Capacity Pricing in Electric Generation Expansion

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

The focus of this thesis is to explore a new mechanism to give added incentive to invest in new capacities in deregulated electricity markets. There is a lot of concern in energy markets, regarding lack of sufficient private sector investment in new capacities to generate electricity. Although some markets are using mechanisms to reward these investments directly, e.g., by governmental subsidies for renewable sources such as wind or solar, there is not much theory to guide the process of setting the reward levels.

The proposed mechanism involves a long term planning model, maximizing the social welfare measured as consumers' plus producers' surplus, by choosing new generation capacities which, along with still existing capacities, can meet demand.

Much previous research in electricity capacity planning has also solved optimization models, usually with continuous variables only, in linear or non-linear programs. However, these approaches can be misleading when capacity additions must either be zero or a large size, e.g., the building of a nuclear reactor or a large wind farm. Therefore, this research includes binary variables for the building of large new facilities in the optimization problem, i.e. the model becomes a mixed integer linear or nonlinear program. It is well known that, when binary variables are included in such a model, the resulting commodity prices may give insufficient incentive for private investment in the optimal new capacities. The new mechanism is intended to overcome this difficulty with a capacity price in addition to the commodity price: an auxiliary mathematical program calculates the minimum capacity price that is necessary to ensure that all firms investing in new capacities are satisfied with their profit levels.

In order to test the applicability of this approach, the result of the suggested model is compared with the Ontario Integrated Power System Plan (IPSP), which recommends new generation capacities, based on historical data and costs of different sources of electricity generation for the next 20 years given a fixed forecast of demand.

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Above all, my parents, Parvaneh and Mohammad, receive my deepest gratitude and love for their dedication and the many years of support during my studies that provided the foundation for this work.

“Success means having the courage, the determination, and the will to become the person you believe you were meant to be”

~George Sheehan

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1 Introduction

Introduction of competitive electricity markets raised lots of concerns in many countries on reliability of these systems for providing enough generating capacity in response to demand growth. Crises such as the 2000-2001 California blackout are sometimes blamed on lack of investments in new generation, due to inefficient market mechanisms, resulting in insufficient revenue for new generators (Taylor & VanDoren, 2001).

Although competitive market instruments are supposed to bring supply and demand to an equilibrium point, where adequate supply meets demand by providing reasonable prices for producers and consumers, electricity markets suffer from market imperfections which are attributed to unique characteristics of such markets. In a theoretical energy market, price signals provide not only sufficient electricity supply, but also an efficient technology mix (Joskow & Tirole, Retail Electricity Competition, 2005b). Unusual electricity market characteristics, causing the market to deviate from its equilibrium point are unpredictability of demand, lack of real time pricing, price volatility, inability to control the path of power flow on transmission lines, non-storability of electricity, shortage intolerance and requirements to balance supply and demand to meet physical constraints such as voltage, frequency and stability (Joskow, 2006).

In order to stabilize electricity markets, system operators set short term market rules and regulations which interfere with "the invisible hand of the market" (Smith, 1776) and therefore cause market disequilibrium. For instance, the competitive market clearing price during peak hours is much higher than what consumers pay in the wholesale market due to time-averaging of wholesale prices for consumers' bills and also due to price caps, enforced by market regulator.

These price caps limit electricity prices to levels below market clearing prices in peak periods, when demand is high. Although generators may make positive short-run operating profits (Revenue minus variable cost), to make the profit in the time horizon of the plan on the investment in generation, they (especially peak generators) rely on very high prices (peak prices) that happens occasionally and therefore a price cap can have a negative impact on their survival. For instance, in 2001, in New England, 93% of energy supply was provided by 55% of power capacity and the rest of energy demand, just 7%, was supplied by 45% of available power capacity (IAEE, 2003). Hence, putting price limitations for that 7% of demand which is usually peak load could discourage new investors, since they are not going to be able to make enough profit during the limited hours of their operation.

Although, this type of market mitigation policy benefits consumers by enforcing lower prices, it doesn't favor suppliers since they might not recover all their capital cost and operating costs of generation. For instance, Table 1-1 demonstrates net energy and ancillary services revenue for a new combustion turbine peaking plant in the PJM market. The annualized 20-year fixed cost for this generator is $70,000/Mw/year; however during none of its operation years, does it make enough profit to recover costs and therefore this investment is considered infeasible (Joskow,2006).

Year	Net Energy and Ancillary Services Revenue ($/MW-Yr)
1999	64445
2000	18866
2001	41659
2002	25622
2003	14544
2004	10453
Average	29265

Table 1-1: Net Revenue for a New Combustion Turbine

Source: (PJM, 2005)

Some advocate abolishing price caps as a solution, but even without a price cap, there can still be a problem of insufficient revenue, due to “lumpiness” of investment in new electric capacity (Scarf, 1994). Therefore, the models of this thesis do not include price caps; they focus on the lumpiness of investment and suggest mechanisms to overcome this problem by introducing two part pricing or capacity payments.

Two part pricing mechanism encompasses two types of payments; market clearing prices and capacity payments. Available literature on this issue is reviewed, and a novel approach for calculating capacity payments is presented to overcome deficiencies of the previous proposals.

The thesis is organized as follows: Section 2 explains the effects of price caps on the capital cost recovery process of electricity generators and further elaborates the existing mechanism of capacity payment to mitigate the effects of price caps. Section 3 reviews literature on difficulties to price binary variables in mixed integer linear programs (MIPs) and methodologies to calculate these prices. Section 4 proposes a mixed integer nonlinear social welfare program (MINLP), and methodologies to calculate capacity payments based on (O’Neill et al.,2005) and (Fuller, 2008) proposals. Furthermore, Section 5 examines the mathematical models introduced in section 4 based on Ontario’s integrated power system plan (IPSP) data. The thesis will be concluded by section 6, in which the thesis is summarized and some directions for further research have been proposed.

2 Economics of Electric Generation Expansion: The Standard View

This chapter further elaborates the standard view of capital cost recovery by electricity generators; the effect of price caps; and the existing practice of capacity payments.

2.1 Cost Recovery

Investment in electricity markets is a risky business, tied up with many uncertainties, which complicate the decision making process of investors. In order to generate electricity in these markets, while assuring a minimum expected rate of return, the investor requires confidence in the prediction of estimates related to factors, such as:

- Hours of operation during the year
- Electricity prices during different load types (peak, intermediate and base)
- Average level of fuel cost
- Elasticity of demand
- Weather conditions
- Maintenance outages and system reliability
- Labor costs

Although there is a high level of uncertainty involved in the predicted estimate of each of the above factors, the decision to invest assumes that free market mechanisms will let investors obtain enough profit, recovering the capital and operating costs of their operation based on an adequate level of demand. If price is always marginal cost of the generator, e.g. a peaker, then it cannot cover its capital cost, however price is sometimes set by demand when the system is at capacity.

Figure 2-1 demonstrates market clearing prices and their contribution to generators' cost recovery. As shown, during base load (Demand 1), the cheaper generating facilities can make a little bit of short-run operating profit (R_{mc1}, the shaded area below P_1) to recover their capital costs, since the market clearing price (P_1) is higher than their marginal cost; however the more expensive and the non-active ones wouldn't be able to make any profit. On the other hand, during peak load (Demand 2), due to scarcity of resources, the market clearing price (P_2) increases, such that all facilities would be able to make a great amount of short-run operating profit (area of R_c and R_{mc2}) to recover their capital costs.

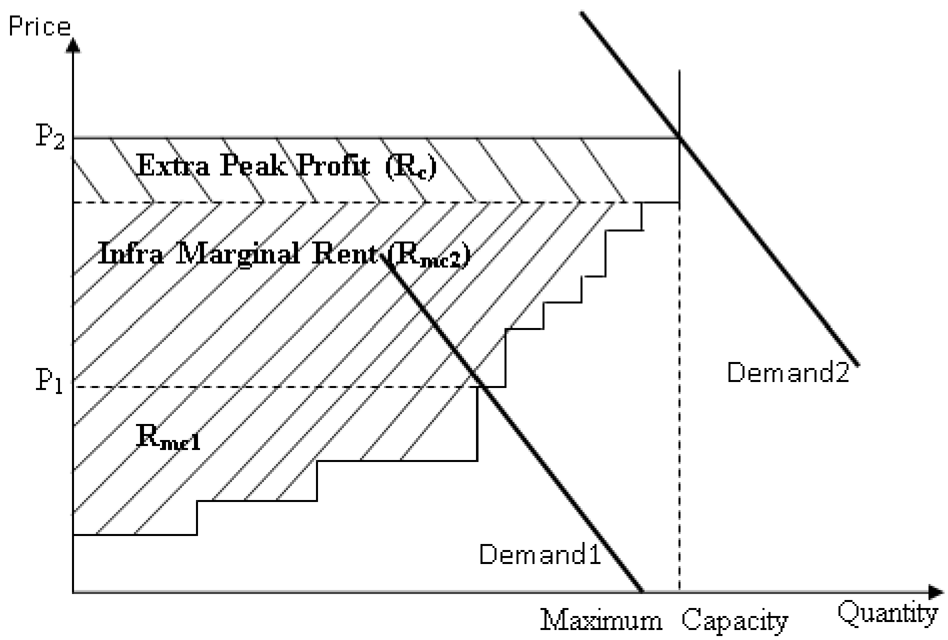


Figure 2-1: Generators' Cost Recovery

Base generators like nuclear and coal facilities which have high capital cost and low operating cost, produce electricity through the year, and therefore would be able to make enough revenue to recover their operating cost and a large part of their capital cost (perhaps all); however peaking generators have low capital cost and high operating cost and operate for a small fraction of time; therefore they would be able to recover all their costs only by high market clearing prices occurring during peak periods.

If some generators cannot recover their capital costs, then they will not be replaced, causing the maximum capacity (vertical line in Figure 2-1) to move to the left, raising the peak price P_2. A long-run equilibrium may result in which all generators earn enough to recover their capital costs.

Although high market clearing prices during peak periods are essential for peaking generators to survive, governmental regulations impose price caps, in order to prevent huge costs on consumers, forbidding new generators from recovering their costs. However, regulators use other instruments such as forward contracts, subsidies and capacity payments to hedge producers against price fluctuations and market rule changes. The next part explains capacity payments in more detail.

2.2 Capacity Payments

Capacity payments are additional payments to producers to recover what is called "missing money" (Carmpton & Stoft, 2006) due to price caps. They are incentive mechanisms, practised in many countries to promote investment in new generation. For instance, electricity producers in England and Wales could earn capacity payments during peak periods until 2001, when this policy was cancelled by the introduction of the New Electricity Trading Arrangements (NETA) (Green & Newbery, 1995). Also, PJM (Pennsylvania New Jersey Maryland) wholesale market has recently introduced a Reliability Pricing Model (RPM) under its long term resource adequacy program to assure the adequate supply of energy in the future by providing additional payments to producers. Before RPM, this market was using another sort of capacity payment, called ICAP (Installed Capacity), which was less payment in a shorter period of time (Chandley, 2008). Other countries like Argentina, Chile and Columbia have also implemented this incentive mechanism.

Although capacity payments may induce new generation, the regulator should be careful not to put a huge burden on consumers by very high electricity prices. Furthermore, in countries with imperfect electricity markets, in which some generators have excessive market power, this incentive mechanism might be abused; since those generators can easily exaggerate the value of lost load (VOLL) and therefore increase the amount of capacity payment (Chuang & Felix, 2000). Due to this problem, capacity payments in England market went so high that they were 20% of total payments to generators (Green & Newbery, 1995).

2.3 Summary

Capacity payments have been used to counter the effects of price caps on the ability of generators to recover all their capital costs. Since these cap prices reduce the prices and hence impact capital cost recovery for generators some policy instruments such as forward contracts and capacity payments have been introduced to incentivize new investments.

In the next chapter, we review literature which suggests another reason for capacity payments: the "lumpiness" of investment in capacity. In mathematical terms, this means that investment in new capacity comes in large, discrete chunks, i.e. it is represented by integer variables other than by continuous variables.

3 Literature Review on Pricing of Integer Activities

A model of a market with some discrete activities and some continuous activities is a mixed integer program (MIP). Prices are often extracted as dual variables when all variables are continuous, but a MIP model presents special difficulties.

The classic work of interpreting dual variables in MIP programs goes back to Gomory's and Baumol's paper, looking into dual prices and their relationship with the marginal cost of adding indivisible sources (Gomory & Baumol, 1960). They introduced a cutting plane methodology (adding new constraints) in order to find a solution to a MIP program and used the dual variables of these constraints to price the cost of integer activities; however these extra prices are related to the choice of additional constraints and wouldn't result in unique answers. Moreover, some integer constraints will have zero prices while they have positive prices in non-integer solutions.

Shapley and Shubik used the dual variables of an assignment problem in a market with indivisible products, which modeled as a two-sided assignment game, to clear the market (Shaley & Shubik, 1972). Their proposal is valid only if the linear programming relaxation solves the integer programming representation of the market as well (O'Neill, 2005).

Williams extended Gomory's and Baumol's work by examining the mathematical and economic properties of LP duality and relating them to integer programming dualities (Williams, 1996). Proving that the dual program, proposed by Gomory and Baumol, doesn't provide optimality, he introduced a more complicated dual problem, satisfying optimality; however it doesn't satisfy complementarity conditions.

In order to resolve missing money problem, in electricity markets, several studies in the literature proposed methodologies to calculate additional capacity payments while maintaining the market equilibrium. Scarf raised the problem of indivisibility and equilibrium prices by suggesting that, lots of activities involve non-convexities or indivisibilities, such as building a new generator, and therefore it makes it difficult to introduce equilibrium prices in such activities (Scarf, 1994).

O'Neill et al. proposed a new two part pricing scheme, promising market equilibrium in markets with non-convexities (O'Neill, 2005). In this proposal, they introduce an auxiliary linear program, with additional constraints in which binary variables are set at their optimal values, derived from the MILP program. Interpreted in the context of electricity investments, the dual variables of these additional constraints are used as capacity payments, which along with commodity prices provide enough incentive for new investors to invest in electricity markets. However, this methodology has been criticized because it discriminates among investors by paying them different additional payments; also sometimes the dual variables of these constraints are negative which makes it difficult to implement in real practices; moreover since the total amount of payments to producers is not equal to the total amount of money collected from consumers, it's not clear where the money comes from in this mechanism (Fuller,2008).

Hogan and Ring discussed another incentive concept in electricity markets, called uplift pricing, which are the additional payments to producers besides market clearing prices (Hogan & Ring, 2003). In order to reduce the burden on consumers for these extra payments, they suggested a minimum uplift pricing scheme while maintaining the social welfare.

Sen and Genc introduced “startup” prices, which are non-negative for built capacities and zero for others (Sen & Genc, 2008). The suggested prices follow a two part pricing scheme and they provided equilibrium although still allowing discrimination among producers.

Finally, Fuller proposed a general definition of equilibrium, to price both continuous and binary variables in a more general form of models than social welfare maximization, to include nonintegrable models as well (Fuller, 2008). The ideas were applied to the short run unit commitment problem in (Fuller, 2009), in which the binary variables represent the on/off status of generators. An efficient way to calculate commodity prices and binary related prices (as payments in proportion to capacities) was derived by Fuller (2009) as a modification of the method of O’Neill et al (2005). The capacity prices are non-negative and nondiscriminatory.

This thesis applies the approaches of (Fuller, 2008) and (Fuller, 2009) to the long run problem of investment in generation capacity, in which the binary variables represent build/don’t build decisions.

4 The Mathematical Model

The proposed model is a mixed integer non-linear social welfare maximization program which maximizes consumers' plus producers' surplus. This model could be implemented by either a market operator or a regulator to identify the near optimal number of permits which could be given to potential investors, the starting time of the developments and also the amount of energy that could be produced by new and old capacities in order to achieve its long term energy supply goals and targets.

Social welfare maximization model is preferred over other mathematical models such as producer's cost minimization or profit maximization programs, since it reimburses new producers by not putting a huge burden on consumers and therefore maintains the market equilibrium. It's important for a regulator to be confident that generators can recover the cost of new investments, and on the other hand it's vital to support consumers by maintaining the energy price at a reasonable level. A social welfare maximization model addresses both of these critical planning issues.

This chapter presents the proposed long term planning model, used to obtain capacity prices. It is a price responsive model with a linear demand function.

4.1 Social Welfare

Social welfare consists of consumers' and producers' surplus. The former is a measurement, indicating the benefit consumers get from using the product minus the money they pay for that, and the latter is the difference between the amount for which a good sells and the minimum amount necessary for generators to be willing to produce electricity (Perloff, 2007).

In order to measure this concept, different methodologies have been suggested. Some economists use the utility functions of consumers and suppliers to calculate the amount of consumption and generation which satisfies both players. However, this approach is not practical, since it's a very complicated task to find utility functions of all individuals in a market. Also, even if an individual's utility functions existed, it would be impossible to compare them, because each person has a different scale to measure his/her happiness or utility function. Therefore we measure social welfare in terms of dollars, which is the amount of money that is enough for both generators and consumers to supply and consume.

In order to measure this concept, the information, obtained from demand and supply curves could be used. The demand curve can be interpreted as a "marginal value" curve for consumers; thus, the integral of this curve gives total value or "benefit" to consumers. In Figure 4-1(b), the area underneath the demand curve, up to point Q demonstrates the total value to consumers for consuming the amount of Q. Consumers' surplus (CS) could be obtained by subtracting expenditures (E),assuming a single price (P), from total value. On the other hand, producers' surplus (PS) is a profit measure, indicated by total revenue minus variable cost (VC), which is the area under the marginal cost curve or the supply curve (Fraser, 2008).

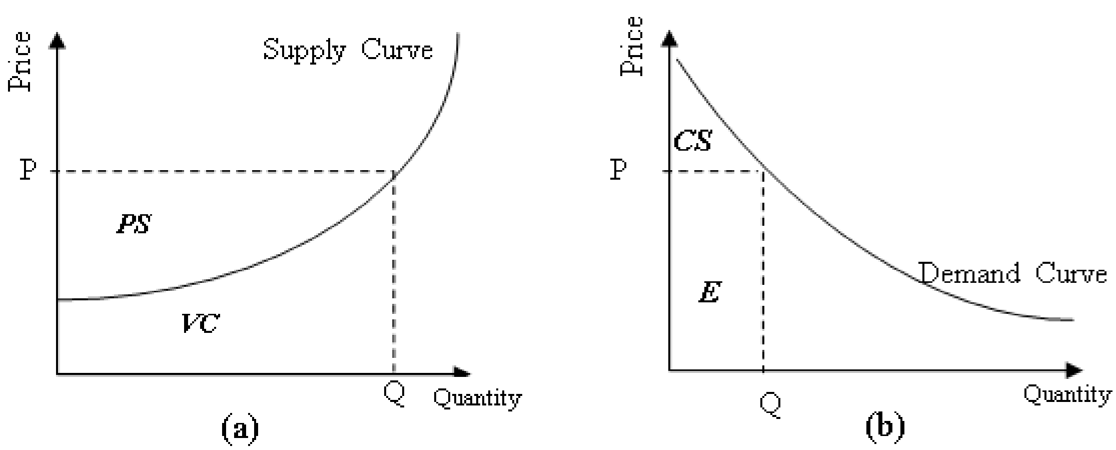


Figure 4-1 : Consumers' and Producers' Surplus

Knowing the producers' and consumers' surplus functions, each evaluated at the same quantity (Q), social welfare is calculated by summation of them. Competitive markets always try to maximize social welfare by finding the equilibrium level of production and price which is the point that supply and demand curves intersect. This equilibrium point has been shown as e_1 in Figure 4-2 while indicating the generation of Q_1 and equilibrium price of P_1. It generates the consumers' surplus of (A+B+C) and producers' surplus of (D+E), resulting in social welfare of (A+B+C+D+E) .In the same figure, if output and price, paid by consumers are different, then the social welfare will be decreased. For instance, in case a generator produces less electricity (Q_2), price increases to P_2, and therefore the consumers' welfare will be reduced to A. On the other side producers' surplus will be B+D, which is reducing the social welfare to the area of (A+B+D).

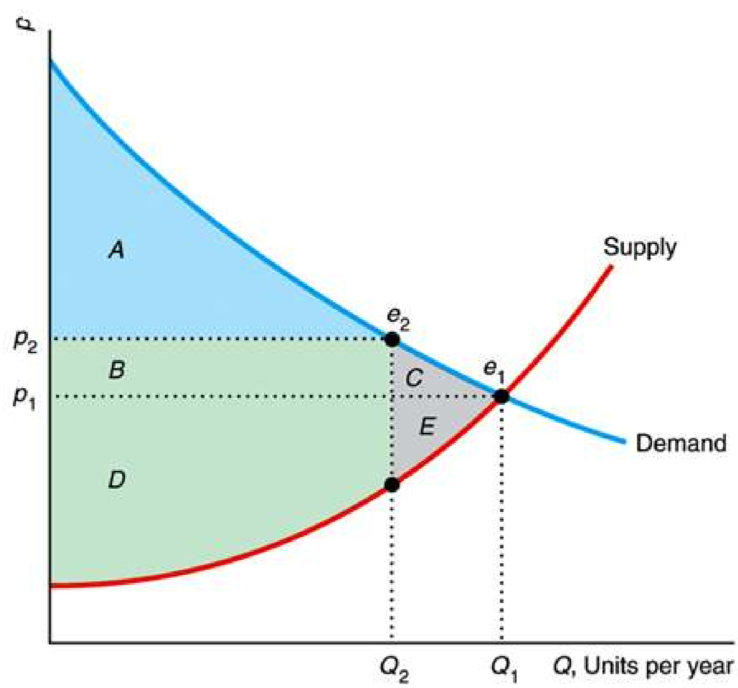


Figure 4-2: Social Welfare
Source: (Perloff, 2007)

4.2 General Formulation (MINLP Model)

In contrast with the common market equilibrium models which consider just continuous variables (Samuelson, 1952), (Gabriel, Kiet, & Zhuang, 2005), the model of this thesis includes both continuous and binary variables which makes a complicated mathematical problem. Formulation 4-1 gives a general overview of the model.

MINLP MODEL

$$Max \sum_{time} Area\ Underneath\ Demand\ Curve$$

$$- \sum_{firm,time} Variable\ \ cost\ of\ old\ facilities$$

$$- \sum_{firm,time} Variable\ and\ capital\ cost\ of\ new\ \ facilities$$

$Subject\ to:$

$Supply\ Constraint$
$Capacity\ Constraint$
$Binary\ Constraint$
$Regulatory\ Constraints$

Formulation 4-1

The following sections describe the variables, objective function and constraints in more detail.

4.2.1 Decision Variables

This section describes important decision variables used in the model, with symbols introduced one by one. For a complete list of indices, variables and parameters please refer to Appendix A.

4.2.1.1 Demand

In this model, demand in period *t* and block *s* (e.g., *s* = base, intermediate or peak) (q_{ts}^D), measured in energy units of MWh, is a decision variable, responsive to price fluctuations. Different demand blocks could be considered in the model: for instance in the next numerical example, three demand blocks of Base, intermediate and peak have been examined. However, depending on the model purposes, more or fewer demand blocks could be used.

The set of demand blocks (S) can be defined by approximating the load duration curve, which is obtained by rearranging the hourly load patterns during a period of time, e.g., a year or 8760 hours, or a month or the percentage of time for which it occurs. The loads are reordered from chronological order to an ordering from highest to lowest. Figure 4-3 shows three load duration curves (LDCs), for the month of February in Ontario, in three different years.

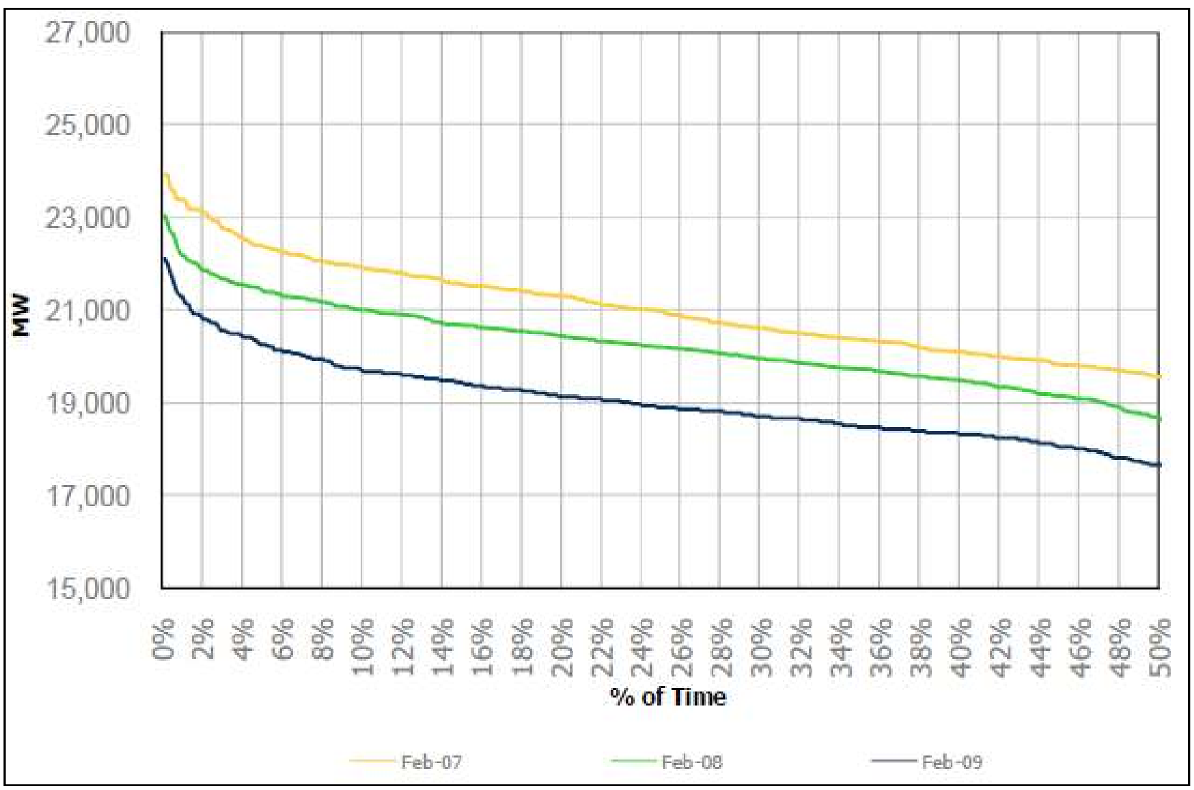


Figure 4-3: Ontario's Load Duration Curve
Source: (IESO, 18 Months Outlook From June 2009 to Dec 2010, 2009)

Usually vertical or horizontal approximations of load duration curve, (Figure 4-4) are used, in order to fit an LDC into a linear programming model (Sherali, 1982).

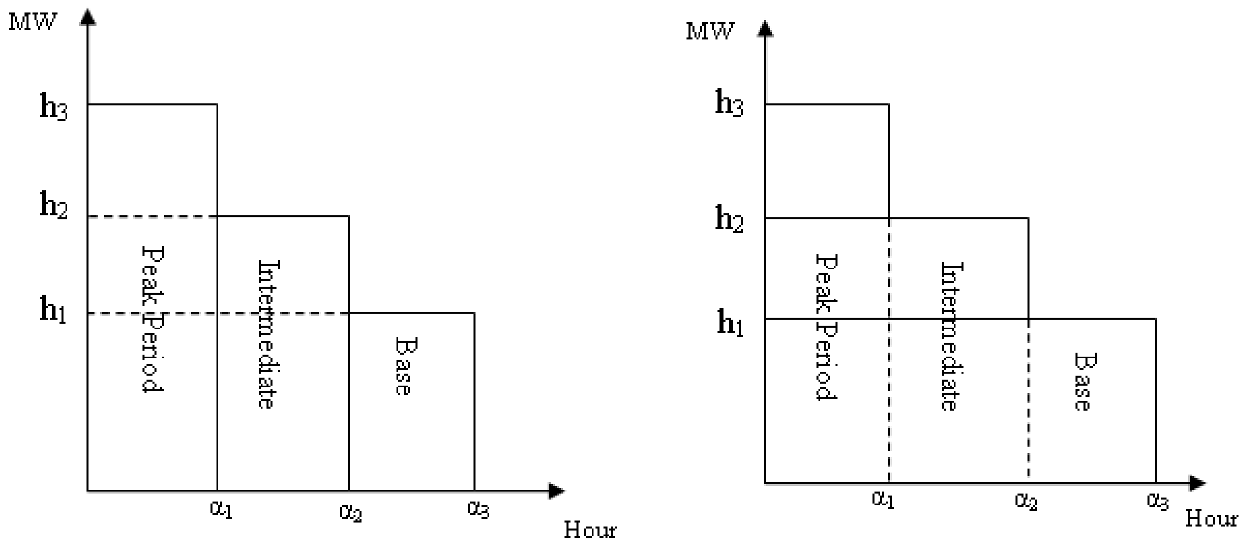


Figure 4-4: Vertical and Horizontal Approximation of Annual Load Duration Curve

In the vertical approximation, the load duration curve is divided by allocating proper time intervals for each block. On the other hand in the horizontal one, the load is divided by assigning appropriate capacity for each horizontal block. The area of each strip is the demand for that block, in MWh. Different methods lead to different ways to define variables in a model. The formulations in this chapter, and the numerical example in chapter 5 use the vertical method to define demand variables.

4.2.1.2 Production level decision variables

The decision variables $(X_{i,t,s})$ and $(exgen_{j,t,s})$ identify the level of production (in power units of MW) from new and existing generators in each time period *t* and each demand block *s*. The value of production from new capacities could be greater than zero only when the binary variable of build/no-build decision $(Z_{i,t})$ is equal to one.

4.2.1.3 Binary decision variable

The binary decision variable ($Z_{i,t}$) identifies when generator *i* should be built. A value of one means the generator should be built in the specific time period *t*. On the other hand, the binary decision variable ($W_{j,t}$) is used to allocate fixed variable cost for existing generator j in time period *t,* only if it is active.

4.2.2 Objective Function

The proposed objective function is a social welfare function as described in section 4.1; however in the proposed model there are other cost terms in addition to variable cost, such as capital cost of construction of new facilities and fixed operating cost. Also, the model consists of many periods while consumers' values and producers' costs are simply added over all periods, with discounting. The supply curve of the model is a step function, with constant supply for each generator up to its capacity. The height of each step is the marginal variable cost of each unit and therefore the area under such a supply curve is the total variable cost. In order to consider fixed costs in producers' cost function, we include them separately in the mathematical model. As illustrated in Figure 4-5, which is a simple case of one period, one commodity, and only continuous variables (no binary variables), the area between demand and supply curves (A+B+C+D+E), which has been derived by subtracting the area underneath the supply curve (F) from the area underneath the linear demand curve is the social welfare for the time period that is illustrated.

At equilibrium point (Eq*) in Figure 4-5, P* and Q* are the market equilibrium price and quantity. If any of these values change, then the social welfare will be reduced. For instance, at point (Eq), the market price will be P and the production level will be Q, therefore the social welfare will be the area of (A+B+D), which is reduced by (C+E). There will be no other points that gives a larger social welfare than the one introduced by (eq*). That's why intersection of supply and demand curves is being called equilibrium point.

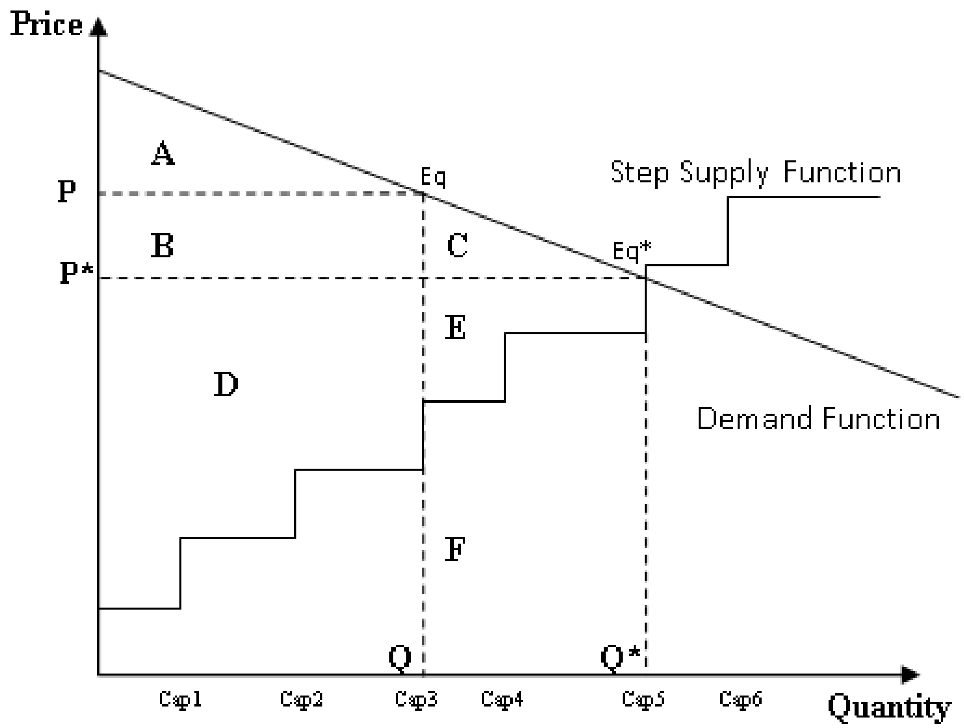


Figure 4-5 : Social Welfare not including fixed cost component

If the inverse demand function is assumed to be the linear function of (4.1), then the total value to consumers - the area underneath - is presented by (4.2).

$$Price(q) = \alpha * Demand + \beta \tag{4.1}$$

$$Total\ Value\ to\ Consumers = \alpha \frac{Demand^2}{2} + \beta * Demand \tag{4.2}$$

Social welfare is the total value to consumers, minus the costs incurred by producers, as given by the following expressions (4.3) to (4.5):

$$\sum_t \sum_s \{(\beta_{t,s} * q_{t,s}^D + {}^1/_2 * \ \alpha_{t,s} * {q_{t,s}^D}^2 - cD_s * q_{t,s}^D)/\,(1+r)^t\} \tag{4.3}$$

$$-\sum_{i}\sum_{t}\sum_{s}\{\, C_{i,t} * X_{i,t,s} * h_s + fuel_{i,t} * X_{i,t,s} * h_s + K_{i,t} * Z_{i,t} + FixedOpC_i * Z_{i,t} * XK_i * \sum_{t\prime \geq t} 1/(1+r)^{t\prime}\} \tag{4.4}$$

$$-\sum_{j}\sum_{t}\sum_{s}\{C_{j,t} * exgen_{j,t,s} * h_s + fuel_{j,t} * X_{j,t,s} * h_s + FixedOpC_j * W_{j,t} * XK_j/(1+r)^t\,\} \tag{4.5}$$

The first part of the objective function (4.3) is the discounted area underneath the electricity demand curve minus the delivery charges, paid by consumers for each period t and demand block s, and the next part (4.4) gives the cost of new generation, including variable cost, fuel cost, capital cost and fixed operating cost. The last part of this objective function (4.5) takes care of variable cost; fuel cost and fixed operating cost associated with existing capacities. In expressions (4.4) and (4.5) parameters$C_{i,t}$,$fuel_{i,t}$, $K_{i,t}$, $C_{j,t}$ and $fuel_{j,t}$ are already discounted, and therefore no explicit discount factor is needed in the model.

4.2.3 Constraints

Since the proposed model is a long term planning program, constraints which are related to short-term operational issues such as ramping time and minimum up and down time are not used. For simplicity, issues related to the transmission system are ignored in the model. Relevant constraints for this model are supply, capacity and regulatory limitations. This section presents these relevant constraints.

Supply Constraint:

Constraint (4.6) forces the amount of supply by new and old generators to be greater than or equal to demand, in each time period, and each demand block. As already mentioned, demand is a price responsive variable. The produced electricity from new generators ($X_{i,t,s}$) and existing generators ($exgen_{j,t,s}$) are in MW, but the demand of $q^D_{t,s}$ is in MWh, therefore the number of hours in each demand block,h_s reconciles the units.

$$\sum_i X_{i,t,s} * h_s + \sum_j exgen_{j,t,s} * h_s \geq q^D_{t,s} \qquad \forall t, s \tag{4.6}$$

Capacity Constraint:

This constraint doesn't allow the amount of production to be greater than the effective potential capacity of each generator. Effective capacity of each generator depends on its capacity and availability factor. Capacity factor is the ratio of a generator's actual energy output in a period of time over its energy output in the same time period if it ran at nameplate power output all the time. For example, capacity factor of a wind turbine could be 70% which means its actual output is 70% of its nameplate output. On the other hand, availability factor is the amount of time that the generator is available for production and is not down due to maintenance or other unexpected problems. For instance, the availability factor of a new wind turbine could be 98%. The following equation illustrates the capacity constraint:

$$X_{i,t,s} \leq XK_i * Capf_i * av_i * \sum_{t\prime\leq t} Z_{i,t\prime} \qquad \forall\, i, t \text{ and } s \tag{4.7}$$

$$exgen_{j,t,s} \leq excap_{j,t} * Capf_j * av_j \qquad \forall\, j, t \text{ and } s \tag{4.8}$$

As shown in (4.7) the amount of production will be zero if the generator is not built until that time period. The same idea will be applied for existing generators in (4.8) except that they`ve been already built and they can produce as needed in each time period and demand block.

Binary Constraints:

Constraint (4.9) is to limit the model to build each generator no more than once during the time horizon of the plan. This constraint has important implications about capacity payments which will be discussed in the next chapter.

$$\sum_{t} Z_{i,t} \leq 1 \qquad \forall\, i \tag{4.9}$$

Also equation (4.10) defines the binary variable of $W_{j,t}$, which allows the fixed operating cost to be equal to zero for existing capacity (j), when it is not active or has been retired.

$$M * W_{j,t} \geq excap_{j,t} \qquad \forall j, t \tag{4.10}$$

Regulatory Constraints:

These types of constraints are different, depending on countries' regulations and long term plans that affect their supply mix. For example, in Ontario's Integrated Power System Plan, there is a requirement to increase the total installed capacity of renewable energy to 15700 MW by 2025 (OPA, 2007).

Other regulatory constraints such as reserve capacity (4.11) and emission limit (4.12) play an important role in various countries' long term plans. The following equations demonstrate these types of constraints:

$$\begin{aligned}\sum_{i} X_{i,t,peak} + \sum_{j} exgen_{j,t,peak} + q^{D}_{t,peak} * reserverate_t / h_{peak} \\ \leq \sum_{i} (XK_i * Capf_i * av_i * \sum_{t' \leq t} Z_{i,t'}) + \sum_{j} (XK_j * Capf_j * av_j) \ \forall t\end{aligned} \tag{4.11}$$

$$\sum_{i,s} e_i * X_{i,t,s} * h_s + \sum_{j,s} e_j * exgen_{j,t,s} * h_s \leq EmissionCap_t \tag{4.12}$$

Equation (4.11) limits the amount of peak production from new and existing capacities and also reserve capacities to be less than or equal to available effective capacity for each time period. If a solution of the model is associated with a lot of risk due to the generation type, for instance renewable energy, then the model should be run under different scenarios with a modified reserve margin to accommodate uncertainties. Equation (4.11) is consistent with the following formula which is used to calculate the reserve margin, considering the expected risk uncertainties (IESO, Ontario Reserve Requirements to Meet NPCC Criteria, 2007):

$$Reserve\ margin(\%) = \frac{Required\ Available\ Capacity - Forecast\ Peak\ Demand}{Forecast\ Peak\ Demand} * 100\%$$

Equation (4.12) limits the amount of emission caused by different types of electricity generators during the time horizon of the plan. In order to lessen the amount of green house gas emission, the parameter $EmissionCap_t$ reduced, causing the generation from coal base or gas base facilities to be decreased and instead wind, hydro and solar capacities to be increased.

4.2.4 Summary of MINLP

To summarize, the MINLP of Formulation 4-1 has objective (4.3), (4.4), (4.5) ,and constraints (4.6), (4.7), (4.8), (4.9), (4.10), (4.11) and (4.12), together with non-negativity constraints for continuous variable and $\{0,1\}$constraints for binary variables.

4.3 Auxiliary Nonlinear Program for Pricing

By solving Formulation 4-1 as detailed in (4.3) to (4.12), the time of construction, amount of production and electricity demand in each time period will be identified. In models with only continuous variables, the dual of supply constraint (4.6) is normally interpreted as the market clearing price (Samuelson, 1952). However, in models that include discrete variables, such as Formulation 4-1, it is possible for an investor to have negative profit, which is one form of disequilibrium that is discussed by O’Neill (O'Neill, 2005). In order to identify capacity payments or reimbursement for binary activities such as building new capacities, O'Neill suggests using an auxiliary NLP problem in which the binary variables become continuous, but they are fixed at their optimal values, obtained from Formulation 4-1. The dual variables associated with the new equality constraints $\gamma^{z}_{i,tt}$, together with the dual variables of the market clearing constraints, are proven to be equilibrium capacity payments for each new facility in each time period (O'Neill, 2005). Also, the equilibrium market clearing prices can be obtained from this formulation, as duals of market clearing constraints. With these capacity payments and the market clearing prices, each producer is content to construct new capacity and operate its units as indicated by the optimal solution of the MINLP of Formulation 4-1.

In equations (4.22) and (4.21), $Z^{*}_{i,t}$ and $W^{*}_{j,t}$ are the binary variables in Formulation 4-1, which are fixed at their optimal values in Formulation 4-2, and therefore make it an NLP model with just continuous variables. The objective function and all the constraints are exactly as described in the previous section. The only differences are the conversion of $Z_{i,t}$ and $W_{j,t}$ from binary to continuous, the inclusion of equality constraints (4.21) and (4.22), and elimination of equations (4.9) and (4.10) due to redundancy.

$$\underset{(q^D_{t,s}\,, X_{i,t,s}\,, exgen_{j,t,s}\,, W'_{j,t}, Z'_{i,t})}{Max}$$

$$\sum_t \sum_s \{(\alpha_{t,s} * q^D_{t,s} + {}^1/_2 * \beta_{t,s} * {q^D_{t,s}}^2 - cD_s * q^D_{t,s}) / (1+r)^t\} \tag{4.13}$$

$$- \sum_i \sum_t \sum_s \{(C_{i,t} * X_{i,t,s} * h_s + fuel_{i,t} * X_{i,t,s} * h_s + K_{i,t} * Z'_{i,t} + FixedOpC_i * Z'_{i,t} * XK_i * \sum_{t\prime \geq t} 1/(1+r)^{t\prime})\} \tag{4.14}$$

$$- \sum_j \sum_t \sum_s \{(C_{j,t} * X_{j,t,s} * h_s + fuel_{j,t} * X_{j,t,s} * h_s + FixedOpC_j * W'_{j,t} * XK_j * \sum_{t\prime \geq t} 1/(1+r)^{t\prime})\} \tag{4.15}$$

Subject to:

$$\sum_i X_{i,t,s} * h_s + \sum_j exgen_{j,t,s} * h_s \geq q^D_{t,s} \qquad \forall t, s \tag{4.16}$$

$$X_{i,t,s} \leq XK_i * Capf_i * av_i * \sum_{t\prime \leq t} Z'_{i,t\prime} \qquad \forall\, i, t \text{ and } s \tag{4.17}$$

$$exgen_{j,t,s} \leq excap_{j,t} * Capf_j * av_j \qquad \forall\, j, t \text{ and } s \tag{4.18}$$

$$\sum_i X_{i,t,peak} + \sum_j X_{j,t,peak} + q^D_{t,peak} * reserverate_t / h_{peak} \leq \sum_i (XK_i * Capf_i * av_i * \sum_{t' \leq t} Z'_{i,t'}) + \sum_j (XK_j * Capf_j * av_j) \; \forall t \tag{4.19}$$

$$\sum_{i,s} e_i * X_{i,t,s} * h_s + \sum_{j,s} e_j * X_{j,t,s} * h_s \leq EmissionCap_t \tag{4.20}$$

$$W'_{j,t} = W^*_{j,t} \qquad \forall\, j, t \tag{4.21}$$

$$Z'_{i,t} = Z^*_{i,t} \qquad \forall\, i, t \qquad (\gamma^Z_{i,t}) \tag{4.22}$$

$$\boldsymbol{q^D_{t,s}\,, X_{i,t,s}\,, exgen_{j,t,s}\,, W'_{j,t}, Z'_{i,t} \geq 0} \tag{4.23}$$

Formulation 4-2

Formulation 4-2 suggests discriminatory capacity payments to each new facility in each time period. However in practice it is easier to implement a non-discriminatory capacity price based on a generator's available capacity in each time period. Another issue with O'Neill's payment is that sometimes the dual variables of O'Neill's equality constraints are negative, meaning that generators should pay consumers to be able to produce, which is hard to be practiced.

In order to overcome the problems with O'Neill's capacity price, this thesis suggests the following LP minimization formulation, which introduces non-discriminatory capacity prices, while avoiding negative profits for generators. In Formulation 4-3, the only decision variables are the capacity prices in different periods $Cappr_t$, which should be chosen to minimize the present worth of total capacity payments to reduce the burden on consumers and hence increase the consumer's welfare. Furthermore, values of binary variables and amount of generation have been fixed at their near optimal values, calculated by Formulation 4-1. On the other hand, market clearing prices have been derived from duals of market clearing constraints in the NLP Formulation 4-2.Also Formulation 4-3 minimizes the present worth of all capacity payments which should be paid to new investors at the time of construction for providing effective installed capacity for the rest of the time horizon of the plan. This payment could be changed in each time period according to available installed capacity. Constraint (4.25) forces the model to calculate capacity payments, such that no loss happens to new generators.

Although, it's important to provide positive profit for producers, it shouldn't impact consumers' welfare, such that they have negative surplus. Constraint (4.26) takes care of this problem by forcing the model to have positive welfare for consumers' while providing a reasonable amount of profit for producers. It's assumed that capacity payments are being made directly by consumers. Also, $Price_{t,s}$ is the discounted price paid to generators, and is obtained as the dual of the market clearing constraint (4.16). In this formulation, capacity payments haven't been considered for existing capacities, since the goal of the model is providing incentives for new investors.

$$\underset{(Cappr_t)}{Min} \sum_i \sum_t (Cappr_t * XK_i * Capf_i * av_i * Z^*{}_{i,t}/(1+r)^t) \tag{4.24}$$

Subject to :

$$\begin{aligned}\sum_{t,s} \{Cappr_t * Z^*{}_{i,t'} * av(i) * capf(i) * XK_i /(1+r)^t + Price_{t,s} * X^*{}_{i,t,s}\} \\ \geq \sum_t \sum_s \{ C_{i,t} * X^*_{i,t,s} * h_s + fuel_{i,t} * X^*_{i,t,s} * h_s \\ + K_{i,t} * Z^*_{i,t} + FixedOpC_i * Z^*_{i,t} * XK_i \\ * \sum_{t' \geq t} 1/(1+r)^{t'}\} \quad \forall i\end{aligned} \tag{4.25}$$

$$\begin{aligned}\sum_s \{\alpha_{t,s} * q^D_{t,s} + .5 * \beta_{t,s} * {q^D_{t,s}}^2 - Price_{t,s} * q^D_{t,s} \\ - \sum_i \{ Cappr_t * XK_i * Capf_i * av_i * Z^*{}_{i,t}\}\} \geq 0 \ \forall t\end{aligned} \tag{4.26}$$

$$\boldsymbol{Cappr_t \geq 0} \tag{4.27}$$

Formulation 4-3

Formulation 4-3 pays a capacity price at the time of construction, and this could put a large burden on consumers, and perhaps violating constraint (4.26). In order to avoid this problem Formulation 4-4 has been suggested, which pays a smaller price for any capacity that exists in every period from the time of its construction and later.

In order to apply this policy, objective function of (4.24) should be replaced by (4.28), which has a sum, over previous time periods during the time horizon of the plan. Since (4.28) is about paying capacity price to a facility that has already been built, the constraint of (4.25) should also be modified as (4.29). Also, in order to force the model to generate no capacity prices during the time periods that no generator has been built, constraint (4.31) has been added to Formulation 4-4. This constraint equates capacity payments to zero in time periods when there is no new construction by defining a large number like M which makes the constraint redundant while there is new construction. It expresses the idea that if there is no need for new capacity in a time period, then there should be enough capacity for that time period. According to basic economic theory, the price of any good that has excess supply should be zero.

Since Formulation 4-4 is more credible, it's the only one which has been illustrated in the numerical example in section 5.

$$Min \sum_{t}\sum_{s}(Cappr_t * XK_i * Capf_i * av_i * \sum_{t' \leq t} Z^*{}_{i,t'})/(1+r)^t \tag{4.28}$$

$$(Cappr_t)$$

$$Subject\ to:$$

$$\sum_{t,s}\{Cappr_t * \sum_{t' \leq t} Z^*{}_{i,t'} * av(i) * capf(i) * XK_i /(1+r)^t + Price_t * X^*{}_{i,t,s}\} \geq \sum_{t}\sum_{s}\{(C_{i,t} * X^*_{i,t,s} * h_s + fuel_{i,t} * X^*_{i,t,s} * h_s + K_{i,t} * Z^*_{i,t} + FixedOpC_i * Z^*_{i,t} * XK_i * \sum_{t' \geq t} 1/(1+r)^{t'}) \quad \forall i \tag{4.29}$$

$$\sum_{s}\{\alpha_{t,s} * q^D_{t,s} + .5 * \beta_{t,s} * {q^D_{t,s}}^2 - Price_{t,s} * q^D_{t,s}\} - \sum_{i}\{Cappr_t * XK_i * Capf_i * av_i * \sum_{t' \leq t} Z^*{}_{i,t'}\} \geq 0 \ \ \forall t \tag{4.30}$$

$$Cappr_t \leq M * Z^*{}_{i,t} \qquad \forall i,t \tag{4.31}$$

$$Cappr_t \geq 0 \tag{4.32}$$

Formulation 4-4

5 Numerical Example (IPSP)

This section examines the proposed mathematical models in chapter 4 based on Ontario's integrated power system plan data and evaluates the viability of calculated capacity prices in promoting energy investments in the next 20 years.

Although Ontario currently has 31,000 MW of electricity capacity, building new generators to meet future demand is important for the province, since 80% of existing power facilities need to be refurbished or replaced by new ones over the next 10 to 15 years (Love, 2008). By phasing out coal fired generation in 2014 and retiring nuclear and gas plants, Ontario will just have half of the necessary energy to provide reliable operation for industries and residents (Electricity Conservation and Suplly Task Force, 2004). Figure 5-1 illustrates the importance of planning for new capacities for the next 20 years.

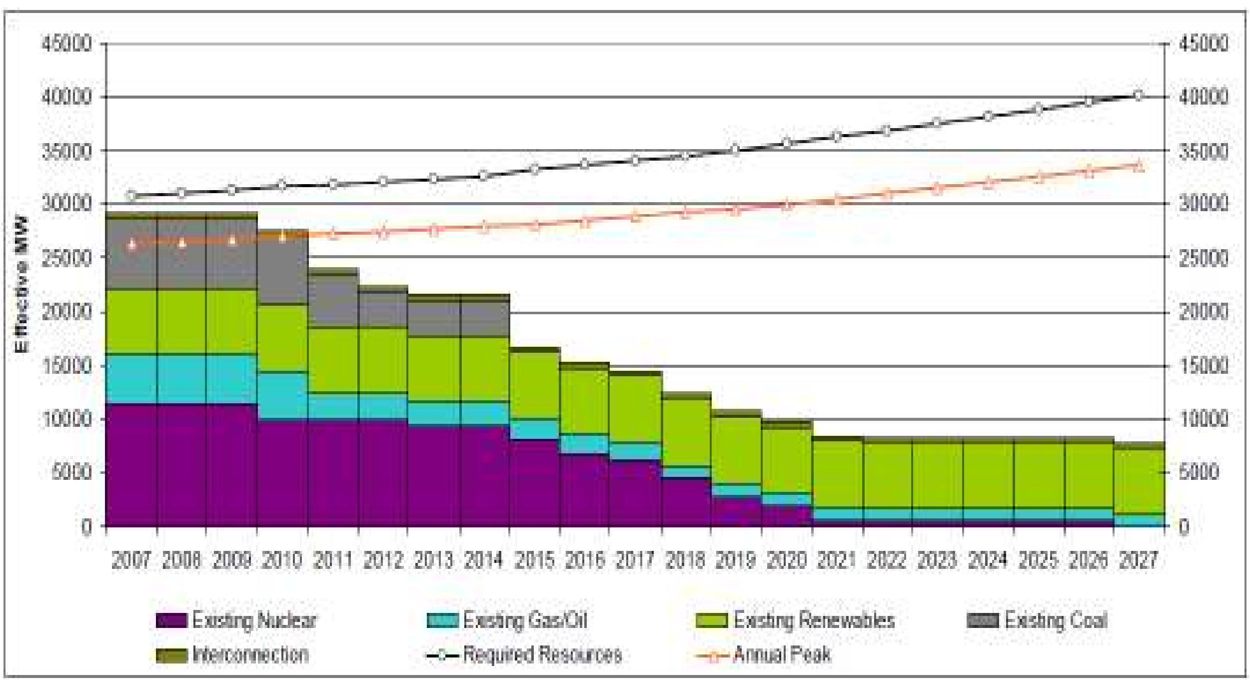


Figure 5-1: IPSP Capacity Requirements
Source: (Ministry of Energy and Infrastructure, 2007)

Data used in this example replicates IPSP's supply resources and cost estimations, however the demand is calculated based on the estimated demand curve for the next 20 years, which has been approximated by available demand elasticity forecasts. Therefore the proposed demand variable is price responsive and changes in each time period according to price fluctuations, while in IPSP, demand is forecasted based on 1.1% average annual growth rate (OPA, Load Forecast – IPSP Reference Energy And Demand Forecast, 2007). This increase rate is approximated by considering historical data, population growth, associated household activities and technological advances.

Another major difference between this numerical example and IPSP is that, IPSP doesn't consider different prices for different demand blocks such as peak, intermediate and base, while the proposed model has the capability of forecasting prices in various demand blocks, and therefore makes it possible to forecast demand during each demand block.

5.1 Data

5.1.1 Supply Mix

In order to satisfy demand, IPSP recommends different sources for generating electricity including Nuclear, gas/oil, wind and hydro. Supply requirements include existing, committed and planned resources. Existing resources are the ones in service as of June 2007, and committed resources are the ones which have signed contracts with Ontario Power Authority (OPA) and will be in service in future years of IPSP time horizon. Planned resources are calculated by the proposed mathematical model, according to the value of demand and existing resources in each time period. Table B-1 in Appendix B , demonstrates existing and committed resources, which have been used in our numerical example.

According to IPSP, coal generators will be shut down by 2014, and will be replaced by renewable resources such as wind and hydro to reduce the amount of emission and therefore comply with Canada's environmental protection obligations such as Kyoto protocol.

In order to promote renewable investments, IPSP considers different mechanisms such as "Standard Offer Program" - SOP, which provides small generators (the ones with less than 10MW nameplate capacity) with a fixed pricing scheme according to the type of their generation. The new revision of this policy is called Ontario's Feed in Tariff program, which has different pricing scheme. However, this numerical example doesn't consider FIT programs in order to identify new capacities; instead it assumes all new capacities follow the same payment mechanism, which is a two part pricing scheme, based on the proposed model in chapter 4.

To calculate new capacities, all the potential wind and hydro sites, including large and small ones and also nuclear and gas projects are given to the mathematical program. The proposed model selects the most economical renewable sites and projects for generating electricity and identifies the most efficient time for constructing these facilities. Table B-2 to Table B-6 in appendix B demonstrate the capacities (XK_i) of potential sites used in this example. XK_i Values in appendix 0 should be multiplied by the parameters of capacity factor ($Capf_i$) and availability factor (av_i), as in (4.7) and (4.8) to find the effective capacities of these sites if they'd be constructed. Capacity factor is the ratio of actual output to the nameplate of the unit (Gipe, 2004). It is high for controllable sources of energy such as nuclear and gas (around 90%) and low for intermittent sources of energy like wind (around 30%). Availability factor is a ratio, indicating the time duration which the unit is available to run and not down due to maintenance or outage. Inversely, this indicator is higher for intermittent sources like wind, which is around

98%, than other sources like nuclear, which is around 90%. In this numerical example, we used the average ratios for different sources of energy. Table B-7 illustrates these ratios.

5.1.2 Demand Variable

In this example demand is a decision variable, responsive to price fluctuations. In contrast, IPSP forecasts demand, based on 1.1% annual growth rate. Three demand blocks of peak, intermediate and base have been used in this example. Table 5-1 shows assumed total hours of year, allocated to each demand block.

Demand block	Hours
Peak	1460
Intermediate	2920
Base	4380

Table 5-1: Demand Blocks

Deploying different demand blocks allows the model to suggest different commodity prices in each interval and hence introduce conservation mechanisms by price discrimination. To the author's knowledge, IPSP doesn't consider price differentiation among various demand blocks, instead it forecasts prices based on a "techno-vert" price scenario (OPA, Load Forecast – IPSP Reference Energy And Demand Forecast, 2007) in Canada's Energy Future report(NEB, 2003), which represents Canada as a country with rapid technological advancements and environmentally friendly society. Table 5-2 illustrates IPSP's price assumptions based on this scenario.

	2005	2010	2015	2020	2025
NEB Techno-Vert Used in CIMS – Reference Forecast					
Residential	12.2	14.4	13.9	13.2	12.3
Commercial	10.8	12.0	11.5	10.9	10.1
Industrial	10.1	11.2	10.6	10.0	9.3
Average	11.0	12.5	12.0	11.4	10.6

Table 5-2: IPSP Electricity Price Assumptions – (2007 cents/kWh)
Source: (OPA, Load Forecast – IPSP Reference Energy And Demand Forecast, 2007)

Since above prices are total unit costs to consumers, including wholesale energy prices plus transmission and distribution fees, delivery charges could be extracted from Table 5-2 by deducting the known value of wholesale price in base year and assuming this value for the rest of the time horizon of the plan. In 2005, the wholesale price for electricity was 5.3 ¢/kWh (IESO, 2005) and therefore the extra delivery charge was considered 5.7 ¢/KWh for intermediate demand block, and 6.7 ¢/KWh and 3.7 ¢/KWh for peak and base loads. This extra cost term is assumed to be constant for the 20 year planning horizon of IPSP.

To estimate demand, the linear curve of (4.1) has been considered. Values of α and β are approximated based on values of demand elasticity, shown in Table B-8 in Appendix B. Demand elasticity indicates the percentage of demand change in response to 1% increase in price. As shown in Table B-8, price increase will affect demand more in later years of IPSP time horizon, because consumers have more time to switch to other sources of electricity like self-generation. In order to calculate elasticity of demand, the following formula is used (Perloff, 2007):

$$\varepsilon = \frac{\Delta Q/Q}{\Delta P/P} = \frac{\Delta Q}{\Delta P}\frac{P}{Q} \tag{5.1}$$

Where $\frac{\Delta Q}{\Delta P}$ is the ratio of change in demanded electricity to the electricity price change, which is the inverse of the slope of inverse demand function. In (4.1), β is the price of electricity when there is no demand and α is the slope of inverse demand function. Knowing the value of demand elasticity (ε), electricity price and demand, the slope of (4.1) could be easily calculated by formulation (5.2).

$$\alpha = \frac{Electricity\ Price}{(\varepsilon * Electricity\ Demand)} \tag{5.2}$$

Table B-9 and Table B-10 illustrate price and demand assumptions used to estimate α by plugging these values in formulation (5.2). Electricity prices used in Table B-9 reflect factors such as economic growth and efficiency of future technologies. Since IPSP forecasts prices for every five years (OPA, 2007), some plausible values have been chosen and prices for the remaining years have been interpolated between those chosen time periods through the time horizon of the plan. Due to economical and technological efficiency of new generators and therefore their reduced cost of generation and their effect on electricity prices a declining trend has been considered for these prices. Also because IPSP's price assumptions are not based on different demand blocks (peak, intermidiate and offpeak), they've been considered as intermidiate prices, and therefore base and peak prices have been estimated by assuming peak prices of 40 \$/MWh more expensive and base prices of 60 \$/MWh cheaper. Demand values in Table B-10 are in units of energy or MWh. Please see Table B-11 in Appendix B for the value of α.

Knowing the values of α, electricity price (Table B-9) and demand (Table B-10), β could be easily calculated by using formulation (5.3). Please refer to Table B-12 for the calculated values of β.

$$\beta = Price - \alpha * (Demand) \tag{5.3}$$

5.1.3 Cost Assumptions

This section demonstrates estimates of capital costs and operating costs (fixed and variable costs, fuels costs) which are associated with generating facilities that have been already in the market or can enter into Ontario's electricity market according to IPSP, such as Nuclear plants, wind farms, hydroelectric plants, gas fired generators and coal plants. Uncertainties about future technology advancements make it very difficult to estimate costs of promising technologies like wind. Various studies have been done on approximating cost of entry for common technologies. These studies include, (Navigant, Evaluation of Cost of New Entry , 2007), (BC Hydro, 2006), (EIA, 2006), (US Energy Information Administration, 2006) and (California Energy Commission, 2003). Since information in (Navigant, Evaluation of Cost of New Entry, 2007) report is more relevant to Ontario's energy market, mostly this report has been used in the numerical example.

5.1.3.1 Capital Costs

Capital cost is referred to as "the depreciation expense incurred by the difference between what is paid for the assets required for a particular capacity and what the assets could be resold for some time after purchase" (Fraser, 2008). In this numerical example it's been assumed that the salvage value of each plant will be zero at the end of its life. There are different methodologies for calculating depreciation, including straight-line method and declining balance method. Moreover, some other methods try to determine depreciation rules, consistent with technological progress, inflation, maintenance cost patterns, user costs and other relevant factors (Baumol, 1971). For simplicity, the straight line depreciation method has been used in order to find the present worth of depreciated value of the capacity during the time horizon of IPSP (20 years) at the time of construction of the generator. In straight line depreciation, the facilities are depreciated by an equal amount each year. Formulation (5.4) shows how total present worth of capital cost ($K_{i,t}$) for each generator in time period (t) has been calculated. $Constcost_i{*}XK_i$ is the total capital cost incurred at the time of construction, multiplying by $\left(\frac{21-t}{age_i}\right)$, where age_i is its age at the end of its life. It allocates a fraction of total capital cost to the years of life that fall within the model's time horizon, and finally the discounting rate calculates the present worth of the capital cost. See the values of generators' life (age_i) and their unit cost of construction ($ConstCost_i$) in Table B-13 and Table B-14.

$$K_{i,t} = Constcost_i * XK_i * \left(\frac{21-t}{age_i}\right)/(1+r)^t \tag{5.4}$$

5.1.3.2 Operating Cost

Two types of operating costs have been considered for generators in IPSP, variable operating costs and fixed operating costs. The former includes fuel cost, labor and raw material, and the latter consists of maintenance and administration costs.

In order to keep up with variable cost growth, the following formula (5.5) has been used to calculate the present worth of the two types of variable cost per unit of generation in each time period t ($C_{i,t}$ and $fuel_{it}$). Please see Table B-15 and Table B-16 for values of the parameters $VarCost_i$ and $growthrate_i$ for C_{it} and $fuel_{it}$.

$$PWVariableCost_{i,t} = VarCost_i * (1 + growth\ rate_i)^{t-1}/(1+r)^t \tag{5.5}$$

On the other hand fixed operating costs occur as soon as the facility is constructed regardless of the amount of production and it's only related to the capacity of generators. In order to incorporate this cost for each facility we use expression (5.6) in (4.4) of the objective function; $Fixed\ OpCost_i$ is given in Table B-15.

$$FixedOpCost_i * Z_{i,t} * XK_i * \sum_{t}^{T} \frac{1}{(1+r)^t} \tag{5.6}$$

5.2 Numerical Example Model

In order to find the near optimal values of Ontario's supply mix and time of construction for new generating facilities, such as nuclear, wind and hydro, the mathematical model, introduced in chapter 4 has been programmed in GAMS (General Algebraic Modeling System). Please refer to appendix 0 for the GAMS code.

The first part of the model is a long term (20 years) mixed integer nonlinear program given in Formulation 4-1, calculating the amount of production from existing generators ($exgen_{j,t,s}$), new capacities ($X_{i,t,s}$), electricity demand in each time period ($q^D_{t,s}$) and also the time of construction for new facilities ($Z_{i,t}$). This model is a large scale optimization problem with 24,340 variables (6,069 binary variables) and 18,786 constraints.

In addition to constraints such as, supply (4.6), capacity (4.7) & (4.8), binary (4.9) & (4.10) and reserve capacity (4.11), the following policy constraints, according to IPSP, have been added to the model:

$$\sum_{Nuc} X_{(Nuc,t,s)} \leq 14000 \quad , \quad \forall t,s \tag{5.7}$$

$$\sum_{Gas} X_{(Gas,t,s)} \leq 10200 \quad , \quad \forall t,s \tag{5.8}$$

$$\sum_{Bio} X_{(Bio,t,s)} \leq 450 \quad , \quad \forall t,s \tag{5.9}$$

$$\sum_{Wind} X_{(Wind,t,s)} \leq 4039 \quad , \quad \forall t,s \tag{5.10}$$

$$\sum_{Hydro} X_{(Hydro,t,s)} \leq 4921 \quad , \quad \forall t,s \tag{5.11}$$

Above equations limit the amount of nuclear, gas, bio, wind and hydro production to 14000, 10200, 450, 2039 and 4921 MW, during the time horizon of IPSP.

Using the SBB (Simple Branch and Bound) solver and the NLP solver of CONOPT, following results, shown in Figure 5-4 to Figure 5-17 have been obtained. SBB uses the branch and bound algorithm, while relaxing the integrality constraints and tightening the bounds on integer variables (Bussieck & Drud, 2001). Relaxed nodes could be solved by any NLP solver, which in this example is CONOPT, which is based on the outer approximation (OA) method. Figure 5-2 shows the collaboration between SBB and NLP solver:

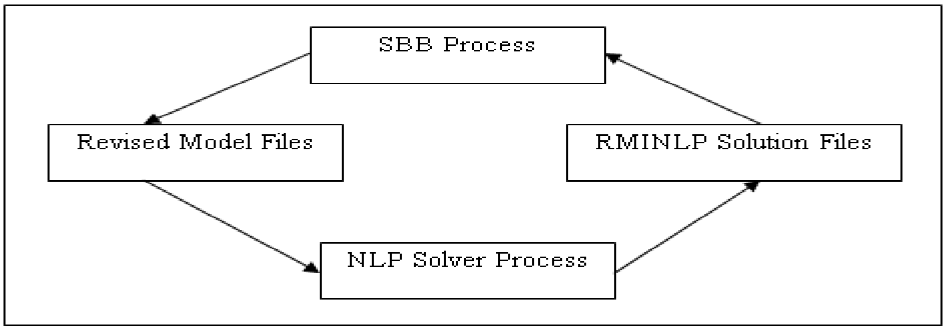


Figure 5-2: SBB and NLP
Source: (Bussieck & Drud, 2001)

As shown, the relaxed MINLP model (RMINLP) will be solved by CONOPT solver, the results will be updated in RMINLP solution files, and then SBB solver uses these solutions to update nodes on the next iteration. The details of SBB algorithm are shown in Figure 5-3.

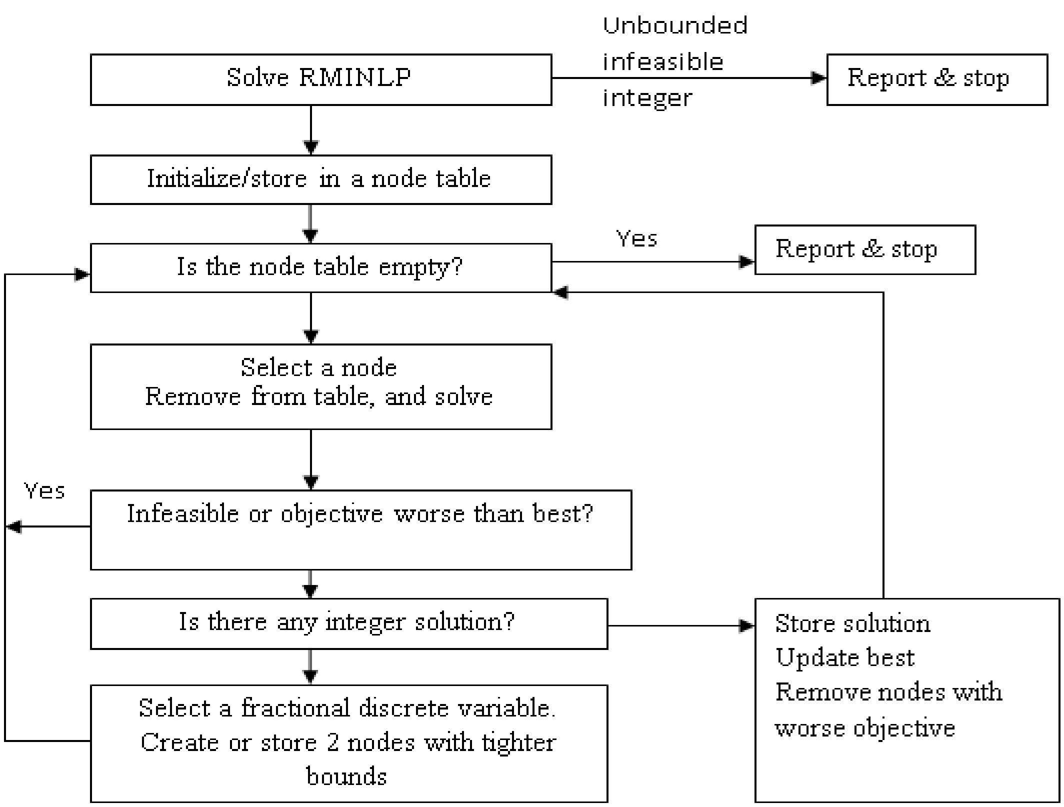


Figure 5-3: SBB Algorithm
SBB Algorithm: Source (Bussieck & Drud, 2001)

As illustrated in Figure 5-3, the relaxed MINLP problem is solved by assuming a starting point, chosen randomly from the initialized node table. In case the RMINLP problem is unbounded or infeasible the program stops. The branch and bound process will start on non-integer answers. If the sub-problem non-linear programs are infeasible or have objectives worse than the best one, then the node will be fathomed and a new one will be selected. Otherwise, if the sub-problem has integer solutions, the solution will be stored and the best objective solution will be updated. However, if the answer is not integer, a fractional discrete variable will be selected and 2 or more nodes with tighter bounds will be created. This process will continue till it reaches the best objective function.

Figure 5-4 demonstrates the amount of generation from existing capacities, which are less than the effective available capacity. Due to nuclear facilities retirement and coal fired generators shut down, the amount of supply from these resources will be reduced through the time horizon of the plan, and hence there is a vital need to construct new facilities to avoid energy shortage. Comparing Figure 5-1 (available existing capacity) and Figure 5-4, it's obvious that most types of existing generators, including nuclear, coal and renewables are being utilized through the time horizon of the plan, however gas facilities are not being used that much, since they have high variable costs and therefore the model intends to produce electricity from other facilities or new sources of generation with lower variable costs.

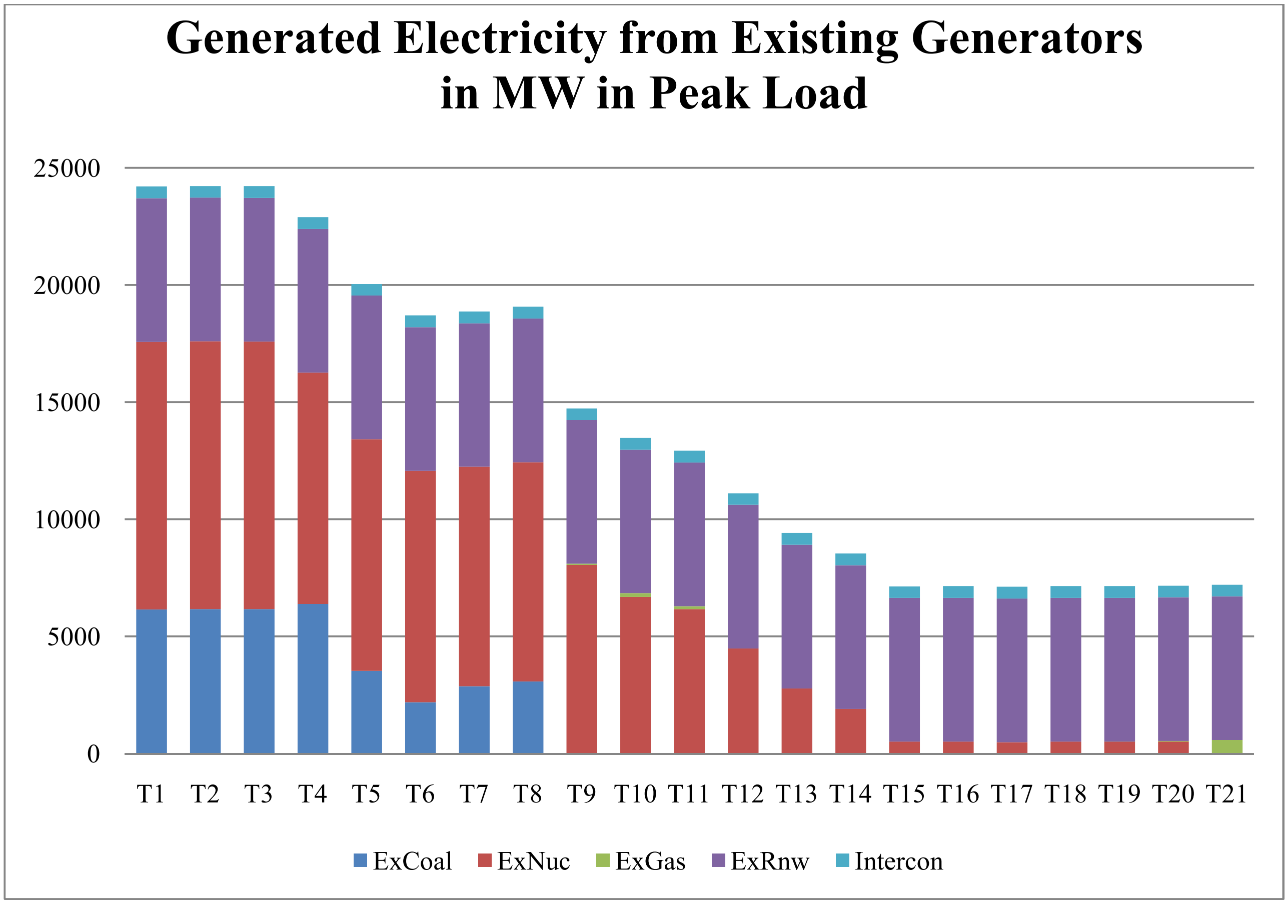


Figure 5-4: Existing Facilities Production

The proposed model chooses new generators, based on their economical efficiency, production capacity and policy limitations. Figure 5-6 illustrates the chronological order of new sources installation. Most of the new generators will be built as soon as coal generators are shut down or have been considered as back up sources of energy.

According to GAMS results Figure 5-5, hydro facilities will be built in the beginning of the time horizon of the plan, because they are the cheapest source, overall, so due to discounting, they are favored to be constructed earlier. Moreover, since gas facilities are coming in lower capacities, they will be built right after coal plants shut down, which causes gradual decrease of energy supply. Also, in order to make up for nuclear plants retirement, new nuclear sources start their production in T9. Wind sources will be built later in the time horizon of the model (T14).

Figure 5-5 illustrates the amount of new capacity in MW for each type of new generation. Each line demonstrates the cumulative nameplate (installed) capacity for each type of generation, for instance hydro capacities won't be higher than 6467 MW through the time horizon of the plan. Also, Figure 5-6 illustrates the accumulative generation of all new facilities in each year. For instance the contribution of new installed capacities is 5312 MW in the beginning of the time horizon and 3268 MW at the end of it. Clearly new facilities have higher installed capacity at the end of the time horizon, to make up for supply shortage caused by retirement or shutdown of existing capacities.

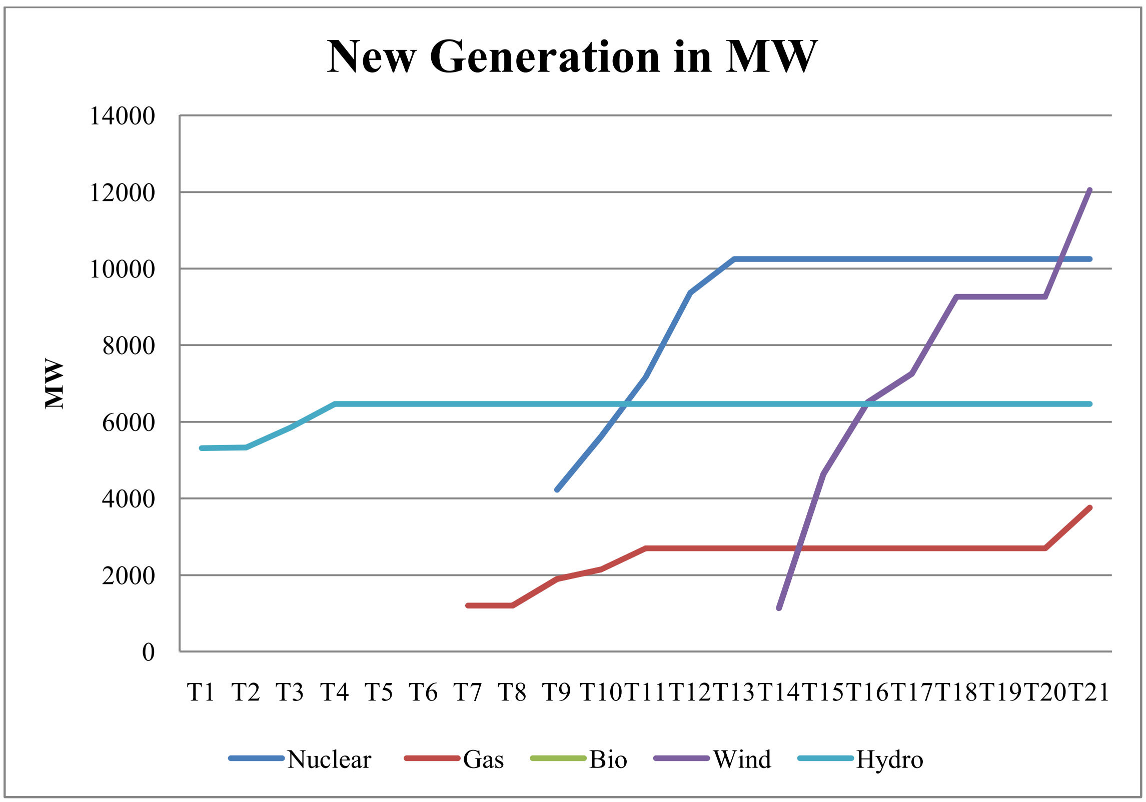
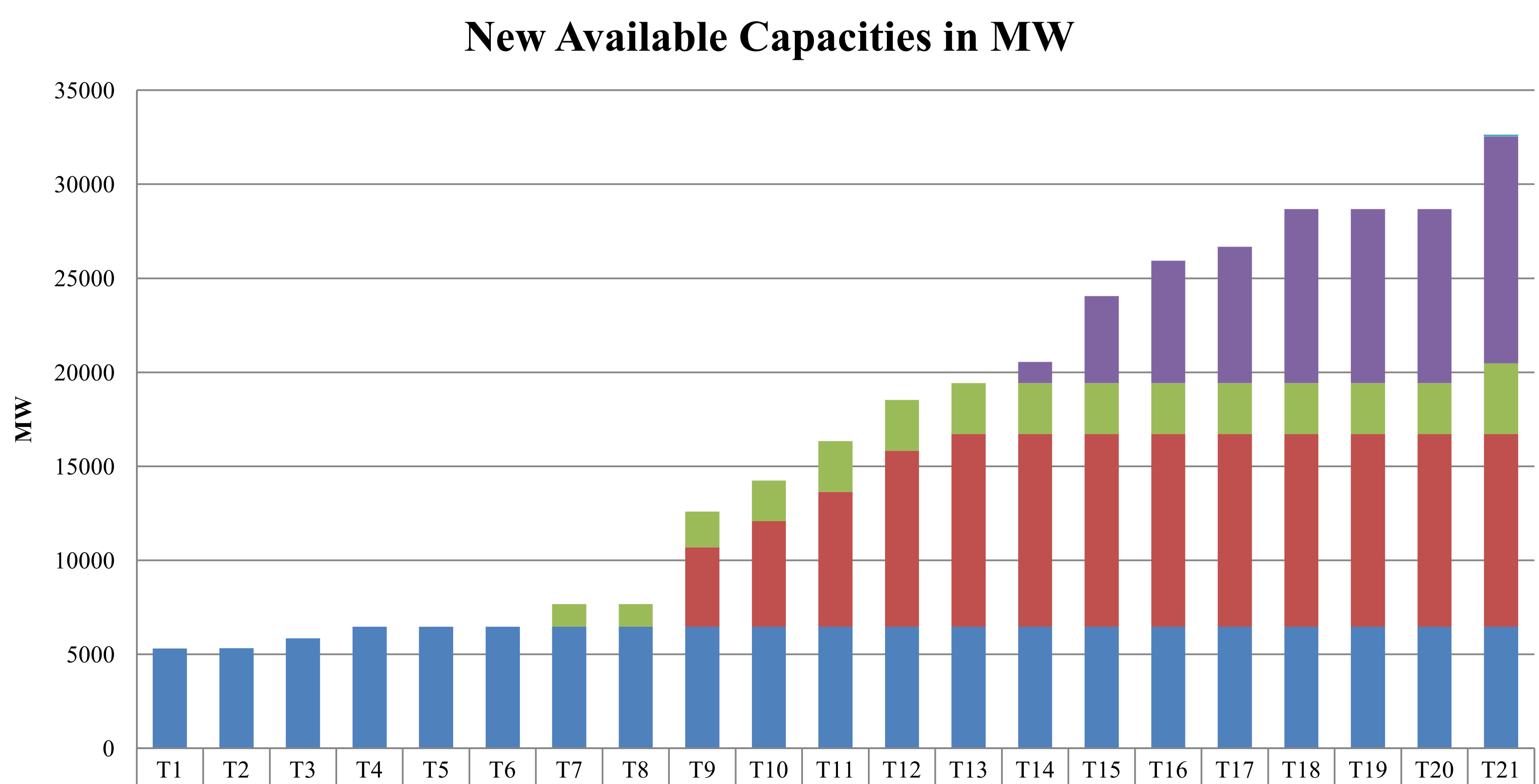


Figure 5-5: New Generation in MW



	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15	T16	T17	T18	T19	T20	T21
Bio																					100
Wind														1134	4636	6509	7258	9268	9268	9268	12056
Gas							1201	1201	1901	2151	2701	2701	2701	2701	2701	2701	2701	2701	2701	2701	3756
Nuclear									4224	5621	7169	9368	10249	10249	10249	10249	10249	10249	10249	10249	10249
Hydro	5312	5331	5852	6467	6467	6467	6467	6467	6467	6467	6467	6467	6467	6467	6467	6467	6467	6467	6467	6467	6467

Figure 5-6: New Available Capacity

Another major result of the model is the forecasted demand for the time horizon of the plan. As already mentioned, demand variable($q_{t,s}^{D}$) is price responsive, while the forecasted demand in IPSP follows 1.1% growth rate, based on historical factors and population growth. Figure 5-7 demonstrates the results for average demand which is the total demand in MWh divided by total annual hours of 8760. As shown, demand increases in the beginning of the time horizon; however it will be reduced shortly after coal fired generators shut down, due to energy price increase, which has been caused right after constructing more expensive generators such as hydro and nuclear. The demand will be fluctuating slightly in later years in response to price variations, but still follows a steady growth especially at the end of the plan's time horizon.

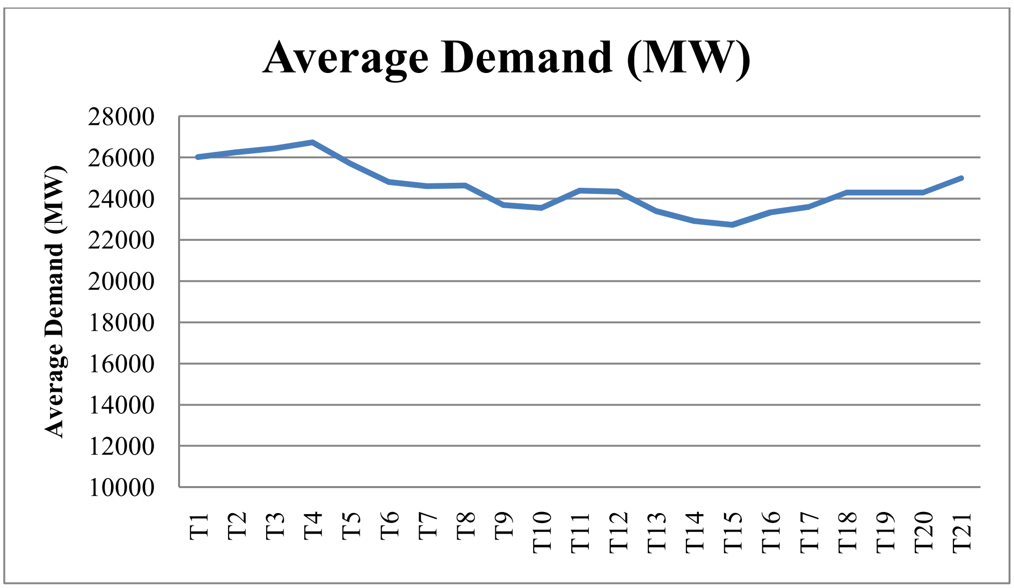


Figure 5-7: Demand Forecast

Since the demand results vary by price fluctuations and IPSP doesn't consider price responsiveness in its forecast, the proposed results are very different from IPSP's. As depicted in Figure 5-8, IPSP's demand follows an increasing trend with constant increase rate of 1.1%.

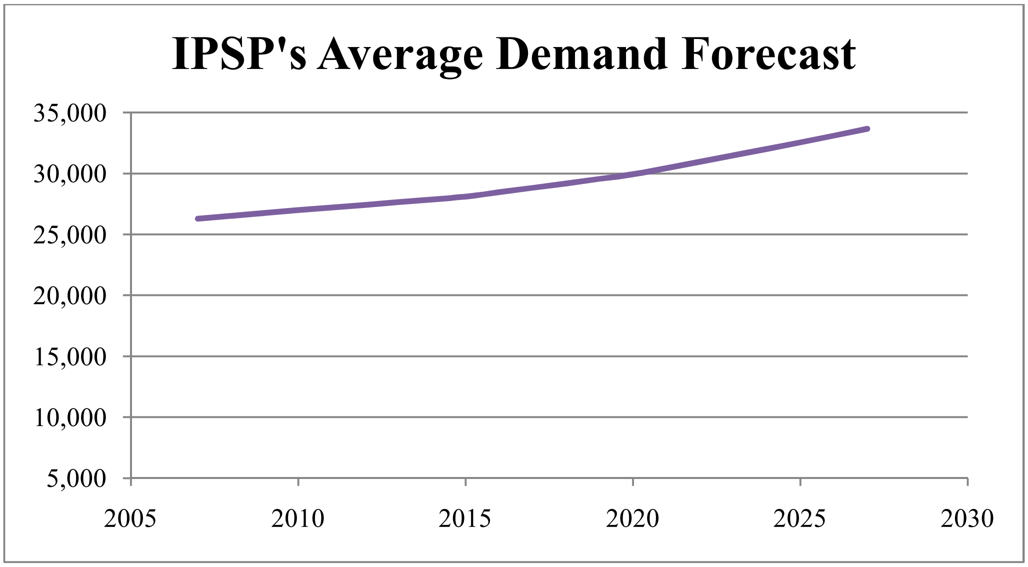


Figure 5-8: IPSP Demand Forecast

Looking at Figure 5-9, one can realize that when prices go high demand goes down and vice versa. For instance, when prices for different demand blocks of peak, intermediate and base increase in T4, demand will be reduced right at the same time period. This will illustrate the value of including price-responsiveness in the model and its importance in future price forecasts and sensitivity analysis.

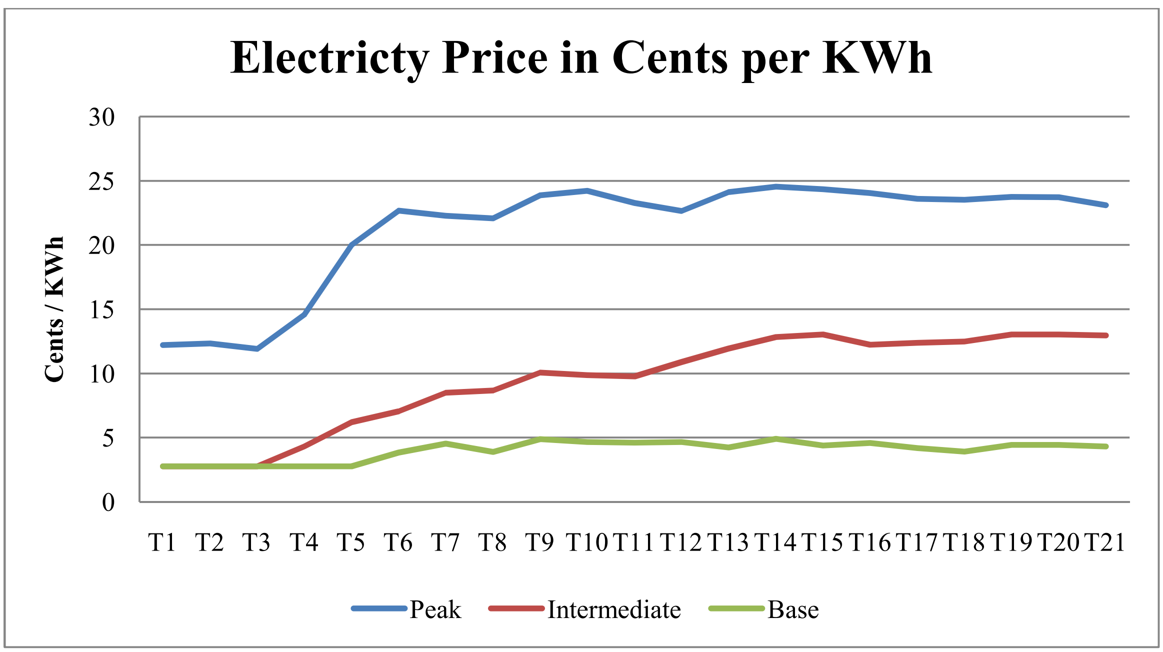


Figure 5-9: Electricity Price

On the other hand, IPSP does another type of analysis in order to forecast prices by performing a sensitivity analysis in respect to cost of electricity to customers and comparing it with a reference forecast that has been introduced in a techno-vert scenario in National Energy Board (NEB) report(NEB, 2003). These prices haven't been categorized based on time of use; however they are close to the intermediate prices, which have been proposed by the numerical example.

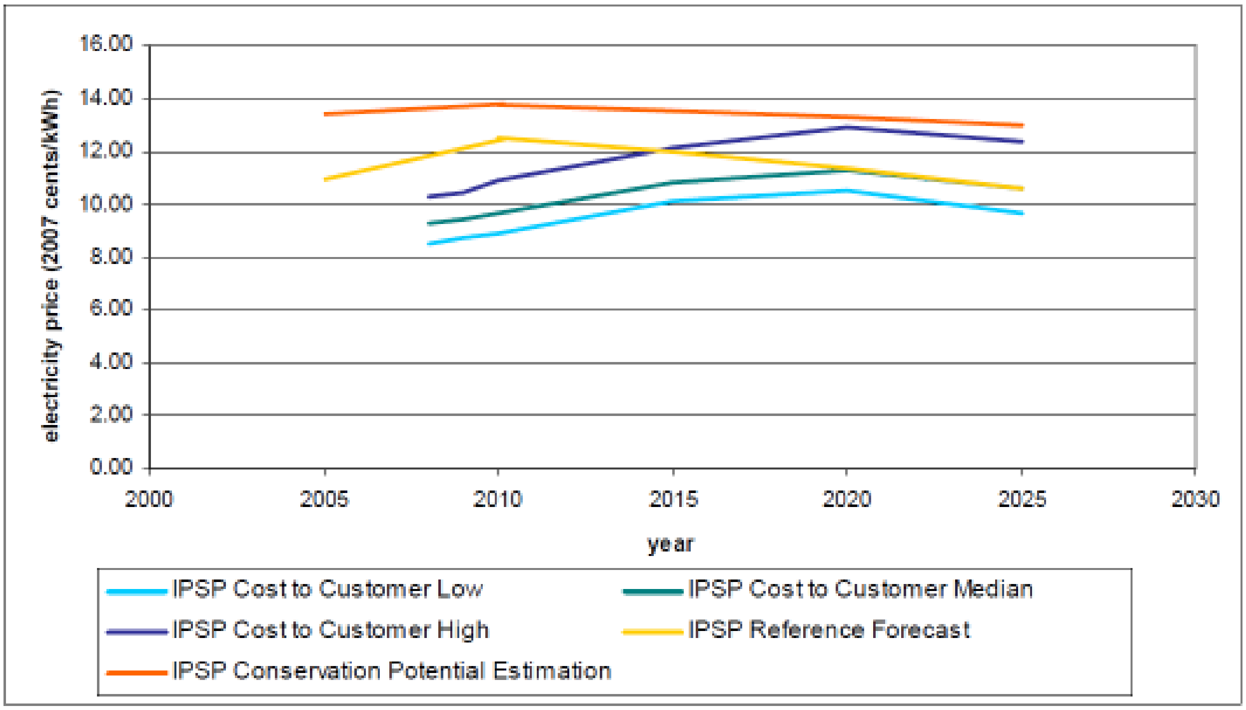


Figure 5-10: IPSP Price Assumptions

Source: (M.K. Jaccard and Associates, 2006)

Finding the values of binary variables, Formulation 4-2 has been used while fixing binary variables $Z_{i,t}$ and $W_{i,t}$ at their obtained optimal values from Formulation 4-1. As discussed in chapter 4, dual variables of supply constraint (4.16) can be considered as market clearing price or commodity price, and the dual variables of constraint (4.22), based on O'Neill's approach (O'Neill, 2005), could be considered as capacity payments to new generators in order to be compensated for their initial costs.

Figure 5-11 clearly shows that some peak or intermediate generators like Gas and Bio will have negative or near zero profit, if they'd be compensated by only market clearing prices, and therefore won't have any incentive to invest in energy sector. On the other hand, some generators like large wind sources would make enough profit to recover their capital costs; however these low profits may not be sufficient enough for investors, given the risks associated with electricity markets.

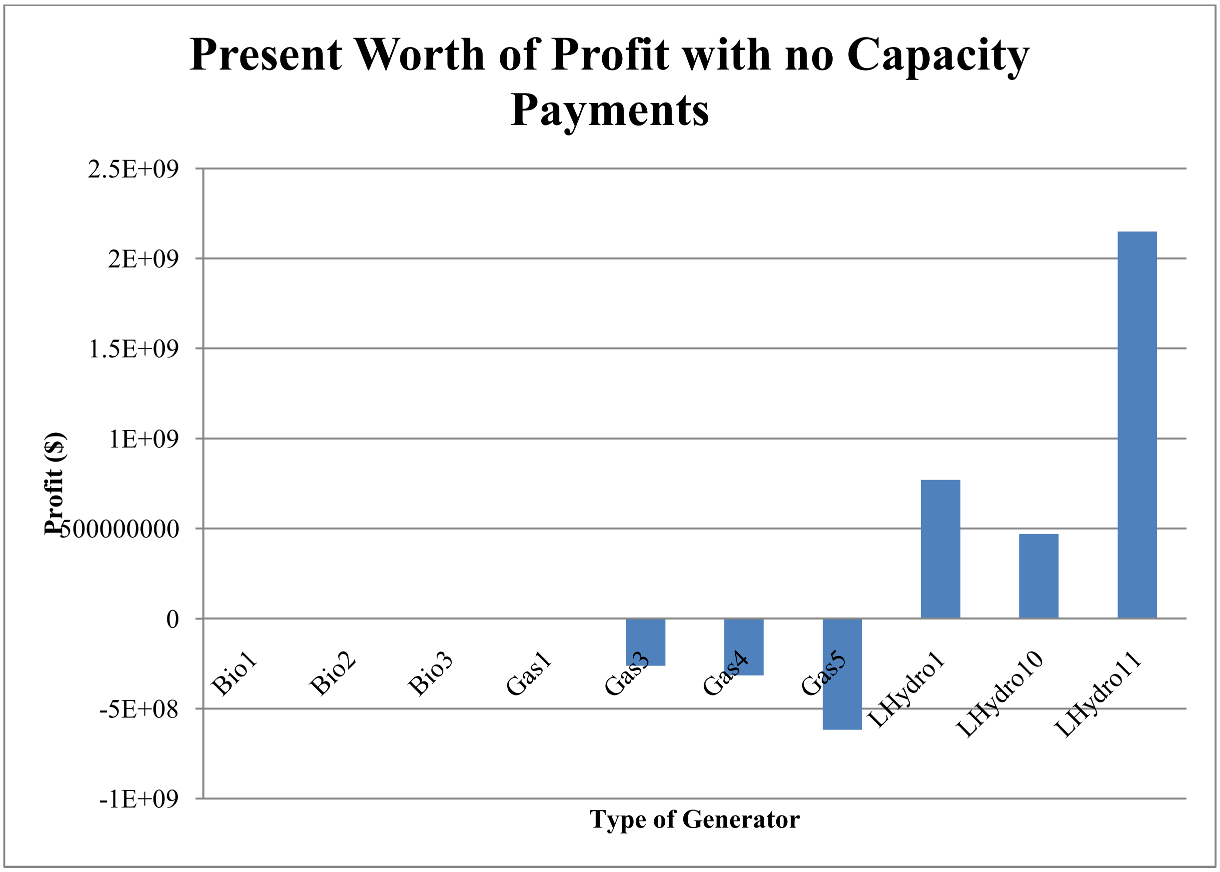


Figure 5-11: PW of Profit with no Capacity Payments

However, base generators make better profit since they always operate, during peak, intermediate and base periods. Figure 5-12 shows the amount of profit that base generators make throughout the time horizon of the plan. As shown, these generators don't encounter any loss, although they have higher capital cost.

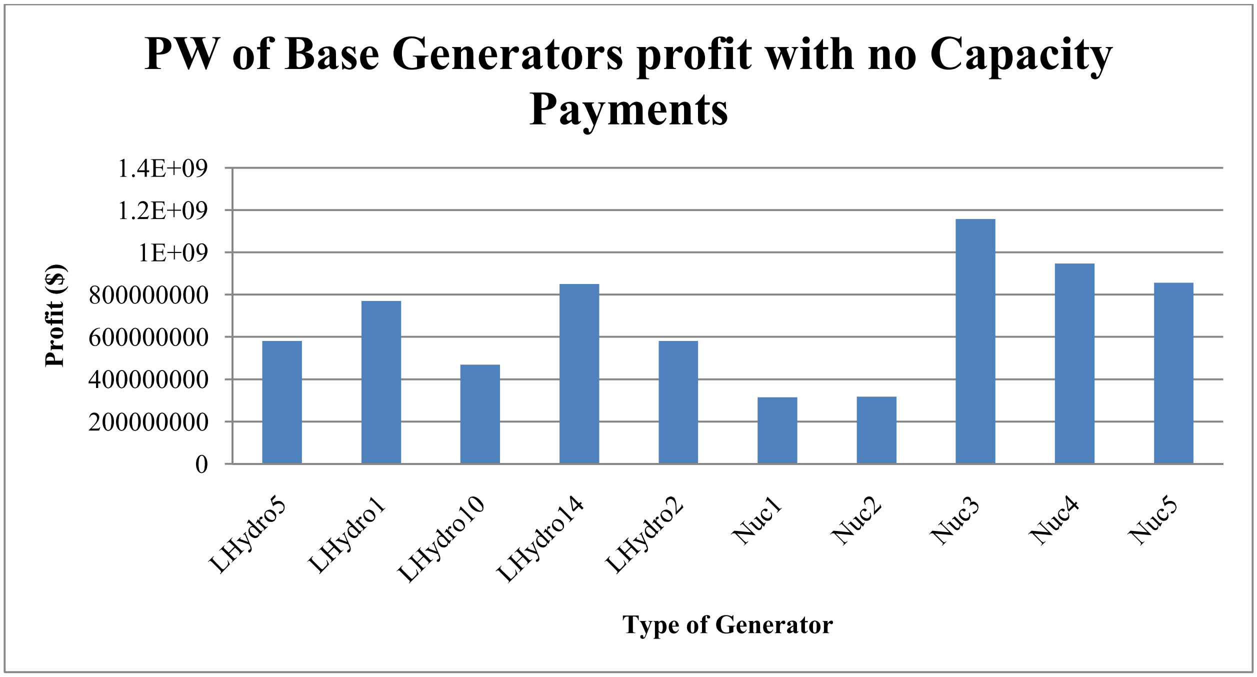


Figure 5-12: Base Generator's Profit with no Capacity Price

In order to give sufficient incentives for new investments, O'Neil's capacity payments of $(\gamma_{i.t}^{z})$, which are dependent on type of generation and time of construction has been introduced. As already described in last chapter, these capacity payments are dual variables of binary constraint (4.22), where the value of the binary variable is one. Although there are dual values for each generator for time periods without any construction, those dual values are not economically important because they represent "prices" that would be multiplied by zero valued variables, for zero revenue, and therefore they haven't been considered in capacity price calculations. They simply show if the generators are offered by those dual values, they still won't have enough incentive to construct new capacities in those periods. Figure 5-13 and Figure 5-14 shows suggested capacity payments for generators in Figure 5-11 and the amount of profit they will make after receiving additional O'Neil's payments.

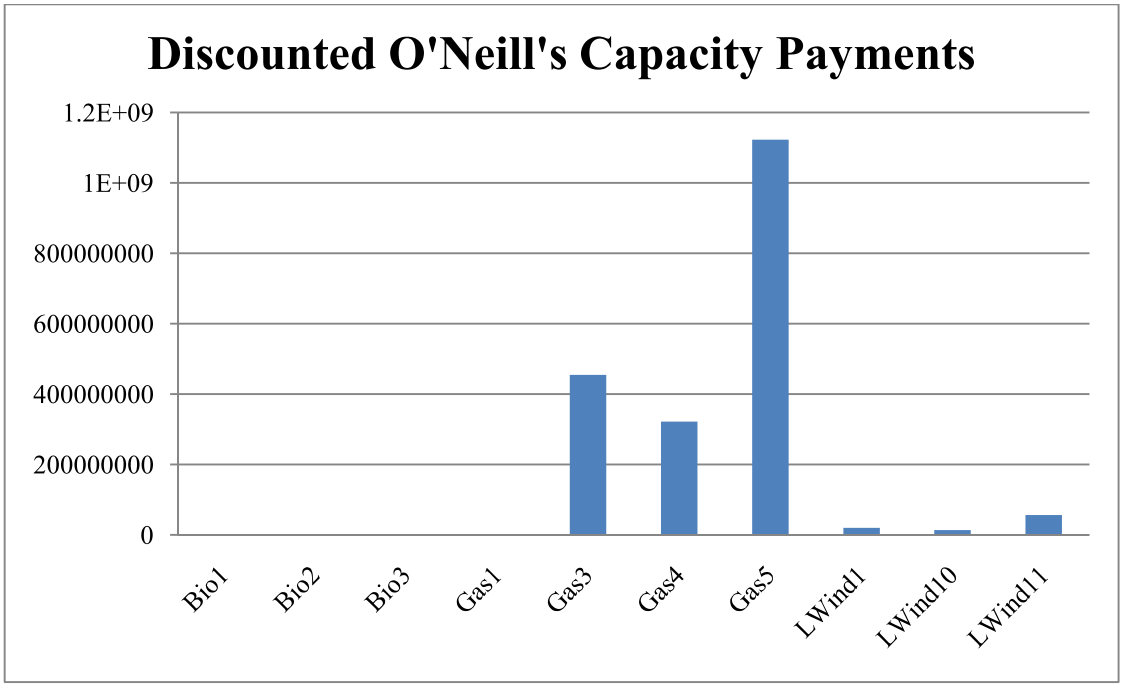


Figure 5-13: Discounted O'Neil's Capacity Payments

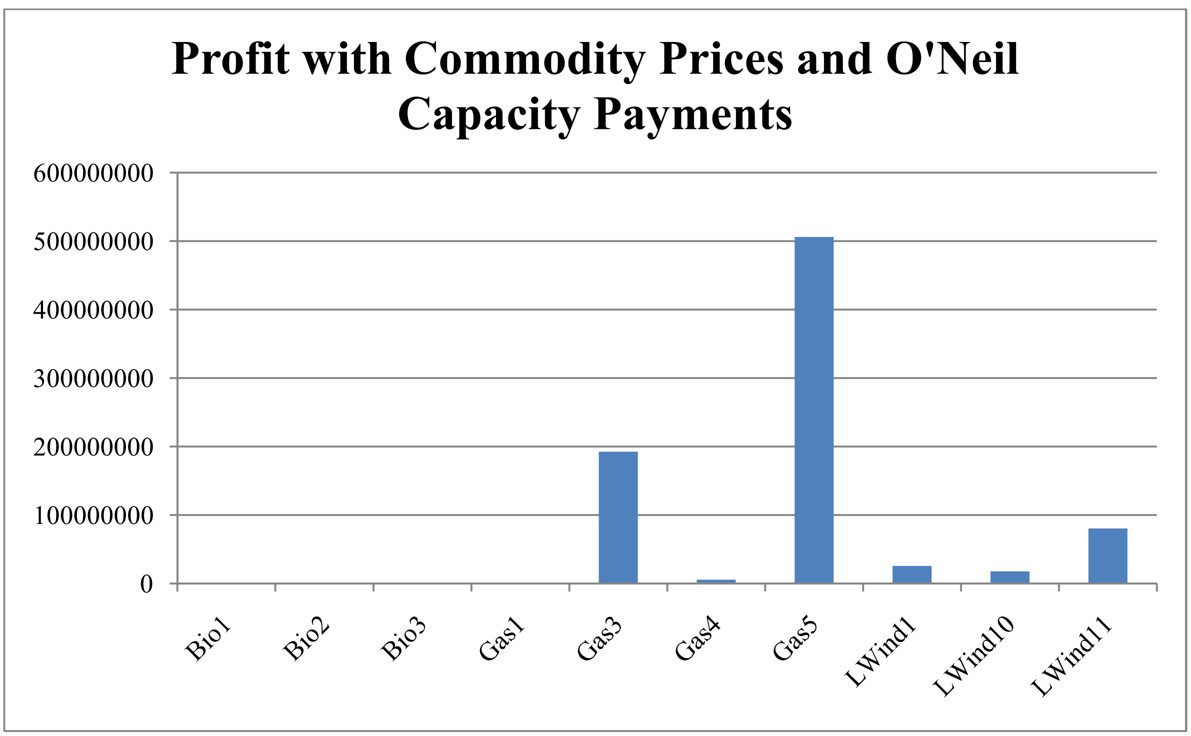


Figure 5-14: Profit with Commodity Prices and O’Neil Capacity Payments

As illustrated in above figures those generators who were making negative profit receive the most capacity payments, such that they can make non-negative profit at the end of the plan’s time horizon. Other generators also receive additional payments; however, it is difficult to give an economic rationale for these payments -- they are simply the mathematical result of the O'Neill method.

As mentioned in chapter 4, O'Neil's capacity prices are discriminatory; since they discriminate among generators and cause some of them to have much higher profit than others. To illustrates payment discriminations, The following figure illustrates O'Neill's capacity payments in ($/MW) instead of lump-sum payments:

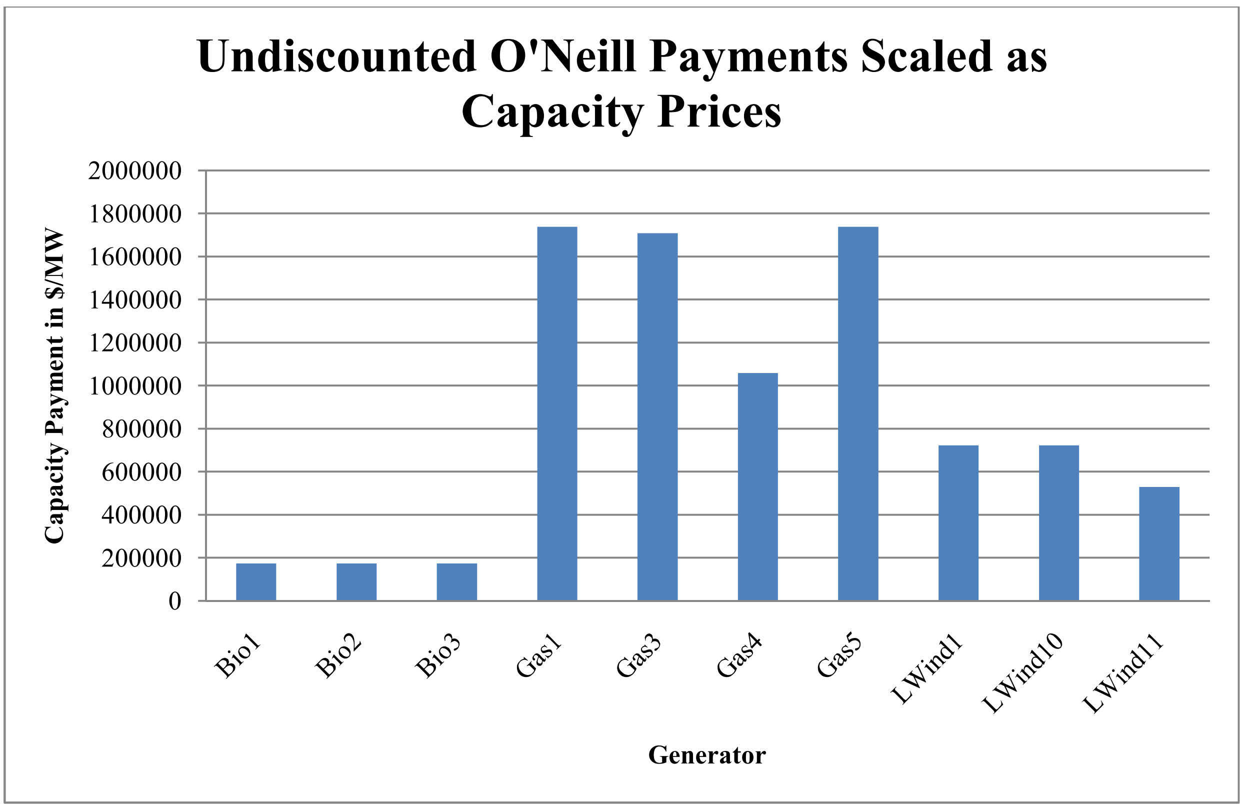


Figure 5-15: Undiscounted O'Neill Payments Scaled as Capacity Prices

In order to overcome these problems, Formulation 4-4 has been proposed, which introduces a non-discriminatory capacity payments in each time period in which there is at least one construction. Figure 5-16 demonstrates these capacity payments:

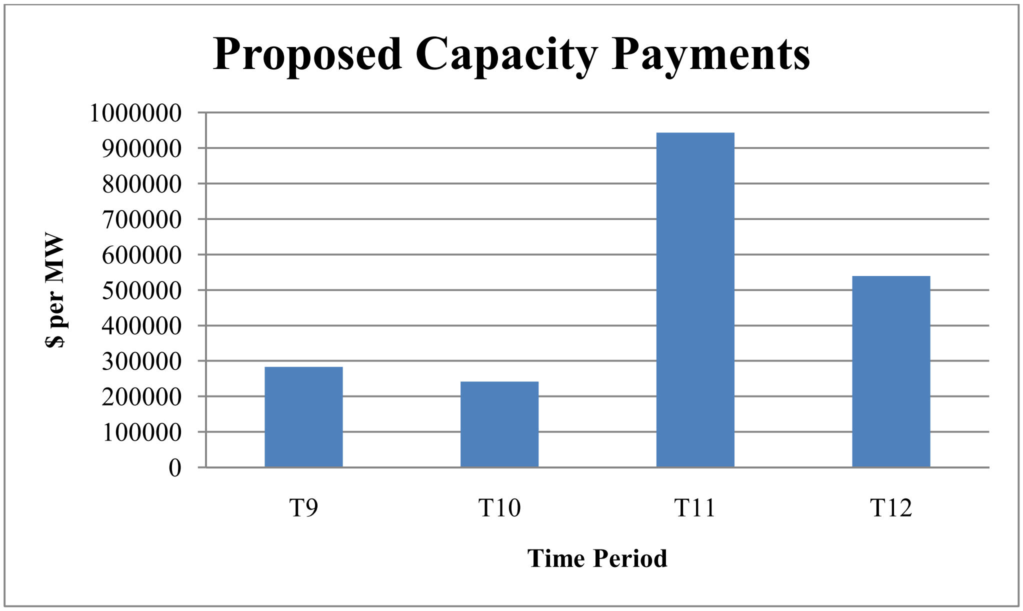


Figure 5-16: Proposed Capacity Payments

The proposed capacity payments result in positive profits for all generators, which have been shown in Figure 5-17 for some selected producers.

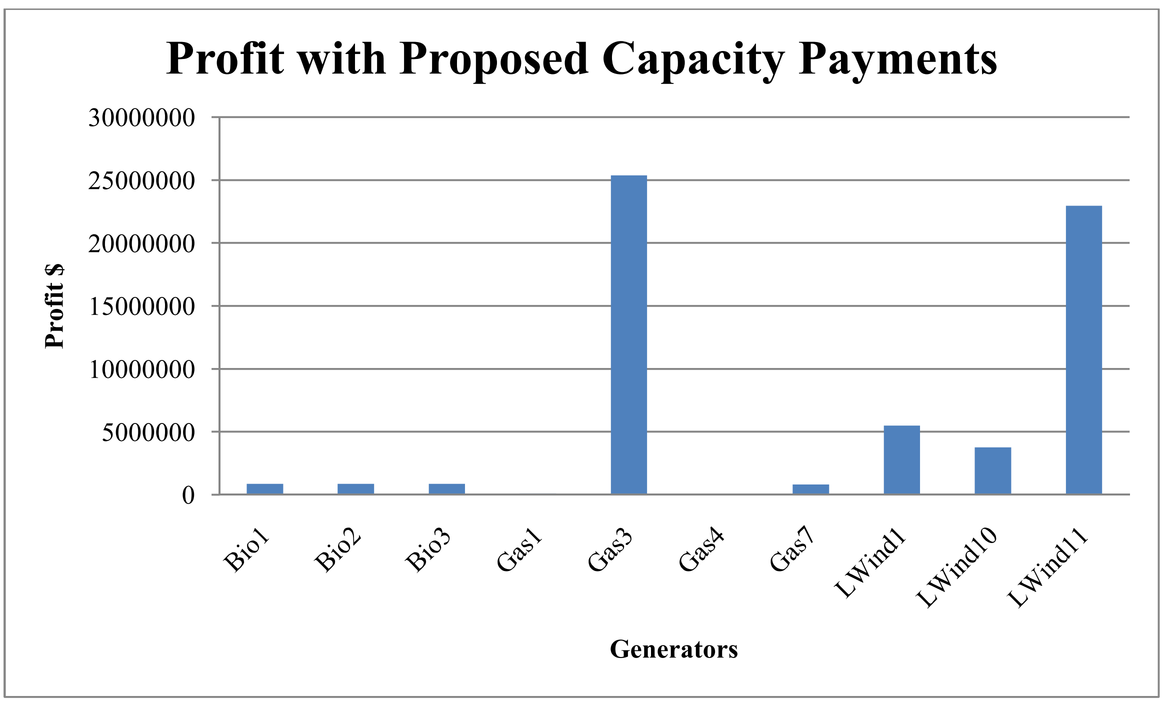


Figure 5-17: Profit with Proposed Capacity Payments

6 Summary and Directions for Future Research

Mechanisms to explore new incentives for investment in the electricity sector play an important role in deregulated electricity markets. Since there is a lot of concern regarding lack of sufficient private sector investment in this area, many solutions such as “Feed in Tariff” or FIT, forward contracts and capacity payments have been proposed. Among these instruments, this thesis investigated capacity payments and developed a methodology to calculate these prices by introducing an MILP model that optimizes overall social welfare.

6.1 Summary of main contributions

6.1.1 Introduced an MINLP Social Welfare Maximization Model

The proposed model includes a long term MINLP social welfare program, maximizing consumers’ plus producers’ surplus, by choosing the amount of generation from new and existing facilities, time of construction and electricity demand in each period. Furthermore, the adequacy of commodity prices which result from the duals of market clearing constraints, have been examined, in order to see whether they produce market equilibrium.

It’s valuable to include price responsiveness in the model, since its characteristics helps regulator in a better forecasting process. Another advantage of the proposed model is forecasting prices based on different demand blocks of peak, intermediate and base, while in IPSP prices have been forecasted regardless of time of the day. Since this model is a long term planning model, computational speed is not an issue. Available MINLP solvers can solve the model in less than an hour; hence the main focus of this thesis has been on modeling the problem.

6.1.2 Capacity Prices

Due to insufficiency of market clearing prices for producers to recover their capital cost, a two part pricing scheme, which pays new generators capacity payments in addition to commodity prices has been introduced.

Previous research in electricity capacity planning have tried to price continuous variables; however there isn't much work done on pricing binary variables like decisions on adding new capacities. Therefore this research included binary variables in the optimization program, based on Ontario's Integrated Power System Plan (IPSP) Data.

6.1.3 Examined O'Neill's Approach in Calculating Capacity Prices

In order to calculate Capacity payments, O'Neill's approach was examined with IPSP data. This approach includes an auxiliary linear or nonlinear program, with additional constraints in which binary variables are set at their optimal values. The dual variables of these constraints are being used as capacity payments, promising that these extra payments provide market equilibrium.

O'Neill's mechanism has been criticized because it discriminates among investors by paying them different additional payments, which makes it difficult to implement in real practice.

6.1.4 Introduced a Novel Approach to Calculate Capacity Payments

To overcome deficiencies with O'Neill's approach, an auxiliary program has been proposed, which calculates non-discriminatory capacity payments. The proposed model is an LP model, minimizing total present worth of all capacity payments while guaranteeing that the new generators have positive profit and also consumers' surplus is greater than zero.

Results demonstrate a strategy to compensate new producers with capacity payments for the next 20 years while meeting the demand with new and existing suppliers. It also compensates producers during the time horizon of the plan regardless of the type of generator. Moreover, the proposed price responsive demand shows a good estimation of future demand, reflecting price fluctuations in the time horizon of the plan.

6.2 Directions for Future Research

Following is a list of suggestions to build on this research:

6.2.1 Extension of Model with Different Technologies

This research can be extended in future by incorporating technologies with continuous capacity additions like distributed generation. Also, the final results of the model may need to be modified according to different countries regulations. For instance to discourage investments in coal generation there could be a penalty term added to the model, decreasing the amount of capacity prices for that special generator.

6.2.2 Incorporating other Policy Mechanisms like FIT

Moreover, this model can be used in calculating other policy instruments like Feed-In-Tariff. Knowing the cost of new generators, the model can estimate a reasonable amount for premiums, in order to encourage investments in electricity markets.

6.2.3 Exploring Mechanisms to Compensate Producers

Since the proposed capacity payments are paid in just a few time periods through the time horizon of the plan, e.g. five time periods in the numerical example, there would be a delay in paying producers. For instance, according to the numerical example, the generators which have been built in the beginning of the plan should wait for a few years, up to t10, to receive their first capacity price.

In order to overcome this problem, the results of the model could be modified to have the capacity prices paid to producers in a form of annuity each year after their construction. However, the effect of this new payment strategy on social welfare should be investigated.

Appendix A

Indices:	
i	Set of new sources of energy (Nuclear, Gas, Bio, Wind and Hydro)
j	Set of old sources of energy (Nuclear, Gas, Coal, Renewable, Interconnection)
t	Time period (Year)
s	Set of Demand blocks (Base, Intermediate, Peak)
Variables	
$X_{i,t,s}$	Level of production from new capacities (in MW)
$Exgen_{j,t,s}$	Level of production from existing and committed capacities (in MW) in time period t and demand block s
$Z_{i,t}$	Binary variable, indicating build or no build decision for generator i in time period t
$W_{j,t}$	Binary Variable, used to allocate fixed variable cost for existing generator j in time period *t,* only if it is active
$q^D_{t,s}$	Demand in time period t and demand block s
Parameters	
$\alpha_{t,s}$	Slope of demand function
$\beta_{t,s}$	Intersection of linear demand function with price axis
cD_s	Cost of delivery charges in demand block s
r	Interest rate
h_s	Allocated hours of each demand block in a year
$C_{i,t}$	Present worth of variable cost of generator i in time period t
$fuel_{i,t}$	Present worth of fuel cost for generator I in time period t
$K_{i,t}$	Present worth of capital cost of building generator I in time period t
$FixedOpC_i$	Present worth of fixed operating cost for generator i
XK_i	Installed capacity of generator i in MW
$C_{j,t}$	Present worth of variable cost of existing generator j in time period t
$fuel_{j,t}$	Present worth of fuel cost for existing generator j in time period t
$FixedOpC_j$	Present worth of fixed operating cost for existing generator j
XK_j	Installed capacity of existing generator j in MW
$Capf_i$	Capacity factor for new generator i
$Capf_j$	Capacity factor for existing generator j
av_i	Availability factor for new generator i
av_j	Availability factor for existing generator j

$reserverate_t$	Reserve margin in time period t
$EmissionCap_t$	Maximum limit of emission in time period t

Appendix B

Contribution of Existing and Committed Resources in Numerical Example

	Contribution of Existing Resources Towards Resource Requirement (MW)										
Year	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11
Nuclear	11419	11419	11419	9879	9879	9879	9363	9363	8050	6686	6170
Gas/Oil	4578	4578	4578	4578	2473	2473	2308	2308	2004	1897	1691
Renewable	6129	6129	6129	6129	6129	6129	6129	6129	6129	6129	6129
Coal	6434	6434	6434	6434	4969	3293	3293	3293	0	0	0
Interconnection	500	500	500	500	500	500	500	500	500	500	500

Year	T12	T13	T14	T15	T16	T17	T18	T19	T20	T21
Nuclear	4487	2792	1911	515	515	515	515	515	515	0
Gas/Oil	1236	1236	1236	1236	1105	1105	1105	1105	1105	1105
Renewable	6129	6129	6129	6129	6129	6129	6129	6129	6129	6129
Coal	0	0	0	0	0	0	0	0	0	0
Interconnection	500	500	500	500	500	500	500	500	500	500

Table B-1: Contribution of Existing Resources

Source: (OPA, Integrated Power System Plan-Exhibit D - Tab 3 - Schedule 1, 2007)

Potential Wind Capacities

Potential Wind Capacities in Ontario										
Site	Lwind1	LWind2	Lwind3	Lwind4	Lwind5	Lwind6	Lwind7	Lwind8	Lwind9	Lwind10
Capacity	48	50	109	130	162	42	200	200	41	33
Site	Lwind11	Lwind12	Lwind13	Lwind14	Lwind15	Lwind16	Lwind17	Lwind18	Lwind19	Lwind20
Capacity	200	85	145	172	33	40	43	50	71	200
Site	Lwind21	Lwind22	Lwind23	Lwind24	Lwind25	Lwind26	Lwind27	Lwind28	Lwind29	Lwind30
Capacity	100	200	85	100	152	69	177	188	192	75
Site	Lwind31	Lwind32	Lwind33	Lwind34	Lwind35	Lwind36	Lwind37	Lwind38	Lwind39	Lwind40
Capacity	75	79	200	60	123	36	66	72	200	125
Site	Lwind41	Lwind42	Lwind43	Lwind44	Lwind45	Lwind46	Lwind47	Lwind48	Lwind49	Lwind50
Capacity	155	200	200	200	200	200	200	119	54	57
Site	Lwind51	Lwind52	Lwind53	Lwind54	Lwind55	Lwind56	Lwind57	Lwind58	Lwind59	Lwind60
Capacity	58	100	200	200	200	200	200	66	76	78
Site	Lwind61	Lwind62	Lwind63	Lwind64	Lwind65	Lwind66	Lwind67	Lwind68	Lwind69	Lwind70
Capacity	200	154	179	200	200	96	200	44	59	88
Site	Lwind71	Lwind72	Lwind73							
Capacity	107	163	187							

Table B-2: Ontario's Potential Wind Capacity

Source: (OPA, Integrated Power System Plan - Exhibit D, Tab 5, Schedule 1, 2007)

Potential Hydro Capacities

Medium Hydro Sites in Ontario

Site	MHydro1	MHydro2	MHydro3	MHydro4	MHydro5	MHydro6	MHydro7	MHydro8	MHydro9	MHydro10
Capacity	1	16	58	12	13	11	126	490	18	14

Site	MHydro11	MHydro12	MHydro13	MHydro14	MHydro15	MHydro16	MHydro17	MHydro18	MHydro19	MHydro20
Capacity	42	17	21	28	48	36	25	12	47	12

Site	MHydro21	MHydro22
Capacity	94	16

Large Hydro Sites in Ontario

Site	LHydro1	LHydro2	LHydro3	LHydro4	LHydro5	LHydro6	LHydro7	LHydro8	LHydro9	LHydro10
Capacity	174	131	295	140	131	135	126	490	370	106

Site	LHydro11	LHydro12	LHydro13	LHydro14	LHydro15
Capacity	485	729	1558	192	250

Table B-3: Ontario's Potential Hydro Capacities

Source: (OPA, Integrated Power System Plan - Exhibit D, Tab 5, Schedule 1, 2007)

Potential Nuclear Facilities

Nuclear Facilities								
Site	Nuc1	Nuc2	Nuc3	Nuc4	Nuc5	Nuc6	Nuc7	Nuc8
Capacity	516	516	2013	1548	1397	1683	1695	881

Table B-4: Ontario's Planned Nuclear Facilities

Source: (OPA, IPSP, Exhibit D, Tab 6, Schedule 1, 2007)

Potential Gas Facilities

Gas Facilities										
Site	Gas1	Gas2	Gas3	Gas4	Gas5	Gas6	Gas7	Gas8	Gas9	Gas10
Capacity	10	586	350	450	850	550	165	304	250	250

Table B-5: Ontario's Planned Gas Facilities

Source: (OPA, IPSP, Exhibit D, Tab 8, Schedule 1, 2007)

Potential Bio-Fuel Facilities

Bio Facilities										
Site	Bio1	Bio2	Bio3	Bio4	Bio5	Bio6	Bio7	Bio8	Bio9	Bio10
Capacity	10	10	10	20	20	13	13	4	35	30
Site	Bio11	Bio12	Bio13	Bio14	Bio15					
Capacity	33	30	16	48	111					

Table B-6: Ontario's Planned Bio Energy Facilities

Source:(OPA, IPSP, Exhibit D, Tab 5, Schedule 1, Attachment 5, 2007)

Capacity Factor and Availability Factor

	Capacity Factor	Availability Factor
Nuclear	0.94	0.9
Gas	0.65	0.97
Bio	0.8	0.85
Wind	0.35	0.98
Hydro	0.75	0.9
Coal	0.89	0.95

Table B-7: Generators' Capacity and Availability Factors

Source: (Navigant, Evaluation of Cost of New Entry , 2007)

Demand Elasticity

Year	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10
Off-Peak	-0.36	-0.38	-0.39	-0.4	-0.41	-0.42	-0.42	-0.43	-0.44	-0.44
Mid-Peak	**-0.28**	-0.29	-0.3	-0.32	-0.33	**-0.34**	-0.34	-0.35	-0.36	-0.36
Peak	-0.22	-0.24	-0.25	-0.28	-0.28	-0.28	-0.28	-0.29	-0.3	-0.31

Year	t11	t12	t13	t14	t15	t16	t17	t18	t19	t20
Off-Peak	-0.45	-0.46	-0.47	-0.48	-0.48	-0.49	-0.49	-0.49	-0.5	-0.5
Mid-Peak	**-0.37**	-0.37	-0.38	-0.39	-0.39	**-0.4**	-0.4	-0.4	-0.41	**-0.41**
Peak	-0.31	-0.32	-0.33	-0.34	-0.35	-0.35	-0.36	-0.36	-0.37	-0.37

Table B-8: Demand Elasticity

Source: (OPA, Load Forecast - IPSP Reference Energy and Demand Forecast, 2007)*

*** Values in Bold are from IPSP, the rest of the values have been approximated.**

Price Assumptions

	Electricity Price assumption ($/MWh)										
retail price	**t1**	**t2**	**t3**	**t4**	**t5**	**t6**	**t7**	**t8**	**t9**	**t10**	**t11**
off peak($/MWh)	65	65	63	62	61	60	58	57	56	55	54
Mid peak ($/Mwh)	**125**	125	123	122	121	**120**	118	117	116	115	**114**
Peak($/MWh)	165	165	163	162	161	160	158	157	156	155	154
retail price	**t12**	**t13**	**t14**	**t15**	**t16**	**t17**	**t18**	**t19**	**t20**	**t21**	
off peak($/MWh)	52	50	49	47	46	45	45	44	44	44	
Mid peak ($/Mwh)	112	110	109	107	**106**	105	105	104	104	104	
Peak($/MWh)	152	150	149	147	146	145	145	144	144	144	

Table B-9: Electricity Price assumptions

Source:(OPA, Load Forecast – IPSP Reference Energy And Demand Forecast, 2007)*

* Values in Bold are from IPSP, the rest of the values have been approximated.

Demand Assumptions

	Demand (MWh)										
Demand	**t1**	**t2**	**t3**	**t4**	**t5**	**t6**	**t7**	**t8**	**t9**	**t10**	**t11**
off-peak Demand - MW	21340	21360	21420	21500	21800	21900	22000	22100	22150	22200	22540
off-peak Demand - MWh	93469200	93556800	93819600	94170000	95484000	95922000	96360000	96798000	97017000	97236000	98725200
mid-peak Demand - MW	21450	21600	21800	22750	23800	23900	24550	24700	24950	25100	26000
mid-peak Demand - MWh	62634000	63072000	63656000	66430000	69496000	69788000	71686000	72124000	72854000	73292000	75920000
Peak Demand - MW	26986	27000	27250	27700	27900	28099	28200	28500	28900	29500	29936
Peak Demand - MWh	39399560	39420000	39785000	40442000	40734000	41024540	41172000	41610000	42194000	43070000	43706560

Demand	**t12**	**t13**	**t14**	**t15**	**t16**	**t17**	**t18**	**t19**	**t20**	**t21**
off-peak Demand - MW	22700	22950	23000	23140	23180	24000	24500	24650	24800	25000
off-peak Demand - MWh	99426000	1.01E+08	1.01E+08	1.01E+08	1.02E+08	1.05E+08	1.07E+08	1.08E+08	1.09E+08	1.1E+08
mid-peak Demand - MW	26480	27000	27180	27500	27700	28000	29000	29450	30000	30500
mid-peak Demand - MWh	77321600	78840000	79365600	80300000	80884000	81760000	84680000	85994000	87600000	89060000
Peak Demand - MW	30000	31000	31500	32000	32563	33000	33677	33700	33800	34000
Peak Demand - MWh	43800000	45260000	45990000	46720000	47541980	48180000	49168420	49202000	49348000	49640000

Table B-10 : Demand Assumptions

Source: (OPA, Load Forecast – IPSP Reference Energy And Demand Forecast, 2007)Demand Curve Parameters

	Demand Curve parameter (α) in $/(MWh)^2						
	t1	**t2**	**t3**	**t4**	**t5**	**t6**	**t7**
α (Base)	-0.0000019	-0.0000018	-0.0000017	-0.0000016	-0.0000016	-0.0000015	-0.0000014
α (Intermediate)	-0.0000071	-0.0000068	-0.0000064	-0.0000057	-0.0000053	-0.0000051	-0.0000048
α (peak)	-0.0000190	-0.0000174	-0.0000164	-0.0000143	-0.0000141	-0.0000139	-0.0000137
	t8	**t9**	**t10**	**t11**	t12	t13	t14
α (Base)	-0.0000014	-0.0000013	-0.0000013	-0.0000012	-0.0000011	-0.0000011	-0.0000010
α (Intermediate)	-0.0000046	-0.0000044	-0.0000044	-0.0000041	-0.0000039	-0.0000037	-0.0000035
α (peak)	-0.0000130	-0.0000123	-0.0000116	-0.0000114	-0.0000108	-0.0000100	-0.0000095
	t15	**t16**	**t17**	**t18**	**t19**	**t20**	**t21**
α (Base)	-0.0000010	-0.0000009	-0.0000009	-0.0000009	-0.0000008	-0.0000008	-0.0000008
α (Intermediate)	-0.0000034	-0.0000033	-0.0000032	-0.0000031	-0.0000029	-0.0000029	-0.0000028
α (peak)	-0.0000090	-0.0000088	-0.0000084	-0.0000082	-0.0000079	-0.0000079	-0.0000078

Table B-11: Demand Curve Parameter α

	Demand Curve parameter (β)						
	t1	t2	t3	t4	t5	t6	t7
β (Base)	245.56	236.05	224.54	217.00	209.78	202.86	196.10
β (Intermediate)	571.43	556.03	533.00	503.25	487.67	472.94	465.06
β (peak)	915.00	852.50	815.00	740.57	736.00	731.43	722.29
	t8	t9	t10	t11	t12	t13	t14
β (Base)	189.56	183.27	180.00	174.00	165.04	156.38	151.08
β (Intermediate)	451.29	438.22	434.44	422.11	414.70	399.47	388.49
β (peak)	698.38	676.00	655.00	650.77	627.00	604.55	587.24
	t15	t16	t17	t18	t19	t20	t21
β (Base)	144.92	139.88	136.84	136.84	132.00	132.00	132.00
β (Intermediate)	381.36	371.00	367.50	367.50	357.66	357.66	357.66
β (peak)	567.00	563.14	547.78	547.78	533.19	533.19	533.19

Table B-12: Demand Curve Parameter β

Facilities Life

	Age of Facilities					
Facilities	Nuclear	Gas	Bio	Wind	Medium Hydro	Large Hydro
Life (Yrs)	30	20	15 & 20	30	80	100

Table B-13: Age of Facilities

Source: (Navigant, Evaluation of Cost of New Entry, 2007)

Generators' Unit Capital Costs

	Unit Capital Cost of Generators ($/KWh)								
Facilities	Nuclear	Gas		Bio		Wind	Hydro	
		SCGT	CCGT	Landfill	Biomass		Medium	Large
Cost($/KWh)	2970	665	1174	2288	2096	1741	2750	2000

Table B-14: Capital Cost Assumptions

Source:(Navigant, Evaluation of Cost of New Entry , 2007)

Generators' Operating Costs

	VarCost$_i$ & Growthrate$_i$ for C_{it} and FixedOpCost$_i$							
Facilities	Nuclear	Gas		Bio		Wind	Hydro	
		SCGT	CCGT	Landfill	Biomass		Medium	Large
Variable Cost($/MWh)	1.5	3.5	2.75	0	4	0	1.5	1.5
Growth rate	0.03	0.05	0.05	0	0.03	0	0.015	0.015
Fixed Operating Costs($/Kw)	89	16	17	140	231	37	25	27

Table B-15: Variable and Fixed Costs Assumptions

Source:(Navigant, Evaluation of Cost of New Entry , 2007)

Generator's Fuel Costs

	VarCost$_i$ & Growthrate$_i$ for fuel$_{it}$							
Facilities	Nuclear	Gas		Bio		Wind	Hydro	
		SCGT	CCGT	Landfill	Biomass		Medium	Large
Variable Cost($/MWh)	6	56	56	0	23	0	0	0
Growth rate	0.02	0.05	0.03	0	0	0	0	0

Table B-16: Fuel Costs Assumptions

Source:(Navigant, Evaluation of Cost of New Entry , 2007)

Appendix C: GAMS Code

```
Set
   I Planned generator /Nuc1*Nuc8 ,
                        Gas1*Gas10,
                        Bio1*Bio15,
                        LWind1*LWind73,
                        SWind1*SWind11,
                        SHydro1*SHydro130,
                        MHydro1*MHydro22,
                        LHydro1*LHydro15/
   Nuc(i) new Nuc capacities /Nuc1*Nuc8/
   Gas(i) new Gas capacities /Gas1*Gas10/
   Bio(i) new Bio capacities /Bio1*Bio15/
   Wind(i) new wind capacities /LWind1*LWind73,SWind1*SWind11/
   Hydro(i) new Hydro capacities
/SHydro1*SHydro130,MHydro1*MHydro22,LHydro1*LHydro15/
   j Existing and committed facilities /ExCoal, ExNuc, ExGas, ExRnw,
Intercon/
   t period /T1*T21/
   s status /Peak,Intermediate, Base/;

alias(t,tt);

**Source(Wind): IPSP D,5,1
**Source(Hydro): IPSP D,5,1
**Source(Nuclear): IPSP D,6,1
**Source(Gas): IPSP D,8,1
**Source(Bio): IPSP D,5,1
Parameter XK(i) Capacity of planned generator i in MW
/Nuc1 516,Nuc2 516,Nuc3 2013,Nuc4 1548,Nuc5 1397,Nuc6 1683,Nuc7 1695,Nuc8
881,

Gas1 1,Gas2 586,Gas3 350,Gas4 450,Gas5 850,
Gas6 550, Gas7 165,Gas8 304,Gas9 250,Gas10 250,

Bio1 10,Bio2 10,Bio3 10, Bio4 20, Bio5 20, Bio6 13, Bio7 13, Bio8 4, Bio9
35,
Bio10 30, Bio11 33, Bio12 30, Bio13 16, Bio14 48, Bio15 111,

LWind1 48,LWind2 50,LWind3 109, LWind4 130, LWind5 162,LWind6 42, LWind7
200,
LWind8 200,LWind9 41,LWind10 33,LWind11 200,LWind12 85,LWind13 145,LWind14
172,
LWind15 33,LWind16 40,LWind17 43,LWind18 50,LWind19 71,LWind20 200,LWind21
100,
LWind22 200,LWind23 85,LWind24 100,LWind25 152,LWind26 69,LWind27 177,
LWind28 188,LWind29 192,LWind30 75,LWind31 75,LWind32 79,LWind33
200,LWind34 60,
```

```
LWind35 123,LWind36 36,LWind37 66,LWind38 72,LWind39 200,LWind40 125,
LWind41 155,LWind42 200,LWind43 200,LWind44 200,LWind45 200,LWind46 200,
LWind47 200,LWind48 119,LWind49 54,LWind50 57,LWind51 58,LWind52 100,
LWind53 200,LWind54 200,LWind55 200,LWind56 200,LWind57 200,LWind58 66,
LWind59 76,LWind60 78,LWind61 200,LWind62 154,LWind63 179,LWind64 200,
LWind65 200,LWind66 96,LWind67 200,LWind68 44,LWind69 59,
LWind70 88,LWind71 107,LWind72 163,LWind73 187,

SWind1 753,SWind2 180,SWind3 168,SWind4 59,SWind5 174,SWind6 113,SWind7
277,
SWind8 0,SWind9 120,SWind10 25,SWind11 919,

**Hydro Info Source : Ontario Water Power Potentials, Hatch Co.
SHydro1 6,SHydro2  1,SHydro3 1,SHydro4 4,SHydro5 10,SHydro6 10,SHydro7 3,
SHydro8 1,SHydro9 11,SHydro10 5,SHydro12 2,SHydro13 4,SHydro15 2,SHydro16
2,
SHydro17 2,SHydro18 1,SHydro19 1,SHydro22 10,SHydro23 6,SHydro24
3,SHydro25 4,
SHydro26 7,SHydro27 5,SHydro28 2,SHydro29 2,SHydro31 9,SHydro32 3,SHydro33
3,
SHydro35 10,SHydro36 7,SHydro37 10,SHydro38 3,SHydro39 1,SHydro40 1,
SHydro41 2,SHydro42 4,SHydro43 7,SHydro45 3,SHydro47 5,SHydro49 8,SHydro50
1,
SHydro51 4,SHydro52 2,SHydro53 7,SHydro54 4,SHydro55 10,SHydro56
8,SHydro57 3,
SHydro58 4,SHydro59 2,SHydro60 4,SHydro61 5,SHydro62 4,SHydro63 7,SHydro64
5,
SHydro65 5,SHydro66 5,SHydro67 5,SHydro68 7,SHydro69 7,SHydro70 5,SHydro71
7,
SHydro72 4,SHydro73 2,SHydro74 1,SHydro75 6,SHydro76 4,SHydro77 5,SHydro78
7,
SHydro80 5,SHydro81 5,SHydro83 2,SHydro84 2,SHydro86 7,SHydro89 2,SHydro90
3,
SHydro91 10,SHydro94 5,SHydro95 7,SHydro96 7,SHydro97 8,SHydro98
9,SHydro99 10,
SHydro109 3,SHydro116 6,SHydro117 2,SHydro118 10,
SHydro121 9,SHydro124 2, SHydro126 2,SHydro127 2,SHydro128 2,SHydro130 3,

MHydro1 1,MHydro2 16,MHydro3 58,MHydro4 12,MHydro5 13,MHydro6 11,MHydro7
85,
MHydro8 85,MHydro9 18,MHydro10 14,MHydro11 42,MHydro12 17,MHydro13 21,
MHydro14 28,MHydro15 48,MHydro16 36,MHydro17 25,MHydro18 12,MHydro19 47,
MHydro20 12,MHydro21 94,MHydro22 16,

LHydro1 174,LHydro2 131,LHydro3 295,LHydro4 140,LHydro5 131,LHydro6 135,
LHydro7 126,LHydro8 490,LHydro9 370,LHydro10 106,LHydro11 485,LHydro12
729,
LHydro13 1558,LHydro14 192,LHydro15 250/;

**source: IPSP D-3_1 table 4 and 7
Table excap(j,t) existing and committed generation capacities in MW
            T1        T2        T3       T4      T5       T6       T7
T8      T9       T10      T11      T12      T13      T14     T15     T16
T17      T18      T19      T20      T21
```

```
ExNuc       11419    11419     11419     9879    9879     9879     9363
9363    8050     6686     6170     4487     2792     1911     515     515
515      515      515      515       0
ExGas       4578     4578      4578      4578    2473     2473     2308
2308    2004     1897     1691     1236     1236     1236     1236    1105
1105     1105     1105     1105     1105
ExRnw       6129     6129      6129      6129    6129     6129     6129
6129    6129     6129     6129     6129     6129     6129     6129    6129
6129     6129     6129     6129     6129
ExCoal      6434     6434      6434      6434    4969     3293     3293
3293     0        0        0        0        0        0        0        0
0        0        0        0        0
Intercon    500      500       500       500     500      500      500
500     500      500      500      500      500      500      500      500
500     500      500      500      500

**Source: IPSP D-3_1, Attachment 1
Parameter r interest rate;
   r = .04;

**Source : S& P Assessment and NAVIGANT report
**Gas gens are either combined cycle gas turbine or CCGT or
**Simple cycle gas turbine or SCGT
Parameter age(i) Accounting life of the generator
/Nuc1*Nuc8 30,
 Gas1*Gas10 20,
 Bio1*Bio8 20, Bio9*Bio15 20,
 LWind1*LWind73 30,
 SWind1*SWind11 30,
 SHydro1*SHydro130 75,
 MHydro1*MHydro22 80,
 LHydro1*LHydro15 100/;

**Source: IPSP D-3-1 Attachment2, Navigant Report,
parameter FixedOpCost(i) Fixed operating cost $ per KW per Year
/Nuc1*Nuc8 89,
Gas1 17,Gas2 22,Gas3 17,Gas4 17,Gas5 17,Gas6 17,Gas7 34,Gas8 34,Gas9
17,gas10 17,
Bio1*Bio8 140, Bio9*Bio15 231,
LWind1*LWind73 37,
SWind1*SWind11 41,
SHydro1*SHydro130 20,
MHydro1*MHydro22  25,
LHydro1*LHydro15 27/;

*Source: IPSP D-3-1 Attachment2, Navigant Report,
parameter constcost(i) Construction cost in $ per kw
/Nuc1*Nuc8 2970,
Gas1 665,Gas2 1413,Gas3 665,Gas4 924,Gas5 665,
Gas6 924, Gas7 1174,Gas8 1174,Gas9 924,gas10 924,
Bio1*Bio8 2288, Bio9*Bio15 2096,
LWind1*LWind73 1741,
SWind1*SWind11 2750,
SHydro1*SHydro130 3700,
```

```
MHydro1*MHydro22 2750,
LHydro1*LHydro15 2000/;

parameter k(i,t) Coefficient of binary variable in $ ;
k(i,t) = constcost(i)*1000* xk(i)*((20-ord(t)+1)/age(i)) /
((1+r)**(ord(t)));

*Source: IPSP D-3-1 Attachment2, Navigant Report,
Parameter varc(i) Variable cost for each producer in $ per MWh
/Nuc1*Nuc8 1.5,
 Gas1 2.75,Gas2 3,Gas3 3.5,Gas4 2.75,Gas5 2.75,Gas6 3.5,
 Gas7 3.5,Gas8 3.5,Gas9 2.75, gas10 2.75,
 Bio1*Bio8 0, Bio9*Bio15 4,
 LWind1*LWind73 0,
 SWind1*SWind11 0,
 SHydro1*SHydro130 1,
 MHydro1*MHydro22 1.5,
 LHydro1*LHydro15 1.5/;

parameter varcgrowth(i) Annual variable cost growth for each type of
generation
/Nuc1*Nuc8 .03,
 Gas1*Gas10 .05 ,
 Bio1*Bio15 0.03,
 LWind1*LWind73 .01,
 SWind1*SWind11 .01,
 SHydro1*SHydro130 .015,
 MHydro1*MHydro22 .015,
 LHydro1*LHydro15 .015/;

parameter c(i,t) PW of variable cost per unit of capacity for each gen in
each time period in $ per MWh;
c(i,t) = varc(i)*((1+varcgrowth(i))** (ord(t)-1))/((1+r)**ord(t));

**source : IPSP D-8 --> gas price : 8 $/MMBTU
**source : Navigant report : Heat rate info
**formula for fuel cost in $/mwh : gas price * heat rate ($/MMBTU *
MMBTU/MWh)
parameter fuel(i) fuel cost for each generator in $ per MWh
/Nuc1*Nuc8 6,
 Gas1 56,Gas2 56,Gas3 56,Gas4 56,Gas5 56,Gas6 56, Gas7 56,Gas8 56,Gas9 56,
gas10 56,
 Bio1*Bio8 0, Bio9*Bio15 23,
 LWind1*LWind73 0,
 SWind1*SWind11 0,
 SHydro1*SHydro130 0,
 MHydro1*MHydro22 0,
 LHydro1*LHydro15 0/;

parameter fuelgrowthn(i) growth rate for fuel for each new generator
/Nuc1*Nuc8 .02,
 Gas1*Gas10 .05 ,
 Bio1*Bio15 0.03,
 LWind1*LWind73 0,
```

```
 SWind1*SWind11 0,
 SHydro1*SHydro130 0,
 MHydro1*MHydro22 0,
 LHydro1*LHydro15 0/;

parameter fueln(i,t) fuel cost for each new generator in $ per MWh;
fueln(i,t)= fuel(i)*(1+fuelgrowthn(i))/((1+r)**ord(t));

Parameter varcc(j) variable cost for each generator in $ per MWh
/ExNuc 1.5,ExGas 2.7, ExRnw .2, ExCoal .5, Intercon 5 /;

parameter varccgrowth(j) annual variable cost growth for each producer
/ExNuc .03,ExGas .05, ExRnw .03,ExCoal .01 , Intercon .02/;

parameter cc(j,t) PW of variable cost per unit of capacity for each
generator in each time period in $ per MWh;
cc(j,t) = varcc(j)*(1+varccgrowth(j))** (ord(t)-1)/((1+r)**ord(t));

parameter FixedOpCc(j) fixed operating cost $ per KW per Year for existing
gen
/ExNuc 89, ExGas 17, ExRnw 30, ExCoal 10, Intercon 0 /;

**source : coal price : Navigant Report, Ontario Wholesale Electricity
Market Price Forecast, 2.5 $/MMBTU
**source : coal gen heat rate : 10800  BTU/kwh
parameter fuele(j) fuel cost for existing generator in $ per MWh
/ExNuc 6,ExGas 56, ExRnw 0, ExCoal 27, Intercon 0/;

parameter fuelgrowthex(j) Fuel growth rate for each new generator
/ExNuc .02,ExGas .05, ExRnw 0, ExCoal .01, Intercon 0/;

parameter fuelex(j,t) fuel cost for each old generator in $ per MWh;
fuelex(j,t)= fuele(j)*(1+fuelgrowthex(j))/((1+r)**ord(t));

**Source: IPSP (Exhibit D-2-1, Attachment 1, Page 8)
**based on this formula (required available capacity - forcasted peak
demand)/forcast peak demand)
parameter reserverate(t) reserve rate in each time period
/T1 .175, T2 .174, T3 .172, T4 .17, T5 .16, T6 .155, T7 .15, T8 .14, T9
.139,
 T10 .137, T11 .14, T12 .141, T13 .142, T14 .144, T15 .14, T16 .13, T17
.125,
 T18 .121, T19 .12, T20 .119, T21 .119/;

parameter av(i) availabity factor-portion of time that the unit is down
for maintenance or outage
/Nuc1*Nuc8 .9,
 Gas1*Gas10 .97,
 Bio1*Bio15 .85,
 LWind1*LWind73 .98,
 SWind1*SWind11 .98,
 SHydro1*SHydro130 .9,
 MHydro1*MHydro22 .9,
 LHydro1*LHydro15 .9/;
```

```
parameter avv(j) availability factor-time the unit is not down for
maintenance outage
/ExNuc .9,ExGas .97, ExRnw .98, ExCoal .95, Intercon 1/;

parameter capf(i) capacity factor - ratio of actual output to 100% of
capacity
/Nuc1*Nuc8 .94,
 Gas1*Gas10 .65,
**Source: IPSP D-5 pg22 Att5
 Bio1*Bio15 .8,
**Source: IPSP D-5 pg9  Att4
 LWind1*LWind73 .35,
 SWind1*SWind11 .30 ,
 SHydro1*SHydro130 .75,
 MHydro1*MHydro22 .75,
 LHydro1*LHydro15 .75/;

parameter capff(j) capacity factor for existing gen- ratio of actual
output to 100% of capacity
/ExNuc .9,ExGas .65, ExRnw .5, ExCoal .89, Intercon 1 /;

**Hours for each status: peak 4 hr/day, interm 8 hr/day, Base 12 hr/day
parameter h(s) number of hours for each load status during
/Peak 1460, Intermediate 2920, Base 4380/;

table b(t,s) representing beta in inverse demand function in $per(mwh)^2
              Peak          Intermediate          Base
T1         -0.0000190       -0.0000071       -0.0000019
T2         -0.0000174       -0.0000068       -0.0000018
T3         -0.0000164       -0.0000064       -0.0000017
T4         -0.0000143       -0.0000057       -0.0000016
T5         -0.0000141       -0.0000053       -0.0000016
T6         -0.0000139       -0.0000051       -0.0000015
T7         -0.0000137       -0.0000048       -0.0000014
T8         -0.0000130       -0.0000046       -0.0000014
T9         -0.0000123       -0.0000044       -0.0000013
T10        -0.0000116       -0.0000044       -0.0000013
T11        -0.0000114       -0.0000041       -0.0000012
T12        -0.0000108       -0.0000039       -0.0000011
T13        -0.0000100       -0.0000037       -0.0000011
T14        -0.0000095       -0.0000035       -0.0000010
T15        -0.0000090       -0.0000034       -0.0000010
T16        -0.0000088       -0.0000033       -0.0000009
T17        -0.0000084       -0.0000032       -0.0000009
T18        -0.0000082       -0.0000031       -0.0000009
T19        -0.0000079       -0.0000029       -0.0000008
T20        -0.0000079       -0.0000029       -0.0000008
T21        -0.0000078       -0.0000028       -0.0000008

table a(t,s) representing alpha in inverse demand function in $ per MWh

           Peak        Intermediate     Base
T1         915.00        571.43        245.56
```

```
T2          852.50        556.03        236.05
T3          815.00        533.00        224.54
T4          740.57        503.25        217.00
T5          736.00        487.67        209.78
T6          731.43        472.94        202.86
T7          722.29        465.06        196.10
T8          698.38        451.29        189.56
T9          676.00        438.22        183.27
T10         655.00        434.44        180.00
T11         650.77        422.11        174.00
T12         627.00        414.70        165.04
T13         604.55        399.47        156.38
T14         587.24        388.49        151.08
T15         567.00        381.36        144.92
T16         563.14        371.00        139.88
T17         547.78        367.50        136.84
T18         547.78        367.50        136.84
T19         533.19        357.66        132.00
T20         533.19        357.66        132.00
T21         533.19        357.66        132.00;

parameter cD(s) Cost of Delivery
/Peak 6.7,Intermediate 5.7, Base 3.7/;

*************************************************************************
***************                                            **************
***************            ORIGINAL PPROBLEM               **************
***************                                            **************
*************************************************************************

Variable Socialwelfare definition of social welfare in 1000s of units;

Positive variables
   X(i,t,s)  electricity produced by new gen in period t and status s in
             MW
   exgen(j,t,s) electricity produced by existing gen in t and status s in
                MW
   qD(t,s)   demand in period t and status s in mwh ;

binary variable
   Z(i,t)  binary var indicating building or not building a new generator
   w(j,t)  binary var indicating active or non active gens in each period

Equations
   Objective definition of present worth of socialwelfare
   Supply(t,s) supply constraint
   Capacity(i,t,s)  Capacity constraint for just new facilities
   NewFacility(i) limiting each gen to be built at most once during time
                  horizon
   exgenbin(j,t) Constraint that sets w equal to zero if excap is shut
                 down
   ExistingSupplyCap(j,t,s) capacity constraint for existing facilities
   reserve(t) reserve capacity
   NucMax  Max capacity of Nuclear generation
```

```
   GasMax  Max capacity of Gas generation
   BioMax  Max capacity of Bio generation
   WindMax Max capacity of Wind generation
   HydroMax Max capacity of Hydro generation;

Objective.. sum((t,s),(a(t,s)*qD(t,s)+(.5*b(t,s)*(qD(t,s)**2)
            -cD(s)*qD(t,s)))/((1+r)**ord(t)))
            -sum((i,t,s), c(i,t)*x(i,t,s)*h(s)+ fueln(i,t)*x(i,t,s)*h(s)
            + K(i,t)* z(i,t)+ FixedOpCost(i)*1000*z(i,t)*xk(i)*
                 (sum(tt$(ord(tt)>=ord(t)), 1/((1+r)**ord(tt)))))
            -sum((j,t,s),cc(j,t)* exgen(j,t,s)*h(s)+FixedOpCc(j)*W(j,t)
                 *1000*excap(j,t)*(1/((1+r)**ord(t)))
            +fuelex(j,t)*exgen(j,t,s)*h(s))- socialwelfare*1000 =e= 0;
Supply(t,s).. sum (i,x(i,t,s)*h(s))+sum(j,exgen(j,t,s)*h(s))-qD(t,s)=g= 0;
Capacity(i,t,s).. x(i,t,s)-XK(i)*capf(i)*av(i)
                         *sum(tt$(ord(tt)<=ord(t)),z(i,tt))=l=0;
Newfacility(i)..  sum(t,z(i,t)) =l= 1;
exgenbin(j,t).. 20000 * w(j,t)=g= excap(j,t);
ExistingSupplyCap(j,t,s).. exgen(j,t,s)- excap(j,t)=l= 0;
reserve(t)..   sum(i, x(i,t,'peak'))+sum(j,exgen(j,t,'peak'))
                         + qD(t,'peak')/h('peak')*reserverate(t)
             -sum(i,xk(i)*capf(i)*av(i)*sum(tt$(ord(tt)<=ord(t)),z(i,tt)))
               -sum(j,excap(j,t)) =l= 0;
NucMax(t,s).. sum(nuc,X(Nuc,t,s))=l=14000;
GasMax(t,s).. Sum(gas,X(Gas,t,s))=l=10200;
BioMax(t,s)..sum(Bio,X(Bio,t,s))=l=450;
WindMax(t,s).. sum(wind,X(Wind,t,s))=l=4039;
HydroMax(t,s).. sum(Hydro,X(Hydro,t,s))=l=4921;

Model Capacityplanning /Objective,Supply,Capacity,Newfacility,exgenbin,
      ExistingSupplyCap,reserve,NucMax,GasMax,BioMax,WindMax,HydroMax/;
options iterlim = 1000000,
        reslim = 1000000;
Solve Capacityplanning maximizing socialwelfare using MINLP;

Parameter NewNucCap(t) Available Nuc Cap During the Time Horizon of IPSP
in MW;
NewNucCap(t) = sum(nuc,sum(tt$(ord(tt)<=ord(t)),Z.l(Nuc,tt)*XK(Nuc)));
$libinclude xlchart NewNucCap

Parameter NewGasCap(t) Available Gas Cap During the Time Horizon of IPSP
                       in MW;
NewGasCap(t) = Sum(gas,sum(tt$(ord(tt)<=ord(t)),Z.l(Gas,tt)*XK(Gas)));
$libinclude xlchart NewGasCap

Parameter NewBioCap Available Bio Cap During the Time Horizon of IPSP in
                    MW;
NewBioCap(t) = sum(Bio,sum(tt$(ord(tt)<=ord(t)),Z.l(Bio,tt)*XK(Bio)));
$libinclude xlchart NewBioCap

Parameter NewWindCap(t) Available Wind Cap During the Time Horizon of IPSP
                        In MW;
NewWindCap(t) = sum(wind,sum(tt$(ord(tt)<=ord(t)),Z.l(Wind,tt)*XK(Wind)));
$libinclude xlchart NewWindCap
```

```
Parameter NewHydroCap Available Hydro Cap During the Time Horizon of IPSP
                     in MW;
NewHydroCap(t)=
       sum(Hydro,sum(tt$(ord(tt)<=ord(t)),Z.l(Hydro,tt)*XK(Hydro)));
$libinclude xlchart NewHydroCap

parameter newgen(t,i) New Generators Production in MW in Peak Load ;
newgen(t,i) = x.l(i,t,"Peak");
$set charttype 52
$libinclude xlchart newgen

parameter exgenMW(t,j) Existing Generators Production in MW in Peak Load;
exgenMW(t,j) = exgen.l(j,t,"Peak");
$set charttype 52
$libinclude xlchart exgenMW

Parameter qDMW(t,s) Demanded Elecricity in MW;
qDMW(t,s)= qD.l(t,s)/h(s);
$libinclude xlchart qDMW

Parameter qDl(t,s) Demanded Elecricity in MWh;
qDl(t,s)= qD.l(t,s);
$libinclude xlchart qDl

parameter construction(i,t) Time of Construction for New Generation i;
construction(i,t) = z.l(i,t);
$libinclude xlchart construction

parameter marketprice(t,s) Market Clearing Price in cents per KWh
undiscounted;
marketprice(t,s)= -supply.m(t,s)*100* ((1+r)**ord(t));

Parameter InvDemand(t,s) Inv Demand Function or Price in cents KWh
undiscounted;
invDemand(t,s)= (a(t,s)+ (b(t,s)*qD.l(t,s)))/10;

display
newnuccap,newgascap,newbiocap,newhydrocap
newgen,
exgenMW,
qDMW,qDl,
Construction,
marketprice,invdemand;
```

```
**************************************************************************
***************                                               ***********
***************           Fixed Binary Variable               ***********
***************                                               ***********
**************************************************************************

parameter Zt(i,t);
Zt(i,t) = Z.l(i,t);

parameter wt(j,t);
wt(j,t) = w.l(j,t);

Variable SocialwelfareT;

Positive variables
   XT(i,t,s)  electricity produced by new gen in period t and status s in
              MW
   exgenT(j,t,s) electricity produced by existing gen in t and status s in
                 MW
   qDT(t,s)   demand in period t and load s in MWh

Equations
   ObjectiveT   definition of present worth of socialwelfare
   SupplyT(t,s) supply  constraint
   CapacityT(i,t,s)  Capacity constraint for new facilities
   ExistingSupplyCapT(j,t,s) capacity constraint for existing facilities
   reserveT(t) reserve capacity;

ObjectiveT..  sum((t,s),(a(t,s)*qDT(t,s)+(.5*b(t,s)*(qDT(t,s)**2))-
              cD(s)*qDT(t,s))
/((1+r)**ord(t)))
              -sum((i,t,s), c(i,t)*xT(i,t,s)*h(s)+
               fueln(i,t)*xt(i,t,s)*h(s)+ K(i,t)* zt(i,t)
               +FixedOpCost(i)*1000*zt(i,t)
               *xk(i)*(sum(tt$(ord(tt)>=ord(t)), 1/((1+r)**ord(tt)))))
               -sum((j,t,s),cc(j,t)* exgenT(j,t,s)*h(s)
               +FixedOpCc(j)*1000*wt(j,t)*excap(j,t)*(1/((1+r)**ord(t)))
              +fuelex(j,t)*exgent(j,t,s)*h(s))- socialwelfareT*1000 =e= 0;
SupplyT(t,s).. sum (i, xT(i,t,s)*h(s))
                 +sum(j,exgenT(j,t,s)*h(s))- qDT(t,s) =g= 0;
CapacityT(i,t,s).. xT(i,t,s)- XK(i)*capf(i)*av(i)
                                * sum(tt$(ord(tt)<=ord(t)),Zt(i,tt))=l=0;
ExistingSupplyCapT(j,t,s).. exgenT(j,t,s)-excap(j,t) =l= 0;
reserveT(t)..   sum(i, xT(i,t,'peak'))+ sum(j,exgenT(j,t,'peak'))
                +(qDT(t,'peak')/h('peak'))*reserverate(t)
            -sum(i,xk(i)*capf(i)*av(i)*sum(tt$(ord(tt)<=ord(t)),zT(i,tt)))
                -sum(j,excap(j,t)) =l= 0;

Model CapacityplanningT   /ObjectiveT,SupplyT,CapacityT
                          ,ExistingSupplyCapT,reserveT/;
Solve CapacityplanningT maximizing socialwelfareT using NLP;

parameter electricitypriceT(t,s) Electricty Price in Cents per KWh
          Undiscounted;
```

```
electricitypriceT(t,s) = -supplyT.m(t,s)*100*((1+r)**(ord(t)));
$libinclude xlchart electricitypriceT

Parameter InvDemandT(t,s) Inv Dem Function (Pr) in cents per KWh
undiscounted;
InvDemandT(t,s)= (a(t,s)+ (b(t,s)*qDT.l(t,s)))/10;
$libinclude xlchart InvDemandT

parameter profitT(i) Total PW of Profit of New Gen i With Only Commodity
Price;
profitT(i)= sum((t,s), -supplyT.m(t,s) * 1000* xT.l(i,t,s)*h(s));


parameter varcostT(i) total cost of producing elec by generator i;
varcostT(i) = sum((t,s), c(i,t)*xT.l(i,t,s)*h(s)+
fueln(i,t)*xt.l(i,t,s)*h(s));

parameter capitalcostT(i) capital cost of generator i;
capitalcostT(i) =  sum((t,s),K(i,t)* zT(i,t)+
FixedOpCost(i)*1000*zt(i,t)*xk(i)
                   *(sum(tt$(ord(tt)>=ord(t)),1/((1+r)**ord(tt)))));

Parameter netprofitT(i) Net Profit of New Generators with no Capacity
                        Price;
netprofitT(i) = profitT(i) - varcostT(i) - capitalcostT(i);
$libinclude xlchart netprofitT


display
electricitypriceT,InvDemandT ,
ProfitT,
varcostT,
capitalcostT,
netprofitT;
```

```
**************************************************************************
****************                                        ****************
****************            Oneill's Approach           ****************
****************                                        ****************
**************************************************************************
******
Variable SocialwelfareO in 1000s of units;

Positive variables
   XO(i,t,s) produced electricity by new gen i during period t and load s
in MW
   exgenO(j,t,s) generation of existing capacity j in period t and load s
in MW
   qDO(t,s)   demand in period t and status s in MWh
   Zo(i,t)   continuous var indicating building a new generator;

Equations
   ObjectiveO    definition of present worth of social welfare
   SupplyO(t,s)  supply constraint
   CapacityO(i,t,s)  Capacity constraint for just new facilities
   NewFacilityO(i) limiting each gen to be built at most once during
timehorizon
   ExistingSupplyCapO(j,t,s) capacity constraint for existing generators
   reserveo(t) reserve capacity
   zOconstraint(i,t) O'Neill's Equality Constraint
   reserveo(t) reserve capacity ;

ObjectiveO.. sum ((t,s),((a(t,s)*qDo(t,s)+
                   (.5*b(t,s)*(qDo(t,s)**2)))-
cD(s)*qDO(t,s))/((1+r)**ord(t)))
             -sum((i,t,s), c(i,t)*xo(i,t,s)*h(s)+
fueln(i,t)*xo(i,t,s)*h(s)
                    +K(i,t)* zo(i,t)+
FixedOpCost(i)*1000*zo(i,t)*xo(i,t,s)
                       *(sum(tt$(ord(tt)>=ord(t)), 1/((1+r)**ord(tt)))))
             -sum((j,t,s),cc(j,t)* exgeno(j,t,s)*h(s)
              +FixedOpCc(j)*1000*wt(j,t)*excap(j,t)*(1/((1+r)**ord(t)))
              +fuelex(j,t)*exgeno(j,t,s)*h(s))- socialwelfareO*1000 =e= 0;
SupplyO(t,s)..sum(i,xo(i,t,s)*h(s))
                 +sum(j,exgeno(j,t,s)*h(s))-qDo(t,s)=g= 0;
CapacityO(i,t,s).. xo(i,t,s)-XK(i)*capf(i)*av(i)*
                   sum(tt$(ord(tt)<=ord(t)),zo(i,tt))=l=0;
NewfacilityO(i)..  sum(t,zo(i,t)) =l= 1;
ExistingSupplyCapO(j,t,s).. exgeno(j,t,s)-excap(j,t) =l= 0;
reserveo(t)..   sum(i, xo(i,t,'peak'))+sum(j,exgeno(j,t,'peak'))
                    +(qDo(t,'peak')/h('peak'))*reserverate(t)
            -sum(i,xk(i)*capf(i)*av(i)*sum(tt$(ord(tt)<=ord(t)),zo(i,tt)))
                -sum(j,excap(j,t)) =l= 0;
zOconstraint(i,t).. zo(i,t) =e= z.l(i,t);

Model CapacityplanningO /ObjectiveO,SupplyO,CapacityO,NewfacilityO,
                        ExistingSupplyCapO,reserveo,zOconstraint/;
Solve CapacityplanningO maximizing socialwelfareO using nlp  ;
```

```
parameter newgenMWhODiff(i,t) produced electricity from new generations in
                             MW;
newgenMWhODiff(i,t) = xO.l(i,t,'peak')-XT.l(i,t,'peak');

parameter exgenMWODiff(j,t) produced electricity from old generations in
                            MW;
exgenMWODiff(j,t) = exgenO.l(j,t,'peak')-exgenT.l(j,t,'peak');

Parameter qDMWhODiff(t,s) demanded elecricity in MWh;
qDMWhODiff(t,s)= qDO.l(t,s)-qDT.l(t,s);

parameter marketpriceODiff(t,s) Electricty Price O'Neil in cents per KWh
                                  undiscounted;
marketpriceODiff(t,s) = -supplyO.m(t,s)*100*((1+r)**(ord(t)))
                         -electricitypriceT(t,s);

parameter cappriceO1(i) O'Neil Cappr Summed over t not Undiscounted ;
cappriceO1(i) = sum(t,zo.l(i,t)*zOconstraint.m(i,t)*1000);
$libinclude xlchart cappriceO1

parameter cappriceO2(i) O'Neil Cappr time of construction Undiscounted ;
cappriceO2(i) = sum(t,zOconstraint.m(i,t)*1000);
$libinclude xlchart cappriceO2

parameter profitO(i) total profit of producing electricity in generator i
                     by selling with commodity price;
profitO(i)= sum((t,s), -supply.m(t,s)* xo.l(i,t,s)*h(s)*1000);

parameter varcostO(i) total cost of producing elec by generator i;
varcostO(i) = sum((t,s), c(i,t)*xo.l(i,t,s)*h(s)+
fueln(i,t)*xo.l(i,t,s)*h(s));

parameter capitalcostO(i) capital cost of generator i;
capitalcostO(i) =  sum((t,s),K(i,t)*
                  zo.l(i,t)+FixedOpCost(i)*1000*zo.l(i,t)
                  *xk(i)*(sum(tt$(ord(tt)>=ord(t)), 1/((1+r)**ord(tt)))));

parameter netprofitO1(i) net profit of each generator considering Oneil
                         cap price as Dual of eq constraint;
netprofitO1(i)= ProfitO(i)-varcostO(i)-capitalcostO(i)+ cappriceO1(i);
$libinclude xlchart netprofitO1

parameter netprofitO2(i) net profit of each generator considering Oneil
                         cap price as Dual of eq constraint for all t;
netprofitO2(i)= ProfitO(i)-varcostO(i)-capitalcostO(i)+ cappriceO2(i) ;
$libinclude xlchart netprofitO2

parameter capprOF(i) suggested cappr by Dr Fuller;
capprOF(i) = sum((t,s),k(i,t) - xk(i)* (-supplyO.m(t,s)- c(i,t)))

display
newgenMWhODiff,exgenMWODiff,qDMWhODiff,marketpriceODiff,cappriceO1,
cappriceO2,profitO,varcostO,capitalcostO,netprofitO1,netprofitO2;
```

```
*************************************************************************
****************        Min Capacity price                  ************
****************                                            ************
****************   S.t generators have profit > 0           ************
*************************************************************************

Parameter qDP(t,s);
qDP(t,s)=qD.l(t,s);

Variable
CapacityPrice Present worth of total capacity price in dollars;

Positive Variable
capprice(t)  capacity prices;

Equations
TotalPWcapprice  define PW of all capacity prices
profit(i)        limits all PW profits to be nonnegative
ConsumerW(t)   Consumer wellfare in each year and each system condition
built(t)  a constraints that forces cappr to zero when there is no
construction;

TotalPWcapprice.. sum((i,t), capprice(t)*xk(i)*capf(i)*av(i)*
                     sum(tt$(ord(tt)<=ord(t)),Zt(i,tt))/((1+r)**(ord(t))))
                   - CapacityPrice=e= 0;
profit(i).. sum(t,capprice(t)*XK(i)*av(i)*capf(i)
                  *sum(tt$(ord(tt)<=ord(t)),Zt(i,tt))/((1+r)**(ord(t))))
           + netprofitT(i) =g= 0;
ConsumerW(t).. sum(s,(((a(t,s)*qDP(t,s)+.5*b(t,s)*(qDP(t,s)**2)))
                   +(1000*supply.m(t,s)*qDP(t,s))
                    -sum(i,capprice(t)*xk(i)*capf(i)*av(i)
                         *sum(tt$(ord(tt)<=ord(t)),Zt(i,tt)))))=g=0;
built(t).. capprice(t)- 1000000*sum(i,zt(i,t))=l=0;

model cappriceP /TotalPWcapprice,profit,ConsumerW,built/;

solve cappriceP using lp minimizing CapacityPrice;

Parameter cappricePP(t) Proposed Capacity Price;
cappricePP(t)= capprice.l(t);
$libinclude xlchart cappricePP

Parameter ProfitP(i) New net Profit;
ProfitP(i) = sum(t,capprice.l(t)*xk(i)*av(i)*capf(i)*

sum(tt$(ord(tt)<=ord(t)),Zt(i,tt))/((1+r)**ord(t)))+netprofitT(i);

parameter consumerwp(t);
consumerwp(t) = sum(s,(((a(t,s)*qDP(t,s)+.5*b(t,s)*(qDP(t,s)**2)))
                   +(1000*supply.m(t,s)*qDP(t,s))
                    -sum(i,capprice.l(t)*xk(i)*capf(i)*av(i)
                         *sum(tt$(ord(tt)<=ord(t)),Zt(i,tt)))));

Display profitP,consumerwp,cappricePP;
```

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