# Upper Bounds to the Capacity of Wireless Networks

by

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# AUTHOR'S DECLARATION

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

# Abstract

In this thesis, I mainly focus on the evaluation of the upper bounds to the capacity of wireless networks. With the consideration of the two measures, the maximal transmission rate for any source-destination pair and the *transport capacity* of wireless networks, I summarize the most recent results to the upper bounds of these two measures first in this thesis. At the same time, I also improve and modify the previous results given in these papers. Moreover, I present a proof to the upper bound of maximal transmission rate with high probability by taking the fading of the channel into account when the full CSI is only known to the receivers. With a simple extension of the result, I derive an upper bound to the transport capacity of wireless networks without full CSI at the receiver side. A linear scaling of the upper bound to transport capacity is also derived when the path loss exponent $\alpha$ is greater than three. Compared with the previous results, it is shown that the upper bound given in this thesis is much better for relatively large $\alpha$ and a minimum distance constraint.

# Acknowledgements

Firstly I would like to thank Professor Mohamed Oussama Damen and Professor Pin-Han Ho for agreeing to read my thesis.

Secondly, I would like to thank my supervisor, Professor Liang-liang Xie, for his continuous encouragement, enthusiasm and instruction. Whenever I was blocked by shorting of ideas and the difficulty in proof, he was always there to help and encourage me. I greatly appreciate the time and effort he has spent on me.

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# Chapter 1

# Introduction

## 1.1 Introduction to Upper Bounds to the Capacity of Wireless Networks

Wired and wireless networks have enabled a wide range of devices to communicate with each other over vast distances. But with the constraint in physical realization of wired networks, people put more efforts on the research of wireless networks. Networks like Ad Hoc networks [1] Figure 1.1, mesh networks and sensor networks, have no wired backbone and all communications share the wireless medium. With the topical interest in wireless networks, people concentrate on deriving different metrics to evaluate the performance of wireless networks. And it is fundamental to determine the capacity of the wireless networks, but this question has not been completely solved even in several very simple scenarios, like relay channel and interference channel [12,Ch.14].

However, still a lot of outstanding work has been done to evaluate the capacity of wireless networks. In the seminal paper written by Gupta and Kumar [1], they initiated the study of scaling laws in large wireless networks. The concept of the *transport capacity* was first introduced which measures the distance weighted total rate that a wireless network can support. It was shown that the transport capacity of a wireless network scales with the square root of the product of the area of the network and the number of the nodes. And another result was obtained that a wireless network with random node distribution and every node chooses its destination randomly, then the uniform communication rate for each source-destination pair scales with $\Theta(\frac{1}{\sqrt{n\log n}})$.

Xie and Kumar [2] confirmed this result from the information theoretic point of view with a strong assumption on the signal attenuation level over distance. They showed that whenever the path loss exponent $\alpha$ of the environment is greater than

***Introduction***

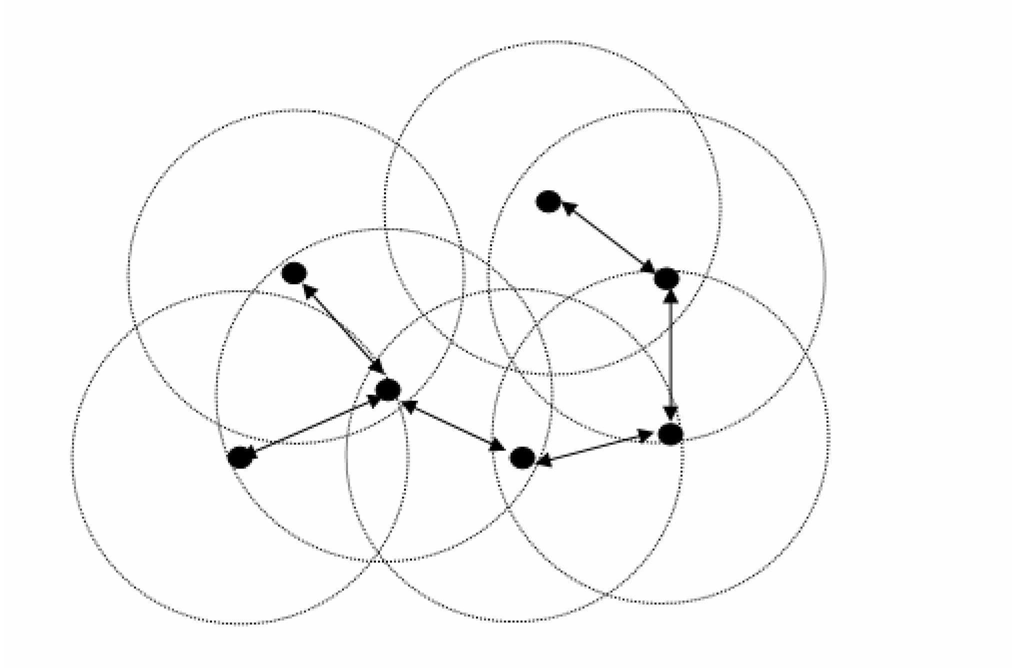

Figure1. 1 Ad Hoc Wireless Network.

four in one dimensional networks or greater than six in two dimensional networks, the network transport capacity is upper bounded by a multiple of the total transmitted power $P_{total}$. And in the following work [8], they further improved the results to lower attenuation region that with random phases of the signal attenuations, the transport capacity of wireless Ad Hoc networks is upper bounded by a multiple of the total power transmitted by all transmission nodes when the path loss exponent $\alpha$ is greater than 4 for two-dimensional networks, and $\alpha$ is greater than 2.5 for one-dimensional networks. In the case that the phases are arbitrary, the transport capacity is upper bounded by a multiple of the total transmission power in the network when $\alpha$ is greater than 5 for two-dimensional networks, and $\alpha$ is greater than 3 for one-dimensional networks. These are the best results known until now, and it narrows the attenuation region where the upper bounds to transport capacity achieved a linear scaling. But in the region of path loss exponents $3 \le \alpha \le 5$ for two dimensional networks, and $2 \le \alpha \le 3$ for one-dimensional networks, it is still an open problem and they have showed that one cannot improve the result with the same technique used in their paper.

These results imply the fact that if all the pairs are chosen randomly without consideration of the relative locations, the common rate R for any pair goes to zero by $1/\sqrt{n}$ for an example of two-dimensional network. Leveque and Telatar [4] showed that when all the nodes are uniformly and independently distributed in the extended network, the maximal achievable rate tends to be zero when the number of nodes goes to infinity. And they proved the results with a weaker assumption on the attenuation level for one dimensional network [5]. In the subsequent paper [6] Ozgur, Leveque and Preissmann generalized the results to any attenuation level $\alpha > 0$, they obtained the communication rate per source-destination pair is bounded by 1/n up to a factor $(\log n)^3$ for one dimensional network and bounded by $\frac{n^{\frac{1}{\alpha+8}}}{n^{1/2}}$ up to a factor of $(\log n)^3$ for two dimensional network. A quick extension to the transport capacity can be obtained from the results that the transport capacity is bounded by n up to a factor of $(\log n)^3$ for one dimensional network and bounded by $n^{1+\frac{1}{\alpha+8}}$ up to a factor of $(\log n)^3$ for two dimensional network with arbitrary vale of $\alpha > 0$.

Then Ozgur, Leveque and Tse [7] considered the channel model with multi-path fading and assumed that both the transmitters and receivers know the full channel state information, an upper bound on the aggregate throughput of extended network is obtained that with random source-destination pairing the total throughput is upper bounded by $K n^{2-\alpha/2+\varepsilon}$ when the path loss exponent $\alpha$ satisfies that $2<\alpha<3$, and is upper bounded by $Kn^{1/2+\varepsilon}$ when the path loss exponent $\alpha$ is greater than three.

In this thesis, I summarize the most recent results about the upper bounds to the capacity of wireless networks for different channel models. At the same time I also present some corrections and improvements of the previous results. Moreover, I also provide a proof to the upper bound of the maximal communication rate per source-destination pair for wireless networks that, with high probability, the maximal transmission rate per source-destination pair goes to zero with the increasing size of the network when $\alpha > 2$ and a linear scaling to the upper bound of the transport capacity is obtained when $\alpha > 3$. Unlike the channel model used in the paper above which I have just mentioned, we assume the full CSI is only known to the receiver side. Moreover, a comparison with the previous results in [9] shows that my upper bound is much better with relatively large path loss and minimum distance constraint.

## 1.2 Thesis Organization

This thesis is organized as follows:

Chapter 2 provides the descriptions of the background on the network modeling and channel modeling.

Chapter 3 discusses the results to the capacity of wireless networks. Specifically, I will mainly focus on the paper [4], [5], [6], [7], [8] [9] and [10]. The basic ideas and fundamental results are discussed in details about the upper bound to maximal communication rate and the transport capacity. A detailed summarization will be given in this chapter.

Chapter 4 presents some corrections and modifications I have made about the previous work. A detailed description will be given.

Chapter 5 provides a new proof to the upper bounds to maximal communication rate for the channel model with multi-path fading with the assumption that the full channel state information is only known to the receiver side. And an extension to the upper bound of the transport capacity is presented in the following. A comparison between my results with the previous results is given.

Chapter 6 concludes the thesis briefly.

# Chapter 2

# System Model

## 2.1 Background

To describe a wireless channel, people mainly focus on the variations of the channel strength over time Figure 2.1 and over frequency. And two kinds of variations of wireless channel are well-known:

- *Small-scale fading*, it is caused by the multiple signal paths between the transmitter and receiver and usually it is frequency dependent.
- *Large-scale fading*, it is caused by the signal attenuation. Usually it is denoted as the path loss of signal and can be expressed as a function of the distance between the transmitter and the receiver. Moreover, it can be also caused by the shadowing of large objects. This kind of fading is typically frequency independent [15].

The distance between transmitter and receiver is very important to evaluate the channel characteristic. As the distance $r$ increases, the electric field decreases as $r^{-1}$ then the power in free space wave decreases as $r^{-2}$ and it is often not valid when there are obstructions in free space propagation. In order to describe the signal attenuation with distance, the parameter called path loss exponent $\alpha$ is used and the power of a signal decays with distance by $r^{-\alpha}$. Another parameter called absorption constant is also presented in some model. The power decays with distance by $e^{-\gamma r}$.

For the *Small-scale fading*, usually it is difficult to handle. For simplicity, people usually use a random phase term to characterize the multi-path fading for each channel. And the Rayleigh fading channel is the simplest probabilistic model. As compared with the wavelength, the distance between reflectors and scatterers are much farther, it is reasonable to assume the phase of each path is uniformly distributed and independent to each other. Then for a large number of independent paths, the channel gain can be modeled as a complex zero-mean Gaussian random variable through Central Limit Theorem. Consequently, the magnitude of the channel gain is a Rayleigh random variable. This model is frequently used with the constraint

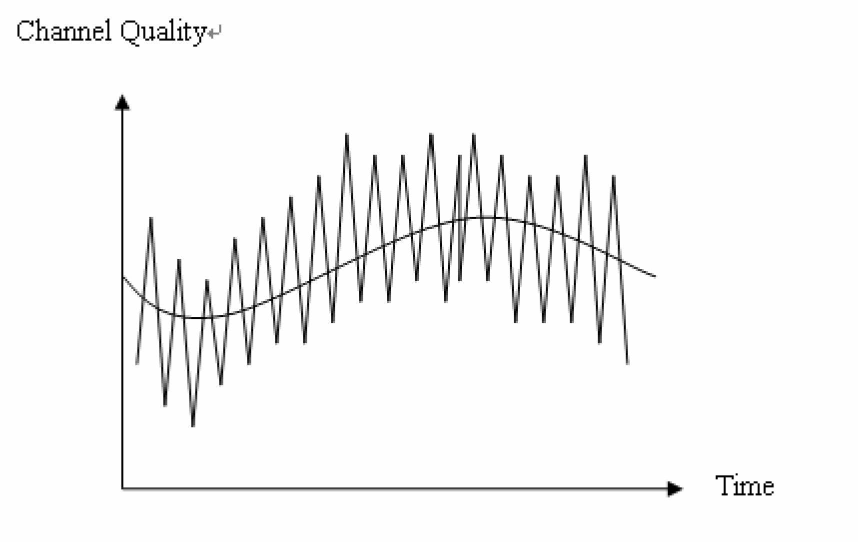


Figure 2. 1 Channel quality varies over multiple time-scales.

that the channel gains are circularly symmetric complex Gaussian random variables.

Usually, there is a standard assumption that an additive noise is included in each channel. And it is quite normal and reasonable to assume all the noises are independent with each other and are identically distributed with a Gaussian distribution. This assumption of AWGN essentially means all the additive noises are from the receiver side which are independent with different receivers. Moreover, there is no relation between the noises and the channel as well as the transmitters. And in practice, this model has been popularly applied to most of the communication situations [15].

## 2.2 Network Model

### 2.2.1 Dense Networks

For a dense network, Figure 2.2, the area is fixed, usually is defined as with area one for simplicity, and the node density is increasing while the number of nodes in the network is increasing, or equivalently, the number of nodes increases in a constant area which leads to the increasing of the node density. In this scenario, it is quite obvious to find that the affect of interference among all the nodes becomes more and more severe with the increasing number of nodes in the network.

*System Model*

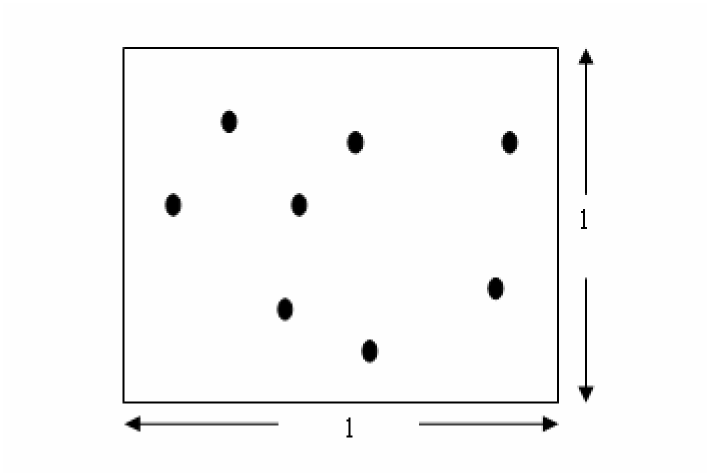


Figure 2. 2 Two dimensional dense network with n nodes and area one

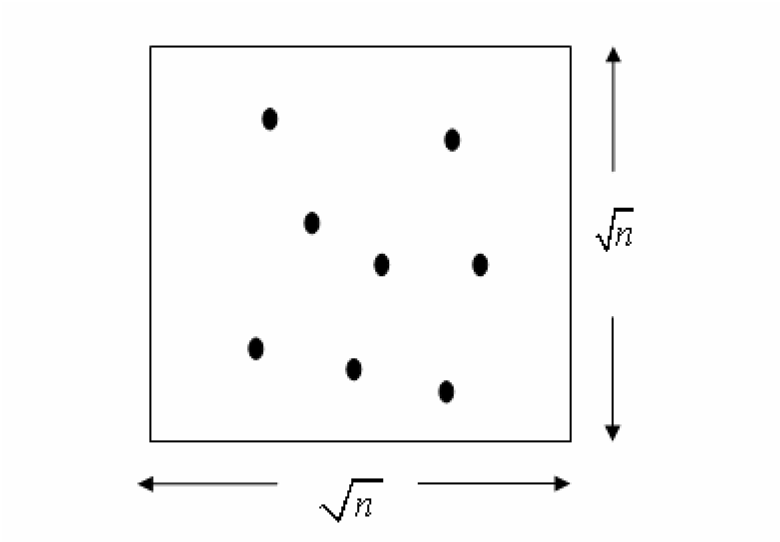


Figure 2. 3 Two dimensional extended network with n nodes and area n

### 2.2.2 Extended Networks

For an extended network Figure 2.3, the node density of the network is fixed, i.e., the number of nodes increases with the area at the same time and the density keeps constant. In this scenario, compared with the dense network, the distance between two nodes is scaled by $\sqrt{n}$, take two-dimensional network for instance, then with the same transmission power, the received power for the intended transmitter decreases

***System Model***

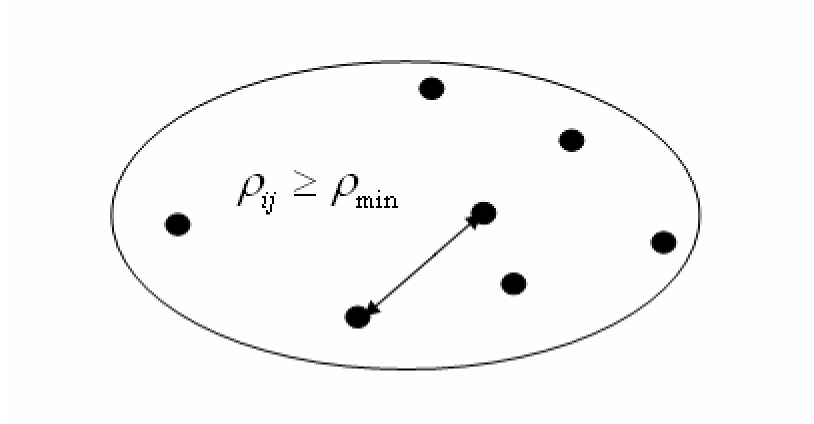


Figure 2. 4 A planar network with the minimum separation $\rho_{\min}$.

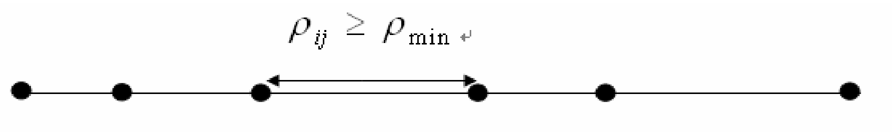


Figure 2. 5 A linear network with the minimum separation $\rho_{\min}$.

by a factor of $n^{-\alpha/2}$. So the extended network can be assumed to be equivalent to the dense network with the average power constraint per node reduced to $P/n^{\alpha/2}$ instead of $P$.

## 2.3 Channel Model

For the channel formulation, people focus on the characterization of the signal attenuation with distance and the fading due to multi-path. Next I will present two different kinds of channel models with and without small scale fading which are with great interest in channel modeling.

### 2.3.1 Channel Model without Consideration of Small Scale Fading

Xie and Kumar [2] firstly considered the problem in *information theoretic* point of view. And their fundamental system model is essentially frequently used in the following work. The model used is called linear network for one-dimensional Figure 2.4 and planar network for two-dimensional Figure 2.5.

Take the planar network for an example, the network model is defined as follows,

A. n nodes uniformly located in the plane.

B. The minimum distance between any two nodes is $\rho_{\min}$, i.e., $\rho_{ij} \geq \rho_{\min}$ for any two nodes i and j in the network, where $\rho_{ij}$ is the distance between i and j.

C. Each node has both a transmitter and a receiver. At each time instant, node i sends signal $X_i(t)$ and receives signal $Y_i(t)$, which satisfies

$$Y_i(t) = \sum_{j \neq i} \frac{e^{-\gamma \rho_{ij}} X_j(t)}{\rho_{ij}^{\delta}} + Z_i(t)$$

and $Z_i(t)$ is the additive noise which is Gaussian independent and identically distributed random variable with zero mean and variance $\sigma^2$. The two constants $\gamma \geq 0$ and $\delta$ >0 are the absorption constant and path loss exponent.

D. Let $P_i$ denote the transmit power used by node i, and the power satisfies the individual power constraint that $P_i \leq P_{ind}$ for any i. At the same time, all the transmitting nodes satisfy the total power constraint that

$$\sum_{i=1}^{n} P_i \leq P_{total} .$$

E. There are several source-destination pairs $(s_l, d_l)$ where $s_l \neq d_l$, and each node can only pick up one node uniformly as its destination.

Essentially, this model is an extension of single user Gaussian additive noise channel to a network version which relates the channel gain to the distance between the transmitter and the receiver. The most special case of the planar network is called regular planar network, for which, all the nodes are located at the cross points of a square lattice. And similarly, the regular linear network is defined as the one-dimensional network with all the nodes distributed in the integer point of a line with the interval length defined as the minimum distance constraint. And usually, it makes sense to assume the minimum distance constraint between any two nodes, since it is provable that with high probability, there exists such a minimum distance in the network. Thus, the regular network just simplifies the problem by knowing that all nodes have fixed locations.

In the paper by Leveque and Telatar [4], Ozgur, Leveque and Preissmann [6], they all considered the same network model and channel model. And they also take a much more simple case into account that there is no absorption in the channel model, i.e., $\gamma = 0$.

### 2.3.2 Channel Model with Consideration of Small Scale Fading

Usually, the small scale fading caused by multi-path can be taken into account by adding a random phase term in the channel model that

***System Model***

$$Y_i(t) = \sum_{j \neq i} \frac{e^{j\theta_{ij}} X_j(t)}{\rho_{ij}^{\delta}} + Z_i(t)$$

where $\theta_{ij}$ is the random phase for the channel from node i to node j caused by the multi-path from i to j. And the random phase term is defined as uniformly distributed in the interval $[0, 2\pi]$. The phase term is considered to be independent with the distance from i to j. Usually a constant term G which represents the gain from the transmitter and receiver is added to the channel model and the carrier wavelength is omitted in the channel model. In the paper [7],[8],[9] and [10], they all have considered the channel model with multi-path fading. And usually, the evaluation of the upper bound to capacity of wireless networks is different due the existence of the small scale fading.

### 2.3.4 Channel State Information

In the channel modeling, it is also important to know whether the information about the channel state is available or not. Usually, there are two different kinds of assumptions that the full channel state information is fully known to both the transmitters and receivers or the full channel state information is only known to the receivers. The difference between the two cases is, if the full channel state information is known to all the transmitters, then all the transmitters can cooperate and distribute the transmission power according to the channel state information. Otherwise, all the transmitters can only transmit with fixed power allocation. And intuitively, it should perform much better with the full information of the channel state known to both transmitters and receivers.

# Chapter 3

# Upper Bounds to the Capacity of Wireless Network

A lot of insightful results have been presented about the capacity of wireless networks in the past few years. People mainly focus on the measure of maximal communication rate and the transport capacity to evaluate the performance of the networks. In the discussion of the two metric, all the techniques are based on the cut set bound which was first introduced by Cover [12]. So in the following, I will present the most recent results about the upper bound to maximal communication rate and the transport capacity based on single cut analysis and multi-cut analysis. And I will also provide two distinct models for different channel characterization which are frequently used. Then the upper bound to the transport capacity over fading channels will be presented in the end.

## 3.1 Cut-Set Bound

Cut set bound is the most frequently used and the only one can be applied to the derivation of upper bounds to the throughput and the transport capacity for wireless networks. With consideration of the spirit of Max-Flow Min-Cut Lemma into information theory, the cut set bound was first introduced in [12]. Then all the analysis of upper bound is based on this general method.

For a general wireless network with multiple transmitters and receivers randomly distributed in an area. Each node has both transmitter and receiver with transmitted variable $X^{(i)}$ and received variable $Y^{(i)}$ for node i. For any two different nodes i and j, they communicate with rate $R^{(ij)}$ and all the messages sent from i to j are independent and uniformly distributed over the message alphabet $\{1,2,\ldots,2^{nR(ij)}\}$. If there exists a

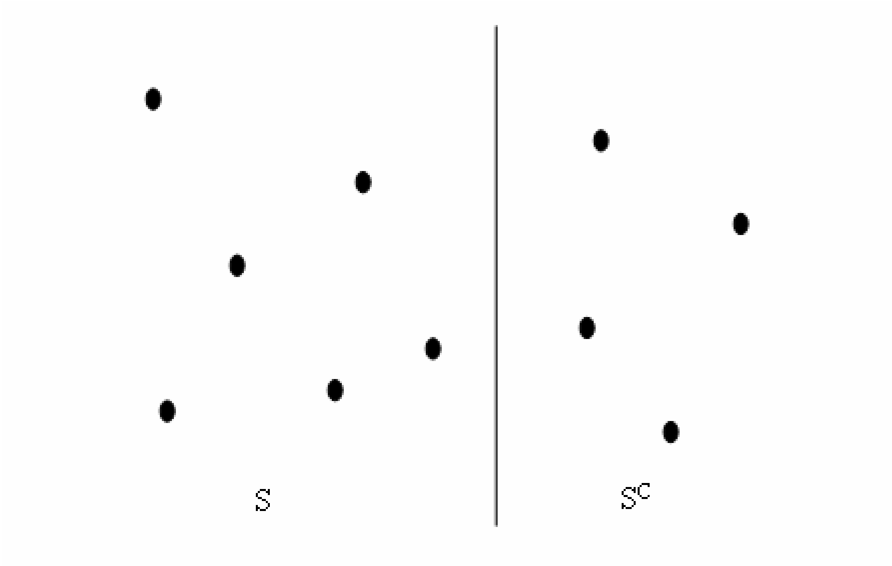


Figure 3. 1 Cut set applied to wireless network

random cut which divides the network into two parts, S and $S^C$ , Figure 3.1, then the total communication rate through the cut from the nodes in S to $S^C$ is upper bounded by the conditional mutual information that

$$\sum_{i \in S, j \in S^C} R^{(ij)} \le I(X_S; Y_{S^C} \mid X_{S^C}) \qquad (1)$$

So the interpretation of the cut-set bound is the total rate of the information flow across the cut is less than or equals to the mutual information between the inputs in one side and the outputs on the other side, conditioned on the inputs on the other side. But this upper bound is not tight for most cases.

This nice upper bound can be simply applied to the analysis of the upper bound of throughput and the transport capacity of wireless network. Basically, people use two different ways which are both direct application of cut set bound. Firstly, consider only one cut which divides the network into two parts. In [4],[5],[6], they consider a deterministic cut that divide the network into two equal parts and in [9], they consider a random cut which also divide the network into two parts. On the other hand, Xie and Kumar first use the multi cut analysis to derive the upper bound which is much more precise. Because there is a nice relationship between the transport capacity and the cut set bound applied to multiple cuts, obviously, the multiple cut analysis can provide a much better result on the transport capacity since now the total communication rates through the cut which divides the network into two equal part and the cut which divides the network into two parts with one node and n-1 nodes are totally different.

The relationship between cut set bound and the transport capacity ensures the accuracy to use multi cut analysis. Moreover, people can use it to make a connection between communication rate and the transport capacity. For one dimensional network as an example, the transport capacity and the multi-cut set bound is related by,

$$C_T = \sum_{i \neq j} R^{(i,j)} \rho_{ij} \tag{2}$$

$$= \sum_{i<j} R^{(i,j)} \rho_{ij} + \sum_{i>j} R^{(i,j)} \rho_{ij} \tag{3}$$

The first part in equation (3) can be represented by

$$\sum_{i<j} R^{(i,j)} \rho_{ij} = \sum_{i<j} R^{(i,j)} \sum_{k=i}^{j} \rho_{kk+1} \tag{4}$$

$$= \sum_{k=1}^{n-1} \rho_{kk+1} \sum_{i=1}^{k} \sum_{j=k+1}^{n} R^{(i,j)} \tag{5}$$

Obviously, $\sum_{i=1}^{k} \sum_{j=k+1}^{n} R^{(i,j)}$ is just the total rate through the $k^{th}$ cut from left to right between the two node k and k+1 which can be represented by $R_{k+}$.

Similarly, the same process can be applied to the second term in equation (3). So the transport capacity can be represented by

$$C_T = \sum_{k=1}^{n} \rho_{kk+1} (R_{k+} + R_{k-}) \tag{6}$$

This result can also be extended to two-dimensional case [6].

## 3.2 Single Cut Analysis to the Capacity of Wireless Networks

For the analysis to the capacity of wireless networks according to cut-set bound, usually there are two kinds of application. The simplest one is the single cut analysis which is defined as there exist only one cut dividing the network into two equal parts.

Then people focus on the total throughput across the single cut and the maximal communication rate across the cut for any source-destination pairs. Although it is not as tight as the multi-cut analysis, it simplifies the problem to evaluate the performance of the network. Several papers have used this method to derive the upper bound to capacity of wireless networks and I will present them in details in the following. Moreover, I will mention two kinds of different channel modeling in the application of the single cut analysis.

### 3.2.1 Upper Bounds to the Capacity of Wireless Networks for the Channel Model without Fading

There is a series of work which concentrate in the scenario that all the nodes are uniformly distributed in the network with constant node density. An information theoretic upper bound to the communication rate is derived that the maximum communication rate goes to zero with the increasing of the network size.

In the first work [4], the model used is called uniformly distributed network. In the d-dimensional region

$$\Omega_n = [-n^{1/d}, n^{1/d}] \times [0, n^{1/d}]^{d-1}$$

with n users uniformly distributed in it. The single cut analysis is applied to the network which divides the region into two equal groups, and there are n/2 nodes in each group. Every node can be either a transmitter or a receiver, so statistically, there are about n/8 communications crossing the cut from left to right hand side. We can only consider these nodes because the deviations from the idealized situation are of order much smaller than n when n goes to infinity with high probability. Moreover, another n additional users which are called “mirror” nodes are introduced in helping relaying communications Figure 3.2. These n additional nodes (with n/2 on each side) not only help the communication, but also bring the nice symmetry to this problem, because with the mirror nodes, there are n nodes on both sides which are symmetrical according to the single cut in middle, then the channel gain matrix G is therefore a symmetric matrix which simplifies the analysis of the capacity.

Based on the equation (1), a simple upper bound for the total communication rates from the left to right hand side can be derived with the relation between $X_S$ and $Y_{S^C}$ that

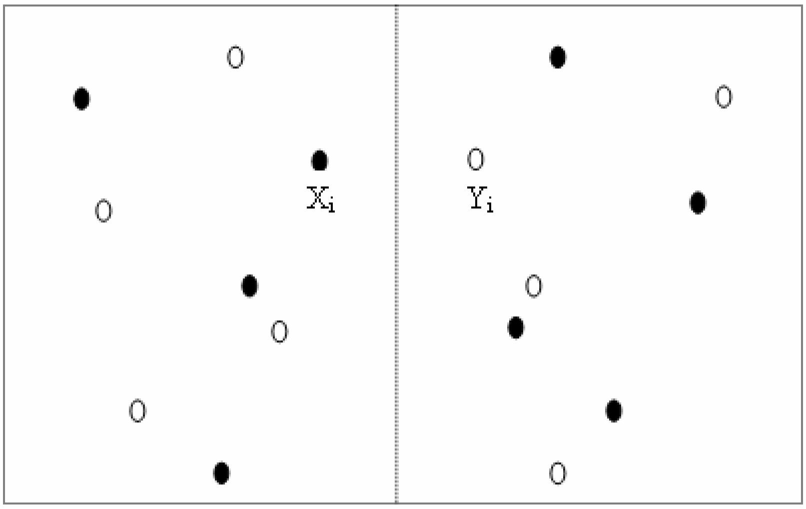


Figure 3. 2 The wireless network with the "mirror" nodes and the white ones represent the original nodes and the black ones represent the mirror nodes.

$$Y_{S^C} = GX_S + Z \qquad (7)$$

where G is the channel matrix with each element $G_{ij}$ is corresponding to the channel gain from node i on the left hand side to node j on the right hand side. So

$$Y_j = \sum_{i=1}^{n} G_{ij} X_i + Z_j$$

As the jointly Gaussian vector achieves the maximum mutual information in equation (1), it can be represented by logarithm function in terms of the channel gain matrix and input variables' autocorrelation matrix, and the basic idea used here is to convert the matrix form to scalar case by Hardmard's inequality which simplifies the calculation. By the symmetry of the channel gain matrix, the whole problem becomes easy by taking use of the majorization theory for the Schur-concave function.

The majorization theory which is not introduced in details in [4], is defined as a vector $X = [x_1, x_2 ... x_n]$ majorizes another vector $Y = [y_1, y_2 ... y_n]$ which is represented by

$$X \succ Y \quad \text{or} \quad [x_1, x_2 ... x_n] \succ [y_1, y_2 ... y_n]$$

if

$$\begin{cases} \sum_{i=1}^{l} x_i \geq \sum_{i=1}^{l} y_i & l = 1, 2 \ldots n - 1 \\ \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i & \end{cases}$$

Intuitively, the majorizaiton argument for vector X and Y means the elements of Y are much closer to each other than the X vector.

The majorization theory for the Schur-concave function is for any Schur-concave function $f(X)$ where the argument X is a vector, then $f(X) \leq f(Y)$ if $X \succ Y$.

So the key point in this proof is just to use the result that the eigenvalues of the channel gain matrix majorizes the diagonal elements. And the diagonal elements of channel gain matrix G is easy to handle since the gain is only related to the distance between any two nodes, so the upper bound can be simply derived. Moreover, all the upper bounds obtained in this paper are held with high probability. And there is a good observation here about how to derive an upper bound with high probability. Basically, they tried to find the expected communication rate due to the randomness of the node location. And based on the expected value of communication rate, if the deviation from the mean value is sublinear to the expected value, then one can say the upper bound will hold for any communication pair with high probability when the number of nodes n goes to infinity.

The result obtained in this paper can be summarized as

$$\begin{cases} R \leq K \dfrac{\log n}{n^{1/d - 1/\alpha}} & \text{almost sure} \quad \text{without absorption} \quad \alpha > d \vee 2(d-2) \\ R \leq K \dfrac{(\log n)^2}{n^{1/d}} & \text{almost sure} \quad \text{with absorption} \quad \beta > 0 \end{cases}$$

Basically, they proved that the maximum transmission rate per communication pair in wireless network tends to be zero with the number of nodes goes to infinity under the condition that $\alpha > d \vee 2(d-2)$ without absorption or for any $\alpha$ with the presence of absorption. But obviously, the upper bound derived here is not tight due to the loose upper bound used to analyze the mutual information, there must exit a better analysis on the capacity with single cut. So the following work [5] and [6] just improved the result.

In [5], Leveque and Preissmann improved the result in one dimensional case for small values of the path loss exponent $\alpha$. The model they used is exactly the same as that defined in [4] with an additional minimum distance assumption that the minimum distance between any two nodes in the network is $\rho_{\min}$. Actually this assumption is quite reasonable, it is because for most random networks there do exit a minimum distance between any two nodes with high probability. With the same technique to upper bound the mutual information, they try to convert it to another matrix form instead of the majorization theory to analyze the upper bound. Although it is much more complicated, it obtains a much better result. The key step in their proof is they upper bounded the mutual information by a logarithm function in terms of channel gain matrix that

$$I(X;Y) \le \max_{P_k>0, \sum_{k=1}^{n} P_k \le nP} \sum_{k=1}^{n} \log(1+P_k\lambda_k^2) \tag{8}$$

$$\le 2\sum_{k=1}^{n} \log(1+\sqrt{nP}\lambda_k) \tag{9}$$

$$= 2\log\det(I+\sqrt{nP}G) \tag{10}$$

where $\lambda_k$ is the k-th eigenvalue of channel gain matrix G. The reason to upper bound the mutual information by equation (10) is to use the nice analytic expression of Cauchy Matrix that

$$\det(D(x_J)) = (\prod_{\substack{i,j\in J \\ i<j}} (x_j - x_i)^2)/(\prod_{i,j\in J} (x_j + x_i)) \tag{11}$$

and the elements of the matrix D($x_J$) is $D_{ij} = \dfrac{1}{x_i + x_j}$ for any i and j.

So as the exact expression of the determinant is known now, the upper bound will be much more accurate based on these analysis, therefore a better upper bound is obtained that

$$R \le \frac{K(\log n)^{3+\varepsilon}}{n}$$

for any $2 \le \alpha \le 4$ and $\varepsilon > 0$.

This upper bound approximately showed that the maximum transmission rate per communication pair goes to zero by 1/n up to a factor of order $(\log n)^{3+\varepsilon}$ with n denoting the number of nodes in the network. This improved the upper bound by $1/n^{1/\alpha}$ compared with the results that derived in the last paper for one dimensional

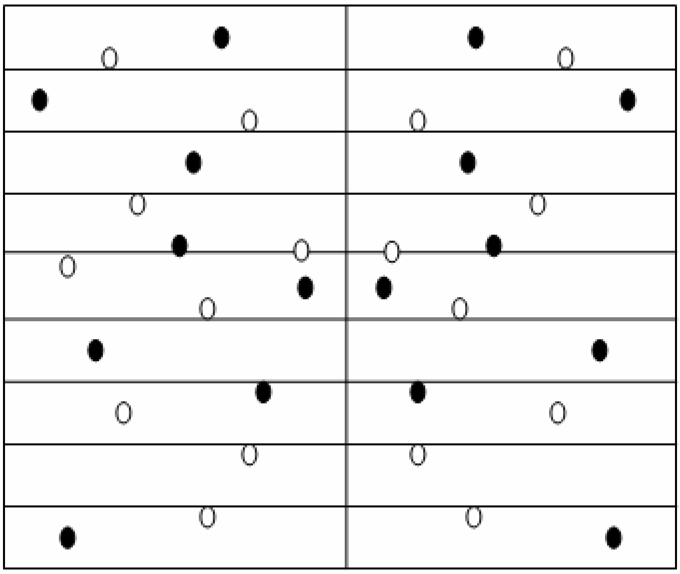


Figure 3. 3 Divide the two dimensional network into horizontal strips with length $\varepsilon$

network. Moreover, in the next paper [6], the result was even extended with a much weaker assumption on the pass loss level that for any value of $\alpha$, the maximal communication rate for each source-destination pair can be bounded above by

$$\begin{cases} R \le K_1 \dfrac{(\log n)^3}{n} & \text{One-dimensional} \\ R \le K_2 \dfrac{n^{\frac{1}{\alpha+8}} (\log n)^3}{\sqrt{n}} & \text{Two-dimensional} \end{cases}$$

and all the results presented here are obtained with high probability.

This upper bound provides a general evaluation on any kind of pass loss level, it was even generalized to the two-dimensional case by dividing the whole network into multiple of strips Figure 3.3.

With this division, approximately in each strip, it is assumed to be a one-dimensional network which has been already upper bounded before. Therefore, this approximation provides an upper bound to the maximal communication rate for two-dimensional network.

Moreover the upper bound can be simply generalized to the upper bound to total aggregate throughput of the network by multiplying the network size n because the total throughput can be represented by

$$T(n) = \sum R_{ij}$$

and there are at most n communication pairs, the total throughput can be simply bounded by

$$\sum R_{ij} \le nR_{\max}$$

And a trivial upper bound to the transport capacity of the wireless network can be obtained by applying the nice relation proved in the section 3.1 that

$$\begin{cases} T_C(n) \le K_3 n(\log n)^3 & \text{One-dimensional} \\ T_C(n) \le K_4 n n^{\frac{1}{\alpha+8}} (\log n)^3 & \text{Two-dimensional} \end{cases}$$

for any path loss exponent $\alpha > 0$ with high probability.

The upper bound implies that the transport capacity of a wireless network grows at most linear with n in one dimensional case, up to a logarithmic factor for any attenuation coefficient. But this upper bound on the transport capacity is trivial, because the essence of the proof is to upper bound the aggregate throughput first and then use this bound for every single cut with n times. Obviously, this bound is loose since the throughput through different cuts is totally different, like the cut which divides the network into two parts by one node on the left hand side and n-1 nodes on the right hand side and the cut which equally divides the network. So the upper bound to the transport capacity can be even improved my multi-cut analysis which has been done in [8], and I will talk about it later.

### 3.2.2 Upper Bounds to the Capacity of Wireless Networks for the Channel Model with Fading

The most recent paper written by Ozgur, Leveque and Tse [7] obtained the upper bounds to the maximum communication rate of wireless network with full CSI known to both the transmitter and receiver. Compared with the previous work introduced in last section, they considered the channel model with multi-path fading which is characterized by a random phase term. They also assume that the full channel state information are known such that the transmitters can cooperate and distribute the total power depending on the channel state information, i.e. the covariance matrix for the

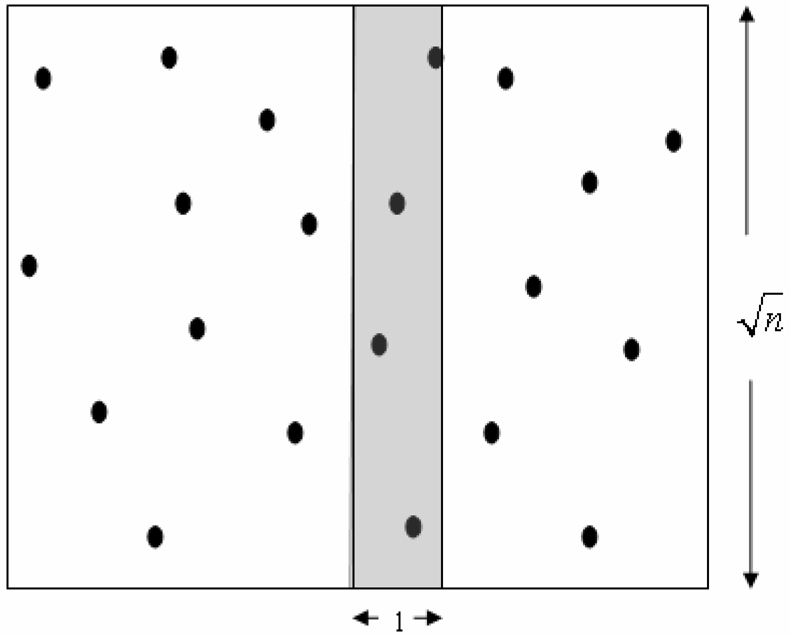


Figure 3. 4 A single cut applied to the wireless network. The right hand side is divided into two parts, the first part includes all nodes near the cut and the second part includes all nodes relatively far away from the cut.

transmitting variables is a function in terms of the channel state information. With the channel model defined in chapter 2.3.2, each node randomly chooses its destination without consideration of the location. For the single cut analysis, the network was divided into two equal parts with the same number of nodes on each side with high probability. For all the receiving nodes on the right hand side, they were divided into another two parts Figure 3.4. The nodes in the first part which are closed to the cut in the middle of the network will have a severe affect on the upper bound when the signal attenuation level is high, so consider the upper bound on the receiving nodes in two different parts separately will result in the better analysis on the upper bound in low attenuation level.

Compared with the previous work [4],[5] and [6], the key step in this work is to analyze the maximal eigenvalue of the scaled version of the channel gain matrix(see proof in [7]).

The upper bound can be derived as following

$$\begin{cases} R \le K' n^{1-\alpha/2+\varepsilon} & 2 \le \alpha < 3 \\ R \le K' n^{-1/2+\varepsilon} & \alpha \ge 3 \end{cases}$$

for any $\varepsilon > 0$ and $K'$ is a constant.

The most important thing of this proof is that they found independent signaling among all the transmitting nodes is enough to achieve the upper bound. Equivalently, with consideration of the random phase of the channel gain caused by the multi-path, there is no need for the transmitting nodes to cooperate or make use of any kind of beamforming. This is an interesting and useful result that it may simplify the analysis of the performance of the network due to the independence. Quite intuitively, this conclusion is due to the random phase introduced into the channel model. Another importance of the proof is that the dominant part of the upper bound of capacity is different for different attenuation level. When the pass loss exponent $\alpha$ is greater than three, then the received power in the cut set bound mainly comes from the power transferred from the nodes near the cut and correspondingly, the received power is mainly from the nodes far away from the cut when $\alpha$ is less than three.

Similar to the previous work, this upper bound can give a trivial upper bound on the transport capacity immediately by applying the nice relationship between multiple cut set bound and the transport capacity,

$$\begin{cases} T_C(n) \le K' n^{2.5-\alpha/2+\varepsilon} & 2 \le \alpha < 3 \\ T_C(n) \le K' n^{1+\varepsilon} & \alpha \ge 3 \end{cases}$$

For this result, the upper bound of the transport capacity is quite closed to a linear scaling, up to a factor of $n^{\varepsilon}$ for any $\varepsilon$ greater than zero when the attenuation factor $\alpha$ is greater than or equals to three.

## 3.3 Multi-cut Analysis to the Capacity of Wireless Networks

As the nice relationship between the multiple cut set bound and the transport capacity, the multi-cut analysis should provide a much better analysis to the upper bound of the transport capacity of wireless network. Xie and Kumar proved the linear scaling of the transport capacity of wireless network with and without multi-path fading for certain range of path loss level in 2006 [8]. Like the model used in [2], a network with n nodes randomly distributed is introduced. For each node, it can be both transmitter and receiver, and the received variable and the transmitted variable can be related by equation (12).

$$Y_j(t) = \sum_{\substack{i \in N \\ i \neq j}} g_{ij} X_i(t) + Z_j(t) \tag{12}$$

Without the "mirror" nodes introduced in the network, the channel gain matrix in this model is quite general in the form that

$$G = \begin{pmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,n} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ g_{n,1} & g_{n,2} & \cdots & g_{n,n} \end{pmatrix}$$

with each element $g_{i,j} = e^{i\theta_{ij}} / \rho_{ij}^{\alpha/2}$ for the channel model with multi-path fading and $g_{i,j} = 1/\rho_{ij}^{\alpha/2}$ for the channel model without multi-path fading.

The idea for the proof is to use the Hardmard's inequality to bound above the mutual information in terms of the channel gain matrix. Then by changing the order of the multiple sum, the linear scaling for the transport capacity can be obtained through strict proof. The results got in this paper are

1) For the channel model with multi path fading

$$EC_T \le \frac{\overline{c}_1(\delta) \log e}{\sigma^2 \rho_{\min}^{\alpha-1}} P_{total}$$

for one dimensional case with the path loss exponent $\alpha > 2.5$ and define $\alpha = 2\delta$ ,the constant $\overline{c}_1(\delta)$ is defined as

$$\overline{c}_1(\delta) = \frac{2(4\delta - 1)(4\delta - 3)^{1/2}}{(4\delta - 5)^{3/2}}$$

For two dimensional network

$$EC_T \le \frac{\overline{c}_2(\delta) \log e}{\sigma^2 \rho_{\min}^{\alpha-1}} P_{total}$$

with the path loss exponent $\alpha > 4$ and the constant $\overline{c}_2(\delta)$ is defined as

$$\bar{c}_2(\delta) = (\frac{2^{2\alpha+2}}{(\frac{\alpha}{2}-1)^{1/2}(\frac{\alpha}{2}-2)} + \frac{2^{2\alpha+4}}{(\frac{\alpha}{2}-1)^{1/2}(\alpha-3)} + \frac{2^{1.5\alpha+5.5}}{(\frac{\alpha}{2}-2)^{1.5}} + \frac{2^{\alpha+1.5}}{(\frac{\alpha}{2}-1)^{0.5}})$$

2) For the channel model without multi path fading

$$C_T \leq \frac{c_1(\delta)\log e}{\sigma^2 \rho_{\min}^{\alpha-1}} P_{total}$$

for one dimensional case with the path loss exponent $\alpha > 3$ and the constant $c_1(\delta)$ is defined as

$$c_1(\delta) = 2(1 + \frac{3}{2\delta-2} + \frac{1}{(\delta-1)(2\delta-3)})$$

For two dimensional network

$$C_T \leq \frac{c_2(\delta)\log e}{\sigma^2 \rho_{\min}^{\alpha-1}} P_{total}$$

with the path loss exponent $\alpha > 5$ and the constant $c_2(\delta)$ is defined as

$$c_2(\delta) = \frac{2^{2\delta+8}}{(\delta-2)^2}$$

Here $\bar{c}_1(\delta)$ and $\bar{c}_2(\delta)$ are two constants in terms of the path loss parameter $\delta = \alpha/2$, $\sigma$ is the standard variance of the complex Gaussian noise, $\rho_{\min}$ is the minimum distance between any two nodes in the network defined in the network model and $P_{total}$ is the total power constraint for the total transmitting power. It can also be represented by $nP_{ind}$ where $P_{ind}$ is the individual power constraint for any transmitting node. It is obvious to find that by simply replacing the $P_{total}$ by $nP_{ind}$, a linear scaling to the transport capacity of wireless network can be obtained immediately. So basically, the results showed the transport capacity of wireless network has a linear scaling behavior in a certain level of path loss attenuation. As it is already proved that the upper bound to the transport capacity do not have a linear scaling when $\alpha < 2$ for one dimensional case and $\alpha < 3$ for two dimensional case, there is a gap for the path loss exponent which is still an open problem.

These upper bounds also have implications for the ergodic the transport capacity of wireless network over fading channels, so it is also with great importance to find the performance of the transport capacity over general fading channels.

## 3.4 Upper Bounds to the Transport Capacity of Wireless Networks over Fading Channels

In the last few sections, the performance of the wireless network has already been evaluated in details. And the best result is the linear scaling for the upper bounds to the transport capacity of wireless network. So what happened to the general fading channels? Is there any affect to the upper bound of the transport capacity for wireless network? Xue, Xie and Kumar analyzed the performance of the transport capacity over fading channel and proved that the linear scaling still holds in a certain power attenuation regime [10].

The model used here is similar to the previous model defined in [8] except the fading process introduced in the network. N communication nodes randomly distributed in a two dimensional network, the model for the communication among them is described as

$$Y_j(t) = \sum_{i \neq j} \frac{\beta e^{-\gamma \rho_{ij}}}{\rho_{ij}^{\delta}} \left( \sum_{l=0}^{\infty} H_{ijl}(t) \cdot X_i(t - \tau_{ij} - l) \right) + Z_j(t) \qquad (13)$$

where $\gamma$ and $\delta$ is the absorption constant and one half of the path loss exponent as defined before. The random process $H_{ijl}(t)$ is the random fading process introduced in the network and the $\tau_{ij}$ is the propagation delay for the signals from node i to node j which can be also defined as $\lfloor \rho_{ij} / \rho_0 \rfloor$ with the distance from node i to node j as the nominator and the distance a signal traveling in one time slot as denominator.

With this channel model, when the $H_{ijl}(t) \equiv 0$ for all $l \geq 1$, the channel is flat fading. And reversely, when the $H_{ijl}(t)$ depends on the $l \geq 1$,i.e. $H_{ijl}(t) > 0$ when $l \geq 1$, the channel is frequency-selective fading. Moreover, when the $H_{ijl}(t)$ depends on t or we say the coherence time is longer, it is slow fading, correspondingly, when the fading parameter is independent from time to time, it means the channel coherence time is relatively small, therefore, the channel is fast fading. Based on this model, the upper bound to the transport capacity can be obtained that

$$C_T^{(n)} \leq c_1 \cdot n \qquad \text{for all n}$$

if the channel is slow fading or flat fading with positive absorption constant or without absorption and the path loss exponent is greater than 6 no matter the full channel state is known or not to both transmitters and receivers.

$$C_T^{(n)} \geq c_2 \cdot n \qquad \text{for all n}$$

if the channel is fast fading or frequency-selective fading with positive absorption constant or without absorption and the path loss exponent is greater than 6 no matter the full channel state is known or not to both transmitters and receivers.

Basically, this result just proved that there is no affect from the fading of the channel to the upper bound of the transport capacity in high attenuation regime or in the case with positive absorption. Or equivalently, the transport capacity cannot grow faster than linearly in the size of the network even if the fading process is known perfectly to all nodes in the network. But if the fading process is independent from time to time which should be the worst case, the transport capacity achieves linear scaling for a specific construction of the node locations.

For a conclusion of the whole chapter, Table 3.1 provides a summarization of the results of upper bound to capacity of wireless networks.

Table 3. 1 Upper bounds to the capacity of wireless networks

| | d=1 | condition | d=2 | Condition |
|---|---|---|---|---|
| Upper bound to communication rate of wireless networks | | | | |
| without fading [6] | $O((\log n)^3 / n)$ | $\alpha > 0$ | $O(n^{\frac{1}{\alpha+8}} (\log n)^3 / \sqrt{n})$ | $\alpha > 0$ |
| with fading [7] | N/A | N/A | $O(n^{1-\alpha/2+\varepsilon})$ | $2 \le \alpha < 3$ |
| | | | $O(n^{-1/2+\varepsilon})$ | $\alpha \ge 3$ |
| Upper bound to the transport capacity of wireless networks | | | | |
| without fading [6] | $O(n(\log n)^3)$ | $\alpha > 0$ | $O(nn^{\frac{1}{\alpha+8}} (\log n)^3)$ | $\alpha > 0$ |
| without fading [8] | $O(n)$ | $\alpha > 3$ | $O(n)$ | $\alpha > 5$ |
| with fading [7] | N/A | N/A | $O(n^{2.5-\alpha/2+\varepsilon})$ | $2 \le \alpha < 3$ |
| | | | $O(n^{1+\varepsilon})$ | $\alpha \ge 3$ |
| with fading [8] | $O(n)$ | $\alpha > 2.5$ | $O(n)$ | $\alpha > 4$ |
| with fading [9] | $O(n)$ without CSI | $\alpha > 2$ | $O(n)$ without CSI | $\alpha > 3$ |
| | $O(n)$ with CSI | $\alpha > 3$ | $O(n)$ with CSI | $\alpha > 5$ |

# Chapter 4

# Improvements to the Previous Results

In this chapter, based on the papers I have read, some improvements and modifications of those results obtained in the paper are presented. Specifically, a mistake in [4] is mentioned and some improvements to the upper bound of total throughput are provided. At the same time, the exact expressions for the constant parts of the upper bound obtained in [7] are given.

## 4.1 Upper Bounds to the Maximal Transmission Rate of Regular Networks: A Correction

In [4], Leveque and Telatar found the upper bounds to the maximal communication rate of regular network Figure 4.1 by taking the majorization argument into account. Although the results seemed quite reasonable, the proof is totally wrong due to a mistaken use of majorization argument.

First let me introduce the definition for majorization argument and the Schur's theorem which are used in [4].

For any two vectors $X$ and $Y$, we say $X$ majorizes $Y$ if the following relations are satisfied

$$\begin{cases} \sum_{i=1}^{l} x_i \geq \sum_{i=1}^{l} y_i & l = 1, 2 \ldots n-1 \\ \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i \end{cases}$$

where $X$ and $Y$ are defined as two $n \times 1$ vectors with the corresponding elements that $X = [x_1, x_2, ... x_n]$ and $Y = [y_1, y_2, ... y_n]$. And we use the notation $X \succ Y$ to represent the fact that $X$ majorizes $Y$. Basically, the majorization argument just

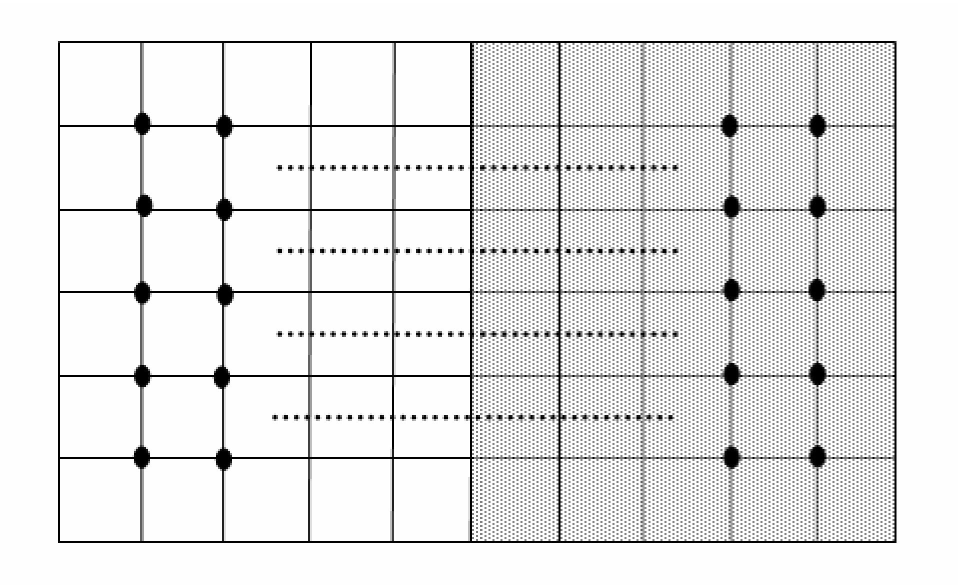

Figure 4. 1 Two dimensional regular networks.

indicate that with the sum of all elements equals to each other, the elements of $Y$ are much closer or distributed in a smaller interval than $X$ if $X \succ Y$.

With the definition for majorization argument, we have the corresponding theorem which is called Schur's Theorem that for any Schur-concave function $f(X)$ where the argument X is a $n \times 1$ vector, then $f(X) \le f(Y)$ if $X \succ Y$.

With these in mind, let me first briefly present the proof to obtain the upper bound to maximal communication rate of any pair for regular networks given in [4]. According to the inequality (3) derived in [4], the total communication rate can be upper bounded by

$$\sum_{i,j=1}^{n} R_{ij} \le \max_{p(X):\sum_{i=1}^{n} E(|X_i|^2) \le nP} I(X_1,...,X_n;Y_1,...Y_n)$$

$$= \max_{Tr(Q) \le nP} \log\det(I + GQG^*)$$

$$\le \max_{P_k \ge 0, \sum_{k=1}^{n} P_k \le nP} \sum_{k=1}^{n} \log(1 + P_k \lambda_k^2) \qquad (14)$$

where G stands for the channel gain matrix, and $Q$ denotes the autocorrelation matrix for the input variables $X_1, X_2, \dots X_n$ and $\lambda_k^2$ is the k-th eigenvalue of the matrix $G^*G$. There is a good observation here that $P_k$ in (14) is not the individual transmission power for transmitting node k due to the unitary transform for matrix $Q$ as following

***Improvements to the Previous Results***

$$\log\det(I+GQG^*)=\log\det(I+QG^*G)$$

based on the fact that $\det(I+AB)=\det(I+BA)$. And as $G^*G$ is Hermitian matrix which can be diagonalized as $U^*\Lambda U$ where U is a unitary matrix and $\Lambda$ is a diagonal matrix with its diagonal values equal to the eigenvalues of matrix $G^*G$. Therefore,

$$\begin{aligned}\log\det(I+QG^*G)&=\log\det(I+QU^*\Lambda U)\\&=\log\det(I+\Lambda^{1/2}UQU^*\Lambda^{1/2})\\&=\log\det(I+\Lambda^{1/2}\tilde{Q}\Lambda^{1/2})\end{aligned}$$

where $\tilde{Q}$ is defined as the matrix $UQU^*$ which can be considered as the unitary transform of matrix $Q$ and these two matrix $Q$ and $\tilde{Q}$ has the following property that

$$tr(Q)=tr(\tilde{Q})$$

Thus, the inequality of (14) is obtained from Hardmard's inequality, where $P_k$ stands for the k^th diagonal element for the matrix $\tilde{Q}$. At the same time, since the trace is still the same as before, so the maximization over $\tilde{Q}$ is therefore equivalent to the maximization over $Q$.

In the proof to get the upper bound of maximal communication rate of regular network, they apply the Schur's theorem and got the following upper bound since sum of logarithm function is a Schur-concave function

$$\sum_{k=1}^{n}\log(1+P_k\lambda_k^2)\le\sum_{k=1}^{n}\log(1+P_k(GG^*)_{kk}) \qquad (15)$$

And they have used the fact that the vector of eigenvalues majorizes the diagonal values of $GG^*$ that

$$\begin{cases}\sum_{i=1}^{l}\lambda_i^2\ge\sum_{i=1}^{l}(GG^*)_{ii} & l=1,2\ldots n-1\\ \sum_{i=1}^{n}\lambda_i^2=\sum_{i=1}^{n}(GG^*)_{ii}\end{cases} \qquad (16)$$

But actually this is not correct since there is also another term $P_k$ in the function which may affect the result. For an simple example, assume n equals to two and $P_1, P_2$ are fixed with $P_1=2P$, $P_2=0$. Then if (15) is correct, we should have

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$$\log(1+2P\lambda_1^2) \leq \log(1+2P(GG^*)_{11}) \qquad (17)$$

And from (16), we know,

$$\begin{cases} \lambda_1^2 \geq (GG^*)_{11} \\ \lambda_2^2 \leq (GG^*)_{22} \end{cases}$$

Then the only possibility for (17) to be correct is the case that $\lambda_1^2 = (GG^*)_{11}$, at this time the equality in (17) can be obtained, otherwise for $\lambda_1^2 > (GG^*)_{11}$, we will have a conclusion that

$$\log(1+2P\lambda_1^2) > \log(1+2P(GG^*)_{11})$$

Even though it is possible to obtain the equality in (17) when $\lambda_1^2 = (GG^*)_{11}$, we can never derive the inequality that

$$\log(1+2P\lambda_1^2) < \log(1+2P(GG^*)_{11})$$

And moreover, generally, $\lambda_1^2$ does not equal to $(GG^*)_{11}$ since the matrix $GG^*$ is not a diagonal matrix almost for all the networks. Therefore, the inequality of (17) is not correct generally.

Basically, they made a mistake in using the Schur's theorem, it works only when the whole argument of the Schur-concave function satisfies the majorization argument. And for (15), the condition given in (16) is not enough to apply Schur's theorem.

Actually, they have used it correctly in [4,eq(6)] that

$$\sum_{k=1}^{n}\log(1+P_k\lambda_k^2) \leq \sum_{k=1}^{n}\log(1+np(GG^*)_{kk}) \qquad (18)$$

since they bound above the $P_k$ by total power constraint $nP$, then with the fact

$$\begin{cases} nP\sum_{i=1}^{l}\lambda_i^2 \geq nP\sum_{i=1}^{l}(GG^*)_{ii} \\ nP\sum_{i=1}^{n}\lambda_i^2 = nP\sum_{i=1}^{n}(GG^*)_{ii} \end{cases}$$

the inequality of (18) holds.

But for inequality of (15), the majorization argument can not be simply applied. And as I proved, the inequality of (15) is not correct even if all the $P_k$ s are fixed.

Thus the upper bound to the maximal communication rate of regular networks is not correct.

## 4.2 Improvements to the Upper Bounds to the Capacity of Wireless Networks with Multi-path Fading

In the paper by Ozgur, Leveque and Tse [7], they got an upper bound to the total throughput for extended network. And I improved the upper bound by a factor of $\log n$ in the case that the path loss exponent $\alpha = 2$ through a much more detailed analysis. Moreover, I also replace the term $n^{\varepsilon}$ by $(\log n)^{3+\varepsilon}$ and derived the exact expression of the constant in the upper bound in terms of $\alpha$ which make the upper bound more accurate in the general case for $\alpha \geq 2$ .

First, let me give a brief introduction to the proof given in their paper. In their proof, basically they first try to upper bound the total aggregate throughput by [7,eq(10)]

$$\sum_{k\in S, i\in D} R_{ik} \leq \max_{\substack{Q(H_1)\geq 0 \\ E(Q_{kk}(H_1))\leq P, \forall k\in S}} E(\log\det(I + H_1 Q(H_1) H_1^*)) + \max_{\substack{Q(H_2)\geq 0 \\ E(Q_{kk}(H_2))\leq P, \forall k\in S}} E(\log\det(I + H_2 Q(H_2) H_2^*)) \quad (19)$$

and they have proved the second term in (19) can be also bounded above by $n^{\varepsilon} P_{tot}(n)$ [7,Lemma 5.2] which is also the dominant term in (19). And $P_{tot}(n)$ is the total received power which can be bounded by

$$P_{tot}(n) \leq 2(\log n)^2 PG \sum_{k_x, k_y}^{\sqrt{n}} d_{k_x, k_y} \quad (20)$$

where $d_{k_x, k_y}$ is defined as

$$d_{k_x, k_y} = \sum_{i_x, i_y = 1}^{\sqrt{n}} \frac{1}{((i_x + k_x - 1)^2 + (i_y - k_y)^2)^{\alpha/2}} \quad (21)$$

Here they have used the idea that simplifying the problem to a regular network and assume that all the nodes are distributed in the integer vertices points of the network Figure 4.2, $(i_x, i_y)$, $(k_x, k_y)$ just denote the node location.

In order to upper bound $P_{tot}(n)$, they provide an upper bound to (21) first which is

*Improvements to the Previous Results*

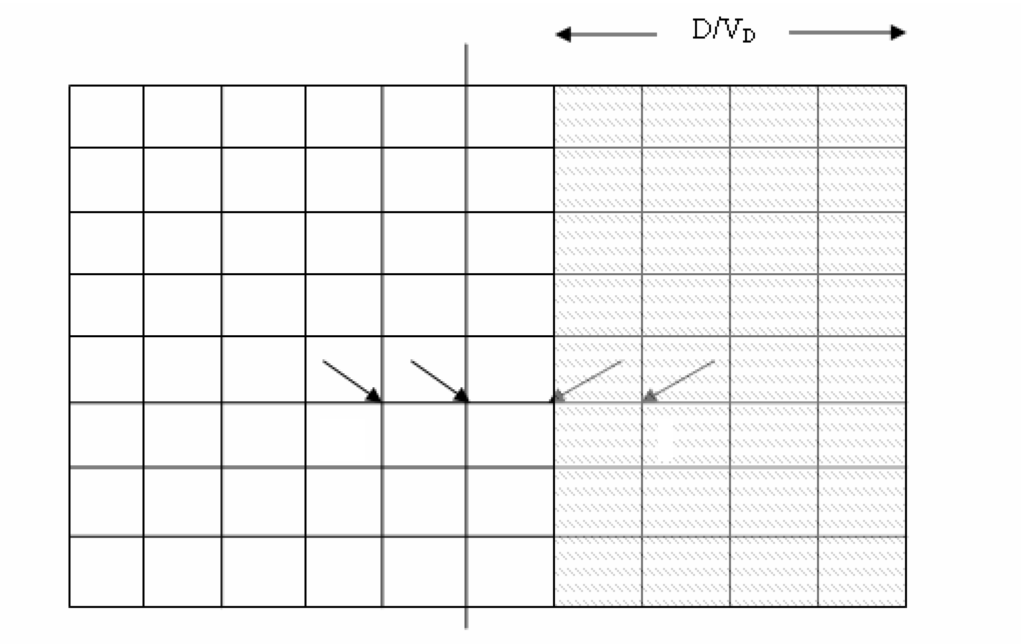


Figure 4. 2 Consider the nodes in one small square are all distributed on the vertices like regular network.

$$d_{k_x,k_y} \le k_x^{-\alpha} + (1+\pi)k_x^{1-\alpha} + \pi \log r \Big|_{k_x}^{3\sqrt{n}} \le K_2^{'} \log n \quad (22)$$

Then for the inequality of (20), when the $\alpha = 2$ the $P_{tot}(n)$ can be upper bounded by

$$P_{tot}(n) \le 2(\log n)^2 PG \sum_{k_x,k_y=1}^{\sqrt{n}} K_2^{'} \log n$$

$$= 2PGK_2^{'} n(\log n)^3$$

But this upper bound, especially for $\alpha = 2$, is not tight since they simply upper bound the $d_{k_x,k_y}$ by $K_2^{'} \log n$ where $K_2^{'}$ should be a function of $k_x$ based on (22) and neglect all the other terms. And actually $P_{tot}$ defined in (20) is a term summed over $k_x$, probably there will be affect from the sum over $k_x$, specially for the $K_2^{'}$ which is function in terms of $k_x$. So I think if we plug (22) into (20) directly without upper bounding it first by $K_2^{'} \log n$, a better upper bound to the total received power will be obtained.

Substitute $d_{k_x,k_y}$ by (22) in (20), then the total received power can be upper bounded by a sum over all $k_x$ s which represents the horizontal component of the k^th node's location

$$P_{tot}(n) \le 2(\log n)^2 PG \sum_{k_x,k_y}^{\sqrt{n}} (k_x^{-\alpha} + (1+\pi)k_x^{1-\alpha} + \pi \log 3\sqrt{n} - \pi \log k_x)$$

$$\le 2\sqrt{n}(\log n)^2 PG(\sum_{k_x=1}^{\sqrt{n}} (k_x^{-\alpha} + (1+\pi)k_x^{1-\alpha} + \pi \log 3\sqrt{n} - \pi \log k_x))$$

It is easy to upper bound the first two terms in this summation when the path loss exponent $\alpha = 2$ by

$$\begin{cases} \sum_{k_x=1}^{\sqrt{n}} k_x^{-2} \le \int_1^{\sqrt{n}} k_x^{-2} dk_x + 1 \le 2 \\ \sum_{k_x=1}^{\sqrt{n}} (1+\pi)k_x^{1-\alpha} \le (1+\pi)(\int_1^{\sqrt{n}} k_x^{-1} dk_x + 1) \end{cases} \quad (23)$$

Here, similar to the analysis in [7], I also used the fact that

$$\sum_{l=0}^{n} \frac{1}{(l+a)^{\gamma}} \le \frac{1}{a^{\gamma}} + \int_0^n \frac{1}{(l+a)^{\gamma}} dl$$

and moreover the last term in this summation can be simplified to

$$\sum_{k_x=1}^{\sqrt{n}} (\pi \log 3\sqrt{n} - \pi \log k_x) = \sum_{k_x=1}^{\sqrt{n}} (\pi \log 3 + \frac{\pi}{2} \log n - \pi \log k_x)$$

$$= \pi\sqrt{n} \log 3 + \frac{\pi}{2}\sqrt{n} \log n - \pi \log \sqrt{n}! \quad (24)$$

From Sterling's series, we have

$$\sqrt{n}! > \sqrt{2\pi\sqrt{n}} (\frac{\sqrt{n}}{e})^{\sqrt{n}}$$

Plug this into (24), it can be upper bounded by $K\sqrt{n}$. Together with (23), the dominant part in this summation comes from (24). So I can have an upper bound for $P_{tot}$ that

$$P_{tot} \le 2PGK' n(\log n)^2$$

Compared this result with the one given in [7], I improved the result by a factor of $\log n$. Although this proof does not improve the result by too much, it still provide an intuition and idea on how can we do better. Sometimes the constant part in scaling is also important and can be done much better with detailed analysis on it.

Actually, the constant parts of the upper bound given in [7] can also be expressed exactly in terms of path loss exponent $\alpha$. And it is also important to evaluate the constant part of the upper bound, since when the network size is large enough, the constant multiplier in the upper bound of capacity also have significant affect to the results. Therefore, I derived the exact expression for constant $K'$ given in [7] in the following.

Consider the equation (32) in [7], plug this into the equation (20), an upper bound can be derived that

1) $\alpha = 2$

$$P_{tot}(n) \le 2PG(\log n)^2 \sum_{k_x,k_y=1}^{\sqrt{n}} k_x^{-\alpha} + (1+\pi)k_x^{1-\alpha} + \pi \log 3\sqrt{n}$$

$$\le 2PG(\log n)^2 \sum_{k_x,k_y=1}^{\sqrt{n}} (\frac{1+1+\pi+\pi\log 3}{\log 2} + \frac{\pi}{2}) \log n$$

$$\le 2PG(\frac{1+1+\pi+\pi\log 3}{\log 2} + \frac{\pi}{2})(\log n)^3 n$$

2) $2 < \alpha < 3$

$$P_{tot}(n) \le 2PG(\log n)^2 \sum_{k_x,k_y=1}^{\sqrt{n}} k_x^{-\alpha} + (1+\pi)k_x^{1-\alpha} + \frac{\pi}{\alpha-2} k_x^{2-\alpha}$$

$$\le 2PG(\log n)^2 \sum_{k_x,k_y=1}^{\sqrt{n}} (1+1+\pi+\frac{\pi}{\alpha-2}) k_x^{2-\alpha}$$

$$\le 2PG(\log n)^2 \frac{1}{3-\alpha}(1+1+\pi+\frac{\pi}{\alpha-2}) n^{2-\alpha/2}$$

3) $\alpha = 3$

$$P_{tot}(n) \le 2PG(\log n)^2 \sum_{k_x,k_y=1}^{\sqrt{n}} k_x^{-3} + (1+\pi)k_x^{-2} + \frac{\pi}{\alpha-2} k_x^{-1}$$

$$\le 2PG(\log n)^2 \sum_{k_x,k_y=1}^{\sqrt{n}} (1+1+\pi+\pi) k_x^{-1}$$

$$\le 2PG(1+\pi)(\log n)^3 \sqrt{n}$$

***Improvements to the Previous Results***

4) $\alpha > 3$

$$P_{tot}(n) \le 2PG(\log n)^2 \sum_{k_x,k_y=1}^{\sqrt{n}} k_x^{-\alpha} + (1+\pi)k_x^{1-\alpha} + \frac{\pi}{\alpha-2}k_x^{2-\alpha}$$

$$\le 2PG(\log n)^2 \sum_{k_x,k_y=1}^{\sqrt{n}} (1+1+\pi+\frac{\pi}{\alpha-2})k_x^{2-\alpha}$$

$$\le 2PG\frac{1}{\alpha-3}(2+\pi+\frac{\pi}{\alpha-2})(\log n)^2\sqrt{n}$$

So the constant in the upper bound given in [7] can be obtained in the exact expression in terms of $\alpha$ that

$$\begin{cases} K' = \dfrac{2+\pi+\pi\log 3}{\log 2} + \dfrac{\pi}{2} & \alpha = 2 \\ K' = \dfrac{1}{3-\alpha}(2+\pi+\dfrac{\pi}{\alpha-2}) & 2 < \alpha < 3 \\ K' = 1+\pi & \alpha = 3 \\ K' = \dfrac{1}{\alpha-3}(2+\pi+\dfrac{\pi}{\alpha-2}) & \alpha > 3 \end{cases}$$

And for my improvement to the case when $\alpha = 2$, the constant part can be also derived as following with the similar idea proved before

$$K' = \pi \log 3e$$

Compared with the previous results obtained in [7], my constant is much smaller than theirs in the case when $\alpha = 2$, so when n is large enough, my upper bound is much better than theirs.

Moreover, it is trivial to get rid of the term $n^{\varepsilon}$ obtained in their upper bound. First, this term, $n^{\varepsilon}$, comes from the proof of Lemma 5.3 in [7] that with high probability, the maximum eigenvalue of $\tilde{H}^*\tilde{H}$ is less than $n^{\varepsilon}$. And they have used the Chebyshev's inequality to upper bound the probability that

$$P(\|\tilde{H}\|^2 \ge n^{\varepsilon}) \le \frac{E(\|\tilde{H}\|^{2m})}{n^{m\varepsilon}} \le \frac{(4(K_1' \log n)^3)^m}{n^{m\varepsilon}}$$

Actually, we can simply replace $n^{\varepsilon}$ by $(\log n)^{3+\varepsilon}$ and the same conclusion will be obtained that with high probability, the maximum eigenvalue of $\tilde{H}^*\tilde{H}$ is less than $(\log n)^{3+\varepsilon}$ that

***Improvements to the Previous Results***

$$P(\|\tilde{H}\|^2 \geq (\log n)^{3+\varepsilon}) \leq \frac{E(\|\tilde{H}\|^{2m})}{(\log n)^{m(3+\varepsilon)}} \leq \frac{(4(K_1^{'}\log n)^3)^m}{(\log n)^{m(3+\varepsilon)}}$$

When n is large enough, this probability goes to zero, or equivalently, with high probability, the maximum eigenvalue of $\tilde{H}^*\tilde{H}$ is less than $(\log n)^{3+\varepsilon}$.

Therefore, with the improvement to the upper bound in the case when $\alpha = 2$, replacing the term $n^{\varepsilon}$ by $(\log n)^{3+\varepsilon}$ and the exact expression for all the constants with different path loss level, all the upper bounds given in [7] can be represented by the following now

$$T(n) \leq \begin{cases} 2PG\pi \log 3e \cdot n(\log n)^{5+\varepsilon} & \alpha = 2 \\ 2PG(2+\pi+\dfrac{\pi}{\alpha-2})\dfrac{1}{3-\alpha} \cdot n^{2-\alpha/2}(\log n)^{5+\varepsilon} & 2<\alpha<3 \\ 2PG(1+\pi)\cdot\sqrt{n}(\log n)^{6+\varepsilon} & \alpha = 3 \\ 2PG(2+\pi+\dfrac{\pi}{\alpha-2})\dfrac{1}{\alpha-3} \cdot \sqrt{n}(\log n)^{5+\varepsilon} & \alpha > 3 \end{cases}$$

# Chapter 5

# Upper Bounds to the Capacity of Wireless Networks without Full CSI

In this chapter, an upper bound to maximal communication rate of wireless network is derived for which the channel state information is known only at the receiver side. Moreover, the multi-path fading affect is also taken into account which can be characterized by a random phase term in channel model. A simple extension to the upper bound of the transport capacity is derived then. Compared with the result given in [9], the same linear scaling of the upper bounds to the transport capacity with high probability when $\alpha > 3$ is obtained with a much simpler proof and an upper bound for the low attenuation level when $\alpha < 3$ is also presented. Moreover, with the same scaling, a comparison among all the constant parts for different upper bounds proved that when the path loss level and the minimum distance constraint are relatively large, the results presented in this thesis are much better.

## 5.1 System Model

Consider a two-dimensional network Figure 5.1 with constant density of nodes which equals unity, or equivalently, there are n nodes uniformly distributed in the two dimensional area with area n, and I divide the area into two equal parts, such that

$$D:[-\frac{\sqrt{n}}{2},\frac{\sqrt{n}}{2}]\times[0,\sqrt{n}] = D_x:[-\frac{\sqrt{n}}{2},0]\times[0,\sqrt{n}] + D_y:[0,\frac{\sqrt{n}}{2}]\times[0,\sqrt{n}]$$

With high probability, there are n/2 nodes in each of the two areas $D_x$ and $D_y$ due to the uniform nodes distribution. Assume the nodes in $D_x$ are the source nodes which want to establish communication with nodes in $D_y$, the destination nodes. Each source node has a maximum transmission power constraint P. Let $x_1, x_2, \ldots\ldots x_{n/2}$

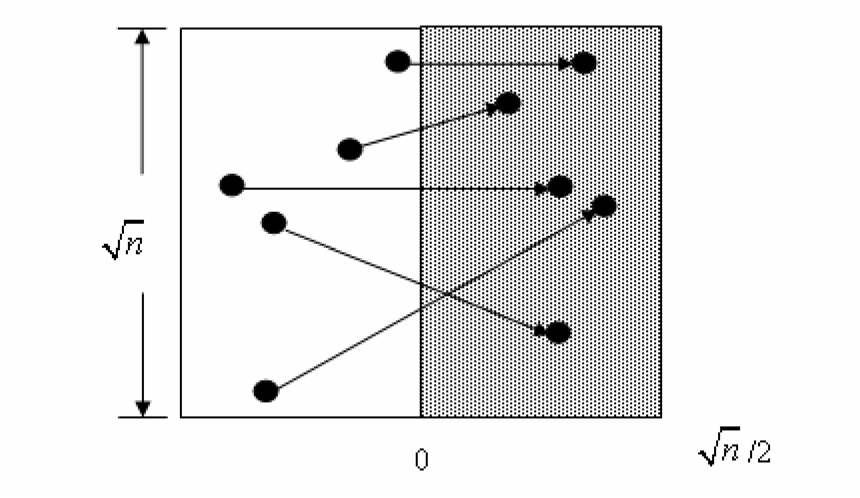


Figure 5. 1 A two-dimensional network divided into two equal parts

in $D_x$ denote the node positions of the source nodes and the corresponding destination nodes be $y_1, y_2 \ldots\ldots y_{n/2}$ in $D_y$.

Define the following relationship between the output and the input

$$Y = GX + Z \qquad (25)$$

where the vector $X=(X_1, X_2, \ldots\ldots X_{n/2})$ is the transmitted variables by source nodes and the vector $Y=(Y_1, Y_2 \ldots\ldots Y_{n/2})$ is the received variables by destination nodes. Z is the additive Gaussian noise which is also a vector with n/2 elements and each element equals to $Z_i$, i=1,2………n/2 which is a Gaussian random variable with zero mean and unit variance. G matrix stands for the channel gain matrix with elements $G_{ij}$ which represents the channel gain from i to j that

$$G_{ij} = \frac{e^{j\theta}}{1 + \left|x_i - y_j\right|^{\frac{\alpha}{2}}}$$

where $\theta$ is independent for different channels which characterizes the multi-path fading of the channel. And $|x_i - y_j|$ represents the Euclidean distance between node i in $D_x$ and node j in $D_y$. There is subtle difference for the channel attenuation model by using $(1+\rho^{\alpha/2})^{-1}$ instead of $\rho^{-\alpha/2}$. All the information is perfectly known to the receiver side but not the transmitter side. And $\alpha$ is the path loss exponent as defined before.

Assume all the transmitting nodes are subjected to an individual power constraint $P$ and let $Q$ denotes the autocorrelation matrix for all these transmitted variables, i.e., $Q = E[XX^T]$.

## 5.2 Upper Bounds to the Maximal Communication Rate

Define the communication rate between the transmitting node i and receiving node j as $R_{ij}$ and for the channel defined in (25), based on the cut set bound, the total transmission rates across the cut is bounded above by the conditional mutual information that

$$\sum_{i\in D_x,\, j\in D_y} R_{ij} \le \max_{EX_i^2 \le P, i=1,2....n/2} E_g(I(X;Y \mid X_{D_y})) \tag{26}$$

where the $X_{Dy}$ denotes the variables transmitted by the nodes in $D_y$. And the expectation is taken with respect to the channel gain which is random due to the random phase term for multi-path fading. From (25) which relates the transmitted variable X to the received variable Y, we can deduce that

$$\begin{aligned} I(X;Y \mid X_{D_y}) &= H(GX+Z \mid X_{D_y}) - H(GX+Z \mid X, X_{D_y}) \\ &= H(GX+Z \mid X_{D_y}) - H(Z) \\ &\le H(GX+Z) - H(Z) \\ &= I(X;Y) \end{aligned}$$

So (26) can be bounded above by

$$\begin{aligned} \sum_{i\in D_x,\, j\in D_y} R_{ij} &\le \max_{EX_i^2 \le P, i=1,2....n/2} E_g(I(X;Y)) \\ &= E_g[\log\det(I_{n/2} + G^* P I_{n/2} G)] \end{aligned} \tag{27}$$

$$\leq E_g \sum_{i=1}^{n/2} \log(1+P\sum_{j=1}^{n/2} G_{ij}^2)$$

The equation (27) is obtained by applying the fact that independent signaling achieves the maximum for the channel with multi-path fading (see proof in [9]). As there are n/2 source-destination pairs, then we can find the communication rate per source-destination pair on average by simply dividing n/2 on both sides of (27) which is denoted by $\overline{R}$

$$\overline{R} \leq \frac{2}{n} E \sum_{i=1}^{n/2} \log(1+P\sum_{j=1}^{n/2} G_{ij}^2)$$

$$\leq E \log(1+\frac{2P}{n}\sum_{i=1}^{n/2}\sum_{j=1}^{n/2} G_{ij}^2)$$

$$\leq \log(1+\frac{2P}{n}\sum_{i=1}^{n/2}\sum_{j=1}^{n/2} \frac{1}{1+|x_i-y_j|^{\alpha}})$$

where the inequalities are obtained through concavity of logarithm function and the Jensen's inequality.

As all the nodes are uniformly distributed in the network, we can derive a communication rate on average by taking expectation over the random nodes positions.

$$E(\overline{R}) \leq E\log(1+\frac{2P}{n}\sum_{i=1}^{n/2}\sum_{j=1}^{n/2} \frac{1}{1+|x_i-y_j|^{\alpha}})$$

$$\leq \log(1+\frac{2P}{n}E\sum_{i=1}^{n/2}\sum_{j=1}^{n/2} \frac{1}{1+|x_i-y_j|^{\alpha}}) \qquad (28)$$

Due to the uniform distribution for all these nodes, the probability density function for the horizontal components of $x_i$ and $y_j$ is $2/\sqrt{n}$ and for the vertical components is $1/\sqrt{n}$. So just consider the expectation term in (28) that

$$\frac{2}{n}E\sum_{i=1}^{n/2}\sum_{j=1}^{n/2} \frac{1}{1+|x_i-y_j|^{\alpha}} =$$

$$\frac{2}{n}\int_{-\sqrt{n}/2}^{0}\int_{0}^{\sqrt{n}}\int_{0}^{\sqrt{n}/2}\int_{0}^{\sqrt{n}} \frac{1}{1+[(x_{i1}-y_{j1})^2+(x_{i2}-y_{j2})^2]^{\alpha/2}}\,dx_{i1}dx_{i2}dy_{j1}dy_{j2} \qquad (29)$$

where ($x_{i1}, x_{i2}$) and ($y_{j1}, y_{j2}$) denote the coordinates of nodes $x_i$ and $y_j$. Then change the variables by defining $x'_{i1} = -x_{i1}$, $x'_{i2} = x_{i2}$, $y'_{j1} = y_{j1}$ and $y'_{j2} = y_{j2} - x_{i2}$, we can rewrite (29) as

$$\frac{2}{n}\int_0^{\sqrt{n}/2}\int_0^{\sqrt{n}}\int_0^{\sqrt{n}/2}\int_{-\sqrt{n}}^{\sqrt{n}}\frac{1}{1+[(x'_{i1}+y'_{j1})^2+{y'_{j2}}^2]^{\alpha/2}}dx'_{i1}dx'_{i2}dy'_{j1}dy'_{j2}$$

$$=\frac{2\sqrt{n}}{n}\int_0^{\sqrt{n}/2}\int_0^{\sqrt{n}/2}\int_{-\sqrt{n}}^{\sqrt{n}}\frac{1}{1+[{x'_{i1}}^2+{y'_{j1}}^2+{y'_{j2}}^2]^{\alpha/2}}dx'_{i1}dy'_{j1}dy'_{j2}$$

For this integral in a three-dimensional domain, we can simply upper bound the integral by expanding the integration range to a half ball with radius $\sqrt{3n}/2$ which contains the original integration range. Then change the variables again by polar coordinates that,

$$\begin{cases} x'_{i1} = r\sin\theta_1\sin\theta_2 \\ y'_{j1} = r\sin\theta_1\cos\theta_2 \\ y'_{j2} = r\cos\theta_1 \end{cases}$$

Such that

$$\frac{2\sqrt{n}}{n}\int_0^{\sqrt{n}/2}\int_0^{\sqrt{n}/2}\int_{-\sqrt{n}}^{\sqrt{n}}\frac{1}{1+[{x'_{i1}}^2+{y'_{j1}}^2+{y'_{j2}}^2]^{\alpha/2}}dx'_{i1}dy'_{j1}dy'_{j2}$$

$$=\frac{2\sqrt{n}}{n}\int_0^{\sqrt{3n}/2}\int_0^{\pi}\int_0^{\pi}\frac{|\det(J_d)|}{1+r^{\alpha}}dr\,d\theta_1 d\theta_2 \qquad (30)$$

where $\det(J_d)$ is the Jacobian matrix due to the change of variables and we have used the determinant of the Jacobian matrix.

$$\det(J_d) = -r^2\sin\theta_1$$

Plug the determinant of the Jacobian matrix into (30), I have the following expression, and it can be further simplified by the following.

$$\frac{2\sqrt{n}}{n}\int_0^{\sqrt{3n}/2}\int_0^{\pi}\int_0^{\pi}\frac{|\det(J_d)|}{1+r^\alpha}drd\theta_1 d\theta_2$$

$$=C\frac{2\sqrt{n}}{n}\int_0^{\sqrt{3n}/2}\frac{r^2}{1+r^\alpha}dr$$

$$\leq C\frac{2\sqrt{n}}{n}\left(\int_0^1 r^2dr+\int_1^{\sqrt{3n}/2}r^{2-\alpha}dr\right)$$

$$=I(r)$$

Then for different values of $\alpha$, an upper bound on $I(r)$ can be obtained as following

$$I(r)=\begin{cases} C_1(\alpha)n^{\frac{2-\alpha}{2}}, & \alpha<2 \\ C_2(\alpha), & \alpha=2 \\ C_3(\alpha)n^{\frac{2-\alpha}{2}}, & 2<\alpha<3 \\ C_4(\alpha)\frac{\log n}{\sqrt{n}}, & \alpha=3 \\ C_5(\alpha)/\sqrt{n}, & \alpha>3 \end{cases}$$

where $C_i(\alpha)$ for i=1,2,3,4,5 is constant which is a function in terms of the path loss exponent $\alpha$. By plugging $I(r)$ into (28), the upper bound on the communication rate on average can be obtained

$$E[\bar{R}]=\begin{cases} C_1'(\alpha)\log n, & \alpha<2 \\ C_2'(\alpha), & \alpha=2 \\ C_3'(\alpha)n^{\frac{2-\alpha}{2}}, & 2<\alpha<3 \\ C_4'(\alpha)\frac{\log n}{\sqrt{n}}, & \alpha=3 \\ C_5'(\alpha)/\sqrt{n}, & \alpha>3 \end{cases}$$

A similar result has been obtained in [11]. With the same channel model and they assume that the channel state information is only known to the receiver side, they also derived an upper bound to the communication rate on average over the random node locations that

$$E[R] \le \begin{cases} O(\ln n) & \alpha < 2 \\ O(\ln(\ln n)) & \alpha = 2 \\ O(n^{1-\alpha/2}) & 2 < \alpha < 3 \\ O(\ln n/\sqrt{n}) & \alpha = 3 \\ O(1/\sqrt{n}) & \alpha > 3 \end{cases}$$

Compared the result with mine, my upper bound is much more tight when the path loss exponent $\alpha = 2$. But this is just an upper bound to the average transmission rate for all the communication pairs. And how can we generalize the results to more general cases or how can we find the upper bound to the maximal transmission rate for each communication pair which is of much more interests? Basically, it is difficult to prove the upper bound for all kinds of random networks, but it is possible to find an upper bound for most of the random networks. My goal is to find an upper bound which bounds above the maximal transmission rate of each communication pair with high probability.

Assume there is a minimum distance constraint $d_{\min}$ between any two nodes in the network, i.e. $\rho_{ij} \ge d_{\min}$ for any i and j. Actually, this assumption is quite reasonable since for most of the networks, there exits a minimum distance between any two nodes in the network. And similar assumptions are also made in [2] and [8]. With this assumption, an upper bound to the maximal transmission rate per communication pair is obtained

$$R \le \log\left(1 + \frac{2P}{n} \int_{-\sqrt{n}/2}^{0} \int_{0}^{\sqrt{n}} \int_{0}^{\sqrt{n}/2} \int_{0}^{\sqrt{n}} \frac{1}{1 + \{d_{\min}^2[(x_{i1} - y_{j1})^2 + (x_{i2} - y_{j2})^2]\}^{\alpha/2}} dx_{i1} dx_{i2} dy_{j1} dy_{j2}\right)$$

With the similar analysis to the average communication rate, the maximal transmission rate R can be upper bounded by

$$R = \begin{cases} C_1'(\alpha, d_{\min}) \log n, & \alpha < 2 \\ C_2'(\alpha, d_{\min}), & \alpha = 2 \\ C_3'(\alpha, d_{\min}) n^{\frac{2-\alpha}{2}}, & 2 < \alpha < 3 \\ C_4'(\alpha, d_{\min}) \dfrac{\log n}{\sqrt{n}}, & \alpha = 3 \\ C_5'(\alpha, d_{\min}) / \sqrt{n}, & \alpha > 3 \end{cases}$$

where the constant $C_i^{'}(\alpha, d_{\min})$ i=1,2,3,4,5 is a function in terms of the path loss exponent $\alpha$ and the minimum distance constraint between any two nodes $d_{\min}$.

This upper bound showed that the transmission rate for any communication pair goes to zero with the number of nodes going to infinity in the case that the path loss exponent $\alpha$ is greater than 2. And compared with the other work which are also based on single cut analysis, this model has taken the multi-path fading into account but the full channel state information is only known to the receiver, or equivalently, the transmitter cannot distribute the transmission power based on different channel state information.

More specifically, all the constants can be calculated through the previous integration actually. And it is also with great interests to get the exact expression of the constant. Similar to the constant obtained in [2] and [8], all of them are functions in terms of the path loss exponent $\alpha$ and the constraint of minimum distance $d_{\min}$ between any two nodes. And I can also evaluate the performance of the constant part with respect to the two parameters. Sometimes it is also important to analyze the constant part, especially when the network size is large enough, such that the constant part will have significant affect to the upper bound to the capacity of wireless networks.

$$\begin{cases} C_1^{'}(\alpha, d_{\min}) = \dfrac{2-\alpha}{2} + \dfrac{4\pi^2 3^{1.5-\alpha/2}}{(3-\alpha)2^{3-\alpha}\log 2d_{\min}^2} \\ C_2^{'}(\alpha, d_{\min}) = \log \dfrac{2\sqrt{3}P\pi^2}{d_{\min}^3} \\ C_3^{'}(\alpha, d_{\min}) = \dfrac{P\pi^2 3^{\frac{3-\alpha}{2}}}{2^{2-\alpha}(3-\alpha)d_{\min}^3} \\ C_4^{'}(\alpha, d_{\min}) = \dfrac{P\pi^2}{d_{\min}^3} \\ C_5^{'}(\alpha, d_{\min}) = \dfrac{2P\pi^2}{3d_{\min}^{3/2}} + \dfrac{2P\pi^2}{(\alpha-3)d_{\min}^{\frac{3}{\alpha}+\alpha-1}} \end{cases} \tag{31}$$

I especially analyze the constant $C_5^{'}(\alpha, d_{\min})$ with respect to $\alpha$ and $d_{\min}$ when $\alpha > 3$. It is obviously to see from the expression in (31) that for some fixed path loss level, the constant decreases with the increasing of the minimum distance constraint and finally it converges to zero with the increasing of the $d_{\min}$. Correspondingly, for fixed minimum distance constraint, the constant also decreases with the increasing of the path loss exponent and it converges to $\dfrac{2P\pi^2}{3d_{\min}^{3/2}}$ with the increasing of $\alpha$. And I will make a comparison of my results with the results given in [9] later.

An trivial extension to the upper bound of total aggregate throughput can be obtained by simply multiplying the number of communication pairs n/2 that

$$T(n) = \begin{cases} C_1'(\alpha, d_{\min}) n \log n, & \alpha < 2 \\ C_2'(\alpha, d_{\min}) n, & \alpha = 2 \\ C_3'(\alpha, d_{\min}) n^{\frac{4-\alpha}{2}}, & 2 < \alpha < 3 \\ C_4'(\alpha, d_{\min}) \sqrt{n} \log n, & \alpha = 3 \\ C_5'(\alpha, d_{\min}) \sqrt{n}, & \alpha > 3 \end{cases}$$

## 5.3 Upper Bounds to the Transport Capacity of Wireless Networks

In the previous section, I considered the upper bounds to the maximal communication rate among all the communication pairs. However, we can simply extend this upper bound to a more general measure, the transport capacity. Like one-dimensional network, we can also derive a nice relation between the transport capacity and cut set bound. Firstly, for a two dimensional network, from the definition of the transport capacity, it can be bounded by

$$\sum_{\substack{i,j \\ i \neq j}} R_{ij} \rho_{ij} \leq \sum_{\substack{i,j \\ i \neq j}} R_{ij} (\rho_{x_{ij}} + \rho_{y_{ij}}) \quad (32)$$

where the $\rho_{x_{ij}}$ and $\rho_{y_{ij}}$ denote the horizontal and vertical components of the distance from i to j. So this inequality in (32) comes from the triangle inequality immediately. Then we can consider the (32) is just the sum of two the transport capacity in one dimensional network. As we have already shown the nice relationship between the transport capacity and cut set bound in Chapter 3 for one-dimensional case, then (32) can be bounded by

$$\sum_{k=1}^{n-1} \rho_{x_{k,k+1}} \Big( \sum_{\substack{i,j \\ i \leq k < j}} R_{ij} + \sum_{\substack{i,j \\ j \leq k < i}} R_{ij} \Big) + \sum_{k=1}^{n-1} \rho_{y_{k,k+1}} \Big( \sum_{\substack{i,j \\ i \leq k < j}} R_{ij} + \sum_{\substack{i,j \\ j \leq k < i}} R_{ij} \Big) \quad (33)$$

The (33) can be viewed as there are n-1 single cuts in both two dimensions, and a trivial upper bound to the transport capacity can be established immediately by bounding above the total throughput through any single cut by the maximal throughput through a single cut, which actually is the total throughput through the cut right in the middle of the network, such that the sum over all k with respect to the corresponding distance between any two neighbor nodes, it gives the total distance in both horizontal and vertical dimensions.

$$\sum_{\substack{i,j \\ i \neq j}} R_{ij}\rho_{ij} \leq 2\rho_x \max_{1\leq k<n}(\sum_{\substack{i,j \\ i\leq k<j}} R_{ij} + \sum_{\substack{i,j \\ j\leq k<i}} R_{ij})$$

$$+2\rho_y \max_{1\leq k<n}(\sum_{\substack{i,j \\ i\leq k<j}} R_{ij} + \sum_{\substack{i,j \\ j\leq k<i}} R_{ij}) \quad (34)$$

As we already got the upper bound to the maximal transmission rate R per communication pair, the maximum communication rate through any of the k-1 cuts can be simply bounded above by $n \cdot R$, and moreover, we can bound $\rho_x$ and $\rho_y$ by the horizontal and vertical length of the network which are all $\sqrt{n}$ for this extended network. So we can get an upper bound to the transport capacity as following

$$T_C(n) \leq \begin{cases} C_1'(\alpha, d_{\min})n^{3/2}\log n, & \alpha < 2 \\ C_2'(\alpha, d_{\min})n^{3/2}, & \alpha = 2 \\ C_3'(\alpha, d_{\min})n^{5/2-\alpha/2}, & 2 < \alpha < 3 \\ C_4'(\alpha, d_{\min})n\log n, & \alpha = 3 \\ C_5'(\alpha, d_{\min})n, & \alpha > 3 \end{cases}$$

Therefore, a linear scaling of the upper bound to the transport capacity is obtained when the path loss exponent $\alpha$ is greater than three and for $\alpha$ equals to three, a linear scaling is achieved up to a factor of $\log n$. And all these upper bound is obtained with high probability.

Now I can compare the result with the previous results given in the paper [9]. With the same channel modeling and the same assumption that the full channel state information is only known to the receiver side as well as there is a minimum distance constraint between any two nodes, they also derived a linear scaling of the transport capacity when $\alpha$ is greater than 3 that

$$T_C(n) \leq \frac{(2\pi + 12)P\zeta(\alpha - 2)}{d_{\min}^{\alpha-1}} n \quad (35)$$

where the function $\zeta(\alpha)$ is defined as $\zeta(\alpha) = \sum_{i=1}^{\infty} i^{-\alpha}$, so the upper bound given in (34) can be written as following, which is also a linear upper bound with respect to the total number of nodes in the network.

$$T_C(n) \leq C \cdot n$$

with the constant C equals to

$$C = \frac{(2\pi + 12)P(\alpha - 2)}{d_{\min}^{\alpha-1}(\alpha - 3)}$$

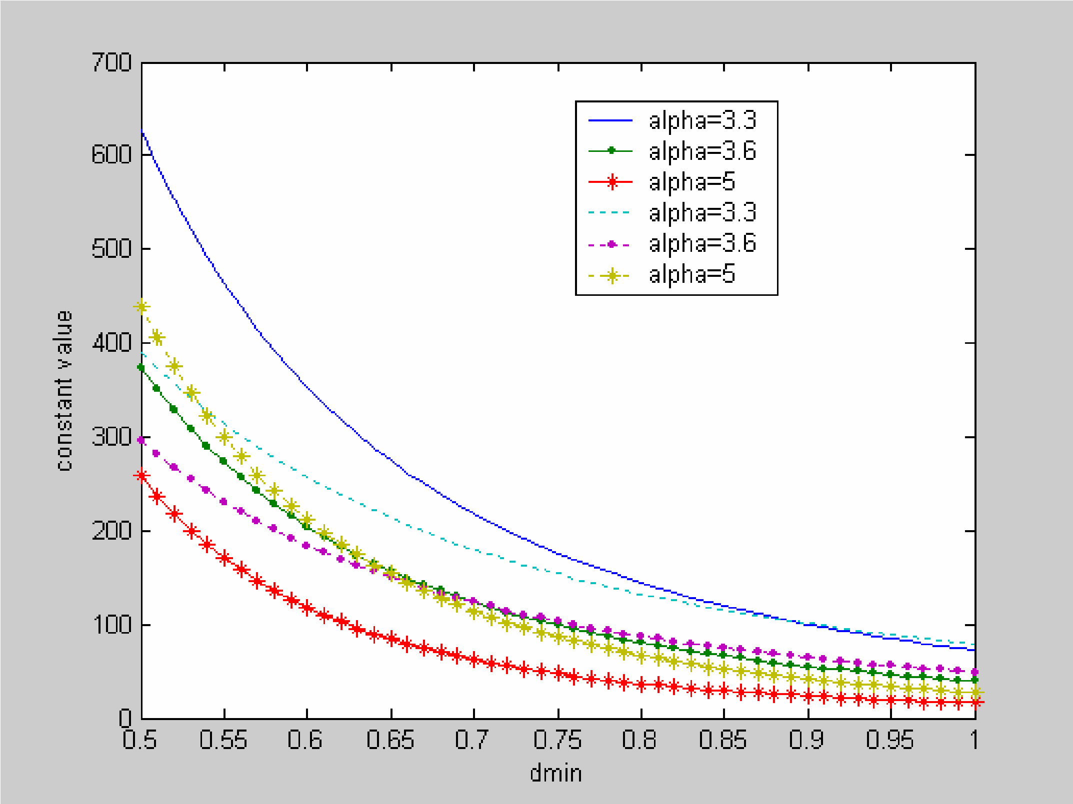


Figure 5. 2 A comparison between the constant in [9] (dotted line) and the constant in my proof (solid line) for some fixed values of $\alpha$

where I have used the definition of the function $\zeta(\alpha)$ which is defined just before that

$$\zeta(\alpha-2)=\sum_{i=1}^{\infty} i^{-\alpha+2}$$

$$\leq \int_{1}^{\infty} i^{-\alpha+2} di + 1$$

$$=\frac{\alpha-2}{\alpha-3}$$

and P is the individual power constraint for each transmitting node.

So with the assumption of the minimum distance between any two nodes and the full channel state information is only known to the receiver side, they also get a linear scaling of the upper bound to the transport capacity in the case when $\alpha$ is greater than three.

Obviously, although we have the same scaling for the transport capacity when $\alpha$ is greater than three, we still have different constant parts, and when n is large enough, the different values of the constant will have different affects to these upper bounds. I

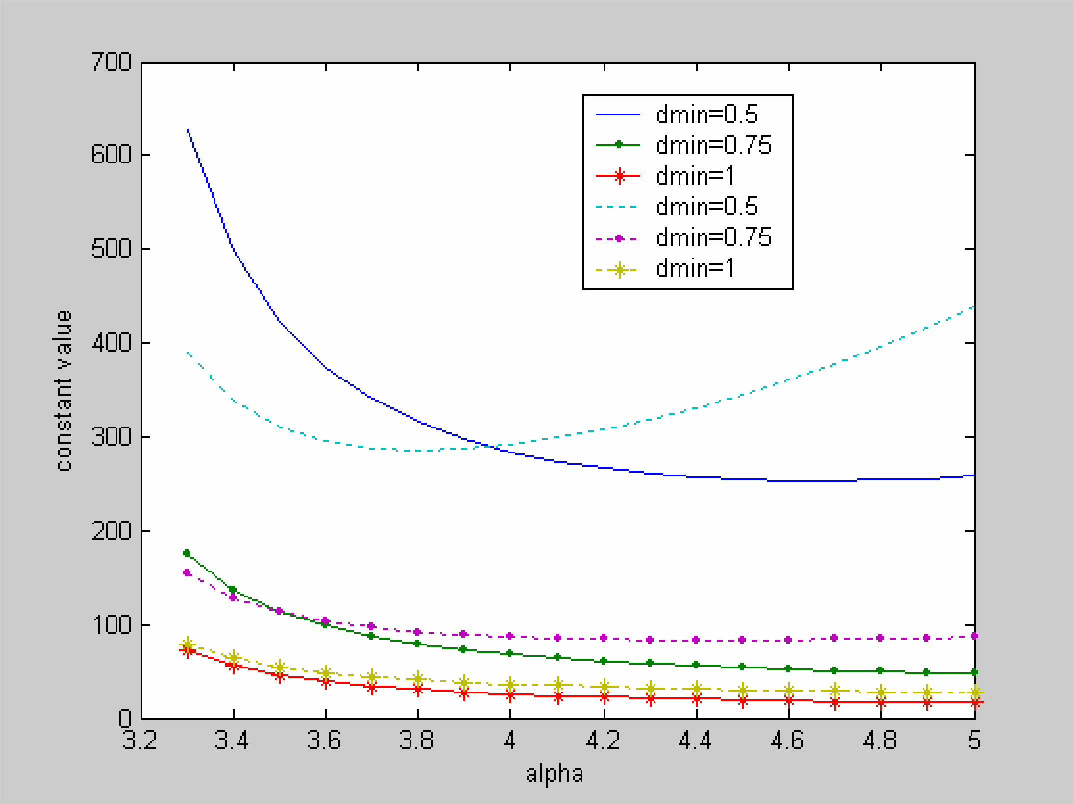


Figure 5. 3 A comparison between constant in [9] (dotted line) and the constant in my proof (solid line) for some fixed values of the minimum distance constraint $d_{\min}$

have made a comparison between the constant part given in [9] and the constant in my proof. From Figure 5.2, it was shown that, for some fixed path loss level, both of our constant decreases with the increasing of $d_{\min}$. But the constant in my proof will be much smaller for relatively large values of $d_{\min}$. If the path loss exponent $\alpha$ is large, for example, it equals to five in Figure 5.2, my constant is almost always smaller than the constant given in [9]. For Figure 5.3, my constant decreases with the increasing of the path loss level for some fixed minimum distance constraint, but their constants will decrease first and increase then with the increasing of the path attenuation level for some fixed minimum distance constraint. And it is obvious that my results are much smaller than their constants especially in the case that the minimum distance constraint $d_{\min}$ is relatively large. For an simple example, my result is almost always smaller than the constant part given in [9] for any value of alpha when the $d_{\min}$ is greater than one in Figure 5.3. Therefore, the upper bound to the transport capacity obtained in my proof is much better than the result given in [9] for large path loss exponent and relatively large minimum distance constraint.

To evaluate the affect from the constant part, I also compare the results given in [8] which are the best results until now with the results given in my proof and the results in [9]. Xie and Kumar derived an upper bound to the transport capacity with consideration of fading but they have assumed that the full channel state information is all known to both of the receivers and transmitters. Moreover, in this channel model characterization, they used $\frac{1}{\rho^{\alpha/2}}$ instead of $\frac{1}{1+\rho^{\alpha/2}}$ defined in my proof and in the paper [9]. So it is helpful to compare the constant part to see the affect from the channel state information in the upper bound. Firstly, the constant obtained in the upper bounds to the transport capacity in [8] are

1) One dimensional case and $\alpha > 3$ with full CSI

$$C = \frac{2(1+\frac{3}{\alpha-2}+\frac{1}{(\alpha/2-1)(\alpha-3)})P\log e}{\sigma^2 d_{\min}^{\alpha-1}}$$

2) Two dimensional case and $\alpha > 4$ with full CSI

$$C = \frac{\left(\frac{2^{2\alpha+2}}{(\frac{\alpha}{2}-1)^{1/2}(\frac{\alpha}{2}-2)}+\frac{2^{2\alpha+4}}{(\frac{\alpha}{2}-1)^{1/2}(\alpha-3)}+\frac{2^{1.5\alpha+5.5}}{(\frac{\alpha}{2}-2)^{1.5}}+\frac{2^{\alpha+1.5}}{(\frac{\alpha}{2}-1)^{0.5}}\right)\log e}{d_{\min}^{\alpha-1}}$$

The constant parts derived in [9] are

1) One dimensional case and $\alpha > 3$ with full CSI

$$C = \frac{4(\frac{\alpha/2}{\alpha/2-1}+\frac{\alpha-2}{\alpha-3}\cdot\frac{1}{2-\alpha/2}-\frac{1}{2-\alpha/2}\cdot\frac{1}{\alpha/2-1})P}{d_{\min}^{\alpha-1}}$$

2) One dimensional case and $\alpha > 2$ with no CSI

$$C = \frac{2\frac{\alpha-1}{\alpha-2}P}{d_{\min}^{\alpha-1}}$$

3) Two dimensional case and $\alpha > 5$ with full CSI

$$C = \frac{2(2\pi+12)^2(\frac{\alpha/2-1}{\alpha/2-2}+\frac{\alpha-4}{\alpha-5}\cdot\frac{1}{3-\alpha/2}-\frac{1}{\alpha/2-2}\cdot\frac{1}{3-\alpha/2})P}{d_{\min}^{\alpha-1}}$$

4) Two dimensional case and $\alpha > 3$ with no CSI

$$C = \frac{(2\pi+12)\frac{\alpha-2}{\alpha-3}P}{d_{\min}^{\alpha-1}}$$

And the constant part in my proof is

1) Two dimensional case and $\alpha > 3$ with no CSI

$$C = \frac{2P\pi^2}{3d_{\min}^{3/2}} + \frac{2P\pi^2}{(\alpha-3)d_{\min}^{\frac{3}{\alpha}+\alpha-1}}$$

With the same assumption that the individual power constraint is one and the variance of the Gaussian additive noise is also one, then compare the results as following and all the graphs are listed in Appendix.

Figure A.1 shows the comparison between the results given in [8] and [9] for one dimensional case and the full channel state information is known to both receivers and transmitters. It shows that if the path loss level is fixed, both of the constants decrease with the increasing of the minimum distance constraint. And the constant given in [8] is much smaller than [9] for relatively small path loss level, like $\alpha = 3.3$ and $\alpha = 3.6$ in Figure A.1. But when the path loss level is relatively high, the constant in [9] is much smaller. In Figure A.2, it shows the comparison between the constant given in [8] with CSI and the constant given [9] without CSI, and the conclusion is totally different that for relatively small path loss level like $\alpha = 3.3$ and $\alpha = 3.6$ in Figure A.2, the constants in [9] are much smaller and when the path loss level is relatively high, like $\alpha = 5$, the constants in [8] is much smaller.

For two dimensional case, as shown in Figure A.3 and Figure A.4, the constants given in [8] with full CSI is very large, usually with the order of $10^5$, so the constants in my proof and [9] when the full CSI is not available are much smaller in this case.

For conclusion, I list all my contributions in Table5.1 with a comparison to the previous results.

And the Table 3.1 can also be rewritten which is presented in Table 5.2 in the following.

Table 5. 1 My contributions with a comparison to the previous results.

| Previous results | My contributions |
|---|---|
| The upper bounds to the maximal communication rate of each source destination pair for regular networks [4]. | It is shown that the proof is not correct. |
| The upper bounds to the total aggregate throughput for wireless networks in the case $\alpha = 2$ that $T(n) \le K' n^{1+\varepsilon} (\log n)^3$ | The upper bounds is improved by $\log n$ that $T(n) \le K' n^{1+\varepsilon} (\log n)^2$ |
| The upper bounds to the total aggregate throughput fro wireless networks that $T(n) \le \begin{cases} K' n^{2-\alpha/2+\varepsilon} & 2 \le \alpha < 3 \\ K' n^{1/2+\varepsilon} & \alpha \ge 3 \end{cases}$ | The exact expression for the constants in the upper bounds in terms of $\alpha$ is derived and $n^{\varepsilon}$ is replaced by $(\log n)^{3+\varepsilon}$ $T(n) \le \begin{cases} 2PG\pi \log 3e \cdot n(\log n)^{5+\varepsilon} \\ 2PG(2+\pi+\frac{\pi}{\alpha-2})\frac{1}{3-\alpha} \cdot n^{2-\alpha/2} (\log n)^{5+\varepsilon} \\ 2PG(1+\pi) \cdot \sqrt{n} (\log n)^{6+\varepsilon} \\ 2PG(2+\pi+\frac{\pi}{\alpha-2})\frac{1}{\alpha-3} \cdot \sqrt{n} (\log n)^{5+\varepsilon} \end{cases}$ for different path loss level that $\alpha = 2$, $2 < \alpha < 3$, $\alpha = 3$ and $\alpha > 3$ |
| The upper bounds to the communication rate on average over the random node location that $E[R] \le \begin{cases} O(\ln n) & \alpha < 2 \\ O(\ln(\ln n)) & \alpha = 2 \\ O(n^{1-\alpha/2}) & 2 < \alpha < 3 \\ O(\ln n / \sqrt{n}) & \alpha = 3 \\ O(1/\sqrt{n}) & \alpha > 3 \end{cases}$ with consideration of fading and the full channel state information is only known to the receivers. | An upper bound to the maximal communication rate for each source destination with consideration of fading and the assumption that the full channel state information is only known to the receiver side is obtained $R = \begin{cases} C_1'(\alpha, d_{\min}) \log n, & \alpha < 2 \\ C_2'(\alpha, d_{\min}), & \alpha = 2 \\ C_3'(\alpha, d_{\min}) n^{\frac{2-\alpha}{2}}, & 2 < \alpha < 3 \\ C_4'(\alpha, d_{\min}) \frac{\log n}{\sqrt{n}}, & \alpha = 3 \\ C_5'(\alpha, d_{\min}) / \sqrt{n}, & \alpha > 3 \end{cases}$ and all the constants are also obtained with exact expression given in (31). |
| The upper bounds to the transport capacity of planar wireless networks with the consideration of fading, minimum distance constraint and there is no full channel state information [9] when $\alpha > 3$ that $T_C(n) \le \frac{P(2\pi+12)(\alpha-2)}{d_{\min}^{\alpha-1}(\alpha-3)} n$ | An upper bound to the transport capacity with same condition and assumption as [9] when $\alpha > 3$ is derived that $T_C(n) \le (\frac{2P\pi^2}{3d_{\min}^{3/2}} + \frac{2P\pi^2}{(\alpha-3)d_{\min}^{\frac{3}{\alpha}+\alpha-1}})n$ It is much tighter with relatively large $\alpha$ and $d_{\min}$ Figure5.4 & 5.5 |

Table 5. 2 New table of upper bounds to the capacity of wireless networks

| | d=1 | condition | d=2 | condition |
|---|---|---|---|---|
| Upper bound to communication rate of wireless networks | | | | |
| without fading [4] | $O((\log n)^3/n)$ | $\alpha > 0$ | $O(n^{\frac{1}{\alpha+8}}(\log n)^3/\sqrt{n})$ | $\alpha > 0$ |
| with fading [7] and my proof | N/A | N/A | $O((\log n)^{5+\varepsilon})$ | $\alpha = 2$ |
| | | | $O(n^{1-\alpha/2}(\log n)^{5+\varepsilon})$ | $2 < \alpha < 3$ |
| | | | $O(n^{-1/2}(\log n)^{6+\varepsilon})$ | $\alpha = 3$ |
| | | | $O(n^{-1/2}(\log n)^{5+\varepsilon})$ | $\alpha > 3$ |
| Upper bound to the transport capacity of wireless networks | | | | |
| without fading [6] | $O(n(\log n)^3)$ | $\alpha > 0$ | $O(nn^{\frac{1}{\alpha+8}}(\log n)^3)$ | $\alpha > 0$ |
| without fading [8] | $O(n)$ | $\alpha > 3$ | $O(n)$ | $\alpha > 5$ |
| with fading [7] and my proof | N/A | N/A | $O(n^{1.5}(\log n)^{5+\varepsilon})$ | $\alpha = 2$ |
| | | | $O(n^{2.5-\alpha/2}(\log n)^{6+\varepsilon})$ | $2 < \alpha < 3$ |
| | | | $O(n(\log n)^{6+\varepsilon})$ | $\alpha = 3$ |
| | | | $O(n(\log n)^{5+\varepsilon})$ | $\alpha > 3$ |
| with fading [8] | $O(n)$ | $\alpha > 2.5$ | $O(n)$ | $\alpha > 4$ |
| with fading [9] and my proof | $O(n)$ without CSI | $\alpha > 2$ | $O(n^{3/2}\log n)$ | $\alpha < 2$ |
| | | | $O(n^{3/2})$ | $\alpha = 2$ |
| | | | $O(n^{5/2-\alpha/2})$ | $2 < \alpha < 3$ |
| | | | $O(n\log n)$ | $\alpha = 3$ |
| | | | $O(n)$ without CSI | $\alpha > 3$ |
| | $O(n)$ with CSI | $\alpha > 3$ | $O(n)$ with CSI | $\alpha > 5$ |

# Chapter 6

# Conclusions and Future Work

This thesis presents some upper bounds to the capacity of wireless networks. With a summarization of the previous insightful results on the upper bounds to the maximal communication rate and the transport capacity of the wireless networks, the improvements to those results are provided. A wrong proof in [4] was pointed out and an improvement to the upper bound of total throughput obtained in [7] is derived. Moreover, the exact expressions of the constants for the upper bounds in [7] are given in this thesis.

A new upper bound to the maximal communication rate for each source-destination pair of wireless networks for which the channel is subject to multi-path fading and the full channel state information is only known to the receiver side is derived in this thesis. The maximal communication rate goes to zero with the increasing of the network size when the path loss exponent $\alpha$ is greater than two. An upper bound to the transport capacity is also presented in this thesis. A linear scaling of the upper bound to the transport capacity when $\alpha$ is greater than three is obtained. Although the same scaling of the upper bound as the previous results given in [9] is obtained in somehow, it still proved that the upper bound derived in this thesis is much better for relatively large path loss level and the minimum distance constraint of the networks. It is because of the different constants derived in the upper bound and the difference among the constant parts will provide great affect to the upper bounds when the number of nodes is large enough.

Based on these works, our future work will mainly focus on the upper bounds to the transport capacity. We will try to narrow the gap of the path loss exponent in which the linear scaling of upper bounds to the transport capacity is obtained. As Xie and Kumar have shown that it is impossible to improve the results based on their method, we need to consider the matrix case directly. But obviously, this will make the problem much more complicated.

# Appendix

# The Comparison of the Constants

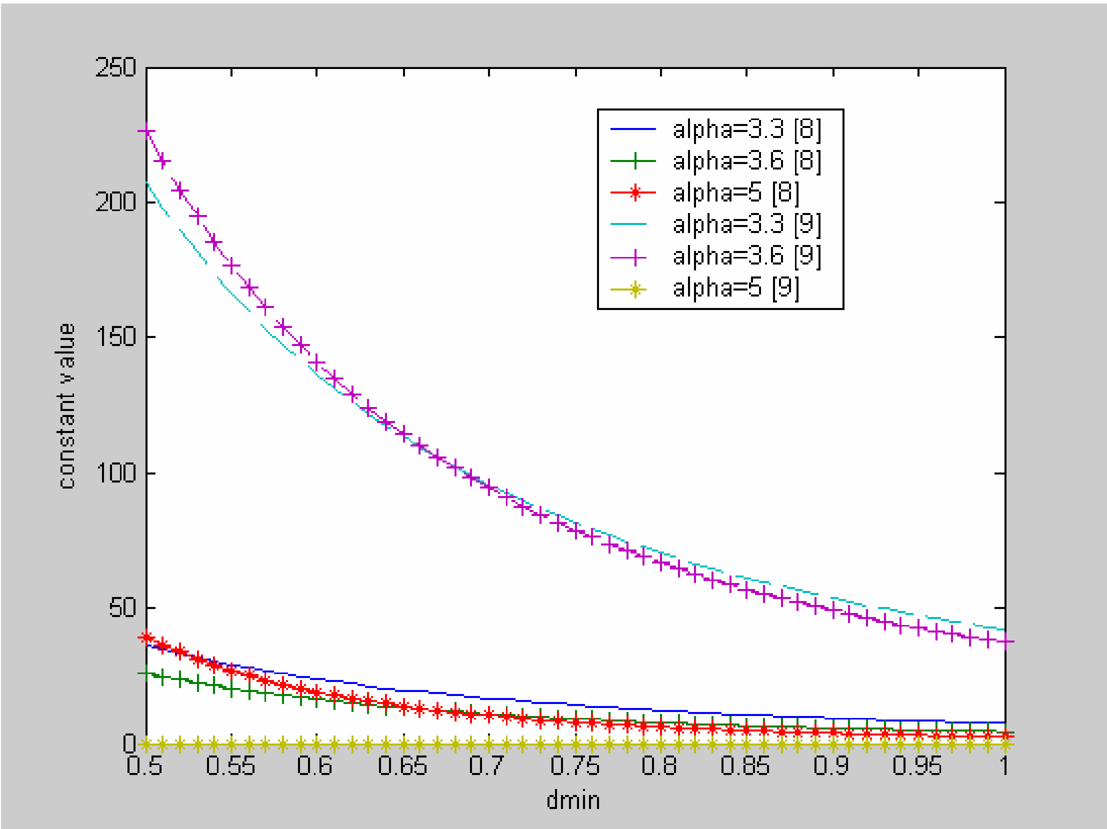


Figure A. 1 The comparison between constant part in [8] (real line) and [9] (imaginary line) with fixed path loss level for one dimensional case with full CSI

***Appendix***

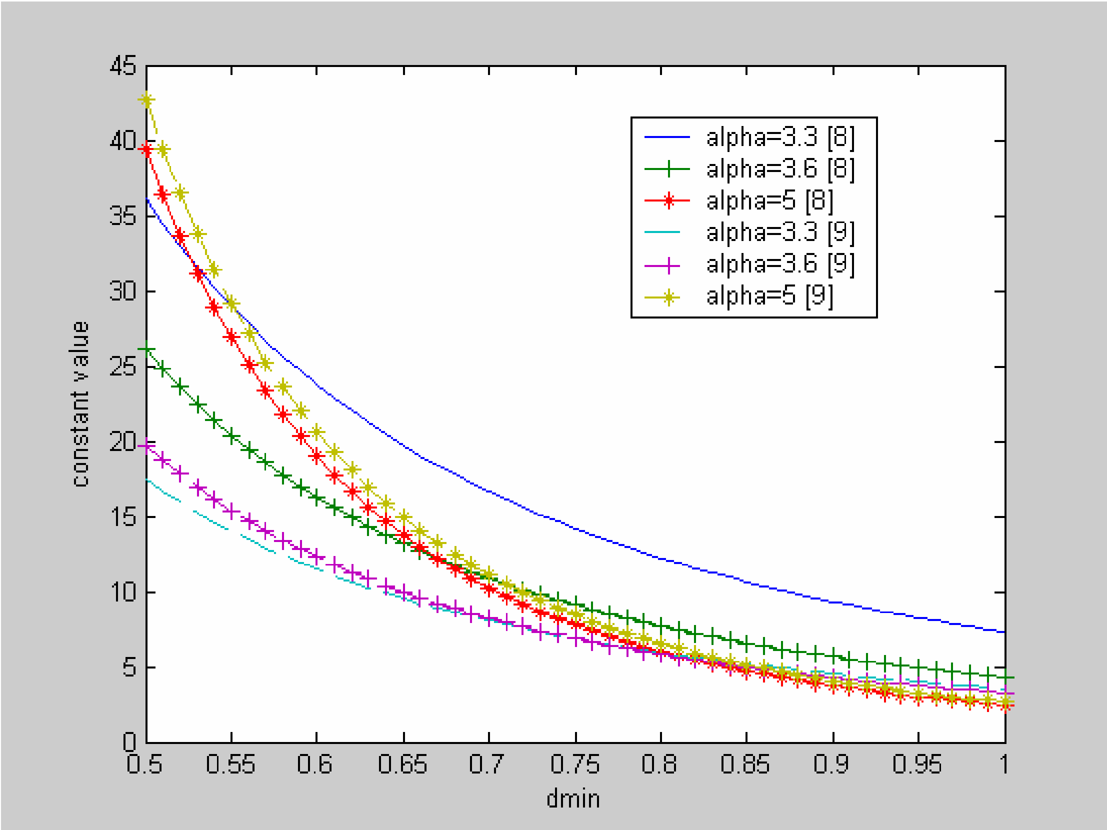


Figure A. 2 The comparison between the constant in [8] with full CSI (real line) and the constant in [9] without full CSI (imaginary line) for fixed path loss level in one dimensional network.

***Appendix***

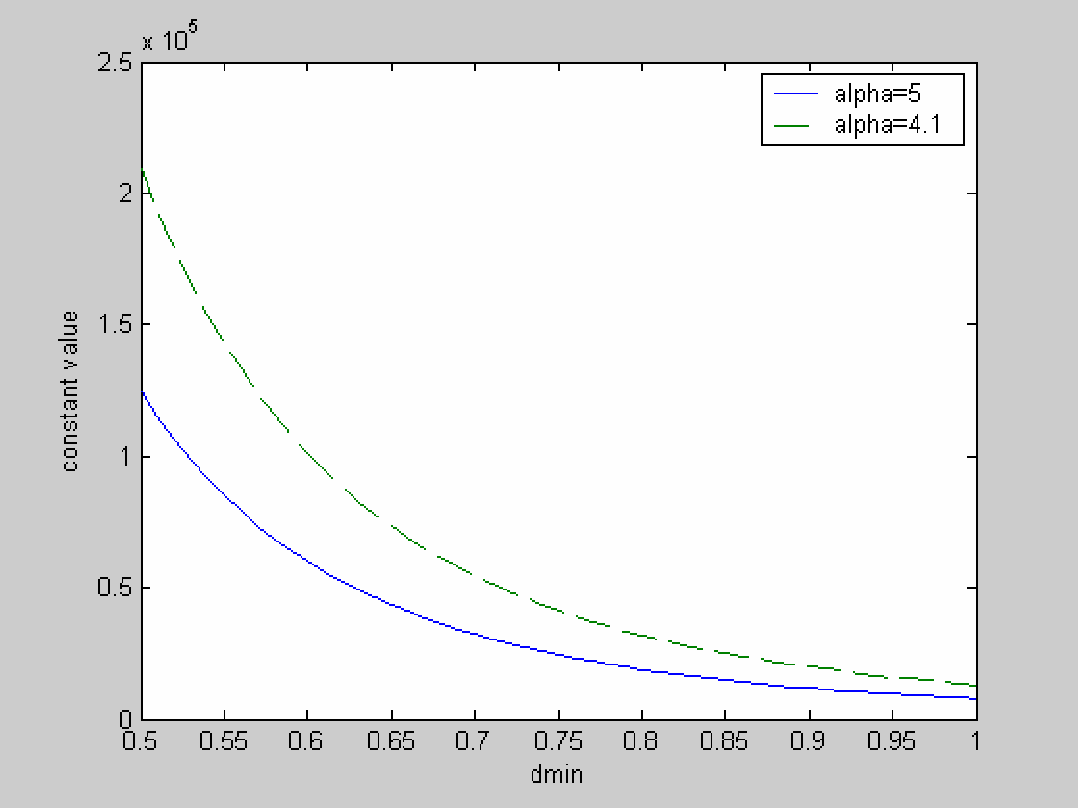


Figure A. 3 The constant in [8] with respect to $d_{\min}$ for some fixed $\alpha$ in two dimensional networks.

***Appendix***

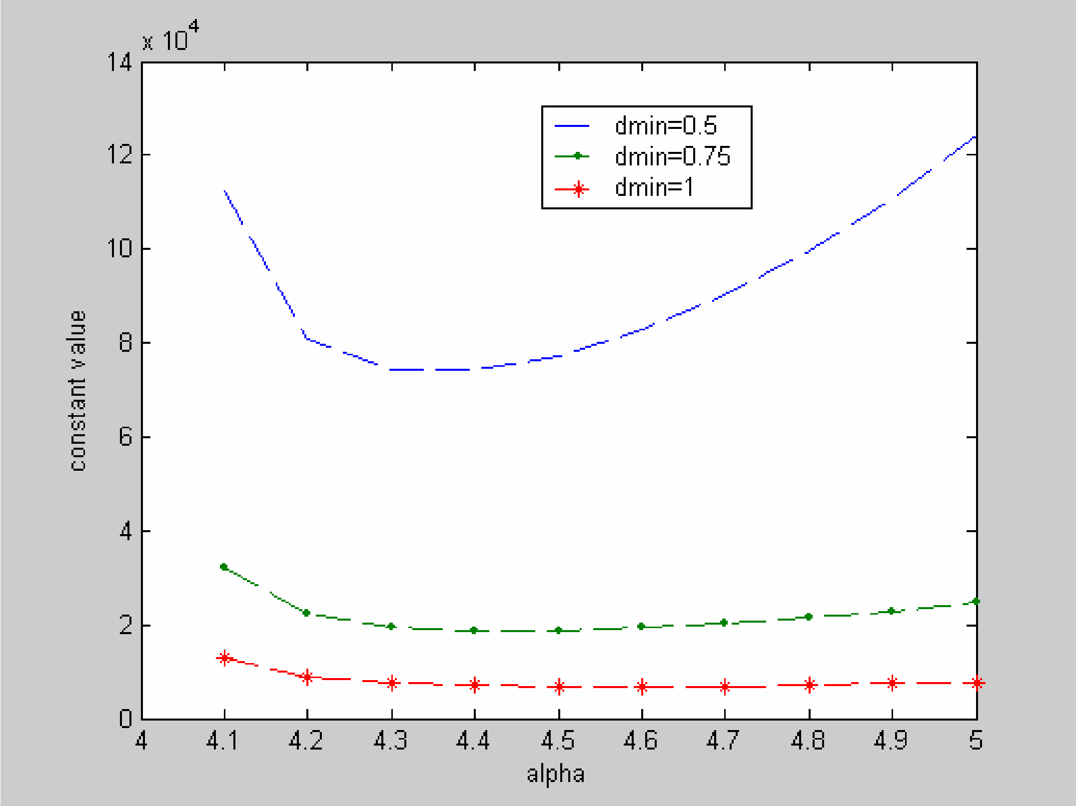


Figure A. 4 The constant in [8] with respect to $\alpha$ for some fixed $d_{\min}$ in two dimensional networks

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