Feasibility of Testing the Event Formalism with Refractive Materials

by

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A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Master of Science
in
Physics

Waterloo, Ontario, Canada, 2022

Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

In this thesis, we evaluate the feasibility of verifying the effect of gravity on the quantum entanglement by simulating gravity's effects with other physical systems. Specifically, we work in the framework of the Event Formalism, a non-standard theory about gravitational decoherence. We consider simulating gravity with acceleration as well as refractive materials. We find that these scenarios can not simulate the effect of gravity on entanglement as imagined by the Event Formalism.

Acknowledgements

I would like to thank my supervisors Prof. Thomas Jennewein and Dr. Agata Branczyk for their mentorship and guidance.

I would also like to thank Paul Oh. I learned a lot from him and it was a pleasure working together.

Dedication

To my parents, Zhang Yingchao and Li Keyi.

And Zheng Kun, who helped me a lot during the thesis writing time.

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Chapter 1

Introduction

Since quantum mechanics and general relativity were discovered, they became the most important theories in physics. Both are successful and have been supported by many experiments. However, the two theories are in conflict about the concept of "locality." General relativity is a local theory, while quantum mechanics is inherently nonlocal due to a phenomenon known as quantum entanglement [7, 8]. Scientists are exploring the overlap area of quantum theory and general relativity and trying to explain the conflict. One of the overlap areas is the gravitational effect on quantum entanglement [1, 9, 10, 11].

Several experiments on quantum entanglement have been performed. However, such experiments all happened in a weak gravity environment (on the earth). Because of this, we could only assume that gravity does not affect quantum entanglement. In recent years, many satellite experiments have also been performed [12, 13]. Satellites can provide more gravitational potential difference so that the effects of gravity on entanglement may show up. However, the potential difference between satellites and the earth are still not strong enough for some of the effects [14]. Higher orbit satellites are needed, which cost more money.

In this thesis, we explore proposals for testing gravitational effects on quantum entanglement using simulated gravity. The idea starts with the Event Formalism proposed by Ralph *et al.* [1, 3, 5], which is a similar concept to the hidden variable that appears in a strong gravity environment. Notice that this hidden variable does not question the existence of entanglement like the hidden variable hypotheses disproved by the Bell experiment [15]; it is just a possible factor that affects correlation. The factor can be integrated into one with the absence of gravity or entanglement, such that whether the factor exists or not, it will not affect our current theories. I will investigate the possibility of detecting the effect of gravity on entanglement in several ways.

In Chapter 3, I will introduce the possibility of experimental testing the Event Formalism and talk about testing it with acceleration. Because acceleration can simulate gravity due to Einstein's equivalence principle. However, the theoretical result shows that it is not possible to generate strong enough effects of the Event Formalism to be detected.

In Chapter 4, I will discuss the possibility of using simulated gravity to test the event formalism. Because I hope that the simulated gravity could be strong enough to test the Event Formalism and can be realized on the earth so that it can cost less than satellite experiments. I will use refractive material as the gravity simulation, since it is a common way. Using an optical medium to mimic gravity is not rare; many experiments on optic black holes or other phenomena have been studied [16, 17, 18, 19]. In this chapter, I will focus on the testing of Event Formalism with static refractive materials and determine whether refractive materials are good choices. The result shows that the effect of the Event Formalism will not be observed in the refractive material.

In Chapter 5, I will introduce another gravitational quantum decoherence theory, gravitational time dilation decoherence, which is a standard theory proposed by Zych*et al.* [20, 21]. I will discuss the possibility of testing the gravitational time dilation decoherence with refractive material. The reason of doing so is that we can study which gravitational properties can (or can not) be simulated by the refractive material by trying to reproduce the phenomenon predicated by standard quantum theory in the refractive material. The result shows that the refractive material can simulate the phase shift caused by gravity but can not simulate the decoherence caused by the Event Formalism.

My research indicates that, testing the Event Formalism on the earth directly or using refractive material is impossible. It also shows the limitation of simulating gravity with refractive material so that can be a reference of testing other effects of gravity on entanglement.

Chapter 2

Background

In this chapter, we review several concepts that will serve as a starting point for the work in this thesis. We begin with an introduction to quantum optics, then come to Event Formalism, discussing both the theoretical proposal as well as a experimental test. We then review some theories behind the existence of the refractive index.

2.1 Introduction to Quantum Optics

Quantum optics is a study that takes classical optics into the quantum world. Classical optics studies classical light, while quantum optics studies quantized light. This includes photons, atoms, and molecules, and the interaction between them, for example, the light-matter interaction. Quantum optics has wide application in experiments in many quantum areas, such as fundamental quantum mechanics, quantum communication, quantum computing, and quantum information.

2.1.1 Wave Packet

Harmonic Oscillator and Fock state

This section is based on [22].

The electromagnetic field can be considered as a harmonic oscillator, which describes the motion with potential energy which has the formula:

$$V(x) = \frac{m\omega^2}{2}x^2. \tag{2.1}$$

The time-independent Schrodinger equation of the harmonic oscillator will be

$$\hat{H}\psi_n = E_n\psi_n = \frac{1}{2m}\left[\hat{p}^2 + (m\omega\hat{x})^2\right]\psi_n, \tag{2.2}$$

where $\hat{p}$ is the momentum operator and $\hat{p} = \frac{\hbar}{i}\frac{d}{dx}$. ψ_n is the eigenstate of Hamiltonian $\hat{H}$, and the corresponding energy eigenvalue E_n is

$$E_n = \hbar\omega(n + \frac{1}{2}), \tag{2.3}$$

where $n = 0, 1, 2, 3...$ is a natural number. ψ_n is also known as Fock state and can be written as $|n\rangle$.

Annihilation and Creation operator

The annihilation and creation operators are also known as ladder operators in simple harmonic oscillation. They can be expressed in Fock state:

$$\hat{a} = \sum_n \sqrt{n+1}|n\rangle\langle n+1|, \tag{2.4}$$

$$\hat{a}^\dagger = \sum_n \sqrt{n+1}|n+1\rangle\langle n|, \tag{2.5}$$

$$[\hat{a}, \hat{a}^\dagger] = 1. \tag{2.6}$$

Light, which is an electromagnetic field, can be described by a harmonic oscillation. For light, the Fock state can be understood as the state of a different number of photons, where n is the photon number. The operator $\hat{a}$ can reduce the photon by -1 so it is called the annihilation operator. In the same way, $\hat{a}^\dagger$ can increase the photon by 1 so it is called the creation operator. People often write a single photon state as $\hat{a}^\dagger|0\rangle$.

The relationship between the position and momentum operators and the annihilation and creation operators are:

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a^\dagger} + \hat{a}), \tag{2.7}$$

$$\hat{p} = i\sqrt{\frac{m\omega\hbar}{2}}(\hat{a^\dagger} - \hat{a}). \tag{2.8}$$

Inserting this relationship (2.8) into (2.2), we can express the Hamiltonian with annihilation and creation operators:

$$H = \hbar\omega(a^\dagger a + 1/2). \tag{2.9}$$

Wave Packet

The wave packet is comprised of several waves that have different frequencies, amplitudes, phases, and wavenumbers. These waves can travel in a wide space but only interfere constructively in a small region. This region changes its position and size over time, and this is the propagation of the wave packet. The shape of the wave packet may change during propagation, and it depends on whether the material has dispersion. Dispersive material has the property of changing the refractive index with frequency. As for non-dispersive material, the shape of the wave packet remains constant, while in dispersive material it will change.

Mode operator

In quantum optics, a monochromatic light wave with exact frequency is named “a mode”, and the annihilation operator we introduced above is a single-mode operator. When we describe a wave packet of plane waves with some spectrum structure, we may mix different modes, so we need a multiple modes operator

$$\hat{a}(x,t) = \int dk G(k) e^{ik(x-t)} \hat{a}_k, \tag{2.10}$$

where k is the wavenumber and $\hat{a}_k$ is the annihilation operator of k, which means that it can annihilate a photon with frequency $1/k$. $G(k)$ is a distribution that describes the spectrum, for example, it can be Gaussian. Here we assume that the speed of light $c = 1$ and the plane waves are traveling in the positive direction.

2.1.2 Displacement Operator

The displacement operator can displace a state in phase space.

Coherent state

A coherent state is an eigenstate of an annihilation operator:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle. \tag{2.11}$$

where eigenvalue α is a complex number. Also, $|0\rangle$ is a coherent state:

$$\hat{a}|0\rangle = 0 = 0 * |0\rangle. \tag{2.12}$$

The coherent state is an important concept in quantum optics experiments; it has many benefits. For example, the coherent state is stable under linear optical operations. Imagine two sources, one light source provides Fock states with 5 photons $|5\rangle$, while one laser source provides coherent states. Due to the loss photons, the light source output is not stable, it may provide $|4\rangle$ sometimes. However, coherent state will keep stable even under the loss of photons, since the coherent state is an eigenstate of annihilation operator.

We can also write the coherent state in terms of Fock states:

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \tag{2.13}$$

Displacement operator

The definition of displacement operator is [22]

$$D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}, \tag{2.14}$$

where α is a complex number.

The displacement operator can move a state in phase space. When it acts on a coherent state α:

$$D(\beta)|\alpha\rangle = e^{iIm(\beta\bar{\alpha})}|\alpha + \beta\rangle. \tag{2.15}$$

The phase moving from α to $\alpha + \beta$ is shown in Figure 2.1

$$D^\dagger(\alpha) a D(\alpha) = a + \alpha \tag{2.16}$$

Recall that the mode operator is made up of ladder operators of different modes. Then the displacement of the mode operator [1] is

$$D(\alpha) = e^{\int dk G(k) e^{ik(x-t)} \hat{a}_k \alpha^* - h.c.}. \tag{2.17}$$

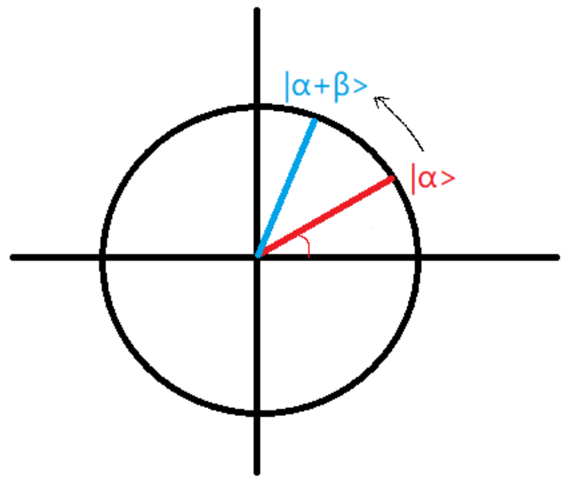

Figure 2.1: phase moving

2.1.3 Coherence

Coherent waves

The coherent wave is a commonly used concept in quantum optics. Coherent waves have identical frequencies and waveforms. When two waves are coherent, they can form stable interference fringes.

The mathematical definition of interference visibility is as follows [23]:

$$\gamma_{12} = \frac{G^{(1)}(x_1, x_2)}{\sqrt{G^{(1)}(x_1, x_1)G^{(1)}(x_2, x_2)}}, \tag{2.18}$$

where

$$G^{(1)}(x_1, x_2) = Tr(\rho \hat{E}^{\dagger}(x_1)\hat{E}(x_2)). \tag{2.19}$$

This mathematical formula describes the spatial coherence since we consider the wave E at different position x. If $|\gamma_{12}| = 1$, the state ρ is complete coherent, while $|\gamma_{12}| = 0$, it is incoherent. We can also study temporal coherence, spectral coherence, and so on, by writing the wave function as other physical quantities.

Coherence time and coherence length of light

This section is based on [24].

Coherence time and coherent length are similar concepts. The light source in an experiment often uses coherence time as a parameter. The concept of coherent time comes from temporal coherence. Temporal coherence describes the correlation between a wave at time t and $t + \tau$, that is, the correlation of it with itself after an amount of time τ. If the coherence significantly decreases after τ, then we call τ the "coherence time."
The coherence time of a wave packet is:

$$\tau = 1/\Delta\nu, \tag{2.20}$$

where $\Delta\nu$ is the bandwidth, which is the width of the spectrum.

The coherence time can indicate how monochromatic a light source is. If a light source is perfectly monochromatic, the coherence time will be infinite, because the waveform does not change after any amount of time, it can interfere with itself perfectly after any period of time. For a non-monochromatic source, the waves with different frequencies and velocities will wander and spread in the process of propagation, so it will make the shape of the wave packet change from tight to loose. After a period of time, the shape changes more obviously, and the coherence decreases. Some experiments require a coherent light source, and the experimental time should be shorter than the coherent time of the source, for example in a Mach-Zehnder interferometer with asymmetric path lengths.
The coherence length is the distance at which the wave propagates in coherent time. In the vacuum, it is

$$L = c\tau = c/\Delta\nu. \tag{2.21}$$

Quantum coherent

Since all objects have wave-like properties in quantum mechanics, they can interfere as well. For example, in Young's double-slit experiment, the electrons passing through the two slits act like the light passing through the two slits. The probability wave of the electrons will form interference fringes on the screen [25].

2.1.4 Beam Splitter

The beam splitter is an optical device that can split light from one mode into two output modes by reflecting and transmitting a portion of the beam.

To understand the effect of beam splitters, it will be convenient if we write the photon state as a creation operator acting on vacuum. As shown in Figure 2.2, the input states are $\hat{a}^\dagger|0\rangle$ and $\hat{b}^\dagger|0\rangle$, the output states are $\hat{c}^\dagger|0\rangle$ and $\hat{d}^\dagger|0\rangle$.

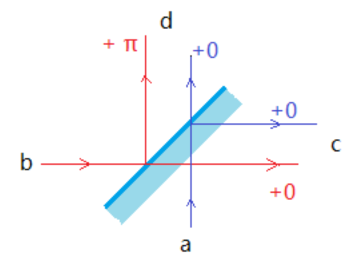

Figure 2.2: Beam Splitter.

Notice that the reflection on one of the surfaces will shift the phase by π.

A beam splitter with transmittance T will give the relation:

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \begin{pmatrix} \sqrt{1-T} & \sqrt{T} \\ \sqrt{T} & -\sqrt{1-T} \end{pmatrix} \begin{pmatrix} \hat{c} \\ \hat{d} \end{pmatrix}. \tag{2.22}$$

2.1.5 Hong–Ou–Mandel effect

This section is based on[26, 27]

The Hong–Ou–Mandel (HOM) effect is a kind of two-photon interference effect with a 50:50 beam splitter. The detectors count the photon number. For convenience of writing,

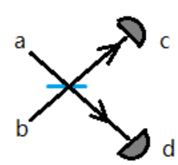

Figure 2.3: HOM effect. To measure the HOM effect, two photons will enter modes a and b of a beam splitter, and the output modes c, d will be observed with photon detectors.

if both detectors click once, we say that they count (1,1). While if one detector clicks twice and the other does not click, we say that they count (0,2) or (2,0). We do not distinguish (0,2) and (2,0) in the following chapter.

Assuming the two input photons are distinguishable, for example, they have orthogonal polarization: $\hat{a}_V$ and $\hat{b}_H$. The effect of the beam splitter is

$$\hat{a}_V = \frac{1}{\sqrt{2}}(\hat{c}_V + \hat{d}_V), \tag{2.23}$$

$$\hat{b}_H = \frac{1}{\sqrt{2}}(\hat{c}_H - \hat{d}_H). \tag{2.24}$$

The output is:

$$\hat{a}_V^\dagger \hat{b}_H^\dagger |0\rangle = \frac{1}{2}(\hat{c}_V^\dagger + \hat{d}_V^\dagger)(\hat{c}_H^\dagger - \hat{d}_H^\dagger)|0\rangle = \frac{1}{2}(\hat{c}_V^\dagger \hat{c}_H^\dagger - \hat{c}_V^\dagger \hat{d}_H^\dagger + \hat{c}_H^\dagger \hat{d}_V^\dagger - \hat{d}_V^\dagger \hat{d}_H^\dagger)|0\rangle. \tag{2.25}$$

The detectors might count (0,2) or (1,1) with a 50:50 probability. The outcome of the detectors is classically correlated.

If the input photons are not distinguishable, the output is as follows:

$$\hat{a}^\dagger \hat{b}^\dagger |0\rangle == \frac{1}{2}(\hat{c}^\dagger \hat{c}^\dagger - \hat{c}^\dagger \hat{d}^\dagger + \hat{c}^\dagger \hat{d}^\dagger - \hat{d}^\dagger \hat{d}^\dagger)|0\ = \frac{1}{2}((\hat{c}^\dagger)^2 - (\hat{d}^\dagger)^2) = \frac{\sqrt{2}|2,0\rangle + \sqrt{2}|0,2\rangle}{2}. \tag{2.26}$$

The detectors will count (0,2) and (2,0), they are quantum correlated.

2.2 Event Formalism

Event Formalism is a hypothesis raised by Ralph *et al.* [1, 3, 5], which describes a gravitational effect on quantum entanglement. The most well-known gravitational effect is curved space time. In a curved space time, an object traveling in different world lines may experience time differences. In Ralph's hypothesis, this time difference is essential because he supposes there exists an operator named event operator that is sensitive to the time difference between the world lines of the entangled pair. The difference between the standard theory and the Event Formalism is that the event operator keeps track of the time experienced by a particle on its past trajectory, and therefore the Event Formalism predicts decoherence which is different from the standard theory's prediction. We have not observed the event operator so far because the gravity on the earth is weak and the space-time is almost flat. In flat space-time, the effect of the event operator will not show up. Up to now, no experiment has proved the event operator is existing or not.

2.2.1 Motivation: Closed Time Curve

Ralph got his idea from the close time curve [2, 21], which allows a system to travel to the past and interact with itself as showed in Figure 2.4 and Figure 2.5.

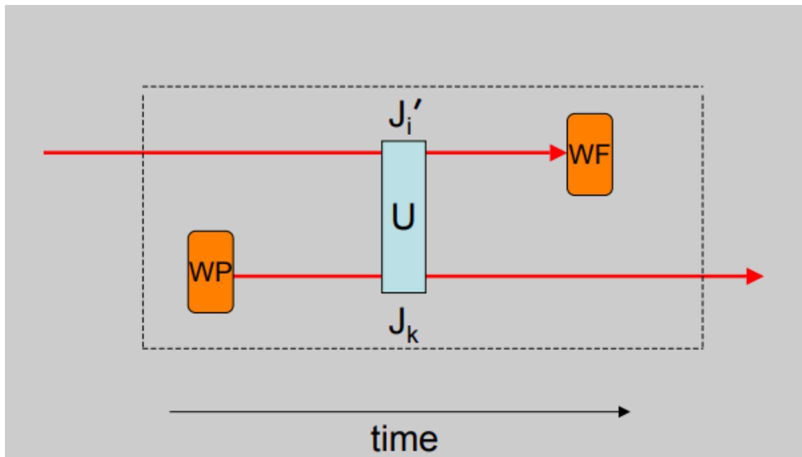

Figure 2.4: a particle interacts with the "younger" itself. *Note.* The figure is from [1]

The interaction was described by a gate U and a part of the output was also the input. To keep a consistent output, Bacon mentioned that the evolution from the input to the output was nonlinear [28]. It made Ralph want to build a non-standard theory that allows the commutator along the geodesic, which is the event formalism.

For time travelers inside the close time curve, it is obvious that they experience different time lengths as the outside. Based on this difference, Ralph constructed the event operator. Besides the close time curve, the curved space time can also make different rest times; this is the reason the event operator has the potential to describe the gravitational effects.

2.2.2 Event Operator

Recall the mode operator 2.1.1 of plane waves is

$$\hat{a}_K(x,t) = \int dk K(k) e^{ik(x-t)} \hat{a}_k. \tag{2.27}$$

where $\hat{a}_k$ is an annihilation operator at frequency k, $K(k)$ is a distribution on k. This equation means we generate waves with multiple modes and a spectrum.

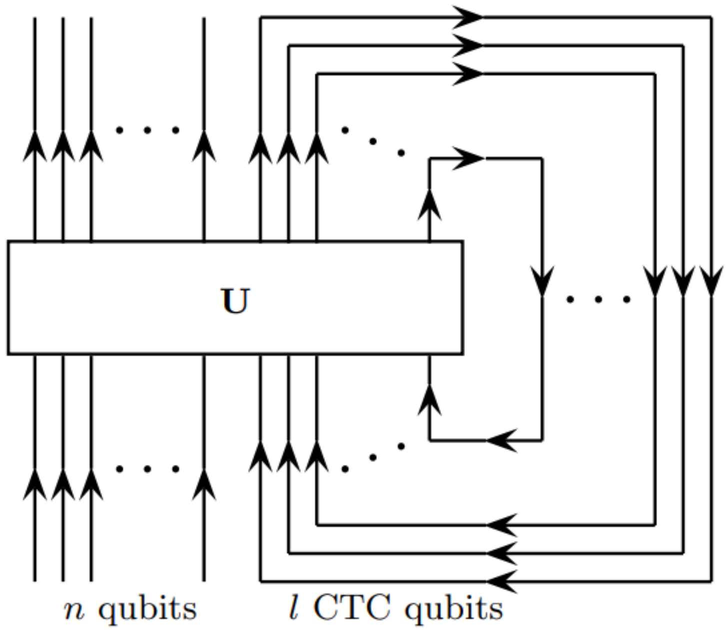

Figure 2.5: particles interacts with the "younger" others. *Note.* The figure is from [2]

Then, imagine a new annihilation operator based on $\hat{a}_K$. We want the new operator not only to depend on the wave number k but also on local time because local time is a manifestation of the gravity effect. Define Δ as the time difference: global time - local time.

To define global time and local time, Ralph has put forward several ideas. The final definition is showed in Figure 2.6. The global time t_d is the propagation time measured by a detector placed at the end of the particle path. Local time is measured by a series of detectors placed along the particle's world line. Each detector measures the time it takes for particles to pass by (from t' to $t^{(n)}$). Then add up all of them, we can get the local time.

The time difference Δ is

$$\Delta = t_d - \int_t^{t_d} ds. \tag{2.28}$$

Where $t_d = \sqrt{-g_{00}|_{x=x_d}} \int_{x_s}^{x_d} \sqrt{\frac{g_{ii}}{-g_{00}}} dx_i$, g_{ii} are spacial component is the global time, x_s

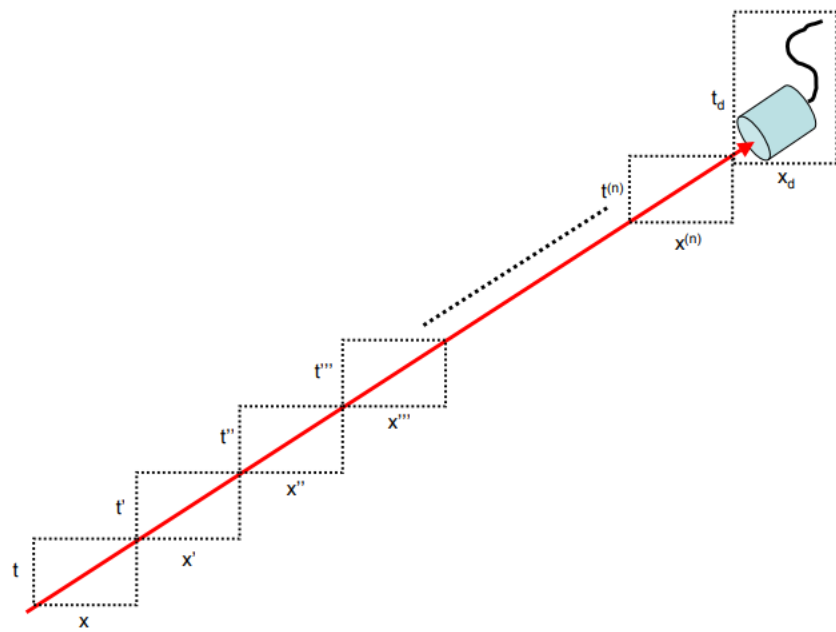

Figure 2.6: Definition of time difference. *Note.* The figure is from [3]

and x_d are the position of the source and the detector. ds is the time that the light passing by the detector, the integral of ds is the local time.

For particles in flat space-time, Δ is zero, while in curved space-time or time travelling in closed time curve, Δ will be nonzero. We also introduce the Fourier Complement of the time difference, which is Ω. With Δ and Ω, the new annihilation operator and event operator are defined as fellows:

$$\bar{a}_K(x, t, \Delta) = \int dk K(k) e^{ik(x-t)} \int d\Omega |K(\Omega)| e^{i\Delta\Omega} \hat{a}_{k,\Omega}. \tag{2.29}$$

The new annihilation operator $\hat{a}_{k,\Omega}$ and the event operator $\bar{a}_K(x, t, \Delta)$ will obey bosonic commutator relation:

$$[\hat{a}_{k,\Omega}, \hat{a}^\dagger_{k',\Omega'}] = \delta(k - k')\delta(\Omega - \Omega'), \tag{2.30}$$

$$[\bar{a}_K(x,t,\Delta), \bar{a}^\dagger_{K'}(x',t',\Delta')] = \frac{[\hat{a}_{k,\Omega}, \hat{a}^\dagger_{k',\Omega'}]}{\int d\Omega |K(\Omega)K'(\Omega)|}. \tag{2.31}$$

If the event operator does exist, it will change correlation under some conditions, and it is what makes the event operator different from the regular mode operator. Let's consider the correlation in 3 possible conditions: (1)in flat space time, (2)classical correlated pair in curved space time, and (3)entangled pair in curved space time.

Condition (1): if two wave packets are in flat space time, in other words, have the same time differences $\Delta = \Delta'$, then we can get

$$[\bar{a}_K(x,t,\Delta), \bar{a}^\dagger_{K'}(x',t',\Delta')] = [\hat{a}_K(x,t), \hat{a}^\dagger_{K'}(x',t')]. \tag{2.32}$$

The event operator acts the same as the mode operator, so the effect of Δ will not show up here.

Condition (2): the two wave packets have different time differences $\Delta \neq \Delta'$ and are classically correlated. Imagine this case: we have classically correlated laser beams, then send one to the flat space time and send the other to a curved space time. In this condition, we use two coherent states to build the classical correlation.

Recall the displacement operator in 2.1.2, which will generate coherent state from the vacuum is

$$D(\alpha) = e^{\bar{a}\alpha^* - \bar{a}^\dagger \alpha}. \tag{2.33}$$

The expected photon number given by the detection can be described by a projection

$$\langle \alpha | \bar{a}^\dagger \bar{a} | \alpha \rangle = \langle 0 | \bar{a}'^\dagger \bar{a}' | 0 \rangle = |\alpha|^2, \tag{2.34}$$

where $\bar{a}'$ is the Heisenberg picture evolution:

$$\bar{a}' = D(\alpha)^\dagger \bar{a} D(\alpha) = \bar{a} + \alpha. \tag{2.35}$$

α is the displacement of photon number. Here we assume that the coherent state we generated and the detector (the state to project on) have the same spectrum $K(k)$. By adjusting the light path, they can also have the same phase, so that α can reach the maximum value α_{max}, or it would be

$$\alpha = \int dk |K(k)|^2 e^{ik\phi} \alpha_{max}, \tag{2.36}$$

where ϕ is the phase different between the coherent state and the detector. Then consider the photon number correlation of the two pumps

$$C = \langle 0|\bar{a'}_1^\dagger \bar{a'}_2^\dagger \bar{a'}_1 \bar{a'}_2|0\rangle = |\alpha_1|^2|\alpha_2|^2, \tag{2.37}$$

where

$$\alpha_1 = \int dk|K(k)|^2 e^{ik\phi_1}\alpha_{max}, \tag{2.38}$$

$$\alpha_2 = \int dk|K(k)|^2 e^{ik\phi_2}\alpha_{max}. \tag{2.39}$$

The phase difference ϕ_1 and ϕ_2 are classical related since they both depends on the phase of the detector. By adjusting the light path, which means we set $\phi_1 = \phi_2$, then C can reach the maximum value $|\alpha_{max}|^4$ We can also see that the correlation C is independent on time difference Δ, the effect of the gravity still not show up.

Condition (3): the two pumps have different Δ and are entangled. The parametric entangling operator, which can generate a time-energy entangled pair from the vacuum (like an entangled version of displacement operator) is

$$U(\alpha) = e^{\alpha\bar{a}_1\bar{a}_2 - \alpha^*\bar{a}_1^\dagger\bar{a}_2^\dagger}. \tag{2.40}$$

The Heisenberg picture evolution is

$$\bar{a'}_1 = U(\alpha)^\dagger \bar{a}_1 U(\alpha) = \cosh\alpha\bar{a}_1 + \sinh\alpha\bar{a}_2^\dagger, \tag{2.41}$$

$$\bar{a'}_2 = U(\alpha)^\dagger \bar{a}_2 U(\alpha) = \cosh\alpha\bar{a}_2 + \sinh\alpha\bar{a}_1^\dagger. \tag{2.42}$$

We can see that $\bar{a}_1$ and $\bar{a}_2$ are mixed together after the evolution, and this is the most important difference from the classical correlation.

The photon number correlator is

$$C = |\alpha|^2 \int\int dk d\Omega |K(k)|^2 e^{ik\phi}|K(\Omega)|^2 e^{i\Omega\Delta}. \tag{2.43}$$

This time, the correlation is decreased by the term $e^{i\Omega\Delta}$. It means gravity can make the entangled pair decoherence if their time difference is nonzero. Notice that C will not reach $|\alpha_{max}|^4$ even after adjusting phases, because of the term $e^{i\Omega\Delta}$.

2.2.3 Satellite Experiment Result

In 2019, Ping Xu and colleagues did a satellite experiment to test the Event Formalism [14]. They considered the gravitational decoherence near the earth. The experiment set up a source on the ground products an entangled photon pair. Send one photon to the satellite, and keep the other one on the earth. The time difference for the one on the earth is neglectable, and the time difference for the one sent to the satellite, caused by gravity, is

$$\Delta = \int_{r_e}^{r_e+h} (\frac{M}{r} - \frac{M}{r_e + h})(1 + \frac{2M}{r} + \frac{r_e^2 \tan^2(90-\theta)}{r^2})^{1/2} dr, \quad (2.44)$$

where r_e is the radius of the Earth, h is the height of the satellite, M is the earth mass in the unit of length, and θ is the altitude angle.

If we assume the spectrum of Ω is Gaussian:

$$K(\Omega) = \sqrt{\frac{\sqrt{2}d_t}{\sqrt{\pi}}} e^{-(\Omega-\Omega_0)^2 d_t^2}, \quad (2.45)$$

where d_t is the coherence length of source. To find the coherence length, we can use the relationship between counting rate and the coherent length in Hong-Ou-Mandel measurement[26]:

$$N_c = 1 - e^{-\frac{\delta t^2}{4d_t^2}}, \quad (2.46)$$

where δt is the difference between the interferometer's two arms. In this experiment, the coherence length $d_t = o.o7mm$. The photon pair are polarization entangled.

The decoherence factor is therefore

$$D = e^{-\frac{\Delta^2}{2d_t^2}}. \quad (2.47)$$

The experimental results did not find evidence of the existence of the event operator, because the theoretical decoherence factor in this satellite experiment is 0.96, which is too close to 1 to be tested given the quality limitations of the experiment. Before the experiment, the laboratory team used Prof. Ralph's old definition of time difference that theoretically can be measured. After the experiment, Ralph corrected the definition. This chapter will introduce the corrected definition.

2.3 Generation of Refractive Index

The speed of light is constant in vacuum, however it can be slowed down by refractive material. The refractive index is the ratio of the slowed speed and the speed in vacuum, it describes the ability of the material to slow down light.
Since the gravity and the refractive material can both slow down the light, several interesting situations allow for one to simulate the other, for example gravitational lensing [29].

Before using the refractive material to simulate gravity, it is necessary to know how the refractive index generates. There are many models to explain the generation of refractive index, but some of them do not make sense. For instance, some think photons are absorbed by atoms in refractive material and emitted again and this process needs time. So, it seems like the material slows down the light. However, atoms emit photons in many different directions, but refracted light always travels in one direction as we can observe in the real world.

I will show two models to explain, one in classical and another one in quantum.

2.3.1 Classical Model of Refractive Index

I want to introduce the classical model given in Feynman's lecture [25] which consider light as electromagnetic wave. And for dielectric material, we consider that it has some free electrons. The basic idea is that, an electromagnetic wave can act on free electrons, and the moving free electrons will produce an additional wave. The slow light is the result of the sum of the original wave and the new wave. light sources in vacuum, the light from the sourcs, which the original wave is

$$E_s = E_0 e^{i\omega(t-z/c)}, \tag{2.48}$$

propagating along z axis.

We now assume the electrons in the material experience damped oscillation along the x-axis, which is the direction of the electric field. The driving force is provided by the incident light. Newton's 2nd Law of Motion with this driving force can be rearranged as

$$m(\ddot{x} + \gamma\dot{x} + \omega_0^2 x) = q_e E_0 e^{i\omega t}, \tag{2.49}$$

where m is the mass and q_e is the charge of an electron, ω_0 is the resonant frequency of the electron. Notice that the electron's position in the material is almost stable, so the original light can be simplified to $E_s = E_0 e^{i\omega t}$

From solving the former equation, we can get the motional function. Of the electron as

$$x = \frac{q_e/m}{-\omega^2 + i\gamma\omega + \omega_0^2} E_0 e^{i\omega t}. \tag{2.50}$$

The material is dielectric. The dipole moment of an atom in the material is

$$p = q_e x = \frac{q_e^2/m}{-\omega^2 + i\gamma\omega + \omega_0^2} E_0 e^{i\omega t}. \tag{2.51}$$

Rewrite it as

$$\vec{p} = \epsilon_0 \alpha(\omega) \vec{E}_s, \tag{2.52}$$

where

$$\alpha(\omega) = \frac{q_e^2/m\epsilon_0}{-\omega^2 + i\gamma\omega + \omega_0^2}. \tag{2.53}$$

α is called the atomic polarizability.

We consider the whole material, not only an atom. If the atom number density is N, the total polarization P will be

$$\vec{P} = Np = \epsilon_0 N \alpha(\omega) \vec{E}_s. \tag{2.54}$$

Consider the isotropic dielectric here, so P and E_s always have the same direction.

From Maxwell equation:

$$\nabla(\nabla \cdot \vec{E}_s) - \nabla^2 \vec{E}_s = -\frac{1}{\epsilon_0 c^2}\frac{\partial^2 \vec{P}}{\partial t^2} - \frac{1}{c^2}\frac{\partial^2 \vec{E}_s}{\partial t^2}, \tag{2.55}$$

$$\nabla^2 \vec{E}_s - \frac{1}{c^2}\frac{\partial^2 \vec{E}_s}{\partial t^2} = -\frac{1}{\epsilon_0}\nabla(\nabla \cdot \vec{P}) + \frac{1}{\epsilon_0 c^2}\frac{\partial^2 \vec{P}}{\partial t^2}. \tag{2.56}$$

To solve the equations, we first try to find a solution propagating along the z-axis. Assume it polarizes along the x-axis:

$$E_x = E_0 e^{i(\omega t - kz)}. \tag{2.57}$$

Knowing that the refractive index is n satisfy $n = kc/\omega$, we rearrange the solution:

$$E_x = E_0 e^{i(\omega t - nz/c)}. \tag{2.58}$$

Assume the material is linear dielectric, the polarization P_x will be dependent on time as $e^{i\omega t}$, so $\frac{\partial^2 P_x}{\partial t^2} = -\omega^2 P_x$. Put this and (2.58) into (2.56):

$$-\frac{n^2}{c^2} E_x + \frac{\omega^2}{c^2} E_x = -\frac{\omega^2}{\epsilon_0 c^2} P_x. \tag{2.59}$$

Recall (2.54) $P_x = \epsilon_0 N \alpha(\omega) E_x$. Insert this into (2.59):

$$n^2 = 1 + N\alpha(\omega) = 1 + \frac{N q_e^2}{m\epsilon_0} \frac{1}{-\omega^2 + i\gamma\omega + \omega_0^2}. \tag{2.60}$$

However, this result is not under the condition for dense material that the surrounding atoms will produce a field that is comparable to E_x. Assume the distance between the atoms is much smaller than the wavelength of the field, this condition can be described by the model: an atom placed in a spherical hole of the dielectric. From this model, we have:

$$E_{local} = E_x + \frac{P_x}{3\epsilon_0}, \tag{2.61}$$

$$P_x = \epsilon_0 N \alpha(\omega) E_{local}. \tag{2.62}$$

Solving the equation for P_x:

$$P_x = \frac{N\alpha(\omega)}{1 - \frac{N\alpha(\omega)}{3}} \epsilon_0 E_x. \tag{2.63}$$

Finally we obtain

$$n^2 = 1 + \frac{N\alpha(\omega)}{1 - \frac{N\alpha(\omega)}{3}}, \tag{2.64}$$

with $\alpha(\omega) = \frac{q_e^2/m\epsilon_0}{-\omega^2 + i\gamma\omega + \omega_0^2}$

2.3.2 Quantum model of Refractive Index

In the classical model, the energy level of the atom is not considered, and we will see how it makes difference in the quantum model. There are several approaches of applicable to slow light. Some of them use dispersion to get slow group velocity. I am going to introduce the model that gives slowed phase velocity. It is raised by W. G. Unruh and R. Schutzhold [30] with Λ energy model of atoms, which means two low energy levels and one excited level.

Assume the two low energy states are $|a\rangle$ and $|b\rangle$ with energy $-\omega_a$ and $-\omega_b$, the excited state $|c\rangle$ has energy 0 and decay constant Γ.

The Lagrangian of electromagnetic field is

$$L_E(x) = \frac{1}{2}(E^2 - B^2) = \frac{1}{2}[(\partial_t A)^2 - (\partial_x A)^2], \tag{2.65}$$

where A is the potential vector.

For each of the atoms, it is in the state $\psi_a|a\rangle + \psi_b|b\rangle + \psi_c|c\rangle$. The Lagrangian of an atomic is

$$L_A = i(\psi_a^*\partial_t\psi_a + \psi_b^*\partial_t\psi_b + \psi_c^*\partial_t\psi_c) + \omega_a\psi_a^*\psi_a + \omega_b\psi_b^*\psi_b + i\Gamma\psi_c^*\psi_c. \tag{2.66}$$

Use dipole approximation for the light-matter interaction. Let M_a, M_b be the transition amplitude, which describe the transition probability from low energy state to high energy state, and let x be the location of the atom.Then the Lagrangian of light-matter interaction:

$$L_I = E(x)(M_a\psi_c^*\psi_a + M_b\psi_c^*\psi_b) + h.c. \tag{2.67}$$

Then we use a strong background field (can be caused by a input field) to place the atoms in the "dark state", where we can cancel the spontaneous emission. Such background field should have the expression:

$$A_0 = \Omega[\frac{\cos\theta}{M_a\omega_a}e^{i\omega_a(t-x)} + \frac{\sin\theta}{M_b\omega_b}e^{i\omega_b(t+x)}] + h.c. \tag{2.68}$$

the background field is constituted by a left-going beam and a right-going beam. θ describes the relative strength of them., and Ω is the average Rabi frequency of the two beams. With rotation wave approximation [31], the non-decaying solution under this background field is

$$\psi_a^0 = e^{i\omega_a(t-x)}\sin\theta, \tag{2.69}$$

$$\psi_b^0 = -e^{i\omega_b(t+x)}\cos\theta, \tag{2.70}$$

$$\psi_c^0 = 0. \tag{2.71}$$

The total electromagnetic field contains the strong background field A_0 and external part:

$$A(t,x) = [\Omega\frac{\cos\theta}{M_a\omega_a} + \Phi_a(t,x)]e^{i\omega_a(t-x)} + [\Omega\frac{\sin\theta}{M_b\omega_b} + \Phi_b(t,x)]e^{i\omega_b(t+x)} + h.c. \tag{2.72}$$

where Φ represents the external part of the electromagnetic field and assume it changes slowly.

The atom state under the total electromagnetic field can be write as

$$\psi_a = e^{i\omega_a(t-x)}(\Phi_a + \sin\theta), \tag{2.73}$$

$$\psi_b = e^{i\omega_b(t+x)}(\Phi_b - \cos\theta), \tag{2.74}$$

$$\psi_c = \Phi_c, \tag{2.75}$$

where Ψ are also change slowly.

Input A, ψ_a, ψ_b and ψ_c into Lagrangian (2.65), (2.66) and (2.67). Consider the effective Lagrangian with second order terms of Φ and Ψ, the Lagrangian of electromagnetic field is

$$L_E \simeq 2i\omega_a\Phi_a^*(\partial_t + \partial_x)\Phi_a + 2i\omega_b\Phi_b^*(\partial_t - \partial_x)\Phi_b. \tag{2.76}$$

The Lagrangian of an atom is

$$L_A \simeq i(\Psi_a^*\partial_t\Psi_a + \Psi_b^*\partial_t\Psi_b + \Psi_c^*\partial_t\Psi_c + \Gamma\Psi_c^*\Psi_c) - i\Omega(\Psi_c^*\Psi_a\cos\theta + \Psi_c^*\Psi_b\sin\theta - h.c.). \tag{2.77}$$

The interaction Lagrangian is

$$L_I \simeq -i\omega_a M_a\sin\theta\Phi_a(x)\Psi_c^* + i\omega_b M_b\cos\theta\Phi_b(x)\Psi_c^* + h.c. \tag{2.78}$$

We obtain the equation of motion from Lagrangian:

$$\begin{aligned} &d_t\Psi_a = -\Omega\cos\theta\Psi_c, \\ &d_t\Psi_b = -\Omega\sin\theta\Psi_c, \\ &d_t\Psi_c = \Omega(\cos\theta\Psi_a + \sin\theta\Psi_b) - \Gamma\Psi_c + \omega_a M_a\sin\theta\Phi_a(x) - \omega_b M_b\cos\theta\Phi_b(x) \end{aligned} \quad . \tag{2.79}$$

$$(\partial_t + \partial_x)(2\Phi_a) = -\rho M_a\sin\theta\Psi_c(x), \tag{2.80}$$

$$(\partial_t - \partial_x)(2\Phi_b) = \rho M_b\cos\theta\sum_i \Psi_c(x_i), \tag{2.81}$$

where ρ is the density of atoms in the media.

Then solve the electromagnetic field Φ from the equation of motion. Remember that Φ and Ψ are changing slowly, we can neglect 2nd order derivative:

$$(\partial_t + \partial_x)\Phi'_a = -\frac{\rho\omega_a M_a^2 \sin^2\theta}{2\Omega^2}(\partial_t\Phi'_a - \partial_t\Phi'_b), \tag{2.82}$$

$$(\partial_t - \partial_x)\Phi'_b = \frac{\rho\omega_b M_b^2 \cos^2\theta}{2\Omega^2}(\partial_t\Phi'_a - \partial_t\Phi'_b), \tag{2.83}$$

where $\Phi'_a = \omega_a M_a \sin\theta\Phi_a$, $\Phi'_b = \omega_b M_b \cos\theta\Phi_b$.

We can choose a θ to simplify the math, make the coefficients equal:

$$\frac{\rho\omega_a M_a^2 \sin^2\theta}{2\Omega^2} = \frac{\rho\omega_b M_b^2 \cos^2\theta}{2\Omega^2} = N. \tag{2.84}$$

Then we can rewrite the field from (2.83) to:

$$(\partial_t^2 - \partial_x \frac{1}{1+2N}\partial_x)\Phi_\pm = 0, \tag{2.85}$$

where $\Phi_\pm = \Phi'_a \pm \Phi'_b$.

Assume the density ρ is constant, then the refractive index is

$$n = \sqrt{2N+1} = \sqrt{\frac{\rho\omega_a M_a^2 \sin^2\theta}{\Omega^2} + 1} = \sqrt{\frac{\rho\omega_b M_b^2 \cos^2\theta}{\Omega^2} + 1}. \tag{2.86}$$

2.4 Commutation Relation Provide by Refractive Material

Using a refractive material to simulate gravity is not only popular in theories and experiments of classical phenomenon, but also in quantum area. Many scientists have used refractive materials to build analogue event horizons and test Hawking radiation [32, 33, 34].

However, using refractive material to test Hawking radiation is not favored by William Unruh, who is the first one to use analogue gravity to observe Hawking radiation [35].

Unruh found that the commutation relation of light slowed by a refractive material is not suitable to simulate some quantum phenomenon [30]. With the model introduced in

2.3.2, the commutators of field in (2.85), which is the electromagnetic field inside refractive material are

$$[\Phi_\pm(t,x),\Phi_\pm(t',x')] = [\Phi_\pm(t,x)^\dagger,\Phi_\pm^\dagger(t',x')] = 0, \tag{2.87}$$

$$[\Phi_+(t,x),\Phi_+^\dagger(t,x')] = \delta(x-x'), \tag{2.88}$$

$$[\Phi_-(t,x),\Phi_-^\dagger(t,x')] = \frac{\delta(x-x')}{1+2N}. \tag{2.89}$$

The equal-time commutators are

$$[\Phi_+(t,x),\Phi_-^\dagger(t,x')] = [\Phi_+^\dagger(t,x),\Phi_-(t,x')] = 0. \tag{2.90}$$

We can compare the above commutators with the commutators of other fields. The scalar field commutators [36] are

$$[\phi(t,x),\phi(t,x')] = [\partial_t\phi(t,x),\partial_t\phi(t,x')] = 0, \tag{2.91}$$

$$[\phi(t,x),\partial_t\phi^\dagger(t,x')] = i\delta(x-x'). \tag{2.92}$$

The commutators of (1+1 dimensional) bosonic Schrodinger field are

$$[\phi(t,x),\phi(t',x')] = [\phi^\dagger(t,x),\phi^\dagger(t',x')] = 0, \tag{2.93}$$

$$[\phi(t,x),\phi^\dagger(t',x')] = \delta(x-x'). \tag{2.94}$$

Comparing the commutators of above three fields, we can see that the commutation relation of electromagnetic field inside refractive material is different from the scalar field and it is bosonic. It should not be used to simulate any system based on scalar field, for example, Black Hole horizon nor Hawking radiation.

In my research, I am questioning if the light inside the refractive material can simulate the light inside gravity and test the Event Formalism. Gravity is scalar field, however, the light inside gravity still has a bosonic commutator, which is not the same condition that were disproved by Unruh and Schutzhold, in other words, it is still meaningful to raise this question.

Chapter 3

Testing the Event Formalism under Angular Acceleration

Gravitational effects on quantum entanglement, such as Event Formalism, are difficult to study experimentally. One reason is that, if it exists at all, the effect will be very weak. To see such a weak effect, one approach is to make use of large differences in gravitational potential. One way to achieve this is by sending light between Earth and satellites orbiting Earth. However, that is a challenging task, especially when it requires a huge amount of financial support. Furthermore, as discussed in Chapter 2.2.3, a recent satellite experiment testing the Event Formalism was inconclusive [14]. Therefore, we propose an simulation approach to test these effects on Earth. In this approach, we consider the simulation of gravity by using acceleration, and investigate whether its effect on entanglement validates the Event Formalism introduced in Chapter 2.

Ultimately, we are interested in the effect of gravity on entanglement, but since it is not practical to create strong gravitational fields on the Earth, we can simulate them by accelerating our experiment setup. This analogue is possible to the connection between gravity and acceleration thanks to Einstein's equivalence principle [37]. The question is, how to design such an experiment? Is the experiment doable?

The experiment proposed in this chapter was inspired by an experiment performed by Fink *et al.* [4], which was designed to find the effect of a constant acceleration on the entanglement. I will consider a modified version of the configuration with the attempt to make it sensitive to the Event Formalism. This new experiment I designed should theoretically allow to show the effect of the Event Operator. However, it requires large scale equipment and a high angular velocity, which makes the experiment practically infeasible.

3.1 Testing effect of acceleration on entanglement

I will first revisit the analysis done by Fink *et al.* [4], where the authors considered two experiments: one experiment applies a free fall and another rotates the setup to observe its effects on polarization entanglement.

The effect was expected to be weak, so large acceleration was required. To ensure completion of the measuring without any optical equipment damaged, they built an ultra-stable entangled photon source. They placed the source and detectors in a crate that could withstand the large force while falling or rotating. In addition, another device is involved to keep the temperature stable. The source and detector are shown in Figure 3.1.

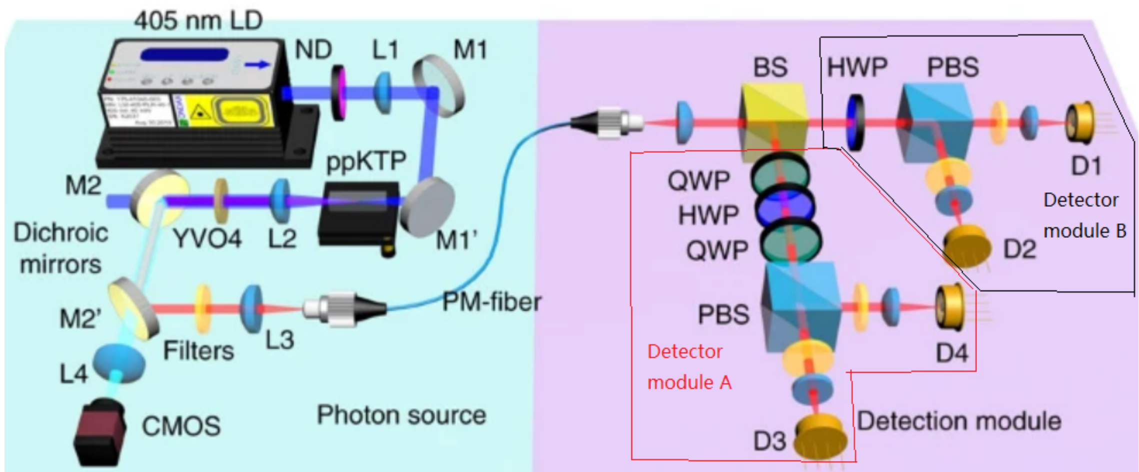

Figure 3.1: The source and detection in the experiment. Note. The figure taken from [4]

The source in their experiment was a nonlinear crystal that generated pairs of polarization-entangled photons, where the state is as follows:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|H_1V_2\rangle - |V_1H_2\rangle), \tag{3.1}$$

where H and V represent the horizontal and vertical polarization states, respectively.

They derived a lower bound for the fidelity of the experimental state with the ideal Bell state $|\Psi\rangle$:

$$F = \langle\Psi|\rho_{exp}|\Psi\rangle \geq \frac{1}{2}(V_{HV} + V_{DA}), \tag{3.2}$$

where V_{HV} and V_{DA} are the visibilities of the H/V and D/A basis. D/A are the diagonal and anti-diagonal basis, respectively. This equation indicates that the similarity between the experimental state and the ideal Bell state can be seen by measuring the visibility.

Four detectors were used to measure the correlation visibility. Detectors 1 and 2 measure Photon 1 on the H/V (or D/A) basis. Detectors 3 and 4 measure Photon 2 on the V/H(or A/D) basis. Then the visibility is

$$V = \frac{N_{13} + N_{24} - N_{14} - N_{23}}{N_{13} + N_{24} + N_{14} + N_{23}}, \tag{3.3}$$

where N_{ij} is the number of coincidences between detectors i and j. A coincidence is detected when the detectors i and j click. Both experiments were performed under the constant acceleration in time and position.

In the free-fall experiment, they dropped the crate which gained a high velocity right before hitting the ground. The stopping acceleration reached $16g$, where $g = 9.80m/s^2$. They collected data from 400,000 signal photon clicks.

In the rotation experiment, their set-up is shown in 3.2A. They placed the crate on the edge of a rotating bar. The source and the detector were about 3m away from the rotation axis. The acceleration of the setup was raised from $1g$ to $30g$ in $5g$ steps, making about 140,000 coincident counts for each acceleration.

They found that compared to the low acceleration ($= 3 \times 10^{-3}$g), the lower bound on Bell-state fidelity did not change much. The mean value was 0.9645. After considering the error caused by temperature fluctuations and other factors, the fidelities for different accelerations were all in the range of the mean value. Therefore no abnormal or unexpected effects were observed in the observed entanglement quality.

3.2 Proposed Event Formalism Test with Acceleration

The experiment described in the previous section was a good approach for testing the gravitational effect on entanglement; however, the experiment was not designed to test the event formalism, because they were placed in the same crate so that both felt the same acceleration, and therefore does not determined that the existence event operator,

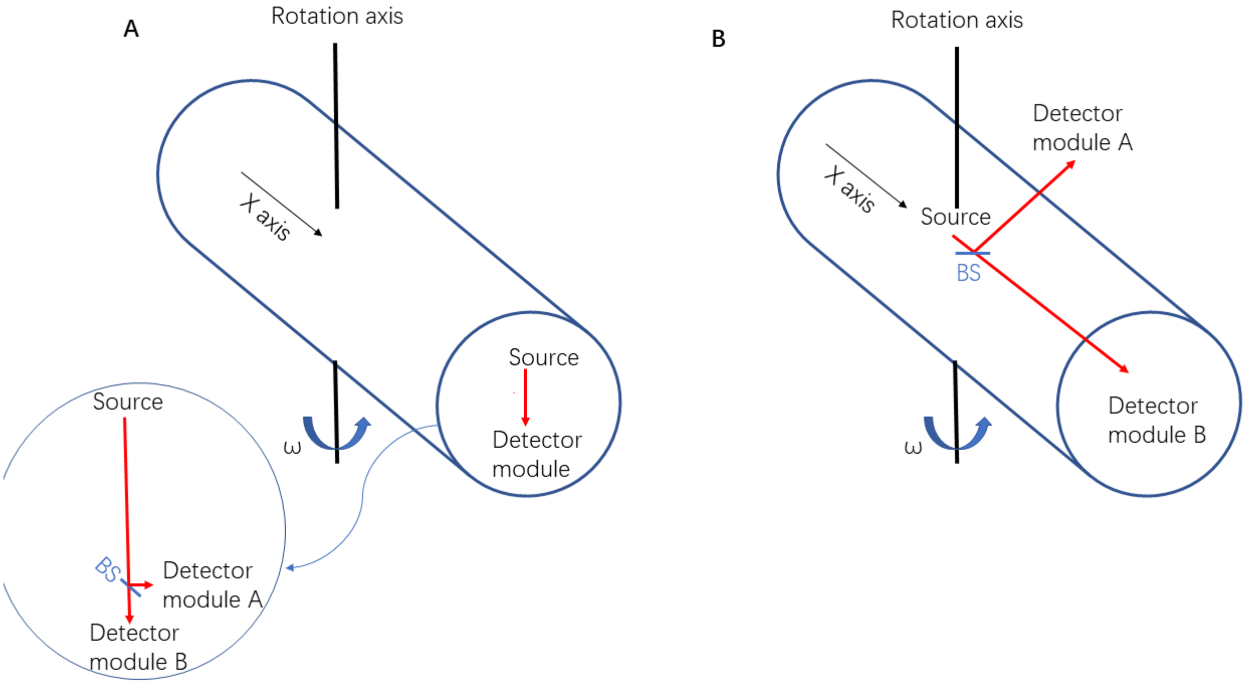

Figure 3.2: Modified setup. A shows the original setup that Fink used. B shows the modified setup introduced in this chapter. The source and the detection module A and B are the same as showed in Figure 3.1. The x axis is rotating with the bars.

nor can it confirm other theories based on changes on gravity [38, 39, 40, 41]. This is the case because the Event Formalism requires each photon from an entangled pair to feel different gravitational fields however. We therefore seek to modify the experiment in the previous section such that each photon experiences different accelerations. Out of the two experiments presented, the rotating setup makes this modification easier. Furthermore, it makes it possible to gain even larger accelerations. We thus focus on the rotating setup in this section.

My proposed modification is to move the photon source and detector module A to the rotation axis and keep the detector module B at the edge of the bar, as shown in Figure 3.2B. As the bar rotates, one photon of the entangled pair is sent from the rotation axis to the edge, while the other photon does not leave the rotation axis, instead traveling along the axis. Thus, the two photons experience different accelerations.

To observe the expected decoherence, we need the time difference Δ in the Event Formalism, which describes how curved the space time is. Recall that the time difference (2.28) is the difference between the global time and the local time along the light path. In the rotating setup, the time difference in the event formalism can be calculated by special relativity. We consider the case with a small angular velocity since the bar's length and angular velocity that can be produced in the laboratory are small.

To calculate the time difference, we need the global time and the local time. Recall that the global time (2.28), which is also the propagation time, is measured by placing a detector at the end of the photon's path. The photon's path in the rotating setup is along the bar. We now want to know the Lorenz factor, so we can use it to calculate the local traveling time between imaginary detectors along the light path. The local time is measured by a series of imaginary detectors placed along x axis. The Lorenz factor [25] for the detector is

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{r^2\omega^2}{c^2}}}, \tag{3.4}$$

where ω is the constant angular velocity, and r is the distance between the detector and the axis in the laboratory frame. The velocity is $r\omega$.

We define the travelling time to be the time that takes a photon to pass by point-like detectors. For the local time, each detector measures the travelling time as the photon passes by that detector, and the local time is the sum of these travelling times. The travelling time in the detector frame is $\Delta r/c\gamma$, where Δr is a short distance located at the detector, and the travelling time is the time that takes a photon to travel this short distance. The local time τ is

$$\tau = \int_0^L \frac{dr}{c}\frac{1}{\gamma}, \tag{3.5}$$

where L is the total length of the bar.

The global time, which is the propagating time measured by the detector at the edge of the bar in the detector's frame:

$$t'_d = \frac{L}{c}\frac{1}{\gamma} \tag{3.6}$$

Insert (3.5) and (3.6) into (2.28), we can get the time difference (result calculated by Mathematica):

$$\Delta = \tau - t'_d = -\frac{L\sqrt{1 - \frac{L^2\omega^2}{c^2}}}{2c} + \frac{\arcsin(\frac{L\omega}{c})}{2\omega}. \tag{3.7}$$

With the time difference, we can derive the decoherence factor $D = e^{-\frac{\Delta^2}{2d_t^2}}$, where d_t is the coherence time of the source. According to [4], the source has about 5% systematic errors. The expected decoherence should be at least larger than 5% to be observed.

3.3 Results

To determine the feasibility of observing the effect of gravity on entanglement as predicted by the Event Formalism, we will consider specific experimental parameters. If we set the length of the bar $L = 10$ m, and the angular velocity $\omega = 30$ rad/s, then the time delay will be $\Delta = 1.11107 \times 10^{-20}$ s, and the decoherence factor is then $D = 1 - \exp(-0.5\Delta^2/d_t^2) = 0.55 \times 10^{-15}$ where d_t in the unit of length is 0.07 mm, which is the same as the source used in the satellite experiment. [14]. While this assumed 10-meter-long bar with angular velocity = 30 rad/s is not easy to be accomplished in the lab, the decoherence is still much too small to be observed. If we want to observe the decoherence, for instance, 10% gives the time difference of 5×10^{-10} s with $d_t = 0.07$ mm.

Other rotating equipment, such as ultracentrifuge, could provide a larger acceleration with a shorter length and a higher angular velocity. To decide whether to choose the longer length or higher angular velocity, I plotted Figure 3.4 and found that a longer length L can increase the time difference more significantly, which helps more in verifying the event operator.

3.4 Discussion

Since a longer rotation bar can make the effect of the Event Formalism more observable and the effect increases nonlinearily with L, I prefer a larger experimental equipment with a longer centrifuge's length is preferred. My estimates show that the length and angular frequency of the setup are too difficult to achieve with current technology, and therefore in conclusion, this experiment is unlikely to be feasible.

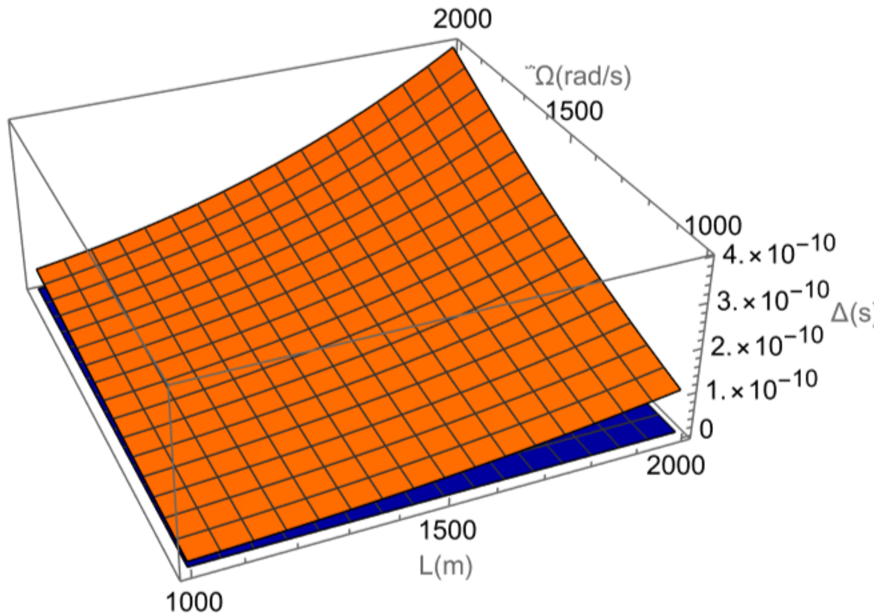

Figure 3.3: Time difference Δ vs L and ω. The figure shows the the region near the point that the decoherence is observable ($\Delta = 5 \times 10^{-10}$ s). The length and the angular are too big to be achieved.

Since it is too difficult to verify the event operator with an accelerating setup, I will explore other ways in the following chapters using refractive index.

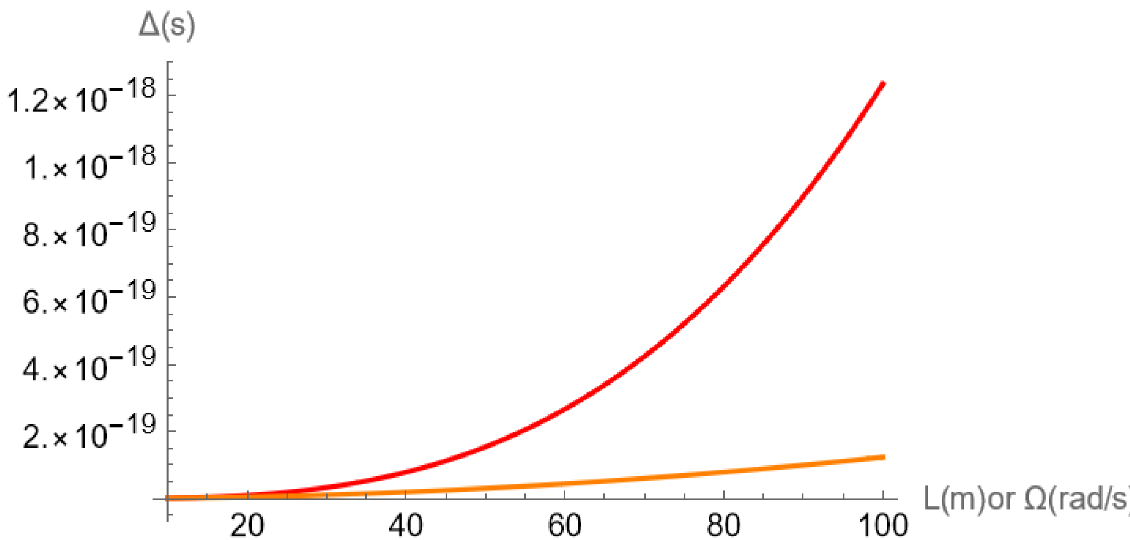

Figure 3.4: Time difference Δ vs L or ω. The figure shows the time difference. Red: Bar length changes from 10m to 100m. Orange: Angular velocity changes from 10 rad/s to 100 rad/s. The time difference is extremely small in this region. We can see that the time difference increases faster as L increases, compared to the increase with ω. The increasing of ω looks almost linear, while the increasing of L is higher order. For example, at L=10 m and ω= 10 rad/s, when L increases 10 times, the time difference will be around 1000 times larger. When ω increases 10 times, the time difference will be around 100 times bigger.

Chapter 4

Testing the Event Formalism with Refractive Index

Simulating the effect of gravity on light using analogue systems has received a lot of attention lately [42]. An example of such an analogue system is an optical medium in which the refractive index, like gravity, slows down light. Simulating gravity with refractive materials can be done in different ways, such as the use of moving materials [43, 44], using nonlinear hyperbolic metamaterials [45], or changing refractive index [34].

In the previous chapter 3, we looked at simulating gravity with acceleration to test the effect of gravity on entanglement. We show that conditions that would lead to observable effects were difficult to engineer in a laboratory setting on the earth. In this chapter, we will consider simulating gravitational effects with refractive materials. As in the previous chapter, we will specifically focus on the Event Formalism. In section 2.4 we already know that the commutation relation we need in the Event Formalism can be realised in refractive materials.

The question is, if it is to test the Event Formalism using refractive materials? Is the refractive material a suitable one? It may be unclear if slowing light is enough to provide evidence for the necessity of Event Formalism, or whether the gravitational effect on quantum entanglement has other properties that cannot be simulated by the refractive material. In this chapter, I will formulate a map between gravity and the refractive index, and then describe an experimental set-up using stationary refractive materials. This setup is chosen because it is easier to be set up in a laboratory. We find that under conditions regularly met in laboratory settings, one would already expect to see an effect. The fact that we have not seen this in existing experiments suggests that either the Event Formalism is wrong or the refractive material should not be used to test the Event Formalism.

4.1 Space-time Metric of Refractive Material

In general relativity, the geometric structure of the universe can be described by its metric. I will introduce a way to map from the metric to the refractive index of a stationary material.

We set the direction of propagation of light to be x^1 and assume that the direction of propagation does not change. For instance, the light travelling straight towards the center of a stationary black hole will not change direction. The refractive index is defined as the ratio of the speed of light in a vacuum c to the speed of light in the material $v = \frac{\mathrm{d}x^1}{\mathrm{d}t}$ [23]:

$$n = \frac{c}{v} = c\frac{\mathrm{d}t}{\mathrm{d}x^1}. \tag{4.1}$$

The propagation of light under general relativity is described in the following way [46]

$$g_{00}(c\mathrm{d}t)^2 + g_{01}(c\mathrm{d}t)\mathrm{d}x^1 + g_{10}\mathrm{d}x^1(c\mathrm{d}t) + g_{11}(\mathrm{d}x^1)^2 + g_{22}(\mathrm{d}x^2)^2 + g_{33}(\mathrm{d}x^3)^2 = 0, \tag{4.2}$$

where g_{ij} is the element of the metric, which describes the space time. Since the light strictly propagates in the x^1 direction, $\mathrm{d}x^2 = \mathrm{d}x^3 = 0$. We solve equations (4.1) and (4.2) and obtain

$$c\mathrm{d}t = \frac{-2g_{01}x^1 \pm \sqrt{(4g_{01}^2 - 4g_{00}g_{11})}\mathrm{d}x^1}{2g_{00}} \tag{4.3}$$

$$c\frac{\mathrm{d}t}{\mathrm{d}x^1} = \frac{-g_{01} \pm \sqrt{(g_{01}^2 - g_{00}g_{11})}}{g_{00}} = n. \tag{4.4}$$

For convenience, we may consider a metric with $g_{01} = 0$ (many space time have this properties, for example, the Schwarzschild metric). We are trying to artificially create a curved space-time, so it is reasonable to describe the setup environment with the Schwarzschild metric. We find:

$$n = \sqrt{\frac{g_{11}}{-g_{00}}}. \tag{4.5}$$

This equation will be our main relation between the refractive index and the curvature of space-time.

According to general relativity, our metric (which contains the refractive index) would be equivalent to the metric that describes the linearized gravity field (where a weak gravity field is considered) [46]:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} + \begin{pmatrix} 0 & & & \\ & \chi & & \\ & & \chi & \\ & & & \chi \end{pmatrix} = \begin{pmatrix} -1 & & & \\ & n^2 & & \\ & & n^2 & \\ & & & n^2 \end{pmatrix}, \tag{4.6}$$

where χ is the susceptibility. We can see that $g_{11} = n^2$, $g_{00} = -1$, therefore

$$\mathrm{d}s^2 = -(c_0 \mathrm{d}t)^2 + n^2(\mathrm{d}x^1)^2. \tag{4.7}$$

This result agrees with others, we get the same map as in [47].

However, we will consider stronger gravity to verify Event Formalism and the effect will therefore become noticeable. We thus consider the Schwarzschild metric elements from general relativity [46]:

$$g_{00} = -\left(1 - \frac{r_s}{r}\right) \tag{4.8}$$

$$g_{11} = \left(1 - \frac{r_s}{r}\right)^{-1} \tag{4.9}$$

We insert (4.9) into (4.5) to find

$$n = \left(1 - \frac{r_s}{r}\right)^{-1} = g_{11} = \frac{1}{-g_{00}}. \tag{4.10}$$

In this way, the metric elements can be expressed by the refractive index. We will apply all these relations in the simulation. Note that air, which refractive index ≈ 1, can simulate flat space time, which is equivalent to an observer being infinitely far away.

4.2 Testing the Event Formalism

We can now create the experimental simulation to verify Event Formalism. Using the metric tensor that we just defined in terms of the refractive index. The Event Formalism studies the possibility of decoherence between an entangled pair experiencing different gravity. We will use a refractive material to generate an artificial Schwarzschild metric, and air to generate an infinitely far region. The setup is shown in Figure 4.1

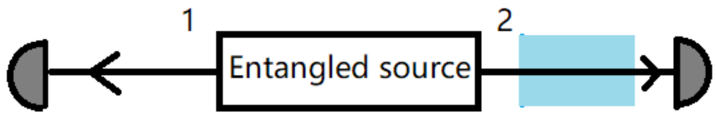

Figure 4.1: Proposed experiment set up to test the Event Formalism. The entangled source provides photon 1 and photon 2. Due to [5], I choose time-energy entangled pair because the decoherence will be easier to be observed. Photon 1 travels to the photon detector through air. Photon 2 travels through a refractive material (blue box). Path 1 would be longer than path 2 to keep the two photons in the same phase while being detected. The detectors record the arriving time
, and we want the rate of coincidence detection.

The first step is to find the time difference Δ between global time and local time [1], and then calculate the decoherence factor, given a material with a refractive index n and its length L. The global time is measured by an observer at the end of both light paths, which is described by [14]

$$t'd = \sqrt{-g_{00|x=x_d}}\sqrt{\frac{g_{11}}{-g_{00}}}x_l, \tag{4.11}$$

where x_d is the position of the observer, x_l is the length of light path in the observer frame, and corresponds to L in the simulation experiment. Note this equation is a simplified version when gravity is constant along the light path (we can assume $x_l \ll r$ because the light path is very short compared to the radius of the Earth, so that gravity does not affect much along the light path), which can be exactly simulated by refractive material, since the refractive index is constant inside the material. The observer at the end of the path is outside the material, in the air. It is equivalent to the observer in flat space time. Therefore, $-g_{00|x=x_d} = 1$.

Next, we insert (4.5) into (4.11) to get:

$$t'd = \frac{nL}{c}. \tag{4.12}$$

The local time is measured by the imaginary observers inside the material, which means that inside the curved space time, it is given by [14]

$$\tau = x_l \left(\sqrt{1 - \frac{r_s}{r}} \right)^{-1}. \tag{4.13}$$

Again, this is the simplified version for constant gravity. Insert (4.10) into (4.13):

$$\tau = \frac{\sqrt{n}L}{c}. \tag{4.14}$$

Then the time difference is

$$\Delta = t'd - \tau = (n - \sqrt{n})\frac{L}{c}. \tag{4.15}$$

Recall that the decoherence factor was defined as (2.47):

$$D = \exp\left(\frac{-0.5\Delta^2}{d_t^2}\right) = \exp\left(-0.5\frac{(n - \sqrt{n})^2 L^2}{d_t^2}\right), \tag{4.16}$$

where d_t is the coherence time of the source. For our analysis, we assume that our source has the same coherence was the one used in the satellite experiment [14], so that the coherence length is 0.07 mm.

If we have $L = 0.5cm$, $n = 1.5$, which is the data for a beam splitter slice, the decoherence factor will be:

$$D = \exp\left(-0.5\frac{((1.5 - \sqrt{1.5})0.5cm)^2}{(0.07mm)^2}\right) \approx 0. \tag{4.17}$$

This corresponds to complete decoherence.

4.3 Discussion

I used the metric tensor derived in terms of the refractive index, and designed an experiment using a beam splitter as the refractive material. Calculation of the expected decoherence factor ($D = 0$) indicated that the photon pair is fully decoherent. However, the beam splitter is a commonly used device in many quantum experiments. If a beam splitter can efficiently make photons decoherent, the decoherence event would have been observed before. Note that the decoherence caused by Event Formalism cannot be canceled by adjusting the light path, refer to (2.43). However, the decoherence caused by non-dispersive materials can be canceled in this way. To do this, we just need to change the length of the light path until the phases of the photons at the detectors are the same. Since no scientist has observed non-canceled decoherence, the simulation experiment failed to support the Event Formalism.

Since the experiment fails to support Event Formalism, I speculate that this is explained in one of two probable ways: 1. The hypothesis of Event Formalism is not plausible. 2. Using dielectric material to test Event Formalism is inappropriate. Regarding the first possible explanation: The Event Formalism is just a hypothesis for a newly defined event operator, which is incorrect. Regarding the second possible explanation: The Event Formalism is directly related to time. Although using dielectric material to simulate a curved space-time is mathematically feasible, it does not mean that the simulated time has the same physical properties as the real world time. Inside the dielectric material, the speed and phase of photons act as if time is stretched, but we cannot be sure whether it is really stretched. In addition, the following question remains unanswered: Is the stretched simulated time enough to test Event Formalism? We will answer these questions in Chapter 5.

Chapter 5

Testing Quantum Decoherence due to Gravitational Time Dilation

If a quantum system is not perfectly isolated, it will be correlated with the environment and the system will decohere. This is called quantum decoherence. One of the recent quantum decoherence theories, is that it could be caused by gravitational time dilation, the topic of which was introduced by Zych *et al.* [20, 21]. Like the Event Formalism, it is based on both relativity and quantum mechanics, however, it only contains standard theory without new assumptions.

Since refractive materials can mimic a curved space-time, can they also mimic the decoherence effect induced by gravitational time dilation? In this chapter, we will study the gravitational time dilation decoherence. I will briefly talk about the possibility to observe the decoherence on satellites. Then I will focus on the decoherence between entangled photons and design a simulation experiment that uses refractive material to simulate the effects of gravity.

By studying this topic, we will try to answer the following questions raised in Chapter 4: (i) Does the refractive material experiment disprove the Event Formalism since it does not show decoherence? (ii) Is the refractive material experiment inappropriate for testing the Event Formalism?

In this chapter we will find that the refractive material can only simulate the phase shift effect of gravitational time dilation. In other words, it is not suitable to test the Event Formalism.

5.1 Gravitational Time Dilation Decoherence

Decoherence caused by gravitational time dilation of massive particles was introduced in Pikovski *et al.* [20]. In order to explain why its not possible to observe a large group of massive particles -for example, chemical compounds- in the superposition of positions [20]. As for nanoparticles, this kind of quantum superposition is still possible [48]. Pikovski and Zych's theory combined relativity and quantum mechanics.

The basic idea that Pikovski *et al.* is proposed as follows. In relativity, if a system with mass (m) has Hamiltonian (H_0) for its time evolution of the internal degrees of freedom, then the total rest mass is

$$m_{tot}c^2 = mc^2 + H_0. \tag{5.1}$$

The gravitational potential energy is $m_{tot}\Phi(x) = m\Phi(x) + H_{int}$, where $\Phi(x)$ is the gravitational potential at position x. The interaction term $H_{int} = \Phi(x)\frac{H_0}{c^2}$. If we also consider the momentum, then we have

$$H_{int} = \left(\Phi(x) - \frac{p^2}{2m^2}\right)\frac{H_0}{c^2}. \tag{5.2}$$

H_{int} describes how gravity couples to the internal energy. For example, if the massive particle is a harmonic oscillator, then the gravitational effect leads to a frequency change: $\omega \to \omega(1 + \frac{\Phi(x)}{c^2})$. This result agrees with the gravitational redshift [49, 50].

Consider a system that is in the superposition of two different gravitational potentials, we can express it as $|\Psi\rangle = \frac{1}{\sqrt{2}}(|x_1\rangle + |x_2\rangle)$ (different potential may be caused by different height), where x_1 and x_2 represent the position where the particles appear. The time evolution generated by H_{int} will lead to decoherence. By theoretically calculating the visibility of the superposition state, Pikovski *et al.* obtained the decoherence time:

$$\tau_{dec} = \sqrt{\frac{2}{N}}\frac{\hbar c^2}{k_B T g \Delta x}, \tag{5.3}$$

where N is the number of particles. We can see that with a large particle number, the decoherence time will be very small. If $N \sim 10^{23}$ and $\Delta x = 10^{-6}$ m, then $\tau_{dec} \approx 10^{-3}$ s. The short decoherence time explains why we do not observe the superposition of a large group of particles in everyday situations.

Next, we will apply the gravitational time dilation to photons. The derivation follows the similar method that Pikovski *et al.* used, but we replace the massive particle's Hamiltonian with the photon's Hamiltonian, and use a Gaussian distributed state instead of thermal state. This allows us to find that the decoherence time for photons has a similar expression as the decoherence time of massive particles they found.

5.2 Photon Decoherence

We will assume that the theory of decoherence by gravitational time dilation shown in Section 5.1 has applications beyond massive particles, and can also apply to photons, because photons also have decoherence under H_{int}, and we will show this decoherence theoretically. This photon decoherence makes it possible to use the refractive material to test the theory.

The internal Hamiltonian for photon is

$$H_0 = \hbar\omega(a^\dagger a + 1/2). \tag{5.4}$$

The interact Hamiltonian is

$$H_{int} = \Phi(x)\frac{H_0}{c^2} = \frac{\Phi(x)}{c^2}\hbar\omega(a^\dagger a + 1/2), \tag{5.5}$$

where $\Phi(x)$ is the gravitational potential at position x.

We consider a photon at the superposition of different gravitational potentials (height), as shown in Figure 5.1. The gravity metric on the earth can be considered as Schwarzschild metric since other objects' gravity are weak comparing to earth's gravity.

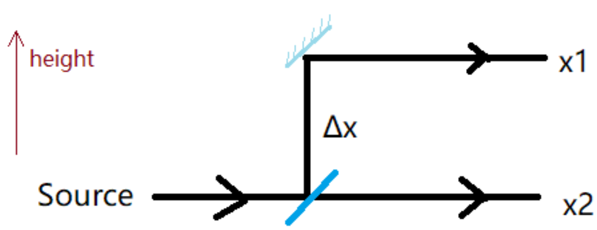

Figure 5.1: A light beam is prepared in a superposition of different height. The beam splitter (blue) generates a superposition of an optical signal along paths x_1 and x_2. If the paths have a height difference Δx, this will cause a superposition of different heights. There is no superposition of spectrum or polarization.

The initial density matrix (after the beam splitter) of this photon is

$$\rho(t=0) = |\psi(0)\rangle\langle\psi(0)| \bigotimes \rho', \tag{5.6}$$

where $|\psi(t=0)\rangle$ represents the path term, it is

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|x_1\rangle + |x_2\rangle). \tag{5.7}$$

ρ describes the internal freedom of the photon. Assume that it has a Gaussian spectrum:

$$\rho' = \int d\omega |G(\omega)|^2 a_\omega^\dagger |0\rangle\langle 0| a_\omega, \tag{5.8}$$

where $|G(\omega)|^2 = \frac{1}{\sqrt{a_0 \pi}} e^{-\frac{(\omega-\omega_0)^2}{a_0}}$. We will see that the path terms cause red shift in the following calculation.

Insert (5.7) and (5.8) into (5.6):

$$\begin{aligned} \rho &= \frac{1}{2}\int d\omega |G(\omega)|^2 a_\omega^\dagger (|x_1, 0\rangle + |x_2, 0\rangle)(\langle x_1, 0| + \langle x_2, 0|) a_\omega \\ &= \frac{1}{2}\int d\omega (|x_1(0,0)\rangle + |x_2(0,0)\rangle)(\langle x_1(0,0)| + \langle x_2(0,0)|) \end{aligned} \tag{5.9}$$

where $|x_i(0,0)\rangle = G(\omega) a_\omega^\dagger (0,0) |x_i, 0\rangle$

In plane wave modes, the mode operator is [1]

$$a_\omega(s,l) = e^{-i\omega(s-l/c)} a_\omega(0,0), \tag{5.10}$$

where s is the local time and l is the local position. The speed of light is always $c = 2.99 * 10^8 m/s$ when measured locally. For convenience, we rewrite $a_\omega(0,0)$ as a_ω in the following equations. With (5.10) we can get the evolution of $|x_i(s,l)\rangle$, which is the photon at (s,l):

$$|x_i(s,l)\rangle = G(\omega) e^{-i\omega(s-l/c)} a_\omega^\dagger |x_i, 0\rangle. \tag{5.11}$$

If we observe the photon at infinite distance from a mass, we need to translate the local time s and local position l into our space time coordinate. The translation will obey [1]:

$$ds = \sqrt{1 - \frac{r_s}{r}} dt, \tag{5.12}$$

$$dl = \left(\sqrt{1 - \frac{r_s}{r}}\right)^{-1} dr, \tag{5.13}$$

where t and r are the time and position at an infinite distance. r_s is the Schwarzschild radius, a constant related to the center mass. For the earth, r_s is very small (about 9mm).

By setting the initial point (0,0) in both (s, l) and (t, r) frame, we can have rewrite (5.12) and (5.13) as

$$\begin{aligned} s &= \sqrt{1-\frac{r_s}{r}}t \\ l &= \left(\sqrt{1-\frac{r_s}{r}}\right)^{-1} r \end{aligned}, \tag{5.14}$$

Assume that the most part of the light path is horizontal, so that the path x_1 has constant height $r = x_1$, and $\sqrt{1-\frac{r_s}{x_1}}$ is also constant. We obtain $\sqrt{1-\frac{r_s}{x_1}} = \eta_1$ for path x_1, and $\sqrt{1-\frac{r_s}{x_2}} = \eta_2$ for path x_2. After inserting η_1, η_2 and (5.14) into (5.11), we obtain the evolution of path x_1:

$$|x_1(t,r)\rangle = G(\omega)e^{-i\omega(\eta_1 t - \frac{r}{\eta_1})}a^\dagger_\omega|x_1, 0\rangle. \tag{5.15}$$

And for path x_2:

$$|x_2(t,r)\rangle = G(\omega)e^{-i\omega(\eta_2 t - \frac{r}{\eta_2})}a^\dagger_\omega|x_2, 0\rangle. \tag{5.16}$$

Next we look at the non-diagonal elements of the density matrix at time t, because they are related to decoherences as the state evolves from a pure superposition state to a mixed state. Assume that the lengths of the two paths are z_1 and z_2, measured by an observer infinitely far away. Then the nondiagonal element is $\rho_{12} = \langle x_1|\rho|x_2\rangle$. The time evolution of the nondiagonal element is

$$\begin{aligned} \rho_{12}(t) &= \frac{1}{2}\langle x_1(0,0)|\left(\int d\omega|G(\omega)|^2 e^{-i\omega(\eta_1 t - \frac{z}{\eta_1})}a^\dagger_\omega|x_1,0\rangle\langle x_2,0|a_\omega e^{i\omega(\eta_2 t - \frac{z}{\eta_2})}\right)|x_2(0,0)\rangle \\ &= \frac{1}{2}\frac{1}{\sqrt{a_0\pi}}\langle x_1(0,0)|\left(\int d\omega e^{-\frac{(\omega-\omega_0)^2}{a_0}} e^{-i\omega(\eta_1 t - \frac{z}{\eta_1})}a^\dagger_\omega|x_1,0\rangle\langle x_2,0|a_\omega e^{i\omega(\eta_2 t - \frac{z}{\eta_2})}\right)|x_2(0,0)\rangle \end{aligned}, \tag{5.17}$$

which can be simplified to

$$\rho_{12}(t, z_1, z_2) = \frac{1}{2}\frac{1}{\sqrt{a_0\pi}}\int d\omega e^{-\frac{(\omega-\omega_0)^2}{a_0}} e^{-i\omega((\eta_1-\eta_2)t - (\frac{z_1}{\eta_1} - \frac{z_2}{\eta_2}))}a^\dagger_\omega|0\rangle\langle 0|a_\omega. \tag{5.18}$$

In order to concentrate on the decoherence caused by the time dilation, we adjust the length of each path to make their propagation times cancel each other out:

$$\frac{z_1}{\eta_1} - \frac{z_2}{\eta_2} = 0. \tag{5.19}$$

The visibility is defined as [20]

$$v(t) = 2|\rho_\psi^{12}(t)| = |Tr_\omega[\rho_{12}(t)]|. \tag{5.20}$$

The result is calculated by Wolfram Mathematica:

$$v(t) = |\frac{1}{\sqrt{a_0 \pi}} \int d\omega e^{-\frac{(\omega-\omega_0)^2}{a_0}} e^{*i\omega(\eta_1-\eta_2)t}| = |e^{-i(\eta_1-\eta_2)t\omega_0 - \frac{(\eta_1-\eta_2)^2 t^2 a_0}{4}}| = e^{-(\eta_1-\eta_2)^2 t^2 a_0/4}. \tag{5.21}$$

If we write the visibility in the form $v(t) = e^{-(t/\tau_{dec})^2}$, then we can define the decoherence time τ_{dec}:

$$\tau_{dec} = \sqrt{\frac{4}{a_0}} \frac{1}{\eta_1 - \eta_2}. \tag{5.22}$$

In the following sections, we will calculate the expected decoherence time to determined if the experiment is doable and if the theory is supported.

5.3 Possible Satellite Experiment

If we perform the experiment on a satellite to observe the decoherence in time, the gravitational potential will be varied along the radial path from the Earth's center such that the evolution of a state will be [21]

$$|\tau\rangle = e^{\frac{-i}{\hbar} \int_\gamma d\tau H_{int}} |\tau(0)\rangle, \tag{5.23}$$

where γ is the path and τ is the proper time along the path. If the path has higher gravitational potential, it will produce faster decoherence. To simplify the equations, we assume that the potential along the whole light path is equal to the potential on the satellite. This assumption is not true since the path between the satellite and the earth has lower potential. This assumption makes the total potential much higher so that it will make the decoherence much stronger than the real condition, which means the decoherence time will be much shorter. However, the following analysis shows that even the upper bound is not strong enough to be observed. With this assumption, we can use the decoherence time in (5.22):

$$\tau_{dec} = \sqrt{\frac{4}{a_0}} \frac{1}{\sqrt{1 - \frac{r_s}{R_{sate}}} - \sqrt{1 - \frac{r_s}{R_E}}}, \tag{5.24}$$

where R_{sate} is the radius of the satellite orbit, and R_E is the radius of the Earth. The decoherence time depends on the spectrum of the source, and the coherence time of the source also depends on the spectrum. One should not confuse the decoherence time with coherence time: the decoherence time describes the density matrix losing its nondiagonal element, becoming a mixed state. While the coherence time is only determined by the source, which is introduced in Chapter 2.1.3. We can express the relationship between the decoherence time and the coherence time, then evaluate whether the experiment is feasible.

The full width at half maximum (FWHM) of gaussian is $2\sqrt{\ln 2 * a_0}$, and the coherence time of the Gaussian source is [24]:

$$\tau = 1/\Delta\omega = \frac{1}{2\sqrt{\ln 2 * a_0}}. \tag{5.25}$$

Rearranging for a_0, then insert (5.25) into (5.24); then we have the relation between decoherence time τ_{dec} and coherence time τ:

$$\tau_{dec} = 4\sqrt{\ln 2}\frac{1}{\sqrt{1-\frac{r_s}{R_{sate}}} - \sqrt{1-\frac{r_s}{R_E}}}\tau. \tag{5.26}$$

If we wanted to do the experiment with a satellite at 36000 km altitude orbit, then $\tau_{dec} = 7.5 \times 10^8 \tau$. The decoherence time would be much longer than the coherence time, which means that it would not be possible to observe the decoherence because the gravitational decoherence will be covered by the decoherence caused by other perturbations. With a larger gravitational potential difference can be larger, the decoherence time could decrease. However, earth orbiting satellites are not able to provide the potential we need (even the sun can not provide enough potential.). Therefore, in the next section, we will discuss the probability of using simulated gravity.

5.4 Consideration of an Experiment using Refractive Material

5.4.1 Proposed Experiment Setup and Expected Outcome

We will consider if we can use refractive index in an optical material to simulate strong gravity and test the gravitational time dilation decoherence. We can test the feasibility

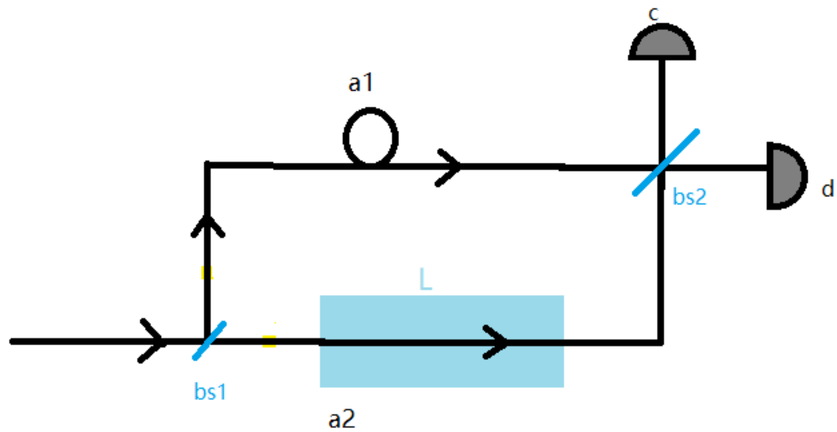

Figure 5.2: Proposed experimental setup to test Gravitational Time Dilation Decoherence. bs_1 and bs_2 are two beam splitters. After bs_1, the photon is in the superposition state of path a_1 and path a_2. Path a_1 has no refractive material, which simulates a path the flat space-time (infinite far away in Schwarzschild space). While the refractive material on path a_2 simulates some other path in Schwarzschild. The superposition of path a_1 and a_2 simulates the height superposition. The detector c and d are single photon detector. We can use single photon interference to find out the visibility.

of using refractive index by testing the decoherence time. The setup of the proposed experiment is shown in Figure 5.2.

First, we want to calculate the expected decoherence time. Recall that for the Schwarzschild metric (4.5):

$$n = \left(1 - \frac{r_s}{r}\right)^{-1}. \tag{5.27}$$

Fellow the similar calculations from (5.10) to (5.15), the light in Schwarzschild will travel like:

$$|\psi\rangle = \int G(\omega) e^{-i\omega((1-\frac{r_s}{r})^{1/2}t-(1-\frac{r_s}{r})^{-1/2}z)} a_\omega^\dagger |0\rangle = \int G(\omega) e^{-i\omega(\eta t - \frac{z}{\eta})} a_\omega^\dagger |0\rangle. \tag{5.28}$$

We can see that:

$$\eta = \left(1 - \frac{r_s}{r}\right)^{1/2} = \frac{1}{\sqrt{n}}. \tag{5.29}$$

For path a_1, the refractive index = 1. We insert $\eta_1 = 1$ and $\eta_2 = 1/\sqrt{n}$ into (5.22), then we can get the expected decoherence time:

$$\tau_{dec} = \frac{4\sqrt{\ln 2}}{(1 - n^{-1/2})}\tau, \tag{5.30}$$

where τ is the coherence time of the source. The limit for an extremely high refractive index is $\tau_{dec} = 3.3\tau$. In the laboratory, the high refractive index could be 3 to 4.

In the experiment, we can test the visibility with single photon interference. From the definition of the decoherence time, (5.21) to (5.22), the expected visibility is:

$$v(t) = e^{-(t/\tau_{dec})^2}. \tag{5.31}$$

The way to find out the visibility with the experiment measurement outcome (the click rates of detectors) is introduced in Appendix B. By recording the measurement outcomes at different time, we can find out the visibility at different time, and compare to the expected $v(t)$.

Recall that the decoherence time (5.30) works under the condition (5.19): $\frac{z_1}{\eta_1} - \frac{z_2}{\eta_2} = 0$. I will determine the lengths of l_1, l_2 and L to satisfy this condition.

Recall that l_2 is the part of path a_2 that is outside the material. This part has the same refractive index as path a_1, so that the time-like dilation will not cause decoherence, but will cause phase shift. To cancel the phase shift, we need to add the same length to path a_1. We can divide path a_1 into two parts; one has length l_2 for canceling phase shift, the other part has length $l_1 - l_2$ for simulating time dilation decoherence, which we can call $l_1 - l_2$ the"effective length". In the following calculation, we only consider the effective length, because we only want to study the decoherence. The effective length for path a_1 is $l_1 - l_2$, for path a_2 is L.

z in (5.19) is the length of the path measured in the flat frame, which is the laboratory frame. Therefore, z is the effective length:

$$z_1 = l_1 - l_2, \tag{5.32}$$

and

$$z_2 = L. \tag{5.33}$$

From (5.19) and (5.29), we have

$$\sqrt{n}L = l_1 - l_2. \tag{5.34}$$

In the experiment, we can choose the material with exact n and make the L, l_1 and l_2 satisfy (5.34) and test if the measurement results satisfy (5.30).

5.4.2 Calculation of Decoherence time for Refractive Material

The simulation experiment has not been done yet, and we now have some doubt that one will observe the decoherence effect. Lets for a moment forget the time dilation and determine the decoherence generated by the experimental set up. We only consider the effective length because only the effective length affects decoherence. Assume the state in path a_1 is

$$\rho_{a_1} = \int d\omega G(\omega) e^{-i\omega(t-l_1/c)} a_\omega^\dagger |a_1, 0\rangle\langle a_1, 0| a_\omega e^{i\omega(t-l_1/c)}. \quad (5.35)$$

The state in path a_2 is

$$\rho_{a_2} = \int d\omega G(\omega) e^{-i\omega(t-l_2/c-nL/c)} a_\omega^\dagger |a_2, 0\rangle\langle a_2, 0| a_\omega e^{i\omega(t-l_2/c-nL/c)}. \quad (5.36)$$

As introduced in 2.1.4, the superposition that is produced by beam splitter 1 is:

$$\begin{aligned}\rho = \frac{1}{2}\int d\omega a_\omega^\dagger (e^{-i\omega(t-l_1/c)}|a_1, 0\rangle + e^{-i\omega(t-l_2/c-nL/c)}|a_2, 0\rangle) \\ (e^{i\omega(t-l_1/c)}\langle a_1, 0| + e^{i\omega(t-l_2/c-nL/c)}\langle a_2, 0|) a_\omega\end{aligned} \quad (5.37)$$

The visibility is:

$$\begin{aligned}V(t) &= 2|\rho_a^{12}(t)| = |Tr_\omega[\rho_{12}(t)]| \\ &= \sum_\omega \langle 0|a_\omega|a_1\rangle\langle a_2|a_\omega^\dagger|0\rangle = \int d\omega |G(\omega)|^2 e^{-i\omega(t-l_1/c)} e^{i\omega(t-l_2/c-nL/c)}.\end{aligned} \quad (5.38)$$

We insert (5.34) into (5.38) (the integral calculated by Mathematica) and obtain:

$$V(t) = \int d\omega |G(\omega)|^2 e^{-i\omega((\sqrt{n}-n)L/c)} = e^{-(\sqrt{n}-n)^2 (L/c)^2 a_0/4} = e^{-\frac{(L/c)^2}{\tau_{dec}^2}}, \quad (5.39)$$

where

$$\tau_{dec} = \frac{2}{\sqrt{a_0}} \frac{1}{\sqrt{n}-n}. \quad (5.40)$$

Using (5.25) for the coherence time, we find that the decoherence time is

$$\tau_{dec} = \frac{4\sqrt{\ln 2}}{(\sqrt{n}-n)} \tau. \quad (5.41)$$

We can see that the decoherence time here is different from the expected value in previous section (5.30): $\tau_{dec} = \frac{4\sqrt{\ln 2}}{(1-n^{-1/2})}\tau$, they are different by a factor of $\sqrt{n}$. Moreover, the visibility in 5.39 is not a function of time, but a function of length. It is not surprising that we observe the decoherence caused by the arm length difference in the HOM measurements many times. The difference between the two decoherence times comes from the gravity simulation: In the previous section, I mapped the refractive index to gravity and applied the gravitational decoherence formula to it, while in this section, I did not perform the simulation and studied the decoherence that occurs in refractive materials. The difference in the decoherence time shows that the map is inappropriate and refractive material cannot reproduce the gravitational time dilation decoherence.

5.5 Try a Different Map

The refractive index cannot simulate the time dilation decoherence with the map I showed because in the simulation experiment, the states $|a_1\rangle$ and $|a_2\rangle$ always have the same time term so that their time terms cancel each other and will not affect the decoherence.

Assume that we find another map from refractive index to gravity. With such a map, we could do the simulation experiment, and the visibility can have the same formula as the visibility in gravitational time dilation decoherence. To do this, the new map should allow interference to happen between the photons at different times, so that the time terms will not cancel each other. Fortunately, the map exists.

For example, let the time evolution for a curved space time be

$$a = e^{-i\omega(\eta t - x/\eta)} a^\dagger |0\rangle, \tag{5.42}$$

then the evolution in a refractive material is

$$a' = e^{-i\omega(t' - nx'/c)} a^\dagger |0\rangle. \tag{5.43}$$

We can make a' equivalent to a by simply setting $t' = \eta t$, $nx'/c = x/\eta$. Notice that n can be any value, it does not need to satisfy (5.29).

The method we use here is space-time translation in Euclidean geometry, it can only simulate the effect of phase shift. For gravitational time dilation decoherence, we can always reproduce the same value of the phase shift that caused by the red shift, by choosing the appropriate t' and x'. Therefore, we can reproduce the decoherence.

5.6 Discussion

In Section 5.4, we introduced a map that lets the refractive material simulate the curved space-time. In this map, the states $|a_1\rangle$ and $|a_2\rangle$ (the two states to interfere) always have the same time terms, because the interference as an event can not have two times (in the same frame). However, this method does not predict the correct decoherence time nor reproduces the correct gravitational time dilation decoherence, which indicates that it does not simulate the gravity appropriately.

While in Section 5.5, we tried another map which is a translation in Euclidean Geometry. This map could only reproduce the phase shift, thus reproducing the correct value of the gravitational time dilation decoherence.

For the decoherence caused by other gravitational effects, especially when the decoherence requires more than a phase shift, the refractive material will not reproduce it. For example, the Event Formalism. Remember that the Event Formalism decoherence effect cannot be canceled by moving the detector (adjusting the phase). Therefore, refractive materials are not suitable for verifying the Event Formalism. It answers the questions we left in the previous chapter.

Chapter 6

Discussion

In this thesis, I studied the Event Formalism and gravitational time dilation decoherence and investigated the feasibility of using the refractive material to test the gravitational effect on entanglement.

The previous satellite experiment [14] and my own analysis show that the earth and the satellite cannot achieve sufficient gravitational potential difference to observe the effect of the Event Formalism. In Chapter 3, I showed that the acceleration experiment on the Earth is also not doable. I selected the rotation acceleration experiment rather than the free-falling acceleration experiment because the former could offer higher acceleration and met the requirement of testing the Event Formalism. However, due to the tough requirement of long rotation radius and high angular velocity, it is not possible to observe the effect of this experiment on the Earth.

Due to the failure of satellite experiment and acceleration experiments, I turned to simulated gravity to study the gravitational effect on entanglement in Chapters 4 and 5. I chose to use the refractive material to simulate gravity. In Chapter 4, I introduced the map from refractive index to gravity and an experiment setup that could show the decoherence caused by the Event Formalism. By theoretical analysis, we showed that we would not observe the Event Formalism decoherence in refractive materials.

The question becomes, is Event Formalism disproved? Or is refractive material suitable to be used in testing the Event Formalism? To answer these questions, I used refractive material to test other gravitational effects on entanglement in Chapter 5. I chose to test the gravitational time dilation decoherence. It is a standard theory, and its effect can be observed through decoherence like the Event Formalism does. I first applied the gravitational time dilation decoherence to the photon. I found that the refractive material cannot reproduce such decoherence. The refractive material can simulate the decoherence caused by phase shift, but cannot simulate other properties of gravity, nor the gravitational effect caused by possible hidden variable (like the Event Formalism). It means that the Event Formalism (or any other gravitational decoherence phenomenon that requires more than a phase shift) and therefore cannot be tested with refractive material. However, more complex ways to create slow light have been realized or will be realized, for example, negative refractive index. The complex refractive materials have the possibility to simulate more properties of gravity.

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APPENDICES

Appendix A

Canceling Dispersion

A.1 Dispersion

The refractive index is a function of frequency ω. This is a characteristic of refractive materials called "dispersion". Since different frequency lights propagate with different velocity, the wave packet spreads while it is propagating through the medium which can causes decoherence.

To describe the dispersion, we use the definition of the group delay α and group-velocity dispersion (GVD) β [54]

$$\alpha = \frac{1}{v_g} = \frac{dk}{d\omega}, \tag{A.1}$$

$$\beta = \frac{d^2k}{d\omega^2} = -v_g^{-2}\frac{dv_g}{d\omega}, \tag{A.2}$$

where k is the wavenumber and v_g is called group velocity. The wavenumber can be expressed as

$$k(\Omega + d\omega) = k(\Omega) + \alpha d\omega + \beta d\omega^2. \tag{A.3}$$

A.2 Nonlocal Dispersion Cancelation

Because I am planning to study using refractive material to test the decoherence caused by the Event Formalism; it is important to make sure that the dispersion decoherence is taken into account. So-Young Baek and his colleagues found a way to cancel the dispersion nonlocally with entangled photons, which could restore the two-photon coincidence more effectively than nonentangled photons [6][55] This method can cancel the dispersion without need of the photon pair to meet again prior to detection.

The experiment set up they used is showed in Figure A.2

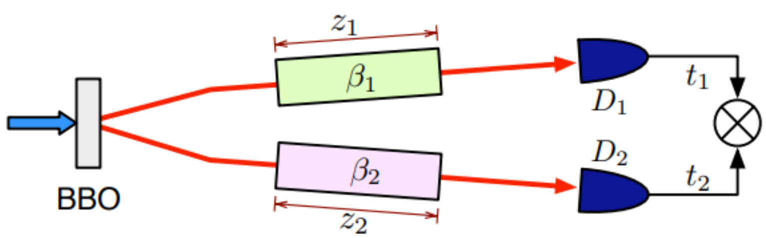

Figure A.1: Note.The figure is from [6]. The two materials have length z_1 and z_2, and GVD dispersion β_1 and β_2

The entangled pair generated by the source is

$$|\Psi\rangle = \int d\omega_1 d\omega_2 F(\omega_1,\omega_2) a^\dagger(\omega_1) a^\dagger(\omega_2)|0\rangle. \tag{A.4}$$

Since they are entangled, $F(\omega_1,\omega_2) \neq F(\omega_1)F(\omega_2)$. The central frequencies of the two photons are Ω_1 and Ω_2. In their experiment, they chose a monochromatic pump as the source

$$|\Psi\rangle = \int d\omega F(\omega) a^\dagger(\Omega_1+\omega) a^\dagger(\Omega_2-\omega)|0\rangle. \tag{A.5}$$

Where $F(\omega) \approx e^{-\gamma(\omega D L)^2}$ and $\gamma = 0.04822$. L is the thickness of BBO crystal and $D = \alpha_1 - \alpha_2$ is the difference between the group delay inside the two materials.

The two-photon coincidence to be measured is the Glauber second-order correlation, which is defined as

$$G^{(2)}(t_1,t_2) = |\langle 0|E_2(t_2)E_1(t_1)|\Psi\rangle|^2, \tag{A.6}$$

where $E_1(t_1) = \int d\omega_1 a(\omega_1) e^{i(k_1 z_1 - \omega_1 t_1)}$ is the electric field at the detector with positive frequency. Recall A.3, we can rewrite the wavenumber k in the equations

$$G^{(2)}(t_1,t_2) = |\int_{-\infty}^{\infty} d\omega F(\omega) e^{i\omega(t_1-t_2)} \times e^{i(\alpha_1 z_1 - \alpha_2 z_2)\omega} e^{i(\beta_1 z_1 + \beta_2 z_2)\omega^2}|^2. \tag{A.7}$$

In Chapter 5, I use the same method showed above to determined the effect of dispersion.

With $F(\omega)$, we obtain

$$G^{(2)}(t_1,t_2) \approx Ce^{-(t_1-t_2-\tau)^2/2\sigma^2}, \tag{A.8}$$

where $C = \frac{\pi}{\sqrt{\gamma^2 D^4 L^4 + (\beta_1 z_1 + \beta_2 z_2)^2}}$, $\tau = \alpha_1 z_1 - \alpha_2 z_2$, and $\sigma^2 = \gamma D^2 L^2 + (\beta_1 z_1 + \beta_2 z_2)^2/\gamma D^2 L^2$ The FWHM of $G^{(2)}$ is

$$\Delta t \approx 2\sqrt{\frac{2\ln 2}{\gamma D^2 L^2}}(\beta_1 z_1 + \beta_2 z_2). \tag{A.9}$$

Consequently, if one of the materials has positive dispersion and the other one has negative dispersion, then they can cancel each other. So-Young Baek and his colleagues' experiment also support the above result.

If the pair is not entangled where $F(\omega_1,\omega_2) = F(\omega_1)F(\omega_2)$, $F(\omega) = e^{-\omega^2/2\sigma_0^2}$, then the state is

$$|\Psi\rangle = |\Psi_1\rangle \otimes |\Psi_2\rangle, \tag{A.10}$$

where $|\Psi_1\rangle = \int d\omega F(\omega) a^\dagger(\Omega_1+\omega)|0\rangle$. The Glauber second-order correlation is:

$$\begin{aligned} G^{(2)}(t_1,t_2) &= |\langle 0|E_2(t_2)|\Psi_2\rangle\langle 0|E_1(t_1)|\Psi_1\rangle|^2 \\ &= |\int d\omega F(\omega) e^{i(\alpha_1 z_1 \omega + \beta_1 z_1 \omega^2 - \omega t_1)} \int d\omega F(\omega) e^{i(\alpha_2 z_2 \omega + \beta_2 z_2 \omega^2 - \omega t_2)}|^2 \\ &\approx Ce^{-\frac{(t_1--t_2-\alpha_1 z_1+\alpha_2 z_2)^2}{2\sigma_T^2}}, \end{aligned} \tag{A.11}$$

where $\sigma_T^2 = 1/\sigma_0^2 + 2\sigma^2(\beta_1^2 z_1^2 + \beta_2^2 z_2^2)$, and C is a constant depends on $\beta_1, \beta_2, z_1, z_2$. The FWHM of $G^{(2)}$ is

$$\Delta t \approx 4\sqrt{\ln 2}\sigma_0\sqrt{(\beta_1^2 z_1^2 + \beta_2^2 z_2^2)}, \tag{A.12}$$

which can not be canceled by positive and negative β

Appendix B

Calculate the Visibility from the Experiment Measurement Outcomes

I will now introduce how the visibility value is obtained from the detector outcomes for the interference studied in Section 2.1.5. Every single photon passing through the interferometer is detected by either detector c or detector d. As introduced in chapter 2.1.5, use $c^\dagger|0\rangle$ to represent detection happens on detector c, and $d^\dagger|0\rangle$ to represent detection happens on detector d.

The state path a_1 is:

$$|a_1\rangle = \int d\omega G(\omega) e^{-i\omega(T-l_1/c)} a_\omega^\dagger |0\rangle, \tag{B.1}$$

where l_1 is the length of path a_1. For a beam splitter, it is $a_\omega^\dagger = \frac{(c_\omega^\dagger + d_\omega^\dagger)}{\sqrt{2}}$, so the state becomes:

$$|a_1\rangle = \int d\omega G(\omega) e^{-i\omega(T-l_1/c)} \frac{(c_\omega^\dagger + d_\omega^\dagger)}{\sqrt{2}} |0\rangle. \tag{B.2}$$

The state path a_2 is:

$$|a_2\rangle = \int d\omega G(\omega) e^{-i(\omega T - kL - \omega l_2/c)} a_\omega^\dagger |0\rangle, \tag{B.3}$$

where L is the length of the refractive material, and l_2 is the length of the path outside the material. For the simulation, it is not necessary to have l_2, but it is impossible to remove l_2 in the experiment. How such an effect can be cancelled is discussed in l_2.

I also consider the dispersion of the material, because it will cause errors in the result. The dispersion makes the refractive index as a function of ω. Here I will consider the first order dispersion. Assume that

$$k(\omega) = (n/c)\omega + \beta(\omega - \omega_0)^2, \tag{B.4}$$

where β is a constant that describes the dispersion. Then the state path a_2 is:

$$|a_2\rangle = \int d\omega G(\omega) e^{-i\omega(T - nL/c - l_2/c) - i\beta(\omega-\omega_0)^2 L} a_\omega^\dagger |0\rangle. \tag{B.5}$$

After the Beam Splitter 2, I have $a_\omega = \frac{(c_\omega^\dagger - d_\omega^\dagger)}{\sqrt{2}}$ that can be rewritten into:

$$|a_2\rangle = \int d\omega G(\omega) e^{-i\omega(T - nL/c - l_2/c) - i\beta(\omega-\omega_0)^2 L} \frac{(c_\omega^\dagger - d_\omega^\dagger)}{\sqrt{2}} |0\rangle. \tag{B.6}$$

Assume state $|a_1\rangle$ is $(1,0)^\top$ and $|a_2\rangle$ is $(0,1)^\top$, then the density matrix of the photon with visibility V is:

$$\rho = \begin{pmatrix} 1/2 & V/2 \\ V/2 & 1/2 \end{pmatrix}, \tag{B.7}$$

where $V = \exp\left[-(t/\tau_{dec})^2\right]$ [20].
The probability of the photon causing a click in detector c is:

$$P(c) = \langle 0|c\rho c^\dagger|0\rangle = \langle c|\frac{1}{2}((|a_1\rangle\langle a_1| + |a_2\rangle\langle a_2|) + \frac{V}{2}(|a_1\rangle\langle a_2| + |a_2\rangle\langle a_1|))|c\rangle. \tag{B.8}$$

We can see that the probability includes V. Next I will express this function with n, L, l_1, l_2 and β.

Insert (B.2) and (B.6) into the first and second term $\frac{1}{2}(|a_1\rangle\langle a_1| + |a_2\rangle\langle a_2|)$ in (B.8), I obtain:

$$\langle c|\frac{1}{2}(|a_1\rangle\langle a_1| + |a_2\rangle\langle a_2|)|c\rangle = \frac{1}{2}(\langle c|a_1\rangle\langle a_1|c\rangle + \langle c|a_2\rangle\langle a_2|c\rangle) = 1/2. \tag{B.9}$$

Then insert (B.2) and (B.6) into the third term in (B.8):

$$\langle c|a_1\rangle\langle a_2|c\rangle = \int d\omega \langle c_\omega|a_1\rangle\langle a_2|c_\omega\rangle \tag{B.10}$$

$$= \int d\omega G(\omega) e^{-i\omega(T-l_1/c)} \langle c_\omega | \frac{(c_\omega^\dagger + d_\omega^\dagger)}{\sqrt{2}} |0\rangle \langle 0| e^{i\omega(T-nL/c-l_2/c)+i\beta(\omega-\omega_0)^2 L} \frac{(c_\omega - d_\omega)}{\sqrt{2}} G^*(\omega)|c_\omega\rangle \tag{B.11}$$

$$= \frac{1}{2} \int d\omega |G(\omega)|^2 e^{i\omega(l_1-nL-l_2)/c+i\beta(\omega-\omega_0)^2 L}. \tag{B.12}$$

Similarly, the fourth term in (B.8) is

$$\langle c|a_1\rangle\langle a_2|c\rangle = \frac{1}{2} \int d\omega |G(\omega)|^2 e^{-i\omega(l_1-nL-l_2)/c-i\beta(\omega-\omega_0)^2 L}, \tag{B.13}$$

which can be reformulated:

$$\frac{V}{2}\langle c|(|a_1\rangle\langle a_2| + |a_2\rangle\langle a_1|)|c\rangle = \frac{V}{2} \int d\omega |G(\omega)|^2 cos[\omega(l_1 - nL - l_2)/c + \beta(\omega-\omega_0)^2 L]. \tag{B.14}$$

The probability is (B.9) + (B.14) and the equation is:

$$P(c) = 1/2 + \frac{V}{2} \int d\omega |G(\omega)|^2 cos[\omega(l_1 - nL - l_2)/c + \beta(\omega-\omega_0)^2 L]. \tag{B.15}$$

Then we can do the experiment and measure the probability of detection by c and determine visibility. If the visibility satisfies (5.30), the gravitational time dilation decoherence theory is proved.