

Predictions of Flexural Behaviour of Built-Up Cold-Formed Steel Sections

by

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A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Master of Applied Science
in
Civil Engineering

Waterloo, Ontario, Canada, 2007

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AUTHOR'S DECLARATION

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

In recent years, light gauge cold-formed steel members have been used extensively in low and mid-rise residential building construction. In cold-formed steel design there are several applications where built-up box girders are used to resist load induced in a structure when a single section is not sufficient to carry the design load. The cold-formed steel box girders may be subjected to eccentric loading when the web of one of the sections receives the load and transfers it through the connection to another section. There may be an unequal distribution of load in built-up girder assemblies loaded from one side. In the current North American Specification for the Design of Cold-Formed Steel Structural Members (CSA-S136-01, 2001), there is no guideline or design equation to calculate the flexural capacity of this type of section. AISI cold-formed steel framing design guide (2002) has recommended that the moment of resistance and inertia of the built-up section are the simple addition of the component parts, based on deflection compatibility of the two sections. However, this design approximation has not been justified by any experimental or numerical study. Very little information was found in literature about this topic.

The objective of this study is the investigation of the flexural behaviour of built-up box girders assembled from cold-formed stud and track sections when subjected to eccentric loading. Finite element analysis is conducted for this purpose, being much more economical than expensive experimental testing. Detailed parametric studies are carried out to identify the factors affecting the flexural capacity of built-up cold-formed steel sections. The parametric results are used to develop a design equation for calculating the flexural capacity of built-up cold-formed steel sections.

Acknowledgements

I am extremely grateful to my supervisors Prof. D. Grierson and Prof. L. Xu for their continuous advice, guidance and encouragement throughout this research. Without their help, the thesis could not be completed. I would also like to thank Dr. S. Walbridge for his valuable comments and suggestions about the research to improve the quality of the work.

I also thank the Dietrich Design Group, for providing the test results used in the study for finite element model calibration.

I would also like to thank Canadian Commonwealth Scholarship Agency for giving me the financial support to come in Canada and pursue my Masters degree in University of Waterloo.

I am tremendously indebted to my first academic supervisor, Dr. Khan Mahmud Amanat, from Bangladesh University of Engineering and Technology, Bangladesh for encouraging me to pursue my research and higher studies in Civil engineering.

I am thankful to my friends Jafar H. Al Bin Mousa and Chandrika Prakash for their help to learn ANSYS software. I am also grateful to my friends Suriyapriya Vallamsundar, Sonika Soor, Elina Atroshchenko, Dan Mao, Guru Prakash, Budhaditya Hazra, Lucia Moosa, Sabrina Anjum and Farzana Afreen, for their encouragement and company which really help me to live in Waterloo alone, away from home.

I wish to express my sincere thanks to my husband Sufian, my parents and my sister Dahlia, for their love, patience, understanding, sacrifice and encouragement during the pursuit of my academic work.

To My Parents

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Chapter 1

Introduction

1.1 General

In steel construction, there are two main categories of steel: hot-rolled steel and cold-formed steel. Hot-rolled steel sections have been used in the construction industry for more than one hundred years. As compared with thicker hot-rolled shapes, thin cold-formed members can be used for relatively light loads and short spans. Unusual sectional configurations can also be produced economically by cold-forming operations. Cold-formed steel structural members are made by cold-forming steel sheets, strips, plates or flat bars in roll forming machines or by press brake operations. The typical thickness of cold-formed steel products ranges from 0.4mm to about 6.4mm, although plates as thick as 25mm can also be cold-formed by some manufacturers. Presently, cold-formed steel sections are being extensively used in airplanes, automobiles, grain storage structures and building structures, to name just a few. They have been used in building construction as early as the 1850's. However, in North America such steel members were not widely used in buildings until the publication of the first edition of the American Iron and Steel Institute (AISI) specification in 1946. The first edition of the unified North American Specification was prepared and issued in 2001(CSA-S136-01, 2001).

Cold-formed steel is currently being used widely in residential and light commercial building constructions instead of wood framing because of the decreasing supply of reasonably priced quality lumber. Besides that, cold-formed steel has the highest strength-to-weight ratio of any building material used in construction today. Cold-formed steel sections are economical, light weight, non-combustible and also recyclable.

1.2 Description of the problem

Cold -formed steel sections such as C-sections with or without lips, I-sections, and hat sections are normally used as flexural members. When single sections are not sufficient for design loads, built-up sections made of back-to-back C-sections or nested C-sections forming a box girder are normally used as flexural members. Figure 1.1 shows a picture of a floor opening where built-up joists are used to carry a heavy load. For the joist assembly shown in Figure 1.1, load from the framing member is directly applied to the web of one member of the built-up joist assembly. As a result, any resistance provided by the other members in the assembly depends on the efficiency of the connection components in transferring load. Cold-formed steel (CFS) built-up box girders may also be subject to torsional moments. There may also be an unequal distribution of load in built-up girder assemblies loaded from one side. The current North American Specification for the Design of Cold-Formed Steel Structural Members (CSA-S136-01, 2001) does not provide any guideline on this issue. The AISI Cold-Formed Steel Framing Design Guide (AISI Cold-Formed Steel Framing Design Guide, 2002) suggests that the moment of resistance and inertia of built-up sections are the simple addition of the component parts. This assumption was made based on the assumption of displacement compatibility among the component parts but has not been confirmed by testing. Unequal load distribution can potentially lead to a reduction in capacity compared to the sum of the capacities of the individual members that make up the assembly. Addressing these problems presents an interesting challenge for the designer, or research is required to understand the flexural behaviour of CFS built-up box girders subjected to eccentric loading.

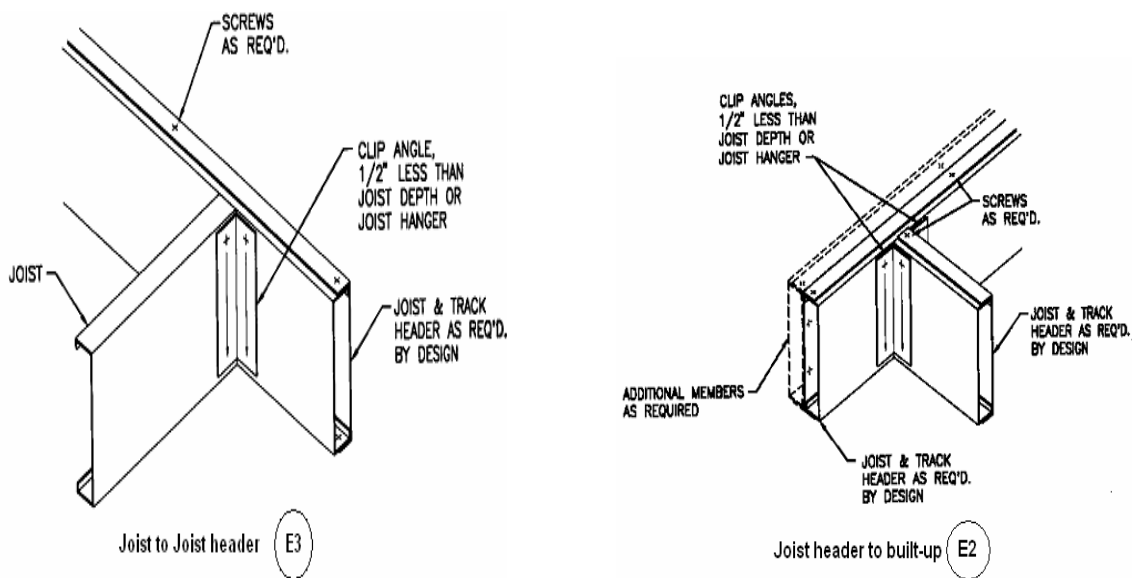
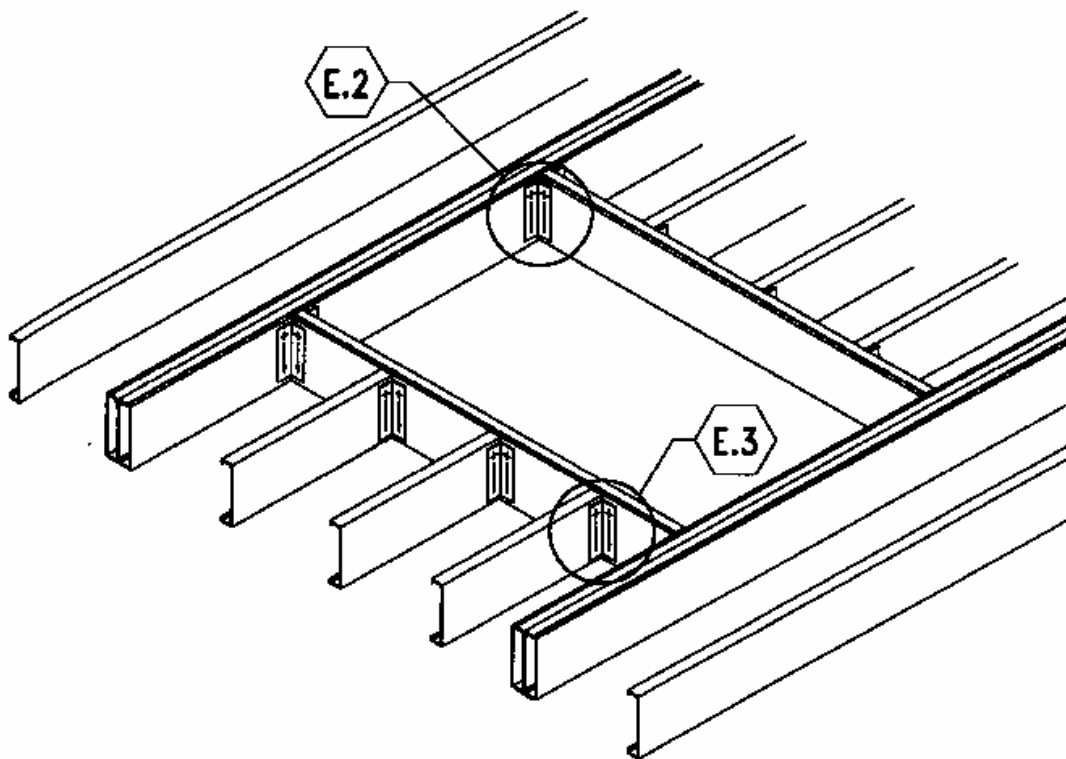


Figure 1.1 Flexural loading condition in cold-formed steel framing (CSSBI low rise residential construction details, 1994)

1.3 Objective of study

The objective of this study is to understand the flexural behaviour of CFS built-up box girders subjected to eccentric loading, and to verify whether the current design practice for calculating the moment capacity of this type of section is conservative or not. The built-up girder studied in this thesis is made from a stud section and a track section. The descriptions of the stud and track sections are given in section 1.4 in this chapter. The box sections were made by connecting both the top and bottom flanges of the stud and track sections using self-drilling screws, as shown in Figure 1.2. The stud section receives the load first and then transfers the load to the track section through the self-drilling screws. A finite element model is developed using ANSYS (version 10) to determine the ultimate moment capacity of CFS built-up box girders. After that, parametric studies are conducted using FEM analysis to identify the factors affecting the moment capacity of this type of section. Based on the results from the parametric studies, a simple equation is developed to calculate the ultimate moment capacities of CFS built-up box girders.

1.4 Terminology

The CFS built-up box sections examined in this study consist of two C-sections, one with stiffening lips and one without. Figure 1.2 shows cross-sections of C-sections with (a) stiffening lips and (b) without stiffening lips. The combination of the C-sections used in built-up assemblies is shown in Figure 1.2(c) and 1.2(d).

In this study, C-sections with lips are called stud sections. C-sections without lips are often used as alignment tracks for channels. For this reason, C-sections without lips are commonly referred to as track sections. The track sections with equal top and bottom flanges are

referred to as track sections, whereas track sections with bottom flanges longer than the top flanges are referred to as rim-track sections.

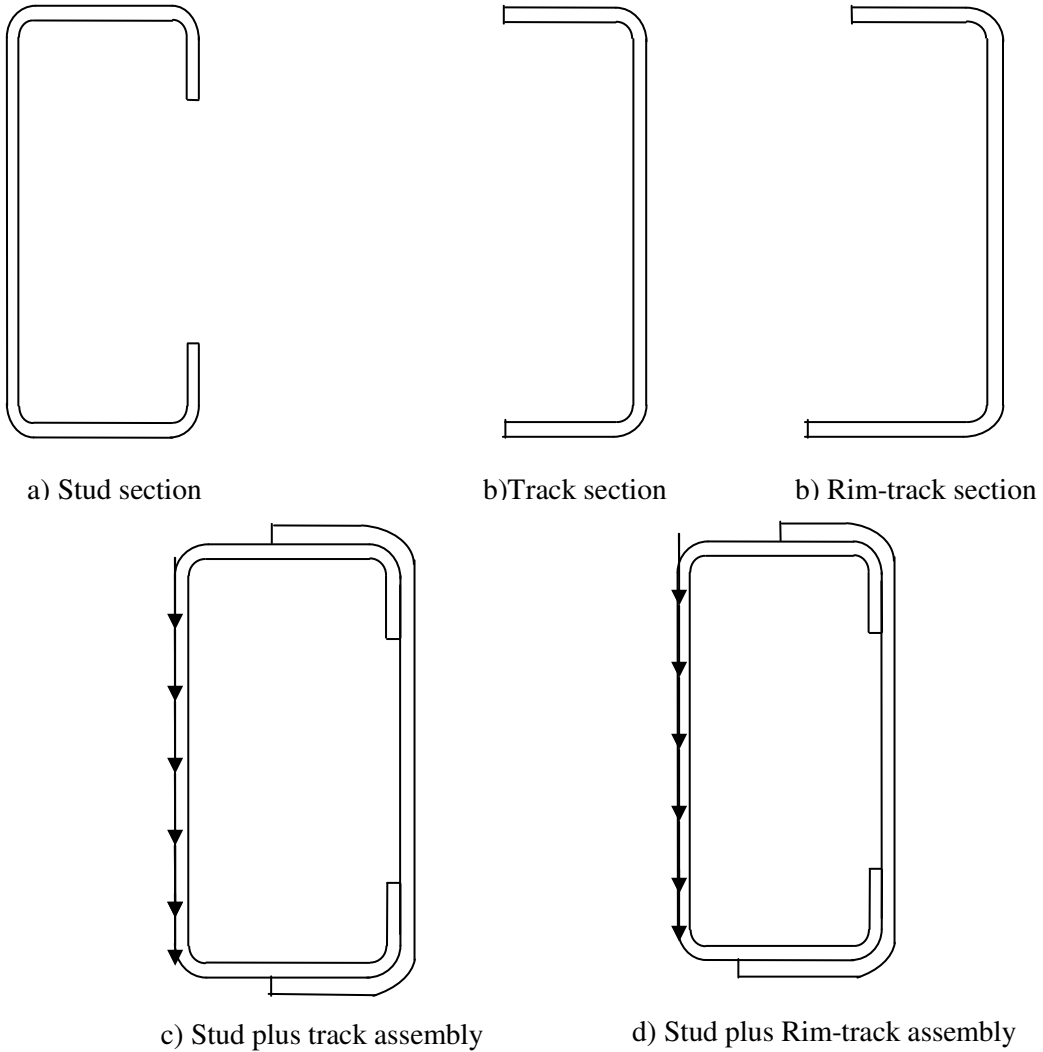


Figure 1.2 Cold-formed steel sections

1.5 Organization of thesis

The first requirement of any study is to review the theoretical background and identify what research has already been done in the field of interest. In Chapter 2, the theoretical

background necessary to understand the behaviour of cold-formed steel sections is discussed, including elastic plate buckling theory, the effective width concept for post buckling analysis of plates, and design expressions for calculating moment capacity. A review of previous experimental work and finite element analysis carried out in the cold-formed steel research area is also discussed in this chapter.

In Chapter 3, the development and verification of the finite element method (FEM) model created by this study to investigate the flexural behaviour of CFS built-up box girders is discussed. The FEM model accounts for the material nonlinearities, geometric nonlinearities and initial geometric imperfections of cold-formed sections. The results from the FEM analysis are compared with the available experimental test results to establish the accuracy of the FEM model. It is demonstrated that the FEM model reliably predicts the ultimate moment capacities of CFS built-up box girders as determined by laboratory tests.

As such, instead of doing expensive testing, the FEM model is used in Chapter 4 to investigate CFS girders configurations that were not tested and to carry out parametric studies. Specifically, the FEM model is used to investigate the influence of section depth, thickness, flange screw spacing and material yield stress on the ultimate moment capacity of CFS built-up sections. The results from the parametric studies are then used to develop an equation for calculating the moment capacity of CFS built-up box girders.

Chapter 5 gives a summary of the work carried out, and presents the conclusions and topics for future work emanating from the study. The appendices list all the calculations required to determine the nominal moment capacities of individual CFS built-up sections.

Chapter 2

Theoretical background and literature review

2.1 Introduction

The purpose of this chapter is to provide some background knowledge on the theoretical development of the design equations for cold-formed steel flexural members. Although the main objective of the research is to investigate the flexural performance of built-up box girders, current design practice and all experimental and numerical studies related to this topic are also discussed later in this chapter.

2.2 Theoretical background

In 1883, Saint Venant derived (Timoshenko and Gere, 1961) the differential equation for a plate subjected to combined bending and tension or compression. The equation is as follows:

$$\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} \left(q + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} - X \frac{\partial w}{\partial x} - Y \frac{\partial w}{\partial y} \right) \quad (2.1)$$

where,

N_x = Force acting in x direction per unit width

N_y = Force acting in y direction per unit width

N_{xy} = Shearing force acting in y direction, per unit width of plate perpendicular to x axis.

X = Body force acting in x direction.

Y = Body force acting in y direction.

q = Pressure loading force in direction of w

D = Flexural rigidity of plate

$$= \frac{Et^3}{12(1-\nu^2)}$$

If the plate is subjected to a compressive edge loading in the x direction and there is no body force and $q = 0$, Eq. (2.1) reduces to:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} \left(N_x \frac{\partial^2 w}{\partial x^2} \right) \quad (2.2)$$

Putting $N_x = -t\sigma_x$, Eq. (2.2) reduces to:

$$D \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) = -t\sigma_x \frac{\partial^2 w}{\partial x^2} \quad (2.3)$$

If the deflection w is independent of y , the solution of this differential equation for hinged

end is: (2.4)

$$\sigma_x t = \frac{\pi^2 E}{12(1-\nu^2)a^2}$$

where, a is the length of the strip in x the direction. This solution is equivalent to the Euler critical buckling load for columns.

In 1891, Bryan (Bryan, 1891) solved the stability problem of plate buckling using the energy method. He considered a simply supported rectangular plate under uniform edge compression in one direction. The deflection surface of the buckled simply-supported plate is represented by a double trigonometric series as follows:

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (2.5)$$

where, a = length of the plate

b = width of the plate

m, n are numbers of half sine waves in the x and y directions, respectively.

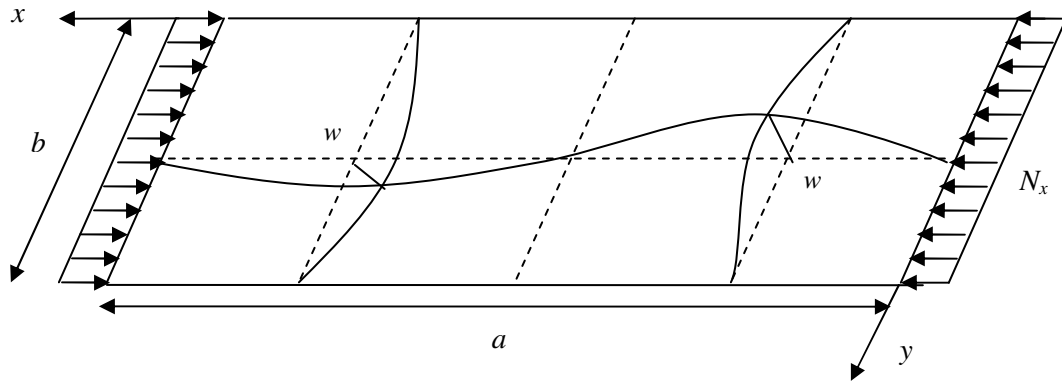


Figure 2.1 Buckled shape of simply supported plate (Timoshenko and Gere, 1961)

The corresponding critical value of the compressive force ($(N_x)_{cr}$) can be determined by integrating Eq. (2.3) using Eq.(2.5), or by considering the energy of the system. The resulting critical value is as follows:

$$(N_x)_{cr} = \frac{\pi^2 D}{a^2} \left(m + \frac{1}{m} \cdot \frac{a^2}{b^2} \right)^2 \quad (2.6)$$

The first factor in this expression represents the Euler load for a strip of unit width and length a . The second factor indicates in what proportion the stability of a continuous plate is greater than the stability of an isolated strip. The magnitude of the factor depends on the magnitude

of the ratio a/b and also the number of half sine waves into which the plate buckles. Eq.(2. 6)

can be rewritten as follows:

$$(N_x)_{cr} = \frac{\pi^2 D}{b^2} \left(m \frac{b}{a} + \frac{1}{m} \cdot \frac{a}{b} \right)^2 \quad (2.7)$$

Substituting in the value of D , and using the notation,

$$K = \left(m \frac{b}{a} + \frac{1}{m} \cdot \frac{a}{b} \right)^2 \quad (2.8)$$

the equation for the critical elastic plate buckling stress F_{cr} is expressed as follows:

$$F_{cr} = \frac{k\pi^2 E}{12(1-\nu^2) \left(\frac{b}{t} \right)^2} \quad (2.9)$$

Values of the plate buckling coefficient for a plate with simply supported edges are shown in Figure 2.2. From this figure it is clear that when a/b ratio is an integer, the value of k becomes 4. The transition from m to $m+1$ half sine waves occurs when the two corresponding curves have equal ordinates. Equating ordinates of the two corresponding curves, the following equation can be derived:

$$\frac{a}{b} = \sqrt{m(m+1)} \quad (2.10)$$

For a long plate Eq.(2.10) becomes:

$$\frac{a}{b} \approx m \quad (2.11)$$

Eq. (2.11) indicates that a very long plate buckles in half sin waves and the length of half sin waves equals approximately the width of plate, and therefore square waves are formed as shown in Figure 2.1. As shown in Figure 2.2, whenever the aspect ratio exceeds 4, k approaches 4. So $k = 4$ can be used for determining the critical buckling stress for a simply supported plate subjected to uniform compression in one direction. For different loading and boundary conditions, the value of k will vary. The values of k for different loading and boundary conditions are listed in Table 2.1 (Yu, 2000).

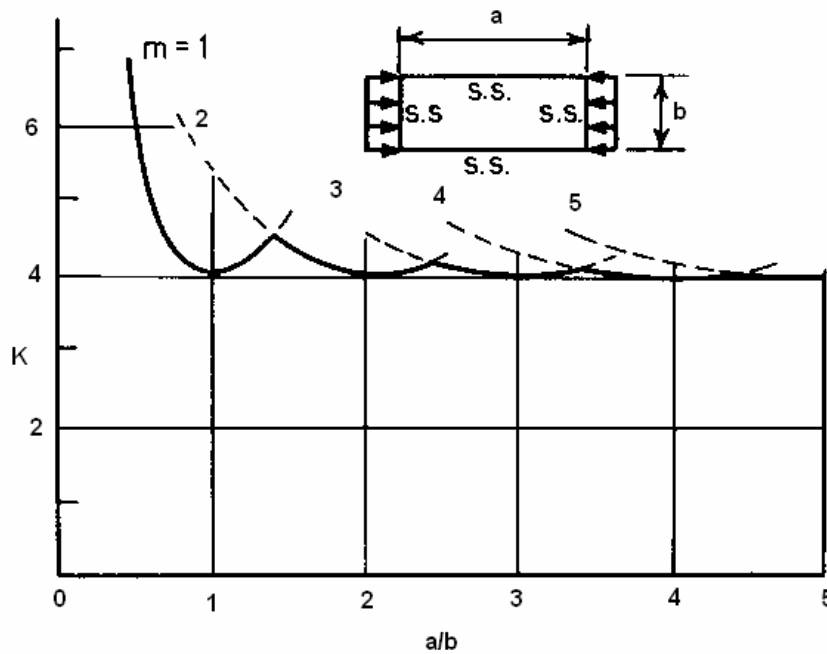


Figure 2.2 Buckling coefficients for flat rectangular plate (Timoshenko and Gere, 1961)

2.3 Effective design width concept

The critical elastic buckling load F_{cr} , does not necessarily indicate the failure of the plate, but distinguishes a load carrying region within which the load carrying mechanism of the plate

changes from one form to another. An additional load can be carried by the element after buckling by means of a redistribution of stress. The additional load carrying capacity developed in the plate beyond F_{cr} is called post buckling capacity and is most pronounced for stiffened compression elements with large width-to-thickness ratios (w/t). Post buckling strength increases with increase of the w/t ratio. The mechanism of the post buckling behaviour of a compression element was discussed by Winter (Winter, 1970) using a square plate model.

The plate is uniformly compressed in one direction and simply supported along the unloaded edges. This plate can be replaced by a model as shown in Figure 2.3 (Winter, 1970), which consists of longitudinal and transverse bars in which the material of the plate is assumed to be concentrated. As soon as the longitudinal bars start deflecting at F_{cr} , the transverse bars in the grid will act as tie rods to counteract the increasing deflection of the longitudinal struts. After F_{cr} is reached, a portion of the pre-buckling load of the centre strip is transferred to the edge strips. The strut closest to the edges continues to resist increasing load with hardly any increasing deflection. As a result the uniformly distributed compression stress in the plate redistributes itself in a manner as shown in Figure 2.4, with the stress being largest at the edges and smallest in the centre. With further increase in load this nonuniformity increases further. The plate fails only when the most highly stressed strips, near the supported edges, begin to yield.

Table 2.1 Plate Buckling Coefficient (Yu, 2000)

Case	Boundary condition	Type of stress	Value of k for long plate
(a)		Compression	4.0
(b)		Compression	6.97
(c)		Compression	0.425
(d)		Compression	1.277
(e)		Compression	5.42
(f)		Shear	5.34
(g)		Shear	8.98
(h)		Bending	23.9
(i)		Bending	41.8

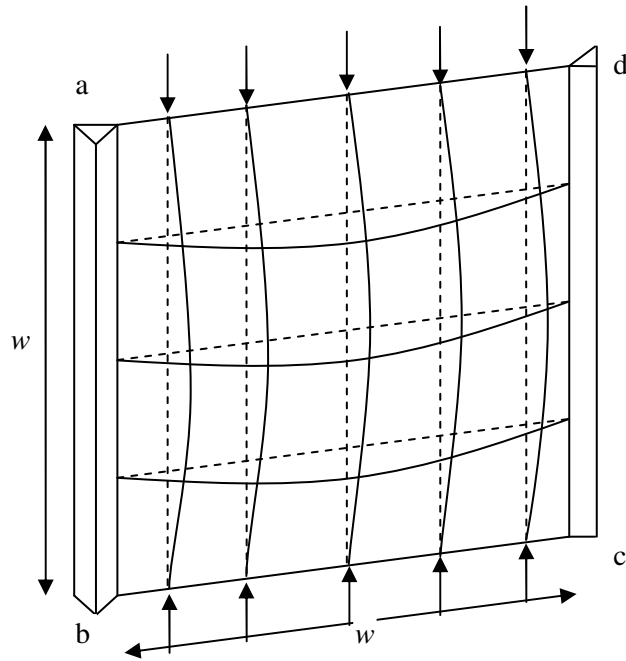


Figure 2.3 Post buckling behaviour of square plate (Winter, 1970)

Post-buckling behaviour of plates can be analyzed by using large deflection theory. In 1910, von Karman first proposed the following differential equation:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{t}{D} \left(\frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right) \quad (2.12)$$

This equation depends on the external stress function F and the deformation w .

As the solution of Eq. (2.12) is complicated, it has limited application in practical design. For this reason, in 1932, von Karman et al. (von Karman et al. 1932) proposed the “Effective width concept”. In this approach, instead of considering the non uniform distribution of stress over the entire width of w , it is assumed that the total load is carried by a fictitious effective width b , subject to a uniformly distributed stress equal to the edge stress f_{max} , as shown in the

Figure 2.4. The width b is selected so that the area under the curve of the actual non uniform stress distribution is equal to the sum of the two parts of the equivalent rectangular shaded area with a total width b and an intensity of stress equal to the edge stress f_{max} , i.e.,

$$\int_0^w f dx = b f_{max} \quad (2.13)$$

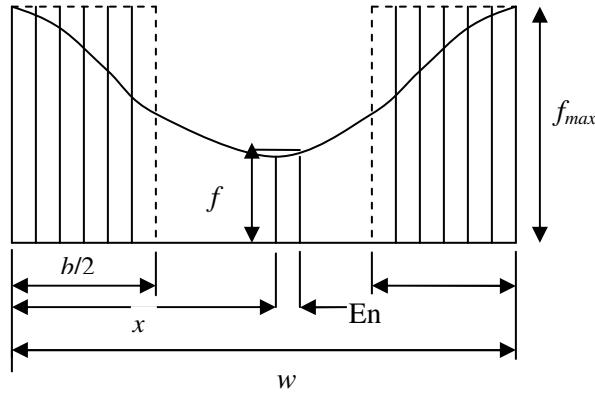


Figure 2.4 Effective width assumed by von Karman.

Rearranging Eq. (2.9) and solving for the plate width b , gives the following equation:

$$b = \frac{\pi t}{\sqrt{12(1 - \nu^2)}} \sqrt{\frac{kE}{\sigma_{cr}}} \quad (2.14)$$

By setting the critical buckling stress σ_{cr} equal to the yield stress, Poisson's ratio $\nu = 0.3$ and factor $k = 4$ (for simply supported edges), Eq.(2.14) reduces to:

$$b = 1.9t \sqrt{\frac{E}{F_y}} \quad (2.15)$$

where b is the equivalent plate width for thickness t .

Based on extensive investigation, Winter (Winter, 1946) proposed that the effective width equation developed by von Karman (von Karman et al. 1932) was not limited to determining

ultimate load, but was also valid for stress levels below the yield stress. Eq.(2.15) can be rewritten as:

$$b_e = Ct \sqrt{\frac{E}{S}} \quad (2.16)$$

where,

b_e = effective width of plate

S = any stress at or below the yield point.

It has been found that the value of coefficient C depends on the nondimensional parameter

$\sqrt{\frac{E}{S} \left(\frac{t}{w} \right)}$ and can be expressed by the following linear equation:

$$C = 1.9 - 1.09 \sqrt{\frac{E}{S} \left(\frac{t}{w} \right)} \quad (2.17)$$

By substituting this expression for C into Eq. (2.16), the following equation is obtained for the equivalent width b_e for a stiffened compression element simply supported along both longitudinal edges:

$$b_e = 1.9t \sqrt{\frac{E}{f_{\max}}} \left[1 - 0.574 \left(\frac{t}{w} \right) \sqrt{\frac{E}{f_{\max}}} \right] \quad (2.18)$$

The first AISI specification (AISI, 1946) for the design of cold form steel, published in 1946, incorporated the effective width concept for a stiffened compression element based on the work of Winter and others. The equation accounted for the effects of various imperfections and out of straightness. During the period from 1946 to 1968, the AISI design

provision for the determination of the effective design width was based on the following equation (AISI, 1946; AISI, 1968):

$$b_e = 1.9t \sqrt{\frac{E}{f_{\max}}} \left[1 - 0.475 \left(\frac{t}{w} \right) \sqrt{\frac{E}{f_{\max}}} \right] \quad (2.19)$$

Based on long time accumulated data and experience, Eq. (2.19) was modified as follows (Winter, 1970):

$$b_e = 1.9t \sqrt{\frac{E}{f_{\max}}} \left[1 - 0.415 \left(\frac{t}{w} \right) \sqrt{\frac{E}{f_{\max}}} \right] \quad (2.20)$$

which can be rewritten in terms of F_{cr}/f_{\max} ratio as follows:

$$\frac{b_e}{w} = \sqrt{\frac{F_{cr}}{f_{\max}}} \left[1 - 0.22 \sqrt{\frac{F_{cr}}{f_{\max}}} \right] \quad (2.21)$$

Therefore, the effective width can be determined as,

$$b_e = \rho w \quad (2.22)$$

$$\text{where } \rho = \frac{1 - \frac{0.22}{\sqrt{\frac{f_{\max}}{F_{cr}}}}}{\sqrt{\frac{f_{\max}}{F_{cr}}}} = \left(1 - \frac{0.22}{\lambda} \right) \leq 1 \quad (2.23)$$

in which,

$$\lambda = \sqrt{\frac{f_{\max}}{F_{cr}}} = \text{slenderness factor.} \quad (2.24)$$

From the relationship between ρ and λ , it is clear that when $\lambda \leq 0.673$, $\rho = 1$.

So, the equation for effective width becomes:

$$b_e = w, \text{ when } \lambda \leq 0.673 \quad (2.25)$$

$$b_e = \rho w \text{ when } \lambda > 0.673 \quad (2.26)$$

The 1986 edition of the AISI specification adopted Eqs. (2.25/6) for determining the effective width for a uniformly compressed stiffened element (AISI, 1986), and these equations are retained in the North American Specification (CSA-S136-01, 2001).

E.A. Miller (Miller, 1943) proposed the following equation for determining the equivalent width of an unstiffened element under uniform compression:

$$b_e = 1.25t \sqrt{\frac{E}{f_{\max}}} \left[1 - 0.333 \frac{t}{w} \sqrt{\frac{E}{f_{\max}}} \right] \quad (2.27)$$

where f_{\max} is the stress in the unstiffened compression element at the supported edge.

As Eq. (2.27) showed considerable scatter, a more conservative equation was proposed by Winter (Winter, 1946) as follows:

$$b_e = 0.8t \sqrt{\frac{E}{f_{\max}}} \left[1 - 0.202 \frac{t}{w} \sqrt{\frac{E}{f_{\max}}} \right] \quad (2.28)$$

Due to lack of extensive experimental verification and concern for excessive out of plate distortion at service load, the effective width concept was not used in the AISI specification prior to 1986 (AISI, 2001). In 1986, Pekoz (Pekoz, 1986) evaluated some experimental data using $k = 0.43$ and concluded that Eq. (2.23) developed for a stiffened compression element

gives a conservative lower bound to the test results for an unstiffened compression element. The effective width design approach for unstiffened elements was adopted for the first time in the 1986 AISI specification, and is retained in the North American Specification.

The compression portion of a flexural member may buckle due to compressive stress developed by bending. Buckling of a simply supported beam subjected to bending was investigated by Timoshenko (Timoshenko and Gere, 1961) and he showed that the theoretical critical buckling stress of a flat rectangular plate under pure bending can be determined by:

$$F_{cr} = \frac{k\pi^2 E}{12(1 - \mu^2) \left(\frac{h}{t}\right)^2} \quad (2.29)$$

where h = flat depth of web, t = thickness of web, and k = plate buckling coefficient.

k depends on the boundary condition, bending stress ratio and edge restraint provided by the beam flange (Yu, 2000). Bending strength of the beam web is affected by the web slenderness ratio h/t , aspect ratio (a/h), bending strength ratio, material properties such as E , F_y , μ , and the interaction between flange and web (Yu, 2000).

As bending strength depends on so many parameters, test results have been used to develop AISI design criteria. Before 1986, the full depth of web with allowable bending stress was used for the design of cold formed steel beam webs. The “effective depth design” concept was first adopted in the 1986 AISI specification based on studies performed by Pekoz (Pekoz, 1986), Cohen and Pekoz (Cohen and Pekoz, 1987).

2.4 Moment capacity of built-up sections: Hot-rolled versus cold-formed steel sections

Cold-formed steel sections such as C-sections with or without lip, I-sections, hat sections, and built-up sections made of back to back C-sections or nested C-sections forming a box girder are normally used as flexural members. The moment resisting capacity of these members is highly influenced by the lateral buckling of the beam. So, flexural members should be braced adequately according to specifications to provide lateral restraint.

The North American Specification (CSA-S136-01, 2001) has included Section C3.1.1, where two procedures for calculating the section strength of laterally supported flexural members are discussed (CSA-S136-01, 2001). Procedure 1 is based on “initiation of material yielding” and Procedure 2 is based on “inelastic reserve capacity”. In procedure 1, the nominal moment capacity of the cross section is the effective yield moment M_y based on effective areas of flanges and webs. The yield moment M_y is defined as the moment at which the outer fibre first attains yields. For a balanced section, the outer fibres for compression and tension reach the yield point at the same time. But for an eccentrically located neutral axis, initiation of yielding can take place either in the compression flange or tension flange. The nominal section strength for the initiation of yielding is calculated by using the following equation:

$$M_n = M_y = S_e F_y \quad (2.30)$$

where F_y = design yield stress, and S_e = elastic section modulus of the effective section.

Effective section modulus is calculated based on the effective width of individual elements of the section under design yield stress.

There is no specified design formula to calculate the moment resistance of built-up sections in the current North American Standard (CSA-S136-01, 2001). Instead, the AISI Cold-Formed Steel Framing Design Guide (AISI, 2002) and CSSBI Lightweight Steel Framing Design Manual (CSSBI, 2006) suggest that the moments of resistance and inertia of built-up sections are the simple addition of the component parts. This assumption was made based on the assumption of deflection compatibility of the two sections but not confirmed by testing.

For laterally braced hot-rolled beam sections, the nominal moment resistance of the section is calculated according to the Clause 13.5 of CSA S16-01 (CSA S16-01, 2003). According to that clause, when both the web and compressive flanges exceed the limits for Class 3 sections, the nominal moment resistance shall be determined in accordance with CSA-S136.

In the current North American specification (CSA-S136-01, 2001), there is no guideline about the minimum and maximum spacing of the connecting screws of built-up flexural members. The current design practice also does not account for the torsion resulting from unequal distribution of load due to eccentric loading condition. In the clause 19.3, CSA S16-01 (CSA S16-01, 2003) specifies requirements for built-up open box type beams and grillages. According to this clause, two or more hot-rolled beams or channels used side by side to form a flexural member shall be connected at intervals of not more than 1500mm.

Through-bolts and separators may be used, provided that, in beams having a depth of 300mm or more, no fewer than two bolts shall be used at each separator locations.

2.5 Review of previous experimental work

A thorough literature review of previous work related to the flexural capacity of built-up box girder members was performed, and very little corresponding information was found.

Serrette (Serrette, 2004) investigated the flexural performance of rafter box beams under eccentric loading as shown in Figure 2.5. The top and bottom tracks did not extend to the bearing support. Their primary function was to tie the joist member together to form a box. Three different box beam configurations were evaluated. The tests revealed that failure of the beams under the eccentric loading condition ultimately resulted from twisting. The analytically computed capacities of the tested box beams were compared with the test values. The cumulative strength of the box beam joist members was computed based on the assumptions that there is no composite flexural action between the box components and that lateral buckling is restrained. The limited test data suggests that the eccentric loading and the mechanism of load transfer from the directly loaded joist member to the adjacent joist member induces twist in the box beam. The edge loaded box beam was able to resist at most 85-90% of its calculated fully braced flexural capacity.

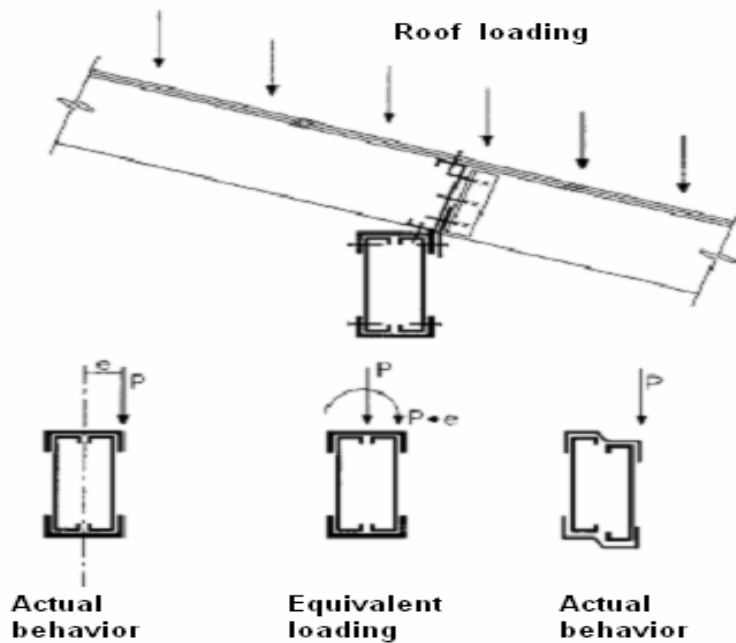


Figure 2.5 Box beam edge loading (Serrette, 2004)

For box type girders constructed of nested C-sections, screws are commonly located along the upper and lower faces of the box. Dietrich Design Group (DDG) (Beshara and Lawson, 2002), performed “Built-Up girder Screw connection Variation Flexural Tests”. The purpose of the tests was to evaluate the impact of varying the location of connection screws on the behaviour of built-up box girders. These test results were used to validate the FEM model of the built-up cold-formed steel beams in Chapter 3 of this study.

All specimens were fabricated from 16-gauge material and all utilized the same inner C-section component, a Dietrich 254 mm (10 in) deep x 16-gauge CSS stud. The CSS section had a 76.2 mm (3 in) flange width with 25.4 mm (1 in) long return lips. The outer C-sections were either unpunched 254 mm (10 in) deep TradeReady rim-track or 254 mm (10 in) deep TSB track. The TradeReady product had a bottom flange width of 63.5 mm (2.5 in), and a

top flange width of 31.8 mm (1.25 in). The TSB track system was symmetric with 31.8 mm (1.25 in) long flanges. The flanges of these track sections did not have return lips. The connections between the girders components were made with #10-16 HWH T-3 self drilling screws placed 300 mm (12 in) on center. When the connection was flange-to-flange, screws were placed approximately 19 mm (3/4 in) from the outer edge of flanges. When the connection was web-to-lip, the screws were centered along the length of the lips. The mechanical properties of the test specimen components were determined from tensile tests according to ASTM A370, 1992. The yield strength and ultimate strengths were recorded for each test coupons, as well as the elongation based on a 50.8 mm (2 in) gauge length.

Each test assembly consisted of two parallel girder specimens with span lengths of 3048 mm (120 in). Two 914.4 mm (3 ft) long cross-member beams that framed into the girder webs through hot-rolled steel angle brackets linked the girders. A single row of #12 self-drilling screws connected each angle bracket to the girder web, defining the two lines of load application. The lines of loading were spaced 813 mm (32 in) apart.

The cross members were formed from 254 mm (10 in) × 12-gauge back-to-back C-section joist members. A load distribution beam spanned between these cross members. Load was applied at the centre of the load distribution beam, loading each cross members equally and creating a region of constant bending moment between the two lines of load application on both specimens. The distance between the girder support and the line of loading was 1118 mm (44 in). An Enerpac hydraulic cylinder was stacked onto a 222 kN (50 kip) capacity interface load cell placed at the centre of the load distribution beam. The hydraulic cylinder

was activated with an electric pump, and reacted against the horizontal crosshead of the reaction frame. Mid-span deflections of each girder specimen were monitored using linear potentiometer displacement transducers. All measurements were recorded using a digital data acquisition system. Load was applied continuously until the specimen displayed increased deflection with no increase in loading. The peak load measured was defined as the test load for each specimen assembly.

The flexural capacity of the built-up girder was determined to investigate the influence of screw connection variation. It was found that the nominal moment capacity from the test was significantly less than the capacity calculated by adding the individual moment capacity of the CSS stud and track sections, as shown in Table 2.2.

Table 2.2 Test results (Beshara and Lawson, 2002)

Test Description	Test No.	Test Load N (lbs)	Tested Moment kN.m (lb.in)	Stud Nominal Moment kN.m (lb.in)	Track Nominal Moment kN.m (lb.in)	AISI Nominal Total Moment kN.m (lb.in)	M_{test}/M_{calc}
CSS stud + rim-track-flange screws (loading stud side)	1	63058 (14176)	17.618 (155937)	15.560 (137723)	6.678 (59108)	22.239 (196831)	0.79
	2	61145 (13746)	17.084 (151203)	15.560 (137723)	6.678 (59108)	22.239 (196831)	0.77
	Average	62101 (13961)	17.351 (153570)	15.560 (137723)	6.678 (59108)	22.239 (196831)	0.78
CSS stud + TSB track-flange screws (loading stud side)	1	62315 (14009)	17.411 (154101)	15.560 (137723)	6.142 (54360)	21.702 (192083)	0.80
	2	62653 (14085)	17.506 (154939)	15.560 (137723)	6.142 (54360)	21.702 (192083)	0.81
	Average	62484 (14047)	17.458 (154520)	15.560 (137723)	6.142 (54360)	21.702 (192083)	0.80

Test Description	Test No.	Test Load N (lbs)	Tested Moment kN.m (lb.in)	Stud Nominal Moment kN.m (lb.in)	Track Nominal Moment kN.m (lb.in)	AISI Nominal Total Moment kN.m (lb.in)	M_{test}/M_{calc}
CSS stud + rim-track lip screw (loading stud side)	1	61986 (13935)	17.318 (153280)	15.560 (137723)	6.678 (59108)	22.239 (196831)	0.78
	2	60723 (13651)	16.966 (150164)	15.560 (137723)	6.678 (59108)	22.239 (196831)	0.76
	Average	61354 (13793)	17.142 (151722)	15.560 (137723)	6.678 (59108)	22.239 (196831)	0.77
CSS stud + TSB track-lip screws (loading stud side)	1	59317 (13335)	16.573 (146681)	15.560 (137723)	6.142 (54360)	21.702 (192083)	0.76
	2	60789 (13666)	16.985 (150331)	15.560 (137723)	6.142 (54360)	21.702 (192083)	0.78
	Average	60055 (13501)	17.411 (154101)	15.560 (137723)	6.142 (54360)	21.702 (192083)	0.77
CSS stud + TSB track-lip screws (loading track side)	1	56701 (12747)	15.843 (140219)	15.560 (137723)	6.142 (54360)	21.702 (192083)	0.73
	2	58810 (13221)	16.431 (145430)	15.560 (137723)	6.142 (54360)	21.702 (192083)	0.76
	Average	57756 (12984)	16.138 (142824)	15.560 (137723)	6.142 (54360)	21.702 (192083)	0.74

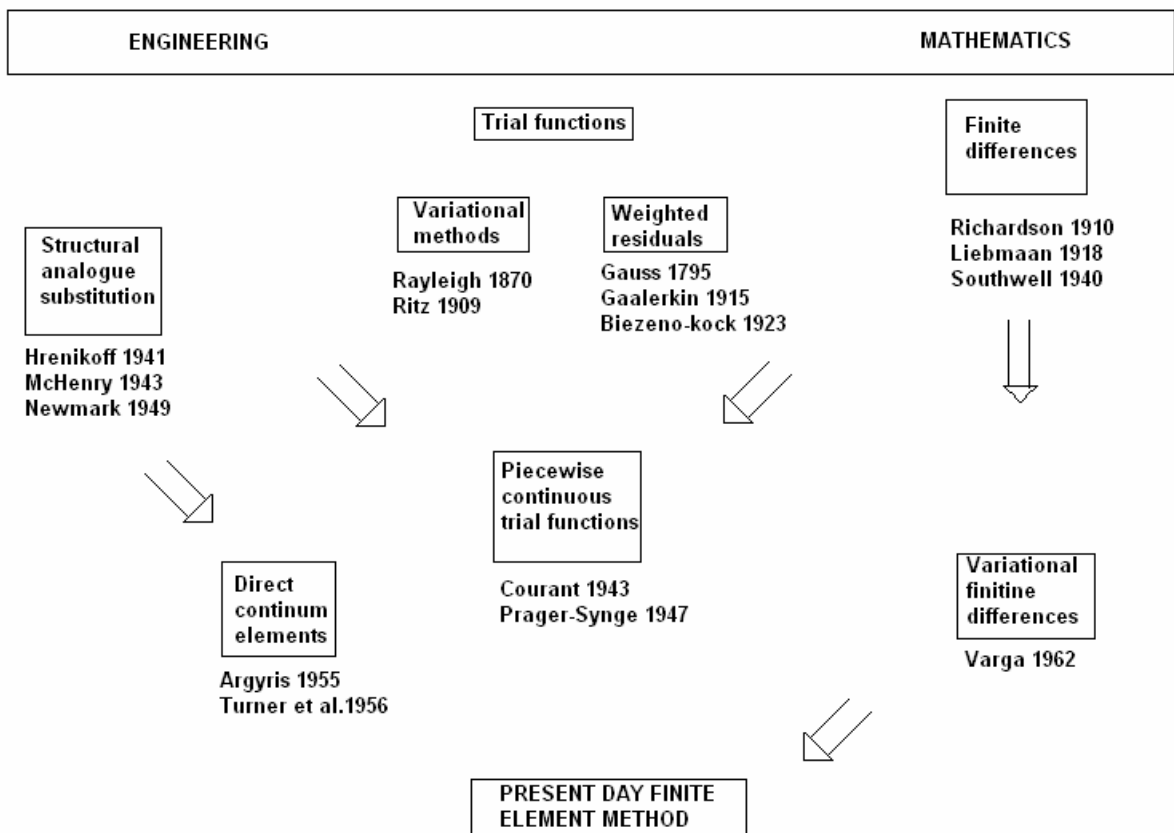
Based on the results of this test series, Beshara and Lawson (Beshara and Lawson, 2002) recommended that the nominal moment capacity of a built-up girder section should be considered equivalent to 75% of the combined nominal capacities of its component members.

2.6 Finite element analysis of thin-walled steel sections

The finite-element method (FEM) originated from the needs for solving complex, structural analysis problems in civil, mechanical and aeronautical engineering. Although much of the mathematical foundation of finite element methods was laid during the 1940's and 1950's, the method began to establish itself as one of the standard tools for modelling and analysis of

physical systems in the 1960's. The development and accessibility of powerful computers during 1960's made it feasible to develop computer-based methods to perform these analyses. It is difficult to determine the origins of the finite element method and the precise moment of its invention. Table 2.3 shows the process of evolution which led to the present concept of the finite element method (Zienkiewicz, 1977).

Table 2.3 Family tree of finite element method (Zienkiewicz, 1977)



Today, finite element analysis is widely used in cold-formed steel research and design. Following on more than 35 years of intense efforts, research continues towards developing robust finite elements for the linear, buckling, geometric nonlinear and material nonlinear

analysis of plate and shell structures. As cold-formed steel sections are subjected to local buckling, web crippling, distortional buckling, and flexural and lateral torsional buckling, depending on different shapes and dimensions, special care has to be taken during modelling to simulate their exact behaviour. A thorough literature review has been conducted to find applications of the finite element method to thin-walled structures (Rasmussen and Hancock, 2000; Sarawit et al., 2003; Chou et al. 2000; Zhang et al., 2007; Hancock, 2003; Shanmugam and Dhanalakshmi, 2001; Graciano and Casanova, 2005; Tryland et al., 2001). Though no paper was found about finite element modelling of built-up beam sections, several papers were found about finite element modelling of cold-formed steel sections. Some of these papers provide very important information about modelling, element selection, mesh density and highly nonlinear simulation techniques.

In 1994, Moreyra and Pekoz (Moreyra and Pekoz, 1994) carried out a finite element analysis of lipped channel flexural members. Their model of the C-section used nine node shell elements and incorporated nonlinear material and geometric properties. Though their primary objective was to investigate the strength of these members, their numerical model also served to highlight problems with the design procedures at that time.

Extensive numerical modelling was carried out by Shafer in 1997 (Shafer, 1997). In this study three specific problems were investigated in detail: a stiffened element with multiple longitudinal intermediate stiffeners in compression, a stiffened element with a longitudinal intermediate stiffener under a stress gradient, and a flexural member with edge stiffened flanges. An ultimate strength design approach was proposed based on the elastic local distortional and overall buckling modes.

Finite element simulations of cold-formed steel C and Z- section beams were done by Yu and Schafer (Yu and Schafer, 2007). A nonlinear finite element model was developed to simulate local and distortional buckling failure of beams using ABAQUS. The beams were modelled using shell elements. Initial geometric imperfections and material nonlinearities were incorporated in the model, but residual stress was not considered. The developed finite element model showed good agreement with the test data for ultimate bending strength. The model was also applied to investigate the effect of moment gradient on distortional buckling. An empirical equation was developed for use in design to predict the increase in the elastic distortional buckling moment due to change in moment gradient.

The finite element method has also been used widely to investigate web crippling behaviour of cold-formed steel sections. Wei-Xin Ren, Shen-En fang and Ben Young (Ren et al., 2006a) carried out a finite element simulation of cold formed steel channels subjected to web crippling under end one flange loading (EOF) and interior one flange loading (IOF) conditions. They also carried out another finite element study to predict the ultimate strength of channel sections under both pure bending and combined bending and web crippling (Ren et al., 2006b). ANSYS was used for the numerical analysis. A 3D model was developed considering material and geometric nonlinearities. Their finite element model of unstiffened channel sections used 4 node shell elements with six degrees of freedom at each node. This shell element is suitable to model thin to moderately thick structures with large deflection, large rotation and large strain nonlinear capabilities. The developed finite element models for web crippling capacity were then verified against the test results and used for an extensive parametric study of different channel dimensions. Modified design formulas based

on plastic mechanisms were also proposed to calculate web crippling strengths within an accepted safety margin.

The finite element results for pure bending and combined bending and web crippling were in good agreement with the experimental results in terms of ultimate loads and moments, failure modes and web load deformation curves. The verified finite element model was used for extensive parametric studies and it was shown that the interaction equations for combined bending and web crippling specified in the North American Specifications are generally conservative for cold-formed steel channels with web slenderness ranging from 7.8 to 108.5.

Chapter 3

Finite Element Modelling

3.1 The finite element method

The finite element method is a numerical technique of solving differential equations describing a physical phenomenon. It is a convenient way to find displacements and stresses of structures at definite physical coordinates called nodes. The structure to be analyzed is discretized into finite elements connected to each other at their nodes. Elements are defined and equations are formed to express nodal forces in terms of the unknown nodal displacements, based on known material constitutive laws. Forces and initial displacements are prescribed as initial conditions and boundary conditions. A global matrix system is assembled by summing up all individual element stiffness matrices and the global vector of unknown nodal displacement values is solved for using current numerical techniques.

3.2 Reasons for finite element study

Until now no research papers are to be found in the literature about the ultimate moment capacity of the cold formed steel (CFS) built-up box girders discussed in chapter 2. The experiments conducted by Dietrich Design Group (Beshara and Lawson, 2002) that were presented in the previous chapter are not sufficient to develop a comprehensive model of the behaviour of the built-up box girder assembly. Numerous physical experiments are required to develop and verify any newly proposed design procedure. With the availability of powerful computers and software, the finite element method is an excellent tool to investigate the behaviour of engineering structures. The accuracy of the results mainly

depends on the finite element model, application of load and the boundary conditions. The model and the boundary conditions should be appropriate to represent the actual member and loading conditions in order to get results close to reality. At present, the finite element method is being used extensively to understand the behaviour of cold-formed steel sections. Finite element analysis has been used to investigate the ultimate moment capacity of flexural members, ultimate load capacity of compression members, web crippling capacity, local, torsional, flexural and distortional buckling modes of different kinds of cold formed steel sections, as discussed in Chapter 2. Many of the papers discussed in Chapter 2 demonstrated that finite element results show very good agreement with test results. Finite element analysis is also more economical than physical testing, especially in parametric studies. So, in lieu of conducting expensive testing, the finite element method has been used in this study to investigate the flexural performance of cold formed steel (CFS) built-up box girders under eccentric loading. In particular, parametric studies have been conducted to determine the factors affecting the moment capacity of CFS built-up box girders.

3.3 Objective of Finite element study

The main objectives of the finite element study are to understand the flexural behaviour and determine the ultimate moment capacity of CFS built-up box sections, and identify the factors that affect the flexural capacity. The established finite element model was first verified with experimental results. A parametric study was then carried out to investigate the influence of section depth, thickness, connection screw spacing and material yield stress on the ultimate moment capacity of CFS built-up box girders.

3.4 General features of finite element modelling

Finite element analysis consists of four steps: creating the geometry of the model, generating a mesh for the solid model (i.e. dividing the model into elements), applying appropriate boundary and loading conditions, and solution. When the meshing of the model and assigning of the material properties are completed, the appropriate load and boundary conditions are applied at the element nodes. Once stiffness equations describing individual elements are constructed, the global stiffness matrix is assembled. The unknown displacement vector to be solved can be symbolically written as follows:

$$\{u\} = [K]^{-1}\{F\} \quad (3.1)$$

where $\{u\}$ is the vector of nodal unknown displacements, $\{F\}$ is the load vector, and $[K]$ is the global stiffness matrix.

Finite element analysis with geometric and material nonlinearities is carried out herein using the software ANSYS (version 10). Details concerning the modelling are discussed in the following sections.

3.4.1 Finite Elements used in FE model

The accuracy of the finite element analysis results highly depends on choosing the appropriate elements to predict the actual behaviour of the structure. In this study, shell, link and contact elements were used to model the CFS built-up box girder.

3.4.1.1 Shell element

Cold-formed steel sections are very thin, with high width to thickness ratio. The ultimate strength of the CFS section is mainly governed by the post buckling strength. The Shell181

element is the most advanced shell element in the ANSYS element library, and it was determined to be the best choice to model the stud and track section. This element is suitable for analyzing thin to moderately thick shell structures. It is a four node element with six degrees of freedom at each node: translations in the x, y and z directions and rotations about the x, y, and z axes, as shown in Figure 3.1. Element formulation is based on logarithmic strain and true stress measures. Element kinematics allow for finite membrane strains. It is well suited for linear, large rotation, and/or large strain nonlinear applications. It can account for changes in shell thickness during nonlinear analysis. It supports uniform reduced integration and full integration. For all options, ANSYS uses five points of integration through the thickness of the shell. In this study the uniform reduced integration option was used for better nonlinear analysis performance. The element can be associated with linear elastic, elastoplastic or hyperelastic material properties. Only isotropic and orthotropic linear elastic properties can be input for elasticity. The von Mises isotropic hardening plasticity models can be invoked. Kinematic hardening plasticity and creep material models are not available for this element.

3.4.1.2 Link element

The ANSYS Link8 element is a two node uniaxial tension, compression element with three degrees of freedom at each node: translations in x, y and z direction, as shown in Figure 3.2. Plasticity, creep, swelling, stress stiffening, and large deflection capabilities are included. As in a pin-jointed structure, no bending of the element is considered. The Link8 element has been used to model the bracing connecting two parallel girders by connecting the mid-nodes of top and bottom flanges.

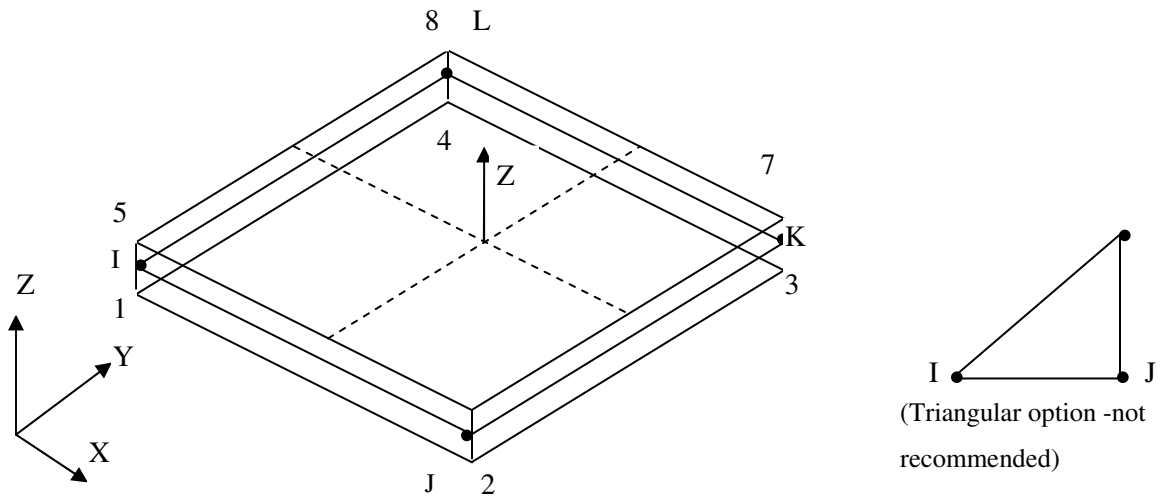


Figure 3.1 Shell181 Finite Strain shell (ANSYS)

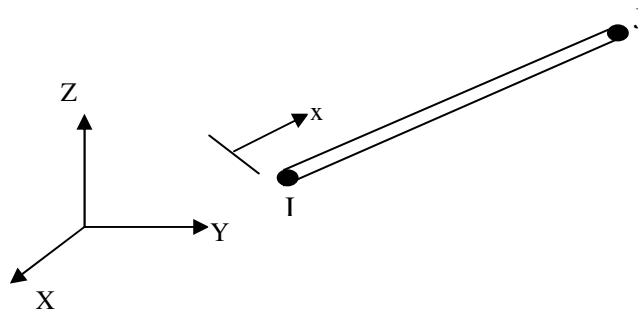


Figure 3.2 link8 element (ANSYS)

3.4.1.3 Contact element

Contact elements have been used to simulate the boundary condition. Contacts are classified as rigid-to-flexible or flexible-to-flexible depending on the relative rigidity between the surfaces in contact. In rigid-to-flexible contact problems, one or more of the contacting surfaces are treated as rigid, which means it has a much higher stiffness relative to the deformable body it contacts. In flexible-to-flexible contact, both (or all) contacting bodies are deformable which means they have similar stiffness. In this study, all the contacts are flexible-to-flexible type contacts. Depending on the geometry and loading of the structure,

contact can be defined as node-to-node, node-to-surface and surface-to-surface. For this model, surface-to-surface flexible-to-flexible contact has been used. In order to define the contact, two types of contact elements are necessary, contact and target. For this model, the two elements used are known in the ANSYS element library as CONTA174 and its target TARGE170.

The CONTA174 element has eight nodes as shown in Figure 3.3. The target element TARGE170 is used to represent various 3D target surfaces for the associated contact elements. This target surface is discretized by a set of target segment elements (TARGE170) as shown in Figure 3.4 and is paired with its associated contact surface via a shared real constant set.

For surface-to-surface contact elements, ANSYS offers several different contact algorithms such as the penalty method, augmented Lagrangian, Lagrange multiplier on contact normal and penalty on tangent, pure Lagrange multiplier on contact normal and tangent, and internal multipoint constraint (MPC). The penalty method uses a contact “spring” to establish a relationship between the two contact surfaces. The spring stiffness is called the contact stiffness. The augmented Lagrangian method (default) is an iterative series of penalty methods. The contact tractions (pressure and frictional stresses) are augmented during equilibrium iterations so that the final penetration is smaller than the allowable tolerance (FTOLN). Compared to the penalty method, the augmented Lagrangian method usually leads to better conditioning and is less sensitive to the magnitude of the contact stiffness. This is the default option in ANSYS and this option is used in this study. For this method, normal and tangential contact stiffnesses are required. The amount of penetration between contact

and target surfaces depends on the normal stiffness. The amount of slip in sticking contact depends on the tangential stiffness. Higher stiffness values decrease the amount of penetration/slip, but can lead to ill-conditioning of the global stiffness matrix and to convergence difficulties. Lower stiffness values can lead to a certain amount of penetration/slip and produce an inaccurate solution. ANSYS provides default values for contact stiffness (FKN, FKT), allowable penetration (FTOLN), and allowable slip (SLTO). For this analysis the default values are used. The contact stiffness was updated automatically after each iteration during the analysis.

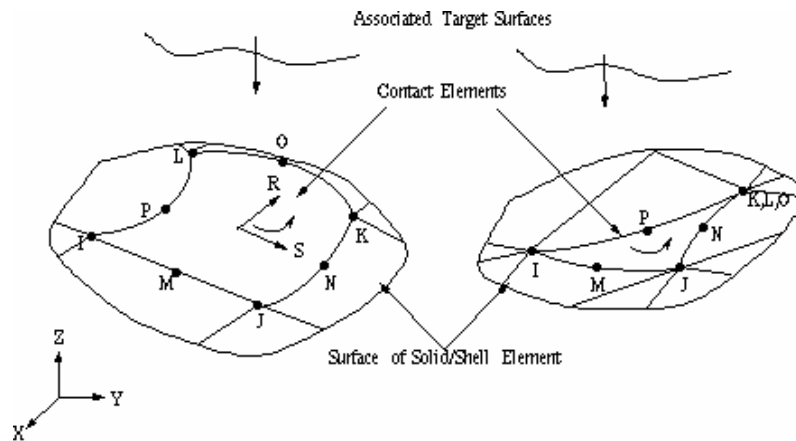


Figure 3.3 CONTA174-3D surface-to-surface contact element (ANSYS)

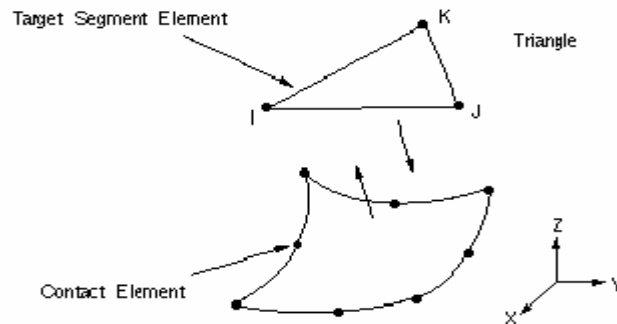


Figure 3.4 TARGE170 target surface element (ANSYS)

3.4.2 Meshing of the model

The model was first developed to simulate the test conducted by Dietrich Design Group (DDG) (Beshara and Lawson, 2002). Instead of simulating the whole test set up, only half of the specimen was modelled taking the advantage of symmetry. The section dimensions were as shown in Figure 3.5 and Table 3.1. The solid geometry of the stud and track sections was modelled as shown in Figure 3.6 and Figure 3.7. After creating the solid geometry, the area was meshed with shell181 elements. A free mesh technique could be used for meshing but it would increase both the number of elements and the computational time. Instead, the model was meshed with two objectives: to create a sufficiently fine mesh to model the essential feature of the deformed shape, and to minimize the number of elements to reduce computation time. The stud sections were perforated sections with holes in the mid depth, as shown in Figure 3.5. The area of the stud section was divided into several areas. The area with holes was meshed using the free mesh command. Near the hole, a fine mesh was created to account for stress concentrations. The maximum size of the shell element was 12.5mm by 10mm. The cold forming process for fabricating members such as C section does not make square corners, but will have some bend radius. The rounded corners avoid manufacturing problems associated with cracking the base steel or metallic coatings along the tension bends, which can occur with sharp corners. The standard inside bend radius is equal to two times the thickness. The finite element model accounted for the corner radius with 3 elements along the bend.

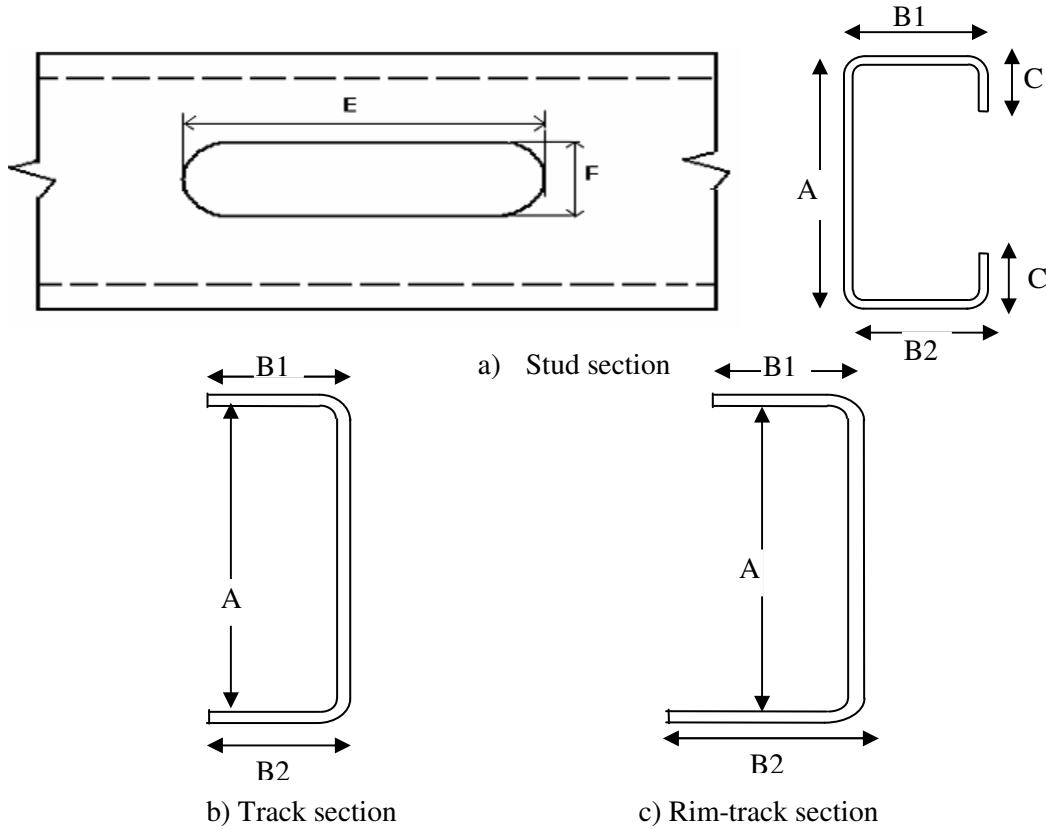


Figure 3.5 Stud and Track section dimension

Table 3.1 Section dimensions (Beshara and Lawson, 2002)

Section	Gauge	Yield stress F_y MPa (ksi)	Thickness mm (in)	Depth A mm (in)	Top Flange B1 mm (in)	Bottom Flange B2 mm (in)	Lip C mm (in)	Hole dimension	
								Depth F mm (in)	Width E mm (in)
Stud section	16	349 (50.6)	1.61 (0.0632)	254 (10)	76.2 (3)	76.2 (3)	25.4 (1)	38.1 (1.5)	101.6 (4.0)
Track section	16	307 (44.5)	1.44 (0.0567)	254 (10)	31.8 (1.25)	31.8 (1.25)			
Rim-track section	16	417 (60.5)	1.39 (0.0547)	254 (10)	31.8 (1.25)	63.5 (2.5)			

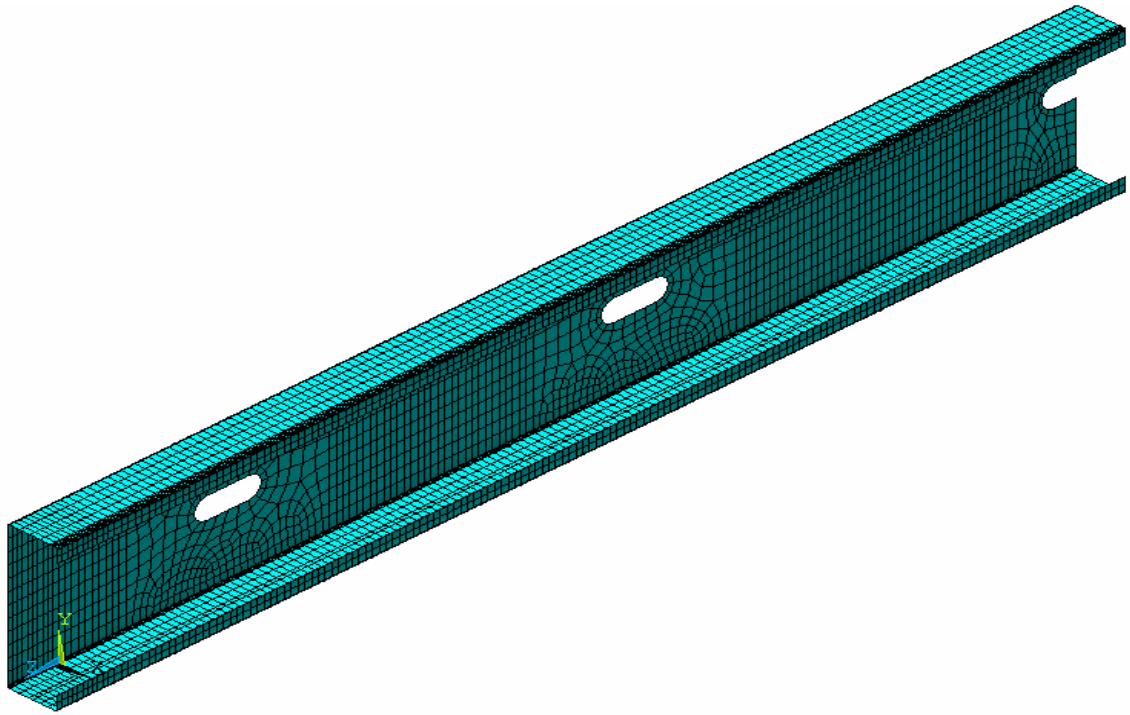


Figure 3.6 Typical mesh of stud section

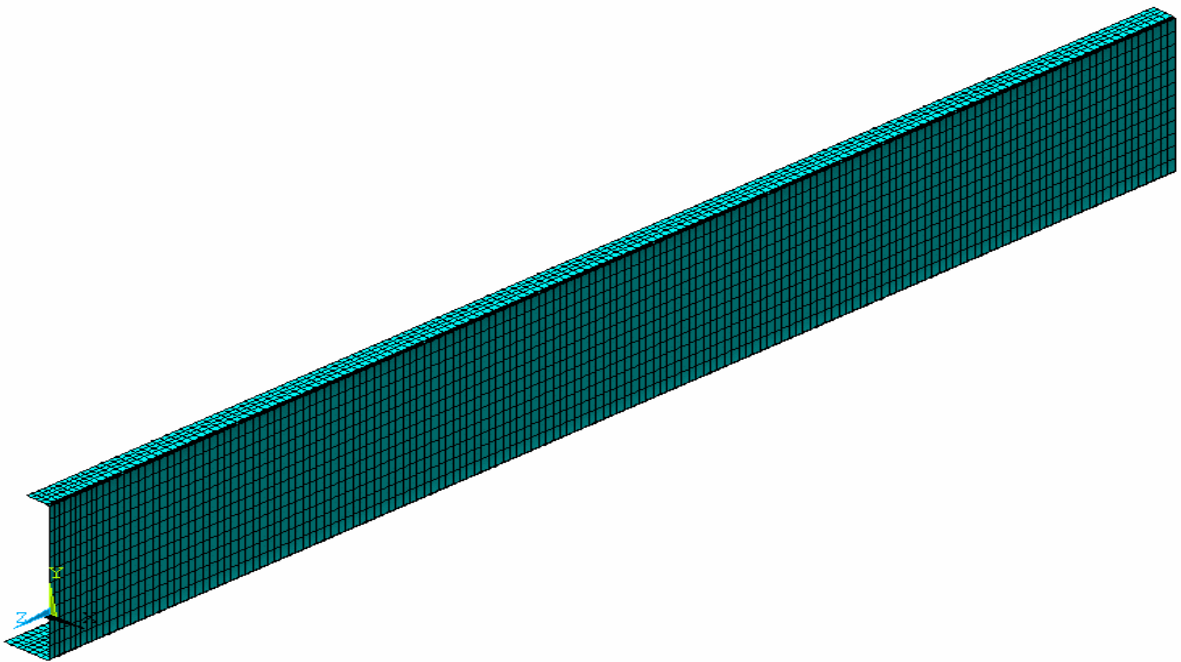


Figure 3.7 Typical mesh of rim-track section

3.4.3 Screw representation

In the experiment, the top and bottom flanges of CFS stud and track sections were connected using self drilling screws with 300mm (12in) spacing. For modelling purposes, the effect of screws has been accounted for by coupling translational and rotational degrees of freedom (DOF) of the nodes in the global x, y and z directions at the exact locations of the screws in the flanges. The size of the screws and the possible failure of the screws are neglected in the FE model.

3.4.4 Modelling End stiffener

In practice and in the test conducted by DDG (Beshara and Lawson, 2002), an end stiffener is used at the support location to avoid web crippling. This was modelled by creating shell181 elements that overlapped the web in the location of the stiffener. Designating it as a contact area and the corresponding area on the web as a target area, a bonded contact was defined to model the influence of the stiffener restraining the deformation of the joist web.

3.4.5 Material properties

Material nonlinearities can be defined as a nonlinear relationship between stress and strain; that is, the stress being a nonlinear function of the strain. Plasticity theories model the material's mechanical response as it undergoes nonrecoverable deformation in a ductile fashion. The ANSYS program can account for many material nonlinearities. Plasticity theory provides a mathematical relationship that characterizes the elastoplastic response of materials. There are three ingredients in the rate-independent plasticity theory, they are: a yield criterion, flow rule and hardening rule. The yield criterion determines the stress level at

which yielding is initiated, the flow rule determines the direction of plastic straining and the hardening rule describes the changing of the yield surface with progressive yielding, so that the conditions (i.e. stress states) for subsequent yielding can be established. Two hardening rules are widely used: work (or isotropic) hardening and kinematic hardening. In work hardening, the yield surface remains centered about its initial centerline and expands in size as the plastic strains develop, whereas kinematic hardening assumes that the yield surface remains constant in size and the surface translates in the stress space with progressive yielding.

Material nonlinearity of cold-formed steel sections has been incorporated by considering a perfectly elastic plastic material obeying von Mises yield criteria. The cold forming process introduces cold work into sections, especially in the corners. As a result, in corner regions the yield stress is increased and the ductility is decreased. The material at corner region may be anisotropic and in addition include residual stresses. Due to lack of test data, addressing these properties in the finite element model is extremely difficult. To make the model simple, the same material properties were adopted for the entire section. As the finite element model was verified with test results found in literature, the material properties were adopted to be consistent with the test report (Beshara and Lawson, 2002). For all the members, young modulus $E = 203000$ MPa (29435 ksi) and Poisson's ratio = 0.3. The yield stress for the stud, rim-track and track sections were taken as 349 MPa (50.6 ksi), 417 MPa (60.5 ksi) and 307 MPa (44.5 ksi), respectively, as mentioned in the test report. Residual stresses were ignored for the model because in this study, the ultimate moment capacity of the CFS built-up box

sections obtained from the FE analysis was compared with the nominal moment of the sections calculated according to CSA-S136-01 without considering the cold work of forming.

3.4.6 Initial Geometric imperfection

Geometric imperfection can be defined as the deviation of a member from its perfect geometry. Imperfections of a member may include bowing, warping and twisting as well as local deviations. Global and local distortion may be present, as a result of the manufacturing process of the plates and due to accidental impacts during the transportation and erection of girders. Initial geometric imperfections influence significantly the ultimate strength of cold-formed steel sections. Chou et al. (Chou et al, 2000) presented a buckling analysis using the finite element method and pointed out that to accurately obtain the ultimate load of a structure that has undergone buckling; the initial geometric imperfections must be considered. Modelling geometric imperfections remains an active research area because of the sensitivity of thin walled structures to the initial geometric imperfection. When precise data for the distribution of geometric imperfections is not available, several modelling approaches have been proposed by Schafer and Pekoz (Schafer and Pekoz, 1998). Initial imperfections can be incorporated into the numerical model by superimposing multiple buckling modes and controlling their magnitudes, using the Fourier transform and spectra. Schafer and Pekoz (Schafer and Pekoz, 1998) also recommended the use of a maximum deviation that is approximately equal to the plate thickness as a simple rule of thumb. DeVille (DeVille, 1996) questioned the conventional procedure of modelling imperfections as a linear combination of factored elastic buckling modes. He suggested that the imperfection be chosen in the shape of the final elastic–plastic collapse pattern.

No measurement was taken to identify the initial geometric imperfection of the CFS sections used in the test conducted by DDG (Beshara and Lawson, 2002). As initial imperfections influence the ultimate load capacity of CFS sections, it is important to consider it in the finite element model. In this study, first eigenvalue buckling analysis was performed on the perfect structure to establish the probable collapse mode using ANSYS. Initial imperfection was incorporated in the model by scaling the first eigenvalue buckling mode shape and then adding it to the perfect geometry such that the maximum imperfection does not exceed the thickness of the section, as proposed by Schafer and Pekoz (Schafer and Pekoz, 1998). Then, a geometrically nonlinear load displacement analysis of the structure containing the imperfection was carried out to determine the ultimate moment capacity.

The response of some structures is highly influenced by the initial imperfections in the original geometry. By adjusting the magnitude of the scaling factors of the various buckling modes, the imperfection sensitivity of the structure can be assessed, but this was not investigated in this study.

3.4.7 Boundary conditions and application of loads

The accuracy of finite element analysis is highly influenced by the simulation of exact boundary conditions. In the test (Beshara and Lawson, 2002), the CFS built-up box girders are laterally restrained at locations 762 mm (30 in) from both end supports. The ends of the beams were not laterally restrained. The top and bottom flanges of the two parallel girders were braced by using stud sections located 152 mm (6 in) and 762 mm (30 in) from the support. The test setup was as shown in Figure 2.6. Instead of modelling the two parallel CFS

built-up box girders, only half of one girder need be modelled with appropriate boundary condition so as to save computational time. Details about boundary conditions are discussed later on in this chapter.

In finite element analysis the loading can be applied in two ways: apply load directly on the model or, instead, impose displacement on the model. In order to simulate the test results, loading was applied in both ways and comparison was made between the results in terms of ultimate moment capacity, load-deformation behaviour, failure modes and stress conditions. In the test setup, the two parallel built-up box girders were connected by two cross girders with clip angles. Six self drilling screws were used for this connection. Thus, the web of stud sections first received the load as shown in Figure 3.8. In the FE analysis when loading was applied directly, a 2890 N (650 lb) load was applied vertically downward on each node at the location of screws attaching the built-up box girder and cross girder as shown in Figure 3.9(a). The loading was applied incrementally by defining the initial load as 347 N (78 lb), with a maximum and minimum load increment of 867 N (195 lb) and 2.9 N (0.65 lb), respectively. When loading was applied as controlled displacement, a 17.78 mm (0.7 in) vertical downward displacement was applied incrementally by defining the initial displacement as 0.35 mm (0.014 in), with a maximum and minimum displacement increment of 1.7 mm (0.07 in) and 0.0003 mm (0.0000145 in), respectively, as shown in Figure 3.9(b).

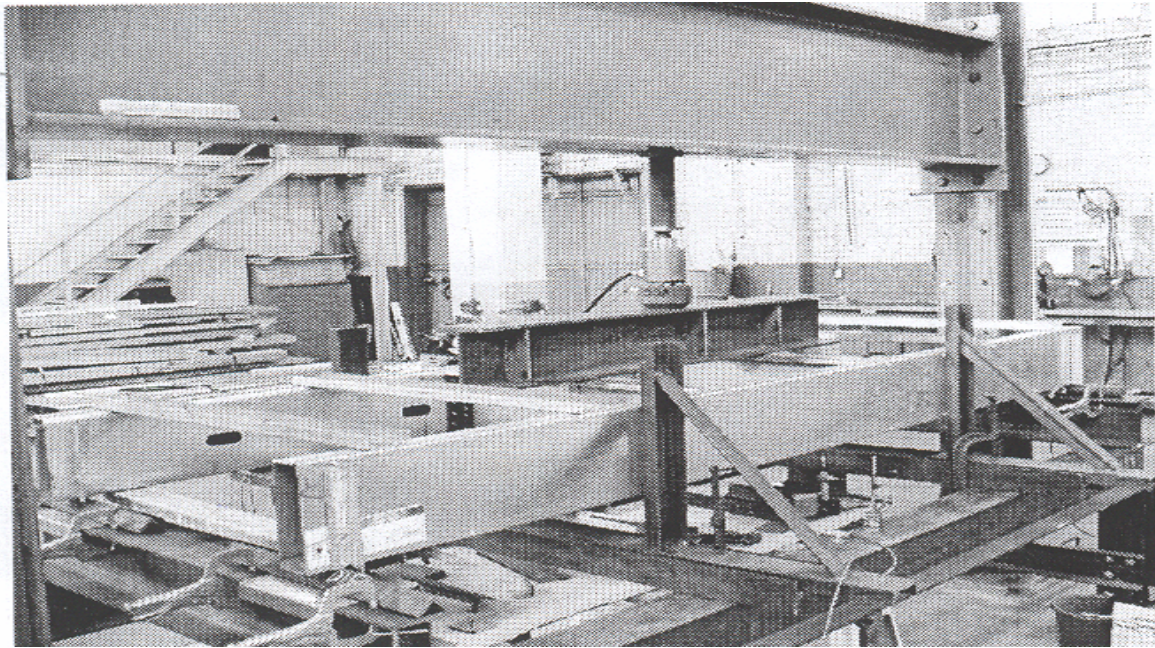
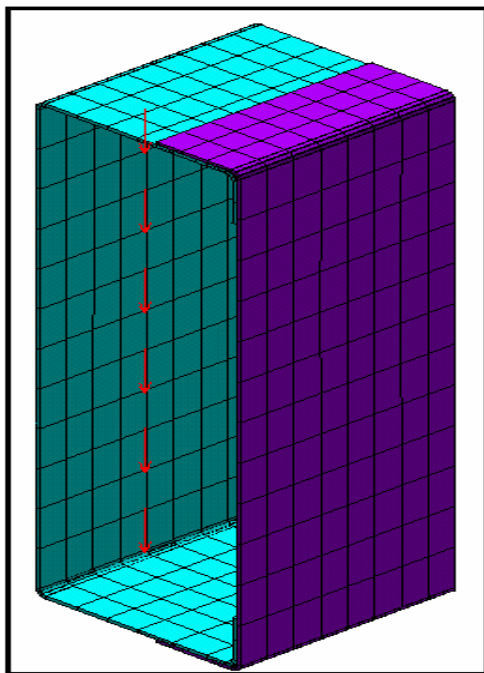
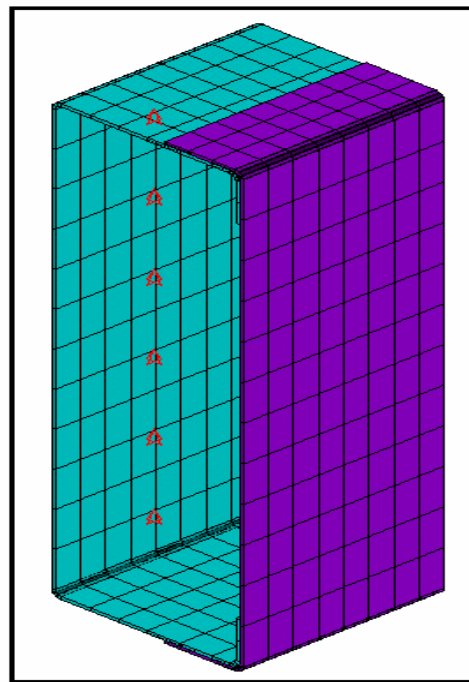


Figure 3.8 Built-up girder test assembly (Beshara and Lawson, 2002)



a) Applied load as force



b) Load applied as controlled displacement

Figure 3.9 Application of load as force or displacement in finite element model

3.4.8 Solution

A large displacement static analysis considering geometric nonlinearities was carried out to determine the ultimate load carrying capacity of the CFS built-up box girders. The stress stiffening effect was included to simulate the local buckling behaviour. Stress stiffening (also called geometric stiffening, incremental stiffening, initial stress stiffening, or differential stiffening) is the stiffening (or weakening) of a structure due to its stress state. This stiffening effect normally needs to be considered for thin structures with bending stiffnesses that are very small compared to axial stiffness, such as cables, thin beams, and shells. This effect also augments the regular nonlinear stiffness matrix produced by large strain or large deflection effects. The effect of stress stiffening is accounted for by generating and then using an additional stiffness matrix, called the “stress stiffness matrix”. The stress stiffness matrix is added to the regular nonlinear stiffness matrix in order to obtain the total stiffness. If membrane stresses become compressive rather than tensile, then terms in the stress stiffness matrix may cancel the positive terms in the regular stiffness matrix and therefore yield a nonpositive-definite total stiffness matrix, which indicates the onset of buckling.

In this study, the Newton-Raphson method was used for solution. The Newton-Raphson solution uses the tangent modulus stiffness corresponding to the previous iteration to calculate the next deformed position. The stiffness continues to be updated until the difference between the load step magnitude and the projected position is within some acceptable tolerance.

A nonlinear analysis requires multiple substeps within each load step so that ANSYS can apply the specified loads gradually and obtain an accurate solution. To apply load (force or

displacement) gradually, a number of substeps was defined in the ANSYS “Solution Controls” dialogue box. The load (Force or displacement) was applied in one step with 50 initial substeps. By default the “Solution Control” is on, which means that ANSYS will monitor how many iterations are required to converge at that substep. If convergence is deemed to be difficult, it will automatically reduce the next increment of load (force or displacement) it takes and add more substeps with the automatic time stepping on options. Minimum and maximum numbers of substeps were equal to 10 and 48000, respectively, when the load was applied as controlled displacement. When the load was applied directly, the minimum and maximum substeps were equal to 20 and 6000 respectively. A default convergence criterion for both force and moment was used for all solutions.

3.5 Investigation of lateral restraint provided by bracing

The ultimate moment capacity of a CFS box girder is highly influenced by the lateral restraint of the girder. Though the box girder was not laterally restrained at the support, the adjacent bracing may have provided restraint to the lateral movement. In order to investigate the effect of bracing, the first half of the test set up was modelled as shown in Figure 3.10. The mid-nodes of both top and bottom flanges of the two parallel girders were connected by Link8 elements at the location of the bracing. The mid-nodes of both the top and bottom flanges located 762 mm (30in) away from end supports were laterally restrained. Only half of one CFS box girder was modelled by constraining the mid-nodes of both the top and bottom flanges in the lateral direction at the location of bracing, as shown in Figure 3.11. A symmetrical boundary condition was defined by setting the translational DOF equal to zero in the direction perpendicular to the symmetry plan ($U_z = 0$) and setting the rotational DOF

equal to zero along the symmetry plan ($Rot_x = 0$ and $Rot_y = 0$) at all the nodes on the symmetry plan, as shown in Figure 3.10 and Figure 3.11. For both cases the translational DOF in the vertical direction ($U_y = 0$) was restrained for all the nodes of the bottom flanges of both the track and stud sections at the support location. In practice, the stud and track sections can not penetrate each other during deformation. Flexible- to-flexible standard contact was used along the overlap between stud and track sections to prevent the penetration of one through the other. Default contact settings in ANSYS were used for all types of contact pairs.

For this case, load was applied as controlled displacement. Nonlinear static analysis was performed considering the material nonlinearity and geometric nonlinearity. Initial geometric imperfection was not considered for simplification.

Load versus mid-span deflection curves were compared for both the cases shown in Figure 3.10 and Figure 3.11. From Figure 3.12, it is clear that the two models predict the same load deflection pattern and also the same ultimate load capacity. From this result, it can be concluded that the bracing is providing full lateral restraint of the two parallel CFS box girders. So, instead of modelling two parallel CFS box girders, only one box girder was modelled by restraining the lateral translational DOF ($U_x = 0$) of mid-nodes of top and bottom flanges at the bracing location.

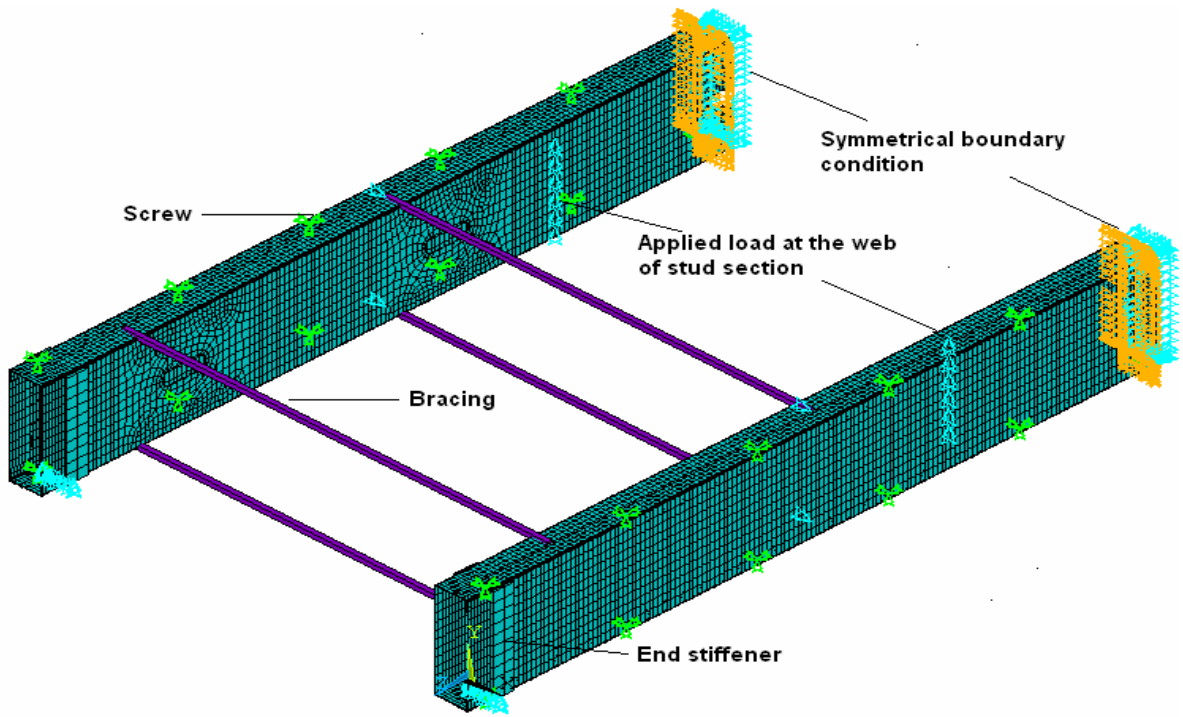


Figure 3.10 Finite element model (Full model)

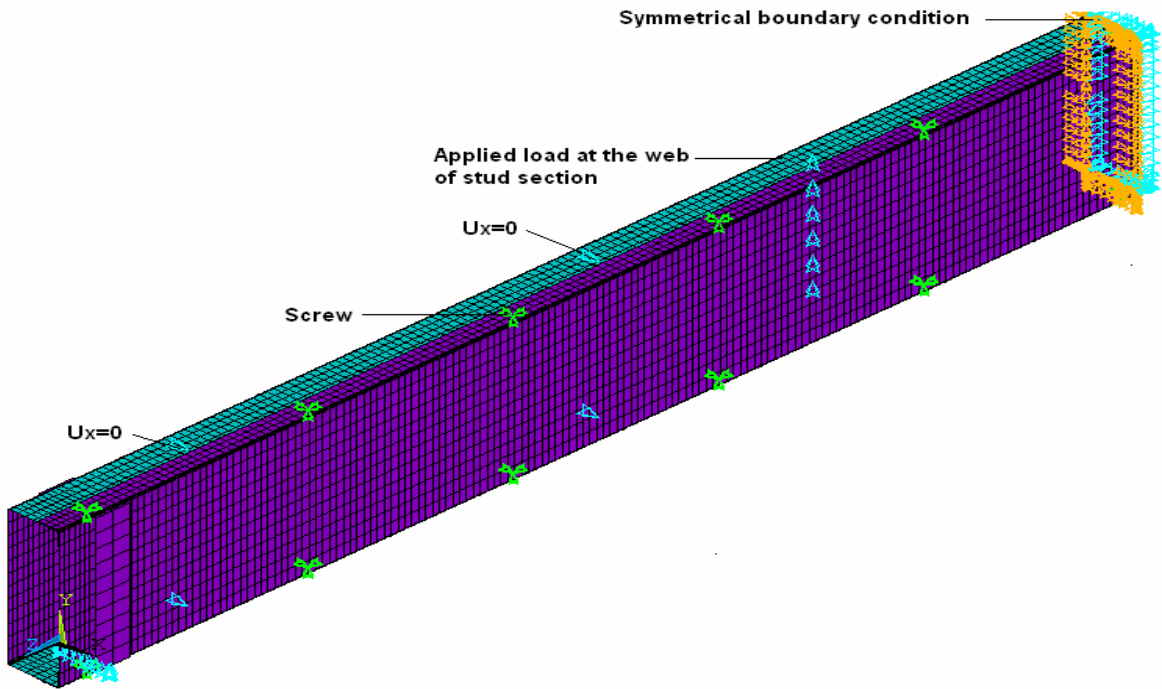


Figure 3.11 Finite Element model (Half model)

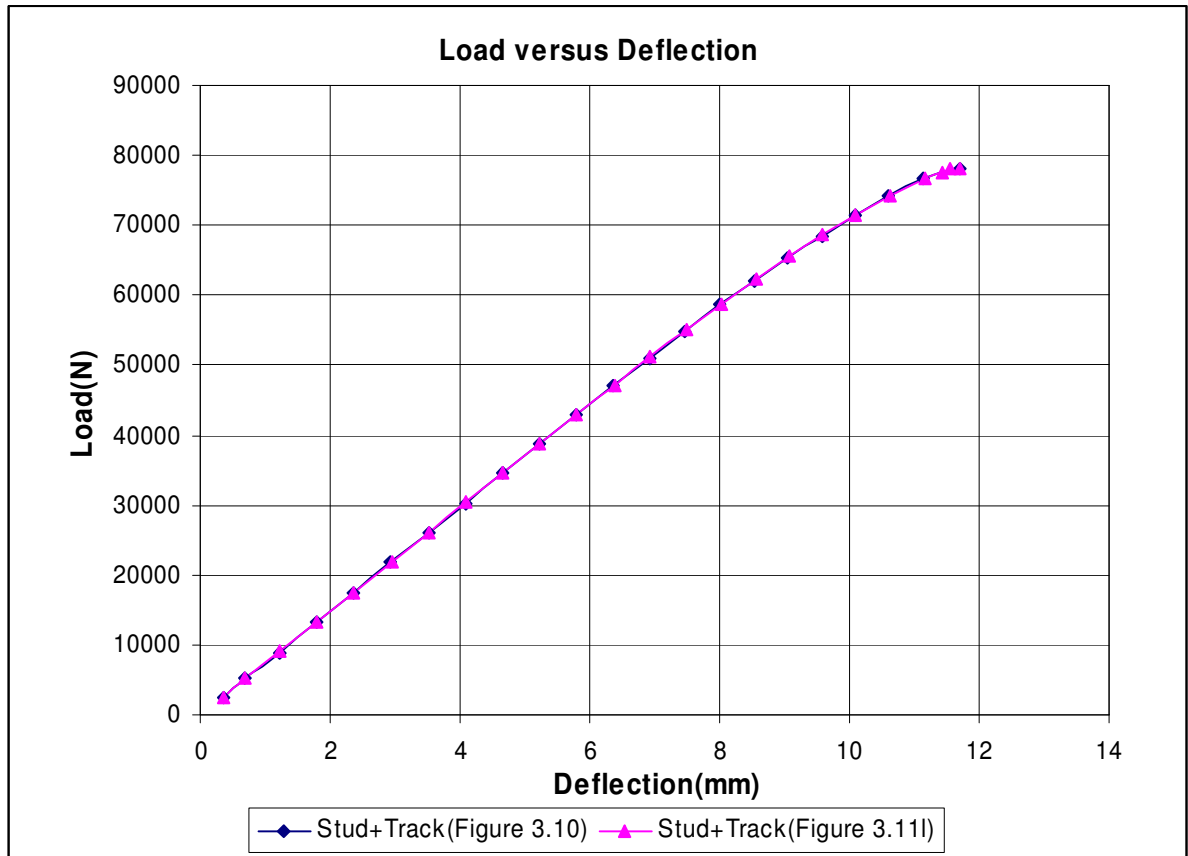


Figure 3.12 Load versus mid-span deflection curve

3.6 Finite element modelling for simulating test

In order to simulate the test results conducted by DDG, (Beshara and Lawson, 2002), only half of one CFS built-up box girder was modelled by using the symmetric boundary conditions. As described in section 3.5 in this chapter, the FE model of the CFS built-up box girders were laterally restrained by constraining the mid-nodes of both top and bottom flanges in the lateral direction ($U_x = 0$) at the location of bracing. Finite element results are highly influenced by appropriate boundary conditions. Simulating exact test boundary conditions is a great challenge. In the tests the CFS box girder was on top of an inverted

angle at one end and a roller on the other end to create the simply supported condition shown in Figure 3.13(a). There was no bearing plate at the support.

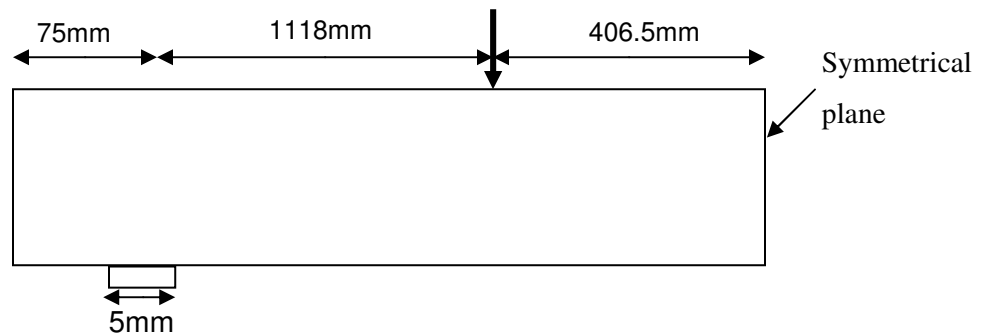
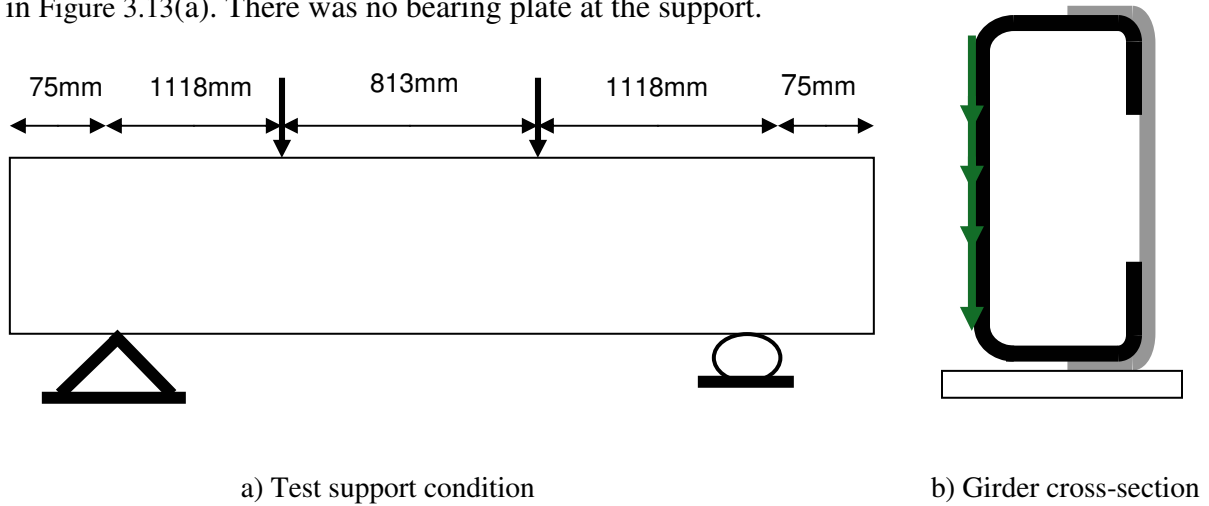


Figure 3.13 End support condition

Due to the cross sectional shape of the built-up section, only the track section was in touch with the support before application of the load (Figure 3.13(b)). The stud section may come in contact with the support during the application of the load. In order to simulate the exact boundary condition, on one end, a plate was modelled with a very small width (5mm) as shown in Figure 3.13(c). The plate was modelled as a 2D surface and then meshed it with shell181 elements as shown in Figure 3.14. The CFS box girder was placed upon it as in the

test. Bonded flexible- to-flexible contact was defined between the plate and the track section. As the stud section can come in contact with the support during application of load, standard flexible-to-flexible contact was also defined between the stud and support plate. The translational DOF of all the nodes of the plates at the support location was restrained in the vertical direction ($U_y = 0$). Geometric imperfections were incorporated by updating the geometry using a scaled first eigenvalue deformed shape so that maximum imperfection does not exceed the thickness of the section. Load (force or controlled displacement) was applied in the same way as discussed in the section 3.4.7 of this chapter. The first eigenvalue buckling shape used as initial geometric imperfection was shown in Figure 3.15. After incorporating the initial geometric imperfections, nonlinear static analysis was performed considering the material nonlinearities and geometric nonlinearities.

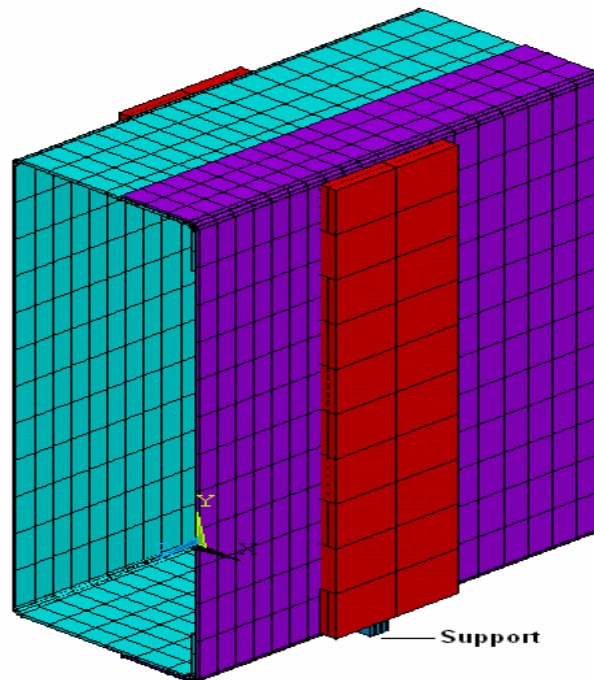


Figure 3.14 Support condition of FE model for simulating test

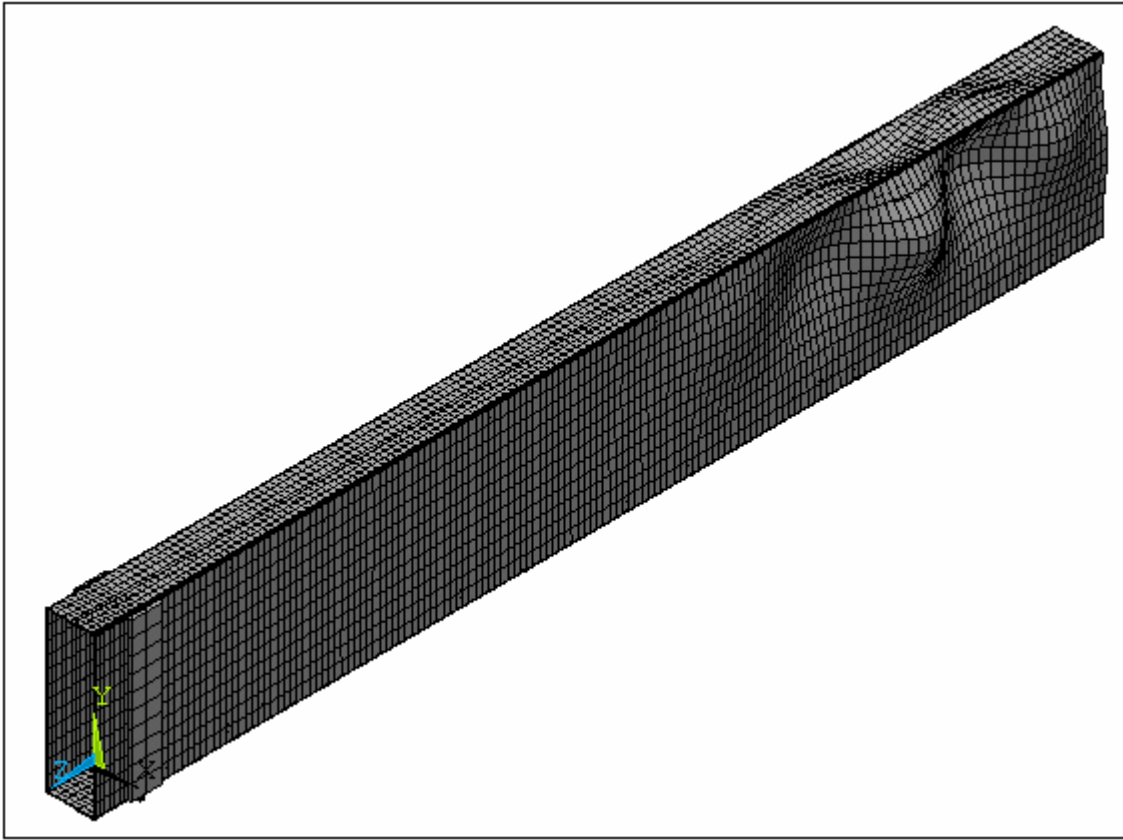


Figure 3.15 Initial geometric imperfection plotted in large scale (35 times)

3.7 Determining ultimate moment capacity from FEM

In order to determine the ultimate load carrying capacity, the load (Force or controlled displacement) was applied in very small increments as discussed in section 3.4.7 of this chapter. Based on the load deflection curves shown in Figure 3.16 and Figure 3.17, it was concluded that the model could not predict the behaviour after reaching the ultimate load capacity due to convergence problems even for very small increments of loading (force or displacement). Convergence problem could not be overcome even using Riks solution method and refining the mesh near the support.

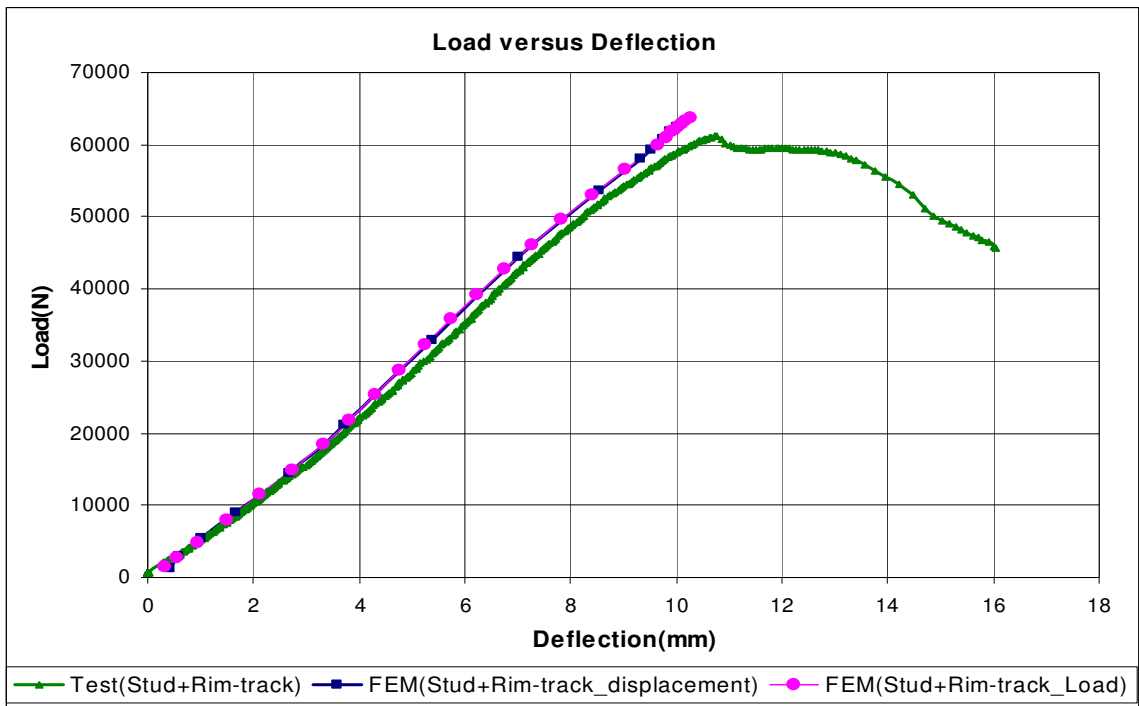


Figure 3.16 Load versus mid-span deflection curve for stud plus rim-track section

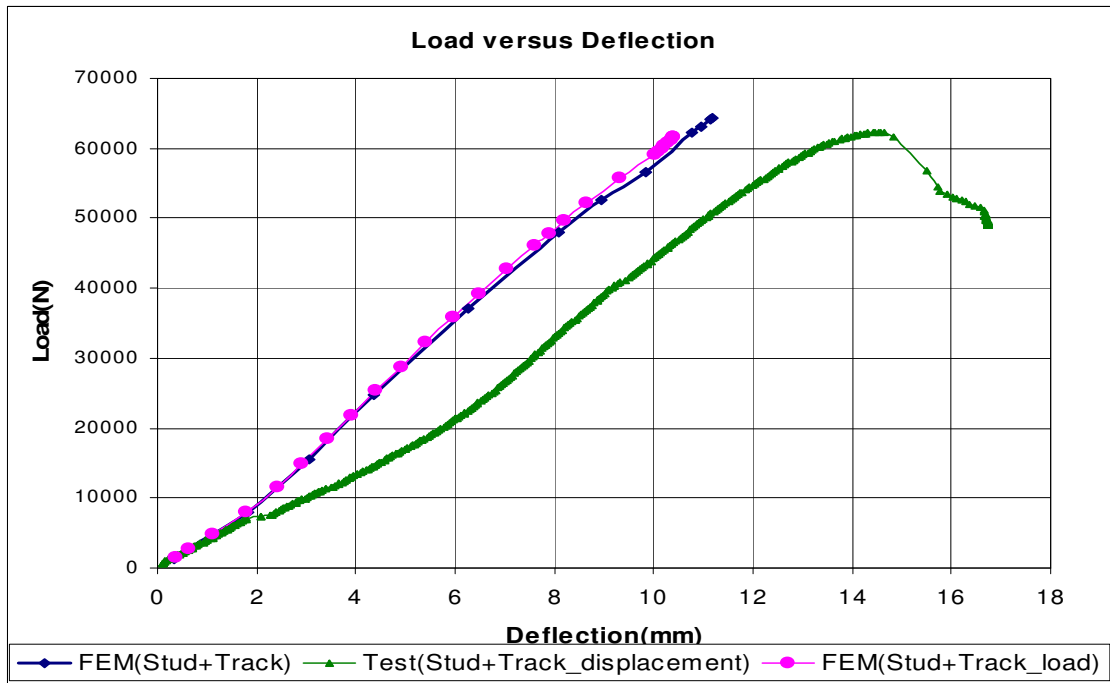


Figure 3.17 Load versus mid-span deflection curve for stud plus track section

Upon investigating the stress and strain condition of the last converged solution, it was found that the material starts yielding in the stud and track sections near support locations shown in Figures 3.18 and 3.19 as the stress to yield stress ratio is equal to unity. Figure 3.20 shows the von Mises total strain, where the maximum strain is equal to 0.319. For the stud sections, the percentage elongation reported in the material coupon test (Beshara and Lawson, 2002) was 27%. So it can be concluded that the CFS box girder has already reached its ultimate load capacity due to material failure at the support location at the last converged solution. As such, the result of the last converged solution was taken as the ultimate load capacity of the CFS built-up box girder.

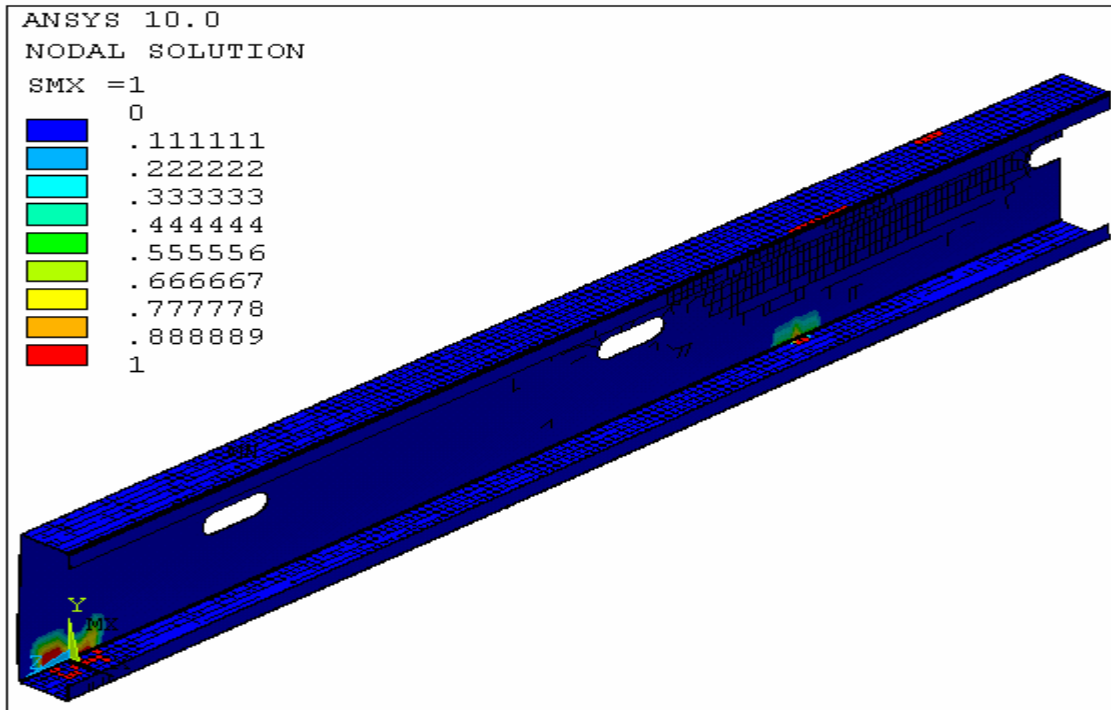


Figure 3.18 Stress state ratio of stud section at ultimate moment capacity (stud plus track assembly)

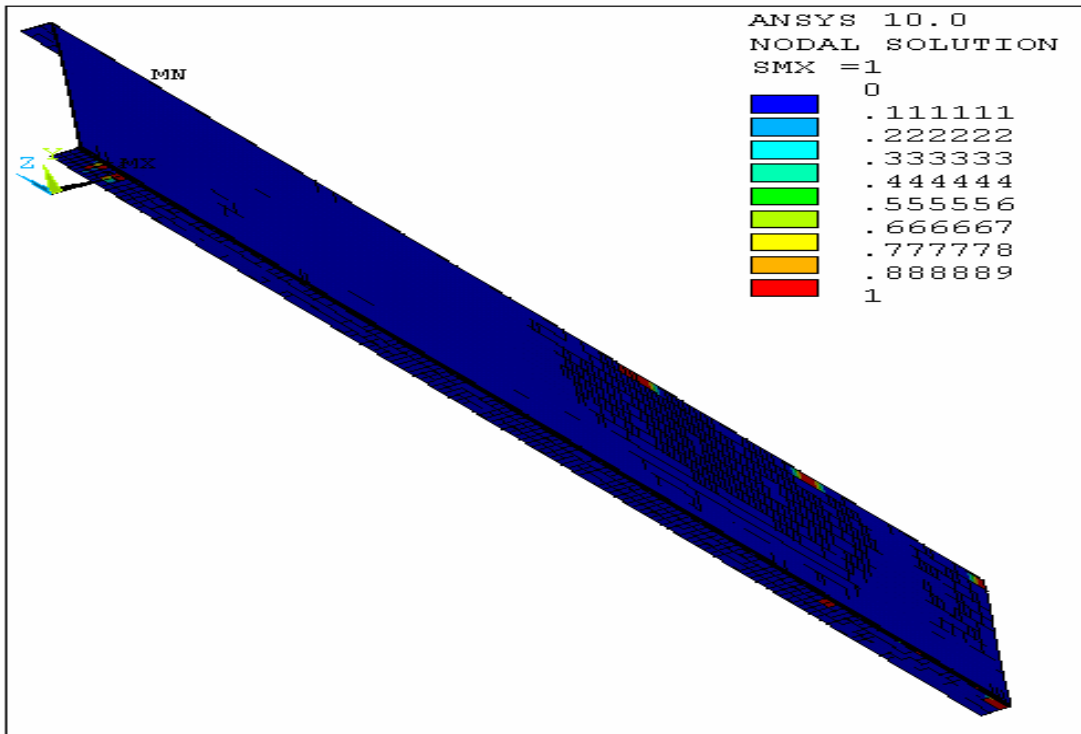


Figure 3.19 Stress state ratio of track section at ultimate moment capacity (stud plus track assembly)

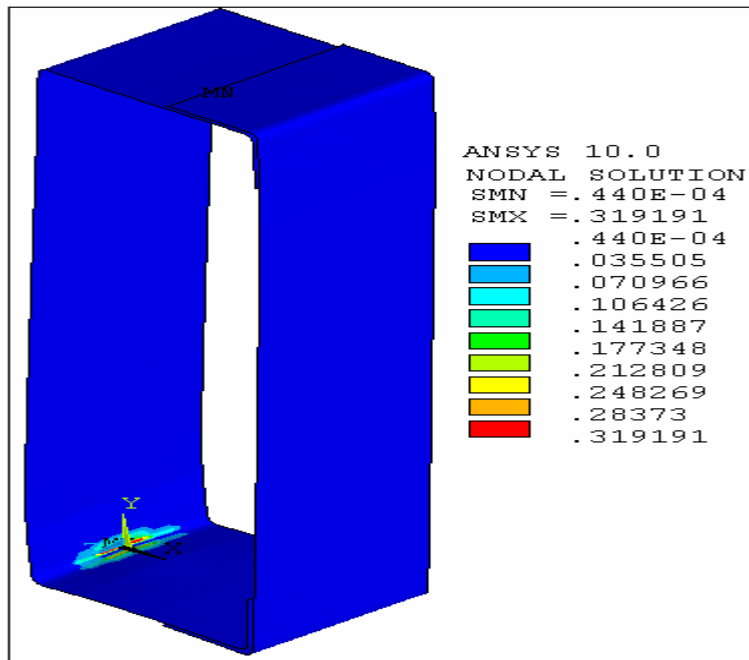


Figure 3.20 von Mises total strain at ultimate moment capacity

3.8 Verifying finite element model

It is very important to be sure that the finite element model is giving reasonable results and can predict the actual behaviour of the structure. The best way to verify the accuracy of the model is to compare the results with test results. The results from finite element simulation were compared with the ultimate moment capacity, deformed shape and load deflection curve obtained from the tests (Beshara and Lawson, 2002). For both of the track assemblies, the ultimate load carrying capacity is almost equal to the test values, as shown in Table 3.2, when the load was applied directly or as controlled displacement. The failure pattern is also similar to the test failure pattern. The upper flange of track section was observed to ripple prior to local flange buckling failure just like the test, as shown in Figures 3.21 and 3.22. The valleys of the ripples were seemed to coincide with the screw locations. Local buckling of the flanges of the track sections in the constant moment region was observed in the model like the tests, as shown in Figures 3.23 to 3.25. The von Mises stress for both type of assemblies are shown in Figures 3.24 through 3.31. These figures indicate that the von Mises stress in the flanges of both stud and track sections reached the material yield stress level in the constant moment region and also at the support locations for both types of loading (force or displacement). Distortion of the built-up girder sections was observed as a consequence of the applied load, as shown in Figure 3.32. The load versus mid-span deflection curves shown in Figures 3.16 and 3.17 have been compared with the test curves. For the stud plus rim-track section, the FE analysis initially predicts the exact same shape as the test. For the stud plus track section, the FE analysis gave a higher stiffness than the test results. Though the model deflection is less than the test value, the ultimate load was almost equal for both track

assemblies. The deflection curves represent well the actual behaviour of the sections. The curves are straight for a very short time and after that they become nonlinear. The curve shows a softer response initially due to the distortion of the girder cross sections. Figures 3.16 and 3.17 also indicate that the load-deflection curve follows the same path for the FE model whether the load is applied directly as force or as a controlled displacement. The ultimate moment capacities for the two cases differ by less than 3%.

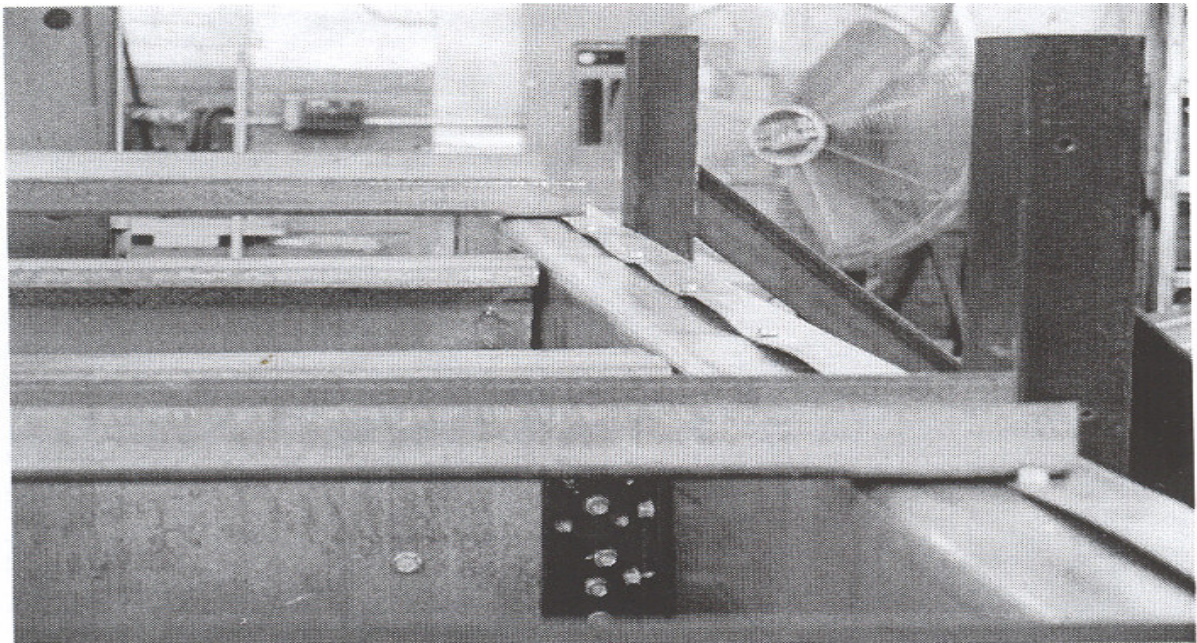


Figure 3.21 Rippled upper compression flange (Beshara and Lawson, 2002)

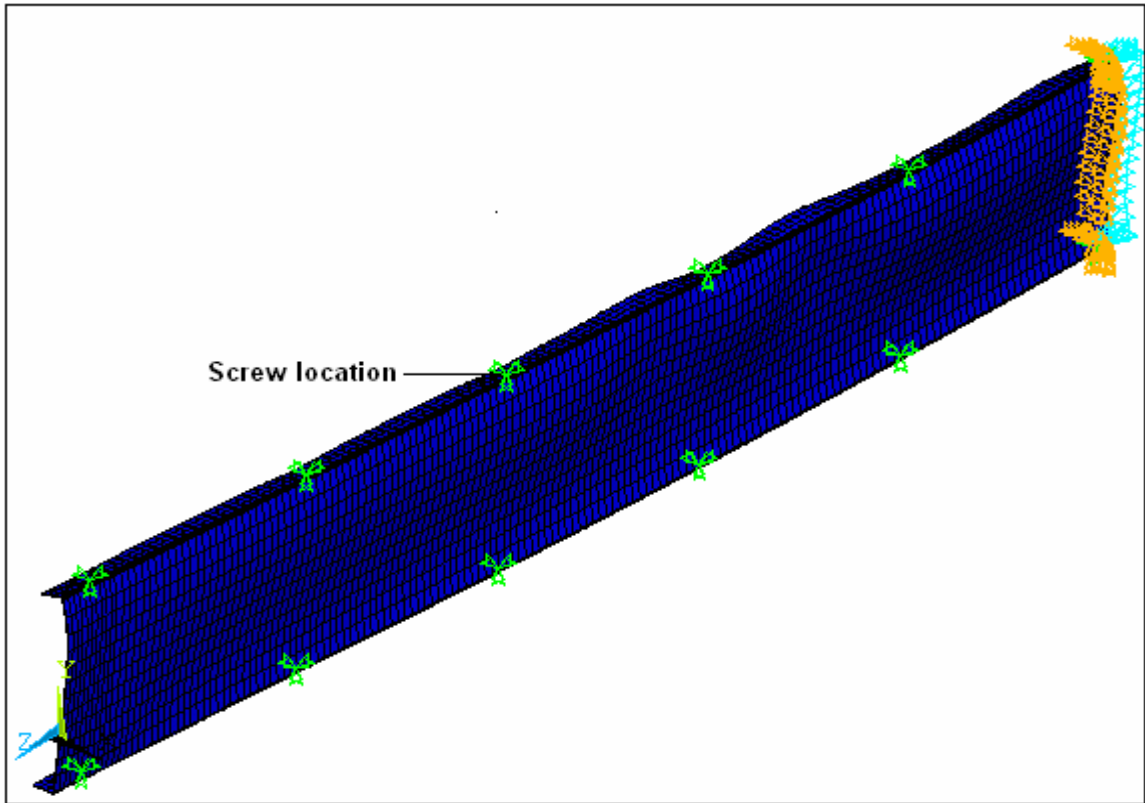


Figure 3.22 Rippled upper compression flange (FEM)

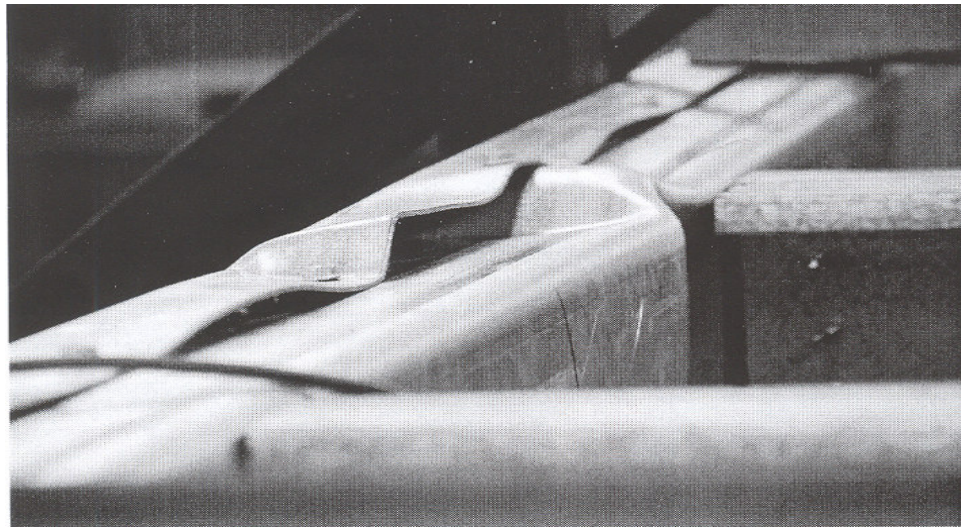


Figure 3.23 Typical Failure (Beshara and Lawson, 2002)

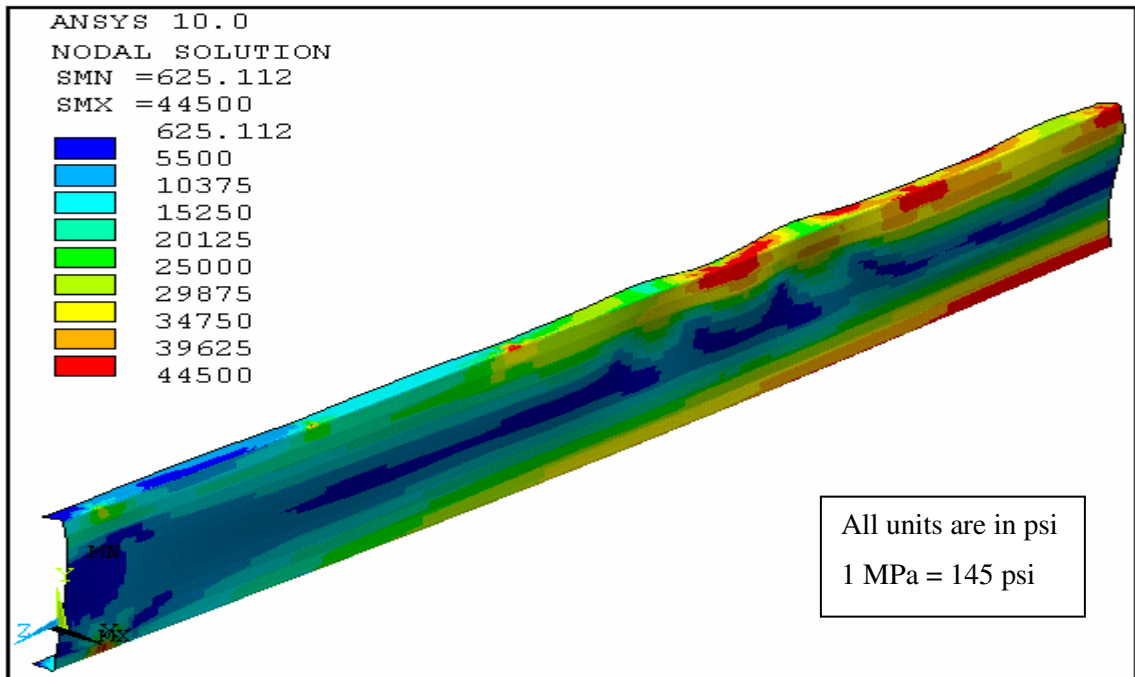


Figure 3.24 von Mises stress of track section at ultimate moment capacity (load applied as controlled displacement, stud plus track assembly)

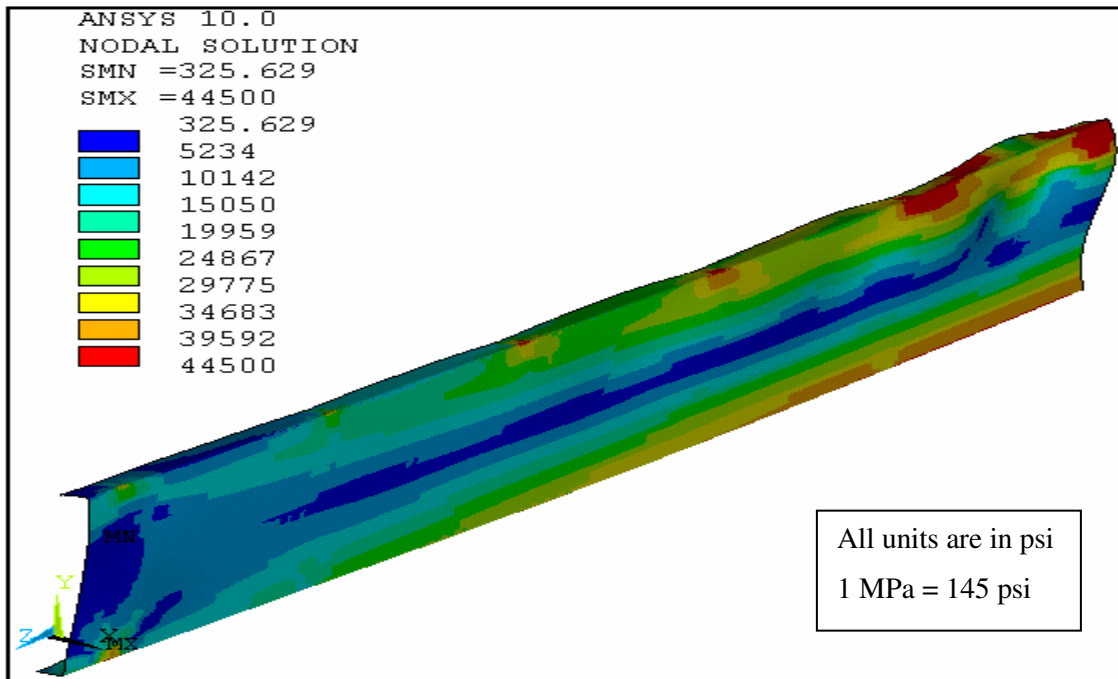


Figure 3.25 von Mises stress of track section at ultimate moment capacity (load applied as force, stud plus track assembly)

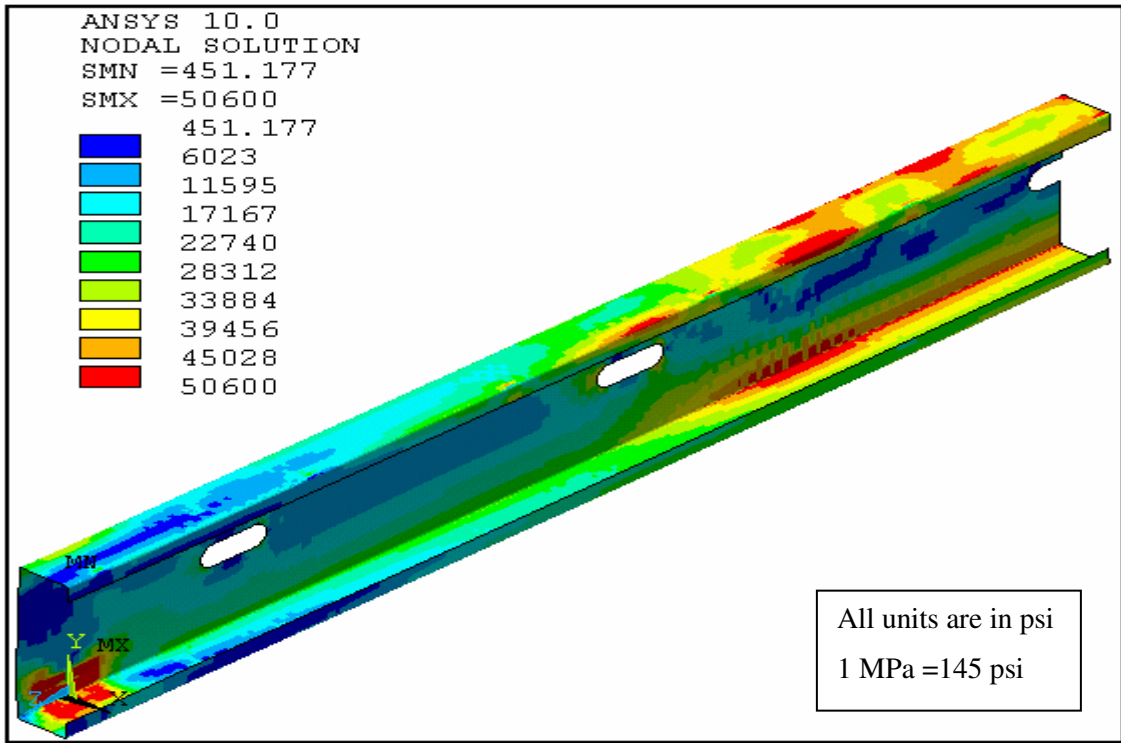


Figure 3.26 von Mises stress of stud section at ultimate moment capacity (load applied as controlled displacement, stud plus track assembly)

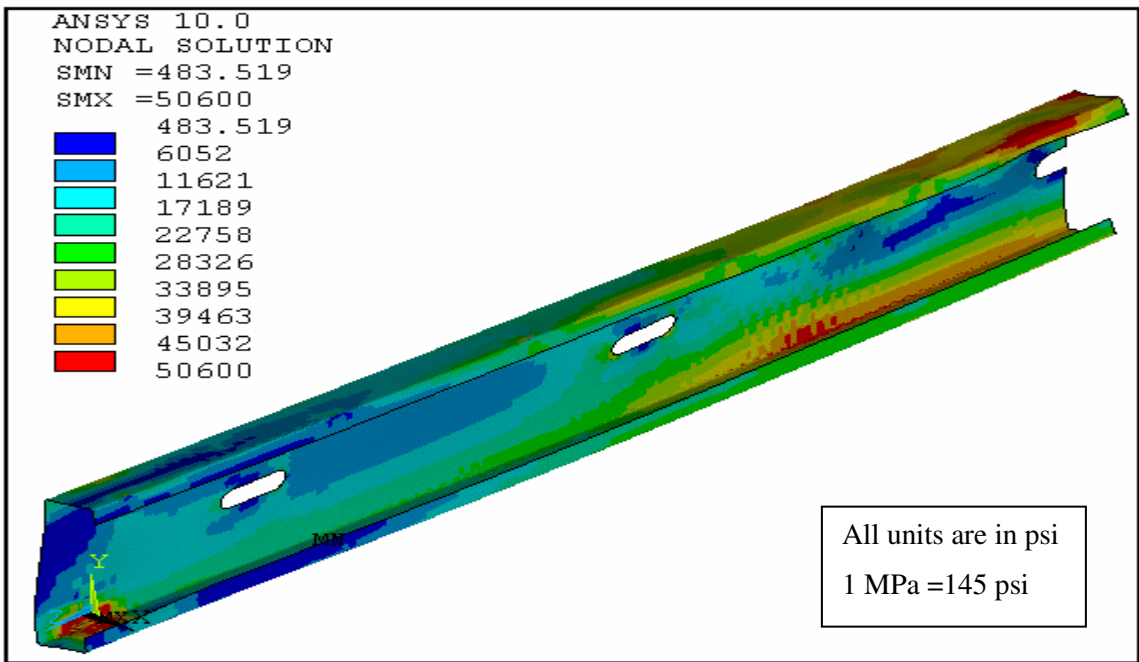


Figure 3.27 von Mises stress of stud section at ultimate moment capacity (load applied as force, stud plus track assembly)

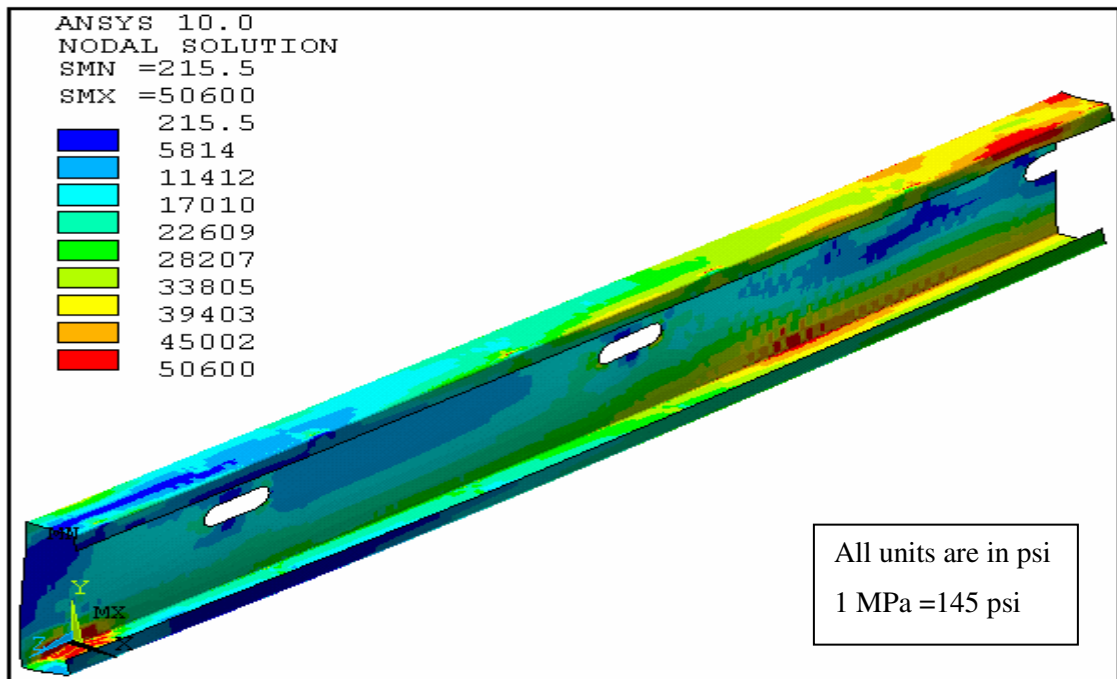


Figure 3.28 von Mises stress of stud section at ultimate moment capacity (load applied as controlled displacement, stud and rim-track assembly)

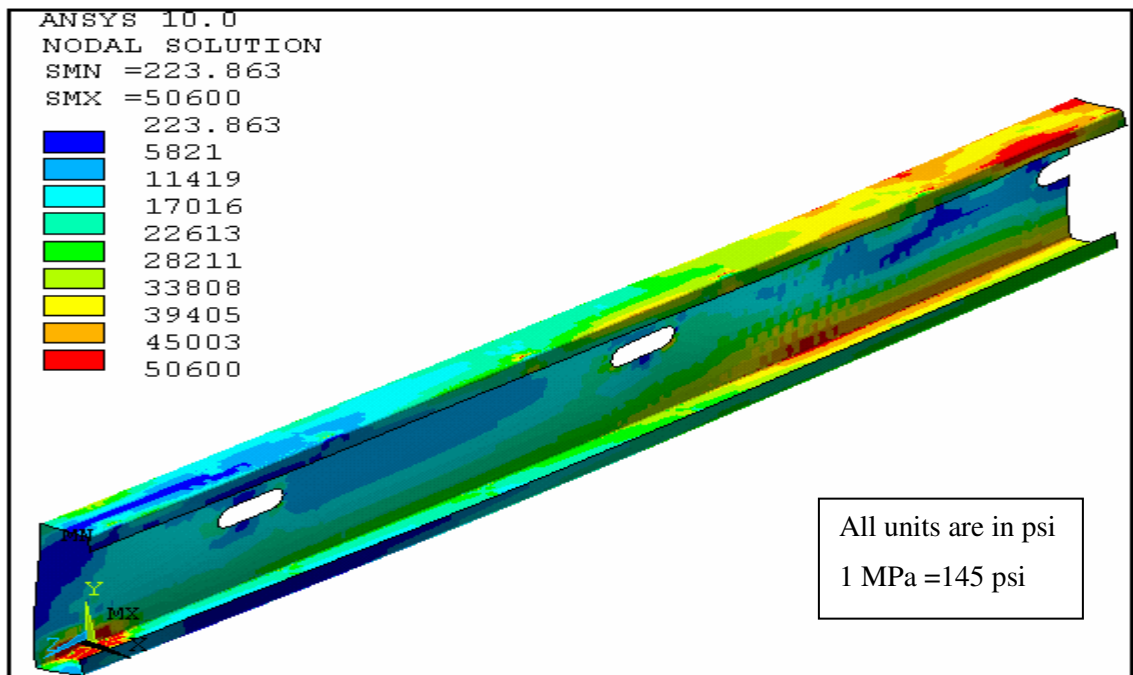


Figure 3.29 von Mises stress of stud section at ultimate moment capacity (load applied as force, stud and rim-track assembly)

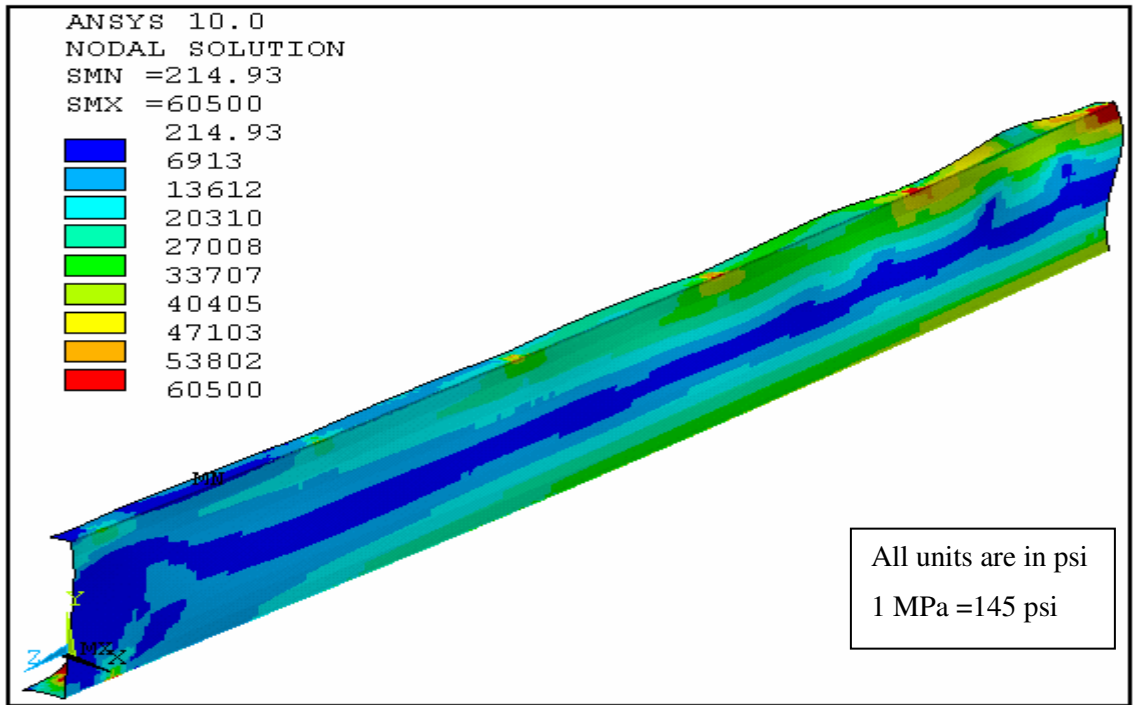


Figure 3.30 von Mises stress of rim-track section at ultimate moment capacity (load applied as controlled displacement, stud and rim-track assembly)

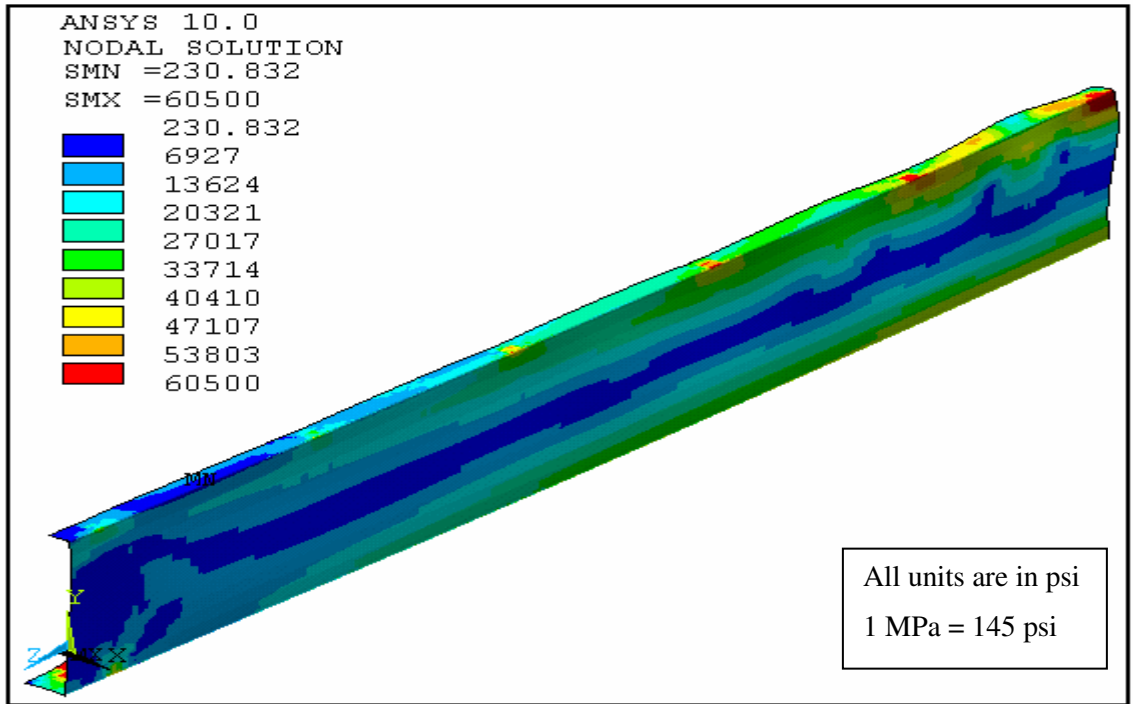
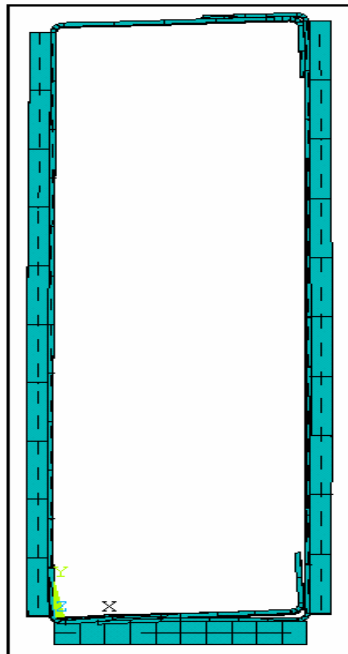
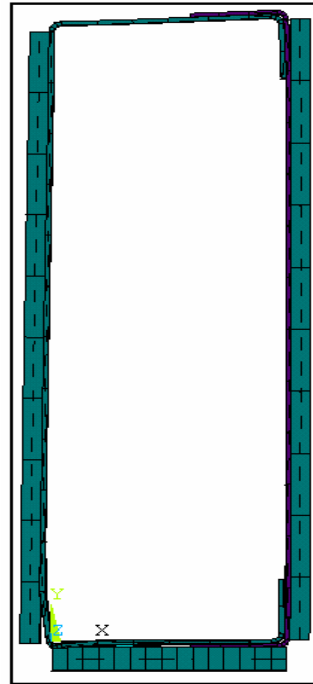


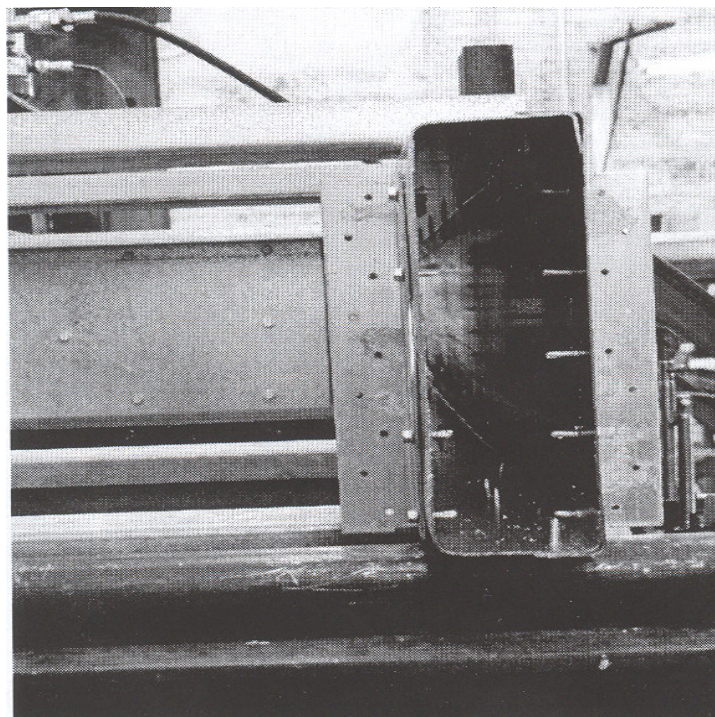
Figure 3.31 von Mises stress of rim-track section at ultimate moment capacity (load applied as force, stud and rim-track assembly)



a) stud plus rim-track section



b) stud plus track section



c) stud plus track section

Figure 3.32 Distorted girder assembly

3.9 Comparison of flexural capacity

The ultimate moment capacity of CFS built-up box girders determined from FE analysis was compared with both the test results and calculated values found according to the Cold-formed Steel Framing Design Guide (AISI Cold-formed steel framing design guide, 2002).

According to the guide, the capacity is the sum of the moment capacity of the two individual sections forming the built-up box. In this study, the nominal moment capacities of the stud, track and rim-track sections were calculated according to North American Specification for the Design of Cold-Formed Steel Structures (CSA-S136-01, 2001) and supplement to CSA-S136-01 (CSA-S136S1-04, 2004). After that, the nominal moment capacities of the built-up sections (M_n) were calculated by adding the capacities of the individual sections. A

MathCAD file was created for this calculation, the details of which calculations are shown in Appendix A. The ultimate moment capacities obtained from finite element analysis (M_{FEM}) with the load applied as a controlled displacement are listed in the third column of Table 3.2. Similar values obtained from the FE analysis when applying the load directly as force are listed in the fourth column of Table 3.2. The moment capacities of the built-up sections obtained from the FE analysis and the test (Beshara and Lawson, 2002) are almost the same for both types of track assemblies. For the stud and rim-track assembly, the ratio M_{FEM}/M_n equals 0.786 and 0.801 when applying the load as controlled displacement or direct force, respectively. For the stud and track assembly, the corresponding M_{FEM}/M_n ratios are 0.848 and 0.811. The ultimate moment capacities vary within 3.8% due to the different way of applying load in the FE model. The ratios M_{FEM}/M_n and M_{test}/M_n are compared in the last three columns of Table 3.2.

Table 3.2 Comparison of FEM and Test results

Description	M_{test} (kN.m)	M_{FEM} (Displacement) (kN.m)	M_{FEM} (Force) (kN.m)	M_n (kN.m)	M_{test}/M_n	M_{FEM}/M_n (displacement)	M_{FEM}/M_n (Force)
Stud + Rim-track	17.351	17.438	17.779	22.187	0.782	0.786	0.801
Stud + Track	17.458	17.984	17.194	21.194	0.824	0.848	0.811

For both cases, the finite element values are very close to the corresponding experimental values (the results vary within an acceptable range of 3.8%).

3.10 Summary

In this chapter, CFS built-up box girders were modelled using the FE analysis software ANSYS (version 10) to simulate the tests conducted by DDG (Beshara and Lawson, 2002). In the tests, no bearing plate was used at the support location. From the FE analysis, it was found that material failure occurred at the support location. The ultimate moment capacity obtained from the FE analysis is very close to the experimental value (within 3.8%). When a bearing plate was not used at the support location, the ultimate moment capacity of the CFS built-up box sections obtained from the FE analysis (M_{FEM}) is equal to 78%-85% of the nominal moment (M_n) calculated according to current design practice, which is quite close to the 78%-82% range predicted by the test results (Beshara and Lawson, 2002).

It is therefore concluded that the ANSYS finite element model reliably predicts the ultimate moment capacities of CFS box girders. Hence, it can be used to carry out parametric studies in order to identify the factors affecting the ultimate moment capacity of these girders.

Chapter 4

Parametric Study

4.1 Introduction

In the Chapter 3, it was shown that a finite element model can predict the ultimate moment capacity of a CFS built-up box girder quite accurately. In the analyzed test (Beshara and Lawson 2002), no bearing plate was used at the support location. From the finite element analysis it was found that material failure was occurring at the support location in the stud and track sections as shown in Figure 4.1, thereby reducing the ultimate moment capacity of the CFS built-up box girder. In practice, there is a requirement for a minimum-width bearing plate at support locations for CFS box girders. In order to investigate the effect of a bearing plate at the support location on the ultimate moment capacity of a CFS built-up box girder, the finite element models were modified to include a bearing plate as shown in Figure 4.2. This chapter presents the results of a parametric study carried out to understand the effect of the height, thickness, screw spacing and material yield stress on the ultimate moment capacity of a CFS built-up box girder.

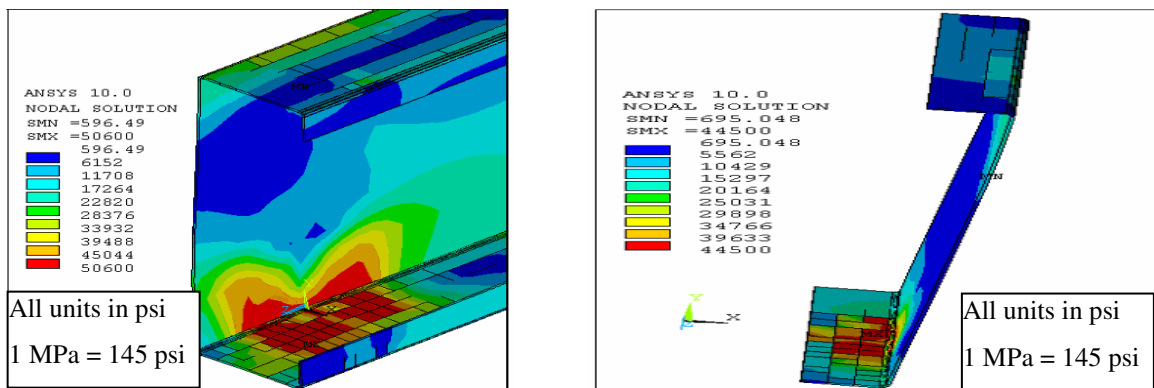


Figure 4.1 Failure at support location (von Mises stress)

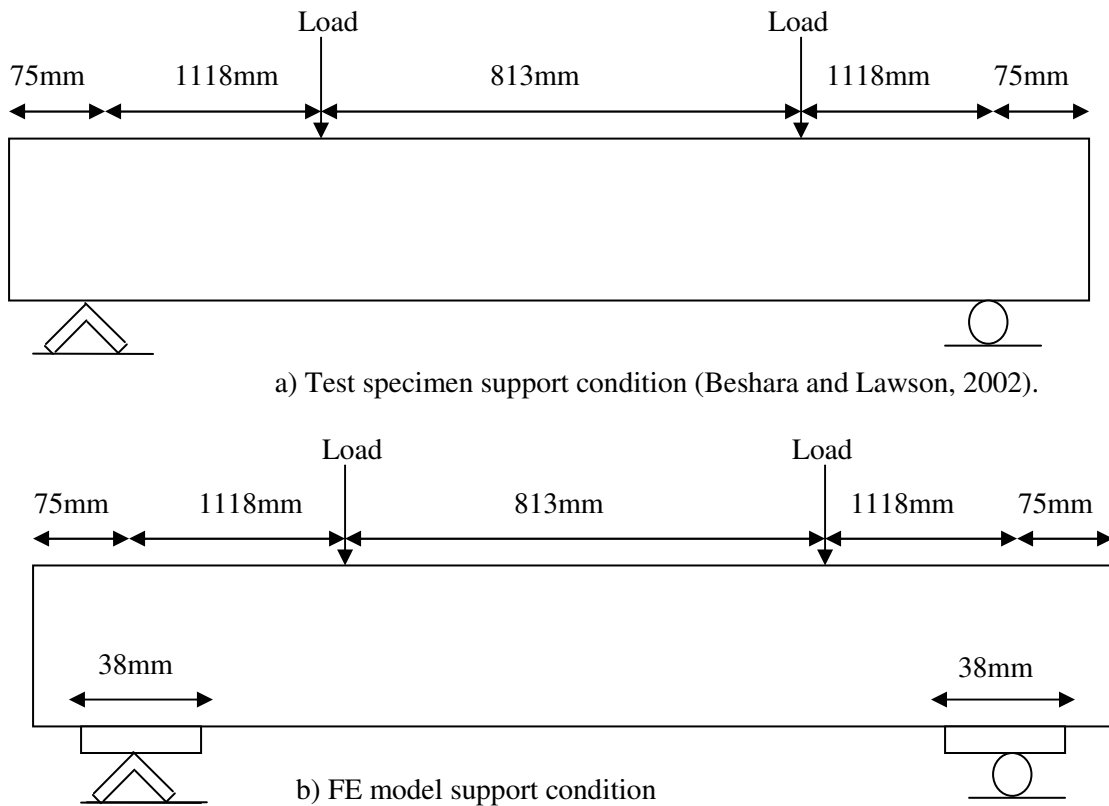


Figure 4.2 Support condition of test specimen and FE model.

4.2 Specimen labelling scheme

An example of the CFS built-up box girder labelling scheme used in this thesis is “254-ST-16-S300-BP-F1”, where: “254” represents the section depth of 254 mm (10 in); “ST” indicates a stud + track section (alternatively, “SRT” indicates a stud + rim-track section); “16” represents the section thickness of 16-gauge (1.44 mm); “S300” represents the flange screw spacing of 300 mm (12 in); “BP” indicates that a bearing plate is used at the support location; “F1” indicates that the yield stresses of stud, track and rim-track sections are 349 MPa (50.6 ksi), 307 MPa (44.5 ksi), and 417 MPa (60.5 ksi), respectively (alternatively, “F2”

indicates that the yield stress of stud, track and rim-track sections is equal to 228 MPa (33 ksi)).

4.3 Effect of bearing plate on the ultimate moment capacity of CFS built-up box girder

4.3.1 Finite element modelling

The box girders assembled with either stud and track sections, or stud and rim-track sections were modelled with a bearing plate at support locations. The stud and track sections were modelled using a mesh identical to that used for the finite element model discussed in Chapter 3. The bearing plate was modelled with dimensions of width = 38 mm (1.5 in) and length = 75 mm (3 in), and the corresponding area was meshed with Shell181 elements (ANSYS element library) as shown in Figure 4.3. The thickness of the bearing plate was taken equal to 10 mm (3/8 in). The self drilling screws connecting the top and bottom flanges of the stud and track sections were spaced at 300 mm, which is consistent with the test report (Beshara and Lawson 2002). The self drilling screws and end stiffener were modelled as discussed in Chapter 3. For all steel sections, young modulus $E = 203000$ MPa (29435 ksi) and Poisson's ratio = 0.3 were used. The yield stress for the stud, rim-track and track sections was taken as 349 MPa (50.6 ksi), 417 MPa (60.5 ksi) and 307 MPa (44.5 ksi), respectively, as mentioned in the test report (Beshara and Lawson, 2002). Residual stresses were ignored. Initial geometric imperfection was incorporated in the model by scaling the first eigenvalue buckling mode shape and adding it to the initial geometry such that maximum imperfection did not exceed the wall thickness of the section, as discussed in Chapter 3.

4.3.2 Boundary condition and application of load

Due to the cross sectional shape, the support bearing plate is initially in contact with the track section only. During application of load the stud section may come in contact with the plate due to deformation of the stud section. As the plate is always in contact with the track section, a flexible-to-flexible bonded contact was defined using the default contact options in ANSYS (ANSYS manual). A flexible-to-flexible standard contact was defined between the plate and the bottom flange of the stud section. The translational DOF of all the nodes of the plates at the location of support along the vertical direction ($U_y = 0$) was restrained as shown in the Figure 4.4. A symmetrical boundary condition was defined by setting the translational DOF equal to zero in the direction perpendicular to the symmetry plan ($U_z = 0$), and by also setting the rotational DOF equal to zero for all the nodes on the symmetry plan ($Rot_x = 0$ and $Rot_y = 0$), as shown in Figure 4.4. By defining $U_x = 0$ at the mid-nodes of both top and bottom flanges, the CFS built-up box girders were laterally restrained at locations 152 mm (6 in) and 762 mm (30 in) from both end supports, as in the test (Beshara and Lawson, 2002).

Loading was again applied in two ways, either as a direct force or as a controlled displacement, as discussed in section 3.4.7 of Chapter 3. For direct loading, a 3336 N (750 lb) load was applied gradually at each node with an initial load of 400 N (90 lb) and a minimum and maximum load increment of 3.34 N (0.75 lb) and 1000 N (225 lb), respectively. When the loading was applied as controlled displacement, the same procedure as discussed in section 3.4.7 of Chapter 3 was followed. For all the results presented in this chapter, elastic eigenvalue buckling analysis was first conducted and scaled displacements associated with the first buckling mode were taken used for the initial geometric

imperfections. Then nonlinear static analysis was carried out accounting for geometric and material nonlinearities to determine the ultimate moment capacity of the CFS built-up box girder.

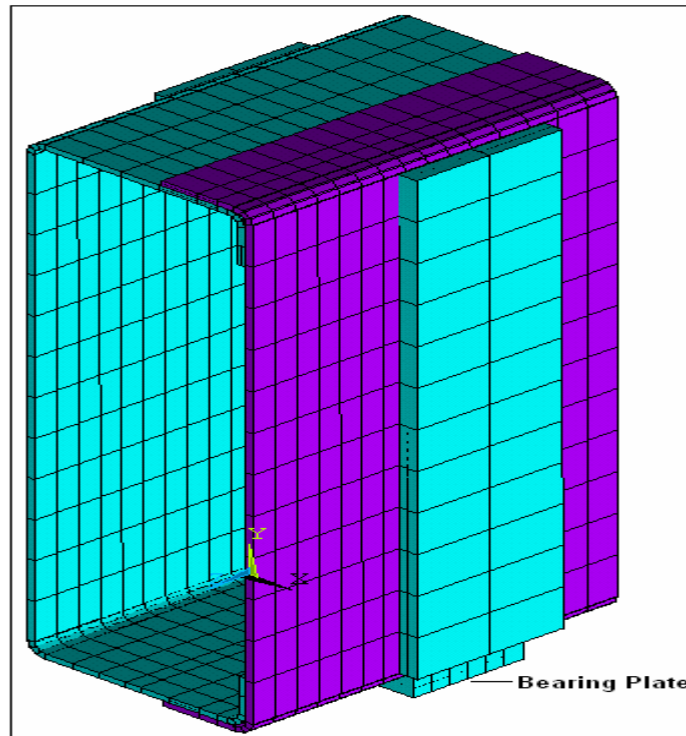


Figure 4.3 Finite element model of bearing plate

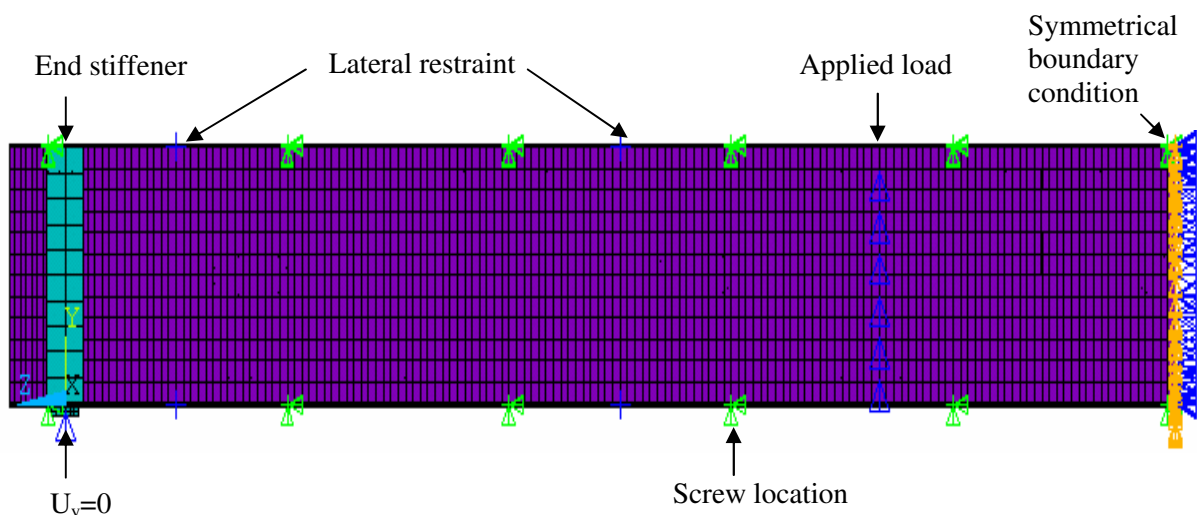


Figure 4.4 Boundary condition of FE model with bearing plate (Right side view)

4.3.3 Effect of support bearing plate

Without a support bearing plate, the CFS box girder's ultimate moment capacity was found to be limited by local material failure in the bottom flanges of both stud and track sections at the support location, as shown in Figure 4.1. Using a bearing plate at the support increased the ultimate load carrying capacity of the CFS built-up box girder because the ultimate capacity was then limited by material failure in the constant moment region, not at the support location. The stress in the top flanges for both stud and track sections reached the material yield stress in the constant moment region. The stress for the bottom flanges of track sections at the support location was below the material yield stress. As the bottom flange of the stud sections was not in contact with the bearing plate initially, the stress at this location reached the material yield stress due to deformation.

The FE results for the two models, obtained by applying the load directly and as controlled displacement, were compared in terms of the ultimate moment capacity, load-deformation behaviour, and stress condition of the model. For both the cases, the upper flange of the track section was observed to ripple prior to local buckling. The valleys of the ripples coincided with the screw locations. Local buckling occurred in the top flanges of the track and rim-track sections in the constant moment region, as shown in Figure 4.5 to 4.10. Local buckling of the top flanges and lip of the stud sections was observed for both models, as shown in Figure 4.11 to 4.14. The von Mises stress shown in Figures 4.7 through 4.14 reached the yield stress of the material in the constant moment region for both models. As the material model used in the simulation is perfectly elastic-plastic, the von Mises stress will not increase beyond the yield stress of the material. As a consequence of the applied load, some

distortion of the built-up box girder cross sections was also observed at the support location for both the models. The stress to yield stress ratio was equal to unity in the top flanges of stud and track sections in the constant moment region and also in the corner of the bottom flange of stud sections near the support location, as shown in Figures 4.15 and 4.18. All the Figures from 4.5 to 4.18 show that the deformation pattern, failure mode and the stress condition are almost the same for both the cases, i.e., whether the load is applied directly or as controlled displacement.

The FEM load versus mid-span deflection curves accounting for the bearing plate are compared with the test curves (Beshara and Lawson, 2002) found without the bearing plate as shown in Figures 4.19 and 4.20. Whether the FE analysis was conducted applying the load directly or as controlled displacement, the load-deflection curves followed the same path, as shown in Figures 4.19 and 4.20 for both cases. However, the FE model with the load applied directly as force could not predict the load-deformation behaviour beyond the ultimate capacity. For both of the models, the curve initially shows softer response due to distortion of the girder cross section. When the stud sections come in contact with the bearing plate, the load deflection curve displays higher stiffness than before as shown in Figures 4.19 and 4.20. The FEM load deflection curve for the stud plus rim-track assembly with bearing plate initially followed the same path as the test curve, as shown in Figure 4.19. After that, the stiffness increases, such that the FEM model with bearing plate shows higher ultimate load capacity than the test without bearing plate. The FEM load deflection curve for the stud plus track assembly with bearing plate initially followed the same path as the test curve without

bearing plate, as shown in Figure 4.20. After that, the FEM curve could not follow the test curve any more.

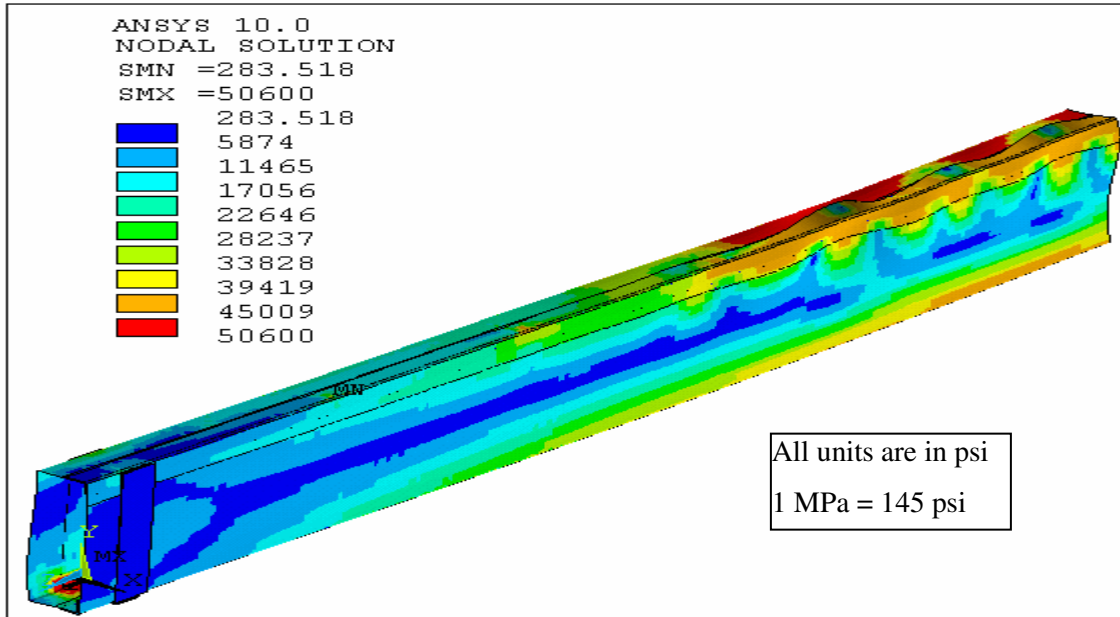


Figure 4.5 von Mises stress on deformed shape at ultimate moment capacity (load applied as controlled displacement, stud plus track assembly)

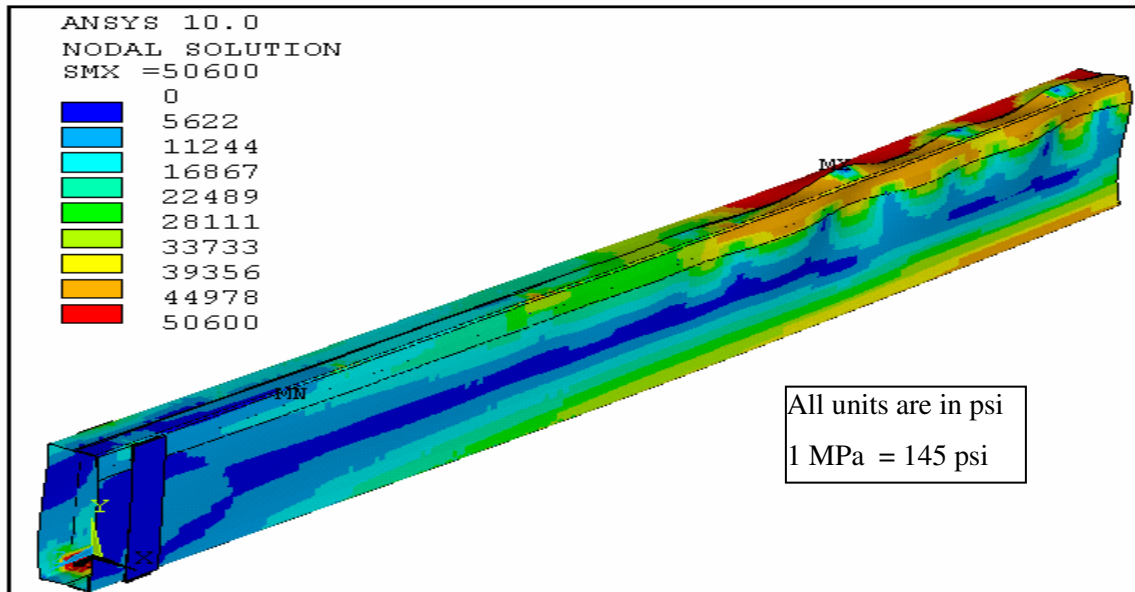


Figure 4.6 von Mises stress on deformed shape at ultimate moment capacity (load applied as force, stud plus track assembly)

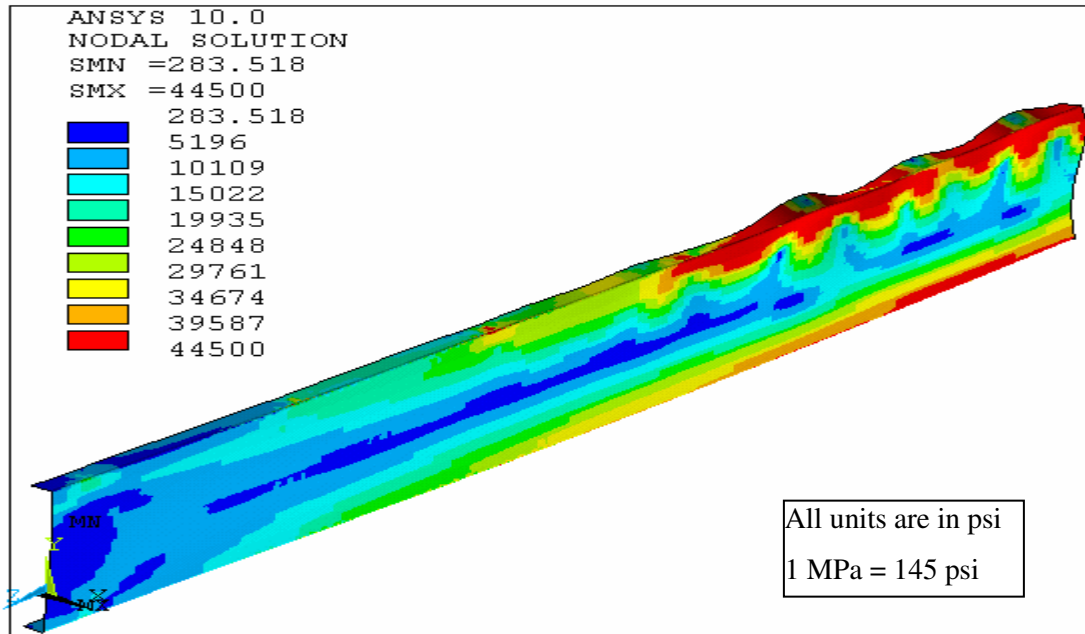


Figure 4.7 von Mises stress on deformed shape at ultimate moment capacity (load applied as controlled displacement, stud plus track assembly)

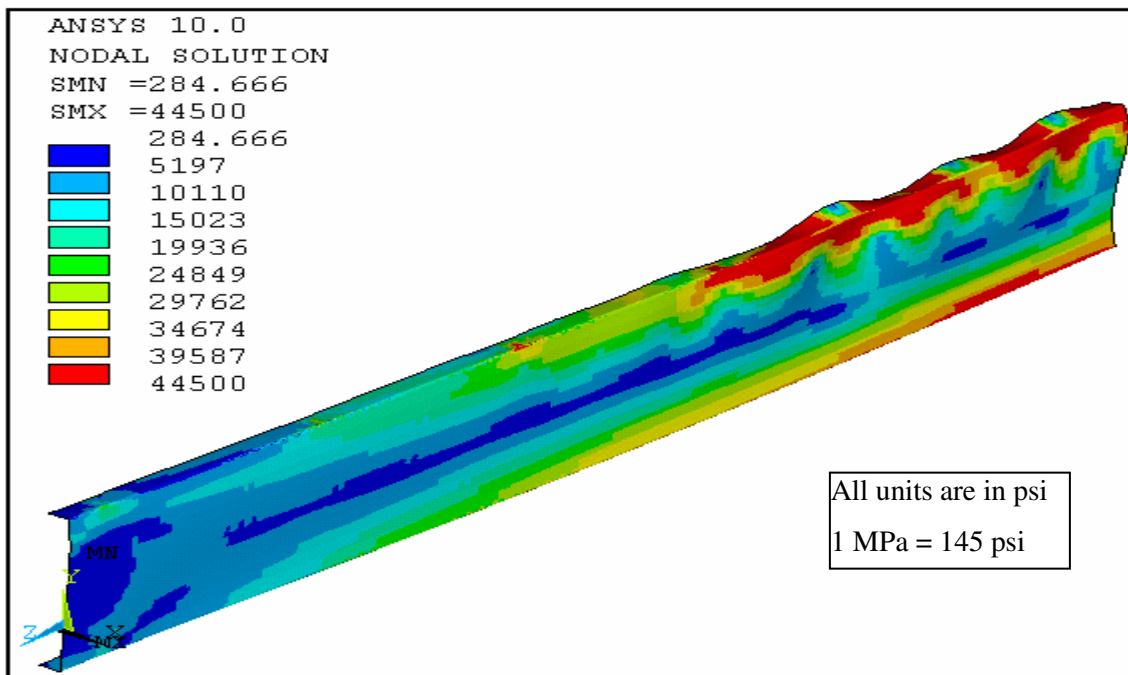


Figure 4.8 von Mises stress on deformed shape at ultimate moment capacity (load applied as force, stud plus track assembly)

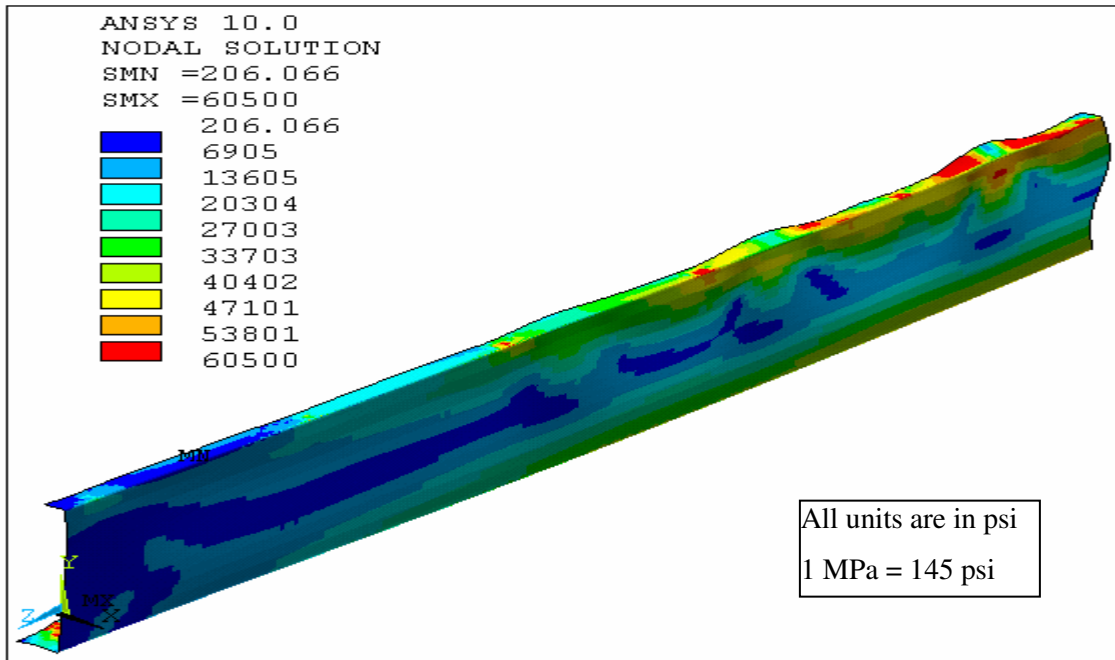


Figure 4.9 von Mises stress of stud sections on deformed shape at ultimate moment capacity (load applied as controlled displacement, stud plus rim-track assembly)

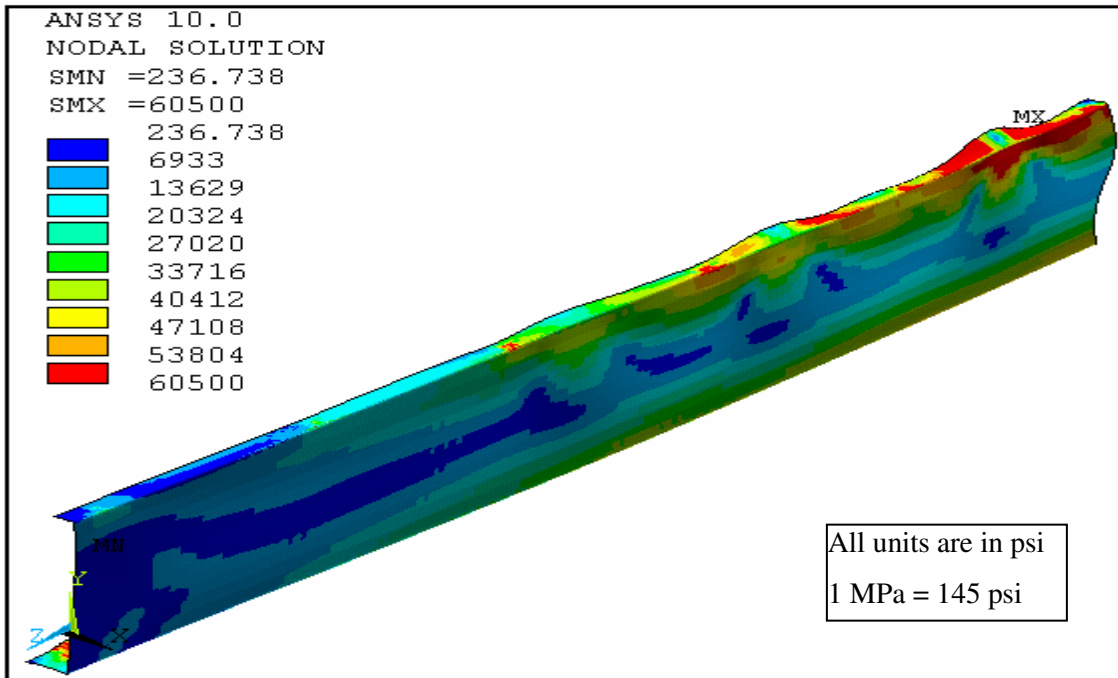


Figure 4.10 von Mises stress of stud sections on deformed shape at ultimate moment capacity (load applied as force, Model stud plus rim-track assembly)

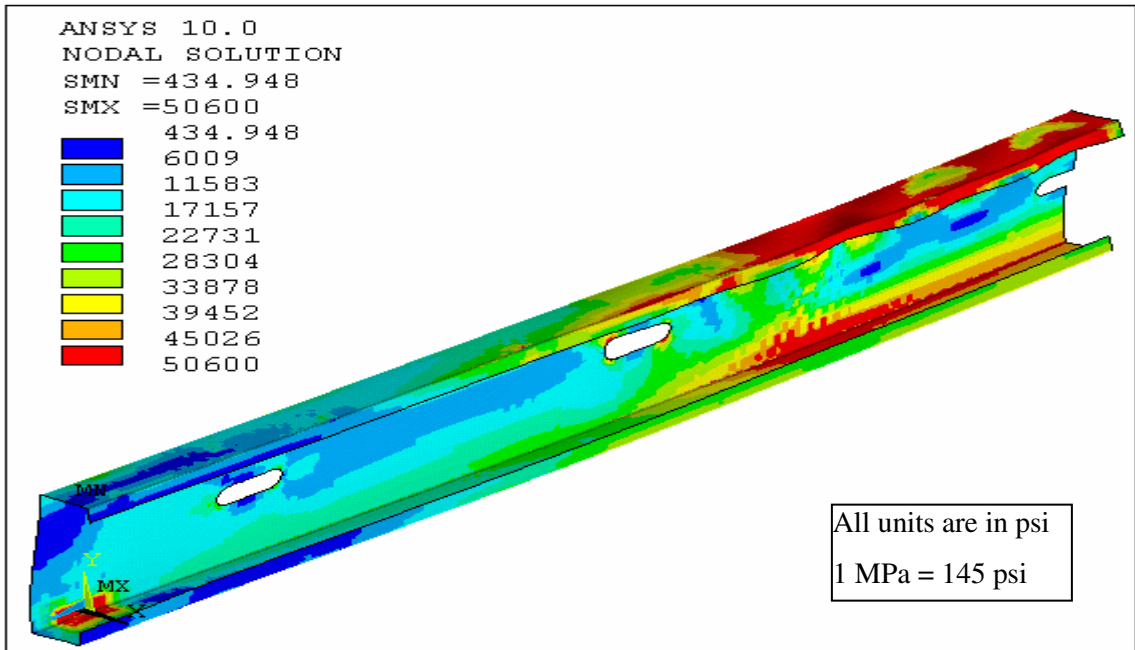


Figure 4.11 von Mises stress of stud sections on deformed shape at ultimate moment capacity (load applied as controlled displacement, stud plus track assembly)

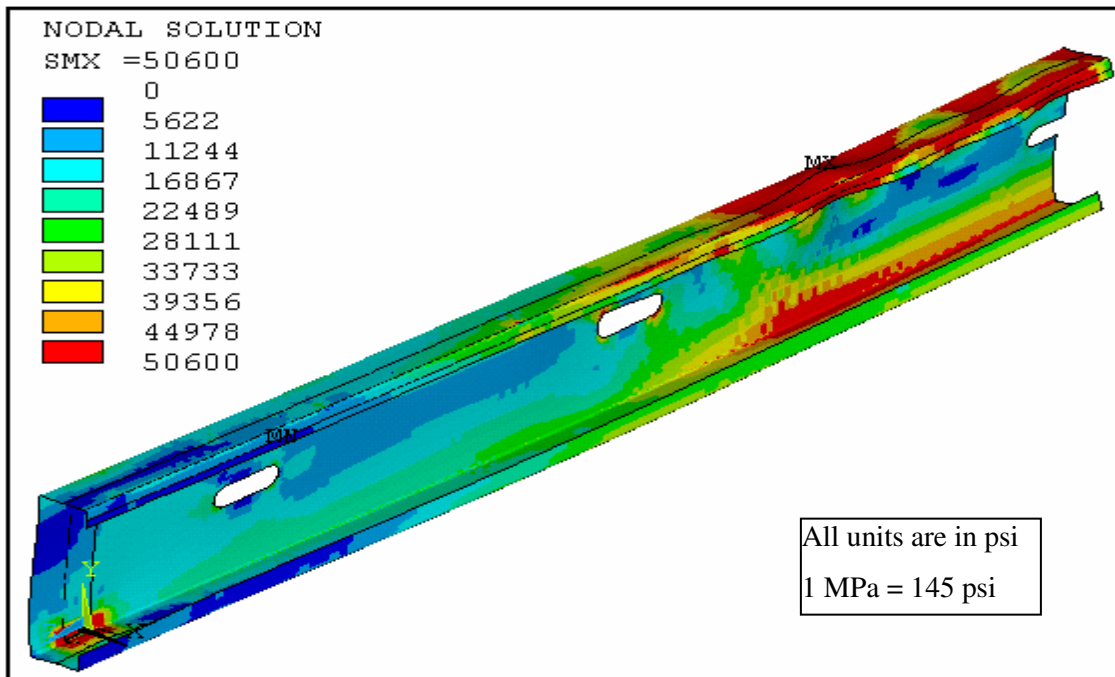


Figure 4.12 von Mises stress of stud sections on deformed shape at ultimate moment capacity (load applied as force, stud plus track assembly)

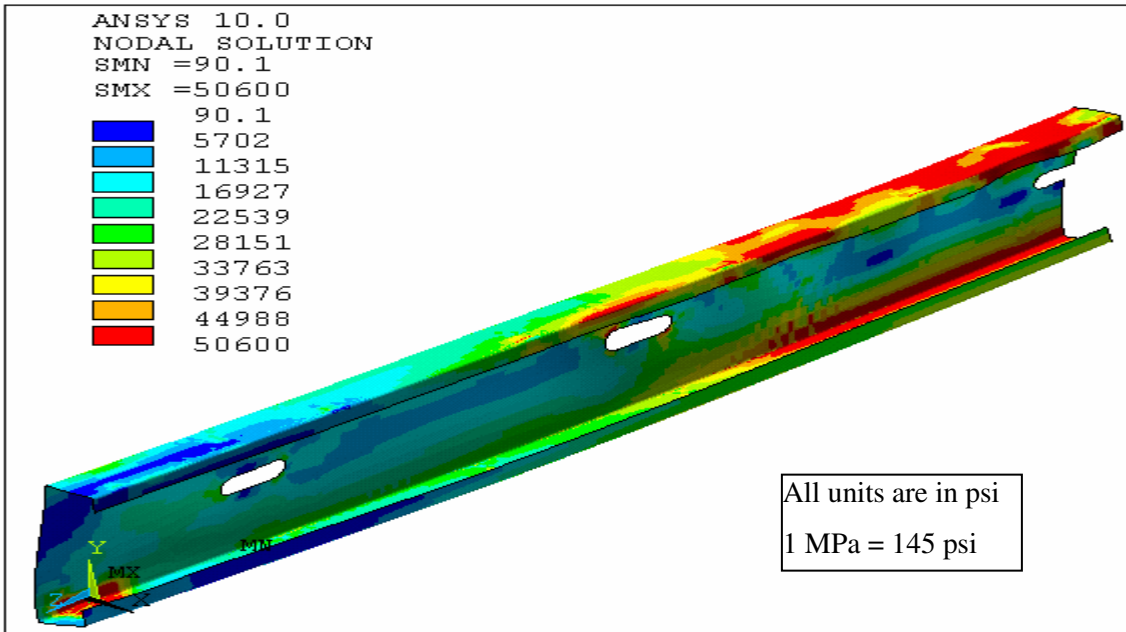


Figure 4.13 von Mises stress of stud sections on deformed shape at ultimate moment capacity
 (load applied as controlled displacement, stud plus rim-track assembly)

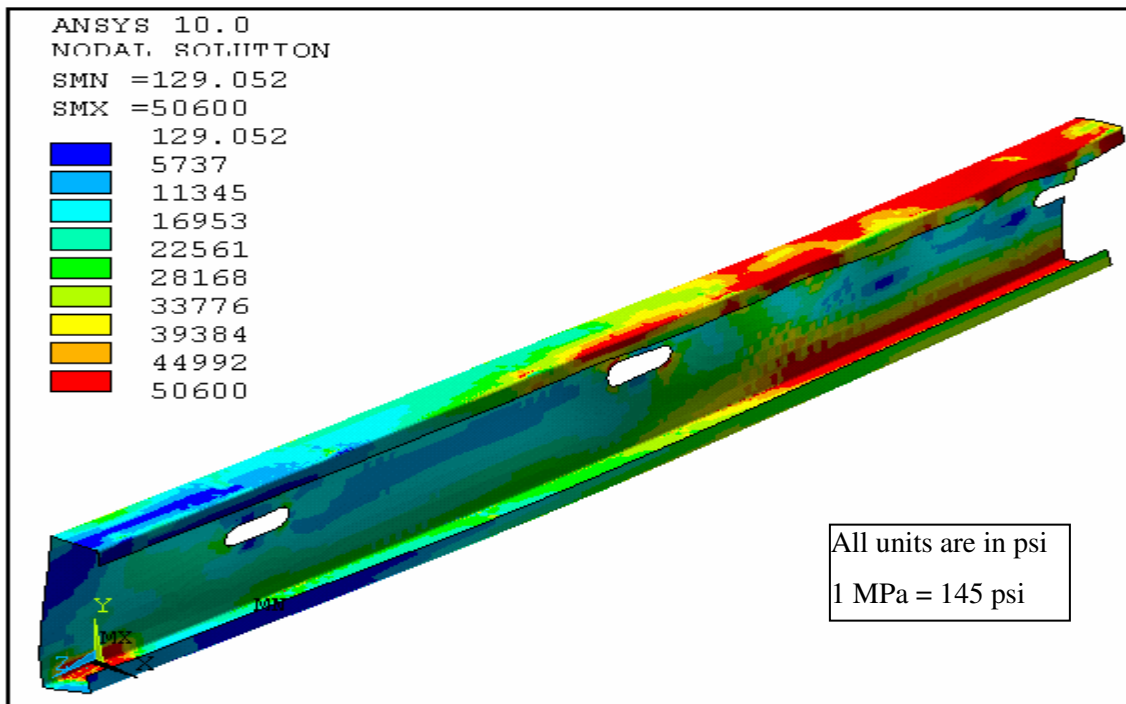


Figure 4.14 von Mises stress of stud sections on deformed shape at ultimate moment capacity
 (load applied as force, stud plus rim-track assembly)

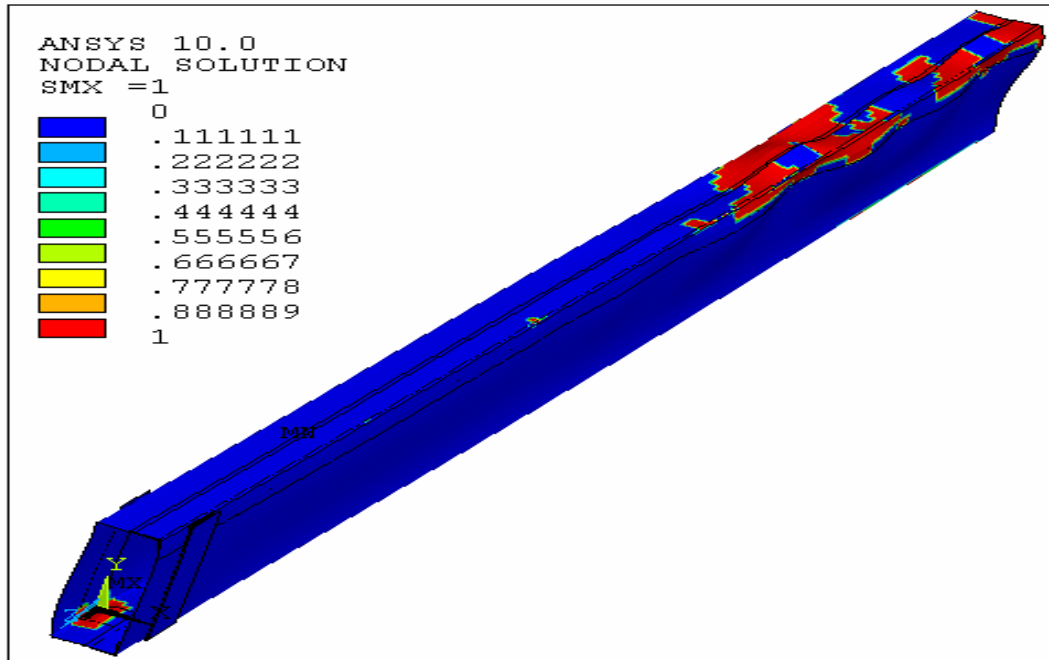


Figure 4.15 Stress state ratio on deformed shape at ultimate moment capacity (load applied as controlled displacement, stud plus track assembly)

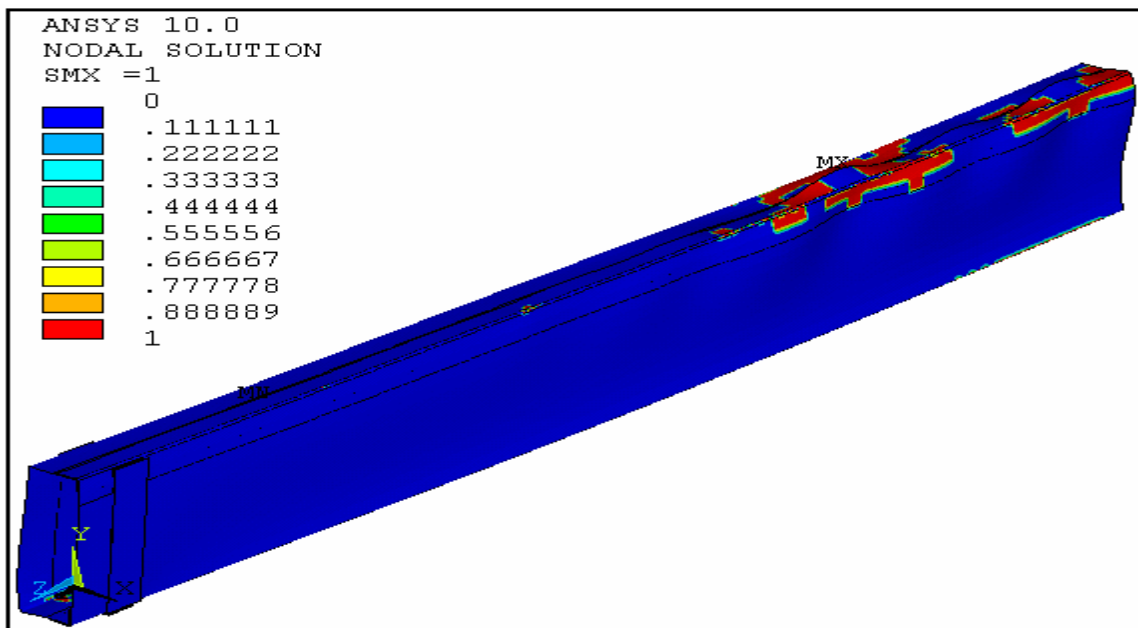


Figure 4.16 Stress state ratio on deformed shape at ultimate moment capacity (load applied directly as force, stud plus track assembly)

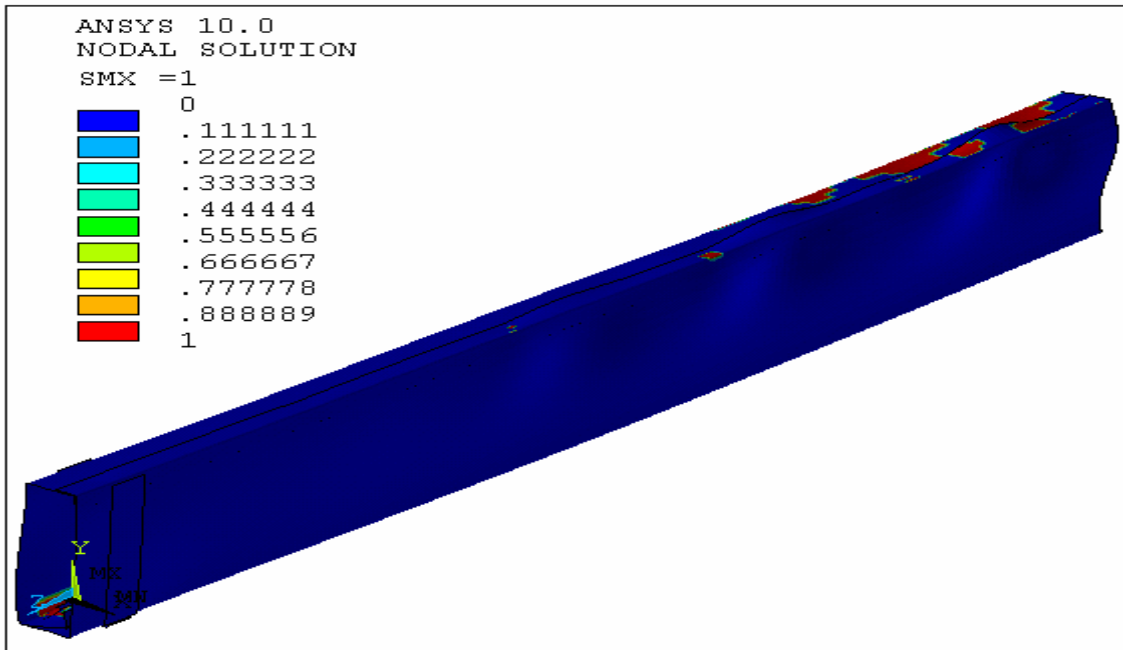


Figure 4.17 Stress state ratio on deformed shape at ultimate moment capacity (load applied as controlled displacement, stud plus rim-track assembly)

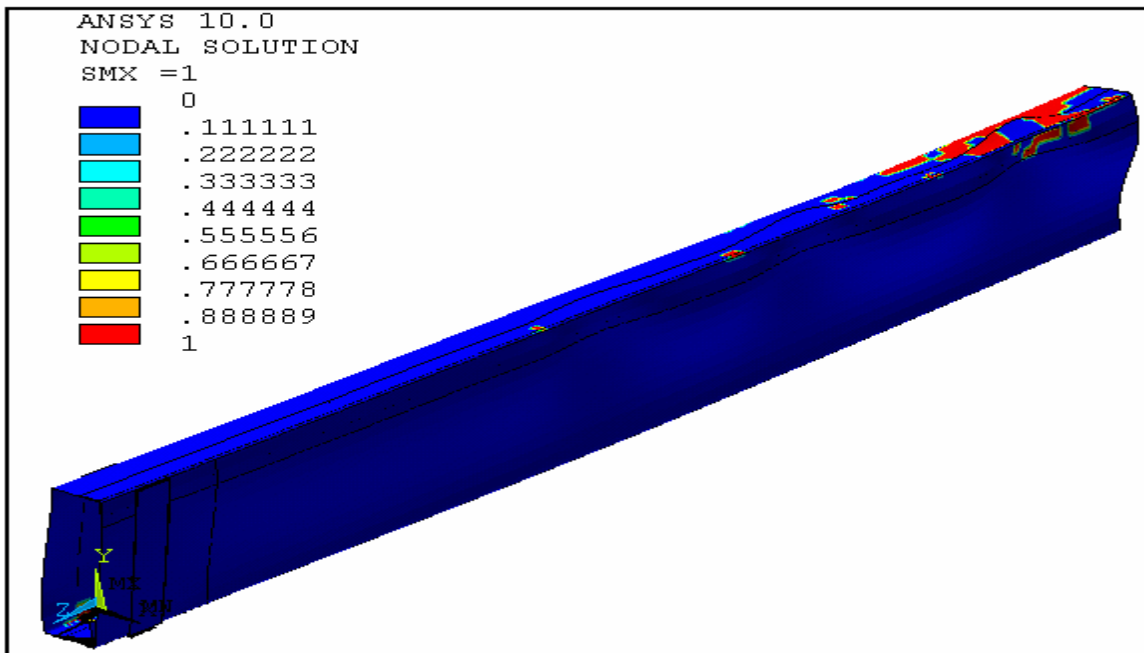


Figure 4.18 Stress state ratio on deformed shape at ultimate moment capacity (load applied directly as force, stud plus rim-track assembly)

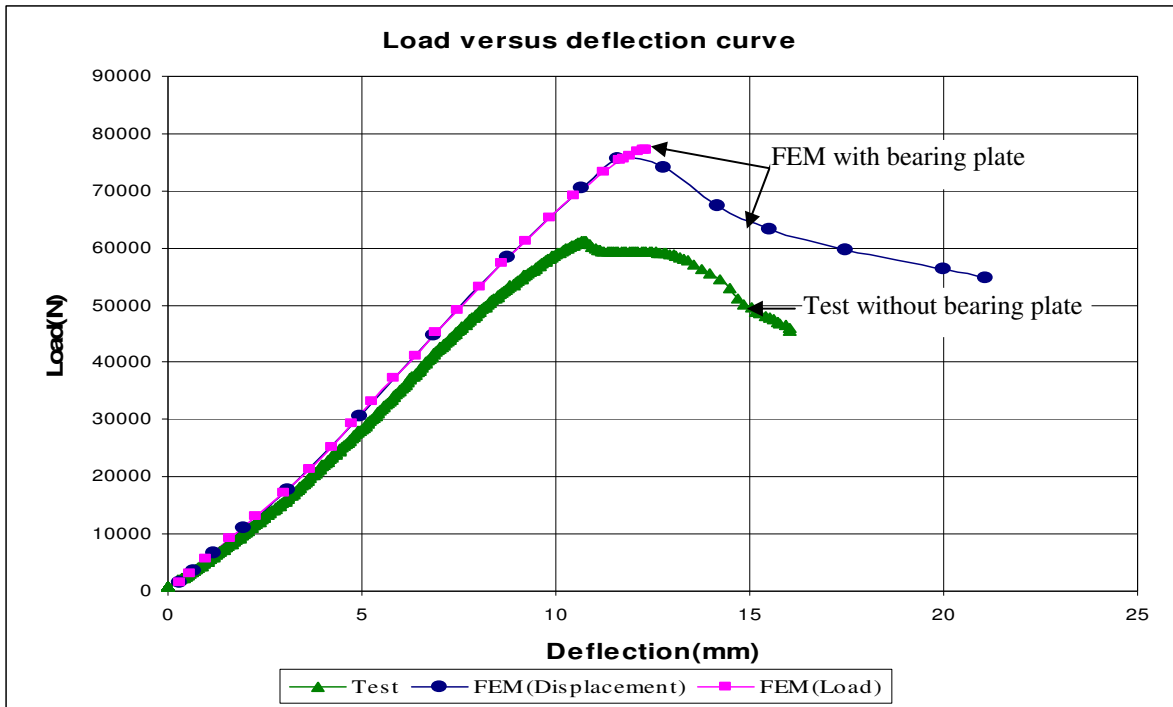


Figure 4.19 Load versus mid-span deflection curve (stud plus rim-track sections)

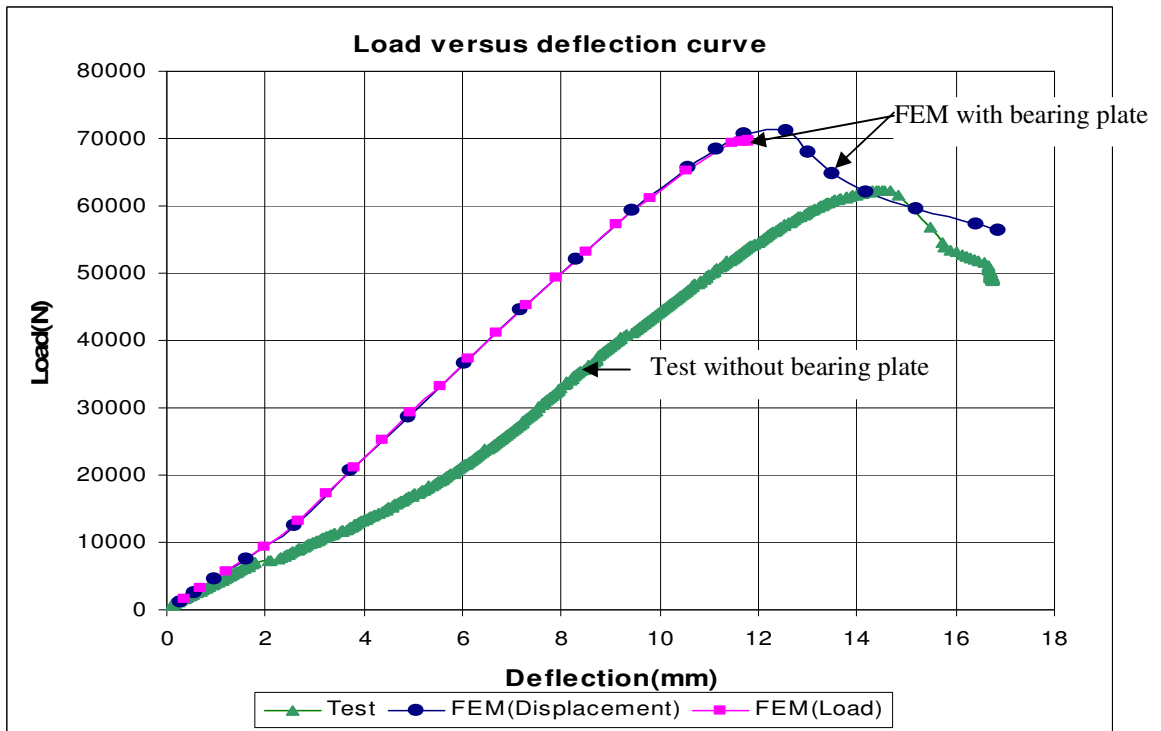


Figure 4.20 Load versus mid-span deflection curve (stud plus track sections)

4.3.4 Comparison of ultimate moment capacity of CFS built-up box girder

The peak loads of the FEM load deflection curves shown in Figure 4.19 and Figure 4.20 were taken as the ultimate load capacities of the CFS built-up box girders, and the corresponding ultimate moment capacities of the CFS built-up box girders were calculated. The ultimate moment capacity predicted from FE analysis (M_{FEM}) was compared with the nominal moment capacity (M_n) calculated assuming addition of the capacity of the two individual sections, and also with the test moment capacity (M_{test}) found by Behsara and Lawson (2002), as shown in Table 4.1. The M_{FEM}/M_n ratio increased from 0.786 to 0.951 for the stud plus rim-track assembly and from 0.848 to 0.938 for the stud plus track assembly when load was applied as controlled displacement because of the use of the bearing plate at the support location, as shown in Table 4.1. When the load was applied directly, the M_{FEM}/M_n ratio for the stud plus rim-track assembly increased from 0.801 to 0.971 while that for stud plus track assembly increased from 0.811 to 0.918.

Table 4.1 Comparison of ultimate moment capacity of CFS built-up box girder

Description	M_{test} (kN.m)	M_{FEM} (Displacement) (kN.m)	M_{FEM} (Force) (kN.m)	M_n (kN.m)	M_{test}/M_n	M_{FEM}/M_n (Displacement)	M_{FEM}/M_n (Force)
Stud+rim-track (no bearing plate)	17.351	17.438	17.779	22.187	0.782	0.786	0.801
Stud+rim-track (with bearing plate)		21.103	21.557	22.187		0.951	0.971
Stud+track (no bearing plate)	17.458	17.984	17.194	21.194	0.824	0.848	0.811
Stud+track (with bearing plate)		19.888	19.466	21.194		0.938	0.918

From the finite element analysis, it can be concluded that if no bearing plate was used, as in the test (Beshara and Lawson, 2002), the ultimate moment capacity of the CFS built-up box girder was 78% to 85% of M_n due to material failure at the support location. By using a bearing plate at the support location, the ultimate moment capacity of the CFS built-up box girder is increased to almost 94% to 97% of the nominal moment capacity calculated according to the current design practice. The FE analysis results for the loading applied as either direct force or controlled displacement showed the same failure pattern and deformation behaviour. The ultimate moment capacities predicted by FE for both cases are within 1.5%-3.8% of each other.

4.4 Parametric Study

Cold-formed steel members possess considerable post-buckling strength, although they are susceptible to local buckling at relatively low loads. The ultimate moment capacities of CFS flexural members depend mainly on the material yield strength and the width-to-thickness ratio of the individual plate elements that form the sections, assuming the members are laterally restrained properly. Built-up box girders are made of CFS stud and track sections connected by self drilling screws in both the top and bottom flanges. The stud section receives the load and transfers it to the track section through the connecting screws. The screw spacing may have a great influence on the ultimate moment capacity of CFS built-up box sections. Parametric studies are necessary to investigate the effects of section height, thickness, yield strength and screw spacing on the ultimate moment capacity of CFS built-up sections. In lieu of doing expensive tests, CFS built-up box girders consisting of stud and regular track sections and supported on bearing plates are investigated herein as extensive

FEM-based parametric study involving a variety of different section dimensions and properties. The four parameters considered for the parametric study are section depth, thickness, material yield strength and screw spacing. The section dimensions of the stud and track sections were chosen based on available sections in the market. The depth of sections considered for the parametric studies were 203 mm (8 in), 254 mm (10 in) and 305 mm (12 in). The section thicknesses were taken to be 18-gauge (1.14 mm), 16-gauge (1.44 mm) and 14-gauge (1.81 mm). The section dimensions of all of the stud and track sections considered in the parametric studies are given in Table 4.2 and Table 4.3. The length of the CFS built-up box girders was taken as 3200 mm (126 in) for the entire parametric studies. The yield strength of the stud and track sections available in the market is 228 MPa (33 ksi). As such, the finite element analyses were carried out using a yield stress equal to 228 MPa (33 ksi) for both the stud and track sections (F2), as well as with material properties similar to the test, as described in section 3.4.5 of Chapter 3 (F1). The labelling used to describe the FE model is same as that discussed in section 4.2 of this chapter.

Initial geometric imperfection, material nonlinearity and geometric nonlinearity were considered in the same way as before. In order to predict the load-deformation curve beyond ultimate capacity, the loading was applied as controlled displacement in the parametric study. The boundary conditions used for the parametric study were identical to those used in the FE model discussed in section 4.3 of this chapter. Extensive finite element analyses were carried out by varying each particular parameter while keeping the others constant. A total of thirty FE analyses were carried out for different section dimensions, material properties and screw

spacing, as shown in Table 4.4. The last two columns of Table 4.4 show the ultimate moment capacity and maximum mid-span deflection at ultimate load obtained from FE analysis.

Table 4.2 Sections dimension of CFS stud sections used in parametric studies.

Description	Thickness mm (in)	Depth mm(in)	Flange	Lip	Hole dimension	
					Depth mm(in)	Width mm(in)
203S76-18	1.14(0.0451)	203(8)	76.2(3)	25.4 (1)	38.1(1.5)	101.6(4)
203S76-16	1.44(0.0566)	203(8)	76.2(3)	25.4 (1)	38.1(1.5)	101.6(4)
203S76-14	1.81(0.0713)	203(8)	76.2(3)	25.4 (1)	38.1(1.5)	101.6(4)
254S76-18	1.14(0.0451)	254(10)	76.2(3)	25.4 (1)	38.1(1.5)	101.6(4)
254S76-16	1.44(0.0566)	254(10)	76.2(3)	25.4 (1)	38.1(1.5)	101.6(4)
254S76-14	1.81(0.0713)	254(10)	76.2(3)	25.4 (1)	38.1(1.5)	101.6(4)
304S76-18	1.14(0.0451)	305(12)	76.2(3)	25.4 (1)	38.1(1.5)	101.6(4)
304S76-16	1.44(0.0566)	305(12)	76.2(3)	25.4 (1)	38.1(1.5)	101.6(4)
304S76-14	1.81(0.0713)	305(12)	76.2(3)	25.4 (1)	38.1(1.5)	101.6(4)

Table 4.3 Sections dimension of CFS track sections used in parametric studies

Description	Thickness mm (in)	Depth mm(in)	Flange mm(in)
203T32-18	1.14(0.0451)	203(8)	32(1.25)
203T32-16	1.44(0.0566)	203(8)	32(1.25)
203T32--14	1.81(0.0713)	203(8)	32(1.25)
254T32-18	1.14(0.0451)	254(10)	32(1.25)
254T32-16	1.44(0.0566)	254(10)	32(1.25)
254T32-14	1.81(0.0713)	254(10)	32(1.25)
305T32-18	1.14(0.0451)	305(12)	32(1.25)
305T32-16	1.44(0.0566)	305(12)	32(1.25)
305T32-14	1.81(0.0713)	305(12)	32(1.25)

Table 4.4 Results of FE parametric studies

FE model description	Depth (A) mm (in)	Thickness (t) mm (in)	Stud designation	Track designation	Flange screw spacing mm (in)	M_{FEM} kN.m	Mid-span deflection mm(in)
			Yield stress F_y MPa(ksi)	Yield stress F_y MPa(ksi)			
203-ST-18-S300-BP-F1	203 (8)	1.14 (0.0451)	203S76-18	203T32-18	300 (12)	9.515	13.64 (0.537)
			349(50.6)	307(44.5)			
203-ST-18-S300-BP-F2	203 (8)	1.14 (0.0451)	203S76-18	203T32-18	300 (12)	7.203	9.98 (0.393)
			228(33)	228(33)			
203-ST-16-S300-BP-F1	203 (8)	1.44 (0.0566)	203S76-16	203T32-16	300 (12)	14.049	14.50 (0.571)
			349(50.6)	307(44.5)			
203-ST-16-S300-BP-F2	203 (8)	1.44 (0.0566)	203S76-16	203T32-16	300 (12)	10.455	10.95 (0.431)
			228(33)	228(33)			
203-ST-14-S300-BP-F1	203 (8)	1.81 (0.0713)	203S76-14	203T32-14	300 (12)	19.545	16.33 (0.643)
			349(50.6)	307(44.5)			
203-ST-14-S300-BP-F2	203 (8)	1.81 (0.0713)	203S76-14	203T32-14	300 (12)	14.237	11.61 (0.457)
			228(33)	228(33)			
254-ST-18-S300-BP-F1	254 (10)	1.14 (0.0451)	254S76-18	254T32-18	300 (12)	11.938	11.05 (0.435)
			349(50.6)	307(44.5)			
254-ST-18-S300-BP-F2	254 (10)	1.14 (0.0451)	254S76-18	254T32-18	300 (12)	9.078	7.47 (0.294)
			228(33)	228(33)			
254-ST-16-S300-BP-F1	254 (10)	1.44 (0.0566)	254S76-16	254T32-16	300 (12)	19.888	12.57 (0.495)
			349(50.6)	307(44.5)			
254-ST-16-S300-BP-F2	254 (10)	1.44 (0.0566)	254S76-16	254T32-16	300 (12)	15.061	9.17 (0.361)
			228(33)	228(33)			
254-ST-14-S300-BP-F1	254 (10)	1.81 (0.0713)	254S76-14	254T32-14	300 (12)	25.548	12.75 (0.502)
			349(50.6)	307(44.5)			
254-ST-14-S300-BP-F2	254 (10)	1.81 (0.0713)	254S76-14	254T32-14	300 (12)	18.468	9.32 (0.367)
			228(33)	228(33)			

FE model description	Depth (A) mm (in)	Thickness (t) mm (in)	Stud designation	Track designation	Flange screw spacing mm (in)	M_{FEM} kN.m	Mid-span deflection mm(in)
			Yield stress F_y MPa(ksi)	Yield stress F_y MPa(ksi)			
305-ST-18-S300-BP-F1	305 (12)	1.14 (0.0451)	305S76-18	305T32-18	300 (12)	15.028	9.25 (0.364)
			349(50.6)	307(44.5)			
305-ST-18-S300-BP-F2	305 (12)	1.14 (0.0451)	305S76-18	305T32-18	300 (12)	11.359	7.21 (0.284)
			228(33)	228(33)			
305-ST-16-S300-BP-F1	305 (12)	1.44 (0.0566)	305S76-16	305T32-16	300 (12)	21.239	9.30 (0.366)
			349(50.6)	307(44.5)			
305-ST-16-S300-F2	305 (12)	1.44 (0.0566)	305S76-16	305T32-16	300 (12)	16.488	7.49 (0.295)
			228(33)	228(33)			
305-ST-14-S300-BP-F1	305 (12)	1.81 (0.0713)	305S76-14	305T32-14	300 (12)	29.860	10.29 (0.405)
			349(50.6)	307(44.5)			
305-ST-14-S300-BP-F2	305 (12)	1.81 (0.0713)	305S76-14	305T32-14	300 (12)	22.466	8.26 (0.325)
			228(33)	228(33)			
203-ST-16-S600-BP-F2	203 (8)	1.44 (0.0566)	203S76-16	203T32-16	600 (24)	9.979	10.46 (0.412)
			228(33)	228(33)			
203-ST-16-S150-BP-F2	203 (8)	1.44 (0.0566)	203S76-16	203T32-16	150 (6)	10.667	11.15(0.439)
			228(33)	228(33)			
254-ST-16-S600-BP-F2	254 (10)	1.44 (0.0566)	254S76-16	254T32-16	600 (24)	14.066	8.41 (0.331)
			228(33)	228(33)			
254-ST-16-S150-BP-F2	254 (10)	1.44 (0.0566)	254S76-16	254T32-16	150 (6)	15.982	9.73 (0.383)
			228(33)	228(33)			
305-ST-16-S600-BP-F2	305 (12)	1.44 (0.0566)	305S76-16	305T32-16	600 (24)	15.698	6.83 (0.269)
			228(33)	228(33)			
305-ST-16-S150-BP-F2	305 (12)	1.44 (0.0566)	305S76-16	305T32-16	150 (6)	16.638	7.75 (0.305)
			228(33)	228(33)			
203-ST-14-S600-BP-F2	203 (8)	1.81 (0.0713)	203S76-14	203T32-14	600 (24)	13.425	11.25 (0.443)
			228(33)	228(33)			

FE model description	Depth (A) mm (in)	Thickness (t) mm (in)	Stud designation	Track designation	Flange screw spacing mm (in)	M_{FEM} kN.m	Mid-span deflection mm(in)
			Yield stress F_y MPa(ksi)	Yield stress F_y MPa(ksi)			
203-ST-14-S150-BP-F2	203 (8)	1.81 (0.0713)	203S76-14	203T32-14	150 (6)	14.696	12.57 (0.495)
			228(33)	228(33)			
254-ST-14-S600-BP-F2	254 (10)	1.81 (0.0713)	254S76-14	254T32-14	600 (24)	17.746	8.92 (0.351)
			228(33)	228(33)			
254-ST-14-S150-BP-F2	254 (10)	1.81 (0.0713)	254S76-14	254T32-14	150 (6)	19.320	10.19 (0.401)
			228(33)	228(33)			
305-ST-14-S600-BP-F2	305 (12)	1.81 (0.0713)	305S76-14	305T32-14	600 (24)	22.512	7.95 (0.313)
			228(33)	228(33)			
305-ST-14-S150-BP-F2	305 (12)	1.81 (0.0713)	305S76-14	305T32-14	150 (6)	23.265	8.99 (0.354)
			228(33)	228(33)			

4.4.1 Parametric study of section depth and thickness variation

FE analyses were performed to investigate the influence of section depth and thickness variation on the ultimate moment capacity of the CFS built-up box girders. For this part of the study, a screw spacing of 300 mm was maintained. For a particular section depth, three different models were created for three different section thicknesses of 1.14 mm (0.0451 in), 1.44 mm (0.0566 in) and 1.81 mm (0.0713 in). All the FE analyses were conducted for two different types of steel material (F1 & F2) as discussed in the previous section. A total of eighteen FE analyses were conducted to investigate the influence of section depth and thickness on the ultimate moment capacity of the CFS built-up box girders.

4.4.1.1 Discussion of section depth and thickness study results

The FE analysis results for the girder models of different depths and thicknesses were compared with each other in terms of moment capacity, deformed shape and load vs. deflection response. Local buckling of top flanges of the track sections were observed for all the models in the constant moment region, as shown in Figures 4.21 to 4.36 (thickness 1.14 mm-1.81 mm, depth 203 mm-305 mm). The buckling of the top flanges and lips of stud sections were also observed in the models with 18-gauge and 16-gauge section thicknesses due to high flange width-to-thickness ratios. As the section thickness increases, local buckling of the top flanges of stud sections was not observed for 14-gauge thickness. The material of the top flanges and also some parts of the bottom flanges of all the models started to yield before the girder reached to its ultimate moment capacity. The stress to yield stress ratios (F/F_y) for all the models are shown on deformed shapes of the girders in Figures 4.21 to 4.36. As the material is perfectly elastic plastic, the maximum stress ratio equals unity indicating yielding of material. Local buckling of the web was also observed in the constant moment region for the models with thickness 1.14 mm-1.44 mm and depth 254 mm-305 mm. Material yielding occurred in the unsupported portion of the bottom flanges of stud sections at the support location. Distortion of the built-up cross sections was observed for all models as a consequence of the applied load.

For the models 305-ST-18-S300-BP-F1, 305-ST-18-S300-BP-F2 and 305-ST-16-S300-BP-F2, the finite element analysis could not converge after the ultimate load level was reached due to the presence of a large strain in the corner of the stud section at the support

location, even for a very small increment of displacement. In those cases the last converged solution was taken as the ultimate load capacity of the built-up box girder.

The ultimate load carrying capacities of the built-up girders were determined from the load versus mid-span deflection curves obtained from the FE analysis as shown in Figures 4.37 to 4.48. According to the AISI Design Guide (AISI Cold-Formed Steel Framing Design Guide, 2002), the nominal moment capacity of CFS built-up box girders is the addition of the nominal moment capacities of the individual CFS sections. In order to determine whether this assumption is conservative or not for each girder model, the ratio between the ultimate moment capacity obtained from FE analysis (M_{FEM}) and the nominal moment capacity of the built-up sections (M_n) was calculated as shown in the last column of Table 4.5.

Table 4.5 Ratio of Ultimate moment capacity obtained from FEM to the calculated M_n of CFS built-up box girder

Model description	Web depth to thickness ratio of track section h/t	M_{FEM} kN.m	CSA-S136-01 nominal moment		Nominal moment of built-up sections M_n kN.m	M_{FEM}/M_n
			M_{stud} kN.m	M_{track} kN.m		
203-ST-18-S300-BP-F1	174.38	9.515	6.741	2.861	9.602	0.991
254-ST-18-S300-BP-F1	173.68	11.938	8.428	3.647	12.075	0.989
305-ST-18-S300-BP-F1	263.07	15.028	10.117	4.438	14.554	1.033
203-ST-16-S300-BP-F1	138.34	14.049	10.318	4.251	14.568	0.964
254-ST-16-S300-BP-F1	173.67	19.888	15.713	5.482	21.195	0.938
305-ST-16-S300-BP-F1	209.0	21.239	15.321	6.712	22.033	0.964
203-ST-14-S300-BP-F1	109.2	19.545	14.357	6.215	20.572	0.950

Model description	Web depth to thickness ratio of track section h/t	M_{FEM} kN.m	CSA-S136-01 nominal moment		Nominal moment of built-up sections M_n kN.m	M_{FEM}/M_n
			M_{stud} kN.m	M_{track} kN.m		
254-ST-14-S300-BP-F1	137.25	25.548	19.268	8.102	27.370	0.933
305-ST-14-S300-BP-F1	165.3	29.860	22.717	10.000	32.717	0.913
203-ST-18-S300-BP-F2	174.38	7.203	5.202	2.359	7.561	0.953
254-ST-18-S300-BP-F2	173.68	9.078	6.535	3.024	9.559	0.950
305-ST-18-S300-BP-F2	263.07	11.359	7.816	3.687	11.503	0.987
203-ST-16-S300-BP-F2	138.34	10.455	7.535	3.462	10.997	0.951
254-ST-16-S300-BP-F2	173.67	15.061	11.168	4.503	15.671	0.961
305-ST-16-S300-BP-F2	209.0	16.488	11.834	5.516	17.350	0.950
203-ST-14-S300-BP-F2	109.2	14.237	10.05	4.976	15.026	0.947
254-ST-14-S300-BP-F2	137.25	18.468	13.589	6.556	20.145	0.917
305-ST-14-S300-BP-F2	165.3	22.466	17.489	8.142	25.631	0.876
					Average	0.954
					Std_Dev	0.034
					Coefficient of variation	0.036

4.4.1.2 Effect of section depth and thickness variation on the ultimate moment capacity

The effect of section depth and thickness variation on the ultimate moment capacity of the built-up girders was investigated. To make a comparison of different load-deflection behaviour due to section depth variation only, the load versus mid-span deflection curves for girders of different depth with all other factors constant were plotted on the same graph, as shown in Figures 4.37 through 4.42. All the curves show nonlinear behaviour after

application of a very small amount of load. The initial nonlinearity of the curve is an indication of the distortion of the stud section at the support location prior to the bottom flange of the stud coming into contact with the bearing plate. The stiffness increases as the stud section comes into contact with the bearing plate. The slope of the load-deflection curve increases with the increase of section depth, as shown in Figures 4.37 to 4.42, indicating an increase in section stiffness. The ultimate moment capacity of the CFS built-up box girders also increases with an increase in section depth.

For different thicknesses of the section, keeping all other factors constant, the load deflection curves for the girder models were plotted in the same graphs in Figure 4.43 through 4.48. Looking at these graphs, it can be seen that the slope of load-deflection curve increases with an increase in section thickness. The ultimate moment capacities of the CFS built-up box girders also increase with an increase in section thickness. The M_{FEM}/M_n ratios shown in Table 4.5 vary from 0.876 to 1.03 for the different section depths and thicknesses, with an average value of 0.954, standard deviation of 0.03 and coefficient of variation of 0.036. The bar charts shown in Figures 4.49 and 4.50 indicate the effect of section depth and thickness on the M_{FEM}/M_n ratio. From these figures it can be observed that for a particular section depth, the M_{FEM}/M_n ratios decrease with an increase in section thickness. For the depth of 305 mm, the effect of thickness variation on the M_{FEM}/M_n ratios is significant and the ratio ranges from 0.876 to 1.033. For the depth of 203 mm, the ratio varies from 0.947 to 0.991, and for the 254 mm depth, these ratios vary from 0.91-0.989. For the 14-gauge thickness, the M_{FEM}/M_n ratio decreases with the increase of section depth from 203 mm to 305 mm and varies from 0.876 to 0.95. For the 16-gauge and 18-gauge thicknesses, the

M_{FEM}/M_n ratio decreases with an increase in depth from 203 mm to 254 mm and varies from 0.93 to 0.962 and 0.95 to 0.99, respectively. The M_{FEM}/M_n ratio increases with the increase of section depth from 254 mm to 305 mm for 16-gauge and 18-gauge thicknesses.

For the section depth of 305 mm with thicknesses of 18-gauge and 16-gauge, and for the section depth of 254 mm with 18-gauge thickness, the web depth to thickness ratio (h/t) is higher than 200. CSA-S136-01 is therefore not applicable to determine the effective width of webs with holes under stress gradient for these sections. In this study, in order to calculate the nominal moment capacity (M_n), the effective width of web with $h/t \geq 200$ was determined according to the Clause B2.4 of CSA-S136-01 (CSA-S136-01, 2001).

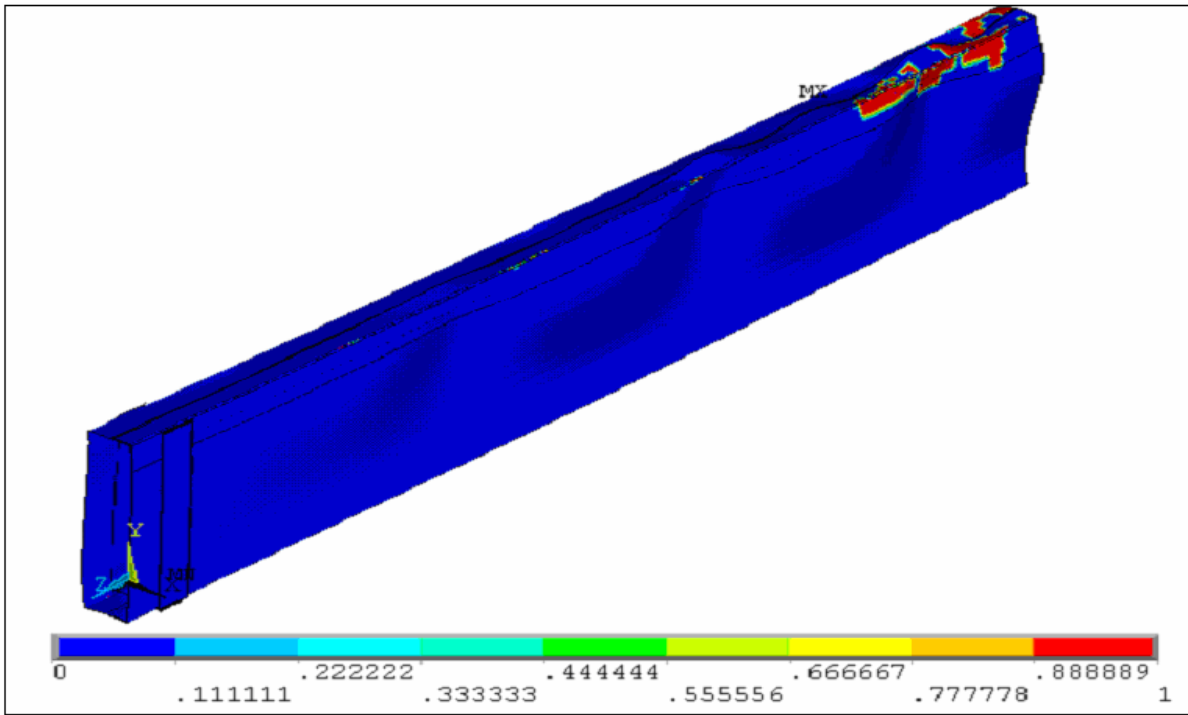


Figure 4.21 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 305-ST-18-S300-BP-F1)

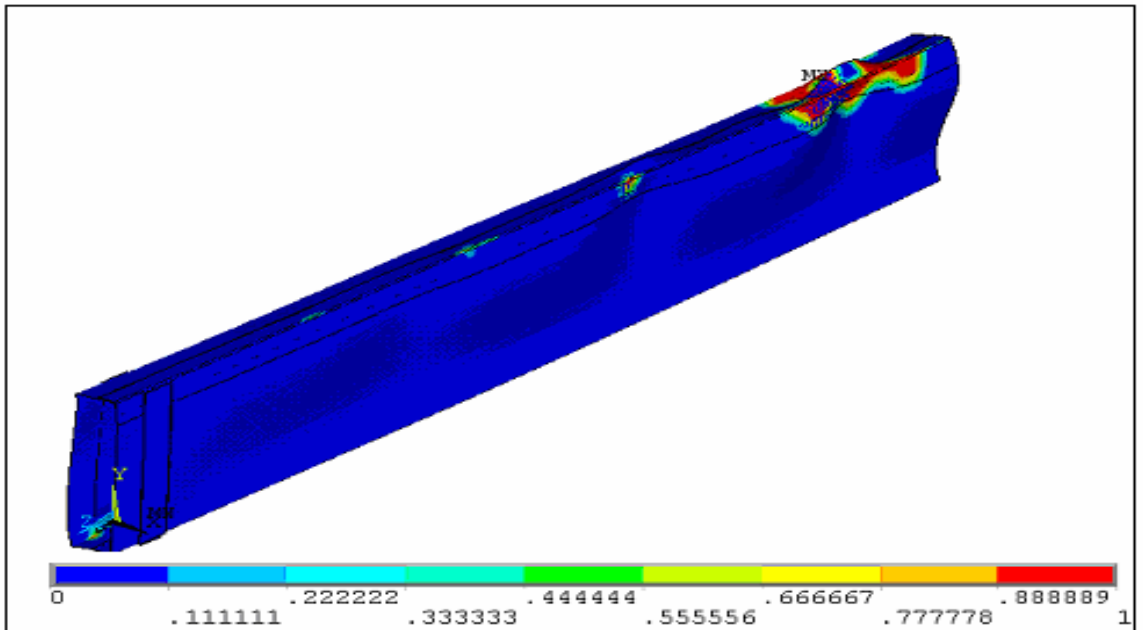


Figure 4.22 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 305-ST-18-S300-BP-F2)

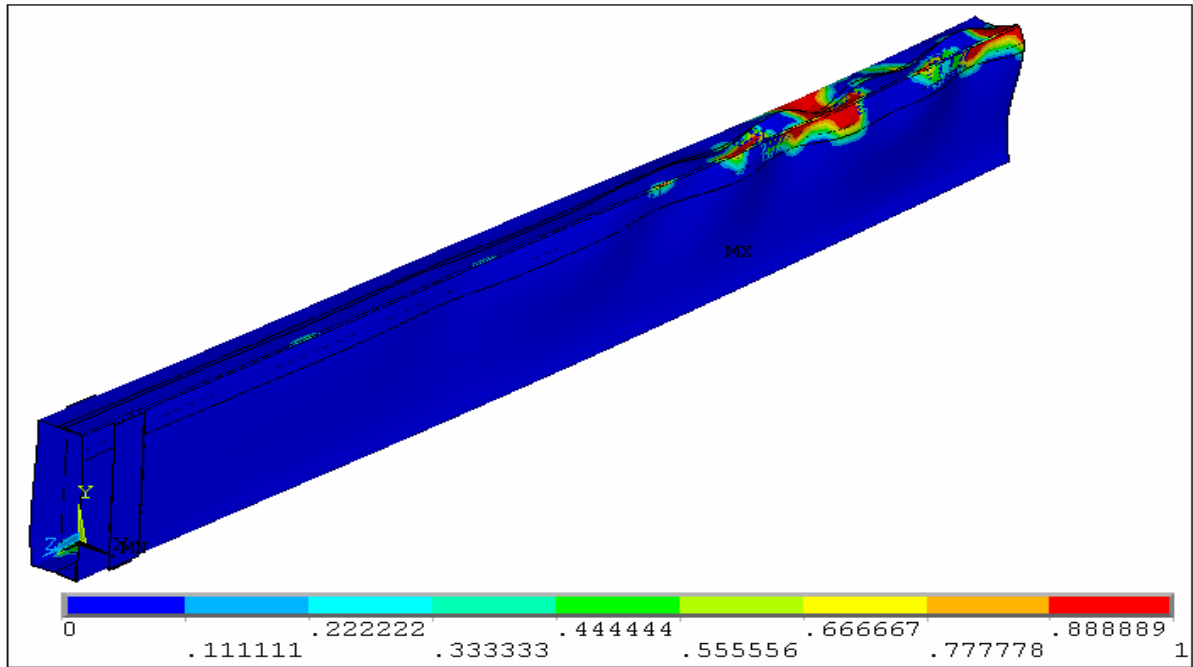


Figure 4.23 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 254-ST-18-S300-BP-F1)

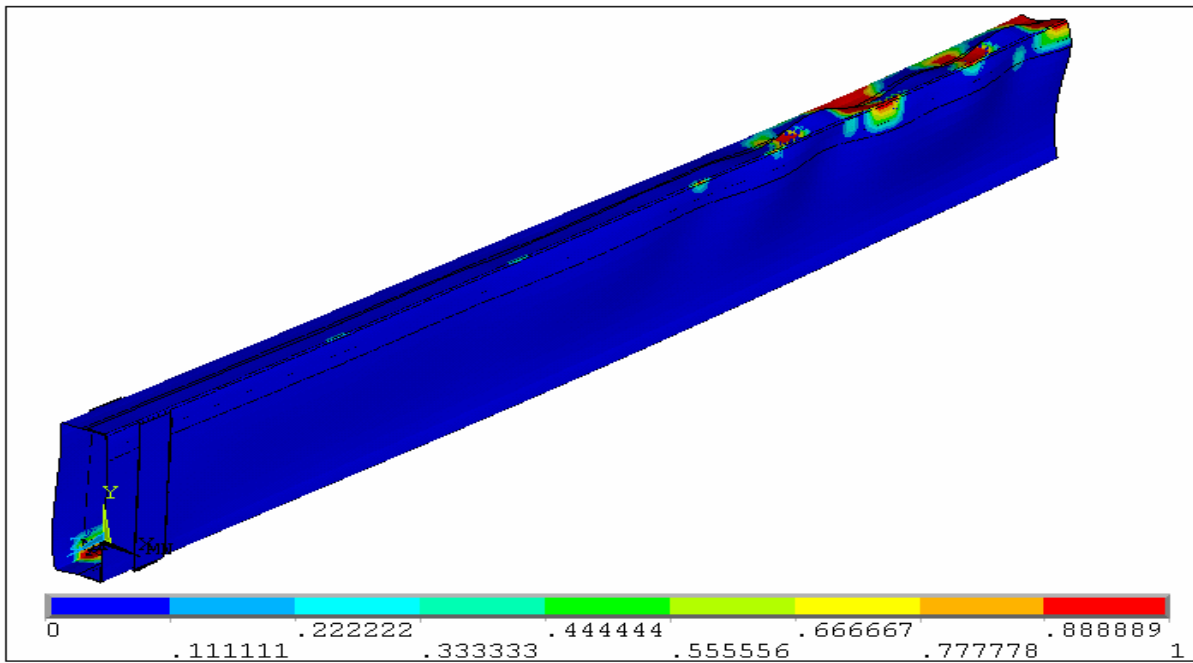


Figure 4.24 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 254-ST-18-S300-BP-F2)

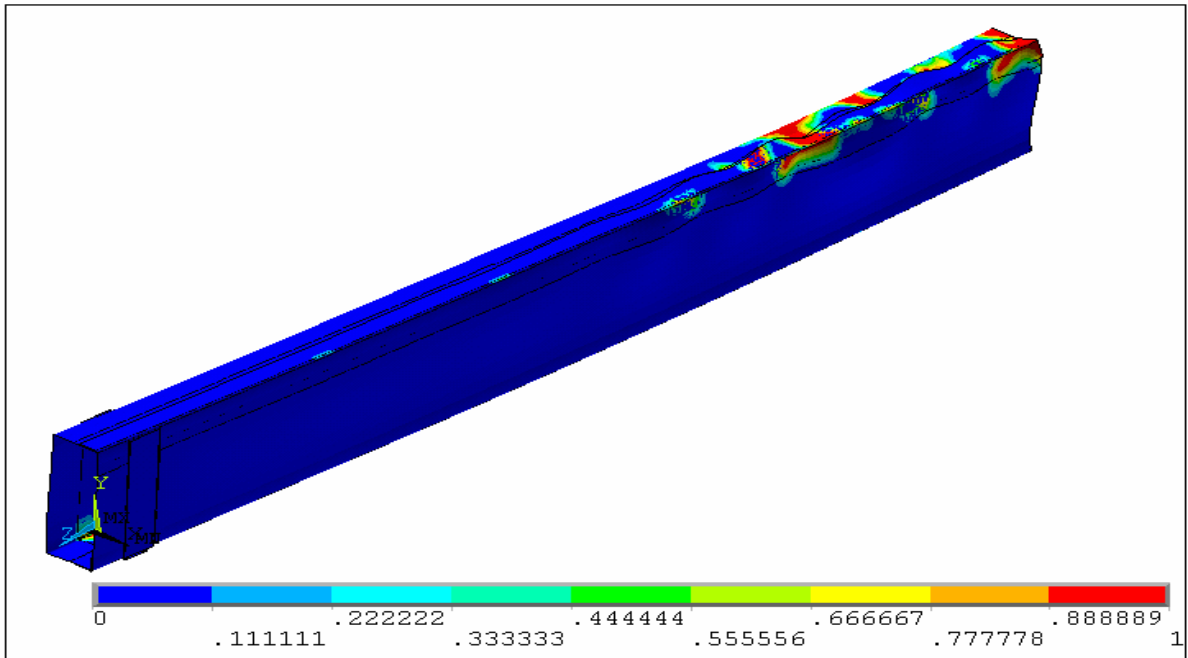


Figure 4.25 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 203-ST-18-S300-BP-F1)

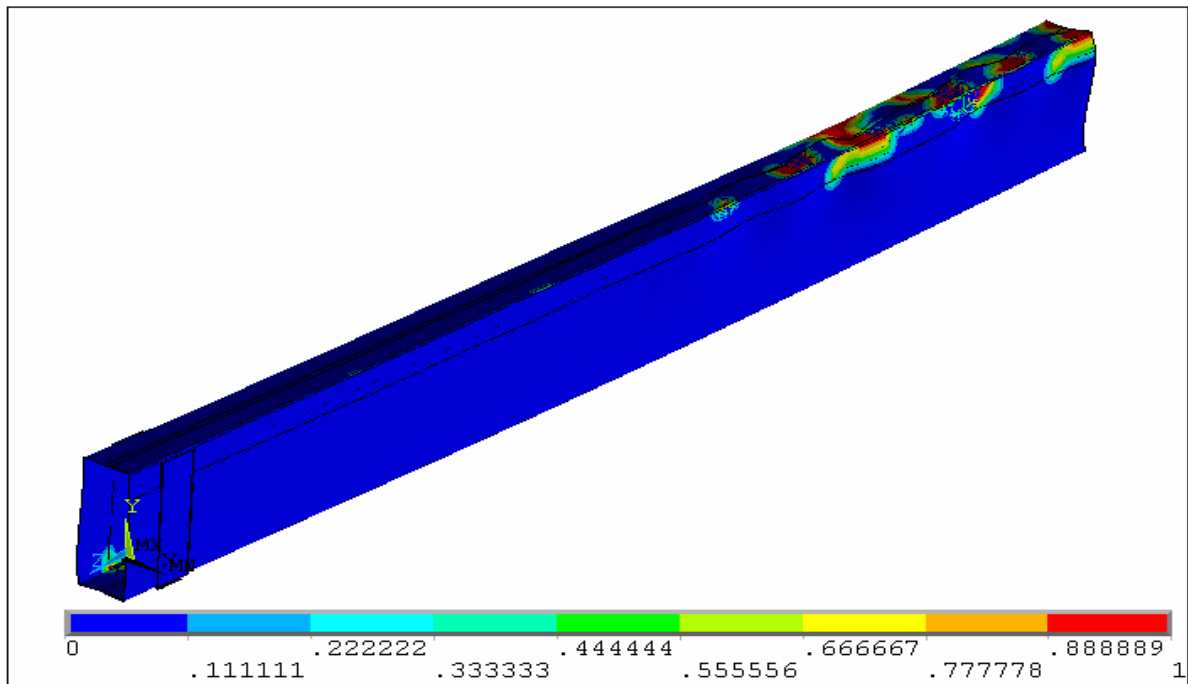


Figure 4.26 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 203-ST-18-S300-BP-F2)

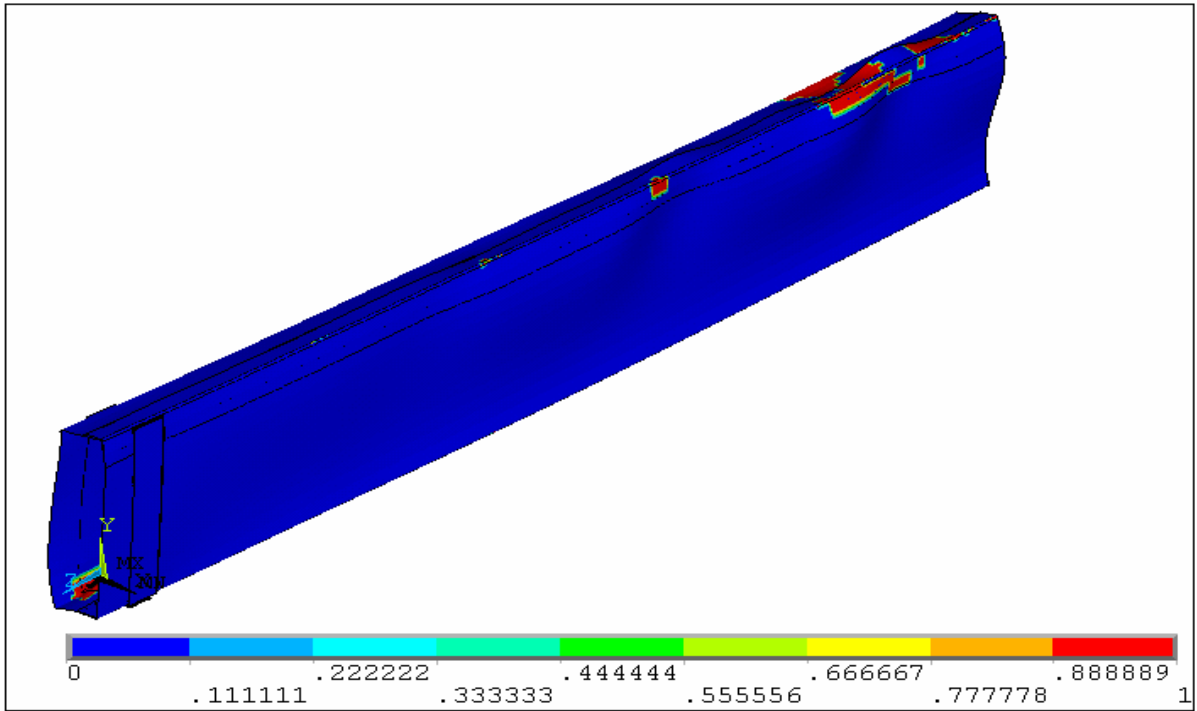


Figure 4.27 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 305-ST-16-S300-BP-F1)

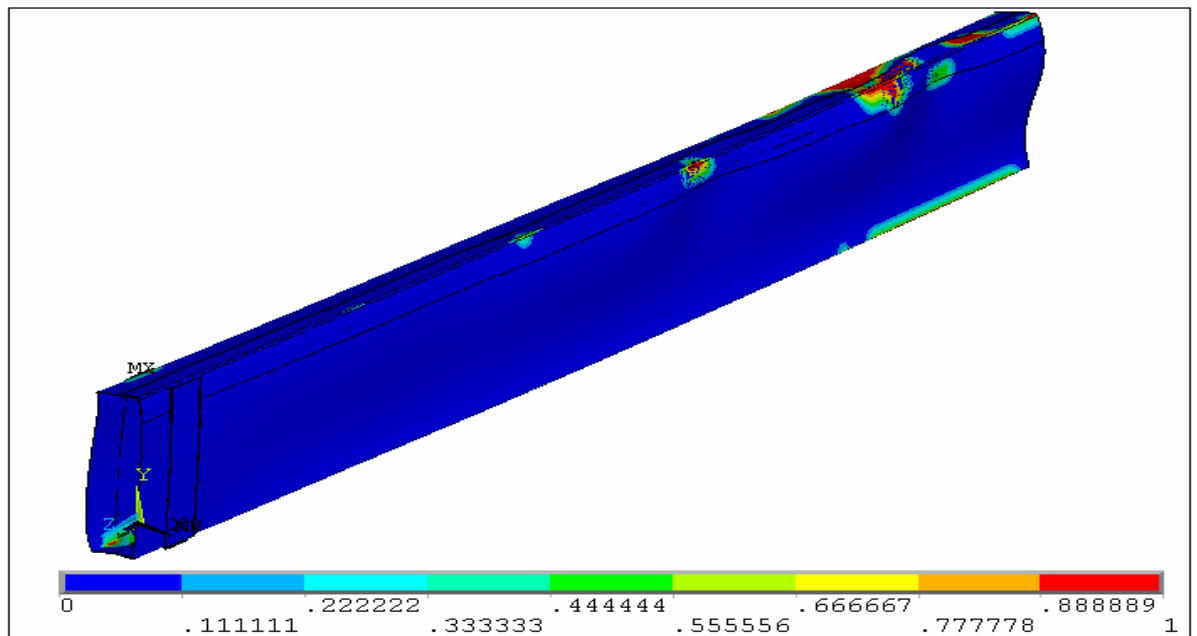


Figure 4.28 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 305-ST-16-S300-BP-F2)

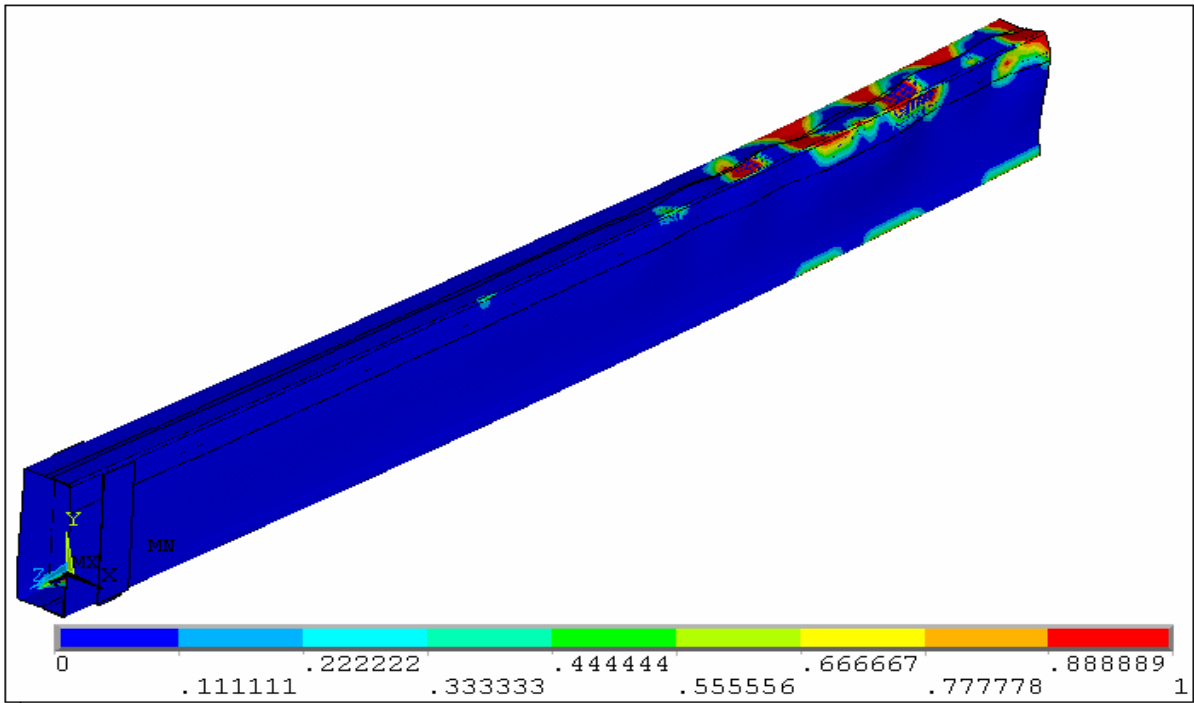


Figure 4.29 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 203-ST-16-S300-BP-F1)

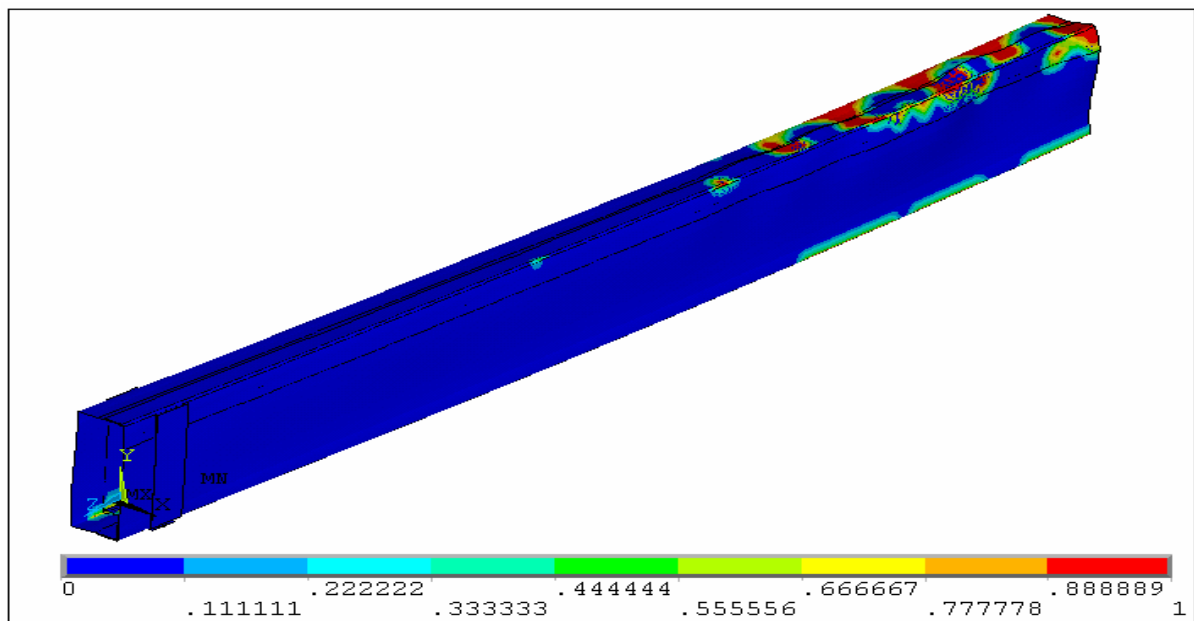


Figure 4.30 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 203-ST-16-S300-BP-F2)

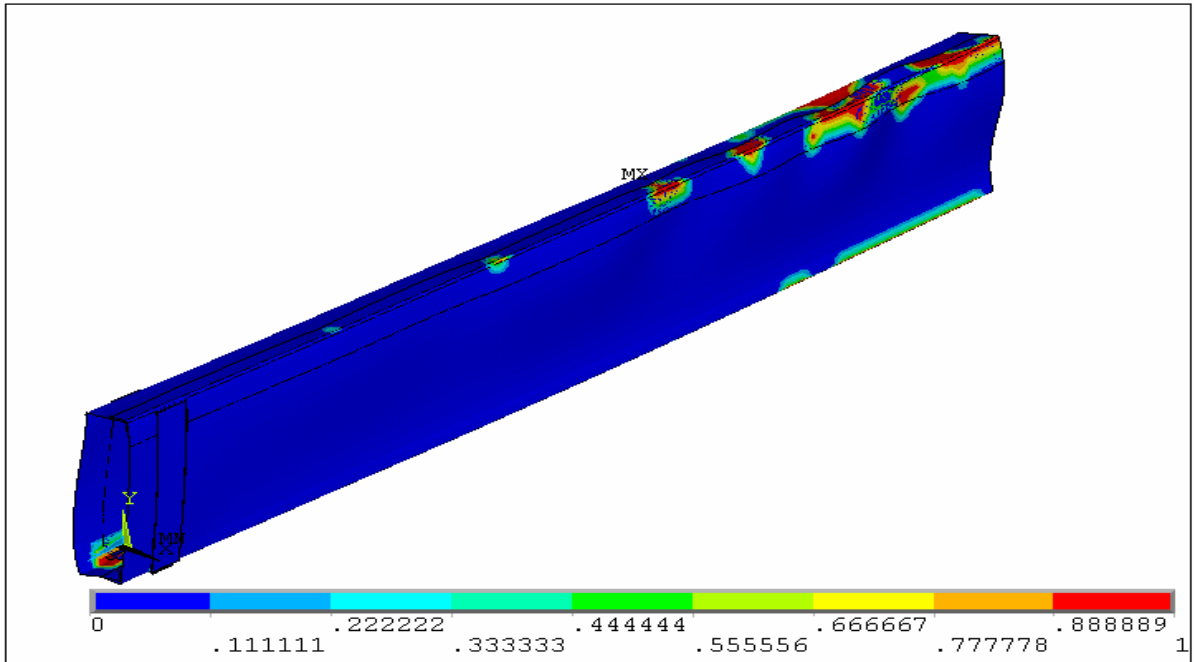


Figure 4.31 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 305-ST-14-S300-BP-F1)

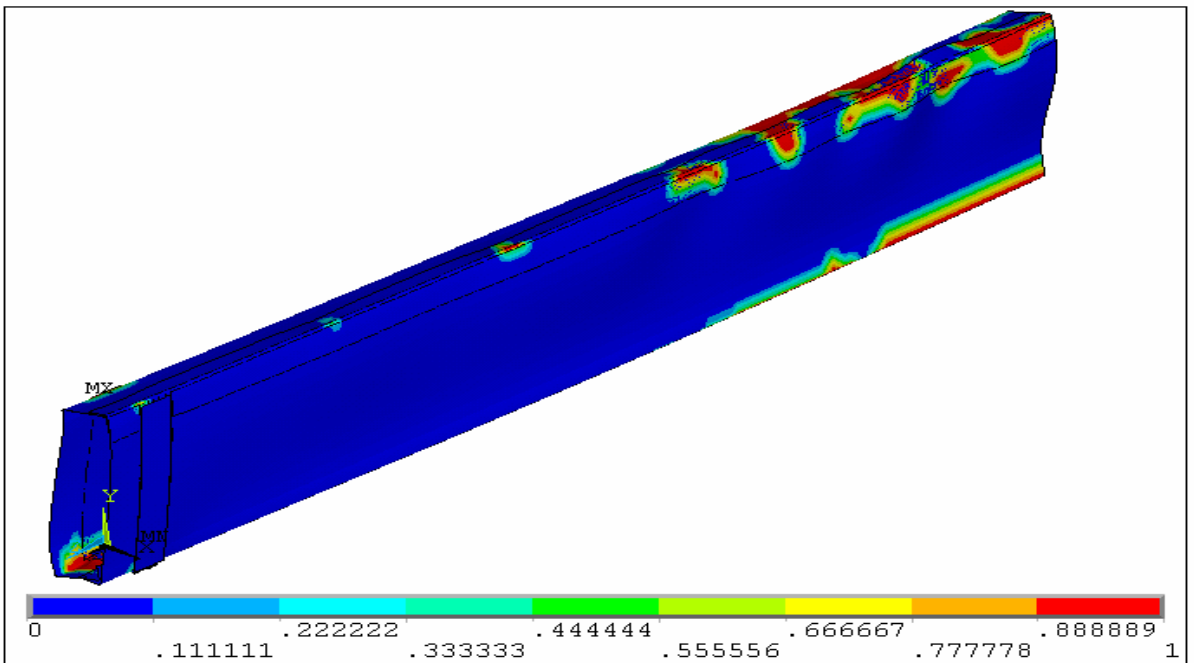


Figure 4.32 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 305-ST-14-S300-BP-F2)

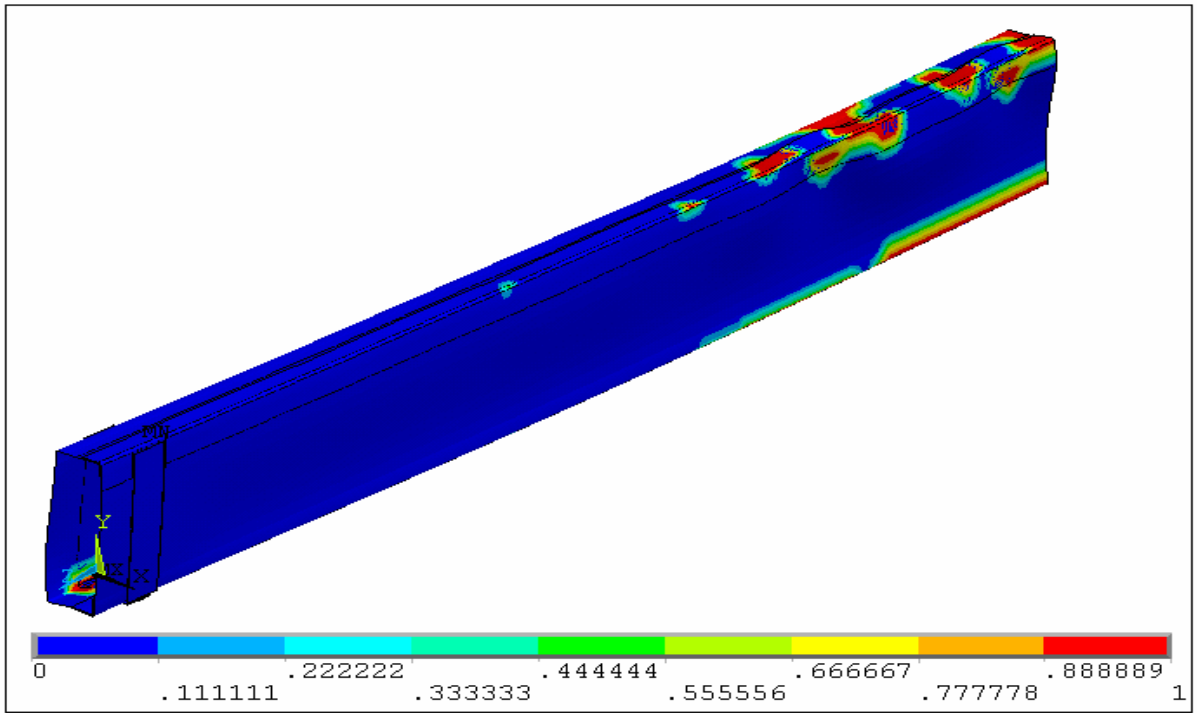


Figure 4.33 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 254-ST-14-S300-BP-F1)

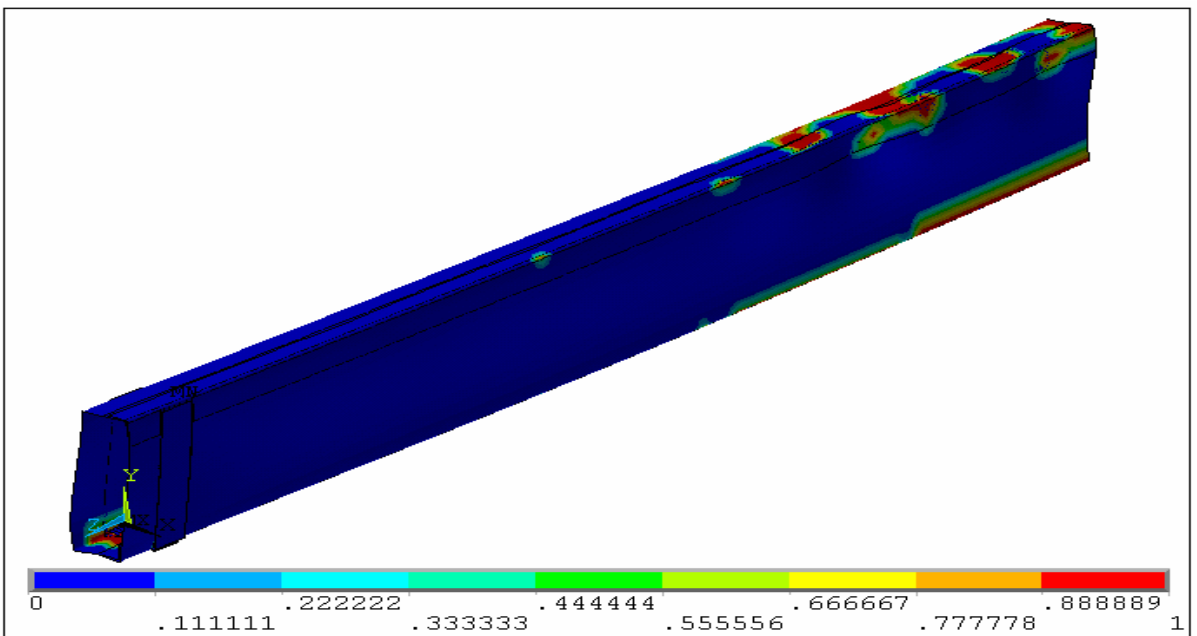


Figure 4.34 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 254-ST-14-S300-BP-F2)

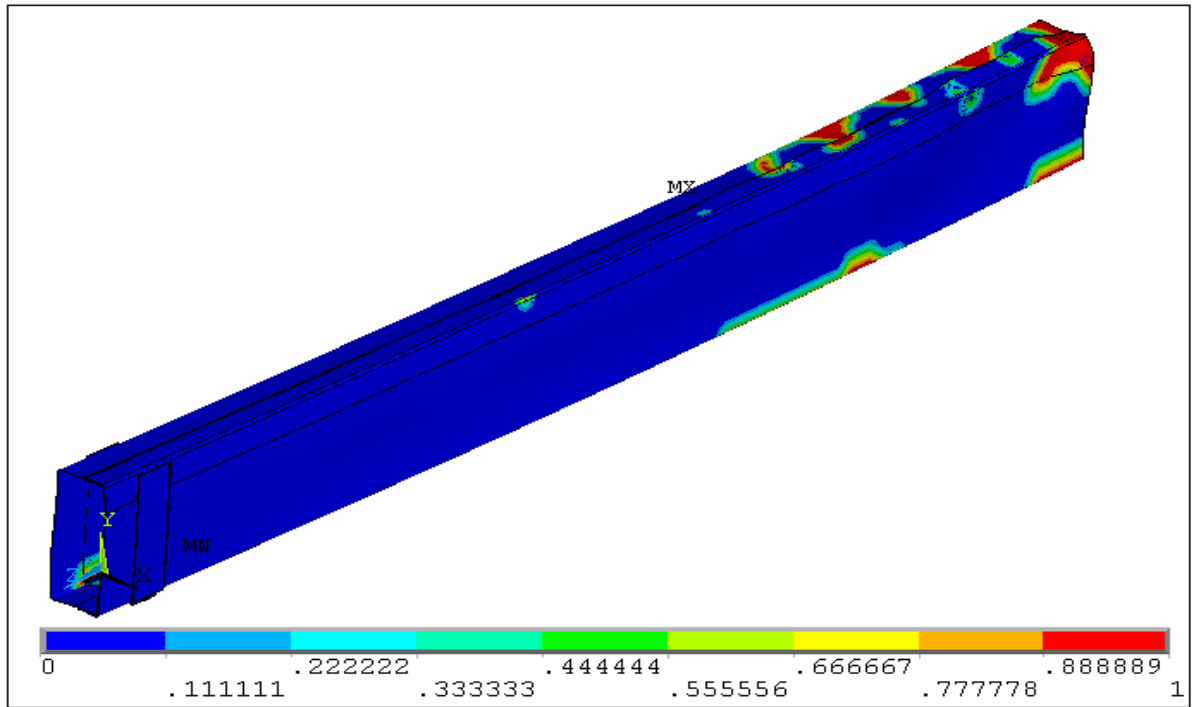


Figure 4.35 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 203-ST-14-S300-BP-F1)

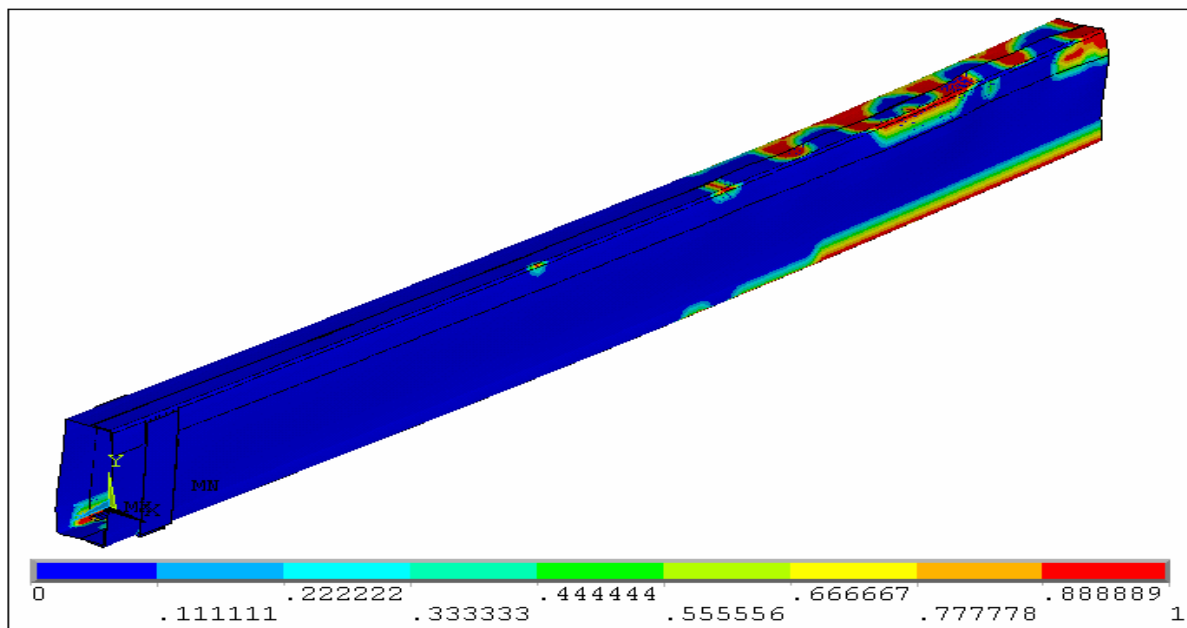


Figure 4.36 Stress state ratio at ultimate moment capacity of the built-up box girder (Model: 203-ST-14-S300-BP-F2)

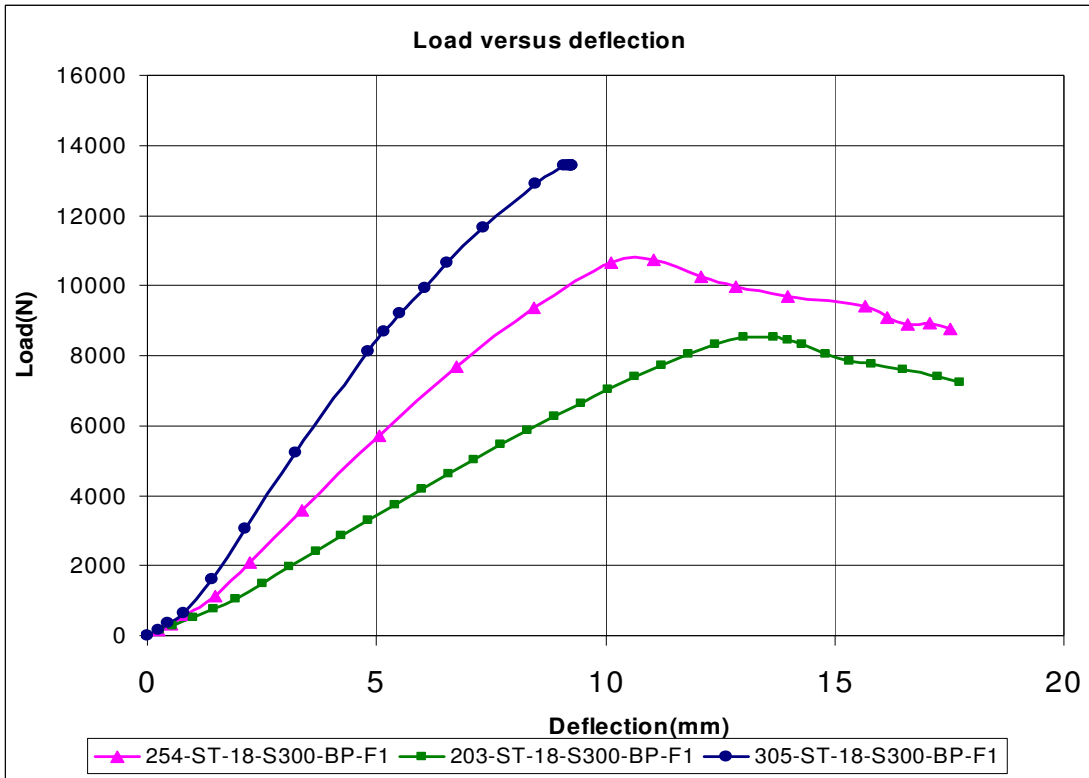


Figure 4.37 Load versus mid-span deflection curve

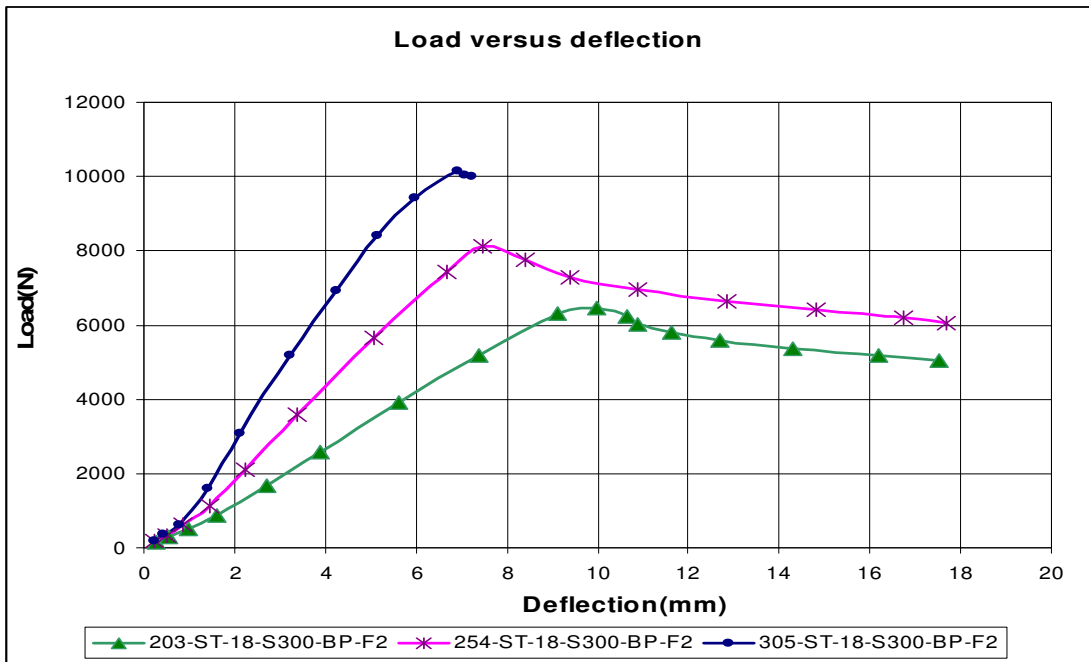


Figure 4.38 Load versus mid-span deflection curve

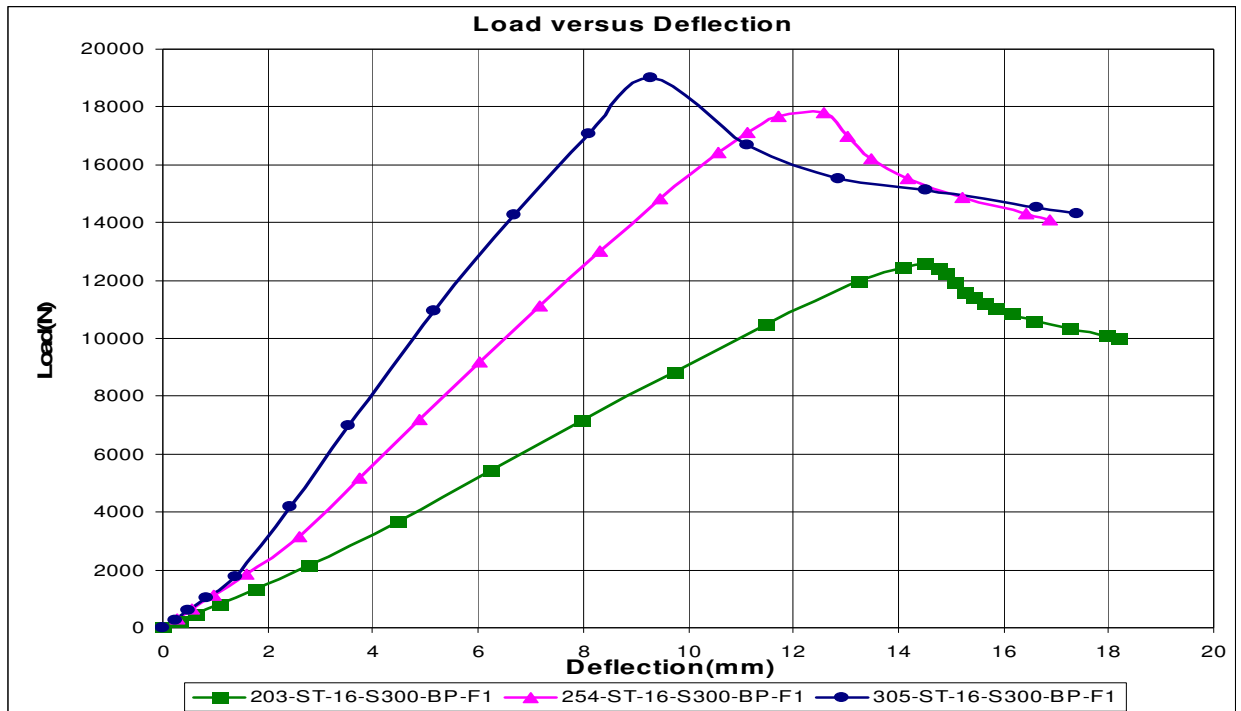


Figure 4.39 Load versus mid-span deflection curve

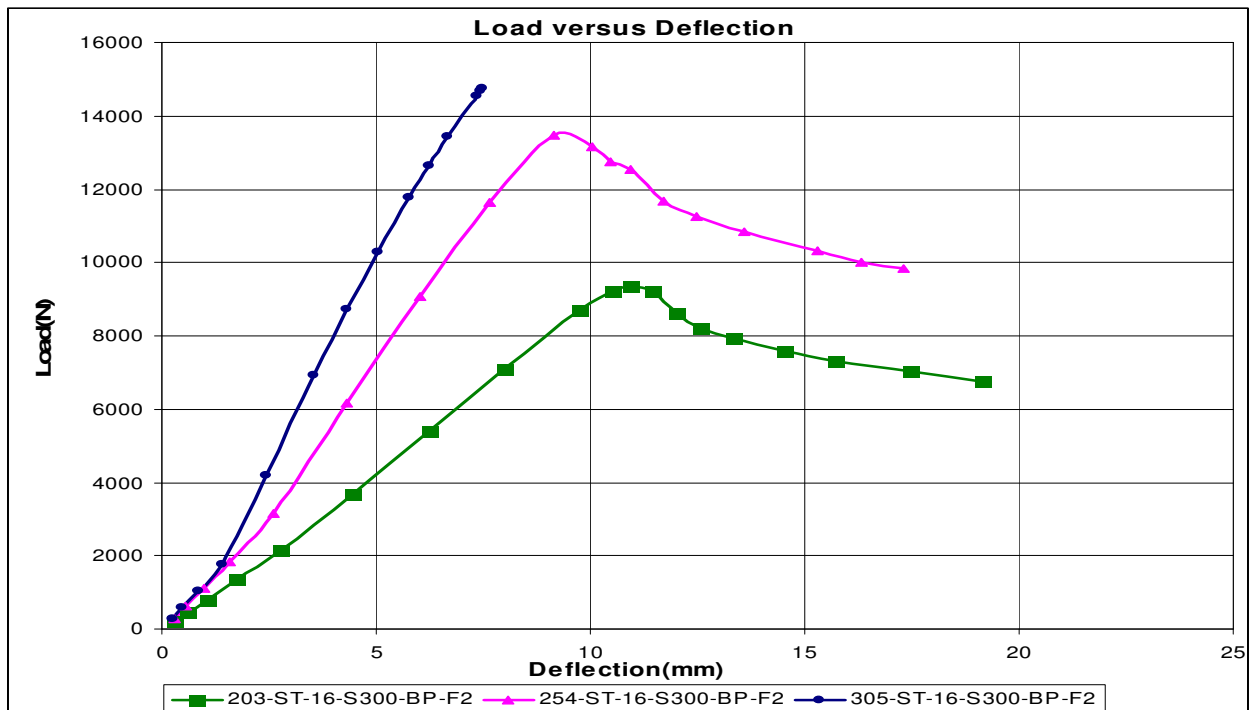


Figure 4.40 Load versus mid-span deflection Curve

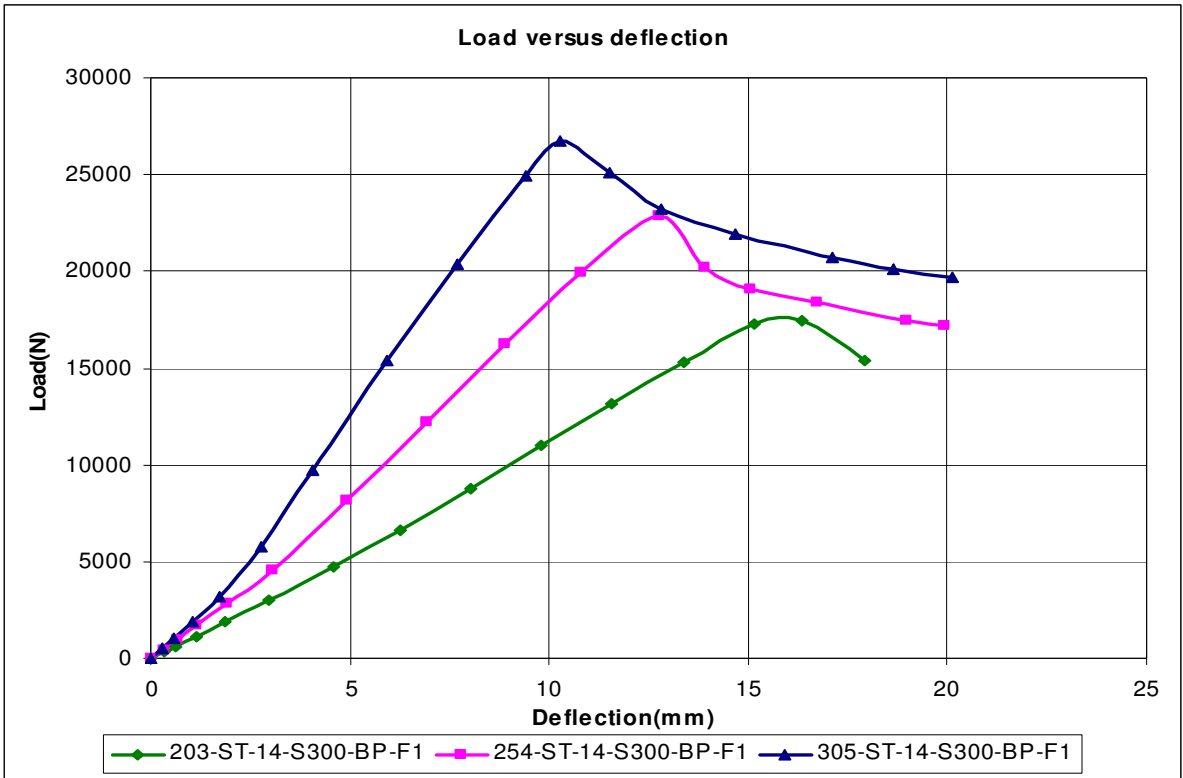


Figure 4.41 Load versus mid-span deflection curve

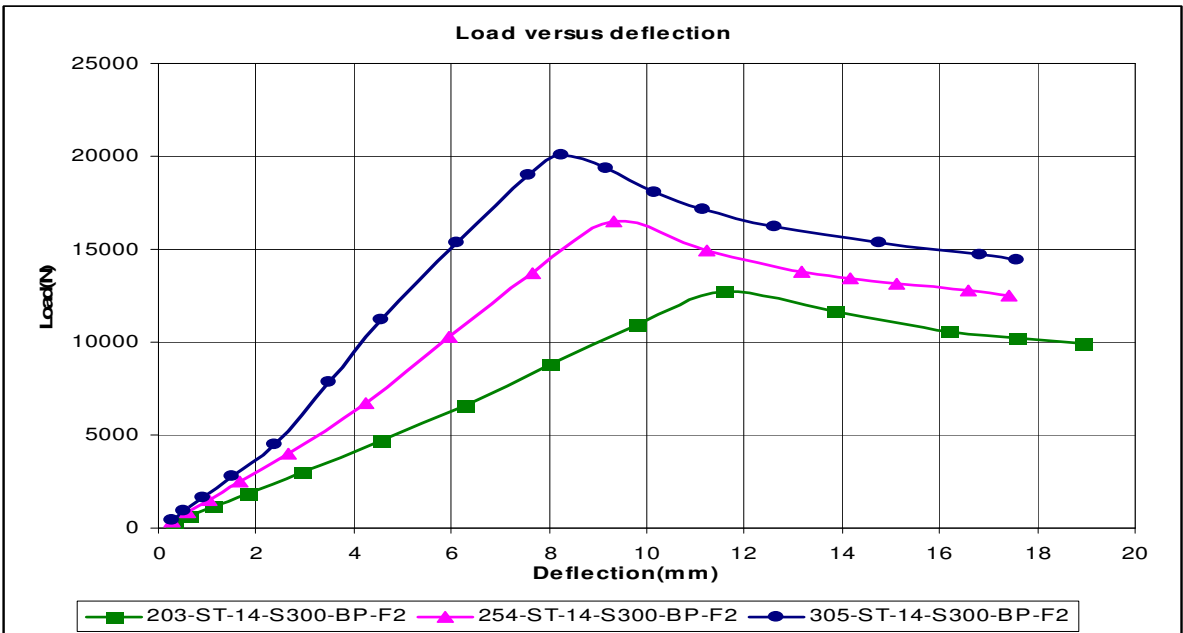


Figure 4.42 Load versus mid-span deflection curve

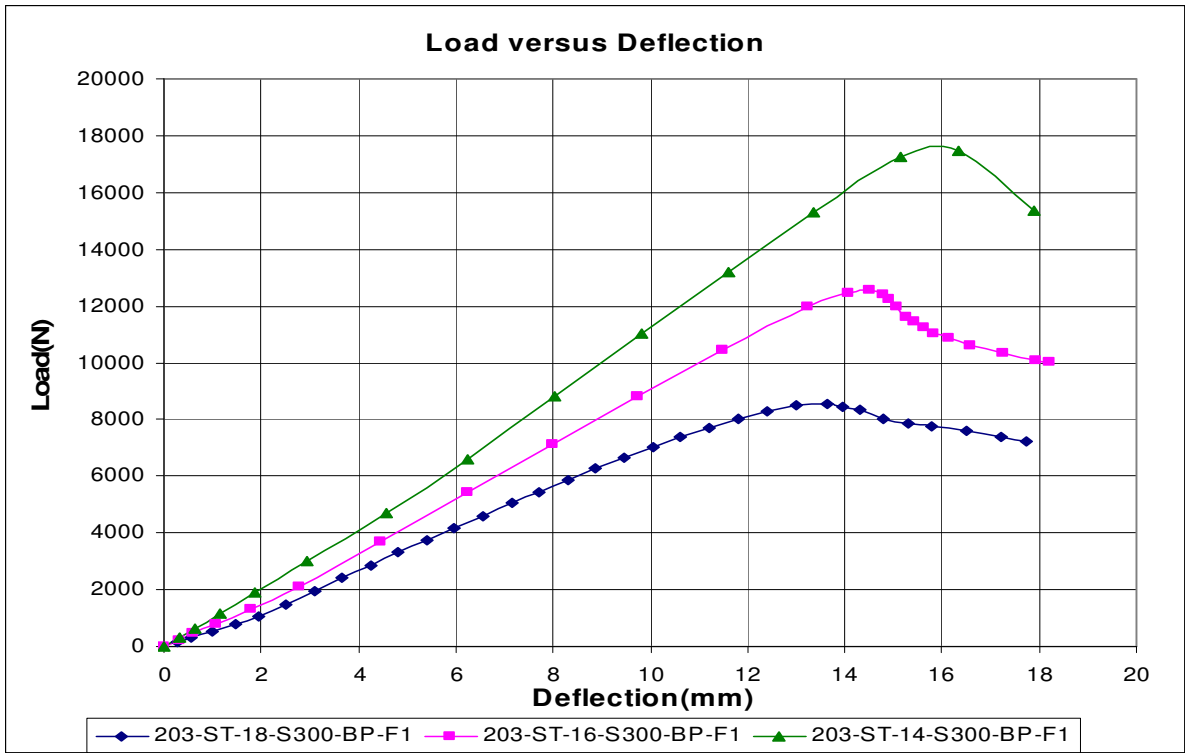


Figure 4.43 Load versus mid-span deflection curve

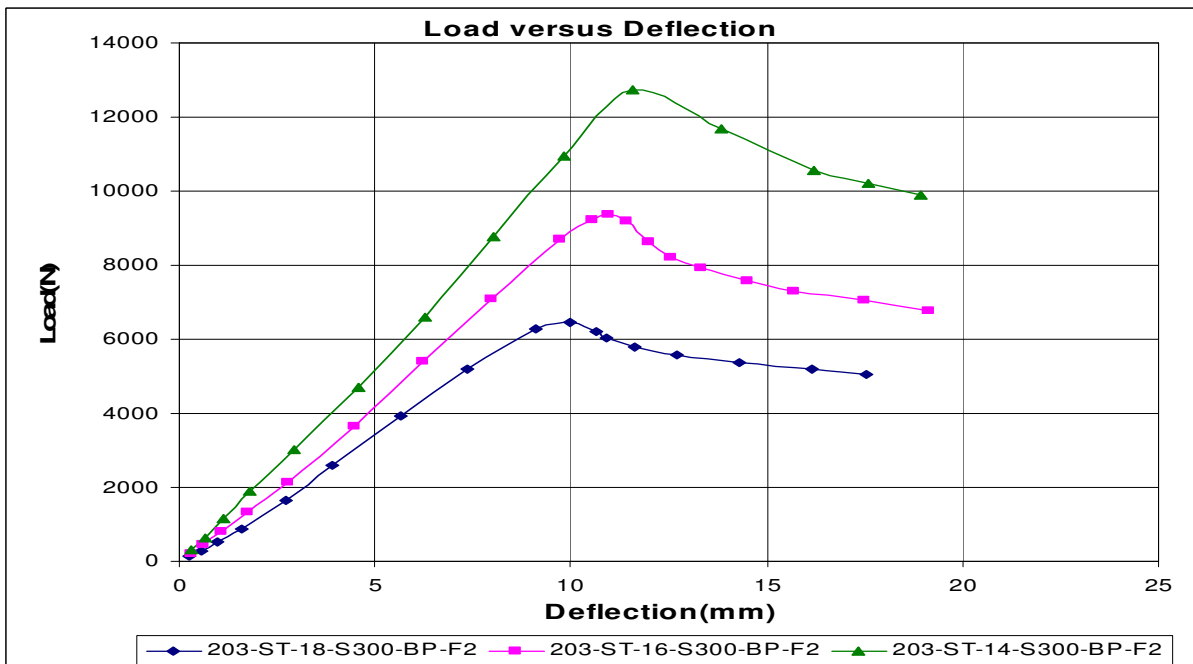


Figure 4.44 Load versus mid-span deflection curve

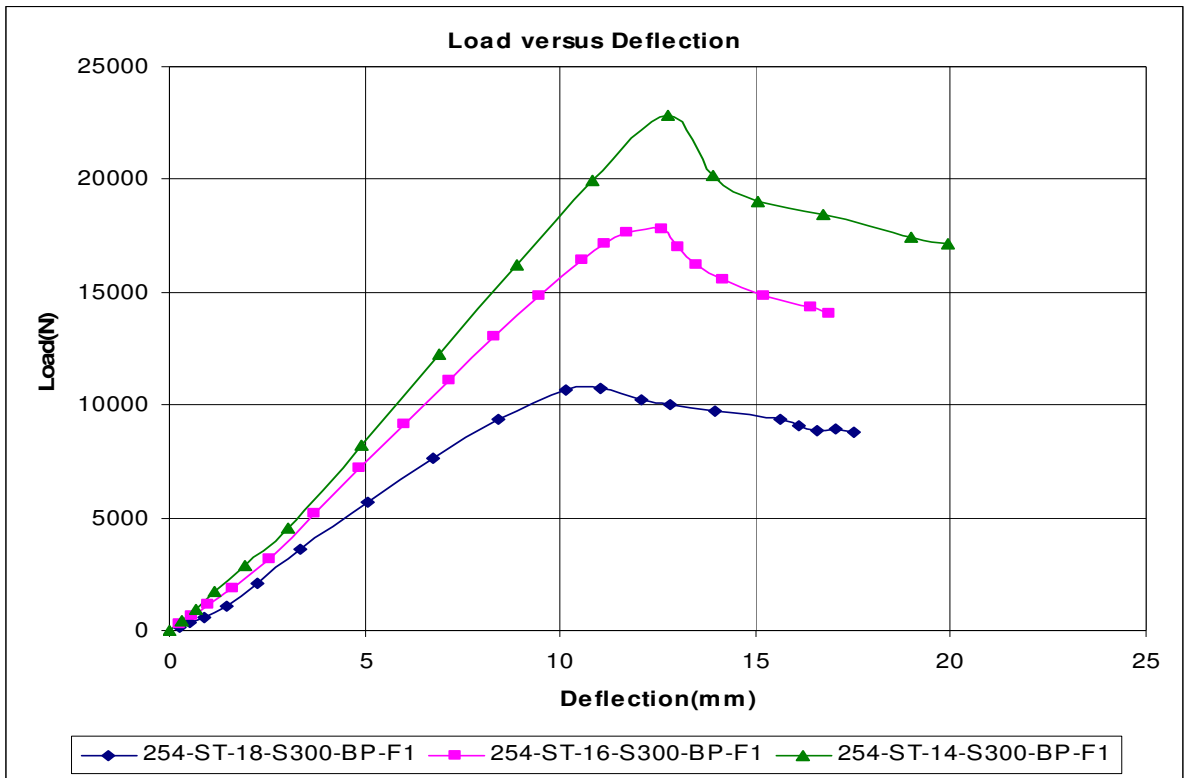


Figure 4.45 Load versus mid-span deflection curve

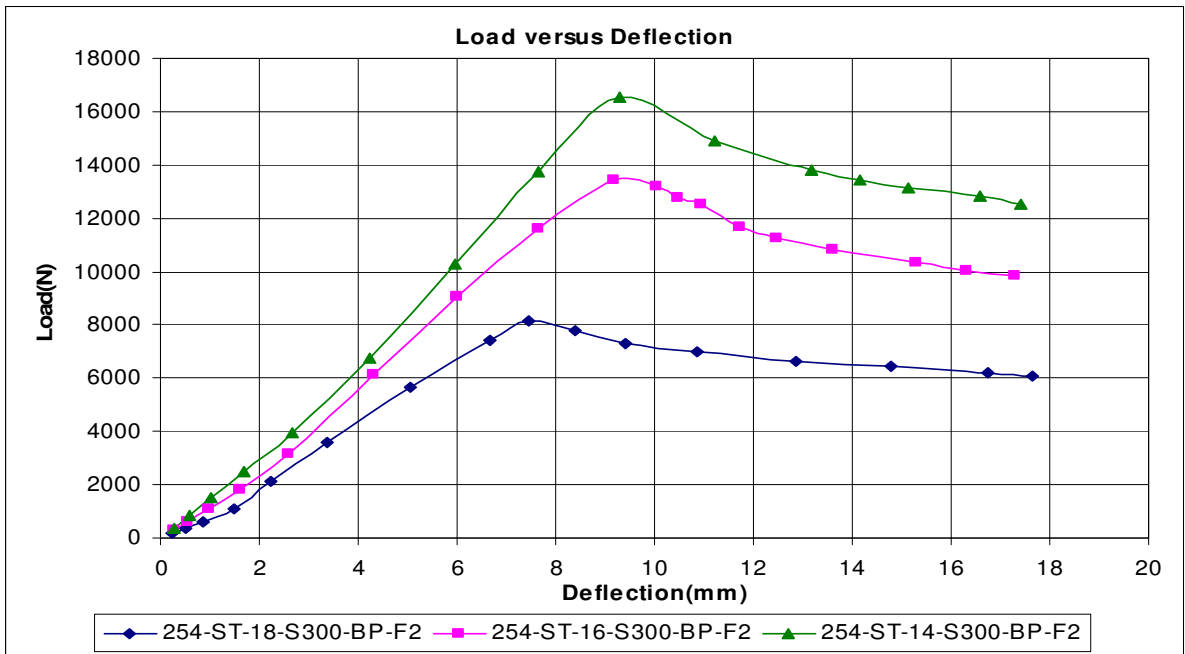


Figure 4.46 Load versus mid-span deflection curve

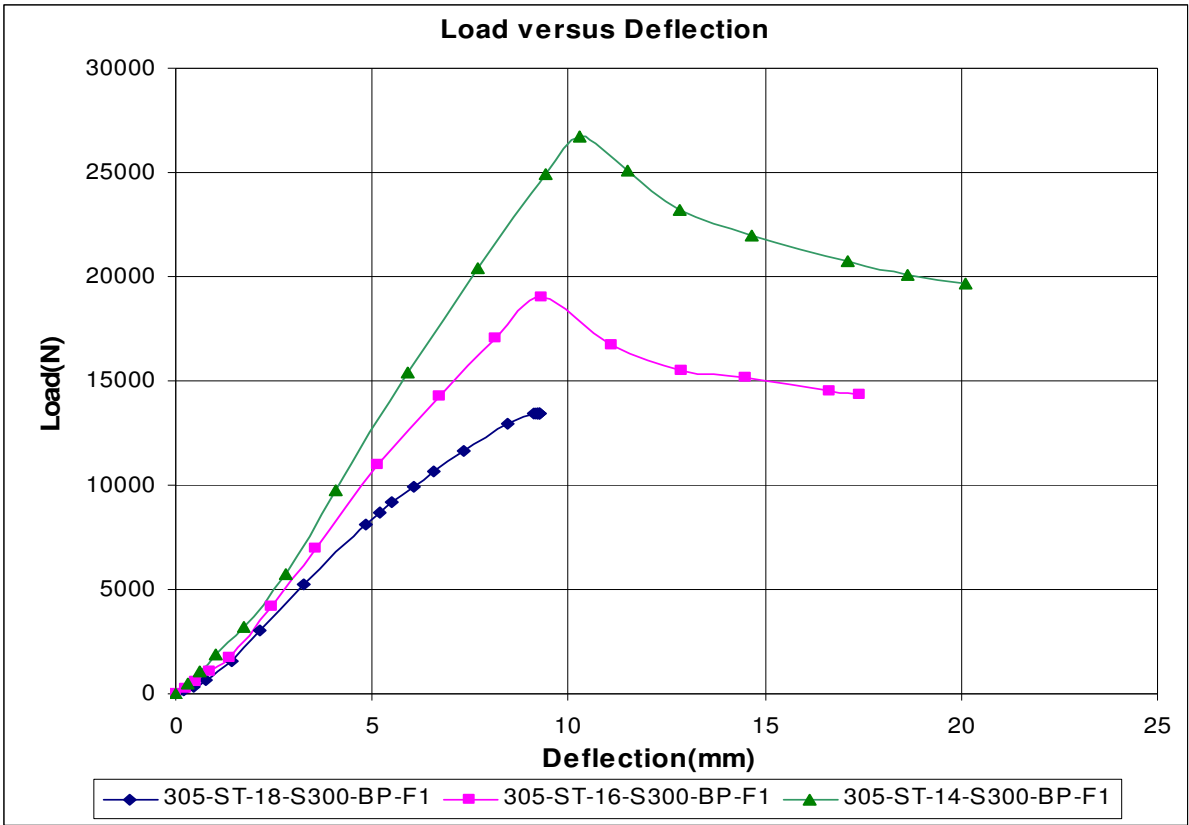


Figure 4.47 Load versus mid-span deflection curve

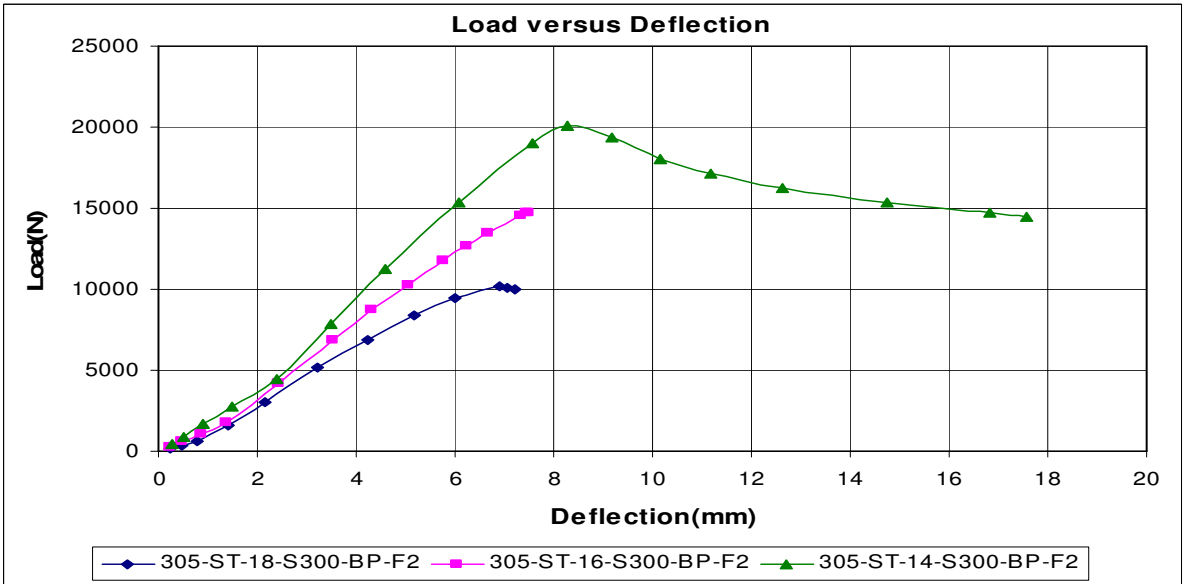


Figure 4.48 Load versus mid-span deflection curve

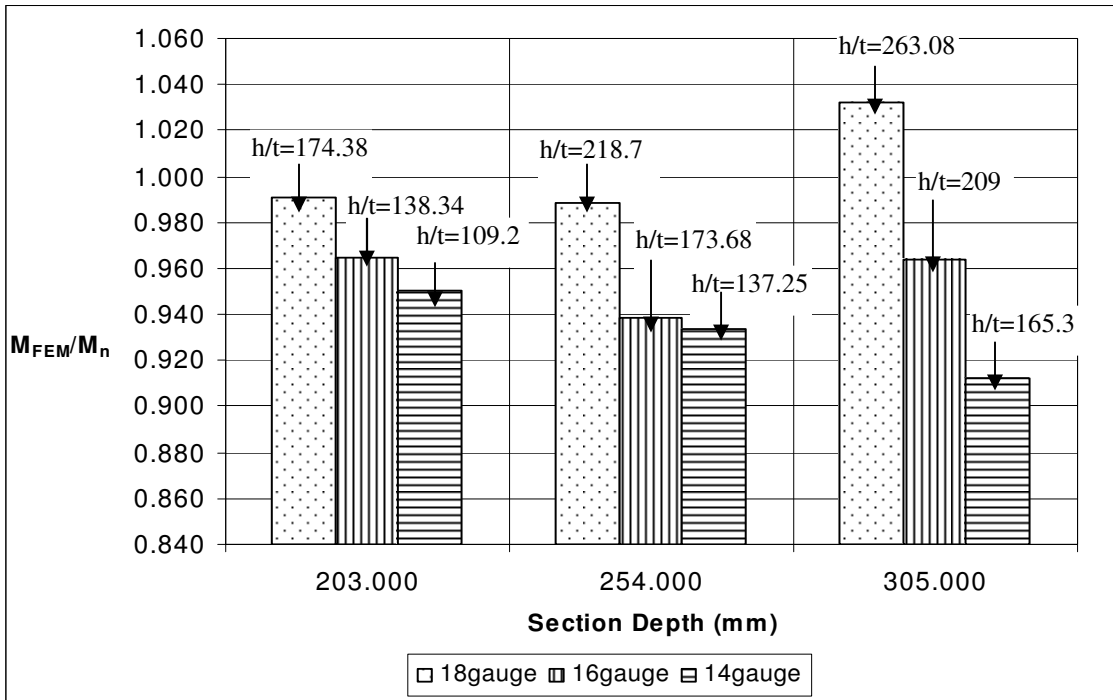


Figure 4.49 M_{FEM}/M_n for CFS built-up box sections of different section depth and thickness
(F_y for stud=349 MPa, for track = 307 MPa)

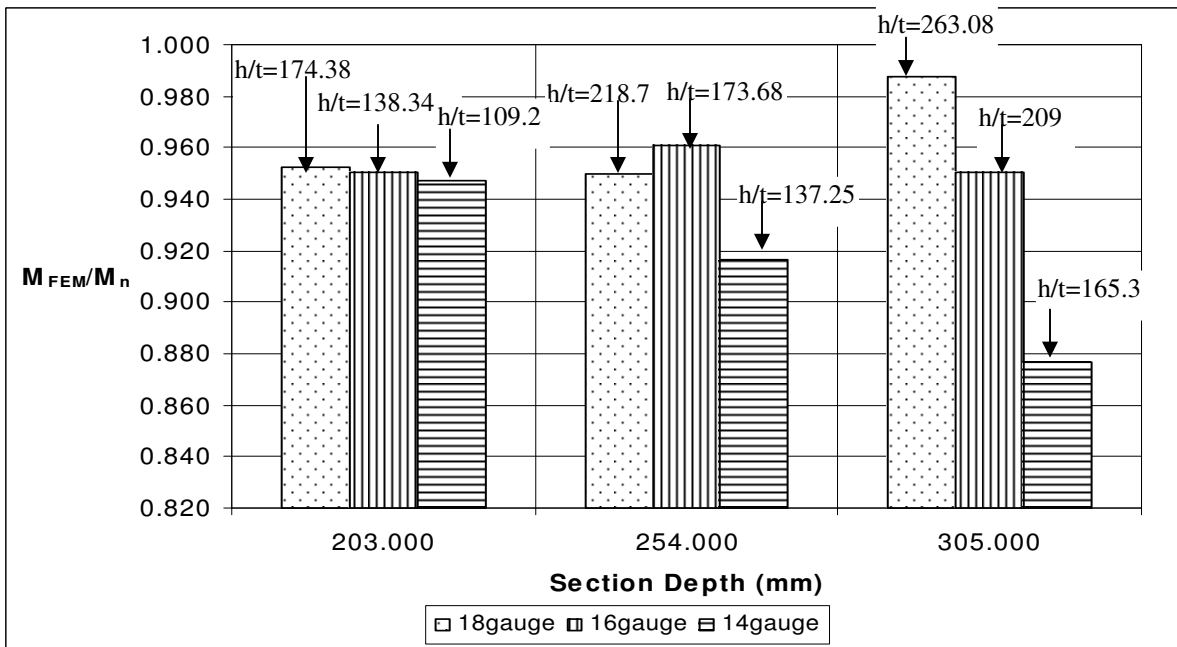


Figure 4.50 M_{FEM}/M_n for CFS built-up box sections of different section depth and thickness
(F_y for both stud and track = 228 MPa)

4.4.2 Parametric studies of flange screw spacing variation

The influence of screw spacing was investigated for screw spacing of 150 mm, 300 mm and 600 mm, while keeping all other factors the same for each finite element analysis. The sections with 16-gauge and 14-gauge thickness were chosen with depths of 305 mm (12 in), 254 mm (10 in) and 203 mm (8 in). A total of eighteen FE analyses were conducted, and the results obtained are shown in Table 4.6.

Table 4.6 Effect of flange screw variation on ultimate moment capacity

Model description	Web depth to thickness ratio of track section h/t	M_{FEM} kN.m	CSA-S136-01 nominal moment		Nominal moment of built-up sections M_n kN.m	M_{FEM}/M_n
			M_{stud} kN.m	M_{track} kN.m		
203-ST-16-S600-BP-F2	138.34	9.979	7.535	3.462	10.997	0.907
203-ST-16-S300-BP-F2	138.34	10.455	7.535	3.462	10.997	0.951
203-ST-16-S150-BP-F2	138.34	10.667	7.535	3.462	10.997	0.970
254-ST-16-S600-BP-F2	173.68	14.066	11.168	4.503	15.671	0.898
255-ST-16-S300-BP-F2	173.68	15.061	11.168	4.503	15.671	0.961
254-ST-16-S150-BP-F2	173.68	15.982	11.168	4.503	15.671	1.020
305-ST-16-S600-BP-F2	209.01	15.698	11.834	5.516	17.350	0.905
305-ST-16-S300-BP-F2	209.01	16.488	11.834	5.516	17.350	0.950
305-ST-16-S150-BP-F2	209.01	16.638	11.834	5.516	17.350	0.959
203-ST-14-S600-BP-F2	109.2	13.425	10.050	4.976	15.026	0.893
203-ST-14-S300-BP-F2	109.2	14.237	10.050	4.976	15.026	0.947
203-ST-14-S150-BP-F2	109.2	14.696	10.050	4.976	15.026	0.978
254-ST-14-S600-BP-F2	137.25	17.746	13.589	6.556	20.145	0.881

Model description	Web depth to thickness ratio of track section h/t	M_{FEM} kN.m	CSA-S136-01 nominal moment		Nominal moment of built-up sections M_n kN.m	M_{FEM}/M_n
			M_{stud} kN.m	M_{track} kN.m		
254-ST-14-S300-BP-F2	137.25	18.468	13588.622	6556.479	20145.101	0.917
254-ST-14-S150-BP-F2	137.25	19.320	13588.622	6556.479	20145.101	0.959
305-ST-14-S600-BP-F2	165.3	22.512	17489.179	8142.440	25631.618	0.878
305-ST-14-S300-BP-F2	165.3	22.466	17489.179	8142.440	25631.618	0.876
305-ST-14-S150-BP-F2	165.3	23.265	17489.179	8142.440	25631.618	0.908
					Average	0.931
					Std_Dev.	0.041
					Coefficient of variation	0.043

From Table 4.6, it is observed that the ultimate moment capacity of the CFS built-up box girders is influenced by the screw spacing in that M_{FEM} decreases as the screw spacing is increased. The effect of screw spacing on the ultimate capacity of CFS built-up box girders was investigated by comparing the M_{FEM}/M_n ratios for different screw spacings with all the other factors held constant. The M_{FEM}/M_n ratios were then plotted with respect to girder depth for different screw spacings for section thicknesses of 16-gauge and 14-gauge, as shown in Figures 4.51 and 4.52, respectively. The M_{FEM}/M_n ratio reduces with an increase in the screw spacing, which means the effectiveness of the built-up section increases with increasing screw spacing. For section depths of 203 mm, 254 mm, and 305 mm, the M_{FEM}/M_n ratio varies from 0.907 to 0.97, 0.898 to 1.02, and 0.905 to 0.959 for the 16-gauge thickness and from 0.893 to 0.978, 0.881 to 0.959 and 0.878 to 0.908 for the 14-gauge, respectively, for the three different screw spacing. For the 600 mm screw spacing, the

ultimate moment capacity of the CFS built-up girder is very low, whereas for the screw spacings of 300 mm and 150 mm, the moment capacities are within 4% of each other, except for the girder with a 254 mm depth and 16-gauge thickness. The average M_{FEM}/M_n ratio is equal to 0.931 and standard deviation is 0.041, as shown in Table 4.6.

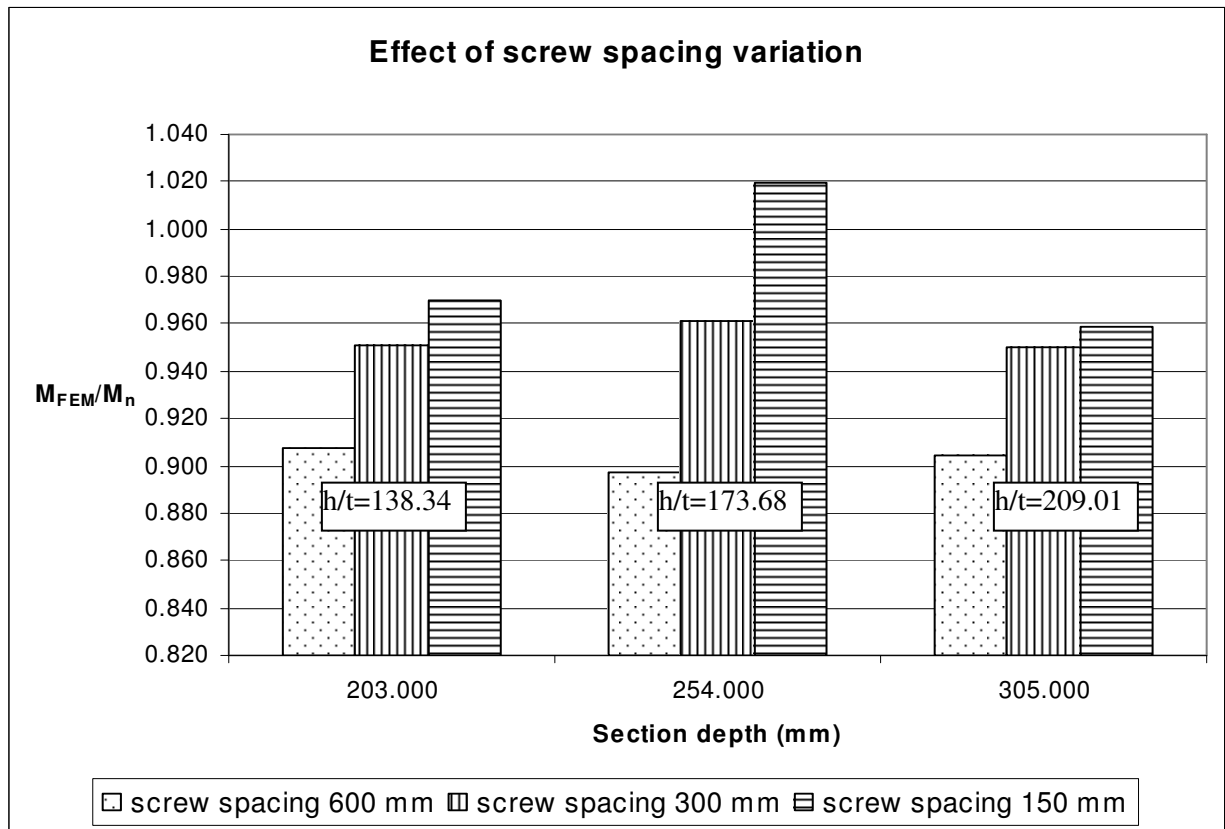


Figure 4.51 Effect of screw spacing variation for 16-gauge material. ($F_y = 228$ MPa)

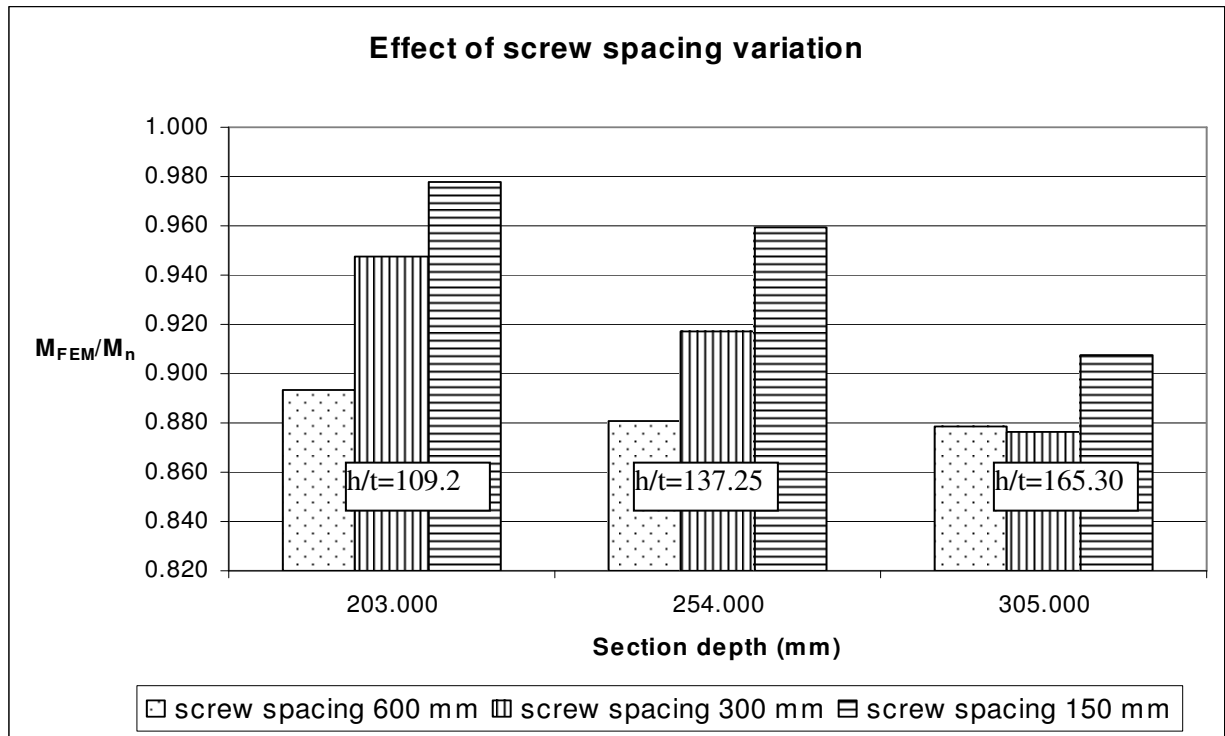


Figure 4.52 Effect of screw spacing variation for 14-gauge material ($F_y = 228$ MPa)

4.4.3 Effect of material yield strength on the ultimate moment capacity

A total eighteen finite element analyses were conducted using two different material properties to investigate the effect of material yield stress on the ultimate moment capacity of CFS built-up box girders. Nine analyses were conducted using a different yield stress for the stud (349 MPa) and track sections (307 MPa), like the test (Beshara and Lawson, 2002).

Another nine analyses were performed using the same yield stress (228 MPa) for both stud and track sections. Figures 4.53 through 4.55 indicate the effect of material yield stress on the load deformation behaviour of the built-up box girders. From these Figures it is clear that decreasing the material yield stress decreases the ultimate moment capacity of the girders.

The slope of the load-deflection curve remains same for the girder models with different

yield stress, which indicates that there is no variation of the stiffness of CFS built-up box sections due to the yield stress variation. The M_{FEM}/M_n ratios are, however, influenced by the material yield stress as shown in Figures 4.56 to 4.58. The M_{FEM}/M_n ratios for all of the girder models decrease with the decrease of material yield stress up to 3%, except for the girder with 254 mm depth and 16-gauge thickness. For the model 254-ST-16-S300-BP, the ratio increases from 0.938 to 0.961 with the decrease of material yield stress, as shown in Figure 4.57.

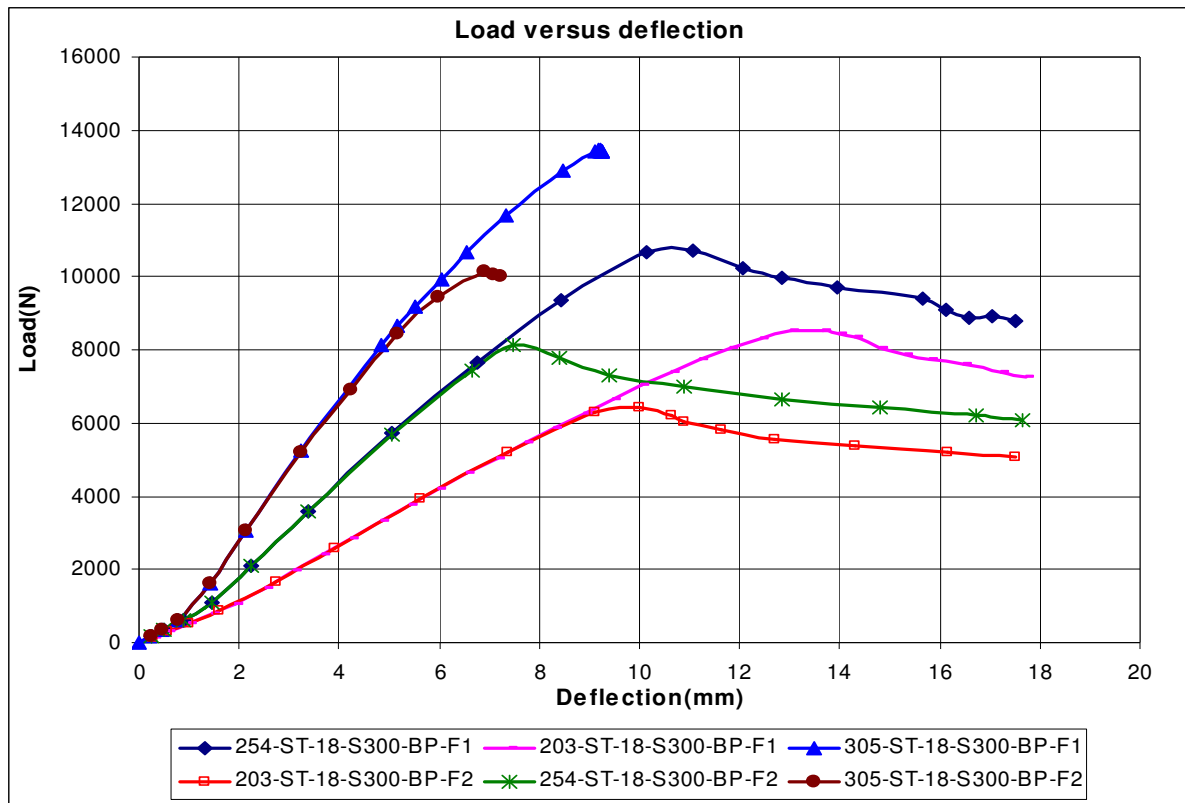


Figure 4.53 Load versus mid-span deflection curve for two types of material

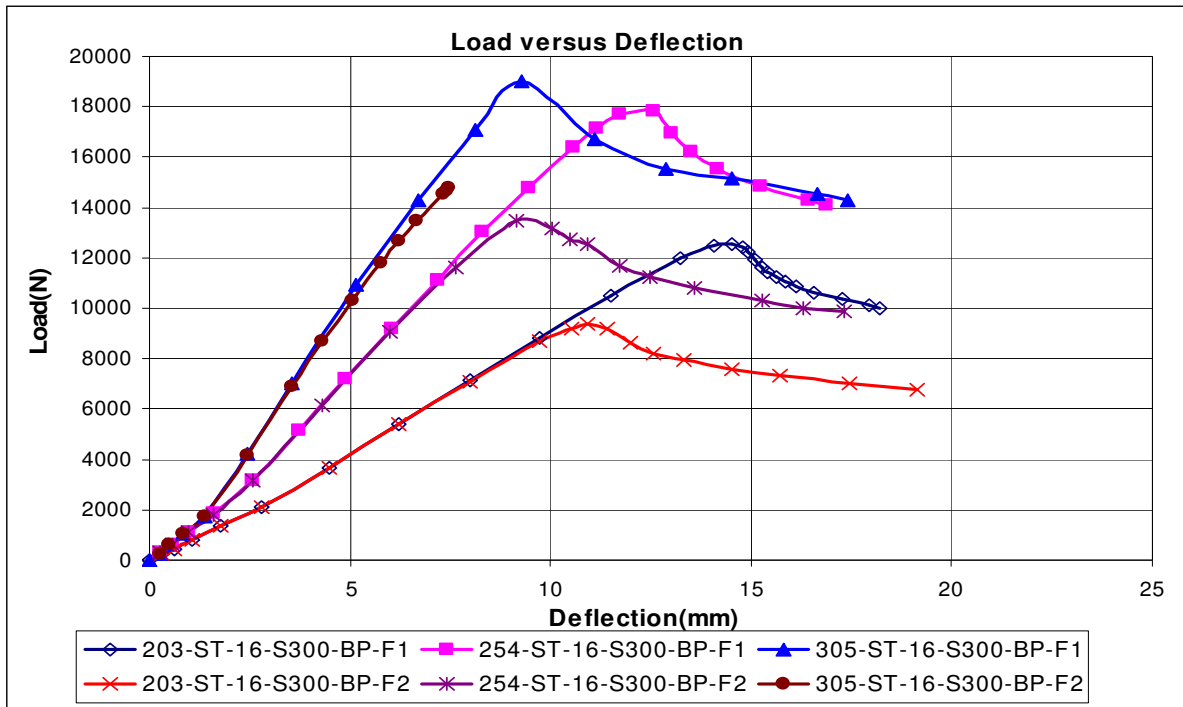


Figure 4.54 Load versus mid-span deflection curve for two types of material

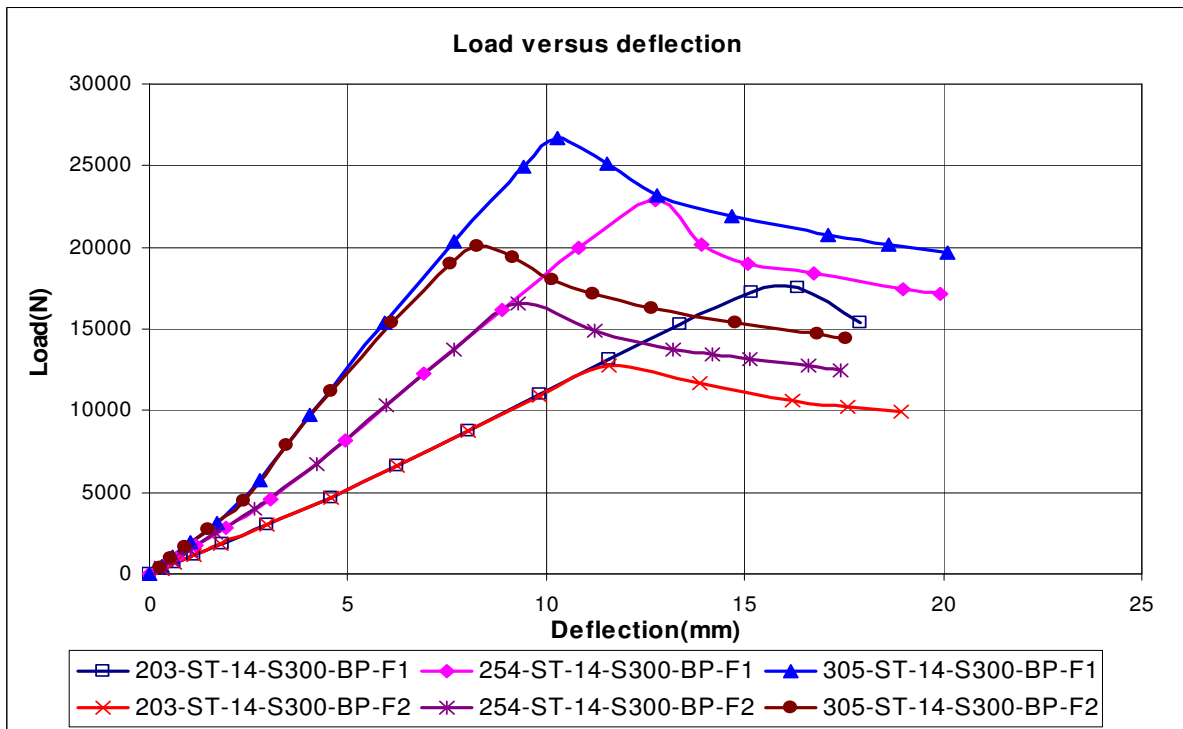


Figure 4.55 Load versus mid-span deflection curve for two types of material

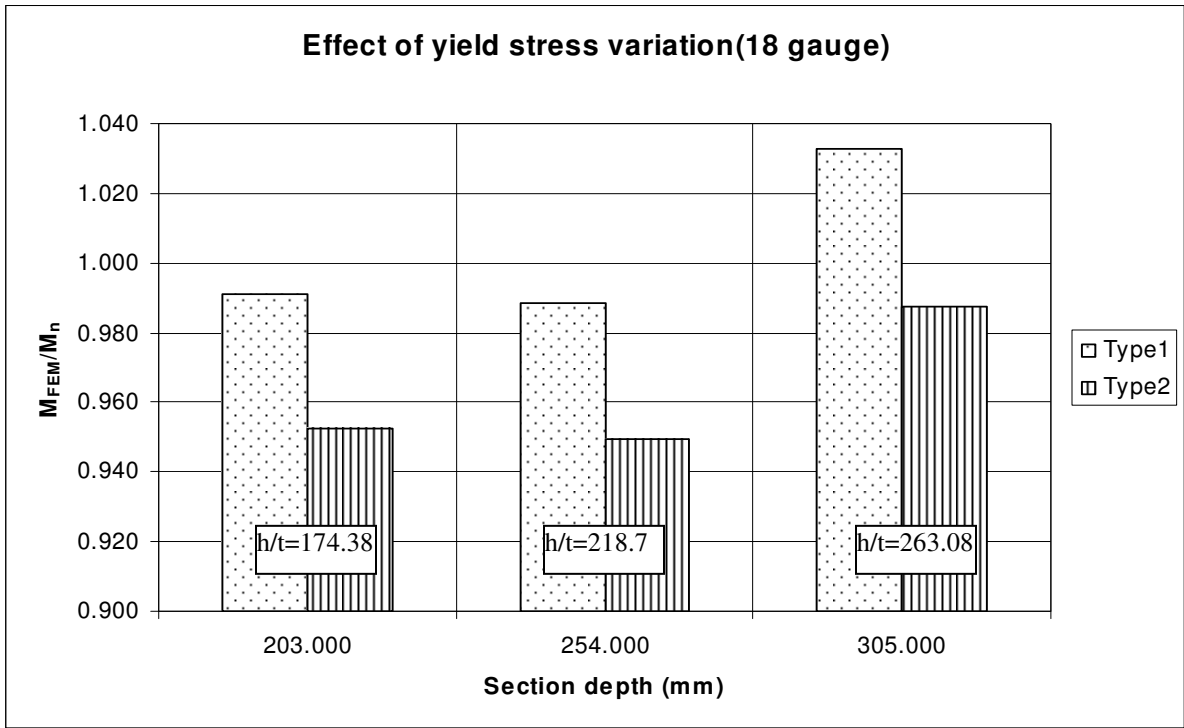


Figure 4.56 Effect of material yield stress on M_{FEM}/M_n ratio

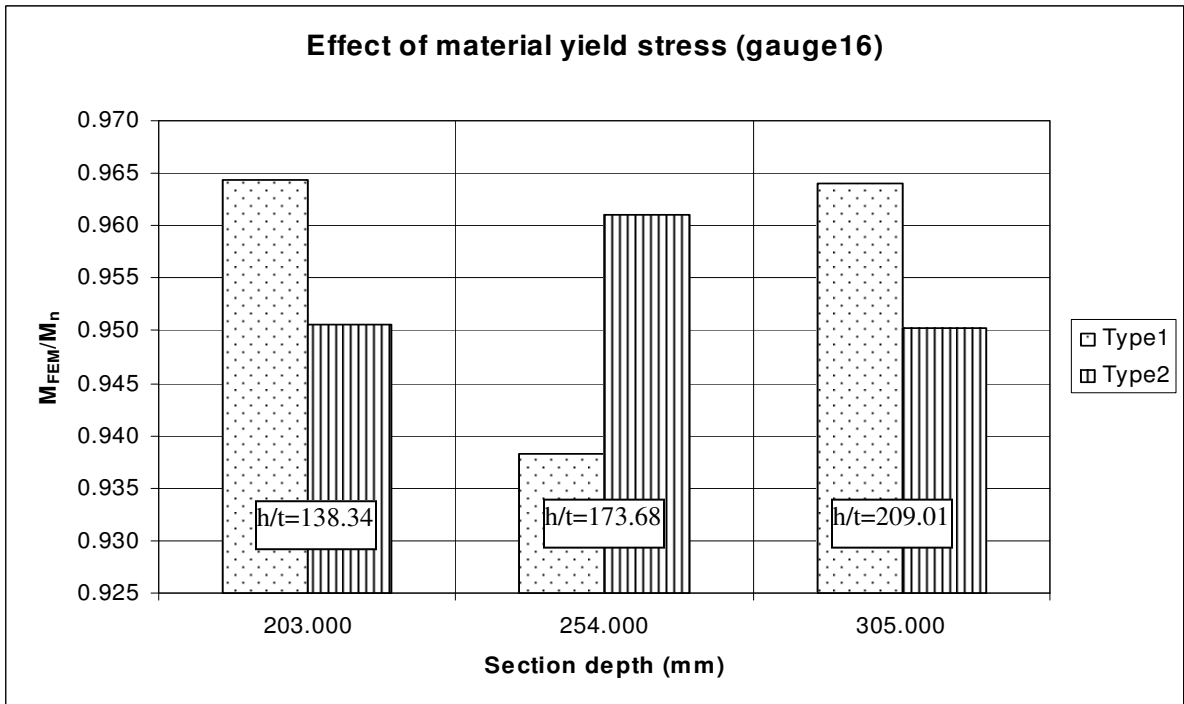


Figure 4.57 Effect of material yield stress on M_{FEM}/M_n ratio

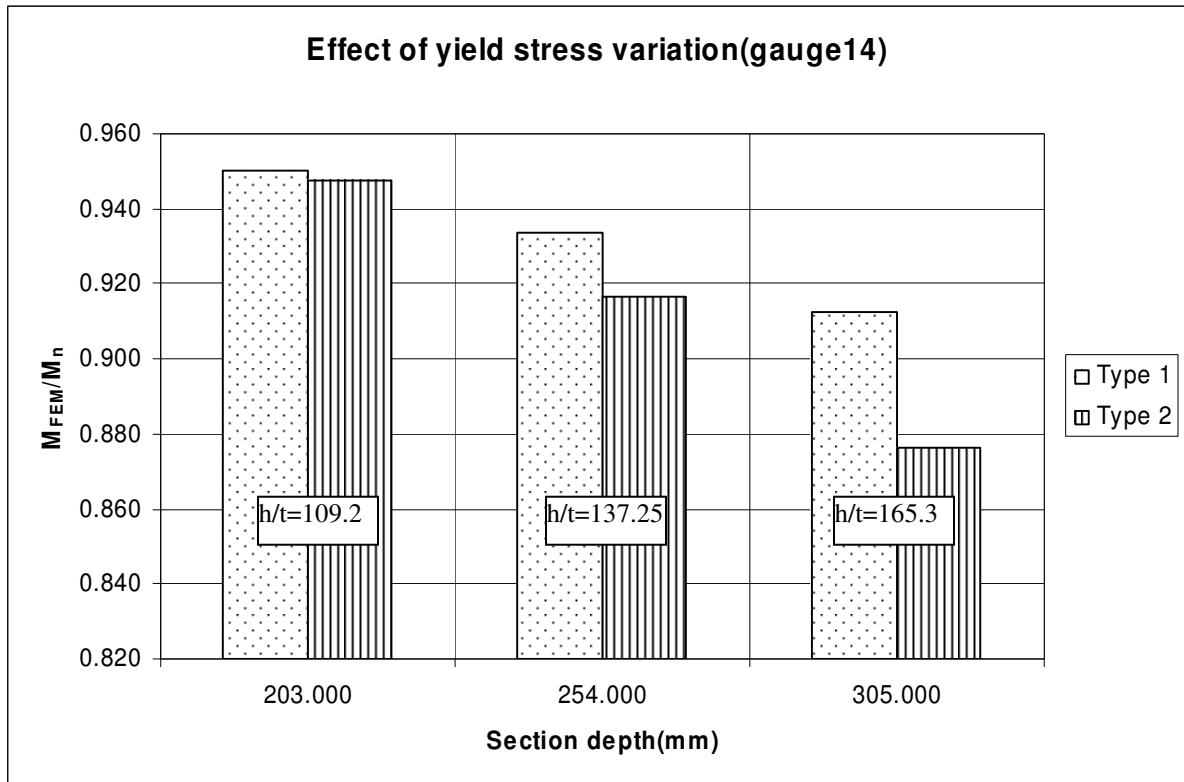


Figure 4.58 Effect of material yield stress on M_{FEM}/M_n ratio

4.5 Prediction of moment capacity

In order to propose a single equation to calculate the moment capacity of CFS built-up box girders, the ultimate moment capacities (M_{FEM}) obtained from FE analysis considering different section depth, thickness, flange screw spacing and material yield stress, were plotted against the nominal moment capacities (M_n) calculated according to the current design practice, as shown in Figure 4.59 for web depth-to-thickness ratio (h/t) less than or equal to 200.

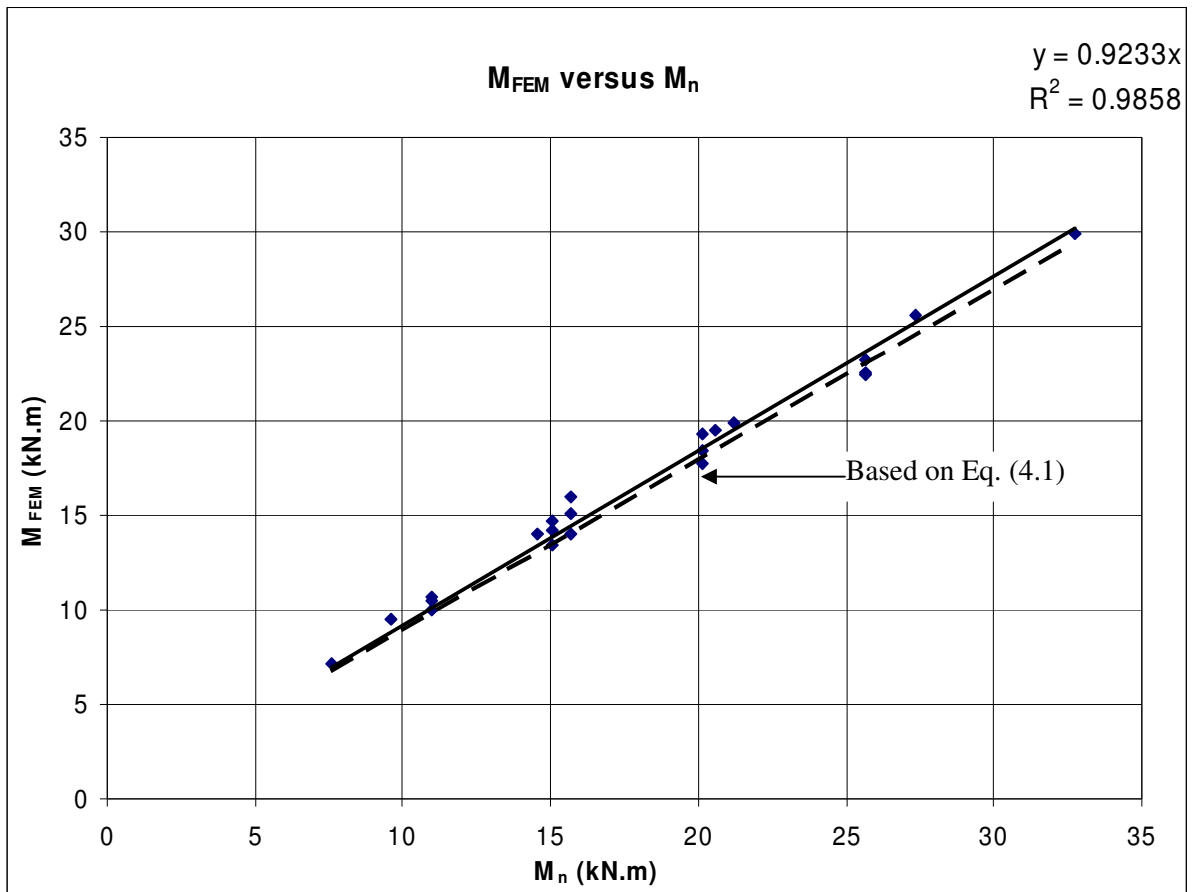


Figure 4.59 M_{FEM} versus M_n ($h/t \leq 200$)

All the points in Figure 4.59 can be seen to fall very close to the line $y = 0.9233x$ with $R^2 = 0.986$, thus confirming the fact that the moment capacities predicted by FE analysis are equal to 92% of the nominal moment capacity calculated according to current design practice. On this basis, it is proposed that:

$$M'_n = \beta M_n \tag{4.1}$$

where M'_n = modified nominal moment capacity of built-up box girder.

M_n = nominal moment capacity of CFS built-up box girder calculated according to current design practice.

β = proposed modification factor = 0.9

The parameter β is a factor of section depth (203 mm-305 mm), thickness (1.14 mm-1.81 mm); screw spacing (150 mm-600 mm) and material yield stress (228 MPa-349 MPa).

The dotted line in the Figure 4.59 was drawn according to the proposed Eq. (4.1) and shows that most of the points are at or above the line. So assuming the value of the parameter β equals 0.9, the nominal moment capacity of the CFS built-up box girder calculated according to Eq. (4.1) is conservative.

Chapter 5

Conclusions

5.1 Summary

Cold-formed built-up sections are used when a single section is not sufficient to carry the design load. In practice, CFS built-up box sections consist of two C-sections connected by self drilling screws through the top and bottom flanges. Load from the framing member may be directly applied to the web of one member of the built-up joist assembly. As a result, any resistance provided by the other members in the assembly depends on the efficiency of the connection components in transferring load. CFS built-up box girders may also be subject to torsion due to loading conditions. The current North American Specification for the Design of Cold-Formed Steel Structural Members (CSA-S136-01, 2001) does not provide any guideline on this issue. The AISI Cold-Formed Steel Framing Design Guide (AISI Cold-Formed Steel Framing Design Guide, 2002) suggests that the moment resistance and inertia of built-up sections are the simple addition of those for the component parts. The objective of this research was to study the flexural behaviour of cold-formed steel built-up box sections subjected to eccentric loading, and to investigate whether the current design practice proposed by the AISI Cold-Formed Steel Framing Design Guide (AISI Cold-Formed Steel Framing Design Guide, 2002) is conservative or not.

A theoretical background and thorough literature review was carried out and presented in Chapter 2, and very little corresponding information was found. Numerous physical experiments are required to develop and verify any newly proposed design procedure. With

the availability of powerful computers and software, the finite element method is an excellent tool to investigate the behaviour of engineering structures. So, in lieu of conducting expensive testing, the finite element method was used by this study to investigate the flexural performance of cold formed steel (CFS) built-up box girders. In particular, parametric studies were conducted to determine the factors affecting the moment capacity of CFS built-up box girders.

The main objectives of the finite element study were to understand the flexural behaviour and determine the ultimate moment capacity of CFS built-up box sections, and to identify the factors which affect the flexural capacity. A finite element model was established and verified through comparison of its results with those from experimental tests reported in the literature (Beshara and Lawson, 2002). It was shown that FE analysis can reliably predict the ultimate moment capacity of CFS built-up box sections.

The loading was applied in the finite element model by two methods: applying the load directly as force and applying the load as controlled displacement to investigate the effect of the method of load application on ultimate moment capacity and failure of the CFS built-up box girder. It was shown that the ultimate moment capacity varies within 1.5%-3.8% for the different methods of load application. The failure mechanism also remains the same regardless of the method of load application. So it can be concluded that the method of load application does not affect the ultimate moment capacity of the CFS built-up box girder under eccentric loading. In the parametric studies, load was applied as controlled displacement instead of force in order to simulate the load-deformation behaviour beyond ultimate capacity.

The ultimate moment capacities of the CFS built-up box girders are highly influenced by the support conditions. In the test (Beshara and Lawson, 2002), no bearing plate was used at the support location and, as a result, material failure occurred at the bearing location due to the high reaction forces. The FE analysis was conducted with a bearing plate at the support location. The FE model showed that by introducing a bearing plate at the support location, the local failure at that region can be minimized and the ultimate moment capacity of built-up box girders can be increased up to 95% of the nominal moment capacity (M_n) calculated according to current design practice.

A parametric study was carried out to investigate the influence of section depth, thickness, connection screw spacing, the use of bearing plate at the support location, and material yield stress on the ultimate moment capacity of CFS built-up box girders. A total of thirty FE studies were carried out for different depth (203 mm-305 mm), thickness (1.41 mm-1.81 mm), flange screw spacing (150 mm-600 mm) and material yield stress (207 MPa-349 MPa). The web slenderness ratio (h/t) varies from 109-263 for different sections. Nonlinear static analysis was carried out considering the material nonlinearities, geometric nonlinearities and initial geometric imperfections to obtain the ultimate moment capacity of CFS built-up box sections under eccentric loading. All the FE models for the different section dimensions showed distortion of the girder cross sections as a consequence of the applied load. The local buckling of the top flange of track sections was also observed at the constant moment region. The moment capacity (M_{FEM}) obtained from the FE analysis was compared with the nominal moment (M_n) calculated according to current design practice. It was found that the nominal moment capacity of CFS built-up box sections is not equal to the simple addition of the

nominal moment capacity of the individual sections. In fact, the ratio M_{FEM}/M_n was found to be less than 1.

From the parametric studies it was found that the ultimate moment capacity of CFS built-up box sections increases with an increase in section depth and thickness. The stiffness of built-up girders also increases with an increase in these parameters. The average M_{FEM}/M_n for built-up sections of different depth (203 mm-305 mm) and different thickness (1.14 mm-1.81 mm) equals 0.954 with a standard deviation of 0.034 and coefficient of variation of 0.036. The ultimate moment capacity of CFS built-up box sections was also influenced by the flange screw spacing. The moment capacity reduces with an increase in screw spacing. The average M_{FEM}/M_n ratio equals 0.931 with a standard deviation of 0.04 and coefficient of variation of 0.043 as flange screw spacing varies from 150mm-600mm. FE studies were also conducted to investigate the effect of the material yield stress (228 MPa-349 MPa) variation on the ultimate moment capacity of CFS built-up sections. It was found that decreasing yield stress of the steel reduces the ultimate moment capacity, however, the stiffness of the CFS built-up box girder remains the same. The M_{FEM}/M_n ratio decreases with the decrease of material yield stress.

For all thirty FE studies (h/t varies from 109-264), considering all the parameters, the average M_{FEM}/M_n ratio equals 0.938 with a standard deviation of 0.04 and coefficient of variation of 0.043. Considering only sections with $h/t \leq 200$, the average M_{FEM}/M_n ratio equals 0.926 with a standard deviation of 0.038 and coefficient of variation of 0.04.

From the results of the parametric studies, it was concluded that the current design practice to determine the moment capacity of CFS built-up box sections under eccentric loading is not conservative.

5.2 Design recommendations

By analyzing the data obtained from the FE model for $h/t \leq 200$ (web slenderness ratio), a modification factor (β) that is a function of section depth, thickness, yield stress and flange screw spacing was proposed to calculate the nominal moment capacity of built-up box sections. This study concluded that the modified nominal moment capacity (M_n') of eccentrically loaded CFS built-up box sections eccentric load can be determined according to the following equation:

$$M_n' = \beta M_n \quad (5.1)$$

Where M_n' = modified nominal moment capacity of built-up box girder.

M_n = nominal moment capacity of CFS built-up box girder calculated by adding the nominal moment capacities of individual sections, as currently specified by the AISI cold-formed steel framing design guide (AISI, 2002).

β = Proposed modification factor = 0.9

The modification factor (β) is valid for built-up box girders made of two C-sections connected through the top and bottom flanges and having a minimum bearing length at the support location equal to 38 mm (1.5 in). The β factor was based on FE results with section

depth of 203 mm-305 mm, thickness of 1.14 mm-1.81 mm, flange screw spacing of 150 mm-600 mm and material yield stress of 228 MPa-349 MPa.

5.3 Recommendations for future work

In the FE analysis, the self drilling screws connecting the top and bottom flanges of the two sections were not modelled. Instead, the screws were represented using a coupling node technique whereby the size of the screw, screw hole, and failure of the screws were neglected. The FE model could be improved by taking direct account of the actual screws and their related properties.

The initial geometric imperfections of the CFS sections were accounted for by scaling the first eigenvalue buckling mode and adding it to the perfect geometry so that the maximum imperfection did not exceed the thickness of the section. Future studies should include a sensitivity analysis to investigate the effect of initial geometric imperfection on the ultimate moment capacity of the CFS built-up box girders.

Even though bearing plates at support locations are recommended in practice, in the tests conducted by DDG (Beshara and Lawson, 2002) the CFS built-up box sections were placed directly on top of a knife-edge support. Based on the FE analysis carried out in this study, it was found that material failure occurred at the support location when no bearing plate was used. By accounting for a bearing plate at the support location, the FE analysis determined that the ultimate moment capacity of the CFS built-up box sections was increased. The author recommends that future tests should be carried out using a bearing plate at the support location, so as to verify the results obtained from the FE analysis in this research.

References

American Iron and Steel Institute (AISI), "Commentary on North American Specification for the Design of Cold-Formed steel structural Members", 2001.

American Iron and Steel Institute (AISI), "Specification for the Design of Cold-Formed Steel Structural Members", New York, NY, 1968.

American Iron and Steel Institute (AISI), "Specification for the Design of Light Gage Cold-Formed Steel Structural Members", New York, NY, 1946.

American Iron and Steel Institute, "Cold -Formed Steel Design Manual", Washington, D.C., 1986

American Iron and Steel Institute, "Cold-Formed Steel Framing design Guide", 2002.

ANSYS version 10.0, Finite Element Analysis Software

Beshara, B., and Lawson, T.J., "Built-Up Girder Screw Connection Variation Flexural Tests", Dietrich Design Group Internal Report, April 2002.

Bryan, G.H., "On the stability of a plane plate under thrusts in its own plane, with applications to the buckling of the sides of a ship" proceedings, London mathematical Society, Vol.22, 1891, pp.54-67

Canadian Sheet Steel Building Institute (CSSBI), "Low-Rise Residential Construction Details", 1994.

Canadian Sheet Steel Building Institute (CSSBI), "Lightweight Steel Framing design

Manual”, 2006.

CAN/CSA-S16-01, “Limit States Design of Steel Structures”, 2003.

Chou S.M., Chai G.B and Ling L., "Finite element technique for design of stud columns", *Thin-Walled Structures*, Vol.37, No.2, 2000, pp. 97-112.

Cohen, J.M., and Pekoz, T.B., "Local Buckling Behaviour of Plate Elements", Research Report, Department of Structural Engineering, Cornell University, 1987.

CSA- S136-01, "North American Specification for the Design of Cold-Formed Steel Structural Members", 2001.

CSA-S136S1-04, "Supplement 2004 to the North American Specification for the Design of Cold-Formed Steel Structural Members", 2004.

De Ville de Goyet, V., "Initial deformed shape-essential data for a nonlinear computation", *Proceedings 2nd international conference on coupled instabilities in metal structures*, 1996, pp.61-68

Graciano, C., and Casanova, E., "Ultimate strength of longitudinally stiffened I girder webs subjected to combined patch loading and bending", *Journal of Construction Steel research*, Vol.61, 2005, pp.93-111

Hancock, G.J., "Cold-formed steel structures", *Journal of construction steel research*, Vol.59, 2003, pp.473-487.

Miller, E.A., "A Study of the Strength of Short, Thin Walled Steel Studs", Master Thesis, Cornell University, 1943.

Moreyra, M.E., and Pekoz, T., "Finite element studies on lipped channel flexural members", Proceedings of the twelfth international specialty conference on cold-formed steel structures, University of Missouri-Rolla, October 1994.

Pekoz, T.B., "Development of a Unified Approach to the design of Cold Formed Steel Members" Report SG-86-4, American Iron and Steel Institute, 1986.

Rasmussen, K.J.R, and Hancock, G.J, " Buckling analysis of thin walled structures: numerical developments and applications", Progress in Structural Engineering and materials, Vol 2, No. 3, November, 2000, pp. 359-368.

Ren, WX, Fang SE, and Young B, " Finite Element Simulation and Design of Cold - Formed steel Channels Subjected to Web Crippling", Journal of Structural Engineering, Vol. 132, No. 12, December, 2006, pp.1967-1975.

Ren, WX, Fang SE, and Young B, " Analysis and Design of Cold-Formed steel channels subjected to combined bending and web crippling", Thin-walled Structures 44, May, 2006, pp. 314-320.

Sarawit, A.T., Kim, Y., Bakker, M.C.M., and Pekoz, T., "The finite element method for thin-walled members-applications", Thin-walled structures, Vol.41, 2003, pp.191-206

Schafer, B.W., and Pekoz, T., "Computational modelling of cold formed steel: Characterizing geometric imperfections and residual stresses", Journal of constructional steel research, Vol.47, 1998, pp.193-210.

Schafer, B.W., "Cold formed steel behaviour and design: Analytical and Numerical modelling of elements and members with longitudinal stiffeners, Phd Thesis, Cornell

University, Ithaca, NY, August, 1997.

Serrette, R.L., " Performance of Edge-Loaded Cold Formed Steel Built-up Box Beams", Practice Periodical on Structural Design and Construction, ASCE, Vol.9, No. 3, 2004, pp. 170-174.

Shanmugam, N.E., and Dhanalakshmi, M., "Design for openings in cold formed steel channel stud columns", Thin-walled structures, Vol.39, 2001, pp.961-981.

Timoshenko, S.P., and Gere, J.M., "Theory of Elastic Stability," 2nd ed., McGraw-Hill, New York, NY, 1961.

Tryland, T., Hopperstad, O.S., and Langseth, M., "Finite element modelling of beams under concentrated loading", Journal of structural engineering, Vol.127, No.2, February, 2001.

von Karman, T., Sechler, E.E., and Donnell, L.H., "The Strength of Thin Plates in Compression", Transactions ASME, Vol.54, APM 54-5, 1932.

Winter, G., "Commentary on the 1968 Edition of the Specification for the Design of Cold-Formed Steel Structural Members," American Iron and Steel institute, 1970.

Winter, G., "Strength of Thin Steel Compression Flanges", ASCE Transactions, Vol. 112, 1946, pp. 527-554.

Yu, C., and Schafer, B.W., " Simulation of cold formed steel beams in local and distortional buckling with applications to the direct strength method", Journal of Construction Steel Research, Vol.63, 2007, pp.581-590

Zhang, Y., Wang, C., and Zhang, Z., "Tests and finite element analysis of pin-ended channel columns with inclined simple edge stiffeners", *Journal of Construction Steel Research*, Vol.63, 2007, pp.383-395.

Zienkiewicz, O.C., 1977 "The finite element method", 3rd edition, McGraw-Hill, Maidenhead.

Appendix A

Sample calculation

Stud sections: 254S76 – 16

$$E := 29435 \text{ ksi} \quad f_y := 50.0 \text{ ksi} \quad \mu := 0.3 \quad t := .0632 \text{ in} \quad h_0 := 10 \text{ in} \quad b_0 := 3 \text{ in}$$

$$r_i := 1.5 \cdot t \quad r_i = 0.095 \text{ in} \quad d_1 := t + r_i \quad D := 1 \text{ in}$$

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d_1 \quad w = 2.684 \text{ in}$$

Gross section properties

Element	L	y	Ly	Ly ²	I
1	0.842	0.579	0.488	0.282	0.050
2	0.397	0.077	0.031	0.002	
3	2.684	0.032	0.085	0.003	
4	9.684	5.000	48.420	242.100	75.680
5	2.684	9.968	26.755	266.706	
6	0.397	9.923	3.939	39.087	
7	0.842	9.421	7.932	74.732	0.050
sum	17.530		87.650	622.913	75.780
	Y _{top}	5.000		I _x	260.443
				I _{xg}	16.460

$$\frac{w}{t} = 42.468 \quad d := D - d_1 \quad I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 3.144 \times 10^{-3} \text{ in}^4 \quad f := f_y$$

$$S := 1.28 \sqrt{\frac{E}{f}} \quad S = 30.872 \quad 0.328 S = 10.126 \quad \frac{w}{t} > 0.328 \cdot S$$

$$I_{a1} := 399 t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3$$

$$I_{a1} = 7.319 \times 10^{-3} \text{ in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \frac{\frac{w}{t}}{S} + 5 \right)$$

$$I_{a2} = 2.604 \times 10^{-3} \text{ in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases}$$

$$I_a = 2.604 \times 10^{-3} \text{ in}^4$$

$$\frac{D}{w} = 0.373 \quad Ri := \begin{cases} \frac{Is}{Ia} & \text{if } \frac{Is}{Ia} < 1 \\ 1 & \text{otherwise} \end{cases} \quad Ri = 1$$

$$n := \begin{cases} 0.582 - \frac{\frac{w}{t}}{4 \cdot S} & \text{if } 0.582 - \frac{\frac{w}{t}}{4 \cdot S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.333$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w}\right) \cdot Ri^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot Ri^n + 0.43 & \text{otherwise} \end{cases}$$

$$\overset{w}{k} := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.387$$

$$Fcr := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w}\right)^2 \quad Fcr = 49.962 \text{ksi} \quad \lambda := \sqrt{\frac{f}{Fcr}} \quad \lambda = 1.006$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.776$$

$$be := \rho \cdot w \quad be = 2.084 \text{in}$$

Lip (Unstiffened compression element)

$$Ytop := 5 \text{in} \quad \overset{w}{d1} := t + ri \quad \overset{w}{f} := fy \quad d = 0.842 \text{in}$$

$$\frac{d}{t} = 13.323 \quad f1 := f \cdot \frac{Ytop - d1}{Ytop} \quad f2 := f \cdot \frac{Ytop - d1 - d}{Ytop}$$

$$f1 = 49.001 \text{ksi} \quad f2 = 40.48 \text{ksi} \quad \psi := \frac{f2}{f1} \quad \psi = 0.826$$

$$\overset{w}{k} := \frac{0.578}{\psi + 0.34} \quad k = 0.496$$

$$\overset{w}{Fcr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad Fcr = 74.292 \text{ksi} \quad \overset{w}{f} := f1$$

$$\lambda_{\text{eff}} := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.812$$

$$\rho_{\text{eff}} := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.898$$

$$ds' := \rho \cdot d \quad ds' = 0.756 \text{in} \quad ds := ds' \cdot Ri \quad ds = 0.756 \text{in}$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 9.684 \text{in} \quad \frac{h}{t} = 153.228 \quad f_{\text{eff}} := f_1 \quad f_{2\text{eff}} := \frac{(h_0 - Y_{\text{top}} - d_1) \cdot f_y}{Y_{\text{top}}}$$

$$f_1 = 49.001 \text{ksi} \quad f_2 = 49.001 \text{ksi}$$

$$\psi_{\text{eff}} := \frac{f_2}{f_1} \quad \psi = 1 \quad k_{\text{eff}} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 27.194 \text{ksi} \quad f_{\text{eff}} := f_1$$

$$\lambda_{\text{eff}} := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.342$$

$$\rho_{\text{eff}} := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.623$$

$$b_{e\text{eff}} := \rho \cdot h \quad b_e = 6.032 \text{in}$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.508 \text{in} \quad b_2 = 3.016 \text{in}$$

$$b_1 + b_2 = 4.524 \text{in} \quad Y_{\text{top}} - d_1 = 4.842 \text{in}$$

$b_1 + b_2 < 4.842.in$ the web is not fully effective.

Moment of Inertia calculation

Trial1	Assume	Ytop=5in			
Element	L	y	Ly	Ly ²	I
1	0.756	0.536	0.405	0.217	0.036
2	0.397	0.077	0.031	0.002	
3	2.084	0.032	0.066	0.002	
4a	1.508	0.912	1.375	1.254	0.286
4b	3.016	3.492	10.532	36.777	2.286
4c	4.842	7.421	35.932	266.655	9.460
5	2.684	9.968	26.755	266.706	
6	0.397	9.923	3.939	39.087	
7	0.842	9.464	7.969	75.416	0.050
sum	16.526		87.005	686.117	12.118
	Ytop	5.265		Ix	240.181
				Ixe	15.179

Trial2

$$Y_{top} := 5.3in \quad f := f_y$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 9.684in \quad \frac{h}{t} = 153.228$$

$$f_1 := f \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 49.092ksi$$

$$f_2 = 43.363ksi$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.883 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 21.126$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h} \right)^2 \quad F_{cr} = 23.938ksi \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.432$$

$$\rho_{\text{w}} := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.591$$

$$b_{\text{e}} := \rho \cdot h \quad b_{\text{e}} = 5.723 \text{ in}$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_{1\text{w}} := \frac{b_{\text{e}}}{3 + \psi}$$

$$b_{2\text{w}} := \begin{cases} \frac{b_{\text{e}}}{2} & \text{if } \psi > 0.236 \\ b_{\text{e}} - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.474 \text{ in} \quad b_2 = 2.862 \text{ in}$$

$$b_1 + b_2 = 4.336 \text{ in} \quad Y_{\text{top}} - d_1 = 5.142 \text{ in}$$

$$b_1 + b_2 < 5.142 \quad \text{the web is not fully effective.}$$

Trail 2	assume	Ycg=5.3in			
Element	L	y	Ly	Ly ²	I
1	0.756	0.536	0.405	0.217	0.036
2	0.397	0.077	0.031	0.002	
3	2.084	0.032	0.066	0.002	
4a	1.474	0.895	1.319	1.181	0.267
4b	2.862	3.869	11.073	42.842	1.954
4c	4.542	7.571	34.387	260.348	7.808
5	2.684	9.968	26.755	266.706	
6	0.397	9.923	3.939	39.087	
7	0.842	9.421	7.932	74.732	0.050
sum	16.038		85.909	685.117	10.115
	Ytop	5.357		Ix	235.057
				Ixe	14.856

Trial 3

$$Y_{\text{top}} := 5.38 \text{ in} \quad f := f_y$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 9.684 \text{ in} \quad \frac{h}{t} = 153.228$$

$$f_{1\text{w}} := f \cdot \frac{Y_{\text{top}} - d_1}{Y_{\text{top}}} \quad f_{2\text{w}} := \frac{(h_0 - Y_{\text{top}} - d_1) \cdot f_y}{Y_{\text{top}}}$$

$$f_1 = 49.114 \text{ ksi}$$

$$f_2 = 41.966 \text{ ksi}$$

$$\psi := \frac{f2}{f1} \quad \psi = 0.854 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 20.464$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 23.188 \text{ksi} \quad f := f1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.455$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.583$$

$$b_e := \rho \cdot h$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.465 \text{in} \quad b_2 = 2.824 \text{in}$$

$$b_1 + b_2 = 4.289 \text{in} \quad Y_{top} - d_1 = 5.222 \text{in}$$

$$b_1 + b_2 < 5.222 \quad \text{the web is not fully effective.}$$

Trial3	Assume	Ycg=5.38in			
Element	L	y	Ly	Ly ²	I
1	0.756	0.536	0.405	0.217	0.036
2	0.397	0.077	0.031	0.002	
3	2.084	0.032	0.066	0.002	
4a	1.465	0.891	1.305	1.162	0.262
4b	2.824	3.968	11.206	44.464	1.877
4c	4.462	7.611	33.960	258.472	7.403
5	2.684	9.968	26.755	266.706	
6	0.397	9.923	3.939	39.087	
7	0.842	9.421	7.932	74.732	0.050
sum	15.911		85.599	684.845	9.628
	Ytop	5.380		lx	233.959
				lx _e	14.786

From trial 3 it is found that $Y_{top}=5.380\text{in}$ which is almost equal to 5.38 in . So no other iteration is necessary.

$$I_{xe} := 14.78619\text{in}^4 \quad Y_{xe} := 5.379878\text{in}$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 2.748\text{in}^3 \quad M_n := S_{xe} \cdot f_y \quad f_y = 50.6\text{ksi}$$

$$M_n = 139.07\text{kip-in}$$

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$$E := 29435\text{ksi} \quad f_y := 50.6\text{ksi} \quad \mu := 0.3 \quad t := 0.045\text{in} \quad h_0 := 10\text{in} \quad b_0 := 3\text{in}$$

$$r_i := 1.5 \cdot t \quad r_i = 0.068\text{in} \quad d_1 := t + r_i \quad D := 1\text{in}$$

Gross section properties

Element	L	y	Ly	Ly ²	I
1	0.887	0.556	0.494	0.275	0.058
2	0.284	0.055	0.016	0.001	
3	2.775	0.023	0.063	0.001	
4	9.775	5.000	48.873	244.363	77.822
5	2.775	9.977	27.682	276.200	
6	0.284	9.945	2.824	28.087	
7	0.887	9.444	8.379	79.127	0.058
sum	17.666		88.330	628.053	77.938
	Ytop	5.000		Ix	264.342
				Ixg	11.922

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d_1 \quad w = 2.774\text{in}$$

$$\frac{w}{t} = 61.519 \quad d := D - d_1 \quad I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 2.625 \times 10^{-3} \text{in}^4 \quad f := f_y$$

$$S_{cr} := 1.28 \sqrt{\frac{E}{f}} \quad S = 30.872 \quad 0.328S = 10.126 \quad \frac{w}{t} > 0.328S$$

$$I_{a1} := 399t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3$$

$$I_{a1} = 7.615 \times 10^{-3} \text{ in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \frac{w}{S} + 5 \right) \quad I_{a2} = 9.688 \times 10^{-4} \text{ in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases}$$

$$I_a = 9.688 \times 10^{-4} \text{ in}^4$$

$$\frac{D}{w} = 0.36 \quad R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases} \quad R_i = 1$$

$$n := \begin{cases} 0.582 - \frac{w}{4S} & \text{if } 0.582 - \frac{w}{4S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.333$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w} \right) \cdot R_i^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot R_i^n + 0.43 & \text{otherwise} \end{cases}$$

$$k := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.448$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w} \right)^2 \quad F_{cr} = 24.237 \text{ ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.445$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.587$$

$$b_e := \rho \cdot w \quad b_e = 1.628 \text{ in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 5in \quad \underline{d1} := t + r_i \quad \underline{f} := f_y \quad d = 0.887in$$

$$\frac{d}{t} = 19.673 \quad f1 := f \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f2 := f \cdot \frac{Y_{top} - d1 - d}{Y_{top}}$$

$$f1 = 49.459ksi \quad f2 = 40.48ksi \quad \psi := \frac{f2}{f1} \quad \psi = 0.818$$

$$\underline{k} := \frac{0.578}{\psi + 0.34} \quad k = 0.499$$

$$\underline{Fcr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad Fcr = 34.297ksi \quad \underline{f} := f1$$

$$\underline{\lambda} := \sqrt{\frac{f}{Fcr}} \quad \lambda = 1.201$$

$$\underline{\rho} := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.68$$

$$ds' := \rho \cdot d \quad ds' = 0.603in \quad ds := ds' \cdot R_i \quad ds = 0.603in$$

Web under stress gradient

$$h := h_0 - 2 \cdot d1 \quad h = 9.774in \quad \frac{h}{t} = 216.729 \quad \underline{f} := f1 \quad \underline{f2} := \frac{(h_0 - Y_{top} - d1) \cdot f_y}{Y_{top}}$$

$$f1 = 49.459ksi \quad f2 = 49.459ksi$$

$$\underline{\psi} := \frac{f2}{f1} \quad \psi = 1 \quad \underline{k} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$\underline{Fcr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad Fcr = 13.593ksi \quad \underline{f} := f1$$

$$\underline{\lambda} := \sqrt{\frac{f}{Fcr}} \quad \lambda = 1.907$$

$$\underline{\rho} := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.464$$

$$b_e := \rho \cdot h \quad b_e = 4.533\text{in}$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.133\text{in} \quad b_2 = 2.267\text{in}$$

$$b_1 + b_2 = 3.4\text{in} \quad Y_{\text{top}} - d_1 = 4.887\text{in}$$

$$b_1 + b_2 < 4.887 \quad \text{the web is not fully effective.}$$

Moment of Inertia calculation

Trial1	Assume	Ytop=5in			
Element	L	y	Ly	Ly ²	I
1	0.603	0.414	0.250	0.103	0.018
2	0.284	0.055	0.016	0.001	
3	1.628	0.023	0.037	0.001	
4a	1.133	0.679	0.770	0.523	0.121
4b	2.267	3.867	8.765	33.891	0.971
4c	4.887	7.444	36.379	270.791	9.728
5	2.775	9.977	27.682	276.200	
6	0.284	9.945	2.824	28.087	
7	0.887	9.444	8.379	79.127	0.058
sum	14.748		85.102	688.723	10.896
	Ytop	5.770		Ix	208.551
				Ixe	9.406

Trial 2

$$Y_{\text{top}} := 5.85\text{in} \quad f := f_y$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 9.774\text{in} \quad \frac{h}{t} = 216.729$$

$$f_1 := f_y \cdot \frac{Y_{\text{top}} - d_1}{Y_{\text{top}}}$$

$$f_2 := \frac{(h_0 - Y_{\text{top}} - d_1) \cdot f_y}{Y_{\text{top}}}$$

$$f_1 = 49.625\text{ksi}$$

$$f_2 = 34.92\text{ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.704 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 17.298$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 9.797 \text{ksi} \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.251$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{\lambda}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.401$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h \quad b_e = 3.918 \text{in}$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} \\ b_e - b_1 \end{cases} \text{ if } \psi > 0.236$$

$$\text{otherwise}$$

$$b_1 = 1.058 \text{in} \quad b_2 = 1.959 \text{in}$$

$$b_1 + b_2 = 3.017 \text{in} \quad Y_{top} - d_1 = 5.737 \text{in}$$

$$b_1 + b_2 < 5.737 \quad \text{the web is not fully effective.}$$

Trail 2 assume $Y_{cg} = 5.85$

Element	L	y	L_y	L_y^2	I
1	0.603	0.414	0.250	0.103	0.018
2	0.284	0.055	0.016	0.001	
3	1.628	0.023	0.037	0.001	
4a	1.058	0.642	0.679	0.436	0.099
4b	1.959	4.871	9.541	46.471	0.627
4c	4.037	7.869	31.768	249.967	5.484
5	2.775	9.977	27.682	276.200	
6	0.284	9.945	2.824	28.087	
7	0.887	9.444	8.379	79.127	0.058
sum	13.515		81.176	680.393	6.285
	Y_{top}	6.006		I_x	199.110
				I_{xe}	8.980

Trial 3

$$Y_{top} := 6.05 \text{ in} \quad f := f_y$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 9.774 \text{ in} \quad \frac{h}{t} = 216.729$$

$$f_1 := f_y \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 49.657 \text{ ksi} \quad f_2 = 32.093 \text{ ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.646 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 16.217$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h} \right)^2 \quad F_{cr} = 9.185 \text{ ksi} \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.325$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.389$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} \\ b_e - b_1 \end{cases} \text{ if } \psi > 0.236$$

$$\text{otherwise}$$

$$b_1 = 1.044 \text{ in} \quad b_2 = 1.903 \text{ in}$$

$$b_1 + b_2 = 2.947 \text{ in} \quad Y_{top} - d_1 = 5.937 \text{ in}$$

$$b_1 + b_2 < 5.937 \text{ in} \quad \text{the web is not fully effective.}$$

Trial3 y=6.05

Element	L	y	Ly	Ly ²	I
1	0.603	0.414	0.250	0.103	0.018
2	0.284	0.055	0.016	0.001	
3	1.628	0.023	0.037	0.001	
4a	1.044	0.635	0.663	0.421	0.095
4b	1.903	5.099	9.702	49.468	0.574
4c	3.837	7.969	30.578	243.661	4.708
5	2.775	9.977	27.682	276.200	
6	0.284	9.945	2.824	28.087	
7	0.887	9.444	8.379	79.127	0.058
sum	13.245		80.131	677.069	5.454
	Ytop	6.050		Ix	197.744
				Ixe	8.918

From trial 3 it is found that Ycg=6.04987in which is almost equal to 6.05 in. So no other iteration is necessary.

$$I_{xe} := 8.91824 \text{ in}^4 \quad Y_{xe} := 6.04987 \text{ in}$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 1.474 \text{ in}^3 \quad M_n := S_{xe} f_y \quad f_y = 50.6 \text{ ksi}$$

$$M_n = 74.591 \text{ kip-in}$$

Stud section: 254S76 – 14

$$E := 29433 \text{ ksi} \quad f_y := 50.6 \text{ ksi} \quad \mu := 0.3 \quad t := 0.0713 \text{ in} \quad h_0 := 10 \text{ in} \quad b_0 := 3 \text{ in}$$

$$r_i := 1.5 \cdot t \quad r_i = 0.107 \text{ in} \quad d_1 := t + r_i \quad D := 1 \text{ in}$$

Gross section properties:

Element	L	y	Ly	Ly ²	I
1	0.822	0.589	0.484	0.285	0.046
2	0.448	0.087	0.039	0.003	
3	2.644	0.036	0.094	0.003	
4	9.644	5.000	48.218	241.088	74.735
5	2.644	9.964	26.341	262.469	
6	0.448	9.913	4.438	43.997	
7	0.822	9.411	7.733	72.778	0.046
sum	17.470		87.348	620.623	74.827
	Ytop	5.000		Ix	258.712
				Ixg	18.446

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d_1 \quad w = 2.643 \text{ in}$$

$$\frac{w}{t} = 37.076 \quad d := D - d_1 \quad I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 3.297 \times 10^{-3} \text{ in}^4 \quad f := f_y$$

$$S := 1.28 \sqrt{\frac{E}{f}} \quad S = 30.872 \quad 0.328 S = 10.126 \quad \frac{w}{t} > 0.328 \cdot S$$

$$I_{a1} := 399 t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3$$

$$I_{a1} = 6.859 \times 10^{-3} \text{ in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \cdot \frac{\frac{w}{t}}{S} + 5 \right) \quad I_{a2} = 3.698 \times 10^{-3} \text{ in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases}$$

$$I_a = 3.698 \times 10^{-3} \text{ in}^4$$

$$\frac{D}{w} = 0.378 \quad R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases} \quad R_i = 0.891$$

$$n := \begin{cases} 0.582 - \frac{\frac{w}{t}}{4 \cdot S} & \text{if } 0.582 - \frac{\frac{w}{t}}{4 \cdot S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.333$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w} \right) \cdot R_i^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot R_i^n + 0.43 & \text{otherwise} \end{cases}$$

$$k := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.249$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w}\right)^2 \quad F_{cr} = 62.87 \text{ ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.897$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{\lambda}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.841$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot w \quad b_e = 2.224 \text{ in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 5 \text{ in} \quad d_1 := t + r_i \quad f_w := f_y \quad d = 0.822 \text{ in}$$

$$\frac{d}{t} = 11.525 \quad f_1 := f \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := f \cdot \frac{Y_{top} - d_1 - d}{Y_{top}}$$

$$f_1 = 48.796 \text{ ksi} \quad f_2 = 40.48 \text{ ksi} \quad \psi := \frac{f_2}{f_1} \quad \psi = 0.83$$

$$k_w := \frac{0.578}{\psi + 0.34} \quad k = 0.494$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 98.978 \text{ ksi} \quad f_w := f_1$$

$$\lambda_w := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.702$$

$$\rho_w := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{\lambda}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.978$$

$$1 \text{ otherwise}$$

$$d_s' := \rho \cdot d \quad d_s' = 0.804 \text{ in} \quad d_s := d_s' \cdot R_i \quad d_s = 0.716 \text{ in}$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 9.643 \text{ in} \quad \frac{h}{t} = 135.252 \quad f_w := f_1 \quad f_w := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 48.796 \text{ ksi} \quad f_2 = 48.796 \text{ ksi}$$

$$\psi := \frac{f2}{f1} \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 34.903 \text{ksi} \quad f := f1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.182$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.688$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h \quad b_e = 6.638 \text{in}$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} \\ b_e - b_1 \end{cases} \text{ if } \psi > 0.236$$

$$\text{otherwise}$$

$$b_1 = 1.66 \text{in} \quad b_2 = 3.319 \text{in}$$

$$b_1 + b_2 = 4.979 \text{in} \quad Y_{top} - d_1 = 4.822 \text{in}$$

$$b_1 + b_2 > 4.822 \text{in} \quad \text{the web is fully effective.}$$

Moment of Inertia calculation

Trial1	Assume	Ytop=5			
Element	L	y	Ly	Ly ²	I
1	0.716	0.536	0.384	0.206	0.036
2	0.448	0.087	0.039	0.003	
3	2.224	0.036	0.094	0.003	
4	9.644	5.000	48.218	241.088	74.735
5	2.644	9.964	26.341	262.469	
6	0.448	9.913	4.438	43.997	
7	0.822	9.411	7.733	72.778	0.046
sum	16.944		87.247	620.544	74.817
	Ytop	5.149		Ix	246.116
				Ixe	17.548

Trial2

$$Y_{top} := 5.15 \text{ in} \quad f := f_y$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 9.643 \text{ in} \quad \frac{h}{t} = 135.252$$

$$f_1 := f \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 48.849 \text{ ksi} \quad f_2 = 45.901 \text{ ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.94 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 22.474$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 32.684 \text{ ksi} \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.223$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.671$$
$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h \quad b_e = 6.469 \text{ in}$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} \\ b_e - b_1 \end{cases} \text{ if } \psi > 0.236$$
$$\text{otherwise}$$

$$b_1 = 1.642 \text{ in} \quad b_2 = 3.234 \text{ in}$$

$$b_1 + b_2 = 4.876 \text{ in} \quad Y_{top} - d_1 = 4.972 \text{ in}$$

$$b_1 + b_2 < 4.972 \quad \text{the web is not fully effective.}$$

Trial2	assume	Ytop=5.15			
Element	L	y	Ly	Ly ²	I
1	0.716	0.536	0.384	0.206	0.036
2	0.448	0.087	0.039	0.003	
3	2.224	0.036	0.094	0.003	
4a	1.642	0.999	1.641	1.640	0.267
4b	3.234	3.533	11.426	40.367	2.819
4c	4.672	7.486	34.972	261.797	8.497
5	2.644	9.964	26.341	262.469	
6	0.448	9.913	4.438	43.997	
7	0.822	9.411	7.733	72.778	0.046
sum	16.849		87.069	683.260	11.665
	Ytop	5.168		Ix	244.978
				Ixe	17.467

Trial 3

$$Y_{top} := 5.175 \text{ in} \quad f := f_y$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 9.643 \text{ in} \quad \frac{h}{t} = 135.252$$

$$f_1 := f \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 48.857 \text{ ksi}$$

$$f_2 = 45.435 \text{ ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.93 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 22.237$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h} \right)^2 \quad F_{cr} = 32.339 \text{ ksi} \quad f_w := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.229$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.668$$

$$b_e := \rho \cdot h$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b1 = 1.639\text{in} \quad b2 = 3.221\text{in}$$

$$b1 + b2 = 4.86\text{in} \quad Y_{\text{top}} - d1 = 4.997\text{in}$$

$b1 + b2 < 5.222$ the web is not fully effective.

Trial3	assume	Ytop=5.175			
Element	L	y	Ly	Ly ²	I
1	0.716	0.536	0.384	0.206	0.036
2	0.448	0.087	0.039	0.003	
3	2.224	0.036	0.094	0.003	
4a	1.639	0.998	1.635	1.632	0.367
4b	3.221	3.565	11.481	40.925	2.785
4c	4.647	7.498	34.843	261.266	8.361
5	2.644	9.964	26.341	262.469	
6	0.448	9.913	4.438	43.997	
7	0.822	9.411	7.733	72.778	0.046
sum	16.808		86.990	683.279	11.595
	Ytop	5.176		Ix	244.648
				Ixe	17.443

From trial 3 it is found that $Y_{cg}=5.175635$ which is almost equal to 5.175 in. So no other iteration is necessary.

$$I_{xe} := 17.44339\text{in}^4 \quad Y_{xe} := 5.175635\text{in}$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 3.37\text{in}^3 \quad M_n := S_{xe} \cdot f_y \quad f_y = 50.6\text{ksi}$$

$$M_n = 170.537\text{kip}\cdot\text{in}$$

Stud section: 203S76 – 18

$$E := 29435 \text{ ksi} \quad f_y := 50.6 \text{ ksi} \quad \mu := 0.3 \quad t := 0.045 \text{ in} \quad h_0 := 8 \text{ in} \quad b_0 := 3 \text{ in}$$

$$r_i := 1.5 t \quad r_i = 0.068 \text{ in} \quad d_1 := t + r_i \quad D := 1 \text{ in}$$

Gross section:

Element	L	y	Ly	Ly ²	I
1	0.887	0.556	0.494	0.275	0.058
2	0.284	0.055	0.016	0.001	
3	2.775	0.023	0.063	0.001	
4	7.775	4.000	31.098	124.392	39.159
5	2.775	7.977	22.133	176.568	
6	0.284	7.945	2.256	17.926	
7	0.887	7.444	6.604	49.160	0.058
sum	15.666		62.664	368.323	39.276
	Ytop	4.000		Ix	156.943
				Ixg	7.078

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d_1 \quad w = 2.774 \text{ in}$$

$$\frac{w}{t} = 61.519 \quad d := D - d_1 \quad I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 2.625 \times 10^{-3} \text{ in}^4 \quad f := f_y$$

$$S := 1.28 \sqrt{\frac{E}{f}} \quad S = 30.872 \quad 0.328 S = 10.126 \quad \frac{w}{t} > 0.328 S$$

$$I_{a1} := 399 t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3$$

$$I_{a1} = 7.615 \times 10^{-3} \text{ in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \cdot \frac{\frac{w}{t}}{S} + 5 \right) \quad I_{a2} = 9.688 \times 10^{-4} \text{ in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases}$$

$$I_a = 9.688 \times 10^{-4} \text{ in}^4$$

$$\frac{D}{w} = 0.36 \quad R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases} \quad R_i = 1$$

$$n := \begin{cases} 0.582 - \frac{\frac{w}{t}}{4S} & \text{if } 0.582 - \frac{\frac{w}{t}}{4S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.333$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w}\right) \cdot R_i^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot R_i^n + 0.43 & \text{otherwise} \end{cases}$$

$$\underline{k} := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.448$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w}\right)^2 \quad F_{cr} = 24.237 \text{ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.445$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.587$$

$$b_e := \rho \cdot w \quad b_e = 1.628 \text{in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 4 \text{in} \quad d_1 := t + r_i \quad f := f_y \quad d = 0.887 \text{in}$$

$$\frac{d}{t} = 19.673 \quad f_1 := f \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := f \cdot \frac{Y_{top} - d_1 - d}{Y_{top}}$$

$$f_1 = 49.174 \text{ksi} \quad f_2 = 37.95 \text{ksi} \quad \psi := \frac{f_2}{f_1} \quad \psi = 0.772$$

$$\underline{k} := \frac{0.578}{\psi + 0.34} \quad k = 0.52$$

$$\underline{F}_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 35.737 \text{ksi} \quad \underline{f} := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.173$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.693$$

$$1 \text{ otherwise}$$

$$ds := \rho \cdot d \quad ds' = 0.615 \text{ in} \quad ds := ds' \cdot R_i \quad ds = 0.615 \text{ in}$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 7.774 \text{ in} \quad \frac{h}{t} = 172.384 \quad f_1 := f_1 \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 49.174 \text{ ksi} \quad f_2 = 49.174 \text{ ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 21.486 \text{ ksi} \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.513$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.565$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h \quad b_e = 4.392 \text{ in}$$

$$\frac{h_0}{b_0} = 2.667 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.098 \text{ in} \quad b_2 = 2.196 \text{ in}$$

$$b_1 + b_2 = 3.294 \text{ in} \quad Y_{top} - d_1 = 3.887 \text{ in}$$

$$b_1 + b_2 < 3.887 \quad \text{the web is not fully effective.}$$

Moment of Inertia calculation

Trial1	assume	Ytop=4in			
Element	L	y	Ly	Ly ²	I
1	0.615	0.420	0.258	0.109	0.019
2	0.284	0.055	0.016	0.001	
3	1.628	0.023	0.037	0.001	
4a	1.098	0.662	0.727	0.481	0.110
4b	2.196	2.902	6.373	18.494	0.883
4c	3.887	5.944	23.104	137.324	4.895
5	2.775	7.977	22.133	176.568	
6	0.284	7.945	2.256	17.926	
7	0.887	7.444	6.604	49.160	0.058
sum	13.654		61.509	400.063	5.965
	Ytop	4.505		Ix	128.943
				Ixe	5.815

Trial 2

$$f := f_y \quad Y_{top} := 4.6in$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 7.774in \quad \frac{h}{t} = 172.384$$

$$f_1 := f \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 49.36ksi \quad f_2 = 36.16ksi$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.733 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 17.867$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h} \right)^2 \quad F_{cr} = 15.996ksi \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.757$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{\lambda}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.498$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h \quad b_e = 3.871in$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.795$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.489$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h$$

$$\frac{h_0}{b_0} = 2.667 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} \\ b_e - b_1 \end{cases} \text{ if } \psi > 0.236$$

$$\text{otherwise}$$

$$b_1 = 1.029 \text{ in} \quad b_2 = 1.9 \text{ in}$$

$$b_1 + b_2 = 2.929 \text{ in} \quad Y_{\text{top}} - d_1 = 4.587 \text{ in}$$

$$b_1 + b_2 < 4.587 \text{ in} \quad \text{the web is not fully effective.}$$

Trial3 assume $Y_{\text{top}}=4.7 \text{ in}$

Element	L	y	Ly	Ly^2	I
1	0.615	0.420	0.258	0.109	0.019
2	0.284	0.055	0.016	0.001	
3	1.628	0.023	0.037	0.001	
4a	1.029	0.627	0.645	0.405	0.091
4b	1.900	3.750	7.125	26.719	0.572
4c	3.187	6.294	20.059	126.246	2.698
5	2.775	7.977	22.133	176.568	
6	0.284	7.945	2.256	17.926	
7	0.887	7.444	6.604	49.160	0.058
sum	12.589		59.135	397.134	3.438
	Y_{top}	4.697		I_x	122.797
				I_{xe}	5.538

From trial 3 it is found that $Y_{cg}=4.697 \text{ in}$ which is almost equal to 4.7 in. So no other iteration is necessary.

$$I_{xe} := 5.53813 \text{ in}^4 \quad Y_{xe} := 4.697 \text{ in}$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 1.179 \text{ in}^3 \quad M_n := S_{xe} \cdot f_y \quad f_y = 50.6 \text{ ksi}$$

$$M_n = 59.661 \text{ kip} \cdot \text{in}$$

Stud section: 203S76 – 16

$$E := 29435 \text{ ksi} \quad f_y := 50.6 \text{ ksi} \quad \mu := 0.3 \quad t := 0.0566 \text{ in} \quad h_0 := 8 \text{ in} \quad b_0 := 3 \text{ in}$$

$$r_i := 1.5 \cdot t \quad r_i = 0.085 \text{ in} \quad d_1 := t + r_i \quad D := 1 \text{ in}$$

Gross section:

Element	L	y	Ly	Ly ²	I
1	0.859	0.571	0.490	0.280	0.053
2	0.356	0.069	0.025	0.002	
3	2.717	0.028	0.077	0.002	
4	7.717	4.000	30.868	123.472	38.297
5	2.717	7.972	21.659	172.660	
6	0.356	7.931	2.823	22.390	
7	0.859	7.429	6.382	47.408	0.053
sum	15.581		62.324	366.215	38.403
	Y _{top}	4.000		I _x	155.321
				I _{xg}	8.791

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d_1 \quad w = 2.717 \text{ in}$$

$$\frac{w}{t} = 48.004 \quad d := D - d_1 \quad I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 2.984 \times 10^{-3} \text{ in}^4 \quad f := f_y$$

$$S_x := 1.28 \sqrt{\frac{E}{f}} \quad S = 30.872 \quad 0.328 S = 10.126 \quad \frac{w}{t} > 0.328 S$$

$$I_{a1} := 399 t^4 \cdot \left(\frac{w}{t} - 0.328 S \right)^3$$

$$I_{a1} = 7.563 \times 10^{-3} \text{ in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \frac{t}{S} + 5 \right) \quad I_{a2} = 1.886 \times 10^{-3} \text{ in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases}$$

$$I_a = 1.886 \times 10^{-3} \text{ in}^4$$

$$\frac{D}{w} = 0.368 \quad R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases} \quad R_i = 1$$

$$n := \begin{cases} 0.582 - \frac{w}{4 \cdot S} & \text{if } 0.582 - \frac{w}{4 \cdot S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.333$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w}\right) \cdot Ri^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot Ri^n + 0.43 & \text{otherwise} \end{cases}$$

$$\underline{k} := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.41$$

$$F_{cr} := \frac{\underline{k} \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w}\right)^2 \quad F_{cr} = 39.365 \text{ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.134$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.711$$

$$be := \rho \cdot w \quad be = 1.931 \text{in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 4 \text{in} \quad \underline{d1} := t + ri \quad \underline{f} := f_y \quad d = 0.859 \text{in}$$

$$\frac{d}{t} = 15.168 \quad f1 := f \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f2 := f \cdot \frac{Y_{top} - d1 - d}{Y_{top}}$$

$$f1 = 48.81 \text{ksi} \quad f2 = 37.95 \text{ksi} \quad \psi := \frac{f2}{f1} \quad \psi = 0.778$$

$$\underline{k} := \frac{0.578}{\psi + 0.34} \quad k = 0.517$$

$$\underline{F_{cr}} := \frac{\underline{k} \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 59.81 \text{ksi} \quad \underline{f} := f1$$

$$\underline{\lambda} := \sqrt{\frac{f}{\underline{F_{cr}}}} \quad \lambda = 0.903$$

$$\rho_{\text{eff}} := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{\lambda}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.837$$

$$1 \text{ otherwise}$$

$$ds' := \rho \cdot d \quad ds' = 0.719 \text{ in} \quad ds := ds' \cdot Ri \quad ds = 0.719 \text{ in}$$

Web under stress gradient

$$h := h0 - 2 \cdot d1 \quad h = 7.717 \text{ in} \quad \frac{h}{t} = 136.343$$

$$f1 := fy \cdot \frac{Y_{\text{top}} - d1}{Y_{\text{top}}} \quad f2 := \frac{(h0 - Y_{\text{top}} - d1) \cdot fy}{Y_{\text{top}}}$$

$$f1 = 48.81 \text{ ksi} \quad f2 = 48.81 \text{ ksi}$$

$$\psi := \frac{f2}{f1} \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h} \right)^2 \quad F_{cr} = 34.347 \text{ ksi} \quad f_{\text{eff}} := f1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.192$$

$$\rho_{\text{eff}} := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{\lambda}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.684$$

$$1 \text{ otherwise}$$

$$b_{e,\text{eff}} := \rho \cdot h \quad b_{e,\text{eff}} = 5.279 \text{ in}$$

$$\frac{h0}{b0} = 2.667 \quad \frac{h0}{b0} \leq 4$$

$$b1 := \frac{b_{e,\text{eff}}}{3 + \psi}$$

$$b2 := \begin{cases} \frac{b_{e,\text{eff}}}{2} \text{ if } \psi > 0.236 \\ b_{e,\text{eff}} - b1 \text{ otherwise} \end{cases}$$

$$b1 = 1.32 \text{ in} \quad b2 = 2.639 \text{ in}$$

$$b1 + b2 = 3.959 \text{ in} \quad Y_{\text{top}} - d1 = 3.858 \text{ in}$$

$$b1 + b2 < 3.858 \quad \text{the web is not fully effective.}$$

Moment of Inertia calculation

Element	L	y	Ly	Ly ²	I
1	0.719	0.501	0.360	0.180	0.031
2	0.356	0.069	0.025	0.002	
3	1.931	0.028	0.055	0.002	
4a	1.320	0.802	1.058	0.848	0.192
4b	2.639	2.681	7.074	18.961	1.532
4c	3.859	5.929	22.878	135.649	4.787
5	2.717	7.972	21.659	172.660	
6	0.356	7.931	2.823	22.390	
7	0.859	7.429	6.382	47.408	0.053
sum	14.756		62.313	398.101	6.594
	Ytop	4.223		Ix	141.543
				Ixe	8.011

Trial 2

$$Y_{top} := 4.23in \quad f := f_y$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 7.717in \quad \frac{h}{t} = 136.343$$

$$f_1 := f_y \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 48.907ksi \quad f_2 = 43.405ksi$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.887 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 21.224$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 30.374ksi \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.269$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.651$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h \quad b_e = 5.027in$$

$$\frac{h_0}{b_0} = 2.667 \quad \frac{h_0}{b_0} \leq 4$$

$$b1 := \frac{be}{3 + \psi}$$

$$b2 := \begin{cases} \frac{be}{2} & \text{if } \psi > 0.236 \\ be - b1 & \text{otherwise} \end{cases}$$

$$b1 = 1.293 \text{ in} \quad b2 = 2.514 \text{ in}$$

$$b1 + b2 = 3.807 \text{ in} \quad Y_{top} - d1 = 4.089 \text{ in}$$

$b1 + b2 < 4.089$ the web is not fully effective.

Trail 2 assume $Y_{top}=4.23$

Element	L	y	Ly	Ly ²	I
1	0.719	0.501	0.360	0.180	0.031
2	0.356	0.069	0.025	0.002	
3	1.931	0.028	0.055	0.002	
4a	1.293	0.788	1.019	0.803	0.180
4b	2.514	2.973	7.474	22.221	1.324
4c	3.629	6.044	21.932	132.560	3.981
5	2.717	7.972	21.659	172.660	
6	0.356	7.931	2.823	22.390	
7	0.859	7.429	6.382	47.408	0.053
sum	14.374		61.728	398.226	5.569
	Y_{top}	4.295		I_x	138.699
				I_{xe}	7.850

Trial 3

$$Y_{top} := 4.32 \text{ in} \quad f := f_y$$

Web under stress gradient

$$h := h_0 - 2 \cdot d1 \quad h = 7.717 \text{ in} \quad \frac{h}{t} = 136.343$$

$$f1 := f_y \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f2 := \frac{(h_0 - Y_{top} - d1) \cdot f_y}{Y_{top}}$$

$$f1 = 48.943 \text{ ksi}$$

$$f2 = 41.446 \text{ ksi}$$

$$\psi := \frac{f2}{f1} \quad \psi = 0.847$$

$$k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 20.292$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h} \right)^2$$

$$F_{cr} = 29.04 \text{ ksi}$$

$$f := f1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.298$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.64$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h \quad b_e = 4.937 \text{ in}$$

$$\frac{h_0}{b_0} = 2.667 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} \\ b_e - b_1 \end{cases} \text{ if } \psi > 0.236$$

$$\text{otherwise}$$

$$b_1 = 1.283 \text{ in} \quad b_2 = 2.469 \text{ in}$$

$$b_1 + b_2 = 3.752 \text{ in} \quad Y_{\text{top}} - d_1 = 4.179 \text{ in}$$

$$b_1 + b_2 < 4.179 \text{ in} \quad \text{the web is not fully effective.}$$

Trial3 assume $y=4.32 \text{ in}$

Element	L	y	L_y	L_y^2	I
1	0.719	0.501	0.360	0.180	0.031
2	0.356	0.069	0.025	0.002	
3	1.931	0.028	0.055	0.002	
4a	1.283	0.783	1.005	0.787	0.176
4b	2.469	3.086	7.618	23.506	1.254
4c	3.509	6.104	21.417	130.733	3.599
5	2.717	7.972	21.659	172.660	
6	0.356	7.931	2.823	22.390	
7	0.859	7.429	6.382	47.408	0.053
sum	14.199		61.343	397.668	5.113
	Y_{top}	4.320		I_x	137.756
				I_{xe}	7.797

From trial 3 it is found that $Y_{cg}=4.32038 \text{ in}$ which is almost equal to 4.32 in. So no other iteration is necessary.

$$I_{xe} := 7.79699 \text{ in}^4 \quad Y_{xe} := 4.32038 \text{ in}$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 1.805 \text{ in}^3 \quad M_n := S_{xe} \cdot f_y \quad f_y = 50.6 \text{ ksi}$$

$$M_n = 91.318 \text{ kip} \cdot \text{in}$$

Stud section: 203S76 – 14

$$E := 29435 \text{ ksi} \quad f_y := 50.6 \text{ ksi} \quad \mu := 0.3 \quad t := 0.0713 \text{ in} \quad h_0 := 8 \text{ in} \quad b_0 := 3 \text{ in}$$

$$r_i := 1.5 \cdot t \quad r_i = 0.107 \text{ in} \quad d_1 := t + r_i \quad D := 1 \text{ in}$$

Gross section properties

Element	L	y	Ly	Ly ²	I
1	0.822	0.589	0.484	0.285	0.046
2	0.448	0.087	0.039	0.003	
3	2.644	0.036	0.094	0.003	
4	7.644	4.000	30.574	122.296	37.213
5	2.644	7.964	21.054	167.680	
6	0.448	7.913	3.543	28.034	
7	0.822	7.411	6.090	45.131	0.046
sum	15.470		61.878	363.433	37.306
	Ytop	4.000		Ix	153.226
				Ixg	10.925

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d_1 \quad w = 2.643 \text{ in}$$

$$\frac{w}{t} = 37.076 \quad d := D - d_1 \quad I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 3.297 \times 10^{-3} \text{ in}^4 \quad f := f_y$$

$$S := 1.28 \sqrt{\frac{E}{f}} \quad S = 30.872 \quad 0.328 S = 10.126$$

$$I_{a1} := 399 t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3$$

$$I_{a1} = 6.859 \times 10^{-3} \text{ in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \frac{\frac{w}{t}}{S} + 5 \right) \quad I_{a2} = 3.698 \times 10^{-3} \text{ in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases}$$

$$I_a = 3.698 \times 10^{-3} \text{ in}^4$$

$$\frac{D}{w} = 0.378 \quad R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases} \quad R_i = 0.891$$

$$n := \begin{cases} 0.582 - \frac{w}{4 \cdot S} & \text{if } 0.582 - \frac{w}{4 \cdot S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.333$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w}\right) \cdot Ri^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot Ri^n + 0.43 & \text{otherwise} \end{cases}$$

$$k_{\text{eff}} := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.249$$

$$F_{cr} := \frac{k_{\text{eff}} \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w}\right)^2 \quad F_{cr} = 62.871 \text{ ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.897$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.841$$

$$b_e := \rho \cdot w \quad b_e = 2.224 \text{ in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 4 \text{ in} \quad d1 := t + r_i \quad f := f_y \quad d = 0.822 \text{ in}$$

$$\frac{d}{t} = 11.525 \quad f1 := f \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f2 := f \cdot \frac{Y_{top} - d1 - d}{Y_{top}}$$

$$f1 = 48.345 \text{ ksi} \quad f2 = 37.95 \text{ ksi} \quad \psi := \frac{f2}{f1} \quad \psi = 0.785$$

$$k_{\text{eff}} := \frac{0.578}{\psi + 0.34} \quad k = 0.514$$

$$F_{cr} := \frac{k_{\text{eff}} \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 102.902 \text{ ksi} \quad f := f1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.685$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.991$$

$$ds' := \rho \cdot d \quad ds' = 0.814\text{in} \quad ds := ds' \cdot Ri \quad ds = 0.726\text{in}$$

Web under stress gradient

$$h := h0 - 2 \cdot d1 \quad h = 7.643\text{in} \quad \frac{h}{t} = 107.202 \quad \underline{f} := f1 \quad \underline{f2} := \frac{(h0 - Y_{\text{top}} - d1) \cdot fy}{Y_{\text{top}}}$$

$$f1 = 48.345\text{ksi} \quad f2 = 48.345\text{ksi}$$

$$\underline{\psi} := \frac{f2}{f1} \quad \psi = 1 \quad \underline{k} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$\underline{Fcr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad Fcr = 55.558\text{ksi} \quad \underline{f} := f1$$

$$\underline{\lambda} := \sqrt{\frac{f}{Fcr}} \quad \lambda = 0.933$$

$$\underline{\rho} := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.819$$

$$\underline{be} := \rho \cdot h \quad be = 6.261\text{in}$$

$$\frac{h0}{b0} = 2.667 \quad \frac{ho}{b0} \leq 4$$

$$b1 := \frac{be}{3 + \psi}$$

$$b2 := \begin{cases} \frac{be}{2} & \text{if } \psi > 0.236 \\ be - b1 & \text{otherwise} \end{cases}$$

$$b1 = 1.565\text{in} \quad b2 = 3.131\text{in}$$

$$b1 + b2 = 4.696\text{in} \quad Y_{\text{top}} - d1 = 3.822\text{in}$$

$$b1 + b2 > 3.822 \quad \text{the web is fully effective.}$$

Moment of Inertia calculation

Trial1	assume	Ytop=4in			
1	0.726	0.541	0.393	0.213	0.032
2	0.448	0.087	0.039	0.003	
3	2.224	0.036	0.079	0.003	
4	7.644	4.000	30.574	122.296	37.213
5	2.644	7.964	21.054	167.680	
6	0.448	7.913	3.543	28.034	
7	0.822	7.411	6.090	45.131	0.046
sum	14.954		61.772	363.360	37.291
	Ytop	4.131		lx	145.488
				lxe	10.373

Web under stress gradient

$$Y_{top} := 4.130724 \text{ in} \quad f := f_y$$

$$h := h_0 - 2 \cdot d_1 \quad h = 7.643 \text{ in} \quad \frac{h}{t} = 107.202$$

$$f_1 := f \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 48.416 \text{ ksi}$$

$$f_2 = 45.214 \text{ ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.934 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 22.332$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h} \right)^2 \quad F_{cr} = 51.697 \text{ ksi} \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.968$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.798$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h \quad b_e = 6.103 \text{ in}$$

$$\frac{h_0}{b_0} = 2.667 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} \\ b_e - b_1 \end{cases} \text{ if } \psi > 0.236$$

$$\text{otherwise}$$

$$b1 = 1.551\text{in} \quad b2 = 3.051\text{in}$$

$$b1 + b2 = 4.603\text{in} \quad Y_{\text{top}} - d1 = 3.952\text{in}$$

$$b1 + b2 > 3.96 \quad \text{the web is fully effective.}$$

$$Y_{\text{xe}} := 4.130724\text{in} \quad I_{\text{xe}} := 10.3733\text{in}^4$$

$$S_{\text{xe}} := \frac{I_{\text{xe}}}{Y_{\text{xe}}} \quad S_{\text{xe}} = 2.511\text{in}^3 \quad M_{\text{n}} := S_{\text{xe}} \cdot f_y \quad f_y = 50.6\text{ksi}$$

$$M_{\text{n}} = 127.069\text{kip}\cdot\text{in}$$

Stud section: 30S76 – 18

$$E := 29435\text{ksi} \quad f_y := 50.6\text{ksi} \quad \mu := 0.3 \quad t := 0.045\text{in} \quad h_0 := 12\text{in} \quad b_0 := 3\text{in}$$

$$r_i := 1.5t \quad r_i = 0.068\text{in} \quad d1 := t + r_i \quad D := 1\text{in}$$

Gross section

Element	L	y	Ly	Ly ²	I
1	0.887	0.556	0.494	0.275	0.058
2	0.283	0.055	0.016	0.001	
3	2.775	0.023	0.063	0.001	
4	11.775	6.000	70.647	423.882	39.159
5	2.775	11.977	33.231	398.028	
6	0.283	11.945	3.383	40.410	
7	0.887	11.444	10.153	116.191	0.058
sum	19.664		117.987	978.788	39.276
	Y _{top}	6.000		I _x	310.143
				I _{xg}	13.987

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d1 \quad w = 2.774\text{in}$$

$$\frac{w}{t} = 61.519 \quad d := D - d1 \quad I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 2.625 \times 10^{-3} \text{in}^4 \quad f := f_y$$

$$S := 1.28 \sqrt{\frac{E}{f}} \quad S = 30.872 \quad 0.328S = 10.126$$

$$I_{a1} := 399t^4 \cdot \left(\frac{w}{S} - 0.328 \right)^3 \quad I_{a1} = 7.615 \times 10^{-3} \text{in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \cdot \frac{w}{S} + 5 \right) \quad I_{a2} = 9.688 \times 10^{-4} \text{ in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases}$$

$$I_a = 9.688 \times 10^{-4} \text{ in}^4$$

$$\frac{D}{w} = 0.36 \quad R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases} \quad R_i = 1$$

$$n := \begin{cases} 0.582 - \frac{w}{4 \cdot S} & \text{if } 0.582 - \frac{w}{4 \cdot S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.333$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w} \right) \cdot R_i^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot R_i^n + 0.43 & \text{otherwise} \end{cases}$$

$$k := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.448$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w} \right)^2 \quad F_{cr} = 24.237 \text{ ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.445$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.587$$

$$b_e := \rho \cdot w \quad b_e = 1.628 \text{ in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 6 \text{ in} \quad d1 := t + r_i \quad f := f_y \quad d = 0.887 \text{ in}$$

$$\frac{d}{t} = 19.673 \quad f1 := f \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f2 := f \cdot \frac{Y_{top} - d1 - d}{Y_{top}}$$

$$f1 = 49.649 \text{ ksi} \quad f2 = 42.167 \text{ ksi} \quad \psi := \frac{f2}{f1} \quad \psi = 0.849$$

$$k := \frac{0.578}{\psi + 0.34} \quad k = 0.486$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 33.407 \text{ksi} \quad f_w := f1$$

$$\lambda_w := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.219$$

$$\rho_w := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{\lambda}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.672$$

$$1 \text{ otherwise}$$

$$ds' := \rho \cdot d \quad ds' = 0.596 \text{in} \quad ds := ds' \cdot Ri \quad ds = 0.596 \text{in}$$

Web under stress gradient

$$h := h0 - 2 \cdot d1 \quad h = 11.774 \text{in} \quad \frac{h}{t} = 261.075 \quad f_w := f1 \quad f_w^2 := \frac{(h0 - Y_{top} - d1) \cdot fy}{Y_{top}}$$

$$f1 = 49.649 \text{ksi} \quad f2 = 49.649 \text{ksi}$$

$$\psi_w := \frac{f2}{f1} \quad \psi = 1 \quad k_w := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 9.367 \text{ksi} \quad f_w := f1$$

$$\lambda_w := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.302$$

$$\rho_w := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{\lambda}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.393$$

$$1 \text{ otherwise}$$

$$be_w := \rho \cdot h \quad be = 4.626 \text{in}$$

$$\frac{h0}{b0} = 4 \quad \frac{h0}{b0} \leq 4$$

$$b1 := \frac{be}{3 + \psi}$$

$$b2 := \begin{cases} \frac{be}{2} \\ be - b1 \end{cases} \text{ if } \psi > 0.236$$

$$\text{otherwise}$$

$$b1 = 1.156\text{in} \quad b2 = 2.313\text{in}$$

$$b1 + b2 = 3.469\text{in} \quad Y_{\text{top}} - d1 = 5.887\text{in}$$

$b1 + b2 < 5.887$ the web is not fully effective.

Moment of Inertia calculation

Element	L	y	Ly	Ly ²	I
1	0.596	0.411	0.245	0.101	0.018
2	0.283	0.055	0.016	0.001	
3	1.628	0.023	0.037	0.001	
4a	1.156	0.691	0.799	0.552	0.129
4b	2.313	4.844	11.203	54.262	1.031
4c	5.887	8.944	52.653	470.912	17.004
5	2.775	11.977	33.231	398.028	
6	0.283	11.945	3.383	40.410	
7	0.887	11.444	10.153	116.191	0.058
sum	15.808		111.720	1080.456	18.240
	Y _{top}	7.067		I _x	309.161
				I _{xe}	13.943

Trial 2

$$Y_{\text{top}} := 7.25\text{in} \quad f := f_y$$

Web under stress gradient

$$h := h_0 - 2 \cdot d1 \quad h = 11.774\text{in} \quad \frac{h}{t} = 261.075$$

$$f1 := f_y \cdot \frac{Y_{\text{top}} - d1}{Y_{\text{top}}} \quad f2 := \frac{(h_0 - Y_{\text{top}} - d1) \cdot f_y}{Y_{\text{top}}}$$

$$f1 = 49.813\text{ksi} \quad f2 = 32.365\text{ksi}$$

$$\psi := \frac{f2}{f1} \quad \psi = 0.65 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 16.279$$

$$F_{\text{cr}} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{\text{cr}} = 6.354\text{ksi} \quad f := f1$$

$$\lambda := \sqrt{\frac{f}{F_{\text{cr}}}} \quad \lambda = 2.8$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.329$$

$$b_e := \rho \cdot h \quad b_e = 3.875 \text{ in}$$

$$\frac{h_0}{b_0} = 4 \qquad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.062\text{in} \qquad b_2 = 1.937\text{in}$$

$$b_1 + b_2 = 2.999\text{in} \qquad Y_{\text{top}} - d_1 = 7.137\text{in}$$

$b_1 + b_2 < 7.137$ the web is not fully effective.

Trial2	assume	Y _{top} =7.25in			
Element	L	y	Ly	Ly ²	I
1	0.596	0.411	0.245	0.101	0.018
2	0.283	0.055	0.016	0.001	
3	1.628	0.023	0.037	0.001	
4a	1.062	0.644	0.684	0.440	0.100
4b	1.937	6.282	12.167	76.429	0.606
4c	4.637	9.569	44.372	424.580	8.310
5	2.775	11.977	33.231	398.028	
6	0.283	11.945	3.383	40.410	
7	0.887	11.444	10.153	116.191	0.058
sum	14.088		104.288	1056.180	9.091
	Y _{top}	7.402		I _x	293.291
				I _{xe}	13.227

Trial 3

$$Y_{\text{top}} := 7.4\text{in} \qquad f := f_y$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \qquad h = 11.774\text{in} \qquad \frac{h}{t} = 261.075$$

$$f_1 := f_y \cdot \frac{Y_{\text{top}} - d_1}{Y_{\text{top}}} \qquad f_2 := \frac{(h_0 - Y_{\text{top}} - d_1) \cdot f_y}{Y_{\text{top}}}$$

$$f_1 = 49.829\text{ksi} \qquad f_2 = 30.683\text{ksi}$$

$$\psi := \frac{f_2}{f_1} \qquad \psi = 0.616 \qquad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \qquad k = 15.668$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 6.115 \text{ksi} \quad f_w := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.854$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.323$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h$$

$$\frac{h_0}{b_0} = 4 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} \\ b_e - b_1 \end{cases} \text{ if } \psi > 0.236$$

$$\text{otherwise}$$

$$b_1 = 1.053 \text{in} \quad b_2 = 1.903 \text{in}$$

$$b_1 + b_2 = 2.956 \text{in} \quad Y_{top} - d_1 = 7.287 \text{in}$$

$$b_1 + b_2 < 7.287 \text{in} \quad \text{the web is not fully effective.}$$

Trial 3 assume $Y_{top} = 7.4 \text{in}$

Element	L	y	Ly	Ly^2	I
1	0.596	0.411	0.245	0.101	0.018
2	0.283	0.055	0.016	0.001	
3	1.628	0.023	0.037	0.001	
4a	1.053	0.639	0.673	0.430	0.097
4b	1.903	6.449	12.271	79.133	0.574
4c	4.487	9.644	43.273	417.312	7.529
5	2.775	11.977	33.231	398.028	
6	0.283	11.945	3.383	40.410	
7	0.887	11.444	10.153	116.191	0.058
sum	13.895		103.283	1051.606	8.277
	Y_{top}	7.433		I_x	292.194
				I_{xe}	13.178

Trial 4

$$Y_{top} := 7.44 \text{in} \quad f_w := f_y$$

Web under stress gradient

$$\begin{aligned} \underline{h} &:= h_0 - 2 \cdot d_1 & h &= 11.774 \text{in} & \frac{h}{t} &= 261.075 \\ \underline{f1} &:= f_y \cdot \frac{Y_{\text{top}} - d_1}{Y_{\text{top}}} & \underline{f2} &:= \frac{(h_0 - Y_{\text{top}} - d_1) \cdot f_y}{Y_{\text{top}}} \\ f1 &= 49.833 \text{ksi} & f2 &= 30.246 \text{ksi} \\ \underline{\psi} &:= \frac{f2}{f1} & \psi &= 0.607 & \underline{k} &:= 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) & k &= 15.513 \\ \underline{F_{cr}} &:= \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h} \right)^2 & F_{cr} &= 6.055 \text{ksi} & \underline{f} &:= f1 \\ \underline{\lambda} &:= \sqrt{\frac{f}{F_{cr}}} & \lambda &= 2.869 \\ \underline{\rho} &:= \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} & \rho &= 0.322 \\ \underline{b_e} &:= \rho \cdot h \\ \frac{h_0}{b_0} &= 4 & \frac{h_0}{b_0} &\leq 4 \\ \underline{b1} &:= \frac{b_e}{3 + \psi} & \underline{b2} &:= \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b1 & \text{otherwise} \end{cases} \\ b1 &= 1.051 \text{in} & b2 &= 1.895 \text{in} \\ b1 + b2 &= 2.945 \text{in} & Y_{\text{top}} - d_1 &= 7.327 \text{in} \\ b1 + b2 &< 7.287 \text{in} & & \text{the web is not fully effective.} \end{aligned}$$

Trial4 assume $y=7.44\text{in}$

Element	L	y	Ly	Ly ²	I
1	0.596	0.411	0.245	0.101	0.018
2	0.283	0.055	0.016	0.001	
3	1.628	0.023	0.037	0.001	
4a	1.051	0.638	0.671	0.428	0.097
4b	1.895	6.493	12.303	79.879	0.567
4c	4.447	9.664	42.977	415.309	7.330
5	2.775	11.977	33.231	398.028	
6	0.283	11.945	3.383	40.410	
7	0.887	11.444	10.153	116.191	0.058
sum	13.845		103.016	1050.348	8.069
	Ytop	7.440		Ix	291.940
				Ixe	13.166

From trial 4 it is found that $Y_{cg}=7.440397\text{in}$ which is almost equal to 7.44 in. So no other iteration is necessary.

$$Y_{xe} := 7.440397\text{in} \quad I_{xe} := 13.16647\text{in}^4$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 1.77\text{in}^3 \quad M_n := S_{xe} \cdot f_y \quad f_y = 50.6\text{ksi}$$

$$M_n = 89.541\text{kip-in}$$

Stud section: 30S76 – 16

$$E := 29435\text{ksi} \quad f_y := 50.6\text{ksi} \quad \mu := 0.3 \quad t := 0.0566\text{in} \quad h_0 := 12\text{in} \quad b_0 := 3\text{in}$$

$$r_i := 1.5t \quad r_i = 0.085\text{in} \quad d_1 := t + r_i \quad D := 1\text{in}$$

Gross section

Element	L	y	Ly	Ly ²	I
1	0.859	0.571	0.490	0.280	0.053
2	0.356	0.069	0.025	0.002	
3	2.717	0.028	0.077	0.002	
4	11.717	6.000	70.302	421.812	134.050
5	2.717	11.972	32.527	389.405	
6	0.356	11.931	4.247	50.673	
7	0.859	11.429	9.818	112.204	0.053
sum	19.581		117.486	974.378	134.156
	Ytop	6.000		Ix	403.618
				I _{xg}	22.845

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d_1$$

$$w = 2.717 \text{ in}$$

$$\frac{w}{t} = 48.004$$

$$d := D - d_1$$

$$I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 2.984 \times 10^{-3} \text{ in}^4$$

$$f := f_y$$

$$S := 1.28 \sqrt{\frac{E}{f}}$$

$$S = 30.872$$

$$0.328 S = 10.126$$

$$\frac{w}{t} > 0.328 S$$

$$I_{a1} := 399 t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3$$

$$I_{a1} = 7.563 \times 10^{-3} \text{ in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \cdot \frac{\frac{w}{t}}{S} + 5 \right)$$

$$I_{a2} = 1.886 \times 10^{-3} \text{ in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases}$$

$$I_a = 1.886 \times 10^{-3} \text{ in}^4$$

$$\frac{D}{w} = 0.368$$

$$R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases}$$

$$R_i = 1$$

$$n := \begin{cases} 0.582 - \frac{\frac{w}{t}}{4S} & \text{if } 0.582 - \frac{\frac{w}{t}}{4S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.333$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w} \right) \cdot R_i^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot R_i^n + 0.43 & \text{otherwise} \end{cases}$$

$$k := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases}$$

$$k = 3.41$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w}\right)^2 \quad F_{cr} = 39.365 \text{ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.134$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{\lambda}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.711$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot w \quad b_e = 1.931 \text{in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 6 \text{in} \quad d_1 := t + r_i \quad f_w := f_y \quad d = 0.859 \text{in}$$

$$\frac{d}{t} = 15.168 \quad f_1 := f \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := f \cdot \frac{Y_{top} - d_1 - d}{Y_{top}}$$

$$f_1 = 49.407 \text{ksi} \quad f_2 = 42.167 \text{ksi} \quad \psi := \frac{f_2}{f_1} \quad \psi = 0.853$$

$$k_w := \frac{0.578}{\psi + 0.34} \quad k = 0.484$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 56.003 \text{ksi} \quad f_w := f_1$$

$$\lambda_w := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.939$$

$$\rho_w := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{\lambda}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.815$$

$$1 \text{ otherwise}$$

$$d_s' := \rho \cdot d \quad d_s' = 0.7 \text{in} \quad d_s := d_s' \cdot R_i \quad d_s = 0.7 \text{in}$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 11.717 \text{in} \quad \frac{h}{t} = 207.014$$

$$f_1 := f_y \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 49.407 \text{ksi} \quad f_2 = 49.407 \text{ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 14.899 \text{ksi} \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.821$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.483$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h \quad b_e = 5.657 \text{in}$$

$$\frac{h_0}{b_0} = 4 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} \\ b_e - b_1 \end{cases} \text{ if } \psi > 0.236$$

$$\text{otherwise}$$

$$b_1 = 1.414 \text{in} \quad b_2 = 2.828 \text{in}$$

$$b_1 + b_2 = 4.243 \text{in} \quad Y_{top} - d_1 = 5.858 \text{in}$$

$$b_1 + b_2 < 5.858 \quad \text{the web is not fully effective.}$$

Moment of Inertia calculation

Trial1	assume	Ycg=6in			
Element	L	y	Ly	Ly ²	I
1	0.700	0.492	0.344	0.169	0.029
2	0.356	0.069	0.025	0.002	
3	1.931	0.028	0.055	0.002	
4a	1.414	0.849	1.200	1.018	0.236
4b	2.828	4.586	12.969	59.477	1.885
4c	5.859	8.929	52.312	467.107	16.756
5	2.717	11.972	32.527	389.405	
6	0.356	11.931	4.247	50.673	
7	0.859	11.429	9.818	112.204	0.053
sum	17.020		113.496	1080.056	18.958
	Ytop	6.669		lx	342.152
				lxe	19.366

Trial 2

$$Y_{top} := 6.85 \text{ in} \quad f := f_y$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 11.717 \text{ in} \quad \frac{h}{t} = 207.014$$

$$f_1 := f_y \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 49.555 \text{ ksi} \quad f_2 = 36.997 \text{ ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.747 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 18.149$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h} \right)^2 \quad F_{cr} = 11.267 \text{ ksi} \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.097$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.427$$

$$b_e := \rho \cdot h \quad b_e = 5.001 \text{ in}$$

$$\frac{h_0}{b_0} = 4 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.335 \text{ in} \quad b_2 = 2.5 \text{ in}$$

$$b_1 + b_2 = 3.835 \text{ in} \quad Y_{top} - d_1 = 6.708 \text{ in}$$

$b_1 + b_2 < 6.708$ the web is not fully effective.

Trail 2 assume Ycg=6.85

Element	L	y	Ly	Ly ²	I
1	0.700	0.492	0.344	0.169	0.029
2	0.356	0.069	0.025	0.002	
3	1.931	0.028	0.055	0.002	
4a	1.335	0.809	1.080	0.874	0.198
4b	2.500	5.600	14.000	78.400	1.302
4c	5.009	9.354	46.851	438.254	10.470
5	2.717	11.972	32.527	389.405	
6	0.356	11.931	4.247	50.673	
7	0.859	11.429	9.818	112.204	0.053
sum	15.763		108.946	1069.982	12.052
	Ytop	6.912		lx	329.028
				lx _e	18.623

Trial 3

$$Y_{top} := 6.93 \text{ in} \quad f := f_y$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 11.717 \text{ in} \quad \frac{h}{t} = 207.014$$

$$f_1 := f_y \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 49.567 \text{ ksi} \quad f_2 = 35.986 \text{ ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.726 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 17.736$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 11.01 \text{ ksi} \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.122$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.422$$

$$b_e := \rho \cdot h \quad b_e = 4.95 \text{ in}$$

$$\frac{h_0}{b_0} = 4 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} \\ b_e - b_1 \end{cases} \text{ if } \psi > 0.236$$

$$b1 = 1.328\text{in} \quad b2 = 2.475\text{in}$$

$$b1 + b2 = 3.803\text{in} \quad Y_{\text{top}} - d1 = 6.788\text{in}$$

$$b1 + b2 < 6.788\text{in} \quad \text{the web is not fully effective.}$$

Trial3	assume	Ytop=6.93			
Element	L	y	Ly	Ly ²	I
1	0.700	0.492	0.344	0.169	0.029
2	0.356	0.069	0.025	0.002	
3	1.931	0.028	0.055	0.002	
4a	1.328	0.806	1.070	0.862	0.195
4b	2.475	5.693	14.089	80.201	1.263
4c	4.929	9.394	46.300	434.950	9.976
5	2.717	11.972	32.527	389.405	
6	0.356	11.931	4.247	50.673	
7	0.859	11.429	9.818	112.204	0.053
sum	15.651		108.474	1068.467	11.516
	Ytop	6.931		Ix	328.154
				Ixe	18.574

From trial 3 it is found that $Y_{cg}=6.930994\text{in}$ which is almost equal to 6.93 in. So no other iteration is necessary.

$$I_{xe} := 18.5735\text{in}^4 \quad Y_{xe} := 6.930994\text{in}$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 2.68\text{in}^3 \quad M_n := S_{xe} \cdot f_y \quad f_y = 50.6\text{ksi}$$

$$M_n = 135.597\text{kip}\cdot\text{in}$$

Stud section: 30S76 – 14

$$E := 29435\text{ksi} \quad f_y := 50.6\text{ksi} \quad \mu := 0.3 \quad t := 0.0713\text{in} \quad h_0 := 12\text{in} \quad b_0 := 3\text{in}$$

$$r_i := 1.5t \quad r_i = 0.107\text{in} \quad d1 := t + r_i \quad D := 1\text{in}$$

Gross section

Element	L	y	Ly	Ly ²	I
1	0.822	0.589	0.484	0.285	0.046
2	0.448	0.087	0.039	0.003	
3	2.644	0.036	0.094	0.003	
4	11.644	6.000	69.861	419.166	131.544
5	2.644	11.964	31.628	378.406	
6	0.448	11.913	5.334	63.542	
7	0.822	11.411	9.377	106.998	0.046
sum	19.470		116.817	968.404	131.636
	Ytop	6.000		Ix	399.137
				Ixg	28.458

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d_1$$

$$w = 2.643 \text{ in}$$

$$\frac{w}{t} = 37.076$$

$$d := D - d_1$$

$$I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 3.297 \times 10^{-3} \text{ in}^4$$

$$f := f_y$$

$$S := 1.28 \sqrt{\frac{E}{f}}$$

$$S = 30.872$$

$$0.328 S = 10.126$$

$$I_{a1} := 399 t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3$$

$$I_{a1} = 6.859 \times 10^{-3} \text{ in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \cdot \frac{t}{S} + 5 \right)$$

$$I_{a2} = 3.698 \times 10^{-3} \text{ in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases}$$

$$I_a = 3.698 \times 10^{-3} \text{ in}^4$$

$$\frac{D}{w} = 0.378$$

$$R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases}$$

$$R_i = 0.891$$

$$n := \begin{cases} 0.582 - \frac{\frac{w}{t}}{4S} & \text{if } 0.582 - \frac{\frac{w}{t}}{4S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.333$$

$$k := \begin{cases} \left[\left(4.82 - 5 \cdot \frac{D}{w} \right) \cdot R_i^n + 0.43 \right] & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ \left[3.57 \cdot R_i^n + 0.43 \right] & \text{otherwise} \end{cases}$$

$$k := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases}$$

$$k = 3.249$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w}\right)^2 \quad F_{cr} = 62.871 \text{ ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.897$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.841$$

$$b_e := \rho \cdot w \quad b_e = 2.224 \text{ in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 6 \text{ in} \quad d_1 := t + r_i \quad f := f_y \quad d = 0.822 \text{ in}$$

$$\frac{d}{t} = 11.525 \quad f_1 := f \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := f \cdot \frac{Y_{top} - d_1 - d}{Y_{top}}$$

$$f_1 = 49.097 \text{ ksi} \quad f_2 = 42.167 \text{ ksi} \quad \psi := \frac{f_2}{f_1} \quad \psi = 0.859$$

$$k := \frac{0.578}{\psi + 0.34} \quad k = 0.482$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 96.562 \text{ ksi} \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.713$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.97$$

$$d_s' := \rho \cdot d \quad d_s' = 0.797 \text{ in} \quad d_s := d_s' \cdot R_i \quad d_s = 0.71 \text{ in}$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 11.643 \text{ in} \quad \frac{h}{t} = 163.303 \quad f := f_1 \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 49.097 \text{ ksi} \quad f_2 = 49.097 \text{ ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 23.942 \text{ ksi} \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.432$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{\lambda}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.591$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h \quad b_e = 6.882 \text{ in}$$

$$\frac{h_0}{b_0} = 4 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.72 \text{ in} \quad b_2 = 3.44 \text{ in}$$

$$b_1 + b_2 = 5.16 \text{ in} \quad Y_{\text{top}} - d_1 = 5.822 \text{ in}$$

$$b_1 + b_2 < 5.822 \text{ in} \quad \text{the web is not fully effective.}$$

Moment of Inertia calculation

Trial1 assume $Y_{cg}=6\text{in}$

Element	L	y	Ly	Ly ²	I
1	0.710	0.533	0.484	0.285	0.030
2	0.448	0.087	0.039	0.003	
3	2.224	0.036	0.094	0.003	
4a	1.720	1.038	1.786	1.854	0.424
4b	3.441	4.280	14.726	63.019	3.395
4c	5.822	8.911	51.877	462.268	16.443
5	2.644	11.964	31.628	378.406	
6	0.448	11.913	5.334	63.542	
7	0.822	11.411	9.377	106.998	0.046
sum	18.278		115.345	1076.380	20.338
	Y_{top}	6.311		I_x	368.809
				I_{xe}	26.296

Web under stress gradient

$$Y_{\text{top}} := 6.45 \text{ in}$$

$$h := h_0 - 2 \cdot d_1 \quad h = 11.643 \text{ in} \quad \frac{h}{t} = 163.303$$

$$f_1 := f_y \cdot \frac{Y_{\text{top}} - d_1}{Y_{\text{top}}} \quad f_2 := \frac{(h_0 - Y_{\text{top}} - d_1) \cdot f_y}{Y_{\text{top}}}$$

$$f_1 = 49.202 \text{ ksi}$$

$$f_2 = 42.14 \text{ ksi}$$

$$\psi := \frac{f2}{f1} \quad \psi = 0.856 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 20.51$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 20.461 \text{ ksi} \quad f := f1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.551$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.553$$

$$1 \text{ otherwise}$$

$$b_e := \rho \cdot h \quad b_e = 6.443 \text{ in}$$

$$\frac{h_0}{b_0} = 4 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.671 \text{ in} \quad b_2 = 3.222 \text{ in}$$

$$b_1 + b_2 = 4.892 \text{ in} \quad Y_{top} - d_1 = 6.272 \text{ in}$$

$$b_1 + b_2 < 6.272 \text{ in} \quad \text{the web is not fully effective.}$$

Trial2 Assume $y_{cg} = 6.45$

Element	L	y	Ly	Ly ²	I
1	0.710	0.533	0.484	0.285	0.030
2	0.448	0.087	0.039	0.003	
3	2.224	0.036	0.094	0.003	
4a	1.671	1.014	1.694	1.717	0.389
4b	3.222	4.839	15.591	75.446	2.787
4c	5.372	9.136	49.076	448.349	12.917
5	2.644	11.964	31.628	378.406	
6	0.448	11.913	5.334	63.542	
7	0.822	11.411	9.377	106.998	0.046
sum	17.560		113.317	1074.750	16.169
	Y _{top}	6.453		I _x	359.650
				I _{xe}	25.643

From trial 2 $Y_{xe} = 6.453 \text{ in}$ which is almost equal to 6.45 in. So further iteration is not necessary.

$$Y_{xe} := 6.453308 \text{ in} \quad I_{xe} := 25.64305 \text{ in}^4$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 3.974 \text{ in}^3 \quad M_n := S_{xe} \cdot f_y \quad f_y = 50.6 \text{ ksi}$$

$$M_n = 201.066 \text{ kip-in}$$

Track section: 254T32 - 16

$$F_y := 44.5 \text{ ksi} \quad h := 10.1134 \text{ in} \quad t := 0.0566 \text{ in} \quad r := 1.5 \cdot t \quad r = 0.085 \text{ in}$$

$$b := 1.25 \text{ in} \quad w := b - 2.5t \quad w = 1.109 \text{ in} \quad E := 29435 \text{ ksi} \quad \mu := 0.3$$

Element	Length	y	LY	Ly ²	I'
1	1.108	0.028	0.031	0.001	
2	0.178	0.070	0.012	0.001	
3	9.830	5.057	49.707	251.353	79.153
4	0.178	10.044	1.788	17.960	
5	1.108	10.085	11.177	112.718	
sum	12.402		62.716	382.033	79.153
ycg	5.057				
lxg	8.168				

Flange under uniform compression:

$$\frac{w}{t} = 19.585 \quad k := 0.43 \quad f := F_y$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 29.824 \text{ ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.222$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.671$$

$$be := \rho \cdot w \quad be = 0.744 \text{ in}$$

Web under stress gradient:

$$\frac{w}{t} := h - 5 \cdot t \quad w = 9.8304 \text{ in} \quad \frac{w}{t} = 173.682 \quad y_{cg} := 5.0567 \text{ in}$$

$$w = 9.8304 \text{ in}$$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f1 = 43.255 \text{ ksi} \quad f2 := f1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f2 = 43.255 \text{ ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 1$$

$$k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 21.166 \text{ ksi} \quad f := f1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.43$$

$$\rho_w := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.592 \quad b_e := \rho \cdot w \quad b_e = 5.818 \text{ in}$$

$$h_o := 10.1134 \text{ in} \quad b_o := 1.25 \text{ in} \quad \frac{h_o}{b_o} = 8.091$$

$$b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1$$

$$b1 = 1.455 \text{ in} \quad b2 = 1.455 \text{ in}$$

$b1 + b2 = 2.909 \text{ in}$ is less than compression portion of web element = 6.4085 in

So the web is not fully effective

Trial1 assume $y_{cg} = 5.0566 \text{ in}$

Element	Length	y	LY	Ly ²	I'
1	0.744	0.028	0.021	0.001	
2	0.178	0.070	0.012	0.001	
3a	1.455	0.869	1.265	1.099	0.257
3b	1.455	4.329	6.299	27.270	0.257
3c	4.915	7.514	36.932	277.512	9.894
4	0.178	10.044	1.788	17.960	
5	1.108	10.085	11.177	112.718	
sum	10.033		57.494	436.561	10.407
y_{cg}	5.730				
l_{xe}	6.663				

Trial 2 $y_{cg} := 5.75 \text{ in}$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 43.405 \text{ ksi} \quad f2 := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5t) \quad f2 = 32.674 \text{ ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.753$$

$$k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 18.275$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 16.117 \text{ ksi} \quad f := f1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.641$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.528 \quad \underline{be} := \rho \cdot w \quad be = 5.187 \text{ in}$$

$$\underline{ho} := 12.1132 \text{ in} \quad \underline{bo} := 1.25 \text{ in} \quad \frac{ho}{bo} = 9.691$$

$$\underline{b1} := \frac{be}{3 + \psi} \quad \underline{b2} := \frac{be}{1 + \psi} - b1$$

$$b1 = 1.382 \text{ in} \quad b2 = 1.577 \text{ in}$$

Trial2 Assume $ycg=5.75$

Element	Length	y	LY	Ly ²	I'
1	0.744	0.028	0.021	0.001	
2	0.178	0.070	0.012	0.001	
3a	1.382	0.833	1.151	0.958	0.220
3b	1.577	4.962	7.824	38.820	0.327
3c	4.222	7.861	33.186	260.867	6.270
4	0.178	10.044	1.788	17.960	
5	1.108	10.085	11.177	112.718	
sum	9.389		55.159	431.325	6.817
ycg	5.875				
lxe	6.469				

Trial 3 $\underline{ycg} := 5.9 \text{ in}$

$$\underline{f1} := \frac{Fy}{ycg} \cdot (ycg - 2.5t) \quad f1 = 43.433 \text{ ksi} \quad \underline{f2} := \frac{Fy}{ycg} \cdot (h - ycg - 2.5t) \quad f2 = 30.712 \text{ ksi}$$

$$\underline{\psi} := \left| \frac{f2}{f1} \right| \quad \psi = 0.707$$

$$\underline{k} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 17.364$$

$$\underline{Fcr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad Fcr = 15.314 \text{ ksi} \quad \underline{f} := f1$$

$$\underline{\lambda} := \sqrt{\frac{f}{Fcr}} \quad \lambda = 1.684$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.516 \quad \underline{be} := \rho \cdot w \quad be = 5.075 \text{ in}$$

$$\frac{ho}{bo} = 9.691$$

$$\underline{b1} := \frac{be}{3 + \psi} \quad \underline{b2} := \frac{be}{1 + \psi} - b1$$

$$b1 = 1.369\text{in} \quad b2 = 1.604\text{in}$$

Trial 3 assume $y_{cg}=5.9$

Element	Length	y	LY	Ly ²	I'
1	0.744	0.028	0.021	0.001	
2	0.178	0.070	0.012	0.001	
3a	1.369	0.826	1.131	0.935	0.214
3b	1.604	5.098	8.177	41.687	0.344
3c	4.072	7.936	32.312	256.422	5.625
4	0.178	10.044	1.788	17.960	
5	1.108	10.085	11.177	112.718	
sum	9.253		54.619	429.724	6.183
y _{cg}	5.903				
I _{xe}	6.436				

$Y_{cg} := 5.90282\text{in}$ So no further iteration is necessary.

$$I_{xe} := 6.43558\text{in}^4 \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad Y_{cg} = 5.903\text{in}$$

$$M_n := F_y \cdot S_{xe} \quad M_n = 48.516\text{kip}\cdot\text{in}$$

Rim Track section

$$F_y := 60.5\text{ksi} \quad h := 10.1094\text{in} \quad t := 0.0547\text{in} \quad r := 2 \cdot t \quad r = 0.109\text{in}$$

$$b := 1.25\text{in} \quad w := b - 2.5t \quad w = 1.113\text{in} \quad E := 29435\text{ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	Ly	Ly ²	I'
1	1.114	0.027	0.030	0.001	
2	0.172	0.067	0.012	0.001	
3	9.836	5.055	49.718	251.307	79.298
4	0.172	10.042	1.725	17.322	
5	2.363	10.082	23.826	240.219	
	13.656		75.311	508.849	79.298
y _{cg}	5.515				
I _{xe}	9.454				

Flange under uniform compression:

$$\frac{w}{t} = 20.352 \quad k := 0.43 \quad f := F_y$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 27.618\text{ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.48$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.575 \quad be := \rho \cdot w \quad be = 0.64 \text{ in}$$

Web under stress gradient:

$$\begin{aligned} w_{\text{eff}} &:= h - 5 \cdot t & w &= 9.8359 \text{ in} & \frac{w}{t} &= 179.815 & y_{cg} &:= 5.514772 \text{ in} \\ w &= 9.8359 \text{ in} \end{aligned}$$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f1 = 59 \text{ ksi} \quad f2 := f1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f2 = 48.905 \text{ ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.829$$

$$k_{\text{eff}} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 19.893$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 16.368 \text{ ksi} \quad f_w := f1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.899$$

$$\rho_{\text{eff}} := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.466 \quad be_{\text{eff}} := \rho \cdot w \quad be = 4.58 \text{ in}$$

$$h_o := 10.1094 \text{ in} \quad b_o := 1.25 \text{ in} \quad \frac{h_o}{b_o} = 8.088$$

$$b1 := \frac{be}{3 + \psi} \quad b2 := \frac{be}{1 + \psi} - b1$$

$$b1 = 1.196 \text{ in} \quad b2 = 1.308 \text{ in}$$

$b1 + b2 = 2.504 \text{ in}$ is less than compression portion of web element = 5.378022 in

So the web is not fully effective

Trial1 assume $y_{cg} = 5.5147 \text{ in}$

Element	Length	y	Ly	Ly ²	I'
1	0.640	0.027	0.018	0.000	
2	0.172	0.067	0.012	0.001	
3a	1.196	0.735	0.879	0.646	0.143
3b	1.308	4.861	6.358	30.904	0.186
3c	4.458	7.744	34.521	267.317	7.382
4	0.172	10.042	1.725	17.322	
5	2.363	10.082	23.826	240.219	
	10.309		67.337	556.409	7.712
y _{cg}	6.532				
l _{xe}	6.797				

Trial 2 $y_{cg} := 6.75\text{in}$

$$f1 := \frac{Fy}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 59\text{ksi} \quad f2 := \frac{Fy}{y_{cg}} \cdot (h - y_{cg} - 2.5t) \quad f2 = 28.884\text{ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.487$$

$$k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 13.555$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 11.153\text{ksi} \quad f := f1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.305 \quad w$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.392 \quad b_e := \rho \cdot w \quad b_e = 3.859\text{in}$$

$$h_o := 10.1094\text{in} \quad b_o := 1.25\text{in} \quad \frac{h_o}{b_o} = 8.088$$

$$b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1$$

$$b1 = 1.107\text{in} \quad b2 = 1.488\text{in}$$

Trial2 assume $y_{cg}=6.75\text{in}$

Element	Length	y	Ly	Ly ²	I'
1	0.640	0.027	0.018	0.000	
2	0.172	0.067	0.012	0.001	
3a	1.107	0.690	0.764	0.527	0.113
3b	1.488	6.006	8.937	53.675	0.275
3c	3.223	8.361	26.946	225.301	2.789
4	0.172	10.042	1.725	17.322	
5	2.363	10.082	23.826	240.219	
	9.164		62.227	537.046	3.177
y _{cg}	6.790				
l _{xe}	6.438				

Trial 3 $y_{cg} := 6.80\text{in}$

$$f1 := \frac{Fy}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 59.283\text{ksi} \quad f2 := \frac{Fy}{y_{cg}} \cdot (h - y_{cg} - 2.5t)$$

$$f2 = 28.227\text{ksi} \quad \psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.476$$

$$k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 13.385$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 11.013 \text{ksi} \quad f := f1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.32$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.39 \quad b_e := \rho \cdot w \quad b_e = 3.837 \text{in}$$

$$h_o := 10.1094 \text{in} \quad b_o := 1.25 \text{in} \quad \frac{h_o}{b_o} = 8.088$$

$$b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1$$

$$b_1 = 1.104 \text{in} \quad b_2 = 1.496 \text{in}$$

$$Y_{xe} := 6.7915 \text{in}$$

So no further iteration is necessary.

$$I_{xe} := 6.4324 \text{in}^4 \quad S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad Y_{xe} = 6.792 \text{in}$$

$$M_n := F_y \cdot S_{xe} \quad M_n = 57.301 \text{kip-in}$$

Track section: 254T32 - 18

$$F_y := 44.5 \text{ksi} \quad h := 10.0902 \text{in} \quad t := 0.045 \text{in} \quad r := 1.5 \cdot t \quad r = 0.068 \text{in}$$

$$b := 1.25 \text{in} \quad w := b - 2.5t \quad w = 1.137 \text{in} \quad E := 29435 \text{ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	LY	Ly ²	I'
1	1.137	0.023	0.026	0.001	
2	0.142	0.055	0.008	0.000	
3	9.865	5.045	49.768	251.087	79.996
4	0.142	10.035	1.421	14.260	
5	1.137	10.068	11.449	115.269	
sum	12.422		62.672	380.617	79.996
ycg	5.045				
lxe	6.514				

Flange under uniform compression:

$$\frac{w}{t} = 25.216 \quad k := 0.43 \quad f := F_y$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 17.991 \text{ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.573 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.547$$

$$b_e := \rho \cdot w \quad b_e = 0.622 \text{in}$$

Web under stress gradient:

$$\frac{w}{t} := h - 5 \cdot t \quad w = 9.8647 \text{in} \quad \frac{w}{t} = 218.729 \quad y_{cg} := 5.045 \text{in}$$

$$w = 9.8647 \text{in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f_1 = 43.505 \text{ksi} \quad f_2 := f_1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f_2 = 43.505 \text{ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 13.346 \text{ksi} \quad f := f_1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.806 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.486$$

$$b_e := \rho \cdot w \quad b_e = 4.798 \text{in}$$

$$h_o := 10.0902 \text{in} \quad b_o := 1.25 \text{in} \quad \frac{h_o}{b_o} = 8.072$$

$$b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1$$

$$b_1 = 1.199 \text{in} \quad b_2 = 1.199 \text{in}$$

$b_1 + b_2 = 2.399 \text{in}$ is less than compression portion of web element = 4.93235in

So the web is not fully effective

Trial1 assme ycg=5.0451

Element	Length	y	LY	Ly ²	I'
1	0.622	0.028	0.018	0.000	
2	0.142	0.055	0.008	0.000	
3a	1.199	0.712	0.854	0.608	0.144
3b	1.199	4.446	5.330	23.696	0.136
3c	4.932	7.511	37.048	278.279	10.000
4	0.142	10.035	1.421	14.260	
5	1.137	10.068	11.449	115.269	
sum	9.373		56.128	432.114	10.280
ycg	5.988				
lxe	4.793				

Trial 2 $\underline{ycg} := 6\text{in}$

$$\underline{f1} := \frac{Fy}{ycg} \cdot (ycg - 2.5t) \quad f1 = 43.664\text{ksi} \quad \underline{f2} := \frac{Fy}{ycg} \cdot (h - ycg - 2.5t) \quad f2 = 29.499\text{ksi}$$

$$\underline{\psi} := \left| \frac{f2}{f1} \right| \quad \psi = 0.676 \quad \underline{k} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 16.76$$

$$\underline{Fcr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad Fcr = 9.32\text{ksi} \quad \underline{f} := f1$$

$$\underline{\lambda} := \sqrt{\frac{f}{Fcr}} \quad \lambda = 2.164 \quad \underline{\rho} := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.415$$

$$\underline{be} := \rho \cdot w \quad be = 4.094\text{in} \quad \underline{ho} := 10.0902\text{in} \quad \underline{bo} := 1.25\text{in} \quad \frac{ho}{bo} = 8.072$$

$$\underline{b1} := \frac{be}{3 + \psi} \quad \underline{b2} := \frac{be}{1 + \psi} - b1 \quad b1 = 1.114\text{in} \quad b2 = 1.33\text{in}$$

Trial2 Assume ycg=6in

Element	Length	y	LY	Ly ²	I'
1	0.622	0.028	0.018	0.000	
2	0.142	0.055	0.008	0.000	
3a	1.114	0.670	0.746	0.500	0.115
3b	1.330	5.335	7.096	37.855	0.190
3c	3.977	7.989	31.775	253.840	5.244
4	0.142	10.035	1.421	14.260	
5	1.137	10.068	11.449	115.269	
sum	8.464		52.512	421.725	5.549
ycg	6.204				
lxe	4.576				

Trial 3 $y_{cg} := 6.25\text{in}$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 43.697\text{ksi} \quad f2 := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5t) \quad f2 = 26.539\text{ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.607 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 15.52$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 8.63\text{ksi} \quad f := f1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.25 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.401 \quad b_e := \rho \cdot w \quad b_e = 3.955\text{in}$$

$$\frac{h_o}{b_o} = 8.072 \quad b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1 \quad b1 = 1.096\text{in} \quad b2 = 1.364\text{in}$$

Element	Length	y	LY	Ly ²	I'
1	0.622	0.028	0.018	0.000	
2	0.142	0.055	0.008	0.000	
3a	1.096	0.661	0.724	0.479	0.110
3b	1.364	5.568	7.595	42.288	0.211
3c	3.727	8.114	30.244	245.387	4.316
4	0.142	10.035	1.421	14.260	
5	1.137	10.068	11.449	115.269	
sum	8.230		51.458	417.684	4.637
y _{cg}	6.253				
I _{xe}	4.536				

$Y_{xe} := 6.252593\text{in}$ So no further iteration is necessary.

$$I_{xe} := 4.535815\text{in}^4 \quad S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad Y_{xe} = 6.253\text{in} \quad M_n := F_y \cdot S_{xe} \quad M_n = 32.282\text{kip-in}$$

Track section: 254T32 - 14

$$F_y := 44.5\text{ksi} \quad h := 10.1426\text{in} \quad t := 0.0713\text{in} \quad r := 2 \cdot t \quad r = 0.143\text{in}$$

$$b := 1.25\text{in} \quad w := b - 2.5t \quad w = 1.072\text{in} \quad E := 29435\text{ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	LY	Ly ²	I'
1	1.072	0.036	0.038	0.001	
2	0.224	0.087	0.020	0.002	
3	9.786	5.071	49.628	251.680	78.099
4	0.224	10.055	2.251	22.636	
5	1.072	10.107	10.832	109.480	
sum	12.377		62.769	383.799	78.099
ycg	5.071				
lxe	10.237				

Flange under uniform compression:

$$\frac{w}{t} = 15.032 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 50.629 \text{ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.938 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.816 \quad b_e := \rho \cdot w \quad b_e = 0.875 \text{in}$$

Web under stress gradient:

$$\frac{w}{t} := h - 5 \cdot t \quad w = 9.786 \text{in} \quad \frac{w}{t} = 137.252 \quad y_{cg} := 5.0713 \text{in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f_1 = 42.936 \text{ksi} \quad f_2 := f_1 \cdot \frac{h - y_{cg} - 2.5t}{y_{cg} - 2.5t} \quad f_2 = 42.936 \text{ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 33.893 \text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.126$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.715 \quad b_e := \rho \cdot w \quad b_e = 6.995 \text{in}$$

$$h_o := 10.1426 \text{in} \quad b_o := 1.25 \text{in} \quad \frac{h_o}{b_o} = 8.114 \quad b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1$$

$$b_1 = 1.749 \text{in} \quad b_2 = 1.749 \text{in}$$

$b_1 + b_2 = 3.498 \text{in}$ is less than compression portion of web element = 4.89305in

So the web is not fully effective

Trial1	ycg=5.0713				
Element	Length	y	LY	Ly ²	I'
1	0.875	0.036	0.031	0.001	
2	0.224	0.087	0.020	0.002	
3a	1.749	1.053	1.841	1.938	0.446
3b	1.749	4.197	7.340	30.805	0.446
3c	4.893	7.518	36.785	276.544	9.762
4	0.224	10.055	2.251	22.636	
5	1.072	10.107	10.832	109.480	
sum	10.786		59.101	441.406	10.654
ycg	5.480				
lxe	9.142				

Trial 2 $ycg := 5.50in$

$$f1 := \frac{Fy}{ycg} \cdot (ycg - 2.5t) \quad f1 = 43.058ksi \quad f2 := \frac{Fy}{ycg} \cdot (h - ycg - 2.5t) \quad f2 = 36.121ksi$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.839 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 20.114$$

$$Fcr := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad Fcr = 28.406ksi \quad f := f1$$

$$\lambda := \sqrt{\frac{f}{Fcr}} \quad \lambda = 1.231 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.667 \quad be := \rho \cdot w \quad be = 6.528in$$

$$ho := 10.1426in \quad bo := 1.25in \quad \frac{ho}{bo} = 8.114 \quad b1 := \frac{be}{3 + \psi} \quad b2 := \frac{be}{1 + \psi} - b1$$

$b1 = 1.701in \quad b2 = 1.85in$

Trial2	Ycg=5.50in				
Element	Length	y	LY	Ly ²	I'
1	0.875	0.036	0.031	0.001	
2	0.224	0.087	0.020	0.002	
3a	1.701	1.029	1.750	1.800	0.410
3b	1.850	4.575	8.464	38.722	0.528
3c	4.464	7.732	34.519	266.908	7.415
4	0.224	10.055	2.251	22.636	
5	1.072	10.107	10.832	109.480	
sum	10.410		57.867	439.548	8.352
ycg	5.559				
lxe	9.000				

Trial 3 $y_{cg} := 5.57\text{in}$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 43.076\text{ksi} \quad f2 := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5t) \quad f2 = 35.107\text{ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.815 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 19.588$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 27.663\text{ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.248$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.66 \quad b_e := \rho \cdot w \quad b_e = 6.46\text{in}$$

$$h_o := 10.1426\text{in} \quad b_o := 1.25\text{in} \quad \frac{h_o}{b_o} = 8.114 \quad b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1$$

$$b1 = 1.693\text{in} \quad b2 = 1.866\text{in}$$

Trial 3	$y_{cg}=5.57\text{in}$				
Element	Length	y	LY	Ly^2	I'
1	0.875	0.036	0.031	0.001	
2	0.224	0.087	0.020	0.002	
3a	1.693	1.025	1.735	1.778	0.404
3b	1.866	4.637	8.653	40.122	0.541
3c	4.394	7.767	34.132	265.107	7.071
4	0.224	10.055	2.251	22.636	
5	1.072	10.107	10.832	109.480	
sum	10.348		57.653	439.125	8.017
y_{cg}	5.572				
I_{xe}	8.979				

$Y_{cg} := 5.571516\text{in}$ So no further iteration is necessary.

$$I_{xe} := 8.97855\text{in}^4 \quad Y_{cg} = 5.572\text{in} \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad M_n := F_y \cdot S_{xe} \quad M_n = 71.712\text{kip-in}$$

Track section: 203T32-18

$$F_y := 44.5\text{ksi} \quad h := 8.0902\text{in} \quad t := 0.045\text{in} \quad r := 1.5t \quad r = 0.068\text{in}$$

$$b := 1.25\text{in} \quad w := b - 2.5t \quad w = 1.137\text{in} \quad E := 29435\text{ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	Ly	Ly ²	I'
1	1.137	0.023	0.026	0.001	
2	0.142	0.055	0.008	0.000	
3	7.865	4.045	31.813	128.689	40.538
4	0.142	8.035	1.138	9.143	
5	1.137	8.068	9.175	74.020	
sum	10.422		42.160	211.853	40.538
ycg	4.045				
lxe	3.691				

Flange under uniform compression:

$$\frac{w}{t} = 25.216 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 17.991 \text{ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.573 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.547 \quad b_e := \rho \cdot w \quad b_e = 0.622 \text{in}$$

Web under stress gradient:

$$\frac{w}{t} = h - 5 \cdot t \quad w = 7.8647 \text{in} \quad \frac{w}{t} = 174.384 \quad y_{cg} := 4.045 \text{in} \quad w = 7.8647 \text{in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f_1 = 43.26 \text{ksi} \quad f_2 := f_1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f_2 = 43.26 \text{ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24 \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2}$$

$$F_{cr} = 20.996 \text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.435$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.59 \quad b_e := \rho \cdot w \quad b_e = 4.639 \text{in}$$

$$h_o := 8.0902 \text{in} \quad b_o := 1.25 \text{in} \quad \frac{h_o}{b_o} = 6.472$$

$$b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1 \quad b_1 = 1.16 \text{in} \quad b_2 = 1.16 \text{in}$$

$$b_1 + b_2 = 2.32 \text{in} \quad \text{is less than compression portion of web element} = 3.93235 \text{in}$$

So the web is not fully effective

Trial1 assume $y_{cg}=4.0451$

Element	Length	y	Ly	Ly^2	I'
1	0.622	0.028	0.018	0.000	
2	0.142	0.055	0.008	0.000	
3a	1.160	0.693	0.804	0.557	0.144
3b	1.160	3.465	4.020	13.928	0.136
3c	3.932	6.011	23.638	142.097	10.000
4	0.142	8.035	1.138	9.143	
5	1.137	8.068	9.175	74.020	
sum	8.295		38.800	239.746	10.280
y_{cg}	4.678				
I_{xe}	3.091				

Trial 2 $y_{cg} := 4.83in$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 43.461ksi \quad f2 := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5t) \quad f2 = 28.998ksi$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.667 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 16.603$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 14.525ksi \quad f := f1$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.73 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.505 \quad b_e := \rho \cdot w \quad b_e = 3.968in$$

$$h_o := 8.0902in \quad b_o := 1.25in \quad \frac{h_o}{b_o} = 6.472 \quad b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1$$

$$b1 = 1.082in \quad b2 = 1.298in$$

Trial2 Assume $y_{cg}=4.83in$

Element	Length	y	Ly	Ly^2	I'
1	0.622	0.028	0.018	0.000	
2	0.142	0.055	0.008	0.000	
3a	1.082	0.654	0.707	0.462	0.106
3b	1.298	4.181	5.427	22.690	0.182
3c	3.147	6.404	20.155	129.070	2.598
4	0.142	8.035	1.138	9.143	
5	1.137	8.068	9.175	74.020	
sum	7.570		36.628	235.386	2.886
y_{cg}	4.839				
I_{xe}	2.753				

$Y_{cg} := 4.838609in$ So no further iteration is necessary.

$$I_{xe} := 2.753072in^4 \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad Y_{cg} = 4.839in \quad M_n := F_y \cdot S_{xe} \quad M_n = 25.32kip \cdot in$$

Track section: 203T32 – 16

$$F_y := 44.5 \text{ksi} \quad h := 8.1132 \text{in} \quad t := 0.0566 \text{in} \quad r := 1.5 \cdot t \quad r = 0.085 \text{in}$$

$$b := 1.25 \text{in} \quad w := b - 2.5t \quad w = 1.109 \text{in} \quad E := 29435 \text{ksi} \quad \mu := 0.3$$

Gross section:

Element	Length	y	Ly	Ly ²	I'
1	1.109	0.028	0.031	0.001	
2	0.178	0.069	0.012	0.001	
3	7.830	4.057	31.764	128.854	40.007
4	0.178	8.044	1.430	11.499	
5	1.109	8.085	8.966	72.490	
sum	10.404		42.203	212.845	40.007
ycg	4.057				
lxe	4.621				

Flange under uniform compression:

$$\frac{w}{t} = 19.585 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 29.824 \text{ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.222 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.671 \quad b_e := \rho \cdot w \quad b_e = 0.744 \text{in}$$

Web under stress gradient:

$$w_w := h - 5 \cdot t \quad w = 7.8302 \text{in} \quad \frac{w}{t} = 138.343 \quad y_{cg} := 4.0566 \text{in}$$

$$w = 7.8302 \text{in}$$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 42.948 \text{ksi} \quad f2 := f1 \cdot \frac{h - y_{cg} - 2.5t}{y_{cg} - 2.5t} \quad f2 = 42.948 \text{ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 1 \quad k_w := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr_w} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 33.361 \text{ksi} \quad f_w := f1 \quad \lambda_w := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.135$$

$$\rho_w := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.71 \quad b_{e_w} := \rho \cdot w \quad b_e = 5.563 \text{in}$$

$$h_o := 8.1132 \text{in} \quad b_o := 1.25 \text{in} \quad \frac{h_o}{b_o} = 6.491 \quad b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1$$

$$b1 = 1.391 \text{in} \quad b2 = 1.391 \text{in}$$

$b1 + b2 = 2.782\text{in}$ is less than compression portion of web element= 3.915in

So the web is not fully effective

Trial1	ycg=4.0566in				
Element	Length	y	Ly	Ly ²	I'
1	0.744	0.028	0.021	0.001	
2	0.178	0.069	0.012	0.001	
3a	1.391	0.837	1.164	0.974	0.139
3b	1.391	3.361	4.675	15.714	0.191
3c	3.915	6.014	23.546	141.609	3.487
4	0.178	8.044	1.430	11.499	
5	1.109	8.085	8.966	72.490	
sum	8.906		39.815	242.289	3.817
ycg	4.471				
Ixe	3.855				

Trial 2 $ycg := 4.56\text{in}$

$$f1 := \frac{Fy}{ycg} \cdot (ycg - 2.5t) \quad f1 = 43.119\text{ksi} \quad f2 := \frac{Fy}{ycg} \cdot (h - ycg - 2.5 \cdot t) \quad f2 = 33.294\text{ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.772 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 18.675$$

$$Fcr := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad Fcr = 25.959\text{ksi} \quad f_w := f1 \quad \lambda := \sqrt{\frac{f}{Fcr}} \quad \lambda = 1.289$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.643 \quad be := \rho \cdot w \quad be = 5.038\text{in}$$

$$ho := 8.1132\text{in} \quad bo := 1.25\text{in} \quad \frac{ho}{bo} = 6.491 \quad b1 := \frac{be}{3 + \psi} \quad b2 := \frac{be}{1 + \psi} - b1$$

$$b1 = 1.336\text{in} \quad b2 = 1.507\text{in}$$

Trial2 Ycg=4.56in

Element	Length	y	Ly	Ly ²	I'
1	0.744	0.028	0.021	0.001	
2	0.178	0.069	0.012	0.001	
3a	1.336	0.810	1.081	0.875	0.199
3b	1.507	3.807	5.736	21.836	0.285
3c	3.412	6.266	21.377	133.946	3.309
4	0.178	8.044	1.430	11.499	
5	1.109	8.085	8.966	72.490	
sum	8.463		38.624	240.649	3.793
ycg	4.564				
Ixe	3.858				

Ycg := 4.5638in So no further iteration is necessary.

$$Ixe := 3.8584\text{in}^4 \quad Sxe := \frac{Ixe}{Ycg} \quad Ycg = 4.564\text{in} \quad Mn := Fy \cdot Sxe \quad Mn = 37.622\text{kip}\cdot\text{in}$$

Track section: 203T32 - 14

$$F_y := 44.5 \text{ ksi} \quad h := 8.1426 \text{ in} \quad t := 0.0713 \text{ in} \quad r := 1.5 \cdot t \quad r = 0.107 \text{ in}$$

$$b := 1.25 \text{ in} \quad w := b - 2.5t \quad w = 1.072 \text{ in} \quad E := 29435 \text{ ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	Ly	Ly ²	I'
1	1.072	0.036	0.038	0.001	
2	0.224	0.087	0.020	0.002	
3	7.786	4.071	31.700	129.058	39.335
4	0.224	8.055	1.803	14.527	
5	1.072	8.107	8.689	70.438	
sum	10.377		42.249	214.026	39.335
ycg	4.071				
lxe	5.800				

Flange under uniform compression:

$$\frac{w}{t} = 15.032 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 50.629 \text{ ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.938 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.816 \quad b_e := \rho \cdot w \quad b_e = 0.875 \text{ in}$$

Web under stress gradient:

$$\frac{w}{t} := h - 5 \cdot t \quad w = 7.786 \text{ in} \quad \frac{w}{t} = 109.202 \quad y_{cg} := 4.0713 \text{ in} \quad w = 7.786 \text{ in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f_1 = 42.552 \text{ ksi} \quad f_2 := f_1 \cdot \frac{h - y_{cg} - 2.5t}{y_{cg} - 2.5t} \quad f_2 = 42.552 \text{ ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 53.542 \text{ ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.891$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.845 \quad b_e := \rho \cdot w \quad b_e = 6.579 \text{ in}$$

$$h_o := 8.1426 \text{ in} \quad b_o := 1.25 \text{ in} \quad \frac{h_o}{b_o} = 6.514 \quad b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1$$

$$b_1 = 1.645 \text{ in} \quad b_2 = 1.645 \text{ in}$$

$$b_1 + b_2 = 3.289 \text{ in} \quad \text{is less than compression portion of web element} = 3.89305 \text{ in}$$

So the web is not fully effective

Trial1	ycg=4.0713				
Element	Length	y	Ly	Ly ²	I'
1	0.875	0.036	0.031	0.001	
2	0.224	0.087	0.020	0.002	
3a	1.645	1.001	1.646	1.647	0.371
3b	1.645	3.249	5.344	17.362	0.371
3c	3.893	6.018	23.428	140.984	4.917
4	0.224	8.055	1.803	14.527	
5	1.072	8.107	8.689	70.438	
sum	9.578		40.961	244.962	5.659
ycg	4.277				
Ixe	5.379				

Trial 2 $ycg := 4.313in$

$$f1 := \frac{Fy}{ycg} \cdot (ycg - 2.5t) \quad f1 = 42.661 \text{ ksi} \quad f2 := \frac{Fy}{ycg} \cdot (h - ycg - 2.5t) \quad f2 = 37.673 \text{ ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.883 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 21.121$$

$$Fcr := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad Fcr = 47.119 \text{ ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{Fcr}} \quad \lambda = 0.952$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.808 \quad be := \rho \cdot w \quad be = 6.291 \text{ in}$$

$$ho := 8.1426 \text{ in} \quad bo := 1.25 \text{ in} \quad \frac{ho}{bo} = 6.514 \quad b1 := \frac{be}{3 + \psi} \quad b2 := \frac{be}{1 + \psi} - b1$$

$$b1 = 1.62 \text{ in} \quad b2 = 1.721 \text{ in}$$

Trial2	Ycg=4.313in				
Element	Length	y	Ly	Ly ²	I'
1	0.875	0.036	0.031	0.001	
2	0.224	0.087	0.020	0.002	
3a	1.620	0.988	1.601	1.582	0.354
3b	1.721	3.453	5.942	20.514	0.425
3c	3.651	6.139	22.414	137.595	4.057
4	0.224	8.055	1.803	14.527	
5	1.072	8.107	8.689	70.438	
sum	9.387		40.500	244.659	4.836
ycg	4.314				
Ixe	5.333				

Ycg := 4.314in So no further iteration is necessary.

$$Ixe := 5.333in^4 \quad Ycg = 4.314in \quad Sxe := \frac{Ixe}{Ycg} \quad Mn := Fy \cdot Sxe \quad Mn = 55.011 \text{ kip}\cdot\text{in}$$

Track section: 305T32 – 18

$$F_y := 44.5 \text{ ksi} \quad h := 12.0902 \text{ in} \quad t := 0.045 \text{ in} \quad r := 1.5 \cdot t \quad r = 0.068 \text{ in}$$

$$b := 1.25 \text{ in} \quad w := b - 2.5t \quad w = 1.137 \text{ in} \quad E := 29435 \text{ ksi} \quad \mu := 0.3$$

Gross section:

Element	Length	y	LY	Ly ²	I'
1	1.137	0.023	0.026	0.001	
2	0.142	0.055	0.008	0.000	
3	11.865	6.045	71.723	433.575	139.184
4	0.142	12.035	1.704	20.511	
5	1.137	12.068	13.724	165.616	
sum	14.422		87.185	619.702	139.184
ycg	6.045				
lxe	10.456				

Flange under uniform compression:

$$\frac{w}{t} = 25.216 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 17.991 \text{ ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.573 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.547 \quad b_e := \rho \cdot w \quad b_e = 0.622 \text{ in}$$

Web under stress gradient:

$$w_w := h - 5 \cdot t \quad w = 11.8647 \text{ in} \quad \frac{w}{t} = 263.075 \quad y_{cg} := 6.045 \text{ in}$$

$$w = 11.8647 \text{ in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f_1 = 43.67 \text{ ksi} \quad f_2 := f_1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f_2 = 43.67 \text{ ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 1 \quad k_w := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 9.226 \text{ ksi} \quad f_w := f_1 \quad \lambda_w := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.176$$

$$\rho_w := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.413 \quad b_{e,w} := \rho \cdot w \quad b_e = 4.902 \text{ in}$$

$$h_o := 12.0902 \text{ in} \quad b_o := 1.25 \text{ in} \quad \frac{h_o}{b_o} = 9.672 \quad b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1$$

$$b_1 = 1.225 \text{ in} \quad b_2 = 1.225 \text{ in}$$

$$b_1 + b_2 = 2.451 \text{ in} \quad \text{is less than compression portion of web element} = 5.93235 \text{ in}$$

Trial1 assume ycg=6.0451

Element	Length	y	LY	Ly ²	I'
1	0.622	0.028	0.018	0.000	
2	0.142	0.055	0.008	0.000	
3a	1.225	0.725	0.888	0.644	0.153
3b	1.225	5.433	6.655	36.154	0.153
3c	5.932	9.011	53.458	481.725	17.398
4	0.142	12.035	1.704	20.511	
5	1.137	12.068	13.724	347.651	
sum	10.425		76.455	886.686	17.704
ycg	7.334				
lxe	15.500				

Trial 2 $ycg := 7.34in$

$$f1 := \frac{Fy}{ycg} \cdot (ycg - 2.5t) \quad f1 = 43.67ksi \quad f2 := \frac{Fy}{ycg} \cdot (h - ycg - 2.5t) \quad f2 = 28.115ksi$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.642 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 16.132$$

$$Fcr := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad Fcr = 6.201ksi \quad f := f1 \quad \lambda := \sqrt{\frac{f}{Fcr}} \quad \lambda = 2.658$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.345 \quad be := \rho \cdot w \quad be = 4.094in$$

$$ho := 12.1132in \quad bo := 1.25in \quad \frac{ho}{bo} = 9.691 \quad b1 := \frac{be}{3 + \psi} \quad b2 := \frac{be}{1 + \psi} - b1$$

$$b1 = 1.124in \quad b2 = 1.37in$$

Trial2 assume ycg=7.34

Element	Length	y	LY	Ly ²	I'
1	0.622	0.028	0.018	0.000	
2	0.142	0.055	0.008	0.000	
3a	1.124	0.675	0.758	0.512	0.118
3b	1.370	6.655	9.117	60.676	0.214
3c	4.637	9.659	44.792	432.632	8.309
4	0.142	12.035	1.704	20.511	
5	1.137	12.068	13.724	165.616	
sum	9.174		70.121	679.948	8.641
ycg	7.644				
lxe	6.883				

Trial 3 $ycg := 7.715in$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 43.85 \text{ksi} \quad f2 := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5t) \quad f2 = 24.586 \text{ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.561 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 14.724$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 5.66 \text{ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.783$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.331 \quad b_e := \rho \cdot w \quad b_e = 3.926 \text{in}$$

$$\frac{h_o}{b_o} = 9.691 \quad b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1 \quad b1 = 1.103 \text{in} \quad b2 = 1.413 \text{in}$$

Trial 3 assume $y_{cg}=7.715$

Element	Length	y	LY	Ly ²	I'
1	0.622	0.028	0.018	0.000	
2	0.142	0.055	0.008	0.000	
3a	1.103	0.664	0.733	0.487	0.112
3b	1.413	7.009	9.903	69.405	0.235
3c	4.262	9.846	41.969	413.237	6.454
4	0.142	12.035	1.704	20.511	
5	1.137	12.068	13.724	165.616	
sum	8.821		68.058	669.256	6.800
y _{cg}	7.716				
I _{xe}	6.808				

$Y_{cg} := 7.7156 \text{in}$ So no further iteration is necessary.

$$I_{xe} := 6.80776 \text{in}^4 \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad Y_{cg} = 7.716 \text{in} \quad M_n := F_y \cdot S_{xe} \quad M_n = 39.264 \text{kip-in}$$

Track section: 305T32 – 16

$$F_y := 44.5 \text{ ksi} \quad h := 12.1132 \text{ in} \quad t := 0.0566 \text{ in} \quad r := 1.5 \cdot t \quad r = 0.085 \text{ in}$$

$$b := 1.25 \text{ in} \quad w := b - 2.5 \cdot t \quad w = 1.109 \text{ in} \quad E := 29435 \text{ ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	Ly	Ly ²	I'
1	1.109	0.028	0.031	0.001	
2	0.178	0.069	0.012	0.001	
3	11.830	6.057	71.651	433.960	137.973
4	0.178	12.044	2.140	25.779	
5	1.109	12.085	13.396	161.891	
sum	14.403		87.231	621.632	137.973
ycg	6.057				
lxe	13.091				

Flange under uniform compression:

$$\frac{w}{t} = 19.585 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 29.824 \text{ ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.222 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.671 \quad b_e := \rho \cdot w \quad b_e = 0.74404 \text{ in}$$

Web under stress gradient:

$$w := h - 5 \cdot t \quad w = 11.8302 \text{ in} \quad \frac{w}{t} = 209.014 \quad y_{cg} := 6.0566 \text{ in} \quad w = 11.8302 \text{ in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f_1 = 43.46 \text{ ksi} \quad f_2 := f_1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f_2 = 43.46 \text{ ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 14.615 \text{ ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.724$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.506 \quad b_e := \rho \cdot w \quad b_e = 5.985 \text{ in}$$

$$h_o := 12.1132 \text{ in} \quad b_o := 1.25 \text{ in} \quad \frac{h_o}{b_o} = 9.691 \quad b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1$$

$$b_1 = 1.496 \text{ in} \quad b_2 = 1.496 \text{ in}$$

$b_1 + b_2 = 2.993 \text{ in}$ is less than compression portion of web element = 5.9151 in

So the web is not fully effective

Trial1					
Element	Length	y	Ly	Ly ²	I'
1	0.744	0.028	0.021	0.001	
2	0.178	0.069	0.012	0.001	
3a	1.496	0.890	1.331	1.184	0.279
3b	1.496	5.309	7.942	42.159	0.279
3c	5.915	9.014	53.320	480.631	17.247
4	0.178	12.044	2.140	25.779	
5	1.109	12.085	13.396	161.891	
sum	11.115		78.162	711.645	17.805
ycg	7.032				
lxe	10.177				

Trial 2 $ycg := 7.047in$

$$f1 := \frac{Fy}{ycg} \cdot (ycg - 2.5t) \quad f1 = 43.46ksi \quad f2 := \frac{Fy}{ycg} \cdot (h - ycg - 2.5t) \quad f2 = 31.098ksi$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.713 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 17.482$$

$$Fcr := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad Fcr = 10.646ksi \quad f := f1 \quad \lambda := \sqrt{\frac{f}{Fcr}} \quad \lambda = 2.024$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda}$$

$$\rho = 0.44$$

$$be := \rho \cdot w$$

$$be = 5.21in$$

$$ho := 12.1132in$$

$$bo := 1.25in$$

$$\frac{ho}{bo} = 9.691$$

$$b1 := \frac{be}{3 + \psi}$$

$$b2 := \frac{be}{1 + \psi} - b1$$

$$b1 = 1.403in$$

$$b2 = 1.638in$$

Trial2 Assume $ycg=7.047$

Element	Length	y	Ly	Ly ²	I'
1	0.744	0.028	0.021	0.001	
2	0.178	0.069	0.012	0.001	
3a	1.403	0.843	1.183	0.997	0.230
3b	1.638	6.228	10.201	63.535	0.366
3c	4.925	9.509	46.831	445.329	9.953
4	0.178	12.044	2.140	25.779	
5	1.109	12.085	13.396	161.891	
sum	10.174		73.785	697.533	10.549
ycg	7.253				
lxe	9.789				

Trial 3 $ycg := 7.29in$

$$f1 := \frac{Fy}{ycg} \cdot (ycg - 2.5t) \quad f1 = 43.636ksi \quad f2 := \frac{Fy}{ycg} \cdot (h - ycg - 2.5t) \quad f2 = 28.578ksi$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 0.655 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 16.375$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 9.972 \text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.092$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.428 \quad b_e := \rho \cdot w \quad b_e = 5.06 \text{in}$$

$$\frac{h_o}{b_o} = 9.691 \quad b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1 \quad b_1 = 1.3846 \text{in} \quad b_2 = 1.6733 \text{in}$$

Trial 3 assume $y_{cg} = 7.29$

Element	Length	y	Ly	Ly ²	I'
1	0.744	0.028	0.021	0.001	
2	0.178	0.069	0.012	0.001	
3a	1.385	0.834	1.154	0.963	0.221
3b	1.673	6.453	10.798	69.686	0.391
3c	4.682	9.631	45.089	434.243	8.551
4	0.178	12.044	2.140	25.779	
5	1.109	12.085	13.396	161.891	
sum	9.948		72.612	692.563	9.163
y _{cg}	7.298				
I _{xe}	9.743				

$Y_{cg} := 7.298 \text{in}$ So no further iteration is necessary.

$$I_{xe} := 9.743 \text{in}^4 \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad Y_{cg} = 7.298 \text{in} \quad M_n := F_y \cdot S_{xe} \quad M_n = 59.41 \text{kip-in}$$

Track section: 305T32 - 14

$$F_y := 44.5 \text{ksi} \quad h := 12.1426 \text{in} \quad t := 0.0713 \text{in} \quad r := 1.5 \cdot t \quad r = 0.107 \text{in}$$

$$b := 1.25 \text{in} \quad w := b - 2.5t \quad w = 1.072 \text{in} \quad E := 29435 \text{ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	Ly	Ly ²	I'
1	1.072	0.036	0.038	0.001	
2	0.224	0.087	0.020	0.002	
3	11.786	6.071	71.557	434.444	136.384
4	0.224	12.055	2.699	32.536	
5	1.072	12.107	12.976	157.095	
sum	14.377		87.289	624.078	136.384
y _{cg}	6.071				
I _{xe}	16.435				

Flange under uniform compression:

$$\frac{w}{t} = 15.032 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 50.629 \text{ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.938 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.816 \quad b_e := \rho \cdot w \quad b_e = 0.875 \text{in}$$

Web under stress gradient:

$$w := h - 5 \cdot t \quad w = 11.786 \text{in} \quad \frac{w}{t} = 165.303 \quad y_{cg} := 6.0713 \text{in}$$

$$w = 11.786 \text{in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f_1 = 43.194 \text{ksi} \quad f_2 := f_1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f_2 = 43.194 \text{ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 23.366 \text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.36$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.616 \quad b_e := \rho \cdot w \quad b_e = 7.266 \text{in}$$

$$h_o := 12.1426 \text{in} \quad b_o := 1.25 \text{in} \quad \frac{h_o}{b_o} = 9.714 \quad b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1$$

$$b_1 = 1.817 \text{in} \quad b_2 = 1.817 \text{in}$$

$$b_1 + b_2 = 3.633 \text{in} \quad \text{is less than compression portion of web element} = 5.89305 \text{in}$$

So the web is not fully effective

Trial1 $y_{cg}=6.0713$

Element	Length	y	Ly	Ly ²	I'
1	0.875	0.036	0.031	0.001	
2	0.224	0.087	0.020	0.002	
3a	1.817	1.053	1.913	2.014	0.500
3b	1.817	4.197	7.626	32.003	0.500
3c	5.893	9.018	53.142	479.230	17.055
4	0.224	12.055	2.699	32.536	
5	1.072	12.107	12.976	157.095	
sum	11.922		78.406	702.881	18.055
y _{cg}	6.577				
l _{xe}	14.636				

Trial 2 $y_{cg} := 6.89\text{in}$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 43.349\text{ksi} \quad f2 := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5t) \quad f2 = 32.773\text{ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.756 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 18.342$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 17.858\text{ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.558$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.551 \quad b_e := \rho \cdot w \quad b_e = 6.497\text{in}$$

$$h_o := 12.1426\text{in} \quad b_o := 1.25\text{in} \quad \frac{h_o}{b_o} = 9.714 \quad b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1$$

$$b1 = 1.73\text{in} \quad b2 = 1.97\text{in}$$

Trial2 $Y_{cg}=6.89\text{in}$

Element	Length	y	Ly	Ly ²	I'
1	0.875	0.036	0.031	0.001	
2	0.224	0.087	0.020	0.002	
3a	1.730	1.043	1.805	1.883	0.432
3b	1.970	5.905	11.633	68.692	0.637
3c	5.074	9.427	47.837	450.966	10.889
4	0.224	12.055	2.699	32.536	
5	1.072	12.107	12.976	157.095	
sum	11.169		77.000	711.175	11.958
y _{cg}	6.894				
I _{xe}	13.712				

$Y_{cg} := 6.894\text{in}$ So no further iteration is necessary.

$$I_{xe} := 13.7123\text{in}^4 \quad Y_{cg} = 6.894\text{in} \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad M_n := F_y \cdot S_{xe} \quad M_n = 88.51\text{kip-in}$$

Stud section : 254S76-18

$$E := 29435 \text{ksi} \quad f_y := 33 \text{ksi} \quad \mu := 0.3 \quad t := 0.045 \text{in} \quad h_0 := 10 \text{in} \quad b_0 := 3 \text{in}$$

$$r_i := 1.5 \cdot t \quad r_i = 0.068 \text{in} \quad d_1 := t + r_i \quad D := 1 \text{in}$$

Gross section

Element	L	y	Ly	Ly ²	I
1	0.887	0.556	0.494	0.275	0.058
2	0.284	0.055	0.016	0.001	
3	2.775	0.023	0.063	0.001	
4	9.775	5.000	48.873	244.363	77.822
5	2.775	9.977	27.682	276.200	
6	0.284	9.945	2.824	28.087	
7	0.887	9.444	8.379	79.127	0.058
sum	17.666		88.330	628.053	77.938
	Y _{top}	5.000		I _x	264.342
				I _{xg}	11.922

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d_1 \quad w = 2.774 \text{in} \quad \frac{w}{t} = 61.519 \quad d := D - d_1 \quad I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 2.625 \times 10^{-3} \text{in}^4 \quad f := f_y \quad S_{ww} := 1.28 \cdot \sqrt{\frac{E}{f}} \quad S = 38.228 \quad 0.328 \cdot S = 12.539 \quad \frac{w}{t} > 0.328 S$$

$$I_{a1} := 399 \cdot t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3 \quad I_{a1} = 3.472 \times 10^{-3} \text{in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \cdot \frac{\frac{w}{t}}{S} + 5 \right) \quad I_{a2} = 7.863 \times 10^{-4} \text{in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases} \quad I_a = 7.863 \times 10^{-4} \text{in}^4$$

$$\frac{D}{w} = 0.36 \quad R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases} \quad R_i = 1$$

$$n := \begin{cases} 0.582 - \frac{\frac{w}{t}}{4 \cdot S} & \text{if } 0.582 - \frac{\frac{w}{t}}{4 \cdot S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.333$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w}\right) \cdot R_i^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot R_i^n + 0.43 & \text{otherwise} \end{cases}$$

$$\begin{aligned} \underline{k} &:= \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} & k &= 3.448 \\ F_{cr} &:= \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w}\right)^2 & F_{cr} &= 24.237 \text{ksi} & \lambda &:= \sqrt{\frac{f}{F_{cr}}} & \lambda &= 1.167 \end{aligned}$$

$$\begin{aligned} \rho &:= \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} & \rho &= 0.695 \\ be &:= \rho \cdot w & be &= 1.929 \text{in} \end{aligned}$$

Lip (Unstiffened compression element)

$$\begin{aligned} Y_{top} &:= 5 \text{in} & \underline{d1} &:= t + r_i & \underline{f} &:= f_y & d &= 0.887 \text{in} \\ \frac{d}{t} &= 19.673 & f_1 &:= f \cdot \frac{Y_{top} - d1}{Y_{top}} & f_2 &:= f \cdot \frac{Y_{top} - d1 - d}{Y_{top}} \\ f_1 &= 32.256 \text{ksi} & f_2 &= 26.4 \text{ksi} & \psi &:= \frac{f_2}{f_1} & \psi &= 0.818 \end{aligned}$$

$$\underline{k} := \frac{0.578}{\psi + 0.34} \quad k = 0.499$$

$$\underline{F_{cr}} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 34.297 \text{ksi} \quad \underline{f} := f_1$$

$$\underline{\lambda} := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.97$$

$$\underline{\rho} := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.797$$

$$ds' := \rho \cdot d \quad ds' = 0.707 \text{in} \quad ds := ds' \cdot R_i \quad ds = 0.707 \text{in}$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 9.774 \text{ in} \quad \frac{h}{t} = 216.729 \quad \underline{f} := f_1 \quad \underline{f_2} := \frac{(h_0 - Y_{\text{top}} - d_1) \cdot f_y}{Y_{\text{top}}}$$

$$f_1 = 32.256 \text{ ksi} \quad f_2 = 32.256 \text{ ksi}$$

$$\underline{\psi} := \frac{f_2}{f_1} \quad \psi = 1 \quad \underline{k} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$\underline{F_{cr}} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 13.593 \text{ ksi} \quad \underline{f} := f_1 \quad \underline{\lambda} := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.54$$

$$\underline{\rho} := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.556 \quad \underline{b_e} := \rho \cdot h \quad b_e = 5.439 \text{ in}$$

$$1 \text{ otherwise}$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} \text{ if } \psi > 0.236 \\ b_e - b_1 \text{ otherwise} \end{cases}$$

$$b_1 = 1.36 \text{ in} \quad b_2 = 2.72 \text{ in}$$

$$b_1 + b_2 = 4.079 \text{ in} \quad Y_{\text{top}} - d_1 = 4.887 \text{ in}$$

$b_1 + b_2 < 4.887$ the web is not fully effective.

Moment of Inertia calculation

Trial 1

Element	L	y	Ly	Ly ²	I
1	0.707	0.466	0.330	0.154	0.029
2	0.284	0.055	0.016	0.001	
3	1.929	0.023	0.043	0.001	
4a	1.360	0.793	1.078	0.855	0.210
4b	2.720	3.640	9.901	36.039	1.677
4c	4.887	7.444	36.379	270.791	9.728
5	2.775	9.977	27.682	276.200	
6	0.284	9.945	2.824	28.087	
7	0.887	9.444	8.379	79.127	0.058
sum	15.833		86.632	691.253	11.702
	Y _{top}	5.472		I _x	228.936
				I _{xe}	10.325

Trial 2 $Y_{top} := 5.66in$ $f := f_y$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 9.774in \quad \frac{h}{t} = 216.729$$

$$f_1 := f_y \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}} \quad f_1 = 32.343ksi \quad f_2 = 24.647ksi$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.762 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 18.466$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 10.459ksi \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.759$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.498$$

$$b_e := \rho \cdot h \quad b_e = 4.863in$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.293in \quad b_2 = 2.431in$$

$$b_1 + b_2 = 3.724in \quad Y_{top} - d_1 = 5.547in$$

$b_1 + b_2 < 5.547$ the web is not fully effective.

Trail 2 assume $Y_{cg} = 5.66$

Element	L	y	L_y	L_y^2	I
1	0.707	0.466	0.330	0.154	0.029
2	0.284	0.055	0.016	0.001	
3	1.929	0.023	0.043	0.001	
4a	1.293	0.759	0.982	0.745	0.180
4b	2.431	4.445	10.805	48.021	1.197
4c	4.227	7.774	32.861	255.450	6.295
5	2.775	9.977	27.682	276.200	
6	0.284	9.945	2.824	28.087	
7	0.887	9.444	8.379	79.127	0.058
sum	14.817		83.922	687.785	7.760
	Y_{top}	5.664		I_x	220.222
				I_{xe}	9.932

From trial 2 it is found that $Y_{cg}=5.664\text{in}$ which is almost equal to 5.66 in. So no other iteration is necessary.

$$I_{xe} := 9.932\text{in}^4 \quad Y_{xe} := 5.663884\text{in}$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 1.754\text{in}^3 \quad M_n := S_{xe} \cdot f_y \quad f_y = 33\text{ksi} \quad M_n = 57.868\text{kip}\cdot\text{in}$$

Stud section: 254S76 – 16

$$E := 29435 \text{ksi} \quad f_y := 33 \text{ksi} \quad \mu := 0.3 \quad t := .0632 \text{in} \quad h_0 := 10 \text{in} \quad b_0 := 3 \text{in}$$

$$r_i := 1.5 \cdot t \quad r_i = 0.095 \text{in} \quad d_1 := t + r_i \quad D := 1 \text{in}$$

Gross section

Element	L	y	Ly	Ly ²	I
1	0.842	0.579	0.488	0.282	0.050
2	0.397	0.077	0.031	0.002	
3	2.684	0.032	0.085	0.003	
4	9.684	5.000	48.420	242.100	75.680
5	2.684	9.968	26.755	266.706	
6	0.397	9.923	3.939	39.087	
7	0.842	9.421	7.932	74.732	0.050
sum	17.530		87.650	622.913	75.780
	Ytop	5.000		Ix	260.443
				Ixg	16.460

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d_1 \quad w = 2.684 \text{in}$$

$$\frac{w}{t} = 42.468 \quad d := D - d_1 \quad I_s := \frac{d^3 \cdot t}{12} \quad I_s = 3.144 \times 10^{-3} \text{in}^4 \quad f := f_y$$

$$S := 1.28 \sqrt{\frac{E}{f}} \quad S = 38.228 \quad 0.328 S = 12.539 \quad \frac{w}{t} > 0.328 S$$

$$I_{a1} := 399 \cdot t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3 \quad I_{a1} = 3.055 \times 10^{-3} \text{in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \frac{\frac{w}{t}}{S} + 5 \right) \quad I_{a2} = 2.118 \times 10^{-3} \text{in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases} \quad I_a = 2.118 \times 10^{-3} \text{in}^4$$

$$\frac{D}{w} = 0.373 \quad R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases} \quad R_i = 1$$

$$n := \begin{cases} 0.582 - \frac{\frac{w}{t}}{4 \cdot S} & \text{if } 0.582 - \frac{\frac{w}{t}}{4 \cdot S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.333$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w}\right) \cdot Ri^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot Ri^n + 0.43 & \text{otherwise} \end{cases}$$

$$\underline{k} := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.387$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w}\right)^2 \quad F_{cr} = 49.962 \text{ ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.813$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.897$$

$$be := \rho \cdot w \quad be = 2.409 \text{ in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 5 \text{ in} \quad \underline{d1} := t + ri \quad \underline{f} := fy \quad d = 0.842 \text{ in}$$

$$\frac{d}{t} = 13.323 \quad f1 := f \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f2 := f \cdot \frac{Y_{top} - d1 - d}{Y_{top}}$$

$$f1 = 31.957 \text{ ksi} \quad f2 = 26.4 \text{ ksi} \quad \psi := \frac{f2}{f1} \quad \psi = 0.826 \quad \underline{k} := \frac{0.578}{\psi + 0.34} \quad k = 0.496$$

$$\underline{F_{cr}} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 74.292 \text{ ksi} \quad \underline{f} := f1 \quad \underline{\lambda} := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.656$$

$$\underline{\rho} := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 1$$

$$ds' := \rho \cdot d \quad ds' = 0.842 \text{ in} \quad ds := ds' \cdot Ri \quad ds = 0.842 \text{ in}$$

Web under stress gradient

$$h := h0 - 2 \cdot d1 \quad h = 9.684 \text{ in} \quad \frac{h}{t} = 153.228 \quad \underline{f} := f1 \quad \underline{f2} := \frac{(h0 - Y_{top} - d1) \cdot fy}{Y_{top}}$$

$$f1 = 31.957 \text{ ksi} \quad f2 = 31.957 \text{ ksi}$$

$$\underline{\psi} := \frac{f2}{f1} \quad \psi = 1 \quad \underline{k} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$\underline{F_{cr}} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 27.194 \text{ ksi} \quad \underline{f} := f1 \quad \underline{\lambda} := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.084$$

$$\rho_w := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{1 - \frac{0.22}{\lambda}}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.735 \quad \rho_w := \rho \cdot h \quad \text{be} = 7.12 \text{in}$$

$$1 \text{ otherwise}$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.78 \text{in} \quad b_2 = 3.56 \text{in}$$

$$b_1 + b_2 = 5.34 \text{in} \quad Y_{\text{top}} - d_1 = 4.842 \text{in}$$

$$b_1 + b_2 > 4.842 \text{in} \quad \text{the web is fully effective.}$$

Moment of Inertia calculation

Trial1					
Element	L	y	Ly	Ly ²	I
1	0.842	0.579	0.488	0.282	0.050
2	0.397	0.077	0.031	0.002	
3	2.084	0.032	0.066	0.002	
4c	9.684	5.000	48.420	242.100	75.680
5	2.684	9.968	26.755	266.706	
6	0.397	9.923	3.939	39.087	
7	0.842	9.464	7.969	75.416	0.050
sum	16.930		87.667	623.596	75.780
	Y _{top}	5.178		I _x	245.416
				I _{xe}	15.510

$$\text{Trial2} \quad Y_{\text{top}} := 5.178 \text{in} \quad f_w := f_y$$

Web under stress gradient

$$h_w := h_0 - 2 \cdot d_1 \quad h = 9.684 \text{in} \quad \frac{h}{t} = 153.228$$

$$f_1 := f \cdot \frac{Y_{\text{top}} - d_1}{Y_{\text{top}}} \quad f_1 = 31.993 \text{ksi} \quad f_2 := \frac{(h_0 - Y_{\text{top}} - d_1) \cdot f_y}{Y_{\text{top}}} \quad f_2 = 29.724 \text{ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.929 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 22.216$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 25.173 \text{ksi} \quad f_w := f1 \quad \lambda_w := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.127$$

$$\rho_w := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.714 \quad b_{e,w} := \rho \cdot h \quad b_e = 6.914 \text{in}$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b1_w := \frac{b_e}{3 + \psi}$$

$$b2_w := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b1 & \text{otherwise} \end{cases}$$

$$b1 = 1.76 \text{in} \quad b2 = 3.457 \text{in}$$

$$b1 + b2 = 5.216 \text{in} \quad Y_{top} - d1 = 5.02 \text{in}$$

$$b1 + b2 > 5.02 \quad \text{the web is fully effective.}$$

$$I_{xe} := 15.51027 \text{in}^4 \quad Y_{xe} := 5.1782 \text{in}$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 2.995 \text{in}^3 \quad M_n := S_{xe} \cdot f_y \quad f_y = 33 \text{ksi} \quad M_n = 98.845 \text{kip-in}$$

Stud section: 254S76-14

$$E := 29433 \text{ksi} \quad f_y := 33 \text{ksi} \quad \mu := 0.3 \quad t := 0.0713 \text{in} \quad h_0 := 10 \text{in} \quad b_0 := 3 \text{in}$$

$$r_i := 1.5 \cdot t \quad r_i = 0.107 \text{in} \quad d1 := t + r_i \quad D := 1 \text{in}$$

Gross section

Element	L	y	Ly	Ly ²	I
1	0.822	0.589	0.484	0.285	0.046
2	0.448	0.087	0.039	0.003	
3	2.644	0.036	0.094	0.003	
4	9.644	5.000	48.218	241.088	74.735
5	2.644	9.964	26.341	262.469	
6	0.448	9.913	4.438	43.997	
7	0.822	9.411	7.733	72.778	0.046
sum	17.470		87.348	620.623	74.827
	Y _{top}	5.000		lx	258.712
				lxg	18.446

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d_1 \quad w = 2.643 \text{ in}$$

$$\frac{w}{t} = 37.076 \quad d := D - d_1 \quad I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 3.297 \times 10^{-3} \text{ in}^4 \quad f := f_y$$

$$S := 1.28 \cdot \sqrt{\frac{E}{f}} \quad S = 38.228 \quad 0.328 \cdot S = 12.539 \quad \frac{w}{t} > 0.328 \cdot S$$

$$I_{a1} := 399 \cdot t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3 \quad I_{a1} = 2.727 \times 10^{-3} \text{ in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \cdot \frac{\frac{w}{t}}{S} + 5 \right) \quad I_{a2} = 3.012 \times 10^{-3} \text{ in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases} \quad I_a = 2.727 \times 10^{-3} \text{ in}^4$$

$$\frac{D}{w} = 0.378 \quad R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases} \quad R_i = 1$$

$$n := \begin{cases} 0.582 - \frac{\frac{w}{t}}{4 \cdot S} & \text{if } 0.582 - \frac{\frac{w}{t}}{4 \cdot S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.34$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w} \right) \cdot R_i^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot R_i^n + 0.43 & \text{otherwise} \end{cases}$$

$$k := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.359$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w} \right)^2 \quad F_{cr} = 65 \text{ ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.713$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.97$$

$$be := \rho \cdot w \quad be = 2.565 \text{in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 5 \text{in} \quad d1 := t + r_i \quad f_{\text{w}} := f_y \quad d = 0.822 \text{in}$$

$$\frac{d}{t} = 11.525 \quad f1 := f \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f2 := f \cdot \frac{Y_{top} - d1 - d}{Y_{top}}$$

$$f1 = 31.824 \text{ksi} \quad f2 = 26.4 \text{ksi} \quad \psi := \frac{f2}{f1} \quad \psi = 0.83 \quad k_{\text{w}} := \frac{0.578}{\psi + 0.34} \quad k = 0.494$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 98.978 \text{ksi} \quad f_{\text{w}} := f1 \quad \lambda_{\text{w}} := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.567$$

$$\rho_{\text{w}} := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 1$$

$$ds' := \rho \cdot d \quad ds' = 0.822 \text{in} \quad ds := ds' \cdot R_i \quad ds = 0.822 \text{in}$$

Web under stress gradient

$$h := h_0 - 2 \cdot d1 \quad h = 9.643 \text{in} \quad \frac{h}{t} = 135.252 \quad f_{\text{w}} := f1 \quad f2_{\text{w}} := \frac{(h_0 - Y_{top} - d1) \cdot f_y}{Y_{top}}$$

$$f1 = 31.824 \text{ksi} \quad f2 = 31.824 \text{ksi}$$

$$\psi_{\text{w}} := \frac{f2}{f1} \quad \psi = 1 \quad k_{\text{w}} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 34.903 \text{ksi} \quad f_{\text{w}} := f1 \quad \lambda_{\text{w}} := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.955$$

$$\rho_{\text{w}} := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.806 \quad be_{\text{w}} := \rho \cdot h \quad be = 7.772 \text{in}$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.943\text{in} \quad b_2 = 3.886\text{in}$$

$$b_1 + b_2 = 5.829\text{in} \quad Y_{\text{top}} - d_1 = 4.822\text{in}$$

$$b_1 + b_2 > 4.822\text{in} \quad \text{the web is fully effective.}$$

Moment of Inertia calculation

Trial1	Assume	Ytop=5			
Element	L	y	Ly	Ly ²	I
1	0.822	0.589	0.484	0.285	0.046
2	0.448	0.087	0.039	0.003	
3	2.565	0.036	0.094	0.003	
4	9.644	5.000	48.218	241.088	74.735
5	2.644	9.964	26.341	262.469	
6	0.448	9.913	4.438	43.997	
7	0.822	9.411	7.733	72.778	0.046
sum	17.391		87.348	620.623	74.827
	Ytop	5.023		Ix	256.741
				Ixe	18.306

$$\text{Trial2 } Y_{\text{top}} := 5.023\text{in} \quad f := f_y$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 9.643\text{in} \quad \frac{h}{t} = 135.252$$

$$f_1 := f \cdot \frac{Y_{\text{top}} - d_1}{Y_{\text{top}}} \quad f_1 = 31.829\text{ksi} \quad f_2 := \frac{(h_0 - Y_{\text{top}} - d_1) \cdot f_y}{Y_{\text{top}}} \quad f_2 = 31.527\text{ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.991 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 23.754$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 34.546\text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.96$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.803 \quad b_e := \rho \cdot h \quad b_e = 7.744\text{in}$$

$$\frac{h_0}{b_0} = 3.333 \quad \frac{h_0}{b_0} \leq 4$$

$$b_{1w} := \frac{b_e}{3 + \psi}$$

$$b_{2w} := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.941 \text{ in} \quad b_2 = 3.872 \text{ in}$$

$$b_1 + b_2 = 5.813 \text{ in} \quad Y_{\text{top}} - d_1 = 4.845 \text{ in}$$

$$b_1 + b_2 > 4.845 \quad \text{the web is fully effective.}$$

$$I_{xe} := 18.3056 \text{ in}^4 \quad Y_{xe} := 5.02256 \text{ in}$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 3.645 \text{ in}^3 \quad M_n := S_{xe} \cdot f_y \quad f_y = 33 \text{ ksi} \quad M_n = 120.274 \text{ kip-in}$$

Stud section: 203S76 - 18

$$E := 29435 \text{ ksi} \quad f_y := 33 \text{ ksi} \quad \mu := 0.3 \quad t := 0.045 \text{ in} \quad h_0 := 8 \text{ in} \quad b_0 := 3 \text{ in}$$

$$r_i := 1.5t \quad r_i = 0.068 \text{ in} \quad d_1 := t + r_i \quad D := 1 \text{ in}$$

Gross section

Element	L	y	Ly	Ly ²	I
1	0.887	0.556	0.494	0.275	0.058
2	0.284	0.055	0.016	0.001	
3	2.775	0.023	0.063	0.001	
4	7.775	4.000	31.098	124.392	39.159
5	2.775	7.977	22.133	176.568	
6	0.284	7.945	2.256	17.926	
7	0.887	7.444	6.604	49.160	0.058
sum	15.666		62.664	368.323	39.276
	Y _{top}	4.000		I _x	156.943
				I _{xg}	7.078

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d_1 \quad w = 2.774 \text{ in} \quad \frac{w}{t} = 61.519 \quad d := D - d_1 \quad I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 2.625 \times 10^{-3} \text{ in}^4 \quad f := f_y \quad S_w := 1.28 \sqrt{\frac{E}{f}} \quad S = 38.228 \quad 0.328S = 12.539 \quad \frac{w}{t} > 0.328S$$

$$I_{a1} := 399t^4 \cdot \left(\frac{w}{t} - 0.328 \right)^3 \quad I_{a1} = 3.472 \times 10^{-3} \text{ in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \frac{w}{t} + 5 \right) \quad I_{a2} = 7.863 \times 10^{-4} \text{ in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases} \quad I_a = 7.863 \times 10^{-4} \text{ in}^4$$

$$\frac{D}{w} = 0.36 \quad R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases} \quad R_i = 1$$

$$n := \begin{cases} 0.582 - \frac{w}{4S} & \text{if } 0.582 - \frac{w}{4S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.333$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w} \right) \cdot R_i^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 R_i^n + 0.43 & \text{otherwise} \end{cases}$$

$$k := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.448$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w} \right)^2 \quad F_{cr} = 24.237 \text{ ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.167$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.695 \quad b_e := \rho \cdot w \quad b_e = 1.929 \text{ in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 4 \text{ in} \quad d1 := t + r_i \quad f := f_y \quad d = 0.887 \text{ in}$$

$$\frac{d}{t} = 19.673 \quad f1 := f \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f2 := f \cdot \frac{Y_{top} - d1 - d}{Y_{top}} \quad f1 = 32.07 \text{ ksi} \quad f2 = 24.75 \text{ ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.772 \quad k := \frac{0.578}{\psi + 0.34} \quad k = 0.52$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 35.737 \text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.947$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.81$$

$$1 \text{ otherwise} \quad ds' := \rho \cdot d \quad ds' = 0.719 \text{in} \quad ds := ds' \cdot R_i \quad ds = 0.719 \text{in}$$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 7.774 \text{in} \quad \frac{h}{t} = 172.384 \quad f := f_1 \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 32.07 \text{ksi} \quad f_2 = 32.07 \text{ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 21.486 \text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.222$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.671 \quad be := \rho \cdot h \quad be = 5.218 \text{in}$$

$$\frac{h_0}{b_0} = 2.667 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{be}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{be}{2} \text{ if } \psi > 0.236 \\ be - b_1 \text{ otherwise} \end{cases}$$

$$b_1 = 1.304 \text{in} \quad b_2 = 2.609 \text{in}$$

$$b_1 + b_2 = 3.913 \text{in} \quad Y_{top} - d_1 = 3.887 \text{in}$$

$$b_1 + b_2 > 3.887 \quad \text{the web is fully effective.}$$

Moment of Inertia calculation

Trial1 assume Ycg=4.0in

Element	L	y	Ly	Ly ²	I
1	0.719	0.472	0.340	0.160	0.031
2	0.284	0.055	0.016	0.001	
3	1.929	0.023	0.043	0.001	
4a	7.775	4.000	31.098	124.392	39.159
5	2.775	7.977	22.133	176.568	
6	0.284	7.945	2.256	17.926	
7	0.887	7.444	6.604	49.160	0.058
sum	14.652		62.491	368.209	39.249
	Ytop	4.265		Ix	140.938
				Ixe	6.356

Trial 2 $f := fy$ $Y_{top} := 4.265in$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 7.774in \quad \frac{h}{t} = 172.384$$

$$f_1 := f \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_1 = 32.128ksi \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot fy}{Y_{top}} \quad f_2 = 28.027ksi$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 0.872 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 20.873$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 18.686ksi \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.311$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.635 \quad b_e := \rho \cdot h \quad b_e = 4.934in$$

$$\frac{h_0}{b_0} = 2.667 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.274in \quad b_2 = 2.467in$$

$$b_1 + b_2 = 3.741in \quad Y_{top} - d_1 = 4.152in$$

$$b_1 + b_2 < 4.152 \quad \text{the web is not fully effective.}$$

Trial2 assume Ycg=4.2649

Element	L	y	Ly	Ly ²	I
1	0.719	0.472	0.340	0.160	0.031
2	0.284	0.055	0.016	0.001	
3	1.929	0.023	0.043	0.001	
4a	1.274	0.779	0.992	0.772	0.172
4b	2.467	2.767	6.825	18.881	1.251
4c	3.622	6.076	22.010	133.732	3.961
5	2.775	7.977	22.133	176.568	
6	0.284	7.945	2.256	17.926	
7	0.887	7.444	6.604	49.160	0.058
sum	14.241		61.219	397.202	5.474
	Ytop	4.299		lx	139.508
				lx _e	6.292

Trial 3 $Y_{top} := 4.37in$ $f := f_y$

Web under stress gradient

$$h := h_0 - 2 \cdot d_1 \quad h = 7.774in \quad \frac{h}{t} = 172.384 \quad f_1 := f \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$$

$$f_1 = 32.149ksi \quad f_2 = 26.56ksi \quad \psi := \frac{f_2}{f_1} \quad \psi = 0.826$$

$$k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 19.833$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 17.755ksi \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.346$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.622 \quad b_e := \rho \cdot h$$

$$1 \text{ otherwise}$$

$$\frac{h_0}{b_0} = 2.667 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} \\ b_e - b_1 \end{cases} \text{ if } \psi > 0.236$$

$$\text{otherwise}$$

$$b_1 = 1.263in \quad b_2 = 2.417in$$

$$b1 + b2 = 3.68\text{in} \quad Y_{\text{top}} - d1 = 4.257\text{in}$$

$$b1 + b2 < 4.257\text{in} \quad \text{the web is not fully effective.}$$

Trial3	y=4.37in				
Element	L	y	Ly	Ly ²	I
1	0.719	0.472	0.340	0.160	0.031
2	0.284	0.055	0.016	0.001	
3	1.929	0.023	0.043	0.001	
4a	1.263	0.744	0.940	0.700	0.091
4b	2.417	3.162	7.641	24.158	0.572
4c	3.517	6.129	21.556	132.108	2.698
5	2.775	7.977	22.133	176.568	
6	0.284	7.945	2.256	17.926	
7	0.887	7.444	6.604	49.160	0.058
sum	14.075		61.530	400.782	3.450
	Ytop	4.372		Ix	135.248
				Ixe	6.100

From trial 3 it is found that $Y_{cg}=4.372\text{in}$ which is almost equal to 4.37 in. So no other iteration is necessary.

$$I_{xe} := 6.099684n^4 \quad Y_{xe} := 4.372n$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 1.395in^3 \quad M_n := S_{xe} f_y \quad f_y = 33\text{ksi} \quad M_n = 46.04\text{kip-in}$$

Stud section: 203S76 – 16

$$E := 2943\text{ksi} \quad f_y := 33\text{ksi} \quad \mu := 0.3 \quad t := 0.0566n \quad h_0 := 8in \quad b_0 := 3in$$

$$r_i := 1.5t \quad r_i = 0.085in \quad d1 := t + r_i \quad D := 1in$$

Gross section

Element	L	y	Ly	Ly ²	I
1	0.859	0.571	0.490	0.280	0.053
2	0.356	0.069	0.025	0.002	
3	2.717	0.028	0.077	0.002	
4	7.717	4.000	30.868	123.472	38.297
5	2.717	7.972	21.659	172.660	
6	0.356	7.931	2.823	22.390	
7	0.859	7.429	6.382	47.408	0.053
sum	15.581		62.324	366.215	38.403
	Ytop	4.000		Ix	155.321
				Ixg	8.791

Stiffened compression flange under uniform compression

$$w := b_0 - 2d1 \quad w = 2.717in \quad \frac{w}{t} = 48.004 \quad d := D - d1 \quad I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 2.984 \times 10^{-3} in^4 \quad f := f_y$$

$$S := 1.28 \sqrt{\frac{E}{f}} \quad S = 38.228 \quad 0.328 S = 12.539 \quad \frac{w}{t} > 0.328 S$$

$$Ia1 := 399 t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3 \quad Ia1 = 3.269 \times 10^{-3} \text{ in}^4$$

$$Ia2 := t^4 \cdot \left(115 \frac{\frac{w}{t}}{S} + 5 \right) \quad Ia2 = 1.533 \times 10^{-3} \text{ in}^4$$

$$Ia := \begin{cases} Ia1 & \text{if } Ia1 < Ia2 \\ Ia2 & \text{otherwise} \end{cases} \quad Ia = 1.533 \times 10^{-3} \text{ in}^4$$

$$\frac{D}{w} = 0.368 \quad Ri := \begin{cases} \frac{Is}{Ia} & \text{if } \frac{Is}{Ia} < 1 \\ 1 & \text{otherwise} \end{cases} \quad Ri = 1$$

$$n := \begin{cases} 0.582 - \frac{\frac{w}{t}}{4S} & \text{if } 0.582 - \frac{\frac{w}{t}}{4S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.333$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w} \right) \cdot Ri^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot Ri^n + 0.43 & \text{otherwise} \end{cases}$$

$$k := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.41$$

$$Fcr := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w} \right)^2 \quad Fcr = 39.365 \text{ ksi} \quad \lambda := \sqrt{\frac{f}{Fcr}} \quad \lambda = 0.916$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.83 \quad be := \rho \cdot w \quad be = 2.254 \text{ in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 4\text{in} \quad d1 := t + r_i \quad f_c := f_y \quad d = 0.859\text{in}$$

$$\frac{d}{t} = 15.168 \quad f1 := f \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f2 := f \cdot \frac{Y_{top} - d1 - d}{Y_{top}}$$

$$f1 = 31.833\text{ksi} \quad f2 = 24.75\text{ksi} \quad \psi := \frac{f2}{f1} \quad \psi = 0.778 \quad k := \frac{0.578}{\psi + 0.34} \quad k = 0.517$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 59.81\text{ksi} \quad f_c := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.73$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ \lambda & \text{otherwise} \end{cases} \quad \rho = 0.957$$

$$ds' := \rho \cdot d \quad ds' = 0.822\text{in} \quad ds := ds' \cdot R_i \quad ds = 0.822\text{in}$$

Web under stress gradient

$$h := h_0 - 2 \cdot d1 \quad h = 7.717\text{in} \quad \frac{h}{t} = 136.343$$

$$f1 := f_y \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f1 = 31.833\text{ksi} \quad f2 := \frac{(h_0 - Y_{top} - d1) \cdot f_y}{Y_{top}} \quad f2 = 31.833\text{ksi}$$

$$\psi := \frac{f2}{f1} \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 34.347\text{ksi} \quad f_c := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.963$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ \lambda & \text{otherwise} \end{cases} \quad \rho = 0.801 \quad be := \rho \cdot h \quad be = 6.184\text{in}$$

$$\frac{h_0}{b_0} = 2.667 \quad \frac{h_0}{b_0} \leq 4$$

$$b1 := \frac{be}{3 + \psi}$$

$$b2 := \begin{cases} \frac{be}{2} & \text{if } \psi > 0.236 \\ be - b1 & \text{otherwise} \end{cases}$$

$$b1 = 1.546 \text{ in} \quad b2 = 3.092 \text{ in}$$

$$b1 + b2 = 4.638 \text{ in} \quad Y_{top} - d1 = 3.858 \text{ in}$$

$$b1 + b2 > 3.858 \quad \text{the web is fully effective.}$$

Moment of Inertia calculation

Trial1

Element	L	y	Ly	Ly ²	I
1	0.822	0.553	0.454	0.251	0.046
2	0.356	0.069	0.025	0.002	
3	2.254	0.028	0.064	0.002	
4c	7.717	4.000	30.868	123.472	38.297
5	2.717	7.972	21.659	172.660	
6	0.356	7.931	2.823	22.390	
7	0.859	7.429	6.382	47.408	0.053
sum	15.081		62.275	366.185	38.396
	Y _{top}	4.129		I _x	147.428
				I _{xe}	8.344

Trial 2 $Y_{top} := 4.129 \text{ in}$ $f := f_y$

Web under stress gradient

$$h := h_0 - 2 \cdot d1 \quad h = 7.717 \text{ in} \quad \frac{h}{t} = 136.343$$

$$f1 := f_y \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f1 = 31.869 \text{ ksi} \quad f2 := \frac{(h_0 - Y_{top} - d1) \cdot f_y}{Y_{top}} \quad f2 = 29.807 \text{ ksi}$$

$$\psi := \frac{f2}{f1} \quad \psi = 0.935 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 22.367$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h} \right)^2 \quad F_{cr} = 32.011 \text{ ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.998$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.781 \quad b_{e,web} := \rho \cdot h \quad b_e = 6.029 \text{ in}$$

$$\frac{h_0}{b_0} = 2.667 \quad \frac{h_0}{b_0} \leq 4$$

$$b1 := \frac{be}{3 + \psi}$$

$$b2 := \begin{cases} \frac{be}{2} & \text{if } \psi > 0.236 \\ be - b1 & \text{otherwise} \end{cases}$$

$$b1 = 1.532\text{in} \quad b2 = 3.014\text{in}$$

$$b1 + b2 = 4.546\text{in} \quad Y_{top} - d1 = 3.987\text{in}$$

$$b1 + b2 > 3.987 \quad \text{the web is fully effective.}$$

$$I_{xe} := 8.34444\text{n}^4 \quad Y_{xe} := 4.129\text{n}$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 2.021\text{in}^3 \quad M_n := S_{xe} \cdot f_y \quad f_y = 33\text{ksi} \quad M_n = 66.691\text{kip}\cdot\text{in}$$

Stud section: 20S76 - 14

$$E := 29435\text{ksi} \quad f_y := 33\text{ksi} \quad \mu := 0.3 \quad t := 0.0713\text{in} \quad h_0 := 8\text{in} \quad b_0 := 3\text{in}$$

$$r_i := 1.5 \cdot t \quad r_i = 0.107\text{in} \quad d1 := t + r_i \quad D := 1\text{in}$$

Gross section

Element	L	y	Ly	Ly2	I
1	0.822	0.589	0.484	0.285	0.046
2	0.448	0.087	0.039	0.003	
3	2.644	0.036	0.094	0.003	
4	7.644	4.000	30.574	122.296	37.213
5	2.644	7.964	21.054	167.680	
6	0.448	7.913	3.543	28.034	
7	0.822	7.411	6.090	45.131	0.046
sum	15.470		61.878	363.433	37.306
	Y _{top}	4.000		I _x	153.226
				I _{xg}	10.925

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d1 \quad w = 2.643\text{in} \quad \frac{w}{t} = 37.076 \quad d := D - d1 \quad I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 3.297 \times 10^{-3} \text{in}^4 \quad f := f_y \quad S := 1.28 \sqrt{\frac{E}{f}} \quad S = 38.228 \quad 0.328 S = 12.539$$

$$I_{a1} := 399 t^4 \cdot \left(\frac{w}{S} - 0.328 \right)^3 \quad I_{a1} = 2.727 \times 10^{-3} \text{in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \frac{w}{S} + 5 \right) \quad I_{a2} = 3.012 \times 10^{-3} \text{in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases} \quad I_a = 2.727 \times 10^{-3} \text{ in}^4$$

$$\frac{D}{w} = 0.378 \quad R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases} \quad R_i = 1$$

$$n := \begin{cases} 0.582 - \frac{t}{4 \cdot S} & \text{if } 0.582 - \frac{t}{4 \cdot S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.34$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w}\right) \cdot R_i^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot R_i^n + 0.43 & \text{otherwise} \end{cases}$$

$$k := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.359$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w}\right)^2 \quad F_{cr} = 65 \text{ ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.713$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.97 \quad b_e := \rho \cdot w \quad b_e = 2.565 \text{ in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 4 \text{ in} \quad d_1 := t + r_i \quad f := f_y \quad d = 0.822 \text{ in}$$

$$\frac{d}{t} = 11.525 \quad f_1 := f \cdot \frac{Y_{top} - d_1}{Y_{top}} \quad f_2 := f \cdot \frac{Y_{top} - d_1 - d}{Y_{top}}$$

$$f_1 = 31.529 \text{ ksi} \quad f_2 = 24.75 \text{ ksi} \quad \psi := \frac{f_2}{f_1} \quad \psi = 0.785$$

$$k := \frac{0.578}{\psi + 0.34} \quad k = 0.514$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 102.902 \text{ ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.554$$

$$\rho_{\omega} := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 1$$

$$ds' := \rho \cdot d \quad ds' = 0.822 \text{ in} \quad ds := ds' \cdot Ri \quad ds = 0.822 \text{ in}$$

Web under stress gradient

$$h := h_0 - 2 \cdot d1 \quad h = 7.643 \text{ in} \quad \frac{h}{t} = 107.202 \quad f_{\omega} := f1 \quad f2_{\omega} := \frac{(h_0 - Y_{top} - d1) \cdot fy}{Y_{top}}$$

$$f1 = 31.529 \text{ ksi} \quad f2 = 31.529 \text{ ksi}$$

$$\psi_{\omega} := \frac{f2}{f1} \quad \psi = 1 \quad k_{\omega} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h} \right)^2 \quad F_{cr} = 55.558 \text{ ksi} \quad f_{\omega} := f1 \quad \lambda_{\omega} := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.753$$

$$\rho_{\omega} := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.94 \quad be_{\omega} := \rho \cdot h \quad be = 7.183 \text{ in}$$

$$\frac{h_0}{b_0} = 2.667 \quad \frac{h_0}{b_0} \leq 4$$

$$b1 := \frac{be}{3 + \psi}$$

$$b2 := \begin{cases} \frac{be}{2} & \text{if } \psi > 0.236 \\ be - b1 & \text{otherwise} \end{cases}$$

$$b1 = 1.796 \text{ in} \quad b2 = 3.592 \text{ in}$$

$$b1 + b2 = 5.387 \text{ in} \quad Y_{top} - d1 = 3.822 \text{ in}$$

$$b1 + b2 > 3.822 \quad \text{the web is fully effective.}$$

Moment of Inertia calculation

Trial1

1	0.822	0.589	0.484	0.285	0.046
2	0.448	0.087	0.039	0.003	
3	2.565	0.036	0.091	0.003	
4	7.644	4.000	30.574	122.296	37.213
5	2.644	7.964	21.054	167.680	
6	0.448	7.913	3.543	28.034	
7	0.822	7.411	6.090	45.131	0.046
sum	15.391		61.875	363.433	37.306
	Y _{top}	4.020		I _x	151.986
				I _{xe}	10.837

Web under stress gradient

$$\begin{aligned}
 Y_{top} &:= 4.02 \text{ in} & f_1 &:= f_y & h &:= h_0 - 2 \cdot d_1 & h &= 7.643 \text{ in} & \frac{h}{t} &= 107.202 \\
 f_1 &:= f \cdot \frac{Y_{top} - d_1}{Y_{top}} & f_1 &= 31.537 \text{ ksi} & f_2 &:= \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}} & f_2 &= 31.208 \text{ ksi} \\
 \psi &:= \frac{f_2}{f_1} & \psi &= 0.99 & k &:= 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) & k &= 23.731
 \end{aligned}$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h} \right)^2 \quad F_{cr} = 54.934 \text{ ksi} \quad f_w := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.758$$

$$\rho_w := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.937 \quad b_{e,w} := \rho \cdot h \quad b_e = 7.159 \text{ in}$$

$$\frac{h_0}{b_0} = 2.667 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.794 \text{ in} \quad b_2 = 3.579 \text{ in}$$

$$b_1 + b_2 = 5.374 \text{ in} \quad Y_{top} - d_1 = 3.842 \text{ in}$$

$$b_1 + b_2 > 3.842 \quad \text{the web is fully effective.}$$

$$Y_{xe} := 4.0202 \text{ in} \quad I_{xe} := 10.8366 \text{ in}^4$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 2.696 \text{ in}^3 \quad M_n := S_{xe} \cdot f_y \quad f_y = 33 \text{ ksi} \quad M_n = 88.952 \text{ kip-in}$$

Stud section: 30S76 – 18

$E := 29435 \text{ ksi}$ $f_y := 33 \text{ ksi}$ $\mu := 0.3$ $t := 0.045 \text{ in}$ $h_0 := 12 \text{ in}$ $b_0 := 3 \text{ in}$
 $r_i := 1.5 \cdot t$ $r_i = 0.068 \text{ in}$ $d_1 := t + r_i$ $D := 1 \text{ in}$
 Gross section

Element	L	y	Ly	Ly ²	I
1	0.887	0.556	0.494	0.275	0.058
2	0.283	0.055	0.016	0.001	
3	2.775	0.023	0.063	0.001	
4	11.775	6.000	70.647	423.882	39.159
5	2.775	11.977	33.231	398.028	
6	0.283	11.945	3.383	40.410	
7	0.887	11.444	10.153	116.191	0.058
	19.664		117.987	978.788	39.276
	Ytop	6.000		Ix	310.143
				Ixg	13.987

Stiffened compression flange under uniform compression

$w := b_0 - 2 \cdot d_1$ $w = 2.774 \text{ in}$ $\frac{w}{t} = 61.519$ $d := D - d_1$ $I_s := \frac{d^3 \cdot t}{12}$
 $I_s = 2.625 \times 10^{-3} \text{ in}^4$ $f := f_y$ $S := 1.28 \sqrt{\frac{E}{f}}$ $S = 38.228$ $0.328 S = 12.539$

$I_{a1} := 399 \cdot t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3$ $I_{a1} = 3.472 \times 10^{-3} \text{ in}^4$

$I_{a2} := t^4 \cdot \left(115 \frac{\frac{w}{t}}{S} + 5 \right)$ $I_{a2} = 7.863 \times 10^{-4} \text{ in}^4$

$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases}$ $I_a = 7.863 \times 10^{-4} \text{ in}^4$

$\frac{D}{w} = 0.36$ $R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases}$ $R_i = 1$

$n := \begin{cases} 0.582 - \frac{\frac{w}{t}}{4 \cdot S} & \text{if } 0.582 - \frac{\frac{w}{t}}{4 \cdot S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$

$$n = 0.333$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w}\right) \cdot R_i^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot R_i^n + 0.43 & \text{otherwise} \end{cases}$$

$$\underline{k} := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.448$$

$$F_{cr} := \frac{\underline{k} \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w}\right)^2 \quad F_{cr} = 24.237 \text{ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.167$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.695 \quad be := \rho \cdot w \quad be = 1.929 \text{in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 6 \text{in} \quad \underline{d1} := t + r_i \quad \underline{f} := f_y \quad d = 0.887 \text{in}$$

$$\frac{d}{t} = 19.673 \quad f1 := f \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f2 := f \cdot \frac{Y_{top} - d1 - d}{Y_{top}}$$

$$f1 = 32.38 \text{ksi} \quad f2 = 27.5 \text{ksi} \quad \psi := \frac{f2}{f1} \quad \psi = 0.849$$

$$\underline{k} := \frac{0.578}{\psi + 0.34} \quad k = 0.486$$

$$\underline{F_{cr}} := \frac{\underline{k} \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 33.407 \text{ksi} \quad \underline{f} := f1 \quad \underline{\lambda} := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.985$$

$$\underline{\rho} := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.789$$

$$ds' := \rho \cdot d \quad ds' = 0.7 \text{in} \quad ds := ds' \cdot R_i \quad ds = 0.7 \text{in}$$

Web under stress gradient

$$h := h_0 - 2 \cdot d1 \quad h = 11.774 \text{in} \quad \frac{h}{t} = 261.075 \quad \underline{f} := f1 \quad \underline{f2} := \frac{(h_0 - Y_{top} - d1) \cdot f_y}{Y_{top}}$$

$$f1 = 32.38 \text{ksi} \quad f2 = 32.38 \text{ksi}$$

$$\psi := \frac{f_2}{f_1} \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 9.367 \text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.859$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.474 \quad b_e := \rho \cdot h \quad b_e = 5.584 \text{in}$$

$$1 \text{ otherwise}$$

$$\frac{h_0}{b_0} = 4 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} \\ b_e - b_1 \end{cases} \text{ if } \psi > 0.236$$

$$\text{otherwise}$$

$$b_1 = 1.396 \text{in} \quad b_2 = 2.792 \text{in}$$

$$b_1 + b_2 = 4.188 \text{in} \quad Y_{\text{top}} - d_1 = 5.887 \text{in}$$

$$b_1 + b_2 < 5.887 \quad \text{the web is not fully effective.}$$

Moment of Inertia calculation

Trial1

Element	L	y	Ly	Ly ²	I
1	0.700	0.463	0.324	0.150	0.029
2	0.283	0.055	0.016	0.001	
3	1.929	0.023	0.043	0.001	
4a	1.396	0.811	1.132	0.918	0.227
4b	2.792	4.604	12.854	59.182	1.814
4c	5.887	8.944	52.653	470.912	17.004
5	2.775	11.977	33.231	398.028	
6	0.283	11.945	3.383	40.410	
7	0.887	11.444	10.153	116.191	0.058
	16.932		113.790	1085.792	19.131
	Y _{top}	6.720		lx	340.222
				lxe	15.344

Trial 2 $Y_{top} := 6.9\text{in}$ $f := f_y$

Web under stress gradient

$h := h_0 - 2 \cdot d_1$ $h = 11.774\text{in}$ $\frac{h}{t} = 261.075$

$f_1 := f_y \cdot \frac{Y_{top} - d_1}{Y_{top}}$ $f_1 = 32.461\text{ksi}$ $f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$ $f_2 = 32.38\text{ksi}$

$\psi := \frac{f_2}{f_1}$ $\psi = 0.735$ $k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi)$ $k = 17.911$

$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2$ $F_{cr} = 6.991\text{ksi}$ $f := f_1$ $\lambda := \sqrt{\frac{f}{F_{cr}}}$ $\lambda = 2.155$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{\lambda}{\lambda} \end{cases} \text{ if } \lambda > 0.673$$

$\rho = 0.417$

1 otherwise

$b_e := \rho \cdot h$ $b_e = 4.906\text{in}$

$\frac{h_0}{b_0} = 4$ $\frac{h_0}{b_0} \leq 4$

$b_1 := \frac{b_e}{3 + \psi}$

$b_2 := \begin{cases} \frac{b_e}{2} \text{ if } \psi > 0.236 \\ b_e - b_1 \text{ otherwise} \end{cases}$

$b_1 = 1.314\text{in}$ $b_2 = 2.453\text{in}$

$b_1 + b_2 = 3.767\text{in}$ $Y_{top} - d_1 = 6.787\text{in}$

$b_1 + b_2 < 6.787$ the web is not fully effective.

Element	L	y	L_y	L_y^2	I
1	0.700	0.463	0.324	0.150	0.018
2	0.283	0.055	0.016	0.001	
3	1.929	0.023	0.043	0.001	
4a	1.314	0.770	1.011	0.779	0.189
4b	2.453	5.674	13.917	78.959	1.230
4c	4.987	9.394	46.848	440.076	10.337
5	2.775	11.977	33.231	398.028	
6	0.283	11.945	3.383	40.410	
7	0.887	11.444	10.153	116.191	0.058
	15.611		108.928	1074.594	11.832
	Y_{top}	6.977		I_x	326.389
				I_{xe}	14.720

Trial 3 $Y_{top} := 7.0\text{in}$ $f := f_y$

Web under stress gradient

$h := h_0 - 2 \cdot d_1$ $h = 11.774\text{in}$ $\frac{h}{t} = 261.075$
 $f_1 := f_y \cdot \frac{Y_{top} - d_1}{Y_{top}}$ $f_1 = 32.468\text{ksi}$ $f_2 := \frac{(h_0 - Y_{top} - d_1) \cdot f_y}{Y_{top}}$ $f_2 = 23.04\text{ksi}$

$\psi := \frac{f_2}{f_1}$ $\psi = 0.71$ $k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi)$ $k = 17.413$

$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2$ $F_{cr} = 6.796\text{ksi}$ $f := f_1$ $\lambda := \sqrt{\frac{f}{F_{cr}}}$ $\lambda = 2.186$

$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \lambda \end{cases}$ if $\lambda > 0.673$ $\rho = 0.411$ $b_e := \rho \cdot h$
 1 otherwise

$\frac{h_0}{b_0} = 4$ $\frac{h_0}{b_0} \leq 4$

$b_1 := \frac{b_e}{3 + \psi}$

$b_2 := \begin{cases} \frac{b_e}{2} \\ b_e - b_1 \end{cases}$ if $\psi > 0.236$ otherwise

$b_1 = 1.306\text{in}$ $b_2 = 2.422\text{in}$

$b_1 + b_2 = 3.728\text{in}$ $Y_{top} - d_1 = 6.887\text{in}$

$b_1 + b_2 < 6.907\text{in}$ the web is not fully effective.

Trial3	Ytop=7.0in				
Element	L	y	Ly	Ly ²	I
1	0.700	0.463	0.324	0.150	0.029
2	0.283	0.055	0.016	0.001	
3	1.929	0.023	0.043	0.001	
4a	1.306	0.766	1.000	0.766	0.186
4b	2.422	5.789	14.021	81.167	1.184
4c	4.887	9.444	46.153	435.855	9.728
5	2.775	11.977	33.231	398.028	
6	0.283	11.945	3.383	40.410	
7	0.887	11.444	10.153	116.191	0.058
	15.472		108.325	1072.569	11.184
	Ytop	7.001		lx	325.349
				lx _e	14.673

From trial 3 it is found that $Y_{cg}=7.0\text{in}$. So no other iteration is necessary.

$Y_{xe} := 7.0\text{in}$ $I_{xe} := 14.67322\text{in}^4$
 $S_{xe} := \frac{I_{xe}}{Y_{xe}}$ $S_{xe} = 2.096\text{in}^3$ $M_n := S_{xe} \cdot f_y$ $f_y = 33\text{ksi}$ $M_n = 69.174\text{kip}\cdot\text{in}$

Stud section: 305S76 – 16

$E := 29435 \text{ksi}$ $f_y := 33 \text{ksi}$ $\mu := 0.3$ $t := 0.0566 \text{in}$ $h_0 := 12 \text{in}$ $b_0 := 3 \text{in}$
 $r_i := 1.5 \cdot t$ $r_i = 0.085 \text{in}$ $d_1 := t + r_i$ $D := 1 \text{in}$

Gross section

Element	L	y	Ly	Ly ²	I
1	0.859	0.571	0.490	0.280	0.053
2	0.356	0.069	0.025	0.002	
3	2.717	0.028	0.077	0.002	
4	11.717	6.000	70.302	421.812	134.050
5	2.717	11.972	32.527	389.405	
6	0.356	11.931	4.247	50.673	
7	0.859	11.429	9.818	112.204	0.053
sum	19.581		117.486	974.378	134.156
	Ytop	6.000		Ix	403.618
				Ixg	22.845

Stiffened compression flange under uniform compression

$w := b_0 - 2 \cdot d_1$ $w = 2.717 \text{in}$ $\frac{w}{t} = 48.004$ $d := D - d_1$ $I_s := \frac{d^3 \cdot t}{12}$
 $I_s = 2.984 \times 10^{-3} \text{in}^4$ $f := f_y$ $S := 1.28 \cdot \sqrt{\frac{E}{f}}$ $S = 38.228$ $0.328 \cdot S = 12.539$ $\frac{w}{t} > 0.328 \cdot S$

$I_{a1} := 399 \cdot t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3$ $I_{a1} = 3.269 \times 10^{-3} \text{in}^4$

$I_{a2} := t^4 \cdot \left(115 \cdot \frac{\frac{w}{t}}{S} + 5 \right)$ $I_{a2} = 1.533 \times 10^{-3} \text{in}^4$

$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases}$ $I_a = 1.533 \times 10^{-3} \text{in}^4$

$\frac{D}{w} = 0.368$ $R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases}$ $R_i = 1$

$n := \begin{cases} 0.582 - \frac{\frac{w}{t}}{4 \cdot S} & \text{if } 0.582 - \frac{\frac{w}{t}}{4 \cdot S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$

$n = 0.333$

$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w} \right) \cdot R_i^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot R_i^n + 0.43 & \text{otherwise} \end{cases}$

$$k := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.41$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w}\right)^2 \quad F_{cr} = 39.365 \text{ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.916$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.83 \quad be := \rho \cdot w \quad be = 2.254 \text{in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 6 \text{in} \quad d1 := t + r_i \quad f := f_y \quad d = 0.859 \text{in}$$

$$\frac{d}{t} = 15.168 \quad f1 := f \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f2 := f \cdot \frac{Y_{top} - d1 - d}{Y_{top}}$$

$$f1 = 32.222 \text{ksi} \quad f2 = 27.5 \text{ksi} \quad \psi := \frac{f2}{f1} \quad \psi = 0.853$$

$$k := \frac{0.578}{\psi + 0.34} \quad k = 0.484$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 56.003 \text{ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.759$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.936$$

$$ds' := \rho \cdot d \quad ds' = 0.804 \text{in} \quad ds := ds' \cdot R_i \quad ds = 0.804 \text{in}$$

Web under stress gradient

$$h := h_0 - 2 \cdot d1 \quad h = 11.717 \text{in} \quad \frac{h}{t} = 207.014$$

$$f1 := f_y \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f1 = 32.222 \text{ksi} \quad f2 := \frac{(h_0 - Y_{top} - d1) \cdot f_y}{Y_{top}} \quad f2 = 32.222 \text{ksi}$$

$$\psi := \frac{f2}{f1} \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 14.899 \text{ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.471$$

$$\rho_w := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.578 \quad \underline{\rho_w} := \rho \cdot h \quad be = 6.776 \text{ in}$$

$$\frac{h_0}{b_0} = 4 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{be}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{be}{2} & \text{if } \psi > 0.236 \\ be - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.694 \text{ in} \quad b_2 = 3.388 \text{ in}$$

$$b_1 + b_2 = 5.082 \text{ in} \quad Y_{\text{top}} - d_1 = 5.859 \text{ in}$$

$$b_1 + b_2 < 5.858 \quad \text{the web is not fully effective.}$$

Moment of Inertia calculation

Trial1

Element	L	y	Ly	Ly ²	I
1	0.804	0.544	0.437	0.237	0.043
2	0.356	0.069	0.025	0.002	
3	2.254	0.028	0.064	0.002	
4a	1.694	0.989	1.675	1.655	0.405
4b	3.388	4.306	14.589	62.819	3.241
4c	5.859	8.929	52.312	467.107	16.756
5	2.717	11.972	32.527	389.405	
6	0.356	11.931	4.247	50.673	
7	0.859	11.429	9.818	112.204	0.053
sum	18.287		115.693	1084.104	20.498
	Y _{top}	6.327		I _x	372.654
				I _{xe}	21.092

$$\text{Trial 2} \quad \underline{Y_{\text{top}}} := 6.48 \text{ in} \quad \underline{f_y} := f_y$$

Web under stress gradient

$$\underline{h_w} := h_0 - 2 \cdot d_1 \quad h = 11.717 \text{ in} \quad \frac{h}{t} = 207.014$$

$$\underline{f_1} := f_y \cdot \frac{Y_{\text{top}} - d_1}{Y_{\text{top}}} \quad f_1 = 32.279 \text{ ksi} \quad \underline{f_2} := \frac{(h_0 - Y_{\text{top}} - d_1) \cdot f_y}{Y_{\text{top}}} \quad f_2 = 27.391 \text{ ksi}$$

$$\underline{\psi} := \frac{f_2}{f_1} \quad \psi = 0.849 \quad \underline{k} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 20.33$$

$$\underline{F_{cr}} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h} \right)^2 \quad F_{cr} = 12.621 \text{ ksi} \quad \underline{f_w} := f_1 \quad \underline{\lambda} := \sqrt{\frac{f_w}{F_{cr}}} \quad \lambda = 1.599$$

$$\rho_w := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{\lambda}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.539 \quad \overline{b_e} := \rho \cdot h \quad b_e = 6.319 \text{ in}$$

$$1 \text{ otherwise}$$

$$\frac{h_0}{b_0} = 4 \quad \frac{h_0}{b_0} \leq 4$$

$$\overline{b_1} := \frac{b_e}{3 + \psi}$$

$$\overline{b_2} := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 1.642 \text{ in} \quad b_2 = 3.159 \text{ in}$$

$$b_1 + b_2 = 4.801 \text{ in} \quad Y_{\text{top}} - d_1 = 6.339 \text{ in}$$

$$b_1 + b_2 < 6.339 \quad \text{the web is not fully effective.}$$

Trail 2 assume $Y_{cg}=6.48$

Element	L	y	Ly	Ly^2	I
1	0.804	0.544	0.437	0.237	0.043
2	0.356	0.069	0.025	0.002	
3	2.254	0.028	0.064	0.002	
4a	1.642	0.963	1.580	1.521	0.369
4b	3.159	4.901	15.481	75.863	2.627
4c	5.379	9.169	49.317	452.198	12.966
5	2.717	11.972	32.527	389.405	
6	0.356	11.931	4.247	50.673	
7	0.859	11.429	9.818	112.204	0.053
sum	17.526		113.495	1082.105	16.058
	Y_{top}	6.476		I_x	363.167
				I_{xe}	20.555

From trial 2 it is found that $Y_{cg}=6.48 \text{ in}$ which is almost equal to 6.476 in. So no other iteration is necessary.

$$I_{xe} := 20.55524 \text{ in}^4 \quad Y_{xe} := 6.476 \text{ in}$$

$$S_{xe} := \frac{I_{xe}}{Y_{xe}} \quad S_{xe} = 3.174 \text{ in}^3 \quad M_n := S_{xe} f_y \quad f_y = 33 \text{ ksi} \quad M_n = 104.744 \text{ kip-in}$$

Stud section: 30S76 – 14

$$E := 29435 \text{ksi} \quad f_y := 33 \text{ksi} \quad \mu := 0.3 \quad t := 0.0713 \text{in} \quad h_0 := 12 \text{in} \quad b_0 := 3 \text{in}$$

$$r_i := 1.5 \cdot t \quad r_i = 0.107 \text{in} \quad d_1 := t + r_i \quad D := 1 \text{in}$$

Gross section

Element	L	y	Ly	Ly ²	I
1	0.822	0.589	0.484	0.285	0.046
2	0.448	0.087	0.039	0.003	
3	2.644	0.036	0.094	0.003	
4	11.644	6.000	69.861	419.166	131.544
5	2.644	11.964	31.628	378.406	
6	0.448	11.913	5.334	63.542	
7	0.822	11.411	9.377	106.998	0.046
sum	19.470		116.817	968.404	131.636
	Ytop	6.000		Ix	399.137
				Ixg	28.458

Stiffened compression flange under uniform compression

$$w := b_0 - 2 \cdot d_1 \quad w = 2.643 \text{in} \quad \frac{w}{t} = 37.076 \quad d := D - d_1 \quad I_s := \frac{d^3 \cdot t}{12}$$

$$I_s = 3.297 \times 10^{-3} \text{in}^4 \quad f := f_y \quad S_x := 1.28 \cdot \sqrt{\frac{E}{f}} \quad S = 38.228 \quad 0.328 \cdot S = 12.539$$

$$I_{a1} := 399 \cdot t^4 \cdot \left(\frac{\frac{w}{t}}{S} - 0.328 \right)^3 \quad I_{a1} = 2.727 \times 10^{-3} \text{in}^4$$

$$I_{a2} := t^4 \cdot \left(115 \cdot \frac{\frac{w}{t}}{S} + 5 \right) \quad I_{a2} = 3.012 \times 10^{-3} \text{in}^4$$

$$I_a := \begin{cases} I_{a1} & \text{if } I_{a1} < I_{a2} \\ I_{a2} & \text{otherwise} \end{cases} \quad I_a = 2.727 \times 10^{-3} \text{in}^4$$

$$\frac{D}{w} = 0.378 \quad R_i := \begin{cases} \frac{I_s}{I_a} & \text{if } \frac{I_s}{I_a} < 1 \\ 1 & \text{otherwise} \end{cases} \quad R_i = 1$$

$$n := \begin{cases} 0.582 - \frac{\frac{w}{t}}{4 \cdot S} & \text{if } 0.582 - \frac{\frac{w}{t}}{4 \cdot S} \geq \frac{1}{3} \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$$n = 0.34$$

$$k := \begin{cases} \left(4.82 - 5 \cdot \frac{D}{w} \right) \cdot R_i^n + 0.43 & \text{if } 0.25 < \frac{D}{w} \leq 0.8 \\ 3.57 \cdot R_i^n + 0.43 & \text{otherwise} \end{cases}$$

$$k := \begin{cases} 4 & \text{if } k > 4 \\ k & \text{otherwise} \end{cases} \quad k = 3.359$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{w}\right)^2 \quad F_{cr} = 65 \text{ksi} \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.713$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.97 \quad be := \rho \cdot w \quad be = 2.565 \text{in}$$

Lip (Unstiffened compression element)

$$Y_{top} := 6 \text{in} \quad d1 := t + r_i \quad f := f_y \quad d = 0.822 \text{in}$$

$$\frac{d}{t} = 11.525 \quad f1 := f \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f2 := f \cdot \frac{Y_{top} - d1 - d}{Y_{top}}$$

$$f1 = 32.02 \text{ksi} \quad f2 = 27.5 \text{ksi} \quad \psi := \frac{f2}{f1} \quad \psi = 0.859$$

$$k := \frac{0.578}{\psi + 0.34} \quad k = 0.482$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad F_{cr} = 96.562 \text{ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.576$$

$$\rho := \begin{cases} 1 - \frac{0.22}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 1$$

$$ds' := \rho \cdot d \quad ds' = 0.822 \text{in} \quad ds := ds' \cdot R_i \quad ds = 0.822 \text{in}$$

Web under stress gradient

$$h := h_0 - 2 \cdot d1 \quad h = 11.643 \text{in} \quad \frac{h}{t} = 163.303 \quad f := f1 \quad f2 := \frac{(h_0 - Y_{top} - d1) \cdot f_y}{Y_{top}}$$

$$f1 = 32.02 \text{ksi} \quad f2 = 32.02 \text{ksi}$$

$$\psi := \frac{f2}{f1} \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 23.942 \text{ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.156$$

$$\rho := \begin{cases} \frac{1 - \frac{0.22}{\lambda}}{\lambda} & \text{if } \lambda > 0.673 \\ 1 & \text{otherwise} \end{cases} \quad \rho = 0.7 \quad b_{e} := \rho \cdot h \quad b_e = 8.153 \text{in}$$

$$\frac{h_0}{b_0} = 4 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 2.038 \text{in} \quad b_2 = 4.076 \text{in}$$

$$b_1 + b_2 = 6.115 \text{in} \quad Y_{top} - d1 = 5.822 \text{in}$$

$$b_1 + b_2 > 5.822 \text{in} \quad \text{the web is fully effective.}$$

Moment of Inertia calculation

Trial1

Element	L	y	Ly	Ly ²	I
1	0.822	0.589	0.484	0.285	0.046
2	0.448	0.087	0.039	0.003	
3	2.565	0.036	0.094	0.003	
4c	11.644	6.000	69.861	419.166	131.544
5	2.644	11.964	31.628	378.406	
6	0.448	11.913	5.334	63.542	
7	0.822	11.411	9.377	106.998	0.046
sum	19.391		116.817	968.404	131.636
	Y _{top}	6.024		I _x	396.309
				I _{xe}	28.257

Web under stress gradient

$$Y_{top} := 6.02 \text{in} \quad h := h_0 - 2 \cdot d1 \quad h = 11.643 \text{in} \quad \frac{h}{t} = 163.303$$

$$f1 := f_y \cdot \frac{Y_{top} - d1}{Y_{top}} \quad f1 = 32.023 \text{ksi} \quad f2 := \frac{(h_0 - Y_{top} - d1) \cdot f_y}{Y_{top}} \quad f2 = 31.804 \text{ksi}$$

$$\psi := \frac{f2}{f1} \quad \psi = 0.993 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 23.823$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{h}\right)^2 \quad F_{cr} = 23.765 \text{ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.161$$

$$\rho_{\text{eff}} := \begin{cases} 1 - \frac{0.22}{\lambda} \\ \frac{1 - \frac{0.22}{\lambda}}{\lambda} \end{cases} \text{ if } \lambda > 0.673 \quad \rho = 0.698 \quad b_{\text{eff}} := \rho \cdot h \quad b_e = 8.13 \text{ in}$$

$$\frac{h_0}{b_0} = 4 \quad \frac{h_0}{b_0} \leq 4$$

$$b_1 := \frac{b_e}{3 + \psi}$$

$$b_2 := \begin{cases} \frac{b_e}{2} & \text{if } \psi > 0.236 \\ b_e - b_1 & \text{otherwise} \end{cases}$$

$$b_1 = 2.036 \text{ in} \quad b_2 = 4.065 \text{ in}$$

$$b_1 + b_2 = 6.101 \text{ in} \quad Y_{\text{top}} - d_1 = 5.842 \text{ in}$$

$$b_1 + b_2 > 5.842 \text{ in} \quad \text{the web is fully effective.}$$

$$Y_{\text{xe}} := 6.024 \text{ in} \quad I_{\text{xe}} := 28.2568 \text{ in}^4$$

$$S_{\text{xe}} := \frac{I_{\text{xe}}}{Y_{\text{xe}}} \quad S_{\text{xe}} = 4.691 \text{ in}^3 \quad M_n := S_{\text{xe}} \cdot f_y \quad f_y = 33 \text{ ksi} \quad M_n = 154.793 \text{ kip-in}$$

Track section: 305T32-18

$$F_y := 33 \text{ ksi} \quad h := 12.0903 \text{ in} \quad t := 0.045 \text{ in} \quad r := 1.5 \cdot t \quad r = 0.068 \text{ in}$$

$$b := 1.25 \text{ in} \quad w := b - 2.5t \quad w = 1.137 \text{ in} \quad E := 29435 \text{ ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	Ly	Ly ²	I'
1.000	1.137	0.023	0.026	0.001	
2.000	0.142	0.055	0.008	0.000	
3.000	11.865	6.045	71.723	433.575	139.184
4.000	0.142	12.035	1.704	20.511	
5.000	1.137	12.068	13.724	165.616	
sum	14.422		87.185	619.702	139.184
ycg	6.045				
lxe	10.456				

Flange under uniform compression:

$$\frac{w}{t} = 25.216 \quad k := 0.43 \quad f := F_y \quad F_{\text{cr}} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{\text{cr}} = 17.991 \text{ ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{\text{cr}}}} \quad \lambda = 1.354 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.618 \quad b_e := \rho \cdot w \quad b_e = 0.703 \text{ in}$$

Web under stress gradient:

$$\begin{aligned} \underline{w} &:= h - 5 \cdot t & w &= 11.8647 \text{in} & \frac{w}{t} &= 263.075 & y_{cg} &:= 6.045 \text{in} \\ w &= 11.8647 \text{in} \end{aligned}$$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f1 = 32.385 \text{ksi} \quad f2 := f1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f2 = 32.385 \text{ksi}$$

$$\underline{\psi} := \left| \frac{f2}{f1} \right| \quad \psi = 1 \quad \underline{k} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$\underline{F_{cr}} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 9.226 \text{ksi} \quad \underline{f} := f1 \quad \underline{\lambda} := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.874$$

$$\underline{\rho} := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.471 \quad \underline{b_e} := \rho \cdot w \quad b_e = 5.589 \text{in}$$

$$h_o := 12.0902 \text{in} \quad b_o := 1.25 \text{in} \quad \frac{h_o}{b_o} = 9.672$$

$$b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1 \quad b1 = 1.397 \text{in} \quad b2 = 1.397 \text{in}$$

$b1 + b2 = 2.795 \text{in}$ is less than compression portion of web element = 5.93235in

So the web is not fully effective

Element	Length	y	Ly	Ly ²	I'
1.000	0.703	0.028	0.020	0.001	
2.000	0.142	0.055	0.008	0.000	
3a	1.397	0.811	1.133	0.919	0.227
3b	1.397	5.347	7.469	39.935	0.227
3c	5.932	9.011	53.458	481.725	17.398
4.000	0.142	12.035	1.704	20.511	
5.000	1.137	12.068	13.724	347.651	
sum	10.850		77.517	890.742	17.852
ycg	7.144				
lxe	16.001				

Trial 2 $\underline{y_{cg}} := 7.34 \text{in}$

$$\underline{f1} := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f1 = 32.493 \text{ksi} \quad \underline{f2} := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5 \cdot t) \quad f2 = 20.85 \text{ksi}$$

$$\underline{\psi} := \left| \frac{f2}{f1} \right| \quad \psi = 0.642 \quad \underline{k} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 16.132$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 6.201 \text{ ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.289$$

$$\rho_w := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.395 \quad b_e := \rho \cdot w \quad b_e = 4.685 \text{ in}$$

$$h_o := 12.113 \text{ in} \quad b_o := 1.25 \text{ in} \quad \frac{h_o}{b_o} = 9.691$$

$$b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1 \quad b1 = 1.287 \text{ in} \quad b2 = 1.567 \text{ in}$$

Trial2 Assume $y_{cg} = 7.34$

Element	Length	y	Ly	Ly ²	I'
1.000	0.703	0.028	0.020	0.001	
2.000	0.142	0.055	0.008	0.000	
3a	1.287	0.756	0.973	0.736	0.178
3b	1.567	6.557	10.274	67.362	0.321
3c	4.637	9.659	44.792	432.632	8.311
4.000	0.142	12.035	1.704	20.511	
5.000	1.137	12.068	13.724	165.616	
sum	9.615		71.495	686.858	8.809
y _{cg}	7.436				
l _{xe}	7.398				

Trial 3 $y_{cg} := 7.455 \text{ in}$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \text{ t}) \quad f1 = 32.501 \text{ ksi} \quad f2 := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5 \text{ t}) \quad f2 = 20.019 \text{ ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.616 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 15.671$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 6.024 \text{ ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 2.323$$

$$\rho_w := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.39 \quad b_e := \rho \cdot w \quad b_e = 4.624 \text{ in} \quad \frac{h_o}{b_o} = 9.691$$

$$b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1 \quad b1 = 1.279 \text{ in} \quad b2 = 1.583 \text{ in}$$

Trial 3 assume $y_{cg}=7.455$

Element	Length	y	L_y	L_y^2	I'
1.000	0.703	0.028	0.020	0.001	
2.000	0.142	0.055	0.008	0.000	
3a	1.279	0.753	0.963	0.724	0.174
3b	1.583	6.664	10.548	70.289	0.331
3c	4.522	9.716	43.941	426.942	7.708
4.000	0.142	12.035	1.704	20.511	
5.000	1.137	12.068	13.724	165.616	
sum	9.508		70.908	684.083	8.213
y_{cg}	7.457				
I_{xe}	7.376				

$Y_{cg} := 7.457 \text{ in}$ So no further iteration is necessary.

$$I_{xe} := 7.375 \text{ in}^4 \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad Y_{cg} = 7.457 \text{ in} \quad M_n := F_y \cdot S_{xe} \quad M_n = 32.64 \text{ kip}\cdot\text{in}$$

Track section: 305T32 – 16

$$F_y := 33 \text{ ksi} \quad h := 12.113 \text{ in} \quad t := 0.056 \text{ in} \quad r := 1.5 \cdot t \quad r = 0.085 \text{ in}$$

$$b := 1.25 \text{ in} \quad w := b - 2.5t \quad w = 1.109 \text{ in} \quad E := 29435 \text{ ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	L_y	L_y^2	I'
1	1.109	0.028	0.031	0.001	
2	0.178	0.069	0.012	0.001	
3	11.830	6.057	71.651	433.960	137.973
4	0.178	12.044	2.140	25.779	
5	1.109	12.085	13.396	161.891	
	14.403		87.231	621.632	137.973
y_{cg}	6.057				
I_{xe}	13.091				

Flange under uniform compression:

$$\frac{w}{t} = 19.585 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 29.824 \text{ ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.052 \quad \rho := \frac{1 - 0.22}{\lambda} \quad \rho = 0.752 \quad b_e := \rho \cdot w \quad b_e = 0.833 \text{ in}$$

Web under stress gradient:

$$w := h - 5 \cdot t \quad w = 11.830 \text{ in} \quad \frac{w}{t} = 209.014 \quad y_{cg} := 6.056 \text{ in} \quad w = 11.830 \text{ in}$$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 32.229\text{ksi} \quad f2 := f1 \cdot \frac{h - y_{cg} - 2.5t}{y_{cg} - 2.5t} \quad f2 = 32.229\text{ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 14.615\text{ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.485$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.574 \quad b_e := \rho \cdot w \quad b_e = 6.786\text{in}$$

$$h_o := 12.1132\text{in} \quad b_o := 1.25\text{in} \quad \frac{h_o}{b_o} = 9.691$$

$$b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1 \quad b1 = 1.697\text{in} \quad b2 = 1.697\text{in}$$

$b1 + b2 = 3.393\text{in}$ is less than compression portion of web element = 5.9151in

So the web is not fully effective

Trial1					
Element	Length	y	Ly	Ly ²	I'
1	0.833	0.028	0.024	0.001	
2	0.178	0.069	0.012	0.001	
3a	1.697	0.990	1.680	1.663	0.279
3b	1.697	5.208	8.838	46.030	0.279
3c	5.915	9.014	53.320	480.631	17.247
4	0.178	12.044	2.140	25.779	
5	1.109	12.085	13.396	161.891	
	11.606		79.410	715.996	17.805
y _{cg}	6.842				
l _{xe}	10.780				

Trial 2 $y_{cg} := 7.047\text{in}$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 32.337\text{ksi} \quad f2 := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5t) \quad f2 = 23.062\text{ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.713 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 17.482$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 10.646\text{ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.743$$

$$\rho_w := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.501 \quad b_e := \rho \cdot w \quad b_e = 5.931 \text{ in}$$

$$h_o := 12.113 \text{ in} \quad b_o := 1.25 \text{ in} \quad \frac{h_o}{b_o} = 9.691$$

$$b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1 \quad b_1 = 1.597 \text{ in} \quad b_2 = 1.865 \text{ in}$$

Trial2	Assume	ycg=7.047			
Element	Length	y	Ly	Ly ²	I'
1	0.833	0.028	0.024	0.001	
2	0.178	0.069	0.012	0.001	
3a	1.597	0.940	1.501	1.411	0.339
3b	1.865	6.115	11.404	69.727	0.541
3c	4.925	9.509	46.831	445.329	9.954
4	0.178	12.044	2.140	25.779	
5	1.109	12.085	13.396	161.891	
	10.684		75.308	704.139	10.834
ycg	7.048				
I _{xe}	10.426				

Y_{cg} := 7.048 in So no further iteration is necessary.

$$I_{xe} := 10.426 \text{ in}^4 \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad Y_{cg} = 7.048 \text{ in} \quad M_n := F_y \cdot S_{xe} \quad M_n = 48.82 \text{ kip} \cdot \text{in}$$

Track section: 305T32 - 14

$$F_y := 33 \text{ ksi} \quad h := 12.1426 \text{ in} \quad t := 0.0713 \text{ in} \quad r := 1.5 \cdot t \quad r = 0.107 \text{ in}$$

$$b := 1.25 \text{ in} \quad w := b - 2.5t \quad w = 1.072 \text{ in} \quad E := 29435 \text{ ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	Ly	Ly ²	I'
1	1.072	0.036	0.038	0.001	
2	0.224	0.087	0.020	0.002	
3	11.786	6.071	71.557	434.444	136.384
4	0.224	12.055	2.699	32.536	
5	1.072	12.107	12.976	157.095	
sum	14.377		87.289	624.078	136.384
ycg	6.071				
I _{xe}	16.435				

Flange under uniform compression:

$$\frac{w}{t} = 15.032 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 50.629 \text{ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.807 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.901 \quad b_e := \rho \cdot w \quad b_e = 0.966 \text{in}$$

Web under stress gradient:

$$w := h - 5 \cdot t \quad w = 11.7861 \text{in} \quad \frac{w}{t} = 165.303 \quad y_{cg} := 6.0713 \text{in} \quad w = 11.7861 \text{in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f_1 = 32.031 \text{ksi} \quad f_2 := f_1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f_2 = 32.031 \text{ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 23.366 \text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.171$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.694 \quad b_e := \rho \cdot w \quad b_e = 8.175 \text{in}$$

$$h_o := 10.1426 \text{in} \quad b_o := 1.25 \text{in} \quad \frac{h_o}{b_o} = 8.114$$

$$b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1 \quad b_1 = 2.044 \text{in} \quad b_2 = 2.044 \text{in}$$

$b_1 + b_2 = 4.088 \text{in}$ is less than compression portion of web element = 5.89305in

So the web is not fully effective

Trial1 $y_{cg}=6.0713$

Element	Length	y	Ly	Ly ²	I'
1	0.966	0.036	0.034	0.001	
2	0.224	0.087	0.020	0.002	
3a	2.044	1.200	2.453	2.945	0.500
3b	2.044	5.049	10.321	52.113	0.500
3c	5.893	9.018	53.142	479.230	17.055
4	0.224	12.055	2.699	32.536	
5	1.072	12.107	12.976	157.095	
sum	12.467		81.645	723.921	18.055
y _{cg}	6.549				
l _{xe}	14.778				

Trial 2 $y_{cg} := 6.65\text{in}$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 32.115\text{ksi} \quad f2 := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5t) \quad f2 = 26.372\text{ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.821 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 19.723$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 19.202\text{ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.293$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.642 \quad b_e := \rho \cdot w \quad b_e = 7.563\text{in}$$

$$h_o := 8.1426\text{in} \quad b_o := 1.25\text{in} \quad \frac{h_o}{b_o} = 6.514$$

$$b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1 \quad b1 = 1.979\text{in} \quad b2 = 2.174\text{in}$$

Trial2 $y_{cg}=6.65\text{in}$

Element	Length	y	Ly	Ly ²	I'
1	0.966	0.036	0.031	0.001	
2	0.224	0.087	0.020	0.002	
3a	1.979	1.168	2.311	2.699	0.646
3b	2.174	5.563	12.094	67.279	0.856
3c	5.314	9.307	49.462	460.348	12.507
4	0.224	12.055	2.699	32.536	
5	1.072	12.107	12.976	157.095	
sum	11.953		79.592	719.959	14.009
y _{cg}	6.659				
I _{xe}	14.542				

$Y_{cg} := 6.659\text{in}$ So no further iteration is necessary.

$$I_{xe} := 14.5423\text{in}^4 \quad Y_{cg} = 6.659\text{in} \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad M_n := F_y \cdot S_{xe} \quad M_n = 72.067\text{kip-in}$$

Track section: 254F32 - 18

$$F_y := 33\text{ksi} \quad h := 10.0902\text{in} \quad t := 0.045\text{in} \quad r := 2 \cdot t \quad r = 0.09\text{in}$$

$$b := 1.25\text{in} \quad w := b - 2.5t \quad w = 1.137\text{in} \quad E := 29435\text{ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	LY	Ly ²	I'
1	1.137	0.023	0.026	0.001	
2	0.142	0.055	0.008	0.000	
3	9.865	5.045	49.768	251.087	79.996
4	0.142	10.035	1.421	14.260	
5	1.137	10.068	11.449	115.269	
	12.422		62.672	380.617	79.996
ycg	5.045				
lxe	6.514				

Flange under uniform compression:

$$\frac{w}{t} = 25.216 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 17.991 \text{ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.354 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.618 \quad b_e := \rho \cdot w \quad b_e = 0.703 \text{in}$$

Web under stress gradient:

$$w := h - 5 \cdot t \quad w = 9.8647 \text{in} \quad \frac{w}{t} = 218.729 \quad y_{cg} := 5.045 \text{in} \quad w = 9.8647 \text{in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f_1 = 32.263 \text{ksi} \quad f_2 := f_1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f_2 = 32.263 \text{ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 13.346 \text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.555$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.552 \quad b_e := \rho \cdot w \quad b_e = 5.447 \text{in}$$

$$h_o := 10.0902 \text{in} \quad b_o := 1.25 \text{in} \quad \frac{h_o}{b_o} = 8.072$$

$$b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1 \quad b_1 = 1.362 \text{in} \quad b_2 = 1.362 \text{in}$$

$b_1 + b_2 = 2.723 \text{in}$ is less than compression portion of web element = 4.93235in

So the web is not fully effective

Element	Length	y	LY	Ly ²	I'
1	0.703	0.028	0.020	0.001	
2	0.142	0.055	0.008	0.000	
3a	1.362	0.794	1.081	0.858	0.211
3b	1.362	5.726	7.799	44.658	0.211
3c	4.932	7.511	37.048	278.279	10.000
4	0.142	10.035	1.421	14.260	
5	1.137	10.068	11.449	115.269	
	9.780		58.827	453.326	10.421
ycg	6.015				
lxe	4.956				

Trial 2 $ycg := 6.015n$

$$f1 := \frac{Fy}{ycg} \cdot (ycg - 2.5t) \quad f1 = 32.381ksi \quad f2 := \frac{Fy}{ycg} \cdot (h - ycg - 2.5t) \quad f2 = 21.739ksi$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.671 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 16.68$$

$$Fcr := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad Fcr = 9.275ksi \quad f := f1 \quad \lambda := \sqrt{\frac{f}{Fcr}} \quad \lambda = 1.868$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.472 \quad be := \rho \cdot w \quad be = 4.658in$$

$$ho := 10.0902in \quad bo := 1.25in \quad \frac{ho}{bo} = 8.072$$

$$b1 := \frac{be}{3 + \psi} \quad b2 := \frac{be}{1 + \psi} - b1 \quad b1 = 1.269in \quad b2 = 1.518in$$

Trial2 Assume $y_{cg}=6.015$

Element	Length	y	LY	Ly ²	I'
1	0.703	0.028	0.020	0.001	
2	0.142	0.055	0.008	0.000	
3a	1.269	0.747	0.948	0.709	0.170
3b	1.518	5.256	7.979	41.936	0.291
3c	3.962	7.996	31.685	253.358	5.185
4	0.142	10.035	1.421	14.260	
5	1.137	10.068	11.449	115.269	
	8.873		53.510	425.532	5.646
y _{cg}	6.031				
I _{xe}	4.892				

Trial 3 $y_{cg} := 6.033\text{in}$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5\text{t}) \quad f1 = 32.383\text{ksi} \quad f2 := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5\text{t}) \quad f2 = 21.576\text{ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.666 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 16.585$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 9.222\text{ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.874$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.471 \quad b_e := \rho \cdot w \quad b_e = 4.646\text{in}$$

$$\frac{h_o}{b_o} = 8.072 \quad b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1 \quad b1 = 1.267\text{in} \quad b2 = 1.521\text{in}$$

Trial 3 assume $y_{cg}=6.033$

Element	Length	y	LY	Ly ²	I'
1	0.703	0.028	0.020	0.001	
2	0.142	0.055	0.008	0.000	
3a	1.267	0.746	0.945	0.706	0.170
3b	1.521	5.271	8.016	42.251	0.293
3c	3.946	8.004	31.588	252.840	5.122
4	0.142	10.035	1.421	14.260	
5	1.137	10.068	11.449	115.269	
	8.858		53.448	425.326	5.585
y _{cg}	6.033				
I _{xe}	4.893				

$Y_{cg} := 6.033\text{in}$ So no further iteration is necessary.

$$I_{xe} := 4.8934\text{in}^4 \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad Y_{cg} = 6.033\text{in} \quad M_n := F_y \cdot S_{xe} \quad M_n = 26.766\text{kip-in}$$

Track section: 254T32 - 16

$$F_y := 33\text{ksi} \quad h := 10.1134\text{n} \quad t := 0.0567\text{n} \quad r := 1.5 \cdot t \quad r = 0.085\text{in}$$

$$b := 1.25\text{in} \quad w := b - 2.5t \quad w = 1.108\text{in} \quad E := 29435\text{ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	Ly	Ly ²	I'
1	1.108	0.028	0.031	0.001	
2	0.178	0.070	0.012	0.001	
3	9.830	5.057	49.707	251.353	79.153
4	0.178	10.044	1.788	17.960	
5	1.108	10.085	11.177	112.718	
sum	12.402		62.716	382.033	79.153
ycg	5.057				
lxe	8.168				

Flange under uniform compression:

$$\frac{w}{t} = 19.546 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 29.943\text{ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.05 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.753 \quad b_e := \rho \cdot w \quad b_e = 0.834\text{in}$$

Web under stress gradient:

$$w_w := h - 5 \cdot t \quad w = 9.8299\text{in} \quad \frac{w}{t} = 173.367 \quad y_{cg} := 5.0567\text{n} \quad w = 9.8299\text{in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f_1 = 32.075\text{ksi} \quad f_2 := f_1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f_2 = 32.075\text{ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 1 \quad k_w := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr_w} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 21.243\text{ksi} \quad f_w := f_1 \quad \lambda_w := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.229$$

$$\rho_w := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.668 \quad b_{e_w} := \rho \cdot w \quad b_e = 6.567\text{in}$$

$$h_o := 10.1134\text{n} \quad b_o := 1.25\text{in} \quad \frac{h_o}{b_o} = 8.091$$

$$b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1 \quad b_1 = 1.642\text{in} \quad b_2 = 1.642\text{in}$$

$$b_1 + b_2 = 3.284\text{in} \quad \text{is less than compression portion of web element} = 6.4085\text{in}$$

So the web is not fully effective

Trial1

Element	Length	y	Ly	Ly ²	I'
1	0.834	0.028	0.024	0.001	
2	0.178	0.070	0.012	0.001	
3a	1.642	0.963	1.581	1.522	0.257
3b	1.642	4.236	6.955	29.459	0.257
3c	4.915	7.514	36.932	277.512	9.894
4	0.178	10.044	1.788	17.960	
5	1.108	10.085	11.177	112.718	
sum	10.497		58.469	439.173	10.407
ycg	5.570				
lxe	7.026				

Trial 2 $ycg := 5.68 \text{ in}$

$$f1 := \frac{Fy}{ycg} \cdot (ycg - 2.5t) \quad f1 = 32.176 \text{ ksi} \quad f2 := \frac{Fy}{ycg} \cdot (h - ycg - 2.5t) \quad f2 = 24.934 \text{ ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.775 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 18.733$$

$$Fcr := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t} \right)^2} \quad Fcr = 16.58 \text{ ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{Fcr}} \quad \lambda = 1.393$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.604 \quad be := \rho \cdot w \quad be = 5.942 \text{ in}$$

$$ho := 12.113 \text{ in} \quad bo := 1.25 \text{ in} \quad \frac{ho}{bo} = 9.691$$

$$b1 := \frac{be}{3 + \psi} \quad b2 := \frac{be}{1 + \psi} - b1 \quad b1 = 1.574 \text{ in} \quad b2 = 1.774 \text{ in}$$

Trial2 Assume $ycg = 5.68$

Element	Length	y	Ly	Ly ²	I'
1	0.834	0.028	0.024	0.001	
2	0.178	0.070	0.012	0.001	
3a	1.574	0.929	1.462	1.358	0.325
3b	1.774	4.793	8.503	40.754	0.465
3c	4.292	7.826	33.586	262.836	6.587
4	0.178	10.044	1.788	17.960	
5	1.108	10.085	11.177	112.718	
sum	9.938		56.551	435.627	7.377
ycg	5.690				
lxe	6.872				

Trial 3 $y_{cg} := 5.6904\text{in}$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 32.178\text{ksi} \quad f2 := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5t) \quad f2 = 24.828\text{ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.772 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 18.663$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 16.52\text{ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.396$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.604 \quad b_e := \rho \cdot w \quad b_e = 5.933\text{in} \quad \frac{h_o}{b_o} = 9.691$$

$$b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1 \quad b1 = 1.573\text{in} \quad b2 = 1.776\text{in}$$

Trial 3 assume $y_{cg}=5.6904$

Element	Length	y	Ly	Ly ²	I'
1	0.834	0.028	0.024	0.001	
2	0.178	0.070	0.012	0.001	
3a	1.573	0.928	1.460	1.355	0.324
3b	1.776	4.802	8.528	40.953	0.467
3c	4.281	7.831	33.526	262.546	6.539
4	0.178	10.044	1.788	17.960	
5	1.108	10.085	11.177	112.718	
sum	9.929		56.516	435.534	7.330
y _{cg}	5.691				
I _{xe}	6.873				

$Y_{cg} := 5.69\text{in}$ So no further iteration is necessary.

$$I_{xe} := 6.873\text{in}^4 \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad Y_{cg} = 5.691\text{in} \quad M_n := F_y \cdot S_{xe} \quad M_n = 39.855\text{kip-in}$$

Track section: 254T32 - 14

$$F_y := 33\text{ksi} \quad h := 10.1426\text{in} \quad t := 0.0713\text{in} \quad r := 2 \cdot t \quad r = 0.143\text{in}$$

$$b := 1.25\text{in} \quad w := b - 2.5t \quad w = 1.072\text{in} \quad E := 29435\text{ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	Ly	Ly ²	I'
1	1.072	0.036	0.038	0.001	
2	0.224	0.087	0.020	0.002	
3	9.786	5.071	49.628	251.680	78.099
4	0.224	10.055	2.251	22.636	
5	1.072	10.107	10.832	109.480	
sum	12.377		62.769	383.799	78.099
ycg	5.071				
lxe	10.237				

Flange under uniform compression:

$$\frac{w}{t} = 15.032 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 50.629 \text{ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.807 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.901 \quad b_e := \rho \cdot w \quad b_e = 0.966 \text{in}$$

Web under stress gradient:

$$w := h - 5 \cdot t \quad w = 9.786 \text{in} \quad \frac{w}{t} = 137.252 \quad y_{cg} := 5.071 \text{in} \quad w = 9.786 \text{in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f_1 = 31.84 \text{ksi} \quad f_2 := f_1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f_2 = 31.84 \text{ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 33.893 \text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.969$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.798 \quad b_e := \rho \cdot w \quad b_e = 7.805 \text{in}$$

$$h_o := 10.1426 \text{in} \quad b_o := 1.25 \text{in} \quad \frac{h_o}{b_o} = 8.114$$

$$b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1 \quad b_1 = 1.951 \text{in} \quad b_2 = 1.951 \text{in}$$

$$b_1 + b_2 = 3.902 \text{in} \quad \text{is less than compression portion of web element} = 4.87845 \text{in}$$

So the web is not fully effective

Trial1 $y_{cg}=5.0713$

Element	Length	y	Ly	Ly ²	I'
1	0.966	0.036	0.034	0.001	
2	0.224	0.087	0.020	0.002	
3a	1.951	1.154	2.251	2.597	0.446
3b	1.951	4.096	7.991	32.729	0.446
3c	4.893	7.518	36.785	276.544	9.762
4	0.224	10.055	2.251	22.636	
5	1.072	10.107	10.832	109.480	
sum	11.281		60.164	443.989	10.654
y _{cg}	5.333				
l _{xe}	9.537				

Trial 2 $y_{cg} := 5.385\text{in}$

$$f_{1w} := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f_1 = 31.908\text{ksi} \quad f_{2w} := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5t) \quad f_2 = 28.063\text{ksi}$$

$$\psi_w := \left| \frac{f_2}{f_1} \right| \quad \psi = 0.88 \quad k_w := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 21.038$$

$$F_{crw} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 29.71\text{ksi} \quad f_w := f_1 \quad \lambda_w := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.036$$

$$\rho_w := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.76 \quad b_{ew} := \rho \cdot w \quad b_e = 7.438\text{in}$$

$$h_{ow} := 8.1426\text{in} \quad b_{ow} := 1.25\text{in} \quad \frac{h_o}{b_o} = 6.514$$

$$b_{1w} := \frac{b_e}{3 + \psi} \quad b_{2w} := \frac{b_e}{1 + \psi} - b_1 \quad b_1 = 1.917\text{in} \quad b_2 = 2.04\text{in}$$

Trial2 $y_{cg}=5.385\text{in}$

Element	Length	y	Ly	Ly ²	I'
1	0.966	0.036	0.031	0.001	
2	0.224	0.087	0.020	0.002	
3a	1.917	1.137	2.179	2.477	0.587
3b	2.040	4.365	8.905	38.869	0.707
3c	4.579	7.675	35.145	269.727	8.003
4	0.224	10.055	2.251	22.636	
5	1.072	10.107	10.832	109.480	
sum	11.022		59.363	443.191	9.298
y _{cg}	5.385				
l _{xe}	9.469				

$Y_{cg} := 5.385\text{in}$ So no further iteration is necessary.

$$I_{xe} := 9.469\text{in}^4 \quad Y_{cg} = 5.385\text{in} \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad M_n := F_y \cdot S_{xe} \quad M_n = 58.026\text{kip-in}$$

Track section: 203T32 – 18

$$F_y := 33\text{ksi} \quad h := 8.0902\text{in} \quad t := 0.045\text{in} \quad r := 1.5 \cdot t \quad r = 0.068\text{in}$$

$$b := 1.25\text{in} \quad w := b - 2.5t \quad w = 1.137\text{in} \quad E := 29435\text{ksi} \quad \mu := 0.3$$

Gross section:

Element	Length	y	Ly	Ly ²	I'
1	1.137	0.023	0.026	0.001	
2	0.142	0.055	0.008	0.000	
3	7.865	4.045	31.813	128.689	40.538
4	0.142	8.035	1.138	9.143	
5	1.137	8.068	9.175	74.020	
sum	10.422		42.160	211.853	40.538
ycg	4.045				
lxe	3.691				

Flange under uniform compression:

$$\frac{w}{t} = 25.216 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 17.991\text{ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.354 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.618 \quad b_e := \rho \cdot w \quad b_e = 0.703\text{in}$$

Web under stress gradient:

$$w := h - 5 \cdot t \quad w = 7.8647\text{in} \quad \frac{w}{t} = 174.384 \quad y_{cg} := 4.045\text{in} \quad w = 7.8647\text{in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f_1 = 32.08\text{ksi} \quad f_2 := f_1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f_2 = 32.08\text{ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 20.996\text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.236$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.665 \quad b_e := \rho \cdot w \quad b_e = 5.23\text{in}$$

$$h_o := 8.0902\text{in} \quad b_o := 1.25\text{in} \quad \frac{h_o}{b_o} = 6.472$$

$$b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1 \quad b_1 = 1.308\text{in} \quad b_2 = 1.308\text{in}$$

$b_1 + b_2 = 2.615\text{in}$ is less than compression portion of web element=3.93235in

So the web is not fully effective

Trial1 assme ycg=4.0451

Element	Length	y	Ly	Ly ²	I'
1	0.703	0.028	0.020	0.001	
2	0.142	0.055	0.008	0.000	
3a	1.308	0.832	1.089	0.906	0.144
3b	1.308	3.391	4.436	15.041	0.136
3c	3.932	6.011	23.638	142.097	10.000
4	0.142	8.035	1.138	9.143	
5	1.137	8.068	9.175	74.020	
sum	8.672		39.503	241.208	10.280
ycg	4.555				
Ixe	3.226				

Trial 2 $\tilde{y}_{cg} := 4.66\text{in}$

$$\tilde{f}_1 := \frac{F_y}{\tilde{y}_{cg}} \cdot (y_{cg} - 2.5\text{t}) \quad f_1 = 32.202\text{ksi} \quad \tilde{f}_2 := \frac{F_y}{\tilde{y}_{cg}} \cdot (h - y_{cg} - 2.5\text{t}) \quad f_2 = 23.493\text{ksi}$$

$$\overset{\sim}{\psi} := \left| \frac{f_2}{f_1} \right| \quad \psi = 0.73 \quad \tilde{k} := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 17.806$$

$$\tilde{F}_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 15.578\text{ksi} \quad \tilde{f} := f_1 \quad \overset{\sim}{\lambda} := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.438$$

$$\overset{\sim}{\rho} := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.589 \quad \tilde{b}_e := \rho \cdot w \quad b_e = 4.633\text{in}$$

$$\tilde{h}_o := 8.0902\text{in} \quad \tilde{b}_o := 1.25\text{in} \quad \frac{h_o}{b_o} = 6.472$$

$$\tilde{b}_1 := \frac{b_e}{3 + \psi} \quad \tilde{b}_2 := \frac{b_e}{1 + \psi} - b_1 \quad b_1 = 1.242\text{in} \quad b_2 = 1.437\text{in}$$

Trial2 Assume ycg=4.66in

Element	Length	y	Ly	Ly ²	I'
1	0.703	0.028	0.020	0.001	
2	0.142	0.055	0.008	0.000	
3a	1.242	0.734	0.911	0.669	0.160
3b	1.437	3.942	5.664	22.324	0.247
3c	3.317	6.319	20.962	132.453	3.043
4	0.142	8.035	1.138	9.143	
5	1.137	8.068	9.175	74.020	
sum	8.120		37.878	238.610	3.449
ycg	4.664				
Ixe	2.951				

$\tilde{Y}_{cg} := 4.664\text{in}$ So no further iteration is necessary.

$$\tilde{I}_{xe} := 2.9506\text{in}^4 \quad \tilde{S}_{xe} := \frac{\tilde{I}_{xe}}{\tilde{Y}_{cg}} \quad \tilde{Y}_{cg} = 4.664\text{in} \quad \tilde{M}_n := F_y \cdot \tilde{S}_{xe} \quad \tilde{M}_n = 20.877\text{kip}\cdot\text{in}$$

Track section: 203T32 – 16

$$F_y := 33\text{ksi} \quad h := 8.1132\text{in} \quad t := 0.0566\text{in} \quad r := 1.5 \cdot t \quad r = 0.085\text{in}$$

$$b := 1.25\text{in} \quad w := b - 2.5t \quad w = 1.109\text{in} \quad E := 29435\text{ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	Ly	Ly ²	I'
1	1.109	0.028	0.031	0.001	
2	0.178	0.069	0.012	0.001	
3	7.830	4.057	31.764	128.854	40.007
4	0.178	8.044	1.430	11.499	
5	1.109	8.085	8.966	72.490	
	10.404		42.203	212.845	40.007
ycg	4.057				
lxe	4.621				

Flange under uniform compression:

$$\frac{w}{t} = 19.585 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 29.824\text{ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.052 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.752 \quad b_e := \rho \cdot w \quad b_e = 0.833\text{in}$$

Web under stress gradient:

$$w_w := h - 5 \cdot t \quad w = 7.8302\text{in} \quad \frac{w}{t} = 138.343 \quad y_{cg} := 4.0566\text{in} \quad w = 7.8302\text{in}$$

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f1 = 31.849\text{ksi} \quad f2 := f1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f2 = 31.849\text{ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 1 \quad k_w := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 33.361\text{ksi} \quad f_w := f1 \quad \lambda_w := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.977$$

$$\rho_w := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.793 \quad b_{e,w} := \rho \cdot w \quad b_e = 6.209\text{in}$$

$$h_o := 8.1132\text{in} \quad b_o := 1.25\text{in} \quad \frac{h_o}{b_o} = 6.491$$

$$b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1$$

$$b1 = 1.552\text{in} \quad b2 = 1.552\text{in}$$

$b_1 + b_2 = 3.105\text{in}$ is less than compression portion of web element = 3.9153in

So the web is not fully effective

Trial1

Element	Length	y	Ly	Ly ²	I'
1	0.833	0.028	0.024	0.001	
2	0.178	0.069	0.012	0.001	
3a	1.552	0.837	1.299	1.087	0.312
3b	1.552	3.361	5.216	17.533	0.312
3c	3.915	6.014	23.546	141.609	3.487
4	0.178	8.044	1.430	11.499	
5	1.109	8.085	8.966	72.490	
	9.317		40.493	244.221	4.110
ycg	4.346				
Ixe	4.094				

Trial 2

$$y_{cg} := 4.40\text{in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f_1 = 31.939\text{ksi} \quad f_2 := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5t) \quad f_2 = 26.788\text{ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 0.839 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 20.111$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 27.954\text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 1.069$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.743 \quad b_e := \rho \cdot w \quad b_e = 5.818\text{in}$$

$$h_o := 8.1132\text{in} \quad b_o := 1.25\text{in} \quad \frac{h_o}{b_o} = 6.491$$

$$b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1 \quad b_1 = 1.516\text{in} \quad b_2 = 1.648\text{in}$$

Trial2 Ycg=4.40in

Element	Length	y	Ly	Ly ²	I'
1	0.83300	0.02830	0.02357	0.00067	
2	0.17772	0.06939	0.01233	0.00086	
3a	1.51600	0.89950	1.36364	1.22660	0.29035
3b	1.64800	3.57600	5.89325	21.07425	0.37298
3c	3.57170	6.18585	22.09400	136.67017	3.79703
4	0.17772	8.04380	1.42958	11.49923	
5	1.10900	8.08490	8.96615	72.49046	
	9.03315		39.78253	242.96223	4.46036
ycg	4.40400				
Ixe	4.088				

Ycg := 4.404in So no further iteration is necessary.

$$I_{xe} := 4.0885\text{in}^4 \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad Y_{cg} = 4.404\text{in} \quad M_n := F_y \cdot S_{xe} \quad M_n = 30.64\text{kip}\cdot\text{in}$$

Track section: 203T32 – 14

$$F_y := 33\text{ksi} \quad h := 8.1426\text{in} \quad t := 0.0713\text{in} \quad r := 1.5 \cdot t \quad r = 0.107\text{in}$$

$$b := 1.25\text{in} \quad w := b - 2.5t \quad w = 1.072\text{in} \quad E := 29435\text{ksi} \quad \mu := 0.3$$

Gross section

Element	Length	y	Ly	Ly ²	I'
1	1.07	0.04	0.04	0.00	
2	0.22	0.09	0.02	0.00	
3	7.79	4.07	31.70	129.06	39.33
4	0.22	8.06	1.80	14.53	
5	1.07	8.11	8.69	70.44	
sum	10.38		42.25	214.03	39.33
ycg	4.07				
lxe	5.80				

Flange under uniform compression:

$$\frac{w}{t} = 15.032 \quad k := 0.43 \quad f := F_y \quad F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 50.629\text{ksi}$$

$$\lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.807 \quad \rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.901 \quad b_e := \rho \cdot w \quad b_e = 0.966\text{in}$$

Web under stress gradient:

$$\frac{w}{t} := h - 5 \cdot t \quad w = 7.7861\text{in} \quad \frac{w}{t} = 109.202 \quad y_{cg} := 4.0713\text{in} \quad w = 7.7861\text{in}$$

$$f_1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5 \cdot t) \quad f_1 = 31.555\text{ksi} \quad f_2 := f_1 \cdot \frac{h - y_{cg} - 2.5 \cdot t}{y_{cg} - 2.5 \cdot t} \quad f_2 = 31.555\text{ksi}$$

$$\psi := \left| \frac{f_2}{f_1} \right| \quad \psi = 1 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 24$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 53.542\text{ksi} \quad f := f_1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.768$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.929 \quad b_e := \rho \cdot w \quad b_e = 7.236\text{in}$$

$$h_o := 8.1426\text{in} \quad b_o := 1.25\text{in} \quad \frac{h_o}{b_o} = 6.514$$

$$b_1 := \frac{b_e}{3 + \psi} \quad b_2 := \frac{b_e}{1 + \psi} - b_1 \quad b_1 = 1.809\text{in} \quad b_2 = 1.809\text{in}$$

$b_1 + b_2 = 3.618\text{in}$ is less than compression portion of web element=3.89305in

So the web is not fully effective

Trial1 $y_{cg}=4.0713$

Element	Length	y	Ly	Ly ²	I'
1	0.97	0.04	0.03	0.00	
2	0.22	0.09	0.02	0.00	
3a	1.81	1.08	1.96	2.12	0.49
3b	1.81	3.17	5.73	18.14	0.49
3c	3.89	6.02	23.43	140.98	3.45
4	0.22	8.06	1.80	14.53	
5	1.07	8.11	8.69	70.44	
sum	10.00		41.66	246.21	4.44
ycg	4.17				
lxe	5.49				

Trial 2 $y_{cg} := 4.18$ in

$$f1 := \frac{F_y}{y_{cg}} \cdot (y_{cg} - 2.5t) \quad f1 = 31.593 \text{ ksi} \quad f2 := \frac{F_y}{y_{cg}} \cdot (h - y_{cg} - 2.5t) \quad f2 = 29.876 \text{ ksi}$$

$$\psi := \left| \frac{f2}{f1} \right| \quad \psi = 0.946 \quad k := 4 + 2 \cdot (1 + \psi)^3 + 2 \cdot (1 + \psi) \quad k = 22.623$$

$$F_{cr} := \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu^2) \cdot \left(\frac{w}{t}\right)^2} \quad F_{cr} = 50.469 \text{ ksi} \quad f := f1 \quad \lambda := \sqrt{\frac{f}{F_{cr}}} \quad \lambda = 0.791$$

$$\rho := \frac{1 - \frac{0.22}{\lambda}}{\lambda} \quad \rho = 0.912 \quad b_e := \rho \cdot w \quad b_e = 7.105 \text{ in}$$

$$h_o := 8.1426 \text{ in} \quad b_o := 1.25 \text{ in} \quad \frac{h_o}{b_o} = 6.514$$

$$b1 := \frac{b_e}{3 + \psi} \quad b2 := \frac{b_e}{1 + \psi} - b1 \quad b1 = 1.801 \text{ in} \quad b2 = 1.851 \text{ in}$$

Trial2 $Y_{cg}=4.18$ in

Element	Length	y	Ly	Ly ²	I'
1	0.97	0.04	0.03	0.00	
2	0.22	0.09	0.02	0.00	
3a	1.80	1.08	1.94	2.10	0.49
3b	1.85	3.25	6.02	19.61	0.53
3c	3.78	6.07	22.98	139.53	4.52
4	0.22	8.06	1.80	14.53	
5	1.07	8.11	8.69	70.44	
sum	9.92		41.49	246.20	5.53
ycg	4.18				
lxe	5.58				

$Y_{cg} := 4.18$ in So no further iteration is necessary.

$$I_{xe} := 5.5803 \text{ in}^4 \quad Y_{cg} = 4.181 \text{ in} \quad S_{xe} := \frac{I_{xe}}{Y_{cg}} \quad M_n := F_y \cdot S_{xe} \quad M_n = 44.044 \text{ kip-in}$$