OPTIMIZATION OF THE OPERATION OF MULTIRESERVOIR SYSTEMS:

A GREAT LAKES CASE STUDY

by

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Abstract

To solve reservoir operation problems it is necessary to take into account that natural processes such as inflows, groundwater contribution, evaporation, rainfall, runoff, which are the inputs to the system, are uncertain events. The methods here proposed consider the stochasticity of the inputs. The uncertainty of events like evaporation, rainfall, groundwater contribution, runoff, is incorporated in either the net inflows or the net basin supplies.

In order to recognize the intrinsic stochastic features of the natural inputs and take them into consideration explicitly, Stochastic Dynamic Programming is employed to generate long-term operation policies. Because of the well-known *"curse of dimensionality"*, that can affect the optimization of large systems, the technique Multi-Level Approximate Aggregation/Decomposition - Stochastic Dynamic Programming (MAM-SDP) Methodology is employed. The performance of this technique can be enhanced by using the suggested alternate approximation to the conditional distribution of the releases from the reservoirs. So far, MAM-SDP performs physical diagnosis to determine the aggregation scheme. A means of applying Principal Components Analysis is presented, therefore adding a different perspective to solving the problem, i.e., statistical decomposition.

This work also aims at obtaining the relationship performance of the system versus its respective variance. To this effect, an extension of the Expected Return-Variance of Return Rule was developed, applied to a multistage decision type of problem. This technique was called Two-Pass Mean-Variance Approach. The algorithm for doing so is described. It was possible to show a significant range of performances and the variances associated with them for the operation of reservoir systems.

This work ends with the application of the techniques above mentioned to a real case study. In it, an alternate closed-loop type operation policy is presented for the North American Great Lakes. These policies and those from the Two-Pass Mean-Variance Approach are then compared with the ones obtained from a simplified model of the actual operation, based on heuristics. Two sets of synthetic Net Basin Supplies for the five lakes are studied, generating two different sets of release policies.

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Dedicated to

Beatriz,

Cloris,

José,

and Luíza.

Table of Contents

List of Tables

List of Illustrations

Chapter 1

Introduction

1. INTRODUCTION

1.1 Introduction

As the world population grows and its water demand become more diversified, it is necessary to develop more efficient ways of planning, designing, and managing those resources. Therefore, those involved in the Water Resources management developed and adopted many different stances of coping with this increasing demand using systemic approaches and optimization techniques.

Regarding the case of reservoir operations, those presented below are among the most used methods in the existing literature. The following list of Linear and Dynamic Programming Methods as presented in Yeh (1982, 1985), Yakowitz (1982) and Pereira and Pinto (1985) is next.

1. LINEAR PROGRAMMING METHODS (LP Models)

* Deterministic LP
* Chance Constrained LP
* Stochastic LP
    * for Markov Processes
    * with recourse
* Benders Decomposition Algorithm

2. DYNAMIC PROGRAMMING MODELS (DP Models)

* Incremental DP (IDP)
* Discrete Differential DP (DDDP)
* Incremental DP and Successive Approximations (DIPS)
* Reliability-Constrained DP
* Differential DP (DDP)
* Progressive Optimality Algorithm

Each of these methods can be used on its own or in combination with others depending on the characteristics of the reservoir system being studied.

When dealing with water resources systems one must be aware of the amount of uncertainty related to the natural processes such as evaporation, rainfall, and temperature. Possible alternatives that try to deal with the uncertainty are the use of expected values, medians or even critical values in a deterministic model. However, under certain conditions, these procedures may lead to a poor performance of the mathematical model proposed, reducing drastically its ability to produce satisfactory results.

Therefore, it is important to recognize the intrinsic stochastic features of the natural stream flows and attempt to set up a model in which these conditions are taken into consideration explicitly. Nonlinearity is another feature that must be regarded along with the stochastic characteristics of the inflows. These two features resulted in the success of the approach originally formulated by Richard Bellman (1957), namely, Dynamic Programming (DP).

The DP approach was very successful because it decomposes highly complex problems, with a great number of variables, into a series of small subproblems that can be solved recursively. Usually, a problem may be solved by DP in a number of different ways. The criterion used to choose among the different ways is to try to identify the one that is more efficient from the computational viewpoint.

Unlike Linear Programming (LP) methods, with many software packages available in the market, a DP problem frequently has to be solved starting from the formulation of a fundamental algorithm, until it is finally coded in a computer. Thus, additional effort and computational skills are required to use this flexible approach. A Stochastic DP method accounts for the probability distribution of the natural streamflow. The cost of a more realistic model is its increased complexity and dimensionality.

This document focuses mostly on one of the available tools that deal with the tradeoff between computational tractability and the accuracy in modeling. It is called Multi-Level Aggregation/Decomposition Methodology using Stochastic DP (MAM-SDP), Ponnambalam and Adams (1993). The major objective of the present work is to assess the existing relationships between the expected returns versus variance of the expected returns when operating multiple reservoir systems. The relationships between the expected costs and their respective variances are studied as well. Once this relationship has been established and validated by means of experimenting with a Single Reservoir System, it is extended to the Multireservoir Case. Also, an analysis of how this rapport affects the reliability of the system is presented. After the literature review and the extensive testing of the methodology proposed, the method is then applied to the North American Great Lakes System, as a case study.

1.2 Scope and Outline

This work concerns the Optimization of the Operation of Multireservoir Systems. The methodology used was Stochastic DP (SDP) because it has been shown to be the most suitable for obtaining closed-loop type policies, Turgeon, (1981). Also, the optimization policies obtained are tested under Stochastic Simulation of the Operation of the Multireservoir System in order to have an accurate evaluation of its performance and shortcomings.

In SDP, our main objective was not only to get the optimal trajectory of controls, but to have these controls as a function of time and storage as well. In order to overcome problems inherent to the well-known *"curse of dimensionality"*, the Aggregation/Decomposition Methodology, Turgeon (1980, 1981) and Ponnambalam (1987), was employed. All the models set up were developed in Matlab®, which is a high level language that allows very concise programming instructions. One of the major tradeoffs, however, is the long computing time, especially within the "for" loops. On the other hand, Matlab® comes with a large library of functions and routines that, along with the concise programming instructions, reduces the coding effort. This reduction in coding effort is an important issue when considering the extent of the present task. Examples were taken from the literature on the subject to test the models. The main purposes of employing these standard test problems are the evaluation of the model's performance and validation of the methodology. Only after subjecting the models to these analyses it was decided to submit them to the real case application. The test-examples were previously used by Chow and Cortez-Rivera (1974), Murray and Yakowitz (1979), Chara and Pant (1984) and Ponnambalam and Adams (1993). During the test part, the probability distribution and discretization scheme suggested in Turgeon (1981) was used. During the real case study, a more refined approach had to be employed. As a means of validating the results, comparison between present operation policies is used.

1.3 Organization of the Chapters

Chapter 2 reviews the existing literature on the subject. The first section presents in a brief fashion the Reservoir Operation Problem, which introduces the problems involving the operation of reservoir systems. After presenting a simple analogy between Inventory Storage Theory and Storage Reservoir Theory, the Deterministic and Stochastic solution approaches are discussed. The differences amongst open-loop and closed-loop type policies, under the light of Control Theory are briefly stated. The mathematical formulations for solving Single and Multiple Reservoir Systems problems using the DP approach are described, as well as the advantages of water resource system model simulation for more precise analysis of the proposed control policies. (See Ponnambalam and Adams (1988).)

In the second section of Chapter 2, the type of policies that are obtained in an operation optimization framework and explain the reason why the work is mostly concerned with a specific kind of optimization is reviewed. The type of policy that is the most suitable to long-term operation is the closed-loop. This is obtained from a Dynamic Programming optimization (deterministic or stochastic) but it is usually computationally expensive. Important factors to consider when employing SDP in the Optimization of Reservoir Systems are the discretization schemes and the choice of a suitable convergence test. The long-term optimization is part of an overall optimization scheme that includes real-time operation optimization, and generally a mid-term operation optimization that functions as a bridge between the two distinct time horizons.

Because the interest is to optimize large systems employing SDP, the third section reviews the various methodologies that use aggregation to reduce the size of a large problem so that it becomes computationally tractable. This review is not comprehensive and it focuses on the methodologies that provide the necessary theoretical background to the MAM-SDP, which is one of the objectives of our study. It starts with the Composite Representation developed by Arvanitidis and Rosing (1970a, 1970b) and the Decomposition Method for Long-Term Scheduling for Reservoir in Series by Turgeon (1981). As MAM-SDP, developed by Ponnambalam and Adams (1993) is the chosen methodology, the review ends with it. Chapter 2 closes with a brief introduction to the Stochastic Extension of the Benders Algorithm. This algorithm provides a feasible implementation for the optimization of the operation of large reservoir systems. This is because it decomposes the global optimization problem into smaller successive one-stage subproblems. Again, the analogy with DP is evident. A common feature to MAM-SDP and the Stochastic Extension of the Benders Decomposition is the possibility to speed up the computations by means of simultaneous execution. As both techniques are used for the optimization of the operation of reservoir systems, the former providing state derived policies and the latter open-loop type policies, they would provide complementary and useful information on the long-term operating policies. Because these calculations can be solved in a parallel processing environment that will allow a significant reduction in computing time.

Chapter 3 presents the proposed method and samples results from the tests conducted using the presented research. It is our purpose to improve the suboptimal policies obtained by the original MAM-SDP and a better approximation to the Conditional Distribution of Releases. Principal Components Analysis has been employed as an additional means to define the aggregation scheme for solving multireservoir operation problems within the MAM-SDP framework. Different possibilities of aggregation are studied and summaries of the results with standard test problems shown. Further, the Expected Return-Variance of Return Rule is employed for multistage decision type of problem and an algorithm for doing so is described. It was possible to present a significant range of performances and the variances associated with them for the operation of reservoir system. It is expected that the methodology shown in this work could be applied successfully to different types of optimization problems involving Stochastic Dynamic Programming.

Chapter 4 presents an alternate closed-loop operation policy for the North American Great Lakes employing the methodologies described in Chapters 2 and 3, the latter developed in this work. Two synthetic sets of Net Basin Supplies for the five lakes are studied and two different sets of policies are suggested. The results are then compared with a simplified version of the actual operation by means of Stochastic Simulation. Once the optimization model is thus validated, and the superiority of state derived long-term operation policies acknowledged, the Two-Pass Mean-Variance Approach is then applied. This approach is used to generate a set of combinations of the optimization of the objective function, i.e., minimization of the annual accumulated distance from the target levels, and their respective variances (in the present case, shown as standard deviation). The implicit tradeoff in reducing the variances of the expected cost is their increase.

Finally, the conclusions and suggestions for extending the present research are in Chapter 5. Most of the extensive data and part of the results obtained for the Case Study are listed in Appendices.

Chapter 2

Literature Review

2. LITERATURE REVIEW

2.1 Reservoir Operation Problem

2.1.1 Introduction

There are several reasons for building a dam and consequently creating reservoir storage. Reservoirs are regulatory devices that act like a "buffer" to an input which is ruled by randomness.

Some major reasons for constructing a reservoir can be listed as:

- creation of water head for hydroelectric power generation;
- regularization of the releases for either hydroelectric power, irrigation purposes, water supply, navigation or improvement of water quality;
- attenuate the effects of big peaks of rainfall that might lead to flooding of downstream areas;
- creation of a place for water based recreation;
- creation of lakes, pools for maintenance of fish and/or wild life in specific areas; and
- a combination of any of the items above in any order or number.

In his classic work, Moran (1959) defines the reservoir problem as a "sui generis" type of Inventory Storage problem because the output (demand, in the Inventory problem) is not considered as random, but rather the input, which is represented by the inflows. As noted by that author, another way of considering the problem is by means of the Theory of Queues. An analogy can be drawn with the reservoir problem regarding the members of the queue that are lining up for service as the inputs and its length as the storage. The release policy can be translated as the service rule .

Both Inventory Storage Theory and Storage Reservoir Theory can be associated with one another by a dual relationship, the former having random output and the latter random input. This dual relationship can be briefly stated as follows. Consider, for the Reservoir Storage case, the very simple annual model, with the variables as explained below.

S_{max} – capacity of dam (maximum storage admissible);
I^t – random input to reservoir during time interval t;
S^{t+1} – storage at the end of time t (i.e, after the release);
R^t – release during time period t,
where $R = min(Q, I^t + S^t)$, Q being a definite quantity.

The variables above can be associated with the following for the Inventory Storage Problem:

S_{max} - store capacity (finite);
I^t - random demand in the time interval t;
R^t - quantity being stored in the time interval t;

Namely, the decision rules for R are given by

1. Release $Q < S_{max}$ if there is at least the amount Q stored in the dam;
2. Release $I^t + S^t$ if $Q > I^t + S^t$

Also, let us define deficit (here signifying storage space available in the dam) as

$$Def^t = S_{max} - S^t; \qquad \text{2.1. 1}$$

which is a random value and it can be reinterpreted as the stock in a store or warehouse for the Inventory Storage case. During each time interval t, there is a demand I^t associated with it. Therefore,

1) if $S_{max} - S^t \geq I^t$, **2.1. 2**

it is possible to satisfy the demand and the stock is given by $S_{max} - I^t - S^t$. From the reservoir operation point of view, this means that it is possible to store the coming inflow and, in the Inventory problem, the space still available in the warehouse would be equal to that amount.

2) if $S_{max} - S^t < I^t$, **2.1. 3**

it is not possible to satisfy the demand and the stock is equal to zero. This event has correspondence with overflow or spill. At the end of each period, the amount of stock renewal can be given as min $(Q, S_{max} - I^t - S^t)$. See the release rule in the previous page. For the cases where it is not possible to satisfy the demand, it should remain under this condition. In the storage case, this is equivalent to spill.

This simple example illustrates the dual relationship between the dam storage problem and the Inventory problem and was first presented by Moran (1959).

Although intuitive and trivial nowadays, it has not always been obvious that if the design supply was of greater magnitude than the average long-term inflow to the reservoir, the problem had no feasible solution. This is because the initial question posed by researchers and engineers alike in the area of water resources management was "*how big must a reservoir be if it is fed by a natural stream, in order to be able to provide a steady supply of water of a prescribed magnitude?*" A relatively more recent preoccupation was that even for an average draught smaller than the average inflow, the reservoir was not always able to continuously provide the supply. The difference between both questions is that while the first poses an unconstrained problem, the second one addresses the issue of reliability.

Klemeš (1987), in his excellent extended review of Applied Storage Reservoir Theory, points out that since the beginnings of the Theory of Storage Reservoir, two fashions of approaching the reservoir problems were clearly defined:

- deterministic approach
- stochastic approach

2.1.2 Deterministic Approach

According to Klemeš (1987), the first rigorous attempt to systematize the Reservoir Storage Theory was established by Rippl in 1883. The major contribution in Rippl's work was to recognize the cumulative effect of two or more consecutive dry years, instead of just taking into account the worst case, or the driest year in record. Besides that, he also set up a solution method that computes the minimum storage requirements for a given specified target draught. Klemeš (1979), describes it as a "backward moving, forward looking recursive sequential maximization". With the necessary adjustments, this method is still in use, and available in software packages as the "sequent peak algorithm".

Although Klemeš (1987) reckons those achievements as Rippl's most important contributions, he is still better known in the present as the creator of the "mass-curve technique" and not as the person who figured out that the best way of representing "the series of reservoir inflows as residual mass curve computed with respect to the desired reservoir draft or target release." An important factor for the fast acceptance of the mass-curve technique by the technical world of that period was the ease of graphical implementation of the method. However, this is not crucial to the method today because computers allow us to perform the calculations in a numerical fashion very efficiently.

Rippl's method has at least two disadvantages which can be summarized as follows:

1. Historical records were used in the methodology but, of course, the future inflows are unknown. This means that just one realization of the stochastic process is used, and important statistical information regarding the process not explicitly considered. As Butcher and Fordham (1970) put it, this is a problem inherent to all deterministic methodologies.

2. The starting point for the reservoir working cycle to generally does not coincide with the starting point of the recorded past inflows. The consequence of this is that the first

computed cycle is generally incomplete, adding more uncertainty on the storage capacity thus obtained.

To attenuate the effects of item 2 mentioned above, two practices were adopted.

- The European School used what was called "two-cycle" computation, making the ending storage equal to the initial one. In this situation there is a closed water balance.
- On the other hand, the American School used to start the computations with full reservoir, as appears in Hazen's paper (1914).

Thomas and Burden (1963) introduced the "two-cycle" along with the well-known "sequent peak algorithm", which is essentially Rippl's method modified. Klemeš (1987) notes very appropriately that even though they acknowledged the originality of the method to Rippl himself, the "sequent peak algorithm" is commonly understood as their original contribution.

Finally, when studying the optimal storage capacity for the large Aswan Dam, aiming at the complete regulation of the Nile River, Hurst (1951) realized that his computations would not point to an asymptotic value for convergence. The availability of long series of recorded inflows showed that the convergence was not possible. The immediate implication is the acknowledgment that it is virtually impossible to construct a failure proof storage reservoir. Hipel and McLeod (1994) present an appraisal of the research developed concerning the Hurst phenomenon and the Hurts statistics. Hurst (1951), when calculating the storage required to yield an average annual discharge employed what is known as the cumulated range. This "is obtained by computing the cumulative sums of the departure of the annual totals from the mean annual total discharge", Hurst (1951). The required storage is the difference between the maximum and the minimum cumulative totals. This range can then be normalized by the general standard deviation, i.e., rescaled and adjusted by means of a constant coefficient. From Hipel and McLeod (1994), the asymptotic formula for identically independently distributed random variables with finite variance is

$$E(\overline{R}_N^{\,*}) = 1.2533N^{1/2}, \qquad \textbf{2.1.4}$$

where

$\overline{R}_N^*$ - cumulated rescaled adjusted range (maximum adjustment to a level of development),

E - expectation operator.

The exponent of N, the size of the available time series, has a limiting value of ½. However, Hurst, for an annual time series of size 690 obtained a much greater value of 0.73. The difference between the limiting value and the one obtained by is called the Hurst phenomenon.

2.1.3 Stochastic Approach

Before briefly reviewing the development of the stochastic approach in the last hundred years, it is advisable to justify the use of stochastic methods instead of the deterministic ones. This is because the uncertainty definitely makes the problems much more difficult to deal with.

The preferred approach used by engineers when designing is to employ either the average values (mean, median) or the critical ones (the worst cases). This is recommended when the variations around these values are of small magnitudes and can be neglected. However, as explained in Loucks, Stedinger, and Haith (1981), and Wagner (1975) it is erroneous to assume that

$$E[f(x_1,\ldots,x_n)] = f(E[x_1],\ldots,E[x_n]) \qquad \textbf{2.1.5}$$

That is to say, the expected value of a nonlinear function (even if the dynamics is linear the existence of storage bounds would make it nonlinear) does not coincide with the mapping of the expected values of the variates. For more details the reader is referred to Fletcher and Ponnambalam (1998).

Contrary to the deterministic approach, which was slowly and steadily developed since the introduction of Rippl's method, the stochastic approach appeared in a sudden fashion. Hazen's paper (1914) pioneered with the introduction of statistical concepts such as risk of failure year, storage-yield-reliability (S-Y-R) and the idea of stochastic simulation for the streamflow series. For this purpose, he used standardized streamflow values, in a sense that they represented the average values for an entire region. In this way, some general common statistical parameters were preserved, producing results comparatively acceptable. Besides, in order to assess different mean flows, he used the coefficient of variation, by normalizing the standard deviations by their respective means.

Suddler (1927) pushed the envelope created by Hazen, improving the computation of the failure periods and the simulation of streamflows using probabilistic models. Barnes (1954) refined Suddler's work using different types of probability distributions to the streamflow data and improving the sampling methodology. Working directly with the probability distribution of inflows, Kritskiy and Menkel (1935) obtained the stationary value of reservoir reliability employing a model with random annual inflows.

Only five years later, in 1940, Savarenskiy (1995) proposed a "numerical sequential scheme for obtaining stationary distribution of storage for the case of random inflows and a given draft" Kritskiy and Menkel immediately incorporated a graphical version of it into their own model. It is of interest to note that, Savarenskiy, an engineer, was the first to define empty reservoir as a state, when computing the transition probabilities, something that became an issue only later, by mathematicians in the search for rigorous representation of empty and full reservoir states.

Moran (1954), an Australian statistician and probabilist, proposed the following matrix formulation for the discretized stationary distribution of storage. The now classical Moran annual model is given by

$\mathbf{p} = \mathbf{P}\mathbf{p}$, where

$\mathbf{p}$ - vector of stationary distribution of storage, **2.1.6**

$\mathbf{P}$ - transition probability matrix, obtained from the distribution of inputs.

One year later, Moran (1955), extended his annual model to an arbitrary number of seasons, where each season has its own matrix of transition probabilities. In contrast with the first model, where a two-step procedure was used (i.e., in the first step only the inputs were taken into account, whereas in the second, only the release), here the net storage changes were considered.

The model is now

$$\mathbf{p} = \mathbf{P}^{(k)}\mathbf{P}^{(k-1)}\ldots\mathbf{P}^{(2)}\mathbf{P}^{(1)}\mathbf{p} \tag{2.1.7}$$

and as the previous one, solved for

$$\sum_{i}^{n} p_i = 1; \tag{2.1.8}$$

with n standing for the number of states

Making

$$\mathbf{P} = \mathbf{P}^{(k)}\mathbf{P}^{(k-1)}\ldots\mathbf{P}^{(2)}\mathbf{P}^{(1)} \tag{2.1.9}$$

and we obtain equation 2.1.5.

One of the ways Moran (1954) proposed for computing equation 2.1.8, multiplying several times **P** by **p** until the former converges to the steady-state values, where all the rows are composed by vectors of equal values, is still practical in the present time with the help of computers. Interestingly enough, this formulation is the analytical analogue of the graphical method used by Kritskiy and Menkel (1995), also in 1940, when adapting Savarenkiy's formulation. These two seminal papers for the theory of stochastic reservoir were translated to English by Klemeš (1995) and recently published. Moran (1959) also stressed out the differences between discrete models, employed to solve applied problems, object of this work, and the more mathematical approaches that use continuous models.

Following Moran's work, Gould (1961) disaggregated the annual flows into monthly ones and Lloyd (1963) and Kaczmarek (1963) solved the operation problem for serially correlated inflows, all situations referring to single reservoir case.

2.1.4 Types of Policy

There are two ways of obtaining control release policies in a reservoir system, either Single Reservoir System or Multireservoir System. First, an introduction to the control theory model is described followed by an analogy with Markov Decision Processes, which is the main interest of this work. Then the types of policies that are possible to obtain from these systems are defined as well as their possible applications.

2.1.4.1 Model Formulation

To obtain control release policies according to the control theory model it is necessary to present the following definitions. Given state space S, control set U, and disturbance set Ω, which are subsets of $\Re_k$, $\Re_m$ and $\Re_n$ respectively, the state of the system at time $t+1$, s^{t+1}, is related to the state of the system at time t, s^t, to the control used at time t, u^t, and to the disturbance at time t, ω^t, through the "law of motion" which is also known as the system equation. It is assumed that the distribution of Ω is given by a probability function $q(\cdot)$. This disturbance ω^t is the realization of the random variable Ω^t.

For a given state s^t, the decision maker or controller takes a decision u^t, from the admissible set of decisions and, as a result, the system obtains a reward, $g^t(s^t,u^t)$. These decisions are taken according to the decision rules which are functions $d^t: S \rightarrow A_s$ defining the kind of action for given states at epoch t. Also, for each $s \in S$, $d^t(s) \in A_s$. Depending on the definition of the problem, it can incur in a cost. The evolution of the process is determined by the disturbance, and consequent decisions and states. The illustration of this process follows.

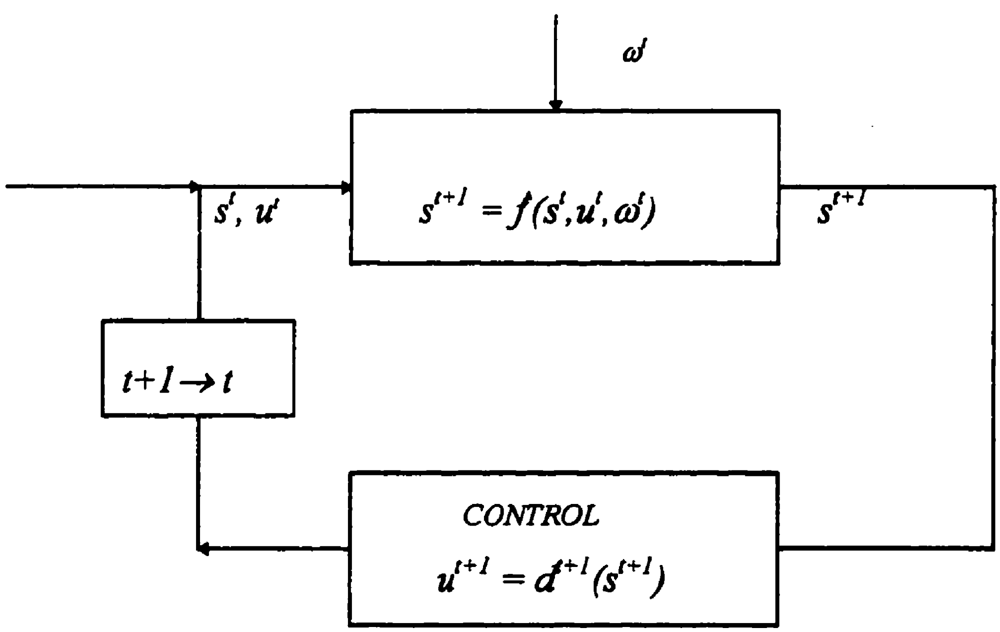


Figure 2.1. 1. Scheme of the Control Decision Process

where f^t is a vector-valued function mapping $S \times U \times \Omega$ into S. Putting the above in a Markov decision process formulation, we get:

Decision epoch:

$$T = \{1, 2, \cdots, N\}, N \leq \infty$$

States:

$$S \subset \Re_k$$

Now, let $A = \cup_{s \in S} A_s$. In other words, the set of allowable actions when the system is in state s is given by A_s. Then, define an action a, $a \in A$. Therefore, for possible actions when the system is in s:

$A_s = U_s$, U_s being the possible decisions (control actions).

Rewards:

$$r^t(s,a) = g^t(s,a),\ t < N,$$
$$r^N(s) = g^N,\ t = N \text{ when } N < \infty$$

Transition probabilities:

$$p^t(j|s,a) = P[\Omega^t \in \{\omega \in \Omega : j = f^t(s,a,\omega)\}] = \sum_{\{\omega \in \Omega : j = f^t(s,a,\omega)\}} q^t(\omega) \qquad \textbf{2.1. 10}$$

The major difference when approaching a problem either as a control problem or as a Markov decision process are the transition probabilities. While they have to be computed in the former, they are decisive for the latter.

2.1.4.2 Open-Loop Policy

Reservoir systems are subject to the periodic behavior of the weather along the year. This behavior results in a cyclic pattern of the inflows. The most simple type of control policy is the one that accounts only for the time period and the given initial condition. This type of policy provides a decision rule in which $d^t = a$, for all $s \in S$ and $a \in A$. It does not take into account the state of system at given time, therefore does not depend on observations to make a decision. Revisions of the policy are not possible once this one is defined. It is easier to implement than others but the policies are usually conservative, thus suboptimal. It is suitable for real-time or short-term decision processes, especially because it enables fast computations, being comparatively less expensive than those mentioned below. Linear and Nonlinear Programming are examples of optimization methods that provide the operator with Open-Loop type policies.

To attenuate the lack of state information in an open-loop type of policy, in repetitive control strategies, some feedback is returned to the system operator who notes the effect of the applied policy and corrects it when needed. The feedback is usually composed of information on the state of the system and prediction of the disturbances. These predictions have to be accurate, what can be accomplished by establishing short time horizons.

2.1.4.3 Adaptive Control

Adaptive Control is also known as closed-loop policy. The optimal decision rules are obtained considering the present state of the system at a given time t. To make a parallel between the open-loop and closed-loop type policies, it suffices to expand the idea of computing the former type for all possible initial conditions. Also, taking into account the state, which is feedback, it adapts the decisions according to the state. If the reservoir is above the expected level, the policy would allow the dispatcher to release more and vice-versa. With the additional decision information, the stabilization of the system for different situations from those initially considered is faster and more efficient.

The computations are much more complex and some kind of compromise is usually required. As a consequence, sub-optimal policies are expected. Although feedback gives robustness to the formulation, a necessary assumption is the complete knowledge of the statistical characteristics of future observations, provided by the probability distribution functions of the disturbances.

2.1.5 Single Reservoir

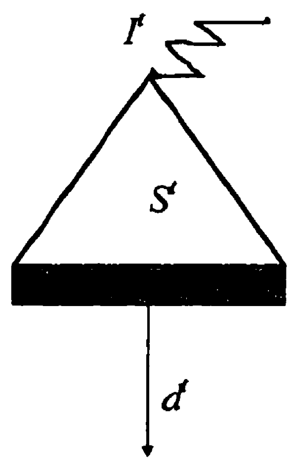


Figure 2.1. 2. System of a Single Reservoir

In a system consisting of a single reservoir, the transfer from one state to the other is represented by the following continuity equation

$$S^t = S^{t-1} + I^t - d^t - e^t \quad \textbf{2.1. 11}$$

subject to the constraints on states or storages:

$$S_{\min} \leq S^t \leq S_{\max} \quad \textbf{2.1. 12}$$

and the constraints on decisions or releases:

$$d_{\min} \leq d^t \leq d_{\max} \quad \textbf{2.1. 13}$$

where

S - storage (state),
I - Inflow (disturbance),
d - Release (decision),
e - all losses, including evapotranspiration, seepage, and others,
t - period.

The accumulated return for a given time interval is given by

$$r(S^{t-1}) = r^t(S^t,d^t) + r^{t+1}(S^{t+1},d^{t+1}) + \dots + r^{T-1}(S^{T-1},d^{T-1}) + r^T(S^T,d^T), \qquad 2.1.14$$

where

S^{t-1} - stands for the state at the beginning of period t,
r^t - return (which can be given either as a reward function or a cost function with respect the decisions or the states or a combination of both),
T - end of the total time period considered.

2.1.6 Multiple Reservoir System

The types of reservoirs that might appear in this work can have the following configurations.

In series:

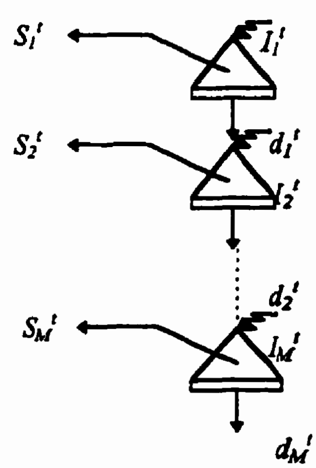


Figure 2.1.3. System of Reservoirs in Series

The transfer equation becomes

$$S_k^{\ t} = S_k^{\ t-1} + I_k^{\ t} + d_{k-1}^{\ t} - d_k^{\ t} - e_k^{\ t} \qquad 2.1.15$$

$d_0^{\ t}$ being a dummy variable with value zero.

In Parallel :

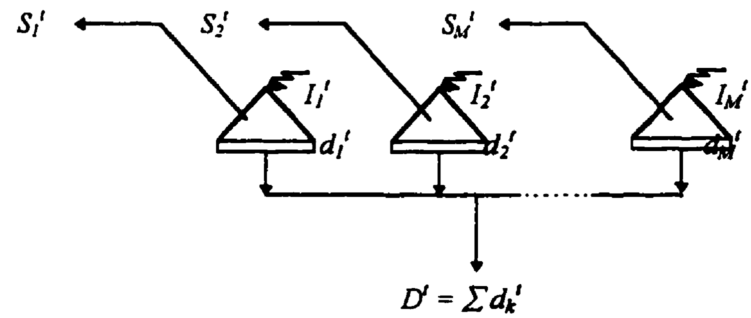


Figure 2.1. 4. System of Reservoirs in Parallel

The transfer equation becomes

$$S_k^{\ t} = S_k^{\ t-1} + I_k^{\ t} - d_k^{\ t} - e_k^{\ t} \qquad 2.1.16$$

A more general type of transfer equation, can be written as follows

$$S_k^{\ t} = S_k^{\ t-1} + \sum_{j=1}^{J} I_{kj}^{\ t} + \sum_{m=1}^{M} d_{k-1_m}^{\ t} - d_k^{\ t} - e_k^{\ t} \qquad 2.1.17$$

where

J - total number of contributing inflows to reservoir;

M - total number of contributing releases to reservoir coming from upstream ones.

Or, of a mixed type (using a configuration that is very common in water resources literature, with small adaptations). This is an arbitrary configuration, it is used here just as an example:

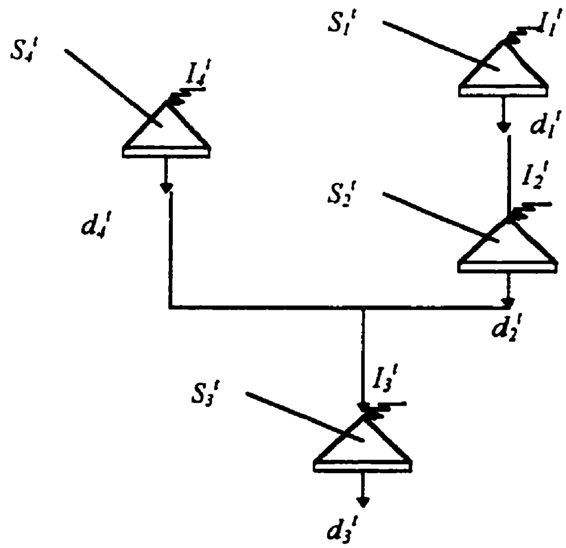


Figure 2.1. 5. System of Reservoirs with Mixed Configuration

And, assuming the rewards are separable by reservoir, the total return for an arbitrary time interval is

$$r(S^{t-1}) = r_1(S_1^{t-1}, d_1^t) + r_2(S_2^{t-1}, d_2^t) + \ldots + r_N(S_M^{t-1}, d_M^t) \quad 2.1.18$$

where

$$r_k(S_k^{t-1}, d_k^t) = r^t(S_k^t, d_k^t) + r^{t+1}(S_k^{t+1}, d_k^{t+1}) + \ldots + r^T(S_k^T, d_k^T) \quad 2.1.19$$

2.1.7 Water Resource Systems Simulation

To evaluate the performance of the reservoir system and its control policies, it is interesting to set up a mathematical model that replicates the desired operation. Using the historical data, is possible to observe how the model would have behaved in the past. However, as the length of the recordings rarely exceeds two hundred years, the historical data is nothing but one of the possible realizations of the stochastic process that generated the disturbances. A more thorough analysis would cover at least some of the all possible processes. Thus, one must obtain the statistical parameters of the process and set up a mathematical model that generates synthetic data with the same stochastic characteristics of the historical one. With long synthetic streamflow sequences one can analyze the response of the system to inputs that might contain longer and more extreme conditions than those recorded in the past, Askew, Yeh, and Hall. (1971b).

There are two reasons for the use of simulation models in Water Resources, namely:

A. To overcome mathematical difficulties in setting up the model, the simplifications assumed are such that the modeler believes it does not correspond to the actual problem.

B. The inherent complexity of the analytical model is so great that makes it mathematically intractable.

The purpose of this work is to analyze the performance of the control policies/decisions under a more demanding environment that tries to emulate, within the limitations imposed by the simplifying assumptions, the operation of the system. Nevertheless, these control policies/decisions were derived from an optimization model as will be described in the next chapters.

The idea of using the Monte Carlo Method, which was termed by von Neumann, in the 1940's, originated from a very simple statistic concept.

Let us suppose the following integral is to be evaluated

$$I = \int_0^1 g(x)dx \qquad 2.1.20$$

Also, let us define the random variable **x**, with uniform distribution in the defined interval, i.e., (0,1). Now, forming the random variable **y** = $g(\mathbf{x})$, the payoff function of the system design.

$$E\{g(\mathbf{x})\} = \int_0^1 g(x)f_x(x)dx = \int_0^1 g(x)dx \qquad 2.1.21$$

It follows that $\eta_y = I$, η_y is the expected value (mean) of y. Where $I = E\{g(\mathbf{x})\}$. E being the expectation operator. A valid manner of evaluating I is repeating the experiment several times, observing the values x_i and computing the respective $g(x_i)$. We can say that

$$I = E\{g(x_i)\} \cong \frac{1}{n}\sum g(x_i); \qquad 2.1.\ 22$$

and n is the number of times the experiment is repeated (frequency interpretation) and the equation above referring to obtaining the estimate of expected value for the random variable y. (See Papoulis, (1991).)

Suppose now that it is possible to reproduce numerically an experiment a large number of times, and $\mathbf{x}$ represents a physical quantity. Also, define x_i as the values that $\mathbf{x}$ assume during this experiment. Then

$$I = E\{g(\mathbf{x})\} \cong \frac{1}{n}\sum g(x_i) \qquad 2.1.23$$

Therefore, after generating synthetic streamflow sequences, it is possible to evaluate and assess the performance of the model without having to recourse to the actual operation. Because a detailed computer program is able to reproduce mathematically most of the complex interactions that occur within the system allowing the assessment of weaknesses and eventual pitfalls. However, as is always necessary to introduce at least some approximations, the modeler must always exercise critical analysis and carefully assess their effects in the numerical results..

The synthetic streamflow generation techniques available allow the researcher to test his model for types of sequences that might not have occurred in the historical records of streamflows but are likely to happen in the foreseeable future. With this regard, it is noteworthy to mention that the recent publication by Hipel and McLeod (1994) covers a broad spectrum of the techniques used to simulate multiperiod and multisite time series. Other interesting contributions are the technical reports from the Water Resources Center, California, by Asfur and Yeh (1971) and the one by Askew and Yeh (1970a). The former, although dated, has a good synthesis of what has been done up to that year while the latter focuses mostly in the simulation of critical periods.

2.2 Stochastic Dynamic Programming

2.2.1 Dynamic Programming

The Dynamic Programming (DP) method, initially formulated by Richard Bellman, is a multistage decision process optimization. According to his wife, Nina Bellman (1989), the author himself said that rather than Dynamic Programming, it is a Theory of Multistage Decision Processes. The reason for the now popular name was the need for funds. As programming has a more practical connotation than theory, it facilitated the access to scarce grants. DP has the advantage of decomposing highly complex problems into a solvable set of subproblems. It has been widely used in water resources systems optimization, especially because it is easy to consider in the DP formulation the intrinsic non-linear and stochastic characteristics of the natural streamflows. Since the method's first appearance in the literature, there were so many possible ways of implementing this formulation, that it has been said that formulating a problem in DP is an art rather than a science. Because of that, the choice of the most convenient formulation is not a trivial task and it usually affects the results obtained.

The variables that appear in this work are of a continuous type, such as reservoir storage levels, inflows, time, and releases. For practical reasons the type of dynamic programming used is the discrete dynamic programming and, the finer the discretization of the continuous variables, the more accurate the results. On the other hand, this procedure will increase the computational burden. Therefore, a compromise between the discretization and the desired accuracy is an important consideration.

Consider the following network. It is required to find an optimal path from node 1 to node 27. There is an associated cost c_{ij} to go from node i to node j. Note that at each node there are five possible states (or nodes) to go in the next stage.

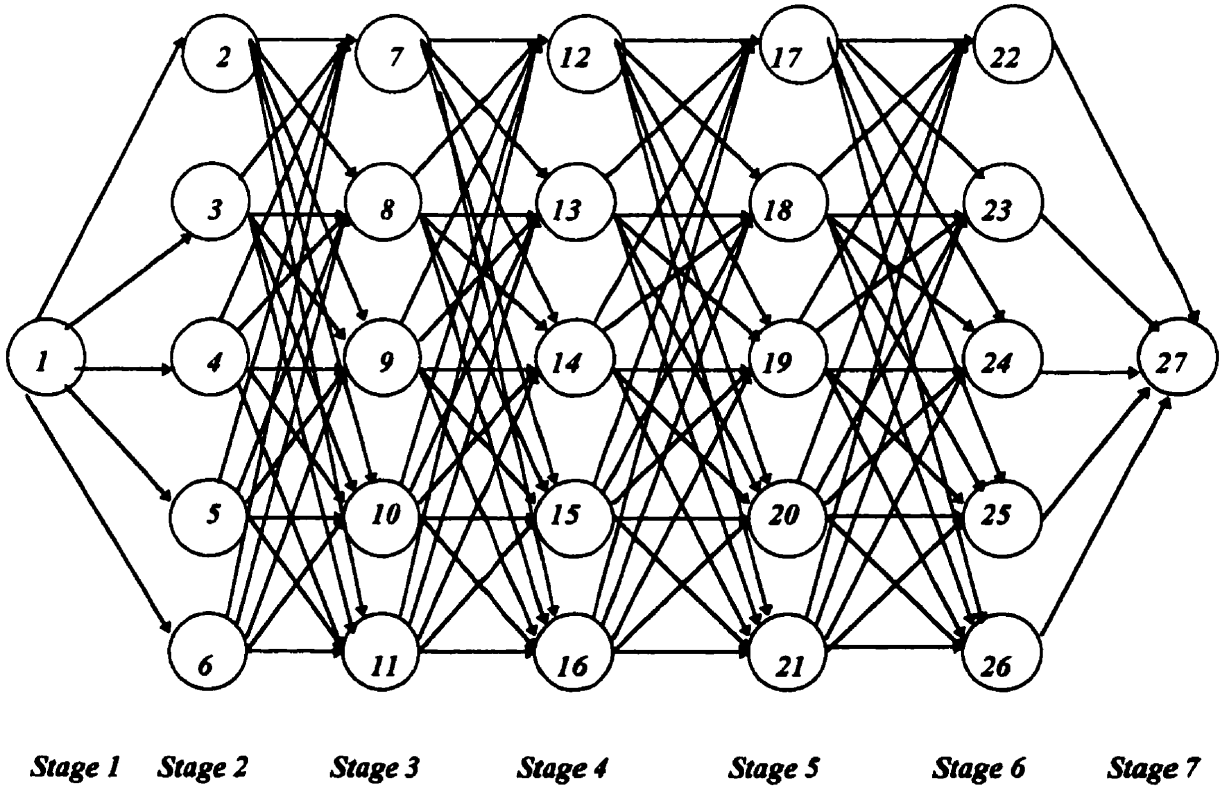


Figure 2.2. 1. Optimal Path Example

If explicit enumeration is used, there are 5^5 (3125) possible paths in the above example and 5 conditions are required for calculating the cost of each path. This would require 5^6 additions (15625) and 5^5-1 comparisons (3124) to determine the optimal path. However, using the DP Methodology, the optimal path can be found using only 110 decision paths. But how?

2.2.2 Properties of Dynamic Programming Applications [adapted from Hillier and Lieberman (1990)]

1. The problem can be divided into stages with a policy decision required at each stage.
   - DP problems require making a sequence of interrelated decisions, and each decision is taken at a specific stage.
   - a natural stage in most dynamic programming problems is the discrete time t at which the decisions are taken..

2. Each stage has a number of states associated with it.
   - a state simply represents the possible condition at which a system can be. It can be finite or infinite.
3. The decision taken at any stage describes how the system moves from the current state to the next one in the following stage.
   - a transition equation that relates current state, decision with the next state as well as any uncontrollable input (disturbance, as in previous Section) is used. For example, in the reservoir case, *storage in the next period = present storage + inflow - decision or* $[state^{t+1}] \rightarrow [state^{t}]$.
   - sometimes it is possible to associate the transition from one stage to the next, going from state i to state j, using probability distribution.

4. The DP Method is designed to find an optimal policy for the overall problem, that is, an optimal decision for each state at each stage will be available at the end of the procedure.
   - decision in period t as a function of state in the beginning of period t for all states and time t.
   - given current state, an optimal policy for the remaining stages is independent of the policy adopted in previous stages. this is called the Principle of Optimality for DP as named by Bellman, in 1957, while working for the Rand Corporation, in California. It states that:

     "The optimal set of decisions in a multistage process has the property that whatever the initial stage, state and decisions are, the remaining decisions, given the current stage, must form a set of optimal decisions for the remaining problem."
   - or, the current state is the result of the past states and decisions. Therefore, the immediate and future optimal decisions must depend only on the current state and should be optimal for the remaining problem starting from the current state.
   - if the condition above is not fulfilled then the problem cannot be set as a dynamic programming problem, because a necessary condition for a problem to be solved by DP is that it can be separable by stages. Another necessary condition to assure the decomposability of the N-stage problem is that the objective function be a monotonically nondecreasing function. If these two conditions hold, then there are sufficient conditions for the decomposition (Nemhauser, 1966).

5. The most common solution procedure is starting the computations from the last stage and then proceeding backwards. In some cases, the forward computation is possible. However, when dealing with Stochastic DP, the backward algorithm is mandatory. This because we need to use the conditional expectation from the decision epoch onward, proceeding forward would require enumeration of all possible outcomes and consequent derivation of the probabilities.

6. A recursive equation that can be used to determine an optimal policy in stage n is available given an optimal policy in stage $n+1$.

The general form for the recursive equation is:

$$f^{*}_{n}(s) = max\{f_n(s_n, d_n)\};$$

which can be understood as **2.2. 1**

$$f^{*}_{n}(s) = \text{immediate benefit} + \text{maximum future benefit (or benefit - to - go)}$$

or

$$f^{*}_{n}(s) = min\{f_n(s_n, d_n)\};$$

which can be understood as **2.2. 2**

$$f^{*}_{n}(s) = \text{immediate cost} + \text{future minimum cost (or cost - to - go)}$$

or

2.2. 3

$$f_n(s_n, d_n) = c_{sx_n} + f^{*}_{n+1}(d_n).$$

Where:

n - index for current *stage* $(n = 1, 2, 3, \dots, N)$;

N - total number of stages;

s_n - current state for stage n;

d_n - decision variable for stage n;

d^* - optimal value of d_n (given s_n!);

c_{sx_n} - immediate cost with decision x_n and current state s_n;

$f_n(s_n, d_n)$ - contribution of stages $n, n+1, \dots, N$ to the objective function if the system starts in <u>state</u> s_n at <u>stage</u> n, the <u>immediate decision</u> is d_n, and <u>optimal decisions</u> are taken subsequently.

Consider the scheme that follows as an illustration for the one reservoir case, at different stages (epochs) t:

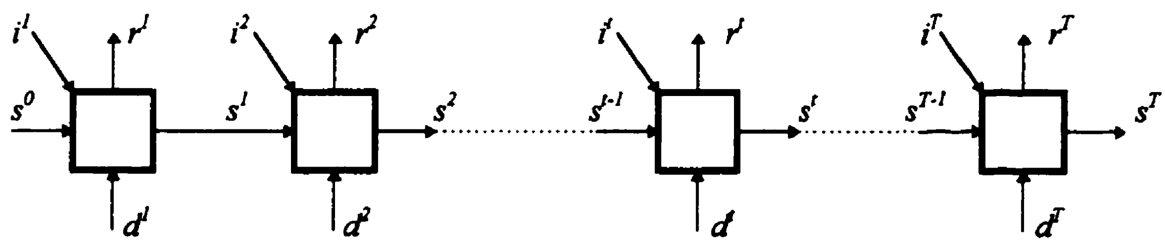


Figure 2.2. 2. General Scheme for DP in Reservoir Operation Problems

The stages are $1, 2, 3, \ldots, t, \ldots, T$ and let us define,

s^0 - initial state of the system;
s^{t-1} - state of the system at the beginning of period t;
d^t - decision taken during stage t;
i_t - input to the system, net natural inflow during period t,
r^t - immediate cost (return or benefit) $\rightarrow g^t(s^{t-1}, d^t)$;
T - final stage.

Now looking at the multistage process:

Given the process is at the current state s^{t-1}, the accumulated optimal return $f^{*t}(s^{t-1})$ is

$$f^{*t}(s^{t-1}) = \underset{d^t, d^{t+1}, \ldots, d^T}{opt} \{r^t + r^{t+1} + \ldots + r^T\}; \qquad 2.2.4$$

Also, for the last stage:

$$f^{*T}(s^{T-1}) = \underset{d^T}{opt}\, r^t = g^T(s^{T-1}, d^T); \qquad 2.2.5$$

And the next one (proceeding backwards):

$$f^{*T-1}(s^{T-2}) = \underset{d^{T-1}}{opt}\{r^{T-1} + f^{*T}(s^{T-1})\}; \tag{2.2.6}$$

Where:

$$s^{T-1} = s^{T-2} + d^{T-1}; \tag{2.2.7}$$

Generalizing:

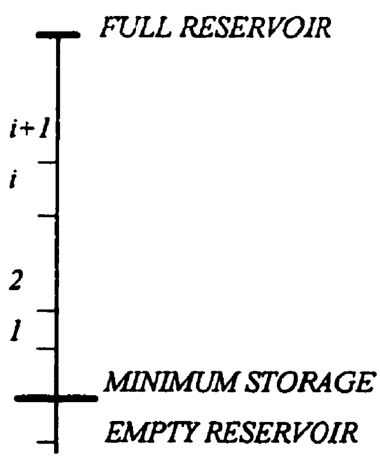
$$f^{*}(s^{t-1}) = \underset{d^{t}}{opt}\{r^{t} + f^{*t-1}(s^{t})\}; \tag{2.2.8}$$

but

$$f^{*t-1}(s^{t}) = \underset{d^{t+1}}{opt}\{r^{t} + r^{t+1} + \cdots + r^{t}\}. \tag{2.2.9}$$

Therefore, $d^t, \ldots, d^T$ decisions are calculated one at a time in each stage.

Continuous variables like reservoir storage, inflow, time, release are used for reservoir problems. Thus, for computational convenience, it is necessary to discretize them. The diagram below represents the discretization of reservoir storages in different levels.



s_i^{t-1} - current state is represented by the index i.

Figure 2.2. 3. Discretization Scheme

For the next stage (period t), similarly, s_j represents the possible states. Thus, in Discrete Dynamic Programming (DDP), the objective function is:

$$f^{*t}(s^{t-1}{}_{i}) = \underset{d^{t}{}_{k}}{opt}\{r^{t} + (s^{t-1}{}_{i}, d^{t}{}_{k}) + f^{*t+1}(s^{t}{}_{j})\}; \qquad \textbf{2.2. 10}$$

Having as transition equation:

$$s^{t}{}_{j} = s^{t-1}{}_{i} + I^{t} - d^{t}{}_{k}; \qquad \textbf{2.2. 11}$$

Where:
s - storage,
I - natural inflow,
d - decision (release).

The basis for the formulation above lies into the separation, or decomposition, of the T-stage problem into T separated subproblems. The concept is applicable only for cases where the problem is separable into stages. The transition equation will establish the relation between the states. The foundation for the decomposition comes from the Principle of Optimality as enunciated by Bellman (1957).

2.2.3 Stochastic/Probabilistic Dynamic Programming

Probabilistic Dynamic Programming (PDP) differs from deterministic DP in that the state at the next stage is not completely determined by the state and policy decision at the current stage. Rather, there is a probability distribution for what the next state will be. However, this probability distribution is completely determined by the state and policy decision at the current stage.

DIAGRAM:

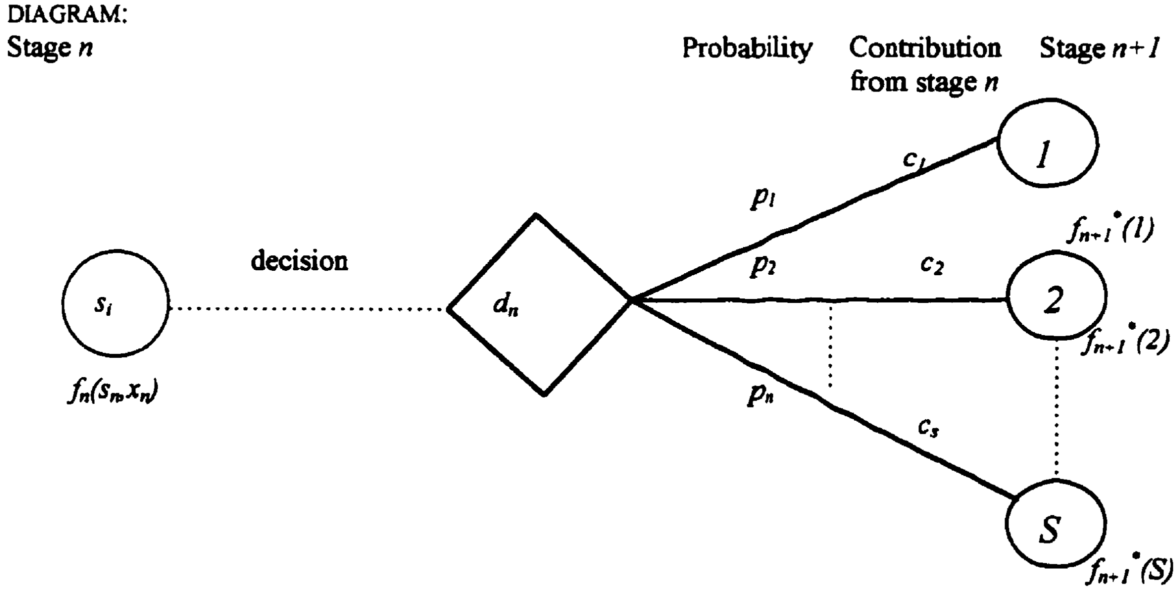


Figure 2.2. 4. Stochastic DP

Where:

s_j - number of possible states at stage $n+1$ $(j = 1,2,\ldots,S)$;

p_{ij} - probability that the system moves from stage n, state s_i with decision d_n to state j in stage $n+1$;

c_{ij} - contribution of stage n to the objective function given that the system goes to state j departing from state i.

For instance, suppose that the objective function is the minimization of the expected sum of the contributions from the individual stages. It follows that:

$$f_n(s_n,d_n) = \sum_{j=1}^{S} p_{ij}[c_{ij} + f_{n+1}(j)], \qquad \text{2.2. 12}$$

with

$$f_{n+1}^*(j) = \min_{x_{n+1}} f_{n+1}(j, d_{n+1}) \qquad \text{2.2. 13}$$

Observation: This minimization is taken over the <u>feasible</u> values of x_{n+1}.

The difference between the probabilistic model and the stochastic one lies in "p_{ij}", where p_{ij} is the probability that the system goes to state j. Given that the state is s_i and the decision d for stage n and $j = 1,2,\ldots,S$.

2.2.4 Stochastic Dynamic Programming

If the natural inflows (I^t) that discharge into a reservoir are Markovian then it is possible to assume them as states, as is the case of the reservoir storages. In the reservoir operation case, it is possible to assume the storage level at the beginning of the period together with the natural inflows during the period as a state. A decision is taken as a function of the current state that will cause a transition to a new state in the next stage. This decision is the release from the reservoir (in a multireservoir case, from the reservoirs). The definition of a Markov Process, as appears in Bartlett (1955), pg. 24, follows:

> *"A Markov Process, ... is a stochastic process for which the values of the random variable* $\mathbf{S}_a$ *(*reservoir storage level at time t_a*) at any set of times* t_a *(a = 1, 2,...,n) depend on the values of* $\mathbf{S}_b$ *at any previous set of times* t_b *(b = 0, -1, -2,...,-j) only through the last available value* $\mathbf{S}_0$. *When the random variable* $\mathbf{S}$ *takes only discrete values, the process is referred to as a Markov chain."*

Note the similarity between the definitions of Markov Processes and the Principle of Optimality of DP, because the latter states that the "immediate and future optimal decisions must depend only on the current state and must remain optimal for the remaining of the problem starting from the current state". (See page 27.)

Another possibility is to assume the natural inflows and storage as distinct states, but that will increase the complexity of the problem. However, with this approach it is possible to account for serial dependence between inflows. In Turgeon (1981) there is a brief review on how to deal with this situation. For the sake of simplicity, it is possible to consider monthly inflows, which have random and seasonal influences, as stochastic but assume them as independent in time. However, this must be acknowledged as an approximation. Transitions are associated with probabilities for a finite number of stages and states. An important assumption regarding the transitions is the independence of the current decisions from the previous ones, otherwise the problem is not separable in stages, violating a necessary assumption for the use of the Principle of Optimality.

In a deterministic approach, the DP algorithm may be solved recursively either in a forward fashion or moving backwards. However, in the stochastic case, it is necessary that the solution is obtained moving backwards along the stages (time periods). The backward recursive equation is analogous to the deterministic one. It has the form of the optimization problem

$$f_i^t(S_i^{t-1}) = \underset{d \in D}{opt}\{r^t + \sum_{j=1}^{n} p^t(i,j,d^t) * f_j^{t+1}(S_j^t)\} \qquad \textbf{2.2. 14}$$

subject to

$$S_j = S_i^{t-1} + I^t - d^t \qquad \textbf{2.2. 15}$$

In the above problem the following notation is used

i, j - states,

$f_i^t(S_i^{t-1})$ - optimal expected return,

d^t - decision (release) over the current period t given it started in state i in period t,

r^t - expected immediate return,

$p^t(i,j,d^t)$ - probability that the system state moves from state i to state j given d^t,

$f_j^{t+1}(S_j^t)$ - assumed known for all j's.

And, in the transition equation (constraints)

S_j^t - storage at the end of period t,

S_i^{t-1} - storage at the beginning of period t,

I^t - net inflow to the reservoir in period t.

Another way of rewriting the equation above is using the routing method. First, from equation 2.2.5, the following is true

$$r_i^t = \sum_{j=1}^{N} p^t(i,j,d^t).g^t(S_i^t,d^t) \quad 2.2.16$$

which is the expression for the expected present return. This implies that equation 2.2.8 has the form

$$\sum_{j=1}^{N} p^t(i,j,d^t)[g^t(S_i^{t-1},d^t)+f(S_j^{t+1})] \quad 2.2.17$$

Also, $p^t(i,j,d^t)$ is the probability that $I^t = I_l^t = S_j^t - S_i^{t-1} + d^t$, l being the state of the discretized inflow. Now, it is possible to rewrite the equation above as

$$\sum_{l=1}^{L} p^t(I^t = I_l^t)[g^t(S_i^t,d^t)+f(S_j^{t+1})] \quad 2.2.18$$

where L is the number of discretized states of the inflow probability distribution.

Extending the formulation above to the multireservoir case, the following optimization problem is solved. Here, the variables of the transition equation have their subscript of discretization suppressed and instead present the subscript for the reservoir indexes as a matter of convenience.

$$F_i^t(S_1^{t-1},S_2^{t-1},\ldots,S_N^{\ t-1}) = \underset{d\in D}{opt}\, E\{r_i^t + F_j^{t+1}(S_1^t,S_2^t,\ldots,S_N^t)\}, \quad 2.2.19$$

subject to

$$S_K^t = S_k^{t-1} + I_k^t + d_{k\in C_k}^{\ t} - d_k^t, \quad 2.2.20$$

Where:

F_i^t - optimal total expected return from the process for state i;

E - expectation operator;

r_i^t - immediate return from state i in period t to state j in period $t+1$ due to the overall decision

$D^t = \{d_1^t, d_2^t, ..., d_N^t\}$;

F_j^t - total accumulated return for state j over $T-(t+1)$ periods;

S_k - storage in the k^{th} reservoir;

d_k - release from k^{th} reservoir;

I_k - net inflow to k^{th} reservoir,

C_k - the set of releases flowing into reservoir k.

The release policies (decisions) obtained for each reservoir of the system is obtained now as a function g of all the storages S_k^{t-1}.

$$
\begin{aligned}
d_1^t &= g(S_1^{t-1}, S_2^{t-1}, \ldots, S_N^{t-1}), \\
d_2^t &= g(S_1^{t-1}, S_2^{t-1}, \ldots, S_N^{t-1}), \\
&\vdots \\
d_N^t &= g(S_1^{t-1}, S_2^{t-1}, \ldots, S_N^{t-1}).
\end{aligned}
\qquad 2.2.21
$$

It is easy to observe that the dimensionality of the problem has increased considerably compared to the one-reservoir system case. In the derivation of the recurrence equations above, it was always assumed the independence between the inflows. This assumption, which yields an elegant mathematical formulation and simplifies the computations, is generally reasonable for time intervals of a month. However, for a real case study, this would not be satisfactory.

The alternate method for the recursive equation of the DP will have the form

$$\sum_j p^t(i,j,\mathbf{d}^t)[g^t(i,\mathbf{d}^t) + F_j^{t+1}] \qquad 2.2.22$$

Note that for the sake of simplicity and coherence with the notation to be used further ahead in this work, the S is dropped as a reference for state and replaced by the more general indices i and j. This is because state will be associated with storage and, in this specific case, it is referred to the state of the system as a whole.

According to Ponnambalam (1987), extending the routing method to the multireservoir case will lead to the following:

$$\sum_{l_1=1}^{L_1}\sum_{l_2=1}^{L_2}\cdots\sum_{l_N=1}^{L_N}\{p_1^{t}(I_1(l_1)|I_2(l_2),\ldots,I_N(l_N))\ldots p_N^{t}(I_N(l_N)|I_1(l_1),\ldots,I_{N-1}(l_{N-1}))\cdot[g^{t}(i,\mathbf{d}^{t})+F_j^{t+1}]\}$$

2.2. 23

which is the expression for the total expected returns over T-$t+1$ stages given decision vector $\mathbf{d}^t$,

where

$L_1, L_2, \ldots, L_N$ - maximum number of discrete states,

$I_1(l_1), I_2(l_2), \ldots, I_N(l_N)$ - natural inflows to corresponding reservoir,

$p_1^t(\cdot|\cdot), \ldots, p_N^t(\cdot|\cdot)$ - conditional probabilities of the discrete inflow states,

$g^t(i\ ,\ \mathbf{d}^t)$ - immediate return, the system being in state i, at the beginning of time t.

$\mathbf{d}^t$ - decision vector.

Ponnambalam (1987) compares and discusses the computational costs of using either the routing or the alternate method, for the single and multireservoir cases. The use of interpolation to obtain the maximum (or minimum) future benefit increases the computational costs, especially because this happens in the innermost loop. In the present work, this approach was used instead of rounding to closest state. To reduce the computational burden that will result in an increase of the computing time, as much preprocessing as possible was used, before entering the internal loops. The consequent tradeoff is the need for more online memory.

2.2.5 Discretization Approaches

The variables involved in Reservoir Operation problems are of continuous type. They are time, storage, inflows, releases, and probabilities. Because the continuous variables present in the storage problem are discretized, the type of SDP used is discrete. A careful approach to discretize all these variables must be taken such that they would represent as closely as possible the natural conditions, although always bearing in mind that this procedure is always an approximation and not the exact reproduction of the real situation. Below, general comments are made about the how they can be discretized and the scheme chosen for the test problems presented further in Chapter 3.

2.2.5.1 Time

For a long-term optimization, the time periods chosen are normally months or seasons. Also, when several consecutive dry periods are likely to happen with no significant values for a single month, it is computationally convenient to aggregate two (or even more) months in just one season. That would help reduce the complexity of the problem without compromising the expected accuracy. In the test problems, there are either 12 stages (months) or less seasons in a cycle (year). For medium-term or short-term optimization, the time steps employed are equal to weeks and days, respectively. For long-term problems, the optimal value of the objective function will be accumulated throughout the whole cycle. For medium-term, short-term and real-time optimizations, the entire period of time is generally considered.

2.2.5.2 Inflows

For the test problems, presented in Chapter 3, the inflows were assumed to take only five discrete values during the stochastic optimization part. For the sake of simplicity, they were assumed to be normally independently distributed around the given mean values, as in Turgeon (1981). To obtain the disturbances that were assumed to be composed by Gaussian noise $(0,\sigma^2)$, the Matlab© random number generator routines were employed. For the real case study, Chapter 4, finer discretization schemes had to be used, and will be discussed with more detail later.

2.2.5.3 Releases

With respect to the releases, or decision variables, a very flexible approach was considered. Due to the variability of sizes of possible feasible releases, especially regarding the aggregate and the non-aggregate parts, the discretization interval varied accordingly. This happened because the difference in scale from a non-aggregate reservoir to an aggregate one is sometimes very significant. In test cases where the problem involved an adaptation of hydropower or irrigation optimization, where the objective function was directly a function of releases, more attention was paid to finer discretization. For the real case study, it is a function of storages and, although obviously still related to the releases, more attention was paid to the storage discretization (for the discussion on aggregation, see the following two chapters, 3 and 4).

2.2.5.4 Storages

As Klemeš (1977) mentions, the discretization of the storages is a concern that has accompanied the evolution of Applied Storage Reservoir Theory. Because the expected accuracy of the results can be directly proportional to the number of discretized states, this concern is more than justifiable. The expected trade-off is naturally a more expensive computation. It is of interest to the author to determine which relationship computational cost/accuracy is the most appropriate without compromising too much to the dimensionality problem.

In the case of a Single Reservoir System, considering the computational power nowadays, this choice does not pose too big a problem. But, when it comes to extending the solution method to the multireservoir problem, it is an issue of careful thought. The computations easily fall to the 'curse of dimensionality'.

As in Klemeš (1977), beside the 'curse of dimensionality', other aspects are also relevant for the choice of the discretization scheme. They can be summarized as below.

- "Optimization is based on comparison, and comparison is meaningful only if the results being compared have at least approximately equal accuracy."

- "The value of an optimality criterion, e.g., expected loss or gain, does not depend on the risk of failure alone but also on the probability of spills and generally on probabilities of all storage states."
- "To acquire better insight into a problem, it is often necessary to extend the range of a certain variable beyond the range of the usual practical interest."
- "An inadequate number of storage states, besides causing a decrease in accuracy, may result in a gradual collapse of the optimization scheme."

That author also adds that that the probabilities of three types of events are important when working with the reservoir problem. They are:

1. the probability of empty reservoir, i.e., risk of failure $P(S=0)$;
2. the probability of full reservoir, i.e., probability of spillage $P(S=S_n)$;
3. the probability that neither of the above occurs, i.e., that the reservoir is able to supply the target release, $P(0<S<S_n)$.

The most common used discretization schemes are listed below, with the help of the diagrams. Two epochs are presented, t and the following one, $t+1$. Δ is the size of the discretization interval and S_j represents the state (storage) of the system. According to each author, w is the minimum required change in the state of the system to make it move to an adjacent stage.

Savarenskiy's

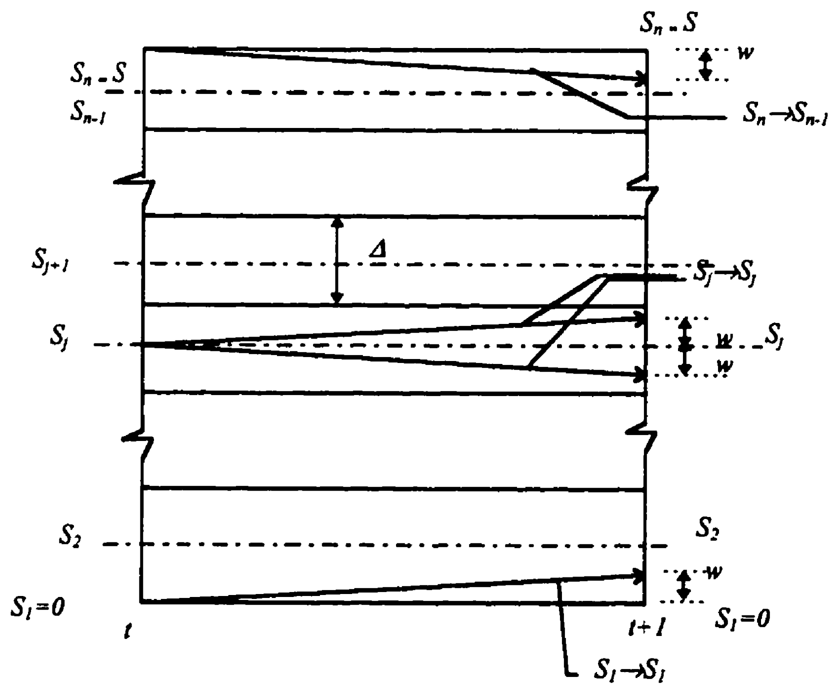


Figure 2.2. 5. Savarenskiy's Discretization Scheme

Moran's

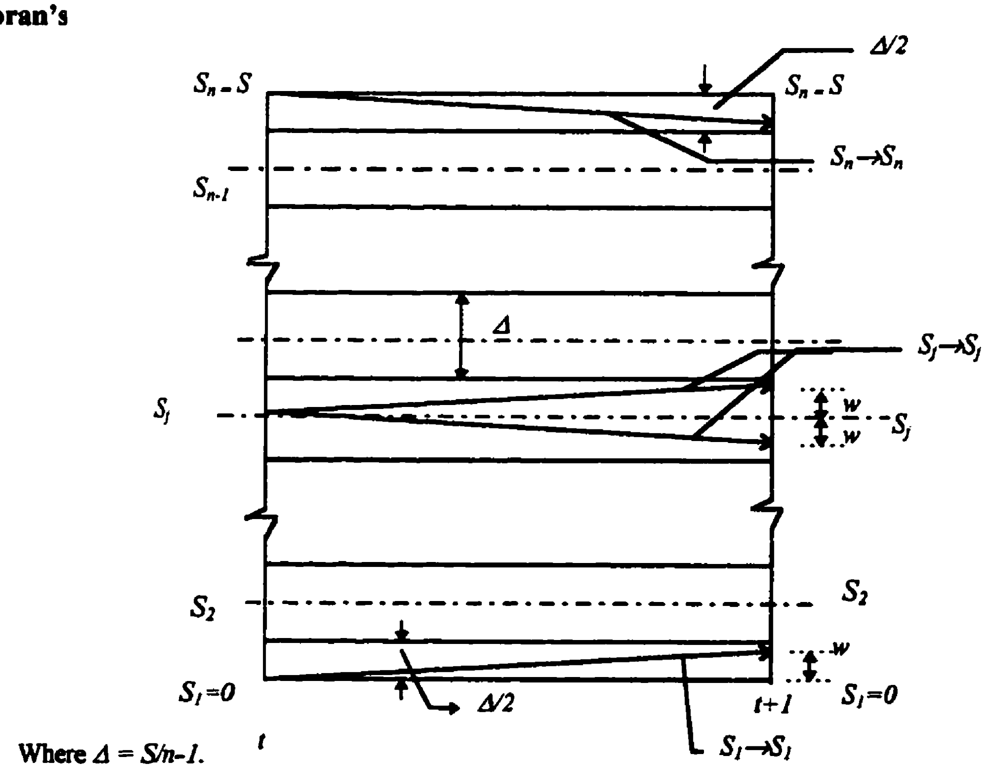


Figure 2.2. 6. Moran's Discretization Scheme

From the diagrams above it is easy to note that the two discretization schemes differ in how they treat the boundaries for the reservoir. In Savarenskiy, when leaving either $S_n = S$ or $S_1 = 0$, any net storage change would cause a transition to a different state, although this may not be true for any of the remaining states. On the contrary, in Moran's scheme, for any of the states, to cause a transition to another state in the next stage, it is necessary to have $w \geq \Delta/2$ which is more consistent than the former one. This causes the boundaries to be "sharp" in Savarenskiy while they may show an error ε, $0 \leq \varepsilon \leq \Delta/2$, in the second.

In his thesis, Ponnambalam (1987) presents an alternative to both schemes above, where the class interval for the boundary states is equal to $\Delta/4$ instead of $\Delta/2$. The adjacent states therefore have class equal to $3\Delta/4$. In this case, while the boundaries are "sharper" than in Moran's work, they still preserve a non-zero class for the boundary states. Also, the Δ interval is smaller than the one computed for Savarenskiy's. The computation of the Δ interval is done according to the following.

Savarenskiy's	Moran's	Ponnambalam's
$\Delta = \dfrac{S}{n-2}$	$\Delta = \dfrac{S}{n-1}$	$\Delta = \dfrac{S}{n-2}$

The differences for the computations using the schemes above are not significant for discretization schemes that are not coarse, i.e., when $n > 6$. However, if reducing the number of states is an important issue, these questions must be addressed. For more details on this discussion, the reader is referred to the works of Klemeš (1977a, 1977b), Doran (1975), and Ponnambalam (1987).

In this work the author introduces extensions of Savarenkiy's, Moran's and Ponnambalam's schemes, such that to the original number of states some more are added to the upper boundary, that is to say, the state equivalent to full reservoir. That will allow the inclusion of the transitions between two or more consecutive stages where spill occurs and would otherwise be considered as infeasible states. Using this approach will avoid the collapse of the solution, what happens when vectors representing the future accumulated costs/rewards are composed uniquely by what would be infeasible states according to the previous aggregation schemes.

Two schemes were derived from those presented before and they are used in this work. They are:

Type 1:

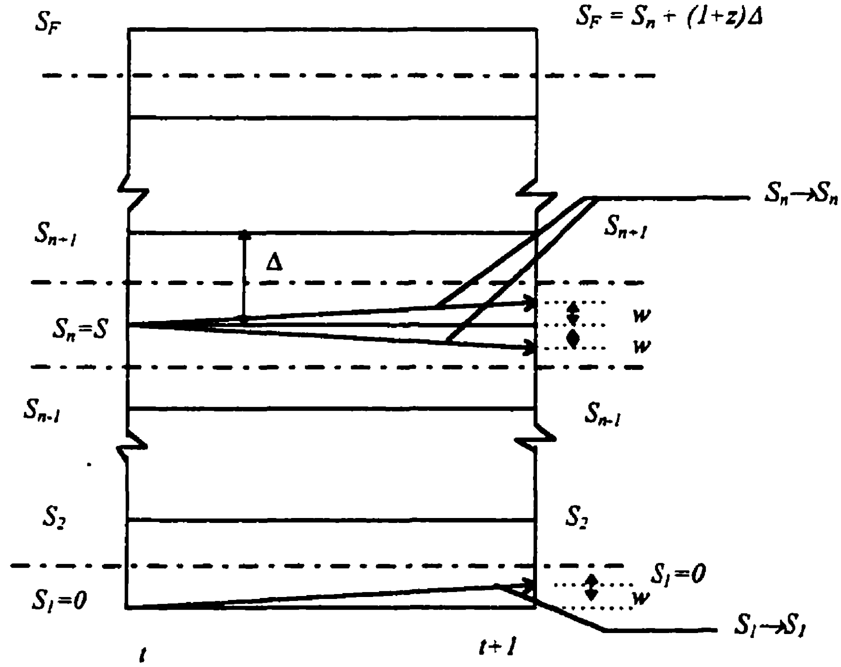
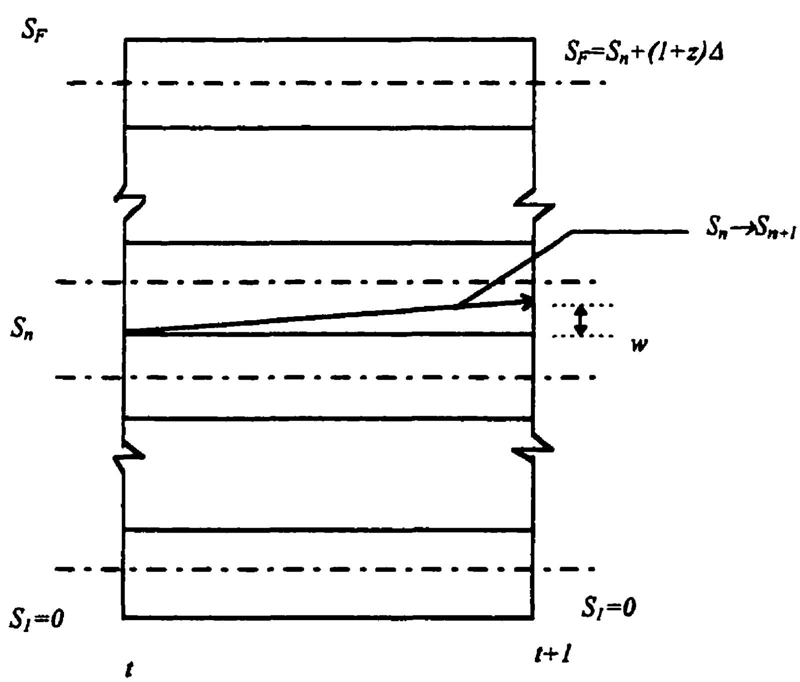


Figure 2.2. 7. Type 1 Discretization Scheme

Type 2:



Where $\Delta = S/(n-1)$ and $0 \leq z \leq 1$.

Figure 2.2. 8. Type 2 Discretization Scheme

The reasons for the introduction of the suggested discretization schemes are two. First, the estimation of non-exact transitions. Instead of approximating the states to the nearest one or interpolating between states that are reached through a non-exact transition, interpolation of the accumulated future cost/benefit was employed. Therefore, the effects of the discretization schemes would only be felt when deriving the conditional distribution of releases, what is necessary for the MAM-SDP methodology (see Section 2.3 and Chapter 4). Second, the use of the above would simplify the computations without compromising the results.

The first proposed scheme, although consistent in sizing the discrete intervals, allows the inclusion of some releases that might occur simultaneously with reservoir overflow or spill. In the second one, the last range before full reservoir has size $\Delta/2$ while the one immediately after has $3\Delta/2$. This brings some inconsistency in terms of sizing but, on the other hand, permits setting the range $S_1 \rightarrow S_n$ in a fashion that comprises the whole storage , i.e., from empty to full reservoir, and their respective expected conditional releases.

The minimum permissible storage level is the lower boundary for the states and the full reservoir level their upper boundary. The non-feasible points were discarded, or values equal to either $-\infty$ or $+\infty$ were assigned to them. Thus, the searchable range is an admissible set of states.

2.2.6 Steady-State Optimization

2.2.6.1 Bellman's Value Iteration Method and Howard's Policy Iteration Method

Howard (1960) presented Value Iteration Method (VIM) and Policy Iteration Method (PIM) to solve the SDP problem. The first one is computationally equivalent to the SDP Method, and requires the same calculation effort. When evaluating PIM, the author points out that the method is not well suited to processes that have a determined termination horizon. Namely, short-term horizon processes.

The objective of the optimization is to get the operation policy such that in the long-term, or steady-state case, the expected returns are maximized, or the losses minimized. As mentioned in the above paragraph, the two different solution methods for obtaining the optimal gain of the cycle are the VIM - in the function space - and the PIM - in the policy space. For more details, see Howard (1960). Because the optimal policy can be used to find the optimal values and vice-versa, one method is called the dual of the other. In both methods, either the functional values or the operating policies are approximated iteratively, therefore they are named methods of successive approximations. As a brief introduction to the methods, the algorithms for both of them are succinctly presented below. For the unichain models, with completely ergodic Markov processes, the Value Iteration algorithm follows.

Step 1 - Select an initial value for the objective function, let us assume $r^0 = 0$. Define tolerance $\varepsilon > 0$ and set t equal to T.

Step 2 - For each $S \in \wp$, this being the set of possible of states the system may assume, compute $f^{t}(S)$ employing the recursive formula

$$f^{t-1}(S) = \underset{d \in D}{opt}\{r^t(S,d) + \sum_{j \in \wp} p(j|S,d) * f^t(j)\} \qquad 2.2.24$$

Step 3 - If $||f^{t} - f|| < \varepsilon$ go to Step 4. Otherwise, decrease t by 1 and return to Step 2.

Step 4 - For each $S \in \wp$, choose

$$d_\varepsilon \in \underset{d \in D}{opt}\{r^t(S,d) + \sum_{j \in \wp} p(j|S,d) * f^t(j)\} \qquad 2.2.25$$

and stop.

Comments:

- For sake of simplicity a simple convergence test was employed in the example above. More sophisticated ones can be employed improving the algorithmic efficiency. For more details, see Putterman (1994).

- Another feature was the use of a non-periodic process. When it is necessary to compute the gain/cost during a cycle with period ω, the convergence criterion then becomes

$$||f^{(\omega-1)t} - f^{\omega t}|| < \varepsilon \qquad 2.2.26$$

The asymptotic gain/cost within one cycle being defined as g and the right hand side of equation

$$f^{t-1}(S) = \underset{d\in D}{opt}\{r^t(S,d) + \sum_{j\in\wp} p(j|S,d) * f^t(j)\} \qquad 2.2.27$$

becoming $g + f^{t}(S)$.

The Policy Iteration Method is easily illuminated by the algorithm shown below.

Step 1 - Value Determination Operation

For a given policy d, solve the equation

$$g + f^{t-1}(S) = r^t(S,d) + \sum_{j\in\vartheta} p(j|S,d) * f^t(j); \qquad 2.2.28$$

for all $f^{t}(S)$ and initially setting $r^0 = 0$.

Step 2 - Policy Improvement Routine

For each state S, find the alternative d^* that yields the optimum for the following equation

$$r^t(S,d^*) + \sum_{j\in\wp} p(j|S,d^*) * f^t(j) \qquad 2.2.29$$

using the $f^{t}(S)$ obtained with the policy from the previous iteration.

Step 3 - If $||d - d^*|| > \varepsilon_d$ make $d = d^*$, $r^t(S,d) = r^t(S,d^*)$, and return to Step 1. Otherwise, go to Step 4.

Step 4 - Terminate the iteration process.

The PIM presents the following properties.

- The solution for the sequential decision process can be obtained by solving a set of linear equations as given by equation 2.1.5.

- The convergence towards the optimal value is monotonic.

- The final policy obtained will provide the optimal solution for the problem, considering that an optimal solution exists, within a reduced number of iterations.

The mathematical proofs for the above are in Howard (1960).

2.2.6.2 White's Bounded Value Iteration Method, [White, (1963)]

The Value Iteration Methods can be subdivided into Bellman's Value Iteration Method and White's Bounded Value Iteration Method. The Policy Iteration Methods have two further classifications which are the one developed by Carton (1963) and Riis (1965), and that of Loucks and Falkson (1970). The first method needs no decomposition either in time or space, guaranteeing an optimal global solution while the latter performs decomposition in time, thus not guaranteeing the global optimal solutions. Both methods mentioned are extensions of Howard's policy iteration method, Howard (1960).

The method employed in this work, for obtaining steady-state policies, was White's Bounded Value Iteration Method, according to the approach proposed by Su and Deininger (1972). This because, in practice, presents quick convergence in gain more often than Bellman's VIM and demands less computational effort.

White's Bounded VIM is easy to understand and simple to implement. It consists of choosing an arbitrary state at the end of the cycle (because the backward recursion is used, the first period is at the end of cycle) and subtract the feasible corresponding gain (accumulated expected return after one cycle) from all the other considered gains at different states. A convergence criterion, ε, is employed to determine when to stop the iterations. When the absolute difference of gains between two cycles is less than the assumed ε, the iterations terminate.

It can be used either in maximization or minimization and the optimal policy is the one at the last iteration. According to the work of Su and Deininger (1972), when convergence in gain is attained, the policies had already converged. It is important to make a proper choice of this arbitrary state. It cannot have zero gain and must be a feasible state. In the situation where these two conditions are not satisfied, the iterations terminate without the desired convergence.

2.2.7 Real - Time Optimization

One of the reasons for incorporating this item in this chapter is that there are marked distinctions between steady-state and real-time, also known as short-term, optimization. They differ in the kind of information provided and the objectives to be attained. Furthermore, for water resources problems, when referring to real-time operations, it is necessary to separate problems with time-steps with a length of 1, 2 or 3 days from the hourly or minute-to-minute operation. While in the latter some uncertainty should be incorporated in the decision model, in the former the problem can be addressed as a deterministic one. (See Datta and Houck (1984) for a discussion on this topic.)

If deterministic or stochastic DP is the chosen optimization tool, a termination horizon must be clearly defined. Starting and ending states are factors that might be included in the input to the problem as well. While a steady-state optimization generally needs more than two cycles to converge, in the real-time optimization, one is enough. In the deterministic case there is choice with regard to the epoch of the start of the optimization: it can be either the current or the ending one. On the other hand, stochastic DP demands that the optimization procedure begins at the termination point.

The real-time optimization models frequently make use of forecasted inflow values. The accuracy of forecasts decreases sensibly as the time lag from the present period increases. As a result, it is commonplace to employ a sliding window with updated forecasts at the beginning of each time step.

The real-time model is usually inserted into an overall model which includes long-term and short-term models. The real-time optimization uses the targets provided by the long-term model and operates within these boundaries. A schematic presentation of the above follows. This schematic presentation was adapted from Yeh (1982).

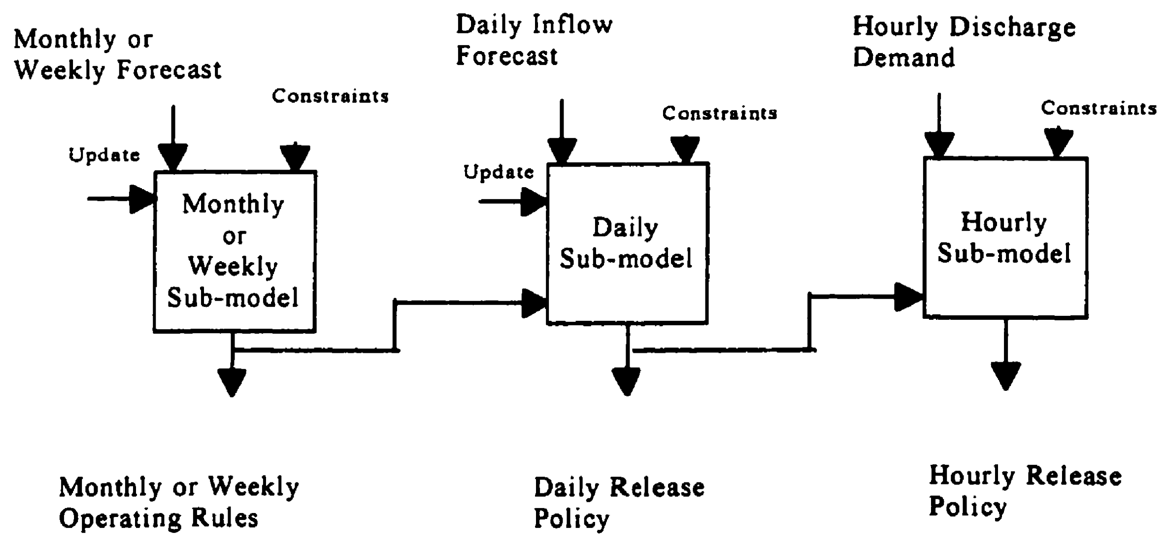


Figure 2.2. 9. Overall Operation Optimization Model

An important aspect of the real-time optimization is the need to consider the time the water takes to travel from one point to another. This feature is commonly disregarded in long-term optimization, where the time step is given either in months or seasons.

The objective function and the operating constraints are normally associated with release (decision) by means of nonlinear relationships. The fastest methods for computation are generally linear optimization methods and, computation speed being an important issue to be addressed in real-time problems, it is necessary to have recourse to piecewise linearization techniques. These techniques are employed to sort out this additional complexity factor providing reliable and fast results, as required by the nature of this optimization. As pointed out before in this work, open-loop type policies, obtained using linear optimization models, suit well this form of operation optimization.

Yeh (1982) applied the overall model as described above to the Central Valley Project (CVP) Case Study. First, a monthly model over a period of one year is set up; second, a daily model over a period of a month; and finally, an hourly model over a period of 24 hours. The CVP consists of 9 reservoirs, 9 power plants, 3 canals and 4 pumping plants. Considering the monthly model as hierarchically placed on the highest position (see explanation in the next Section), its outputs, that are monthly ending storages and monthly releases serve as inputs to the model situated just below that position, i.e., the daily model. The same can be said with respect to the daily and hourly model. When it is the case, weekly models are employed as well.

2.3 Aggregation/Decomposition Methodology

Stochastic Dynamic Programming (SDP) is a very desirable method for long-term optimization in water resources because it provides closed-loop type policies. However, when dealing with multireservoir systems, the problem may become unsolvable owing to size. The present computer technology allows the solution of large problems, unsolvable some years ago, in a reasonable time. Nevertheless, the scale of some problems in the real world still remains beyond the capability of the present machines or are computationally too expensive to be dealt with in its original size.

This leads to the interest in using methods that, although they may not yield the global optimal policies, could improve the actual operating policies and be of feasible implementation. In a field where a fraction of a percentage in the annual expected gains sometimes represents millions of dollars, any possible improvement is not to be disregarded. And, for some of the test cases, it is possible to show that the sub-optimal policy obtained is close to the optimal one. For instance, in reservoir systems where the output has a direct relation with the inputs, the average annual inflow serves a benchmark. Another possible fashion is computing the deterministic linear programming optimal return employing mean values for the natural inflows, and use it as a reference for the results of the MAM-SDP optimization with uncertainty added, i.e., a stochastic problem.

The crucial idea of Dynamic Programming, which is the decomposition of a highly complex problem into a series of small manageable subproblems was further extended to produce the Aggregation/Decomposition Methodology. Imagine it, when coupled with Dynamic Programming, as a nested decomposition.

2.3.1 Review of the Previously Existing Methods

The following items present the fashions adopted in the literature for solving large water resources optimization problems. The chosen domains to be decomposed are time and space.

2.3.2 Decomposition in Time

One of the ways to reduce the size of a problem is to decompose it in the time domain. For example, it is possible to perform separately two types of optimization: the steady-state or long-term optimization and the real-time optimization. In this case, the boundaries assumed for the latter are the policies obtained for the steady-state situation. It is performed in this way because the time interval employed in long-term optimization is usually coarse for the real-time objective. The previous section of this chapter, Stochastic Dynamic Programming, reviewed one of the methods employed to this purpose.

2.3.3 Hierarchical Optimization

Similar to the overall model presented by Yeh (1982) are those employed by Unny et al. (1981) and Ponnambalam (1987). Unny et al. (1981) used an ordered multiple level optimization model vertically arranged hierarchically. As was the case with Yeh's model, the output from a higher optimization level becomes the input for the hierarchically inferior one. The major difference comes from the fact that the interdependence between adjacent levels is executed by means of intervention and performance feedback.

The table and the figure below might help clarify the model development and main ideas involved.

Optimization Model	Type	Horizon Length	Policy Period
Long-term Optimization (LTO)	Multiple period	Several years	Half month based on calendar dates
Medium Term Optimization (MTO)	Multiple period	Several climatological seasons (1 - 2 years)	One week based on a working week
Short-term Optimization (LTO)	Multiple period	Several weeks	One week based on a working week
Real Time Optimization	Single period	One day	—

Table 2.3.1. Hierarchical Model Structure for a Multiple Reservoir Hydroelectric System

The LTO model, placed on top of the hierarchical echelon, is run only once to yield the long-term optimization goals. Therefore, the authors defined it as static for given configuration, objective function, system constraints and so forth. Its output is the long-term rule curves that will be the reference for the system operation. Those curves are given as a function of the system state, here defined as the stored volume of water. The optimization approach employed was Implicit Stochastic Optimization.

The MTO model sets the optimal operating policies for the next weeks, under uncertain inflow conditions. The data to the model is hydrologic data biased by recent past information. The optimization approach chosen is once more Implicit Stochastic Optimization, using DP as a tool. Croley (1974) used a similar approach called rolling schedule.

For the STO model, a dynamic and multi-period optimization is chosen. The length of the planning horizon is equal to the length of policy season for the upper and adjacent optimization level.

The RTO uses one step ahead inflow forecasts to execute the operation of the system. Of all the optimization levels, this is the only one that is actually executed by the operator. The physical transformations in the state of the system are defined by this optimization level. It makes use of the policy information from the higher levels and operates within the boundaries defined by the constraints.

The methodology above was applied to a large hydroelectric system in the Saguenay-Lac Saint Jean in Québec Province, Canada. The system is composed by three major reservoirs, one minor reservoir, and six power plants having an installed capacity of 2687 MW.

We find in the literature approaches that are similar to the one used by Unny et al. (1981). They are those from Bras, Buchanan and Curry (1983) and Wang and Adams (1983) and adapt the same rolling schedule idea. They both have the steady-state policies as terminal boundaries for the real-time optimization. The major difference between the two methodologies mentioned above is the fact that while Bras et al. (1983) define his state vector as storage and inflow from previous periods, the second one defined the state vector solely as storages. The probability distribution for the inflows is repeatedly updated as new information is received from the system.

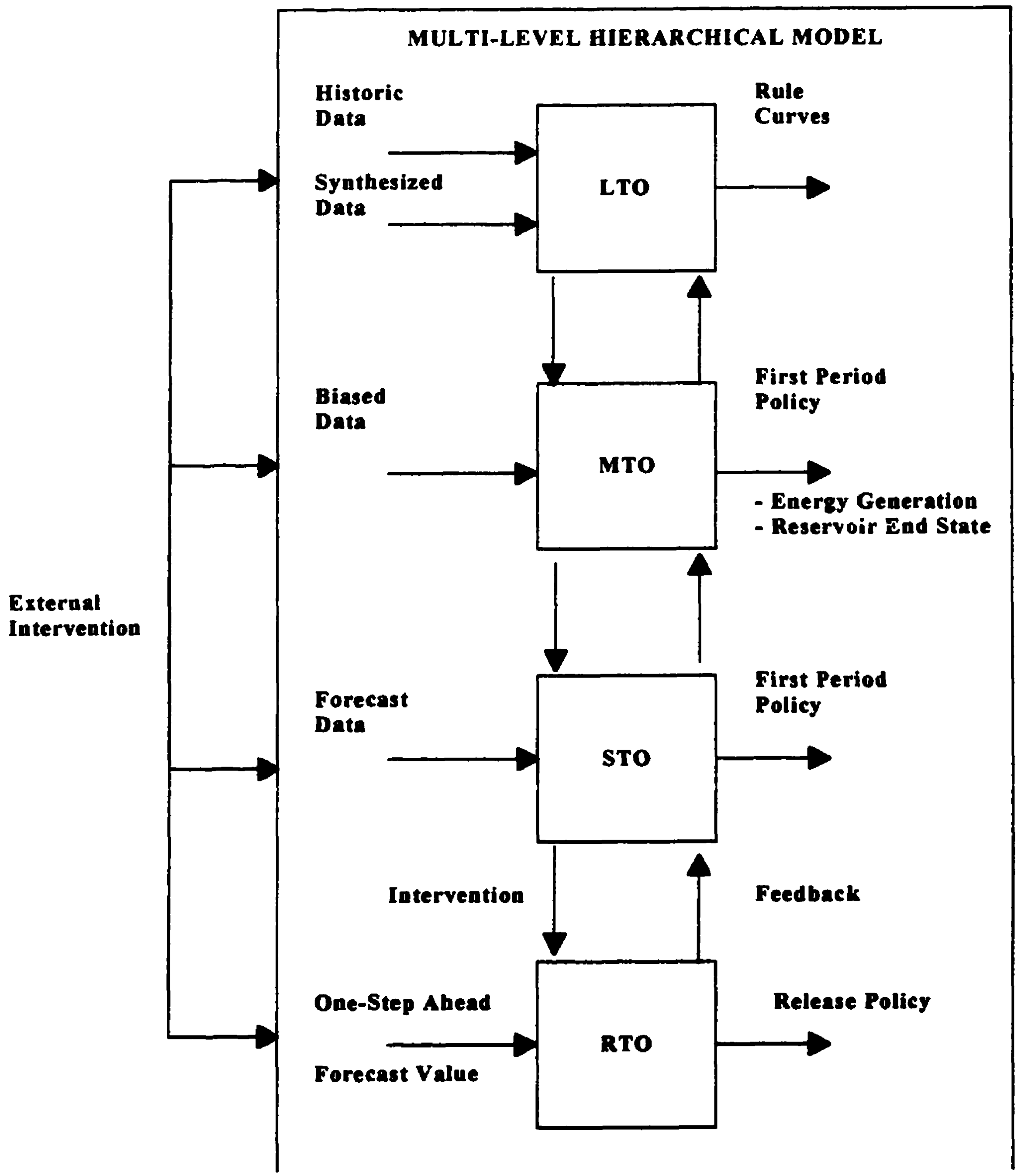


Figure 2.3. 1. Hierarchical Model for Multireservoir System

Ponnambalam (1987) suggested a coordination methodology for real-time optimization which attempts to combine the advantages of the previous methods. The reasons for combining the previous methods and the use of open-loop policy are replicated here as stressed by that author:

1. An open-loop-feedback policy type facilitates the inclusion of the time lag of upstream releases.
2. It permits latitude in defining the time-step, i.e., can be as short as needed.
3. And, it uses the boundaries defined by the upper level model.

According to Ponnambalam (1987), the open-loop-feedback type policy allows the inclusion of time lags in a multireservoir model, what would be computationally very expensive in a closed-loop type.

Figure 2.3. 2. Coordination between steady-state and mid-term optimization and Figure 2.3. 3. Real-time optimization model, are presented below with the coordination methodology.

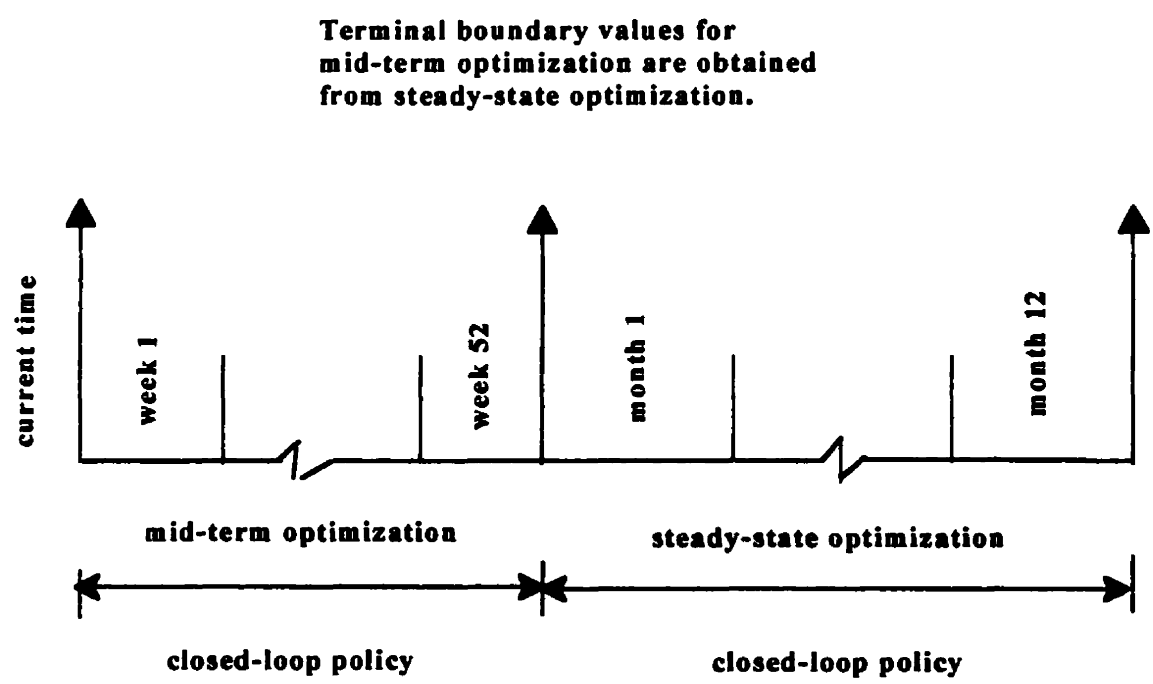


Figure 2.3. 2. Coordination between steady-state and mid-term optimization

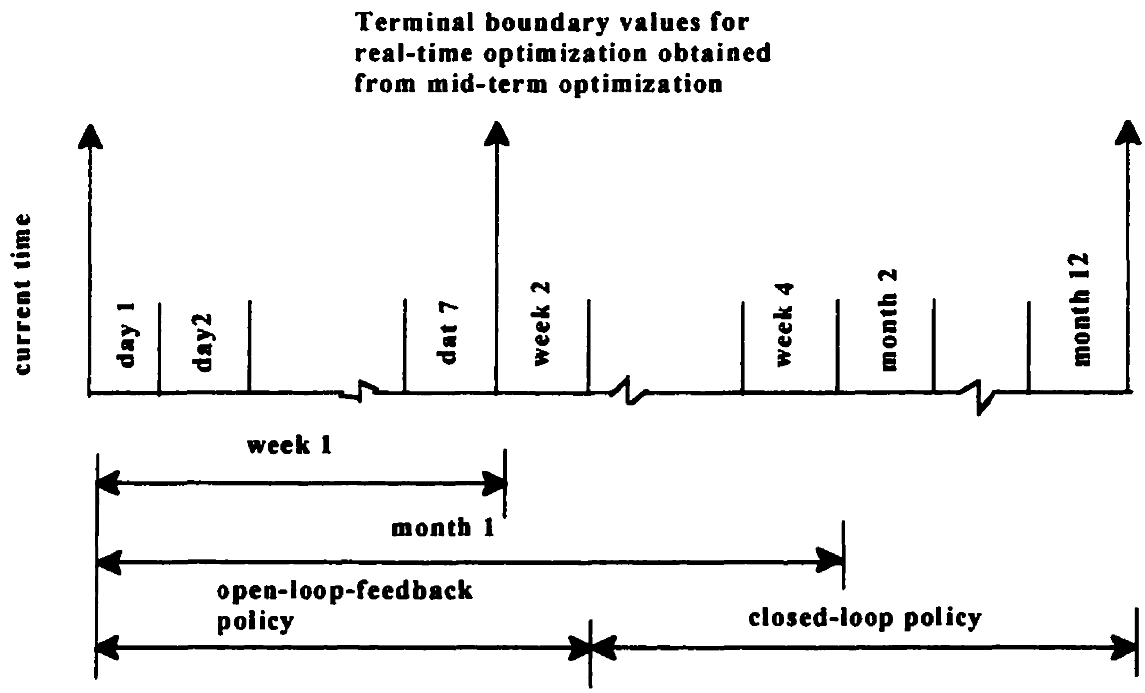


Figure 2.3. 3. Real-time optimization model

As in Unny et al. (1981), a hierarchical optimization is utilized starting from the highest level, steady-state optimization, to the lowest one, real-time optimization. The mid-term optimization is then placed adjacent to both, having time-step equal to one week. The other time-steps are month for the long-term (steady-state) optimization and daily for the real-time. It is suggested that the mid-term optimization be performed at the end of each week. Thus, for the real-time one, the target releases are defined beforehand. Further in the present chapter follows the explanation of how the aggregation/decomposition methodology is adopted for the steady-state optimization of a multireservoir system.

During real-time optimization, inflow forecasts are used to obtain the daily release policies. One reason to proceed in such a fashion is that daily forecasts normally present an acceptable accuracy for reservoir operation purposes. After each time-step with 24 hour length the information is updated and the time horizon shortened in equal duration. Ponnambalam (1987) employed this hierarchical framework in his case study, the Parambikulam-Aliyar Project, in India.

While Ponnambalam (1987), Unny et al. (1981) and Yeh (1982) did not choose explicit SDP for their real-time optimization, it is this author opinion that a stochastic DP single reservoir operation optimization could be adopted to all three methodologies. The major advantage is certainly the possibility of obtaining a policy that is given with respect to the state of the single reservoir. The need for fast computations is not compromised because the optimizations are performed for a single reservoir at a time, what decreases significantly the calculation time and would allow the inclusion of time-lags between the reservoirs. It should be added that, as the targets are pre-determined in a multireservoir system optimization, the loss in accuracy due to the use of just one reservoir is acceptable when considered the uncertainties originated from the use of forecast inflows.

2.3.4 Decomposition in space

This type of decomposition can be done in two ways: using approximate models or using hierarchical analysis. In the first alternative the policies obtained are sub-optimal, but near optimal. In the second one, the system is solved as a whole but in a restricted manner for some problems, also generating sub-optimal policies.

2.3.4.1 Composite Representation

This method, developed by Arvanitidis and Rosing (1970a, 1970b) uses the composite representation to attain the optimal monthly total generation of hydropower. As of the work presented here, the tool utilized is stochastic dynamic programming.

The composite model captures the main features and capabilities of the system in question. It transforms, for each and every hydropower plant of the system, the inflows and releases into their potential energy equivalents. The same is done with respect to the stored water. The stored water is converted into its at-site and downstream generating capability. The composite model "*aggregates*" the potential energy and, according to the mass balance equation, stores, releases and receives it.

To build a model which will represent the multireservoir system into a single reservoir system, it is necessary to have the generation function to convert the released potential energy into actual potential energy and the statistical description of the potential inflows.

By means of this composite model, which is a simplified version of a complex multireservoir system, it is possible to perform stochastic dynamic programming optimization that otherwise may not be solvable due to the number of states considered.

2.3.4.2 Getting the Potential Energy:

A conversion factor is associated to each hydropower plant. This is because at different locations, water may have different hydropower potentials. Therefore, the conversion factor relates the potential to generate hydroelectric power with the amount of water available at a specific hydroplant. It is called the *H/K* of the hydroplant, and gives the megawatts that can be generated by an outflow of $1000\ ft^3/s$. Evidently, this is an approximation of the reality. The net head, which is one of the determining factors in the amount of hydropower generated, may present significant variations. Thus, some error is introduced. Therefore, from Arvanitidis and Rosing (1970a):

> *"The potential energy of water at a particular hydroproject is obtained by multiplying the volume of water by the sum of the conversion factors of at-site and downstream plants."*

$$PE^t = \sum_{j=1}^{n} V_j^t \left(\sum H/_k\right)_j, \qquad 2.3.1$$

Where:

PE^t - total potential energy stored in the system at the beginning of month t in *MWd* (*1 MWd* is the potential energy accumulated during 1 day by an inflow of *1 MW*),

V_j^t - volume of water in storage at plant j at the beginning of month t in *ksfd* (*1 kdsfd* is the volume of water accumulated during *1* day by a continuous inflow of $1\ kft^3/s$),

$\Sigma(H/K)_j$ - sum of all downstream conversion factors, including the one at plant j, in $MW/kft^3/s$,

n - number of plants in the system.

For the *MW* inflow

$$X^t = \sum_{j=1}^{n} x_j^t (H/K)_j^t, \qquad 2.3.2$$

Where:

X^t - average MW into the system during month t,

x_j^t - base power flow[1] above plant j during month t in kft^3/s,

$(H/K)_j^t$ - conversion factor at plant j in *MW* per kft^3/s during month t.

The *MW* outflow from the system is

$$Q^t = \sum_{j=1}^{n} q_j^t (H/K)_j^t, \qquad 2.3.3$$

Where:

Q^t - average *MW* outflow from the system during month t,

q_j^t - average outflow from plant j during month i in kft^3/s.

Finally, the following energy balance equation must be satisfied

$$PE^t = PE^t + (X^t - Q^t) * d^t, \qquad 2.3.4$$

where d^t is the number of days in month t.

[1] The base power flow is the observed stream flow adjusted for the effects on flow regulation by reservoir control, irrigation programs, and evaporation losses.

2.3.4.3 Obtaining the Composite Generation Function:

Although this item is beyond the scope of the present work, the main idea is mentioned below. In a more detailed work, dealing with hydropower generation, the setting of the composite generation function needs careful attention.

A more accurate composite generation function would relate the actual generation to the *MW* outflow. And, as noted previously, the net head may vary significantly through the operation. The Composite Generation Function can be obtained, for a real situation, simulating the reservoir for, let us say, *1000* years, and after that relating the storage levels to the conversion factor. For this simulation, a detailed mathematical model of the physical system must be set. If it is the multireservoir case, the whole system must be considered altogether.

2.3.4.4 Conclusions

Arvanitidis and Rosing (1970a, 1970b) presented in their work the important idea of establishing a composite model of a complex multireservoir hydroelectric system. Fundamental to this is the concept of the potential energy which allows the aggregation of several single reservoir systems into one composite model. Their model was employed to determine the optimal operation of the hydroelectric power system in the Pacific Northwest (*PNW*). The dimensionality problem caused by the multireservoir system was alleviated by means of this technique. Further extensions to this work follow.

2.3.5 Decomposition Method for Long-Term Scheduling of Reservoirs in Series

This method, developed by André Turgeon (1981), determines the monthly operating policy of a power system of reservoirs in series. The randomness of the inflows are considered and it overcomes the dimensionality problem by transforming an optimization problem of *N* state variables into *N-1* problems of two state variables. The optimization method is stochastic dynamic programming. The method requires that the objective function be a separable function of the energy generation from each plant because it decomposes the global problem in a series of two-level ones The technique is applicable to either weekly or monthly operating policies.

2.3.5.1 Problem Formulation

The n-reservoir system is in series and the numbering of the installations is done from upstream to downstream. See scheme below.

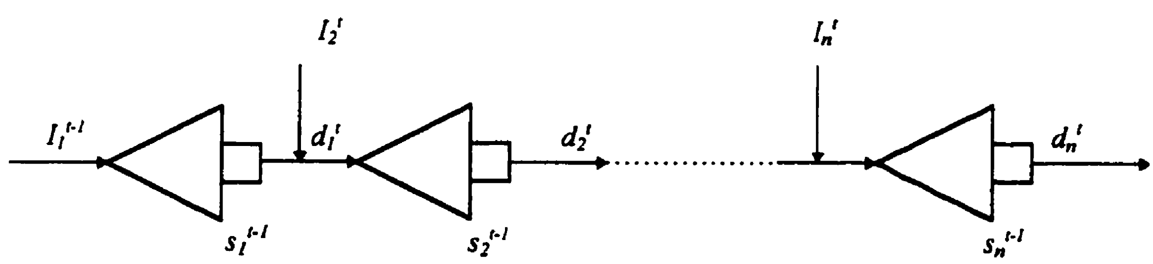


Figure 2.3. 4. Multiple Reservoir System

Where:

$$I_j^t = a_j^t + b_j^t I_{j-1}^t + \xi_j^t, \quad j = 2, \ldots n, \tag{2.3.5}$$

I_j^t - random variable representing the natural inflow in hcm (*1 hcm = 10^6 cubic meters*) to reservoir j in month t;

a_j^t and b_j^t - constants;

ξ_j^t - statistically independent random variable;

d_j^t - discharge from reservoir j in month t in hcm;

s_j^t - content of reservoir j at the end of month t in hcm.

To determine the long-term optimal discharges, it is necessary to obtain the d's such that

$$\max \sum_j \{V_j(s_j^t) + \sum_t \phi^t E[H_j(s_j^{t-1}, d_j^t)]\}, \tag{2.3.6}$$

subject to the inequality constraints

$$0 \le s_i^t \le \bar{s}_i^t, \text{ where } s_i^t \text{ is the capacity of the } i^{th} \text{ reservoir;} \tag{2.3.7}$$

$d_i^t \geq 0,$

and subject to the equality constraints (continuity equation)

$$s_j^t = s_j^{t-1} + I_j^t + d_{j-1}^t - d_j^t, \quad \text{for } j = 1,\ldots,n \text{ and } t = 1,\ldots,T. \qquad \textbf{2.3. 8}$$

In the above formulae the following notation is used:

$V_j(s_j^t)$ - expected value of the water remaining in reservoir j at the end of the last month studied;
$H_j(s_j^{t-1}, d_j^t)$ - generation of plant j in month t in MWh,
(Obs: The generation is a function of the discharge d_j^t and the water head. In this simplification is just a function of the storage level of the downstream reservoir and the j^{th} reservoir);
ϕ^t- value (in dollars) of a MWh produced anywhere on the river in month t, assumed independent of the amount produced;
E - stands for the expected value operator.

2.3.5.2 Global Feedback Solution

In the case of a multireservoir system, with $n \leq 4$, then it is possible to obtain a global feedback solution by means of straight dynamic programming.

The recursive equation, solved backwards is

$$Z_0^{t+1}(s_1^t, s_2^t, \ldots, s_n^t) = V_1(s_1^t) + \ldots + V_n(s_n^t) \qquad \textbf{2.3. 9}$$

The equation above is solved until the minimum is reached

$$Z_0^t(s_1^{t-1}, s_2^{t-1}, \ldots, s_n^{t-1}) = \min_{d^t \in D^t} E\{\phi^t \sum_j H_j(s_j^{t-1}, d_j^t) + Z_0^{t+1}(s_1^t, s_2^t, \ldots, s_n^t)\}$$

2.3. 10

The set $d_j^{\,t}(s_1^{\,t-1}, s_2^{\,t-1}, \ldots, s_n^{\,t-1})$, for $j = 1,2,\ldots,n$; being the solution of the recursion above. Note that: $d^t = \{d_1^{\,t}, d_2^{\,t}, \ldots, d_n^{\,t}\}$, with d^t satisfying the constraints. The expectation is taken with respect to the inflows, I_j^t, for the ranges already defined.

2.3.5.3 Decomposition

It was mentioned that, for a large n, the problem is excessively large, using a great amount of CPU time and memory. An approximate solution applied can be:

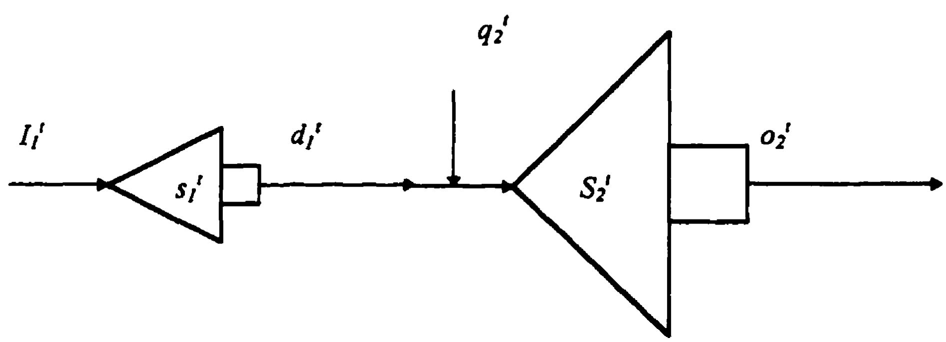


Figure 2.3. 5. Multiple Reservoir System with n-1 reservoirs aggregated

The second reservoir is a composite model of reservoirs *2* to *n*, according to physical aggregation methodology presented in Arvanitidis and Rosing (1970a). With this sort of solution, the release policy would be a function of only two variables, namely, s_j^{t-1} and S_{j+1}^{t-1} where S_{j+1}^{t-1} is a new variable, namely, the total energy content of the downstream reservoirs, $j+1, j+2, \ldots, n$. Therefore, the release policies are determined separately, by means of solving *n-1* dynamic programming problems, each with only two state variables.

To determine the release policy for reservoir *1* (one), a composite model of reservoirs *2* to *n*, based on the approach used by Arvanitidis and Rosing, is set. With these two reservoirs, using dynamic programming, a release policy, which is a function of s_1^{t-1} and S_2^{t-1}, is obtained. After this, the expected releases obtained from the first reservoir, along with their joint conditional probabilities, are used as input in the next stage of the decomposition. These releases are conditional on the storages.

In the next stage, the release policy for reservoir *2*, which is a function of s_2^{t-1} and S_3^{t-1}, is obtained. To this effect, first it is necessary to get the conditional probability distribution of releases from reservoir *1* (one), conditional on s_1^{t-1} and S_2^{t-1} and second to set another composite model for the remaining downstream reservoirs, i.e., $s_3, s_4, \ldots s_n$. The procedure is repeated until the *n-1* reservoir is reached.

2.3.5.4 Composite Model

Using the procedure presented by Arvanitidis and Rosing, firstly a fixed conversion factor is computed, *'MWH/hcm'* and secondly the total amount of stored water converted in potential energy.

The following inequality is used

$$S_{i+1}^t = \sum_{j=i+1}^{n} \sum_{m=j}^{n} h_m s_j^t \leq \sum_{j=i+1}^{n} \sum_{m=j}^{n} h_m \bar{s}_j = \bar{S}_{i+1} \qquad \textbf{2.3. 11}$$

Where:

S_{i+1}^t - total potential energy stored in reservoirs $i+1$ to n in period t;

h_m - fixed conversion factor *'MWH/hcm'* at site m.

The inflow of potential energy (q) to reservoirs $i+1$ to n can be computed by

$$q_{i+1}^t = \sum_{j=i+1}^{n} \sum_{m=j}^{n} h_m I_j^t \qquad \textbf{2.3. 12}$$

and the potential outflow of energy by

$$o_{i+1}{}^{t} = \sum_{j=i+1}^{n} h_j d_j{}^{t} \qquad \textbf{2.3. 13}$$

It is necessary to point out that the outflow of potential energy from the composite reservoir may differ from the actual energy generated. This because of the approximations introduced due to the fixed conversion factor.

In order to have a generation function that would reflect more closely the behavior of the composite reservoir it is possible to do the following. Simulate the operation of the whole system with the known operating policy for, let us say, at least *500* years, getting for each period, the relation between the actual generation, $g_{i+1}{}^{k}$, and the stored potential energy, $S_{i+1}{}^{t-1}$, and the potential outflow of energy, $o_{i+1}{}^{t}$. To do this, the formulae presented for the composite model can be used.

Turgeon (1981), in his paper, also presents an algorithm for solving this type of problem and applies it to a numerical example, a case of a system of four reservoirs for hydropower generation, configured in series. The solution obtained for these problems is not the global feedback solution, but, in fact, a sub-optimal one. An important feature is the fact that the processing time increases only linearly with the number of reservoirs because it does not involve iterations. Due to the characteristics above, large systems in series that could not yet be solved by means of a global-feedback solution can at least be solved.

2.3.6 Multi-Level Approximate Aggregation/Decomposition - Stochastic Dynamic Programming (MAM-SDP) Methodology

This methodology combines the characteristics of the generalized aggregation/decomposition methodology technique with multi-level dynamic programming. Therefore, problems not solvable by straight dynamic programming can be solved. Although it does not assure the global optimal policy, it allows the optimization of very large systems, generating closed-loop type policies that could not be obtained in a straight manner. Great advantages of the methodology are its applicability to any type of configuration (i.e., reservoirs in series, parallel or any combination of both), does not require a separable objective function by reservoir and its ability to solve very large multireservoir systems. As was the case with the previous technique, by Turgeon, the processing time increases only linearly with the number of reservoirs and is not iterative.

2.3.6.1 Methodology

The algorithm proposed by Ponnambalam and Adams (1993) is a heuristic one. No mathematical proof to justify its application is available yet. However, as shown in the mentioned paper and in the tests performed in the current work, sub-optimal policies, close to the optimal ones, are attainable.

2.3.6.2 Problem Formulation:

The transition of an n-reservoir system is given by

$$\underline{s}^t = G\,[\underline{s}^{t-1}, \underline{d}^t, \underline{x}^t, \underline{e}^t], \text{ for all } t. \qquad \textbf{2.3.14}$$

Where

$\underline{s}^t$ - vector of reservoir storage levels, $[s_1^t, s_2^t, \ldots, s_n^t]$ at the beginning of period $t+1$,

G - n-dimensional vector-valued continuity function,

$\underline{d}^t$ - release decision vector, $[d_1^t, d_2^t, \ldots, d_{n'}^t]$, where n' can be $\geq n$,

$\underline{x}^t$ - vector of natural inflows to the n reservoirs, $[x_1^t, x_2^t, \ldots, x_n^t]$,

$\underline{e}^t$ - vector of seepage and evaporation losses from the n reservoirs, $[e_1^t, e_2^t, \ldots, e_n^t]$.

The backward dynamic programming equation is

$$f^t(s^{t-1}) = \max E\,\{C^t(s^t, d^t) + f^{t+1}(s^t)\},\ t = T, \ldots, 1. \qquad \textbf{2.3.15}$$

Subject to:

- the transition equation,
- the inequality constraints on the storage level:

$s_i^t\ min \le s_i^t \le s_i^t\ max,$

- the inequality constraints on releases:

$d_i^t\ min \le d_i^t \le d_i^t\ max,$

for all i, t.

In the above problem the following notation is used:

$C^t\ (d^t, s^t)$ - scalar-valued system benefits during the time-period t,

$f^t(s^{t-1})$ - total expected benefits from period t to the end of horizon T, given that the system state is represented by s^{t-1},

E - expectation operator, taken with respect to the conditional probability distribution $p^t[(s^t|s^{t-1});d]$[2].

Now, applying the Aggregation/Decomposition-Dynamic Programming technique to overcome the dimensionality problem, common to multireservoir systems, for the i^{th} reservoir in an n-reservoir system, the following problem is obtained (see also Figure 2.3. 4 and Figure 2.3. 5)

$$f^t(s_i^{t-1}, S_{i+1}^t) = \max_{d_i^t, o_{i+1}^t} E\{H(s_{i+1}^{t-1}, S_{i+1}^{t-1}, d_i^t, o_{i+1}^t) + f^{t+1}(s_i^t, S_{i+1}^t)\}, \quad \text{for} \quad t = T, \ldots, 1;$$

2.3. 16

subject to

$s_i^t = s_i^{t-1} + I_i^t + d_{i-1}^t(.|s_i^{t-1} + S_{i+1}^{t-1} = S_i^{t-1}) - d_i^t,$

$S_{i+1}^t = S_{i+1}^{t-1} + q_{i+1}^t + c \cdot d_i^t - o_{i+1}^t,$

$s_i\ min \le s_i \le s_i\ max,$ **2.3. 17**

$S_{i+1}\ min \le S_{i+1} \le S_{i+1}\ max,$

for all t.

Where:

---

[2] Observation: $p^t[(s^t|s^{t-1});d^t)]$ is derived from the joint distribution of all inflows I^t for a given release decision d^t and the system transition equation.

s_i^t - reservoir storage level of the i^{th} reservoir,

S_{i+1}^t - aggregated[3] reservoir storage of reservoirs $i+1$ to n, at the beginning of time period $t+1$,

$d_{i-1}^t(.|S_i^{t-1})$ - release from upstream reservoir $i-1$ conditioned on the aggregate storage S_i^{t-1} determined at the $i-1^{th}$ stage of optimization,

o_{i+1}^t - aggregated release decision corresponding to the aggregated reservoir $i+1$,

c^* - conversion factor for the releases from the upstream reservoir to reflect the potential benefits available from the aggregated system of reservoirs,

$H(s_i^{t-1}, S_{i+1}^{t-1}, d_i^t, o_{i+1}^t)$ - immediate benefit in time period t[4],

E - expectation operator taken with respect to the conditional distribution corresponding to the current decision vector derived using the system equation and the joint probability distribution of the inflow I_i^t to i^{th} reservoir, the aggregated inflow q_{i+1}^t to the aggregated reservoir $i+1$ and any release from the upstream reservoir d_{i-1}^t[5].

As is the case with Turgeon method, the optimization stages start at $i=1$ and go up to $n-1$ stages, n being the total number of reservoirs. This methodology is a combination of the generalized A/D-DP with the multi-level incremental dynamic programming (MIDP) technique proposed by Nopmongcol and Askew (1976). The authors named it multi-level approximate model dynamic programming (MAM-DP, MAM-SDP for the stochastic case). The methodology provides sub-optimal policies because, as the optimization stages are performed, the policies become more and more local. For instance,

$$
\begin{aligned}
&d_1^t = f(s_1, S_2^{t-1}), \text{ where } S_2^t = h(s_2^{t-1}, s_3^{t-1}, \dots, s_n^{t-1}),\\
&d_2^t = f(s_2, S_3^{t-1}), \text{ where } S_3^t = h(s_3^{t-1}, s_4^{t-1}, \dots, s_n^{t-1}),\\
&\vdots \\
&d_{n-1}^t = f(s_{n-1}^{t-1}, s_n^{t-1}),\\
&d_n^t = f(s_{n-1}^{t-1}, s_n^{t-1}).
\end{aligned}
\qquad \mathbf{2.3.18}
$$

Observation: h in above is an aggregation function.

[3] The aggregation is performed using a linear operator as explained in Section 3.

[4] This is only an approximate function of the real benefit.

[5] d_{i-1}^t is obtained as a function of S_i^t in the previous optimization stage.

In contrast, in the global optimal solution, all d_i^t, for $i=1$ to n, will be given as a function of $s_1^{t-1}, s_2^{t-1},..., s_n^{t-1}$.

Another reason for the sub-optimality of the policies is the need to consider the independence between the releases obtained at the previous optimization stage and the inflows during the stochastic dynamic programming part. It is not difficult to realize that, as the releases are a function of storages, and these depend on the inflows, that this assumption does not replicate the reality. However, this assumption is necessary for implementation purposes.

In this work and in the description of the methodology so far, only the 2-level A/D-SDP was presented. Nevertheless, this is only the first step in applying the MAM-SDP. In case the results obtained with this level are not satisfactory, one of the possible alternatives is to increase the level of the algorithm. For example, in a 3-level algorithm, two reservoirs are not aggregated and *k-2* are aggregated. For this situation, a three state (not considering the inflows as state) stochastic dynamic programming optimization has to be performed. The complexity of the optimization increases demanding more CPU time and memory.

2.3.6.3 Validation

It was mentioned so far that the results obtained with the methodology are sub-optimal. But how to evaluate the degree of sub-optimality? For test problems, like those presented in the next section, it is possible to make a comparison with results previously available in the literature, obtained with other methodologies. For some of them, it is not difficult to calculate the upper bounds of the objective function (or lower bounds, if minimization) and estimate how the methodology performs. For some other test cases, when the size of the reservoir system is not greater than 3 or 4 reservoirs, it is possible to compute the global optimal results by means of straight stochastic DP and use them as a reference for comparison. This kind of validation is not carried out in the present work.

However, for some real problems, when it is not feasible to compute the global optimal results by means of SDP, one way of evaluating the performance of the methodology is to compare it with the operating policy currently in use. The study case, presented in Chapter 4 is an example of the aforementioned. To this effect, it is necessary to use stochastic simulation.

2.3.7 Stochastic Extension of the Benders Decomposition

The brief review of this Decomposition Approach is based on the paper presented by Pereira and Pinto (1985). In Stochastic Linear Programming Problems (SLP), dimensionality is one of the major difficulties in solving problems of large systems of reservoirs. The authors of this paper propose a way of obtaining an approximation to the optimum at a reduced computational cost. The description of the approach starts with its deterministic version and is further extended to the stochastic case in the final part.

In their work, the authors implement an algorithm which determines the most economical generation decision for each plant of a hydrothermal system. An small example is explained in detail and the model tested with a Case Study with 37 reservoirs of the Brazilian system.

2.3.7.1 Benders Decomposition Principle

Let us assume a two-stage optimization problem,

$$\begin{aligned} &\min c^T x + d^T y \\ &\text{subject to} \\ &Ax \geq b \\ &Ex + Fy \geq g \end{aligned} \qquad \textbf{2.3. 19}$$

Where:

x - vector of decision variables in the first stage,

c - vector of objective function coefficients,

A - matrix of constraint coefficients,

b - vector of right-hand side constraints,

T - transpose operator.

Now, given a feasible solution x^*, to the first-stage problem

$$Ax^* \geq b, \quad 2.3.20$$

For the second stage, the optimization problem becomes

$$\begin{aligned} &\min d^T y \\ &\text{subject to} \\ &Fy \geq g - Ex^* \end{aligned} \quad 2.3.21$$

Where:
y - vector of decision variables in the second stage,
d - vector of objective function coefficients,
F and E - matrices of constraint coefficients,
g - vector of right-hand side constraints.

At this moment, two observations are necessary:

1. the decisions x^*, taken in the first stage, affect the second-stage constraints,
2. for any given vector of decisions x^*, it is assumed that the second-stage problem is always feasible.

How the decomposition is performed

Assuming an optimal decision vector y^* for the second-stage optimization problem, equation 2.3.21 will appear as follows

$$\begin{aligned} &\alpha(x) = \min d^T y^* \\ &\text{subject to} \\ &Fy \geq g - Ex^* \end{aligned} \quad 2.3.22$$

where $\alpha(x)$ is a function of the first-stage decision, x. Therefore, rewriting the set of equations 2.3.19 as

$$\begin{aligned}&\min c^T x + \alpha(x)\\&\text{subject to}\\&Ax \geq b\end{aligned} \qquad \textbf{2.3. 23}$$

Moving back to Section 2.2 of this Chapter, and relating the expression above with the DP Approach to optimization:

1. $c^T x$ embodies the immediate cost of the objective function,
2. $\alpha(x)$ provides the information about the future cost of the decision vector x.

The Benders Decomposition Technique is essentially a methodology to construct a convex polyhedron which is represented by the function $\alpha(x)$. The process is iterative and starts with an approximation of $\alpha(x)$, called $\hat{\alpha}(x)$, a lower bound for the function.

2.3.7.2 Derivation of the Benders Decomposition

Using the Benders Decomposition, the following function is to be approximated

$$\begin{aligned}&\alpha(x) = \min d^T y\\&\text{subject to}\\&F(y) \geq g - E(x),\end{aligned} \qquad \textbf{2.3. 24}$$

in problem defined by 2.3.19.

Its dual is

$$\begin{aligned}&\text{maximize } \Pi(g - Ex)\\&\text{subject to}\\&\Pi F \leq d,\end{aligned} \qquad \textbf{2.3. 25}$$

where Π is defined as the vector of the dual variables, namely, the Simplex multiplier. The set of constraints represented by equations $(\Pi F \leq d)$ bears independence with respect to the decisions taken during the first stage. Furthermore, the region defined by these equations represents a convex polyhedron allowing the use of linear programming techniques to solve it.

Thus, the two-stage optimization problem can be rewritten as below

$$\begin{aligned}
&\text{minimize } c^T x + \alpha \\
&\text{subject to} \\
&Ax \geq b' \\
&\pi^1 (g - Ex) - \alpha \leq 0 \\
&\pi^2 (g - Ex) - \alpha \leq 0 \\
&\vdots \\
&\pi^p (g - Ex) - \alpha \leq 0
\end{aligned} \qquad \textbf{2.3. 26}$$

The original problem is now redefined in terms of the decision vector x and the scalar variable α. According to Geoffrion (1970), a natural approach to solve these equations is by means of relaxation. This is because it avoids the need of determining in advance all the vectors π^i, $i = 1, \ldots, p$. This set may be a very large one, but only those of the following type are of interest

$$\pi^i (g - Ex) = \alpha \qquad \textbf{2.3. 27}$$

2.3.7.3 Stochastic Extension

As the concern is in tackling stochastic optimization problems, especially because of the randomness associated to the natural inflows, the following extension of the Benders algorithm is to be used.

Let us assume, as is the case in the test problem, that the vector g, in equations 2.3.24 can take on five values, associated with five probabilities. Equations 2.3.19 are now of the form

$$
\begin{aligned}
&\min c^T x + p_1 d^T y_1 + p_2 d^T y_2 + p_3 d^T y_3 + p_4 d^T y_4 + p_5 d^T y_5 \\
&\text{subject to} \\
&Ex + Fy_1 \geq g_1 \\
&Ex + Fy_2 \geq g_2 \\
&Ex + Fy_3 \geq g_3 \\
&Ex + Fy_4 \geq g_4 \\
&Ex + Fy_5 \geq g_5
\end{aligned}
\qquad \textbf{2.3. 28}
$$

Thus, using the methodology just described, first select a feasible decision set x^*, such that the constraint set for the first stage is satisfied. Next, in the second stage, search for y_j^*, $j = 1,\ldots,5$ that optimize

$$
\begin{aligned}
&\min p_1 d^T y_1 + p_2 d^T y_2 + p_3 d^T y_3 + p_4 d^T y_4 + p_5 d^T y_5 \\
&\text{subject to} \\
&Fy_1 \geq g_1 - Ex^* \\
&Fy_2 \geq g_2 - Ex^* \\
&Fy_3 \geq g_3 - Ex^* \\
&Fy_4 \geq g_4 - Ex^* \\
&Fy_5 \geq g_5 - Ex^*
\end{aligned}
\qquad \textbf{2.3. 29}
$$

Now, let us split the optimization problem given by equations 2.3.29 into five optimization subproblems. An interesting feature to note is that they are independent one from another. So, the following set of equations can be stated

$$
\begin{aligned}
&w_1 = \min d^T y_1 \\
&\text{subject to} \\
&Fy_1 \geq g_1 - Ex^*
\end{aligned}
\qquad \textbf{2.3. 30}
$$

$$
\begin{aligned}
&w_2 = \min d^T y_2 \\
&\text{subject to} \\
&Fy_2 \geq g_2 - Ex^*
\end{aligned}
\qquad \textbf{2.3. 31}
$$

$$w_3 = \min d^T y_3$$
subject to **2.3. 32**
$$Fy_3 \geq g_3 - Ex^*$$

$$w_4 = \min d^T y_4$$
subject to **2.3. 33**
$$Fy_4 \geq g_4 - Ex^*$$

$$w_5 = \min d^T y_5$$
subject to **2.3. 34**
$$Fy_5 \geq g_5 - Ex^*$$

The values w_j, $j = 1,\dots,5$ are multiplied by their respective probabilities. Needless to say, $\Sigma_j p_j = 1$.

The two-stage problem can now be rewritten as

$$\min c^T x + \overline{\alpha}(x)$$
subject to **2.3. 35**
$$Ax \geq b$$

which is very similar to the one just obtained for the deterministic case, the only difference being $\overline{\alpha}(x)$, the *expected value of the optimal solution* for each x.

As in Pereira and Pinto (1985), the analogy with stochastic dynamic programming formulation should be stressed. In their algorithm, for this specific stochastic case, the Benders cut follows

$$\overline{w}^i + \overline{\pi}^i E(x^i - x) - \alpha \leq 0 \qquad \textbf{2.3. 36}$$

where $\overline{w}^i = \sum_j p_j w_j^{\,i}$, or its expected value. The same applies to the Simplex multipliers, $\overline{\pi}^i$, i.e., $\overline{\pi}^i = \sum_j p_j \pi_j^{\,i}$.

2.3.7.4 Conclusion

The Benders Decomposition algorithm provides a feasible implementation for the optimization of the operation of large reservoir systems. This because it decomposes the global optimization problem into smaller successive one-stage subproblems. Once more, the analogy with DP is evident. Another important feature worth mentioning is that as the set of constraints in the Stochastic Extension are independent one from another, the calculations can be sped up by means of simultaneous execution. Performing these calculations in parallel allows significant reduction in computational time.

2.4 Summary

Section 1 of Chapter 2 reviews the Reservoir Operation Problem. After presenting a simple analogy between Inventory Storage Theory and Storage Reservoir Theory, the Deterministic and Stochastic solution approaches were discussed. The differences amongst open-loop and closed-loop type policies, under the light of Control Theory were briefly stated. The mathematical formulation for Single and Multiple Reservoir Systems problems were shown, as well as the advantages of water resource system model simulation for more precise analysis of the proposed control policies.

In the second section the type of policies that can be obtained for reservoir operation optimization were reviewed. The type of policy that is the most suitable to long-term operation is the closed-loop. This is obtained from a Dynamic Programming optimization (deterministic or stochastic) but it is usually computationally expensive. Important factors to consider when employing SDP in the Optimization of Reservoir Systems are the discretization schemes and the choice of a suitable convergence test. The long-term optimization is part of an overall optimization scheme that includes real-time operation optimization, and generally a mid-term operation optimization that bridges the gap between them.

Section 2.3 reviews various methodologies that use aggregation to reduce the size of a large problem so that it becomes computationally tractable. This review focused mostly on those that were the backbone to the MAM-SDP, which is one of the tools of this study. It starts with the Composite Representation developed by Arvanitidis and Rosing (1970a, 1970b) and the Decomposition Method for Long-Term Scheduling for Reservoir in Series by Turgeon (1981). A brief introduction to Benders Decomposition Approach that was applied to a water resources problem, as presented by Pereira and Pinto (1985), brings this review to a closure.

Chapter 3

Proposed Methodology and Tests Performed

3. PROPOSED METHODOLOGY AND TESTS PERFORMED

3.1 Introduction

It is well known to any decision maker that a good decision is the one that not only provides an optimal return but is also robust. A robust decision in a multistage decision process is the one that copes better with inputs that present large variance and consequently high uncertainty. Those decisions must be less sensible to input values that are placed distant from their expected mean. It is not rare to find cases where he is willing to accept less performance in exchange for higher operation reliability.

In the present study, those events that are affected by meteorological conditions are considered as uncertain. For instance, for the tests performed, natural inflows, and in the case study, Net Basin Supplies, as defined further in Chapter 4. Wets (1996) presents an interesting review of the techniques available to help the decision maker within the context of decision making under uncertainty.

According to that author, the main components of a stochastic programming model are (adapted from Wets (1996)):

- a decision vector that must satisfy certain constraints, $d \in D$;
- a random variable I whose value will only be observed after d has been selected;
- a cost or benefit as a result of the decision taken.

The above can be translated into the following, according to what was described so far:

1. cost or benefit function of the familiar form,

$$r^t + f^{t-1}(s^t, d) = f^t(s^{t-1}, d) \qquad 3.1$$

and r^t, the immediate expected costs/benefits are associated with the decision d and the future costs/benefits, $f^{t}(s^t,d)$.

2. the probability distribution P of the random variable I;
3. the decision criteria, roughly:
   - an appraisal function, $v : \Re \rightarrow \Re$, of the following form

$$\min_d E\{v[f(I,d)]\},\quad \text{or} \quad \max_d E\{v[f(I,d)]\},\quad \text{with } d \in D; \tag{3.2}$$

   - probability constraints of the form

$$prob[f(I,d) \leq 0] \leq \alpha,\quad \text{or} \quad prob[f(I,d) \geq 0] \leq \alpha, \text{ and} \tag{3.3}$$

   - constraints on the variance of $f(\cdot)$ that appear in the appraisal function v,

   - multicriteria, amongst others.

So far, the major concern in the optimization parts were either maximization or minimization of the objective function. Nevertheless, it is of importance for the decision maker to establish a threshold or, in some other cases, a range of probability of failure of the operation and associate them to an expected return or vice-versa. The normal procedure in the literature is to include this probability of failure, α, in the set of constraints. The insertion of probabilistic constraints defines the model as a chance constrained one.

This work aims at improving the suboptimal policies that are obtained with the proposed MAM-SDP methodology, compute the variances for the expected return for the test cases, i.e., Single Reservoir and Multiple Reservoir Systems (test cases with 3 and 4 reservoirs) and after that, suggest ways of reducing the mentioned variances. It is also shown empirically that the well used paradigm that the higher the variance, which is a measure of spread or diffusion around the expected values, the higher the probability of failure for the system. The first experiment was the application of Principal Components Analysis (PCA) to MAM-SDP. As the MAM-SDP author mentions in Ponnambalam (1987) and Ponnambalam and Adams (1989,1996), this method is based on heuristics and this analysis proposes to assess different forms of possibly rearranging the aggregation scheme and how this scheme might influence the final results, namely, the computed expected returns or cost and their respective variances. Of course, this type of study was not performed for a Single Reservoir System. PCA is employed as a means of evaluating which component of the reservoir system is responsible for the greater portion of the total variance of returns/cost of the system. Once this is established, this specific reservoir is set to be optimized in the first stage of MAM-SDP optimization. Proceeding in this fashion, the inherent suboptimality due to local policies for this specific reservoir is avoided. Because it is then optimized against the rest of the entire system, the policies will no longer be local, although they may still be suboptimal. Furthermore, a way of implementing the well-known Expected Return-Variance of Return plots for multistage decision processes using SDP is presented. This is the major asset of this thesis. Before applying the methodologies presented in this chapter in the Study Case, Chapter 4, they were extensively tested with some standard reservoir systems that appeared previously in the literature. The next section describes them and, as the methodologies are explained, the main results, shown. These results had to be summarized for the sake of presentation but they were always evaluated by means of simulation models employing stochastic inputs.

3.2 Test Problems and Preliminary Results

To assess the performance of the models here developed and validate the results obtained, they were compared with those from already extensively studied standard test problems, using the described methodology or employing problems already solved by other researchers. This section will focus in presenting these problems and the first stage of the study, i.e., the comparison between our results and those obtained by others. Once the models were thus validated, they were then extended to apply the proposed methodology. Three types of problems with increasing order of complexity were used:

- Single Reservoir System,
- Three Reservoir System, and
- Four Reservoir System.

The Three Reservoir System differs from the Four Reservoir not only in size but in its configuration too. While the former is a system in series, the latter has a configuration of the mixed type. The major reason for the inclusion of the Single Reservoir is to present the reader with a system where it is possible to evaluate the Two-Pass Mean-Variance Approach without aggregation considerations and therefore allow a better understanding of the results obtained with the technique.

3.2.1 Single Reservoir System

The same problem appeared in Fletcher (1995). The values are all presented in units of volume. Because this is a test problem, all the variables were already transformed to volume, including discharges, that are given as volume per month. The relevant data follow.

Discretization	Probabilities	Values for the ordinates
1	0.0668	-1.83
2	0.2417	-0.89
3	0.3830	0.00
4	0.2417	0.89
5	0.0668	1.83

Table 3 - 1 Discretization of probabilities for the inflows

Note that for the stochastic inflows it was assumed a discretized Normal distribution with values that are the same as those that appear in Turgeon (1981).

The next table presents the average monthly inflows and return coefficients. These coefficients were obtained as the average power generated per month per unit of volume of water released from the reservoir.

Month	Average Inflow	Return Coefficients
January	3.4	1.4
February	3.7	1.1
March	5.0	1.0
April	5.0	1.0
May	7.0	1.2
June	6.5	1.8
July	6.0	2.5
August	5.5	2.2
September	4.3	2.0
October	4.2	1.8
November	4.0	2.2
December	3.7	1.8

Table 3 - 2 Average Monthly Inflow and Return Coefficients

The constraints on storage and releases are listed next. From the maximum and minimum storage it is possible to determine the active storage.

Month	Maximum Storage	Minimum Storage	Maximum Release	Minimum Release
January	8.0	1.0	4.0	0.0
February	8.0	1.0	4.0	0.0
March	8.0	1.0	6.0	0.0
April	8.0	1.0	6.0	0.0
May	8.0	1.0	7.5	0.0
June	8.0	1.0	12.0	0.0
July	8.0	1.0	8.5	0.0
August	8.0	1.0	8.5	0.0
September	8.0	1.0	6.0	0.0
October	8.0	1.0	5.0	0.0
November	8.0	1.0	4.0	0.0
December	8.0	1.0	4.0	0.0

Table 3 - 3 Maximum and Minimum Monthly Storage Capacity and Maximum and Minimum Releases from the Reservoir

The figure below presents the scheme of the single reservoir system:

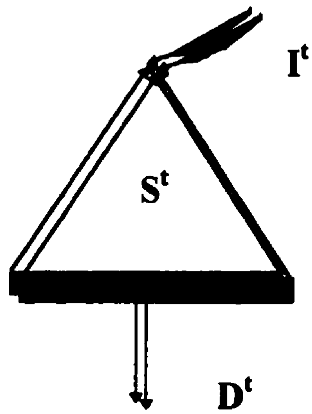


Figure 3 - 1 Scheme of the Single Reservoir System

3.2.2 Objective Function

The aim of the optimization is to maximize a performance index, PI, which gives the total return due to power generation of the single reservoir system.

Therefore,

$$PI = \max \sum_{t=1}^{12} c^t d^t \qquad 3.4$$

Where:

c^t - return from power generation per unit of discharge from reservoir in month t,

d^t - discharge released from reservoir in month t;

subject to the continuity equation:

$$s^t = s^{t-1} + I^t - d^t; \qquad 3.5$$

Two approaches were used to solve this problem. The first one, called Case A, considers penalties for spills occurring during the optimization part. As spills are undesired waste of resources it is assumed that when it happens the system loses them to the exterior. This is quite useful when dealing with systems consisting of more than a single reservoir because what is lost by an upstream reservoir might be conveyed and used by a downstream one with the consideration of adjusting the storage levels above the reservoir capacity at the present period. Another interesting feature is the avoidance of a transition from one state to another that would be infeasible otherwise. The approach can be extended to include states below a specified minimum, provided that this is not equivalent to an empty reservoir. As it is necessary to consider more states than in the usual procedure, the computation becomes more expensive. The second one, called Case B, only takes into account states occurring within the boundaries of the reservoir capacity.

3.2.3 Three Reservoir System

The following test problem appeared in Ponnambalam (1987) and was used to validate the long-term, steady-state operation model then presented. It was based on an originally existing reservoir system with a few alterations in the input data. The main objective of the system was the maximization of the annual accumulated releases for irrigation purposes. Because of the characteristics of the system, only the releases from the most downstream reservoir are conveyed to irrigation. The one year cycle was split into five different seasons, and the two most upstream reservoirs are fed with natural inflows. These were assumed independently, identically and normally distributed. Another assumption was having a coefficient of correlation equal to one between them and natural inflow to reservoir 2 could be regressed from natural inflow to reservoir 1 by the relation: $I_2^t = 0.5036847\ I_1^t$. No natural inflow is present for reservoir 3. A second test model was derived from this one, with their capacities modified. The objective function employed is the same as the one used by the author, namely, the maximization of the discharges from reservoir 3. The other input data follow.

Season	Average Inflow	Standard Deviation
1	10.25	3.15
2	210.27	44.90
3	133.51	43.16
4	73.60	20.86
5	76.39	12.88

Table 3 - 4 Mean and Standard Deviation of Natural Inflow to Reservoir 1

Problem 1			Problem 2		
Reservoir 1	Reservoir 2	Reservoir 3	Reservoir 1	Reservoir 2	Reservoir 3
150.00	400.00	45.00	200.00	200.00	200.00

Table 3 - 5 Active Storages for Problems 1 and 2

The capacities for the channels leaving the reservoirs are the same for all channels in both problems, i.e., 200.00 units of volume per season. Another assumption regards the coefficients of return, the same for all reservoirs and seasons, and its weight is equal to one throughout the year cycle.

3.2.3.1 Objective Function

$$PI = \max \sum_{t=1}^{5} c_3 d_3^{t} \qquad \mathbf{3.6}$$

where

c_3^t - return from irrigation per unit of discharge from reservoir *i* in month *t*,

d_3^t - discharge released from reservoir *i* in month *t*;

subject to the continuity equations:

$$s_1^t = s_1^{t-1} + I_1 - d_1^t;$$
$$s_2^t = s_2^{t-1} + I_2^t + d_1^t - d_2^t; \quad (3.7)$$
$$s_3^t = s_3^{t-1} + d_2^t - d_3^t;$$

Figure 3 - 2 below shows a schematic representation of the Three Reservoir System, Problem 1 and Problem 2.

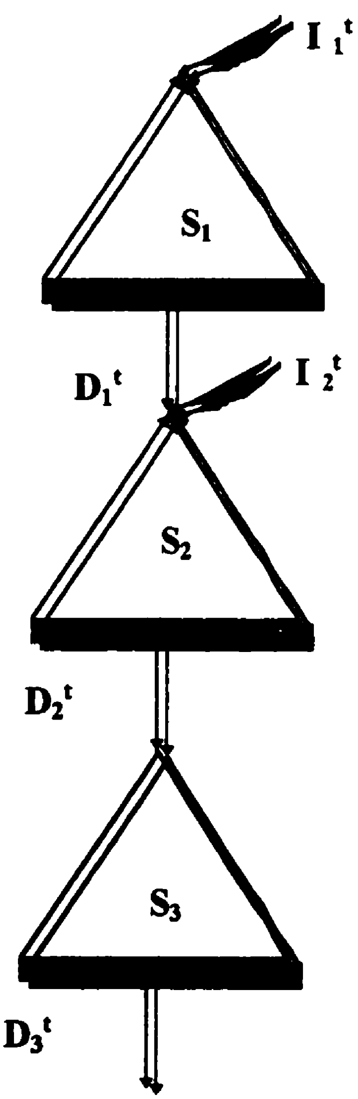


Figure 3 - 2 Scheme of Three Reservoir System in Series

Ponnambalam (1987) reports obtaining, for 10-state discretization, a global maximum return (using straight SDP) of 745.56 units of volume (*u.v.*) of expected annual releases from the system for Problem 1. Using the aggregation technique he now obtained 745.54 *u.v.* which is practically the same value. For Problem 2, the same author reported 748.30 *u.v.* and 748.31 *u.v.* respectively. The results were computed after 1000 year stochastic simulation. The value presented in the "Average Annual Input" row serves as a benchmark for the upper boundary for the optimization, because it is the maximum possible annual return for the operation of the system. Next, the preliminary results are presented for these test problems. For a system that has irrigation as the main purpose, the event failure of the type deficit has precedence over overflow. The same observation applies to hydropower generation systems.

Problem 1

Average Annual Releases

Reservoir 1	498.74
Reservoir 2	751.87
Reservoir 3	751.87

Average Annual Input	756.61
Average Annual Output	751.87
Difference	4.74

Probabilities of Failure

	DEFICIT	OVERFLOW
Reservoir 1	0.0088	0.0430
Reservoir 2	0.0006	0.0000
Reservoir 3	0.0006	0.0000

Problem 2

Average Annual Releases

Reservoir 1	502.93
Reservoir 2	744.07
Reservoir 3	744.07

Average Annual Input	756.61
Average Annual Output	744.07
Difference	12.54

Probabilities of Failure

	DEFICIT	OVERFLOW
Reservoir 1	0.0074	0.0048
Reservoir 2	0.0000	0.1018
Reservoir 3	0.0000	0.0000

The results above aimed at validating the routines employed to solve these two test problems. All the values above were computed after 1000 year stochastic simulation with time-step equal to one month. With respect to performance index the results were similar to those reported by Ponnambalam (1987) but are not exactly the same. While that author had a better performance with the type 2 problem, in this work problem 1 has the advantage over problem 2. The explanation for this better performance of type 1 problem is due to the size of reservoir 2 in its configuration. As it is the double in size with respect to problem type 2, it was more able to accommodate the inflow disturbances. The reader is reminded that only reservoirs 1 and 2 are fed by natural inflows. For the results shown, reservoir 3 has no actual participation in the regulation, its role is limited to reproduce the outflows from the previous one. In fact, the release policies obtained for reservoirs 2 and 3 are exactly the same. In terms of performance of the system, it could be removed from it with no effect in the regulation. Of course, this is probably not the case with the real system and is certainly a consequence of the approximations employed in setting up the case, mostly for the inflow discretization and its probability distribution.

Another feature worth mentioning is that the optimization of the operation of these systems required the inclusion of states that are located out of the maximum storage bounds. That is to say that, to have feasible states in the SDP optimization, it was necessary to consider states where spills were taken into account. Otherwise the optimization routine would not converge to a result.

With respect to probabilities of failure the results merely reflect the importance of the size of the reservoirs. Because in type 1 problem reservoir 1 is smaller than in type 2 and the inflows are same, it is more prone to have deficit and overflow situations. The opposite happens in reservoir 2, i.e., as it is larger, it presents null probability of overflow for type 1 while for type 2 it is around 10 %. The probability of deficit, although very small, it is still present because of the higher probability of the same kind of event in reservoir 1.

3.2.4 Four Reservoir System

One of the problems used for testing the MAM-SDP model incorporates a combined configuration, that is, series and parallel. Figure 3 - 3 shows a schematic representation of the system. Because of its characteristics, although small in size, it is a very complicated case to solve. It is a four reservoir system, with only two natural inflows. These inflows are spatially correlated, namely, it was assumed that both are located in the same catchment area, and no serial correlation was considered.

The problem, in the fashion that appears in this work, has also been solved by Chow and Cortez-Rivera (1974), Murray and Yakowitz (1979), Chara and Pant (1984), and Fletcher et al. (1994). The problem originally appeared in the literature in Chow and Cortez-Rivera (1974) and became a benchmark for water resources optimization of the operation of reservoir systems. As cited in Ponnambalam and Adams (1989), the main characteristics of this problem are having a linear objective function and the benefit coefficients c_i^t, for each reservoir i, being a function of time t. A careful comparison between the data that appear in Chow and Cortez-Rivera (1974) and some of the other references will show that although there are some minor differences between the monthly inflows and the capacities of Reservoir 1, the comparisons of the maximum benefit are made with the one obtained by referred authors The same data that appeared in that technical report is reproduced below. Note that in Ponnambalam and Adams (1989) this four reservoir problem has been solved only deterministically. For the first time, the results of applying MAM-SDP to the four reservoir problem are presented here.

3.2.4.1 Input to the Problem

Month	Inflow to Reservoir 1 - I_1	Inflow to Reservoir 2 - I_2
January	0.75	1.00
February	2.00	2.00
March	2.00	3.00
April	4.00	3.50
May	3.50	2.50
June	3.00	2.00
July	2.50	1.25
August	1.30	1.25
September	1.20	0.75
October	1.00	1.75
November	0.75	1.00
December	0.40	0.50

Table 3 - 6 Average Values for the Inflows per Month in units of volume

Month	Reservoir 1	Reservoir 2	Reservoir 3	Reservoir 4
January	18.00	8.00	15.00	12.00
February	17.00	8.00	15.00	12.00
March	15.00	8.00	15.00	12.00
April	15.00	8.00	15.00	10.00
May	15.00	8.00	15.00	9.00
June	12.00	8.00	15.00	8.00
July	12.00	8.00	15.00	8.00
August	15.00	8.00	15.00	9.00
September	17.00	8.00	15.00	10.00
October	18.00	8.00	15.00	10.00
November	18.00	8.00	15.00	12.00
December	18.00	8.00	15.00	12.00

Table 3 - 7 Maximum Admissible Storage Capacities of the Reservoirs per Month in units of volume

The minimum permissible storage volume per reservoir throughout the year is 1.0 unit of volume.

Month	Reservoir 1	Reservoir 2	Reservoir 3	Reservoir 4
January	4.50	4.50	8.00	4.00
February	4.50	4.50	8.00	4.00
March	4.50	4.50	8.00	4.00
April	4.50	4.50	8.00	4.00
May	4.50	4.50	8.00	4.00
June	4.50	4.50	8.00	4.00
July	4.50	4.50	8.00	4.00
August	4.50	4.50	8.00	4.00
September	4.50	4.50	8.00	4.00
October	4.50	4.50	8.00	4.00
November	4.50	4.50	8.00	4.00
December	4.50	4.50	8.00	4.00

Table 3 - 8 Maximum Admissible Releases from the Reservoirs per Month in units of volume

The minimum permissible release per reservoir throughout the year is 0.005 unit of volume.

Month	Reservoir 1	Reservoir 2	Reservoir 3	Reservoir 4
January	1.4	1.0	2.6	1.1
February	1.1	1.0	2.9	1.0
March	1.0	1.2	3.6	1.0
April	1.0	1.8	4.4	1.2
May	1.2	2.5	4.2	1.8
June	1.8	2.2	4.0	2.5
July	2.5	2.0	3.8	2.2
August	2.2	1.8	4.1	2.0
September	2.0	2.2	3.6	1.8
October	1.8	1.8	3.1	2.2
November	2.2	1.4	2.7	1.8
December	1.8	1.1	2.5	1.4

Table 3 - 9 Return due to Power Generation per unit of volume Released per Reservoir and per Month (coefficient c)

3.2.4.2 Objective Function

The aim of the optimization is to maximize a performance index, PI, which gives the total return due to power generation of the entire system of four reservoirs. Therefore,

$$PI = \max \sum_{i=1}^{4} \sum_{t=1}^{12} c_i^t d_i^t \qquad 3.8$$

where

c_i^t - return from power generation per unit of discharge from reservoir i in month t,

d_i^t - discharge released from reservoir i in month t;

subject to the continuity equations:

$$
\begin{aligned}
s_1^{\,t} &= s_1^{\,t-1} + I_1 - d_1^{\,t}; \\
s_2^{\,t} &= s_2^{\,t-1} + d_1^{\,t} - d_2^{\,t}; \\
s_3^{\,t} &= s_3^{\,t-1} + d_2^{\,t} + d_4^{\,t} - d_3^{\,t}; \\
s_4^{\,t} &= s_4^{\,t-1} + I_2^{\,t} - d_4^{\,t}
\end{aligned}
\qquad 3.9
$$

3.2.4.3 Scheme of the Four Reservoir System

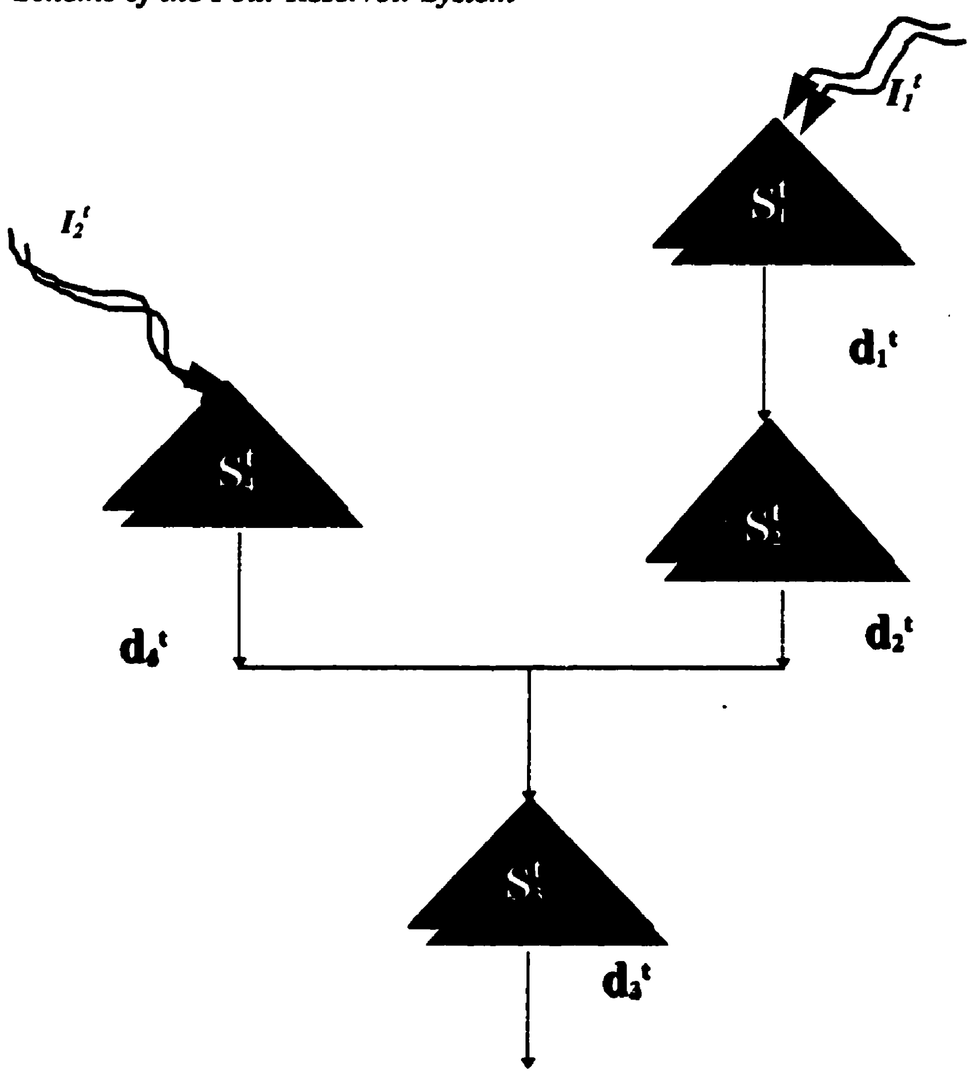


Figure 3 - 3 Scheme of the 4 Reservoir System

3.2.4.4 Results Obtained with MAM-SDP

The following tables present sample results from applying MAM-SDP (2-level) to the four reservoir problem.

3.2.4.4.1 General Information

Next is presented the full optimization with the application of the standard Two-Level MAM-SDP scheme for definition of release targets. Standard scheme stands for aggregation performed from upstream to downstream. The figures presenting the three stages for this kind of configuration are shown next.

First Stage of Aggregation/Decomposition:

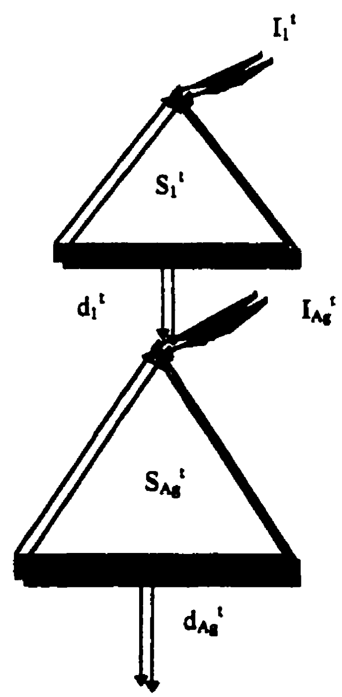


Figure 3 - 4 First Stage Scheme

Second Stage of Aggregation/Decomposition:

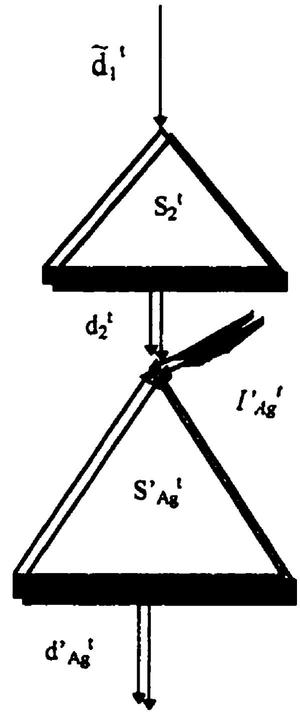


Figure 3 - 5 Second Stage Scheme

Third Stage of Aggregation/Decomposition:

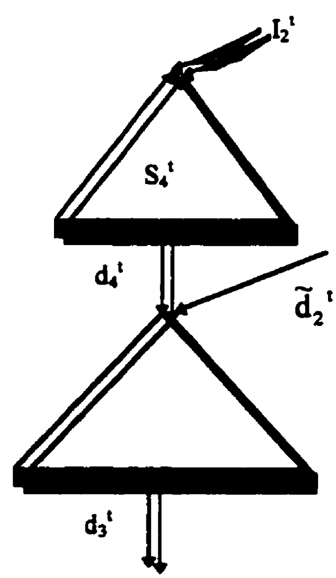


Figure 3 - 6 Third Stage Scheme

Coefficient of Variation = 0.30

Average Annual Return:

Reservoir 1	40.83
Reservoir 2	46.31
Reservoir 3	172.67
Reservoir 4	38.29
System	298.11

Total Input	42759.00
Total Output	42719.00
Difference	39.41

Probabilities of Failure:

	DEFICIT	OVERFLOW
Reservoir 1	0.0001	0.0018
Reservoir 2	0.0042	0.0000
Reservoir 3	0.0001	0.0000
Reservoir 4	0.0033	0.0016

Coefficient of Variation = 1.00

Average Annual Return:

Reservoir 1	38.97
Reservoir 2	42.95
Reservoir 3	143.19
Reservoir 4	26.29
System	251.40

Total Input	46019.00
Total Output	39301.00
Difference	6717.50

Probabilities of Failure:

	DEFICIT	OVERFLOW
Reservoir 1	0.8009	0.0024
Reservoir 2	0.6499	0.0001
Reservoir 3	0.3386	0.0000
Reservoir 4	0.0665	0.2519

Where the coefficient of variation of the normally distributed inflows are given as a percentage of the mean value as presented in Chara and Pant (1984) and Murray and Yakowitz (1979). It is necessary to note that during the stochastic simulation the normal distribution had to be truncated to avoid the presence of negative inflows. This occurred because the random number generator produced negative values. However, during the optimization part no negative values were registered allowing the use of the normal distribution with no truncation. The same observations are valid for the previous test cases.

The results were obtained after a stochastic simulation was performed maintaining the same standard deviations (disturbance) for the inflows as those employed for the stochastic optimization. Note that generally it is not easy to predict the effect of discretization. However, it is anticipated that the policies generally have worse variance in benefit than those obtained with finer discretization policies.

Also, one should observe that reservoir 3 has four specific characteristics that make it different from the others. And, as reservoir 2, no natural inflow. It has:

1. the highest return per unit of volume discharged;
2. the largest active storage;
3. the largest admissible releases;
4. a location characteristic, it is the most downstream reservoir.

From these items, it is easy to deduct that it is more efficient to discharge from it than from the others. Another circumstance to consider for the location characteristic is the fact that in the aggregation scheme set up it was chosen as one of the last reservoirs to be optimized. As a result, as the decomposition is carried out, and the decisions become more local, the two last reservoirs will be most affected by this limitation. To test this within the MAM-SDP formulation the possibilities are twofold. In the combined configuration, parallel and in series, rearrange the aggregation scheme. In any type of configuration scheme, increase the level of the algorithm.

As a level greater than 3 or 4 is not very tractable computationally in sequential processing, it would be of interest to consider high performance computing, including parallel processing for this purpose.

Remarks:

1. The problem was also solved using the standard Linear Programming routines of Matlab® Optimization Toolbox in deterministic fashion as a means of defining an upper bound for the SDP optimization. The optimum values for the objective function obtained during the LP optimization were PI = 313.24 for the case where no starting and ending points were previously defined. For the same case as the one presented in Chara and Pant (1984) the PI was 309.40.

2. Although the policy was obtained for a Deterministic LP model, these results can be used as a starting point for a stochastic optimization.

3.3 MAM-SDP with Approximate Conditional Probabilities

The following suggestion of extending MAM-SDP came from empirical results. Part of the suboptimality of the results obtained when applying the technique was originated from the fact that not all the states were tested during the stochastic DP with the joint conditional probabilities for the releases. These conditional probabilities are obtained once the optimization is completed and the system is simulated. Because they are obtained from steady-state conditions, the transformations do not include many states. Add to this the approximate character of the aggregate reservoir. The first contradictory results appeared when, as the coefficient of variation increased, and consequently the uncertainty, the performance of the system improved. For instance, when computing the Four Reservoir System, in the so-called extended model, where spills were considered as states, for a coefficient of variation of 0.30 the Performance Index reached 264.34 and for a coefficient equal of 1.00, with much more uncertainty added, 283.64. The second evidence was again improved performance, now when testing different aggregation schemes that used these type of probabilities in less stages. Although the coefficient of variation was the same, the improvement was considerable. The affirmations above are illustrated by presenting a sample of the probabilities obtained with different coefficient of variations for the same epoch, in the present case, the month of January. These samples were collected from the Four Reservoir System Test Case. These are the Conditional Probabilities for the releases coming from Reservoir 1 after the first stage of MAM-SDP. In this case a very small coefficient of variation for the inflows was used, namely, 0.05. Therefore, releases from it are conditional on the state of Reservoir 1 versus the rest of the system. The first graph shows the conditional probability surface as is obtained just after the simulation, while the second one, the transformed one.

The transformation was accomplished by means of a function that resets the matrix of Conditional Distribution Probability of Releases with respect to the storages. It assumes that the infeasible regions (represented by zeroes) can be replaced by the relations "greater than" or "less than", therefore diffusing the probability region up to the boundaries. In this work a linear transformation was used for the sake of simplicity and speed of computation. As an example it follows the transformation for a simple and small matrix of probabilities.

$$A_{joint} = \begin{matrix} 0.9500 & 0.0200 & 0.0000 \\ 0.0100 & 0.0100 & 0.0000 \\ 0.0100 & 0.0000 & 0.0000 \end{matrix} \rightarrow A_{marginal} = \begin{matrix} 0.9794 & 0.6667 & 0.6667 \\ 0.0103 & 0.1667 & 0.1667 \\ 0.0103 & 0.1667 & 0.1667 \end{matrix}$$

Where:

A_{joint} - Joint Conditional Probability of Releases,

$A_{marginal}$ - Approximate Marginal Probability of Releases.

Please note that, for column vectors that present the Marginal Probability of Releases, the conditional probabilities are already preprocessed for the SDP optimization, i.e., while $\Sigma_{ij} A_{joint} = 1$; now the following is true: $\Sigma_i A_{marginal.(j-1)} = \Sigma_i A_{marginal.(j-2)} = \Sigma_i A_{marginal.(j-3)} = 1$. Not only for this specific case but for the most of the steps where this was possible, preprocessing was employed as a means of reducing the computation time. The previous example was just an illustration and was not used in the actual computations for the test problems.

Figure 3 - 7 presents one of the joint conditional probabilities of the releases for reservoir 1 as computed by the standard MAM-SDP. The probabilities have the shape of spikes and this characteristic is reinforced when the inflows have little noise. Figure 3 - 8 presents the preprocessed Marginal Probabilities. A linear transformation was considered sufficient for an initial approximation, especially because of the other simplifications involved. The following tables show the original probabilities and those derived from them. Note that the knots in the mesh were connected using linear relationships.

From:

State	1	2	3	4	5	6
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.7410	0.0020	0.0000	0.0000
3	0.0000	0.0000	0.2570	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

To:

State	1	2	3	4	5	6
1	0.0400	0.0400	0.0400	0.0244	0.0244	0.0244
2	0.1920	0.1920	0.1920	0.1951	0.1951	0.1951
3	0.1920	0.1920	0.1920	0.1951	0.1951	0.1951
4	0.1920	0.1920	0.1920	0.1951	0.1951	0.1951
5	0.1920	0.1920	0.1920	0.1951	0.1951	0.1951
6	0.1920	0.1920	0.1920	0.1951	0.1951	0.1951

Therefore, graphically, from:

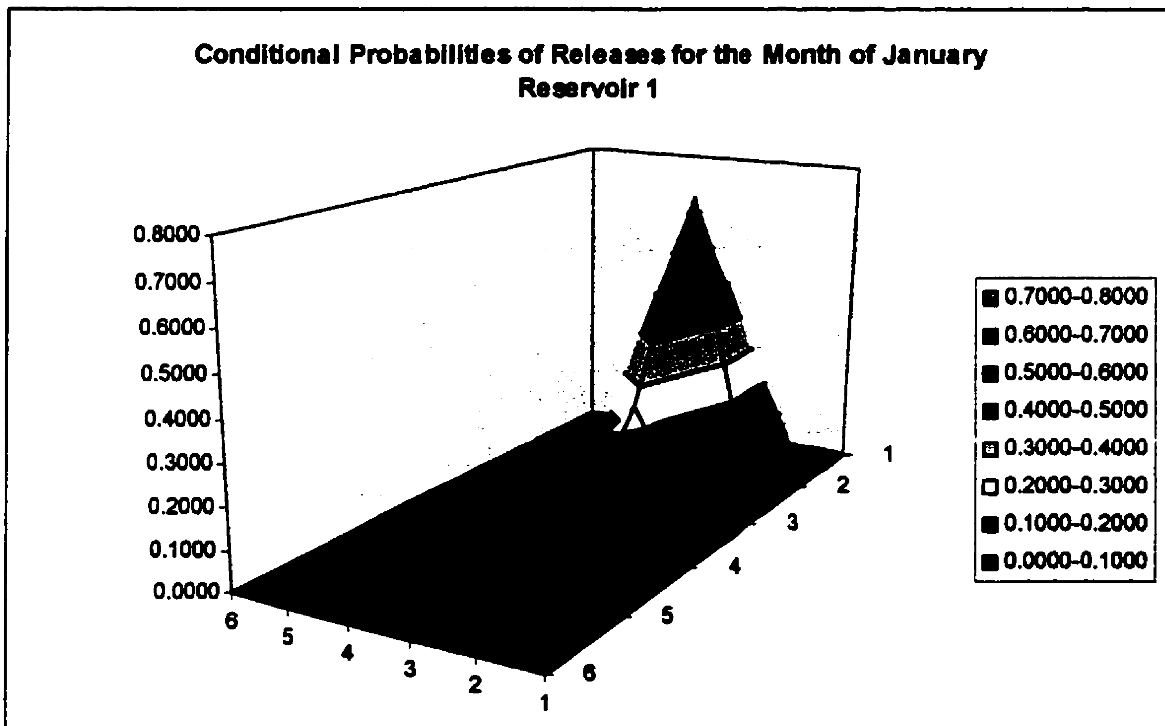


Figure 3 - 7 Conditional Probabilities for the Releases from Reservoir 1

To:

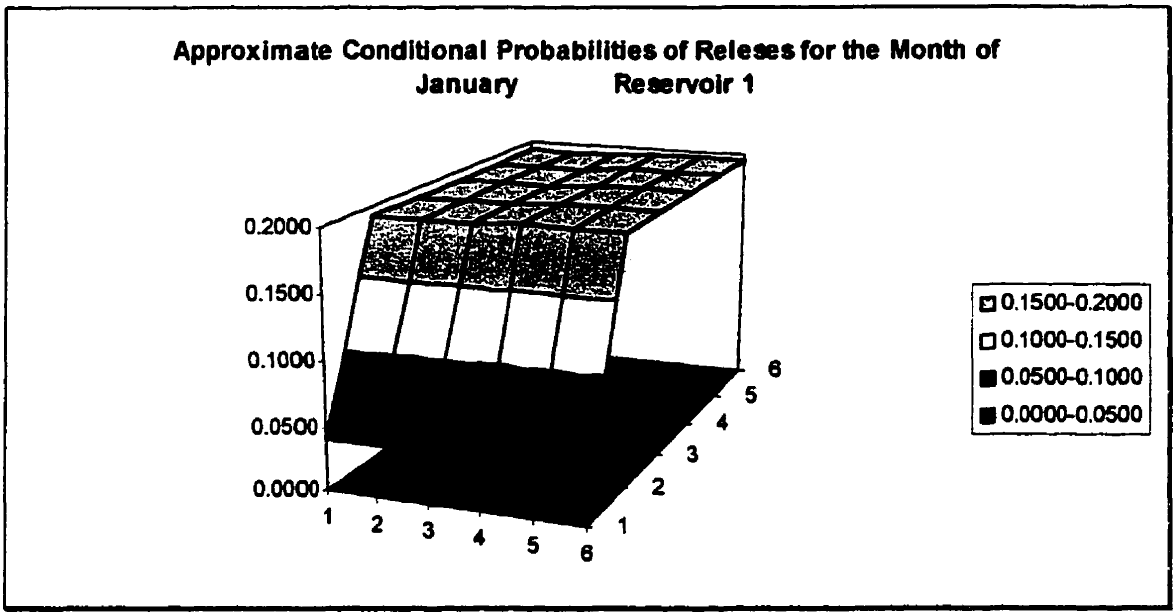


Figure 3 - 8 Marginal Probabilities for the Releases from Reservoir 1

3.3.1 Main Results with the Four Reservoir System Test Problem

3.3.1.1 Coefficient of Variation: 0.30

Table 3 - 10 shows the results obtained for the MAM-SDP according to what was described. The need for adjustment of policies stems from the fact that some of the states present release policy equal to zero. When this happens for the last states of the system it easily falls into what is known as a trapping state, once in it, the system does not leave to another state. Other corrections were also made, more specifically interpolation for intermediate release policies with values equal to zero between two other values with clear tendencies. For example, if state 1 had release policy equal to 20, state 2, zero and state 3, 40; state 2 would end up with release policy equal to 30, after linear interpolation. The decision to employ this technique was due to the realization that it really improved the performance of the method in a substantial way.

	System	Reservoir 1	Reservoir 2	Reservoir 3	Reservoir 4
Perf. Index	264.34	42.17	38.19	153.62	30.36
Std. Dev.	30.21	4.53	6.14	15.46	5.07
Std. Dev. %	11.43	10.74	16.08	10.06	16.70
Prob. Prob. Spill	-	0.00	0.21	0.06	1.29
Prob. Def.	-	8.44	2.53	0.00	0.00

Table 3 - 10 Case 1 - MAM-SDP (with adjustment of release policies)

For the Turgeon's method there was no need to use adjustment of policies and the models were set as described in his 1981 paper. With the exception of the modules dedicated to the computation of the conditional distribution of the releases the rest of the model is exactly the same as the one used above. Isolating these factors allowed us to a meaningful comparison between their performance. The performance of the system was significantly better than with the method above, even with respect to the standard deviation of the returns, which are smaller.

	System	Reservoir 1	Reservoir 2	Reservoir 3	Reservoir 4
Perf. Index	293.67	41.67	44.71	167.71	39.58
Std. Dev.	21.56	3.56	3.89	11.95	3.18
Std. Dev. %	7.34	8.54	8.70	7.13	8.03
Prob. Spill	-	0.15	0.00	0.00	0.89
Prob. Def.	-	0.05	0.01	0.00	0.02

Table 3 - 11 Case 2 - Turgeon Method (no adjustment of release policies)

MAM-SDP using a different approximation to the conditional distribution of releases performed just slightly better than the one from Turgeon's but at the cost of higher standard deviation. Also, when considering that the upper bounds for the Performance Index for this type of system ranges around 310 units for the deterministic problem, this one, with a coefficient of variation of 30 % performs nicely, here considered all the approximations used in the aggregation part.

	System	Reservoir 1	Reservoir 2	Reservoir 3	Reservoir 4
Perf. Index	295.28	40.06	45.81	169.42	39.99
Std. Dev.	24.31	4.60	4.82	12.50	3.98
Std. Dev. %	8.23	11.48	10.52	7.38	9.95
Prob. Spill	-	0.01	4.17	0.43	0.38
Prob. Def.	-	1.64	0.94	0.01	0.05

Table 3 - 12 Case 3 - MAM-SDP (with marginal distribution of releases)

Table 3 - 13 presents a test with the use of penalties for spills, i.e., the inclusion of states that include spills to the storage levels. For this level of uncertainty, the model performance was very slightly inferior to the one shown in Table 3 - 12 not only with respect to PI, but its standard deviation was higher too.

	System	Reservoir 1	Reservoir 2	Reservoir 3	Reservoir 4
Perf. Index	294.84	40.04	45.76	169.16	39.88
Std. Dev.	25.02	4.70	5.04	12.78	3.94
Std. Dev. %	8.49	11.74	11.01	7.55	9.88
Prob. Spill	-	0.01	0.00	1.31	0.37
Prob. Def.	-	1.63	1.00	0.00	0.06

Table 3 - 13 Case 4 - MAM-SDP (as Case 3 plus considering penalties for the spills)

3.4 Principal Components Analysis

In the previous chapter, Literature Review, the focus of the attention concentrated in presenting the aggregation methodologies developed by Ponnambalam and Adams (1989), MAM-SDP, and Turgeon (1981), decomposition method. It was pointed out that, as stressed by the author, Ponnambalam's algorithm was based on heuristics and the solutions obtained, suboptimal.

MAM-SDP allows the modeler to employ different levels of aggregation until a suitable one is found. However, in terms of practical applications, the use of more than 3 or four states would produce very expensive computations, time wise. A paper by Archibald, McKinnon, and Thomas (1997) presents an interesting suggestion of employing 3-level aggregation scheme where, with the obvious exception of the reservoir located at the most upstream position within the system in consideration, the drawback of having local policies is averted by always taking into account the entire system. It must be reminded that this possibility was already present in Ponnambalam's paper just mentioned above. In both methods presented by Turgeon and Ponnambalam the aggregation scheme always starts by the most upstream reservoir and proceed downward in the system until the last two reservoirs are optimized in the last stage. But while this is one of the range of possible approaches, it does not necessarily have to be so. For instance, let us consider a system composed of five reservoirs in series. Also, that the capacity of the most upstream one is greater than the sum of the capacities of the rest of the system. Even intuitively, the importance of the most upstream reservoir within the system is evident. And having local policies might heavily affect the performance of the system. Thus, the definition of the importance of each component of the system must be clearly defined and taken into consideration when establishing the aggregation scheme. Saad and Turgeon (1988) applied Principal Components Analysis (PCA) to define the reduced model of the system.

Their method will be briefly reviewed and then a similar version of it, to be applied to MAM-SDP, will be presented. Alternative aggregation schemes to the "top down" sequence are proposed having in mind their effect on the objective function, and the consequent spread around its mean values.

Saad and Turgeon (1988) developed a method where the implicit approach is used "to reduce the number of state variables in the problem, and the explicit approach is used to find the optimal solution of the reduced problem". As cited in their paper, aggregation, and consequent reduction of the size of the problem, is possible if the state variables are interdependent. It has the major advantage of considering explicitly the stochastic nature of the disturbances, e.g., natural inflows, net basin supplies and, what may be viewed as an intrinsic weakness, the need for interdependency between the states, is generally present in reservoir systems, which are usually located in the same river, catchment area, or groundwater aquifer.

The solution method presented by them is succinctly described below.

Step 1: Generate m synthetic flow sequences with the same length using the statistical information existing in the historical flows. The minimum suggested number of sequences is 30 but a higher one is advisable, for the sake of accuracy.

Step 2: The optimization problem is solved deterministically for each of the synthetic flow sequences, generating m deterministic solutions. (Here the possible optimization methods are not specified, the reader is referred to the original paper for further information.)

Step 3: Perform Principal Components Analysis and determine whether the system can be modeled with fewer variables and the most appropriate scheme of doing so.

The formal justification for employing PCA comes from the following reasoning. It is necessary to find a vector ξ such that its elements are uncorrelated and are linear functions of the vector of observations **x**. Therefore,

$$\xi = \mathbf{lx} \qquad 3.10$$

For the elements of ξ to be uncorrelated, following must be true

$$E(\xi_i \xi_j) = E\{\sum_{k=1}^{p} l_{ik} \sum_{m=1}^{p} l_{jm} x_m\} = 0,\ i \neq j \qquad 3.11$$

The equation above translates into

$$\sum_{k,m=1}^{p} l_{ik} l_{jm} E(x_k x_m) = \sum_{k,m=1}^{p} l_{ik} l_{jm} c_{km} = 0 \qquad 3.12$$

where c_{km} is the covariance of x_k and x_m. The index p refers to the number of variables. Also, we are interested in obtaining an orthogonal transformation, which will lead us to

$$\mathbf{l}\mathbf{l}' = \mathbf{I} \qquad 3.13$$

and **I** is the identity matrix. Rewriting equation 3.7, we obtain

$$E(\xi\xi') = E\{\mathbf{lx}(\mathbf{lx})'\} = E\{\mathbf{lxx}'\mathbf{l}'\} = \mathbf{lcl}' = \Lambda \qquad 3.14$$

Matrix Λ is a diagonal matrix where its nonzero elements are the variances of ξ's. From the previous equation, $\mathbf{cl}' = \mathbf{l}'\Lambda$ and therefore

$$|\mathbf{c} - \lambda\mathbf{I}| = 0 \qquad 3.15$$

where λ are the eigenvalues and the columns of **l**, the eigenvectors.

The uncorrelated, orthogonal linear transform ξ are the Principal Components. When the **x**'s present normal distribution of the probabilities, they are not only uncorrelated but independent as well. The next and final step is to rearrange the principal components in decreasing order, the first component corresponds to the variable with the largest variance, the second component with the variable with the second largest variance, and so on. The expression Principal Components Analysis comes from the fact that the sum of the eigenvalues corresponds to the sum of the distances from their center of gravity. Thus, the ratio $\lambda_i/\Sigma\lambda_i$ gives the relative participation of the λ_i in the total variation.

Based on what was exposed so far, a way of defining the aggregation scheme in which the policies from the reservoir that accounts for the most of the objective function variance are optimized was presented. It should be reinforced that variance is a measure of dispersion, an undesirable feature for the operation of the reservoir system. In doing so, the aim is to have a greater control on the component of the system that accounts for the greater portion of the variance.

The next page has the flowchart of the proposed way of applying PCA to MAM-SDP with the objective of reducing the variance of the system components that account for the major part of it. It differs from the algorithm suggested by Saad and Turgeon (1988) in the following:

1. The normal MAM-SDP optimization is performed even before a definition of the most appropriate way of aggregating the system. In the previous method, implicit optimization is employed. In this case, once the operation policies are set up by standard MAM-SDP, the stochastic simulation of the system is performed to evaluate them. The assessment of the operation features like stochastic input, adherence to stipulated constraints, performance, rate of failure, amongst others, is better executed in this fashion. Nonetheless, the computation cost is higher.

2. Instead of analyzing the principal component per optimization epoch, the entire cycle is used. One of the reasons for this is having a long-term optimization involved, instead of short-term. The other is having to consider the aggregation scheme as a whole and not per epoch.

ITERATIVE DETERMINATION OF MAM-SDP AGGREGATION

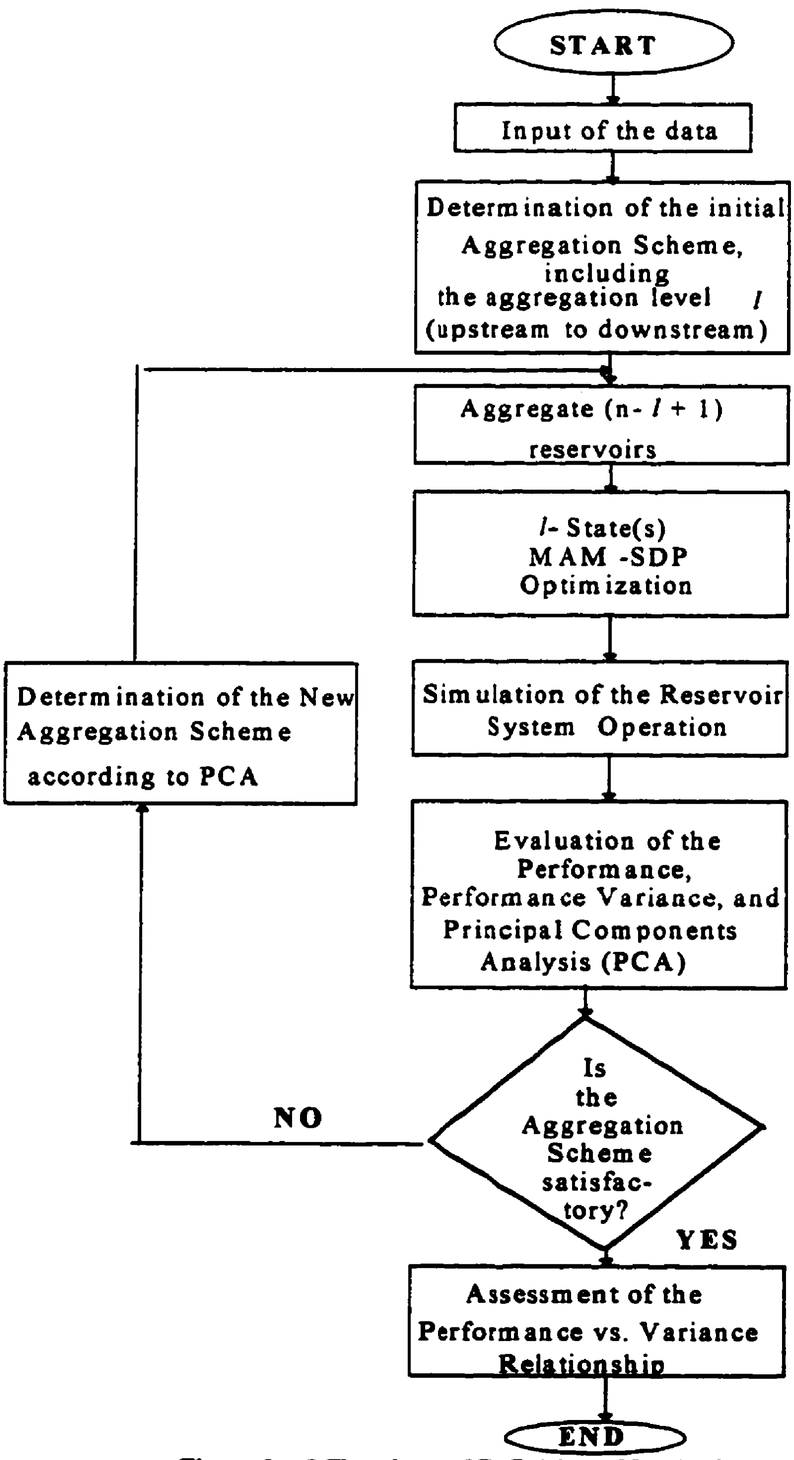


Figure 3 - 9 Flowchart of Definition of MAM-SDP Aggregation Scheme

3.4.1 Test Problems and Results

3.4.1.1 Three Reservoir System:

3.4.1.1.1 Problem 1:

Season	Return			Storages		
	Reservoir 1	Reservoir 2	Reservoir 3	Reservoir 1	Reservoir 2	Reservoir 3
1	10.32	0.00	89.68	7.69	92.31	0.00
2	11.42	0.00	88.58	71.40	28.60	0.00
3	56.47	43.53	0.00	44.61	55.39	0.00
4	72.38	27.62	0.00	11.64	88.36	0.00
5	13.84	0.00	86.16	14.62	85.38	0.00
Cycle Average	32.89	14.23	52.88	29.99	70.01	0.00

Table 3 - 14 Analysis of Variance using Principal Components Analysis Applied to Problem 1 of Three Reservoir System (first test).

From Table 3 - 14 above it is easily noticeable that when considering the observations, in the present case, storages, the Principal Component of the system is reservoir 2. Because reservoir 2 is located in the middle of the system, 2-level MAM-SDP is not applicable to it, the 3-level being already global optimization for this specific case. When considering the returns, reservoir 3 is the Principal Component with 52.88 %, although not significantly with respect to the rest of the system considered as a whole which adds up to 47.12 %. Table 3 - 15 presents MAM-SDP performing the aggregation from bottom to top. This expression means that in the first stage, reservoirs 1 and 2 are aggregated while the release policy is obtained for reservoir 3 alone. In the second stage, reservoirs 1 and 2 are optimized and reservoir 3 is not considered. It should be observed by the reader the existing difference with the results from the previous table, i.e., reservoir 3 had previously no importance for the variance in the storages while in the second case it accounts for the most of it.

Season	Return			Storages		
	Reservoir 1	Reservoir 2	Reservoir 3	Reservoir 1	Reservoir 2	Reservoir 3
1	3.60	8.22	88.17	45.85	33.95	20.20
2	1.00	7.05	91.95	3.00	1.82	95.18
3	0.36	2.10	97.54	15.06	1.85	83.09
4	5.44	0.68	93.88	17.41	2.61	79.97
5	3.09	20.78	76.12	20.72	16.56	62.71
Cycle Average	2.70	7.77	89.53	20.41	11.36	68.23

Table 3 - 15 Analysis of Variance using Principal Components Analysis Applied to Problem 1 of Three Reservoir System (second test)

Figure 3 - 10 below plots the accumulated expected returns for the cycle against their respective variances. Because the original problem concerned irrigation, the only objective was to maximize the output from the most downstream reservoir. Figure 3 - 11 presents a more general picture of the system as a whole as well as per reservoir. In the second case, the standard deviation was used as the measure of dispersion. In graph below, the blue diamonds represent the physically based aggregation scheme (standard) while the pink squares, the one derived from the statistical decomposition. The variances per reservoir are more or less equivalent but when the entire system is taken into account, the standard scheme presents a much better figure, i.e., much less. When considering solely the output from reservoir 3, which is the objective of the optimization, the top down is still superior, although not as significant.

Analyzing the results above the consequent weak conclusion is that the physical diagnosis, here typified by MAM-SDP, performs better than the statistical decomposition. And if so, under which conditions this occur? Unfortunately, for the three reservoir system a fixed value for the coefficient of variation for the inflows was employed, therefore is possible to state that for the present case the physical diagnosis provided better results.

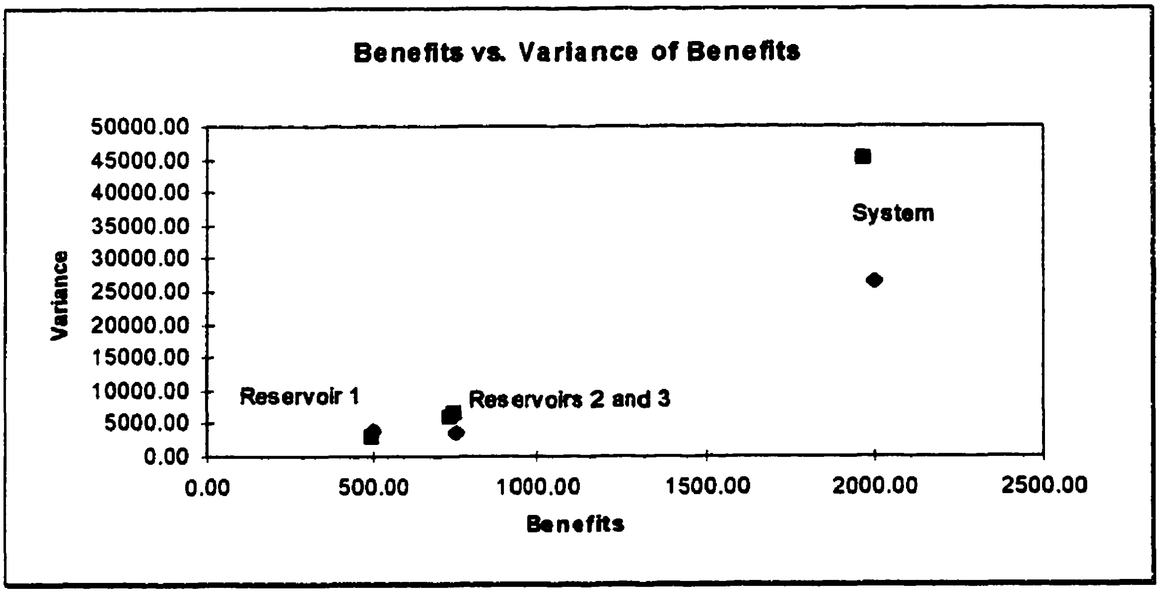


Figure 3 - 10 Performance versus Variance of the Returns, Three Reservoir System, Problem 1

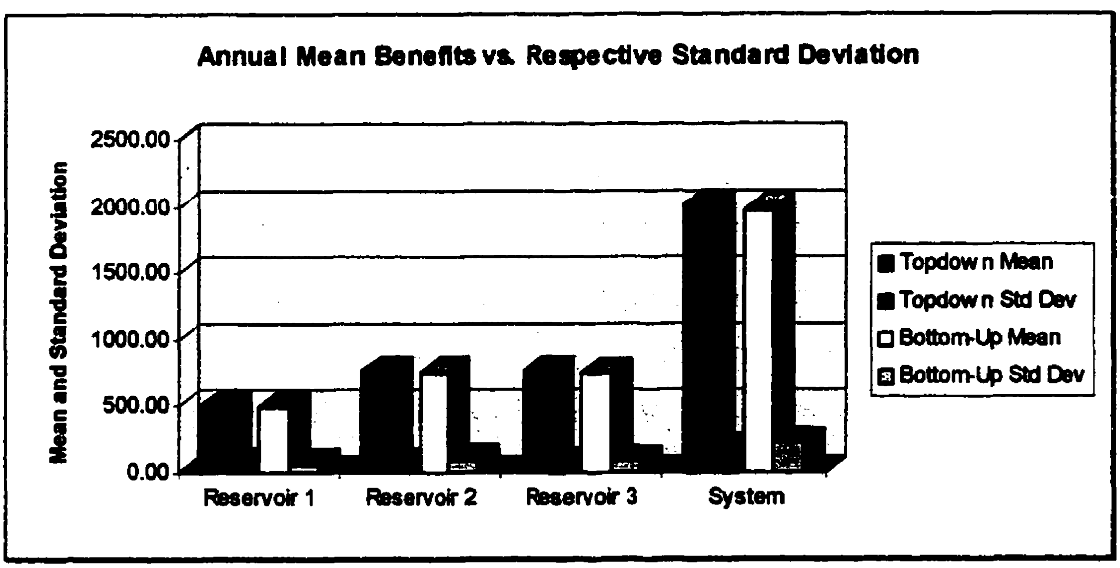


Figure 3 - 11 Average Annual Return and Respective Standard Deviation, Three Reservoir System, Problem 1

3.4.1.1.2 Problem 2:

	Return			Storage		
Season	Reservoir 1	Reservoir 2	Reservoir 3	Reservoir 1	Reservoir 2	Reservoir 3
1	8.47	0.00	91.53	5.91	94.09	0.00
2	29.75	0.00	70.25	74.51	25.49	0.00
3	38.40	61.60	0.00	56.44	43.56	0.00
4	43.13	56.87	0.00	17.70	82.30	0.00
5	27.45	0.00	72.55	15.65	84.35	0.00
Cycle Average	29.44	23.69	46.87	34.04	65.96	0.00

Table 3 - 16 Analysis of Variance using Principal Components Analysis Applied to Problem 2 of Three Reservoir System (first test)

Repeating the comments made for Problem 1, and regarding Table 3 - 16 the Principal Component of the system is once more reservoir 2 with respect to the storages. When considering the returns, reservoir 3 is the Principal Component, but its influence is now smaller than half of the total variation of the system, meaning not significantly with respect to rest of the system together, considering that reservoirs 1 and 2 account for 53.13 %. Table 3 - 17 presents MAM-SDP performing the aggregation from bottom to top. As before, the reader can note the differences with the results from the previous table, e.g., reservoir 3 had previously no importance for the variance in the storages whereas in the second case it accounts for the most of it

	Return			Storage		
Season	Reservoir 1	Reservoir 2	Reservoir 3	Reservoir 1	Reservoir 2	Reservoir 3
1	3.60	8.22	88.17	45.85	33.95	20.20
2	1.00	7.05	91.95	3.00	1.82	95.18
3	0.36	2.10	97.54	15.06	1.85	83.09
4	5.44	0.68	93.88	17.41	2.61	79.97
5	3.09	20.78	76.12	20.72	16.56	62.71
Cycle Average	2.70	7.77	89.53	20.41	11.36	68.23

Table 3 - 17 Analysis of Variance using Principal Components Analysis Applied to Problem 2 of Three Reservoir System (second test)

The graphs below present first the relationship between the accumulated sum of the mean releases for the cycle, that is to say, the objective function for this optimization, with respective variances, while the second one the more general picture of the system as whole and per reservoir. The notation regarding the aggregation schemes is the same as in the previous situation. Also, the same conclusions noted from the previous case are valid here, i.e., variances per reservoir are more or less equivalent between both. However, when the entire system is taken into account the top down aggregation scheme presents better results in terms of variance of the returns. The returns themselves are more or less equivalent, but the variance is significantly smaller. When considering just the output from reservoir 3, the top down is still superior, and not as significant.

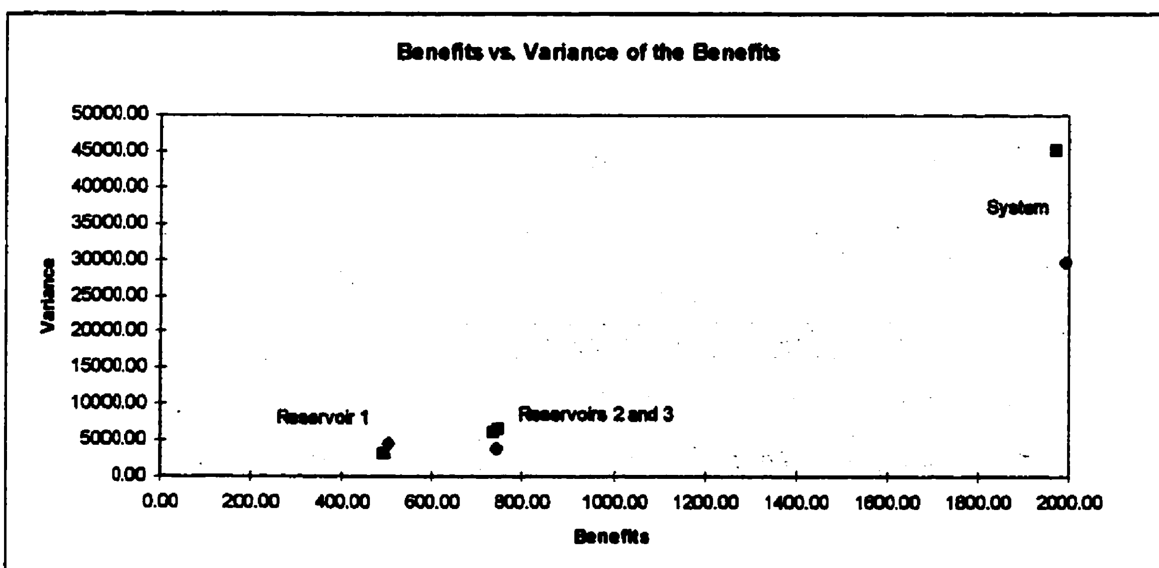


Figure 3 - 12 Performance versus Variances of the Returns, Three Reservoir System, Problem 2

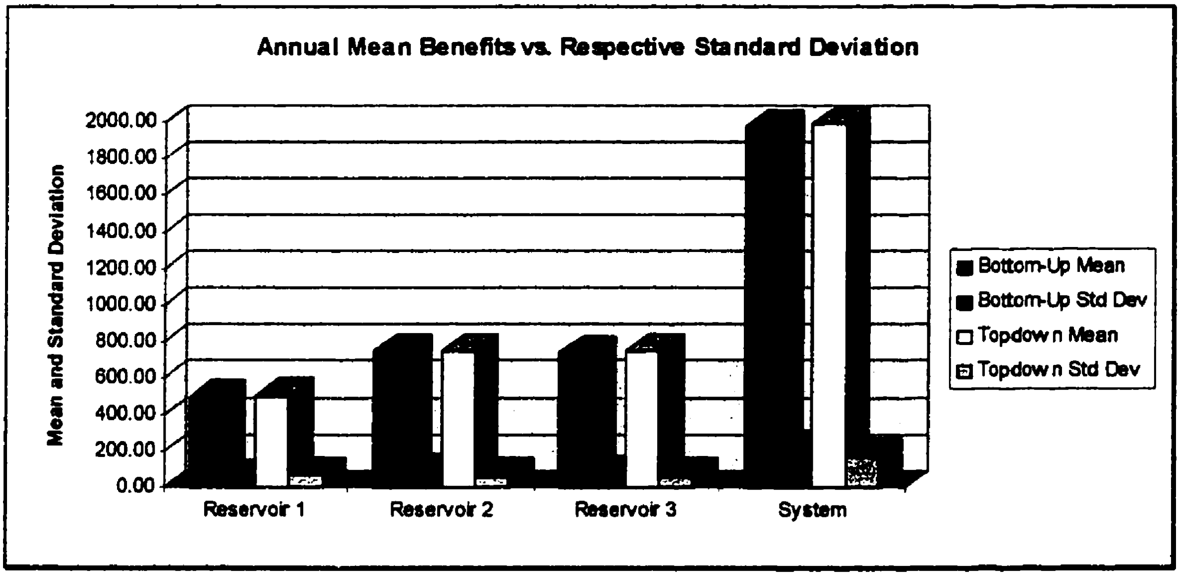


Figure 3 - 13 Average Annual Return and Respective Standard Deviation, Three Reservoir System, Problem 2

3.4.1.2 Four Reservoir System:

For the following test problems, the results obtained with two different coefficients of variation for the natural inflows, 0.30 and 1.00 are analyzed. In the previous situation, the 3 reservoir system, the configuration of the system varied (basically storage capacity) and kept the equivalent inflow pattern. Now, the original configuration is maintained and the behavior of the system and how PCA is affected for different coefficients of variation for the inflows is under scrutiny. As happened to the previous example, PCA was applied after running the model using the physical diagnosis aggregation scheme. Then the system was once more re-aggregated, optimized the resulting Two-Level MAM-SDP model and, both results finally compared.

3.4.1.2.1 Coefficient of Variation = 0.30

3.4.1.2.2 Using the standard aggregation scheme:

	Return					Storages			
Month	Res 1	Res 2	Res 3	Res 4	Month	Res 1	Res 2	Res 3	Res 4
January	0.00	0.00	0.22	99.78	January	70.53	21.54	6.75	1.18
February	0.00	0.00	5.20	94.80	February	13.78	29.54	0.76	55.91
March	0.00	0.04	1.31	98.65	March	3.71	8.68	33.51	54.09
April	2.93	10.85	22.59	63.64	April	12.62	35.42	1.30	50.66
May	2.92	3.39	53.90	39.78	May	12.49	31.75	1.39	54.36
June	1.60	2.53	27.28	68.60	June	4.61	1.42	34.65	59.32
July	0.34	1.50	18.43	79.73	July	3.06	1.89	20.87	74.18
August	0.81	1.71	2.93	94.55	August	6.24	4.53	20.99	68.24
Septemb	15.16	2.03	0.10	82.71	Septemb	10.42	2.73	21.33	65.52
October	6.50	10.55	1.34	81.61	October	6.12	2.47	23.61	67.81
Novemb	0.05	0.59	3.24	96.12	Novemb	17.96	7.84	1.15	73.05
Decemb	0.00	0.00	0.00	100.00	Decemb	74.52	21.77	2.77	0.95
Cycle Average	2.53	2.77	11.38	83.33	Cycle Average	19.67	14.13	14.09	52.11

Table 3 - 18 Analysis of Variance using Principal Components Analysis Applied to Four Reservoir System (first test)

Observing the table above and considering the storages, the Principal Component of the system is reservoir 4 albeit its dominance with respect to rest of the system cannot be assumed as total, because it accounts for a little more than 50 %. This type of analysis is a little subjective in a strict sense. A component is considered as dominant when it accounts for at least around 80 to 90 % of the total variance of the system. When considering the returns, reservoir 4 also appears as the Principal Component, this time in a very significant fashion with respect to the rest of the system. Reservoir 2 and reservoir 3 receive only controlled outflows from upstream reservoirs, what explains their smaller participation. Table 3 - 19 presents MAM-SDP performing the aggregation now starting from reservoir 4. Although reservoirs 1 and 2 are located in parallel with reservoir 4, in the aggregate scheme they will appear downstream from it. In the second stage, reservoir 1 is optimized and reservoir 4 contribution is added as conditional discharge to the system. It should be observed by the reader the sharp difference existing when compared with the results from the previous table, e.g., reservoir 3 has its importance increased significantly with respect to returns.

3.4.1.2.3 Using an aggregation scheme that starts with reservoir 4:

	Return					Storage			
Month	Res 1	Res 2	Res 3	Res 4	Month	Res 1	Res 2	Res 3	Res 4
January	0.40	0.59	24.76	74.25	January	2.07	0.83	4.79	92.31
February	0.01	0.58	6.73	92.68	February	0.01	0.44	11.14	88.41
March	3.39	0.37	26.21	70.03	March	0.00	9.33	2.33	88.33
April	0.00	0.00	14.70	85.30	April	3.10	0.57	43.64	52.69
May	5.01	0.66	29.52	64.82	May	2.17	0.91	25.26	71.66
June	0.38	5.77	39.24	54.61	June	1.04	3.37	24.38	71.20
July	1.09	0.11	6.24	92.56	July	0.51	1.14	17.10	81.26
August	5.34	1.19	23.44	70.03	August	0.96	0.29	11.32	87.43
September	0.72	1.34	39.68	58.27	September	1.51	2.10	0.20	96.19
October	0.09	0.69	1.26	97.96	October	0.25	1.71	8.41	89.63
November	0.17	0.35	19.20	80.28	November	1.45	0.63	5.42	92.50
December	0.00	0.09	28.61	71.30	December	0.37	1.53	5.18	92.92
Cycle Average	1.38	0.98	21.63	76.01	Cycle Average	1.12	1.91	13.26	83.71

Table 3 - 19 Analysis of Variance using Principal Components Analysis Applied to Four Reservoir System (second test)

Figure 3 - 14 presents the relationship between the accumulated mean for the cycle and its respective variance. As the original problem concerned hydropower generation from all the reservoirs belonging to the system, the objective is the maximization of a separable function, differing from the 3 Reservoir System previously presented. Then Figure 3 - 15 presents a more general picture of the system as whole as well as per reservoir. This feature was not very important in the previous problem but this is no longer the case, in view of the different type of objective function. In this Four Reservoir System Case, the standard deviation is the measure of dispersion. In graph below, the blue points represent the standard aggregation scheme, starting by reservoir 1, while the pink ones, starting by reservoir 4. The standard deviations per reservoir are more or less equivalent but when the entire system is taken into account the standard aggregation scheme

presents a much better figure with higher returns for equivalent standard deviation. Table 3 - 20, presenting the coefficient of variation for the average annual return follows the graphs.

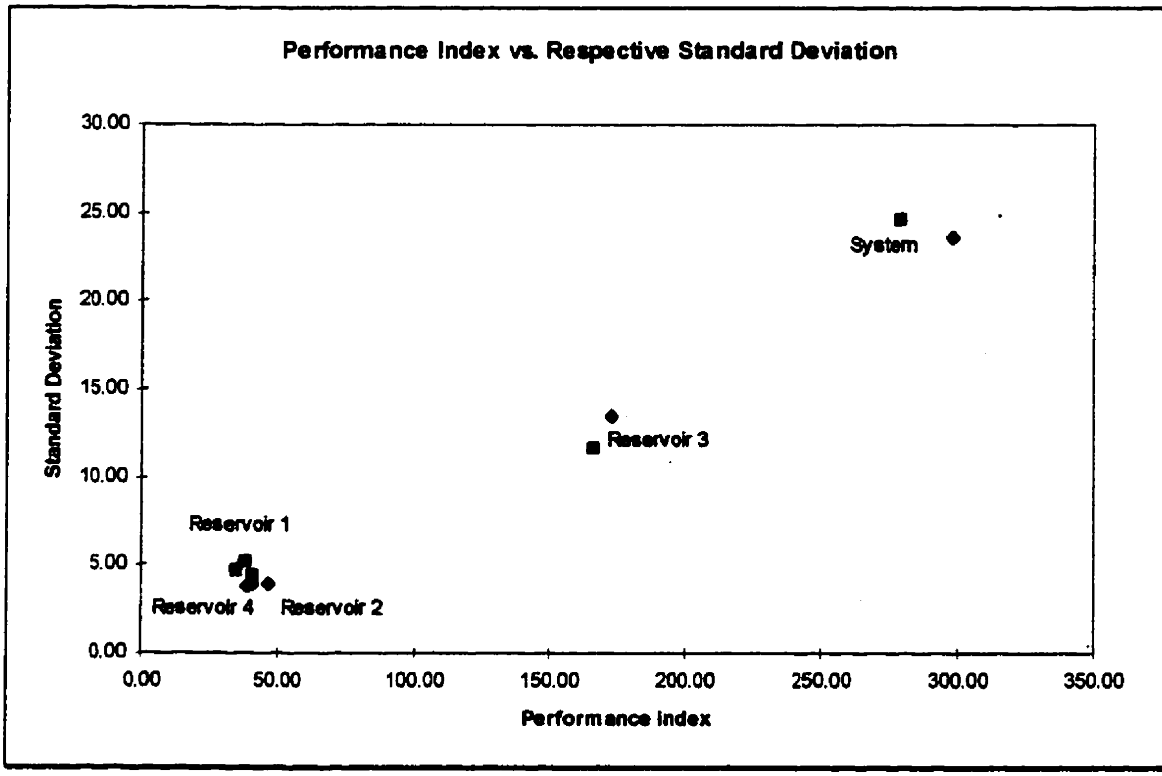


Figure 3 - 14 Performance Index versus Standard Deviation, Four Reservoir System

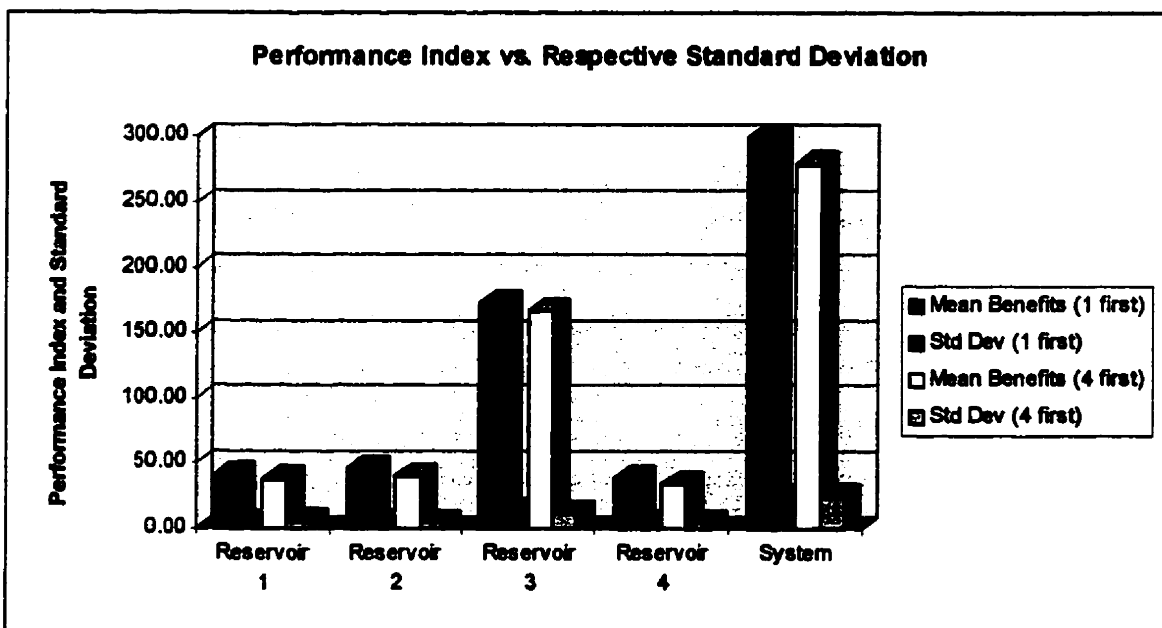


Figure 3 - 15 Performance Index and Standard Deviation, Four Reservoir System

The ratio standard deviation/average performance index per reservoir and for the system as a whole, also referred to as coefficient of variation, is shown below.

	Standard MAM-SDP	**Starting by Reservoir 4**
Reservoir 1	0.10	0.14
Reservoir 2	0.08	0.11
Reservoir 3	0.08	0.07
Reservoir 4	0.10	0.14
System	0.08	0.09

Table 3 - 20 Coefficient of Variation for the Annual Return, per Reservoir and for the System

3.4.1.2.4 Coefficient of Variation = 1.00

3.4.1.2.4.1 Using the standard aggregation scheme:

	Return					Storage			
Month	Res 1	Res 2	Res 3	Res 4	Month	Res 1	Res 2	Res 3	Res 4
January	0.00	0.00	7.89	92.11	January	0.00	0.37	12.91	86.72
February	0.10	0.00	0.00	99.90	February	0.00	1.60	5.52	92.87
March	0.00	0.32	6.52	93.16	March	0.00	2.23	5.25	92.53
April	0.00	0.97	15.06	83.97	April	0.00	26.33	8.59	65.08
May	0.07	0.31	8.35	91.27	May	25.07	0.80	4.81	69.32
June	0.56	10.73	88.71	0.00	June	1.76	2.23	4.69	91.32
July	1.56	18.22	80.22	0.00	July	3.70	1.86	11.13	83.30
August	1.55	0.31	19.87	78.27	August	3.02	9.29	23.96	63.73
September	2.71	0.87	11.92	84.50	September	12.53	6.16	22.32	58.99
October	1.72	5.15	10.99	82.14	October	1.67	5.15	26.93	66.24
November	6.46	1.74	23.38	68.43	November	0.15	3.31	18.65	77.89
December	2.15	2.99	0.10	94.76	December	0.00	0.15	20.17	79.68
Cycle Average	1.41	3.47	22.75	72.38	Cycle Average	3.99	4.96	13.74	77.31

Table 3 - 21 Principal Components Analysis Applied to Four Reservoir System (first test)

From the previous table and considering the storages, the Principal Component of the system is reservoir 4. It is interesting to observe that when comparing with the previous case (coefficient of variation equal to 0.30), its dominance with respect to rest of the system now appears to be much more significant, accounting for roughly ¾ of the total variance, or 77.31 %. When considering the returns, reservoir 4 is once more the Principal Component, with a relative participation slightly smaller than that of the observations. Once more, only reservoirs 2 and 3 receive controlled outflows from upstream reservoirs, thus their smaller participation. Table 3 - 22 presents MAM-SDP performing the aggregation now starting from reservoir 4. The considerations previously expressed for the coefficient of variation of 0.30 regarding the manner the aggregation was executed are exactly the same for the present case. The mentioned sharp difference no longer exists.

3.4.1.2.5 Using an aggregation scheme that starts with reservoir 4:

	Return					Storage			
Month	Res 1	Res 2	Res 3	Res 4	Month	Res 1	Res 2	Res 3	Res 4
January	0.00	0.00	6.62	93.38	January	0.00	0.00	97.11	2.89
February	0.00	0.10	5.96	93.94	February	0.00	0.52	7.63	91.85
March	0.00	0.33	8.68	90.99	March	0.00	0.63	13.54	85.83
April	0.00	0.25	10.93	88.82	April	0.00	17.78	2.98	79.24
May	0.00	0.44	6.87	92.68	May	0.00	17.41	2.09	80.50
June	0.00	1.00	6.51	92.49	June	0.00	15.60	1.97	82.44
July	0.00	1.14	17.13	81.73	July	0.00	1.64	21.89	76.47
August	0.00	0.81	10.90	88.29	August	0.00	34.80	63.56	1.63
September	0.00	0.59	17.22	82.20	September	0.00	20.97	3.01	76.02
October	0.00	56.93	33.13	9.94	October	0.00	30.37	4.87	64.76
November	0.00	3.14	23.07	73.79	November	0.00	35.49	60.93	3.59
December	0.00	1.20	11.59	87.21	December	0.00	1.16	7.12	91.72
Cycle Average	0.00	5.50	13.22	81.29	Cycle Average	0.00	14.70	23.89	61.41

Table 3 - 22 Principal Components Analysis Applied to Four Reservoir System (second test)

Figure 3 - 16 presents the relationship between the accumulated mean for the cycle and respective variance. a more general picture of the system as whole as well as per reservoir. In the

second case, the standard deviation is the measure of dispersion. In graph below, the blue points represent the standard aggregation scheme, starting by reservoir 1, while the pink ones, starting by reservoir 4. However, there is no clear winner in the case of the standard aggregation scheme versus the one suggested by statistical decomposition, because a smaller return has led to smaller standard deviation. Table 3 - 23 which presents the coefficient of variation for the average annual return follows the graphs.

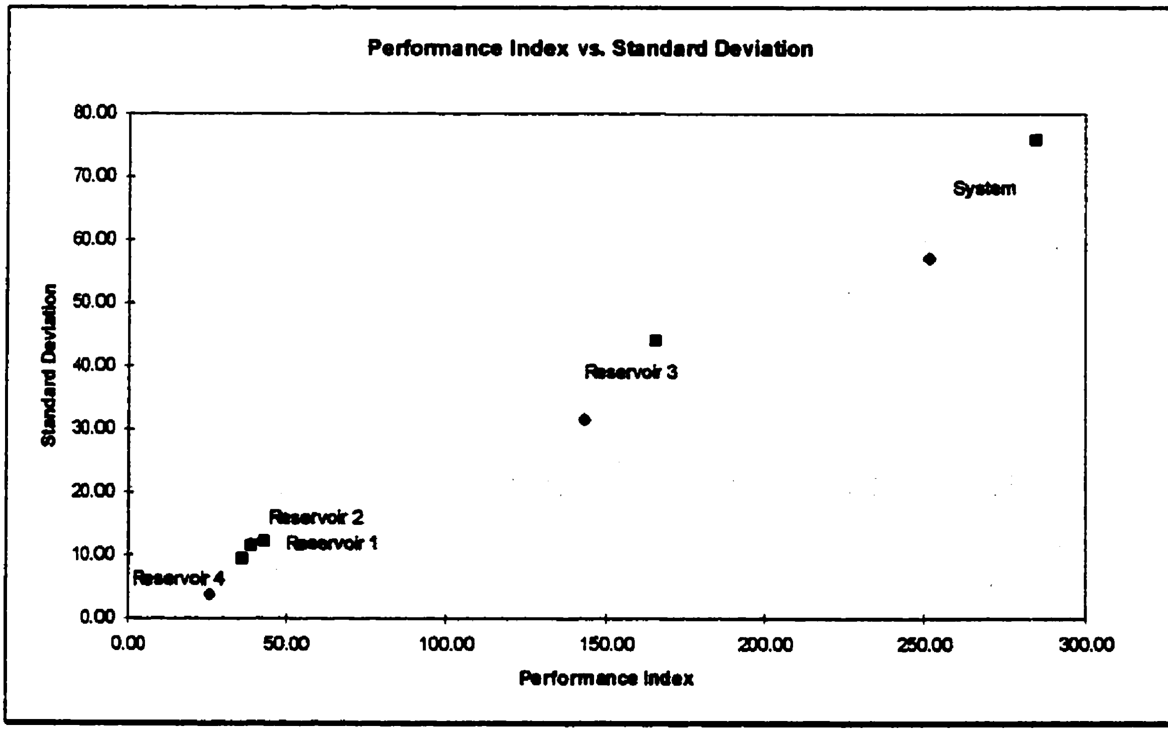


Figure 3 - 16 Performance Index versus Standard Deviation, Four Reservoir System

Another weak conclusion can be drawn from the comparison of the graph above with its similar for coefficient of variation of 0.30. The aggregation scheme configuration as suggested by the statistical (PCA) decomposition is less sensitive to changes in the coefficient of variation, i.e., it responds better to increase in disturbance in the inflows than the standard aggregation scheme, originated from physical diagnosis. One of the reasons for this better behavior can be from having the policies from reservoir 4 less local than those from the standard aggregation scheme.

Another possibility within the MAM-SDP framework is to operate reservoir 4 using the policies from the statistical decomposition model, i.e., reservoir 4 versus the rest of the aggregated system, and the other 3 reservoirs with the policies from the physical diagnosis model. However, all the above conclusions must be further studied and other tests should be conducted.

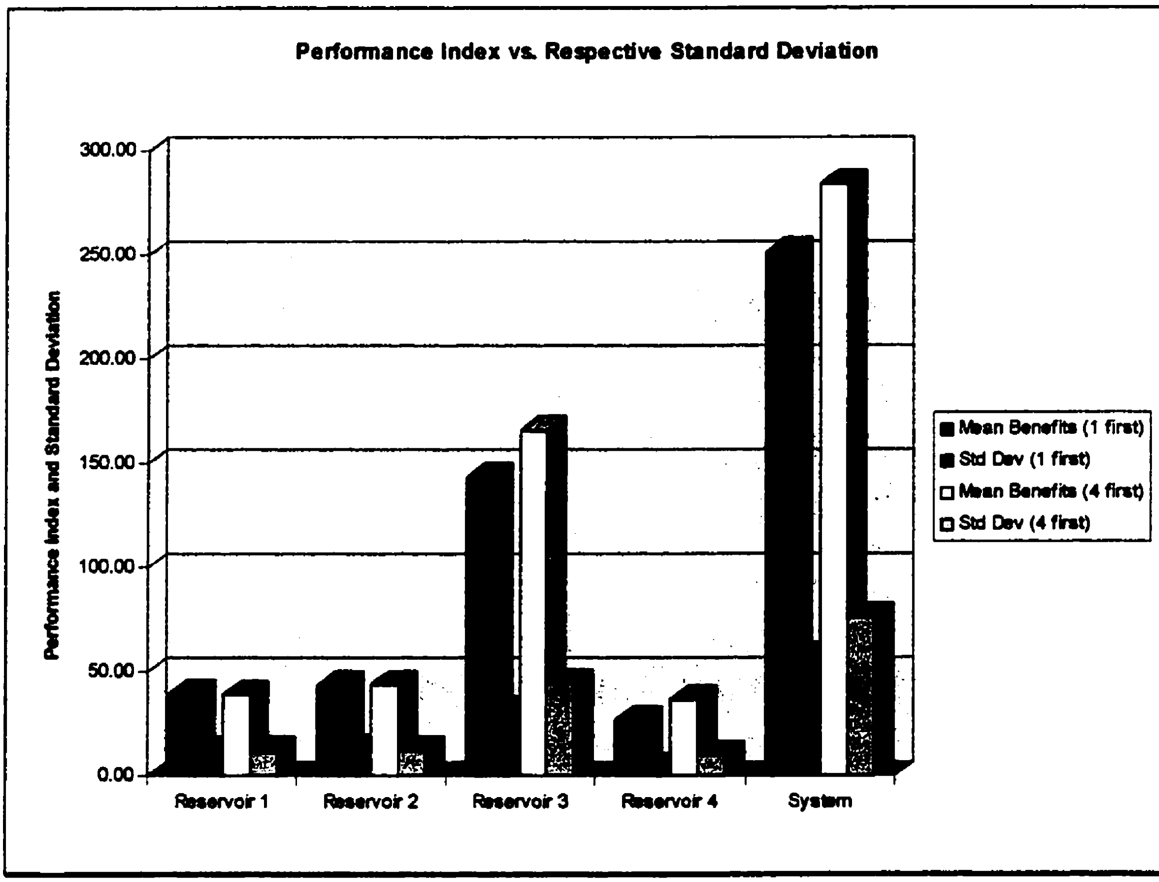


Figure 3 - 17 Performance Index and Standard Deviation, Four Reservoir System

The ratio standard deviation/average performance index per reservoir and for the system as a whole is shown below.

	Standard MAM-SDP	Starting by Reservoir 4
Reservoir 1	0.30	0.30
Reservoir 2	0.29	0.28
Reservoir 3	0.22	0.27
Reservoir 4	0.15	0.26
System	0.23	0.27

Table 3 - 23 Coefficient of Variation for the Annual Return, per Reservoir and for the System

In this section the use of physical and statistical decomposition to search an improved aggregation scheme in the optimization of the operation of large reservoir systems was shown. The physical diagnosis, herein referred to as the standard MAM always performs the aggregation starting from the top of the system and proceeding towards its bottom. The use of statistical decomposition, by means of PCA, aims at adding more information to the aggregation procedure and might as well just confirm the correctness of the application of the standard MAM. The MAM-SDP is used as a starting point for the definition of the final aggregation scheme and consequent optimization of the system. A weak conclusion derived from the previous study indicates that the aggregation scheme suggested by the application of statistical decomposition might lead to release policies that are more robust, i.e., less sensitive, to higher levels of uncertainty in the inputs to the system. This initial conclusion, however, should be substantiated by further tests and studies. The most important conclusion that can be drawn from the tests performed so far is the realization that the aggregation scheme as prescribed by the physical diagnosis is not the only possible way of aggregating a multireservoir system. Other fashions of aggregation can be used as well. In the following section there is another way of extending the idea of including variance/standard deviation, measures of dispersion of the expected returns or expected costs in the steady-state optimization of large multistage decision processes.

3.5 Expected Return-Variance of Return Rule

According to Ziemba and Vickson (1975) the major concepts of the mean-variance analysis, or expected return-variance of return analysis, can be found in works dated as early as in the 1930's. Thanks to Harry Markowitz's approach (see Markowitz (1952, 1959)) these concepts could then be employed and their basic ideas mathematically implemented since the publication of the Markowitz 1952 seminal paper. The Portfolio Selection approach can summarized simply by quoting Ziemba and Vickson (1975), page 203:

> *"... that the investor should limit consideration to those portfolios that are mean-variance efficient, i.e., those portfolios for which there does not exist an alternative portfolio that has at least as high a mean and lower variance."*

Note that this also means reducing risk for the same expected benefit. The presentation that follows is based on Markowitz's approach to perform Portfolio Selection as presented in his 1952 paper and 1959 book. The main concepts will be briefly introduced and then proceed to proposing a new manner of tackling multistage decision problems.

Let us define

- X_i - relative amount that an investor applies in a given security i, $0 \leq X_i \leq 1$, $\Sigma X_i = 1$,
- R_i - return on the i^{th} security,
- ${}_i$ - expected value of R_i,
- ${}_{ij}^{2}$ - covariance between R_i and R_j, with σ_{ii}^{2} being the variance of R_i.

Then, the yield R on the whole portfolio is

$$R = \sum_{i=1}^{N} R_i X_i \qquad 3.16$$

Where R_i and consequently R are random variables, whereas X_i, as chosen by the investor, is not random. And, because X_i is the percentage of the investor's assets which are allocated to the i^{th} security, $\Sigma X_i = 1$. It should be noted that $X_i \geq 0$, *for all i* (in economic terms, this means the exclusion of short term sales). As the equation above shows, the return R on the portfolio is a weighted sum of random variables. From that the expected return E from the entire portfolio is

$$E = \sum_{i=1}^{N} X_i \mu_i \qquad 3.17$$

with variance defined as

$$V = \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_{ij}^{2} X_i \qquad 3.18$$

An important assumption in both equations above are the fixed probability beliefs for the X's, (μ_i, σ_i^2). Thus, the investor has a choice of different combinations of E and V depending on his choice of portfolio. The picture below depicts the set of all possible combinations of E and V.

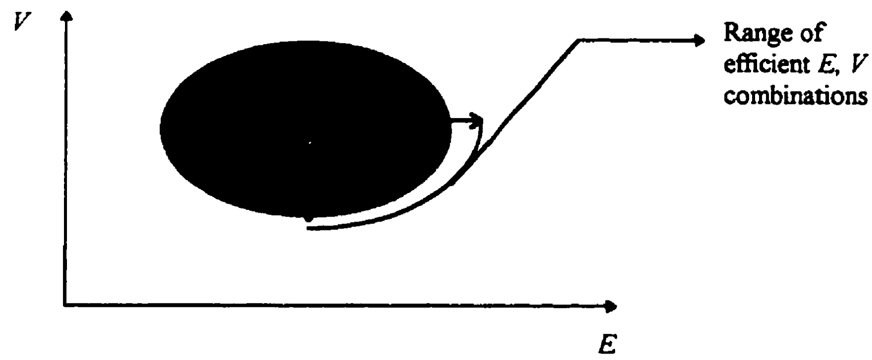


Figure 3 - 18 E-V Combinations

The efficient (E, V) combinations, for portfolio analysis, are those with minimum V for a given E or maximum E for a given V. However, calculating such an efficient frontier in the optimization of multireservoir system is an extremely complex problem. How to solve this problem is described next. The reader is reminded that the analysis of Figure 3 - 18 considers only the maximization of the expected benefits. Where minimization of costs is considered, the efficient (E, V) combinations are those with minimum E for a given V or maximum V for a given E, in other words, they would be located in the North to East region of the graph.

In the present work, the computation of efficient combinations of (E, V) for the steady-state optimization of the operation of reservoir systems using dynamic programming unlike in most of the other works currently employed, is proposed. Once the decision maker is provided with the efficient possible combinations, he will then decide which one is the most suitable for his type of operation. It should be stressed that albeit the application throughout this document is oriented toward reservoir systems operations, the method can be used to other type of problems.

For Markowitz (1952), the possible uses of the expected return-variance of return rule are:

- To explain the well established investment behavior of investment diversification (underlined by the famous maxim, "don't put all eggs in one basket"),
- To suggest guidelines for investment options

The author, himself, extends the application of the rule to differentiate between "investment" and "speculative behavior". This is a kind of behavior that may be attractive to some investors acting like bettors willing to take chances. But it is certainly not what one would expect from a serious (i.e., cautious) investor, or returning to the original case, the decision maker within the framework of a multistage process.

Therefore, the policy with the highest expected return or minimum expected cost may not be the one with least uncertainty of return/cost. On the other hand, the most reliable one might provide too small a return. A plot of the intermediate points, or possible linear combinations, having as parameters the expected returns and variances of returns could show the other options.

Depending on the sort of objective (or objectives), safety or risk reduction may be more attractive than a higher likely return. Thus, a more conservative policy would be appropriate and vice-versa. Below, Markowitz (1959), page 201, is quoted:

> *"Usually Monte Carlo techniques and techniques of mathematical programming are thought of as competitive: the former is used when the problem is too complicated for the latter. In the present case, however, the two techniques of Monte Carlo and those of quadratic programming are required to provide portfolios with [variance] reasonably close to the minimum [variance] attainable for given E."*

The fundamental idea of the method here proposed is to use SDP, MAM-SDP, concurrently with Monte Carlo simulation of the operation of the reservoir systems, so that the resulting procedure combines implicit and explicit techniques. The Monte Carlo simulation has its importance in obtaining the expected return or expected cost after the first optimization step. Once this is accomplished, a bi-objective optimization is performed, implicitly by adding the minimization of the standard deviation to the original single objective. Using varying weights for either of them in the same fashion as an investor partition its wealth in different portfolios, and keeping $\sum X_i = 1$, it is then possible to plot the existing linear combinations of (E, V). Another feature that is employed in economic analysis, but needs further consideration for the present purposes is the use of semi-variance (or its square root) instead of variance when the original objective function is the maximization of the return.

The reason for choosing standard deviation as an additional element in the optimization process is the need to employ a measure of dispersion of the probability distribution with the same unit as the original objective. Tobin (1965) considered the main advantages of its use the following:

1. If the central tendency of the probability distribution can be expressed by means of the mathematical expectation, the standard deviation is a natural measurement of its spread.

2. The standard deviation of return or cost of a compound objective function can be readily available from the standard deviations and correlations of the constituent factors of the return/cost function.

However, there are two sides to risk, gain and loss. If the chances of loss are minimized, there is still a price to pay: it is a considerable sacrifice of the chance for gain. The plot the results will follow the common fashion for Portfolio Selection, the E-σ diagram.

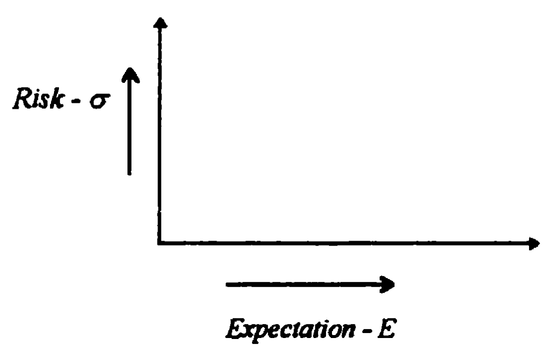


Figure 3 - 19 Model of Graph of Expectation versus Risk

With the diagram above there is a desired simplification to decision maker, because the ranking of his options will now be limited to only two numbers, i.e., mathematical expectations and standard deviations, and not on the entirety of the probability distribution of returns or costs.

So far the E-V (or E-σ) approach, was presented considering a static situation. If this approach is now transported to a multiperiod decision process, the decision taken in each period must follow the E-V criteria. For more details, the reader is referred to Tobin (1965), who probably was the first one to attempt to extend the Mean-Variance Approach to a multiperiod process .When applying the *E-V* criteria, each decision at any period is independent from the previous ones, therefore allowing the application of the Principle of Optimality and, consequently, use the SDP recursion to solve the problem. As the most suitable combination of return-risk (to the decision maker) is generally not known in advance, it is therefore convenient to compute it to a discrete set of linear combinations of E-σ. Mossin (1968) presents a general method of solution for multiperiod processes that employs a backward-recursive procedure. This procedure did not account for serial correlation for returns in different periods and becomes difficult to manage for long periods.

3.6 Two-Pass Mean-Variance Approach

The flowchart that follows is quite simple and straight forward to understand but there is a major difficulty in its implementation. How to know the mean and consequently the standard deviation (the square root of the variance) to perform the necessary computations? These are measurements that are usually obtained after computing the release policies during the optimization part and employing them to evaluate the system actual performance by means of simulation. Thus, the use of stochastic simulation to compute the necessary measurements followed by their insertion in the optimization is the suggested approach. The proposed methodology is then a mixture of the explicit and implicit approaches and has two passes. The algorithm for the method follows.

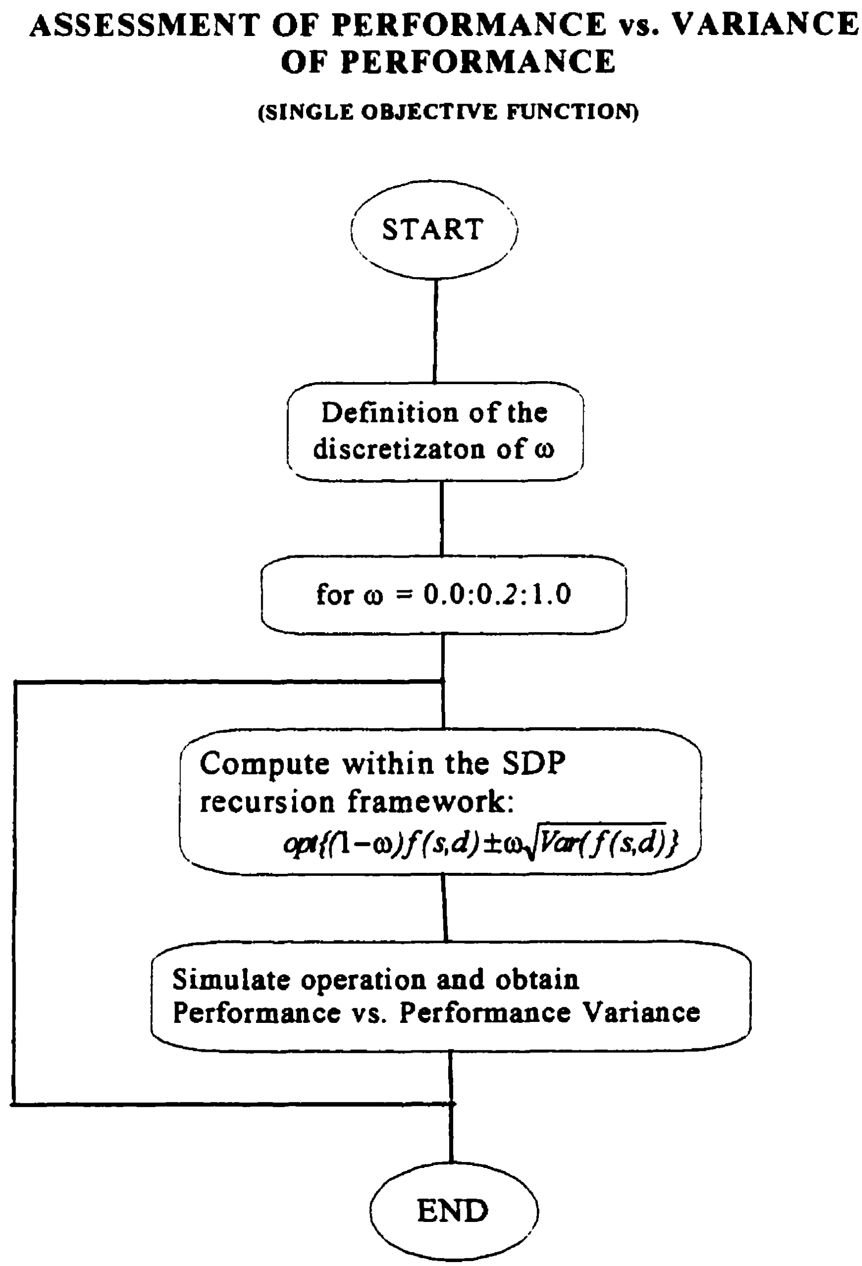


Figure 3 - 20 Flowchart of Two-Pass Mean-Variance Approach

The use of $\omega = 0.2$ in the flowchart above is just a suggestion. The size of the step is dependent on the type of problem and behavior of the *E-V* plot.

3.6.1 Algorithm for Two-Pass Mean-Variance Approach

Step 1: Set initially $\omega = 0$. From that, obtain policies that optimize the objective function (either maximization or minimization).

Step 2: Using the state-derived policies obtained from the optimization part, simulate the operation of the system until obtaining steady-state conditions.

Step 3: Get the estimates of the expected values for the decisions for all optimization (here, SDP) stages.

Step 4: Proceed to a new optimization with various levels of control for the dispersion around the estimates of the expected values (represented by either variance or standard deviation), the higher the ω, the lower the level of dispersion. Now, the expected immediate return r'', has the following form

$$r''(s,d) = E((1-\omega)r'(s,d)) \pm \omega\{Var[r'(s,d)]\}^{1/2} \qquad \textbf{3.19}$$

Note that the variance in the expression above is computed using the previously obtained mean values.

Step 5: Perform as many simulations as the number of ω discrete values, obtain the new estimates of the expected returns/costs and the variances (standard deviation) and plot the results in *E-V* (E, σ) diagrams.

It must be added that although it is not possible to use exactly the same algorithm to perform real-time optimization, a similar procedure can be executed using the forecasted inflows for a short horizon. This idea was not further developed but its implementation will not be difficult. The quality of the results will be highly dependent on the accuracy of the forecasts. Next, the Two-Pass Mean-Variance Approach is applied to the test problems as discussed earlier.

3.6.2 Test Results

The just described methodology was then tested using the test problems already presented. The first step of the validation was employing a single reservoir system, which allowed us to concentrate exclusively in the relationship performance/standard deviation without having to be concerned with contributions to the variance coming from the application of different aggregation schemes.

3.6.2.1 Single Reservoir System

3.6.2.1.1 RESULTS FOR CASE A

The following test problem considers penalties for the present spills and therefore uses the adjustment of the storage levels above the reservoir capacity at each present period, as explained in the discretization of storages, Chapter 2, Section 2. For better visualization of how the control of dispersion affects the Performance Indexes and respective Standard Deviations, several coefficients of variation for the inflows within the same graph were plotted. Although the control level ω is used during the optimization part of the method, the same is not true for the simulation part. That is to say that the different policies that are obtained for different coefficient of variations are assessed under the same constraints during the simulation part.

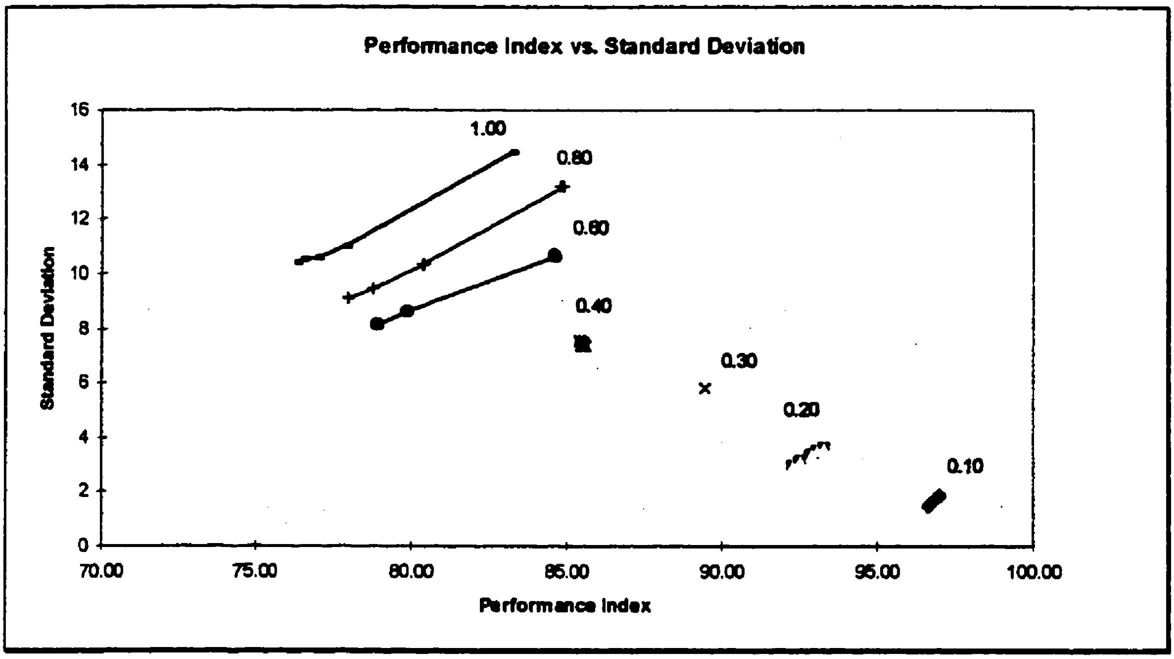


Figure 3 - 21 Performance Index versus Respective Standard Deviation, Single Reservoir System, Several Coefficients of Variation

The coefficients of variation studied for the present case are 0.10, 0.20, 0.30, 0.40, 0.60, 0.80 and 1.00. Some patterns are easily noticeable from Figure 3 - 21. As uncertainty grows in a direct proportion to the coefficient of variation, the expected return (here represented by the Performance Index) decreases. To mention the increase in range of the possible combinations of E and σ as the uncertainty increases. With the exception of the coefficients of variations of 0.30 and 0.40, the extension of the ranges grow monotonically.

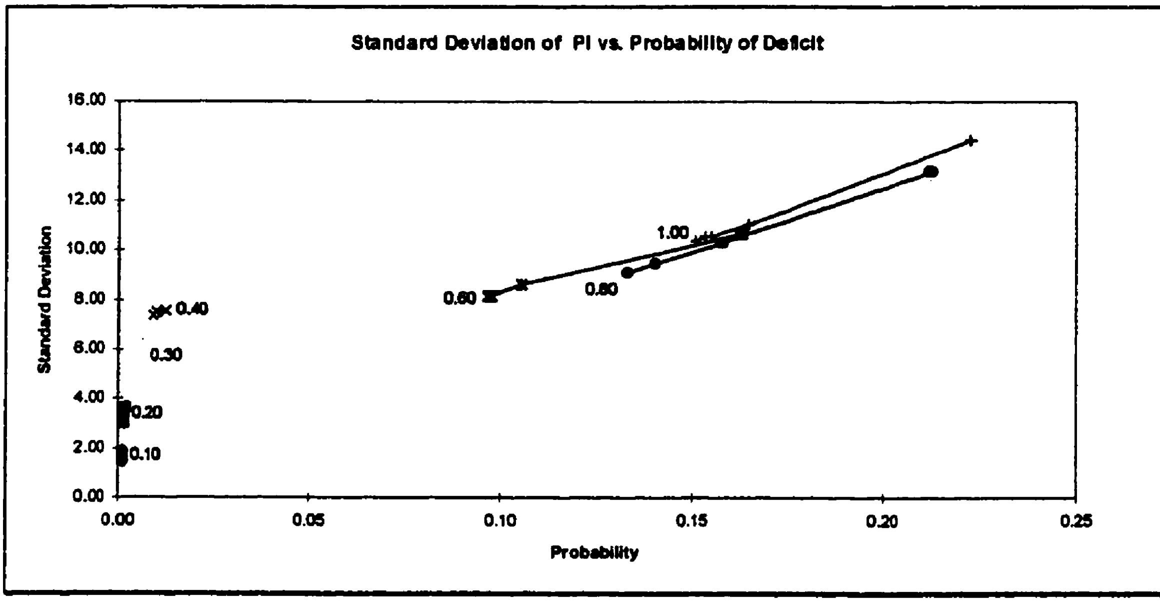


Figure 3 - 22 Standard Deviation of Performance Index versus Probability of Deficit, Single Reservoir System, Several Coefficients of Variation

Figure 3 - 22 shows the influence of uncertainty in the probability of failure of the operation of the system, here represented by the probability of deficit. It is also evident the direct relationship between the standard deviation of the return and the associated risk of failure. As is the case with all the simulations in this work, the operation is not allowed to go below the boundary of the minimum storage. (For this specific problem, the minimum storage is equal to zero, but usually is not necessarily so.) However, if the optimization policy has to be corrected to keep the storage trajectories within the specified boundaries, it is assigned as a failure of the operation policy. The method is very effective for higher uncertainty, for instance, the probabilities of deficit for the coefficient of variation equal to 1.00, the probabilities of deficit range from 15.07 to 22.28 %.

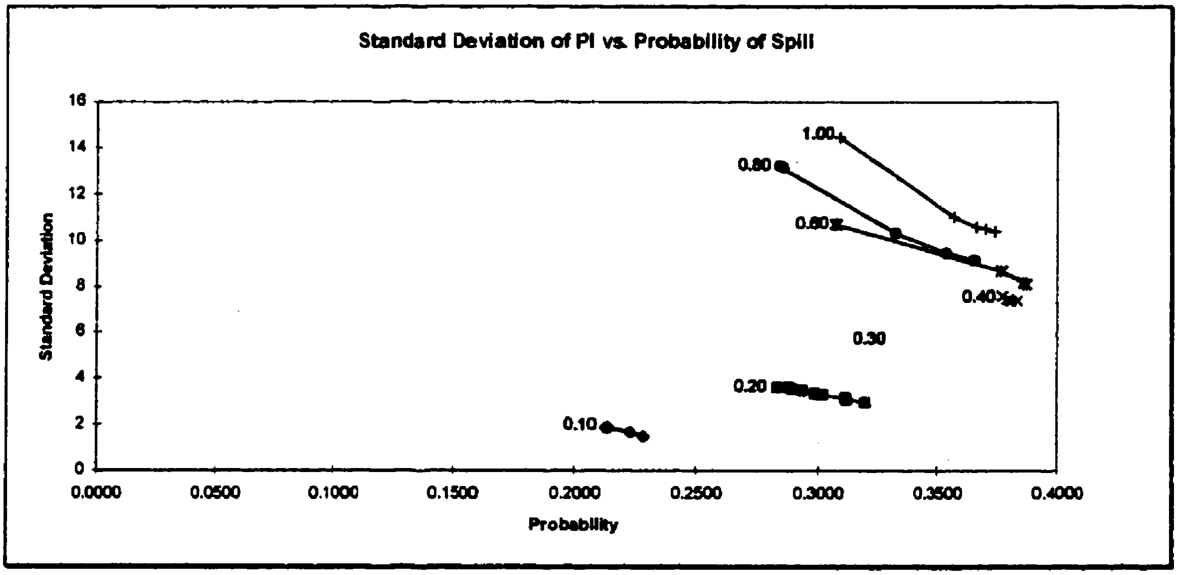


Figure 3 - 23 Standard Deviation of Performance Index versus Probability of Spill, Single Reservoir System, Several Coefficients of Variation

In reservoir operation, due to the uncertainty in the inflows, there is always a trade-off between spill and deficit. To keep a higher head and maintain a secure flow the reservoir has be as full as possible. Figure 3 - 23 is just the confirmation of that as can be inferred from the negative inclination of the lines of the coefficient of variation.

3.6.2.1.2 RESULTS FOR CASE B

With the exception of no consideration of penalties for present spills, the other preliminary observations regarding Case A are valid for Case B. The coefficients of variation studied for the present case are exactly the same as before, i.e., 0.10, 0.20, 0.30, 0.40, 0.60, 0.80 and 1.00, for the sake of comparison. The same patterns are again easily noticeable from Figure 3 - 24. Because of the non-inclusion of the penalties for the spills, the operation is more susceptible to the uncertainty, therefore the E-σ ranges slightly longer. The following ones, Figure 3 - 25 and Figure 3 - 26, repeat the behavior of Case A and was is mentioned for that regarding the methodology is valid for this one as well.

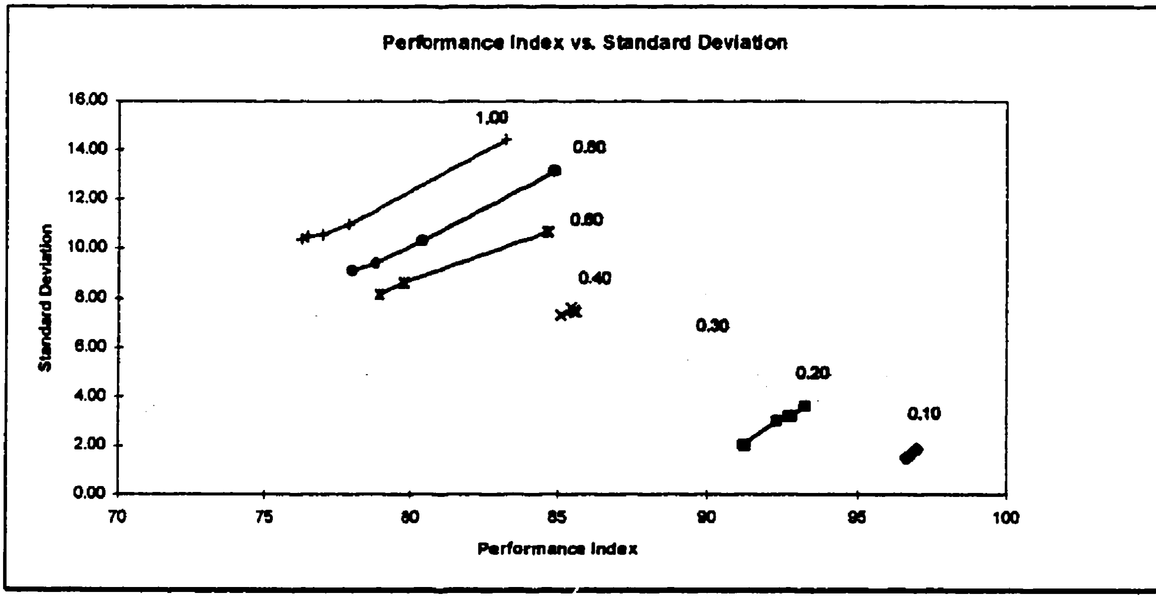


Figure 3 - 24 Performance Index versus Respective Standard Deviation, Single Reservoir System, Several Coefficients of Variation

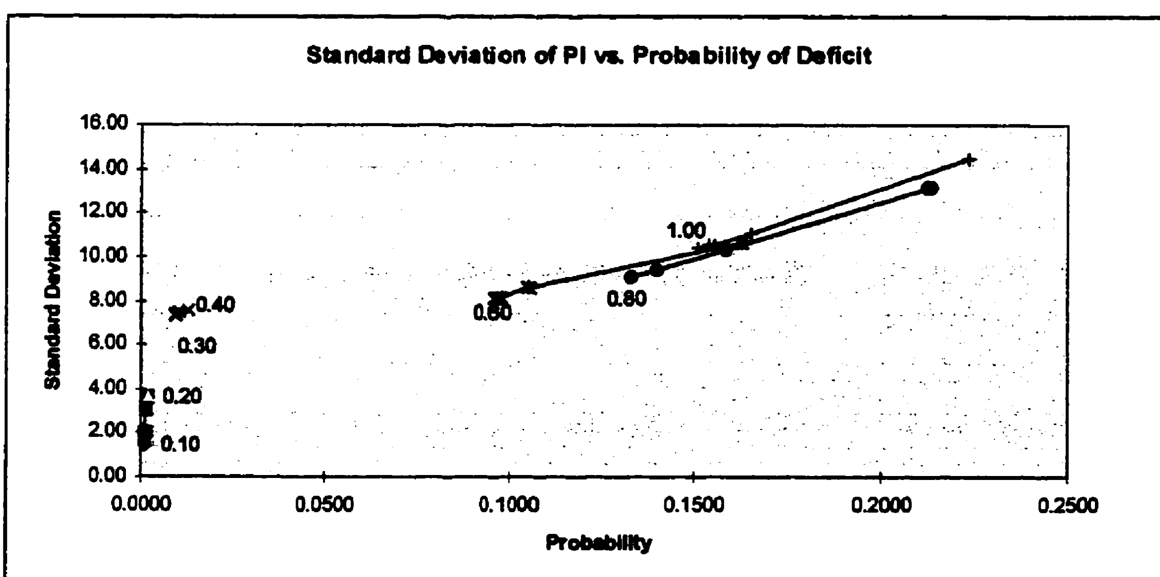


Figure 3 - 25 Standard Deviation versus Probability of Deficit, Single Reservoir System

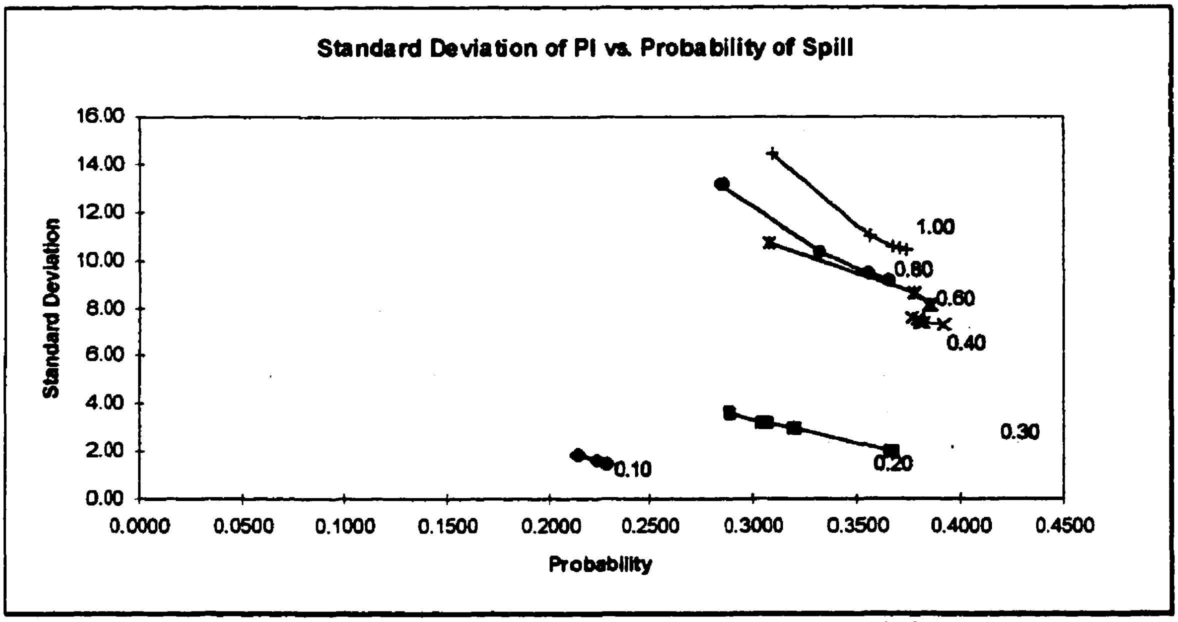


Figure 3 - 26 Standard Deviation versus Probability of Spill, Single Reservoir System

The major objective of the use of the single reservoir system as a test-problem in this chapter is the validation of the Two-Pass Mean-Variance Approach. When dealing with large systems, with more than one reservoir, other factors come into play in the assessment of the E-σ relationship and sometimes interfere with the principal idea. Using several coefficient of variations was very useful not only to reinforce the knowledge that the increased uncertainty in the input is transferred to the expected return and confirm its association with the probability of failure, but to show that the proposed methodology is very effective for higher levels of this uncertainty. This because the decision maker should be offered a tool that would allow him to examine the trade-off existing between the performance and its associated risk.

3.6.3 Multiple Reservoir Systems

Once the methodology was validated for the single reservoir system, it was then applied to multiple reservoir systems and in conjunction with the PCA technique described earlier in this chapter. The tests problems were those presented in the beginning of this chapter, and comprise systems with 3 and 4 reservoir. The former in series and the second of mixed configuration.

3.6.3.1 Three Reservoir System

For a better visualization of the possibilities of combining Two-Pass Mean-Variance Approach and different aggregation schemes, the operation of the reservoirs was first optimized using two aggregation schemes, the standard one, using the physical diagnosis, i.e., from upstream to downstream and the other one starting from bottom up. Once the release policies were obtained and the steady-state operation simulated, the average release targets were computed. Then the Two-Pass Mean-Variance Approach was applied using the release targets previously calculated for each type of aggregation scheme. The points in the plots were connected in increasing order for the value of ω and their general tendency to resemble the corresponding efficient frontier of the hypothetical Figure 3 - 18 is evident.

3.6.3.1.1 Problem 1

3.6.3.1.1.1 Case 1: Using the targets obtained from the standard aggregation scheme

To assess how the methodology behaves under different set of policies the following experiment was conducted. First, the operation of the reservoir system was optimized using the standard aggregation scheme. With this set of policies, the Two-Pass Mean-Variance Approach was then employed for both aggregation schemes, the standard one and the other possibility. For this 3 reservoir system, there are only two options, already analyzed during the PCA section, either start from the most upstream reservoir and proceed the aggregation to the bottom of the system or vice-versa. The Two-Pass Mean-Variance Approach with the policies obtained from the standard aggregation scheme follows.

The standard aggregation scheme is shown in blue in Figure 3 - 27 which presents the relationship between the Performance Index (PI) and its standard deviation for different values of ω. The other aggregation scheme starts from the most downstream reservoir to the most upstream one. The color for this aggregation scheme is brown.

The plot of Figure 3 - 27 shows that for this case, the application of targets initially obtained from the standard aggregation scheme has better performance when the Two-Pass Mean-Variance Approach is aggregated in the same fashion. For ω's varying from 0.00 to 0.60 the

Performance Index remains relatively constant while its respective Standard Deviation is reduced substantially. For instance, for ω equal to 0.00 is 61.02, 8.12 % of the total PI. For ω equal to 0.60 it is 47.66, representing 6.34 % a reduction of roughly 22 % in the Standard Deviation without practically any reduction in the performance of the operation of the system. When looking at the other aggregation scheme the behavior of the standard deviation is equivalent but for higher levels.

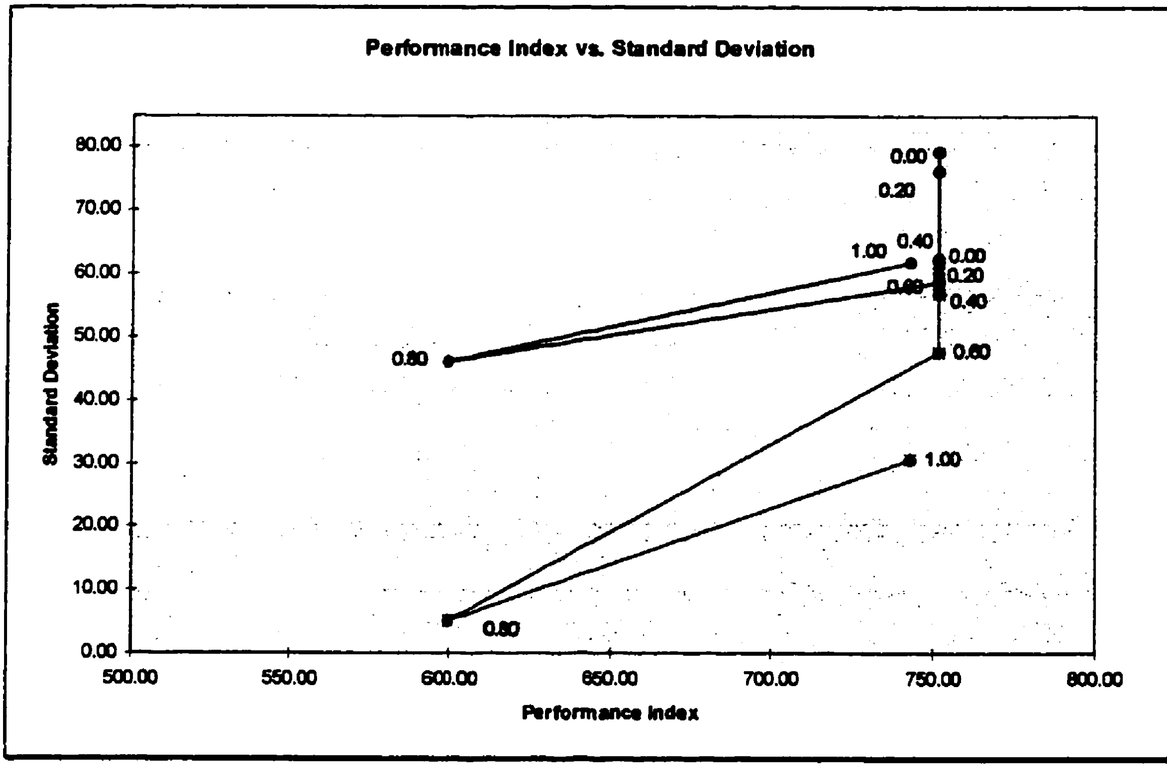


Figure 3 - 27 Performance Index versus Standard Deviation, Three Reservoir System, Problem 1

3.6.3.1.1.2 Case 2: Using the targets obtained from the 'bottom-to-top' aggregation scheme

The continuation of the previous test now uses the other possible set of policies. Again, the first step is the optimization of the operation of the reservoir system, but now using an aggregation scheme that isolates reservoir 3 at the first stage of the aggregation. With this set of policies, the Two-Pass Mean-Variance Approach was then applied for both aggregation schemes, the standard one and the other possibility.

The standard aggregation scheme is shown in blue in the following graph, presenting the relationship between the PI and its standard deviation. The other aggregation scheme starts from the most downstream reservoir to the most upstream one. The color for this aggregation scheme is pink.

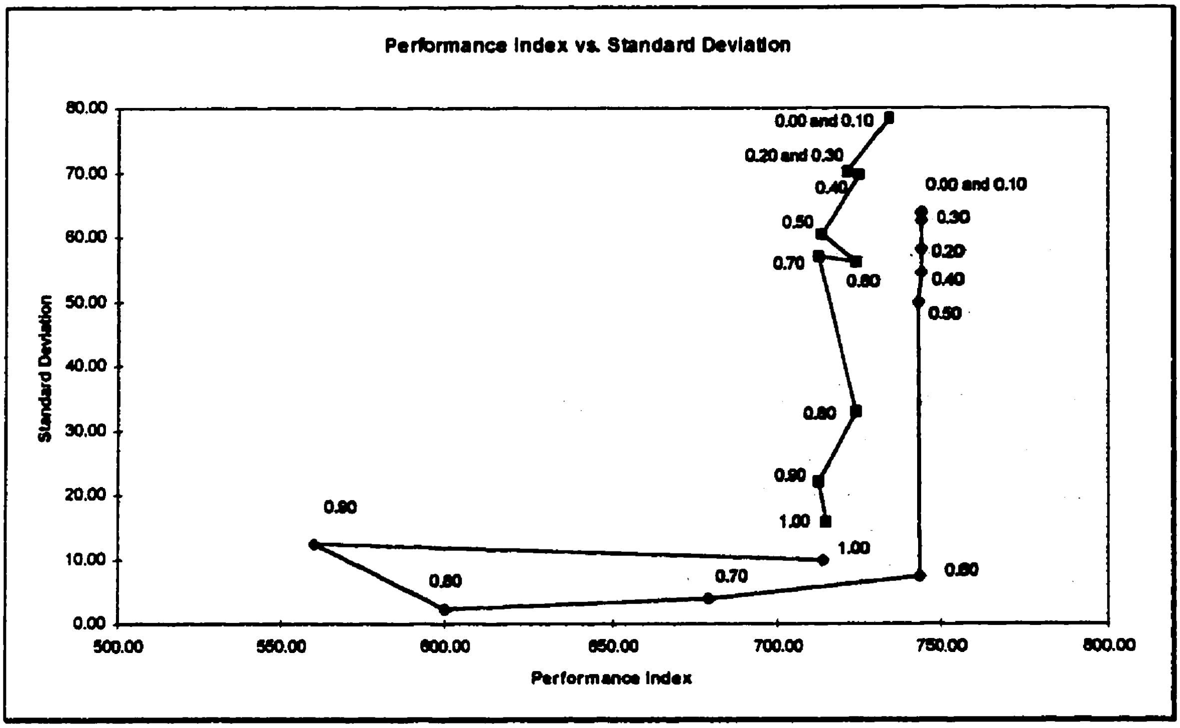


Figure 3 - 28 Performance Index versus Standard Deviation, Three Reservoir System, Problem 1

Figure 3 - 28 shows that for the application of targets obtained from the bottom up aggregation scheme also performs better when the Two-Pass Mean-Variance Approach is aggregated in the standard fashion. However, the performance of the operation is slightly inferior than for the previous example. But, once more, for ω's varying from 0.00 to 0.60 the Performance Index remains relatively constant while its respective Standard Deviation is reduced in a significant manner. For the sake of comparison with the other test, for ω equal to 0.00 is 63.72, 8.56 % of the total PI. For ω equal to 0.60 it is a mere 7.34, representing only 0.99 % a reduction of around 88 % in the Standard Deviation values with very small reduction in the performance of the operation of the system. These results were so surprising that more points were added to the plot to have a better appreciation of the reduction. When looking at the other aggregation scheme the behavior of the standard deviation reduction is somewhat erratic but the monotonic trend persists now at an inferior performance level.

3.6.3.1.2 Problem 2

3.6.3.1.2.1 Case 1: Using the targets obtained from the standard aggregation scheme

The same experiment as for Problem 1 is repeated using the other similar problem. Once more, the operation of the reservoir system was optimized using the standard aggregation scheme. After obtaining this set of policies, the Two-Pass Mean-Variance Approach was conducted for both aggregation schemes, the standard one and the other possibility. The Two-Pass Mean-Variance Approach was optimized with the policies obtained from the standard aggregation scheme.

The standard aggregation scheme is shown in blue in Figure 3 - 29 presenting the relationship between the Performance Index (PI) and respective standard deviation under different values of ω. The other aggregation scheme starts from the most downstream reservoir to the most upstream one. The color for this aggregation scheme is pink.

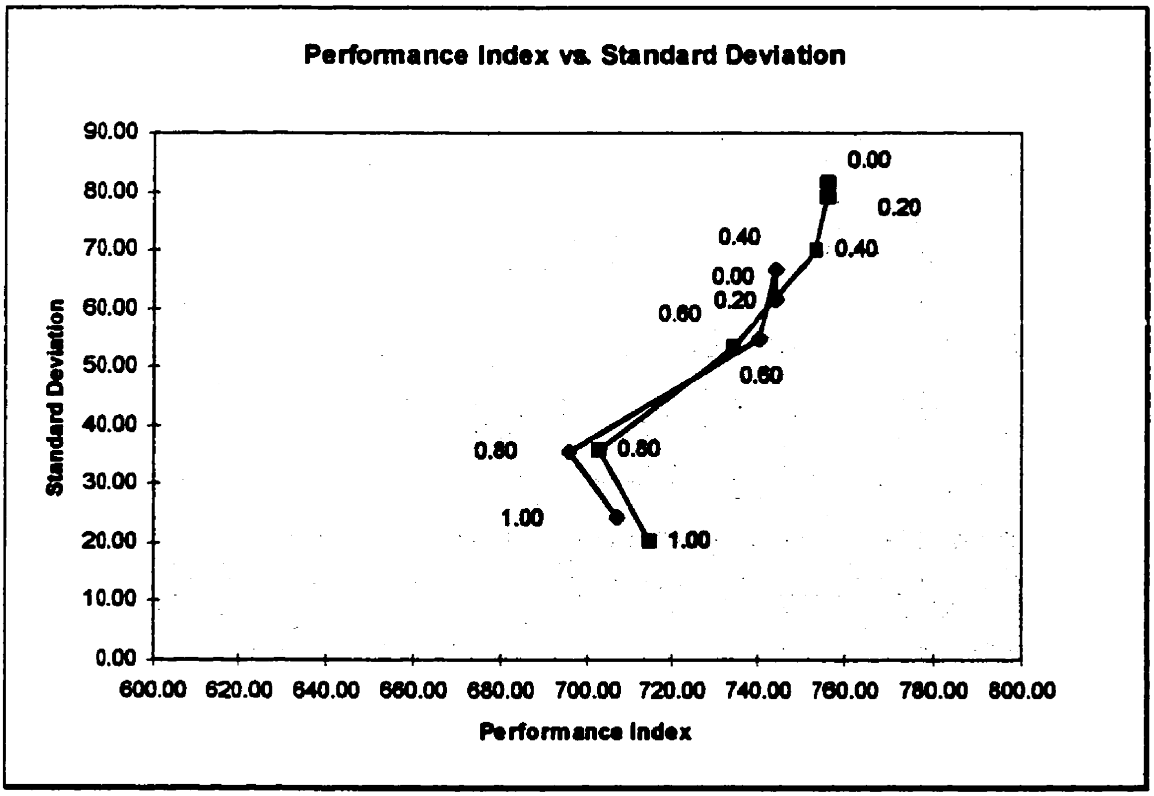


Figure 3 - 29 Performance Index versus Standard Deviation, Three Reservoir System, Problem 2

The plot of Figure 3 - 29 shows that for this case there is a clear trade-off between PI and its respective Standard Deviation. For the bottom up policies better performance is obtained with the increase in the Standard Deviation values. When the PI ranges are equivalent the behavior of both graphs are similar. What is the most interesting to note is that for the set of policies obtained for the standard aggregation scheme, the performance achieved using the other aggregation scheme is better. This better performance, however, comes with the cost of higher uncertainty in returns.

3.6.3.1.2.2 Case 2: Using the targets obtained from the 'bottom-to-top' aggregation scheme

Now follows the other possible set of policies. The optimization of the operation of the reservoir system uses the aggregation scheme that isolates reservoir 3 at the first stage of the aggregation. With this set of policies, the Two-Pass Mean-Variance Approach for both aggregation schemes was computed.

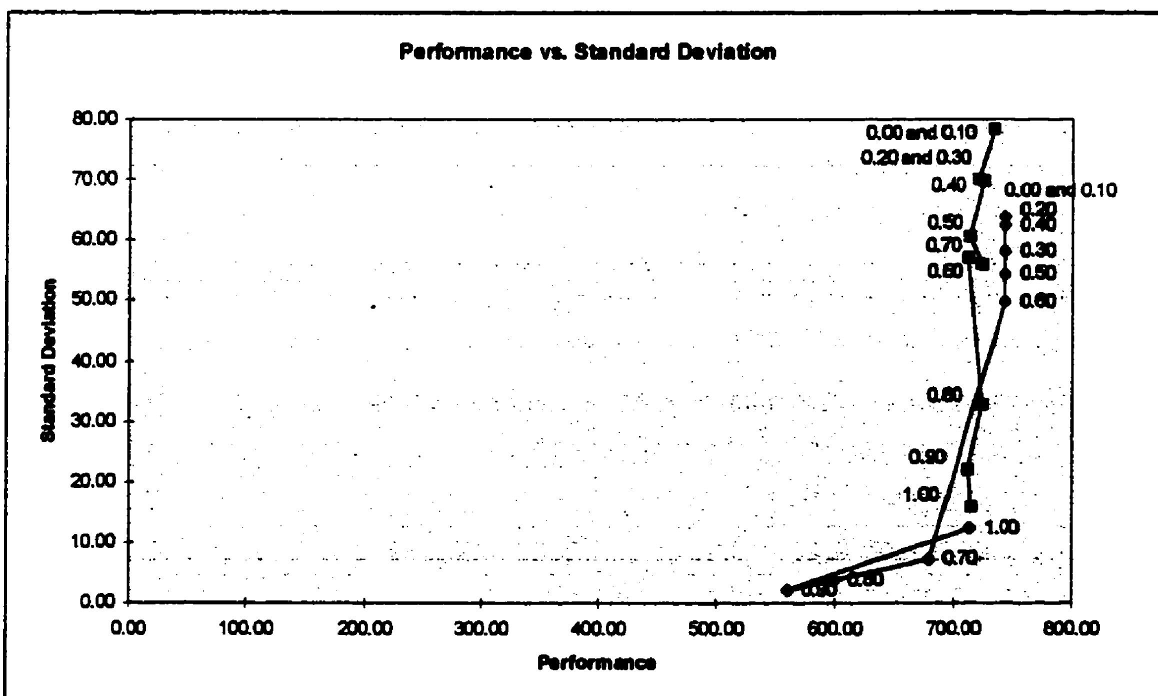


Figure 3 - 30 Performance Index versus Standard Deviation, Three Reservoir System, Problem 2

The standard aggregation scheme is shown in blue in the above graph, presenting the relationship between the PI and its standard deviation. The other aggregation scheme is shown in pink.

Figure 3 - 30 shows that the standard aggregation scheme performs better than the other one because for higher PI's there are equivalent values for their respective Standard Deviation and the preference for better performance for the same risk is clear. For some PI ranges, the Standard Deviation is equivalent, with the plots overlapping and behaving in a similar fashion. However, the standard aggregation scheme still provides further reduction of the risk but at higher cost or diminishing returns.

3.6.3.2 Four Reservoir System

3.6.3.2.1 Coefficient of Variation = 1.00

3.6.3.2.1.1 Case using the targets obtained from the standard aggregation scheme

In the test presented below the system was first optimized employing the aggregation scheme suggested by the physical diagnosis. Once the targets were obtained in this way, the Two-Pass Mean-Variance Approach was used. As additional testing, these targets were employed not only for the physical diagnosis aggregation scheme but the one from the statistical decomposition as well. For clearer visualization, both plots appear in the same graph. The standard aggregation scheme is shown in blue in following Figure 3 - 31 and, according to the format used so far, presenting the relationship between the PI and its standard deviation. The other aggregation scheme starts from reservoir 4, that is say, the one that is located on the left side of the configuration of the system. The color for this aggregation scheme is pink. The next test has high disturbance levels for the natural inflows, a coefficient of variation equal to 1.00 was the chosen one.

The reader will notice that the aggregation scheme that maintains global release policies for reservoir 4, the one shown in pink, has a better performance than the other one. It achieves higher performances obviously with the onus of higher standard deviation and reaches ranges with lower standard deviation with lower benefits. This reinforces the previous conclusions of having the aggregation scheme as suggested by the statistical decomposition as more effective than the other one from physical diagnosis. Moreover, the behavior of the E-σ plot for this aggregation scheme is closer to the theoretical curve, as seen in Figure 3 - 18.

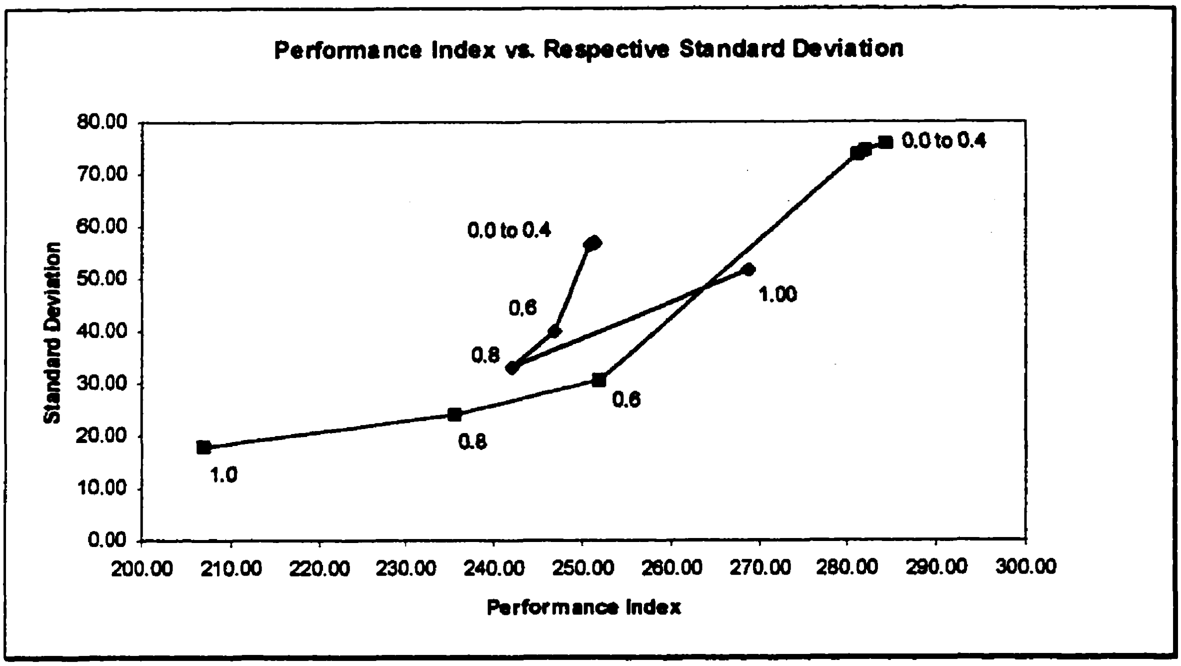


Figure 3 - 31 Performance Index versus Standard Deviation, Four Reservoir System

With the four reservoir system test above the methodology testing was completed with satisfactory results and now this work proceeds to its application to a real case study scenario. This case is in Chapter 4 and will submit the proposed methodology to a much more demanding and complex problem that those presented so far in order to further validate the present research.

3.7 Summary

This chapter presented the proposed methodologies and samples from the tests effectuated during the research period. The first part shows the improvements in the suboptimal policies obtained by the original MAM-SDP with the use of Approximate Conditional Distribution of Releases. The next part describes the theoretical background of Principal Components Analysis and how to apply it to define the most convenient aggregation scheme for solving multireservoir problems within the MAM-SDP framework. Two possibilities were studied and the main results obtained with standard test problems shown. The third and final part of this chapter is the presentation of the Expected Return-Variance of Return Rule. The original methodology was conceived for static models, and some extensions were already developed for dynamic models. An algorithm to solve multistage decision type problems is presented. As was the case of the two previous parts, the methodology was extensively tested and a summary of the results were presented.

Chapter 4

North American Great Lakes Case Study

4. NORTH AMERICAN GREAT LAKES CASE STUDY

4.1 Introduction

The North American Great Lakes System is composed of five large lakes and a smaller one in between, totaling six. Figure 4 - 1 depicts the location map in North America. They will be treated here as only five. They are listed below, in upstream to downstream order:

- Lake Superior
- Lakes Michigan-Huron
- Lake St. Clair
- Lake Erie
- Lake Ontario

Lakes Michigan and Huron present practically the same water level due to the physical characteristics of their connection, the Straits of Mackinac. As a consequence, for the purpose of this study, they will be considered hydraulically and hydrologically as just one lake, the Michigan-Huron. All the lakes, except Lake Erie and Lake St. Clair, are deep have very large surface areas and are able to store enormous quantities of water. On the other hand, the channels and rivers that connect them have a very limited capacity of water transport, if compared to the sheer capacity of storage. Lake St. Clair might be minute in size, but its location between Lakes Michigan-Huron and Lake Erie makes it quite important hydraulically. Figure 4 - 2 shows the Great Lakes profiles and illustrates well what was just described.

The water supplied to the Great Lakes is originated directly from rain, and snow and the drainage within the watershed. Because of the extensive area they cover, the portion of water lost due to evaporation from the surface is generally very significant. Any calculation of the hydrologic balance must consider this very fact.

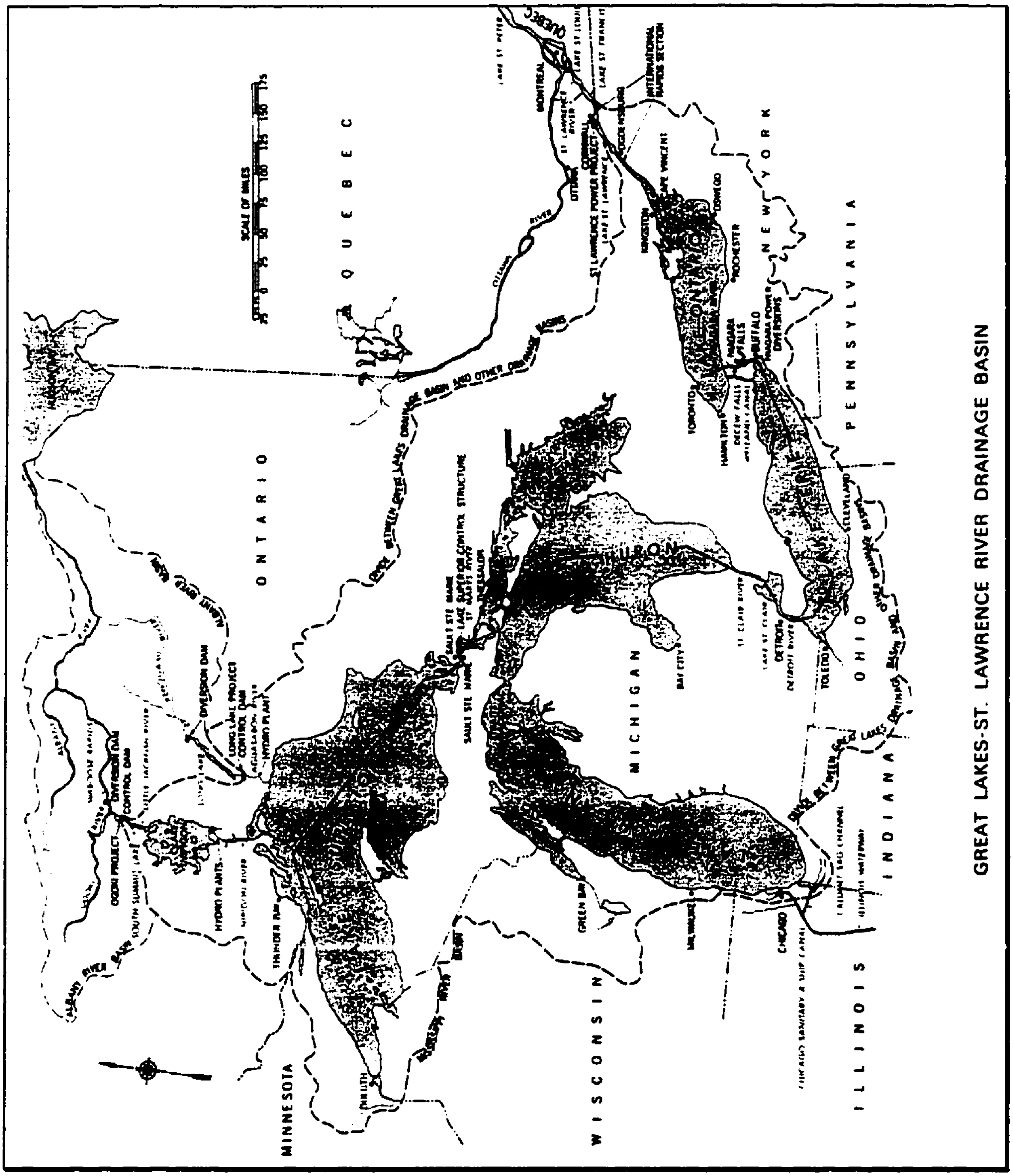


Figure 4 - 1 Location Map

Source: Regulation of Great Lakes Water Levels, Report to the IJC, December 1973.

Usually, the lakes are able to store the mean net supplies of water without an excessive change in water levels. However, long periods of drought or abundant precipitation, coupled with limited capacity of their connections may cause the levels to fluctuate beyond what would be considered the desired threshold to the water users and the inhabitants of the region.

The lakes are situated in Canada and the United States, their regulation subject to the approval of both governments, and the bi-national committee, the International Joint Commission (IJC) which coordinates the regulation of Great Lakes level variations. This was done in accordance to the Article IX of the Boundary Waters Treaty of 1909.

Much of the region surrounding the lakes is densely populated and this population relies heavily on the stored water for hydroelectric energy generation, commercial navigation, water supply, recreation, etc. Besides, a factor of utmost importance to be taken into consideration for any lakes regulation policy is the fact that, many people live on the lakeshores.

Because of the above, on October 7, 1964, the Governments of Canada and the United States submitted to the IJC a Reference where they expressed their desire that the Commission study and determine possible actions in order to improve the range of water levels fluctuations with beneficial effects to the following 10 interest groups.

1. Municipal infrastructure (such as water intakes and sewage outfalls)
2. Commercial navigation
3. Hydropower generation
4. Industrial and commercial facilities (that have docks and/or process lakes water)
5. Agriculture
6. Residential shore property (riparian)
7. Fish, wildlife and other environmental considerations
8. Commercial fisheries
9. Recreation and tourism (including recreational boating)
10. Native North Americans

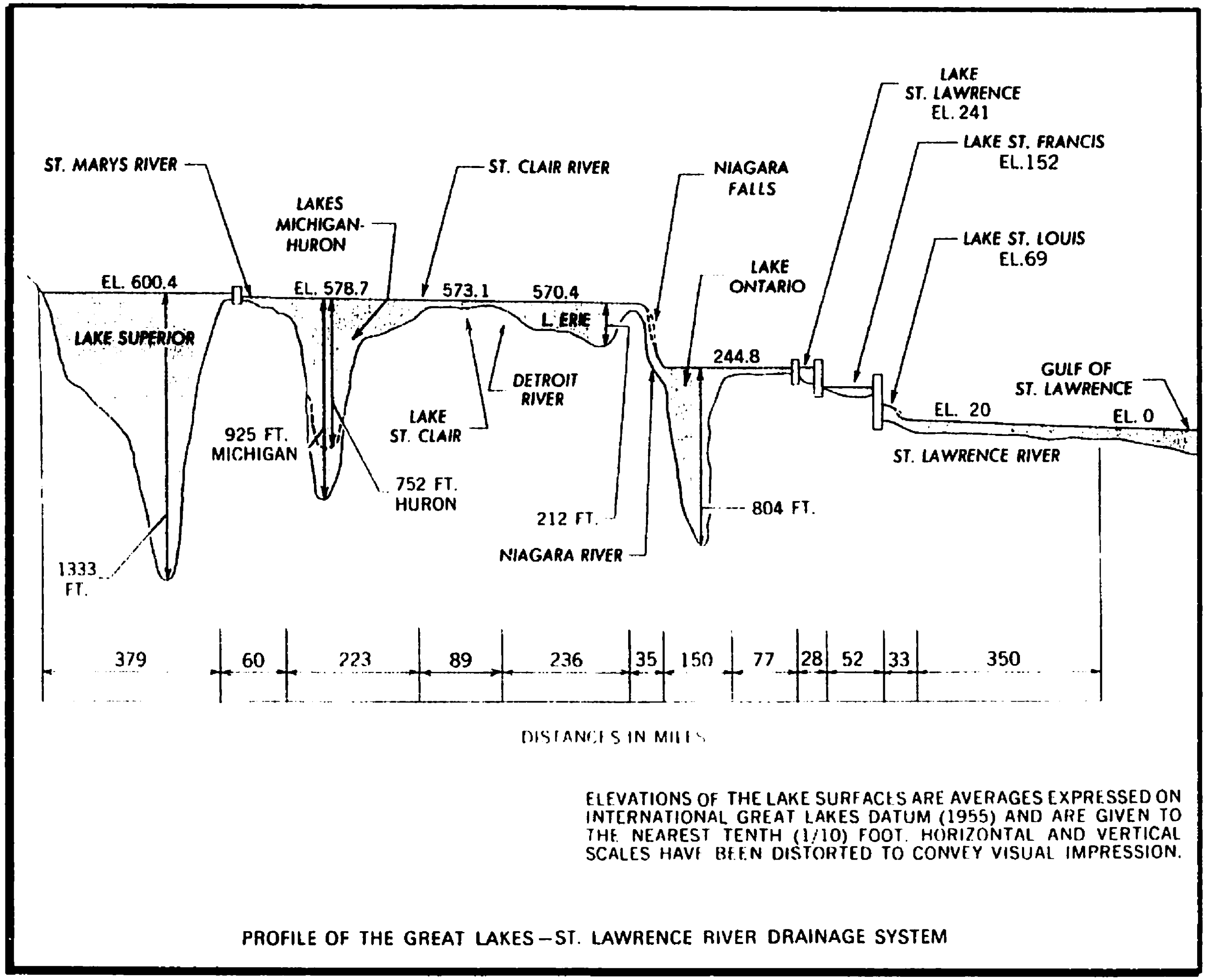


Figure 4 - 2 Great Lakes Profile

Source: Regulation of Great Lakes Water Levels, Report to the IJC, December 1973.

As one would expect, these uses of the lake water depend mostly on the magnitude of the outflows from the connecting rivers and canals along with the lakes levels. The studies so far have shown that it is not possible to completely solve the problem of high water levels in such a manner that all the prospective users will be completely satisfied at the same time. Nevertheless, it is feasible to adopt measures that would alleviate some of the users from the privation caused by them without causing further troubles to others.

4.2 Great Lakes Region

4.2.1 Geographic description and climate

The Great Lakes-St. Lawrence River system is located along the 45th parallel, bordering the Provinces of Ontario and Quebec in Canada; and the states of Minnesota, Wisconsin, Illinois, Michigan, Ohio, Pennsylvania, and New York in the United States.

Table 4 - 1 that follows shows the physical and main hydrologic features for each of the Great Lakes, as is presented in *"Regulation of Great Lakes Water Levels, Report to the International Joint Commission by the International Great Lakes Board (Under the Reference of October 7, 1964)"*, dated December 7, 1973.

The areas located north and west of Lake Superior and north of Lake Huron consist of low mountains, hills, many lakes and marshes and are known as the Laurentian Uplands. This area is not extensively cultivated, is sparsely populated, composed mainly by forests. The Central Lowlands have a different type of topography, consisting of flatlands.

An interesting feature to mention is that the lakes were formed by a process similar to the one that contributed to form the stores of groundwater. As the Wisconsin stage glacier retreated, the melting of the ice filled in the huge cavities left by the glaciers. A rough estimate of the volume of water stored would mount to 5,475 cubic miles in an area of 94,000 square miles. Important to stress as well, is the fact that only a small portion of the immense amount of stored water is affected by climatic variation which will induce the experienced range of volume

(Historical Record: 1860-1972)

All Elevations are presented in feet IGLD (1955)

	Lake Superior	Lake Michigan	Lake Huron	Lake St. Clair[1]	Lake Erie	Lake Ontario
Elevation of Low Water Datum Monthly Elevations	600.00	576.80	576.80	571.70	568.60	242.80
- Average	600.39	578.70	578.70	573.09	570.41	244.77
- Maximum	602.06	581.94	581.94	575.70[2]	572.76[2]	248.06
-Minimum	598.23	575.35	575.35	569.86	567.49	241.45
- Range of Stage	3.80	6.60	6.60	5.80	5.30	6.60
Range, Winter Low to Summer High (Monthly)						
- Average	1.10	1.10	1.10	1.80	1.50	1.90
-Maximum	1.90	2.20	2.20	3.30	2.70	3.50
-Minimum	0.40	0.10	0.10	0.90	0.50	0.70
Recorded Monthly Outflows[3] (*cfs*) Outlet	St. Marys	Str. Mackinac	St. Clair	Detroit	Niagara	St. Lawrence
-Average	75,400	52,000[4]	187,900	188,900	202,300	239,700
-Maximum	127,000	-	242,000[2]	- [5]	256,000[2]	314,000[2]
- Minimum	40,900	-	99,000	100,000[5]	116,000	154,000
Average Outflow in Inches on Total Drainage Basin [6]	12.40	11.10	11.10	11.10	10.20	10.50
Drainage Areas (sq. mi.)						
- Land Area[7]	49,300	45,600	51,800	6,100	23,600	27,200[9]
- Water Surface Area[9]	31,700	22,300	23,000	400	9,900	7,600[9]
Storage Capacity per ft. Depth (*cfs-months*)	337,000	481,000		5,000	105,000	80,000

Table 4 - 1 Great Lakes Physical and Hydrologic Data

1) Lake St. Clair elevations were available only for the period starting in 1898 and ending in the collection of present data, 1972.
2) Maxima set in 1973: Lake St. Clair 576.23 (June), Lake Erie 573.51 (June). Outflows: St. Clair River 245,000; Niagara River 265,000; St. Lawrence River 350,000.
3) Outflows include the effects of diversions.
4) Approximate.
5) Insufficient records available.
6) Drainage basin includes land and water surface areas.
7) Land areas include the total drainage area to the outlet of the upstream lake.
8) Water areas do not include areas of connecting channels.
9) Includes area down the St. Lawrence Power Project at Cornwall.

and, as a consequence, levels change. According to IGBLC 1975-76, Appendix 11, pp. 7-81, only around 1.4% of the total volume is affected.

The Great Lakes area is marked by four very distinct seasons with a great deal of temperature variations. Because of the extensive water area they cover, the lakes have great influence on the climate of the region. They act by regulating the exchange of heat, absorbing it during the day, and liberating it during the night, reducing the daily range of temperature variation. Although there is no significant difference in the amount of precipitation from one month to another, the types and sources of precipitation are very diversified.

4.2.2 Hydrologic Background

Lake Superior is located at the highest elevation and discharges into Lake Huron by St. Marys River. Lake Michigan, as mentioned in the introduction, connects with Lake Huron, through the Straits of Mackinac. Due to nature of this connection, it is possible to encounter reverse flow between both lakes, i.e., Michigan and Huron, that allows the level stabilization to occur very rapidly. This is the only situation where the reverse flow can physically happen in the region.

Lake Huron empties to Lake Erie passing first by Lake St. Clair. The connections are the St. Clair River and Detroit River, respectively. The Niagara River conveys Lake Erie outflows to Lake Ontario. This last one has a distinct situation from the rest of the five lakes, being situated an average of 325 feet below the upper ones. The reader is reminded that the average level difference between Lake Superior and Lake Erie differ by a mere 30 to 35 feet. Lake Ontario outflows are discharged through the St. Lawrence River. Figure 4 - 2 presents this situation.

The excess water from Lake Superior goes to Lake Michigan-Huron, and similarly from Michigan-Huron to St. Clair, from St. Clair to Erie and from Erie to Ontario. The time this excess water takes to travel from one lake to the other is different from the time it takes when it goes through the connecting channels. As is the case with long-term optimization, this difference in traveling time is neglected in the present work.

Since 1922 Lake Superior has its outflows regulated. It should be observed that the mean regulated monthly outflows from it are greater than the historical unregulated ones and registered before that date. As a consequence, Lake Michigan-Huron had its water levels variations increased, the same happening to the average water levels. However, since the implementation of Regulation Plan 1977, discussed later, and the consequent inclusion of Lakes Michigan-Huron water levels in the decision process, Lake Superior elevation has been kept in the upper half of what is considered as its natural range of elevation. There is an economic reason for this: when Lakes Michigan-Huron and Erie have their levels reduced, the property values of the area along the shore line increased. Also, hydroelectric power generation and navigation on the St. Marys River benefit from higher levels in Lake Superior. And there is practically no influence in the power generation at Niagara Falls.

There is a 20 foot head difference between Lake Michigan-Huron and Lake Superior. Because of that, any regulation of the former will not influence the levels in the latter. However, the same cannot be said with respect to Lakes Erie and St. Clair, that are highly dependent on the levels in Lake Michigan-Huron.

4.3 Great Lakes Water Levels and Flows Variation

The official level reference, accepted by both countries, Canada and the United States, is the International Great Lakes Datum - 1955 (IGLD - 1955). An important simplifying assumption in the present work, which eases the computations, is the non-inclusion of the earth's crust movement. This crustal movement affects the measured water levels with respect to time. Disregarding the occurrence of this physical phenomenon means assuming a constant reference for water level fluctuations throughout the study period.

Another necessary assumption is the use of average levels. Level oscillation in lakes can vary very rapidly with the water waves movements having periods ranging from one to ten seconds. Another type of movement, also disregarded, is the rocking motion called seiching, provoked by wind set up. This type of frequency analysis is beyond the scope of the present study because they have little influence in the lake levels management in the long-term.

The natural factors that contribute to the variations in water levels are listed below.

- Precipitation
- Evaporation
- Runoff
- Groundwater
- Ice Retardation for the connecting channel flows, causing flow retardation.
- Aquatic Growth (Weed Retardation), causing the same retardation problem as above.
- Meteorological Disturbances like winds, atmospheric pressure (disregarded).
- Tides (disregarded)
- Crustal Movement (disregarded)

From the list above, only the first six factors influenced the data used in the present work. Moreover, as explained further in this chapter, when discussing the Net Basin Supply approach, the first four, that are difficult to measure because of the uncertainties involved, are combined into just one, the Net Basin Supply (NBS).

As one might expect, the lake levels suffer variation when the net difference between the input and the output of water is different from zero. When greater than zero, there is an increase in the lake level. And when less than zero the lake level decreases.

And the artificial factors that contribute to the variations in water levels are:

- Dredging
- Diversions
- Consumptive use of lakes water
- Regulation

Dredging is not taken into account in this work, therefore it was assumed that the connecting channels have a static configuration along the study time. For the diversions, their locations and respective amount is summarized below.

Average monthly diversions, to and from the lakes, in *tcfs-month*. This notation, means an average flow of 1000 cubic feet per second during one month. It can be easily converted into volume.

1. To Lake Superior
   - Long Lac and Ogoki diversions - 5.0 *tcfs-month*
2. From Lake Michigan
   - Chicago Sanitary and Ship Canal - 3.2 *tcfs-month*
3. From Lake Erie
   - Welland Canal diversion, from lake Erie and into lake Ontario - 7.0 *tcfs-month*

Diversion	*Lake Superior*	*Lakes Michigan-Huron*	*Lake Erie*	*Lake Ontario*
Long Lake and Ogoki	0.00	+0.37	+0.23	0.00
Chicago	-	-0.23	-0.14	0.00
Welland Canal	-	-0.10	-0.32	0.00

Table 4 - 2 Effect of the Major Diversions on Lakes Levels (ft.)

The present regulation plans were designed in such a way that the diversions and their effects on lakes levels were accommodated. Any suggested alternate plan must do the same and this Case Study makes no exception. The consumptive uses for lakes water were neglected because they present varying rates from one year to another and the projection of their increase for the future is not very well determined as of the publication of "Regulation of Great Lakes Water Levels", 1973. Although this might affect the results for the operation, that it is beyond the scope of this research. Besides, once these values are clearly specified, they can easily be inserted in any model.

The following figure, Figure 4 - 3, illustrates the configuration of the system and served as a reference for this work, presenting the regulated and unregulated flows from the lakes as well as the existing diversions.

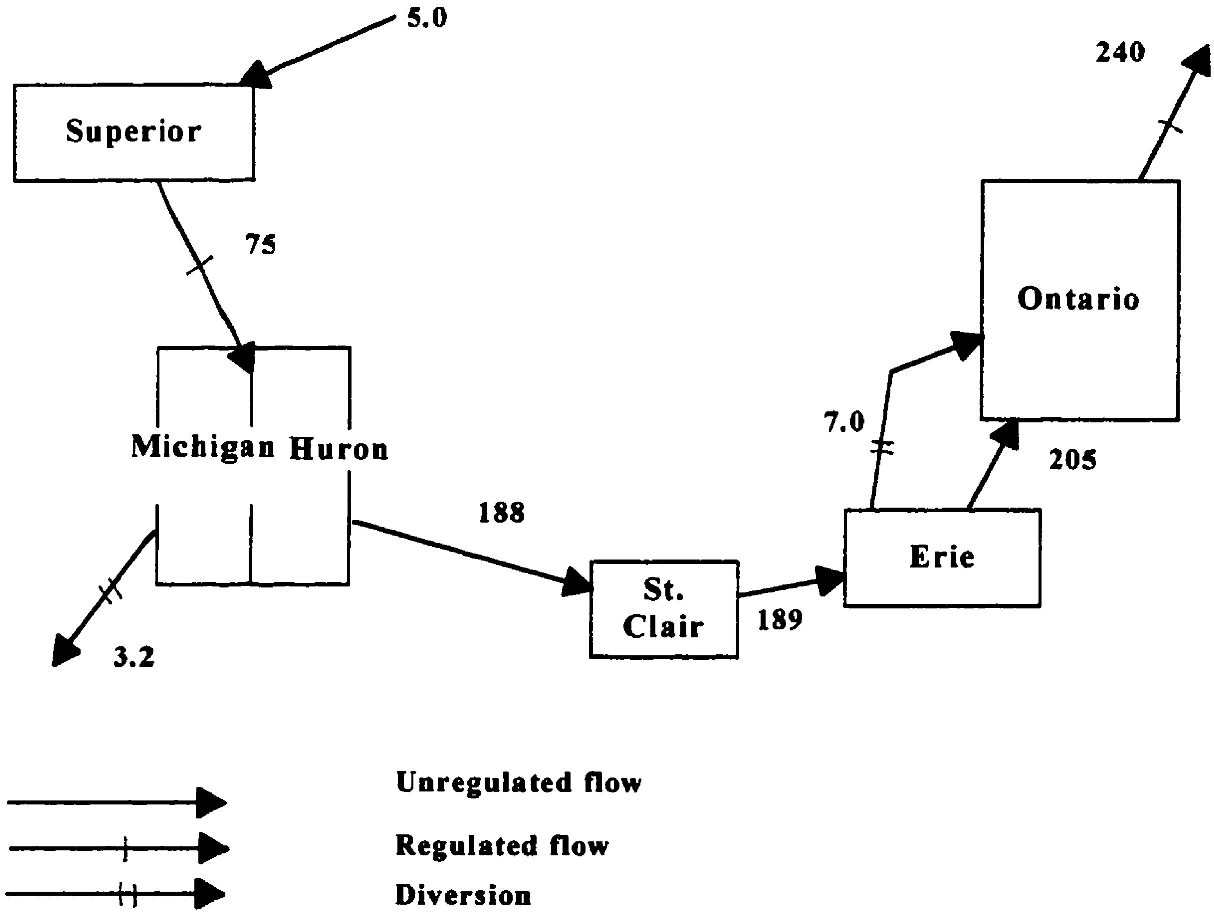


Figure 4 - 3 Schematic Diagram of the Great Lakes System Showing Outflows and Significant Diversions

4.3.1 Hydrologic data

4.3.1.1 Recorded data

The data used in the present work was obtained from "Regulation of Great Lakes Water Levels", Appendix B, Vol. 2, 1973. They are records of levels, flows and diversions dating from the period starting in January 1900 and ending in June 1973. That study, which reached its conclusions in December 1973, although dated, considered the data "uniformly consistent" and has the agreement of the Federal Agencies from both countries. It presented the necessary amount of information that enabled the author to develop and to test the optimization and simulation models. The results obtained from the simulation model using derived policies were compared with those from the simulation of the operation, described further in this Chapter in an attempt to emulate the operation as the IJC prescribes it.

4.3.1.2 Derived data

The Net Basin Supply, a parameter that was derived from the data above, was also used. As the name implies, it provides the information on the total net supply to the basin and below the two existing approaches to establish a hydrologic database are explained. From it, the hydrologist may know the amount of water that the lake receives from precipitation from within the watershed, including its surface area subtracted from the loss caused by evaporation and condensation. The approaches are based in the following equation, differing in the choice of recorded parameters to be employed. The hydrologic budget can be written as:

$$\Delta S = (I - O)\Delta t + (P - E)A_1 + R\Delta t + G_{net} - D\Delta t \qquad 4.1$$

Also, ΔS is defined as equal to ΔHA_l, a linear relationship, applicable in the present case.

Where, defining the time step Δt equal to one month, we can define:

ΔS - monthly variation of storage on lake from beginning to end of period in *cfs-months*;

ΔH - monthly change of lake level in feet;

A_l - lake area in *cfs-months* per foot depth;

I - average monthly inflow from inflow connecting channel in *cfs*;

O - average monthly outflow from outflow connecting channel in *cfs*;

P - direct precipitation onto lake in feet (areal average);

E - evaporation from lake in feet (areal average);

R - basin runoff in *cfs*, excludes the portion that enters the lakes through the mainstem connecting channel;

G_{net} - net inflow in *cfs-months*, contribution from groundwater sources;

D - diversion out of lake in *cfs*.

Rearranging Equation 4.1.

$$(P-E)A_1 + R\Delta t + G_{net} = \Delta H A_1 + (O-I)\Delta t + D\Delta t \qquad \textbf{4. 2}$$

Each side of Equation 4. 2 can be used to compute the NBS. Depending on which side is employed, there is either the rainfall-runoff technique or the lake response technique. The former is defined as

$$NBS = (P-E)A_1 + R\Delta t + G_{net} \qquad \textbf{4. 3}$$

while the latter is defined as

$$NBS = \Delta H A_1 + (O-I)\Delta t + D\Delta t \qquad \textbf{4. 4}$$

Both techniques above have a considerable margin of error, but for the Great Lakes case, the lake response method is easier and more reliable because of the kind of uncertainty embedded in the runoff-rainfall method. Precipitation, evaporation, runoff and groundwater contribution measurements are far more difficult to be effective for lakes with the size of those that compose the Great Lakes system than levels, areas, outflows, inflows and diversion measurements. Figure 4 - 3 presents a schematic view of the hydrologic phenomena just described.

The tables presenting the monthly historical values for the NBS for the five lakes are presented below. All values are given in *tcfs-months*. They were obtained for the period ranging from January 1900 to June 1973

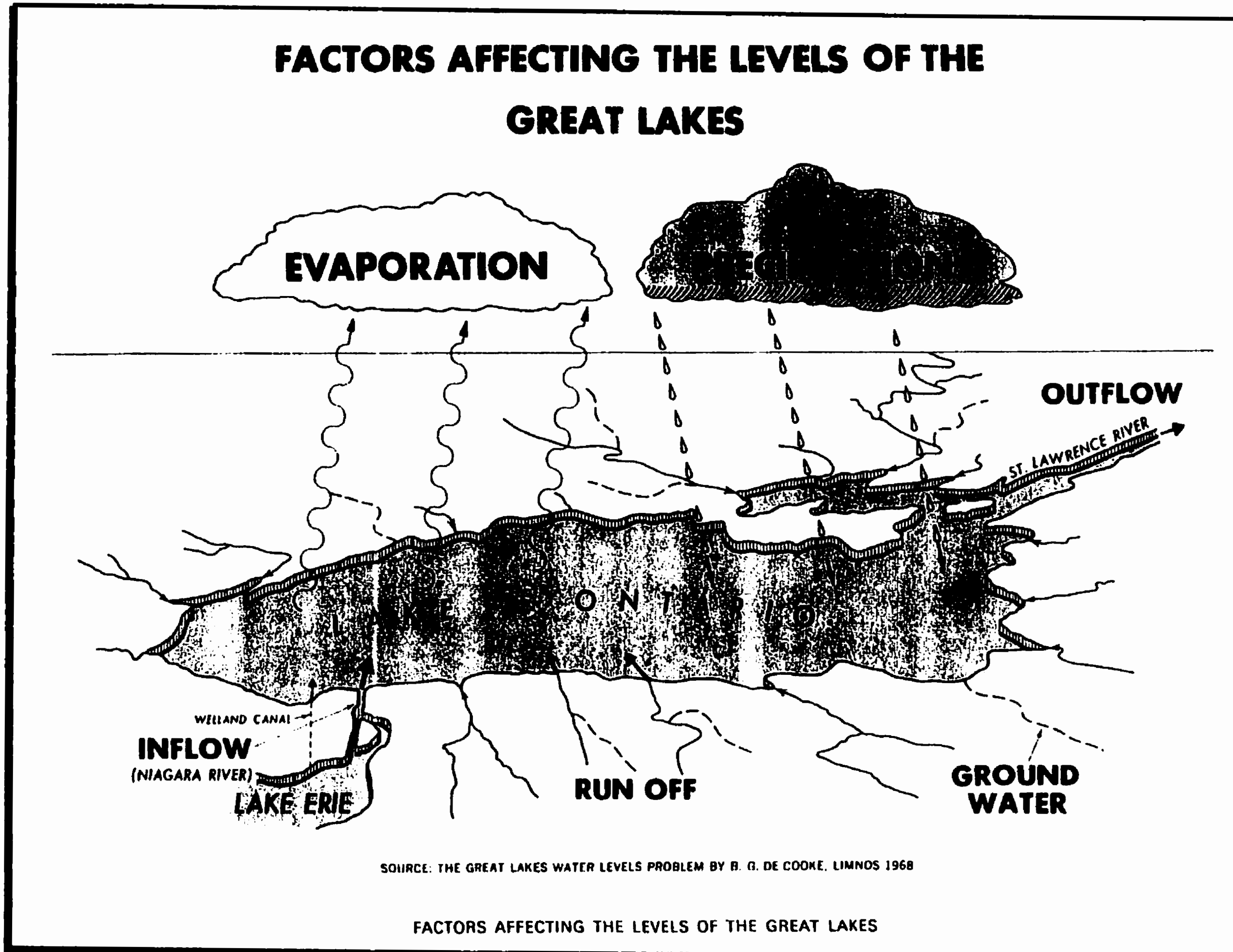


Figure 4 - 4 Schematic Presentation of the Hydrologic Phenomena Affecting the Great Lakes

Source: Regulation of Great Lakes Water Levels, Report to the IJC, December 1973.

Month	*Lake Superior*	*Lakes Michigan-Huron*	*Lake St. Clair*	*Lake Erie*	*Lake Ontario*
January	-12.99	55.18	6.15	22.99	31.91
February	8.99	86.78	6.76	29.77	35.55
March	42.73	178.89	8.34	68.88	73.41
April	148.59	283.74	7.81	65.42	92.31
May	191.50	255.61	6.46	44.19	60.00
June	158.72	208.31	4.50	26.81	41.26
July	131.32	132.15	3.75	2.16	24.74
August	100.23	30.26	2.14	-15.92	7.34
September	75.32	29.19	1.68	-22.48	2.48
October	36.97	49.97	1.60	-24.95	5.19
November	14.92	-1.16	1.73	-9.47	16.36
December	-24.40	25.62	4.33	11.58	22.55

Table 4 - 3 Historical Mean Values for the Net Basin Supplies to the lakes (values in *tcfs-month*)

Month	*Lake Superior*	*Lakes Michigan-Huron*	*Lake St. Clair*	*Lake Erie*	*Lake Ontario*
January	25.13	52.44	7.15	37.91	22.67
February	27.53	44.84	6.79	28.18	18.61
March	41.88	74.10	7.16	30.76	27.23
April	51.06	84.79	7.60	31.32	28.74
May	59.12	82.08	6.36	25.91	25.55
June	52.24	64.38	4.58	20.62	18.12
July	39.82	54.18	4.60	17.35	14.80
August	42.01	59.91	3.87	14.28	12.58
September	53.03	68.23	3.50	19.29	12.29
October	44.93	71.21	3.29	19.68	16.85
November	39.49	63.29	3.41	21.22	18.38
December	29.22	63.65	5.79	27.67	20.73

Table 4 - 4 Historical Standard Deviation Values for the Net Basin Supplies to the lakes (values in *tcfs-month*)

4.3.2 Basis-of-Comparison

During the period that the data were recorded, the lakes regime and the hydraulic conditions of the connecting channels changed. Therefore, the registered data has the influence of these changes and cannot be considered as part of a stationary process. The IJC Report and its data assumed some constant conditions, called the Basis-of-Comparison, in such a way that it is possible to make the necessary evaluations within a controlled environment. With the conditions defined as stated below, the historical NBS were routed through the system and the monthly mean levels and flows were computed. To allow meaningful comparison with the operation as defined by the Commission, for the case of the present work, the synthetic NBS (see discussion further in this Chapter) were also routed through the conditions used to define the Basis-of-Comparison. That means that these conditions were valid either for the simulation of the present operation, and for the testing of the proposed release policies. They are:

1. The values for the diversions to and from the lakes assume constant monthly values as previously presented in this Chapter.
2. Lake Superior is operated in accordance with the 1977 Regulation Plan. This is the operation plan used by the International Lake Superior Board of Control (ILSBC).
3. 1962 outlet conditions for Lake Huron.
4. 1953 outlet conditions for Lake Erie.
5. Lake Ontario is operated in accordance with the 1958A Plan. The IJC Reports use the 1958D Plan, where the basic rule flow, computed as 1958A Plan, is further submitted to a series of adjustments, based on a index called Supply Indicator, that serves as an indication of future supplies. If it points to a situation where there is a significant amount of positive supplies more outflow is released. And vice-versa. These adjustments are not considered in the present work. The main reason for that is that although Plan 1958D is presently in operation, it must be noted that depending on the interests involved and physical conditions like levels, outflows, etc., the authorities can exercise discretion to modify the rules of operation and this happens with a certain frequency. Plan 1958D can be seen rather like the guideline for operation than the exact rules of operation.

The first part of Appendix A, entitled LakSim (acronym for Lakes Simulation), has the algorithms with the explanation of how the present operation simulation was set up. Appendix B reproduces the findings and conclusions from the 1973 IJC report that served as a reference for the approach used in this work. These findings and conclusions influenced not only the present operation simulation, but also the aggregation and optimization parts, here included with the performance evaluation simulation. Table 4 - 5 lists the levels and ranges used as the Basis-of-Comparison for the five lakes. All values are presented in *ft*.

Level	Lake Superior	Lakes Michigan-Huron	Lake St. Clair	Lake Erie	Lake Ontario
Maximum	601.86	581.59	576.56	573.63	247.32
Minimum	598.68	575.13	570.84	568.02	241.66
Range	3.18	6.46	5.72	5.61	5.66

Table 4 - 5 Maximum and Minimum Suggested Levels and Ranges

During all the simulations, whenever the tentative lake level at the end-of-period was situated outside this desired range, which is specified in Table 4 -5, it was considered as an occurrence of exceedance. Next, they were recorded to obtain the probabilities of exceedance. These probabilities refer to the inability of the release policy to maintain the lakes levels within the specified range, without further adjustments. However, because the real simulation conditions were reproduced, the outflow was corrected in order to maintain the lake level within the desired boundaries. These corrections did not violate the outflows constraints.

4.3.3 Regulation Plans

4.3.3.1 General Information

According to the Levels Reference Study, Annex 3, 1993, there were two approaches used to determine the regulation plans for that analysis. The first one, as will be noted in the next two sections, employed a traditional hydrologic approach, by setting up rule curves that aim at satisfying an ensemble of hydrologic specifications. The process consists of simulating several conditions of levels and outflows for the upstream and downstream lakes and, by routing the monthly (or weekly) NBS through them, doing the necessary adjustments until a satisfactory output is obtained. This process is basically trial-and-error. This is the methodology currently under use. The second approach, considers the Great Lakes System as a whole and proceeds the optimization of the operation in such a way that the benefits from the entire set of interests, a multiobjective function, are maximized. They employed value functions that assigned a value within the range 0-1 to outflows or levels. Zero being the most preferred situation and one being the least preferred situation. The major difficulty with this approach is the definition of the value functions, that are fuzzy values and thus generating fuzzy benefits. As written in the above mentioned report, they are a "wish list", defined in "the context of not knowing the impacts or consequences of such water level and flow regimes, and not knowing if it is possible to achieve such regimes." (See referred Study, page 6-5).

For the present work, the objective of the first type of operation plan was adopted. In it, the most important issue is the maintenance of some desired levels of operation. This type of objective was employed for the simulation of the present operation and for the optimization part. The desired levels of operation are simply the average levels obtained by routing the historical NBS through the system subject to the Basis-of-Comparison conditions. The present release policies for Lake Superior consider the states of Lake Superior and Lakes Michigan-Huron, while for Lake Ontario only its levels are taken into account. In the current work, during the optimization phase, however, a fundamental difference from that approach was introduced. The operation policies are now computed considering the state of the system as a whole. As the system presents controls only for the lakes situated at the extreme locations of the system, the policies are defined solely for Lakes Superior and Ontario.

Now will follow the description of how the present policies were obtained for the International Joint Commission. Policies devised using historical NBS are satisfactory for the past sequence of supplies. If critical periods of droughts or excess supply of water of greater amplitude than those registered so far occur, it is probable that the policies would not be satisfactory. Because of that, synthetic supplies with the same statistical properties as the historical ones were generated, but with much longer duration. Once more, it is important to have specified stationary conditions to generate the stochastic process. Using the simulation approach as described in the first paragraph, it was necessary to test an empirical combination of policies for several hundred trial plans. To reduce the computational effort, first just the historical sequences were used. After selecting a set of the best policies, the synthetic series were used to obtain the final ones. By adopting a stochastic strategy to solve the problem, it is expected that this calculation effort is reduced and that the operation policies thus obtained, scientifically formulated.

There are three major reasons for adopting the strategy of assuming the stationarity of flows and trying to maintain the resultant historical mean levels as the most appropriate for operation objectives.

1. Even though it is possible to make some inferences about the future expected water supplies, they still have embedded a great deal of uncertainty. The historical levels were already tested and, as of the present moment, considered as acceptable (although not preferable) by most of the interest groups.
2. It is very difficult to put economical values to some desired features of operation such as recreation and aesthetics.
3. It is practically impossible to promote equitable allocation of potential benefits amongst the different, and sometimes conflicting, groups of interest.

The structure of the hydrologic response model along with the regulation plan can be summarized as is presented in Figure 4 - 5, according to Loucks et al. (1987).

4.3.3.2 Lake Superior Regulation

Lake Superior Regulation is called Regulation Plan 1977. In October 1979, it replaced the IGLLB plan SO-901 and is just an update of this previous plan. Before SO-901, where SO stands for Superior and Ontario regulation, there existed the Modified Rule of 1949, which was in operation during the 1950s and 1960s. The latter considered only Lake Superior levels for the determination of its outflows, while the other two take Lakes Michigan-Huron level into consideration. As a result, it was expected that the benefits of this inclusion would extend to the lakes located downstream of Lakes Michigan-Huron, Lakes St. Clair and Erie. However, as pointed by Loucks et al. (1987), it never became clear whether this really happened, with two conflicting opinions about it. Plan 1977 is established in such a way that the natural Lake Superior outflows are emulated, already considering the Long Lac-Ogoki diversion. With the inclusion of Lakes Michigan-Huron levels in the regulation plan, the excess water is stored in Lake Superior. The consequence is an increase in its average level that would be beneficial for hydroelectric energy generation and navigation in the St. Marys River. The lowering of average levels in the Middle Lakes would improve the shoreline property values without compromising the energy generation at Niagara Falls. The critics to this Plan state that, in the long-term, this increase in shoreline property values will not take place because the levels downstream Lake Superior will gradually return to its previous situation. My opinion coincides with this point of view, because once the new average level is attained for Lake Superior, the long-term average outflows would remain the same, and consequently the increase in the expected benefits are transient and not stationary. However, if only hydropower generation and navigation in Lake Superior are the objectives, the benefits would increase.

The basic outflow for Lake Superior can be obtained from the equation below.

$$Q_{basic} = Q_{AvSup} + A[(S_t - S_j) - (H_t - H_j)R0_j] \quad 4.5$$

Where:

Q_{basic} - basic rule flow.

A - statistically derived constant - adopted the "best" one, as defined in "Regulation of Great Lakes Water Levels", Appendix B, Lake Regulation, 7 December 1973, p.B-40, equal to 200 *tcfs/ft*. For more details, follow explanation below on how this value was set.

Q_{AvSup} - historical average outflow, computed from the data collection period as recommended in the above mentioned report.

$$R0_j^2 = \frac{Var[S_j]}{Var[H_j]}$$

$Var[S_j]$ and $Var[H_j]$ - estimated monthly historical variances for water levels at lakes Superior and Michigan-Huron respectively.

S_j, H_j - target levels for lakes Superior and Michigan-Huron, also defined as the estimated monthly historical averages.

S_t, H_t - Beginning-of-Period (BOP) water levels.

The adjustment to the historical average outflows comes from the simple linear relationship, where the present hydraulic head is normalized by the monthly standard deviation of the BOP water levels:

$$\frac{S_i - S_j}{\sigma_S} = \frac{H_i - H_j}{\sigma_H} \qquad 4.6$$

The objective of the relationship above is to try to approximate the relative water levels from both lakes to the same situation as the one registered by the average historical ones and within the natural ranges.

A is an empirically derived parameter expressing the rate of adjustment to the initially calculated flow from lake Superior. It adjusts the speed of bringing the relative levels between lake Superior and lakes Michigan-Huron into balance. The value below was established based on computer simulations.

Other tested values were 50,000; 100,000 and 300,000 *cfs/ft*. The chosen one presented the best economical impact on lakes Superior, Michigan-Huron and Erie considering the multiobjective function during the study period of 1900-1967.

$R0$ is the relationship between the standard deviations of lakes Superior and Michigan-Huron. As the parameter A, it was employed in the computation of the rule flow.

Month	R0
January	0.3706
February	0.4254
March	0.4546
April	0.4480
May	0.5166
June	0.4625
July	0.4878
August	0.4445
September	0.4196
October	0.4106
November	0.4059
December	0.4274

Table 4 - 6 Monthly values for the *R0* parameter

Once the rule outflow is computed, its value is compared with the maximum and minimum allowable limitations and, if these constraints are not satisfied, these limiting values are assumed. There is an exception, however. The minimum outflow is 55 *tcfs-months* but if the rule outflow falls within the range 55 to 65 *tcfs-months*, it is automatically adjusted to 55 to allow Lake Superior levels to increase. Another kind of criteria is also specified, such as the change in flow rate from one month to another must not exceed 30 *tcfs-months*, in absolute value. This limitations aims at maintaining a regular flow rate, reducing the deviation from the average values.

Plan 1977 also includes the computation of the gate settings based on the forecast of NBS for the next few months in order to reduce the number of gate movements during the year. For example, the gate setting defined at the beginning of December is valid until the month of April. In this work, the gate settings computations are not carried out, nor are the predicasts, the name given to "predicate" forecasts, that are predictions effectuated using just mean values for the net supplies.

4.3.3.3 Middle Lakes Routing

Lakes Michigan-Huron, St. Clair and Erie are directly connected and no regulation is exercised. Thus, the flow between them depends on their relative levels, i.e., difference between upstream and downstream lake levels and the physical characteristics of their connection. The connecting channels are St. Clair and Detroit rivers. The outflow in the St. Clair river is a function of both lakes Michigan-Huron and St. Clair and, in Detroit river, of lakes St. Clair and Erie. The outflow from this last one, through the Niagara river, affects its levels. This information must, therefore, be taken into account, as well. Because of the abrupt hydraulic jump between lakes Erie and Ontario, levels from the latter do not affect the upstream levels. If all this information is assembled, the resulting system of equations has six unknowns and six equations to be solved. For sake of simplicity, the approach already derived for the Middle Lakes Routing in the report by Loucks et al. (1987) was used. The derivation of the system of equations has the following reasoning.

For the relationship that described the steady nonuniform flow for St. Clair and Detroit rivers the authors of that report employed the one from GLBC 1975-76, Appendix B, p. 56 and they remark its similarity to the theoretical formula for flow over a submerged, broad-crested weir.

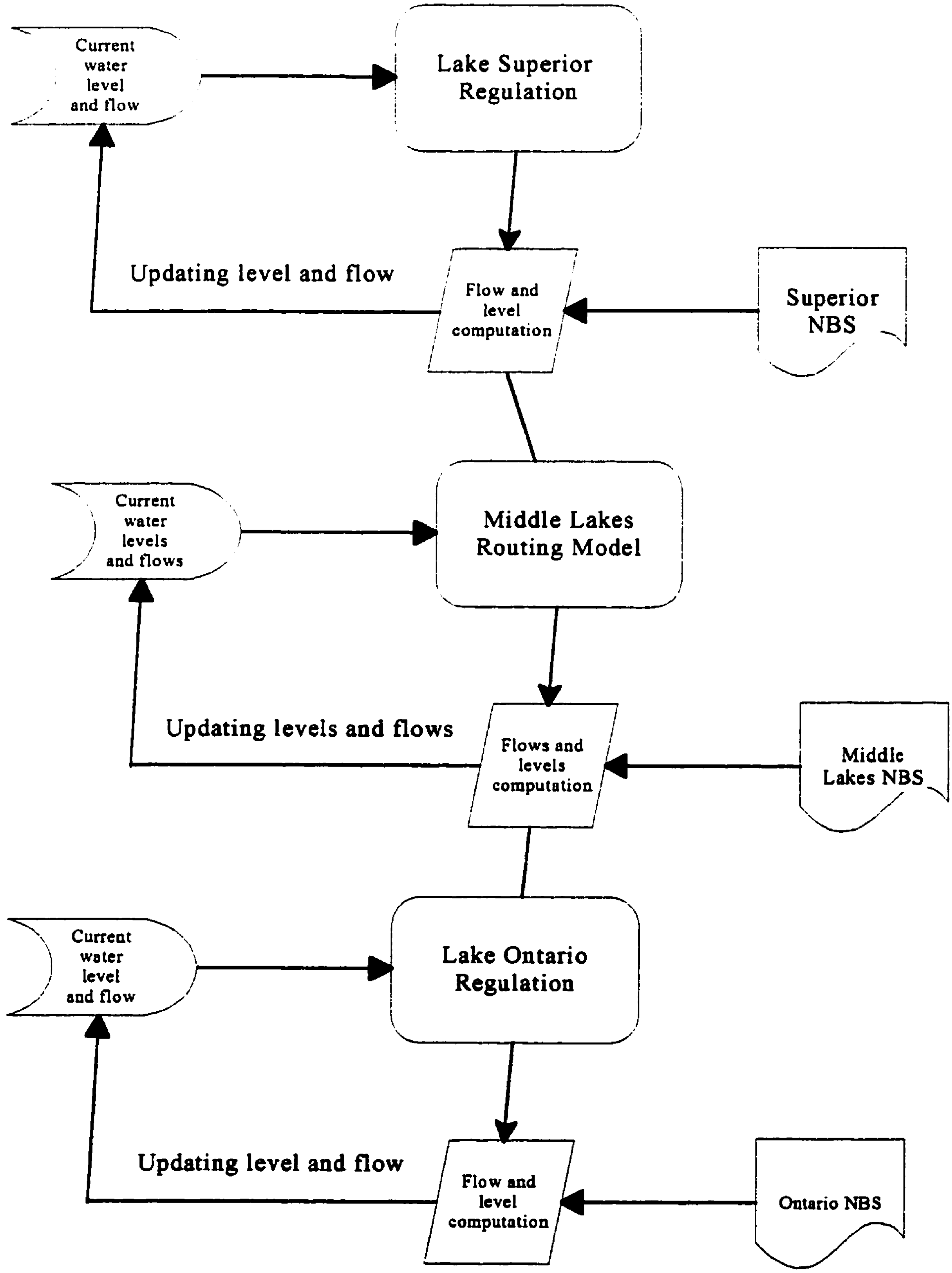


Figure 4 - 5 Great Lakes Regulation and Hydrologic Response Model

$$Q = C'AH^{1/2} = C''ByH^{1/2} \qquad 4.7$$

Where:

A - average area of the flow;

B - average width of the flow;

y - mean depth;

H - effective upstream head;

C' and C'' - constants that depend on the channel geometry and its roughness, values that vary according to the system of units employed.

Let us assume that

$$y = \frac{h_1 + h_2}{2}, \qquad 4.8$$

$h1$ and $h2$ being the elevations of the water surfaces for the upstream and downstream lakes respectively.

Replacing the y in formula 4-7 we obtain

$$Q = C(\frac{h_1 + h_2}{2})^2 (h_1 - h_2)^{1/2} \qquad 4.9$$

The formula above assumed that the width of the flow is a linear function of depth and it is called a two-stage discharge relation. It is named two-stage because it considers the stages in both upstream and downstream lakes.

For the Niagara river, the one-stage relationship gives

$$Q = C(h_1 - h_2)^{3/2} \qquad 4.10$$

where the C 's are determined empirically by a fitting technique. It is of import to note that in this work the values for these constants are different from those employed in the just mentioned report. Those that were set up specifically for the data in hand, that consider exactly the same stations for the flow measurements, were the chosen ones. They were obtained from IGLLB, 1973, App. B, V.2, p.B-9.

The so-called constants *kapa* assumed the following values:

- For St. Clair river:

kapa1 = 73.5150

- For Detroit river:

kapa2 = 177.2860

- For Niagara river:

kapa3 = 3665.0000

For Lakes Michigan-Huron it is necessary to consider the regulated outflow from Lake Superior, the NBS for that month, the diversion of flow into the Chicago Sanitary and Ship canal. This value has been fixed, in an agreement between the two countries, remaining constant for the figure presented before. For Lake St. Clair, the smallest one of all five, receives the nonregulated outflow from Lakes Michigan-Huron, its NBS and produces nonregulated outflow. Lake Erie reproduces the previous situation, with the inclusion of the Welland Canal diversion. This diversion is assumed as a constant addition to the Lake Erie outflow. From the relationships presented above, six nonlinear equations are derived. To solve this system of nonlinear equations, linearization is adopted, and the standard time step considered for Lakes Superior and Ontario, a month, is decomposed in 40 sub-steps. After testing, it was observed that an iteration cycle of 5 repetitions was enough to guarantee convergence in each sub-step.

Appendix A presents the algorithm for the Middle Lakes Routing problem. As they remain unregulated for the proposed operation policies, the same algorithm can be employed in the second part.

4.3.3.4 Lake Ontario Regulation

As there is no lake downstream of Lake Ontario, the computation of the basic rule flow uses just its own levels and Lake Erie's. The relationship that appears below is shown in "Regulation of Great Lakes Water Levels", December 1973. Originally, this formula refers to a time step equal to one week, but here it was employed to solve for a monthly time step. The adjustments performed with the help of the supply indicators as it is prescribed in Plan 1958 D, which is currently in use, were disregarded.

The following relationship was employed to compute the basic rule flow

$$Q_{basic} = 200\left(\frac{(EOP_{ont} - Tg_{ont})80 + (EOP_{erie} - Tg_{erie})105}{185}\right) + AvOut_{ont} \qquad \textbf{4.11}$$

where

Q_{basic} - basic rule flow,

EOP_{ont} - monthly end-of-period Ontario water level,

Tg_{ont} - target water level for lake Ontario, given by the estimate of historical mean level,

EOP_{erie} - monthly end-of-period Erie water level,

Tg_{ont} - target water level for lake Erie, given by the estimate of historical mean level, and

$AvOut_{ont}$ - long-term mean pre-project[1] lake Ontario outflow.

Having the basic rule outflow, its value is compared with the maximum and minimum allowable limitations and, if these constraints are not satisfied, the necessary adjustments are made. Appendix C presents the information regarding these constraints. As was the case with Lake Superior regulation, the changes in flow rate from one week to another must not exceed 20 *tcfs-months*, in absolute value. This figure was adopted for the maximum allowable monthly variation. The intendments of this measure are similar to those in Lake Superior. The reader is referred to Appendix A for more details on this regulation implementation.

[1] The outlet conditions are those existing before the regulation of lake Ontario, i.e., before the year of 1955.

4.4 Definition of the Problem

In the introduction of this chapter it was mentioned that the intent of the Reference submitted to the governments of both countries was to improve the range of water levels fluctuations in such a manner that the interest groups would benefit from it. Or, at least, that the improvement of the conditions for one group would not result in the worsening of the present situation for the others.

To be consistent with the approach already utilized in the IJC studies, given that the major concern of this application is the validation of an experimental methodology, and facilitate the comparison between an existing rule of operation and a new proposed one, the conditions stipulated by the existing agreements, materialized as the Basis-of-Comparison, are those considered as valid in this Case Study. In other words, they were the same for the simulation of the routing of the synthetic NBS for the five lakes either in the case of the present regulation (as just described in the previous section) and the new proposed one.

It was also pointed out that it is very difficult to measure the real benefits obtained from improvements regarding recreation, aesthetic and environmental considerations, along with other similar benefits that are important and generate returns to the society as whole but with fuzzy benefits, hard to evaluate with precision. As dealing with this type of fuzzy values will add more uncertainty to a stochastic problem and is not directly connected to the main purpose of this research, it was decided to opt for more well-defined designs, the maintenance of the IJC's objectives as stated in the 1973 report, "Regulation of the Great Lakes Water Levels". As a result, they were defined as the attempt to reproduce the average estimated mean levels that would have occurred in the 1900-1973 period, had the static configuration of the Great Lakes System been as defined in the Basis-of-Comparison.

Some simplifying assumptions were introduced in setting the simulation of the operation. For instance, the use of monthly time steps for Lake Ontario and Lake Erie instead of quarter monthly time steps, and the Lake Ontario regulation as presented in "Regulation of Great Lakes Water Levels", App. B, 1973, p. B-40. To avoid the comparison of different types of operation, first the system operation is simulated by routing the historical NBS through the five lakes and connecting channels. These results are then compared with the same operation procedure for the synthetic data generated as explained further ahead. With this procedure, two models that emulate more closely the stochastic process that generated the Net Basin Supplies were validated. A practical implementation aspect that was taken into consideration was the need to use pre-existent Matlab® routines, especially those from the System Identification Toolboxes, because of the compatibility with the rest of the routines developed for this work and immediate availability.

In "Levels Reference Study", Annex 3, 1993, there is the information that the autoregressive moving average (ARMA) processes, ARMA (1,1) and ARMA (2,0) had good performance in modeling the NBS. But once the processes were modeled as univariate, they had to be linked together due to the spatial correlation existent between the lakes. Yevjevich (1975) and Buchberger (1992) used a multivariate ARMA (2,0) to model the processes with respect to time and spatial correlation.

In 1991 the U.S. Army Corps of Engineers modeled the NBS for each lake (univariate) as an ARMA (1,1) process. Loucks (1989) used Contemporaneous ARMA (CARMA) processes for this purpose and the above mentioned study also experimented with the ARMA (1,1) processes for the upper lakes and ARMA (2,0) for the lower ones. Because it only accounts for lags equal to zero, hence the contemporaneous in its name, it is much faster and easier to implement and generate the synthetic NBS than most multivariate processes. This work is constrained by the fact that the Matlab© Identification Toolbox has available only multivariate autoregressive models and univariate ARMA ones. Therefore, the following ones were tested:

- ARMA (1,1), five univariate models.
- ARMA (2,0), five univariate models.
- Multivariate AR models
  - ◊ Contemporaneous AR models of orders 1 and 2.
  - ◊ Contemporaneous Periodical AR model of order 1.
  - ◊ AR model with lower triangular input matrix of order 1.

From the models above, the contemporaneous multivariate seasonal AR model of order 1 and the contemporaneous multivariate periodical AR model of order 1 were those that more closely approximated to the historical NBS data. As additional testing, outputs like lakes levels and outflows that were obtained after the simulation of the operation were considered as well, and both models performed satisfactorily. Although some of the other models presented similar performance, the suggestion presented in Hipel and McLeod (1994) was followed and Occam's razor was applied, i.e., those with less complexity involved were chosen. Appendix C presents a summary of the inputs for the simulation and the main parameters involved in the system operation. In the following pages, are added tables with the operation results for the simulation of both the historical NBS and the synthetic ones.

The objective function evaluated during the simulations was defined according to the following relationship:

$$\text{Cost} = \sum_{t=1}^{T}\sum_{l=1}^{L}\sqrt{(S_l^t - \overline{S}_l^t)^2}, \quad \text{for } t = 1,\ldots,12; \text{ and } l = 1,\ldots,L. \qquad \textbf{4. 12}$$

Where:

S_l^t - average level of lake l in month t;

$\overline{S}_l^t$ - estimated average monthly level of lake l in month t, after routing the historical NBS through the lakes employing the BOC.

This information can be obtained from "Regulation of Great Lakes Water Levels", Lake Regulation, App. B, V. 1, Coordinated Basic Data, 1973. Also, the average level is defined as:

$$S_l^t = \frac{BOP + EOP}{2} \qquad \textbf{4. 13}$$

As one might expect, the objective of the optimization was the minimization of the relationship defined by equation 4. 12, but rewritten as follows:

$$\text{Expected Cost} = \sum_{t=1}^{T}\sum_{l=1}^{L} E[\sqrt{(S_l^t - \overline{S}_l^t)^2}\,] \qquad \textbf{4. 14}$$

The use of the expectation operator above is due to the fact that for a specific release policy, given a BOP level, and a stochastic NBS, the average level becomes a random variable.

4.5 Aggregation Scheme

The Great Lakes system presents control structures in only two of its components, namely, Lake Superior and Lake Ontario. Therefore, to avoid local policies for the latter, the aggregation was performed considering in the first stage, Lake Superior versus an aggregated lake downstream that comprised Lakes Michigan-Huron, St. Clair, Erie and Ontario. In the second stage, Lakes Superior, Michigan-Huron, St. Clair and Erie were aggregated in one upstream lake and then a two-level MAM-SDP optimization, with Lake Ontario located downstream of it, was performed. For both controls the policies are global, but still approximate. This approach was called *"Whole System Optimization"*.

On the other hand, another approach where Lake Ontario is optimized as an independent part of the system, was also set up. This is basically the approach utilized by the present regulation, however in a heuristic fashion. As this lake is situated roughly 330 *ft.* below the average level of the other upstream ones, its backwater does not affect the levels of none of them. The first stage of aggregation, however, remains as described in the above paragraph. This one was called *"Lake Ontario Independent"*. There is a schematic representation of stages 1 and 2 for the *"Whole System Optimization"* shown below and for the second approach, stage 1 is repeated and the average monthly outflow from Lake Erie obtained after simulation and fed it into the single reservoir optimization model. The most notable consequence is the sensible reduction of the computation time.

Another feature worth mentioning is the aggregation of the releases. As already said, only Lakes Superior and Ontario are regulated. The standard fashion of aggregating does not consider unregulated outflows. Therefore, the aggregated outflows were computed as a function of the states of Lake Superior versus the rest of the system and Lake Ontario versus the rest of the system after 1000 year simulation using the current operation. These preprocessed data were then fed into the SDP optimization. As one might conclude, these relations are not the same as those that will be computed using MAM-SDP, or state-derived policies. A natural continuation of the present work would now obtain the relations for the optimization after the first optimization is performed and the simulation executed, and fed them once more to the optimization, i.e., repeat at least one more iteration. The present computations did not take this into consideration.

Appendix A has a succinct description of the algorithms employed to perform the optimization of the Case Study. The reader is referred to it for more details on the implementation.

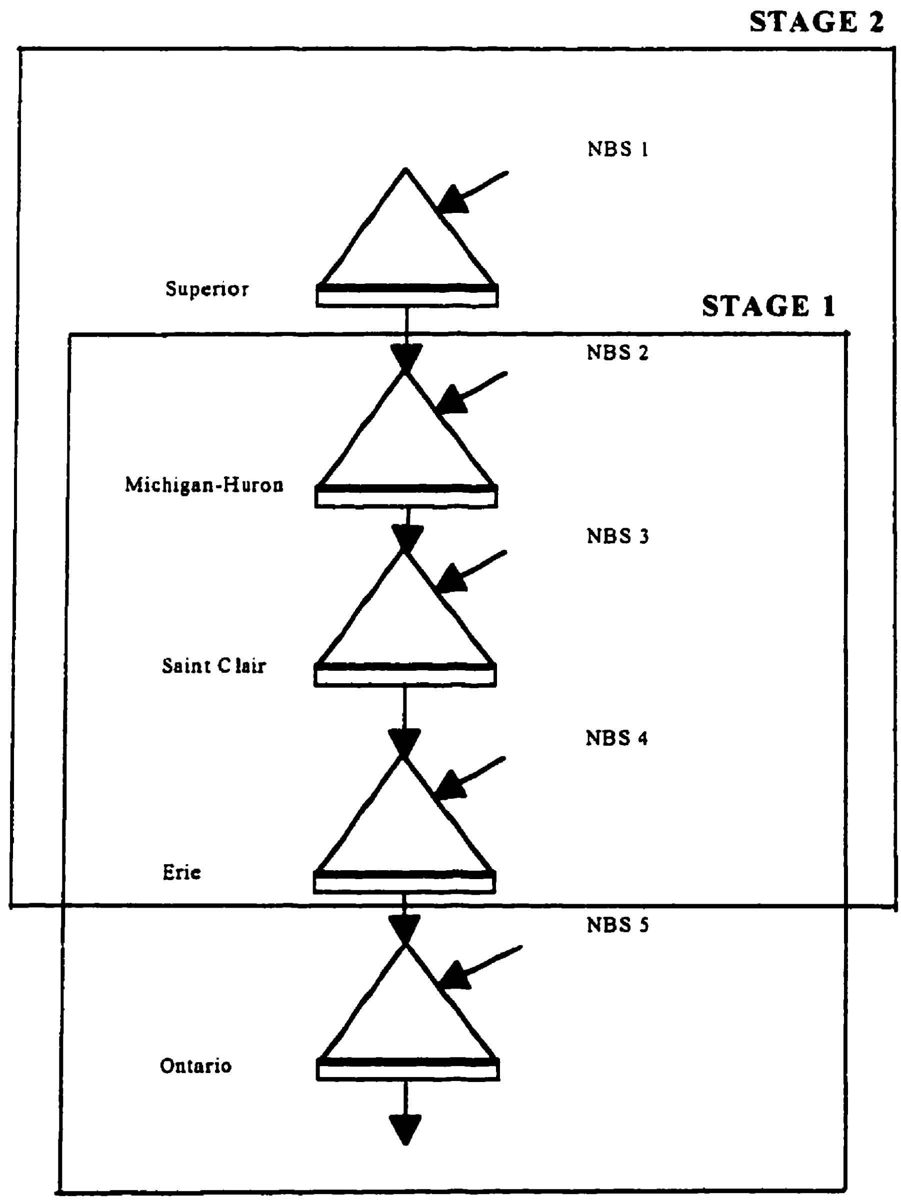


Figure 4 - 6 Schematic Representation of the Great Lakes Aggregation Approach

4.6 Validation of the Simulation Model

The validation of the 1000 year synthetic data was performed not only by analyzing the time series but comparing parameters like cost, storage levels, outflows and spills obtained either routing the historical NBS and the synthetic ones. The suggestion presented in one of the IJC's reports and respective methodology were followed.

4.6.1 Results for the Simulation of the Great Lakes Operation as defined by the IJC

The following graphs present the:

- Results for the Simulation of the Operation Using the Historical NBS
- Results for the Simulation of the Operation Using 1000 Year Synthetic Data Generated by a Contemporaneous Multivariate Seasonal Autoregressive Model of Order 1 (ARX 1 Model)
- Results for the Simulation of the Operation Using 1000 Year Synthetic Data Generated by a Contemporaneous Multivariate Periodical Autoregressive Model of Order 1 (PARX 1 Model)

Annual Average Cost (Objective Function):

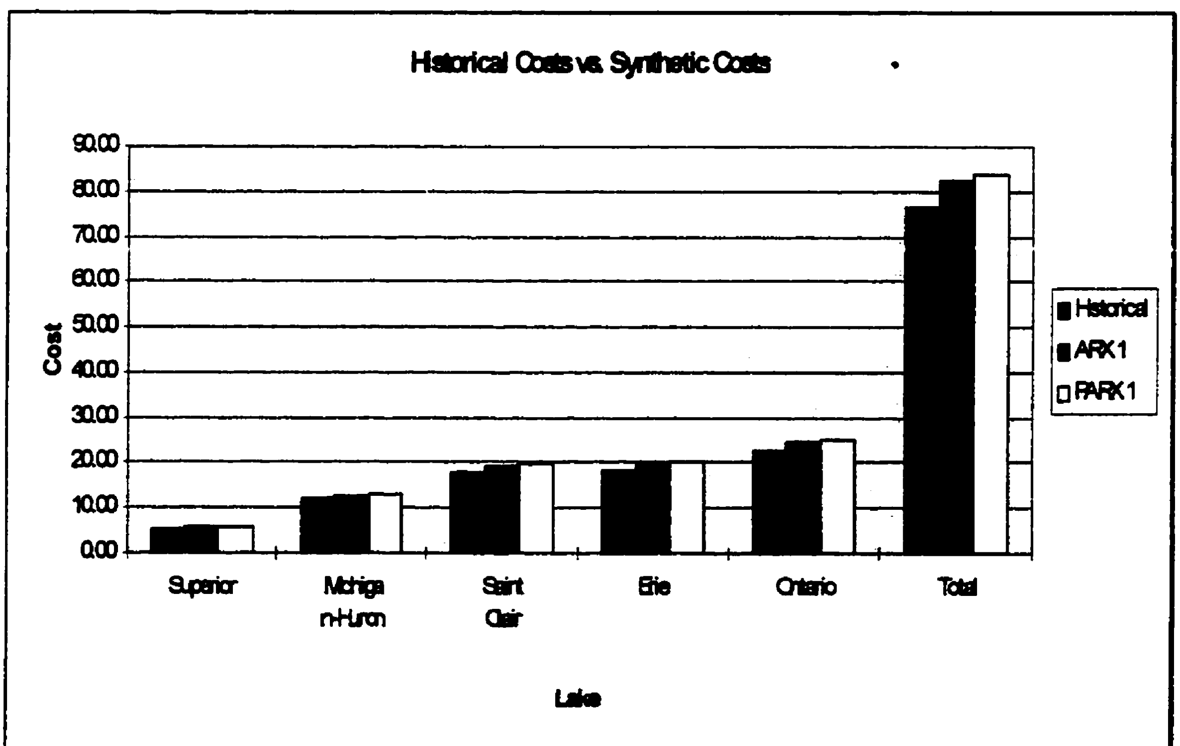


Figure 4 - 7 Comparison between Costs Using Historical and Costs Using Synthetic NBS

Monthly Average Cost (Objective Function) with respective Standard Deviations:

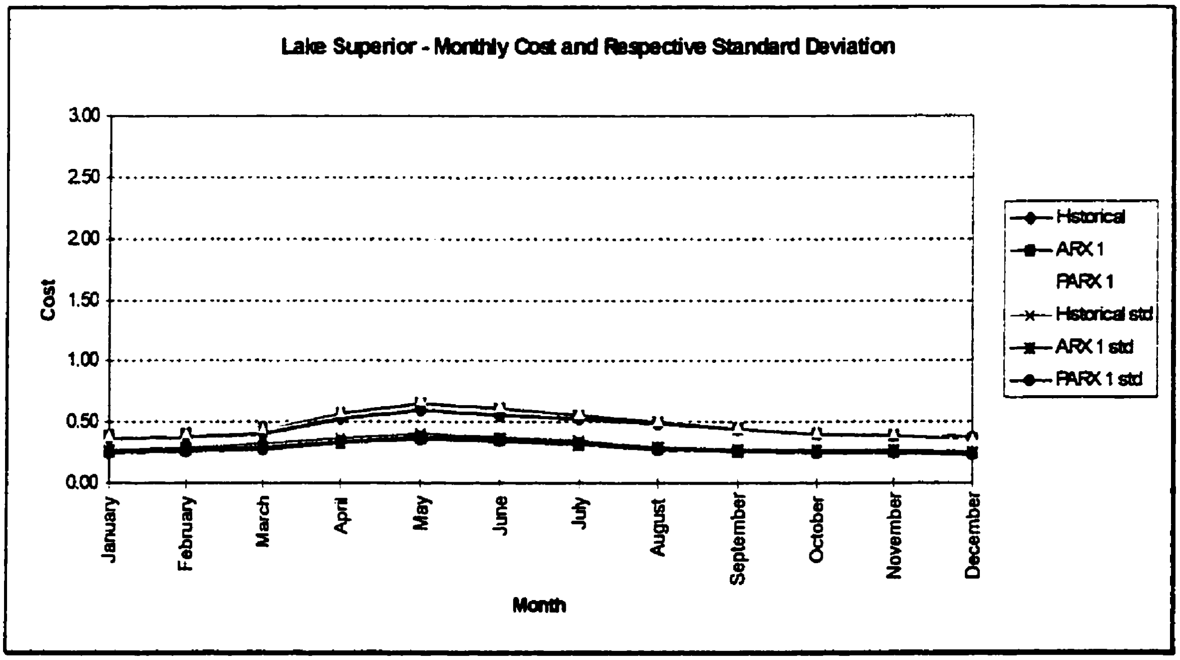


Figure 4 - 8 Comparison between Monthly Costs Using Historical and Synthetic NBS for Lake Superior

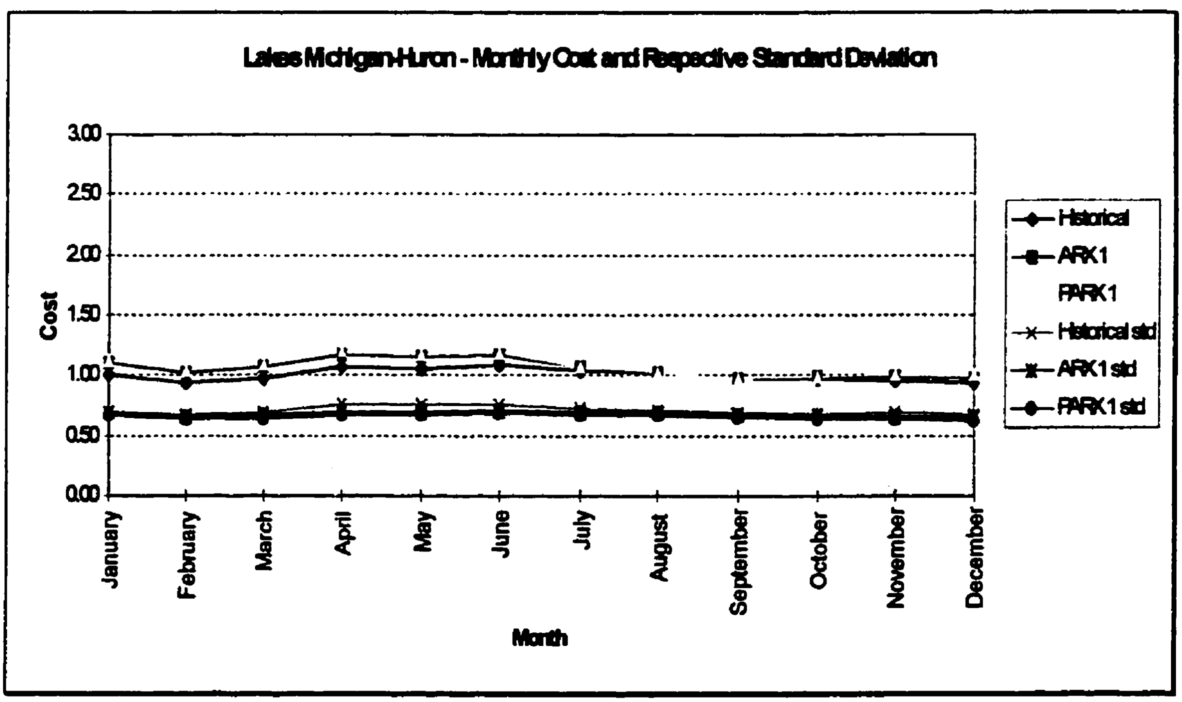


Figure 4 - 9 Comparison between Monthly Costs Using Historical and Synthetic NBS for Lakes Michigan-Huron

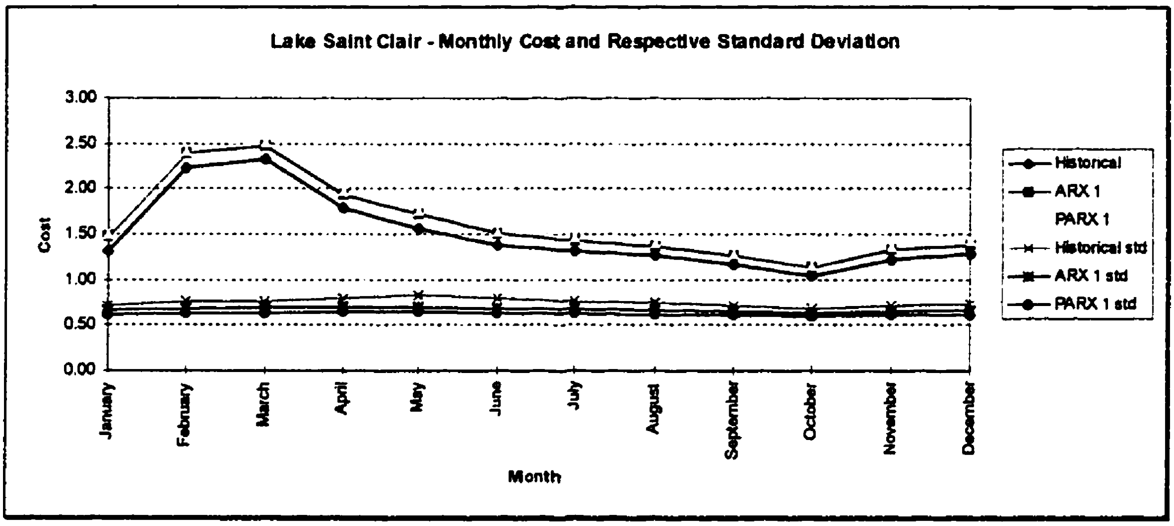


Figure 4 - 10 Comparison between Monthly Costs Using Historical and Synthetic NBS for Lake St. Clair

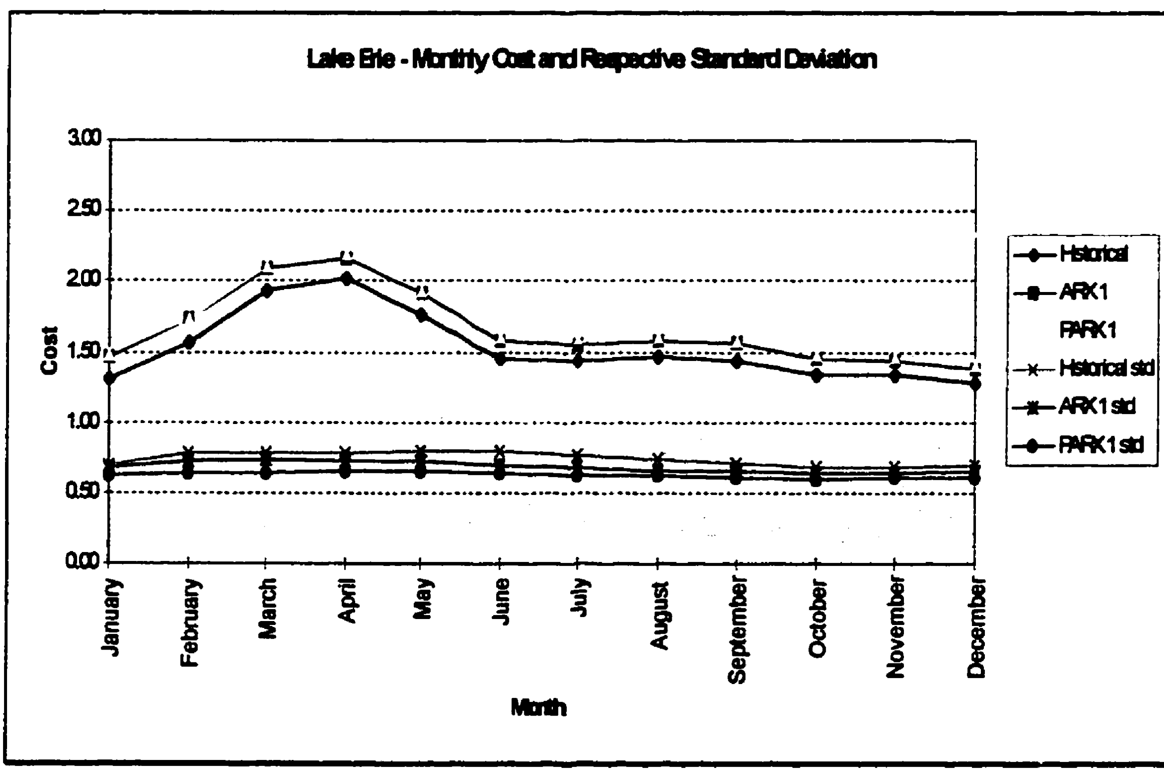


Figure 4 - 11 Comparison between Monthly Costs Using Historical and Synthetic NBS for Lake Erie

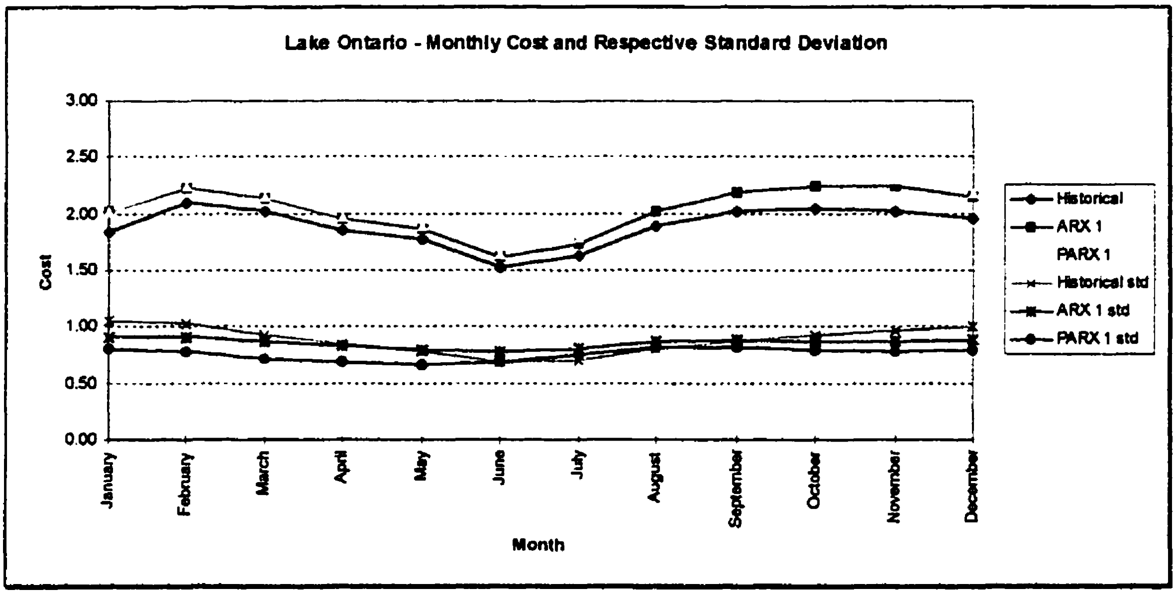


Figure 4 - 12 Comparison between Monthly Costs Using Historical and Synthetic NBS for Lake Ontario

The reader is reminded to note that with the exception of Lake Superior and, in a smaller scale, Lakes Michigan-Huron, the operational costs are higher for the synthetic NBS than for the historical ones. Although it is not expected that the costs be exactly the same, one would expect to have them closer to the historical ones than what the graphs have suggested. The justification for these results is that the present regulation attempts the lowering of the levels of the four most downstream lakes by the increase of Lake Superior levels. The former ones, located in areas more densely populated than Lake Superior and consequently with higher average cost of the shoreline area, would benefit from this kind of policy. However, this is a transient solution. Once Lake Superior establishes itself in a new and higher level, the levels of the other four lakes would tend to return to their original situation, especially those of the unregulated ones. In the long run, the total output from the system, represented by total outflows, must be more or less equivalent to the total input to the system, represented by total NBS.

In the next pages the average levels obtained during the 1000 year simulation of the operation are presented. The reader is referred to Appendix E, Great Lakes Outflows for additional information regarding the referred parameter and further details on the historical and long-term operation for the Great Lakes.

Average Monthly Storage Levels:

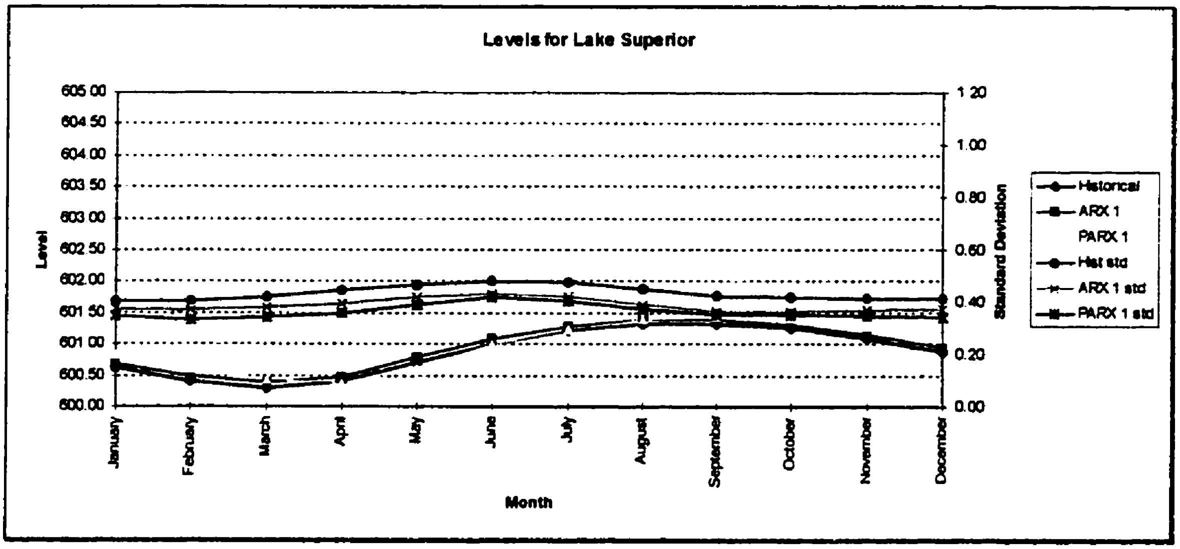


Figure 4 - 13 Lake Superior - Monthly Average Levels and Respective Standard Deviation

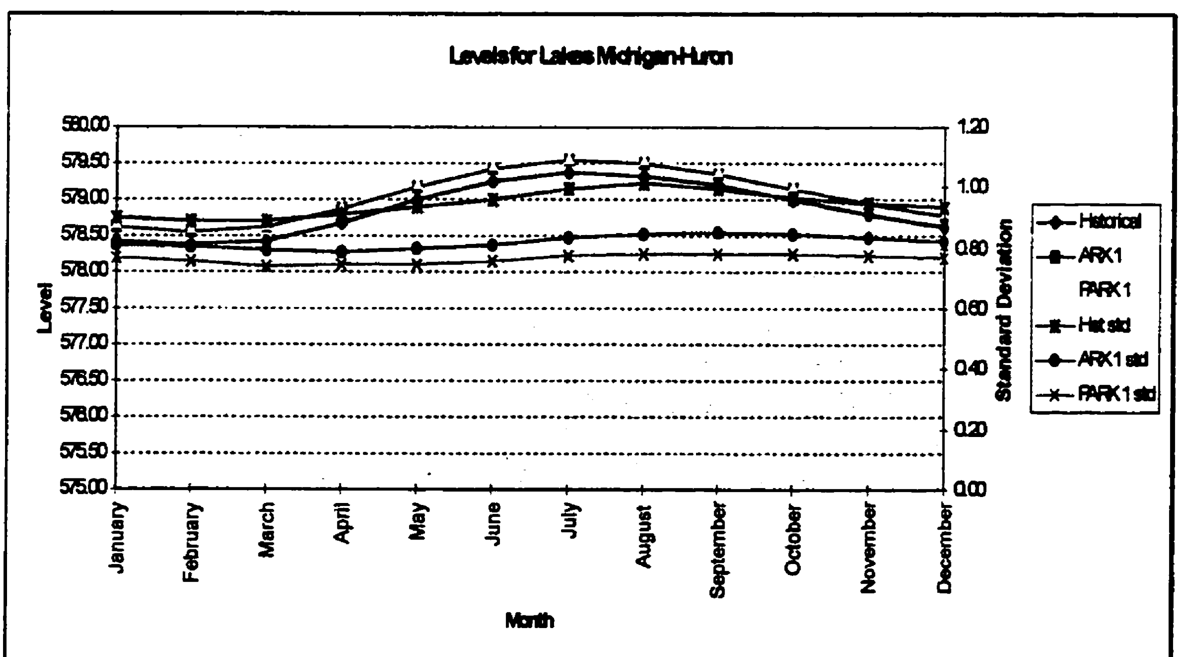


Figure 4 - 14 Lakes Michigan-Huron - Monthly Average Levels and Respective Standard Deviation

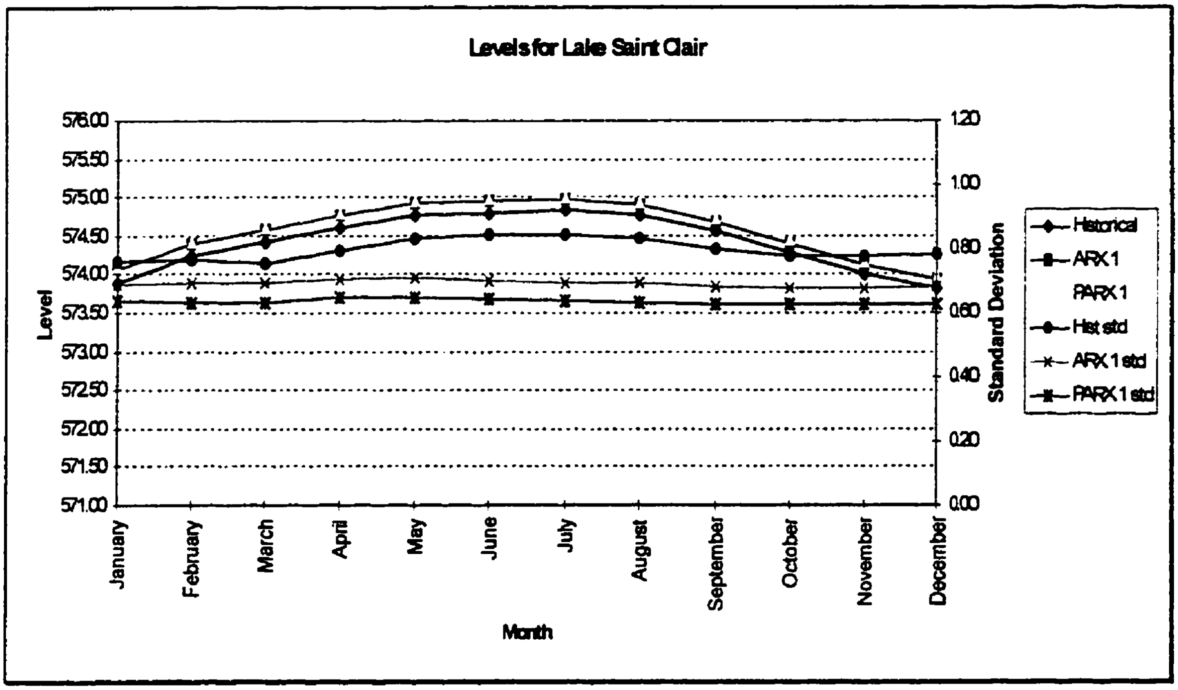


Figure 4 - 15 Lake St. Clair - Monthly Average Levels and Respective Standard Deviation

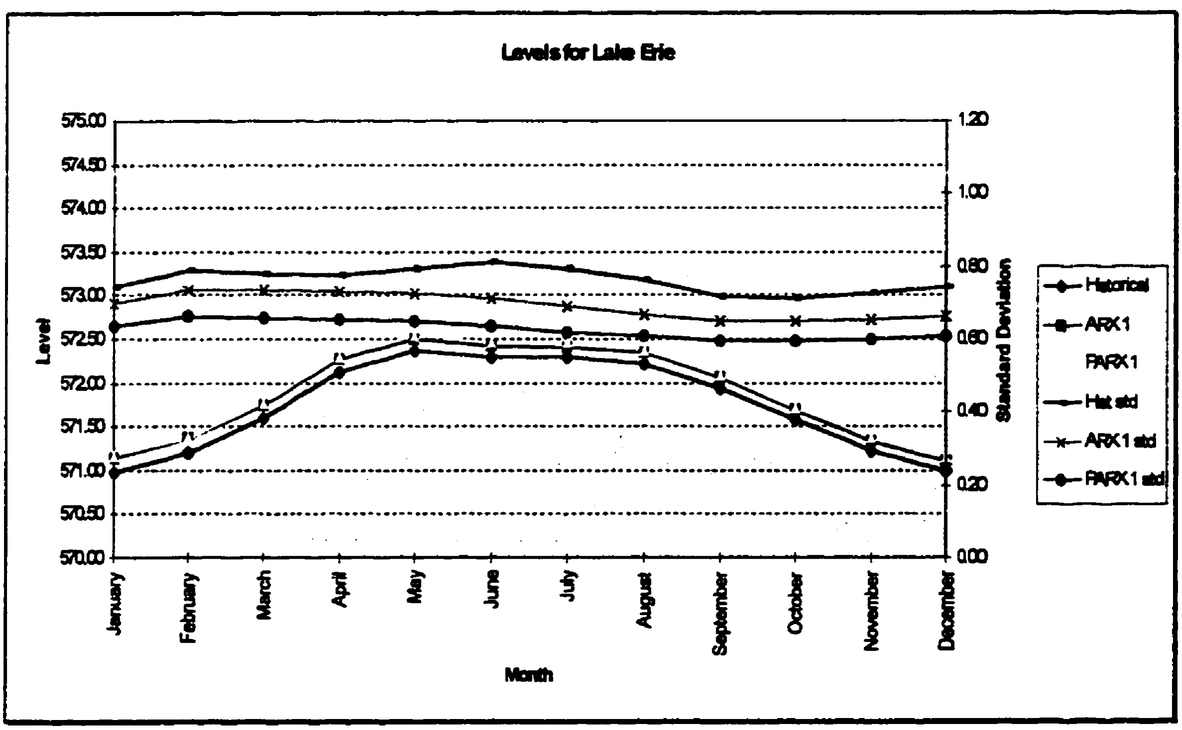


Figure 4 - 16 Lake Erie - Monthly Average Levels and Respective Standard Deviation

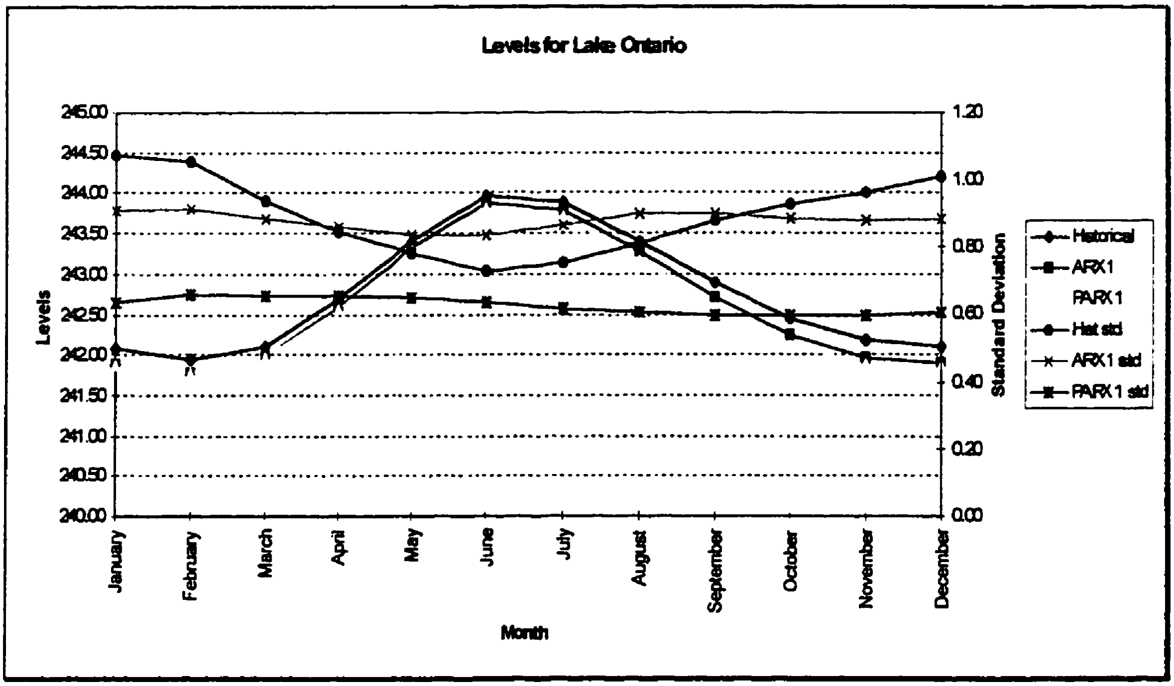


Figure 4 - 17 Lake Ontario - Monthly Average Levels and Respective Standard Deviation

From Figure 4 - 13, the registered average levels for Lake Superior obtained by routing the historical and synthetic NBS are equivalent, with no significant discrepancies. The standard deviations for them present the characteristic that those for the synthetic NBS are smaller than the ones for the historical values. This is a consequence of the use of longer series for the synthetic ones, the operation of the system for the long-term has steady-state behavior. This feature is clearly visible in the other graphs as well.

As Lakes Michigan-Huron, St. Clair and Erie are not controlled and therefore have similar behavior, the comments about all the three lakes will take place at the same time. It is possible to observe that the trajectories for the average levels obtained with the synthetic NBS are systematically above those obtained with the historical ones. The previous observation about the transient nature of the present policy is now recalled. Figure 4 - 14, Figure 4 - 15 and Figure 4 - 16 illustrate well this fact, and the reader is reminded to note that the levels obtained with the historical NBS have the same pattern of behavior (or trajectory) than the long term ones but are located in a lower level.

Figure 4 - 17 presents the average levels for Lake Ontario and how the transient lowering of the levels in the previous three upstream lakes affects its levels. As expected, the average levels

for the historical inputs are above the long-term ones. The aimed reduction in levels for Lakes Michigan-Huron, St. Clair and Erie generates a transient increase in the levels in Lake Ontario, what will probably benefit the navigation conditions in the connecting channels. However, this situation is not stationary and will not subsist in the long-term. It should be added that, because this lake receives the contributions form all the others that are located upstream and, differently from the three immediately above, is controlled, the existing gap between the registered standard deviation for the levels obtained with historical and synthetic NBS is widened if compared with the previous cases.

Finally, the major intent for simulating with three different types of NBS, one historical and two synthetic, is the validation of the synthetic ones. With the acknowledgment of the features specified above, the use of the synthetic series is validated for the goals of this research, i.e., firstly optimize the operation for the Great Lakes System and obtain state-derived operating policies and secondly apply the Two-Pass Mean-Variance Approach evaluating both studies by means of long-term simulation.

4.7 Comparative Results

4.7.1 IJC's Operation vs. Steady-State Optimization using Aggregation

Proposed Steady-State Optimizations using Aggregation Techniques in Two Stages

As an initial step for the application of the Two-Pass Mean-Variance Approach to the case study, the general stochastic long-term optimization of the operation for the Great Lakes using DP was performed. This problem was approached in two different fashions as described in the following paragraphs. The second approach is used not only to provide another perspective to a complex problem, but to serve as validation of the first one, considered as theoretically better.

First Approach:

This first approach is the most appropriate for the aggregation methodology, i.e., the application of the aggregation methodologies to the entire system and obtain, with that, policies that are as global as possible. To overcome the inherent difficulty of having local policies for Lake Ontario, where one of the only two controls of the system is located, a combination of the aggregation schemes that were used in statistical decomposition was employed. In the fashion that is presented below the release policies are global for Lake Ontario and for Lake Superior, each being optimized with respect to the rest of the system. We then have:

- Stage 1: Lake Superior vs. Rest of the System
- Stage 2: Rest of the System vs. Lake Ontario

Second Approach:

The second approach tried to use as much as possible of the optimization scheme presently used for the operation of the lakes. For Lake Ontario, the present operation uses the levels for Lake Ontario and Lakes Michigan-Huron to obtain the long-term release policy. The present approach, however, is equivalent to the one just described, aiming at obtaining release policies based on the aggregate state of the four most downstream lakes. However, for Lake Ontario the release policies are computed using only its state . These release policies are, therefore, independent from the rest of the system. According to the IJC's reports , the justification for this approach is the abrupt level difference between the rest of the system and Lake Ontario. While for the present operation either for Lake Superior and Lake Ontario the release policies are based on heuristics, in the present approach they are optimized and the policies are state-derived by means of SDP. The approach has also two stages, namely:

- Stage 1: Lake Superior vs. Rest of the System
- Stage 2: Lake Ontario Considered Independent

For the sake of simplicity and not divert the reader from the main topic of the thesis, that is to say, the use of aggregation methods to solve steady-state optimization of large systems employing variance control of objectives in multistage decision processes, the results presented here below are only the ones obtained from the Multivariate Contemporaneous Autoregressive Model of Order 1 (MCAR 1) time series (referred to in the previous section as the ARX 1 model) for the Net Basin Supplies for the five considered lakes with length equal to 1000 years.

During the experimental part, the same was done with the other model, i.e., Periodical Multivariate Contemporaneous Autoregressive Model of Order 1. The results are equivalent to those presented below, with the exception that during the simulation part the policies are not as efficient as in the model referred to above, with performance slightly inferior to it. Nevertheless, in both cases, the results are superior to those obtained from the simplified version of the IJC model.

It is necessary to mention that the variables were converted to volumes in order to proceed to the optimization part. Once this was done, the impact on lakes levels due to the same storage volume was dependent on each lake volume/lake surface relationship as specified in the IJC Reports.

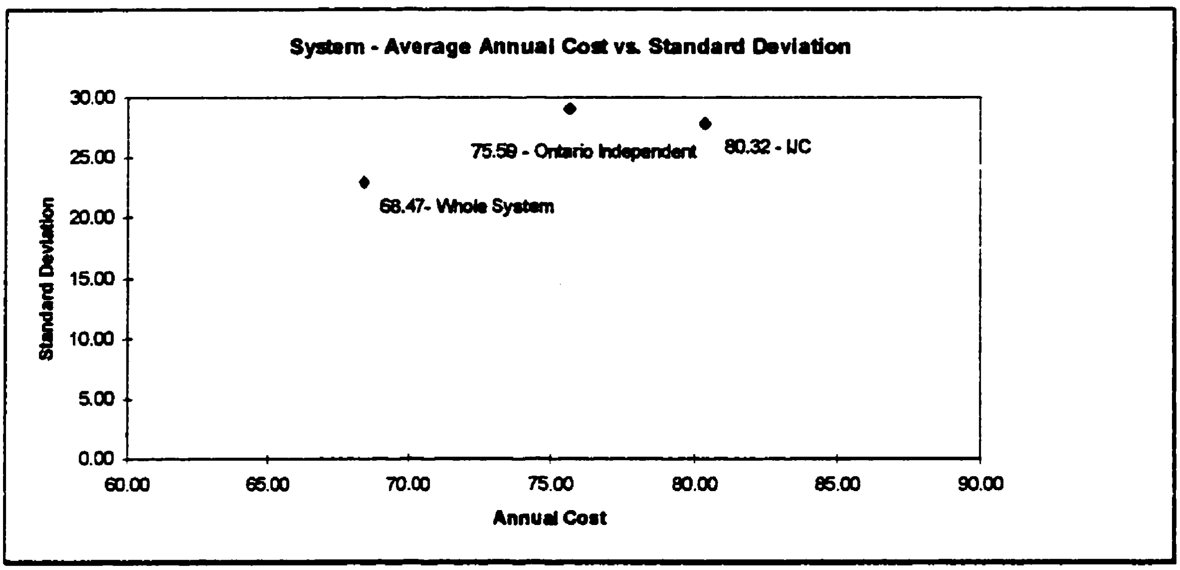


Figure 4 - 18 Comparison Between Annual Costs of Operation for the Entire System

The reader already noticed the evident superiority of state-derived policies not only in terms of operational cost reduction, but in better standard deviation for these costs too. While the IJC's policies are less efficient in terms of overall performance, the standard deviation for their costs are slightly inferior than the ones having Lake Ontario considered as an independent part of the system. Next figures will compare the cost function for each lake belonging to the system in a separate fashion.

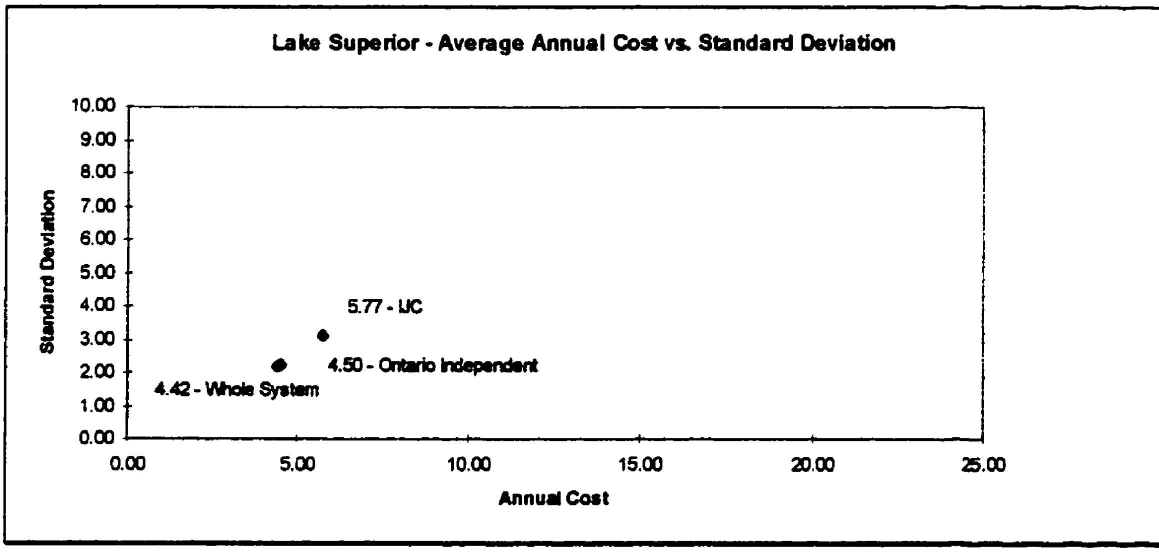


Figure 4 - 19 Comparison Between Annual Costs of Operation for Lake Superior

For Lake Superior, as the state-derived policies are identical, the operational costs are equivalent and, as expected, fare well in comparison to those based on heuristics with a reduction in the expected costs of around 23 %. The reduction in their standard deviations were significant as well.

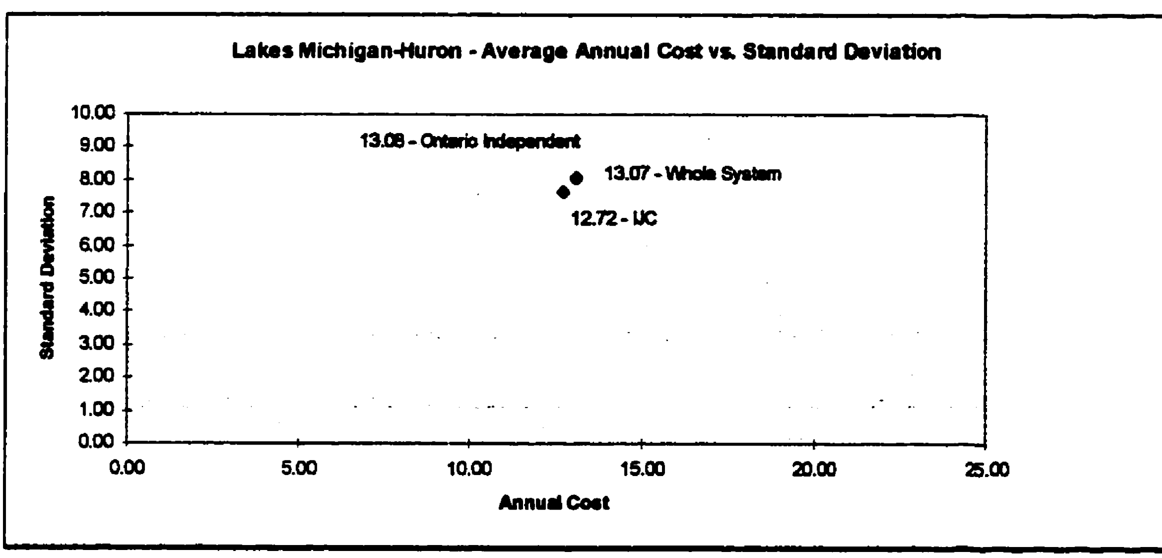


Figure 4 - 20 Comparison Between Annual Costs of Operation for Lakes Michigan-Huron

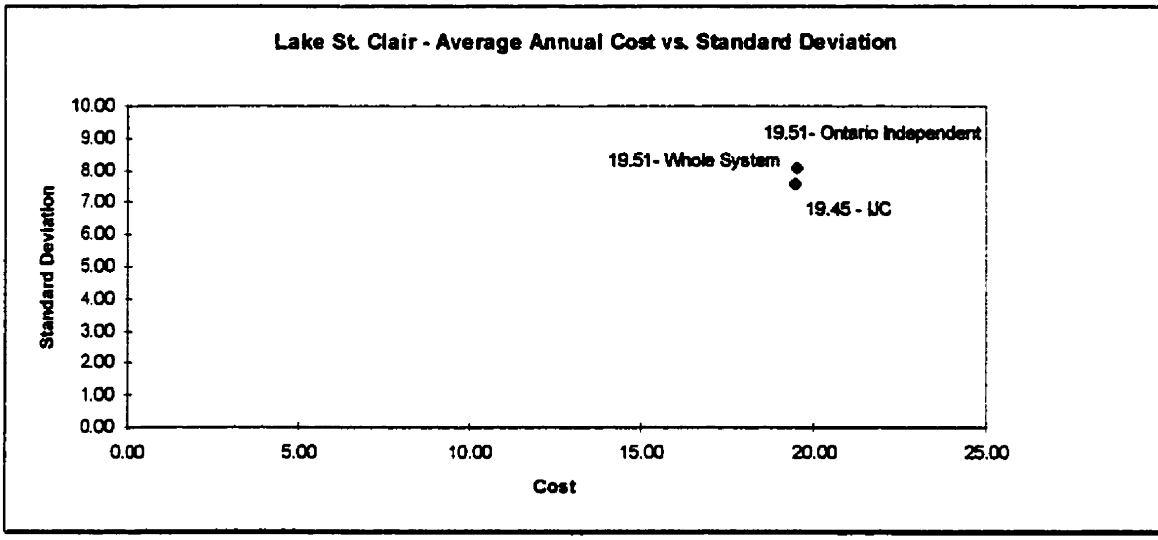


Figure 4 - 21 Comparison Between Annual Costs of Operation for Lake St. Clair

For Lakes Michigan-Huron, St. Clair and Erie, the IJC's policies perform better than the other two. Although the differences are not very large for any of them, the state-derived policies perform worst for Lake Erie. These results can be credited to one of the heuristics employed in the operation according to the IJC, i.e., the limitation of monthly releases differences from Lake Superior. When these monthly differences present values higher than a threshold, the rule outflow is reduced the maximum allowed difference, thus keeping the outflows very steady from one month to another. And as all three lakes have no control, reducing the monthly flow difference has a direct impact on the levels variation for Lakes Michigan-Huron, St. Clair and Erie.

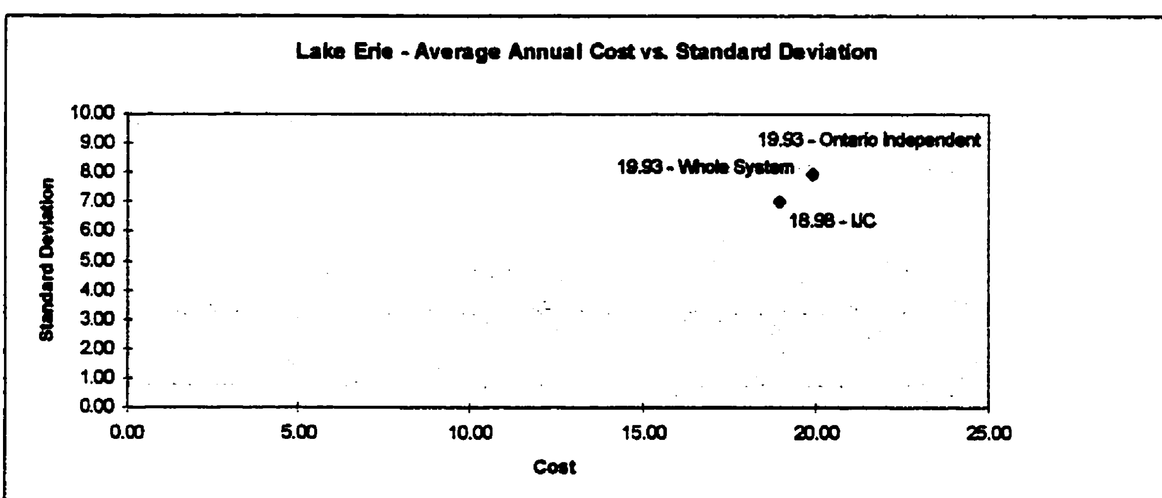


Figure 4 - 22 Comparison Between Annual Costs of Operation for Lake Erie

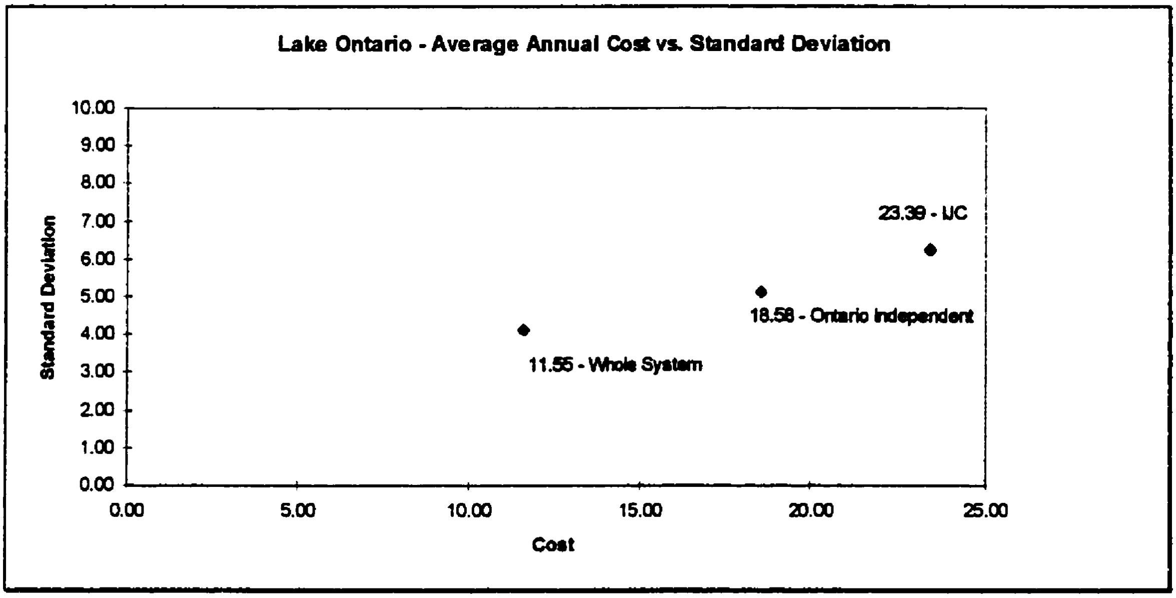


Figure 4 - 23 Comparison Between Annual Costs of Operation for Lake Ontario

Figure 4 - 23 shows the effect on the cost of state-derived policies for Lake Ontario and the substantial improvement that can be obtained with them. However, it is necessary to remember that while the IJC simulation model constructed for the present work has a time-step equal to one month, the original model has a time-step of a week. This model is a simplified one, without all the fine adjustments of the original one. In a more refined comparison, it would be necessary to build a link between the state-derived long-term policies and medium-term policies by the use of a hierarchical model and only then, have a better evaluation of the real improvement. Nevertheless, the Lake Ontario Independent state-derived policies, employed as another comparison reference, performed better than the IJC rules, but much worse than the ones considering Lake Ontario against the rest of the entire system.

The following graphs will present the average monthly levels obtained after the stochastic simulation and their trajectories location with respect to the IJC targets. Concerning the levels graphs, the reader's attention should be called for the targets trajectories and their relative position with respect to the average trajectories for the levels. For the four most upstream lakes they are systematically located below them. For Lake Ontario, the opposite is true for the IJC's policies and those state-derived considering the entire system. This evidence seems to concur with the aforementioned observations that mostly the targets for Lakes Michigan-Huron, St. Clair and Erie are not placed in a realistic position for the present structure of controls.

Average Monthly Levels:

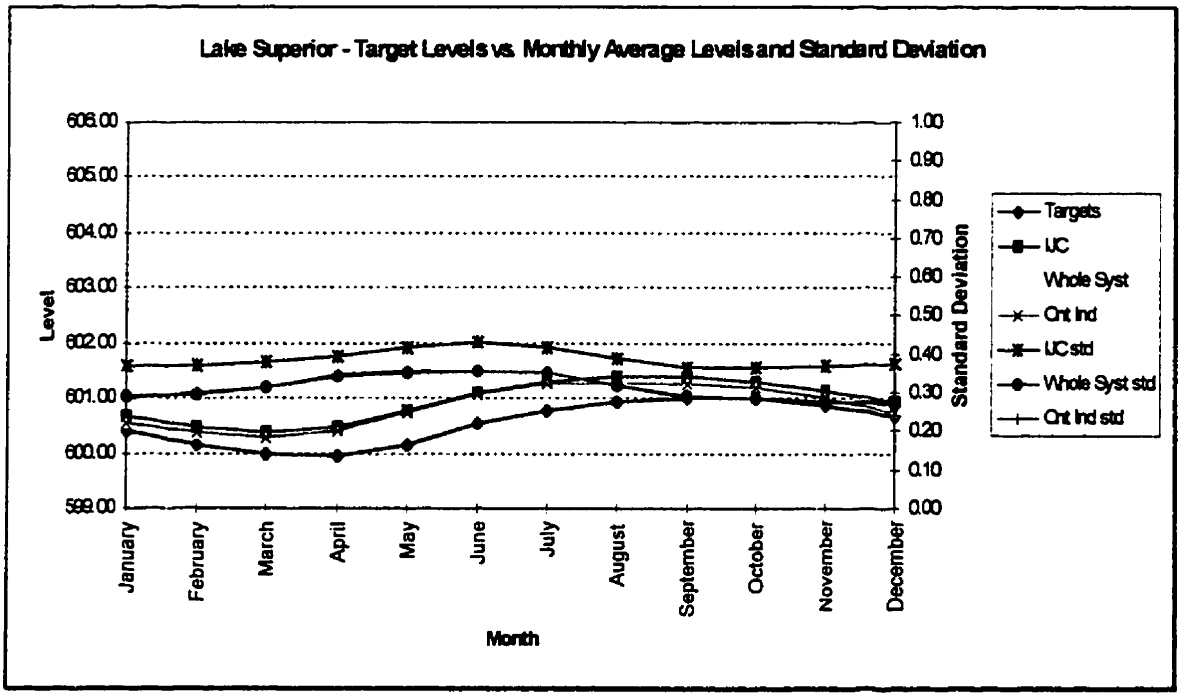


Figure 4 - 24 Lake Superior - Monthly Average Levels for Different Types of Release Policies and Respective Standard Deviation

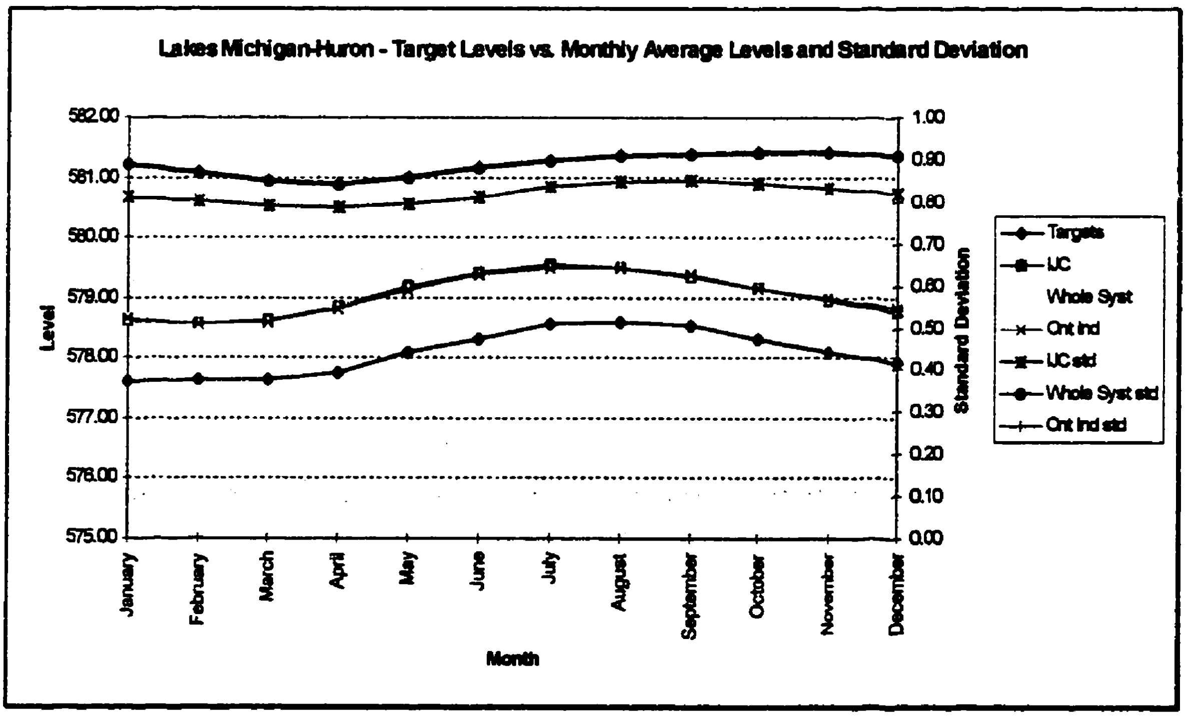


Figure 4 - 25 Lakes Michigan-Huron - Monthly Average Levels for Different Types of Release Policies and Respective Standard Deviation

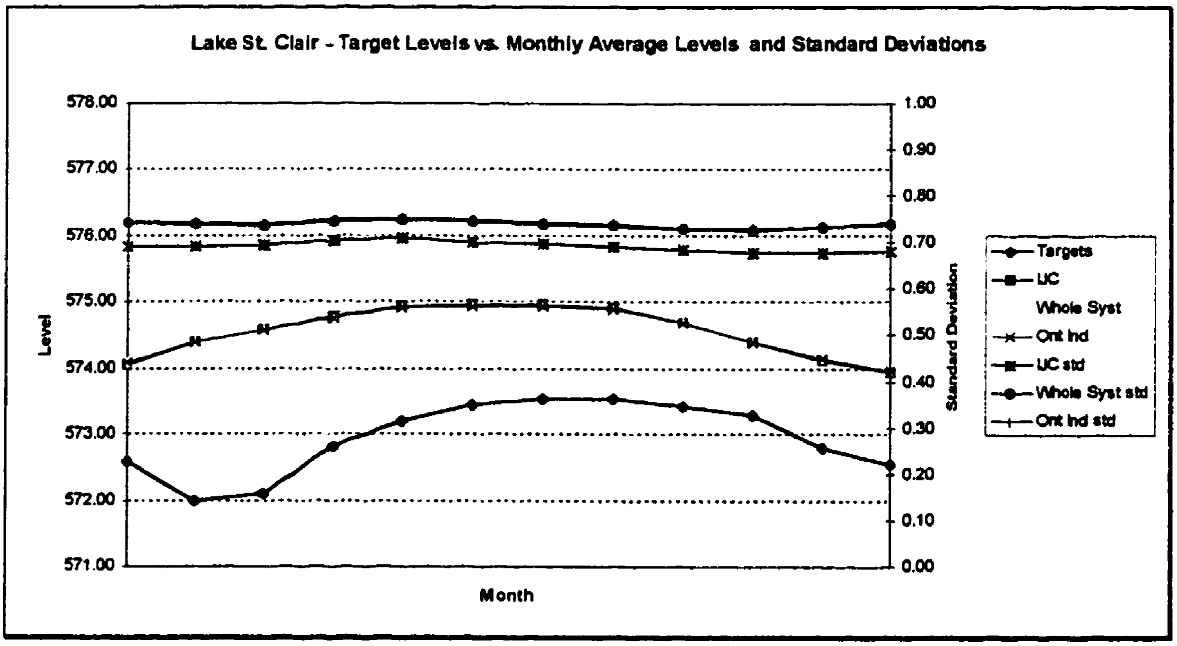


Figure 4 - 26 Lake St. Clair - Monthly Average Levels for Different Types of Release Policies and Respective Standard Deviation

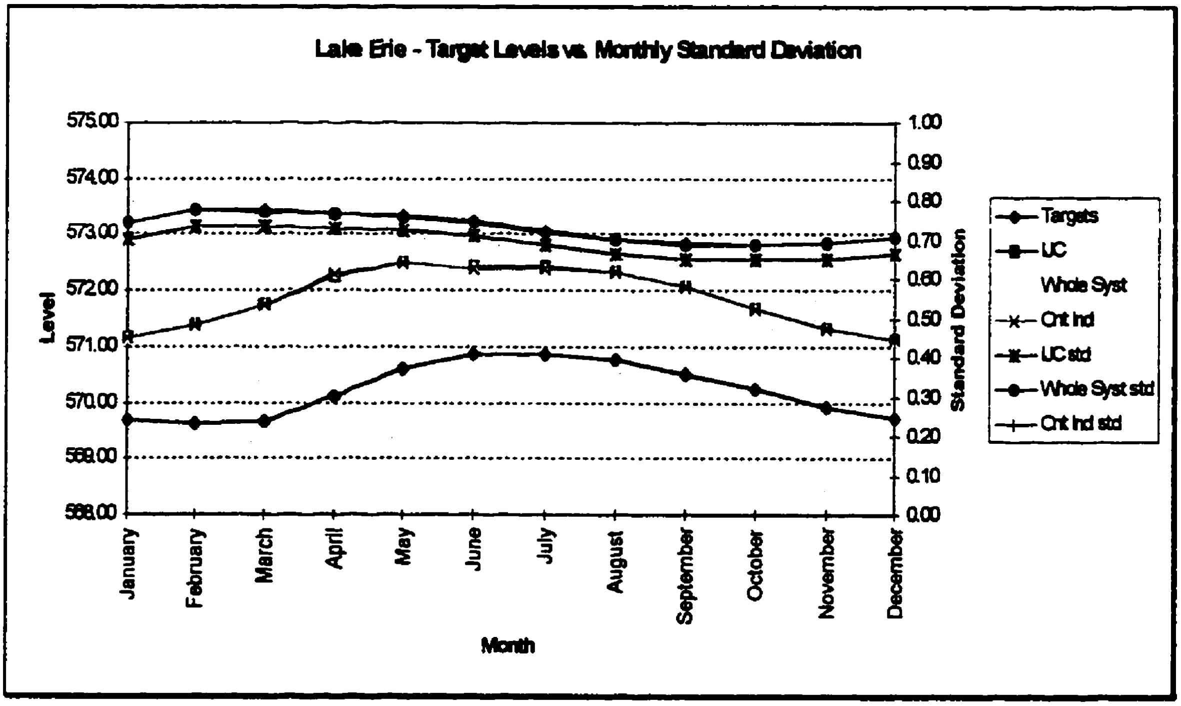


Figure 4 - 27 Monthly Average Levels for Different Types of Release Policies and Respective Standard Deviation

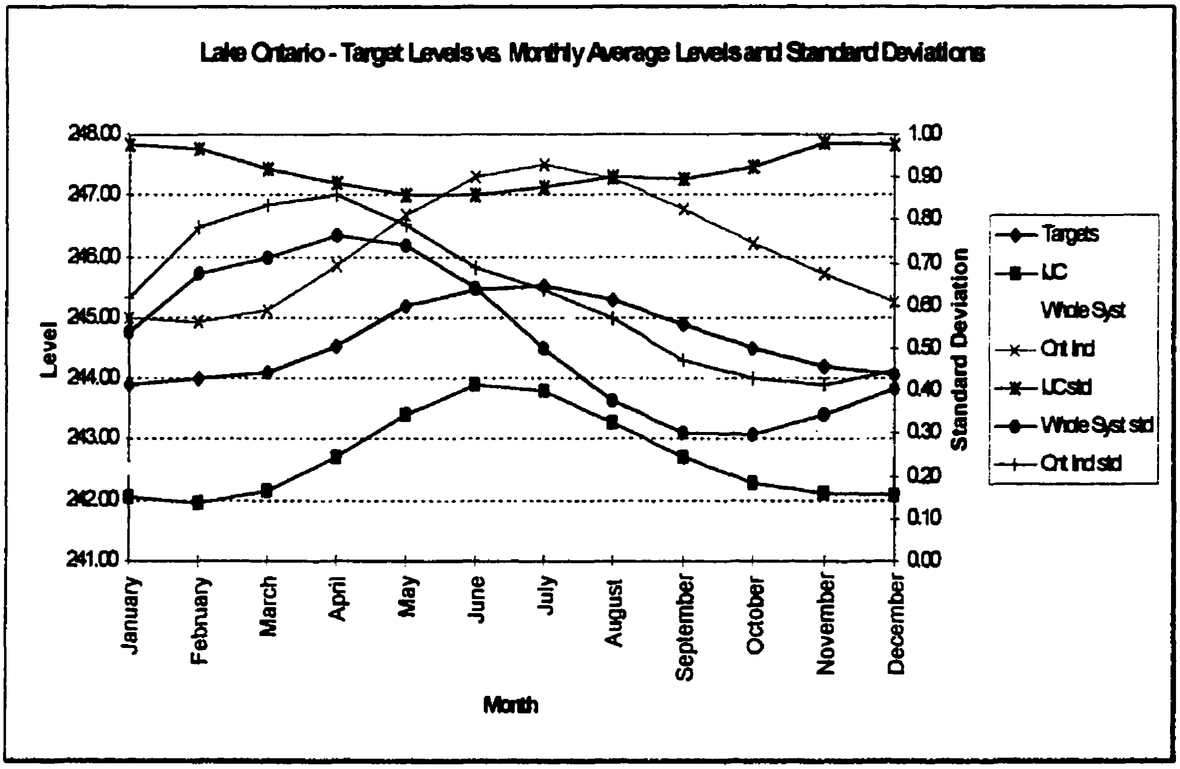


Figure 4 - 28 Lake Ontario - Monthly Average Levels for Different Types of Release Policies and Respective Standard Deviation

From Figure 4 - 28 it is interesting to note that the average levels resulting from the use of the policies considering Lake Ontario Independent are always above the other two types of policies, what may affect negatively the hydropower generation at the Niagara plants. These values are relatively small but their influence is not positive. After these comparative results, additional details referring to the operation of the system are shown below. As has been the case so far, all results were obtained after 1000 year simulation of the long-term operation for the lakes system.

Probabilities of Exceeding the Minimum and Maximum Specified Levels:

While the IJC's operating rules are better for Lake Erie and far superior when compared with *"Ontario Independent"* for Lake Ontario, in general, the *"Whole System"* optimization is better than the other two types of policies when considering the Probability of Exceeding the Minimum Specified Level. Now, considering exceeding the maximum level specified, IJC's policies are far worse than the other two, with *"Ontario Independent"* presenting a better performance. In this specific case, the tradeoff is clear.

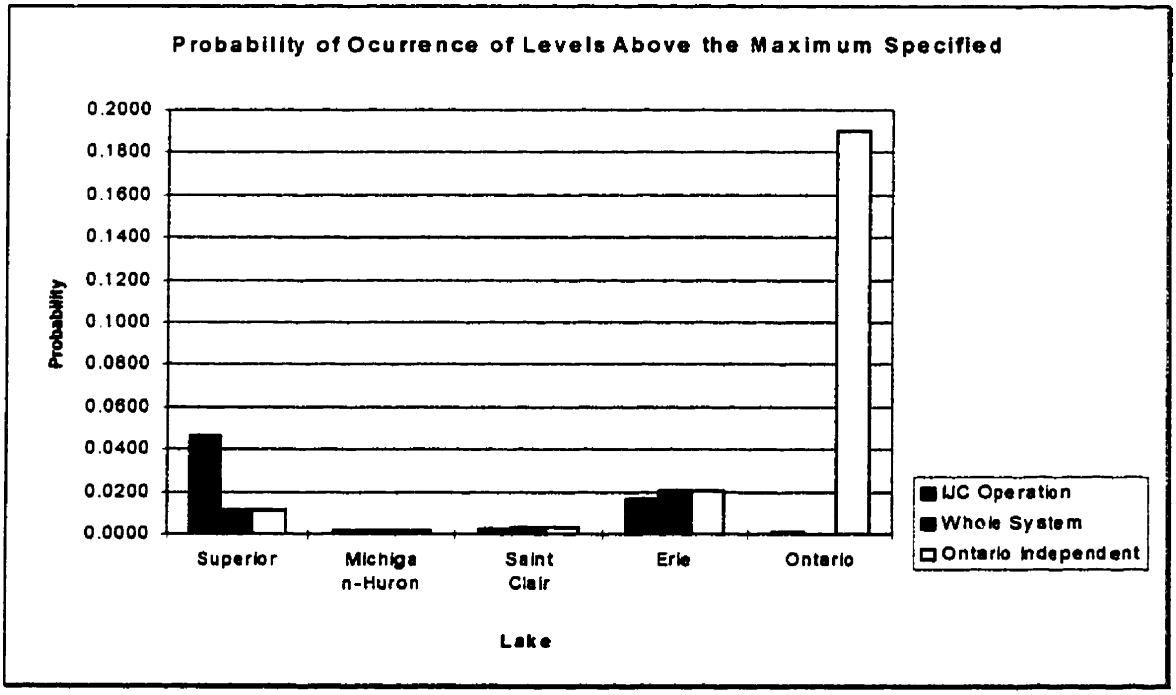


Figure 4 - 29 Probabilities of Exceedance for Levels for the Five Lakes - Above the Maximum Specified

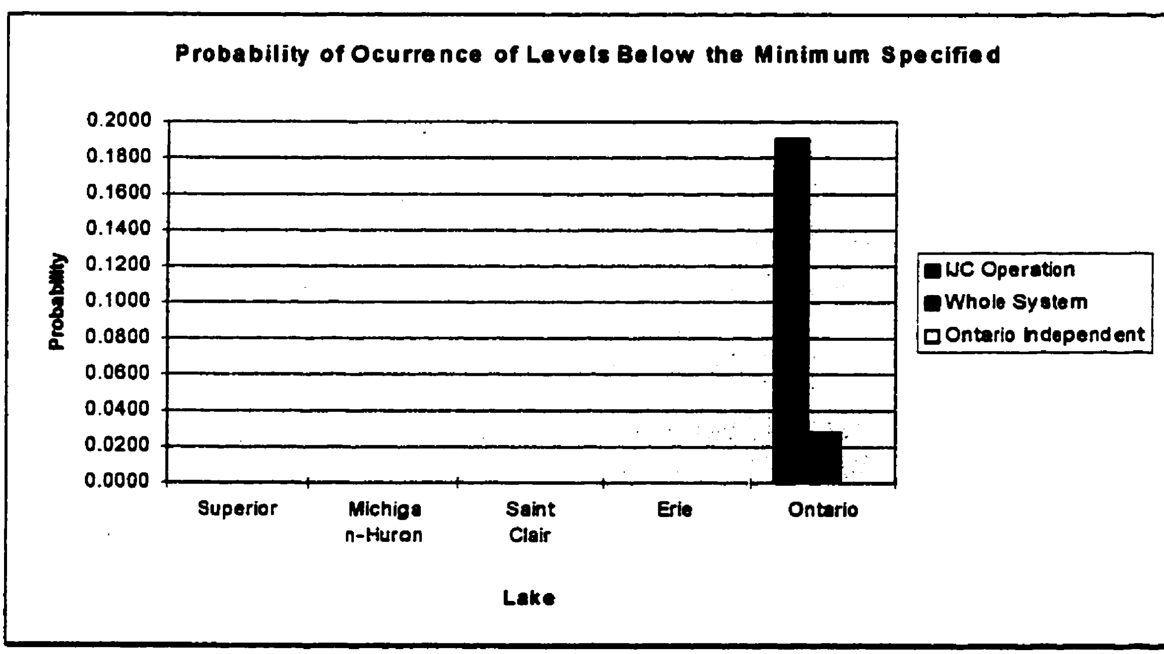


Figure 4 - 30 Probabilities of Exceedance for Levels for the Five Lakes - Above the Maximum Specified

Appendix E presents the average monthly outflows for the cases presented so far. There is a tradeoff between the maintenance of the levels as close as possible to the established levels targets and the outflows. Namely, as the original objective function incorporates outflow targets (what is not true in the present optimization, only storage levels were taken into account) there is much more latitude in the current study than in those that had them included. It is possible to mention the works of Seifi and Hipel (1998), Sadjad (1997) and Fletcher and Ponnambalam (1998) as having this bi-objective optimization, i.e., a much more constrained problem.

Now, a sample of the Release Policies Graphs is presented. For the "*Whole System Optimization*" are shown two characteristic months of the year, one representing the dry season and the other the wet season. As it is possible to represent the entire year in a single graph, for "*Ontario Independent*" all the surfaces are presented.

Lake Superior:

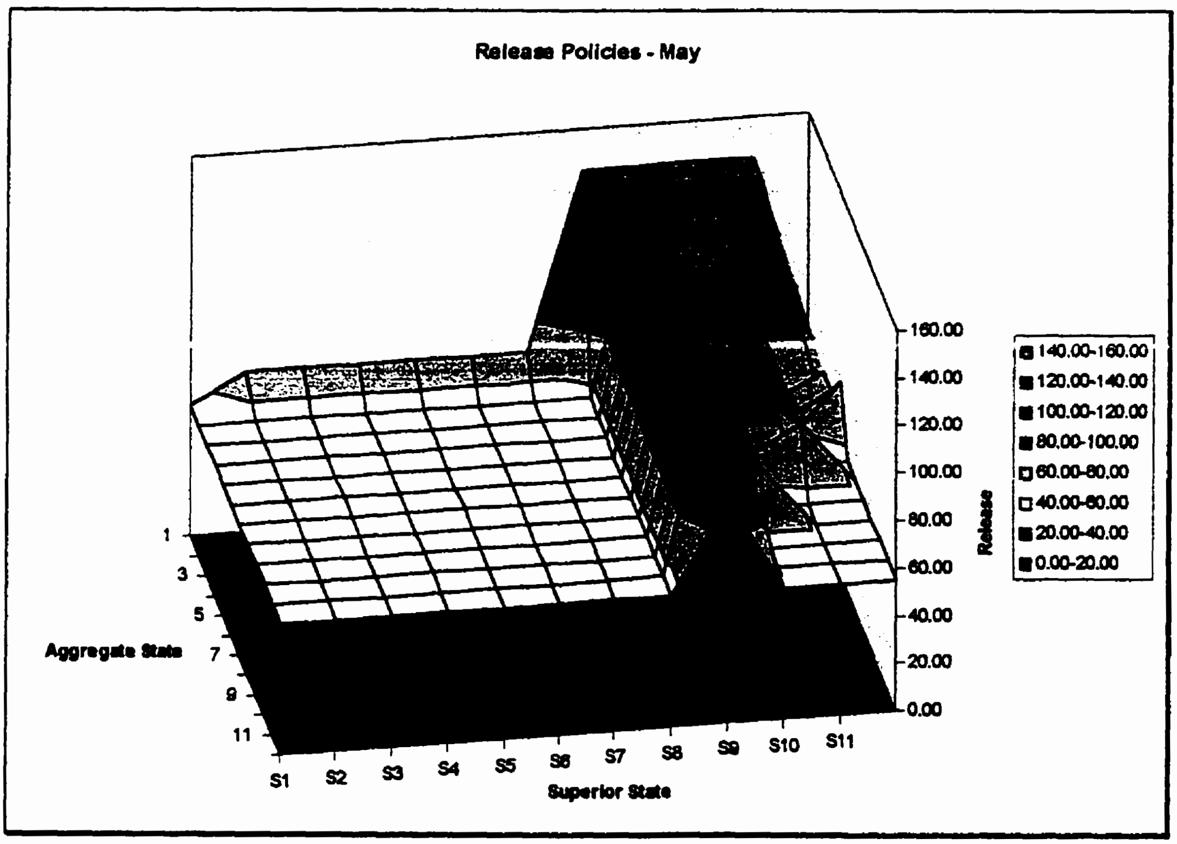


Figure 4 - 31 Release Policy for the Month of May for Lake Superior vs. Rest of the System Aggregated

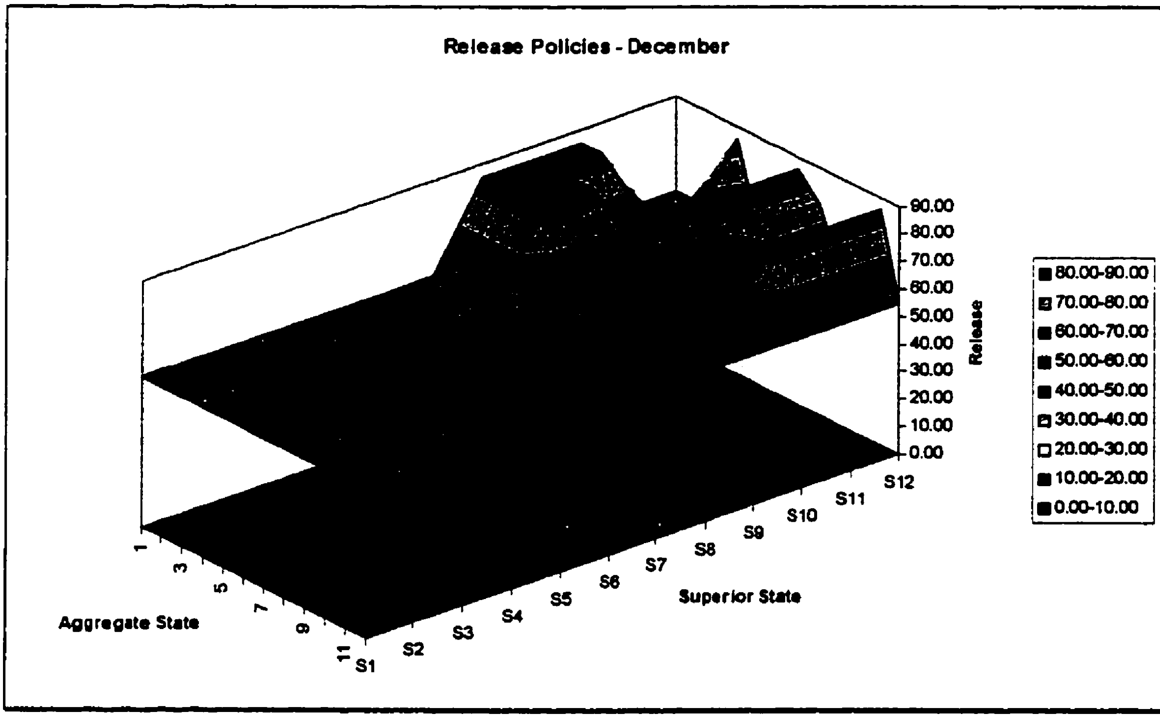


Figure 4 - 32 Release Policy for the Month of December for Lake Superior vs. Rest of the System Aggregated

Lake Ontario:

1. Whole System Optimization

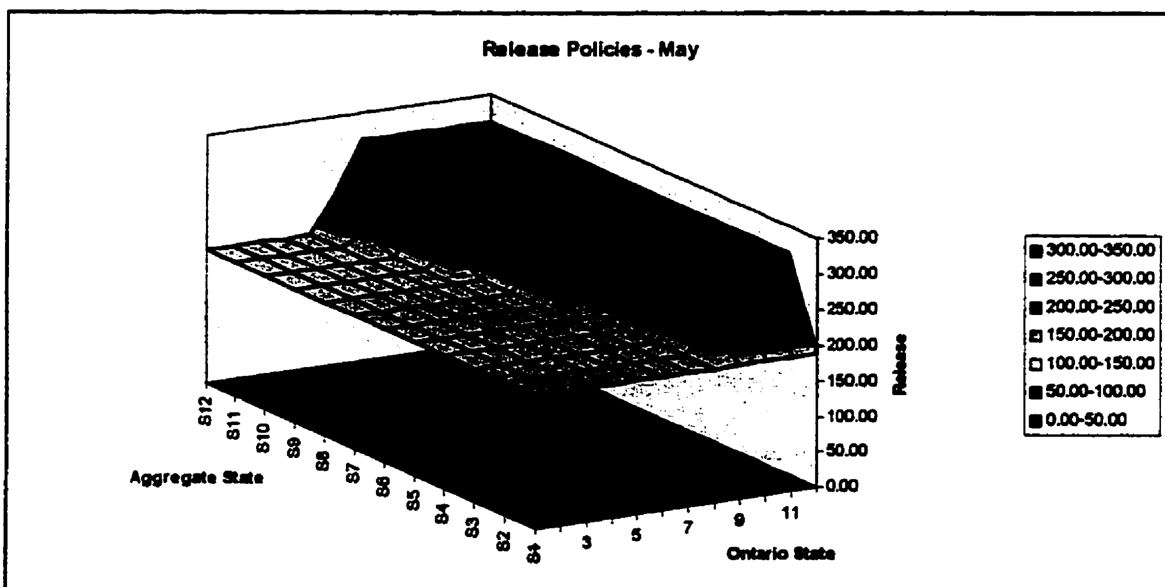


Figure 4 - 33 Release Policy for the Month of May for Lake Ontario vs. Rest of the System Aggregated

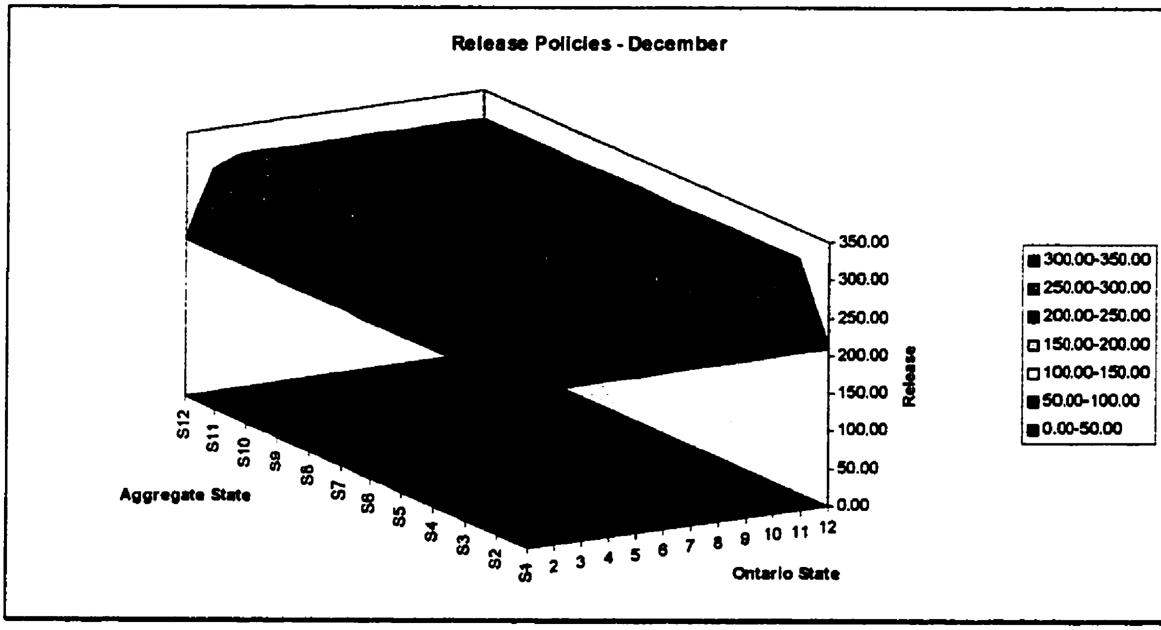


Figure 4 - 34 Release Policy for the Month of December for Lake Ontario vs. Rest of the System Aggregated

2. Lake Ontario considered independent

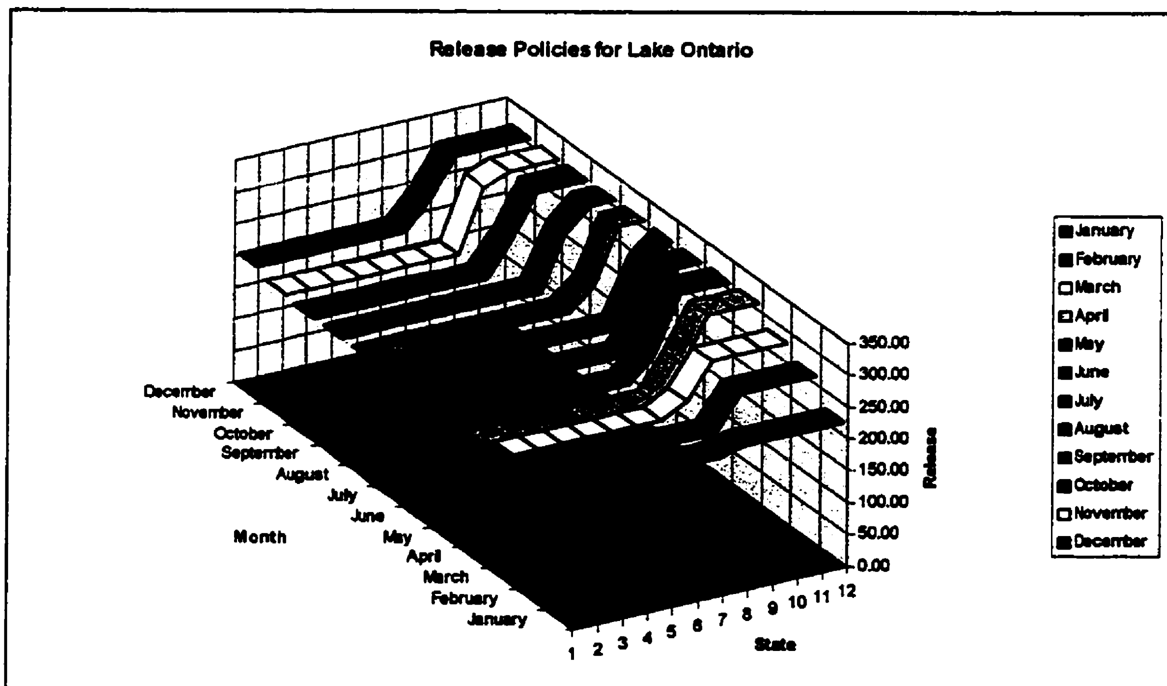


Figure 4 - 35 Release Policies for Lake Ontario Considered Independent from the Rest of the System

4.8 Steady-State Optimization Using Aggregation and the Proposed Methodology, Two-Pass Mean-Variance Approach

Because most of the discussion on the application of the proposed methodology was already made, the only thing that should be remarked is the difference between the objective function analyzed so far, maximization of benefits and the one for the Great Lakes Case, which has the minimization of costs. The presentation of the formulation for the proposed methodology follows:

First, let us define

$$Dist_l^t = \sqrt{(S_l^t - \bar{S}_l^t)^2}, \text{ for } t = 1,\ldots,T \text{ and } l = 1,\ldots,L. \qquad 4.15$$

Therefore, the objective function for the optimization becomes the minimization of the absolute cost of operating the lakes beyond, above or below, the target levels as specified by the IJC regulations. Thus,

$$\min \ E[Dist_l^t] \qquad 4.16$$

subject to the constraints as specified previously.

However, in the Two-Pass Mean-Variance Approach the objective function becomes

$$\text{Expected Cost} = \sum_{t=1}^{T}\sum_{l=1}^{L}[(1-\omega)*E(Dist_l^t)+\omega*\sigma(Dist_l^t)]; \qquad 4.17$$

Where:

$0 \le \omega \le 1.$

4.8.1 Results with the Proposed Methodology

Below the results obtained by applying the proposed Two-Pass Mean-Variance Approach are presented. The original objective function for the Great Lakes Case is a bi-objective one. Not only storages are included in it, but outflows as well. For the case presented further, which experiments with storage control, only the storage variations were taken into account as objectives. The minimization of the distance from the specified storage targets is the principal and single objective. In setting up these storage targets (levels) the IJC recommendation was followed, namely, attempt to maintain the levels at the average level that would have occurred historically if the historical Net Basin Supplies were routed through the specified static configuration of the lakes and their connecting channels. With this information, a Two-Level MAM-SDP Optimization was conducted. From that, the proposed methodology was applied. However, bearing in mind that the next step for the work would be the bi-objective optimization, the routines for this kind of optimization are already available. After running this model and defining the relative importance that should be given to each of the objectives, it will be possible to extend the suggested methodology for this type of multiobjective optimization. As the objective function has now a sum of random variables, the correlation between storages and outflows must be computed and included in it. The values shown in the following tables and graphs were already converted to *feet* (levels) and thousand of cubic feet per second, *tcfs* (outflows).

4.8.2 Whole System Considered

Graphs presenting the relationship Average Costs vs. Standard Deviation for the Costs

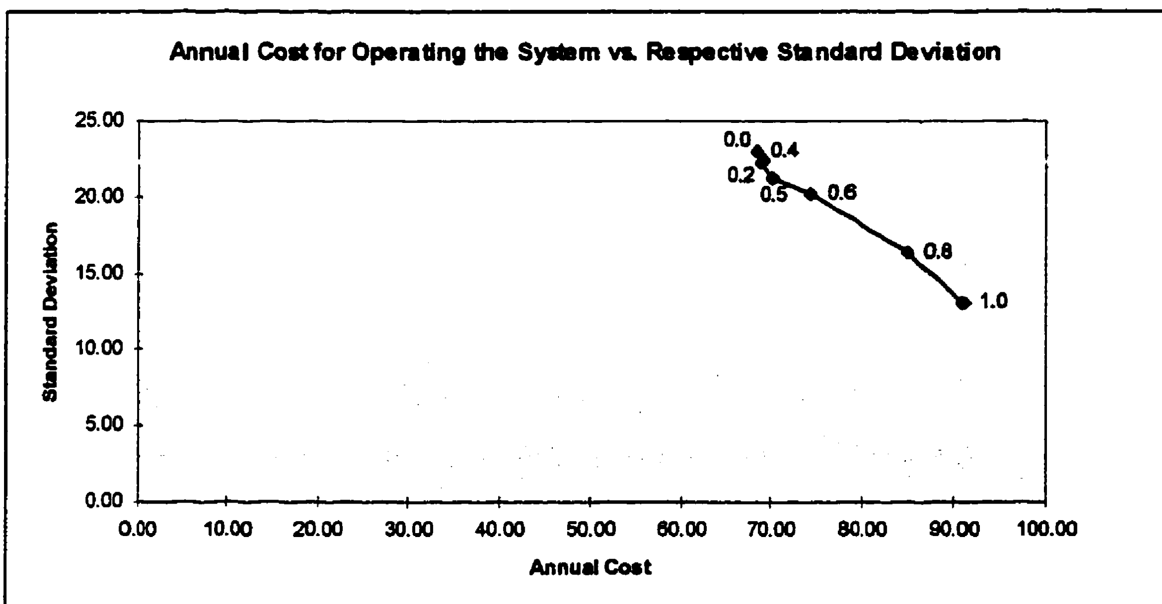


Figure 4 - 36 Annual Cost vs. Respective Standard Deviation for Operating the System Under Different ω' s

Observation: The annual cost (deviation from the storage targets) is represented as the absolute sum of the monthly distance from the desired level during the period of a year.

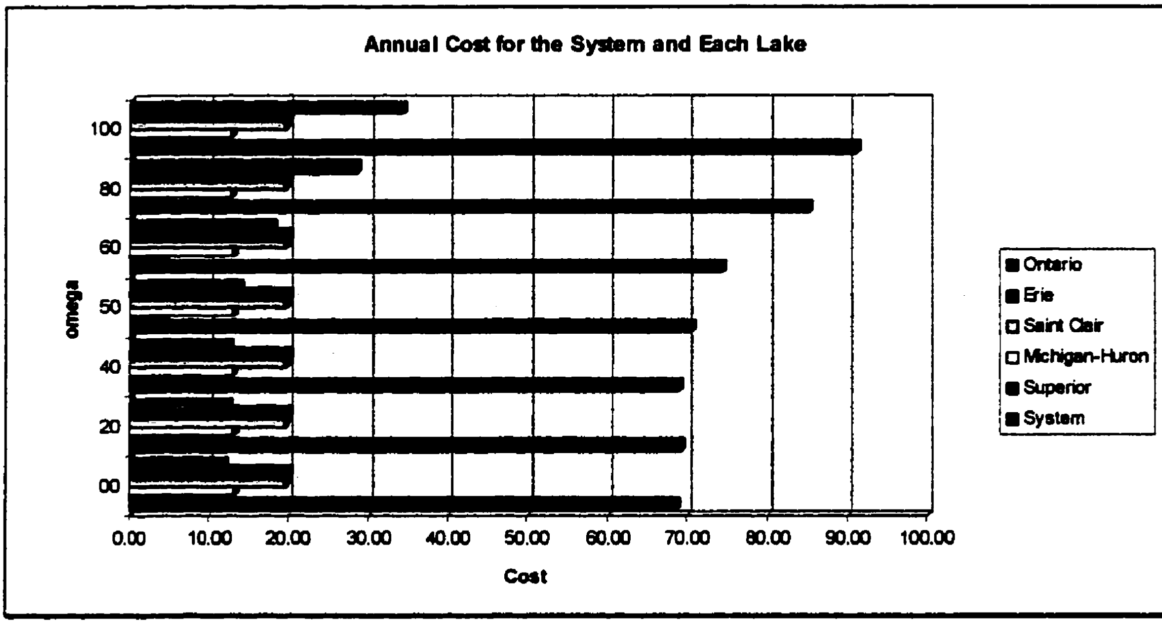


Figure 4 - 37 Annual Cost for Operating the System Under Different ω's for the System and Per Lake

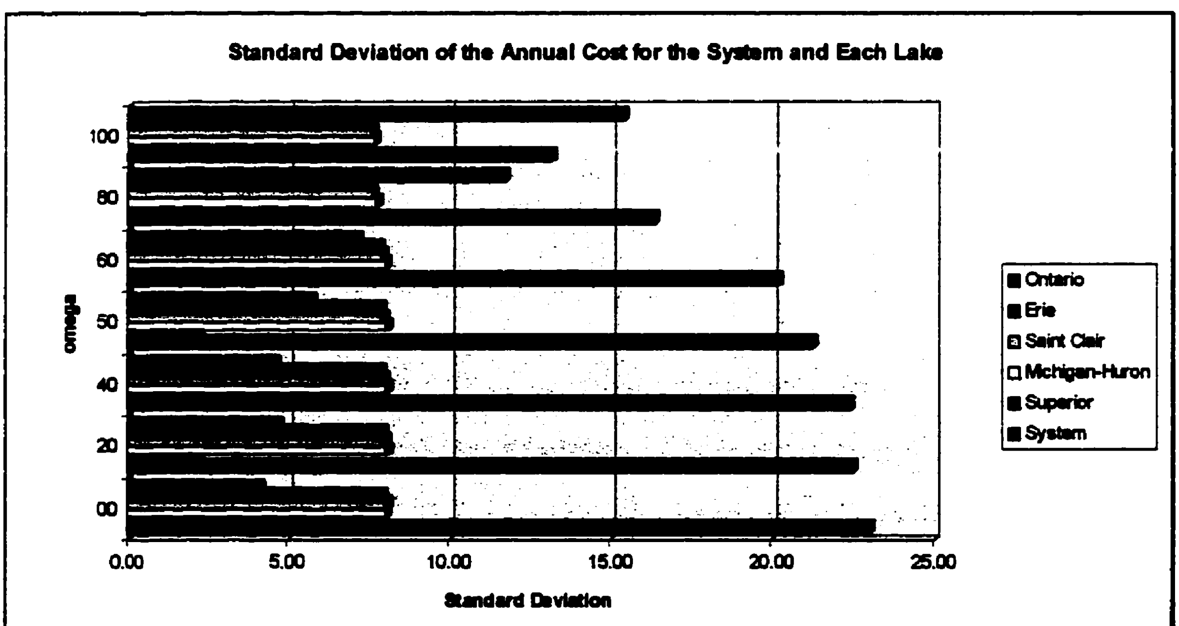


Figure 4 - 38 Standard Deviation for the Annual Cost for Operating the System Under Different ω's for the System and Per Lake

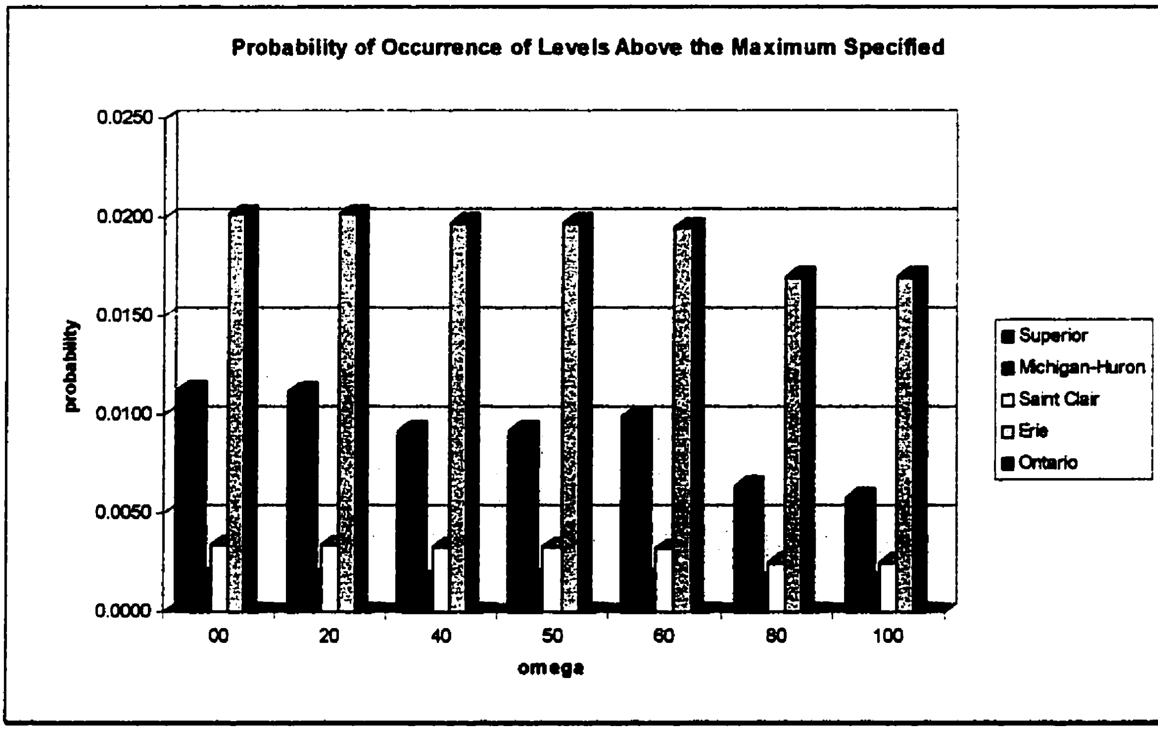


Figure 4 - 39 Probabilities of Exceedance for Levels for the Five Lakes for Different ω's - Above the Maximum Specified

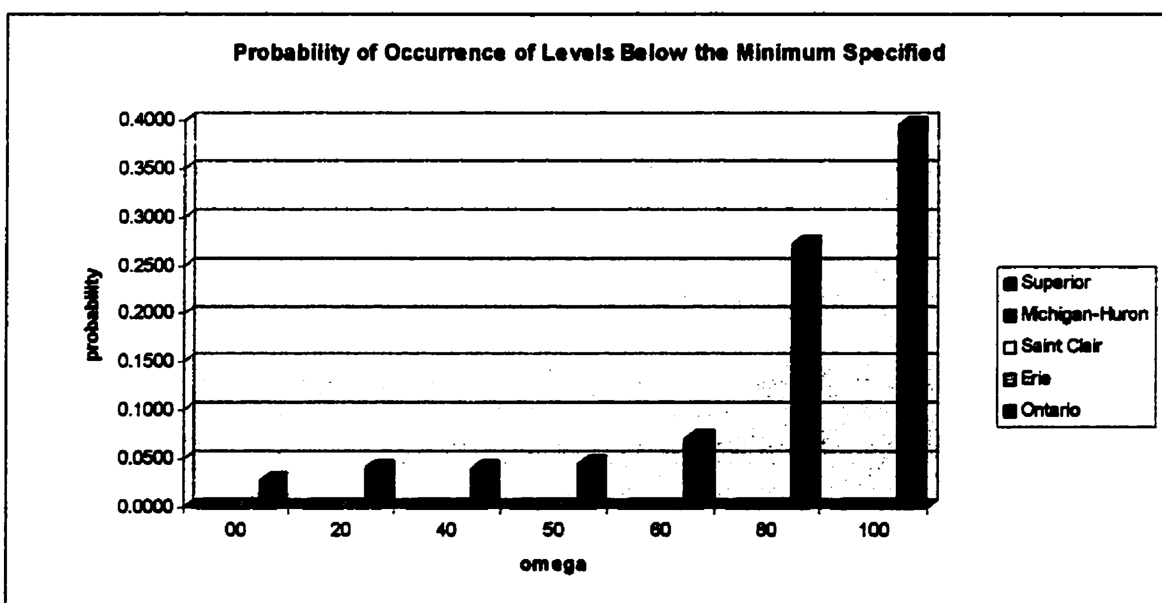


Figure 4 - 40 Probabilities of Exceedance for Levels for the Five Lakes for Different ω's - Below the Minimum Specified

Figure 4 - 36 presents the main results for the application of the Two-Pass Mean-Variance Approach to the Great Lakes Operation Problem as described so far. The main purpose of the methodology is to reduce the risks involved in the systems operation. This is effectuated by reducing the standard deviation of the costs involved, which usually has a tradeoff in increasing the costs themselves. For this case study, the similarity between the upper part (region between the North and East or IV Quadrant) of the theoretical curve as suggested by Markowitz (1952) is remarkable. As their standard deviations increase, the costs themselves decrease. The lower observed cost is 68.47 *feet* with respective standard deviation of 22.98 *feet* for ω equal to zero. To reduce the standard deviation to 13.12 *feet* it is necessary to accept an increase in the costs to 90.94 *feet* for ω equal to 1.00. In other words a reduction of 42.91 % in the standard deviation values will incur in an increase of 24.71 % in the costs. How acceptable this is in terms of operation is a task that belongs to the decision maker. Nevertheless, there are several intermediate situations that might be interesting as well. The main purpose is not to specify which relationship performance/risk is the most acceptable for the decision maker, but to offer a range of possibilities.

Figure 4 - 37 and Figure 4 - 38 show the same parameters, Annual Cost and Respective Standard Deviation, but in separate bar graphs to facilitate the reader the visualization of the individual contribution each lake makes to the costs and standard deviation, total contribution and be able to compare them. It is interesting to note that in Figure 4 - 38, the individual standard deviation for the operational costs for Lake Ontario, for ω equal to 1.00, has a greater figure than the standard deviation for the costs for the system as a whole. Figure 4 - 39 and Figure 4 - 40 present the probabilities of occurrence of levels above and below the specified thresholds respectively. For the first one, levels above the maximum specified, the general trend is to have a reduction in these probabilities for Lakes Superior, Michigan-Huron, St. Clair and Erie while these probabilities are not significant for Lake Ontario. On the other hand, there is an increase for the probabilities of this lake having levels below the minimum specified, which can be qualified as very high for ω's equal to 0.80 and 1.00. The other four lakes do not present a significant value for these probabilities.

4.8.3 Lake Ontario Considered Independent

Graphs presenting the relationship Average Costs vs. Standard Deviation for the Costs

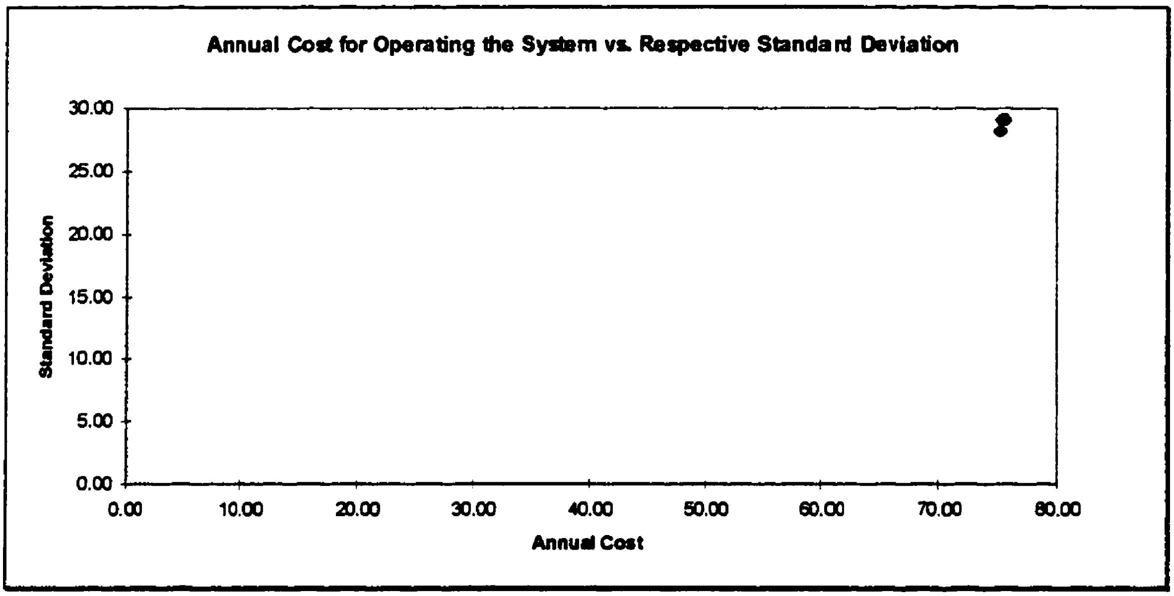


Figure 4 - 41 Annual Cost vs. Respective Standard Deviation for Operating the System Under Different ω' s

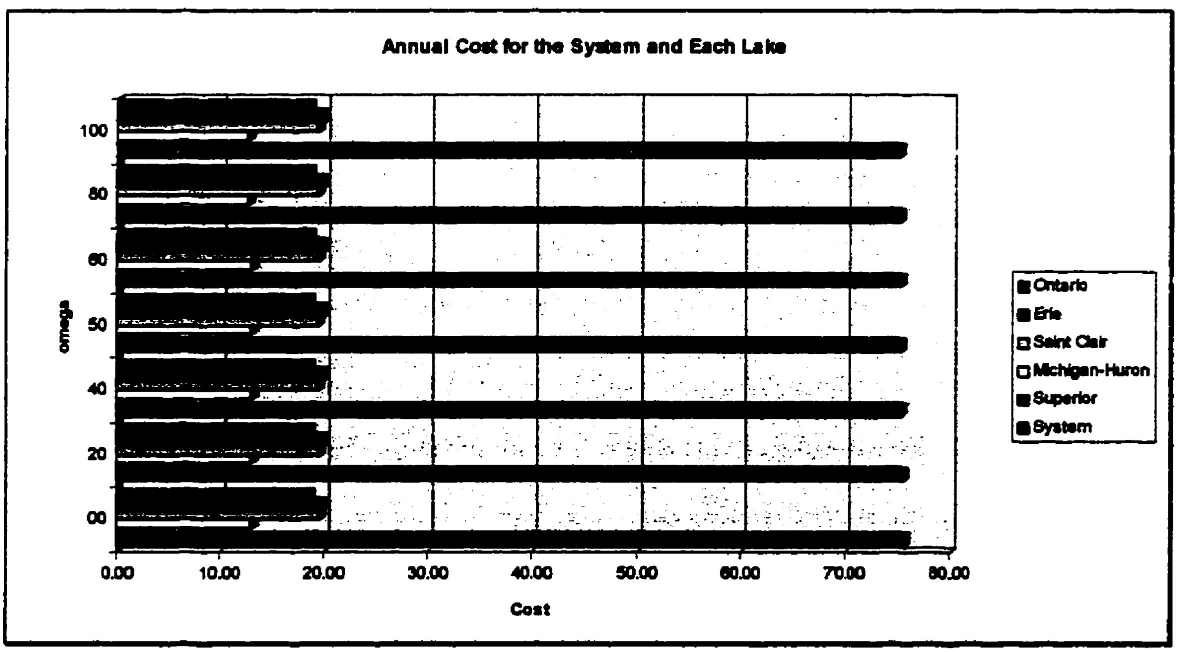


Figure 4 - 42 Annual Cost for Operating the System Under Different ω's for the System and Per Lake

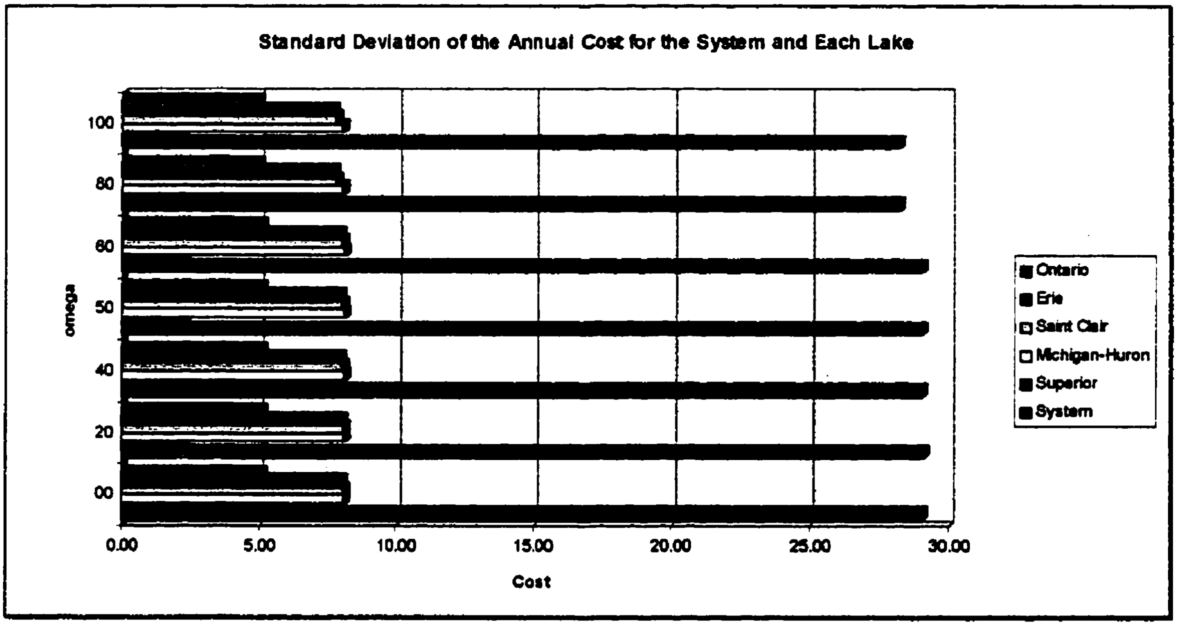


Figure 4 - 43 Standard Deviation for the Annual Cost for Operating the System Under Different ω's for the System and Per Lake

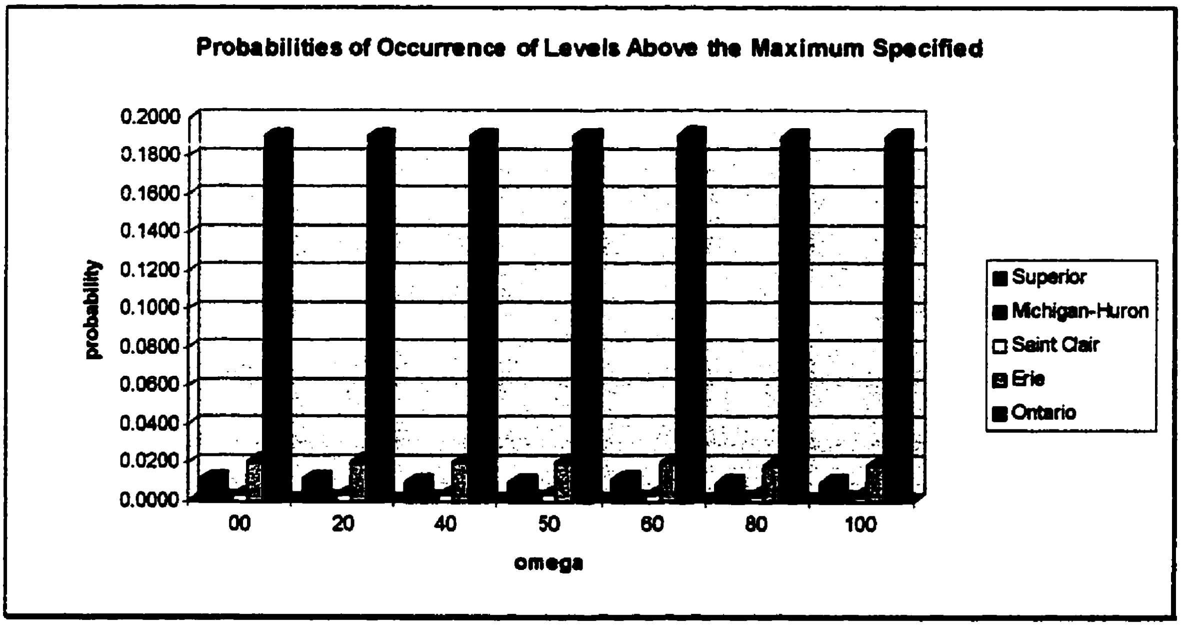


Figure 4 - 44 Probabilities of Exceedance for Levels for the Five Lakes for Different ω's - Above the Maximum Specified

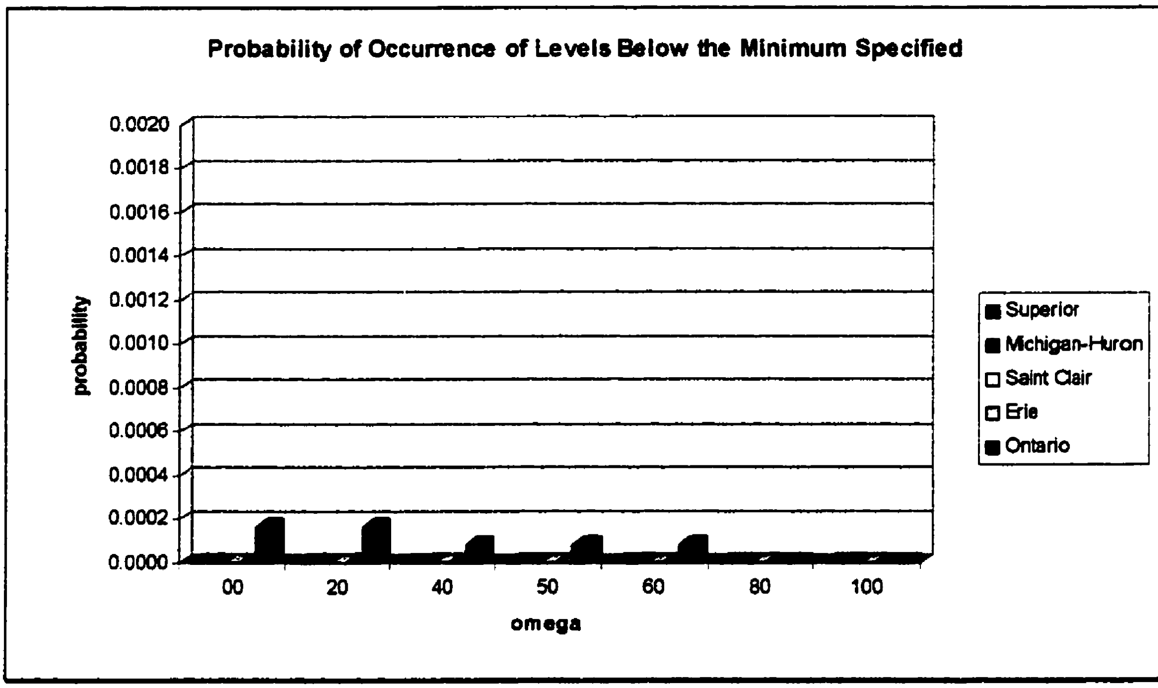


Figure 4 - 45 Probabilities of Exceedance for Levels for the Five Lakes for Different ω's - Below the Minimum Specified

The application of the proposed methodology to the second approach of solving the Great Lakes Operation Problem did not present any significant improvement to the policies obtained with the optimization. The relationship cost/standard deviation did not show a noticeable range of possibilities of operation and the same can be said for the costs, their standard deviations and probabilities of exceedance. As the Two-Pass Mean-Variance Approach is set up in an aggregated fashion for the system as whole, its implementation has certainly suffered from the independence assumption used for Lake Ontario.

The results have shown that the *"Whole System Optimization"*, used in conjunction with the Two-Pass Mean-Variance Approach, performed quite well. The optimization considered on its own already improving the present heuristic operation with regard to the same single objective and the Two-Pass Mean-Variance Approach presenting an interesting range of possible annual costs versus respective standard deviations that can be used in the definition of the most appropriate release policies.

4.9 Summary

This Chapter presented an application of the proposed Two-Pass Mean-Variance Methodology as described in Chapter 3 to the Great Lakes System. The five lakes considered in this study were Lake Ontario, Lakes Michigan-Huron (hydrologically considered as one single lake), Lake St. Clair, small in size but important operation wise, Lake Erie and Lake Ontario. The major characteristic of the system is having only two controls, located in the extreme components of it, in Lake Superior, the most upstream and in Lake Ontario, the most downstream. First, the present operation as defined in the IJC's reports is used to validate two of the studied synthetic data. Once this is done, the aggregation of the components and the optimization of the system is performed. As the original methodology was developed for maximization of the expected returns, some alternatives were proposed. Also, because the system consists of lakes instead of artificial reservoirs with full control, other techniques, different from those presented in the previous chapters were introduced and can be seen in more detail in Appendix A, Great Lakes Case Study Algorithms, mostly in the aggregation of the outflows for the connecting channels. Finally, after defining the expected deviation from the storage targets, which are the same as those specified for the IJC operation, the methodology was successfully employed for the optimization of the system as a whole.

Chapter 5

Conclusions and Further Research

5. CONCLUSIONS AND FURTHER RESEARCH

5.1 Objectives Statement

The main objectives of this work are:

- assess the performance of the Multi-Level Approximate Aggregation Decomposition - Stochastic Dynamic Programming (MAM-SDP) formulation, and propose enhancements to it,
- search for new methodologies that could improve the performance of the MAM-SDP,
- based on the previous study, propose a framework of research that will aim at obtaining better results for the Stochastic Optimization of Large Reservoir Systems that will include information on the variance of the expected returns/costs. Also, these results must include information not only on the maximization of return/minimization of costs but offer a range of choice, as in portfolio selection procedures.
- provide the decision maker with a tool that would enable him to choose amongst different combinations of expected return-variance of return or expected cost-variance of cost.
- provide the decision maker with information on the expected rate of failure of the operation of the stochastic system and possible alternative options of operation.

The MAM-SDP Methodology is applicable to any sort of configuration, namely, reservoirs in series, in parallel or any form of combination of both. It generates a closed-loop type policy, a desired feature when optimizing the operation of reservoir systems. It performs explicit optimization, and allows the computation of release policies for large reservoir systems. Furthermore, there is no need for assumptions regarding the convexity or smoothness of the objective functions. Very large systems can be optimized within the limits of computing tractability.

On the other hand, the results are hardly optimal, especially because the policies become gradually local as the calculations move toward the most downstream reservoir. The independence assumption of the conditional distribution of the releases with respect to the storages and the inflows is another source of error. This source of error was not analyzed in detail in this research and should be subject to further studies.

Therefore, the author analyzed the next steps of the research, bearing in mind the main objectives stated at the beginning.

5.2 Accomplished Research Work

1) **Improved the performance for the MAM-SDP model**

◊ The use of the approximate conditional distribution of probabilities for the releases has shown that it improves the performance of the MAM-SDP model by considering states otherwise not included in the SDP search. The tradeoff is a much longer computing time because the vectors of the probabilities, initially very sparse ones, become dense.

2) **Obtained the variances in the MAM-SDP model**

◊ In the Steady-State Case, Stochastic Dynamic Programming yields the mean values for the benefits. It was of interest to produce a measure of spread from these expected benefits. The variances for three types of multiple reservoir systems were computed, i.e., 3 and 4 reservoir systems, and evaluated the expected return and its variance. To this purpose, several combinations of coefficients of variation for the inflows, with varying levels of uncertainty, were utilized for the 4 reservoir system. For the 3 reservoir system two different combinations of storage capacities were employed under the same inflow pattern.

◊ The computation of the variances was done for a single reservoir system as well. Several coefficients of variation were employed to produced different levels of uncertainty of the natural inflows. With the use of a single reservoirs has the advantage of obtaining this type of relationship isolated from other influences such as configuration of the system, different capacities between reservoirs, relative importance of returns within the system, and location of reservoirs.

3) Application of Principal Components Analysis

◊ PCA was applied to multiple reservoir systems in order to classify the reservoirs in a system according to their relative contribution to the variance of the storage and returns for the entire system. To check the consistency of the approach, several tests were conducted and the results, analyzed using the test problems already mentioned. PCA was here considered as a tool for statistical decomposition of the reservoir system. With this supplemental information, the aggregation can be carried out under a different scheme than the one offered by physically based schemes. From the tests performed, it was shown that PCA can be employed as an additional tool in the decomposition step of the MAM-SDP since it provides a means of obtaining reduced variance for the costs or returns. The technique seems to be more effective for systems with high levels of disturbance in the inputs while the physical diagnosis seems to offer a better performance for low levels.

4) Proposition of Different Aggregation Schemes in MAM-SDP

◊ Using the results from item 3), new aggregation schemes were proposed and tested. From the results obtained in the tests in Chapter 3, it is possible to evaluate the reduction in the variance of the expected return and levels of reliability. One feature to mention is the tradeoff between the reduction of variance and performance of the operation of the system. The conventional aggregation scheme, or aggregating the reservoirs from upstream to downstream usually yields the higher returns for the objective function when the disturbance levels for the inputs are low. However, this higher benefit is, for greater values of uncertainty for the inflows, associated with higher variance of the returns. As it is not possible to quantify these levels a priori it was necessary to define the operating policies for the system for different aggregation schemes and then assess their performance. A good indicator for the performance of the system is its performance after the first stage of MAM-SDP optimization.

5) Application of the Expected Return-Variance of Return Rule

- ◊ It has been of interest to those involved in the optimization of the operation of reservoir systems to include the control of the variance of returns (or costs) in their objective function. Up to the present time, the results were not very encouraging because of the difficulty of dealing with two objectives, i.e., optimization of returns and minimization of variance, simultaneously. In this work, the fundamental ideas presented in Markowitz 1952 paper, were applied to the SDP problem, and specifically to Multiple Reservoir Systems. The solution encountered was to split the optimization into two steps, therefore proceeding a Two-Pass Optimization. The first step is concerned solely with the optimization of returns (or costs) while the second one consists of several optimizations obtaining combinations of control of variance (or standard deviation) and maximization of returns. These different combinations also provide different levels of reliability.

- ◊ The suggested method can be used in real-time optimization as well, with the substitution of the Expected Return by the forecasted one. Its accuracy, however, will depend on the accuracy of the forecast.

6) Applied the above-mentioned to the North American Great Lakes Case Study.

- ◊ The methodologies described in items 4 and 5, and reviewed in detail in Chapter 3, were then applied to the Great Lakes System. This system is composed by six lakes, Lake Superior, Lake Michigan, Lake Huron, Lake St. Clair, Lake Erie and Ontario. Although the PCA was not employed, the use of different aggregation schemes than the conventional one is a direct consequence of this study. This system has two peculiarities:

  - * only the most upstream and the most downstream lakes are controlled.
  - * Lake St. Clair dimensions are much smaller than the other five lakes, reducing drastically its capacity of absorbing the levels variations and affecting the optimization and simulation parts.

First, the system was optimized using aggregation schemes and state-derived release policies were obtained. The simulation of these policies, when compared to the simulation of a simplified version of the IJC heuristic operation policies, has shown better performance for the use of long-term (steady-state) state-derived policies. It should be added that the objective function for this optimization is the minimization of the accumulated annual distance from the target storage levels, differing from the test cases, which were all referring to maximization of the expected return. Therefore, the Two-Pass Mean-Variance Approach was then extended to minimization of costs and corresponding variances. The methodology was successfully implemented for the long-term optimization with monthly time-steps and the results were very encouraging. Besides, two types of optimization approaches were suggested and two kinds of synthetic data analyzed. Regarding the two types of optimization approaches, the one that considers the system as whole performed better than the one that views Lake Ontario as an independent part of the system.

5.3 Further Research

- **Improve the computing times**

    - The major hindrance with the present work was the long time the computations took to process. One of the reasons for that was the use of the Matlab® environment. Two extensions are proposed. The use of other environments like Fortran, C and C++ and experiment with parallel computations.

- **Improve the quality of the synthetic data for the Great Lakes System Study Case**

    - Due to software limitations only two multivariate models were used to generate the synthetic Net Basin Supplies for the Great Lakes Case. These models were able to yield only autoregressive models. A more detailed analysis using multivariate mixed moving-average and autoregressive (ARMA) models would probably provide synthetic data of better quality.

◊ **Parallelization of the Second Pass of Two-Pass Mean-Variance Approach**

◊ It is possible and feasible to perform simultaneously the selection of the optimal returns in the two-level SDP for all the weights used for the Two-Pass Mean Variance Approach. This is the only optimization part where it is necessary to include this information, and for the rest of the nested loops in SDP the computations are exactly the same. Thus, parallelizing the selection of the optimal returns with the respective policies would speed up considerably the computation time for the methodology here suggested. The evaluation of the performance of the policies, done by means of simulation can be parallelized.

◊ **Further implementation for the North American Great Lakes Case Study**

◊ As it is needed to validate the proposed methodology by comparing the operation policies with those from an already tested one, the same configuration for the Great Lakes as the one currently in use was adopted. However, it is possible to extend the research applying either the methods described in this work, to the Great Lakes now considering full control for the system, that is to say, having all the lakes controlled.

◊ For the present computations, the expected outflows for the aggregate upstream and downstream lakes as obtained from the simulation as described by the heuristic operation from IJC's reports were employed. The continuation for the present work would be to employ the aggregated outflows derived from the *"Whole System Optimization"* Approach. Then, not only proceed to the optimization part once more, but to use these results for the implementation of the Two-Pass Mean-Variance Approach.

◊ It is important to add the minimization of the distance from the target discharges in a multiobjective optimization framework. The weights, or relative importance given to each of both objectives, must be defined carefully. This can be easily implemented by assuming different linear combination for the weights and presenting the results. Then, having the results of weighing levels against discharges, the choice of the most appropriate ones must be made by means of an agreement between all the parts with interests involved. The code to perform these computations is already written and the results should be published shortly.

◊ The multiobjective optimization that would add the discharge targets to the already computed storage level targets must also include the correlation between the storages and discharges. In a multiobjective optimization the impact of the variables in the objective function that present covariance between themselves should be studied carefully. For the lakes case, this is very intuitive because levels and discharges from the lakes are intimately correlated. The reduction of levels variance in Lake Superior is associated with an increase of the variance for the discharges from the same lake. Moreover, as the three intermediate lakes are not controlled, this affects directly the levels variance for the three downstream ones. It would be of interest to investigate further the existing correlation patterns between the levels and discharges for the entirety of the system, i.e., the five lakes. For a more thorough assessment on how the policies would benefit the interest groups, it would be necessary to evaluate the policies for a full controlled system, that is to say, by adding controls to lakes Michigan-Huron, St. Clair and Erie.

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Appendices

Appendix A

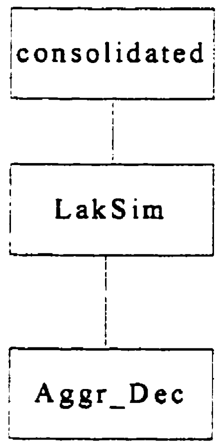
North American Great Lakes Study Case

Algorithms employed to Aggregate and perform the Steady-State Optimization

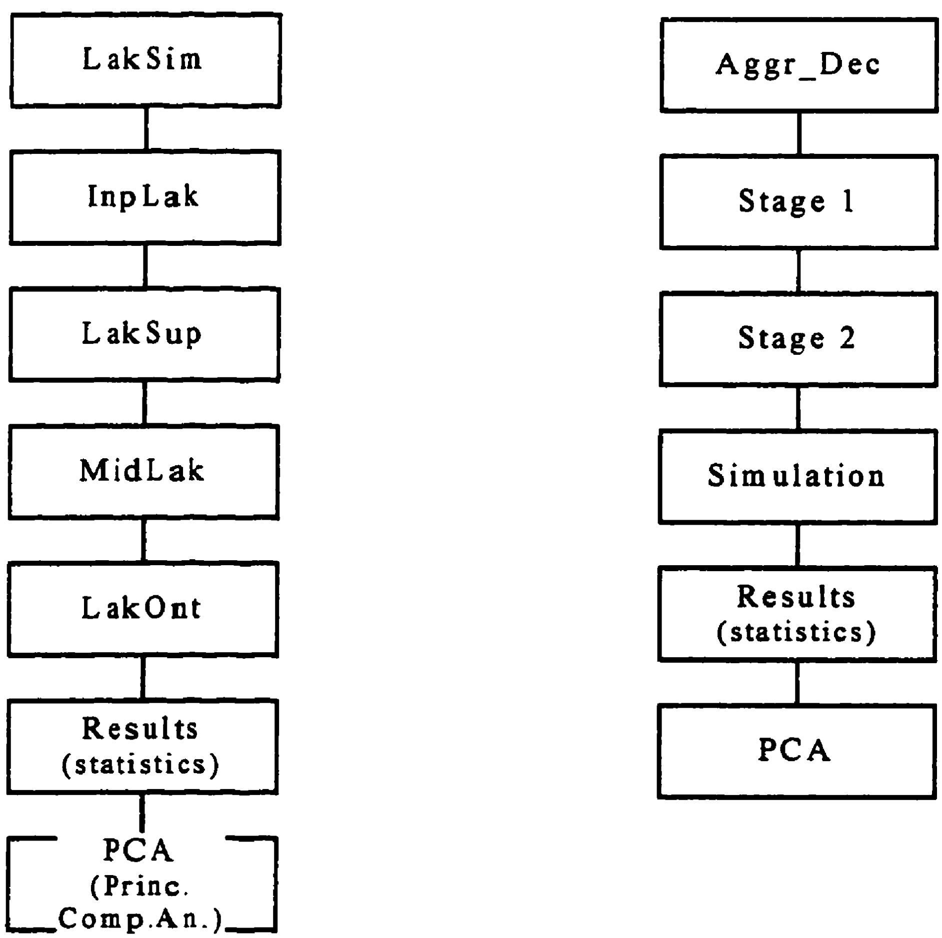
Great Lakes Subroutine Chart

Organization of programs

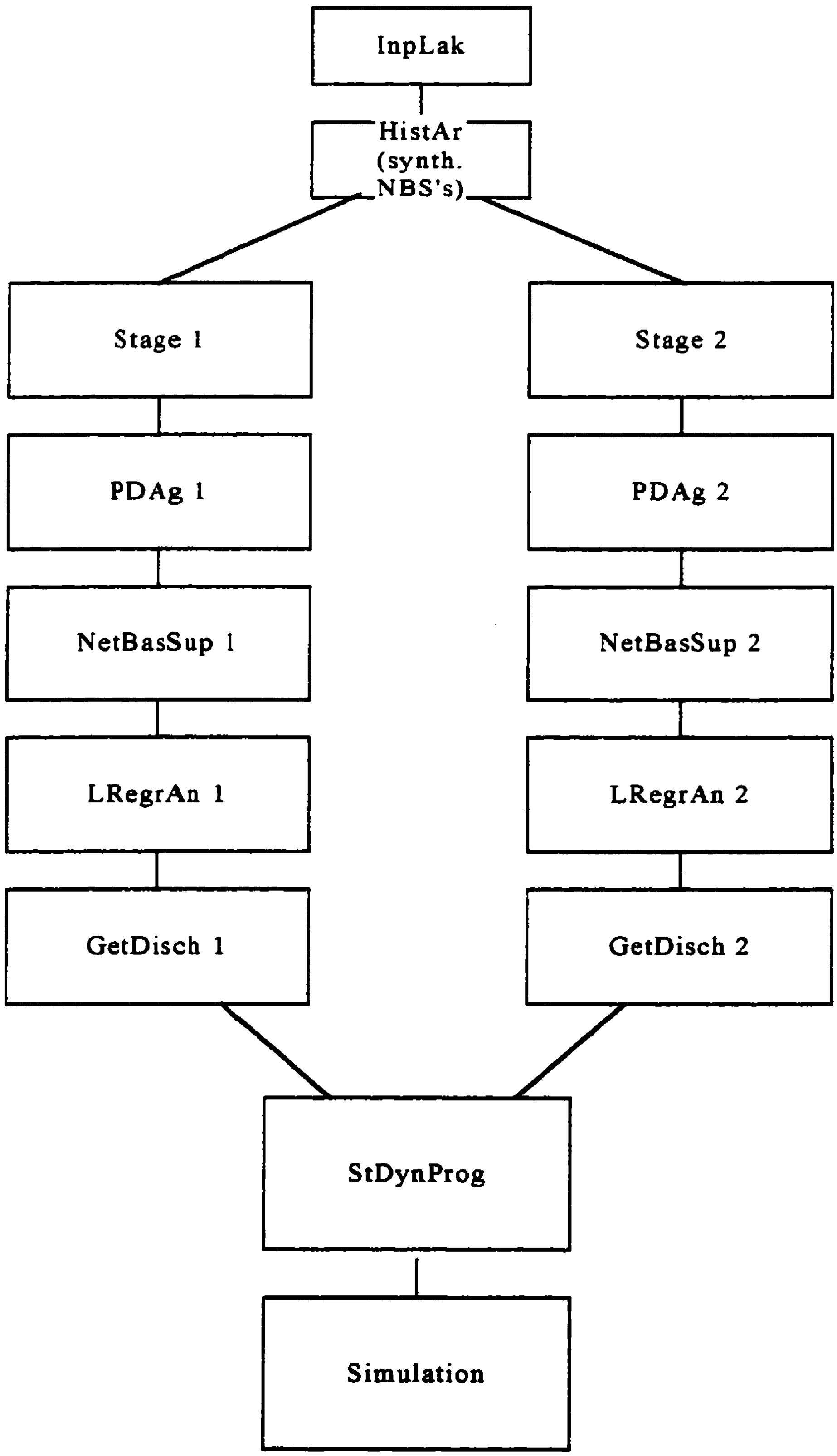
First hierarchical level:



Second hierarchical level:



Third hierarchical level:



Observation: The computation of the Principal Components Analysis in the Study Case was performed for checking purposes only. As there are only two regulated controls the use of it for determining the most appropriate combination for the aggregation becomes unnecessary.

Great Lakes Algorithm:

Main Driver → consolidated

calls two subroutines → LakSim

→ Aggr_Dec

Subroutine InpLak

⇒ Provides the input of the general data, according to what follows.

⇒ Monthly recorded and derived data

- Beginning-of-Period (BOP) levels (ft.)
- Mean monthly outflows (tcfs-month)
- Mean monthly NBS (tcfs-month)

The data above refer to lakes Superior, Michigan-Huron, St. Clair, Erie and Ontario. The collection period is from 1900 to 1973 (IGLD 1955). All information comes from the IJC report, "Regulation of Great Lakes Water Levels", December 1973, available at the UW Library, Davis Centre. Most of the data is in Appendices to the main volume.

⇒ From this information, the 1000 year synthetic NBS is obtained for the deseasonalized data as described in Hipel and McLeod (1994), p. 465. The time series are computed in two formulations:

1. Multivariate autoregressive model with lag 1 (AR1).
2. Periodical multivariate autoregressive model with period equal to one year (PAR1).

* Matlab routines employed are from the System Identification Toolbox. Main functions are: arx, randn, and idsim.
* Means and standard deviations for the NBS's, including two aggregate lakes (preprocessing - 4 most downstream lakes and 4 most upstream lakes) are calculated as well.
* The subroutine that computes the synthetic data is called HistAR.

⇒ Reading the relationship between an increase (or decrease) of one foot in lake elevation and the corresponding average monthly discharge. The relationship is linear because of the large area of lakes and relative small variations in lake levels.

⇒ Constants derived from the Manning coefficients employed in the Middle Lakes Routing Model.

◊ kapa1 - 73.515

◊ kapa2 - 177.286

◊ kapa3 - 3665

The values above were obtained from "Regulation of Great Lakes Water Levels", Appendix B, Vol. 2, 7 December 1973, p. B-9.

⇒ Definition of maximum and minimum flow limitations.

◊ Obtained from "Regulation of Great Lakes Water Levels", Appendix G, 7 December 1973, pp. G-42, G-114.

⇒ Definition of maximum and minimum level limitations.

◊ Values obtained from "Levels Reference Study, Great Lakes - St. Lawrence River Basin", Levels Reference Study Board, March 31, 1993. These are values used as Basis-of-Comparison, and were used to compute the probabilities of exceedance, defined as the occurrence of levels below or above these references.

⇒ Definition of range of operation as the difference between the maximum and minimum levels.

⇒ Definition of connecting channel ice and weed retardation. Given in mean historical values, in tcfs-month. Assumed the deterministic values, given the approximation character of those measurements.

- ◊ for lake St. Clair
- ◊ for Detroit River
- ◊ for Niagara River (only one that includes weed retardation)
- ◊ for Ontario outlet

Subroutine LakSim

⇒ Performs simulation of the monthly operation for the 5 Lakes as defined by the IJC in its reports. The historical data for the Net Basin Supplies (NBS), registered levels, registered outflows are also taken from theses reports.

⇒ Data input for the 1000 year monthly simulation and the general input comes from the following subroutine.

⇒ calls InpLak, which has the historical information.

⇒ Definition of starting levels for the simulation. Adopted those from "Basis-of-Comparison, Great Lakes - St. Lawrence River System, NOAA Technical Memorandum, ERL GLREL-79", by D.H. Lee (ed.), April 93.

- ◊ Lake Superior - 600.39 ft.
- ◊ Lakes Michigan-Huron - 577.92 ft.
- ◊ Lake St. Clair - 572.84 ft.
- ◊ Lake Erie - 569.99 ft.
- ◊ Lake Ontario - 244.07 ft.
- * These levels refer to the month of January.

⇒ Assumed diversions, to and from the lakes, used fixed monthly values, in tcfs-month.

1. To lake Superior
   - Long Lac and Ogoki diversions - 5.0 tcfs-month
2. From lake Michigan
   - Chicago Sanitary and Ship Canal - 3.2 tcfs-month
3. From lake Erie
   - Welland Canal diversion - 7.0 tcfs-month

⇒ call three subroutines with the simulation of

- Lake Superior (controlled)
- Middle Lakes; Michigan-Huron, St. Clair, and Erie (uncontrolled)
- Lake Ontario (controlled)

⇒ Computation of the statistics like mean, maximum and minimum values for levels and outflows, existence or not of spills, probabilities of exceedance

⇒ Computing Principal Component Analysis (PCA), based on the objective function, namely, annual cumulative sum of monthly deviations from the storage targets, given in levels (ft.).

Subroutine LakSup

⇒ Simulates the present operation of Lake Superior.

⇒ Computation of Lake Superior rule flow for the compensating works according to Regulation Plan 1977. Based on the algorithm presented by E. Loucks et al., in "Diversion of Great Lakes Water, Part 1: Hydrologic Impacts", February 1987. Appendix 4, p. 59.

⇒ Initialization

◊ Computation of the maximum possible outflow from the compensating works.

$$Outflow_{max} = (2601.0 * (0.944x - 560.292)\hat{}1.5) * 0.001$$

The result given in tcfs-month, where x is the average level (for sake of simplicity, it was assumed the level at BOP). From "Great Lakes Diversions and Consumptive Uses", Appendix B - Computer Models. Report to the IJC, September 1981.

⇒ First Step - Computing the basic rule flow with the balance equation:

$$Q_{basic} = Q_{AvSup} + A[(S_t - S_j) - (H_t - H_j)R_j]$$

where:

Q_{basic} - basic rule flow,

A - statistically derived constant - adopted the "best" one, as defined in "Regulation of Great Lakes Water Levels", Appendix B, Lake Regulation, 7 December 1973, p.B-40, equal to 200 tcfs/ft.

Q_{AvSup} - historical average outflow, computed from the data collection period,

$R_j^2 = \dfrac{Var[S_j]}{Var[H_j]}$ - S_j and H_j are the estimated monthly historical variances for water levels at lakes Superior and Michigan-Huron respectively,

S_j, H_j - target levels for lakes Superior and Michigan-Huron, also defined as the estimated monthly historical averages.

S_t, H_t - BOP water levels.

The adjustment to the historical average outflows comes from the simple linear relationship:

$$\frac{S_t - S_j}{\sigma_S} = \frac{H_t - H_j}{\sigma_H}$$

The relationship above tries to bring the relative water levels from both lakes to the same situation with regard to the average historical ones and within the natural ranges.

The constant A is obtained from computer simulation, aiming at obtaining the best performance considering the total benefit from the multiple objectives that are power generation, navigation, both lakes and Erie shores. The other tested values were 50, 100 and 300 *tcfs/ft*.

⇒ Second Step - Checking the flow limitations

◊ The difference from previous month outflow must not exceed 30 *tcfs-month*.

- ◊ The absolute maximum and minimum summer outflow is computed adding 65 *tcfs-month* to the computed maximum allowable outflow.
- ◊ The contingency plan states that if the outflow is less than 65 *tcfs-month*, only 55 must be released in order to speed up the reservoir replenishment process.

⇒ Third Step - Use of the balance equation to compute the End-of-Period (EOP) level. For sake of simplicity, no average water levels were employed. If the maximum or minimum level constraints were violated, no restriction was imposed, but the occurrence was registered for later computation of the probabilities of exceedance.

Subroutine MidLak

⇒ Performs the Middle Lakes, i.e., Michigan-Huron, St. Clair, and Erie, Routing Model. As all three have uncontrolled outflows, these and the levels are obtained from a non-linear relationship composed by a system of six equations and six unknowns.

I. Inputs to the problem
   A. BOP lake elevations
      1. $L1_{mh}$ - Lake Michigan-Huron water level,
      2. $L1_{sc}$ - Lake St. Clair water level,
      3. $L1_{er}$ - Lake Erie water level.
   B. Water supplies to the lakes
      1. Tot_{mh} - Lake Michigan-Huron total water supply,
      2. NBS_{sc} - Lake St. Clair NBS,
      3. NBS_{er} - Lake Erie NBS.
   C. Monthly ice and weed retardation
      1. Ret_{mh} - St. Clair River ice retardation,
      2. Ret_{sc} - Detroit River ice retardation,
      3. Ret_{er} - Niagara River ice and weed retardation.

II. Output
   A. $L2_{mh}$, $L2_{sc}$, $L2_{er}$ - EOP lake elevations,
   B. Out_{mh}, Out_{sc}, Out_{er} - monthly mean outflows

C. $Diff_{mh}$, $Diff_{sc}$, $Diff_{er}$ -difference between BOP and EOP lake elevations for the current month.

III. Constants used

A. *wASup* - volume of water equivalent to one foot of water over the lake Superior area (given in *tcfs-month*),

B. *wAMH* - volume of water equivalent to one foot of water over lake Michigan-Huron area (*tcfs-month*),

C. *wAStC* - volume of water equivalent to one foot of water over the lake St. Clair area (*tcfs-month*),

D. *wAErie* - volume of water equivalent to one foot of water over the lake Erie area (*tcfs-month*),

E. *wAOnt* - volume of water equivalent to one foot of water over the lake Ontario area (*tcfs-month*).

F. kapa1 - constants derived using connecting channel geometry, Manning coefficients, roughness, and configuration for St. Clair River,

G. kapa2 - same as above for Detroit River,

H. kapa3 - same as above for Niagara River.

I. N - number of time steps in a month, which is the simulation time step. Used 40.

J. It - fixed number of allowed iterations to converge. Used 5, although other researchers used only 3. But differently from ours, their simulation time step for lakes Erie and Ontario was equivalent to a week, or a quarter of a month.

IV. Set of non-linear equations employed:

A.
$$L2_{mh} = L1_{mh} + \frac{Tot_{mh}}{N * wAMH}$$
$$L2_{sc} = L1_{sc} + \frac{NBS_{sc}}{N * wAStC}$$
$$L2_{er} = L1_{er} + \frac{NBS_{er}}{N * wAErie}$$

B. Iterate *for j = 1:It*

$$Out_{mh} = kapa1 * (\frac{1}{2}(L1_{mh} + L2_{mh} - L1_{sc} - L2_{sc}))^{1/2} * (\frac{1}{4}(L1_{mh} + L2_{mh} + L1_{sc} + L2_{sc}) - 542.0$$

$$Out_{sc} = kapa2 * (\frac{1}{2}(L1_{sc} + L2_{sc} - L1_{er} - L2_{er}))^{1/2} * (\frac{1}{2}(L1_{sc} + L2_{sc}) - 548.49)^2 - Ret_{sc}$$

$$Out_{er} = kapa3 * (\frac{1}{2}(L1_{er} + L2_{er}) - 556.25)^{3/2} + 7000 - Ret_{er}$$

$$L2_{mh} = L1_{mh} + \frac{Tot_{mh} - Out_{mh}}{N * wAMH}$$

$$L2_{sc} = L1_{sc} + \frac{Out_{mh} + NBS_{sc} - Out_{sc}}{N * wAStC}$$

$$L2_{er} = L1_{er} + \frac{Out_{sc} + NBS_{er} - Out_{er}}{N * wAErie}$$

end of iteration

C. The EOP water levels are reached after N steps of the above iteration and the mean outflows were computed from the average of the N values obtained in iteration It.

V. The outflows are checked against the maximum and minimum historically registered extreme values, that are assumed to be the references for the 1000 year simulation.

VI. The EOP water levels are checked against the maximum and minimum historically registered extreme values, that are assumed to be the references for the 1000 thousand year simulation and for the computation of probabilities of exceedance.

- The 7000 *cfs-month* figure that appears in one of the equations refers to the diversion from Lake Erie.

- All the values are given in *cfs-month*, instead of *tcfs-month*, that were the rule for the rest of the computations.

Subroutine LakOnt

Performs the computation of Lake Ontario rule flow according to Plan 1958A, i.e., it does not take into consideration the normalization as prescribed by Plan 1958D, which is currently in operation.

⇒ First Step - Computing the basic rule flow using the relationship present in "Regulation of Great Lakes Water Levels", Appendix B, Lake Regulation, report to the IJC, Vol. 1 (of 3), p. B-40, 7 December 1973.

The following relationship was employed to compute the basic rule flow

$$Q_{basic} = 200\left(\frac{(EOP_{ont} - Tg_{ont})80 + (EOP_{erie} - Tg_{erie})105}{185}\right) + AvOut_{ont}$$

where

Q_{basic} - basic rule flow,

EOP_{ont} - monthly end-of-period Ontario water level,

Tg_{ont} - target water level for lake Ontario, given by the estimate of historical mean level,

EOP_{erie} - monthly end-of-period Erie water level,

Tg_{ont} - target water level for lake Erie, given by the estimate of historical mean level,

$AvOut_{ont}$ - long term mean pre-project[1] lake Ontario outflow.

⇒ Second Step - Check for flow limitations

1. The difference in discharge from previous month must not exceed 20 *tcfs-month*.
2. Check against allowed maximum and minimum outflows.
3. If necessary, proceed to the necessary adjustments

⇒ Third Step - Use of balance equation to compute the EOP level. . For sake of simplicity, no average water levels were employed. If the maximum or minimum level constraints were violated, no restriction was imposed, but the occurrence was registered for later computation of the probabilities of exceedance.

Subroutine Aggr_Dec

⇒ This subroutine defines the main inputs to the optimization part of the Great Lakes Case Study and then calls the subroutines with the Two-Stage SDP. Finally, the performance of the derived release policies are assessed by long-term simulation, with 1000 year length on a monthly time step.

[1] The outlet conditions are those existing before the regulation of lake Ontario, i.e., before the year of 1955.

⇒ Inputs

1. Number of seconds in each month,
2. Number of discrete intervals for the NBS's,
3. Number of discrete intervals for the state vector (storage),
4. Number of discrete intervals for the accessory outside state vector, defined for states located beyond the normal range of operation, as recommended by the IJC,
5. Number of discrete intervals for the decision vector (release).

⇒ First Step - call the subroutine that performs the first stage two-level optimization, Stage1.

⇒ Second Step - call the subroutine that performs the second stage two-level optimization, Stage 2.

⇒ Third Step - call the subroutine that performs the 1000 year simulation of the system on a monthly basis, using the derived operation policy.

⇒ Fourth Step - Computation of the performance statistics.

Subroutine Stage 1

⇒ Performs the aggregation of lakes Michigan-Huron, St. Clair, Erie and Ontario. Transforms all units in volume, the discharges being volume-month. Lake Superior whose rule curve will be based on its state and the state of the aggregate downstream reservoir.

⇒ Calls subroutines which provide the input to the optimization and are described in the following pages. They are:

1. PDAg1
2. NetBasSup1
3. LRegrAn1
4. GetDisch1

⇒ Definition of the aggregation coefficients, that will be used in the objective function. This function minimizes the square root of the squared distances from the target levels, measured in feet. Because, as mentioned above, the working unit is volume, transformation is needed and influences the choice the aggregation coefficients.

⇒ The major assumption adopted in this case is that all the aggregated reservoirs are proportionally equally filled (or emptied), in level terms. In other words, the rate of increase or decrease in water levels is the same for all the aggregated reservoirs.

⇒ In this case, we transformed the concept of potential energy to potential storage. While in Turgeon, Arvanitidis and Rosing and Ponnambalam, the coefficients were function of the hydroelectric plant or irrigation capability, directly related to the discharges from the reservoirs, here they are function of their capacity of storing water. The coefficients, as was the case with the previous authors, are functions of the physical features of the system.

⇒ Following the idea above, the release from lake Superior, the most upstream and non-aggregated reservoir, has to be aggregated when entering the aggregated part of the system. However, as the water will travel through the connecting channels, now aggregated in just a large one, it is necessary to account for the physical characteristics of all four downstream channels. Therefore, a linear relationship between the monthly maximum allowable outflows, normalized by the same feature from lake Superior outlet is employed.

⇒ Call the Two-Level SDP optimization.

Subroutine PDAg1

⇒ Computes the joint conditional distribution of probabilities of the Great Lakes synthetic NBS's in a specific month. There are two types of routines, one the ARX1, i.e., multi-variate autoregressive model with monthly time lag equal to 1, and another for the PARX1, with a time lag equal to the year cycle.

⇒ First Step - Aggregate the NBS for the four most downstream lakes.

⇒ Second Step - Obtain the maximum and the minimum values from the synthetic NBS's for lake Superior and the aggregated one. Done on a monthly basis.

⇒ Third Step - Get the ranges of variation and subdivide them into classes.

⇒ Fourth Step - Define the search bounds for the classes.

⇒ Fifth Step - Obtain the conditional frequencies and median values for the NBS's for each of the class as defined above and for each and every month.

⇒ Sixth Step - With the information obtained in the previous step, compute the conditional probabilities. These can be defined as follows. Given that in a given month the NBS for lake Superior is in a specified class, get the probabilities of the occurrence of all classes in the next epoch according to what was established in the time series model. For instance, in an autoregressive model of lag 1, in the next month. In a periodical ARX1, for the same month in the next year.

Subroutine NetBasSup1

⇒ This subroutine simplifies the information on conditional distribution of probabilities computed in the previous step of the calculations.

⇒ It gets the probabilities of the expected NBS's, reducing the size of the conditional probabilities and expected means matrices to a more manageable size. Basically, it is another pre-processing step focusing at diminishing the computing time for the two-level SDP routine. From the just mentioned matrices it gets the expected values from the medians and their conditional probabilities. Transforms matrices of conditional probabilities into vectors.

Subroutine LRegrAn1

⇒ Performs the regression analysis for the NBS for the aggregated lake. Lake Superior is the chosen independent variable because of its upstream situation and import in the optimization part. The release policies computed in the first stage of the optimization will eventually be used in its operation while the policies for the aggregated lake are not used. Only its aggregated state has practical utility.

⇒ The linear regression coefficients are obtained for each and every month of the year after deseasonalizing the synthetic time series data.

⇒ The assumption valid here is the same for the rest of the work. Because all five lakes are in the same catchment area and therefore subject to similar climatic influences, we consider high spatial correlation between them. This assumption reduces the complexity of the problem affecting positively the computation time without jeopardizing the precision of the optimization.

Subroutine GetDisch1

⇒ This subroutine computes the aggregated discharge from the aggregated downstream lake. This approximation has to be done because of the four lakes that compose the aggregated one, only one, lake Ontario is regulated. The three others are unregulated. Thus, they are directly dependent on the discharges from Superior and their difference in water levels. An important simplifying assumption here, already employed in this work, is the consideration of all downstream lakes proportionally equally filled.

⇒ For the given discrete set of levels and discharges from lake Superior and from each of the lakes that compose the aggregated downstream one, the nonlinear system of equations for the middle lakes is solved and the basic rule flow from lake Ontario is computed using the algorithms explained in subroutines MidLak and LakOnt. These discharges and levels are then aggregated and from this information we obtain the aggregate levels and discharges from the aggregated lake conditional on the levels and discharges from lake Superior. Here is where the importance of the assumption mentioned in the paragraph above appears. If that consideration is not observed the aggregated states and discharges do not match with those used in the two-level SDP optimization.

⇒ In the test program, only this type of rule flow was used. For a more sophisticated model, once the release policies are obtained and lake Superior and Ontario have their operation policies defined by the state of the entire system, it would be advisable to repeat the optimization part, using the just obtained policy of operation. The optimization part becomes an iterative process too.

Subroutine StDynProg

⇒ Computes the steady-state long-term operation policies for two reservoirs, i.e., the two-level optimization. For the Great Lakes Case, one of the reservoirs, either the most downstream or the most upstream is always aggregated. One of the reasons for that is the fact that only lake Superior and lake Ontario are presently regulated and are placed in the extremities of the system. Lakes Michigan-Huron, St. Clair and Erie are unregulated. Another reason is that with this approach we avoid having local policies for lake Ontario, what would be the case in the previous methodology. This is possible because of the specific configuration of the Great Lakes System..

⇒ The convergence to the optimal values is facilitated in this algorithm with the use of White's Iteration Method. Another feature of the algorithm is having the reservoirs disposed in series.

⇒ Two subroutines make use of this one. They are Stage1 and Stage2. The SDP routine is adapted to suit the specific computational demands of both of them and most of its functionality shares common parts.

⇒ Initialization

Assignment of the states and decision vectors. One characteristic of this subroutine is that only one reservoir has discharge policy to be analyzed. The other reservoir, always the aggregated one, has its release dependent on the state and discharge of the other one and its own state. The simplifying assumption here is having this conditional discharge deterministic instead of stochastic. This reduces drastically the computation time and as this discharge comes from the aggregated reservoir, it has already been approximated in other computational instances.

⇒ For the optimization of lake Superior, when its range of the states goes beyond its full capacity, the system is allowed to spill and this amount of water is conveyed by the connecting channel to the downstream reservoir. However, we assumed that the aggregated reservoir, composed by the non-regulated ones in its majority, this event is not possible.

⇒ The SDP optimization proceeds similarly for Stage2, the major difference being the aggregated reservoir placed upstream and the non-aggregated one, lake Ontario being the most downstream one.

Subroutine Stage 2

⇒ Performs the aggregation of lakes Superior, Michigan-Huron, St. Clair, and Erie. Transforms all units in volume, the discharges being volume-month. Lake Ontario rule curve will be based on its state and the state of the aggregated upstream reservoir.

⇒ Calls subroutines which provide the input to the optimization and are described in the following pages. They are:

1. PDAg2
2. NetBasSup2
3. LRegrAn2
4. GetDisch2

Obs: Because the procedure for the subroutines above is quite equivalent to that described in the set for Stage 1 we will omit their explanation. The difference is that here the four most upstream lakes are aggregated and Lake Ontario is the one to have its release policy defined with respect to the state of the system.

⇒ Definition of the aggregation coefficients, that will be used in the objective function. This function minimizes the square root of the squared distances from the target levels, measured in feet. Because, as mentioned above, the working unit is volume, transformation is needed and influences the choice the aggregation coefficients.

⇒ The major assumption adopted in this case is that all the aggregated reservoirs are proportionally equally filled (or emptied), in level terms. In other words, the rate of increase or decrease in water levels is the same for all the aggregated reservoirs.

⇒ Contrary to what happened in Stage 1, the release from the aggregated lake, the most upstream reservoir, has to be disaggregated when entering the non-aggregated part of the system. However, as the water has travelled through the aggregated connecting channels, now in just a large one, it is necessary to account for the physical characteristics of the single downstream channel. Therefore, a linear relationship between the monthly maximum allowable outflows, normalized by the aggregated capacity of the connecting channels for the four upstream ones, is employed.

⇒ Call the Two-Level SDP optimization.

Simulation:

Once the obtained the state-derived release policies the operation of the Great Lakes is simulated for long-term operation, using time-step equal to one month during 1000 years. The results presented in Chapter 4 are all computed from this simulation results.

Appendix B

North American Great Lakes Study Case

Approach Justification

Appendix B - Justification for the methodology employed:

From "Regulation of Great Lakes Water Levels", main report, December 1973, in Section 14, we adopted the findings and conclusions from that work group to fundament our approach to solving the Great Lakes Case Study. The main items are reproduced below.

Findings

1. There are three categories of water level fluctuations on the Great Lakes: short period, seasonal and long term.
2. The large storage capacities and restricted outflow characteristics of the Great Lakes are highly effective in providing a naturally regulated system.
3. The mean levels and outflows of the lakes will change progressively with time as a result of:
   3.1. The steadily increasing consumptive use of water in the basin, and
   3.2. The nearly imperceptible movement of the earth's crust in the region of the Great Lakes basin.
4. To the extent that the lakes already possess a high degree of natural regulation and artificially regulated by means of the works at the outlets of Lake Superior and Lake Ontario, only small improvements are practicable without costly regulatory works and remedial measures.
5. A new regulation plan for Lake Superior, SO-901 (a plan that maintains regulation only for the most upstream and downstream lakes, Superior and Ontario), can be expected to yield small long-term average annual net benefits to the system at minimal cost.
6. Two preliminary plans for the combined regulation of Lakes Superior, Erie and Ontario exhibit favorable benefit-cost ratios.
7. Regulation of Lakes Michigan-Huron by construction of control works and dredging of channels at their outlet. Combined with the regulation of Lakes Superior and Ontario, would not provide benefits commensurate with costs.
8. Regulation of all five lakes, employing existing control works for Lakes Superior and Ontario and newly constructed works for Lakes Michigan-Huron and Lake Erie, would not provide benefits commensurate with costs.

9. The physical dimensions of the St. Lawrence River are not adequate to accommodate the record supplies to Lake Ontario received in 1972-1973 and at the same time satisfy all the criteria and other requirements of the IJC orders of approval for the regulation of Lake Ontario.
10. Construction of works in the St. Clair and Detroit Rivers to compensate hydraulically for the remaining effect of the 25 and 27 foot navigation project would result in increased shoreline damage from higher lake levels.
11. Better and faster determination of basin hydrologic response will allow improvement in regulation.
12. The most promising measures for minimizing future damages to shore property interests are strict land use zoning and structural setback requirements.

Conclusions:

1. Small net benefits to the Great Lakes system would be achieved by a new regulation plan for Lake Superior which takes into consideration the levels of both Lake Superior and Lakes Michigan-Huron.
2. Regulation of Lakes Michigan-Huron by the construction of works in the St. Clair and Detroit Rivers does not warrant any further consideration.
3. Further study is needed of the alternatives for regulating Lake Erie and improving regulation of Lake Ontario, taking into account the full range of supplies received to date.
4. The hydrologic monitoring network of the Great Lakes basin should be progressively improved.
5. Appropriate authorities should act to institute land use zoning and structural setback requirements to reduce future shoreline damage.

Appendix C

North American Great Lakes Study Case

Input Data

Appendix C - Input Data to the Great Lakes Case Study:

For the Simulation as reported by the IJC and Optimization and Simulation as defined in this work:

Total Number of Years for the historical time series: 74 (1900-1973 up to the month of June)
Total Number of Years for the synthetic time series: 1000
Number of Epochs (months) considered in a year cycle: 12

Average monthly flow assumed for the diversions (*tcfs*):

- Diversion to Lake Superior: 5.000
- Diversion from Lake Michigan: 3.200
- Diversion from Lake Erie: 7.000

Historical Mean Values for the Net Basin Supplies to the lakes:

Month	*Lake Superior*	*Lakes Michigan-Huron*	*Lake Clair*	*Saint Lake Erie*	*Lake Ontario*
January	-12.99	55.18	6.15	22.99	31.91
February	8.99	86.78	6.76	29.77	35.55
March	42.73	178.89	8.34	68.88	73.41
April	148.59	283.74	7.81	65.42	92.31
May	191.50	255.61	6.46	44.19	60.00
June	158.72	208.31	4.50	26.81	41.26
July	131.32	132.15	3.75	2.16	24.73
August	100.22	49.97	2.14	-15.92	7.34
September	75.32	29.19	1.68	-22.48	2.48
October	36.97	-1.16	1.60	-24.95	5.19
November	14.92	30.26	1.73	-9.47	16.36
December	-24.40	25.62	4.33	11.58	22.55

Historical Standard Deviation Values for the Net Basin Supplies to the lakes :

Month	*Lake Superior*	*Lakes Michigan-Huron*	*Lake Saint Clair*	*Lake Erie*	*Lake Ontario*
January	25.13	52.44	7.15	37.91	22.67
February	27.53	44.84	6.79	28.18	18.61
March	41.88	74.10	7.16	30.76	27.23
April	51.06	84.79	7.60	31.32	28.74
May	59.12	82.09	6.36	25.91	25.55
June	52.24	64.38	4.58	20.62	18.12
July	39.82	54.18	4.60	17.35	14.80
August	42.02	59.91	3.87	14.28	12.58
September	53.03	68.23	3.50	19.29	12.29
October	44.93	71.21	3.29	19.68	16.85
November	39.49	63.29	3.41	21.22	18.38
December	29.22	63.65	5.79	27.67	20.73

The two types of Synthetic Time Series employed in this case:

1. Multivariate Contemporaneous Autoregressive (MCAR) Model of Order 1. The corresponding matrix of the delay order follows. :

	Lake Superior	*Lakes Michigan-Huron*	*Lake St. Clair*	*Lake Erie*	*Lake Ontario*
Lake Superior	1	0	0	0	0
Lake Michigan-Huron	0	1	0	0	0
Lake St. Clair	0	0	1	0	0
Lake Erie	0	0	0	1	0
Lake Ontario	0	0	0	0	1

The corresponding coefficients for the polynomial in the delay operator of the order as defined by the table above were computed considering the normalization by the standard deviation as described in Hipel and McLeod (1994), pg. 465. These coefficients were calculated using least-squares estimates and for the disturbance it was assumed Gaussian noise with $\mu = 0.00$. The AR parameter matrix, Φ_1, will be a diagonal one having the elements below as the non-zero elements:

Lake Superior	*Lakes Michigan-Huron*	*Lakes St. Clair*	*Lake Erie*	*Lake Ontario*
0.1477	0.2344	0.5382	0.1929	0.2808

From the coefficients above the Synthetic Mean Values for the Net Basin Supplies to the lakes were:

Month	*Lake Superior*	*Lakes Michigan-Huron*	*Lake St. Clair*	*Lake Erie*	*Lake Ontario*
January	-13.17	56.79	6.21	25.20	32.81
February	9.34	87.51	6.83	29.74	35.57
March	40.32	177.43	8.26	68.54	73.12
April	147.89	287.12	8.01	66.03	93.36
May	190.33	253.88	6.70	43.79	59.93
June	161.09	208.54	4.42	25.82	40.64
July	129.90	133.15	3.83	2.29	24.78
August	100.08	48.19	2.32	-15.56	5.08
September	74.64	28.76	1.59	-21.94	2.92
October	35.36	-2.47	1.62	-25.34	7.02
November	14.45	30.63	1.77	-9.91	16.25
December	-23.47	28.31	4.37	12.28	22.73

Synthetic Standard Deviation Values for the Net Basin Supplies to the lakes :

Month	*Lake Superior*	*Lakes Michigan-Huron*	*Lake St. Clair*	*Lake Erie*	*Lake Ontario*
January	25.06	51.19	7.12	37.31	22.66
February	27.84	44.74	6.61	28.11	19.00
March	40.41	73.54	6.79	30.26	27.30
April	50.54	84.81	7.47	30.72	28.52
May	57.65	81.97	6.36	26.25	24.37
June	52.19	64.44	4.40	20.41	17.86
July	40.55	55.49	4.72	16.91	14.95
August	43.33	59.16	3.90	14.17	12.07
September	51.23	66.08	3.38	18.30	12.29
October	45.26	72.53	3.32	19.75	17.23
November	38.85	62.20	3.40	21.17	18.16
December	29.58	63.46	5.81	28.35	20.97

Mean Monthly NBS and respective Standard Deviations for the Aggregate Lakes used in the Aggregation Method. Aggregated Lake 1 is composed by Lakes Michigan-Huron, St. Clair, Erie, and Ontario. Aggregated Lake 2 by Lakes Superior, Michigan-Huron, St. Clair, and Erie:

	Aggregated Lake 1		*Aggregated Lake 2*	
Month	*Mean*	*Standard Deviation*	*Mean*	*Standard Deviation*
January	121.00	94.23	75.02	90.67
February	159.64	79.61	133.42	80.49
March	327.35	113.47	294.56	115.42
April	454.53	125.42	509.06	134.99
May	364.30	115.99	494.70	133.03
June	279.41	89.46	399.86	110.20
July	164.05	77.77	269.17	92.74
August	41.97	76.00	135.03	96.02
September	11.33	85.47	83.05	109.27
October	-21.12	97.41	9.17	111.82
November	38.73	88.76	36.94	99.71
December	67.70	99.16	21.50	99.52

In the optimization part, first stage, the aggregated NBS for the downstream lake was given by means of multivariate linear regression that associated them with lake Superior values. The values employed are shown below.

Stage 1 : Monthly regression coefficients

Month	*A1*	*A0*
January	0.9336	0.3861
February	0.8650	0.0373
March	0.8772	-0.0043
April	0.9769	0.3533
May	0.9124	0.0436
June	0.9440	-0.3631
July	1.0652	0.2249
August	0.9461	0.0531
September	0.9447	0.1092
October	0.9943	-0.0127
November	1.0806	0.0128
December	0.9571	0.1424

Stage 2 : Monthly regression coefficients

Month	*A1*	*A0*
January	1.5207	0.0790
February	1.5735	0.0919
March	1.6299	-0.2185
April	1.6744	0.0291
May	1.6756	-0.0380
June	1.6804	0.1034
July	1.7704	0.0003
August	1.7508	0.2240
September	1.5857	-0.1968
October	1.7895	-0.1535
November	1.7990	-0.0077
December	1.7344	0.2441

2. Periodical Multivariate Contemporaneous Autoregressive Model of Order 1. The difference from the model above stems from the fact that instead of having just one MCAR model for the entire cycle, 12 MCAR's , each one relating to one month of the year, are generated. The corresponding input matrices follow, but as they are the same for all the months, we will show just one, that is exactly the same as the previous one:

	Lake Superior	*Lakes Michigan-Huron*	*Lake St. Clair*	*Lake Erie*	*Lake Ontario*
Lake Superior	1	0	0	0	0
Lake Michigan-Huron	0	1	0	0	0
Lake St. Clair	0	0	1	0	0
Lake Erie	0	0	0	1	0
Lake Ontario	0	0	0	0	1

For more information on these types of time series models the reader is referred to Hipel and McLeod (1994).

The corresponding coefficients for the polynomial in the delay operator of the order as defined by the table above were also computed considering the normalization by the standard deviation as mentioned in the previous item, but the computations were repeated for each month of the year. The criteria for the calculation of the coefficients and the disturbances is the same as in item 1. The table of the coefficients follows.

	Lake Superior	*Lakes Michigan-Huron*	*Lake St. Clair*	*Lake Erie*	*Lake Ontario*
January	0.0786	0.0072	0.1814	0.1536	0.0653
February	-0.0035	-0.1799	0.2047	0.2143	-0.0628
March	0.0216	0.1086	0.4163	-0.0262	-0.0408
April	0.0808	0.1011	0.2258	-0.1911	-0.1298
May	0.0989	0.0187	0.3009	0.0585	-0.0547
June	0.0989	0.1716	0.6235	0.0741	0.1779
July	-0.1152	0.2498	0.4849	0.0346	0.1649
August	0.0138	-0.0319	0.5592	-0.0186	0.0791
September	-0.0425	-0.1688	0.5242	0.1383	0.0028
October	0.0722	-0.0564	0.3614	0.0749	0.2130
November	-0.0630	-0.0217	0.2645	0.0708	0.2314
December	-0.1294	0.1170	0.1458	0.0245	0.0769

The use of the coefficients above led to the following Synthetic Mean Values for the Net Basin Supplies to the lakes:

Month	*Lake Superior*	*Lakes Michigan-Huron*	*Lake St. Clair*	*Lake Erie*	*Lake Ontario*
January	-13.45	54.1182	5.9859	22.10	30.92
February	7.66	86.30	7.24	28.67	34.67
March	42.94	177.84	8.37	68.64	72.78
April	149.51	287.04	7.87	66.50	92.58
May	191.80	257.53	6.61	44.90	61.15
June	157.36	208.12	4.08	27.22	41.78
July	131.11	132.26	3.67	2.47	25.14
August	101.06	49.92	1.91	-16.29	7.20
September	77.97	29.81	1.79	-22.71	2.96
October	38.06	0.13	1.43	-25.07	4.99
November	16.16	30.88	1.71	-9.20	16.11
December	-24.72	24.30	4.50	11.11	22.59

Synthetic Standard Deviation Values for the Net Basin Supplies to the lakes :

Month	*Lake Superior*	*Lakes Michigan-Huron*	*Lake St. Clair*	*Lake Erie*	*Lake Ontario*
January	25.77	52.78	7.19	36.5613	22.0903
February	27.86	46.04	6.70	28.40	19.29
March	42.64	76.76	7.66	31.47	27.90
April	50.76	82.98	7.65	31.32	28.59
May	58.47	85.18	6.43	24.88	25.21
June	51.87	66.28	4.57	20.57	18.93
July	39.82	53.03	4.27	17.41	14.6
August	40.90	60.71	3.68	14.34	12.66
September	53.39	67.77	3.29	19.14	12.23
October	46.43	69.81	3.29	19.66	16.48
November	38.22	63.68	3.40	20.31	18.23
December	29.33	63.79	6.00	27.97	20.63

Mean Monthly NBS and respective Standard Deviations for the Aggregate Lakes used in the Aggregation Method. Aggregated Lake 1 is composed by Lakes Michigan-Huron, St. Clair, Erie, and Ontario. Aggregated Lake 2 by Lakes Superior, Michigan-Huron, St. Clair, and Erie:

	Aggregated Lake 1		*Aggregated Lake 2*	
Month	*Mean*	*Standard Deviation*	*Mean*	*Standard Deviation*
January	113.13	98.9059	68.7547	90.38
February	156.89	81.63	129.88	81.32
March	327.63	124.96	297.79	134.27
April	453.99	122.21	510.92	132.35
May	370.18	122.61	500.83	135.19
June	281.20	93.02	396.78	113.19
July	163.55	76.67	269.52	84.01
August	42.74	76.87	136.60	98.33
September	11.86	85.28	86.87	116.10
October	-18.52	92.55	14.54	111.15
November	39.50	87.81	39.55	97.23
December	62.51	100.58	15.20	103.06

In the optimization part, second stage, the aggregated NBS for the upstream lake was given by means of multivariate linear regression that associated them with the lake Ontario values. Below we show the values employed.

Other information:

To change the level of each of the following lakes by one foot in a one month period, the necessary amount of water to be stored or withdrawn is:

Lake Superior	337,800 *cfs-months*
Lakes Michigan-Huron	480,800 *cfs-months*
Lake St. Clair	4,600 *cfs-months*
Lake Erie	105,200 *cfs-months*
Lake Ontario	80,000 *cfs-months*

Flow retardation due to ice formation and aquatic growth:

Below are presented the assumed values of flow retardation due to ice formation, ice jamming and aquatic growth (weed retardation). The last item applies only to Niagara river. Values in *tcfs-months*.

Month	*St.Clair River*	*Detroit River*	*Niagara River*	*St. Louis River*	*Lake Ontario Outlet*
January	30.25	12.10	40.00	32.11	6.77
February	37.68	10.27	47.00	25.19	9.87
March	16.29	4.32	34.00	7.61	5.49
April	3.15	1.16	49.00	4.47	0.00
May	0.19	0.00	0.00	0.00	0.00
June	0.00	0.00	15.00	0.00	0.00
July	0.00	0.00	51.00	0.00	0.00
August	0.00	0 00	39.00	0.00	0.00
September	0.00	0.00	26.00	0.00	0.00
October	0.00	0.00	16.00	0.00	0.00
November	0.00	0.00	4.00	0.00	0.00
December	3.75	5.33	0.00	10.35	0.42

Data for the lakes regulation:

Maximum and minimum flow limitations for the four most upstream lakes

Observation: Used maximum and minimum observed values as described in "Regulation of Great Lakes Water Levels", report to the IJC, Appendix G, 1973.

Maximum flow limitations. Values in tcfs-months.

◊ From lake Superior. Those values were used only during the optimization part. As demonstrated later on, during the simulation of the present operation, the maximum output is given as a function of the level. To obtain the values that follow, we computed the maximum possible outflow from the given formula assuming the highest admissible level, i.e., before the compensating works start to spill.

◊ From Michigan-Huron. See reference above, page G-42. Situation in Saint Clair River according to 1970 hydraulic conditions.

◊ From lake Saint Clair. Situation in Detroit River according to 1970 hydraulic conditions.

◊ From lake Erie. See referred appendix, page G-114. Situation in Niagara River. Assumed maximum during period 1900-1967, as specified in the regulations.

Month	*St. Marys River*	*St. Clair River*	*Detroit River*	*Niagara River*	*Lake Ontario Outlet*
January	85.00	210.00	210.00	251.00	220.00
February	85.00	210.00	210.00	251.00	260.00
March	85.00	210.00	210.00	251.00	280.00
April	85.00	210.00	210.00	251.00	310.00
May	142.38	210.00	210.00	251.00	310.00
June	142.38	210.00	210.00	251.00	310.00
July	142.38	210.00	210.00	251.00	310.00
August	142.38	210.00	210.00	251.00	310.00
September	142.38	210.00	210.00	251.00	310.00
October	142.38	210.00	210.00	251.00	310.00
November	142.38	210.00	210.00	251.00	310.00
December	85.00	210.00	210.00	251.00	310.00

Minimum flow limitations. Values in *tcfs-months*.

◊ From lake Superior. Those values were used only during the optimization part.

◊ From Michigan-Huron. See reference above, page G-42. Situation in Saint Clair River according to 1970 hydraulic conditions.

◊ From lake Saint Clair. Situation in Detroit River according to 1970 hydraulic conditions.

◊ From lake Erie. See referred appendix, page G-114. Situation in Niagara River. Assumed minimum during period 1900-1967, as specified in the regulations.

Month	*St. Marys River*	*St. Clair River*	*Detroit River*	*Niagara River*	*Lake Ontario Outlet*
January	55.00	152.00	152.00	116.00	210.00
February	55.00	152.00	152.00	116.00	207.00
March	55.00	152.00	152.00	116.00	204.00
April	55.00	152.00	152.00	116.00	188.00
May	55.00	152.00	152.00	116.00	188.00
June	55.00	152.00	152.00	116.00	190.00
July	55.00	152.00	152.00	116.00	193.00
August	55.00	152.00	152.00	116.00	193.00
September	55.00	152.00	152.00	116.00	193.00
October	55.00	152.00	152.00	116.00	193.00
November	55.00	152.00	152.00	116.00	198.00
December	55.00	152.00	152.00	116.00	210.00

Assumed target levels for the lakes:

Values computed according to what is presented in "Great Lakes Diversions and Consumptive Uses", Appendix B, Computer Models, 1981. This document specifies that the "desired" targets for the lakes levels are the estimates of the mean historical ones. Therefore, we computed the average ones for the basic data, presented in "Regulation of Great Lakes Water Levels", Appendix B, Coordinated Basic Data, Vol. 2, 1973.

The period, already mentioned, is from 1900 to 1973. As the information for the year of 1973 was incomplete, going until the month of June, the averages are for 1900-73 from January to June and 1900-72 to the remaining months.

Month	*Lake Superior*	*Lakes Michigan-Huron*	*Lake St. Clair*	*Lake Erie*	*Lake Ontario*
January	600.40	577.61	572.60	569.69	243.90
February	600.16	577.64	572.00	569.64	244.02
March	600.00	577.63	572.10	569.66	244.12
April	599.94	577.75	572.83	570.11	244.54
May	600.15	578.10	573.21	570.60	245.19
June	600.54	578.33	573.45	570.85	245.46
July	600.79	578.57	573.55	570.87	245.51
August	600.95	578.61	573.55	570.76	245.29
September	601.01	578.53	573.44	570.51	244.90
October	600.99	578.31	573.30	570.24	244.49
November	600.86	578.09	572.81	569.91	244.21
December	600.69	577.92	572.56	569.74	244.07

Maximum and Minimum suggested levels

Either in the simulation of the Present Operation and in the Proposed one, it was expected that the levels of the lakes remain within the reference values shown below. Whenever it was not possible to maintain the desired range of operation, due to drought or high net basin supplies, that moment was considered as an exceedance. Therefore, we have two types of exceedances: above the desired level and below it. Because of the peculiar characteristics of the lakes operation, levels above the desired ones do not necessarily imply in spill exception made to Lake Superior.

Level	Lake Superior	Lakes Michigan-Huron	Lake St. Clair	Lake Erie	Lake Ontario
Maximum	601.86	581.59	576.56	573.63	247.32
Minimum	598.68	575.13	570.84	568.02	241.66
Range	3.18	6.46	5.72	5.61	5.66

State Discretization:

For optimization purposes, a minimum level was tentatively found to serve as a reference. The levels shown below represent the empty reservoir state for the five lakes. The justification for that is the minimal quantity of volume that affects level and flow variations when relative to the absolute total volume considered. The stochastic dynamic programming procedure requires that all the variables assume the same unit. Therefore, it was established that volume, given in *tcf,* became the working unit For the sake of consistency, the decision is the volume discharged in the time-step, a month.

Lake Superior	*Lake Michigan*	*Lake St. Clair*	*Lake Erie*	*Lake Ontario*
596.136	569.962	566.264	563.532	237.132

Note: Values given in *ft*.

Also, it was admitted that the volumes assumed values beyond those corresponding to the maximum allowable level. If only the desirable ranges were employed, the optimization would not converge because of occurrence of infeasible states. This is necessary to consider when all the optimal paths pass through a state that is technically considered as infeasible. In the first optimization stage, for lake Superior, the amount of volume that spilled was considered as externalized from the system and, as a result, lost. For the other lakes, oscillation beyond the maximum level, was not considered. In the second stage, the upstream aggregated lake, that includes Lake Superior, did not externalize the spill, due to the relative contribution of Lake Superior to the total aggregated volume. It can be inferred from the table below that when all the levels are at the minimum desirable level, shown above, the lake already has accumulated some minimum volume. There is some margin of operation beyond the maximum desirable, too. Had not these measures taken, the iterations would not converge.

First Stage:

◊ *Lake Superior*

Month	Volume at Desirable Level	Minimum Volume at Desirable Level	Maximum Total Capacity Considered in the Optimization
January	2301.70	5178.90	6329.70
February	2097.50	4719.50	5768.20
March	2301.70	5178.90	6329.70
April	2227.50	5011.80	6125.50
May	2301.70	5178.90	6329.70
June	2227.50	5011.80	6125.50
July	2301.70	5178.90	6329.70
August	2301.70	5178.90	6329.70
September	2227.50	5011.80	6125.50
October	2301.70	5178.90	6329.70
November	2227.50	5011.80	6125.50
December	2301.70	5178.90	6329.70

◊ *Aggregated Lake:*

Month	Volume at Desirable Level	Minimum Volume at Desirable Level	Maximum Total Capacity Considered in the Optimization
January	8946.40	20129.00	24603.00
February	8152.80	18344.00	22420.00
March	8946.40	20129.00	24603.00
April	8657.80	19480.00	23809.00
May	8946.40	20129.00	24603.00
June	8657.80	19480.00	23809.00
July	8946.40	20129.00	24603.00
August	8946.40	20129.00	24603.00
September	8657.80	19480.00	23809.00
October	8946.40	20129.00	24603.00
November	8657.80	19480.00	23809.00
December	8946.40	20129.00	24603.00

Second Stage:

◊ *Aggregated Lake:*

Month	Volume at Desirable Level	Minimum Volume at Desirable Level	Maximum Total Capacity Considered in the Optimization
January	10278.00	23125.00	28264.00
February	9366.00	21074.00	25757.00
March	10278.00	23125.00	28264.00
April	9946.00	22379.00	27352.00
May	10278.00	23125.00	28264.00
June	9946.00	22379.00	27352.00
July	10278.00	23125.00	28264.00
August	10278.00	23125.00	28264.00
September	9946.00	22379.00	27352.00
October	10278.00	23125.00	28264.00
November	9946.00	22379.00	27352.00
December	10278.00	23125.00	28264.00

◊ *Lake Ontario:*

Month	*Volume at Desirable Level*	*Minimum Volume at Desirable Level*	*Maximum Total Capacity Considered in the Optimization*
January	970.22	2183.00	2668.10
February	884.16	1989.30	2431.40
March	970.22	2183.00	2668.10
April	938.93	2112.60	2582.00
May	970.22	2183.00	2668.10
June	938.93	2112.60	2582.00
July	970.22	2183.00	2668.10
August	970.22	2183.00	2668.10
September	938.93	2112.60	2582.00
October	970.22	2183.00	2668.10
November	938.93	2112.60	2582.00
December	970.22	2183.00	2668.10

Appendix D

North American Great Lakes Study Case

Transition Probabilities for the NBS

Appendix D - Transition Probabilities for the Net Basin Supplies:

These transition Probabilities were computed only for the 2-level optimization part, i.e., always considering one lake aggregated and one not. Below are presented the two studied cases: the MCAR 1 and the Periodical MCAR 1. They are subdivided into two other subparts, Stage 1 and Stage 2. For instance, in Stage 1, the maximum and minimum values registered for the synthetic NBS for Lake Superior are subdivided into 9 classes and their median values is then employed with its respective probability of occurrence. The same is done for the aggregated NBS for the downstream lake that includes Lakes Michigan-Huron, St. Clair, Erie and Ontario. In Stage 2, the order is reversed, i.e., the aggregated lake now is composed by Lakes Superior, Michigan-Huron, St. Clair and Erie and the downstream lake is Lake Ontario.

MCAR 1:

Lake Superior

January

Probabilities	0.0000	0.0010	0.0040	0.0040	0.0020	0.0010	0.0000	0.0000	0.0000
	0.0020	0.0050	0.0170	0.0180	0.0100	0.0040	0.0000	0.0000	0.0000
	0.0000	0.0060	0.0370	0.0560	0.0500	0.0170	0.0000	0.0000	0.0000
	0.0020	0.0090	0.0540	0.1030	0.0680	0.0330	0.0060	0.0000	0.0000
	0.0020	0.0080	0.0440	0.0900	0.0930	0.0380	0.0100	0.0010	0.0000
	0.0010	0.0050	0.0220	0.0480	0.0420	0.0210	0.0030	0.0000	0.0000
	0.0000	0.0000	0.0070	0.0190	0.0160	0.0120	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0010	0.0050	0.0020	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	-48.60	-33.11	-4.12	26.06	51.85	0.00	0.00	0.00
	-64.41	-50.38	-25.65	2.51	20.63	44.37	0.00	0.00	0.00
	0.00	-47.08	-21.51	3.19	26.41	45.86	0.00	0.00	0.00
	-64.52	-43.93	-20.23	0.37	24.38	47.07	71.01	0.00	0.00
	-73.39	-42.64	-20.67	0.84	25.75	46.37	75.15	116.80	0.00
	-88.55	-51.87	-21.27	4.63	25.61	48.00	73.79	0.00	0.00

0.00	0.00	-26.75	-2.94	23.93	54.49	0.00	0.00	0.00
0.00	0.00	0.00	9.62	25.59	50.02	0.00	0.00	0.00
0.00	0.00	0.00	3.78	0.00	0.00	0.00	0.00	0.00

February

Probabilities

0.0010	0.0010	0.0020	0.0010	0.0020	0.0000	0.0000	0.0000	0.0000
0.0020	0.0040	0.0070	0.0080	0.0060	0.0070	0.0000	0.0000	0.0000
0.0050	0.0190	0.0250	0.0670	0.0410	0.0180	0.0070	0.0020	0.0010
0.0050	0.0280	0.0580	0.1070	0.0830	0.0430	0.0150	0.0010	0.0000
0.0010	0.0100	0.0520	0.0890	0.0800	0.0330	0.0190	0.0020	0.0000
0.0000	0.0090	0.0130	0.0410	0.0310	0.0240	0.0100	0.0000	0.0000
0.0000	0.0000	0.0030	0.0060	0.0080	0.0020	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values

-60.21	-27.00	8.32	37.76	56.20	0.00	0.00	0.00	0.00
-76.68	-26.58	2.02	31.13	62.91	87.26	0.00	0.00	0.00
-48.98	-29.72	4.15	27.06	59.11	87.50	120.24	139.25	168.33
-55.09	-24.09	6.49	30.34	61.13	88.02	118.46	145.27	0.00
-61.19	-22.69	0.54	28.52	58.99	89.67	117.99	154.92	0.00
0.00	-31.04	2.22	32.97	60.78	88.75	121.01	0.00	0.00
0.00	0.00	9.71	39.19	69.02	86.56	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	113.12	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

March

Probabilities

0.0000	0.0020	0.0010	0.0020	0.0060	0.0020	0.0010	0.0000	0.0000
0.0000	0.0040	0.0140	0.0190	0.0190	0.0090	0.0050	0.0010	0.0000
0.0030	0.0050	0.0240	0.0430	0.0510	0.0220	0.0080	0.0040	0.0000
0.0020	0.0140	0.0380	0.0780	0.0930	0.0590	0.0310	0.0040	0.0000
0.0020	0.0040	0.0320	0.0610	0.0720	0.0520	0.0260	0.0020	0.0000
0.0020	0.0050	0.0120	0.0240	0.0350	0.0310	0.0130	0.0050	0.0000
0.0000	0.0020	0.0030	0.0070	0.0240	0.0120	0.0040	0.0000	0.0000

	0.0000	0.0010	0.0000	0.0020	0.0010	0.0000	0.0010	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000

Median Values	0.00	49.49	83.80	121.81	157.40	187.12	212.90	0.00	0.00
	0.00	27.08	87.01	128.38	152.23	193.63	223.67	255.98	0.00
	14.03	39.98	83.00	123.01	157.39	188.54	215.49	273.47	0.00
	0.86	47.13	85.01	123.66	154.29	190.81	228.85	263.74	0.00
	0.34	49.26	89.14	120.38	159.19	192.58	231.41	261.07	0.00
	5.10	31.65	84.21	117.65	152.79	188.31	223.30	260.16	0.00
	0.00	34.05	97.06	114.89	156.93	196.12	224.35	0.00	0.00
	0.00	51.92	0.00	115.28	162.18	0.00	222.77	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	187.04	0.00	0.00	0.00

April

Probabilities	0.0000	0.0040	0.0030	0.0020	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0040	0.0120	0.0160	0.0040	0.0010	0.0000	0.0000	0.0000
	0.0070	0.0180	0.0280	0.0440	0.0200	0.0070	0.0000	0.0000	0.0000
	0.0070	0.0260	0.0610	0.0600	0.0570	0.0220	0.0030	0.0000	0.0000
	0.0070	0.0200	0.0750	0.1030	0.0590	0.0310	0.0030	0.0020	0.0010
	0.0040	0.0210	0.0410	0.0620	0.0440	0.0110	0.0050	0.0000	0.0000
	0.0030	0.0060	0.0200	0.0250	0.0250	0.0080	0.0010	0.0010	0.0000
	0.0000	0.0010	0.0020	0.0060	0.0030	0.0030	0.0010	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	107.21	134.05	192.51	0.00	0.00	0.00	0.00	0.00
	0.00	102.07	144.26	186.43	226.61	287.22	0.00	0.00	0.00
	62.56	105.67	151.96	194.60	231.28	281.34	0.00	0.00	0.00
	56.78	104.92	147.67	197.24	230.69	283.73	320.20	0.00	0.00
	66.21	102.59	150.48	196.69	237.70	279.82	322.23	370.60	405.33
	60.78	108.69	150.90	200.11	230.25	280.55	320.15	0.00	0.00
	75.73	119.26	145.81	197.77	241.88	288.79	316.17	386.79	0.00
	0.00	108.91	144.63	197.39	238.21	279.22	314.34	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

May

Probabilities

0.0010	0.0000	0.0050	0.0130	0.0060	0.0030	0.0000	0.0000	0.0000
0.0000	0.0090	0.0160	0.0230	0.0310	0.0170	0.0020	0.0020	0.0000
0.0010	0.0130	0.0350	0.0540	0.0790	0.0460	0.0110	0.0030	0.0000
0.0030	0.0080	0.0500	0.0830	0.0940	0.0450	0.0310	0.0040	0.0000
0.0000	0.0100	0.0290	0.0540	0.0480	0.0470	0.0190	0.0040	0.0010
0.0000	0.0010	0.0040	0.0180	0.0210	0.0300	0.0080	0.0010	0.0000
0.0000	0.0000	0.0030	0.0050	0.0030	0.0010	0.0000	0.0010	0.0000
0.0000	0.0000	0.0000	0.0010	0.0000	0.0010	0.0010	0.0000	0.0000
0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values

25.60	0.00	95.99	141.47	170.06	211.21	0.00	0.00	0.00
0.00	60.06	99.45	134.67	165.92	212.17	240.90	285.05	0.00
-8.09	60.45	100.95	134.63	172.14	201.72	249.70	295.05	0.00
25.24	44.35	101.64	133.44	171.95	210.19	246.21	278.87	0.00
0.00	61.86	101.81	133.43	174.97	206.80	240.19	288.31	313.99
0.00	35.57	99.52	132.51	173.81	212.53	257.54	287.26	0.00
0.00	0.00	109.19	142.27	160.59	216.14	0.00	287.43	0.00
0.00	0.00	0.00	145.92	0.00	224.96	271.71	0.00	0.00
0.00	0.00	73.77	0.00	0.00	0.00	0.00	0.00	0.00

June

Probabilities

0.0000	0.0000	0.0030	0.0010	0.0010	0.0000	0.0000	0.0000	0.0000
0.0020	0.0050	0.0120	0.0130	0.0060	0.0020	0.0010	0.0000	0.0000
0.0010	0.0160	0.0360	0.0420	0.0340	0.0090	0.0050	0.0000	0.0000
0.0050	0.0260	0.0560	0.0870	0.0530	0.0190	0.0040	0.0010	0.0000
0.0040	0.0180	0.0570	0.0910	0.0810	0.0230	0.0060	0.0010	0.0010
0.0000	0.0130	0.0310	0.0650	0.0560	0.0170	0.0070	0.0010	0.0000
0.0000	0.0030	0.0150	0.0250	0.0220	0.0070	0.0000	0.0000	0.0000
0.0000	0.0000	0.0030	0.0030	0.0060	0.0030	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000

Median Values

0.00	0.00	106.23	131.88	175.38	0.00	0.00	0.00	0.00
31.77	69.20	88.28	126.87	154.79	183.90	232.34	0.00	0.00
29.96	67.12	94.65	129.04	159.90	181.52	219.28	0.00	0.00
25.95	62.59	97.96	127.62	155.58	194.33	219.13	263.95	0.00
16.25	63.26	95.83	126.30	155.58	187.32	225.13	255.73	279.01
0.00	64.75	96.03	127.04	158.27	190.24	220.88	254.88	0.00
0.00	65.75	94.20	125.12	160.91	196.41	0.00	0.00	0.00
0.00	0.00	94.42	124.09	155.51	199.78	0.00	0.00	0.00
0.00	0.00	0.00	0.00	168.91	0.00	0.00	0.00	0.00

July

Probabilities

0.0000	0.0020	0.0040	0.0030	0.0020	0.0010	0.0000	0.0000	0.0000
0.0020	0.0070	0.0100	0.0260	0.0200	0.0150	0.0000	0.0010	0.0000
0.0020	0.0050	0.0380	0.0630	0.0540	0.0420	0.0060	0.0020	0.0010
0.0010	0.0190	0.0430	0.0770	0.0940	0.0650	0.0240	0.0040	0.0000
0.0010	0.0110	0.0390	0.0570	0.0720	0.0590	0.0180	0.0030	0.0000
0.0000	0.0000	0.0060	0.0210	0.0320	0.0150	0.0050	0.0010	0.0000
0.0000	0.0000	0.0000	0.0070	0.0070	0.0080	0.0010	0.0000	0.0000
0.0000	0.0000	0.0000	0.0010	0.0010	0.0000	0.0010	0.0000	0.0000
0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values

0.00	13.21	49.69	79.93	105.76	131.60	0.00	0.00	0.00
-14.96	22.58	49.24	78.89	110.24	147.99	0.00	198.84	0.00
-6.24	16.65	50.64	78.46	111.46	139.28	180.08	215.69	229.47
-27.41	17.63	47.89	77.12	110.61	142.34	171.99	210.52	0.00
-39.64	17.45	49.14	79.23	108.11	139.21	171.47	218.64	0.00
0.00	0.00	53.34	80.73	111.40	148.64	179.28	215.32	0.00
0.00	0.00	0.00	82.94	108.66	141.52	170.38	0.00	0.00
0.00	0.00	0.00	81.70	111.86	0.00	195.75	0.00	0.00
0.00	0.00	0.00	69.69	0.00	0.00	0.00	0.00	0.00

August

Probabilities

0.0000	0.0010	0.0000	0.0000	0.0040	0.0010	0.0000	0.0000	0.0000

0.0010	0.0030	0.0090	0.0120	0.0130	0.0060	0.0000	0.0000	0.0000
0.0010	0.0050	0.0230	0.0490	0.0380	0.0180	0.0050	0.0010	0.0000
0.0020	0.0190	0.0410	0.0630	0.0780	0.0390	0.0100	0.0040	0.0000
0.0030	0.0050	0.0530	0.0690	0.0810	0.0530	0.0130	0.0040	0.0010
0.0010	0.0050	0.0290	0.0560	0.0610	0.0370	0.0120	0.0040	0.0000
0.0000	0.0010	0.0060	0.0160	0.0150	0.0120	0.0050	0.0000	0.0000
0.0000	0.0000	0.0020	0.0030	0.0030	0.0030	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000

Median Values	0.00	-12.44	0.00	0.00	88.58	113.60	0.00	0.00	0.00
	-69.19	-27.25	17.51	46.51	101.19	134.73	0.00	0.00	0.00
	-53.27	-22.24	16.45	49.40	94.61	130.58	171.04	218.03	0.00
	-86.17	-17.93	16.55	53.46	89.16	125.64	168.21	207.92	0.00
	-56.48	-20.44	18.29	52.73	90.00	129.75	172.37	214.59	233.57
	-72.78	-35.90	17.88	54.44	90.06	129.15	164.91	207.18	0.00
	0.00	-7.90	12.85	53.30	91.28	125.30	159.07	0.00	0.00
	0.00	0.00	9.98	55.75	86.92	130.75	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	158.59	0.00	0.00

September

Probabilities	0.0000	0.0020	0.0010	0.0030	0.0020	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0050	0.0110	0.0130	0.0100	0.0000	0.0000	0.0000	0.0000
	0.0020	0.0050	0.0280	0.0570	0.0410	0.0240	0.0050	0.0010	0.0000
	0.0020	0.0230	0.0500	0.0780	0.0690	0.0350	0.0100	0.0010	0.0000
	0.0030	0.0160	0.0470	0.0830	0.0760	0.0490	0.0160	0.0030	0.0000
	0.0040	0.0060	0.0300	0.0450	0.0490	0.0260	0.0080	0.0010	0.0000
	0.0000	0.0010	0.0050	0.0130	0.0100	0.0130	0.0010	0.0020	0.0010
	0.0000	0.0000	0.0020	0.0020	0.0030	0.0040	0.0010	0.0010	0.0000
	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	-38.97	-7.00	18.82	58.08	0.00	0.00	0.00	0.00
	0.00	-54.37	-7.79	20.12	54.19	0.00	0.00	0.00	0.00
	-73.97	-50.73	-15.50	21.04	58.35	88.14	133.83	146.70	0.00

-92.83	-42.23	-11.37	21.02	56.91	88.87	117.63	145.48	0.00
-92.35	-37.49	-6.78	22.57	53.16	87.81	118.27	167.11	0.00
-80.42	-39.41	-10.22	23.76	55.72	86.29	125.28	163.86	0.00
0.00	-42.96	-6.10	23.00	57.42	86.77	111.08	171.41	180.54
0.00	0.00	-14.02	11.97	57.59	81.56	119.48	149.29	0.00
0.00	0.00	0.00	32.87	0.00	0.00	0.00	0.00	0.00

October

Probabilities	0.0000	0.0000	0.0040	0.0060	0.0010	0.0000	0.0000	0.0000	0.0000
	0.0030	0.0070	0.0110	0.0200	0.0110	0.0060	0.0000	0.0000	0.0000
	0.0020	0.0150	0.0370	0.0510	0.0440	0.0210	0.0030	0.0010	0.0000
	0.0050	0.0240	0.0520	0.0750	0.0830	0.0430	0.0100	0.0030	0.0000
	0.0030	0.0110	0.0420	0.0760	0.0750	0.0380	0.0130	0.0010	0.0010
	0.0010	0.0050	0.0220	0.0470	0.0430	0.0270	0.0060	0.0000	0.0000
	0.0000	0.0030	0.0050	0.0090	0.0150	0.0090	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0040	0.0030	0.0020	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	0.00	-29.23	4.53	49.11	0.00	0.00	0.00	0.00
	-81.47	-52.76	-19.92	2.88	37.80	64.03	0.00	0.00	0.00
	-76.81	-54.99	-26.54	4.18	32.95	62.15	87.20	116.20	0.00
	-83.46	-50.39	-23.54	5.04	33.46	61.47	97.14	127.69	0.00
	-78.52	-50.06	-21.53	5.03	33.52	63.36	92.45	118.43	142.86
	-82.63	-59.62	-22.52	6.61	34.00	62.43	88.03	0.00	0.00
	0.00	-49.93	-20.96	3.97	27.50	60.41	0.00	0.00	0.00
	0.00	0.00	0.00	-0.90	33.61	61.55	0.00	0.00	0.00
	0.00	0.00	0.00	4.16	0.00	0.00	0.00	0.00	0.00

November

Probabilities	0.0000	0.0020	0.0030	0.0030	0.0030	0.0020	0.0010	0.0000	0.0000
	0.0010	0.0070	0.0080	0.0190	0.0180	0.0080	0.0040	0.0000	0.0000
	0.0040	0.0110	0.0250	0.0410	0.0540	0.0320	0.0050	0.0010	0.0000
	0.0010	0.0150	0.0410	0.0800	0.0890	0.0390	0.0180	0.0060	0.0000

0.0030	0.0100	0.0370	0.0730	0.0810	0.0490	0.0180	0.0040	0.0000
0.0010	0.0050	0.0150	0.0360	0.0420	0.0360	0.0100	0.0000	0.0010
0.0000	0.0020	0.0050	0.0030	0.0110	0.0080	0.0020	0.0010	0.0000
0.0000	0.0000	0.0000	0.0000	0.0020	0.0030	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000

Median Values	0.00	-76.21	-61.69	-39.55	-17.24	13.76	26.60	0.00	0.00
	-98.35	-75.12	-57.59	-36.06	-12.93	3.79	25.11	0.00	0.00
	-102.19	-81.65	-58.01	-37.61	-17.94	8.26	26.65	44.23	0.00
	-93.78	-79.49	-60.22	-35.82	-17.16	3.55	23.21	52.21	0.00
	-96.00	-81.96	-56.44	-34.91	-15.07	6.68	31.13	54.31	0.00
	-94.53	-79.11	-59.96	-36.23	-17.06	3.44	31.15	0.00	64.61
	0.00	-81.71	-52.96	-36.87	-17.05	2.23	34.24	47.27	0.00
	0.00	0.00	0.00	0.00	-18.13	5.36	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	2.08	0.00	0.00	0.00

December

Probabilities	0.0000	0.0010	0.0010	0.0030	0.0030	0.0020	0.0000	0.0000	0.0000
	0.0000	0.0020	0.0040	0.0180	0.0160	0.0100	0.0020	0.0000	0.0000
	0.0000	0.0070	0.0310	0.0330	0.0360	0.0220	0.0040	0.0010	0.0000
	0.0070	0.0100	0.0460	0.0620	0.0790	0.0370	0.0110	0.0030	0.0000
	0.0020	0.0210	0.0470	0.0840	0.0840	0.0380	0.0210	0.0030	0.0000
	0.0020	0.0090	0.0270	0.0550	0.0450	0.0280	0.0100	0.0010	0.0010
	0.0010	0.0050	0.0080	0.0150	0.0200	0.0040	0.0050	0.0000	0.0000
	0.0000	0.0010	0.0020	0.0050	0.0030	0.0010	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000

Median Values	0.00	-58.98	-36.14	-21.71	-2.47	16.32	0.00	0.00	0.00
	0.00	-58.07	-34.29	-22.96	-4.79	16.99	39.99	0.00	0.00
	0.00	-54.09	-38.49	-20.82	-4.44	18.98	31.33	56.20	0.00
	-80.05	-59.13	-40.02	-22.59	-3.05	14.58	31.31	45.14	0.00
	-74.55	-55.51	-38.83	-21.27	-3.81	13.63	34.84	53.07	0.00
	-71.90	-56.90	-40.83	-22.44	-4.05	14.30	30.72	46.92	63.64

-75.35	-64.77	-38.31	-24.71	-2.07	22.52	37.51	0.00	0.00
0.00	-52.76	-37.53	-25.28	-7.84	9.63	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	26.73	0.00	0.00

Aggregated Lake

January

Probabilities	0.0000	0.0000	0.0000	0.0010	0.0010	0.0000	0.0000	0.0000	0.0000
	0.0010	0.0020	0.0080	0.0100	0.0050	0.0010	0.0000	0.0000	0.0000
	0.0000	0.0010	0.0180	0.0390	0.0400	0.0160	0.0020	0.0000	0.0000
	0.0000	0.0060	0.0430	0.1530	0.1460	0.0430	0.0090	0.0000	0.0000
	0.0000	0.0010	0.0280	0.1000	0.1200	0.0750	0.0160	0.0010	0.0000
	0.0000	0.0010	0.0040	0.0170	0.0420	0.0340	0.0020	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0020	0.0060	0.0030	0.0020	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	0.00	0.00	71.76	178.23	0.00	0.00	0.00	0.00
	-119.55	-56.79	32.33	107.70	173.45	227.99	0.00	0.00	0.00
	0.00	-18.21	26.70	111.34	187.84	261.40	339.89	0.00	0.00
	0.00	-48.77	29.56	115.15	179.64	254.40	332.80	0.00	0.00
	0.00	-58.63	39.54	112.38	182.77	256.33	317.64	395.89	0.00
	0.00	-18.97	48.23	121.42	192.86	264.12	367.77	0.00	0.00
	0.00	0.00	0.00	111.91	204.80	250.71	327.01	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	291.14	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

February

Probabilities	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0030	0.0030	0.0040	0.0000	0.0010	0.0000	0.0000	0.0000
	0.0000	0.0060	0.0250	0.0370	0.0270	0.0050	0.0010	0.0000	0.0000
	0.0010	0.0090	0.0770	0.1210	0.0850	0.0230	0.0060	0.0000	0.0000
	0.0000	0.0080	0.0590	0.1450	0.1020	0.0410	0.0050	0.0000	0.0000

	0.0000	0.0050	0.0140	0.0410	0.0720	0.0350	0.0060	0.0000	0.0000
	0.0000	0.0000	0.0030	0.0080	0.0100	0.0090	0.0010	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	0.00	0.00	253.66	0.00	0.00	0.00	0.00	0.00
	0.00	115.69	142.14	292.94	0.00	530.79	0.00	0.00	0.00
	0.00	86.63	185.32	268.99	387.74	533.39	613.06	0.00	0.00
	9.49	110.90	205.01	291.65	384.43	483.99	590.88	0.00	0.00
	0.00	109.24	192.06	292.63	404.57	487.70	621.09	0.00	0.00
	0.00	109.01	183.48	295.57	395.76	505.11	619.50	0.00	0.00
	0.00	0.00	164.62	309.93	394.18	472.69	590.87	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	511.65	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

March

Probabilities	0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0010	0.0050	0.0190	0.0030	0.0030	0.0000	0.0000	0.0000
	0.0000	0.0080	0.0400	0.0670	0.0470	0.0180	0.0010	0.0000	0.0000
	0.0010	0.0110	0.0470	0.1350	0.1200	0.0390	0.0030	0.0010	0.0000
	0.0000	0.0020	0.0350	0.0920	0.1100	0.0440	0.0130	0.0000	0.0000
	0.0000	0.0010	0.0090	0.0300	0.0430	0.0290	0.0030	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0030	0.0090	0.0060	0.0000	0.0010	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	0.00	0.00	0.00	516.41	0.00	0.00	0.00	0.00
	0.00	186.30	317.94	404.10	461.03	618.22	0.00	0.00	0.00
	0.00	149.41	307.32	387.99	505.59	629.59	712.39	0.00	0.00
	34.63	164.62	285.40	402.62	509.51	629.70	743.29	852.21	0.00
	0.00	161.73	290.28	402.83	505.24	628.82	727.72	0.00	0.00
	0.00	205.43	305.66	407.12	500.75	636.01	767.19	0.00	0.00
	0.00	0.00	0.00	420.86	522.60	626.26	0.00	865.03	0.00

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

April

Probabilities	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0010	0.0030	0.0060	0.0050	0.0070	0.0010	0.0000	0.0000	0.0000
	0.0000	0.0080	0.0340	0.0490	0.0380	0.0060	0.0010	0.0000	0.0000
	0.0010	0.0070	0.0610	0.1310	0.1070	0.0320	0.0070	0.0000	0.0000
	0.0000	0.0070	0.0330	0.1140	0.1100	0.0590	0.0080	0.0020	0.0000
	0.0000	0.0000	0.0110	0.0410	0.0480	0.0350	0.0040	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0050	0.0070	0.0040	0.0040	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0010	0.0010	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	0.00	200.16	0.00	0.00	0.00	0.00	0.00	0.00
	-9.51	74.28	247.75	339.40	393.67	551.02	0.00	0.00	0.00
	0.00	104.49	222.81	309.75	411.28	523.82	646.45	0.00	0.00
	17.36	94.15	209.31	311.50	412.92	506.11	633.94	0.00	0.00
	0.00	100.15	226.23	321.91	418.96	523.86	621.91	728.51	0.00
	0.00	0.00	234.03	334.71	418.51	519.43	633.81	0.00	0.00
	0.00	0.00	0.00	359.53	421.34	523.47	608.45	0.00	0.00
	0.00	0.00	0.00	267.44	388.01	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

May

Probabilities	0.0000	0.0000	0.0010	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0010	0.0090	0.0090	0.0060	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0080	0.0270	0.0470	0.0460	0.0180	0.0000	0.0000	0.0000
	0.0000	0.0060	0.0480	0.1310	0.1010	0.0510	0.0080	0.0010	0.0000
	0.0000	0.0030	0.0350	0.1050	0.1180	0.0480	0.0090	0.0000	0.0000
	0.0000	0.0010	0.0070	0.0380	0.0540	0.0280	0.0080	0.0010	0.0000
	0.0000	0.0010	0.0010	0.0040	0.0070	0.0050	0.0060	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0020	0.0000	0.0000	0.0000	0.0000

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values

0.00	0.00	172.49	224.56	0.00	0.00	0.00	0.00	0.00
0.00	70.11	144.32	237.93	318.14	0.00	0.00	0.00	0.00
0.00	82.01	164.74	227.44	314.43	384.98	0.00	0.00	0.00
0.00	59.28	152.01	230.62	306.06	392.51	473.41	582.00	0.00
0.00	91.92	154.43	233.45	314.56	391.03	459.26	0.00	0.00
0.00	74.34	160.29	254.37	311.12	388.12	468.64	545.35	0.00
0.00	55.06	113.62	254.96	332.63	401.19	466.54	0.00	0.00
0.00	0.00	0.00	0.00	303.01	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

June

Probabilities

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0020	0.0030	0.0080	0.0060	0.0010	0.0000	0.0000	0.0000
0.0010	0.0060	0.0230	0.0400	0.0490	0.0090	0.0000	0.0000	0.0000
0.0010	0.0080	0.0420	0.1010	0.1320	0.0420	0.0090	0.0000	0.0000
0.0000	0.0050	0.0220	0.0920	0.1300	0.0710	0.0140	0.0000	0.0000
0.0000	0.0000	0.0090	0.0360	0.0540	0.0400	0.0090	0.0020	0.0000
0.0000	0.0000	0.0000	0.0060	0.0130	0.0080	0.0040	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0020	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	-14.33	45.01	111.08	167.35	248.58	0.00	0.00	0.00
-101.34	-16.02	53.84	118.79	175.57	245.16	0.00	0.00	0.00
-96.34	-10.28	48.12	118.84	177.45	252.94	316.27	0.00	0.00
0.00	-37.76	48.40	116.77	184.31	254.41	329.35	0.00	0.00
0.00	0.00	55.05	118.55	191.85	258.43	323.65	410.98	0.00
0.00	0.00	0.00	129.27	204.67	272.59	316.47	0.00	0.00
0.00	0.00	0.00	0.00	0.00	250.20	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

July

Probabilities

0.0000	0.0000	0.0000	0.0020	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0010	0.0010	0.0050	0.0110	0.0030	0.0000	0.0000	0.0000
0.0000	0.0010	0.0110	0.0330	0.0370	0.0160	0.0000	0.0010	0.0000
0.0010	0.0020	0.0200	0.0790	0.1200	0.0530	0.0070	0.0010	0.0000
0.0000	0.0000	0.0210	0.0980	0.1460	0.0940	0.0230	0.0020	0.0000
0.0000	0.0010	0.0080	0.0340	0.0600	0.0510	0.0180	0.0010	0.0000
0.0000	0.0000	0.0010	0.0050	0.0180	0.0090	0.0030	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0010	0.0010	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values

0.00	0.00	0.00	-52.89	0.00	0.00	0.00	0.00	0.00
0.00	-170.57	-79.26	-3.63	38.87	104.26	0.00	0.00	0.00
0.00	-192.12	-96.97	-37.68	41.04	114.04	0.00	239.59	0.00
-260.94	-161.60	-88.76	-21.61	44.56	107.23	181.22	257.91	0.00
0.00	0.00	-85.74	-18.98	46.22	114.94	178.04	265.55	0.00
0.00	-149.21	-90.63	-22.04	46.59	121.73	189.94	261.75	0.00
0.00	0.00	-74.93	-12.13	39.76	139.81	223.17	0.00	0.00
0.00	0.00	0.00	0.00	62.55	130.92	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

August

Probabilities

0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0050	0.0000	0.0000	0.0000	0.0000	0.0000
0.0010	0.0080	0.0110	0.0200	0.0170	0.0050	0.0000	0.0000	0.0000
0.0000	0.0120	0.0470	0.0800	0.0780	0.0350	0.0030	0.0010	0.0000
0.0030	0.0080	0.0520	0.1320	0.1220	0.0640	0.0120	0.0000	0.0000
0.0000	0.0030	0.0220	0.0610	0.0720	0.0460	0.0210	0.0020	0.0000
0.0000	0.0000	0.0030	0.0130	0.0160	0.0130	0.0060	0.0000	0.0000
0.0000	0.0000	0.0000	0.0010	0.0020	0.0020	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values

0.00	0.00	-74.73	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	-50.78	0.00	0.00	0.00	0.00	0.00
-238.95	-171.10	-85.07	-36.35	46.16	109.49	0.00	0.00	0.00
0.00	-178.43	-102.09	-29.93	43.10	110.68	193.21	238.76	0.00
-225.55	-181.85	-93.50	-27.54	37.76	115.92	180.52	0.00	0.00
0.00	-148.86	-90.98	-20.72	43.52	109.31	180.25	248.81	0.00
0.00	0.00	-91.26	-18.51	47.59	113.36	166.68	0.00	0.00
0.00	0.00	0.00	-34.43	70.78	112.53	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

September

Probabilities

0.0000	0.0000	0.0010	0.0020	0.0010	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0100	0.0100	0.0090	0.0020	0.0000	0.0000	0.0000
0.0000	0.0030	0.0240	0.0440	0.0510	0.0080	0.0060	0.0000	0.0000
0.0030	0.0120	0.0500	0.1040	0.1010	0.0340	0.0070	0.0010	0.0000
0.0010	0.0070	0.0290	0.1030	0.1030	0.0510	0.0130	0.0000	0.0000
0.0000	0.0050	0.0150	0.0410	0.0680	0.0280	0.0080	0.0000	0.0000
0.0000	0.0000	0.0010	0.0130	0.0150	0.0120	0.0000	0.0010	0.0000
0.0000	0.0000	0.0000	0.0030	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values

0.00	0.00	-203.75	-78.82	-1.68	0.00	0.00	0.00	0.00
0.00	0.00	-160.27	-76.47	47.21	129.69	0.00	0.00	0.00
0.00	-218.29	-142.77	-73.11	8.10	102.05	178.92	0.00	0.00
-323.52	-230.19	-147.28	-62.63	17.65	94.50	173.49	258.72	0.00
-302.24	-240.35	-145.71	-63.95	21.57	108.00	194.99	0.00	0.00
0.00	-231.20	-151.18	-54.98	15.14	102.38	172.51	0.00	0.00
0.00	0.00	-158.38	-62.45	15.25	95.67	0.00	251.12	0.00
0.00	0.00	0.00	-37.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

October

Probabilities

0.0000	0.0010	0.0000	0.0010	0.0020	0.0000	0.0000	0.0000	0.0000

0.0000	0.0010	0.0050	0.0160	0.0040	0.0010	0.0000	0.0000	0.0000
0.0000	0.0060	0.0180	0.0620	0.0370	0.0050	0.0020	0.0000	0.0000
0.0030	0.0090	0.0520	0.1170	0.1060	0.0270	0.0050	0.0010	0.0000
0.0010	0.0060	0.0370	0.1310	0.1200	0.0460	0.0060	0.0010	0.0000
0.0000	0.0010	0.0090	0.0480	0.0510	0.0190	0.0060	0.0010	0.0000
0.0000	0.0000	0.0020	0.0080	0.0130	0.0100	0.0010	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0020	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	-183.07	0.00	35.93	49.30	0.00	0.00	0.00	0.00
	0.00	-160.07	-99.89	-2.58	68.17	137.34	0.00	0.00	0.00
	0.00	-163.76	-66.40	-6.97	83.91	148.94	249.67	0.00	0.00
	-250.89	-159.41	-75.83	4.58	78.82	161.57	265.48	345.14	0.00
	-229.98	-156.53	-70.66	8.56	79.67	162.67	238.55	338.37	0.00
	0.00	-147.73	-82.41	0.77	79.81	174.46	243.79	364.71	0.00
	0.00	0.00	-67.56	11.30	89.69	178.80	251.77	0.00	0.00
	0.00	0.00	0.00	0.00	103.50	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

November

Probabilities	0.0000	0.0000	0.0010	0.0020	0.0000	0.0010	0.0000	0.0000	0.0000
	0.0010	0.0040	0.0070	0.0080	0.0020	0.0020	0.0000	0.0000	0.0000
	0.0000	0.0040	0.0340	0.0490	0.0300	0.0060	0.0000	0.0000	0.0000
	0.0020	0.0120	0.0760	0.1380	0.1210	0.0290	0.0040	0.0010	0.0000
	0.0000	0.0100	0.0390	0.1310	0.1000	0.0480	0.0070	0.0000	0.0000
	0.0000	0.0020	0.0140	0.0340	0.0330	0.0210	0.0040	0.0000	0.0000
	0.0000	0.0000	0.0020	0.0100	0.0040	0.0040	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0020	0.0000	0.0010	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	0.00	-96.40	34.72	0.00	227.46	0.00	0.00	0
	-222.22	-125.25	-84.76	16.31	142.52	218.77	0.00	0.00	0
	0.00	-125.16	-48.06	31.48	113.33	208.92	0.00	0.00	0

-221.13	-124.54	-48.07	39.57	120.68	214.21	322.93	386.99	0
0.00	-142.19	-38.56	40.03	125.41	214.80	326.54	0.00	0
0.00	-124.69	-33.07	49.77	124.89	207.98	304.90	0.00	0
0.00	0.00	-30.68	56.40	130.63	199.25	0.00	0.00	0
0.00	0.00	0.00	43.87	0.00	218.85	0.00	0.00	0
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0

December

Probabilities	0.0000	0.0000	0.0000	0.0020	0.0010	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0050	0.0120	0.0120	0.0020	0.0000	0.0010	0.0000
	0.0000	0.0050	0.0220	0.0740	0.0580	0.0130	0.0010	0.0000	0.0000
	0.0010	0.0100	0.0480	0.1380	0.1360	0.0360	0.0050	0.0000	0.0000
	0.0010	0.0080	0.0290	0.1190	0.0980	0.0280	0.0070	0.0000	0.0000
	0.0000	0.0020	0.0110	0.0460	0.0330	0.0200	0.0000	0.0000	0.0000
	0.0000	0.0020	0.0010	0.0080	0.0030	0.0010	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	0.00	0.00	119.92	146.66	0.00	0.00	0.00	0.00
	0.00	0.00	-13.53	72.08	166.86	239.39	0.00	451.11	0.00
	0.00	-95.50	3.99	86.88	179.20	246.31	406.09	0.00	0.00
	-199.38	-79.70	-4.56	83.55	173.41	261.79	342.17	0.00	0.00
	-194.63	-90.39	5.16	83.44	176.83	266.50	361.02	0.00	0.00
	0.00	-129.89	-9.82	77.52	172.71	279.71	0.00	0.00	0.00
	0.00	-134.80	13.69	77.84	155.04	299.52	0.00	0.00	0.00
	0.00	0.00	0.00	78.75	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

To improve the speed of computations, each of the monthly probability matrix above was consolidated in just one vector of conditional probabilities as presented below:

NBS1	-20.88	-18.55	19.37	20.05	34.68	32.28	39.37	71.84
	27.74	71.20	84.37	96.58	106.75	120.62	127.12	0.00

	374.58	368.75	376.92	397.08	409.85	408.97	421.36	357.53
	350.22	442.46	454.69	486.77	506.97	495.48	520.25	558.76
	382.50	402.60	421.58	429.20	441.25	490.25	417.06	573.71
	324.50	293.78	328.04	324.51	339.67	354.35	347.51	378.08
	180.36	241.23	258.62	269.68	271.19	298.69	307.87	347.58
	203.32	163.66	184.68	187.30	207.39	214.20	223.53	222.93
	28.41	29.65	90.44	80.01	100.97	97.31	144.37	152.27
	-9.90	1.14	21.33	39.07	52.30	55.17	48.17	65.56
	-84.01	-74.65	-72.31	-65.28	-55.24	-51.66	-48.02	-10.47
	-36.17	-25.40	-36.37	-37.97	-34.26	-35.46	-38.80	-59.84
PrNBS1	0.0120	0.0560	0.1660	0.2750	0.2860	0.1420	0.0540	0.0090
	0.0070	0.0340	0.1850	0.3400	0.2860	0.1280	0.0190	0.0000
	0.0140	0.0710	0.1600	0.3190	0.2510	0.1270	0.0520	0.0050
	0.0090	0.0370	0.1240	0.2360	0.3010	0.1880	0.0890	0.0160
	0.0280	0.1000	0.2420	0.3180	0.2120	0.0830	0.0130	0.0030
	0.0050	0.0410	0.1430	0.2510	0.2820	0.1900	0.0720	0.0150
	0.0120	0.0810	0.2130	0.3270	0.2600	0.0800	0.0230	0.0030
	0.0060	0.0440	0.1400	0.2560	0.2820	0.2050	0.0550	0.0120
	0.0080	0.0390	0.1630	0.2680	0.2930	0.1690	0.0460	0.0130
	0.0110	0.0580	0.1740	0.2950	0.2600	0.1510	0.0410	0.0090
	0.0140	0.0650	0.1730	0.2890	0.2750	0.1460	0.0320	0.0050
	0.0100	0.0520	0.1340	0.2550	0.3000	0.1780	0.0580	0.0120

Now, employing the information above, and consistent with the aggregation methodology as described by Turgeon (19) and briefly reviewed in Chapter 4, the aggregated NBS is therefore utilized in linear regression fashion with respect to Lake Superior. The monthly linear regression coefficients are:

	A1	A0
January	0.9336	0.3861
February	0.8650	0.0373
March	0.8772	-0.0043

April	0.9769	0.3533
May	0.9124	0.0436
June	0.9440	-0.3631
July	1.0652	0.2249
August	0.9461	0.0531
September	0.9447	0.1092
October	0.9943	-0.0127
November	1.0806	0.0128
December	0.9571	0.1424

The corresponding NBS for the aggregated lakes becomes:

472.10	480.30	613.40	615.80	667.20	658.80	683.60	797.60	545.40
409.10	516.60	549.20	579.40	604.50	638.90	654.90	340.50	1023.40
1532.20	1517.80	1537.90	1587.60	1619.00	1616.90	1647.40	1490.10	1843.50
1212.70	1436.30	1466.00	1543.80	1592.70	1564.90	1624.90	1718.30	363.70
755.60	792.50	827.40	841.40	863.50	953.50	819.10	1106.70	416.20
489.50	439.80	495.20	489.50	514.00	537.80	526.70	576.20	672.80
143.90	268.20	303.80	326.40	329.40	385.60	404.40	485.50	156.70
15.80	-50.00	-15.10	-10.80	22.50	33.80	49.30	48.30	-321.60
-206.60	-204.70	-108.90	-125.30	-92.30	-98.00	-23.90	-11.40	-117.10
-283.80	-260.10	-216.90	-179.00	-150.70	-144.50	-159.50	-122.30	-238.70
-196.60	-173.50	-167.70	-150.30	-125.50	-116.70	-107.70	-15.00	24.20
304.80	339.30	304.10	299.00	310.90	307.00	296.30	228.80	650.50

Periodical MCAR 1:

Lake Superior

January

Probabilities	0.0000	0.0000	0.0020	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0050	0.0080	0.0090	0.0010	0.0030	0.0000	0.0000

0.0010	0.0030	0.0140	0.0220	0.0400	0.0310	0.0100	0.0020	0.0000
0.0000	0.0040	0.0270	0.0521	0.0741	0.0410	0.0240	0.0070	0.0000
0.0010	0.0060	0.0410	0.0681	0.0711	0.0591	0.0280	0.0060	0.0010
0.0010	0.0100	0.0220	0.0531	0.0490	0.0571	0.0210	0.0040	0.0000
0.0000	0.0030	0.0110	0.0240	0.0250	0.0200	0.0070	0.0040	0.0000
0.0000	0.0000	0.0020	0.0020	0.0120	0.0060	0.0010	0.0020	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000

Median Values	0.00	0.00	-2.66	0.00	-14.19	0.00	0.00	0.00	0.00
	0.00	0.00	-14.19	-8.23	-13.42	-5.18	-45.46	0.00	0.00
	-27.84	13.25	-14.86	-17.76	-6.29	-8.84	-16.25	-27.92	0.00
	0.00	-2.34	-7.42	-18.08	-12.75	-7.48	-15.85	-19.72	0.00
	-19.73	-10.05	-9.66	-13.81	-12.94	-1.56	-11.24	-17.31	-59.24
	-45.95	-23.90	-17.86	-12.53	-5.26	-16.21	-5.51	-17.60	0.00
	0.00	6.07	-15.90	-10.70	-5.28	-15.44	-15.53	-3.21	0.00
	0.00	0.00	-21.97	16.26	-16.11	-6.78	-27.84	-8.08	0.00
	0.00	0.00	0.00	0.00	0.00	-0.13	0.00	0.00	0.00

February

Probabilities	0.0000	0.0000	0.0000	0.0010	0.0040	0.0010	0.0000	0.0000	0.0000
	0.0000	0.0010	0.0040	0.0070	0.0070	0.0060	0.0020	0.0000	0.0010
	0.0000	0.0040	0.0080	0.0260	0.0290	0.0250	0.0080	0.0030	0.0000
	0.0000	0.0090	0.0260	0.0591	0.0591	0.0470	0.0210	0.0070	0.0040
	0.0020	0.0050	0.0270	0.0621	0.0801	0.0601	0.0310	0.0090	0.0000
	0.0030	0.0040	0.0250	0.0531	0.0611	0.0400	0.0250	0.0030	0.0030
	0.0010	0.0040	0.0060	0.0190	0.0270	0.0270	0.0120	0.0030	0.0020
	0.0000	0.0010	0.0060	0.0030	0.0080	0.0060	0.0020	0.0000	0.0000
	0.0000	0.0000	0.0010	0.0020	0.0020	0.0040	0.0000	0.0010	0.0000

Median Values	0.00	0.00	0.00	7.60	-9.00	39.52	0.00	0.00	0.00
	0.00	-35.95	0.28	-4.87	-3.67	5.53	-13.47	0.00	39.52

0.00	24.25	3.48	7.95	-2.55	14.29	15.59	19.02	0.00
0.00	16.65	-0.14	8.76	14.53	5.90	-1.06	-0.78	18.43
12.32	8.96	5.90	13.08	9.42	8.74	2.30	16.65	0.00
16.82	31.62	15.25	15.47	2.54	3.78	-2.26	-3.17	16.13
16.65	0.90	2.40	17.02	8.76	-3.67	16.04	57.79	5.33
0.00	-18.97	29.72	-3.67	-11.36	17.16	29.59	0.00	0.00
0.00	0.00	39.52	30.91	-9.00	-1.75	0.00	52.76	0.00

March

Probabilities	0.0000	0.0000	0.0000	0.0020	0.0000	0.0010	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0020	0.0090	0.0080	0.0040	0.0040	0.0000	0.0000
	0.0000	0.0000	0.0080	0.0150	0.0160	0.0200	0.0100	0.0020	0.0000
	0.0010	0.0080	0.0100	0.0571	0.0811	0.0551	0.0160	0.0020	0.0010
	0.0010	0.0100	0.0210	0.0671	0.1001	0.0831	0.0260	0.0070	0.0000
	0.0010	0.0070	0.0210	0.0551	0.0811	0.0631	0.0190	0.0040	0.0000
	0.0000	0.0020	0.0050	0.0240	0.0220	0.0200	0.0050	0.0020	0.0010
	0.0000	0.0000	0.0040	0.0020	0.0070	0.0040	0.0000	0.0010	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0010	0.0000	0.0000

Median Values	0.00	0.00	0.00	16.94	0.00	45.84	0.00	0.00	0.00
	0.00	0.00	41.76	55.56	36.60	44.13	33.18	0.00	0.00
	0.00	0.00	61.65	28.56	38.65	48.01	11.59	86.57	0.00
	30.30	29.24	20.94	37.77	41.49	49.34	71.99	90.86	111.90
	3.70	49.30	28.11	30.36	44.39	43.04	62.76	47.36	0.00
	58.48	70.79	51.98	47.28	31.55	44.55	37.61	21.11	0.00
	0.00	36.89	28.43	46.21	39.72	43.49	7.97	12.27	-26.96
	0.00	0.00	50.70	50.83	12.22	14.59	0.00	28.43	0.00
	0.00	0.00	0.00	0.00	36.13	0.00	45.84	0.00	0.00

April

Probabilities0.	0.0000	0.0000	0.0000	0.0010	0.0010	0.0000	0.0000	0.0000	0.0000
	0.0010	0.0010	0.0050	0.0060	0.0100	0.0030	0.0010	0.0010	0.0000

	0.0010	0.0060	0.0130	0.0280	0.0290	0.0140	0.0040	0.0000	0.0010
	0.0000	0.0080	0.0280	0.0661	0.0841	0.0460	0.0120	0.0060	0.0010
	0.0000	0.0070	0.0230	0.0731	0.0931	0.0761	0.0270	0.0080	0.0010
	0.0000	0.0060	0.0170	0.0571	0.0631	0.0480	0.0200	0.0050	0.0010
	0.0000	0.0000	0.0070	0.0140	0.0180	0.0220	0.0040	0.0040	0.0000
	0.0000	0.0000	0.0020	0.0050	0.0100	0.0060	0.0010	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0010	0.0000	0.0030	0.0000	0.0000	0.0000

Median Values	0.00	0.00	0.00	145.81	3.43	0.00	0.00	0.00	0.00
	166.43	169.12	131.21	150.89	116.58	130.63	182.57	139.30	0.00
	3.43	113.24	145.81	136.96	153.92	145.23	117.75	0.00	94.81
	0.00	170.33	137.89	144.88	155.56	162.06	147.59	139.64	97.44
	0.00	171.25	133.47	147.90	156.63	144.58	153.95	129.70	121.12
	0.00	185.25	137.11	161.24	153.50	145.62	156.18	142.32	137.91
	0.00	0.00	182.57	130.10	176.56	132.51	64.78	113.19	0.00
	0.00	0.00	167.25	137.11	135.40	135.01	98.22	0.00	0.00
	0.00	0.00	0.00	145.81	0.00	130.63	0.00	0.00	0.00

May

Probabilities	0.0000	0.0000	0.0010	0.0030	0.0010	0.0010	0.0010	0.0000	0.0000
	0.0000	0.0020	0.0060	0.0060	0.0060	0.0090	0.0010	0.0000	0.0000
	0.0020	0.0030	0.0200	0.0260	0.0420	0.0150	0.0090	0.0000	0.0000
	0.0020	0.0060	0.0300	0.0691	0.0911	0.0480	0.0210	0.0020	0.0000
	0.0020	0.0130	0.0370	0.0841	0.0821	0.0651	0.0220	0.0030	0.0010
	0.0010	0.0040	0.0200	0.0581	0.0601	0.0290	0.0140	0.0010	0.0000
	0.0000	0.0010	0.0030	0.0220	0.0240	0.0160	0.0030	0.0010	0.0010
	0.0000	0.0010	0.0000	0.0010	0.0030	0.0020	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0010	0.0010	0.0000	0.0000

Median Values	0.00	0.00	250.57	169.80	183.22	128.34	113.55	0.00	0.00
	0.00	207.17	198.60	197.95	200.39	169.80	129.69	0.00	0.00
	208.32	198.59	181.19	188.39	198.25	195.79	208.69	0.00	0.00

228.63	240.12	181.92	184.81	183.45	186.67	195.06	170.13	0.00
175.95	181.01	188.98	191.15	192.89	185.77	180.30	206.63	262.24
258.97	207.23	198.54	183.33	186.43	187.08	184.94	213.74	0.00
0.00	143.15	240.61	187.51	193.25	181.93	179.44	136.04	188.98
0.00	165.42	0.00	113.55	188.98	155.78	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	188.98	169.80	0.00	0.00

June

Probabilities	0.0000	0.0000	0.0000	0.0010	0.0020	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0040	0.0060	0.0040	0.0000	0.0000	0.0000
	0.0010	0.0040	0.0100	0.0170	0.0240	0.0290	0.0030	0.0000	0.0000
	0.0010	0.0010	0.0270	0.0711	0.0771	0.0571	0.0080	0.0010	0.0010
	0.0000	0.0070	0.0320	0.0821	0.1351	0.0761	0.0190	0.0040	0.0000
	0.0000	0.0020	0.0160	0.0541	0.0841	0.0470	0.0190	0.0060	0.0000
	0.0010	0.0000	0.0030	0.0110	0.0220	0.0120	0.0040	0.0010	0.0000
	0.0000	0.0000	0.0000	0.0040	0.0040	0.0030	0.0010	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	0.00	0.00	254.68	249.18	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	163.79	180.00	215.28	0.00	0.00	0.00
	141.71	133.09	164.67	137.12	160.63	154.58	178.07	0.00	0.00
	184.05	61.75	174.11	142.52	161.42	163.60	154.21	156.39	222.84
	0.00	186.29	172.22	152.35	168.13	156.13	161.83	139.24	0.00
	0.00	187.48	141.35	146.50	150.46	168.87	175.68	202.00	0.00
	97.13	0.00	140.26	152.50	171.63	190.02	165.49	120.26	0.00
	0.00	0.00	0.00	148.35	255.78	186.23	217.18	0.00	0.00
	0.00	0.00	0.00	0.00	217.18	0.00	0.00	0.00	0.00

July

Probabilities	0.0000	0.0000	0.0000	0.0040	0.0040	0.0010	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0050	0.0130	0.0090	0.0100	0.0040	0.0010	0.0010

0.0000	0.0040	0.0150	0.0410	0.0490	0.0270	0.0090	0.0010	0.0010
0.0040	0.0120	0.0380	0.0861	0.0791	0.0571	0.0150	0.0110	0.0000
0.0040	0.0150	0.0320	0.0861	0.0551	0.0370	0.0190	0.0020	0.0000
0.0010	0.0070	0.0410	0.0521	0.0330	0.0270	0.0080	0.0010	0.0000
0.0000	0.0040	0.0120	0.0140	0.0170	0.0080	0.0040	0.0000	0.0000
0.0000	0.0010	0.0040	0.0060	0.0030	0.0020	0.0000	0.0010	0.0000
0.0000	0.0000	0.0000	0.0010	0.0000	0.0010	0.0000	0.0000	0.0000

Median Values	0.00	0.00	0.00	115.88	114.61	112.51	0.00	0.00	0.00
	0.00	0.00	153.30	123.06	126.12	107.14	117.29	136.85	82.90
	0.00	145.23	150.46	129.78	130.85	116.35	133.76	78.40	173.87
	139.31	110.86	142.53	132.06	122.60	134.87	141.37	130.86	0.00
	124.95	141.76	136.70	128.72	134.76	123.79	141.66	105.17	0.00
	30.52	136.93	130.85	132.16	129.78	130.86	130.53	89.10	0.00
	0.00	89.72	159.43	107.27	116.28	116.42	134.82	0.00	0.00
	0.00	147.11	111.72	150.05	126.12	145.61	0.00	118.30	0.00
	0.00	0.00	0.00	112.51	0.00	110.92	0.00	0.00	0.00

August

Probabilities	0.0000	0.0000	0.0000	0.0000	0.0020	0.0010	0.0020	0.0000	0.0000
	0.0000	0.0000	0.0020	0.0030	0.0100	0.0130	0.0050	0.0020	0.0000
	0.0010	0.0020	0.0140	0.0350	0.0350	0.0250	0.0100	0.0010	0.0000
	0.0000	0.0060	0.0250	0.0390	0.0761	0.0390	0.0160	0.0070	0.0000
	0.0030	0.0150	0.0410	0.0521	0.0891	0.0711	0.0270	0.0070	0.0010
	0.0010	0.0070	0.0250	0.0581	0.0591	0.0450	0.0130	0.0040	0.0000
	0.0000	0.0050	0.0120	0.0190	0.0260	0.0120	0.0080	0.0020	0.0010
	0.0000	0.0000	0.0030	0.0020	0.0080	0.0050	0.0050	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0010	0.0010	0.0000	0.0000	0.0000

Median Values	0.00	0.00	0.00	0.00	80.01	141.60	127.67	0.00	0.00
	0.00	0.00	54.31	67.26	94.73	96.77	94.31	53.08	0.00
	112.73	134.23	98.80	102.96	100.60	87.04	100.03	103.08	0.00

0.00	125.82	90.67	99.50	91.61	102.96	124.43	76.49	0.00
37.84	77.48	105.03	105.72	101.00	100.60	98.15	73.72	41.19
99.50	103.08	92.52	91.80	104.80	104.18	82.53	91.66	0.00
0.00	99.50	95.79	90.86	103.19	101.06	111.61	99.67	152.41
0.00	0.00	122.00	58.31	88.13	99.50	154.00	0.00	0.00
0.00	0.00	0.00	0.00	141.60	94.31	0.00	0.00	0.00

September

Probabilities	0.0000	0.0000	0.0000	0.0000	0.0010	0.0010	0.0010	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0030	0.0020	0.0020	0.0000	0.0010	0.0000
	0.0000	0.0000	0.0010	0.0100	0.0200	0.0150	0.0060	0.0020	0.0010
	0.0000	0.0020	0.0060	0.0290	0.0571	0.0571	0.0200	0.0050	0.0010
	0.0010	0.0020	0.0210	0.0460	0.1171	0.1071	0.0370	0.0050	0.0010
	0.0010	0.0020	0.0190	0.0581	0.0921	0.0651	0.0390	0.0010	0.0010
	0.0000	0.0010	0.0060	0.0250	0.0390	0.0270	0.0120	0.0040	0.0010
	0.0000	0.0010	0.0020	0.0060	0.0060	0.0030	0.0010	0.0020	0.0000
	0.0010	0.0000	0.0000	0.0000	0.0020	0.0010	0.0000	0.0010	0.0000

Median Values	0.00	0.00	0.00	0.00	13.43	84.23	75.74	0.00	0.00
	0.00	0.00	0.00	123.20	20.75	91.48	0.00	109.35	0.00
	0.00	0.00	51.82	43.10	77.19	59.79	55.76	74.91	113.33
	0.00	61.64	82.32	75.61	77.64	75.23	81.60	107.56	109.82
	76.73	74.38	91.82	93.84	75.61	76.71	71.62	94.32	59.12
	103.90	69.56	87.60	73.56	77.19	76.63	76.44	89.49	86.99
	0.00	145.00	79.76	89.49	84.04	58.94	92.04	41.33	4.73
	0.00	53.20	61.97	58.93	97.00	63.50	77.64	79.99	0.00
	28.07	0.00	0.00	0.00	103.71	13.43	0.00	75.74	0.00

October

Probabilities	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0010	0.0010	0.0010	0.0000	0.0000	0.0000

0.0000	0.0000	0.0000	0.0040	0.0090	0.0090	0.0030	0.0010	0.0000
0.0000	0.0000	0.0050	0.0210	0.0450	0.0521	0.0160	0.0050	0.0010
0.0010	0.0010	0.0060	0.0450	0.0931	0.0981	0.0480	0.0060	0.0020
0.0000	0.0020	0.0050	0.0490	0.1051	0.1021	0.0551	0.0160	0.0010
0.0000	0.0000	0.0080	0.0230	0.0340	0.0571	0.0190	0.0050	0.0010
0.0000	0.0000	0.0010	0.0020	0.0090	0.0150	0.0050	0.0030	0.0010
0.0000	0.0000	0.0000	0.0000	0.0030	0.0020	0.0010	0.0000	0.0000

Median Values	0.00	0.00	103.14	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	103.14	3.58	42.08	0.00	0.00	0.00
	0.00	0.00	0.00	86.48	16.23	14.22	2.07	79.65	0.00
	0.00	0.00	48.89	55.56	43.03	33.64	41.34	-12.43	121.22
	90.77	-21.82	68.26	33.17	33.36	35.79	40.12	54.17	-9.78
	0.00	64.85	33.07	40.28	37.55	34.20	42.08	10.96	-63.63
	0.00	0.00	55.92	17.11	43.14	33.17	47.34	-1.31	112.14
	0.00	0.00	3.58	40.02	67.84	14.41	48.89	42.08	-89.88
	0.00	0.00	0.00	0.00	3.58	70.94	126.12	0.00	0.00

November

Probabilities	0.0000	0.0000	0.0000	0.0020	0.0010	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0010	0.0020	0.0060	0.0020	0.0010	0.0000	0.0010
	0.0010	0.0010	0.0020	0.0100	0.0120	0.0150	0.0100	0.0020	0.0000
	0.0000	0.0020	0.0120	0.0400	0.0631	0.0651	0.0280	0.0050	0.0000
	0.0000	0.0030	0.0100	0.0631	0.1061	0.0861	0.0450	0.0050	0.0000
	0.0010	0.0040	0.0210	0.0691	0.0811	0.0571	0.0190	0.0050	0.0010
	0.0000	0.0030	0.0050	0.0240	0.0410	0.0300	0.0140	0.0000	0.0010
	0.0010	0.0000	0.0010	0.0040	0.0070	0.0030	0.0010	0.0010	0.0000
	0.0000	0.0000	0.0000	0.0010	0.0020	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	0.00	0.00	61.79	-50.23	0.00	0.00	0.00	0.00
	0.00	0.00	36.00	10.55	52.57	65.10	77.75	0.00	5.92
	78.65	58.56	31.67	4.25	5.58	4.93	25.47	-22.73	0.00

0.00	58.02	5.25	18.30	12.22	10.79	21.82	49.19	0.00
0.00	15.78	-0.85	8.57	19.92	9.71	16.83	43.61	0.00
-2.87	30.51	-0.08	15.87	9.37	16.25	20.24	64.66	4.62
0.00	15.11	20.74	15.60	5.38	28.78	11.65	0.00	19.73
99.22	0.00	5.92	39.06	40.47	-21.48	94.81	20.74	0.00
0.00	0.00	0.00	58.92	7.22	0.00	0.00	0.00	0.00

December

Probabilities	0.0000	0.0000	0.0020	0.0010	0.0040	0.0040	0.0000	0.0000	0.0000
	0.0010	0.0030	0.0030	0.0070	0.0150	0.0120	0.0030	0.0040	0.0000
	0.0000	0.0030	0.0110	0.0350	0.0470	0.0260	0.0070	0.0040	0.0000
	0.0020	0.0130	0.0300	0.0631	0.0801	0.0571	0.0110	0.0030	0.0020
	0.0070	0.0150	0.0440	0.0791	0.0861	0.0480	0.0170	0.0050	0.0000
	0.0010	0.0050	0.0310	0.0501	0.0561	0.0200	0.0080	0.0030	0.0000
	0.0000	0.0070	0.0070	0.0200	0.0080	0.0050	0.0030	0.0000	0.0000
	0.0000	0.0020	0.0050	0.0060	0.0030	0.0020	0.0010	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0010	0.0010	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	0.00	30.53	-49.59	-14.66	-44.01	0.00	0.00	0.00
	-76.64	-13.94	0.37	-11.36	-24.08	-17.62	-16.51	-14.68	0.00
	0.00	-11.36	-32.81	-18.56	-15.14	-22.47	-40.24	-18.56	0.00
	25.82	-17.79	-26.17	-24.08	-25.10	-19.31	-24.78	-8.77	-23.51
	-14.17	-17.01	-24.67	-33.42	-19.52	-22.39	-7.99	-76.64	0.00
	-26.95	-14.00	-21.32	-16.19	-20.23	-29.10	-28.31	-39.21	0.00
	0.00	-34.49	-39.21	-13.97	-32.20	-11.36	-4.69	0.00	0.00
	0.00	-23.55	-34.49	-36.69	3.38	-4.70	-26.95	0.00	0.00
	0.00	0.00	0.00	-53.53	47.63	0.00	0.00	0.00	0.00

Aggregated Lake

January

Probabilities

0.0000	0.0000	0.0000	0.0000	0.0020	0.0000	0.0020	0.0000	0.0000
0.0000	0.0010	0.0030	0.0100	0.0090	0.0030	0.0040	0.0010	0.0000
0.0010	0.0020	0.0130	0.0270	0.0230	0.0180	0.0090	0.0010	0.0000
0.0010	0.0100	0.0230	0.0611	0.0761	0.0460	0.0190	0.0070	0.0010
0.0000	0.0070	0.0200	0.0721	0.0861	0.0751	0.0320	0.0080	0.0030
0.0020	0.0050	0.0210	0.0440	0.0711	0.0551	0.0200	0.0050	0.0020
0.0000	0.0010	0.0070	0.0170	0.0320	0.0190	0.0080	0.0040	0.0010
0.0000	0.0010	0.0020	0.0090	0.0090	0.0040	0.0050	0.0000	0.0000
0.0000	0.0000	0.0010	0.0000	0.0070	0.0010	0.0000	0.0000	0.0000

Median Values

0.00	0.00	0.00	0.00	-14.00	0.00	122.43	0.00	0.00
0.00	180.61	130.10	111.28	129.13	116.76	157.22	209.49	0.00
48.51	113.15	119.94	130.10	120.77	128.17	173.20	168.09	0.00
129.13	94.60	124.82	90.66	126.50	128.08	155.98	118.15	139.51
0.00	184.76	105.09	148.51	98.39	107.99	87.38	143.91	0.48
135.47	45.72	151.53	131.31	98.47	107.99	125.70	27.85	110.50
0.00	50.87	139.51	122.91	139.19	117.92	93.16	127.23	-70.84
0.00	79.10	87.81	129.13	171.11	143.56	122.08	0.00	0.00
0.00	0.00	48.69	0.00	117.92	0.48	0.00	0.00	0.00

February

Probabilities

0.0000	0.0000	0.0000	0.0010	0.0010	0.0000	0.0010	0.0000	0.0000
0.0000	0.0010	0.0040	0.0110	0.0120	0.0040	0.0020	0.0000	0.0000
0.0010	0.0020	0.0080	0.0380	0.0360	0.0260	0.0070	0.0000	0.0010
0.0000	0.0070	0.0270	0.0691	0.0891	0.0561	0.0200	0.0080	0.0000
0.0010	0.0090	0.0360	0.0821	0.0891	0.0641	0.0200	0.0020	0.0000
0.0010	0.0090	0.0300	0.0541	0.0601	0.0370	0.0130	0.0020	0.0000
0.0000	0.0030	0.0090	0.0220	0.0140	0.0110	0.0020	0.0010	0.0000
0.0000	0.0000	0.0020	0.0030	0.0050	0.0050	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0020	0.0000	0.0000	0.0000	0.0000

Median Values

0.00	0.00	0.00	148.60	211.88	0.00	147.78	0.00	0.00
0.00	144.41	138.61	135.53	148.97	209.10	232.78	0.00	0.00

190.27	241.70	128.16	144.95	148.97	149.61	165.97	0.00	232.78
0.00	153.61	171.59	155.54	143.51	152.80	166.81	120.87	0.00
228.25	184.70	148.38	160.70	168.65	146.96	142.28	129.77	0.00
81.16	122.34	156.04	150.20	144.41	158.56	162.44	152.12	0.00
0.00	147.09	168.73	147.78	134.78	142.04	140.23	210.10	0.00
0.00	0.00	142.36	178.54	176.06	147.78	0.00	0.00	0.00
0.00	0.00	0.00	0.00	148.60	0.00	0.00	0.00	0.00

March

Probabilities	0.0000	0.0000	0.0000	0.0010	0.0010	0.0010	0.0000	0.0000	0.0000
	0.0000	0.0010	0.0000	0.0080	0.0030	0.0050	0.0010	0.0000	0.0000
	0.0000	0.0020	0.0040	0.0130	0.0160	0.0220	0.0080	0.0020	0.0000
	0.0000	0.0020	0.0130	0.0290	0.0511	0.0551	0.0220	0.0060	0.0010
	0.0010	0.0040	0.0170	0.0541	0.0891	0.0681	0.0370	0.0140	0.0000
	0.0020	0.0060	0.0140	0.0430	0.0781	0.0851	0.0410	0.0130	0.0000
	0.0000	0.0000	0.0150	0.0290	0.0400	0.0340	0.0230	0.0020	0.0000
	0.0000	0.0000	0.0010	0.0060	0.0100	0.0150	0.0080	0.0030	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0020	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	0.00	0.00	50.52	435.51	153.21	0.00	0.00	0.00
	0.00	350.49	0.00	217.52	330.04	260.59	265.26	0.00	0.00
	0.00	380.30	327.64	255.25	354.83	327.40	342.73	190.71	0.00
	0.00	490.93	279.94	358.23	325.91	310.07	281.77	324.53	356.26
	400.30	324.14	329.05	321.32	312.99	342.84	335.72	333.15	0.00
	373.62	400.56	339.81	311.52	346.42	322.66	285.17	360.61	0.00
	0.00	0.00	336.27	340.69	320.81	326.33	286.08	379.47	0.00
	0.00	0.00	413.78	287.22	323.54	339.73	312.31	400.36	0.00
	0.00	0.00	0.00	0.00	153.21	0.00	0.00	0.00	0.00

April

Probabilities	0.0000	0.0010	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0020	0.0070	0.0100	0.0050	0.0010	0.0010	0.0000

0.0000	0.0040	0.0040	0.0130	0.0350	0.0210	0.0120	0.0020	0.0000
0.0000	0.0070	0.0210	0.0480	0.0601	0.0440	0.0200	0.0060	0.0020
0.0010	0.0110	0.0280	0.0641	0.0771	0.0691	0.0350	0.0110	0.0010
0.0010	0.0030	0.0190	0.0360	0.0691	0.0521	0.0430	0.0070	0.0000
0.0000	0.0040	0.0120	0.0350	0.0290	0.0330	0.0060	0.0020	0.0000
0.0000	0.0000	0.0020	0.0090	0.0120	0.0040	0.0020	0.0020	0.0010
0.0000	0.0000	0.0010	0.0000	0.0030	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	572.95	0.00	242.86	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	350.63	516.09	416.50	387.11	327.18	617.71	0.00
	0.00	365.58	490.65	434.55	448.24	434.84	438.84	477.42	0.00
	0.00	410.14	481.95	449.64	447.84	454.55	448.05	470.07	601.05
	555.70	543.87	428.69	442.98	458.90	456.89	471.04	423.25	368.97
	462.10	396.51	481.95	454.54	432.93	435.10	470.95	357.47	0.00
	0.00	416.41	432.89	429.05	429.35	493.51	486.75	565.18	0.00
	0.00	0.00	469.62	442.99	401.76	444.96	582.61	602.85	356.32
	0.00	0.00	572.95	0.00	372.23	0.00	0.00	0.00	0.00

May

Probabilities	0.0000	0.0000	0.0000	0.0020	0.0050	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0010	0.0030	0.0060	0.0150	0.0070	0.0020	0.0010	0.0000
	0.0010	0.0040	0.0090	0.0240	0.0430	0.0190	0.0120	0.0050	0.0020
	0.0010	0.0060	0.0250	0.0470	0.0661	0.0420	0.0270	0.0080	0.0000
	0.0020	0.0060	0.0260	0.0601	0.0651	0.0761	0.0300	0.0110	0.0000
	0.0030	0.0050	0.0310	0.0541	0.0651	0.0511	0.0180	0.0070	0.0020
	0.0000	0.0040	0.0150	0.0240	0.0300	0.0230	0.0070	0.0030	0.0000
	0.0000	0.0020	0.0020	0.0120	0.0150	0.0080	0.0030	0.0010	0.0000
	0.0000	0.0000	0.0010	0.0000	0.0000	0.0030	0.0000	0.0000	0.0000

Median Values	0.00	0.00	0.00	438.91	436.87	0.00	0.00	0.00	0.00
	0.00	545.44	352.04	368.05	420.78	436.87	430.88	349.79	0.00
	453.96	365.91	312.18	418.49	372.16	442.59	382.84	330.20	405.36

277.50	384.71	340.35	354.36	362.58	381.83	407.10	349.72	0.00
483.49	328.11	420.01	396.95	330.85	372.39	345.49	337.68	0.00
337.63	181.42	361.13	378.04	381.59	389.30	415.10	412.80	504.05
0.00	329.65	301.86	382.40	374.08	406.96	442.22	494.84	0.00
0.00	310.91	458.41	398.54	436.89	338.72	372.21	545.44	0.00
0.00	0.00	418.25	0.00	0.00	489.51	0.00	0.00	0.00

June

Probabilities	0.0000	0.0000	0.0000	0.0000	0.0030	0.0000	0.0000	0.0000	0.0000
	0.0010	0.0010	0.0020	0.0060	0.0050	0.0040	0.0000	0.0000	0.0000
	0.0000	0.0050	0.0220	0.0150	0.0280	0.0170	0.0060	0.0000	0.0000
	0.0000	0.0050	0.0190	0.0701	0.0791	0.0561	0.0080	0.0020	0.0010
	0.0020	0.0040	0.0280	0.0791	0.0961	0.0721	0.0280	0.0070	0.0000
	0.0000	0.0010	0.0130	0.0440	0.0841	0.0711	0.0220	0.0060	0.0000
	0.0000	0.0010	0.0050	0.0190	0.0280	0.0130	0.0140	0.0040	0.0000
	0.0000	0.0000	0.0010	0.0040	0.0090	0.0050	0.0030	0.0010	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0040	0.0000	0.0000	0.0000	0.0000

Median Values	0.00	0.00	0.00	0.00	346.07	0.00	0.00	0.00	0.00
	173.52	326.16	360.54	319.05	339.10	315.84	0.00	0.00	0.00
	0.00	253.58	297.52	290.48	238.08	306.24	330.83	0.00	0.00
	0.00	169.90	266.35	293.09	251.51	285.36	201.76	266.38	256.66
	177.69	317.91	268.95	280.91	256.94	292.73	298.64	335.71	0.00
	0.00	200.28	295.60	261.80	266.35	256.66	316.67	285.81	0.00
	0.00	266.83	193.23	308.23	280.03	294.56	227.98	342.09	0.00
	0.00	0.00	173.52	305.36	304.14	353.56	267.62	370.99	0.00
	0.00	0.00	0.00	0.00	353.56	0.00	0.00	0.00	0.00

July

Probabilities	0.0000	0.0010	0.0000	0.0000	0.0010	0.0010	0.0000	0.0000	0.0000
	0.0010	0.0020	0.0070	0.0050	0.0100	0.0040	0.0020	0.0000	0.0000

0.0010	0.0050	0.0090	0.0160	0.0330	0.0170	0.0090	0.0020	0.0010
0.0000	0.0080	0.0230	0.0330	0.0541	0.0390	0.0100	0.0020	0.0010
0.0010	0.0070	0.0220	0.0501	0.1011	0.0831	0.0400	0.0120	0.0000
0.0000	0.0060	0.0190	0.0400	0.0771	0.0651	0.0330	0.0090	0.0020
0.0000	0.0050	0.0080	0.0170	0.0320	0.0370	0.0170	0.0020	0.0010
0.0000	0.0000	0.0020	0.0060	0.0080	0.0080	0.0060	0.0010	0.0000
0.0000	0.0000	0.0000	0.0000	0.0050	0.0000	0.0000	0.0010	0.0000

Median Values	0.00	159.13	0.00	0.00	161.19	187.36	0.00	0.00	0.00
	46.17	161.19	169.96	114.84	198.38	198.70	225.11	0.00	0.00
	281.14	194.54	159.69	164.43	200.30	165.71	165.22	197.56	225.12
	0.00	160.03	198.79	164.92	165.22	176.74	167.86	174.40	114.84
	36.81	160.70	162.51	170.06	180.53	164.35	178.03	196.11	0.00
	0.00	201.62	177.76	191.00	168.67	168.17	168.11	158.64	179.67
	0.00	183.03	203.88	198.99	162.47	160.28	184.95	249.59	160.03
	0.00	0.00	192.57	143.89	148.11	198.38	237.24	170.23	0.00
	0.00	0.00	0.00	0.00	153.62	0.00	0.00	161.19	0.00

August

Probabilities	0.0000	0.0000	0.0010	0.0000	0.0000	0.0010	0.0010	0.0000	0.0000
	0.0000	0.0000	0.0020	0.0020	0.0000	0.0050	0.0020	0.0020	0.0000
	0.0000	0.0000	0.0070	0.0170	0.0310	0.0270	0.0120	0.0050	0.0010
	0.0010	0.0010	0.0290	0.0490	0.0761	0.0360	0.0270	0.0080	0.0000
	0.0000	0.0030	0.0230	0.0691	0.0901	0.0811	0.0330	0.0080	0.0030
	0.0010	0.0010	0.0220	0.0531	0.0641	0.0430	0.0280	0.0030	0.0000
	0.0000	0.0010	0.0170	0.0260	0.0360	0.0230	0.0140	0.0010	0.0000
	0.0010	0.0040	0.0020	0.0070	0.0090	0.0030	0.0040	0.0000	0.0000
	0.0000	0.0000	0.0020	0.0010	0.0010	0.0000	0.0020	0.0000	0.0000

Median Values	0.00	0.00	102.56	0.00	0.00	-3.09	-22.98	0.00	0.00
	0.00	0.00	184.43	76.76	0.00	74.26	-22.98	-29.02	0.00
	0.00	0.00	-27.21	61.61	30.63	72.84	25.42	17.52	57.70

-58.64	17.52	84.03	46.53	55.17	24.97	24.11	98.40	0.00
0.00	53.57	25.20	55.36	40.18	29.60	51.24	79.50	110.08
-77.20	17.63	40.96	36.89	42.37	63.11	52.98	-3.09	0.00
0.00	-0.28	42.64	13.47	61.17	42.68	55.36	-66.22	0.00
-0.21	48.38	-77.20	0.63	-0.28	45.02	17.52	0.00	0.00
0.00	0.00	-22.98	-3.09	102.56	0.00	197.39	0.00	0.00

September

Probabilities	0.0000	0.0000	0.0010	0.0010	0.0030	0.0010	0.0000	0.0010	0.0000
	0.0010	0.0000	0.0020	0.0040	0.0140	0.0110	0.0020	0.0020	0.0000
	0.0000	0.0050	0.0080	0.0220	0.0260	0.0250	0.0120	0.0120	0.0000
	0.0010	0.0050	0.0190	0.0501	0.0611	0.0490	0.0280	0.0120	0.0020
	0.0010	0.0060	0.0250	0.0581	0.0841	0.0521	0.0280	0.0110	0.0030
	0.0020	0.0070	0.0270	0.0511	0.0501	0.0440	0.0290	0.0080	0.0000
	0.0020	0.0030	0.0170	0.0300	0.0290	0.0240	0.0160	0.0030	0.0010
	0.0000	0.0030	0.0100	0.0100	0.0150	0.0090	0.0030	0.0000	0.0000
	0.0000	0.0000	0.0020	0.0000	0.0040	0.0040	0.0000	0.0020	0.0000

Median Values	0.00	0.00	32.50	-15.87	-4.65	101.93	0.00	-42.84	0.00
	29.62	0.00	14.38	6.06	29.93	-27.04	3.06	81.49	0.00
	0.00	-72.56	29.93	-20.40	32.50	-4.06	-18.10	-15.44	0.00
	-46.94	10.37	56.78	0.22	-3.03	5.70	-1.46	49.86	-110.82
	68.97	-26.25	16.27	6.19	-1.41	15.24	8.97	9.50	-14.07
	4.15	-18.86	0.70	2.33	2.39	-5.74	29.62	5.86	0.00
	-36.34	-16.33	-22.66	-18.63	1.30	28.06	8.11	45.46	-6.62
	0.00	28.93	-12.02	-3.47	22.00	18.00	-56.45	0.00	0.00
	0.00	0.00	-42.84	0.00	32.50	-4.65	0.00	101.93	0.00

October

Probabilities	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0020	0.0000	0.0000
	0.0000	0.0010	0.0000	0.0090	0.0090	0.0030	0.0030	0.0010	0.0000

	0.0000	0.0030	0.0050	0.0250	0.0340	0.0130	0.0110	0.0010	0.0000
	0.0010	0.0050	0.0210	0.0440	0.0631	0.0571	0.0220	0.0040	0.0000
	0.0020	0.0060	0.0290	0.0551	0.0891	0.0731	0.0440	0.0060	0.0000
	0.0000	0.0050	0.0200	0.0480	0.0721	0.0591	0.0250	0.0050	0.0010
	0.0000	0.0030	0.0110	0.0270	0.0300	0.0230	0.0160	0.0050	0.0010
	0.0000	0.0000	0.0020	0.0120	0.0030	0.0040	0.0040	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0010	0.0010	0.0000	0.0000	0.0000
Median Values	0.00	0.00	0.00	-68.05	0.00	0.00	-36.43	0.00	0.00
	0.00	74.99	0.00	-26.21	-68.05	-71.72	31.76	167.06	0.00
	0.00	-17.95	-51.78	-62.42	-24.55	-4.08	78.51	-44.13	0.00
	74.99	-71.85	-4.54	-22.61	-36.18	-12.73	-53.69	-23.14	0.00
	98.80	-44.10	-11.19	-39.63	-17.95	-20.54	-34.45	-25.61	0.00
	0.00	-3.72	25.09	-24.32	-36.66	-21.38	-28.19	-20.54	-143.14
	0.00	2.86	-46.27	-35.22	-63.49	-7.85	5.43	12.01	3.66
	0.00	0.00	-37.95	-48.84	26.88	-82.77	-18.42	0.00	0.00
	0.00	0.00	0.00	0.00	-60.14	-68.05	0.00	0.00	0.00

November

Probabilities	0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0010	0.0010	0.0070	0.0120	0.0020	0.0000	0.0000	0.0000
	0.0010	0.0040	0.0140	0.0280	0.0320	0.0250	0.0050	0.0010	0.0000
	0.0000	0.0040	0.0340	0.0751	0.0821	0.0420	0.0260	0.0010	0.0000
	0.0000	0.0050	0.0290	0.0871	0.1181	0.0661	0.0300	0.0010	0.0000
	0.0000	0.0060	0.0260	0.0440	0.0721	0.0290	0.0100	0.0010	0.0010
	0.0000	0.0020	0.0050	0.0190	0.0230	0.0230	0.0030	0.0010	0.0000
	0.0000	0.0000	0.0000	0.0020	0.0020	0.0010	0.0010	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000
Median Values	0.00	0.00	0.00	0.00	61.26	0.00	0.00	0.00	0.00
	0.00	3.90	-31.06	77.41	61.26	79.62	0.00	0.00	0.00
	80.88	68.88	0.13	3.13	57.43	18.39	51.34	72.22	0.00

0.00	52.12	55.92	24.21	30.31	65.73	26.93	55.83	0.00
0.00	16.76	55.47	58.91	32.70	41.94	6.94	-99.33	0.00
0.00	27.46	10.85	36.88	30.31	48.77	17.07	355.23	52.72
0.00	-49.63	3.90	3.10	29.02	52.72	19.85	101.17	0.00
0.00	0.00	0.00	117.38	130.66	77.41	61.26	0.00	0.00
0.00	0.00	0.00	61.26	0.00	0.00	0.00	0.00	0.00

December

Probabilities	0.0000	0.0000	0.0000	0.0000	0.0030	0.0020	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0010	0.0010	0.0060	0.0070	0.0000	0.0010	0.0000
	0.0000	0.0000	0.0080	0.0130	0.0150	0.0170	0.0090	0.0010	0.0000
	0.0020	0.0020	0.0080	0.0330	0.0581	0.0531	0.0150	0.0070	0.0010
	0.0020	0.0030	0.0170	0.0511	0.1021	0.0801	0.0310	0.0150	0.0020
	0.0000	0.0050	0.0170	0.0410	0.0771	0.0811	0.0450	0.0100	0.0020
	0.0010	0.0010	0.0060	0.0300	0.0360	0.0350	0.0240	0.0060	0.0000
	0.0000	0.0000	0.0020	0.0030	0.0190	0.0110	0.0100	0.0010	0.0000
	0.0000	0.0000	0.0000	0.0010	0.0060	0.0030	0.0000	0.0000	0.0000

Median Values	0.00	0.00	0.00	0.00	98.48	44.89	0.00	0.00	0.00
	0.00	0.00	168.31	98.48	120.86	73.48	0.00	124.25	0.00
	0.00	0.00	97.40	92.36	-2.30	124.25	59.31	81.46	0.00
	98.30	-39.29	65.31	69.23	76.74	68.39	98.48	79.43	-49.50
	126.31	123.13	49.22	78.72	72.16	62.22	69.23	42.43	-26.91
	0.00	18.38	60.39	52.05	50.08	85.10	79.43	75.84	-45.24
	92.36	23.41	52.27	79.79	90.50	68.39	78.56	105.26	0.00
	0.00	0.00	-154.36	-28.65	90.04	73.48	86.91	123.78	0.00
	0.00	0.00	0.00	124.25	13.72	98.48	0.00	0.00	0.00

Appendix E

North American Great Lakes Study Case

Outflows

Appendix E - Great Lakes Outflows

Average Outflows in the Lakes Connecting Channels

1. Results for the Simulation of the Great Lakes Operation as defined by the IJC, Comparison Between Historical and Synthetic NBS:

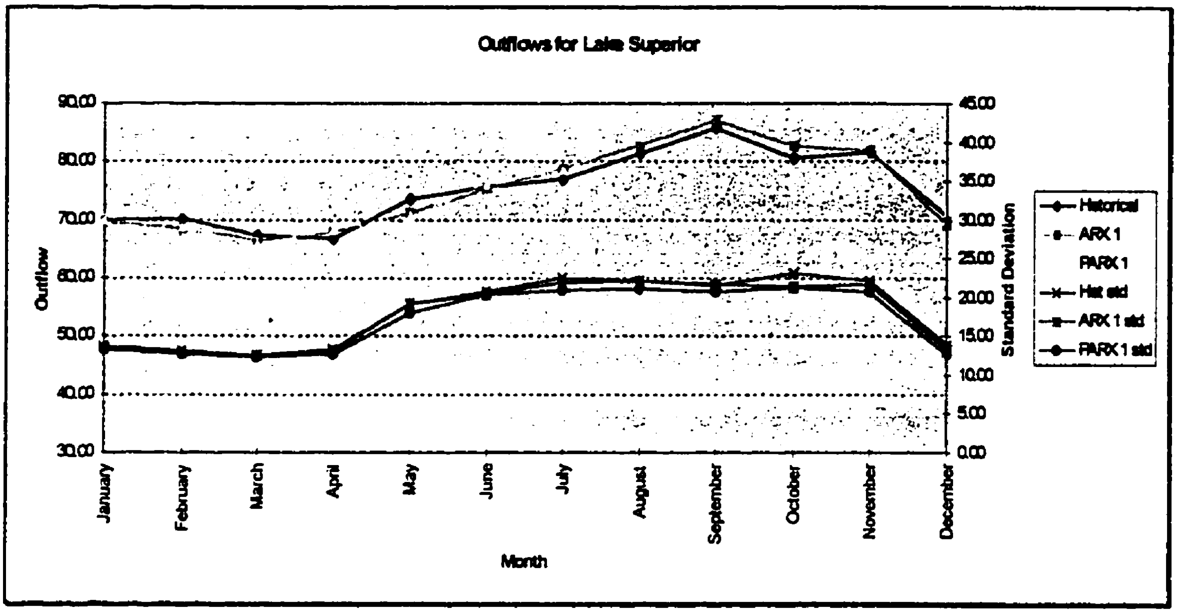


Figure E - 1 Lake Superior, Simulation with Historical and Synthetic Net Basin Supplies

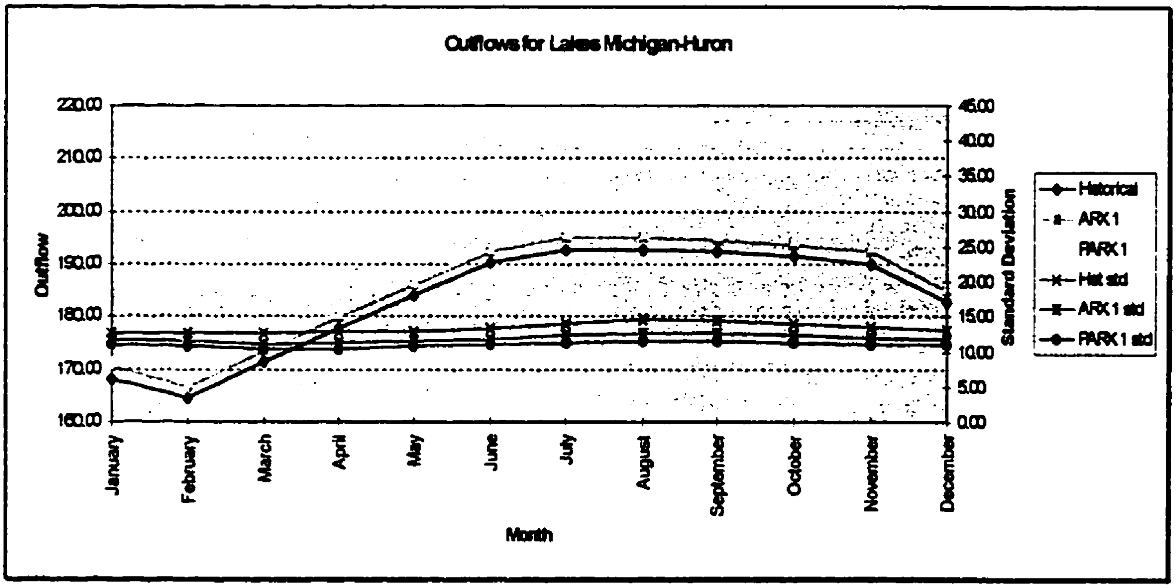


Figure E - 2 Lakes Michigan -Huron, Simulation with Historical and Synthetic Net Basin Supplies

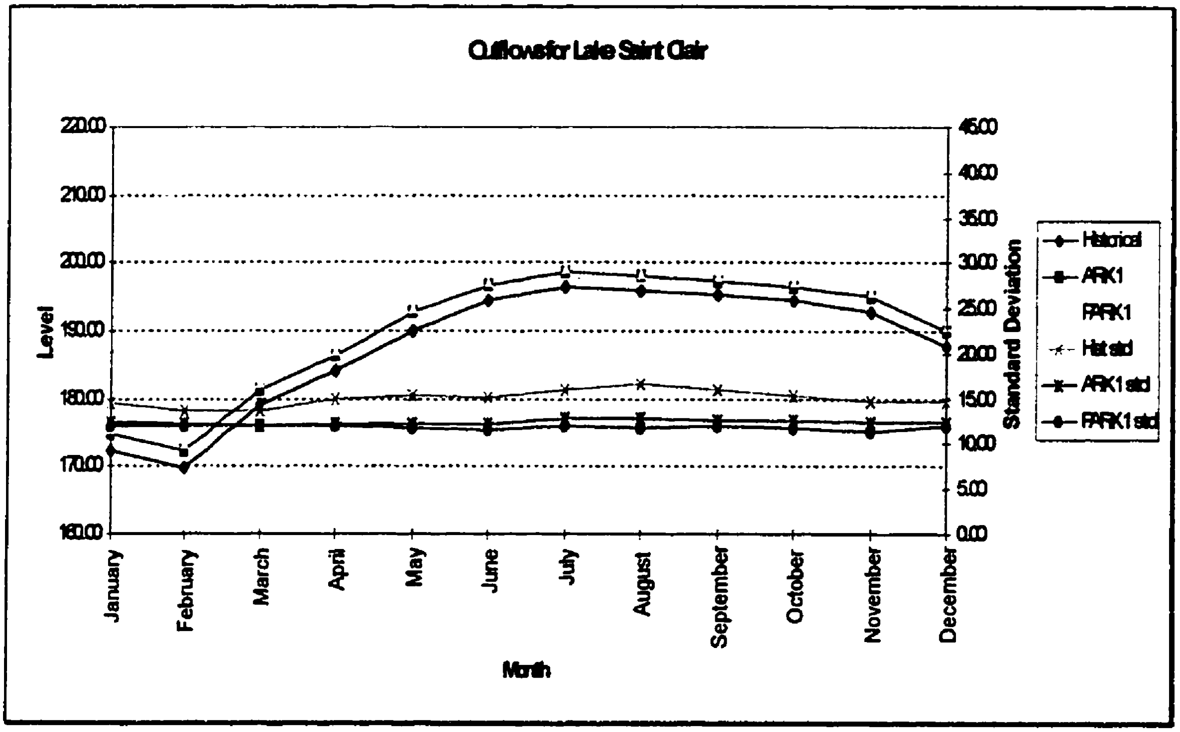


Figure E - 3 Lake St. Clair, Simulation with Historical and Synthetic Net Basin Supplies

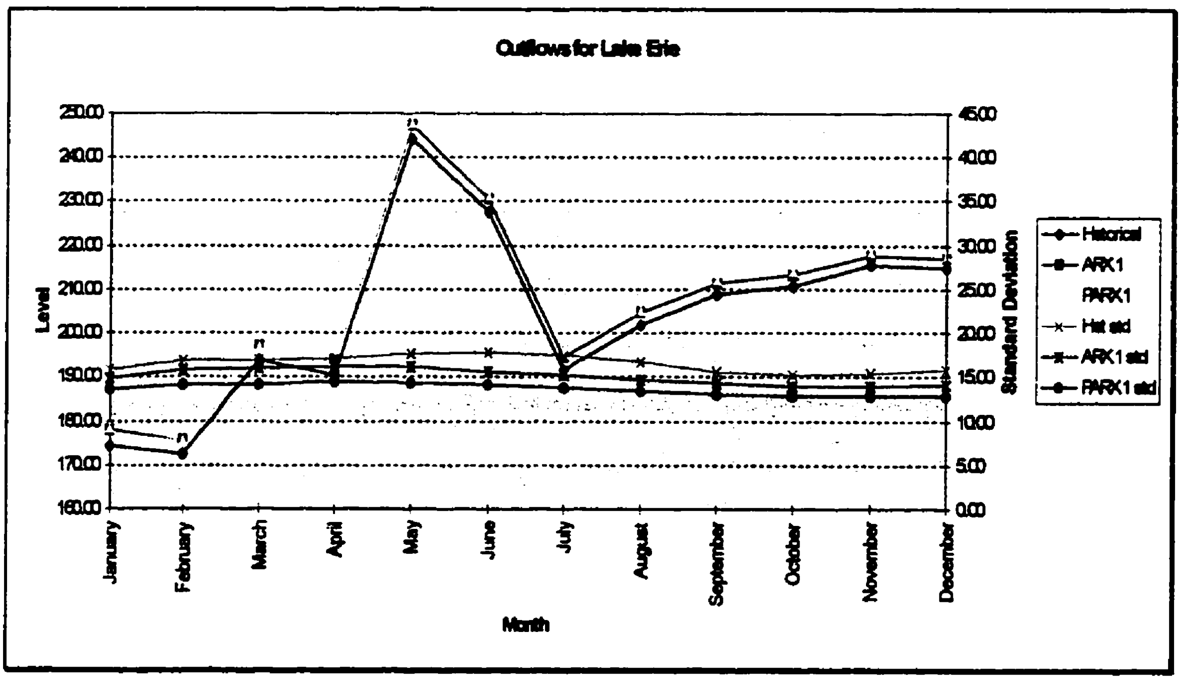


Figure E - 4 Lake Erie, Simulation with Historical and Synthetic Net Basin Supplies

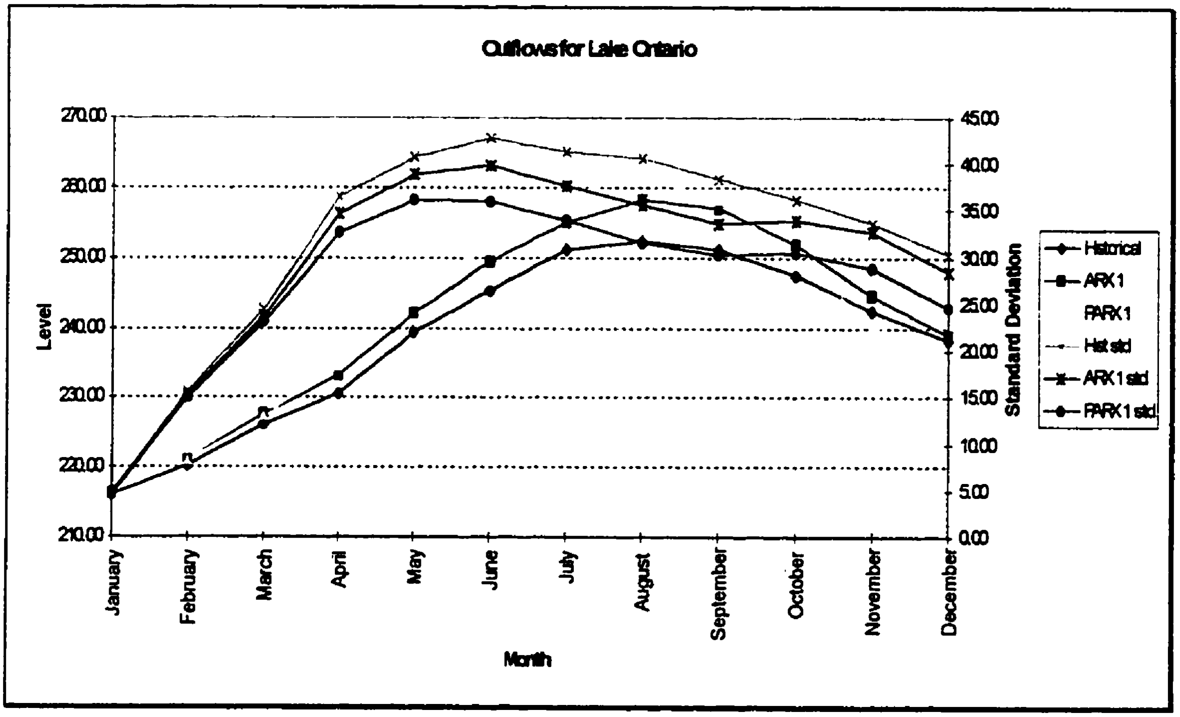


Figure E - 5 Lake Ontario, Simulation with Historical and Synthetic Net Basin Supplies

2. Comparison between IJC's Operation and Steady-State Optimization using aggregation:

Results obtained for Synthetic NBS

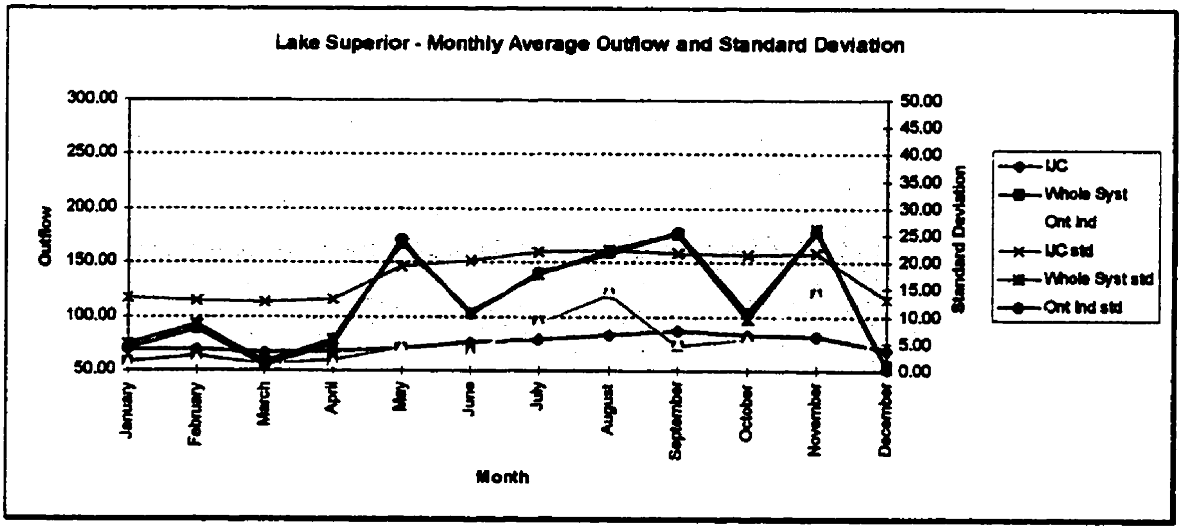


Figure E - 6 Lake Superior, Comparison Between IJC and Optimization Policies

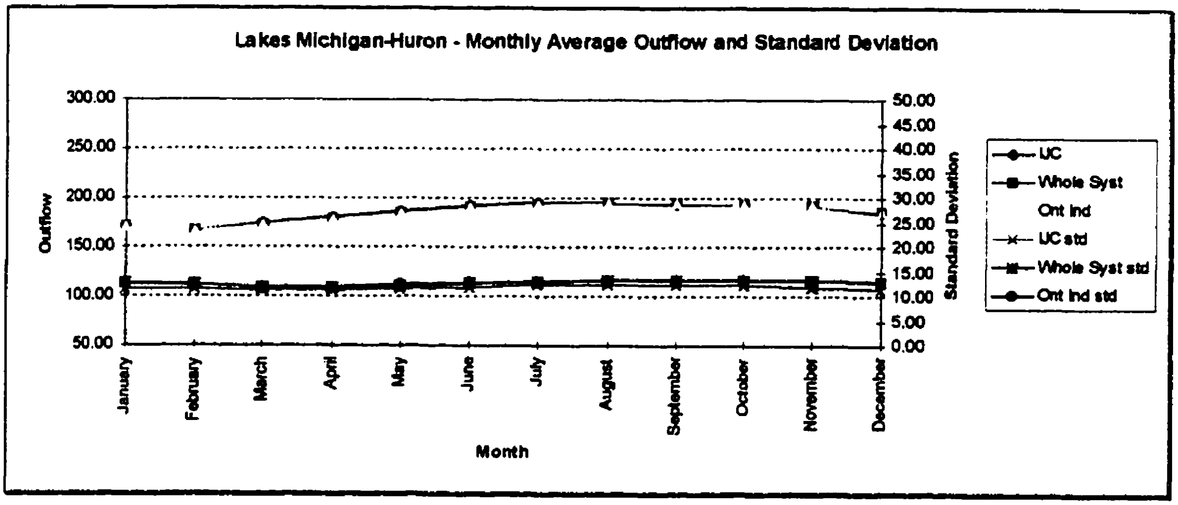


Figure E - 7 Lakes Michigan-Huron, Comparison Between IJC and Optimization Policies

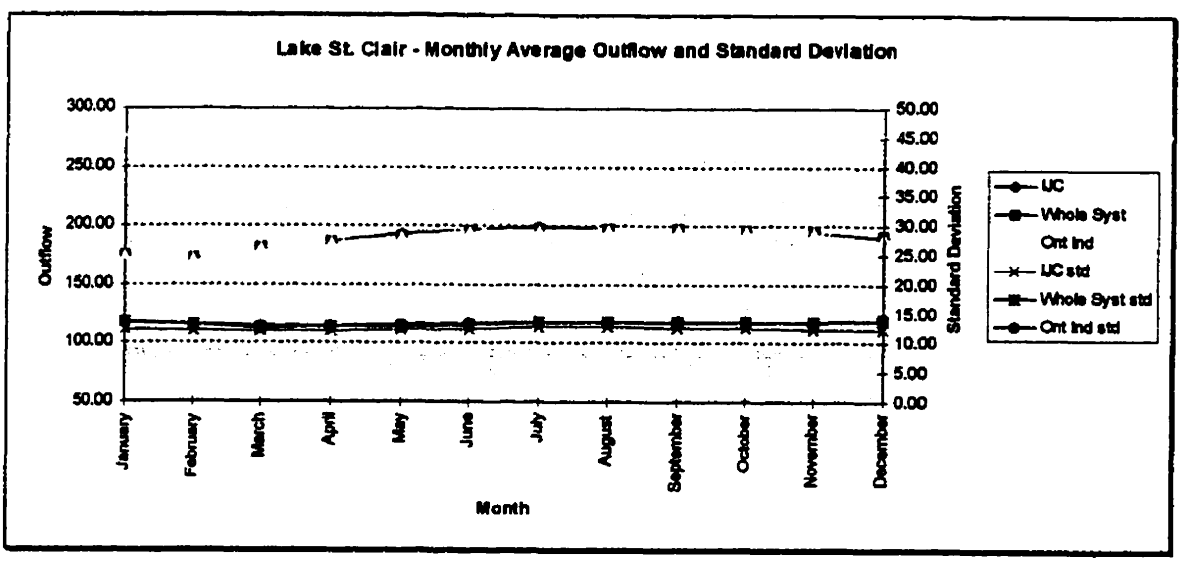


Figure E - 8 Lake St. Clair, Comparison Between IJC and Optimization Policies

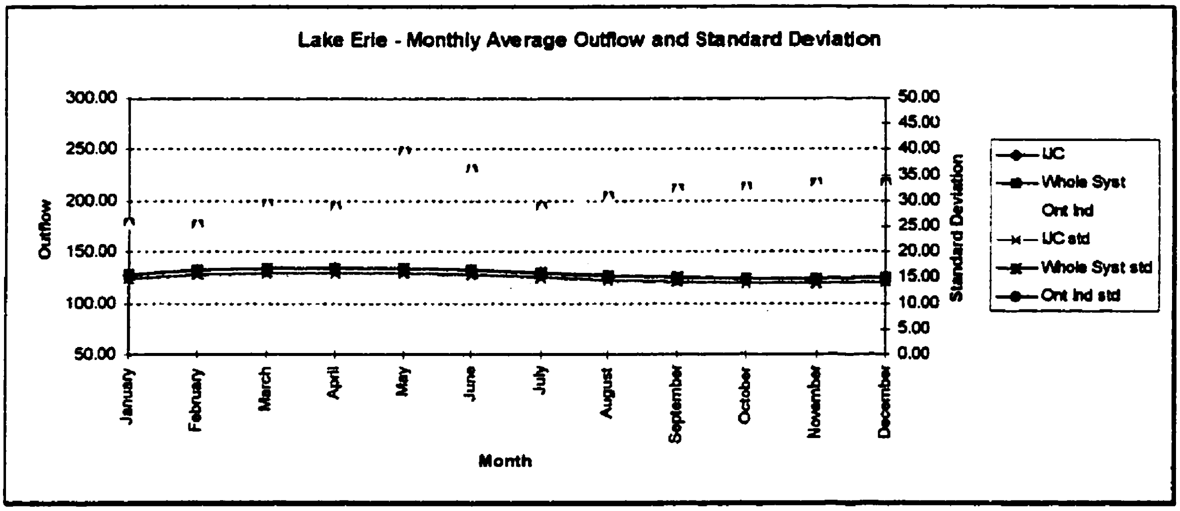


Figure E - 9 Lake Erie, Comparison Between IJC and Optimization Policies

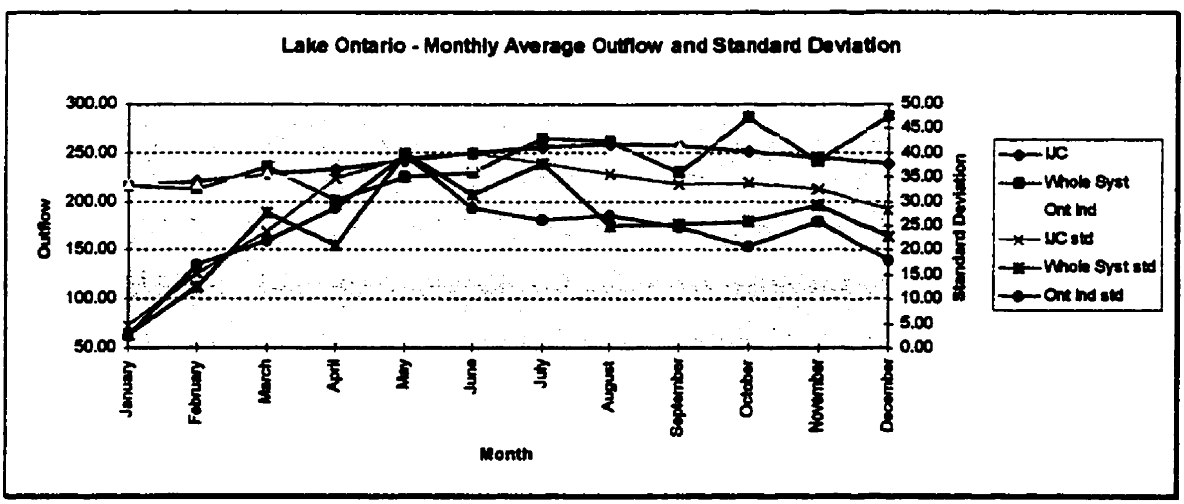


Figure E - 10 Lake Ontario, Comparison Between IJC and Optimization Policies