Optimal Node Placement in Multiple Relay Wireless Networks

by

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A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Master of Applied Science
in
Electrical and Computer Engineering

Waterloo, Ontario, Canada, 2008

Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

This thesis explores the optimal node placement for linear Gaussian multiple relay networks of an arbitrary size and with one source-destination pair. Consider the model that signal attenuation grows in magnitude as $\frac{1}{d^\delta}$ with distance d in the low attenuation regime (path loss exponent $\delta < 3/2$). Under the condition that the minimum achievable rate from source to destination is maintained, we derive upper bounds of node placement with the incoherent and coherent coding schemes, and examine the optimal power assignment related to the node placement with the coherent coding scheme. We prove that the farthest distance between two adjacent nodes is bounded even for an infinite total number of relay nodes, and closed-form formulas of the bounds are derived for both the coding schemes. Furthermore, the distance from the source to the destination is of the same order as the total number of nodes, given the path loss exponent $\delta > \frac{1}{2}$ under the incoherent coding scheme and $\delta > 1$ with coherent relaying with interference subtraction coding scheme. Conditioned on a conjecture based on the simulation results, we also provide heuristic upper bounds, which are a little tighter than the strictly proved bounds. The bounds provided in this thesis can serve as a helpful guideline for the relay extension problem in practical network implementation.

Acknowledgements

I am deeply indebted to many people for their support during the course of my Master study. First and foremost, I am really grateful to my advisor, Professor Liang-Liang Xie, for his insightful guidance, great patience, and generous support. His passion for research and exceptional knowledge of the field has always been the inspiration to me.

I would like to thank Professor Pin-Han Ho and Professor Zhou Wang for reading my thesis, as well as for their helpful comments that help to develop this work.

Special thanks are given to Dr. Haiquan Wang for his invaluable help and insightful interaction. I also wish to extend my appreciation to all friends, Xiaoyu Chu, Bin Lin, Kun Shi, and Xiugang Wu for their friendship.

Finally, I give thanks to my husband, Xiang Yu, for his love and encouragement throughout my study. I could not finish this journey without him.

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Abbreviations

CRIS	Coherent Relaying with Interference Subtraction
IRIS	Incoherent Relaying with Interference Subtraction
LGMRN	linear Gaussian Multiple Relay Network
AWGN	Additive White Gaussian Noise
MAC	Multiple Access Channel
BC	Broadcast Channel
DMC	Discrete Memoryless Channel
DF	Decode-and-Forward
CF	Compress-and-Forward

Chapter 1

Introduction

With the fast expansion of wireless communications applications, the demand for more efficient and cost effective wireless communication networks is motivating the research community to put great efforts in exploring the cooperation among a multitude of network nodes/terminals.

When cooperation strategies are considered in a wireless network with even a few nodes, the situation becomes very complicated. There are many possible functions that a node can simultaneously serve, such as relaying, broadcasting, interference canceling, etc [1]. Relaying, as an elementary and important mode of cooperation that has been explored for 40 years, has become a hot topic along with the advancement in network information theory. The study on relay networks provides a great deal of guidance to the application of cooperation among the wireless nodes/terminals.

1.1 Motivations and Objectives

How should a pair of source and destination nodes communicate through multi-terminal wireless networks? One possibility is to select a sequence of nodes forming the path from source to destination and then relay the packets from node to node until the destination is reached. Without exploring the cooperation between nodes, the traditional way of relaying along such a path is that each node fully decodes the packets and retransmits to the next node. While doing the regeneration of the packets, all interferences are treated as noise. Accompanying this mode of information transfer, which is called "multihop mode" in [2], there are many protocols addressing the interference related problem, such as medium access control (MAC) protocol, power control protocol, etc. Along with the advancement of network information theory, it is clear that exploring the interference instead of avoiding its happening is important to improve the achievable rate of multi-terminal wireless relay networks ([3], [2]).

To our best knowledge, there is no study on the fundamental limits of the relay node placement yet. Few works have taken advantage of cooperative relaying while addressing the node placement problem. In [4], Lin et. al. examines the relay station placement with cooperative scheme in a practical setting, a two-relay scenario. There are some studies exploring the node placement problem of relay sensor networks, such as [5], [6], and [7]. But these works mainly explore node placement or related issues under the conventional non-cooperative "multi-hop" mode.

Thus motivated, this study is targeted at understanding the fundamental limits of node placement problem in linear Gaussian multiple relay network (LGMRN) with cooperative relaying schemes. Two advanced cooperative coding schemes utilized in this study are coherent relaying with interference subtraction (CRIS) and incoherent relaying with interference subtraction (IRIS) coding schemes [1]. The fundamental limits to be pursued will provide a foundation for practical system design.

1.2 Contributions

The thesis, based on the result of the achievable rate region for the Gaussian multiple relay network in Xie and Kumar's previous work, i.e., Theorem 3.11 of [1], addresses the optimal node placement problem for the LGMRN. Given a set of relay nodes on a straight line, our objective is to find out the optimal positions for the whole set of relay nodes in order to transmit data to the furthest distance under the condition that the minimum rate requirement is satisfied at each node up to the destination.

This node placement problem is studied with two related coding schemes, i.e., CRIS and IRIS.

We prove that, under a minimum transmission rate requirement, the distance between any adjacent 2 nodes in an LGMRN is bounded and the total relayed distance from source to destination is of the same order as the number of nodes in the LGMRN for both the coding schemes. Being more specifically, the bounds of the distance between adjacent nodes are $1 + (\frac{P}{P_{min}^{rec}} \frac{2\delta}{2\delta-1})^{\frac{1}{2\delta}}$ and $1 + \left(\frac{P}{P_{min}^{rec}}\right)^{\frac{1}{2\delta}} \left(\frac{\delta}{\delta-1}\right)^{\frac{1}{\delta}}$ for CRIS and IRIS schemes respectively, where P is the individual power constraint of source and each of the relay nodes, P_{min}^{rec} is the minimum received power at destination node required to meet the minimum transmission rate requirement, and δ is the path loss exponent used in the attenuation model. Strict proofs are given for these bounds. Conditioned on a conjecture based on the simulation results, we also provide heuristic bounds which are a little tighter than the above bounds.

Simulations are presented after the theorems and proofs. There is also some comparison between the simulations for both situations. The purpose is to compare that how efficient one is over the other, thus the guidance can be provided when the tradeoff is considered between the transmission rate (or the distance could be reached, equivalently) and the practical cost when the relaying is implemented.

1.3 Organization

This thesis continues as follows.

Chapter 2 reviews the basic concepts and related research works on relay channels. We present relay channel models, such as the Discrete Memoryless Channel (DMC), the Gaussian relay channel and some capacity and achievable rate results developed so far. We also provide the background on the node placement problem for the relay channel.

In Chapter 3 and 4, we study the node placement problem by presenting the theorems, proofs and corresponding simulation results. Chapter 3 is mainly for the node placement problem with the IRIS coding scheme. Chapter 4 is focused on the node placement problem with the CRIS coding scheme. We will also compare the theoretical and simulation results corresponding to these two coding schemes in this chapter. The conclusion of the thesis and some discussions about the future research are presented in Chapter 5.

Chapter 2

Background and Literature Review

In this chapter, we review the basic concepts of relay channels. We present several relay channel models, such as the discrete memoryless relay channel and the Gaussian relay channel. We also provide some background about the coding schemes, related research results on capacity and achievable rate region, and node placement problem for the relay channel.

Due to the broadcasting nature of wireless networks, there are many possibilities of cooperative strategies among the wireless terminals, such as relaying, broadcasting, interference canceling, etc. Relaying is a primary type of cooperation and has been one of the basic topics of the multi-user information theory for 40 years.

In a relay network, generally, the relay nodes may have their own information to transmit and also help forwarding information from other nodes. This work considers the case that relay nodes solely assist the transmission from source to destination.

2.1 Overview of Relay Channels

A model of relay channels was first introduced by Van de Meulen in his pioneering work [8], [9]. The simplest case studied is the three nodes scenario where there is one node that functions purely as a relay to assist the transmission between the source and destination nodes. Two fundamental coding strategies, namely, Decode-and-Forward (DF) and Compress-and-Forward (CF), were developed in [10] by Cover and El Gamal, though the strategies got their names later in [11]. In the last decade, many works have been done on the multiple relay channel and the achievable rate and transport capacity has been explored in [1], [3], [12], [13] and [14], etc.

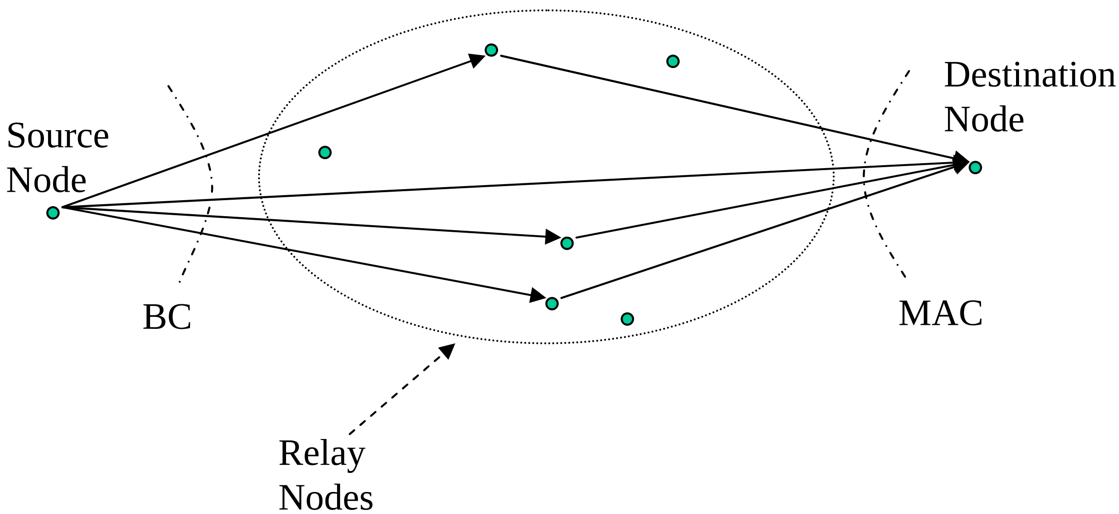

Figure 2.1: A general relay model

Fig. 2.1 shows a general relay channel model with one source and one destination. All the nodes lie in a 3-dimensional Euclidean space. There are one or more relay node(s) helping the transmission from source to destination. It is a combination of the broadcast channel (BC), which is from source to relays and destination, and multiple access channel (MAC), which is from source and relays to destination.

The capacity of the channel is upper bounded by the max-flow min-cut bound [10], [15], which has become the standard tool for bounding the capacity regions [16](p.445).

For example, for a single-relay channel, which can be formulated by

$$(\mathfrak{X}_0 \times \mathfrak{X}_1, p(y, y_1/x_{0,x_1}), \mathfrak{Y} \times \mathfrak{Y}_1)$$

where the source send x_0, the relay receives y_1 and sends x_1, the destination receives y, then, the capacity is bounded above by

$$C \leq \sup_{p(x_0,x_1)} min\{I(X_0, X_1; Y), I(X_0; Y, Y_1/X_1)\}. \quad (2.1)$$

The first term in (2.1) upper bounds the maximum rate of information transfer X_0 and X_1 from transmitters of source and relay to destination receiver Y (Multiple Access Channel); the second term bounds the rate from X_0 to Y and Y_1 (Broadcast Channel), but the destination should decode the relay signal X_1 before decoding X_0, which contributes to the conditioning term X_1 in $I(X_0; Y, Y_1/X_1)$. The proof can be found in [10].

Discrete memoryless relay channel model:

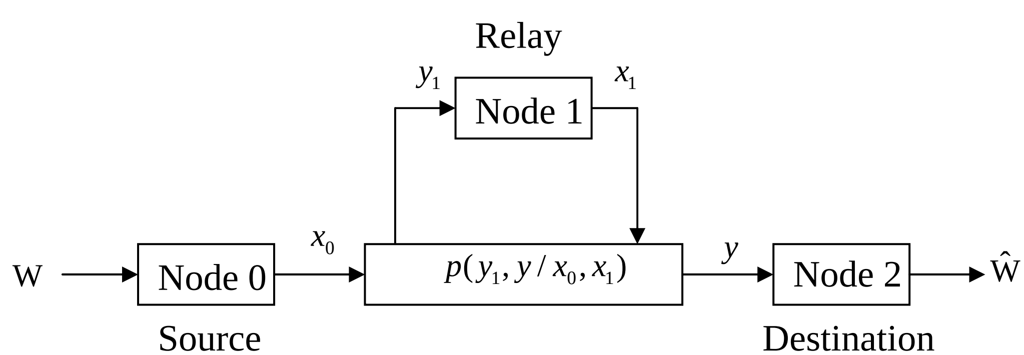

Figure 2.2: A discrete memoryless relay channel model.

In the simplest DMC (Discrete Memoryless Channel) relay model introduced in ([8]), as shown in Fig. 2.2, there are three nodes: 0, 1 and 2, which are source, relay and destination respectively. W is the original message. $\hat{W}$ is the estimated message at the destination end. x_0 and x_1 are the inputs sent by the source and relay nodes. y_1 is the output sent to relay. y is the output sent to the destination by the channel. The inputs and outputs are all discrete random variables in the discrete channel model. If we denote the transmitter alphabets of node 0 and 1 as $\mathcal{X}_0$ and $\mathcal{X}_1$, and the receiver alphabets of node 1 and 2 as $\mathcal{Y}_1$ and $\mathcal{Y}$, the channel can be described as $(\mathcal{X}_0 \times \mathcal{X}_1, p(y_1, y/x_0, x_1), \mathcal{Y}_1 \times \mathcal{Y})$, where $p(\cdot, \cdot/x_0, x_1)$ is a collection of probability distributions on $\mathcal{Y}_1 \times \mathcal{Y}$ for every pair of $(x_0, x_1) \in \mathcal{X}_0 \times \mathcal{X}_1$. The source sends to the channel input x_0 according to the specific message it need to transmit. relay gets y_1 and sends the input x_1 after processing y_1.

The channel is memoryless in the sense that $(Y_i, Y_{1,i})$ depends on the past only through the current transmitted symbols $(X_{0,i}, X_{1,i})$.

For the single-relay DMC described above, the best achievable rates so far are still those proved 20 years ago in [10]:

$$R < \max_{p(x_0,x_1)} \min\{I(X_0; Y_1/X_1), I(X_0, X_1; Y)\}. \tag{2.2}$$

The above rate could not be achieved by simply multihop. Instead, the destination needs to make use of both inputs from the source and the relay in order to achieve (2.2). A special discrete memoryless relay channel even constructed where no reliable transmission from source to destination if no relay helps in [8].

The degraded relay channel implies that the destination receives a degraded version of the relay received signal for the single-relay channel described above. The signal y_1 that the relay received is better than the signal y that the destination received. Thus the relay can cooperate with the source to transmit information to the destination. The other case, in which the relay received y_1 is worse than y, is less interesting, because the relay can not contribute new information to the destination. It is called a reversely degraded relay channel and the relay still can facilitates the transmission of x_0 by sending x_1.

Cover and El Gamal presented that the right hand side of (2.2) is the capacity of the degraded relay channel in [10].

The relay channel $(\mathfrak{X}_0 \times \mathfrak{X}_1, p(y, y_1/x_{0,x_1}), \mathfrak{Y} \times \mathfrak{Y}_1)$ is said to be degraded if $X_0 \rightarrow (X_1; Y_1) \rightarrow Y$ form a Markov chain.

Equivalently, a relay channel is degraded if $p(y/y_1, x_0, x1) = p(y/y_1; x_1)$, which makes

$$p(y, y_1/x_0, x_1) = p(y_1/x_0, x_1)p(y/y_1, x_1) \tag{2.3}$$

holds.

Hence, due to the degradedness, $I(X_0; Y, Y_1/X_1) = I(X_0; Y_1/X_1)$.

In the following, we present more details about the model of Gaussian relay channel.

Gaussian relay channel:

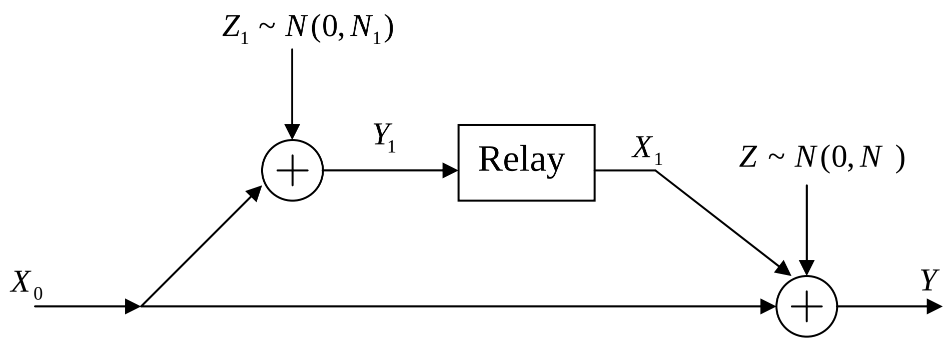

Figure 2.3: Gaussian single-relay channel

A Gaussian single-relay channel (see Figure 2.3) is modeled by

$$Y_1 = X_0 + Z_1, \tag{2.4}$$

$$Y = X_0 + X_1 + Z, \tag{2.5}$$

where Z_1 and Z are independent Gaussian noise . For the general Gaussian single-relay channel, the capacity is still unknown. Through an interesting study [17] about the Gaussian parallel relay channel, it was shown that there exists no unifying optimal coding scheme because the asymptotically optimal coding scheme dramatically depends on the relative locations of the nodes.

Only for the degraded Gaussian relay channel, the capacity is known for some cases (see [10] p.380-381 and [18]). In [10], The degradedness of the Gaussian single

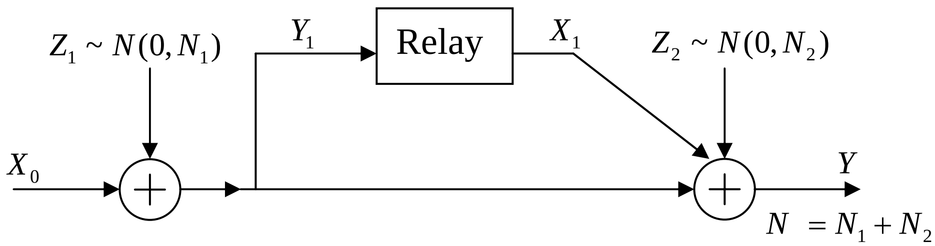

Figure 2.4: Degraded Gaussian single-relay channel

relay channel means it can satisfy (2.3), i.e., $X_0 \rightarrow (X1; Y1) \rightarrow Y$ form a Markov chain.

A degraded Gaussian single-relay channel (as shown in Figure 2.4) is modeled by

$$Y_1 = X_0 + Z_1, \tag{2.6}$$

$$Y = X_0 + Z_1 + X_1 + Z_2, \tag{2.7}$$

The receiver Y is a degraded version of the receiver Y_1 conditioning on X_1.

If we denote the power of the source and relay as P_0 and P_1 respectively, the channel capacity is

$$C = \max_{0 \leq \alpha \leq 1} \min \left\{ S\left(\frac{P_0 + P_1 + \sqrt{(1 - \alpha P P_1)}}{N_1 + N} \right), S\left(\frac{\alpha P}{N_1} \right) \right\} \tag{2.8}$$

where $S(x) := \frac{1}{2} \log(1 + x)$ is the Shannon function.

Multiple level relay channel:

When it involves a multitude of relay nodes, the problem becomes complicated. Recently, problems of multiple-level relay channels attracted more research efforts along with the increased interest in wireless networks and network information theory.

Gupta and Kumar demonstrated an achievable rate region result for a fairly general multi-level relay channel in [3], where the coding scheme of [10] was extended.

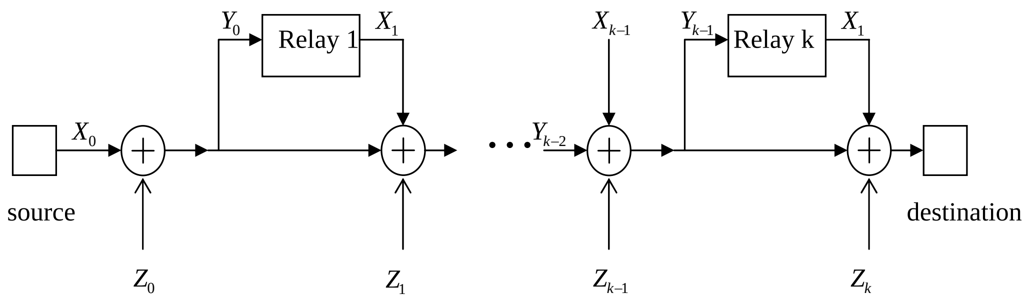

Figure 2.5: Degraded Gaussian K-relay channel

The result holds for both the discrete memoryless and the additive white Gaussian noise (AWGN) channel. Xie and Kumar [1] proposed a new coding scheme, namely, Coherent Relaying and Interference Subtraction(CRIS), for the Gaussian multi-level relay channel. They also proved an achievable rate region which is higher than those proved in [3].

Figure 2.5 depicts a degraded Gaussian multiple relay channel which has one source-destination pair and K relays. It is a special case of the general gaussian multiple relay channel. The achievable rate for general gaussian multiple relay channel presented in Xie and Kumar [1](see Theorem 3.11) actually is the capacity for the physically degraded gaussian multiple relay channel. The proof can be found in [18].

Due to the complexity of relay channels, not much about the capacity for a general relay channel has been known even only one relay gets involved. Transport capacity, as a very helpful and effective metric instead of capacity, has been explored from almost a decade ago [12] and it underwent thorough examination in Xie and Kumar's works [1], [13], [14].

2.2 Coding Schemes for Relaying

Two fundamental coding strategies, Decode-and-Forward (DF) and Compress-and-Forward (CF) for single-relay model, were developed by Cover and El Gamal [10] (see Theorems 1 and 6), though they got these names later in [11].

There are 3 different DF strategies developed so far, as reviewed by Kramer et. al. in [19]:

a) irregular encoding/successive decoding;

b) regular encoding /sliding window decoding;

c) regular/backward decoding.

The irregular encoding is different from the regular encoding by using codebooks of different sizes.

What used in [10] (Theorem 1) is irregular encoding (Block Markov superposition encoding) and successive decoding. King developed the other 2 strategies in [20]. The irregular encoding/successive decoding strategy was extended to degraded relay networks by Aref in [15]. These strategies were generalized to multiple relay channels in subsequent works in [11], [12] and [3].

Xie and Kumar extended regular encoding/sliding window decoding for multiple relays in [1] and the result was extended to discrete memoryless channels in [13]. It also has been shown that their scheme achieves better rates than what were used in [3] and [15].

In this thesis, theorems and simulations are based on the condition of using both CRIS scheme and IRIS scheme. Both coding schemes are explained in the following.

CRIS coding scheme:

An advanced new coding scheme called coherent relaying with interference subtraction (CRIS) for general Gaussian relay network was proposed by Xie and Kumar in [1]. An explicit achievable rate formula was obtained also. Later, this coding scheme and achievable rate formula were extended to the discrete memoryless channel in [13].

Here we present the basic idea of the CRIS scheme. CRIS scheme uses regular encoding/sliding window decoding strategy. The channel using this scheme has better achievable rate than using other schemes developed in [3], [10], [15] and [16] .

To see how this coding scheme is applied, consider the simplest case first: a single-relay channel.

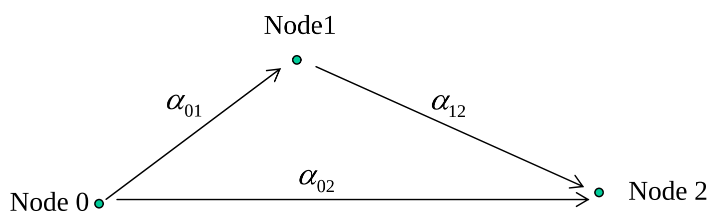

Figure 2.6: A single relay channel

As fig. 2.6 shows, there is a source node 0, relay node 1, and destination node 2. The α_{01}, α_{02} and α_{12} denote the corresponding signal attenuation factors. Based on the block-Markov coding scheme, the transmission time is partitioned into equal blocks. The first and the last blocks need special treatment. In each intermediate node, Node 0 divide its power into $P_{01} = \theta P_0$ and $P_{02} = (1 - \theta)P_0$, $0 \leq \theta \leq 1$. P_{01} is used to inform relay node 1 what message the source node intents to transmit in the next block. It can achieve any rate R that satisfies

$$R \leq S\left(\frac{\alpha_{01}^2 \theta P_0}{\sigma^2}\right). \tag{2.9}$$

The other part P_{02} is dedicated to the destination node and it cooperates with the total power of relay node 1 since node 1 has already known the intention of node 0 from the previous block. In other words, node 0 and node 1 (using part of its power P_{02}) coherently transmit to the destination node. The signal that the destination node 2 receives contains 3 components: the coherent cooperation of node 0 and 1, the signal send by node 0 intended to node 1, and the noise with power $(\alpha_{02}\sqrt{(1-\theta)P_0} + \alpha_{12}\sqrt{P_1})^2$, $\alpha_{02}^2\theta P_0$, and σ^2 respectively. The intuition for the decoding at destination node 2 is that, at the end of each block, node 2 takes the first part of the present block and the second part of the previous block as signal simultaneously and decodes. The first part of the previous block is deducted because it is interference to the present block and its content is known at the end of the previous block. This is also the reason that this coding scheme is called coherent relaying with interference subtraction).

For a multiple relay channel with one source and one destination as shown in Figure 2.7, the CRIS scheme applies similarly as in the one relay channel, though it appears much more complicated.

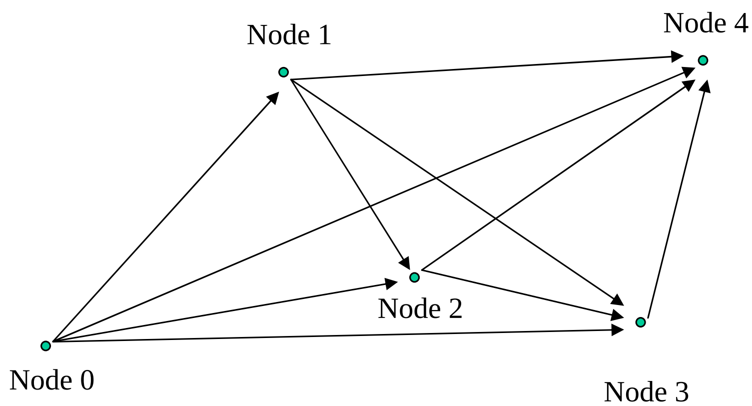

Figure 2.7: Single-source single-destination multiple relay channel

Let's consider a relay network with $n + 1$ nodes, which is not necessary to be a linear network, or even planar network. This network has one source-destination pair, denoted as node 0 and n, all the intermediate nodes denoted as node 1 to $n-1$. The higher numbered nodes are called "downstream nodes" but it is not necessary that they are the ones closer to the destination. The attenuation from node i to node j is known as α_{ij} and there is an i.i.d. additive noise $N(0, \sigma^2)$ at each receiver. Each node i has a power constraint P_{ind}. Source node and each relay node (except the last relay node, which contributes all of its power to the destination) divides its power into different portions for the downstream nodes respectively.

The block-Markov coding scheme will be used also. The whole transmission time is divided to equal blocks and the first n-1 and the last n-1 blocks will be treated specially. In each of the intermediate time blocks, each node (include the source node 0 and every relay node) uses explicit parts of its total power for the immediate next node and every node after that. The coherence cooperation lies in the coherent transmission, e.g. node i dedicate a portion of its power P_{ij} for the transmission to a certain downstream node $j (0 \leq j \leq k-1)$, coherently transmit to node j with other nodes using P_{kj}, $0 \leq k < j$ but $k \neq i$. For decoding, each intermediate node and the destination node do the decoding only when it has received all related signals and determines the message directly. This coding scheme generally achieves higher rate than the scheme proved in [3].

Remarks: For each of the intermediate nodes, it can be replaced with a group of nodes and the coding scheme still can apply.

IRIS coding scheme:

The IRIS scheme using interference subtraction but no coherent transmission among the upstream nodes j $(0 \leq j \leq k-1)$ as they each devotes the power to transmit to the next downstream node. The incoherent relaying scheme does not do coherent transmission of the CRIS scheme but keeps the interference subtraction part while decoding.

It results in a lower achievable rate for the relay network compared with the CRIS scheme but higher achievable rate than the conventional "multi-hop" scheme. The information is transmitted from source to destination as in a multihop mode, but the key difference is that the interference from downstream nodes are subtracted from the received signal when the decoding is performed at every node because the information transmitted among the downstream nodes is already known by upstream nodes.

The tradeoff here is that the incoherent scheme demands significantly less coding and decoding complexity and cost than that of applying coherence among the relaying nodes.

2.3 The Achievable Rate Theorem

For Gaussian multiple relay networks, regular encoding/sliding window decoding, named CRIS coding scheme in [1], is currently the preferred substitution of multi-hopping in the sense that it achieves the best rates in the simplest way.

Theorem 3.11 of [1] states: Consider a Gaussian multiple relay channel with a single source-destination pair using the CRIS coding scheme. There are $n+1$ nodes, sequentially denoted by 0, 1, ..., n, with 0 as the source and n as the destination, the rest serving as $n-1$ relays. α_{ij} stands for the attenuation from node i to node j. There are also i.i.d. additive Gaussian noise $N(0,\sigma^2)$ at each receiver. Then the following inequality is achievable from 0 to n:

$$R < \min_{1 \leq j \leq n} S\left(\frac{1}{\sigma^2}\sum_{k=1}^{j}\left(\sum_{i=0}^{k}\alpha_{ij}\sqrt{P_{ik}}\right)^2\right) \tag{2.10}$$

where $P_{ik} \geq 0$ satisfies $\sum_{k=i+1}^{n} \leq P_i$.

The nodes need not lie on a straight line or a plane. The proof can be found in [1].

2.4 Node Placement

There have been a few related studies to the relay node placement problem. Nevertheless, in most of these studies, e.g., [5], [21] and [6], the main focus is on the wireless sensor networks with a few relay nodes helping a nearby sensor node using the conventional noncooperative “multi-hop” mode. In [4], a practical relay station placement problem is examined in a dual-relay network with cooperative relaying scheme. In this thesis, we study a cooperative coding scenario for the node placement problem in a Gaussian multiple relay networks with arbitrary size. In other words, we explore the ultimate limits of the node placement in LGMRN with advanced cooperative relaying schemes.

For the attenuation model, we utilize $\alpha = \frac{e^{-\gamma d}}{d^{2\delta}}$, where d is the distance. In the low attenuation regime, the path loss exponent $\delta < \frac{3}{2}$ and absorption constant

$\gamma = 0$. With two different but related cooperative coding schemes, we place the nodes sequentially to the right on a straight line as long as the minimum transmission rate is maintained. We examine the limit of the largest distance between adjacent nodes and the source-destination distance can be covered when the total number of nodes goes to infinity.

More specifically, we prove that, under a minimum rate requirement, the source-destination distance is of the same order as n and the distance between two adjacent nodes is upper bounded even when the total number of nodes, each possesses the same individual transmit power, goes to infinity with incoherent relaying scheme-IRIS ($\delta > \frac{1}{2}$) or with coherent relaying scheme-CRIS ($\delta > 1$) in an LMGRN.

This chapter discussed the background of relay channels and the node placement problem. In the next Chapter, we will present our theorem of an upper bound for the node placement problem with IRIS coding schemes, followed by corresponding simulation results.

Chapter 3

Node Placement for the LGMRN using IRIS Scheme

In this chapter we study the node placement problem for an LGMRN that uses an with the IRIS coding scheme. We first present theoretic results with the IRIS coding scheme, which are bounds for the distance between adjacent nodes and the maximum relay distance. Simulations are then conducted to demonstrate the theorem.

3.1 Node Placement with the IRIS Scheme

This section formulate the node placement problem with the IRIS coding scheme and presents two theoretic results on the distance between adjacent nodes and the source-destination distance.

3.1.1 System Setting

Consider a Gaussian multiple relay network, as shown in fig. 3.1, where all nodes lie on a straight line.

This linear network consists of $n+1$ nodes, which include one source-destination pair, denoted as node 0 and n. All the intermediate nodes denoted as nodes 1 to $n-1$.

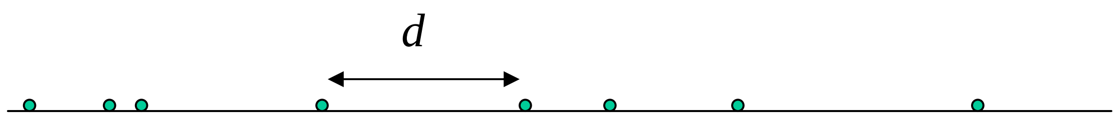

Figure 3.1: A linear multiple relay network

The source node represents a transmitter, the destination is a receiver, and each of the intermediate relay nodes has both a transmitter and a receiver. Consider the low attenuation regime, i.e., the absorption constant $\gamma = 0$ and the path loss exponent $\delta < \frac{3}{2}$, while using the simplistic model of signal attenuation $\alpha = \frac{e^{-d\gamma}}{d^{\delta}}$ over distance d. Let d_{ij} denote the distance between nodes i and j, make it simpler. Denote d_i as the distance between the ith node and the $i+1$th node. The attenuation from node i to node j is

$$\alpha_{ij} = \frac{1}{d_{ij}^{\delta}} = \frac{1}{(\sum_{k=i}^{j-1} d_k)^{\delta}}.$$

There is a constraint on P for each node's individual power P_{ind}. At the receiver end of each node, there is an i.i.d. additive white noise $N(0, \sigma^2)$.

3.1.2 IRIS Coding Scheme for LGMRN

Consider the above linear Gaussian multiple relay network using the IRIS coding scheme. The IRIS coding scheme employs interference subtraction without coherent transmission among the upstream nodes j $(0 \leq j \leq k-1)$ as each of them devote its power to transmit to the next downstream node. The relay nodes are placed in a way that the achievable rate from the source to the destination is no less than a required value. Without loss of generality, we assume $P_i \leq P$, where P_i is the power of the ith node. Denote the actual received power at the destination as P^{rec}. Denote R_{min} as the minimum achievable rate from the source to the destination since there is a minimum rate requirement of the system. Therefore, accordingly the received power at the destination as P^{rec} is assumed to be greater than a minimum of P^{rec}_{min}, which corresponds to the minimum achievable rate as the Shannon function defines, i.e., $R_{min} = S(\frac{P^{rec}_{min}}{\sigma^2})$, where σ^2 is the power of the Gaussian noise.

According to Theorem 3.11 in [1], which is for CRIS scheme, we derive the achievable rate formula for IRIS:

$$\begin{aligned} R &< \min_{1\le j\le n} S\left(\frac{1}{\sigma^2}\sum_{i=1}^{j}\alpha_{i,n}^2 P_i\right) \\ &= \min_{1\le j\le n} S\left(\frac{1}{\sigma^2}\sum_{k=1}^{j}\frac{P_k}{(\sum_{i=k-1}^{n-1} d_i)^{2\delta}}\right). \end{aligned} \tag{3.1}$$

For the case of $j = n$, we have

$$R_{min} \le R < S\left(\frac{1}{\sigma^2}\sum_{i=1}^{n}\alpha_{i,n}^2 P_i\right). \tag{3.2}$$

Correspondingly, we have the following inequalities for the received power at the destination node:

$$P^{rec}_{min} \le P^{rec} = \sum_{i=1}^{n}\frac{P_i}{(\sum_{l=i}^{n} d_l)^{2\delta}}, \tag{3.3}$$

3.1.3 Theorem and Proof of the Bound

Using the above formulas of the achievable rate from source to destination and the received power at the destination, we develop the following theoretical results:

Theorem 3: Consider a Gaussian linear relay network with $n+1$ nodes, where n might be infinite. Let the source node denoted as 0, the destination node denoted as n and the intermediate nodes denoted as 1 to $n-1$ sequentially. With incoherent relaying and interference subtraction strategy, place the relays, i.e. all the intermediate nodes, in a manner that the minimum transmission rate is achieved from source to each of the downstream nodes and the destination. Given the path loss exponent $\delta > 1/2$, as the number of nodes $n+1$ goes to infinity,

i) the distance between any two adjacent nodes d_i is bounded above by $1 + (\frac{P}{P_{min}^{rec}} \frac{2\delta}{2\delta - 1})^{\frac{1}{2\delta}}$;

ii) the total distance between source and destination nodes $\sum_{i=1}^{n} d_i = \Theta(n)$.

Proof: First let's prove that all d_i's $(0 \le i \le n-1)$ are bounded for even infinite total number of nodes $n+1$ given $\delta > 1/2$.

Assume there exists a finite number j such that d_j is unbounded. Let $m \le d_j < m+1$, where m is a positive integer. Under this assumption, m should be unbounded also.

Since it is always preferable to move the nodes to the right as much as possible in order to maximize the source-destination distance, we construct an extreme situation for the worst case as the following:

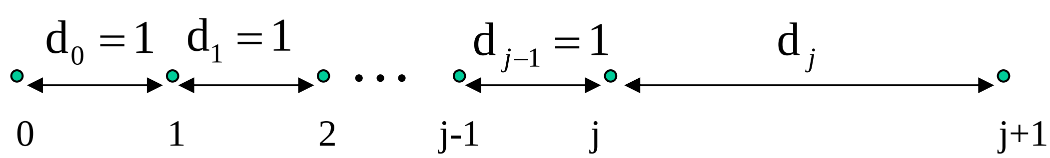

Figure 3.2: A constructed network to maximize the d_j

.

As Figure 3.2 shows, place all the nodes in a way that $d_i = 1$ $(0 \le i \le j-1)$ and place node $j+1$ to the furthest point possible as long as the minimum required rate still can be attained. The received power at the node $j+1$, which should be greater or equal to the minimum received power corresponding to the minimum required rate.

From inequality (3.3), we have

$$
\begin{aligned}
P_{min}^{rec} \leq P_j^{rec} &< \sum_{k=1}^{j} \frac{P_k}{(\sum_{i=k}^{n} d_i)^{2\delta}} \\
&< P\left(\sum_{k=1}^{j} \frac{1}{(\sum_{i=k}^{j} d_i)^{2\delta}}\right) \\
&= P\left(\sum_{k=1}^{j} \frac{1}{(j-k+d_j)^{2\delta}}\right)
\end{aligned} \tag{3.4}
$$

as all $P_k \leq P$, where P is the constraint of individual power that each nodes possesses. The equation follows from $d_i = 1$ $(0 \leq i \leq j-1)$.

Combining $m \leq d_j < m+1$ and the inequality (3.4), we have

$$
\begin{aligned}
P_{min}^{rec} \leq P_j^{rec} &< P\left(\sum_{k=1}^{j} \frac{1}{(j-k+d_j)^{2\delta}}\right) \\
&\leq P\left(\sum_{k=1}^{j} \frac{1}{(j-k+m)^{2\delta}}\right) \\
&= P\left(\sum_{i=m}^{j+m-1} \frac{1}{i^{2\delta}}\right)
\end{aligned} \tag{3.5}
$$

When j goes to infinity, $\sum_{i=m}^{j+m-1} \frac{1}{i^{2\delta}}$ is a special case of Riemann zeta-function $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}}$ ([22], also see Appendix A). We have

$$
\begin{aligned}
\sum_{n=m}^{\infty} \frac{1}{n^{\alpha}} &< \frac{1}{m^{\alpha}} + \int_{m}^{\infty} \frac{1}{x^{\alpha}} dx \\
&= \frac{\alpha}{(\alpha-1)m^{\alpha}}.
\end{aligned} \tag{3.6}
$$

which is bounded given $\alpha > 1$.

Therefore,

$$
P_{min}^{rec} \leq P_j^{rec} < P\frac{2\delta}{(2\delta-1)m^{2\delta}}, \tag{3.7}
$$

given $\delta > \frac{1}{2}$.

Apparently, when δ is given ($\delta > \frac{1}{2}$), $P_j^{rec} \to 0$ if m is unbounded, which will lead to the situation that the received power tends to 0 so that it cannot meet the requirement of the minimum received power. Thus, the assumption that the distance d_j could be unbounded can not satisfy the requirement of the network. Therefore, we proved that the distance between adjacent nodes is bounded.

From the inequality (3.7), we can derive

$$m < \left(\frac{P}{P_{min}^{rec}} \frac{2\delta}{2\delta - 1}\right)^{\frac{1}{2\delta}}, \tag{3.8}$$

From the assumption $m \leq d_j < m + 1$, we have

$$d_j < 1 + \left(\frac{P}{P_{min}^{rec}} \frac{2\delta}{2\delta - 1}\right)^{\frac{1}{2\delta}}, \tag{3.9}$$

even for the worst case. This is an upper bound for the distance between adjacent nodes.

Thus, for all $0 \leq i \leq j$,

$$1 \leq d_i < 1 + \left(\frac{P}{P_{min}^{rec}} \frac{2\delta}{2\delta - 1}\right)^{\frac{1}{2\delta}}, \tag{3.10}$$

where the right hand side is a finite number given $\delta > \frac{1}{2}$.

Furthermore,

$$n + 1 < \sum_{i=0}^{n} d_i < (n + 1)\left(1 + \left(\frac{P}{P_{min}^{rec}} \frac{2\delta}{2\delta - 1}\right)^{\frac{1}{2\delta}}\right), \tag{3.11}$$

which means that the total distance from source to destination is of the same order as n, i.e., $\sum_{i=1}^{n} d_i = \Theta(n)$. □

3.1.4 A Heuristic Bound

Through the simulation result (cf. section 3.2), it is safe to make a conjecture that the sequence of d_i is monotonically increasing. The proof of this conjecture could be an interesting problem for future research. Based on the conjecture, we attain a tighter upper bounded for the distance between adjacent nodes, which is

$$\left(\frac{P}{P_{min}^{rec}}\frac{2\delta}{2\delta-1}\right)^{\frac{1}{2\delta}},$$

where P is the constraint of individual power, P_{min}^{rec} is minimum received power required corresponding to the minimum required rate, path loss exponent $\delta > \frac{1}{2}$. We outline the proof in the following:

Assume there exists an integer N, which could be very large, such that for every $i > N$, d_i is not bounded.

From inequality (3.3), we have

$$\begin{aligned} P_{min}^{rec} &< \sum_{k=1}^{n-1} \frac{P_k}{(\sum_{i=k}^{n-1} d_i)^{2\delta}} \\ &< P\left(\sum_{k=1}^{n-1} \frac{1}{(\sum_{i=k}^{n-1} d_i)^{2\delta}}\right) \end{aligned} \tag{3.12}$$

as all $P_i \le P$, where P is the constraint for individual power that each node possesses.

Using the heuristic property of monotonically increasing for the sequence of d_i, we have

$$\sum_{i=k}^{n} d_i \ge \begin{cases} \sum_{i=N}^{n} d_i & \text{if } k \le N\ . \\ (n-k+1)d_N & \text{if } k > N. \end{cases}$$

The inequality (3.12) then turns into

$$P_{min}^{rec} < P\left\{\frac{N-1}{(\sum_{i=N}^{n} d_i)^{2\delta}} + \frac{1}{d_N^{2\delta}}\sum_{m=1}^{n-N}\frac{1}{m^{2\delta}}\right\}. \tag{3.13}$$

The first term of the right hand side in the inequality (3.13) becomes

$$\frac{N-1}{(\sum_{i=N}^{n} d_i)} \to 0$$

since n goes to infinity and $\sum_{i=N}^{n} d_i$ is infinite.

Thus,

$$P_{min}^{rec} < P\left\{\frac{1}{d_N^{2\delta}}\sum_{m=1}^{n-N}\frac{1}{m^{2\delta}}\right\}. \tag{3.14}$$

Using the result derived from the Riemann Zeta-function ([22], also cf. Appendix A) that

$$\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}} < \frac{\alpha}{\alpha - 1},$$

given $\alpha > 1$, where $\frac{\alpha}{\alpha-1}$ is a finite number, we have

$$\frac{1}{d_N^{2\delta}} \left(\sum_{m=1}^{n-N} \frac{1}{m^{2\delta}} \right) \to 0,$$

if there exists N such that d_N is unbounded given $\delta > 1/2$.

Thus, given a specific δ ($\delta > 1/2$), the minimum received power P^{rec} at the destination node tends to 0 if our assumption of unbounded d_n's existence is true, which is contrary to the requirement that the minimum rate being achieved from source to destination.

Hence, d_N is bounded for any number of nodes given $\delta > 1/2$, if the minimum source-destination rate is required in the multiple relay channel.

Moreover, we can derive from the inequality (3.14) and (3.1.4) that

$$d_N < \left(\frac{P}{P_{min}^{rec}} \frac{2\delta}{2\delta - 1} \right)^{\frac{1}{2\delta}},$$

which means for any value of N, the distance d_N is bounded above by a finite number which is related to the path loss exponent δ, the minimum required rate from source to each downstream node and the individual power constraint.

Apparently,

$$n < \sum_{i=0}^{n-1} d_i < n \left(\frac{P}{P_{min}^{rec}} \frac{2\delta}{2\delta - 1} \right)^{\frac{1}{2\delta}},$$

which also means the total distance from source to destination is on the order of n, i.e., $\sum_{i=1}^{n} d_i = \Theta(n)$ for any number of total nodes $n + 1$ given $\delta > 1/2$. □

The heuristic upper bound for the distance between adjacent nodes is 1 less than the upper bound we strictly proved above. For example, if $\delta = 1.25$, $P_{min}^{rec} = P = 1$, then, the heuristic upper bound is 1.50 and the strictly proved upper bound is 2.50 for the distance between adjacent nodes.

Remark: Intuitively, the heuristic upper bound of d_i corresponds to the situation that there are infinite number of relay nodes. When i is large enough, every d_i possess a same value d^* and is arbitrarily close to this heuristic upper bound $(\frac{P}{P_{min}^{rec}} \frac{2\delta}{2\delta-1})^{\frac{1}{2\delta}}$ we derived above.

3.2 Simulations of Node Placement with IRIS Scheme

In this section, we present the methods and results of the simulation for LMGRN using IRIS coding scheme given a minimum rate requirement.

Consider a linear Gaussian relay networks using IRIS coding strategy in the low attenuation regime. For computing what is the farthest distance that the destination node can be placed given the requirement that a minimum rate from source to destination is maintained, we can use a recursive algorithm to solve the problem.

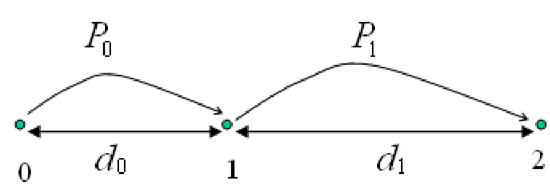

Figure 3.3: A 3-nodes linear relay network using IRIS.

Fig. 3.3 shows the simplest case of a linear relay network, where is only one relay node helping the transmission from node 0 (source node) to node 2 (destination node). Node 0 doesn't need to divide its power to two parts dedicated to the transmission to node 1 and node 2 respectively while the IRIS coding strategy is used.

While applying the IRIS scheme discussed above, the node placement problem is to solve

$$P_{min}^{rec} \le P^{rec} < \min_{1\le j\le n} \sum_{i=1}^{j-1} \alpha_{ij}^2 P_i. \tag{3.15}$$

When we use the model of signal attenuation $\alpha_{ij} = \frac{1}{d_{ij}^{\delta}}$ in the low attenuation regime, the above expression can be written as

$$P_{min}^{rec} \le P^{rec} < \min_{1\le j\le n} \sum_{i=1}^{j-1} \frac{P_i}{(\sum_{l=i}^{j-1} d_l)^{2\delta}} \tag{3.16}$$

Under the assumption of each node occupying the same power, one need to find the optimal value of all d_i's (total number is n) that maximize the source-destination distance. The computing complexity is not high if we compare with the situation using CRIS scheme. Especially in the recursive algorithm I developed, one only need to find one optimal value of the d_{n-1} at each recursion. For the three modes case, only the point the farthest point that node 2 can be placed is needed to be computed. The dual question is, if we normalize the distance from source to destination, which point is the optimal one to place node 1 to maximize the achievable rate from source to destination.

Just as what was clarified in section 2.3, for node placement problem in a linear network using IRIS scheme, the computing of the power assignment of each nodes is not needed, thus it is relatively easy to apply the recursive algorithm for solving the node placement problem. Since no power assignment is needed, every time when a new node, denoted as $n+1$, is added to the end of the system as the present destination, where just like the previous destination get a transmitter attached and become a relay node, all the terms in the minimization expression (3.16) need not being changed to fit in the new system. Only be one new term

$$\sum_{i=0}^{n-1} \frac{P_i}{(\sum_{l=i}^{n-1} d_i)^{2\delta}},$$

should be added to this expression.

The algorithm applied for getting the sequence of optimal d_i's is as following:

Step 1. Initiation: Start from a 3-node scenario. Let $P_0 = P_1 = 1$ and $d_0 = 1$, thus the received power P^{rec} at node 1 is also 1 and it is set as the minimum received power P^{rec}_{min}. using exhaustive search to find the farthest point that node 2 can be placed, i.e., d_2.

Step 2. Add another node at the end of the linear network (total number of nodes $n+2$), using

$$P^{rec}_{min} < \sum_{i=0}^{n} \frac{P_i}{(\sum_{l=i}^{n} d_l)^{2\delta}},$$

to search for the farthest d_n if the total node number is $n+1$.

Step 3. Repeat step 2 until reach the desired number n.

The following figures are drawn according to the result calculated using the above recursive algorithm. The individual power of each node, the distance d_0, and the minimum achievable rate are normalized.

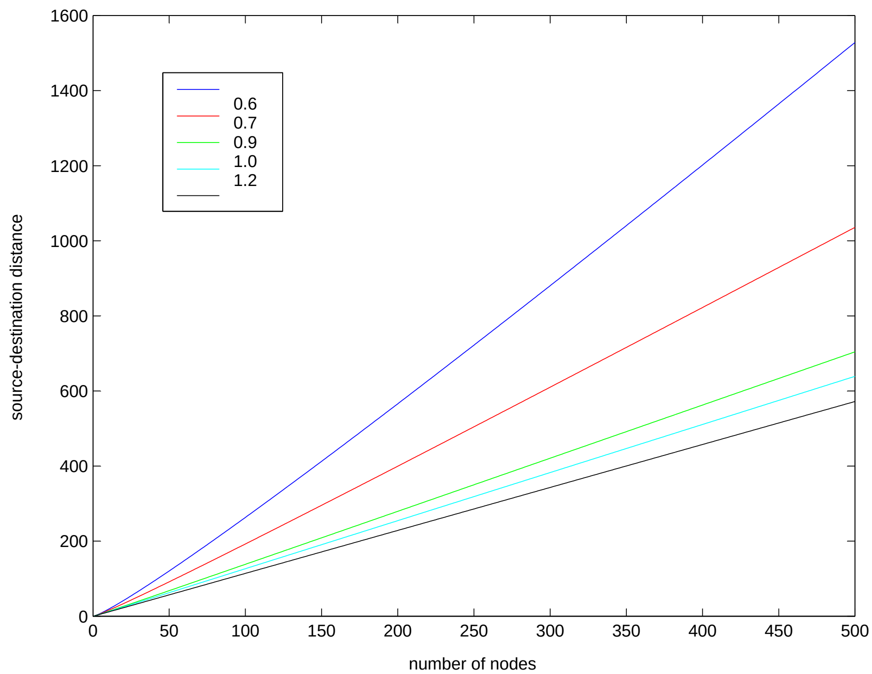

Figure 3.4: The maximal source-destination distances vs. the number of nodes under the condition of $\delta = 0.6, 0.7, 0.9, 1, 1.2$.

In fig. 3.4, the horizontal axis is the number of nodes, which is up to 500, and the vertical is the distance between source and destination, given the distance from node 0 to node 1 is 1. This figure shows how the distance from source to destination grows when the number of nodes increases. According to **Theorem 3**, if the path loss exponent $\delta > 1/2$, the source-destination distance is of the same order as the number of nodes. $\delta = 0.6, 0.7, 0.9, 1, 1.2$ are chosen for the recursive algorithm.

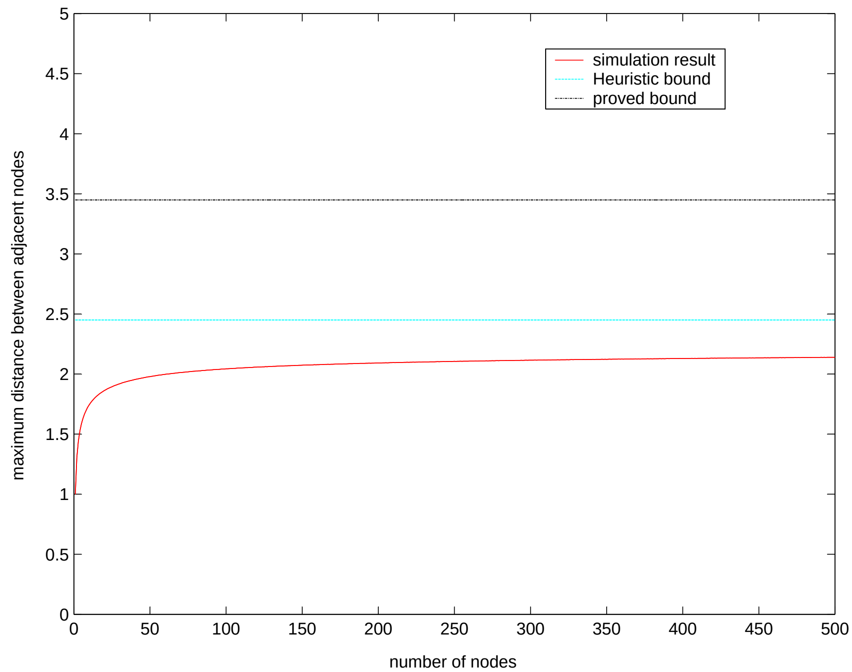

Figure 3.5: The maximum distance between adjacent nodes d_i and two bounds given $\delta = 0.7$.

Fig. 3.5 depicts the simulation result of the maximum distance between adjacent nodes, the heuristic bound and the strictly proved bound. We calculated d_i for up to 500 nodes.

From the above figures we can see, the result is quite consistent with the theoretical result we proved. Moreover, the heuristic bound provides helpful insight for understanding the improvement that the IRIS scheme can achieve.

Chapter 4

Node Placement for LGMRN with CRIS Scheme

In this chapter, First we deform the achievable rate formula for the CRIS case, then we examine the bound for the distance between adjacent nodes using a similar approach as in Chapter 3. Finally, we present algorithms and results for both optimal(up to 4-nodes case) and suboptimal (up to 28-nodes case) simulations. We also make some comparison between the simulation results for the network with IRIS and CRIS coding scheme.

4.1 Node Placement with the CRIS Scheme

We present the theorem and the proof in the following. The system Setting is linear Gaussian multiple relay channel similar as what we used in the last chapter. The difference is the CRIS coding scheme applied here, which causes the node placement problem much more complicated than in the IRIS case. The reason is, with the CRIS coding scheme, the power assignment at source and each relay node is necessary to implement the coherent transmission under this situation.

4.1.1 Theorem and Proof of the Bound

Consider a Gaussian linear relay network with $n+1$ nodes in the low attenuation regime, i.e., the absorption constant $\gamma = 0$ and the path loss exponent $\delta < \frac{3}{2}$. There is only one source-destination pair.

In the following theorem that we develop for the scenario using CRIS scheme, the statement is similar as what we presented in the case of the IRIS scheme, but the proof starts with the deformation of the achievable rate formula to make the proofing applicable.

Theorem 4: Consider a linear Gaussian multiple relay channel with the CRIS coding scheme. There are n+1 nodes lies on a straight line and each node possesses the same power constraint P. Place the relay nodes in a manner that the minimum rate is achieved from source to destination. Given $\delta > 1$, for any number of total nodes $n+1$, we have the following two results:

i) The distance between any two adjacent nodes d_i is upper bounded by $1 + \left(\frac{P}{P_{min}^{rec}}\right)^{\frac{1}{2\delta}} \left(\frac{\delta}{\delta-1}\right)^{\frac{1}{\delta}}$, where P is the constraint of individual power, P_{min}^{rec} is the minimum received power required corresponding to the minimum required rate,

ii) the distance between source and destination is on the order of n, i.e., $\sum_{i=1}^{n} d_i = \Theta(n)$.

Proof: First we prove inductively that

$$\sum_{k=1}^{j} \left(\sum_{i=0}^{k-1} \alpha_{ij} \sqrt{P_{ik}} \right)^2 < \left(\sum_{k=1}^{j} \alpha_{k-1,j-1} \sqrt{P_{k-1}} \right)^2 \quad (4.1)$$

For a linear network with n+1 nodes, it is easy to prove

$$\alpha_{0,n}^2 P_{01} + \left(\alpha_{0,n} \sqrt{P_{02}} + \alpha_{1,n} \sqrt{P_{12}} \right)^2 < \left(\alpha_{0,n} \sqrt{P_{01} + P_{02}} + \alpha_{1,n} \sqrt{P_{12}} \right)^2 . \quad (4.2)$$

The LHS (left hand side) of the inequality (4.2) is

$$\alpha_{0,n}^2 (P_{01} + P_{02}) + \alpha_{1,n}^2 P_{12} + 2\alpha_{0,n}\alpha_{1,n} \sqrt{P_{02} P_{12}}.$$

The RHS (right hand side) of (4.2) is

$$\alpha_{0,n}^2(P_{01}+P_{02})+\alpha_{1,n}^2P_{12}+2\alpha_{0,n}\alpha_{1,n}\sqrt{(P_{01}+P_{02})P_{12}}.$$

Apparently, LHS $<$ RHS, i.e. (4.2) holds. Thus, for the case of j=3, inequality (4.1) holds.

Suppose that inequality (4.1) holds for $j=n$, we have

$$\sum_{k=1}^{n}\left(\sum_{i=0}^{k-1}\alpha_{i,n}^2\sqrt{P_{ik}}\right)^2<\left(\sum_{k=1}^{n}\alpha_{k-1,n}\sqrt{\sum_{l=k}^{n}P_{k-1,l}}\right)^2, \tag{4.3}$$

where $P_{k-1}=\sum_{l=k}^{n}P_{k-1,l}$.

We need to prove

$$\sum_{k=1}^{n+1}\left(\sum_{i=0}^{k-1}\alpha_{i,n}\sqrt{P_{ik}}\right)^2<\left(\sum_{k=1}^{n+1}\alpha_{k-1,n}\sqrt{\sum_{l=k}^{n}P_{k-1,l}}\right)^2 \tag{4.4}$$

where $P_{k-1}=\sum_{i=k}^{n+1}P_{k-1,i}$.

Using the assumed inequality (4.3), the LHS of (4.4) is less or equal to

$$\left(\sum_{k=1}^{n}\alpha_{k-1,n}\sqrt{\sum_{l=k}^{n-1}P_{k-1,l}}\right)^2+\left(\sum_{i=0}^{n}\alpha_{i,n}\sqrt{P_{i,n+1}}\right)^2.$$

Now we need to show

$$\left(\sum_{k=1}^{n-1}\alpha_{k-1,n}\sqrt{\sum_{l=k}^{n-1}P_{k-1,l}}\right)^2+\left(\sum_{i=0}^{n}\alpha_{i,n}\sqrt{P_{i,n+1}}\right)^2\leq\left(\sum_{k=1}^{n}\alpha_{k-1,n}\sqrt{\sum_{l=k}^{n}P_{k-1,l}}\right)^2 \tag{4.5}$$

If we unfold the square of all the terms in the above inequality, and compare the LHS and RHS, we see that we need to prove

$$\begin{aligned}
&\alpha_{k,n}\alpha_{m,n}\sqrt{\sum_{l=k}^{n-1} P_{k-1,l}}\sqrt{\sum_{l=m}^{n-1} P_{m-1,l}} + \alpha_{k,n}\alpha_{m,n}\sqrt{P_{k-1,n-1}}\sqrt{P_{m-1,n-1}} \\
&\leq \alpha_{k,n}\alpha_{m,n}\sqrt{\sum_{l=k}^{n} P_{k-1,l}}\sqrt{\sum_{l=m}^{n} P_{m-1,l}} \qquad (4.6)
\end{aligned}$$

where

$$k = 1, 2, ..., n-1, m = 1, 2, ..., n-1,$$

$$\sum_{l=k}^{n} P_{k-1,l} = \sum_{l=k}^{n-1} P_{k-1,l} + P_{k-1,n}$$

and

$$\sum_{l=m}^{n} P_{k-1,l} = \sum_{l=m}^{n-1} P_{m-1,l} + P_{m-1,n}.$$

In the inequality (4.6), the $\alpha_{k,n}\alpha_{m,n}$ can be canceled from both the LHS and RHS sides, so we only need to compare the square root terms.

Using the fact

$$\sqrt{ab} + \sqrt{cd} \leq \sqrt{(a+c)(b+d)}. \qquad (4.7)$$

it is clear that the inequalities (4.6) and (4.5) hold.

Also, we have

$$\alpha_{k,n}\alpha_{n,n+1}\sqrt{P_{k-1,n}}\sqrt{P_{n-1,n}} \leq \alpha_{k,n}\alpha_{n,n+1}\sqrt{\sum_{l=k}^{n-1} P_{l,n}}\sqrt{P_{n-1,n}} \qquad (4.8)$$

where $k = 1, 2, ..., n-2$. This inequality holds apparently.

Combine (4.8) with (4.5), it is proved that the inequality (4.1) holds.

Now let's prove that all d_i's ($0 \leq i \leq n-1$) are bounded for even infinite total number of nodes given $\delta > 1$. This part of the proof is similar as the proof in section 3.1.3.

Assume there exists a finite number j such that d_j is unbounded. Let $m \leq d_j < m+1$, where m is a positive integer. Under the assumption, m should be unbounded also.

We also construct the worst case situation as we did for the proof of **Theorem 3**. As Figure 3.2 shows, place all the nodes in a way that $d_i = 1$ ($0 \leq i \leq j-1$) and place node $j+1$ to the furthest point possible as long as the minimum required rate can be achieved. The received power at the node $j+1$, which should be greater or equal to the minimum received power corresponding to the minimum required rate.

From Theorem 3.11 in Xie and Kumar's work [1], we have inequality (2.2). Substituting α_{ij} by $\frac{1}{d_{ij}^{\delta}}$ and rewrite (2.2), that is

$$P_j^{rec} < \sum_{k=1}^{n} \left(\sum_{i=0}^{k-1} \frac{\sqrt{P_{ik}}}{(\sum_{l=i}^{n} d_l)^{\delta}} \right)^2, \tag{4.9}$$

for the received power at node j.

Do similar substitution in the inequality (4.1), we have

$$\sum_{k=1}^{j} \left(\sum_{i=0}^{k-1} \frac{\sqrt{P_{ik}}}{(\sum_{l=i}^{j-1} d_l)^{\delta}} \right)^2 < \left(\sum_{k=1}^{j} \frac{\sqrt{P_{k-1}}}{(\sum_{l=k-1}^{j-1} d_l)^{\delta}} \right)^2, \tag{4.10}$$

From (4.9) and (4.10), we have

$$P_j^{rec} < \left(\sum_{k=1}^{j} \frac{\sqrt{P_{k-1}}}{(\sum_{l=k-1}^{j-1} d_l)^{\delta}} \right)^2, \tag{4.11}$$

Consider the minimum received power requirement and the individual power constraint $P_i \leq P$ ($0 \leq i < j$),

$$\begin{aligned}
P_{min}^{rec} \le P_j^{rec} &< P\left(\sum_{k=1}^{j}\frac{1}{(\sum_{l=k-1}^{j-1} d_l)^{\delta}}\right)^2 \\
&\overset{1}{=} P\left(\sum_{k=1}^{j}\frac{1}{(j-k+d_{j-1})^{\delta}}\right)^2 \\
&\overset{2}{<} P\left(\sum_{k=1}^{j}\frac{1}{(j-k+m)^{\delta}}\right)^2,
\end{aligned} \tag{4.12}$$

where $\overset{1}{=}$ follows from $d_i = 1$ $(0 \le i \le j-1)$ and $\overset{2}{<}$ is due to $m \le d_j < m+1$.

Using the fact of Riemann zeta-function ([22], and cf. Appendix A) that

$$\begin{aligned}
\sum_{n=m}^{\infty}\frac{1}{n^{\alpha}} &< \frac{1}{m^{\alpha}} + \int_{m}^{\infty}\frac{1}{x^{\alpha}}dx \\
&= \frac{\alpha}{(\alpha-1)m^{\alpha}},
\end{aligned} \tag{4.13}$$

we have

$$P_{min}^{rec} \le P_j^{rec} < P\left(\frac{\delta}{(\delta-1)m^{\delta}}\right)^2, \tag{4.14}$$

Given $\delta > 1$, if m is unbounded as we assumed, P_j^{rec} will tend to 0 which will lead to the situation that the received power can not meet the minimum received power requirement.

Thus, the assumption that the distance d_j could be unbounded is not satisfying the requirement of the network. Therefore, we proved that the distance between adjacent nodes is bounded.

From the inequality (4.14), we can derive

$$m < \left(\frac{P}{P_{min}^{rec}}\right)^{\frac{1}{2\delta}}\left(\frac{\delta}{\delta-1}\right)^{\frac{1}{\delta}}, \tag{4.15}$$

From the assumption $m \le d_j < m+1$, we have

$$d_j < 1 + \left(\frac{P}{P_{min}^{rec}}\right)^{\frac{1}{2\delta}} \left(\frac{\delta}{\delta - 1}\right)^{\frac{1}{\delta}}, \tag{4.16}$$

even for the worst case.

Hence, for all $0 \le i \le j$,

$$1 \le d_i < 1 + \left(\frac{P}{P_{min}^{rec}}\right)^{\frac{1}{2\delta}} \left(\frac{\delta}{\delta - 1}\right)^{\frac{1}{\delta}}, \tag{4.17}$$

where the right hand side is a finite number given $\delta > 1$.

Moreover,

$$n + 1 < \sum_{i=0}^{n} d_i < (n+1)\left[1 + \left(\frac{P}{P_{min}^{rec}}\right)^{\frac{1}{2\delta}} \left(\frac{\delta}{\delta - 1}\right)^{\frac{1}{\delta}}\right], \tag{4.18}$$

which means that the total distance from source to destination is on the order of n, i.e., $\sum_{i=1}^{n} d_i = \Theta(n)$. □

4.1.2 A Heuristic Upper Bound

Similar to the conjecture introduced in Section 3.1.4, we assume that the sequence of d_i is monotonically increasing. Therefore, we attain a tighter upper bounded for the distance between adjacent nodes, which is $\left(\frac{P}{P_{min}^{rec}}\right)^{\frac{1}{2\delta}} \left(\frac{\delta}{\delta-1}\right)^{\frac{1}{\delta}}$, where P is the constraint of individual power, P_{min}^{rec} is minimum received power required corresponding to the minimum required rate, path loss exponent $\delta > 1$. In the following, we briefly prove this heuristic bound.

Proof: Assume there exists a finite number N, for all $n \ge N$, d_n is unbounded.

From inequalities (4.1) and (4.9), we have

$$\begin{aligned} P_{min}^{rec} \le P^{rec} &< \sum_{k=1}^{n} \left(\sum_{i=0}^{k-1} \frac{\sqrt{P_{ik}}}{(\sum_{l=i}^{n} d_l)^{\delta}} \right)^2 \\ &< \left(\sum_{i=0}^{n} \frac{\sqrt{P_{i-1}}}{(\sum_{k=i}^{n} d_k)^{\delta}} \right)^2 \\ &\le P \left(\sum_{i=0}^{n} \frac{1}{(\sum_{k=i}^{n} d_k)^{\delta}} \right)^2 \end{aligned} \tag{4.19}$$

as all $P_i \leq P$, where P is the constraint for individual power that the nodes possess.

Using the conjecture that all d_i's are monotonically increasing, we have the relation that

$$\sum_{i=k}^{n} d_i \geq \begin{cases} \sum_{i=N}^{n} d_i & \text{if } k \leq N \ , \\ (n-k+1)d_N & \text{if } k > N. \end{cases}$$

then, inequality 4.19 turns into

$$P_{min}^{rec} < P \left(\frac{N-1}{(\sum_{i=N}^{n} d_i)^{\delta}} + \frac{1}{d_N^{\delta}} \sum_{m=1}^{n-N} \frac{1}{m^{\delta}} \right)^2 . \tag{4.20}$$

The first term of the right hand side in the parenthesis of inequality (4.20)

$$\frac{N-1}{(\sum_{i=N}^{\infty} d_i)} \rightarrow 0$$

since $\sum_{i=N}^{\infty} d_i$ goes to infinity.

Hence, we have

$$P_{min}^{rec} < P \left(\frac{1}{d_N^{\delta}} \sum_{m=1}^{n-N} \frac{1}{m^{\delta}} \right)^2 . \tag{4.21}$$

Using the fact (cf. Appendix A) that

$$\sum_{m=1}^{\infty} \frac{1}{m^{\alpha}} < \frac{\alpha}{\alpha - 1}$$

for $\alpha > 1$ [22], we have

$$\frac{1}{d_N^{\delta}} \left(\sum_{m=1}^{n-N} \frac{1}{m^{\delta}} \right) \rightarrow 0$$

if d_N is unbounded for a finite N given $\delta > 1$.

This means the right hand side of inequality (4.20) tends to 0 if d_n is unbounded for all $n \geq N$.

Thus, given a specific δ ($\delta > 1$), the received power P^{rec} at the destination node tends to 0 if our assumption of the existence of the unbounded d_n is true, which is contrary to the requirement that the minimum rate being achieved from source to destination. Hence, d_N is bounded for any number of nodes given $\delta > 1$, if the minimum source-destination transmission rate is required in the LMGRN with CRIS scheme.

Moreover, from inequality 4.21, we have

$$d_N < \left(\frac{P}{P_{min}^{rec}}\right)^{\frac{1}{2\delta}} \left(\frac{\delta}{\delta - 1}\right)^{\frac{1}{\delta}}, \tag{4.22}$$

which means for any value of N, the distance d_N is bounded by a finite number which is related to the path loss exponent δ, minimum required received power, and the individual power constraint.

Apparently,

$$n < \sum_{i=0}^{n-1} d_i < n \cdot \left(\frac{P}{P_{min}^{rec}}\right)^{\frac{1}{2\delta}} \left(\frac{\delta}{\delta - 1}\right)^{\frac{1}{\delta}},$$

which also means the total distance from source to destination is on the order of n, i.e., $\sum_{i=1}^{n} d_i = \Theta(n)$ for any number of total nodes $n + 1$ given $\delta > 1$. □

Again, we compute an example of the upper bound for the distance between adjacent nodes. Choose the same $\delta = 1.25$, $P_{min}^{rec} = P = 1$ as we did for the IRIS scheme, we get the heuristic bound 3.62 and the strictly proved bound 4.62 for the LMGRN with CRIS scheme.

Remark: The heuristic upper bound is based on our conjecture of an monotonically increasing sequence of $\{d_i\}$. It is 1 less than the upper bound we strictly proved above. Meanwhile, the heuristic bound perfectly matches with our simulation results, as to be presented later in this chapter.

4.2 Simulations

In this section, I present the simulation algorithm and numerical results for linear Gaussian relay network using CRIS coding scheme, which was explained in section 2.1, given a minimum rate requirement. The problem of the optimal relay nodes placement and power assignment are studied. A suboptimal method is also developed to decrease the computing complexity of finding the optimal solution.

For the Gaussian linear network using CRIS scheme, the situation becomes more complicated than that use the IRIS scheme. Each node i divides its power P_i into a group of $P_{ij}(j > i)$ for every downstream node. The power assignment and the distances between adjacent node introduce large amount of variables to deal with in the optimization problem for networks with node number greater than 3.

4.2.1 Three Nodes Scenario

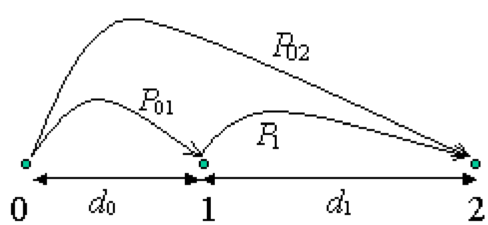

Figure 4.1: A 3-nodes relay network using CRIS scheme

In Fig. 4.1 shows the simplest case, a three modes Gaussian linear network using CRIS scheme. There are three modes denoted 0, 1,and 2. Node 0 is the source, node 1 is the relay helping the source node 0 to transmit to destination node 2. d_0 and d_1 are the distances between the nodes. Node 0 divide its power P_0 into two parts P_{01} and P_{02}, dedicated to the transmission to node 1 and 2 respectively. P_1 is the power of node 1 and it is dedicated to node 2 totally since there is only one downstream node to node 1.

Apply the achievable rate formula 2.2, we have

$$R < \min \left\{ S\left(\frac{1}{\sigma^2}\alpha_{01}^2 P_{01}\right), S\left(\frac{1}{\sigma^2}\left(\alpha_{02}^2 P_{01} + (\alpha_{02}\sqrt{P_{02}} + \alpha_{12}\sqrt{P_1})^2\right)\right)\right\}$$

Substitute $\frac{1}{d_0^\delta}$, $\frac{1}{d_1^\delta}$ and $\frac{1}{(d_0+d_1)^\delta}$ for α_{01}, α_{12} and α_{02}, and omit the Gaussian formula S, our object function, which aims to maximize achievable rate, for the optimization becomes

$$\max_{d_0, P_{01}} \min \left\{ \frac{P_{01}}{d_0^{2\delta}}, \frac{P_{01}}{(d_0+d_1)^{2\delta}} + \left(\frac{\sqrt{P_{02}}}{(d_0+d_1)^\delta} + \frac{\sqrt{P_1}}{d_1^\delta}\right)^2 \right\}$$

where $d_0 + d_1$ and P_0 are given.

for the three modes case: optimal placement of nodes 1 when the distance from node 0 to node 2 is normalized to 1.

Without loss of generality, we normalize the distance from source(node 0) to destination(node 1), and the power P_0, P_1 to 1. Let $d_0 = x$, $P_{01} = \theta P_0$, thus $d_1 = 1-x$, $P_{02} = (1-\theta)P_0$, where $0 < \theta, x < 1$. The above expression now is

$$\max_{\theta, x} \min \left\{ \frac{\theta}{x^{2\delta}}, \theta + (\sqrt{(1-\theta)} + \frac{1}{(1-x)^\delta})^2 \right\},$$

which can be further simplified as

$$\max_{\theta, x} \min \left\{ \frac{\theta}{x^{2\delta}}, 1 + \frac{1}{(1-x)^{2\delta}} + \frac{2\sqrt{1-\theta}}{(1-x)^\delta} \right\}, \tag{4.23}$$

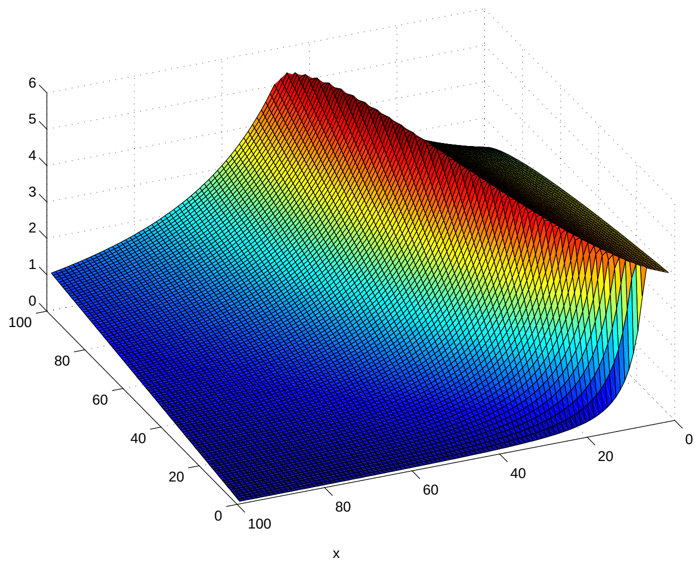

Figure 4.2: The surface of achievable rate in a three-nodes LMGRN.

Fig. 4.2 shows the 3-dimensional curved surface for

$$\min\left\{\frac{\theta}{x^{2\delta}}, 1 + \frac{1}{(1-x)^{2\delta}} + \frac{2\sqrt{1-\theta}}{(1-x)^{\delta}}\right\},$$

while $\delta = 1.25$. The x,y and z axis of this figure are x, θ and the corresponding value of the minimization function respectively.

Let

$$f_1(\theta, x) = \frac{\theta}{x^{2\delta}}, \quad (4.24)$$

$$f_2(\theta, x) = 1 + \frac{1}{(1-x)^{2\delta}} + \frac{2\sqrt{1-\theta}}{(1-x)^{\delta}}. \quad (4.25)$$

If we fix θ, i.e., fix the power assignment of the source node, $f_1(x)$ is monotonically decreasing, $f_2(x)$ is monotonically increasing. If we fix x, i.e., fix the distances between these nodes, $f_1(\theta)$ is monotonically increasing, $f_2(\theta)$ is monotonically decreasing.

We apply a recursive algorithm for finding the optimal θ and x for the three-nodes scenario. The exhaustive search can also be used to do it also since there are only 2 variables in the three modes scenario.

The recursive algorithm is as follows:

step 1: Initialization: Normalize the power of node 0 and 1 and the distance between node 0 and 2. let $x = 0.5$, or other value between 0 and 1 can be taken as the starting point of x.

step 2: Using equations (4.24) and (4.25) compute f_1 and f_2, record the smaller value of them as $\min\{f_1, f_2\}$. **step 3:** Keep the value of x, add small step size to θ until $\min\{f_1, f_2\}$ won't increase. **step 4:** Add a small step size(can be positive or negative) to x, repeat step 2 and 3. If the $\min\{f_1, f_2\}$ improves, keep adding the same step size to x and repeat steps 2 and 3 until there is no more improvement to $\min\{f_1, f_2\}$. Halt.

Then, the corresponding θ and x will be the optimal value we are looking for. The result of optimal θ and x, which are $x \approx 0.42$ and $\theta \approx 0.80$ when the path loss exponent $\delta = 1.25$.

There is another approach for solving this problem. Mathematically, the optimal pair of (θ, x) satisfies $f_1(\theta, x) = f_2(\theta, x)$. From this equation, we can derive the relation of $x = g(\theta)$.

Define function

$$\begin{aligned} h(\theta) &:= \max_{\theta,x} \min \{f_1(\theta, x), f_2(\theta, x)\} \\ &= \max_{\theta,x} \min \{f_1(\theta, g(\theta)), f_2(\theta, g(\theta))\} \end{aligned} \tag{4.26}$$

Thus, getting the derivative of the function $h(\theta)$ and let it be 0, we can get the optimal θ^*. Through $x = g(\theta)$, we can find the corresponding optimal x^*, and get the optimal θ^* from the relation $x = g(\theta)$.

4.2.2 Four Nodes Scenario

Though the above approach seems simple to get the optimal value of x and θ, the situation changes dramatically for the problem of more nodes. As the following figure shows, when there are 4 nodes in the network, we need to deal with 5 variables for the optimization problem.

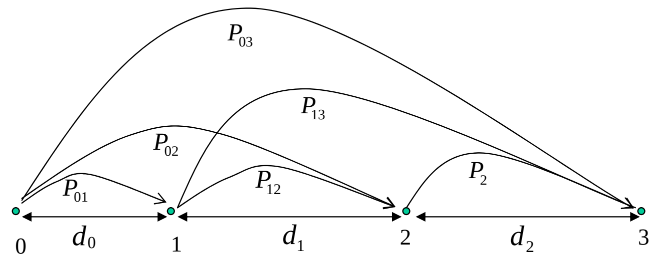

Figure 4.3: A 4-nodes linear network.

Fig. (4.3) depicts a 4-nodes linear network using CRIS scheme. Node 0 divides its power into 3 parts dedicated to node 1 to 3 respectively; node 1 divides its power into 3 parts dedicated to node 2 and 3; node 2 does not need to divide its power since there is only one downstream node to it. After normalize the individual power and the total distance from node 0 to 3, there are 5 variables needed to be determined in order to solve this optimization problem.

According to 2.10, the achievable rate formula for a 4-nodes network is

$$R < \min\{f_1, f_2, f_3\}, \tag{4.27}$$

where

$$\begin{aligned} h_1 &:= S\left(\frac{1}{\sigma^2}\alpha_{01}^2 P_{01}\right), \\ h_2 &:= S\left(\frac{1}{\sigma^2}\left(\alpha_{02}^2 P_{01} + (\alpha_{02}\sqrt{P_{02}} + \alpha_{12}\sqrt{P_{12}})^2\right)\right), \\ h_3 &:= S\left(\frac{1}{\sigma^2}\left(\alpha_{03}^2 P_{01} + (\alpha_{03}\sqrt{P_{02}} + \alpha_{13}\sqrt{P_{12}})^2 + (\alpha_{03}\sqrt{P_{03}} + \alpha_{13}\sqrt{P_{13}} + \alpha_{23}\sqrt{P_{23}})^2\right)\right). \end{aligned} \tag{4.28}$$

Substitute $\frac{1}{d_{ij}^{\delta}}$ for $\alpha_{ij} (0 \leq i < j \leq 3)$, and omit the Gaussian function S, our object function, which aims to maximize achievable rate, becomes

$$\max\min\{f_1, f_2, f_3, \}, \tag{4.29}$$

where the maximization is over d_i and P_{ij} $(0 \le i < j \le 3)$ and

$$\begin{aligned}
f_1 &:= \frac{P_{01}}{d_0^{2\delta}},\\
f_2 &:= \frac{P_{01}}{(d_0+d_1)^{2\delta}} + \left(\frac{\sqrt{P_{02}}}{(d_0+d_1)^{\delta}} + \frac{\sqrt{P_{12}}}{d_1^{\delta}}\right)^2,\\
f_3 &:= \frac{P_{01}}{(d_0+d_1+d_2)^{2\delta}} + \left(\frac{\sqrt{P_{02}}}{(d_0+d_1+d_2)^{\delta}} + \frac{\sqrt{P_{12}}}{(d_1+d_2)^{\delta}}\right)^2\\
&\quad + \left(\frac{\sqrt{P_{03}}}{(d_0+d_1+d_2)^{\delta}} + \frac{\sqrt{P_{13}}}{(d_1+d_2)^{\delta}} + \frac{\sqrt{P_2}}{d_2^{\delta}}\right)^2.
\end{aligned} \tag{4.30}$$

Without loss of generality, we normalize the distance from source (node 0) to destination (node 2), and the power P_0, P_1 and P_2 to 1. Let $P_{01} = \theta_{01}$, $P_{02} = \theta_{02}$, $P_{12} = \theta_{12}$, thus $d_2 = 1 - d_0 - d_1$, $P_{03} = 1 - \theta_{01} - \theta_{02}$, $P_{13} = 1 - \theta_{12}$, where θ_{01}, θ_{02} are positive.

Now we have

$$\begin{aligned}
f_1 &:= \frac{\theta_{01}}{d_0^{2\delta}},\\
f_2 &:= \frac{\theta_{01}}{(d_0+d_1)^2\delta} + \left(\frac{\sqrt{\theta_{02}}}{(d_0+d_1)^{\delta}} + \frac{\sqrt{\theta_{12}}}{d_1^{\delta}}\right)^2,\\
f_3 &:= \theta_{01} + \left(\sqrt{\theta_{02}} + \frac{\sqrt{\theta_{12}}}{(1-d_0)^{\delta}}\right)^2 + \left(\sqrt{1-\theta_{01}-\theta_{02}} + \frac{\sqrt{1-\theta_{12}}}{(1-d_0)^{\delta}} + \frac{1}{(1-d_0-d_1)^{\delta}}\right)^2.
\end{aligned} \tag{4.31}$$

The expression (4.29) now is

$$\max_{\theta_{ij}, d_i} \quad \min\{f_1, f_2, f_3\},$$

where $(0 \le i < j \le 2)$.

By examining the changing direction of the power assignment from the 3-nodes case to the 4-nodes case, we can see that, in the 4-nodes case, the power and distance ratio between the first three modes will only differ a little from the 3-nodes case. Thus, the search conducted for the optimal set of θ_{ij} and d_i $(0 \le i < j \le 2)$ starts from the result of the 3-nodes scenario. Firstly, we use two constants c_1 and c_2 to scale the power and the distance of results from the 3-nodes case respectively. Then, based on the approximation value by scaling, a recursive algorithm is used to find the optimal results for the 4-nodes case.

Step 1: Initialization: Normalize the power of node 0, 1 and 2, and the distance between node 0 and 3 to 1. The optimal result of 3-nodes case is $\theta^* = 0.80$, $x^* = 0.42$ from above algorithm for 3-nodes case. Using c_1 and c_2 to scale the power and the distance assignment for 3-nodes, i.e., let

$$\begin{aligned}
\theta_{01} &= c_1\theta^* = 0.8c_1, \\
\theta_{02} &= c_1(1-\theta^*) = 0.2c_1, \\
\theta_{12} &= c_1, \\
d_0 &= c_2x^* = 0.42c_2, \\
d_1 &= c_2(1-x^*) = 0.58c_2,
\end{aligned}$$

Now there are only two variables, which are c_1 and c_2, left in the optimization problem. Using the algorithm for 3-nodes case, we can find optimal $c_1^* = 0.86$ and $c_2^* = 0.60$. Substitute c_1^* and c_2^* back into the θ_{ij} and d_i $(0 \le i < j \le 2)$, we have the starting value for these variables.

Step 2: Select a searching step size, e.g. 0.02. Using a 5-layer loop, compute $\min\{f_1, f_2, f_3\}$ in the 5-dimensional ball which has a radius 10 times of the stepsize, centered at starting point of the θ_{ij} and d_i $(0 \le i < j \le 3)$. Keep the set of θ_{ij} and d_i $(0 \le i < j \le 3)$ corresponding to the maximum $\min\{f_1, f_2, f_3\}$.

Step 3: Shorten the stepsize and repeat **Step 2**, get a new set θ_{ij} and d_i $(0 \le i < j \le 3)$ corresponding to the maximum $\min\{f_1, f_2, f_3\}$.

Step 4: Repeat Step 2 until reached desired stepsize. Halt.

Using the above algorithm, the optimal θ_{ij} and d_i $(0 \le i < j \le 3)$ attained for $\delta = 1.25$ are:
$\theta_{01} = 0.734$, $\theta_{02} = 0.194$, $\theta_{12} = 0.820$, $d_0 = 0.263$, and $d_1 = 0.344$.

Since the source-destination distance d_{02} is normalized to 1, we can see from above result that $d_0 = 0.263$, $d_1 = 0.344$, and $d_2 = 0.393$ are in an increasing order.

4.2.3 Recursive Power Updating: Suboptimal Strategy for Multiple Nodes Scenario

In this section, we present a simulation strategy more practical for the multi-relay nodes scenario.

For the simulation of the network using IRIS scheme, the recursive algorithm we used in the last chapter is adding a node every time to compute for more nodes scenario. Due to the nature of the achievable rate formula corresponding to CRIS scheme, every time there is a new node added in as the new destination, the power assignment of all the previous nodes and the distances between every pair of adjacent nodes will all change if the optimal setting is needed. The recursive algorithm we used in the simulation for the linear network using IRIS scheme is not appropriate anymore.

Thus, a recursive power updating strategy, a suboptimal approach, is developed here for making the problem easier. This strategy make an recursive algorithm possible for the linear network using CRIS scheme. It is applied as the following procedure:

The recursive individual power updating procedure:

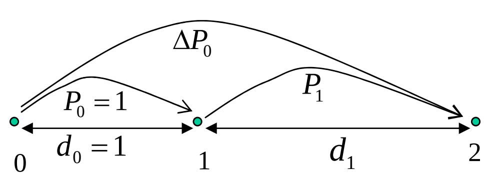

Figure 4.4: Power updating strategy for a 3-nodes network.

Step 1: Normalize the individual power of each node to 1. Normalize the distance between node 0 to node 1 as 1. Calculate the received power at node 1 and set it as the required minimum received power for each destination node as each of them are added at the end of the line.

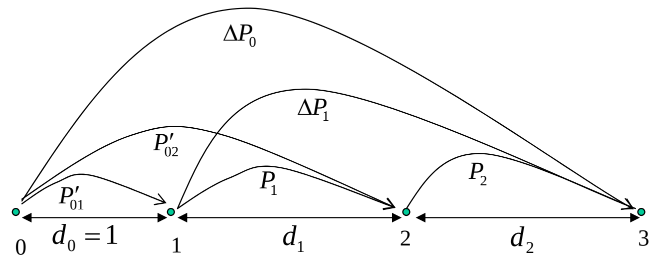

Figure 4.5: Power updating strategy for a 4-nodes network.

Step 2: Add the 3rd node after node 1 at a distance x away from node 1. As what is shown in Fig. 4.4, let node 0's power updated as $P_0 = P_0 + \Delta P_0$ and let $\Delta P_0 + P_1 = 1$. The task now is to find optimal power assignment between ΔP_0 and P_1 instead of finding $P_{01} + P_{02} = 1$ and letting $P_1 = 1$ in the optimal setting. Using recursive steps to find the best ΔP_0 that maximizes d_1. Record ΔP_0 as P'_{02} and the furthest d_1 that still satisfies the minimum rate requirement. Also, let $P'_{01} = P_1$.

Step 3: As Fig. 4.5 shows, add the 4th node at the end of the line, let node 0's power updated as $P_0 = P'_{01}+P'_{02}+\Delta P_0$, let node 1's power updated as $P_1 = P_1+\Delta P_1$, and let $\Delta P_0 + \Delta P_1 + P_2 = 1$. The task now is to find optimal power assignment between ΔP_0, ΔP_1 and P_2 instead of finding $P_{01}, P_{02}, P_{03} = 1, P_{12}$ and P_{13} that satisfy $P_{01} + P_{02} + P_{03} = 1$, $P_{12} + P_{13} = 1$ and letting $P_2 = 1$ in the optimal setting. Using recursive steps to find the best ΔP_0 and ΔP_1that maximizes d_2 while keeping d_0 and d_1's value from the last step. Record ΔP_0 as P'_{03}, and ΔP_1 as P'_{13}, and the furthest d_2 that still satisfies the minimum rate requirement. Also, let $P'_{12} = P_1$.

Step 4: Now do the similar thing as in Step 3 whenever add a new node at the end of the line as the new destination. Suppose the last node is n. Keep all the previous value of P_{ij} $(0 \leq i \leq n-1, i \leq j \leq n-1)$ and d_i $(0 \leq i \leq n-1)$. Let $\sum_{i=0}^{n-2} \Delta P_i + P_{n-1} = 1$, then find the optimal assignment among them corresponding to the furthest d_{n-1} that still satisfies the minimum rate requirement. Also, update all the P'_{ij} as calculated in this step.

Step 5: repeat Step 4 until reach the desired number of nodes of n or the source-

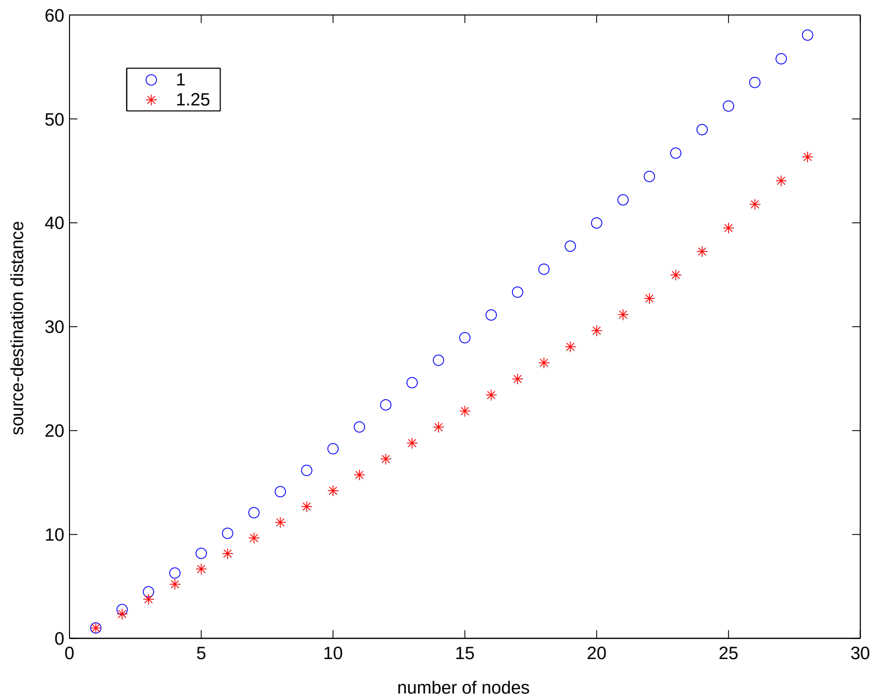

Figure 4.6: Simulation result of using CRIS scheme given $\delta = 1$ and $\delta = 1.25$.

destination distance according to the request.

As a new node is added and each individual power is updated, the received power at the destination node can always be met without change the distances between upstream nodes.

The simulation has been done for the linear Gaussian relay network under the CRIS scheme under two conditions: one is given the path loss exponent $\delta = 1$, the other is given $\delta = 1.25$. Both results are shown in the figure (4.6).

4.3 Comparison between Results with IRIS, CRIS and Multi-hop Schemes

Some comparison between the schemes of CRIS, IRIS and conventional "multi-hop" are presented in this section. The "multi-hop" relaying herein means that each relay node use Decode-and-Forward coding scheme to decode then retransmit the information to the next node (can be relay or destination). No cooperation amongs the nodes in the network.

The main purpose of this comparison is to show that with the same kind of interference subtraction strategy, how the effect of coherent relaying scheme differ from the incoherent scheme or the conventional "multi-hop" scheme. we did prove that the node placement bounds for a Gaussian linear relay network using IRIS or CRIS are quite similar given the path loss exponent $\delta > 1$, but through the simulation, we can see more clearly how much the coherent scheme can be better than the incoherent scheme or the "multi-hop" scheme. This comparison can provide significant guidance for applications, since there is the trade-off between the higher cost or the lower achievable rate, since the devices need to be equipped with beamforming function, which is necessary for utilize the coherent scheme,

In the following figure 4.7, the result for the CRIS scheme is calculated by using the power update strategy. For a fair comparison, the result of updated individual power for each node is used for the calculation for the IRIS scheme, though it is not needed because the computing complexity is low.

Fig. 4.7 shows under IRIS, CRIS and "multi-hop" schemes, how the total distance from source to destination changes when the number of nodes increases, given the path loss exponent $\delta = 1.25$. For the "multi-hop" scheme, since there is no cooperation, the distance between adjacent nodes keeps as the constant 1 as we normalized.

From this 2 figure we can see, as we have proved, the distance from source to destination, which is the summation of the distances between adjacent nodes of the linear network, grows linearly with the number of the nodes, i.e., $\sum_{i=0}^{\infty} = \Theta(n)$.

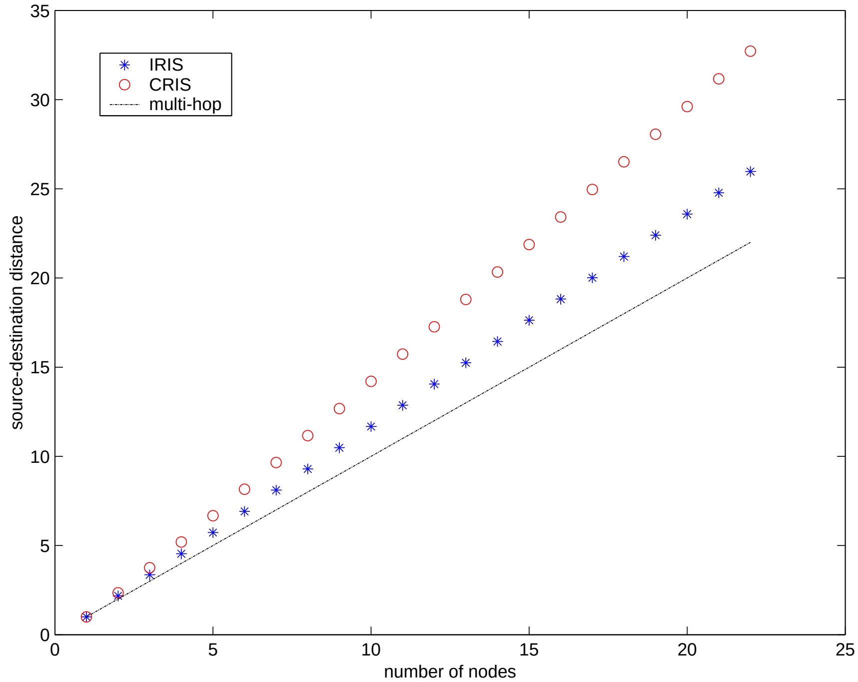

Figure 4.7: Comparison of the source-destination distance under 3 schemes given $\delta = 1.25$

The figure also shows apparently the advantage of the CRIS scheme and IRIS scheme over the "multi-hop" , which presented here as the longer source-destination distance given the same individual power and minimum achievable rate requirement.

Chapter 5

Conclusions and Future Research

This chapter concludes the thesis with a summary of contributions and presents a few thoughts on future research.

5.1 Conclusions

5.1.1 Upper Bounds with CRIS and IRIS Schemes

We consider the LGMRN in the low attenuation regime ($\delta < \frac{3}{2}$) using the attenuation model of $\alpha = \frac{1}{d^{\delta}}$, where δ is the path loss exponent and d is the distance. For the LGMRN with IRIS and CRIS coding schemes, upper bounds for the distance between adjacent nodes are presented and proved under the condition of a minimum required achievable rate.

For the LGMRN with IRIS scheme, the upper bound for the distance between adjacent nodes is $1 + (\frac{P}{P^{rec}_{min}} \frac{2\delta}{2\delta-1})^{\frac{1}{2\delta}}$ if $\delta > \frac{1}{2}$. For the LGMRN with CRIS scheme, the upper bound for the distance between adjacent nodes is $1 + \left(\frac{P}{P^{rec}_{min}}\right)^{\frac{1}{2\delta}} \left(\frac{\delta}{\delta-1}\right)^{\frac{1}{\delta}}$ if $\delta > 1$. We also proved that the corresponding total distance from source to destination is of the same order as the total number of nodes for both cases.

5.1.2 Simulations

Thorough simulations are conducted for demonstrating the above theoretical results.

For the LGMRN with IRIS scheme, the simulation is relatively easy to proceed because the incoherent relaying scheme does not need each upstream node to divide its power to all the downstream nodes. We simulate the relay placement problem for up to 500 nodes based on the same condition as the theorem.

For the LGMRN with IRIS scheme, the simulation is quite complicated since the power assignment of each node for the downstream nodes is necessary to implement the coherent transmission. For the optimal power assignment and node placement problem, we conducted simulations for three-nodes and four-nodes scenarios. We also developed a suboptimal power updating procedure to simplify the problem while maintaining the main property of the problem. Using the suboptimal power updating procedure, we simulate the power assignment and node placement problem for up to 28 nodes.

Comparing the theoretical and simulation results, which are well consisted with each other, we can say we attained tight upper bounds for the LGMRN node placement problem.

5.2 Future Research

As a part of future work, we would like to examine the scenario for a planar network, i.e., the nodes lies on a 2 dimensional plane. Also, the result in this thesis can be extended to the scenario that each relay node be placed by a group of nodes.

Also, result for the situation when $\delta = 1$, which corresponds to the ideal inverse square law, can be another interesting point in our future research.

Application prospects: in relay placement problem for sensor networks, how to make use of the relaying with CRIS or IRIS coding scheme to improve the resource efficiency, including power and lifetime trade-off, etc.

Appendix A

Upper Bound for Riemann Zeta-function

Riemann zeta-function $\zeta(\alpha)$ is defined by the following infinite series:

$$\zeta(\alpha) = \sum_{n=1}^{\infty} \frac{1}{n^{\alpha}}, \tag{A.1}$$

for values of α, which is a complex variable, with real part greater than one, and then analytically continued to all complex $\alpha \neq 1$. This Dirichlet series converges for all real values of α greater than one [22] .

The summation $\sum_{m=1}^{\infty} \frac{1}{m^{\alpha}}$ can be bounded above by an integration

$$\sum_{m=1}^{\infty} \frac{1}{m^{\alpha}} < 1 + \int_{1}^{\infty} \frac{1}{x^{\alpha}} dx. \tag{A.2}$$

$\int_{1}^{\infty} \frac{1}{x^{\alpha}} dx = \frac{1}{\alpha - 1}$ is finite as long as $\alpha > 1$, .

Example: if $\alpha = 1.25$,

$$1 + \int_{1}^{\infty} \frac{1}{x^{\alpha}} dx = 1 + \frac{\alpha}{\alpha - 1} = 5. \tag{A.3}$$

Corresponding simulation result: $\sum_{m=1}^{10^6} \frac{1}{m^{\alpha}} = 4.47$

We could see that the upper bound attained by integration is quite tight.

We also can derive

$$\begin{aligned} \sum_{n=m}^{\infty} \frac{1}{n^\alpha} &< \frac{1}{m^\alpha} + \int_m^\infty \frac{1}{x^\alpha} dx \\ &= \frac{\alpha}{(\alpha - 1)m^\alpha}, \end{aligned} \tag{A.4}$$

which we used in Chapter 3 and 4 for the proofs.

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