings from our research are important to evaluate whether the drone resupply system is applicable to practical problems and the impact of the problem settings on the solution performance. The computational experiments showed that the drone resupply system outperforms the vehicle only system with respect to vehicle travel time, service time and cost over several different settings, and also is more environmentally friendly. The experiments also revealed that neither the drone nor the vehicle is fully utilized in the drone resupply system, which is an encouraging observation for possible future research. That is, the drone resupply system outperforms the conventional delivery systems even when the system is not fully utilized. It is possible to increase the utilization by changing the system setting, such that multiple drones resupply a single vehicle. A possible future research direction might be to decide the service zones and the zones, i.e., vehicles, that the drone resupplies or even to integrate the decision regarding the minimum number of vehicles and drones required to the routing problem. A more utilized system should further highlight the advantages of the drone resupply delivery system, as such setting could allow the same amount of demand to be fulfilled with fewer resources compared to the conventional delivery systems. Note that, the computational results showed that the drone resupply system is feasible for the given service zone, as the flight duration of the drone is in practical endurance range for drones. However, to extend the work to consider service zones as a variable, the travel distance of the drone must be constrained by the flight range in the problem definition. Likewise, another direction might be to consider the impact of the location of the distribution center on the feasibility of the drone resupply delivery setting. Since the delivery service strategies discussed here are either rather new in practice or even not achievable with conventional delivery systems, we considered a dynamic and deterministic environment as a starting point. An apparent direction for future research is to introduce exploitable stochastic information about future requests.

Chapter 4

Dynamic and Stochastic Last-Mile Delivery with Outsource

In this chapter, we introduce the motivation for the dynamic last-mile delivery problem with outsourcing. We review and discuss the relevant research in the literature. We then present the formal definition and the problem formulation of the problem.

4.1 Introduction

E-retailers experience several challenges due to ever-growing customer expectations with regards to delivery options, such as free delivery services and personally selected delivery time windows (Allen et al. (2018)). One of the challenges under these market conditions is to fulfill the demand with the available resources in times of fluctuations. Demand fluctuations might result in significant peaks during the season such as Christmas, or singular events such as Black Friday and Cyber Monday in the United States (Allen et al. (2016)). A global-scale demand peak is observed in e-retailing during the COVID-19 pandemic (OECD (2020)). These demand fluctuations require strategies that improve the resilience of the distribution network. One approach considered here when the total demand exceeds the available resources is to outsource a portion of the transportation operations to other logistics service providers. For example, the e-retailing giant Amazon transfers a portion of the demand to local postal services to be less dependent on parcel operators (Janjevic and Winkenbach (2020)). Besides ensuring the feasibility of delivery in cases when the demand exceeds the available resource capacity, outsourcing might also be an economically convenient option in several cases. For instance, in cases of isolated locations, far from others and the distribution center, having both the options to outsource to a common carrier and to serve with a vehicle from the private fleet, might introduce opportunities for reducing distribution costs.

Research in the transportation literature includes several fundamental optimization problems that address the search for efficient strategies in practical applications. Chu (2005) introduced the first routing problem that evaluates the impact of outsourcing. Recently, the attention around the subject in the literature has been growing. Nonetheless, the research is still rather scarce compared to the relevance of the problem in practice and the existing research does not address the requirements of the recent problems arising from e-retailers.

In this research, we investigate the impact of outsourcing in same-day delivery services in last-mile delivery systems. In this set-up, there is a fleet of private vehicles available to serve orders of the customer where each order defines a service request. At the beginning of each planning period, a subset of requests is available and due the same period. Throughout the period, customers dynamically place orders, which are due either the current or the next period depending on the time customer placed the order. That is, a request that is placed at the current period might either be served at the current or postponed to the next period. If the request is not served at the current period, then it might either be served with a private vehicle or outsourced to a common carrier at the next period. There are two main decisions to consider: (1) whether to serve a request at the current period or the next period with a private vehicle or to outsource to a common carrier (2) routes of the vehicles. Here, an important thing is that the outsourcing option is considered during the planning phase, instead of outsourcing the remaining, unfulfilled requests as a result of the plan, like in existing research in the literature. Thus, we are able to comment on the impact of outsourcing on a same-day last-mile delivery problem.

The research problem under consideration is dynamic, by the definition of the same-day delivery problems. That is, customers dynamically place same-day orders anytime during the day and as a result, a subset of the problem information is uncertain. Considering the relevance of the same-day delivery services in practice, we suggest that exploitable stochastic information might be gathered for the dynamically revealed information. Furthermore, our problem is a multi-period problem, in which customers place orders over several days, with the option to postpone delivery from one period to another. To the best of our knowledge, in this research, we are the first to introduce outsourcing in the literature of dynamic multi-period vehicle routing problems with probabilistic information (DVRPP). We refer to the corresponding research problem as the dynamic and stochastic multi-period vehicle routing problem with partial outsourcing (DS-MVRPPO). To fulfill customers' orders, we consider a fleet of homogeneous uncapacitated private vehicles and a common carrier that charges fixed cost per unit outsourced. We first discuss the deterministic case of the problem and present a mixed-integer linear programming model for the corresponding problem. Then, to capture the stochastic elements, we define recourse actions on the deterministic problem and propose a multi-stage stochastic programming model for the DS-MVRPPO. We also propose a dynamic stochastic hedging approach that uses a heuristic method to define the delivery plan at each stage based on the information from sample scenarios (Hvattum et al. (2006)). We conduct detailed computational experiments to observe the impact of postponing and outsourcing requests for different problem settings. We suggest two planning approaches for the assignment decisions. The first approach suggests outsourcing the remaining requests and to consider just the new set of requests that are placed in between periods at each period while the second approach chooses between outsourcing and postponing based on the stochastic information on future requests and the cost at each period. Furthermore, we also suggest two planning approaches for the route decisions. The first approach does not consider any outsourcing option during the planning phase and instead focuses on maximizing the number of requests that can be served with the private vehicle. On the other hand, the second one chooses between the options of outsourcing to a common carrier and serving with a private vehicle during the planning phase. These different planning approaches allow us to provide detailed conclusions about the impact and tradeoffs of postponing and outsourcing requests in last-mile delivery problems.

The remaining of this chapter is organized as follows. In Section 4.2, we review the research relevant to the problem under consideration and discuss our contribution to the literature. In Section 4.3, we present a formal definition and canonical model of the DS-MVRPPO. Then, in the next chapter, we discuss the proposed solution approaches and the conclusions of the computational experiments.

4.2 Literature Review

In this section, we review the literature related to the dynamic and stochastic vehicle routing problem with partial outsourcing introduced in Section 4.1. We first extend the brief literature review on its deterministic counterpart from Section 2.2.2 to include the particular literature on the dynamic and stochastic vehicle routing problems in Section 4.2.1. Then, in Sections 4.2.2 and 4.2.3, we review the literature on multi-period routing and partial outsourcing, as the problem under consideration relates to research from both pieces of literature.

4.2.1 Dynamic and Stochastic Routing

The dynamic and stochastic routing problems (SDVRPs) are extensions of their deterministic counterparts mentioned in Section 2.2.2. Here, in contrast with the former literature, exploitable stochastic knowledge is available on the information that reveals dynamically during the decision process. In SDVRPs, routing decisions are made in anticipation of this dynamically revealed stochastic information on future requests, to improve the solution performance. Accordingly, to design ever more efficient routes, the research in the literature has expanded to design solution methods that can integrate potential information on the future to the decision model. In the following, we first present the categorization of SDVRPs and methods proposed to solve SDVRPs in the literature based on Soeffker et al. (2022). Here, problems are categorized according to the application areas, sources of stochastic information and solution methods are categorized according to exploitation and use of information model in the decision model. Then, we introduce our problem in this categorization and present the review of literature under the same categorization.

Applications areas of the SDVRPs are extensive. For instance, the last-mile same-day delivery problems, dial-a-ride problems, ride-sharing problems and health-care services. Nonetheless, these problems might be categorized under three broad areas, such as transportation of goods, transportation of passengers and provision of services. Furthermore, even among the different application areas, each of these problems has common characteristics in terms of settings, such as resources, demand and environment. That is, in every SDVRP, routing of the resources is required to fulfill the demand in an environment. Any of these three dimensions might be the source of uncertainty in the dynamic problem. Disruptive events like malfunctions or accidents are instances of stochasticity in resources, in terms of vehicle availability. Stochasticity in demand occurs in the form of uncertainty about when and where a customer requests, how much a customer requests or the time required to serve the request. Finally, the weather is an instance of stochasticity in the environment, in terms of travel times. The review of Soeffker et al. (2022) shows that the literature in SDVRP focuses on the uncertainty around the demand, through considering stochastic requests, demands and service times.

The SDVRP might be described as an interconnected sequence of iterations between information model realizations and decision model instances (Soeffker et al. (2022)). Following, solution methods for SDVRPs might be categorized according to exploitation and use of information model in the decision model. According to this categorization, there exists three categories as descriptive, predictive and prescriptive methods. Descriptive methods consider just the observed realizations of the information model in the current state. On the other hand, predictive methods analyze and consider the characteristics of the information model in the decision model. Furthermore, prescriptive methods analyze both the information model and the interaction between the information and decision model. Predictive and prescriptive methods might further be categorized according to the stage in which the information is considered, as to whether it is considered externally to derive the method or internally during the implementation of the model. It is important to note that there exists no one dominant method applicable to all the SDVRPs and in fact, the fitness of a method depends on the characteristics of the information and decision models (Soeffker et al. (2022)).

The SDVRP under consideration is based on an application that delivers goods to customers that are revealed dynamically. That is, according to prior categorization, we consider an application from the area of transportation of goods with stochastic requests. Note that, the categorization differs between stochastic requests and demand quantities such that the prior refers to the uncertainty about when and where a customer requests besides the uncertainty about the demand quantities. Table 4.1 summarizes the research in the literature from the last decade that also considers the same problem settings and presents different solution methods proposed for the corresponding problem according to the categorization of Soeffker et al. (2022). Following the above-mentioned categorization of the solution methods, the solution method proposed in our research is a predictive and internal method, which is also the common approach in the literature based on the below review. That is, our solution method considers the potential information on the future during the decision-making process, unlike in the descriptive methods; however, does not consider the potential changes in the decision-making process, unlike in the prescriptive methods. Furthermore, the potential impact of the future is assessed in the decision model and not approximated outside, unlike in external methods. There are two common strategies considered for the predictive and internal methods in the literature, which are scenario-based approaches and post-decision rollout algorithms. The solution method proposed here is a scenario-based approach known as the multiple-scenario approach (MSA). These methods allow detailed short-term anticipation; however, generally computationally intractable for long-term anticipation due to the significant effort required to solve the scenarios in detail. Nevertheless, through computational experiments, the heuristic algorithm proposed to solve scenarios is shown to be an effective approach. Hence, the solution method proposed in this research is considered to be a suitable match with the requirements of the problem under consideration.

Table 4.1: Review of the SDVRP literature with common characteristics

Reference	Application	Stochastic	Approach
Lorini et al. (2011)	g	r, t	-
Azi et al. (2012)	g	r	ii - b
Ferrucci et al. (2013)	g	r	ii - b
Ghannadpour et al. (2013))	g	r, t	-
Ferrucci and Bock (2014)	g	r, t, v	ii - a
Albareda-Sambola et al. (2014)	g	r	ii - b
Sarasola et al. (2016)	g	r, d	ii - b
Billing et al. (2018)	g	r	ii - b
Klapp et al. (2018)	g	r	ii - b
Zou and Dessouky (2018)	g	r	ii - b
Arslan et al. (2018)	g	r, d	-
Soeffker et al. (2019)	g	r	iii - b
Brinkmann et al. (2019)	g	r, d	ii - b
Ulmer et al. (2018)	g	r	iii - b
Ulmer and Thomas (2018)	g	r	iii - b
Ulmer et al. (2019)	g	r	iii - b
Voccia et al. (2019)	g	r	ii - b
Joe and Lau (2020)	g	r	iii - b
Ulmer (2020b)	g	r	iii - b
Ulmer and Thomas (2020)	g	r	iii - b
Chen et al. (2022)	g	r	iii - b

Application: good (g), passenger (p), service (s)

Stochastic: request (r), demand (d), time (t), vehicle (v)

Approach: descriptive (i), prescriptive (ii), predictive (iii) - external (a), internal (b)

4.2.2 Multi-Period Routing

The literature in vehicle routing focuses on the decision problems over a single period, in which the operations start at the beginning of the planning period and terminates at the end of the same period, once the orders in the vehicles are delivered to the customers (Larrain et al. (2019)). That is, each period is independent of the others and so, the period corresponds to the planning horizon. However, besides the single period problems, there are several practical applications in which operations repeat with a certain period over the planning horizon, such as the collection of waste or replenishment of vending machines. The corresponding routing problems are known as the periodic vehicle routing problem (PVRP) in the literature. If the periodicity of the service is not required, i.e., routes at each period depend on others but are not repeated, then the resulting problem corresponds to a special case of PVRP, known as the multi-period vehicle routing problem (MVRP). In the other words, the MVRP is a variant of the PVRP in which delivery frequency for all customers is one (Wen et al. (2010)). In the following, we review the limited research on the literature of the MVRP that also considers either or both dynamic and stochastic features of the problem.

Angelelli et al. (2007) are first in the literature to include the decision to postpone orders in a dynamic multi-period setting. In their research problem, orders of customers must be fulfilled at either the current or the next period. That is, the routes of the next period depend on the route of the current period, since customers might be postponed from one period to another. Furthermore, their problem considers a dynamic and deterministic setting such, that the customers reveal dynamically throughout the planning horizon and decisions have to be made with no knowledge on the future orders. The authors refer to the corresponding problem as the dynamic multi-period routing problem (DMPRP) and focus rather on the dynamic nature of the problem by considering a simple routing problem with a single uncapacitated vehicle.

Then, Angelelli et al. (2009) generalize the previous research to account for different settings. In their extension, authors categorize orders based on the order placement and due times. According to this categorization, orders that are placed throughout the period of planning are referred to as online orders while the orders that are known prior to the planning are as offline. Furthermore, orders that are due the current period are referred to as unpostponable while orders due the next period are as postponable. Note that, we also refer to this categorization in our problem definition. The authors assume orders that are neither selected to be fulfilled in the current period nor postponed to the next period are transferred to an alternative service at a high cost. In our research, we consider the option to outsource during planning, along with the options to select and postpone, instead of outsourcing the remaining. For their dynamic problem, authors propose different short-term routing strategies based on different look-ahead periods. Angelelli et al. (2009) suggest a further extension to investigate the cases of capacitated vehicles and time windows.

Following, Wen et al. (2010) extend the research of Angelelli et al. (2007) to consider both fleet of capacitated vehicles and different service time windows for customers. However, unlike the previous and our research, their research does not differentiate between unpostponable and postponable for online orders, since all dynamically revealed orders are assumed to be due earliest by a time in the next period. In other words, no same period orders are accepted. Wen et al. (2010) present a mixed-integer linear programming model and propose a rolling horizon method based on a heuristic to solve the problem.

Albareda-Sambola et al. (2014) further extend the problem from a deterministic to a stochastic setting and refer to the corresponding problem as the dynamic multi-period vehicle routing problem with probabilistic information (DVRPP). The authors consider exploitable probabilistic information regarding the future service requests of customers. Albareda-Sambola et al. (2014) propose an adaptive policy in which the set of customers to be served during a period is determined based on geographical compatibility of order locations. Ulmer et al. (2018) also consider a similar problem but refer to it as multi-period dynamic vehicle routing problem stochastic service requests (MDRPSR) instead. Ulmer et al. (2018) propose an anticipatory policy based on approximate dynamic programming. In both Albareda-Sambola et al. (2014) and Ulmer et al. (2018), all orders of the customers are either selected for the current period or postponed and so the option to reject, as in

Angelelli et al. (2009), or to outsource, as in this research under consideration, is not considered.

4.2.3 Outsourcing

In distribution logistics, it is possible to either serve a customer by using the vehicles from the private fleet or assign the customer to a common carrier. The latter case is also known as outsourcing and is considered to be beneficial when the customer is located in an isolated location, far from other customer locations and the distribution center, or the order of the customer cannot be accommodated in a vehicle from the private fleet. As a result, outsourcing of infeasible or unprofitable customers is considered in the design of several practical distribution services. The corresponding routing problem is a generalization of the classical vehicle routing problem and is known as the vehicle routing problem with private fleet and common carriers (VRPPC) in the literature. The VRPPC consists of decisions regarding both clustering and routing (Gahm et al. (2017)). The first decision is to select one of the two service modes for each customer. Then, the second decision is to route the vehicles in the private fleet to serve the customers according to the first decision. Despite the relevance of the problem in distribution logistics, the VRPPC has received scarce attention in the literature. In the following, we review the literature around the VRPPC that considers outsourcing.

Chu (2005) is the first in the literature to examine the VRPPC. In their research problem, a fleet of heterogeneous capacitated vehicles is routed to deliver orders of known customers, from a single depot. In Chu (2005), unlike in our problem, no service time windows are considered for the customers. Each customer is served exactly once by one vehicle from either the private fleet or common carrier. Chu (2005) formulate the corresponding problem with an integer linear programming model and propose a solution approach based on the savings heuristic procedure of (Clarke and Wright (1964)). Bolduc et al. (2008) show that the VRPPC can be formulated as a heterogeneous VRP. The authors also present a heuristic approach based on perturbation which improves the solutions presented in Chu

(2005). Following, Côté and Potvin (2009) propose a tabu search heuristic which outperforms the solution approach from Bolduc et al. (2008) for the case of homogeneous vehicles. Furthermore, Potvin and Naud (2011) build upon the tabu search heuristic from Bolduc et al. (2008) and present improved solutions for both homogeneous and heterogeneous vehicles.

Gahm et al. (2017) introduce a generalization of the VRPPC which presents a realistic cost function to calculate the cost of common carriers and also considers additional rental options. The authors refer to the corresponding problem as the vehicle routing problem with a private fleet, multiple common carriers offering volume discounts, and rental options (VRPPCdR). In the VRPPCdR, the common carrier's cost function is defined based on the demand volume and travel and accounts for unit discounts. The authors propose a heuristic solution approach based on variable neighbourhood search.

Dabia et al. (2019) present another generalization of the VRPPC which includes both practical cost structures and service time windows. In their generalization of the VRPPC, the authors define a cost function with a discount structure such that the unit cost to outsource an order depends on the total number of orders outsourced to the common carrier. The authors refer to the corresponding problem as the rich VRPPC (RVRPPC) and present the first known exact method to solve a class of VRPPC. Through several examples from computational experiments, Dabia et al. (2019) also present managerial insights regarding the impacts of outsourcing on the routing decisions and also the impacts of problem parameters on outsourcing decisions.

Baller et al. (2020) consider another generalization in which customers can be served by both a private vehicle and a common carrier, instead of assigning each customer to exactly one service mode as in the VRPPC. In their research problem, customers are not allowed to be served by more than one vehicle from the private fleet of heterogeneous vehicles. In our research problem, we also implicitly allow split deliveries between private fleet and common carrier, as we exclusively define requests for each order of each customer. However, unlike their research, we allow split deliveries among the vehicles of the private fleet as well. The authors consider service time windows for the customers and assume fixed cost per unit outsourced. Baller et al. (2020) refer to the corresponding problem as vehicle routing problem with partial outsourcing (VRPPO) and propose an exact solution method based on the branch-and-price-and-cut algorithm. Computational experiments compare the solutions for VRPPO and VRPPC following Dabia et al. (2019) and present insights regarding the impact of partial outsourcing.

4.3 Problem Framework

In this section, we present the problem under consideration. First, we introduce the dynamic and stochastic multi-period vehicle routing problem with partial outsourcing (DS-MVRPPO) in Section 4.3.1. Then, in Section 4.3.2, we present a modelling framework for the DS-MVRPPO through a Markov decision process (MDP) model.

4.3.1 Problem Definition

The DS-MVRPPO is defined as follows. A set of orders must be delivered to customers over a planning horizon of T periods. Each order of each customer defines a service request. At the beginning of each period, a subset of the requests is available, while the remaining reveals dynamically over time. This problem is dynamic due to this nature of the service requests. Furthermore, to be able to integrate potential information on these future requests into the current decision, we consider exploitable stochastic information on dynamically revealing service requests. The service requests that are known at the beginning are referred to as offline requests and the ones that are placed during the planning period are as online requests. Each request is further differentiated with respect to the time window in which the order must be delivered to the customers. That is, the order from an unpostponable request must be delivered to the customer in the same period as it is placed while the order from a postponable request might also be postponed to the next period.

Each service request must be fulfilled by one of the three following options: (1) selected for the current period (2) postponed to the next periods (3) outsourced. Requests that are selected for the current or postponed to the next period are served with a vehicle from the private fleet while outsourced requests are transferred to a common carrier once the decision is finalized. Once the request is selected to be fulfilled during the current period, the corresponding decision is final; however, postponement and outsourcing decisions are not finalized until the end of the planning period. Accordingly, since the unpostponable requests can neither be postponed nor be outsourced by definition, unpostponable requests must be selected to be fulfilled during the current period. On the other hand, postponable requests might also be postponed or outsourced based on the cost. That is, considering the requests of the next period, if it is more cost-effective not to fulfill a postponable request during the current period, it is either postponed or outsourced. This option to postpone requests from the current period to the next indicates that this problem is multi-period. Furthermore, since the same customer is allowed to be served with both private vehicle and common carrier for different orders based on our definition of request, we also consider partial outsourcing.

A set of homogeneous uncapacitated vehicles are available at the distribution center to deliver orders to the customers. Note that, even though vehicles are uncapacitated, service time windows enforce a limit on the number of requests that can be served with each vehicle. Each vehicle might do multiple trips throughout the period, each starting and ending at the distribution center at each period. Besides ensuring feasible plans for unpostponable requests, returning to the distribution center to fulfill requests during the same period, instead of postponing or outsourcing, might also result in savings in terms of cost for postponable requests.

The problem is to define the plan, that assigns the requests and routes the vehicles, with the minimum total cost. The total cost comprises three components; the setup and operational costs of the private vehicles and the cost of the common carrier. Note that, the cost of the common carrier does not depend on the route of the common carrier and does not benefit from a flat rate or a discount structure.

4.3.2 Decision Model

Markov decision processes (MDPs) connect applications to models by representing the decision environment through system dynamics Powell (2019). We also refer to the MDP modelling framework to provide a precise description of the DS-MVRPPO. The MDP model is characterized by the following elements.

State Space

We consider a finite planning horizon in which decisions are made at predetermined time intervals, that are also referred as stages. The state of the system at a stage defines the information required to decide on the assignment of the requests to the subsets, such that the subsets of requests selected for the current, postponed to the next period or outsourced, alongside the routes of the vehicles to fulfill the requests at the corresponding stage. Let the state of the system at stage e be denoted as S_e. Further, let the subset of service requests that have not been fulfilled until stage e be denoted as $C_e = C_e^u \cup C_e^p$, in which C^u and C^p represent the subset of unpostponable and postponable requests respectively. The attributes of the request $i \in C_e$ are denoted with the tuple $(\rho_e^i, \tau_e^i, a_e^i, b_e^i)$. Here, ρ_e^i and τ_e^i represent the location of the customer and time of the order from the request i. Further, $[a_e^i, b_e^i]$ represent the service time window of $i \in C_e$. Let the set of vehicles in the private fleet be denoted as K. That is, K represent the vehicle to be routed at stage e. The attributes of the vehicle in $k \in K$ are denoted with the tuple $(\mathrm{P}_e^k, \mathrm{T}_e^k, \gamma_e^k)$. Here, P_e^k represents the location of the vehicle k which might also be the distribution center while T_e^k represents the last time vehicle k was at the distribution center. The subset of orders from $i \in C_e$ that are loaded to vehicle k at stage $e-1$ but not delivered to the customer until the beginning of the stage e is represented with γ_e^k. Following, the state of the system S is shown with the tuple of information $((\rho_e, \tau_e, a_e, b_e), (\mathrm{P}_e, \mathrm{T}_e, \gamma_e))$.

Action Space

At each stage e, an action is chosen based on a policy function $X^\pi(S_e) = ((x_e, y_e, z_e), p_e)$, that generates the feasible decisions $((x_e, y_e, z_e), p_e)$ for state S_e such that π presents the information about the function. An action in each stage includes decisions regarding the assignment of the requests and the route of the private vehicles. Let the assignment of the requests that are selected to be delivered during stage e be denoted with the set $x_e \subseteq C_e$, such that $x_e^k \subseteq x_e$ represents the subset of requests that are selected to be delivered with vehicle $k \in K$. Furthermore, let the assignments of the requests that are postponed and outsourced be denoted with the sets y_e and z_e, such that $y_e, z_e \subseteq C_e$. Then, let the route of the private vehicles be denoted with the set p_e, such that elements of p_e represent the times that the private vehicles fulfilled the requests in x_e. Here, p_e^i denotes the times that the request $i \in x_e$ is fulfilled. Note that, at each stage, since the state of the system (S_e) changes over time, the routes are designed just for the stage and so for the current period. Further, the route of the common carrier is not considered per the problem definition.

$$\{x_e, y_e, z_e \subseteq C_e, \quad p_e \subseteq \mathbb{R}^+ \ | \tag{4.1}$$

$$x_e \cup y_e \cup z_e = C_e, \tag{4.2}$$

$$x_e \cap y_e = x_e \cap z_e = y_e \cap z_e = \emptyset, \tag{4.3}$$

$$x_e \supseteq C_e^u, \tag{4.4}$$

$$y_e \subseteq C_e^p, \quad z_e \subseteq C_e^p, \tag{4.5}$$

$$p_e^i \in [a_i, b_i], \quad \forall i \in x_e \tag{4.6}$$

Constraint (4.2) and (4.3) indicate that all requests in C_e must either be assigned to a private vehicle to be served during current or next periods or be transferred to a common carrier. Constraint (4.4) ensures that the unpostponable requests must be served at the current period while Constraint (4.5) ensures that the postponed and outsourced requests are postponable. Constraint (4.6) ensures the service time windows.

Transition Function

Once the action is chosen, then based on the deterministic transition function the state of system changes from S_e to post-decision state $S_e^{((x,y,z),p)}$. In $S_e^{((x,y,z),p)}$, the subset of requests that are fulfilled according to x_e in stage e are eliminated from C_e. The attributes of the request $i \in C_e$ in $S_e^{((x,y,z),p)}$ remains the same as in S_e. Furthermore, if vehicle $k \in K$ did not serve requests during stage e, then in $S_e^{((x,y,z),p)}$ also the attributes remains the same as in in S_e. Otherwise, the attributes of the vehicle $k \in K$ are updated according to p_e. Note that, the transition from S_e to $S_e^{((x,y,z),p)}$ is deterministic since the system changes in response to a chosen decision.

However, the transition from the post-decision state $S_e^{((x,y,z),p)}$ to the next pre-decision state is probabilistic, since the requests are dynamically revealing in between stages e and $e+1$ and the regarding information is only known probabilistically at the time of the decision. Note that, the stage $e+1$ is triggered at a predetermined time interval. The requests and the corresponding tuple of information that are revealed with the triggering of stage $e+1$ are also referred to as exogenous information. The exogenous information constitutes W_e the realization of the random variables regarding the dynamic requests. Here, the probability of transition between states is assumed to be known and independent. Then, given the post-decision state $S_e^{((x,y,z),p)}$ and the realization of the exogenous information W_e, the system transitions to the next pre-decision state $S_e + 1$. In S_{e+1}, the attributes of the requests in C_e still remains unchanged from $S_e^{((x,y,z),p)}$ to $S_{(e+1)}$ and the attributes of the new requests are introduced according to W_e.

Reward Function

An action is associated with a cost. According to $((x_e, y_e, z_e), p_e)$, let the total of the setup and operational costs of the private vehicle $k \in K$ be denoted as f_k^T and outsource cost of the common carrier be denoted as q^T. Then, the cost function $R(S_e, ((x_e, y_e, z_e), p_e))$ is as follows.

$$R(S_e, ((x_e, y_e, z_e), p_e)) = \sum_{k \in K} f_k^T + q^T \tag{4.7}$$

Further, let the expected cost from stage e to the final stage in the planning horizon T be denoted as $r(S_e)$. Then, $r^\pi(S_e)$ is as follows.

$$r^\pi(S_e) = \mathbb{E}^\pi \sum_{e \in T} R(S_e, X^\pi(S_e)) \tag{4.8}$$

The objective is to find an optimal policy ϕ^* such that $r^{\pi^*}(S_e) \leq r^\pi(S_e)$ for each π among the set of possible policies Π.

The abovementioned model demonstrates that the dynamic problem considered in our research suffers from the curse of dimensionality because of the size of the state and action spaces. The uncertainty regarding the requests results in remarkably large state space. Further, an extensive range of feasible actions must be defined for each state in each stage. Hence, in the practical instances of the problem, it is computationally intractable to calculate the expected cost in Objective (4.8) for each state, let alone find the optimal policy over each action, with the traditional approach that uses Bellman equation Powell (2019) and dynamic programming. Following, we propose to use a stochastic look-ahead approximation as an alternative methodology to sequentially solve Objective (Constraint (4.8)) to find an optimal policy. A multi-stage programming model and a progressive hedging algorithm are proposed in Chapter 5 as alternative approaches to solving the sequential decision problems. Note that, the proposed approaches do not depend on the assumptions about the distributions of the stochastic variables.

Chapter 5

Solution of the Last-Mile Delivery Problem with Outsource

In this chapter, we present a multi-stage stochastic programming model for the dynamic and stochastic multi-period vehicle routing problem with partial outsourcing (DS-MVRPPO) from Section 4.3. We also present a dynamic stochastic hedging heuristic approach to address the practical instances of the problem. We discuss the impact of postponing and outsourcing requests in last-mile delivery systems. We discuss the conclusions of the research and future directions.

5.1 Multi-Stage Stochastic Programming Model

In this section, we first present a mathematical model for the static and deterministic case of the problem (MVRPPO). Then, we introduce a multi-stage stochastic programming model of the DS-MVRPPO, in which the recourse actions are defined based on the decision definitions of MVRPPO.

It is important to first discuss the MVRPPO, as the multi-stage stochastic model of DS-MVRPPO is based on the decisions introduced in the model of MVRPPO. The following model is an extension of the mathematical model in Wen et al. (2010). Different from that in Wen et al. (2010), the following accounts for the same-period requests. That is, it also considers different time windows during the same period. Also, to fulfill the same-period requests that are placed later in the period, the model allows vehicles to return to the distribution center multiple times throughout the period. This is especially important to be able to extend the problem to a dynamic setting. Furthermore, unlike the MVRP model in Wen et al. (2010), the model considers the option to outsource requests.

The MVRPPO is defined over the planning horizon of $T \in \{1, 2, \ldots, h\}$ as follows. Note that, here in the static and deterministic problem, the values of all the following parameters are available and known from the beginning of the T.

Let the set of requests be denoted as $C \in \{1, 2, \ldots, n\}$ and the distribution center as 0. Each request $i \in C$ has a service time window $[a_i, b_i]$ in which it must be fulfilled with either a private vehicle or a common carrier. The time the customer placed the request $i \in C$ is denoted with τ_i where $\tau_i \leq a_i$. Note that, request $i \in C$ might be picked up from the distribution center prior to a_i but not to τ_i. Let the set of homogeneous uncapacitated vehicles in the private fleet be denoted as $K \in \{1, 2, \ldots, m\}$. Each vehicle has a periodic service time window $[a_0, b_0]$, which corresponds to the time when the route of each vehicle might start and must end at the distribution center. Travel time between nodes i and $j \in N$ is denoted with s_{ij} and defined to also include service time at $i \in N$. Operational cost between i and $j \in N$ and vehicle setup cost for $k \in K$ are denoted with c_{ij} and f_k. Note that, s_{ij} and c_{ij} satisfy triangle inequality. The cost to outsource request $i \in C$ is z, regardless of the request or the total requests outsourced. The decisions to be considered are whether to serve a request $i \in C$ with a private vehicle either at the current or the next (postpone) periods or to transfer to a common carrier (outsource) and the route for each vehicle in K at each period in T. The objective is to determine a feasible plan with minimum total cost which includes setup and operational costs of the vehicles in the private fleet and outsource the cost of the common carrier.

The decision variables of the problem are y_{ikt}, such that $y_{ikt} = 1$ if request $i \in C$ is served with a private vehicle $k \in K$ at period $t \in T$ and $y_{ikt} = 0$ if otherwise it is outsourced to a common carrier to be delivered at period $t+1 \in T$. The route of the vehicle $k \in K$ at period $t \in T$ is defined together with variables u_{ijkt} and v_{ijkt}. Finally ,p_i and q_i denote the time that request $i \in C$ is picked up from the distribution center and delivered to the customer, respectively. Following, the MVRPPO is formulated with a mathematical model as follows.

Sets

$T = \{1, 2, ...h\}$: Set of periods

$C = \{1, 2, ..., n\}$: Set of requests

$N = \{0, 1, ..., n\}$: Set of all nodes with the distribution center

$K = \{1, 2, ..., m\}$: Set of vehicles

Parameters

τ_i : Time of request $i \in C$

$[a_i, b_i]$: Service time window of request $i \in N$

s_{ij}, c_{ij} : Travel time, cost from request i to $j \in N$

f_k : Setup cost of vehicle $k \in K$

z_i : Outsource cost for customer $i \in C$

Decision Variables

$$y_{ikt} = \begin{cases} 1, & \text{if request } i \in C \text{ is served with vehicle } k \in K \text{ at period } t \in T, \\ 0, & \text{otherwise} \end{cases}$$

$$u_{ijkt} = \begin{cases} 1, & \begin{array}{l}\text{if request } i \in C \text{ is picked up from the distribution center following } j \in C \\ \text{with vehicle } k \in K \text{ at period } t \in T\end{array}, \\ 0, & \text{otherwise} \end{cases}$$

$$v_{ijkt} = \begin{cases} 1, & \begin{array}{l}\text{if request } i \in C \text{ is delivered following } j \in N \\ \text{with vehicle } k \in K \text{ at period } t \in T\end{array}, \\ 0, & \text{otherwise} \end{cases}$$

$$p_i = \text{time when request } i \in C \text{ is picked up from the distribution center}$$

$$q_i = \text{time when request } i \in C \text{ is delivered}$$

$$F_i = \text{cost to serve request } i \in C$$

Mathematical Model

$$\min \qquad Q(y,u,v) = \sum_{i\in N} F_i + \sum_{k\in K} f_k \sum_{t\in T}(1 - \sum_{i\in N} y_{ikt}) + z_i(n - \sum_{t\in T}\sum_{k\in K}\sum_{i\in N} y_{ikt}) \tag{5.1}$$

$$\text{s.t. } F_i \geq c_{ji} \cdot v_{ijkt} \qquad \forall i,j \in N, k \in K, t \in T \tag{5.2}$$

$$F_i \geq (c_{j0} + c_{0i}) \cdot u_{hjkt} \cdot v_{ijkt} \qquad \forall h,i,j \in N, k \in K, t \in T \tag{5.3}$$

$$\sum_{t\in T}\sum_{k\in K} y_{ikt} \leq 1 \qquad \forall i \in C \tag{5.4}$$

$$\sum_{i\in N} v_{ijkt} = y_{jkt} \qquad \forall j \in C, k \in K, t \in T \tag{5.5}$$

$$\sum_{j\in C} v_{0jkt} = 1 \qquad \forall k \in K, t \in T \tag{5.6}$$

$$\sum_{i\in N} v_{ihkt} - \sum_{j\in N} v_{hjkt} = 0 \qquad \forall h \in C, k \in K, t \in T \tag{5.7}$$

$$\sum_{i\in C} v_{i0kt} = 1 \qquad \forall k \in K, t \in T \tag{5.8}$$

$$\sum_{j\in N} v_{ijkt} = \sum_{j\in C} u_{ijkt} \qquad \forall i \in C, k \in K, t \in T \tag{5.9}$$

$$p_i \geq q_j + (s_{j0} \cdot u_{ijkt}) \qquad \forall i, j \in C, k \in K, t \in T \tag{5.10}$$

$$q_i \geq q_j + (s_{ji} \cdot v_{ijkt}) \qquad \forall h \in C, i, j, \in N, k \in K, t \in T \tag{5.11}$$

$$q_i \geq q_j + ((s_{j0} + s_{0i}) \cdot u_{hjkt} \cdot v_{ijkt}) \qquad \forall h \in C, i, j, \in N, k \in K, t \in T \tag{5.12}$$

$$p_i \geq \tau_i \qquad \forall i \in C, t \in T \tag{5.13}$$

$$q_i \geq a_i \qquad \forall i \in N, t \in T \tag{5.14}$$

$$q_i \leq b_i \qquad \forall i \in N, t \in T \tag{5.15}$$

Objective (5.1) minimizes the total cost of the private fleet and the common carrier. Constraint (5.4) states that not all requests have to be served with a private vehicle, as it is also possible to outsource a request. In case the request $i \in C$ is served with the private vehicle $k \in K$ at period $t \in T$, then Constraint (5.5) ensures that the node i is included in the route of k at period t. For each vehicle $k \in K$, Constraint (5.6) to Constraint (5.8) define the well-known flow balance constraints on v_{ijkt} at each period.

That is, the route of each vehicle $k \in K$ must start from the distribution center, leave each node $h \in N$ to which it arrives, then return to the distribution center at the end of its route at each $t \in T$. The routes in Constraint (5.7) define the sequence with which vehicles serve requests and do not include returns to the distribution center. In other words, variable v_{ijkt} defines the route without considering the pick-up requirements. However, if request $i \in C$ is served with vehicle $k \in K$ at period $t \in T$ ($y_{ikt} = 1$) and request is placed at a time τ_i which is later than the time when k starts its route from the distribution center, then k must return to the distribution center. Constraint (5.9) ensures that the order from $i \in C$ must be picked up from the distribution center for it to be delivered to the customer. Constraint (5.10) states the time when vehicle $k \in K$ picks up the request $i \in C$ while Constraint (5.11) and Constraint (5.12) together state the time when k delivers i. Constraint (5.11) corresponds to the case in which k delivers i immediately following the delivery of request $j \in C$, whereas Constraint (5.12) corresponds to the case in which k picks up a request from the distribution center in between the delivery of request j and i. Hence, Constraint (5.11) sums up the travel time from node j to i (s_{ji}) and q_j to calculate q_i while in Constraint (5.10) the travel from node j to i also includes return to the distribution center and time is defined as $s_{j0} + s_{0i}$. Constraint (5.13) states that the request $i \in C$ cannot be picked up prior to the time customer placed the order and Constraint (5.14) and (5.15) ensure that the request $i \in C$ delivered with the respect to the service time windows.

The aforementioned mathematical model is static and deterministic since it assumes that all of the parameters for the horizon are available and known at the time of planning. This model is further extended to a multi-stage stochastic programming model to capture the dynamic and stochastic elements from the problem under consideration.

First, to capture the stochastic elements of the problem, assume that there exists single known moment $\theta \in [a_0, b_0]$ at which the values of all parameters that are previously only known probabilistically are now available deterministically. That is, all of the dynamic requests are revealed at θ. Let $C \in \{1, 2, \ldots, n\}$ and $N = C \cup \{0\}$ be defined as per the deterministic model. Then, let the set of customers that are revealed at θ be denoted as

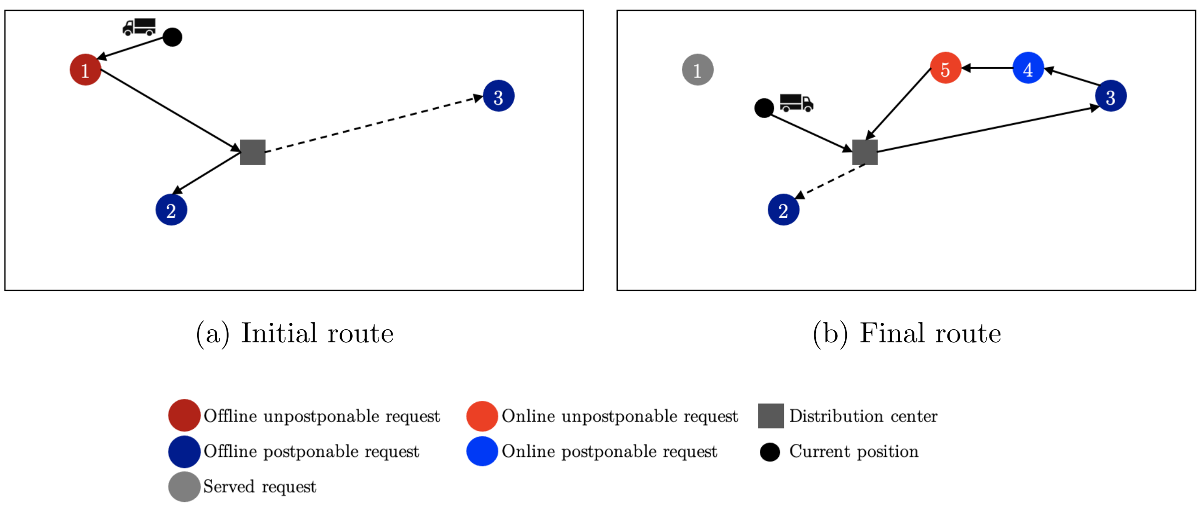
$\hat{C} \in \{n+1, n+2, ..., n+n_2\}$ and let $N^+ = C^+ \cup \{0\}$ indicate the set of all nodes that are known at θ such that $C^+ = C \cup \hat{C}$. Furthermore, time and service time window of request $i \in C^+$ are denoted with τ_i and $[a_i, b_i]$. The travel time and cost between nodes i and $j \in N^+$ are denoted with s_{ij} and c_{ij} and defined as per the previous section. Let $F = \{\hat{C}, \hat{\tau}, \hat{a}, \hat{b}, \hat{s}, \hat{c}\}$ represent the future events to be revealed at θ. Here, F is a stochastic vector since elements of this vector are only known probabilistically prior to θ. Let Ω denote the set of scenarios that approximate the future events at θ and $F(\hat{\omega}) = \{\hat{C}, \hat{\tau}, \hat{a}, \hat{b}, \hat{s}, \hat{c}\}$ be a particular realization such that $\hat{\omega} \in \Omega$.

For a single known moment θ, the problem is modelled as a two-stage stochastic programming model. A stage is defined as a sequence of the revelation of new information followed by a decision and formed at θ. The optimal decision includes both first and second stage decisions, which is known as the policy that minimizes the total cost of first stage decisions and the expected cost of second stage decisions for a given probability distribution. The first stage decisions define a partial plan based on the requests available until θ. The deterministic model corresponds to the model of such a partial plan. Then, the second stage decisions, often known as recourse actions, define the set of actions that ensure a feasible final plan with the new requests revealed at θ. The approximate impact of each recourse action for all possible realizations of scenarios in Ω must be computed to then find the optimal policy. To better understand the stochastic model presented in the following, Figures 5.1 and 5.2 illustrates some of the possible realizations from Ω and corresponding recourse actions. Note that, these recourse actions are defined in response to the first-stage decisions in the deterministic problem, based on the modelling approach of Hvattum et al. (2006).

Figure 5.1 illustrates a partial plan in which $C \in \{1, 2, 3\}$. According to the partial plan from the first stage, requests 1 and 2 are selected for the current period $t \in T$ and request 3 is postponed to the next period $t+1 \in T$ to be served with private vehicle $k \in K$ ($y_{1kt} = y_{2kt} = y_{3k(t+1)} = 1$). Request 1 is already loaded to the vehicle prior to time of planning; however, request 2 needs to be picked up from the distribution center to be delivered during the period t. Figure 5.2a illustrates the route at which the vehicle k picks up ($u_{21kt} = 1$) and delivers ($v_{21kt} = 1$) request 2 at period t in the partial plan. At θ, when the vehicle is en route to the distribution center, requests 4 and 5 arrive so $\hat{C} = \{4, 5\}$, according to scenario $\omega_1 \in \Omega$. Following the recourse actions, request 4 and 5 are selected to be delivered with vehicle k at period t ($y^+_{4kt} = y^+_{5kt} = 1$), which were not considered in the partial plan. Furthermore, request 2, which was selected for t in the partial plan ($y^-_{2kt} = 1$), is postponed to period $t+1$ ($y^+_{2k(t+1)} = 1$) and request 3, which was postponed to $t+1$ in the partial plan ($y^-_{3k(t+1)} = 1$), is selected for period t ($y^+_{3kt} = 1$) now following the recourse actions. Figure 5.1b illustrates the final route of vehicle k at period t.

These recourse actions on the partial route of vehicle k at period t are represented in the stochastic model as follows: $u^-_{21kt} = u^+_{20k(t+1)} = u^-_{30k(t+1)} = u^+_{31kt} = u^+_{41kt} = u^+_{51kt} = 1$ and $v^-_{21kt} = v^+_{20k(t+1)} = v^+_{31kt} = v^+_{54kt} = v^+_{43kt} = 1$.

Figure 5.1: Changes on the route of k at t following the recourse actions for ω_1.



(a) Initial route

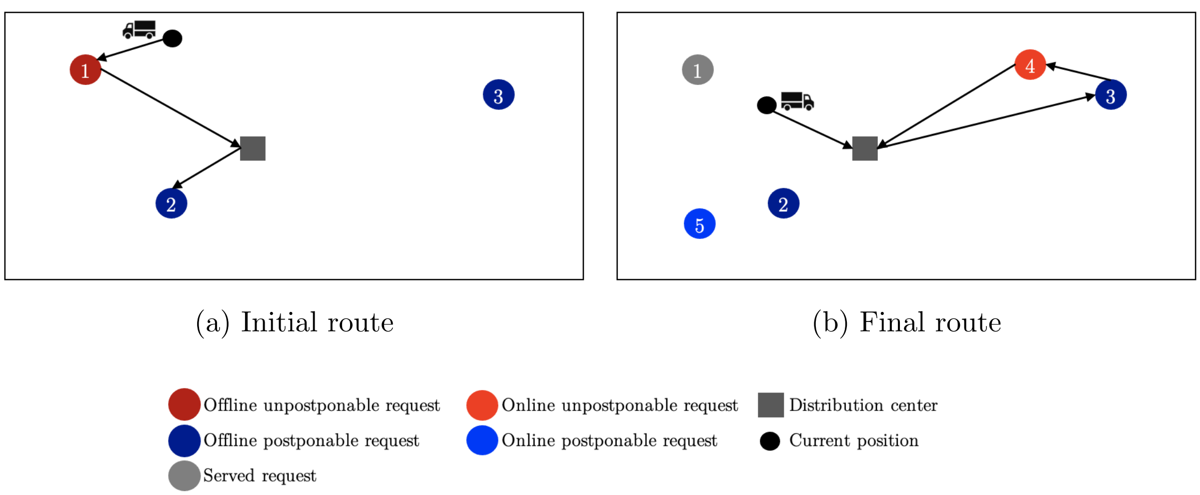
(b) Final route

Figure 5.2 illustrates a partial plan in which $C \in \{1, 2, 3\}$. According to the partial plan, requests 1 and 2 are selected for the current period $t \in T$ be served with private vehicle $k \in K$ and request 3 outsourced to common carrier ($y_{1kt} = y_{2kt} = 1$, $y_{3kt} = 0 | k \in K, t \in T$). Again, request 1 is loaded to vehicle k at the time of planning; however, request 2 is not.

Figure 5.1a illustrates the route at which the vehicle k picks up ($u_{21kt} = 1$) and delivers ($v_{21kt} = 1$) request 2 at period t in the partial plan. At θ, when the vehicle k is en route to the distribution center again, requests 4 and 5 arrive so $\hat{C} = \{4, 5\}$, according to scenario $\omega_2 \in \Omega$. Then, recourse actions are defined such that, request 4, which was not considered in the partial plan, is selected to be delivered with vehicle k at period t ($y^+_{4kt} = 1$), and request 5, which was not considered in the partial plan, is outsourced to a common carrier. Furthermore, request 2, which was selected for t in the partial plan ($y^-_{2kt} = 1$), is outsourced and request 3, which was outsourced in the partial plan, is selected to be delivered with vehicle k at period t ($y^+_{3kt} = 1$) now following the recourse actions. Figure 5.2b illustrates the final route of vehicle k at period t following the recourse actions.

These recourse actions on the partial route of vehicle k at period t are represented in the stochastic model as follows: $u^-_{21kt} = u^+_{31kt} = u^+_{41kt} = 1$ and $v^-_{21kt} = v^+_{31kt} = v^+_{43kt} = 1$.

Figure 5.2: Changes on the route of k at t following the recourse actions for ω_2.



(a) Initial route

(b) Final route

Based on the deterministic model and the initial assumption that there exists a single known moment θ at which all of the parameters are revealed, the aforementioned recourse actions and two-stage stochastic programming model is defined as follows.

Decision Variables

$$y^+_{ikt} = \begin{cases} 1, & \text{if request } i \in C \text{ is served with vehicle } k \in K \text{ at period } t \in T \text{ in recourse} \\ & \text{but not in partial solution,} \\ 0, & \text{otherwise} \end{cases}$$

$$y^-_{ikt} = \begin{cases} 1, & \text{if request } i \in C \text{ is served with vehicle } k \in K \text{ at period } t \in T \text{ in partial} \\ & \text{but not in recourse solution,} \\ 0, & \text{otherwise,} \end{cases}$$

$$y^R_{ikt} = \quad \text{Recourse on request } i \in C \text{ such that } y^R_{ikt} = y^+_{ikt} - y^-_{ikt}$$

$$u^+_{ijkt} = \begin{cases} 1, & \begin{array}{l}\text{if request } i \in C \text{ is picked up from the distribution center following } j \in C \\ \text{with vehicle } k \in K \text{ at period } t \in T \text{ in recourse but not in partial solution,}\end{array}, \\ 0, & \text{otherwise} \end{cases}$$

$$u^-_{ijkt} = \begin{cases} 1, & \begin{array}{l}\text{if request } i \in C \text{ is picked up from the distribution center following } j \in C \\ \text{with vehicle } k \in K \text{ at period } t \in T \text{ in partial but not in recourse solution,}\end{array}, \\ 0, & \text{otherwise} \end{cases}$$

$$u^R_{ijkt} = \quad \begin{array}{l}\text{Recourse on the sequence vehicle } k \in K \text{ at period } t \in T \text{ picks up requests} \\ \text{such that } u^+_{ijkt} + u^-_{ijkt}\end{array}$$

$$v^+_{ijkt} = \begin{cases} 1, & \begin{array}{l}\text{if request } i \in C \text{ is delivered following } j \in N \\ \text{with vehicle } k \in K \text{ at period } t \in T \text{ in recourse but not in partial solution,}\end{array} \\ 0, & \text{otherwise} \end{cases}$$

$$v^-_{ijkt} = \begin{cases} 1, & \begin{array}{l}\text{if request } i \in N \text{ is delivered following } j \in N \\ \text{with vehicle } k \in K \text{ at period } t \in T \text{ in partial but not in recourse solution,}\end{array} \\ 0, & \text{otherwise} \end{cases}$$

$$v^R_{ijkt} = \quad \begin{array}{l}\text{Recourse on the sequence vehicle } k \in K \text{ at period } t \in T \text{ delivers requests} \\ \text{such that } u^+_{ijkt} + u^-_{ijkt}\end{array}$$

$p^+_i =$ time when request $i \in C$ is picked up from the distribution center following the recourse

$q^+_i =$ time when request $i \in C$ is delivered following the recourse

Mathematical Model

$$\min \quad \mathbb{E}[Q(y,u,v) + R(y,u,v,\hat{\omega})] \tag{5.16}$$

$$\text{s.t.} \ (5.2) - (5.15) \tag{5.17}$$

$$\sum_{t \in T} \sum_{k \in K} (y_{ikt} + y^R_{ikt}) \leq 1 \qquad \forall i \in \tilde{C} \tag{5.18}$$

$$\sum_{i \in \tilde{N}} v^t_{ihk} + v^R_{ijkt} = y_{ikt} + y^R_{jkt} \qquad \forall j \in \tilde{C}, k \in K, t \in T \tag{5.19}$$

$$\sum_{i \in \tilde{N}} (v^t_{ihk} + v^R_{ihkt}) - \sum_{j \in \tilde{N}} (v^t_{hjk} + v^R_{hjkt}) = 0 \qquad \forall h \in \tilde{C}, k \in K, t \in T \tag{5.20}$$

$$\sum_{j \in \tilde{N}} (v_{ijkt} + v^R_{ijkt}) = \sum_{j \in \tilde{C}} (u_{ijkt} + u^R_{ijkt}) \qquad \forall i \in \tilde{C}, k \in K, t \in T \tag{5.21}$$

$$p^+_i \geq q^+_j + (s_{j0} \cdot (u_{ijkt} + u^R_{ijkt})) \qquad \forall i, j \in \tilde{C}, k \in K, t \in T \tag{5.22}$$

$$q^+_i \geq q^+_j + (s_{ji} \cdot (u_{hjkt} + u^R_{hjkt}) \cdot (v_{ijkt} + v^R_{ijkt})) \qquad \forall h \in \tilde{C}, i, j, \in \tilde{N}, k \in K, t \in T \tag{5.23}$$

$$q^+_i \geq q^+_j + ((s_{j0} + s_{0i}) \cdot (u_{hjkt} + u^R_{hjkt}) \cdot (v_{ijkt} + v^R_{ijkt})) \qquad \forall h \in \tilde{C}, i, j, \in \tilde{N}, k \in K, t \in T \tag{5.24}$$

$$p^+_i \geq \tau_i \qquad \forall i \in C, t \in T \tag{5.25}$$

$$q^+_i \geq a_i \qquad \forall i \in N_2, t \in T \tag{5.26}$$

$$q^+_i \leq b_i \qquad \forall i \in N_2, t \in T \tag{5.27}$$

$$\Theta \cdot u^-_{ijkt} \leq p_i \qquad \forall i \in \tilde{C}, j \in \tilde{N}, k \in K, t \in T \tag{5.28}$$

$$\Theta \cdot v^-_{ijkt} \leq q_i \qquad \forall i \in \tilde{C}, j \in \tilde{N}, k \in K, t \in T \tag{5.29}$$

$$y^-_{ikt} \leq y_{ikt} \qquad \forall i \in \tilde{C}, k \in K, t \in T \tag{5.30}$$

$$u^-_{ijkt} \leq u_{ijkt} \qquad \forall i \in \tilde{C}, j \in \tilde{N}, k \in K, t \in T \tag{5.31}$$

$$v^-_{ijkt} \leq v_{ijkt} \qquad \forall i \in \tilde{C}, j \in \tilde{N}, k \in K, t \in T \tag{5.32}$$

Constraint (5.18) - (5.21) state that conditions from Constraint (5.4) - (5.9) must also fulfilled following the recourse actions. Constraint (5.28) and (5.29) ensure that if an order of a customer is already picked up or delivered by the time of the recourse actions (θ), then it cannot be retracted. Finally, Constraint (5.30) - (5.32) ensure that recourse actions do not retract an action that did not happen in the partial phase. That is, if request $i \in C$ was not served with vehicle $k \in K$ at period $t \in T$ ($y_{ikt} = 0$), then y^{-}_{ikt} must also be zero by definition. Note that, because y^{+}_{ikt} and y^{-}_{ikt}, u^{+}_{ijkt} and u^{-}_{ijkt}, v^{+}_{ijkt} and v^{-}_{ijkt} always appear simultaneously respectively as in the form of y^{R}_{ikt}, u^{R}_{ijkt} and v^{R}_{ijkt}, any two being equal to one has the same impact as being equal to zero in the model. Hence, it is implicit in the model as is that either one of the two actions or no recourse happens and so enforcing such constraint is redundant. Constraints (5.18) - (5.32) allow non-anticipativity by stating that all the current routing decisions, besides the ones that are already implemented at the time of the decision, might be changed completely, without being limited to any anticipations about the future realizations, as a response to the dynamically revealing information.

The two-stage stochastic programming model presented here is based on the assumption that there exists a single known moment θ at which all parameters are revealed, which is not suitable to properly capture the dynamics of the successive information disclosure in the DS-MVRPPO. That is, the aforementioned two-stage model only approximates the problem by aggregating the entire future into a single known moment. To thoroughly capture the dynamic nature of the DS-MVRPPO, the stochastic model must be extended to a multi-stage setting. The extension from two-stage to multi-stage setting allows more information to be integrated into the decision process, and so to improve the solution performance, at the expense of computational effort. A possible approach to make such an extension is to define a stage for each time a new request arrives, which is only known probabilistically. However, instead of defining a stochastic variable for the number of stages in the model, we suggest using w prespecified time intervals of the same duration. Here, the number of time intervals equals the number of stages, and so the terms time interval and stage are used interchangeably. With prespecified stages, the two-stage model might be extended to a multi-stage model by introducing additional variables and constraints for each stage. The extension procedure is similar to that of the two-stage model from the deterministic model and so the details of the multi-stage model are not discussed again in this section.

5.2 Dynamic Stochastic Hedging Algorithm

In this section, we present a heuristic approach to address the practical instances of the DS-MVRPPO.

Finding an optimal solution to a stochastic program as in Section 5.1 might be computationally quite challenging, as it is basically $w \cdot |\Omega|$ times bigger than its deterministic counterpart. The exact solution methods in the literature are capable of solving two-stage stochastic programs of the vehicle routing problems for just a few customers Gendreau et al. (1995). In a multi-stage problem with rather extensive recourse actions, even the calculation of the cost function in the objective might be quite challenging. Therefore, to be able to address the practical instances of the problem, we also suggest a heuristic solution approach known as dynamic stochastic hedging heuristic by Hvattum et al. (2006).

The dynamic stochastic hedging heuristic (DSHH) is an approach based on scenario sampling and progressive hedging Løkketangen and Woodruff (1996). The heuristic consists of two sub-algorithms that operate at different levels but are linked with a feedback loop. The sub-algorithm at the outer level addresses the dynamic component of the problem by iteratively updating the plan at each stage while the one at the inner level considers the stochastic component through the use of scenario sampling to define the pickup and delivery plan for each stage. Sections 5.2.1 and 5.2.2 summarize the phases of the algorithms from outer to inner level in order.

5.2.1 Outer Algorithm

For Algorithm 2, assume the planning horizon is divided into w prespecified, equal duration time intervals $I_1, \cdots, I_w$, which corresponds to the stages in multi-stage model from **??**. The assumption suggests that the dynamic problem over the entire horizon H is broken into w static but still stochastic problems. That is, no new information reveals in between stages; however, the complete set of requests is still uncertain. The problem at each stage includes the subset of requests that are available at the beginning of the corresponding stage and potential information about the future requests. The objective of each problem is the same as the objective of the DS-MVRPPO, to find the plan with minimum (expected) cost to serve all requests both are known from the beginning of the stage (deterministic) and will reveal in the future (stochastic). Details about the set of requests considered at each problem are further discussed in the following section, through the sampling of the scenarios. Each problem is solved iteratively at the inner level algorithm according to Algorithm 4.

Note that, at stage e, the algorithm generates delivery plans for the time between I_e and $I_{(e+1)}$, considering the fact that available information is updated at the beginning stage $e+1$. That is because, once new information is revealed at stage $e+1$, the plan starting at from $I_{(e+1)}$ might change. Nonetheless, it is straightforward to extend the algorithm to generate a delivery plan for the entire horizon.

Algorithm 2: Outer Algorithm

$w \leftarrow$: number of time intervals (stages)
$I_1, \cdots, I_w \leftarrow$: time intervals in the planning horizon
$C_0 = \emptyset \leftarrow$: set of requests at the beginning of the planning horizon
for all e such that $1 \leq e \leq w$
  do
  $C_e^- \leftarrow$: subset of requests in C_{e-1} that are not served in I_{e-1}
  $\hat{C}_e \leftarrow$: set of requests revealed between intervals I_{e-1} and I_e
  $C_e = C_e^- \cup \hat{C}_e \leftarrow$: set of requests available in I_e
  Define the plan for I_e for requests C_e following Algorithm 4
end for
return Final plan for the horizon

5.2.2 Inner Algorithm

Algorithm 4 defines a delivery plan for the stochastic problems from Algorithm 2 based on the solutions of the sample scenarios. The algorithm consists of three main phases, which iterates at each stage according to Algorithm 2, as follows.

Phase 1: Generating Sample Scenarios

In the initial phase, the algorithm generates sample scenarios to guide the search of a delivery plan for I_e. The sampling of scenarios is a commonly proposed approach to solve stochastic problems with infinitely many scenarios with a computationally reasonable effort. The approach is shown to converge to the optimal solutions as the size of the samples increase. Here, ω sample scenarios are generated at stage I_e. Note that, there exists a trade-off between the solution performance and computational effort. Hence, considering this trade-off, a finite sample size ω must be chosen for the scenarios. Each scenario in each stage includes all the same deterministic requests but differs with regards to future requests. Here, future requests are sampled based on stochastic information. The set of deterministic requests includes the set of requests that are revealed at the beginning of the current stage and the subset of previous requests that are still not served. The latter corresponds to the subset of requests that are not selected, so either postponed or outsourced, in the previous stages. The future requests are generated based on the distributions of the stochastic variables in F. Note that, for all sample scenarios in Ω, at the beginning of the planning horizon, for all private vehicles in K, the initial location is the distribution center. Then, once the vehicles start serving requests, the initial location for each vehicle $k \in K$ is updated as the last location vehicle k is at or en route to at the end of the previous stage. Finally, all scenarios have an equal probability of occurrence.

Phase 2: Selecting Requests

Once the sample scenarios are generated in the first phase, then in the second phase, corresponding problems can be solved as a deterministic problem using an insertion heuristic. Algorithm 2 summarizes the steps of the insertion heuristic algorithm. The algorithm evaluates all feasible positions at the routes of all private vehicles and chooses the route that incurs a minimum cost to insert the request. If the cost of insertion to the route is less than the unit outsourcing cost, the algorithm chooses to serve the request with the corresponding route (vehicle and position). Otherwise, if the cost is more than the unit outsourcing cost, the algorithm chooses to outsource the request. Note that, here the outsource cost is considered to be fixed for unit outsourced and does not benefit from flat rates or discount structures. Nonetheless, the algorithm and selection procedure presented here might be applied without changing the aforementioned steps but just updating the unit cost with the appropriate cost function.

Algorithm 3: Insertion Heuristic

$z \leftarrow$: unit outsource cost
$K \leftarrow$: set of vehicles in the private fleet
$V_k \leftarrow$: route of vehicle $k \in K$ at I_e
$C_e \leftarrow$: set of requests available in I_e according to sample scenario ω
for all $i \in C_e$
    for all $k \in K$
        for all $p \in V_k$
         do
            $\hat{c}_i^{kp} \leftarrow$: Cost of inserting request i following p to the route of k
        end for
    end for
    $\hat{c}_i^* = \min_{k \in K}(\min_{p \in V_k}(\hat{c}_i^{kp})) \leftarrow$: Cost of serving request i with private vehicle
end for
if $\hat{c}_i^* \leq$ z **then**
    Insert request i following the corresponding p at V_k to serve with private vehicle
else
    Transfer request i to common carrier
end if
return Solution to sample scenario ω

From the solutions of the sample scenarios, the algorithm selects the subset of requests that are often served with private vehicle $k \in K$ during stage I_e (R_e). The subset R_e is defined as the set requests that are served with $k \in K$ during I_e in at least ν_1% of the sample solutions. Then, the algorithm solves the sample scenarios again while enforcing the requests in R_e to be served with vehicle $k \in K$ during stage I_e. Requests from C_e are included into R_e until a termination criterion is met. The criterion is defined as the case in which there remains no request that is served with $k \in K$ during I_e in at least ν_2% of the sample solutions but still not included in R_e. Note that, enforcing all at least ν_2%

common requests at once is not the same as the iterative procedure here, as the selection of requests at each iteration depends on the selections from previous iterations. That is, once R_e is enforced in the solution, then the routes and also the subset of at least $\nu_2\%$ common requests might change.

Phase 3: Routing Requests

In the last phase, the algorithm defines the delivery plan for I_e. With the requests in R_e enforced, the algorithm solves the deterministic problem for the requests in C_e again using an insertion heuristic.

Algorithm 4: Inner Algorithm

```
ω ←: number of sample scenarios
C_e ←: set of customers available at the beginning of interval I_e
R_e = ∅ ←: subset of customers from C_e
Generate ω sample scenarios that includes C_e and sample requests
while termination criterion do
    Solve sample scenarios such that R_e is forced in the solution of all scenarios
    Find the requests c ∈ C_e \ R_e that are often served with vehicle k ∈ K
    during interval I_e
    R_e := R_e ∪ c
end while
for all k ∈ K do
    Determine the route for I_e while enforcing R_e
end for
return Plan for the current interval
```

5.3 Computational Experiments

In this section, we first present the design of the computational experiments in Section 5.3.1 and then in Section 5.3.2, discuss the computational results.

5.3.1 Instance Generation

For computational experiments, we investigate a planning problem defined over $h = 10$ periods, such that the service operates from 8:00 AM to 6:00 PM at each period. We evaluate a service network in which the requests are distributed over an area of 100×100 km^2 in the Euclidean plane based on Solomon (1987). We consider uniformly distributed (R) and clustered (C) settings. The coordinate of the distribution center is that of the corresponding setting. Furthermore, the coordinates of requests are randomly selected from the corresponding setting. The distance between two coordinates is the Euclidean distance. The service time window of requests are represented by interval $[a, b]$ in which a is set to the time the request is placed and b is generated according to a uniform distribution $[e, l]$, such that e and l represent the lower and upper bound of the latest due time, respectively. We consider the values of e and l to be 2 and 4 stages following a respectively, i.e. $e = a + 2 \times stage$ and $l = a + 4 \times stage$. We consider a private fleet of m homogeneous uncapacitated vehicles with an average speed of 45 km/h. There is a setup cost for each private vehicle, besides the operational cost that depends on the time en route. We set the setup cost $f = 1000$ units per vehicle. Furthermore, we set $c_{ij} = v \times s_{ij}$, so that the total operational cost is proportional to the total amount of time the vehicles are on the route and we choose v as 60. An outsource cost z is charged for each request transferred to a common carrier.

We use the Poisson process with parameter λ to represent the random order placements of customers. Hence, interarrival time between requests follows exponential distribution with $\mu = {}^1\!/\!_\lambda$. We consider $\lambda \in \{50, 100\}$ per period. Furthermore, we consider $m \in \{1, 3, 5, 7\}$ and $f \in \{25, 50, 75\}$ during the computational experiments, to evaluate the significance of the size of the private fleet and the outsource cost of the common carrier in the routing and outsourcing decisions. Table 5.1 summarizes the settings of the problem parameters.

Table 5.1: Settings of problem parameters in the computational experiments

Description	Parameter	Setting (Unit)
Number of Periods:	h	$h = 10$
Service Area:	Euclidean Solomon (1987)	100×100 (km^2)
Travel Distance:	c	Euclidean
Travel Time:	s	$c = \frac{s}{45}$
Service Time:	Uniform $[e, l]$	$[e = 2, l = 4]$ (stage)
Time Window:	$[a, b]$	[Request Time, Uniform $[e, l]$] (stage)
Request Placement:	Poisson(λ)	$\lambda = \{50, 100\}$ per period
Private Fleet:	Uncapacitated m vehicles	$m = \{1, 3, 5, 7\}$
Cost Component:	[Setup (f), Operational ($v \times s$), Outsource (z)]	$[1000, v = 60, \{15, 35, 55\}]$

For each setting, we generate 10 different instances. That is, we investigate a total of 480 instances over 48 different problem settings. Table 5.2 summarizes the characteristics of the problems at each setting.

Table 5.2: Characteristics of customers for instances for different rates.

λ	Number of Requests (min, max, avg)	Percentage of Requests	
		Unpostponable	Postponable
50	(39.0, 65.0, 51.4)	(50.9, 74.4, 62.1)	(25.6, 49.0, 37.9)
100	(89.0, 110.0, 99.8)	(54.1, 67.7, 62.2)	(29.2, 45.9, 37.8)

The following summarizes the settings of the solution algorithm: $w = 4$ time intervals (stages) in each period, i.e. stages of 9,000 seconds, $\omega = 10$ sample scenarios for each stage in which request are generated based on Poisson(λ), $\nu_1 = 50\%$ and $\nu_2 = 75\%$.

We consider two different planning approaches to evaluate the impact of postponing requests: (1) Assigning without postponement ($\mathrm{c^s}$), suggests outsourcing the remaining requests and considering just the new set of requests that are placed in between periods at the beginning of each period (2) Assigning with postponement ($\mathrm{c^m}$), suggests to chooses between outsourcing and postponing based on the stochastic information on future requests and the cost at each period. Furthermore, we consider two different planning approaches to evaluate the impact of outsourcing requests: (1) Routing without outsourcing ($\mathrm{r^{min}}$), does not consider outsourcing option during the planning phase and first focuses on maximizing the number of requests that can be served with the private vehicle, then outsources remaining unfulfilled requests (2) Routing with outsourcing ($\mathrm{r^{max}}$), considers both the options of outsourcing to a common carrier and serving with a private vehicle during the planning phase.

As the performance indicators, we consider the following: (1) Vehicle utilization, as the percentage of the total en-route time over total available service time (2) Percentages of the total unfulfilled, delivered and outsourced requests over total requests (3) Percentages of the outsourced and postponed requests over postponable requests (4) Percentages of the cost components over total costs.

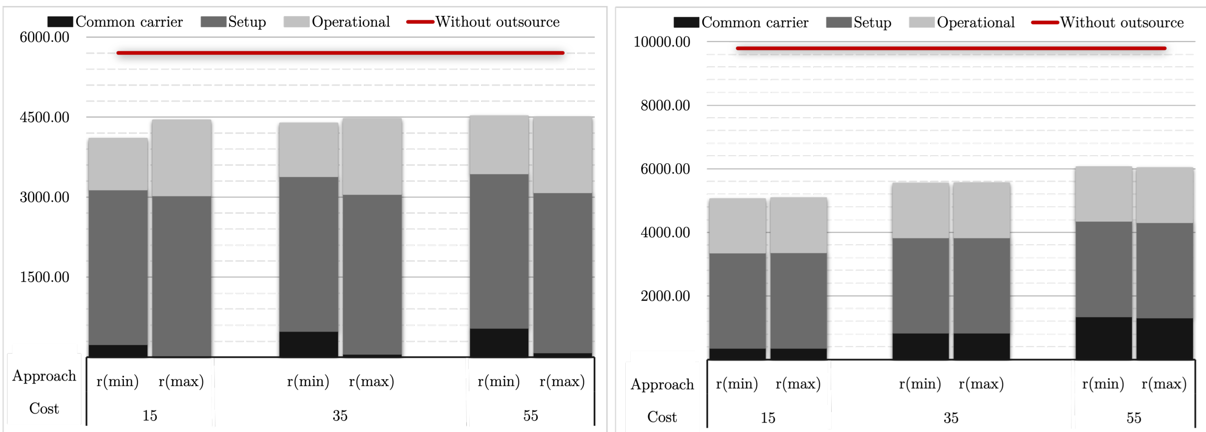
5.3.2 Computational Results

We present the computational results based on first the approach on postponement, i.e., assigning with ($\mathrm{c^m}$) and without ($\mathrm{c^s}$) postponement, and then the approach on outsourcing, i.e., routing with ($\mathrm{r^{min}}$) or without ($\mathrm{r^{max}}$) outsourcing. Section 5.3.2 and Section 5.3.2 present the results without and with postponement in order. We present the results for routing with and without outsourcing together at each section. For the results with outsourcing, we categorize each result based on the outsourcing cost. We represent the results without outsourcing with an outsourcing cost of *max*, by referring to the fact that here the focus is on maximizing the number of requests that can be served with the private vehicles. We discuss the impact of problem parameters on the performance of different planning approaches by addressing the foregoing performance indicators.

Without Postponement

Tables 5.3 and 5.4 summarize the results without postponement. First, let us note that considering the outsource option, either with $\mathrm{r}^{\mathrm{min}}$ or $\mathrm{r}^{\mathrm{max}}$, is profitable for all problem settings without postponement according to Figure 5.3. On average, the cost to fulfill all requests decreases by 15.3% and 23.8% in R and C instances respectively, even for the highest outsource cost. This difference increases to 23.7% and 31.9% as the outsource cost decreases to the lowest. Following, as expected, the impact of outsource cost (z) on the percentage of requests outsourced is rather straightforward. That is, the increase z, decreases the percentage of requests outsourced and so, increases the utilization of the fleet. The impact of z in percentage outsourced is more significant in R and lower rates (λ). On average, percentage outsourced decreases by (28.7%, 22.7%) and (22.4%, 8.9%) from lowest to highest z value in R and C instances for $\lambda = (50, 100)$ respectively. Note that, the average decrease in percentage outsourced decreases as the rate (λ) increases in the preceding. This relationship also follows for the impact of λ on percentage outsourced in $\mathrm{r}^{\mathrm{min}}$. In other words, in $\mathrm{r}^{\mathrm{min}}$, the increase in λ decreases the percentage of customers outsourced. Percentage outsourced decreases by 0.5% and 8.5% from $\lambda = 50$ to $\lambda = 100$ on average in R and C instances respectively. This is explained by the increase in the percentage of the service area that must be covered with the private vehicles, since the number of unpostponable customers also increases, once the rate increases. In this setting, it is expected for the private vehicles to be in close distance to more of the postponable customers. Therefore, fulfilling the demand with a private vehicle instead of outsourcing turns into a cost-efficient option for more of the (postponable) customers. On the other hand, in $\mathrm{r}^{\mathrm{max}}$, the relationship is the opposite and the increase in λ increases the percentage outsourced. The percentage outsourced decreases by 49.4% and 59.6% from $\lambda = 100$ to $\lambda = 50$ on average in R and C instances respectively. This is also straightforward following the definition of the corresponding planning approach. That is, because the outsourcing option is not considered during the routing phase, once the capacity of the private fleet is fully utilized, the remaining is outsourced and the remaining increases, once the rate increases.

Figure 5.3: Cost for different outsourcing costs and approaches without postponement



(a) Random with $(\lambda, m) = (50, 3)$ (b) Clustered with $(\lambda, m) = (50, 3)$

Noting that considering outsourcing is profitable, we then evaluate the cost performance of different planning strategies about outsourcing. A significantly important finding is that the r^{min} is the predominantly cost-efficient planning approach. For 79.2% of both the R and C instances, r^{min} generates less costly plans compared to r^{max}. Considering the outsourcing option during the routing phase, rather than ignoring it until the end of the period, decreases the cost of the plan by 4.43% and 4.47% on average in R and C instances respectively. Note that, the preceding percentage values also include the plans with unfulfilled requests; however, the conclusion also follows even once such plans are ignored. As a matter of fact, the percentage cost advantage increases by 2.8% and 2.7% on average in R and C instances respectively, once unfulfilled instances are ignored. Furthermore, the decrease in outsource cost (z) increases the cost advantage of r^{min} over r^{max}. As a result, it is expected that r^{min} follows to be the profitable approach under a discount structure in which the unit outsource cost decreases as the number of units

Table 5.3: Percentages of fulfillment and cost component without postponement for random setting.

λ	$[m, z]$	Utilization	Number of Requests (%)					Minimum Insertion Cost (%)				Maximum Utilization Cost (%)			
				Total		Postponable		Common Carrier	Private Fleet		Total Cost	Common Carrier	Private Fleet		Total Cost
			Unfulfilled	Delivered	Outsourced	Outsourced	Postponed		Setup	Operational			Setup	Operational	
50	[1, 15]	96.16	18.73	51.74	29.54	81.38	0.00	12.70	55.36	31.94	1806.46	12.69	55.29	32.03	1808.76
	[1, 35]	96.54	18.73	51.74	29.54	81.38	0.00	25.32	47.29	27.39	2114.76	25.32	47.29	27.39	2114.76
	[1, 55]	96.54	18.73	51.74	29.54	81.38	0.00	34.76	41.31	23.93	2420.76	34.76	41.31	23.93	2420.76
	[1, max]	96.54	18.73	51.74	29.54	81.38	0.00								
	[3, 15]	55.32	0.00	69.69	30.31	83.51	0.00	5.75	70.77	23.49	4098.07	0.51	67.54	31.95	4441.72
	[3, 35]	57.49	0.00	73.36	26.64	73.40	0.00	11.02	66.16	22.82	4383.34	1.17	67.09	31.74	4471.72
	[3, 55]	62.30	0.00	81.08	18.92	52.13	0.00	11.92	64.12	23.97	4522.97	1.83	66.64	31.53	4501.72
	[3, max]	78.85	0.00	97.10	2.90	7.98	0.00								
	[5, 15]	44.08	0.00	68.53	31.47	86.70	0.00	5.24	74.94	19.82	4670.13	0.00	73.62	26.38	5705.09
	[5, 35]	46.10	0.00	72.20	27.80	76.60	0.00	10.14	70.39	19.47	4972.15	0.00	73.62	26.38	5705.09
	[5, 55]	51.20	0.00	80.69	19.31	53.19	0.00	10.73	68.29	20.98	5125.16	0.00	73.62	26.38	5705.09
	[5, max]	59.73	0.00	100.00	0.00	0.00	0.00								
	[7, 15]	44.08	0.00	68.53	31.47	86.70	0.00	5.24	74.94	19.82	4670.13	0.00	73.62	26.38	5705.09
	[7, 35]	46.10	0.00	72.20	27.80	76.60	0.00	10.14	70.39	19.47	4972.15	0.00	73.62	26.38	5705.09
	[7, 55]	51.20	0.00	80.69	19.31	53.19	0.00	10.73	68.29	20.98	5125.16	0.00	73.62	26.38	5705.09
	[7, max]	59.73	0.00	100.00	0.00	0.00	0.00								
100	[1, 15]	99.24	31.67	33.63	34.71	91.71	0.00	24.97	47.03	28.00	2126.44	24.97	47.03	28.00	2126.44
	[1, 35]	99.24	31.67	33.63	34.71	91.71	0.00	43.71	35.28	21.01	2834.44	43.71	35.28	21.01	2834.44
	[1, 55]	99.24	31.67	33.63	34.71	91.71	0.00	54.96	28.23	16.81	3542.44	54.96	28.23	16.81	3542.44
	[1, max]	99.24	31.67	33.63	34.71	91.71	0.00								
	[3, 15]	84.64	0.20	75.98	23.82	62.95	0.00	7.46	61.37	31.17	4888.09	5.12	60.24	34.64	4980.03
	[3, 35]	85.94	0.20	76.47	23.33	61.66	0.00	15.48	55.76	28.75	5380.00	11.18	56.39	32.43	5320.03
	[3, 55]	87.75	0.20	77.75	22.06	58.29	0.00	21.27	51.57	27.15	5817.09	16.52	53.00	30.48	5660.03
	[3, max]	95.83	0.20	83.14	16.67	44.04	0.00								
	[5, 15]	50.85	0.00	70.00	30.00	79.27	0.00	6.57	71.59	21.84	6984.51	0.16	67.89	31.94	7364.38
	[5, 35]	52.97	0.00	72.75	27.25	72.02	0.00	12.87	66.12	21.01	7562.14	0.38	67.75	31.87	7380.38
	[5, 55]	59.47	0.00	82.16	17.84	47.15	0.00	12.86	64.23	22.92	7784.96	0.59	67.60	31.80	7396.38
	[5, max]	78.41	0.00	99.22	0.78	2.07	0.00								
	[7, 15]	45.25	0.00	70.49	29.51	77.98	0.00	5.86	74.03	20.10	7699.10	0.00	71.74	28.26	8642.12
	[7, 35]	48.91	0.00	74.51	25.49	67.36	0.00	11.16	68.68	20.15	8153.22	0.00	71.74	28.26	8642.12
	[7, 55]	55.36	0.00	83.14	16.86	44.56	0.00	11.25	66.62	22.13	8406.13	0.00	71.74	28.26	8642.12
	[7, max]	65.65	0.00	100.00	0.00	0.00	0.00								

Table 5.4: Percentages of fulfillment and cost component without postponement for clustered setting.

λ	$[m, z]$	Utilization	Number of Requests (%)					Minimum Insertion Cost (%)				Maximum Utilization Cost (%)			
				Total		Postponable		Common Carrier	Private Fleet		Total Cost	Common Carrier	Private Fleet		Total Cost
			Unfulfilled	Delivered	Outsourced	Outsourced	Postponed		Setup	Operational			Setup	Operational	
50	[1, 15]	97.14	22.35	43.53	34.12	86.14	0.00	24.15	47.63	28.22	2099.51	24.15	47.63	28.22	2099.51
	[1, 35]	97.27	22.35	44.12	33.53	84.65	0.00	42.62	36.03	21.35	2775.51	42.62	36.03	21.35	2775.51
	[1, 55]	97.27	22.35	44.12	33.53	84.65	0.00	53.86	28.97	17.17	3451.51	53.86	28.97	17.17	3451.51
	[1, max]	97.27	22.35	44.12	33.53	84.65	0.00								
	[3, 15]	62.76	0.00	71.37	28.63	72.28	0.00	7.01	59.38	33.62	5052.52	7.02	59.02	33.95	5082.76
	[3, 35]	63.71	0.00	71.57	28.43	71.78	0.00	15.03	54.13	30.84	5542.60	14.99	53.97	31.05	5558.76
	[3, 55]	68.47	0.00	77.65	22.35	56.44	0.00	22.25	49.53	28.23	6057.40	21.69	49.71	28.60	6034.76
	[3, max]	86.11	0.00	91.57	8.43	21.29	0.00								
	[5, 15]	43.57	0.00	65.49	34.51	87.13	0.00	5.00	68.63	26.37	7285.40	1.12	65.50	33.38	7634.15
	[5, 35]	44.99	0.00	69.41	30.59	77.23	0.00	9.75	64.50	25.75	7751.81	2.57	64.53	32.89	7748.15
	[5, 55]	49.57	0.00	76.86	23.14	58.42	0.00	13.99	61.16	24.85	8175.63	3.99	63.60	32.42	7862.15
	[5, max]	64.63	0.00	98.24	1.76	4.46	0.00								
	[7, 15]	43.57	0.00	65.49	34.51	87.13	0.00	4.97	74.28	20.74	8750.22	0.08	70.65	29.28	9766.92
	[7, 35]	44.99	0.00	69.41	30.59	77.23	0.00	9.97	69.32	20.71	9232.28	0.18	70.57	29.25	9776.92
	[7, 55]	49.57	0.00	76.86	23.14	58.42	0.00	12.46	66.52	21.01	9620.60	0.28	70.50	29.22	9786.92
	[7, max]	59.28	0.00	100.00	0.00	0.00	0.00								
100	[1, 15]	98.75	34.12	31.25	34.63	92.10	0.00	14.16	54.24	31.61	1843.81	13.94	54.34	31.72	1840.13
	[1, 35]	98.75	34.12	31.25	34.63	92.10	0.00	27.43	45.83	26.75	2182.13	27.43	45.83	26.75	2182.13
	[1, 55]	98.75	34.12	31.25	34.63	92.10	0.00	37.26	39.62	23.12	2524.13	37.26	39.62	23.12	2524.13
	[1, max]	98.75	34.12	31.25	34.63	92.10	0.00								
	[3, 15]	94.36	3.69	72.13	24.18	64.31	0.00	5.04	68.99	25.98	4348.68	1.40	65.01	33.59	4614.43
	[3, 35]	94.98	3.69	71.93	24.39	64.85	0.00	10.90	64.46	24.64	4654.22	3.20	63.82	32.97	4700.43
	[3, 55]	94.99	3.59	71.31	25.10	66.76	0.00	12.90	61.74	25.36	4859.47	4.94	62.68	32.38	4786.43
	[3, max]	95.88	3.59	72.03	24.39	64.85	0.00								
	[5, 15]	64.03	0.00	75.10	24.90	66.21	0.00	4.75	75.51	19.74	5562.04	0.21	71.91	27.88	6535.93
	[5, 35]	66.53	0.00	77.87	22.13	58.86	0.00	9.29	71.43	19.28	5879.87	0.48	71.71	27.81	6553.93
	[5, 55]	67.72	0.00	78.69	21.31	56.68	0.00	10.64	68.87	20.49	6098.26	0.75	71.52	27.73	6571.93
	[5, max]	84.95	0.00	94.16	5.84	15.53	0.00								
	[7, 15]	46.54	0.00	70.29	29.71	79.02	0.00	4.75	75.51	19.74	5562.04	0.00	73.76	26.24	7049.55
	[7, 35]	49.79	0.00	73.05	26.95	71.66	0.00	9.29	71.43	19.28	5879.87	0.00	73.76	26.24	7049.55
	[7, 55]	52.65	0.00	77.66	22.34	59.40	0.00	10.64	68.87	20.49	6098.26	0.00	73.76	26.24	7049.55
	[7, max]	69.07	0.00	99.49	0.51	1.36	0.00								

outsourced increases. That is because, r_{min} outperforms r_{max} even for the highest unit outsource cost and the difference increases as the unit cost decreases, which is the case in a discount structure. This result is intuitive, considering the fact that one of the main differences between r_{min} and r_{max} is that the former approach favours outsourcing compared to the latter and the discount structure causes outsourcing to be more beneficial at higher volumes.

With Postponement

In Section 5.3.2, we showed that considering outsourcing option during routing is the most advantageous planning approach in terms of the cost. Here, we further discuss the potential savings of postponing customers from one period to another. In other words, we evaluate the following three options: a (postponable) request might (1) either be delivered with a private vehicle during the period that it is placed, (2) or outsourced to a common carrier (3) or postponed to be delivered with a private vehicle during the following period. We answer the following two questions: (1) with postponement, which planning approach is more profitable, still $\mathrm{r}^{\mathrm{min}}$ as in without postponement or $\mathrm{r}^{\mathrm{max}}$ in this case. (2) with (c^{m}) or without postponement (c^{s}), which approach is more profitable in overall.

We first compare the results of $\mathrm{r}^{\mathrm{min}}$ and $\mathrm{r}^{\mathrm{max}}$ for c^{m} in Figure 5.4. According to $\mathrm{r}^{\mathrm{min}}$, we consider that, if it is more profitable to do so, requests are either outsourced or postponed at the end of each period in the planning horizon. On the other hand, in $\mathrm{r}^{\mathrm{max}}$, we do not consider the outsourcing option during the routing, as in the case without postponement. Note that, in the case without postponement, since each period starts fresh with just the new requests, requests remaining at the end of each period had to be outsourced. However here, the remaining requests are instead postponed at the end of each period, except for the final period, at which the remaining requests are outsourced.

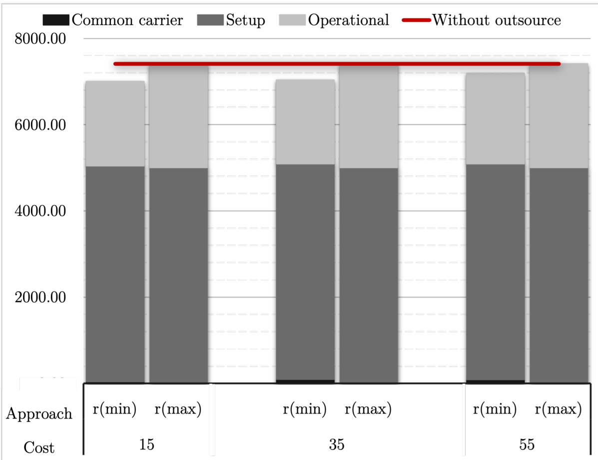
Table 5.5: Percentages of fulfillment and cost component with postponement for random setting.

λ	z	Utilization	Number of Requests (%)					Minimum Insertion Cost (%)				Maximum Utilization Cost (%)			
				Total		Postponable			Private Fleet				Private Fleet		
			Unfulfilled	Delivered	Outsourced	Outsourced	Postponed	Common Carrier	Setup	Operational	Total Cost	Common Carrier	Setup	Operational	Total Cost
50	[1, 15]	95.61	16.99	55.02	28.96	79.79	0.00	12.51	55.60	31.89	1798.65	2.12	61.53	36.34	1625.09
	[1, 35]	96.40	24.32	68.53	18.92	52.13	0.00	17.85	52.05	30.10	1921.38	4.82	59.84	35.34	1671.09
	[1, 55]	97.68	33.78	69.11	14.86	40.96	0.00	21.07	49.76	29.16	2009.56	7.37	58.24	34.39	1717.09
	[1, max]	98.43	39.19	56.37	4.44	12.23	73.94								
	[3, 15]	68.68	0.19	97.30	2.51	6.91	67.02	0.46	70.49	29.05	4255.75	0.17	66.94	32.89	4481.42
	[3, 35]	67.78	0.00	97.49	2.51	6.91	62.23	1.07	70.33	28.60	4265.60	0.39	66.79	32.82	4491.42
	[3, 55]	71.84	0.00	97.49	2.51	6.91	42.55	1.64	68.73	29.63	4364.68	0.61	66.65	32.74	4501.42
	[3, max]	81.88	0.00	99.03	0.97	2.66	6.91								
	[5, 15]	45.42	0.00	96.72	3.28	9.04	75.53	0.45	78.23	21.32	5624.68	0.00	73.62	26.38	5705.09
	[5, 35]	46.71	0.00	97.10	2.90	7.98	67.55	0.90	77.40	21.69	5813.66	0.00	73.62	26.38	5705.09
	[5, 55]	51.01	0.00	97.10	2.90	7.98	39.89	1.48	75.43	23.09	5568.02	0.00	73.62	26.38	5705.09
	[5, max]	59.73	0.00	100.00	0.00	0.00	0.00								
	[7, 15]	44.14	0.00	96.72	3.28	9.04	75.53	0.44	78.72	20.85	5843.79	0.00	73.62	26.38	5705.09
	[7, 35]	45.76	0.00	97.10	2.90	7.98	67.55	0.89	77.76	21.35	5915.38	0.00	73.62	26.38	5705.09
	[7, 55]	51.01	0.00	97.10	2.90	7.98	39.89	1.48	75.43	23.09	5568.02	0.00	73.62	26.38	5705.09
	[7, max]	59.73	0.00	100.00	0.00	0.00	0.00								
100	[1, 15]	98.53	34.02	41.67	28.82	76.17	0.00	21.70	49.21	29.09	2032.18	3.09	60.68	36.23	1648.10
	[1, 35]	99.18	43.73	47.55	21.57	56.99	0.00	32.56	42.28	25.16	2365.09	6.93	58.27	34.79	1716.10
	[1, 55]	98.87	47.45	46.27	20.29	53.63	0.00	41.68	36.61	21.72	2731.72	10.48	56.05	33.47	1784.10
	[1, max]	99.52	60.00	36.67	3.33	8.81	81.61								
	[3, 15]	94.67	2.16	95.98	1.86	4.92	55.44	0.60	63.39	36.01	4732.55	0.78	62.45	36.77	4803.61
	[3, 35]	94.62	3.43	94.71	1.86	4.92	58.55	1.39	62.90	35.71	4769.59	1.80	61.81	36.39	4853.61
	[3, 55]	93.91	2.84	94.51	2.65	6.99	55.70	3.07	62.00	34.93	4838.84	2.80	61.18	36.02	4903.61
	[3, max]	98.12	2.75	94.80	2.45	6.48	46.63								
	[5, 15]	65.56	0.00	97.45	2.55	6.74	69.17	0.56	71.37	28.07	7005.86	0.00	67.46	32.54	7411.78
	[5, 35]	64.95	0.00	97.45	2.55	6.74	64.25	1.29	71.03	27.68	7039.61	0.00	67.46	32.54	7411.78
	[5, 55]	70.09	0.00	98.43	1.57	4.15	47.41	1.22	69.53	29.24	7190.71	0.00	67.46	32.54	7411.78
	[5, max]	80.39	0.00	100.00	0.00	0.00	2.07								
	[7, 15]	50.68	0.00	97.45	2.55	6.74	75.13	0.45	76.34	23.21	8645.74	0.00	71.74	28.26	8642.12
	[7, 35]	51.24	0.00	97.45	2.55	6.74	59.07	1.06	75.67	23.27	8589.47	0.00	71.74	28.26	8642.12
	[7, 55]	58.40	0.00	98.92	1.08	2.85	42.75	0.74	73.50	25.76	8162.94	0.00	71.74	28.26	8642.12
	[7, max]	65.65	0.00	100.00	0.00	0.00	0.00								

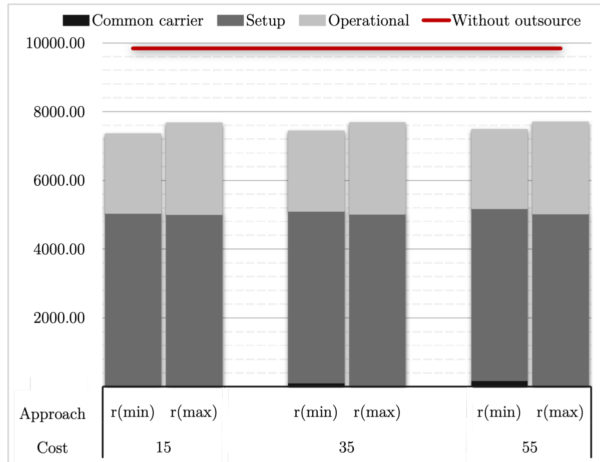
Table 5.6: Percentages of fulfillment and cost component with postponement for clustered setting.

λ	z	Utilization	Number of Requests (%)					Minimum Insertion Cost (%)				Maximum Utilization Cost (%)			
				Total		Postponable		Common Carrier	Private Fleet		Total Cost	Common Carrier	Private Fleet		Total Cost
			Unfulfilled	Delivered	Outsourced	Outsourced	Postponed		Setup	Operational			Setup	Operational	
50	[1, 15]	96.67	21.76	49.80	31.57	79.70	0.00	13.26	54.90	31.84	1821.53	1.85	61.76	36.39	1619.19
	[1, 35]	96.41	32.94	65.69	18.04	45.54	0.00	16.94	52.62	30.44	1900.48	4.22	60.27	35.51	1659.19
	[1, 55]	97.39	38.24	66.86	14.51	36.63	0.00	20.44	50.22	29.34	1991.34	6.47	58.85	34.67	1699.19
	[1, max]	98.20	43.53	52.55	3.92	9.90	75.74								
	[3, 15]	73.95	0.39	95.49	4.12	10.40	61.39	0.72	68.77	30.51	4362.61	0.39	64.91	34.70	4622.09
	[3, 35]	74.88	0.20	95.49	4.31	10.89	56.44	1.74	67.80	30.46	4424.85	0.90	64.57	34.53	4646.09
	[3, 55]	77.85	0.98	95.29	3.73	9.41	55.45	2.32	66.58	31.10	4505.78	1.41	64.24	34.35	4670.09
	[3, max]	89.12	0.00	97.65	2.35	5.94	22.28								
	[5, 15]	49.54	0.00	95.69	4.31	10.89	76.24	0.52	76.69	22.79	6389.35	0.02	71.29	28.68	6592.53
	[5, 35]	50.15	0.00	95.69	4.31	10.89	72.77	1.19	75.95	22.85	6451.38	0.05	71.27	28.68	6594.53
	[5, 55]	51.24	0.00	97.65	2.35	5.94	65.35	1.06	75.67	23.27	6211.09	0.08	71.25	28.67	6596.53
	[5, max]	67.06	0.00	99.80	0.20	0.50	4.95								
	[7, 15]	43.34	0.00	95.69	4.31	10.89	76.24	0.47	78.99	20.54	7089.08	0.00	73.76	26.24	7049.55
	[7, 35]	44.30	0.00	98.04	1.96	4.95	72.77	0.49	78.61	20.90	7123.64	0.00	73.76	26.24	7049.55
	[7, 55]	46.84	0.00	95.69	4.31	10.89	62.38	1.78	76.67	21.55	6782.44	0.00	73.76	26.24	7049.55
	[7, max]	59.28	0.00	100.00	0.00	0.00	0.00								
100	[1, 15]	98.64	36.58	38.63	29.51	78.47	0.00	21.35	49.41	29.24	2023.83	3.45	60.51	36.04	1652.53
	[1, 35]	98.56	43.75	46.62	21.31	56.68	0.00	31.39	43.12	25.50	2319.37	7.69	57.85	34.45	1728.53
	[1, 55]	97.76	46.72	43.14	22.03	58.58	0.00	42.70	36.11	21.18	2769.05	11.58	55.42	33.00	1804.53
	[1, max]	99.26	59.94	36.17	3.89	10.35	79.84								
	[3, 15]	97.95	13.52	83.30	3.18	8.45	58.04	0.97	62.38	36.66	4809.57	1.06	62.30	36.65	4815.72
	[3, 35]	98.11	9.73	87.70	2.56	6.81	54.77	1.80	61.81	36.39	4853.49	2.44	61.43	36.13	4883.72
	[3, 55]	98.36	10.86	86.17	2.97	7.90	59.95	3.24	60.85	35.91	4930.03	3.78	60.58	35.64	4951.72
	[3, max]	98.04	11.37	85.14	3.48	9.26	54.77								
	[5, 15]	77.03	0.00	97.23	2.77	7.36	54.22	0.55	68.01	31.44	7351.45	0.10	65.18	34.73	7671.47
	[5, 35]	77.69	0.00	96.93	3.07	8.17	50.41	1.41	67.24	31.34	7435.66	0.23	65.09	34.68	7681.47
	[5, 55]	76.91	0.00	96.72	3.28	8.72	46.05	2.35	66.81	30.83	7483.44	0.36	65.01	34.64	7691.47
	[5, max]	88.80	0.00	99.49	0.51	1.36	11.99								
	[7, 15]	57.83	0.00	96.21	3.79	10.08	67.03	0.59	73.80	25.61	9349.81	0.02	70.15	29.83	9835.51
	[7, 35]	57.50	0.00	95.90	4.10	10.90	59.67	1.49	73.25	25.27	9420.31	0.04	70.14	29.82	9837.51
	[7, 55]	61.83	0.00	95.70	4.30	11.44	44.41	2.45	71.15	26.40	9416.71	0.06	70.13	29.82	9839.51
	[7, max]	70.87	0.00	99.90	0.10	0.27	1.09								

Figure 5.4: Cost for different outsourcing costs and approaches with postponement



(a) Random with $(\lambda, m) = (100, 5)$

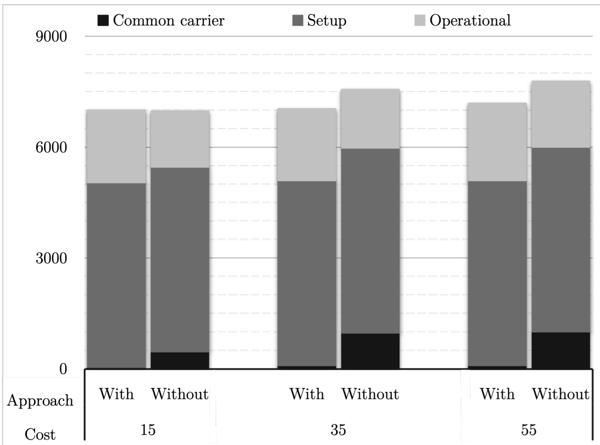


(b) Clustered with $(\lambda, m) = (100, 5)$

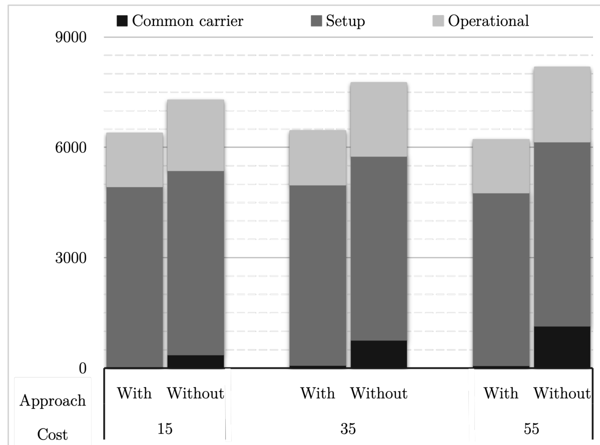
Regardless, the relationship between $\mathrm{r}^{\mathrm{min}}$ and $\mathrm{r}^{\mathrm{max}}$ in terms of percentage outsourced and cost still follows that of in c^{s}. That is, $\mathrm{r}^{\mathrm{min}}$ results in higher outsource percentages and lower costs compared to $\mathrm{r}^{\mathrm{max}}$ for 58.3% of R and 66.7% of C instances. The cost advantage of $\mathrm{r}^{\mathrm{min}}$ decreases compared to that of in c^{s}; however, the difference between the percentage of unfulfilled requests increases in favour of $\mathrm{r}^{\mathrm{min}}$. That is, considering the outsourcing option during the routing phase, rather than ignoring it until the end of the period, decreases both the cost and number of unfulfilled requests in c^{m}.

Following, another significantly important finding is that postponing customers from one period to another makes the plan further cost efficient. The cost advantage of postponing increases as the unit cost of outsourcing (z) increases. For R, postponing with the highest z value decreases the cost by 6.7% and 11.5% for $\mathrm{r}^{\mathrm{min}}$ and $\mathrm{r}^{\mathrm{max}}$ respectively. For C, the cost decreases by 8.0% and 12.8% for $\mathrm{r}^{\mathrm{min}}$ and $\mathrm{r}^{\mathrm{max}}$ respectively. Likewise, the increase in rate (λ) increases the cost advantage. For R, the decrease in cost from $\lambda = 50$ to $\lambda = 10$ increases by 12.0% and 6.4% for $\mathrm{r}^{\mathrm{min}}$ and $\mathrm{r}^{\mathrm{max}}$ respectively. For C, the increase is by 10.0% and 6.1% for $\mathrm{r}^{\mathrm{min}}$ and $\mathrm{r}^{\mathrm{max}}$ respectively.

Figure 5.5: Cost with and without postponement



(a) Random with $(\lambda, m) = (50, 5)$



(b) Clustered with $(\lambda, m) = (50, 5)$

5.4 Conclusion and Future Research

In this research, we introduced the dynamic and stochastic multi-period vehicle routing problem with partial outsourcing (DS-MVRPPO). The DS-MVRPPO addresses the challenges in same-day delivery problems from last-mile delivery systems. We presented the formal definition and a Markov decision process model of the dynamic and stochastic problem. We discussed the deterministic case of the problem over which we defined recourse actions. Following, we proposed a multi-stage stochastic programming model of the problem. We also proposed a dynamic stochastic hedging heuristic to address the practical instances of the DS-MVRPPO. The proposed approach uses the common features from the solutions of sample scenarios to determine the assigns and routes at each stage while iteratively constructing the delivery plan. We proposed two planning approaches to evaluate the impact of postponing requests; assigning with and without postponement, and two planning approaches o evaluate the impact of outsourcing requests; routing with and without outsourcing. These different planning approaches allowed us to provide detailed conclusions about the impact of postponing and outsourcing requests in last-mile delivery problems.

The computational experiments showed that assigning with postponement and routing with outsourcing indeed decreases the cost. In assigning without postponement (c^{s}), as the outsource cost increases the percentage outsourced decreases while routing with outsourcing ($\mathrm{r}^{\mathrm{min}}$) and increases while routing without outsourcing ($\mathrm{r}^{\mathrm{max}}$). Furthermore, the decrease in outsource cost increases the cost advantage of $\mathrm{r}^{\mathrm{min}}$ over $\mathrm{r}^{\mathrm{max}}$ in this setting. As a result, it is expected that $\mathrm{r}^{\mathrm{min}}$ follows to be the profitable approach under a discount structure in which the unit outsource cost decreases as the number of units outsourced increases. In assigning with postponement (c^{m}), the relationship between $\mathrm{r}^{\mathrm{min}}$ and $\mathrm{r}^{\mathrm{max}}$ still follows the same trend as in c^{s}, with respect to percentage outsourced and cost. Furthermore, the percentage outsourced with $\mathrm{r}^{\mathrm{min}}$ decreases compared to c^{s}. An apparent extension to our research is to consider the impact of the solution approach on the solution performance. Further, an interesting future research direction is to evaluate the performance of given planning strategies for outsource cost with a discount structure.

Chapter 6

Conclusion and Future Research

Motivated by the challenges arising from the growth in the demand for e-retailing, and so the last-mile delivery services, we investigate efficient routing strategies for the problem. The resulting routing problem is of importance both in practice and transportation science and logistics literature. On the one hand, the e-retailers, and so the last-mile delivery services, might benefit from the strategies and observations based on optimization techniques. On the other hand, the resulting problem corresponds to a case of the vehicle routing problem, which is a well-known and critical problem in logistics literature.

In Chapter 2, we analyzed an alternative vehicle and drone integrated delivery system, in which drones are responsible for resupplying the vehicle with dynamically revealing orders while vehicles are delivering orders to the customers. The drone resupply delivery system suggests an encouraging approach for the use of drones in delivery systems, by addressing almost all of the expectations without posing an additional challenge. We discussed the contribution of the research under consideration along with the position of the research in the literature. Following the detailed description of the problem, we formulated a special case with a single vehicle and a single drone in Chapter 2. In Chapter 3, we proposed a solution approach and conducted computational experiments to evaluate the performance of the drone resupply system over the conventional vehicle only delivery systems. We provided detailed observations with the respect to the operational time, cost and carbon emissions levels for several different practical settings. An apparent extension is to integrate stochastic information about the dynamically revealing requests into the decision model, to improve solution performance. Furthermore, the computational results indicated that it is possible to increase the number of requests that the vehicle serves in the drone resupply system, as both the utilization and endurance of the vehicle and drone allow so. To design ever more efficient strategies, an encouraging research direction might be deciding the service zones and the zones, i.e., vehicles, that the drone resupplies or integrating the decision regarding the minimum number of vehicles and drones required to the routing problem.

In Chapter 4, we explored the impact of outsourcing in last-mile same-day delivery problems. Besides ensuring the resilience of the distribution network in times of demand fluctuations, outsourcing might also introduce opportunities for reducing distribution costs. We discussed the motivation for the research under consideration and introduced the mathematical model of the problem in Chapter 4. In Chapter 5, we presented two different solution approaches. Following, we presented different planning strategies and provided detailed observations on the impact of postponing and outsourcing requests. The computational results indicated that outsourcing results in more profitable plans for all planning strategies. Here, considering the outsourcing decision during the planning phase, instead of outsourcing remaining requests, increases the cost advantage of outsourcing. Furthermore, postponing requests from one to another decreases the percentage of the unfulfilled requests and the cost, compared to otherwise. An interesting future research direction is to evaluate the performance of the given planning strategies for outsource cost with a discount structure.

In the context of dynamic last-mile delivery problems, an interesting research direction is to evaluate the strategic decisions, such as the number of vehicles, size of the service region, number and locations of the distribution centers. We considered these decisions as problem parameters and provided detailed observations on the impact of these parameters on the solution performance in our problems. Our observations suggested that some of these parameters have a significant impact on the resulting operational routing decision and that it is valuable to evaluate these strategic decisions more in detail together with operational decisions. Future research might further evaluate the impact of problem parameters, such as the number and locations of the distribution centers, size of the service area, the requests, on the solution by considering some of these parameters as decisions in the problem definition. Note that, such changes require also to design a new solution approach. Furthermore, in our research, we addressed same-day delivery problems with dynamic requests. However, in the context of the last-mile delivery problems, there are several other dynamic components, such as travel time and service time. Considering that the literature on dynamic service times is especially scarce, it might be helpful to include such sources of dynamism in future research to better represent the problem. Future research might also consider different sources for the stochastic information. Here, in our research, we considered the source of the stochasticity to be the demand and we assumed the capacity of the resources to be deterministic. It might be worthwhile to further evaluate the strategies considered in our research under a setting in which both the demand and the resources are stochastic in the delivery problem.

Besides the research direction based on the problems discussed here, it is also possible to contribute to the literature by building on the solution approaches from our research. Here, we focused on introducing efficient routing strategies for the dynamic last-mile delivery and presented an elaborate sensitivity analysis on the parameters of the corresponding delivery problem. We referred to the literature during the design of the solution approaches. The computational results showed that the proposed routing strategies have significant advantages over several problem settings. Note that, the solutions of the dynamic problems might be highly impacted by the solution parameters, so defining the best solution setting requires elaborate research. Furthermore, a solution setting that is performing well for given problem parameters might not show the same performance under different parameters. However, considering our encouraging findings, future researches might consider improving the performance of these solutions through either tuning parameters of our solution approach, such as the number of periods and scenarios, or implementing other solution approaches, such as simulation optimization.

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