A Decentralised Transactive Energy Market Considering Physical System Constraints

by

Colton Pankhurst

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Increasing levels of Distributed Energy Resources (DERs) are expected to play a key role in achieving global electricity decarbonisation goals, providing both a challenge and an opportunity for the electricity industry. Conventional approaches such as Net Energy Metering (NEM) have been questioned regarding their effectiveness in properly rewarding DERs, and larger efforts around the integration of DERs into wholesale markets do not address potential value streams at the distribution system level. Local energy markets leveraging direct Peer-to-Peer (P2P) trading have been proposed as a solution, which can increase prosumer participation in lower cost and more reliable supply of energy to consumers. Many approaches have been proposed to determine the optimal dispatch of distributed resources; however, a gap remains in the research to date on how to efficiently allow for prosumer decision autonomy while ensuring that the physical layer of the power system is considered.

This thesis proposes a decentralised transactive solution that retains prosumer negotiation and decision autonomy, while using network operator and market determined prices to allocate limited system resources for a feasible, locally optimal system state. Peer-to-Utility (P2U) transactions are added to existing P2P energy frameworks to obtain transactive local peer decision criteria considering Peer-Centric (PC) and System-Centric (SC) objectives. Peers are able to interact with wholesale electricity market derived prices through P2U transactions, allowing for consideration of net export value in welfare maximising decisions. The proposed approach includes a split transaction fee pricing mechanism for virtual prosumer interactions that considers the networks characteristics such as topology and operational constraints to ensure consideration of the physical layer in peer decision making. In addition to pricing mechanisms for coupling the virtual and physical layers, a congestion clearing process is proposed, which coordinates with the decentralised transaction matching process and the Network Usage Charges (NUCs) to ensure efficient allocation of network capacity.

Previously reported distribution networks are used to compare the transaction decisions, economic performance, and system performance of the proposed solution with existing approaches. The results demonstrate the effectiveness of the proposed method in ensuring system feasible, locally optimal transaction sets with prioritisation of local peers.

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List of Abbreviations

ADMM	Alternating Direction Method of Multipliers
CAISO	California Independent System Operator
DER	Distributed Energy Resource
DLMP	Distribution Locational Marginal Price
FERC	Federal Energy Regulatory Commission
IIS	Irreducible Inconsistent Subsystem
ISF	Injection Shift Factor
NEM	Net Energy Metering
NUC	Network Usage Charge
OPF	Optimal Power Flow
P2P	Peer-to-Peer
P2U	Peer-to-Utility
PC	Peer-Centric
PTDF	Power Transfer Distribution Factor
\mathbf{SC}	System-Centric
VPP	Virtual Power Plant
VSC	Voltage Sensitivity Coefficient

Nomenclature

Indices and Sets

Ω^*	Set of matched peer-to-peer transactions
Ω_n	Set of peer-to-peer transactions for peer n
$b \in \mathcal{B}$	Set of buses
b_w/s_w	Indices of buying/selling peers for trade w
$l\in\mathcal{L}$	Set of feeders
$n \in \mathcal{N}$	Set of peers
$\mathcal{N}^b/\mathcal{N}^s$	Set of buying/selling peers
o_l/r_l	Indices of originating/receiving-end nodes of distribution feeder l
$w\in \Omega$	Set of all peer-to-peer transactions
Parameters	
Γ_n	Proportion of P2P demand for peer n
$\Delta \rho$	Trade negotiation price increment [\$/MWh]
Φ^l_{ij}	Power transfer distribution factor for feeder l corresponding to trade b tween peers i, j [p.u.]
ν τ α	

- e-
- Injection shift factor of feeder l for peer n Ψ_l^n
- $\sigma_n^{u,f}$ Binary parameter = 1, if utility transactions are fixed, and 0 otherwise

σ_w	Binary parameter $= 1$, if trade is cleared, and 0 otherwise
ς_w	Binary parameter $= 1$, if trade is blocked, and 0 otherwise
B_b	Susceptance of bus b [p.u.]
C^w	Wholesale electricity market price [\$/MWh]
D_b^Q	Reactive power demand at bus b [MVAr]
$\underline{D}_n/\overline{D}_n$	Min/max demand by peer n [MW]
E^{ϵ}	Transaction fee increment [\$/MWh]
E^Z	Network usage charge [\$/MWh]
G_b	Conductance of bus b [p.u.]
$\underline{P}_n/\overline{P}_n$	Min/max generation by peer n [MW]
$\underline{P}^g_b/\overline{P}^g_b$	Min/max real power output limit of utility generator at bus b [MW]
\overline{p}^w	Standard peer-centric trade size [MW]
$\underline{Q}^g_b / \overline{Q}^g_b$	Min/max reactive power output limit of utility generator at bus $b \ [\mathrm{MVAr}]$
R_l	Resistance of distribution feeder l [p.u.]
S_l	Apparent flow limit of distribution feeder l [p.u.]
T_n^g	Feed in tariff rate for peer n [\$/MWh]
T_n^d	Utility supply tariff rate for peer n [\$/MWh]
$\underline{V}_b/\overline{V}_b$	Min/max limit on the voltage magnitude at bus b [p.u.]
X_l	Reactance of distribution feeder l [p.u.]
Z_{ij}^{th}	The venin impedance between peers i, j [p.u.]
Variables	
Δu^{ij}	Voltage sensitivity coefficient of bus b for a trade between peers i_{i}

$\Delta \nu_b$	voltage sensitivity coefficient of bus b for a trade between peers i, j
$\epsilon_n^{u,g}/\epsilon_n^{u,d}$	Transaction fee for selling/buying to/from the utility for peer $n~[\mathrm{MWh}]$

ϵ_w	Transaction fee for trade w [\$/MWh]
δ_b	Voltage angle of bus b [rads]
λ_b	Distribution locational marginal price at bus $b~[\mathrm{MWh}]$
ρ_w^b/ρ_w^s	Buying/selling price for trade w [\$/MWh]
a_l	Squared current flow of distribution feeder l [p.u.]
C_w	Network usage charge for trade w [\$/MWh]
d_n	Total power bought by peer $n [MW]$
d_n^p	Power bought by peer n from peers [MW]
d_n^u	Power bought by peer n from the utility [MW]
$d_n^{u,f}$	Fixed power bought from the utility by peer $n [MW]$
f_l^P/f_l^Q	Active/reactive power flow of distribution feeder l [p.u.]
p_n	Total power sold by peer n [MW]
p_n^p	Power sold by peer n to peers [MW]
p_n^u	Power sold by peer n to the utility [MW]
$p_n^{u,f}$	Fixed power sold to the utility by peer $n [MW]$
p_0^g	Net active power injection from the wholesale market at the root node [MW]
p_b^g/q_b^g	Active/reactive power output of the utility generator at bus $b \; [{\rm MW}/{\rm MVAr}]$
p_w^b/p_w^s	Trade size for buyer/seller of trade w , a linear variable for system-centric approaches, and a binary variable for peer-centric approaches = \overline{p}^w , if trade is selected, and 0 otherwise [MW]
q_n	Reactive power output of peer n [MVAr]
v_b	Squared nodal voltage magnitude of bus b [p.u.]
Functions	

 Υ_n Value of surplus energy for peer n [\$/MWh]

- $C_b^u(\cdot)$ Cost function of utility generator at bus b [\$/MWh]
- $C_n(\cdot)$ Cost function of selling peer n [\$/MWh]
- $U_n(\cdot)$ Utility function of buying peer n [\$/MWh]

Chapter 1

Introduction

1.1 Motivation

The electricity system is undergoing substantial changes with the growing demand for clean electricity to support electrification and net-zero emission targets. Increasing penetration of renewables at the bulk power system level are challenging conventional operating practices and are requiring new services to ensure reliability and resiliency [1]. While the need for change is becoming increasingly obvious at the wholesale level, a similar transformation is projected to occur at the distribution system level with the accelerated adoption of Distributed Energy Resources (DERs) to support decarbonisation [2].

DERs provide both a challenge and an opportunity for the electricity industry. The widespread adoption of DERs could quickly over-burden the infrastructure and challenge existing distribution utility practices if sufficient consideration is not given to their integration. Time sensitive peak loads, reversed power flows, and resource visibility are just a few of the major issues that need to be addressed for cost-effective DER integration [3]. While DERs present an opportunity to unlock flexibility and value stacking from price responsive prosumers when efficient incentive mechanisms are available [4], significant questions remain regarding how prosumers may participate in the overall energy system and markets.

At the wholesale level, work is underway to develop DER paricipation models in existing markets for products including energy, capacity, and ancillary services. In the US, much of this work is related to the 2020 Federal Energy Regulatory Commission (FERC) Order No. 2222, which seeks to remove barriers preventing DERs from fair and level participation in wholesale markets. This rule requires that regional grid operators under the jurisdiction

of FERC revise their tariffs to provide models, accounting for physical and operational characteristics of aggregated DERs of at least 100 kW [5].

Value stacking opportunities for DERs and prosumers at the distribution system level are still in the research and demonstration phase, lacking a consistent framework and value realisation mechanism. The status quo for the monetisation of DERs, primarily distributed solar, to date, has been through Net Energy Metering (NEM) programs, which allow DER owners to receive direct credit for local generation against their energy usage. While these programs have increased the uptake of DERs due to the favourable returns offered, NEM has faced challenges from utilities over the past several years regarding inequitable subsidisation and failure to represent the actual value of DERs to the electricity system. NEM subisides were quantified in [6] for 16 US utilities, indicating subsidies up to \$100/customer/month for DER owners. Alternative options to NEM and the transition pathways to get to those options are currently being explored by various regulators, utilities, and advocacy groups; however, a consensus has not yet been reached.

Many jurisdictions are exploring structures and entities that are best suited to supporting DER and net-zero emission targets, such as distribution system operators, load serving entities, local energy markets, and transactive energy [7, 8, 9]. Local energy markets and transactive energy systems have been proposed as a solution to provide value stacking opportunities for DERs. These solutions can be used to support reliable system operation while maintaining transactive energy principles such as privacy through implementations leveraging blockchain [10]. Demonstration and pilot projects for the design of local and transactive energy markets have been growing, for example, the Brooklyn Microgrid, the York Region Non Wires Alternative Project, and the CoordiNet project [11, 12, 13]. Although general architectures are being explored to understand value streams and pilot projects focused on specific products are onging, a complete framework aligning Peer-to-Peer (P2P) transactions with network constraints is needed, and is thus the focus of this thesis.

1.2 Literature Review

1.2.1 Transactive Energy

The GridWise Architecture Council defines transactive energy in [14] as "a system of economic and control mechanisms that allows the dynamic balance of supply and demand across the entire electrical infrastructure using value as a key operational parameter."

Transactive energy seeks to enable voluntary active participation of prosumers in energy markets through the design of value discovery mechanisms. These mechanisms should represent the economic or engineering value associated with specific transactions, while assuring interoperability and stability [14]. Common transactive principles include, for example, scalability, transparency, non-discrimination, prosumer decision autonomy, and product differentiation.

Peer-to-Peer Trading

Transactive energy is a generalisation of P2P trading, and as such can leverage existing approaches in the P2P literature. Transactive frameworks can be categorised into Peer-Centric (PC) and System-Centric (SC) approaches depending on the objective of the coordination mechanism [15]. SC approaches utilise a central coordinator responsible for the system welfare maximising dispatch of all local resources according to their costs, marginal utility, and operational constraints. On the other hand, PC approaches allow for peers to select the locally optimal set of transactions according to their private costs, marginal utility, and constraints. The elements making up a framework can be further categorised within the virtual and physical layer. The virtual layer includes elements such as the market and pricing mechanism, and the physical layer includes the actual power flows and network limitations of the underlying system [16]. In proposing a feasible transactive framework, both the virtual and physical layers must be aligned.

System-Centric Approaches

Much of the existing literature on P2P markets relies upon a SC approach [16]. These methodologies attempt to maximise overall system welfare through a central coordination mechanism. In [17], the concept of a Virtual Power Plant (VPP) is presented as an approach for the integration of DERs. A portfolio of DERs are aggregated into a controllable VPP, which has a single set of operating parameters and can participate in wholesale energy markets as a typical transmission connected generator. This proposed model can participate in both commercial and technical activities, where commercial activities are concerned with market participation in the virtual layer, and technical activities deal with system management of the physical layer. Although the VPP model allows for aggregate control and participation of decentralised DERs at the wholesale market level, mechanisms for consideration of the distribution physical layer are not included, limiting real world practicality at scale without additional work. An approach utilising Optimal Power Flow (OPF) techniques for the consideration of physical layer constraints directly in the DER dispatch is proposed in [18]. The method uses a distributed Alternating Direction Method of Multipliers (ADMM) approach to solve a series of OPF problems for dispatch of local resources. The results demonstrate the potential for ADMM-based approaches in real-world settings; however, the objective of central system optimisation violates transactive principles such as consumer choice and control, and requires sharing sensitive cost and marginal welfare functions with a central coordinator.

A cooperative Stackelberg game is proposed in [19], where the utility leads and the peers respond. The utility is assumed to have contracts with a number of peers allowing for increased energy prices during peak hours to incentivise local P2P trading. Peers respond to the utility determined energy prices by participating in a coalition formation game, consisting of a double auction market for willing peers. Peers who do not participate in the initial double auction may choose to satisfy their demand through another coalition, trading at the mid-market price of the previously cleared P2P double auction and the feed-in-tariff price. The proposed approach demonstrates the potential value of P2P trades in avoiding peak energy prices, but peers are unable to provide differentiated products in the double auction clearing process and the impact of P2P trades on the physical layer is not considered.

In [20], a continuous double auction is used where bids and asks are paired according to the associated price and submission times. To account for the physical layer, sensitivity coefficients are derived for losses, voltages, and feeder flows. For each incrementally matched trade, voltage and feeder congestion are evaluated to assess the feasibility and determine whether to block injection from specific peers. If a trade is approved, the cost associated with the change in network constraints is assigned to the trade participants. Although the proposed method ensures consideration of the physical layer in the clearing of trades, it fails to introduce a visible price signal to peers and does not account for physical network usage in the clearing order. Retroactively applied network fees can impact the economic viability of trades after already being accepted, and the proposed clearing order can result in an inefficient set of cleared trades as the clearing order is decoupled from physical layer constraints.

Peer-Centric Approaches

In a PC approach, peers retain decision making autonomy to select the optimal set of transactions for their own objectives. This process ensures the security of peer information, limits communication overhead, and allows for individual product differentiation.

Distributed optimisation solutions have been proposed that enable PC decision making, while addressing concerns of scalability and privacy to arrive at a feasible, system optimal state [21]. In [22], a market design is proposed consisting of both P2P trade and a distribution market for ancillary services. Consideration of the physical layer is included with the use of grid usage prices derived from the Distribution Locational Marginal Price (DLMP) components, including loss compensation, voltage support, and congestion management. A distributed optimisation ADMM approach is used with the objective of maximising the social welfare of all peers. Since only the proxy variables of the optimisation are shared, sensitive peer and system information is kept private. A blockchain-based energy management platform is proposed in [23], which considers a decomposed OPF problem using ADMM for the physical layer. Bilateral trading is enabled in the virtual layer, with smart contracts responsible for enforcing agreed upon transactions. Previous ADMM techniques are improved upon in [24] using a fast ADMM technique for reduced computational time, with the additional consideration of a reputation-based product and a Network Usage Charge (NUC) for transactions. In [25], distributed optimal operation of distribution grids using ADMM is achieved with P2P transactions limited to neighbouring agents, and nodal agents responsible for securing ancillary services to respect local system constraints. These ADMM approaches rely on the assumption of cooperative, non-strategic, and rational agents for convergence, limiting the potential extension to real world applications [22, 25].

Decentralised PC approaches relying on direct negotiation present a potential solution allowing for scalability with non-collaborative agents. For example, a decomposition of the SC P2P problem with a novel primal-dual gradient method for decentralised clearing is presented in [26]. The proposed method uses linear Power Transfer Distribution Factors (PTDFs) to consider the network flow constraints with an associated feeder utilisation charge. Local buyer and seller problems are solved decentrally in an iterative process using Lagrangian multipliers associated with each individual trade. Bilateral trading is enabled while ensuring privacy of information and product differentiation; however, the coupling between the physical and virtual layer is only approximate.

A multi-leader, multi-follower Stackelberg game approach is considered in [27] for noncollaborative P2P energy trading. Sellers compete in a noncooperative game to determine the Nash equilibrium solution for energy prices offered to the buyers, while buyers participate in an evolutionary game with the seller prices as input and the evolutionary equilibrium state as output. Interaction between the seller and buyer games is in the form of a Stackelberg game, which requires an interative distributed algorithm to reach the final equilibrium. Results demonstrate the convergence of the various games with consideration of both demand response and battery energy storage resources. However, the proposed games do no not include any pricing or coordination mechanisms for consideration of the physical layer.

Bilateral contract networks are used in [28] to facilitate P2P trading while satisfying full substitutability for establishing a stable outcome. Real-time and forward markets allow peers to secure upstream and downstream contracts to maximise local utility. Forward markets are determined according to expectations for real-time energy price and local demand, while the real-time market has no uncertainty. Contract prices are negotiated using an iterative process consisting of all peers selecting their locally optimal set of contracts. After the optimal set of contracts has been selected by all participants, contracts which were selected by the associated buyer but not the associated seller have their prices incremented by a fixed amount. This contract selection and price increment process is repeated until no price changes occur, indicating a stable set of contracts. The proposed process achieves a stable solution which satisfies full substitutability; however, consideration of the physical layer is not included in contract selection or the iterative price-adjustment process.

In [29], prosumer preferences are considered with the introduction of energy classes. Non-financial characteristics of buyers and sellers are included in the decomposed energy management problem, solved using ADMM. The paper considers three potential energy classes with associated prioritisation by peers, including green energy, subsidised energy, and grid energy. Receding horizon model predictive control is included for prosumers to consider predicted future wholesale electricity prices in their decision making criteria. Battery storage is included in the model to allow prosumers to respond more meaningfully to predicted prices and real-time demand. The results demonstrate successful consideration of non-financial energy classes for a test system. Consideration of the physical layer is not included in peer decision making.

A relaxed consensus-innovation approach adapted for bilateral trading is proposed in [30]. The method allows for product and price differentiation while ensuring data privacy and local control. Agent cost functions consist of a quadratic production cost component and a linear bilateral trading cost to consider alternative criteria such as, for example, associated emissions, distance, or community sources. The bilateral trading cost is generic by design and could be applied to consider physical factors such as the electrical trading distance; however, the proposed implementation focuses on the virtual layer without coupling to the physical layer.

1.2.2 Physical Layer Representations

Additional mechanisms are necessary to ensure that virtual peer transactions consider physical layer limits for achieving both a feasible and optimal solution. Thus, in [31], a decentralised clearing method is proposed that makes use of a network utilisation fee based on fixed electrical distances. PTDFs are used to calculate the power transfer distance between each buyer and seller. Power losses and network utilisation fees are assigned to P2P trades and accounted for in the proposed decentralised algorithm which retains prosumer information privacy. The introduced pricing mechanism accounts for the fixed network topology; however, it does not consider the dynamic network usage or whether the proposed set of trades are feasible in the physical layer.

For consideration of network losses and dynamic grid constraints, [32] explores various loss allocation methods with additional representation of system operators. Optimisation problems for the transmission system operator, the distribution system operator, and the market operator are combined into an equivalent optimisation problem for which the Nash equilibrium can be determined. The equivalent optimisation problem includes power flow constraints for ac feeders using PTDFs, and considers linear losses as a function of the feeder flows. Pricing elements for losses and flow constraints are determined from dual variables of the equivalent optimisation problem. Two potential loss allocation policies are considered, including a socialised loss method where losses are assigned according to the number of active trades, and an individual loss method where trades are directly allocated the losses produced by their flows. A decentralised trading mechanism to ensure prosumer decision autonomy is not considered.

In [33], the authors propose an efficient direct negotiation and peer matching process with a loss based transaction fee. The proposed trading method allows for direct negotiation and trade selection without central clearing. Contribution to losses by individual trades are determined and assigned using the associated PTDFs. Although some consideration of the network is included in the PTDF derived transaction fees, the proposed method includes no consideration of the actual network flows or constraints for a feasible solution and does not consider potential revenues from interaction with the wholesale market. In the case of an infeasible set of trades, out of market actions would be necessary to ensure a reliable system is maintained.

Paper [15] extends the peer matching process of [28] with the addition of a trade NUC derived from DLMPs to consider physical system feasibility. The NUC is calculated as the difference in buyer and seller DLMPs obtained from the OPF. Buyers and sellers select their locally optimal set of trades and negotiate trade prices through an iterative decentralised matching process. NUCs are updated after each iteration to ensure that the

updated DLMPs are accounted for. An incremental penalty mechanism is introduced to disincentivise infeasible trade sets since the DLMPs cannot be calculated for an infeasible system. The proposed penalty mechanism is a fixed increment of NUCs for all trades included in the infeasible set. For trades included in a feasible trade set, all prior penalties are removed and the NUC is determined according to the DLMPs. The proposed solution introduces consideration of the physical layer and addresses potentially infeasible sets; however, the penalty mechanism is inefficient as it does not consider the proportional impacts of trades and is purely virtual given that it is removed once a trade is included in any feasible set. Additionally, the P2P approach does not allow for decision autonomy to select the optimal proportion of P2P trades to maximise local welfare, and revenues from system exports are limited.

A bilevel prosumer framework for decentralised bilateral trading is proposed in [34], where a NUC based on DLMPs and electrical distance elements is considered. Prosumers participate in a day-ahead market according to projections for generation, market prices, and demand, then participate in a real-time market for final balancing of local resources. Prosumers are modeled using multiagent theory and consist of several individual DERs such as, for example, distributed generation, flexible loads, energy storage, and conventional loads. Prosumers first determine the locally optimal set of actions, and then participate in bilateral negotiation to maximise utility in the P2P market. Network charges consist of two pricing elements, an operation network charge, and an investment network charge. Operation network charges are derived using the DLMPs from the OPF solution and are calculated using the same method as [15]. Investment network charges are determined using the Thevenin impedance electrical distance of trades and are used for recovering costs for upgrading feeders. The two pricing methods for considering network costs introduce a more accurate pricing signal than a single mechanism. However, the proposed method does not consider a potentially infeasible trade set for which the DLMPs cannot be calculated, and does not allow peers to interact with the wholesale electricity market price at the interface of the distribution and transmission system.

1.2.3 Discussion

As seen from the literature review, previously reported SC approaches require knowledge of individual prosumer utility and cost functions, presenting challenges regarding privacy and transparency, and removing prosumer decision autonomy. Although distributed approaches such as ADMM have frequently been used in the literature for increased scalability of large optimisation problems, these solution approaches require central assumptions which may not hold in practical implementations. PC approaches have been proposed to achieve locally optimal solutions while respecting transactive energy principles. These approaches generally focus on the virtual layer of the transactive market in proposing decentralised algorithms to facilitate P2P negotiation and economic transactions. Most of the works do not include any representation of the physical layers, limiting the practicality of these approaches at scale as local transactions begin to more significantly impact physical system constraints. Some authors have proposed pricing mechanisms for consideration and allocation of network based fees such as those associated with network losses or constraints. Methods relying on fixed PTDFs or dynamically determined DLMPs have been explored and shown to influence prosumer decision making; however, the existing solutions are either partially inefficient or require additional out of market processes, rather than serving as a complete transactive framework. Additionally, a mechanism allowing peers to interact with the wholesale electricity market price in addition to directly with peers has not been explored in detail with physical layer considerations.

1.3 Research Objectives

Based on the issues identified in the literature review, the main objectives of this thesis are:

- Design an effective pricing signal determined by the network operator for coupling of virtual peer transaction decision making with fixed and dynamic network elements to ensure efficient and feasible usage of network resources.
- Develop a market mechanism allowing for locally optimal peer decisions with consideration of the wholesale electricity market price aligned with the physical layer.
- Present a comprehensive, decentralised transactive energy market model which ensures an efficient, feasible, and PC solution including network operator derived price signals for coupling of the virtual and physical market layers, and without the need for additional out of market processes.

1.4 Thesis Outline

The remainder of this thesis is organised as follows: Chapter 2 presents a review of necessary background material, including a relevant OPF formulation, transactive market layer concepts, and previously proposed P2P frameworks, which form the foundation of the proposed transactive approaches. Chapter 3 details the proposed SC and PC transactive market frameworks, where all critical sub-components of the proposed decentralised PC transactive framework are described and integrated into a comprehensive approach. In this chapter, simulation results are presented and the transaction decisions, economic outcomes, and network performance of the proposed model are compared against relevant existing approaches. Finally, Chapter 4 summarises the thesis content and main conclusions, reviews the key contributions, and identifies potential areas for future work.

Chapter 2

Background

This section covers the related background material necessary for this thesis. The underlying branch flow second-order-cone ac OPF model used for physical network representation is presented. Next, the physical and virtual layers used for categorisation of market components is also discussed. The reference P2P model is then presented, including all core sub-components with a discussion of potential deficiencies. The derivation of PTDFs and Voltage Sensitivity Coefficients (VSCs) are presented for use in the proposed transaction pricing mechanism.

2.1 Optimal Branch Power Flow

Consideration of the physical network is critical in ensuring the proposed optimal set of transactions are feasible. An efficient OPF model is therefore necessary to accomodate the potentially large number of nodes present in a distribution network.

The second-order-cone ac OPF model of [35] is selected due to its efficient performance for both mesh and radial distribution networks. The model employs an angle and conic relaxation to obtain a convex problem. The optimal power flow from the perspective of the utility is as follows:

$$\max_{\Xi^{\text{Dist}}} \quad O^{\text{Dist}} := \sum_{b \in \mathcal{B}} (-C_b^u(p_b^g)) - C^w p_0^g \tag{2.1a}$$

s.t.
$$0 = f_{l|o_l=b}^P - \sum_{l|r_l=b} (f_l^P - a_l R_l) - p_b^g - p_{n=b} + d_{n=b} + G_b v_b \quad \forall b \in \mathcal{B}$$
(2.1b)

$$0 = f_{l|o_l=b}^Q - \sum_{l|r_l=b} (f_l^Q - a_l X_l) - q_b^g - q_{n=b} + D_b^q - B_b v_b \ \forall b \in \mathcal{B}$$
(2.1c)

$$S_l^2 \ge (f_l^P)^2 + (f_l^Q)^2 \quad \forall l \in \mathcal{L}$$

$$(2.1d)$$

$$S_{l}^{2} \ge (f_{l}^{P} - a_{l}R_{l})^{2} + (f_{l}^{Q} - a_{l}X_{l})^{2} \quad \forall l \in \mathcal{L}$$
(2.1e)

$$v_{r_l} = v_{o_l} - 2(R_l f_l^P + X_l f_l^Q) + a_l (R_l^2 + X_l^2) \quad \forall l \in \mathcal{L}$$

$$(2.1f)$$

$$v_{o_l} \ge \frac{(f_l^r)^2 + (f_l^*)^2}{a_l} \quad \forall l \in \mathcal{L}$$

$$(2.1g)$$

$$\underline{P}_{b}^{g} \le p_{b}^{g} \le \overline{P}_{b}^{g} \quad \forall \, b \in \mathcal{B}$$

$$(2.1h)$$

$$Q_b^g \le q_b^g \le \overline{Q}_b^g \quad \forall \, b \in \mathcal{B} \tag{2.1i}$$

$$\underline{V}_b^2 \le v_b \le \overline{V}_b^2 \ \forall b \in \mathcal{B}$$
(2.1j)

$$p_0^g \ge 0 \tag{2.1k}$$

where $\Xi^{\text{Dist}} = \{p_b^g, q_b^g, p_0^g, q_n, v_b, a_l, f_l^P, f_l^Q\}$, with the parameters, variables, and functions being defined in the Nomenclature Section for this and other models presented in this thesis.

The objective function O^{Dist} in (2.1a) considers the welfare of the system utility, including the cost of utility generation $C_b^u(p_b^g)$ and the cost of energy purchased from the wholesale market $C^w p_0^g$. Peer welfare and cost of generation are considered in a separate formulation to allow for simple separation in the case of PC solutions where peer welfare is only considered locally. Eqs. (2.1b) and (2.1c) consider the nodal active and reactive power balance respectively. Forward and backward branch flow limits are enforced with (2.1d) and (2.1e). The nodal voltages and feeder flows are linked in (2.1f), and the problem is convexified with the second-order-conic constraint (2.1g). The active and reactive power limits for utility generators are included in (2.1h) and (2.1i), and the squared nodal voltage limits are enforced in (2.1j). Lastly, (2.1k) is included to restrict the ability to export power to the wholesale market at a profit. This last equation is necessary to ensure comparable results between SC and PC formulations, since the peers in the PC P2P formulation are not exposed directly to the wholesale market. The model assumes that peer reactive power output q_n is controlled by the system operator to minimise balancing costs. In practice, a product could be designed to incentivise the offering of reactive power support by local resources.

The model in [35] eliminates voltage and current angles as part of an angle relaxation. This results in a second-order-cone problem; however, the voltage angles may be necessary for use in the derivation of VSCs. A centralised angle recovery method is proposed in [35] which is proven effective for radial networks. In the case of meshed networks, recovery criteria must be checked to ensure the angles are recoverable, or an alternative network model may be used.

2.2 Transactive Market Concepts

Transactive energy markets consist of several individual elements which interact to create a comprehensive framework. In the literature, elements are frequently categorised within the virtual and physical layers [16]. The virtual layer elements make up the platform in which peers are able to select and negotiate their optimal set of transactions, and in which financial transactions occur. Key elements of this layer include the market mechanism responsible for detailing how peers participate in the market and the clearing mechanism determining the final set of selected transactions according to peer preferences.

The physical layer is the physical network in which power flows between peers. The power flows in this physical network are dictated by the constraints of the system and the injected energy at each node. The physical communication network between the virtual platform and peers is also included in the physical layer, although this is not the focus of this thesis. The virtual layer and physical layer are decoupled by default and require an intentional coordination mechanism to ensure a mutually feasible solution is obtained. The virtual layer must communicate the net nodal injections resulting from the virtual transactions, and the physical layer must exchange a price signal representing the physical network constraints for consideration in virtual decision making.

Comprehensive frameworks integrate several elements considering both the virtual and physical layer to ensure an efficient and feasible solution. These frameworks can be categorised into SC and PC approaches depending on the coordination mechanism and objective function [15]. SC approaches rely upon a centralised implementation, with a central coordinator responsible for considering the cost and utility curves of system resources to maximise system welfare. An alternative to SC approaches is a PC approach, which seeks to allow peers to make their locally optimal decisions in a decentralised manner. Decentralised decision making and negotiation between peers maintains transactive principles of scalability, decision autonomy and privacy. A decentralised PC approach for a comprehensive transactive framework is the focus of this thesis.

2.3 Peer-to-Peer Trading

The P2P trading problem seeks to determine a welfare maximising set of trades between peers to meet a certain portion of demand. As discussed in the literature review, this can be achieved through SC or PC approaches. SC approaches require the central utility to determine the system optimal set of trades using knowledge of peer utility and cost functions. Although this achieves the globally optimal solution, it violates certain principles of transactive energy such as prosumer decision autonomy and prosumer privacy. PC approaches require appropriate pricing mechanisms to ensure a locally optimal solution, which may differ from the globally optimal solution.

Complete SC and PC P2P trading implementations are proposed in [15], which form the foundation for the proposed novel decentralised transactive framework in this thesis. The key sub-components of these implementations are discussed in the following subsections. These implementations are strictly P2P, meaning that a fixed proportion of overall demand is met by P2P trades while the remainder is met by utility supply. The potential for choosing between peer or utility supply according to local welfare optimality is not considered in the reference models of [15].

2.3.1 System-Centric Peer-to-Peer Trading

The SC model consists of two perspectives, the peer and the central coordinating entity. Peer welfare is determined according to the utility of demand and cost of supply, while welfare of the central coordinator is determined by the cost of balancing the system. The first part of the model, which accounts for the peer welfare resulting from P2P transactions, is as follows [15]:

$$\max_{\Xi^{\text{Peer}}} O^{\text{Peer}} := \sum_{n \in \mathcal{N}^b} U_n(d_n) - \sum_{n \in \mathcal{N}^s} C_n(p_n)$$
(2.2a)

s.t.
$$\mathcal{N}^b \cap \mathcal{N}^s = \emptyset$$
 (2.2b)

$$U_n(d_n) = \begin{cases} \Upsilon_n(d_n - \underline{D}_n) & \text{if } d_n \ge \underline{D}_n \\ -\infty & \text{otherwise} \end{cases} \quad \forall n \in \mathcal{N}^b$$
(2.2c)

$$\underline{P}_n \le p_n \le \overline{P}_n \quad \forall n \in \mathcal{N}^s \tag{2.2d}$$

$$\underline{D}_n \le d_n \le \overline{D}_n \quad \forall \, n \in \mathcal{N}^b \tag{2.2e}$$

$$p_n = \sum_{w \in \Omega_n} p_w^s \quad \forall \, n \in \mathcal{N}^s \tag{2.2f}$$

$$d_n^p = \sum_{w \in \Omega_n} p_w^b \quad \forall \, n \in \mathcal{N}^b \tag{2.2g}$$

$$d_n^p = d_n \Gamma_n \ \forall \, n \in \mathcal{N}^b$$
(2.2h)

$$p_w^b = p_w^s \ \forall \, w \in \Omega \tag{2.2i}$$

where $\Xi^{\text{Peer}} = \{d_n, p_n, p_w^b, p_w^s\}$. Equation (2.2a) is the total welfare for all peers, accounting for the cost of selling peers $C_n(p_n)$ and the utility of buying peers $U_n(d_n)$. Constraint (2.2b) requires that each peer is strictly a buyer or seller for each time period. The marginal utility Υ_n of incremental demand above the minimum demand \underline{D}_n for peer n is defined in (2.2c). Constraints (2.2d) and (2.2e) limit the peer supply and demand, and (2.2f) defines the total energy sold by peer n as the sum of all P2P trades where s_w is equal to n. Equations (2.2g) and (2.2h) establish the total demand for peer n as the sum of P2P trades where b_w is equal to n, and the fixed proportion of total demand d_n supplied by peers. The parameter Γ_n can be adjusted in a range of [0, 1] to modify the amount of demand supplied by P2P trading. This approach assumes a fixed proportion of P2P demand according to cost and utility functions. Constraint (2.2i) ensures coordination between the buyer and seller trade sizes for trade w.

The welfare of the central balancing entity is considered using the ac OPF (2.1) described in Section 2.1. Considering both the peer and utility welfare components, the SC P2P trading formulation is as follows:

$$\max_{\Xi^{\text{Peer}} \bigcup \Xi^{\text{Dist}}} O^{\text{Peer}} + O^{\text{Dist}}$$
(2.3a)

s.t. Eqs. (2.1b)-(2.1k) Network constraints (2.3b)

Eqs. (2.2b)–(2.2i) Peer constraints (2.3c)

2.3.2 Peer-Centric Peer-to-Peer Trading

The PC approach to P2P trading decouples peer and utility decision making. Ensuring coordination between the virtual and physical layers of the peer and utility requires the addition of a physical layer based NUC, which is described first. The PC peer decision

making criteria with consideration of the NUC is presented next. The peer matching process enabling decentralised negotiation between peers is explained and finally, the overall implementation of the PC P2P approach is described.

System Derived Network Usage Charge

An important component of the proposed PC framework in [15] is the NUC c_w for trade w. This pricing mechanism links the virtual and physical layers of the P2P system by providing a price signal derived from dynamic system conditions. The NUC is derived using the DLMPs resulting from (2.1), and is calculated as follows:

$$c_w = \begin{cases} (\lambda_{b_w} - \lambda_{s_w})/2 & \forall w \in \Omega^* & \text{if OPF is feasible} \\ c_w + \epsilon & \forall w \in \Omega^* & \text{otherwise} \end{cases}$$
(2.4)

Recognising that peers may select an infeasible set of trades in which the DLMPs cannot be calculated, a small penalty increment ϵ is used. All trades included in the proposed infeasible optimal set Ω^* have their NUC increased by the same fixed increment. In the scenario that the optimal set is feasible, all trades in the optimal set have their NUC set to the default DLMP based value, and any prior penalty increments are removed for those selected trades. This approach introduces an approximate coupling mechanism between the virtual layer and physical layer to support convergence to a feasible solution.

Although the incremental penalty supports convergence to a feasible solution, there are two major issues with the proposed approach. Firstly, the penalty is purely virtual and relies upon non-strategic participants. The NUC c_w is only paid for a given trade w if it is included in the final optimal, stable, and feasible set Ω^* . However, in the case of a feasible set, all incremental penalties are removed and the final NUC paid consists of only the DLMP component. A strategic participant could recognise these penalties are removed for any trade in the final optimal set and choose to select their preferred set of trades regardless of any virtual penalty increments. The second issue is due to the general application of the penalty increment. All trades in the infeasible, optimal set have a fixed penalty applied, regardless of their proportional impact on the actual network usage within the system. Trades which could be beneficial to the system may become sub-optimal in the local peer optimisation due to the presence of these virtual penalties, even if that trade could reduce network congestion. This results in an inefficient price signal which may penalise trades reducing congestion and may not ensure covergence to a locally optimal, stable solution.

Peer-Centric Decision Formulation

In the PC formulation, peers select the subset of all possible trades which maximises local welfare. Peers must determine the optimal total power bought d_n , and determine the least cost set of trades to supply the P2P component of demand d_n^p . With the addition of the NUC mechanism proposed in [15], peer decision making criteria for selection of the locally optimal set of trades is as follows:

$$\Omega_n^* = \begin{cases} \arg \max_{\Omega_n} \{ \sum_{w \in \Omega_n} (\rho_w^s - c_w) p_w^s - C_n(p_n) \} \quad \forall n \in \mathcal{N}^s \\ \arg \max_{\Omega_n} \{ U_n(d_n) - \sum_{w \in \Omega_n} (\rho_w^b + c_w) p_w^b \} \quad \forall n \in \mathcal{N}^b \end{cases}$$
(2.5a)

s.t.
$$p_w^b, p_w^s \in \{0, \overline{p}^w\} \quad \forall w \in \Omega$$
 (2.5b)

Eqs.
$$(2.2b)-(2.2h)$$
 Peer constraints $(2.5c)$

The peer selection result is a locally optimal set of P2P trades Ω_n^* , which ensures $d_n^p = d_n \Gamma_n$. The trade specific NUC c_w is critical to ensuring peers consider the cost of network usage in their decision making, since the utility system optimisation is now completely decoupled. Similar to the SC formulation, [15] assumes a predetermined proportion of total power bought is supplied by P2P transactions rather than allowing peers to determine the welfare optimal proportion. Note the absence of the coordinating constraint (2.2i) since the optimal trade sizes are decentrally determined, and the addition of (2.5b), which ensures p_w^b and p_w^s are binary variables equal to \overline{p}^w , if trade w is selected, and 0 otherwise for the PC scenario.

Peer Matching Process

The PC trading architecture enables peers to keep control of how they would like to best use their own resources; however, a process is needed to facilitate peer negotiation and ensure a stable solution is achieved. Thus, as proposed in [28] and further developed in [15], the peer matching process in Algorithm 1 satisfies full substitutability for establishing a stable outcome with direct bilateral negotiation.

Algorithm 1 Peer Matching Process [15, 28]

1: Initialization: 2: for $w \in \Omega$ do $\rho_w^b, \rho_w^s \leftarrow 0$ 3: $\Lambda_w := \{\rho_w^b, \rho_w^s\}, \ \forall w \in \Omega$ 4: Adjustment: 5: **do** $\Lambda_w^{old} \leftarrow \Lambda_w, \ \forall \ w \in \Omega$ 6: for $n \in \mathcal{N}$ do 7: $\Omega_n^* \leftarrow \Omega_n^* (2.5)$ 8: for $w \in \Omega$ do 9: if $p_w^b = \overline{p}^w$ and $p_w^s \neq \overline{p}^w$ then 10: if $\rho_w^b > \rho_w^s$ then 11: $\rho_w^s \leftarrow \rho_w^s + \Delta \rho$ 12:else 13:14: $\rho_w^b \leftarrow \rho_w^b + \Delta \rho$ 15: while $\Lambda_w^{old} \neq \Lambda_w, \ \forall w \in \Omega$ 16: return $\Omega^* := \bigcup_{w \in \Omega} (w \mid p_w^b = p_w^s = \overline{p}^w)$

All trades are first initialized with a buying and selling price of zero. The set of trade price pairs Λ_w is defined to determine when a stable solution has been obtained. The price negotiation and matching process consists of an iterative process stopping when $\Lambda_w^{old} = \Lambda_w$, indicating that no price changes occured in the most recent iteration. Peers select their optimal set of trades to maximise their local welfare according to (2.5). Individual trade prices are then incremented according to the following criteria:

- 1. If trade w is selected by buyer b_w , but not by seller s_w , the trade prices are incremented as below:
 - (a) If the buyer trade price ρ_w^b is greater than the seller trade price ρ_w^s , then the seller trade price is incremented by the fixed trade price increment $\Delta \rho$.
 - (b) Otherwise, the buyer trade price ρ_w^b is incremented by the fixed trade price increment $\Delta \rho$.
- 2. Otherwise, the trade prices remain unchanged.

Separate buyer and seller trade prices are necessary to facilitate negotiation since trade prices are monotonically increasing. Buyer trade prices will always be equal to or greater than the seller trade price, and equal to the seller trade price for all mutually selected trades. Beginning at a point that the trade prices are equal, if the buyer selects trade w but the seller does not, the price of ρ_w^b is increased. This first iteration assesses the willingness of buyer b_w to pay a higher trade price. If the now incremented trade remains in the optimal set of trades of b_w , $\Omega_{b_w}^*$, then in the next iteration, ρ_w^s will be increased and the seller s_w will have the option to choose the trade at the higher price. The seperate buyer and seller trade prices therefore allow for assessing the buyer's willingness to pay more prior to increasing the potential price received by the seller, avoiding a scenario where the new price is revealed to the seller but the buyer is no longer willing to pay that price. The result of the negotiation process is a mutually optimal stable set of trades Ω^* which consists of all trades w present in both the optimal buyer set of trades $\Omega_{b_w}^*$ and the optimal seller set of trades $\Omega_{s_w}^*$.

Implementation

After determining the stable set of optimal trades Ω^* , (2.1) is solved separately by the utility for balancing and to assess the trade set feasibility. If the trade set is infeasible, NUCs are incremented as described prior. If the trade set is feasible, the peer matching is checked to ensure the solution is stable after any prior NUC updates. If the matching is unchanged, a stable solution has been obtained, otherwise the NUCs are updated according to the system DLMPs. The complete PC P2P trading process proposed in [15] is illustrated in Figure 2.1, where the functions are grouped by the entity responsible for their completion.

2.4 Network Sensitivity Coefficients

Network sensitivity coefficients are used to determine changes to the network state as a result of a change in network inputs, which is typically a change in nodal power injections. The pricing mechanism proposed in this thesis is based on two existing sensitivity coefficients to determine the proportional contribution of individual trades to network constraints, i.e., the PTDF and the VSC, which are described next.



Figure 2.1: PC P2P trading solution [15].

2.4.1 Approximate Power Transfer Distribution Factors (PTDFs)

Contributions to feeder flows for individual transactions are required as part of the proposed transaction fee and congestion clearing process in this thesis. Approximate PTDFs are derived using the method described in [20]. Thus, suppose p_n , the energy sold and hence injected by peer n, is varied by a small amount Δp_n , and $\Delta f_l^{P,n}$ is the corresponding change in the active power flow of feeder l, one can define an Injection Shift Factor (ISF) for feeder l with respect to a change in active power injection from peer n as follows:

$$\Psi_l^n := \frac{\partial f_l^{P,n}}{\partial p_n^P} \approx \frac{\Delta f_l^{P,n}}{\Delta p_n^P} \tag{2.6}$$

To calculate the ISFs, let $\widetilde{X} = \text{diag}\{X_l\}$ be the diagonal matrix whose entries are the

susceptance X_l of feeder l, and the feeder incidence matrix denoted by $A = [..., a_l, ...]'$, where a_l is a vector in which the $o(l)^{th}$ entry is 1 and the $r(l)^{th}$ entry is $-1 \forall l \in L$. Hence, using dc approximations described in [20], one can obtain:

$$\Psi_l = \widetilde{X}_l A X^{-1} \tag{2.7}$$

where \widetilde{X}_l is the row in \widetilde{X} corresponding to feeder l, $X = A'\widetilde{X}A$, and $\Psi_l = [\Psi_l^1, ..., \Psi_l^N]'$. Using the ISFs, the PTDF for feeder l with respect to a trade with seller peer i and buyer peer j can then be calculated as follows:

$$\Phi_{ij}^l = \Psi_l^i - \Psi_l^j \tag{2.8}$$

These dc approximate PTDFs are solely dependent upon the network parameters and are therefore fixed regardless of the dynamic network state. This is acceptable for a typical radial distribution network and is suitable for use in assigning transaction fees according to contribution to the flows in congested feeders, and thus can be used in the clearing of these feeders.

2.4.2 Voltage Sensitivity Coefficients (VSCs)

In addition to considering the proportional contribution of individual transactions to feeder flow constraints, the proposed method for dynamic transaction fees requires a VSC to determine the proportional contribution to nodal voltage constraints. The first step of deriving this sensitivity coefficient is to recover the voltage angles δ_b at each node of a feasible OPF result as the second-order cone ac OPF model utilises an angle relaxation [35]. The centralised angle recovery algorithm described in [35], which is guaranteed to ensure angle recovery for radial systems, is described next.

To recover the voltage angles, the phase angle difference β_l across feeder l is calculated from Ξ^{Dist} and is defined as follows:

$$\beta_l = \angle [v_{o_l} - (R_l - jX_l)(f_l^P + jf_l^Q)] \quad \forall l \in \mathcal{L}$$

$$(2.9)$$

where the phase angle difference vector is defined as $\beta = [..., \beta_l, ...]'$, which is then used with the feeder incidence matrix A to obtain the voltage angle vector δ as follows:

$$\delta = A^{-1}\beta \tag{2.10}$$

Upon recovering the voltage angles, VSCs are computed using the Jacobian associated with the Newton Rhapson method for the relaxed system. The Jacobian matrix J can be calculated from the power flow equations and is of the form:

$$J = \begin{bmatrix} \begin{bmatrix} \frac{\partial P_b}{\partial \delta_b} \\ \\ \frac{\partial Q_b}{\partial \delta_b} \end{bmatrix} \begin{bmatrix} \frac{\partial P_b}{\partial |V_b|} \\ \\ \frac{\partial Q_b}{\partial |V_b|} \end{bmatrix}$$
(2.11)

where P_b and Q_b are the active and reactive power injections at bus b, and $|V_b|$ and δ_b are the voltage magnitude and voltage angle at bus b, repectively. Inverting the Jacobian allows one to directly obtain the VSCs for both active and reactive power injection. This thesis only considers the impact on nodal voltages from the real power injection associated with individual transactions, and therefore defines the VSC $\Delta \nu_b^{ij}$ for the voltage at bus bfor an active power transaction between seller peer i and buyer peer j as follows:

$$\Delta \nu_b^{ij} = \Delta \nu_b^i - \Delta \nu_b^j \tag{2.12}$$

Calculating the inverse Jacobian for large systems is computationally complex and so an alternative method for determining nodal VSCs could be used, such as in [36].

2.5 Summary

This chapter covered necessary background material for this thesis. The optimal branch flow was presented first, defining the central balancing authority objective and all necessary physical layer power flow relationships. Next, the typical categorisation of transactive market elements into the virtual and physical layer were introduced. The SC and PC P2P models from [15] in which the proposed techniques are based were then detailed, including the critical sub-components of the NUC, the decision formulation, and the peer matching process. Finally, the PTDF and VSC network sensitivity coefficients used in the proposed transaction fee were described.
Chapter 3

Proposed Decentralised Peer-Centric Transactive Framework

Transactive energy markets are an extension of the P2P concept, enabling not only P2P transactions but considering the complete relationship between prosumers and utility to derive maximum benefits from DERs. Rather than only considering P2P transactions, transactive energy can also consider Peer-to-Utility (P2U) transactions. In the transactive framework, peers do not have a fixed proportion of demand to be met by P2P sources and instead can select the welfare optimising mix of P2P and P2U transactions.

Similar to the P2P implementation, transactive implementations can take a SC or PC approach. The SC transactive market formulation is first introduced as a baseline for the globally optimal solution when centrally controlled. Next, a decentralised PC transactive framework is proposed consisting of novel transaction fee and congestion clearing mechanisms to achieve efficient and locally optimal solutions. Economic, transaction, and network results are compared between the various models to assess the performance of the proposed decentralised PC framework.

3.1 System-Centric Transactive Market

The conventional approach to power system operations is a SC approach, in which the central coordinating authority is aware of the operating ranges and associated bidding curves of market participants. The general SC transactive market represents the globally optimal solution for the complete distribution system. It is assumed that the central coordinating authority, typically the utility, is aware of the peer cost and marginal utility curves to be used in solving the central optimisation problem. Although the utility is assumed to be the central coordinating authority in the general case of this thesis, aggregators responsible for the global optimisation of peer decisions could act as an alternative. Modifications to the SC objective function may be required depending on the information available to the central coordinating authority.

To extend the NLP model to being fully transactive, the following modifications to (2.2) can be made:

$$\max_{\Xi^{\text{TPeer}}} O^{\text{TPeer}} := \sum_{n \in \mathcal{N}^b} (U_n(d_n) - T_n^d d_n^u) + \sum_{n \in \mathcal{N}^s} (T_n^g p_n^u - C_n(p_n))$$
(3.1a)

s.t.
$$\mathcal{N}^b \cap \mathcal{N}^s = \emptyset$$
 (3.1b)

$$U_n(d_n) = \begin{cases} \Upsilon_n(d_n - \underline{D}_n) & \text{if } d_n \ge \underline{D}_n \\ -\infty & \text{otherwise} \end{cases} \quad \forall \ n \in \mathcal{N}^b$$
(3.1c)

$$p_n = p_n^p + p_n^u \quad \forall \, n \in \mathcal{N}^s \tag{3.1d}$$

$$d_n = d_n^p + d_n^u \quad \forall n \in \mathcal{N}^b \tag{3.1e}$$

$$\underline{P}_n \le p_n \le \overline{P}_n \quad \forall \, n \in \mathcal{N}^s \tag{3.1f}$$

$$\underline{D}_n \le d_n \le \overline{D}_n \quad \forall \, n \in \mathcal{N}^b \tag{3.1g}$$

$$p_n^p = \sum_{w \in \Omega_n} p_w^s \quad \forall \, n \in \mathcal{N}^s \tag{3.1h}$$

$$d_n^p = \sum_{w \in \Omega_n} p_w^b \quad \forall \, n \in \mathcal{N}^b \tag{3.1i}$$

$$p_w^b = p_w^s \ \forall w \in \Omega \tag{3.1j}$$

where $\Xi^{\text{TPeer}} = \{d_n, d_n^p, d_n^u, p_n, p_n^p, p_n^u, p_w^b, p_w^s\}$. To consider the option of P2U transactions, (3.1d) and (3.1e) are added. The objective function is modified to include T_n^g and T_n^d , which are the fixed P2U tariff rates associated with selling energy to and buying energy from the utility. The utility may adjust these P2U rates to account for cost of supply or to incentivise specific behaviour through, for example, time-of-use pricing mechanisms. With these additions, the SC formulation introduced in Section 2.3.1 is now generally transactive rather than limited solely to P2P.

The OPF model (2.1) for the distribution utility welfare can also be modified to consider

P2U transactions as follows:

$$\max_{\Xi^{\text{Dist}}} \quad O^{\text{TDist}} := \sum_{b \in \mathcal{B}} (-C_b^u(p_b^g)) - C^w p_0^g + \sum_{n \in \mathcal{N}} (T_n^d d_n^u - T_n^g p_n^u)$$
(3.2a)

s.t. Eqs.
$$(2.1b)-(2.1j)$$
 Network constraints $(3.2b)$

where the utility welfare is modified to consider the revenue from selling P2U energy to buyer peers and the cost of purchasing P2U energy from selling peers. Additionally, constraint (2.1k) limiting the total system injection at the utility root bus p_0^g to only positive values is removed to allow for the net export of energy to the wholesale market. With the addition of P2U transactions, a wholesale market derived price is now available to peers, ensuring comparable results between the PC and SC transactive frameworks even with net exports, and enabling surplus local energy to be sold at the wholesale market rate to increase overall system welfare. This results in the following SC transactive formulation:

$$\max_{\Xi^{\text{TPeer}} \bigcup \Xi^{\text{Dist}}} \quad O^{TPeer} + O^{TDist} = \sum_{n \in \mathcal{N}^b} U_n(d_n) - \sum_{n \in \mathcal{N}^s} C_n(p_n) - \sum_{b \in \mathcal{B}} C_b^u(p_b^g) - C^w p_0^g \quad (3.3a)$$

s.t. Eqs.
$$(2.1b)-(2.1j)$$
 Network constraints $(3.3b)$

Eqs. (3.1b)-(3.1j) Transactive peer constraints (3.3c)

3.2 Network Usage Charge (NUC)

A critical deficiency of the PC P2P formulation in [15] is the approximate coupling mechanism for the virtual and physical layers as discussed in Section 2.3.2. The purpose of the coupling mechanism is to ensure consideration of the cost of network usage for transactions in the virtual trade selection process. However, the NUCs can be split into fixed and dynamic price components rather than a single integrated price. Fixed costs are associated with the fixed network topology and are therefore proportional to the path of a potential transaction. The dynamic costs are a result of the necessary actions taken to balance and achieve a feasible system, and can be attributed to the impact on system constraints from individual transactions. Fixed infrastructure costs not suited for recovery by volumetric charges should be addressed through alternative pricing mechanisms. Hence, rather than a single pricing mechanism as proposed in [15], a split mechanism is proposed in this thesis which individually accounts for the fixed and dynamic elements of network usage costs. This mechanism introduces improved price visibility for peers to respond to specific network usage by individual transactions. The fixed cost component is described in Section 3.2.1, while the dynamic cost component is detailed in Section 3.2.2.

3.2.1 Fixed NUC

To consider the proportional fixed network cost element of transactions, the equivalent impedance between buyer and seller node is used by the transacton platform to determine a buyer-seller pair specific NUC. This electrical distance derived NUC can then be used to determine the power distance of each transaction for allocating the fixed costs in a proportional manner, and are determined here using the distance based Thevenin Impedance Method proposed for P2P trades in [37].

The Thevenin impedance Z_{ij}^{th} between nodes i, j and associated NUC for trade w, as well as the node specific P2U trading prices T_n^g and T_n^d can be calculated as follows:

$$Z_{ij}^{th} = Z_{ii} + Z_{jj} - Z_{ij} - Z_{ji} \quad \forall i, j \in \mathcal{N}$$

$$(3.4a)$$

$$c_w = Z^{th}_{s_w b_w} E^Z \quad \forall \, w \in \Omega \tag{3.4b}$$

$$T_n^g = C^w - Z_{n1}^{th} E^Z \quad \forall n \in \mathcal{N}$$
(3.4c)

$$T_n^d = C^w + Z_{n1}^{th} E^Z \quad \forall n \in \mathcal{N}$$
(3.4d)

where Z_{ij} is the element in the i^{th} row and j^{th} column of the Z_{bus} matrix. The transaction platform is responsible for setting the constant scalar E^Z , which scales Z_{ij}^{th} to obtain the individual trade NUC. This method allows E^Z to be scaled as necessary to cover the fixed costs of network transactions while distributing those costs according to the electrical distance of the transaction paths. Since peers are now exposed to the wholesale market price C^w through P2U transactions, the peer specific feed in tariff T_n^g and demand tariff T_n^d are defined as the difference of the wholesale price and the NUC associated with the electrical distance between the peer and the interface between the transmission and distribution system or root node 1, i.e., Z_{n1}^{th} .

3.2.2 Dynamic Network Transaction Fee

While the fixed network costs associated with transactions are covered by the NUC c_w , an additional mechanism is required to represent peer willingess to pay for dynamic network usage. The method proposed in [15] introduces an incremental penalty assigned to all individual trades included in an infeasible optimal set; hence, rather than the general approach of treating all trades in the infeasible set as equal, a new approach is proposed here utilising derived network factors to introduce a price signal accounting for a peer's willingess to pay for network usage.

Flow Constraints

The optimal set of PC transactions may violate feeder flow limits due to decoupling of the virtual and physical layers. To ensure consideration of congestion in the PC trade selection, the dynamic pricing mechanism should introduce a transaction fee element proportional to an individual transaction's usage of limited network resources. The contributions to congestion for individual transactions are determined using the simplified PTDFs proposed in [20] and described in Section 2.4.1, resulting in the following sensitivity factors:

$$\Phi_{ij}^l = \Psi_l^i - \Psi_l^j \tag{3.5}$$

Voltage Constraints

In the scenario that the proposed set of optimal transactions results in an infeasible system due to voltage constraint violations, a network sensitivity based mechanism is necessary to penalise trades contributing to those violations. This process is described in Section 2.4.2, resulting in the following sensitivity coefficients:

$$\Delta \nu_b^{ij} = \Delta \nu_b^i - \Delta \nu_b^j \tag{3.6}$$

Dynamic Transaction Fee Assignment

The central coordinator, generally the utility, is responsible for assessing the feasibility of the optimal set of trades and updating proportional transaction fees using the above derived sensitivity coefficients. A transaction fee assignment process is proposed in Algorithm 2 to update individual transaction fees in the case of an infeasible set of locally optimal trades.

Algorithm 2 Transaction Fee Assignment Process

1: feederState, nodeState = model. ComputeIIS() 2: $v_b, \delta_b = \text{RelaxedOPF}(\text{feederState, nodeState})$ 3: $\Delta \nu_b^{ij} = \text{VSC}(v_b, \delta_b)$ 4: Flow fees: 5: for $l \in feederState$ do for $w \in \Omega$ do 6: $\epsilon_w \leftarrow \epsilon_w + E^\epsilon \Phi^l_{s_w b_w} feederState[l]$ 7: for $n \in \mathcal{N}$ do 8:
$$\begin{split} \epsilon_n^{u,g} &\leftarrow \epsilon_n^{u,g} + E^{\epsilon} \Phi_{n,0}^l feederState[l] \\ \epsilon_n^{u,d} &\leftarrow \epsilon_n^{u,d} + E^{\epsilon} \Phi_{0,n}^l feederState[l] \end{split}$$
9: 10:11: Voltage fees: 12: for $b \in nodeState$ do for $w \in \Omega$ do 13: $\epsilon_w \leftarrow \epsilon_w + E^{\epsilon} \Delta \nu_b^{s_w b_w} nodeState[b]$ 14:15: return ϵ

In the scenario that a set of negotiated transactions is infeasible, an Irreducible Inconsistent Subsystem (IIS) is computed from the network model. The IIS is a subset of the constraint and variable bounds with the properties that the model is still infeasible, and that if a single constraint or bound is removed, the system becomes feasible. The result of the IIS computation is a set of feeder and node index value pairs, *feederState*, *nodeState*, for which the proposed optimal set of trades violates their associated contraints. These sets have the following form:

$$feederState[l] = \begin{cases} 1 & \text{if } f_l > S_l \\ -1 & \text{if } f_l < -S_l \quad \forall l \in L \\ null & \text{otherwise} \end{cases}$$
(3.7a)

$$nodeState[b] = \begin{cases} 1 & \text{if } v_b > \overline{V}_b^2 \\ -1 & \text{if } v_b < \underline{V}_b^2 \\ null & \text{otherwise} \end{cases} \quad \forall b \in B$$
(3.7b)

The function RelaxedOPF(·) is the OPF (2.1) where all violated feeder and nodal constraints are relaxed to recover the system voltage angles. Violated feeders and nodes are iterated through to update individual transaction fees. Each P2P trade w has an associated transaction fee ϵ_w and each prosumer n has P2U transaction fees $\epsilon_n^{u,g}$ and $\epsilon_n^{u,d}$

for selling and buying respectively. For the derivation of sensitivity coefficients of P2U transactions, it is assumed that the utility root node is at the interface of the distribution and transmission system, i.e., b = 1. These transaction fees are incremented by a fixed quantity E^{ϵ} , which is scaled by the relevant sensitivity coefficient. In cases where a trade reduces a constraint violation, applicable transaction fees are reduced and can become negative to incentivise constraint reducing trades. The introduction of transaction fees, proportional to sensitivity coefficients, ensures that the dynamic value of network usage is considered in the PC selection process.

3.3 Congestion Clearing Process

While the above noted transaction fees introduce a price signal representing a peer's willingness to pay for limited network resources, an additional clearing mechanism is necessary to ensure that marginal congestion is cleared efficiently. Thus, transaction fees are incremented until the proposed set of optimal transactions is feasible and stable. However, an issue might arise where as a result of incrementing the transaction fees for using a congested feeder, a block of trades could become suboptimal, resulting in under utilisation of the previously congested feeder and deadweight loss of welfare. An example of this scenario can be considered for the network in Figure 3.1, in which there is a buyer at Bus 1, sellers at Buses 2 and 3, and fixed NUCs are ignored. Without considering feeder limits, the optimal solution would be 100 kW of supply purchased from Seller 2 at a price of \$10/MWh, whereas considering feeder limits, the optimal solution would be 50 kW of supply purchased from both Seller 2 and 3 at a price of \$20/MWh. Considering dynamic transaction fees and no additional clearing mechanism, the PC solution process would take the following steps:

- 1. Iteration 0 of the solution process would begin, consisting of the following sub-steps:
 - (a) Transaction matching would run with transaction fees at a starting price of \$0/MWh. This scenario is equivalent to no consideration of feeder limits, resulting in 100 kW of matched trades between Buyer 1 and Seller 2.
 - (b) The OPF would be run to assess the feasibility of the proposed set of transactions.
 - (c) Since the total flow over Feeder 1 is 100 kW and exceeds the flow limit, an infeasible set of trades is detected and the process in Algorithm 2 would be run to update the transaction fees for all trades impacting the constrained feeder.

- (d) Peers are made aware of the updated transaction fees, and the initial iteration ends.
- 2. The next iteration would begin and repeat while the transaction fee is less than \$10/MWh, consisting of the following sub-steps:
 - (a) Peers complete transaction matching with the updated set of transaction fees to make their locally optimal set of decisions. As long as the transaction fee for trades with Seller 2 are less than \$10/MWh, Buyer 1 will choose to trade 100 kW with Seller 2.
 - (b) The OPF would be run to assess the feasibility of the proposed set of transactions.
 - (c) Since the system is still infeasible due to the transactions exceeding Feeder 1 flow limits, the transaction fees are increased again.
 - (d) Peers are made aware of the updated transaction fees, and the current iteration ends.
- 3. The final iteration is run after repeating Step 2 until the updated transaction fees for trades between Buyer 1 and Seller 2 exceed \$10/MWh, at which point the following sub-steps occur:
 - (a) Peers complete transaction matching considering transaction fees exceeding \$10/MWh for trades between Buyer 1 and Seller 2. Since the effective trade price seen by Buyer 1 is now greater than the \$20/MWh of Seller 3, Buyer 1 chooses to purchase the full 100 kW from Seller 3, and none from Seller 2.
 - (b) The OPF would be run to assess the feasibility of the proposed set of transactions.
 - (c) The set of proposed transactions are now feasible according to the feeder flow limits.
 - (d) The process ends and the peers act upon their mutually selected trades.

The transaction results for the described process are displayed in Table 3.1 to demonstrate the peer behaviour as the transaction prices increase. In this table, an iteration corresponds to a completed solution iteration in which locally optimal transactions are negotiated, the feasibility of the proposed transactions is checked, and transaction fees are updated as needed. The steps included in the table correspond to the generalised steps described above for the PC solution process and are specific to the example system in



Figure 3.1: Congestion example network.

Iteration	Step	Transaction Fee [\$/MWh]	$p_2 \; [kW]$	p_3 [kW]
0	1	0.0	100	0
1	2	2.1	100	0
2	2	4.2	100	0
3	2	6.3	100	0
4	2	8.4	100	0
5	3	10.5	0	100

Table 3.1: Congestion example results without congestion clearing.

Figure 3.1. The iteration number increases for each completed solution process, whereas the steps change when the increasing transaction fee initiates the next step of the overall example process. A transaction increment of \$2.1/MWh is assumed for the transaction fee increment. Observe that Peer 1 prefers to purchase the full 100 kW of energy from Peer 2 when the transaction fee is less than \$10/MWh; however, as soon as the transaction fee exceeds that amount, Peer 1 prefers to buy solely from Peer 3, since the effective buying price would be \$20.5/MWh in iteration 5.

Note that due to the decoupling of the virtual and physical layers in the PC solution, the transaction fee increases at a proportionally determined fixed increment, rather than being set to a precise value at which the effective marginal cost of both resources are equal for a congested feeder. This can result in the overstepping of the transaction fee in the example, where prior to an increment Seller 2 is completely prioritised, and after the increment Seller 3 is completely prioritised. This results in a deadweight loss of welfare where the lower cost Seller 2 is not dispatched to the full system potential, which is addressed next.

A new congestion clearing process is proposed in Algorithm 3 to address the aforementioned market inefficiencies. This process runs when the system state changes from infeasible to feasible, indicating that transaction fees have been incremented such that all feeder congestions are resolved. Once the congestion clearing process is complete, all transactions implicating the last cleared feeders are fixed at an optimal level and must remain in that state for the remainder of the negotiation process. To achieve this, σ_w and ς_w are introduced as clearing and blocking parameters respectively for P2P transactions. When σ_w is set to 1, this trade must be selected by the relevant peers, and when ς_w is set to 1, this transaction must not be selected by the relevant peers. Similarly, $\sigma_n^{u,f}$, $p_n^{u,f}$, and $d_n^{u,f}$ are introduced for P2U trades. For P2U transactions which implicate the cleared feeder, $\sigma_n^{u,f}$ is set to 1, requiring that $p_n^u = p_n^{u,f}$ and $d_n = d_n^{u,f}$ for the remainder of the negotiation.

The clearing process for each feeder transitioning from infeasible to feasible is as follows:

- 1. A flow variable is initialized to monitor the available feeder capacity using Φ_{ij}^l for each cleared transaction.
- 2. Transactions selected in the last set of optimal trades are considered to maximise utilisation of the congested feeder while ensuring locally optimal decisions. Transactions are cleared such that the flow variable is updated and the associated congestion clearing parameters are updated. Transaction fees for cleared transactions are reversed to the prior iteration amount to ensure that the cleared transactions are locally optimal. The clearing order is as follows:
 - (a) All P2P and P2U transactions reducing congestion in the constrained feeder are cleared first.
 - (b) P2P transactions increasing congestion are cleared in ascending order of NUC c_w while capacity remains.
 - (c) Any remaining capacity is allocated to P2U transactions in ascending order of NUC c_w until all available capacity has been allocated.
 - (d) Once the available capacity has been allocated, all remaining trades in the last optimal set are blocked.
- 3. Any trades which were not in the last optimal set, but which impact the flow of the congested feeder are blocked.

In the clearing process for P2U transactions increasing congestion, P2U peer sales are considered first, and then P2U peer purchases. The flow variable is first updated to reflect

Algorithm 3 Congestion Clearing Process

```
1: for l \in feederStateOld do
              flow \leftarrow 0
  2:
              for w \in \Omega^*_{old} do
  3:
                 if \Phi^l_{s_w b_w} feederStateOld[l] < 0 then
  4:
                     \begin{aligned} flow &\leftarrow flow + \Phi^l_{s_w b_w} \overline{p}^w \\ \epsilon_w &\leftarrow \epsilon_w - E^{\epsilon} \Phi^l_{s_w b_w} feederStateOld[l] \end{aligned} 
  5:
  6:
                    \sigma_w \leftarrow 1
  7:
  8:
              for n \in \mathcal{N} do
                 if \Phi_{n,0}^l feederStateOld[l] < 0 then
  9:
                    \begin{aligned} flow &\leftarrow flow + \Phi_{n,0}^{l} p_{n,old}^{u} \\ \epsilon_{n}^{u,g} &\leftarrow \epsilon_{n}^{u,g} - E^{\epsilon} \Phi_{n,0}^{l} feederStateOld[l] \end{aligned}
10:
11:
                    \begin{array}{c} \stackrel{''}{p_n^{u,f}} \leftarrow \stackrel{''}{p_{n,old}^{u}} \\ \sigma_n^{u,f} \leftarrow 1 \end{array} 
12:
13:
                 if \Phi_{0,n}^{l} feederStateOld[l] < 0 then
14:
                     \begin{aligned} & flow \leftarrow flow + \Phi_{0,n}^{l} d_{n,old}^{u} \\ & \epsilon_{n}^{u,d} \leftarrow \epsilon_{n}^{u,d} - E^{\epsilon} \Phi_{0,n}^{l} feederStateOld[l] \\ & d_{n}^{u,f} \leftarrow d_{n,old}^{u} \end{aligned} 
15:
16:
17:
                    \sigma_n^{u,f} \leftarrow 1
18:
              for w \in \Omega^*_{old} do
19:
                 if \Phi_{s_w b_w}^l feederStateOld[l] > 0 then
20:
                     flow \stackrel{s_w v_w}{\leftarrow} flow + \Phi^l_{s_w b_w} \overline{p}^w
21:
                    if flow < S_l then
22:
                       \epsilon_w \leftarrow \epsilon_w - E^{\epsilon} \Phi^l_{s_w b_w} feederStateOld[l]
23:
24:
                        \sigma_w \leftarrow 1
                    else
25:
26:
                        \varsigma_w \leftarrow 1
              for w \notin \Omega^*_{old} do
27:
                 if \Phi_{s_w b_w}^l \neq 0 then
28:
29:
                     \varsigma_w \leftarrow 1
            CongestionClearingP2U()
30:
31: return \epsilon, \sigma, \varsigma, g^{u,f}, d^{u,f}
```

Algorithm 4 CongestionClearingP2U()

1: for $n \in \mathcal{N}$ do if $\Phi_{n,0}^l feederStateOld[l] > 0$ then 2: $flow \leftarrow flow + \Phi_{n,0}^{l} p_{n,old}^{u}$ $\epsilon_{n,0}^{u,g} \leftarrow \epsilon_{n}^{u,g} - E^{\epsilon} \Phi_{n,0}^{l} feederStateOld[l]$ 3: 4: $\sigma_n^{u,f} \leftarrow 1$ 5: if $flow < S_l$ then 6: $p_n^{u,f} \leftarrow p_{n,old}^u$ 7:else 8: $p_n^{u,f} \leftarrow p_{n,old}^u - (flow - S_l)$ $flow \leftarrow S_l$ 9: 10: if $\Phi_{0,n}^{l} feederStateOld[l] > 0$ then 11: $\begin{aligned} & flow \leftarrow flow + \Phi^l_{0,n} d^u_{n,old} \\ & \epsilon^{u,d}_n \leftarrow \epsilon^{u,d}_n - E^{\epsilon} \Phi^l_{0,n} feederStateOld[l] \\ & \sigma^{u,f}_n \leftarrow 1 \end{aligned}$ 12:13:14: if $flow < S_l$ then 15: $d_n^{u,f} \leftarrow d_{n,old}^u$ 16:17:else $d_{n,old}^{u,f} \leftarrow d_{n,old}^u - (flow - S_l)$ 18: $flow \leftarrow S_l$ 19:

the flow of the congested feeder when considering the next marginal P2U block, followed by the reversing of transaction fees to the prior iteration, and finally the capacity of the feeder is checked to ensure flow constraints are not violated. This process is sequentially repeated for peers until all congested feeder capacity has been allocated, or until no pending P2U transactions remain.

The order of clearing to determine which transactions are accepted and which are blocked is as follows:

- 1. P2P trades are cleared first, with remaining capacity allocated to P2U transactions.
- 2. Transactions within the P2P or P2U pool are cleared in ascending order of the associated transaction NUC c_w .

This ordering prioritises P2P trades above P2U transactions, and further prioritises transactions according to the fixed NUC, which is proportional to the electrical distance of the transaction. The trade set Ω^*_{old} and set of peers \mathcal{N} are assumed to be ordered according to the above logic. This method seeks to maximise welfare; however, a fairness based approach could be implemented with an alternative allocation of the limited capacity. With this process, the congested feeders are cleared, utilising all available capacity, with all transactions impacting those feeders fixed at a specified amount.

The example in Figure 3.1 is repeated here using the aforementioned congestion clearing process. The PC solution process would now take the following updated steps:

- 1. Iteration 0 of the solution process would begin, consisting of the same sub-steps as originally introduced.
- 2. The next iteration would begin and repeat while the transaction fee is less than \$10/MWh, consisting of the same original sub-steps.
- 3. After repeating Step 2 such that the transaction fees between Buyer 1 and Seller 2 exceed \$10/MWh, the updated sub-steps would be followed:
 - (a) Peers complete transaction matching considering transaction fees exceeding \$10/MWh for trades between Buyer 1 and Seller 2. Since the effective trade price seen by Buyer 1 is now greater than the \$20/MWh of Seller 3, Buyer 1 chooses to purchase the full 100 kW from Seller 3, and none from Seller 2.
 - (b) The OPF would be run to assess the feasibility of the proposed set of transactions.

- (c) The set of proposed transactions are now feasible according to the feeder flow limits. The transition from infeasible to feasible indicates that congestion clearing is necessary, and the aforementioned congestion clearing process is run.
- (d) Peers are made aware of the clearing and blocking parameters resulting from the congestion clearing process, and the reversed transaction fees for trades impacting the congested feeder.
- 4. The final iteration with consideration of the congestion clearing results is run with the following sub-steps:
 - (a) Peers complete transaction matching considering the congestion clearing results. The congestion clearing process clears 50 kW of trades between Buyer 1 and Seller 2, and blocks the remaining 50 kW of potential trades. Buyer 1 purchases the remaining 50 kW of supply from Seller 3.
 - (b) The OPF would be run to assess the feasibility of the proposed set of transactions.
 - (c) The set of proposed transactions are feasible, and since the prior iteration was also feasible, congestion clearing is not run.
 - (d) The process ends and the peers act upon their mutually selected trades.

The results for each iteration of the process are shown in Table 3.2. The same descriptions for iterations and steps included for Table 3.1 are used. Note that in iteration 5, the increased transaction fees lead to the congestion in Feeder 2 being resolved, leading to a transition from an infeasible state to a feasible state. This transition indicates the potential need for the congestion clearing process. The transaction fees for trades impacting the congested Feeder 2 are reversed to their values of \$8.4/MWh in iteration 4, since this represents the last set of trades and associated transactions fees which maximise feeder utilisation with locally optimal decisions. All those trades between Buyer 1 and Seller 2 are fixed in the congestion clearing process, with the full capacity of 50 kW cleared and the remaining potential 50 kW of supply blocked. Iteration 6 resumes with all potential trades between Buyer 1 and Seller 2 either cleared or blocked, requiring the remaining 50 kW to be purchased from Seller 3. The result of the process is the maximal utilisation of the congested feeder, while respecting peer decision autonomy with cleared trades consisting only of those which were previously determined locally optimal.

Iteration	Step	Transaction Fee [\$/MWh]	$p_2 [\mathrm{kW}]$	$p_3 [kW]$
0	1	0.0	100	0
1	2	2.1	100	0
2	2	4.2	100	0
3	2	6.3	100	0
4	2	8.4	100	0
5	3	10.5	0	100
6	4	8.4	50	50

Table 3.2: Congestion example results with congestion clearing.

3.4 Peer-Centric Transactive Decision Formulation

Similar changes required for a transactive SC model are required in the PC decision formulation. In addition to considering P2U transactions and welfare based determination of P2P share, the fixed NUC, dynamic transaction fees, and congestion variables must be considered in selecting a peer's optimal set of trades. Thus, the following is the MINLP transactive PC decision criteria:

$$\Omega_n^* = \begin{cases} \arg \max_{\Omega_n} \{ \sum_{w \in \Omega_n} (\rho_w^s - c_w) p_w^s + (T_n^g - \epsilon_n^{u,g}) p_n^u - C_n(p_n) \} \ \forall n \in \mathcal{N}^s \\ \arg \max_{\Omega_n} \{ U_n(d_n) - (T_n^d + \epsilon_n^{u,d}) d_n^u - \sum_{w \in \Omega_n} (\rho_w^b + c_w + \epsilon_w) p_w^b \} \ \forall n \in \mathcal{N}^b \end{cases}$$
(3.8a)

s.t.
$$p_w^b \wedge p_w^s = \overline{p}^w \quad \forall w | \sigma_w = 1$$
 (3.8b)

$$p_w^b \wedge p_w^s = 0 \quad \forall \, w | \, \varsigma_w = 1 \tag{3.8c}$$

$$p_n^u = p_n^{u,f} \ \forall \, n | \, \sigma_n^{u,f} = 1 \tag{3.8d}$$

$$d_n^u = d_n^{u,f} \quad \forall \, n | \, \sigma_n^{u,f} = 1 \tag{3.8e}$$

$$p_w^b, p_w^s \in \{0, \overline{p}^w\} \ \forall w \in \Omega$$

$$(3.8f)$$

Eqs.
$$(3.1b)-(3.1i)$$
 Peer constraints $(3.8g)$

The optimal set of trades for peer n now fully considers the fixed NUC c_w , the dynamic transaction fees ϵ_w , $\epsilon_n^{u,g}$, and $\epsilon_n^{u,d}$, and the congestion clearing variables σ_w , ς_w , $\sigma_n^{u,f}$, $p_n^{u,f}$, and $d_n^{u,f}$. It is of note that the transaction fee ϵ_w for trade w is fully paid by the buyer b_w , and as such does not appear in the selection criteria of seller s_w . Equations (3.8b) and (3.8c) are added to ensure that all P2P trades implicated by the congestion clearing

process are either accepted or rejected respectively. Equations (3.8d) and (3.8e) ensure that all P2U trades implicated by the congestion clearing process are fixed at their cleared values.

3.5 Enhanced Transaction Matching Process

The matching process is the most significant computational component of the solution methodology, requiring multiple iterations as individual trade prices increase until a stable set of trades is reached. The matching process proposed in [15] and seen in Figure 2.1 requires a complete price renegotiation each time the NUC is updated, since the set of trade prices is reset to zero for all trades. A modified matching process is proposed in this thesis accounting for the peer optimal set criteria of (3.8), which reduces the computational needs of completely solving for a stable solution after a variable change. The updated matching process is described in Algorithm 5.

Algorithm 5 Transaction Matching Process
1: for $n \in \mathcal{N}$ do
2: $\Omega_n^* \leftarrow \Omega_n^* (3.8)$
3: for $w \in \Omega$ do
4: if $p_w^b = \overline{p}^w$ and $p_w^s \neq \overline{p}^w$ then
5: if $\rho_w^b > \rho_w^s$ then
6: $\rho_w^s \leftarrow \rho_w^s + \Delta \rho$
7: else
8: $\rho_w^b \leftarrow \rho_w^b + \Delta \rho$
9: $\Omega^* := \bigcup_{w \in \Omega} (w \mid p_w^b = p_w^s = \overline{p}^w)$
10: return Ω^*

The transaction matching process now becomes a single increment of the trade prices, rather than resetting trade prices to zero upon initialization and resolving for a stable solution. This process is repeated until a stable set of trades is obtained, at which point the need for either transaction fee assignment or congestion clearing is assessed.

3.6 Comprehensive Decentralised Peer-Centric Framework

The proposed individual elements consisting of the fixed NUC, dynamic network transaction fee, congestion clearing process, and simplified peer matching process are integrated into a single proposed transactive solution process as shown in Figure 3.2. The solution process is as follows:

- 1. The set of possible transactions are initialized for all peers.
- 2. A single iteration of the transaction matching process is run, in which peers select their optimal set of transactions and transaction prices are updated.
- 3. The transaction matching process is repeated until no change in matches have occurred, ensuring a stable solution.
- 4. The utility runs the OPF (2.1) to assess system feasibility. If the system is infeasible, the transaction fee assignment process is run and transaction matching resumes from the most recent transaction state.
- 5. If the OPF is feasible, a change in feasibility is checked. If a change in feasibility from infeasible to feasible occurred, the congestion clearing process is run for the cleared feeders and transaction matching resumes from the most recent transaction state.
- 6. If the OPF is feasible and the feasibility is unchanged, a final optimal, feasible, and stable solution has been obtained.

This process results in a decentralised, locally optimal set of transactions for all peers for a single time step, and would need to be repeated for each dispatch or contracting period. The corresponding full implementation is provided in [38].

3.7 Simulation Results

In this section, the performance of the proposed PC method is compared against the SC approach and the PC approach in [15], and the proposed SC method. The reference system models consisting of a 15-bus and 33-bus network are described in Section 3.7.1. Economic and system performance results are presented and discussed for the 15-bus network in Section 3.7.2, and for the 33-bus network in Section 3.7.3. For all proposed PC results, it is assumed that $E^Z = 1$.



Figure 3.2: PC proposed transactive solution process.

3.7.1 Reference System Models

Previously reported distribution networks are used to compare the economic and system performance of the proposed solution with existing approaches. These networks include the 15-bus network from [39], and a modified 33-bus network from [40] and [25], which are illustrated in Figure 3.3. The complete network parameters can be found in Appendix A for the 15-bus network and in Appendix B for the 33-bus network. The 15-bus system includes 2 local sellers and the utility bus for balancing at Bus 1. The marginal cost of Peer 1 is \$50/MWh, and the marginal cost of Peer 12 is \$10/MWh. As the 15-bus network



Figure 3.3: Distribution test networks with peers: (a) 15-bus P2P grid from [39], and (b) 33-bus grid from [40] and [25] with additional local supply.

is a P2P network, all peers meet their demand through P2P transactions. The 33-bus network includes 6 local sellers and a utility bus for balancing and P2U transactions at Bus 1. Peers are free to determine whether to transact with peers or with the utility. The complete modifications to the peer demand and cost curves for the 33-bus network are described in Appendix B.

3.7.2 15-Bus P2P Trading

To compare the performance of the proposed PC approach with the existing methods in [15], the network in Figure 3.3a is used, where the complete system parameters are described in Appendix A. P2U transactions in the proposed implementation are restricted to ensure comparable results with the existing P2P approaches, and Γ_n is set to 1 for all peers in the existing models, requiring that all peer demand be met through P2P trades.

The set of selected trades for each approach are shown in Figure 3.4. The existing SC set of optimal trades for the 15-bus network utilises 100% of the capacity of Feeder 11, as seen in Figure 3.4a, since the cost of supply from Peer 12 is lower than Peer 1 in the central optimisation. Both selling peers trade across the entire network, with all buying peers other than Peer 14 receiving energy from both Peer 1 and Peer 12. As discussed in Section 2.3.2, the existing PC implementation's consideration of the physical layer uses approximate, non-proportional fees. This results in an under utilisation of Peer 12 with only 228 kW of a possible 255 kW of supply, shown in Figure 3.4b. The addition of the congestion clearing process in the proposed PC method ensures maximal utilisation of priority feeders, and therefore Peer 12 optimises output within system constraints. In addition to increasing feeder utilisation, the proposed transaction fee incentivises trading with least electrical distance peers as seen in Figure 3.4c. Thus, the limited capacity of Peer 12 is traded with the least distance peers, rather than trading across the entire network.

A core component of the existing and proposed PC solutions is the respective approach to ensuring consideration of network limits in the decentralised matching of transactions. To compare the relative performance of each solution, the feeder utilisation and nodal voltages resulting from the optimal solutions are considered, as illustrated in Figure 3.5. As noted above, the non-proportional fees considered in the existing PC solution result in transactions which underutilise high priority feeders. On the otherhand, the proposed pricing mechanism and congestion clearing process ensure that Feeder 11 utilisation aligns with the SC approach, as seen in Figure 3.5a. All other feeders have a dynamic network transaction fee component of zero, since the proposed optimal solutions in the iterative process do not exceed any other feeder limits. In general, utilisation of the various feeders is almost identical for the proposed PC and existing SC approaches, whereas the existing PC approach consistently under utilises available feeder capacity.

The resulting nodal voltages, presented in Figure 3.5b, can be used as an approximate indicator of the effective co-optimisation of active and reactive power. The utility is responsible for determining the utility and peer generator reactive power setpoints which minimise costs. Due to the central co-optimisation process of the SC approach, nodal voltage are lowest in this case, indicating the most efficient usage of reactive power resources to support optimal active power trades. The existing PC approach results in the furthest nodal voltages from the optimal voltage values obtained in the SC technique due to the approximate consideration of the physical layer, whereas the proposed PC approach is closer. Although a price signal for reactive power is introduced in the case of voltage constraint violations, there is no price signal representing the marginal value or cost in either of the PC local peer decision criteria. Ensuring consideration of the reactive power for local peer

Table 3.3: 15-bus network economic performance in [\$/h].

Model	Buyer	Seller	Transaction	Cost of
	costs	revenues	fees	generation
Existing SC	80.94	70.8	10.15	71.25
Existing PC	79.46	79.01	0.45	72.26
New PC	82.76	82.28	0.48	71.27

decision making could improve nodal voltage results.

Table 3.3 compares the performance of the models in the virtual layer. The economic indicators considered are calculated as follows:

$$\sum_{w \in \Omega^*} (\rho_w^b + c_w + \epsilon_w) p_w^b \tag{3.9a}$$

$$\sum_{w\in\Omega^*} (\rho_w^s - c_w) p_w^s \tag{3.9b}$$

$$\sum_{w \in \Omega^*} (2c_w + \epsilon_w) \tag{3.9c}$$

$$\sum_{b \in \mathcal{B}} C_b^u(p_b^g) + C^w p_0^g + \sum_{n \in \mathcal{N}^s} C_n(p_n)$$
(3.9d)

where (3.9a) defines the buyer costs, (3.9b) the seller revenues, (3.9c) the transaction fees, and (3.9d) the cost of generation. Since buyers in this network are assumed to not consider marginal utility above their minimum demand, the welfare is captured in the cost of generation, which includes both peer owned generators and the cost of balancing by the utility. Sub-optimal usage of Peer 12 results in the highest cost of generation for the existing PC solution, whereas the proposed PC solution is almost identical in cost of generation to the SC solution in [15]. System usage costs are comparable for both PC implementations and significantly less than the SC case, ensuring increased revenues from energy sales for peers; the value of E^Z could be adjusted to ensure sufficient recovery of dynamic operational costs. Buyer costs are highest in the proposed PC method as the cleared trading prices are centered around the cost of the marginal resource, which for the considered scenario is Peer 1. The penalty mechanism in the existing PC approach introduces a decoupling of trading prices from actual willingness to pay by peers, resulting in a lower buyer cost but also a lower seller revenue.

3.7.3 33-Bus Transactive Market

The proposed transactive models extend the standard P2P problem by including the option of P2U transactions, which are considered here. Performance of the proposed SC and PC models are assessed using a modified version of the 33-bus network from [25], which includes 6 net selling peers and 26 buying peers meeting their demand through the transactive market. The generator cost functions and feeder flow limits from [25] are adjusted to ensure increased competition between peers and utility for the specified demand. The test system data is presented in Appendix B.

The resulting transactions of the proposed PC solution are shown in Figure 3.6. The optimal PC transactions display significant diversity, with all selling peers involved in either P2P or P2U transactions. Peers 25 and 31 are limited in their ability to transact due to congestion on Feeders 24 and 30. Peers prioritise least electrical distance transactions first, confirming the effectiveness of the proposed transaction fee mechanism. For example, Peer 6 prefers selling to Peers 7 and 8 over Peers 9, 10, and 11, since the closer peers are able to offer higher trading energy prices with lower transaction fees. Peers 9, 10, and 11 must satisfy their demand through P2U transactions as they represent the least cost remaining option. In the proposed SC approach, the centralised optimisation is able to directly decide whether transactions are P2P or P2U. The SC technique chooses all transactions to be carried out via P2U since the bilateral agreement constraint (eq:SCPeersub8) is only applicable for P2P transactions, providing greater flexibility to manage system balancing than with P2P transactions.

The generator node dispatch points and marginal costs are displayed in Table 3.4. These results can be used to observe and analyse discrepancies in generator dispatch resulting from the varying approaches. As a central solution, the SC approach can maximise total system exports with consideration of the full DLMP at each node, rather than the fixed NUC used in the proposed PC model to account for electrical distance of a particular transaction. This difference in approaches results in total exports of 2.944 MW for the SC approach, compared to only 2.235 MW for the PC approach. The electrical distance method introduces some overall system inefficiencies compared to the centralised approach. System performance results shown in Figure 3.7 can be used to understand these results with respect to feeder utilisation. As seen in Figure 3.7a, feeder utilisation for Feeder 17 is at 100% for the SC case, and less for the PC case. From the SC perspective, the marginal cost of Peer 18 is only \$7.158/MWh compared to a potential export price of \$7.65/MWh. Even with consideration of network losses and constraints, this price discrepancy ensures that Peer 18 is dispatched to the maximum setpoint while respecting the limit of Feeder 17. The PC introduction of electrical distance based fixed transaction charges results in

Node	New SC		New PC	
	Dispatch	Marginal cost	Dispatch	Marginal cost
	[MW]	[MWh $]$	[MW]	[%/MWh]
1	-2.944	7.650	-2.235	7.650
2	2.500	7.450	2.500	7.450
6	1.265	7.651	1.040	7.642
18	0.967	7.158	0.840	7.150
22	0.694	7.511	0.360	7.458
25	1.987	7.019	1.999	7.020
31	1.219	7.498	1.180	7.494

Table 3.4: 33-bus generator node dispatch points and marginal costs.

Peer 18 only trading with Peers 12-17, with a final dispatch that under utilises the valuable supply of Peer 18, and the available capacity of Feeder 17. Similar behaviour is observed for Peers 6, 22, and 31, where as a result of the additional fixed transaction fee charge, the dispatch points are lower in the PC approach than in the SC approach.

In general, feeder utilisation is similar in the two cases. Although the PC introduction of a split transaction fee mechanism allows for consideration of the physical layer in locally optimal transaction decisions, the approach cannot consider the network to the same degree as a centrally optimised approach. The most significant feeder utilisation discrepancies are due to the variance in dispatch for Peers 18 and 22, which arise for the reasons discussed above. As expected, all nodal voltages are within the acceptable operating ranges, as seen in Figure 3.7b, but the central optimisation of the SC approach allows for direct consideration of reactive power alongside active power, resulting in nodal voltages that are consistently higher in the SC case than in the PC case.

The total generation, demand, exports, and losses are shown in Table 3.5 for the two approaches. Total peer generation is less in the PC approach than in the the SC approach due to reduced exports when considering locally optimal welfare rather than system wide welfare. Losses are also lower in the PC case; however, whereas total generation in the PC case is reduced by 8.3% with respect to the SC generation, losses are just reduced 5.1% with respect to the SC losses. This difference can be attributed to the location of the additional generation and the co-optimisation of reactive and active power in the SC approach.

Table 3.6 shows the economic results of the two solutions. With the additional consid-

Table 3.5: 33-bus generation and demand results [MWh].

Model	Total	Total	Total	Total
	generation	demand	export	losses
New SC	8.632	5.610	2.944	0.078
New PC	7.919	5.610	2.235	0.074

eration of P2U transactions, the economic indicators are calculated as follows:

$$\sum_{n \in \mathcal{N}^b} (T_n^d + \epsilon_n^{u,d}) d_n^u + \sum_{w \in \Omega^*} (\rho_w^b + c_w + \epsilon_w) p_w^b$$
(3.10a)

$$\sum_{w\in\Omega^*} (\rho_w^s - c_w) p_w^s + \sum_{n\in\mathcal{N}^s} (T_n^g - \epsilon_n^{u,g}) p_n^u$$
(3.10b)

$$\sum_{w \in \Omega^*} (2c_w + \epsilon_w) p_w^b + \sum_{n \in \mathcal{N}^b} (\epsilon_n^{u,d} + Z_{n1}^{th} E^Z) d_n^u + \sum_{n \in \mathcal{N}^s} (\epsilon_n^{u,g} + Z_{n1}^{th} E^Z) p_n^u$$
(3.10c)

$$\sum_{n \in \mathcal{N}^s} C_n(p_n) \tag{3.10d}$$

$$\sum_{b\in\mathcal{B}} C_b^u(p_b^g) + C^w p_0^g + \sum_{n\in\mathcal{N}^s} C_n(p_n)$$
(3.10e)

where (3.10a) defines the buyer costs, (3.10b) the seller revenues, (3.10c) the transaction fees, (3.10d) the cost of generation, and (3.10e) the net system cost.

The cost of generation only includes the cost of generation for peers, whereas the system cost includes net injection at the utility bus at wholesale price C^w and is thus the net cost of the system as a whole. Note that in the proposed SC solution, peers buy or sell according to their net injection at the nodal DLMP. All transactions occur with the utility and hence the DLMP transaction cost method described in Section 2.3.2 is used. The proposed SC solution results in selling peers receiving less revenue from transactions than their local cost. This occurs as the transactions are optimised to minimise total system cost, where additional revenues from exporting to the wholesale market are not directly accounted for in the peer transaction revenue. A mechanism to distribute the total revenue from wholesale sales across the selling peers would be necessary to ensure they are made whole. The proposed PC implementation, on the other hand, ensures selling peers directly receive revenue for both P2P and P2U trades. Selling peers will never select a set of trades such that the local cost of generation exceeds the local revenue. The proposed PC method ensures a greater allocation of revenue to the peers, rather than being

	1	I	1		
Model	Buyer	Seller	Transaction	Cost of	System
	\cos ts	revenues	fees	generation	$\cos t$
New SC	42.73	52.99	12.86	62.94	40.41
New PC	43.5	59.21	1.58	57.6	40.5

Table 3.6: 33-bus network economic performance in [\$/h].

captured in system fees. The net system cost is comparable between the two methods, as expected, demonstrating that the proposed PC solution is still economically efficient while maintaining transactive and decentralised principles.

The economic results for buyers and sellers are shown in Tables 3.7 and 3.8 respectively. The peer specific results can be used to provide further clarity on the differences in costs and revenues for peers resulting from the SC and PC approaches. These costs, revenues, and transaction fees are calculated according to the previously noted equations, except only for the particular peer rather than for all peers. Buyer costs are generally comparable between the SC and PC scenarios. This is expected since the value of surplus energy is not considered for peers, resulting in consistent demand for both approaches, and since energy prices are centered on the marginal resource in the system, which is the wholesale electricity market price for the system considered. More significant differences are observed when comparing the revenues, costs, and transaction fees of seller peers. Seller peers are generally dispatched at higher levels due to direct consideration of potential export revenues in the SC solution, leading to higher generation costs for peers. Although the overall system welfare increases in the SC approach, the effect of using DLMPs for determining revenues can be seen in comparing the seller revenues and transaction fees. Depressed DLMPs for Peers 18 and 25 result in significant transaction fees, and DLMP derived revenues far below the actual local costs. This same scenario does not arise in the PC case, and although seller revenues are higher for some non-congested peers in the SC approach, the PC approach avoids the need for out of market settlements.

3.8 Summary

This chapter presented a decentralised PC transactive energy approach which ensures a stable and feasible solution. The P2P SC approach proposed in [15] was extended to a generalised transactive SC formulation with additional consideration of P2U transactions and the wholesale electricity market price. A split pricing mechanism for consideration of

Node	New SC		New PC		
	Buyer	Transaction	Buyer	Transaction	
	\cos ts	fees	\cos ts	fees	
3	1.370	-0.007	1.390	0.013	
4	1.831	-0.005	1.856	0.020	
5	0.917	-0.001	0.928	0.010	
7	3.068	0.008	3.103	0.023	
8	3.084	0.024	3.148	0.028	
9	0.929	0.011	0.955	0.037	
10	0.930	0.012	0.967	0.049	
11	0.698	0.009	0.720	0.032	
12	0.930	0.012	0.966	0.054	
13	0.925	0.007	0.960	0.048	
14	1.848	0.012	1.869	0.069	
15	0.919	0.001	0.917	0.029	
16	0.913	-0.005	0.914	0.026	
17	0.901	-0.017	0.883	0.007	
19	1.372	-0.005	1.378	0.001	
20	1.364	-0.013	1.384	0.016	
21	1.361	-0.016	1.382	0.014	
23	1.364	-0.013	1.395	0.018	
24	6.283	-0.143	6.515	0.131	
26	0.919	0.001	0.928	0.004	
27	0.919	0.001	0.930	0.006	
28	0.920	0.002	0.946	0.028	
29	1.841	0.005	1.876	0.028	
30	3.064	0.004	3.073	0.033	
32	3.154	-0.059	3.199	0.007	
33	0.902	-0.016	0.920	0.008	

Table 3.7: 33-bus buyer peer economic performance in [\$/h].

Node		New SC		New PC		
	Seller Transaction		Generation	Seller	Transaction	Generation
	revenues	fees	costs	revenues	fees	costs
2	19.075	0.050	18.313	19.081	0.044	18.313
6	9.677	0.000	9.646	8.048	0.061	7.926
18	4.060	3.335	6.891	6.276	0.234	5.985
22	5.212	0.097	5.174	2.736	0.030	2.674
25	5.823	9.381	13.832	14.908	0.390	13.913
31	9.143	0.183	9.080	8.992	0.077	8.788

Table 3.8: 33-bus seller peer economic performance in [\$/h].

the physical layer in decentralised peer negotiation was proposed, which accounts for both the fixed and dynamic network characteristics. To address potential issues with coordination between the decentralised negotiation and efficient capacity allocation, a congestion clearing process was described which introduces additional parameters to ensure optimal network utilisation. The transactive SC formulation proposed in this thesis and the P2P PC decision criteria from [15] were utilised to obtain a transactive PC formulation for peer transaction decisions. These individual elements were integrated into a comprehensive decentralised PC framework with coordination between the virtual and physical transactive layers.

Simulations of the previously reported approach and the approaches proposed in this thesis were carried out on a 15-bus and 33-bus network. The transactions, economic, and network results were compared to assess the performance of the proposed methods. The transaction fee mechanism and congestion clearing process included in the PC approach ensured efficient and optimal allocation of limited network capacity, ensuring comparable results for the 15-bus network to the existing SC approach and improved performance compared to the existing SC approach. For the 33-bus network, a transactive baseline was determined with the proposed SC approach additionally considering P2U transactions and net system exports. Overall welfare of the transactive PC approach was similar to the SC approach and ensured a feasible solution was obtained. Total dispatch was higher in the SC case due to system level optimisation and direct consideration of the wholesale electricity price, whereas the PC approach required out of market settlements to ensure peers were kept economically whole due to the disconnect between DLMP determined transaction rates and system revenue from exports. On the other hand, the PC approach allowed for

direct consideration of system export value through P2U transactions, avoiding the need for additional settlement procedures and ensuring a feasible, stable set of locally optimal transactions.





Figure 3.4: P2P transactions for the 15-bus system: (a) existing SC, (b) existing PC, and (c) proposed PC approaches.







(b)

Figure 3.5: Network results for the 15-bus system: (a) feeder utilisation, and (b) nodal voltage.



Figure 3.6: Transactions for the 33-bus system using the proposed PC method.



(b)

Figure 3.7: Network results for the 33-bus system: (a) feeder utilisation, and (b) nodal voltage.

Chapter 4

Conclusion

4.1 Summary

This thesis extended existing P2P market formulations to develop a general transactive market design for both SC and PC objectives. Gaps in the existing literature were identified indicating a need for decentralised, PC approaches which retain transactive energy principles such as, for example, security and decision autonomy. The background material necessary for this thesis was presented in Chapter 2, including the second-order ac OPF formulation, a description of transactive market layers, and the complete SC and PC P2P approaches from [15], which formed the foundation of the proposed transactive approaches.

The P2P models described in the background were extended to generally transactive approaches in Chapter 3, with the consideration of P2U transactions in addition to P2P transactions. The SC P2P reference model was extended with transactive decision criteria to obtain a general SC transactive energy market. To address identified issues with the physical coupling mechanism in the reference PC P2P approach, a split pricing mechanism was introduced, which separately accounts for the transaction fees associated with the fixed network topology and the dynamic system conditions. A congestion clearing process was proposed that coordinates with the previously proposed transaction matching process to ensure efficient allocation of limited network capacity. The decision criteria introduced for the transactive SC approach were integrated with the existing PC P2P decision criteria to obtain a PC transactive decision formulation for local selection of optimal trades. These individual components were integrated into a cohesive, decentralised PC framework, which determines a locally optimal, feasible set of transactions. Simulation results for the existing and proposed approaches were presented and compared for a 15-bus and 33-bus network [39, 40, 25].

The main conclusions of this thesis are as follows:

- The proposed PC approach can ensure feasible, stable, and locally optimal solutions while maintaining transactive principles. It achieved similar performance to the SC approaches and improved performance compared to the existing PC approach.
- The proposed split transactive fee mechanism is an effective price signal for peers to introduce coupling between the physical and virtual layers. The fixed network topology component ensured prioritisation of least electrical distance transactions and assigned volumetric costs according to proportional network usage. The dynamic fee component represented the economic value of congestion and supported efficient allocation of network capacity.
- An additional clearing process is necessary for marginal transactions in a decentralised negotiation approach to ensure efficient allocation of network capacity. The proposed congestion clearing process is successful in ensuring high utilisation of congested feeders with priority according to electrical distance based principles.
- PC approaches can ensure locally acceptable transaction sets without additional out of market settlements, whereas SC approaches utilising DLMP pricing may require additional distribution of revenues to ensure selling peer revenues exceed their local costs and reflect their contribution to system welfare maximisation.

4.2 Contributions

The main contributions of this thesis are the following:

- Transactive SC and PC formulations were proposed allowing for consideration of net exports, and both P2P and P2U transactions. These formulations allow for privacy and peer autonomy with consideration of transactive pricing signals for locally optimal transaction decisions.
- A split transaction fee mechanism was proposed consisting of a fixed electrical distance based component accounting for network topology, and a dynamic sensitivity

coefficient based component accounting for physical network constraints. This mechanism was shown to effectively couple the virtual and physical transactive market layers to obtain feasible solutions.

- A congestion clearing process was proposed which coordinates with a decentralised transaction negotiation and matching process to maximise high priority feeder usage. The process considers peer decisions and electrical distance of transactions to ensure a stable and efficient solution.
- The proposed transactive PC approach achieves a stable, feasible, and locally optimal set of transactions with comparable economic and network performance to a SC approach while maintaining transactive principles.

The transactive formulations and results reported in Chapter 3 have been submitted to the IEEE Transactions on Smart Grid [41].

4.3 Future Work

As extensions to the research presented in this thesis, the following areas may be explored for future work:

- The proposed transactive formulations allow for the consideration of the value of surplus energy with the function Υ_n ; however, the value of surplus is assumed as zero for the networks and simulations presented. Considering price responsiveness of buyers in the transactive process could allow for further exploration of local optimality.
- Extend the solution horizon to multiple time periods to include some stochastic considerations of demand, generation and prices in selecting optimal transactions. Furthermore, a day-ahead market may be necessary in addition to a real-time market to allow for forward contracts mitigating risk.
- With the extended solution horizon, studying the potential of resource types with intertemporal considerations in a transactive market. For example, resources such as battery storage, electric vehicles, or electric water heaters could provide increased price responsiveness and revenue opportunities for peers.
- Introduce additional market products and pricing mechanisms to unlock value stacking for a wider range of DER capabilities. Potential products could include, for example, reserves, demand response, or reactive power compensation.

• Investigate the potential for collaborative PC aggregate bids into wholesale electricity markets, rather than treating the transactive market as a price taker.
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APPENDICES

Appendix A

15-Bus Distribution Network Parameters

Table A.1: 15-bus network seller parameters [39, 15].

b/n	$[\underline{P}_n, \overline{P}_n]$	$[Q_b^g, \overline{Q}_b^g]$	Cost Coefficients	
	[MW]	[MVAr]	$[\mathrm{MW^2h}]$	[%/MWh]
1	[0, 2.5]	[-2.0, 2.0]	0	50
12	[0, 3.5]	[-0.4, 0.4]	0	10

le	A.2:	15-bus network	bus data
	b/n	$[\underline{D}_n, D_b^Q]$	B_b
		[MW, MVAr]	[p.u.]
	1	[0.0, 0.0]	0.0
	2	[0.794, 0.1855]	0.0011
	3	[0.0, 0.0]	0.0028
	4	[0.021, 0.0084]	0.0024
	5	[0.018, 0.0043]	0.0004
	6	[0.030, 0.0073]	0.0008
	7	[0.022, 0.0055]	0.0006
	8	[0.022, 0.0019]	0.0006
	9	[0.024, 0.0059]	0.0012
_	10	[0.023, 0.0142]	0.0004
	11	[0.022, 0.0065]	0.0004
_	12	[0.0, 0.0]	0.0001
_	13	[0.622, 0.1291]	0.0001
	14	[0.002, 0.0008]	0.0002
_	15	[0.023, 0.0083]	0.0001

Table A.2: 15-bus network bus data [39].

Lä	able	A.5:	19-0	ous netwo	ork leeder	uata [59
	l	o_l	r_l	R_l	X_l	S_l
				[p.u.]	[p.u.]	[MVA]
	1	1	2	0.001	0.12	2
	2	2	3	0.0883	0.1262	0.256
	3	3	4	0.1384	0.1978	0.256
	4	4	5	0.0191	0.0273	0.256
	5	5	6	0.0175	0.0251	0.256
	6	6	7	0.0482	0.0689	0.256
	7	4	8	0.0407	0.0582	0.256
	8	8	9	0.0523	0.0747	0.256
	9	8	10	0.01	0.0143	0.256
	10	10	11	0.0241	0.0345	0.256
	11	11	12	0.0103	0.0148	0.256
	12	1	13	0.001	0.12	1
	13	13	14	0.1559	0.1119	0.204
	14	14	15	0.0953	0.0684	0.204

Table A.3: 15-bus network feeder data [39].

Appendix B

33-Bus Distribution Network Parameters

Table D.I. 55 Sub benef parameters [26].					
Bus b and	Capacity		Cost Coefficients		
peer n	$[\underline{P}_n, \overline{P}_n]$	$[\underline{Q}_{b}^{g}, \overline{Q}_{b}^{g}]$			
	[MW]	[MVAr]	$[\text{MW}^2h]$	[%/MWh]	
2	[0, 2.5]	[-2.0, 2.0]	0.05	7.2	
6	[0, 3.5]	[-3.0, 3.0]	0.02	7.6	
18	[0, 1.6]	[-1.0, 1.0]	0.03	7.1	
22	[0, 2.3]	[-2.0, 2.0]	0.08	7.4	
25	[0, 2.9]	[-2.5, 2.5]	0.03	6.9	
31	[0, 2.5]	[-2.0, 2.0]	0.04	7.4	

Table B.1: 33-bus seller parameters [25].

b/n	$[\underline{D}_n, D_b^Q]$	B_b
	[MW, MVAr]	[p.u.]
1	[0.0, 0.0]	0.0
2	[0.0, 0.0]	0.0
3	[0.180, 0.080]	0.0
4	[0.240, 0.160]	0.0
5	[0.120, 0.060]	0.0
6	[0.0, 0.0]	0.0
7	[0.400, 0.200]	0.0
8	[0.400, 0.200]	0.0
9	[0.120, 0.040]	0.0
10	[0.120, 0.040]	0.0
11	[0.090, 0.060]	0.0
12	[0.120, 0.070]	0.0
13	[0.120, 0.070]	0.0
14	[0.240, 0.160]	0.0
15	[0.120, 0.020]	0.0
16	[0.120, 0.040]	0.0
17	[0.120, 0.040]	0.0
18	[0.0, 0.0]	0.0
19	[0.180, 0.080]	0.0
20	[0.180, 0.080]	0.0
21	[0.180, 0.080]	0.0
22	[0.0, 0.0]	0.0
23	[0.180, 0.100]	0.0
24	[0.840, 0.400]	0.0
25	[0.0, 0.0]	0.0
26	[0.120, 0.050]	0.0
27	[0.120, 0.050]	0.0
28	[0.120, 0.040]	0.0
29	[0.240, 0.140]	0.0
30	[0.400, 1.200]	0.0
31	[0.0, 0.0]	0.0
32	[0.420, 0.200]	0.0
33	[0.120, 0.080]	0.0

Table B.2: 33-bus network bus data $[40,\,25].$

					L
l	o_l	r_l	R_l	X_l	S_l
			[ohm]	[ohm]	[MVA]
1	1	2	0.0922	0.0470	6.0
2	2	3	0.4930	0.2511	6.0
3	3	4	0.3660	0.1864	6.0
4	4	5	0.3811	0.1941	6.0
5	5	6	0.8190	0.7070	6.0
6	6	7	0.1872	0.6188	1.5
7	7	8	0.7114	0.2351	1.5
8	8	9	1.0300	0.7400	1.5
9	9	10	1.0440	0.7400	1.5
10	10	11	0.1966	0.0650	1.5
11	11	12	0.3744	0.1238	1.5
12	12	13	1.4680	1.1550	1.0
13	13	14	0.5416	0.7129	1.0
14	14	15	0.5910	0.5260	1.0
15	15	16	0.7463	0.5450	1.0
16	16	17	1.2890	1.7210	1.0
17	17	18	0.7320	0.5740	1.0
18	2	19	0.1640	0.1565	1.0
19	19	20	1.5042	1.3554	1.0
20	20	21	0.4095	0.4784	1.0
21	21	22	0.7089	0.9373	1.0
22	3	23	0.4512	0.3083	2.0
23	23	24	0.8980	0.7091	2.0
24	24	25	0.8960	0.7011	2.0
25	6	26	0.2030	0.1034	1.0
26	26	27	0.2842	0.1447	1.0
27	27	28	1.0590	0.9337	1.0
28	28	29	0.8042	0.7006	1.0
29	29	30	0.5075	0.2585	1.0
30	30	31	0.9744	0.9630	1.0
31	31	32	0.3105	0.3619	1.0
32	32	33	0.3410	0.5302	1.0

Table B.3: 15-bus network feeder data [40].