

A Review and Analysis of the Sustainability and Equity
Of Social Security Adjustment Mechanisms

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A thesis

presented to the University of Waterloo

in fulfillment of the

thesis requirement of the degree of

Doctor of Philosophy

in

Actuarial Science

Waterloo, Ontario, Canada, 2008

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AUTHOR'S DECLARATION

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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ABSTRACT

This thesis examines stabilizing mechanisms in social security retirement systems (“SSRS”), especially those purporting to be automatic balancing mechanisms (“ABM”). It develops a consistent approach to identifying whether an ABM is robust, partial or transitory and establishes a terminology to classify balancing mechanisms. Both financial and equitable balances are considered in assessing whether an ABM achieves balance. Families of definitions of equity are presented and a benchmark by which to measure equitable balance is defined and applied. The balancing mechanisms of Canada, Germany, Japan and Sweden are described, evaluated and classified. None of these mechanisms are found to be robust.

This thesis provides a critical analysis of an approach referred to as integration to financial markets and the approach is found to be deficient. In analyzing the Swedish SSRS a critical error in the way assets are calculated is identified and a suitable correction is proposed. A further weakness in the application of the Swedish ABM is identified that means that once an imbalance occurs, balance is unlikely to be restored. The thesis also discusses some of the unusual characteristics of the steady-state contribution rate calculation for the Canadian SSRS and shows that although it has limited application and does not appear to depend on any actuarial principle, the steady-state contribution rate calculation creates a tension between the near and distant future, which is a factor in achieving financial balance over a seventy-five year horizon. With respect to the balancing mechanism in the Canadian SSRS, the thesis proposes a change in how the mechanism is defined so that the mechanism would be robust, within certain ranges.

ACKNOWLEDGEMENTS

I wish to thank my immediate family, Pixie, Robert and Emily, without their support and sacrifice this venture would not have been possible. I also acknowledge the contribution of my parents who extolled the desirability of a university education starting in my childhood days.

I gratefully acknowledge the assistance and encouragement provided by my supervisor, Robert L. Brown, from the admission process to the thesis completion, he was always prompt to respond and to provide helpful advice.

I also acknowledge the constructive advice provided by the examiners, Robert Brown, Eli Donkar, Mary Hardy, Ranjini Jha and Christiane Lemieux. Christiane was accommodating in receiving my phone calls from whatever country I happened to be working in. Thank you also to Harry Panjer for the interest he showed in my progress.

The production of a thesis is physical work. Thank you to Joan Hatton for all the hours spent in typing and formatting. Thank you also to Mary Lou Dufton who guided me through the regulatory aspects of completion and was always ready to jump in and assist when some situation arose that threatened to postpone my expected completion date.

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List of Abbreviations

The following abbreviations are used throughout the thesis. These abbreviations may be used in the singular or the plural, as required by the context. There are also symbols defined in certain subsections. They are not listed here but are defined in the subsection in which they are used.

ABM	automatic balancing mechanism
Act	Canada Pension Plan Act (in Canadian context)
CPP	Canada Pension Plan
Current Rule	used in CPP context and defined in subsection 7.5.1
CWB	covered wage bill
DB	defined benefit
DC	defined contribution
DI	Disability Insurance (in U.S. social security context)
EPI	Employees' Pension Insurance Scheme (in Japanese context)
Equitable Rule	used in CPP context and defined in subsection 7.5.1
FDC	funded defined contribution (or financial defined contribution)
Financial Rule	used in CPP context and defined in subsection 7.5.1
GDP	Gross Domestic Product
IFM	integration to financial markets
ILO	International Labour Organization
NDC	notional defined contribution (or non-financial defined contribution)
NP	National Pension Scheme (in Japanese context)
OAS	Old Age, Survivors Insurance (in U.S. social security context)
OASDI	Old Age, Survivors, and Disability Insurance

OLG	overlapping generations
SSRS	social security retirement systems
TFR	total fertility rate
UN 2002	United Nations 2002 populations projections
U.S.	United States

1.0 Chapter Overview

This chapter presents an overview of this thesis. It enumerates important characteristics of social security retirement systems (“SSRS”). The automatic balancing mechanism (“ABM”) in the Swedish SSRS is one that most people refer to when discussing ABM. It is described.

This thesis assesses ABM with respect to whether they are automatic and whether they achieve financial balance and/or equitable balance. A review of the literature regarding the financial and equitable balance of the Swedish ABM is presented.

This chapter lists the important contributions that this research makes to the study of ABM and SSRS. The chapter concludes with an outline of what will be presented in subsequent chapters.

1.1 Introduction

It is inherent to SSRS that some guarantees will be imbedded. In this thesis, guarantee is being used in the sense of contractual undertaking or implied promise. It is recognized that even that which is guaranteed may not be delivered as originally promised and that governments may have reserved the right to revise or change the terms, the tax and benefit structures.

For example, in a traditional defined benefit (“DB”) partially funded SSRS, such as exists in Canada, a benefit based on an established formula is guaranteed to be paid for a retiring participant’s life. Moreover, the pension is adjusted to offset the erosion of purchasing power due to inflation, which is another type of guarantee. There may also be guaranteed death benefits both before and after retirement. The contribution schedule may be presented as fixed or guaranteed. Or, for example, in a fully funded defined contribution (“DC”) SSRS, such as exists in Chile, there are guarantees in that the accumulated contribution balances at retirement are converted to income, which is guaranteed to be paid for the retiring participant’s remaining lifetime. These are just examples of a few of the guarantees that may exist. Guarantees will be discussed more fully later.

SSRS are long-term undertakings. In the most limited form, there is a commitment to the generation of participants able to work for whatever is defined to be the period of full service that such participants, when they retire, will receive benefits determined in some stated manner for the rest of their lives. For some participants such an undertaking may last at least seventy years, e.g., forty years during the service period establishing full benefit entitlement and thirty years during the period when benefits are received. For

other participants of this generation such an undertaking may be shorter or longer. But typically the establishment of an SSRS is not merely a commitment to current generations; it is viewed as an undertaking to future generations as well. As such an SSRS is a long-term undertaking, potentially with an indefinite horizon.

There are many factors that may impact the ability of an SSRS to deliver on its guarantees over a long horizon. Increasing longevity has been an important factor affecting the cost of lifetime pension guarantees. The rate of inflation impacts the cost of providing inflation-adjusted (referred to as “indexed”) pensions. Investment returns are a factor in partially funded schemes and whenever an interest rate is assumed in determining the value of a guarantee or the amount to be guaranteed. Within an SSRS with guarantees, each individual participant cannot be responsible for fully backing or supporting his or her own guarantee, some pooling or insurance will apply. The participant may pay some risk premium that is more or less than the actual value of the participant’s guarantee but each participant does not pay only the exact value of his or her guarantee, there is some pooling of risk associated with providing guarantees. As such, demographic factors and changes to those demographic factors will affect the ability of the system to deliver on the guarantees. Depending on what benefits the SSRS includes, there may be other factors affecting the ability to deliver on guarantees such as: incidence of disability and rates of recovery (if disability guarantees exist); family composition and survivor mortality (if survivorship guarantees exist); participation rates and work history (if there is not a one-to-one link between benefits and work history). Finally, and perhaps most importantly, the assumptions made in establishing the SSRS will affect its ability to deliver on its guarantees. If the contribution schedule is unlikely to support the generous level of benefits guaranteed, except in the most optimistic scenarios, the ability of the SSRS to deliver on its guarantees is more likely to be challenged than if the contribution schedule and benefit levels have been set using realistic assumptions.

Due to:

- the existence of guarantees,
- the long-term horizon of the SSRS, and
- the many factors which may affect the guarantees, all of which may change in the long term,

it is likely that all SSRS will experience stress at some time. This stress may manifest itself in different ways, or observers, especially if they are not impartial, may identify the stress more strongly in relation to certain components. For example, the stress may be stated as a funding deficiency, or as a need to increase contribution rates, or as a requirement to cut benefits, or as a need to raise the retirement age to be eligible for full benefits. Reforms that address the stress by changing some parameter, such as the contribution rate or retirement age, are referred to as parametric reforms. But structural reform may also be the response to stress. In a structural reform, the structure of the SSRS is changed. For example, in 1981 Chile replaced its DB SSRS by a system of fully funded DC accounts (“FDC”). Or, for example in 1999, Sweden replaced its DB SSRS by a combined system of notional defined contribution accounts (“NDC”) and FDC accounts. These are both examples of structural reform.

Because SSRS are likely to require adjustment, at some point, to respond to the stress experienced, it is reasonable to attempt to describe the characteristics that the most acceptable adjustments to an SSRS should have. These characteristics will be discussed in detail in Chapter 2. Certain SSRS have adopted automatic balancing mechanisms in order to respond to the stresses and maintain balance.

Automatic balancing mechanisms are relatively scarce, perhaps non-existent, as will be discussed. However, there is an immediate appeal to an effective balancing mechanism that makes the necessary adjustments in response to stresses that have occurred. The focus of this thesis is automatic balancing mechanisms (“ABM”). The most famous example of an ABM occurs in the Swedish SSRS. Accordingly, this ABM will be described in this chapter and the literature regarding this ABM reviewed. In the body of this thesis, mechanisms used by SSRS in four countries (Sweden, Japan, Germany, Canada) that might, at least at first glance, be considered ABM are reviewed and critiqued and an assessment made whether the mechanism can be truly considered an ABM. In preparation for such an assessment, there are several preliminary steps.

The first step is to develop terminology that may be used to classify ABM depending on whether or not they are automatic and on how well they succeed in achieving balance. This is done in Chapter 2. The next step is to describe more fully the concept of balance among and within cohorts. In Chapter 4, the literature on generational accounting and intergenerational equity is reviewed and the relevant aspects for making an assessment of balance are identified. In Chapter 5, various families of definitions of equity are discussed and an approach is specified that can be used to determine how well equitable balance is achieved among and within cohorts. Vidal-Melia et al. (2008) state that the existence of an ABM goes hand in hand with the prior calculation of a financial solvency indicator or sustainability indicator. The objective is to assess whether the adjustment mechanism succeeds in achieving balance only in specified circumstances, such as the expected demographics for a specified horizon for a particular country, or whether the adjustment mechanism is robust, i.e., able to adjust to stresses regardless of the demographics or economics or however remote the likelihood of occurrence of the demographic or economic stress may be. This standard of robustness is very high and this thesis finds that none of the ABM are robust, according to this definition. In the penultimate chapter, the results of the analysis, modeling and assessment of adjustment mechanisms are summarized and observations made regarding the types of controls that a robust ABM might include. The final chapter identifies fields outside of SRSS to which this research might be applied and suggests areas for future related research.

1.2 Literature Review Regarding the ABM in the Swedish NDC

1.2.1 Description of the ABM

The most famous of SSRS that has an ABM is Sweden, so it will be described and discussed in this opening chapter. The Swedish SSRS is a complex system with many components, most of which have been commented on in the literature. This subsection

briefly outlines the Swedish system identifying components to be included in any subsequent analysis.

The Swedish SSRS requires an aggregate contribution from the employee and on behalf of the employee by the employer of 18.5% of the employee's pay. Of this contribution 16% is allocated to the NDC part of the system and the remaining 2.5% is allocated to the FDC part of the system. The FDC part will be excluded from further discussion and analysis. An important characteristic of the NDC part is that the contribution rate is, and will remain, fixed.

Within the SRSS there is a guaranteed minimum pension. The excess of the guaranteed minimum pension over the pension provided by the NDC is paid for through general taxes and hence is outside the fixed-contribution NDC component. As such, it will be excluded from further discussion and analysis. (Although it will enter the analysis of whether the ABM is successful in achieving balance).

There are also certain disability and survivors' benefits within the SSRS that are paid for through general taxes. As such they will be excluded from further discussion and analysis.

Within the NDC, the contributions are credited notionally to the account of the contributor. The credit is notional because benefits paid to pensioners are financed using contribution income and any contribution income that exceeds pension payments is deposited in a buffer fund which is invested. However, the actual investment earnings on the buffer fund are not credited to the contributors' accounts. Annually, the notional balances in the contributors' accounts are credited (notionally) with "interest" at the rate of growth in average wage or income of the population over the past year, calculated on a per capita basis, based on a three-year moving average.

At retirement, the contributor's notional account balance is used to purchase the contributor's initial pension. The pension is calculated using an annuity factor that incorporates a 1.6% per annum discount (interest) rate and a mortality factor that is reflective of the mortality improvement by cohort. Table 1 from Settergren (2003) shows the projected annuity factors at age 65. This table also shows the projected increase in life expectancy at age 65. The postponement of the age of full benefit entitlement to reflect increases in life expectancy is one component of the mechanism to achieve balance. An alternative to having one's pension be less than the pension at age 65 of someone in the 1940 birth cohort with an equivalent notional balance at age 65 would be to postpone retirement beyond age 65. The table also shows the period of postponement necessary to equate the initial pension amount to that available at age 65 to the 1940 birth cohort.

Table 1: Effect of Projected Life Expectancy on Annuity Factor, Pension Levels or Retirement Age

		Age 65	Reduction In	Retirement Age	Remaining
Birth		Projected	Pension Due to	To Neutralize	Life
Cohort	Year	Annuity	Increased Life	Increased Life	Expectancy
Born	Reaches 65	Factor	Expectancy	Expectancy	At Age 65
1940	2005	15.7	0%	Age 65	18 years & 6 months
1945	2010	16.1	2%	+ 4 months	+ 6 months
1950	2015	16.4	4%	+ 7 months	+ 11 months
1955	2020	16.7	6%	+ 10 months	+ 16 months
1960	2025	17.0	7%	+ 13 months	+ 20 months
1965	2030	17.2	9%	+ 16 months	+ 24 months
1970	2035	17.4	10%	+ 18 months	+ 28 months
1975	2040	17.7	11%	+ 21 months	+ 32 months
1980	2045	17.9	12%	+ 23 months	+ 35 months
1985	2050	18.0	13%	+ 25 months	+ 38 months
1990	2055	18.2	14%	+ 26 months	+ 41 months

Once pensions commence they are adjusted annually by the increase in the average wage per capita, the same rate that is used to adjust the notional accounts of contributors (taking into account the 1.6% discount factor already incorporated in the pension calculation). Hence, if the increase in the average wage per capita exceeds the rate of inflation by 1.6% then the pensioner's pension is adjusted to fully reflect the rate of inflation. If the increase in the average wage per capita less the rate of inflation is more (less) than 1.6% then the pensioner's pension is adjusted by more (less) than the rate of inflation.

The changing annuity factor by birth cohort to reflect increased cohort life expectancy is a component of the balancing mechanism. There is another component of the system referred to as the automatic balance mechanism. To understand how this mechanism works it is necessary to understand how the system calculates its financial balance.

Two weighted average ages are calculated. The expected income-weighted age of income earners represents the single age of income earners (contributors), such that if all earnings (contributions) were made at that age, it would be equivalent to the income-weighted age of income earners. The expected pension-weighted age of pensioners represents the single age of pensioners, such that if all pension payments were made at that age, it would be equivalent to the pension-weighted age of pensioners. The absolute value of the number of years' difference between these two ages is called the expected turnover duration.

Expected turnover duration will be analyzed critically in Section 6.1.3. A very simplistic explanation of this concept of expected turnover duration is the following. If the population of contributors and pensioners were in a steady state, then the expected

income-weighted age of the contributors represents the age at which a single contribution equivalent to the contribution for all other ages (weighted by income) would be made and the expected pension-weighted age of the pensioners represents the age that a lump sum pension equivalent to the pensions at all other ages (weighted by pension) would be paid. Then the period between the two ages represents the time between receipt of the single contribution and payment of the lump sum pension, i.e., the expected turnover duration.

The contribution asset is defined as one year's contributions multiplied by the expected turnover duration. The total assets equal the contribution asset plus any assets in the buffer fund.

The total pension liabilities of the NDC component are defined as the accumulated notional account balances of non-pensioners plus the sum over all pensioners of the product of their current pension times their remaining life expectancy. The liability in respect of non-pensioners is consistent with how an actuary would define the liability in a DC plan. The liability in respect of pensioners is only consistent with how an actuary would define the liability if the future (interest) discount rate were zero percent.

To determine if there is financial balance, a balance ratio is calculated. The balance ratio is total assets divided by total pension liabilities. When the balance ratio is less than 1 then the automatic balance mechanism is triggered, affecting the next and subsequent years until financial balance is restored. Pensions are no longer increased by the rate of increase in the average income. Instead they are increased by the rate of increase in average income multiplied by the balance ratio. Similarly, the interest rate credited to notional accounts is no longer the rate of increase in the average income; rather it is the rate of increase in the average income multiplied by the balance ratio. To summarize, when the total assets are less than the total pension liabilities, there is no change to the contribution rate but the rate at which notional accounts are credited is reduced (from the level that would have applied if the balance mechanism had not been triggered) and the rate of indexing of pensions is reduced below the rate of increase in the average income.

It is expected that the application of the automatic balance mechanism will rectify the financial balance over time. After the automatic balance mechanism has been triggered and it is subsequently determined that the balance ratio exceeds 1 then indexing of pensions and crediting of (notional) interest to notional accounts continues at the product of the balance ratio and the rate of increase in average income until the pension liability attains the level it would have been at if indexing had been (continuously) at the rate of increase in the average income.

1.2.2 Critique of the ABM Based on Financial Balance

Valdes Prieto (2000) uses relatively simple models to derive formulae to analyze the financial stability of various ways that notional account pension systems could operate. He concludes that where the contribution rate is constant forever, the notional interest rate credited is based on the growth rate of the average covered wage and the pensions

paid are increased by the growth of the average wage, all aspects which characterize the Swedish NDC (except that the pension is calculated using a 1.6% discount rate), then the system does not exhibit automatic financial stability unless the number of contributors remains constant forever. To show this, he defines N_t to represent the number of contributors at time t , R_t to represent Revenue at time t , where $R_t = \sum_i \theta Y_t^i$ and θ is the contribution rate (a constant) and Y_t^i represents the income of individual i at time t . He defines Expenditure at time t , E_t , as $E_t = \sum_j P_t^j$ where P_t^j represents the pension at time t of pensioner j and P_t^j is indexed by the growth in the average wage, which rate

for period $t+1$ is $\frac{1}{N_{t+1}} \sum_k Y_{t+1}^k$. Then $E_{t+1} = \left(\frac{\theta_t}{\theta_{t+1}} \right) \left(\frac{N_t}{N_{t+1}} \right) R_{t+1}$ and since

$\theta_t = \theta_{t+1} = \theta$, $E_{t+1} = \frac{N_t}{N_{t+1}} R_{t+1}$. So automatic financial stability only applies when the number of contributors remains constant. Because the number of contributors is unlikely to remain constant, a stabilizing mechanism like Sweden's automatic balance mechanism is required in order to achieve financial stability.

McGillivray (2005) believes that cohort mortality improvement after the cohort's pensioners have annuitized, in excess of what was incorporated in the annuity factor, has the potential to affect financial stability. Such excess mortality improvement must be borne by any buffer funds or be mediated using the ABM.

Letzner et al. (2004) observes that "when the work force decreases, the average income growth can be higher than the growth rate of the total wage bill and benefits and pension rights will grow faster than the contribution base from which benefits are paid". In such a situation, financial balance can only be achieved by the use of buffer funds or the ABM.

However, when the working population is shrinking dramatically, as is projected for the Japanese population through 2050, according to Ono (2007), the automatic balance mechanism may not be able to achieve financial balance. Ono claims that using the Japanese population projections, a career average pension formula and applying the ABM methodology applicable to the Swedish NDC, the system still exhausts all available assets (i.e., requiring an external cash infusion) by 2045. Without applying the ABM, Ono claims the assets would run out by 2038. Ono concludes that "the [automatic balance] mechanism works efficiently to some extent, it has its own limit under the decreasing population because it does not assume the rate of annual decrease in the discount rate". Although the plan analyzed by Ono is not the same as the Swedish plan design, his observation that the ABM may not operate to ensure financial stability in the presence of a decreasing fertility rate that is already below the population replacement rate is an important one to analyze. At the International Actuarial Association Pension Benefits and Social Security Colloquium in May 2007 in Helsinki (where Ono presented

his findings), Settergren observed that he had not seen the calculations but, based on the results, he doubted that the Swedish ABM had been properly applied. This aspect will be discussed in subsequent chapters.

Cichon (2005) states that a central problem will be negative or low economic growth rates that could potentially be triggered by a contraction of the labour force. He considers such analysis beyond the scope of that paper and does not elaborate.

Barr (2004a) argues that in SSRS a pension credit earned is only a claim on future production. Extending this argument to the extreme, i.e., in a society where all were pensioners and there was no production, such pension claims would be worthless. Sweden's ABM would not be able to produce financial balance in such an extreme scenario; although, if Ono's work is believed, the decline in production due to a declining fertility rate does not have to be as extreme as the example above.

1.2.3 Critique of the ABM Based on Equitable Balance

Given that a full discussion of equitable balance does not take place until Chapter 5, it is not possible at this juncture to distinguish the relative merits of various possible definitions of equitable balance. The critiques presented in this subsection present the various authors' views on equitable balance without assessment. It is recognized that different authors may be using different definitions of equitable balance.

Settergren (2003) offers several ways in which the system is equitable. One way is by use of a constant contribution rate – all workers (and pensioners) have contributed at the same rate. A second way is by adjusting the amount of pension received in order to account for projected increases in cohort life expectancy. A third method is to define intergenerational balance or fairness as having a constant ratio of present value of pension benefits over present value of contributions for all birth cohorts, using the growth in average income as the discount factor. He claims that an NDC system achieves a very stable cohort benefit/contribution ratio. A fourth indication of inter-generational balance or fairness is a relatively stable ratio of average pension over average income, which he calls the pension level.

Analyzing the Swedish NDC in the context of typical European demographics, Cichon (2005) thinks that it is likely that pensions will continue to decline from generation to generation. A conclusion shared by Scherman (2003), Hagberg et al. (2002), and McGillivray (2005). Cichon (2005) states further that after the automatic balance mechanism has been triggered and even if financial balance is achieved so that the mechanism no longer is operating, there will be distortions. This may occur because there is no correction to pensions that were adjusted, for the period of adjustment. It may also occur because in bringing pensions back on track for indexing, account balances of non-pensioners may be overcompensated for the loss during the period when the adjustment applied. Cichon considers all of these effects examples of lack of equitable balance.

Barr (2004a) labels the Swedish NDC with its constant contribution rate and various balancing adjustments, an actuarial relationship. Contrary to those who argue that benefits derived from actuarial relationships are equitable, he would argue that state pensions might achieve greater equity through redistributive assistance. Even if it were acknowledged that consumption smoothing had been achieved equitably, there may be other social welfare goals that are not achieved equitably such as poverty relief and distributional objectives. Letzner et al. (2004) also take a broader social welfare view of equity arguing that to view generational fairness only from a perspective of the contribution rate and the rate of return fails to take into account changes in society and societal values. They argue that greater equity might be achieved in certain circumstances, such as when pensions are deemed inadequate, by raising the contribution rate.

McGillivray (2005) thinks a more equitable and transparent reform approach in the face of increasing longevity is to gradually increase the normal retirement age. He believes that the Swedish system will lead indirectly to an increase in the retirement age but that by using such a complex approach to achieve financial balance, winners and losers will be created. Further, when the ABM operates to reduce the indexing of pensions, he thinks it is possible that pensioners will suffer decreases in their standard of living compared to the rest of the population, which is inequitable. Hagberg et al. (2002) also express concern that the standard of living of pensioners will decline relative to the standard previously experienced by Sweden's elderly - a standard they categorize proudly as among the highest in the world.

Finally, with respect to equity, McGillivray (2005) raises the question of the manner in which the Swedish reform was introduced. He asks whether public pension reform should be decided by responsible and visionary political leaders who rise above partisan and ideological issues and reach a compromise consensus on the basis of sound technical advice and inputs from shareholders, or whether reform should be framed by shareholders who are inevitably constrained by parochial perspectives.

Könberg (2004) believes that the new Swedish system is fair. He would respond to McGillivray's question with a question of his own "Is broad political consensus undemocratic?" He claims the reform was reached through an agreement between five political parties representing more than 85% of the voters.

Könberg sites two main principles that underlie the design of the system, which make it fair. The life-income principle requires that contributions result in pension rights. With a fixed contribution rate DC approach, the same pension rights as a percentage of pay are given to all contributors. He would consider unfair a system in which contribution rates could be raised without giving any new pension rights. The adjustment principle requires that pension rights and the pensions should follow the development of the economy and life expectancy.

In summary, there is no consensus on whether the system attains an equitable balance. Much depends on your definition of equity, which in turn is imbedded in a value system. This issue will be discussed in detail in Chapter 5.

From the commentary, it would appear that there are scenarios based on labour market contraction and slowing or negative economic growth in which financial balance is not achieved. As such the system cannot be considered robust. There are opposing arguments regarding whether the system achieves an equitable balance. Until the systematic discussion of this issue in Chapter 5, the most we can conclude is that the Swedish ABM is not robust. Settergren (2003) states that the existence of uninsurable risks such as a development of the contribution base at a slower rate than the growth in average income or changes in life expectancy may make it impossible to achieve the dual goal: financial and inter-generational balance. The analysis provided later in this thesis will provide an assessment of how well the Swedish ABM achieves this dual goal.

1.3 Outline of Contributions

This research concerns SSRS and specifically those with ABM. The objective of this thesis is to contribute to the literature on SSRS and ABM in the following areas:

- In Chapter 2, it establishes a terminology to classify balancing mechanisms.
- In Chapter 3, it provides a critical analysis of an approach proposed by Valdes-Prieto referred to as Integration to Financial Markets and the approach is found to be deficient.
- In Chapter 5, it presents families of definitions of equity and presents a benchmark by which to measure equitable balance. In Chapter 7, it illustrates how this benchmark could be applied to four countries SSRS that differ in significant respects.
- In Chapter 6, it identifies a critical error in the way in which assets are calculated by the Swedish SSRS in determining financial status and proposes a suitable correction. Also in Chapter 6, it identifies a weakness in the manner of applying the ABM in the Swedish system, so that once an imbalance occurs, balance is unlikely to be restored.
- In Chapter 6, it discusses some of the unusual characteristics of the steady-state contribution rate calculation for the Canadian SSRS and shows that although it has limited application and does not appear to depend on any actuarial principle it creates a tension between the near and distant future, which is a factor in achieving financial balance over a seventy-five year horizon.
- In Chapters 6 and 7, it provides an analysis of the balancing mechanism in the Canadian SSRS and proposes a change in how the mechanism is defined so that the mechanism would be robust, within certain ranges.

1.4 Outline of Thesis

This first chapter sets the stage for the further work in this thesis by introducing SSRS and ABM. The Swedish ABM is the most famous of all ABM, so it is described and the literature with respect to it reviewed. The contributions of this thesis are outlined.

Chapter 2 develops terminology to be used to classify purported ABM. It discusses the desirable characteristics of an ABM and distinguishes among the classifications of robust, partial and transitory. It outlines the approach to be used in reviewing the ABM of the four countries (Sweden, Japan, Germany and Canada).

Chapter 3 is divided into Part A and Part B. Part A provides background information necessary for the arguments and research developed subsequently in the thesis. Topics in Part A include: a description of the main assumptions affecting the financial status of an SSRS; a general discussion of stochastic optimal control and a positioning of the balancing process problem within a stochastic optimal control framework; a brief overview of the overlapping generations model which, in some form is often used in the analysis of SSRS; a description of different methods of indexing benefits after retirement; a discussion of demographic adjustments with respect to life expectancy and fertility. Part B presents a number of topics that are related to this research that will not be considered further. The topics in Part B are presented to provide greater understanding of the position of this thesis within the broader area of research on SSRS. It is not necessary to read Part B to understand the main arguments of this thesis.

Chapters 4 and 5 discuss various concepts of equity and by the end of Chapter 5 a benchmark to measure equitable balance within the countries' ABM is presented. In Chapter 4, the methodology of generational accounting developed by Auerbach, Gokhale and Kotlikoff is described. Based on the results of the generational accounting analysis, conclusions are frequently drawn regarding generational equity. Chapter 4 argues that an uneven burden of taxes as shown by the generational accounts does not necessarily mean there is generational inequity. In Chapter 5, it is shown that the economic concept of Pareto efficiency should not be equated with equity. This chapter presents and discusses six families of definitions of equity. It concludes by presenting the method of determining equity to be used in this thesis.

Chapter 6 begins the analysis of the balance achieved by the four countries' ABM. It considers financial balance. The approach used is to describe the ABM, define the objective for financial balance to be achieved, and then analyze whether financial balance is likely to be achieved in all situations. Significant financial analysis has been done by the countries' actuaries, economists and governmental agencies, as well as by certain other commentators and transnational bodies. Such analysts have access to vast amounts of data, complex models and significant resources. The approach of this thesis is to review the published analysis rather than to perform independent modeling. None of the ABM are found to be robust from the perspective of financial balance. The Canadian ABM is found to provide partial financial balance; moreover, an adjustment to the

Canadian ABM is proposed that would make it robust from the perspective of financial balance.

Chapter 7 continues the analysis of the balance achieved by the four countries' ABM by considering equitable balance. The mechanism used by Sweden is found to provide partial equitable balance but the equitable balance of the mechanisms in Germany, Japan and Canada is considered to be transitory. The adjustment to the Canadian mechanism proposed in Chapter 6 is analyzed and found to be lacking with respect to equitable balance, in certain circumstances. Further adjustments are proposed to the Canadian mechanism so that it will achieve financial and equitable balance, i.e., will be robust, within certain ranges.

Chapter 8 summarizes the results of the analysis in Chapters 6 and 7, using the terminology presented in Chapter 2. It describes the characteristics of mechanisms and controls that an ABM should possess in order to be robust and the role that extra-systematic flows may play in achieving balance, but notes that the existence of such extra-systematic flows, unless they are treated as loans, undermines the efficiency of the ABM.

Chapter 9 summarizes the conclusions of this thesis. It presents the contributions of this thesis to this area of research. It identifies the implications of this work, other areas where it might be applied and topics for further research.

A list of abbreviations is included at the end of this thesis.

Chapter 2 TERMINOLOGY

2.0 Chapter Overview

This chapter discusses the characteristics of balancing mechanisms and identifies those considered to be essential. It presents terminology to be used to classify ABM. It describes the method to be used to distinguish among ABM that are robust, partial or transitory. It also introduces the concept of natural balance and contends that an ABM that did no more than achieve natural balance would not be doing an appropriate job of balancing.

2.1 Essential Characteristics of Balancing Mechanisms

Chapter 1 outlined how SSRS that are long-term undertakings and that contain guarantees are likely to encounter stresses over time. ABM have been created to enable the SSRS to respond to the stress automatically and to achieve balance. Vidal-Melia et al. (2008) state that the ABM is a selection of predetermined measures set by law to be applied immediately as required by the solvency indicator in order to re-establish the solvency or financial sustainability of pay-as-you-go pension systems through successive application. For a balancing mechanism to operate effectively, it should possess the following characteristics:

- Gradual – the adjustments take place over a period of time without any sudden significant adjustment being borne by any single participant or cohort of participants
- Equitable – the adjustments should be borne by cohorts of participants in a manner that each cohort would consider fair, i.e., although the adjustment may be viewed negatively, it should be implemented so that the method of implementation is not viewed as being distributed unevenly among cohorts
- Sustainable – the adjustments should be sufficient to deal with the stress for a reasonable period of time on some reasonable set of assumptions not merely a temporary fix which simply delays the stress for a short period of time.

These three characteristics are essential to an acceptable adjustment mechanism and an adjustment mechanism that displays these characteristics will be referred to as a “balancing mechanism”. “Balancing” carries multiple meanings. It provides financial balance so that as a result of the adjustment, the SSRS is sustainable. It also refers to balance among and within cohorts, i.e., no cohort is treated inequitably relative to others and members of a cohort are treated fairly. It also provides balance through the transition. There is no sudden significant change affecting any cohort.

There are also some characteristics of an adjustment mechanism that may not be essential but might be considered ideal.

- Automatic – Adjustments would occur as stresses develop in order to keep the SSRS in balance.
- Transparent – The nature of the adjustment would be open and understandable by all cohorts.

A few comments on the characteristic of transparency are warranted. As will be elaborated in the next subsection, the most desirable of all ABM is one that is robust. If a robust ABM could be constructed that were also transparent that would be an improvement over a robust ABM that was not transparent. However, given a robust ABM, i.e., an adjustment mechanism that automatically adjusted to stresses, in a manner that was gradual and equitable in its treatment of cohorts and members within cohorts, and which produced financially sustainable results, such a mechanism would be considered successful. So transparency is an ideal rather than an essential characteristic. Moreover, given the complexity of SSRS and the relatively small amount of time and the limited amount of interest the public appears willing to expend in understanding adjustment mechanisms of SSRS, an adjustment mechanism is unlikely to be transparent to all even though it might be transparent to a group with specialized knowledge pertaining to SSRS and SSRS adjustment mechanisms. Accordingly, transparency will not be a required component in the determination of whether an SSRS has a robust ABM, a partial ABM or no ABM at all (i.e., is transitory), as explained further in the next section.

2.2 Classification of Purported Automatic Balancing Mechanisms

This section introduces and explains the terminology used to classify ABM. Turner (2008) categorizes automatic adjustment mechanisms on two dimensions: frequency of adjustment; the event that causes the adjustment to be made. These dimensions are not required for the analysis in this thesis but can be applied to the ABM presented.

With respect to purported ABM, one may distinguish between those that make an adjustment according to set formulae and procedures without any intervention or approval by authorities, and those which require an approval by authorities before they are triggered in response to certain defined warning indicators. The former shall be referred to as “mechanistic” and the latter as “discretionary”.

In developing a SSRS it is common and not unreasonable to describe its operation assuming a stationary population and a non-decreasing economy, for which the income distribution is not changing. However, if the system follows a stationary population model indefinitely, and the economy remains non-decreasing and the income distribution is not changing, there should be no need to trigger an ABM. An SSRS that was developed for a stationary population with a non-decreasing economy for which the income distribution is not changed that required an ABM would either represent a poorly designed system or result from benefits being granted in respect of a period prior to system initiation.

The critical cases where an ABM is required are those where stresses are placed on the system. Typically, the stress will be of an economic or demographic nature. The economic stresses may result from a shrinking economy or from an economy in which the income distribution is changing. For example, if the population is stationary but the

economy shrinks either the citizens must pay more taxes or the contributors must pay more social security contributions in order to maintain pensions at the same level or pensions must be reduced or some combination of the foregoing adjustments is required. Or, for example, if the income distribution changes such that the share of income of older workers increases relative to the share of income of younger workers and the pension is directly related to income, then the actuarial present value of the pensions earned increases, even though the nominal amount of pensions earned may be the same as under the previous income distribution.

The demographic stresses arise from an increase in the ratio of pensioners to contributors. Such an increase may result from an increase in the number of pensioners, for example, due to improving mortality rates leading to longer longevity. Or an increase in the ratio of pensioners to contributors may be attributable to a decrease in the number of contributors. This may be attributable to a decline in the fertility rate, which is not accompanied by an offsetting increase in net migration, or to a decrease in the labour force participation rate.

Another question to be addressed in evaluating an ABM is the time period over which it should be assessed. Valdés-Prieto (2000) focuses on whether a system of notional accounts can provide financial equilibrium in the short run and concludes that it cannot. He suggests that the short run might be considered to be the period between elections. He concludes that with certain restrictions on the benefit formula there will be a tendency towards financial equilibrium in the long run where both demography and the economy are in a growing steady state. The countries examined in this thesis may have growing or steady-state economies but they all have aging populations so the demographics are neither growing nor steady-state.

For the purpose of this thesis, the critical assessment period will be the long run. Although there is no universal definition of the long run, many would consider a seventy-five year time horizon as a reasonable approximation to the long run. The actuaries for both U.S. Social Security and the Canada Pension Plan are required to provide financial projections using a seventy-five year horizon. Some would see disadvantages in using such a short horizon, arguing that it may conceal trends in shortfalls, is arbitrary, and ignores what happens to system finances outside the valuation period (President's Commission to Strengthen Social Security, 2001). The projections of financial position of the Japanese SSRS use a ninety-five year horizon; although, the same disadvantages that were leveled against the seventy-five year horizon could be made concerning a ninety-five year horizon. In discussing SSRS, Cichon et al. (2004) state that it takes at least seven decades for a scheme to mature (and some would argue that due to ongoing changes to schemes, they never do mature). An evaluation of an ABM over the long run requires a period where the scheme has had a chance to mature after the adjustment. Seventy-five years will be used as the evaluation period.

In Chapter 6, the ABM of the four countries will be evaluated with respect to whether they achieve financial balance. Because significant financial analysis involving vast amounts of data and complex models have been performed by the countries' actuaries,

economists and governmental agencies, as well as by certain other commentators and transnational bodies, this analysis will be used to analyze financial balance. If the system is able to adjust to stresses regardless of the demographics or economics, however remote the likelihood of occurrence, it will be considered robust. This is a very high standard. If the system achieves balance in most circumstances other than those considered extreme and quite unlikely, it will be considered to achieve partial balance. If the system does not achieve financial balance even in relatively likely circumstances it will be considered transitory.

The other evaluation measure of the ABM (besides financial balance) is balance within and among generations. Such balance is difficult to define and a number of possibilities will be considered in Chapter 5 before identifying the definition to be used for this evaluation exercise. This evaluation measure will be referred to as “equitable balance”.

Using this evaluation measure, a similar terminology will be applied to equitable balance as is applied to financial balance. If the system achieves equitable balance regardless of the demographic and economic stresses however remote the likelihood of occurrence, it will be considered robust. If the system achieves equitable balance in most circumstances other than those considered extreme and quite unlikely, it will be considered to achieve partial balance. If the system does not achieve equitable balance even in relatively likely circumstances it will be considered transitory.

The final classification of purported ABM will be:

- Completely robust, if they achieve both financial and equitable balance over the long run
- Robust on one dimension of balance that will be included in the label, if they achieve either financial and equitable balance over the long run, but not both
- Partial, if they achieve either partial financial or partial equitable balance in the long run
- Transitory, if they achieve not even partial financial or partial equitable balance in the long run.

The following table shows the methods of classification and how purported ABM may be labeled.

Table 2: Methods of Classification of Balancing Mechanisms

ABM	ABM Achieves		Label
	Financial Balance	Equitable Balance	
Triggered			
Automatically	Robust	Robust	Mechanistic Completely Robust
	Robust	Partial or Transitory	Mechanistic Financially Robust
	Partial or Transitory	Robust	Mechanistic Equitably Robust
	Partial	Partial or Transitory	Mechanistic Partial
	Partial or Transitory	Partial	Mechanistic Partial
	Transitory	Transitory	Mechanistic Transitory
After Approval	Robust	Robust	Discretionary Completely Robust
	Robust	Partial or Transitory	Discretionary Financially Robust
	Partial or Transitory	Robust	Discretionary Equitably Robust
	Partial	Partial or Transitory	Discretionary Partial
	Partial or Transitory	Partial	Discretionary Partial
	Transitory	Transitory	Discretionary Transitory

The standard for robustness has been set high on purpose. Some proponents of ABM, such as Settergren (2003) with respect to the Swedish ABM, claim that the mechanisms work without limitation. Because so many SSRS are experiencing aging populations that are subjecting the SSRS to severe demographic stress, there are many countries and politicians in those countries that would welcome a solution that would provide balance, without limitation. By setting a high standard for robustness, this thesis provides a warning that an adjustment mechanism that is claimed to provide balance without limitations should be carefully and critically examined; in the absence of critical examination, the adopter of the ABM may find that it has received a Trojan horse rather than a robust ABM. None of the ABM examined in this thesis are found to be robust.

2.3 Natural Balance

This section defines natural balance for different types of SSRS.

In a FDC SSRS where the benefit entitlement of the individual is equal to the individual's account balance, the SRSS is always in natural balance, by definition. This will be referred to as natural balance. In such a situation an ABM is not required. Therefore, such SSRS will not be considered in this thesis.

In the pure paygo DB SSRS natural balance occurs if annual benefit entitlements are defined to be equal to the annual amount of contributions received. This is the situation for the U.S. Social Security system in the event that the trust funds are exhausted, as described in subsection 3.10.1.

In a partially funded DB SSRS the concept of natural balance does not arise as long as funds exist. If no funds are exist, then natural balance would be defined as it was defined above for a pure paygo DB SSRS.

For a NDC, because it is paygo, and may or may not have cash assets, the definition of natural balance would be as it was for partially funded DB SSRS. While there are cash assets the concept of natural balance does not arise. Once the cash assets are exhausted, natural balance exists if annual benefits payable are equal to annual contributions received.

The definition of natural balance will be discussed further in Chapter 6 when the objectives of financial balance for the SSRS in the countries are described. It is the contention of this thesis that an ABM that continued to make adjustments until a state of natural balance occurred would not be doing an appropriate job of balancing. To be effective, an ABM should achieve balance at a level higher than natural balance.

2.4 Chapter Conclusion

This chapter has elaborated on the characteristics of balancing mechanisms and distinguished between characteristics considered essential and those considered desirable although not essential. It has described how the terms robust, partial and transitory will be applied with respect to financial and equitable balance. In Table 2 terminology has been presented to classify purported ABM. An explanation has been provided for the choice of a high standard for robustness. The concept of natural balance was introduced. It is the contention that an ABM that did no more than achieve natural balance would not be doing an appropriate job of balancing.

Chapter 3 BACKGROUND

3.0 Chapter Overview

This chapter is divided into Part A and Part B. Part A provides background information necessary for the arguments and research developed subsequently in the thesis. Part B presents a number of topics that are related to this research but that will not be considered further in this thesis. The topics in part B are presented to provide greater understanding of the position of this thesis within the broader area of research on SSRS. It is not necessary to read Part B to understand the main arguments of this thesis.

Part A begins with a description of the main assumptions affecting the financial status of an SSRS. It then presents a general discussion of stochastic optimal control and argues that in order to be an effective ABM, a stochastic optimal control framework is required. The next subsection identifies those assumptions or controls that will be considered in assessing ABM in this thesis. Subsection four presents an overview of the overlapping generations model. Subsection five discusses different approaches adopted by SSRS for indexing benefits after retirement. The final two subsections of Part A discuss demographic adjustments with respect to life expectancy and fertility.

Part B presents briefly topics related to this research that will not be considered further. It begins with a review of the provisions and the administrative approaches in SSRS to deliver disability, unemployment and ancillary benefits. It continues with a discussion of other factors that may create imbalance that are not considered further. Three adjustment approaches are discussed, that of the U.S., that of Brazil, and a method referred to as integration to financial markets. Finally, SSRS are subject to a variety of economic forces that are interrelated. Empirical data illustrating the lack of stability of these interrelationships, even over long time periods, is presented.

PART A

3.1 Main Assumptions Affecting the Financial Status of an SSRS

An SSRS is a highly complex dynamic system. Simplistically, its financial status is determined by comparing its assets to its liabilities.

In an SSRS that is solely defined contribution with funded individual accounts such a determination is relatively straightforward – the total assets are equal to the total liabilities, by definition. Such a system is in natural balance. In practice there are few if any, SSRS that are solely defined contribution with funded individual accounts. For example, the Chilean SSRS comes close to being this type. However, because life annuities that are indexed for price inflation are the payout form for benefits, there is an uncertain liability that must be valued. Because there is no asset that exactly replicates such a liability, the present value of the liabilities may change in a different amount from

period to period than do the assets. Moreover, because Chile had a defined benefit system prior to switching to defined contribution with funded individual accounts, there are transition issues. Recognition bonds promised to be equal in value to certain defined benefit liabilities provide financial balance since assets equal liabilities; however, the true value of the defined benefit liabilities in question is uncertain.

In a defined benefit SSRS there are many factors about which assumptions could be made in order to calculate the value of liabilities. For example, with respect to the U.S. Social Security System, assumptions are made regarding three categories of factors: demographic, economic, program (Trustees, 2008). The categories of factors will be discussed in the following three subsections.

3.1.1 Demographic Factors

Broadly, the critical demographic measure expresses the relationship between the number of pensioners and the number of workers contributing to support (pay pensions of) the pensioners. The three main demographic factors affecting this relationship are fertility rates, mortality rates and net migration rates.

The fertility rate plays a large role in determining this relationship for a long period of time, albeit on a lagged basis, e.g., the number of babies born in year t is an important determinant of the number of contributors in years $t+18$ to $t+65$ and also an important indicator of the number of pensioners in the years beyond $t+65$.

Another important factor is mortality rates and the general trend of reductions in mortality rates (as countries develop). If the age of full-benefit entitlement remains constant and there is no change to retirement assumptions, there will be upward pressure on the relationship between the number of pensioners and the number of contributors, because pensioners will be expected to live longer.

A third factor that may affect this demographic measure is net migration. Due to the typical qualification rules for pension benefits that require a number of years of contributory service in order to qualify for benefits, in the short term, net migration affects the number of covered workers but not the number of pensioners. If net migration is positive, there will be more covered workers. If net migration is negative, there will be fewer covered workers. In the longer term, the net migration factor affects the system similarly to the fertility rate, although usually not with such a long lag. Migrants who have contributed may reach the age of full benefit entitlement and become pensioners.

Two potential differences between fertility and migration, other than the length of the lag until becoming a covered worker, are the following. The state (or the system) may be able to exert greater influence over the migration rate than over the fertility rate, i.e., it is easier to decide on the number of immigrants to target and to take the enabling actions to admit them than it is to target a positive number of births. Secondly, depending on the country, immigrants may be more likely to leave the country to which they have

immigrated than are those that are born in the country – this may be especially true if there are significant numbers of illegal immigrants that are unable to change their status to legal immigrant. This latter situation may increase the number of covered workers without resulting in these workers becoming pensioners at a later date.

3.1.2 Economic Factors

Whereas there were three main demographic factors about which to make assumptions, there are considerably more economic factors requiring assumptions. In fully and truly paygo SSRS in which benefits are defined solely by the contributions available to pay benefits, natural balance exists. Unless the SSRS is fully and truly paygo, the rate of interest to be earned on excess assets will be an important assumption. In developing assumptions regarding such a rate it is necessary first to consider the eligible investments and investment policy. In this regard, the assumptions for the U.S. Social Security system are straightforward, in that the assets are assumed to be invested in special debt instruments of equivalent credit quality to U.S. government securities with horizons ranging up to 15 years. Canada, by contrast, in addition to fixed income commitments, has assets managed by an independent board that invests in many different asset classes including publicly-traded equity and private equity, both Canadian and foreign. Developing models and assumptions to forecast accurately the returns on such diverse asset classes is challenging.

Because the benefits in most SSRS are adjusted, at least to some extent, by price changes, it is necessary to make assumptions regarding the rate of price inflation. Generally, there is available historical data to support time-series analysis to develop such a forecast. However, because benefits are adjusted for price inflation, at least to some extent, it is necessary to separate asset returns into a real component and a component due to price inflation in order to be able to identify the net assets available for benefits and the real rate of return.

Wage growth is another important factor to be estimated. Contributions are a function of wages, so wage growth has an effect on contribution income. Benefits are also typically a function of wages, so the benefits to be paid in the future will be determined by the wage growth assumptions. Some SSRS adjust pensions in payment using a formula that incorporates the rate of wage growth. Some countries use gross rates and others use net rates, where net is defined differently, depending on the country.

Wage growth may be a single assumption or it may be decomposed into a number of components, each of which must be forecast. Such a decomposition increases the complexity. For example, in developing the rate of growth in real earnings per worker, the U.S. Social Security approach is to develop assumptions regarding the following components: change in productivity, the ratio of total compensation to GDP, the ratio of covered wages to total compensation, the change in the hours worked, the ratio of the GDP deflator to the consumer price index.

Labour force participation rates and unemployment rates are also important economic assumptions that will affect contributions and eventually benefits.

3.1.3 Program Factors

Program factors refer to assumptions required to determine benefits based on program provisions. The extent to which such assumptions are required will vary by SSRS depending on the complexity of the program provisions. All SSRS will require assumptions about number of retirements by age. But this assumption will vary by SSRS not only because there are different ages of full-benefit entitlement but also because there are different benefit adjustment provisions if retirement occurs at other than the age of full-benefit entitlement. The size of such adjustments may affect retirement elections.

Many SSRS provide benefits to individuals who did not contribute to the system and yet who are entitled to benefits. For example, the program may provide benefits to a surviving spouse or dependent children on the death of the covered worker. Data with respect to such benefit recipients may not be captured until the death of the covered worker. Hence in making financial projections, assumptions must be made about the relevant characteristics of potential beneficiaries.

A related assumption in programs that provide benefits to the spouse of the member is the probability of being married. In programs that provide spousal entitlements for periods of marriage, assumptions regarding rates of divorce and length of marriage will be required. In programs that provide benefits to same-sex spouses, further refinement of assumptions will be required.

Where program provisions provide different rates or benefit levels depending on earnings, for example, or impose limitations or maximums, assumptions will be required. For example, if contributions are only required to a limit that is a function of the average wage, an assumption regarding the distribution of wages up to this level will be required. Or if there are different benefit rates on earnings up to certain bend points, earnings distributions must be developed.

If the program provides special benefits and/or credits for periods of disability, assumptions regarding incidence of disability and the probability of recovery from disability will be required. Such assumptions are frequently age and gender dependent. Or if the program provides credit for periods when not in covered employment, for example, for parenting, assumptions will be required.

3.1.4 Mathematical Relationship Among Factors in Paygo SSRS

The following formulas from Thompson (2005) show the inter-relationships among some of the demographic, economic and program factors in a paygo SSRS.

Paygo contribution rate = system dependency ratio * average replacement rate

where average replacement rate = $\frac{\text{average benefit}}{\text{average wage}}$

and system dependency ratio = $\frac{\text{retirement age population} * \text{reciency rate}}{\text{working age population} * \text{labour force participation rate} * \text{employment rate} * \text{coverage rate}}$

Note that: average wage, labour force participation rate and employment rate are all economic factors; retirement age population and working age population are demographic factors; average benefit, reciency rate and coverage rate are program factors. Thompson (2005) simplifies system dependency ratio by observing that if the percentage of the retirement age population receiving benefits is approximately equal to the percentage of the working age population working or looking for work, the reciency rate, labour force participation rate, and coverage rate drop out leaving:

System dependency ratio = $\frac{\text{retirement age population}}{\text{working age population} * \text{employment rate}}$

3.2 Brief Review of Stochastic Optimal Control

Stochastic optimal control has been used to analyze problems faced by insurance companies (e.g., Taylor 2002, Cadenillas et al. 2006) and by defined benefit pension plans (e.g., Taylor 2002, Boulier et al. 1995, Boulier et al. 1996, Cairns 1995, Cairns 1996).

It is then an easy step to see that the stochastic optimal control framework is applicable to SSRS; although, a literature review did not indicate that this has been done.

Taylor (2002) states that a typical set up for a stochastic optimal control analysis is as follows:

- There exists a financial system subject to stochastic obligations
- To be funded by contributions whose magnitude may vary from time to time, at the discretion of the contributor
- Contributions in excess of the amount required to immediately settle obligations accumulate in a fund as assets
- Assets are invested according to some policy, frequently involving at least one risky asset
- The problem is then to avoid insolvency and contributions may be adjusted as necessary depending on the returns on assets and the obligations to be settled
- The desired solution is one that selects the values of the contribution rate and the investment in risky assets in an optimal manner at each point of time, according to some loss function.

This typical set up is applicable, to some extent, to most SSRS, other than those that are FDC. With respect to the second point in the list, most SSRS do not give discretion to the contributor regarding the magnitude of the contribution, however, the contribution rate may be changed for the whole system periodically. With respect to the investment policy regarding any (temporarily) excess assets, some SSRS invest in risky assets and others do not (excluding the credit risk of the country). SSRS considered in this thesis may also adjust benefit obligations and program provisions in addition to selecting the contribution rate and investment strategy mentioned in the last point in Taylor's list.

Such stochastic optimal control problems are typically very difficult to solve. Without a number of simplifying assumptions they do not permit a closed form solution. See Bjork (2005) for an example of a special case involving simplifying assumptions. Typically techniques of dynamic programming are required to approximate the solution, if one exists. The stochastic control problems posed by SSRS are even more complicated and it can be challenging to even specify the loss function.

Taylor (2002) provides an example of a benefit plan for which the function of interest is solvency ratio. The control law permits the adjustment of the contribution rate and the allocation to risky assets. He finds two "optimal points", 39% solvency ratio and 88% solvency ratio. On further analysis he finds the 39% solvency ratio is stable. In Taylor's example, the liabilities are calculated at a fixed discount rate and are a set amount. Since the solvency ratio is the ratio of assets to liabilities and liabilities are determined, the solvency ratio is effectively a function of assets. Since the two factors defining assets are the contribution rate and the allocation to risky investments, the control law in the example seems appropriately structured.

In SSRS the situation is unlikely to be so simple or capable of such a simple definition. While solvency ratio may be an important function, the liabilities are not determined and unchanging; rather they are dependent on many factors, such as those described in section 3.1. The types of controls that might be applied would include: adjusting the contribution rate; adjusting the benefit level; adjusting the age of full benefit entitlement; investing any excess funds; extra-systematic flows. A complex control law would be required. If a sufficiently complex control law were developed, it is unlikely that the stochastic control problem would have a closed form solution. This subject will be expanded further in Chapter 6.

3.3 Factors to be Considered in This Thesis

In Chapter 6 the key relationships used to establish financial balance will be described, in both words and symbols, for the four countries' balancing mechanisms. It will be shown there, that none of the four countries uses a stochastic control framework.

The thrust of this thesis is to examine whether the balancing mechanisms are robust, partial or transitory. Accordingly, the approach taken is to determine if there are circumstances in which the mechanisms would fail to achieve financial balance.

As described in section 3.1 there may be a large number of assumptions that must be made, over long time horizons. There could always be some deviation from assumptions – in fact, in practice, it is likely there will be deviations from all assumptions at some time over the long horizon.

In assessing the balancing mechanisms of the four countries, the first step will be to define what it will mean for that system to achieve or not to achieve financial balance. Then certain assumptions considered particularly important for financial balance will be identified. Since the SSRS have different definitions of financial balance and have different program provisions, these assumptions vary by SSRS. Having defined financial balance and the assumptions to be considered, the balancing mechanisms will be assessed on whether they achieve or do not achieve financial balance when there are deviations from the assumptions selected.

This approach should be clearer in Chapter 6 when the specific objective of financial balance and the assumptions to be considered are defined for each of the four countries. We provide an example for the sake of illustration. As is described in subsection 3.10.1 for the U.S. system, if financial imbalance is defined as being “unable to pay benefits”, then that system can never be considered to lack financial balance. The reason is that the provisions of the program define that benefits may not exceed the amount of funds available to pay the benefits, i.e., natural balance exists. However, if some minimum level of benefits were included in the definition of financial balance, e.g., financial imbalance occurs if available assets are insufficient to pay at least 90% of scheduled benefits, then the system might lack balance. To continue this example, let us assume that all the assumptions made by the U.S. Social Security system are the relevant ones. Then on the seventy-five year horizon, even assuming that all those assumptions are realized, in 2041 the assets are projected to be sufficient to pay less than 80% of the scheduled benefits, i.e., by our hypothetical definition of financial balance, the system would not be in balance. The conclusion would be that the system as designed is not robust.

3.4 Overlapping Generations Model

3.4.1 Brief Introduction

A very powerful and commonly used model among economists in analyzing economic growth is the Overlapping Generations (“OLG”) model. Peter Diamond first described the two-period version, described here. Many variations on the model are possible by changing certain assumptions and such variations produce different results. The OLG model has been used to analyze social security systems (see for example Blanchard et al., 1989).

In perhaps the simplest version of the OLG model, people live for two periods. In the first period of their life, i.e., as the “young”, they are born, work, consume part of their

income, and save some income for the subsequent period. In the second part of their life, i.e., as the “old”, they don’t work but they consume all of their savings from the first period and the earnings on those savings, which have grown with investment income from the first period at rate r , and die.

Let the population grow from one period to the next at rate n , then a golden rule economy is said to be one in which the return on capital is equal to the rate of population growth (Blanchard et al., 1989).

It is common to add to this model the rate of time preference, from the perspective of either a decentralized economy or a centralized planner. In this case, for relatively small values of the parameters, the modified golden rule economy is one in which the return on capital is equal (approximately) to the sum of the rate of time preference and the rate of population growth.

In both the golden rule and the modified golden rule economies the economy moves to a steady state where the capital per unit of labour is constant and output, consumption and saving are optimal.

In this simplified theoretical modified golden rule economy, the introduction of a fully funded social security system has no impact on capital accumulation, since accounts earn the rate of return on capital r . The impact of the introduction of a pay-go social security system is less certain because the accounts are effectively credited with the rate of population growth, n . If the old generation all have a bequest motive, i.e., intend to save sufficiently, through reduced consumption, to leave some capital to their children, then this theoretical model predicts no change in capital accumulation, because the additional social security benefits paid by the young for the old generation are returned to the young by the old as an additional bequest. However, in the absence of a bequest motive, the prediction depends on whether the return on capital is higher or lower than the population growth rate but generally the theory implies a decrease in capital accumulation (Blanchard et al., 1989).

This background may prove useful in understanding the assumptions and dynamics underlying the ABM developed by certain SSRS, specifically the concepts of steady state, internal rates of return and growth rates.

3.4.2 Internal Rate of Return of Paygo SSRS

There has been much discussion about whether a partially funded SSRS with the excess funds invested in capital markets is more efficient than a paygo SSRS. To better understand this issue, consider an OLG model operating on a pure paygo basis, and, as Sinn (2000) does, define the following terms:

In period t there are N_t^Y young workers and N_t^P pensioners; each young person contributes C_t in period t , each pensioner receives P_t in period t , so on pure paygo

$$N_t^Y C_t = N_t^P P_t \quad (1)$$

on the OLG model, this period's contributors are the next period's pensioners

$$N_t^Y = N_{t+1}^P \quad (2)$$

In pure paygo the implicit debt of the system is $N_t^P P_t$ and this must be serviced

Suppose the rate of return in the capital markets for an investment made in t and paid in $t+1$ is r_{t+1}

and the rate of return within the system for an investment made in t and paid $t+1$ is i_{t+1}

Then the implicit saving S_t by a young person to provide his or her pension in $t+1$ is

$$S_t = \frac{P_{t+1}}{1+r_{t+1}} \quad (3)$$

If $C_t > S_t$ then there is also an implicit tax T_t , the tax share of contributions is

$$\tau_t = \frac{T_t}{C_t} \quad (4)$$

$$C_t = S_t + T_t \quad (5)$$

We can now derive the following relationships

$$C_t = \frac{P_{t+1}}{1+i_{t+1}} \quad (6)$$

rearranging (6) and substituting (1)

$$i_{t+1} = \frac{C_{t+1} N_{t+1}^Y}{C_t N_t^Y} - 1 \quad (7)$$

substituting (5), (3) and (7) into (4) yields

$$\tau_t = \frac{r_{t+1} - i_{t+1}}{1+r_{t+1}} \quad (8)$$

Equation (7) shows that the internal rate of return of the system is determined by the rate of growth in the contribution volume. Since this analysis is of a paygo SSRS the internal rate of return of the system could also be expressed as the rate of growth in the pension volume. Equation (8) shows that when the rate of return in capital markets is greater than the internal rate of return of the system, there is an implicit tax on contributors.

However, if the internal rate of return of the system exceeds the rate of return in capital markets then there is a “negative tax” or subsidy to contributors.

Sinn (2000) states that in the long-run the growth rate of aggregate pensions will not be able to exceed the growth rate of GDP which, in a steady state, will be less than the rate of return in capital markets, i.e., the internal rate of return will be less than the market rate of interest. However, this relationship need not hold in practice in any year and sometimes for a number of consecutive years.

For example, in the early years of the Canada Pension Plan, the large baby boom generation was beginning to enter the work force. There was strong growth in the contribution volume, resulting in a high internal rate of return, that exceeded the market rate of interest. In such a situation, paygo was an efficient basis. However, by the mid-1980s when the baby boom generation was participating in the labour force almost fully, the growth rate of the contribution volume was much lower, and the internal rate of return became less than the market rate of return. In such a situation, partial funding is more efficient than pure paygo. Partial funding was adopted in the 1997 amendments to the Canada Pension Plan.

3.5 Indexing of Benefits

A barely adequate initial pension will only maintain its adequacy over a pensioner’s lifetime if it is adjusted periodically to reflect changes in prices and/or wages since commencement. In this section the rationale for using a price adjustment or a wage adjustment is discussed.

In the OLG model in steady state, a golden rule economy operates with the real rate of interest equal to the sum of the growth rate of the population and the growth rate of labour – augmenting productivity. This rate of interest is equal to the rental rate on capital. Assuming there are constant returns to scale, in a steady state, i.e.

- the rental rate on capital
equals
- the rate of wage increase
equals
- the sum of the growth rate of the population plus the growth rate of labour-augmenting productivity.

Since the main rates, i.e., interest, rental rate on capital, and wage rate, are increasing at the same rate, the position of an individual whose income is not increasing at such a rate is changing, from one period to the next, in relation to others in the country.

Following this logic, the argument for increasing social security pensions by the growth rate in average wages is as follows. During the contributory period, the worker is experiencing wage increases, approximately equal to the increase in the average wage, and such worker’s contributions, in a pay-go SSRS, are being used to pay benefits to

pensioners. In the worker's retirement period, unless this worker's social security benefits are increased in line with changes in the average wage, the former worker's relative position in society is deteriorating. However, by virtue of the former worker having contributed to social security to provide benefits to other workers, this former worker should be entitled to maintain his/her same relative position in society. Furthermore, the contributions of the workers are increasing, generally, in relation to the increase in average wages, so in a steady state pay-go SSRS, there should be sufficient increase in contributions annually to pay pensions to retired workers that are increased at the rate of increase in average wages.

A refinement to this argument is to use the increase in disposable income rather than the increase in gross wages. This approach is also designed so that pensioners' relative position in retirement will remain the same as during their working period. However, this approach recognizes that disposable income is a better measure of position than gross wages, especially if the country's tax system is progressive so that the tax rates are higher on higher levels of income than on lower levels of income. In general, workers who are contributing will have higher levels of income than pensioners will have. In a progressive tax system, the rate of increase in contributors' disposable income will tend to be less than the rate of increase in gross wages. Since pensioners tend to have lower incomes, in a progressive tax system, the rate of increase in contributors' disposable income will tend to be less than the rate of increase in pensioners' disposable income. To prevent pensioners from receiving greater increases to their disposable incomes than contributors are receiving, pensions are increased by the average rate of increase in disposable income of contributors.

The argument for increasing pensions by the rate of increase in prices, as measured by some appropriate cost-of-living index is as follows. It is widely observed that the expenses to maintain a comparable standard of living are less after retirement than while working prior to retirement. For example, Sakamoto (2005) reports that, in Japan, to maintain a comparable living standard, the average consumption of households of aged couples to the average disposable income of the active labour force is 50%, although this may change as the socio-economic environment changes. There are various possible explanations for this phenomenon, including:

- the consumption needs of the aged are less because they have already made significant capital expenditures that have lasting value and because they may be less active and as such, they do not need to consume so much;
- they have a reduction in expenses related to employment such as for traveling to work, clothing associated with work, payroll taxes for workers such as social security contributions;
- the elderly receive various age-related subsidies such as age deductions for income taxes, discounts on banking and public transit fees due to age, special health benefits based on age such as drug, physician or hospital expenses.

Accordingly, the retirees may be able to maintain an equivalent life style on a lesser income. If their initial pension is adequate, then its adequacy can be maintained by adjusting it periodically for the change in the cost-of-living associated with a basket of goods and services that measures approximately the expenses of the elderly.

Some countries adjust pensions based on the change in a wage index, other countries adjust pensions based on a change in a price index, and some countries use some combination of the change in a price index and the change in a wage index. See, for example, Whitehouse (2006) for a listing of various OECD countries' practices.

This discussion is intended to provide some rationale for the different practices. It is not intended to specify the one and only appropriate practice or even to opine on whether there is a correct practice. However, in some discussions of equity, individuals will adopt one of these approaches to indexing pensions as being the appropriate basis and then argue that adjustments using other than that "appropriate basis" are inequitable. The argument may be valid within the assumption that a certain basis is the appropriate basis, but I do not accept such an assumption in assessing equity. However, I would agree that a barely adequate social security pension that is not indexed, if there are both wage and prices increases, will not maintain its adequacy, and that is inequitable.

3.6 Adjustments for Life Expectancy

As observed by Whitehouse (2007a), it is hard to see why people that have benefitted by living longer should not bear at least some of the cost of this longer life. So far reference has been made to two countries' SSRS, i.e., Sweden and Brazil, that make changes to the benefits at retirement to reflect changes in life expectancy. This is a type of balancing mechanism that attempts to create some form of balance between the period of work and contribution or tax payment and the period of retirement and benefit receipt. Turner (2008) classifies countries' method of adjustment for life expectancy into two categories: those that correct for the percentage increase in life expectancy; those that correct for the percentage increase in the present value of benefits caused by the increase in life expectancy. He places countries such as Brazil, Finland and Portugal in the first category and countries such as Sweden, Italy, Norway, Poland, Latvia in the second category. From an actuarial perspective, if the discount rate is zero per cent (which is often assumed) then the two methods are the same.

Such methods achieve financial balance, in the sense of limiting the benefits paid. They also provide a type of equitable balance across cohorts, since each cohort receives benefits for the same expected period of time or of the same expected present value (after suitable adjustments for salary and economic differences across cohorts). However, such methods do not achieve individual equity since the individual is expected to contribute for a longer period of time (since the period to full-benefit retirement age has been extended) for the equivalent initial level of benefits received by earlier cohorts.

An alternative method of adjusting for changes in life expectancy would be to maintain a constant ratio of expected period during which contributions will be made up until the age of full benefit entitlement to the expected period of benefit receipt after full-benefit retirement age (or some similar ratio, such as the inverse of the above, or such as the ratio of the expected period of benefit receipt to the expected period of life from work commencement, etc.). Such an approach maintains individual equity across cohorts and within cohorts. Such an approach would also maintain financial balance. According to Turner (2008) such an approach was proposed in the United Kingdom in discussions on how the full-benefit retirement age should be adjusted; however, in the discussions and implementation it was discarded for a simpler approach of scheduled increases in the retirement age. This author is unaware of this approach having been adopted by any country; although, Whitehouse (2007a) states that France has begun a tightening of the qualifying conditions for the public pension, such that after 2012, the ratio of the period of pension receipt to the period of working will be kept constant.

Using an OLG model, the assumption that life length is deterministic and increasing, a legislative approach implying that all living generations have an influence on the social security scheme and have a veto power; Anderson (2006) claims to show that in a paygo scheme the complete solution to increasing longevity cannot be obtained by indexing the retirement age and the consumption of the older generation to longevity. Anderson (2006) claims consumption of the younger generation is also reduced. This has important implications for the Swedish ABM that will be described in subsection 6.1.2.

3.7 Adjustments for Fertility

For a pay-go SSRS to remain sustainable without adjustment it is important that the population of contributors increase at least as fast as the population of pensioners. An important factor determining the rate of increase in contributors is the total fertility rate (“TFR”). For developed countries, at current levels of normal mortality, a TFR of approximately 2.10 children per woman of childbearing age is considered sufficient to provide for population replacement. With the exception of Mexico, United States and Iceland, most developed countries are experiencing TFR that are well below the population replacement level. The TFR of a selection of developed countries is shown in the following table.

Table 3: Total Fertility Rates in 2003 in Selected Countries

Country	TFR 2003
Australia	1.75
Canada	1.53
France	1.88
Germany	1.34
Iceland	1.99
Italy	1.26
Japan	1.29
Mexico	2.20
Spain	1.31
Sweden	1.71
United Kingdom	1.71
U.S.A.	2.04

Source: OECD (www.oecd.org/dataoecd/21/57/38181878.xls)

Andersen (2006) labels a change in fertility a backward looking problem and states that a change in fertility cannot be undone. Some scholars, such as Sinn (2000), are not as fatalistic as Andersen (2006) and have argued that to make SSRS financially sustainable and more equitable, generations that do not have a TFR sufficiently high to replace themselves should have their social security adjusted in some manner. The adjustment might be in the form of higher contributions or reduced benefits.

As espoused by Sinn (2000), the theory is one of capital production. In retirement one requires either money capital in order to provide a pension or human capital, in the sense of workers producing goods and services and willing to pay contributions to provide pensions. The creation of this generation of workers, i.e., the human capital, is to a certain extent the responsibility of the retiree cohort. If the retiree cohort experienced a TFR at or above the replacement level, then there is some likelihood that there will be sufficient human capital to contribute to pay retirees' pensions.

None of the countries examined has an adjustment mechanism that is directly related to the TFR. How such an adjustment could work in practice is unclear. Some of the issues to consider would include how to define cohorts for the measurement of TFR and the adjustment of pensions; what a suitable adjustment to pensions would be should the TFR be below the replacement rate; whether adjustments would be required if the TFR was above the replacement rate.

Alho et al. (2006) describe an approach that they refer to as fertility-dependent prefunding. In Finland, there is some prefunding of the defined benefit pensions. The extent of prefunding does not affect the benefit levels. Alho et al. (2006) propose that the standard prefunding formula be multiplied by a factor for each cohort that estimates its

size at working age to the estimated size of all working age cohorts. The factor is as follows:

$$b(i,t) = B(t-i) / \sum_{j=0}^{i-1} w(j,i)B(t-j-1)$$

where $b(i,t)$ is the adjustment factor in year t for those age i

$B(t-i)$ is size of cohort $t-i$

$w(j,i)$ are weights calculated so that they approximate the shares of the various cohorts in the working-age population when the funding cohort (those in age i at t) has retired

and $w(j,i) \geq 0$ add up to 1 for each i

If the funding cohort is bigger than the younger cohorts (on average) then b exceeds 1 and funding is increased, whereas if the funding cohort is smaller than the younger cohorts (on average) then b is less than 1 and funding is decreased. This is a novel suggestion worthy of future consideration, but it is not considered further in this thesis.

PART B

3.8 Disability, Unemployment and Ancillary Benefits

The focus of this thesis is on SSRS, and specifically on earnings-related pensions. However, most SSRS provide benefits for other contingencies. It is common to include disability provisions, to recognize that a disabled worker cannot have earnings on which to earn pensions but said worker should not suffer a reduction in pensions for a disability, typically beyond the worker's control. Some systems, such as in Germany, extend this argument to unemployment, and others such as Canada, extend protection to certain types of unemployment, for example a period of unemployment to give birth or to raise young children. Moreover, the pension that the participant is entitled to at retirement may have a different (actuarial present) value depending on whether the retiree has a spouse.

In certain countries in Europe, the SSRS is used as a workforce transition device (Holzmann, 2006). Even though there may exist a relatively high retirement age to qualify for a retirement pension, there may be an employment practice that is condoned whereby workers below the retirement age for social security pensions are transitioned to that age by treating them as disabled. Marin (2006) estimates that 91.4% of Mexican disability benefit recipients are not disabled and shows that disability rates by country vary considerably in the age 55-59 and age 60-64 age bands. Marin (2006) considers

most of this variation to be due to the accepted practice by country with respect to using disability benefits as a retirement transition device.

It is beyond the scope of this thesis to consider the relative merits of these various ancillary benefits and these employment transition practices. In assessing equity, only the situations of contributors and pensioners will be considered.

3.9 Factors That May Create Imbalance That Are Not Considered

In addition to the factors described in section 3.8, there are other factors or provisions that could affect financial balance that will be excluded from subsequent analysis. A SSRS may contain a variety of provisions designed on redistributive rather than insurance principles. It may also adopt practices for social and/or administrative reasons that add additional cost that have not been incorporated directly into the contribution formula. The inclusion of such provisions and practices may affect the financial balance of the system and may make it more difficult to develop an ABM that will counter the effect of such provisions and practices and restore financial balance. A list of a number of the provisions and practices of this type often included in SSRS follows with a brief description of each. Generally, such provisions and practices create a situation in which the value of benefits created is not directly related to the amount of contributions made.

- (1) The creation of earnings ranges for the purpose of determining contributions where benefits earned do not relate directly to the earnings on which contributions are required, e.g., contributions may not be required on the first \$X of earnings but benefits are earned on all earnings.
- (2) The calculation of benefits based on only a portion of the earnings history, even though contributions are paid based on lifetime-covered earnings, e.g., if the pension formula is based on final average earnings.
- (3) The imposition of benefit maximums in relation to a different level of earnings than the maximum earnings for determining maximum contributions.
- (4) The provision for early retirement on a subsidized basis under specified conditions, (and much less commonly subsidized postponed retirement benefits that are greater than actuarial equivalent value), i.e., most defined benefit SSRS reduce the benefits on early retirement by less than an actuarially equivalent reduction.
- (5) The provision of credit for years of service when contributions were not made, e.g., in respect of certain years of parenting.
- (6) The provision of benefits calculated on a unisex basis, i.e., traditionally defined benefit pension formulas do not differentiate by the gender of the participant and defined contribution and NDC pensions are often calculated using unisex annuity factors.
- (7) The inclusion of survivor benefits on an actuarial inequivalent basis, e.g., in defined benefit pension formulas providing the same level of initial pension regardless of spousal relationship but providing a continuing pension to a surviving spouse or surviving dependent children.

- (8) The inclusion of special provisions in the event of disability, e.g., providing continuing credit for years of disability even though no contributions are required.
- (9) Failure to adjust pensions to recognize increases in life expectancy, i.e., if in a defined benefit plan the normal retirement age remains fixed but life expectancy increases, the system will move away from financial balance unless the contributions are increased.

Legros (2006) and in an earlier related article, Guerin and Legros (2002), discuss a number of these provisions and draw the distinction between actuarial fairness at the margin and actuarial fairness on average. Under actuarial fairness at the margin, as used by these authors, all the factors affecting an individual's choice with respect to a provision would be included in determining actuarial equivalence. For example, rather than having a set scale of early retirement reductions, the calculation would consider the future wage-earning prospects if the prospective retiree were to remain in the labour force, the individual weights and discount factors the prospective retiree would apply with respect to work and labour tradeoffs and possibly an allowance for myopia regarding future life expectancy and future inflation. Certainly such a calculation would be impossible to administer within an SSRS and would also likely be impossible to calculate since there is so much information that must be gathered regarding an individual's circumstances and preferences. Although these authors refer to this as actuarial fairness at the margin, I would view this more as an attempt by an economist or central planner to get the optimum from an economic system. I would not refer to this as actuarial fairness.

Under these authors' definition of actuarial fairness on average, the measure is whether the value of benefits is equivalent to the value of contributions, on some set of reasonable assumptions. This is very close to the approach used to prepare the list of provisions and practices that may add additional cost to the system beyond what is provided for directly in the contribution rate basis. However, Legros (2006) takes this definition further than this in the following example. She writes that in a defined contribution or an NDC system where the annuity factors are not adjusted to incorporate the fact that longer life expectancy is closely correlated with higher income, then those with higher incomes are receiving a benefit of greater value than are those of lesser income, because the same annuity factor is used to convert the accumulated contribution balances into pensions. In this instance, I think Legros takes the definition of actuarial fairness too far, as will be explained below.

Quiesser et al. (2006) distinguish between actuarial fairness and actuarial neutrality. For these authors, actuarial fairness requires the present value of lifetime contributions to equal the present value of lifetime benefits. Actuarial neutrality requires the present value of accrued pension benefits for working an additional year be the same as in the year before. As defined by these authors actuarial neutrality is certainly an individual concept. These authors err in assuming that actuarial fairness must also be determined on an individual basis. Actuaries frequently perform calculations, design plans and assess fairness on a group basis.

First, actuaries do not typically speak of actuarial fairness but rather of actuarial equivalence. It may be a subtle distinction but for actuaries the focus is on determining a financially equivalent value on the basis of a set of reasonable assumptions. If the actuary has studied the mortality experience and trends likely to be applicable to a particular group and if the annuity factors have been determined using this information, then financial equivalence will be achieved. It may well be true that there is a sense of intragenerational inequity between those with higher incomes and those with lower incomes. It is also true that if the actuary took such correlations into account in developing the annuity basis such inequities would be mitigated, at least in part. But regardless of whether such adjustments are made, if the actuary has developed the appropriate tables, the financial balance of the system should be unaffected.

Actuarial equivalence is about financial balance at a system level. It is a collective measure. Within accepted actuarial practice, occasionally modified by social considerations, actuaries have identified factors of collectives that, in the absence of other more relevant information, determine prospective life expectancy, factors such as gender, smoking behaviour, presence of ill health such as heart disease or cancer, income, etc. Within an SSRS most of these factors are not permitted to be used in calculating benefits, not for actuarial reasons but for social reasons. Even without refined tools and considerations, the actuary is able to determine actuarially equivalent values for the collective. They may not be actuarially fair, either at the margin or on average, as defined by Legros (2006) and Guerin and Legros (2002), but they are actuarially equivalent and that is what matters for financial balance.

In conclusion, the list of provisions and practices at the beginning of this section will affect the financial balance of the system. To the extent that appropriate actuarial techniques are used to value these provisions and to adjust the contribution rates accordingly, the SSRS is more likely to be in financial balance and an ABM will be more likely to operate effectively.

3.10 Adjustment Approaches Not Considered

3.10.1 U.S. Old Age, Survivors, and Disability Insurance

There is much concern regarding the financial sustainability of the United States social security system (“OASDI”). The 2008 Trustees Report indicates that assuming no changes, on the basis of a valuation over a 75-year horizon:

- There is an actuarial deficit of 1.70% of covered taxable payroll; and
- in 2017 projected cash inflows will be less than projected cash outflows; and
- in 2025 the DI Trust Fund will be exhausted; and
- in 2042 the OAS Trust Fund will be exhausted. (Trustees, 2008).

This does sound like a financial situation worthy of attention. However, the OASDI contains a form of automatic balancing mechanism that ensures financial viability. The law is written to provide that benefits paid may not exceed funds available. Hence, if no

changes were made and if assumptions and projections were realized, in 2025 when there were insufficient funds to pay DI benefits at the then-current level, the DI benefits would be adjusted so as not to exceed available funds. A similar adjustment would occur in 2042 in respect of OAS payments when OAS benefits, at the then-current level, are projected to exceed the available funds. According to information provided by Stephen Goss, Chief Actuary for social security, in a personal communication, the law does not prescribe how benefits are to be adjusted in such a situation and the responsibility for the determination would lie with the Commissioner who was appointed at the time.

From a financial perspective, this is truly an automatic balancing mechanism since it ensures that benefits do not exceed funds available. From an equity perspective, it is impossible to assess this mechanism since it is not known or knowable how benefits would be determined. Accordingly this mechanism will not be considered further.

3.10.2 Brazilian Adjustment Mechanism

Brazil's social security system is a defined benefit pay-go contributory system that includes retirement pensions. In 1999 it adopted a benefit formula that provides for pensions commencing in the year to be adjusted to take account of changes in life expectancy.

According to Pinheiro (1999), after the five-year phase-in period, the basic pension is defined as:

$$Sb = M \times f$$

where Sb is the pension

M is the average of 80% of the best years' earnings on which contributions were made since 1994 to the date of retirement (1994 earnings are substituted for earnings in the years of contributions prior to 1994), i.e., one-fifth of the earnings that pertain to the years of lowest earnings on which contributions were made are excluded in calculating the average

$$f = \frac{Tc \times a}{Es} \times \left(1 + \frac{ld + Tc \times a}{100} \right)$$

And f is an adjustment factor

Tc is the length of time contributions have been made, in years (plus 5 years in the case of females)

a is the contribution rate, currently 31%

Es is the unisex expectation of life in years at the retirement age

ld is retirement age in years

This factor is calculated annually and published by the Instituto Brasileiro de Geografia e Estatística in a two dimensional table showing time of contribution on the vertical axis and retirement age on the horizontal axis. To understand the f factor, note that $Tc \times a$ is a factor that when applied to a year's earnings provides an estimate of total contributions without interest (or treating the rate of interest credited on contributions as zero). Dividing this quantity by E_s , expectation of life, provides an estimate of the pension provided by each dollar of contributions, treating the interest discount rate as zero. The term in brackets does not have an intuitive meaning, but it may be thought of as a way to increase pensions to reflect the time at which they start. So, for example, for someone with 30 years of contributions who retired at age 65 the pension would be 2.95% higher than for someone with 30 years of contributions who retired at age 60.

The particularly interesting feature of this approach is that the initial defined benefit pensions are adjusted to reflect unisex life expectancy at the age of retirement in the year of retirement. Vidal-Melia et al. (2008) report that in its 2005 pension reform, Finland introduced a life expectancy coefficient to be applied to its defined benefit pension that adjusts the amount of pensions in payment as longevity changes. Although the approach adopted by Brazil and Finland adjusts the present value of the pension liability at retirement, this mechanism does not directly relate to the financing of the SSRS. As such it is an unsophisticated balancing mechanism. Because the Swedish system contains an adjustment for changing life expectancy, this type of mechanism will be considered in the analysis of the Swedish system. It is worth noting though that both the Brazilian and Finnish systems are defined benefit whereas the Swedish system is notional defined contribution. Hence we see that a mechanism that adjusts for life expectancy at retirement could be applied to different plan designs.

3.10.3 An Integration to Financial Markets Approach

Valdes-Prieto (2006) proposes an approach for an SSRS he describes as integration to financial markets ("IFM"). In this subsection, this approach is described, analyzed critically, and reasons are provided for not considering it further in this thesis. The IFM approach is quite complicated. A brief description of the details related to this discussion follows:

- The starting point is an SSRS that is solvent. As defined by Valdes-Prieto, a solvent pension plan is one that does not require financial support from its sponsor, nor will pay a profit to it, in present expected value.
- The type of pension plan analyzed is one that relates benefits to lifetime earnings, requires contributions at rate θ_0 of earnings, operates on a notional account, pay-go basis, and pays a pension of β percent of earnings.
- The preliminary balance sheet of the plan would show no assets, in pure pay-go, and would have liabilities calculated using the benefit formula and the salary and contribution history of current members, including pensioners.

- However, if all future generations are considered, there is a net hidden asset equal to the present value of all future contributions less the present value of all future benefits. This asset arises from the definition of a solvent plan.
- The existence of a hidden asset means there is a hidden tax τ , such that the true required contribution rate θ' plus the hidden tax is equal to the actual contribution rate θ_0 , i.e., $\theta_0 = \theta' + \tau$. The first step is to make the hidden tax explicit.
- In a steady-state pay-go SSRS, the rate credited to contributions is the sum of the rate of population growth and the rate of labour-augmenting capital, g , which is less than the return on physical capital, r .
- Through legislative means, establish that the future net flow of contributions in excess of benefits is an asset of the plan and that this asset has enforceable property rights. Then securitize this net cash flow through Covered Wage Bill (CWB) securities that pay a dividend in each period, that is sufficient along with θ' to pay the pension payments that are due.
- Now θ_0 earns g in total, but θ' could be credited with r , since it is the amount of actual contributions which could be invested in physical assets and used to pay future benefits when due. The CWB securities could be priced in financial markets. Given that they were legitimate assets, they would also command a rate payable on physical capital as determined by financial markets. The CWB securities would have a payment profile, like equity, with the return varying with fluctuations in fertility, labour force participation, and real earnings trends.
- Accounts would be established using a mutual fund approach, so that contributions would create shares and at any time the share value would be determined as the total value of assets divided by the total number of shares outstanding. This guarantees the system is always in financial balance. Another way to ensure financial balance would be to hire a sponsor that guarantees wage-indexed annuity benefits.

The method described has appeal. Not only does it ensure financial stability at all times, but it also creates a method of linking the rates credited to accounts to the rates available in financial markets while retaining a system of notional accounts using a pay-go approach. Unfortunately, it suffers from several difficulties, as explained below.

- First, although the definition of solvency is appealing, it is slippery. Valdes- Prieto's argument uses both finite horizons for an illustration and an infinite horizon to make his case. To see the difficulty, consider a debt D on which interest is accruing at rate i , so that payments of iD are required each period. If this debt must be repaid in n periods and the discount factor is v then the expected present value is:

$$viD + v^2iD + \dots + v^niD + v^nD = \frac{\{v - v^{n+1}\}}{1 - v} iD + v^nD \quad (9)$$

However, if we let n to go to infinity, because v is less than 1, then the expected present value is:

$$\frac{v \overbrace{1}^n}{1-v} iD \quad (10)$$

To apply this to Valdes-Prieto's case, D is the value of benefits granted to previous generations in excess of their contributions. If that debt is required to be repaid over n periods then (9) shows that not only must the debt be serviced but also it must be repaid. However, if the time horizon is infinite, all that is required is to service the debt, as shown in (10). By using an infinite horizon, on Valdes-Prieto's definition of solvency, a plan that paid benefits falling due and just serviced its debt would be considered solvent, i.e., the plan would not require financial support from its sponsor, nor will it pay profit to it, in present expected value.

However, this is not the traditional meaning of solvency. In traditional parlance, one would require that the present expected value be calculated over a finite horizon using formula (9). Suppose that this requirement is made and that the pension plan had a level contribution rate. Then during the first n years the contribution rate must be sufficient to pay benefits as they fall due and to amortize the debt; thereafter, such a contribution rate will be higher than that required to pay benefits, i.e., by the periodic amortization payment. Accordingly, the concept of a level contribution rate and a solvent plan are inconsistent unless an infinite horizon is used. If an infinite horizon is used, the definition of solvency is inconsistent with traditional parlance because it permits the non-repayment of an initial debt, i.e., an initial debt can be created to pay social security benefits to the founding generation, that debt need only be serviced and never be repaid, and the plan is solvent.

- Second, Valdes-Prieto is no doubt correct that social security systems with level contribution rates with legacy benefits must charge a contribution rate higher than the rate required to pay the benefits being earned by the current members, if the system is to continue unchanged indefinitely. Using the steady-state growth model the pay-go rate credited to contributions is g . If the contribution rate required to provide the current members benefits is θ' , then, in theory, it could be invested to earn a rate of return of r . Even if the remaining hidden tax could be made explicit and endowed with enforceable property rights, it is a leap to argue that the return on such an asset would approximate the return earned in financial markets.

An example will help to clarify this point. The Canada Pension Plan is considered to be actuarially sound over a 75-year horizon. It has a contribution rate of 9.9%, say 10%, and it has been estimated that the level

contribution rate required over a current member's career to pay that member's benefits is approximately 7%, i.e., the hidden tax is approximately 3%. Valdes-Prieto suggests, for example, that the real rate of interest might be 3%. Then we can write the following equation where g is the rate of growth of the population plus the rate of growth of labour-augmenting productivity and i is the rate that may be credited to the hidden tax, per unit of salary:

$$\theta_0 (1+g)^T = \theta' (1+r)^T - \tau (1+i)^T$$

i.e.,

$$.10 (1+g)^T = .07 (1.03)^T - .03 (1+i)^T$$

$$i = \frac{-.0021 + .10g}{.03}$$

So unless g is greater than 2.1%, i is negative and i is only 3%, i.e., the return on financial markets, if g is greater than 3%, which Valdes-Prieto argues is too high. Valdes-Prieto's argument can be criticized along similar lines to how proponents of financial economics have criticized pension plan actuaries that valued the liabilities by anticipating the equity risk premium. If, in fact, the correct rate of interest to credit on θ_0 is g then, at the same level of risk, it cannot be possible to credit θ' with some rate $r > g$ and the balance $\theta_0 - \theta'$ with the same rate $r > g$, regardless of how the contributions are sliced and repackaged, using property rights and securitization. The only way that this could happen is if there is a mispricing of risk. Such risk could take a variety of forms such as credit risk that future generations will pay the required contributions or default risk that the property rights may not be fully enforceable.

- Finally, it is true that Valdes-Prieto's approach brings financial stability, in the sense that assets equal liabilities. But this is not due to his definition of solvency, making the hidden tax explicit, creation of CWB and enforceable property rights, or integration to financial markets. Financial stability arises from two mechanisms:
 - The use of mutual fund accounting.
 - The reliance on a sponsor to guarantee wage-indexed annuities.

The first mechanism automatically guarantees that assets equal liabilities because the liabilities are defined by the total value of all the shares and the value of each share is determined by dividing the total value of the assets by the total number of shares. By convention, total assets equal total liabilities. This does not say anything about the adequacy of pensions. Financial markets rise and fall, over time, in a relatively unpredictable manner, and the value of pensions that are integrated with financial markets could be expected to similarly rise and fall. A social security system is comprised of many individuals who have short horizons. While one may argue that over long time horizons the general trend of markets is upward, producing real returns,

designing a social security system that passes the full exposure of financial market volatility to these individuals is a poor design. Even in Chile, the home country of Valdes Prieto, the social security system using funded defined contribution accounts does not pass full exposure to financial markets to the participants, requiring instead minimum rates of return, within a range, to be credited to accounts.

The second mechanism would be a way to guarantee the ongoing adequacy of pensions, if the initial pension level P were adequate, since it provides wage-indexed benefits. However, such a sponsor is more likely to be a theoretically desirable construct than a realistic possibility in any economy of significance, for the following reasons. First, such a sponsor would have to be well capitalized with an indefinitely long future lifetime. Not only is it difficult to identify such entities, but it is extremely difficult to identify any such entity that would be large enough and sufficiently well capitalized to provide a financial guarantee for a country's social security system. Second, if an appropriate entity were identified how would it hedge its exposure to wage-indexed benefits? In the absence of appropriate hedging instruments, how secure is a "guarantee"?

The IFM approach proposed by Valdes Prieto creates financial stability through mutual fund accounting, which would expose participants to inappropriate levels of risk exposure and may result in inadequate pensions, or through the use of a sponsor that provides guarantees that are not realistically attainable. As such, this approach to automatic balancing will not be considered further for this thesis.

3.11 Some Financial Rates

The models discussed and the assumptions underlying many projections assume relatively stable relationships among key financial rates. In practice, these relationships are not stable over annual measurement periods and may even include apparent idiosyncrasies over longer periods, such as ten years. For the purpose of illustrating this point, consider the financial rates in the following table excerpted from a Canadian Institute of Actuaries report (2008).

Table 4: Average Annual Percentage Rates Of Change/Return

10-Year Period	Consumer Price Index	Wage and Salary Index	GDP per Employed	Canada Long Bonds	Common Stock Index	US Common Stocks in Canadian \$
1958-1967	2.13	4.31	4.76	2.02	11.33	13.92
1968-1977	6.61	9.45	9.47	5.66	5.71	3.69
1978-1987	7.15	6.90	7.25	10.04	16.01	17.18
1988-1997	2.65	2.90	3.45	13.29	10.99	19.09

Source: Canadian Institute of Actuaries, Report on Canadian Economic Statistics 1924-2007, Tables 2A, 2B.

Examples of what might be considered idiosyncrasies include the following:

- In the 1958-1967 period, the extent to which the return on common stocks, both Canadian and US, exceeded the returns on Canada Long Bonds.
- In the 1968-1977 period, the return on Canadian stocks exceeded the return on US stocks, but both these returns were less than the increase in Consumer Price Index.
- In the 1978-1987 period, the increase in the Consumer Price Index exceeded the increase in the Wage and Salary Index.
- In the 1988-1997 period, the return on Canada Long Bonds exceeded the return on the Canadian Common Stock Index and exceeded 13% annually over a ten-year period.

3.12 Chapter Conclusion

Part A of this chapter has provided background information necessary for the arguments and research developed subsequently in this thesis. Section 3.1 described the types of demographic, economic and program factors that affect the financial position of an SSRS. Section 3.2 argues that maintaining financial balance in an SSRS is a stochastic control process, but due to the complex nature of SSRS, a complex control law would be required and there is unlikely to be a straightforward solution to the problem. Subsection 3.3 identified the assumptions and controls to be considered in this thesis when assessing

ABM. Section 3.4 presented an overview of the OLG model. Section 3.5 discussed different approaches adopted by SSRS for indexing benefits after retirement, which will be important in understanding the methods used by different countries. Sections 3.6 and 3.7 discussed demographic adjustments with respect to life expectancy and fertility, the two demographic factors considered in this thesis.

Part B of this chapter presented a number of topics that are related to this research that will not be considered further in this thesis. The topics were presented to provide greater understanding of the position of this thesis within the broader area of research on SSRS. Of particular importance was the analysis in subsection 3.10.3 of the integration to financial markets approach proposed by Valdes-Prieto (2006), which showed that approach to be deficient. This is one of the contributions of this thesis.

4.0 Chapter Overview

Generational accounting is an approach used to assess the relative burden of taxes borne by different generations to support current policies, assuming no change in policy. Frequently those who perform a generational accounting analysis argue, or imply, that an uneven burden of taxes means that generational equity does not exist. This chapter introduces generational accounting. It then considers generational equity. It argues that an uneven burden of taxes as shown by the generational accounts does not necessarily mean there is generational inequity.

4.1 Generational Accounting

This chapter describes the approach referred to as generational accounting, discusses the connections and differences between generational accounting and generational equity.

Credit for the methodology of generational accounting is often given to Auerbach, Gokhale and Kotlikoff. The methodology was developed circa 1990 and has been applied to the accounts of more than 22 developed nations.

Generational accounting is based on the budget constraints of all levels of a nation's governments. Assuming that existing policies and programs continue indefinitely, it requires that the present value of future net tax payments of current and future generations be sufficient to cover the present value of future government consumption as well as to service the government's initial net indebtedness. A set of generational accounts is established for each birth year to the present and an additional single generational account is established in respect of future unborn generations. The generational accounts reflect taxes paid less transfers received with government expenditures on health care and education being treated as transfers. Social security taxes, contributions and payments are included in the accounts. However the accounts do not impute to particular generations the value of the government's purchases of goods and services. Therefore the accounts do not show the complete net benefit or burden any generation receives in full (Auerbach et al., 1999).

If the present value of the generational accounts for the unborn generations, assuming the continuation of current programs and policies, is not exactly equal to the net present value of future consumption plus initial indebtedness less the present value of the generational accounts of those already born, then current programs and policies are considered unsustainable. Potential actions to make programs and policies sustainable include tax increases, consumption reductions or some combination of the two.

Auerbach et al. (1999) provide the following formula:

$$\sum_{k=t-D}^t N_{t,k} + (1+r)^{-(k-t)} \sum_{k=t+1}^{\infty} N_{t,k} = \sum_{s=t}^{\infty} G_s (1+r)^{-(s-t)} - W_t^g$$

more correctly the formula should be written as:

$$\sum_{k=t-D}^t N_{t,k} + \sum_{k=t+1}^{\infty} (1+r)^{-(k-t)} N_{t,k} = \sum_{s=t}^{\infty} G_s (1+r)^{-(s-t)} - W_t^g$$

where $N_{t,k} = \sum_{s=K}^{k+D} T_{s,k} p_{s,k} (1+r)^{-(k-s)}$ and $K = \max(t, k)$ is a generational account for the generation born in the year k . k runs from $t-D$ (those aged D , the maximum length of life, in year 0) to t (those born in year 0).

$T_{s,k}$ stands for the projected average net tax payment to government in the year s by a member of the generation born in year k .

$p_{s,k}$ stands for the number of surviving members of the cohort in year s who were born in year k .

G_s is the value of government consumption in year s .

W_t^g denotes the government's net wealth in year t .

r is the interest rate used to discount the future tax, consumption and transfer payments. Auerbach et al. (1999) say that common practice is to use a constant rate per annum and to present a range of results using a number of different interest rates. Typically, when Auerbach et al. (1999) present a single result, a rate of 5% per annum has been used.

Since the choice of interest/discount rate could affect materially the results, some further discussion is warranted. Auerbach et al. (1999) discuss the rationale regarding the use of a risk-free rate or a risk-adjusted rate.

If one were to assume that the program and policies would be delivered forever into the future then one might choose the risk-free rate as the appropriate discount rate. However there are several difficulties with such an approach. First, the appropriate risk-free rate does not likely exist for all desired maturities. If a proxy for the risk-free rate is the government bond yield curve, then for most countries, rates would not be available for cash flows more than 30 years from the present. Second, some programs and policies are adjusted based on inflation and economic growth; hence, the discount rate should be able to reflect inflation or economic growth, as appropriate. There are even fewer government bonds available with these characteristics from which to derive discount rates. Third, the promises of many governments, particularly over long time horizons, are far from risk-free. On the one hand, use of such a government's bond yield rate would not be a risk-free rate. On the other hand, one could argue that the appropriate

discount rate in a country where the programs and policies have a positive probability of default or revision is not the risk-free rate.

What then about recognizing that a country's tax revenues and expenditures are uncertain and choosing a discount rate that includes some provision for uncertainty, a risk-adjusted rate. Auerbach et al. (1999) acknowledge that is the correct approach but rightly point out some of the difficulties in identifying the appropriate adjustment for risk. First, different adjustments may be appropriate with respect to revenues, consumption and transfers. Second, how should one account for the potential difference in perspective of each generation? Auerbach et al. (1999) present the example of two generations whose income is negatively correlated which might arise due to relative productivity shocks where the first generation supplied capital and the second generation supplied labour. Both generations might perceive lower net tax burdens but the risk-adjustment each generation perceives as appropriate may differ. It is a relatively difficult task to assess the risk-adjustment of a current generation with respect to current taxes and expenditures. It is a more difficult task to assess the risk-adjustments of a current generation with respect to future taxes and expenditures. It is an even more difficult task to assess the risk-adjustment of future generations with respect to future taxes and expenditures. In the context of such uncertainty, the standard practice of discounting using a range of rates is understandable and defensible.

With respect to the four countries that are analyzed in this thesis – Canada, Germany, Japan and Sweden – Kotlikoff et al. (1999) present information shown in the following table:

**Table 5: International Comparisons of Generational Accounting:
Alternative Ways to Achieve Generational Balance**

Country	Cut in Government Purchases		Cut in Government Transfers		Increase in All Taxes		Increase in Income Tax	
	A	B	A	B	A	B	A	B
Canada	0.0	0.1	0.0	0.1	0.0	0.1	0.0	0.2
Germany	21.1	25.9	17.6	14.1	9.5	9.5	29.5	29.5
Japan	26.0	29.5	28.6	25.3	15.5	15.5	53.6	53.6
Sweden	37.6	50.5	22.6	18.9	16.1	15.6	42.9	41.9

Note: Table entries are percentage adjustments needed to achieve generational balance. In the columns labeled “A”, education is treated as government consumption. In the columns labeled “B”, education expenditure is treated as government transfers and distributed by age groups.

Although the calculations were performed in the late 1990's so that the precise figures may be somewhat outdated, the numbers are sufficiently large (or small in the case of Canada) to draw some relevant conclusions. Canada's programs and policies are almost in balance and hence sustainable. The programs and policies of Germany, Japan and Sweden are not sustainable and will require tax increases and/or government

consumption and transfer reductions to be made sustainable. Although these calculations do not identify social security as a separate component, given that social security is a significant component in the accounts, based on the information in the table, it is not surprising that Germany, Japan and Sweden have all adopted measures to “balance” social security. In 1997, Canada took measures to reform its Canada Pension Plan, designed to achieve balance, and from the table it would appear that, in aggregate, Canada’s programs and policies are sustainable.

The methodology of generational accounts requires numerous assumptions, substantial amounts of data and complex calculations. Nonetheless, it does provide useful insight regarding the sustainability of programs and policies.

4.2 Introduction to Generational Equity

It is an easy step to conclude that if the generational accounts indicate that the programs and policies are unsustainable then they are not fair. This subsection discusses this conclusion and argues that while it is an easy conclusion to draw it is not necessarily correct.

A simple argument to lead to the conclusion that generational equity is lacking given that the generational accounts show that the policies and programs are unsustainable is the following:

1. The generational accounts show that the current programs and policies are unsustainable.
2. For the programs and policies to be sustainable either taxes must be increased or consumption reduced or some combination of the two.
3. If taxes are increased, the burden of such tax increases will be borne more heavily by future generations of taxpayers than by past generations of taxpayers.
4. If consumption is reduced, the burden of such consumption reductions will be borne more heavily by future generations of consumers than by past generations of consumers.
5. Based on 3 and 4 there will be some future generations that bear a heavier burden (of tax increases or consumption reductions) than was borne by some past generations.
6. Bearing a heavier burden than another generation is inequitable.
7. Therefore if the generational accounts show that current programs and policies are unsustainable then generational equity is lacking.
8. Consequently a condition for generational equity to exist is that the generational accounts indicate that the programs and policies are sustainable.

Since this thesis is focused on SSRS, the foregoing argument will be analyzed in that context with an indication of how the analysis could be generalized to the full range of programs and policies. However, before analyzing this argument, I present similar types of arguments that are made to define generational equity in the SSRS context.

In Chapter 1, four arguments of Settergren (2003) were presented on why he thinks the Swedish system is equitable. Three of the arguments have similarities to the argument that a condition for equity is that the generational accounts indicate that programs and policies are sustainable. Those three arguments are summarized below:

1. The system is equitable because of the use of a constant contribution rate – all workers (and pensioners) have contributed at the same rate.
2. Intergenerational balance or fairness may be defined as having a constant ratio of the present value of pension benefits over present value of contributions for all birth cohorts, using growth in average income as the discount factor.
3. Intergenerational balance or fairness is a relatively stable ratio of average pension over average income (referred to as the “pension level”).

Let us expand on these arguments in turn to identify the similarities to the argument regarding the connection between the sustainability of programs and policies under generational accounts and generational equity.

1. A constant contribution rate is equivalent to stating that tax rates need not be adjusted. However, this argument makes no connection between contributions and benefits received. It makes the statement that equity is achieved by having everyone contribute at the same rate. The proponents of the argument regarding the connection between generational equity and the sustainability of programs and policies under generational accounting do not make such a bold claim. They require current programs and policies to continue without change to taxes (or contribution rates) and without change to consumption. They would not claim, as Settergren does, that equity is achieved by maintaining constant taxes (or contributions) regardless of the consumption.

It is a strange sense of fairness that Settergren proposes – paying the same contributions regardless of the benefits received. In analyzing generational equity, the weaker claim that a condition for generational equity is that programs and policies be sustainable according to the generational accounts will be analyzed.

2. Let us try to re-express a constant ratio of present value of pension benefits over present value of contributions for all birth cohorts, using the growth in average income as the discount factor in the context of the methodology of generational accounts. For each birth cohort, calculate the following ratio.

$$\frac{\sum_{\text{all born in } t} \sum_{s=r}^{\omega} {}_tP_r (1+u)^{s-r} (1+w)^{t-s}}{\sum_{\text{all born in } t} \sum_{s=e}^r c_s {}_tI_e (1+w)^{s-e} (1+w)^{t-s}}$$

where ω is the oldest age
 r is the retirement age
 ${}_tP_r$ is the pension at retirement for those born in year t
 u is the average rate of increase in pensions
 w is the rate of growth in average income
 e is the age of entering the work force
 ${}_tI_e$ is the income at entry for those born in year t
 c_s is the contribution rate in year s
and $t < e < r < \omega$

Let us assume that the programs and policies are the ones that define the pension benefits and contributions and that the pension benefits are defined as consumption or transfers and that contributions are defined as taxes. Then ignoring the initial indebtedness of the government, the generational accounts reflect the difference in the present value of taxes to be paid less the present value of future consumption and transfers. Unless this difference is constant for all future generations, the programs are unsustainable. But this result can be re-arranged to be expressed as Settergren has:

$$k = PV \text{ taxes} - PV \text{ consumption and transfers}$$

where k is constant for all generations. Dividing by PV consumption and transfers and rearranging the terms gives

$$1 + \frac{k}{PV \text{ consumption and transfers}} = \frac{PV \text{ taxes}}{PV \text{ consumption and transfers}} = r$$

But if r is constant then PV consumption and transfers must be constant and so also must PV taxes.

Hence Settergren's formulation is equivalent, provided the discount rate is equivalent. Settergren uses the growth in average income as a discount factor. How does this compare to the range of discount rates or single constant rate used by proponents of generational accounting? Settergren (2003) says the underlying assumption is that the system is in steady state with respect to both income and mortality. Moreover, Settergren states that an advantage of this approach is that it is simple and avoids projections. As a consequence of this assumption, the discounting to calculate present values is at a constant single rate that is consistent with the approach used by Auerbach et al. (1999). Hence this formulation of intergenerational equity is very similar to the formulation used by those who examine the sustainability of programs and policies under generational accounting.

3. Let us try to re-express Settergren's relatively stable ratio of average pension to average income in the language of generational accounting. Once again, assume that the rules regarding pensions and contributions form the programs and policies and that pensions represent consumption and contributions represent taxes. In this

context contributions are a fixed percentage of average income. For a population in a steady state where rates of mortality, income (and consequently pension increase) remained unchanged over time, we have:

$$C = \frac{\text{average pension}}{\text{average income}} = \frac{\frac{1}{\omega - r} \sum_{s=r}^{\omega} P_r (1 + u)^{s-r}}{\frac{1}{r - e} \sum_{s=e}^r I_e (1 + w)^{s-e}}$$

where ω is the oldest age
 r is the retirement age
 P_r is the pension at retirement
 u is the average rate of increase in pensions
 e is the age of entering the work force
 I_e is the income at entry
 w is the rate of growth in average income

On these assumptions the above ratio is only constant if $\omega - r$ and $r - e$ are constant, i.e. if the period of labour-force participation and the period for which a pension is received are constant. Settergren (2003) observes that variations in pension level result primarily from changes in life expectancy, but that increasing life expectancy should result in individuals working longer which will help maintain a constant ratio and that the Swedish system adjusts pensions for changes in life expectancy, which makes the ratio more stable.

However, in the language of generational accounts, it is necessary to work with the present value of taxes (or contributions) not the average value and to work with the present value of consumption and transfers (or pensions) not the average value. Note that where the contribution rate is a constant percentage of income, income may be used as a proxy for contributions. A constant pension level will not be an equivalent formulation to the generational accounting approach unless the discount rate is zero percent. Zero percent is a single constant rate and arguably one of the rates in a possible range of discount rates but it is a very special case. The following analysis will use the more general formulation of generational accounting and focus on definition two of generational equity used by Settergren.

4.3 Analysis of Generational Accounting Approach to Generational Equity

To restate the argument to be examined, Settergren (2003) would say that generational equity arises from having a constant ratio of the present value of pension benefits to the present value of contributions for all birth cohorts, using the growth in average income as the discount factor. Proponents of generational accounting would make the weaker claim that if the present value of net taxes less the present value of net consumption and transfers is constant by generation, generational equity is achieved.

To assess generational equity for a SSRS, it is necessary to recognize the context of the assessment, namely that a SSRS is a form of insurance arrangement. SSRS provide income during retirement years in return for contributions (or taxes) paid during the working years. To assess the equity of insurance arrangements, it is common practice to assign utility functions to the contracting parties, in this case the individual and the state. The utility function of the state may be linear so that the present value of a dollar of pension benefit paid must be matched by the present value of a dollar of contribution collected, in aggregate, for the arrangement to be considered equitable to the state. However, individuals seldom demonstrate linear utility functions over the whole range of outcomes. Individuals are typically risk averse resulting in utility functions that are concave downward (i.e., their first derivatives are positive and their second derivatives negative). In such circumstances, there can be a range of outcomes where it may be considered to be of mutual benefit for the parties to contract. Such a range of outcomes, if they are mutually beneficial, would be considered fair or equitable.

The existence of different utility functions for contracting parties, some of which are concave downward, underlies many insurance arrangements. Such insurance arrangements are considered equitable even though the present value of benefits received by one party may be less than the present value of contributions paid. Due to its risk-averse nature, the risk-averse party may be prepared to pay a premium to avoid or reduce the impact of an outcome that is considered undesirable and consider the payment of such a premium equitable, even if the premium (and the expected value of any other costs) is greater than the expected value of benefits.

Another way of viewing this same point is to consider that the contracting parties each discount the future cash flows using different discount rates. In the presence of different discount rates, it is possible that one party will perceive that it is receiving a premium while the other party will consider the contract “premium-free” or fair or equitable. The possibility of such a situation was suggested in the earlier example from Auerbach et al. (1999) in the discussion of the appropriate discount rate to use for generational accounting. In that example, it was suggested that if the relative productivity of capital and labour supplied by generations was negatively correlated then from the perspective of each generation, the burden might be reduced and a higher discount rate would be used. Although Auerbach et al. (1999) admit this possibility, they recognize the practical difficulties in attempting to model different discount rates by generation. However, our purpose here is not to derive a practical approach to discounting but to develop a coherent theory of generational equity.

If one admits that in insurance arrangements, two parties may enter into a contract in which the present value of future contributions exceeds the present value of future benefits for one party and that such an agreement will be considered fair and equitable provided that the other party believes that it is increasing its utility by entering the agreement or believes that the present value of future contributions equals or is less than the present value of future benefits on the discount rates it uses, then the requirement that

policies and programs be considered unsustainable is not a sufficient condition for the arrangements to be considered to lack equity.

4.4 Chapter Conclusion

This chapter has introduced the concepts of generational accounting and generational equity. Generational accounting is an approach used to assess the relative burden of taxes borne by different generations to support current policies, assuming no change in policy. The chapter has argued that an uneven burden of taxes as shown by the generational accounts does not necessarily mean there is generational inequity. In Chapter 5, families of definitions of generational equity will be considered.

5.0 Chapter Overview

In this chapter six families of definitions of equity are presented and discussed. At the conclusion of this presentation, the method of determining equity to be used in this thesis to analyze the ABM is identified and a discussion of this method to analyze equity is presented.

Before presenting the six families of definitions of equity, the chapter begins with a discussion of the economic concept of Pareto efficiency and shows that this concept should not be equated with equity. The chapter also discusses transition considerations to show that because it may take decades for a change to an SSRS to be fully incorporated, individuals are required to evaluate conditions to decide on whether a condition is equitable in the face of (almost) continuous change.

5.1 Pareto Efficiency and Equity

In assessing questions of equity, it is often necessary to compare the situations of different individuals who may have diverse and conflicting preferences. Utility theory provides a general framework for such a comparison, because utility is a measure of the individual's satisfaction. Utility theory becomes hard to apply in practice for a number of reasons including:

- Individuals have difficulty quantifying their level of utility.
- Individuals may not exhibit rationality in comparing items with differing utility measures.
- There is no universally accepted standard method of categorization of utility measures that can be used to compare utility among members.
- Even if there were such a standard universal categorization, it would still not be clear what weight to apply to the utility of different individuals.
- It is not clear which individuals' utility should be considered when making an assessment, e.g., should as yet unborn generations' utility be considered? This might be important if the yet unborn generations are expected to pay taxes based on the decision taken.

These considerations aside, various methods have been proposed for calculating a single aggregate utility measure to determine a social outcome for a group. Kreps (1990) lists the following:

1. Equal weights to each individual's utility, i.e., pure utilitarianism.
2. Some other set of non-equal weights of the individual's utility, e.g., if the ship is sinking, women and children are loaded into the life rafts first and

the crew leaves the sinking ship last - this may be referred to as weighted utilitarianism.

3. Weights may be applied based on social position, so, for example, those in society who are worse off receive a heavier weight - this carries a measure of social justice with it.
4. All the weight is given to the worst off member of the social group - this is often referred to as being a Rawlsian approach.

Another concept used in economics to assess options is that of Pareto optimality. A particular set of choices is considered to be Pareto optimal if any change in the choices would make at least one individual worse off.

It is important not to equate equity with Pareto optimality, which is an economic construct to measure efficiency. For example, one can imagine a situation of great inequality where a dictator enjoys great advantages to the detriment of his/her countrymen. There are a number of alternative distributions of wealth that most people would consider more equitable, but since all of those redistributions would involve reducing, at least to a certain extent, the wealth of the dictator, none of the alternative distributions would be Pareto optimal.

In the situation to be considered, where we are comparing the equity of the application of an ABM, we will be concerned with choices/alternatives that are less attractive for some than the status quo, i.e., these balancing mechanisms operate when the status quo approach has produced or will lead to financial deficits so some reductions (of benefits) or increases (of contributions) are required. Hence, in our context, none of the approaches will be considered Pareto optimal; however, these approaches might be able to be considered equitable.

For the purpose of comparing alternatives, a weighted utilitarian approach is adopted. The rationale is that the system includes both contributors and pensioners and both of their situations should be considered in assessing the equity of the application of the mechanism. That being said, to assess equity, we should also assess the capability of the individual to adjust to the proposed alternative distribution. For a social security system that relates benefits to earnings, it is reasonable to give consideration to the situation of all who have had earnings covered by the system, be they current contributors or pensioners. It is argued that pensioners are generally less able to adjust to a redistribution of the system's wealth because, on average, they tend to be heavily dependent on their social security pension for their livelihood or life style and because they have a shorter time horizon over to which to adjust. Hence in assessing the equity of the various mechanisms, a weighted utilitarian approach will be adopted with greater weight being given to pensioners.

5.2 Transition Considerations

Cichon et al. (2004) state that it takes at least seven decades for a SSRS mature. From the perspective of an individual participant, many of whom may not participate for as long as

seven decades, the scheme may be viewed as always being in transition, i.e., not mature. If, from the individual participant's perspective, the system is viewed as being in continuous or ongoing transition, it adds some rationality to the adoption of an ABM, which is a type of transition mechanism.

The individual's perspective of continuous transition is different in some significant ways from the creators' or designers' perspective. Their perspective tends to be of a steady state and of the desirable level of benefits and provisions in a mature system. For a creator/designer of a system, a financial imbalance which moves the system out of steady state is a shock to the system, whereas the individual participant may view the imbalance as another step in the system's evolution. Because, from the individual's perspective, the system appears to be always in transition, the individual participant may recognize that there is an ongoing process of intergenerational sharing and that it is unrealistic to assess the equality of the system based on whether each participant gets out of the system an equivalent amount to what he/she contributes. However, in the creators'/designers' mature steady-state it is reasonable to do such an assessment. Chen et al. (1997) discuss many factors such as gender, marital status, earnings level, birth cohort, that a creator/designer would wish to consider in trying to do such an assessment.

Hence, the individual participant is more likely to compare his/her situation, i.e., contributions and entitlements, to other participants participating in the system during his/her lifetime, or a shorter period. The individual participant judges equity on a relative basis rather than comparing his/her own benefits to contributions ratio, i.e., using an absolute basis.

Because the individual participant judges over a short comparison period, he/she is more likely to judge equity based on his/her situation compared to that of his/her family members, such as parents or children. So, for example, a "second generation" participant may accept paying a contribution rate higher than what is technically required to provide for the payment of his/her own pension because he/she recognizes that a parent, a member of the "first generation" received benefits of greater value than contributions made. Analogously, the second-generation member may recognize that if his/her generation has reproduced, in aggregate, at less than the population fertility replacement rate, then in a pay-go system, a financial burden is placed on the third generation. Recognizing this, the second generation may be willing to accept some of the financial burden. The purpose of this argument is to suggest that individual participants may:

- recognize that the system is continually evolving;
- understand that intergenerational sharing is occurring;
- judge equity by comparison with other participants in a short time frame, perhaps as long as the individual's lifetime; and
- accept as equitable arrangements under which the individual receives benefits whose value is less than the contributions made.

5.3 Definitions of Equity

As indicated in Chapter 1, there are many possible definitions of equity, some of which are completely incompatible with each other. Six possible families of definitions are considered in this subsection and two definitions are identified for further consideration and evaluation in this thesis. The six families of definitions are described and the reasons for their inclusion or exclusion explained.

1. “Hardline-No Changes” – This definition of equity may be paraphrased as saying “There are certain provisions of the social security act and these provisions form a binding contract with the individual, abiding by the exact terms of the contract is equitable and any change to the terms of the contract without the mutual agreement of both parties to the contract is inequitable”. For the purposes of this thesis, the equity of ABM is being considered. By definition, the ABM is only applied in situations where operating the system as it has operated in the past is no longer sufficient to maintain financial balance so some change is required. For anyone who had earned any benefits under the social security system prior to the introduction of the ABM, the introduction and application of the ABM would violate the “Hardline No Changes” definition of equity. Therefore, without disputing the validity of this definition of equity, it is not informative to evaluate ABM using this definition since, by definition, the application of the ABM would be inequitable.
2. “Paying the same contribution rate” or “Paying the same contributions” – These two definitions are slight variations on the same definition, so they will be treated as a single definition. Palmer (2005) states that a definition of fairness across generations is that the percentage of individual earnings transferred between generations (in a paygo arrangement) is the same for individuals in all generations. There is a simple appeal to the notion that if everyone has paid the same contribution rate or that if the contributions for a particular earnings history are the same among participants with this earnings history, then an equitable arrangement exists. However, this definition of equity does not have any relation to benefits. Hence, two participants could receive benefits of considerably different amounts and yet, if they paid the same contribution rate or the same contributions, it would be considered equitable. It seems that for equity to be achieved there needs to be some consideration of the value of benefits received. Furthermore, in response to Palmer, as successive generations live longer on average than their predecessors then either more contributions are paid for benefits of the same value if the retirement age is adjusted for life expectancy or more benefits are received, even though the contribution rate is the same – it is hard to see the inherent fairness in such an approach. Moreover, on this definition, if the ABM operated solely to change (reduce) benefits but did not change the contribution rate or contributions paid then the ABM would be equitable by definition. Conversely, any ABM that modified the contribution rate or the contributions paid would be considered inequitable regardless of the effect

it had on benefits. In this case, I would dispute the validity of the definition, but it is being discarded because it is not informative in assessing equity.

3. “Having a constant relationship between the present value of pensions and the present value of contributions across cohorts” – There is a family of definitions of equity here. The key elements of this family of definitions are:
 - both pensions and contributions are considered
 - there is a comparison across cohorts
 - a present value is calculated for the purpose of the comparison.

Some components of this family, which must be selected, that would affect the calculations but would still result in a definition within the same family of definitions are:

- the relationship to choose, i.e., one might choose the ratio of the present value of pensions to the present value of contributions or the present value of contributions to the present value of pensions or some standardized measure such as the ratio of the difference between the present value of contributions and the present value of benefits to the present value of contributions.
- A discount rate must be selected for the present value calculations. There are many possible choices for the discount rate, such as growth in the average income proposed by Settergren (2003) or an interest rate available in capital markets or a risk-free interest rate or some arbitrary, yet appropriate, rate.

This family of definitions seems to be valid; although, the choice of discount rate could affect the validity of the definition. This will be one measure used to assess equity of the ABM. Since, for the purpose of assessment, a specific definition must be considered, I will use a constant ratio of the present value of pension benefits over the accumulated value of contributions for all birth cohorts using growth in the average income as the discount factor, which is similar to the definition Settergren (2003) states.

Settergren (2003) also indicates that a high degree of intergenerational fairness could be achieved if there were a stable ratio of average pension to average income. Note that in a system with a fixed contribution rate that is a constant percentage of income, such a system belongs to this family of definitions. By calculating the present values at a zero discount rate, and instead of using the present value of contributions, using the present value of contributions divided by the contribution rate, i.e., the present value of income at a zero discount rate, the ratio is of a present value of pension to a linear transformation of contributions.

4. “The lifetime net benefit rate is zero for all cohorts” – The U.S. Department of the Treasury (2007) has defined the lifetime net benefit rate as the present value of net Social Security benefits (benefits less taxes) as a percentage of the present value of the individual’s lifetime wages. For this discussion, tax is used

interchangeably with contribution rate. Hence, the lifetime net benefit rate is a definition within the family defined in 3. The reason that this definition is discussed separately here, is that the definition regarding equity is formulated from the perspective of generational accounting, i.e., unless the lifetime net benefit rate is zero, there is intergenerational inequity. If the lifetime net benefit rate is positive then that individual/cohort will receive greater benefits than were paid for by its taxes or, in other words, some of its benefits must be paid by other generations' taxes which is considered an intergenerational subsidy that is unfair. Similarly, if the lifetime net benefit rate is negative, then that individual is receiving less benefits than taxes paid and its taxes may be used to subsidize another generation which is also considered unfair. Hence to avoid subsidies and be fair, the lifetime net benefit rate must be zero.

Although this definition is a member of the family of definitions defined in 3, which I will consider further, I reject this definition for further consideration because it is too restrictive. First, as noted earlier, it may take seventy or more years for a social security system to cease to be in a transition period and to become mature. During the transition period, the lifetime net benefit rate will be positive for some individuals who are beneficiaries of the introduction of or amendment to the social security system and will be negative for some individuals whose contributions or taxes must be used to subsidize the beneficiaries; hence, throughout the transition period, the system could not be considered equitable, by definition.

Note that a version of this definition of equity is incorporated in the argument for the inherent fairness of a system of individual social security accounts, which, in the extreme version, would permit individuals to contribute, invest their contributions (and any other employer contributions), and then receive benefits only to the extent to which there were a positive balance in their account to provide such benefits; moreover, if on the contributor's death there were still a positive balance in the contributor's account it would be transferred to the contributor's beneficiary. Although such an approach provides a hardnosed version of individual fairness, it does not rely on any insurance principles of risk pooling and so is not an appropriate approach to apply to a social security system. By their nature, social security systems are insurance systems, which involve risk pooling and risk transfers. They are not established to provide an administrative mechanism to receive an individual's contributions, do record keeping (and perhaps investment), and administer payments until the contributions are exhausted. One might quibble with these last few sentences by pointing to the example of social security in Chile, and other Latin American countries, that have fully funded defined contribution social security systems. However, even in such systems, there is some risk pooling, e.g., requiring a guaranteed minimum rate of interest be credited or requiring an annuity be purchased. Therefore, this definition of equity is rejected as invalid for SSRS with an ABM.

5. “Equity requires achieving certain social welfare goals“ – as in Barr (2004). As noted in the previous discussion, there is no question that social security systems are designed to go beyond the strictest version of individual equity to achieve some social welfare goals through risk pooling and risk transfer. Rather than attempt to challenge the validity of this definition, I will describe the approach taken and how it addresses, to some extent, this definition. Note that this definition is not well specified. The social welfare goals to be achieved are not listed. Does it mean providing a minimum pension? Does it mean requiring lifetime annuities with survivor benefits? Does it mean that benefits must be adjusted for inflation? None of these questions are answered and there are many more possibilities and questions that could be posed.

One way to answer these questions would be to look at Convention No. 102 of the International Labour Conference that defines minimum standards of social security. This convention was first adopted in 1952 as a result of thoughtful discussion and negotiation among international representatives of national governments, employers and labour organizations. As such, it is a standard worthy of consideration. Briefly, it specifies that a male worker with a wife who has earnings at the level of the ordinary male labourer and who has 30 years of employment, should be entitled to an annual pension in old age of at least 40% of average annual wages, valorized to time of calculation, and indexed after retirement. While both the establishment of a minimum standard and the level of standard are laudable achievements, fewer than one quarter of the countries have complied with this convention. Canada is one of the countries that has not complied. Hence to apply this standard may be to set the bar too high (whether or not it be a relevant level to set the bar).

Barr (2004b), for example, and other scholars believe that social security systems need to have some redistributive provisions in order to achieve their social welfare goals. For example, for workers whose earnings are disrupted due to disability, there should be special provisions, and, for example, for individuals whose pension is inadequate (however defined) there should be programs to provide supplementary income. To simplify this analysis, I will assume that in addition to a contributory earnings-related social security system that I will analyze, that there are other redistributive programs designed to meet specific social welfare goals for those who require assistance and qualify. Furthermore, I will analyze SSRS that have level contribution rates and that can be considered to provide the same level of benefit per unit of contribution. This will narrow the range of considerations regarding whether or not equity is achieved.

It is worth noting that in all of the four countries’ SSRS (except Canada) that I consider there is a level contribution rate on eligible earnings and that the same level of benefit per unit of contribution is promised. Even in Canada, this is almost the case, because contributions are not paid on the first \$3,500 of earnings, a relatively small percentage of earnings for all but low-paid workers. It is also worth noting that this approach has possibly eliminated some valid considerations

of equity. For example, the U.S. social security system has a level contribution rate on eligible earnings but provides a different level of benefit per unit of contributions, based on earnings ranges, i.e., lower earnings receive a higher level of benefit per unit of contributions than do higher earnings. In fact, one could describe the U.S. SSRS formula as progressive and argue that progressivity is an aspect of (intragenerational) equity. Consideration of such a benefit formula is beyond the analysis of this thesis but is a valid area for future analysis.

6. “Having an equivalent change in the expected utility among cohorts” – This definition is not a way of measuring equity prior to a change but it is a way of measuring the equity of a change. In the context of this thesis in which the equity of the operation of an ABM is being considered, it is appropriate to consider definitions of equity that measure the impact of a change. However, this definition is a relative definition of equity, evaluating the impact of a change, as opposed to an absolute definition of equity.

A prime difference in this family of definitions of equity, from any of the other definitions considered, is that it introduces the notion of expected utility. This changes the perspective from simply measuring the amount of the change, expressed in some form (such as a ratio), and then comparing across cohorts, to one of considering the impact of the change using the cohort’s utility measure. In this regard, it is arguable that a change in the future level of expected pension of $x\%$ may have quite a significantly different impact on expected utility if the change in expected pension is a long way in the future with much time to adjust lifestyle, savings plans, etc., as would be the case for a 20-year old, than if the change is immediate as in the case of a 75-year old pensioner.

To measure the effect of a change in expected utility requires at least the following:

- a utility function which can be used to evaluate the impact of an action
- a method of weighting the changes in utility for various periods in order to compute a simple measure of expected utility
- a way of comparing the expected utility before and after the change

In keeping with the methodology adopted which is to make as few assumptions as possible, I will make the following assumptions regarding the utility function without going so far as to specify the precise characteristics of the utility function:

- in assessing the impact of a change, the individual does a comparison, quite likely informal and not highly quantitative, of the expected value of the “item under consideration”, both before and after the change
- the expected value is calculated using discount rates
- for our particular investigation, the “item under consideration” is a standardized measure of the present value of future pension benefits. To standardize the measure, I will calculate the ratio of the present value of pension benefits to the accumulated value of contributions.
- the utility curve is upward-sloping and concave. However, given that the annual changes that will occur as a result of applying the ABM are likely

to be relatively small, I will treat the relevant part of the utility curve as being constant, i.e., there is no change in the utility of the alternatives to be considered. A ratio of expected utility before and after the change will be calculated to minimize the effect of the different utility functions.

With respect to what are the appropriate discount rates to use, I propose the following approach. A general rule of thumb is that a well-functioning economy with well-developed capital markets should permit investors to achieve a real rate of return of 3% per annum over the long term. Provided that an individual is assessing an investment over the long term, it is a reasonable assumption to discount future income at a real rate of 3% per annum. Consumers, in our case pensioners or future pensioners, view their prospective pension like an investment. It is reasonable to apply the discount rate that reflects time preference. For my calculations, I will assume that the long term is defined as the minimum of the individual's years of life expectancy and 30 years. So, for example, an individual age 20 with a life expectancy of 60 years would treat the long term as 30 years whereas an individual age 75 with a life expectancy of 9 years would treat the long term as 9 years.

In terms of discount rates, if the long term is at least 30 years, it is reasonable to use a real rate of return of 3%; however, for much longer terms a higher discount rate may be used and for much shorter horizons a lower discount rate may be used. The relevant considerations in developing this approach are:

- in capital markets, the normal shape of the yield curve for risk-free bonds is upward sloping by term to maturity
- individuals place higher present value on events that will affect them immediately or in the near future. A higher present value means that the future income to be received closer to the present is discounted at a lower rate than future income to be received further into the future is discounted.

I propose to evaluate income streams using the following discount rates:

Table 6: Personal Discount Rates

Period (current age x)	Discount Rate	
	For Period of Life Expectancy	Thereafter
$65 - x > 30$	4%	4%
$30 \geq 65 - x > 20$	3%	3%
$20 \geq 65 - x > 10$	2%	3%
$10 \geq 65 - x > 0$	1%	3%
$0 \geq 65 - x$	0%	3%

In calculating the period, 65 is used as a proxy for the normal age at retirement. So in this model, individuals determine how far they are from the age at which the pension will commence and adjust their discount rate accordingly. This decreasing discount rate, as

time to normal age at retirement decreases, is consistent with the theory that individuals are risk averse, which is consistent with a concave-shaped utility function.

3% is used as the hypothetical normal long-term discount rate. When individuals are considering a time period in excess of their life expectancy, they revert to 3%, if they were below that level, or remain at a higher level if already at a higher discount rate.

Using this table of discount rates, in Chapter 7 I will evaluate the ratio of the following quantity (for the Canadian SSRS):

$$\frac{\text{Present Value of Pension}}{\text{Accumulated Value of Contributions to 65}}$$

after-change to before-change.

To assess equity I will compare this ratio across cohorts. It is probably too much to demand that this ratio be equal across cohorts. For equity to exist, I will require that ratios be not more than 5% different in any one year and be not more than 10% different in any five-year period.

5.4 Methodology

There are at least three approaches that might be adopted to attempt to evaluate the properties and robustness of the provisions or mechanisms to be considered and tested, as follows:

1. Make a number of assumptions regarding the economy, population growth, etc. so that a model can be adopted that admits of a closed form solution.
2. Make fewer assumptions than above, but not so many that a closed form solution exists, and then attempt to use dynamic programming techniques to draw conclusions.
3. Make fewer assumptions than in 2, use population projections and examine the results.

For this thesis, for the purpose of evaluating equitable balance, approach 3 has been adopted. Reasons for adopting this approach include:

- By making sufficient assumptions to be able to use approach 1 or 2, there is the danger that relevant factors will be excluded or modified as a result of the assumptions made, that will jeopardize the validity of any conclusions drawn.
- For approach 1 to be viable, it is generally necessary to assume a steady state, whereas the provisions to be tested only apply in situations when steady state conditions do not hold; consequently, approach 1 is not appropriate for the purpose.
- Population projections from reliable sources are available through 2050. If the mechanisms are demonstrated to be equitable over such a time horizon, many

- observers would consider such a time frame to be sufficient, for practical purposes. On the other hand, if the mechanisms are demonstrated to be inequitable over such a time horizon, the conclusions would be valid.
- For many observers, especially those who are not pure mathematicians, a demonstration by applying the mechanisms to actual population projections is a more understandable, acceptable and convincing demonstration than an approach using models and mathematical techniques.

5.5 Chapter Conclusion

This chapter began with a discussion of the economic concept of Pareto efficiency and showed that this concept should not be equated with equity. The chapter then made the important point that because of the long period time that may be required for a change to be fully incorporated in an SSRS, from the perspective of the individual conditions must be evaluated in the face of (almost) continuous change.

Six families of definitions of equity were presented and discussed and the definition of equity to be used in this thesis was determined. This definition measures the ratio of the present value of pension to the accumulated value of contributions at age 65, after-change to before change. Using this approach the effect on cohorts can be compared. Provided that the adjustment is less than 5% in one year and less than 10% over five years, equity will be considered to exist. With this definition, the characteristics of a robust ABM set out in Chapter 2 are complete.

6.0 Chapter Overview

In this chapter, the stabilizing mechanisms of the four countries on which this thesis focuses are described. The countries are Canada, Germany, Japan and Sweden. Next the objective of financial balance is determined. Within this context, the mechanisms are analyzed to determine whether they are designed to be a solution to a stochastic optimal control problem. All of the mechanisms use a set of mechanical rules. It is demonstrated that the application of these rules is insufficient to ensure financial balance. Because significant financial analysis has been done by the countries' actuaries, economists and governmental agencies, as well as by certain other commentators and transnational bodies, this analysis is referred to. Since such analysts have access to vast amounts of data, complex models and significant resources, the approach of this thesis is to review the published analysis rather than to perform independent modeling. A significant problem in the manner used by Sweden to calculate assets is identified, quantified, and an approach suggested to correct the error. The unusual properties of the steady-state contribution rate calculation in the Canadian SSRS are analyzed.

6.1 Sweden

6.1.1 Sweden's ABM

The ABM for Sweden's NDC was described in Chapter 1. To determine if the approach used is a formulation of a stochastic optimal control problem, define the following:

Buffer Fund (assuming no positive contribution flow)

$F_t(t, I_t)$ is the amount of the buffer fund which commenced at F_0 and varies
 $F_t \geq 0$ with time t and the investment earnings I_t

Return on Buffer Fund for period $(0, t)$

$I_t = F_0 \{ \alpha_1 dB_t + (1 - \alpha_1) dS_t \}$ where α_1 is allocated to risk-free investments B and
 $(1 - \alpha_1)$ is allocated to risky investments S where rr
 $dB = rrBdt$ is the return on the risk-free asset and S follows a
 $dS = \alpha Sdt + \sigma SdW$ Wiener process with mean α and variance σ^2

Contributions

$C_t(k, nc_t, w_{nc_t})$ contributions at time t depend on contribution rate k which is
 $C_t \geq 0$ constant and number of contributors nc_t and average wage of the
 $nc_t \geq 0$ contributors w_{nc_t}
 $w_{nc_t} > 0$

For sc_t , suspended contributors at time t , $w_{sc_t} = 0$

Accounts

$A_t(A_{t-1}, C_t, T_t, r_t)$ Account at time t depends on account at time $t-1$,
 $A_t \geq 0$ contributions at time t , the withdrawals T_t , and the
 $C_t \geq 0$ rate credited to accounts r_t
 $T_t \geq 0$
 $r_t \in \mathcal{R}_n$

Consumption

$P_t(np_t, P_{np_{t-1}}, \pi_t, e_t, D_t)$ Pensions at time t depend on np_t the number of pensioners at time
 t , the average pension the previous year $P_{np_{t-1}}$, the increase rate in
 $np_t \geq 0$ the pension π_t , the number of new entrants to pension status e_t
and pensioners deceased D_t , where \ddot{a}_t is the annuity factor to
convert entrants' accounts to pensions
 $P_{np_{t-1}} > 0$
 $P_{e_t} = \frac{A_t}{\ddot{a}_t}$
 $\pi_t \in \mathcal{R}^n$

Control law (ignoring situation where balance ratio goes below 1 and then increases above 1)

U_t depends on balance ratio
$$\begin{cases} U_t = 1 & \text{if } \frac{\text{Assets}_{t-1}}{\text{Liabilities}_{t-1}} \geq 1 \\ U_t = \frac{\text{Assets}_{t-1}}{\text{Liabilities}_{t-1}} & \text{if } 0 < \frac{\text{Assets}_{t-1}}{\text{Liabilities}_{t-1}} < 1 \end{cases}$$

Crediting Rate

$$r_t = \left\{ \frac{\sum_{nc_t} w_{nc_t}}{nc_t} \right\} \left\{ \frac{\sum_{nc_{t-1}} w_{nc_{t-1}}}{nc_{t-1}} \right\}^{-1} U_t$$

$$\pi_t = r_t - .016 U_t$$

Assets

$F_t + C_t(TD_t)$ sum of buffer fund plus sum of one year's contribution multiplied by the turnover duration TD_t

where

$$TD_t = \left| \frac{\sum_{c_x} C_{c_x} X_{c_x}}{C_t} - \frac{\sum_{p_x} P_{p_x} X_{p_x}}{P_t} \right|$$

is the absolute value of the weighted average of

contributors age c_x with contributions C_{c_x} of their age x minus the weighted average for pensioners P_x with pensions of P_{p_x} of their age x .

Liabilities

$\sum_{nc_t} A_t + \sum_{np_t} P_t e_{p_x}$ sum of the accounts for contributors and suspended contributors

and sum of the pension accounts for pensioners multiplied by their remaining life expectancy.

Comparing this to the traditional formulation of the stochastic optimal control problem we note that:

- it is a stochastic process as obligations increase in relation to accounts and life expectancy, where accounts are dependent on contributions and the growth in the average wage
- the contribution rate is constant
- the control law is defined deterministically.

The ABM of the Swedish NDC is relying on a deterministic control law to maintain the system in balance. In effect, the problem reduces to one of determining whether the Swedish system is self-financing given a constant contribution rate, a deterministic control law and a stochastic process for obligations based on the growth in average wages, i.e., the stochastic process is dependent on the Swedish economy.

6.1.2 Objective of Financial Balance - Sweden

There are two methods of adjustment used in the Swedish SSRS to attempt to achieve financial balance. The first is the adjustment of the age of full-benefit entitlement to reflect expected increases in life expectancy. According to the work of Anderson (2006), in a paygo social security system, it is not sufficient to adjust the retirement age and the consumption of the elderly in proportion to longevity increases, in order to obtain balance. It is also necessary to increase the contribution rate for the young. Because in the Swedish SSRS, the contribution rate is fixed, it is necessary to have a second adjustment. The second is the ABM, which applies when the ratio of assets to liabilities is less than one. It reduces the nominal rate of return credited to the contributors' accounts and rate of increase applied to pensions paid.

The Swedish SSRS defines a minimum benefit. If the pension benefit falls below this minimum benefit level then a supplemental pension is paid from general revenues to increase the total pension from all sources to the minimum benefit level. For the balancing mechanism to be considered to work effectively, financial balance should be achieved without any pensioner who became entitled at retirement to a pension above the minimum level, ever having such pensioner's payment from the system fall below the minimum benefit level. The reason for this condition is that if it is not met then the Swedish SSRS and its balancing mechanisms will achieve financial balance by definition. This results because the SSRS could always pay pension benefits with available contributions and if those pension benefits were below the minimum benefit level then they would be supplemented from general revenues. In the extreme, everyone would receive the minimum benefit from the contributions and general revenues. But such a balance should not be attributed to the effectiveness of the balancing mechanism but rather to the failure of the mechanism to achieve balance, i.e., continuing to reduce benefits until they equal contributions is not a balancing mechanism but the absence of an effective mechanism. Hence, in assessing the effectiveness of the mechanisms in the Swedish SSRS, it is necessary to determine whether it will achieve balance without reducing pensions that were higher than the minimum benefit at retirement, to below the minimum benefit level, i.e., balance will be achieved without extra-systematic flows.

6.1.3 Analysis of Swedish Balance Sheet

As explained previously in subsection 1.2.1 and in subsection 6.1.1 with the aid of mathematical symbols, the balance sheet used in the NDC portion of the Swedish system to calculate the financial position in order to determine if the ABM is to be applied defines assets and liabilities as follows: Total assets equal the contribution asset plus any assets in the buffer fund where the contribution asset is defined as one year's contributions multiplied by the expected turnover duration, which is defined as the absolute value of the number of years difference between the expected income-weighted age of income earners and the expected pension-weighted age of pensioners, both represented as a single age. Total pension liabilities are defined as the accumulated

notional account balances of non-pensioners plus the sum over all pensioners of the product of their current pension times their remaining life expectancy.

In this subsection, I will show that this method of defining the balance sheet systematically over-states the value of the assets and therefore the balance ratio. Moreover, as a consequence of this over-statement, the application of the ABM does not achieve financial balance. I suggest a modification to the formula to improve the definition.

Let us start by considering how the balance sheet would be defined in an actuarial valuation using a closed group approach. Assets would equal the assets in the buffer fund plus the present value of contributions to be paid by non-pensioners. Liabilities would equal the accumulated notional account balances of non-pensioners plus the present value of contributions to be paid by non-pensioners plus the present value of remaining pensions to be paid to pensioners. In a closed group approach, only pensioners and non-pensioners that were part of the SSRS at the valuation date would be included. Let us compare this formulation to the balance sheet defined by the Swedish system.

First, note that the present value of contributions to be paid by non-pensioners appears in the traditional closed group valuation balance sheet in both the assets and liabilities, so it can be deducted from both terms without affecting the financial position. Next, consider the calculation of the present value of remaining pensions to be paid. If this were computed at a zero percent interest rate, then it would be equal to the sum over all pensioners of the level of pension by pensioner multiplied by the pensioner's remaining life expectancy. On this basis, pension liabilities, net of the present value of future contributions, are equal to pension liabilities as defined in the Swedish system; however, pension assets, net of the present value of future contributions, are equal to the assets in the buffer fund. This differs from the assets in the Swedish system by the contribution asset.

The definition of the actuarial financial balance in the closed group seems correct. If the system were terminated at the valuation date, the assets available to pay all the accumulated liabilities, which are the accumulated value of the notional accounts and the present value of pensions to the pensioners, would only be the assets in the buffer fund. So the Swedish SSRS includes an additional amount of assets equal to the contribution asset. How is this inclusion to be rationalized?

Because the SSRS is not being terminated at the valuation date and because the country can rely on future contributors or taxpayers to continue to pay to provide for the liabilities, the Swedish system adds contributions to be paid in the future.

In a steady state, which the Swedish system assumes, there is some rationale for this approach. Even though new contributions create new and additional liabilities, new contributions may be used to pay pensions because in a steady state there will be an equally large successor group of contributors to pay the pensions being earned through contributions by the current group of contributors. In other words, if the net value of the

closed group liability can be maintained then, in a steady state, each group of contributors' pensions is paid by the subsequent group of contributors and there is no change to the net financial position. But does the contribution asset make the appropriate adjustment?

The expected turnover duration is the absolute value of the difference between the expected income-weighted age of income earners and the expected pension-weighted age of pensioners. The rationale is that the current contributions (along with the buffer fund) can be used to pay pensions because the current contributors (on average) will not receive their pension until the expected duration has elapsed. In a steady state, there should be a sufficient contribution to pay their pensions. The rationale is appealing but it is flawed.

The problem is that the rationale implicitly assumes that the contributions of one group can be used to pay the pensions of the predecessor group and that those contributions will be paid from the expected turnover duration. But that is not the case. Contributions will only be paid by the current contributors until their normal retirement age, approximately age 65. (Or if greater refinement is desired, calculate contributions up to the expected retirement age of contributors, which could be determined, for example, by using the scheduled age for full benefit entitlement for the contributing cohort but adjusted for expected retirements at other than the age of full benefit entitlement by applying retirement-age weights based on an analysis of the retirement experience of the pensioners). The expected turnover duration provides for contributions to be made for a period of time up to the pension-weighted average age. During the time period between the normal retirement age and the pension-weighted average age, it is not the current contributor group who will be paying pensions for the predecessor group; it is the group following the current contributor group. In other words, the assets to pay the pensioners are coming from both the group of current contributors and the group of not-yet contributors.

If we think back to the closed group valuation, we included the present value of future contributions in both the asset and liability values. This represents the appropriate portion of liability that the present contributors could expect the successor contributors to bear in a steady state pay-go model; however, the present value of contributions only would include contributions to be paid to the contributors' normal retirement age.

Because the expected turnover duration includes the years beyond the normal retirement age up to the pension-weighted average age of pensions, it overstates the contribution asset. Because the assets are overstated, when and if the ABM is triggered, the financial position is much worse than is represented and the amount of the annual adjustment via the ABM is less than required to rectify the financial imbalance. Moreover, because the assets implicitly include some contributions from a future group of contributors after the current group of contributors, when there is a negative imbalance, due to demographics or economic factors, there will be insufficient uncounted contributions from the future group of contributors available to rectify the imbalance.

To get a sense of the magnitude of the misstatement of the assets, consider the following. Settergren (2003) states the expected turnover duration is approximately 33 years, the difference between age 76 and age 43, the pension-weighted average age and the income-weighted average age respectively. If normal retirement age is approximately age 65, then the correct expected turnover duration is approximately 22 years. Hagberg (2006) estimates that the 2006 buffer fund assets are approximately 11.75% of total assets. So combining these figures and the adjustment, assets are overstated by approximately 41.60%. On this basis the ABM ought to have been triggered already.

Next, notice that once triggered, the ABM is unlikely to restore balance, because its application still results in liabilities greater than assets. When liabilities exceed assets, the stated method of adjustment is to multiply the liabilities by the ratio of assets to liabilities in order to adjust liabilities. But the adjustment is not applied in this manner. The ratio of assets to liabilities is multiplied by the rates to credit to the accounts of actives and the rates to increase the pensions. Such an application of the adjustment results in new liabilities that still exceed assets. Even if all economic and demographic variables develop as expected in the next period, liabilities will still exceed assets and the mechanism must be applied again. However again, adjusted liabilities will still exceed assets, and so it will continue. Using the notation developed earlier.

$$\text{When unadjusted liabilities} = \sum_{\substack{nc_t \\ sc_t}} A_t + \sum_{np_t} P_x e_{p_x} > F_t + C_t(TD_t) = \text{Assets}_t$$

Then for balance we want

$$\text{Assets}_t = \text{Adjusted Liabilities}_t = \text{Unadjusted Liabilities}_t \times U_t$$

$$\text{But} \quad \text{Adjusted Liabilities} = \sum_{\substack{nc_t \\ sc_t}} \text{adjusted } A_t + \sum_{np_t} \text{adjusted } P_x e_{p_x}$$

$$\text{and} \quad \text{adjusted } A_t = \text{unadjusted } A_t(A_{t-1}, C_t, T_t, \text{adjusted } r_t)$$

$$\text{and} \quad \text{adjusted } P_x = \text{unadjusted } P_x(np_t, P_{np_{t-1}}, \text{adjusted } \pi_t, e_t, D_t)$$

$$\text{so} \quad \text{Adjusted Liabilities} > \text{Unadjusted Liabilities}_t \times U_t$$

$$\text{and} \quad \text{Adjusted Liabilities} > \text{Assets}$$

so financial balance is not achieved.

Recall that we require as a minimum for the ABM to be considered to be operating effectively that any pensioner whose initial pension exceeds the minimum benefit will not, at a later date, find, as a result of the operation of the ABM that the pension is less than the minimum benefit. But there is no assurance of this for initial pensions that are only slightly above the minimum benefit. These pensions have been determined using a

1.6% interest rate to calculate the annuity factor. If the average rate of real wage growth is 1.6%, in the absence of the ABM being applied, the pensions will be increased by the rate of inflation. This is the same rate to be applied to the minimum benefit. However, if the rate of average wage growth is less than 1.6% then the amount of deficiency will be deducted from the rate of inflation and the rate credited to pensions will be reduced further. No such adjustments apply to the minimum benefit that continues to increase annually by the rate of inflation. Therefore, it could easily be the case that with a few years of average wage growth less than 1.6% that an initial pension that commenced slightly above the minimum benefit would have increased less rapidly than the minimum benefit so that in the absence of supplement, it would be less than the minimum benefit. This point alone is sufficient to conclude that the Swedish ABM does not even achieve partial financial balance. It would be economic factors that would cause such a situation to occur.

Finally, there is no direct financial connection between the rate of increase in average annual income per capita for contributors and the amount of indexing that can be provided to pensioners. To illustrate this point, assume that the real rate of increase in average wages is 1.6 percent, the rate that has been incorporated in the pensions of pensioners. If 16% (the contribution rate) of the volume of income increase is exactly equal to the volume of pension increase, then, on a paygo basis, the system is balanced financially. But it is highly improbable that this balance would exist, given the number of unrelated parameters, and if it did exist in any year, it is highly unlikely that it would persist. It is extremely difficult and highly conjectural to project future rates of income growth, but changes in population are more predictable. The following figures from the United Nations 2002 population projections (“UN 2002”) provided by the International Labour Organization (“ILO”) show how the population ages 15-64, a proxy for the working population of contributors, is expected to decrease from 2010 onward while the population age 65 and over, a proxy for the pensioner population, is expected to increase in all years until 2045. With such changing demographics, it is unlikely that a persistent financial balance can be achieved.

Table 7: Swedish Population Projections

Year	Population (in thousands)	
	Ages 15 – 64	Age 65 and Over
2005	5,805	1,576
2010	5,826	1,723
2015	6,646	1,924
2020	5,509	2,051
2025	5,354	2,158
2030	5,167	2,274
2035	4,980	2,375
2040	4,823	2,417
2045	4,701	2,386
2050	4,555	2,353

Legros (2006) states that when the contribution rate is fixed, as in the case in NDC systems, the indexing rule will try to provide an automatic stabilizing effect. She concludes that Sweden's system is the only one that does this successfully. For the reasons, presented in this subsection, this author disagrees that Sweden's system succeeds in providing automatic stabilization (balance).

6.2 Japan

6.2.1 Japan's Pension System – Greatly Abbreviated

The Japanese pension system is highly complex, with many components that have been subject to various reforms over the past two decades. The Japanese society is aging relatively rapidly due to the combination of increasing life expectancy and a total fertility rate that has been below the replacement rate of approximately 2.10 (per female aged 15 to 49) since 1975 and has continued to decline to 1.3 in 2000 (Sakamoto, 2005). The impact of an aging society has been to exert pressure on the financing and stability of Japan's pay-go defined benefit social security system resulting in amendments to the contribution rate, government subsidies, retirement age, benefit indexation formulas and system structure. For the purpose of this thesis, only a specific component of the system will be analyzed. The analysis will be generally applicable to other components of the system. For a fuller description of the system and reforms see Sakamoto (2005).

For the purpose of this analysis, I will consider the social security pensions available to those employed in the private sector not covered by an occupational pension plan. Such employees are covered by the National Pension Scheme (NP) that provides a basic flat-rate pension benefit proportionate to covered months divided by 480 months (maximum 1) and by the Employees' Pension Insurance Scheme (EPI) that provides earnings-related pensions for the covered period. The annual accrual rate for EPI is 0.5481 percent of average earnings, revalued to age 65. After age 65, benefits are indexed to increases in the Consumer Price Index. The pensionable age for the flat rate part is 65 and the pensionable age for the earnings-related part is 60 but it will be increased gradually to reach 65 in 2025 for men and in 2030 for women.

The EPI receives contributions from employees and employers and is required to transfer funds to pay the NP and EPI pensions in proportion to the covered pensioners. Under the 2004 reform the contribution rate was set at 13.58%, then raised to 13.934% in October 2004, thereafter it is to be raised by 0.354% in September every year until it reaches the ultimate rate of 18.3% in September 2017, when it becomes fixed. Funds held by the EPI are invested in various securities including bonds, Japanese equities and foreign equities.

In addition to contributions and investment income, the government also subsidizes the basic pension. In 2004 the subsidy rate was $\frac{1}{3}$. It is scheduled to be $\frac{1}{2}$ in 2009 and to increase to that level in three steps.

With this very abbreviated background on a specific component of the social security system, the description of the automatic balancing mechanism in subsection 6.2.2 will have a context.

6.2.2 Japan's Automatic Balancing Mechanism

Even with the various reforms to increase contributions, raise the government subsidy and extend the pensionable age, it is not forecast that financial stability will be achieved. The 2004 reform of Japan's system introduced an ABM, which is described in this subsection. The ABM is a method of reducing earnings-related benefits from the EPI until financial equilibrium is achieved. There are two prongs to the benefit reduction: the one adjusts the initial level of benefits earned by adjusting the factor for revaluing average earnings; the second adjusts the indexation of pensions being paid. However, the contribution rate remains fixed according to a set schedule.

Prior to the introducing an ABM into the system, average wages for the purpose of benefit determination were revalued in respect of the improvement in active workers' disposable income, until the beneficiary attains the age of 65. After the age of 65 the benefit was indexed to the increase in the Consumer Price Index.

Recognizing that an increasing ratio of pensioners to active workers increases the costs of a pay-go system and is de-stabilizing, Japan introduced an ABM that incorporates adjustments for the factors creating this increasing ratio. In Japan's case, this increasing ratio is attributable both to a declining active work force due to below replacement level fertility rates and to increased life expectancy. Thus the traditional adjustments, discussed in the foregoing paragraph, are modified by the rate of decline of active contributors. The traditional revaluation of earnings or indexation of benefits is modified by subtracting a modifier from the traditional increase factor. The modifier is equal to the rate of decline of active participants in social security pension schemes plus the yearly rate of increase in life expectancy at age 65. For completeness of description, should financial balance be achieved through the application of the ABM then the system reverts to indexation without the modifier. An approximation for the increase in life expectancy is used, namely a constant adjustment of 0.3 percent per year.

This may be expressed in mathematical symbols, in respect of the EPI, as follows:

Pensions in year t

New pensions in year t in respect of pensioner x ${}_xP_k^j$ is pension payable in year k for worker x retiring in year j

$${}_xP_t^t = \sum_{i=1}^t \alpha {}_xw_i \prod_{j=i}^{t-1} [1 + k_{j+1} - b_{j+1}]$$

${}_xw_i$ is gross wage in year i for

worker x

α is accrual rate

b_t is balancing factor in year t

Pensions payable in year t for pensioner x retiring in year k for $k < t$

$${}_xP_t^k = {}_xS_t^k {}_xP_k^k \prod_{j=k}^{t-1} [1 + cp_{j+1} - b_{j+1}]$$

k_j is the average increase in disposable income of active workers in j over year $j-1$
 ${}_xS_t^k$ is a survival function – the probability a life age x who retires in year k survives to year t
 cp_j is average increase in consumer prices in year j over $j-1$

Total pensions in year t

P_t is pensions in year t
 C_t is contributions in year t

$$P_t = \sum_{\text{all } x} \left\{ {}_xP_t^t + \sum_{s=k}^{t-1} {}_xP_t^s \right\}$$

c_t is contribution rate in year t

Contributions in year t

G_t is government subsidy in year t

$$C_t = \sum_{x=NC_t} c_t {}_xW_t$$

Government subsidy (from 2009)

NC_t is number of active workers in year t

$$G_t = C_t$$

Balancing Factor

$$b_{t+2} = \left[\text{Lesser of } \begin{cases} 0 \\ \left(\frac{NC_t}{NC_{t+1}} - 1 \right) \end{cases} \right] + .003$$

6.2.3 Objective of Financial Balance – Japan

For Japan the contribution rate for the EPI is scheduled to increase from 2004 until it reaches its ultimate level of 18.3% in 2017. Financial balance will be considered to be maintained provided the replacement rate is at least 50%. The balancing mechanism will be considered to achieve financial balance if all pensioners have a replacement rate of at least 50%.

6.2.4 Analysis of Japan's ABM

According to projections, the application of this ABM will gradually bring the system into financial balance by reducing the initial replacement rate for pensioners and by reducing the amount of increases in pensions. However, in order to achieve financial balance, the replacement rate was projected to drop to 45%. Because it was deemed that 50% was the ratio of the average consumption of aged couples to the average amount of disposable income of the active labour force, the reform imposed a minimum replacement rate of 50% (Sakamoto, 2005). Accordingly, if the projections are correct, some further adjustments to the system will be required to achieve financial balance. In assessing financial stability and making projections a 95-year time horizon is used.

Based on these projections the automatic balancing mechanism will not achieve financial balance at a benefit replacement rate considered adequate by the Japanese. The mechanism is a manner of reducing benefits according to a defined approach, without political intervention, but it does not achieve financial balance. It is anticipated that a review of the system resulting in revisions to contributions and/or benefits will be required. But the introduction of this mechanism will postpone the time for such review and revisions. Since on the Japanese own projections, the system is not likely to achieve financial balance, the mechanism can only be considered to achieve a transitory balance. It is primarily demographic factors that cause this to occur.

One problem with this mechanism is that the parameters linking the contributions and the benefit reductions are not closely enough associated with the objective of financial balance. First, the rate of benefit modification is determined by the rate of increase (decrease) in income adjusted for an increase in life expectancy. These factors are indicators of the financial pressures on the SSRS but they are not good measures of the SSRS ability to pay benefits at a level that maintains a 50% replacement rate. The rate of increase (decrease) in gross income volume is a proxy for the available contributions increase (decrease) in the year, since contributions are a percentage of income. The factor to reflect increases in life expectancy is an indication of how much longer benefits may be expected to be paid than was previously expected. However, neither of these factors is a good measure of the contributions available (not available) to pay pension increases (decreases) in the year nor an indicator of the replacement rate. The contributor population is projected to decline over time and the group of benefit recipients is projected to continue to increase until 2045, when it is projected to begin to decline slightly. See the following table, based on the UN 2002 projections as supplied

by the ILO, that shows how the group ages 15-64, a proxy for contributors, is projected to decrease and the group ages 65 and over, a proxy for pensioners, is projected to increase until 2045. With such significant demographic changes occurring not only to the numbers of contributors and pensioners but also in the rate of change in contributors and pensioners, as shown in Table 8, the actual decreases in contributions measured by a rate of gross income change with a fixed life expectancy adjustment could not be expected to equal the paygo cost of benefit reductions and the system could not be expected to achieve at least a 50% replacement rate.

Table 8: Japanese Population Projections

Year	Population (in thousands)		Rate of Change	
	Ages 15 – 64	Age 65 and Over	Ages 15 – 64	Age 65 and Over
2005	84,854	25,176	n/a	n/a
2010	81,908	28,683	-.0347	.1393
2015	77,638	33,056	-.0521	.1525
2020	75,050	35,338	-.0333	.0690
2025	73,209	36,070	-.0245	.0207
2030	70,471	36,748	-.0374	.0188
2035	66,570	37,824	-.0554	.0293
2040	61,317	39,798	-.0789	.0522
2045	57,187	40,474	-.0674	.0170
2050	53,964	40,077	-.0564	-.0098

6.3 Germany

6.3.1 The German Public Pension System

This section describes briefly the component of the German public pension system that applies to employees excluding civil servants and the self-employed, after the reform package, proposed by the Rurup Commission in 2003, was put into law in 2004. It does not include a description of revised tax measures that were introduced subsequently that are described by Toft (2007). These reforms are frequently referred to as the Rurup Reform. The system had undergone significant change in 2001, referred to as the Riester Reform after the Secretary of Labour at the time. This description is based on an article by Borsch-Supan et al. (2006).

The reforms have transformed a pay-go career-average defined benefit retirement insurance system that provided a high level of net income replacement, approximately 70%, to a worker who had earned average lifetime earnings and participated for 45 years to a notional defined contribution (NDC) system look-alike. Net replacement rate was based on disposable income defined as gross income net of income tax. Approximately

70% of the budget of the retirement insurance system is financed by contributions administered like a payroll tax, levied equally on employers and employees. The remaining 30% of the budget is financed by earmarked indirect taxes and a subsidy from the federal government. The insurance system distinguishes five types of old-age pensions: normal; long service life; women; older disabled; unemployed. Only “normal” does not have an earnings test, so it will be the focus of this analysis. The “normal” pension is available at age 65 after 5 years of service. Benefits are computed on a lifetime basis and adjusted according to the type of pension and the retirement age, as follows:

$$P_{t,i} = EP_i \times SY_i \times AF_i \times PV_t$$

where in respect of pensioner i :

$P_{t,i}$ is the annual value of a pension in year t

EP_i are earnings points expressed as a multiple of the average annual contribution in each historical working year, e.g., one EP corresponds to average earnings, 0.5 EP corresponds to 50% of average earnings, two EP corresponds to earnings twice as large as earnings for the year

SY_i comprise years of active contributions plus years of credit where no contributions were made, e.g., in respect of military service, parent’s time in respect of a child’s education, etc., and notably years of unemployment

AF_i is an adjustment factor that is 1 for the normal old-age pension

PV_t represents the current pension value in year t and is the crucial link between workers’ earnings and pensioners’ benefits. It is indexed to the annual changes in the level of wages and salaries, net of pension contributions.

The mechanism follows the following formula:

$$PV_t = PV_{t-1} \left(\frac{AGI_{t-2}}{AGI_{t-3}} \right) \left(\frac{1 - \delta_{t-2} - \tau_{t-2}}{1 - \delta_{t-3} - \tau_{t-3}} \right) \left(\left(1 - \frac{PQ_{t-2}}{PQ_{t-3}} \right) \alpha + 1 \right)$$

where PV_t , PV_{t-1} are current pension value in year t and year $t-1$ respectively

AGI_{t-2} , AGI_{t-3} represent average gross income in year $t-2$ and year $t-3$ respectively

δ_{t-2} , δ_{t-3} represent the assumed contribution rate to supplementary pensions in year $t-2$ and year $t-3$ respectively, which is gradually increased from 0.5% in 2003 to 4%, which is the ultimate level, in 2009 and thereafter

τ_{t-2} , τ_{t-3} represent the contribution rate to social security in year $t-2$ and year $t-3$ respectively

PQ_{t-2} , PQ_{t-3} represent the ratio of the number of pensioners to the sum of the number of contributors and the unemployed in year $t-2$ and year $t-3$ respectively

α is the sustainability parameter that is set (at least for some time) at 0.25, which shares the burden of the adjustment between pensioners and workers

In the 1992 reform, normal retirement age was increased to age 65. Further increases to normal retirement age, which are not directly linked to life expectancy, are scheduled. The 1992 reform also anchored benefits to net wages of social security taxes and savings rather than to gross wages, which provides a stronger link between pensioners' benefits and workers' wages.

The Riester Reform changed the nature of the system by introducing a funded pension component. It also introduced measures to control costs and provide for stable pension levels. Contributions must remain below 20% until 2020 and below 22% until 2030 and the net replacement rate for pension benefits in relation to lifetime earnings must stay above 67% until 2030. Net replacement rate is based on gross earnings less income tax and social security contributions, which effectively reduces the target replacement rate. To compensate for the drop in net replacement rate (from approximately 70%), a system of tax-assisted supplementary (funded) pensions, using a DC approach, was created. It is not mandatory to contribute and the uptake in participation has been slow (although the change is relatively recent so it is too early to make any conclusive assessment regarding the popularity of this reform).

The Rurup Reform introduced two significant provisions to enhance the financial stability of the system. It provides for a gradual increase in the normal retirement age from 65 to 67 years, in monthly steps from 2011 to 2035. It also introduced a sustainability factor that is a part of the mechanism that modifies pension benefits in relation to the system dependency ratio.

Let us consider how this mechanism works. To begin, assume a steady state population and a mature system so $\delta_{t-2} = \delta_{t-3}$ and $\tau_{t-2} = \tau_{t-3}$. In a steady state, population $PQ_{t-2} = PQ_{t-3}$ so the formula reduces to $PV_t = PV_{t-1} \left(\frac{AGI_{t-2}}{AGI_{t-3}} \right)$. This means that the current pension value is adjusted by the rate of increase in average gross income (wages) on a lagged basis. The lag was introduced to provide sufficient time to assemble the data and do the computations.

Note $AGI_{t-2} \left(\delta_{t-2} - \tau_{t-2} \right)$ represents average income in year $t-2$ net of deemed pension saving, i.e., both through supplementary pensions and social security. So if the

contributions to pensions increase, then the increase in the value of current pensions is dampened, so that pensioners share some of the pain of a more costly pension system.

Note also that PQ introduces a demographic dependency ratio of the number of pensioners to the number of non-pensioners, i.e., the contributors plus the unemployed. If this ratio increases year over year, then the ratio of $\frac{PQ_{t-2}}{PQ_{t-3}}$ is greater than 1 so there is a reduction in the otherwise calculated increase in the current pension value, because this term appears in the formula as $\left(1 - \frac{PQ_{t-2}}{PQ_{t-3}}\right)$.

Note that this demographic dependency ratio is not only affected by retirement practices and life expectancy but also by labour force dynamics such as retirement practices and participation rates. However, the full amount of this reduction is not applied – it is modified by a sustainability parameter α . In effect, α shares the burden of the adjustment between pensioners and workers. If α were 1, then pensioners would bear the whole burden of the adjustment. If α were 0, then pensioners would not bear any impact of a change in demographic dependency ratio. (α is now equal to 0.25).

Whitehouse (2007a) states that the German system is unique because it adjusts the point value, PV_t , even after the pension has commenced. Thus the adjustment affects the entitlements of both current and future pensioners. Whitehouse (2007a) has calculated that 3/8 of financial effects of life-expectancy changes fall on pensioners and 5/8 on contributors.

Legros (2006) refers to this mechanism to adjust the value of the current pension as the reverse spring mechanism. If the current pension value increases by more than the system can support financially, then the contribution rate would have to rise in a subsequent year. The consequence of a contribution increase in a subsequent year would be that the increase in current pension value subsequently would be moderated, i.e., if the contribution rate, τ , is raised in $t-2$ and given that $\tau_{t-2} \geq \tau_{t-3}$ then the ratio

$\left(\frac{1 - \delta_{t-2} - \tau_{t-2}}{1 - \delta_{t-3} - \tau_{t-3}}\right)$ is less than 1 and PV_t is less than PV_{t-1} (all other things being equal).

(However, contribution rates are scheduled and could only change by legislative action, so Legros' description is somewhat idealistic.)

6.3.2 Objective of Financial Balance – Germany

The reform measures in Germany required that the contribution rate must remain below 20% until 2020 and below 22% until 2030, according to a contribution increase schedule. They require also that the net replacement rate for pension benefits in relation to lifetime earnings, including the pension to be provided from an assumed maximum contribution each year to private pensions, must stay above 67% until 2030. The balancing

mechanism will achieve financial balance if the combined replacement rate from pensions payable by social security and from deemed maximum contributions to provide pensions stays above 67% until 2030, given the contribution schedule.

6.3.3 Analysis

The German system contains some important features that will produce greater financial stability, as follows:

- the gradual increase in the normal retirement age which will reduce the financial pressure created by increasing life expectancy
- the method of adjusting the value of a current pension point shares the burden of any financial balance between workers and contributors in the following ways:
 - (1) in the absence of other factors, the pension's value would increase by the rate of increase in average wages, on a lagged basis, net of (deemed) pension contributions; hence, if the contribution rate increases then the rate of increase in the current pension value is dampened
 - (2) by including a factor that is based on the demographic dependency ratio, the rate of pension value increase is dampened when the demographic dependency ratio is increasing, on a lagged basis; thus reducing the burden measured in amount of pensions as the demographic dependency ratio measured in numbers of participants is increasing the burden
 - (3) the sustainability parameter shares the adjustment between pensioners and workers; at present, pensioners bear 25% of the adjustment but the sustainability parameter could be changed.

Are these components sufficient to create an automatic balancing mechanism? Raising the normal retirement age has a significant impact on financial balance; however, the increase in the normal retirement age is defined by a fixed schedule and is not related directly to life expectancy changes, so this component does not provide automatic balance. Moreover, because the first scheduled increase in age is not until 2011, it was not included in the current legislation. In other words, further legislative action will be required.

Suppose that due to increasing longevity, after the normal retirement age of 67 is in place, that it is not a sufficiently high age to maintain financial balance. Would the other components restore financial balance automatically?

In this situation, one would expect that the demographic dependency ratio, year over year, would be increasing, which would reduce the amount of the increase in pensions. Moreover, to achieve financial stability, the contributions could be increased which in turn would dampen the increase in pension value. There is no question that both these factors operate in the right direction towards restoring the financial balance. However, there is no assurance that financial balance will be restored, for several reasons:

- the demographic dependency ratio includes the unemployed in the numerator. If the number of unemployed is increasing, the demographic dependency ratio might

remain unchanged but the financial position of the system would be deteriorating because there would be less in contributions in aggregate, due to the large number unemployed, and there may be increased pensions to pay because average gross income might continue to increase. A similar argument would apply if the demographic dependency ratio were stable or decreasing, year over year, but the proportion of the contribution base that were unemployed was increasing.

- Any required increases in contribution rates do not occur automatically. Legislation would have to be passed to change the contribution rate, so this part of the mechanism does not work automatically.
- The sustainability factor is a useful device for sharing the burden of financial imbalance between pensioners and workers; however, the sustainability parameter is set at 0.25 and does not adjust automatically as the financial position changes.
- The amount of replacement ratio is not included in the application of the ABM so there is no assurance that the desired replacement rate will be maintained.

The German system identifies several components that are likely to affect the financial position and includes them in its adjustment mechanism. However, some of the components are fixed and so there is no assurance that automatic financial balance will be achieved. By setting a tight range for contribution rates and also setting a desired replacement rate, if the economy does not perform in the required manner to provide sufficient wage growth and adequate employment to overcome demographic change then the only component in the mechanism that can provide a necessary adjustment is the pension available from the individual savings accounts. If the accounts have sufficiently high investment earnings then the target replacement rate could be achieved. The return on the individual accounts is not part of the defined balancing mechanism.

Furthermore, just as for the balancing mechanisms of the Swedish and Japanese SSRS, rates of change in gross income and in dependency are used to determine benefit reductions. To achieve financial balance in a paygo SSRS, it is necessary that the change in contribution income equal the change in benefit outflow. The use of rates of change provides only an approximation, at best. In countries such as Germany, Sweden and Japan where the number of contributors is projected to decrease for many years and the number of pensioners is projected to increase for many years, if such an approximation produces equality in any year, it is unlikely that equality would persist in subsequent years, i.e., the approximation is neither strong nor persistent. The following table based on the UN 2002 projections supplied by the ILO, show how Germany's population ages 15-64, a proxy for contributors, is projected to decrease, and how the population ages 65 and over, a proxy for pensioners, is projected to increase until 2035, after which it is projected to decline.

Table 9: German Population Projections

Year	Population (in thousands)	
	Ages 15 – 64	Age 65 and Over
2005	55,279	15,351
2010	54,624	16,672
2015	54,479	17,132
2020	53,147	18,193
2025	51,101	19,534
2030	48,203	21,536
2035	45,561	23,137
2040	44,402	23,036
2045	43,457	22,456
2050	42,043	22,121

Moreover, in the German SSRS, success of the ABM requires achievement of at least a 67% replacement rate. In reviewing projections for the German system, Toft (2007) concludes that if all assumptions are realized the system will provide the desired replacement rates within the scheduled contribution rates. However, he views the assumptions as optimistic and he finds it hard to find a “basis for this optimism” (Toft, 2007). The projections are based on a long-term nominal wage growth of almost 3 per cent and a rate of inflation of 1.5 percent leading to an expected 1.5 percent annual increase in real earnings. Toft (2007) states that real net earnings have been stagnant and may even have fallen a few percentage points since 1991. Toft (2007) also expresses reservations about whether contributors will contribute the maximum amount to private pensions, which is a necessary assumption to maintain the replacement rate. He thinks this is a particularly questionable assumption for low-income earners.

Borsch-Supan et al. (2003) state that an important task of reform is to find a formula which is capable of providing a systematic response to the kind of unforeseeable changes which we must always live with in the labour market. Although they see the sustainability factor as elegant, they see it as merely a way of taking a weighted average between placing the burden of adjustment on pensioners through benefit reductions and placing the burden of adjustment on contributors through increased contributions. Borsch-Supan et al. (2003) conclude that the burden which demographic change imposes on the pay-as-you-go system cannot be eliminated by reform. Moreover, Borsch-Supan et al. (2006) note the governmental commission’s projections incorporate scheduled increases in the retirement age which were not legislated, so even those projections cannot be expected to be realized as the system is currently defined.

Given Toft’s (2007) and Borsch-Supan’s et al. (2003, 2006) observations, as well as the analysis of the changing demographics, the balancing mechanism is judged to achieve transitory financial balance. It is a combination of economic and demographic factors that cause this to occur. Based on this author’s readings, the sustainability factor was carefully selected to fit the German situation and requires a great many assumptions to be

realized. Its complexity may mask the benefit reductions that are likely to occur, particularly when assumed contributions to voluntary accounts with hypothetical earnings are included in calculating replacement ratios.

6.4 Canada

6.4.1 Canada Pension Plan

The Canada Pension Plan (CPP) is a defined benefit earnings-related pension plan, currently requiring contributions from employers and their employees, and from the self-employed of 9.9% of covered earnings, in aggregate, which is approximately equal to, although slightly higher than, the steady-state contribution rate, determined over a 75 year horizon. The Canada Pension Plan Act (the “Act”) contains a provision that has some similarities to an ABM. Briefly, this Act provides that if the actuary has recommended that the contribution rate be increased, which would occur if the contribution rate were less than the required steady-state contribution rate, and if the federal finance minister, after consultation with the provincial finance ministers, is not able to make a recommendation, then the following changes will occur:

- the contribution rate will increase by 50% of the increase recommended by the actuary; and
- the benefits will be frozen for three years, the time until the next actuary’s report, by treating the cost-of-living adjustment factor as 1 so no increase in benefit is prescribed.

In the classification terminology adopted, this provision would be considered “discretionary” as opposed to “automatic”, since it requires an action not to be taken, i.e., the finance ministers failing to agree to an amendment. The provision has a direct impact on the financial position of the CPP since it increases contributions (without increasing benefits because it is a defined benefit formula) and it freezes benefits, thus limiting liabilities and reducing the present value of the liabilities and thereby reducing the steady-state contribution rate. Does it ensure financial balance?

Based on the current demographics of the plan, it would be expected to bring the plan into actuarial balance. The Act provides that the maximum annual increase in the contribution rate for each of employees and employers is 0.1%. The Chief Actuary of the CPP, Jean-Claude Menard, has estimated, based on current provisions and demographics, that the financial impact of a benefit freeze is equivalent to a contribution increase of 0.15% of covered wages. So, for example, if the actuary’s report recommended a 0.20% contribution rate increase to the aggregate contribution rate from 9.9% to 10.1% and if the requisite ministerial agreement regarding an amendment was not achieved, then the balancing provision would apply. The aggregate contribution rate would increase to 10.0% and the benefits would be frozen. The financial impact of these actions would be an increase in contributions of 0.1% of covered wages plus a decrease in liabilities of 0.15% of covered wages for a total financial impact of 0.25% of covered wages. Since the required contribution increase recommended by the actuary was 0.20% of covered

wages, there would be a net financial gain or improvement of 0.05% of covered wages. The financial deficit would be more than rectified.

This is the situation based on current demographics but there is no certainty that these provisions would always have such an impact, for two reasons:

- First, the reason that the application of the provisions overcame the financial imbalance in the example is because the benefit freeze had a greater value than the value of one-half the required contribution increase. This situation need not necessarily obtain.
- Second, our considerations thus far regarding the CPP have been with respect to its financial position considering only the impact of contributions and changes to the value of liabilities. If the CPP were pure pay-go, this would be the complete analysis. However, the CPP is accumulating an investment fund that is invested in markets and from which the CPP actuary anticipates an investment return in assessing the financial position and determining the required contribution rate. The system is partially funded. It is possible the investments of the CPP investment fund might not achieve the rate of return assumed by the actuary for a number of years. Hence, even if the balancing provisions were activated, and even if they should be sufficient on their own to restore balance, as in the example, it is possible that poor investment performance of the CPP investment fund might keep the CPP from ever achieving financial balance. Note that the balancing provisions only relate the balancing mechanisms to contributions and benefits and not to investment performance.

The operation of the mechanism will be analyzed in more depth in subsection 6.4.3. As will be discussed further, the steady-state contribution rate is defined as the minimum rate such that the asset-expenditure ratio is the same in the thirteenth and sixty-third years after the valuation date. (These apparently odd dates occur because the valuation is in respect of the three-year period following the valuation date and the asset-expenditure ratios are compared at the tenth and sixtieth years following the end of the current valuation period).

In mathematical symbols, the balancing mechanism, ignoring administrative expenses, may be described as follows:

- A_t = assets end of year t
- C_i = contributions in year i
- P_i = pensions payable in year i
- r_i = investment return in year i
- A_0 = assets at inception

Assets at end of year t

$$A_t = \sum_{i=1}^t (C_i - P_i) \prod_{j=i}^t (1 + r_j) + A_0 \prod_{j=1}^t (1 + r_j)$$

S_t = those surviving to year t

Asset-Expenditure Ratio

Year $V + 13$

V is year of valuation

$$AER_{V+13} = \frac{1}{P_{V+14}} \left\{ \sum_{i=V+1}^{V+13} (C_i - P_i) \prod_{j=i}^{V+13} (1 + r_j) + A_V \prod_{j=1}^{V+13} (1 + r_j) \right\}$$

AER_t is asset-expenditure ratio in year t

Year $V + 63$

$$AER_{V+63} = \frac{1}{P_{V+64}} \left\{ \sum_{i=V+1}^{V+63} (C_i - P_i) \prod_{j=i}^{V+63} (1 + r_j) + A_V \prod_{j=1}^{V+63} (1 + r_j) \right\}$$

Contributions in year t

C_t is contributions in year t

$$C_t = \sum_{\text{all } x} [\text{minimum}({}_x w_t, Y_t) - 3,500] c_t$$

c_t is contribution rate in year t

w_t^x is wage rate in year t for worker x (treat ${}_x w_t = 3,500$ if ${}_x w_t \leq 3,500$)

Y_t is YMPE in year t

Pensions in year t

${}_x P_k^j$ is pension payable in year k for worker age x retiring in year j

New in year t in respect of worker x retiring in year t

$${}_x P_t^t = .25 n_x \left[\frac{Y_t + \dots + Y_{t-5}}{5} \right]$$

n_x is number of years of credited

service at the maximum pension level for worker x

payable in year t due to retirements prior to year t by worker x

${}_xS_t^j$ is a survival function – the probability a life age x who retires in year j survives to year t

$$\sum_{j=1}^{t-1} P_t^j {}_xS_t^j \prod_{S=j}^{t-1} (1 + cp_{S+1})$$

cp_S is rate credited in respect of consumer price growth in year S

Total pensions in year t

$$P_t = \sum_{\text{all } x} \left\{ {}_xP_t^t + \sum_{j=1}^{t-1} P_t^j {}_xS_t^j \prod_{S=j}^{t-1} (1 + cp_{S+1}) \right\}$$

Contribution Rate in year t

$$c_t = \text{greater of } \begin{cases} .0990 \\ C, \text{ a constant } C \text{ such that for all } t > V, C \text{ is the minimum contribution rate so that} \\ AER_{V+13} = AER_{V+63} \end{cases}$$

Adjustment Mechanism in year $t+1$

If c_t greater than .0990 and finance ministers cannot agree on action then

$$c'_{t+1} = \frac{c_{t+1} - .0990}{2}$$

and $cp_j = 0$ for $j = t+1, t+2, t+3$.

6.4.2 Objective of Financial Balance – Canada

For Canada, if the actuary's report has determined that the required contribution rate over a 75-year horizon needs to be increased to achieve financial balance, and the Federal Finance Minister after consultation with the provincial finance ministers, has not recommended the action to be taken, then the balancing mechanism is triggered. Financial balance is achieved if the balancing mechanism will bring the contribution rate to be at least equal to the steady-state contribution rate determined by the actuary in some subsequent report.

6.4.3 Analysis of the Canadian Balancing Mechanism

It is difficult to conceive of how the Canadian SSRS could ever be much out of financial balance given its very strong financial position and the good governance provisions it has adopted. For the purpose of analysis, different levels of imbalance and assumptions are made.

In 1997, legislated increases in the contribution rate were adopted to raise the total contribution rate in a number of annual steps to 9.90%. This steady-state contribution rate, as it is referred to, was the appropriate rate at the time the legislated increases were adopted. As at December 31, 2006, 9.90% exceeds the required steady-state contribution rate of 9.82% (Office of the Chief Actuary, 2007). Moreover, the legislation provides that any improvements to the CPP be fully funded on an incremental basis, through an adjustment to the contribution rate. A seventy-five year horizon is used in the actuarial report and the steady-state contribution rate requires that the asset-expenditure ratio be equal in the tenth year and the sixtieth year after the three-year review period covered by the valuation, i.e., at years thirteen and sixty-three after the valuation date.

This is not to say that the contribution rate may not change from its current level. This is certainly possible. The rate calculated is based on many assumptions. Over a seventy-five year horizon, there are certain to be deviations from assumptions, both positive and negative. However, given that reports are prepared at least tri-annually, it is unlikely that there could be a significant move in the steady-state contribution rate from one report to the next. Also, it is possible that the action agreed upon to adjust the contribution rate might be less than the amount recommended in the actuarial report, which could contribute to a worsening of the financial position of the plan. Nonetheless, it is unlikely that the plan's financial position would be significantly out of balance.

With this preamble, it is important to recognize that the purpose of this thesis is neither to assess the financial status of any country's SSRS nor to opine on how that financial status could change. Purposes of this thesis include to describe and examine the operation of balancing mechanisms in the SSRS of certain countries, to evaluate and classify those balancing mechanisms.

In the event that the Canadian SSRS was not in financial balance and the actuarial report recommended that the contribution rate be increased, and the finance ministers failed to agree on a course of action, the balancing mechanism would be triggered. The balancing mechanism stipulates that the contribution rate would be increased by one half the recommended amount and the indexing of benefits to pensioners would be frozen for three years (ignoring refinements regarding the maximum increase at a time and rounding rules). All other things being as assumed, if the value of the freezing of the indexing of benefits to pensioners for three years is less than one-half the value of the recommended increase, then financial balance would not be achieved within three years. Of course, this process would repeat itself (if the finance ministers could still not agree on the action to be taken) at the time of the next actuarial review if the contribution rate were still less

than the recommended amount. If the amount of the freeze of indexation of pension benefits for three years was still less than one half of the recommended increase in the contribution rate, then, all else being equal, the system would still not be in financial balance. The process would be repeated at the next actuarial review, and so on.

Some would conclude that in practice the system would come back into financial balance. I would not disagree with such an assessment; however, from the perspective of classifying this balancing mechanism, it is necessary to be rigorous. This mechanism does not ensure that financial balance will be achieved.

Let us take an extreme example. Suppose that inflation had been eliminated and that it were acceptable for the actuary to assume an inflation assumption of 0% for the future. Extrapolating from the Actuarial Report as at December 31, 2006, the adoption of such an assumption would mean that the steady-state contribution rate might be approximately 10.45%. In this scenario of no inflation, the freezing of indexation of benefits for three years has no value at all. Every time the valuation is repeated, the freezing of indexation of benefits would have no value. Which raises the paradoxical type of question asked by Zeno many centuries ago: How can one get all the way to the required rate by only going one-half of the way there every three years? We know that the sum of the series $1/2 + 1/4 + 1/8 + \dots$ is equal to one in the limit, but if we stop before the limit it is less than one. Nonetheless over a seventy-year horizon, with an adjustment every three years to go one-half the way, the series $\frac{1}{2} + \frac{1}{4} + \dots + \left(\frac{1}{2}\right)^{25}$ is 1 for all intents and purposes.

But one might think that if all else is as assumed, that the steady-state contribution rate would remain as a fixed target. This is unlikely to be the case because of the way that the steady-state contribution rate is defined, as explained in the following subsection.

6.4.3.1 Steady-state Contribution Rate

The steady-state contribution rate is defined (effectively) as the minimum rate such that the asset-expenditure ratio is the same in the thirteenth and sixty-third years after the valuation date. It is highly unlikely that the asset-expenditure ratio would be constant throughout the seventy-five-year period or even over the period from the thirteenth year to the sixty-third year. If the critical asset-expenditure ratios are not constant, then the steady-state contribution rate may change, from valuation to valuation, even though all else is as assumed.

Selected asset-expenditure ratios, as presented in the actuarial report at December 31, 2003 are shown in the following table to provide a clearer appreciation of how these ratios vary by year (Office of the Chief Actuary, 2007). The years 2025 to 2033 are shown because this is a period of relative stability, even though the paygo rate exceeds the steady-state contribution rate, due to the expected income from investments. Between 2050 and 2060 the paygo rate exceeds the steady-state contribution rate sufficiently that

the asset-expenditure ratio begins to decline. It is not apparent that the asset-expenditure ratios are equal in 2016 and 2066, but differences are due to rounding.

**Table 10: Selected Asset-Expenditure Ratios Canada Pension Plan
December 31, 2003 Valuation**

Year	Asset-Expenditure Ratio Based on 9.8% Rate	PayGo Rate
2004	3.08	8.27
2013	4.75	8.97
2014	4.87	9.04
2015	4.99	9.13
2016	5.09	9.22
2017	5.19	9.31
2025	5.53	10.42
2029	5.53	10.84
2033	5.53	11.03
2040	5.60	11.07
2050	5.62	11.29
2060	5.46	11.52
2075	5.38	11.32

Moreover, the likelihood that all assumptions will be realized is small. In this regard, the steady-state contribution rate moves in odd ways. Consider what happens to the steady-state contribution rate when the investment returns are better than assumed, all else being equal. In such a situation, the asset-expenditure ratio at the thirteenth year will be higher than expected, which may require the steady-state contribution rate to increase, in order for the asset-expenditure ratios after the thirteenth and sixty-third years to be equal. Conversely, when investment returns are less than expected, all else being equal, the asset-expenditure ratio at the thirteenth year will be less than expected. This may require that the steady-state contribution rate be decreased so that the asset-expenditure ratios after the thirteenth and sixty-third years will be the same. This is not logical.

Because of the various manners in which the steady-state contribution rate is determined, it is worthy of further examination. In our analysis let us assume that the return on assets and the rate of inflation are both non-negative.

First, note that in a paygo equilibrium scenario, the typical starting point for OLG analysis, the constant contribution rate k is determined by the following ratio.

$$\frac{\text{Pension Payments in the Year}}{\text{Taxable Wages in the Year}}$$

Without any initial assets, if the steady-state rate is set equal to the paygo rate k , then the asset-expenditure ratio in years thirteen and sixty-three will be equal and will be equal to zero.

Next, note that in a growing population scenario where the annual taxable wage base grows faster than annual pension payments, if there were no initial assets, then there is no steady-state contribution rate. This may be demonstrated by defining the paygo rate in year t by k_t , as follows:

$$\frac{\text{Pension Payments}_t}{\text{Taxable Wages}_t}$$

In an expanding population, $k_t > k_{t+1}$ for all t . If the steady-state rate, ss_r , were chosen to be greater than or equal to k_t , then there would be increasing excesses of $(ss_r - k_t)$ for all t greater than the initial t and, in the absence of negative investment returns, the asset-expenditure ratio in year thirteen would not equal the asset-expenditure ratio in year sixty-three.

If in this scenario the initial assets were greater than zero, it might be possible to find a ss_r that is less than k_t such that the asset-expenditure ratios in year thirteen and year sixty-three were equal. This is possible because the steady-state rate could be less than the paygo rate initially because the available assets could be used to supplement the contributions in order to pay the pension payments. On the right set of assumptions, the asset-expenditure ratios at years thirteen and sixty-three might be equal. In a growing population scenario, it is uncertain how long this delicate equality between asset-expenditure ratios in the thirteenth and sixty-third years after the valuation could be maintained. Certainly it could not be maintained once assets at the valuation date were zero.

Finally, consider an aging population scenario. In this scenario $k_t < k_{t+1}$ for all t . In the absence of any initial assets, it is possible to find $ss_r > k_t$ such that, on certain assumptions, the asset-expenditure ratio in year thirteen will equal the asset-expenditure ratio in year sixty-three. In other words, in an aging population scenario choose a contribution rate higher than the paygo rate. This will permit the development of a fund of assets that can be used to mitigate the impact of increasing paygo rates, which will occur in an aging population scenario.

This analysis provides some interesting considerations. First, the Canada Pension Plan is expected to face an aging population scenario indefinitely into the future, so although this methodology is not universally applicable, it is suitable for the situation likely to be encountered. Second, in an aging population scenario, this methodology requires the steady-state rate to be higher than the paygo rate. In 1997, Canada took the necessary legislative action to increase the contribution rate above the paygo rate. If the other countries, Sweden, Japan and Germany, were to take action to raise their contribution rates above the paygo rate they would provide an additional margin to help

in stabilizing their systems. Having a margin or pool of assets that can be invested, provides an additional dimension with which to attempt to achieve balance. However, there are two marked differences between Canada and the other countries. First, Canada acted at an earlier stage before its senior dependency ratios were as severe as the other countries. See Table 11.

Table 11: Senior Dependency Ratios 2005

Country	Population Aged 65 and over	Population Aged 15-64	Senior Dependency Ratio
Canada	4,229,882	22,202,220	0.1905
Germany	15,350,900	55,279,490	0.2777
Japan	25,175,600	84,853,990	0.2967
Sweden	1,576,408	5,825,291	0.2715

Second, the replacement ratio from the Canadian system is far less than from any of the other three countries (Whitehouse, 2007a), so the level to which the contribution rate needed to be raised was far lower on an absolute basis, in Canada than any of the other countries. As it currently stands, Canada has taken the action required, yet its contribution rate is lower than any of the other countries. Finally, while there does not seem to be any actuarial principle behind why the asset-expenditure ratios in years thirteen and sixty-three should be compared, and should be equal; however, by performing such a calculation there is a greater tension or alignment between the near and distant future, over a seventy-five year horizon. This means that this approach is more likely to bring financial stability over a seventy-five year horizon, than is the type of approach used in the other countries that looks at a certain ratio in the year, or a lagged basis.

6.4.3.2 Conclusion with Respect to Financial Balance

In view of the potential for the steady-state contribution rate to move in an uncertain manner, the rigorous definition of balance will be maintained in this thesis, even though in practice it is likely that balance would be achieved. Therefore, with respect to financial balance, the provisions of the CPP will be classified as partial. Because the steady-state contribution rate equates asset-expenditure ratios at two future points, it is a combination of economic and demographic factors that would cause the mechanism not to be robust.

It would be a simple matter to redefine the balancing mechanism so that financial balance would be achieved. For example, in the event that the finance ministers could not agree on the action to be taken when the actuarial report recommended an increase in the contribution rate, the balancing mechanism might be defined as follows:

- If the value of the freeze in indexing for three years is greater than one half the recommended increase in the contribution rate, then impose a proportionate freeze on indexing for three years such that it has a value equal to one half the recommended increase in the contribution rate and increase the contribution rate by one half the recommended amount;
- If the value of the freeze in indexing for three years is less than one half the recommended increase in the contribution rate, then freeze the indexing for three years and increase the contribution rate by the difference between the recommended contribution rate and the value of the freeze in indexing for three years expressed as a contribution rate.

Such a rule would be robust, with respect to financial balance. In Chapter 6 this rule will be examined with respect to equitable balance and a modification proposed that would still be considered robust with respect to financial balance but which would also be considered robust with respect to equitable balance, within certain ranges.

6.5 Stochastic Control Revisited

In section 3.2 it is stated that a stochastic optimal control approach could be used to maintain financial balance in an SSRS; however, because SSRS are highly complex systems, complex control laws would be required. Now that the stochastic dynamics of the various countries' SSRS have been described, it is possible to elaborate on this point, with an illustration using the Canadian system.

For CPP, many assumptions and calculations are required in order to determine if there is a loss; however, the outcome of the calculations could be represented rather simply by an indicator function, such that:

$$I = \begin{cases} 1 & \text{if the steady-state contribution} \\ & \text{rate exceeds the contribution rate} \\ 0 & \text{otherwise} \end{cases}$$

The controls are an adjustment to contributions and benefits in such a way that the steady-state rate will exceed the contribution rate. In any given year, the contribution rate has been determined so it may be considered fixed, but the steady-state contribution rate is a complex random variable. The steady-state contribution rate is (effectively) the lowest rate such that the asset-expenditure ratio in the thirteenth and sixty-third years after the valuation date are equal. Expenditures will be driven by demographic, economic and program factors, such as those described in section 3.1. Assets will be affected by the contribution rates, expenditures, and the investment policy. The control law in Canada, in the default situation, adjusts contributions and benefits, both of which will affect the asset-expenditure ratios. However, to do the necessary calculations requires projections of many variables over at least a sixty-three year period and many assumptions to do those projections.

It is not surprising that the ABM in the four countries do not try to solve a stochastic control problem directly. What is surprising though, is the choice of controls selected by the systems. For example, in both Sweden and Japan a requirement should be that any adjustments maintain pensions above the minimum benefit. Since this is a requirement one would expect it to be incorporated in the controls. But that is not the case.

6.6 Important Characteristics to Achieve Financial Balance

The review of the four countries' SSRS provides some important conclusions about the characteristics of mechanisms if they are to attain financial balance under changing demographics.

In a paygo system, to maintain financial balance, contribution income must equal benefit outflow, and so change in contribution income must equal change in benefit outflow. None of the mechanisms reviewed have a direct link between contribution income and benefit outflow.

The factors affecting the contribution income are the gross income, subject to contributions, of the contributory group and the contribution rate. The factors affecting the benefit outflow are the benefit payments to each pensioner and the number of pensioners. Writing this relationship in the form of an equation, to show the change between year t and $t+1$ year, and rearranging produces an informative result.

$$\Delta \text{ income} = AI_{t+1}NC_{t+1} - AI_tNC_t$$

$$\Delta \text{ outflow} = AP_{t+1}NP_{t+1} - AP_tNP_t$$

$$\text{so} \quad \left\{ \frac{AI_{t+1}NC_{t+1}}{AI_tNC_t} - 1 \right\} = \left\{ \frac{AP_tNP_t}{AI_tNC_t} \right\} \left\{ \frac{AP_{t+1}NP_{t+1}}{AP_tNP_t} - 1 \right\}$$

where AI_k represents average contributory income per contributor in year k

NC_k represents number of contributors in year k

AP_k represents average pension per pensioner in year k

NP_k represents number of pensioners in year k

The last equation shows that the rate of change in gross contributory income from t to $t+1$ minus 1, is equal to the ratio of the pension outflow to the contribution income in year t multiplied by the rate of change in pension outflow from t to $t+1$ minus 1.

The Japanese and German adjustment mechanisms come close to incorporating these components, but do not include them completely. The Japanese mechanism adjusts

pensions based on a proxy for the change in the contribution volume but the mechanism does not have a component that represents the ratio of the pension outflow in year t to the contribution volume in year t . The German mechanism adjusts pensions based on a proxy for the change in contribution volume and for the change in the number of pensioners to the total potential contributory group, i.e., both contributors and the unemployed. However, this latter adjustment is with respect to numbers and not to dollar flows. If the pensions per capita and the income per capita were constant, the two ratios would be equivalent; however, neither pensions per capita nor income per capita are constants. Furthermore, the German system adjusts the demographic ratio by a sustainability parameter, currently 25%, which moves the balance further away from equivalence. Finally, for reasons of administration, the German system uses ratios based on lagged data.

A further technical point is that the adjustment rates may be calculated by addition rather than by multiplication, viz., the application of the Japanese mechanism. For relatively small adjustments that are close to 1, which most of these adjustments are, these two methods are approximately the same; however, it is worth noting that this is only an approximate mathematical relationship. Similarly, the Swedish mechanism only makes an approximate adjustment. It multiplies the rates by the adjustment factor rather than multiplying the accounts or the pensions by the adjustment factor. For example, if the rate of real wage growth is 0.016, the rate of inflation is 0.025 and the adjustment factor is 0.99, to achieve balance, the adjustments should be $0.99 (1 + 0.016 + 0.025) = 1.03059$ to the accounts and $0.99 (1 + 0.025) = 1.01475$ to the pensions. The actual adjustment would be $1 + 0.99 (0.016 + 0.025) = 1.04059$ and $1 + 0.99 (0.025) = 1.02475$ respectively, so the resulting liabilities are higher than they should be if the adjustment had been applied correctly. Once again, for relatively small adjustments that are close to 1, which most of these adjustments are expected to be, the two methods are approximately equal. However, over successive periods of adjustment, there is a compounding effect that could become significant.

In a partially funded SSRS, the pension payments may be met by contribution income and by drawing on assets. The amount drawn from assets could be a portion or all of the investment income or some portion of the assets when investment income, along with contribution income, is insufficient to pay pensions. The equation for balance may be written as follows, in respect of the year t .

$$\text{Income} = AI_t NC_t + k_t A_t \quad \text{where } k_t \text{ represents the draw-down rate in year } t$$

$$\text{and } A_t \text{ represents the assets in year } t$$

$$\text{Outflow} = AP_t NP_t$$

Equating Income and Outflow and rearranging

$$\frac{AI_t NC_t}{AP_t NP_t} = 1 - k_t \left(\frac{A_t}{AP_t NP_t} \right) \quad \text{where } AER_t = \frac{A_t}{AP_t NP_t}$$

$$\frac{1}{D_t R_t} = 1 - k_t AER_t \quad \text{and } R_t = \frac{NP_t}{NC_t}$$

$$\text{and } D_t = \frac{AP_t}{AI_t}$$

The foregoing equation may be interpreted as: the product of the inverse of the replacement rate and the inverse of the senior dependency ratio is equal to 1 minus the draw-down rate multiplied by the asset-expenditure ratio. Let us examine these relationships.

If the asset-expenditure ratio is constant and the draw-down rate is constant then the replacement rate must be adjusted as the dependency rate changes, i.e., if the dependency rate is increasing, benefits must be reduced.

Alternatively, if benefits are not to be reduced and the dependency rate is increasing then $1 - k_t AER_t$ would be declining, which implies $k_t AER_t$ is increasing. This term may be increasing because the draw-down rate is increased or because the ratio of assets to expenditures is increasing or both. If the asset-expenditure ratio is constant then the draw-down ratio is given by:

$$k_t = \frac{1}{AER_t} \left\{ 1 - \frac{1}{D_t R_t} \right\}$$

on our assumptions AER_t is constant say a and R_t is constant say c , so

$$k_t = \frac{1}{a} \left\{ 1 - \frac{1}{D_t c} \right\}$$

and we have an expression for the draw-down rate in terms of the dependency ratio.

6.7 Chapter Conclusion

In this chapter the stabilizing mechanisms of the four countries were described. For each country their objective of financial balance was determined. Using financial analysis performed by actuaries, economists and governmental agencies, the countries' ABM were analyzed.

A significant problem with the manner of calculating assets in the Swedish determination of financial balance was identified. This is an important contribution of this thesis. The magnitude of the error was quantified and a suggestion made on how to correct the definition of assets.

None of the SSRS of Sweden, Japan or Germany was considered to even achieve partial financial balance. The Canadian system is in a strong financial and governance position so it is unlikely to encounter financial difficulty. Nonetheless it was concluded that the balance achieved by applying the mechanism would only be considered partial.

Subsection 6.4.3.1 analyzed the unusual features of the calculation of the steady-state contribution rate in the Canadian SSRS and explained why it was easier for Canada to adopt such a mechanism than it would be for the other three countries to try to adopt it. Although the mechanism has limited application and does not appear to rely on any specific actuarial principle, it does create a tension between financial ratios in the near and distant future, which does provide greater financial stability over a seventy-five year horizon, in an aging population scenario. This is a contribution of this thesis.

Section 6.5 expanded on the observations in section 3.2 regarding the complexity of setting up and solving a stochastic control problem for an SSRS, because of the complexity of SSRS. It used Canada as an illustration. Section 6.6 discussed some important characteristics to maintain financial balance.

7.0 Chapter Overview

In this chapter the definition of equity described in Chapter 5 is applied to each of the four countries' balancing mechanisms. Numerical results are presented. Using Canada as an example, calculations are performed to show how various rules could be tested. The mechanism used by Sweden is found to provide partial equitable balance, because the burden of adjustment is borne by both contributors and pensioners. Even though in Canada the burden is borne by both these groups, the share borne by pensioners is considered inequitable. The equitable balance of the mechanisms in Germany and Japan is considered to be transitory.

7.1 Background

In this chapter, the results of calculations are presented to assess whether the balancing mechanisms of the four countries can be expected to achieve equitable balance, using the measure of equitable balance described in Chapter 5. This measure is repeated here for ease of reading. The following quantity is calculated using the discount rates in Table 12.

$$\frac{\text{Present Value of Pension}}{\text{Accumulated Value of Contributions to 65}}$$

This quantity is calculated both after-change and before-change and the ratio of after-change to before-change is then calculated. If this ratio is not more than 5% different in one year (i.e., is greater than .95) and is not more than 10% different in any five-year period (i.e., is greater than .90), then the adjustment will be considered equitable.

Table 12: Personal Discount Rates Repeated

Period (current age x)	Discount Rate	
	For Period of Life Expectancy	Thereafter
$65 - x > 30$	4%	4%
$30 \geq 65 - x > 20$	3%	3%
$20 \geq 65 - x > 10$	2%	3%
$10 \geq 65 - x > 0$	1%	3%
$0 \geq 65 - x$	0%	3%

Before performing any calculations it is necessary to examine the nature of the adjustment mechanisms in the four countries. In Japan, the contribution rates are set so all of the adjustment affects the benefit rates. With such an approach equity can only be achieved if the adjustments are sufficiently gradual. Since the adjustments are driven by a demographic ratio $\frac{NC_t}{NC_{t+1}} - 1$ (plus a life expectancy adjustment), the analysis examines how close the demographic ratio is to failing to preserve equity (i.e., a ratio of after-change to before-change less than .95).

For Germany, similar types of calculations are performed. Although, commentators (for example, Legros, 2006) suggest that Germany could change the contribution rates, the contribution rates are set and may only be changed by the German legislature. Accordingly, the analysis of Germany examines how close the demographic ratio and the ratio of change in gross income is to the .95 measure, at which point equity would not be achieved.

The analysis of Sweden is more complicated than for Japan and Germany. As with those countries, the contribution rate in Sweden is fixed, but in Sweden, there are two types of adjustments. The first is an ongoing increase in the age of full benefit entitlement to reflect life expectancy improvements. The second is a reduction in the increases in benefits and a reduction in the rate credited to accounts, when the system's liabilities exceed its assets, as Sweden defines those terms. The first adjustment is borne fully by those not yet retired. The present value of pension remains approximately the same but the accumulated value of contributions to receive such a pension is increased. In terms of the equity measure, because the accumulated value of contributions to 65 is used, it remains unchanged but the present value of pensions decreases, due to the delay in age for full benefit entitlement. The second adjustment is borne primarily by the pensioners. Their accumulated contributions remain unchanged but their pensions, and therefore the present value of their pensions, is not increased as much as anticipated. The calculations examine how gradual the adjustments to the benefit rates are likely to be.

For Canada, the situation is quite different from the other three countries. The adjustment involves both contribution rate increases and benefit freezes. This provides a good basis to compare how the burden of adjustment is shared between individuals of different ages and between contributors and pensioners. Section 6.3 provides a good illustration of the application of the approach to determining equity.

7.1.1 Data and Assumptions

Because of gaps in the available data, there are limits to the extent to which the calculations can be performed, especially for Sweden, Japan and Germany. Accordingly, the calculations identify conditions in which equitable balance would not exist.

The population data used has been extracted from the UN 2002 projections, as supplied by the ILO, using the constant fertility variant projection. It includes population

projections by country at five-year intervals from 2005 to 2050. It is presented in summary form, so, for example, the projected population is shown in certain age groups such as population below age 15, population aged 15-64, population aged 65 or over. Using this data and assuming that normal retirement age, but not the age of full benefit entitlement, will be age 65 and that the age group 15-64 is a proxy for the working age population, an indication of the demographic pressures projected to be experienced by the four countries' systems can be modeled.

Where economic assumptions are required, the assumptions underpinning the Swedish SSRS are used, i.e., expected annual real wage growth per capita of 1.6 percent and an average annual inflation rate of 2.5 percent (Settergren, 2003).

The population projection data for the four countries is shown in the following four tables.

Table 13: Sweden Population Profile

SWEDEN	CONSTANT FERTILITY VARIANT PROJECTION (population in thousands)									
	2005	2010	2015	2020	2025	2030	2035	2040	2045	2050
Total population	8870.987	8857.510	8836.992	8813.714	8760.226	8651.354	8489.467	8296.446	8093.141	7888.250
Male population	4395.021	4393.364	4386.148	4374.069	4342.763	4281.232	4193.567	4092.923	3989.730	3886.794
Female population	4475.966	4464.146	4450.844	4439.645	4417.463	4370.122	4295.900	4203.523	4103.411	4001.456
Sex ratio (per 100 females)	98.192	98.144	98.546	98.523	98.309	97.966	97.618	97.369	97.230	97.134
Median age (years)	41.092	42.625	44.086	45.277	45.958	46.603	46.607	48.860	50.225	50.619
Age distribution										
Population below 15	1489.288	1308.460	1266.707	1253.705	1247.626	1209.773	1135.103	1057.389	1006.625	981.092
Population aged 15 – 64	5805.291	5825.747	5646.169	5508.892	5354.351	5167.290	4979.603	4822.525	4700.96	4554.658
Population aged 60 or over	2136.331	2347.784	2486.538	2613.052	2755.18	2882.694	2943.180	2918.719	2883.637	2922.886
Population aged 65 or over	1576.408	1723.303	1924.116	2051.117	2158.249	2274.291	2374.761	2416.532	2385.556	2352.500
Population aged 80 or over	497.106	505.869	504.541	532.955	635.700	757.101	806.693	839.681	890.292	945.264

Table 14: Japan Population Profile

JAPAN	CONSTANT FERTILITY VARIANT PROJECTION (population in thousands)									
	2005	2010	2015	2020	2025	2030	2035	2040	2045	2050
Total population	128221.3	128548	127755.6	125952.3	123412.9	120406.5	117058.9	113350.6	109276.5	104842.7
Male population	62623.34	62606.14	61997.63	60865.26	59379.38	57692.93	55881.43	53951.39	51922.25	49814.54
Female population	65597.96	65941.89	65757.95	65087.02	64033.5	62713.53	61177.45	59399.19	57354.25	55028.12
Sex ratio (per 100 females)	95	94.941	94.282	93.514	92.732	91.994	91.343	90.828	90.529	90.526
Median age (years)	42.8	44.294	46.069	48.092	50.194	51.829	53.159	54.151	54.782	55.147
Age distribution										
Population below 15	18191.71	17957.11	17061.82	15564.21	14133.75	13187.37	12665.75	12235.5	11615.44	10801.39
Population aged 15 – 64	84853.99	81907.64	77637.54	75049.85	73209.09	70470.85	66569.56	61316.71	57187.38	53964.43
Population aged 60 or over	33636.35	38658.32	41505.57	42702.43	43761.83	45122.17	47314.18	48099.74	47805.97	46544.46
Population aged 65 or over	25175.6	28683.29	33056.22	35338.22	36070.04	36748.24	37823.58	39798.36	40473.67	40076.84
Population aged 80 or over	6136.541	7675.563	9847.012	11426.13	13168.29	15603.27	16485.18	16350.9	15452.9	17053.42

Table 15: Germany Population Profile

GERMANY	CONSTANT FERTILITY VARIANT PROJECTION (population in thousands)									
	2005	2010	2015	2020	2025	2030	2035	2040	2045	2050
Total population	82496.43	82407.57	82113.57	81559.83	80659.43	79442.89	77967.39	76286.06	74438.53	72459.42
Male population	40327.02	40282.34	40082.84	39750.52	39217.58	38512.78	37674.39	36728.07	35698.5	34631.8
Female population	42169.41	42125.23	42028.74	41809.32	41441.85	40930.11	40293	39558	38740.03	37827.62
Sex ratio (per 100 females)	95.631	95.625	95.375	95.076	94.633	94.094	93.501	92.846	92.149	91.552
Median age (years)	41.988	44.036	45.944	47.039	47.586	48.099	48.847	49.691	50.191	50.527
Age distribution										
Population below 15	11866.04	11111.33	10503.29	10220.23	10025.08	9704.078	9269.509	8848.254	8525.712	8295
Population aged 15 – 64	55279.49	54623.78	54478.53	53146.9	51100.78	48202.85	45561.17	44402.15	43456.66	42043.31
Population aged 60 or over	20563.78	21195.9	22503.32	24099.88	26323.25	28109.27	28136.34	27670.95	27431.4	27323.42
Population aged 65 or over	15350.90	13372.46	17131.75	18192.71	19533.58	21535.96	23136.71	23035.66	22456.15	22121.11
Population aged 80 or over	3430.216	3998.297	4434.063	5457.32	5949.84	5916.995	6445.035	7246.134	8461.118	9259.5

Table 16: Canada Population Profile

CANADA	CONSTANT FERTILITY VARIANT PROJECTION (population in thousands)									
			2015	2020	2025	2030	2035	2040	2045	2050
Total population	32059.24	33280.36	34443.36	35516.91	36449.63	37198.23	37732.79	38082.72	38294.36	38418.16
Male population	15871.96	16477.52	17046.28	17560.3	17992.57	18323.37	18544.97	18680.7	18760.18	18810.36
Female population	16187.28	16802.84	17397.08	17956.61	18457.06	18874.86	19187.81	19402.02	19534.18	19607.8
Sex ratio (per 100 females)	95.052	98.064	97.984	97.793	97.483	97.078	96.65	96.282	96.038	95.933
Median age (years)	38.836	40.353	41.555	42.609	43.593	44.537	45.418	46.201	46.565	46.681
Age distribution										
Population below 15	5627.14	5364.371	5364.496	5430.214	5471.279	5438.48	5356.291	5290.927	5276.479	5292.21
Population aged 15 – 64	22202.22	23163.63	23468.67	23495.91	23267.49	22985.03	23039.31	23146.84	23167.88	23076.18
Population aged 60 or over	5762.204	6734.287	7848.716	9126.587	10377.06	11136.47	11634.77	11994.89	12293.63	12570.04
Population aged 65 or over	4229.882	4752.353	5610.198	6590.792	7710.855	8774.722	9337.185	9644.954	9850.007	10049.77
Population aged 80 or over	1115.955	1283.223	1414.872	1537.512	1784.766	2236.718	2712.823	3214.917	3630.689	3714.087

7.2 Sweden

Since Sweden has a NDC SSRS, a starting point is to check if the demographics and the fixed contribution rate can be reasonably expected to sustain the benefit rate. The cost of the benefits will vary depending on the length of participation, contributory history and service. The following table shows the expected real accumulation per unit of starting salary at age 65, assuming a continuous contributory period from different commencement ages. This accumulation can then be converted to an annuity using the annuity factors shown in Chapter 1, that vary by birth cohort attaining age 65 (Settergren, 2003).

Table 17: Accumulations at Age 65

Age of Commencement of Continuous Contributory Service	Real Accumulation at Age 65 Per Unit of Starting Salary
15	57.3028
20	42.1856
25	30.6732
30	21.9540
35	15.3926

Converting these real accumulations using the annuity factors and dividing by the projected real salary at age 65 provides the following replacement rates (assuming the NDC had been in effect for a whole career for a person retiring in the year shown).

Table 18: Sweden Replacement Rates at Age 65

Replacement Rates at Age 65					
	Age of Commencement of Continuous Contributory Service				
Year Reaching Age 65	15	20	25	30	35
2005	.5096	.4586	.4076	.3567	.3057
2010	.4969	.4472	.3975	.3478	.2981
2015	.4878	.4390	.3902	.3415	.2927
2020	.4790	.4311	.3832	.3353	.2874
2025	.4706	.4235	.3765	.3294	.2824
2030	.4651	.4186	.3721	.3256	.2791
2035	.4598	.4138	.3678	.3218	.2759
2040	.4520	.4068	.3616	.3164	.2712
2045	.4469	.4022	.3575	.3128	.2682
2050	.4444	.4000	.3556	.3111	.2667

From the table it can be seen that the projected replacement rates decline by year of reaching age 65 for all ages of commencement of contributory service. This occurs by design. Sweden's SSRS plans to raise the age of full benefit entitlement, according to a set schedule, to adjust, at least partially, for increasing life expectancy; accordingly, the projected replacement rates at age 65, (which is no longer the age of full benefit entitlement), continue to fall. The replacement rates are also lower by increasing age of commencement of continuous service, regardless of year of attainment of age 65. The explanation relates to the defined contribution nature of the plan. The later one commences the period of contributory service, the lower the projected accumulation at age 65 and the lower the replacement rate.

These replacement rates were applied to the demographic projections to show what the required contribution rate would be, if the system were pure paygo, i.e., no buffer or reserve funds were available. If the contribution rate exceeds .1600 then the replacement rate is not sustainable by the current contribution formula, and the ABM would need to be applied.

Table 19: Sweden Paygo Contribution Rate

PAYGO CONTRIBUTION RATE										
Age at Commencement of Contributory Service	Year of Attaining Age 65									
	2005	2010	2015	2020	2025	2030	2035	2040	2045	2050
15	.1384	.1495	.1700	.1829	.1946	.2093	.2236	.2317	.2319	.2336
20	.1245	.1346	.1530	.1646	.1751	.1883	.2012	.2085	.2087	.2103
25	.1107	.1196	.1360	.1463	.1557	.1674	.1788	.1854	.1855	.1869
30	.0969	.1047	.1190	.1280	.1362	.1465	.1565	.1622	.1624	.1635
35	.0830	.0897	.1020	.1097	.1168	.1256	.1341	.1390	.1392	.1402

From the table it can be seen that regardless of the age of commencement of contributory service, the required contribution rate is below .1600 through 2010. Further, the higher the assumed age of commencement of contributory service, the longer the required contribution rate is below .1600. From the population data, it is not possible to do a weighted blend of commencement ages; although, in any population one would expect there to be many different commencement ages. However, these calculations are sufficient to establish that the contribution rate has been set at an adequate level, when combined with the scheduled increases in age of full benefit entitlement, to be sustainable for a number of years, without triggering the ABM, should the economic assumptions underlying the program be realized. Consequently, should the ABM be triggered sooner, it will likely be due to the economic assumptions not being realized.

Should the ratio of system assets to liabilities fall below one, the ABM would be triggered. However, the expected changes to the crediting rate for pensioners are likely to be small. Consequently, the measure of equitable balance described in Chapter 5

would be met. On this analysis, the Swedish ABM could be considered equitable. This analysis will be considered further in section 7.6.

In preparation for the analysis in section 7.6, it is important to recognize that on the equity measure described in section 5.3, the ratio for a non-pensioner remains constant, even though the ABM is applied. This occurs because the pension is ultimately based on the accumulated balance using the set annuity divisor. When the ABM is applied, the rate credited to accounts is adjusted producing a lower accumulation and a lower pension; however, because the equity measure is a ratio of the value of pension to an accumulation, this ratio remains constant.

The same relationship does not hold for retirees, whose pensions are reduced by the application of the ABM but whose accumulated contributions remain unchanged. Hence, using this equity measure, all the adjustment is experienced by pensioners.

In assessing the relative shares of adjustment borne by contributors and pensioners due to the adjustment of the age of full benefit entitlement and the application of the ABM, the adjustment of the age of full benefit entitlement appears to have the greater impact. The adjustment to the age of full benefit entitlement is a response to demographic stresses. The Swedish mechanism will be considered to achieve partial equitable balance.

7.3 Japan

Japan's SSRS has a fixed schedule of contribution rates, so any adjustments required to maintain financial balance are borne by pensioners through an adjustment to their benefit indexation. The adjustment is calculated based on the rate of annual increase (decrease) in the total wage volume and a set adjustment of .3 percent in respect of increasing life expectancy. The rate of annual increase (decrease) in the total wage volume can be decomposed into the product of the rate of annual increase (decrease) in the average wage per covered worker multiplied by the rate of annual increase (decrease) in the number of covered workers (Sakamoto, 2008). Throughout the projection period from 2005-2050, the number of covered workers declines monotonically. The rate of decline from one five-year date to the next is shown in the following table. Also shown is the rate of decline with the life expectancy adjustment. It will be noted that these rates of decline exceed 5%, i.e., the adjustment factor is less than .9500 in one-half of the periods, and all periods from 2035 on.

Table 20: Japan Covered Workers and Life Expectancy Adjustment

Year of Calculation	Covered Workers and Life Expectancy Adjustment	
	Ratio of Covered Workers	Ratio of Covered Workers Less Life Expectancy Adjustment
2010	.9653	.9623
2015	.9479	.9449
2020	.9667	.9637
2025	.9755	.9725
2030	.9626	.9596
2035	.9446	.9416
2040	.9211	.9181
2045	.9327	.9297
2050	.9436	.9406

In assessing equity using the measure described in Chapter 5, if the decrease is more than 5%, i.e., the adjustment factor is less than .9500, then the adjustment might be considered inequitable. The adjustments in the last column of the table above need to be further adjusted by the average change in salary per covered worker. Although it is possible that this change will be sufficiently positive that the total adjustment factor would never be less than .9500, it does also seem possible that this would not always be the case and that the total adjustment factor might be less than .9500. The Japanese economy and its workers have experienced very low inflation and very low salary growth rates for at least 15 years. What will transpire in the future is a matter of speculation. Whether low rates will continue or whether there will be higher rates of wage growth as the covered worker group declines is beyond the scope of this thesis. Nonetheless, the adjustments required by the Japanese SSRS due to declining numbers of covered workers and increasing life expectancy may result in inequitable adjustments. The equitable balance achieved by this mechanism is considered transitory, due to the combination of demographic and economic factors.

7.4 Germany

In Germany, the contribution schedule is fixed, so any adjustments to rectify financial imbalances are made to pensioners' benefits. The formula for adjustments in year t , involves two ratios that will be considered in this section. One is a ratio of pensioners to covered people (including the unemployed), in year $t - 2$ to year $t - 3$, multiplied by a sustainability parameter, which is currently set at .25. The other is approximated by the ratio of the annual rate of change in gross income of year $t - 2$ to year $t - 3$. The following table shows the change in senior dependency ratio from one five-year period to the next. Given that the first ratio includes the unemployed in covered persons, the senior dependency ratio is a relatively good proxy for the ratio under consideration, since in the German system the unemployed are included in the denominator of the dependency ratio.

The last column shows the approximate ratio of the change in gross income required in combination with the change in the senior dependency ratio to obtain an overall adjustment factor of .9500, i.e., a 5% adjustment.

Table 21: Germany Demographic and Wage Adjustments

DEMOGRAPHIC AND WAGE ADJUSTMENTS		
(1) Year of Calculation (<i>t</i>)	(2) Ratio of Seniors Dependent Ratio <i>t</i> to Seniors Dependent Ratio <i>t</i> - 5	(3) Required Ratio of Change in Gross Income When Multiplied by Column (2) Gives .9500
2010	.9752	.9741
2015	.9924	.9572
2020	.9779	.9715
2025	.9708	.9785
2030	.9578	.9919
2035	.9658	.9836
2040	.9946	.9552
2045	1.0010	.9491
2050	.9955	.9543

From the table, if we look at 2030, it would only take a very slight decline in lagged ratio of gross income when combined with a change in the senior dependency ratio, similar to what is shown, to create an adjustment to pensions of more than 5%. Using the equity measure described in Chapter 5, this would be considered inequitable. Hence, the adjustment mechanism in the German SSRS has the potential to be considered inequitable and will only be considered to achieve transitory equitable balance, due to the combination of demographic and economic factors.

7.5 Canada

In Canada, if the finance ministers could not agree on a course of action, after an actuarial valuation indicated action was required to restore financial balance, both the contribution rate and the rate of indexation of pensions would be adjusted. The extent to which the burden is shared between contributors and pensioners would depend on the demographics at the time of the required adjustment. As noted in Chapter 6, based on current demographics, the burden is borne more heavily by pensioners than by contributors. Given that the number of pensioners is only a small fraction of the number of contributors, the burden per pensioner is more significant than the burden per contributor. However, the CPP is on a sound financial basis now so a comparison based on the burden per pensioner and per contributor today, does not prove the formula is inequitable. But there is the potential that it would be inequitable, if applied.

It is also worth noting that if the adjustment provision were triggered and it were applied for a number of years, there would be considerations of intragenerational equity, if the equity measure in Chapter 5 were used. Older pensioners, especially those who died during the adjustment period, would be less affected than younger pensioners who lived well beyond the adjustment period. Also younger contributors who paid a higher contribution rate for a longer period, without any increase in pension, would bear a heavier burden than older contributors.

In the following subsections, numerical calculations for different age-sex groups are presented to provide greater understanding of the equitableness of the Canadian adjustment mechanism.

7.5.1 Further Analysis of Canadian Balancing Mechanism

In subsection 6.4.3, it is argued that, in the extreme, the balancing mechanism is not robust because the adjustment to the contribution rate is only one half of the required amount but the adjustment to freeze the indexing to pensioners may be less than one half of the required amount (referred to as the “Current Rule”). A simple refinement (referred to as the “Financial Rule”) is suggested to achieve financial balance, namely:

- If the value of the freeze in indexing for three years is greater than one half the recommended increase in the contribution rate, then impose a proportionate freeze on indexing for three years such that it has a value equal to one half the recommended increase in the contribution rate and increase the contribution rate by one half the recommended amount;
- If the value of the freeze in indexing for three years is less than one half the recommended increase in the contribution rate, then freeze the indexing for three years and increase the contribution rate by the difference between the recommended contribution rate and the value of the freeze in indexing for three years expressed as a contribution rate.

Such a rule would be robust, with respect to financial balance. However, as will be shown, the rule is partial, as opposed to robust, with respect to equitable balance, depending on the extent of the adjustment required and the age of the contributors and pensioners. After a few alternative adjustments are analyzed, a further refinement is proposed to produce a rule that is robust with respect to both financial and equitable balance, within certain ranges.

For this analysis, the measure of equity will be the change in the following ratio, which was described in section 5.3, namely:

$$\frac{\text{Present Value of Pension}}{\text{Accumulated Value of Contribution to 65}}$$

The calculations are performed using the discount rates shown in Tables 6 and 12, which are repeated below for convenience. These discount rates are designed to reflect

differences in utility depending on the proximity of pension payments. The ratios will be calculated for males and females aged 30, 40, 50, 60, all considered to be still contributing and also for males and females aged 70, 80, considered to be pensioners. Retirement at age 65 is assumed. The ultimate CPP valuation assumption of 2.5% inflation and 3.8% nominal wage growth, both on the best-estimate basis, are used. Salaries are assumed to be at least the maximum for CPP benefit calculation purposes and contributions are assumed to be made on all salary up to the maximum (ignoring the Year's Basic Exemption).

Table 22: Personal Discount Rates Repeated Again

Period (current age x)	Discount Rate	
	For Period of Life Expectancy	Thereafter
$65 - x > 30$	4%	4%
$30 \geq 65 - x > 20$	3%	3%
$20 \geq 65 - x > 10$	2%	3%
$10 \geq 65 - x > 0$	1%	3%
$0 \geq 65 - x$	0%	3%

For the purpose of calculating the ratios the UN 2002 projections for ultimate mortality of the general population are used. These tables are considered by the UN as appropriate for the global population in the year 2100. By that time, it is assumed that mortality rates will have ceased improving. The tables have an upper age of life of 100 for both males and females. The following table provides an indication of the life expectancy using these tables. As such they appear reasonable for the purpose of making comparisons. Slight variations in the figures are due to rounding.

**Table 23: Life Expectancy Based on UN 2002 Projections
(Ultimate Table)**

Age x	Life Expectancy at Age x	
	Male	Female
Birth	82.44	87.34
30	53.11	57.84
40	43.29	47.98
50	33.79	38.30
60	24.84	28.84
65	20.64	24.28
70	16.68	19.90
80	9.84	12.10

(i) Scenario 1: Current Rule - Small Adjustment

Current Contribution Rate: 9.9%

Recommended Contribution Rate: 10.10%

Adjustment: Can be accomplished in one adjustment

Results

Table 24: Canada - Current Rule Small Adjustment

Age	Males			Females		
	Ratio Before Change	Ratio After Change	Reduction Factor	Ratio Before Change	Ratio After Change	Reduction Factor
30	1.1136	1.0371	0.9911	1.3391	1.3274	0.9913
40	1.2464	1.2381	0.9934	1.5170	1.5069	0.9934
50	1.4013	1.3955	0.9959	1.7286	1.7215	0.9959
60	1.6324	1.6300	0.9986	2.0033	2.0004	0.9986
70	1.4262	1.3296	0.9322	1.8982	1.7679	0.9314
80	0.8021	0.7509	0.9362	1.1868	1.1085	0.9341

Analysis

For pensioners the reduction factor is less than 0.95 so the test of equity is not met. Equitable balance is at most transitory because the pension freeze, an economic factor, is too large.

(ii) Scenario 2: Current Rule - Large Adjustment

Current Contribution Rate: 9.9%

Recommended Contribution Rate: 10.30%

Adjustment: Must be accomplished in two adjustments

Results

Table 25: Canada – Current Rule Large Adjustment

Age	Males			Females		
	Ratio Before Change	Ratio After Change	Reduction Factor	Ratio Before Change	Ratio After Change	Reduction Factor
30	1.1136	1.0913	0.9799	1.3391	1.3124	0.9801
40	1.2464	1.2278	0.9851	1.5170	1.4944	0.9851
50	1.4013	1.3883	0.9908	1.7286	1.7127	0.9908
60	1.6324	1.6274	0.9970	2.0033	1.9972	0.9970
70	1.4262	1.2534	0.8788	1.8982	1.6617	0.8754
80	0.8021	0.7167	0.8935	1.1868	1.0326	0.8701

Analysis

For pensioners the reduction is factor is less than 0.95 so the test of equity is not met. Equitable balance is at most transitory because the pension freeze, an economic factor, is too large.

(iii) Scenario 1: Financial Rule - Small Adjustment

Current Contribution Rate: 9.9%

Recommended Contribution Rate: 10.10%

Adjustment: Accomplished in one adjustment

Results

Table 26: Canada – Financial Rule Small Adjustment

Age	Males			Females		
	Ratio Before Change	Ratio After Change	Reduction Factor	Ratio Before Change	Ratio After Change	Reduction Factor
30	1.1136	1.1037	0.9911	1.3391	1.3274	0.9913
40	1.2464	1.2381	0.9934	1.5170	1.5069	0.9934
50	1.4013	1.3955	0.9959	1.7286	1.7215	0.9959
60	1.6324	1.6300	0.9986	2.0033	2.0004	0.9986
70	1.4262	1.3608	0.9541	1.8982	1.8100	0.9535
80	0.8021	0.7674	0.9568	1.1868	1.1338	0.9553

Analysis

The Financial Rule makes a sufficient adjustment to the Current Rule to achieve equitable balance.

(iv) Scenario 2: Financial Rule - Large Adjustment

Current Contribution Rate: 9.9%

Recommended Contribution Rate: 10.30%

Adjustment: Accomplished in one adjustment

Results

Table 27: Canada – Financial Rule Large Adjustment

Age	Males			Females		
	Ratio Before Change	Ratio After Change	Reduction Factor	Ratio Before Change	Ratio After Change	Reduction Factor
30	1.1136	1.0891	0.9780	1.3391	1.3098	0.9782
40	1.2464	1.2259	0.9836	1.5170	1.4921	0.9836
50	1.4013	1.3869	0.9897	1.7286	1.7113	0.9899
60	1.6324	1.6265	0.9964	2.0033	1.9961	0.9964
70	1.4262	1.3296	0.9322	1.8982	1.7679	0.9314
80	0.8021	0.7509	0.9362	1.1868	1.1085	0.9340

Analysis

This rule improves the equity for pensioners by putting more of the burden of adjustment on contributors; however, for pensioners, the reduction factor is less than 0.95 so the test of equity is not met.

To provide greater equity than the Financial Rule, define the following Equitable Rule:

- For existing pensioners indexing for years four to six after the date of adjustment (for contributors) will be reduced by up to 1%. If the value of a 1% reduction in indexing is greater than one half the recommended increase in the contribution

- rate, then impose a proportionate reduction to the 1% proposed reduction for years four to six such that it has a value equal to one half the recommended increase in the contribution rate and increase the contribution rate by one half the recommended amount;
- If the value of a 1% reduction in indexing for existing pensioners for years four to six is less than one half the recommended increase in the contribution rate, then reduce the indexing by 1% for years four to six and increase the contribution rate by the difference between the recommended contribution rate and the value of the reduction in indexing expressed as a contribution rate.

(v) Scenario 1: Equitable Rule - Small Adjustment

Current Contribution Rate: 9.9%

Recommended Contribution Rate: 10.10%

Adjustment: Accomplished in one adjustment

Results

Table 28: Canada – Equitable Rule Small Adjustment

Age	Males			Females		
	Ratio Before Change	Ratio After Change	Reduction Factor	Ratio Before Change	Ratio After Change	Reduction Factor
30	1.1136	1.0100	0.9876	1.3391	1.3226	0.9877
40	1.2464	1.2349	0.9907	1.5170	1.5030	0.9907
50	1.4013	1.3932	0.9942	1.7286	1.7186	0.9942
60	1.6324	1.6291	0.9980	2.0033	1.9993	0.9980
70	1.4262	1.3761	0.9648	1.8982	1.8282	0.9631
80	0.8021	0.7795	0.9719	1.1868	1.1494	0.9685

Analysis

Compared to the Financial Rule, the Equitable Rule makes a further reduction to the contributors and improves the reduction of the pensioners. All reduction factors are at least 0.95 so equitable balance is achieved.

(vi) Scenario 2: Equitable Rule - Large Adjustment

Current Contribution Rate: 9.9%

Recommended Contribution Rate: 10.30%

Adjustment: Accomplished in one adjustment

Results

Table 29: Canada – Equitable Rule Large Adjustment

Age	Males			Females		
	Ratio Before Change	Ratio After Change	Reduction Factor	Ratio Before Change	Ratio After Change	Reduction Factor
30	1.1136	1.0806	0.9703	1.3391	1.2995	0.9705
40	1.2464	1.2187	0.9778	1.5170	1.4833	0.9778
50	1.4013	1.3817	0.9861	1.7286	1.7045	0.9861
60	1.6324	1.6244	0.9951	2.0033	1.9936	0.9951
70	1.4262	1.3761	0.9648	1.8982	1.8282	0.9631
80	0.8021	0.7795	0.9719	1.1868	1.1494	0.9685

Analysis

Compared to the Financial Rule, the Equitable Rule makes a further reduction to the contributors and improves the reduction of pensioners. All reduction factors are at least 0.95 so equitable balance is achieved.

7.5.2 Limits to the Equitable Rule

With the Equitable Rule, the reduction factor applied to pensioners is limited. It will not reduce further as greater aggregate adjustments are required. The burden of any such greater adjustments must be borne by contributors. With respect to the distribution of the burden of adjustment among contributors, it is heavier on younger contributors than on older contributors because the higher contribution rate must be paid for a longer period of time in order to get the same pension (since the pension is defined by formula unrelated to contributions). At some point, if the aggregate adjustment required were sufficiently large, then the reduction factor for contributors would be below 0.95 and equitable balance would not be achieved. Such a situation could be partially mitigated by having different contribution rates for contributors of different ages at the time of adjustment. (Practically, it is unlikely that such a politically challenging and administratively complex approach would be implemented.) The following table shows the impact on the reduction factors of limiting the adjustments for contributors to certain amounts. The maximum freeze to indexing for existing pensioners remains at 1% for years four to six.

Table 30: Canada – Equitable Rule Size of Contribution Adjustment to Maintain Equity by Age Group

Age	Contribution Rate Increased by	Males			Females		
		Ratio Before Change	Ratio After Change	Reduction Factor	Ratio Before Change	Ratio After Change	Reduction Factor
30	0.50%	1.1136	1.0657	0.9569	1.3391	1.2816	0.9571
40	0.75%	1.2464	1.1870	0.9523	1.5170	1.4447	0.9523
50	1.25%	1.4013	1.3320	0.9506	1.7286	1.6432	0.9506
60	3.00%	1.6324	1.5648	0.9586	2.0033	1.9204	0.9586
70	n/a	1.4262	1.3676	0.9648	1.8982	1.8282	0.9631
80	n/a	0.8021	0.7795	0.9719	1.1868	1.1494	0.9685

In summary, the Equitable Rule is only effective in achieving equity for contributors and pensioners of all ages within certain limits. Beyond those limits, some further adjustment to the parameters of the system, i.e., the contribution rate, benefit formula, retirement age for full benefit entitlement, is required. The adjustment for pensioners has been designed to enhance equity by limiting the amount of the adjustment to 1% and by lagging the adjustment to the fourth to sixth years after the time of adjustment. Both of these factors would make for an easier adjustment by pensioners.

7.6 Rethinking Equity

When considering equitable balance, if one party, namely pensioners, is asked to bear the full burden of adjustments, it is difficult to see that there is equitable balance. As has been argued earlier, pensioners are poorly situated to adjust to reductions in their income, both because their pension often represents a large portion of their total income and also because they have few options available to adjust their income and must make the adjustments on the expenditure side, which may be difficult. Both Germany and Japan have set the schedule of contribution rates and adjustments are borne by pensioners. This does not appear to be an equitable balance.

The situation with Sweden is slightly different. Raising of the age for full benefit entitlement is a way of sharing the burden between contributors and those eligible to retire. (Those eligible to retire are one subclass of contributors; however, such individuals have a choice of continuing to contribute longer in order to receive the full pension or ceasing to contribute and receiving a pension that has been reduced to reflect the new age of full benefit entitlement). Barring disability or unemployment, those eligible to retire who have not yet retired do have the means to adjust to an increase in the age of full benefit entitlement, i.e., to a reduction in the pension amount at age 65. Such people can continue to work, thereby increasing their aggregate lifetime income and shortening the expected period on which they must live on a pension. So this provision in the Swedish SSRS is equitable.

However, as the analysis in section 7.2 showed, once the ABM is triggered, on the measure of equity described in Chapter 5, pensioners bear the full burden of adjustment. This is not equitable.

It would be desirable from the perspective of equitable balance that both contributors and pensioners bear some of the adjustment and contributors who are more able to adapt than pensioners bear a larger share of the adjustment.

7.7 Chapter Conclusion

In this chapter, there was a thorough demonstration of how the definition of equity described in Chapter 5 could be applied to the Canadian system. The current rule was analyzed, as well as the financial rule proposed in Chapter 6. Both these rules place too great a share of the burden of adjustment on pensioners; although, the financial rule is an improvement over the current rule. A further variation of the financial rule, referred to as the equitable rule, was proposed. Using this equitable rule, the Canadian mechanism would be robust, within certain ranges. This is a contribution of this thesis.

The chapter also examined various numerical ratios for the other three countries' SSRS. In all these three countries, the contribution rate is fixed so the burden of adjustment is borne by the pensioners. Japan and Germany were not considered to achieve equity. Because Sweden also has a component in its system that adjusts the age for full benefit entitlement, a portion of the burden of adjustment is borne by contributors. Sweden is judged to achieve partial equitable balance.

8.0 Chapter Overview

This chapter begins by summarizing the results of the analysis in Chapters 6 and 7, using the terminology presented in Chapter 2. It then describes the characteristics of mechanisms and controls that an ABM would possess in order to be robust. The chapter discusses the role that extra-systematic flows may play in achieving balance but notes that the existence of such extra-systematic flows, unless they are treated as loans, undermines the efficiency of the ABM.

The characteristics of a robust ABM are identified. The final subsection of the chapter presents a practical approach to applying a balancing mechanism, based on setting acceptable limits with respect to contribution rates and benefit replacement ratios. An extraordinary adjustment to the system and/or to the mechanism would be made when the system approaches the point at which it will violate the acceptable limits.

8.1 Summary Regarding the Four Countries' Mechanisms

The following table summarizes the analysis of the balancing mechanisms in the four countries and applies the classification terminology presented in Chapter 2.

Table 31: Classification of Balancing Mechanisms

Does The Balancing Mechanism				
Country	Work Automatically	Achieve Financial Balance	Achieve Equitable Balance	Classification
Canada	No	Partial	Transitory	Discretionary Partial
Germany	Yes	Transitory	Transitory	Mechanistic Transitory
Japan	Yes	Transitory	Transitory	Mechanistic Transitory
Sweden	Yes	Transitory	Partial	Mechanistic Partial

The conclusion is that none of the four countries' systems examined has a balancing mechanism that is automatic and robust in achieving both financial and equitable balance.

8.2 Characteristics of Mechanisms and Controls

This chapter considers characteristics that a balancing mechanism might possess in order to be considered automatic and robust, both financially and equitably.

1. To be automatic, the mechanism must be triggered based on some defined measure or hurdle, without the requirement of further decisions by lawmakers or the system's administrators. Hence, a requirement is that a measure or hurdle be defined and that the data necessary to calculate the measure or hurdle be obtainable. A further requirement is that when the measure or hurdle indicates that a balancing action is required that the required action is determined without any further decisions.
2. To achieve financial balance over an indefinite, and possibly infinite, time horizon is a problem in stochastic optimal control. As an initial starting point, if the contribution rates, benefit levels and retirement ages have been set so that the system is financially sustainable over a long horizon, such as 75 years, on a consistent basis on some set of plausible economic and demographic assumptions, the system will be better placed to remain in financial balance when adjusted by an appropriate balancing mechanism. Setting the contribution rates, benefit levels and retirement ages to produce a contingency reserve (or surplus) on some set of plausible economic and demographic assumptions would produce greater likelihood of financial balance being maintained. However, if these rates, levels and ages produce too large a contingency reserve, it is likely that the system would not meet the requirement of achieving equitable balance.

Assuming that the rates, levels and ages have been set to maintain balance over a long horizon such as 75 years, on some set of plausible economic and demographic assumptions, then the system will be more resilient to shocks or deviations from the assumptions, the more controls are available to make adjustments. In the examination of the four countries' systems we observed mechanisms designed to adjust system outflows, i.e., through benefit payments, and to adjust system inflows, i.e., through contributions or taxes. Such mechanisms may respond to economic factors, demographic factors or both. These controls observed in the four systems and the country where observed were the following:

- (i) Adjust the contribution rate – Canada
- (ii) Adjust the benefit level – Canada, Germany, Japan, Sweden
- (iii) Adjust the retirement age – Sweden (Germany and Japan did some scheduled increases but it is not an ongoing feature)
- (iv) Invest reserve funds – Canada
- (v) Extra-systematic flows – Germany through government subsidy, Japan through government subsidy and minimum benefit, Sweden through minimum benefit. Supplemental pensions to raise pensions to the minimum benefit level are paid by government, i.e., using general tax revenues, in both Japan and Sweden.

The following table shows how these items affect inflows and outflows and whether they are a response to economic or demographic factors.

Table 32: Controls and Responsiveness

Control	In Response To			
	Economic Factors		Demographic Factors	
	Adjust Inflows	Adjust Outflows	Adjust Inflows	Adjust Outflows
(i) Adjust the contribution rate	✓		✓	
(ii) Adjust the benefit level		✓		✓
(iii) Adjust the retirement age			✓	✓
(iv) Invest reserve funds	✓	✓		
(v) Extra-systematic flows	✓	✓	✓	✓

The chart shows the limitations of the controls. Extra-systematic flows can be used to adjust inflows and outflows in response to economic or demographic factors. All the other controls can adjust inflows or outflows or can adjust for economic factors or demographic factors but not for all such flows and factors. To have the greatest flexibility within the control system, and greater likelihood of achieving financial balance, it is desirable to be able to adjust all flows in response to both factors. Although extra-systematic flows can be an important and useful way of achieving financial balance, a balancing mechanism which requires extra-systematic flows would not be considered an effective balancing mechanism, i.e., the assessment of the effectiveness of the balancing mechanism is from the perspective of the system and whether balance is achieved without resort to extra-systematic flows.

3. To achieve equitable balance requires the sharing of the burden of adjustment between cohorts of contributors and cohorts of pensioners. In the foregoing table, we might think of those representing the inflows as contributors and those representing the outflows as pensioners. A control mechanism that only makes adjustments in respect of contributors or only makes adjustments in respect of pensioners would not be considered balanced. Hence controls (i) and (ii), if adopted without some other control, will mean that the system cannot be considered to maintain equitable balance. Furthermore, controls (iii) and (iv) could be designed to be equitable controls since they can affect both contributors and pensioners. Of course, control (v) could be designed to affect all flows and so achieve equitable balance; however, as noted, because balance is maintained due to an extra-systematic flow, the balancing mechanism would not be considered effective (when judged from the perspective of the system).
4. In considering whether to adopt control (iii) or control (iv) one must consider the effect that demographics is having on the system and compare the expected

“demographic return” to the expected “economic return”. If changing demographics is affecting the system, adjusting the retirement age for full benefit entitlement can be used as a financial control and can be designed as an equitable control. As noted in Chapter 3, periodically adjusting the retirement age to maintain a constant ratio of the expected period during which contributions will be made up until the age of full benefit entitlement to the expected period of benefit receipt after full-benefit retirement age (or some similar ratio) maintains individual equity across cohorts and within cohorts. It has the further equitable aspect that all adjustments that affect an individual are known before the individual becomes a pensioner. It is easier (and I would argue more equitable) to adjust one’s retirement plans based on a change in the age of full benefit entitlement while one is not retired than it is to adjust to a change in benefits once one has retired.

As discussed in Chapter 3, in determining whether control (iv) is likely to be effective, it is important to compare the expected economic return on investments in capital markets to the expected return attainable (due to demographics) if the excess funds are used by the economy to generate employment opportunities and economic growth. Hence, although control (iv) is designed to capitalize on economic factors, its desirability and viability is dependent on a comparison of the returns available due to both economic and demographic factors.

8.3 Extra-Systematic Flows

As stated, a balancing mechanism that requires extra-systematic flows is considered ineffective. However, for the benefit of fuller discussion, a description of some of the extra-systematic flows used may be informative.

In Japan, the basic pension is subsidized by the government (i.e., by the current taxpayers) by $\frac{1}{2}$ (in 2009). In Germany, the government subsidy is approximately 30%. In both countries, this is a significant subsidy that reduces the extent of the reduction in benefits (since the contribution rate is prescribed). It also broadens the burden for financing the social security system to the general taxpayer. If the general tax system has been designed to be equitable, which most countries would claim is their objective, then using the general tax system to bear a portion of the contribution burden is an equitable approach.

Greater equity might be achieved if both benefits were reduced and contributions were increased, in combination with a subsidy, as this would require contributors to bear some of the adjustment directly (not just through the general tax system). Given that a social security system is a complex and dynamic system, greater flexibility and resiliency could be achieved with a varying amount of government subsidy that was targeted at some expected level. For example, with respect to Japan, if it were established that the expected level of government subsidy was $\frac{1}{2}$ but the actual amount of subsidy in any year were to be in a range, say of $\frac{2}{5}$ to $\frac{3}{5}$, and if over a 10-year period the subsidy had not

been in aggregate approximately $\frac{1}{2}$, some adjustment to benefits and contributions would be made. Such an adjustment would provide a more flexible response to economic and demographic downturns that might last for several years but which might correct over a ten-year time frame, e.g., recession, stagflation, stalled productivity growth, or a sluggish economy accompanied by increased unemployment.

A second type of extra-systematic flow, used by both Japan and Sweden, is the establishment of a minimum benefit that is paid for through general revenues. The minimum benefit is set at a level deemed adequate. Then if the social security pension is less than this level, the difference between the minimum benefit level and the social security pension is paid as an additional or supplemental pension, using general revenues.

The existence of such a minimum benefit plays an important social role in ensuring adequate minimum pensions. Sakamoto (2008) argues that a minimum benefit guarantee is an essential component in a system where benefits are being reduced. But the existence of a minimum benefit makes it harder to assess the effectiveness of any balancing mechanism. The repeated application of the balancing mechanism may cause social security pensions to be reduced below the minimum benefit level but concerns regarding the effectiveness of the balancing mechanism may not be raised because pensioners receive at least the minimum pension, which is deemed adequate. However, there is a subtle transfer occurring from a self-financing social security system to a social security system delivering inadequate benefits that requires transfers from public taxes to make the benefits adequate. This is a concern with the Swedish ABM raised by Scherman (2007).

Moreover there are considerations of intergenerational and intragenerational equity. The portion of the pension paid as a minimum benefit supplement is financed from general revenues. This passes the burden to all taxpayers but typically those working, who are also contributing to social security, will pay the greatest amount of taxes. So once benefits fall to the minimum benefit level, any further adjustments are paid by all taxpayers and the benefit recipient does not experience any further reduction in benefits; predominantly a form of intergenerational transfer. From an intragenerational perspective, those whose benefits fall to the minimum benefit level are treated differently than those whose benefits are above the minimum benefit level, when benefit reductions are required. The former group maintains its level of income whereas the latter group receives a benefit reduction.

Hence, while one may argue strongly for the existence of a minimum benefit from an adequacy perspective, the existence of a minimum benefit may hide the ineffectiveness of the balancing mechanism and create issues of both intergenerational and intragenerational equity.

8.4 A Mechanistic Adjustment Within Limits

The balancing mechanisms of the four countries do not employ stochastic controls but rather make adjustments according to prescribed rules when some measure or hurdle so indicates. The simplicity of the rules provides ease of application and is more readily understandable by the public. However, none of the mechanisms reviewed is sufficient to ensure balance will be achieved. If such simple mechanisms are to be employed, a pragmatic approach is to define limits within which the mechanism will operate, without any revision to the system. These limits could be defined in terms of contribution level and benefit replacement rate. These are both items on which societal input could be sought. The levels for both acceptable contributions and replacement rates will vary by society (or country).

This approach was adopted by both Japan and Germany when the most recent reforms described were implemented. In Japan, the contribution rate to the EPI (beginning in 2004) and National Pension Scheme (beginning in 2005) are to be increased in steps from 2005 until 2017 when the rate reaches its ultimate level. Based on the financial analysis, the replacement rate, based on disposable income of pensioners to disposable income of contributors, will gradually decline in accordance with these economic and demographic assumptions. According to Sakamoto (2008), the 2004 actuarial projections indicated that the replacement rate would stabilize in 2023 at 50.2%, which was considered acceptable. Further, it was decided that a replacement rate below 50 percent was unacceptable so accommodation measures were introduced, including introduction of a minimum benefit at the 50% level, suspension of modified indexing between actuarial reviews if the minimum 50% replacement level might be violated, initiation of a scheme review. According to Borsch-Supan et al. (2006) in the case of the German reforms, the sustainability parameter was set at .25, which on the basis of projections should be sufficient to meet the dual objective of keeping the contribution rate under 20% until 2020 and under 22% until 2030 and maintaining an adequate replacement rate, approximately 67%. These contribution rate levels were considered acceptable after analyzing the impact on replacement rates and determining that they were not projected to be much different than the current replacement levels when all aspects of the revised SSRS were considered, including assumptions regarding high participation in the voluntary savings component (Borsch-Supan et al., 2006). Toft (2007) raised questions of the realism of the projections particularly in view of recent economic performance.

Using data for Lithuania, Alho et al. (2006) demonstrate how stochastic population simulations combined with an OLG model that assumes perfect foresight from the agents could be used to demonstrate the probability that reform proposals would require the contribution rate to be higher than a set level or that the replacement rate would fall below a set level. This type of analysis might be used by countries considering social security reforms in order to gain insight into the likelihood of the sustainability of the reform proposal. Although the work presented involves stochastic methods and appears quite sophisticated, Alho et al. (2006) consider it as a first step toward a more comprehensive model in which future uncertainty could be handled in a more advanced manner. Perhaps such an advancement would be to define a steady-state contribution rate

in the way the Canadian system does. This would create a tension between the near and the distant future that may enhance the likelihood of maintaining financial balance over a seventy-five year horizon. A further advancement might be to use a stochastic optimal control approach.

8.5 Chapter Conclusion

This chapter summarizes the results of the analysis in Chapters 6 and 7 and using the terminology presented in Chapter 2 categorizes the four countries' mechanism. Sweden is mechanistic partial, Canada is discretionary partial and both Germany and Japan are labeled mechanistic transitory. The chapter then described the characteristics of mechanisms and controls that an ABM would possess in order to be robust.

All countries' mechanism, except Canada's, involves extra-systematic flows. Such flows may help in achieving financial balance but may change the share of the burden between contributors and pensioners by including general taxpayers. The existence of such extra-systematic flows, unless they are treated as loans, undermines the efficiency of the ABM.

The final subsection of the chapter presented a practical approach to applying a balancing mechanism, based on setting acceptable limits for contribution rates and benefit rates. An extraordinary adjustment to the system and/or mechanism is required when the system approaches the point at which it will violate acceptable limits.

9.0 Chapter Overview

This chapter summarizes the conclusions of this thesis. It presents the contributions of this thesis to this area of research. These contributions were listed in Chapter 1. The thesis concludes by identifying the implications of this work and other areas in which it might be applied. Topics for further research are listed.

9.1 Conclusions

The conclusions of this thesis are as follows:

- (1) Managing an SSRS that does not have fully funded individual accounts, in a changing environment, is a complex, dynamic problem. One approach to the problem would be to formulate it as a stochastic control problem.
- (2) Likely due to the complexity of not only the solution but even the formulation and specification of the stochastic optimal control problem, none of the international SSRS analyzed uses a stochastic control approach.
- (3) If the contribution rates, benefit levels and retirement ages have been set at a level that financial balance may be sustained, on some reasonable set of economic and demographic assumptions, the balancing mechanism will be in a better starting position to maintain financial balance.
- (4) Adjusting the retirement age for full benefit entitlement to maintain a constant ratio of the expected contributory period to the expected period of benefit receipt, or some similar ratio, can be an effective method of maintaining financial and equitable balance, when the SSRS is impacted by changing demographics.
- (5) When the rate of return available in capital markets exceeds the internal rate of return on a paygo SSRS, there are some financial benefits to be gained through partial funding.
- (6) Since it typically takes at least seven decades for a system change to mature, from the perspective of individual participants in the SSRS, the system is in continual transition and this is the normal state.
- (7) Because individual participants perceive the system to be in continual transition, some changes to the system that affect their rights (benefit entitlements) and obligations (contribution requirements) may be considered equitable.

- (8) Even though gradual adjustments to participants' rights and obligations may be considered equitable because they fall within acceptable quantitative limits, if such adjustments only affect pensioners, after they have retired, they would be considered inequitable because pensioners are least able to adapt to such changes.
- (9) Adjustments are more equitable if they are borne by contributors. They may also be equitable if their impact is shared between contributors and pensioners. However, where there is a sharing, the greater share should be borne by contributors recognizing that they are more able to adapt to such an adjustment, both because they are working and also because they have a longer time horizon for adaptation. Such considerations can be incorporated in the assessment of equity, by using different personal discount rates, which vary by time to retirement.
- (10) In their current form the adjustment mechanisms of Germany and Japan are inequitable because the contribution rate is fixed and the adjustment is borne primarily by pensioners after retirement. Although the contribution rate is fixed in the Swedish system because the age for full benefit entitlement is adjusted based on life expectancy, there is a sharing of the burden of adjustment between contributors and pensioners; hence, Sweden is judged to be partially equitable. The adjustment mechanism in Canada, should the politicians not be able to agree on a course of action, is inequitable, based on current demographics, since it places a heavier burden on pensioners than on contributors.
- (11) The control mechanisms adopted by Canada, Germany, Japan and Sweden are simple to calculate and apply. They are easy to communicate, which is an advantage. If such simple control mechanisms are to be used, then it would be desirable to do projections, on various assumptions, ideally using stochastic methods, to determine the likely impact on the contribution rate and the benefit replacement rate. Acceptable contribution rates and replacement rates should be determined, from time to time. When the contribution rate or the benefit level would violate the acceptable limit, a review of the SSRS should be conducted and suitable adjustments made.
- (12) The approach to determining the assets of the SSRS adopted by Sweden, in which a contribution asset is defined based on the turnover duration, where the turnover duration includes a period beyond which contributions will be made by the current contributing generation, is flawed. Although it may be viable on certain idealistic steady state assumptions, it is not appropriate in a non-steady state world since it overstates the assets. It is important that this inherent flaw in this approach be recognized and corrected. The Swedish model is widely studied and may be copied. For example, Boada-Penas et al. (2008) adopt this approach in preparing the balance sheet of the Spanish SSRS; hence, overstating that system's financial position.

- (13) Because participants may assign different utility values to current contributions and to future benefit payments, participants (and different generations of participants) may consider the SSRS to be equitable even if the net lifetime benefit is not zero.

9.2 Summary of Contribution

This thesis makes the following contributions to the literature on SSRS and ABM:

- In Chapter 2, it establishes a terminology to classify balancing mechanisms.
- In Chapter 3, a critical analysis of an approach proposed by Valdes-Prieto referred to as Integration to Financial Markets is presented and the approach is found to be deficient.
- In Chapter 5, families of definitions of equity and a benchmark to measure equitable balance are presented. In Chapter 7, it is illustrated how this benchmark could be applied to four countries' SSRS that differ in significant aspects.
- In Chapter 6, a critical error is identified in the way in which assets are calculated by the Swedish SSRS in determining financial balance and a suitable correction is proposed. Also in Chapter 6, a weakness in the manner of applying the ABM in the Swedish system is identified. Due to this weakness, once liabilities exceed assets, it is unlikely that applying the ABM will restore balance.
- In Chapter 6, some of the unusual characteristics of the steady-state contribution rate calculation for the Canadian SSRS were identified. Although this approach was shown to have limited application and does not appear to depend on any actuarial principle, it creates a tension between the near and distant future, which is a factor in achieving financial balance over a seventy-five year horizon.
- In Chapters 6 and 7, an analysis of the balancing mechanism in the Canadian SSRS is presented and a proposed change to the ABM is presented that would make the mechanism robust, within certain ranges.

9.3 Implications, Applications and Topics for Further Research

Because none of the SSRS analyzed uses a stochastic control process, it is possible that the balancing mechanism will not work at some point to maintain financial balance. To avoid misleading the population and those who may copy the approach, it is important that this be communicated. Both the Germans and the Japanese have set limits with respect to contribution levels and/or benefit replacement rates that are considered acceptable and they conduct regular actuarial reviews of their SSRS to determine how they are performing. Some Swedes (Settergren, 2003) maintain that their ABM will maintain financial balance regardless of future economic and demographic developments. This claim is misleading and should be rectified.

This research has application to all SSRS, except those that only have fully funded individual accounts, since they are likely to experience economic and demographic

pressures which will challenge the ability of the SSRS to deliver its promised benefits for the specified contributions.

This research could also be applied to private pension plans, i.e., those outside the realm of social security. It is not likely to be applicable where the pension plan has been established by a commitment made by one party that is considered irrevocable, e.g., the situation of single employer pension plans that have been voluntarily established and which were not subject to union negotiations. However, where there has been union negotiations and/or where there is some form of risk sharing in the pension deal, it may be fruitful to consider how to incorporate a balancing mechanism and how it might operate when contribution levels and benefit replacement rates are within acceptable limits. Cui et al. (2005) present some interesting analysis of the application of risk-absorbing mechanisms to hybrid pension plans that involves the sharing of adjustments by contributors and pensioners. Other types of pension arrangements that include some form of risk sharing include negotiated plans, matching contribution defined benefit plans, negotiated contribution defined benefit plans, jointly-sponsored pension plans, and multi-employer pension plans.

This research might be extended to any arrangement where certain benefits are to be delivered for a fixed budget financed by taxes. For example, with respect to the funding of school boards' budgets, which is relatively rigid, one might establish certain prerequisites that would be reduced automatically if there were a financial imbalance or one might establish how class sizes would be increased automatically in response to a financial imbalance.

Topics for further research include:

- Consideration of how SSRS should be adjusted to recognize the impact of TFR, particularly when a cohort's TFR is below the population replacement rate.
- Consideration of how adjusting the age for full benefit entitlement to maintain a constant ratio of the expected contributory period to the expected benefit period might be applied to other programs than SSRS. For example, jurisdictions such as Ontario provide a drug benefit to those residents age 65 and older that is paid from general revenues. This age of entitlement might be raised as average life expectancy increases.
- Use of dynamic programming to investigate stochastic controls that might be adopted by an SSRS. Examples of stochastic controls to be considered would include: adjusting the age for full benefit entitlement; partial funding with capital market investments and the appropriateness of various investment asset classes; contribution increases when certain economic and/or demographic measures or hurdles are met; benefit reductions when certain economic and/or demographic measures or hurdles are met; the use of government subsidies as loans when deviations are within a specified range and are considered temporary.
- Development of more advanced techniques to enhance the stochastic modeling of system outcomes within certain contribution rate and benefit replacement rate ranges.

- Investigation of living expenses and life style among retirees, and compared to contributors, to provide guidance of what type of indexing of pensions may be most appropriate.
- Investigation of the utility functions of contributors and pensioners, and to the extent possible, quantification of those functions, to be better able to assess the equity of proposed adjustments.
- Development of a conceptual model for analysis that does not rely on steady-state assumptions.
- Greater analysis of empirical data leading to a better understanding of the economic interactions and interrelationships among variables such as: GDP growth, rate of salary and wage growth, rate of consumer price index growth, rate of productivity; GDP growth, return on common stock, return on long bonds, equity risk premium; GDP growth, interest rate levels, demographic profile.
- Analysis to see if an actuarial principle could be developed to justify a calculation of the type done in Canada to determine a steady-state contribution rate.

9.4 Chapter Conclusion

This thesis developed a terminology for classifying ABM. On examination of four countries' ABM, none was found to be robust. This may be largely because the definition of robustness requires the ABM to achieve both financial and equitable balance over a seventy-five year horizon, regardless of the demographics or economics or however remote the likelihood of occurrence of the demographic or economic stress may be. Nonetheless, this is an important point that may be overlooked (example, Settergren, 2003). The chapter summarized the contributions of this thesis and identified areas for further research.

Bibliography

Aaron, H. 1966. The Social Insurance Paradox. *The Canadian Journal of Economics and Political Science / Revue canadienne d'Economie et de Science politique* 32, 371-374.

Alho, J.M., J. Lassila and T. Valkonen 2006. Demographic Uncertainty and Evaluation of Sustainability of Pension Systems. *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*, R. Holzmann and E. Palmer, Eds. The World Bank, Washington, D.C., 95-112.

Ambachtsheer, K. 2006. Why We Need A Pension Revolution. International Centre for Pension Management Working Paper. Toronto, Canada.

Andersen, T.M. 2006. Increasing Longevity and Social Security Reforms. CESifo Working Paper No. 1789, 1-16.

Applebaum, Joseph A. 1979. Some Effects of Fully Funding OASDI. *Actuarial Note 97*. U.S. Department of Health, Education, and Welfare, Social Security Administration. Washington, D.C., 1-5.

Arenas De Mesa, A. 2005. Fiscal and Institutional Considerations of Pension Reform: Lessons Learned from Chile. *A Quarter Century of Pension Reform in Latin America and the Caribbean: Lessons Learned and Next Steps*, C.A. Crabbe, Ed. Inter-American Development Bank, Washington, D.C., 83-125.

Arenas de Mesa, A., D. Bravo, J.R. Behrman, O.S. Mitchell and P. E. Todd. 2006. The Chilean Pension Reform Turns 25: Lessons from the Social Protection Survey. Pension Research Council Working Paper. The Wharton School, University of Pennsylvania, Philadelphia, PA.

Auerbach, A.J., L.J. Kotlikoff, and W. Leibfritz, Eds. 1999. *Generational Accounting around the World*. The University of Chicago Press, Chicago and London.

Barr, N. 2004a. Notional defined contribution pensions: What they can do, and what they can't. *Scandinavian Insurance Quarterly* 2004, 203-214.

----- . 2004b. *The Economics of the Welfare State Fourth Edition* Oxford University Press, Oxford, Great Britain, 408.

-----, 2000. Reforming Pensions: Myths, Truths, and Policy Choices. IMF Working Paper WP/00/139. International Monetary Fund, Washington, D.C.

Barrientos, A. 1998. *Pension Reform In Latin America*. Ashgate Publishing Ltd, England, 265.

Barro, R.J. 1974. Are Government Bonds Net Wealth? *The Journal of Political Economy* 82, 1095-1117.

Becker, G.S., E.L. Glaeser and K.M. Murphy. 1999. Population and Economic Growth. *The American Economic Review* 89, 145-149.

Bethell, T.N., Ed. 2000. *Insuring the Essentials: Bob Ball on Social Security*. The Century Foundation Press, New York, U.S.A.

Bjork, T. 2005. *Arbitrage Theory in Continuous Time* Oxford University Press, Great Clarendon Street, Oxford, U.K., 466.

Blanchard, O.J. and S. Fischer. 1989. *Lectures on Macroeconomics*. The MIT Press, Cambridge, Massachusetts; London, England; 650.

Boada-Penas, M., S. Valdes-Prieto and C. Vidal-Melia. 2008. *The Actuarial Balance Sheet For Pay-As-You-Go Finance: Solvency Indicators For Spain And Sweden*. 2182. CESifo, Germany.

Board of Trustees, Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds. 2008. The 2008 Annual Report of the Board of Trustees of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds. U.S. Government Printing Office, Washington, D.C.

Bongaarts, J. 1999. Fertility decline in the developed world: Where will it end? *The American Economic Review* 89, 256-260.

Borsch-Supan, A., A. Reil-Held and C.B. Wilke. 2003. How to make a Defined Benefit System Sustainable: The "Sustainability Factor" in the German Benefit Indexation Formula. Discussion Paper No. 37, Mannheim Institute for the Economics of Aging, Mannheim University, 1-34.

Borsch-Supan, A.H. and C.B. Wilke. 2005. Shifting Perspectives: German Pension Reform. *Intereconomics* 40(5), 248-253.

-----2006. The German Public Pension System: How It Will Become an NDC System Look-Alike. *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*, R. Holzmann and E. Palmer, Eds. The World Bank, Washington, D.C., 573-610.

Borzutsky, S. 1998. Chile: The Politics of Privatization. *Do Options Exist? The Reform of Pension and Health Care Systems in Latin America*, M.A. Cruz-Saco and C. Mesa-Lago, Eds. University of Pittsburgh Press, Pittsburgh, PA, 35-55.

Boulier, J., S. Michel and V. Wisnia. 1996. Optimizing Investment and Contribution Policies of a Defined Benefit Pension Fund. *Sixth AFIR Colloquium*, 593-607.

Boulier, J., E. Trussant and D. Florens 1995. A Dynamic Model For Pension Funds Management. *Fifth AFIR Colloquium*, 361-384.

Brooks, S.M. and R.K. Weaver. 2006. Lashed to the Mast? The Politics of NDC Pension Reform. *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*, R. Holzmann and E. Palmer, Eds. The World Bank, Washington, D.C., 345-385.

Brown, R.L. 2006. Actuarial Issues In The Design of An Optimal Social Security System. http://www.actuaries.org/PBSS/Colloquia/Helsinki/Home_EN.cfm.

-----1997. In Defense of Pay-As-You-Go (Paygo) Financing of Social Security. *North American Actuarial Journal* 1, 1-20.

-----1996. An Actuary's Perspective. *Social Security: What Role for the Future?* P.A. Diamond, D.C. Lindeman and H. Young, Eds. National Academy of Social Insurance, Washington, D.C., 21-43.

Buiter, W.H. 1977. Generational Accounts, Aggregate Saving and Intergenerational Distribution. *Economica* 64, 605-626.

Cadenillas, A., T. Choulli, M. Taksar and L. Zhang. 2006. Classical And Impulse Stochastic Control For The Optimization Of The Dividend And Risk Policies Of An Insurance Firm. *Mathematical Finance* 16, 181-202.

Cairns, A.J.G. 1996. Continuous-Time Pension-Fund Modelling. *Proceedings of the 6th AFIR Colloquium* 1, 609-624.

-----, 1995. Pension funding in a stochastic environment: the role of objectives in selecting an asset allocation strategy. *Proceedings of the 5th AFIR International Symposium 1*, 429-453.

Canadian Institute of Actuaries. 2008. *Report on Canadian Economic Statistics 1924 - 2007*. Canadian Institute of Actuaries, Ottawa, Canada.

-----, 1995. *Troubled Tomorrows - The Report of the Canadian Institute of Actuaries' Task Force on Retirement Savings*. Canadian Institute of Actuaries, Ottawa, Canada.

Casey, B.H. 2004. Evaluating Pension Reform. *Scandinavian Insurance Quarterly*, 111-121.

Chen, Y. and S.C. Goss. 1997. Are Returns On Payroll Taxes Fair? *Social Security in the 21st Century*, E.R. Kingson and J.H. Schulz, Eds. Oxford University Press, New York, New York, U.S.A., 76-90.

Cichon, M. 2005. Balanced Notional Defined Contribution Schemes: A new “geist” in old bottles? *Scandinavian Insurance Quarterly* 2005, 174-190.

Cichon, M., W. Scholz, A. Van De Meerendonk, K. Hagemeyer, F. Bertranou and P. Plamondon. 2004. *Financing Social Protection* International Labour Organization, Geneva, 663.

Cogan, J.F. and O.S. Mitchell. 2003. Perspectives from the President’s Commission on Social Security Reform. *The Journal of Economic Perspectives* 17, 149-172.

Congressional Budget Office. 1999. *Social Security Privatization: Experiences Abroad*. Washington, D.C.

-----, 1998. *Social Security And Private Saving: A Review Of The Empirical Evidence*. Washington, D.C.

Cox Edwards, A. 2001. *Social Security Reform and Women’s Pensions*. The World Bank Development Research Group Poverty Reduction and Economic Management Network, Washington, D.C.

Cruz-Saco, M.A. and C. Mesa-Lago, Eds. 1998. *Do Options Exist? The Reform Of Pension And Health Care Systems In Latin America*. University of Pittsburgh Press, Pittsburgh, PA.

Cui, J., F. de Jong and E. Ponds. 2005. The value of intergenerational transfers within funded pension schemes. International Centre For Pension Management Discussion Paper. Toronto, Canada.

Cutler, D.M. 1993. Generational Accounting: Knowing Who Pays, and When, for What We Spend. *National Tax Journal* 46, 61-67.

Daykin, C. 2007. The Implications Of Longevity For Risk-Sharing In Public And Private Pension Schemes. PowerPoint Presentation. Helsinki, Finland.

-----, 2006. *The Challenge of Ageing: Pension Reform, International Trends and Future Imperatives*. Politeia Social Science Research Series No. 9, Surrey, United Kingdom.

De Menil, G., F. Murtin and E. Sheshinski. 2006. Planning for the optimal mix of paygo tax and funded savings. *Journal of Pension Economics and Finance* 5, 1-25.

Department for Work and Pensions. 2006. Security in Retirement: Towards a New Pensions System. The Stationary Office, Norwich, United Kingdom.

Diamond, P. 1996. Generational accounts and generational balance: An assessment. *National Tax Journal* 49, 597-607.

Ehrentraut, O., M. Heidler and B. Raffelhuschen. 2005. En Route to Sustainability: History, Status Quo and Future Reforms of the German Public Pension Scheme. *Intereconomics* 40(5), 254-257.

Enders, W. and H.E. Lapan. 1982. Social Security Taxation and Intergenerational Risk Sharing. *International Economic Review* 23, 647-658.

Favreault, M. M., J.H. Goldwyn, K.F. Smith, L.H. Thompson, C.E. Uccello and S.R. Zedlewski. 2004. *Reform Model Two Of The President's Commission To Strengthen Social Security: Distributional Outcomes Under Different Economic And Behavioral Assumptions*. Center for Retirement Research at Boston College, Chestnut Hill, MA.

Fehr, H. and C. Haberman. 2004. *Pension Reform and Demographic Uncertainty: The Case of Germany*. University of Wuerzburg, Wuerzburg, Germany.

Feinberg, E.A. and A. Shwartz. 1999. Constrained Dynamic Programming with Two Discount Factors: Applications and an Algorithm. *Transactions On Automatic Control* 44, 628-631.

-----, 1994. Markov Decision Models with Weighted Discounted Criteria. *Mathematics of Operations Research* 1, 152-168.

Feldstein, M. 1974. Social Security, Induced Retirement, and Aggregate Capital Accumulation. *The Journal of Political Economy* 82, 905-926.

Flemming, J.S. 1977. Optimal Payroll Taxes And Social Security Funding: Some transitional problems for a closed economy. *Journal of Public Economics* 7, 329-349.

Gokhale, J. 2007. The Connection between Wage Growth and Social Security's Financial Condition. *Policy Analysis*, 1-16.

-----, 1998. *Generational Equity and Sustainability in U.S. Fiscal Policy*. Federal Reserve Bank of Cleveland, Cleveland, OH.

Gordon, R.J. 1999. U.S. Economic Growth Since 1870: One Big Wave? *The American Economic Review* 89, 123-128.

Goss, S.C. 1996. Comment. *Social Security: What Role for the Future?* P.A. Diamond, D.C. Lindeman and H. Young, Eds. National Academy of Social Insurance, Washington, D.C., 67-78.

Government of Canada. 1997. Canada Pension Plan Act. 113.1-118 (2).

Gramlich, E.M. 2004. Rules for Assessing Social Security Reform. <http://www.federalreserve.gov/BoardDocs/Speeches/2004/20040812/default.htm>

Greg A. and B. Wasow. 2004. *Twelve Reasons Why Privatizing Social Security is a Bad Idea*. The Century Foundation.

-----, 2001. *What Would REALLY Happen Under Social Security Privatization? Part III: IRAs and 401(k)s You Can Not Control or Leave to Heirs*. The Social Security Network, New York, NY.

Guerin, J. and F. Legros. 2002. Neutralite Actuarielle: Un Concept Elegant Mais Delicat A Mettre En Oeuvre. *Revue D'Economie Financiere* 14, 1-12.

Hagberg, J. 2006. How NDC Social Security changes the balance of risk - the Swedish example. Unpublished Paper. Sweden.

- Hagberg, J. and E. Wohlner. 2002. The Market for Social Insecurity: A shady pension reform entices economic elites and clouds the future of Sweden's elderly. *Scandinavian Insurance Quarterly* 2002, 333-340.
- Holzmann, R. 2006. Toward A Coordinated Pension System In Europe. *Pension reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*, R. Holzmann and E. Palmer, Eds. The World Bank, Washington, D.C., 225-265.
- Holzmann, R. and R. Hinz. 2005. *Old-Age Income Support in the 21st Century: An International Perspective on Pension Systems and Reform*. The World Bank, Washington, D.C., 232.
- Holzmann, R. and E. Palmer Eds. 2006. *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*. The World Bank, Washington, D.C.
- Hu, S.C. 1979. Social Security, the Supply of Labor, and Capital Accumulation. *The American Economic Review* 69, 274-283.
- International Labour Organization. 1952. *Convention No. 102 Convention concerning Minimum Standards of Social Security*. International Labour Organization, Geneva, Switzerland.
- International Social Security Association. 2003. *Ageing and Social Security: Ten Key Issues*. International Social Security Association, Geneva.
- James, E., A.C. Edwards and R. Wong. 2003. The gender impact of pension reform. *Journal of Pension Economics and Finance* 2, 181-219.
- King, I. 2002. *A Simple Introduction to Dynamic Programming in Macroeconomic Models*. University of Auckland, Auckland, New Zealand.
- Konberg, B. 2004. The New Swedish Pension System – a Fair and Sustainable Model. *Scandinavian Insurance Quarterly* 2004, 15-20.
- Kotlikoff, L.J. 1997. Reply To Diamond's and Cutler's Reviews of Generational Accounting. *National Tax Journal* 50, 303-314.
- Kotlikoff, L.J. and B. Raffelhuschen, B. 1999. Generational Accounting Around the Globe. *The American Economic Review* 89, 161-166.

Kremer, M. and D. Chen. 1999. Income-Distribution Dynamics with Endogenous Fertility. *The American Economic Review* 89, 155-160.

Kreps, D.M. 1990. *A Course in Microeconomic Theory*. Princetown University Press, Princeton, New Jersey, 850.

Lang, A.C. 2000. Saving Social Security, The World...and Your Souls. PowerPoint Presentation, U.S.A.

Legros, F. 2006. NDCs: A Comparison of the French and German Point Systems. *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*, R. Holzmann and E. Palmer, Eds. The World Bank, Washington, D.C., 203-222.

Letzner, P. and O. Tippelmann. 2004. A German perspective of the new Swedish public pension system. *Scandinavian Insurance Quarterly*, 315-325.

Levine, P.B., O.S. Mitchell and J.W. Phillips. 1999. *Worklife Determinants Of Retirement Income Differentials Between Men And Women*. National Bureau Of Economic Research, Cambridge, MA.

Levine, R. 1999. Law, Finance, and Economic Growth. *Journal of Financial Intermediation* 8, 8-35.

Lindert, P.H. 2004. *Growing public: social spending and economic growth since the eighteenth century* 1. Cambridge, Cambridge, UK; New York, 377.

Little, B. 2008. Fixing the Future: How Canada's usually fractious governments worked together to rescue the Canada Pension Plan. Canada.

Lundberg, K., D. Mikula and O. Settergren. 2007. *Distribution of Surpluses, and the Role of Bookkeeping in the Swedish Public Pay-As-You-Go Pension Scheme*. Swedish Social Insurance Agency, Sweden.

Marin, B. 2006. A Magic All-European Pension Reform Formula: Selective Comments. *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*, R. Holzmann and E. Palmer, Eds. The World Bank, Washington, D.C., 266-292.

- Marmor, T.R. and J.L. Mashaw. 2006. Understanding Social Insurance: Fairness, Affordability, And The 'Modernization' Of Social Security And Medicare. *Health Affairs* 25, w114-w134.
- McCandless, Jr., T.G. and N. Wallace. 1991. *Introduction to Macroeconomic Theory: An Overlapping Generations Approach*. Harvard University Press, Cambridge, Massachusetts; London, England; 372.
- McGillivray, W.R. 2005. Reflections on Notional Defined Contributions Public Pension Schemes. *Scandinavian Insurance Quarterly*, 219-228.
- Mehle, R.W. 2001. Statement By The Honorable Roger W. Mehle, Executive Director Federal Thrift Investment Board Before The President's Commission To Strengthen Social Security. The President's Commission To Strengthen Social Security, Washington, D.C. www.csss.gov.
- Mesa-Lago, C. 1989. *Ascent to Bankruptcy: Financing Social Security in Latin America*. University of Pittsburgh Press, Pittsburgh, PA, 290.
- , 1978. *Social Security in Latin America: Pressure Groups, Stratification, and Inequality*. University of Pittsburgh Press, Pittsburgh, PA, 212.
- , 1994. *Changing Social Security in Latin America: Toward Alleviating the Social Costs of Economic Reform*. Lynne Rienner Publishers, Inc., Boulder & London, 351.
- Mitchell, O.S. and S.P. Zeldes. 1996. Social Security Privatization: A Structure for Analysis. *The American Economic Review* 86, 363-367.
- Miyazato, N. 2004. Pension Reform in Sweden and Implications for Japan. *The Japanese Journal of Social Security Policy* 3, 10-16.
- Munnell, A. 2001. Strengthening Social Security: Testimony before The President's Commission to Strengthen Social Security. The President's Commission To Strengthen Social Security. Washington, D.C. www.csss.gov.
- Nataraj, S. and J.B. Shoven. 2003. Comparing the Risks of Social Security with and without Individual Accounts. *The American Economic Review* 93, 348-353.
- Oeppen, J. and J.W. Vaupel. 2002. Broken limits to Life Expectancy. *Science* 296, 1029-1031.

Office of the Chief Actuary. 2007a. Actuarial Report on the Canada Pension Plan as at December 31, 2006. Minister of Public Works and Governmental Services, Ottawa, Canada.

----- . 2007b. Optimal Funding of the Canada Pension Plan: Actuarial Study No. 6. Office of the Superintendent of Financial Institutions Canada, Ottawa, Canada.

Ono, M. 2007. Applying Swedish "Automatic Balance Mechanism" to Japanese Population. Unpublished Paper. Tokyo, Japan.

Packard, T.G. 2002. Are there positive incentives from privatizing social security? A panel analysis of pension reform in Latin America. *Journal of Pension Economics and Finance* 1, 89-109.

Palacios, R. and R. Rofman. 2001. *Annuity markets and benefit design in multipillar pension schemes: Experience and lessons from four Latin America countries*. The World Bank, Washington, D.C.

Palmer, E. 2005. The Way Forward for European Pension Schemes. *Intereconomics* 40(5), 268-272.

Penner, R.G. 2007. *International Perspectives on Social Security Reform*. The Urban Institute Press, Washinton, D.C.

Phelps, E. 1961. The Golden Rule of Accumulation: A Fable for Growthmen. *The American Economic Review* 51, 638-643.

Pinera, J. 2006. *Chile's Social Security Lesson For The U.S.* Cato Institute, Washington, D.C.

Pinheiro, V.C. 1999. A Nova Regra de Calculo dos Beneficios: o Fator Previdenciario. *Informe de Previdencia Social* 11, 1-16.

Plamondon, P., A. Drouin, G. Binet, M. Cichon, W. McGillivray, M. Bédard, and H. Perez-Montas. 2002. *Actuarial practice in social security*. International Labour Organization, Geneva, 513.

President's Commission to Strengthen Social Security. 2001. *Strengthening Social Security and Creating Personal Wealth for All Americans*. U.S. Government Press, Washington, D.C. www.csss.gov.

- Queisser, M. and E.R. Whitehouse. 2006. Neutral or Fair? Actuarial Concepts in Pension-System Design. Social, Economic and Migration Working Paper No. 40, OECD, Paris, 1-40.
- Raffelhuschen, B. 1999. Generational Accounting in Europe. *The American Economic Review* 89, 167-170.
- Reno, V.P. and J. Lavery. 2007. Social Security and Retirement Adequacy. *Social Security Brief*, 1-12.
- Romer, D. 1996. *Advanced Macroeconomics*. The McGraw-Hill Companies Inc., New York; St. Louis; San Francisco; 540.
- Sakamoto, J. 2008. *Roles of the Social Security Pension Schemes and the Minimum Benefit Level under the Automatic Balancing Mechanism*. 125. Nomura Research Institute, Japan.
- . 2005. *Japan's Pension Reform*. The World Bank, Washington, D.C.
- Samuelson, P.A. 1975. The Optimum Growth Rate for Population. *International Economic Review* 16, 531-538.
- . 1975. Optimum Social Security in a Life-Cycle Growth Model. *International Economic Review* 16, 539-544.
- . 1958. An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money. *The Journal of Political Economy* 66, 467-482.
- Sargent, T.J. 1987. *Dynamic Macroeconomic Theory*. Harvard University Press, Cambridge, Massachusetts; London, England; 369.
- Scarth, W. 2007. *A Note on Income Distribution and Growth*. McMaster University, Hamilton, Canada.
- Scherman, K.G. 2007. The Swedish NDC system - A critical assessment. PowerPoint Presentation. Helsinki, Finland.
- . 2004. The pension reform debate. *Scandinavian Insurance Quarterly* 2004, 97-98.

-----, 2003. The Swedish pension reform: a good model for other countries? *Scandinavian Insurance Quarterly*, 304.

Schieber, S.J. 2001. Social Security: Budgetary Tradeoffs and Transition Costs, Testimony before the President's Commission to Strengthen Social Security. The President's Commission To Strengthen Social Security, Washington, DC. www.csss.gov.

Scholz, W., M. Cichon and K. Hagemeyer. 2000. *Social Budgeting* International Labour Organization, Geneva, 328.

Scweizer, M. 2001. From actuarial to financial valuation principles. *Insurance: Mathematics and Economics* 28, 31-47.

Settergren, O. 2003. Financial and Inter-generational Balance? An introduction to how the new Swedish pension system manages conflicting ambitions. *Scandinavian Insurance Quarterly*, 99.

Sherraden, M. 2001. Assets and the Poor: Implications for Individual Accounts and Social Security, Invited Testimony to the President's Commission on Social Security, The President's Commission To Strengthen Social Security, Washington, D.C. www.csss.gov.

Shiller, R.J. 2003. The Implications Of Financial And Nonfinancial Risk For Public Pension Reform: Social Security and Individual Accounts as Elements of Overall Risk-Sharing. *The American Economic Review* 93, 343-347.

Sinn, H. 2000. Why a Funded System is Useful and Why it is Not. *Journal of International Tax and Public Finance* 7, 389-410.

Smith, G. 2000. Macroeconomics Lecture Notes. 124-212.

Solow, R.M. 1956. A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics* 70, 65-74.

Sombrotto, V.R. 2001. Testimony of Vincent R. Sombrotto, President of the National Association of Letter Carriers, AFL-CIO Before the President's Commission to Strengthen Social Security. The President's Commission To Strengthen Social Security, Washington, D.C. www.csss.gov.

Steuerle, C.E. 2001. Major Issues In Social Security Reform, Statement before the President's Commission to Strengthen Social Security. The President's Commission to Strengthen Social Security, Washington, D.C. www.csss.gov.

Takayama, N. and Y. Kitamura. 1999. Lessons from Generational Accounting in Japan. *The American Economic Review* 89, 171-175.

Taylor, G. 2002. Stochastic control of funding systems. *Insurance: Mathematics and Economics* 30, 323-350.

Thogersen, O. 2006. Intergenerational Risk Sharing by Means of Pay-as-you-go Programs – An Investigation of Alternative Mechanisms. CESifo Working Paper No. 1759, Category 1: Public Finance. Bergen, Norway.

Thompson, L.H. 2005. Establishing Basic Pension Parameters. *Workable Pension Systems: Reforms in the Caribbean*. P. Desmond Brunton and Pietro Masci, Eds. Inter-American Development Bank, Washington, D.C. and Caribbean Development Bank, St. Michael, Barbados, W.I., 415-434.

TIAA-CREF. 2006. Focusing on You: TIAA-CREF 2005 Annual Report. TIAA-CREF, New York, U.S.A.

Toft, C. 2007. The Transformation of the German Public Pension System and the Future Pension Benefit Level. Paper. Helsinki, Finland. www.uni-kassel.de/~toft/.

Turner, J.A. 2007. Autopilot: Self-Adjusting Mechanisms for Sustainable Retirement Systems. <http://retirement2020.soa.org/self-adjustment-mechanisms8.pdf>.

-----, 2008. Sustainable Social Security: Automatic Adjustments for Restoring Solvency. Private Communication.

U.S. Department of the Treasury. 2008. *Social Security Reform: Benchmarks For Assessing Fairness And Benefit Adequacy*. 3. U.S. Department of the Treasury, Washington, D.C.

-----, 2007. *Social Security Reform: A Framework For Analysis*. 2. U.S. Department of the Treasury, Washington, D.C.

Valdes-Prieto, S. 2006. A Market Method to Endow NDC Systems with Automatic Financial Stability. *Pension Reform: Issues and Prospects for Non-Financial Defined*

Contribution (NDC) Schemes, R. Holzmann and E. Palmer, Eds. The World Bank, Washington, D.C., 149-164.

-----, 2000. The Financial Stability of Notional Account Pensions. *Scandinavian Journal of Economics* 102, 395-417.

Vidal-Melia, C., M. del Carmen Boado-Penas and O. Settegren. 2008. Automatic Balance Mechanisms In Pay-As-You-Go Pension Systems. <http://ssrn.com/abstract=1132686>

Wagener, A. 2003. Pensions as a portfolio problem: fixed contribution rates vs. fixed replacement rates reconsidered. *Journal of Population Economics* 16, 111-134.

Walker, A. 2003. The Policy Challenges of Population Ageing. SEDAP Research Paper No. 108. McMaster University, Hamilton, Canada.

Weicher, J.C. 1999. Some Income-Measurement Issues and Their Policy Implications. *The American Economic Review* 89, 29-33.

Weil, D.N. 1999. Why Has Fertility Fallen Below Replacement in Industrial Nations, and Will it Last? Population Growth, Dependency, and Consumption. *The American Economic Review* 89, 251-255.

Whitehouse, E. 2007a. Life-Expectancy Risk and Pensions: Who Bears the Burden? OECD Social Employment and Migration Working Papers No. 60. Paris, France.

-----, 2007b. Pensions Panorama. The World Bank. Washington, D.C., 234.

-----, 2006. New indicators of 30 OECD countries' pension systems. *Journal of Pension Economics and Finance* 5, 275-298.

Williamson, J.B. 2005. *An Update on Chile's Experience with Partial Privatization and Individual Accounts*. AARP, Washington, D.C.

Williamson, J.B., D.M. Watts-Roy and E.R. Kingson, Eds. 1999. *The Generational Equity Debate*. Columbia University Press, New York. U.S.A.

Williamson, J.B. and M. Williams. 2003. *The Notional Defined Contribution Model: An Assessment Of The Strengths And Limitations Of A New Approach To The Provision Of Old Age Security*. Center for Retirement Research at Boston College, Chestnut Hill, MA.

Wolf, J.A. 2001. Testimony of James A. Wolf President, TIAA-CREF Retirement Services before the President's Commission to Strengthen Social Security. The President's Commission To Strengthen Social Security, Washington, D.C. www.csss.gov.

World Bank. 1994. *Averting The Old Age Crisis*. Oxford University Press, Inc., New York, NY, 402.