Fundamental Limits of Rate-Constrained Multi-User Channels and Random Wireless Networks

by

Hengameh Keshavarz

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Doctor of Philosophy
in
Electrical and Computer Engineering

Waterloo, Ontario, Canada, 2008

Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

This thesis contributes toward understanding fundamental limits of multi-user fading channels and random wireless networks. Specifically, considering different samples of channel gains corresponding to different users/nodes in a multi-user wireless system, the maximum number of channel gains supporting a minimum rate is asymptotically obtained.

First, the user capacity of fading multi-user channels with minimum rates is analyzed. Three commonly used fading models, namely, Rayleigh, Rician and Nakagami are considered. For *broadcast channels*, a power allocation scheme is proposed to maximize the number of active receivers, for each of which, a minimum rate $R_{\min} > 0$ can be achieved. Under the assumption of independent Rayleigh fading channels for different receivers, as the total number of receivers n goes to infinity, the maximum number of active receivers is shown to be arbitrarily close to $\ln(P \ln n)/R_{\min}$ with probability approaching one, where P is the total transmit power. The results obtained for Rayleigh fading are extended to the cases of Rician and Nakagami fading models. Under the assumption of independent Rician fading channels for different receivers, as the total number of receivers n goes to infinity, the maximum number of active receivers is shown to be equal to $\ln(2P \ln n)/R_{\min}$ with probability approaching one. For broadcast channels with Nakagami fading, the maximum number of active receivers is shown to be equal to $\ln(\frac{\omega}{\mu} P \ln n)/R_{\min}$ with probability approaching one, where ω and μ are the Nakagami distribution parameters. A by-product of the results is to also provide a power allocation strategy that maximizes the total throughput subject to the rate constraints. In *multiple-access channels*, the maximum number of simultaneous active transmitters (i.e. user capacity) is obtained in the many user case in which a minimum rate must be maintained for all active users. The results are presented in the form of scaling laws as the number of transmitters increases. It is shown that for all three fading distributions, the user capacity scales double logarithmically in the number of users and differs only by constants depending on the distributions. We also show that a

scheduling policy that maximizes the number of simultaneous active transmitters can be implemented in a distributed fashion.

Second, the maximum number of active links supporting a minimum rate is asymptotically obtained in a wireless network with an arbitrary topology. It is assumed that each source-destination pair communicates through a fading channel and destinations receive interference from all other active sources. Two scenarios are considered: 1) Small networks with multi-path fading, 2) Large Random networks with multi-path fading and path loss. In the first case, under the assumption of independent Rayleigh fading channels for different source-destination pairs, it is shown that the optimal number of active links is of the order $\log N$ with probability approaching one as the total number of nodes, N, tends to infinity. The achievable total throughput also scales logarithmically with the total number of links/nodes in the network. In the second case, a two-dimensional large wireless network is considered and it is assumed that nodes are Poisson distributed with a finite intensity. Under the assumption of independent multi-path fading for different source-destination pairs, it is shown that the optimal number of active links is of the order N with probability approaching one. As a result, the achievable per-node throughput obtained by multi-hop routing scales with $\Theta(\frac{1}{\sqrt{N}})$.

Acknowledgements

The author acknowledges her debt to those who have helped along the way and influenced the formation of this thesis. It is a pleasant aspect that I have now the opportunity to express my sincere gratitude for all of them.

In particular, I wish to express my gratitude to my co-supervisors, Professor Ravi Mazumdar and Professor Liang-Liang Xie for their continued encouragement and invaluable suggestions during this work. Their overt enthusiasm and integral view on research and their mission for providing only high-quality work and not less, has made a deep impression on me. I owe them a lot of gratitude for having me shown this way of research. They could not even realize how much I have learned from them. Without their overt enthusiasm, guidance, and compassion, I could not have completed this dissertation.

I wish to thank Professor Mazumdar for accepting me as a graduate student after changing my former supervisor, for providing me with exciting opportunities and passionate encouragement throughout my Ph.D. program, and for introducing me to Professor Xie. Not only has he been an enthusiastic and innovative teacher, but also an inspiring and dedicated mentor.

I am also grateful to Professor Xie. He has been an active listener who also takes the time to give advice. He has taught me how to understand difficult concepts, to approach and identify a research problem, and to precisely carry out a research plan to accomplish any goal.

I wish to warmly thank the members of my dissertation committee, Professor Massimo Franceschetti, Professor Mary Thompson, Professor Patrick Mitran, and Professor Amir K. Khandani for taking the time to read my thesis and providing me with insightful questions and constructive suggestions.

I am very thankful to Professor Rahul Roy from Indian Statistical Institute for helpful discussions that made this work better. My thanks also go to the staff of the Graduate Office in the Department of Electrical and Computer Engineering. Specially, I would like to thank Wendy Boles for helping me Compassionately during the process of switching my supervisor. I also thank Wendy Gauthier (Now, in the Department of Chemical Engineering) for making my graduate admission process so joyful.

Finally, I want to thank my family. The encouragement and support from my husband, Javad Ahmadi-Shokouh, is a powerful source of inspiration and energy. One of the best experiences of our partnership throughout this period was the birth of our sweet little son, Kian, who adds a joyful dimension to my life. I feel a deep sense of gratitude for my parents who formed a part of my vision and taught me the good things that really matter in life. I am really thankful for their never-ending support.

To my parents,

two great teachers, who have continuously supported me all the way since the first step I took in my life to the last step toward completing my education.

To my husband, Javad,

who has been a great source of motivation and inspiration. Without his support, I could not have completed this dissertation.

and

To my sweet little son, Kian,

who gives me the joy of being a mother.

Contents

List of Figures

Standard Notations

Notation	Definition
$f(n) = o(g(n))$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$
$f(n) = O(g(n))$	$\liminf_{n\to\infty} \frac{f(n)}{g(n)} > 0$
$f(n) = \Omega(g(n))$	$\limsup_{n\to\infty} \frac{f(n)}{g(n)} < \infty$
$f(n) \sim g(n)$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1$
$f(n) = \Theta(g(n))$	$f(n) = O(g(n));\ g(n) = O(f(n))$
$\mathbf{1}_{[A]}(\cdot)$	Indicator function for event A
$\mathbb{P}(A)$	Probability of event A
$\mathbf{E}(x)$	Expected value of random variable x
$\mathrm{Var}(x)$	Variance of random variable x
$\lfloor\cdot\rfloor$	Floor operator
$\Phi(\cdot)$	Normal distribution function
$\Gamma(\cdot)$	Gamma function
$\gamma(\cdot,\cdot)$	Lower incomplete gamma function
$\Gamma(\cdot,\cdot)$	Upper incomplete gamma function

Chapter 1

Introduction

"I have acquired that much knowledge to realize I do not know anything."

-Ibn Sina (Avicenna)

Nowadays, Wireless communications plays an important role in our daily life, although it has been a topic of study since the 1960's. Two fundamental aspects of wireless communications make it challenging and interesting. First of all, the time-varying nature of the underlying channel due to small-scale and large-scale fading is one of the most significant problems in designing wireless communication systems. Secondly, unlike wired communications in which each transmitter-receiver pair is isolated and can be thought of as a point-to-point link, wireless communication users suffer from interference made by all other active users sharing the same transmission medium. A lot of effort has been done to improve the performance of wireless communication systems in the presence of *fading* and *interference* and this field is still attracting many researchers. In this thesis, some multi-user wireless systems are analyzed in the presence of fading and interference.

This chapter contains of some background knowledge used throughout this thesis. First of all, two notions of Shannon capacity are defined and the corresponding capacity regions for fading broadcast and multiple-access channels are reviewed, Then, rate-constrained fading broadcast channels are introduced and their capacity region is characterized. Finally, some basic features of wireless networks are reviewed.

1.1 Capacity Regions of Multi-User Channels

Two notions of Shannon capacity have been developed for multi-user channels: *ergodic capacity* and *outage capacity*. Ergodic capacity and outage capacity are two different performance measures. Ergodic capacity deals with long-term rates averaged over all fading states and takes advantage of channel variations by allocating higher transmission rates to users with strong channels, while outage capacity achieves a constant rate in all non-outage fading states subject to an outage probability.

Considering random communication channel h, the channel capacity is defined as

$$C(h) = \max_{f_X(x)} I(X;Y)$$

where $I(X;Y)$ denotes mutual information between received signal Y and transmitted signal X, and $f_X(x)$ represents the transmitted signal's probability density function (pdf). As the channel capacity is a function of channel gains, it is random. Considering Gaussian fading channels, ergodic capacity and outage capacity are defined as follows.

Definition 1.1.1 $x\%$ outage capacity is the data rate, R, that can be supported with $x\%$. That is,

$$\mathbb{P}(R > C(h)) \leq x\%$$

Zero-outage capacity refers to outage capacity with $x = 0$.

Definition 1.1.2 Ergodic capacity is given by

$$C = \mathbf{E}\left(C(h)\right)$$

In a multi-user channel, the capacity is not a scalar and we are dealing with the capacity region.

In [1], the ergodic capacity region of fading multiple-access channels with Gaussian noise is characterized. In [1], it is shown that each point on the boundary of the region can be achieved by successive decoding. Moreover, the optimal rate and power allocation in each fading state can be explicitly obtained in a greedy manner. The solution can be viewed as the generalization of the water-filling construction for single-user channels to multiple-access channels with arbitrary number of users, and exploits the underlying polymatroid structure of the capacity region. In [2], the ergodic capacity region of an M-user fading broadcast channel is derived for code division (CD), time division (TD), and frequency division (FD), assuming that both the transmitter and the receivers have perfect channel side information (CSI). It is shown in [2] that by allowing dynamic resource allocation, TD, FD, and CD without successive decoding have the same ergodic capacity region, while optimal CD has a larger region. Optimal resource allocation policies are obtained for these different spectrum-sharing techniques. A simple sub-optimal policy is also proposed in [2] for TD and CD without successive decoding that results in a rate region quite close to the ergodic capacity region.

In [3], the outage capacity region of fading broadcast channels is derived, assuming that both the transmitter and the receivers have perfect CSI. This capacity region and the associated optimal resource allocation policies are obtained for CD with and without successive decoding, for TD, and for FD. It is shown in [3] that in an M-user broadcast system, the outage capacity region is implicitly obtained by deriving the outage probability region for a given rate vector. Given the required rate of each user, a strategy which bounds the outage probability region is presented for different spectrum-sharing techniques. The corresponding optimal power allocation scheme is a multi-user generalization of the threshold-decision rule for a single-user fading channel. Also discussed is a simpler minimum common outage probability problem under the assumption that the broadcast channel is either not used at all when fading is severe or used simultaneously for all users. In [4], the outage capacity region of an M-user fading multiple-access channel is derived under the assumption of perfect CSI at the transmitters and the receiver. Given a required rate and average power constraint for each user, a successive decoding strategy and a power allocation policy are proposed in [4] that achieve points on the boundary of the outage probability region. The scenario where an outage must be declared simultaneously for all users (common outage) and when outages can be declared individually (individual outage) for each user are discussed.

1.2 Rate-Constrained Multi-User Channels

Many businesses are now seeking the collaboration and productivity benefits offered by mobile voice and video. To support these applications, information technology organizations must design a wireless network that is multimedia ready. In fact, health-care companies and retail industries, in particular, are already deploying voice and video for mission-critical applications. In health-care, some examples include voice communications between hospital staff, physicians and nurses, as well as sharing imaging files such as X-ray and Magnetic Resonance Imaging (MRI) fies between staff members. In retail, these applications include Push-To-Talk applications for employee collaboration and streaming media for digital display units in stores. When voice and video are added to a wireless network, a number of challenges arise. Because voice and video are latency-sensitive, they require higher levels of priority, predictability, and reliability than data applications. The same packet loss that would not significantly affect a data file can be completely disruptive to a voice call or video stream [7]. Hence, all users need to transmit/receive information with a rate greater than a threshold. In this section, broadcast channels as a simple example of multi-user wireless systems are chosen and some technical aspects of supporting a minimum rate in such a system are introduced.

1.2.1 Broadcast Channels with Minimum Rates

For a system with delay-sensitive data, neither the ergodic capacity nor the outage capacity is optimal because it is not desirable to shut off users for a long period of time. In [6], ergodic capacity is maximized subject to minimum rate requirements for all users in all fading states in a broadcast channel.

In a rate-constrained broadcast channel, all receivers must support a minimum rate. Hence, some transmit power is used to maintain the minimum rates in all fading states, while the remaining power is used to maximize the average sum rate. In this case, users are never completely cut off due to the minimum rate requirement. However, the amount of power allocated to each user still depends on its channel variations. Users having good channel conditions receive more extra power and are able to transmit with higher rates, while less transmit power is allocated to those receivers experiencing poor channel conditions.

1.2.2 Minimum-Rate Capacity Region

In [6], the minimum-rate capacity region of K-user broadcast channels is defined as the region of all achievable average rate vectors subject to an average power constraint $\bar{P}$ and minimum-rate constraints $\mathbf{R}^* = (R_1^*, R_2^*, \cdots, R_K^*)$. Throughout this thesis, vectors are indicated by bold face letters. The minimum-rate constraint basically forces each receiver's instantaneous rate to be at least equal to the minimum rate in all fading states. Assume $R_j(n)$ and R_j denote respectively the instantaneous and the average rate of user j at time slot n. Hence, according to the minimum-rate constraint, $R_j(n) \geq R_j^*, j = 1, \cdots, K, \forall n$. Note that we are dealing with slow or block fading channels that are assumed to be constant during each time block. Let $\mathcal{C}_{min}(\mathcal{P})$ denotes the set of achievable average rates in excess of the minimum rates for power policy $\mathcal{P}$ [6]

$$\mathcal{C}_{min}(\mathcal{P}) = \{R_j : R_j^* \leq R_j \leq \mathbf{E}_n \left(R_j(\mathbf{P}(n))\right); j = 1, \cdots, K\} \tag{1.1}$$

and $R_j(\mathbf{P}(n))$ is given by

$$R_j(\mathbf{P}(n)) = \log \left(1 + \frac{P_j(n)}{n_j + \sum_{k=1}^{K} P_k(n) \mathbf{1}_{[n_j > n_k]}} \right)$$

where $P_j(n)$ denotes transmitted power allocated to the jth receiver at time n, and n_j refers to the background noise variance at receiver j. As (1.1) indicates, $\mathcal{C}_{min}(\mathcal{P})$ does not include rates below the given minimum rates. To ensure that the minimum-rate constraints are satisfied, the set of feasible power policies must be tightly restricted. Define $\mathcal{F}$ as the set of all power policies satisfying the minimum-rate constraints and the average power constraint $\bar{P}$ in each fading state [6]

$$\mathcal{F} \equiv \left\{ \mathcal{P} : \mathbf{E}_n \left(\sum_{k=1}^{K} P_k(n) \right) \leq \bar{P},\ R_j(\mathbf{P}(n)) \geq R_j^* \quad \forall j, n \right\} \tag{1.2}$$

Definition 1.2.1 The minimum-rate capacity region of a fading broadcast channel with perfect CSI at the transmitter and receivers, average power constraint $\bar{P}$, and the minimum-rate constraints $\mathbf{R}^* = (R_1^*, R_2^*, \cdots, R_K^*)$ is [6]

$$\mathcal{C}_{min}(\mathcal{P}, \mathbf{R}^*) = Co \left(\bigcup_{\mathcal{P} \in \mathcal{F}} \mathcal{C}_{min}(\mathcal{P}) \right) \tag{1.3}$$

where Co denotes the convex hull operation. Achievability of this region can be proved using achievability of the ergodic capacity region and time sharing arguments.

The minimum-rate capacity region is basically a combination of the ergodic capacity and zero-outage (i.e. the probability of outage is zero) capacity regions. In the

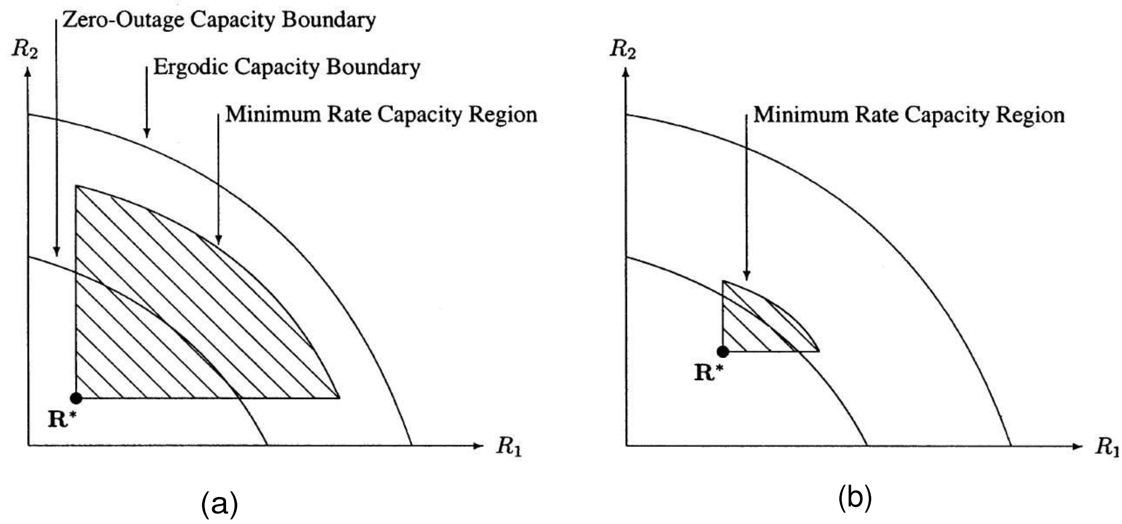


Figure 1.1: Ergodic, zero-outage, and minimum-rate capacity regions for (a) small minimum rates, and (b) large minimum rates [6]

minimum-rate capacity region, a fraction of transmitted power is used to maintain the minimum rates in all fading states, while the remaining power is used to achieve higher rates in excess of the minimum rates. As the minimum rates must be maintained in all fading states, the minimum rates vector should be located inside the zero-outage capacity region. Let $\mathcal{C}_{ergodic}$ and $\mathcal{C}_{zero}$ denote the ergodic capacity and zero-outage capacity regions respectively. It can be shown that [6]

$$\mathcal{C}_{zero} \subseteq \mathbf{Boundary}\{\mathcal{C}_{min}(\mathcal{P}, \mathbf{R}^*)\} \subseteq \mathcal{C}_{ergodic}$$

Figure 2.3 shows this relationship for two minimum rates vectors. It can be seen that increasing minimum rates results in a smaller set of achievable rates because a large fraction of transmitted power is required to maintain the minimum rates for all receivers. If the minimum rates of all users are zero, the minimum-rate capacity region is the same as the ergodic capacity region. As Figure 2.3 indicates, the minimum rates vector is located in the zero-outage capacity region; therefore, it is achievable in all fading states with probability one.

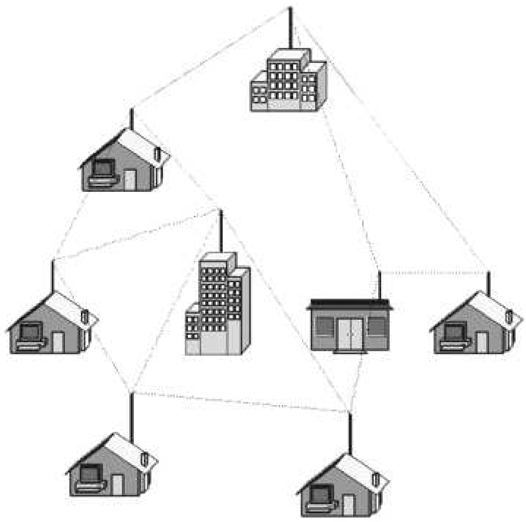


Figure 1.2: A wireless ad-hoc network

1.3 Wireless Ad-Hoc Networks

Wireless ad-hoc networks (e.g. Figure 1.2) are wireless networks without any infrastructure or decentralized wireless networks. In this network, each node is capable of transmitting information to other nodes in the network and selecting the nodes forwarding data to their destinations is made dynamically based on the network connectivity. This is in contrast to wired networks in which routers perform the routing task or to managed wireless networks (i.e. wireless networks with an infrastructure) in which a base station or an access point manages communication among the nodes.

1.3.1 Network Architecture

A wireless ad-hoc network is a collection of autonomous nodes or terminals communicating with each other by forming a multi-hop (see Figure 1.3) radio network and maintaining connectivity in a decentralized manner. Since the nodes communicate over wireless links, they have to contend with the effects of radio communication, such as noise, fading, and interference. In addition, the links typically have less bandwidth than those in a wired network. Each node in a wireless ad-hoc network

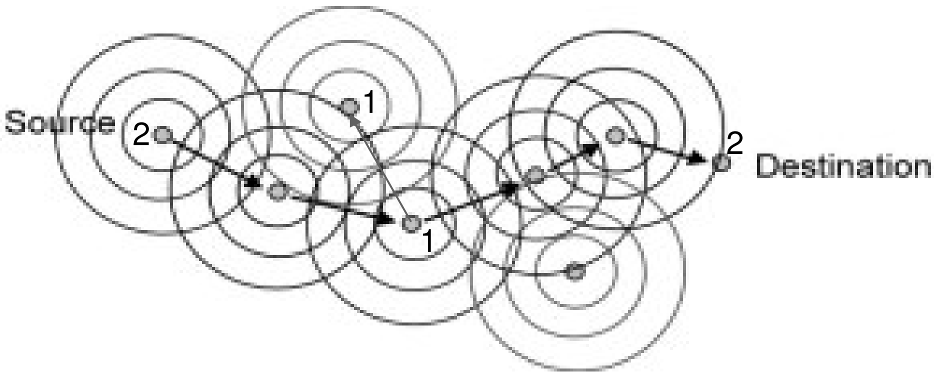


Figure 1.3: Single-hop (source 1 to destination 1) and multi-hop (source 2 to destination 2) transmissions in a wireless ad-hoc network

functions as both a host and a router, and the control of the network is distributed among the nodes. The network topology is in general dynamic, because the connectivity among the nodes may vary with time due to node departures, new node arrivals, and the possibility of having mobile nodes. Hence, there is a need for efficient routing protocols to allow the nodes to communicate over multi-hop paths consisting of possibly several links in a way that does not use any more of the network ”resources” than necessary.

1.3.2 Applications of Wireless Ad-Hoc Networks

The decentralized nature of wireless ad-hoc networks makes them suitable for a variety of applications where having central nodes is impossible or central nodes can't be relied on. Ad-hoc wireless networks can be further classified into the following categories based on their applications:

- Mobile ad-hoc networks
- Sensor networks
- Mesh networks

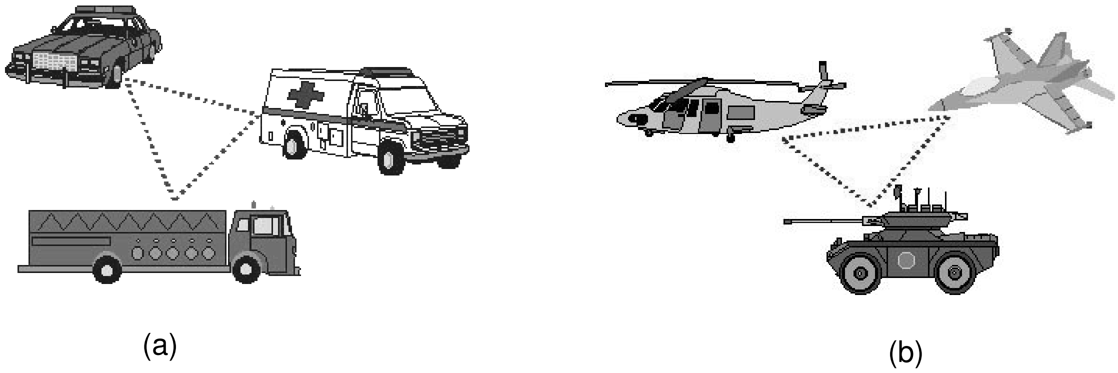


Figure 1.4: Mobile ad-hoc networks (MANETs) for (a) emergency/rescue operations, and (b) military applications [8]

A. Mobile Ad-Hoc Networks

In the next generation of wireless communication systems, there will be a need for the rapid deployment of independent mobile users. Significant examples include establishing survivable, efficient, dynamic communication for emergency/rescue operations (e.g. Figure 1.4.a), disaster relief efforts, and military networks (e.g. Figure 1.4.b). Such network scenarios cannot rely on centralized and organized connectivity, and can be conceived as applications of mobile ad-hoc networks (MANETs). A MANET is an autonomous collection of mobile users that communicate over relatively bandwidth constrained wireless links. Since the nodes are mobile, the network topology may change rapidly and unpredictably over time. The network is decentralized, where all network activity including discovering the topology and delivering messages must be executed by the nodes themselves; in other word, routing functionality will be incorporated into mobile nodes [8].

B. Sensor Networks

A wireless ad hoc sensor network consists of a number of sensors spread across a geographical area. Each sensor has wireless communication capability and some level of intelligence for signal processing and networking of the data. Some examples of sensor networks are the following [8]:

1. Military sensor networks to detect and gain as much information as possible

about enemy movements, explosions, and other phenomena of interest.

2. Sensor networks to detect and characterize Chemical, Biological, Radiological, Nuclear, and Explosive (CBRNE) attacks and material.
3. Sensor networks to detect and monitor environmental changes in plains, forests, oceans, etc.
4. Traffic sensor networks to monitor vehicle traffic on highways or in congested parts of a city.
5. Surveillance sensor networks for providing security in shopping malls, parking garages, and other facilities.
6. Parking lot sensor networks to determine which spots are occupied and which are free.

The above list suggests that sensor networks offer certain capabilities and enhancements in operational efficiency in civilian applications as well as assist in the national effort to increase alertness to potential terrorist threats.

The basic goals of a sensor network generally depend upon the application, but the following tasks are common to many networks [8]: **1)** Determine the value of some parameter at a given location (e.g. in an environmental network, the temperature, atmospheric pressure, amount of sunlight, and the relative humidity at a number of locations), **2)** Detect the occurrence of events of interest and estimate parameters of the detected events (e.g in the traffic sensor network, detecting a vehicle moving through an intersection and estimate the speed and direction of the vehicle), **3)** Classify a detected object (e.g. in a traffic sensor network, a vehicle crossing the intersection is a car, a mini-van, or a bus), **4)** Track an object (e.g. in a military sensor network, track an enemy tank). In these four tasks, an important requirement of the sensor network is that the required data be disseminated to the proper end users. In some cases, there are fairly strict time requirements on this communication. For instance, the detection of an intruder in a surveillance network should be immediately communicated to the police so that action can be taken.

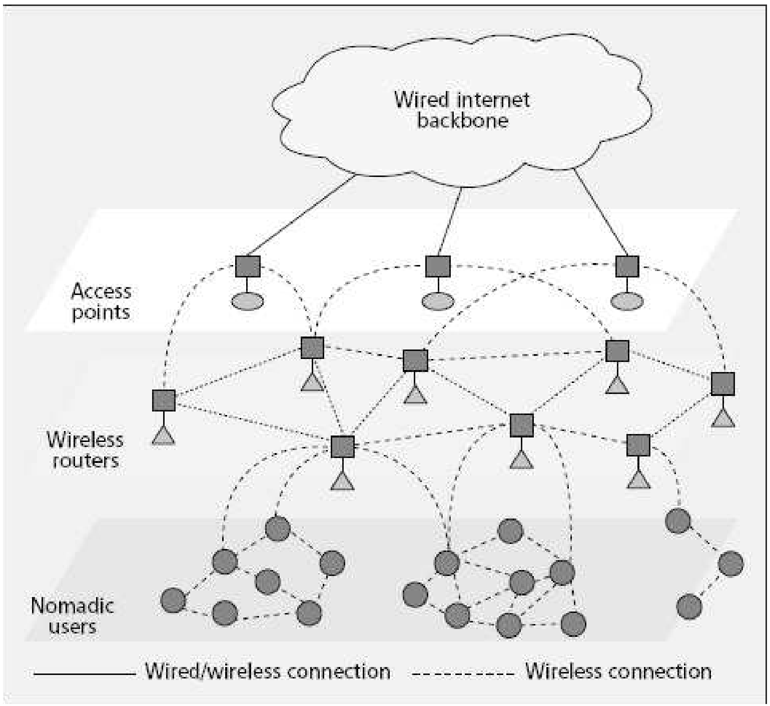


Figure 1.5: Three-level architecture for wireless mesh networks [9]

B. Mesh Networks

A mesh network is a wireless network made up of radio nodes organized in a mesh topology. A wireless mesh network is a fully wireless network that employs multi-hop communications to forward traffic to and from wired Internet entry points. Different from flat ad-hoc networks, a mesh network introduces a hierarchy in the network architecture with the implementation of wireless routers providing wireless transport services to data traveling from users to either other users or access points (access points are special wireless routers with a high-bandwidth wired connection to the Internet backbone) [9]. Figure 1.5 shows a three-level architecture for wireless mesh networks. Several emerging and commercially interesting applications of wireless mesh networks are the following [9]:

- Integrated public transportation systems
- Public safety
- public Internet access

1.4 Thesis Objective and Outline

The objective of this thesis is on analyzing some fundamental limits of fading multi-user channels and random wireless networks. Specifically, considering different samples of channel gains corresponding to different users/nodes in a multi-user wireless system, the maximum number of channel gains supporting a minimum rate is asymptotically obtained.

First, the user capacity of fading multi-user channels with minimum rates is analyzed. Three commonly used fading models, namely, Rayleigh, Rician and Nakagami are considered. For *broadcast channels*, a power allocation scheme is proposed to maximize the number of active receivers, for each of which, a minimum rate can be achieved. In *multiple-access channels*, the maximum number of simultaneous active transmitters (i.e. user capacity) is obtained in the many user case in which a minimum rate must be maintained for all active users.

Second, the maximum number of active links supporting a minimum rate is asymptotically obtained in a wireless network with an arbitrary topology. It is assumed that each source-destination pair communicates through a fading channel and destinations receive interference from all other active sources. Two scenarios are considered: 1) Small networks with multi-path fading, 2) Large Random networks with multi-path fading and path loss. In the first case, independent Rayleigh fading channels for different source-destination pairs are assumed. In the second case, a two-dimensional large wireless network is considered and it is assumed that nodes are Poisson distributed with a finite intensity.

The rest of the thesis is organized as follows: In Chapter 2, the user capacity of rate-constrained fading broadcast channels is obtained. Chapter 3 presents the same analysis for fading multiple-access channels with minimum rates. In Chapter 4, a random wireless ad-hoc network is considered and the maximum number of active source-destination pairs is asymptotically achieved for small and large networks. Finally, Chapter 5 concludes thesis contributions and presents future work.

Chapter 2

User Capacity of Rate-Constrained Broadcast Channels

In a broadcast system where the transmitter can allocate different portions of its total transmit power to different receivers according to their channel states, there is a basic trade-off between the total throughput and the minimum rate achievable for all the receivers. To increase the total throughput, it is always favorable to allocate more power to receivers with better channel states, while in order to increase the minimum rate, obviously, more power should be allocated to receivers with worse channel states. In this chapter, a power allocation scheme is proposed to maximize the number of active receivers (i.e. user capacity) in broadcast channels, for each of which, a minimum rate $R_{\min} > 0$ can be achieved. Three commonly used fading distributions, namely, Rayleigh, Rician, and Nakagami, are considered and the user capacity of rate-constrained broadcast channels is asymptotically analyzed.

2.1 Literature Review

In a dynamic environment, where the channel states are time-varying, opportunistic power allocation schemes can be exploited to increase the total throughput while maintaining an average rate constraint for each receiver. The basic idea is to adapt the power allocation to the variations of the channel states. The transmission rate for a receiver is increased when its channel state becomes better, thus higher rates can be achieved using less power. However, in delay-sensitive applications, it may not be admissible for a receiver to wait too long before its rate increases. Basically, this raises an issue of the trade-off between ergodic capacity and outage capacity, for which, extensive studies have been given in [2, 3, 6] in the context of broadcast channels.

In a rate-constrained broadcast channel, all receivers must maintain a minimum rate. The idea of broadcast channels with minimum rates is originally proposed in [6] and the capacity region and the optimal power allocation scheme for a block fading broadcast channel with minimum rates are derived. As mentioned in Section 1.2.2, [6] presents the relationship between the minimum-rate capacity region with the ergodic and zero-outage capacity regions. Other papers dealing with single-antenna or multi-antenna rate-constrained broadcast channels are as follows: In [10], the optimal transmit strategy is studied for a multi-antenna Gaussian broadcast channel in which each user requires a specific rate. These rate requirements correspond to the respective service the receiver is using. This problem leads to the non-degraded multi-antenna Gaussian broadcast channel. An algorithm is proposed to fulfill the rate requirements of all receivers and maximizes the minimum rate factor which is defined as the quotient of the available rate and the required rate for each user. The proposed algorithm balances the rate factors until an equilibrium is reached. In [11], a joint power and rate allocation scheme is proposed for the downlink wireless data services in Code Division Multiple Access (CDMA) networks. The goal is on maximizing the total utility while maximizing the utility of each mobile user. A distributed allocation algorithm is presented based on dynamic pricing. The algorithm is composed of three processes, user selection, power allocation and rate allocation. Moreover, the optimization model and algorithm considering users'minimum-rate constraints are established to provide Quality of Service (QoS) for mobile users. In [12], Dynamic Resource Allocation (DRA) has been developed to improve the total throughput by taking the benefit of channel variations among users in a multi-carrier system. For practical use, it is important for DRA algorithms to be both fair and efficient. In [12], resource allocation algorithms are measured in terms of fairness and efficiency and then, a new scheduling algorithm (called the MRR algorithm) considering users' QoS provision is proposed. The MRR algorithm is designed to meet individual users minimum required rate while maximizing fairness and efficiency of the whole system. In [13], the optimal solution is presented to the problem of allocating bandwidth and power across users for downlink transmission in wireless systems when multiple users can be scheduled for transmission simultaneously. Maximum and minimum rate per user constraints and a maximum rate per unit bandwidth constraint are included in the formulation. When only the constraint of a maximum rate per unit bandwidth is imposed, [13] shows that scheduling at most two users simultaneously is sufficient for optimality. In [14], a method having a low computational complexity is proposed for fast broadcasting with minimum rate constraints, suited for transmissions over Orthogonal Frequency Division Multiplexing (OFDM) channels. Results obtained by computer simulations show that the novel scheme performs close to the optimum throughput. In [15], a downlink resource allocation problem is considered in an OFDM system. The resource allocation problem is modeled as a cooperative game where a fairness criterion is enforced in the bargaining outcome of the game. Given a minimum rate requirement for each user, Nash bargaining model ensures all users to attain their minimum rate requirements. If a set of maximum rate requirement is also provided, Raiffa-Kalai-Smorodinsky bargaining model regulates the bargaining outcome to consider both the minimum and maximum requirements of all users. The main interest of the cooperative game is to achieve a Pareto optimal outcome. In [15], a reduced complexity algorithm is proposed to achieve transmission rates as close as possible to the Pareto optimal rates. In [16], the downlink of a multi-cell OFDMA system is considered. Rate Adaptive Optimization is investigated with a minimum-rate constraint in presence of co-channel interference and a low-complexity sub-carrier allocation scheme is proposed. A particular procedure provides limitation of co-channel interference by dynamically adapting the sub-carrier reuse factor. A rate requirement violation threshold is introduced to decide whether or not the interference limitation procedure is to be used. In [17], downlink user capacity is asymptotically derived in a single-cell with successive interference cancellation using order statistics. It is assumed that all the users have a common target date rate. In [17], user capacity is defined as the expected number of simultaneously active receivers and a complicated expression in terms of the normal distribution function is presented for the downlink user capacity. In [18], the weighted sum rate problem is solved for an OFDM broadcast channel under a sum power constraint, if minimum rates have to be guaranteed in each fading state and perfect CSI is assumed at the base station and the receivers. The problem is subdivided into two problems. First, the problem of feasibility is tackled, which occurs since the system is power limited and not all required rates might be supportable. Subsequently, the optimal resource allocation in case of feasibility is derived. In [19], the aforementioned results are also extended to Multiple-Input Multiple-Output (MIMO) OFDM broadcast channels. In [20], optimal resource allocation is proposed for parallel Gaussian broadcast channels. In other words, the maximization of a weighted sum of rates is studied where a rate constraint over all parallel Gaussian channels has to be met for each user in each time instant with limited sum power. The derived algorithms can be interpreted as primal-dual algorithms, where one has an appealing interpretation as rate water-filling. In [21], scheduling in a broadcast channel based on partial channel state information at the transmitter is carried out in an opportunistic way, where several orthogonal beams are randomly generated at the transmitter to simultaneously deliver several users with their intended data. Within a more practical perspective of the opportunistic systems, [21] presents a transmission scheme where a minimum rate per user is required for each scheduled user. This minimum rate is demanded by each user to properly decode and manage its received signal, which stands as a possible QoS indicator for the system behaviour. Then, [21] considers an imperfect CSI situation where robust schemes are required to meet the QoS restrictions. Two robust opportunistic transmission philosophies are presented through a power allocation over the transmitting beams, and they are efficiently solved via convex optimization tools. In [22], the problem of maximizing the overall spectral efficiency is investigated for the downlink of multi-user OFDM systems while maintaining users' QoS requirements, including bit error rate and individual minimum rate requirements. Under the assumption of equal power allocation, an efficient algorithm is proposed to obtain the suboptimal solution of the resource allocation. In this algorithm, first, some positive multipliers are introduced, one for each user, according to their minimum rate constraints, and then a parallel subcarrier-and-bit allocation scheme is designed using these multipliers with low complexity. As providing multimedia services is particularly challenging in wireless networks such as high speed downlink packet access (HSDPA) systems, [23] presents a generalized scheduling algorithm which allows controlling over users' fairness as well as balancing fairness-throughput trade-offs. Furthermore, in [23], it is shown that the generalized algorithm can well support a minimum rate for delay-sensitive services, which is important in providing QoS for multimedia services over HSDPA. In [24], the impact of Space Division Multiple Access (SDMA) on access layer channel allocation is captured. This impact obtains different twists in Time Division Multiple Access (TDMA), CDMA and OFDM due to different natures of co-channel and cross-channel interference and different interactions of user spatial channel characteristics with system channels, namely, time slots, codes and sub-carriers. In [24], heuristic algorithms are proposed for channel allocation, downlink beamforming and transmit power control so as to increase total provisioned system rate and provide QoS to users in the form of minimum rate guarantees.

2.2 Motivation and Objective

In this chapter, we consider a power allocation scheme with a minimum-rate constraint $R_{\min} > 0$. Since for a fixed $R_{\min}$, in a time-varying fading environment, it may not be always possible for all receivers to achieve this minimum rate simultaneously, we propose a scheme to maximize the number of active receivers, for each of which, such a minimum rate can be supported, while allocating no power to the other inactive receivers.

By adjusting the value of $R_{\min}$, different trade-offs between the total throughput and the delay can be achieved. Specifically, by increasing $R_{\min}$, transmitted power is shared among fewer receivers with relatively better channel states, resulting in higher total throughput; However, this also results in delay for more inactive receivers; therefore, longer delay for each receiver on average. On the other hand, choosing $R_{\min}$ small enough, it is possible to make it simultaneously achievable for all the receivers, resulting in no delay for any receiver; However, it may be too costly to let receivers at extremely bad channel states transmit data.

While the number of supportable active receivers depends on the specific channel states, the asymptotic behavior is analyzed when the total number of receivers n is large for three commonly used fading distributions, namely, Rayleigh, Rician, and Nakagami. These fading distributions cover the commonly used models for wireless communication channels. For example, if there are multiple indirect paths between transmitter and receiver, with no distinct dominant path, Rayleigh fading is appropriate from the central limit theorem. If there is a dominant component, say line-of-sight (LOS), in addition to indirect paths, the Rician distribution is appropriate. Nakagami fading occurs in the case of relatively large delay-time spreads, with different clusters of reflected waves. Within any one cluster, the delay times are approximately equal for all waves, and as a result the envelope of each cumulated cluster signal is Rayleigh distributed. Since the average time delay differs significantly between clusters, Nakagami fading follows from a sum of multiple Rayleigh-faded signals [25].

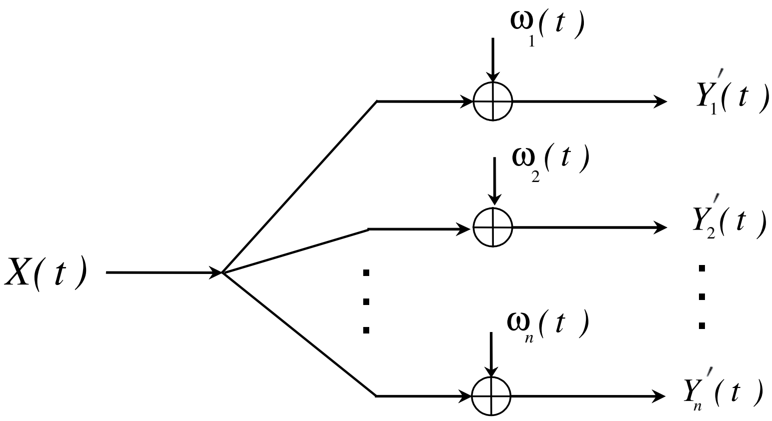


Figure 2.1: A fading broadcast channel

The rest of this chapter is organized as follows: In Section 2.3, the broadcast channel model is introduced. Section 2.4 presents the proposed power allocation scheme. In Section 2.5, the user capacity of rate-constrained fading broadcast channels is asymptotically analyzed for Rayleigh, Rician, and Nakagami fading models. Finally, simulation results are shown in Section 2.6.

2.3 System Model

Consider a broadcast channel with one transmitter and n receivers with the following channel model in the time block $t = 1, 2, \ldots, T$:

$$Y_i(t) = g_i X(t) + Z_i(t), \quad i = 1, 2, \ldots, n, \tag{2.1}$$

where $X(t) \in \mathbb{C}$ is the signal sent by the transmitter, and $Y_i(t) \in \mathbb{C}$ is the signal received by receiver i. Noise $Z_i(t) \in \mathbb{C}$, $i = 1, \ldots, n$, $t = 1, \ldots, T$ are assumed to be independent and identically distributed (iid) complex Gaussian distributed according to $\mathcal{CN}(0, 1)$. The channel gains $g_i \in \mathbb{C}$, $i = 1, \ldots, n$ are assumed to be constant during this time block, and known to the transmitter and all the receivers.

Equivalently, as shown in Figure 2.1, the model (2.1) can be written as

$$\begin{aligned} Y_i'(t) &= X(t) + Z_i(t)/g_i, \quad i = 1, 2, \ldots, n \\ &= X(t) + \omega_i(t) \end{aligned} \tag{2.2}$$

where noise $Z_i(t)/g_i$ is still complex Gaussian distributed, but with variance $1/|g_i|^2$.

Let $N_i = 1/|g_i|^2$. Without loss of generality, assume that $N_1 \leq N_2 \leq \cdots \leq N_n$. It is well known [26, Sec.14.6] that the broadcast channel (2.2) is stochastically degraded, and the capacity region is given by

$$R_i < \ln\left(1 + \frac{P_i}{\sum_{j=1}^{i-1} P_j + N_i}\right), \quad i = 1, \ldots, n \tag{2.3}$$

where R_i is the achievable rate for receiver i, to which, the power $P_i \geq 0$ is allocated by the transmitter under the total transmit power constraint $\sum_{i=1}^n P_i = P$.

2.4 Power Allocation

Different rates can be achieved by different power allocation schemes in (2.3). To increase the total throughput, $\sum_{i=1}^n R_i$, it is always favorable to allocate more power to receivers with smaller N_i, as demonstrated by the following lemma.

Lemma 2.4.1 For any two power allocation schemes $\{P_i, i = 1, \ldots, n\}$ and $\{P_i', i = 1, \ldots, n\}$ in (2.3), where for some $1 \leq i_1 < i_2 \leq n$ and $\Delta > 0$, $P_{i_1}' = P_{i_1} + \Delta$, and $P_{i_2}' = P_{i_2} - \Delta$, and $P_i = P_i'$ for any $i \notin \{i_1, i_2\}$, the following inequality always holds:

$$\sum_{i=1}^n \ln\left(1 + \frac{P_i}{\sum_{j=1}^{i-1} P_j + N_i}\right) \leq \sum_{i=1}^n \ln\left(1 + \frac{P_i'}{\sum_{j=1}^{i-1} P_j' + N_i}\right) \tag{2.4}$$

where "=" holds if and only if $N_{i_1} = N_{i_1+1} = \cdots = N_{i_2}$.

Proof 2.4.1 By induction, we only need to prove the case when $i_2 = i_1 + 1$, for which, (2.4) is equivalent to

$$\sum_{i=i_1}^{i_1+1} \ln\left(1+\frac{P_i}{\sum_{j=1}^{i-1} P_j + N_i}\right) \leq \sum_{i=i_1}^{i_1+1} \ln\left(1+\frac{P_i'}{\sum_{j=1}^{i-1} P_j' + N_i}\right)$$

which is equivalent to

$$\begin{aligned}\left(1+\frac{P_{i_1}}{\sum_{j=1}^{i_1-1} P_j + N_{i_1}}\right)\left(1+\frac{P_{i_1+1}}{\sum_{j=1}^{i_1-1} P_j + P_{i_1} + N_{i_1+1}}\right) \leq \\ \left(1+\frac{P_{i_1}+\Delta}{\sum_{j=1}^{i_1-1} P_j + N_{i_1}}\right)\left(1+\frac{P_{i_1+1}-\Delta}{\sum_{j=1}^{i_1-1} P_j + P_{i_1} + \Delta + N_{i_1+1}}\right)\end{aligned}$$

which is equivalent to

$$\frac{\sum_{j=1}^{i_1} P_j + N_{i_1}}{\sum_{j=1}^{i_1} P_j + N_{i_1+1}} \leq \frac{\sum_{j=1}^{i_1} P_j + N_{i_1} + \Delta}{\sum_{j=1}^{i_1} P_j + N_{i_1+1} + \Delta}$$

which holds obviously for any $\Delta > 0$ and $N_{i_1} \leq N_{i_1+1}$, where "=" holds if and only if $N_{i_1} = N_{i_1+1}$. □

In order to maximize the total throughput, all power should be allocated to the best receiver, which has the maximum channel gain $|g_1|$, or the minimum equivalent noise variance N_1. However, in order to maintain a trade-off between the throughput and delay, the following power allocation scheme is considered:

$$\max\{m\} \tag{2.5}$$

$$\text{subject to} \quad \ln\left(1+\frac{P_1}{N_1}\right) \geq R_{\min} \tag{2.6}$$

$$\ln\left(1+\frac{P_i}{\sum_{j=1}^{i-1} P_j + N_i}\right) = R_{\min}, \quad 2 \leq i \leq m \tag{2.7}$$

$$\sum_{i=1}^{m} P_i = P \tag{2.8}$$

where $R_{\min} > 0$ (in nats) is a pre-set minimum-rate constraint for all active receivers.

The reason for setting "=" instead of "≥" in (2.7) is that once the minimum rate is satisfied, any redundant power should be given to the best receiver in order to maximize the total throughput, as implied by Lemma 2.4.1.

A simple algorithm to solve the optimization problem (2.5)-(2.8) is as follows: First, the maximum m can be determined by recursively defining P_i', $i = 1, 2, \ldots$, according to the following equations:

$$R_{\min} = \ln\left(1 + \frac{P_i'}{\sum_{j=1}^{i-1} P_j' + N_i}\right), \quad i = 1, 2, \ldots, \tag{2.9}$$

until some integer m such that $\sum_{i=1}^{m} P_i' \leq P$ but $\sum_{i=1}^{m+1} P_i' > P$, or $m = n$.

Then, after the maximum m is determined, the optimal power allocation can be obtained by letting $P_i = 0$ for $i = m+1, \ldots, n$, and choosing P_i, $i = m, m-1, \ldots, 2$ recursively according to the following equations:

$$R_{\min} = \ln\left(1 + \frac{P_i}{P - \sum_{j=i}^{m} P_j + N_i}\right), \quad i = m, \ldots, 2,$$

and at last, setting $P_1 = P - \sum_{j=2}^{m} P_j$.

Obviously, with fixed P and $R_{\min}$, the maximum number of active receivers completely depends on the equivalent noise variance $N_i = 1/|g_i|^2$, $i = 1, \ldots, n$. When the channel gains g_i obey some statistical distribution, asymptotic behavior of the maximum m can be determined when the total number of receivers n becomes large.

2.5 Asymptotic Analysis

Let M_n denote the maximum number of simultaneous active receivers (out of n receivers) that can be supported with a rate greater than or equal to R_{min}. Note, M_n is random which depends on the channel gains. Assuming the total number of receivers is large enough, the distribution of M_n can be obtained using the central limit theorem. In this section, some characteristics of this distribution is analyzed.

2.5.1 Rayleigh Fading

Consider independent Rayleigh fading channels for different receivers, i.e., the gains g_i, $i = 1, \ldots, n$ are independent realizations of the complex Gaussian distribution $\mathcal{CN}(0, 1)$. We have the following theorem.

Theorem 2.5.1 Under the assumption of independent Rayleigh fading channels for different receivers with the gain $g_i \sim \mathcal{CN}(0, 1)$ and for any $\epsilon > 0$, the maximum number of active receivers M_n determined by (2.5)-(2.8) is bounded as:

$$\mathbb{P}(\lfloor \nu(n) - \epsilon \rfloor \leq M_n \leq \nu(n) + \epsilon) \to 1, \quad \text{as } n \to \infty, \tag{2.10}$$

where n denotes the total number of receivers, and

$$\nu(n) \triangleq \ln(P \ln n)/R_{\min}. \tag{2.11}$$

Proof 2.5.1 Consider the broadcast channel (2.1), with the independent gains $g_i \sim \mathcal{CN}(0, 1)$, for $i = 1, \ldots, n$. For the equivalent model (2.2), the noise variance $N_i = 1/|g_i|^2$ is of the following distribution function:

$$\begin{aligned} F(y) &= \mathbb{P}(N_i < y) = \mathbb{P}(1/|g_i|^2 < y) = \mathbb{P}(|g_i|^2 > 1/y) \\ &= \int_{1/y}^{\infty} e^{-x} dx = e^{-\frac{1}{y}}, \text{ for } y > 0. \end{aligned}$$

For any fixed $N_0 > 0$, we can characterize the number of "good" channels with the equivalent noise variance N_i less than N_0 as the following. Let $p_0 = F(N_0) = e^{-\frac{1}{N_0}}$. Then, with probability p_0, a channel is good. Consider Bernoulli sequence

$$x_i = \begin{cases} 1, & \text{with probability } p_0 \\ 0, & \text{with probability } 1 - p_0 \end{cases}$$

for $i = 1, 2, \ldots, n$. Then, the number of good channels has the same distribution as $M_n = \sum_{i=1}^{n} x_i$, which satisfies the binomial distribution $B(n, p_0)$.

Now, consider the following power allocations for the m best receivers.

$$P_i = \frac{c}{\alpha^{m-i}}, \text{ for } i = 1, \ldots, m,$$

where $\alpha = e^{R_{\min}} > 1$, and $c = (1 - 1/\alpha)P$. It is easy to check that the total power constraint is satisfied. That is,

$$\sum_{i=1}^{m} \frac{c}{\alpha^{m-i}} = c\frac{1-(1/\alpha)^m}{1-1/\alpha} \le c\frac{1}{1-1/\alpha} = P.$$

If $\max_{1\le i\le m} N_i \le P/\alpha^m$, we have the following uniform lower bound for Signal-to-Interference-plus-Noise Ratios (SINR's) at all these m receivers. For $i = 1$,

$$\frac{P_1}{N_1} \ge \frac{c/\alpha^{m-1}}{P/\alpha^m} = \alpha - 1,$$

and for any $i = 2, \ldots, m$,

$$\begin{aligned} \frac{P_i}{\sum_{j=1}^{i-1} P_j + N_i} &\ge \frac{c/\alpha^{m-i}}{\sum_{j=1}^{i-1} c/\alpha^{m-j} + P/\alpha^m} \\ &= \frac{1/\alpha^{m-i}}{\frac{(1/\alpha)^{m-i+1}-(1/\alpha)^m}{1-1/\alpha} + \frac{(1/\alpha)^m}{1-1/\alpha}} = \alpha - 1. \end{aligned}$$

Hence, the minimum-rate constraint is satisfied for all these m receivers, since

$$\ln\left(1 + (\alpha - 1)\right) = \ln\alpha = R_{\min}.$$

Next, we show that for any $\epsilon > 0$, if $m \le \nu(n) - \epsilon$, $\max_{1\le i\le m} N_i \le P/\alpha^m$ holds with probability approaching one as n tends to infinity. Let $N_0 = P/\alpha^m$. Then,

$$\begin{aligned} p_0 = F(N_0) &= \exp\left(-\frac{\alpha^m}{P}\right) \\ &\ge \exp\left(-\frac{\alpha^{\nu(n)-\epsilon}}{P}\right) \\ &= \exp\left(-\alpha^{-\epsilon}\ln n\right) = n^{-\lambda}, \end{aligned}$$

where $\lambda = \alpha^{-\epsilon} < 1$. Since $m \leq \nu(n) - \epsilon$ and $np_0 \geq n^{1-\lambda}$, it can be seen that as $n \to \infty$,

$$\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n} \sim \frac{n^2 p_0^2}{2np_0} = \frac{np_0}{2} \geq \frac{n^{1-\lambda}}{2} \to \infty. \tag{2.12}$$

As $m - 1 \leq np_0$, the Chernoff bound for independent Poisson trials can be used as [27, page 70]:

$$\mathbb{P}(M_n \leq m - 1) \leq \exp\left(-\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n}\right).$$

or

$$\mathbb{P}(M_n \geq m) \geq 1 - \exp\left(-\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n}\right). \tag{2.13}$$

Hence, by (2.13), $\max_{1 \leq i \leq m} N_i \leq P/\alpha^m$ with probability approaching one as $n \to \infty$.

Therefore, we proved that as $n \to \infty$, with probability approaching one, there are at least $M_n = \lfloor \nu(n) - \epsilon \rfloor$ good channels satisfying the minimum rate.

Next, we prove the upper bound; in other words, $M_n \leq \nu(n) + \epsilon$ holds with probability approaching one. First, we show that for any $\delta > 0$, for sufficiently large m, the best receiver should have the equivalent noise variance $N_1 \leq P_\delta/\alpha^m$, with $P_\delta \triangleq P + \delta$. Otherwise, if $\min_{1 \leq i \leq n} N_i > P_\delta/\alpha^m$, by the minimum-rate constraint,

$$\frac{P_i}{\sum_{j=1}^{i-1} P_j + N_i} \geq \alpha - 1, \quad \text{for } i = 1, 2, \ldots, m,$$

we have

$$P_1 \geq (\alpha - 1)N_1 > (\alpha - 1)P_\delta/\alpha^m,$$

and inductively, for $i = 2, \ldots, m$,

$$\begin{aligned} P_i &\geq (\alpha - 1)\left(\textstyle\sum_{j=1}^{i-1} P_j + N_i\right) \\ &> (\alpha - 1)\left(\textstyle\sum_{j=1}^{i-1}(\alpha - 1)P_\delta/\alpha^{m-j+1} + P_\delta/\alpha^m\right) \\ &= (\alpha - 1)P_\delta/\alpha^{m-i+1}, \end{aligned}$$

which violates the total-power constraint since

$$\sum_{i=1}^{m} P_i > \sum_{i=1}^{m} (\alpha - 1) P_\delta / \alpha^{m-i+1} = (1 - 1/\alpha^m) P_\delta > P$$

for sufficiently large m.

Therefore, to show that

$$\mathbb{P}(M_n \le \nu(n) + \epsilon) \to 1,$$

or

$$\mathbb{P}(M_n > \nu(n) + \epsilon) \to 0,$$

we only need to show that

$$\mathbb{P}(N_1 \le P_\delta / \alpha^{\nu(n)+\epsilon}) \to 0.$$

Let $p_1 = F(P_\delta / \alpha^{\nu(n)+\epsilon})$. Then, $(1 - p_1)^n$ is the probability that all the receivers have equivalent noise variance greater than $P_\delta / \alpha^{\nu(n)+\epsilon}$. Hence,

$$\mathbb{P}(N_1 \le P_\delta / \alpha^{\nu(n)+\epsilon}) = 1 - (1 - p_1)^n, \tag{2.14}$$

which tends to zero if and only if

$$\left(1 - \exp\left(-\frac{\alpha^{\nu(n)+\epsilon}}{P_\delta}\right)\right)^n \to 1. \tag{2.15}$$

Since

$$\left(1 - \exp\left(-\frac{\alpha^{\nu(n)+\epsilon}}{P_\delta}\right)\right)^{\exp\left(\frac{\alpha^{\nu(n)+\epsilon}}{P_\delta}\right)} \to e^{-1},$$

(2.15) holds if

$$n \cdot \exp\left(-\frac{\alpha^{\nu(n)+\epsilon}}{P_\delta}\right) = n \cdot \exp\left(-\frac{P\alpha^\epsilon \ln n}{P + \delta}\right) \to 0, \tag{2.16}$$

which holds by choosing $\delta < (\alpha^{\epsilon} - 1)P$. $\square$

Corollary 2.5.1 The lower and upper tail distribution in (2.10) are given by

$$\mathbb{P}(M_n < \lfloor \nu(n) - \epsilon \rfloor) = o\left(\exp\left(-\frac{n^{1-\lambda}}{2+\tilde{\sigma}}\right)\right) \tag{2.17}$$

and

$$\mathbb{P}(M_n > \nu(n) + \epsilon) = o\left(n^{1-\frac{1}{\lambda(1+\tilde{\sigma})}}\right) \tag{2.18}$$

where $\lambda \stackrel{\Delta}{=} e^{-\epsilon R_{\min}} < 1$, and $\tilde{\sigma} > 0$ can be arbitrarily small.

Proof 2.5.2 Following the proof of Theorem 2.5.1, especially noting (2.13), to prove (2.17), we only need to show that for $m = \lfloor \nu(n) - \epsilon \rfloor$,

$$\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n} \geq \frac{n^{1-\lambda}}{2+\tilde{\sigma}}, \text{ for sufficiently large } n,$$

which actually follows from (2.12) with the following modification

$$\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n} \geq \frac{n^2 p_0^2}{(2+\tilde{\sigma})np_0}, \text{ for sufficiently large } n.$$

To prove (2.18), noting (2.14), we have

$$\begin{aligned}
\mathbb{P}(M_n > \nu(n) + \epsilon) &\leq \mathbb{P}(N_1 \leq P_\delta / \alpha^{\nu(n)+\epsilon}) \\
&= 1 - \left(1 - \exp\left(-\frac{\alpha^{\nu(n)+\epsilon}}{P_\delta}\right)\right)^n \\
&= O\left(n \cdot \exp\left(-\frac{\alpha^{\nu(n)+\epsilon}}{P_\delta}\right)\right) \\
&= O\left(n \cdot \exp\left(-\frac{P \ln n}{\lambda(P+\delta)}\right)\right) \\
&= o\left(n^{1-\frac{1}{\lambda(1+\tilde{\sigma})}}\right),
\end{aligned}$$

where, $\tilde{\sigma} > 0$ can be arbitrarily small, since $\delta > 0$ can be arbitrarily small. $\square$

Remark 2.5.1 Theorem 2.5.1 states that the number of active receivers is close to $\nu(n)$ with high probability. Actually, for any $\epsilon < \frac{1}{2}$, there are at most two integers during the range $\lfloor \nu(n) - \epsilon \rfloor \leq m \leq \nu(n) + \epsilon$. An interesting observation of the equation (2.11) is that the number of active receivers will almost double by halving $R_{\min}$, with the total power P and the total number of receivers n fixed.

Basically, Theorem 2.5.1 states a double logarithmic scaling law. That is, the maximum number of active receivers scales double logarithmically with the total number of receivers. This is a rather slow scaling, and is basically determined by the tail of the Rayleigh distribution. Comparatively, (2.11) can also be written as

$$\nu(n) = (\ln P + \ln \ln n)/R_{\min}$$

which shows that the maximum number of active receivers scales logarithmically with the total transmit power, and as remarked before, is inversely proportional to the minimum-rate constraint.

Remark 2.5.2 According to Theorem 2.5.1, there are about $\nu(n)$ active receivers, for each of which, a minimum rate $R_{\min}$ can be achieved. Hence, the total throughput scales at least as

$$\nu(n) R_{\min} = \ln(P \ln n). \tag{2.19}$$

It is interesting to compare (2.19) with the maximum achievable total throughput when all the power is allocated to the best receiver, which can be shown to be upper bounded with probability approaching one by

$$\ln(1 + \beta P \ln n) \tag{2.20}$$

where the constant $\beta > 1$ can be arbitrarily close to one.

Proof 2.5.3 First, it follows from (2.14)-(2.16) that for any $0 < \delta < (\alpha^{\epsilon} - 1)P$

$$\mathbb{P}(N_1 \leq P_\delta / \alpha^{\nu(n)+\epsilon}) \to 0.$$

Hence,

$$\mathbb{P}(N_1 > P_\delta/\alpha^{\nu(n)+\epsilon}) \to 1.$$

Since

$$P_\delta/\alpha^{\nu(n)+\epsilon} = (P+\delta)/\alpha^{\nu(n)+\epsilon} = (\alpha^\epsilon P - \eta)/\alpha^{\nu(n)+\epsilon}$$

where $\eta = (\alpha^\epsilon - 1)P - \delta > 0$ can be arbitrarily small, the maximum achievable total throughput is upper bounded with probability approaching one as

$$\begin{aligned}
\ln\left(1+\frac{P}{N_1}\right) &< \ln\left(1+\frac{P}{P_\delta/\alpha^{\nu(n)+\epsilon}}\right) && (2.21)\\
&= \ln\left(1+\frac{P\alpha^{\nu(n)+\epsilon}}{\alpha^\epsilon P-\eta}\right) && (2.22)\\
&= \ln\left(1+\beta\alpha^{\nu(n)}\right) && (2.23)\\
&= \ln\left(1+\beta P\ln n\right) && (2.24)
\end{aligned}$$

where $\beta = \frac{\alpha^\epsilon P}{\alpha^\epsilon P - \eta} > 1$ can be arbitrarily close to one. □

Clearly, as n increases, the difference between (2.19) and (2.20) decreases to $\ln\beta$, which can be made arbitrarily small. The essential reason for such a negligible difference is that for large n, the gains of the best $\nu(n)$ receivers are very close to each other. It should also be pointed out that the smaller $\ln\beta$ is, the slower the probability converges to one, as can be seen from the proof.

Besides Rayleigh fading, one can also consider Rician and Nakagami fading models. The analytic techniques developed for the Rayleigh distribution can be similarly applied.

2.5.2 Rician Fading

Consider independent Rician fading channels for different receivers; in other words, channel gains g_i, $i = 1, \ldots, n$ are independent realizations of the complex Gaussian distribution $\mathcal{CN}(\mu, 2)$.

Theorem 2.5.2 Under the assumption of independent Rician fading channels for different receivers with channel gains $|g_i| \sim Rice(1, \mu)$, $i = 1, \ldots, n$ and for any $\epsilon > 0$, the maximum number of active receivers M_n is bounded as:

$$\mathbb{P}(\lfloor \nu_1(n) - \epsilon \rfloor \leq M_n \leq \nu_1(n) + \epsilon) \to 1, \quad \text{as } n \to \infty, \tag{2.25}$$

where

$$\nu_1(n) \triangleq \ln(2P \ln n)/R_{\min}. \tag{2.26}$$

Remark 2.5.3 In Theorem 2.5.2, the channel gain variance equals two because the resulting distribution (i.e. the non-central Chi-square distribution with two degrees of freedom) is easy to work with; however, this theorem can be easily generalized to any arbitrary variance by normalization.

Proof 2.5.4 Consider the broadcast channel (2.1) with independent gains $g_i \sim \mathcal{CN}(\mu, 2)$, for $i = 1, \cdots, n$; as a result, $|g_i| \sim Rice(1, \mu)$ and $|g_i|^2 \sim \mathcal{NC}\chi_2^2(\mu^2)$ (i.e. non-central Chi-square distribution with two degrees of freedom) with the cumulative distribution function

$$F_{\mathcal{NC}\chi_2^2}(x; 2, \mu^2) = \sum_{j=0}^{\infty} e^{-\mu^2/2} \frac{(\mu^2/2)^j}{j!} \frac{\gamma(j+1, x/2)}{\Gamma(j+1)}$$

where $\Gamma(a)$ and $\gamma(a, x)$ are defined as

$$\begin{aligned} \Gamma(a) &= \int_0^{\infty} t^{a-1} e^{-t} dt \\ \gamma(a, x) &= \int_0^{x} t^{a-1} e^{-t} dt \end{aligned} \tag{2.27}$$

and

$$\lim_{x \to \infty} \gamma(a, x) = \lim_{x \to \infty} [\Gamma(a) - \Gamma(a, x)] = \Gamma(a). \tag{2.28}$$

Furthermore, if a is an integer,

$$\Gamma(a,x) = (a-1)!e^{-x}\sum_{k=0}^{a-1}\frac{x^k}{k!} \tag{2.29}$$

For the equivalent model (2.2), the noise variance $N_i = 1/|g_i|^2$ is of the following distribution function:

$$F(y) = \mathbb{P}(N_i < y) = \mathbb{P}(1/|g_i|^2 < y) = \mathbb{P}(|g_i|^2 > 1/y). \tag{2.30}$$

Hence, for any fixed $N_0 > 0$, we can characterize the number of "good" channels with the equivalent noise variance N_i less than N_0 as the following. Let $p_0 = F(N_0)$. Then, a channel is good with probability p_0. Consider a Bernoulli sequence

$$x_i = \begin{cases} 1, & \text{with probability } p_0 \\ 0, & \text{with probability } 1-p_0 \end{cases} \tag{2.31}$$

for $i = 1, 2, \ldots, n$. Then, the number of good channels has the same distribution as $M_n = \sum_{i=1}^{n} x_i$, which satisfies the binomial distribution $B(n, p_0)$.

Now, for any integer m, consider the following power allocation for the m best receivers:

$$P_i = \frac{c}{\alpha^{m-i}}, \text{ for } i = 1, \ldots, m, \tag{2.32}$$

where $\alpha = e^{R_{\min}} > 1$, and $c = (1-1/\alpha)P$. As shown in the previous section, using this power allocation in the broadcast channel, the total power and the minimum-rate constraints are satisfied.

Next, we show that for any $\epsilon > 0$, if $m \leq \nu_1(n) - \epsilon$, $\max_{1\leq i\leq m} N_i \leq P/\alpha^m$ holds with probability approaching one as n tends to infinity. Let $N_0 = P/\alpha^m$ and $\lambda = \alpha^{-\epsilon} < 1$. Then, using 2.28 and 2.29,

$$\begin{aligned} p_0 &= F(N_0) = 1 - F_{\mathcal{NC}\chi_2^2}(\alpha^m/P; 2, \mu^2) \\ &\geq 1 - F_{\mathcal{NC}\chi_2^2}(\alpha^{\nu_1(n)-\epsilon}/P; 2, \mu^2) \end{aligned}$$

$$
\begin{aligned}
&= 1 - e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)}\, \gamma\,(j+1, \lambda \ln n) \\
&= 1 - e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)}\, [\Gamma(j+1) - \Gamma\,(j+1, \lambda \ln n)] \\
&= e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)}\, \Gamma(j+1, \lambda \ln n) \\
&= n^{-\lambda} e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{(\mu^2/2)^j}{j!} \sum_{k=0}^{j} \frac{(\lambda \ln n)^k}{k!} \qquad (2.33)
\end{aligned}
$$

It is clear that as $n \to \infty$,

$$
\begin{aligned}
\frac{1}{2p_0} \frac{(np_0 - m + 1)^2}{n} \sim \frac{n^2 p_0^2}{2np_0} &= \frac{np_0}{2} \qquad (2.34) \\
&\geq \frac{n^{1-\lambda}}{2} e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{(\mu^2/2)^j}{j!} \sum_{k=0}^{j} \frac{(\lambda \ln n)^k}{k!} \to \infty.
\end{aligned}
$$

As $m - 1 \leq np_0$, the Chernoff bound on the sum of Poisson trials can be used as

$$
\mathbb{P}(M_n \geq m) \geq 1 - \exp\left(-\frac{1}{2p_0} \frac{(np_0 - m + 1)^2}{n} \right). \qquad (2.35)
$$

Hence, the probability of $\max_{1 \leq i \leq m} N_i \leq P/\alpha^m$ approaches one as $n \to \infty$.

Therefore, we proved that with probability approaching one, there are at least $M_n = \lfloor \nu_1(n) - \epsilon \rfloor$ good channels for which the minimum-rate constraint is satisfied.

Next, we prove $M_n \leq \nu_1(n) + \epsilon$ holds with probability approaching one. First, for any $\delta > 0$ and for sufficiently large m, the best receiver should have the equivalent noise variance $N_1 \leq P_\delta/\alpha^m$, with $P_\delta := P + \delta$. Otherwise, if $\min_{1 \leq i \leq n} N_i > P_\delta/\alpha^m$, as shown for Rayleigh fading channels, the total power constraint or the minimum-rate constraint is violated.

Therefore, to show that

$$
\mathbb{P}(M_n \leq \nu_1(n) + \epsilon) \to 1,
$$

we only need to show that

$$\mathbb{P}(N_1 \leq P_\delta/\alpha^{\nu_1(n)+\epsilon}) \to 0.$$

Let $p_1 = F(P_\delta/\alpha^{\nu_1(n)+\epsilon})$. Then, $(1-p_1)^n$ is the probability that all the receivers have equivalent noise variance greater than $P_\delta/\alpha^{\nu_1(n)+\epsilon}$. Hence,

$$\mathbb{P}(N_1 \leq P_\delta/\alpha^{\nu_1(n)+\epsilon}) = 1 - (1-p_1)^n, \tag{2.36}$$

which tends to zero if and only if

$$\begin{aligned}
\left(1 - F(P_\delta/\alpha^{\nu_1(n)+\epsilon})\right)^n &= F_{\mathcal{NC}\chi_2^2}(\alpha^{\nu_1(n)+\epsilon}/P_\delta; 2, \mu^2)^n \\
&= \left(1 - e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)} \Gamma\left(j+1, \frac{P \ln n}{\lambda P_\delta}\right)\right)^n \\
&= (1-h(n))^n \to 1.
\end{aligned}$$

As $n \to \infty$, $h(n) \to 0$ and $(1-h(n))^{h^{-1}(n)} \to e^{-1}$. Hence, (2.36) tends to zero if

$$\begin{aligned}
n\,h(n) &= n e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)} \Gamma\left(j+1, \frac{P \ln n}{\lambda P_\delta}\right) \qquad (2.37) \\
&= n e^{-\mu^2/2} \left(\sum_{j=0}^{c \ln n} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)} \Gamma\left(j+1, \frac{P \ln n}{\lambda P_\delta}\right) \right. \\
&\quad \left. + \sum_{j=c \ln n}^{\infty} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)} \Gamma\left(j+1, \frac{P \ln n}{\lambda P_\delta}\right) \right) \to 0.
\end{aligned}$$

where $c < \frac{P}{\lambda P_\delta}$ is selected such that the following expansion of the incomplete gamma function for large x can be applied to the first summation [28, page 263].

$$\Gamma(a,x) \sim e^{-x} x^{a-1} \left(1 + \frac{a-1}{x} + \frac{(a-1)(a-2)}{x^2} + \cdots \right) \tag{2.38}$$

Hence, using $j! \geq (j/2)^{j/2}$ and defining $j_0 = 2\left(\frac{\mu^2 P}{P - \lambda P_\delta}\right)^2$, the first summation in

(2.37),

$$
\begin{aligned}
n\,h_1(n) &= ne^{-\mu^2/2}\sum_{j=0}^{c\ln n}\frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)}\,\Gamma\left(j+1,\frac{P\ln n}{\lambda P_\delta}\right) \\
&\sim n^{1-\frac{P}{\lambda P_\delta}}e^{-\mu^2/2}\sum_{j=0}^{c\ln n}\frac{(\frac{\mu^2 P\ln n}{2\lambda P_\delta})^j}{j!\,j!} \\
&\le n^{1-\frac{P}{\lambda P_\delta}}e^{-\mu^2/2}\sum_{j=0}^{j_0}\frac{\left(\frac{\mu^2 P\ln n}{2\lambda P_\delta}\right)^j}{\left(\sqrt{\frac{j}{2}}\right)^j j!} + n^{1-\frac{P}{\lambda P_\delta}}e^{-\mu^2/2}\sum_{j=j_0}^{\infty}\frac{\left(\frac{\mu^2 P\ln n}{2\lambda P_\delta}\right)^j}{\left(\sqrt{\frac{j_0}{2}}\right)^j j!} \\
&\sim n^{1-\frac{P}{\lambda P_\delta}}e^{-\mu^2/2}\sum_{j=0}^{\infty}\frac{\left(\frac{(P-\lambda P_\delta)\ln n}{2\lambda P_\delta}\right)^j}{j!} \\
&= e^{-\mu^2/2}\,n^{\frac{1}{2}(1-\frac{P}{\lambda P_\delta})}\to 0 \qquad (2.39)
\end{aligned}
$$

which holds by choosing $\delta < (\alpha^\epsilon - 1)P$. Using Stirling's approximation for sufficiently large $c\ln n$, the second summation in (2.37),

$$
\begin{aligned}
n\,h_2(n) &= ne^{-\mu^2/2}\sum_{j=c\ln n}^{\infty}\frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)}\,\Gamma\left(j+1,\frac{P\ln n}{\lambda P_\delta}\right) \\
&= n^{1-\frac{P}{\lambda P_\delta}}e^{-\mu^2/2}\sum_{j=c\ln n}^{\infty}\frac{(\mu^2/2)^j}{j!}\sum_{k=0}^{j}\frac{\left(\frac{P\ln n}{\lambda P_\delta}\right)^k}{k!} \\
&\sim ne^{-\mu^2/2}\sum_{j=c\ln n}^{\infty}\frac{(\mu^2/2)^j}{j!} \\
&\sim ne^{-\mu^2/2}\sum_{j=c\ln n}^{\infty}\frac{1}{j^j} \\
&\le ne^{-\mu^2/2}\int_{c\ln n}^{\infty}\frac{dx}{x^x} \\
&\le ne^{-\mu^2/2}\frac{1}{(c\ln n)^{(c\ln n-2)}}\int_{c\ln n}^{\infty}\frac{dx}{x^2} \\
&= ne^{-\mu^2/2}\frac{c\ln n}{(c\ln n)^{(c\ln n)}} \\
&\sim ne^{-\mu^2/2}\frac{1}{(c\ln n)^{(c\ln n)}}\,. \qquad (2.40)
\end{aligned}
$$

which tends to zero since

$$\frac{\ln n}{(c\ln n)\ln(c\ln n)} \to 0. \tag{2.41}$$

Hence, according to (2.39) and (2.40), (2.37) holds by choosing $\delta < (\alpha^{\epsilon} - 1)P$. $\square$

2.5.3 Nakagami Fading

The results obtained for Rayleigh fading channels can be also extended to Nakagami fading channels with a constant shift which is a function of the minimum rate and distribution parameters. Consider independent Nakagami fading channels for different receivers.

Theorem 2.5.3 Under the assumption of independent Nakagami fading channels for different receivers with channel gains $|g_i| \sim Nakagami(\mu, \omega)$, $i = 1, \ldots, n$ and for any $\epsilon > 0$, the maximum number of active receivers M_n is bounded as:

$$\mathbb{P}(\lfloor \nu_2(n) - \epsilon \rfloor \leq M_n \leq \nu_2(n) + \epsilon) \to 1, \quad \text{as } n \to \infty, \tag{2.42}$$

where

$$\nu_2(n) \triangleq \ln(\frac{\omega}{\mu} P \ln n)/R_{\min}. \tag{2.43}$$

Proof 2.5.5 In Nakagami fading channels, the cumulative distribution function of $|g_i|^2$ is given by

$$F(x; \mu, \omega) = \frac{\gamma(\mu, \frac{\mu}{\omega}x)}{\Gamma(\mu)}$$

where μ denotes the shape parameter and ω controls distribution spread.

Defining Bernoulli random variable (2.31) and using power allocation (2.32), as shown for Rayleigh fading, the sum-power and minimum-rate constraints are satisfied. Now, we show that for any $\epsilon > 0$ and integer m, if $m \leq \nu_2(n) - \epsilon$, $\max_{1\leq i\leq m} N_i \leq P/\alpha^m$ holds with probability approaching one. Let $N_0 = P/\alpha^m$.

Using (2.38),

$$
\begin{aligned}
p_0 = 1 - F(\alpha^m/P;\mu,\omega) &= 1 - \frac{\gamma(\mu, \frac{\mu\alpha^m}{\omega P})}{\Gamma(\mu)} \\
&\geq 1 - \frac{\gamma(\mu, \frac{\mu\alpha^{\nu_2(n)-\epsilon}}{\omega P})}{\Gamma(\mu)} \\
&= \frac{\Gamma(\mu, \lambda \ln n)}{\Gamma(\mu)} \\
&\sim \frac{n^{-\lambda}(\lambda \ln n)^{\mu-1}}{\Gamma(\mu)} \qquad (2.44)
\end{aligned}
$$

Based on the fact $\ln n = o(n^\epsilon)$ for any $\epsilon > 0$, it is clear that as $n \to \infty$,

$$
\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n} \sim \frac{np_0}{2} \geq \frac{n^{1-\lambda}(\lambda \ln n)^{\mu-1}}{2\,\Gamma(\mu)} \to \infty. \qquad (2.45)
$$

As $m - 1 \leq np_0$, the Chernoff bound can be used as

$$
\mathbb{P}(M_n \geq m) \geq 1 - \exp\left(-\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n}\right). \qquad (2.46)
$$

Therefore, we proved that with probability approaching one, there are at least $M_n = \lfloor \nu_2(n) - \epsilon \rfloor$ good channels for which the minimum-rate constraint is satisfied.

Next, we prove the upper bound; in other words, $M_n \leq \nu_2(n) + \epsilon$ holds with probability approaching one. The same as Rayleigh fading channels, to show that

$$
\mathbb{P}(M_n \leq \nu_2(n) + \epsilon) \to 1,
$$

we only need to show that

$$
\mathbb{P}(N_1 \leq P_\delta/\alpha^{\nu_2(n)+\epsilon}) \to 0.
$$

Let $p_1 = F(P_\delta/\alpha^{\nu_2(n)+\epsilon})$. Then, $(1-p_1)^n$ is the probability that all the receivers have equivalent noise variance greater than $P_\delta/\alpha^{\nu_2(n)+\epsilon}$. Hence,

$$
\mathbb{P}(N_1 \leq P_\delta/\alpha^{\nu_2(n)+\epsilon}) = 1 - (1 - p_1)^n, \qquad (2.47)
$$

which tends to zero if and only if

$$\begin{aligned}
\left(1 - F(P_\delta/\alpha^{\nu_2(n)+\epsilon})\right)^n &= F(\alpha^{\nu_2(n)+\epsilon}/P_\delta;\mu,\omega)^n \\
&= \left(1 - \frac{\Gamma(\mu, \frac{P}{\lambda P_\delta}\ln n)}{\Gamma(\mu)}\right)^n \\
&= \left(1 - \tilde{h}(n)\right)^n \to 1. \qquad (2.48)
\end{aligned}$$

Using (2.38), (2.48) holds if

$$\begin{aligned}
n\tilde{h}(n) &= n\frac{\Gamma(\mu, \frac{P}{\lambda P_\delta}\ln n)}{\Gamma(\mu)} \\
&\sim n^{1-\frac{P}{\lambda P_\delta}} \frac{\left(\frac{P}{\lambda P_\delta}\ln n\right)^{\mu-1}}{\Gamma(\mu)} \to 0 \qquad (2.49)
\end{aligned}$$

which holds by choosing $\delta < (\alpha^\epsilon - 1)P$. □

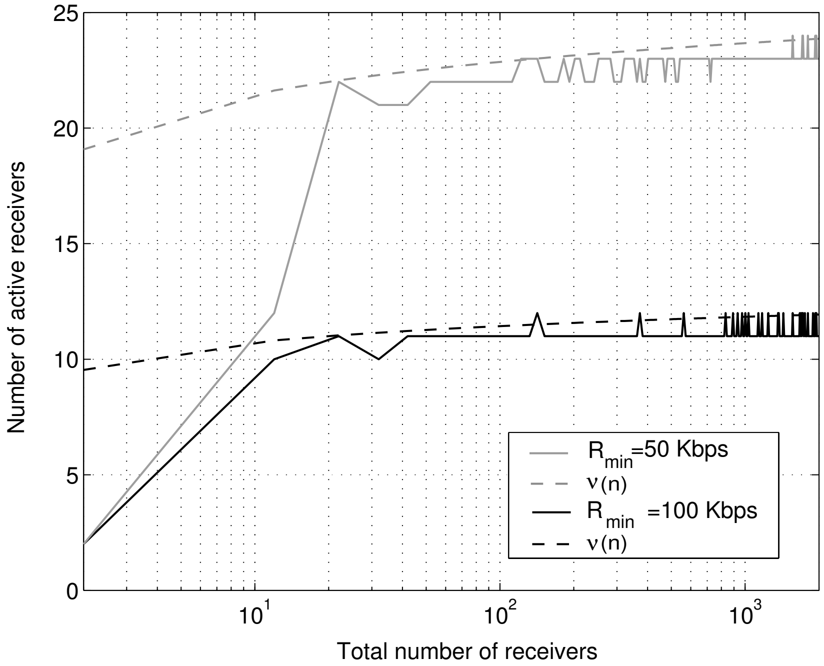
Remark 2.5.4 From Theorems 2.5.2 and 2.5.3, it can be seen that the total throughput scales at least as $\ln\ln n$. This result can be compared to the broadcast channels sum capacity upper-bounded as $\ln(\beta \ln n)$ where the constant $\beta > 1$ can be arbitrarily close to one. The proof is identical to the one for remark 2.5.2. Clearly, as n increases, the difference decreases to $\ln\beta$, which can be made arbitrarily small. That is, a set of rates arbitrary close to the boundary of the capacity region can be achieved. It should thus be noted that the total throughput scaling laws are the same for Rayleigh, Rician, and Nakagami distributions modulo some constants that depend on the distributions. The reason is that the scaling law only depends on the distribution tail which decays exponentially.

2.6 Simulation Results

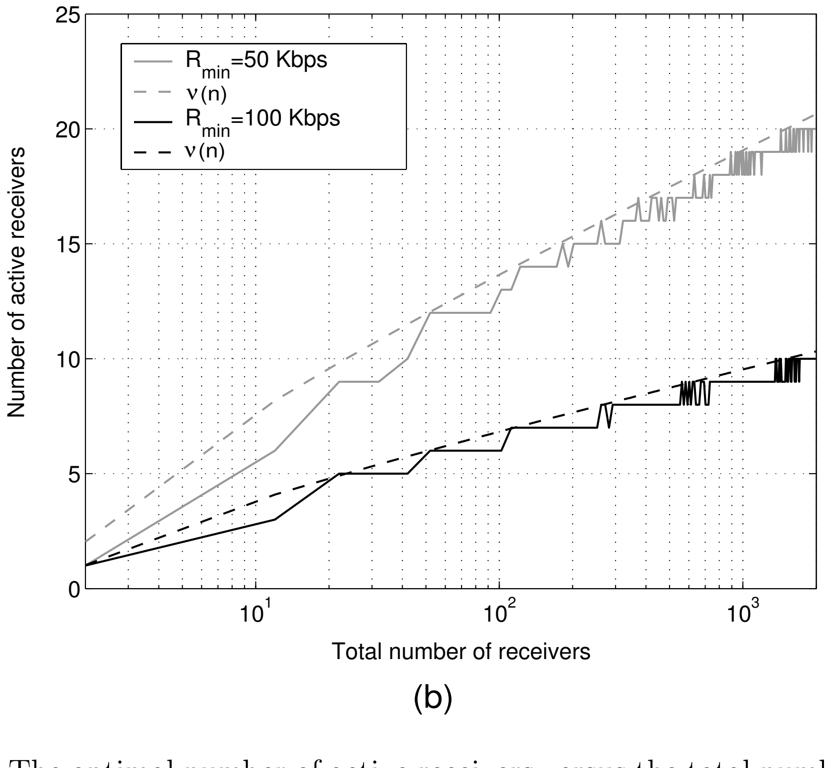
Consider a system with noise variance $\sigma^2 = 1$, and channel bandwidth $B = 50K$ samples/second. Then, a transmission rate of $100K$ bits per second is equivalent to $100K/2B = 1$ bits per sample. For Rayleigh fading channels, $|h_i|^2 \sim \mathcal{NC}\chi_2^2(\mu^2)$, and for Rician fading channels, $|h_i|^2 \sim$ Exponential(1).

Figure 2.2 shows the optimal number of active receivers versus the total number of users for both fixed ($P = 10^4$, or equivalently, SNR$=$ 40 dB for model (2.1)) and linearly increasing ($P = n$, or equivalently, SNR$=$ $10 \log_{10} n$ dB) transmit power. The value of $\nu(n)$ given by (2.11) is also indicated in Figure 2.2. As shown in Figure 2.2 and mentioned in Remark 2.5.1, the number of active receivers is almost doubled as $R_{\min}$ is halved. For further illustration, Figure 2.3 shows that as the total number of users increases, the estimate of the number of active receivers is sharply concentrated around the theoretical value. Figure 2.3 is sketched by 10000 simulation runs. The optimal number of active receivers versus different $R_{\min}$ for fixed SNR$=$ 40dB and $n = 1000$ is shown in Figure 2.4, where the curve of $\nu(n)$ is also drawn.

Figure 2.5 shows the optimal number of active receivers versus the total number of receivers for Rician fading broadcast channels with $R_{\min} = 50, 100$ Kbps and SNR$=$ 40 dB at the transmitter. In Figure 2.5.a, $\mu = 2$ and in Figure 2.5.b, $\mu = 0.8$. The value of $\nu_1(n)$ given by (2.26) is also indicated in Figure 2.5. As shown in Figure 2.5, the number of active receivers is almost doubled as $R_{\min}$ is halved. The same relationship also exists for Nakagami fading broadcast channels.

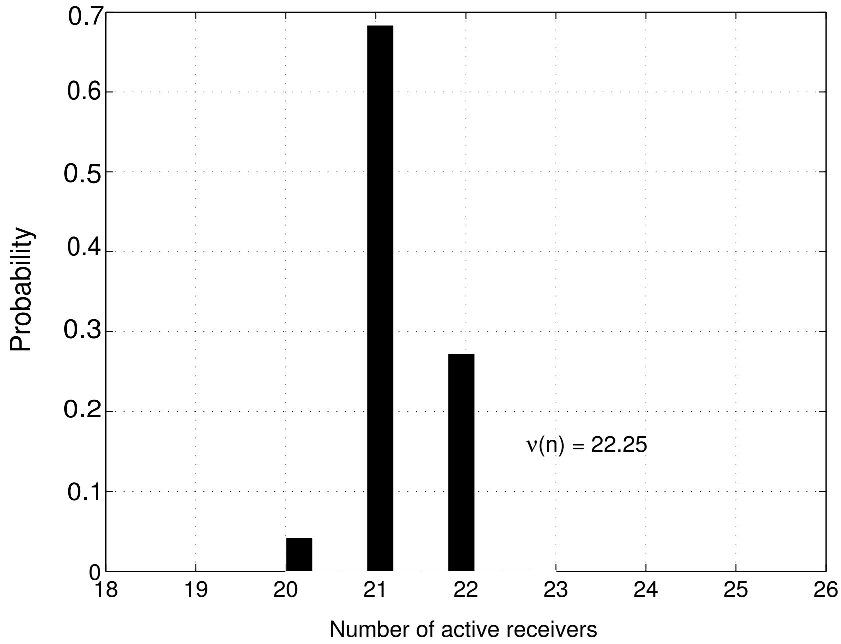


(a)

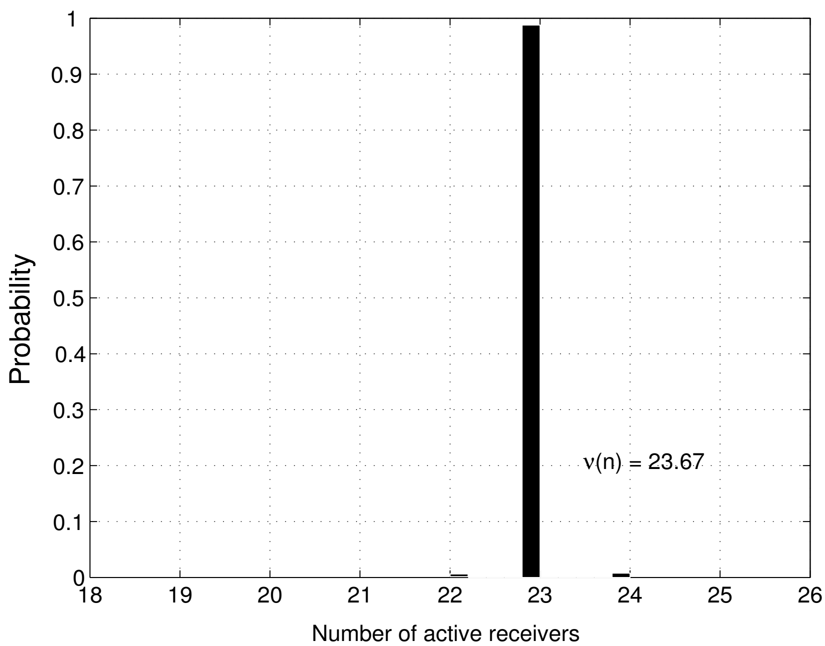


(b)

Figure 2.2: The optimal number of active receivers versus the total number of users for Rayleigh fading broadcast channels and $R_{\min} = 50, 100\,\mathrm{Kbp}$, (a) Fixed total transmit power: $P = 10^4$, or equivalently, $\mathrm{SNR} = 40\,\mathrm{dB}$, (b) Linearly increasing transmit power: $P = n$, or equivalently, $\mathrm{SNR} = 10\log_{10} n\,\mathrm{dB}$.



(a)



(b)

Figure 2.3: The histogram of the number of active receivers for Rayleigh fading, $R_{\min} = 50\,\text{Kbps}$, $\text{SNR} = 40\,\text{dB}$, and (a) $n = 30$, and (b) $n = 1000$.

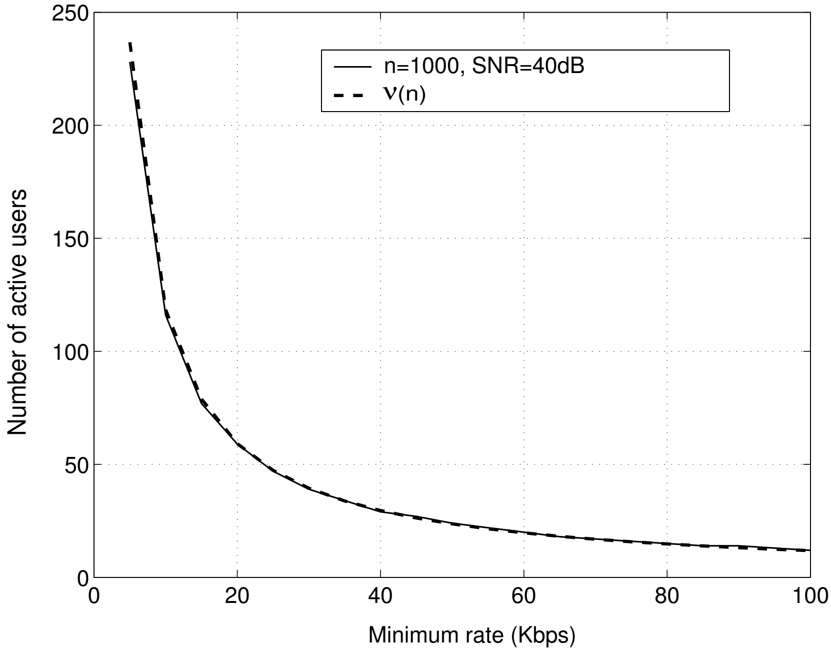
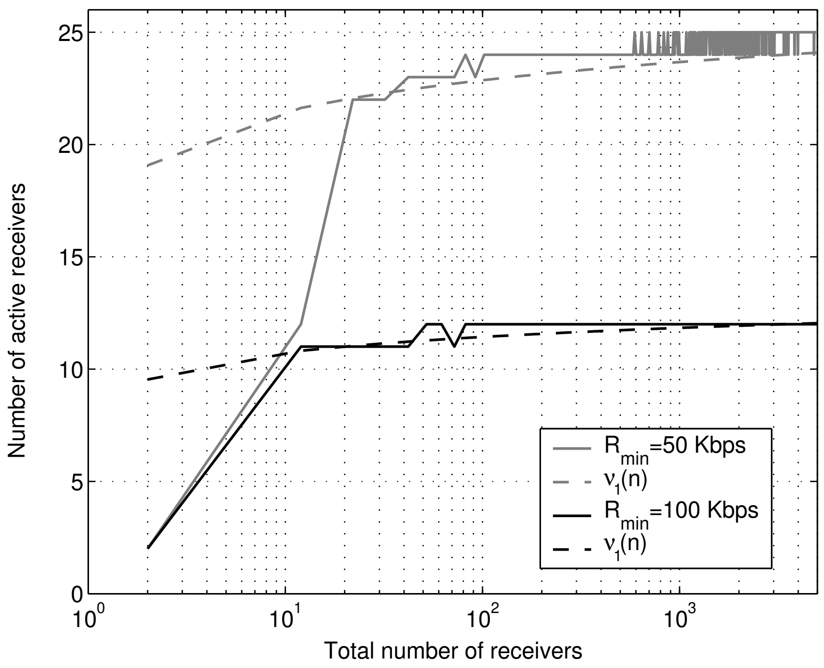
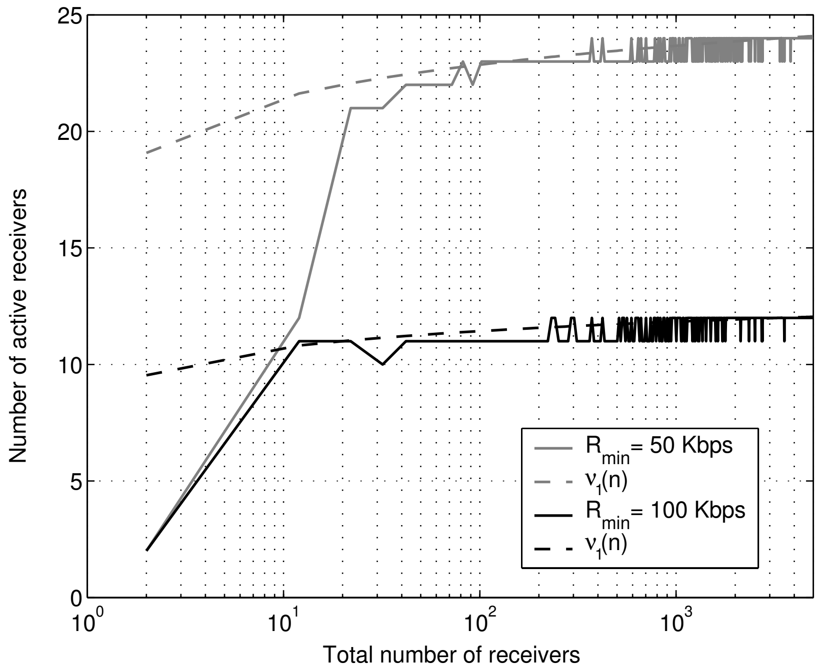


Figure 2.4: The optimal number of active receivers versus the minimum rate for Rayleigh fading, n=1000 and $\mathrm{SNR} = 40\,\mathrm{dB}$.



(a)



(b)

Figure 2.5: The optimal number of active receivers versus the total number of receivers for Rician fading broadcast channels with $R_{\min} = 50, 100\,\text{Kbps}$, $\text{SNR} = 40\,\text{dB}$ at the transmitter, (a) $\mu = 2$, and (b) $\mu = 0.8$.

Chapter 3

User Capacity of Rate-Constrained Multiple-Access Channels

In a wireless environment, channel gains vary dynamically and users experience different fading conditions. As a result, some users have high channel gains while other users experience poor channel conditions. Delay-sensitive applications such as video and voice need users to maintain a minimum rate. Due to limited transmission power, it is not always possible for all users to maintain a minimum rate. A reasonable strategy is to allow users with good channel conditions to be active while others remain silent during each time slot. This is often referred to as opportunistic scheduling. In a multiple-access channel, it is desirable to have an opportunistic scheduling policy that maximizes the number of active transmitters satisfying the minimum-rate constraint. In this chapter, the results presented in Chapter 2 for broadcast channels are extended to fading multiple-access channels.

3.1 Literature Review

A multiple-access channel consists of several transmitters communicating to a single receiver. The capacity region and optimal scheduling schemes for single-antenna or multi-antenna multiple-access channels are well studied. The properties of the capacity region in the case of deterministic time-invariant multiple-access channels with additive white Gaussian noise was initially given in [29], where it was shown that the solution has an interesting interpretation as a multi-user water-filling method. The extension to random channels was then derived in [1], whereas the generalization to MIMO flat-fading channels was given in [30].

In rate-constrained multiple-access channels, all active transmitters maintain a minimum rate. The results presented in [6] for broadcast channels with minimum rates are extended to multiple-access channels in [31] using the duality (see [32]) of broadcast and multiple-access channels. Precisely, the minimum-rate capacity region, optimal power allocation, and the optimal decoding order are obtained in [31] for rate-constrained multiple-access channels. Some papers addressing single-antenna or multi-antenna multi-user systems with minimum-rate constraints in the uplink are as follows: In [33], optimal power and rate allocation policies that maximize the weighted sum rate while satisfying the minimum-rate and average-power constraints are obtained for fading multiple-access channels. The highest allocated rate corresponds to the user having the highest weight and channel gain. In [34], a method is proposed to compute each user's power and codes for wideband multiple-access channels in order to maximize the rates of all the users, under the constraint of maximum total (rather then individual) available power, guaranteeing a desired rate profile. In [34], it is shown that under which conditions this optimization problem admits a unique set of rates. Then, a simple iterative strategy is proposed to compute the capacity region under the constraint that total power in the network is bounded, but each user can adapt its power. In [35], the problem of transceiver design with individual rate constraints is investigated for multi-user MIMO systems. Linear processing with two design goals is considered: one is to maximize the minimum rate per user under a total-power constraint, and the other is to minimize total transmit power while maintaining certain rate requirements. The optimization is carried out in an alternating manner in both virtual uplink and downlink channels in [35]. Each iteration contains the optimization of uplink power allocation, and uplink and downlink Minimum Mean Square Estimation (MMSE) receive filters.

3.2 Motivation and Objective

In multiple-access channels, the total throughput increases by the number of active transmitters. As in the downlink case, although delay-sensitive applications need transmitters to maintain a minimum rate, it is not always possible for all users to keep this minimum rate due to limited transmission power.

In this chapter, user capacity of fading multiple-access channels in which a minimum rate must be maintained for all active transmitters is asymptotically analyzed. The joint decoding scheme is used at the receiver since it is well known that this decoding scheme maximizes the total throughput. In this case, messages sent by all transmitters are simultaneously decoded at the receiver. Note that because of joint decoding at the receiver and individual power constraints at each transmitter, the duality of broadcast and multiple-access channels can not be simply used to extend the results of chapter 2 to the uplink, although the asymptotic results follow the same scaling law. Three fading distributions, namely, Rayleigh, Rician, and Nakagami are considered. While the number of active transmitters in each slot depends on the specific channel states, the asymptotic behavior can be precisely characterized when the total number of transmitters n is large.

The rest of this chapter is organized as follows: In Section 3.3, the system model is introduced. In Section 3.4, the scheduling policy maximizing the number of active transmitters is proposed. Section 3.5 presents the asymptotic analysis of the user capacity for Rayleigh, Rician, and Nakagami fading channels. In Section 3.6, the effect of path loss on the user capacity scaling law is analyzed. Section 3.7 presents implementation issues. Finally, simulation results are shown in Section 3.8.

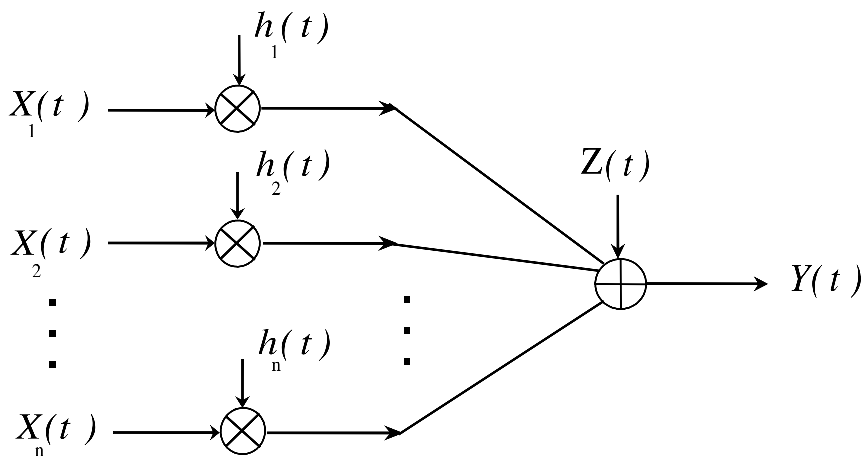


Figure 3.1: A fading multiple-access channel

3.3 System Model

Consider a multiple-access channel shown in Figure 3.1 with one receiver and n transmitters. The receiver and all transmitters are each equipped with single antenna and are assumed to know channel state information perfectly. Then, the received signal is represented by

$$Y(t) = \sum_{i=1}^{n} h_i(t)\, X_i(t) + Z(t) \tag{3.1}$$

where $X_i(t)$ denotes the ith user's transmitted signal, $Y(t)$ refers to the received signal, $Z(t) \sim \mathcal{CN}(0, \sigma^2)$, and $h_i(t)$ denotes the time-varying channel gain of the path from transmitter i to the receiver and t is the time index. Note that $h_i,\ i = 1, \ldots, n$ are assumed to be constant during each time block. Without loss of generality, assume $|h_1| \leq |h_2| \leq \cdots \leq |h_n|$. Joint decoding is exploited at the receiver. This decoding scheme maximizes the sum rate and achieves a set of rates satisfying the following conditions:

$$R_i \;\leq\; \ln\left(1 + \frac{P_i\,|h_i|^2}{\sigma^2}\right); \qquad i = 1, \ldots, n \tag{3.2}$$

$$R_i + R_j \leq \ln\left(1 + \frac{P_i |h_i|^2 + P_j |h_j|^2}{\sigma^2}\right); \qquad i,j = 1,\ldots,n \tag{3.3}$$

$$\vdots$$

$$\sum_{i=1}^{n} R_i \leq \ln\left(1 + \frac{\sum_{i=1}^{n} P_i |h_i|^2}{\sigma^2}\right) \tag{3.4}$$

where R_i and P_i denote the ith user's achievable rate (in nats) and transmitted power respectively.

3.4 Scheduling Policy

As in the case of broadcast channels, to decrease delay in a multiple-access channel, a minimum rate constraint could be considered for all active transmitters. That is, each transmitter maintains a minimum rate or remains silent during each time slot. Due to time-varying channel states and limited transmitted power, it is not always possible for all transmitters to keep minimum rate R_{min}. Hence, the following scheduling policy is proposed to maximize the number of active transmitters.

$$\max\{m\} \tag{3.5}$$

$$\text{subject to} \quad R_i \geq R_{\min}, \quad i = n-m+1,\ldots,n \tag{3.6}$$

$$P_i = P, \quad i = n-m+1,\ldots,n. \tag{3.7}$$

That is, users with high channel gains are allowed to transmit data while other transmitters are inactive. As messages sent by all active transmitters are decoded simultaneously, each user's signal is not affected by interference from other active transmitters. Hence, all users are allowed to transmit data with maximum power. For simplicity, it is assumed that all transmitters have the same power constraint; however, different individual power constraints can be considered without much difficulty.

With fixed P and $R_{\min}$, the maximum number of active transmitters completely depends on the channel gains h_i, $i = 1,\ldots,n$. In general, these are not known a priori since they are random. However, when the distribution of channel gains is known, the asymptotic behavior of the maximum number of active users can be obtained when the total number of transmitters is large enough. This asymptotic behavior is determined even without knowledge of the exact channel gains for each transmitter, although channel gains are required for scheduling active transmitters.

3.5 Asymptotic Analysis

Let M_n denote the maximum number of simultaneous active transmitters (out of n transmitters) that can be supported with a rate greater than or equal to R_{min}.

3.5.1 Rayleigh Fading

Consider independent Rayleigh fading channels for different transmitters; in other words, the channel gains h_i, $i = 1, \dots, n$ are independent realizations of the complex Gaussian distribution; as a result, $|h_i|^2$, $i = 1, \dots, n$ are independent realizations of the exponential distribution.

Theorem 3.5.1 Under the assumption of independent Rayleigh fading channels for different transmitters with channel gains $h_i \sim \mathcal{CN}(0, 1)$, $i = 1, \dots, n$, for any $\epsilon > 0$, the maximum number of active transmitters, M_n, satisfies

$$\mathbb{P}\left(\lfloor \tilde{\nu}(n) - \epsilon \rfloor \ \leq M_n \leq \tilde{\nu}(n) + \epsilon\right) \to 1, \quad \text{as } n \to \infty, \tag{3.8}$$

where n is the total number of transmitters, and

$$\tilde{\nu}(n) \;\; = \;\; \frac{1}{R_{min}} \ln\left(\frac{P\tilde{\nu}(n)}{\sigma^2} \ln n\right). \tag{3.9}$$

Remark 3.5.1 In Theorem 3.5.1, the channel gain variance equals one; however, this result can be easily generalized to any arbitrary variance by normalization. That is, dividing the channel gain by its variance results in a channel gain with the normal distribution.

Proof 3.5.1 Consider the multiple-access channel (3.1) with independent channel gains $h_i \sim \mathcal{CN}(0,1)$, for $i = 1, \ldots, n$; as a result, $|h_i|^2 \sim$ Exponential(1). For any fixed $h_0 > 0$, we can characterize the number of "good" channels with $|h_i|^2$ greater than h_0 as the following. Let $p_0 = 1 - \mathbb{P}(|h_i|^2 \leq h_0) = e^{-h_0}$. That is, with probability p_0, a channel is good (i.e. a user can be activated). Consider a Bernoulli sequence:

$$x_i = \begin{cases} 1, & \text{with probability } p_0 \\ 0, & \text{with probability } 1 - p_0 \end{cases} \tag{3.10}$$

for $i = 1, 2, \ldots, n$. Then, the number of transmitters having good channels has the same distribution as $M_n = \sum_{i=1}^{n} x_i$, which satisfies the Binomial distribution $B(n, p_0)$.

For any large integer m, if $\min_{n-m+1 \leq i \leq n} |h_i|^2 \geq \sigma^2 e^{mR_{min}}/(mP)$,

$$\begin{aligned} \ln\left(1 + \frac{P|h_i|^2}{\sigma^2}\right) &\geq \ln\left(1 + \frac{e^{mR_{min}}}{m}\right) \geq R_{min} \quad ; \quad i = 1, \cdots, m \\ \ln\left(1 + \frac{P|h_i|^2 + P|h_j|^2}{\sigma^2}\right) &\geq \ln\left(1 + 2\frac{e^{mR_{min}}}{m}\right) \geq 2R_{min} \quad ; \quad i, j = 1, \cdots, m \\ &\vdots \\ \ln\left(1 + \frac{P\sum_{i=1}^{m} |h_i|^2}{\sigma^2}\right) &\geq \ln\left(1 + e^{mR_{min}}\right) \geq mR_{min}. \end{aligned} \tag{3.11}$$

Clearly, based on (3.2)-(3.4), the minimum-rate constraint is satisfied for all m active transmitters.

Next, we show that if $m \leq \tilde{\nu}(n) - \epsilon$, $|h_{n-m+1}|^2 \geq \sigma^2 e^{mR_{min}}/(mP)$ holds with probability approaching one. Let $h_0 = \sigma^2 e^{mR_{min}}/(mP)$. Then,

$$\begin{aligned} p_0 = 1 - \mathbb{P}(|h_i|^2 \leq h_0) = \exp(-h_0) &= \exp\left(-\frac{\sigma^2 e^{mR_{min}}}{mP}\right) \\ &\stackrel{*}{\geq} \exp\left(-\frac{\sigma^2 e^{R_{min}(\tilde{\nu}(n)-\epsilon)}}{P(\tilde{\nu}(n)-\epsilon)}\right) \\ &= \exp\left(-\lambda \ln(n) \frac{\tilde{\nu}(n)}{\tilde{\nu}(n)-\epsilon}\right) \sim n^{-\lambda} \end{aligned} \tag{3.12}$$

where $\lambda = e^{-\epsilon R_{min}} < 1$. Inequality $*$ holds for any sufficiently large m and n. Since $m \leq \tilde{\nu}(n) - \epsilon$ and $np_0 \geq n^{1-\lambda}$, it can be readily seen that as $n \to \infty$,

$$\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n} \sim \frac{n^2 p_0^2}{2np_0} = \frac{np_0}{2} \geq \frac{n^{1-\lambda}}{2} \to \infty. \tag{3.13}$$

As $m - 1 \leq np_0$, using the Chernoff bound on the sum of independent Poisson trials,

$$\mathbb{P}(M_n \geq m) \geq 1 - \exp\left(-\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n}\right). \tag{3.14}$$

Thus, from (3.13), we proved that as $n \to \infty$, with probability approaching one, there are at least $M_n = \lfloor \tilde{\nu}(n) - \epsilon \rfloor$ transmitters for which the minimum rate constraint is satisfied and can be activated.

Next, we prove $M_n \leq \tilde{\nu}(n) + \epsilon$ holds with probability approaching one. Consider a multiple-access channel with m active transmitters. First, we show that for any $\delta > 0$, the best transmitter should have the channel gain $|h_n|^2 \geq h_0' = \sigma^2 e^{mR_{min}}/(mP_\delta)$, with $P_\delta = P + \delta$. Otherwise, if $\max_{n-m+1 \leq i \leq n} |h_i|^2 < h_0'$,

$$\begin{aligned} \ln\left(1 + \frac{\sum_{i=n-m+1}^{n} P|h_i|^2}{\sigma^2}\right) &< \ln\left(1 + \frac{mPh_0'}{\sigma^2}\right) \\ &= \ln\left(1 + \frac{Pe^{mR_{min}}}{P_\delta}\right) \\ &\sim \ln\left(\frac{Pe^{mR_{min}}}{P_\delta}\right) < mR_{min} \end{aligned} \tag{3.15}$$

which violates (3.4). Hence, to show that

$$\mathbb{P}(M_n \leq \tilde{\nu}(n) + \epsilon) \to 1,$$

we only need to show that

$$\mathbb{P}\left(|h_n|^2 \geq \frac{\sigma^2 e^{(\tilde{\nu}(n)+\epsilon)R_{min}}}{P_\delta(\tilde{\nu}(n) + \epsilon)}\right) \to 0.$$

Define $p_1 = 1 - \mathbb{P}(|h_i|^2 \leq h_0') = e^{-h_0'}$ with $h_0' = \sigma^2 e^{(\tilde{\nu}(n)+\epsilon)R_{min}}/(P_\delta(\tilde{\nu}(n) + \epsilon))$.

The probability that all the transmitters have channel gains less than $\acute{h_0}$ equals $(1-p_1)^n$. Hence,

$$\mathbb{P}\left(|h_n|^2 \geq \frac{\sigma^2 e^{(\tilde{\nu}(n)+\epsilon)R_{min}}}{P_\delta(\tilde{\nu}(n)+\epsilon)}\right) = 1-(1-p_1)^n, \tag{3.16}$$

which tends to zero if and only if

$$\left(1-\exp(-\acute{h_0})\right)^n = \left(1-\exp\left(-\frac{\sigma^2 e^{(\tilde{\nu}(n)+\epsilon)R_{min}}}{P_\delta(\tilde{\nu}(n)+\epsilon)}\right)\right)^n \to 1. \tag{3.17}$$

Since

$$\left(1-\exp\left(-\frac{\sigma^2 e^{(\tilde{\nu}(n)+\epsilon)R_{min}}}{P_\delta(\tilde{\nu}(n)+\epsilon)}\right)\right)^{\exp\left(\frac{\sigma^2 e^{(\tilde{\nu}(n)+\epsilon)R_{min}}}{P_\delta(\tilde{\nu}(n)+\epsilon)}\right)} \to e^{-1},$$

(3.17) holds if

$$\begin{aligned} n\cdot\exp\left(-\frac{\sigma^2 e^{(\tilde{\nu}(n)+\epsilon)R_{min}}}{P_\delta(\tilde{\nu}(n)+\epsilon)}\right) &= n\cdot\exp\left(-\frac{P\ln(n)\,\tilde{\nu}(n)}{\lambda P_\delta(\tilde{\nu}(n)+\epsilon)}\right) \\ &\sim n^{1-\frac{P}{\lambda P_\delta}} \to 0 \end{aligned}$$

which holds by choosing $\delta < (e^{\epsilon R_{min}}-1)P$. As $\tilde{\nu}(n)$ is given by a nonlinear fixed-point equation, finding a closed-form expression for this function is complicated. However, $\tilde{\nu}(n)$ can be computed by iterative fixed-point algorithms. $\square$

Remark 3.5.2 In chapter 2, the maximum number of active receivers in broadcast channels with $Z(t) \sim \mathcal{CN}(0,\sigma^2)$ was shown to be given by

$$\frac{1}{R_{min}}\ln\left(\frac{P}{\sigma^2}\ln n\right) \tag{3.18}$$

where P denotes total transmitted power. In multiple-access case with each transmitter having power P, $P\tilde{\nu}(n)$ is total transmitted power of the system. Thus, the result has the the same interpretation as for broadcast channels, since (3.9) can be also written as

$$\frac{1}{R_{min}}\ln\left(\frac{P_{total}}{\sigma^2}\ln n\right) \tag{3.19}$$

where P_{total} denotes total transmitted power in the system.

We can also use the results mentioned in Theorem 3.5.1 to characterize the convergence rate which is of importance in determining how large n should be in order to have an accurate estimate of M_n. These are given by the rate of decay of the upper and lower tail distribution of M_n.

Corollary 3.5.1 The lower and upper tail distribution of M_n satisfy

$$\mathbb{P}(M_n < \lfloor \tilde{\nu}(n) - \epsilon \rfloor) = o\left(\exp\left(-\frac{n^{1-\lambda}}{2+\tilde{\sigma}}\right)\right) \tag{3.20}$$

and

$$\mathbb{P}(M_n > \tilde{\nu}(n) + \epsilon) = o\left(n^{1-\frac{1}{\lambda(1+\tilde{\sigma})}}\right) \tag{3.21}$$

where $\lambda = e^{-\epsilon R_{min}} < 1$, and $\tilde{\sigma} > 0$ can be arbitrarily small.

Proof 3.5.2 To prove (3.20), we only need to show that for $m = \lfloor \tilde{\nu}(n) - \epsilon \rfloor$,

$$\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n} \geq \frac{n^{1-\lambda}}{2+\tilde{\sigma}} \tag{3.22}$$

As $m = o(n)$, for sufficiently large n,

$$\frac{(np_0)^2}{(2+\tilde{\sigma})np_0} \leq \frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n} \leq \frac{(np_0)^2}{2np_0}$$

Hence, with the following modification, (3.22) is proved.

$$\frac{(np_0)^2}{(2+\tilde{\sigma})np_0} = \frac{n^{1-\lambda}}{2+\tilde{\sigma}}$$

To prove (3.21), noting (3.16),

$$\begin{aligned}\mathbb{P}(M_n > \tilde{\nu}(n) + \epsilon) &\leq \mathbb{P}\left(|h_n|^2 \geq \frac{\sigma^2(e^{(\tilde{\nu}(n)+\epsilon)R_{min}})}{P_\delta(\tilde{\nu}(n)+\epsilon)}\right) \\ &= 1 - \left(1 - \exp\left(-\frac{\sigma^2(e^{(\tilde{\nu}(n)+\epsilon)R_{min}})}{P_\delta(\tilde{\nu}(n)+\epsilon)}\right)\right)^n\end{aligned}$$

$$
\begin{aligned}
&= O\left(n \cdot \exp\left(-\frac{P\ln(n)\,\tilde{\nu}(n)}{\lambda P_\delta(\tilde{\nu}(n)+\epsilon)}\right)\right)\\
&= O\left(n^{1-\frac{P}{\lambda P_\delta}}\right)\\
&= o\left(n^{1-\frac{1}{\lambda(1+\tilde{\sigma})}}\right).
\end{aligned}
$$

□

3.5.2 Rician Fading

Consider independent Rician fading channels for different transmitters; in other words, channel gains g_i, $i = 1, \ldots, n$ are independent realizations of the complex Gaussian distribution $\mathcal{CN}(\mu, 2)$.

Theorem 3.5.2 Under the assumption of independent Rician fading channels for different transmitters with channel gains $h_i \sim \mathcal{CN}(\mu, 2)$, $i = 1, \ldots, n$ and for any $\epsilon > 0$, the maximum number of active transmitters, M_n is bounded as

$$
\mathbb{P}(\lfloor \tilde{\nu}_1(n) - \epsilon \rfloor \le M_n \le \tilde{\nu}_1(n) + \epsilon) \to 1, \quad \text{as } n \to \infty, \tag{3.23}
$$

where n is the total number of transmitters, and

$$
\tilde{\nu}_1(n) = \frac{1}{R_{min}} \ln\left(\frac{2P\tilde{\nu}_1(n)}{\sigma^2}\ln n\right). \tag{3.24}
$$

Remark 3.5.3 In Theorem 3.5.2, the channel gain variance equals two because the resulting distribution (i.e. the non-central Chi-square distribution with two degrees of freedom) is easy to work with; however, this theorem can be easily generalized to any arbitrary variance by normalization.

Proof 3.5.3 Consider multiple-access channel (3.1) with independent gains $h_i \sim \mathcal{CN}(\bar{\mu}, 2)$, for $i = 1, \cdots, n$; as a result, $|h_i| \sim Rice(1, \bar{\mu})$ and $|h_i|^2 \sim \mathcal{NC}\chi_2^2(\mu^2)$ (i.e. non-central Chi-square distribution with two degrees of freedom) with the cumulative distribution function

$$F_{\mathcal{NC}\chi_2^2}(x;2,\mu^2) = \sum_{j=0}^{\infty} e^{-\mu^2/2}\frac{(\mu^2/2)^j}{j!}\,\frac{\gamma(j+1,x/2)}{\Gamma(j+1)}$$

where $\gamma(a,x)$ and $\Gamma(a)$ are defined as

$$\begin{aligned}\Gamma(a) &= \int_0^{\infty} t^{a-1}e^{-t}dt \\ \gamma(a,x) &= \int_0^{x} t^{a-1}e^{-t}dt \end{aligned} \tag{3.25}$$

and

$$\lim_{x\to\infty}\gamma(a,x) = \lim_{x\to\infty}\left[\Gamma(a)-\Gamma(a,x)\right] = \Gamma(a). \tag{3.26}$$

Furthermore, if a is an integer,

$$\Gamma(a,x) = (a-1)!e^{-x}\sum_{k=0}^{a-1}\frac{x^k}{k!} \tag{3.27}$$

Defining Bernoulli random variable (3.10), we show that for any $\epsilon > 0$, if $m \leq \tilde{\nu}_1(n) - \epsilon$, $\min_{n-m+1\leq i\leq n}|h_i|^2 \geq \sigma^2 e^{mR_{min}}/(mP)$ holds with probability approaching one. Let $h_0 = \sigma^2 e^{mR_{min}}/(mP)$. Then,

$$\begin{aligned} p_0 = 1-\mathbb{P}(|h_i|^2 \leq h_0) &= 1 - F_{\mathcal{NC}\chi_2^2}(h_0;2,\mu^2) \\ &= 1 - F_{\mathcal{NC}\chi_2^2}(\frac{\sigma^2 e^{mR_{min}}}{mP};2,\mu^2) \\ &\overset{\diamond}{\geq} 1 - F_{\mathcal{NC}\chi_2^2}(\frac{\sigma^2 e^{R_{min}(\tilde{\nu}_1(n)-\epsilon)}}{P(\tilde{\nu}_1(n)-\epsilon)};2,\mu^2) \\ &= 1 - e^{-\mu^2/2}\sum_{j=0}^{\infty}\frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)}\,\gamma\left(j+1,\lambda\ln n\frac{\tilde{\nu}_1(n)}{\tilde{\nu}_1(n)-\epsilon}\right) \\ &\sim 1 - e^{-\mu^2/2}\sum_{j=0}^{\infty}\frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)}\,\gamma\left(j+1,\lambda\ln n\right). \end{aligned} \tag{3.28}$$

Inequality $\diamond$ holds for any sufficiently large m and n. Using (3.26) and (3.27), it is clear that as $n \to \infty$,

$$\begin{aligned} p_0 &\geq 1 - e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)} \left[\Gamma(j+1) - \Gamma\left(j+1, \lambda \ln n\right)\right] \\ &= n^{-\lambda} e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{(\mu^2/2)^j}{j!} \sum_{k=0}^{j} (\lambda \ln n)^k \end{aligned}$$

and

$$\frac{1}{2p_0} \frac{(np_0 - m + 1)^2}{n} \approx \frac{np_0}{2} \geq \frac{n^{1-\lambda}}{2} e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{(\mu^2/2)^j}{j!} \sum_{k=0}^{j} (\lambda \ln n)^k \to \infty. \quad (3.29)$$

Hence, using (3.14), we proved that as $n \to \infty$, with probability approaching one, there are at least $M_n = \lfloor \tilde{\nu}_1(n) - \epsilon \rfloor$ transmitters for which the minimum rate constraint is satisfied.

Next, we prove $M_n \leq \tilde{\nu}_1(n) + \epsilon$ holds with probability approaching one. Similar to the proof of Theorem 3.5.1, we only need to show that

$$\mathbb{P}\left(|h_n|^2 \geq \frac{\sigma^2(e^{(\tilde{\nu}_1(n)+\epsilon)R_{min}})}{P_\delta(\tilde{\nu}_1(n)+\epsilon)}\right) \to 0$$

which tends to zero if and only if

$$\begin{aligned} n\,g(n) &= n\,(1 - F_{\mathcal{NC}\chi_2^2}(\frac{\sigma^2(e^{(\tilde{\nu}_1(n)+\epsilon)R_{min}})}{P_\delta(\tilde{\nu}_1(n)+\epsilon)}; 2, \mu^2)) \\ &= n\,e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)} \Gamma\left(j+1, \frac{P \ln n}{\lambda P_\delta} \frac{\tilde{\nu}_1(n)}{\tilde{\nu}_1(n) - \epsilon}\right) \\ &\sim n\,e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)} \Gamma\left(j+1, \frac{P \ln n}{\lambda P_\delta}\right) \\ &= n e^{-\mu^2/2} \left(\sum_{j=0}^{c \ln n} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)} \Gamma\left(j+1, \frac{P \ln n}{\lambda P_\delta}\right) \right. \\ &\qquad \left. + \sum_{j=c \ln n}^{\infty} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)} \Gamma\left(j+1, \frac{P \ln n}{\lambda P_\delta}\right)\right) \to 0. \end{aligned} \quad (3.30)$$

Assume $c < \frac{P}{\lambda P_\delta}$ is selected such that (2.38) can be applied to the first summation.

Hence, using $j! \geq (j/2)^{j/2}$ and defining $j_0 = 2\left(\frac{\mu^2 P}{P-\lambda P_\delta}\right)^2$, the first summation in (3.30),

$$
\begin{aligned}
n\, g_1(n) &= ne^{-\mu^2/2} \sum_{j=0}^{c\ln n} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)} \Gamma\left(j+1, \frac{P\ln n}{\lambda P_\delta}\right) \\
&\sim n^{1-\frac{P}{\lambda P_\delta}} e^{-\mu^2/2} \sum_{j=0}^{c\ln n} \frac{(\frac{\mu^2 P\ln n}{2\lambda P_\delta})^j}{j!\, j!} \\
&\leq n^{1-\frac{P}{\lambda P_\delta}} e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{\left(\frac{\mu^2 P\ln n}{2\lambda P_\delta}\right)^j}{\left(\sqrt{\frac{j}{2}}\right)^j j!} \\
&\leq n^{1-\frac{P}{\lambda P_\delta}} e^{-\mu^2/2} \sum_{j=0}^{j_0} \frac{\left(\frac{\mu^2 P\ln n}{2\lambda P_\delta}\right)^j}{\left(\sqrt{\frac{j}{2}}\right)^j j!} \\
&\quad + \quad n^{1-\frac{P}{\lambda P_\delta}} e^{-\mu^2/2} \sum_{j=j_0}^{\infty} \frac{\left(\frac{\mu^2 P\ln n}{2\lambda P_\delta}\right)^j}{\left(\sqrt{\frac{j_0}{2}}\right)^j j!} \\
&\sim n^{1-\frac{P}{\lambda P_\delta}} e^{-\mu^2/2} \sum_{j=0}^{\infty} \frac{\left(\frac{(P-\lambda P_\delta)\ln n}{2\lambda P_\delta}\right)^j}{j!} \\
&= e^{-\mu^2/2}\, n^{\frac{1}{2}(1-\frac{P}{\lambda P_\delta})} \to 0
\end{aligned} \tag{3.31}
$$

which holds by choosing $\delta < (e^{\epsilon R_{min}} - 1)P$. For sufficiently large $c\ln n$, the second summation in (3.30),

$$
\begin{aligned}
n\, g_2(n) &= ne^{-\mu^2/2} \sum_{j=c\ln n}^{\infty} \frac{(\mu^2/2)^j}{j!\,\Gamma(j+1)} \Gamma\left(j+1, \frac{P\ln n}{\lambda P_\delta}\right) \\
&= n^{1-\frac{P}{\lambda P_\delta}} e^{-\mu^2/2} \sum_{j=c\ln n}^{\infty} \frac{(\mu^2/2)^j}{j!} \sum_{k=0}^{j} \frac{\left(\frac{P\ln n}{\lambda P_\delta}\right)^k}{k!} \\
&\leq ne^{-\mu^2/2} \sum_{j=c\ln n}^{\infty} \frac{(\mu^2/2)^j}{j!}\,.
\end{aligned} \tag{3.32}
$$

Using Stirling's approximation, the right-hand side of (3.32) is asymptotically equal to

$$
\begin{aligned}
ne^{-\mu^2/2} \sum_{j=c\ln n}^{\infty} \frac{(e\,\mu^2/2)^j}{j^j\sqrt{2\pi j}} &\sim ne^{-\mu^2/2} \sum_{j=c\ln n}^{\infty} \frac{1}{j^j} \\
&\leq ne^{-\mu^2/2} \int_{c\ln n}^{\infty} \frac{dx}{x^x} \\
&\leq ne^{-\mu^2/2} \frac{1}{(c\ln n)^{(c\ln n-2)}} \int_{c\ln n}^{\infty} \frac{dx}{x^2} \\
&= ne^{-\mu^2/2} \frac{c\ln n}{(c\ln n)^{(c\ln n)}} \qquad (3.33)
\end{aligned}
$$

which tends to zero since

$$
\frac{\ln n + \ln\ln n}{(c\ln n)\ln(c\ln n)} \sim \frac{\ln n}{(c\ln n)\ln(c\ln n)} \to 0. \qquad (3.34)
$$

Hence, according to (3.31) and (3.34), (3.30) holds by choosing $\delta < (e^{\epsilon R_{min}} - 1)P$.
$\square$

3.5.3 Nakagami Fading

The results presented in this chapter for Rayleigh and Rician fading models can be extended to the Nakagami distribution as follows.

Theorem 3.5.3 Under the assumption of independent Nakagami fading channels for different transmitters with channel gains $|h_i| \sim Nakagami(\mu, \omega)$, $i = 1, \ldots, n$ and for any $\epsilon > 0$, the maximum number of active transmitters, M_n, is bounded as

$$
\mathbb{P}(\lfloor \tilde{\nu}_2(n) - \epsilon \rfloor \leq M_n \leq \tilde{\nu}_2(n) + \epsilon) \to 1, \quad \text{as } n \to \infty, \qquad (3.35)
$$

where n is the total number of transmitters, and

$$
\tilde{\nu}_2(n) = \frac{1}{R_{min}} \ln\left(\frac{\omega P \tilde{\nu}_2(n)}{\mu\,\sigma^2} \ln n\right). \qquad (3.36)
$$

Proof 3.5.4 In Nakagami fading channels, the cumulative distribution function of

$|h_i|^2$ is given by

$$F(x;\mu,\omega)=\frac{\gamma(\mu,\frac{\mu}{\omega}x)}{\Gamma(\mu)}$$

where μ denotes the shape parameter and ω controls distribution spread. Defining Bernoulli random variable (3.10), we show that for any $\epsilon>0$, if $m\leq\tilde{\nu}_2(n)-\epsilon$, $\min_{n-m+1\leq i\leq n}|h_i|^2\geq\sigma^2 e^{mR_{min}}/(mP)$ holds with probability approaching one. Let $h_0=\sigma^2 e^{mR_{min}}/(mP)$. Then,

$$\begin{aligned}
p_0=1-\mathbb{P}(|h_i|^2\leq h_0) &\geq 1-F\left(\frac{\sigma^2 e^{R_{min}(\tilde{\nu}_2(n)-\epsilon)}}{P(\tilde{\nu}_2(n)-\epsilon)};\mu,\omega\right)\\
&= 1-\frac{\gamma\left(\mu,\lambda\ln n\frac{\tilde{\nu}_2(n)}{\tilde{\nu}_2(n)-\epsilon}\right)}{\Gamma(\mu)}\\
&\sim 1-\frac{\gamma(\mu,\lambda\ln n)}{\Gamma(\mu)}\\
&= \frac{\Gamma(\mu,\lambda\ln n)}{\Gamma(\mu)}\\
&\sim \frac{n^{-\lambda}\left(\lambda\ln n\right)^{\mu-1}}{\Gamma(\mu)}. \qquad (3.37)
\end{aligned}$$

Using the fact $\ln n=o(n^\epsilon)$ for any $\epsilon>0$, it is clear that as $n\to\infty$,

$$\frac{1}{2p_0}\frac{(np_0-m+1)^2}{n}\approx\frac{np_0}{2}\geq\frac{n^{1-\lambda}\left(\lambda\ln n\right)^{\mu-1}}{2\Gamma(\mu)}\to\infty. \qquad (3.38)$$

Hence, by (3.14), we proved that as $n\to\infty$, with probability approaching one, there are at least $M_n=\lfloor\tilde{\nu}_2(n)-\epsilon\rfloor$ transmitters for which the minimum rate constraint is satisfied.

Next, we prove the upper bound; in other words, $M_n\leq\tilde{\nu}_2(n)+\epsilon$ holds with probability approaching one. Similar to the proof of Theorem 3.5.1, we only need to show that

$$\mathbb{P}\left(|h_n|^2\geq\frac{\sigma^2(e^{(\tilde{\nu}_2(n)+\epsilon)R_{min}})}{P_\delta(\tilde{\nu}_2(n)+\epsilon)}\right)\to 0.$$

which tends to zero if and only if

$$\begin{aligned} n\,\hat{g}(n) &= n\left(1 - F\left(\frac{\sigma^2(e^{(\tilde{\nu}_2(n)+\epsilon)R_{min}})}{P_\delta(\tilde{\nu}_2(n)+\epsilon)}\right)\right) \\ &= n\left(1 - \frac{\gamma\left(\mu, \frac{P\ln n}{\lambda P_\delta}\frac{\tilde{\nu}_2(n)}{\tilde{\nu}_2(n)+\epsilon}\right)}{\Gamma(\mu)}\right) \\ &\sim n\left(1 - \frac{\gamma\left(\mu, \frac{P\ln n}{\lambda P_\delta}\right)}{\Gamma(\mu)}\right) \\ &= n\,\frac{\Gamma\left(\mu, \frac{P}{\lambda P_\delta}\ln n\right)}{\Gamma(\mu)} \\ &\sim \frac{n^{1-\frac{P}{\lambda P_\delta}}\left(\frac{P}{\lambda P_\delta}\ln n\right)^{\mu-1}}{\Gamma(\mu)} \to 0. \end{aligned} \tag{3.39}$$

which holds by choosing $\delta < (e^{\epsilon R_{min}} - 1)P$. $\square$

Thus, the result presented in Theorem 3.5.1 is valid for Rician and Nakagami fading channels with a constant shift; in other words, the maximum number of active transmitters scales double logarithmically with the total number of transmitters in Rician and Nakagami fading multiple-access channels.

3.6 Path-Loss Effect on the Scaling Laws

In Section 3.5, the channel model for each transmitter-receiver pair only consists of a multi-path fading term. Accounting for both multi-path and path loss, the channel between transmitter i and the receiver is determined by

$$|h_i|^2 = |f_i|^2\, d_i^{-\alpha}$$

where d_i denotes the distance between transmitter i and the receiver, $|f_i|^2 \sim$ Exponential(1); $i = 1, \ldots, n$, and α represents the path-loss exponent. In this section, it is shown that the user capacity scaling laws do not change in the presence of path loss if $d_{min} \leq d_i \leq d_{max}$; $i = 1, \cdots, n$, where d_{min} and d_{max} denote respectively the minimum and the maximum distance between any transmitter and the receiver. Note that this assumption is valid in many wireless systems including cellular communications and small wireless ad-hoc networks in which the path-loss term is bounded. Hence, Theorem 4.5.1 changes as follows.

Theorem 3.6.1 Under the assumption of independent Rayleigh fading channels for different transmitters with channel gains $h_i \sim \mathcal{CN}(0,1)$; $i = 1, \ldots, n$ and $d_{min} \leq d_i \leq d_{max}$; $i = 1, \cdots, n$, for any $\epsilon > 0$, the maximum number of active transmitters, M_n satisfies:

$$\mathbb{P}\{\lfloor \nu_L(n) - \epsilon \rfloor \leq M_n \leq \nu_H(n) + \epsilon\} \to 1, \quad \text{as } n \to \infty, \tag{3.40}$$

where n is the total number of transmitters, and

$$\nu_L(n) = \frac{1}{R_{min}} \ln\left(\frac{P\nu_L(n)}{\sigma^2 d_{max}^{\alpha}} \ln n\right) \tag{3.41}$$

$$\nu_H(n) = \frac{1}{R_{min}} \ln\left(\frac{P\nu_H(n)}{\sigma^2 d_{min}^{\alpha}} \ln n\right). \tag{3.42}$$

Proof 3.6.1 Adding the path-loss term changes the channel distribution. Hence, in the proof of Theorem 4.5.1, only those parts dealing with the channel distribution must be modified. Assuming $d_{min} \leq d_i \leq d_{max}$, the activation probability, p_0, is calculated as

$$\begin{aligned} p_0 = \mathbb{P}(|f_i|^2 d_i^{-\alpha} > h_0) &\geq \mathbb{P}\left(|f_i|^2 > d_{max}^{\alpha} h_0\right) \\ &= \exp\left(-d_{max}^{\alpha} h_0\right) \\ &= \exp\left(-\frac{d_{max}^{\alpha}\sigma^2 e^{mR_{min}}}{mP}\right) \\ &\geq \exp\left(-\frac{d_{max}^{\alpha}\sigma^2 e^{R_{min}(\nu_L(n)-\epsilon)}}{P(\nu_L(n)-\epsilon)}\right) \\ &= \exp\left(-\lambda \ln(n) \frac{\nu(n)}{\nu(n)-\epsilon}\right) \sim n^{-\lambda}. \end{aligned} \tag{3.43}$$

Hence, as $n \to \infty$, with probability approaching one, there are at least $M_n = \lfloor \nu_L(n) - \epsilon \rfloor$ transmitters with $|h_i|^2 \geq \sigma^2 e^{mR_{min}}/(mP)$, for which the minimum rate constraint is satisfied and can be activated.

To prove the upper bound, considering $d_{min} \leq d_i \leq d_{max}$, we have

$$\mathbb{P}\left(|h_n|^2 \geq \frac{\sigma^2 e^{(\nu_H(n)+\epsilon)R_{min}}}{P_\delta(\nu_H(n)+\epsilon)}\right) \leq \mathbb{P}\left(|f_n|^2 \geq \frac{d_{min}^\alpha \sigma^2 e^{(\nu_H(n)+\epsilon)R_{min}}}{P_\delta(\nu_H(n)+\epsilon)}\right).$$

Hence, we only need to show

$$\mathbb{P}\left(|f_n|^2 \geq \frac{d_{min}^\alpha \sigma^2 e^{(\nu_H(n)+\epsilon)R_{min}}}{P_\delta(\nu_H(n)+\epsilon)}\right) \to 0. \tag{3.44}$$

(3.44) holds if

$$\begin{aligned} n \cdot \exp\left(-\frac{d_{min}^\alpha \sigma^2 e^{(\nu_H(n)+\epsilon)R_{min}}}{P_\delta(\nu_H(n)+\epsilon)}\right) &= n \cdot \exp\left(-\frac{P\ln(n)\,\nu_H(n)}{\lambda P_\delta(\nu_H(n)+\epsilon)}\right) \\ &\sim n^{1-\frac{P}{\lambda P_\delta}} \to 0 \end{aligned}$$

which holds by choosing $\delta < (e^{\epsilon R_{min}} - 1)P$.

As a result, if the path-loss term is bounded, the user capacity of Rayleigh fading multiple-access channels still scales double logarithmically with the total number of transmitters and the difference is only a constant. However, in this case, the upper bound does not meet the lower bound and the gap depends on the physical size of the system. If the path-loss term is not bounded, the user capacity scaling laws will be different depending on the transmitters distribution in the space which determines the distribution of the path-loss term. The aforementioned result can be similarly extended to Rician and Nakagami fading multiple-access channels.

3.7 Implementation Issues

The implication of the result is that in a scenario with a large number of transmitters, with high probability the maximum number of simultaneous transmissions in a slot will be of the order $\ln\ln n$. A natural question is whether it is possible to find a scheduling scheme that will maximize the number of simultaneous transmitters while maintaining the minimum rate. We now address this issue.

A scheduling policy activating transmitters can be easily implemented in a distributed fashion. Consider a multiple-access channel with a sufficiently large number of users (e.g. $n \geq 50$ based on simulation results shown in Section 3.8). Suppose each transmitter knows the noise variance at the receiver (i.e. σ^2) and its channel gain by either applying channel estimation algorithms directly or receiving channel state information through a feedback from the receiver. Note that the maximum number of active transmitters can be calculated as the fixed-point of:

$$m = \frac{1}{R_{min}} \ln \left(\frac{Pm}{\sigma^2} \ln n \right).$$

Thus, the transmitter compares its channel gain with threshold h_0 (see (3.11)) given by

$$h_0 = \frac{\sigma^2 e^{mR_{min}}}{mP}.$$

If the channel gain is above the threshold, the transmitter becomes active; otherwise, it remains inactive during the current time slot. In the scenario of a large number of users, following such a policy will lead to selecting the optimum number of transmitters with high probability.

3.8 Simulation Results

Consider a system with noise variance $\sigma^2 = 1$, and channel bandwidth $B = 50K$ samples/second. For the Rayleigh fading distribution, $|h_i|^2 \sim \chi_2^2$, and for the Rician fading distribution, $|h_i|^2 \sim \mathcal{NC}\chi_2^2(\mu^2)$.

For Rayleigh fading, Figure 3.2 shows the optimal number of active transmitters versus the total number of transmitters for SNR$=$ 20 dB at each transmitter. The value of $\tilde{\nu}(n)$ given by (3.9) is calculated by an iterative fixed-point algorithm and shown in Figure 3.2. As shown in Figure 3.2, the number of active transmitters is almost doubled as $R_{\min}$ is halved. To illustrate the convergence rate, the histogram of the number of active transmitters is shown in Figure 3.3 for SNR$=$ 20dB, $R_{min} =$

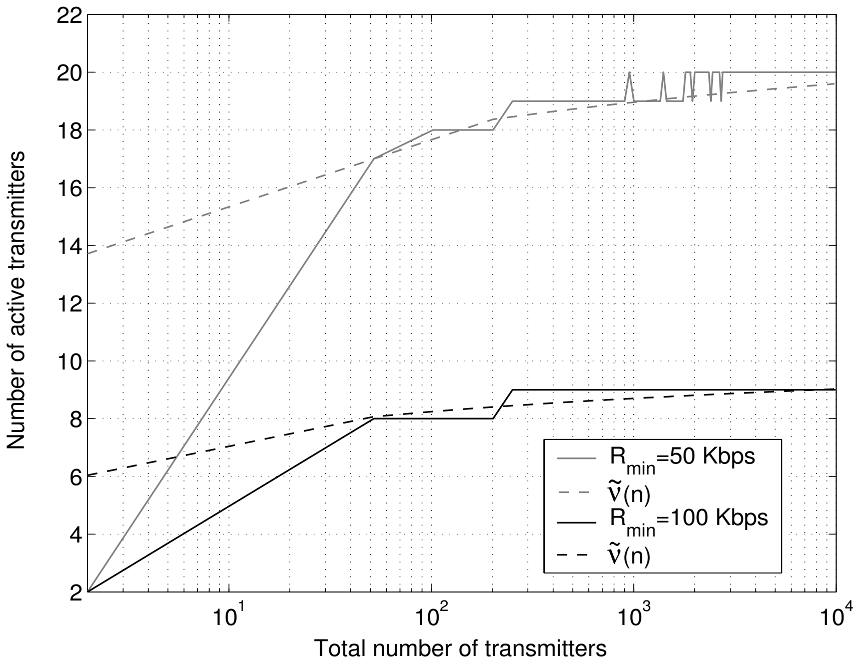


Figure 3.2: The optimal number of active transmitters versus the total number of transmitters for Rayleigh fading, $R_{\min} = 50, 100\,\text{Kbps}$ and SNR=20 dB at each transmitter.

50 Kbps, and $n = 20, 5000$. It is clear that the convergence rate goes to zero as n tends to infinity. Figure 3.3 is sketched by 5000 simulation runs. For a further comparison, the optimal number of active users versus different $R_{\min}$ for SNR$=$ 20dB and $n = 5000$ is shown in Figure 3.4, where the curve of $\tilde{\nu}(n)$ is also drawn.

For Rician fading, Figure 3.5 shows the optimal number of active transmitters versus the total number of transmitters for the Rician fading distribution for $\mu = 0.8, 2$ and for SNR$=$ 20 dB at each transmitter. The value of $\tilde{\nu}_1(n)$ given by (3.24) is also indicated in Figure 3.5. Similar to Rayleigh fading channels, the number of active transmitters shown in Figure 3.5 is almost doubled as $R_{\min}$ is halved.

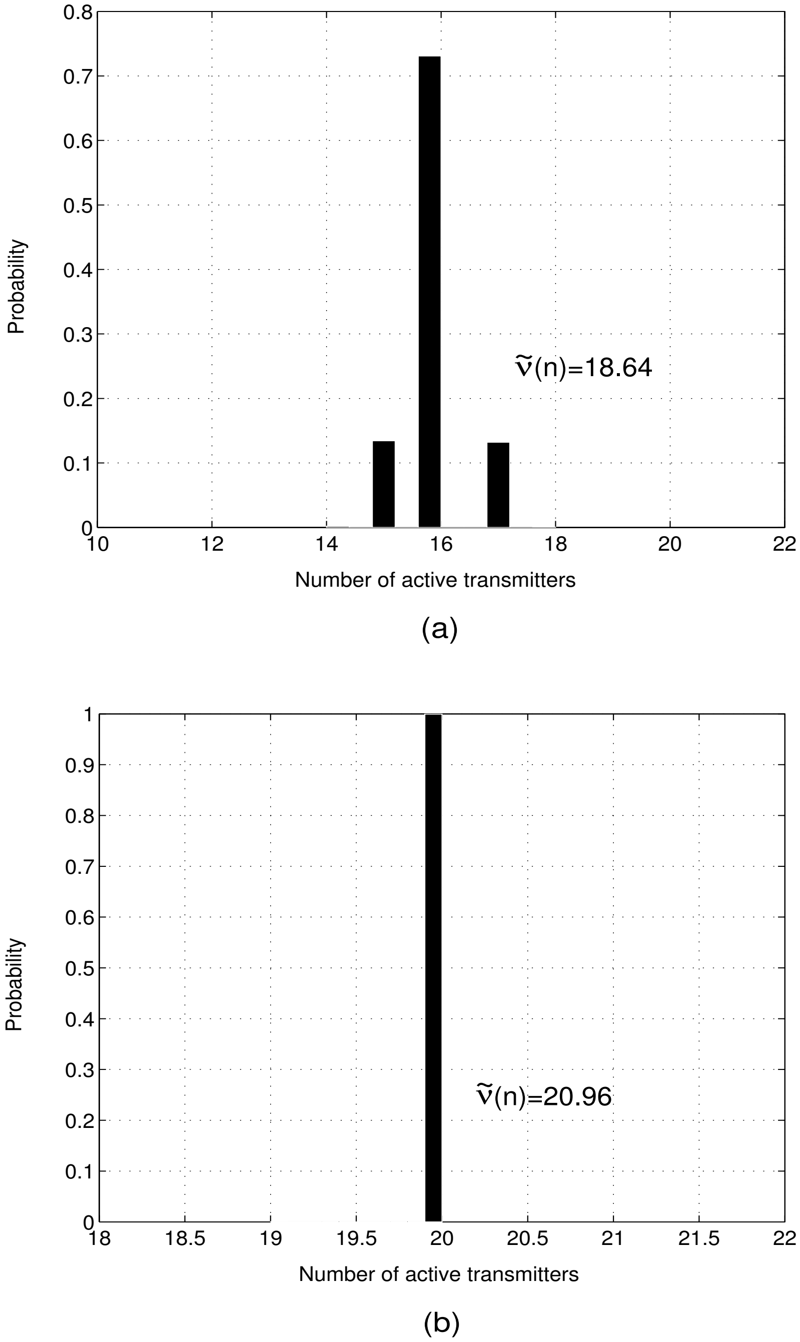


Figure 3.3: The histogram of the number of active transmitters for Rayleigh fading, $R_{\min} = 50\,\text{Kbps}$, $\text{SNR} = 20\,\text{dB}$, (a) $n = 20$, and (b) $n = 5000$.

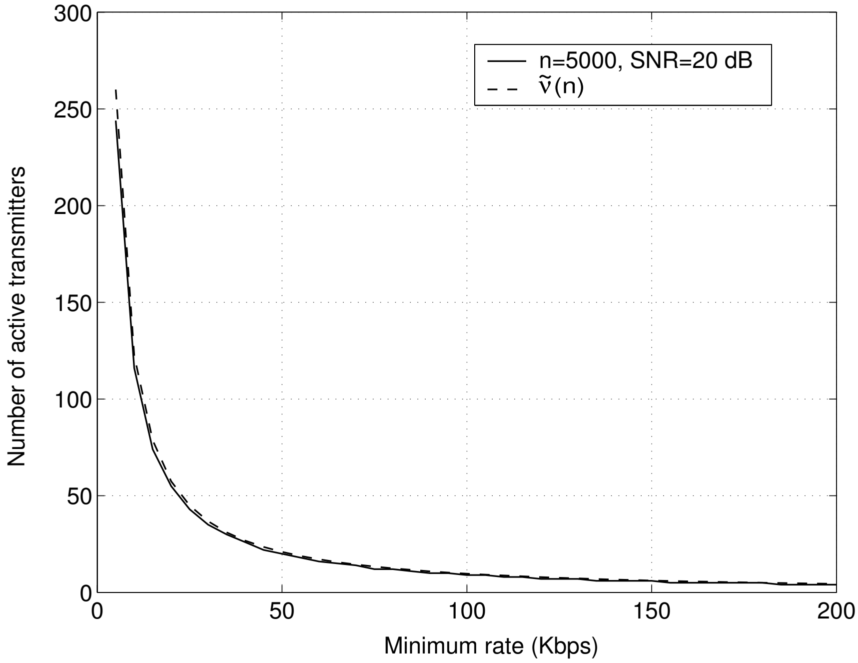
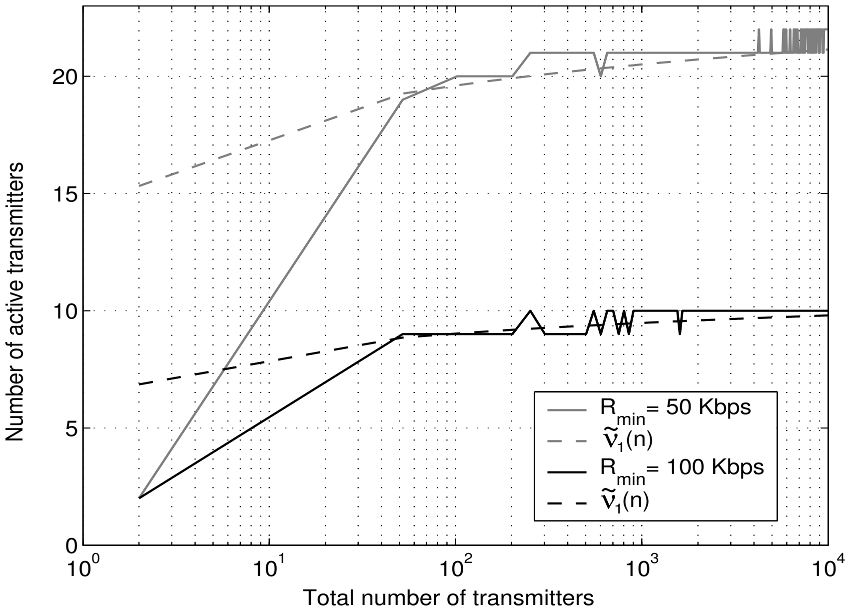
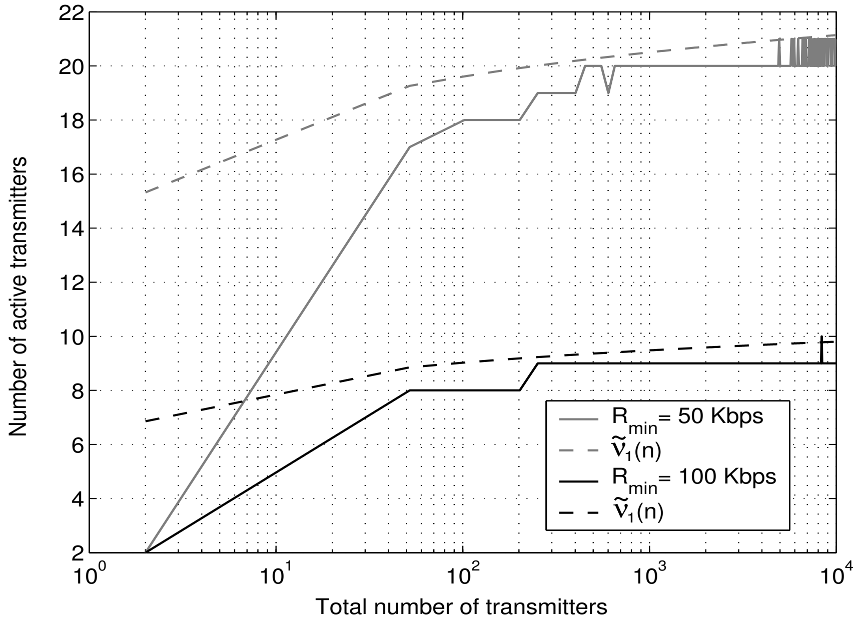


Figure 3.4: The optimal number of active transmitters versus the minimum rate for Rayleigh fading, $n = 5000$, and $\mathrm{SNR} = 20\,\mathrm{dB}$.



(a)



(b)

Figure 3.5: The optimal number of active transmitters versus the total number of transmitters for Rician fading, $R_{\min} = 50, 100\,\mathrm{Kbps}$ and $\mathrm{SNR} = 20\,\mathrm{dB}$ at each transmitter, (a) $\mu = 2$, and (b) $\mu = 0.8$.

Chapter 4

Rate-Constrained Random Wireless Networks

One of the important issues in understanding the performance of wireless ad-hoc networks is capacity. As the capacity region of general wireless networks is still unknown, scaling laws are useful to understand their performance limits. In this chapter, the ideas presented in previous chapters for fading multi-user channels are generalized to wireless ad-hoc networks. In particular, the maximum number of active links supporting a minimum rate is obtained in a random wireless network with an arbitrary topology when the number of nodes is large. It is assumed that each source-destination pair communicates through a fading channel. Two scenarios are considered: 1) Small networks with multi-path fading, 2) Large Random networks with multi-path fading and path loss. A by-product of these results is per-node throughput scaling laws for random wireless networks.

4.1 Literature Review

After the pioneering work by Gupta and Kumar [36], many researchers have tried to consider more realistic situations and present tighter throughput bounds. However, different network and channel models result in differing conclusions.

Assuming a power-law path-loss model for each source-destination pair channel, [36] and [37] show that the per-node throughput scales with $O(\frac{1}{\sqrt{N}})$ in both arbitrary and random wireless networks, where N denotes the total number of nodes in the network. In [38], it is shown that for the relatively high-attenuation case, the transport capacity scales as $O(N)$. In particular, the transport capacity is $\Theta(N)$, for regular planar networks where the nodes are situated at integer lattice sites in a square. In a low-attenuation regime, there exist networks that can provide unbounded transport capacity for fixed total power, yielding zero-energy-priced communication. When nodes lie on a straight line, there are networks which can even attain super-linear scaling $\Theta(N^{\theta})$ for $\theta < 2$. In [39], the capacity scaling of extended wireless networks is studied with an emphasis on the low-attenuation regime and it is shown that in the absence of small-scale fading, the low-attenuation regime does not behave significantly different from the high-attenuation regime.

Introducing multi-path fading effects, in [40], each channel gain is a product of a path-loss term and a non-negative random variable modeling multi-path fading and having an exponentially-decaying tail. In this case, the achievable per-node throughput scales with $O\left(\frac{1}{\sqrt{N(\log N)^3}}\right)$. In [41], upper bounds on the transport capacity of wireless networks are derived. The bounds obtained are solely dependent on the geographic locations and power constraints of the nodes. In [42], under the assumption of having only a mild time-average type of bound on the multi-path fading process, it is shown that the transport capacity can grow no faster than $O(N)$, even when the CSI is available non-causally at both transmitters and receivers. This assumption includes common models of stationary ergodic channels, constant frequency-selective channels, flat rapidly-varying channels, and flat slowly-varying channels. In the second assumption set, which essentially features an independence, time average of expectation, and a non-zero fading process, it is shown in [42] how to achieve transport capacity of $\Omega(N)$ even when the CSI is unknown to both transmitters and receivers, provided that every node has an appropriately nearby node. This assumption set includes common models of independent and identically distributed (iid) channels, constant flat channels, and constant frequency-selective channels. In [43], the authors assume that the channel gains are drawn independently and identically distributed from a given probability density function (pdf). As particular examples, [43] shows that the throughput scaling law of the Rayleigh fading channel is logarithmic and if the given pdf obeys a power law decay, almost linear throughput can be obtained. As can be seen, the last result is substantially different from the one obtained for a geometric power-decay network in [36] and [37]. The reason is that although inter-node distances in a random network can be assumed independent (note that this assumption is valid for some network models including (4.1)), they are not identically distributed. Ignoring this fact leads to the unrealistic linear throughput scaling law. In [44], the same channel model as in [40] is considered and it is shown that for a path-loss exponent $\alpha > 2$ and any absorption modeled by exponential attenuation, a per-node throughput of the order $\Omega(\frac{1}{\sqrt{N}})$ is achievable.

4.2 Motivation and Objective

In this chapter, a wireless network is considered in which all active links are required to provide a minimum rate. This is motivated by the requirements of delay-sensitive applications. Due to limited transmitted power and interference from other active source-destination pairs, it is not always possible for all nodes to keep this minimum rate. Hence, we allow nodes with good channel conditions to be active while others remain silent during each time slot. Thus, an on/off power allocation scheme can be exploited to maximize the number of active links while maintaining the minimum-rate constraint.

The objective of this chapter is on analyzing the maximum number of active links supporting a minimum rate in random wireless networks. Whereas [40] and [44] consider a square-shape network of area N and divide it into small sub-squares with a particular link activation scheme based on a node exclusive model, in this chapter, this assumption is relaxed and a wireless network with an arbitrary topology is assumed. Two different scenarios are considered: 1) *Small networks with multi-path fading*, 2) *Large Random networks with multi-path fading and path loss.* In the first case, due to the small size of the network, a single-hop routing strategy can be exploited. Under the assumption of independent Rayleigh fading channels for different source-destination pairs, the optimal number of active links is asymptotically obtained. In the second case, a large wireless network is considered and it is assumed that nodes are Poisson distributed with a finite intensity. Similar to the channel model in [40] and [44], fading channels between different source-destination pairs are modeled by product of path-loss and multi-path fading terms. Unlike many papers in the literature, the multi-path fading distribution is assumed to be arbitrary with a finite mean and variance. Under the assumption of independent multi-path fading for different source-destination pairs, the optimal number of active links is achieved as the network area goes to infinity.

The rest of this chapter is organized as follows: In Section 4.3, the wireless network model is introduced. In Section 4.4, the problem is formulated. Section 4.5 presents asymptotic results for both small networks with multi-path fading and large random networks with both multi-path fading and path loss.

4.3 Wireless Network Model

Consider a wireless network with N nodes located randomly in the plane. It is assumed that source i is connected to destination i through a fading channel. Throughout this chapter, by sources and destinations, we mean transmitting nodes and receiving nodes respectively. Destinations are conventional receivers without multi-user detectors; in other words, no broadcast or multiple-access channel is embedded in the network. Every node has a receiver and a transmitter but it cannot

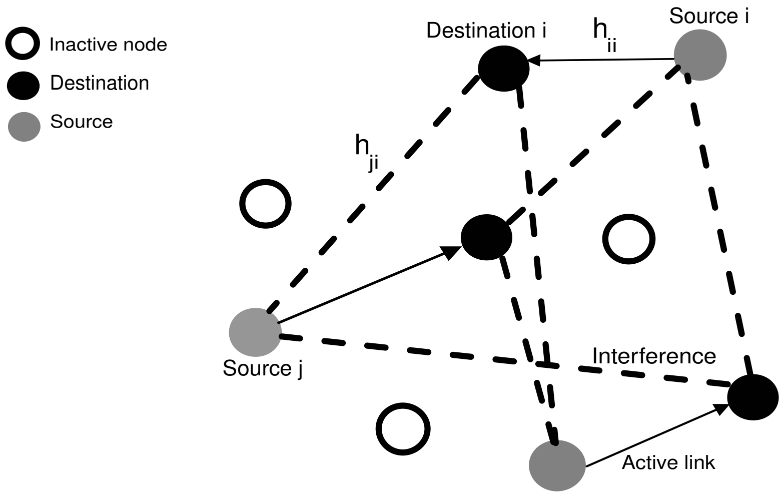


Figure 4.1: A wireless network with active links ($-$) and interference channels ($--$)

transmit and receive signals simultaneously. The nodes are randomly paired into $n = \lfloor N/2 \rfloor$ source-destination pairs without any consideration on their respective locations. Nodes transmit signals with maximum power of P or remain silent during each time slot. Then, the received signal at node i, $Y_i(t)$, is given by

$$Y_i(t) = h_{ii}(t)\, X_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{m} h_{ji}(t)\, X_k(t) + Z_i(t) \tag{4.1}$$

where $h_{ii}(t)$ denotes the link fading channel (i.e. the fading channel between source i and destination i), $h_{ki}(t)$ represents an interference channel for destination i, m refers to the number of active links, and $Z_i(t) \sim \mathcal{CN}(0, \sigma^2)$ represents background noise at node i. Hence, the achievable rate (in nats) of link i can be written as

$$R_i \;\leq\; \ln \left(1 + \frac{P\,|h_{ii}|^2}{\sigma^2 + \sum_{\substack{j=1 \\ j \neq i}}^{m} P\,|h_{ji}|^2} \right) \tag{4.2}$$

4.4 Problem Formulation

In delay-sensitive applications, each active link needs to support a minimum rate. Due to limited transmitted power and interference from other active source-destination pairs, it is not always possible for all nodes to keep this minimum rate. Hence, we allow nodes with good channel conditions to be active while others remain silent during each time slot. Consider the wireless network (4.1). Without loss of generality, assume $|h_{11}| \leq \cdots \leq |h_{nn}|$. In this case, the maximum number of active links supporting the minimum rate is given by the following optimization problem.

$$\max\{m\} \tag{4.3}$$

$$\text{subject to} \qquad R_i \geq R_{\min}, \quad i = n-m+1, \ldots, n \tag{4.4}$$

where m denotes the maximum number of active links. Clearly, with fixed P and $R_{\min}$, the maximum number of active links completely depends on the channel gains $|h_{ji}|^2;\ i, j = 1, \ldots, \lfloor N/2 \rfloor$. When the channel gains obey some statistical distribution, asymptotic behavior of the maximum m can be determined when the total number of links is large enough.

4.5 Asymptotic Analysis

Let M_n denote the maximum number of simultaneous active links (out of n links) that can be supported with a rate greater than or equal to R_{min}. In this section, the distribution of M_n and its features are asymptotically obtained using the central limit theorem and Cramer's theorem.

4.5.1 Small Networks with Multi-Path Fading

We first consider small networks where fading rather than path loss is important. This corresponds to situations within a single-cell in which path loss between any pair of nodes is bounded and thus, can be ignored.

Consider a wireless network with an arbitrary topology and assume that there are independent Rayleigh fading channels between different source-destination pairs; in other words, the channel gains h_{ij}; $i, j = 1, \ldots, \lfloor N/2 \rfloor$ are independent realizations of the complex Gaussian distribution; as a result, $|h_{ij}|^2$; $i, j = 1, \ldots, \lfloor N/2 \rfloor$ are independent realizations of the exponential distribution.

Theorem 4.5.1 Under the assumption of independent Rayleigh fading channels for different source-destination pairs with channel gains $h_{ij} \sim \mathcal{CN}(0,1)$; $i, j = 1, \ldots, \lfloor N/2 \rfloor$, and for any $\epsilon > 0$ arbitray close to zero, the maximum number of active links, M_n, determined by (4.3)-(4.4), is bounded as

$$\lim_{n\to\infty} \mathbb{P}\left(\lfloor \beta_1(n) \rfloor \leq M_n \leq \beta_2(n)\right) = 1, \tag{4.5}$$

where

$$\beta_1(n) = \frac{\ln n}{(1+\epsilon)\, e^{R_{\min}} - (1-2\epsilon)} \tag{4.6}$$

$$\beta_2(n) = \frac{\ln n}{(1-\epsilon) e^{R_{\min}} - (1+2\epsilon)}. \tag{4.7}$$

Proof 4.5.1 Consider the wireless network (4.1) with independent channel gains $h_{ij} \sim \mathcal{CN}(0,1)$, for $i, j = 1, \ldots, \lfloor N/2 \rfloor$; as a result, $|h_{ij}|^2 \sim \text{Exponential}(1)$. For any activation threshold $h_0 > 0$, the number of "good" link channels can be characterized with $|h_{ii}|^2$ greater than h_0 as follows. Let $p_0 = 1 - \mathbb{P}(|h_{ii}|^2 \leq h_0) = e^{-h_0}$. That is, with probability p_0, a link can be activated. Consider Bernoulli sequence

$$x_i = \begin{cases} 1, & \text{with probability } p_0 \\ 0, & \text{with probability } 1 - p_0 \end{cases} \tag{4.8}$$

for $i = 1, 2, \ldots, n$. Then, the number of activated links has the same distribution as $M_n = \sum_{i=1}^{n} x_i$, which satisfies the Binomial distribution $B(n, p_0)$.

Let $h_0 = m\left((1+\epsilon)e^{R_{\min}} - (1-\epsilon)\right)$ for any integer m. Then, we show that

$\min_{n-m+1\leq i\leq n}|h_{ii}|^2 \geq h_0$ holds with probability approaching one if $m \leq \beta_1(n)$.

$$\begin{aligned} p_0 = 1 - \mathbb{P}(|h_{ii}|^2 \leq h_0) &= \exp(-h_0) \\ &= \exp\left(-m\left((1+\epsilon)e^{R_{\min}} - (1-\epsilon)\right)\right) \\ &\geq \exp\left(-\beta_1(n)\left((1+\epsilon)e^{R_{\min}} - (1-\epsilon)\right)\right) \\ &= \exp\left(-\frac{(1+\epsilon)e^{R_{\min}} - (1-\epsilon)}{(1+\epsilon)e^{R_{\min}} - (1-2\epsilon)}\log n\right) \\ &= \exp(-\eta \ln(n)) = n^{-\eta} \end{aligned} \quad (4.9)$$

where $\eta = \frac{(1+\epsilon)e^{R_{\min}}-(1-\epsilon)}{(1+\epsilon)e^{R_{\min}}-(1-2\epsilon)} < 1$. Since $m \leq \beta_1(n)$ and $np_0 \geq n^{1-\eta}$, it is clear that

$$\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n} \sim \frac{n^2p_0^2}{2np_0} = \frac{np_0}{2} \geq \frac{n^{1-\eta}}{2} \to \infty\ ; \quad \text{as } n \to \infty. \quad (4.10)$$

Moreover, as $m \leq np_0 + 1$, the Chernoff bound on the sum of independent Poisson trials can be used as

$$\mathbb{P}(M_n \geq m) \geq 1 - \exp\left(-\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n}\right). \quad (4.11)$$

Thus, we proved that as n goes to infinity, with probability approaching one, there are at least $M_n = \lfloor \beta_1(n) \rfloor$ link channels with $|h_{ii}|^2 \geq h_0$.

Next, we show that the minimum-rate constraint is satisfied asymptotically almost surely for m channel gains if $\min_{n-m+1\leq i\leq n}|h_{ii}|^2 \geq h_0$. For any large integer m and $\epsilon > 0$ small, define event A_n as

$$A_n \triangleq \left\{\omega : \left|\frac{1}{m-1}\sum_{\substack{k=1\\k\neq i}}^{m}|h_{ki}|^2 - \mathbf{E}\left(|h_{11}|^2\right)\right| < \epsilon\right\}. \quad (4.12)$$

From Cramer's theorem [45],

$$\mathbb{P}(A_n^c) = \mathbb{P}\left(\left|\frac{1}{m-1}\sum_{\substack{k=1\\k\neq i}}^{m}|h_{ki}|^2 - \mathbf{E}\left(|h_{11}|^2\right)\right| \geq \epsilon\right) \sim e^{-(m-1)I(\epsilon)} \quad (4.13)$$

where $I(x)$ is called the rate function and defined as

$$\begin{aligned}
I(x) &= \sup_{\theta>0}\left[\theta x - \lambda(\theta)\right] = x - \ln x - 1 \\
\lambda(\theta) &= \ln E(e^{\theta x}) = \ln \frac{1}{1-\theta}\ ; \quad 0<\theta<1.
\end{aligned}$$

Define

$$X_n \triangleq \ln\left(1+\frac{P\left|h_{ii}\right|^2}{\sigma^2+\sum\limits_{\substack{k=n-m+1\\k\neq i}}^{n} P\left|h_{ki}\right|^2}\right). \tag{4.14}$$

Then,

$$\begin{aligned}
&X_n(\omega)\mathbf{1}_{[\min_{n-m+1\le i\le n}|h_{ii}|^2\ge h_0]}(\omega) = \\
&\qquad\left(X_n(\omega)\mathbf{1}_{[A_n]}(\omega)+X_n(\omega)\mathbf{1}_{[A_n^c]}(\omega)\right)\mathbf{1}_{[\min_{n-m+1\le i\le n}|h_{ii}|^2\ge h_0]}(\omega).
\end{aligned} \tag{4.15}$$

Note that according to (4.11), $\min_{n-m+1\le i\le n}|h_{ii}|^2 \ge h_0$ holds with probability approaching one. Clearly, on A_n, we have

$$(m-1)\left(\mathbf{E}[|h_{11}|^2]-\epsilon\right) \le \sum_{\substack{k=n-m+1\\k\neq i}}^{n} \left|h_{ki}\right|^2 \le (m-1)\left(\mathbf{E}[|h_{11}|^2]+\epsilon\right).$$

From (4.13), for any $\epsilon>0$, $\mathbb{P}(A_n^c)$ goes to zero exponentially as $m\to\infty$. Hence, for channel gains satisfying $|h_{ii}|^2\ge h_0$, on A_n, we have,

$$\ln\left(1+\frac{P\left|h_{ii}\right|^2}{\sigma^2+\sum\limits_{\substack{k=n-m+1\\k\neq i}}^{n} P\left|h_{ki}\right|^2}\right) \ \ge\ \ln\left(1+\frac{Ph_0}{\sigma^2+P(m-1)\left(\mathbf{E}(\left|h_{11}\right|^2)+\epsilon\right)}\right)$$

$$
\begin{aligned}
&= \ln\left(1+\frac{Pm\left((1+\epsilon)e^{R_{\min}}-(1-\epsilon)\right)}{\sigma^2+P(m-1)(1+\epsilon)}\right) \\
&\sim \ln\left(1+e^{R_{\min}}-\frac{1-\epsilon}{1+\epsilon}\right) \geq R_{\min}\,.
\end{aligned} \tag{4.16}
$$

Now, for $\epsilon > 0$ small, noting that $X_n \leq \ln(1+\frac{\ln n}{\sigma^2})$ with high probability since it can be shown that

$$
\lim_{n\to\infty} \mathbb{P}\left(|h_{ji}|^2 \leq \ln n\right) = 1,
$$

we have

$$
\mathbf{E}\left(X_n\,\mathbf{1}_{[A_n^c]}(\omega)\right) = \mathbf{E}(X_n)\,\mathbb{P}(A_n^c) \leq \ln\ln n\, e^{-c\ln n} \to 0\ ; \quad \text{as } n\to\infty
$$

where c is a constant depending on ϵ and R_{min}. Hence,

$$
\mathbb{P}\left(\liminf_{n\to\infty} X_n \geq R_{min}\right) = 1.
$$

Clearly, based on (4.2), the minimum-rate constraint is satisfied for these m channel gains. As a result, we proved that as $n \to \infty$, with probability approaching one, there are at least $M_n = \lfloor \beta_1(n) \rfloor$ link channels with $|h_{ii}|^2 \geq h_0$, for which the minimum rate constraint is satisfied and can be activated.

Next, we prove $M_n \leq \beta_2(n)$ holds with high probability. Considering m active links, first of all, we show the best active link should have channel gain $|h_{nn}|^2 \geq h_0^{'} = m\left((1-\epsilon)e^{R_{\min}}-(1+\epsilon)\right)$; otherwise, if $|h_{nn}|^2 < h_0^{'}$, on A_n,

$$
\begin{aligned}
\ln\left(1+\frac{P\,|h_{ii}|^2}{\sigma^2+\displaystyle\sum_{\substack{k=n-m+1\\k\neq i}}^{n} P\,|h_{ki}|^2}\right) &< \ln\left(1+\frac{Ph_0^{'}}{\sigma^2+P(m-1)\left(\mathbf{E}(|h_{11}|^2)-\epsilon\right)}\right) \\
&= \ln\left(1+\frac{Pm\left((1-\epsilon)e^{R_{\min}}-(1+\epsilon)\right)}{\sigma^2+P(m-1)(1-\epsilon)}\right) \\
&\sim \ln\left(1+e^{R_{\min}}-\frac{1+\epsilon}{1-\epsilon}\right) < R_{\min}
\end{aligned} \tag{4.17}
$$

which violates (4.2). Note that the same probabilistic argument as the one in the lower bound proof can be also used here to show that channel gains belong to the set A_n with probability approaching one.

Hence, to show that

$$\mathbb{P}(M_n \le \beta_2(n)) \to 1,$$

or

$$\mathbb{P}(M_n > \beta_2(n)) \to 0,$$

we only need to show that

$$\mathbb{P}\left(|h_{nn}|^2 \ge \beta_2(n)\left((1-\epsilon)e^{R_{\min}} - (1+\epsilon)\right)\right) \to 0.$$

Define $p_1 = 1 - \mathbb{P}(|h_{ii}|^2 \le \acute{h_0}) = \exp(-\acute{h_0})$. The probability that all links have channel gains less than $\acute{h_0}$ equals $(1-p_1)^n$. As $\acute{h_0} = \beta_2(n)\left((1-\epsilon)e^{R_{\min}} - (1+\epsilon)\right)$,

$$\mathbb{P}\left(|h_{nn}|^2 \ge \beta_2(n)\left((1-\epsilon)e^{R_{\min}} - (1+\epsilon)\right)\right) = 1 - (1-p_1)^n \tag{4.18}$$

which tends to zero if and only if

$$\left(1 - \exp(-\acute{h_0})\right)^n = \left(1 - \exp\left(-\beta_2(n)\left((1-\epsilon)e^{R_{\min}} - (1+\epsilon)\right)\right)\right)^n \to 1. \tag{4.19}$$

Since

$$\left(1 - \exp(-\acute{h_0})\right)^{\exp\left(\acute{h_0}\right)} \to e^{-1},$$

(4.19) holds if

$$\begin{aligned} n \cdot \exp\left(-\beta_2(n)\left((1-\epsilon)e^{R_{\min}} - (1+\epsilon)\right)\right) &= n \cdot \exp\left(-\frac{\left((1-\epsilon)e^{R_{\min}} - (1+\epsilon)\right)}{\left((1-\epsilon)e^{R_{\min}} - (1+2\epsilon)\right)} \ln n\right) \\ &= n^{1-\gamma} \to 0 \end{aligned} \tag{4.20}$$

which holds as $\gamma = \frac{\left((1-\epsilon)e^{R_{\min}} - (1+\epsilon)\right)}{\left((1-\epsilon)e^{R_{\min}} - (1+2\epsilon)\right)} > 1.$ $\square$

Corollary 4.5.1 *(Convergence rate)* Based on the Berry-Esseen theorem [46], the probability distribution of M_n converges asymptotically to the normal distribution. Precisely,

$$\begin{aligned}
\sup_x \left| \mathbb{P}\left(\frac{M_n - np_0}{\sqrt{np_0(1-p_0)}} \leq x \right) - \Phi(x) \right| &\leq \frac{c}{\sqrt{np_0}\,(1-p_0)^{\frac{3}{2}}} \\
&\leq \frac{c}{\sqrt{n^{1-\eta}}\,(1-p_0)^{\frac{3}{2}}} \qquad (4.21)
\end{aligned}$$

where c is a purely numerical constant.

Remark 4.5.1 Note that using Corollary 4.5.1, higher-order moments of M_n can be easily calculated. In fact, Theorem 4.5.1 and Corollary 4.5.1 together say that as $n \to \infty$, the distribution of the normalized M_n converges at a rate less than $\frac{1}{\sqrt{n^{1-\eta}}}$ to the normal distribution concentrated between $\lfloor \beta_1(n) \rfloor$ and $\beta_2(n)$.

Remark 4.5.2 According to Theorem 4.5.1, the total throughput of the wireless network is lower-bounded as

$$R_{sum} \geq \frac{R_{\min}}{e^{R_{\min}} - 1} \log n \qquad (4.22)$$

In [47]-[48], a rate-constrained single-hop wireless network with Rayleigh fading channels is considered. An upper bound on the maximum number of active links is calculated as

$$m < \frac{\log n}{R_{\min}} \qquad (4.23)$$

Based on the threshold-based link activation strategy (TBLAS) presented in [48], the maximum number of active links and the total throughput are given by

$$\begin{aligned}
m_{TBLAS} &= \frac{\log n}{e^{R_{\min}} - 1} \\
R_{TBLAS} &= \frac{R_{\min}}{e^{R_{\min}} - 1} \log n \qquad (4.24)
\end{aligned}$$

Although the maximum number of active links achieved by the TBLAS is equal to the one obtained by Theorem 4.5.1, it can be seen that the upper bound presented in [48] is not very tight. [48] also presents a centralized double threshold-based link activation strategy (DTBLAS) to reach the upper bound in (4.23); however, they cannot provide closed-form expressions for optimal thresholds and numerically show that DTBLAS reaches this upper bound at $R_{\min} = 0$ or ∞ which are not practical. Hence, Theorem 4.5.1 compared to [48] has two advantages: First of all, a tighter upper bound is provided. Second, the upper bound meets the lower bound with probability approaching one.

4.5.2 Large Random Networks with Multi-Path Fading and Path Loss

Consider an extended wireless network consisting of N nodes Poisson-distributed with finite intensity λ. Model path loss with a simple attenuation function $d_{ji}^{-\alpha}$, where d_{ji} denotes the Euclidean distance between source i and destination i, and α represents the path-loss exponent. To model multi-path effect, independent fading channels between different source-destination pairs are considered. Accounting both multi-path and path loss, the channel between source i and destination i is determined by

$$|h_{ji}|^2 = |f_{ji}|^2 \, d_{ji}^{-\alpha}.$$

For Poisson-distributed nodes, the pdf of the distance between an arbitrary node and its kth nearest neighbor in a two-dimensional network is given by [49]

$$f_{d_k}(x) = e^{-\lambda\pi x^2} \frac{2(\lambda\pi x^2)^k}{x\Gamma(k)} \tag{4.25}$$

and the corresponding cumulative distribution function (cdf) is written as [49]

$$F_{d_k}(x) = 1 - \frac{\Gamma(k, \lambda\pi x^2)}{k!}\,. \tag{4.26}$$

If node j is the kth nearest neighbor of node i, for simplicity, d_k^α is substituted for d_{ji}^α. Then, the pdf of $d_k^{-\alpha}$ is calculated as

$$f_{d_k^{-\alpha}}(x) = e^{-\lambda\pi x^{-2/\alpha}} \frac{2(\lambda\pi x^{-2/\alpha})^k}{x\alpha\Gamma(k)}. \tag{4.27}$$

The expected value and the variance of d_k^α are also given by [49]

$$\mathbf{E}(d_k^\alpha) = \left(\frac{1}{2\pi\lambda}\right)^{\frac{\alpha}{2}} (k)_{\frac{\alpha}{2}} \tag{4.28}$$

$$\mathrm{Var}(d_k^\alpha) = \left(\frac{1}{2\pi\lambda}\right)^{\alpha} \frac{\Gamma(k)\Gamma(k+\alpha) - \Gamma^2(k+\frac{\alpha}{2})}{\Gamma^2(k)} \tag{4.29}$$

where the Pochhammer sequence $(k)_q$ is calculated by the series expansion [50]

$$(k)_q = k^q \left(1 - O(1/k)\right).$$

Theorem 4.5.2 *(Interference Bound)* In a two-dimensional large wireless network of Poisson-distributed nodes with a finite intensity and for $\alpha > 2$, interference from m source nodes at any arbitrary destination node i, denoted by $I_{i,m}$, is bounded almost surely as m goes to infinity. That is,

$$\mathbb{P}\left(\lim_{m\to\infty} I_{i,m} < \infty\right) = 1. \tag{4.30}$$

Proof 4.5.2 Assume there are m active links (i.e. m source nodes transmitting data), where $m \leq n$ can be any large integer. Based on (4.2), interference at arbitrary node i is given by

$$I_{i,m} = P \sum_{\substack{j=n-m+1 \\ j\neq i}}^{n} |h_{ji}|^2 = P \sum_{\substack{j=n-m+1 \\ j\neq i}}^{n} |f_{ji}|^2 \, d_{ji}^{-\alpha} \tag{4.31}$$

As the path-loss and multi-path fading terms are independent, using the Kol-

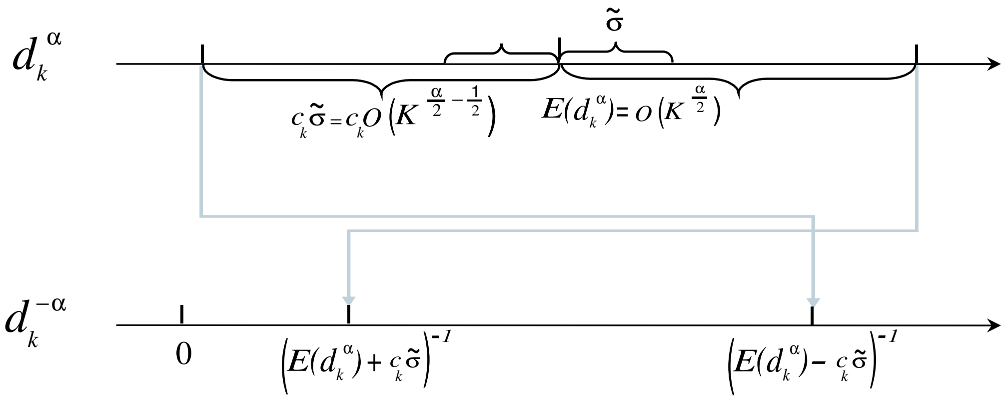


Figure 4.2: Concentration of random variable $d_k^{-\alpha}$ as $k \to \infty$

mogorov convergence criterion [46, page 286], as $m \to \infty$,

$$\sum_{\substack{j=n-m+1 \\ j\neq i}}^{n} \mathrm{Var}(|h_{ji}|^2) < \infty \quad \Rightarrow \quad \mathbb{P}\left(\sum_{\substack{j=n-m+1 \\ j\neq i}}^{n} \left(|h_{ji}|^2 - \mathbf{E}(|h_{ji}|^2)\right) < \infty \right) = 1. \tag{4.32}$$

Considering $\mathbf{E}(|f_{ji}|^2) = \mu_f < \infty$ and $\mathrm{Var}(|f_{ji}|^2) = \sigma_f^2 < \infty$ and

$$\sum_{\substack{j=n-m+1 \\ j\neq i}}^{n} \mathrm{Var}(|h_{ji}|^2) = (\mu_f^2 + \sigma_f^2) \sum_{\substack{j=n-m+1 \\ j\neq i}}^{n} \mathrm{Var}(d_{ji}^{-\alpha}) + \sigma_f^2 \sum_{\substack{j=n-m+1 \\ j\neq i}}^{n} \mathbf{E}^2(d_{ji}^{-\alpha}) \ ,$$

we need to show that as $m \to \infty$,

$$\begin{cases} \displaystyle\sum_{\substack{j=n-m+1 \\ j\neq i}}^{n} \mathrm{Var}(d_{ji}^{-\alpha}) < \infty \\ \displaystyle\sum_{\substack{j=n-m+1 \\ j\neq i}}^{n} \mathbf{E}^2(d_{ji}^{-\alpha}) < \infty. \end{cases} \tag{4.33}$$

The worse-case interference happens when interferers are the first to the $(m-1)$th nearest neighbors of node i. Hence, it is sufficient to show that

$$\begin{cases} \sum_{k=1}^{m-1} \mathrm{Var}(d_k^{-\alpha}) < \infty \\ \sum_{k=1}^{m-1} \mathbf{E}^2(d_k^{-\alpha}) < \infty. \end{cases} \tag{4.34}$$

Applying Stirling's approximation to (4.29),

$$\mathrm{Var}(d_k^{\alpha}) \sim \left(\frac{1}{2\pi\lambda}\right)^{\alpha} \frac{\alpha^2}{2}\, k^{\alpha-1} \;; \quad k \to \infty.$$

Let $\tilde{\sigma} \triangleq \sqrt{\mathrm{Var}(d_k^{\alpha})}$. For any $\epsilon > 0$ arbitrary close to zero, if $c_k = O(k^{\frac{1}{2}-\epsilon})$,

$$c_k\tilde{\sigma} = c_k\, O(k^{\frac{\alpha}{2}-\frac{1}{2}}) = o(k^{\frac{\alpha}{2}}) = o(\mathbf{E}(d_k^{\alpha})).$$

Then, it can be shown that d_k^{α} is concentrated between $[\mathbf{E}(d_k^{\alpha}) - c_k\tilde{\sigma}, \mathbf{E}(d_k^{\alpha}) + c_k\tilde{\sigma}]$ with probability approaching one as $k \to \infty$. That is, using Chebyshev's inequality [46]

$$\mathbb{P}\left(|d_k^{\alpha} - \mathbf{E}(d_k^{\alpha})| > c_k\tilde{\sigma}\right) \leq \frac{1}{c_k^2} \to 0 \;; \quad \text{as } k \to \infty.$$

As Figure 4.2 illustrates,

$$\mathbf{E}(d_k^{-\alpha}) \leq \frac{1}{\mathbf{E}(d_k^{\alpha}) - c_k\tilde{\sigma}} \sim \frac{1}{\mathbf{E}(d_k^{\alpha})} = (2\pi\lambda)^{\frac{\alpha}{2}}\, k^{-\frac{\alpha}{2}} \;; \quad \text{as } k \to \infty.$$

Hence, for $2 < \alpha \leq 4$, as $m \to \infty$,

$$\sum_{k=1}^{m-1} k^{-\alpha} < \sum_{k=1}^{m-1} k^{-\frac{\alpha}{2}} < \infty \quad \Rightarrow \quad \sum_{k=1}^{m-1} \mathbf{E}^2(d_k^{-\alpha}) < \sum_{k=1}^{m-1} \mathbf{E}(d_k^{-\alpha}) < \infty. \tag{4.35}$$

As Figure 4.2 indicates, the variance of $d_k^{-\alpha}$ can be also upper-bounded as

$$\mathrm{Var}(d_k^{-\alpha}) \leq \frac{1}{4}\left(\frac{1}{\mathbf{E}(d_k^{\alpha}) - c\tilde{\sigma}} - \frac{1}{\mathbf{E}(d_k^{\alpha}) + c_k\tilde{\sigma}}\right)^2 \sim \left(\frac{c_k\tilde{\sigma}}{\mathbf{E}^2(d_k^{\alpha})}\right)^2 = O\left(k^{-\alpha-2\epsilon}\right);$$

therefore,

$$\sum_{k=1}^{m-1} \mathrm{Var}(d_k^{-\alpha}) < \infty \ ; \quad \text{as } m \to \infty. \tag{4.36}$$

Then, using (4.32), (4.35), and (4.36),

$$\mathbb{P}\left(\lim_{m\to\infty} \sum_{\substack{j=1 \\ j\neq i}}^{m} \left(|h_{ji}|^2\right) < \infty \right) = 1. \tag{4.37}$$

$\square$

Let M_n denote the maximum number of simultaneous active links (out of n links) that can be supported with a rate greater than or equal to R_{min}.

Theorem 4.5.3 In a large wireless network of N Poisson-distributed nodes with finite intensity λ, under the assumption of independent fading channels for different links, and for any $\epsilon > 0$ arbitrarily close to zero, the maximum number of active links supporting the minimum rate is bounded as

$$\lim_{n\to\infty} \mathbb{P}\left(\lfloor p_0 - \epsilon \rfloor n \leq M_n \leq n\right) = 1$$

Proof 4.5.3 Consider the wireless network (4.1) with independent channel gains h_{ji}; $i, j = 1, \ldots, \lfloor N/2 \rfloor$. Suppose node j is the kth nearest neighbor of node i and $|f_{ji}|^2$; $i, j = 1, \ldots, \lfloor N/2 \rfloor$ is drawn from a given distribution with a finite mean and variance.

For any fixed activation threshold $h_0 > 0$, the number of "good" link channels can be characterized with $|h_{ii}|^2$ greater than h_0 as follows. Let $p_0 = \mathbb{P}(|h_{ii}|^2 \geq h_0)$. That is, with probability p_0, a link can be activated. Consider Bernoulli sequence 4.8. Then, the number of links having good channels has the same distribution as $M_n = \sum_{i=1}^{n} x_i$, which satisfies the Binomial distribution $B(n, p_0)$.

Define $C_m \triangleq \sup_i I_{i,m}$; for $i = n-m+1, \cdots, n$. According to Theorem 4.5.2,

$$\mathbb{P}\left(\lim_{m\to\infty} C_m < \infty\right) = 1.$$

Let $h_0 = (\sigma^2 + C_m)(e^{R_{min}} - 1)/P$. For any sufficiently large integer m, we show that if $\min_{n-m+1\leq i\leq n} |h_{ii}|^2 \geq h_0$, the minimum-rate constraint is satisfied for these m channel gains. That is,

$$\log\left(1 + \frac{P\,|h_{ii}|^2}{\sigma^2 + \sum\limits_{\substack{j=n-m+1\\ j\neq i}}^{n} P\,|h_{ji}|^2}\right) \geq \log\left(1 + \frac{Ph_0}{\sigma^2 + C_m}\right) = R_{\min}. \quad (4.38)$$

Clearly, based on (4.2), the minimum-rate constraint is satisfied for these m channel gains.

Then, we show that for any $\epsilon > 0$ arbitrary close to zero, if $m \leq (p_0 - \epsilon)n$, $\min_{n-m+1\leq i\leq n} |h_{ii}|^2 \geq h_0$ holds with probability approaching one. Using (4.26) and (3.27) and substituting x_f for $\left(|f_{ii}|^2/h_0\right)^{\frac{1}{\alpha}}$,

$$\begin{aligned} p_0 = \mathbb{P}(|h_{ii}|^2 \geq h_0) &= \mathbb{P}(|f_{ii}|^2 \cdot d_{ii}^{-\alpha} \geq h_0) \qquad (4.39)\\ &= \mathbf{E}\left(\mathbb{P}\left(d_{ii} \leq \left(\frac{|f_{ii}|^2}{h_0}\right)^{\frac{1}{\alpha}}\right)\right)\\ &= \mathbf{E}\left(1 - \frac{\Gamma(k, \lambda\pi x_f^2)}{k!}\right)\\ &= \mathbf{E}\left(1 - \frac{e^{-\lambda\pi x_f^2}}{k}\sum_{l=0}^{k-1}\frac{(\lambda\pi x_f^2)^l}{l!}\right) > 0. \end{aligned}$$

Now, as $n \to \infty$,

$$\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n} \geq \frac{(n\epsilon + 1)^2}{2np_0} \sim \frac{n\epsilon^2}{2p_0} \to \infty. \quad (4.40)$$

As $m - 1 \leq np_0$, the Chernoff bound on the sum of independent Poisson trials can be used as

$$\mathbb{P}(M_n \geq m) \geq 1 - \exp\left(-\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n}\right) \tag{4.41}$$

which approaches one based on (4.40). Thus, we proved as $n \to \infty$, with probability approaching one, there are at least $\lfloor p_0 - \epsilon \rfloor n$ good channels with $|h_{ii}|^2 \geq h_0$, for which the minimum rate constraint is satisfied. □

Corollary 4.5.2 *(Convergence rate)* Based on the Berry-Esseen theorem, the probability distribution of M_n converges asymptotically to the normal distribution. That is,

$$\sup_x \left| \mathbb{P}\left(\frac{M_n - np_0}{\sqrt{np_0(1-p_0)}} \leq x\right) - \Phi(x) \right| \leq \frac{c}{\sqrt{np_0}\,(1-p_0)^{\frac{3}{2}}} \tag{4.42}$$

where c is a purely numerical constant.

Remark 4.5.3 All moments and statistics of M_n can be easily calculated using Corollary 4.5.2. In fact, Theorem 4.5.3 and Corollary 4.5.2 together say that as $n \to \infty$, the distribution of the normalized M_n converges at a rate less that $\frac{1}{\sqrt{n}}$ (Note that p_0 does not depend on n.) to the normal distribution concentrated between $\lfloor p_0 - \epsilon \rfloor n$ and n.

Remark 4.5.4 Note that Theorems 4.5.2 and 4.5.3 are proved for any multi-path fading distribution satisfying the following conditions:

1. $\mathbf{E}(|f_{ji}|^2) < \infty$,

2. $\mathrm{Var}(|f_{ji}|^2) < \infty$,

In [40] and [44], to bound the capacity of wireless ad-hoc networks, it is assumed that the multi-path fading distribution has an exponentially-decaying tail (e.g. commonly used fading distributions, namely, Rayleigh, Rician and Nakagami). Here, this assumption is relaxed and the results hold for any distribution satisfying the aforementioned conditions. In fact, the interference bound and the throughput scaling laws are generalized to a variety of distributions with a finite mean and variance.

Corollary 4.5.3 *(Throughput scaling law)* In a two-dimensional large wireless network of N Poisson-distributed nodes with a finite intensity and for $\alpha > 2$, under the assumption of independent fading channels for different links, the achievable per-node throughput obtained by multi-hop routing scales with $\Theta\left(\frac{1}{\sqrt{N}}\right)$.

Proof 4.5.4 According to Theorem 4.5.3, almost all links can be activated simultaneously in a large random wireless network with fading channels. In general, to transmit information from each node to its final destination, single-hop or multi-hop routing can be selected. However, for n random samples of channel gains drawn from a fading distribution having an exponentially decaying tail, including Rayleigh, Rician, and Nakagami,

$$\lim_{n\to\infty} \mathbb{P}\left(|f_{ji}|^2 \leq \log n\right) = 1.$$

Hence, as Theorem 4.5.1 indicates, only $\log n$ multi-path fading gains are large enough to support a single-hop transmission. For $n-\log n$ channel gains, multi-path fading gains cannot compensate signal attenuation due to path loss and information needs to be sent in multiple hops to its final destination. Note that at each time slot, some destination nodes are just relaying information to other neighboring nodes in the network.

In [37], it is shown that the average number of hops is $O(\sqrt{N})$ in a square-shaped wireless network with side dimension $\sqrt{N}$. We show that the same average number of hops holds for the wireless network (4.1). For example, consider N nodes Poisson-distributed and a circle of radius $\sqrt{N}$. The circle contains N nodes with probability approaching one and nodes are uniformly distributed inside the circle. Hence, the average distance between any node pair is of the order $\sqrt{N}$. Based on (4.28), the average distance between a node and its nearest neighbor equals $O(1)$. Hence, using the nearest neighbor routing protocols, the average number of hops required to reach from any arbitrary source node to its final destination is $O(\sqrt{N})$.

Denoting the maximum possible transmission rate by $R_{max} < \infty$, based on Theorem 4.5.3, the achievable per-node throughput obtained by multi-hop routing, $T(N)$, is bounded as

$$\begin{cases} T(N) = \frac{1}{N} R_{sum} \geq \frac{1}{N}\, \Omega(\sqrt{N})\, R_{\min} \\ T(N) = \frac{1}{N} R_{sum} \leq \frac{1}{N}\, \Omega(\sqrt{N})\, R_{\max} \end{cases} \quad \Rightarrow \quad T(N) = \Theta\left(\frac{1}{\sqrt{N}}\right) \tag{4.43}$$

□

Comparing the aforementioned result with the one in [44], the achievable per-node throughput has the same asymptotic order; however, Corollary 4.5.3 also provides a tight upper bound on the achievable throughput of rate-constrained wireless networks using multi-hop routing. Recently, using the laws of physics in communication channels, [52] obtains an upper bound on the per-node communication rate that all nodes can achieve simultaneously and it shows that the per-node throughput is upper-bounded by

$$O\left(\frac{(\log n)^2}{\sqrt{n}}\right).$$

Hence, in terms of asymptotic ordering, Corollary 4.5.3 also confirms the upper bound presented in [52].

Remark 4.5.5 According to Theorem 4.5.2, interference power is bounded; therefore, the central limit theorem cannot be invoked to conclude interference is Gaussian. Hence, the Shannon's capacity formula only provides a lower bound on the rate and the throughput upper bound presented in Corollary 4.5.3 is only an achievable rate.

Chapter 5

Conclusion and Future Work

This thesis addresses the asymptotic and probabilistic analysis of the maximum number of active users/links supporting a minimum rate in a multi-user wireless system. In this chapter, all contributions mentioned throughout the thesis are summarized and some directions for future work are presented.

5.1 Research Contributions

This thesis presents a novel idea and analysis in the context of multi-user channels and random wireless networks. Particularly, the thesis contributions are as follows:

- **Rate-constrained Broadcast Channels:** A power allocation scheme is proposed to maximize the number of active receivers, for each of which, a minimum rate $R_{\min} > 0$ can be achieved. Three fading distributions, namely, Rayleigh, Rician, and Nakagami are considered. Under the assumption of independent Rayleigh fading channels for different receivers, as the total number of receivers n goes to infinity, the maximum number of active receivers is shown to be arbitrarily close to $\ln(P \ln n)/R_{\min}$ with probability approaching one, where P is the total transmit power. The results obtained for Rayleigh fading are extended to the cases of Rician and Nakagami fading models. Under the assumption of independent Rician fading channels for different receivers, as the total number of receivers n goes to infinity, the maximum number of active receivers is shown to be equal to $\ln(2P \ln n)/R_{\min}$ with probability approaching one. For broadcast channels with Nakagami fading, the maximum number of active receivers is shown to be equal to $\ln(\frac{\omega}{\mu} P \ln n)/R_{\min}$ with probability approaching one, where ω and μ are the Nakagami distribution spread and shape parameters respectively. A by-product of the results is to also provide a power allocation strategy that maximizes the total throughput subject to the rate constraints.

- **Rate-constrained Multiple-Acess Channels:** User capacity of fading multiple-access channels in which a minimum rate must be maintained for all active transmitters is asymptotically analyzed. The joint decoding scheme is used at the receiver since it is well known that this decoding scheme maximizes the total throughput. Three fading distributions, namely, Rayleigh, Rician, and Nakagami are considered. Under the assumption of independent Rayleigh fading channels for different transmitters, the maximum number of active users, $\tilde{\nu}(n)$, is shown to be arbitrarily close to $\frac{1}{R_{min}} \ln\left(\frac{P\tilde{\nu}(n)}{\sigma^2} \ln n\right)$ with probability approaching one as the total number of users n goes to infinity, where P denotes each transmitter's power and σ^2 is the background noise variance. As it can be seen, the number of active transmitters is given by a non-linear fixed-point equation. Under the assumption of independent Rician fading channels for different transmitters, the maximum number of active users, $\tilde{\nu}_1(n)$, is shown to be equal to $\frac{1}{R_{min}} \ln\left(\frac{2P\tilde{\nu}_1(n)}{\sigma^2} \ln n\right)$ with probability approaching one. For multiple-access channels with Nakagami fading, the maximum number of active transmitters, $\tilde{\nu}_2(n)$, is shown to be equal to $\frac{1}{R_{min}} \ln\left(\frac{\omega P\tilde{\nu}_2(n)}{\mu\,\sigma^2} \ln n\right)$ with probability approaching one.

- **Rate-constrained Random Wireless Networks:** The maximum number of active links supporting a minimum rate is asymptotically obtained in a wireless network with an arbitrary topology. It is assumed that each source-destination pair communicates through a fading channel and destinations receive interference from all other active sources. Two scenarios are considered: 1) Small networks with multi-path fading, 2) Large Random networks with multi-path fading and path loss. *In the first case*, under the assumption of independent Rayleigh fading channels for different source-destination pairs, it is shown that the optimal number of active links is of the order $\log N$ with probability approaching one as the total number of nodes, N, tends to infinity. The achievable total throughput also scales logarithmically with the total number of links/nodes in the network. Comparing to [48], the analysis presented in this thesis has two advantages: First of all, a tighter upper bound is provided. Second, the upper bound meets the lower bound with probability approaching one. *In the second case*, a two-dimensional large wireless network is considered and it is assumed that nodes are Poisson distributed with a finite intensity. Under the assumption of independent multi-path fading for different source-destination pairs, it is shown that the optimal number of active links is of the order N with probability approaching one. As a result, the achievable per-node throughput scales with $\Theta(\frac{1}{\sqrt{N}})$ bits per second with a multi-hop routing strategy. This result complements those in [44] on the asymptotic throughput of multi-hop wireless networks and have the following contributions: 1) The assumption of the network topology is relaxed. 2) The multi-path fading distribution does not need to have an exponential tail; in other words, the results hold for any distribution with a finite mean and variance. 3) In [44], interference at any arbitrary node is upper-bounded by a constant times $\log N$. Here, it is shown that interference from all active sources at any arbitrary destination is bounded. 4) The proof presented in Section 4.5.2 is only based on the probability theory and is simpler.

5.2 Future Research Directions

The work presented in this thesis answered some questions and perhaps raises new questions. At this point, some ideas that can be of interest for future research are as follows:

- **MIMO Multi-User Channels and Wireless Networks**: In this thesis, all transmitters and receivers are equipped with single antenna. It is well-studied that using multiple antennas at transmitters and receivers increases the channel capacity. Hence, the MIMO rate-constrained multi-user channels and random wireless networks can be considered and the maximum number of simultaneously active users/nodes can be asymptotically analyzed. Note that this extension is not very straightforward in general as we are now dealing with channel matrices instead of scalar channel gains that can be simply sorted.

- **Path-Loss Effect in Rate-Constrained Multi-User Channels**: To analyze the user capacity of broadcast and multiple-access channels, only the multi-path fading effect is considered in Chapters 2 and 3. To model the communication channel more precisely, the path-loss effect should be also taken into account. If it is assumed that the distance between each transmitter-receiver pair, d, is bounded as $d_{min} \leq d \leq d_{max}$, it can be easily shown that the user capacity scaling laws do not change for rate-constrained multi-user channels. Note that this assumption is valid in small wireless networks and cellular communications in which the distance is limited by the cell size. However, in the scenarios that this assumption is violated, the user capacity scaling laws change as the channel distribution and statistics are now different.

- **Three-Dimensional Rate-Constrained Wireless Networks**: A two-dimensional random network of Poisson-distributed nodes is considered in Chapter 4. If the nodes are assumed to be Poisson-distributed inside a three-dimensional region, the distribution of inter-node distances will change.

Hence, the maximum number of active links supporting the minimum rate and the per-node throughput may have different scaling laws.

- **Embedding Multi-User Coding in Wireless Networks**: In Chapter 4, all transmitters and receivers are only capable of point-to-point coding. In other words, no broadcast and multiple-access channel is embedded in the wireless network. If each node is equipped with a multi-user encoder/decoder, the network analysis will be more complicated. However, one can assume a particular number of multi-user channels are embedded in the network and investigate the problems mentioned in Chapter 4.

- **Interference Bound for Other Wireless Networks**: One of the contributions mentioned in Chapter 4 is the interference bound presented in theorem 4.5.2 for two-dimensional large random wireless networks of Poisson-distributed nodes with a finite intensity. The similar interference bound can be also calculated for other wireless networks (e.g. cognitive networks) with different assumptions and topologies.

List of Publications

1. H. Kehavarz, L.-L. Xie, and R. R. Mazumdar, “On the optimal number of active receivers in fading broadcast channels,” *IEEE Trans. Information Theory*, vol. 54, no. 3, pp. 1323-1327, March 2008.

2. H. Kehavarz, L.-L. Xie, and R. R. Mazumdar, “User Capacity of fading multi-user channels with a minimum rate constraint,” *IEEE Intern. Conf. Communications (ICC)*, pp. 4649-4653, Beijing, China, May 2008.

3. H. Keshavarz, L.-L. Xie, and R. R. Mazumdar, ”User Capacity of Rician and Nakagami fading broadcast channels,” *IEEE Global Telecommunications Conf. (GLOBECOM)*, New Orleans, USA, November 2008. (Accepted)

4. H. Keshavarz, L.-L. Xie, and R. R. Mazumdar, ”User capacity scaling laws for fading multiple-access channels,” *IEEE Trans. Wireless Communications*, March 2008. (Submitted)

5. H. Keshavarz, R. R. Mazumdar, and L.-L. Xie, “Maximum Number of Active Links in Wireless Networks with Fading Channels”, *Information Theory and Applications (ITA08)*, San Diego, USA, January 2008.

6. H. Keshavarz, R. R. Mazumdar, and L.-L. Xie, ”On the number of active links in rate-constrained random wireless networks,” *IEEE Conf. Computer Communications (INFOCOM)*, Rio de Janeiro, Brazil, 2009. (Submitted)

References

[1] D. N. C. Tse and S. V. Hanly "Multiaccess fading channels-Part I: Polymatroid structure, optimal resource allocation and throughput capacities", *IEEE Trans. Information Theory*, vol. 44, no. 7, pp. 2796-2815, November 1998.

[2] L. Li and A. J. Goldsmith, "Capacity and optimal resource allocation for fading broadcast channels-Part I: Ergodic Capacity," *IEEE Trans. Information Theory*, vol. 47, no. 3, pp. 1083-1102, March 2001.

[3] L. Li and A. J. Goldsmith, "Capacity and optimal resource allocation for fading broadcast channels-Part II: Outage Capacity," *IEEE Trans. Information Theory*, vol. 47, no. 3, pp. 1103-1127, March 2001.

[4] L. Li, N. Jindal, and A. J. Goldsmith, "Outage capacities and optimal power allocation for fading multiple-access channels," *IEEE Trans. Information Theory*, vol. 51, no. 4, pp. 1326-1347, April 2005.

[5] S. Hanly and D. N. C. Tse, "Multiaccess fading channels-Part II: Delay-limited capacities," *IEEE Trans. Information Theory*, vol. 44, no. 7, pp. 28162831, November 1998.

[6] N. Jindal and A. J. Goldsmith, "Capacity and optimal power allocation for fading broadcast channels with minimum rates," *IEEE Trans. Information Theory*, vol. 49, no. 11, pp. 2895-2909, November 2003.

[7] Cisco Systems, Inc., http://www.cisco.com/en/US/netsol/ns767/net_brochures_list

[8] National Institute of Standards and Technology, http://w3.antd.nist.gov

[9] R. Bruno, M. Conti, and E. Gregori, “Mesh networks: Commodity Multihop ad hoc networks,” *IEEE Communications Magazine*, pp. 123-131, March 2005.

[10] E. Jorswieck and H. Boche, “Rate balancing for the multi-antenna Gaussian broadcast channel,” *IEEE Intern. Symp. Spread Spectrum Techniques and Applications (ISSSTA)*, pp. 545-549, Prague, Czech Republic, September 2002.

[11] Z. Yin and J. Xie, “Joint power and rate allocation for the downlink in wireless CDMA data networks,” *IEEE Intern. Symp. Personal, Indoor and Mobile Radio Communications (PIMRC)*, pp. 326-330, Beijing, China, September 2003.

[12] P. Song and L. Cai, “Multi-user subcarrier allocation with minimum rate requests for downlink OFDM packet transmission,” *IEEE Vehicular Technology Conf. (VTC)*, pp. 1920-1924, Milan, Italy, May 2004.

[13] K. Kumaran and H. Viswanathan, “Joint power and bandwidth allocation in downlink transmission,” *IEEE Intern. Symp. Information Theory*, pp. 216, Chicago, USA, July 2004.

[14] B. Stender, C. Huppert, and G. Richter, “Fast broadcasting,” *Intern. Symp. Wireless Communication Systems (ISWCS)*, pp. 408-411, Valencia, Spain, September 2006.

[15] T. K. Chee, C.-C. Lim, and J. Chooi, “A cooperative game theoretic framework for resource allocation in OFDMA systems,” *IEEE Singapore Intern. Conf. Communication systems (ICCS)*, Singapore, October 2006.

[16] C. Lengoumbi, P. Godlewski, and P. Martins, “Dynamic subcarrier reuse With rate guaranty in a downlink multicell OFDMA system,” *IEEE Intern. Symp. Personal, Indoor and Mobile Radio Communications (PIMRC)*, Helsinki, Finland, September 2006.

[17] S. Boppanna and J. M. Shea, “Downlink user capacity of cellular systems: TDMA vs. dirty paper coding,” *IEEE Intern. Symp. Information Theory (ISIT)*, pp. 754-758, Seattle, USA, July 2006.

[18] T. Michel and G. Wunder, "Minimum rates scheduling for OFDM broadcast channels," *IEEE Intern. Conf. Acoustics, Speech and Signal Processing (ICASSP)*, pp. IV 41-44, Toulouse, France, May 2006.

[19] G. Wunder and T. Michel, "Minimum rates scheduling for MIMO OFDM broadcast channels," *IEEE Intern. Symp. Spread Spectrum Techniques and Applications (ISSSTA)*, pp. 510-514, Manaus, Brazil, August 2006.

[20] G. Wunder and T. Michel, "Optimal resource allocation for parallel Gaussian broadcast channels: Minimum rate constraints and sum power minimization," *IEEE Trans. Information Theory*, vol. 53, no. 12, pp. 4817-4822, December 2007.

[21] N. Zorba and A. I. Perez-Neira, "Robust multibeam opportunistic schemes under quality of service constraints," *IEEE Intern. Conf. Communications (ICC)*, pp. 5371-5376, Glasgow, Scotland, June 2007.

[22] W. Xu, C. Zhao, P. Zhou, and Y. Yang, "Efficient adaptive resource allocation for multiuser OFDM systems with minimum rate constraints," *IEEE Intern. Conf. Communications (ICC)*, pp. 5126-5131, Glasgow, Scotland, June 2007.

[23] C. Shuping, L. Huibinu, H. Jing, Z. Dong, and K. Asimakis, "Generalized scheduler providing multimedia services over HSDPA," *IEEE Intern. Conf. Multimedia and Expo (ICME)*, pp. 927-930, Beijing, China, July 2007.

[24] I. Koutsopoulos and L. Tassiulas, "The Impact of Space Division Multiplexing on Resource Allocation: A Unified Treatment of TDMA, OFDMA and CDMA," *IEEE Trans. Communications*, vol. 56, no. 2, pp. 260-269, February 2008.

[25] M. Pätzold, *Mobile Fading Channels*, John Wiley & Sons, Inc., New York, 2002.

[26] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, Second Edition, John Wiley & Sons, Inc., New York, 2006.

[27] R. Motwani and P. Raghavan, *Randomized Algorithms*, Cambridge University Press, 1995.

[28] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions: with Formulas, Graphs, and Mathematical Tables*, Dover Publications, New York, 1965.

[29] R. S. Cheng and S. Verdu, "Gaussian multiaccess channels with ISI: Capacity region and multiuser Water-Filling", *IEEE Trans. Information Theory*, vol. 39, no. 3, pp. 773-785, May 1993.

[30] P. Viswanath and D. N. C. Tse, "Asymptotically Optimal Water-Filling in Vector Multiple-Access Channels" *IEEE Trans. Information Theory*, vol. 47, no. 1, pp. 241-267, January 2001.

[31] G. J. Foschini and S. Verdú, "Multiantenna Channels: Capacity, Coding and Signal Processing", *DIMACS Workshop on Signal Processing for Wireless Transmission*, Rutgers University, October 2002.

[32] N. Jindal, S. Vishwanath, and A. Goldsmith, "On the duality of Gaussian multiple-access and broadcast channels," *IEEE Trans. Information Theory*, vol. 50, no. 5, pp. 768-783, May 2004.

[33] M. Goyal, V. Sharma, and A. Kumar, "Optimal power allocation for multiaccess fading channels with minimum rate guarantees," *IEEE Intern. Conf. Personal Wireless Communications*, pp. 20-24, New Delhi, India, December 2002.

[34] S. Barbarossa and G. Scutari, "Optimal rate allocation for multiple access wideband systems with rate profile constraint and adaptive power control," *IEEE Workshop on Signal Processing Advances in Wireless Communications*, pp. 115-119, Rome, Italy, June 2003.

[35] S. Shi, M. SChubert, and H. Boche, "Capacity balancing for multiuser MIMIO systems," *IEEE Intern. Conf. Acoustics, Speech and Signal Processing (ICASSP)*, pp. III 397-III 400, Honolulu, Hawaii, USA, 2007.

[36] P. Gupta and P. R. Kumar, "The capacity of wireless networks", *IEEE Trans. Information Theory*, vol. 46, no. 2, pp. 388-404, March 2000.

[37] M. Franceschetti, O. Dousse, D. N. C. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory", *IEEE Trans. Information Theory*, vol. 53, no. 3, pp. 1009-1018, March 2007.

[38] L.-L. Xie and P. R. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation", *IEEE Trans. Information Theory*, vol. 50, no. 5, pp. 748-767, May 2004.

[39] A. Özgür, O. Lévêque, and E. Preissmann, "Scaling laws for one- and two-dimensional random wireless networks in the low-attenuation regime", *IEEE Trans. Information Theory*, vol. 53, no. 10, pp. 3573-3585, October 2007.

[40] S. Toumpis and A. J. Goldsmith, "Large wireless networks under fading, mobility, and delay constraints", *IEEE Conf. Computer Communications (INFOCOM)*, pp. 609-619, Hong Kong, China, March 2004.

[41] A. Jovičić, P. Viswanath, and S. R. Kulkarni, "Upper bounds to transport capacity of wireless networks", *IEEE Trans. Information Theory*, vol. 50, no. 11, pp. 2555-2565, November 2004.

[42] F. Xue, L.-L. Xie, P. R. Kumar, "The transport capacity of wireless networks over fading channels", *IEEE Trans. Information Theory*, vol. 51, no. 3, pp. 834-847, March 2005.

[43] R. Gowaikar, B. Hochwald, and B. Hassibi, "Communication over a wireless network with random connections", *IEEE Trans. Information Theory*, vol. 52, no. 7, pp. 2857-2871, July 2006.

[44] Y. Nebat, “A lower bound for the achievable throughput in large random wireless networks under fixed multipath fading”, *Intern. Symp. Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt)*, pp. 1-10, Boston, USA, April 2006.

[45] A. Dembo and O. Zeitouni, *Large Deviations Techniques and Applications*, Second Edition, Springer, 1998.

[46] A. Gut, *Probability: A Graduate Course*, Springer, 2005.

[47] M. Ebrahimi, M. A. Maddah-Ali, and A. K. Khandani, “Interference-limited versus noise-limited communication over dense wireless networks,” *10th Canadian Workshop on Information Theory*, pp. 172-175 , Edmonton, Canada, June 2007.

[48] M. Ebrahimi, M. A. Maddah-Ali, and A. K. Khandani, “Rate-constrained wireless networks with fading channels: Interference-limited and noise-limited regimes,” *Submitted to IEEE Trans. Information Theory*, October 2007.

[49] M. Haenggi, “On distances in uniformly random networks”, *IEEE Trans. Information Theory*, vol. 51, no. 10, pp. 3584-3586, October 2005.

[50] R. L. Graham, D. E. Knuth, and O. Patashnik, *Concrete Mathematics: A Foundation for Computer Science*, Second Edition, Reading, Massachusetts: Addison-Wesley, 1994.

[51] V. K. Rohatgi, *An Introduction to Probability Theory and Mathematical Statistics*, John Wiley & Sons, Inc., New York, 1976.

[52] M. Franceschetti, M. D. Migliore, and P. Minero, “The capacity of wireless networks: Information-theoretic and physical limits”, *45th Annual Allerton Conf. Communication, Control, and Computing (Allerton07)*, Monticello, USA, September 2007.