

Estimating Effectiveness of Countermeasures Based on Multiple Sources: Application to Highway-Railway Grade Crossings

By

Peter Young-Jin Park

A thesis

Presented to the University of Waterloo

in fulfillment of the

thesis requirement for the degree of

Doctor of Philosophy

in

Civil Engineering

Waterloo, Ontario, Canada, 2007

© Peter Young-Jin Park, 2007

AUTHOR'S DECLARATION FOR ELECTRONIC SUBMISSION OF A THESIS

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

To provide an adequate level of safety at grade crossings, Transport Canada has allocated several millions annually to prevent collisions at grade crossings through the implementation of countermeasures, such as train-actuated warning devices and track devices. Railway companies and provincial agencies have also provided additional support to improve safety at highway-railway grade crossings.

One of technical challenges in estimating safety effect of countermeasures at highway-railway grade crossing is an extremely rare occurrence of collisions. Given that the collision process is random with significant variation over time and space, it is hard to judge whether a specific crossing is safe or safer than other crossings solely based on the number of collisions in a given year. Decision makers are also required to make difficult decisions on safety investment accounting for uncertainty in effectiveness of countermeasures. The level of uncertainty is even higher when there is lack of observed collision data before and after the implementation of specific countermeasures.

This study proposes a Bayesian data fusion method which overcomes these limitations. In this method, we used previous research findings on the effect of a given countermeasure, which could vary by jurisdictions and operating conditions, to obtain a priori inference on its expected effects. We then used locally calibrated models, which are valid for a specific jurisdiction, to provide better estimates of the countermeasure effects. Within a Bayesian framework, these two sources were integrated to obtain the posterior distribution of the countermeasure effect. The outputs provided not only the expected collision response to a

specific countermeasure, but also its variance and corresponding probability distribution for a range of likely values. Some numerical examples using Canadian highway-railway grade crossing data illustrate how the proposed method can be used to predict the effects of prior knowledge and data likelihood on the estimates of countermeasure effects.

Acknowledgement

I would like to express my deepest appreciation to my thesis supervisor, Prof. Frank Saccomanno, for his inspiring, unwavering and thoughtful support over the past four years.

I have to confess that I am indebted.

I gratefully acknowledge helpful guidance and advice of Prof. Liping Fu, Prof. Bruce Hellinga, and Prof. Ian Savage. Their feedback was invaluable. I am also very thankful to Prof. David Matthews for his contribution to my model development.

I would like to dedicate this thesis to my beloved wife Catherine Eun-Sook and my two daughters Sun and Joo for their unconditional support and encouragement. Because of them, I was able to complete this work.

Table of Contents

1	INTRODUCTION	1
1.1	Background.....	1
1.2	Methodology.....	5
1.3	Research Objectives	7
1.4	Organization	8
2	LITERATURE REVIEW.....	10
2.1	Representing Countermeasure Effect.....	10
2.2	Methods for Estimating Effect of Countermeasures	11
2.2.1	Cross-Sectional Statistical Models.....	11
2.2.2	Before-After Models	14
2.2.3	Propensity Score Method	29
2.2.4	Bayesian Safety Assessment Framework.....	32
2.2.5	Summary of Methodologies	34
2.3	Type of Countermeasures.....	36
2.3.1	Crossing Closure/Grade Separation.....	36
2.3.2	Improving Crossing Geometry	37
2.3.3	Upgrading Traffic Control Devices.....	38
2.3.4	Summary of Published Countermeasure Effects	50
3	MODEL FRAMEWORK.....	52
3.1	Bayesian Data Fusion Method.....	53

3.2	Priors.....	56
3.3	Data Likelihood.....	62
4	DEVELOPMENT OF THE MODEL COMPONENTS	64
4.1	Description of Dataset.....	64
4.1.1	Canadian Inventory Data.....	64
4.1.2	Collision Occurrence Data.....	66
4.2	Development of Priors.....	67
4.3	Development of Data Likelihood.....	71
4.3.1	Factor/Cluster Collision Prediction Model.....	72
4.3.2	Stratified Collision Prediction Model.....	80
4.3.3	Collision Prediction Model with Group Indicators	92
5	Estimating Effectiveness of Selected Countermeasures.....	99
5.1	Effectiveness of Elimination of Whistle Prohibition.....	99
5.2	Effectiveness of Upgrading Warning Devices from Flashing Lights to Gates ..	113
5.3	Effectiveness of Upgrading Warning Devices from Signboards to Gates	115
5.4	Effectiveness of Four Quadrant Gates.....	116
5.5	Effectiveness of Reducing Maximum Train Speed.....	117
5.6	Effectiveness of Multiple Countermeasures.....	118
6	UNCERTAINTY IN BAYESIAN DATA FUSION.....	120
6.1	Uncertainty Inherent in Type of Distribution.....	120
6.2	Uncertainty Inherent in Priors	129
6.2.1	Uncertainty in Selecting Different Priors	129

6.2.2	Uncertainty in the Choice of Relative Weights	130
6.3	Uncertainty in the Choice of Different Countermeasures	135
7	CONCLUSIONS	139
7.1	Major Contributions	139
7.2	Contributions in Development of Data Likelihoods.....	140
7.3	Recommendations for Future Analysis	141
	REFERENCES	142

List of Tables

Table 1.1 Highway-Railway Crossing Collisions Statistics	2
Table 2.1 Estimated <i>CMF</i> from Different Before-After Models	27
Table 2.2 Summary of Methodologies to Estimate <i>CMF</i>	35
Table 2.3 Published <i>CMF</i> for Various Countermeasures.....	40
Table 3.1 Certainty Level of Previous Study	58
Table 4.1 Estimated Priors for Improvement from Signboards to Flashing Lights	68
Table 4.2 Estimated Priors for Different Countermeasures	70
Table 4.3 Summary of Variables for Factor/Cluster Collision prediction Model.....	75
Table 4.4 Cluster-Specific Collision Prediction Models Based on Factor/Cluster Analyses .	79
Table 4.5 Selected Control Factors in Canadian Inventory Data	81
Table 4.6 Summary of Variables for Stratification Model.....	87
Table 4.7 Class-specific Collision Prediction Models.....	91
Table 4.8 Reorganized Factors for Collision Prediction Model with Group Indicators.....	96
Table 4.9 Collision Prediction Models with Group Indicators.....	98
Table 5.1 The 1 st Sample Crossing Attributes for Example Calculation.....	100
Table 5.2 The 2 nd Sample Crossing Attributes for Example Calculation.....	110
Table 5.3 The 3 rd Sample Crossing Attributes for Example Calculation	113
Table 5.4 The 3 rd Sample Crossing Attributes for Example Calculation	116
Table 5.5 The 4 th Sample Crossing Attributes for Example Calculation	118
Table 6.1 Sample Mean and Variance of Prior and Data Likelihood.....	122
Table 6.2 Comparison between Normal and Beta Posterior Distribution	125

Table 6.3 Various Relative Weights (W_i) for the Prior Estimates	131
Table 6.4 Estimated Mean of Priors on Selected Countermeasures based on the Different Weighting Schemes	132
Table 6.5 Estimated Standard Errors of Priors on Selected Countermeasures based on the Different Weighting Schemes	132

List of Figures

Figure 2.1 Example Collisions per Year	17
Figure 2.2 Regression-to-the-Mean Phenomenon	18
Figure 2.3 Various Degree of Regression-to-the-Mean	21
Figure 2.4 Effectiveness of Countermeasure by the Before-After Models	24
Figure 2.5 Typical Applications of Passive Traffic Control Devices	42
Figure 2.6 Typical Pavement Markings at Highway-Railway Crossings	43
Figure 2.7 Typical Grade Crossing Illuminations	44
Figure 2.8 Typical Flashing Lights	46
Figure 2.9 Typical Two-Quadrant Gate System	47
Figure 2.10 Plan View of Four-Quadrant Gate System	48
Figure 2.11 Example of 2-Quadrant Gate with Median Separator	49
Figure 3.1 Modeling Framework.....	56
Figure 4.1 Factor/Cluster Modeling Framework.....	76
Figure 4.2 Stratification Modeling Framework.....	82
Figure 4.3 Hypothetical Tree Structure	83
Figure 4.4 RPART Result on the Basis of Control Factors.....	86
Figure 4.5 Group Indicator Modeling Framework.....	93
Figure 4.6 Hierarchical Tree Structure on the basis of RPART Method	97
Figure 6.1 Cumulative Density Functions based on the Two Different Distributions	127
Figure 6.2 Posterior Normal Cumulative Distributions based on Different Priors	130

Figure 6.3 Estimated Mean of Priors on Selected Countermeasures based on the Different Weighting Schemes	133
Figure 6.4 Estimated Standard Errors of Priors on Selected Countermeasures based on the Different Weighting Schemes	133
Figure 6.5 Posterior Normal Cumulative Distributions using Different Weighting Schemes.	134
Figure 6.6 Posterior Beta Cumulative Distributions using Different Weighting Schemes.	135
Figure 6.7 Uncertainties in Different Countermeasures	137

1 INTRODUCTION

1.1 Background

Historically, railways have been constructed across the existing highways or roads at grade to avoid the high construction cost of grade separation (Tustin et al. 1986). With development of the road network, this has led to a large number of highway-railway grade crossings. In Canada, there are more than 30,000 highway-railway grade crossings including both public and private crossings. In the US, there are more than 300,000 crossings (Saccomanno et al., 2004). As railway and highway traffic volumes increase, motor vehicle users, pedestrians and railway passengers/crews are exposed to high risk of crashes at highway-railway grade crossings. In the past ten years (1994-2003), there were a total of 2,987 crossing collisions at both public and private grade crossings in Canada, resulting in 437 fatalities and 500 serious injuries (Table 1.1). These numbers represent approximately one out of every three collisions at highway-railway grade crossings resulted in casualties and highlight a readily identifiable problem that needs attention.

Many attempts have been made over the past several decades to reduce the risk of collisions at grade crossings. Transport Canada has allocated approximately \$7 million annually to prevent collisions at grade crossings mainly through the implementation of various safety interventions, including train-actuated (i.e. active) warning devices and track devices to provide adequate warnings. Railway companies and province agencies also provided additional support to improve grade crossings safety. In addition, recently a large

number of crossings have been closed or separated. For instance, Transport Canada amended the Railway Safety Act in 1999 and developed the Grade Crossing Closure Program. This program has been initiated by the recognition that closing passive railway crossings (i.e. crossings with signs only) in Canada will improve the safety of the rail system. As a result of the amendment, the number of crossing collisions was gradually decreased in Canada over the past ten years (Table 1.1). However, as stated, because of the constantly increasing train and traffic volumes at grade crossings, more effort is needed to achieve consistent reductions in collisions at grade crossings.

Table 1.1 Highway-Railway Crossing Collisions Statistics (Drouin 2004)

Category	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	Sum
Collision Frequencies	364	346	365	307	273	283	263	278	261	247	2,987
Fatalities	57	79	46	32	39	37	33	41	46	27	437
Serious Injuries	55	56	69	60	43	45	33	47	42	50	500

The safest protection will be afforded by crossing elimination, including grade separation and crossing closure. However, for example grade separation is not sometimes feasible if railway and highway volumes are very low. Even where traffic volumes are high, there may be situations where grade separation proves to be too costly in terms of more circuitous travel and pedestrian inconvenience. Given the fact the grade crossings are an inevitable part of the railway and highway network, decision-makers need to find ways of making crossings safer. They need to provide cost-effective countermeasures that maintain

grade crossing safety within a tolerable level. The pursuit of safety at grade crossings can be expressed in terms of providing answers to two fundamental questions:

- 1) Where scarce safety funds should be directed? Which crossings have the highest risk of collisions, such that some form of safety intervention is justified?
- 2) What countermeasures should be considered to enhance safety at “hotspots (i.e. crossings with unacceptable risks)” in a cost effective and practicable manner?
(Saccomanno et al. 2006)

This research investigates and develops models to estimate the effect of different types of countermeasures on collision reduction at specific crossings with certain geometric and traffic attributes. To provide insights into cost-effective countermeasures, it is important that these models yield accurate estimates of expected collision reduction.

Technical challenges involving these tasks come from the peculiar characteristics in collisions at highway-railway grade crossings, and one of which would be the extremely rare nature of collisions (on average less than 0.1 collisions/year/crossing; Saccomanno et al., 2004). Furthermore, the collision process is random with significant variation over time and space, resulting the regression-to-the-mean (RTM) bias in evaluating countermeasure effect. In summary, the estimation of countermeasure effects has been plagued by a number of problems, including (but not limited to):

- 1) Lack of observational before and after data concerning specific countermeasures
- 2) Jurisdictional and reporting biases

- 3) Random fluctuations in collision data, resulting regression-to-the-mean (RTM) bias
- 4) Rare events (too many zero collisions)
- 5) Poor statistical model specification

On the other hand, the search for cost-effective countermeasures is a two stage process:

- 1) Which countermeasures should be considered given the nature of collisions at hotspot crossings? This stage involves an investigation of the causes and consequences of collisions expected at a given hotspot, based on experience and sound engineering judgment, supplemented by an in-depth analysis of historical collision attributes.
- 2) What are the expected collision reduction effects of selected countermeasures, applied to a specific hotspot crossing or group of hotspot crossings?

This research aims at providing insights into how to develop models that yield accurate and reliable estimates of countermeasure effects.

From an economic perspective, public monies should be spent on those safety interventions that will bring the highest safety benefit at given crossings per dollar spent. Safety benefits can be measured in the reduced number of collisions after safety intervention. A practical and analytical challenge involving this issue emerges from the multitude of tasks required to ascertain the effect of these countermeasures individually and/or in combination on reducing collisions at grade crossings. In many cases, decision

makers do not know in advance which countermeasures will be the most necessary and effective before they actually implement them. Therefore, it is an essential subject of safety studies to determine which safety countermeasures are most effective for reducing collisions of the crossings of interest.

In many cases, because of the limited time and budget issues, decision makers cannot afford conducting new and costly studies to find out the best countermeasure among many different candidate countermeasures for hotspot crossings. Instead, they may review past studies to identify similar countermeasures that have been successful to resolve their local and regional problems. Unfortunately, these approaches may not yield reliable practical results, since many previous studies normally focused on the implementation of a single or a couple of countermeasures at a time for a specific railway and highway environment. On the contrary, decision makers must know the effects of various types of countermeasure to choose the most suitable countermeasures for resolving their local crossing problems.

1.2 Methodology

To assess the potential collision reduction effects of selected countermeasures for highway-railway grade crossings, the following methods were used: 1) engineering judgment supplemented by simple statistical analysis of the historical collision data, 2) cross-sectional model analysis, and 3) before and after model analysis.

Transport Canada and the US Federal Railroad Administration (FRA) have used all of these three methods. However, there are a number of unresolved issues in existing methods that have hampered our ability to accurately predict collision reduction effect for countermeasures applied to different types of crossings. For example, a conventional cross-sectional model developed by FRA predicts the number of collisions at given crossings based on various crossing attributes. This model is adopted to evaluate the selected countermeasure effect based on the estimated collision reduction (FRA 2002). While this type of model may be appropriate for predicting collisions at crossings, it may not be suitable for investigating the effect of countermeasures. In general, the conventional cross-sectional models are hampered by a number of unresolved statistical issues inherent in observational collision dataset, including input co-linearity, omitted factor issue, etc. These issues will be addressed in this thesis, as a basis for developing a new model to assist decision-makers in assessing which countermeasures to implement for a given crossing safety problem.

This study proposes a Bayesian data fusion method to overcome the aforementioned challenges in estimating the countermeasures effect. In this modeling framework, we make use of previous research findings on the effect of a given countermeasure, which could vary by jurisdictions and operating conditions, to obtain some a priori inference on its expected effects. We then use locally calibrated collision prediction models by using Canadian crossing inventory and collision dataset, which are valid for the particular jurisdiction of our interest, to develop current best estimated effect of the countermeasure. These two sources are then systematically integrated under the proposed Bayesian framework to

obtain the posterior distribution of the countermeasure effectiveness. The outputs provide information not only on the expected collision response to a specific countermeasure but also a variance that can represent the expected collision within a specific range of likely values.

1.3 Research Objectives

This study has four basic objectives:

- 1) Review existing collision prediction models applied to both highway and railway sectors, and examine basic application issues for highway-railway grade crossings.
- 2) Develop a new method using the Bayesian framework for collision prediction and countermeasure assessment, with the analysis of uncertainty inherent in the estimated effect. This objective requires the development of countermeasure effect “priors” and “data likelihoods” and the estimates of “posterior” countermeasure effects along with their means, variance, and probability density functions.
- 3) Apply the proposed model to the Canadian grade crossing inventory and collision occurrence data. Assess the range of countermeasure effects for different grade crossing attributes, and establish probability density functions for the estimates.

- 4) Describe practical case studies for the evaluation of countermeasures with focus on uncertainty analysis.

1.4 Organization

This study develops a modeling framework using different sources of data to identify and estimates the countermeasure effect that aims to reduce collisions at highway-railway grade crossings. The contents of each chapter are;

- 1) Chapter 2 describes a review of existing methods for estimating the effect of countermeasures as well as predicting collisions at highway-railway grade crossings. The chapter also summarizes the results from the existing studies about the effect of countermeasures and their applications among different jurisdictions and regions, including Canada and the US.
- 2) Chapter 3 introduces the framework of the proposed model to estimate the effectiveness of different countermeasures to improve grade crossing safety.
- 3) Chapter 4 describes the data used in this study to develop the proposed model and the development of model components. The model is developed and validated using Canadian grade crossing data.
- 4) Chapter 5 explains the application of the proposed model to specific crossings and evaluates the effects of the selected countermeasures.
- 5) Chapter 6 explains the analysis of uncertainty inherent in evaluating countermeasures using the proposed Bayesian data fusion method.

- 6) Chapter 7 summarizes the main findings and conclusions of the research, and provides the recommendation for further work. The chapter also describes the main contributions of the work and a decision-support platform for improving safety at grade crossings.
- 7) Appendices contain the outputs of collision prediction models developed in this study as well as some collision prediction models developed in the past studies.

2 LITERATURE REVIEW

A wide variety of statistical methods have been proposed in the literature to estimate the countermeasure effect. In this chapter we review the most popular methods in evaluating countermeasures. These methods include the cross-sectional statistical model and the before-and-after method, and so on. The advantage and disadvantage of each method are also discussed. These are followed by the formal definition of countermeasure effect.

2.1 Representing Countermeasure Effect

Laughland et al. (1975) introduced the concept of Collision Modification Factor (*CMF*) to reflect the safety benefits associated with different countermeasures and to represent the expected changes in collisions after the implementation of countermeasures. The *CMF* can be expressed as the ratio of the expected (or observed) number of collisions after to before a countermeasure is introduced at a given site, such that:

$$CMF_i = 1 - \left(\frac{N_{Bi} - N_{Ai}}{N_{Bi}} \right) = \frac{N_{Ai}}{N_{Bi}} \quad (2.1)$$

where, N_{Bi} and N_{Ai} represent the number of estimated (or observed) collisions per year at a site before (or without) and after (or with) a safety countermeasure ‘*i*’, respectively, and $CMF_i \in [0, \infty)$.

In the above expression, the estimated *CMF* does not produce any negative values. The value greater than 1.0 reflects that the number of collisions increases after a

countermeasure is introduced. A value less than 1.0 reflects a reduction in the number of collisions.

The FHWA developed a series of *CMF* for two-lane rural highways (Harwood et al. 2000, Zegeer et al. 1992). The forthcoming Highway Safety Manual (Hughes et al. 2004, Harkey et al. 2005) will provide a series of *CMF* to reflect the effect of different design and operational strategies applied to highways.

In the highway-railway grade crossing field, the term *CMF* has not been used extensively. Instead, many researchers have preferred to use the expected reduction in collisions resulting from a given safety intervention or countermeasure (Farr 1987, Federal Railroad Administration 2002, Saccomanno and Lai 2005). While these two terms are similar, the term *CMF* will be used to be consistent with the road safety research convention in this study.

2.2 Methods for Estimating Effect of Countermeasures

Over the past several decades, various collision prediction models have been developed to estimate the effectiveness of countermeasures in transportation studies, including expert judgments, before-after and cross-sectional models. In this section, these models are reviewed.

2.2.1 Cross-Sectional Statistical Models

Researchers in transportation safety fields have applied conventional statistical models, such as regression models, to predict the changes in the number of collisions at a given site

after the introduction of countermeasures. These cross-sectional models investigate the differences in safety among different sites, which did not experience any major changes within the period of analysis.

In the highway safety field, Council and Stewart (1999) and Zegeer and Council (1995) applied cross-sectional models to evaluate different types of road safety countermeasures. More recently, a functional type of *CMF* for horizontal curves in two-lane rural highways has been included in the US Highway Safety Manual (Hughes et al. 2004). The *CMF* was originally developed by Zegeer et al. (1992) based on a conventional regression technique.

In the highway-railway grade crossing field, Schoppert and Hoyt (1968) investigated earlier collision prediction models, including the Peabody Dimmick Model (1941), the New Hampshire Index (1971) and NCHRP Hazard Index (1968). Coleman and Stewart (1976) also developed collision prediction models for grade crossings. All these models lack descriptive capabilities due to their limited number of explanatory variables. They also plagued by a number of statistical problems, including including co-linearity, poor statistical significance, and parametric biases.

Compared to the above earlier models, the US DOT Model (Farr 1987, Federal Railroad Administration 2002, Mengert 1980) comprehensively addresses explanatory variables that may influence safety at highway-railway crossings. However, the model also failed to resolve the co-linearity issues among the explanatory variables. Moreover, some factors in the US DOT model were not readily available in Canadian crossing inventory

dataset. For example, the number of through trains per day during daylight and the number of highway lanes are examples of two inputs in the US DOT model that were not available in Canadian crossing inventory dataset. On the other hand, several interesting factors that may be important in explaining the Canadian grade crossing collisions, such as whistle prohibition and track angle, were not included in the US DOT model. As pointed out by Saccomanno et al. (2004), since there are significant differences in the inventory data structure between Canada and the US, it is hard to apply the US DOT model to Canadian crossings.

More recently, Austin and Carson (2002) developed collision prediction models using negative binomial expressions based on the US crossing inventory and collision dataset. Their model is much simpler than the previous US DOT models and therefore it easier to interpret the model results. The model used an “Instrumental Variable” technique mainly to overcome the co-linearity issues in conventional cross-section models. However, the effect of several factors is still counter-intuitive. For example, the presence of stop signs, flashing lights or bells was found to increase the predicted collision frequency; findings that contradict conventional wisdom as to the expected effect of these countermeasures. Appendix A includes all the aforementioned cross-sectional collision prediction models.

A number of methodological issues need to be resolved before utilizing cross-sectional models for estimating the effect of selected countermeasures on collisions at specific grade crossings. These include:

- 1) Lack of statistical significance of explanatory variables.

- 2) Presence of co-linearity leading to counter-intuitive results.
- 3) Failure to interaction effects in countermeasure mix.
- 4) Jurisdictional reporting biases introduced when transferring models to areas or time periods which were not part of data used in calibration.

Recently, more sophisticated multi-stage cross-sectional models have been developed and applied to safety analysis of Canadian grade crossing (Saccomanno and Lai 2005, Park and Saccomanno 2005a, Park and Saccomanno, 2005b). These models attempt to resolve many of the above issues associated with the conventional single stage cross-sectional models. Multi-stage models first classify the crossing data into groups with similar physical and operational attributes. Separate collision prediction models are then developed for each group. The three multi-level models developed for the Canadian grade crossing data by Park and Saccomanno (2005a, 2005b) and Saccomanno and Lai (2005) will be discussed in details in Chapter 4 within the context of the proposed Bayesian prediction model introduced in this thesis.

2.2.2 Before-After Models

Before-after models have been widely used to estimate countermeasure effects in the transportation safety field. The approach analyzes the sites with only one or more improvements, while other characteristics are remained the same. In this model structure, the effect of a given countermeasure is determined by comparing predicted or observed

number of collisions after the countermeasure is introduced to the number of collisions had there been no countermeasure (Hauer 1997, Persaud 2001).

Two types of before-after models have been commonly cited in the literature: naïve and empirical Bayesian (EB) models. One of the major problems associated with the ‘naïve’ before-after model is regression-to-the-mean (RTM) bias. This refers to the situation where safety countermeasures are normally applied to those sites with a high number of observed collisions. The subsequent reduction in collisions following the countermeasure is then assigned fully to the countermeasure effect. However, given the random nature of collisions, the frequency of collisions is more likely to drop from previous high levels notwithstanding the introduction of countermeasures. As noted by Council et al. (1980), the average collision frequency approaches to the mean over the long term in spite of high frequencies of collisions in certain years. This phenomenon is commonly referred to Regression-to-the-Mean (RTM) bias.

- **Regression to the Mean (RTM)**

RTM bias is a form of treatment selection bias, arising when the classical statistical assumption of random sampling is violated (Park and Saccomanno 2007, Pendleton 1991). The phenomenon is a principle stating that of related observations, and selecting those where the first observation is either higher or lower than the average, the expected value of the second is closer to the long term and/or population mean than the observed value of the first. This does not necessarily mean that if the first observation taken is above the average (or below the average), the second will always move towards the population average, but that there is a tendency to do so. When safety analysts estimate countermeasure effects via

a before-and-after model without properly taking into account RTM bias, they may not observe an actual effect of countermeasures.

To illustrate, we first refer to a hypothetical collision example at a highway intersection from Council et al. (1980). As illustrated in Figure 2.1, the average number of collisions over the 10-year period (1969-1978) is assumed to be 20 with some fluctuation in values 8 to 32 collisions/year. If we introduce a safety countermeasure in 1973 in response to the large number of collisions experienced in 1972 (i.e. 32 collisions), the estimated percentage of collision reduction for a given 2-year study period (1972-1974) via a simple before-and-after model at the end of 1974 would be estimated as 50% [= $(32-16)/32 \times 100$] since the observed number of collisions was 16 in 1974.

It is reasonable to believe that some portion of the collision reduction has been introduced due to the intervention of the safety countermeasure. However, given that the long-term average number of collisions is approximately 20 collisions per year, we can recognize that much of the collision reduction has been essentially generated not because of the intervention of a safety countermeasure but because of the effect of RTM. A significant amount of collision reduction would have occurred in the absence of any change in intervention at the site. To simplify this hypothetical illustration, the number of collisions in 1972 (i.e. 32 collisions) has been assumed to be a good estimate of the long-term average collisions per year at the site in the before-countermeasure period in the absence of the actual introduction of a countermeasure. Similarly, the number of collisions in 1974 (i.e. 16 collisions) was assumed to be the best estimate of the long term average collisions per year

in the after-countermeasure period with the introduction of the actual safety countermeasure.

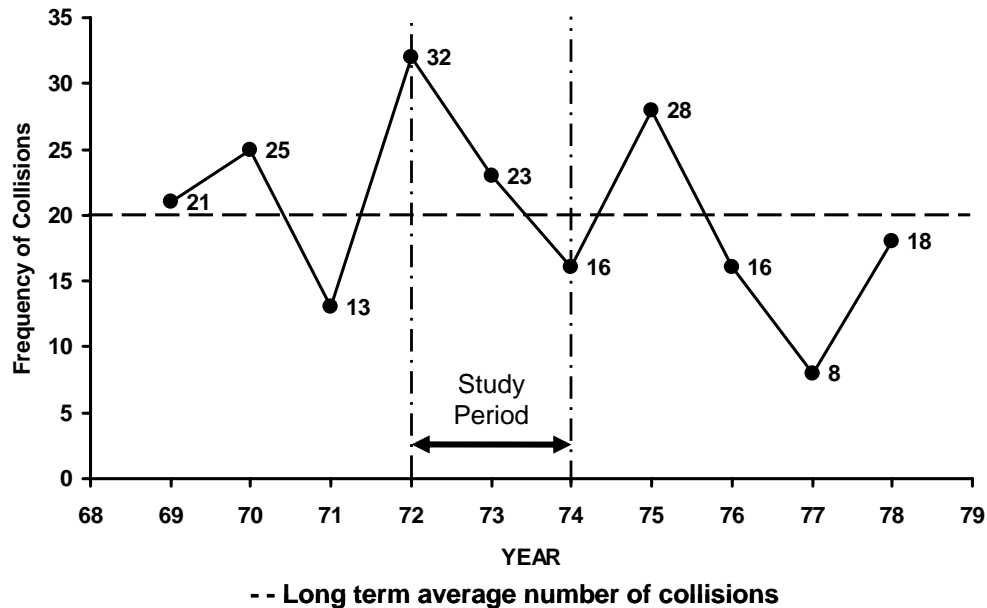


Figure 2.1 Example Collisions per Year (Council et al. 1980)

To clarify why the RTM bias occurs, we consider a second hypothetical collision frequency distributions as shown in Figure 2.2. The bell-shaped distributions in the figure are for demonstration purposes and do not represent the genuine distribution associated with collision frequencies at this site.

Presume that we selected the highest 10% of the sites (e.g. at grade crossings) based on the collision history of the sites at time period ‘ t_1 ’ [i.e. the shaded part of the Figure 2.2-a)] for which we introduce a given countermeasure. What would be the chance that the exactly same sites will once again constitute the same highest 10% in collision frequency distribution in a future time period ‘ t_2 ’. The answer is very low or unlikely. Perhaps, some

of the sites will still in the same highest 10% at future time period 't₂', but many the sites will no longer be in the same highest 10% at time period 't₂'. Even though just a few sites that belonged to the highest 10% at time period 't₁' are not included in the same percentage group in time period 't₂', the mean value of the selected sites will tend to the population mean at time period 't₂'. As we can see in Figure 2.2-b), the same argument is possible on the other extreme (i.e. selecting sites in the lowest 10% of the collision history for time period 't₁'). The degree of the movement in the average number of collisions between the time period 't₁' and the time period 't₂' represents the degree of RTM.

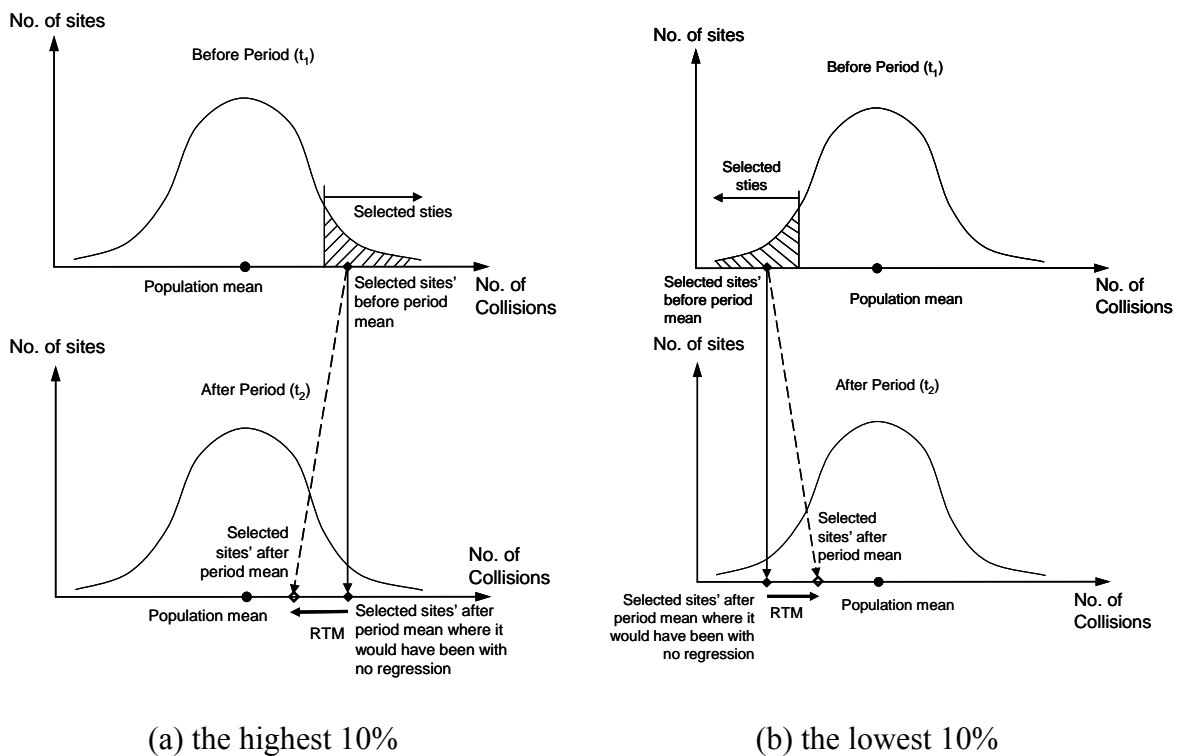


Figure 2.2 Regression-to-the-Mean Phenomenon

In the upper part of the Figure 2.2 (a), the average numbers of collisions at time period "t₁" by the selected sites is considerably higher than the overall population mean [or

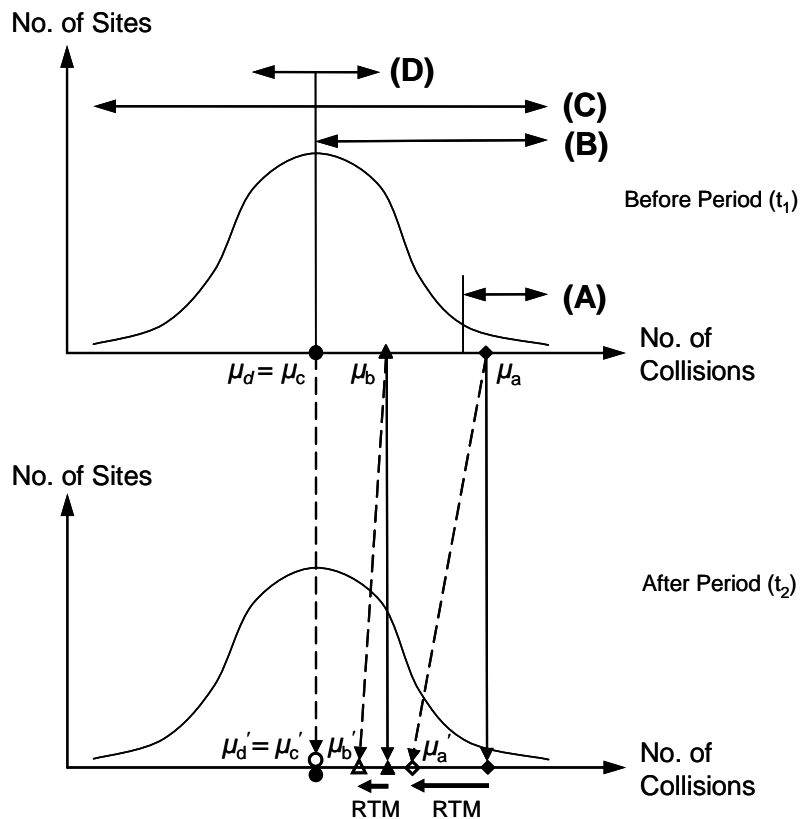
lower for sample in Figure 2.2 (b)]. In the absence of RTM bias, we would expect same collision reduction effect for the countermeasure and crossing attributes in the future time period 't₂'. Inasmuch as the collision occurrence is by nature a rare random event and RTM is inevitable, the collision frequencies at these selected sites in the future time period would be lower than indicated above notwithstanding safety countermeasure intervention.

Figure 2.3 demonstrates the various degree of RTM phenomenon. Again, the graph does not reflect the actual distribution of collision frequency. In the upper part of the figure, ' μ_a ', ' μ_b ', ' μ_c ', and ' μ_d ' represent the average number of collisions in before time period 't₁' based on the information from the sample sites selected from the range (A), (B), (C), and (D), respectively. The ' μ_a' ', ' μ_b' ', ' μ_c' ', and ' μ_d' ', on the other hand, represent the average number of collisions in the after time period ('t₂') using the same sites' information that were selected in the before time period ('t₁'). If there were no RTM bias, the selected sites' before-period (t₂) average numbers of collisions (i.e. μ_a , μ_b , μ_c , and μ_d) would be exactly same as the same sites' after-period (t₁) average values (see the black circle, black triangle, and black diamond in the figure).

The degree of RTM bias would be the highest for those sites with highest before period collision history as in range (A). The degree of RTM would be relatively lower for sites in range (B), which represents the sites with the collision frequencies between the modest and the highest. If we select sites randomly from the entire collision frequency distribution [i.e. range (C)], RTM bias would not be a problem since the mean number of collisions at the sample sites would be identical to the mean value of the population either in the before period or in the after period. From this hypothetical figure, we notice that if

we select sites for treatment more and more randomly, then more and more the degree of the RTM bias will be reduced.

When new safety countermeasures (e.g. ITS technology) become available, there is an understandable desire for the engineers to try out these new countermeasures on their worst clients or sites with highest collision history. These are sites in range (A) in Figure 2.3. However, because of the confounding effect of the RTM bias, the genuine effect of this new countermeasure would be difficult to estimate and an unexpectedly strong collision reduction effect is obtained (Hauer 1986). In simple terms, the collision reduction effects could be wrongly attributed to the countermeasure, where much of the effect could be due to RTM. It is worthwhile to note that, Morton and Torgerson (2005) suggested to evaluate new countermeasures based on a 'samples' from range (D) (e.g. the average number of collisions) in order to reduce the RTM bias. Their suggestion is based on the experiences in the field of clinical experiments.



- △ ◇ ; Selected sites' after period mean
- ▲ ◆ ; Selected sites' before period mean = Selected sites' after period mean where it would have been with no regression

Figure 2.3 Various Degree of Regression-to-the-Mean

- **Naïve and Empirical Bayesian Before-and-After Models**

The naïve before-after model ascribes the full reduction in collisions to the countermeasure being considered. Since it fails to consider nature of the non-random assignment of countermeasure and RTM bias, the application of 'naïve' before-after analysis tends to over-estimate the countermeasure effect.

A number of transportation safety researchers have suggested using a more reliable statistical technique known as Empirical Bayesian (EB) method. The application of the EB method attempts to avoid the over-estimation caused by RTM bias (Abbess et al. 1981, Wright et al. 1988, Mountain et al. 1992).

For instance, in the highway safety field, Al-Masaeid (1997), Bahar et al. (2004), Elvik et al. (2001), Lyon et al. (2005), and Persaud et al. (2001) have employed the EB before and after analysis to assess the impact of selected treatments and to produce more reliable *CMF* with the reduced RTM bias. For the application to highway-railway grade crossings, Hauer and Persaud (1987) conducted a representative EB before-after study using US data to demonstrate the effectiveness of selected warning devices, such as flashing lights and gates. More recently, Park and Saccomanno (2007) examined the applicability of this method to Canadian grade crossing dataset. They noted that the Canadian data may not be suitable for this type of approach because of the lack of observed collisions in the crossing data, a problem referred to as “too many zero collisions”.

To predict collisions in the before period, the EB technique is employed to combine the actual observed number of collisions at the site with the expected number of collisions at the same sites as obtained from statistical prediction models. This procedure is described mathematically as follows (Hauer 1997):

$$E\{\kappa|K\} = \gamma \cdot E\{\kappa\} + (1-\gamma) \cdot K \quad (2.2)$$

where,

$E\{\kappa|K\}$ = the EB adjusted estimate of the expected number of collisions per year at the study site before countermeasure implemented.

$E\{\kappa\}$ = the expected number of collisions per year at the study site from collision prediction models (e.g. negative binomial models) before countermeasure implemented.

K = the actual observed number of collisions per year at the same site before countermeasure implemented.

$$\gamma = \frac{1}{1 + \frac{Var\{\kappa\}}{E\{\kappa\}}}, \text{ the weight factor estimated as a function of } E\{\kappa\} \text{ and } Var\{\kappa\}.$$

If we assume that the observed collisions for a site (K) is Poisson-distributed and the expected number of collisions for the site (κ) are gamma-distributed, the distribution of the entire probability distribution of $\kappa|K$ becomes Negative Binomial (NB) distribution. In this case, we obtain an over-dispersion parameter (ϕ) to represent the degree to which variance in collision frequencies deviate from Poisson assumption, as such;

$$\gamma = \frac{1}{1 + \frac{E\{\kappa\}}{\phi}} \quad (2.3)$$

After obtaining the EB estimate, conversion factors are applied (e.g. traffic volume changes) to predict the number of collisions at the same site for the after-period assuming no countermeasure introduced. The countermeasure effect is simply represented by the difference between this (e.g. volume and time, etc.) adjusted EB estimates and the observed number of collisions in the after period.

Figure 2.4 illustrates the difference in the estimated effectiveness of a countermeasure between the naïve and the EB before and after models. As stated, a before-and-after model requires to 1) estimate what was the expected number of collisions of each treated site in the before period, and 2) to predict how the estimates in 1) would have changed in the after period due to changes in all other relevant factors (e.g. traffic volume, etc.) if there was no treatment. In this example, the duration of before period and the after period are assumed to be the same (e.g. 1 year in each stage for a range 2 years). A strong assumption of constancy of all other relevant factors with collisions (e.g. AADT, number of daily trains, vehicle and train fleet, driver demography, weather, etc.) during the entire study period is also introduced to simplify this illustration.

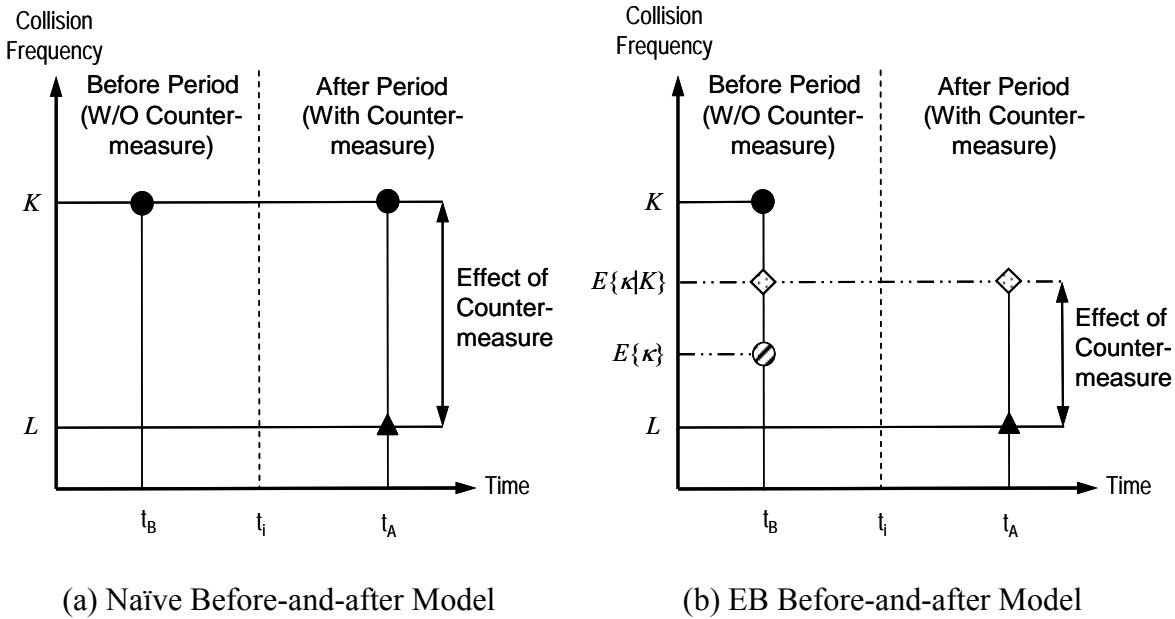


Figure 2.4 Effectiveness of Countermeasure by the Before-After Models

Figure 2.4 (a) shows the estimated effectiveness of safety countermeasure by means of the naïve before-and-after model. In the figure, a year ‘ t_i ’ represents the year that a given

countermeasure is introduced. The ‘ K ’ and ‘ L ’ represents the observed number of collisions in one year before and after the countermeasure introduction, respectively. Since we assumed the constant status of every other relevant factor during the entire study period, the very best estimate of the expected number of collisions in the after period without the safety intervention would be the “ K ”. As noted above, the naïve before-after model attributes the full reduction in collisions to the countermeasure being introduced. As a result, the estimated collision reduction would be represented by the ratio between ‘ K ’ and ‘ L ’ based on the Equation 2.1. However, as we depicted in Figure 2.2 and 2.3, if the site has been selected for the safety intervention due to its abnormally high number of collisions in before period, the estimated effect of the countermeasure will be hard to be isolated from the confounding RTM bias. As a result, the estimated effect would be higher than that it should be.

Figure 2.4 (b) shows how the empirical Bayesian technique contributes to reduce the over-estimation problems in evaluating countermeasure under the framework of EB before and after model. As indicated by Equation 2.2, the safety of a site is estimated using two sources of information: 1) information from sites that have the same characteristics (i.e. $E\{\kappa\}$), 2) information from the actual site the method is being applied (i.e. K). The $E\{\kappa\}$ reflects the selected group mean based on the characteristics of sites (i.e. covariates, input factors) in a collision prediction model. The EB technique pull down the higher value of ‘ K ’, which was considered as the best safety estimate for the before period in the naïve before and after model, toward the lower group mean value (i.e. $E\{\kappa\}$). This ‘regressing’ toward the mean is exactly what is achieved from Equation 2.2 (namely $E\{\kappa|K\}$). Again,

since we assumed all other factors remain constant during a given study period, $E\{\kappa|K\}$ becomes the best estimate of the expected number of collisions in the after period following the introduction of the countermeasure. The estimated effectiveness of countermeasure via an EB before and after model is lower than suggested from a naïve approach. As a result, some of the RTM bias has been removed.

In highway-railway grade crossing field, Hauer and Persaud (1987) introduced the EB before-after model to estimate the effectiveness of selected warning devices. They compared their study results to the previous study findings, which were developed based on a naïve before-after model, and found consistent over-estimation of the estimates in the previous studies. They asserted that the higher estimates were due to the unadjusted RTM bias inherent in the historical data.

Table 2.1 summarizes their study findings. Table 2.1 summarizes several CMF values for three countermeasure applied to grade crossings in the US. The two approaches illustrated are five naïve before and after models and one EB before and after model introduced by Hauer and Persaud (1987). For instance, for the estimated effectiveness of upgrading from signs to flashing lights, the collision reduction percentages from the five naïve before and after models were estimated between 64 and 71%. On the other hand, the result from an EB before and after model was only 51%. The over-estimation by the naïve approach for this specific countermeasure was about between 13 and 20%. Similar patterns have been found on the estimated effectiveness involving the other two countermeasures (i.e. upgrading from signs to 2-quadrant gates, upgrading from flashing lights to gates).

Table 2.1 Estimated *CMF* from Different Before-After Models (Hauer 2005)

Researchers	California P.U.C. (1974)*	Morrisey (1981)	Coleman (1982)	Eck and Halkias (1985)	Farr and Hitz (1985)	Hauer and Persaud (1987)
Modeling Approach	Naïve before-after					EB before-after
From Signs to Flashing Lights	0.36 (64)**	0.35 (65)	0.29 (71)	0.31 (69)	0.30 (70)	0.49 (51)
From Signs to 2-Quadrant Gates	0.12 (88)	0.16 (84)	0.18 (82)	0.16 (84)	0.17 (83)	0.21 (79)
From Flashing Lights to 2-Quadrant Gates	0.34 (66)	0.36 (64)	0.31 (69)	0.28 (72)	0.28 (72)	0.40 (60)

Note) * represents the published year of the study; ** represents the collision reduction percentage.

In general, there are two major advantages in using an EB before and after approach to estimate countermeasure effects. The EB model:

1) Reduces or eliminates much of the RTM bias inherent in the naïve approach, and yields more reliable estimates of *CMF*.

2) Takes into account changes in traffic volume in before-after study period. This traffic volume change is used as a surrogate variable to represent the observed and unobserved factors of study sites, such as the unobserved weather conditions.

The EB before and after approach has been adopted as a main safety evaluation tool by a number of US and Canadian agencies, such as Interactive Highway Safety Design Model and the Highway Safety Manual. (Harkey et al. 2005, Harwood et al. 2000, Hughes et al. 2004)

Notwithstanding its relative success in resolving RTM bias, EB before-after models still have some shortcomings as follows:

1) EB before-after models require large amounts of data, such as yearly-based exposures over a given study period. This increases the cost and time of the analyses. However, researchers often do not have the resources to collect the necessary inputs for their analysis and are therefore forced to produce results with incomplete inputs. For example, the highway-railway grade crossing inventory databases in Canada and the US do not contain the yearly-based exposures (e.g. yearly-based traffic volumes) for individual crossings. This makes difficult to use an EB approach.

2) EB before-after models require a strong assumption that the effect of all unobserved factors can be explained simply by changes in traffic volume between the before and after periods. However, there may be more factors (other than traffic volumes) related to collision frequency independently or in combination with other factors - for example reporting biases. While the EB before-after analysis itself does not require this assumption, most researchers consider traffic volume as the only input factor.

3) It does not estimate the effects of several countermeasures simultaneously. The model has been applied to consider one countermeasure at a time.

4) It produces an average effect rather than tailored effect for crossings of interest. Usually, decision makers are required to resolve problems targeted at specific sites (e.g. crossings). In that case, the average effectiveness of a countermeasure may not be enough to resolve their isolated issues. Evaluating individual countermeasures for local application using before-after models will require an extensive amount of time and resources, and is therefore unachievable.

The Canadian crossing dataset, which is the major source of analysis in this study, does not contain the necessary data to develop accurate EB before-after models. Moreover, the collision occurrences experienced at Canadian grade crossings for the past ten years are extremely rare events (i.e. less than 0.1 collisions/ year/crossing). Without considering this problem, the results can produce substantial bias in the estimate of *CMF* and this will hamper the reliability of EB before-after results (Lord 2006, Park and Saccomanno 2006).

2.2.3 Propensity Score Method

Often previous researchers have treated countermeasures as the exogenous variables in their modeling expression. However, some of the previous researchers including Kim and Washington (2006) have attempted to address the countermeasure selection bias by introducing the concept of endogeneity and to account for endogenous relationships to better understand the true effects on collisions of various kinds of countermeasures. Based on the accident analysis results using the highway intersection data from the state of Georgia, they argued that the often reason of the inconsistent results in the evaluation of the same countermeasure is the lack of the control for the potential endogeneity problem between collision rates and countermeasures. In highway-railway grade crossing field, Austin and Carson (2002) recognized that the presence of warning devices (e.g. flashing lights, gates) is potentially endogenous to collision rates because collision rate is often used to warrant the installation of warning devices such as flashing lights or gates at grade crossings.

To remove countermeasure selection bias, we need to know how specific sites (e.g., grade crossings) are selected for different types of countermeasure. However, the aforementioned before-and-after and cross-sectional models do not specifically address the question as to why specific crossings have been selected for countermeasure. As a result, their estimates are biased by non-randomness in the countermeasure selection process. One such problem is referred to as RTM bias, which results from the fact that countermeasures are normally applied to high collision history crossings. The countermeasure selection bias still can occur if there is a systematic bias in the selection of crossings for improvements. For instance, if all the selected sites experienced more than a certain amount of train speed or a minimum number of tracks at a specific crossing, these selection criteria could also produce a systematic selection bias.

In an observational study including a traffic safety study, randomized experiments are commonly prohibited. As a result, as noted previously observational data are often “contaminated” by sampling bias. To avoid such bias, researchers have proposed to randomize observation sites (i.e. crossings) based on site specific values called the “propensity scores (PS)”. PS estimates the likelihood of a given site being treated and these scores are used to identify the sites with equal likelihood of being treated (Rosenbaum and Rubin 1983, 1984).

The propensity score is defined as the conditional probability of countermeasure for a given site (i.e. crossing) attributes as follows:

$$e(\mathbf{x}) = \Pr(Z_i = 1|\mathbf{x}) \tag{2.4}$$

where,

$e(\mathbf{x})$ = scalar vector of propensity score.

Z_i = countermeasure indicator ($Z_i = 1$ if treated, $0 =$ otherwise)

\mathbf{x} = vector of pre-treatment attributes

Rosenbaum and Rubin (1983) established analytically that:

$$\mathbf{x} \perp Z_i | e(\mathbf{x}) \quad (2.5)$$

Equation 2.5 suggests that the vector \mathbf{x} is conditionally independent of countermeasure (Z_i) given propensity scores [$e(\mathbf{x})$]. As a result, for a given propensity score, each individual observation has the same probability of being treated.

The average countermeasure effect (δ) can be estimated as:

$$\delta = E\{E(Y_i | \mathbf{x}, Z_i = 1) - E(Y_i | \mathbf{x}, Z_i = 0)\} \quad (2.6)$$

where,

Y_i = outcome variable (e.g. the number of collisions after treatment period)

The propensity score [$e(\mathbf{x})$] is commonly estimated from a cross-sectional logistic expression of the form as follows:

$$\log \left[\frac{\Pr(Z_i = 1 | \mathbf{x})}{1 - \Pr(Z_i = 1 | \mathbf{x})} \right] = \beta_0 + \beta_1 \mathbf{x} \quad (2.7)$$

where, β_0, β_1 = parameters of pre-countermeasure covariates.

Currently, this method has been routinely used in a number of observational studies in various disciplines, including medicine, economics, finance, and education (D'Agostino 1998, Dehejia and Wahba 2002, Yanovitzky 2005). In transportation studies, Aul and Davis (2006) applied this PS method to estimate the effect of traffic signal installation on highway intersections.

In highway-railway grade crossing field, Park and Saccomanno (2007) used this method to estimate the effect of different warning devices at Canadian crossings. Two types of warning devices with passive signs were considered for crossings: flashing lights and gates. They found that the PS model reduced diverse systematic selection bias, including RTM bias. PS method takes into account various treatment selection criteria (e.g. exposure, train speed, track type, collision history) when the propensity scores are estimated. In generally, the PS method is relatively simple to apply and effective in reducing countermeasure selection bias, including RTM bias.

The PS method has not been adapted and tested in any major transportation study with exception of Aul and Davis (2006) and Park and Saccomanno (2007). Similar to the before-after models, this method can produce the effect of only one countermeasure at a time.

2.2.4 Bayesian Safety Assessment Framework

Bayesian Safety Assessment Framework (B-SAF) has been developed to statistically combine experts' opinions and findings of previous study to assess the effectiveness of the

given countermeasure. This method combines experts' "subjective" knowledge and judgment with "objective" information obtained from empirical studies to yield meaningful "posterior" collision estimates (Clarke and Sarasua 2003, Melcher et al. 2001, Washington et al. 2002).

Since the B-SAF method does not require any formal experiments or analyses to estimate the effectiveness of a specific countermeasure, the method can be a useful tool for safety engineers to estimate countermeasure effects that have not yet been applied in real world. For these estimates, we rely on expert judgment concerning the possible effect of the untested countermeasure. However, this approach has the following limitations:

1) While expert opinions have been commonly used in transportation safety applications to produce *CMF* of countermeasures (e.g. Harwood et al. 2000, Harkey 2005), it is hard to convince that the experts' judgment is consistent and sound. Especially, the results from this method rely heavily on experts' opinions and the availability of qualified professionals. However, the information obtained from experts varies by experts' expertise, and experience.

2) Combining experts' opinions with the information obtained from past studies is challenging. More specifically, experts' opinions are assumed to follow discrete (ordinal) distributions, and the previous knowledge from literature is assumed to follow continuous distributions. Combining these two different types of distributions is valid only if the discrete (ordinal) distribution can be converted to the continuous distribution and merged with another continuous distribution.

Recently, Washington and Oh (2006) used this method to rank various countermeasures based on the estimated safety benefits at grade crossings. Since they recognized the inherent limitation in this method, they did not recommend this method to produce any quantified *CMF* of countermeasures.

2.2.5 Summary of Methodologies

There are several methods to estimate the effectiveness of countermeasures (e.g. *CMF*) in terms of the changes in the number of collisions after a countermeasure is implemented. These methods differ in theoretical foundation and principles, and have advantages and disadvantages, as summarized in Table 2.2. Unfortunately, it seems that there is no flawless model that provides accurate *CMF*, especially for collision data with zero collisions at a majority sites. Therefore, we need a model that can provide more formal information regarding the uncertainty inherent in the estimated *CMF* of a given countermeasure.

Table 2.2 Summary of Methodologies to Estimate *CMF*

Model Type	Advantage	Disadvantage
EB Before-after Model	<ul style="list-style-type: none"> • Well established theoretical background • Have a long history of successful applications in transportation field • Reduces the RTM bias 	<ul style="list-style-type: none"> • Requires a great amount of dataset for conducting appropriate analysis • May not be suitable for analysis of the dataset with too many zero collisions • Produces effectiveness of one countermeasure at a time • Produces only the average effectiveness of countermeasures
Multi-stage Cross-sectional Model	<ul style="list-style-type: none"> • Reduces the co-linearity problems and relevant statistical issues inherent in single-stage cross-sectional models • Estimates several countermeasures effects simultaneously • Can provide sensitivity analysis of countermeasures • Can estimate varying effect of countermeasures based on the attributes of given sites 	<ul style="list-style-type: none"> • Requires relatively high level of understanding of statistics, therefore it is difficult to develop • May produce inconsistent results among different multi-stage models • May not be a main-stream approach in transportation safety field
Propensity Score Method	<ul style="list-style-type: none"> • Long history of successful applications in many other disciplines, including medicine, economics, etc. • Relatively easy to analyze using many statistical software. • Reduce systematic selection bias • Takes into account various treatment selection criteria 	<ul style="list-style-type: none"> • Lack of application in transportation safety field • Estimates effect of one countermeasure at a time • Produces only the average effect of countermeasures
Bayesian Safety Assessment Framework	<ul style="list-style-type: none"> • Can assess safety countermeasures that have not been applied in real world • Can produce results with relatively small amount of dataset • Consider subjective experts' opinions in assessing countermeasures effect 	<ul style="list-style-type: none"> • Requires high-level understanding of statistics • Difficult to obtain sound expert opinions, and therefore hard to estimate reliable <i>CMF</i>

2.3 Type of Countermeasures

This section reviews various countermeasures cited in literature such as Manuals of Uniform Traffic Control Devices for Canada (TAC 1998) and the US (FHWA 2003), and the Canadian Road/Railway Grade Crossing Detailed Safety Assessment Field Guide (Transport Canada 2005).

There are generally three different types of countermeasures that engineers can use to make crossings safer: 1) crossing closure or grade separation, 2) improving crossing geometry, and 3) upgrading traffic control devices.

2.3.1 Crossing Closure/Grade Separation

Grade crossing closure and grade separation will have the same effectiveness of countermeasures because the both countermeasures can prevent entire collisions between train and motor-vehicle by eliminating exposures at a crossing. However, these two countermeasures have distinct characteristics.

Obviously, the grade separation will require higher construction cost than any other countermeasures due to potential crossing relocation. On the other hand, the crossing closure might be the lowest cost countermeasure that requires nothing to install or change physically. The crossing closure may divert traffic to other crossings and would increase the exposure and associated collision potential at the corresponding crossings. Therefore, even though crossing closure can eliminate potential collisions at a specific crossing by removing exposure at the crossing, it is unclear whether the overall collisions in the entire

railway network can be actually reduced. Moreover, as indicated by Russell (1981), the crossing closure may evoke a strong resistance by the local community since it requires local drivers to use other route than the existing convenient route.

In spite of the aforementioned issues that require to be considered before implementing crossing closure/grade separation, these two countermeasures would be the only countermeasures that can prevent the 100% of collisions between trains and motor-vehicles (Mironer et al. 2000).

2.3.2 Improving Crossing Geometry

A wide variety of geometrical improvements could be introduced as safety countermeasures at grade crossings. They include smoothing the horizontal and/or vertical alignment of the approaching road, changing the crossing intersecting angle, and improving road surface conditions. Improving the sight distance by modifying crossing geometry is also a well-known approach for reducing the collision potential at grade crossings. However, it is practically impossible to evaluate the effectiveness of each of these modification strategies due to lack of observational data. In this study, we will consider two general categories of countermeasures involving geometric improvement: a) sight distance improvement, and b) pavement condition improvement.

For instance, Gan et al. (2005) reported the effect of sight distance improvement at intersections that is currently applied in selected United States, including Missouri ($CMF =$

0.60) and Arizona ($CMF = 0.93$). Similarly, they also reported the CMF of pavement improvement at grade crossings [e.g. Arizona ($CMF = 0.80$)].

In fact, any changes in crossing geometry would change the sight distance and pavement conditions. For example, if track angle is changed, then the sight distance and pavement condition should be also affected. In this case, the improvement of track angle would be a surrogate variable that can measure the impact of improvement in crossing geometry.

2.3.3 Upgrading Traffic Control Devices

The main purpose of traffic control devices is to provide appropriate warning to drivers using various visual and/or audible devices and to assist drivers in taking proper actions to avoid collisions at crossings. Traffic control devices can be further categorized into passive and active devices, as described in the following sections.

1) Passive Traffic Control Devices

Passive traffic control devices such as signage and pavement markings provide static messages of warning, guidance and, in some instances, mandatory action for vehicle drivers. The TAC's Manual of Uniform Traffic Control Devices (1998) describes various control devices to enhance safety at grade crossings. Figures 2.5~2.7 illustrates several passive warning devices. As shown in the figures, each control device can be used individually or in combination.

Agent et al. (1996) provided various *CMF* of different passive control devices in Kentucky. They suggested 0.55 for installing a “Yield Sign” and 0.65 for installing a “Stop Sign” at highway intersections.

It should be noted that, in this study, the *CMF* of passive warning devices for highway intersections are assumed to be the same as the *CMF* of the same devices for highway-railway grade crossings. For example, the Oregon DOT suggested 0.51 as a *CMF* for installing “Stop Ahead Sign” at highway intersections. Basically, the performance of passive sign is expected to be the same for the drivers at both intersections. Similarly, the estimated *CMF* of “Stop Line Sign” at a highway intersection was assumed to be the same as the *CMF* of stop line sign at a grade crossing. As a result, a total of six different passive control devices, including illumination and pavement markings, have been considered in this study (Table 2.3). Appendix B describes the specific role and criteria of all relevant passive warning devices.

Table 2.3 Published CMF for Various Countermeasures

Countermeasure		Sources		Number of Sources
		US State Regulation	Other Studies	
Crossing Elimination				
Grade Separation/Closure		0.00(Alaska)*	0.00(Mironer et al. 2000)	2
Traffic Control Devices				
Passive	Yield Sign	1.37(Arizona)* 0.55(Kentucky)** 0.55(Missouri)* 0.77(New York)*		4
	Stop Sign	0.81(Arizona)* 0.50(Idaho)* 0.65(Kentucky)** 0.65(Missouri)* 0.47(Montana)* 0.80(Texas)		6
	Stop Ahead Sign	0.75(Alaska)* 0.70(Kentucky)** 0.51(Oregon)***		3
	Stop Line Sign	0.75(Kentucky)** 0.75(Missouri)* 0.66(New York)*		3
	Illumination (Lighting)	0.75(Alaska)* 0.40(Idaho)* 0.70(Kentucky)** 0.40(Missouri)*		4
	Pavement Markings	0.75(Alaska)* 0.44(Arizona)* 0.90(Idaho)* 0.90(Indiana) † 0.85(Kentucky)** 0.85(Missouri)* 0.85(Oklahoma)*		7
Active	From Signs to Flashing Lights	0.25(Alaska)* 0.62(Arizona)* 0.23(Idaho)* 0.50(Iowa)* 0.35(Kentucky)** 0.35(Missouri)*	0.36(California P.U.C. 1974)‡ 0.35(Morrissey 1981) 0.31(Eck and Halkias 1985) 0.30(Farr and Hitz 1985) 0.49(Hauer and Persaud 1987)	11
	From Signs to 2Q-Gates	0.10(Alaska)* 0.04(Arizona)* 0.13(Idaho)* 0.25(Kentucky)** 0.25(Missouri)* 0.58(Vermont)*	0.12(California P.U.C. 1974)‡ 0.16(Morrissey 1981) 0.16(Eck and Halkias 1985) 0.17(Farr and Hitz 1985) 0.31(Hauer and Persaud 1987)	11
Active	From Flashing Lights to 2Q-Gates	0.10(Alaska)* 0.20(Arizona)* 0.25(Missouri)*	0.34(California P.U.C. 1974)‡ 0.36(Morrissey 1981) 0.28(Eck and Halkias 1985) 0.28(Farr and Hitz 1985)	8

			0.55(Hauer and Persaud 1987)	
	From 2Q-Gates to 2Q-Gates with Median Separation		0.20(FRA 2001) 0.25(with Channelization, Federal Register 2003) 0.20(with Median Barriers, Federal Register 2003) 0.70(Mironer et al. 2000)	4
	From 2Q-Gates to 4Q-Gates		0.18(without vehicle presence detection, Federal Register 2003) 0.23(with vehicle presence detection, Federal Register 2003) 0.08(with median separation, Federal Register 2003) 0.18(FRA 2001) 0.60(Mironer et al. 2000)	5
	Installing Traffic Signal\$	0.40(Alaska)* 0.58(Arizona)* 0.35(Kentucky)**	0.33(McGee et al. 2003)	4
	Whistle (Train and Wayside Horn)		0.31(Florida State, Rapoza 1999) 0.62(Other State, Rapoza 1999) 0.47(Farnham)	3
Geometry				
	Improving Sight Distance\$\$	0.75(Alaska)* 0.25(Alaska)* 0.93(Arizona)* 0.68(Idaho)* 0.65(Iowa)* 0.70(Kentucky)** 0.60(Missouri)* 0.70(Montana)* 0.75(Oklahoma)* 0.62(Oregon)***		10
	Improving Pavement Condition	0.10(Alaska)* 0.80(Arizona)* 0.66(Indiana) †		3
Enforcement				
	Posted Speed Limit	0.80(Indiana) † 0.80(Kentucky)** 0.80(Missouri)*		3
	Photo/Video Enforcement		0.28(FRA 2001) 0.15(McKeever 1998) 0.36(Caird et al. 2002)	3

Source)

*: Gan et al. (2005)

** : Agent et al. (1996)

***: ODOT (2006)

†: Tarko and Kanodia (2004)

‡: Cited in Hauer and Persaud (1987)

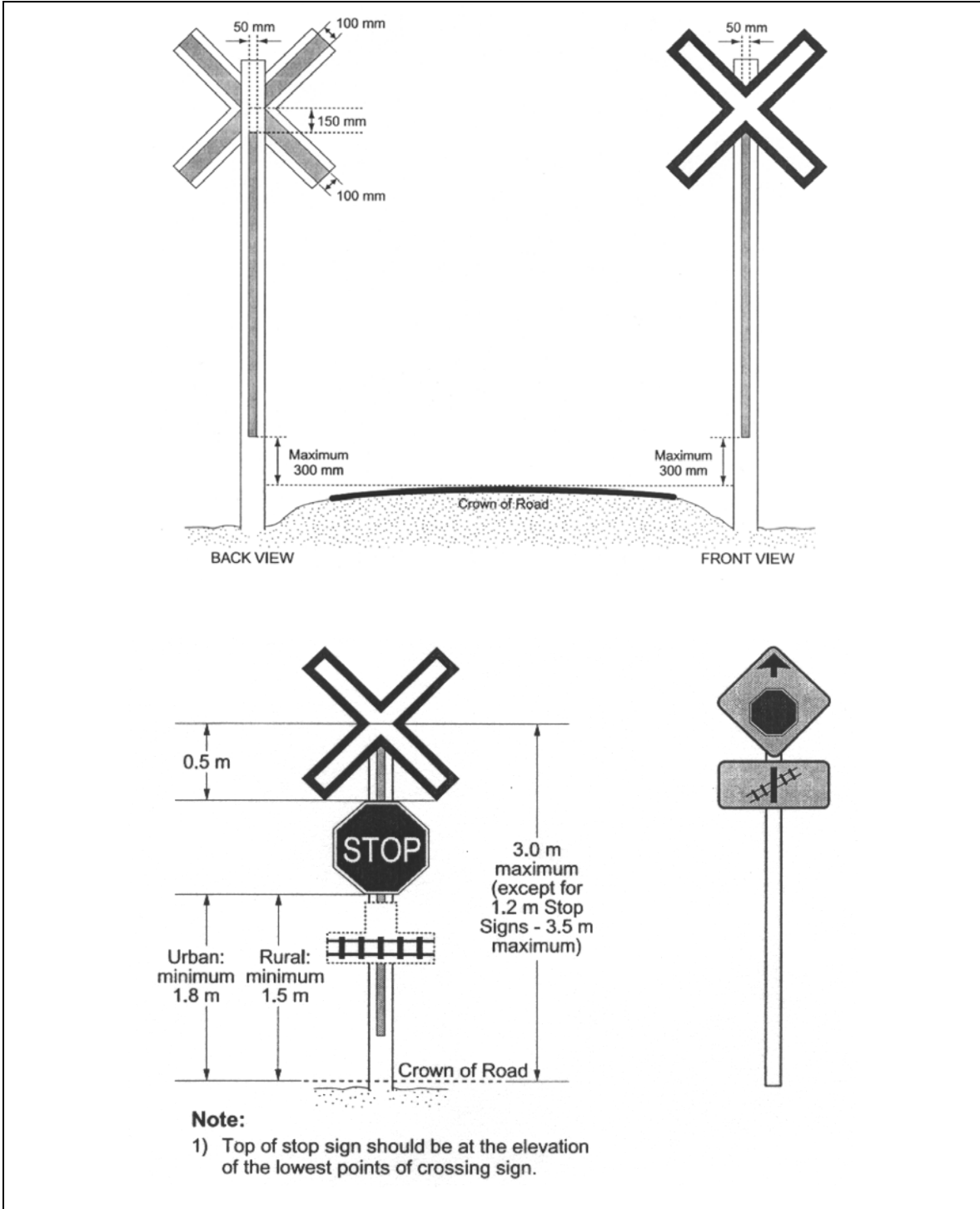


Figure 2.5 Typical Applications of Passive Traffic Control Devices (Transport Canada 2005)

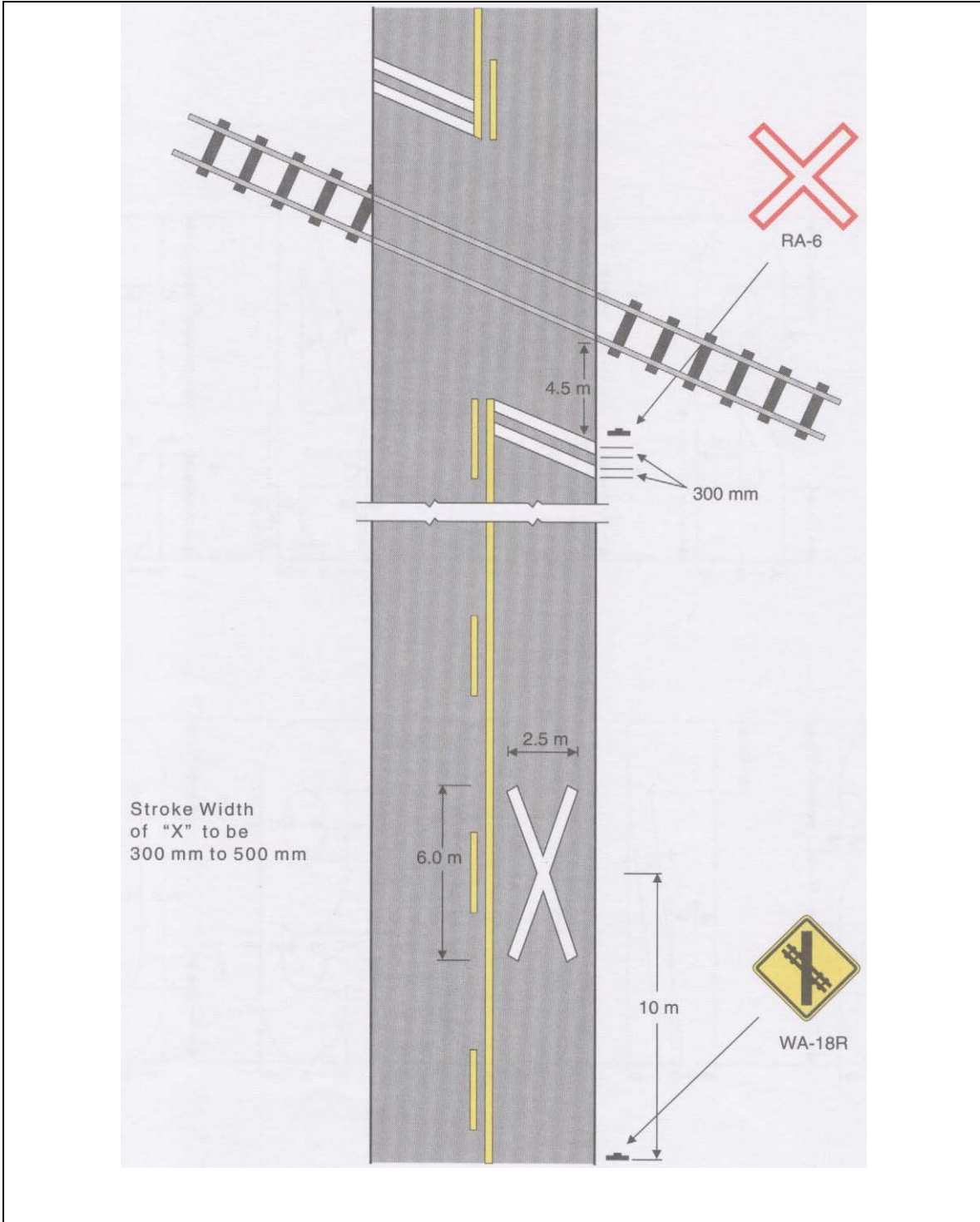
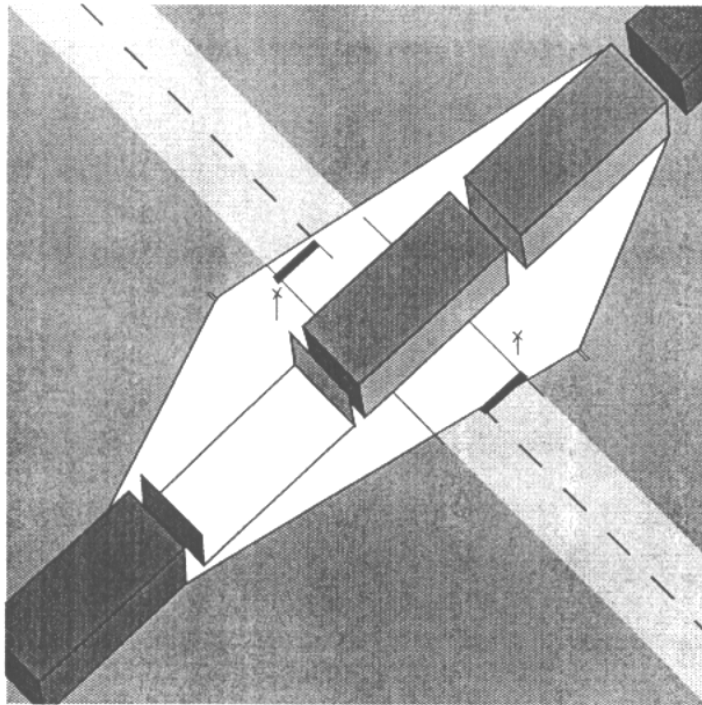
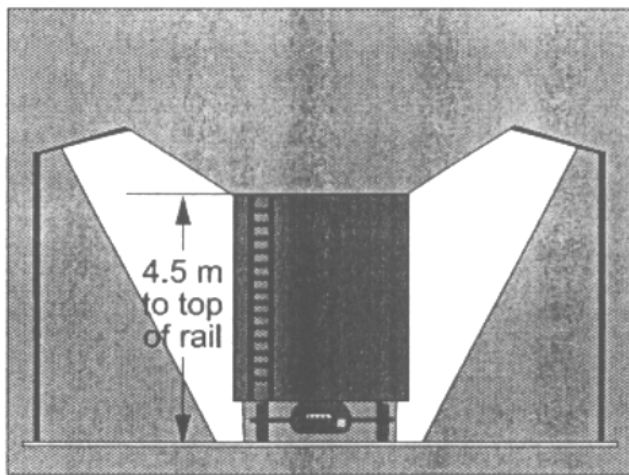


Figure 2.6 Typical Pavement Markings at Highway-Railway Crossings (TAC, 1998)



Plan View



Height to be covered by luminaire

Figure 2.7 Typical Grade Crossing Illuminations (Transport Canada 2005)

2) Active Traffic Control Devices

Active traffic control devices represent all traffic control devices that are activated when train is detected, such as flashing lights and gates. Active traffic control devices are also supplemented with the signs and pavement markings that are used for passive traffic control.

Flashing lights, either post-mounted or cantilevered, are the basic active warning devices used to inform highway users of the approach of a train to a grade crossing. As shown in Figure 2.8, flashing lights are commonly supported by passive traffic control devices, such as cross-bucks and/or warning bells. Cantilevered flashing lights are usually installed at the location where post-mounted flashing lights are ineffective due to a given roadway environment. As shown in Table 2.3, many researchers estimated the effects of installing flashing lights over the existing passive control devices, and the estimated values of *CMF* vary in different studies. These inconsistent results may come from the differences in the dataset or methodologies used in the studies.

Automatic gates provide an additional level of control and are normally used in conjunction with flashing lights. The gate arms are usually reflectorized and fully cover the approaching highway to prevent motor vehicles from circumventing the gate and entering the crossing. The gates and flashing lights are activated together.

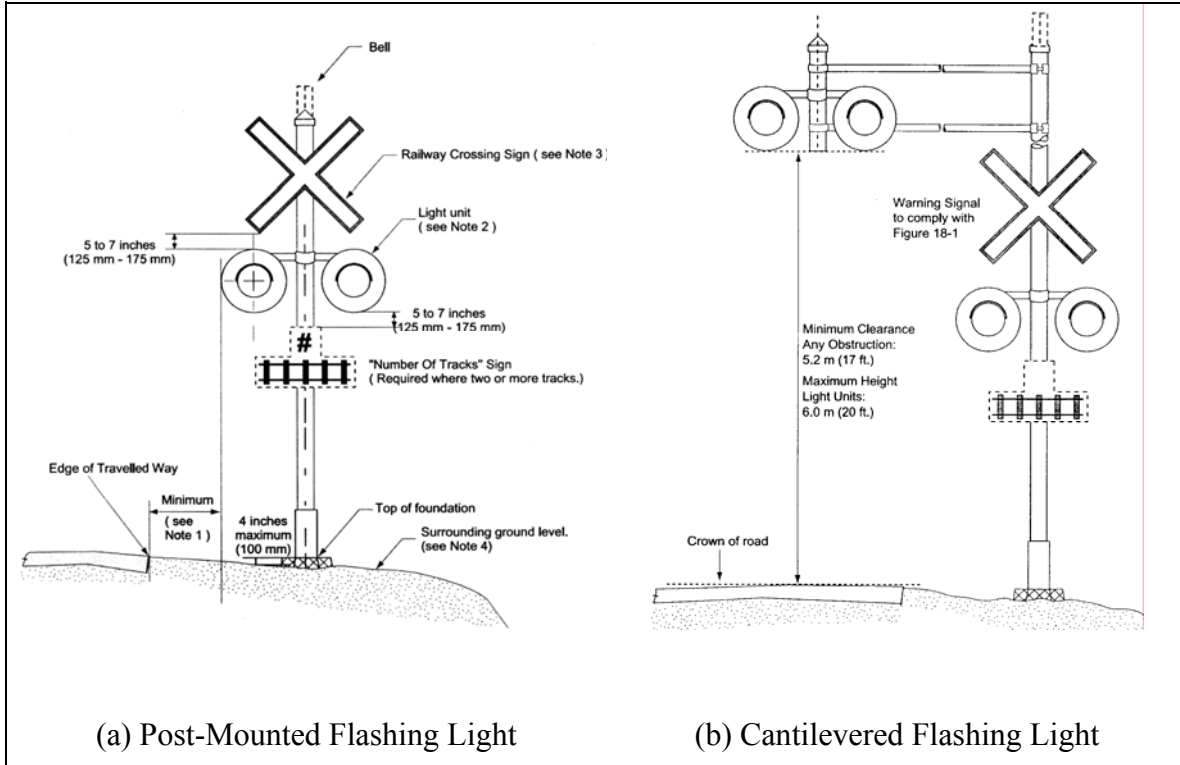


Figure 2.8 Typical Flashing Lights (Transport Canada 2005)

Farr and Hitz (1985), for instance, provided 0.17 as the effectiveness of upgrading warning devices from passive signboards to 2-quadrant gates. The value is estimated using a naïve before-after study. On the other hand, Hauer and Persaud (1987) applied an EB before-after model and they estimated a lower value (0.31) of *CMF* for the same improvement. The US DOT also suggests a *CMF* for the active warning devices applying to their local jurisdictions. The results are summarized in Table 2.3.

In Canada, the current practice is to use 2-quadrant gates with dual gate arms, which block motor vehicles in each direction (as shown in Figure 2.9). Recently, 4-quadrant gates are being used in the US, which block the crossing from both directions, and prevent drivers from crossing between the lowered barriers (FHWA 2003), as shown in Figure 2.10.

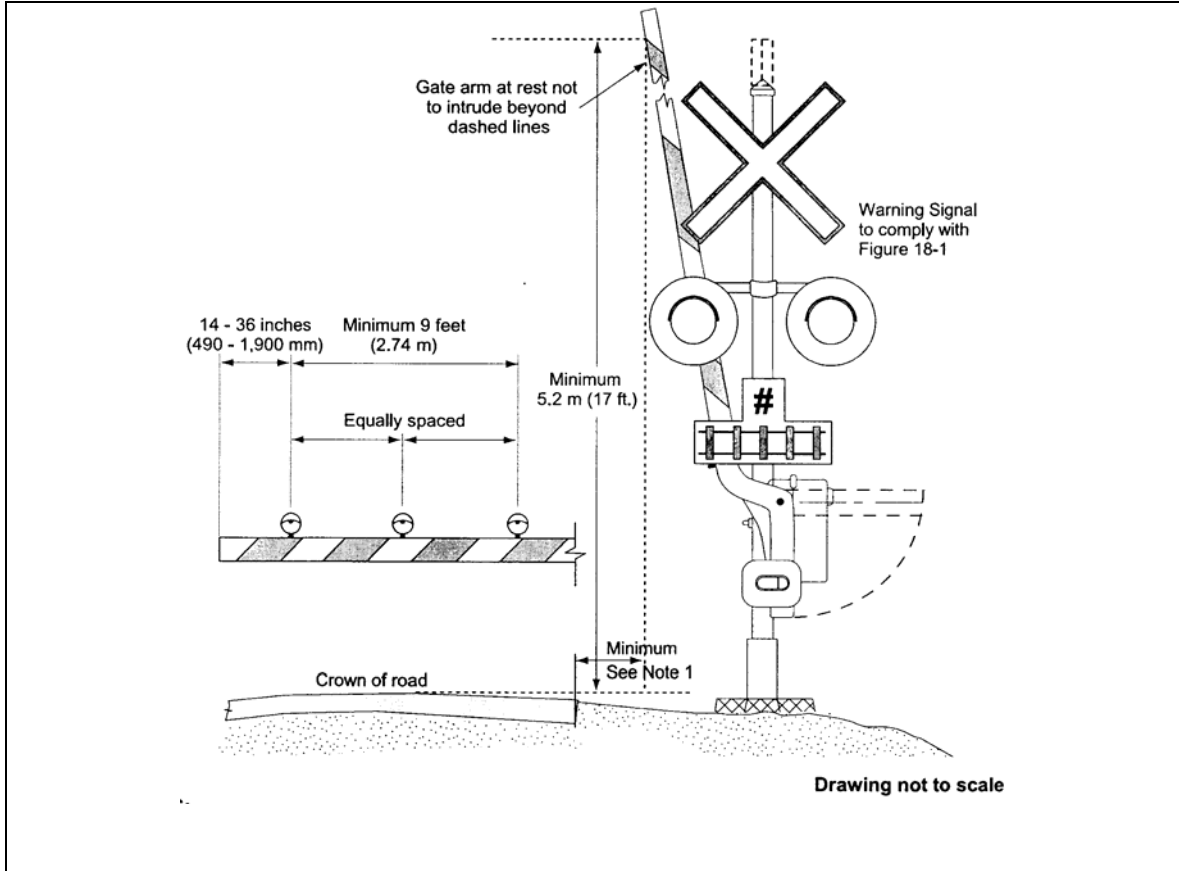


Figure 2.9 Typical Two-Quadrant Gate System (Transport Canada 2005)

As shown in Table 2.3., the US Federal Railroad Administration (FRA) (Federal Register 2003) suggested the expected *CMF* for the improvement from conventional 2-quadrant to 4-quadrant gates. However, it should be noted here that the estimated *CMF* for the 4-quadrant gates is different from the conventional *CMF* in Equation 2.1. FRA estimated the values based on the expected changes in the number of gate violation for specific crossings before and after the introduction of the 4-quadrant gates. The gate violation was used as a surrogate measure of the number of collisions. Given that the 4-quadrant gates have very short history of implementation, and grade crossing collision occurs very rarely, the use of gate violation for representing the effectiveness of this

specific countermeasure might be unavoidable. However, further researches are needed to investigate the accurate *CMF* for this countermeasure when more collision data are available in the future.

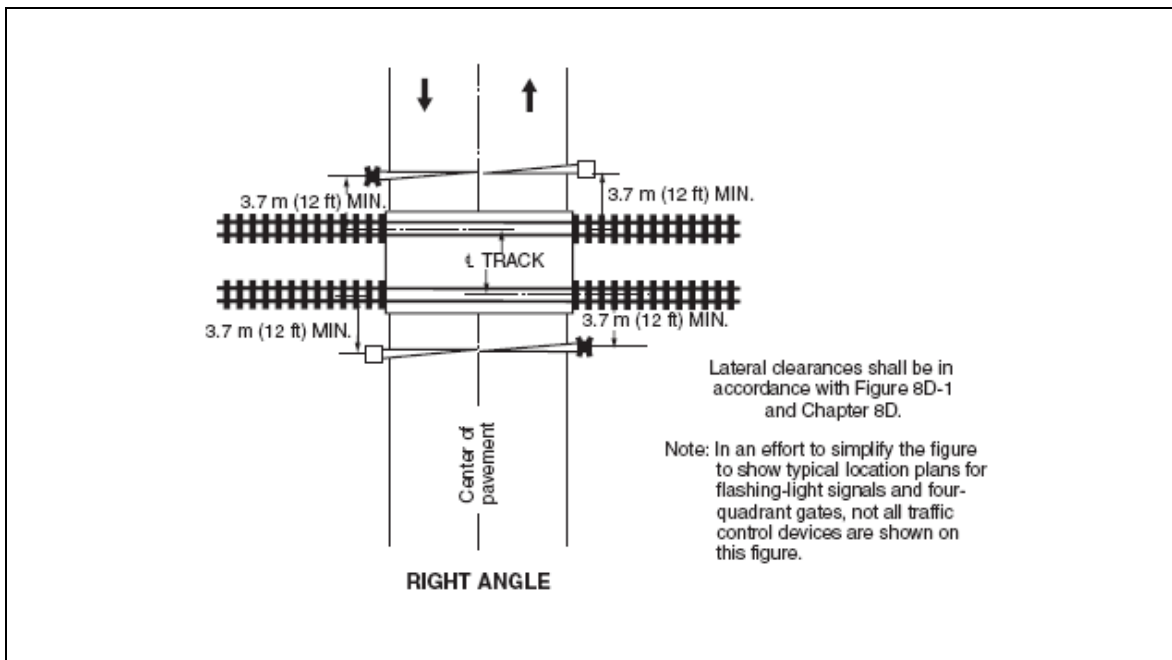


Figure 2.10 Plan View of Four-Quadrant Gate System (FHWA 2003)

3) Median Separation/Traffic Channelization Devices

Crossings with 2-quadrant gates are still risky because drivers can cross the centerline and easily pass the gate. This type of violation can be prevented by using aforementioned 4-quadrant gates. However, the 4-quadrant gates can also introduce a new danger such that vehicles can be trapped between the two lowered gates in the path of an oncoming train. An effective alternative to 4-quadrant gates is installing median separation devices at the road approaches. A significant reduction in the number of motor vehicle violations at grade crossings have been observed when compared to conventional 2-quadrant gates. While the

safety benefits of using median separation devices are expected to be higher, the observed evidence has not been found. In addition, centre median barriers are expected to cost less than a 4-quadrant gate system. Figure 2.11 shows an example of flexible traffic separators currently installed at a highway-railway grade crossing in Central Florida (Ko et al. 2003).



Figure 2.11 Example of 2-Quadrant Gate with Median Separator (Ko et al. 2003)

4) Audible Warning Systems

The use of audible alarms such as train horns or wayside horns can be an effective way of warning motorists and pedestrians of the impending arrival of a train and reducing collisions at highway-railway grade crossings. Based on the collision experience in Florida, Coifman and Bertini (1997) reported that the nighttime collision rate had been tripled during the first five years after whistle was prohibited. FRA found similar results that the

collision rate increased at the selected crossings during the whistle prohibition period (Rapoza et al., 1999). Saccomanno and Lai (2005) recently suggested that the elimination of whistle prohibition would reduce the number of collisions by 26% on average.

5) Photo/Video Enforcement System

Photo/video enforcement systems can be used to prevent and/or reduce two types of traffic violations: vehicle speeding and red-light running. A recent US study (FRA 2001) reported that photo/video enforcement combined with a legal fine/penalty system has shown to be an effective alternative to the conventional enforcement system. The *CMF* of this enforcement system was estimated to be 0.28 based on the changes in driver violation before and after implementation of the enforcement system.

Carroll and Warren (2002) investigated the effectiveness of red light camera enforcement at highway intersections. They suggested that the photo/video enforcement at highway-railway grade crossings would display similar effectiveness since the system works in the same way as the red light cameras at highway intersections.

2.3.4 Summary of Published Countermeasure Effects

In this section we summarize *CMF* for different types of countermeasures as reported in the literature. The values summarized in Table 2.3 were obtained from a number of studies in Canada and the US. In this study, we have assumed that the Canadian and American experience is close enough to justify the assertion that countermeasure effects come from the same statistical population. The *CMF* and sources are summarized in columns 3 and 4.

Initially, a total of 94 different studies were investigated to provide the effectiveness of 18 different countermeasures that could be applied to enhance safety at the highway-railway grade crossings. Later, the information on the *CMF* will be used as a priori information on the effectiveness of countermeasures before analyzing this study dataset.

3 MODEL FRAMEWORK

Frequently transportation engineers are required to make difficult safety investment decisions in the face of uncertainty concerning the cost and effects of countermeasures applied to specific locations or group of crossings. For highway-railway grade crossings this problem is aggravated by the lack of observed before-and-after collision and exposure data to provide empirical inference on the impact of countermeasure(s) for a given mix of crossing attributes.

The model introduced in this Chapter uses Bayesian data fusion to overcome the limitation associated with traditional collision prediction models for the estimation of countermeasure effects. In this approach, we use the findings from past studies concerning expected countermeasure effects, which could vary by jurisdictions and operating conditions, to obtain a priori inference concerning these effects. We then use locally calibrated models, which are valid for explaining variation in collision occurrence at a particular location or jurisdiction, to obtain current best “data likelihood estimates” of countermeasure effects. These two sources of estimates (priors and data likelihood) are then integrated using Bayesian data fusion to obtain the expected best posterior estimates of countermeasure effects along with their corresponding probability distribution.

Since posterior estimates are linked to unique Bayesian posterior probability distribution, the estimates are obtained not only for the expected collision response to a given countermeasures but also their corresponding variance and percentiles for a range of

likely values. The theoretical rationale of Bayesian data fusion is discussed in the following section.

3.1 Bayesian Data Fusion Method

Bayesian data fusion permits the combination of countermeasure effects from different independent sources (models and observational) with estimates obtained from a formal statistical analysis of the grade crossing data. Similar approach has been suggested recently by El Faouzi (2006), Melcher et al. (2001) and Washington and Oh (2006). Their approach is different in treating prior knowledge and data likelihood functions from this study approach.

In this study, our aim is to obtain “posterior” estimates (θ_i) of the probability for a specific countermeasure effects applied to a specific crossing i with a given mix of crossing attributes. The posterior expression is of the form (Migon and Gamerman 1999, Lee 2004):

$$P_i(\theta|\mathbf{x}) \propto P_i(\theta)P_i(\mathbf{x}|\theta) \quad (3.1)$$

where,

θ = Countermeasure effect (*CMF*) for a specific crossing

\mathbf{x} = Estimates from Canadian collision prediction models

$P_i(\theta)$ = Prior probabilities of θ from past studies

$P_i(\mathbf{x}|\theta)$ = Probability of observing the sample data given that a statement about the value of a parameter is true (i.e. objective or current best knowledge);

$P_i(\theta | \mathbf{x}) =$ Posterior probability of θ give \mathbf{x} .

Equation 3.1 assumes that the effect of a given countermeasure is best treated as a random variable with a unique probability distribution. Since these estimates are obtained from independent sources commonly of an empirical nature, we assume that the estimates of countermeasure effects are normally distributed, with a given mean, variance, and probability distribution. As noted by Lee (2004), according to the central limit theorem observations that have a built-in estimation error are likely to reflect a normal distribution. If the distribution of multiple source estimates on the priors and data likelihoods is normal, the posterior estimates will also be normal. We note that this assumption of normal distribution is purely for computational convenience although the other distributions can be equally considered in the proposed data fusion method. However, the other distributions, may require more computationally intensive procedures of use, such as Markov Chains Monte Carlo (MCMC) techniques. As noted by Washington and Oh (2006), a more flexible beta distribution, which provides for non-symmetric countermeasure response can also be considered in establishing reliable and practical posterior probability distribution. The impact of different distribution will be explored in Chapter 6.

If we let the estimate from one of the data likelihoods of countermeasure effects as an experimental result x_1 with probability $P_1(x_1|\theta)$, we can estimate the posterior distribution using Equation 3.1. Repeating the experiment to obtain another experimental result x_2 with probability $P_2(x_2|\theta)$, we estimate the probability $P_i(\theta|x_2x_1) \propto P(\theta)P_2(x_2|\theta)P_1(x_1|\theta)$. Generalizing this procedure for n different experiments, Migon and Gamerman (1999) derived the expression for the posterior probability for n experiments as:

$$P(\theta | \mathbf{x}_n, \mathbf{x}_{n-1}, \dots, \mathbf{x}_1) \propto P(\theta) \left[\prod_{i=1}^n P_i(\mathbf{x}_i | \theta) \right] \quad (3.2)$$

The technical challenge is to obtain posterior probability distributions by integrating multiple distributions using Equation 3.2. From the Bayesian theorem and assuming normality in both the prior [$\theta \sim N(\mu, \tau^2)$] and data likelihood distributions [$l \sim N(x, \sigma^2)$], Lee (2004) and Migon and Gamerman (1999) demonstrated that we can combine the effect means and variances to produce a normal posterior distribution [$\theta|x \sim N(\mu_0, \tau_1^2)$] such that:

$$\tau_1^2 = (\tau^{-2} + \sigma^{-2})^{-1} \quad (3.3)$$

$$\mu_0 = (\tau^{-2}\mu + \sigma^{-2}\mathbf{x})\tau_1^2 \quad (3.4)$$

$$\omega = \frac{\tau^{-2}}{\tau^{-2} + \sigma^{-2}} \in (0,1) \quad (3.5)$$

where, ω reflects the relative information contained in the prior with respect to its corresponding posterior information. We can re-write Equation 3.4 to incorporate this weight factor in the source estimates as:

$$\mu_0 = \omega \cdot \mu + (1 - \omega) \cdot \mathbf{x} \quad (3.6)$$

Equation 3.6 reflects the weighted mean of prior and likelihood means, and has been discussed at length in the literature by several safety researchers (Hauer 1997, Persaud 2001). Figure 3.1 depicts the proposed modeling framework adopted in this study.

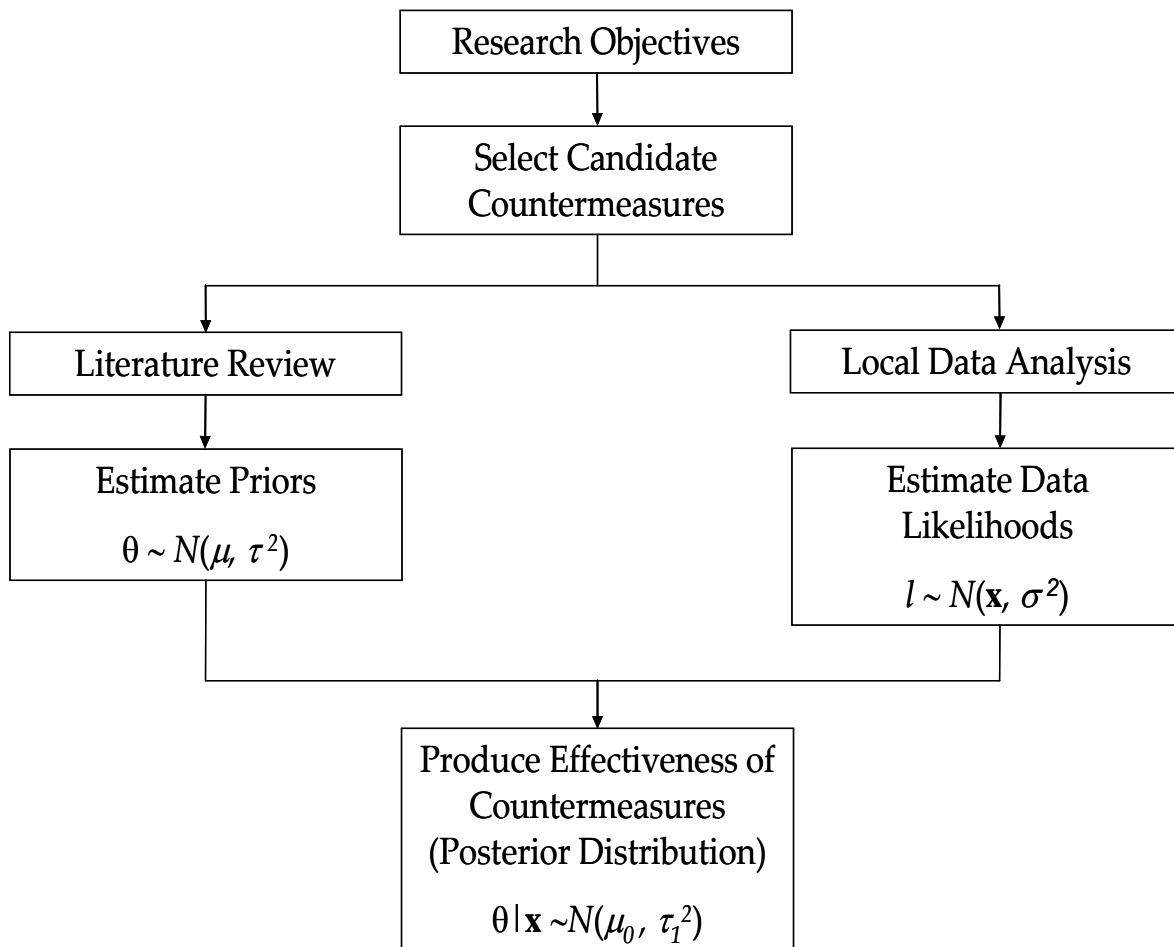


Figure 3.1 Modeling Framework

3.2 Priors

As noted by Melcher et al. (2001), estimates of countermeasure effects based on previous studies represent a “first order a priori” belief concerning the effect of specific countermeasure applied to a given location in the absence of a formal location-specific analysis of the experimental data. Since it is assumed that each source yields a separate “independent” estimated effect, these estimates can be represented by a unique “a priori”

probability distribution. In this study, we use historical knowledge from the past studies (grade crossing and highway safety) regarding similar countermeasures effect based on the different jurisdictional experiences.

Most of these sources are based on research using US data. In this study, we have assumed that the effects of countermeasures at grade crossing are almost the same in Canada and the US. While prior estimates are assumed to be independent, their accuracy is subject to the reliability and strength of the method being used to predict collisions. Some methods improve the shortcomings of the other methods. The study result based on an advanced method would need to be given higher weights when obtaining the “combined” a priori effect.

In this study, we follow the approach suggested by Harkey (2005) and Washington and Oh (2006) to establish the relative weights of countermeasure effects based on the perceived merits of different model types. In general, we obtained the relative model weights as the inverse ranking of the level of certainty summarized in Table 3.1 for different types of analysis methods.

As pointed out by Harkey (2005) and Washington and Oh (2006), the potential bias due to the differences in region and time of studies should also be considered in assigning previous studies into different level of certainties. For instance, Harkey (2005) investigated literatures from various regions, including North America, Europe, and Australia. He recommended of using at least 20% of North American experiences to represent the effect of a specific countermeasure. He also suggested using only “post-1980” studies because the

studies older than 25 years may not reflect the recent developments in trains/vehicles and changes in driver behaviors. Thus, we decided to exclude the California P.U.C. study (1974) with estimates summarized previously in Table 2.3.

Table 3.1 Certainty Level of Previous Study (Harkey et al. 2005)

Level of certainty (<i>i</i>)	Brief description of study methodology	Relative weight (<i>W_i</i>)
1. High	A rigorous before-after model that incorporated the current best study design and statistical analysis method. At this time, the empirical Bayesian (EB) before-after model represents the best available approach.	1.00
2. Medium-High	A before-after model with sound statistical method (but not EB before-after model) or cross-sectional models with rigorous expert judgment. Combination of study results using rigorous Meta-analysis.	0.50
3. Medium-Low	Cross-sectional models with controlling for other factors statistically or less rigorous before-after models (e.g. naïve before-after models).	0.33
4. Low	Before-after or cross-sectional models in which modeling technique were questionable.	0.25

The mean combined countermeasure effect from the past studies is obtained using a weighted expression of the form:

$$\mu_j = \frac{\sum W_{ij} \cdot CMF_{ij}}{\sum W_{ij}} \quad (3.7)$$

where,

μ_j = weighted average effectiveness of countermeasure *j* from all available sources

CMF_{ij} = effectiveness of countermeasure *j* in level of certainty *i*

W_{ij} = relative weight for countermeasure *j* in level of certainty *i*

The term μ_j adopted in Equation 3.7 reflects the weighted average countermeasure effect from past studies. It is basically a multiplier applied to a previous collision rate to yield a new adjusted collision rate.

To obtain the prior distributions for countermeasure effects, we need to obtain a mean number of collisions as well as the variance associated with this mean value. The estimates of the means are routinely provided in the various sources. Unfortunately, the estimates of *CMF* variance are scarce and many sources fail to provide empirical estimates of variance for different countermeasure effects.

Washington and Oh (2006) determined the variance (or standard deviation) of countermeasures based on the level of certainty of studies. Their approach was to assign a predetermined value of variance for each study based on the study's level of certainty. Basically, they assigned larger variance to the study at lower level of certainty and vice versa. Their approach is convenient since no additional analysis is required to approximate a variance of *CMF* for each study. However, there is no theoretical rationale to believe that a study at lower certainty level will produce larger variance/standard deviation than a study at higher certainty level.

In the absence of specific information on countermeasure, the variance of *CMF* for a given prior source, we suggest the following five-step procedure to obtain approximate estimates of the *CMF* variance.

- 1) Obtain the mean countermeasure effect (μ_j) as well as the standard deviation (τ_j) for countermeasure j from all sources that provide these two pieces of information.

2) Estimate the “coefficient of variation” for countermeasure j using an expression of the form (Johnson 1994):

$$CV_j = \frac{\tau_j}{\mu_j} \cdot 100 \quad (3.8)$$

where,

CV_j = coefficient of variation for the countermeasure j

τ_j = standard deviation of the countermeasure j

μ_j = *CMF* of the countermeasure j

3) Obtain the average CV for countermeasures at the same level of certainty i (as shown in Table 3.1).

4) Apply the average CV obtained from the method being used regardless of type of countermeasure and estimate its associated standard deviation for the countermeasure by using Equation 3.8.

5) Assign relative weights in Table 3.1 to the individual countermeasures and combine the estimated standard deviation to obtain weighted average standard deviation for a specific countermeasure j using Equation 3.9.

$$\tau_j = \frac{\sum W_{ij} \cdot \tau_{ij}}{\sum W_{ij}} \quad (3.9)$$

where,

τ_j = weighted average standard deviation of countermeasure j from all available sources

τ_{ij} = standard deviation of countermeasure j in level of certainty i

W_i = relative weight for countermeasure j in level of certainty i

It should be noted that the formal “Meta Analysis” approach provided in literature integrates findings from multiple studies and utilizes the same expression as Equations 3.7 and 3.9 to estimate the weighted average and variance of existing findings (Hunter and Schmidt 1990). But a major difference concerns the estimation of relative study weights from previous studies. Furthermore, the “Meta Analysis” method requires a number of inputs from each study, including sample size, published year, omitted factors, and even the number of researchers. In highway safety studies, White (2002) investigated the effects of 30 different safety countermeasures using Meta analysis. However, the effects of only 5 different countermeasures were obtained due to the lack of necessary input information. Unfortunately, the input information required for a rigorous Meta analysis of grade crossing countermeasure effects was not available from the previous studies cited in Table 2.3. Our aim in this analysis is to produce estimates of effectiveness for as many countermeasures as possible; hence a formal Meta analysis was not employed in this study. The proposed approach remains practicable since the priors can be easily updated or altered should better results become available from future studies concerning specific countermeasures.

3.3 Data Likelihood

Prior estimates may not be reliable because they are study specific and limited in reflecting the full gamut of crossing-specific factors that we would expect to influence collisions at different locations in different jurisdictions. In this analysis, we require an in-depth investigation of the relationship between crossing attributes and collisions as reflected in the Canadian database. The estimated *CMF* from these collision prediction models can best represent the “objective” or current information on grade crossing collisions as well as attributes within Canadian jurisdictions. From the Bayesian perspective, we refer to this type of inference as “data likelihood”.

In this study, we employed three different statistical models based on independent studies carried out by Saccomanno and Lai (2005) and Park and Saccomanno (2005a, 2005b) using different types of models. These models were developed using the collision data at Canadian grade crossings and used multi-stage cross-sectional approaches to solve the problems associated with the conventional cross-sectional models noted in section 2.2.2. Brief descriptions of these models are as follows;

- 1) Saccomanno and Lai (2005) introduced a three-stage cross-sectional model to predict collisions. They grouped crossings into five different clusters with similar crossing attributes based on a sequential factor/cluster analysis and then developed cluster-based collision prediction models by employing negative binomial collision prediction expressions. Since the crossing attributes in individual clusters are assumed to be

homogeneous, the expected change in the number of collisions for a given countermeasure can be used to assess its effect vis-à-vis safety enhancement.

2) Park and Saccomanno (2005a) introduced a data partitioning method (i.e. RPART) to eliminate the impact of different control factors, which can influence collisions but are difficult for engineers to control directly (e.g. jurisdictional factors). The authors assigned individual crossings into homogeneous groups of crossings in terms of control factors, and then developed a series of statistical models to predict collisions and estimate countermeasure effects.

3) Park and Saccomanno (2005b) included higher-order interaction terms in their prediction model. The authors employed a data partitioning method to account for complex interactions, which were not captured in the conventional cross-sectional modeling procedure. The prediction models yield the expected number of collisions before and after a given countermeasure is introduced at each crossing. From these results, we can estimate the *CMF* values for each countermeasure.

A more in-depth discussion of these data likelihood models is provided in Chapter 4, which addresses the overall prediction model components and calibration/validation results.

4 DEVELOPMENT OF THE MODEL COMPONENTS

4.1 Description of Dataset

This section briefly describes the inventory and collision data used for collision prediction models at grade crossings.

4.1.1 Canadian Inventory Data

This database that is administered by Transport Canada is called IRIS (Integrated Rail Information System) and contains an inventory of 29,507 grade crossings for all regions in Canada with information on crossing geometric and traffic attributes as well as types of warning devices. IRIS includes six types of data:

1) Location Data

Each crossing in the dataset contains site information, which indicates its location in the street, city or town, and province or territory. In summary, there are 4,074 crossings in Alberta, 2,185 crossings in British Columbia, 3,161 crossings in Manitoba, 1,291 crossings in New Brunswick, 9 crossings in Newfoundland, 16 crossings in the Northwest Territories and Nunavut, 809 crossings in Nova Scotia, 7,357 crossings in Ontario, 1 crossing in Prince Edward Island, 4,127 crossings in Quebec, and 6,469 crossings in Saskatchewan, and 8 crossings in the Yukon.

2) Warning Device Type

There are nine different types of warning devices in the database: namely flashing light signals and bell, flashing light signals and bells with gate, traffic lights, wigwags, signals and bell, manual gates, and reflectorized signboard. For the purpose of the analysis, the nine different types of warning devices are integrated into three different classes. These are: signs only, flashing lights, and gates.

3) Grade Crossing Type

Five different types of grade crossings are identified in the dataset, namely public automated, public passive, private, farm, and grade separation. In this study, only public grade crossings (automated or passive) have been considered for further analysis.

4) Highway Characteristics

The database contains information on highway geometric characteristics at grade crossings, including highway surface material, road surface width, road type and posted road speed. Road type was classified into arterial, collector, bicycle path, farm road, local, low volume road, pedestrian path, private access, snowmobile path, and unopened road. These road types are grouped into three different classes (i.e. arterial or collector, local road, and other roads). Surface materials include asphalt, concrete, gravel and other. In the actual analysis, only two different classes are used: paved or unpaved.

5) Railway Characteristics

In the database, the information on the number of tracks, track angle, track type, maximum train speed, and train whistle prohibition, etc. is included. All this information can be considered as the railway attributes.

6) Traffic Characteristics

Traffic characteristics represent information on the average annual daily traffic (AADT) and number of daily trains passing a crossing. Later, exposure term that represents the product of these two traffic characteristics is estimated and utilized in the subsequent analysis.

4.1.2 Collision Occurrence Data

The collision occurrence database, referred to as RODS (Rail Occurrence Database System), is administered by the Canadian Transportation Safety Board (TSB). There are 2,905 collisions in the database during the period of 1993-2001 for the 29,507 nation-wide crossings.

Collision occurrence in RODS is classified into five different packages: “Basic Collision Information” such as collision report number, road condition, and railway characteristics, etc.; “Driver and Occupancy Information” such as driver gender, driver age, and the number of occupants in the vehicle involved collisions, etc.; “Vehicle Information” such as vehicle types, status of car window closure, etc.; “Collision Types” such as train struck vehicle, vehicle struck train, etc.; and “Severity Consequence Information” such as

the number of fatalities, the number of injuries, etc. In this study, only the frequency of collisions at each crossing has been used to develop collision prediction models and later to estimate the effectiveness of a certain countermeasure.

The above IRIS and RODS dataset will be used later to develop data likelihood distributions to describe the local crossing characteristics and will be combined with the prior distribution. The following section is devoted to a thorough explanation of how the prior distribution has been estimated and then the method to estimate the data likelihood distribution will be investigated.

4.2 Development of Priors

This section describes the development of prior distributions for the mean and variance of *CMF* from different sources of information and data. As noted in the previous chapter, *CMF* refers to a collision modification factor for individual countermeasures.

A literature review was conducted to find the quantitative *CMF* and associated uncertainty measures (i.e. variance, standard errors, and confidence intervals) for each countermeasure. Individual study estimates were weighted based on the perceived merits of the adopted study approach (refer to Table 3.1).

Table 4.1 contains the mean (μ) and standard deviation (τ) of *CMF* of a sample countermeasure (i.e. upgrading signboards to flashing lights) from different past studies. For instance, based on a naïve before-and-after approach, Morrissey (1980) estimated *CMF* values of 0.35 and 0.04 as the mean and standard deviation of *CMF* respectively for this

countermeasure. A total of ten previous studies reported *CMF* values for this countermeasure. Among these ten studies, nine studies employed a naïve before-and-after approach, with a medium-low weighting. Only one study employed an EB before-and-after approach and this was assigned a high weight. Equation 3.7 was adapted to estimate the weighted average of these differently weighted studies, and yielded a *CMF* value of 0.46 for upgrading from signs to flashing lights.

Table 4.1 Estimated Priors for Improvement from Signboards to Flashing Lights

Level of Certainty	μ	τ	<i>CV</i>	Literature
High (1.00)*	0.49	0.1709	34.88	Hauer and Persaud (1987)
Medium-Low (0.33)*	0.35	0.0400	11.43	Morrissey (1980)
	0.31	0.0160	5.16	Eck and Halkias (1985)
	0.29	0.0231	7.97	Farr and Hitz (1985)
	0.25	0.0205	8.19	Alaska State**
	0.62	0.0507	8.19	Arizona State**
	0.23	0.0188	8.19	Idaho State**
	0.50	0.0409	8.19	Iowa State**
	0.35	0.0286	8.19	Kentucky State***
	0.35	0.0286	8.19	Missouri State**
Average for High Level Studies	0.49	0.1709	-	-
Average for Medium-Low Level Studies	0.36	0.0297	-	-
Weighted Average for Prior Distribution	0.46	0.1356	-	-

Note: * Relative Weight; ** Gan et al. (2005); *** Agent et al. (1996)

As noted in section 3.2, the main challenge becomes estimation of standard deviations of *CMF* for countermeasures with few sources. For this countermeasure, only three previous studies (i.e. Morrissey 1980, Eck and Halkias 1985, and Farr and Hitz 1985) in Table 4.1 provided standard deviation estimates in their reports. From Morrissey (1980),

CV was estimated as 11.43 based on Equation 3.8. The CV's were estimated to be 5.16 and 7.97 from Eck and Halkias (1985) and Farr and Hitz (1985), respectively. The average CV from these sources was calculated as 8.19.

By applying this average CV to the other six studies available for the same level of certainty, we obtained the standard deviation (τ) for these studies. For instance, the Alaska State DOT reported a mean *CMF* value of 0.25 without reporting the standard deviation. Using the Equation 3.8, we estimated this standard deviation to be 0.0205.

A more challenging exercise is estimating the standard deviation of *CMF* for the studies with high levels of certainty, since these types of studies have not been well documented in the literature. Moreover, for the EB before-and-after approach there are no studies that can be utilized for approximating CV, especially in the field of highway-railway grade crossing. Accordingly, in this study, two different studies in the highway safety field (Persaud et al. 2001, 2003) were utilized to produce a substitute CV for the EB before-and-after approach. The rationale here is that decision comes from an idea that the estimated CV in the same level of certainty would be similar than it in the different level of certainty. As pointed out by Johnson (1994), the CV represents the standard deviation as a percentage of the mean and the value can be used to represent the precision of a given dataset.

It should be noted that the procedure to approximate the standard deviation might not be adequate enough to provide a necessary input for a prior distribution due to the lack of defensible scientific base. But it should also be recognized that this procedure is still

valid since a rough inference would be better than an arbitrary decision without conducting any estimation practice.

Based on previous studies in highway-railway grade crossing studies, we were able to obtain the weighted mean and standard deviation of *CMF* for 18 different countermeasures. These results are summarized in Table 4.2 along with the number of studies or sources on which each estimate is based. These represent the historical information or a priori belief as to the effect on collision reduction from the introduction of a specific countermeasure. This is in the absence of any analyses involving the local collision data.

Table 4.2 Estimated Priors for Different Countermeasures

Number	Countermeasures	μ	τ	#.of previous studies
1	Grade Separation/Closure	0.0000	0.0000	2
2	Yield Sign	0.8100	0.0723	4
3	Stop Sign	0.6467	0.0577	6
4	Stop Ahead Sign	0.6533	0.0583	3
5	Stop Line Sign	0.7200	0.0642	3
6	Illumination(Lighting)	0.5625	0.0502	4
7	Pavement Markings	0.7914	0.0706	7
8	From Signs to Flashing Lights	0.4578	0.1356	10
9	From Signs to 2Q-Gates	0.2833	0.0864	10
10	From Flashing Lights to 2Q-Gates	0.4738	0.1489	7
11	From 2Q-Gates to 2Q-Gates with Median Separation	0.3375	0.0301	4
12	From 2Q-Gates to 4Q-Gates	0.2540	0.0227	5
13	Installing Traffic Signal	0.3583	0.1776	4
14	Elimination of Whistle Prohibition	0.4671	0.0417	3
15	Improve Sight Distance	0.6630	0.0591	10
16	Improve Pavement Condition	0.5200	0.0464	3
17	Posted Speed Limit	0.8000	0.0714	3
18	Photo/Video Enforcement	0.2471	0.0220	3
			Total =	91

For these results, a total of 91 sources were utilized to obtain a priori countermeasure effects. Based on these studies, if grade separation or closure is not considered (logically these two countermeasures should yield a 100% collision reduction), we can speculate that historically the strongest countermeasure effect would be associated with the changes in warning devices from 2- to 4-Quadrant Gates and the installation of Photo/Video enforcement. Both countermeasures reduce collisions by about 75% (i.e. $CMF = 0.25$). On the other hand, the weakest effect was found to be the introduction of yield signs ahead of grade crossings. The expected collision reduction for this countermeasure was estimated to be about 19% (i.e. $CMF = 0.81$).

4.3 Development of Data Likelihood

Prior information may be flawed because it fails to reflect the full gamut of crossing-specific attributes that explain collisions at a given crossing over different periods of time. For this analysis, we require an in-depth investigation of the relationship between crossing attributes and collisions as reflected in the Canadian database. From the Bayesian perspective, we refer to this type of inference as “data likelihood”. The estimated CMF from these collision prediction models represents the objective or current information for grade crossing collisions as well as attributes within Canadian jurisdictions.

In large part because of biases resulting from co-linearity in the model inputs, and absence of important factors in the model resulting from lack of statistical significance, many existing prediction models have failed to represent the full spectrum of relevant

factors that explain variation in collision frequency at grade crossings (Saccomanno and Lai 2005).

In this study three different statistical models based on independent studies of the Canadian grade crossing data carried out by Saccomanno and Lai (2005) and Park and Saccomanno (2005a, 2005b) are used to obtain data likelihood estimates of *CMF*. These three multi-stage collision prediction models are expected to reduce unexplained variation in the *CMF* estimates as suggested by the priors alone. We begin with detailed descriptions of these models.

4.3.1 Factor/Cluster Collision Prediction Model

1) Background of Factor/Cluster Collision Prediction Model

Saccomanno and Lai (2005) developed a three-stage collision prediction model using standardized variables of crossing attributes. The first stage of their model involves a factor analysis of the geometric and traffic attributes for grade crossings in RODS/IRIS data. They found four particular factors that represent the main features of the crossing inventory data. The estimated four factors are orthogonal (i.e. not collinear) to each other and provide a unique explanation of variation in collision frequency expressions. Factor scores were obtained from an expression of the following form (Comrey and Lee 1992):

$$F_{jk} = \beta_{j1} Z_{1k} + \beta_{j2} Z_{2k} + \dots + \beta_{jn} Z_{nk} \quad (4.1)$$

where,

F_{jk} = standardized factor score for crossing k on factor j

Z_{nk} = standardized value for crossing k on variable n

β_{jn} = factor coefficient for factor j on variable n

The estimated factor scores for each grade crossing were used as “seed points” in a subsequent cluster analysis to determine groups of crossings with similar crossing attributes. Since we used standardized variables, the estimated factor scores are dimensionless in nature.

As mentioned, the second stage of this procedure is cluster analysis to group grade crossings into different clusters with similar geometric and traffic attributes. These clusters were obtained using Euclidean distance measures for each of the four factor scores. After examining the spatial distribution of grade crossings within and between the clusters, Saccomanno and Lai (2005) determined five unique clusters. Even though a certain degree of subjectivity is associated with factor and cluster interpretation, these analyses provide exclusive insights into the physical properties of crossings reflected in the inventory dataset. As a result of factor/cluster analysis, a homogeneous group of grade crossings in terms of their attributes can be successfully obtained. Equation 4.2 was used to determine the Euclidean distance from each crossing to each cluster. Finally, the cluster membership for each crossing is determined based on the nearest distance rule.

$$d_{iA} = \sqrt{\sum_{k=1}^4 (x_{ik} - x_{Ak})^2} \quad (4.2)$$

where,

d_{iA} = Euclidean distance between crossing i and the center of cluster A

x_{ik} = the k^{th} factor score for crossing

x_{Ak} = mean of the k^{th} factor score for cluster A

Appendix F gives all the related matrices involving factor and cluster analysis and an example calculation by courtesy of Saccomanno and Lai (2005).

Even though all crossings in the same cluster are expected to behave in a similar manner in regard to their expectation of collisions, some variation is still evident among crossings within the same cluster. This variation will be taken into account statistically by developing cluster-specific collision prediction models.

It should be noted that the cluster-specific collision prediction models are modified and recalibrated to obtain a variance-covariance matrix among parameters for each model. The matrices are not reported in Saccomanno and Lai's original paper. But it is necessary to approximate variances of various *CMF* in this study. More detailed illustration of the use of variance-covariance matrices will be discussed in chapter 5.

Initially, Saccomanno and Lai used a sample of crossing dataset for their models since they split crossing dataset into two random samples: One sample to calibrate the model, and the other one to validate it. Since as they reported that the model satisfied the validation process, this study utilized the complete crossing dataset to re-develop cluster-specific collision prediction models (refer to Figure 4.1). As a result, the developed model in this study is somewhat different from the original expressions in the literature. All the variables adapted in "factor/cluster analysis" are provided in Table 4.3.

Table 4.3 Summary of Variables for Factor/Cluster Collision prediction Model

Factors	Variable Levels	Variable Type	Coding Description or Measuring Unit
1.Warning Devices	3	Nominal	Signboards = Reference level*; Flashing Lights = 1 or 0*; Gates = 1 or 0*
	3	Nominal	Signboards = 1; Flashing Lights = 2; Gates = 3
2.Surface Width	Scale Value	Continuous	Meter
3.Surface Type	2	Nominal	Paved surface =; Otherwise = 0.
4.Road Type	2	Nominal	Arterial = 1; Otherwise = 0
5.Track Number	Scale Value	Count	Number
6.Track Angle	Scale Value	Continuous	Degree
7.Whistle Prohibition	2	Nominal	Whistle Prohibition = 1; Whistle Operation = 0
8. Mainline	2	Nominal	Mainline = 1; Otherwise = 0
9. Daily Vehicles (AADT)	Scale Value	Count	Number
10. Daily Trains	Scale Value	Count	Number
11.Posted Road Speed	Scale Value	Continuous	Kilometer per hour
12. Maximum Train Speed	Scale Value	Continuous	Mile per hour
13. Exposure	Scale Value	Continuous	AADT × Daily Train*
14.Collision Frequency	Scale Value	Continuous	Collisions per year

Note: * Coding Description Only for Collision Prediction Model

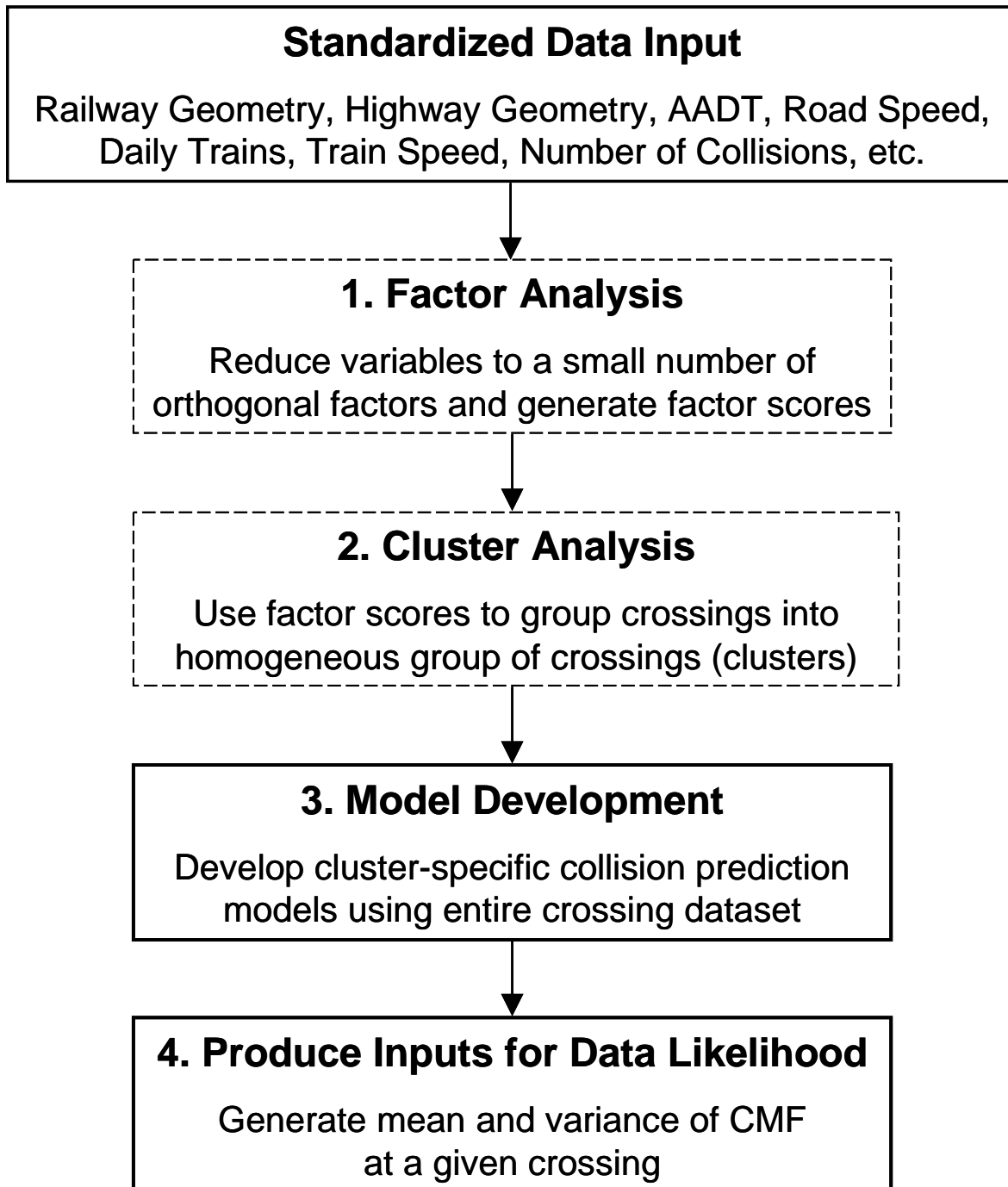


Figure 4.1 Factor/Cluster Modeling Framework

2) Development of Cluster-Specific Collision Prediction Models

Currently, the negative binomial (NB) model is the most common expression to represent the expected number of collision frequencies at given sites including grade crossings since the model takes into account the over-dispersion issues inherent in collision datasets. The NB model is obtained by adding a Gamma-distributed error to the conventional Poisson model, and therefore it is also known as the Poisson-Gamma model. The NB model form is:

$$\mu_i = \exp(\beta \cdot \mathbf{x}_i + \zeta_i) = \exp(\beta \cdot \mathbf{x}_i) \cdot \exp(\zeta_i) \quad (4.3)$$

where,

μ_i = the expected number of collisions at a given crossing i

\mathbf{x}_i = a vector of explanatory variables

β = a vector of estimated coefficients

$\exp(\zeta_i)$ = a gamma-distributed error term with mean 1 and variance α^2

The addition of this term allows the variance to differ from the mean as:

$$\text{var}(y_i) = E(y_i)[1 + \alpha \cdot E(y_i)] = E(y_i) + \alpha \cdot E(y_i)^2 \quad (4.4)$$

where, α = inverse over-dispersion parameter

As a result, the NB model overcomes the limitation in the conventional Poisson regression models of assuming equal mean and variance [i.e. $E(y_i) = \text{var}(y_i)$]. In fact, the Poisson regression model is a limiting model of this NB model when α is equal to 0. As a result, the distinction between these two models is determined by the estimated value of α , which determines the degree of dispersion in the predictions. If the estimated α is

significantly different from 0, a NB expression is more appropriate for the dataset than a Poisson expression. Cameron and Trivedi (1998) suggested the functional form of the NB model as the following expression:

$$P(y_i | \mathbf{x}_i) = \frac{\Gamma(y_i + \alpha^{-1})}{y_i! \Gamma(\alpha^{-1})} \cdot \left(\frac{\alpha^{-1}}{\alpha^{-1} + \mu_i} \right)^{\alpha^{-1}} \cdot \left(\frac{\mu_i}{\alpha^{-1} + \mu_i} \right)^{y_i} \quad (4.5)$$

where,

$\Gamma(\cdot)$ = a gamma function

The SAS GENMOD procedure was utilized to develop the cluster-specific collision prediction models. The complete outputs of the models are shown in Appendix C and Table 4.4 illustrates the selected outputs of each model.

As discussed, the expected *CMF* and corresponding variance from this model will be employed as an input to data likelihoods. Chapter 5 will demonstrate the application of this model using selected crossing samples to produce necessary inputs for data likelihood.

Table 4.4 Cluster-Specific Collision Prediction Models Based on Factor/Cluster Analyses

Variables	Coding Scheme	Cluster 1		Cluster 2		Cluster 3		Cluster 4		Cluster 5	
		Coef.	Std.Errs.	Coef.	Std.Errs.	Coef.	Std.Errs.	Coef.	Std.Errs.	Coef.	Std.Errs.
Flashing Lights (FL)	FL = 1; Otherwise = 0	-0.383	0.326	-0.994	0.188	NA	NA	NA	NA	-0.580	0.212
Gates (GT)	GT = 1; Otherwise = 0	-0.840	0.293	-1.448	0.344	NA	NA	NA	NA	-1.492	0.324
Active Warning Signs	FL or GT = 1; sign = 0	N/A	N/A	N/A	N/A	-1.119	0.137	-1.222	0.208	N/A	N/A
Max. Train Speed	Miles/hour	-	-	0.016	0.004	0.011	0.003	-	-	-	-
Track Angle	Degree	-	-			-0.009	0.006	-	-	-	-
Whistle Prohibition (WP)	WP = 1; Otherwise =0	-	-	1.152	0.334	-	-	-	-	0.807	0.164
Ln(Exposure)	Ln(AADT× Daily Trains)	0.358	0.040	0.387	0.043	0.461	0.032	0.441	0.037	0.497	0.059
Intercept		-4.057	0.373	-4.967	0.290	-4.537	0.481	-4.309	0.306	-6.071	0.525
Dispersion (α)		0.794	0.190	1.112	0.312	0.361	0.213	0.973	0.216	1.614	0.328

Note: N/A = not-available; - = statistically insignificant at 90% confidence level

4.3.2 Stratified Collision Prediction Model

1) Background of the Stratified Collision Prediction Model

Although a number of factors are known to contribute to collisions at highway-railway grade crossings, the mixed effects of the control factors and other countermeasures on collision occurrence are less well explored. In this section, control factors reflect general environmental factors which cannot be altered by decision makers. Representative examples of control factors are weather and jurisdiction (e.g. region, country, province or state) where the crossing is located. Also, from a practitioner's viewpoint, they cannot alter the existing level or function of railways and highways for the sole purpose of reducing collisions at a specific crossing. In fact, hazardous crossings are scattered all over railway and highway networks, therefore it is inefficient to treat the level or function of railways and highways as countermeasures to mitigate the collisions at crossings. Instead, we need to implement and/or improve practical countermeasures, such as flashing lights or posted speed limits.

In this section, we describe a stratified collision prediction model developed by Park and Saccomanno (2005a) for the Canadian grade crossing data. This model is stratified to assess the effects of countermeasures on collision occurrence while the effects of selected "control factors" remain constant. After stratifying the crossings by selected control factors, we estimate the effect of countermeasures on collision reduction by the fitted models for each class of crossings.

The Canadian grade crossing inventory contains three potential control factors as shown in Table 4.5. If a collision prediction expression is obtained for each class in this Table, we would have a total of 12 (= $3 \times 2 \times 2$) different prediction models. However, there may not be a sufficient number of observations (i.e. crossings) for each class to permit such class-specific model development, and this may reduce the number of prediction expressions we develop using this approach.

Table 4.5 Selected Control Factors in Canadian Inventory Data

Factors	Variable Level	Variable Type	Coding Description
1.Highway Class	3	Nominal	Arterial/Collector = 1; Local Road = 0; Other Road Types = -1
2.Track Type	2	Nominal	Mainline = 1; Otherwise (e.g. switching line) = 0
3.Track Number	2	Nominal	Multiple Tracks = 1; Single Track = 0

A number of previous studies have attempted to control collision prediction by stratifying the crossing inventory data according to selected variables and developing separate prediction models for each class of crossing (Farr 1987). In general, these types of models tend to be rather arbitrary in nature when choosing data partitioning criteria. As such, they fail to account for co-linearity problems that arise from the relationship between control and engineering or decision factors. Therefore, a systematic tool is necessary to decide the number of classes for collision predictions as well as to identify statistically valid relationships between engineering factors (i.e. countermeasures) and collisions for each class.

This section has three particular objectives;

- Suggest a valid way to control the effect of control factors in collision prediction. In this study, we make use of the tree-based data stratification method for this purpose.
- Develop a set of collision prediction models for each class of crossings based on the data stratification results.
- After neutralizing the effect of control factors by the data stratification, generate necessary inputs for the data likelihood to evaluate the relationship between a set of important countermeasures and a collision frequency.

Our primary concern is how we can eliminate the control factor effects from the collision prediction expressions and acquire unbiased parameters of countermeasure inputs. This is the main issue of the stratified collision prediction model.

Figure 4.2 depicts the stratification modeling framework.

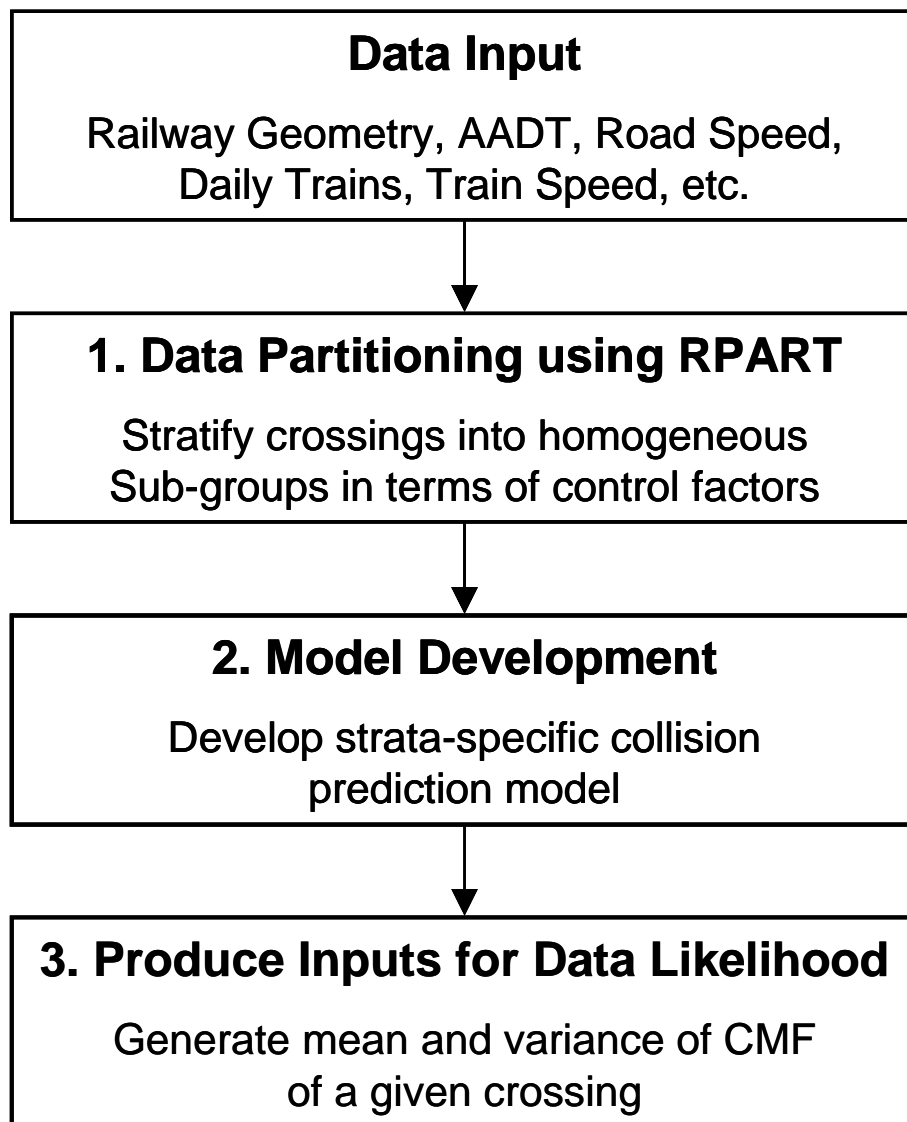


Figure 4.2 Stratification Modeling Framework

To address the above issues, Park and Saccomanno (2005a) utilized a Recursive Partitioning method (RPART) to stratify the study dataset into homogeneous sub-classes in terms of the control factor. RPART is similar to the technique applied in the Classification and Regression Trees (CART) method developed by Brieman et al. (1983). Zhang and Bracken (1996) and Cocchi et al. (2002) have applied the RPART method for the stratification purpose. Stewart (1996) also used the same method to identify complex data structures among the explanatory variables in road safety studies. Hakkert et al. (1996) and Lau and May (1989) also presented CART applications in the highway safety field. The

models presented in this section are based on a sample of 6,014 public crossings from the combined RODS/IRIS database with 1,546 collisions over a 9-year period (1993-2001).

RPART is a non-parametric technique and if the model response is categorical RPART produces classification trees. If the model response is continuous, RPART produces regression trees. Finally if the model response is assumed to be a count/ratio in nature and the model uses Poisson regression trees (Therneau and Atkinson 1997). In this study, the model response (i.e. collision rate) is assumed to be a count/ratio in nature.

In simple terms, RPART splits a sample into binary sub-samples on the basis of the response to a splitting question requiring a binary (yes or no) answer. Figure 4.3 depicts a hypothetical hierarchical tree structure used in RPART. Depending on the answer to the question (yes or no), the sample at a higher level is split into two (left and right) lower level sub-samples. When a split occurs, the split sub-samples end up either in a splitting point or in a rectangular box. The rectangular box represents a terminal node, while the splitting points represent a non-terminal or internal node. Terminal nodes cannot be split further. Internal nodes, on the other hand, are subject to further splitting at lower levels of the tree.

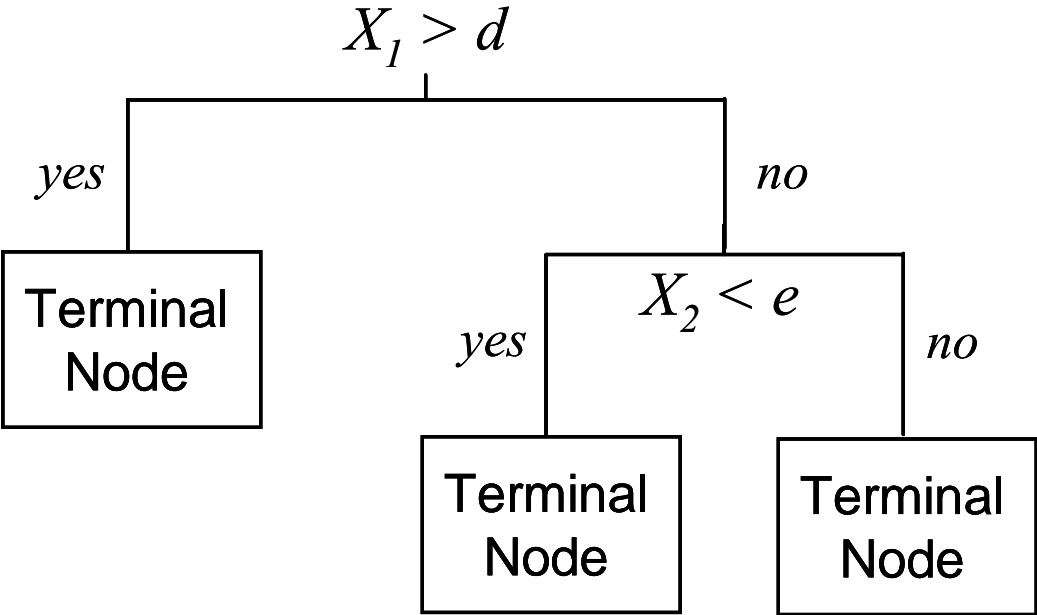


Figure 4.3 Hypothetical Tree Structure

The goal of the conventional regression tree in the RPART method is partitioning data (using binary splits), into relatively homogeneous terminal nodes with the minimized quantity of impurities (i.e. uncertainties or randomness) within the nodes. In the conventional regression tree, sums-of-squares within the nodes represent the aggregation of impurities for the nodes (Brieman et al., 1983).

As stated, the collision occurrence is assumed to follow the Poisson distribution, and the sums-of-squares are not very robust measures for the count/ratio based regression tree. Thus, the likelihood ratio is the simplest substitute for event-based regression trees. The Poisson regression tree in RPART has a different definition of the impurity measures from conventional regression trees. The procedure of building a Poisson regression tree is as follows (Therneau and Atkinson 1997):

- RPART performs all splits on each of the explanatory variables (starting with the root node), applies a predefined node impurity measure to each split, and determines the reduction in impurity that is achieved.
- RPART then selects the best split by applying a goodness-of-split criteria and partitioning the dataset into left and right sub-nodes.
- Because RPART is recursive, it repeats step 1) and 2) for each non-terminal node resulting in the largest possible tree. The change in the impurity (i.e. deviance) of node t of each split s can be estimated using the following expression:

$$\Delta D(s, t) = D(t_C) - D(t_L) - D(t_R) \quad (4.6)$$

where,

$D(t_C)$ = impurity at current node t

$D(t_L)$ = impurity at the left sub-node t_L

$D(t_R)$ = impurity at the right sub-node t_R

From the series of splits generated by a variable at a node, choose the split that maximizes the change in the impurity of the current node. Therefore, the best split is the split with the highest $\Delta D(s, t)$.

Under the likelihood ratio (LR) criterion, the impurity of the node is measured by within-node deviance, which is defined as:

$$D(t) = \sum \left[y_i \log \left(\frac{y_i}{\hat{\lambda} t_i} \right) - (y_i - \hat{\lambda} t_i) \right] \quad (4.7)$$

where,

y_i = the observed event count for observation i ,

t_i = the baseline measure (e.g. index of the time and space),

$\hat{\lambda} = \sum y_i / \sum t_i$ = the overall observed event rate.

The impurity measure has the property that $D(t_C) \geq D(t_L) + D(t_R)$. This property implies that the current estimated impurity is greater than or equal to the estimated impurity of the nodes (i.e. left and right sub-node) created by the current split. As the splits grow, further improvement becomes negligible at a certain point due to the lack of data for further splitting, or response data that are very close in value for the rest of the trees. RPART performs its tree-building until it produces a largest size tree. Generally, a k -fold cross validation strategy is used for determining the optimal size of the tree structure. In brief, RPART seeks the smallest tree with minimum cross-validation estimation error. Detailed tree-building algorithms are discussed in Breiman et al. (1983).

Three factors in Table 4.5 were employed to stratify the study dataset into homogeneous sub-classes in the RPART exercise. The collision rate was used for the response variable in the Poisson regression trees. In a Poisson regression tree, the terminal nodes (i.e. the rectangles in Figure 4.4) represent homogeneous sub-classes according to the control factors.

Figure 4.4 shows that the value of the estimated deviance (i.e. impurity) in each terminal node (i.e. 1, 2, 3 and 4) is much smaller than the root node's (i.e. I). The sum of the deviances in the four terminal nodes ($150.393 + 74.298 + 733.371 + 598.364 = 1,556.426$) is also smaller than the root node ($1,647.527$). Therefore, the stratified models

using the class-based crossings information can describe the effect of more homogeneous groups compared to the un-stratified models using the entire crossings dataset.

It was found that the highway class and the track number are important for explaining deviance in collisions at highway-railway grade crossings. However, the track type (i.e. mainline or not) did not contribute to reducing the impurities in this study dataset, and thus it is not shown in the tree. Therefore, the track type is not considered further in the stratification process. The four classes of crossings in Figure 4.4 are described as follows:

- Crossings which are included in the first class represent the crossings at arterial or collector roads.
- Crossings which are included in the second class represent the crossings with multiple tracks at local or other road types.
- Crossings which are included in the third class represent the crossings with a single track at local roads.
- Crossings which are included in the last class represent the crossings with a single track on other road types.

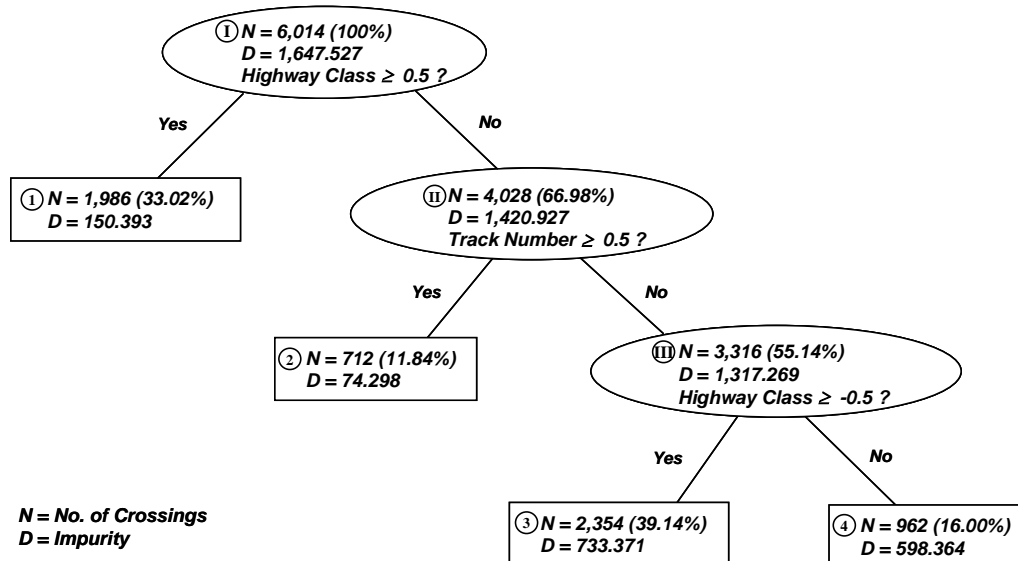


Figure 4.4 RPART Result on the Basis of Control Factors

2) Development of a Collision Prediction Model for Each Class

Table 4.6 describes the variables for stratification models. As usual, a categorical explanatory variable with j levels is included in the model as a set of $j-1$ dummy variables. For example, since Warning Devices has $j = 3$ levels, we included 2 dummy variables (i.e. Flashing Lights and Gates) as explanatory variables in collision prediction. In this case, the level that is excluded becomes the reference level (i.e. Signboards), and the coefficients that are included are interpreted relative to the reference level.

Table 4.6 Summary of Variables for Stratification Model

Factors	Variable Levels	Variable Type	Coding Description or Measuring Unit
1.Warning Devices	3	Nominal	Signboards = Reference level; Flashing lights = 1 or 0; Gates = 1 or 0
2.Extra Warning Devices	2	Nominal	Extra warning devices such as an extra bell or an auxiliary light = 1; Otherwise = 0
3.Surface Type	2	Nominal	Paved Surface = 1; Otherwise = 0
4.Track Angle	2	Nominal	Perpendicular Track-Angle = 1; Otherwise = 1
5.Surface Width	Scale Value	Continuous	Meter
6.Whistle Prohibition	2	Nominal	Whistle Prohibition = 1; Operation = 0
7.Posted Road Speed	Scale Value	Continuous	Kilometers per Hour
8.Time-table Train Speed	Scale Value	Continuous	Kilometers per Hour

Park and Saccomanno (2005a) developed a number of different collision prediction models using a generalized linear regression technique (i.e. Poisson regression). One of their major objectives was to compare the un-stratified models developed on the basis of both control and engineering factors (i.e. variables in Table 4.5. and Table 4.6) to stratified models developed on the basis of engineering factors only (i.e. variables in Table 4.6). They showed that un-stratified models fail to resolve co-linearity issues between control and engineering factors, and therefore produced biased parameters. For instance, an un-stratified model considering all three control factors (i.e. Model 1a in Park and Saccomanno's work) showed counter-intuitive results. The model implies that if the

number of tracks is increased from single to multiple tracks, then the collision rate is reduced. Increasing the number of tracks, however, will increase the passing time of a vehicle over a crossing, therefore the chances of collision should increase. Based on the multiple comparisons between un-stratified and stratified models, they concluded that it is hard to isolate the countermeasures effect from the mixed effect between control and engineering factors in the un-stratified models.

In this thesis, based on the experience by Park and Saccomanno (2005a), we will only use stratified models to produce the second inputs for the data likelihood. However, it should be noted that the stratification models employed in this study are somewhat different from the original expressions in the literature since we recalibrated the models for the following reasons:

- The need to obtain the variance-covariance matrix among parameters for approximating the variances of *CMF* (this will be described in more detail in Chapter 5 of this study)
- The need to reflect nonlinearity in exposure on collision prediction models

The recalibrated stratification models are produced using the NB regression technique, except the model in “Class 4” that used a Poisson expression. These expressions are used to evaluate the effect of countermeasures while taking into account a given mix of control factors.

The Poisson model assumes equal mean and variance, as such: $E(y_i) = \text{var}(y_i)$. As discussed in section 4.3.1, the Poisson model is a limiting form of the NB model with a dispersion parameter (α) reflecting the ratio of NB to Poisson variance that approaches zero. As a result, the Class 4 model in Table 4.7 does not contain any dispersion parameter since the parameter failed to pass a significant test at a 90% confidence level. The NB expression was used for the other models in Table 4.7.

Several notable aspects of the stratification model are:

- The five models in Table 4.7 show the coefficients which are statistically significant at least at a 90% confidence level. The completed outputs of these models including variance-covariance matrices are included in Appendix D.
- The probable advantage of the stratified models over the un-stratified models is that the effect of countermeasures can be estimated after controlling for control factors by the tree-based stratification. In terms of the stratified models based on individual class, the control factors are kept constant within each class. Therefore, even though stratified models may not greatly improve the accuracy of collision prediction, we can still assure that they are more theoretically valid for representing the effects of countermeasures.
- The “Model for Overall Class” in Table 4.7 contains no generic intercept, but the model has class-specific (i.e. a crossing membership determined by highway class and track number) intercepts. For instance, if a crossing intersects with an arterial or collector road, three regression coefficients (i.e. CI02, CI03 and CI04) are simultaneously set equal to zero. The resultant contains only the coefficient of CI01, and that value represents the intercept in the collision prediction model for the crossings on arterial or collector roads. Therefore, the estimated collision frequency at a crossing depends on which class the crossing belongs to.
- The two types of stratified models (i.e. “Models for Individual Classes” and “Model for Overall Class”) have advantages and disadvantages. In the case of stratified models for individual classes, the effects of the same countermeasures are different for separate classes of crossings. For example, the effect of upgrading the warning device from signboards to gates is different for the crossings intersecting with arterial or collector roads than the crossings intersecting with local or other road types. This result is practically appealing.
- On the other hand, for the crossings intersecting with arterial or collector roads, five variables including four countermeasures are statistically significant based on the Class 1 collision prediction model. Thus, for instance, the effect of surface type at the crossings cannot be estimated using this model. For the Class 4 model, only two

countermeasures are statistically significant at a 90% confidence level and therefore applicable for the crossings in the class. On the other hand, the stratified model for overall class provided practically useful output in that five countermeasures passed a significant test at the 90% confidence level. If decision makers want to know the effectiveness of warning devices for a crossing in class 4, instead of relying on the model for Class 4, the Overall Class model can be used to speculate the effect of the countermeasure for the crossings in Class 4. As a result, the model for the overall class can be used to represent the overall effectiveness of individual countermeasures for all types of crossings. This is done by eliminating biases caused by the control factors listed in Table 4.5 if the effects are not captured by the four models on the basis of individual classes.

Table 4.7 Class-specific Collision Prediction Models

Variables	Coding Skim	Models for Individual Classes								Model for overall Class	
		Model for Class 1		Model for Class 2		Model for Class 3		Model for Class 4		Coef.	Std.Errs.
		Coef.	Std.Errs.	Coef.	Std.Errs.	Coef.	Std.Errs.	Coef.	Std.Errs.		
Flashing Lights (FL)	FL = 1; Otherwise = 0	-0.677	0.147	-0.571	0.235	-0.983	0.131	-	-	-0.756	0.084
Gates (GT)	GT = 1; Otherwise = 0	-0.899	0.185	-0.601	0.205	-1.250	0.236	-	-	-1.004	0.114
Surface Type	If Paved = 1; Unpaved = 0	-	-	-0.254	0.155	-0.222	0.124	-	-	-0.112	0.067
Whistle Prohibition (WP)	If WP = 1; Otherwise = 0	0.294	0.114	-	-	0.827	0.174	1.409	0.780	0.373	0.085
Max. Train Speed	km/hour	0.002	0.001	-	-	0.007	0.002	0.011	0.005	0.004	0.001
Ln (Exposure)	Ln (AADT× Daily Trains)	0.345	0.030	0.358	0.048	0.366	0.033	0.290	0.077	0.355	0.020
CI01		n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	-3.867	0.173
CI02		n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	-4.004	0.172
CI03		n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	-3.965	0.150
CI04		n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	-4.388	0.169
Intercept		-3.797	0.266	-3.821	0.368	-4.190	0.241	-4.789	0.421	n.a.	n.a.
Dispersion (α)		0.633	0.114	0.236	0.180	0.439	0.154	n.a.	n.a.	0.543	0.082

Note) Model for Class 1 represents the crossings at arterial or collector roads (NB model)

Model for Class 2 represents the crossings at local or other road types with multiple tracks (NB model)

Model for Class 3 represents the crossings at local roads with a single track (NB model)

Model for Class 4 represents the crossings at other road types with a single track (Poisson model)

Model for Overall Class represents the complete crossings (NB model)

4.3.3 Collision Prediction Model with Group Indicators

1) Background of Collision Prediction Model with Group Indicators

In Chapter 2, a peer review of previous collision prediction models was conducted to find out appropriate model structure for collision predictions, specifically to reduce co-linearity among explanatory variables. None of the previous models developed for highway/railway crossings formally considered the interaction effects among the explanatory variables. To illustrate this point we cite the research by Farr (1987), Tustin et al. (1986), Federal Railroad Administration (2002), and Austin and Carson (2002).

In the previous section, we have attempted to control collision prediction by stratifying the crossing inventory data according to selected control factors and developing separate prediction models for each class of crossing. Our primary interest here is to explore interaction effects among countermeasures, rather than considering control factors as separate and distinctive inputs into the prediction model. In order to achieve these purposes, a Recursive Partitioning (i.e. RPART) method has been employed once again to systematically consider the interaction effects among various explanatory variables. This section is based on a previous study by Park and Saccomanno (2005b), and the modeling framework is given in Figure 4.5.

2) Development of Collision Prediction Model

Given the large number of variables and their potential interactions, the analyst is left with little guidance as to which interactions to specify and which to leave out. The problem can be computationally involving. Not only would specifying all of these interactions be time-consuming and impractical, it fails to account for interaction effects that merely represent spurious rather than real-effects (Washington 2000). Therefore, an analyst needs a systematic tool to identify higher-order interactions in the larger databases, such as collision databases, that would contribute explanatory power to existing regression collision prediction models.

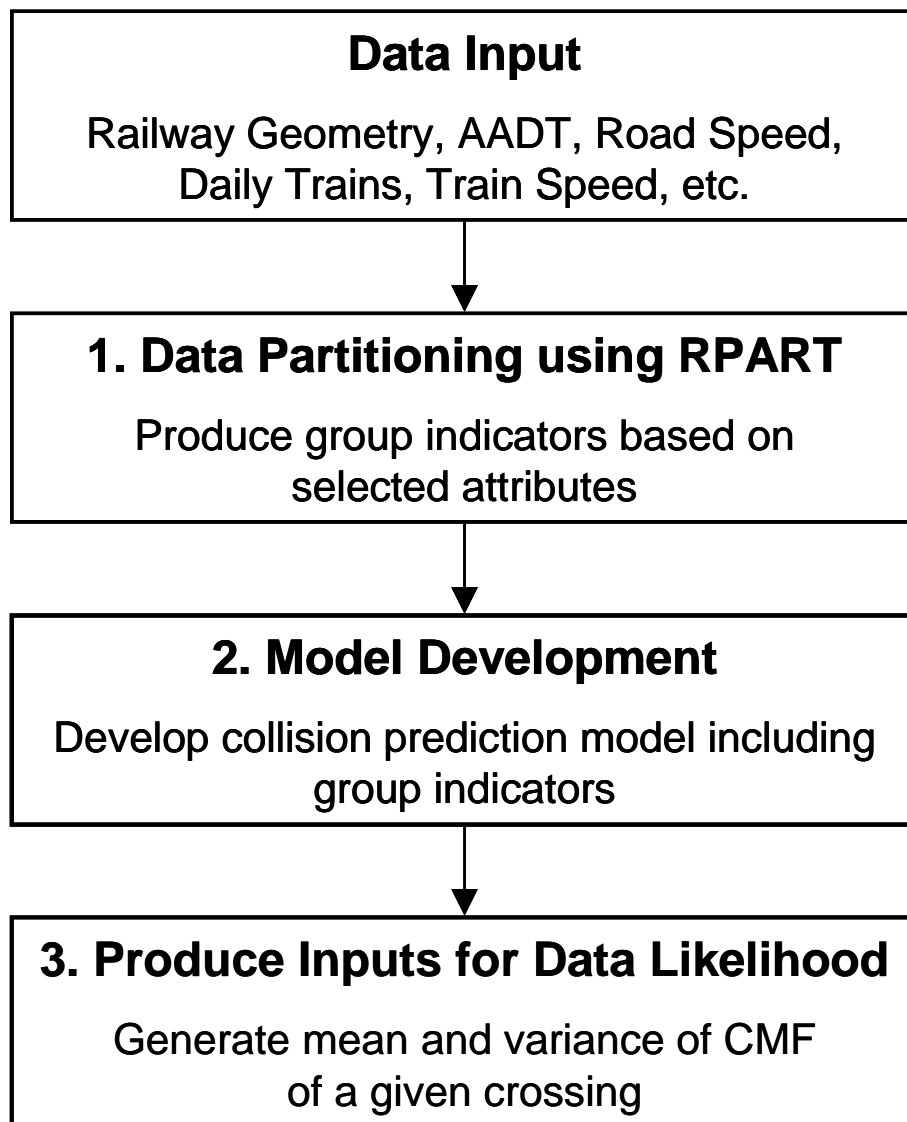


Figure 4.5 Group Indicator Modeling Framework

To reflect the complex relationships among the variables in larger datasets, researchers have adopted data mining techniques (Conerly et al. 2000). Specifically data mining is a process that seeks to discover meaningful correlations, patterns and trends in attributes by sifting through large amounts of data stored in repositories using pattern recognition and statistical techniques. In this section, RPART (one of the commonly applied techniques in data mining) is used to identify important explanatory variables and their interactions. RPART is a non-parametric technique that can select those variables and their interactions that are most important in determining an outcome or response variable.

As discussed in section 4.3.2, The Poisson regression tree technique is used in this application since the response variable is a count/ratio variable.

Because of the mixed variable types in the RODS/IRIS database (e.g. nominal, categorical, scalar, etc.), and the relatively large levels in some categorical variables (e.g. the initial road type contains ten different levels and so on), a re-organization of variables was required before performing supplementary analysis. Also, the selection of cut-off points for the categorization of ratio variables required a systematic way of that exercise. In this study, a number of Poisson regression trees were developed by using factors noted in Table 4.8. Individual factors were applied as single explanatory variables for each tree structure, and split criteria in every internal node were utilized to establish cut-off points for each factor. As a result, the original multi-levels for the categorical variables were systematically combined, and the thresholds for the ratio variables were estimated. Table 4.8 gives a summary of the crossing inventory measurements which were used for the subsequent analysis.

In this research, a total of eleven factors (except exposure and number of collisions) were employed to reflect any possible main and interaction effects among the explanatory variables. Once a hierarchical Poisson regression tree was developed, the internal node splitting rules were transformed into the group indicator variables to reflect interactions in the prediction models. For instance, crossings in sub-node (5) of Figure 4.6 reflect third-order interactions, as represented by a binary coding value of 1 or 0. That is, if the installed warning devices for a crossing are either flashing lights or gates, intersected by an arterial/collector or a local road that is unpaved, then the group indicator takes on a value of 1; otherwise, it takes on a value of 0. Similarly, the group indicator concerning the sub-node (3) describes a second-order interaction, wherein the variable takes a value of 1 if and only if a crossing is equipped with a sign with the lowest train operating speeds; or 0 otherwise.

The splits near the root node [i.e. the node (I)] reflect primary effects (i.e. the nodes (II) and (III), therefore their effects influence a large proportion of the crossings). Splits farther from the root node reflect higher-order interactions and apply to a smaller number of

crossings. The percentage values in Figure 4.6 reflect the proportion of crossings in each splitting node. A total of nineteen different group indicators were obtained from the hierarchical Poisson regression trees in this analysis. These group indicators represent six second-order interactions, four third-order interactions, six fourth-order interactions, and three fifth-order interactions.

In this section, we developed a collision prediction model with group indicators using a NB expression to produce the final input for data likelihood. We note that this study's results are rather different from the original expressions in a previous study by Park and Saccomanno (2005b). This is because we recalibrated the models to obtain the variance-covariance matrix among parameters for approximating the variances of *CMF* and to reflect nonlinearity in exposure on collision prediction models. The summary of the NB collision prediction model is given in Table 4.9 and Appendix E.

The collision prediction model considers only three factors as its main effects for predicting collision frequency (i.e. warning devices, train speed, and exposure). In the process of growing the hierarchical Poisson tree, the splitting criteria over 19 internal nodes were transformed to group indicators. Of these indicator variables, three indicators passed a statistical test with a 90% confidence level. The reason why all indicator variables were not found to be statistically significant is that, while the Poisson regression tree-growing procedure in RPART ensures that splitting maximizes the reduction in impurity at a given node, it fails to ensure that the difference in deviance between sub-nodes is statistically significant. However, it should be recognized that four more factors, such as Surface Type (GI08), Track Number (GI08), Track Angle (GI11, GI13), and Posted Road Speed (GI13), can be taken into account by introducing these three group indicators in the collision prediction expression.

Table 4.8 Reorganized Factors for Collision Prediction Model with Group Indicators

Factors (Variable Type)	Original Levels	Combined Levels	Coding Description
1.Warning Devices (Nominal) *	10	3	-1 for crossings with signs, 0 for crossings with flashing lights, and 1 for crossings with gates
2.Extra Warning Devices(Nominal) †	10	2	1 for crossings with extra warning devices such as an extra bell or an auxiliary light; 0 for otherwise.
3.Highway Class (Nominal) †	10	3	-1 for crossings with other roads types, 0 for crossings with local road, and 1 for crossings with arterial/collector.
4.Surface Type (Nominal)	13	2	1 for crossings with paved surface; 0 for otherwise.
5.Surface Width (Continuous) †	Scale Value	3	-1 for crossings whose surface width is under 8.5m, 0 for crossings whose surface width is between 8.5m and 13.5m, 1 for crossings whose surface width is above 13.5m
6.Track Angle (Continuous)	Scale Value	2	1 for crossings with perpendicular track-angle; 0 for otherwise.
7.Track Type (Nominal) †	15	2	1 for crossings in mainline; 0 for otherwise (e.g. crossings in switching line).
8.Whistle Prohibition (Nominal)	2	2	1 for crossings with whistle operation, 0 for prohibition.
9.Track Number (Continuous) †	Scale Value	2	1 for crossings with multiple tracks; 0 for otherwise (i.e. single track).
10.Posted Road Speed(Continuous) †	Scale Value	4	-2 for crossings with the lowest level posted speed under 47 km/h, -1 for crossings with medium level posted speed between 48 km/h and 75 km/h, 1 for crossings with moderate level posted speed between 76 km/h and 85 km/h, and 2 for crossings with the highest level posted speed over 85 km/h.
11.Time-table Train Speed(Continuous) †	Scale Value	3	-1 for crossings with the lowest level train speed under 36 km/h, 0 for crossings with time-table train speed between the lowest and the highest level, and 1 for crossings with the highest level train speed above 92 km/h.
12.Exposure (Continuous) ‡	Scale Value	Scale Value	Product of train daily volume and vehicle daily volume passing a crossing.
13.Number of Collisions (Ratio) §	Scale Value	Scale Value	The number of collisions at grade crossings over a nine-year period between 1993 and 2001.

Note1) *: The warning device type was re-organized by the criterion of Saccomanno et al. (2004).

Note2) † : RPART was applied for these factors to systematically reduce the measurements level.

Note3) ‡: Exposure was introduced by Schultz (1965), and the variable was used as the baseline measure for RPART procedure.

Note4) §: The response variable for RPART procedure.

Note5) Factor 1~2: Warning Devices; Factor 3~9; Geometric Attributes; Factor 10~11: Traffic Characteristics; Factor 12: Exposure.

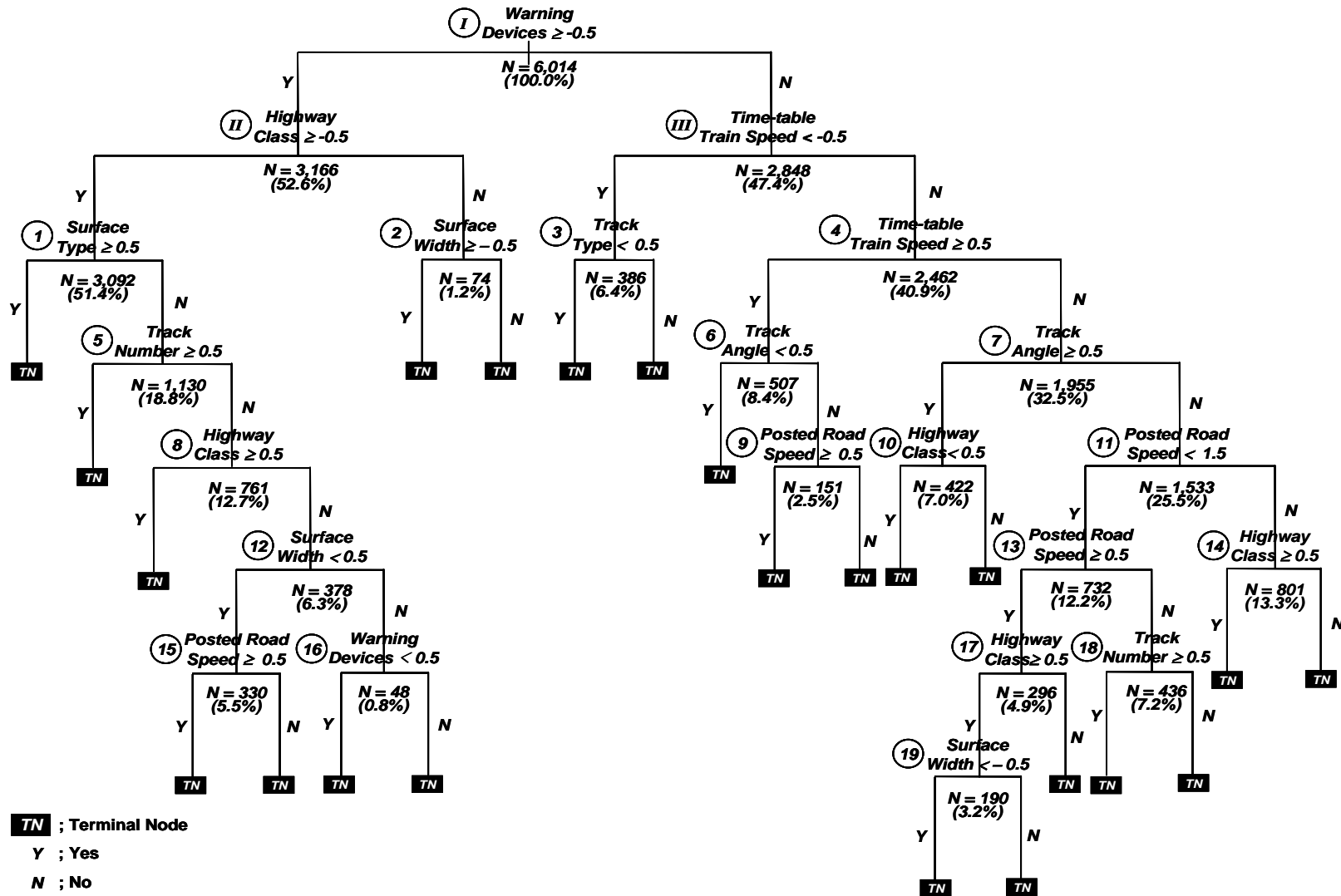


Figure 4.6 Hierarchical Tree Structure on the basis of RPART Method

Table 4.9 Collision Prediction Models with Group Indicators

Variables	Coding Scheme	Coef.	Std. Errs.
Flashing Lights (FL)	FL = 1; Otherwise = 0	-0.728	0.096
Gates (GT)	GT = 1; Otherwise = 0	-0.912	0.118
Max. Train Speed (MTS)	Medium Level MTS: $36 \leq \text{MTS} \leq 92$ km/h = 1; Otherwise = 0	0.274	0.086
	High Level MTS: $\text{MTS} > 92$ km/h = 1; Otherwise = 0	0.316	0.092
Ln(Exposure)	Ln(AADT×DailyTrain)	0.422	0.019
GI08	C11 takes value 1 if a crossing is installed with active warning devices (flashing lights or gates), on arterial or collector or local roads, with paved surface, with multiple tracks; Otherwise = 0	0.144	0.087
GI11	C11 takes value 1 if a crossing is installed with signs, with medium level train speed, with non-perpendicular track angle; Otherwise = 0	0.409	0.127
GI13	C13 takes value 1 if a crossing is installed with signs, with medium level train speed, with non-perpendicular track angle, with posted speed under 85km/h; Otherwise = 0	-0.234	0.140
Intercept		-4.609	0.170
Dispersion (α)		0.554	0.083

5 Estimating Effectiveness of Selected Countermeasures

In this section, several numerical examples are provided to evaluate different types of countermeasures at selected crossings. This application demonstrates how the proposed model can be used to estimate countermeasure effect for specific crossings. For the purpose of illustration four different countermeasures are considered:

- Introducing “whistle operation” at two crossings with different attributes where whistles are currently prohibited by jurisdictional noise bylaws
- Upgrading “warning devices” from flashing lights to 2-quadrant gates
- Introducing “4-quadrant gates” where 2-quadrant gates are presently installed
- Reducing “maximum train speed”
- Introducing “multiple countermeasures” simultaneously at a given grade crossing

5.1 Effectiveness of Elimination of Whistle Prohibition

A summary of crossing attributes for this example is provided in Table 5.1. It should be noted that the coding description in Table 5.1 follows the coding scheme for the factor/cluster collision prediction model in Table 4.3. The coding scheme of the other two models will require a few different strategies for the same crossing attributes. For instance, the collision prediction models with group indicators have only categorical variables in its explanatory variables, and therefore the coding scheme should be changed on the basis of Table 4.8. Table 4.6 contains the coding description of the stratified collision prediction model.

We first obtain estimates of *CMF* (priors) from previous published sources assuming they are normally distributed. In Table 4.2, we already provided the necessary parameters [i.e. mean (μ) and standard deviation (τ)] for generating prior normal density functions. For instance, the mean and standard deviation of the *CMF* of eliminating whistle prohibition based on three past studies are about 0.467 ± 0.0417 (i.e. representing a 53.3% mean reduction in collisions) at a given crossing.

The *CMF* represents the historical a priori belief for this countermeasure before conducting any data analysis. This serves as an input into Equation 3.2 to yield the posterior distribution of the “whistle operation effect”.

Secondly, we obtain three data likelihood estimates on the elimination of whistle prohibition countermeasure based on the three collision prediction models developed for the Canadian crossing data as introduced in Chapter 4.

Table 5.1 The 1st Sample Crossing Attributes for Example Calculation

Crossing Attributes	Data Description and Coding	
Warning Devices	Flashing Light	1 (2*)
Road Surface Width	ft (m)	15 (4.572)
Surface Material	Asphalt (Paved)	1
Road Type	Arterial	1
Track Number	Single	1
Track Angle	70 Degrees	70
Whistle	Prohibition	1 (Before)
	Operation	0 (After)
Mainline or Non-mainline	Mainline	1
AADT	15,000*	
Daily Trains	12*	
Exposure	180,000	
Posted Highway Speed Limit	km/hr	50
Max. Train Speed	miles/hr (km/hr)	10 (16)

Note)*: Only for Factor/Cluster Analysis (Refer to Table 4.3)

1) Data Likelihood based on the Factor/Cluster Collision Prediction Model

We first report the results of the factor/cluster analysis model provided in section 4.3.1 and illustrated in Figure 4.1. The subsequent steps are followed:

- Step 1: Transfer explanatory variables into standardized variables using the following expression:

$$Z_{ij} = \frac{x_{ij} - \bar{x}_j}{\sigma_j} \quad (5.1)$$

where,

Z_{ij} = standardized value for crossing i on variable j

x_{ij} = value for crossing i on variable j

\bar{x}_j = mean of variable j (refer to Table F.1 in Appendix F)

σ_j = standard deviation of variable j (refer to Table F.1 in Appendix F)

For instance, the original value of “Daily Trains” for this example crossing is 12, and therefore the standardized value is calculated as such;

$$Z_{ij} = \frac{12 - 9.3741}{11.1340} = 0.2358$$

Calculated values of Z_{ij} for individual variables are also presented in Table F.1.

- Step 2: Calculate Factor Scores

In this exercise, we employ Equation 4.1. Table F.2 provides the necessary inputs and outputs for this calculation, including factor score coefficients (β_{ik}). For example, the first factor score for this crossing can be estimated as follows;

$$\begin{aligned} \text{Factor Score 1} &= 0.5034 \times 0.2560 + (-1.1144) \times 0.2443 + 1.2793 \times 0.2664 + 1.6585 \\ &\times 0.3066 + (-0.4111) \times 0.0235 + 0.0033 \times 0.0130 + 3.1807 \times 0.1137 + 0.2727 \times (-0.0743) \\ &+ 3.4127 \times 0.2587 + 0.2358 \times (-0.0059) + (-0.4407) \times 0.1321 + (-1.4899) \times (-0.0753) = \\ &1.9731 \end{aligned}$$

Similarly, we obtain a second, third, and fourth factor score for this crossing. The estimated values are estimated as -0.4980, 1.0671, and -0.1470 respectively.

- Step 3: Determine Cluster Membership

In this exercise, we employ the procedure given by Equation 4.2. Table F.3 contains the cluster center information regarding four factor scores. For example, the distance from this sample crossing to cluster 1 is calculated as follows:

$$\text{Distance to Cluster 1} = [(1.9731-0.5949)^2 + (-0.4980-1.8202)^2 + (1.0671 - 1.0418)^2 + (-0.1470 - 0.2106)^2]^{1/2} = 2.7207$$

Following the same procedure, the distances to Clusters 2, 3, 4, and 5 were estimated as 3.0198, 2.8972, 2.5894, and 1.0888 respectively. Our basic aim here is to determine cluster membership based on the minimal Euclidean distance. Cluster 5 is suggested as the cluster membership for this specific crossing.

- Step 4: Calculate the expected number of collisions before the elimination of the whistle prohibition by using cluster-specific collision prediction model from Table 4.4.

After conducting a factor/cluster analysis, we found out that this sample crossing belongs to Cluster 5 before countermeasure was applied. By applying the Cluster 5 model in Table 4.4 and Appendix C, we obtain the expected number of collisions before the elimination of whistle prohibition, as:

$$N_{bi} = \exp[-6.071 - 0.580 \cdot 1 - 1.492 \cdot 0 + 0.807 \cdot 1 + 0.497 \cdot \ln(15000 \cdot 12)] \approx 1.186$$

where,

N_{bi} = the number of estimated collisions at a crossing before (or without) a safety countermeasure ‘ i ’. Here, the countermeasure is whistle operation.

- Step 5: Calculate the expected number of collisions after the elimination of whistle prohibition by using the cluster-specific collision prediction model

For this exercise, we repeat factor/cluster analysis (i.e. step 1 ~ step 3) once again to determine cluster membership of the modified crossing after the elimination of whistle prohibition. After conducting factor/cluster analysis once again, the given crossing after countermeasure was found to belong to “Cluster 4”. Detailed numeric information is given

in Table F.4 ~ F.6 of Appendix F. The expected number of collisions after eliminating whistle prohibition at this specific crossing was estimated to be:

$$N_{ai} = \exp[-4.309 - 1.222 \cdot 1 + 0.441 \cdot \ln(15000 \cdot 12)] \approx 0.823$$

where,

N_{ai} = the number of estimated collisions at a crossing after (or with) a safety countermeasure ‘ i ’. Once again, here the countermeasure is whistle operation.

- Step 6: Estimate the CMF based on the expected number of collisions before and after introducing the countermeasure

If we slightly modify Equation 2.1 in order to reflect the concept of expectation, and to match the notation of Equation 3.6, it becomes:

$$\mathbf{x}_{ijk} = E\{CMF_{ijk}\} = \frac{E\{N_{Aijk}\}}{E\{N_{Bijk}\}} \quad (5.2)$$

where,

\mathbf{x}_{ijk} = an input (i.e. a mean) for data likelihood from collision prediction model k (i.e. in this case, the factor/cluster model) for countermeasure j at grade crossing i

$E\{CMF_{ijk}\}$ = the expected mean value of CMF from collision prediction model k for countermeasure j at grade crossing i

$E\{N_{Bijk}\}, E\{N_{Aijk}\}$, = the expected number of collisions from the collision prediction model k at grade crossing i before and after introducing countermeasure j , respectively (i.e. the output of Steps 4 and 5)

By applying Equation 5.2, we obtain the expected mean value of CMF from the factor/cluster model for “the elimination of whistle prohibition” at this example crossing, as such:

$$\mathbf{x}_{ijk} = E\{CMF_{ijk}\} = \frac{E\{N_{Aijk}\}}{E\{N_{Bijk}\}} = \frac{0.823}{1.186} \approx 0.693$$

As a result, according to the factor/cluster model, we can expect a 30.7% $[(1-0.693) \times 100]$ reduction in collisions after a whistle is introduced in the crossing from Table 5.1.

Next we obtain the variance of CMF_{ijk} . Since $E\{N_{Aijk}\}$ and $E\{N_{Bijk}\}$ were estimated from two cluster-specific different collision prediction models on the basis of different samples of crossings, we can assert that these two expected values are independent of each other. Based on a delta method, which uses a truncated Taylor series expansion of random variables, Hauer (1997) established that the approximate variance of CMF :

$$\sigma_{ijk}^2 = Var\{CMF_{ijk}\} \approx E\{CMF_{Aijk}\}^2 \left[\frac{Var\{N_{Aijk}\}}{[E\{N_{Aijk}\}]^2} + \frac{Var\{N_{Bijk}\}}{[E\{N_{Bijk}\}]^2} \right] \left/ \left[1 + \frac{Var\{N_{Bijk}\}}{[E\{N_{Bijk}\}]^2} \right]^2 \right. \quad (5.3)$$

where,

σ_{ijk}^2 = an input (i.e. a variance) for data likelihood from collision prediction model k (i.e. in this case factor/cluster model) for countermeasure j at grade crossing i

$Var\{CMF_{ijk}\}$ = the variance of CMF from collision prediction model k for countermeasure j at grade crossing i

$Var\{N_{Bijk}\}, Var\{N_{Aijk}\}$ = the variance of expected number of collisions from collision prediction model k at grade crossing i before and after introducing countermeasure j , respectively

Since in this example the NB expression was assumed, the $Var\{N_{Aijk}\}$ and $Var\{N_{Bijk}\}$ can be estimated on the basis of Equation 4.4 and cluster-specific inverse over-dispersion parameters (i.e. α_k) in Table 4.4, as such:

$$Var\{N_{Aijk}\} = E\{N_{Aijk}\} + \alpha_k \cdot E\{N_{Aijk}\}^2 = 0.823 + 0.973 \cdot 0.823^2 \approx 1.480$$

$$Var\{N_{Bijk}\} = E\{N_{Bijk}\} + \alpha_k \cdot E\{N_{Bijk}\}^2 = 1.186 + 1.614 \cdot 1.186^2 \approx 3.457$$

Applying these numbers to Equation 5.3, the end result of variance of CMF is estimated:

$$\sigma_{ijk}^2 \approx 0.693^2 \left[\frac{1.480}{0.823^2} + \frac{3.457}{1.186^2} \right] / \left[1 + \frac{3.457}{1.186^2} \right]^2 = 0.432^2 \approx 0.186$$

As a result, the expected mean value of CMF for the data likelihood obtained from the factor/cluster model is assumed to follow $N(0.693, 0.432^2)$. This represents an expected effectiveness of elimination of whistle prohibition inherent in the factor/cluster model for the given crossing with attributes in Table 5.1.

2) Data Likelihood based on the Stratified Collision Prediction Model

In this exercise, the stratified collision prediction model in section 4.3.2 is employed to produce an independent data likelihood value. For the same example crossing in Table 5.1, we apply the model framework illustrated in Figure 4.2.

- Step 1: Determine Class Membership based on Control Factors

Compared to the previous factor/cluster collision prediction model, the crossing membership is easily determined by considering control factors in this sample crossing. Although the sample crossing is assumed to intersect with an arterial highway regardless of the countermeasure status, in this analysis, the given crossing belongs to Class 1 in both before and after countermeasure states.

- Step 2: Calculate the expected number of collisions before eliminating whistle prohibition by using the stratified collision prediction model

By applying the Class 1 prediction model in Table 4.6 and Appendix D, we obtain the expected number of collisions before eliminating whistle prohibition, as such:

$$N_{bi} = \exp[-3.797 - 0.677 \cdot 1 - 0.899 \cdot 0 + 0.294 \cdot 1 + 0.002 \cdot 16 + 0.345 \cdot \ln(15000 \cdot 12)] \approx 1.027$$

where,

N_{bi} = the number of estimated collisions at a crossing before (or without) a safety countermeasure 'i'. The countermeasure is whistle operation in this case.

- Step 3: Calculate the expected number of collisions after elimination of whistle prohibition by using the stratified collision prediction model

Contrary to the previous factor/cluster model, the class membership in the stratified model will not be affected by the countermeasure status since the stratification is based on the control factors that are irrelevant to countermeasures. As a result, the expected number of collisions after eliminating whistle prohibition at this specific crossing is estimated as:

$$N_{ai} = \exp[-3.797-0.677 \cdot 1-0.899 \cdot 0+0.294 \cdot 0+0.002 \cdot 16+0.345 \cdot \ln(15000 \cdot 12)] \approx 0.765$$

where,

N_{ai} = the number of estimated collisions at a crossing after (or with) a safety countermeasure 'i'. Once again, the countermeasure is whistle operation.

- Step 4: Estimate CMF based on the expected number of collisions before and after introducing the countermeasure

In this exercise, Equation 5.2 is adapted, such that:

$$\mathbf{x}_{ijk} = E\{CMF_{ijk}\} = \frac{E\{N_{Aijk}\}}{E\{N_{Bijk}\}} = \frac{0.765}{1.027} \approx 0.745$$

As a result, based on the stratified collision prediction model, we can expect a 25.5% reduction in collisions after whistle prohibition is eliminated at the given crossing (as per Table 5.1).

A major challenge is how to estimate the variance of CMF_{ijk} . As the same model was used for estimating the expected number of collisions both before and after the countermeasure, the assumption of independence between the two estimates is no longer valid. This is a different situation compared to the previous calculation using the factor/cluster collision prediction model. In this case, we must consider covariance among the parameters involved.

To obtain the $Var\{CMF_{ijk}\}$, once again a delta method that approximates variance of random variables has been employed (Benjamin and Cornell 1970, Sampson 2006, Xu and Long 2005). Since NB and Poisson collision prediction models can be expressed as exponential functions, we can convert the expressions to a linear combination of parameters, such that:

$$\mathbf{x}_k = E\{CMF_k\} = \frac{E\{N_{Ak}\}}{E\{N_{Bk}\}} = \frac{\exp(\hat{\beta}_{Ak} X_{Ak})}{\exp(\hat{\beta}_{Bk} X_{Bk})} = \exp(\hat{\beta}_{Ak} X_{Ak} - \hat{\beta}_{Bk} X_{Bk}) \quad (5.3)$$

where,

$\hat{\beta}_{Bk}, \hat{\beta}_{Ak}$ = the estimated vector of parameters in model k before and after countermeasure states, respectively. We can let $\hat{\beta}_{Bk} = \hat{\beta}_{Ak} = \hat{\beta}_k$

X_{Bk}, X_{Ak} = the vector of explanatory variables of model k before and after countermeasure states, respectively.

Taking the logarithm on both sides of Equation 5.3, we can obtain a linear combination of parameters as follows:

$$\log(\mathbf{x}_k) = \log(\exp(\hat{\beta}_{Ak} X_{Ak} - \hat{\beta}_{Bk} X_{Bk})) = \hat{\beta}_{Ak} X_{Ak} - \hat{\beta}_{Bk} X_{Bk} = \hat{\beta}_k (X_{Ak} - X_{Bk}) \quad (5.4)$$

We can obtain \mathbf{x}_k as:

$$\mathbf{x}_k = \exp(\log(\mathbf{x}_k))$$

By definition of the delta method we approximate $Var\{\mathbf{x}_k\}$ using a first-order Taylor series expansion, such that;

$$Var\{\exp(\log(\mathbf{x}_k))\} = Var\{\log(\mathbf{x}_k)\} \cdot \{\exp(\log(\mathbf{x}_k))\}^2 = Var\{\log(\mathbf{x}_k)\} \cdot \{\mathbf{x}_k\}^2 \quad (5.5)$$

where,

$$Var\{\log(\mathbf{x}_k)\} = (X_{Ak} - X_{Bk}) \cdot Var(\hat{\beta}_k) \cdot (X_{Ak} - X_{Bk})^T$$

As a result, the estimated $Var\{\log(\mathbf{x}_k)\}$ is as follows:

$$\begin{pmatrix} Prm1 & Prm2 & Prm3 & Prm4 & Prm5 & Prm6 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} Prm1 & Prm2 & Prm3 & Prm4 & Prm5 & Prm6 \\ Prm1 & 0.07074 & -0.006822 & -0.001511 & 0.01198 & 0.007228 & -0.000101 \\ Prm2 & -0.006822 & 0.0009024 & -0.001697 & -0.003057 & -0.001280 & 2.9896E-6 \\ Prm3 & -0.001511 & -0.001697 & 0.02163 & 0.02126 & 0.001376 & -0.000014 \\ Prm4 & 0.01198 & -0.003057 & 0.02126 & 0.03415 & -0.001132 & -0.000072 \\ Prm5 & 0.007228 & -0.001280 & 0.001376 & -0.001132 & 0.01296 & 0.0000268 \\ Prm6 & -0.000101 & 2.9896E-6 & -0.000014 & -0.000072 & 0.0000268 & 1.4377E-6 \end{pmatrix} \cdot \begin{pmatrix} Prm1 & 0 \\ Prm2 & 0 \\ Prm3 & 0 \\ Prm4 & 0 \\ Prm5 & -1 \\ Prm6 & 0 \end{pmatrix} \approx 0.013$$

The notation of parameters [e.g. Prm1 = Intercept, Prm2 = $Ln(\text{exposure})$] is presented at Appendix D.

It is interesting to note that the estimated $Var\{\log(\mathbf{x}_k)\}$ is equal to the square of the estimated standard errors corresponding to the whistle prohibition variable (i.e. $0.114^2 = 0.013$) in Table 4.7. Finally, the $Var\{\mathbf{x}_k\}$ ($= 0.013 \times 0.745^2 \approx 0.085^2$) as well as the mean of \mathbf{x}_k (i.e. 0.745) has been approximated based on the stratified collision prediction model, and used as the second input to data likelihood.

3) Data Likelihood based on the Collision Prediction Model with Group Indicators

The collision prediction model with group indicators in section 4.3.3 will be used to produce the third data likelihood input used in this analysis applied to the same crossing (as per Table 5.1). The model framework is given in Figure 4.5. Contrary to the previous two collision prediction models, this model does not require a previous determination of crossing membership, since it uses a single prediction expression for all crossings.

We note that in this specific example we did not use the third model to produce a data likelihood input, since the model failed to explain variation in collision for this specific countermeasure. It might be theoretically rational to include 1.0 as the mean value of the data likelihood estimate (i.e. collision modification factor) of the model. Practically, however it would not be reasonable to assume that the 0% collision reduction effect will be achieved after the implementation of the proposed countermeasure (i.e. the elimination of the whistle prohibition) at a grade crossing. The value is so unlikely. Moreover, there is no straight-forward way of estimating the corresponding variance of this countermeasure unless we keep the countermeasure as an exploratory variable in a statistical modeling expression. As a result, based on the two point estimates from the two collision prediction models, we estimate the data likelihood distribution for this countermeasure effect. In fact, this situation illustrates one of the merits in the proposed approach. If we can estimate at least one CMF and its corresponding variance, we can produce the data likelihood associated with a specific crossing to generate the posterior distribution.

4) Estimating Data Likelihood based on the estimated means and variances

We obtained $P_1(\mathbf{x}_1|\theta) = N(0.693, 0.432^2)$, and $P_2(\mathbf{x}_2|\theta) = N(0.745, 0.085^2)$ from two collision prediction models. The reason of considerable difference in the estimated

variances of the two data likelihood models is resulted from the unique characteristic in the Factor/Cluster collision prediction model. The estimated variance is amplified since the cluster membership of this example crossing has been changed from the Cluster 5 to the Cluster 4 after the introduction of this particular countermeasure. In this example case, the estimated variance by the Factor/Cluster collision prediction model represents a kind of the between-group variance rather than the within-group variance. Therefore, we expect that if a crossing remains in the same cluster after the introduction of a countermeasure, the estimated variance will be similar to those based on the other two data likelihood prediction models. We will see this case at the next example calculation in Chapter 5.2.

Next the data likelihood can be estimated by using Equations 3.2, 3.3, and 3.4, such that:

$$\tau_1^2 = (0.432^{-2} + 0.085^{-2})^{-1} \approx 0.083^2$$

$$\mu_0 = (0.432^{-2} \cdot 0.693 + 0.085^{-2} \cdot 0.745) \cdot 0.083^2 \approx 0.745$$

As a result, the estimated data likelihood distribution for this crossing follows $N(0.745, 0.083^2)$, and this value reflects our current best knowledge regarding the expected effect of whistle operation at this specific crossing. We can compare this to a historical a priori value for the same countermeasure that was given in Table 4.2 as $N(0.467, 0.042^2)$.

Consequently, given the prior [i.e. $N(0.467, 0.042^2)$] and the data likelihood [i.e. $N(0.745, 0.083^2)$] distributions, we can produce the posterior distribution by applying Equations 3.3, and 3.4. The results are:

$$\tau_1^2 = (0.042^{-2} + 0.083^{-2})^{-1} \approx 0.037^2$$

$$\mu_0 = (0.042^{-2} \cdot 0.467 + 0.083^{-2} \cdot 0.745) \cdot 0.037^2 \approx 0.520$$

If we wish to represent the contribution of prior distribution to the posterior distribution, we can use the Equation 3.5 to yield a Bayesian weight factor ω , such that:

$$\omega = \frac{0.042^{-2}}{0.042^{-2} + 0.083^{-2}} \approx 0.800$$

As a result, the expected reduction in collisions at this grade crossing was estimated to be about 48% after the elimination of whistle prohibition. The contribution of prior information (ω) to the posterior distribution [$N(0.520, 0.037^2)$] has been estimated to be about 80%.

The effectiveness of the same countermeasure (i.e. the elimination of whistle prohibition) can be estimated using different crossing attributes. Table 5.2 shows the crossing attributes for a second sample crossing. We note that the difference in attributes between the crossings from Tables 5.1 and 5.2 is the road type. The second grade crossing is located on a local road rather than an arterial road as in the first case.

Again, the same prior distribution [i.e. $N(0.467, 0.042^2)$] in Table 4.2 will be employed for this illustration. Just like the previous example calculation, the data likelihood effects were obtained for each of the three prediction models introduced in Chapter 4. As discussed in the previous section, the factor/cluster model initially requires the estimation of factor scores to determine cluster membership of each crossing. To shorten the illustration, in this section we will not describe how cluster membership is determined, but simply indicate which cluster is involved. In this example calculation, the given crossing in Table 5.2 belongs to Cluster 5 in both the before-and-after countermeasure states. Again, the coding description in Table 5.2 follows the coding scheme of factor/cluster analysis in Table 4.3, and therefore the coding scheme should be changed to reflect the coding method of each collision prediction model.

Table 5.2 The 2nd Sample Crossing Attributes for Example Calculation

Crossing Attributes	Data Description and Coding	
Warning Devices	Flashing Light	1 (2*)
Road Surface Width	ft (m)	15 (4.572)
Surface Material	Asphalt (Paved)	1
Road type	Local	0
Track Number	Single	1
Track Angle	70 Degree	70
Whistle	Prohibition	1 (Before)
	Operation	0 (After)

Mainline or Non-mainline	Mainline	1
AADT	15,000*	
Daily Trains	12*	
Exposure	180,000	
Posted Highway Speed Limit	km/hr	50
Max. Train Speed	mile/hr (km/hr)	10 (16)

Note)*: Only for Factor/Cluster Analysis (Refer to Table 4.3)

By applying the Cluster 5 expression in Table 4.4, we obtain CMF values for “the Elimination of Whistle Prohibition”. For this exercise we use an expression of the form:

$$\begin{aligned}
 x_{ijk} &= \frac{E\{N_{Aijk}\}}{E\{N_{Bijk}\}} = \frac{\exp(-6.071 + 0.497 \cdot \ln(15,000 \cdot 12) - 0.580 \cdot 1 - 1.492 \cdot 0 + 0.807 \cdot 0)}{\exp(-6.071 + 0.497 \cdot \ln(15,000 \cdot 12) - 0.580 \cdot 1 - 1.492 \cdot 0 + 0.807 \cdot 1)} \\
 &= \frac{0.529}{1.186} \approx 0.446
 \end{aligned}$$

Based on the factor/cluster model, for similar crossings in Cluster 5 we can expect a 55.4% reduction in collisions with the elimination of whistle prohibition (Table 5.2). Inasmuch as the same collision prediction model has been used to predict the number of collisions before and after the countermeasure, the independency assumption is not applicable to this case. As noted previously, the delta method has been applied to approximate $Var\{\mathbf{x}_k\}$ by $Var\{\log(\mathbf{x}_k)\} \cdot \{\mathbf{x}_k\}^2$. The necessary variance-covariance matrix among individual parameters of the “Cluster 5” model is obtained throughout the model calibration process and provided in Appendix C. The estimated variance is simply equal to the square of the estimated standard errors corresponding to the whistle prohibition variable in Table 4.4, which can be stated as $(0.164)^2 \approx 0.0268$. The approximated variance of CMF becomes $(0.446)^2 \cdot 0.027 \approx 0.005$ (i.e. standard errors ≈ 0.073). As a result, the range of estimates for input into the data likelihood effect on collisions following the elimination of whistle prohibition at this specific crossing is $N(0.446, 0.073^2)$.

By applying the same procedure to Class 3 crossings in Table 4.7, we obtained the range of estimates [i.e. $N(0.437, 0.076^2)$] for the input into the data likelihood. The necessary information including a variance-covariance matrix for this calculation is provided in Appendix C. As noted earlier, we did not use the third Canadian collision

prediction model, since it could not explain variation in collisions for this countermeasure. Therefore, based on the above two point estimates, we estimated the data likelihood distribution.

Since we obtained $P_1(\mathbf{x}_1|\theta) = N(0.446, 0.073^2)$, and $P_2(\mathbf{x}_2|\theta) = N(0.437, 0.076^2)$, then the data likelihood can be estimated by using Equations 3.3 and 3.4, such that:

$$\tau_1^2 = (0.073^{-2} + 0.076^{-2})^{-1} \approx 0.0028$$

$$\mu_0 = (0.076^{-2} \cdot 0.437 + 0.073^{-2} \cdot 0.446) \cdot 0.0028 \approx 0.442$$

As a result, the estimated data likelihood distribution for this crossing becomes $N(0.442, 0.053^2)$. This represents the current best knowledge concerning the expected effect of elimination of whistle prohibition for this specific crossing. Consequently, given the prior [i.e. $N(0.467, 0.042^2)$] in Table 4.2 and the estimated data likelihood [i.e. $N(0.442, 0.053^2)$] distributions, the posterior distribution from Equations 3.3 and 3.4 becomes:

$$\tau_1^2 = (0.042^{-2} + 0.053^{-2})^{-1} \approx 0.0011$$

$$\mu_0 = (0.042^{-2} \cdot 0.467 + 0.053^{-2} \cdot 0.442) \cdot 0.0011 \approx 0.457$$

The Equation 3.5 used to yield a Bayesian weight factor ω as follows:

$$\omega = \frac{0.042^{-2}}{0.042^{-2} + 0.053^{-2}} \approx 0.616$$

As a result, the expected reduction in collisions at this grade crossing was estimated to be about 54.3% after the elimination of whistle prohibition. We note that the contribution of prior information (ω) to the posterior distribution [$N(0.457, 0.033^2)$] is about 62% of the total estimated reduction in expected collisions at this crossing.

These results from the above two examples do not necessarily mean that the elimination of whistle prohibition gives rise to higher benefits for crossings at local roads compared to crossings on arterial roads. It should be emphasized that these results are legitimate only for the crossings with the same attributes summarized in Table 5.1 and 5.2. Nevertheless, the example outlined here appears to be useful in that it provides a

reasonably accurate estimate of *CMF* for specific crossing attributes and a specific countermeasure.

5.2 Effectiveness of Upgrading Warning Devices from Flashing Lights to Gates

We consider a second countermeasure example dealing with the introduction of 2-Quadrant Gates to a given crossing currently equipped with Flashing Lights. For this exercise, the crossing attributes in Table 5.3 are adopted. These are exactly the same attributes as the crossing in Table 5.1, except for the status of warning devices. The whistle prohibition is assumed to be in effect for both before and after countermeasure conditions.

From Table 4.2, first we obtained the necessary parameters [i.e. the mean (μ) and the variance (σ^2)] for the prior normal density function. The mean and standard deviation of the *CMF* of upgrading flashing lights to gates based on ten past studies was found to be 0.474 ± 0.149 (i.e. representing 52.6% of expected collision reduction). Again, the prior distribution represents the historical a priori value for this countermeasure before conducting any data analysis involving this specific crossing.

Table 5.3 The 3rd Sample Crossing Attributes for Example Calculation

Crossing Attributes	Data Description and Coding	
	Warning Devices	Flashing Light
	Gates	1 (3*) (After)
Road Surface Width	ft (m)	15 (4.572)
Surface Material	Asphalt (Paved)	1
Road type	Arterial	1
Track Number	Single	1
Track Angle	70 Degree	70
Whistle	Prohibition	1
Mainline or Non-mainline	Mainline	1
AADT	15,000*	
Daily Trains	12*	
Exposure	180,000	
Posted Highway Speed Limit	km/hr	50
Max. Train Speed	mile/hr (km/hr)	10 (16)

Note)*: Only for Factor/Cluster Analysis (Refer to Table 4.3)

The given crossing in Table 5.3 belongs to Cluster 5 after conducting factor/cluster analysis in both before and after countermeasure states. By applying the Cluster 5 expression in Table 4.4, we obtain one point estimate of data likelihood for “Upgrading Warning Devices”. For this exercise we use an expression of the form:

$$\begin{aligned} x_{ijk} &= \frac{E\{N_{Aijk}\}}{E\{N_{Bijk}\}} = \frac{\exp(-6.071 + 0.497 \cdot \ln(15,000 \cdot 12) - 0.580 \cdot 0 - 1.492 \cdot 1 + 0.807 \cdot 1)}{\exp(-6.071 + 0.497 \cdot \ln(15,000 \cdot 12) - 0.580 \cdot 1 - 1.492 \cdot 0 + 0.807 \cdot 1)} \\ &= \frac{0.476}{1.186} \approx 0.402 \end{aligned}$$

As a result, a 55.4% reduction in collisions is expected by upgrading warning devices from flashing lights to gates for this sample crossing. Since the independency assumption is not appropriate to this case, as we illustrated in the previous section the delta method has been employed to approximate $Var\{\mathbf{x}_k\} = Var\{\log(\mathbf{x}_k)\} \cdot \{\mathbf{x}_k\}^2$. The corresponding variance-covariance matrix of the “Cluster 5” model is provided in Appendix C.

The estimated $Var\{\log(\mathbf{x}_k)\} = (X_{Ak} - X_{Bk}) \cdot Var(\hat{\beta}_k) \cdot (X_{Ak} - X_{Bk})^T$ becomes:

$$\begin{pmatrix} \text{Prm1} & \text{Prm2} & \text{Prm3} & \text{Prm4} & \text{Prm5} \\ 0 & 0 & -1 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \text{Prm1} & \text{Prm2} & \text{Prm3} & \text{Prm4} & \text{Prm5} \\ \text{Prm1} & 0.27575 & -0.02986 & 0.03751 & 0.08755 & -0.01181 \\ \text{Prm2} & -0.02986 & 0.003534 & -0.006500 & -0.01223 & 0.0003082 \\ \text{Prm3} & 0.03751 & -0.006500 & 0.04494 & 0.04680 & -0.008857 \\ \text{Prm4} & 0.08755 & -0.01223 & 0.04680 & 0.10475 & -0.01409 \\ \text{Prm5} & -0.01181 & 0.0003082 & -0.008857 & -0.01409 & 0.02678 \end{pmatrix} \cdot \begin{pmatrix} \text{Prm1} & 0 \\ \text{Prm2} & 0 \\ \text{Prm3} & -1 \\ \text{Prm4} & 1 \\ \text{Prm5} & 0 \end{pmatrix} \approx 0.056$$

The notation of parameters [e.g. Prm1 = Intercept, Prm2 = $\ln(\text{exposure})$] is presented at Appendix C. It should be noted that the estimated $Var\{\log(\mathbf{x}_k)\}$ is now different from the variance (i.e. the square of the estimated standard errors) corresponding to either flashing lights or gates in Table 4.4, since the covariance of these two variables has been taken into account for this mixed countermeasure. Finally, the $Var\{\mathbf{x}_k\} = 0.056 \times 0.402^2 \approx 0.095^2$ as well as the mean of \mathbf{x}_k (i.e. 0.402) can be approximated based on the Cluster 5 collision prediction model.

In terms of the stratified model, the sample crossing belongs to the Class 1 collision prediction model, since this crossing is located on an arterial highway. By applying the same method, we obtain the second point estimates for data likelihood: $N(0.801, 0.092^2)$. Also, the

third point estimate, $N(0.833, 0.069^2)$ is estimated on the basis of the third collision prediction model in section 4.3.3. Inasmuch as these three point estimates were obtained from the three different collision prediction models, the first two of these three point estimates should be combined using Equations 3.3 and 3.4. After obtaining the combined estimate, the resultant estimate is combined once again with the third point estimate based on the same approach to yield a data likelihood distribution. In this exercise, the order of computing will not affect the result. As such, a posterior distribution [i.e. $N(0.693, 0.045^2)$, $\omega = 0.09$] is estimated. The mean reflects a 30.7% reduction in expected collisions at grade crossing subject to upgrading warning devices from flashing lights to 2-quadrant gates. We note that 9% of this reduction can be explained by the prior distribution.

For this specific crossing, the elimination of whistle prohibition which is normally considered to be a supplementary countermeasure, produces higher safety benefits than the upgrade in warning device (i.e. Flashing Lights to 2Q-Gates). Had we tried to infer the safety benefits solely based on a priori belief, then the effectiveness of these two countermeasures is quite similar to each other....or about 53% of collision reduction for both (refer to Table 4.2). This result does not necessarily mean that elimination of whistle prohibition gives rise to higher benefits than warning device upgrades for all crossing types. It simply states that for a crossing with the given attributes in this example, the elimination of whistle prohibition may lead to higher collision reduction benefits than a more costly installation of gates. As pointed out earlier, the proposed approach produces a tailored *CMF* based on the crossing attributes.

5.3 Effectiveness of Upgrading Warning Devices from Signboards to Gates

A third application example deals with the introduction of 2-Quadrant Gates to the given crossing currently equipped with passive signs (cross-bucks). For this exercise we use the sample crossing in Table 5.4, which considers the same crossing attributes in Table 5.3 except for the status of warning devices.

In Table 4.2, the prior distribution for this countermeasure was determined to be $N(0.283, 0.086^2)$ based on ten previous reported studies. This suggests a 72% reduction in expected collisions. Data likelihood estimates for this specific countermeasure were obtained based on three models: the values are $N(0.225, 0.073^2)$, $N(0.407, 0.075^2)$, and $N(0.402, 0.048^2)$. From Equations 3.2 through 3.6, we obtained the posterior distribution for this countermeasure. The resultant distribution is $N(0.350, 0.033^2)$ for a value of α equal to 0.144. This countermeasure resulted in a 65% reduction in expected number of collisions with about 11.1% of this reduction being explained by the priors.

Table 5.4 The 3rd Sample Crossing Attributes for Example Calculation

Crossing Attributes	Data Description and Coding	
	Warning Devices	Signboards
	Gates	1 (3*) (After)
Road Surface Width	ft (m)	15 (4.572)
Surface Material	Asphalt (Paved)	1
Road Type	Arterial	1
Track Number	Single	1
Track Angle	70 Degree	70
Whistle	Prohibition	1
Mainline or Non-mainline	Mainline	1
AADT	15,000*	
Daily Trains	12*	
Exposure	180,000	
Posted Highway Speed Limit	km/hr	50
Max. Train Speed	mile/hr (km/hr)	10 (16)

Note)*: Only for Factor/Cluster Analysis (Refer to Table 4.3)

5.4 Effectiveness of Four Quadrant Gates

Some safety countermeasures, such as 4-quadrant gates or photo/video enforcement have not yet been introduced in Canadian inventory data. As a result, we cannot make any meaningful inference from the data likelihood model. In this case, instead of employing Bayesian data fusion we recommend relying solely on the prior estimates to represent

countermeasure effects. This appears to be a logical step until additional data likelihood inferences are available for the Canadian data.

For illustration purpose, let us suppose that Canadian decision makers want to know the safety gains by installing 4-quadrant gates at a crossing where presently 2-quadrant gates are installed. As mentioned, none of the collision prediction models in section 4.3 explain the variance of collisions by introducing the 4-quadrant gates because of the absence of information in Canadian inventory data. Therefore, the best inference for the countermeasure effect is simply the output of the prior distribution in Table 4.2 corresponding to a range of values $N(0.254, 0.023^2)$. As a result, the expected collision reduction following this countermeasure is about 74.6%.

In a similar vein, the effect of several countermeasures in Table 4.2 will be directly used to represent the final safety benefits corresponding to the countermeasures, including the installation of additional passive signboards (e.g. stop ahead sign), lighting, traffic signals, and photo/video enforcement.

5.5 Effectiveness of Reducing Maximum Train Speed

If we do not have a priori knowledge about a given countermeasure but we have current knowledge from the data, we can produce estimates of countermeasure effects based on the result from three different collision prediction models. In this section, the crossing attributes in Table 5.5 that are slightly different from the crossing in Table 5.2 will be used to illustrate the collision reduction effect of reducing maximum train speeds by 20 miles/hour.

After conducting factor/cluster analysis for the crossing in Table 5.4, we determined that this crossing belongs to Cluster 5 in both before and after states. As noted from Table 4.4, the Cluster 5 collision prediction model could not estimate the safety benefit by reducing train speed since this model does not include this variable in its prediction expression. On the other hand, the 2nd and 3rd collision prediction models in section 4.3.2 and 4.3.3 can estimate the necessary inputs for data likelihood. From these models we obtained $P_2(\mathbf{x}_2|\theta) = N(0.796,$

0.042^2) and $P_3(\mathbf{x}_3|\theta) = N(0.761, 0.066^2)$. The combined data likelihood estimate was found to be $N(0.786, 0.036^2)$ from Equations 3.3 and 3.4. A 20.4% reduction in the expected number of collisions was obtained for this sample crossing subject to a reduction of 20 miles/hour in the maximum train speed.

Table 5.5 The 4th Sample Crossing Attributes for Example Calculation

Crossing Attributes	Data Description and Coding	
Warning Devices	Flashing Light	1 (2*)
Road Surface Width	ft (m)	15 (4.572)
Surface Material	Asphalt (Paved)	1
Road Type	Local	0
Track Number	Single	1
Track Angle	70 Degree	70
Whistle	Prohibition	1
Mainline or Non-mainline	Mainline	1
AADT	15,000*	
Daily Trains	12*	
Exposure	180,000	
Posted Highway Speed Limit	km/hr	50
Max. Train Speed	mile/hr (km/hr)	30 (48) (Before)
	mile/hr (km/hr)	10 (16) (After)

Note)*: Only for Factor/Cluster Analysis (Refer to Table 4.3)

5.6 Effectiveness of Multiple Countermeasures

Equation 5.2 defines the CMF for a single countermeasure. In practice, several countermeasures can be introduced simultaneously at a given crossing. To estimate the combined effect of multiple countermeasures, we need to know the degree of interaction among these countermeasures. Such information is rarely available in practice and we assume independence among countermeasures (Shen et al. 2004). Under this assumption, the combined CMF of n countermeasures can be approximated using the following Equation:

$$E\{CMF_M\} = E\{CMF_1\} \times E\{CMF_2\} \times \dots \times E\{CMF_M\} \quad (5.6)$$

where,

$E\{CMF_M\}$ = the expected CMF of all n multiple countermeasures.

Note that if data are available, the assumption of independence could be validated empirically (Lord and Bonneson, 2006).

Benjamin and Cornell (1970) suggested the approximation method to estimate the expectation and variance of products among mutually independent random variables. In our case, CMF regarding different countermeasures are random variables. The approximate combined effect becomes:

$$Var\{CMF_1 \cdot CMF_2 \cdots CMF_n\} = \sum_{i=1}^n \left(\prod_{\substack{j=1 \\ j \neq i}}^n E\{CMF_j\}^2 \right) Var\{CMF_i\} \quad (5.7)$$

For instance, if $n = 2$

$$Var\{CMF_1 \cdot CMF_2\} = E\{CMF_1\}^2 \cdot Var\{CMF_2\} + E\{CMF_2\}^2 \cdot Var\{CMF_1\} \quad (5.8)$$

In section 5.1, whistle operation (i.e. CMF_1) was introduced to a crossing with attributes listed in Table 5.1. The estimated effect was found to be $N(0.520, 0.037^2)$. On the other hand, the effect of upgrading warning devices (i.e. flashing lights to gates, CMF_2) for the same crossing was found to be $N(0.693, 0.045^2)$ in section 5.2. If a decision maker wishes to implement these two countermeasures simultaneously, the combined effect can be estimated using Equation 5.6 and 5.8, such that:

$$E\{CMF_{1,2}\} = 0.520 \times 0.693 \approx 0.360$$

$$Var\{CMF_1 \cdot CMF_2\} = 0.520^2 \times 0.045^2 + 0.615^2 \times 0.037^2 = 0.033^2$$

The estimated CMF from the two countermeasures applied to this same crossing becomes $N(0.360, 0.033^2)$, representing 64 % reduction in the expected number of collisions.

6 UNCERTAINTY IN BAYESIAN DATA FUSION

This chapter addresses the uncertainty associated with the countermeasure effect. As noted by Button and Reilly (2000) and Leeming and Saccomanno (1994), it is not possible to obtain perfectly accurate point estimates using a statistical collision prediction model. The point estimate is represented by the expected value of CMF . Previous researchers often used a variance to assist the point estimate and to produce a range of values. The range of values indicates the uncertainty with the estimated CMF values. However, in this study, we prefer to the probability distributions (i.e. probability density functions) rather than a range of values of point estimates to represent countermeasure effects. Since the probability distributions will produce not only a range of values of a certain CMF estimate but also the likelihood of the estimate. The probability of countermeasure effects may lead us to different conclusions compared to a range of point estimates.

We also address the uncertainty with the estimates through the investigation of the input variables or relevant assumptions in Bayesian data fusion. For instance, the assumption of normality in CMF distributions may not be appropriate to describe the unknown CMF distributions. The uncertainties inherent in several different sources for priors may also hamper the development of the rigorous posterior CMF distributions. Probably, the weighting scheme itself in the prior estimates would be another source of uncertainty that may affect the reliability of the posterior estimates. Moreover, the posterior CMF distribution is a form of probability distribution and therefore it may contain a range of collision reduction effect that is unlikely to be materialized in real world. The following section investigates uncertainties inherent in the various input variables and/or assumptions.

6.1 Uncertainty Inherent in Type of Distribution

In this study, we have employed a normal density function to represent both the prior and posterior distributions. The Bayesian formulation applied to the normal distributions combines CMF from the multiple sources without using computationally intensive Markov Chains Monte Carlo (MCMC) method suggested by a number of researchers, including

Gelman et al. (2004). As noted by Lee (2004), the central limit theorem suggests that the observations (in our case the *CMF*) with errors can be assumed to be normally distributed. However, in reality the observation may not follow the normal distribution that has a symmetrical shape. Other distributions could be considered that relax our assumption of symmetry in the values of *CMF*. For instance, previous researchers (e.g. Clarke and Sarasua 2003, Washington and Oh 2006) suggested using a beta distribution to represent the prior and posterior distribution. They assert that the beta distribution is flexible enough to represent *CMF* since it does not require a strong symmetry assumption for both prior and posterior distributions.

In this section, beta distribution was used in the Bayesian fusion method. Since our objective is to produce a varying *CMF* based on grade crossing attributes rather than estimate the average effects for each countermeasure, an analytical method is adapted to combine different beta prior and data likelihood estimates. For illustration purpose, we assume that the estimated prior and data likelihood distribution follows beta distribution.

Similar to the normal distribution, the beta distribution is defined by only two shape parameters r and s , both of which are greater than zero (Harlow et al. 1997, Iversen 1984):

$$\Pr(X = x; r, s) = \frac{(r + s - 1)!}{(r - 1)!(s - 1)!} X^{r-1} (1 - X)^{s-1} \quad (6.1)$$

where,

$$\mu = \frac{r}{r + s} \text{ (i.e. mean of beta distribution)} \quad (6.2)$$

$$\sigma^2 = \frac{\mu(1 - \mu)}{r + s + 1} = \frac{rs}{(r + s)^2 (r + s + 1)} \text{ (i.e. variance of beta distribution)} \quad (6.3)$$

By solving for r and s :

$$r = \mu \left[\frac{\mu(1 - \mu)}{\sigma^2} - 1 \right] \quad (6.4)$$

$$s = [1 - \mu] \left[\frac{\mu(1 - \mu)}{\sigma^2} - 1 \right] \quad (6.5)$$

As pointed out by Harlow et al. (1997), the simplicity of the beta distribution is that the posterior beta parameters are additive functions of the beta prior and beta likelihood parameters, such that:

$$r_{posterior} = r_{prior} + r_{data\ likelihood} \quad (6.6)$$

$$s_{posterior} = s_{prior} + s_{data\ likelihood} \quad (6.7)$$

After obtaining the posterior beta parameters (i.e. $r_{posterior}$, $s_{posterior}$), the mean and the variance of the posterior distribution can be estimated using the Equations 6.2 and 6.3. A numerical example is as follows.

From the second example in section 5.1, the means and variances of prior and data likelihood distribution were obtained after introducing whistle countermeasure at the crossing in Table 5.2. They were:

Table 6.1 Sample Mean and Variance of Prior and Data Likelihood

	Prior	Factor/Cluster Model (D1)	Stratified Model (D2)
Mean (μ)	0.4671	0.4460	0.4373
Variance (σ^2)	0.0417 ²	0.0730 ²	0.0763 ²

In this example, we assume that the prior and data likelihood distributions follows the beta distribution. First, the beta data likelihood distribution is estimated based on Equations 3.2 and 6.2 ~ 6.7 as shown in the following steps:

- Step 1: Estimate r and s for the Factor/Cluster Model output using Equations 6.4 and 6.5:

$$r_{D1} = \mu_{D1} \left[\frac{\mu_{D1}(1-\mu_{D1})}{\sigma_{D1}^2} - 1 \right] = 0.4460 \left[\frac{0.4460(1-0.4460)}{0.0730^2} - 1 \right] \approx 20.2403$$

$$s_{D1} = [1 - \mu_{D1}] \left[\frac{\mu_{D1}(1-\mu_{D1})}{\sigma_{D1}^2} - 1 \right] = [1 - 0.4460] \left[\frac{0.4460(1-0.4460)}{0.0730^2} - 1 \right] \approx 25.1395$$

- Step 2: Estimate r and s for the Stratified Model output using Equations 6.4 and 6.5:

$$r_{D2} = \mu_{D2} \left[\frac{\mu_{D2}(1 - \mu_{D2})}{\sigma_{D2}^2} - 1 \right] = 0.4373 \left[\frac{0.4373(1 - 0.4373)}{0.0763^2} - 1 \right] \approx 18.0406$$

$$s_{D2} = [1 - \mu_{D2}] \left[\frac{\mu_{D2}(1 - \mu_{D2})}{\sigma_{D2}^2} - 1 \right] = [1 - 0.4373] \left[\frac{0.4373(1 - 0.4373)}{0.0763^2} - 1 \right] \approx 23.2097$$

- Step 3: Obtain $r_{data\ likelihood}$ and $s_{data\ likelihood}$ using Equations 6.6 and 6.7:

$$r_{data\ likelihood} = 20.2403 + 18.0406 = 38.2809$$

$$s_{data\ likelihood} = 25.1395 + 23.2097 = 48.3492$$

- Step 4: Estimate the expected mean (\mathbf{x}) and variance (σ^2) of the data likelihood distribution:

$$x_{data\ likelihood} = \frac{38.2809}{38.2809 + 48.3492} \approx 0.4419$$

$$\sigma_{data\ likelihood}^2 = \frac{0.4419(1 - 0.4419)}{38.2809 + 48.3492 + 1} = 0.0531^2$$

Since the beta distribution is assumed for the estimated prior [i.e. $B(0.4671, 0.0417^2)$] and data likelihood distribution [i.e. $B(0.4419, 0.0531^2)$], the beta posterior distribution is obtained as follows:

- Step 1: Estimate r_{prior} and s_{prior} , as such:

$$r_{prior} = \mu_{prior} \left[\frac{\mu_{prior}(1 - \mu_{prior})}{\sigma_{prior}^2} - 1 \right] = 0.4671 \left[\frac{0.4671(1 - 0.4671)}{0.0417^2} - 1 \right] \approx 66.4985$$

$$s_{prior} = [1 - \mu_{prior}] \left[\frac{\mu_{prior}(1 - \mu_{prior})}{\sigma_{prior}^2} - 1 \right] = [1 - 0.4671] \left[\frac{0.4671(1 - 0.4671)}{0.0417^2} - 1 \right] \approx 75.8531$$

- Step 2: $r_{data\ likelihood}$ and $s_{data\ likelihood}$ is already estimated 38.2809 and 48.3492, respectively.

- Step 3: Obtain $r_{posterior}$ and $s_{posterior}$, as such:

$$r_{posterior} = 66.4985 + 38.2809 = 104.7795$$

$$s_{posterior} = 75.8531 + 48.3492 = 124.2022$$

- Step 4: Estimate the expected mean and variance of CMF , as such:

$$\mu_{posterior} = \frac{104.7795}{104.7795 + 124.2022} \approx 0.4576$$

$$\sigma_{posterior}^2 = \frac{0.4576(1 - 0.4576)}{104.7795 + 124.2022 + 1} = 0.0329^2$$

As a result, the estimated CMF follows $B(0.4576, 0.0329^2)$.

In the previous sections 5.1 and 5.1, three numerical examples were provided to show how we can obtain the normal posterior distributions after introducing a specific countermeasure to a given crossing. In this section, instead of assuming normality in distributions, a more flexible beta distribution is assumed for CMF . We assess the differences in the CMF estimates for the normal and beta assumptions.

Table 6.2 shows the comparison results of the estimation based on the normal and beta posterior distributions. It was found that there was no significant difference in the CMF outputs from these two different distributions. Therefore, it may be asserted that developing accurate prior and/or data likelihood distributions are more important than the method of combining the two distributions for producing reliable posterior distributions. Obviously, another type of distributions (e.g. lognormal distributions) can also describe posterior distributions. But in that case, applying simulation techniques (e.g. MCMC method) would be more appropriate to produce the posterior distributions than using cumbersome analytical data fusion technique.

Table 6.2 Comparison between Normal and Beta Posterior Distribution

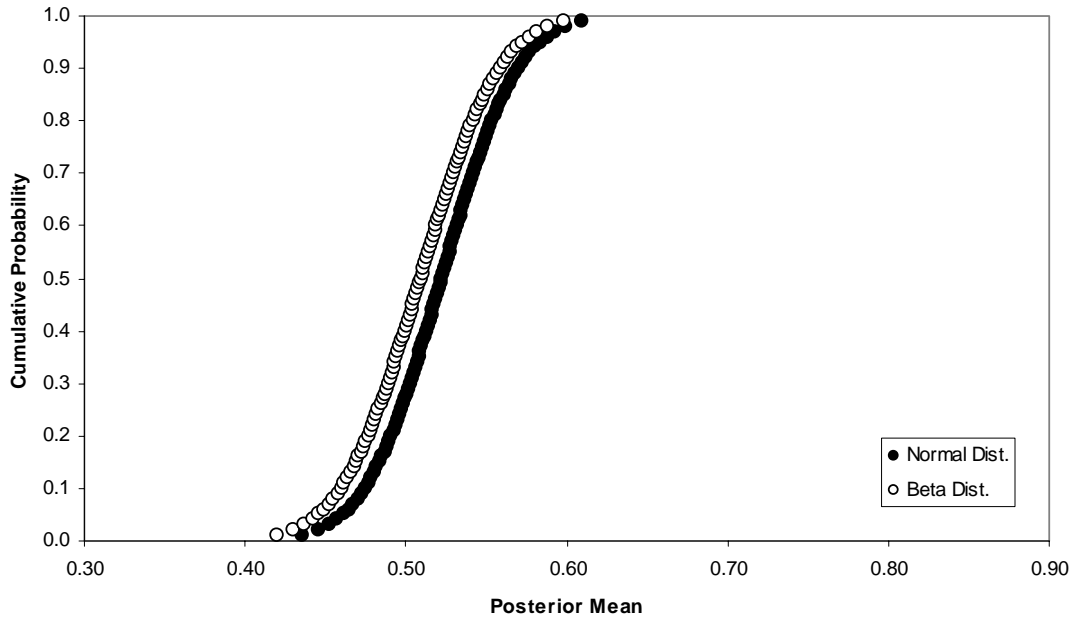
Distribution	Countermeasure	Elimination of Whistle Prohibition		Upgrading Warning Devices	
				Flashing Lights to Gates	Signboards to Gates
	Crossing Attributes at	1) Table 5.1	2) Table 5.2	3) Table 5.1	4) Table 5.4
Normal	Mean (μ)	0.522	0.4574	0.693	0.350
Beta		0.509	0.4576	0.647	0.360
Normal	Standard Errors (σ)	0.037	0.0327	0.045	0.033
Beta		0.038	0.0329	0.052	0.034

Figure 6.1 provide the results of a comparison between the normal and beta cumulative posterior distributions and their corresponding parameters, respectively. The followings were observed:

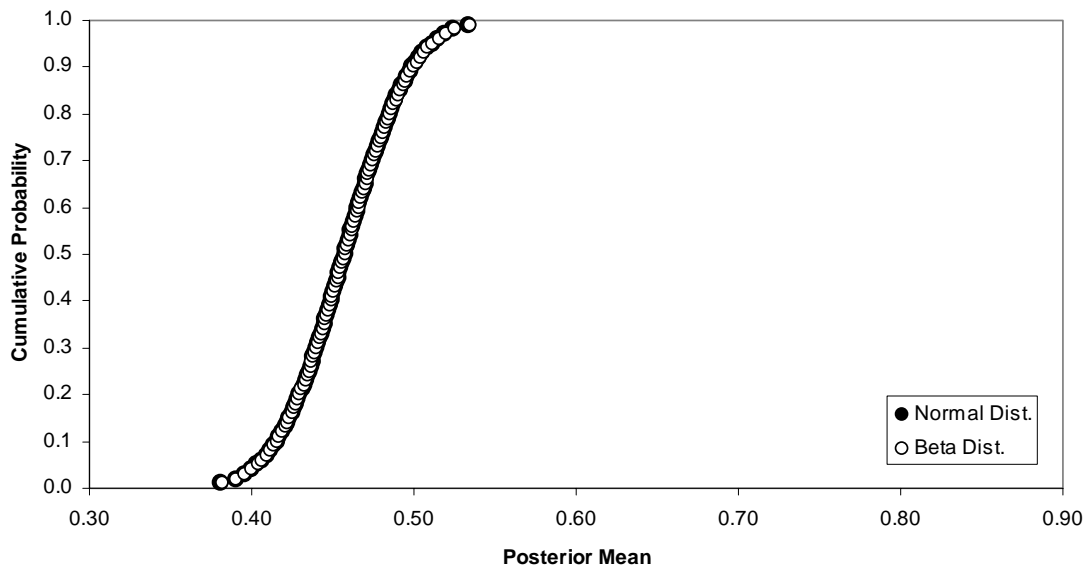
- For the elimination of whistle prohibition, the two cumulative distributions are almost identical and produce the same percentile for a range of *CMF* values. For instance, the 5, 25, 50, 75, and 95 percentile values of the two distributions in Figure 6.1(b) are estimated for *CMF* values of 0.404, 0.435, 0.457, 0.480, and 0.512, respectively. As a result, there is a 5% chance that the estimated *CMF* from the elimination of whistle prohibition is under 0.404, suggesting more than a 60% reduction in the number of collisions. Similarly, there is a 5% chance that we can obtain less than a 49% reduction for the same countermeasure.
- Contrary to the elimination of whistle prohibition, a notable discrepancy is observed in the cumulative distribution for the upgrading of warning devices from flashing lights to gates [Figure 6.1(c)]. For instance, the 5th percentile value of *CMF* based on the cumulative normal distribution is 0.619, representing about a 38.1% reduction in collisions. The same percentile value for the cumulative beta is 0.559, representing about a 44.1% reduction in collisions. The 95th percentile values for the normal and beta distributions are estimated to be 0.768 (i.e. a 23.2% collision reduction) and

0.731 (i.e. a 26.9% collision reduction) respectively for the normal and beta cumulative distributions. As we depicted in Figure 6.1(c), from the normal distribution we can estimate the probability of getting *CMF* values smaller than 0.6 as 0.02 (2%). On the other hand, we obtain 0.185 (18.5%) from the beta distribution. As a result, if we determine the *CMF* estimates based on the normal distribution rather than the beta distribution to represent the effectiveness of the upgrade from flashing lights to gates, a more conservative (lower safety dividend) result are obtained.

- Although the two distributions result in similar *CMF* values, it is interesting to note that the point estimates from the normal distribution tend to over-estimate the countermeasure effect of upgrading warning device from signs to gates as compared to the results based on the beta distribution [Figure 6.1-(d)]. This contradicts the results from the previous application examples listed above. Again, the 5th percentile value of *CMF* based on the cumulative normal distribution is 0.296, representing about a 70.4% reduction in collisions. For the beta distribution, the 5th percentile value is 0.305, representing about a 69.5% reduction in the number of collisions. The 95th percentile values for the normal and beta cumulative distributions are estimated to be 0.404 (i.e. a 59.6% collision reduction) and 0.416 (i.e. a 58.4% collision reduction), respectively. Contrary to the previous example, if we determine the *CMF* estimates based on the beta distribution rather than the normal distribution to represent the effect of upgrading from signs to gates, a more conservative (lower safety benefit) estimate is obtained in spite of small difference.

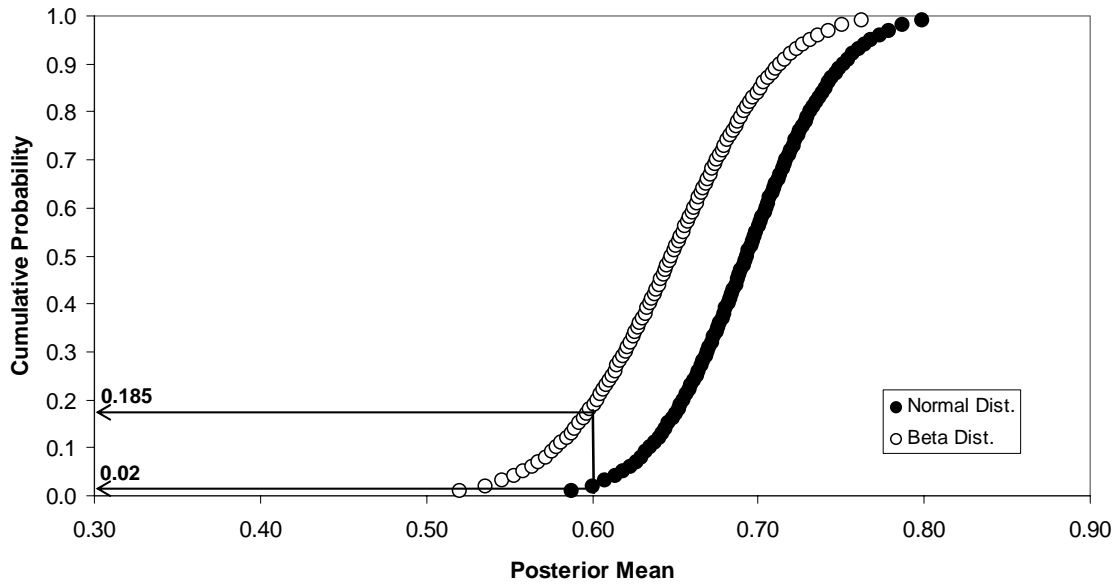


(a) *CMF* for the Elimination of Whistle Prohibition at Crossing in Table 5.1

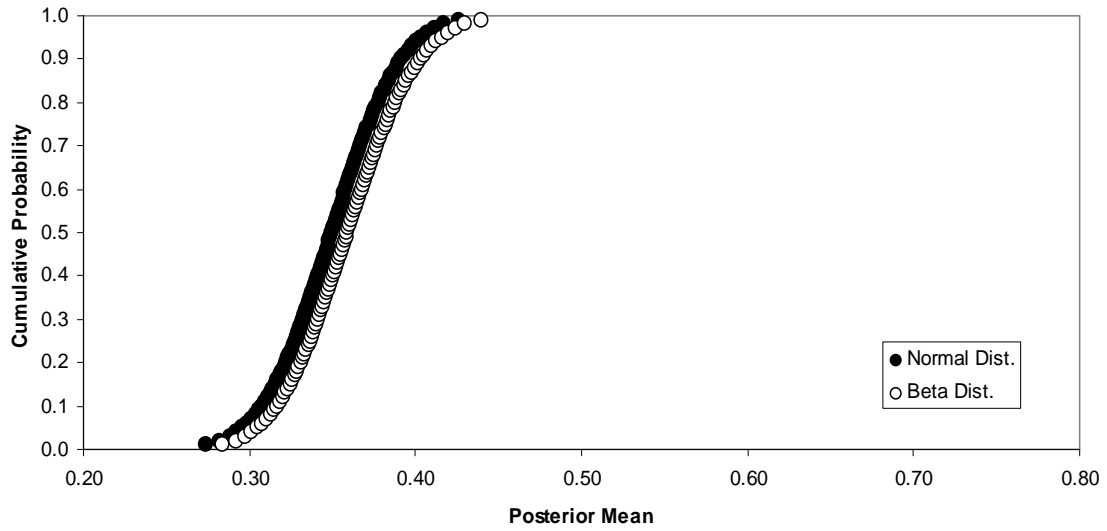


(b) *CMF* for the Elimination of Whistle Prohibition at Crossing in Table 5.2

Figure 6.1 Cumulative Density Functions based on the Two Different Distributions



(c) *CMF* for the Upgrading Warning Device from Flashing Lights to Gates at Crossing in Table 5.3



(d) *CMF* for the Upgrading Warning Device from Signboards to Gates at Crossing in Table 5.4

Figure 6.1 Continued

6.2 Uncertainty Inherent in Priors

6.2.1 Uncertainty in Selecting Different Priors

As stated in section 3.2, the estimated prior distribution reflects the uncertainty in previous studies based on the methodology and/or data adopted in each study. In general, heavier weight in the priors was given to the studies that used more reliable methods, such as an EB before-after model. As a result, the posterior distribution based on the weighted average (i.e. priors in Table 4.2) is likely to produce similar result with the posterior distribution based solely on the finding of the most reliable study.

Figure 6.2 shows the different posterior distributions based on the findings from these studies. For example, suppose that the crossing attributes are as shown in Table 5.3 and the countermeasure is an upgrade from flashing lights to gates. If we use the result (i.e. mean = 0.100, std. dev. = 0.008) from the Alaska study (see Table 2.3) as the only information for priors, the resulting posterior distribution becomes very similar to the States' findings (i.e. mean = 0.117, std. dev. = 0.008) even after combining with the data likelihood information. We note that the finding of the Alaska study was significantly different from the findings of other studies, and it was produced via less reliable study method (i.e. a naïve before-after model). As a result, the estimated posterior distribution is quite different from the posterior distribution based on the weighted average values. On the other hand, the posterior distribution based on the EB before-and-after study by Hauer and Persaud (1989) (i.e. mean = 0.550, std. dev. = 0.192) shows almost identical result with the posterior distribution based on the weighted average. Clearly a heavier weight was assigned to this study based on the weighting scheme in Table 3.1. From this example, we can speculate that the weighting scheme suggested in this study yields reliable posterior distribution by giving more weights to the more reliable individual study findings. However, it should be noted that the estimated posterior distribution (mean = 0.693, std. dev. = 0.045) in this study varies by the crossing attributes. Therefore, the posterior distribution is different from the overall effect of the countermeasure suggested by Hauer and Persaud (1989).

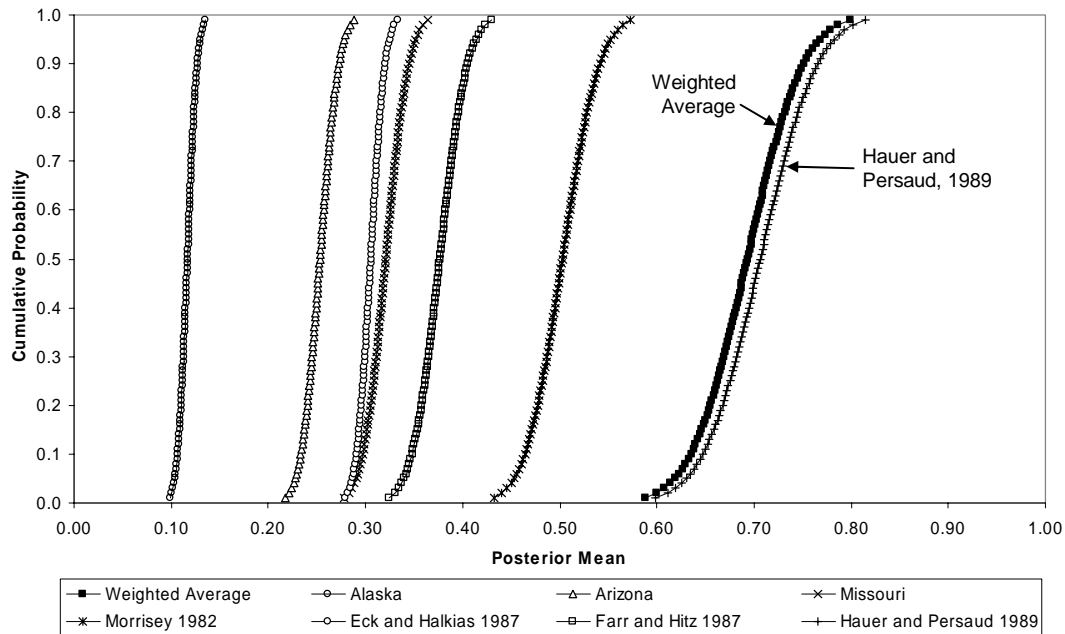


Figure 6.2 Posterior Normal Cumulative Distributions based on Different Priors

6.2.2 Uncertainty in the Choice of Relative Weights

This study used the weighted average effectiveness of countermeasures to represent the prior estimates of countermeasures effect. The weights were selected to reflect the “perceived” relative reliability of the various methods adopted in individual studies. Basically, the better the method the more faith we have in its estimate and the higher the weight. As stated in section 3.2, the reliability ranking of individual study methods was originated from a road safety study by Harkey et al. (2005) and this study followed their suggestion to estimate the relative weights (W_i) for different prediction models. However, since the Harkey et al’s reliability ranking is heavily relying on the expert judgment, their result might be plagued by a certain amount of bias because of the subjectivism inherent in the experts’ decision process.

This section is devoted to look into the effect of relative weights on the estimated effectiveness of countermeasures by introducing different weights in order to investigate the potential uncertainty in prior estimates. Table 6.3 contains the four different weighting schemes that are used in this comparison analysis. They are: 1) Proposed Weight (i.e. the

same as the one in Table 3.1), 2) 0.25 Interval Weight, 3) 0.20 Interval Weight, and 4) Equal Weight. In general, compared to the other weighting scheme, the proposed weighting scheme gives rise to the heaviest weight to the highest level study in reliability ranking than any other weighting scheme. On the other hand, the equal weight virtually does not admit that there is difference in the reliability among the different prediction models.

Table 6.3 Various Relative Weights (W_i) for the Prior Estimates

Level of Certainty	1) Proposed Weight	2) 0.25 Interval Weight	3) 0.20 Interval Weight	4) Equal Weight
1. High	1.000	1.000	1.000	1.000
2. Medium High	0.500	0.750	0.800	1.000
3. Medium Low	0.333	0.500	0.600	1.000
4. Low	0.250	0.250	0.400	1.000

It should be noted here that the varying weighting schemes will only affect to the prior estimates that are developed according to the previous study findings with different levels of reliability. As a result, only 6 out of the 18 total countermeasures in Table 4.2 have been influenced by the different weighting schemes and produced different values of estimates. Table 6.4 and 6.5 and Figure 6.3 and 6.4 shows the estimated mean (μ) and standard errors (τ) of the prior estimates based on the suggested 4 different weighting schemes.

In general, the effect of the weights on the estimated mean of priors is not significant. In particular, the prior mean estimates regarding the Whistle Prohibition shows practically no differences according to the different weighting schemes. However, if we ignore the possible differences in reliability of individual study methods (i.e. equal weight), the result is somewhat different from the results by the other weighting schemes. For example, the upgrade from Flashing Lights to 2Q-Gates produces about 7.6% higher collision reduction effect than it based on the proposed weighting scheme. Consequently, the proposed method produces more conservative results on the estimated countermeasure effects than other weighting schemes that were considered in Table 6.3.

Perhaps, more important results would be the comparison results of standard errors among the different weighting schemes. Table 6.5 and Figure 6.4 show that the proposed weighting scheme produces larger uncertainties in the mean estimates of priors by allowing larger variances (e.g. standard errors). The proposed weighting scheme produces more conservative results not only on the mean estimates but also on the variance estimates. As a result, although the proposed weighting scheme contains a certain degree uncertainty mainly due to the subjectivism inherent in the decision process, the engineers can still obtain more conservative results about the estimated countermeasure effects since the proposed weighting scheme contributes to reducing the excessive conviction on the prior estimates.

Table 6.4 Estimated Mean of Priors on Selected Countermeasures based on the Different Weighting Schemes

Number	Countermeasures	Proposed Weight	0.25 Interval Weight	0.20 Interval Weight	Equal Weight
8	From Signs to Flashing Lights	0.4578	0.4470	0.4417	0.4256
9	From Signs to 2Q-Gates	0.2833	0.2744	0.2700	0.2567
10	From Flashing Lights to 2Q – Gates	0.4738	0.4483	0.4356	0.3975
13	Traffic Signal	0.3583	0.3678	0.3725	0.3867
14	Whistle Prohibition	0.4671	0.4667	0.4670	0.4675
18	Photo/Video Enforcement	0.2471	0.2633	0.2520	0.2350

Table 6.5 Estimated Standard Errors of Priors on Selected Countermeasures based on the Different Weighting Schemes

Number	Countermeasures	Proposed Weight	0.25 Interval Weight	0.20 Interval Weight	Equal Weight
8	From Signs to Flashing Lights	0.1356	0.1239	0.1180	0.1003
9	From Signs to 2Q-Gates	0.0864	0.0792	0.0756	0.0647
10	From Flashing Lights to 2Q – Gates	0.1489	0.1346	0.1275	0.1060
13	Traffic Signal	0.1776	0.1623	0.1546	0.1316
14	Whistle Prohibition	0.0417	0.0416	0.0417	0.0417
18	Photo/Video Enforcement	0.0220	0.0235	0.0225	0.0210

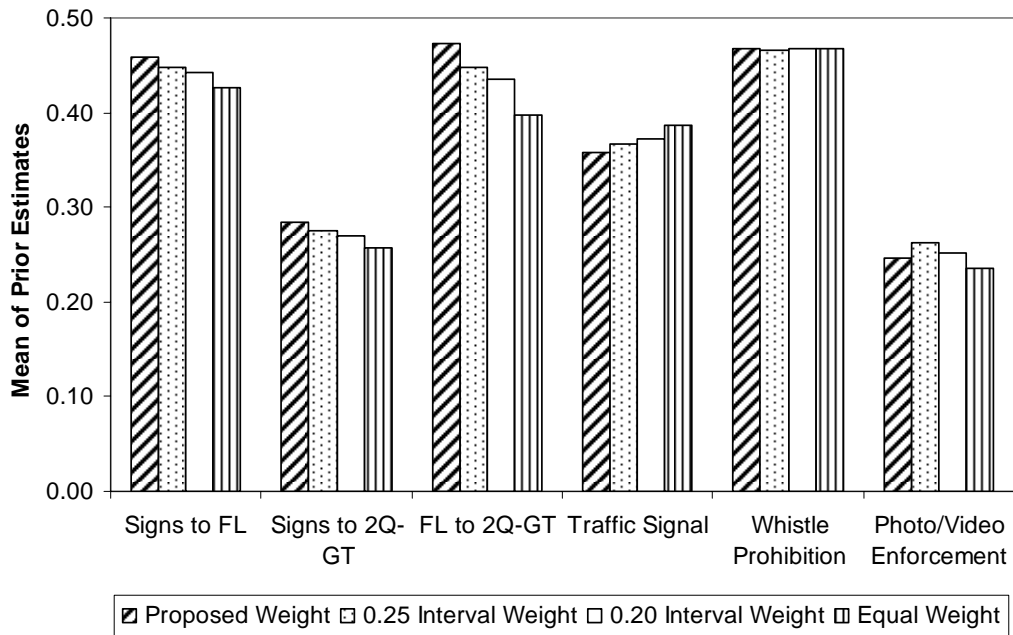


Figure 6.3 Estimated Mean of Priors on Selected Countermeasures based on the Different Weighting Schemes

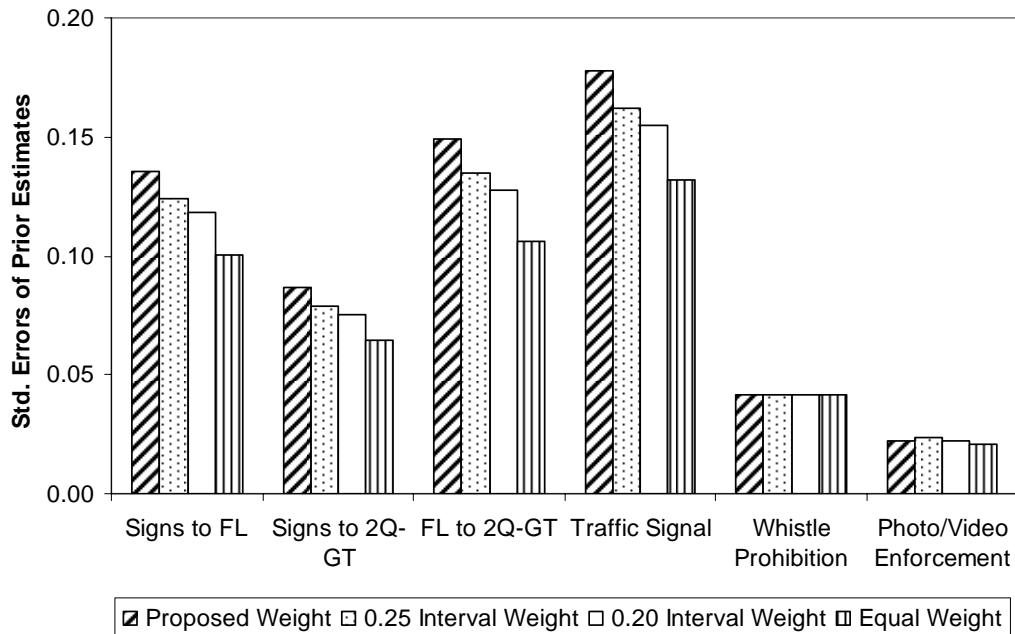


Figure 6.4 Estimated Standard Errors of Priors on Selected Countermeasures based on the Different Weighting Schemes

Figure 6.5 and 6.6 illustrates the estimated posterior distributions on the effectiveness of upgrading from flashing lights to gates using the crossing attributes in Table 5.1. As expected, the proposed weighting scheme produces the most conservative results by estimating the smallest value of effectiveness. For example, the 5th percentile value from the proposed weighting scheme is estimated as 0.619, representing about 38.1% reduction in collisions. The same percentile value for the equal weighting scheme is 0.591, representing about 40.9% collision reduction effect.

Although the proposed weight scheme produced 7.6% higher priors estimates than it from the equal weighting scheme, the difference in posterior estimates is only 2.8%. Moreover, the other two weighting schemes (i.e. 0.25 and 0.20 interval weighting schemes) produce very small amount of discrepancies in the estimated posterior distributions. It is also found that there is no difference in patterns due to the form of prior and posterior distributions (i.e. normal or beta distributions).

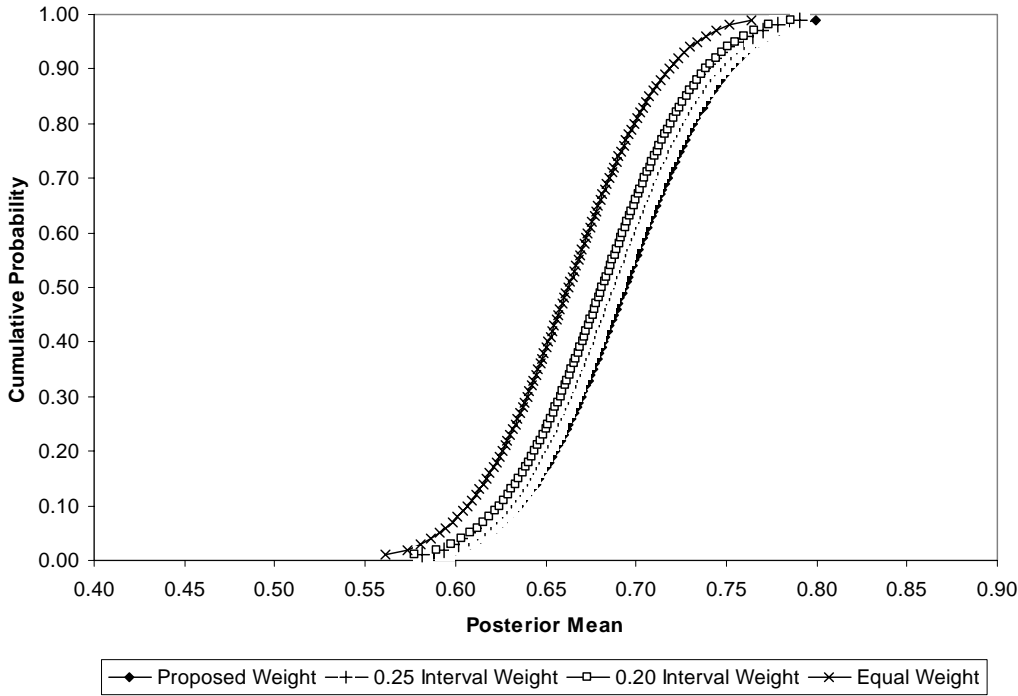


Figure 6.5 Posterior Normal Cumulative Distributions using Different Weighting Schemes.

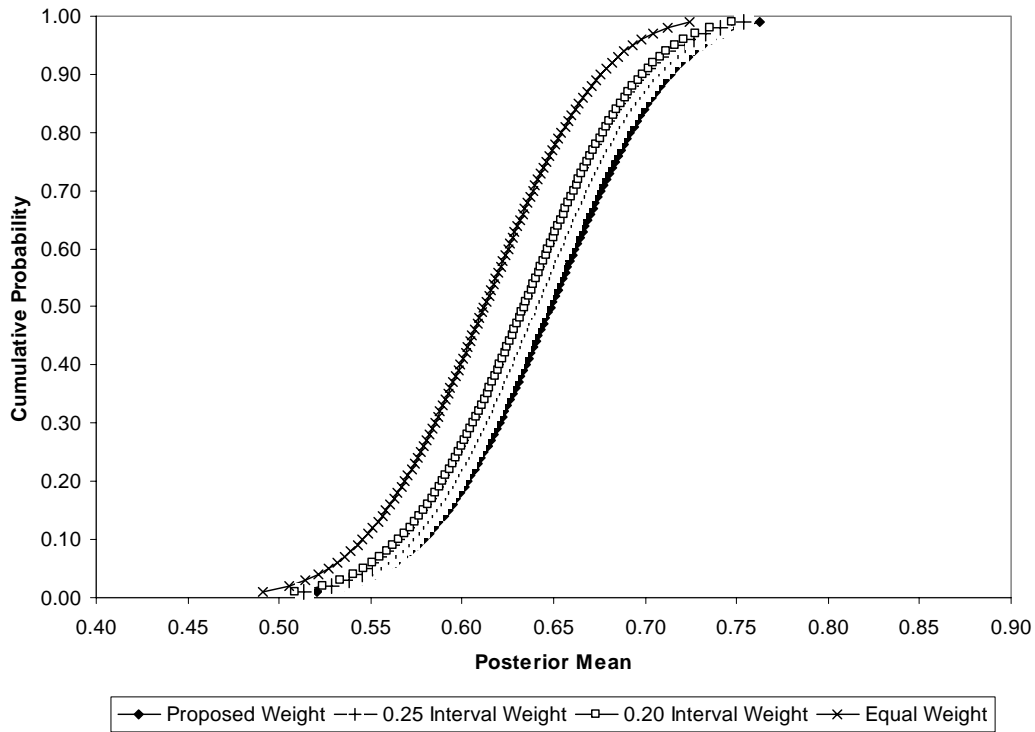


Figure 6.6 Posterior Beta Cumulative Distributions using Different Weighting Schemes.

6.3 Uncertainty in the Choice of Different Countermeasures

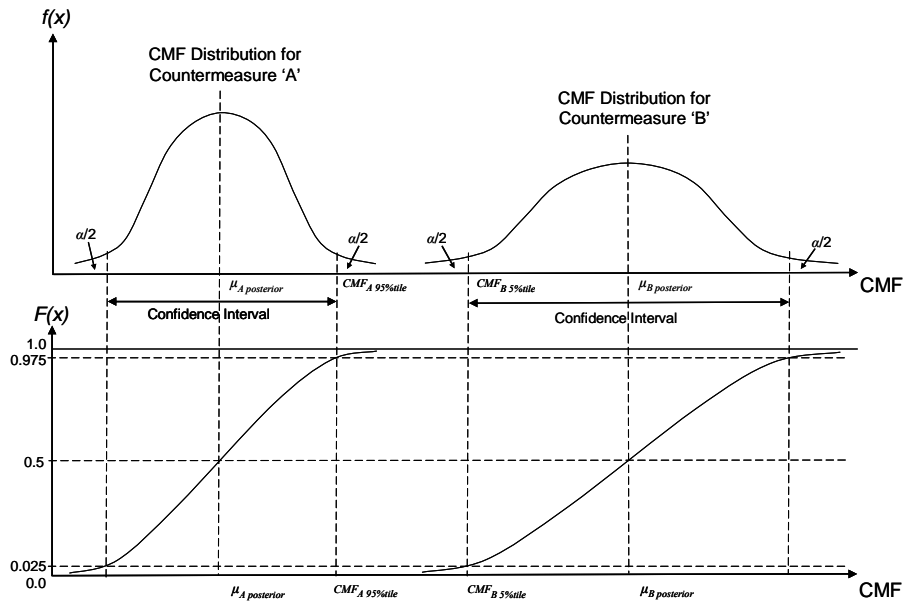
In many instances, decision makers are required to choose one of many countermeasures to reduce collision risk at a specific crossing. For instance, they can consider the installation of photo/video enforcement or 4-quadrant gate. Their decision will be based on the posterior distribution of the two different countermeasures. Hypothetical graphs in Figure 6.7 illustrate a dominant condition by Countermeasure A over Countermeasure B in terms of the mean value of *CMF*.

For instance, over the entire range of posterior distribution in Figure 6.7 (a), there is no overlapping between the two posterior distributions. Therefore, determining a suitable countermeasure based on the mean value of each countermeasure is valid if we have

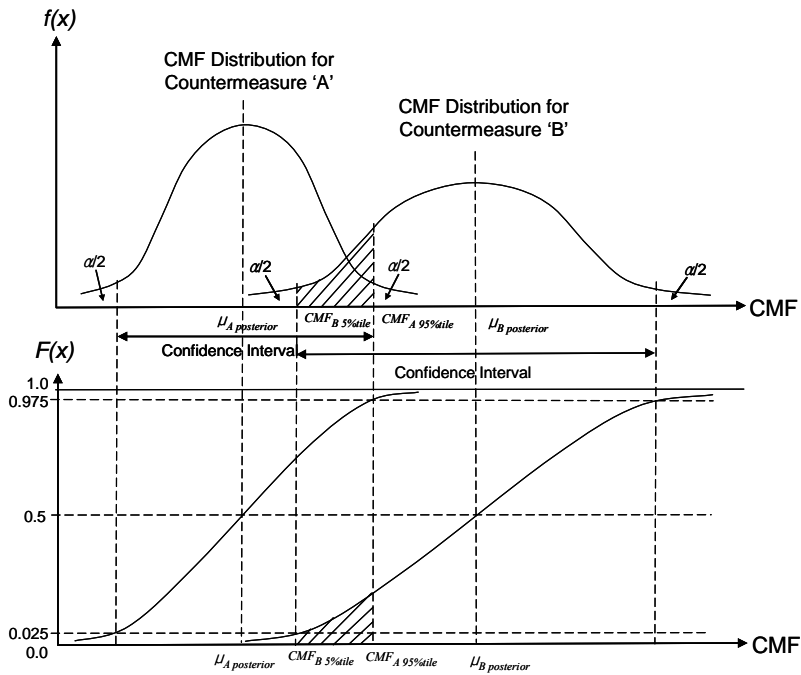
unlimited budget to utilize. As a result, Countermeasure A will be always more effective in reducing collision than Countermeasure B.

In Figure 6.7 (b), we note that Countermeasure A shows a stronger safety effect than Countermeasure B for the mean. However, there is a small chance [i.e. $P(a < X < b)$ in posterior distribution B] that Countermeasure B is more effective than Countermeasure A. As a result, decision makers could not be certain that Countermeasure A give rise to higher collision reduction effects than Countermeasure B applied the same crossing. This is despite the fact that the mean effect for countermeasure A is higher than for countermeasure B. These results underscore the need to consider uncertainty in the estimated countermeasure effects. The crux of the decision making process is to understand the probability that a conclusion reached on the effect of a given countermeasure applied to a specific crossing can be erroneous.

A numerical example is described in Figure 6.7 (c). The example calculation in Table 6.2 was reused in this illustration. The third and the fifth column of Table 6.2 contain the effects of two countermeasures: 1) elimination of whistle prohibition and 2) upgrading flashing lights to gates. Those estimates were calculated based on a crossing attributes in Table 5.1. In section 5.2, based on the mean value of the *CMF*, we already mentioned that the elimination of whistle prohibition is more effective countermeasure than the upgrading the flashing lights to gates for the given grade crossing in Table 5.1. A more precise analysis was conducted using the same example to detect any potential erroneous decision. Assuming the normality in *CMF* distribution, the 97.5th percentile value in the distribution of elimination of whistle prohibition is estimated as 0.596 (i.e. 40.4% of collision reduction effect). On the other hand, the 2.5th percentile value in the distribution of upgrading warning device is estimated as 0.604 (i.e. 39.6% of collision reduction effect) [Figure 6.7 (c)] and this value still shows slightly less safety benefit than that from the alternative countermeasure. There is no overlapping between the two posterior distributions as shown in Figure 6.7 (a). Although this result is only valid for the crossings with given attributes in this example, we can conclude that the elimination of whistle prohibition is more effective than upgrading warning device at a 95% confidence level.

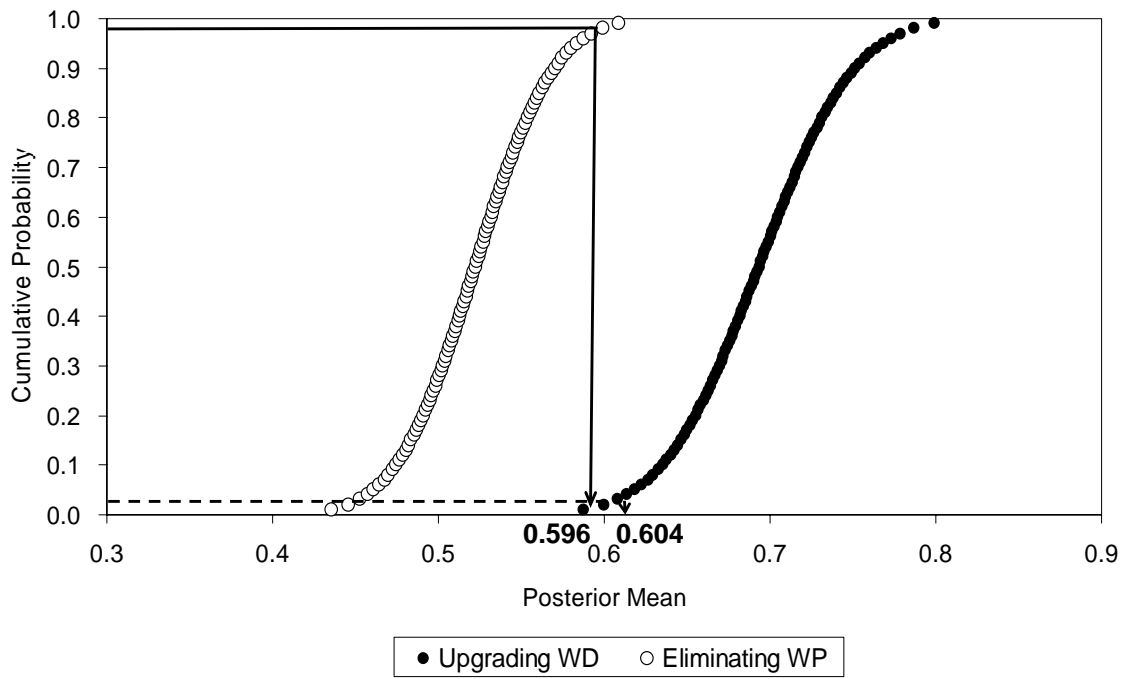


(a) No Overlapping between the Two Posterior Distributions



(b) Partial Overlapping between the Two Posterior Distributions

Figure 6.7 Uncertainties in Different Countermeasures



(C) Numerical Example Using Sample Calculations in Table 6.2

Figure 6.7 Continued

7 CONCLUSIONS

Bayesian data fusion requires two important sources of information to obtain statistical estimates of countermeasure effects, a priori and data likelihood inputs. The approach suggested in this study has a number of practical advantages in evaluating the safety effect of countermeasures applied to different types of highway-railway grade crossings.

7.1 Major Contributions

This study has made several important contributions in estimating the effect of countermeasures.

First, the proposed Bayesian data fusion method incorporates the results from previous studies of countermeasure effects into the analysis of Canadian grade crossings collision experience. Many of the existing collision prediction methods failed to take into account the previous study findings.

Second, the proposed method estimates *CMF* by objectively weighting the *CMF* estimates obtained from past studies on the basis of published model or the reliability of the approach. The use of weights contributes to produce more reliable prior estimates of countermeasure effects.

Third, inasmuch as we used three different collision prediction models to obtain data likelihood estimates, the proposed method yields countermeasure effects that are more reflective of a larger array of confounding factors than is possible from a single model. This reduces problems of misspecification commonly associated with these types of models.

Fourth, the proposed method provides tailored information concerning the effect of countermeasures applied to a specific crossing of interest. This method uses data likelihood as an input, based on crossing-specific collision prediction models for Canadian collision occurrence and inventory data.

Fifth, the method takes into account the uncertainty in the model estimates of countermeasure effects. Output is reported in terms of means, variance, and corresponding

probability distributions. The uncertainty analysis predicts the effect of countermeasures in the form of probabilities that specific unexpected threshold values are exceeded.

Clearly, the Bayesian data fusion method proposed in this study has an advantage in evaluating countermeasures at a regional or local level of problem definition. Based on the estimates by the proposed model, decision makers can make more effective decisions concerning countermeasures in the face of uncertainty. The model produces tailored estimates of effect for countermeasures and represents a noteworthy benefit of this research.

7.2 Contributions in Development of Data Likelihoods

As explained in Chapter 4, we developed multi-stage cross-sectional statistical models that yield reliable estimates of collision frequency at grade crossings. A few key contributions in development of data likelihoods are summarized as follows:

First, cluster-specific collision prediction models developed by Saccomanno and Lai (2005) were modified and re-calibrated. The four factors (i.e. latent variables) were used to reduce dimensionality problems in large dataset, such as IRIS/RODS by the factor analysis. Cluster analysis has been conducted to classify the crossings with similar attributes into a group. As a result, the cluster-specific collision prediction models can produce more reliable estimates of changes in the expected number of collisions after the implementation of countermeasures.

Second, a tree-based data partitioning method (RPART) was effectively used for stratifying dataset based on control factors. Four classes were systematically determined such that the control factor characteristics are homogeneous in each class. Changes in the expected collision rate following specific countermeasures depend on which of four classes a crossing belongs to. The RPART method was used to estimate the effect of interesting countermeasures after eliminating the potential biases from the control factors. Therefore, the coefficients in stratified collision prediction models are more reliable than those in conventional single-stage statistical models.

Third, RPART method was used to identify potentially important interactions and non-additive effects among the explanatory variables. It is expected that the reliability of collision prediction models are improved by adding group indicators, representing interactions among the explanatory input factors, compared to the model without group indicators.

7.3 Recommendations for Future Analysis

A number of research tasks are recommended for future work:

First, in this study, we only considered the expected reduction in the number of collisions by the implementation of specific countermeasures at specific crossings. However, it is suggested that the expected collision severity should be considered in addition to the expected frequency of collisions. Collision severity is needed in order to estimate the overall cost incurred by the collisions at highway-railway grade crossings. The overall cost can be used in a benefit-cost analysis to evaluate the economic feasibility of any specific countermeasures at a given crossing.

Second, more sophisticated methods for estimating more reliable prior distributions of countermeasure effects should be considered. Therefore, the identification and collection of additional data for producing more reliable priors are needed to enhance the quality and confidence of the estimated prior distributions.

Third, the integration of the proposed method into a decision support system is needed to support practitioners for resolving the specific safety problems in their local areas. The decision support system can provide a range of *CMF* (considering uncertainty) as well as the expected *CMF* after the implementation of specific countermeasures at given crossings.

REFERENCES

1. Abbess, C.R., Jarrett, D.F., and C.C. Wright, Accidents at Blackspots: Estimating the Effectiveness of Remedial Treatment, with Special Reference to the 'Regression-To-Mean' Effect, *Traffic Engineering and Control*, Vol. 22(10), 1981, pp. 535-542.
2. Agent, K.R., Stamatiadis, N., and S. Jones, Development of Accident Reduction Factors, Research Report, KTC-96-13, Kentucky Transportation Center, College of Engineering, University of Kentucky, 1996.
3. Al-Masaeid, H.R., Performance of safety evaluation methods, *Journal of Transportation Engineering*, Vol. 123, No. 5, 1997, pp. 364-369.
4. Aul, N., and Davis, G.A., Using a Propensity Score Matching Method and a Hybrid Bayesian Method to Estimate Crash Modification Factors of Signal Installation, Presented at the 85th Annual Meeting of the Transportation Research Board, Washington, D.C., 2006.
5. Austin, R.D., and J.L. Carson, An Alternative Accident Prediction Model for Highway-rail Interfaces. *Accident Analysis and Prevention* 34, 2002, pp. 31-42.
6. Bahar, G., Mollett, C., Persaud, B., Lyon, C., Smiley, A., Smahel, T., and H. McGee, Safety evaluation of permanent raised pavement markers. National Cooperative Highway Research Program, NCHRP Report 518, TRB, National Research Council, Washington, D.C., 2004.
7. Benjamin, J.R., and C.A. Cornell, *Probability, Statistics, and Decision for Civil Engineers*, McGraw-Hill, Inc., 1970.
8. Breman, L., Friedman, J.H., Olshen, R.A., and C.J. Stone, *Classification and Regression Trees*, Belmont, Wadsworth, Calif., 1983.
9. Button, N.P., and P.M. Reilly, Uncertainty in Incident Rates for Trucks Carrying Dangerous Goods, *Accident Analysis and Prevention* 32, 2000, pp. 797-804.

10. Cameron, A.C., and P.K. Trivedi, Regression Analysis of Count Data, Economic Society Monograph No.30, Cambridge University Press, England, 1998.
11. Carroll, A.A., and J.D. Warren, The Use of Photo Enforcement at Highway-Rail Grade Crossings in the U.S., Presented at the 82nd Annual Meeting of the Transportation Research Board, Washington, D.C., 2002.
12. Clarke, D. B., and W.A. Sarasua, Fatal Crashes on Rural Secondary Highways. Columbia, South Carolina Department of Transportation, Final Report for Research Project SPR-608, FHWA-SC-03-04, 2003.
13. Cocchi, D., Fabrizi, E., Raggi, M., and C. Trivisano, Regression tree based stratification: an application to the analysis of the Italian Post Enumeration Survey, The National Institute of Social Research SFI-SURVEY, Copenhagen, Denmark, Proceedings of the International Conference on Improving Surveys, 2002.
14. Coifman, B., and R.L. Bertini, Median Light Rail Crossings: Accident Causation and Countermeasures, California Path Working Paper, UCB-ITS-PWP-97-13, Institute of Transportation Studies, University of California at Berkeley, 1997.
15. Coleman, J., and G.R. Stewart, Investigation of Accident Data for Railroad-Highway Grade Crossings, Journal of the Transportation Research Record 611, TRB, National Research Council, Washington, D.C., 1976, pp. 60-67.
16. Comrey, A.L., and H.B. Lee, A first course in factor analysis, Hillsdale, N.J., 1992.
17. Conerly, M., Gray, B., Kevin, B., and Edward, M., Data mining and visualization of the alabama accident database. The University of Alabama, Department of Management Science and Statistics, UTCA Project Number 99113, 2000.
18. Coulombre, R., Poage, J., Farr, E., and J. Hitz, Summary of the Department of Transportation Rail-Highway Crossing Accident Prediction Formulas and Resource Allocation Model, Final Report, DOT-TSC-FRA-82-1, U.S. Department of Transportation, Federal Highway Administration, Transportation Systems Center, Cambridge, MA., 1982.

19. Council, F., and J. Stewart, Safety Effects of Conversion of Rural Two-lane to Four-lane Roadways based on Cross-Sectional Models, *Journal of the Transportation Research Record*, No. 1665, TRB, National Research Council, Washington, D.C., 1999, pp. 35-43.
20. Council, F.M., Reinfurt, D.W., Campbell, B.J., Roediger, F.L., Carroll, C.L., Dutt, A.K., and J.R. Dunham, *Accident Research Manual, Final Report*, FHWA-RD-80-016, Highway Safety Research Center, University of North Carolina, 1980.
21. D'Agostino, R.B. Jr. Tutorial in Biostatistics: Propensity Score Methods for Bias Reduction in the Comparison of a Treatment to a Non-randomized Control Group. *Statistics in Medicine* 17, 1998, pp.2265-2281.
22. Davis, G.A., Accident Reduction Factors and Causal Inference in Traffic Safety Studies: a Review. *Accident Analysis and Prevention* 32, 2000, pp. 95-109.
23. Dehejia, R.H., and S.W. Wahba. Propensity Score-Matching Method for Nonexperimental Causal Studies. *The Review of Economics and Statistics* 84(1), 2002, pp.151-161.
24. Drouin, G., *Direction 2006: Saving Lives along Canada's Railways*. Ottawa, ON, Transport Canada, 2004.
25. Eck, R.W., and J.A. Halkias. Further Investigation of The Effectiveness of Warning Devices at Rail-Highway Grade Crossings. *Transportation Research Record* 1010, TRB, National Research Council, Washington, D.C., 1985, pp 94-101
26. El Faouzi, N.-E., 2006. Bayesian and Evidential Approaches for Traffic Data Fusion: Methodological Issues and Case Study, Presented at the 85th Transportation Research Board Meeting, Washington, D.C.
27. Elvik, R., Amundsen, F.H., and F. Hofset, Road Safety Effects of Bypasses, *Journal of the Transportation Research Record* 1758, TRB, National Research Council, Washington, D.C., 2001, pp. 13-20.

28. Farnham, W.L., Guarded Crossings: An In-depth Analysis of the Most Effective Railroad Crossing Protection, available at <http://www.angelsontrack.org/guardedcrossings.doc>, accessed January 2006.
29. Farr, E. H., and J. S. Hitz. Effectiveness of Motorist Warning Devices at Rail-highway Crossings. Final Report, FHWA/RD-85/015, U.S. Department of Transportation, Federal Highway Administration, Transportation Systems Center, Cambridge, MA., 1985.
30. Farr, E. H., Summary of the DOT Rail-Highway Crossing Resource Allocation Procedure – Revised, Federal Railroad Administration, U.S. Department of Transportation, Cambridge MA, Transportation System Center, 1987.
31. Federal Highway Administration, Manual on Uniform Traffic Control Devices for Streets and Highways, Part 8: Traffic Controls for Highway-Rail Grade Crossings, US Department of Transportation, 2003 Edition.
32. Federal Railroad Administration, GradeDec 2000 version 2.0 Reference Manual, Office of Policy and Program Development, U.S. Department of Transportation, Cambridge MA, Transportation System Center, 2002.
33. Federal Railroad Administration, North Carolina “Sealed Corridor” Phase I, U.S. DOT Assessment Report, Report to Congress, Office of Railroad Development, Federal Railroad Administration , U.S. Department of Transportation, 2001.
34. Gan, A., Shen, J., and A. Rodriguez, Update of Florida Crash Reduction Factors and Countermeasures to Improve the Development of District Safety Improvement Projects, Final Report, FDOT 99700-3596-119, Lehman Center for Transportation Research, Florida International University, Miami, 2005.
35. Gelman, A., Carlin, J.B., Stern, H.S., and D.B. Rubin, Bayesian Data Analysis, 2nd Ed., CHAPMAN&HALL/CRC, 2004.
36. Hakkert, A.S., Hoeherman, I., and A. Mensah, Levels of Safety on Interurban Roads. Journal of the Transportation Research Record 1553, TRB, National Research Council, Washington, D.C., 1996, pp 95-102.

37. Harkey, D., Crash Reduction Factors for Traffic Engineering and Intelligent Transportation System Improvements: State-of-Knowledge Report, NCHRP Research Result Digest 299, Interim Report, NCHRP Project 17-25, National Cooperative Highway Research Program Project, Washington, D.C., 2005.
38. Harlow, L.L., Mulaik, S.A., and H.S. James, What if there were no significant tests?, Lawrence Erlbaum Associates, Inc., 1997.
39. Harwood, D.W., Council, F.M., Hauer, E., Hughes, W.E., and A. Vogt, Prediction of the Expected Safety Performance of Rural Two-Lane Highways, Final Report, FHWA-RD-99-207. Federal Highway Administration, U.S. Department of Transportation, Washington, D.C., 2000.
40. Hauer, E, and B.N. Persaud, How to Estimate the Safety of Rail-Highway Grade Crossings and the Safety Effects of Warning Devices. Journal of Transportation Research Record 1114, TRB, National Research Council, Washington, D.C., 1987, pp. 131-140.
41. Hauer, E. Observational Before-After Studies in Road Safety. Pergamon Publication, England, 1997.
42. Hauer, E. On the Estimation of the Expected Number of Accidents, Accident Analysis and Prevention, Accident Analysis and Prevention 18 (1), 1986, pp. 1-12.
43. Hughes, W., Eccles, K., Harwood, D., Potts, I., and E. Hauer, Development of a Highway Safety Manual, Contractor's Final Report, NCHRP Web Document 62 (NCHRP Project 17-18[4]), National Cooperative Highway Research Program Project, Washington, D.C., 2004.
44. Hunter, J.E., and F.L. Schmidt, Methods of Meta-Analysis: Correcting Error and Bias in Research Findings, SAGE Publication, Inc. California, 1990.
45. Iversen, G.R., Bayesian Statistical Inference, Sage Publications, Inc., 1984.
46. Johnson, R.A. Miller and Freund's Probability and Statistics for Engineers, Fifth Edition, Prentice-Hall, Inc., 1994.

47. Kim, D.G., and S., Washington, S. The Significance of Endogeneity Problems in Crash Models: An Examination of Left-Turn Lanes in Intersection Crash Models, *Accident Analysis and Prevention* 38, 2006, pp. 1094-1100.
48. Ko, B., Courage, K., and M. Willis, Development and Deployment of a Portable Highway-Railroad Grade Crossing Surveillance System, Volume II: Video Based Studies of Flexible Traffic Separators at Highway-Railroad Grade Crossings, Final Report, UFTRC 891-2, Prepared for the Florida Department of Transportation Rail Office, Transportation Research Center, University of Florida, 2003.
49. Lai, X., Analysis of Highway-Railway Grade Crossing Collisions and Countermeasures, Master Thesis, University of Waterloo, Ontario, Canada, 2004.
50. Lau, Y-K., and A.D. May, Accident Prediction Model Development for Unsignalized Intersections, Final Report, FHWA/CA/TE/89-3, Federal Highway Administration, U.S. Department of Transportation, Sacramento, CA, 1989.
51. Laughland, J., L. Haefner, J. Hall, and D. Clough. NCHRP Report 162: Methods for Evaluating Highway Safety Improvements. National Cooperative Highway Research Program, Transportation Research Board, Washington, D.C., 1975.
52. Lee, P.M., Bayesian Statistics: An Introduction, 3rd Ed., A Hodder Arnold Publication, 2004.
53. Leeming, D.G., and F.F. Saccomanno, Use of Quantified Risk Assessment in Evaluating the Risks of Transporting Chlorine by Road and Rail. *Journal of Transportation Research Record* 1430, TRB, National Research Council, Washington, D.C., 1994, pp. 27-35.
54. Lord, D., and J.A. Bonneson, Role and Application of Accident Modification Factors (AMFs) within the Highway Design Process, Presented at the 85th Annual Meeting of the Transportation Research Board, Washington, D.C., 2006.
55. Lord, D., Modeling Motor Vehicle Crashes using Poisson-gamma Models: Examining the Effects of Low Sample Mean Values and Small Sample Size on the Estimation of

- the Fixed Dispersion Parameter, *Accident Analysis and Prevention*, Vol. 38(4), 2006, pp. 751-766.
56. Lyon, C., Haq, A., Persaud, B.N., and S.T. Kodama, Safety Performance Functions for Signalized Intersections in Large Urban Areas: Development and Application to Evaluation of Left-Turn Priority Treatment, *Journal of the Transportation Research Record* 1908, Washington, D.C., 2005, pp. 165-171.
 57. Melcher, D.J., Dixon, K.K., Washington, S., and C.-H. Wu, Feasibility of “Subjective” Engineering Assessments of Road Safety Improvements: Bayesian Analysis Development, *Journal of the Transportation Research Record* 1758, TRB, National Research Council, Washington, D.C., 2001, pp 36-43.
 58. Mengert, P., Rail-Highway Crossing Hazard Prediction Research Results, Final Report, FRA-RRS-80-02, U.S. Department of Transportation, Federal Railroad Administration, Washington, D.C., 1980.
 59. Migon, H.S., and D. Gamerman, *Statistical Inference: An Integrated Approach*, A Hodder Arnold Publication, 1999.
 60. Mironer, M., Coltman, M, and R. McCown, Assessment of Risks for High-Speed Rail Grade Crossings on the Empire Corridor, Final Report, DOT/FRA/RDV-00/05, Federal Railroad Administration, U.S. Department of Transportation, 2000.
 61. Morrissey, J., The Effectiveness of Flashing Lights and Flashing Lights with Gates in Reducing Accident Frequency at Public Rail-Highway Crossings 1975-1978, Final Report, FRA-RRS-80-005, U.S. Department of Transportation, Federal Railroad Administration, Washington, D.C., 1980.
 62. Morton, V., and Torgerson, D.J., Regression to the Mean: Treatment Effect without the Intervention, *Journal of Evaluation in Clinical Practice*, 11(1), 2005, pp. 59-65.
 63. Mountain, L., Fawaz, B., and L. Sineg, The Assessment of Changes in Accident Frequencies on Link Segments: A Comparison of Four Models, *Traffic Engineering and Control*, Vol 33(8), 1992, pp. 429-431.

64. Park, Y.-J. P., and F.F. Saccomanno, Reducing Treatment Selection Bias for Estimating Treatment Effects Using Propensity Score Method, American Society of Civil Engineers, Vol. 133, No. 2, 2007, pp. 112-118.
65. Park, Y.-J., and F.F. Saccomanno, Collision Frequency Analysis Using Tree-Based Stratification, Journal of the Transportation Research Record 1908, TRB, National Research Council, Washington, D.C., 2005a, pp. 121-129.
66. Park, Y.-J., and F.F. Saccomanno, Evaluating Factors Affecting Safety at Highway-Railway Grade Crossings, Journal of the Transportation Research Record 1918, TRB, National Research Council, Washington, D.C., 2005b, pp. 1-9.
67. Pendleton, O.J. Application of New Accident Analysis Methodologies Volume I – General Methodology, Final Report FHWA-RD-90-091, Federal Highway Administration, U.S. Department of Transportation, Washington, D.C., 1991.
68. Pendleton, O.J. A Systemwide Methodology for Evaluating Highway Safety Studies, Final Report FHWA-RD-92-049, Federal Highway Administration, U.S. Department of Transportation, Turner-Fairbank Highway Research Center, Mclean, Virginia, 1992.
69. Persaud, B., and C. Lyon, Empirical Bayes Before-after Safety Studies: Lessons Learned from Two Decades of Experience, Presented at the 85th Annual Meeting of the Transportation Research Board, Washington, D.C., 2006.
70. Persaud, B., Relating the Effect of Safety Measures to Expected Number of Accidents, Accident Analysis and Prevention, Vol. 18(1), 1986, pp. 63-70.
71. Persaud, B.N., Retting, R.A., Garder, P.E., and D. Lord, Safety effect of roundabout conversions in the united states: empirical bayes observational before-after study, Journal of the Transportation Research Record 1751, TRB, National Research Council, Washington, D.C., 2001, pp. 1-8.
72. Persaud, B.N., Statistical Methods in Highway Safety Analysis, NCHRP Synthesis of Highway Practice 295, Transportation Research Board, National Research Council, 2001.

73. Rapoza, A.S., Raslear, T.G., and E.J. Rickley, Railroad Horn Systems Research, Final Report, DOT/FRA/ORD-99/10, U.S. Department of Transportation, Federal Railroad Administration, Washington, D.C., 1999.
74. Rosenbaum, P.R., and D.B. Rubin. Reducing Bias in Observational Studies Using Subclassification on the Propensity Score. *Journal of American Statistical Association* 79, 1984, pp 516-524.
75. Rosenbaum, P.R., and D.B. Rubin. The Central Role of the Propensity Score in Observational Studies for Casual Effects. *Biometrika* 70, 1983, pp 41-55.
76. Rummel, R.J., *Applied Factor Analysis*, Northwest University Press, 1970.
77. Russell, E.R., New Perspectives on Closure of Railroad-Highway Grade Crossings as a System Improvement, *Journal of the Transportation Research Record* 802, TRB, National Research Council, Washington, D.C., 1981, pp. 80-85.
78. Saccomanno, F. F., Fu, L., Ren, C., and L. Miranda, Identifying Highway-Railway Grade Crossing Hotspots: Phase 1. Department of Civil Engineering, University of Waterloo, Waterloo, ON, 2004.
79. Saccomanno, F.F., Fu, L., Park, Y.-J. P., and Shen, H., A Decision Support Tool for Prioritizing Safety Improvement Programs at High-Risk Grade Crossings: Part II: Estimating Countermeasure Effectiveness, Interim Report, Prepared for Transportation Development Centre, Safety and Security Division, Transport Canada, Department of Civil Engineering, University of Waterloo, 2006.
80. Saccomanno, F.F., and X. Lai, A Model for Evaluating Countermeasures at Highway-Railway Grade Crossings, *Journal of the Transportation Research Record* 1918, 2005, pp. 18-25.
81. Sampson, D.B. The Delta Method, Lecture Note, available at <http://oregonstate.edu/instruct/fw431/sampson/LectureNotes/DeltaMethod/DeltaMethod.htm>, accessed Jan. 2006.

82. Schoppert, D.W., and D.W. Hoyt, Factors Influencing Safety at Highway-Rail Grade Crossings. National Cooperative Highway Research Program, NCHRP Report 50, Highway Research Board, 1968.
83. Schultz, T.G. Evaluation of Safety at Railroad-Highway Grade Crossings. Project No: C-36-59F, Joint Highway Research Project, Purdue University, Lafayette, Indiana., 1965
84. Shen, J., and A. Gen. Development of Crash Reduction Factors: Methods, Problems and Research Needs. Journal of the Transportation Research Record 1840, National Research Council, Washington, D.C., 2003, pp. 50-56.
85. Stewart, J.R. Applications of Classification and Regression Tree Methods in Roadway Safety Studies. Journal of the Transportation Research Record 1542, National Research Council, Washington, D.C., 1996, pp. 1-5.
86. Therneau, T.M., and E.J. Atkinson, An Introduction to Recursive Partitioning Using the RPART Routines. Technical Report No. 61, Department of Health Sciences Research, Division of Biostatistics, Mayo Foundation, 1997.
87. Transport Canada, Canadian Road/Railway Grade Crossing Detailed Safety Assessment Field Guide, UNB Transportation Group, 2005.
88. Transportation Association of Canada, Manual of Uniform Traffic Control Devices for Canada, Fourth Edition, 1998.
89. Tustin B.H., Richards, H., McGee, H., and R. Patterson. Railroad-railway grade crossing handbook. 2nd Ed., Report FHWA-TS-86-215, Federal Highway Administration, U.S. Department of Transportation, Washington, D.C., 1986.
90. Washington, S., Interactively specified tree-based regression: theory and trip generation example. ASCE Journal of Transportation Engineering, 126 (6), 2000, pp. 481-491.
91. Washington, S., and Oh, J., Bayesian methodology incorporating expert judgment for ranking countermeasure effectiveness under uncertainty: Example applied to at grade

- railroad crossings in Korea, *Accident Analysis and Prevention* 38 (2), 2006, pp. 234-247.
92. Washington, S., Dixon, K., White, D. and Wu, C-H. E. Investigation of Fatal Motor Vehicle Crashes on Two-Lane Rural Highways in Georgia. Forest Park, Georgia, Georgia Department of Transportation, Final Report, FHWA-GA-02-9905, 2002.
 93. White, D.J., Development of a Methodology for identifying Effective Countermeasures in Regional Safety Management Programs Using a Bayesian Safety Assessment Framework (B-SAF), Ph.D. Dissertation, Georgia Institute of Technology, 2002.
 94. Wikipedia, the Free Encyclopedia, Regression toward the Mean, available at [http://http://en.wikipedia.org/wiki/Regression_toward_the_mean](http://en.wikipedia.org/wiki/Regression_toward_the_mean), accessed Oct. 2006.
 95. Wright, C.C., Abbess, C., and D.F. Jarrett, Estimating, Estimating the Regression-To-Mean Effect Associated with Road Accident Black Spot Treatment: Towards A More Realistic Approach, *Accident Analysis and Prevention*, Vol. 20(3), 1988, pp. 199-214.
 96. Xu, J., and J.S. Long. Confidence Intervals for Predicted Outcomes in Regression Models for Categorical Outcomes, *Stata Journal*, Vol. 5 (4), 2005, pp. 537-559.
 97. Yanovitzky, I., E. Zanutto, and R. Hornik. Estimating Casual Effect of Public Health Education Campaigns Using Propensity Score Methodology. *Evaluation and Program Planning* 28, 2005, pp.209-220.
 98. Zegeer, C.V, Stewart, J.R., Council, F.M., Reinfurt, D.W., and E. Hamilton. Safety Effects of Geometric Improvements on Horizontal Curves, *Journal of the Transportation Research Record* 1356, TRB, National Research Council, Washington, D.C., 1992, pp 11-19.
 99. Zhang, H.P., and M.B. Bracken, Tree-based, Two-stage Risk Factor Analysis for Spontaneous Abortion. *American Journal of Epidemiology*, 144 (10), 1996, pp. 989-996.

100. Zegeer, C.V. and F.M. Council. Safety Effects Associated with Cross-Sectional Elements, Journal of the Transportation Research Record 1512, TRB, National Research Council, Washington, D.C., 1995, pp. 29-36.

Appendix A: Cross-Sectional Collision Prediction Models

- **Peabody Dimmick Formula (1941)**

The Peabody Dimmick formula (a.k.a. Bureau of Public Roads formula) was developed to estimate the number of collisions at grade crossings in rural areas (Tustin et al. 1986), and the form of the formula is;

$$A_5 = \frac{1.28 \times V^{0.170} \times T^{0.151}}{P^{0.171}} + K \quad (\text{A.1})$$

Where,

A5 = the expected number of collisions in 5 years.

V = annual average daily traffic (AADT);

T = average daily train traffic

P = protection coefficient; K = additional parameter

Each value of different parameters is determined by a given monograph, and uses for estimating the expected number of collisions.

- **Schoppert and Hoyt Model (a.k.a. NCHRP 50 Model; 1968)**

They stratified crossings into several different sub-groups according to the crossings' selected attributes, such as highway volume or type of warning devices. Then, they developed a series of collision prediction models for crossings in each sub-group, as such;

Table A.1 Summary of Schoppert and Hoyt Model

Warning Devices	Highway Volume below 500/day		Highway Volume greater than 500/day	
	Urban	Rural	Urban	Rural
Crossbucks	$X_1 = 38.90 X_{10}$		$X_1 = 30.57 X_{10}$	$X_1 = 30.35 X_{10}$
Stop signs	$X_1 = X_{10}(45.13 + 2.51 X_7 + 13.5 X_6)$		$X_1 = 11.44 X_{10}$	
Wigwags	$X_1 = X_{10}(6.06 + 0.02 X_5 + 0.40 X_7)$			
Flashing Lights	$X_1 = 3.23 X_{10}$	$X_1 = 9.30 X_{10}$	$X_1 = 3.23 X_{10}$	$X_1 = 9.30 X_{10}$
2-Quadrant Gates	$X_1 = 3.23 X_{10}$	$X_1 = 1.93 X_{10}$	$X_1 = 3.23 X_{10}$	$X_1 = 1.93 X_{10}$

where,

X_1 = collisions per year, scaled by 100;

X_2 = average daily traffic, ADT;

X_3 = trains per day;

X_5 = angle of crossing, acute angle measured in degrees;

X_6 = total number of highway lanes

X_7 = maximum absolute approach gradient within 100ft of crossings;

X_{10} = probability of coincidental vehicle and train arrival, or

$$\frac{X_3}{86,400} (1 - e^{-X_2/86,400})$$

- **New Hampshire Index (1971)**

Tustin et al. (1986) also cited this model in their work. The form of the index is;

$$HI = V \times T \times P_f \tag{A.2}$$

where,

HI = hazard index; V = annual average daily traffic (AADT)

T = average daily train traffic; P_f = protection factor (e.g. 0.1 for automatic gates; 0.6 for flashing lights; 1.0 for signboards)

Various versions of the New Hampshire Index were developed, including;

$$HI = V \times 2T_f \times T_s \times [(SD \times AN \times NTR)/4] \tag{A.3}$$

$$HI = V \times T \times [(TT \times TTR \times SD \times AN \times AL \times L \times G \times VSD \times W \times LT)/100] \tag{A.4}$$

$$HI = (V_f \times P_f \times T)/(TR \times TN \times T_f \times HS \times G \times SD \times AN) \tag{A.5}$$

where,

HI = hazard index;

AL = factor for highway alignment;

AN = factor of approach angle; G = factor for approach grade;

HS = factor for highway speed; L = factor for number of lanes;

LT = factor for local interference; NTR = Factor of number of tracks;

P_f = protection factor; SD = factor of sight distance;

T = average daily train traffic; T_f = number of fast trains;

TN = factor for number of night trains;

TR = factor for number and type of tracks;

TT = factor for type of train movement;

TTR = factor for type of tracks;

V = annual average daily traffic, AADT;

V_f = factor for annual average daily traffic;

VSD = factor for vertical sight distance;

W = factor for crossing width

- **Coleman and Stewart Model (1976)**

Before developing collision prediction models, they also stratified crossings into different sub-groups according to the number of tracks (single or multiple), the location (urban or rural), and the type of warning devices (automatic gates, flashing lights, etc.). The developed models are;

$$\text{Log}_{10}A = C_0 + C_1 \text{Log}_{10}V + C_2 \text{Log}_{10}T + C_3 (\text{Log}_{10}T)^2 \quad (\text{A.6})$$

where,

A = average number of collisions per crossing-year;

V = weighted average daily traffic volume for the N crossings (the weights are the number of years of available collision data for each of the N crossings);

T = the weighted average train volume for the N crossings (the weights are the number of years of available collision data for each of the N crossings).

Table A.2 Coefficients of Coleman and Stewart Model

Item	Warning Devices	C ₀	C ₁	C ₂	C ₃
Single-track Urban	Automatic Gates	-2.17	0.16	0.96	-0.35
	Flashing Lights	-2.85	0.37	1.16	-0.42
	Crossbucks	-2.38	0.26	0.78	-0.18
	Other Active	-2.13	0.30	0.72	-0.30
	Stop Signs	-2.98	0.42	1.96	-1.13
	None	-2.46	0.16	1.24	-0.56
Single-track Rural	Automatic Gates	-1.42	0.08	-0.15	0.25
	Flashing Lights	-3.56	0.62	0.92	-0.38
	Crossbucks	-2.77	0.40	0.89	-0.29
	Other Active	-2.25	0.34	0.34	-0.01
	Stop Signs	-2.97	0.61	-0.02	0.29
	None	-3.62	0.67	0.22	0.26
Multiple-track Urban	Automatic Gates	-2.58	0.23	1.30	-0.42
	Flashing Lights	-2.50	0.36	0.68	-0.09
	Crossbucks	-2.49	0.32	0.63	-0.02
	Other Active	-2.16	0.36	0.19	0.08
	Stop Signs	-1.43	0.09	0.18	0.16
	None	-3.00	0.41	0.63	-0.02
Multiple-track Rural	Automatic Gates	-1.63	0.22	-0.17	0.05
	Flashing Lights	-2.75	0.38	1.02	-0.36
	Crossbucks	-2.39	0.46	-0.50	0.53
	Other Active	-2.32	0.33	0.80	-0.35
	Stop Signs	-1.87	0.18	0.67	-0.34
	None	NA*	NA*	NA*	NA*

Note) * Insufficient data

- **US Department of Transportation Collision Prediction Models**

US Department of Transportation (US DOT) developed collision prediction models by using nonlinear multiple regression techniques. The models have been revised several times. In this study, we will look into three representative versions of US DOT models.

1) US DOT Collision Prediction Model by Mengert (1980)

Originally Mengert (1980) developed three different collision prediction models of grade crossings based on the type of warning devices and Farr (1981) summarized the models. They are;

Collision Prediction Model for Crossings with Passive Signboards

$$H = 0.389 \text{ EXP } (2X_1)$$

where,

$$X_1 = 0.74982 \text{ HVOL}_1 + 0.19474 \text{ Log}_{10} (\text{DT}+1) + 0.17491 \text{ MAIN TRACKS} + 0.1780 \text{ HWY PAVED} + 0.045405 \text{ POP} - 0.13139 \text{ FC} \quad (\text{A.7})$$

$$\text{HVOL}_1 = -0.13711 + 0.38069 h - 0.66800 h^2 - 0.19171 h^3 \quad (\text{A.8})$$

$$h = -3.0264 + 1.1580 \text{ Log}_{10} (\text{T}+1) + 0.48654 \text{ Log}_{10} (\text{C}+1) - 0.22122 [\text{Log}_{10} (\text{T}+1)]^2 \quad (\text{A.9})$$

Collision Prediction Model for Crossings with Flashing Lights

$$H = 1.084 \text{ EXP} (2X_2)$$

where,

$$X_2 = 1.0422 \text{ HVOL}_2 + 0.13737 \text{ MAIN TRACKS} - 0.097584 [\text{Log}_{10} (\text{T}+1)]^2 + 0.018064 \text{ LANES} - 0.036259 \text{ Log}_{10} (\text{DT}+1) + 0.018944 \text{ POP} \quad (\text{A.10})$$

$$\text{HVOL}_2 = 2.8395 + 0.75477 \text{ Log}_{10} (\text{T}+1) + 0.083292 [\text{Log}_{10} (\text{C}+1)]^2 \quad (\text{A.11})$$

Collision Prediction Model for Crossings with 2-Quadrant Gates

$$H = 0.820 \text{ EXP} (2X_3)$$

where,

$$X_3 = -0.83656 + 0.74849 \text{ HVOL}_3 + 0.19139 \text{ MAIN TRACKS} + 0.093829 \text{ LANES} \quad (\text{A.12})$$

$$\text{HVOL}_3 = -1.9674 + 0.18621 \text{ Log}_{10} (\text{T}+1) \text{ Log}_{10} (\text{C}+1) \quad (\text{A.13})$$

where,

H = expected number of collisions per year;

T = number of trains per day;

C = number of highway vehicles per day;

DT = number of day thru-trains per day;

MAIN TRACKS = number of main tracks;

HWY PAVED = 1 if highway paved, 0 if not paved;

POP = population – tens digit of the functional classification of road crossing;

FC = units digit of functional classification of road over crossing;

LANES = number of traffic lanes;

EXP (X) = natural base e (2.71828), raised to the power (X)

2) US DOT Collision Prediction Model by Coulombre et al. (1982)

Coulombre et al.'s US DOT collision prediction model consists of two primary equations: a basic prediction equation containing crossing characteristics and a second equation incorporating collision history as an explicit factor, as such;

The basic equation;

$$a = K \times EI \times MT \times DT \times HP \times MS \times HT \times HL \quad (\text{A.14})$$

where,

a = non-normalized collision prediction, collision per year;

K = formula constant;

EI = factor for exposure index;

MT = factor for number of main tracks;

DT = factor for number of through trains per day during daylight;

HP = factor for highway paved factor;

MS = factor for maximum timetable speed;

HL = factor for number of highway lanes;

HT = factor for highway type;

The input factors to calculate the basic collision prediction equations (Coulombre et al. 1982) are summarized in Table A.3.

Table A.3 Factors for the US DOT Collision Prediction Model

Factor	Description	Warning Device		
		Passive	Flashing Lights	Gates
K	Formula Constant	0.002268	0.003646	0.001088
EI	Exposure Index Factor	$[(c \cdot t + 0.2)/0.2]^{0.3334}$	$[(c \cdot t + 0.2)/0.2]^{0.2953}$	$[(c \cdot t + 0.2)/0.2]^{0.3116}$
DT	Day Thru Trains Factor	$[(d + 0.2)/0.2]^{0.1336}$	$[(d + 0.2)/0.2]^{0.0470}$	1.0
MS	Maximum Speed Factor	$e^{0.0077ms}$	1.0	1.0
MT	Main Tracks Factor	$e^{0.2094mt}$	$e^{0.1088mt}$	$e^{0.2912mt}$
HP	Highway Paved Factor	$e^{-0.6160(hp-1)}$	1.0	1.0
HL	Highway Lanes Factor	1.0	$e^{0.1380(hl-1)}$	$e^{0.1036(hl-1)}$
HT	Highway Type Factor	$e^{-0.1000(ht-1)}$	1.0	1.0

where,

c = number of highway vehicles per day;

t = number of trains per day;

mt = number of main tracks;

d = number of through trains per day during daylight;

hp = highway paved (yes = 1, no = 2);

ms = maximum timetable speed at crossing, miles per hour;

hl = number of highway lanes;

ht = highway type factor (e.g. interstate = 1, urban freeway and expressway =2).

The equation (A.15) combines the output of equation (A.14) to the collision history at the corresponding crossings, as such;

$$B = \frac{T_0}{T_0 + T}(a) + \frac{T}{T_0 + T} \left(\frac{N}{T} \right) \quad (A.15)$$

where,

B = weighted average collisions between a and N/T (accident/year);

T = number of years of collision history (suggested by 5 years);

N = number of observed collision in T years ($T = 5$ years);

T_0 is the formula weighting factor defined as $T_0 = 1/(0.05 + a)$

It is interesting to note that if there was no collision history in a given 5 years, then the final prediction B will be equal to the output from the basic equation (A.14).

Berg (1986) compared the first and the second US DOT collision prediction models and showed substantial differences in the estimates between the two models. He asserted that the second model yields counter-intuitive results and therefore the first model is superior to the second one.

3) US DOT Collision Prediction Model in GradeDec 2000 Ver.2 (FRA 2002)

Recently, the Federal Railroad Administration (FRA) developed GradeDec 2000 program as a decision support tool for use by state and local authorities (FRA 2002). The program utilizes the latest version of US DOT collision prediction model. The model is similar to the second US DOT model except for some additional changes.

First, the basic equation (A.14) does not contain the HT term. The expression and the relevant factors have been changed, as such;

$$a = K \times EI \times MT \times DT \times HP \times MS \times HL \quad (\text{A.16})$$

As for the second US DOT model, equation (A.15) is used to combine the model estimate to the collision history. After obtaining the weighted average collisions (B), the adjustment factor in Table A.3 is considered, as such;

$$A = 0.7159 \times B \text{ for crossings with passive warning devices}$$

$$A = 0.5292 \times B \text{ for crossings with flashing lights}$$

$$A = 0.4921 \times B \text{ for crossings with gates}$$

$$A = 0.4921 \times \text{Tech Factor} \times B \text{ for crossings with new technology}$$

An unique contribution of the third model is in that it considers the impact of varying exposure using the expression (A.17);

$$\text{Exp} = 1.35 \times \text{EF} \times \text{AADT} \times \text{TV} \quad (\text{A.17})$$

where,

Exp = base year daily exposure with time-of-day correlation, effective daily exposures;

EF= time-of-day exposure correlation factor;

AADT = average annual daily traffic on the highway at the crossing;

TV = average daily trains at the crossing.

Table A.4 Factors for the US DOT Collision Prediction Model

Factor	Type of Grade Crossings			
	Passive	Flashing Lights	Gates	New Technology
K	0.0006938	0.0003351	0.0005745	0.0001915
EI	$[(\text{Exp}+0.2)/0.2]^{0.37}$	$[(\text{Exp}+0.2)/0.2]^{0.4106}$	$[(\text{Exp}+0.2)/0.2]^{0.2942}$	$[(\text{Exp}+0.2)/0.2]^{0.2942}$
DT	$[(d+0.2)/0.2]^{0.1781}$	$[(d+0.2)/0.2]^{0.1131}$	$[(d+0.2)/0.2]^{0.1781}$	$[(d+0.2)/0.2]^{0.1781}$
MS	$e^{0.0077\text{ms}}$	1.0	1.0	1.0
MT	1.0	$e^{0.1917\text{mt}}$	$e^{0.1512\text{mt}}$	$e^{0.1512\text{mt}}$
HP	$e^{-0.5966(\text{hp}-1)}$	1.0	1.0	1.0
HL	1.0	$e^{0.1826(\text{hl}-1)}$	$e^{0.142(\text{hl}-1)}$	$e^{0.142(\text{hl}-1)}$
Adj.	0.7159	0.5292	0.4921	0.4921×Tech Factor





- **Austin and Carson Model (2002)**


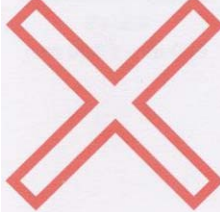

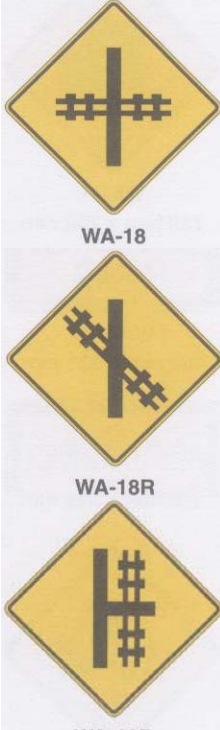
Austin and Carson developed collision prediction models by using negative binomial expression. Compare to the previous US DOT models, this model is simpler and easier to interpret. They used an “Instrumental Variable” technique to overcome the limitation in conventional cross-section models. The table A.5 shows the variables and associated statistics in their model.


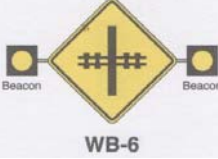


Table A.5 Austin and Carson Collision Prediction Model

Independent Variable	Coefficient	Std. Error	t-statistic
Constant	-6.719	0.136	-49.498
Traffic Characteristics			
Number of Nightly Trough Trains	0.039	0.005	8.236
Maximum Time Table Speed	0.021	0.002	12.828
Number of Main Tracks	0.484	0.064	7.556
Number of Traffic Lanes	0.170	0.031	5.418
AADT in Both Directions	3.59E-05	3.77E-06	9.524
Roadway Characteristics			
Highway Paved or Gravel	0.295	0.090	3.259
Crossing Characteristics			
Surface, Sectional	0.260	0.071	3.684
Surface, Full Wood Plank	0.312	0.074	4.233
Pavement Markings: Stop Lines	0.747	0.073	10.196
Probability of a Stop Sign	19.615	2.174	9.024
Probability of a Gate	-2.974	0.202	-14.687
Probability of Flashing Lights	1.075	0.182	5.922
Probability of a Highway Traffic Signal	-114.447	23.651	-4.839
Probability of Bells	0.649	0.170	3.820
Log Likelihood Function	-7,127.55		
Restricted Log Likelihood	-7,166.86		
Number of Observations	80,962		

Appendix B: Passive Traffic Control Devices for Highway-railway Grade Crossings (Transportation Association of Canada 1998)

Traffic Control Devices	Indication of Need	Symbol
A2.2.1 Stop Sign (RA-1)	The Stop sign indicates to drivers that they must stop their vehicles completely before entering the intersection area. TAC (1998) indicated that the Stop signs are warranted as an interim measure at a railway crossing which is scheduled for automatic protection or as required by the railway authority. The physical characteristics of the intersection, the collision experience, or travel speeds may require that a Stop sign be supplemented by a Stop Ahead sign (WB-1).	 <p>RA-1</p>
A3.6.1 Stop Ahead Sign (WB-1)	The Stop Ahead sign indicates the presence of a Stop sign (RA-1) ahead. Limited visibility due to conditions such as horizontal and vertical curves, parked vehicles, foliage, and/or high vehicle speeds should be considered in determining the need for these signs. In some cases, the advance sign may be used due to poor performance of the Stop sign.	 <p>WB-1</p>
A2.10.1 Stop Line Sign (RC-4)	The Stop Line sign indicates the point at which drivers approaching a traffic control device must stop their vehicles. The sign should be used where the location of the stop line is non-standard, or where the required stopping position may not be obvious to drivers.	 <p>RC-4R</p>
A2.2.2 Yield Sign (RA-2)	The Yield sign indicates to drivers that they must yield the right-of-way, stopping if necessary, before entering the intersection, and must not proceed until it is safe to do so. A Yield sign may be supplemented by a Yield Ahead sign (WB-2). Note: In the US (FHWA 2003), as the discretion of the responsible State or local highway agency, STOP or Yield signs may be used at highway- railway grade crossings that have two or more trains per day and are without automatic traffic control devices	 <p>RA-2</p>

<p>A3.6.2 Yield Ahead Sign (WB-2)</p>	<p>The Yield Ahead sign indicates the presence of a Yield sign (RA-1) ahead. Limited visibility due to conditions such as horizontal and vertical curves, parked vehicles, foliage, and/or high vehicle speeds should be considered in determining the need for these signs. In some cases, the advance sign may be used due to poor performance of the Yield sign.</p>	 <p>WB-2</p>
<p>A2.2.3 Railway Crossing Sign (RA-6)</p>	<p>The railway crossing sign indicates to drivers that they must yield the right-of-way, stopping if necessary, before entering the railway crossing area and must not proceed until it is safe to do so. The Railway Crossing sign is in the form of an “X”. Both crosspieces of the “X” are 1200 mm by 200 mm and they intersect at a right angle</p>	 <p>RA-6</p>
	<p>The supplementary tab sign (RA-6S) must be used with the Railway Crossing Sign where there are two or more tracks at the crossing. This tab sign is in the form of an inverted “T”, where the minor leg displays a numeral corresponding to the number of tracks, and where the major leg graphically depicts a railway track.</p>	 <p>RA-6S</p>
<p>A3.4.2 Railway Crossing Ahead Signs (WA-18, WA-19, WA-20)</p>	<p>The Railway Crossing Ahead sign is used to warn drivers in advance of all at grade railway crossings. Situations exist where a major road and a rail line, which are parallel and in close proximity, intersect a minor road, such that insufficient distance is available on the minor road between the railway crossing and the major road for proper sitting of the WA-18 sign. In such a situation the WA-18 on the minor road between the major road and the railway crossing is replaced by the WA-19 or WA-20, installed on the major road in advance of the intersection, facing both directions of traffic on the major road. Note: In the US, these signs are designated as Advance Warning Signs and shall be used on each highway in advance of every highway-rail grade crossing except under specified restrictions as provided in the US MUTCD (FHWA 2003).</p>	 <p>WA-18 WA-18R WA-19R</p>

		 <p>WA-20R</p>
A3.6.6 Prepare To Stop At Railway Crossing Sign (WB-6)	<p>The Prepare to Stop at Railway Crossing sign indicates to drivers in advance of a railway crossing that there is a high probability of having to stop for the railway crossing signals ahead</p> <p>The sign should only be used where traffic engineering studies have indicated that this sign is warranted. Factors which should be considered include: (a) train and vehicle speeds; (b) train volumes; (c) traffic volumes, particularly heavy trucks; (d) visibility; (e) highway grades; and (f) collision experience.</p>	 <p>WB-6</p>
	A yellow backboard (WB-6 Optional) may be used in conjunction with the basic sign. The backboard is the preferred design for an overhead sign.	 <p>WB-6 (OPTIONAL)</p>
A.3.2.5 Advisory Speed Tab Sign (WA-7S)	The Advisory Speed tab sign may be used in conjunction with standard warning signs. It is not used alone. It is installed immediately below the warning sign, and on the same post. The speed should be in multiples of 10 km/h.	 <p>WA-7S</p>
C3.3 Approaches to Railway Crossings	Pavement markings may be placed on a paved approach to a railway crossing where extra emphasis may be needed. These markings are not sufficient warning by themselves and must always be used in conjunction with signs and other devices. The markings must be white.	Refer to Figure 2.2
C4.1.3 Railway Crossing Symbols	Typical pavement markings at a railway crossing is “X” symbol.	
Illumination (FHWA, US MUTCD 2003 Ed.)	If an engineering study is conducted and if the engineering study determines that better nighttime visibility of the train and the highway-rail grade crossing is needed (for example, where a substantial amount of railroad operation is conducted at night, where train speeds are low and highway-rail grade crossings are blocked for long periods, or crash	Refer to Figure 2.3

	history indicates that drivers experience difficulty in seeing trains or traffic control devices during hours of darkness), then illumination should be installed at and adjacent to the highway-rail grade crossing.	
--	---	--

Note: Class R signage, such as RA-1 and RC-4, indicate a Regulatory Sign; Class W signage, such as WB-1 and WB-2, indicate a Warning Sign.

Appendix C: Cluster Specific Collision Prediction Models

1) Collision Prediction Model for Crossings in Cluster 01

Cluster01 NB Model with Flashing Lights and Gates 1
22:50 Monday, April 17, 2006

The GENMOD Procedure

Model Information

Data Set LAI.CLUSTER01_V7_01
Distribution Negative Binomial
Link Function Log
Dependent Variable acc_occ

Number of Observations Read 1045
Number of Observations Used 1045

Parameter Information

Parameter	Effect
Prm1	Intercept
Prm2	lnexp
Prm3	awd_sign
Prm4	awd_gate

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	1041	731.4191	0.7026
Scaled Deviance	1041	731.4191	0.7026
Pearson Chi-Square	1041	1051.5226	1.0101
Scaled Pearson X2	1041	1051.5226	1.0101
Log Likelihood		-659.8633	

Algorithm converged.

Estimated Covariance Matrix

	Prm1	Prm2	Prm3	Prm4	Dispersion
Prm1	0.13933	-0.01138	-0.03034	-0.01334	-0.000017
Prm2	-0.01138	0.001583	-0.003785	-0.006146	0.0001173
Prm3	-0.03034	-0.003785	0.10647	0.07225	-0.002300
Prm4	-0.01334	-0.006146	0.07225	0.08590	-0.001195
Dispersion	-0.000017	0.0001173	-0.002300	-0.001195	0.03602

Cluster01 NB Model with Flashing Lights and Gates 2
22:50 Monday, April 17, 2006

The GENMOD Procedure

Analysis Of Parameter Estimates

Parameter	DF	Estimate	Standard Error	Wald	95% Confidence Limits	Chi-Square	Pr > Chi Sq
Intercept	1	-4.0566	0.3733	-4.7882	-3.3250	118.11	<.0001
lnexp	1	0.3577	0.0398	0.2797	0.4357	80.85	<.0001
awd_sign	1	-0.3830	0.3263	-1.0225	0.2566	1.38	0.2405
awd_gate	1	-0.8396	0.2931	-1.4141	-0.2652	8.21	0.0042
Dispersion	1	0.7937	0.1898	0.4217	1.1656		

NOTE: The negative binomial dispersion parameter was estimated by maximum likelihood.

2) Collision Prediction Model for Crossings in Cluster 02

Cluster02 NB Model with Flashing Lights and Gates 11
11:07 Tuesday, April 18, 2006

The GENMOD Procedure

Model Information

Data Set LAI.CLUSTER02_V7_01
Distribution Negative Binomial
Link Function Log
Dependent Variable acc_occ

Number of Observations Read 2274
Number of Observations Used 2274

Parameter Information

Parameter	Effect
Prm1	Intercept
Prm2	lnexp
Prm3	awd_sign
Prm4	awd_gate
Prm5	tnmxspdm
Prm6	whistle

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	2268	947.6605	0.4178
Scaled Deviance	2268	947.6605	0.4178
Pearson Chi-Square	2268	2485.3550	1.0958
Scaled Pearson X2	2268	2485.3550	1.0958
Log Likelihood		-808.9254	

Algorithm converged.

Estimated Covariance Matrix

	Prm1	Prm2	Prm3	Prm4	Prm5	Prm6	Dispersion
Prm1	0.08436	-0.009471	0.02139	0.04210	-0.000459	-0.003039	-0.003318
Prm2	-0.009471	0.001888	-0.005565	-0.007543	-0.000029	-0.001554	0.0000411
Prm3	0.02139	-0.005565	0.03532	0.03056	0.0000696	-0.005875	-0.001202
Prm4	0.04210	-0.007543	0.03056	0.11852	-0.000120	-0.006540	0.003318
Prm5	-0.000459	-0.000029	0.0000696	-0.000120	0.0000127	0.0002413	0.0000674
Prm6	-0.003039	-0.001554	-0.005875	-0.006540	0.0002413	0.11134	0.003852
Dispersion	-0.003318	0.0000411	-0.001202	0.003318	0.0000674	0.003852	0.09711

Cluster02 NB Model with Flashing Lights and Gates 12
11:07 Tuesday, April 18, 2006

The GENMOD Procedure

Analysis Of Parameter Estimates

Parameter	DF	Estimate	Standard Error	Wald	95% Confidence Limits	Chi-Square	Pr > Chi Sq
Intercept	1	-4.9670	0.2904	-5.5362	-4.3977	292.45	<.0001
lnexp	1	0.3869	0.0434	0.3018	0.4721	79.32	<.0001
awd_sign	1	-0.9943	0.1879	-1.3627	-0.6259	27.99	<.0001
awd_gate	1	-1.4483	0.3443	-2.1230	-0.7735	17.70	<.0001
tnmxspdm	1	0.0161	0.0036	0.0091	0.0231	20.49	<.0001
whistle	1	1.1519	0.3337	0.4979	1.8059	11.92	0.0006
Dispersion	1	1.1124	0.3116	0.5016	1.7232		

NOTE: The negative binomial dispersion parameter was estimated by maximum likelihood.

3) Collision Prediction Model for Crossings in Cluster 03

```
. nbreg acc_occ lnexp awd tnmxspdm trkangle, nol og
Negati ve binomi al regressi on                               Number of obs = 4040
                                                             LR chi 2(4) = 243.09
                                                             Prob > chi 2 = 0.0000
Log Li kel i hood = -1267.6421                               Pseudo R2 = 0.0875
```

acc_occ	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
lnexp	.4606274	.032122	14.34	0.000	.3976695	.5235854
awd	-1.119466	.1371104	-8.16	0.000	-1.388198	-.8507348
tnmxspdm	.0105388	.0028503	3.70	0.000	.0049523	.0161253
trkangle	-.009173	.00558	-1.64	0.100	-.0201096	.0017636
_cons	-4.536707	.4810155	-9.43	0.000	-5.47948	-3.593934
/lnal pha	-1.019457	.5916986			-2.179165	.1402513
alpha	.3607909	.2134795			.113136	1.150563

```
Li kel i hood-rati o test of al pha=0: chi bar2(01) = 3.72 Prob>=chi bar2 = 0.027
```

```
. matrix list e(V)
symmetri c e(V)[6,6]
      acc_occ: lnexp      acc_occ: awd      acc_occ: tnmxspdm      acc_occ: trkangle      acc_occ: _cons      lnal pha: _cons
acc_occ: lnexp      .00103182
acc_occ: awd      -.0023881      .01879925
acc_occ: tnmxspdm      -2.677e-06      -4.878e-06      8.124e-06
acc_occ: trkangle      -.00001495      .00005446      -1.978e-06      .00003114
acc_occ: _cons      -.00503333      .00708499      -.00018094      -.0023377      .23137589
lnal pha: _cons      .00173449      -.00672211      9.221e-06      -.00007495      -.00359859      .35010725
```

```
. matrix list e(b)
e(b)[1,6]
      acc_occ: lnexp      acc_occ: awd      acc_occ: tnmxspdm      acc_occ: trkangle      acc_occ: _cons      lnal pha: _cons
y1      .46062745      -1.1194662      .01053877      -.00917297      -4.5367075      -1.0194567
```

```
. matrix v_test=e(b)*e(V)*e(b)'
```

```
. matrix list v_test
symmetri c v_test[1,1]
      y1
y1      5.1947823
```

```
. log close
log type: text
closed on: 14 Apr 2006, 13:50:21
```

4) Collision Prediction Model for Crossings in Cluster 04

Cluster04 NB Model with Active Warning Devices 17
16:41 Tuesday, April 18, 2006

The GENMOD Procedure

Model Information

Data Set LAI.CLUSTER04_V7_01
Distribution Negative Binomial
Link Function Log
Dependent Variable acc_occ

Number of Observations Read 1988
Number of Observations Used 1988

Parameter Information

Parameter	Effect
Prm1	Intercept
Prm2	lnexp
Prm3	awd

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	1985	1110.1724	0.5593
Scaled Deviance	1985	1110.1724	0.5593
Pearson Chi-Square	1985	2110.1224	1.0630
Scaled Pearson X2	1985	2110.1224	1.0630
Log Likelihood		-982.2619	

Algorithm converged.

Estimated Covariance Matrix

	Prm1	Prm2	Prm3	Dispersion
Prm1	0.09373	-0.009136	-0.007702	0.001155
Prm2	-0.009136	0.001334	-0.003429	0.0002801
Prm3	-0.007702	-0.003429	0.04330	-0.004044
Dispersion	0.001155	0.0002801	-0.004044	0.04673

Cluster04 NB Model with Active Warning Devices 18
16:41 Tuesday, April 18, 2006

The GENMOD Procedure

Analysis Of Parameter Estimates

Parameter	DF	Estimate	Standard Error	Wald	95% Confidence Limits	Chi-Square	Pr > ChiSq
Intercept	1	-4.3092	0.3061	-4.9092	-3.7092	198.12	<.0001
lnexp	1	0.4408	0.0365	0.3692	0.5124	145.60	<.0001
awd	1	-1.2216	0.2081	-1.6295	-0.8138	34.47	<.0001
Dispersion	1	0.9735	0.2162	0.5498	1.3972		

NOTE: The negative binomial dispersion parameter was estimated by maximum likelihood.

5) Collision Prediction Model for Crossings in Cluster 05

Cluster05 NB Model with Flashing Lights and Gates 23
16:41 Tuesday, April 18, 2006

The GENMOD Procedure

Model Information

Data Set LAI.CLUSTER05_V7_01
Distribution Negative Binomial
Link Function Log
Dependent Variable acc_occ

Number of Observations Read 1098
Number of Observations Used 1098

Parameter Information

Parameter	Effect
Prm1	Intercept
Prm2	lnexp
Prm3	awd_sign
Prm4	awd_gate
Prm5	whistle

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	1093	580.0278	0.5307
Scaled Deviance	1093	580.0278	0.5307
Pearson Chi-Square	1093	1284.7855	1.1755
Scaled Pearson X2	1093	1284.7855	1.1755
Log Likelihood		-538.2695	

Algorithm converged.

Estimated Covariance Matrix

	Prm1	Prm2	Prm3	Prm4	Prm5	Dispersion
Prm1	0.27575	-0.02986	0.03751	0.08755	-0.01181	-0.000129
Prm2	-0.02986	0.003534	-0.006500	-0.01223	0.0003082	0.0002781
Prm3	0.03751	-0.006500	0.04494	0.04680	-0.008857	-0.002986
Prm4	0.08755	-0.01223	0.04680	0.10475	-0.01409	-0.002953
Prm5	-0.01181	0.0003082	-0.008857	-0.01409	0.02678	-0.000893
Dispersion	-0.000129	0.0002781	-0.002986	-0.002953	-0.000893	0.10764

Cluster05 NB Model with Flashing Lights and Gates 24
16:41 Tuesday, April 18, 2006

The GENMOD Procedure

Analysis Of Parameter Estimates

Parameter	DF	Estimate	Standard Error	Wald	95% Confidence Limits	Chi-Square	Pr > Chi Sq
Intercept	1	-6.0712	0.5251	-7.1005	-5.0420	133.67	<.0001
lnexp	1	0.4971	0.0594	0.3806	0.6136	69.91	<.0001
awd_sign	1	-0.5804	0.2120	-0.9960	-0.1649	7.50	0.0062
awd_gate	1	-1.4916	0.3237	-2.1260	-0.8573	21.24	<.0001
whistle	1	0.8074	0.1636	0.4867	1.1281	24.34	<.0001
Dispersion	1	1.6142	0.3281	0.9711	2.2572		

NOTE: The negative binomial dispersion parameter was estimated by maximum likelihood.

Appendix D: Class Specific Collision Prediction Models

1) Collision Prediction Model for Crossings in Class 01

Collision Prediction Model for Class01
09:41 Wednesday, April 19, 2006 1

The GENMOD Procedure

Model Information

Data Set TDC02.GI01_01
Distribution Negative Binomial
Link Function Log
Dependent Variable acc_occ acc_occ

Number of Observations Read 1986
Number of Observations Used 1986

Parameter Information

Parameter	Effect
Prm1	Intercept
Prm2	Inexposure
Prm3	awd_sign
Prm4	awd_gate
Prm5	whistle
Prm6	tnmxspdk

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	1980	1463.1375	0.7390
Scaled Deviance	1980	1463.1375	0.7390
Pearson Chi-Square	1980	2012.9466	1.0166
Scaled Pearson X2	1980	2012.9466	1.0166
Log Likelihood		-1305.5640	

Algorithm converged.

Estimated Covariance Matrix

	Prm1	Prm2	Prm3	Prm4	Prm5	Prm6	Dispersion
Prm1	0.07074	-0.006822	-0.001511	0.01198	0.007228	-0.000101	-0.000597
Prm2	-0.006822	0.0009024	-0.001697	-0.003057	-0.001280	2.9896E-6	0.0001298
Prm3	-0.001511	-0.001697	0.02163	0.02126	0.001376	-0.000014	-0.001332
Prm4	0.01198	-0.003057	0.02126	0.03415	-0.001132	-0.000072	-0.000577
Prm5	0.007228	-0.001280	0.001376	-0.001132	0.01296	0.0000268	-0.000128
Prm6	-0.000101	2.9896E-6	-0.000014	-0.000072	0.0000268	1.4377E-6	4.1556E-6
Dispersion	-0.000597	0.0001298	-0.001332	-0.000577	-0.000128	4.1556E-6	0.01309

Collision Prediction Model for Class01
09:41 Wednesday, April 19, 2006 2

The GENMOD Procedure

Analysis Of Parameter Estimates

Parameter	DF	Estimate	Standard Error	Wald	95% Confidence Limits	Chi-Square	Pr > Chi Sq
Intercept	1	-3.7969	0.2660	-4.3182	-3.2756	203.79	<.0001
Inexposure	1	0.3448	0.0300	0.2859	0.4037	131.76	<.0001
awd_sign	1	-0.6773	0.1471	-0.9656	-0.3891	21.21	<.0001
awd_gate	1	-0.8994	0.1848	-1.2616	-0.5372	23.69	<.0001
whistle	1	0.2936	0.1138	0.0705	0.5167	6.65	0.0099
tnmxspdk	1	0.0021	0.0012	-0.0002	0.0045	3.14	0.0762
Dispersion	1	0.6328	0.1144	0.4086	0.8571		

NOTE: The negative binomial dispersion parameter was estimated by maximum likelihood.

2) Collision Prediction Model for Crossings in Class 02

Collision Prediction Model for Class02

09:41 Wednesday, April 19, 2006 3

The GENMOD Procedure

Model Information

Data Set TDC02.GI02_01
 Distribution Negative Binomial
 Link Function Log
 Dependent Variable acc_occ acc_occ

Number of Observations Read 712
 Number of Observations Used 712

Parameter Information

Parameter	Effect
Prm1	Intercept
Prm2	lnexposure
Prm3	awd_sign
Prm4	awd_gate
Prm5	pave

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	707	527.5398	0.7462
Scaled Deviance	707	527.5398	0.7462
Pearson Chi-Square	707	700.3836	0.9906
Scaled Pearson X2	707	700.3836	0.9906
Log Likelihood		-438.6314	

Algorithm converged.

Estimated Covariance Matrix

	Prm1	Prm2	Prm3	Prm4	Prm5	Dispersion
Prm1	0.13524	-0.01648	0.008535	0.02967	0.009927	-0.006381
Prm2	-0.01648	0.002295	-0.003007	-0.006086	-0.002239	0.0008864
Prm3	0.008535	-0.003007	0.05545	0.02473	-0.004834	-0.002455
Prm4	0.02967	-0.006086	0.02473	0.04191	0.001636	-0.002810
Prm5	0.009927	-0.002239	-0.004834	0.001636	0.02403	0.0006762
Dispersion	-0.006381	0.0008864	-0.002455	-0.002810	0.0006762	0.03250

Collision Prediction Model for Class02

09:41 Wednesday, April 19, 2006 4

The GENMOD Procedure

Analysis Of Parameter Estimates

Parameter	DF	Estimate	Standard Error	Wald	95% Confidence Limits	Chi-Square	Pr > Chi Sq
Intercept	1	-3.8206	0.3677	-4.5414	-3.0998	107.94	<.0001
lnexposure	1	0.3579	0.0479	0.2640	0.4518	55.80	<.0001
awd_sign	1	-0.5711	0.2355	-1.0326	-0.1096	5.88	0.0153
awd_gate	1	-0.6012	0.2047	-1.0025	-0.2000	8.62	0.0033
pave	1	-0.2540	0.1550	-0.5579	0.0498	2.68	0.1013
Dispersion	1	0.2360	0.1803	-0.1173	0.5893		

NOTE: The negative binomial dispersion parameter was estimated by maximum likelihood.

3) Collision Prediction Model for Crossings in Class 03

Collision Prediction Model for Class03 5
09:41 Wednesday, April 19, 2006

The GENMOD Procedure

Model Information

Data Set TDC02.GI03_01
Distribution Negative Binomial
Link Function Log
Dependent Variable acc_occ acc_occ

Number of Observations Read 2354
Number of Observations Used 2354

Parameter Information

Parameter	Effect
Prm1	Intercept
Prm2	lnexposure
Prm3	awd_sign
Prm4	awd_gate
Prm5	pave
Prm6	whistle
Prm7	tnmxspdk

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	2347	1385.6501	0.5904
Scaled Deviance	2347	1385.6501	0.5904
Pearson Chi-Square	2347	2291.4804	0.9763
Scaled Pearson X2	2347	2291.4804	0.9763
Log Likelihood		-1138.0755	

Algorithm converged.

Collision Prediction Model for Class03 6
09:41 Wednesday, April 19, 2006

The GENMOD Procedure

Estimated Covariance Matrix

	Prm1	Prm2	Prm3	Prm4
Prm1	0.05803	-0.006647	0.01224	0.02354
Prm2	-0.006647	0.001121	-0.001821	-0.002779
Prm3	0.01224	-0.001821	0.01711	0.01430
Prm4	0.02354	-0.002779	0.01430	0.05557
Prm5	-0.000982	-0.001223	-0.003452	-0.004153
Prm6	0.002380	-0.001478	-0.002941	-0.009760
Prm7	-0.000178	-3.107E-6	-0.000058	-0.000133
Dispersion	-0.002123	0.0001893	-0.001832	0.002146

Estimated Covariance Matrix

	Prm5	Prm6	Prm7	Dispersion
Prm1	-0.000982	0.002380	-0.000178	-0.002123
Prm2	-0.001223	-0.001478	-3.107E-6	0.0001893
Prm3	-0.003452	-0.002941	-0.000058	-0.001832
Prm4	-0.004153	-0.009760	-0.000133	-0.002146
Prm5	0.01537	0.0006086	0.0000814	0.0007410
Prm6	0.0006086	0.03045	0.0000934	-0.000441
Prm7	0.0000814	0.0000934	2.7793E-6	0.0000157
Dispersion	0.0007410	-0.000441	0.0000157	0.02367

Analysis Of Parameter Estimates

Parameter	DF	Estimate	Standard Error	Wald	95% Confidence Limits	Chi - Square	Pr > Chi Sq
Intercept	1	-4.1902	0.2409	-4.6623	-3.7180	302.54	<.0001
lnexposure	1	0.3658	0.0335	0.3002	0.4314	119.37	<.0001
awd_sign	1	-0.9828	0.1308	-1.2391	-0.7264	56.46	<.0001
awd_gate	1	-1.2501	0.2357	-1.7122	-0.7881	28.12	<.0001
pave	1	-0.2224	0.1240	-0.4654	0.0206	3.22	0.0729
whistle	1	0.8270	0.1745	0.4850	1.1690	22.46	<.0001
tnmxspdk	1	0.0071	0.0017	0.0039	0.0104	18.26	<.0001
Dispersion	1	0.4388	0.1538	0.1373	0.7403		

NOTE: The negative binomial dispersion parameter was estimated by maximum likelihood.

4) Collision Prediction Model for Crossings in Class 04

Collision Prediction Model for Class04

11
09:41 Wednesday, April 19, 2006

The GENMOD Procedure

Model Information

Data Set TDC02.GI04_01
 Distribution Poisson
 Link Function Log
 Dependent Variable acc_occ acc_occ

Number of Observations Read 962
 Number of Observations Used 962

Parameter Information

Parameter	Effect
Prm1	Intercept
Prm2	lnexposure
Prm3	whistle
Prm4	tnmxspdk

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	958	344.6305	0.3597
Scaled Deviance	958	344.6305	0.3597
Pearson Chi-Square	958	877.0370	0.9155
Scaled Pearson X2	958	877.0370	0.9155
Log Likelihood		-240.7701	

Algorithm converged.

Estimated Covariance Matrix

	Prm1	Prm2	Prm3	Prm4
Prm1	0.17723	-0.01571	0.01971	-0.000956
Prm2	-0.01571	0.005905	-0.02278	-0.000176
Prm3	0.01971	-0.02278	0.60778	0.0009949
Prm4	-0.000956	-0.000176	0.0009949	0.0000219

Analysis Of Parameter Estimates

Parameter	DF	Estimate	Standard Error	Wald	95% Confidence Limits	Chi-Square	Pr > Chi Sq
Intercept	1	-4.7893	0.4210	-5.6144	-3.9642	129.42	<.0001
lnexposure	1	0.2896	0.0768	0.1390	0.4402	14.20	0.0002
whistle	1	1.4093	0.7796	-0.1187	2.9373	3.27	0.0706
tnmxspdk	1	0.0114	0.0047	0.0022	0.0205	5.90	0.0152
Scale	0	1.0000	0.0000	1.0000	1.0000		

NOTE: The scale parameter was held fixed.

5) Collision Prediction Model for Crossings with Class-Specific Intercept

Collision Prediction Model for Class Based Intercept 13
 09:41 Wednesday, April 19, 2006

The GENMOD Procedure

Model Information

Data Set TDC02.STATIFICATION_V07_01
 Distribution Negative Binomial
 Link Function Log
 Dependent Variable acc_occ

Number of Observations Read 6014
 Number of Observations Used 6014

Parameter Information

Parameter	Effect
Prm1	Intercept
Prm2	lnexposure
Prm3	awd_sign
Prm4	awd_gate
Prm5	pave
Prm6	whistle
Prm7	tnmxspdk
Prm8	gi 01
Prm9	gi 02
Prm10	gi 03
Prm11	gi 04

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	6004	3675.8845	0.6122
Scaled Deviance	6004	3675.8845	0.6122
Pearson Chi-Square	6004	5808.8171	0.9675
Scaled Pearson X2	6004	5808.8171	0.9675
Log Likelihood		-3138.6191	

Algorithm converged.

The GENMOD Procedure

Estimated Covariance Matrix

	Prm2	Prm3	Prm4	Prm5	Prm6	Prm7
Prm2	0.0003880	-0.000636	-0.001139	-0.000251	-0.000517	3.5426E-7
Prm3	-0.000636	0.007086	0.006313	-0.001092	0.0001356	-0.000011
Prm4	-0.001139	0.006313	0.01307	-0.000773	-0.001584	-0.000037
Prm5	-0.000251	-0.001092	-0.000773	0.004432	0.0000423	0.0000169
Prm6	-0.000517	0.0001356	-0.001584	0.0000423	0.007207	0.0000179
Prm7	3.5426E-7	-0.000011	-0.000037	0.0000169	0.0000179	7.5802E-7
Prm8	-0.002837	0.001854	0.006716	-0.000702	0.002475	-0.000052
Prm9	-0.002627	0.002927	0.006649	-0.000495	0.002032	-0.000056
Prm10	-0.002441	0.002858	0.008169	-0.000647	0.001941	-0.000054
Prm11	-0.001919	0.003682	0.008248	-0.000222	0.001121	-0.000062
Di spersi on	0.0000883	-0.000561	-0.000141	-0.000042	0.0000128	3.6824E-6

Estimated Covariance Matrix

	Prm8	Prm9	Prm10	Prm11	Di spersi on
Prm2	-0.002837	-0.002627	-0.002441	-0.001919	0.0000883
Prm3	0.001854	0.002927	0.002858	0.003682	-0.000561
Prm4	0.006716	0.006649	0.008169	0.008248	-0.000141
Prm5	-0.000702	-0.000495	-0.000647	-0.000222	-0.000042
Prm6	0.002475	0.002032	0.001941	0.001121	0.0000128
Prm7	-0.000052	-0.000056	-0.000054	-0.000062	3.6824E-6
Prm8	0.02987	0.02581	0.02344	0.01829	-0.000680
Prm9	0.02581	0.02968	0.02167	0.01744	-0.000883
Prm10	0.02344	0.02167	0.02256	0.01641	-0.000699
Prm11	0.01829	0.01744	0.01641	0.02867	-0.000719
Di spersi on	-0.000680	-0.000883	-0.000699	-0.000719	0.006749

Analysis of Parameter Estimates

Parameter	DF	Estimate	Standard Error	Wald	95% Confidence Limits	Chi - Square	Pr > Chi Sq
Intercept	0	0.0000	0.0000	0.0000	0.0000	.	.
lnexposure	1	0.3546	0.0197	0.3160	0.3932	324.11	<.0001
awd_si gn	1	-0.7563	0.0842	-0.9213	-0.5913	80.72	<.0001
awd_gate	1	-1.0043	0.1143	-1.2283	-0.7802	77.15	<.0001
pave	1	-0.1120	0.0666	-0.2425	0.0184	2.83	0.0924
whi stle	1	0.3731	0.0849	0.2067	0.5395	19.32	<.0001
tnmxspdk	1	0.0036	0.0009	0.0019	0.0053	17.34	<.0001
ci 01	1	-3.8672	0.1728	-4.2059	-3.5284	500.65	<.0001
ci 02	1	-4.0043	0.1723	-4.3420	-3.6666	540.18	<.0001
ci 03	1	-3.9654	0.1502	-4.2598	-3.6710	696.87	<.0001
ci 04	1	-4.3880	0.1693	-4.7198	-4.0561	671.61	<.0001
Di spersi on	1	0.5428	0.0822	0.3818	0.7038		

The GENMOD Procedure

NOTE: The negative binomial dispersion parameter was estimated by maximum likelihood.

Appendix E: Collision Prediction Model with Group Indicators

Collision Prediction Models with Interactions
21:07 Tuesday, April 18, 2006 ¹

The GENMOD Procedure

Model Information

Data Set	TDC01.INTERACTIONS
Distribution	Negative Binomial
Link Function	Log
Dependent Variable	acc_occ accident_occurrence

Number of Observations Read	6014
Number of Observations Used	6014

Parameter Information

Parameter	Effect
Prm1	Intercept
Prm2	lnexp
Prm3	awd_sign
Prm4	awd_gate
Prm5	tnmedspd
Prm6	tnhghspd
Prm7	gi 08
Prm8	gi 11
Prm9	gi 13

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	6005	3677.8944	0.6125
Scaled Deviance	6005	3677.8944	0.6125
Pearson Chi-Square	6005	5891.8791	0.9812
Scaled Pearson X2	6005	5891.8791	0.9812
Log Likelihood		-3145.0221	

Algorithm converged.

Collision Prediction Models with Interactions
21:07 Tuesday, April 18, 2006 ²

The GENMOD Procedure

Estimated Covariance Matrix

	Prm1	Prm2	Prm3	Prm4	Prm5
Prm1	0.02903	-0.002774	0.002002	0.008204	-0.006825
Prm2	-0.002774	0.0003460	-0.000776	-0.001445	0.0004266
Prm3	0.002002	-0.000776	0.009258	0.008106	-0.002190
Prm4	0.008204	-0.001445	0.008106	0.01391	-0.003541
Prm5	-0.006825	0.0004266	-0.002190	-0.003541	0.007470
Prm6	-0.008066	0.0004825	-0.000749	-0.003443	0.005284
Prm7	-0.000351	0.0001200	-0.002325	-0.001289	-0.000580
Prm8	-0.007227	0.0004799	0.004379	0.003137	-0.002947
Prm9	0.001170	-0.000145	0.0003180	0.0006121	-0.000180
Dispersion	-0.000654	0.0000983	-0.000569	-0.000223	0.0000855

Estimated Covariance Matrix

	Prm6	Prm7	Prm8	Prm9	Di spersi on
Prm1	-0.008066	-0.000351	-0.007227	0.001170	-0.000654
Prm2	0.0004825	0.0001200	0.0004799	-0.000145	0.0000983
Prm3	-0.000749	-0.002325	0.004379	0.0003180	-0.000569
Prm4	-0.003443	-0.001289	0.003137	0.0006121	-0.000223
Prm5	0.005284	-0.000580	-0.002947	-0.000180	0.0000855
Prm6	0.008552	-0.000950	0.0001767	-0.000203	0.0001542
Prm7	-0.000950	0.007588	0.0002815	-0.000048	0.0001552
Prm8	0.0001767	0.0002815	0.01620	-0.008820	-0.000036
Prm9	-0.000203	-0.000048	-0.008820	0.01962	0.0001178
Di spersi on	0.0001542	0.0001552	-0.000036	0.0001178	0.006827

Analysis Of Parameter Estimates

Parameter	DF	Estimate	Standard Error	Wal d	95% Confidence Limits	Chi - Square	Pr > Chi Sq
Intercept	1	-4.6091	0.1704	-4.9431	-4.2752	731.87	<.0001
lnexp	1	0.4222	0.0186	0.3858	0.4587	515.32	<.0001
awd_sigr	1	-0.7284	0.0962	-0.9170	-0.5399	57.32	<.0001
awd_gate	1	-0.9118	0.1179	-1.1429	-0.6806	59.78	<.0001
tnmedspd	1	0.2735	0.0864	0.1041	0.4429	10.01	0.0016
tnhgspd	1	0.3163	0.0925	0.1351	0.4976	11.70	0.0006
gi 08	1	0.1438	0.0871	-0.0270	0.3145	2.72	0.0989
gi 11	1	0.4092	0.1273	0.1598	0.6587	10.34	0.0013
gi 13	1	-0.2345	0.1401	-0.5090	0.0400	2.80	0.0941
Di spersi on	1	0.5538	0.0826	0.3919	0.7158		

NOTE: The negative binomial dispersion parameter was estimated by maximum likelihood.

APPENDIX F. ESTIMATING *CMF* FOR DATA LIKELIHOODS

1. Eliminating Whistle Prohibition

1) Factor/Cluster Model based on a Study by Saccomanno and Lai (2005)

- Estimating Collision Frequency before Eliminating Whistle Prohibition

Step 1: Obtain Standardized Variables Z_{ij}

Table F.1 Standardized Variables

Variable	Mean	Std. Deviation	Z_{ij}
Warning Devices	1.6404	0.7144	0.5034
Road Surface Width	10.6259	5.4323	-1.1144
Surface Material	0.3793	0.4852	1.2793
Road Type	0.2666	0.4422	1.6585
Track Number	1.2390	0.5813	-0.4111
Track Angle	69.9359	19.5532	0.0033
Whistle Prohibition	0.0900	0.2861	3.1807
Mainline	0.9308	0.2538	0.2727
AADT	1582.6377	3931.6533	3.4127
Daily Trains	9.3741	11.1340	0.2358
Posted Speed Limit	59.3586	21.2361	-0.4407
Max. Train Speed	40.9324	20.7620	-1.4899

Step 2: Calculate Factor Scores for Each Crossing

Table F.2 Factor Score Coefficients and Factor Scores

Variable	Factor Score Coefficients (β_{ik})				Factor Score Coefficients (β_{ik}) \times Standardized Variable (Z_{ij})			
	F1	F2	F3	F4	F1 \times Z_{ij}	F2 \times Z_{ij}	F3 \times Z_{ij}	F4 \times Z_{ij}
Warning Devices	0.2560	0.2156	-0.0754	0.0674	0.1289	0.1085	-0.0380	0.0339
Road Surface Width	0.2443	-0.0502	0.0445	-0.1578	-0.2723	0.0559	-0.0496	0.1759
Surface Material	0.2664	-0.1126	-0.0944	0.2331	0.3408	-0.1440	-0.1208	0.2982
Road Type	0.3066	0.0287	-0.3382	0.1030	0.5085	0.0476	-0.5609	0.1708
Track Number	0.0235	0.1962	0.2956	0.0914	-0.0097	-0.0807	-0.1215	-0.0376
Track Angle	0.0130	-0.0271	-0.0456	0.9356	0.0000	-0.0001	-0.0001	0.0031
Whistle Prohibition	0.1137	0.0400	0.3903	-0.1643	0.3616	0.1272	1.2414	-0.5226
Mainline	-0.0743	0.2439	-0.2242	-0.0167	-0.0203	0.0665	-0.0611	-0.0046
AADT	0.2587	-0.0471	0.0996	-0.0873	0.8829	-0.1607	0.3399	-0.2979
Daily Trains	-0.0059	0.3741	0.1731	-0.0601	-0.0014	0.0882	0.0408	-0.0142
Posted Speed Limit	0.1321	0.0565	-0.5273	-0.0281	-0.0582	-0.0249	0.2324	0.0124
Max. Train Speed	-0.0753	0.3904	-0.1105	-0.0239	0.1122	-0.5816	0.1646	0.0356
Factor Scores					1.9731	-0.4980	1.0671	-0.1470

Step 3: Determine Cluster Membership

Table F.3 Cluster Center for Four Factor Scores and Cluster Membership

	Cluster Center for 5 Clusters				
	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Factor Score 1	0.5949	-0.4174	-0.6568	0.7940	1.2764
Factor Score 2	1.8202	0.0135	-0.1827	0.1293	-1.3208
Factor Score 3	1.0418	-0.2617	0.0698	-1.0575	1.2085
Factor Score 4	0.2106	-1.3206	0.4721	0.4909	-0.0909
Distance to Cluster Center	2.7207	3.0198	2.8972	2.5894	1.0888
Cluster Membership					×

- Estimating Collision Frequency After Eliminating Whistle Prohibition

Step 1: Obtain Standardized Variables Z_{ij}

Table F.4 Standardized Variables

Variable	Mean	Std. Deviation	Z_{ij}
Warning Devices	1.6404	0.7144	0.503359
Road Surface Width	10.6259	5.4323	-1.114427
Surface Material	0.3793	0.4852	1.2793
Road Type	0.2666	0.4422	1.6585
Track Number	1.2390	0.5813	-0.4111
Track Angle	69.9359	19.5532	0.0033
Whistle Prohibition	0.0900	0.2861	-0.3146
Mainline	0.9308	0.2538	0.2727
AADT	1582.6377	3931.6533	3.4127
Daily Trains	9.3741	11.1340	0.2358
Posted Speed Limit	59.3586	21.2361	-0.4407
Max. Train Speed	40.9324	20.7620	-1.4899

Step 2: Calculate Factor Scores for Each Crossing

Table F.5 Factor Score Coefficients and Factor Scores

Variable	Factor Score Coefficients (β_{ik})				Factor Score Coefficients (β_{ik}) \times Standardized Variable (Z_{ij})			
	F1	F2	F3	F4	F1 \times Z_{ij}	F2 \times Z_{ij}	F3 \times Z_{ij}	F4 \times Z_{ij}
Warning Devices	0.2560	0.2156	-0.0754	0.0674	0.1289	0.1085	-0.0380	0.0339
Road Surface Width	0.2443	-0.0502	0.0445	-0.1578	-0.2723	0.0559	-0.0496	0.1759
Surface Material	0.2664	-0.1126	-0.0944	0.2331	0.3408	-0.1440	-0.1208	0.2982
Road Type	0.3066	0.0287	-0.3382	0.1030	0.5085	0.0476	-0.5609	0.1708
Track Number	0.0235	0.1962	0.2956	0.0914	-0.0097	-0.0807	-0.1215	-0.0376
Track Angle	0.0130	-0.0271	-0.0456	0.9356	0.0000	-0.0001	-0.0001	0.0031
Whistle Prohibition	0.1137	0.0400	0.3903	-0.1643	-0.0358	-0.0126	-0.1228	0.0517
Mainline	-0.0743	0.2439	-0.2242	-0.0167	-0.0203	0.0665	-0.0611	-0.0046
AADT	0.2587	-0.0471	0.0996	-0.0873	0.8829	-0.1607	0.3399	-0.2979
Daily Trains	-0.0059	0.3741	0.1731	-0.0601	-0.0014	0.0882	0.0408	-0.0142
Posted Speed Limit	0.1321	0.0565	-0.5273	-0.0281	-0.0582	-0.0249	0.2324	0.0124
Max. Train Speed	-0.0753	0.3904	-0.1105	-0.0239	0.1122	-0.5816	0.1646	0.0356
Factor Scores					1.5757	-0.6379	-0.2971	0.4273

Step 3: Determine Cluster Membership

Table F.6 Cluster Center for Four Factor Scores and Cluster Membership

	Cluster Center for 5 Clusters				
	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Factor Score 1	0.5949	-0.4174	-0.6568	0.7940	1.2764
Factor Score 2	1.8202	0.0135	-0.1827	0.1293	-1.3208
Factor Score 3	1.0418	-0.2617	0.0698	-1.0575	1.2085
Factor Score 4	0.2106	-1.3206	0.4721	0.4909	-0.0909
Distance to Cluster Center	2.9738	2.7300	2.3082	1.3349	1.7582
Cluster Membership				×	

APPENDIX G. ESTIMATED PRIORS WITH DIFFERENT WEIGHTING SCHEME

Table G.1 Estimated Priors with Equal Weight

Number	Countermeasures	μ	τ
1	Grade Separation/Closure	0.0000	0.0000
2	Yield Sign	0.8100	0.0723
3	Stop Sign	0.6467	0.0577
4	Stop Ahead Sign	0.6533	0.0583
5	Stop Line Sign	0.7200	0.0642
6	Illumination(Lighting)	0.5625	0.0502
7	Pavement Markings	0.7914	0.0706
8	From Signs to Flashing Lights	0.4256	0.1003
9	From Signs to 2Q-Gates	0.2567	0.0647
10	From Flashing Lights to 2Q-Gates	0.3975	0.1060
11	From 2Q-Gates to 2Q-Gates with Median Separation	0.3375	0.0301
12	From 2Q-Gates to 4Q-Gates	0.2540	0.0227
13	Installing Traffic Signal	0.3867	0.1316
14	Elimination of Whistle Prohibition	0.4675	0.0417
15	Improve Sight Distance	0.6630	0.0591
16	Improve Pavement Condition	0.5200	0.0464
17	Posted Speed Limit	0.8000	0.0714
18	Photo/Video Enforcement	0.2350	0.0210

Table G.2 Estimated Priors with 0.25 Interval Weight

Number	Countermeasures	μ	τ
1	Grade Separation/Closure	0.0000	0.0000
2	Yield Sign	0.8100	0.0723
3	Stop Sign	0.6467	0.0577
4	Stop Ahead Sign	0.6533	0.0583
5	Stop Line Sign	0.7200	0.0642
6	Illumination(Lighting)	0.5625	0.0502
7	Pavement Markings	0.7914	0.0706
8	From Signs to Flashing Lights	0.4470	0.1239
9	From Signs to 2Q-Gates	0.2744	0.0792
10	From Flashing Lights to 2Q-Gates	0.4483	0.1346
11	From 2Q-Gates to 2Q-Gates with Median Separation	0.3375	0.0301
12	From 2Q-Gates to 4Q-Gates	0.2540	0.0227
13	Installing Traffic Signal	0.3678	0.1623
14	Elimination of Whistle Prohibition	0.4667	0.0416
15	Improve Sight Distance	0.6630	0.0591
16	Improve Pavement Condition	0.5200	0.0464
17	Posted Speed Limit	0.8000	0.0714
18	Photo/Video Enforcement	0.2633	0.0235

Table G.3 Estimated Priors with 0.20 Interval Weight

Number	Countermeasures	μ	τ
1	Grade Separation/Closure	0.0000	0.0000
2	Yield Sign	0.8100	0.0723
3	Stop Sign	0.6467	0.0577
4	Stop Ahead Sign	0.6533	0.0583
5	Stop Line Sign	0.7200	0.0642
6	Illumination(Lighting)	0.5625	0.0502
7	Pavement Markings	0.7914	0.0706
8	From Signs to Flashing Lights	0.4417	0.1180
9	From Signs to 2Q-Gates	0.2700	0.0756
10	From Flashing Lights to 2Q-Gates	0.4356	0.1275
11	From 2Q-Gates to 2Q-Gates with Median Separation	0.3375	0.0301
12	From 2Q-Gates to 4Q-Gates	0.2540	0.0227
13	Installing Traffic Signal	0.3725	0.1546
14	Elimination of Whistle Prohibition	0.4670	0.0417
15	Improve Sight Distance	0.6630	0.0591
16	Improve Pavement Condition	0.5200	0.0464
17	Posted Speed Limit	0.8000	0.0714
18	Photo/Video Enforcement	0.2520	0.0225