Unmanned Aerial Vehicle Traffic Network Design with Risk Mitigation

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

As unmanned aerial vehicle (UAV) technology becomes more robust and widespread, more and more retail companies are seeing UAVs as a suitable alternative to ground-based transportation to deliver their packages. As a result, there has been an abundance of OR research focused on UAV utilization for last-mile delivery. Due to the size and mobility of UAVs, most of this research considers UAV movement within a shortest path or Euclidean shortest path context. While this may be plausible if drone usage remains sparse, this framework will not be possible as drone utilization ramps up to the levels required to satisfy the levels of package demand expected in the coming decades. Furthermore, none of this prior research (to our knowledge) suggests using risk inherent with UAV travel to influence their proposals from a logistical and/or modelling perspective. As a solution to this problem, our industry partner AirMatrix proposes that UAV travel be restricted to transportation networks situated above the streets of population centres. We propose a bi-objective network selection model for drone delivery which minimizes risk while maximizing the amount of satisfied demand subject to budgetary constraints. We discuss the factors that affect UAV risk and what metrics can be used to effectively reduce those factors from a modelling perspective. We propose a two-stage stochastic variant of the model and additional problem requirements to reflect practical operational requirements and design goals. Using sample average approximation, we show that a deterministic solution is effectively as good as an associated stochastic solution. We conduct testing on a region of suburban Miami to evaluate how different risk objectives perform with respect to network, path, arc, and performance metrics.

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Chapter 1

Introduction

With the rise in popularity and research into commercial drone technology over the course of the last decade, different companies have been considering the use of commercial drones to suit their supply chain needs [16] [39]. Drones or unmanned aerial vehicles (UAVs) offer a number of benefits in package transportation that most ground-based vehicles do not share. Drones are unmanned, can fly autonomously, and have a smaller impact on the environment when compared to the average ground-based delivery vehicle [13]. As a result of these benefits, UAVs are seen as a viable addition and potential alternative to ground-based delivery vehicles in last-mile delivery.

While the inclusion of UAVs may have a significant impact on modern day package delivery, a number of obstacles still stand in the way of widespread usage. For one, given that the level of demand in door-to-door package delivery is expected to increase dramatically over the coming decades, appropriate UAV legislation is necessary to facilitate the seamless fulfillment of large demand volumes [27]. Furthermore, there needs to be some entity (or entities) responsible for regulating and overseeing UAV traffic. The literature has already coined the term UAV Traffic Management (or UTM) to define any and all systems that manage UAV traffic. While initially it may be intuitive to apply already existing Air Traffic Management (ATM) rules and regulations to drones, there are many key differences that make managing UAV traffic more difficult than managing the aerial vehicles that the ATM currently deals with (airplanes, helicopters, etc.) [10]. These differences include operating in a space that has greater proximity to buildings and people on the ground [1], a considerable increase in the number of arrival and departure points [20], a higher traffic density [35], and vehicles acting autonomously or being controlled by an operator not physically present in the UAV. These differences, among many others, lead to a lot of debate regarding what rules should be set to regulate UAV traffic.

A key question that arises in the deployment of UAVs for package delivery is how to properly structure the airspace to facilitate UAV traffic [5]. Some have suggested little to no restrictions in how the airspace is structured. In this vein, a UAV operator can fly a drone to a sufficient minimum altitude and then utilize whatever path they would like to reach their destination. Avoiding any ground infrastructure (buildings, telephone wires, etc.) as well as other aviation (planes, helicopters, other drones, etc.) would become the responsibility of the operators and their drones. Given that detect-and-avoid technology is still in its infancy [5] and that most origin-destination flight paths will contain sections where the operator cannot see the drone (known as beyond visual line of sight or BVLOS), the likelihood and frequency of collisions in this type of airspace would be high (at least for the foreseeable future). Furthermore, air traffic is already heavily regulated and while the current regulations for drone utilization may be relaxed to some extent, it is unlikely that drones will face less regulation than current widely used aviation.

A second suggestion would be to allow for UAV operators to choose their preferred flight path and send their proposed path to an appropriate UTM manager for review. The manager must ensure the path follows the rules and regulations of the airspace and then they can either approve, deny, or alter the path based on those regulations. While this is a step in the right direction with regards to improving safety, the responsibility of finding a path with a sufficiently low amount of risk is still put on the operators. Many of these operators will not have the access to 3D maps nor the risk assessment ability necessary to determine an optimal path in this regard. Also, as the number of flights increases, this approval process will be difficult to scale up from the perspective of the UTM manager(s).

As a solution to this problem, our industry partner AirMatrix proposes the existence of a transportation network that UAVs will be regulated to travel on. In this scenario, an operator provides their origin, their destination and any other relevant information (type of drone, speed, weight of package, etc.) to their associated UTM manager and an optimal round trip (either in terms of minimal risk and/or minimal battery consumption) is found on this network [32]. This gives the UTM manager(s) more control of the traffic in the airspace that they oversee. The UAV operators are no longer responsible for determining proper paths for their drones and the approval process will become easier and quicker.

AirMatrix further proposes that this network for a given population centre be confined to the street network. That is to say, the horizontal layout of this network is a subnetwork of the already existing street network for a given population centre. The benefits associated with drones utilizing this network include a reduced risk of contacting ground infrastructure, a reduced risk associated with flying directly over people, a reduced probability of flying over private property [11], and an easier palatability of drone transportation from a public standpoint. While capacity becomes a large issue as we scale up the number of UAVs on this network, this issue can be mitigated by incorporating multiple layers of the same 2D network on top of one another. In this vein, a drone would initially depart from its origin by flying vertically to a given layer. It would utilize the horizontal network associated with this layer on its way to its destination, potentially changing layers if necessary. Once it has arrived at its destination, it would descend vertically to drop off the package or release the package via a rope down to the ground. Once the package has successfully been delivered, the drone would return to its origin, potentially utilizing the same path back. While the details as to how this routing will take place are important, these details will not be fleshed out in this thesis.

Under the assumption that we are confining all UAV traffic to a given city's street network, the main question from a network configuration standpoint becomes determining which roads and intersections should be included in this UAV subnetwork. While having access to the complete road network would clearly be preferable to any proper subnetwork, this is likely to be impractical, unnecessary, and costly. Many roads are unable to be utilized as they exist on private property or in controlled airspace. A lot of roads would see little to no utilization either as a result of demand or more optimal roads being available. There will also likely be some preliminary costs and ongoing costs associated with including any given road on this UAV network. In particular, the costs associated with obtaining and updating 3-dimensional maps of the streets chosen to be in the UAV network is the primary concern of AirMatrix. If a road is deemed unavailable for use or is determined to go unused, it can easily be removed from consideration. What makes this problem nontrivial is the existence of a limit on how much can be spent in the construction of a UAV network. We refer to this limit as the budget. Given a specific budget, we can optimize for which road segments and intersections will be included in the UAV network relative to factors such as risk incurred, distance travelled, and the amount of satisfied demand.

In this thesis, we propose a variant of the network design problem that finds an optimal subnetwork of a given street network for the purpose of UAV traffic utilization. This model considers several factors such as arc risk, arc length, expected demand and budgetary constraints. We consider a range of risk measures, objectives, and associated constraints to account for different preferences in setting the service network. A comparative study between various risk measures is also included.

The rest of this thesis is organized as follows. Chapter 2 provides some of the relevant OR and non-OR literature on UAV research and relevant problems to the one discussed in this thesis. Chapter 3 presents the problem definition and model. Chapter 4 discusses the key factors that contribute to UAV risk and gives measures for how we can quantify the risk inherent in a given network. Chapter 5 presents the stochastic variant of the problem and how it can be simplified to a deterministic one in certain settings. Chapter 6 gives a range of additional requirements that a UTM manager may be inclined to add in the design of their UAV network along with the necessary alterations to the model to ensure those requirements are met. Chapter 7 gives testing results from both a deterministic and stochastic framework on a sample network based on suburban Miami. Chapter 8 offers a summary of what has been done in this thesis and some suggestions for future research directions.

Chapter 2

Literature Review

In Chapter 1, we proposed the necessity for the existence of an entity (or entities) responsible for overseeing and regulating UAV traffic. The term UAV Traffic Management (or UTM) was used to define one of these entities. Papers [20] and [10] make thorough cases for the need of a dedicated UTM. For the purposes of this thesis, a UTM would be responsible for constructing and maintaining the UAV network infrastructure, approving flight requests, ensuring flight requests respect the rules and regulations set by the appropriate government body, and supplying optimal routes aided by routing algorithms to pilots. There are a number of important questions regarding what other responsibilities UTMs should have and how they would work in practice. Most of these questions lie outside the scope of this thesis. We encourage those interested in learning about UTMs in more detail to read the surveys presented in [20] and [33]; both of which describe key UTM concepts, terminology, and information.

Various initiatives and proposals for UAV traffic management and airspace design are surveyed in [24] and [5]. The authors of [5] define the physical structure of urban airspace to refer to the position and size of airspace elements such as flying trajectories, tubes, corridors, and layers, as well as their associated rules of operation. In this context, our industry partner AirMatrix is proposing structuring the urban airspace as a horizontal 2-dimensional network in which each arc must be above an already existing road within the urban area. Furthermore, a drone flying in this urban area must fly at some approved altitude above the roads in this constructed network. The rules set for which altitudes specific drones would fly at and the decision-making process for why drones would potentially change altitudes mid-flight are more an issue of legislation and routing. Therefore, these discussions are omitted from the remainder of this thesis.

The qualifications for what technological requirements drones must have to fly in urban airspace is also an important question that will not be fleshed out in this thesis. Some of these key qualifications are discussed in [38]. We do however assume that all drones utilizing this constructed network have sufficient vertical take-off/landing (VTOL) and hovering capabilities. This is so the drones can change altitudes easily, deliver packages easily, stop in the event of a potential collision, and take-off/land without the need of substantial ground infrastructure. At the interest of the reader, important terminology, UAV classifications, and design requirements are detailed in [16].

The flying above roads assumption appears in the literature about airspace design in a few notable places. One of the concepts given by NASA [19] proposes dividing urban airspace into multiple layers with each layer consisting of a particular airspace structure situated above the streets of said urban area. The study tested three different airspace structures, each with a different level of UAV freedom: sky-lanes, sky-tubes, and sky-corridors. Sky-lanes hold the closest resemblance to the structure being proposed in this thesis. They found that while sky-lanes offer the safest and least complex environment, they also led to higher delays and less capacity. These are problems that we hope will be mitigated with the utilization of a sufficient number of layers and more efficient routing models.

The authors of [11] also focus on flying above roads, observing it provides separation, reduces privacy concerns, and enables ground vehicle coordination. However, they point out that conflicts may be triggered by limited flexibility and large volumes of disparate drones. To evaluate the airspace design, they develop one-way and two-way airspace configurations with altitude levels segmented by turning/through traffic and heading direction. They simulate air traffic over all the streets of Manhattan and measure intrusions (separation violations) and conflicts (predicted separation violations). The goals in [11] are more Operations Research oriented and implementation focused. Costs, regulatory restrictions for flight approvals, UTM involvement, and 3-dimensional maps illustrating the necessary geospatial features of urban areas must also be incorporated to ensure the networks chosen are viable today and in the future.

The authors of [36] developed a network selection model for the purposes of UAV traffic subject to budgetary constraints. Since this work also originated through working with AirMatrix, their problem has a similar foundation to the one proposed in this thesis. The goal of their model however is to reduce the congestion caused by UAV traffic and does not formally consider UAV associated risk.

Operations Research literature focused on drone-specific applications is abundant. Surveys are provided in [31], [28], [25] and [29]. Many routing-based applications have been studied including routing delivery drones, coordinating drone and truck deliveries, routing drones for inspecting physical infrastructure, and routing drones for surveillance purposes. Many routing-based applications assume drones fly directly point-to-point (via shortest path or Euclidean shortest path), and few consider the need for flight-approval or flight delay caused by waiting for other traffic. For more information, see the surveys in [7] and [21]. Other problems specialized to drones include the location of facilities such as delivery depots or drone charging infrastructure (see [8], [30] and [17]), and assigning tasks to drones based on capabilities. Although these works are not directly related to the problem at hand, they provide insights into how conventional models can be modified to accommodate the unique capabilities and requirements for drones.

In this thesis, the problem at hand is that of network design. Considering Operations Research literature dedicated to the design of networks for drone traffic, there are a few papers of note. The authors of [4] investigate strategic network design for drone parcel delivery from the point of view of an e-retailer accounting for technological limitations, government regulation, and customer behaviour. The authors of [18] design a drone traffic network accounting for lane capacity and charging needs and use distributionally robust modeling to deal with demand uncertainty. However, the optimization objectives are coverage focused and the model does not account for flight risk directly. Although research specific to drone networks is limited, designing drone traffic networks shares many similarities with the well-studied problems of designing traffic networks for other transportation modes such as cars, planes, and trains. For an early review of network design in the context of transportation planning, see [26]. A more recent survey is provided in [12] while [37] focuses on uncertainty. [9] is a comprehensive recent book on the topic.

An existing problem in operations research literature that most closely resembles the problem at hand is the network design with service requirements (NDSR) problem originally formulated in [2]. A generalization of the fixed-charge multicommodity network design problem, the NDSR problem seeks to minimize the costs associated with selecting arcs and routing. Furthermore, the arcs are selected in such a way that every commodity k utilizes an origin-destination path that satisfies all service metrics relative to the parameters set for commodity k. Also referred to as the network design with routing requirements

(NDRR) problem, further work on this problem is detailed in [3]. The problem discussed in this thesis serves as a variant of the NDSR problem with key differences that are deemed important when constructing a UAV network. Firstly, if a given arc is selected, the opposite arc must also be selected, and a fixed cost is associated with selecting both arcs. We are assuming that if a given street segment is chosen to be part of the subnetwork, UAV traffic can flow on said street segment in both directions and there is no extra costs associated with making said street segment bidirectional as opposed to just one-way. Secondly, the NDSR problem assumes that all demand must be satisfied whereas the model in this thesis does not make the same assumption. We instead opt to maximize the amount of satisfied demand as an objective. Lastly, the notion of arc routing costs will be omitted as it is assumed that the UAV operators will be taking on these costs and not the UTM. In addition to these key adjustments, extra requirements and their associated constraints are suggested and described in Chapter 6.

Chapter 3

Problem Definition and Modelling

In this Chapter, we propose a bi-objective model for the purpose of finding a subnetwork of a street network for UAV traffic subject to budgetary constraints. This model aims to minimize some desired risk metric (the different metrics are given in Section 4.2) while maximizing the amount of satisfied demand. The problem is formally defined, associated notation is given, and all the relevant assumptions are listed.

3.1 Problem Definition

Let $G = (N, A)$ be a symmetric directed graph. The graph G is assumed to represent an already existing road network of a city or section of a city that is being considered for drone delivery use. The set of nodes N consists of intersections and drone departure/arrival points. The set of arcs A consists of the streets and street segments that connect these nodes. We require that G be symmetric since a selected street segment can be used by a drone in both directions; even when it is one-way for ground-based traffic.

We define K to be the set of set of commodities on the network G. Each commodity $k \in K$ has an origin O_k, destination D_k and a non-negative demand d_k which is a quantitative indication of the traffic between O_k and D_k. For the UAV setting, d_k denotes the expected number of drones moving between O_k and D_k over the course of a given period of time. Without loss of generality, we assume that for all $k \in K$, the index associated with node O_k is less than the index associated with node D_k. Since all roads on the subnetwork are bidirectional, a directed feasible path from O_k to D_k exists if and only if a directed feasible path from D_k to O_k exists. For the remainder of this thesis, the terms pairs and node pairs will be used interchangeably with the term commodities.

An arc (i, j) has an associated length l_{ij} which measures the distance a drone would have to travel to traverse arc (i, j) and this value is symmetric ($l_{ij} = l_{ji}$). There is a cost c_{ij} associated with undirected arc (i, j) which is incurred if either directed arc (i, j) or (j, i) is in the UAV subnetwork. Parameter ρ_{ijk} is a quantitative indication of the risk a drone associated with pair k incurs when present on arc (i, j) and this value is symmetric ($\rho_{ijk} = \rho_{jik}$). Note that parameter ρ_{ijk} is intended to be used as a measure of how risky traversing arc (i, j) is per unit length. Therefore, the risk a drone associated with pair k incurs when traversing the entirety of arc (i, j) is $\rho_{ijk} \times l_{ij}$. An arc with different risk values at different points along its length should be segmented in such a way that each new arc has approximately the same risk value at each point along its length.

The goal is to find a symmetric directed network $G^* = (N^*, A^*)$ (where $N^* \subseteq N$ and $A^* \subseteq A$) such that the total cost of the network does not exceed budget B. To balance the trade-offs between demand fulfillment and risk, we develop a bi-objective model that minimizes risk measure $\mathcal{R}(\boldsymbol{x})$ while maximizing the amount of satisfied demand. We say that the demand for a given pair (O_k, D_k) can be considered satisfied only if there exists a feasible path from O_k to D_k on the chosen subnetwork G^*. An $O_k - D_k$ path is considered feasible if the distance a drone travels when going from O_k to D_k is at most $\delta_k \times SP_k$ where SP_k is the length of a shortest path from O_k to D_k and $\delta_k \geq 1$ is a scalar that signifies the maximum allowable deviation in length from a shortest path for pair k.

Any pair with satisfied demand will be referred to as a satisfied pair. Given UAV usage is still in its infancy, the presence of any sort of arc or node capacities is difficult to justify. For this reason, there will be no capacity constraints in the model, though suitable model alterations are discussed in Section 6.5. Without capacities on the arcs or nodes, no commodity associated with a satisfied pair has any incentive to utilize multiple origin-destination paths nor do they have any incentive to ship a partial amount of their demand. Therefore, we require that all demand for a given pair must be routed along the same path, i.e., there is no splitting of flows among multiple paths. We also require that no pair will have their demand partially satisfied. Either a pair will have all their demand satisfied or none of their demand satisfied.

3.2 Model

In this section, we describe a mathematical model that solves the problem described in the previous section. The sets, parameters, and decision variables in Table 3.1 will be used in the model.

Table 3.1: Notation Summary

Sets and Parameters			
G	A symmetric directed network	ρ_{ijk}	The risk all drones associated with pair k incur when present on arc (i, j)
N	The set of nodes in G	l_{ij}	The length of arc (i, j)
A	The set of directed arcs in G	c_{ij}	The cost associated with including the undirected arc (i, j) in the UAV subnetwork
K	The set of commodities/node pairs $\{(O_k, D_k) : O_k < D_k\}$	B	The budget; the maximum allowable total cost of undirected arcs in the UAV subnetwork
O_k	The origin associated with node pair k	SP_k	The sum of the lengths of the arcs in a shortest $O_k - D_k$ path
D_k	The destination associated with node pair k	δ_k	The maximum percentage of SP_k in distance that a drone associated with pair k can travel
d_k	The demand of pair k		
Decision Variables			
x_{ijk}	A binary variable which indicates if drones associated with pair k traverse arc (i, j)	z_{ij}	A binary variable which indicates if undirected arc (i, j) is included in the UAV subnetwork

Based on this notation, the bi-objective model is defined as follows.

$$\text{minimize} \quad \mathcal{R}(\boldsymbol{x}) \tag{3.1}$$

$$\text{maximize} \quad \sum_{k \in K} \sum_{\{j:\ (O_k, j) \in A\}} d_k x_{O_k jk} \tag{3.2}$$

subject to

$$\sum_{\{j:\ (i,j) \in A,\ j \neq O_k\}} x_{ijk} = \sum_{\{j:\ (j,i) \in A,\ j \neq D_k\}} x_{jik} \qquad \forall k \in K,\ \forall i \in N \setminus \{O_k, D_k\} \tag{3.3}$$

$$\sum_{\{j:\ (O_k,j) \in A\}} x_{O_k jk} \leq 1 \qquad \forall k \in K \tag{3.4}$$

$$x_{ijk} \leq z_{ij} \qquad \forall k \in K,\ \forall (i,j) \in A:\ i < j \tag{3.5}$$

$$x_{jik} \leq z_{ij} \qquad \forall k \in K,\ \forall (i,j) \in A:\ i < j \tag{3.6}$$

$$\sum_{(i,j) \in A:\ i<j} c_{ij} z_{ij} \leq B \tag{3.7}$$

$$\sum_{(i,j) \in A} l_{ij} x_{ijk} \leq \delta_k SP_k \qquad \forall k \in K \tag{3.8}$$

$$x_{ijk} \in \{0,1\} \qquad \forall k \in K,\ \forall (i,j) \in A \tag{3.9}$$

$$z_{ij} \in \{0,1\} \qquad \forall (i,j) \in A:\ i < j \tag{3.10}$$

The first objective (3.1) minimizes the selected risk metric. The second objective (3.2) maximizes the amount of satisfied demand. Constraints (3.3) ensure that flow conservation is enforced at each node for each pair and that if a drone leaves its origin node, it must reach its destination node. Constraints (3.4) ensure that a drone leaving its origin can only utilize a single arc incident to its origin. Constraints (3.4) are necessary to ensure that the demand associated with a satisfied node pair is only contributing to the amount of satisfied demand in objective (3.2) once. Otherwise, if multiple feasible arc-disjoint $O_k - D_k$ paths are present on the subnetwork, the demand associated with pair k could be counted more than once. Constraints (3.5) and (3.6) ensure that if an undirected arc (i, j) is traversed by a drone (either from node i to node j or vice versa), then arcs (i, j) and (j, i) must be included in A^*. Constraint (3.7) ensures that the sum of the costs of the arcs in the chosen subnetwork does not exceed the chosen budget. Constraints (3.8) ensure that for every pair $k \in K$, the distance travelled by a drone associated with pair k when going from O_k to D_k is at most $\delta_k \times SP_k$. Constraints in (3.9) and (3.10) ensure that x_{ijk} and z_{ij} are binary. Note that as a result of constraints (3.5) and (3.6), each z_{ij} is bounded below by 0 and must be at least 1 if undirected arc (i, j) is included in the subnetwork. It follows that the constraints (3.10) may be dropped if preferred.

Given how difficult the problem can be to solve as the number of nodes, arcs and commodities increase, any steps taken to reduce the number of decision variables and constraints prior to model execution is key. Any arcs deemed ineligible, too risky and/or unlikely to be used should be removed during the network selection stage. Furthermore, since drones can only travel so far on a single charge, any node pair whose associated nodes are too far away from each other should be omitted from the model. For example, suppose L_k is the maximum distance a drone associated with pair k can travel on a single charge. If $SP_k > L_k/2$, then pair k should be omitted from the model since drones associated with pair k will not be able to make an $O_k - D_k$ round trip without running out of battery. Furthermore, δ_k should be chosen in such a way that $\delta_k \leq L_k/(2 \times SP_k)$ so that the distance of a round trip selected for a given pair k is not so long that a drone associated with pair k runs out of battery mid-flight.

Chapter 4

UAV Traffic Risk Assessment and Metrics

The key metric that we will be using when determining a UAV subnetwork and the paths that UAVs will utilize is risk. For the purposes of the problem defined in Section 3.1, risk is treated as a quantitative indication of both the likelihood that a given drone will contact another object (whether that be ground infrastructure, people on the ground, other aircrafts, etc.) and the measure of damage that will be created if a collision were to occur (damage to the drone, damage to public infrastructure, bodily harm, etc.). To work with risk from a modelling perspective, one must ask what risk factors have any effect on UAV travel and how we can effectively quantify them to use in the model.

4.1 Risk Assessment

To get a better idea of what factors are most important when considering UAV risk, we can refer to the legislative body in charge of setting the rules and regulations for UAV travel. In Canada, that body is known as the Remotely Piloted Aircraft Systems Task Force of Canada. This body defines a process for obtaining a Special Flight Operations Certificate (SFOC) which is required for the vast majority of UAV operations performed on Canadian soil. While we will not be going into the process of application and approval for a SFOC in this thesis (those interested can refer to [15]), we will briefly describe how risk is measured in the Joint Authorities for Rulemaking of Unmanned Systems (JARUS) Specific Operational Risk Assessment (SORA) process as it is integral to obtaining approval for a SFOC and because it will play a key role in UAV network configuration from a UTM manager's perspective (please refer to [14] for more information).

The Operational Risk Assessment (ORA) of any operation is signified by a number indicating the overall risk as a score from 1 to 6 (as a Roman numeral). This score is referred to as the Specific Assurance and Integrity Level (SAIL). The SAIL of a given operation is calculated as a function of the operation's Ground Risk Class (GRC) and Air Risk Class (ARC). We note that the SAIL of a multi-section operation is obtained by finding the maximum SAIL amongst all its sections. For urban travel constrained to the street network, the SAIL of a given origin-destination path is the maximum SAIL amongst all of its arcs. The ground risk class is primarily a function of the size of the UAV and the population density of the area situated below the UAV. The air risk class is primarily a function of the class of the airspace, its altitude, and its proximity to controlled airspace. While the SAIL for an arc is a simplification of the total risk that a drone traversing it incurs, it is a necessary metric for obtaining a SFOC for an operation that contains said arc in its flight path.

A more detailed value of the risk that a given UAV incurs partially reflects what is in the SORA process but includes other key factors as well. Some of these factors include proximity to ground infrastructure, signal strength, electromagnetic radiation from nearby buildings, and the weather. Some of these factors are unique to the UAV executing the operation. For example, its ability to withstand harsh weather, its ability to withstand electromagnetic radiation, its ability to detect and avoid collisions, its size, its speed, and the size/weight/contents of its delivered package all affect risk to some degree. This is why we have defined the risk parameter ρ to be not just a function of the arc's parameters, but of the drone's parameters as well. We also steer away from the notion of having the risk of a path simply be the level of risk associated with the riskiest segment of the path. We instead treat risk as a value that a drone incurs as it travels from its origin to its destination.

4.2 Risk Metrics

Since there are many suitable ways to measure the risk that exists in a given UAV network, we list six risk metrics below that may be of particular interest to a UTM manager.

4.2.1 Total Risk

Total risk is the total amount of risk incurred by all drones travelling from their respective origins to their respective destinations. It is the most trivial risk metric but also the easiest to utilize and interpret. A UTM manager can minimize the total risk incurred by minimizing summation (4.1) given by

$$\mathcal{R}(\boldsymbol{x}) = \sum_{k \in K} \sum_{(i,j) \in A} d_k \rho_{ijk} l_{ij} x_{ijk}. \tag{4.1}$$

4.2.2 Total Risk Deviation

While total risk may be a valuable metric because of its simplicity, total risk does not necessarily give a complete picture as to how risky a given network is. For example, suppose we have a chosen subnetwork $G^* \subset G$ and two node pairs k_1 and k_2 where each pair has a single unit of demand ($d_{k_1} = d_{k_2} = 1$) and we would like to satisfy the demand for one of these pairs. The smallest risk path for pair k_1 on G has three units of risk and this path exists on G^*. The smallest risk path for pair k_2 on G has a single unit of risk but this path does not exist on G^* whereas a feasible O_{k_2}-D_{k_2} path with two units of risk does. If the objective is to minimize total risk, we will prioritize satisfying the demand of pair k_2, which contributes two units of risk, over satisfying the demand of pair k_1, which contributes three units of risk. Therefore, we are opting to satisfy the demand of a pair that must utilize a path that is twice as risky as what is possible over the demand of a different pair which can utilize its smallest risk path.

When we opt to minimize total risk, the model does not consider the minimum amount of risk necessary to satisfy the selected pairs and how much the total risk deviates from that value. To obtain a more complete picture, it may be more valuable to use a metric that does not just prioritize satisfying the pairs that contribute the least amount of risk, but also prioritizes satisfying pairs that can utilize their smallest risk paths or can utilize paths that have close to the same amount of risk as their smallest risk paths. Therefore, we introduce a risk metric that we are calling *the deviation from minimum total risk relative* *to the set of satisfied pairs* or *the total risk deviation* for short. We define the total risk deviation to be the ratio of total risk incurred amongst all satisfied pairs to the total risk incurred if said satisfied pairs were to utilize their smallest risk paths. In this vein, if the total risk deviation is 1 or 100%, that means that every satisfied pair on the network is utilizing their associated smallest risk path. If the total risk deviation is 1.2 or 120%, that means that there is a 20% increase in total risk on the network in comparison with a network where all satisfied pairs utilize their respective smallest risk paths.

We define SR_k as the total risk incurred by a drone associated with pair k when traversing a smallest risk $O_k - D_k$ path. Total risk deviation is calculated by finding the total risk and dividing that value by the sum of all $d_k \times SR_k$ values over all satisfied pairs $k \in K$. A UTM manager can minimize the total risk deviation, denoted by σ, by minimizing (4.2), and adding constraint (4.3) given by

$$\mathcal{R}(\boldsymbol{x}) = \sigma, \tag{4.2}$$

$$\sum_{k\in K} \sum_{(i,j)\in A} d_k \rho_{ijk} l_{ij} x_{ijk} \leq \sigma \left(\sum_{k\in K} d_k SR_k \left(\sum_{\{j:\ (O_k,j)\in A\}} x_{O_k jk} \right) \right). \tag{4.3}$$

4.2.3 Maximum Arc and Segment Risk

While the previous metrics aim to reduce risk from a path-based outlook, it may be within a UTM manager's interest to reduce risk by focusing on the arcs of the network instead. For example, a UTM manager may want to minimize the maximum amount of risk flowing on any undirected arc in the network. A UTM manager can minimize the maximum arc risk, denoted by γ, by minimizing (4.4) and adding constraints (4.5) given by

$$\mathcal{R}(\boldsymbol{x}) = \gamma, \tag{4.4}$$

$$\sum_{k \in K} d_k \rho_{ijk} l_{ij} (x_{ijk} + x_{jik}) \leq \gamma, \qquad \forall (i, j) \in A : \; i < j. \tag{4.5}$$

Recall that the risk a drone incurs when traversing an arc is a function of its length. That is to say that the longer an arc is, the more risk that a drone incurs when traversing it. With the above approach, we could end up with demand flowing across shorter riskier arcs being prioritized over demand flowing across longer less risky arcs. With this in mind, it may be interesting to minimize the maximum amount of risk flowing on any arc segment in the network instead. A UTM manager can minimize the maximum segment risk, denoted by Γ, by minimizing (4.6) and adding constraints (4.7) given by

$$\mathcal{R}(\boldsymbol{x}) = \Gamma, \tag{4.6}$$

$$\sum_{k \in K} d_k \rho_{ijk} (x_{ijk} + x_{jik}) \leq \Gamma, \qquad \forall (i, j) \in A : \; i < j. \tag{4.7}$$

Note that this approach ultimately amounts to minimizing the maximum amount of risk per unit length flowing across any undirected arc in the network and so we only need to remove the l_{ij} parameters from constraints (4.5) to obtain constraints (4.7).

4.2.4 Arc and Segment Risk Variance

It may be within a UTM manager's interest to not simply minimize the maximum amount of risk flowing on each undirected arc but minimize how much the risk flowing on each undirected arc varies from the average. A UTM manager can minimize the sum of the arc risk variance values by minimizing summation (4.8) given by

$$\mathcal{R}(\boldsymbol{x}) = \sum_{\{(i,j)\in A:\ i<j\}} \left(\sum_{k\in K} d_k \rho_{ijk} l_{ij} (x_{ijk} + x_{jik}) - \frac{\sum_{k\in K} \sum_{(a,b)\in A} d_k \rho_{abk} l_{ab} x_{abk}}{\sum_{(a,b)\in A:\ a<b} 1} \right)^2. \quad (4.8)$$

By a similar argument made for the previous risk metric, it may be of interest to minimize how much the risk flowing on each arc segment varies from the average. A UTM manager can minimize the sum of the segment risk variance values by minimizing summation (4.9) given by

$$\mathcal{R}(\boldsymbol{x}) = \sum_{\{(i,j)\in A:\ i<j\}} l_{ij} \left(\sum_{k\in K} d_k \rho_{ijk} (x_{ijk} + x_{jik}) - \frac{\sum_{k\in K} \sum_{(a,b)\in A} d_k \rho_{abk} l_{ab} x_{abk}}{\sum_{(a,b)\in A:\ a<b} l_{ab}} \right)^2. \quad (4.9)$$

It may be more applicable to minimize the risk variance amongst only the arcs or segments that exist in the optimal subnetwork as opposed to all arcs or segments in the entire network. Associated summations (4.10) and (4.11) given by

$$\mathcal{R}(\boldsymbol{x}) = \sum_{\{(i,j)\in A:\ i<j\}} z_{ij} \left(\sum_{k\in K} d_k \rho_{ijk} l_{ij} (x_{ijk} + x_{jik}) - \frac{\sum_{k\in K} \sum_{(a,b)\in A} d_k \rho_{abk} l_{ab} x_{abk}}{\sum_{(a,b)\in A:\ a<b} z_{ab}} \right)^2, \quad (4.10)$$

$$\mathcal{R}(\boldsymbol{x}) = \sum_{\{(i,j)\in A:\ i<j\}} l_{ij} z_{ij} \left(\sum_{k\in K} d_k \rho_{ijk} (x_{ijk} + x_{jik}) - \frac{\sum_{k\in K} \sum_{(a,b)\in A} d_k \rho_{abk} l_{ab} x_{abk}}{\sum_{(a,b)\in A:\ a<b} l_{ab} z_{ab}} \right)^2, \quad (4.11)$$

can be minimized instead. Due to the added complexity involved when we use the summations associated with the chosen subnetwork (the z_{ab} variable in the denominators make the functions inside the brackets nonlinear), we will opt to use the summations associated with the whole network when minimizing the sum of the arc or segment risk variance values.

Chapter 5

Stochastic Demand

Up until this point, we have been treating demand as a deterministic quantity. That is to say, for any pair of nodes (O_k, D_k) in the network, we know the exact amount of demand, in the form of parameter d_k, from one node O_k to the other node D_k. This approach is certainly practical in some cases. For example, a likely customer may give a UTM exact figures for how many packages it is expecting to deliver to each delivery node in a given network for a given month or year. A UTM could also obtain past shipping data to get a good idea of how many deliveries will take place between all node pairs in the future. With that being said, a deterministic framework does have its limits. Seeing as how no deliveries will be requested until the subnetwork is chosen and the appropriate UAV network infrastructure is built, the demand values for each node pair at the model execution stage will always have at least some level of uncertainty. Therefore, a UTM manager may prefer to use a model that takes a stochastic approach with respect to demand.

5.1 Stochastic Model

Before we introduce the stochastic variant of the model, we need some new notation. We define S to be the set of possible demand scenarios. Each scenario $s \in S$ has an associated probability of occurrence p^s. We define a new demand parameter d_k^s which is the demand of node pair k under scenario s. We also define a new flow variable x_{ijk}^s which is a binary variable which indicates if drones associated with pair k traverse arc (i, j) under scenario s. Based on this notation, the stochastic variant of the bi-objective model is defined as follows.

$$\text{minimize} \quad \mathcal{R}^S(\boldsymbol{x}) \tag{5.1}$$

$$\text{maximize} \quad \sum_{s \in S} \sum_{k \in K} \sum_{\{j:\ (O_k,j) \in A\}} p^s d_k^s x_{O_k jk}^s \tag{5.2}$$

subject to

$$\sum_{\{j:\ (i,j) \in A,\ j \neq O_k\}} x_{ijk}^s = \sum_{\{j:\ (j,i) \in A,\ j \neq D_k\}} x_{jik}^s \quad \forall s \in S,\ \forall k \in K,\ \forall i \in N \setminus \{O_k, D_k\} \tag{5.3}$$

$$\sum_{\{j:\ (O_k,j) \in A\}} x_{O_k jk}^s \leq 1 \quad \forall s \in S,\ \forall k \in K \tag{5.4}$$

$$x_{ijk}^s \leq z_{ij} \quad \forall s \in S,\ \forall k \in K,\ \forall (i,j) \in A:\ i < j \tag{5.5}$$

$$x_{jik}^s \leq z_{ij} \quad \forall s \in S,\ \forall k \in K,\ \forall (i,j) \in A:\ i < j \tag{5.6}$$

$$\sum_{(i,j) \in A:\ i<j} c_{ij} z_{ij} \leq B \tag{5.7}$$

$$\sum_{(i,j) \in A} l_{ij} x_{ijk}^s \leq \delta_k SP_k \quad \forall s \in S,\ \forall k \in K \tag{5.8}$$

$$x_{ijk}^s \in \{0,1\} \quad \forall s \in S,\ \forall k \in K,\ \forall (i,j) \in A \tag{5.9}$$

$$z_{ij} \in \{0,1\} \quad \forall (i,j) \in A:\ i < j \tag{5.10}$$

A UTM manager may minimize expected total risk by minimizing summation (5.11) given by

$$\mathcal{R}^S(\boldsymbol{x}) = \sum_{s\in S}\sum_{k\in K}\sum_{(i,j)\in A} p^s d_k^s \rho_{ijk} l_{ij} x_{ijk}^s, \tag{5.11}$$

and may minimize expected total risk deviation by minimizing summation (5.12) and adding constraints (5.13) given by

$$\mathcal{R}^S(\boldsymbol{x}) = \sum_{s\in S} p^s \sigma^s, \tag{5.12}$$

$$\sum_{k\in K}\sum_{(i,j)\in A} d_k^s \rho_{ijk} l_{ij} x_{ijk}^s \leq \sigma^s \left(\sum_{k\in K} d_k^s SR_k \left(\sum_{\{j:\ (O_k,j)\in A\}} x_{O_k jk}^s\right)\right), \quad \forall s \in S. \tag{5.13}$$

A UTM manager may minimize expected maximum arc risk by minimizing summation (5.14) and adding constraints (5.15) given by

$$\mathcal{R}^S(\boldsymbol{x}) = \sum_{s\in S} p^s \gamma^s, \tag{5.14}$$

$$\sum_{k\in K} d_k^s \rho_{ijk} l_{ij} (x_{ijk}^s + x_{jik}^s) \leq \gamma^s, \qquad \forall s \in S,\ \forall (i,j) \in A:\ i < j, \tag{5.15}$$

and may minimize expected maximum segment risk by minimizing summation (5.16) and adding constraints (5.17) given by

$$\mathcal{R}^S(\boldsymbol{x}) = \sum_{s\in S} p^s \Gamma^s, \tag{5.16}$$

$$\sum_{k\in K} d_k^s \rho_{ijk} (x_{ijk}^s + x_{jik}^s) \leq \Gamma^s, \qquad \forall s \in S,\ \forall (i,j) \in A:\ i < j. \tag{5.17}$$

Lastly, a UTM manager may minimize the expected sum of the arc risk variance values by minimizing summation (5.18) given by

$$\mathcal{R}^S(\boldsymbol{x}) = \sum_{s \in S} \sum_{\{(i,j) \in A:\ i<j\}} p^s \left(\sum_{k \in K} d_k^s \rho_{ijk} l_{ij} (x_{ijk}^s + x_{jik}^s) - \frac{\sum_{k \in K} \sum_{(a,b) \in A} d_k^s \rho_{abk} l_{ab} x_{abk}^s}{\sum_{(a,b) \in A:\ a<b} 1} \right)^2, \tag{5.18}$$

and may minimize the expected sum of the segment risk variance values by minimizing summation (5.19) given by

$$\mathcal{R}^S(\boldsymbol{x}) = \sum_{s \in S} \sum_{\{(i,j) \in A:\ i<j\}} p^s l_{ij} \left(\sum_{k \in K} d_k^s \rho_{ijk} (x_{ijk}^s + x_{jik}^s) - \frac{\sum_{k \in K} \sum_{(a,b) \in A} d_k^s \rho_{abk} l_{ab} x_{abk}^s}{\sum_{(a,b) \in A:\ a<b} l_{ab}} \right)^2. \tag{5.19}$$

5.2 Sample Average Approximation

Considering the domain of the underlying discrete demand distributions can be vary large, enumerating all possible realizations will result in an excessively large model. For this reason, we opt to apply sample average approximation (SAA) to obtain a viable solution as well as lower and upper bounds on the optimal cost. Sample average approximation is a method for approximating a two-stage stochastic programming model using a discrete set of scenarios [23]. These scenarios are generated by sampling from the probability distributions governing the uncertain parameters. The uncertain parameters in this case is the demand values of the node pairs. As the sample size increases, the result is shown to asymptotically converge to the true optimal solution [22]. Large sample sizes however can become too difficult to solve in a reasonable amount of time. For this reason, smaller sample sizes that can be solved efficiently are used. With these samples, statistical estimates of the lower and upper bounds of the objective function value can be obtained.

To compute the upper bounds on the objective for a given instance of the problem, the stochastic model is solved M times, each time using N independent demand scenarios [34]. From this we obtain M candidate solutions, $\boldsymbol{z}^1, \boldsymbol{z}^2, \ldots, \boldsymbol{z}^M$, where $\boldsymbol{z}^i$ is an optimal subnetwork obtained for sample $i \in \{1, 2, \ldots, M\}$, with associated objective function values $\eta^1, \eta^2, \ldots, \eta^M$. To obtain the upper bound on the objective function, we initially calculate the mean ($\overline{\eta}$) and the variance ($\overline{\sigma}^2_{N,M}$) of the objective function values $\eta^1, \eta^2, \ldots, \eta^M$ and use those values to calculate the upper bound UB. The means, variances and upper bound are calculated as

$$\overline{\eta} = \frac{1}{M}\sum_{m=1}^{M} \eta^m, \tag{5.20}$$

$$\overline{\sigma}^2_{N,M} = \frac{1}{M-1}\sum_{m=1}^{M}(\eta^m - \overline{\eta})^2, \tag{5.21}$$

$$UB = \overline{\eta} + t_{\alpha,M-1}\frac{\overline{\sigma}_{N,M}}{\sqrt{M}}. \tag{5.22}$$

Note that $t_{\alpha,M-1}$ is the α-critical value of the t-distribution with $M-1$ degrees of freedom. For example, if $M = 10$ and a 95% confidence level is desired, $t_{5,9} = 1.833$.

The lower bound on the true objective function value of a given candidate solution z^i is obtained by running the stochastic model with N' scenarios (N' is assumed to represent the true probability distribution) in such a way that the subnetwork is fixed as z^i. While N' may be large, we can decompose the overall problem into N' subproblems. Note that optimizing for any objective in the model subject to a fixed subnetwork can be done very quickly. We denote the objective function value of a given subproblem s as $\phi_s(\boldsymbol{z}^m, s)$. The estimate of the true objective value of the second stage problem, denoted as $\phi(\boldsymbol{z}^m)$, is computed as

$$\phi(\boldsymbol{z}^m) = \frac{1}{N'} \sum_{s=1}^{N'} \phi_s(\boldsymbol{z}^m, s).$$

The value of the true objective function $\overline{\eta}^m$ of candidate solution $\boldsymbol{z}^m$, its variance $\sigma^2(\boldsymbol{z}^m)$, and lower bound η_U^m are calculated as

$$\overline{\eta}^m = \sum_{(i,j)\in A: i<j} c_{ij} z_{ij}^m + \phi(\boldsymbol{z}^m), \tag{5.23}$$

$$\sigma_{N'}^2(\boldsymbol{x}^m) = \frac{1}{N'-1} \sum_{s=1}^{N'} \left[\phi_s(\boldsymbol{z}^s, s) - \phi(\boldsymbol{z}^s)\right]^2, \tag{5.24}$$

$$\eta_U^m = \overline{\eta}^m + z_\alpha \frac{\sigma_{N'}(\boldsymbol{z}^m)}{\sqrt{N'}}, \tag{5.25}$$

where z_α is the α-critical value of the standard normal distribution. We then use the best lower bound among the candidate solutions to be the lower bound generated by the SAA algorithm. In other words, the lower bound generated by the SAA algorithm LB is calculated as

$$LB = \max_{m\in\{1,\ldots,M\}} \eta_U^m. \tag{5.26}$$

5.3 Total Risk Reduction Method

We give a useful theorem below that reduces the stochastic variant of the model into a deterministic one when minimizing total risk. Suppose we have a set of demand scenarios S where each scenario has an associated probability of occurrence p^s. Consider the following stochastic version of the model (which we will call STO) with the objective of minimizing a weighted sum of expected total risk and expected satisfied demand ($\alpha, \beta \geq 0$).

$$\text{[STO]:} \quad \text{minimize} \quad \alpha \left(\sum_{s \in S} \sum_{k \in K} \sum_{(i,j) \in A} p^s d_k^s \rho_{ijk} l_{ij} x_{ijk}^s \right) - \beta \left(\sum_{s \in S} \sum_{k \in K} \sum_{\{j:\ (O_k,j) \in A\}} p^s d_k^s x_{O_k jk}^s \right) \tag{5.27}$$

subject to (5.3), (5.4), (5.5), (5.6), (5.7), (5.8), (5.9), (5.10)

We first propose a useful Lemma.

Lemma 5.3.1. *There exists an optimal solution* $\boldsymbol{x}^*$ *to STO such that for all* $k \in K$ *and all* $(i, j) \in A$, ${x_{ijk}^s}^* = 1\ \forall s \in S$ *or* ${x_{ijk}^s}^* = 0\ \forall s \in S$.

Proof. Clearly STO is feasible since $\mathbf{0}$ is a feasible solution. Let $\boldsymbol{x'}$ be an optimal solution of STO and let A' be the set of directed arcs in the subnetwork associated with the optimal solution $\boldsymbol{x'}$. Suppose we restrict STO by fixing the z_{ij} variables so that $z_{ij} = 1$ if and only if $(i, j) \in A'$ for all $(i, j) \in A$ such that $i < j$. The objective cost of any optimal solution of this restricted model is necessarily the same as the objective cost of solution $\boldsymbol{x'}$ in STO. Since the subnetwork is fixed, constraints (5.5), (5.6), (5.7), and (5.10) can be removed and the model decomposes into $|S| \times |K|$ identical subproblems SP(s, k), one for each $s \in S$

and $k \in K$, in the following manner.

$$
\begin{aligned}
[\text{SP}(s,\,k)]:\quad \text{minimize} \quad & \sum_{(i,j)\in A'} \alpha\rho_{ijk} l_{ij} x^s_{ijk} - \sum_{\{j:\ (O_k,j)\in A'\}} \beta x^s_{O_k jk} \\
\text{subject to} \quad & \sum_{\{j:\ (i,j)\in A',\ j\neq O_k\}} x^s_{ijk} = \sum_{\{j:\ (j,i)\in A',\ j\neq D_k\}} x^s_{jik} \quad \forall i \in N \setminus \{O_k, D_k\} \\
& \sum_{\{j:\ (O_k,j)\in A'\}} x^s_{O_k jk} \leq 1 \\
& \sum_{(i,j)\in A'} l_{ij} x^s_{ijk} \leq \delta_k SP_k \\
& x^s_{ijk} \in \{0,1\} \qquad \forall (i,j) \in A'
\end{aligned}
$$

It can be seen that the optimal solution(s) of subproblem SP(s, k) is independent of the scenario $s \in S$. In particular, the optimal solution of SP(s, k) is the vector associated with the smallest risk $O_k - D_k$ path on A' if the product of α and the total risk of said path is less than β and the optimal solution is $\mathbf{0}$ if the product is greater than β. Both solutions are optimal if the product is equal to β. It follows that we can construct a feasible solution $\boldsymbol{x^*}$ to STO that consists of all the optimal solutions of the SP(s, k)'s and A' in such a way that cost($\boldsymbol{x^*}$) = cost($\boldsymbol{x'}$) and for all $k \in K$ and all $(i,j) \in A$, either ${x^s_{ijk}}^* = 1\ \forall s \in S$ or ${x^s_{ijk}}^* = 0\ \forall s \in S$.

□

We now introduce a deterministic model DET which serves as a simplification of model

STO and will be used in Theorem 5.3.5.

$$\text{[DET]:} \quad \text{Minimize} \quad \alpha \left(\sum_{k \in K} \sum_{(i,j) \in A} \left(\sum_{s \in S} p^s d_k^s \right) \rho_{ijk} l_{ij} x_{ijk} \right) - \beta \left(\sum_{k \in K} \sum_{\{j:\ (O_k,j) \in A\}} \left(\sum_{s \in S} p^s d_k^s \right) x_{O_k jk} \right) \tag{5.28}$$

$$\text{subject to} \quad (3.3), (3.4), (3.5), (3.6), (3.7), (3.8), (3.9), (3.10)$$

We also introduce the following definition.

Definition 5.3.2. *Suppose $\boldsymbol{x^*}$ is a solution to DET and $\boldsymbol{x'}$ is a solution to STO. We say $\boldsymbol{x^*}$ and $\boldsymbol{x'}$ are associated if ${x_{ijk}}^* = {x_{ijk}^s}'$ for all $s \in S$, $k \in K, (i,j) \in A$, and ${z_{ij}}^* = {z_{ij}}'$ for all $(i,j) \in A$ such that $i < j$.*

Remarks 5.3.3 and 5.3.4 follow directly from Definition 5.3.2 and the respective objective functions of DET and STO.

Remark 5.3.3. *Every solution in DET has an associated solution in STO and every solution in STO of the form described in Lemma 5.3.1 has an associated solution in DET.*

Remark 5.3.4. *The objective cost of two associated solutions is the same.*

Theorem 5.3.5. *Suppose $\boldsymbol{x^*}$ is a solution to DET, $\boldsymbol{x'}$ is a solution to STO, and $\boldsymbol{x^*}$ and $\boldsymbol{x'}$ are associated solutions. Then $\boldsymbol{x^*}$ is an optimal solution to DET if and only if $\boldsymbol{x'}$ is an optimal solution to STO.*

Proof. ($\Rightarrow$): Suppose $\boldsymbol{x^*}$ is an optimal solution to DET. The feasibility of $\boldsymbol{x'}$ in STO follows trivially from the feasibility of $\boldsymbol{x^*}$ in DET. Suppose for the sake of contradiction that $\boldsymbol{y'}$ is the optimal solution of STO and the objective cost of $\boldsymbol{y'}$ is less than $\boldsymbol{x'}$. Furthermore, suppose solution $\boldsymbol{y'}$ is of the form described in Lemma 5.3.1 which we know exists as a result of said lemma. By Remark 5.3.3, $\boldsymbol{y'}$ has an associated solution $\boldsymbol{y}^*$ in DET. Therefore, as a result of Remark 5.3.4, cost($\boldsymbol{y}^*$)=cost($\boldsymbol{y'}$)<cost($\boldsymbol{x'}$)=cost($\boldsymbol{x}^*$) which contradicts the optimality of $\boldsymbol{x}^*$ in DET. It follows that $\boldsymbol{x'}$ is an optimal solution to STO.

($\Leftarrow$): The proof is nearly identical to the proof for the other direction. $\square$

Therefore, any stochastic model of the form STO may be simplified to a deterministic model where the demand d_k for each node pair k is the weighted sum of the node pair's demands across all demand scenarios $\sum_{s \in S} p^s d_k^s$. We will be referring to this single scenario constructed from all scenarios in S as the aggregated scenario for the remainder of this thesis. It can be verified that the model STO can be generalized to include any and all of the additional requirements and modifications listed in Chapter 6 (except capacities on the arcs or nodes detailed in Section 6.5) and the previous theorem would still apply. Note that if required demand is included in the model (see Section 6.1), each demand scenario would have to have the same required and unrequired pairs for the Theorem 5.3.5 to still apply. That is to say that no node pair can be required in a given demand scenario and unrequired in another.

Chapter 6

Additional Requirements

Given the wide variety of additional requirements that a UTM manager may opt to include when constructing a UAV network, we list some in this Chapter that may be of particular interest. Some of these requirements are found in most multi-commodity network flow research such as ensuring the demand of some subset of all commodities is satisfied (Section 6.1), and that capacities attributed to arcs and/or nodes are not violated (Section 6.5). Some requirements are slight variations or additions to requirements already present in the problem such as path risk deviation constraints (Section 6.3), optimizing for distance focused objectives (Section 6.2), and costs associated with including nodes in the chosen subnetwork (Section 6.5). Some requirements are considered within the context of ground-based traffic such as one-way streets (Section 6.6), and the energy used and/or risk incurred by a vehicle when turning as opposed to moving in a straight line (Section 6.8). Lastly, the addition of one-way streets and/or capacity constraints may not allow drones to fly back to their origin via their departing path ensuring the need for return paths to be considered (Section 6.7).

6.1 Required Demand

In the model, the existence of a feasible path on the subnetwork for any node pair is optional. It is worth noting however, that there exist scenarios in which some (or potentially all) of the node pairs listed have demand that we are required to satisfy. For example, some origin-destination pairs may be associated with a retail company that a UTM has an existing business relationship with. As a business exchange with this company, the UTM manager would ensure that some subset of the customers associated with this retail company are reachable on the subnetwork. In this vein, all pairs that must be satisfied (which we denote as required pairs) could be associated with existing customers of the UTM whereas the pairs that are not required to be satisfied (which we denote as unrequired pairs) are opportunities for potential growth.

With the inclusion of required demand, set K would be redefined as a disjoint union of two sets K^r and K^u where K^r is the set of node pairs that must have their demand satisfied and K^u is the set of node pairs that have some demand that we could potentially satisfy, but are not required to do so. If a UTM manager opts to include required demand in the model, constraints (3.4) would be replaced with constraints (6.1) and (6.2) given by

$$\sum_{\{j:\ (O_k,j)\in A\}} x_{O_k jk} = 1, \qquad \forall k \in K^r, \tag{6.1}$$

$$\sum_{\{j:\ (O_k,j)\in A\}} x_{O_k jk} \leq 1, \qquad \forall k \in K^u. \tag{6.2}$$

Constraints (6.1) ensure that every drone associated with a pair in K^r leaves its origin whereas constraints (6.2) accomplish the same task as constraints (3.4) for all unrequired pairs. Since all required demand must be satisfied, the second objective (3.2) can be simplified by replacing the set K under the first summation with the set K^u so that the objective now maximizes the demand satisfied amongst only the unrequired pairs.

6.2 Total Distance Travelled and its Variations

It is worth mentioning that total distance travelled (and its variations) is a viable metric and a UTM manager may be inclined to minimize this value as an objective or set an upper bound on this value as a constraint. We have opted to leave this value and its variations out of the model, with the exception of setting an upper bound on the length of each given origin-destination path, for three key reasons. Firstly, we feel that evaluating risk, both in the construction of a subnetwork and in the determination of the origin-destination paths, is more important for UAV traffic given how inherently risky it is relative to other pre-established modes of transportation. Secondly, given that the risk incurred by a drone traversing an arc is a function of the arc's length, it is often the case that enforcing drones to utilize less risky paths leads to them utilizing shorter paths as well. Thirdly, when determining the effect that minimizing total distance as an objective has on the model, the results would be similar in terms of significance and analysis to that of the effect that minimizing total risk has on the model.

If a UTM manager would prefer to minimize total distance travelled, summation (6.3) given by

$$\sum_{k \in K} \sum_{(i,j) \in A} d_k l_{ij} x_{ijk}, \tag{6.3}$$

would be minimized. If a UTM manager would prefer to minimize total distance deviation, denoted by τ, then variable τ would be minimized and constraint (6.4) given by

$$\sum_{k\in K}\sum_{(i,j)\in A} d_k l_{ij} x_{ijk} \leq \tau \left(\sum_{k\in K} d_k SP_k \left(\sum_{\{j:\ (O_k,j)\in A\}} x_{O_k jk}\right)\right), \tag{6.4}$$

would be added. There are no suitable alternatives for adjusting the maximum risk and risk variance objectives to a distance travelled context.

6.3 Path Risk Deviations

Constraints (3.8) in the model are there to ensure that no origin-destination path on the chosen subnetwork is too long. The main purpose of these constraints is for the benefit of the UAV operators. If a given operator is told that they must utilize a path that is say twice or three times as long as the shortest path to their destination simply because that is the shortest path that exists on the chosen subnetwork, that operator will likely abstain from utilizing the UTM's subnetwork. One could also make a similar argument for path risk. If an operator must fly their drone down a path that is significantly riskier than their preferred path, they may also abstain from utilizing the UTM's subnetwork. Therefore, it may be within a UTM's interest to not only consider the feasibility of an origin-destination path as a function of its length, but also a function of its total risk.

We define Δ_k to be the maximum percentage of SR_k in risk that a drone associated with pair k can incur. If a UTM manager opts to include path risk deviation constraints in the model, then constraints (6.5) given by

$$\sum_{(i,j)\in A} \rho_{ijk} l_{ij} x_{ijk} \leq \Delta_k SR_k, \qquad \forall k \in K, \tag{6.5}$$

would be added. Constraints (6.5) ensure that for every pair $k \in K$, the risk incurred by a drone associated with pair k is at most $\Delta_k \times SR_k$.

6.4 Node Costs

Many of the same costs that are associated with including arcs in a UAV subnetwork could also apply to including nodes in said subnetwork. For example, the costs associated with obtaining 3D maps and the overseeing of traffic in the context of intersections could be comparable to that of roads. Furthermore, there may be extra costs associated with including a particular arrival/departure point in the subnetwork with regards to building appropriate take-off/landing infrastructure, and establishing potential contracts with the customer associated with said arrival/departure point.

We define ν_i to be the cost associated with including node i in the subnetwork and n_i to be the binary decision variable which indicates if node i is in the subnetwork. If a UTM manager opts to include node costs in the model, constraint (3.7) would be replaced with constraints (6.6), (6.7), and (6.8) given by

$$\sum_{(i,j)\in A:\ i<j} c_{ij} z_{ij} + \sum_{i \in N} \nu_i n_i \leq B, \tag{6.6}$$

$$2z_{ij} \leq n_i + n_j, \qquad \forall (i,j) \in A:\ i < j, \tag{6.7}$$

$$n_i \in \{0,1\}, \qquad \forall i \in N. \tag{6.8}$$

Constraint (6.6) ensures that the sum of the costs of the undirected arcs and nodes in the chosen subnetwork does not exceed the chosen budget. Constraints (6.7) ensure that any node incident to an arc included in the subnetwork is also included in the subnetwork.

Constraints (6.8) ensure that all the n_i's are binary. Like constraints (3.10), constraints (6.8) can be dropped if preferred.

6.5 Arc and Node Capacities

As is typical in multi-commodity network flow problems, arcs will have finite capacity and capacity constraints should be incorporated into any associated model. Given the physical size and the broader range of mobility of UAVs, arc capacity is much less of an issue for UAV traffic when compared to other pre-established modes of transportation. Nonetheless, as separation standards become more relaxed and UAV traffic scales up, incorporating arc capacity constraints could become necessary.

If a UTM manager opts to include arc capacities in the model, we would define C_{ij} to be the maximum amount of demand allowed to utilize undirected arc $(i, j) \in A$ and constraints (6.9) given by

$$\sum_{k \in K} d_k(x_{ijk} + x_{jik}) \leq C_{ij}, \qquad \forall (i, j) \in A : \ i < j, \tag{6.9}$$

would be added. Constraints (6.9) ensures that the amount of demand on each given undirected arc does not exceed its capacity. A UTM manager may alternatively opt to place an upper bound on the number of origin-destination pairs utilizing a given undirected arc simply by replacing each d_k with a 1 in constraints (6.9) and defining each C_{ij} accordingly.

A UTM manager can also set capacity constraints for the amount of demand allowed to flow through nodes in the network. Let D_i be the maximum amount of demand allowed to flow through node $i \in N$. If a UTM manager opts to include node capacities in the model, we would add constraints (6.10) given by

$$\sum_{\{k \in K:\ i \neq O_k\}} \sum_{\{j:\ (i,j) \in A\}} d_k x_{ijk} \leq D_i, \qquad \forall i \in N, \tag{6.10}$$

to the model. Constraints (6.10) ensures that the amount of demand flowing through each given node does not exceed its capacity. A UTM manager may alternatively opt to place an upper bound on the number of origin-destination pairs utilizing a given node as a transshipment node simply by replacing each d_k with a 1 in constraints (6.10) and defining each D_i accordingly. Note that constraints (6.10) limit the amount of demand flowing through each node and do not limit how much demand departs from or arrives at any given node.

6.6 One-Way Streets

One-way streets are often used to help regulate the flow of ground-based traffic. As drone traffic scales up, it is possible that allowing drones to move only one direction down a given set of streets can help regulate the flow of UAV traffic as well. For example, when conducting tests with both a one-way and two-way street design, the authors of [11] found that safety, which was determined by the amount of times different drones would have to avoid conflict with each other, increased operating under a one-way system when compared to operating under a two-way system.

We define s_{ij} as 2 if undirected arc $(i, j) \in A$ can facilitate two-way traffic and 1 if it can only facilitate one-way traffic. We also define ζ_{ij} to be the binary decision variable that indicates if there exist any drones utilizing directed arc $(i, j) \in A$. If a UTM manager opts to include one-way streets in the model, then constraints (6.11), (6.12), and (6.13) given by

$$x_{ijk} \leq \zeta_{ij}, \qquad \forall k \in K,\ \forall (i,j) \in A, \tag{6.11}$$

$$\zeta_{ij} + \zeta_{ji} \leq s_{ij} z_{ij}, \qquad \forall (i,j) \in A:\ i < j, \tag{6.12}$$

$$\zeta_{ij} \in \{0,1\}, \qquad \forall (i,j) \in A, \tag{6.13}$$

would replace constraints (3.5) and (3.6) in the model. Constraints (6.11), (6.12), and (6.13) together accomplish the same task as constraints (3.5) and (3.6) while also ensuring that undirected arcs that must have traffic flowing in only one direction are required to do so (constraints (6.13) can be dropped if preferred). Note that if constraints (6.12) are added to the model, constraints (3.10) can no longer be dropped from the model. We would need to ensure the z_{ij}'s are binary because otherwise, only half the cost of a selected undirected arc (i,j) could contribute to the budget if $s_{ij} = 2$ and only one of ζ_{ij} or ζ_{ji} is equal to 1.

Suppose the direction of traffic for a given undirected arc is predetermined. For example, traffic flowing from node i to node j is permitted but not the other way around for a particular undirected arc $(i,j) \in A$. By setting the associated ζ variable (in this case, ζ_{ji}) to 0, any and all traffic flow on the arc will be required to go in the predetermined direction.

6.7 Return Paths

Based on how the problem is defined in Section 3.1, the existence of a feasible path in one direction for a given pair ensures the existence of a feasible path in the opposite direction. For this reason, finding return paths is unnecessary since any drone can utilize the exact same path following package delivery to arrive back at its origin. If arc capacities, node capacities and/or one-way streets are incorporated into the model, this no longer becomes a valid assumption.

If a UTM manager opts to include return paths in the model, then for every commodity $k \in K$, we must introduce a commodity $\overline{k}$ that is associated with the return trip. That is to say that for every commodity $k \in K$ with origin O_k and destination D_k, there exists an associated commodity $\overline{k} \in K$ with origin D_k and destination O_k. To ensure that a satisfied commodity k has a feasible return path on the chosen subnetwork, we require that the demand of $\overline{k}$ be satisfied. In other words, the demand of commodity k is satisfied if and only if the demand of commodity $\overline{k}$ is satisfied. Constraints (6.14) given by

$$\sum_{\{j:\ (O_k,j)\in A\}} x_{O_k j k} = \sum_{\{j:\ (D_k,j)\in A\}} x_{D_k j \bar{k}}, \qquad \forall k \in K, \tag{6.14}$$

would then be added to the model. Constraints (6.14) ensures that a drone can leave its origin and arrive at its destination if and only if it can leave its destination and arrive back at its origin. For the sake of consistency, the demand d_k for each $k \in K$ should be halved to account for the fact that half of the demand is associated with the trip from O_k to D_k (commodity k) while the other half is associated with the trip from D_k to O_k (commodity $\overline{k}$).

6.8 Arc-to-Arc Risk and Distance

The model considers the distance travelled (which goes hand in hand with battery usage) and the risk incurred by a given drone only with respect to particular quantities held by the arcs that it traverses. It is worth considering the effect that turning from a given arc to another has on risk incurred and distance travelled. When approaching a turn, a drone would need to slow down, execute the turn safely and then accelerate to its original speed. It is logical to assume that a path consisting of multiple turns (particularly sharp turns) utilizes more energy for a drone than a path that is a straight line. Furthermore, turning seems inherently riskier for a drone as it is a more difficult maneuver and the chance of failure from an execution standpoint seems greater in comparison to just flying directly forward. Also logistically speaking, similar to how left turns are more dangerous then right turns in a ground-based vehicle context (in the regions where vehicles drive on the right side of the road) due to potential collisions with oncoming traffic, certain turns might be more dangerous than others in the presence of other UAV traffic.

If a UTM manager opts to include arc-to-arc risk and/or distance into account, we would define λ_{ijmk} to be the battery usage by a drone associated with pair k going from arc (i,j) to arc (j,m) and Λ_{ijmk} to be the risk incurred by a drone associated with pair k going from arc (i,j) to arc (j,m). We would also define y_{ijmk} to be the binary decision variable which indicates if a drone associated with pair k traverses arc (i,j) followed by arc (j,m). Constraints (3.8) would be replaced with constraints (6.15), (6.16), and (6.17) given by

$$\sum_{(i,j)\in A} l_{ij}x_{ijk} + \sum_{(i,j)\in A} \sum_{\{m:\ (j,m)\in A\}} \lambda_{ijmk}y_{ijmk} \leq \delta_k SP_k, \qquad \forall k \in K, \tag{6.15}$$

$$y_{ijmk} \geq x_{ijk} + x_{jmk} - 1, \quad \forall k \in K,\ \forall (i,j),\ (j,m) \in A, \tag{6.16}$$

$$y_{ijmk} \in \{0,1\}, \qquad \forall k \in K,\ \forall (i,j),\ (j,m) \in A. \tag{6.17}$$

Constraints (6.15) ensure that path length deviation constraints are satisfied in this new arc-to-arc context. Note that the calculation of SP_k must now be modified to take arc-to-arc distance into account. Constraints (6.16) ensure that if a drone associated with pair k traverses a given arc (i, j) followed by arc (j, m), then its associated y_{ijmk} variable is 1. Constraints (6.17) ensure that all the y_{ijmk}'s are binary.

Total risk objective (4.1) and the LHS of total risk deviation constraint (4.3) would have to be replaced with summation (6.18) given by

$$\sum_{k \in K} \sum_{(i,j) \in A} d_k \rho_{ijk} l_{ij} x_{ijk} + \sum_{k \in K} \sum_{(i,j) \in A} \sum_{\{m:\ (j,m) \in A\}} d_k \Lambda_{ijmk} y_{ijmk}. \tag{6.18}$$

The maximum risk and risk variance objectives would not change under this new approach. Path risk deviation constraints (Section 6.3) can be added by including constraints (6.19) given by

$$\sum_{(i,j) \in A} \rho_{ijk} l_{ij} x_{ijk} + \sum_{(i,j) \in A} \sum_{\{m:\ (j,m) \in A\}} \Lambda_{ijmk} y_{ijmk} \leq \Delta_k SR_k, \qquad \forall k \in K, \tag{6.19}$$

in the model. Like the SP_k values in constraints (6.15), the SR_k values in constraints (6.19) and the RHS of total risk deviation constraint (4.3) would have to be recalculated to take arc-to-arc risk into account.

Chapter 7

Testing and Results

In this Chapter, we aim to understand how the stochastic and deterministic variants of the model perform in a given real world setting. The street network associated with a region of suburban Miami was chosen for this setting and 1000 demand scenarios were randomly generated using the neighbourhood Walmart as a shared origin node. The dataset and demand scenarios are further explained in Section 7.1. We discuss an important problem reduction technique for both the deterministic and stochastic variants of the model in Section 7.2. To better understand how the stochastic variant compares with the deterministic variant, we use SAA to find lower bounds on the objective function using both variants. This process is explained, and the corresponding results are given in Section 7.3. We also aim to understand how the different risk objectives compare to each other with respect to some of the key metrics of UAV network construction and path routing. The results of executing the model with each objective under nine different testing instances is displayed and analyzed in Section 7.4. Lastly, we offer graphs of the Pareto fronts of the model with total risk and total risk deviation objectives subject to three different budgets to understand how these risk objectives behave when the amount of satisfied demand is increased.

7.1 Underlying Network and Demand Scenarios

All testing for the model was conducted using the road network from part of suburban Miami, Florida. All of the nodes, arcs and arc lengths were obtained from OpenStreetMap via OSMNX. By downloading all the roads in the selected regions that have a "motorway," "trunk," "primary," "secondary," or "tertiary" tag, we obtain the road network illustrated in Figure 7.1. After cleaning the data by removing unnecessary nodes with degree two and merging nearby arcs with identical endpoints into single arcs, we obtain the network consisting of 262 nodes, 399 undirected arcs, and approximately 229 km of road length illustrated in Figure 7.2. Note that curved roads are straightened in this cleaned network purely for visual purposes and the original arc lengths are maintained. We also note that if a node separating two arcs is removed and the associated arcs are merged into one, the length of the new resulting arc is the sum of the lengths of the two merged arcs.

The cost value c_{ij} associated with an undirected arc (i, j) is chosen to be the same value as its length l_{ij}. Each risk parameter ρ_{ijk} was obtained using population density data. The road network used exists primarily within the seven census designated places (CDPs) depicted in Figure 7.3. These seven CDPs are Kendall West, Kendale Lakes, Tamiami, Westchester (University Park CDP has been merged into Westchester CDP), Westwood Lakes, Sunset, and Kendall (the CDP in the south-east corner of the map). The risk value ρ_{ijk} associated with arc (i, j) for all node pairs $k \in K$ is the population density of the CDP that arc (i, j) exists in (in 1000's of people per square km) multiplied by some random integer from $\{1, ..., 10\}$ to account for the variance in other risk factors amongst different arcs. The population density associated with all the arcs west of Kendall West and Tamiami is considered to be the population density of Kendall West and Tamiami, respectively. Furthermore, the population density associated with an arc that exists on the border of two CDPs or an arc that goes from one CDP into another, is set to be the average of the population densities of the two associated CDPs. Thus, the population densities of The Crossings CDP (which borders Kendale Lakes CDP to the south), Olympia Heights CDP (which borders Westwood Lakes CDP to the east) and Sweetwater CDP (which borders Westchester CDP to the north) are also incorporated for the few arcs in the network sitting on the boundaries of one of these CDPs. The population densities for all the CDPs were obtained from [6].

A total of 1000 demand scenarios were created for testing, each having the exact same 80 origin-destination pairs. The neighbourhood Walmart (identified by a green star in Figure 7.4) was chosen to be the origin for all pairs and 80 nodes were chosen at random (identified by red triangles in Figure 7.4) to be the destinations. Each of the 80 chosen origin-destination pairs in each demand scenario was given a random integer from $\{1, ..., 10\}$ for its demand. The testing instances were chosen to signify a business partnership between a customer (in this case, the neighbourhood Walmart) and an associated UTM. In this vein, the UTM aims to find an optimal UAV subnetwork that minimizes some chosen risk metric as much as possible while fulfilling as much demand as possible to the 80 destinations that represent the regions of Walmart's customers. Different demand scenarios with varying demand values are created to signify the uncertainty in how many Walmart packages will be delivered to any given destination over the course of a certain period of time.

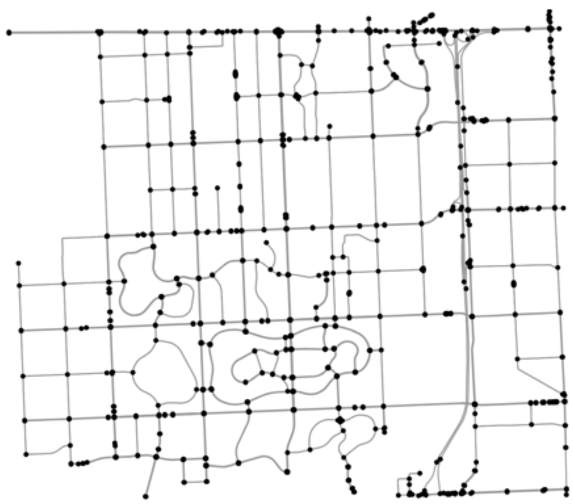

Figure 7.1: The road network for suburban Miami using data from OpenStreetMap

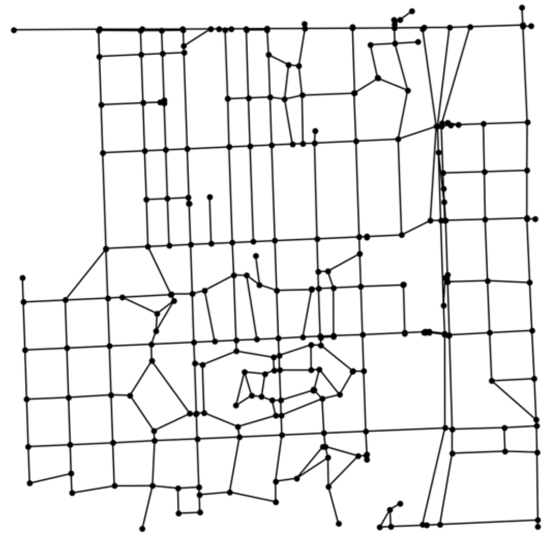

Figure 7.2: The network obtained from OpenStreetMap after cleaning the data

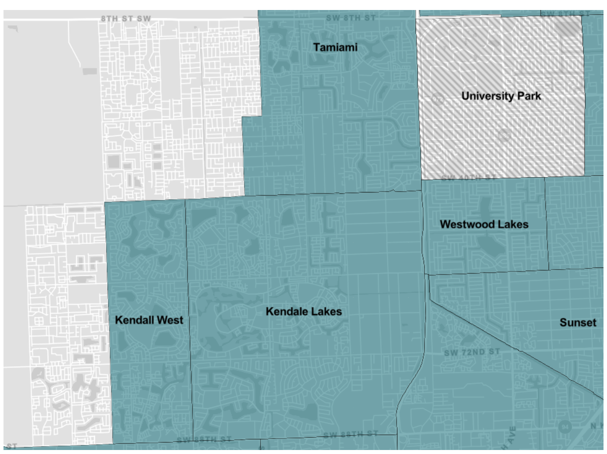

Figure 7.3: A picture of the seven primary CDPs that contain the underlying road network

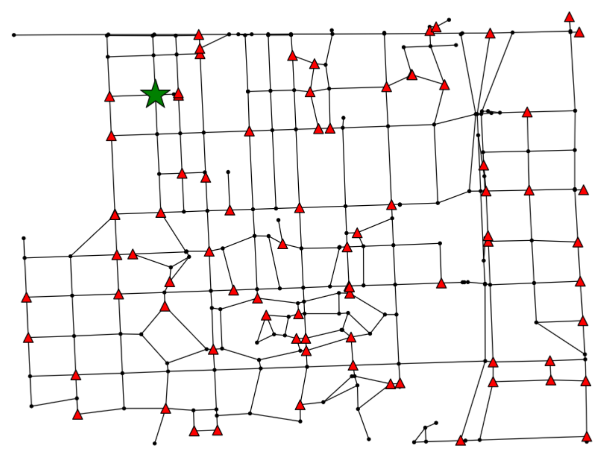

Figure 7.4: The network with origin (green star) and destinations (red triangles) highlighted

7.2 Preprocessing and Graph Reduction

Whether implementing the deterministic or stochastic variant of the model, there is a problem reduction technique detailed in [3] that we have found to be significantly effective at reducing the runtimes of the model. Suppose $SP_{a,b}$ is the length of a shortest path between nodes a and b on graph G and let $A_k = \{(i,j) \in A : SP_{O_k,i} + l_{ij} + SP_{j,D_k} \leq \delta_k SP_k\}$. As a result of constraints (3.8) in the deterministic variant of the model, an arc (i,j) exists in some feasible path for a pair $k \in K$ if and only if $(i,j) \in A_k$. It follows that if $(i,j) \notin A_k$, the associated variable x_{ijk} is necessarily 0 for all feasible solutions to the model and therefore, every instance of variable x_{ijk} can be omitted from the model. The constraint in (3.5) or (3.6) associated with variable x_{ijk} can also be removed. Furthermore, if a given node i is not incident to any arc in A_k for some $k \in K$, then the constraint associated with node i and pair k in (3.3) can also be omitted. Lastly, if for a given undirected arc (i,j), neither (i,j) nor (j,i) is in $\cup_{k \in K} A_k$, then the variable z_{ij} is necessarily 0 for all feasible solutions and every instance of z_{ij} can be omitted. A similar process can be applied using constraints (5.8) to remove associated variables and constraints in the stochastic variant of the model. Unless δ_k values are unusually large or omitted completely, this technique can remove a considerable number of variables and constraints from the model with little preprocessing computation.

Consider the origin-destination pair k highlighted in Figures 7.5, 7.6, 7.7, and 7.8. Each arc in each Figure is an indication that one or both flow variables associated with the given undirected arc needs to be created for the given pair k and path length deviation δ_k. When $\delta_k = 100\%$, only the 12 flow variables associated with arcs on the shortest $O_k - D_k$ path need to be created. When we increase δ_k to 120% and 150%, we see an increase to 117 and 341 flow variables, respectively. Even when we set a fairly loose path length deviation of

200% for pair k, we still only need to create 562 flow variables out of the 798 that would have to created otherwise.

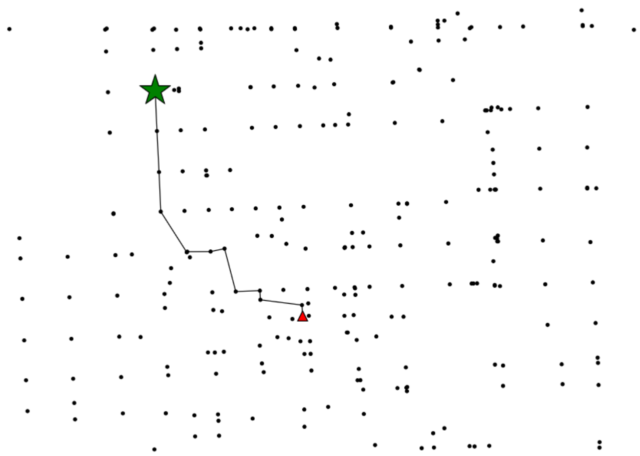

Figure 7.5: $\delta_k = 100\%$, 12 flow variables

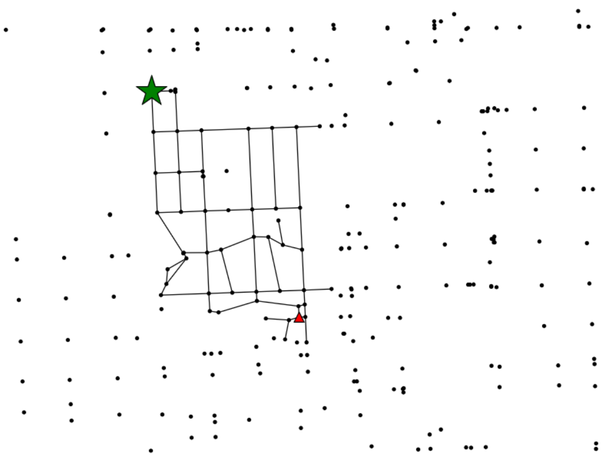

Figure 7.6: $\delta_k = 120\%$, 117 flow variables

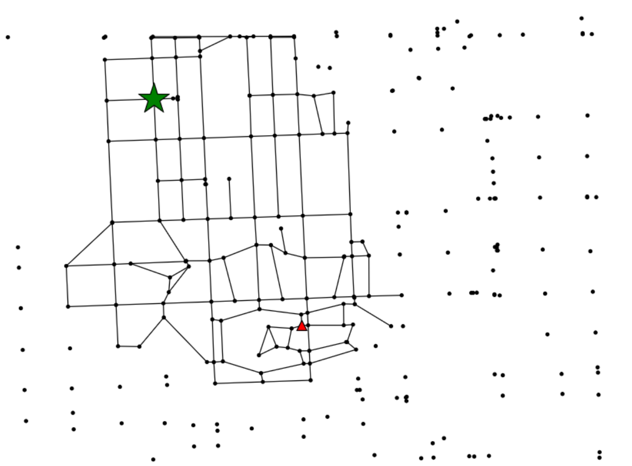

Figure 7.7: $\delta_k = 150\%$, 341 flow variables

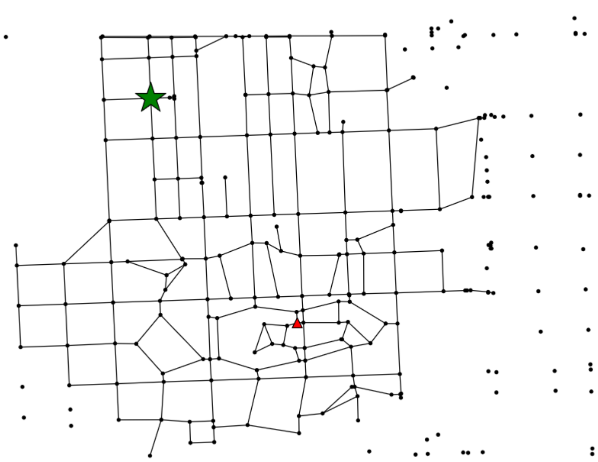

Figure 7.8: $\delta_k = 200\%$, 562 flow variables

To implement this method, the lengths of the shortest paths from the origin node and destination node of every pair $k \in K$ to every other node must be computed. All testing in this Chapter has been run with this implementation in place.

7.3 Stochastic Model Results

Before we look at results using the deterministic variant of the model, we first consider the stochastic variant. As a result of Theorem 5.3.5, we know that if the risk objective is total risk, then the $|S|$ demand scenarios can be replaced with a single aggregated demand scenario and in turn, we can reduce the stochastic problem into a deterministic one. While this theorem does not necessarily hold for any of the other risk objectives, it is worth exploring if the optimal subnetwork obtained via running the deterministic model with this aggregated scenario gives us effectively the same results for the stochastic problem as the optimal subnetwork obtained via stochastic modelling.

To see if that is the case, we use a model that maximizes demand and sets an upper bound on total risk deviation via the ϵ-constraint method. We are applying SAA using 10 samples of 10 demand scenarios (in particular, the first 100 of the 1000 randomly generated scenarios) giving us 10 optimal subnetworks associated with the 10 samples. We are also finding an optimal subnetwork associated with the aggregated scenario. We use all of the subnetworks (the 10 generated from the 10 samples along with the one generated from the aggregated scenario) and all of the 1000 scenarios to obtain lower bounds on the objective function of the stochastic model (this process is explained in more detail in Section 5.2). We note that we are using an α-critical value of 1.645, the value of the standard normal distribution with a 95% confidence level. We run these tests for 12 instances illustrated in Table 7.1. Each instance is determined by its budget in km (denoted by B), its path length deviation percentages for all pairs (denoted by PLD) and its maximum allowable total risk deviation percentage (denoted by MRD). For each testing instance, we give the best lower bound generated via SAA (denoted by SAA-LB), the lower bound generated via the aggregated scenario (AGG-LB), the difference between the two bounds (denoted by Diff) and the percentage gap between the two bounds (denoted by %Gap).

Table 7.1: SAA Results

B	PLD	MRD	SAA-LB	AGG-LB	Diff	%Gap
60	150	120	409.098	409.098	0	0
60	150	105	393.600	391.674	1.927	0.492
60	120	120	399.611	399.611	0	0
60	120	105	388.222	385.947	2.275	0.589
50	150	120	360.971	357.614	3.358	0.939
50	150	105	344.424	342.023	2.401	0.702
50	120	120	355.583	354.841	0.742	0.209
50	120	105	339.140	335.870	3.270	0.974
40	150	120	301.060	301.075	-0.014	0.005
40	150	105	289.588	288.122	1.465	0.509
40	120	120	299.485	298.841	0.645	0.216
40	120	105	284.442	280.912	3.530	1.257

We can see in Table 7.1 that the lower bounds produced via the aggregated scenario are only slightly worse than the ones produced via SAA. At worst, there is a 1.257% gap between the two bounds for the last instance. For one testing instance, the subnetwork associated with the aggregated scenario performs slightly better then the best subnetwork generated through SAA. While this does not necessarily imply that the subnetwork obtained via this aggregated scenario will be the exact same as the optimal subnetwork obtained via stochastic modelling, we can make the assessment that the subnetwork gives a solution that is close enough to optimality that we can reduce any stochastic model with many demand scenarios into a deterministic one with the associated aggregated scenario. We also note that the bounds obtained via SAA perform this well only under the most generous of circumstances. Whenever demand scenarios differed in node pairs even slightly, the lower bounds obtained via the aggregated scenario consistently outperformed SAA.

7.4 Deterministic Model Results

Using the previously mentioned 1000 demand scenarios, the aggregated scenario described in Section 5.3 was used for all tests with the deterministic variant. For each metric detailed below, a table illustrating the results of the model using each of the six objectives for each of the nine testing instances will be displayed. Each testing instance is defined by its budget in km (denoted by B) and the minimum amount of demand that must be satisfied as a percentage of the total demand (denoted by MSDP). All testing instances use a path length deviation of 120% for all node pairs.

The results can be broken into four sections; results focused on the subnetwork as a whole, results focused on the origin-destination paths, results focused on the undirected arcs and segments, and results focused on the performance of the model. For simplicity, all results (except for the performance results) will be displayed as a ratio to the result for the total risk objective. For example, if the result for a given metric, objective, and instance is 1.5, that means that the associated result is 50% larger than the result for the same metric when applying the total risk objective to the same testing instance. To save space in the tables and in analysis, the six objectives previously defined in Section 4.2 are denoted by TR (total risk), TRD (total risk deviation), MAR (maximum arc risk), MSR (maximum segment risk), ARV (arc risk variance) and SRV (segment risk variance). We define K^* to be the set of pairs in K which have their demand satisfied. A maximum runtime of one hour was given for each objective and testing instance, and the best solution obtained via branch and bound was used for the results below. Due to the difficulty of obtaining initial solutions for the ARV and SRV objectives, the optimal solution for objective TR was used as an incumbent solution for each testing instance with these two objectives.

We first analyze the subnetwork results in Table 7.3 with the associated metrics defined in Table 7.2.

Table 7.2: Subnetwork Results Notation

Acronym	Definition	Equation
TR/SD	The total risk incurred divided by the amount of satisfied demand	$\frac{\sum_{k\in K^*}\sum_{(i,j)\in A} d_k\rho_{ijk}l_{ij}x_{ijk}}{\sum_{k\in K^*} d_k}$
TD/SD	The total distance flown divided by the amount of satisfied demand	$\frac{\sum_{k\in K^*}\sum_{(i,j)\in A} d_k l_{ij}x_{ijk}}{\sum_{k\in K^*} d_k}$
TRpUD	The total risk incurred per unit distance travelled	$\frac{\sum_{k\in K^*}\sum_{(i,j)\in A} d_k\rho_{ijk}l_{ij}x_{ijk}}{\sum_{k\in K^*}\sum_{(i,j)\in A_k} d_k l_{ij}x_{ijk}}$
TRD	The total risk deviation	$\frac{\sum_{k\in K^*}\sum_{(i,j)\in A} d_k\rho_{ijk}l_{ij}x_{ijk}}{\sum_{k\in K^*} d_k SR_k}$
C	The sum of the costs of all the undirected arcs in the chosen subnetwork	$\sum_{(i,j)\in A^*} c_{ij}$

Objective TR performs the best overall. TR performs best in TR/SD (best in every instance), best in TD/SD (best in 7 out of 9 instances), second best in TRpUL (second in 6 out of 9 instances and first in the other 3) and second best in TRD (consistently second only to objective TRD). Objective TRD performs best in TRD (best in all instances) and best in TRpUD (best in 7 out of 9 instances). While objective TR prioritizes selecting pairs based on how much risk they contribute to the subnetwork as a whole, objective TRD is far more selective in the pairs that it chooses to satisfy leading to paths that are less risky relative to their length as well as their associated pair's smallest risk path.

Both MAR and MSR objectives perform the worst in every metric with little to no separation between the two for any of the chosen metrics. It seems that by prioritizing the risk on the arc carrying the most risk, certain pairs must utilize riskier paths from

Table 7.3: Subnetwork Results

B	MSDP	Obj	TR/SD	TD/SD	TRpUD	TRD	C	B	MSDP	Obj	TR/SD	TD/SD	TRpUD	TRD	C	B	MSDP	Obj	TR/SD	TD/SD	TRpUD	TRD	C
40	60	TRD	1.090	1.143	0.953	0.979	0.998	60	80	TRD	1.064	1.064	1.000	0.991	0.996	80	100	TRD	1.000	1.000	1.000	1.000	1.000
		MAR	1.274	1.126	1.131	1.154	0.999			MAR	1.473	1.126	1.309	1.373	0.994			MAR	1.328	1.052	1.263	1.328	0.998
		MSR	1.177	1.022	1.151	1.145	0.999			MSR	1.363	1.075	1.269	1.306	1.000			MSR	1.353	1.045	1.295	1.353	0.997
		ARV	1.015	1.003	1.012	1.015	0.999			ARV	1.067	1.022	1.044	1.042	0.999			ARV	1.108	1.014	1.093	1.108	0.999
		SRV	1.052	1.003	1.049	1.052	1.000			SRV	1.079	0.963	1.120	1.059	1.000			SRV	1.110	0.999	1.111	1.110	0.998
40	40	TRD	1.179	1.191	0.989	0.978	1.073	60	60	TRD	1.247	1.333	0.935	0.988	1.271	80	80	TRD	1.091	1.096	0.995	0.992	1.115
		MAR	1.658	1.413	1.173	1.278	1.198			MAR	1.613	1.407	1.147	1.303	1.321			MAR	1.604	1.238	1.296	1.381	1.265
		MSR	1.512	1.306	1.157	1.262	1.196			MSR	1.669	1.366	1.222	1.368	1.343			MSR	1.580	1.237	1.277	1.377	1.263
		ARV	1.141	1.036	1.101	1.136	1.203			ARV	1.120	1.023	1.095	1.118	1.318			ARV	1.113	1.030	1.080	1.098	1.222
		SRV	1.106	1.019	1.086	1.098	1.185			SRV	1.125	1.001	1.125	1.115	1.335			SRV	1.175	1.011	1.163	1.139	1.262
40	20	TRD	1.690	1.514	1.116	0.991	1.643	60	40	TRD	1.206	1.202	1.004	0.978	1.314	80	60	TRD	1.279	1.349	0.947	0.988	1.424
		MAR	2.483	2.008	1.236	1.265	2.309			MAR	1.607	1.385	1.160	1.259	1.712			MAR	1.818	1.529	1.188	1.363	1.765
		MSR	2.527	2.026	1.247	1.280	2.266			MSR	1.776	1.439	1.234	1.334	1.769			MSR	1.718	1.370	1.255	1.325	1.778
		ARV	1.099	1.003	1.096	1.099	1.252			ARV	1.140	1.044	1.092	1.125	1.375			ARV	1.133	1.038	1.092	1.116	1.362
		SRV	1.129	1.039	1.087	1.107	1.429			SRV	1.155	1.097	1.053	1.088	1.538			SRV	1.252	1.133	1.105	1.137	1.790

their respective origins to their respective destinations to avoid utilizing said arc. The metrics TR/SD, TD/SD, TRpUD and TRD all increase for objectives MAR and MSR as a byproduct of this behavior. This is a pattern we will be seeing consistently throughout the results of this section.

Both objectives ARV and SRV perform better than TRD in TR/SD (better in 6 out of 9 instances) and TD/SD (better in 8 out of 9 instances) but perform worse than both TRD and TR in TRpUD (worse in 8 out of 9 instances) and TRD (worse in all instances). Since objectives ARV and SRV contain the summation for total risk in their respective objective functions, it is expected that these objectives perform decently well in TR/SD and TD/SD. Also, much like how objectives MAR and MSR make certain pairs utilize riskier paths to avoid utilizing the arc with the most risk, it seems that objectives ARV and SRV do that to some extent as well to avoid overloading some arcs with a lot of risk.

While the cost of the subnetwork is not necessarily an important metric since it is set as a constraint (the cost of the subnetwork can be made as large or as small as a UTM manager chooses), there is a relationship worth highlighting in the results. Objective TR consistently needs a less costly subnetwork to obtain optimality than the other objectives, particularly as MSDP is smaller relative to a fixed budget B. Objectives MAR and MSR on the other hand consistently produce an optimal subnetwork near the budget regardless of the value of MSDP. These objectives will spread out the demand across as much of the network as possible to reduce the amount of risk present on the arc or segment with the most risk.

Next, we will analyze the path results in Table 7.5 with the associated metrics defined in Table 7.4. Objective TR performs the best or close to best in all the path associated metrics. Objective TR consistently produces networks whose satisfied paths are shorter (best in MLP and ALP in 6 out of 9 instances) and less risky (best in MRP and ARP in

Table 7.4: Path Results Notation

Acronym	Definition	Equation
MLP	The length of the longest satisfied O-D path	$\max_{k \in K^*} \sum_{(i,j) \in A} l_{ij} x_{ijk}$
ALP	The average length among all satisfied O-D paths	$\operatorname{avg}_{k \in K^*} \sum_{(i,j) \in A} l_{ij} x_{ijk}$
MRP	The risk incurred by a drone traversing the riskiest satisfied O-D path	$\max_{k \in K^*} \sum_{(i,j) \in A} \rho_{ijk} l_{ij} x_{ijk}$
ARP	The average risk incurred by a drone traversing a satisfied O-D path	$\operatorname{avg}_{k \in K^*} \sum_{(i,j) \in A} \rho_{ijk} l_{ij} x_{ijk}$
ARpULP	The average risk incurred per unit length by a drone traversing a satisfied O-D path	$\operatorname{avg}_{k \in K^*} \frac{\sum_{(i,j) \in A} \rho_{ijk} l_{ij} x_{ijk}}{\sum_{(i,j) \in A} l_{ij} x_{ijk}}$
MRDP	The maximum deviation in risk among all satisfied O-D paths	$\max_{k \in K^*} \frac{\sum_{(i,j) \in A} \rho_{ijk} l_{ij} x_{ijk}}{SR_k}$
ARDP	The average deviation in risk among all satisfied O-D paths	$\operatorname{avg}_{k \in K^*} \frac{\sum_{(i,j) \in A} \rho_{ijk} l_{ij} x_{ijk}}{SR_k}$

Table 7.5: Path Results

B	MSDP	Obj	MLP	ALP	MRP	ARP	ARpULP	MRDP	ARDP	B	MSDP	Obj	MLP	ALP	MRP	ARP	ARpULP	MRDP	ARDP	B	MSDP	Obj	MLP	ALP	MRP	ARP	ARpULP	MRDP	ARDP
40	60	TRD	1.157	1.144	1.174	1.090	0.931	0.804	0.978	60	80	TRD	1.085	1.064	1.102	1.064	0.985	0.796	0.988	80	100	TRD	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		MAR	1.058	1.125	1.393	1.274	1.133	1.827	1.182			MAR	1.235	1.126	2.137	1.473	1.269	1.567	1.337			MAR	1.147	1.052	1.748	1.327	1.239	1.785	1.305
		MSR	1.015	1.022	1.319	1.177	1.145	1.827	1.153			MSR	1.195	1.074	1.895	1.363	1.241	1.880	1.285			MSR	1.020	1.045	1.566	1.353	1.273	1.857	1.332
		ARV	1.000	1.003	1.000	1.015	1.025	1.827	1.036			ARV	1.000	1.022	1.298	1.067	1.060	1.827	1.067			ARV	1.000	1.014	1.141	1.108	1.116	1.785	1.137
		SRV	1.000	1.003	1.063	1.052	1.058	1.827	1.065			SRV	0.808	0.964	1.256	1.079	1.085	1.108	1.052			SRV	1.000	0.999	1.330	1.109	1.127	1.785	1.129
40	40	TRD	1.250	1.191	1.861	1.178	0.974	0.795	0.982	60	60	TRD	1.973	1.334	1.432	1.247	0.951	0.795	0.987	80	80	TRD	1.267	1.097	1.279	1.091	0.985	0.795	0.990
		MAR	1.951	1.418	2.365	1.664	1.216	1.827	1.301			MAR	1.889	1.408	2.244	1.615	1.120	1.615	1.268			MAR	1.420	1.238	2.072	1.603	1.274	1.827	1.349
		MSR	1.778	1.307	2.259	1.512	1.172	1.880	1.267			MSR	1.934	1.368	2.614	1.671	1.178	1.615	1.310			MSR	1.286	1.237	1.948	1.578	1.252	1.827	1.337
		ARV	1.133	1.036	1.207	1.140	1.138	1.827	1.186			ARV	1.178	1.023	1.351	1.120	1.143	1.900	1.178			ARV	0.999	1.030	1.355	1.113	1.108	1.827	1.135
		SRV	1.126	1.019	1.085	1.107	1.124	1.827	1.141			SRV	1.070	1.001	1.402	1.125	1.160	1.900	1.164			SRV	0.983	1.011	1.459	1.175	1.136	1.193	1.124
40	20	TRD	1.707	1.513	2.479	1.689	1.042	0.910	0.994	60	40	TRD	1.403	1.204	1.861	1.208	0.975	0.795	0.982	80	60	TRD	1.973	1.351	1.633	1.279	0.958	0.795	0.987
		MAR	2.230	2.010	2.776	2.482	1.166	1.670	1.245			MAR	1.948	1.389	2.507	1.610	1.152	1.827	1.237			MAR	2.190	1.531	2.582	1.820	1.159	1.600	1.308
		MSR	2.031	2.044	3.247	2.554	1.171	1.550	1.225			MSR	1.964	1.444	2.695	1.782	1.197	1.827	1.290			MSR	1.769	1.371	2.341	1.720	1.238	1.565	1.305
		ARV	1.000	1.003	1.000	1.100	1.091	1.460	1.116			ARV	1.159	1.044	1.308	1.140	1.115	1.827	1.162			ARV	1.384	1.038	1.661	1.134	1.125	1.827	1.159
		SRV	1.264	1.034	1.138	1.126	1.124	2.090	1.156			SRV	1.619	1.097	1.974	1.154	1.047	1.239	1.086			SRV	2.172	1.133	2.083	1.251	1.114	1.175	1.127

all 9 instances) in comparison with the other objectives. Furthermore, objective TR only performs worse than TRD with respect to ARpULP (best for a single instance and second best in all other instances), MRDP (second best in all instances) and ARDP (second best in all instances). We see again that TRD performs the best in the risk deviation and risk per unit length associated metrics (best in ARpULP, MRDP and ARDP for all but one instance) but not as great for the total risk and total distance related metrics (MLP, ALP, MRP and ARP).

Once again, objectives MAR and MSR perform the worst across all path-based metrics for almost all testing instances. In fact, we see 100% and even 200% increases in metrics MLP, ALP, MRP and ARP relative to objective TR for some testing instances. The ARV and SRV objectives perform similarly with respect to the path-based metrics as what we have seen for the metrics in Table 7.3. They typically perform second to TR in the total distance and total risk associated metrics (MLP, ALP, MRP and ARP) and worse than both TR and TRD (and occasionally worse than objectives MAR and MSR) in the risk deviation and risk per unit length associated metrics (ARpULP, MRDP and ARDP).

Next, we will analyze the arc and segment results in Tables 7.7 and 7.8 with the associated metrics defined in Table 7.6. As expected, objective MAR performs best for every testing instance of MAR with MSR performing second best (second best in 6 out of 9 instances) and SRV performing third best. Objective SRV performs more or less on par with objective ARV with respect to metric MAR but surprisingly performs slightly better. We see objective MSR perform best for every testing instance of MSR, but MAR performs behind both ARV and SRV. Objective ARV surprisingly performs better than SRV in MSR in all but two instances. Objective TR performs second worst while TRD performs worst in both MAR and MSR.

Note that the calculation for average amount of risk present among all undirected arcs

Table 7.6: Arc and Segment Results Notation

Acronym	Definition	Equation
MAR	The maximum amount of risk present among all undirected arcs in the subnetwork	$\max_{(i,j)\in A^*:\, i<j} \sum_{k\in K^*} d_k \rho_{ijk} l_{ij} (x_{ijk} + x_{jik})$
AARA	The average amount of risk present among all undirected arcs	$\operatorname{avg}_{(i,j)\in A:\, i<j} \sum_{k\in K^*} d_k \rho_{ijk} l_{ij} (x_{ijk} + x_{jik})$
AARS	The average amount of risk present among all undirected arcs in the subnetwork	$\operatorname{avg}_{(i,j)\in A^*:\, i<j} \sum_{k\in K^*} d_k \rho_{ijk} l_{ij} (x_{ijk} + x_{jik})$
ARVA	The sum of the arc risk variance values among all undirected arcs	(4.8)
ARVS	The sum of the arc risk variance values among all undirected arcs in the subnetwork	(4.10)
MSR	The maximum amount of risk present among all arc segments in the subnetwork	$\max_{(i,j)\in A^*:\, i<j} \sum_{k\in K^*} d_k \rho_{ijk} (x_{ijk} + x_{jik})$
ASRA	The average amount of risk present among all arc segments	$\operatorname{avg}_{(i,j)\in A:\, i<j} \sum_{k\in K^*} d_k \rho_{ijk} (x_{ijk} + x_{jik})$
ASRS	The average amount of risk present among all arc segments in the subnetwork	$\operatorname{avg}_{(i,j)\in A^*:\, i<j} \sum_{k\in K^*} d_k \rho_{ijk} (x_{ijk} + x_{jik})$
SRVA	The sum of the segment risk variance values among all arc segments	(4.9)
SRVS	The sum of the segment risk variance values among all arc segments in the subnetwork	(4.11)

Table 7.7: Arc Results

B	MSDP	Obj	MAR	AARA	AARS	ARVA	ARVS	B	MSDP	Obj	MAR	AARA	AARS	ARVA	ARVS	B	MSDP	Obj	MAR	AARA	AARS	ARVA	ARVS
40	60	TRD	1.125	1.090	1.101	1.238	1.248	60	80	TRD	0.958	1.064	1.033	1.063	1.059	80	100	TRD	1.000	1.000	1.000	1.000	1.000
		MAR	0.551	1.274	1.358	1.094	0.894			MAR	0.453	1.473	1.553	1.298	1.083			MAR	0.447	1.328	1.436	1.031	0.871
		MSR	0.666	1.177	1.282	1.122	0.994			MSR	0.752	1.363	1.437	1.213	1.050			MSR	0.779	1.353	1.445	1.164	1.020
		ARV	0.969	1.015	1.025	0.965	0.941			ARV	0.855	1.067	1.067	0.922	0.878			ARV	0.660	1.108	1.102	0.806	0.739
		SRV	0.888	1.052	1.097	0.966	0.905			SRV	0.771	1.078	1.103	0.819	0.741			SRV	0.659	1.110	1.097	0.808	0.743
40	40	TRD	1.045	1.179	1.146	1.296	1.281	60	60	TRD	1.218	1.273	1.022	1.461	1.540	80	80	TRD	1.061	1.107	1.054	1.179	1.188
		MAR	0.472	1.658	1.401	1.247	0.913			MAR	0.471	1.612	1.374	1.248	0.996			MAR	0.430	1.604	1.458	1.213	1.020
		MSR	0.650	1.512	1.326	1.120	0.842			MSR	0.620	1.668	1.374	1.382	1.158			MSR	0.841	1.579	1.426	1.253	1.085
		ARV	0.684	1.141	0.953	0.686	0.566			ARV	0.690	1.120	0.900	0.718	0.656			ARV	0.708	1.113	0.933	0.695	0.650
		SRV	0.685	1.107	1.007	0.711	0.583			SRV	0.719	1.125	0.904	0.712	0.645			SRV	0.707	1.175	0.985	0.717	0.655
40	20	TRD	1.657	1.695	1.211	2.274	2.421	60	40	TRD	1.085	1.240	1.000	1.339	1.394	80	60	TRD	1.124	1.279	0.957	1.295	1.356
		MAR	0.538	2.483	1.257	1.664	1.140			MAR	0.472	1.607	0.959	0.994	0.884			MAR	0.471	1.817	1.171	1.214	1.054
		MSR	0.652	2.527	1.189	1.990	1.691			MSR	0.717	1.775	1.068	1.244	1.115			MSR	0.537	1.718	1.026	1.213	1.164
		ARV	0.781	1.099	0.916	0.775	0.676			ARV	0.684	1.140	0.852	0.662	0.581			ARV	0.721	1.133	0.888	0.675	0.602
		SRV	0.782	1.132	0.924	0.809	0.710			SRV	0.680	1.155	0.812	0.694	0.641			SRV	0.749	1.251	0.791	0.698	0.667

Table 7.8: Segment Results

B	MSDP	Obj	MSR	ASRA	ASRS	SRVA	SRVS	B	MSDP	Obj	MSR	ASRA	ASRS	SRVA	SRVS	B	MSDP	Obj	MSR	ASRA	ASRS	SRVA	SRVS
40	60	TRD	1.125	1.072	1.083	1.243	1.279	60	80	TRD	0.958	1.052	1.022	1.078	1.055	80	100	TRD	1.000	1.000	1.000	1.000	1.000
		MAR	1.015	1.237	1.318	1.126	0.795			MAR	0.836	1.368	1.443	1.303	0.955			MAR	0.706	1.273	1.377	1.018	0.790
		MSR	0.626	1.001	1.091	0.980	0.709			MSR	0.464	1.159	1.221	0.973	0.624			MSR	0.464	1.225	1.308	0.976	0.714
		ARV	0.969	1.004	1.014	0.949	0.894			ARV	0.855	1.116	1.116	0.945	0.869			ARV	0.690	1.187	1.180	0.855	0.742
		SRV	0.875	0.988	1.030	0.930	0.812			SRV	0.771	1.040	1.063	0.773	0.620			SRV	0.690	1.121	1.108	0.822	0.697
40	40	TRD	1.058	1.205	1.172	1.301	1.319	60	60	TRD	1.218	1.257	1.010	1.605	1.868	80	80	TRD	1.061	1.065	1.014	1.159	1.200
		MAR	0.865	1.792	1.514	1.602	1.118			MAR	0.931	1.587	1.352	1.465	1.243			MAR	0.835	1.507	1.370	1.246	1.028
		MSR	0.470	1.490	1.306	1.177	0.646			MSR	0.487	1.464	1.206	1.276	0.880			MSR	0.430	1.334	1.205	1.026	0.746
		ARV	0.708	1.261	1.053	0.778	0.567			ARV	0.849	1.165	0.936	0.744	0.658			ARV	0.708	1.157	0.970	0.735	0.664
		SRV	0.883	1.140	1.038	0.739	0.532			SRV	0.969	1.098	0.882	0.679	0.556			SRV	0.806	1.106	0.927	0.635	0.504
40	20	TRD	2.151	1.724	1.231	1.735	1.836	60	40	TRD	1.232	1.217	0.982	1.213	1.296	80	60	TRD	1.124	1.262	0.944	1.379	1.600
		MAR	1.011	2.799	1.417	2.057	1.629			MAR	0.708	1.681	1.003	1.095	0.912			MAR	0.921	1.729	1.115	1.289	1.139
		MSR	0.644	2.630	1.238	2.003	1.321			MSR	0.469	1.661	1.000	1.194	0.913			MSR	0.487	1.568	0.937	1.109	0.956
		ARV	0.781	1.210	1.008	0.815	0.648			ARV	0.708	1.249	0.934	0.743	0.631			ARV	0.726	1.171	0.918	0.705	0.607
		SRV	0.782	1.133	0.925	0.800	0.715			SRV	0.883	1.195	0.840	0.692	0.617			SRV	0.968	1.201	0.759	0.639	0.596

(AARA) is identical to the calculation of total risk divided by $|A|$. This means the ratio-based results for metric AARA are almost identical to the ratio-based results for metric TR/SD. Therefore, objective TR performs best in all instances, the risk variance objectives perform second best, followed by objective TRD, and the maximum risk objectives perform worst. We see similar results with metric ASRA with objective TR performing best (best in 8 out of 9 instances), objectives SRV and ARV performing second best (SRV performed noticeably better than ARV in 8 out of 9 instances) followed by objective TRD, and then objectives MAR and MSR perform the worst (MSR performed better than MAR in all instances). We see a slightly different dynamic when only subnetwork arcs and segments are considered. The separation between how the results for objective TR compares with the other objectives is much smaller. For example, objective TR performs worse than ARV and/or SRV in 6 out of 9 instances for AARS and worse in 5 out of 9 instances for ASRS. This is not necessarily surprising since the risk variance objectives prioritize the risk on subnetwork arcs and segments more heavily than objective TR does.

With respect to the risk variance metrics for all arcs and segments (ARVA and SRVA), we see that ARV and SRV perform the best while TR performs next best. With respect to the risk variance metrics for only subnetwork arcs or segments (ARVS and SRVS), we see that ARV and SRV perform the best for their respective metrics (other than the first instance where both MAR and MSR outperform ARV and SRV). This is a good sign if the goal is to minimize the sum of the arc or segment risk variance values over only the subnetwork arcs or segments because we can use the risk variance objectives for all arcs or segments which is much easier to solve from a modelling perspective. We see objectives MAR and MSR perform much better with respect to the risk variance metrics for only the used arcs and segments than they did with respect to the risk variance metrics for the whole network. Objective MSR especially performs well consistently performing second best in

SRVS. We note that SRV performs better than ARV in metric ARVA when the budget is 60 km and the MSDP is 80% and 60%. As we will see below, the risk variance objectives do not run to optimality within the time limit, and it seems that (at least for these two instances) that the best solution found for objective SRV is much closer to optimality than the best solution found for objective ARV.

Lastly, we analyze the performance results in Table 7.9. Note that the MIP Gap is obtained from Gurobi and is calculated as |ObjValue − ObjBound|/ObjValue. Objectives TR and TRD are the only objectives that run to optimality in every testing instance. Objective TR never takes longer than 8 seconds while objective TRD never takes longer than 7 minutes. Objectives MAR and MSR run to optimality roughly half the time. In particular, these objectives reach optimality (or near optimality) in 6 out of 9 instances. We only see relatively large MIP gaps for these objectives when the MSDP values are at their highest relative to each budget (upwards of 15.18% and 24.41% respectively when the budget is 40 km and the MSDP is 60%). Neither of the risk variance objectives reached optimality for any of the testing instances. The MIP Gaps ranged between 15% and 35%, and the time limit of one hour was reached for every instance with these two objectives. As previously mentioned when analyzing the arc and segment results, this lead to the SRV objective performing better than objective ARV in the ARVA metric when the budget was 60km, and the MSDP was 80 and 60. Note that the MIP gap for ARV is 5% and 2% higher than the MIP gap for SRV at these two instances, respectively.

To conclude, objectives TR and TRD outperform the other objectives with respect to the subnetwork results, path-based results, and performance results. We see that by minimizing total risk, we can obtain a subnetwork that prioritizes risk and distance reduction across all arcs in the network and across all origin-destination paths. By minimizing total risk deviation, we are prioritizing node pairs that can utilize less risky paths relative to

Table 7.9: Performance Results

B	MSDP	Obj	Runtime	ObjVal	ObjBound	MIP Gap	B	MSDP	Obj	Runtime	ObjVal	ObjBound	MIP Gap	B	MSDP	Obj	Runtime	ObjVal	ObjBound	MIP Gap
40	60	TR	7.02	16971.20	16971.20	optimal	60	80	TR	3.76	26155.22	26155.22	optimal	80	100	TR	2.72	37630.74	37630.74	optimal
		TRD	418.35	100.05	100.05	optimal			TRD	296.95	100.01	100.01	optimal			TRD	89.27	101.38	101.38	optimal
		MAR	3600	922.65	782.63	15.18%			MAR	3600	1137.85	1054.83	7.30%			MAR	3600	1449.08	1363.83	5.88%
		MSR	3600	1307.47	988.35	24.41%			MSR	3600	1452.13	1323.11	8.89%			MSR	3600	1876.68	1664.54	11.30%
		ARV	3600	9425458.31	6592722.14	30.05%			ARV	3600	18058576.99	11534804.61	36.13%			ARV	3600	26760177.19	19805827.01	25.99%
		SRV	3600	13820478.57	9250146.19	33.07%			SRV	3600	23063534.42	15746886.07	31.72%			SRV	3600	40726623.47	27091173.26	33.48%
40	40	TR	5.30	8837.77	8837.77	optimal	60	60	TR	3.12	16556.48	16556.48	optimal	80	80	TR	5.37	26082.11	26082.11	optimal
		TRD	76.49	100.00	100.00	optimal			TRD	79.28	100.00	100.00	optimal			TRD	79.50	100.00	100.00	optimal
		MAR	882.24	544.64	544.64	optimal			MAR	3600	789.03	788.98	0.01%			MAR	3600	1078.74	1078.62	0.01%
		MSR	3600	675.84	673.92	0.28%			MSR	877.03	1017.71	1017.71	optimal			MSR	3600	1345.22	1344.99	0.02%
		ARV	3600	2451379.88	2056480.07	16.11%			ARV	3600	6580058.04	5303803.93	19.40%			ARV	3600	13564707.77	10423083.17	23.16%
		SRV	3600	3362562.16	2755750.26	18.05%			SRV	3600	8652766.54	7098566.44	17.96%			SRV	3600	18245973.24	13759061.59	24.59%
40	20	TR	4.11	2763.73	2763.73	optimal	60	40	TR	4.04	8837.77	8837.77	optimal	80	60	TR	3.10	16556.48	16556.48	optimal
		TRD	4.60	100.00	100.00	optimal			TRD	39.55	100.00	100.00	optimal			TRD	41.63	100.00	100.00	optimal
		MAR	20.13	253.36	253.36	optimal			MAR	16.43	544.64	544.64	optimal			MAR	3600	788.98	788.98	< 0.01%
		MSR	253.81	378.40	378.40	optimal			MSR	15.76	673.76	673.76	optimal			MSR	221.01	1017.71	1017.71	optimal
		ARV	3600	417180.12	352714.85	15.45%			ARV	3600	2364047.52	1975001.72	16.46%			ARV	3600	6179495.39	5179419.52	16.18%
		SRV	3600	586505.93	480759.03	18.03%			SRV	3600	3152306.87	2595720.82	17.66%			SRV	3600	8151806.06	6805555.59	16.51%

their respective smallest risk paths.

The maximum risk and risk variance objectives only perform best in the arc and segment results and even then, they only really perform individually well in the metrics that they are defined to minimize. The maximum risk objectives minimize the maximum arc or segment risk in the network in such a way that encourages origin-destination pairs to utilize much longer and riskier paths than what is necessary. While the risk variance objectives performed better than the maximum risk objectives across most metrics, they are much more difficult to solve and still do not perform as well as objectives TR and TRD in several key metrics.

If a UTM manager wants to ensure the level of demand and/or risk present on any part of their network is below some value, it would likely be more productive to use capacity constraints (given in Section 6.5) with a TR or TRD objective rather than using any of the maximum risk or risk variance objectives. If a UTM manager is still inclined to use one of these objectives, it is worth noting that we saw better results with the segment associated variants than the arc associated variants. The segment associated variants not only performed noticeably better in the segment associated metrics, but only performed slightly behind or in some cases, better than the arc associated variants with respect to the arc associated metrics. In conclusion, objectives TR and TRD produce subnetworks that are the most beneficial from the UTM manager and customer perspective and should be prioritized over the other risk objectives listed in this thesis.

To obtain insight into how the total risk and total risk deviation of a subnetwork increases as the amount of satisfied demand increases, we give two graphs which illustrate the Pareto fronts of the model with total risk and total risk deviation objectives subject to three different budgets. In each graph, the Pareto fronts associated with budgets of 40 km, 60 km, and 80 km are given for the selected objective. The path length deviations are set at 120% for all node pairs.

First, we analyze the Pareto fronts for objective TR relative to satisfied demand in Figure 7.9. The fronts associated with 40km, 60km, and 80km consist of 1200, 2050, and 2527 non-dominated solutions, respectively. As expected, the total risk incurred gradually

Figure 7.9: Pareto Fronts for the Model using Total Risk as an Objective

increases as the amount of satisfied demand increases. Each noticeable horizontal gap between points (usually) signifies the number of satisfied destinations increasing by one. The cluster of points between these gaps signifies the model satisfying a certain destination (which has slightly more demand but also incurs slightly more risk) in favour of a previously satisfied destination. Note that the slope of the graph increases as the satisfied demand increases. Early on, the destinations that incur the least amount of risk are satisfied. Typically, these are the destinations nearest to the shared origin. As more destinations are reachable on the subnetwork, the destinations that are left incur more risk to include than the ones before. This becomes evident at the end of each line where the risk increases significantly with each solution. In particular, the total risk associated with the 40km and 60km models trend upwards significantly when looking at their last few non-dominated solutions.

Next, we analyze the Pareto front for TRD relative to satisfied demand in Figure 7.10. The fronts associated with 40km, 60km, and 80km consist of 41, 111, and 91 non-dominated solutions, respectively. The graph is noticeably less dense than the previous graph. This

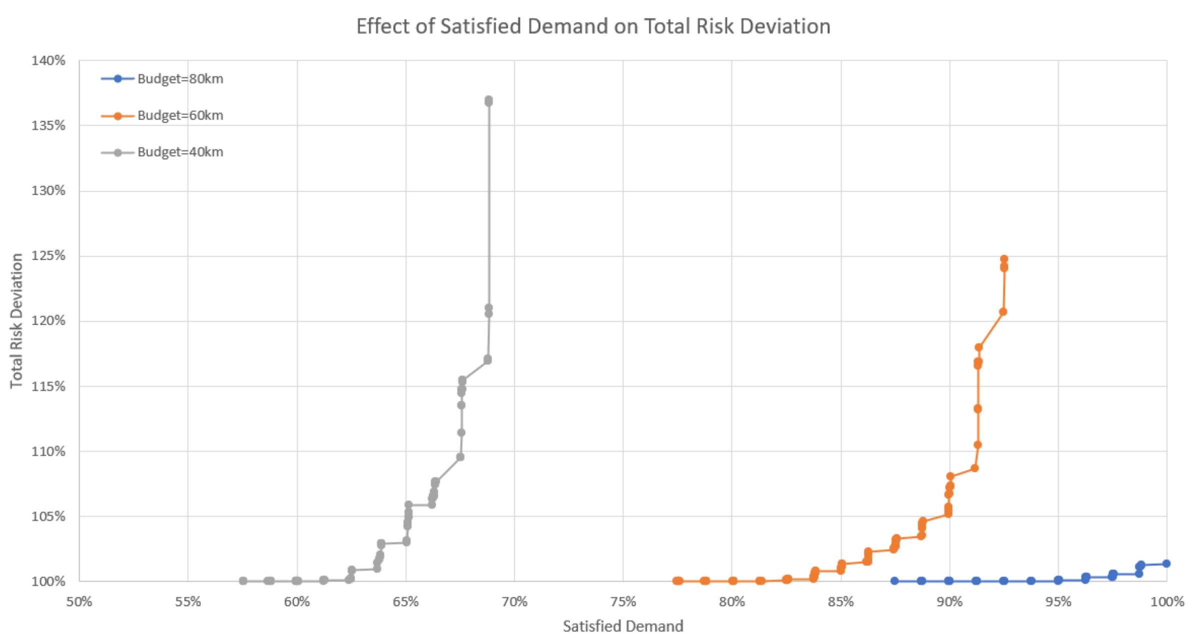

Figure 7.10: Pareto Fronts for the Model using Total Risk Deviation as an Objective

is partially because roughly 57.5%, 77.4%, and 87.5% of the demand is associated with pairs that can utilize their respective smallest risk paths for budgets 40km, 60km, and 80km, respectively. Only when the amount of satisfied demand exceeds these values, do we get some amount of positive risk deviation. The gaps and clusters of points have the same significance as they do for the Pareto fronts associated with objective TR. We do see a much sharper increase in the slope as the satisfied demand increases in comparison with TR. Initially, we can satisfy an extra $5-10\%$ demand with less than a 1% increase in risk deviation for the three models. It is the last 4% of satisfied demand that we see a drastic increase in TRD from around 103% to around 137% for the model associated with a budget of 40km. We see a similar jump in the model associated with a budget of 60km from around 105% to around 125% with only about a 3% increase in satisfied demand. At these respective maximums, the subnetworks are configured in such a way that they satisfy as much demand as possible while forcing a substantial number of pairs to utilize significantly riskier paths as a result. A budget of 80km is large enough to satisfy all demand with only a roughly 1.5% increase in TRD so we do not see the same jump in TRD that we do for the other budgets.

If a UTM manager would like to maximize as much satisfied demand as possible while ensuring that the TRD is exactly 100%, using path risk deviations (see Section 6.3) of 100% for all node pairs is more efficient than using TRD as an objective. If the level of satisfied demand that they seek is above this threshold (for this dataset, these thresholds sit at approximately 57.5%, 77.4%, and 88.5%, respectively), then minimizing TRD as an objective becomes a better way to ensure that the network is as risk adverse as it could be.

Chapter 8

Conclusion and Future Research

As drone usage continues to increase, more time and resources need to be allotted to establishing the proper infrastructure necessary to meet the large-scale demand expected in the coming decades. While there are numerous questions to be answered from a logistical perspective, the question that remains central to large-scale UAV transportation is how to properly structure the airspace. One such solution to this problem is to have a network situated above city streets that drones will be required to fly on.

Within this framework, we present a network selection model that minimizes some chosen risk metric while maximizing satisfied demand subject to a given budget. Risk is discussed within the context of UAV travel and six risk metrics are given. Additional requirements are also given for a UTM manager to tailor the UAV network to their liking. A two-stage stochastic model is presented, and a method is given to reduce said stochastic model to a deterministic problem for certain instances.

Numerical testing was conducted on both the deterministic and stochastic variants of the model. Sample average approximation was used to show that solving a deterministic variant of the model with a single aggregated demand scenario performs effectively as well as solving a stochastic variant with many demand scenarios. Several model instances were run using each risk objective and it was shown that the total risk and total risk deviation metrics vastly outperform the other metrics from both a UTM and UAV operator perspective.

Future research can be devoted to expanding the 2-dimensional framework of the network to one that is 3-dimensional where different altitudes are taken into consideration. As data is accumulated and becomes more readily available, quantitative values for other risk factors mentioned in Chapter 4 can be incorporated into the model. As detect-and-avoid technology improves, a variant of the problem can be studied where UAVs must fly within a network structure in certain regions of a city (densely populated regions with tall buildings) and can fly within a shortest path or Euclidean shortest path context in other regions of a city (less densely populated regions with more wide-open space).

References

[1] B. S. Ali. Traffic management for drones flying in the city. *International Journal of Critical Infrastructure Protection*, 26:100310, September 2019.

[2] A. Balakrishnan, G. Li, and P. Mirchandani. Optimal network design with end-to-end service requirements. *Operations Research*, 65(3):729–750, March 2017.

[3] A. Balakrishnan, T. L. Magnanti, P. Mirchandani, and R. T. Wong. Network Design with Routing Requirements. In T. G. Crainic, M. Gendreau, and B. Gendron, editors, *Network Design with Applications to Transportation and Logistics*, pages 209–253. Springer Cham, Cham, Switzerland, 2021.

[4] G. Baloch and F. Gzara. Strategic Network Design for Parcel Delivery with Drones Under Competition. *Transportation Science*, 54:204–228, January 2020.

[5] A. Bauranov and J. Rakas. Designing airspace for urban air mobility: A review of concepts and approaches. *Progress in Aerospace Sciences*, 125:100726, August 2021.

[6] United States Census Bureau. Quickfacts: Miami city, florida. https://www.census.gov/quickfacts/fact/map/miamicityflorida,=miamidadecountyflorida/POP060220, 2020. Accessed: 05 December 2022.

[7] S. H. Chung, B. Sah, and J. Lee. Optimization for drone and drone-truck combined operations: A review of the state of the art and future directions. *Computers & Operations Research*, 123:105004, November 2020.

[8] T. Cokyasar. Optimization of battery swapping infrastructure for e-commerce drone delivery. *Computer Communications*, 168:146–154, February 2021.

[9] T. G. Crainic, M. Gendreau, and B. Gendron, editors. *Network Design with Applications to Transportation and Logistics*. Springer Cham, Cham, Switzerland, 2021.

[10] C. Decker and P. Chiambaretto. Economic policy choices and trade-offs for Unmanned aircraft systems Traffic Management (UTM): Insights from Europe and the United States. *Transportation Research Part A: Policy and Practice*, 157:40–58, March 2022.

[11] M. Doole, J. Ellerbroek, V. L. Knoop, and J. M. Hoekstra. Constrained Urban Airspace Design for Large-Scale Drone-Based Delivery Traffic. 8(2):38, February 2021.

[12] R. Z. Farahani, E. Miandoabchi, W.Y. Szeto, and H. Rashidi. A review of urban transportation network design problems. *European Journal of Operational Research*, 229(2):281–302, September 2013.

[13] M. A. Figliozzi. Lifecycle modeling and assessment of unmanned aerial vehicles (Drones) CO2e emissions. *Transportation Research Part D: Transport and Environment*, 57:251–261, December 2017.

[14] Joint Authorities for Rulemaking on Unmanned Systems. Pre Defined Risk Assessment, PDRA-05 for Aerial Work operations. http://67.217.59.217/jarus/wp-content/uploads/2023/06/jar_20_doc_PDRA-05_edition-1.0.pdf, 2022.

[15] Remotely Piloted Aircraft Systems Task Force. Remotely Piloted Aircraft Systems Operational Risk Assesement. https://tc.canada.ca/en/aviation/reference-centre/advisory-circulars/advisory-circular-ac-no-903-001, 2021.

[16] M. Hassanalian and A. Abdelkefi. Classifications, applications, and design challenges of drones: A review. *Progress in Aerospace Sciences*, 91:99–131, 2017.

[17] I. Hong, M. Kuby, and A. T. Murray. A range-restricted recharging station coverage model for drone delivery service planning. *Transportation Research Part C: Emerging Technologies*, 90:198–212, May 2018.

[18] W. Hou, T. Fang, Z. Pei, and Q.-C. He. Integrated design of unmanned aerial mobility network: A data-driven risk-averse approach. *International Journal of Production Economics*, 236:108131, June 2021.

[19] D.-S. Jang, C. A. Ippolito, S. Sankararaman, and V. Stepanyan. Concepts of Airspace Structures and System Analysis for UAS Traffic flows for Urban Areas. In *AIAA Information Systems-AIAA Infotech @ Aerospace*, January 2017.

[20] T. Jiang, J. Geller, D. Ni, and J. Collura. Unmanned Aircraft System traffic management: Concept of operation and system architecture. *International Journal of Transportation Science and Technology*, 5(3):123–135, October 2016.

[21] I. Khoufi, A. Laouiti, and C. Adjih. A Survey of Recent Extended Variants of the Traveling Salesman and Vehicle Routing Problems for Unmanned Aerial Vehicles. *Drones*, 3(3):66, August 2019.

[22] S. Kim, R. Pasupathy, and S. G Henderson. A Guide to Sample Average Approximation. In M. C. Fu, editor, *Handbook of Simulation Optimization*, pages 207–243. Springer New York, New York, NY, 2015.

[23] A. J. Kleywegt, A. Shapiro, and T. Homem-De-Mello. The Sample Average Approximation Method for Stochastic Discrete Optimization. *SIAM Journal on Optimization*, 12(2):479–502, February 2002.

[24] U.-J. Lee, S.-J. Ahn, D.-Y. Choi, S.-M. Chin, and D.-S. Jang. Airspace Designs and Operations for UAS Traffic Management at Low Altitude. *Aerospace*, 10(9):737, 2023.

[25] G. Macrina, L. Di Puglia Pugliese, F. Guerriero, and G. Laporte. Drone-aided routing: A literature review. *Transportation Research Part C: Emerging Technologies*, 120:102762, November 2020.

[26] T. L. Magnanti and R. T. Wong. Network Design and Transportation Planning: Models and Algorithms. *Transportation Science*, 18(1):1–55, February 1984.

[27] R. Merkert and J. Bushell. Managing the drone revolution: A systematic literature review into the current use of airborne drones and future strategic directions for their effective control. *Journal of Air Transport Management*, 89:101929, October 2020.

[28] M. Moshref-Javadi and M. Winkenbach. Applications and Research avenues for drone-based models in logistics: A classification and review. *Expert Systems with Applications*, 177:114854, September 2021.

[29] A. Otto, N. Agatz, J. Campbell, B. Golden, and E. Pesch. Optimization approaches for civil applications of unmanned aerial vehicles (UAVs) or aerial drones: A survey. *Networks*, 72(4):411–458, March 2018.

[30] R. Pinto and A. Lagorio. Point-to-point drone-based delivery network design with intermediate charging stations. *Transportation Research Part C: Emerging Technologies*, 135:103506, February 2022.

[31] S. Poikonen and J. F. Campbell. Future directions in drone routing research. *Networks*, 77(1):116–126, September 2020.

[32] N. Pongsakornsathien, S. Bijjahalli, A. Gardi, A. Symons, Y. Xi, R. Sabatini, and T. Kistan. A Performance-Based Airspace Model for Unmanned Aircraft Systems Traffic Management. *Aerospace*, 7(11):154, October 2020.

[33] T. Prevot, J. Rios, P. Kopardekar, J. E. Robinson III, M. Johnson, and J. Jung. UAS Traffic Management (UTM) Concept of Operations to Safely Enable Low Altitude Flight Operations. In *16th AIAA Aviation Technology, Integration, and Operations Conference*, June 2016.

[34] A. Shapiro. Sample Average Approximation. In S. I. Gass and M. C. Fu, editors, *Encyclopedia of Operations Research and Management Science*, pages 1350–1355. Springer US, Boston, MA, 2013.

[35] A. Straubinger, R. Rothfeld, M. Shamiyeh, K.-D. Büchter, J. Kaiser, and K. O. Plötner. An overview of current research and developments in urban air mobility – Setting the scene for UAM introduction. *Journal of Air Transport Management*, 87:101852, August 2020.

[36] L. Stuive and F. Gzara. Airspace Network Design for Urban UAV Traffic Management with Congestion. submitted. https://uwaterloo.ca/scholar/sites/ca.scholar/files/fgzara/files/nd_with_congestion_for_utm_10-23.pdf.

[37] L. Tong, X. Zhou, and H. J. Miller. Transportation network design for maximizing space–time accessibility. *Transportation Research Part B: Methodological*, 81:555–576, November 2015.

[38] J. Xu. *Design Perspectives on Delivery Drones.* RAND Corporation, Santa Monica, CA, 2017.

[39] W. Yoo, E. Yu, and J. Jung. Drone delivery: Factors affecting the public's attitude and intention to adopt. *Telematics and Informatics*, 35(6):1687–1700, September 2018.