Analysis of Islamic Stock Indices

by

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A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Master of Mathematics
in
Actuarial Science

Waterloo, Ontario, Canada, 2009

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

In this thesis, an attempt is made to build on the quantitative research in the field of Islamic finance. Firstly, univariate modelling using special GARCH-type models is performed on both the FTSE All World and FTSE *Shari'ah* All World indices. The AR(1) + APARCH(1,1) model with standardized skewed student-t innovations provided the best overall fit and was the most successful at VaR modelling for long and short trading positions. A risk assessment is done using the Conditional Tail Expectation (CTE) risk measure which concluded that in short trading positions the FTSE *Shari'ah* All World index was riskier than the FTSE All World index but, in long trading positions the results were not conclusive as to which is riskier. Secondly, under the Markowitz model of risk and return the performance of Islamic equity is compared to conventional equity using various Dow Jones indices. The results indicated that even though the Islamic portfolio is relatively less diversified than the conventional portfolio, due to several investment restrictions, the *Shari'ah* screening process excluded various industries whose absence resulted in risk reduction. As a result, the Islamic portfolio provided a basket of stocks with special and favourable risk characteristics. Lastly, copulas are used to model the dependency structure between the filtered returns of the FTSE All World and FTSE *Shari'ah* All World indices after fitting the AR(1) + APARCH(1,1) model with standardized skewed student-t innovations. The t copula outperformed the others and a demonstration of forecasting using the copula-extended model is done.

Acknowledgements

Firstly, I would like to thank *Allah* for using me as a tool to impart the Islamic and mathematical knowledge present in this thesis. The good of this thesis comes from *Allah* alone and whatever wrong it may contain comes from myself for which I seek *Allah's* forgiveness.

Secondly, I would like to thank my parents Imtiaz and Fatima Mohammed for all the sacrifices they continue to make, to ensure I receive the best in this life and the life hereafter.

Last but not least, I would like to thank my supervisor, Ken Seng Tan, for his time, patience and advice which proved to be very helpful and also all those who have contributed in some form or other.

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Chapter 1

Introduction

From the inception of the first Islamic bank in Egypt in 1963 the concept of Islamic finance has indeed raised some eyebrows[1]. Recently, Islamic finance has been growing at a spectacular rate of more than 15% annually with Islamic banks from China to the United States of America managing funds to the tune of USD $200 billion. Standard and Poor's rating service believes the potential market for Islamic financial services is close to USD $4 trillion for 2008 and into the near future. The growth of Islamic finance has clearly outshone the growth found in conventional banks[2]. To compliment this growth, the Muslim population (currently 1.5 billion) is expected to reach 2.5 billion by 2020 (which will account for 30% of the world's population) and in the next few years 40%-50% of the Muslim population's total savings will be in the hands of Islamic banks (See The Banker (2007))[3]. To briefly introduce Islamic finance, two questions need to be answered: "Why Islamic finance?" and "What is Islamic finance?". Following, some research that has been conducted in this area is mentioned and the goal of this thesis is stated.

1.1 Why Islamic Finance?

To appreciate the answer to this question the thought process of a Muslim must be understood. Islam teaches Muslims that every aspect of their lives must be based on their religious teachings; these teachings are believed to be the direct orders from *Allah* (God) providing absolute guidance to human beings. It is believed that no

[1]Other developments include: First modern Islamic bank (Dubai Islamic Bank) founded in 1975, Accounting and auditing organization for Islamic financial institutions (AAOIFI) established in 1991, Islamic financial services board (IFSB) established in Malaysia in 2002.

[2]Overview of Islamic Finance - Grail Research (2007).

[3]Currently, the two largest Islamic banks are Al Rajhi and Kuwait Finance House whose Assets, Deposits and Net Profits in FY2006 were USD $49,491 million and USD $35,222 million respectively. ABN Amro, Barclays, BNP Paribas, Citi Islamic Investment Bank, Deutsche Bank, HSBC Amanah, Llyods TSB and Standard Chartered are some of the leading conventional banks that offer Islamic financial services worldwide.

question can be asked for which the *Qur'an* (religious scripture Muslims are commanded to follow) does not provide an answer and clear guidance on that subject matter. This is observed in the following *Qur'anic* quotations where *Allah* says:

This is the Book; in it is guidance sure without doubt to those who fear Allah. (Qur'an 2:2)

Certainly We have revealed the Book to you in Truth, for (instructing) mankind. He, then, that receives guidance benefits his own soul: But he that strays injures his own soul... (Qur'an 39:41).

As a result, the dealings of Muslims must always be in accordance with the religious teachings that are prescribed - including the way finance and business is conducted. Conventional financial etiquette fails to meet the Islamic *Shari'ah*[4] requirements on business and trade. Therefore, the need for an alternative system, which is *Shari'ah* compliant, is necessary. The next section explains how conventional financial etiquette contradicts the Islamic *Shari'ah* by highlighting the differences between Islamic and conventional finance. It should be noted that the ideology of the Islamic system emphasizes the need for ethical, moral, social and religious dimensions in order to promote equity and fairness across the society in a holistic manner. Hence, the system can only be fully appreciated in the context of Islamic teachings such as wealth distribution and social and economic justice.

1.2 What is Islamic Finance?

Firstly, to clear up any misconceptions about the legitimacy of business in Islam, *Allah* says in the *Qur'an*:

...Allah has permitted trade... (Qur'an 2:275)

...let there be amongst you traffic and trade by mutual good-will... (Qur'an 4:29).

Hence, business is permissible for Muslims. Simply put, Islamic finance is coined to financial activities that adhere to the Islamic *Shari'ah*. The major differences between Islamic and conventional finance sprout from the four fundamentals of Islamic finance which are: freedom from *Gharar* (excessive uncertainty), freedom from *Maysir* (gambling), freedom from unethical investments (any investment that goes against any *Shari'ah* principle. For example, investments in alcohol, pork, pornography and any other investment deemed unethical by the *Shari'ah*) and freedom from *Riba* (interest payment on debt, usury). Abundant detailed literature exists on each of these concepts (See Warde (2000) and El-Gamal (2006)).

[4]The laws by which Muslims adhere to which are derived from the *Qur'an* and the authentic sayings of Prophet Muhammad (May the peace and blessings of *Allah* be upon him).

A more detailed description of *Riba* is given in the next section since it is the most important of the four fundamentals. The restrictions imposed on Islamic financial activity results in differences among the types of contracts an Islamic financial system will allow and those offered in mainstream finance. Most conventional derivative instruments and insurance contracts will not be viable due to the presence of *Gharar*. A call option that is used purely for speculation on the price of the underlying contains elements of *Maysir* and would be unlawful in Islam. Also, interest payments on bonds and mortgages are not allowed since these contracts are not free from *Riba*. Given these restrictions, many Islamically-sound financial contracts have surfaced. *Murabahah* (cost-plus financing), *Musharakah* & *Mudarabah* (forms of partnerships), *Bay' as-Salam* (similar to conventional forward contracts), *Ijarah* (resembles a leasing agreement), *Sukuk* (Islamic bond) and *Takaaful* (Islamic insurance) are some of the common contracts (See Iqbal and Mirakhor (2007) for more details and other Islamic contracts).

1.3 *Riba*

Riba comes from the Arabic root word *raba*, meaning to increase. Many Islamic jurists define the forbidden *Riba* as trading two goods of the same kind in different quantities, where the increase is not a proper compensation. According to the *Shari'ah*, *Riba* is: "The premium that must be paid by the borrower to the lender along with the principal amount as a condition for the loan or for an extension in the duration of the loan." Four characteristics which define the prohibited interest rate are:

- It is positive and fixed *ex-ante*
- It is tied to the time period and the amount of the loan
- Its payment is guaranteed regardless of the outcome or the purposes for which the principal was borrowed
- The state apparatus sanctions and enforces its collection

The prohibition of *Riba* is found in several verses of the *Qur'an*:

Those who devour usury will not stand except as stands one whom the Evil One by his touch hath driven to madness. That is because they say: "Trade is like usury," but Allah hath permitted trade and forbidden usury. Those who after receiving direction from their Lord, desist, shall be pardoned for the past; their case is for Allah (to judge); but those who repeat (the offence) are companions of the fire; they will abide therein (forever). (Qur'an 2:275)

O ye who believe! Fear Allah, and give up what remains of your demand for usury, if ye are indeed believers. (Qur'an 2:278)

O ye who believe! Devour not usury, doubled and multiplied; but fear Allah; that ye may (really) prosper. (Qur'an 3:130)

That which ye lay out for increase through the property of (other) people, will have no increase with Allah: but that which ye lay out for charity, seeking the countenance of Allah, (will increase): it is these who will get a recompense multiplied. (Qur'an 30:39).

The rationale for the prohibition of *Riba* has been discussed by many Islamic scholars. On the economic side, some argue that the existence of *Riba* results in an uneven distribution of wealth throughout society by providing vehicles for the rich to get richer and the poor to get poorer. Others argue that the modern economic system has not yet supplied any justification for the existence of even the necessity of interest rates. On the spiritual side, some argue that human wisdom and understanding is limited when compared to the Knowledge of *Allah* as *Allah* says in the *Qur'an*:

It is He who hath created for you all things that are on earth; then He turned to the heaven and made them into seven firmaments. And of all things He hath perfect knowledge. (Qur'an 2:29).

Therefore, any argument posed to fully understand the rationale of the prohibition of *Riba* may never yield any optimal comprehension.

The prohibition of giving and receiving *Riba* is not confined to Islam, but extends to other religions and ideologies of the historic and modern world. In Judaism, the Old Testament clearly prohibited dealing with interest. It uses the Hebrew word *neshekh* which, similar to *Riba*, refers to any gain. The following are some quotations from the Old Testament that prohibit usury:

If thou lend money to any of my people that is poor by thee, thou shalt not be to him as an usurer, neither shalt thou lay upon him usury. (Exodus 22:25)

Take thou no usury of him, or increase: but fear thy God; that thy brother may live with thee. (Leviticus 25:36)

Thou shalt not lend upon usury to thy brother; usury of money, usury of victuals, usury of any thing that is lent upon usury. (Deuteronomy 23:19)

In Christianity, at the Council of *Nicaea* in 325, usury was made illegal. In 1179, Pope Alexander III excommunicated usurers, which was a very harsh punishment at that time. Some Biblical references that speak against usury include:

He that putteth not out his money to usury, nor taketh reward against the innocent. He that doeth these things shall never be moved. (Psalms 15:5)

He that hath not given forth upon usury, neither hath taken any increase, that hath withdrawn his hand from iniquity, hath executed true judgement between man and man... (Ezekiel 18:8)

In thee have they taken gifts to shed blood; thou hast taken usury and increase, and thou hast greedily gained of thy neighbours by extortion, and hast forgotten me, saith the Lord GOD. (Nehemiah 5:7)

In Hinduism, the *Vedic* texts mentions the *kusidin* (usurer) several times. Also, in the later *Sutra* texts more details and references to interest are found. Vasishtha, who was a well-known Hindu law-maker, made a law forbidding the higher castes of *Brahmanas* (Hindu priests) and *Kshatriyas* (warriors) from being usurers or lenders at interest. In Buddhism, the *Jatakas* refers to the giving and taking of interest in a disparaging manner saying: "hypocritical ascetics are accused of practicing it." Even great philosophers like Plato, Aristotle and Cato condemned usury. For example, Cato in his *De Re Rustica* said: "And what do you think of usury?" – "What do you think of murder?"

In modern economics, the legitimacy of interest is not questioned; rather it is considered as an integral part of the economic system. However, occassionally a few challenge its purpose. For example, a successful merchant in the early twentieth century named Silvio Gesell condemned interest. He complained that his sales were more dependent on the price of money (interest) than consumers' need or product quality. Also, Margrit Kennedy, a German economist, criticized the need for interest and called for interest-free and inflation-free money.

1.4 Brief Literature Review

Amidst the excitement and explosive growth enjoyed by the Islamic financial sector, there has been very little academic research. Numerous papers and books have been written which clearly explain the system of Islamic finance in a qualitative nature (See Elgari (2003), Obaidullah (1999), Rosly (2005), Archer and Karim (2007), Billah (2003), Iqbal and Mirakhor (2007)).

On the quantitative side, Hassan (2002) examines market efficiency and the time-varying risk-return relationship for the Dow Jones Islamic Market Index (DJIMI) from 1996 to 2000. He found the returns of the DJIMI were normally distributed with good market efficiency. Hakim and Rashidian (2004) examine the relationship between the DJIMI, Wilshire 5000 Index and the risk-free rate represented by the yield on the three month treasury bill from 1999 to 2002. They found the DJIMI was not correlated with neither the Wilshire 5000 Index nor the yield on the three month treasury bill which they concluded resulted in an Islamic index influenced by factors independent from the broad market or interest rates. Hussein (2004) investigates whether the returns of the FTSE Global Islamic Index is significantly different from the returns of the FTSE All World Index between 1996 and 2003. He finds the screening process undergone by the companies in the FTSE All World Index, before they qualify to be enlisted on the FTSE Global Islamic Index, has no negative impact on the performance of the FTSE Global Islamic Index. Al-Zoubi and Maghyereh (2007) compared the relative risk performances of the Dow Jones Islamic Index (DJIS) and the Dow Jones World Index (DJIM). They showed that the Value-at-Risk (VaR) is greater for DJIM World than for DJIS Islamic. This implied the DJIS Islamic was less risky than the broader market basket of stocks, DJIM World, when portfolio risk is measured by VaR.

1.5 My Contribution

In this thesis, an attempt is made to build on the quantitative research in the Islamic financial field. The data used are all equity-based, since other forms of Islamic financial data proved difficult to find. In Chapter 2, univariate financial time series analyses are done with focus on sufficiently modelling the long memory property of stock market returns. Model fit and performance are assessed with special attention given to the model's ability to be used for risk management purposes. In Chapter 3, a comparison between the performance of Islamic and conventional equity portfolios is done using a well-known risk-return model. Chapter 4 introduces the actuarial tool known as the copula which will be used to model the dependency structure between Islamic and conventional return series and Chapter 5 concludes this thesis.

Chapter 2

Univariate Modelling

2.1 Defining Value-at-Risk (VaR)

In this chapter, VaR estimates for long and short trading positions will be used to access model fit and investment risk. Hence, it is only appropriate that VaR be defined. VaR is the most widely used statistic in the world of financial risk management. It is popular because it summarizes the loss exposure in a given time horizon with some level of confidence, α, into a single number, VaR_α. Statistically, VaR_α for a given distribution is the associated theoretical quantile at $\alpha\%$ for that distribution. Mathematically,

$$VaR_{1-\alpha} = inf\,\{l \in \Re : F_L(l) \geq 1-\alpha\}$$

for short trading positions and

$$VaR_\alpha = sup\,\{l \in \Re : F_L(l) \leq \alpha\}$$

for long trading positions where L is a random variable with distribution function F_L.

2.2 Financial Time Series

A financial time series is a collection of periodic historical observations/changes of a certain risk factor. Log-return on equities, indices, exchange rates and commodity prices are by no means an exhaustive list of common risk factors. In this chapter, a variety of differenced logarithmic stock price series are used. For notational purposes let P_t represent the price of a given index at time t ($t = 0, 1, 2, \ldots, n$, where n is the number of observations for that index). It follows that $r_t = (\ln P_t - \ln P_{t-1})\,100$ is the compounded return (in %) at time t ($t = 1, 2, \ldots, n-1$) for the price index defined by the stochastic process P_t. All preceding analysis is done on the logarithmic return series, r_t, with granularity set as daily.

2.2.1 Empirical Analysis of Financial Time Series

Extensive research has led to the discovery of many special characteristics of financial time series (See Taylor (1986), Alexander (2001), Tsay (2002) and Zivot and Wang (2003)). In the econometric world, these special characteristics have become so famous they are referred to as *stylized facts*. These *stylized facts* include:

- r_t exhibits little serial correlation but is not independent and identically distributed (iid)
- $|r_t|$ or r_t^2 exhibit noticeable serial correlation
- Conditional expected returns are close to zero
- Volatility is not constant over time, it varies
- r_t is leptokurtic
- Extreme values of r_t appear in clusters

A variety of modelling techniques have been developed which focus on capturing the effects of these *stylized facts*. The most commonly referred to process used in modelling financial time series is undoubtedly the autoregressive conditional heteroscedastic (ARCH) process originally proposed by Engle (1982). Bollerslev (1986) later generalized the ARCH process to the generalized autoregressive conditional heteroscedastic (GARCH) process. These processes together with various extensions and/or variations performed exceedingly well in modeling the *stylized facts* present in financial return series (See Bollerslev, Engle and Nelson (1994), Ding, Granger and Engle (1993), Bhattacharyya *et al* (2008), Shephard (1996), Glosten, Jagannathan and Runkle (1993), Fornari and Mele (1997), Yang, Härdle and Nielsen (1999) and Bühlmann and McNeil (2002)).

2.2.2 Statistical Analysis of Financial Time Series

The procedure of Giot and Laurent (2003) will be implemented to fit the univariate models to the financial return series and is as follows:

1. ***Preliminary Data Analysis***. A variety of descriptive statistics and diagrams are formulated for a given data set. They are used to uncover the properties (paying close attention to the *stylized facts* of financial time series) embedded in the data so that appropriate model selection takes place.

2. ***Model Estimation***. With guidance from the previous step, a variety of models are fit to the data and their parameters estimated.

3. ***Model Performance***. Each model's performance in modelling the financial return time series is assessed by standard misspecification tests together with its accuracy in forecasting the one-day-ahead long and short VaR. The latter becomes particularly important from a risk management point of view.

2.3 The Data

Daily prices for the FTSE All World (FTSEAW) and FTSE *Shari'ah* All World (FTSESAW) indices were collected from 22 September 2003 to 22 January 2009. This gave 1393 realizations of P_t which were transformed by (2.1) to give 1392 log return observations[1].

2.3.1 Methodology of FTSESAW

The FTSESAW selection universe is the large and mid-cap stocks of the FTSEAW index. These securities undergo a screening process to determine whether the company is acceptable as an Islamic investment. The screening process is two-fold. Firstly, any companies whose core business activities involve alcohol, pork-related products, conventional financial services or entertainment (night clubs, casinos, pornography, etc.) are excluded. In addition, defense and weapons and tobacco or tobacco-related products are excluded, even though they are not strictly forbidden under the *Shari'ah*. After the ethical cleansing of companies that profit from unislamic business activities, the remaining companies are screened using several financial ratios. This process is intended to filter out companies with unacceptable debt levels and/or those who earn impure interest (*Riba*) income. According to the companies' financial ratios the company must have:

- Debt less than 33% of total assests
- Cash and interest bearing items less than 33% of total assets
- Accounts receivable and cash less than 50% of total assets
- Total interest and non compliant activities income below 5% of total revenue

The FTSESAW index has been fully certified as *Shari'ah* compliant through the issue of a *Fatwa* (Islamic legal opinion) by the Yasaar Research Inc. scholars who represent all of the major *Shari'ah* schools of thought.

2.4 Presenting the Models

As stated before, the financial return time series to be modelled is given by,

$$r_t = (\ln P_t - \ln P_{t-1})\, 100,\ t = 1, 2, \ldots, n-1 \tag{2.1}$$

where P_t for $t = 0, 1, 2, \ldots, n$ represents the price index. Previous research, as discussed earlier, has highlighted several *stylized facts* akin to financial return time

[1]For details on FTSE indexes see http://www.ftse.com/Indices.

series. One important fact is that they exhibit serial autocorrelation in the first and second moments. As a result, an initial AR(p) structure is fitted to capture the serial autocorrelation in the first moment. Therefore,

$$\Phi(L)(r_t - \mu) = \epsilon_t \tag{2.2}$$

where $\Phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p$ is an AR lag polynomial of order p with backshift operator L. The resulting conditional mean μ_t is given by

$$\mu_t = \mu + \sum_{j=1}^{p} \phi_j (r_{t-j} - \mu). \tag{2.3}$$

To model ϵ_t we consider the RiskMetrics model (See J.P. Morgan (1996)) and the Asymmetric Power ARCH (APARCH) model (See Ding *et al* (1993)) with normal, student-t and skewed student-t innovations. These models are introduced below including their associated VaR computations since their ability to forecast the one-day-ahead VaR in both long and short trading positions is used for model assessment.

RiskMetrics. This model is equivalent to an Integrated GARCH (IGARCH) model with fixed ARCH and GARCH coefficients. In this specification

$$\epsilon_t = \sigma_t z_t \tag{2.4}$$

where z_t is iid N(0,1) and

$$\sigma_t^2 = (1 - \lambda)\,\epsilon_{t-1}^2 + \lambda \sigma_{t-1}^2 \tag{2.5}$$

with $\lambda = 0.94$ for daily data. Let z_α and $z_{1-\alpha}$ represent the left and right quantiles of the normal distribution at $\alpha\%$, respectively. Then the one-day-ahead VaR computed in $t-1$ is $\mu_t + z_\alpha \sigma_t$ for long trading positions and $\mu_t + z_{1-\alpha}\sigma_t$ for short trading positions where the conditional standard deviaiton is given by (2.5).

Normal APARCH. One extension of the GARCH model is the normal APARCH which encompasses several GARCH specifications. A normal APARCH(p, q) model is given by (2.4) where z_t is iid N(0,1) and

$$\sigma_t^\delta = \omega + \sum_{i=1}^{q} \alpha_i \left(|\epsilon_{t-i}| - \gamma_i \epsilon_{t-i}\right)^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta. \tag{2.6}$$

ω ($\omega > 0$), α_i ($\alpha_i \geq 0$ for $i = 1, 2, \ldots, q$), γ_i ($-1 < \gamma_i < 1$ for $i = 1, 2, \ldots, q$), β_j ($\beta_j \geq 0$ for $j = 1, 2, \ldots, p$) and δ ($\delta \geq 0$) are new parameters. Here, δ plays the role of a Box-Cox transformation of σ_t while γ_i reflects the so-called leverage effect. In practice, low-order GARCH models achieve the same effect in modelling the *stylized facts* as high-order ARCH models in a more parsimonious manner. As a result the APARCH(1,1) model is considered where

$$\sigma_t^\delta = \omega + \alpha_1 \left(|\epsilon_{t-1}| - \gamma_1 \epsilon_{t-1}\right)^\delta + \beta_1 \sigma_{t-1}^\delta. \tag{2.7}$$

It has been found that a positive (negative) value of γ_1 suggests that past negative (positive) shocks have a more significant impact on current conditional volatility than past positive (negative) shocks (See Black(1976)). For a more extensive discussion on the properties of APARCH models see He and Teräsvirta (1999a,b). Let z_α and $z_{1-\alpha}$ represent the left and right quantiles of the normal distribution at $\alpha\%$, respectively. Then the one-day-ahead VaR computed in $t-1$ is $\mu_t + z_\alpha \sigma_t$ for long trading positions and $\mu_t + z_{1-\alpha}\sigma_t$ for short trading positions where the conditional standard deviaiton is given by (2.7).

Student-t APARCH. It is well known that financial return time series data is leptokurtic (fat-tailed) and therefore the thin-tailed normal distribution provides a poor fit to the distribution tails. This has serious risk management implications where the distribution tails are of extreme importance, for example when modelling VaR. To correct for this, consider the student-t APARCH(1,1) given by (2.4) where z_t is iid $t(0,1,\nu)$ and σ_t defined as in (2.7). Let $st_{\alpha,\nu}$ and $st_{1-\alpha,\nu}$ represent the left and right quantiles of the standardized student-t distribution with estimated degrees of freedom ν at $\alpha\%$, respectively. Then the one-day-ahead VaR computed in $t-1$ is $\mu_t + st_{\alpha,\nu}\sigma_t$ for long trading positions and $\mu_t + st_{1-\alpha,\nu}\sigma_t$ for short trading positions where the conditional standard deviaiton is given by (2.7).

Skewed Student-t APARCH. Together with excess kurtosis, financial return time series data also exhibit skewness. To facilitate this asymmetric characteristic the skewed student-t density of Lambert and Laurent (2001) is used. The skewed student-t APARCH is given by (2.4) where σ_t is defined by (2.7) and the innovations z_t is iid standardized skewed student-t distributed with

$$f\left(z|\xi,\nu\right) = \begin{cases} \frac{2}{\xi+\frac{1}{\xi}} sg\left[\xi\left(sz+m\right)|\nu\right] & \text{if } z < -\frac{m}{s}, \\ \frac{2}{\xi+\frac{1}{\xi}} sg\left[\frac{sz+m}{\xi}|\nu\right] & \text{if } z \geq -\frac{m}{s}, \end{cases} \tag{2.8}$$

where $g[\cdot|\nu]$ is the unit variance symmetrical student-t density, ξ ($\xi > 0$) is the asymmetry coefficient defined such that the ratio of probability masses above and below the mean is

$$\xi^2 = \frac{Pr\left(z \geq 0|\xi\right)}{Pr\left(z < 0|\xi\right)}, \tag{2.9}$$

m is the mean of the non-standardized skewed student-t distribution given by

$$m = \frac{\Gamma\left(\frac{\nu-1}{2}\right)\sqrt{\nu-2}}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(\xi - \frac{1}{\xi}\right) \tag{2.10}$$

and s^2 is the variance of the non-standardized skewed student-t distribution given by

$$s^2 = \left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2. \tag{2.11}$$

In the model estimation procedure, $\log\xi$ is estimated since its sign reveals the direction of skewness[2]. The third moment is positive (negative) and the density is

[2]Since $f\left(z_t|\frac{1}{\xi},\nu\right) = f\left(-z_t|\xi,\nu\right)$ with respect to the zero mean.

skewed to the right (left) if $\log \xi > 0$ (< 0). The quantile function $skst^*_{\alpha,\nu,\xi}$ of a non-standardized skewed student-t density as given in Lambert and Laurent (2000) is

$$skst^*_{\alpha,\nu,\xi} = \begin{cases} \frac{1}{\xi} st_{\alpha,\nu} \left[\frac{\alpha}{2}\left(1+\xi^2\right)\right] & \text{if } \alpha < \frac{1}{1+\xi^2}, \\ -\xi st_{\alpha,\nu} \left[\frac{1-\alpha}{2}\left(1+\xi^{-2}\right)\right] & \text{if } \alpha \geq \frac{1}{1+\xi^2}, \end{cases} \tag{2.12}$$

where $st_{\alpha,\nu}$ is the unit variance student-t density quantile function. Thus, the standardized skewed student-t quantile function becomes

$$skst_{\alpha,\nu,\xi} = \frac{skst^*_{\alpha,\nu,\xi} - m}{s}. \tag{2.13}$$

Let $skst_{\alpha,\nu,\xi}$ and $skst_{1-\alpha,\nu,\xi}$ represent the left and right quantiles of the standardized skewed student-t distribution with ν degrees of freedom and asymmetry coefficient ξ at α%, respectively. Then the one-day-ahead VaR computed in $t-1$ is $\mu_t + skst_{\alpha,\nu,\xi}\sigma_t$ for long trading positions and $\mu_t + skst_{1-\alpha,\nu,\xi}\sigma_t$ for short trading positions where the conditional standard deviaiton is given by (2.7).

2.5 Estimation and Analysis

The 3-step procedure outlined in Section 2.2.2 is performed on the FTSEAW and FTSESAW indices. After model fitting and performance assessment the best model is selected for each series. The analysis was done using the G@RCH 5.1 module of OxMetrics 5.1 Enterprise (See Laurent and Peters (2002)).

2.5.1 Preliminary Data Analysis

Descriptive Statistics

Table 2.1 shows that both series are statistically similar. They are negatively skewed with strong leptokurtic properties. As a result, there are high values for the Jarque-Bera statistics implying definite non-normality of both return series. The Box-Pierce Q-statistics on the raw residuals reveals some autocorrelation in the first moment for both series. There is strong autocorrelation in the second momment of both series as indicated by the Box-Pierce Q-statistics on the squared returns. Also, Engle's LM ARCH tests (See Engle (1982)) conclude that both series exhibit ARCH effects. Already, several of the *stylized facts* mentioned earlier have been observed. Notice, if risk was measured by standard deviation of returns and reward by the mean of returns then the FTSESAW index outperforms the FTSEAW index for the period 22 September 2003 to 22 January 2009.

	Stock Indices	
	FTSEAW	FTSESAW
Mean	-0.0060	0.0050
Standard Deviation	1.0735	1.0634
Maximum	8.8821	9.1897
Minimum	-7.2819	-7.5990
Skewness	-0.6444(0.0000)	-0.4571(0.0000)
Excess Kurtosis	14.4970(0.0000)	14.9860(0.0000)
Jarque-Bera	12286(0.0000)	13075(0.0000)
Q(20)	106.626(0.0000)	94.1105(0.0000)
$Q^2(20)$	3218.69(0.0000)	3022.69(0.0000)
ARCH(5)	177.29(0.0000)	177.36(0.0000)

Table 2.1: Summary statistics of the FTSEAW and FTSESAW raw returns with p-values in parentheses.

Descriptive Plots

The descriptive plots verify the observations made from the descriptive statistics. Figures 2.1 and 2.2 show that the returns of both series fluctuate such that volatility is not constant over time with extreme values appearing in clusters. There is a clear exaggeration in the volatility post-2008 for both series. This is due to the current global financial crisis and in Chapter 3 the repercussions of this highly volatile period on Islamic indices will be discussed. The correlograms of the raw residuals in Figures 2.3 and 2.4 show some serial correlation in the raw returns with significant values at lags 1 and 2 for both series. Hence, an AR(2) structure is first considered to capture the serial correlation in the conditional mean. However, ϕ_2 was found to be insignificant for both series, so the simpler AR(1) model was used. The correlograms for $|r_t|$ and r_t^2 in Figures 2.3 and 2.4 verify the strong serial correlation in the second moment of both series. The density plots in Figures 2.5 and 2.6 show the kernel estimate of the density function of the returns distribution compared to the corresponding normal density. These plots together with the QQ plots against the normal distribution confirm that both series are not normally distributed since skewness and leptokurtic properties are present. In light of these preliminary findings, GARCH-type models which have been very successful in capturing the *stylized facts* of financial time series are considered.

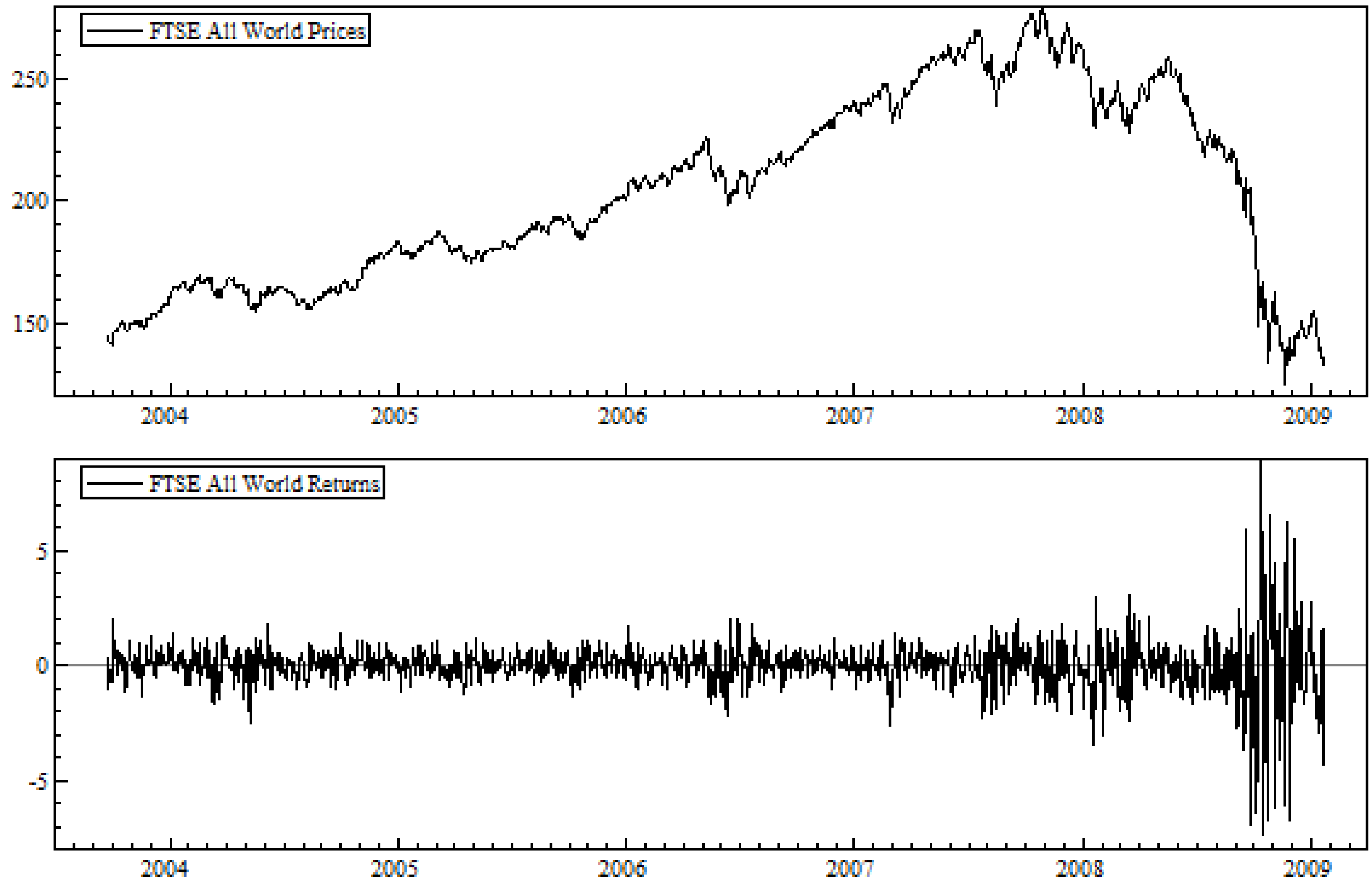

Figure 2.1: Price and return series plots for FTSEAW.

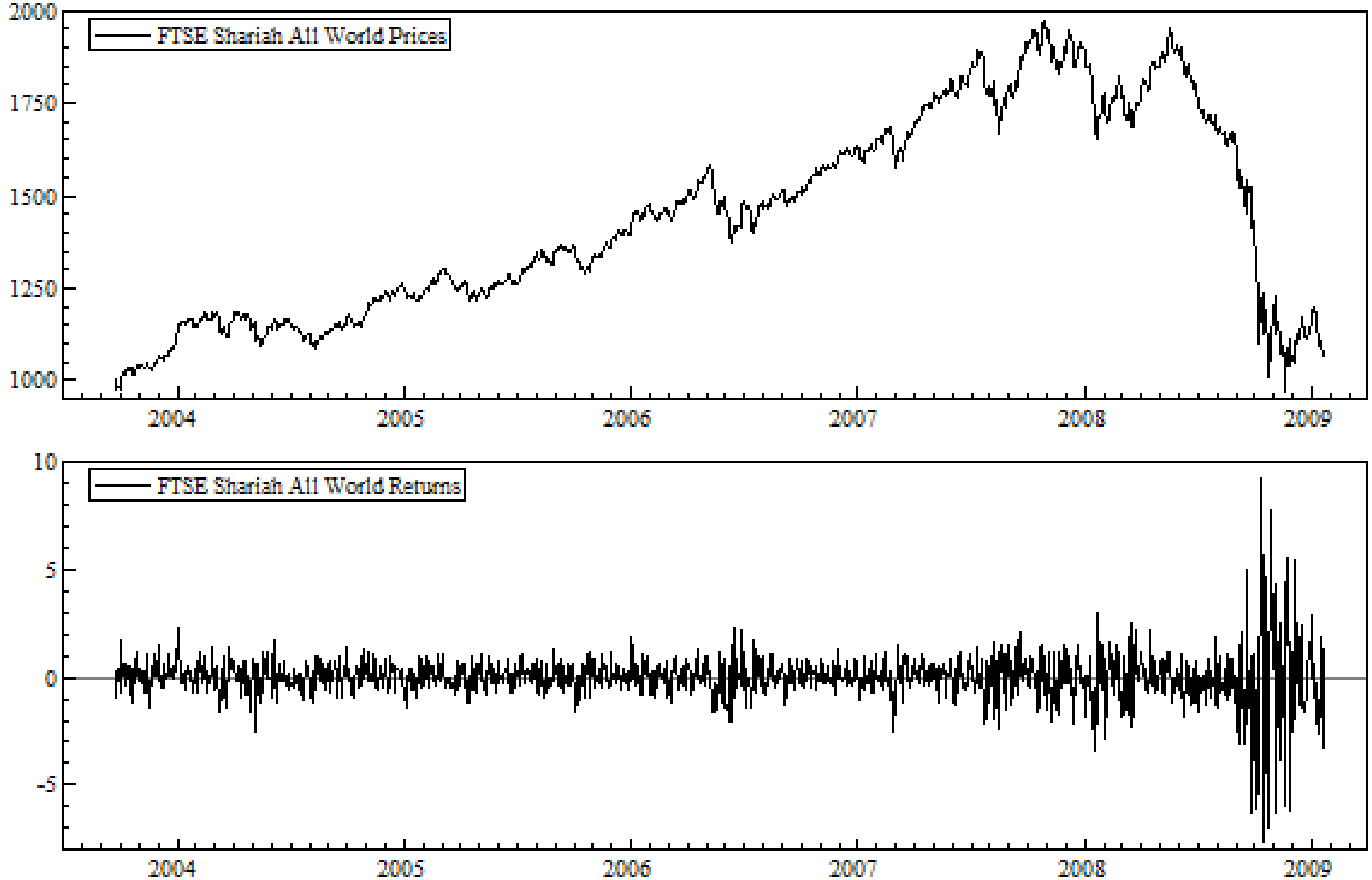

Figure 2.2: Price and return series plots for FTSESAW.

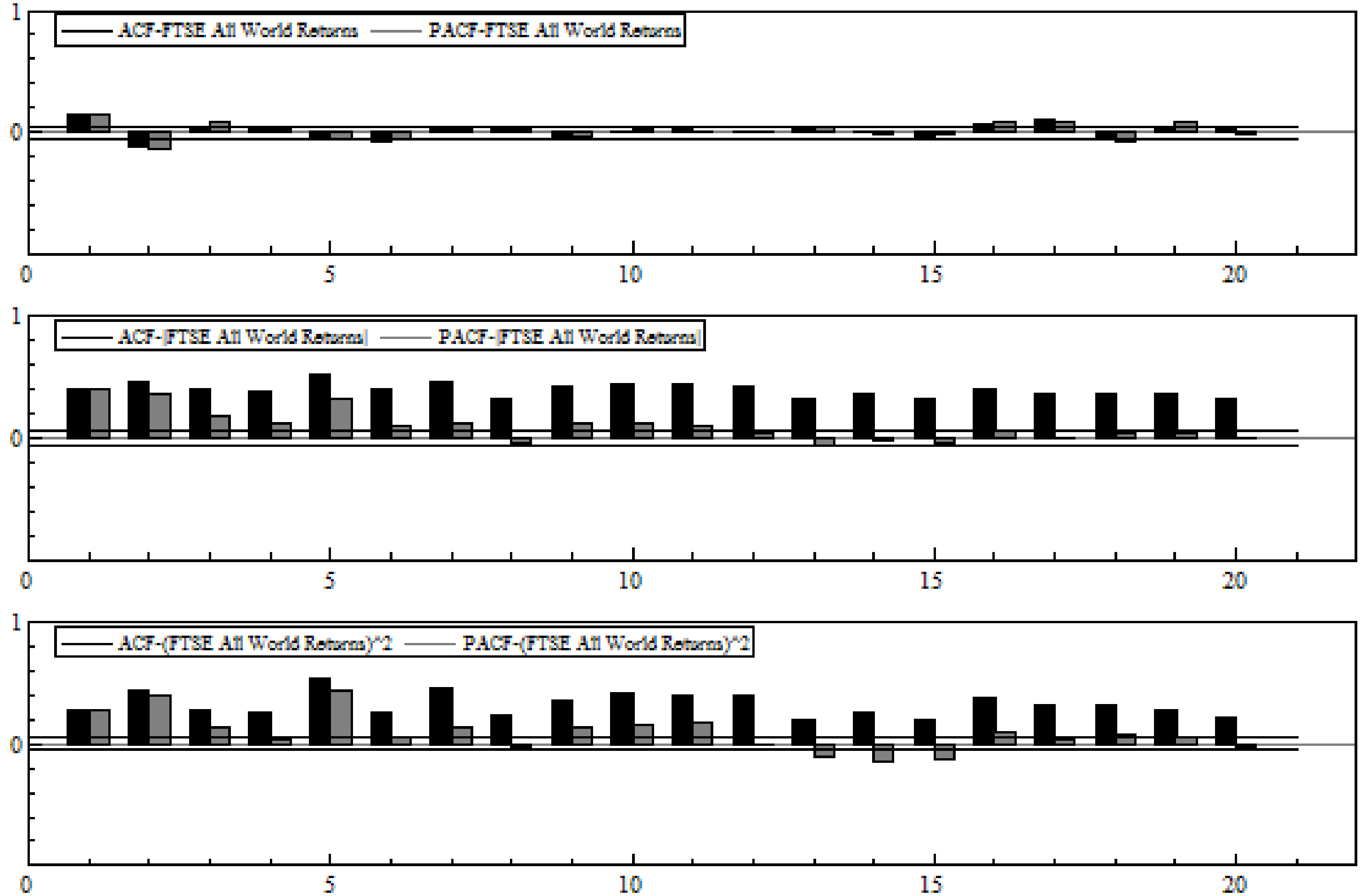

Figure 2.3: Correlograms of raw, absolute and squared FTSEAW returns.

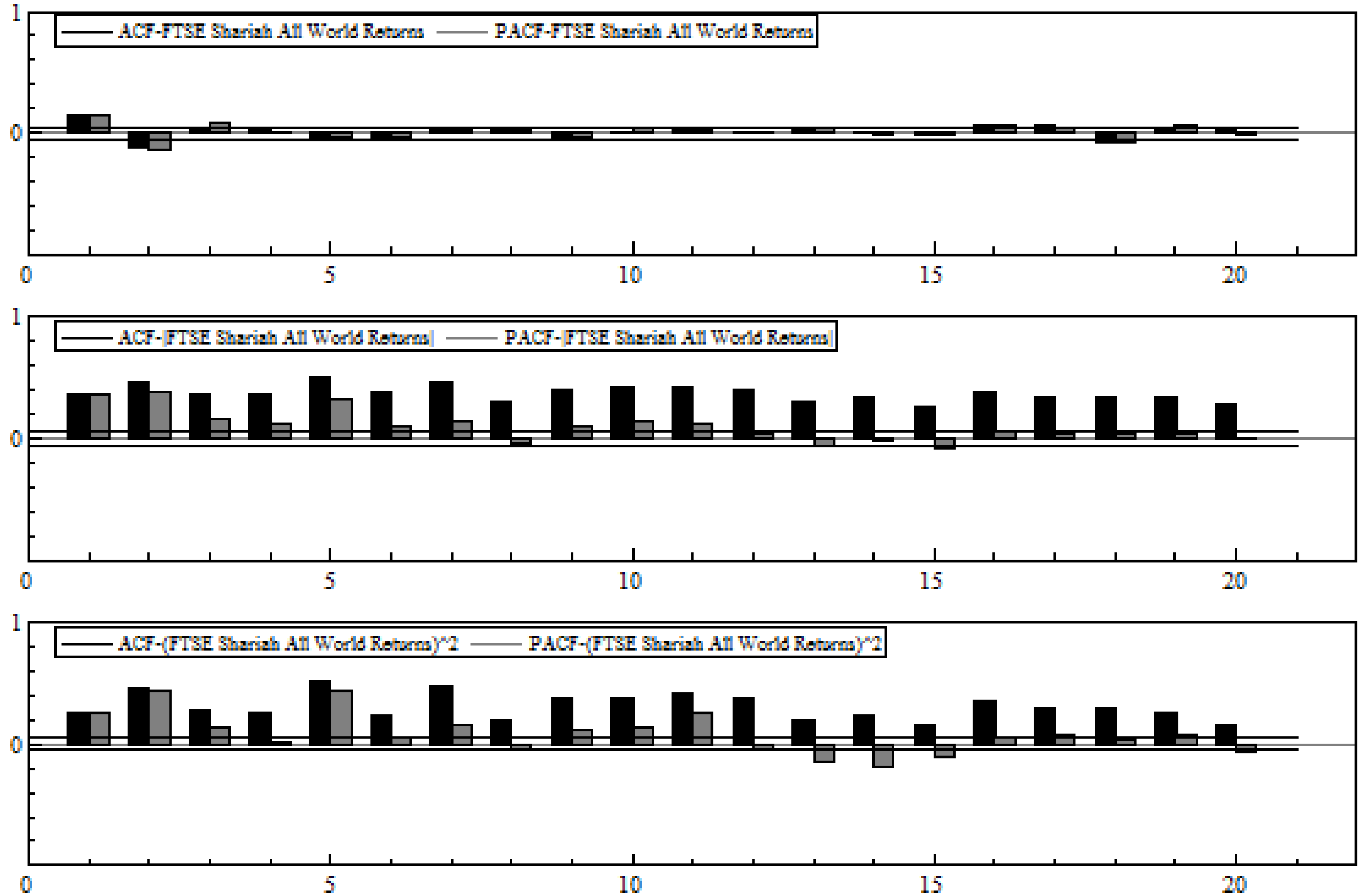

Figure 2.4: Correlograms of raw, absolute and squared FTSESAW returns.

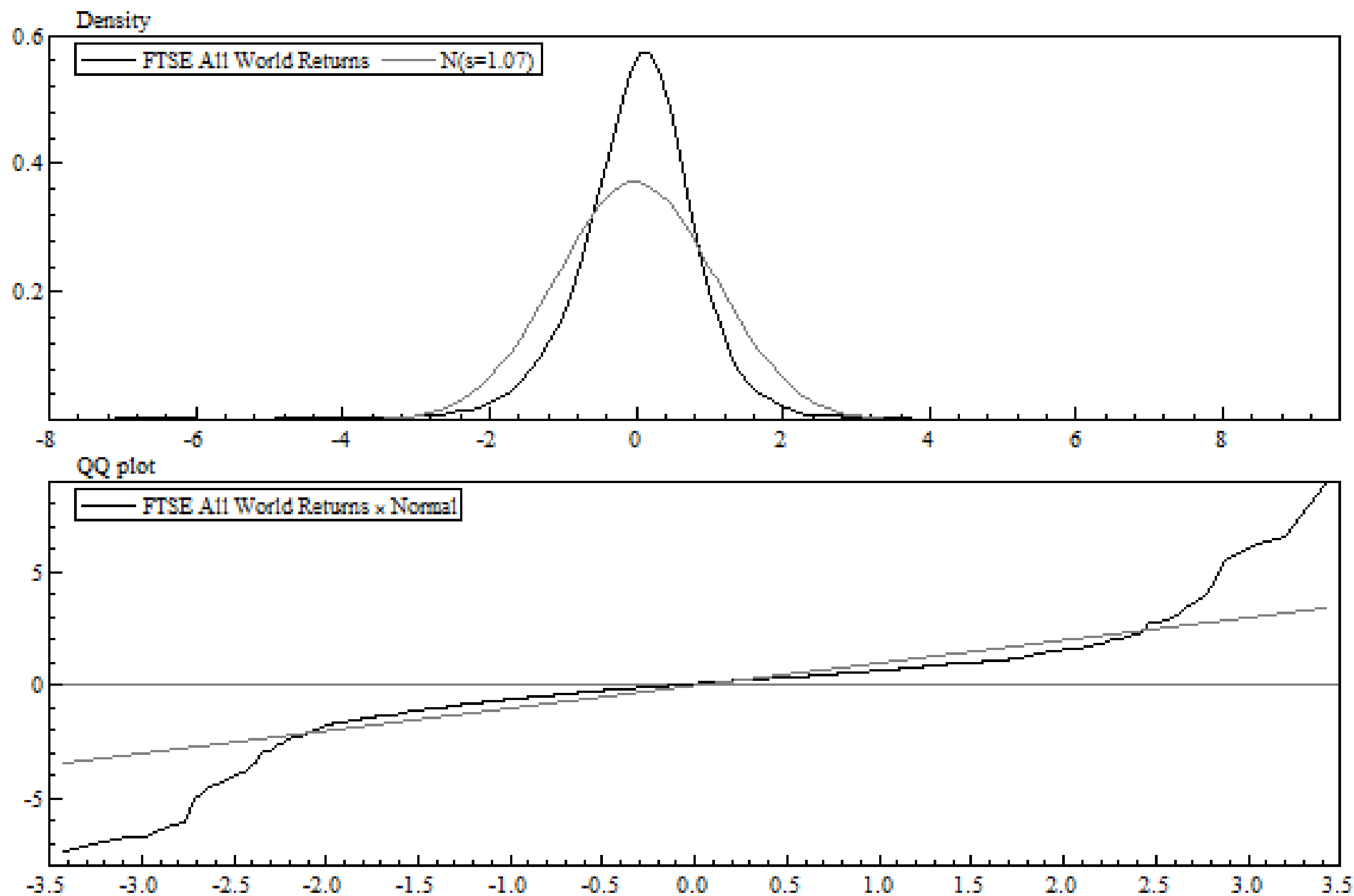

Figure 2.5: Density and QQ-Plots against the normal distribution for FTSEAW returns.

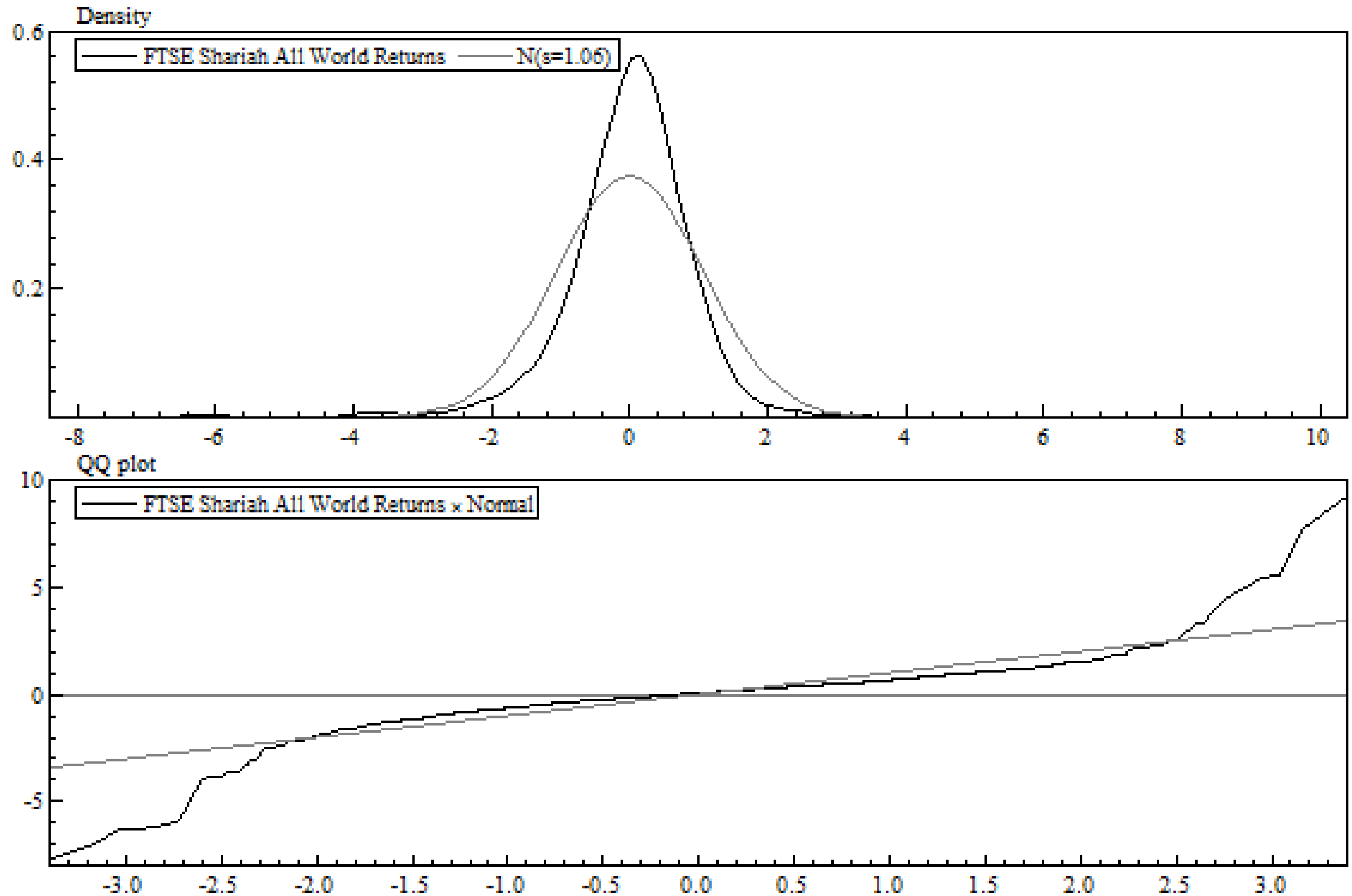

Figure 2.6: Density and QQ-Plots against the normal distribution for FTSESAW returns.

2.5.2 Model Estimation

Tables 2.3 and 2.4 reveal several key properties of both series. Firstly, the autoregressive effect in the volatility's specification is strong, $\beta_1 > 0.88$ for FTSEAW and FTSESAW, which suggests significant serial correlation. Secondly, γ_1 is positive and significant which means there is a leverage effect for negative returns in the conditional variance specification. Thirdly, δ is mostly significantly different from 2 but not 1 which implies modelling the conditional standard deviation is more relevant than modelling the conditional variance using a GARCH process. Finally, $\log \xi$ is negative and significant which suggests that the densities are skewed to the left and the asymmetric student-t distribution may be needed to fully model both return series.

	AR(1) + RiskMetrics - Parameters	
	FTSEAW	FTSESAW
μ	0.0568(0.0243)	0.0546(0.0259)
ϕ_1	0.1795(0.0300)	0.1898(0.0306)
λ	0.94	0.94

Table 2.2: Parameter estimates for the AR(1) + RiskMetrics model with the standard errors given in parentheses.

	FTSEAW - AR(1) + APARCH(1,1) - Parameters		
	Normal	Student-t	Skewed Student-t
μ	0.0261(0.0202)	0.0346(0.0195)	0.0284(0.1607)
ϕ_1	0.1799(0.0264)	0.1786(0.0257)	0.1716(0.0266)
ω	0.0184(0.0046)	0.0183(0.0047)	0.0194(0.0050)
α_1	0.0616(0.0101)	0.0664(0.0115)	0.0659(0.0123)
β_1	0.8954(0.0214)	0.89980(0.0192)	0.8997(0.0192)
γ_1	0.9349(0.1599)	0.9868(0.0605)	0.9932(0.0368)
δ	1.5492(0.3859)	1.3820(0.2900)	1.3654(0.2862)
$\log \xi$	-	-	-0.0925(0.0394)
ν	-	14.0835(5.6768)	14.7738(5.9795)

Table 2.3: FTSEAW parameter estimates under the AR(1) + APARCH(1,1) models with the standard errors given in parentheses.

	FTSESAW - AR(1) + APARCH(1,1) - Parameters		
	Normal	Student-t	Skewed Student-t
μ	0.0239(0.0219)	0.0306(0.0213)	0.0234(0.0210)
ϕ_1	0.1950(0.0270)	0.1932(0.0266)	0.1887(0.0272)
ω	0.0211(0.0057)	0.0207(0.0057)	0.0217(0.0060)
α_1	0.0582(0.0111)	0.0638(0.0108)	0.0633(0.0110)
β_1	0.8864(0.0309)	0.8962(0.0246)	0.8977(0.0240)
γ_1	0.7697(0.2888)	0.9352(0.2001)	0.9652(0.1462)
δ	1.8037(0.5550)	1.4790(0.3667)	1.4347(0.3468)
$\log \xi$	-	-	-0.0810(0.0419)
ν	-	11.9673(3.8617)	11.9504(3.8518)

Table 2.4: FTSESAW parameter estimates under the AR(1) + APARCH(1,1) models with the standard errors given in parentheses.

2.5.3 Model Performance

Misspecification Tests

The results of the misspecification tests presented in Tables 2.5, 2.6, 2.7 and 2.8 indicate the serial correlation in the first and second moments for both series have been successfully captured as observed by the Box-Pierce Q-statistics on the standardized and squared standardized residuals. The results of Engle's LM ARCH test reveals that neither series has any remaining ARCH effects that needs to be modelled. For both series, the residual-based diagnostics (RBD) results for conditional heteroscedasticity of Tse (2002) concludes the simple RiskMetrics specification of the conditional variance is unsatisfactory. On the other hand, the APARCH-type models are preferred as they do not leave any significant heteroscedasticity in the standardized residuals.

The adjusted Pearson goodness-of-fit test, which compares the empirical distribution of the innovations to the theoretical one, concludes that the normal distribution under the RiskMetrics model gives a poor fit. However, the normal, student-t and skewed student-t under the APARCH(1,1) model gives a much better fit. Using the ranking criteria of maximizing the log likelihood and minimizing the Akaike Information Criteria (AIC), the AR(1) + APARCH(1,1) model with skewed student-t innovations performs the best since it fully captures the *stylized facts* present in both series. The FTSESAW p-values for the misspecification tests were generally higher than those for the FTSEAW. This means that the models performed better in capturing the effects present in the FTSESAW than those present in the FTSEAW.

	AR(1) + RiskMetrics - Misspecification Tests	
	FTSEAW	FTSESAW
Log Likelihood	-1526.0180	-1564.4660
AIC	2.1954	2.2507
Skewness	-0.2340(0.0004)	-0.1564(0.0171)
Excess Kurtosis	1.1742(0.0000)	1.1907(0.0000)
Jarque-Bera	92.6680(0.0000)	87.9130(0.0000)
Q(20)	23.3550(0.2221)	19.1438(0.4476)
$Q^2(20)$	28.1770(0.0594)	21.8817(0.2373)
ARCH(5)	2.1133(0.0614)	1.5833(0.1617)
P(40)	59.2069(0.0117,0.0200)	41.3908(0.2849,0.3667)
RBD(5)	14.0587(0.0152)	11.4839(0.0426)

Table 2.5: Results of misspecification tests for the AR(1) + RiskMetrics model with the p-values given in parentheses.

	AR(1) + APARCH(1,1) + Normal - Misspecification Tests	
	FTSEAW	FTSESAW
Log Likelihood	-1484.8740	-1525.6400
AIC	2.1435	2.2021
Skewness	-0.2706(0.0000)	-0.1617(0.0137)
Excess Kurtosis	0.9582(0.0000)	1.0974(0.0000)
Jarque-Bera	70.2330(0.0000)	75.9150(0.0000)
Q(20)	19.0495(0.4537)	16.8306(0.6013)
$Q^2(20)$	21.7098(0.2451)	16.0203(0.5911)
ARCH(5)	1.6091(0.1545)	0.4187(0.8360)
P(40)	42.1379(0.1084,0.3368)	29.6092(0.5881,0.8612)
RBD(5)	8.5587(0.1280)	2.1175(0.8327)

Table 2.6: Results of misspecification tests with p-values given in parentheses for the AR(1) + APARCH(1,1) model with normal innovations.

	AR(1) + APARCH(1,1) + Student-t - Misspecification Tests	
	FTSEAW	FTSESAW
Log Likelihood	-1476.5380	-1514.0480
AIC	2.1330	2.1869
Skewness	-0.2662(0.0000)	-0.1367(0.0371)
Excess Kurtosis	1.0372(0.0000)	1.3293(0.0000)
Jarque-Bera	78.8330(0.0000)	106.8200(0.0000)
Q(20)	18.8684(0.4653)	17.2344(0.5740)
$Q^2(20)$	20.9718(0.2808)	15.1033(0.6549)
ARCH(5)	1.6220(0.1511)	0.3969(0.8512)
P(40)	40.9885(0.1082,0.3833)	34.1494(0.3187,0.6906)
RBD(5)	8.6361(0.1245)	2.4555(0.7832)

Table 2.7: Results of misspecification tests with p-values given in parentheses for the AR(1) + APARCH(1,1) model with student-t innovations.

	AR(1) + APARCH(1,1) + Skewed Student-t - Misspecification Tests	
	FTSEAW	FTSESAW
Log Likelihood	-1473.8120	-1511.9980
AIC	2.1305	2.1853
Skewness	-0.2687(0.0000)	-0.1386(0.0346)
Excess Kurtosis	1.0088(0.0000)	1.3068(0.0000)
Jarque-Bera	75.7750(0.0000)	103.5100(0.0000)
Q(20)	19.2101(0.4434)	17.3965(0.5630)
$Q^2(20)$	21.4712(0.2563)	15.3007(0.6412)
ARCH(5)	1.6550(0.1425)	0.4306(0.8275)
P(40)	32.6552(0.3377,0.7533)	33.0575(0.3201,0.7369)
RBD(5)	9.0274(0.1080)	2.7163(0.7436)

Table 2.8: Results of misspecification tests with p-values given in parentheses for the AR(1) + APARCH(1,1) model with skewed student-t innovations.

VaR Backtesting

From a risk managers' perspective, when choosing an appropriate model for the return series, the model's ability to accurately model VaR is very important. Thus, testing the performance of these models in modelling VaR in both long and short trading positions is necessary. To access each model's performance in modelling VaR, the Kupiec LR test (See Kupiec (1995)) is used. In this test, the failure rate for long (short) trading positions is equal to the percentage of negative (positive) returns smaller (larger) than the long (short) trading position one-step-ahead VaR. The hypotheses $H_0 : f = \alpha$ versus $H_1 : f \neq \alpha$ is then tested, where f is the failure rate which is estimated using the empirical failure rate. Tables 2.9, 2.10, 2.11 and 2.12 show that for both series the AR(1) + APARCH(1,1) model with skewed student-t innovations passes the Kupiec LR tests at all quantiles for both long and short trading positions. Although the AR(1) + APARCH(1,1) model with student-t innovations passes the Kupiec LR tests for the FTSESAW returns, the p-values in the short trading positions are much weaker than those for the AR(1) + APARCH(1,1) model with skewed student-t innovations. Therefore, with respect to accurately modelling VaR in long and short trading positions, the best choice for both series is the AR(1) + APARCH(1,1) model with skewed student-t innovations.

	FTSEAW - Long Trading Positions				
α	5%	2.5%	1%	0.5%	0.25%
RiskMetrics	0.0668	0.0005	0.0073	0.0033	0.0044
Normal APARCH	0.8637	0.3004	0.0448	0.0827	0.2205
Student-t APARCH	0.5117	0.2310	0.4226	0.9879	0.3877
Skewed Student-t APARCH	0.9608	0.5882	0.8021	0.0896	0.3877

Table 2.9: P-values of the Kupiec LR test for the FTSEAW for long trading positions.

	FTSEAW - Short Trading Positions				
α	5%	2.5%	1%	0.5%	0.25%
RiskMetrics	0.5921	0.8377	0.7739	0.7087	0.4441
Normal APARCH	0.5675	0.6261	0.4142	0.0896	0.3877
Student-t APARCH	0.5675	0.2271	0.0004	0.0896	0.3877
Skewed Student-t APARCH	0.8637	0.5882	0.8021	0.0896	0.3877

Table 2.10: P-values of the Kupiec LR test for the FTSEAW for short trading positions.

	FTSESAW - Long Trading Positions				
α	5%	2.5%	1%	0.5%	0.25%
RiskMetrics	0.0118	0.0215	0.0037	0.0013	0.0044
Normal APARCH	0.8637	0.8904	0.0138	0.1571	0.7852
Student-t APARCH	0.5117	0.8377	0.7739	0.7087	0.3877
Skewed Student-t APARCH	0.7477	0.5064	0.5963	0.2214	0.3877

Table 2.11: P-values of the Kupiec LR test for the FTSESAW for long trading positions.

	FTSESAW - Short Trading Positions				
α	5%	2.5%	1%	0.5%	0.25%
RiskMetrics	0.9608	0.8904	0.5842	0.6995	0.4441
Normal APARCH	0.3413	0.3055	0.9828	0.7087	0.7919
Student-t APARCH	0.3413	0.1636	0.1565	0.0896	0.3877
Skewed Student-t APARCH	0.9411	0.7553	0.9828	0.7087	0.3877

Table 2.12: P-values of the Kupiec LR test for the FTSESAW for short trading positions.

Conditional Tail Expectation (CTE)

In addition to VaR, many risk managers are interested in the CTE, which is a coherent risk measure, unlike VaR. The CTE represents the average loss given that losses exceed VaR. In other words, when we exceed VaR we are interested in how bad do things get. Mathematically,

$$CTE_{1-\alpha} = E\left[L|L > VaR_{1-\alpha}\right]$$

for short trading positions and

$$CTE_{\alpha} = E\left[L|L < VaR_{\alpha}\right]$$

for long trading positions where L is the loss random variable. Although CTE is not used as a tool to rank models' performance, it is considered in order to compare the riskiness between the FTSEAW and FTSESAW indices.

Tables 2.14 and 2.16 show that the CTE values of the FTSESAW return series, after Monte Carlo simulations from each model, are genrally larger than the CTE values of the FTSEAW return series for short trading positions. This suggests, in short trading positions, the FTSESAW index is riskier than the FTSEAW index under the CTE risk measure. Based on the CTE risk measure, Tables 2.13 and 2.15 show that at quantiles $\alpha = 2.5\%$ and $\alpha = 5\%$ the FTSESAW index is generally riskier than the FTSEAW index but at $\alpha = 1\%$ the reverse observation is made. Hence, in long trading positions the results are not conclusive as to which index is the riskier investment.

	FTSEAW - Long Trading Positions		
α	5%	2.5%	1%
RiskMetrics	-1.8095	-1.9902	-2.2987
Normal APARCH	-2.0145	-2.2610	-2.8268
Student-t APARCH	-1.9626	-2.2343	-2.9654
Skewed Student-t APARCH	-2.0173	-2.3340	-3.1111

Table 2.13: CTE for the FTSEAW for long trading positions.

	FTSEAW - Short Trading Positions		
α	5%	2.5%	1%
RiskMetrics	1.4921	1.7264	2.2554
Normal APARCH	1.6564	1.8318	2.9216
Student-t APARCH	1.6423	1.8622	3.0162
Skewed Student-t APARCH	1.6307	1.8441	2.6467

Table 2.14: CTE for the FTSEAW for short trading positions.

	FTSESAW - Long Trading Positions		
α	5%	2.5%	1%
RiskMetrics	-1.8114	-1.9481	-2.2667
Normal APARCH	-2.0851	-2.6025	-2.6524
Student-t APARCH	-1.9281	-2.3405	-2.5190
Skewed Student-t APARCH	-2.0490	-2.5408	-2.7683

Table 2.15: CTE for the FTSESAW for long trading positions.

	FTSESAW - Short Trading Positions		
α	5%	2.5%	1%
RiskMetrics	1.6057	1.9612	2.3371
Normal APARCH	1.7227	2.1570	2.5554
Student-t APARCH	1.7174	1.9857	3.0936
Skewed Student-t APARCH	1.6444	2.1539	2.5554

Table 2.16: CTE for the FTSESAW for short trading positions.

2.5.4 Model Selection

To summarize, after fitting an AR(1) structure to the conditional mean the serial correlation in the first moment of both series was captured. The APARCH(1,1) specification of the conditional variance outperformed the RiskMetrics specification in modelling the serial correlation in the second moment of both series. The adjusted Pearson goodness-of-fit test together with the ranking criterion of log likelihood and AIC concluded that the AR(1) + APARCH(1,1) model with skewed student-t innovations provided the best fit for both series. This model also proved to be the most successful at VaR modelling. Therefore, for the FTSEAW and FTSESAW return series the AR(1) + APARCH(1,1) model with skewed student-t innovations performed the best overall.

Chapter 3

Islamic Equity Performance

Does Islamic equity outperform conventional equity? Does the screening process undergone by stocks before they qualify for inclusion in the Islamic stock index have any negative impact on the index's performance? Hussein (2004) reports the screening process undergone by the companies in the FTSE All World index before they qualify to be enlisted in the FTSE Global Islamic Index has no negative impact on the performance of the FTSE Global Islamic index. In this chapter, to analyze the index's performance the Markowitz model of risk and return is considered.

3.1 The Markowitz Model

According to the Markowitz model (See Markowitz (1959)), over a given period, the risk of an asset is measured by its variance and its return is measured by its mean. Consider a universe of N assets with returns $R_1, R_2, \ldots, R_N$. Let σ_{ij} represent the covariance between R_i and R_j and Σ be the $N \times N$ variance-covariance matrix. Now consider an investor who invests 100% of his wealth into these N assets with w_i invested in asset i. Immediately the *budget constraint* becomes

$$w_1 + w_2 + \cdots + w_N = 1 \Leftrightarrow \vec{w}^T \vec{e} = 1 \tag{3.1}$$

where

$$\vec{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{pmatrix}$$

and $\vec{e}$ is a vector of N ones.

The portfolio return R_p is given by

$$R_p = \sum_{i=1}^{N} w_i R_i \tag{3.2}$$

and the expected portfolio return and variance are

$$\mu_p = E(R_p) = \sum_{i=1}^{N} w_i E(R_i) = \sum_{i=1}^{N} w_i \mu_i = \vec{w}^T \vec{\mu} \tag{3.3}$$

and

$$\sigma_p^2 = Var(R_p) = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j Cov(R_i, R_j) = \vec{w}^T \Sigma \vec{w} \tag{3.4}$$

where

$$\vec{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN} \end{pmatrix}.$$

The Markowitz model gives the risk-return argument whereby any rational investor will minimize σ_p^2 for a given expected portfolio return μ_p or maximize μ_p for a given σ_p^2. These two statements can be transformed into the following equivalent optimization problems:

- **Optimization Problem 1**

$$\underset{w_i, i \in 1,2,\ldots,N}{min} \vec{w}^T \Sigma \vec{w}$$

$$\text{subject to } \vec{w}^T \vec{\mu} = \mu_{given} \text{ and } \vec{w}^T \vec{e} = 1$$

- **Optimization Problem 2**

$$\underset{w_i, i \in 1,2,\ldots,N}{max} \vec{w}^T \vec{\mu}$$

$$\text{subject to } \vec{w}^T \Sigma \vec{w} = \sigma_{given}^2 \text{ and } \vec{w}^T \vec{e} = 1$$

All solutions to optimization problems 1 and 2 represented by $(\mu_p, \sigma_{given}^2)$ and $(\mu_{given}, \sigma_p^2)$ give a set of portfolios called the *efficient set*. They can be plotted on the X-Y plane and this graph is referred to as the *efficient frontier* for the given N assets. The portfolios that lie on this *efficient frontier* are the options available to the rational investor since they dominate all other portfolios, according to the risk-return argument.

In some cases, as in the Islamic financial world, the concept of *short selling* assets is prohibited. *Short selling* asset i is defined as borrowing asset i, selling it, repurchasing it at the end of the period and finally returning it to its owner. With this added constraint two more equivalent optimization problems arise:

- **Optimization Problem 3**

$$\min_{w_i, i \in 1,2,\ldots,N} \vec{w}^T \Sigma \vec{w}$$

$$\text{subject to } \vec{w}^T \vec{\mu} = \mu_{given}, \vec{w}^T \vec{e} = 1 \text{ and } w_i \geq 0 \ \forall \ i \in 1, 2, \ldots, N$$

- **Optimization Problem 4**

$$\max_{w_i, i \in 1,2,\ldots,N} \vec{w}^T \vec{\mu}$$

$$\text{subject to } \vec{w}^T \Sigma \vec{w} = \sigma^2_{given}, \vec{w}^T \vec{e} = 1 \text{ and } w_i \geq 0 \ \forall \ i \in 1, 2, \ldots, N$$

These optimization problems can be used to formulate sets of mean-variance efficient portfolios for Islamic and conventional assets. This gives several *efficient frontiers* which will be used to access the Islamic equity's performance in comparison to conventional equity.

3.2 The Data

Data was collected for 5 Islamic Dow Jones indices (Dow Jones US Islamic (DJUSI), Dow Jones UK Islamic (DJUKI), Dow Jones Canada Islamic (DJCAI), Dow Jones Europe Islamic (DJEUI) and Dow Jones Asia/Pacific Islamic (DJAPI)) and 5 conventional Dow Jones indices (Dow Jones US (DJUS), Dow Jones UK (DJUK), Dow Jones Canada (DJCA), Dow Jones Europe (DJEU) and Dow Jones Pacific (DJP)) from the Global Financial database for the period 8 September 1999 to 23 January 2009. This yielded 2414 realizations of P_t which were transformed by (2.1) to give 2413 log-return observations. These indices partitioned most of the Islamic and conventional financial world into five main geographical areas namely, the United States of America, the United Kingdom, Canada, Europe and the Asian and Pacific regions.

3.2.1 Descriptive Plots and Statistics of Conventional Indices

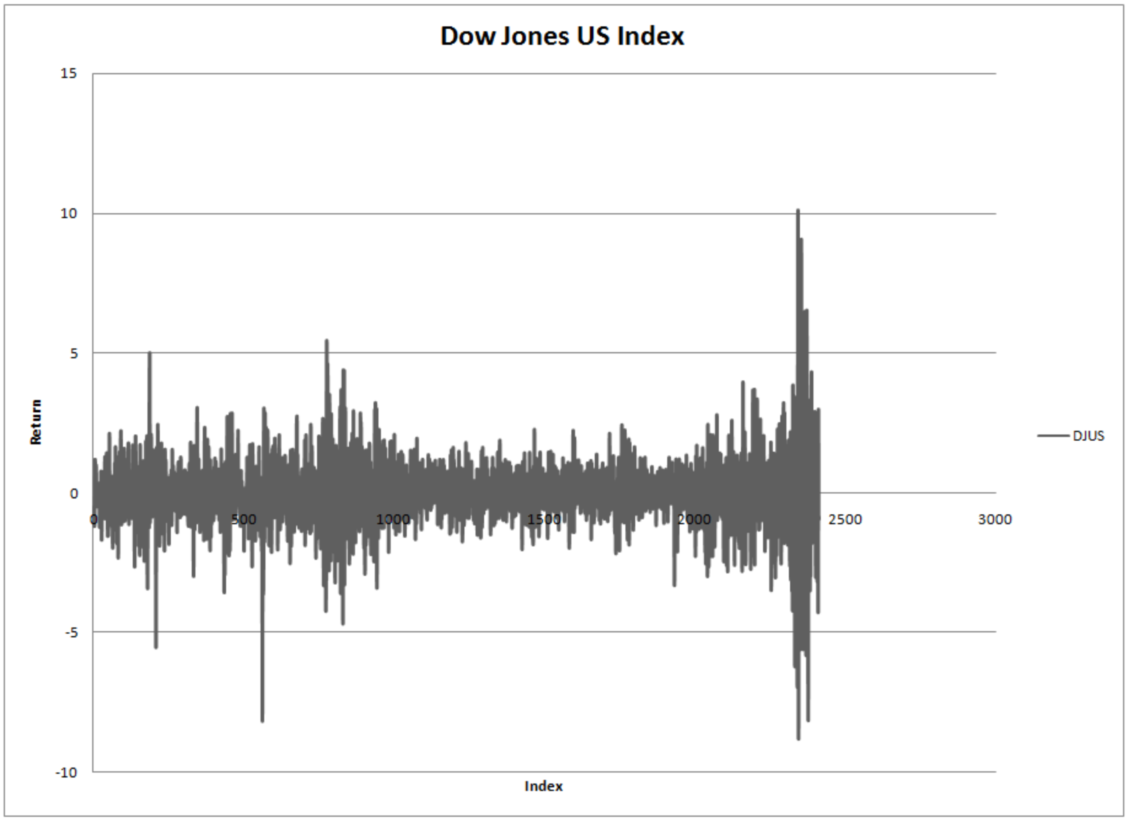

Figure 3.1: Return series of DJUS.

DJUS returns	
Mean	-0.0059
Standard Deviation	1.2526
Maximum	10.0891
Minimum	-8.6950
Skewness	-0.1765
Excess Kurtosis	7.8064

Table 3.1: Summary statistics for DJUS returns.

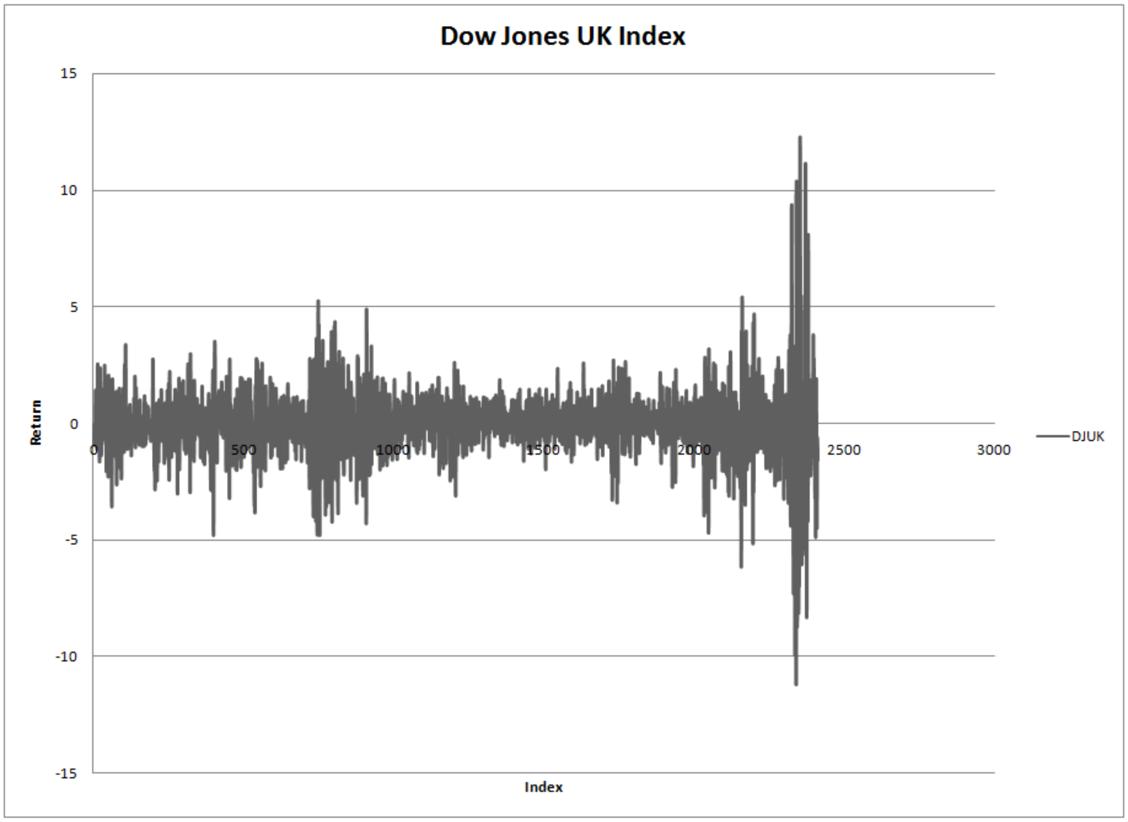

Figure 3.2: Return series of DJUK.

DJUK returns	
Mean	-0.0219
Standard Deviation	1.3867
Maximum	12.2515
Minimum	-10.9428
Skewness	-0.1038
Excess Kurtosis	12.1121

Table 3.2: Summary statistics for DJUK returns.

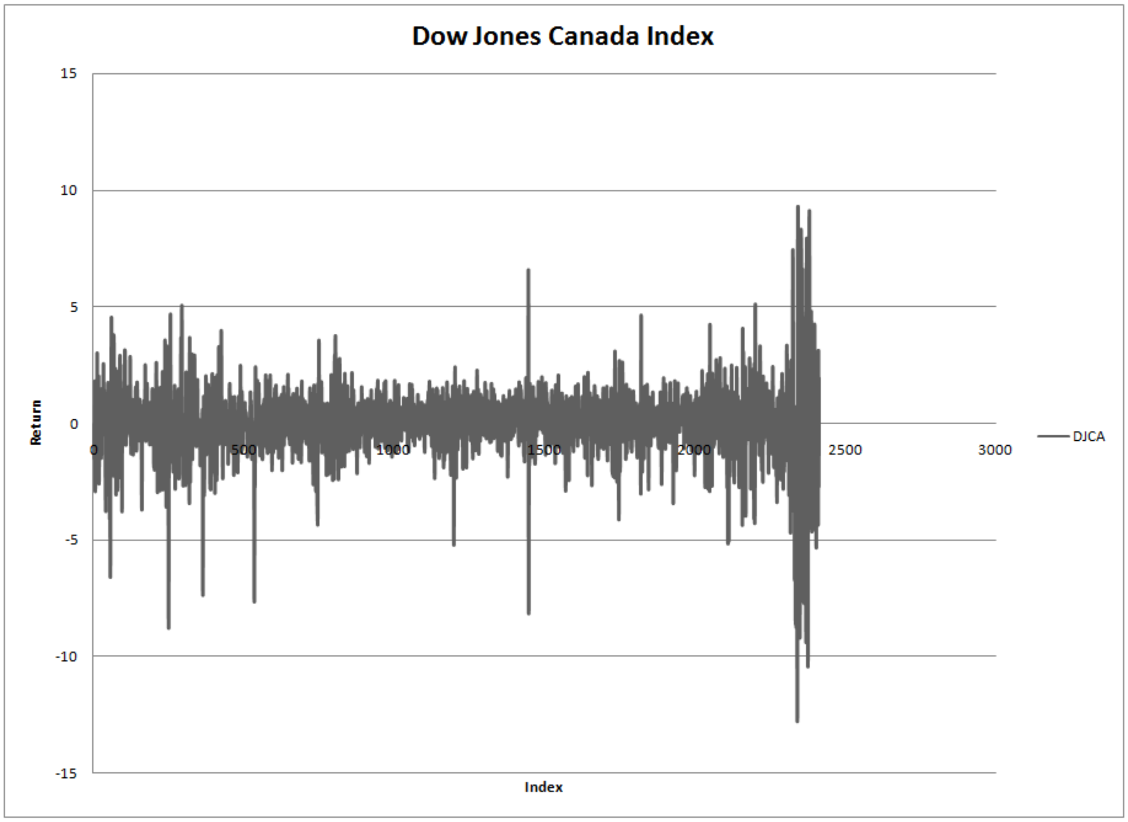

Figure 3.3: Return series of DJCA.

DJCA returns	
Mean	0.0027
Standard Deviation	1.4693
Maximum	9.1245
Minimum	-12.3635
Skewness	-0.8769
Excess Kurtosis	10.8133

Table 3.3: Summary statistics for DJCA returns.

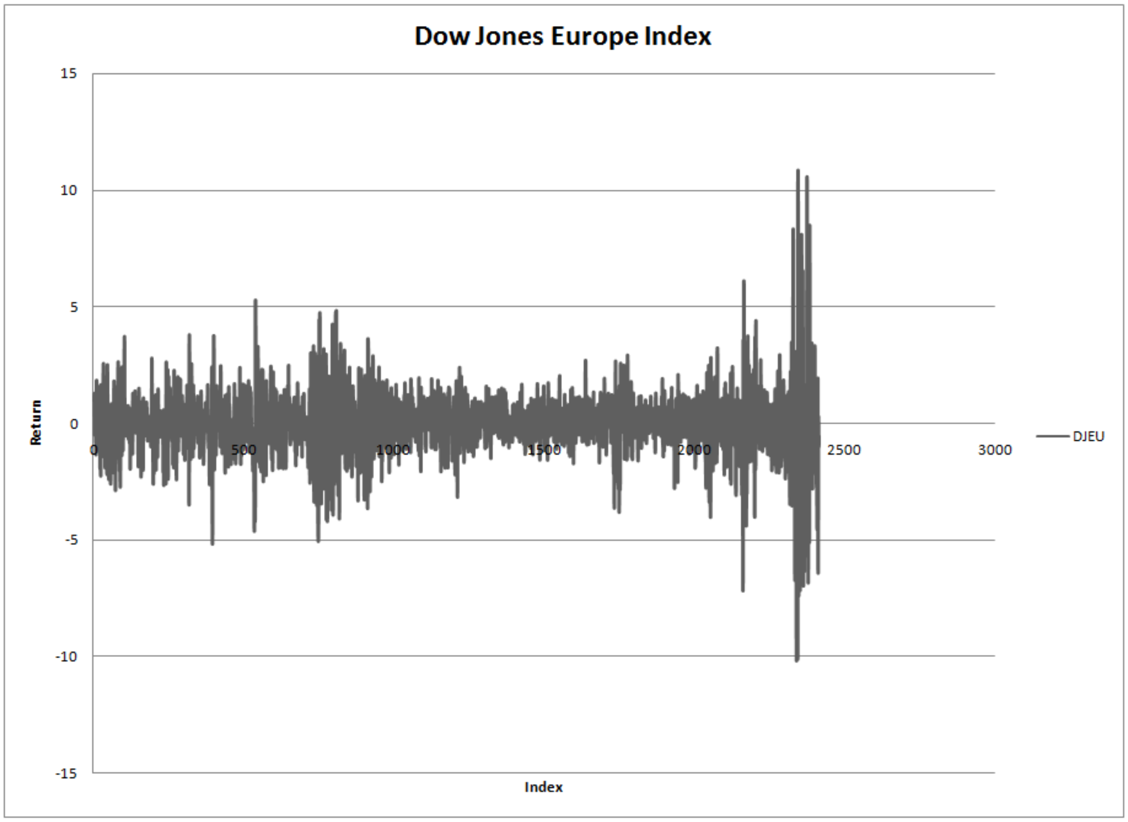

Figure 3.4: Return series of DJEU.

DJEU returns	
Mean	-0.0177
Standard Deviation	1.3788
Maximum	10.5123
Minimum	-10.1614
Skewness	-0.2242
Excess Kurtosis	9.2831

Table 3.4: Summary statistics for DJEU returns.

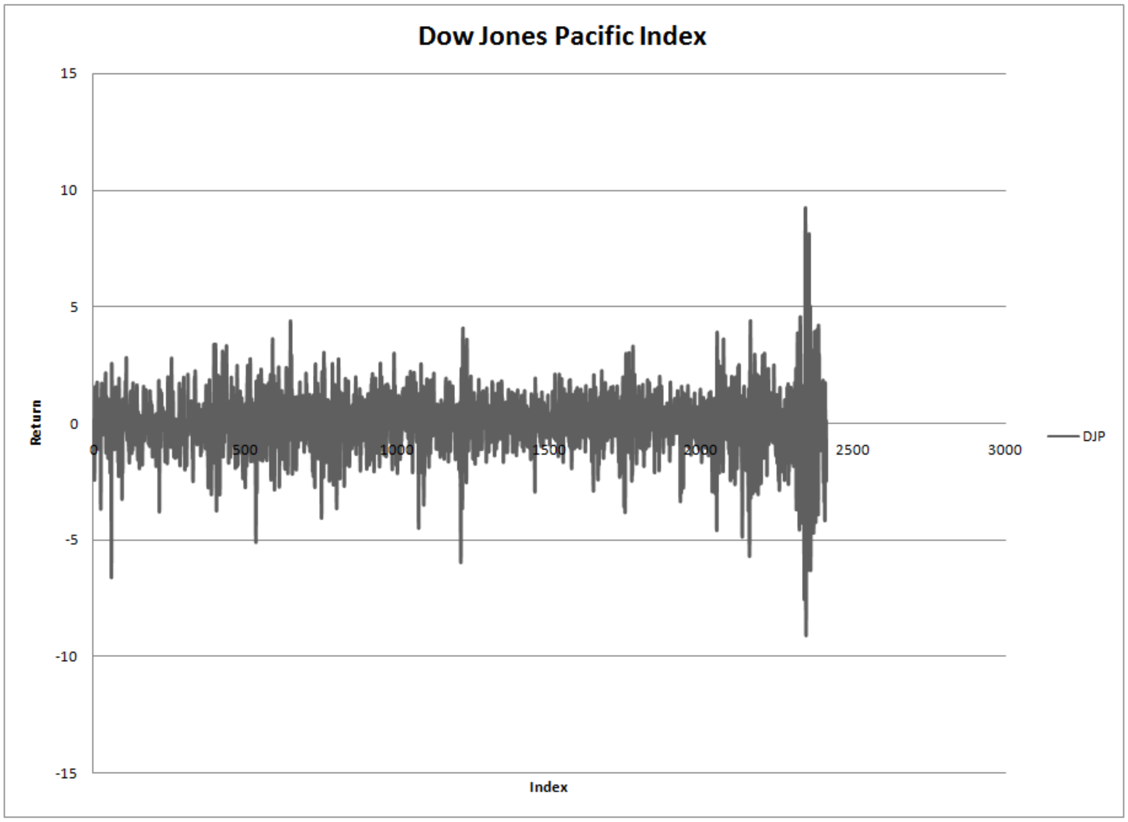

Figure 3.5: Return series of DJP.

DJP returns	
Mean	-0.0161
Standard Deviation	1.2616
Maximum	9.2007
Minimum	-9.0928
Skewness	-0.4657
Excess Kurtosis	5.7517

Table 3.5: Summary statistics for DJP returns.

3.2.2 Descriptive Plots and Statistics of Islamic Indices

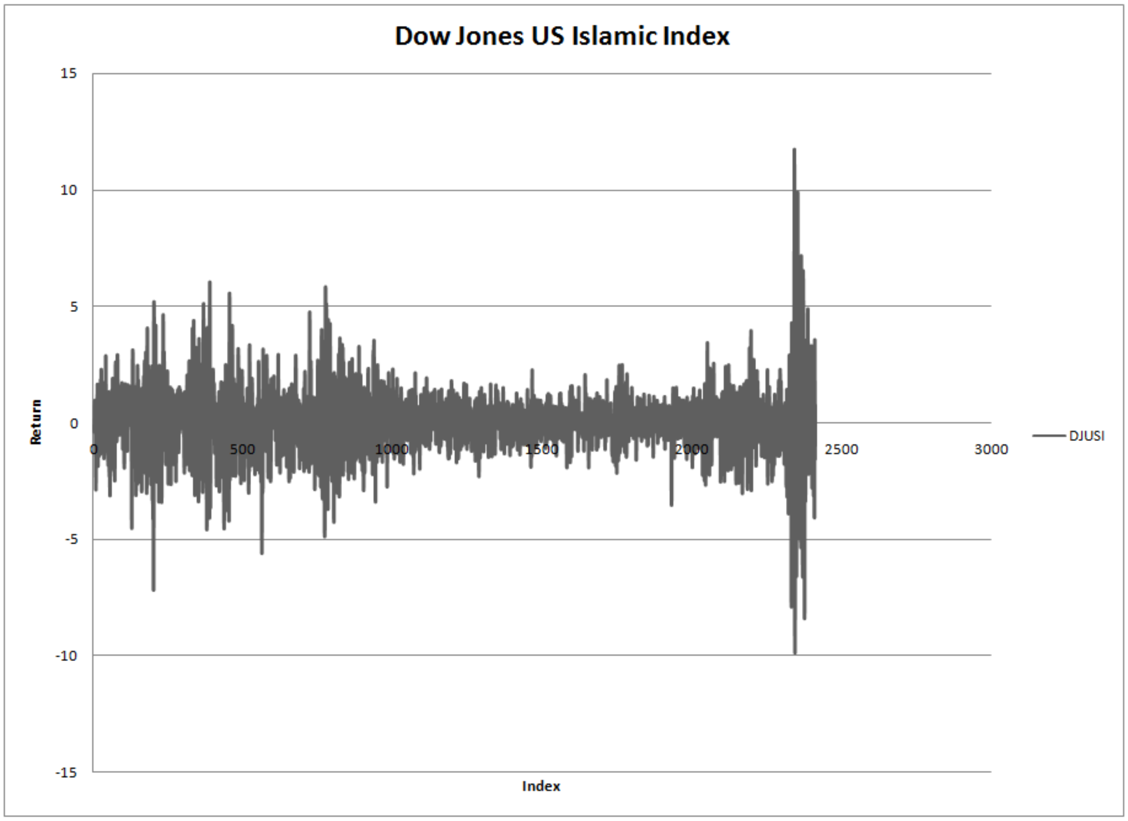

Figure 3.6: Return series of DJUSI.

DJUSI returns	
Mean	-0.0167
Standard Deviation	1.4243
Maximum	11.7405
Minimum	-9.6970
Skewness	0.0387
Excess Kurtosis	6.6542

Table 3.6: Summary statistics for DJUSI returns.

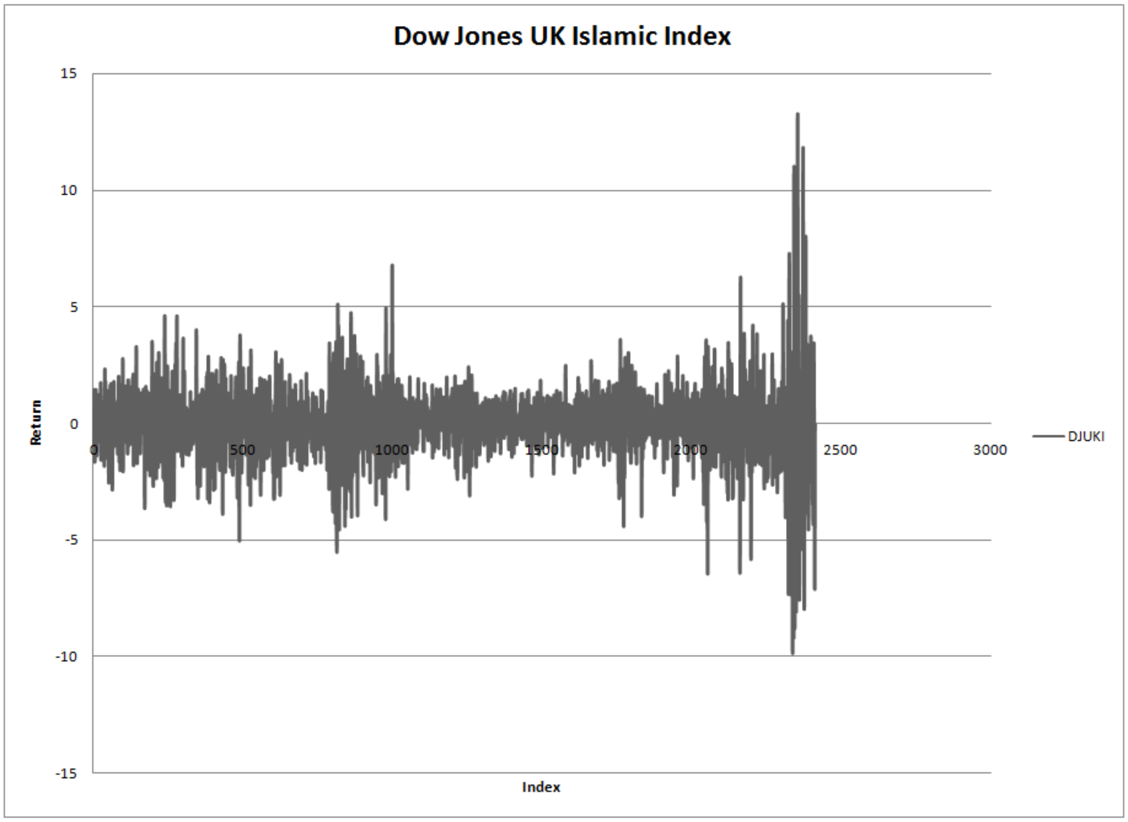

Figure 3.7: Return series of DJUKI.

DJUKI returns	
Mean	-0.0141
Standard Deviation	1.5151
Maximum	13.2847
Minimum	-9.8481
Skewness	-0.0031
Excess Kurtosis	9.3684

Table 3.7: Summary statistics for DJUKI returns.

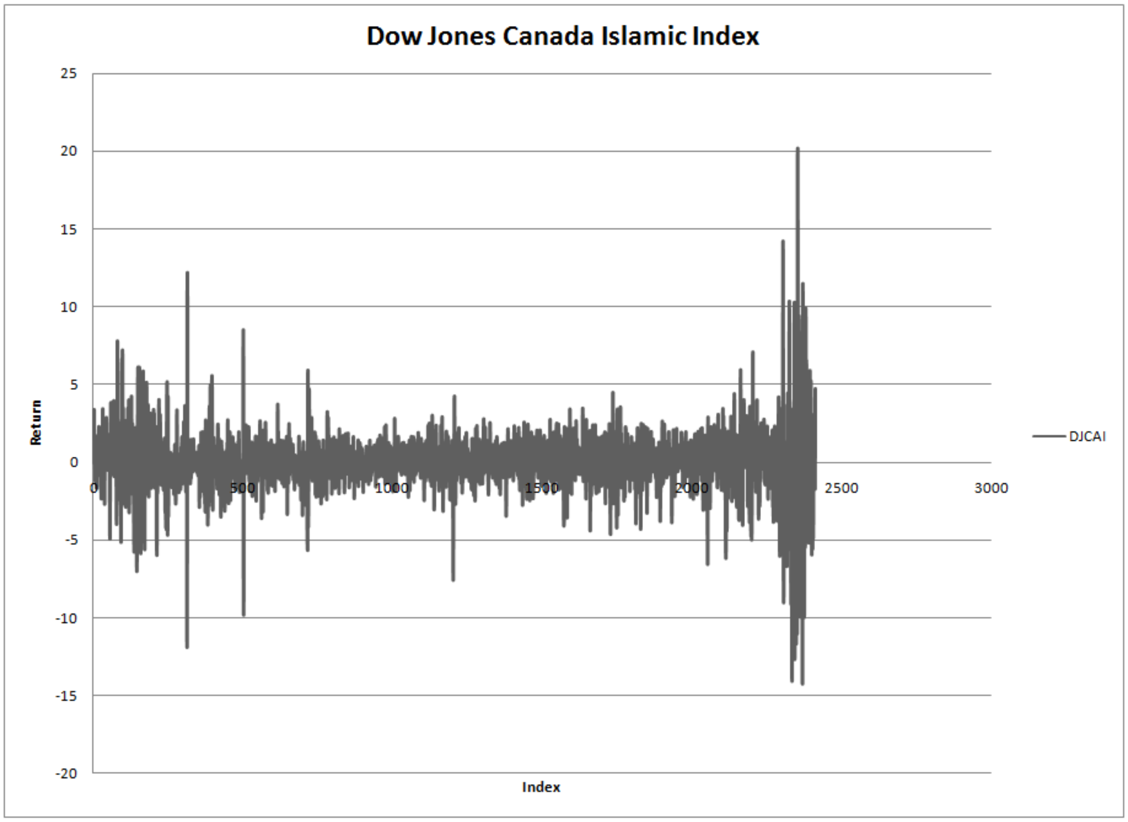

Figure 3.8: Return series of DJCAI.

DJCAI returns	
Mean	0.0336
Standard Deviation	1.9823
Maximum	20.1344
Minimum	-14.0729
Skewness	-0.0862
Excess Kurtosis	13.4719

Table 3.8: Summary statistics for DJCAI returns.

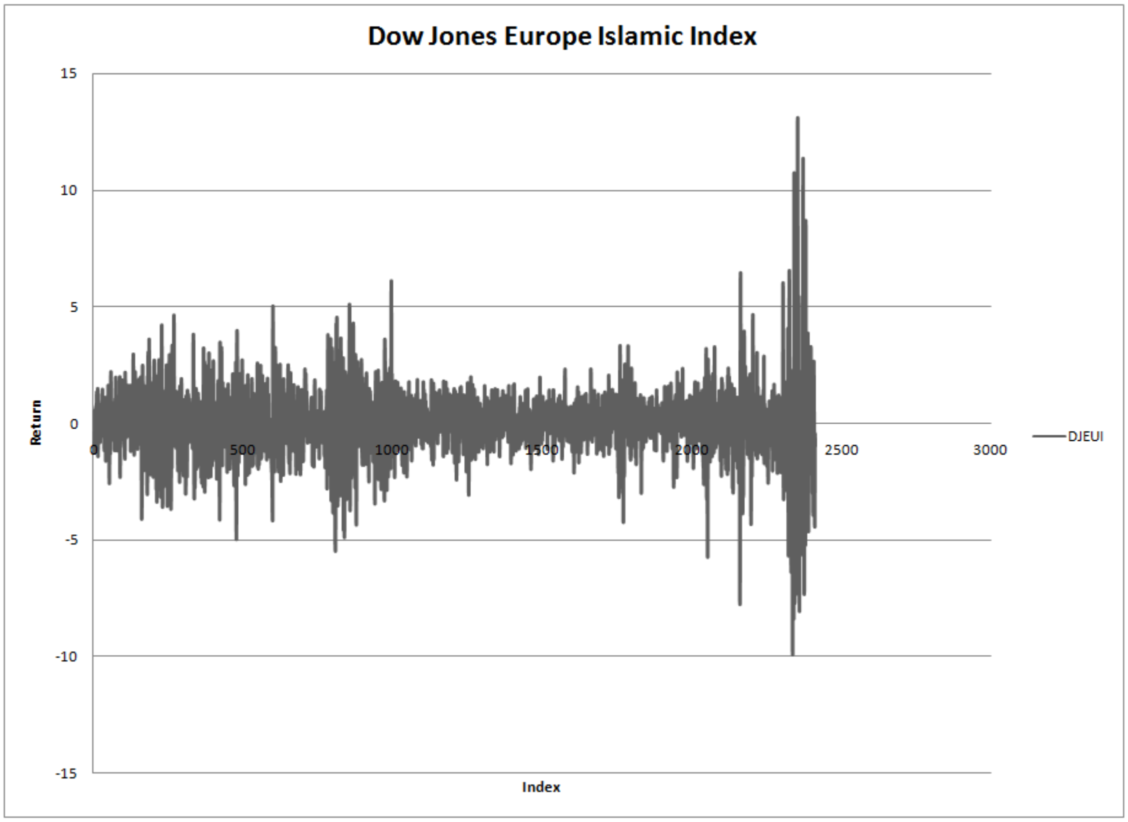

Figure 3.9: Return series of DJEUI.

DJEUI returns	
Mean	-0.0092
Standard Deviation	1.4754
Maximum	13.1075
Minimum	-9.9084
Skewness	0.1085
Excess Kurtosis	9.1837

Table 3.9: Summary statistics for DJEUI returns.

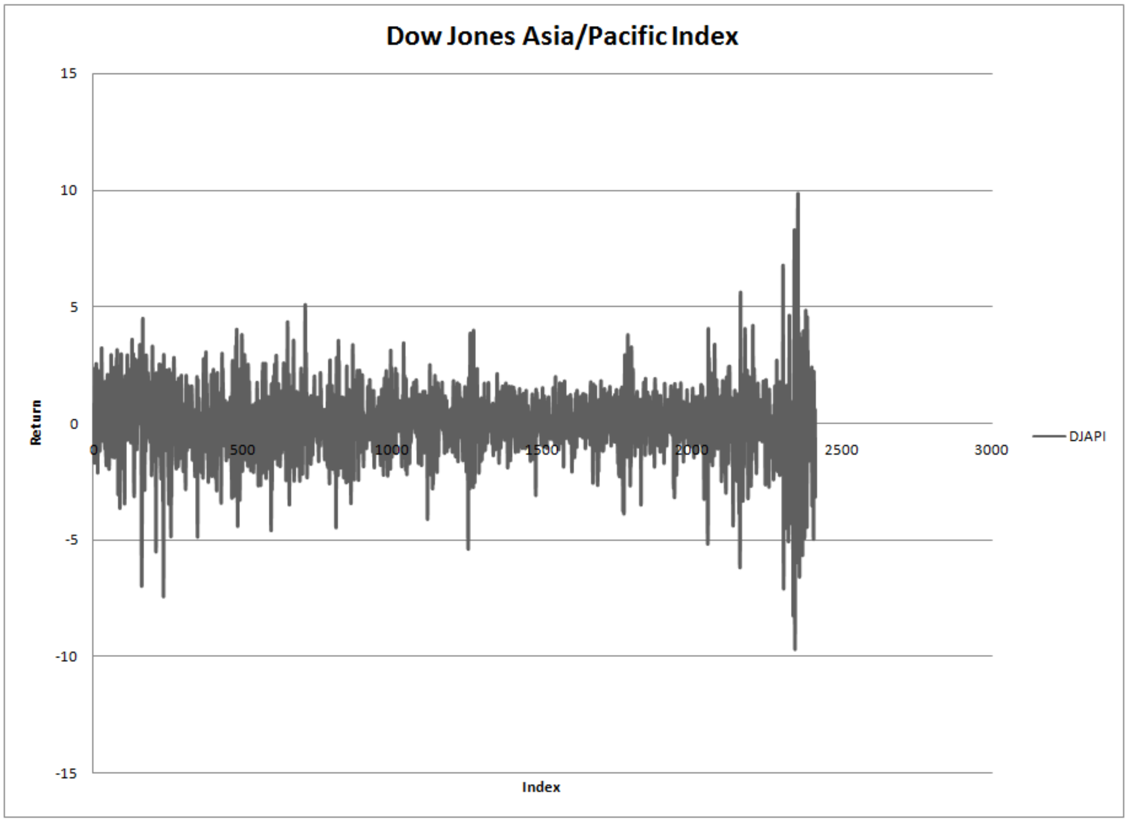

Figure 3.10: Return series of DJAPI.

DJAPI returns	
Mean	-0.0022
Standard Deviation	1.4243
Maximum	9.6905
Minimum	-9.6876
Skewness	-0.3596
Excess Kurtosis	5.3714

Table 3.10: Summary statistics for DJAPI returns.

3.3 Risk-Return Performance

To analyze the performance of Islamic equity under the Markowitz risk-return model three portfolios in four different scenarios are considered. The first portfolio is purely conventional with $N = 5$ assets: DJUS, DJUK, DJCA, DJEU and DJP. The second portfolio is purely Islamic with $N = 5$ assets: DJUSI, DJUKI, DJCAI, DJEUI and DJAPI. The third portfolio is a mixture containing all $N = 10$ assets. The four scenarios considered are:

- Investing with *short selling*
- Investing without *short selling*
- Investing in a period of extreme market volatility using only data from 2008-2009
- Investing without considering the extremely volatile period of 2008-2009

The analysis was done using EXCEL and the solver function was used to compute all constrained optimization problems. To derive the *efficient frontiers*, the vector of means $\vec{\mu}$ and the variance-covariance matrix Σ for each portfolio is needed. For notational purposes, let the subscripts I, C and M denote the Islamic, conventional and mixed portfolios with an additional 1, 2 or 3 representing all the data, the data without 2008-2009 observations and the data with 2008-2009 observations only. This gives

$$\vec{\mu}_{I,1} = \begin{pmatrix} -0.0167 \\ -0.0141 \\ 0.0336 \\ -0.0092 \\ -0.0022 \end{pmatrix}, \Sigma_{I,1} = \begin{pmatrix} 2.0279 & 0.3496 & 0.2518 & 0.2723 & 0.3340 \\ 0.3496 & 2.2946 & 1.0488 & 0.9110 & 0.4835 \\ 0.2518 & 1.0488 & 3.9279 & 1.0000 & 0.5672 \\ 0.2723 & 0.9110 & 1.0000 & 2.1759 & 0.7327 \\ 0.3340 & 0.4835 & 0.5672 & 0.7327 & 2.0277 \end{pmatrix},$$

$$\vec{\mu}_{I,2} = \begin{pmatrix} 0.0016 \\ 0.0161 \\ 0.0695 \\ 0.0219 \\ 0.0283 \end{pmatrix}, \Sigma_{I,2} = \begin{pmatrix} 1.5507 & 0.0600 & -0.0114 & -0.0203 & 0.0017 \\ 0.0600 & 1.5648 & 0.1903 & 0.0807 & 0.0325 \\ -0.0114 & 0.1903 & 2.4208 & 0.1584 & 0.0313 \\ -0.0203 & 0.0807 & 0.1584 & 1.5182 & 0.3274 \\ 0.0017 & 0.0325 & 0.0313 & 0.3274 & 1.5456 \end{pmatrix},$$

$$\vec{\mu}_{I,3} = \begin{pmatrix} -0.1647 \\ -0.2585 \\ -0.2567 \\ -0.2605 \\ -0.2483 \end{pmatrix}, \Sigma_{I,3} = \begin{pmatrix} 5.8551 & 2.6472 & 2.3282 & 2.5924 & 2.9750 \\ 2.6472 & 8.1180 & 7.8982 & 7.5439 & 4.0560 \\ 2.3282 & 7.8982 & 15.9979 & 7.7106 & 4.8120 \\ 2.5924 & 7.5439 & 7.7106 & 7.4127 & 3.9348 \\ 2.9750 & 4.0560 & 4.8120 & 3.9348 & 5.8512 \end{pmatrix},$$

$$\vec{\mu}_{C,1} = \begin{pmatrix} -0.0059 \\ -0.0219 \\ 0.0027 \\ -0.0177 \\ -0.0161 \end{pmatrix}, \Sigma_{C,1} = \begin{pmatrix} 1.5683 & 0.2908 & 0.2923 & 0.2938 & 0.0870 \\ 0.2908 & 1.9220 & 1.0075 & 0.0177 & -0.0595 \\ 0.2923 & 1.0075 & 2.1578 & 0.0725 & 0.0766 \\ 0.2938 & 0.0177 & 0.0725 & 1.9003 & 0.6304 \\ 0.0870 & -0.0595 & 0.0766 & 0.6304 & 1.5910 \end{pmatrix},$$

$$\vec{\mu}_{C,2} = \begin{pmatrix} 0.0146 \\ 0.0148 \\ 0.0362 \\ 0.0181 \\ 0.0108 \end{pmatrix}, \Sigma_{C,2} = \begin{pmatrix} 1.0787 & -0.0193 & 0.0280 & -0.0148 & 0.0850 \\ -0.0193 & 1.1778 & 0.3595 & -0.0311 & 0.0907 \\ 0.0280 & 0.3595 & 1.3244 & -0.0102 & 0.0799 \\ -0.0148 & -0.0311 & -0.0102 & 1.2522 & 0.0554 \\ 0.0850 & 0.0907 & 0.0799 & 0.0554 & 1.1565 \end{pmatrix},$$

$$\vec{\mu}_{C,3} = \begin{pmatrix} -0.1693 \\ -0.3146 \\ -0.2637 \\ -0.3033 \\ -0.2274 \end{pmatrix}, \Sigma_{C,3} = \begin{pmatrix} 5.4400 & 2.7091 & 2.3500 & 2.7012 & 0.2585 \\ 2.7091 & 7.7574 & 6.0851 & 0.3128 & -0.7184 \\ 2.3500 & 6.0851 & 8.7201 & 0.6464 & -0.1212 \\ 2.7012 & 0.3128 & 0.6464 & 6.9740 & 3.1894 \\ 0.2585 & -0.7184 & -0.1212 & 3.1894 & 5.0068 \end{pmatrix},$$

$$\vec{\mu}_{M,1} = \begin{pmatrix} -0.0167 \\ -0.0141 \\ 0.0336 \\ -0.0092 \\ -0.0022 \\ -0.0059 \\ -0.0219 \\ 0.0027 \\ -0.0177 \\ -0.0161 \end{pmatrix}, \Sigma_{M,1} = \begin{pmatrix} 2.0279 & 0.3496 & 0.2518 & 0.2723 & 0.3340 & 1.2180 & 0.3246 & 0.3408 & 0.3238 & 0.1315 \\ 0.3496 & 2.2946 & 1.0488 & 0.9110 & 0.4835 & 0.3125 & 0.6805 & 0.5370 & 0.0503 & -0.0369 \\ 0.2518 & 1.0488 & 3.9279 & 1.0000 & 0.5672 & 0.2250 & 0.6955 & 0.9558 & 0.1847 & 0.0573 \\ 0.2723 & 0.9110 & 1.0000 & 2.1759 & 0.7327 & 0.2970 & 0.6514 & 0.5613 & 0.0017 & -0.0341 \\ 0.3340 & 0.4835 & 0.5672 & 0.7327 & 2.0277 & 0.3278 & 0.3306 & 0.3565 & 0.3124 & 0.0469 \\ 1.2180 & 0.3125 & 0.2250 & 0.2970 & 0.3278 & 1.5683 & 0.2908 & 0.2923 & 0.2938 & 0.0870 \\ 0.3246 & 0.6805 & 0.6955 & 0.6514 & 0.3306 & 0.2908 & 1.9220 & 1.0075 & 0.0177 & -0.0595 \\ 0.3408 & 0.5370 & 0.9558 & 0.5613 & 0.3565 & 0.2923 & 1.0075 & 2.1578 & 0.0725 & 0.0766 \\ 0.3238 & 0.0503 & 0.1847 & 0.0017 & 0.3124 & 0.2938 & 0.0177 & 0.0725 & 1.9003 & 0.6304 \\ 0.1315 & -0.0369 & 0.0573 & -0.0341 & 0.0469 & 0.0870 & -0.0595 & 0.0766 & 0.6304 & 1.5910 \end{pmatrix},$$

$$\vec{\mu}_{M,2} = \begin{pmatrix} 0.0016 \\ 0.0161 \\ 0.0695 \\ 0.0219 \\ 0.0283 \\ 0.0132 \\ 0.0136 \\ 0.0355 \\ 0.0178 \\ 0.0101 \end{pmatrix}, \Sigma_{M,2} = \begin{pmatrix} 1.5507 & 0.0600 & -0.0114 & -0.0203 & 0.0017 & 0.6889 & 0.0066 & 0.0589 & -0.0329 & 0.0782 \\ 0.0600 & 1.5648 & 0.1903 & 0.0807 & 0.0325 & 0.0280 & 0.0209 & -0.0230 & 0.0240 & -0.0244 \\ -0.0114 & 0.1903 & 2.4208 & 0.1584 & 0.0313 & 0.0013 & -0.0014 & -0.0322 & 0.0584 & 0.0468 \\ -0.0203 & 0.0807 & 0.1584 & 1.5182 & 0.3274 & 0.0187 & 0.0361 & 0.0223 & -0.0282 & 0.0179 \\ 0.0017 & 0.0325 & 0.0313 & 0.3274 & 1.5456 & 0.0189 & -0.0008 & 0.0552 & 0.0385 & 0.0000 \\ 0.6889 & 0.0280 & 0.0013 & 0.0187 & 0.0189 & 1.0790 & -0.0191 & 0.0274 & -0.0145 & 0.0851 \\ 0.0066 & 0.0209 & -0.0014 & 0.0361 & -0.0008 & -0.0191 & 1.1789 & 0.3616 & -0.0308 & 0.0920 \\ 0.0589 & -0.0230 & -0.0322 & 0.0223 & 0.0552 & 0.0274 & 0.3616 & 1.3265 & -0.0096 & 0.0819 \\ -0.0329 & 0.0240 & 0.0584 & -0.0282 & 0.0385 & -0.0145 & -0.0308 & -0.0096 & 1.2508 & 0.0558 \\ 0.0782 & -0.0244 & 0.0468 & 0.0179 & 0.0000 & 0.0851 & 0.0920 & 0.0819 & 0.0558 & 1.1563 \end{pmatrix},$$

and

$$\vec{\mu}_{M,3} = \begin{pmatrix} -0.1647 \\ -0.2585 \\ -0.2567 \\ -0.2605 \\ -0.2483 \\ -0.1606 \\ -0.3084 \\ -0.2622 \\ -0.3044 \\ -0.2344 \end{pmatrix}, \Sigma_{M,3} = \begin{pmatrix} 5.8551 & 2.6472 & 2.3282 & 2.5924 & 2.9750 & 5.4634 & 2.8435 & 2.5719 & 3.1552 & 0.4644 \\ 2.6472 & 8.1180 & 7.8982 & 7.5439 & 4.0560 & 2.5661 & 5.9259 & 4.9843 & 0.1839 & -0.3767 \\ 2.3282 & 7.8982 & 15.9979 & 7.7106 & 4.8120 & 1.9806 & 6.2264 & 8.8444 & 1.1111 & -0.1639 \\ 2.5924 & 7.5439 & 7.7106 & 7.4127 & 3.9348 & 2.4994 & 5.5370 & 4.8371 & 0.1628 & -0.3121 \\ 2.9750 & 4.0560 & 4.8120 & 3.9348 & 5.8512 & 2.7779 & 2.9265 & 2.7149 & 2.4442 & 0.4672 \\ 5.4634 & 2.5661 & 1.9806 & 2.4994 & 2.7779 & 5.4900 & 2.7426 & 2.3846 & 2.7323 & 0.2689 \\ 2.8435 & 5.9259 & 6.2264 & 5.5370 & 2.9265 & 2.7426 & 7.8279 & 6.1356 & 0.3168 & -0.7267 \\ 2.5719 & 4.9843 & 8.8444 & 4.8371 & 2.7149 & 2.3846 & 6.1356 & 8.7891 & 0.6501 & -0.1307 \\ 3.1552 & 0.1839 & 1.1111 & 0.1628 & 2.4442 & 2.7323 & 0.3168 & 0.6501 & 7.0509 & 3.2228 \\ 0.4644 & -0.3767 & -0.1639 & -0.3121 & 0.4672 & 0.2689 & -0.7267 & -0.1307 & 3.2228 & 5.0556 \end{pmatrix}.$$

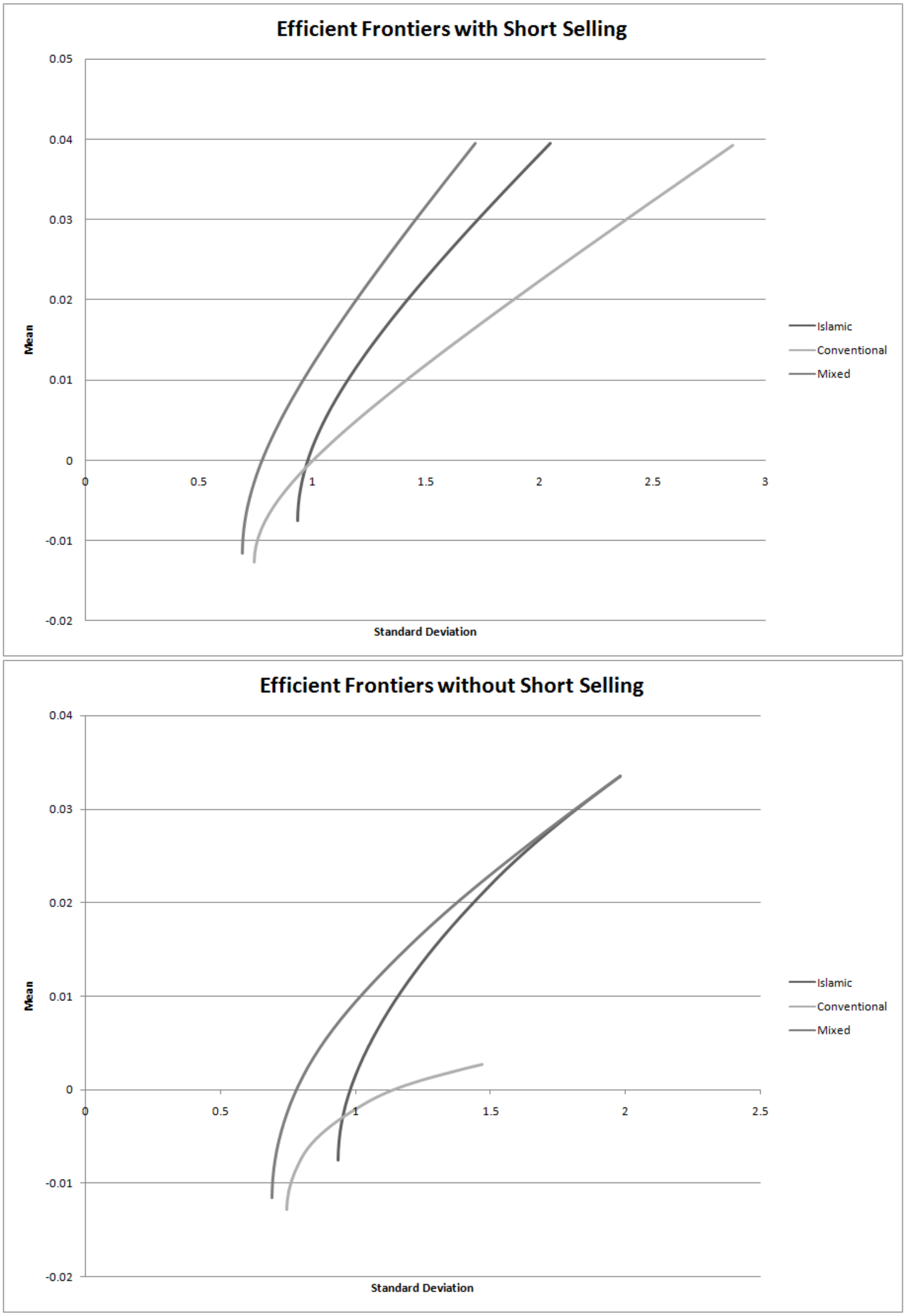

Figure 3.11: Efficient frontiers with and without short selling.

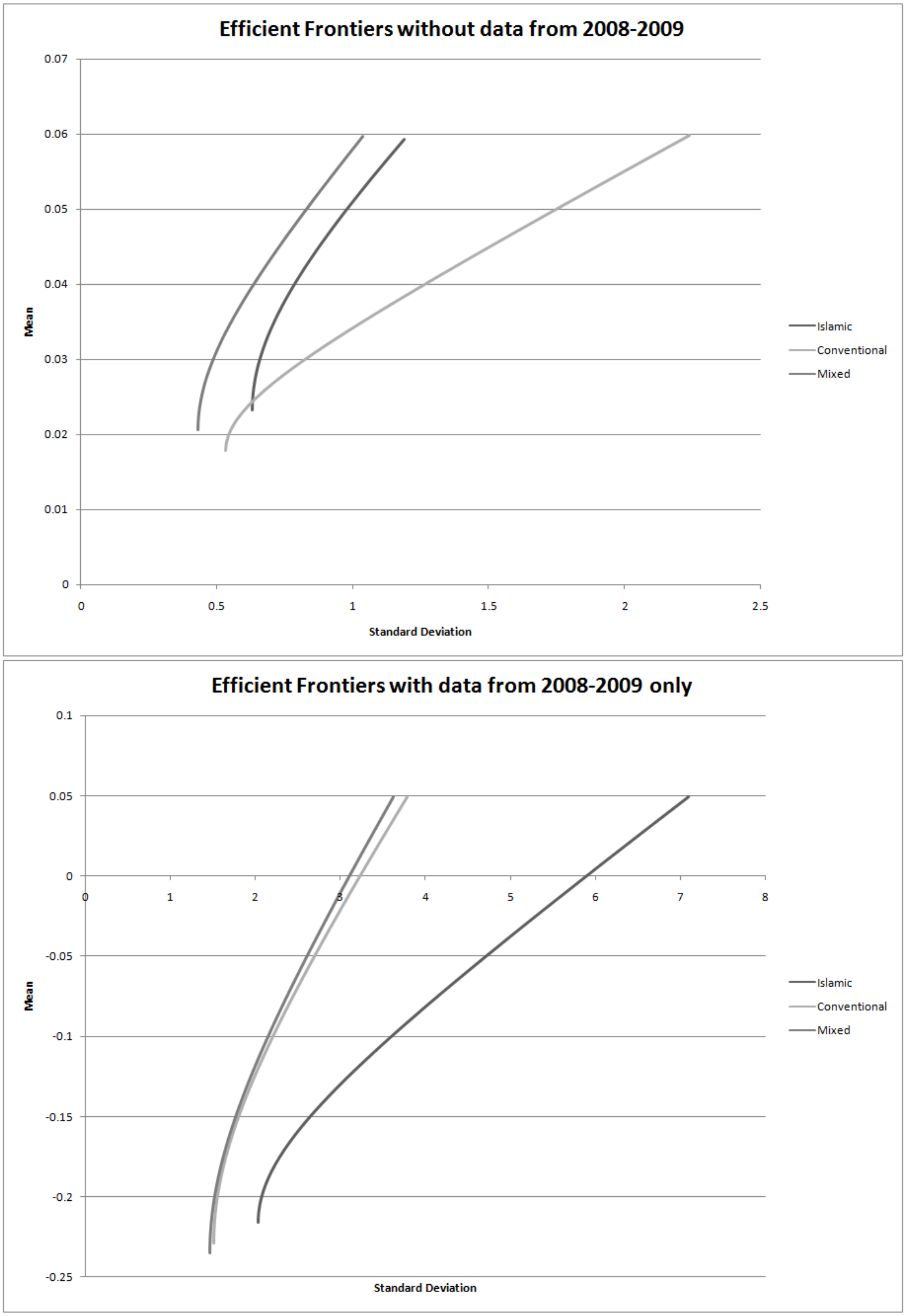

Figure 3.12: Efficient frontiers without 2008-2009 data and with 2008-2009 data only.

From Figure 3.11 the *efficient frontiers* with *short selling* allowed for higher attainable returns and thus dominated the *efficient frontiers* with the *short selling* constraint for all 3 portfolios. Also, with and without *short selling*, the mixed portfolio dominates the purely Islamic and purely conventional portfolios. This is expected since the mixed portfolio is more diversified and more diversity reduces the standard deviation and hence lowers the risk for every given level of return. The dominance of the mixed portfolio is observed in all 4 scenarios. Another key observation is that with or without *short selling* the Islamic portfolio generally dominates its conventional counterpart except for a small set of portfolios around the Islamic minimum-variance portfolio. This suggests that even though the Islamic portfolio is relatively less diversified than the conventional portfolio, due to several investment restrictions, the *Shari'ah* screening process excluded various industries whose absence resulted in risk reduction. As a result, the Islamic portfolio provided a basket of stocks with special and favourable risk characteristics under the Markowitz model.

Figure 3.12 shows that the *efficient frontiers* undergo an upward-left shift when 2008-2009 data is left out. This indicates that the markets in 2008-2009 were highly volatile with periods of low returns. This extreme volatility is also observed in the return plots in Figures 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9 and 3.10. Without 2008-2009 data the dominance of the Islamic portfolio over its conventional counterpart continues. However, from the *efficient frontiers* with 2008-2009 data only there is an interesting observation. The conventional portfolio now dominates the Islamic portfolio. From an investors viewpoint this suggests, when the markets go bad, in the sense of extreme volatility and low returns, the conventional portfolio is the better investment.

To summarize, after analyzing the data under the Markowitz model of risk and return, the Islamic portfolio outperformed the conventional portfolio except during the period of 2008-2009. Due to diversification benefits, the mixed portfolio always dominated the other portfolios. The added constraint of no *short selling* reduced the attainable returns for given risk levels for each portfolio without affecting the hierarchy of *efficient frontier* dominance. Therefore, there is no conclusive evidence that the screening process undergone by stocks in the Dow Jones Islamic indices has a negative impact on the index's performance in comparison to the Dow Jones conventional indices. However, in times of extreme market activity the Islamic portfolio does suffer more than its conventional counterpart.

Chapter 4

Bivariate Copula Extension to Univariate Modelling

In Chapter 1, when modelling the FTSEAW and FTSESAW indices the final model selected was an AR(1) specification for the conditional mean and an APARCH(1,1) specification for the conditional variance with standardized skewed student-t innovations. This chapter extends this model by fitting a bivariate copula to the AR(1) + APARCH(1,1) filtered returns using R statistical software[1].

4.1 Filtered Returns

Let $\hat{z}_t$ represent the filtered returns of the return series r_t where

$$\hat{z}_t = \frac{r_t - \hat{\mu}_t}{\hat{\sigma}_t}, t = 1, 2, \ldots, 1392 \tag{4.1}$$

with

$$\hat{\mu}_t = \hat{\mu} + \hat{\phi}_1 \left(r_{t-1} - \hat{\mu} \right) \tag{4.2}$$

and

$$\hat{\sigma}_t^{\hat{\delta}} = \hat{\omega} + \hat{\alpha}_1 \left(|\epsilon_{t-1}| - \hat{\gamma}_1 \epsilon_{t-1} \right)^{\hat{\delta}} + \hat{\beta}_1 \hat{\sigma}_{t-1}^{\hat{\delta}}. \tag{4.3}$$

$\hat{\mu}$, $\hat{\phi}_1$, $\hat{\omega}$, $\hat{\alpha}_1$, $\hat{\gamma}_1$, $\hat{\delta}$ and $\hat{\beta}_1$ are the parameter estimates attained after fitting the AR(1) + APARCH(1,1) model with standardized skewed student-t innovations to the return series[2]. Let $\hat{\vec{z}}_{AW}$ and $\hat{\vec{z}}_{SAW}$ represent the filtered returns for the FTSEAW and FTSESAW return series respectively.

Figure 4.1 indicates that there is some positive correlation between $\hat{\vec{z}}_{AW}$ and $\hat{\vec{z}}_{SAW}$. It is this dependence the copula will be used to model.

[1] R is a free statistical software package that can be downloaded from http://cran.r-project.org.

[2] Some of these parameter estimates may differ from those in Chapter 1 since R uses a slightly different estimation algorithm than the G@RCH 5.1 module in OxMetrics 5.1.

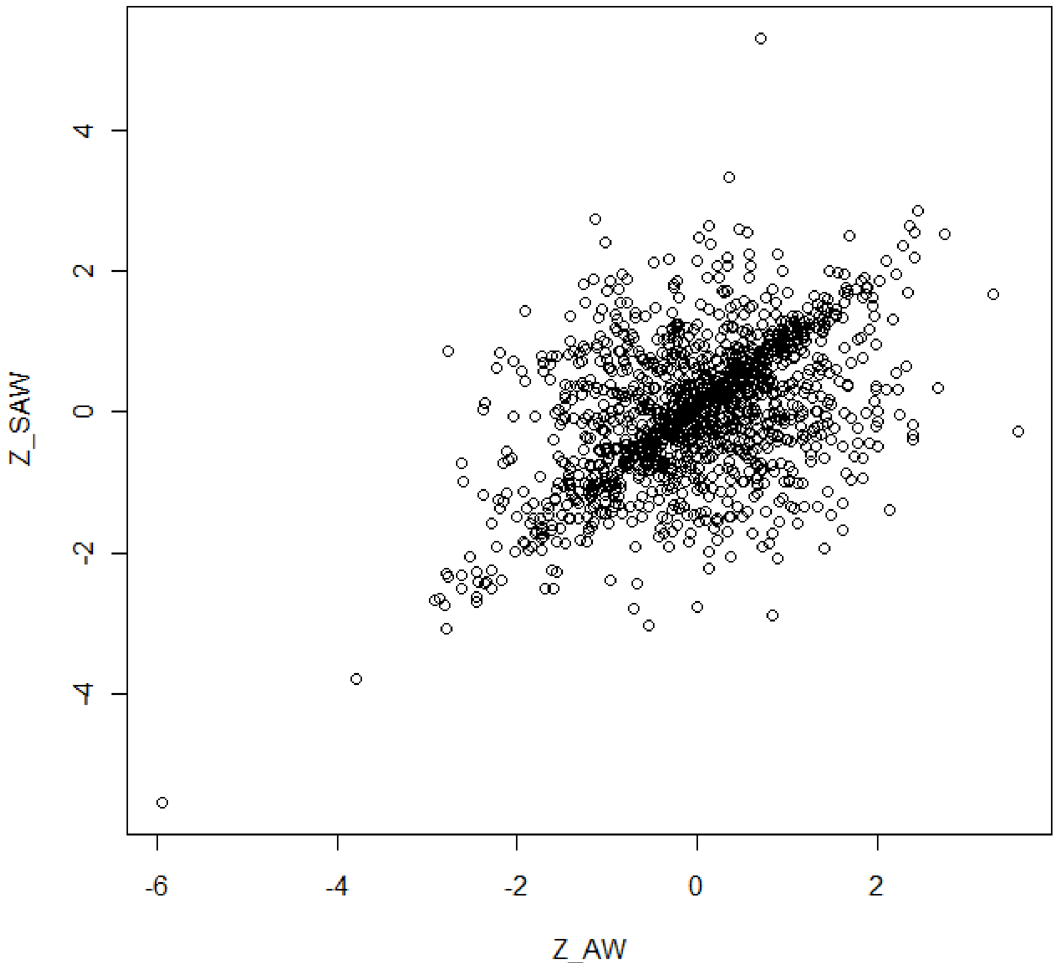

Figure 4.1: Plot of $\hat{\tilde{z}}_{SAW}$ against $\hat{\tilde{z}}_{AW}$.

4.2 Copula

One way of capturing multivariate dependence is through the multivariate modelling tool known as the copula. A d-dimensional copula is a multivariate distribution with Uniform(0,1) marginals and distribution function

$$C(u_1, u_2, \ldots, u_d) = Pr(U_1 \leq u_1, U_2 \leq u_2, \ldots, U_d \leq u_d) \tag{4.4}$$

where $U_1, U_2, \ldots, U_d$ are Uniform(0,1) random variables. Since every continuous random variable can be transformed to be uniform over (0,1) by its distribution function, copulas are able to model the multivariate dependence structure without affecting the marginal distributions. Sklar's Theorem (See Sklar (1959)) showed their exists a unique copula for random variables with continuous distribution functions.

Sklar's Theorem

Suppose $X_1, X_2, \ldots, X_d$ are random variables with continuous distribution functions $F_1, F_2, \ldots, F_d$ and joint distribution function F, then there exists a unique copula C (a distribution function on $[0,1]^d$ with uniform marginals) such that for all $\vec{x} = (x_1, x_2, \ldots, x_d)^T \in \Re^d$:

$$F(x_1, x_2, \ldots, x_d) = C\left(F_1(x_1), F_2(x_2), \ldots, F_d(x_d)\right). \tag{4.5}$$

Conversely, given any distribution functions $F_1, F_2, \ldots, F_d$ and copula C, F defined through (4.5) is a d-variate distribution function with marginals $F_1.F_2, \ldots, F_d$.

4.3 Copula Fitting

After fitting the AR(1) + APARCH(1,1) model recover the residuals or filtered returns $\hat{\vec{z}}_{AW}$ and $\hat{\vec{z}}_{SAW}$ using (4.1). Table 4.1 gives the initial values for r_0 and σ_0 which are needed when calculating $\hat{\mu}_1$ and $\hat{\sigma}_1$ for both series. The marginal distributions of $\hat{\vec{z}}_{AW}$ and $\hat{\vec{z}}_{SAW}$, from Chapter 1, are standardized skewed student-t. Therefore, copula models of the type

$$F\left(z_{AW}, z_{SAW}; \theta\right) = C\left(F_{AW}(z_{AW}), F_{SAW}(z_{SAW}); \theta\right) \tag{4.6}$$

are fitted where C is a copula function, θ is the dependence parameter of the copula and F_{AW} and F_{SAW} are the standardized skewed student-t distribution functions. The parameters of the copula and marginal distributions are estimated by maximizing the log likelihood function. The gaussian and t copulas which belong to the elliptical copula class and the Frank, Clayton and Gumbel single parameter copulas which belong to the Archimedean copula class are considered for capturing the dependence between $\hat{\vec{z}}_{AW}$ and $\hat{\vec{z}}_{SAW}$.

The gaussian and t copulas are derived from the multivariate normal and multivariate t distributions. They have no simple closed form expressions, but like other multivariate distributions with continuous marginals an implicit copula can be extracted. On the other hand, the three Archimedean copulas considered do have simple closed form representations. Their bivariate versions are:

- **Frank Copula**

$$C_\theta^{Fr}(u, v) = -\frac{1}{\theta} \ln \left[1 + \frac{(\exp^{-\theta u} - 1)(\exp^{-\theta v} - 1)}{\exp^{-\theta} - 1}\right]$$

- **Clayton Copula**

$$C_\theta^{Cl}(u,v) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{-1/\theta}$$

- **Gumbel Copula**

$$C_\theta^{Gu}(u,v) = \exp^{-\left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{1/\theta}}$$

4.3.1 Estimation and Analysis

	Initial Values	
	FTSEAW	FTSESAW
r_0	0.0312	0.0272
σ_0	0.3921	0.3467

Table 4.1: Initial values are taken from a GARCH(1,1) specification with normal innovations.

	Estimation results - Copula				
	Gaussian	t	Frank	Clayton	Gumbel
ρ	0.4502(0.0215)	0.4967(0.0261)	-	-	-
ν	-	1.8601(0.1792)	-	-	-
θ	-	-	3.1408(0.1944)	0.8316(0.0651)	1.4545(0.0371)

Table 4.2: Parameter estimates for the copula with the standard errors given in parentheses.

	Estimation results - Marginals			
	FTSEAW		FTSESAW	
	ξ	ν	ξ	ν
Gaussian	0.9391(0.0354)	22.4067(10.1015)	0.9603(0.0370)	15.3018(4.6907)
t	0.9635(0.0317)	11.5714(2.8497)	0.9795(0.0325)	11.2142(2.6874)
Frank	0.9107(0.0353)	17.6723(6.7122)	0.9310(0.0373)	13.8829(4.0466)
Clayton	0.8902(0.0337)	18.3066(6.0079)	0.9152(0.0349)	16.6116(5.2073)
Gumbel	0.9805(0.0363)	15.5197(5.1885)	0.9923(0.0378)	11.9017(2.9827)

Table 4.3: Parameter estimates for the marginal distribution functions with the standard errors given in parentheses.

Copula Assessment	
Copula	Log Likelihood
Gaussian	-3779.323
t	-3691.269
Frank	-3787.78
Clayton	-3750.259
Gumbel	-3773.821

Table 4.4: Maximized log likelihood after fitting each copula to the filtered returns.

Table 4.4 indicates that the t copula has the largest log likelihood value. Hence, based on maximizing the log likelihood function, the t copula provides the best fit to the dependence structure between $\hat{\tilde{z}}_{AW}$ and $\hat{\tilde{z}}_{SAW}$.

4.4 Forecasting

One of the main reasons for financial time series modelling is to make forecasts. The ability to predict future outcomes results in making better decisions today. For example, risk managers may use VaR predictions to understand the future risk exposure in several situations. If the predicted risk exceeds their risk apetite, they can attempt to remove themselves from that position and as a result prevent great losses. Another example is stock market speculators. They do forecasting in order to predict which trades and positions are most profitable.

This section demonstrates how predictions are made from the copula-extended APARCH-type model, as compared to the univariate APARCH-type model. The H-step ahead forecasts from the univariate model are

$$\hat{r}_{1392+h} = \hat{\mu}_{1392+h} + \hat{\epsilon}_{1392+h}, \ h = 1, 2, \ldots, H \tag{4.7}$$

where

$$\hat{\epsilon}_{1392+h} = \hat{\sigma}_{1392+h}\hat{z}_{1392+h}, \tag{4.8}$$

$$\hat{\mu}_{1392+h} = \hat{\mu} + \hat{\phi}_1 \left(\hat{r}_{1392+h-1} - \hat{\mu}\right), \tag{4.9}$$

$$\hat{\sigma}_{1392+h} = \left(\hat{\omega} + \hat{\alpha}_1(|\hat{\epsilon}_{1392+h-1}| - \hat{\gamma}_1\hat{\epsilon}_{1392+h-1})^{\hat{\delta}} + \hat{\beta}_1\hat{\sigma}^{\hat{\delta}}_{1392+h-1}\right)^{1/\hat{\delta}} \tag{4.10}$$

and $\hat{z}_{1392+h}$ are generated from iid standardized skewed student-t distributions with parameters $\hat{\nu}$ and $\hat{\xi}$. $\hat{\mu}$, $\hat{\phi}_1$, $\hat{\omega}$, $\hat{\alpha}_1$, $\hat{\gamma}_1$, $\hat{\beta}_1$, $\hat{\delta}$, $\hat{\nu}$ and $\hat{\xi}$ are estimated by fitting the AR(1) + APARCH(1,1) model with standardized skewed student-t innovations to the FTSEAW and FTSESAW return series.

The copula-extended APARCH-type model generates $\hat{z}^*_{1392+h}$ for both the FTSEAW and FTSESAW together, taking into consideration the dependency structure between $\hat{\vec{z}}_{AW}$ and $\hat{\vec{z}}_{SAW}$. This is done using an alternative representation of (4.6) given by

$$C(u_1, u_2) = F\left(F_{AW}^{-1}(u_1), F_{SAW}^{-1}(u_2)\right) \tag{4.11}$$

where $\hat{u} = (u_1, u_2)^T \in [0,1]^2$. The H-step ahead forecasts from the bivariate copula-extended model are

$$\hat{r}^*_{1392+h} = \hat{\mu}^*_{1392+h} + \hat{\epsilon}^*_{1392+h}, \;\; h = 1, 2, \ldots, H \tag{4.12}$$

where

$$\hat{\epsilon}^*_{1392+h} = \hat{\sigma}^*_{1392+h}\hat{z}^*_{1392+h}. \tag{4.13}$$

$\hat{\mu}^*_{1392+h}$ and $\hat{\sigma}^*_{1392+h}$ are given by (4.9) and (4.10) with $\hat{r}_{1392+h-1}$, $\hat{\epsilon}_{1392+h-1}$ and $\hat{\sigma}_{1392+h-1}$ replaced by $\hat{r}^*_{1392+h-1}$, $\hat{\epsilon}^*_{1392+h-1}$ and $\hat{\sigma}^*_{1392+h-1}$.

The 10-step ahead forecasts for the FTSEAW and FTSESAW return series using the univariate and copula-based models are shown in Figures 4.2 and 4.3. The t copula was used to model the dependency between $\hat{\vec{z}}_{AW}$ and $\hat{\vec{z}}_{SAW}$ since it gave the best fit in Section 4.3.1.

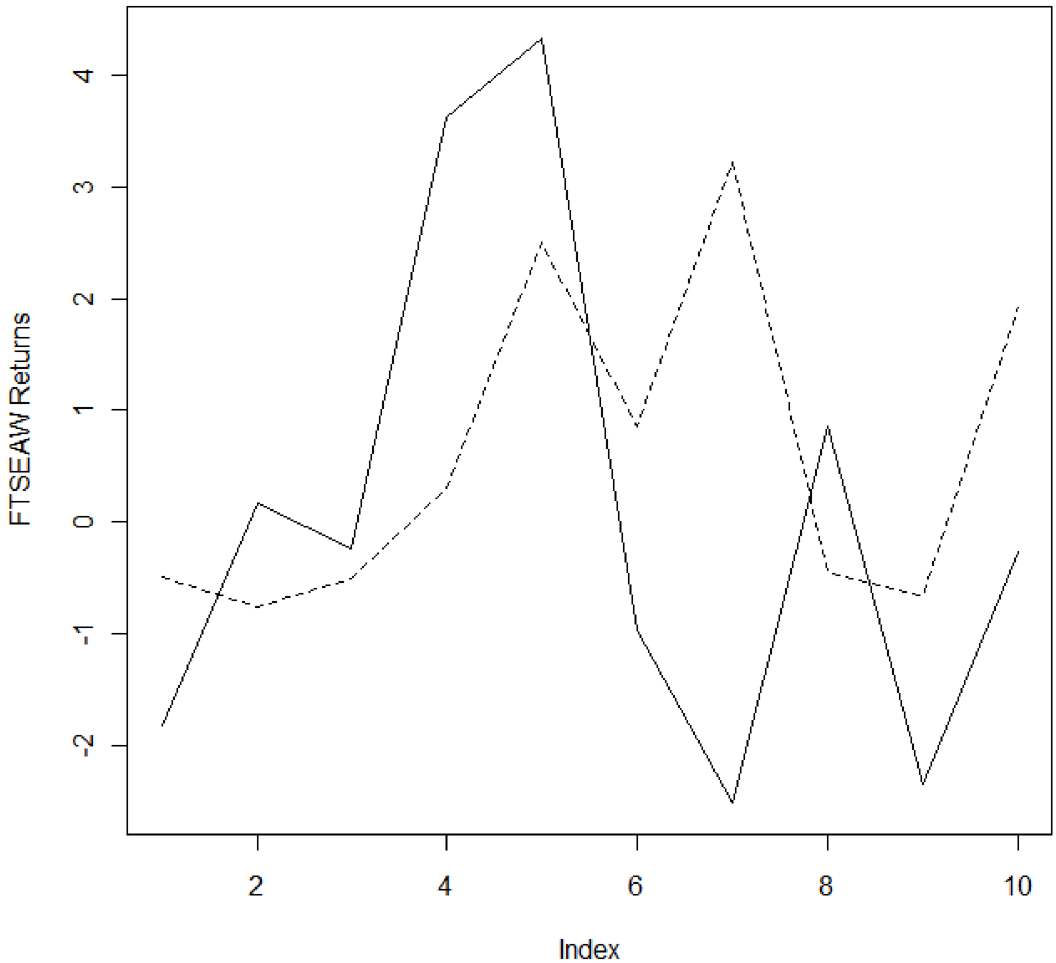

Figure 4.2: FTSEAW 10-step ahead forecasts. The solid line represents the univariate APARCH-type forecasts and the dashed line are the t copula-based forecasts.

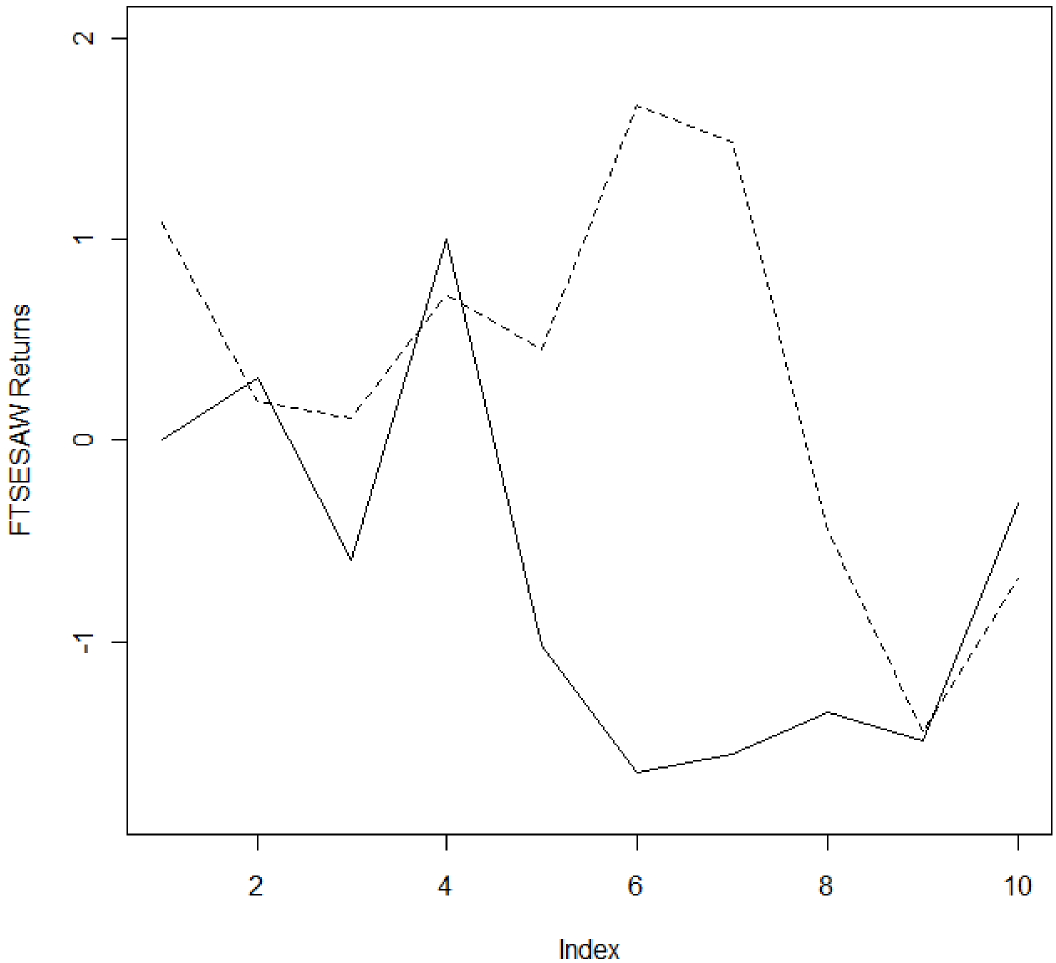

Figure 4.3: FTSESAW 10-step ahead forecasts. The solid line represents the univariate APARCH-type forecasts and the dashed line are the t copula-based forecasts.

Chapter 5

Conclusion

The need for Islamic finance is unquestionable. This ethically driven financial system thrives on the Islamic teachings of wealth distribution, and social and economic justice. Congruent ideologies are observed in other religions such as Judaism, Christianity, Hinduism and Buddhism. Hence, the need for ethical finance is not circumscribed to Muslims. Even if it was confined to Muslims, the expected Muslim population by 2020 is 2.5 billion (which will account for 30% of the world's population) and The Banker (2007) indicates that in the coming years, 40%-50% of the Muslim population's total savings will be in the hands of Islamic banks. Thus, it is expected that this financial sector will continue to grow and expand worldwide. The impressive growth enjoyed in the past however, has slowed down on account of regulatory problems. These problems have resulted in inconsistency and non-uniformity across different Islamic financial institutions unlike in conventional financial institutions, where uniformity is attained through various standards set by various regulatory bodies. Therefore, an appeal is made for the existing Islamic regulatory bodies to coalesce so that growth is no longer hindered by non-uniformity. Another problem faced by the Islamic financial industry is the absence of quantitative academic research. The purpose of this thesis was to contribute to Islamic empirical research by analyzing the log return series of several Islamic stock indices. Firstly, univariate modelling of the FTSE All World and FTSE *Shari'ah* All World return series was done using special GARCH-type models. Secondly, an assessment of the performance of Islamic indices under the Markowitz model was done using various Dow Jones indices. Lastly, copulas were used to model the dependency between the FTSE All World and FTSE *Shari'ah* All World indices.

After model fitting and testing, the AR(1) + APARCH(1,1) model with standardized skewed student-t innovations provided the best fit for both the FTSE All World and FTSE *Shari'ah* All World return series. This model was also the most successful at VaR modelling for both long and short trading positions making it attractive from a risk manager's perspective. Under the CTE risk measure, in short trading positions, the Islamic index was riskier due to higher CTE values. However, in long trading positions the results were not conclusive as to which is the riskier investment.

The *efficient frontier* analysis under the Markowitz model of risk and return concluded that the Islamic Dow Jones portfolio was generally less risky than the conventional Dow Jones portfolio, except during the highly volatile period of 2008-2009. However, a mixture of both portfolios always outperformed the individual portfolios due to diversification benefits. The added constraint of no *short selling* did not affect the hierarchy of portfolio dominance, but for every portfolio, it reduced the possible attainable returns. As a result, there was no decisive eveidence that suggested the screening process undergone by Islamic Dow Jones indices has a negative impact on the indices performance in comparison to the unrestricted conventional Dow Jones indices.

Lastly, after fitting the AR(1) + APARCH(1,1) model with standardized skewed student-t innovations to the FTSE All World and FTSE *Shari'ah* All World return series, it was observed that the filtered returns exhibited some positive correlation. The gaussian, t, Frank, Clayton and Gumbel copulas were used to capture this dependency and based on maximizing the log likelihood function, the t copula performed the best.

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