A Robust Optimization Approach to the Self-scheduling Problem Using Semidefinite Programming

by

Jason Conrad Landry

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Master of Applied Science
in
Management Sciences

Waterloo, Ontario, Canada, 2009

© Jason Conrad Landry 2009

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

In deregulated electricity markets, generating companies submit energy bids which are derived from a self-schedule. In this thesis, we propose an improved semidefinite programming-based model for the self-scheduling problem. The model provides the profit-maximizing generation quantities of a single generator over a multi-period horizon and accounts for uncertainty in prices using robust optimization. Within this robust framework, the price information is represented analytically as an ellipsoid. The risk-adversity of the decision maker is taken into account by a scaling parameter. Hence, the focus of the model is directed toward the trade-off between profit and risk. The bounds obtained by the proposed approach are shown to be significantly better than those obtained by a mean-variance approach from the literature. We then apply the proposed model within a branch-and-bound algorithm to improve the quality of the solutions. The resulting solutions are also compared with the mean-variance approach, which is formulated as a mixed-integer quadratic programming problem. The results indicate that the proposed approach produces solutions which are closer to integrality and have lower relative error than the mean-variance approach.

Acknowledgements

I would like to extend my sincere gratitude to my supervisor, Dr. Miguel Anjos. His guidance and support helped shape the direction of this research. It was an honor to have worked under his supervision. I would also like to thank my readers, Dr. David Fuller and Dr. Guillermo Bautista, for their constructive feedback.

I would also like to thank the Natural Sciences and Engineering Research Council of Canada and the University of Waterloo for their generous financial support.

Most importantly, I am forever indebted to my beloved family for their ongoing care and encouragement.

Contents

List of Tables ix

List of Figures x

1 Introduction 1

2 Electricity Market Basics 4

3 Nomenclature 7

3.1 Unit Commitment Problem 7

3.1.1 Indices 7

3.1.2 Decision Variables 7

3.1.3 Parameters 8

3.2 Self-scheduling Problem 8

3.2.1 Indices 8

3.2.2 Decision Variables 9

3.2.3 Parameters 9

4 Literature Review **10**

4.1 The Unit Commitment Problem 10

4.2 Solution Methods 12

4.2.1 Priority List 12

4.2.2 Meta-heuristics 13

4.2.3 Dynamic Programming 14

4.2.4 Mixed-integer Linear Programming 15

4.2.5 Lagrangian Relaxation 19

4.2.6 Semi-definite Programming 21

4.3 The Self-scheduling Problem 23

4.3.1 Risk Characterization 24

4.3.2 Expected Value 25

4.3.3 Mean-Variance 26

4.3.4 Stochastic Programming 27

4.3.5 Conditional Value-at-risk 28

4.3.6 Discussion of Solution Methodologies 28

5 Proposed Methodology **30**

5.1 Robust Optimization 30

5.1.1 Derivation of the Robustness Constraint 31

5.2 Proposed Semidefinite Programming Model 32

5.2.1 Modeling Start-up and Shutdown Constraints 33

5.2.2 Modeling Ramping Constraints 33

5.2.3 Proposed Model 34

5.3 Improving Solution Quality 36

5.3.1 Triangle Inequalities 36

5.3.2 Branch-and-Bound 36

6 Results and Discussions 40

6.1 Problem Instances 40

6.2 Baseline Model 42

6.3 Strength of the SDP Relaxation 43

6.4 Performance of the Branch-and-bound Algorithm 46

6.5 Solution Characteristics 48

6.6 Effect of Problem Size on Performance 51

6.7 Example Based on IESO Data 51

6.8 Summary of Results 57

7 Conclusions and Future Work 59

References 59

APPENDICES 65

A 66

List of Tables

6.1 Quality of the SDP Relaxation 44

6.2 Quality of the MIQP Relaxation 44

6.3 Comparison of Fractional Unit Commitment Variables 45

6.4 Performance of the Branch-and-Bound Algorithm with Depth-first Node Selection 47

6.5 Performance of the Branch-and-Bound Algorithm with Best-Bound Node Selection 47

6.6 Computational Results for Solving the MIQP using CPLEX 48

6.7 Scheduling Differences Between Low and High Levels of Risk Aversion 49

6.8 Quality of the SDP Relaxation with $n_p = 48$ 51

6.9 Computational Results using SDP with $n_p = 48$ 52

6.10 Computational Results using MIQP with $n_p = 48$ 52

6.11 Quality of the SDP Relaxation for the IESO instance 53

6.12 Quality of the MIQP Relaxation for the IESO instance 53

6.13 Comparison of Fractional Unit Commitment Variables for the IESO Instance 54

6.14 Computational Results using SDP for the IESO Instance 55

6.15 Computational Results using MIQP for the IESO Instance 55

6.16 Scheduling Differences for the IESO Instance 57

A.1 Technical Characteristics of the Generating Unit 66

A.2 Cost Characteristics of the Generating Unit 66

A.3 Expected Values of the Market Clearing Prices 67

A.4 Estimate of Covariance Matrix 68

A.5 IESO Hourly Ontario Energy Price 69

A.6 IESO Day-ahead Price Forecasts 70

A.7 Expected Values of the Market Clearing Prices (IESO) 71

A.8 Estimate of Covariance Matrix (IESO) 72

List of Figures

4.1 Piecewise Linear Production Cost 16

4.2 Stepwise Start-up Cost Function 17

6.1 Comparisons of Efficient Frontiers 50

6.2 Efficient Frontier for the IESO Data 56

Chapter 1

Introduction

During the past two decades, the regulatory framework and organization of the electricity industry has undergone drastic restructuring in many countries. Historically, the monopoly position of utilities enabled them to transfer their costs to customers through regulated tariffs, thus lessening the incentive to improve efficiency [39]. Deregulation and restructuring involved unbundling vertically integrated utilities, as well as introducing competition in the wholesale generation and retailing markets. The prevailing sentiment is that competition promotes efficiency and cost savings.

In order to accommodate deregulation while maintaining the robustness of the traditional market, it was necessary that an independent entity assume the responsibility of ensuring system reliability. The independent system operator (ISO) emerged as a system mediator in many competitive markets where its primary function is to ensure that operational constraints are enforced when dispatching generation. In order to promote fair competition, the ISO also serves to separate ownership of the electricity grid from control of it. The evolution of electricity markets has been mostly reactive, but is slowly converging to a universally accepted configuration [30].

The dynamics of market development are interesting in the sense that two different perspectives need to be understood, namely engineering and economics. The electricity that reaches each customer is the result of a tremendous amount of planning and coordination. Engineers are responsible for overseeing this process from production through to delivery, which requires a profound understanding of the unique properties of power systems. The physical properties of electricity make it impossible to store economically in large quantities. This fact heavily influences production planning since an instantaneous match between supply and demand is needed. Capacity is further constrained by the limitations of the transmission network. These same properties that complicate production and transmission also make electricity a unique commodity. As such, the economic theory on which deregulation is based needs to account for these characteristics.

Deregulated electricity markets rely on the unit commitment (UC) problem as a mechanism to ensure effective operation. This problem, which is used extensively by centralized operators and individual generating firms alike, may be formulated as a large-scale mixed-integer program (MIP). It accounts for several system-wide and generator-specific operating constraints, usually over a large portfolio of generating units. Over the short-term, the solution provides an optimal generating schedule for electric utilities by identifying which generators should operate when, and what quantity of electricity each should produce. From the perspective of the ISO, the objective is to minimize cost subject to meeting demand and adhering to operational constraints. In a competitive marketplace, the system operator neither owns nor operates generation facilities. Now the generating companies are not limited to cost recovery and the strict obligation to serve customers, but compete against each other for the right to serve the electricity demand of the system. As a result, each attempts to maximize their profitability by strategically committing their generating capacity through bilateral engagements or centralized auctions.

Competition has introduced drastic changes in the scheduling of electric generators. Within the deregulated framework, a market-clearing entity allocates the overall demand for electricity within the market among the competing firms. Individual companies must assign value to their generating capacity in order to determine the price at which they are willing to supply electricity. Determining the optimal production quantities for each generator, while accounting for the stochastic nature of electricity prices, plays an important part in this process and is known as the self-scheduling problem. Slight modifications to the traditional UC formulation provide a basis for solving this problem. Having solved the self-scheduling problem, an individual supplier can derive an appropriate bidding strategy for the daily electricity market. Decision makers must not only quantify the risk resulting from fluctuations in electricity prices, but they are also required to find a suitable compromise between profit maximization and risk aversion.

This thesis adopts a different approach for solving the self-scheduling problem than those found in the literature by addressing the problem using two relatively recent developments in mathematical programming, namely robust optimization (RO) and semidefinite programming (SDP). Robust optimization is well suited for dealing with uncertain data, but to our knowledge has not been applied to the self-scheduling problem. Using this approach, it is possible to model the uncertainty in electricity prices in a way that can be easily solved within a SDP framework. Although there are instances of the use of SDP in the power systems literature, we have made improvements to solution quality by implementing a branch-and-bound algorithm along with the introduction of additional constraints that more accurately model the operating and cost characteristics of the generator. The main contribution of this thesis is the design and implementation of a robust optimization approach for solving the self-scheduling problem using SDP within a branch-and-bound algorithm.

This thesis is organized as follows. Chapter 2 provides a general overview of the economic framework of electricity markets. The notation used throughout the thesis is given in Chapter 3. In Chapter 4, the unit commitment and self-scheduling problems are defined, and several approaches found in literature for solving these problems are discussed. A relaxation of the self-scheduling problem is formulated as an SDP problem in Chapter 5. This formulation accounts for the uncertainty in electricity prices through robust optimization. Chapter 5 also outlines the proposed branch-and-bound algorithm for improving the solution quality of the initial relaxation. Results from three case studies are described and analyzed in Chapter 6. Finally, Chapter 7 provides some relevant conclusions, as well as directions for future research.

Chapter 2

Electricity Market Basics

The key features that must be considered in order to study and model electricity markets are the ISO, the market operator (MO), and the market participants. The ISO coordinates supply and demand while ensuring operational feasibility, along with system security and reliability [30]. The MO is responsible for collecting bids from market participants in order to determine the market clearing price and production quantities. In most markets, the functions of the ISO and the MO are undertaken by a single entity. The electricity market in New Zealand is an example of a market which separates these functions by contracting them to independent service providers [8]. The market participants include the generating companies (GENCOs), the distribution companies (DISCOs), the transmission companies (TRANSCOs), and the customers [13]. GENCOs are electricity-producing utilities that compete to meet the power demands of the system. The DISCOs are major load-serving entities, who sell electricity to the end customer. Certain large-scale commercial or industrial entities are also considered to be load-serving entities. An energy service company buys and sells electricity; in this sense, it acts as both a generation company and a distribution company. TRANSCOs own and operate the transmission wires that transport electricity from generators to customers [5]. The market participants strive to meet their goals, which most likely consist of improving their market position. Over the short-term, this might consist of maximizing profits, whereas long-term goals may include expanding current facilities.

The ISO not only establishes day-ahead and real-time markets for electricity, ancillary services and transmission, but it also monitors and mitigates market power. In the day-ahead market, the participants pledge one day in advance to provide or consume a specific amount of electricity on an hourly basis over a 24-hour period. Real-time markets serve to set the real-time market price of electricity, as well as to compensate for differences in scheduled generation and actual load [30]. Ancillary services are essential for reliable system operation. Two examples of ancillary services are capacity reserves and voltage support. Proposed schedules for power generation and ancillary services must also account for the nature of the power transmission system and its limitations. Initial settlements are made in the day-ahead market and the settlements are finalized according to real-time market conditions [30].

Two distinct market designs that are used to clear electricity markets are the pool and bilateral models. In an electricity pool, the operator collects bids from supply-side and demand-side entities and combines them into aggregate curves. The bidding process consists mainly of supply-side bids, with demand-side bidding only permitted in certain markets [30]. Currently, many customers face a fixed retail price for electricity instead of the actual market value at any moment. Under these conditions, demand is largely inelastic. The intersection point of these curves sets the market clearing price, as well as the market clearing quantity. Once the market price and quantity are established, the market clearing entity uses the individual bid curves to determine the amount of power awarded to each supplier. The first trial schedule usually has adjustments for plant dynamics but does not account for transmission or security constraints. Following this procedure, the ISO ensures the feasibility and reliability of the resulting schedule by focusing on reserve capacity and location. Finally, each supplier performs a traditional unit commitment to determine the scheduling of its generating assets [15]. When electricity is traded bilaterally, generating and distribution companies establish price contracts for the supply and consumption of electricity in the bilateral market. Units self-dispatch in order to meet their contractual obligations, but may be required to redispatch for system reasons. Market participants can deviate from the quantities they agreed to consume or produced. However, any such deviation is susceptible to financial penalty determined by a balancing mechanism. After a period of time, participants no longer have the opportunity to trade with each other and the system operator takes over in order to ensure the real-time balancing of supply and demand, as well as system security [15]. Hybrid markets exist where elements from both the pool and bilateral designs are mixed.

In Ontario, the Independent Electricity System Operator (IESO) is responsible for the daily operation of the market. It operates an electricity pool by accepting bids, which include incremental electricity costs and capacity limits, from electricity generators and importers. The demand forecast is met by successively dispatching the lowest priced offers until sufficient supply is available. Once the bidding window closes, the spot price for electricity is set to be the incremental cost of the last generator dispatched to meet the demand [31].

Chapter 3

Nomenclature

The nomenclature used throughout this thesis is given in this chapter. Since two distinct problems are discussed, we present the notation for the both problems separately. Unless otherwise noted, the length of a period is 1 hour.

3.1 Unit Commitment Problem

3.1.1 Indices

i: Generator ($i \in 1 \ldots n_g$)
j: Period ($j \in 1 \ldots n_p$)
l: Block of the piecewise linear production cost function
t: Interval of the staircase start-up cost function

3.1.2 Decision Variables

c_{ij}^{d}: Shutdown cost of unit i in period j (\$)
c_{ij}^{p}: Production cost of unit i in period j (\$)
c_{ij}^{u}: Start-up cost of unit i in period j (\$)
p_{ij}: Production quantity for unit i in period j (MW)
$\overline{p}_{ij}$: Maximum available power output of unit i in period j (MW)
t_{ij}^{on}: Number of periods unit i has been online prior to the shutdown in period j
t_{ij}^{off}: Number of periods unit i has been offline prior to the start-up in period j
u_{ij}: Unit commitment variable for unit i in period j (1 if it is online, 0 otherwise)

3.1.3 Parameters

A_i: Coefficient of the piecewise linear production cost function of unit i (\$)
C_i: Shutdown cost of unit i (\$)
D_j: Power demand in period j (MW)
F_{li}: Slope of block l of the piecewise linear production cost function of unit i (\$/MW)
K_i^t: Cost of the interval t of the staircase start-up cost function of unit i (\$)
$\underline{P}_i$, $\overline{P}_i$: Minimum and maximum generation capacity limits of unit i (MW)
n_g: Number of units
n_p: Number of periods
ND_i: Number of intervals of the staircase start-up cost function of unit i
NL_i: Number of segments of the piecewise linear production cost function of unit i
R_j: Reserve requirement in period j (MW)
RD_i: Ramp-down limit of unit i (MW/h)
RU_i: Ramp-up limit of unit i (MW/h)
SD_i: Shutdown ramp limit of unit i (MW/h)
SU_i: Start-up ramp limit of unit i (MW/h)
$\bar{t}_i$, $\underline{t}_i$: Minimum up and down times of unit i (h)
T_{li}: Upper limit of block of the piecewise linear production cost function of unit i
α_i: Constant production cost coefficients of unit i (\$)
β_i: Linear production cost coefficients of unit i (\$/MWh)
γ_i: Quadratic production cost coefficients of unit i (\$/MW^{2}h)
δ_{lij}: Power produced in block l of the piecewise linear production cost function of unit i in period j (MW)
λ_j: Lagrangian multipliers for the demand constraints in period j
μ_J: Lagrangian multipliers for the reserve constraints in period j

3.2 Self-scheduling Problem

3.2.1 Indices

j: Period ($j \in 1 \ldots n_p$)
s: Scenario
t: Interval of the staircase start-up cost function

3.2.2 Decision Variables

c_j^d: Shutdown cost in period j (\$)
c_j^p: Production cost in period j (\$)
c_j^u: Start-up cost in period j (\$)
p_j: Production quantity in period j (MW)
t_j^{on}: Number of periods the unit has been online prior to the shutdown in period j
t_j^{off}: Number of periods the unit has been offline prior to the start-up in period j
u_j: Unit commitment variable in period j (1 if it is online, 0 otherwise)

3.2.3 Parameters

C: Shutdown cost (\$)
K^t: Cost of the interval t of the staircase start-up cost function (\$)
$\underline{P}$, $\overline{P}$: Minimum and maximum generation capacity limits (MW)
n_p: Number of periods
ND: Number of intervals of the staircase start-up cost function
RD: Ramp-down limit (MW/h)
RU: Ramp-up limit (MW/h)
SD: Shutdown ramp limit (MW/h)
SU: Start-up ramp limit (MW/h)
$\bar{t}$, $\underline{t}$: Minimum up and down times (h)
α, β, γ: Production cost coefficients (\$),(\$/MWh), (\$/MW^{2}h)
β, κ: Risk aversion parameters
λ_j: Electricity price in period j (\$/MWh)

Chapter 4

Literature Review

The UC problem has long played a crucial role in the operation of power systems and continues to be a key component. A huge amount of research has been devoted to solving this problem. Much of this effort extends naturally to the self-scheduling problem because they share much of the same problem structure. In this chapter, we first describe in detail the UC problem and discuss several of the classical approaches proposed in the literature for solving it. Having built an understanding of the traditional approaches, we next define the self-scheduling problem and present four methods for addressing the stochastic nature of electricity prices. The chapter concludes with a brief comparison of these four approaches with the aim of motivating the contribution of this thesis.

4.1 The Unit Commitment Problem

The traditional UC problem is often formulated as a non-linear mixed-integer program. The non-linearity arises in the cost functions, whereas the integer variables reflect the online and offline states of the generating units. Any such problem of realistic size is large in scale and requires significant computational time to solve. For example, an independent system operator faces a UC problem consisting of approximately 200 generators to be scheduled over 24 hour-long periods. Even for modestly sized problems, the computational burden of solving to proven optimality is prohibitive. As a result, exact solution methodologies are seldom applied and practitioners often settle for near-optimal solutions. Considerable effort has been spent developing methods that improve the quality of solutions because even small improvements are likely to translate into substantial cost savings.

The objective function of the basic UC problem is the minimization of the production cost, start-up cost, and shutdown cost. Generally, the production cost is expressed by a quadratic function of the power output. The start-up cost is assumed to be an exponential function of the time that the unit is offline prior to start-up. The shutdown cost is constant and often ignored because its magnitude is insignificant relative to the other costs.

The constraints on the UC schedule that must be satisfied are the system demand, reserve requirements, generation limits, ramping rate limits, and the minimum up and down times. These constraints are classified by two types. The first type is concerned with the system output or capacity requirements for each time period. These constraints require that the demand and spinning reserves be met and are referred to as coupling constraints. Spinning reserves are unused capacity which is readily available to ensure a continuous balance between load and generation at all times. The second type of constraints is imposed on the individual generating units; these are regarded as local constraints. The generation limits restrict the amount of electricity offered by a given generator as a result of its physical generating capacity. Ramping rates restrict the change in output levels between subsequent periods. The minimum up and down time constraints ensure that once the decision is made to start-up or shutdown a generator, it must remain in that state for a pre-specified period of time. The earlier representations of the minimum-time constraints were non-linear and mixed-integer. With the exception of the capacity limits, all of these constraints are inter-temporal, and as such require knowledge of the operating history of each unit.

It should be noted that the basic formulation does not account for network flow constraints. The inclusion of such constraints would require a physical characterization of electricity. A DC power flow approximation produces fairly good results for economic dispatch purposes but an accurate representation of system behavior is only obtainable through a non-linear AC flow model. Unfortunately, the AC flow model significantly increases the complexity of the problem because it takes into account both active and reactive power components. The most notable network characteristics that rely on this distinction are transmission line limits, transformer capacities, and voltage limits. Since reactive power is necessary for voltage stability, generators are often required to run solely on the basis of their reactive capacity. Therefore, neglecting to include the reactive power component in the formulation of the UC model may result in an infeasible solution [29]. In practice, there is a distinction between unit commitment and power flow. The UC model is concerned with resource scheduling, whereas the optimal power flow (OPF) problem considers transmission limitations. These two requirements are often not satisfied simultaneously and the UC and OPF problems are solved iteratively until a satisfactory solution is obtained.

4.2 Solution Methods

The importance of the UC problem is highlighted by the vast amount of literature that has been dedicated to it over the past 50 years. Researchers have spent considerable effort developing methods that decrease computational times and improve solution quality. Certain methods attempt to accurately model the situations that are found in practice while others are focused more toward finding simplified ways for obtaining viable solutions. A comprehensive literature review of solution techniques for the UC problem is presented in [34] and [46]. In this section, we provide a brief description of several common methodologies which include: priority lists, dynamic programming, mixed-integer linear programming, Lagrangian relaxation, and semidefinite programming. Meta-heuristics have also been widely applied to the UC problem. A review of three meta-heuristic approaches is included in this section.

4.2.1 Priority List

The simplest approach for UC consists of creating a priority list of the units. Such a list prioritizes the units according to their full-load average production cost. In each period, the units are committed in a predetermined order until sufficient capacity is available to cover system demand and security requirements. Additional measures are included within the algorithm to take into account the inter-temporal nature of the minimum time constraints, as well as time-dependent start-up and shutdown costs. Most priority list-based methods produce sub-optimal commitment schedules because they only search a small subset of potential schedules [46].

4.2.2 Meta-heuristics

Tabu search (TS) is based on the principles of intelligent problem-solving [33]. The distinguishing characteristic of this approach is that it avoids entrapment at a local optimum by preserving a list of recently visited solutions in short-term memory. TS has been used to solve the UC problem in [26]. The two main components of this algorithm are the tabu list consisting of restricted moves, and the aspiration level which is a measure of solution quality. At each iteration, a set of feasible schedules that span as much of the solution space as possible is generated. A trial solution is selected to be the basis of a future iteration if it is not part of the tabu list or if its aspiration level is superior to that assigned earlier for the same move. Throughout the search, the overall best solutions are noted and the algorithm terminates once a maximum number of iterations is reached.

Simulated annealing (SA) can be applied to large-scale UC problems. This approach is analogous to the annealing process in material sciences whereby a solid is heated to a high temperature and subjected to stepwise cooling. An SA approach to the UC problem is presented in [25], where a set of feasible solutions is generated by modifying a selected feasible solution. A candidate solution is accepted if its objective function value is superior to the previous one. Otherwise, the candidate solution is chosen with a certain probability which is given by an expression dependent on the difference in objective function values. As the number of iterations increases, the probability of selecting a candidate with a worst objective function value is decreased by adjusting a control parameter. The algorithm terminates once the control parameter reaches a given threshold.

Genetic algorithms are a class of meta-heuristics that emulate the inheritance characteristics of humans in order to generate feasible solutions to an optimization model. In [38], the authors present a genetic algorithm approach for solving the UC problem. This approach evaluates solutions, often termed creatures, and assigns them a fitness which reflects their quality. The solutions with acceptable fitness are chosen to procreate, thus forming children that explore a different portion of the solution space and replace less fit creatures. With respect to the UC problem, the parent schedules are arrays of binary data which represent the unit commitment of generators over a finite time horizon. An element is assigned a value of 1 if the generator is online for the period or a value of 0 otherwise. Each row in the array indicates the commitment decisions for a specific generator; whereas, each column corresponds to a period in the planning horizon. The genetic algorithm uses two techniques to produce children from parent schedules. A standard mutation swaps portions of the parent schedules, whereas an intelligent mutation spots and changes the inefficient 010 and 101 combinations. These combinations are inefficient because they entail bringing a unit online or offline for a single period, respectively. The advantage of using this procedure is that solution time increases only linearly with the number of units and periods.

4.2.3 Dynamic Programming

Dynamic programming (DP) was the first optimization-based technique applied to the UC problem [34]. The problem can be represented by a series of commitment decisions processed at each period, thus eliminating the need to schedule the units over the entire horizon simultaneously. The search can be conducted in a forward or backward direction. A forward DP algorithm obtains solutions by working forward from the initial conditions toward the final period in the planning horizon. Such a forward approach is advantageous in the context of the UC problem because the operating history of each unit makes it possible to incorporate the start-up cost and the minimum time constraints. The backward DP approach is similar to the forward approach, but works from the final period toward the beginning of the horizon. In the forward DP process [35], each period qualifies as a stage and the associated states correspond to combinations of units. The economic dispatch of each unit commitment combination is solved using linear programming (LP) or another efficient optimization technique. The objective consists of choosing the states at each stage which result in the lowest-cost strategy. First, it is necessary to define the search range and the possible combinations of units for each hour. Next, the least cost strategy for transitioning from one combination to another in a forward direction is identified using the following recursion formula :

$$F_{COST}(J,K) = \min_{\{L\}} \{P_{COST}(J,K) + S_{COST}(J-1, L:J,K) + F_{COST}(J-1,L)\}. \quad (4.1)$$

When the K^{th} combination provides a feasible dispatch in period J, $P_{COST}(J,K)$ defines the production cost required to meet the electricity demand, while $S_{COST}(J-1, L : J,K)$ denotes the transition cost associated with any changes in unit commitment between periods $J-1$ and J. If the least cumulative cost necessary to arrive to period $J-1$ for each combination in the set L is given by $F_{COST}(J-1,L)$, then the least total cumulative cost for arriving to K through a combination in L is equivalent to the sum of $P_{COST}(J, K)$, $S_{COST}(J-1, L : J, K)$, and $F_{COST}(J-1, L)$. The minimum total cost for arriving to the K^{th} combination, $F_{COST}(J, K)$, may be determined by evaluating equation (4.1) over all L. The same procedure is applied in subsequent periods until the end of the planning horizon is reached. Given the combination with the least total cumulative cost in the final period, the schedule for the entire planning horizon is reconstructed in a backward fashion.

Used appropriately, the DP approach greatly reduces the number of schedules generated. When the search range is not exhaustive, global optimality is not guarantied because there is no mechanism in place to ensure implicit enumeration. As a result, the choice of an appropriate search range greatly influences the solution quality, as well as the computational time. If the search range is large, the computational burden of arriving to the lowest total cost schedule is prohibitive. Although a smaller search range decreases the solution time, it increases the likelihood that high quality solutions will be overlooked. Methods that limit the search range without significantly sacrificing solution accuracy are proposed in [36].

4.2.4 Mixed-integer Linear Programming

Mixed-integer linear programming (MILP) techniques have improved substantially during the last decade. Reductions in solution times are partly the result of more sophisticated formulations that better approximate the convex hull of integer points within the LP relaxation. Another factor influencing computation time is the development of efficient branch-and-cut algorithms. Several commercially available software packages have the capability to effectively handle large-scale instances [6].

The MILP formulation in [6] accurately models the non-linear aspects of the UC problem with a single set of binary variables. The quadratic production cost is approximated as a piecewise linear function, while the exponential start-up cost is discretized by a stepwise function. Figures 4.1 and 4.2 depict the associated cost curves. The linear approximation of the production costs has the following analytic

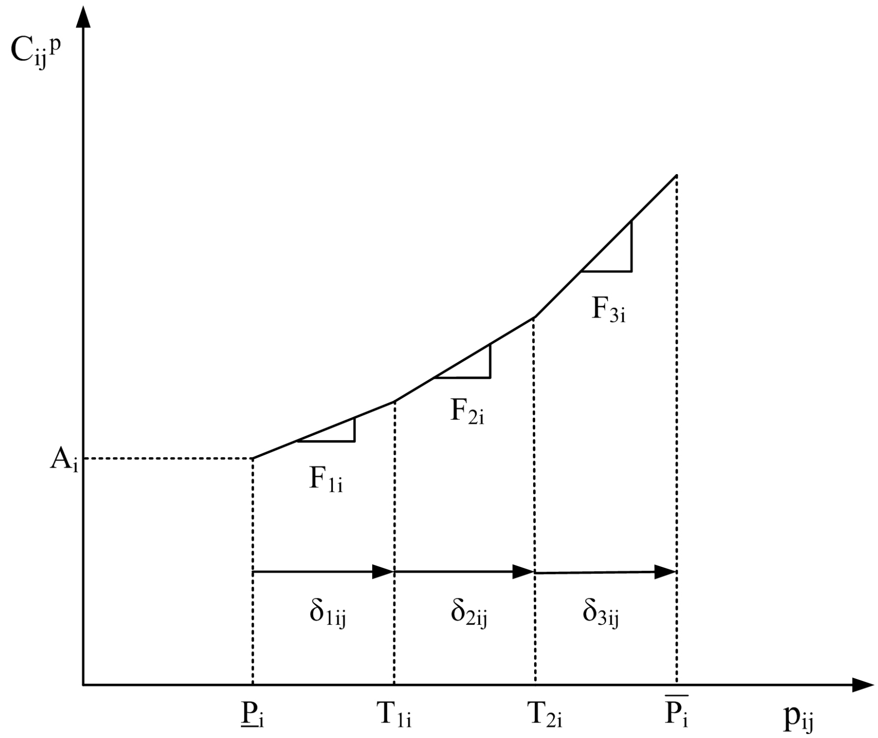

Figure 4.1: Piecewise Linear Production Cost

representation:

$$c_{ij}^{p} = A_i u_{ij} + \sum_{l=1}^{NL_i} F_{li}\delta_{lij} \qquad \forall i\ \forall j \tag{4.2}$$

$$p_{ij} = \sum_{l=1}^{NL_i} \delta_{lij} + \underline{P}_i u_{ij} \qquad \forall i\ \forall j \tag{4.3}$$

$$\delta_{1ij} \leq T_{1i} - \underline{P}_i \qquad \forall i\ \forall j \tag{4.4}$$

$$\delta_{lij} \leq T_{li} - T_{l-1,i} \qquad \forall i\ \forall j\ \forall l = 2 \ldots NL_i - 1 \tag{4.5}$$

$$\delta_{NL_i ij} \leq \overline{P}_i - T_{NL_i-1,i} \qquad \forall i\ \forall j \tag{4.6}$$

$$\delta_{lij} \geq 0 \qquad \forall i\ \forall j\ \forall l = 1 \ldots NL_i \tag{4.7}$$

where $A_i = \alpha_i + \underline{P}_i\beta_i + \underline{P}_i^2\gamma_i$.

The start-up and shutdown costs have the following mixed-integer linear formu-

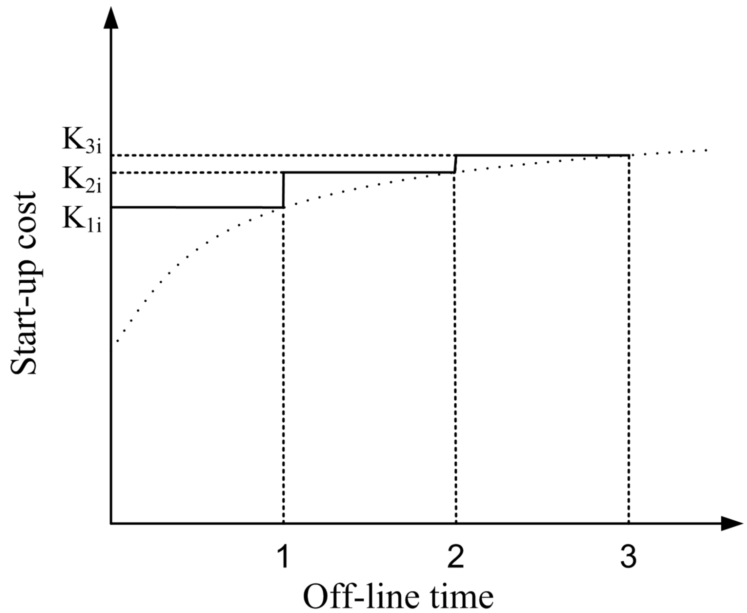

Figure 4.2: Stepwise Start-up Cost Function

lations:

$$c_{ij}^u \geq K_i^t \left[u_{ij} - \sum_{n=1}^{t} u_{i,j-n} \right] \qquad \forall i \; \forall j \; \forall t = 1 \dots ND_i \tag{4.8}$$

$$c_{ij}^d \geq C_i \left[u_{i,j-1} - u_{ij} \right] \qquad \forall i \; \forall j \; \forall t = 1 \dots ND_i \tag{4.9}$$

$$c_{ij}^u \geq 0 \qquad \forall i \; \forall j \tag{4.10}$$

$$c_{ij}^d \geq 0 \qquad \forall i \; \forall j. \tag{4.11}$$

The formulation of generator specific constraints in [6] precisely models the available spinning reserves in each period. The generation limits are defined by the following inequalities:

$$\underline{P}_i u_{ij} \leq p_{ij} \leq \overline{p}_{ij} \qquad \forall i \; \forall j \tag{4.12}$$

$$0 \leq \overline{p}_{ij} \leq \overline{P}_i u_{ij} \qquad \forall i \; \forall j. \tag{4.13}$$

These constraints effectively bound the power output of each generating unit. However, they include an extra variable $\overline{p}_{ij}$ which reflects the upper bound on power output enforced by the ramping constraints. The ramping constraints are formulated as follows:

$$\overline{p}_{ij} \leq p_{i,j-1} + RU_i u_{i,j-1} + \\ SU_i \left[u_{ij} - u_{i,j-1}\right] + \overline{P}_i \left[1 - u_{ij}\right] \qquad \forall i \; \forall j \qquad (4.14)$$

$$\overline{p}_{ij} \leq \overline{P}_i u_{i,j+1} + SD_i \left[u_{ij} - u_{i,j+1}\right] \qquad \forall i \; \forall j = 1 \ldots n_p - 1 \qquad (4.15)$$

$$p_{i,j-1} - p_{ij} \leq RD_i u_{ij} + \\ SD_i \left[u_{i,j-1} - u_{ij}\right] + \overline{P}_i \left[1 - u_{i,j-1}\right] \qquad \forall i \; \forall j. \qquad (4.16)$$

They not only treat the basic ramp-up limits (4.14) and ramp-down limits (4.16) but they also model start-up ramp rates (4.14) and shutdown ramp rates (4.15). Through the addition of the variable $\overline{p}_{ij}$, these constraints capture the amount of available spinning reserves, which is given by the difference between $\overline{p}_{ij}$ and p_{ij}. A mixed-integer linear formulation of the minimum up and down times found in [24] is given by the following set of constraints

$$\sum_{\tau=j}^{a(\bar{t})} u_{i\tau} \geq \left(u_{ij} - u_{i,j-1}\right) b\left(\bar{t}\right) + \delta\left(j-1\right) a_{i0} \qquad \forall i \; \forall j \qquad (4.17)$$

$$\sum_{\tau=j}^{a(\underline{t})} \left(1 - u_{i\tau}\right) \geq \left(u_{i,j-1} - u_{ij}\right) b\left(\underline{t}\right) + \delta\left(j-1\right) b_{i0} \qquad \forall i \; \forall j \qquad (4.18)$$

where the terms $a\left(\cdot\right)$, $b\left(\cdot\right)$, a_{i0}, and b_{i0} are defined as follows

$$a\left(z\right) = \min\left\{j + z - 1, n_p\right\} \quad b\left(z\right) = \min\left\{z, n_p - j + 1\right\}$$

$$a_{i0} = u_{i1} u_{i0} \times \max\left\{0, \bar{t} - t_{i0}^{\text{on}}\right\} \quad b_{i0} = \left(1 - u_{i1}\right)\left(1 - u_{i0}\right) \times \max\left\{0, \underline{t} - t_{i0}^{\text{off}}\right\}.$$

The commitment status of the units in early periods may be fixed due to their operational history. When a unit changes state in a later period, it must remain in the new state for at least $\bar{t}_i - 1$ or $\underline{t}_i - 1$ additional periods. If the number of remaining periods is less than the minimum times, the unit is only required to remain in the state until the end of the planning horizon. The constraints (4.17) and (4.18), along with nature of the $a\left(\cdot\right)$ and $b\left(\cdot\right)$ terms, ensure that these characteristics are met.

The complete mixed-integer linear optimization problem is expressed as follows:

$$\textbf{[UC-MILP]} \min \sum_{i=1}^{n_g} \sum_{j=1}^{n_p} c_{ij}^p + c_{ij}^u + c_{ij}^d \tag{4.19}$$

$$\text{s.t.} \sum_{i=1}^{n_g} p_{ij} = D_j \qquad \forall j \tag{4.20}$$

$$\sum_{i=1}^{n_g} \overline{p}_{ij} \geq D_j + R_j \qquad \forall j \tag{4.21}$$

$$p_{ij} \in \pi_{ij} \qquad \forall i \ \forall j \tag{4.22}$$

$$u_{ij} \in \{0, 1\} \qquad \forall i \ \forall j. \tag{4.23}$$

where the c_{ij} terms are defined by (4.2)-(4.11) and $p_{ij} \in \pi_{ij}$ satisfies the local constrains (4.12)-(4.18).

4.2.5 Lagrangian Relaxation

As presented in [UC-MILP], the problem is neither separable by unit nor by period. However, decomposition may be achieved by relaxing either the coupling or inter-temporal constraints. In practice, the coupling constraints can be relaxed by adding penalty terms to the objective function. The resulting unit specific subproblems are easier to evaluate than the aggregate problem. A lower bound to the [UC-MILP] formulation is obtained by relaxing the demand constraint (4.20) and the reserve constraint (4.21) with the Lagrangian multipliers λ_j and μ_j. The objective function resulting from this operation is:

$$z = \min \sum_{i=1}^{n_g} \sum_{j=1}^{n_p} \left\{ c_{ij}^p + c_{ij}^u + c_{ij}^d \right\} + \sum_{j=1}^{n_p} \left\{ \lambda_j \left(\sum_{i=1}^{n_g} p_{ij} - D_j \right) + \mu_j \left(\sum_{i=1}^{n_g} \overline{p}_{ij} - D_j - R_j \right) \right\}. \tag{4.24}$$

The remaining constraints may be partitioned so that n_g subproblems are defined, each corresponding to an individual unit. Observing that the terms $\lambda_j D_j$, $\mu_j D_j$, and $\mu_j R_j$ in (4.24) remain constant, we obtain the following subproblems:

$$\textbf{[UCLR-SP]}_i \ z_i = \min \sum_{j=1}^{n_p} c_{ij}^p + c_{ij}^u + c_{ij}^d + \lambda_j p_{ij} + \mu_j \overline{p}_{ij} \tag{4.25}$$

$$\text{s.t.} \ p_{ij}, u_{ij} \in \pi_{ij} \qquad \forall j,$$

The main challenge arises in determining the best values for the multipliers. Since (4.24) provides a lower bound for the original problem, it is desirable that any adjustments to the multipliers cause the objective function to increase. These values can be obtained from the optimal solution to the following Lagrangian dual:

$$\textbf{[UCLR-MP]} \max \sum_{j=1}^{n_p} \lambda_j D_j + \mu_j \left(D_j + R_j\right) + \sum_{i=1}^{n_g} z_i(\lambda_i, \mu_i) \tag{4.26}$$

$$\text{s.t. } \lambda_j \text{ free}, \mu_j \leq 0 \qquad \forall j$$

which is a max-min problem with the z_i terms equal to the solution of (4.25).

Unfortunately, the maximization of the Lagrangian dual is not easily achieved due to its non-differentiable nature. Furthermore, the optimal production schedule resulting from its solution is unlikely to be demand-feasible. One approach for solving the Lagrangian dual is the sub-gradient method. The sub-gradient method iteratively updates the Lagrangian multipliers by moving in the direction of the sub-gradient. With respect to the UC problem, the sub-gradient vector corresponds to the mismatch between the output and demand requirements. An effective sub-gradient algorithm for solving the Lagrangian dual problem is given in [47]. The method used to update the multipliers is

$$\begin{bmatrix} \lambda^{k+1} \\ \mu^{k+1} \end{bmatrix} = \begin{bmatrix} \lambda^{k} \\ \mu^{k} \end{bmatrix} + \delta^k \frac{\theta^k}{\|\theta^k\|} \tag{4.27}$$

where θ^k is the sub-gradient defined by

$$\theta^k = \begin{bmatrix} D - \sum_{i \in I} p_i \\ D + R - \sum_{i \in I} u_i \overline{P}_i \end{bmatrix} \tag{4.28}$$

and $\delta^k = 1/\left(\alpha + \beta k\right)$, with α, β as positive constants.

A second method for solving the dual problem consists of adding cutting planes to the Lagrangian master problem until its objective function value converges to the value obtained by solving the subproblems. In [22], the authors present an interior-point cutting-plane algorithm which is well-equipped to solve the problem. Their approach uses the analytic center of a localization set as an improved dual solution. The set corresponds to the area enclosed by a cutting-plane approximation, the lower and upper bounds of the dual variables, and the lower bounds to the dual objective function. The iterative procedure of estimating the dual solution from the analytic center continues until it satisfies a stopping criterion. The advantage of using interior-point cutting-plane methods over other alternatives for evaluating non-differentiable problems is that they are quicker to converge and free from sensitive parameter tuning.

Once the optimal solution to the relaxed problem is determined from the Lagrangian dual, additional processing is often necessary in order to find a feasible solution to the original problem. Since the multipliers indicate the marginal price of an additional unit of output, increasing their values will make it more attractive for units to generate electricity in that particular period. Therefore, load deficits may be corrected by increasing their respective multipliers. This logic is the foundation of the heuristic developed in [47] to search for a demand-feasible solution. Following the search for a feasible solution, the final step of the process consists of fixing the binary variables and then solving the economic dispatch problem, which is a linear program.

4.2.6 Semi-definite Programming

The general notion behind SDP dates back to the middle of the 20th century. However, the approach only gained popularity during the 1990s with the development of interior-point methods for solving general convex optimization problems. SDP is best described as an extension of LP with a matrix variable and positive semidefiniteness in lieu of vector variables and non-negativity constraints. A large portion of the research into solving the UC problem with SDP approaches has been conducted in [24] and their conclusions indicate that SDP relaxations are a promising alternative for solving UC problems.

SDP deals with solving mathematical programs that are cast in the following primal form:

$$\textbf{[SDP]} \min C \cdot X \tag{4.29}$$

$$\text{s.t. } A_i \cdot X = a_i \qquad \forall i = 1 \dots m \tag{4.30}$$

$$X \succcurlyeq 0. \tag{4.31}$$

The matrix variable X is symmetric and positive semidefinite, which implies that all its eigenvalues are non-negative. The equality constraints on X are evaluated in a fashion analogous to the inner product for vectors. Such an inner product is mathematically defined as $A \cdot X = \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} X_{ij}$. A number of software packages based on interior-point algorithms are available to solve SDPs. Other methods have also been developed to solve large-scale SDP problems. These include low-rank factorization, augmented Lagrangian, and spectral bundle methods [28].

Following the procedure presented in [24], the UC problem is reformulated using SDP by observing two relationships. First, it is noted that a $\{0,1\}$ variable can be represented by equating the square of this variable to itself. Hence, the unit commitment constraint (4.23) is reformulated as

$$u_{ij} \in \{0,1\} \Leftrightarrow u_{ij}^2 = u_{ij}v, \text{where } v = 1. \tag{4.32}$$

The addition of the auxiliary variable v makes both terms quadratic. Second, the following relationship holds

$$p_j = u_j p_j \Leftrightarrow p_{ij}^2 = u_{ij}p_{ij}^2. \tag{4.33}$$

Taking these facts into consideration, a simplified UC problem with constant startup cost and no system-wide reserve is formulated as a quadratic-objective quadratic-constraints problem:

$$\textbf{[UC-QQP]} \min \sum_{i=1}^{n_g}\sum_{j=1}^{n_p} u_{ij}^2\alpha_i + u_{ij}p_{ij}\beta_i + p_{ij}^2\gamma_i \tag{4.34}$$

$$\text{s.t.} \sum_{i=1}^{n_g} u_{ij}p_{ij} = D_j \qquad \forall j \tag{4.35}$$

$$u_{ij}^2\underline{P}_i - u_{ij}p_{ij} \leq 0 \qquad \forall i \forall j \tag{4.36}$$

$$u_{ij}p_{ij} - u_{ij}^2\overline{P}_{ij} \leq 0 \qquad \forall i \forall j \tag{4.37}$$

$$u_{ij}p_{ij} - u_{i,j-1}p_{i,j-1} \leq RU_i \qquad \forall i \forall j \tag{4.38}$$

$$u_{i,j-1}p_{i,j-1} - u_{ij}p_{ij} \leq RD_i \qquad \forall i \forall j \tag{4.39}$$

$$s_{ij}(u) \leq b_{ij} \qquad \forall i \forall j \tag{4.40}$$

$$u_{ij}^2 - u_{ij}v = 0 \qquad \forall i \forall j \tag{4.41}$$

$$v - 1 = 0 \tag{4.42}$$

where $s_{ij}(u) \leq b_{ij}$ represents the minimum up and down time constraints in the form $a_{ij}^{(1)}u_{i1}^2 + \ldots + a_{ij}^{(n_p)}u_{in_p}^2 \leq b_ij$, whose coefficients a_{ij} and b_{ij} are determined from (4.17)-(4.18).

In order to solve the UC-QQP, it is necessary to express the problem in SDP format. This is achieved by first grouping all the decision variables corresponding to a single generator into a column vector $x_i = \left[p_{i1}\ u_{i1} \ldots p_{in_p}\ u_{in_p}\ v\right]^T$. A symmetric matrix, which is positive semidefinite and has rank one, is obtained by taking the outer-product of the vector x_i with itself:

$$X_i = x_i x_i^T = \begin{pmatrix} p_{i1}p_{i1} & p_{i1}u_{i1} & \cdots & p_{i1}p_{in_p} & p_{i1}u_{in_p} & p_{i1}v \\ p_{i1}u_{i1} & u_{i1}u_{i1} & \cdots & u_{i1}p_{in_p} & u_{i1}u_{in_p} & u_{i1}v \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ p_{in_p}p_{i1} & p_{in_p}u_{i1} & \cdots & p_{in_p}p_{in_p} & p_{in_p}u_{in_p} & p_{in_p}v \\ u_{in_p}p_{i1} & u_{in_p}u_{i1} & \cdots & u_{in_p}p_{in_p} & u_{in_p}u_{in_p} & u_{in_p}v \\ vp_{i1} & vu_{i1} & \cdots & vp_{in_p} & vu_{in_p} & vv \end{pmatrix}.$$

From within each of the individual matrices X_i, it is possible to identify all the terms needed to construct the local constraints (4.36) - (4.42) and define the contribution of each generator to the overall cost (4.34). The individual blocks must be aggregated into a single block-diagonal matrix because the coupling constraint (4.35) combines the production quantities over all units.

Algorithms for solving SDP do not force the rank of the matrix variable to equal one. When this property is not enforced, the binary variables may take on fractional values between zero and one. As such, the formulation is actually a relaxation of the original problem. In [24], the authors suggest using cutting planes or simple rounding heuristics to improve the solution obtained by the SDP relaxation.

4.3 The Self-scheduling Problem

The deregulation of the power industry has introduced drastic changes in the scheduling of electric generators. A centralized operator is no longer solely responsible for committing assets on a cost-minimizing basis. Now, individual generating companies compete against each other for the right to serve the electricity demand of a given system. As a result, each company strives to maximize its own profits by submitting optimal bidding strategies to the market-clearing entity. Decision makers must not only quantify the risk resulting from fluctuations in electricity prices, but they are also required to find a suitable compromise between profit maximization and risk aversion. In order to successfully manage risk exposure, new procedures are required to appropriately account for the forces that drive the market.

The solution to the self-scheduling problem provides the optimal production quantities for a generator over a specified time horizon, while accounting for the stochastic nature of prices. Having solved the self-scheduling problem, the generating company can derive an appropriate bidding strategy for the day-ahead market. The problem is simplified by assuming that the producers are price-takers in a pool-based electricity market. In this case, the coordination among units has no influence on market-clearing prices and each generator can be modeled separately. This section reviews four self-scheduling models used under these assumptions. More sophisticated equilibrium models [20], [16] and mixed-integer programs [12] exist to study firms that exert significant market power.

4.3.1 Risk Characterization

The proper forecasting of electricity prices plays an important role in developing good bidding strategies. Since electricity prices are highly volatile, the application of straightforward forecasting methods which are prevalent in other commodity markets is often prone to large forecasting errors [40]. The distinct market characteristics that cause such large fluctuations include fuel price volatility, load uncertainties, forced outages, network congestion, and the strategic behavior of large firms. Moreover, electricity differs from other commodities because it cannot be stored in large quantities by practical means. This property necessitates an instantaneous match between supply and demand, which follows yearly, monthly, and daily cycles. Classical methods used to forecast electricity prices include regression and state-space models, while more modern techniques include expert systems, evolutionary programming, fuzzy systems, adaptive neural networks, and various combinations of these approaches. Neural network-based approaches are the most popular among those listed due to their clarity, ease of implementation, and good performance [40]. With such an approach, it is necessary to identify input variables that influence market prices. The model parameters are then conditioned using data sets until they accurately predict market conditions. Thereafter, the model can be used to forecast electricity prices given the current state of the input variables. A detailed treatment of price forecasting is excluded from this thesis. Instead, we assume that an effective forecasting technique is already in place which provides reasonable estimations of the market-clearing prices for the entire planning window.

Within this framework, the overall risk associated with a generation schedule can be attributed to the mutual dependence between revenues obtained in each period. Choosing to produce electricity in periods whose prices move in opposite directions reduces the volatility in revenue resulting from the generation schedule. The price in each period is a random variable; therefore, an average squared deviation between the actual and expected prices for period i is given by the variance σ_{ii}. Furthermore, since the prices in each period are not independent, the covariance σ_{ij} between the prices in periods i and j measures their common variance. A covariance matrix Σ consisting of the individual σ_{ii} and σ_{ij} terms not only establishes the uncertainty in the expected value of a single price, but also how strongly the prices in two periods follow each other, or move in opposite directions.

If historical data is available on the estimated price of electricity as well as the actual values leading up to day d, it is possible to estimate the covariance matrix of day d as

$$\Sigma = \frac{1}{D} \sum_{i=1}^{D} \left(\Lambda_i^{true} - \Lambda_i^{est}\right) \left(\Lambda_i^{true} - \Lambda_i^{est}\right)^T \tag{4.43}$$

where D is an appropriate number of days for which data is available and $\Lambda = \left[\lambda_1, \ldots, \lambda_{n_p}\right]^T$. Unfortunately, this relatively simplistic approach for generating covariance matrices is inadequate for dealing with the complexities of electricity prices. Problems arise due to characteristics such as non-constant mean and variance, multiple seasonality, high volatility, and high percentage of unusual prices [10]. In order to deal with the seasonality effects and potential outliers, the following exponential weighted moving-average approximation is presented in [10]:

$$\Sigma = (1-\nu) \sum_{i=1}^{D} \alpha^{i-1} \left(\Lambda_{D-i+1}^{true} - \Lambda_{D-i+1}^{est}\right) \left(\Lambda_{D-i+1}^{true} - \Lambda_{D-i+1}^{est}\right)^T \tag{4.44}$$

where ν is a smoothing constant between 0 and 1. The main idea is that the contribution of each price vector to the overall discovery of the covariance matrix decays exponentially as they are taken from further in the past. Therefore, the seasonality effects and outliers have less impact [10].

4.3.2 Expected Value

In [9], the authors develop a simple bidding rule which is based on the solution of the self-scheduling problem, as well as a price-forecast. The market-clearing price for a given hour can be represented as a random variable that follows a certain probability distribution and is characterized by an expected value and variance. Using the expected marginal prices for each time period, the respective production quantities p_j are determined by solving the following profit-maximizing problem:

$$\max \sum_{j=1}^{n_p} \left(\hat{\lambda}_j p_j - c_j^p \right) \tag{4.45}$$
$$\text{s.t. } p_j \in \pi_j.$$

In the above equation, the total profit is determined by summing the differences between expected revenue and the production cost c_j^p for each hour j. Furthermore, the production quantities must be contained within the feasible operating region denoted by π_j. The bidding strategy is derived by using a confidence interval to represent the range of values that most likely contains the true market-clearing price. Pending the outcome of the self-scheduling problem, the generating company offers to supply its available energy in one or two blocks of power as follows:

- If production is uneconomical, the bidding curve should consist of a single block valued at the upper limit of the confidence interval.
- If production is within the operating limits, the bidding curve should be a step function which increases in price from the lower limit to the upper limit of the confidence interval at the optimal production quantity.
- If production is at maximum capacity, the bidding curve should consist of a single block valued at the lower limit of the confidence interval.

This bidding rule ensures that the generator will almost always be dispatched according to the result of the self-scheduling problem. The probability that this happens is reflected in the confidence interval.

4.3.3 Mean-Variance

The self-scheduling formulation using expected values ignores risk altogether. A more comprehensive treatment of self-scheduling under price uncertainty is presented in [10]. The model adopts a methodology based on the classical portfolio optimization approach found in finance. Given a series of hourly prices, statistical techniques can be used to estimate the covariance matrix Σ, which reflects the mutual dependence between prices. By adding a term that accounts for the variance in total revenue to the initial problem, we obtain the following mixed-integer quadratic program:

$$\max \sum_{j=1}^{n_p} (\hat{\lambda}_j p_j - c_j^p) - \beta \sum_{i=1}^{n_p} \sum_{j=1}^{n_p} \sigma_{ij} p_i p_j \tag{4.46}$$
$$\text{s.t. } p_j \in \pi_j \qquad \forall j.$$

The penalty parameter β is used to adjust the trade-off between maximizing expected profit and minimizing risk. Solving the problem for several values of β produces the efficient frontier, which is composed of a set of non-dominated points corresponding to the maximum expected profit at a given level of risk.

4.3.4 Stochastic Programming

The uncertainty in market-clearing prices can be characterized by a set of discrete price scenarios, along with the probability of occurrence for each. This observation is the foundation of a stochastic programming approach to the self-scheduling problem. The scenarios act to approximate the continuous probability distributions of electricity prices. As such, a larger number of scenarios results in a more accurate representation of the distribution. Unfortunately, the numerical tractability of the problem suffers with an increasing number of scenarios. Thus, it becomes important to select an adequate number of scenarios in order to properly approximate future uncertainties while ensuring that the computational requirements are realistic [1].

In [41], the authors derive a two-stage process for solving the self-scheduling problem. They define three price scenarios (low price, normal price, and high price) and associate probabilities of occurrences w_s to each. Given these expectations, a price scenario tree is generated for the entire planning horizon. The solution to the optimization problem selects the path that maximizes the expected profit over all future price scenarios while meeting the operational constraints of the problem. This methodology gives rise to the following mathematical program:

$$\textbf{[SS-SP]} \max \sum_{s=1}^{n_s} w_s \sum_{j=1}^{n_p} \left\{ \lambda_j^s \sum_{i=1}^{n_g} p_{ij}^s u_{ij}^s - \sum_{i=1}^{n_g} \left(c_{ij}^{ps} + c_{ij}^{us} \right) \right\} \tag{4.47}$$
$$\text{s.t. } p_{ij}^s, u_{ij}^s \in \pi_{ij} \qquad \forall i \; \forall j.$$

The problem can be efficiently solved using Lagrangian relaxation, where the subproblems are mixed-integer stochastic programs solved using stochastic dynamic programming.

4.3.5 Conditional Value-at-risk

Another self-scheduling approach, provided by Jabr (2005), is based on conditional value-at-risk (CVaR). For a given probability of occurrence, it is unlikely that the loss from expected profit will exceed a certain value, know as value-at-risk (VaR). Although the confidence level provides an indication of the likelihood of observing a loss greater than VaR, the magnitude of any such deviation is unknown. A more rigorous technique involves using CVaR. CVaR represents the average loss from the expected profit given that the observed profit is less than a predetermined threshold t. The average profit resulting from this loss is known as the conditional robust profit and an approximation of this value is given by

$$\tilde{F}\left(p_j, t\right) = t + \frac{1}{N\left(1-\beta\right)} \sum_{k=1}^{N} \left[\lambda_j^k p_j - c_j - t\right]^- \tag{4.48}$$

where β is the risk aversion parameter and $[z]^- = \min\{0, z\}$. This optimization problem uses N samples in order to approximate the probability distribution of the marginal prices. When the total profit resulting from a sample falls below a threshold t, the difference contributes to the discovery of the conditional robust profit in the objective function.

4.3.6 Discussion of Solution Methodologies

The solution methodologies outlined in this section vary in terms of their computational efficiency and analytic rigor. The approach based on expected values is the least computationally demanding of the self-scheduling models described above because the hourly electricity prices are treated as being deterministic. The disadvantage of this type of analysis is that it ignores the potential losses which arise due to discrepancies between forecasted and actual prices [41]. The mean-variance approach takes uncertainty into account through the addition of a variance term in the objective function. As a result, it is multi-objective in nature and produces a final solution that is dependent on the risk tolerance of the decision maker. An attractive feature of this formulation is that it reduces to a quadratic programming problem. However, the objective function value has no practical significance because expected profit and variance have different units and different meanings. The stochastic programming model outputs a single solution that is affected by the number of scenarios and how they are defined. The accuracy of this approach improves as the number of scenarios increases, but the resulting program quickly becomes intractable. Quantitative models are required in order to produce sound estimates of the covariance matrix, as well as to define price scenarios. The CVaR approach does not require such models, but instead takes samples from historical data to build a probability density function of the electricity prices since it lacks a closed-form expression. In this case, the results are highly sensitive to the degree to which the random sample is representative of the actual electricity prices. The proposed model tackles the self-scheduling problem from yet another angle, but shares some of same properties as these methodologies. We present the proposed model in Chapter 5.

Chapter 5

Proposed Methodology

The self-scheduling problem can be perceived as a price-based adaptation of the UC problem. Solving it often requires pairing traditional solution techniques with methods for risk management. In this chapter, we discuss an approach for modeling uncertain mathematical programs known as robust optimization (RO) and derive the robust counterpart of the self-scheduling problem using SDP. SDP was selected since the robustness constraint can be reduced to a second-order cone which can easily be handled by a number of the SDP solvers. The remainder of the chapter discusses measures taken to improve the quality of the initial solution. These include a set of triangle inequalities for strengthening the relaxation, as well as a branch-and-bound algorithm for obtaining integer-feasible solutions.

5.1 Robust Optimization

The classical approach in mathematical programming is to develop models using point estimates to represent input data. In some circumstances, this practice is seriously flawed since real-world data is not always precise. When parameters are uncertain, slight deviations from the prescribed values may significantly deteriorate solution quality and even feasibility. Therefore, it is necessary to include concessions while designing solution approaches in order to lessen the influence of data uncertainty. To this degree, a solution is said to be robust if it is immune to data uncertainty [4].

RO has been an area of ongoing research since the 1970s. Within the realm of linear programming, it deals with problems of the general form

$$\begin{aligned} &\min\ \mathbf{c^T x} \\ &\text{s.t.}\ \ \mathbf{Ax} \geq \mathbf{b} \qquad\qquad\qquad \forall\,(\mathbf{A},\mathbf{b}) \in U \\ &\qquad \mathbf{x} \geq \mathbf{0} \end{aligned}$$

where the constraints must be satisfied for any realization of $(\mathbf{A}, \mathbf{b})$ in the uncertainty set U. In a seminal paper on RO, Soyster [43] addresses the case where each column of the constraint matrix is known to belong to a given convex set. The resulting program is linear and its solution is feasible for all data that belong to the set. Unfortunately, this approach is extremely conservative since every coefficient takes on its worst-case value. A more general approach follows the work presented in [3] and uses ellipsoid uncertainty as an approximation to more complicated uncertainty sets. The choice of an ellipsoid stems from the computational tractability of the resulting conic quadratic problem. In addition, such a representation reflects the unlikelihood that all the coefficients are simultaneously at their worst values. For this reason, it is inherently less conservative than the linear approach [3].

5.1.1 Derivation of the Robustness Constraint

In order to model the self-scheduling problem using a RO framework, it is necessary to define an uncertainty set containing the hourly electricity prices. A simple way to model the uncertainty is to consider bounds on the prices in each period $\lambda_j^{low} \leq \lambda_j \leq \lambda_j^{high}$. Since λ is permitted to vary, the resulting term $\lambda^T p$ in the objective function is non-convex which is a hard problem to solve. An alternative strategy is to take advantage of the underlining statistical properties of the data in order to obtain a reasonable uncertainty set. Analogous to the derivation of a robust portfolio selection model in [2], we allow the hourly prices to vary within an ellipsoid defined by:

$$\lambda \in \{\hat{\lambda} + \kappa \Sigma^{1/2} \delta \text{ s.t. } \parallel \delta \parallel_2 \leq 1\}. \tag{5.1}$$

The ellipsoid is centered at $\hat{\lambda}$ and the total variation in every direction is initially limited by a vector δ whose length is at most one. The magnitude of the permissible variation is further scaled by $\Sigma^{1/2}$, which reflects the covariance between hourly prices. An additional parameter κ is used to adjust the risk aversion properties of the model. When κ is set to zero, the hourly prices revert back to their expected values. Increasing the value of κ expands the uncertainty set, thus allowing more drastic price outcomes.

In order to deal with this definition, we first define an artificial variable q that is bounded from above by the revenue $\lambda^T p$ resulting from any vector of hourly prices contained within the ellipsoid. Under this stipulation, a bound on revenue is established by substituting the expression for the ellipsoid (5.1) into the revenue term

$$\hat{\lambda}^T p + \kappa \delta^T \Sigma^{1/2} p \geq q, \text{ where } \| \delta \|_2 \leq 1. \tag{5.2}$$

The goal is to derive an expression for the upper bound on q corresponding to the lowest revenue among all possible outcomes. It happens that we can evaluate the value of δ for which the left-side of (5.2) is minimized. Observing that

$$\delta^T \Sigma^{1/2} p \geq - \| \Sigma^{1/2} p \|_2 \quad \text{whenever} \quad \| \delta \|_2 \leq 1,$$

we can deduce that the worst-case value of δ is $\delta = \frac{-\Sigma^{1/2} p}{\|\Sigma^{1/2} p\|_2}$. Therefore, the optimal bound for the worst-case revenue is obtained from

$$\hat{\lambda}^T p - \kappa \| \Sigma^{1/2} p \|_2 \geq q \tag{5.3}$$

which is a second-order cone constraint of the general form $z \geq \sqrt{y^T y}$, with $z = \frac{\hat{\lambda}^T p - q}{\kappa}$ and $y = \Sigma^{1/2} p$. It is equivalent to the positive semidefinite constraint

$$\begin{pmatrix} zI & y \\ y^T & z \end{pmatrix} \succcurlyeq 0$$

where I is an identity matrix with diagonal entries equal to one and zeros elsewhere. This equivalence follows from a general result called the Schur Complement Theorem [2].

Theorem 1 (Schur Complement Theorem) *Suppose* $X = \begin{pmatrix} A & B \\ B^T & C \end{pmatrix}$ *and* A *is positive definite. Then* X *is positive semidefinite is and only if* $C - B^T A^{-1} B$ *is positive semidefinite.*

5.2 Proposed Semidefinite Programming Model

The UC problem formulation in [24] forms the foundation of the proposed model. Following this procedure, any quadratic polynomial of the decision variables p and u can be represented using the matrix variable. This characteristic grants significant flexibility in formulating constraints for the model.

5.2.1 Modeling Start-up and Shutdown Constraints

One important aspect missing from the formulation in [24] is the inclusion of start-up and shutdown costs. The linearized versions of these costs (4.14)-(4.16) can be modified to fit within the SDP framework. The quadratic counterparts of these constraints for the purpose of the self-scheduling problem are as follows:

$$c_j^u \geq K^t \left[u_j^2 - \sum_{n=1}^{t} u_{j-n}^2 \right] \qquad \forall j \; \forall t = 1 \ldots ND_i \tag{5.4}$$

$$c_j^d \geq C \left[u_{j-1}^2 - u_j^2 \right] \qquad \forall j \tag{5.5}$$

$$c_j^u \geq 0 \qquad \forall j \tag{5.6}$$

$$c_j^d \geq 0 \qquad \forall j \tag{5.7}$$

where c^u and c^d are vector variables whose treatment within the context of SDP will be discussed in Section 5.2.3.

5.2.2 Modeling Ramping Constraints

A more accurate representation of ramping rates than what is given by (4.38)-(4.39) includes start-up and shutdown ramp rates. If we remove the variables $\overline{p_j}$ that represent the amount of available spinning reserves from constraints (4.14) - (4.16) and make the appropriate substitutions, the ramping rate constraints become

$$u_j p_j \leq u_{j-1} p_{j-1} + RU u_{j-1}^2 + SU \left[u_j^2 - u_{j-1}^2 \right] + \overline{P} \left[1 - u_j^2 \right] \qquad \forall j \tag{5.8}$$

$$u_j p_j \leq \overline{P} u_{j+1}^2 + SD \left[u_j^2 - u_{j+1}^2 \right] \qquad \forall j = 1 \ldots n_p - 1 \tag{5.9}$$

$$u_{j-1} p_{j-1} - u_j p_j \leq RD u_j^2 + SD \left[u_{j-1}^2 - u_j^2 \right] + \overline{P} \left[1 - u_{j-1}^2 \right] \qquad \forall j. \tag{5.10}$$

Spinning reserves are not included in the model since they belong to a separate market which has different price characteristics. However, the analysis performed can be extended to the market for energy reserves.

5.2.3 Proposed Model

The proposed model is obtained by bringing together elements from sections 4.2.6, 5.1.1, 5.2.1 and 5.2.2. The model in its entirety is

$$\max\ q - \sum_{j=1}^{n_p} u_j^2\alpha + u_j p_j \beta + p_j^2 \gamma + c_j^u + c_j^d \tag{5.11}$$

$$\text{s.t.}\ \frac{\sum_{j=1}^{n_p} \hat{\lambda}_j p_j}{\kappa} - \frac{q}{\kappa} \geq \sqrt{\sum_{i=1}^{n_p} (\sum_{j=1}^{n_p} \Sigma_{ij}^{1/2} p_j)^2} \tag{5.12}$$

$$c_j^u \geq K^t \left[u_j^2 - \sum_{n=1}^{t} u_{j-n}^2 \right] \qquad \forall j\ \forall t = 1 \dots ND_i \tag{5.13}$$

$$c_j^d \geq C \left[u_{j-1}^2 - u_j^2 \right] \qquad \forall j \tag{5.14}$$

$$u_j^2 \underline{P} - u_j p_j \leq 0 \qquad \forall j \tag{5.15}$$

$$u_j p_j - u_j^2 \overline{P} \leq 0 \qquad \forall j \tag{5.16}$$

$$u_j p_j \leq u_{j-1} p_{j-1} + RU u_{j-1}^2 + SU \left[u_j^2 - u_{j-1}^2 \right] + \overline{P} \left[1 - u_j^2 \right] \qquad \forall j \tag{5.17}$$

$$u_j p_j \leq \overline{P} u_{j+1}^2 + SD \left[u_j^2 - u_{j+1}^2 \right] \qquad \forall j = 1 \dots n_p - 1 \tag{5.18}$$

$$u_{j-1} p_{j-1} - u_j p_j \leq RD u_j^2 + SD \left[u_{j-1}^2 - u_j^2 \right] + \overline{P} \left[1 - u_{j-1}^2 \right] \qquad \forall j \tag{5.19}$$

$$\sum_{\tau=j}^{a(\overline{t})} u_\tau \geq \left(u_j^2 - u_{j-1}^2 \right) b\left(\overline{t} \right) + \delta\left(j-1 \right) a_0 \qquad \forall j \tag{5.20}$$

$$\sum_{\tau=j}^{a(\underline{t})} \left(1 - u_\tau^2 \right) \geq \left(u_{j-1}^2 - u_j^2 \right) b\left(\underline{t} \right) + \delta\left(j-1 \right) b_0 \qquad \forall j \tag{5.21}$$

$$u_j^2 - u_j v = 0 \qquad \forall j \tag{5.22}$$

$$v - 1 = 0 \tag{5.23}$$

where the terms $a(\cdot)$, $b(\cdot)$, a_0, and b_0 are defined in Section 4.2.4. The objective function consists of maximizing profit, which is the difference between the revenue and the total of the production cost, start-up cost, and shutdown cost. The revenue is established by the robustness constraint (5.12), while the start-up and shutdown costs are given by constraints (5.13) and (5.14), respectively. Each generator is also subjected to local constraints which include: generation limits (5.15)-(5.16), ramping rates (5.17)-(5.19), and minimum up and down times (5.20)-(5.21). Lastly, the binary constraints (5.22) reflect the nature of the unit commitment variables and the auxiliary constraint (5.22) defines an auxiliary variable.

We now express the problem in the standard form (4.29) which is necessary in order to solve it using available software. Recall from Section 4.2.6 that the positive semidefinite matrix variable was obtained by taking the outer-product of the vector $x = \begin{bmatrix} p_1 \, u_1 \ldots p_{n_p} \, u_{n_p} \, v \end{bmatrix}^T$ with itself. However, the inclusion of the robustness constraint, along with the start-up and shutdown costs, has introduced non-negative scalar variables into the formulation. Furthermore, the strict equality of the constraints in standard form necessitates the addition of a slack variable s to each of the m inequalities. After adding these variables, the following positive semidefinite matrix is obtained:

$$X = \begin{pmatrix} xx^T & 0 & 0 & 0 & 0 \\ 0 & q & 0 & 0 & 0 \\ 0 & 0 & C^u & 0 & 0 \\ 0 & 0 & 0 & C^d & 0 \\ 0 & 0 & 0 & 0 & S \end{pmatrix}$$

$$\text{where } xx^T = \begin{pmatrix} p_1p_1 & p_1u_1 & \cdots & p_1p_{n_p} & p_1u_{n_p} & p_1v \\ p_1u_1 & u_1u_1 & \cdots & u_1p_{n_p} & u_1u_{n_p} & u_1v \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ p_{n_p}p_1 & p_{n_p}u_1 & \cdots & p_{n_p}p_{n_p} & p_{n_p}u_{n_p} & p_{n_p} \\ u_{n_p}p_1 & u_{n_p}u_1 & \cdots & u_{n_p}p_{n_p} & u_{n_p}u_{n_p} & u_{n_p}v \\ vp_1 & vu_1 & \cdots & vp_{n_p} & vu_{n_p} & vv \end{pmatrix},$$

$$C^u = \begin{pmatrix} c_1^u & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & & c_{n_p}^u \end{pmatrix}, \; C^d = \begin{pmatrix} c_1^d & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & & c_{n_p}^d \end{pmatrix}, \text{ and } S = \begin{pmatrix} s_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & & s_m \end{pmatrix}.$$

Using the above definition of X, the objective function (5.11) and the constraints (5.12)-(5.23) are transformed into coefficient matrices.

The resulting matrix variable X is block-diagonal and consists of a $n_p \times n_p$ block corresponding to xx^T, as well as a $(2n_p + m + 1) \times (2n_p + m + 1)$ block of diagonal entries. With respect to the second, the block is positive semidefinite if and only if each diagonal element is non-negative. As such, the individual non-negativity constraints are omitted because they are implied by the positive semidefiniteness of the decision variable. SDP solvers are designed to exploit block structure and type, which greatly increases their computational efficiency.

5.3 Improving Solution Quality

Generally, the solution to the SDP relaxation includes a number of variables that violate the binary constraints. When this happens, the objective function value provides an upper bound on the optimal solution of a maximization problem. Simple heuristic approaches can be employed to produce feasible solutions whose objective function values provide lower bounds, and the quality of these solutions is determined from the relative gap between the lower and upper bounds. In some instances, the size of the gap may warrant an improvement of the bounds. The addition of triangle inequalities can strengthen the initial relaxation; whereas, branch-and-bound offers a systematic approach for discovering feasible solutions, while at the same time improving the bounds.

5.3.1 Triangle Inequalities

A straightforward way to improve the SDP relaxation is to add triangle inequalities. Within the context of the self-scheduling problem, these inequalities are:

$$u_i + u_j - u_i u_j \leq 1 \qquad \forall i \; \forall j > i. \tag{5.24}$$

There exist $\binom{n_p}{2}$ such inequalities. If the value of $u_i u_j$ is actually the product of the two variables, the inequalities always hold. However, the observed value of elements within the decision matrix may not correspond to their analytical representations. For example, the entry corresponding to $u_1 u_2$ is not necessarily equivalent to the product of u_1 and u_2 unless the rank of the matrix is one. As a result, the positive semidefiniteness of the decision variable is not sufficient to ensure this equivalence and X may violate one or more triangle inequalities despite being feasible for the SDP relaxation.

5.3.2 Branch-and-Bound

Within the self-scheduling problem, there are 2^{n_p} possible unit commitment combinations, each of which results in a problem that would have to be solved in order to determine the associated electricity generation levels. It is impossible to explore all of these combinations in practice, but implicit enumeration can be achieved using branch-and-bound. Branching occurs when two or more nodes are created by splitting a problem into a set of subproblems, which are further constrained and whose union covers the same set of candidate solutions as the parent node. With respect to a maximization problem, bounding computes an upper bound for the problem by comparing the upper bounds for each subset of solutions, whereas a lower bound is obtained from the objective function value of the best feasible solution. Given this information, the algorithm eliminates large subsets of candidate solutions that are known to be non-optimal. The optimal solution is identified when no subproblems remain that have the potential to improve the objective function. A general description of the branch-and-bound algorithm is:

Step 1: Initialization Solve the SDP relaxation of the self-scheduling problem at the root node without any variables fixed. If the solution is integer-feasible, then the SDP relation provides the optimal solution to the original problem. Else create two subproblems and proceed to Step 2.

Step 2: Termination If the set of active subproblems is empty or the relative gap between the upper and lower bounds is less than a given tolerance, then $X_{incumbent}$, which corresponds to the best feasible solution found, is optimal or sufficiently close to optimality. Else proceed to Step 3.

Step 3: Problem Selection and Relaxation Select and delete a problem from the list of active subproblems. Solve the SDP relaxation of the problem to obtain a solution X_i^* with an objective function value z_i^R. If the solution is fractional, the objective function value z_i^R is an bound for that partition. Proceed to Step 4.

Step 4: Fathoming The node is fathomed when one of the following conditions is met:

1. The solution is feasible for the original problem. The lower bound is updated if the objective function value z_i^R is greater than the previous best solution. In this case, the new solution X_i^* becomes the incumbent $X_{incumbent}$. Go to Step 2.
2. The objective function value z_i^R is less than the value of the current lower bound. Go to Step 2.
3. The problem at the node is infeasible. Go to Step 2.

Step 5: Partitioning If none of the fathoming conditions are met, then create two or more subproblems and add them to the set of active subproblems. The upper bound for the original problem is updated if the maximum of all the upper bounds for each live branch in the tree decrease. The maximum value of each branch is given by the objective function value of the last node in that branch. Go to Step 2.

The specifics of any branch-and-bound algorithm must address strategies for variable and node selection. The easiest way to partition a parent node is to generate two subproblems by branching on a fractional variable and setting that variable equal to zero or one. The degree of infeasibility is determined by evaluating the difference between the variable and its square:

$$u_j - u_j^2 = \epsilon_j \qquad \forall j.$$

Variables with ϵ less than a predetermined tolerance are considered integral. During the partitioning process, branching variables must be selected in order to create children nodes. Since the choice of branching variable may affect the running time of the algorithm, we test four approaches for selecting branching variables among the fractional unit commitment variables. These approaches are defined as follows:

Random Pick: Selects a branching variable randomly from the set of non-integral variables.

Most Infeasible: Selects to branch on the variable with the largest value of ϵ.

Least Infeasible: Selects to branch on the variable with the smallest value of ϵ.

Most Profitable: Selects to branch on the variable corresponding to the period with the highest expected price.

Fixing a unit commitment variable to zero or one is problematic due to the structure of the matrix variable. Setting $u_j = 0$ forces every element in the rows and columns associated with $p_j p_j$ and $u_j u_j$ equal to zero; whereas, setting $u_j = 1$ causes the rows and columns associated with $u_j u_j$ and v^2 to be linearly dependent. These singularities necessitate matrix reduction operations at each node. The operations consist of removing the two empty rows and columns when $u_j = 0$ and consist of removing the linearly dependent row and column corresponding to $u_j u_j$ when $u_j = 1$. If $u_j = 1$, the coefficient matrices must be modified in order to reflect the fact that the unit is committed. In this case, it is necessary to adjust the constant term b_i in each constraint to account for the fact that coefficients corresponding to $u_j u_j$, $u_j v$, and $v u_j$ become constants. Also, since the remaining terms in the deleted row and column are meant to depend on the product of u_j and another decision variable, it is necessary to modify the coefficients of the variables in the row and column associated with v^2 to account for these contributions. Some constraints may become redundant following these operations and they must be removed.

The decision of which problem to solve among the set of active subproblems dictates the total number of nodes explored before optimality is reached. More specifically, it affects the possibility of improving the incumbent solution, along with the chance to fathom nodes more quickly. Two of the most common strategies are the depth-first and best-bound approaches.

Depth-first: The most recently created subproblem is selected. The goal is to find a feasible solution as quickly as possible by fixing a large number of variables.

Best-bound: The subproblem with the lowest bound is selected. The rationale is that it is more difficult to fathom nodes with small objective function values because they are unlikely to exceed the upper bound. It also helps to reduce the gap between bounds as quickly as possible.

In Chapter 6, we will compare the performance of each branch-and-bound configuration in order to determine the best performing rules for node and variable selection.

Chapter 6

Results and Discussions

In this chapter, three test instances are used to analyze the performance of the proposed methodology relative to that of the mean-variance approach described in Section 4.3.3. First, we compare the strength and solution times of the two relaxations using a 24-period test case from the literature. Using the same instance, we test the various branch-and-bound configurations for the SDP approach to determine which performs best. Having selected the best branch-and-bound strategy, we continue by plotting the efficient frontiers of both solution methodologies and compare their solution times for eight levels of risk aversion. The second and third test cases extend the results by subjecting the model to a larger instance consisting of 48 periods and an instance based on real-world data from the Ontario electricity market.

6.1 Problem Instances

The numerical example found in [10] is used to evaluate the quality of the SDP relaxation and the performance of the branch-and-bound algorithm. The problem consists of the day-ahead scheduling of a single generator over a 24-hour planning horizon. The technical and costs data of the generator is given in Table A.1 and Table A.2 of Appendix A. We only consider the scheduling of a unit with these operational characteristics. An example of the parameters used to describe the uncertainty in electricity prices is also provided in [10]. In this paper, three months of price information from the electricity market in mainland Spain was used to generate price forecasts and a covariance matrix. The price forecasts are presented in Table A.3 while an estimate of the covariance matrix is given in Table A.4, both of which are included in Appendix A.

A larger problem instance consisting of 48 periods was generated by replicating each period in Table A.3 and Table A.4 of Appendix A. Using this approach, every two consecutive periods are indistinguishable. For example, if we consider only the first two periods, the following relationships hold:

$$\hat{\lambda}_1 = \hat{\lambda}_2$$
$$\sigma_{1i} = \sigma_{i1} = \sigma_{2j} = \sigma_{j2} \qquad \forall i = 1 \dots n_p \; \forall j = 1 \dots n_p.$$

The resulting 48-period planning horizon is consistent with the electricity markets in the United Kingdom and Australia, where the markets are cleared in half-hour intervals. In order to correctly model the shorter time periods, it is necessary to account for the time dependency of certain operational characteristics. This requires modifying the minimum up and down times, as well as the ramp-up and ramp-down rates, by a factor of two to reflect the shorter time periods. In addition, we divide the cost coefficients and expected prices by a factor of two since they are expressed on a per megawatt hour basis but the production in each period occurs over a half-hour.

In the province of Ontario, the IESO publishes detailed price information pertaining to the market it oversees [32]. Two reports of particular interest are the Hourly Ontario Energy Price (HOEP) and the Day-ahead Price Forecast. The HOEP is the price paid to self-scheduling generators for the electricity they produce, whereas the Day-ahead Price Forecast is derived from publicly available data which includes day-ahead forecasts of demand and the availability of generation capacity, as well as price information from surrounding markets. It is published at 17:00 EST the day prior, but only for Monday to Friday. Five weeks of data spanning between September 1, 2008 and October 3, 2008 was gathered and is provided in Table A.5 and Table A.6 of Appendix A. Using this real-world data, we construct a second problem instance consisting of 24 periods. An approximation of the covariance matrix is obtained from equation (4.44) with $\nu = 0.98$ and $D = 24$. These values were shown to produce a good estimate of the covariance matrix in [10] and are selected here. The price forecasts and an estimate of the covariance matrix are found in Table A.7 and Table A.8 of Appendix A.

6.2 Baseline Model

The mean-variance approach presented in [10] is used as a baseline for analyzing the computational efficiency and solution quality of the proposed model. This approach is selected for comparative purposes because it considers profit and risk simultaneously, which is an important property of the proposed framework. In both models, a covariance matrix of electricity prices is used to measure the uncertainty in expected profit. The mean-variance approach quantifies risk through the addition of a variance term in the objective function, whereas the robust optimization approach uses a covariance matrix to define an uncertainty set.

The formulation of the mean-variance model used in this chapter is as follows:

$$\max \sum_{j=1}^{n_p} \left[\lambda_j^{est} p_j - c_j^p - c_j^u - c_j^d\right] - \beta \sum_{i=1}^{n_p} \sum_{j=1}^{n_p} \sigma_{ij} p_i p_j \tag{6.1}$$

$$\text{s.t. } c_j^p = \alpha u_j + \omega p_j + \gamma p_j^2 \qquad \forall j \tag{6.2}$$

$$c_j^u \geq K\left[u_j - u_{j-1}\right] \qquad \forall j \tag{6.3}$$

$$c_j^d \geq C\left[u_{j-1} - u_j\right] \qquad \forall j \tag{6.4}$$

$$u_j \underline{P} - p_j \leq 0 \qquad \forall j \tag{6.5}$$

$$p_j - u_j \overline{P} \leq 0 \qquad \forall j \tag{6.6}$$

$$p_j \leq p_{j-1} + RU u_{j-1} + SU\left[u_j - u_{j-1}\right] + \overline{P}\left[1 - u_j\right] \qquad \forall j \tag{6.7}$$

$$p_j \leq \overline{P} u_{j+1} + SD\left[u_j - u_{j+1}\right] \qquad \forall j = 1 \ldots n_p - 1 \tag{6.8}$$

$$p_{j-1} - p_j \leq RD u_j + SD\left[u_{j-1} - u_j\right] + \overline{P}\left[1 - u_{j-1}\right] \qquad \forall j \tag{6.9}$$

$$\sum_{\tau=j}^{a(\bar{t})} u_\tau \geq \left(u_j - u_{j-1}\right) b\left(\bar{t}\right) + \delta\left(j-1\right) a_0 \qquad \forall j \tag{6.10}$$

$$\sum_{\tau=j}^{a(\underline{t})} \left(1 - u_\tau\right) \geq \left(u_{j-1} - u_j\right) b\left(\underline{t}\right) + \delta\left(j-1\right) b_0 \qquad \forall j \tag{6.11}$$

$$p_j, c_j^u, c_j^d \geq 0 \qquad \forall j \tag{6.12}$$

$$u_j \in \{0, 1\} \qquad \forall j, \tag{6.13}$$

where the $a(\cdot)$, $b(\cdot)$, a_0, and b_0 terms are defined in Section 4.2.4. The objective function consists of maximizing profit and minimizing variance. The addition of the risk aversion parameter, β, allows for the simultaneous treatment of these multiple objectives. Moreover, increasing its value shifts the emphasis from profit to risk. Similar to the robust approach, an efficient frontier is defined by solving the problem for various levels of risk. The unit-specific constraints correspond to those given in Section (5.2.3) for the robust formulation. They include the production cost (6.2), the start-up and shutdown costs (6.3) - (6.4), the generation limits (6.5) - (6.6), the ramping rates (6.7) - (6.9), the minimum time constraints (6.10) - (6.11), the non-negativity constraints (6.12), and the binary constraints (6.13). Since the variance term is quadratic and the unit commitment variables are binary, the problem is formulated as a mixed-integer quadratic programming (MIQP) problem.

The proposed formulation and algorithm were modeled in MATLAB [27], and the SDPT3 [44] solver was invoked to solve the SDP subproblems. The MIQP formulation of the mean-variance model was implemented in GAMS [11] and solved using CPLEX [17] solver. Both solution approaches were tested on a Sun workstation with 16 processors and 32 GB of RAM.

6.3 Strength of the SDP Relaxation

The problem instance in [10] was solved using the SDP relaxation of the proposed formulation and the MIQP relaxation of the mean-variance model. The solution quality and computation efficiency of both relaxations are given in Table 6.1 and Table 6.2 for several levels of risk. The values for the scaling parameters κ and β were chosen such that the solutions correspond to the same set of non-dominated integer-feasible points. It should also be noted that the definitions of the objective functions differ between approaches. Therefore, we cannot draw definitive conclusions based on the magnitudes of these results.

The results presented above indicate that the SDP relaxation produces a tighter upper bound on the optimal solution but takes approximately 20 times longer to solve than the MIQP relaxation. The strength of the SDP relaxation is especially evident in the least risk-averse schedules. In both cases, the absolute gap tends to increase as the schedules become less risky. The relative gap is significantly large in some instances. However, none of the triangle inequalities are violated in any of the instances. The objective function value only represents the expected profit of the schedule when the risk aversion parameter is 0. As such, the relative error does not reflect the quality of the schedules in terms of their expected profit. The

Table 6.1: Quality of the SDP Relaxation

Kappa	Solution Times (s)	Function Value Upper Bound	Function Value Optimal	Absolute Gap	Relative Error (%) (From Optimal)	Violated Inequalities
0.001	5.76	29238	29203	35	0.1%	0
6	5.22	21927	21858	69	0.3%	0
12	5.67	15453	15320	133	0.9%	0
18	5.66	9746	9499	247	2.6%	0
24	5.66	5151.2	4446	706	15.9%	0
30	7.14	1390.6	450	941	209.3%	0
36	6.19	-1077.6	-2973	1896	63.8%	0
42	5.40	-2185	-5315	3130	58.9%	0

Table 6.2: Quality of the MIQP Relaxation

Beta	Solution Times (s)	Function Value Upper Bound	Function Value Optimal	Absolute Gap	Relative Error (%) (From Optimal)
0	0.31	31273	29204	2069	7.08%
0.0025	0.30	28582	25458	3124	12.27%
0.006	0.29	25572	21317	4255	19.96%
0.0098	0.29	22983	17765	5218	29.37%
0.0166	0.30	19440	13120	6320	48.17%
0.022	0.27	17282	10650	6632	62.28%
0.034	0.27	13704	6556	7147	109.02%
0.082	0.26	6153	63	6091	9727.17%

Table 6.3: Comparison of Fractional Unit Commitment Variables

Hour	Unit Commitment							
	SDP	MIQP	SDP	MIQP	SDP	MIQP	SDP	MIQP
	$\kappa = 0.001$	$\beta = 0$	$\kappa = 12$	$\beta = 0.006$	$\kappa = 24$	$\beta = 0.0166$	$\kappa = 36$	$\beta = 0.034$
1	1.000	0.973	0.688	0.971	0.286	0.971	0.217	0.972
2	0.000	0.206	0.042	0.133	0.192	0.274	0.217	0.343
3	0.000	0.114	0.000	0.071	0.000	0.000	0.000	0.000
4	0.000	0.114	0.000	0.071	0.000	0.000	0.000	0.000
5	0.000	0.114	0.000	0.071	0.000	0.000	0.000	0.000
6	0.000	0.114	0.000	0.071	0.000	0.036	0.003	0.085
7	0.000	0.133	0.000	0.211	0.000	0.231	0.012	0.273
8	0.000	0.138	0.000	0.211	0.000	0.231	0.012	0.273
9	0.000	0.174	0.000	0.211	0.000	0.159	0.012	0.103
10	0.000	0.275	0.000	0.211	0.000	0.159	0.001	0.103
11	0.934	0.471	0.000	0.282	0.000	0.159	0.001	0.103
12	1.000	0.665	0.889	0.536	0.461	0.401	0.001	0.294
13	1.000	0.896	1.000	0.728	1.000	0.521	0.001	0.330
14	1.000	0.896	1.000	0.728	1.000	0.521	0.262	0.330
15	1.000	0.896	1.000	0.728	1.000	0.596	0.262	0.478
16	1.000	0.896	1.000	0.775	1.000	0.650	0.439	0.478
17	1.000	0.911	1.000	0.775	1.000	0.650	0.439	0.478
18	1.000	0.911	1.000	0.775	1.000	0.656	0.913	0.555
19	1.000	0.911	1.000	0.775	1.000	0.656	0.913	0.555
20	1.000	0.911	1.000	0.758	1.000	0.612	0.758	0.487
21	1.000	0.911	1.000	0.758	1.000	0.612	0.758	0.487
22	1.000	0.922	1.000	0.758	0.953	0.612	0.318	0.487
23	1.000	0.922	1.000	0.758	0.147	0.501	0.000	0.375
24	1.000	0.678	1.000	0.589	0.147	0.501	0.000	0.375

unusually high percent differences are mainly the consequence of optimal values that approach zero.

The difference in quality is further highlighted by the fractional values of the unit commitment variables shown in Table 6.3. Despite having relaxed the binary condition on the unit commitment variables, the solution to the SDP relaxation consists almost exclusively of integer feasible variables at small values of κ. At these low values, the robustness constraint has little influence on the solution space and there is a large incentive to produce in the peak evening hours. The fractional values at the transitions between on-line and off-line states are a consequence of the ramping rates. With increasing values of κ, the robustness constraint plays a larger role and the worst-case electricity prices tend to decrease, making it more attractive to commit generating capacity in fewer periods. Under these circumstances, the minimum time constraints add to the degradation of the integrality of the unit commitment variables. Once the number of periods in which electricity generation is profitable falls below a threshold, the integrality of the unit commitment variables may be compromised in order to satisfy the minimum time constraints. As the profit margins decrease, the pressure becomes more pronounced and the fractional values tend to approach 0. The impact of the minimum time constraints is evidence of the complexity of the self-scheduling problem. However, compared with the solution to the MIQP relaxation, the SDP relaxation has a higher degree of integer-feasibility.

This distinction is more evident at low values of κ. From a practitioner's point of view, it would be straightforward to apply a simple heuristic in order to obtain a good quality self-schedule starting with the solution to the SDP relaxation.

6.4 Performance of the Branch-and-bound Algorithm

The self-scheduling problem formulations contain 48 independent decision variables, half of which are binary variables. Solving this problem to a global optimal involves searching a large number of nodes during the solution process. The difference between the solution times of the mixed-integer problem and its relaxation in Section 6.3 highlights the complexity introduced by the binary requirements. A comparison of the solution times and the number of nodes evaluated for each of the branch-and-bound configurations outlined in Section 5.3.2 is given in Table 6.4 and Table 6.5 for eight different values of κ. Selecting the most infeasible unit commitment variable leads to the lowest average solution time among all branching strategies irregardless of the node selection techniques. The difference between the solution time of this strategy and the best performing strategy is significant in only three of the sixteen instances. Furthermore, the best-bound strategy for node selection terminates in less time on average than the depth-first approach under each of the four branching rules. Therefore, unless otherwise noted, we select the most infeasible variable and apply the best-bound node selection strategy within our branch-and-bound algorithm.

The computational results for the MIQP formulation solved using CPLEX are presented in Table 6.6. At low values of κ, the SDP formulation requires approximately two to three times more computational time than the alternative. This disparity becomes even more significant as the risk aversion parameters increase. For large values of κ, the difference in solution times exceeds a factor of 10. Despite having higher solution times, the proposed method requires solving fewer subproblems before terminating. Therefore, any reductions in the solution times of individual subproblems will improve the performance of the algorithm.

A noteworthy factor driving the computational efficiency of many solution methodologies is warm-starting. This technique reduces the number of iterations needed to arrive at the optimal solution of a subproblem by using the solution to a previously solved subproblem as a starting point. The potential for improvement is

Table 6.4: Performance of the Branch-and-Bound Algorithm with Depth-first Node Selection

kappa	Random		Most-infeasible		Least-infeasible		Most Profitable	
	Time (s)	Nodes	Time (s)	Nodes	Time (s)	Nodes	Time (s)	Nodes
0.001	18.18	3	18.28	3	18.32	3	18.13	3
6	28.8	5	28.92	5	28.86	5	28.68	5
12	42.87	7	43.08	7	56.13	9	43.04	7
18	55.63	9	41.35	7	53.92	9	41.94	7
24	271.1	43	80.81	13	156.82	25	261.7	39
30	294.34	41	101.65	15	367.42	55	269.17	37
36	360.13	53	392.84	57	477.52	73	813.84	117
42	1270	195	1022.2	149	864.64	129	1719.6	243
AVERAGE	**292.63**	**44.50**	**216.14**	**32.00**	**252.95**	**38.50**	**399.51**	**57.25**

Table 6.5: Performance of the Branch-and-Bound Algorithm with Best-Bound Node Selection

kappa	Random		Most-infeasible		Least-infeasible		Most Profitable	
	Time (s)	Nodes	Time (s)	Nodes	Time (s)	Nodes	Time (s)	Nodes
0.001	18.70	3	18.17	3	18.22	3	18.25	3
6	28.89	5	29.08	5	28.68	5	28.74	5
12	48.71	8	43.28	7	55.74	9	35.87	6
18	48.66	8	41.15	7	54.01	9	44.21	7
24	186.23	30	67.26	11	142.14	23	145.03	22
30	268.28	39	100.73	15	359.00	54	269.86	37
36	399.59	61	262.42	38	449.91	69	575.69	83
42	550.48	84	254.46	38	856.78	127	880.92	130
AVERAGE	**193.69**	**29.75**	**102.07**	**15.50**	**245.56**	**37.38**	**249.82**	**36.63**

Table 6.6: Computational Results for Solving the MIQP using CPLEX

Beta	Exp. Profit ($)	Std. Deviation	Time (s)	Nodes
0	29205	1252	9.68	16
0.0025	29056	1200	9.46	28
0.006	27543	1019	13.63	42
0.0098	26046	919	17.22	58
0.0166	21829	724	19.86	90
0.022	19963	651	18.38	94
0.034	16093	530	21.69	109
0.082	5390	255	18.00	96

most considerable for the depth-first approach since the subsequent problems in the branch-and-bound tree differ by a single variable. Unfortunately, incorporating this technique into an SDP-based algorithm is a topic of ongoing research. As a result, each subproblem must be solved from scratch, thus greatly increasing the computational burden of the algorithm as a whole.

6.5 Solution Characteristics

Solving the self-scheduling problem for varying levels of risk aversion produces the efficient frontier illustrated in Figure 6.1. The plot depicts the trade-off between expected profit and risk, where risk is quantified by the standard deviation of the generation schedules. The results indicates that the exact solutions to the SDP and MIQP formulations trace out the same efficient frontier. This fact is interesting since robust optimization is notorious for being overly conservative. From the efficient frontier, it is observed that the expected profit increases as the standard deviation also increases. The most risky schedule ($\kappa = 0.001$) has an expected profit of $29,205, whereas the expected profit from a more conservative schedule ($\kappa = 42$) is equal to $5,382.

Figure 6.1 also depicts the efficient frontiers of the relaxed solutions. The resulting curves further support the observations regarding the strength of relaxations given in Section 6.3. As previously noted, the SDP relaxation provides an excellent approximation to the exact solution at low values of κ and diverges as the risk aversion increases. With regards to the MIQP formulation, the relaxation is worst at moderate levels of risk and improves as the risk aversion increases, where it eventually outperform the SDP relaxation when κ and β are large.

Table 6.7 shows the scheduling differences between low and high levels of risk aversion. With increasing risk aversion, producers are more reluctant to offer generation capacity. In general, the risk averse producer remains on-line for fewer periods, and for those periods in which units remain committed, the production levels are lower.

Table 6.7: Scheduling Differences Between Low and High Levels of Risk Aversion

Hour	Power (MW)		Hour	Power (MW)	
	$\kappa = 0.001$	$\kappa = 42$		$\kappa = 0.001$	$\kappa = 42$
1	160	150	13	290	0
2	0	141	14	294	0
3	0	0	15	294	0
4	0	0	16	294	0
5	0	0	17	294	0
6	0	0	18	294	170
7	0	0	19	294	196
8	0	0	20	294	185
9	0	0	21	294	160
10	0	0	22	294	0
11	170	0	23	287	0
12	230	0	24	237	0

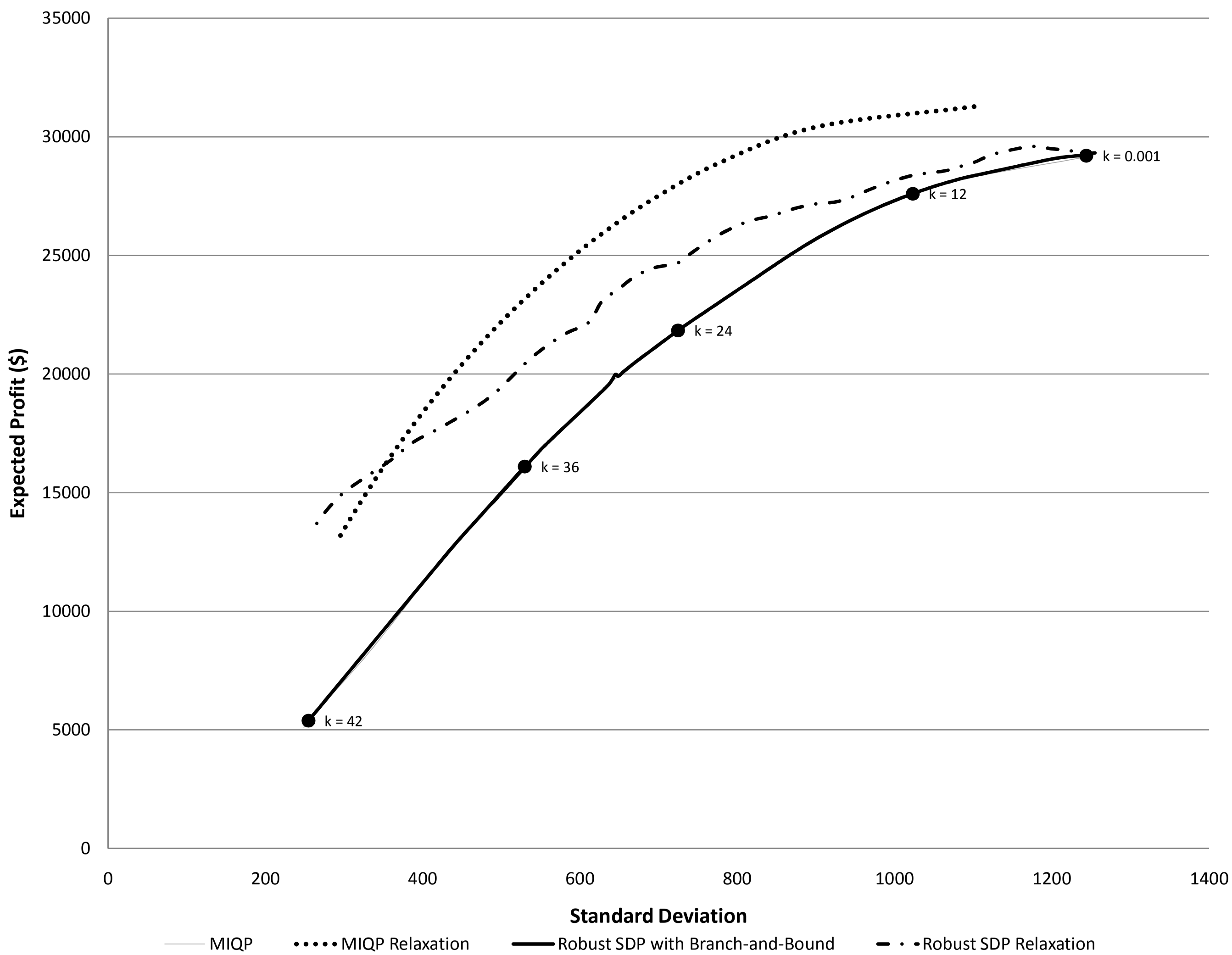

Figure 6.1: Comparisons of Efficient Frontiers

Table 6.8: Quality of the SDP Relaxation with $n_p = 48$

Kappa	Solution Times (s)	Function Value ($) Upper Bound	Optimal	Absolute Gap ($)	Relative Error (%) (From Optimal)
0.001	20.2	29264	29253	11	0.0%
6	16.1	15517	15348	169	1.1%
12	17.4	5323	4385	938	21.4%
18	16.5	-731	-3059	2328	76.1%
24	17.8	-2271	-7195	4924	68.4%

6.6 Effect of Problem Size on Performance

In order to evaluate the effect of the problem size on performance, a larger instance was created by replicating the number of periods. Table 6.8 summarizes the performance of the SDP relaxation for the instance with $n_p = 48$. The results indicate that increasing the number of periods also increases the time needed to solve the subproblems. For this particular test instance, doubling the number periods increases the solution times by a factor of about three. Furthermore, the quality of the SDP relaxation is worse both in terms of absolute and relative gaps than what is presented in Table 6.1, where $n_p = 24$.

Table 6.9 and Table 6.10 show the computational results stemming from both exact solution methodologies. There is a large difference between the solution times presented in these tables and their 24-period counterparts, with several entries more than an order of magnitude larger. However, this fact is not surprising since the number of unit commitment combinations increases exponentially with the number of periods. Another consequence of the larger instance is the increase in computational effort required to construct the input matrices, as well as to perform the matrix reduction operations. These costly procedures greatly deteriorate the performance of the SDP formulation and force its solution times well above those of the commercially available software.

6.7 Example Based on IESO Data

In this section, we apply the robust formulation to a numerical example derived from the IESO data. The example was subjected to many of the same analyses as

Table 6.9: Computational Results using SDP with $n_p = 48$

kappa	Optimal Value	Expected Profit ($)	Standard Deviation	Solution Time (s)			Nodes
				Build	Solve	Total	
0.001	29253	29255	2513	52	72	124	4
6	15348	27802	2076	100	113	212	7
12	4385	21862	1456	832	690	1522	44
18	-3059	14326	966	2709	1862	4571	108
24	-7195	5894	545	2169	1606	3775	90

Table 6.10: Computational Results using MIQP with $n_p = 48$

beta	Optimal Value	Expected Profit ($)	Standard Deviation	Solution Time (s)	Nodes
0	29255	29255	2527	34	39
0.00144	21597	27809	2077	60	104
0.0041	13166	21878	1458	126	304
0.00932	5632	14322	966	161	379
0.0219	-590	5898	546	93	170

Table 6.11: Quality of the SDP Relaxation for the IESO instance

Kappa	Solution Times (s)	Function Value Upper Bound	Function Value Optimal	Absolute Gap	Relative Error (%) (From Optimal)	Violated Inequalities
0.001	7.72	68793	67021	1772	2.6%	0
1	5.68	43942	42424	1518	3.6%	0
2	5.86	21659	20149	1510	7.5%	0
3	5.60	6047	3411	2636	77.3%	0
4	5.64	-809	-4827	4018	83.2%	0
5	6.13	-2451	-8293	5842	70.4%	0
6	5.44	-3134	-11114	7980	71.8%	0

Table 6.12: Quality of the MIQP Relaxation for the IESO instance

Beta	Solution Times (s)	Function Value Upper Bound	Function Value Optimal	Absolute Gap	Relative Error (%) (From Optimal)
0	0.44	69735	67045.451	2690	4.0%
$2.085 \cdot 10^5$	0.34	57169	54415	2754	5.1%
$5.135 \cdot 10^5$	0.41	45008	39624	5384	13.6%
$1.563 \cdot 10^4$	0.34	26790	17806	8984	50.5%
$2.715 \cdot 10^4$	0.33	18780	9906	8874	89.6%
$8.8 \cdot 10^4$	0.32	6543	-1191	7734	649.3%
0.001	0.47	5616	-2148	7764	361.4%

were conducted in Sections 6.3 - 6.5. The quality of the bounds and the solution times for both formulations are reported in Table 6.11 and Table 6.12 for seven levels of risk aversion. From these tables, the SDP relaxation is shown to provide a tighter upper bound on the optimal solution than the MIQP relaxation; however, the difference between the relative errors is less significant than in Section 6.3. At $\kappa = 3$ and $\beta = 1.563 \cdot 10^{-4}$, the relative error given by the SDP relaxation actually exceeds the value of its counterpart. Table 6.13 lists the values of the unit commitment variables. At low levels of risk aversion, the results obtained by both methodologies are either 0 or 1 in most periods. The number of unit commitment variables that take non-integer values increases when κ and β are large.

The computational results of the exact solution methodologies are given in Table

Table 6.13: Comparison of Fractional Unit Commitment Variables for the IESO Instance

Hour	SDP $\kappa = 0.001$	MIQP $\beta = 0$	SDP $\kappa = 2$	MIQP $\beta = 5.135 \cdot 10^5$	SDP $\kappa = 4$	MIQP $\beta = 2.715 \cdot 10^4$	SDP $\kappa = 6$	MIQP $\beta = 0.001$
1	0.342	0.968	0.296	0.968	0.264	0.968	0.217	0.968
2	0.171	0.000	0.188	0.000	0.200	0.000	0.217	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.195	0.000	0.016	0.000	0.000	0.000	0.000
6	0.727	0.773	0.727	0.617	0.198	0.226	0.016	0.167
7	1.000	1.000	1.000	0.975	0.744	0.803	0.063	0.487
8	1.000	1.000	1.000	0.975	0.620	0.803	0.063	0.487
9	1.000	1.000	1.000	0.975	0.620	0.379	0.063	0.228
10	1.000	1.000	1.000	0.903	0.202	0.324	0.000	0.080
11	1.000	1.000	1.000	0.903	0.202	0.319	0.000	0.080
12	1.000	1.000	1.000	0.903	0.202	0.319	0.050	0.113
13	1.000	1.000	1.000	0.824	0.202	0.319	0.050	0.121
14	1.000	1.000	1.000	0.661	0.202	0.319	0.050	0.121
15	1.000	1.000	0.942	0.661	0.202	0.319	0.050	0.121
16	1.000	1.000	0.858	0.709	0.365	0.319	0.082	0.178
17	1.000	1.000	0.809	0.709	0.064	0.233	0.015	0.056
18	1.000	1.000	0.998	0.633	0.064	0.233	0.015	0.056
19	1.000	1.000	0.998	0.633	0.157	0.233	0.015	0.056
20	1.000	1.000	0.998	0.633	0.157	0.233	0.008	0.056
21	1.000	1.000	0.998	0.633	0.169	0.296	0.033	0.178
22	0.963	0.853	0.940	0.633	0.169	0.296	0.033	0.178
23	0.931	0.853	0.211	0.489	0.000	0.264	0.033	0.178
24	0.175	0.213	0.000	0.069	0.000	0.000	0.000	0.000

6.14 and Table 6.15. These results again show the discrepancy in computation time between the two approaches. Figure 6.2 plots the efficient frontier for the example. Any point along this curve corresponds to an optimal solution, which is the consequence of a specified level of risk aversion. Four κ-values are identified along the frontier, and the production schedules which provide these solutions are presented in Table 6.16. They exhibit the same properties that were previously noted in Section 6.5. The main conclusion being that expected profit and standard deviation are positively correlated, whereas output is negatively correlated. A decision maker will select, according to their preferences, the schedule that best balances profit and risk.

Table 6.14: Computational Results using SDP for the IESO Instance

Kappa	Exp. Profit ($)	Std. Deviation	Solution Time (s)	Nodes
0.001	67045	25011	34.76	12
1	66411	23987	22.98	8
2	59095	19473	31.36	10
3	32202	9597	130.82	44
4	24635	7366	224.69	70
5	5891	2837	165.05	62
6	5673	2798	152.52	58

Table 6.15: Computational Results using MIQP for the IESO Instance

Beta	Exp. Profit ($)	Std. Deviation	Solution Time (s)	Nodes
0	67045	25501	0.97	2
$2.085 \cdot 10^5$	66411	23987	0.74	1
$5.135 \cdot 10^5$	59095	19473	0.75	25
$1.563 \cdot 10^4$	32202	9597	0.92	150
$2.715 \cdot 10^4$	24636	7366	0.98	136
$8.8 \cdot 10^4$	5891	2837	0.93	99
0.001	5793	2818	5.78	44

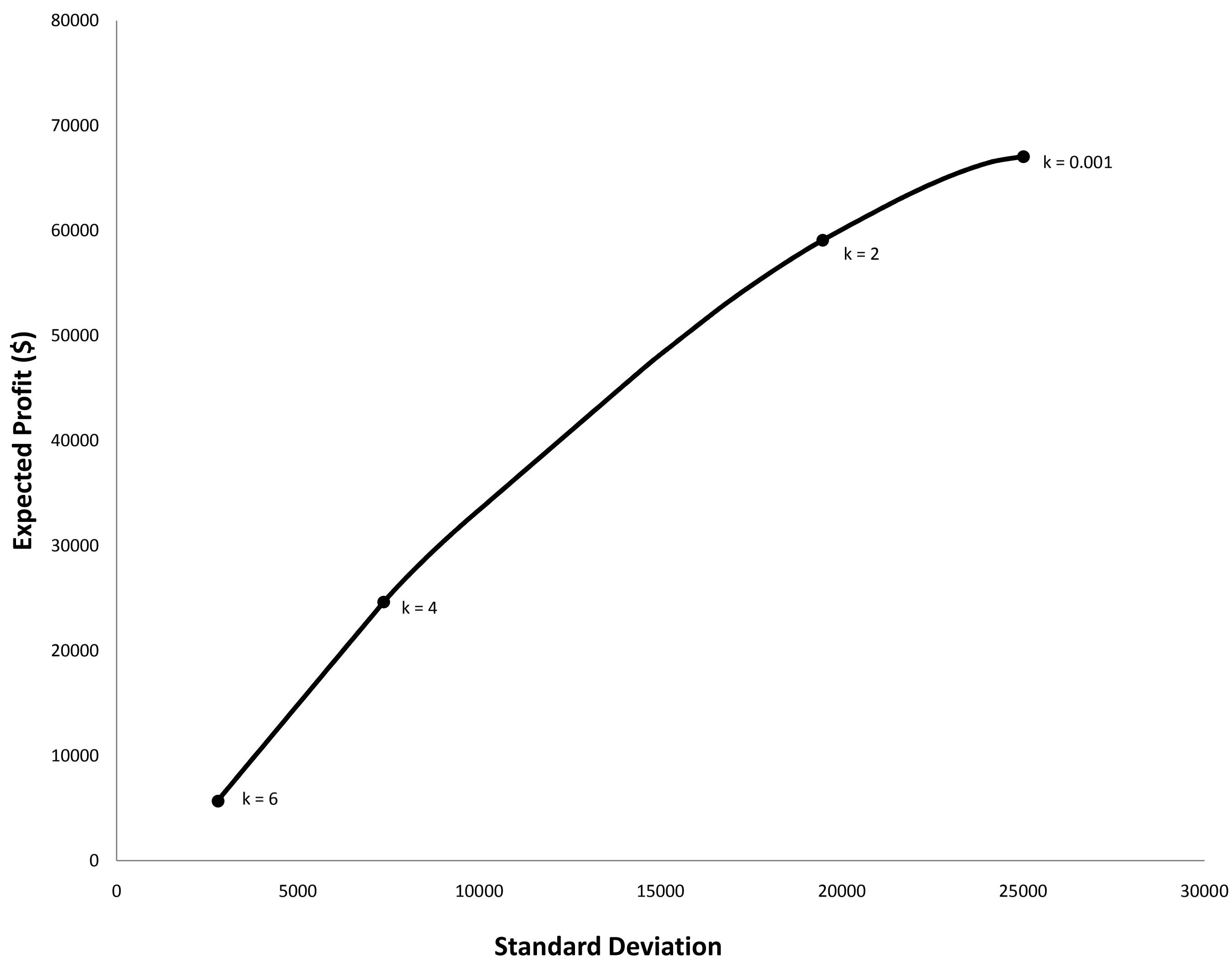

Figure 6.2: Efficient Frontier for the IESO Data

Table 6.16: Scheduling Differences for the IESO Instance

Hour	Power [MW]				Hour	Power [MW]			
	$\kappa = 0.001$	$\kappa = 2$	$\kappa = 4$	$\kappa = 6$		$\kappa = 0.001$	$\kappa = 2$	$\kappa = 4$	$\kappa = 6$
1	150	150	150	150	13	294	269	154	0
2	0	0	0	0	14	294	219	113	0
3	0	0	0	0	15	260	193	112	0
4	0	0	0	0	16	294	213	160	0
5	0	0	0	0	17	294	163	0	0
6	170	170	170	112	18	294	213	0	0
7	230	230	230	157	19	294	239	0	0
8	290	290	180	112	20	294	189	0	0
9	294	294	161	112	21	260	210	0	0
10	294	294	112	0	22	210	160	0	0
11	294	294	112	0	23	160	0	0	0
12	294	265	123	0	24	0	0	0	0

6.8 Summary of Results

In this chapter, the robust self-scheduling approach was compared to the mean-variance approach presented in [10]. The focus of the analysis was directed at the strength and solution times of the SDP relaxation, along with the characteristics of the optimal solutions and the computational efficiency of the branch-and-bound algorithm. The results show that despite producing identical self-schedules, the methods differ significantly in terms of the strength of the relaxations and their computational requirements.

At low levels of risk aversion, the SDP relaxation was found to produce significantly better bounds than the MIQP relaxation at the expense of an increase in solution times. Moreover, the relative errors in the objective function values of the SDP relaxation were generally more stable for all levels of risk aversion than the MIQP formulation. The strength of the SDP relaxation was highlighted by the fact that the unit commitment variables obtained were closer to being binary than those provided by the MIQP relaxation. The distinction was less evident at low uncertainty. In most cases, simply rounding the non-binary variables would produce high quality solutions for the SDP relaxation, while it was not necessarily the case for the MIQP relaxation.

An analysis of eight branch-and-bound configurations indicated that the lowest solution times resulted from the combination of selecting nodes based on the breadth-first strategy and branching on the most integer-infeasible variable. This configuration consistently outperformed the others. It was also found that the solution at the root node of the branch-and-bound tree could not be improved by adding cuts since none of the triangle inequalities were violated. The results indicated that the solution times for the SDP formulation were significantly higher than for the MIQP formulation despite having to solve fewer branch-and-bound nodes before terminating. It was also noted that increasing the number of periods had a drastic impact on solutions times and significantly deteriorated the quality of the relaxations.

In conclusion, the results indicate that both approaches produce the same production schedules at each profit level and consequently the same efficient frontiers. Furthermore, a comparison of the relaxations show that the robust approach produces solutions which are closer to integrality and have lower relative error than the mean-variance approach. However, the significantly higher solution times for the SDP formulation limit its potential within a branch-and-bound algorithm.

Chapter 7

Conclusions and Future Work

In this thesis, we present a robust optimization approach for solving the self-scheduling problem using semidefinite programming (SDP). The addition of a robust constraint introduces a profit term within the objective function which accounts for the risk associated with the uncertainty in electricity prices. As such, it provides decision makers with the ability to quantify the trade-off between profit and risk. In order to obtain integer-feasible solutions, we develop and implement a branch-and-bound algorithm. The proposed approach was tested using three problem instances and different values of κ. When compared to the mean-variance approach from the literature, it provides significantly tighter upper bounds for small values of κ. The quality of the relaxation is further highlighted by the degree to which the unit commitment variables adhere to the integer constraints. The main drawback of the proposed model is the high computation time requirements, which increases substantially as the number of periods increase.

Future work should incorporate bilateral contracts and emission caps into the model. The addition of these constraints would require aggregating the entire generation portfolio, as well as extending the market horizon. Furthermore, the framework could be extended to simultaneously consider markets for ancillary services, such as spinning reserves. Also, a more accurate representation of the time dependency of the start-up and shutdown costs is required. Finally, strategies for improving the efficiency of the branch-and-bound algorithm should be investigated. The computational time required to construct the constraint matrices could be reduced by programming the model using C code. Improvements would also be realized by incorporating warm starts instead of solving the problem from scratch at each node.

References

[1] K. Abhyankar, T. Fogarty, J. Kimber, A. Lin, S. Seo, and S. Takriti. Deterministic and stochastic models for the unit commitment problem. *IMA preprint series*, 1586(9):1–29, 1998. 27

[2] MF Anjos. Semidefinite Programming: Models, Algorithms and Computation. Course notes for MSCI 700, Fall 2008. 31, 32

[3] A. Ben-Tal and A. Nemirovski. Robust solutions of uncertain linear programs. *Operations Research Letters*, 25(1):1–14, 1999. 31

[4] D. Bertsimas and M. Sim. The price of robustness. *Operations Research*, pages 35–53, 2004. 30

[5] K. Bhattacharya, MHJ Bollen, and JE Daalder. *Operation of restructured power systems*. Springer, 2001. 4

[6] M. Carrion and JM Arroyo. A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem. *IEEE Transactions on power systems*, 21(3):1371–1378, 2006. 15, 17

[7] GW Chang, YD Tsai, CY Lai, and JS Chung. A practical mixed integer linear programming based approach for unit commitment. In *IEEE Power Engineering Society General Meeting, 2004*, pages 221–225, 2004.

[8] New Zealand Electricity Commission. About the Commission. URL: http://www.electricitycommission.govt.nz/aboutcommission. Retrieved: January 19, 2010. 4

[9] AJ Conejo, FJ Nogales, and JM Arroyo. Price-taker bidding strategy under price uncertainty. *IEEE transactions on power systems*, 17(4):1081–1088, 2002. 25

[10] AJ Conejo, FJ Nogales, JM Arroyo, and R. Garcia-Bertrand. Risk-constrained self-scheduling of a thermal power producer. *IEEE Transactions on Power Systems*, 19(3):1569–1574, 2004. 25, 26, 40, 41, 42, 43, 57

[11] GAMS Development Corporation. General Algebraic Modeling System. URL: http://www.gams.com/. Retrieved: October 12, 2009. 43

[12] S. de la Torre, J.M. Arroyo, A.J. Conejo, and J. Contreras. Price maker self-scheduling in a pool-based electricity market: a mixed-integer LP approach. *IEEE Transactions on Power Systems*, 17(4):1037–1042, 2002. 24

[13] A. Debs, C. Hansen, and YC Wu. Development of an electric energy market simulator. In BF Hobbs, MH Rothkopf, RP O'Neill, and H. Chao, editors, *The Next Generation of Electric Power Unit Commitment Models*. 4

[14] R. Fuentes-Loyola and VH Quintana. Medium-term hydrothermal coordination by semidefinite programming. *IEEE Transactions on Power Systems*, 18(4):1515–1522, 2003.

[15] C. Harris. *Electricity markets : pricing, structures and economics*. John Wiley, 2006. 5

[16] BE Hobbs. Linear complementarity models of Nash-Cournot competition in bilateral and POOLCO power markets. *IEEE Transactions on Power Systems*, 16(2):194–202, 2001. 24

[17] ILOG. CPLEX. URL: http://www.ilog.com/products/cplex/. Retrieved: October 12, 2009. 43

[18] RA Jabr. Robust self-scheduling under price uncertainty using conditional value-at-risk. *IEEE Transactions on Power Systems*, 20(4):1852–1858, 2005.

[19] SA Kazarlis, AG Bakirtzis, and V. Petridis. A genetic algorithm solution to the unit commitment problem. *IEEE Transactions on Power Systems*, 11(1):83–92, 1996.

[20] T. Li and M. Shahidehpour. Strategic Bidding of Transmission-Constrained GENCOs with Incomplete Information. *IEEE Transactions on Power Systems*, 20(1):437, 2005. 24

[21] PG Lowery. Generating unit commitment by dynamic programming. *IEEE Transactions on Power Apparatus and Systems*, pages 422–426, 1966.

[22] M. Madrigal and VH Quintana. Interior-point/cutting-plane algorithm to solve the dual unit commitment problem - on dual variables, duality gap, and cost recovery. In BF Hobbs, MH Rothkopf, RP O'Neill, and H. Chao, editors, *The Next Generation of Electric Power Unit Commitment Models*. 20

[23] M. Madrigal and VH Quintana. Semidefinite programming relaxations for {0, 1}-power dispatchproblems. In *IEEE Power Engineering Society Summer Meeting, 1999*, volume 2, 1999.

[24] M. Madrigal and VH Quintana. Semidefinite programming relaxations for {0, 1}-power dispatchproblems. Technical report, 2000. 18, 21, 22, 23, 32, 33

[25] AH Mantawy, YL Abdel-Magid, and SZ Selim. A simulated annealing algorithm for unit commitment. *IEEE Transactions on Power Systems*, 13(1):197–204, 1998. 13

[26] AH Mantawy, YL Abdel-Magid, and SZ Selim. Unit commitment by tabu search. *IEE Proceedings-Generation, Transmission and Distribution*, 145(1):56–64, 1998. 13

[27] MathWorks. MATLAB and Simulink for Technical Computing. URL: http://www.mathworks.com/. Retrieved: October 12, 2009. 43

[28] R.D.C. Monteiro. First-and second-order methods for semidefinite programming. *Mathematical Programming*, 97(1):209–244, 2003. 22

[29] CE Murillo-Sanchez and RJ Thomas. Thermal unit commitment with nonlinear AC power flow network model. In BF Hobbs, MH Rothkopf, RP O'Neill, and H. Chao, editors, *The Next Generation of Electric Power Unit Commitment Models*. 12

[30] RP O'Neill. Regulatory evolution, market design and unit commitment. In BF Hobbs, MH Rothkopf, RP O'Neill, and H. Chao, editors, *The Next Generation of Electric Power Unit Commitment Models*. 1, 4, 5

[31] Independent Energy System Operator. How the wholesale price is determined. URL: http://www.ieso.ca/imoweb/siteShared/wholesale$_price.asp?sid = ic.$6

[32] Independent Energy System Operator. Market data. URL: http://www.ieso.ca/imoweb/marketdata/marketData.asp. Retrieved: August 7, 2009. 41

[33] I.H. Osman and J.P. Kelly. *Meta-heuristics: theory & applications*. Kluwer Academic Pub, 1996. 13

[34] NP Padhy. Unit commitment-a bibliographical survey. *IEEE Transactions on power systems*, 19(2):1196–1205, 2004. 12, 14

[35] CK Pang and HC Chen. Optimal short-term thermal unit commitment. *IEEE Transactions on Power Apparatus and Systems*, 95(4 Part 1):1336–1346, 1976. 14

[36] CK Pang, GB Sheble, and F. Albuyeh. Evaluation of dynamic programming based methods and multiple area representation for thermal unit commitments. *IEEE Transactions on Power Apparatus and Systems*, pages 1212–1218, 1981. 15

[37] W.B. Powell. *Approximate Dynamic Programming: Solving the curses of dimensionality*. Wiley-Interscience, 2007.

[38] CW Jr. Richter and GB Sheble. Building and Evaluating Genco Bidding Strategies and Unit Commitment Schedules with Genetic Algorithms. In BF Hobbs, MH Rothkopf, RP O'Neill, and H. Chao, editors, *The Next Generation of Electric Power Unit Commitment Models*. 13

[39] G. Rothwell, T. Gomez, M. Schetzen, and TG San Roman. *Electricity economics: regulation and deregulation*. John Wiley, 2003. 1

[40] M. Shahidehpour, H. Yamin, and Z. Li. *Market operations in electric power systems: forecasting, scheduling, and risk management*. Wiley-IEEE Press, 2003. 24

[41] GB Shrestha, S. Kai, and L. Goel. An efficient stochastic self-scheduling technique for power producers in the deregulated power market. *Electric Power Systems Research*, 71(1):91–98, 2004. 27, 28

[42] W.L. Snyder, H.D. Powell, and J.C. Rayburn. Dynamic programming approach to unit commitment. *IEEE Transactions on Power Systems*, 2(2):339–348, 1987.

[43] AL Soyster. Convex programming with set-inclusive constraints and applications to inexact linear programming. *Operations Research*, pages 1154–1157, 1973. 31

[44] KC Toh, RH Tutuncu, and MJ Todd. On the implementation and usage of SDPT3 a Matlab software package for semidefinite-quadratic-linear programming, version 4.0. URL: http://www.math.nus.edu.sg/ mattohkc/guide4-0-draft.pdf. Retrieved: October 12, 2009. 43

[45] C. Triki, P. Beraldi, and G. Gross. Optimal capacity allocation in multi-auction electricity markets under uncertainty. *Computers and Operations Research*, 32(2):201–217, 2005.

[46] HY Yamin. Review on methods of generation scheduling in electric power systems. *Electric Power Systems Research*, 69(2-3):227–248, 2004. 12

[47] F. Zhuang and FD Galiana. Towards a more rigorous and practical unit commitment by Lagrangian relaxation. *IEEE Transactions on Power Systems*, 3(2):763–773, 1988. 20, 21

APPENDICES

Appendix A

Table A.1: Technical Characteristics of the Generating Unit

Minimum power[MW]	Maximum power[MW]	Start-up ramping limit[MW/h]	Shut-down ramping limit[MW/h]
112	294	170	160
Ramping-up limit[MW/h]	Ramping-down limit[MW/h]	Minimum up-time[hours]	Minimum down-time[hours]
60	50	4	4

Table A.2: Cost Characteristics of the Generating Unit

Fixed [$/h]	Linear [$/MWh]	Quadratic [$/MWh]	Start-up [$/start-up]	Shutdown [$/shutdown]
1150	18	0.035	1038	56

Table A.3: Expected Values of the Market Clearing Prices

Hour	Price [$/MWh]	Hour	Price [$/MWh]
1	33.31	13	41.05
2	26.53	14	41.61
3	22.16	15	38.98
4	23.10	16	39.74
5	22.60	17	42.02
6	23.15	18	42.09
7	24.65	19	40.74
8	24.75	20	38.80
9	25.50	21	39.63
10	27.58	22	46.14
11	31.60	23	39.04
12	35.60	24	33.68

Table A.4: Estimate of Covariance Matrix

Hour	1	2	3	4	5	6	7	8	9	10	11	12
1	1.60	-0.40	0.40	0.14	0.10	0.09	0.10	0.58	0.27	0.40	0.09	0.10
2	-0.40	0.37	0.08	0.00	0.05	0.05	0.06	-0.10	-0.05	-0.05	-0.12	0.01
3	0.40	0.08	0.61	0.08	0.20	0.16	0.08	0.37	0.07	0.18	-0.29	0.08
4	0.14	0.00	0.08	0.32	0.09	0.15	0.11	0.10	0.06	-0.01	-0.07	-0.05
5	0.10	0.05	0.20	0.09	0.25	0.15	0.23	0.43	0.13	0.07	-0.11	-0.10
6	0.09	0.05	0.16	0.15	0.15	0.19	0.31	0.32	0.18	0.10	-0.11	-0.05
7	0.10	0.06	0.08	0.11	0.23	0.31	1.49	1.18	0.85	0.16	-0.18	-0.19
8	0.58	-0.10	0.37	0.10	0.43	0.32	1.18	1.76	0.83	0.55	0.01	-0.11
9	0.27	-0.05	0.07	0.06	0.13	0.18	0.85	0.83	1.21	0.35	0.18	0.07
10	0.40	-0.05	0.18	-0.01	0.07	0.10	0.16	0.55	0.35	0.72	0.27	0.24
11	0.09	-0.12	-0.29	-0.07	-0.11	-0.11	-0.18	0.01	0.18	0.27	0.90	0.14
12	0.10	0.01	0.08	-0.05	-0.10	-0.05	-0.19	-0.11	0.07	0.24	0.14	0.42
13	0.27	-0.05	0.10	0.09	0.12	0.08	-0.20	0.09	0.11	0.24	0.27	-0.03
14	0.19	-0.08	0.12	-0.06	-0.07	-0.06	-0.17	0.09	-0.06	0.17	0.34	0.25
15	-0.14	0.04	0.15	0.09	0.00	-0.01	-0.40	-0.38	-0.29	-0.20	-0.11	-0.06
16	0.08	0.05	0.16	0.08	0.17	0.14	0.32	0.43	0.27	0.12	-0.20	0.01
17	0.23	0.02	0.29	0.08	0.19	0.13	0.09	0.39	0.10	0.15	0.00	-0.05
18	0.06	0.17	0.18	0.02	0.15	0.12	0.24	0.27	0.11	0.11	-0.08	0.00
19	0.19	-0.05	0.06	0.01	0.04	0.04	0.00	0.07	0.02	0.08	0.00	0.05
20	-0.16	0.07	0.03	0.00	0.01	-0.04	-0.29	-0.14	-0.15	-0.01	0.12	0.09
21	0.02	-0.20	-0.11	-0.04	-0.15	-0.17	-0.65	-0.56	-0.39	-0.20	0.03	-0.07
22	-0.15	0.02	-0.32	-0.02	-0.32	-0.28	-1.03	-1.03	-0.66	-0.05	0.44	0.33
23	-0.19	-0.15	-0.13	0.06	-0.10	-0.12	-0.71	-0.59	-0.33	-0.23	0.18	-0.09
24	-0.01	0.10	-0.10	-0.05	-0.15	-0.12	-0.26	-0.44	-0.25	-0.15	0.01	0.20

Hour	13	14	15	16	17	18	19	20	21	22	23	24
1	0.27	0.19	-0.14	0.08	0.23	0.06	0.19	-0.16	0.02	-0.15	-0.19	-0.01
2	-0.05	-0.08	0.04	0.05	0.02	0.17	-0.05	0.07	-0.20	0.02	-0.15	0.10
3	0.10	0.12	0.15	0.16	0.29	0.18	0.06	0.03	-0.11	-0.32	-0.13	-0.10
4	0.09	-0.06	0.09	0.08	0.08	0.02	0.01	0.00	-0.04	-0.02	0.06	-0.05
5	0.12	-0.07	0.00	0.17	0.19	0.15	0.04	0.01	-0.15	-0.32	-0.10	-0.15
6	0.08	-0.06	-0.01	0.14	0.13	0.12	0.04	-0.04	-0.17	-0.28	-0.12	-0.12
7	-0.20	-0.17	-0.40	0.32	0.09	0.24	0.00	-0.29	-0.65	-1.03	-0.71	-0.26
8	0.09	0.09	-0.38	0.43	0.39	0.27	0.07	-0.14	-0.56	-1.03	-0.59	-0.44
9	0.11	-0.06	-0.29	0.27	0.10	0.11	0.02	-0.15	-0.39	-0.66	-0.33	-0.25
10	0.24	0.17	-0.20	0.12	0.15	0.11	0.08	-0.01	-0.20	-0.05	-0.23	-0.15
11	0.27	0.34	-0.11	-0.20	0.00	-0.08	0.00	0.12	0.03	0.44	0.18	0.01
12	-0.03	0.25	-0.06	0.01	-0.05	0.00	0.05	0.09	-0.07	0.33	-0.09	0.20
13	0.48	-0.01	0.08	0.01	0.21	0.02	0.11	0.06	0.08	0.01	0.21	-0.15
14	-0.01	0.60	0.06	-0.04	0.03	-0.01	0.02	0.15	0.06	0.20	0.12	0.17
15	0.08	0.06	0.43	-0.07	0.11	-0.13	0.04	0.13	0.29	0.20	0.40	-0.05
16	0.01	-0.04	-0.07	0.39	0.15	0.19	0.09	0.08	-0.24	-0.22	-0.08	-0.04
17	0.21	0.03	0.11	0.15	0.31	0.07	0.11	0.08	-0.05	-0.12	0.05	-0.22
18	0.02	-0.01	-0.13	0.19	0.07	0.41	-0.07	0.05	-0.27	-0.19	-0.25	0.07
19	0.11	0.02	0.04	0.09	0.11	-0.07	0.21	0.02	0.00	0.01	0.04	-0.01
20	0.06	0.15	0.13	0.08	0.08	0.05	0.02	0.24	0.03	0.32	0.23	0.09
21	0.08	0.06	0.29	-0.24	-0.05	-0.27	0.00	0.03	0.66	0.35	0.56	-0.02
22	0.01	0.20	0.20	-0.22	-0.12	-0.19	0.01	0.32	0.35	1.71	0.45	0.26
23	0.21	0.12	0.40	-0.08	0.05	-0.25	0.04	0.23	0.56	0.45	0.84	-0.03
24	-0.15	0.17	-0.05	-0.04	-0.22	0.07	-0.01	0.09	-0.02	0.26	-0.03	0.60

Table A.5: IESO Hourly Ontario Energy Price

Date	Hour											
	1	2	3	4	5	6	7	8	9	10	11	12
01-Sep-08	28.47	4.45	2.37	4.10	6.53	16.59	15.03	27.73	40.77	43.00	64.87	76.41
02-Sep-08	23.59	4.38	4.14	31.94	39.42	41.32	43.62	46.73	61.98	68.78	67.58	110.35
03-Sep-08	39.53	21.17	12.99	28.07	36.86	42.47	44.50	60.63	77.29	83.62	89.55	95.08
04-Sep-08	35.17	33.82	30.15	26.72	24.37	41.64	37.83	43.49	52.72	59.35	71.21	79.92
05-Sep-08	36.09	37.87	35.84	37.70	39.57	41.17	58.65	69.47	60.32	79.33	76.06	71.50
08-Sep-08	36.18	25.84	28.09	31.09	39.36	41.02	39.10	51.42	68.48	82.40	107.40	85.69
09-Sep-08	34.88	40.95	34.08	39.07	40.91	56.73	77.53	64.36	48.38	51.03	68.31	68.61
10-Sep-08	35.08	4.47	3.59	4.63	32.49	28.24	50.57	46.97	43.05	41.09	57.13	46.48
11-Sep-08	23.03	6.54	3.60	4.48	37.12	35.61	34.72	39.29	59.49	75.61	66.63	67.46
12-Sep-08	7.40	4.63	22.24	4.48	13.78	47.50	48.08	44.73	61.51	68.25	67.07	65.62
15-Sep-08	31.37	9.71	28.31	15.58	11.38	36.92	38.71	63.17	69.82	86.17	80.66	50.21
16-Sep-08	4.84	4.59	5.48	4.58	8.84	31.63	21.84	35.25	38.17	30.13	35.55	35.80
17-Sep-08	6.87	4.75	5.13	4.73	12.99	38.86	35.36	41.11	42.06	46.98	57.62	85.04
18-Sep-08	28.29	32.89	35.34	35.07	31.46	39.17	36.36	42.04	44.06	74.08	59.14	43.63
19-Sep-08	9.91	4.94	4.95	4.66	16.70	40.84	44.69	47.09	56.13	64.80	65.34	133.63
22-Sep-08	5.47	6.33	7.87	6.05	36.56	41.11	41.17	43.25	47.02	42.16	42.13	53.32
23-Sep-08	31.80	4.88	4.90	2.77	14.79	40.08	38.10	40.93	42.91	44.17	43.48	61.71
24-Sep-08	8.66	3.61	3.79	24.35	26.58	40.96	39.47	41.73	41.19	41.70	42.72	52.10
25-Sep-08	5.12	4.81	6.44	5.33	9.84	42.55	39.24	56.63	67.87	56.45	49.14	56.03
26-Sep-08	39.81	34.40	34.06	32.93	31.57	41.51	38.26	51.82	56.30	45.77	53.17	50.91
29-Sep-08	4.94	17.68	4.38	1.81	27.22	48.77	49.98	62.16	53.82	57.27	55.27	68.31
30-Sep-08	11.26	33.04	15.84	11.27	20.27	45.59	54.79	117.28	90.94	71.95	80.44	62.72
01-Oct-08	18.50	16.33	3.74	5.88	23.58	42.89	46.84	44.89	46.35	45.26	44.92	70.58
02-Oct-08	35.33	5.20	5.07	32.68	30.55	58.73	40.08	42.25	43.53	47.49	45.32	53.29
03-Oct-08	34.56	15.17	3.65	20.62	38.23	40.12	46.25	46.65	46.72	49.96	48.00	45.01

	Hour											
	13	14	15	16	17	18	19	20	21	22	23	24
01-Sep-08	79.36	73.58	52.85	65.45	83.05	75.58	75.93	118.23	82.17	47.90	42.56	39.64
02-Sep-08	100.12	111.09	103.58	118.97	214.00	133.65	104.57	99.84	78.59	67.79	56.42	42.40
03-Sep-08	98.51	100.69	103.61	105.87	104.57	102.70	105.10	101.86	90.28	79.55	53.11	43.02
04-Sep-08	91.19	80.92	70.67	83.25	78.76	75.27	63.60	72.04	45.42	51.33	44.59	36.36
05-Sep-08	63.42	71.00	60.03	49.47	61.68	52.35	43.06	43.08	44.55	39.71	53.50	40.39
08-Sep-08	68.72	66.77	55.89	54.96	65.96	42.69	63.21	68.40	40.87	37.26	38.76	28.22
09-Sep-08	62.59	42.94	47.00	43.55	76.86	42.71	41.86	43.02	40.22	11.47	7.03	20.26
10-Sep-08	45.66	44.97	43.20	43.21	42.17	42.18	42.01	43.80	40.13	7.22	3.32	2.33
11-Sep-08	79.48	68.22	63.36	65.78	108.73	43.33	45.67	44.66	53.47	44.44	34.10	27.65
12-Sep-08	59.84	42.26	40.50	49.08	78.53	61.66	76.90	52.46	40.57	47.44	58.62	38.51
15-Sep-08	81.05	69.18	40.98	40.38	38.12	38.07	37.54	42.12	43.43	32.27	36.90	31.89
16-Sep-08	37.01	38.67	39.20	40.20	41.82	41.33	43.60	47.71	39.34	23.80	20.43	6.15
17-Sep-08	102.05	55.69	41.81	40.97	40.90	39.88	43.89	40.97	45.82	45.69	47.97	35.79
18-Sep-08	85.59	42.30	42.63	45.29	52.45	42.89	65.24	58.46	50.93	43.76	38.90	24.81
19-Sep-08	80.86	84.08	79.01	73.62	71.39	43.00	59.16	80.91	42.33	38.39	40.26	26.06
22-Sep-08	67.25	61.71	81.62	103.76	62.03	44.25	65.10	48.92	43.34	37.72	45.55	36.49
23-Sep-08	64.58	61.77	71.17	74.73	56.02	60.43	61.98	52.84	47.25	39.71	33.20	9.00
24-Sep-08	53.86	67.38	65.77	74.27	70.15	58.42	65.49	68.11	55.58	40.16	37.56	15.78
25-Sep-08	65.17	70.07	69.84	62.57	54.40	51.46	59.36	63.66	55.08	42.31	43.11	22.12
26-Sep-08	62.00	68.22	54.33	50.89	44.46	46.63	44.73	47.36	41.40	37.53	43.96	34.54
29-Sep-08	87.29	72.64	52.98	76.58	86.48	61.82	84.10	130.70	45.13	40.43	25.53	28.97
30-Sep-08	42.66	38.66	36.91	39.54	40.44	40.10	44.56	39.95	30.95	51.18	37.30	16.43
01-Oct-08	43.20	36.16	33.15	30.50	20.56	5.86	42.19	45.95	47.74	41.51	41.78	12.71
02-Oct-08	44.21	43.39	40.13	42.03	44.18	55.23	130.46	93.68	51.33	54.10	33.98	32.24
03-Oct-08	42.73	40.74	39.88	40.60	40.05	35.51	41.51	43.47	46.27	46.77	42.19	21.64

Table A.6: IESO Day-ahead Price Forecasts

	Hour											
Date	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
01-Sep-08	22.21	13.13	13.02	12.59	17.61	13.26	21.33	30.35	42.48	43.40	49.03	56.48
02-Sep-08	30.18	28.38	25.37	26.42	29.39	37.07	51.08	51.74	65.58	75.12	74.14	80.38
03-Sep-08	40.87	37.20	34.36	29.48	35.77	38.13	50.26	62.24	71.30	84.07	90.29	76.77
04-Sep-08	36.83	32.53	28.30	26.59	28.77	37.45	47.00	52.90	59.89	61.51	64.65	65.70
05-Sep-08	31.04	28.75	27.26	27.21	28.87	37.26	46.48	46.34	50.68	51.63	53.62	58.78
08-Sep-08	30.65	28.08	22.89	23.20	29.03	34.50	46.55	50.83	56.15	60.22	60.75	59.70
09-Sep-08	27.61	28.12	25.70	24.75	31.20	37.32	52.17	50.20	52.61	50.60	49.12	51.16
10-Sep-08	25.79	23.41	21.11	20.74	30.03	37.04	43.61	43.23	42.66	40.46	44.79	42.30
11-Sep-08	21.68	20.91	14.40	19.18	23.03	35.68	44.13	42.63	45.79	47.49	48.65	52.25
12-Sep-08	23.29	19.70	16.72	19.97	25.67	36.68	45.59	43.33	47.94	48.42	49.53	56.84
15-Sep-08	25.68	27.53	14.72	18.51	24.74	36.12	47.79	47.71	52.80	56.37	57.03	56.43
16-Sep-08	20.99	17.79	13.33	16.29	24.21	34.09	47.07	43.24	48.46	54.72	50.26	50.57
17-Sep-08	25.63	21.83	13.17	21.56	25.50	36.39	43.05	50.01	53.09	52.75	55.81	55.24
18-Sep-08	19.85	15.52	10.61	12.23	19.06	28.57	36.85	41.40	46.91	51.00	49.98	48.39
19-Sep-08	27.30	22.25	20.12	22.67	21.72	36.88	47.90	51.87	57.60	63.50	65.76	67.98
22-Sep-08	16.45	14.12	12.64	13.60	21.07	35.71	41.66	40.85	48.48	52.68	56.36	56.67
23-Sep-08	22.33	16.88	13.04	12.55	23.83	34.93	43.14	42.99	49.81	57.91	55.54	61.25
24-Sep-08	20.03	16.89	15.54	15.32	28.58	42.52	49.20	52.96	55.32	57.41	60.99	61.05
25-Sep-08	21.27	14.99	12.90	14.70	18.81	29.32	40.36	41.59	47.31	49.80	52.85	54.21
26-Sep-08	26.48	19.51	17.56	19.52	24.42	35.51	43.16	43.11	54.93	52.84	56.32	60.95
29-Sep-08	17.94	15.88	14.71	14.24	21.43	34.14	43.76	43.62	51.67	57.06	60.11	59.96
30-Sep-08	19.92	14.25	14.38	13.15	24.83	37.83	45.79	50.98	54.92	61.05	54.28	56.69
01-Oct-08	25.82	17.84	15.12	16.50	25.37	36.31	53.15	55.14	49.89	49.63	54.78	48.36
02-Oct-08	24.75	17.46	15.83	16.42	20.56	33.74	50.02	50.44	46.25	49.21	52.04	45.41
03-Oct-08	21.88	16.22	15.19	17.57	25.98	37.83	55.98	49.20	47.16	49.88	53.62	52.65
	Hour											
	13	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
01-Sep-08	60.05	72.91	74.02	77.40	82.93	60.87	48.29	48.12	54.61	43.95	41.69	25.10
02-Sep-08	100.47	104.08	105.87	99.22	97.37	91.17	91.41	86.92	80.33	58.80	57.15	44.93
03-Sep-08	90.53	95.07	89.77	84.12	76.78	80.95	80.45	75.01	72.62	56.02	47.20	41.29
04-Sep-08	75.45	71.74	73.32	64.67	64.90	63.79	66.89	66.55	54.17	40.71	39.61	39.61
05-Sep-08	60.86	59.21	48.97	52.21	51.63	40.81	47.44	46.54	41.04	40.08	37.87	37.61
08-Sep-08	68.58	60.28	59.43	57.49	56.23	53.34	49.90	52.94	43.61	32.60	32.14	32.71
09-Sep-08	55.23	52.06	47.23	44.78	43.38	37.30	44.52	48.11	38.00	31.40	33.60	32.00
10-Sep-08	48.13	47.29	39.49	39.91	35.73	34.60	42.32	44.17	34.89	27.65	27.96	22.83
11-Sep-08	59.30	53.58	49.14	44.92	51.11	45.28	51.64	58.68	43.81	34.64	30.24	22.97
12-Sep-08	57.68	49.90	42.40	44.45	46.50	33.75	42.47	45.71	38.50	32.23	32.05	23.73
15-Sep-08	62.68	53.85	50.61	50.09	51.04	46.72	51.56	59.36	43.96	33.65	31.36	30.00
16-Sep-08	54.92	47.92	43.14	46.45	44.93	38.35	46.57	51.96	41.25	32.35	32.45	27.45
17-Sep-08	62.14	59.39	54.18	54.45	55.68	52.19	57.18	61.32	49.97	34.80	30.15	21.66
18-Sep-08	54.57	43.53	38.19	42.47	48.73	38.59	47.33	49.36	37.93	32.82	28.20	22.27
19-Sep-08	69.31	58.47	50.35	52.96	58.69	42.85	51.13	56.16	46.14	33.87	29.93	31.77
22-Sep-08	63.82	54.49	45.01	51.06	59.03	46.94	52.26	59.43	45.76	33.49	30.78	21.72
23-Sep-08	65.78	59.60	57.77	57.22	64.35	50.76	56.47	59.65	45.82	36.28	33.66	23.29
24-Sep-08	71.87	68.68	60.19	60.46	62.85	53.98	62.55	65.69	47.64	36.04	33.90	23.22
25-Sep-08	63.87	58.69	52.50	55.93	60.53	46.65	62.73	59.98	42.82	35.82	31.32	26.15
26-Sep-08	64.78	59.57	53.16	54.02	57.18	41.42	53.03	50.48	40.03	32.09	31.47	25.37
29-Sep-08	67.87	57.87	49.16	51.20	56.89	48.57	57.90	62.22	45.59	34.96	31.20	30.23
30-Sep-08	57.43	48.22	44.85	45.98	50.71	50.24	70.50	56.07	40.76	36.44	33.63	29.24
01-Oct-08	49.32	48.97	40.41	51.43	43.23	49.79	47.04	48.93	35.43	32.44	30.62	23.68
02-Oct-08	46.58	38.60	35.99	46.38	42.74	44.64	48.00	45.60	33.74	34.01	27.77	23.77
03-Oct-08	47.28	39.89	36.18	52.15	44.41	44.39	50.67	47.17	42.22	38.46	34.49	23.82

Table A.7: Expected Values of the Market Clearing Prices (IESO)

Hour	Price [$/MWh]	Hour	Price [$/MWh]
1	21.88	13	47.28
2	16.22	14	39.89
3	15.19	15	36.18
4	17.57	16	52.15
5	25.98	17	44.41
6	37.83	18	44.39
7	55.98	19	50.67
8	49.2	20	47.17
9	47.16	21	42.22
10	49.88	22	38.46
11	53.62	23	34.49
12	52.65	24	23.82

Table A.8: Estimate of Covariance Matrix (IESO)

Hour	1	2	3	4	5	6	7	8	9	10	11	12
1	41.60	21.67	19.88	26.93	14.81	-3.34	10.73	0.80	-1.09	13.17	18.17	-24.27
2	21.67	74.07	49.46	37.29	14.60	-0.99	30.35	30.43	3.83	6.04	7.01	-46.82
3	19.88	49.46	60.86	31.61	4.41	0.21	16.83	19.16	8.55	28.59	24.29	-46.15
4	26.93	37.29	31.61	54.68	23.55	7.93	12.13	5.37	-5.76	7.04	5.49	-27.41
5	14.81	14.60	4.41	23.55	36.88	7.49	12.67	4.15	1.33	14.08	8.77	5.39
6	-3.34	-0.99	0.21	7.93	7.49	30.28	3.85	15.50	8.73	11.79	14.56	28.25
7	10.73	30.35	16.83	12.13	12.67	3.85	39.70	29.69	8.49	12.68	14.89	-5.10
8	0.80	30.43	19.16	5.37	4.15	15.50	29.69	93.17	50.90	37.08	46.57	-1.95
9	-1.09	3.83	8.55	-5.76	1.33	8.73	8.49	50.90	48.26	45.80	45.36	7.96
10	13.17	6.04	28.59	7.04	14.08	11.79	12.68	37.08	45.80	87.89	78.72	25.37
11	18.17	7.01	24.29	5.49	8.77	14.56	14.89	46.57	45.36	78.72	110.29	47.36
12	-24.27	-46.82	-46.15	-27.41	5.39	28.25	-5.10	-1.95	7.96	25.37	47.36	156.53
13	-2.00	-15.04	2.92	-12.04	1.47	16.51	-3.28	-5.49	0.74	37.02	33.56	54.55
14	-1.95	-18.00	-8.53	-5.71	8.40	6.91	-8.15	3.20	10.34	21.47	11.49	35.29
15	-10.94	-13.35	-14.35	-7.23	20.23	8.41	3.06	1.53	4.51	2.10	-18.35	19.06
16	-19.16	-31.50	-31.90	-14.45	24.25	12.15	-5.87	-4.82	-0.96	-7.23	-20.73	32.24
17	-20.39	-75.97	-67.63	1.16	50.57	32.56	-5.92	-8.12	8.60	30.15	31.36	123.45
18	-2.21	-32.87	-24.22	5.38	6.39	15.45	-1.67	-1.34	-1.41	-0.47	0.93	27.55
19	3.36	-39.22	-31.32	8.86	15.14	42.20	-15.91	-31.63	-8.49	3.33	-0.44	44.70
20	0.43	-28.43	-46.32	-4.98	2.41	37.26	-13.43	-12.81	-6.74	-3.76	12.31	76.69
21	7.88	-15.17	-15.20	0.12	1.62	12.11	-6.75	-8.03	1.45	8.54	5.33	14.75
22	-13.88	-16.34	-13.38	-2.71	0.33	17.91	-19.64	0.95	13.82	13.91	3.17	33.18
23	-18.36	-10.24	6.99	-5.61	-3.81	12.21	-15.34	1.22	16.21	27.07	9.08	25.89
24	-0.02	0.59	10.42	2.85	3.40	4.03	-1.24	-9.86	2.35	15.12	3.28	1.79

Hour	13	14	15	16	17	18	19	20	21	22	23	24
1	-2.00	-1.95	-10.94	-19.16	-20.39	-2.21	3.36	0.43	7.88	-13.88	-18.36	-0.02
2	-15.04	-18.00	-13.35	-31.50	-75.97	-32.87	-39.22	-28.43	-15.17	-16.34	-10.24	0.59
3	2.92	-8.53	-14.35	-31.90	-67.63	-24.22	-31.32	-46.32	-15.20	-13.38	6.99	10.42
4	-12.04	-5.71	-7.23	-14.45	1.16	5.38	8.86	-4.98	0.12	-2.71	-5.61	2.85
5	1.47	8.40	20.23	24.25	50.57	6.39	15.14	2.41	1.62	0.33	-3.81	3.40
6	16.51	6.91	8.41	12.15	32.56	15.45	42.20	37.26	12.11	17.91	12.21	4.03
7	-3.28	-8.15	3.06	-5.87	-5.92	-1.67	-15.91	-13.43	-6.75	-19.64	-15.34	-1.24
8	-5.49	3.20	1.53	-4.82	-8.12	-1.34	-31.63	-12.81	-8.03	0.95	1.22	-9.86
9	0.74	10.34	4.51	-0.96	8.60	-1.41	-8.49	-6.74	1.45	13.82	16.21	2.35
10	37.02	21.47	2.10	-7.23	30.15	-0.47	3.33	-3.76	8.54	13.91	27.07	15.12
11	33.56	11.49	-18.35	-20.73	31.36	0.93	-0.44	12.31	5.33	3.17	9.08	3.28
12	54.55	35.29	19.06	32.24	123.45	27.55	44.70	76.69	14.75	33.18	25.89	1.79
13	87.37	23.92	-0.65	15.03	35.08	11.25	20.12	38.16	15.99	21.54	22.85	27.52
14	23.92	38.20	26.26	34.67	40.65	17.53	14.06	28.72	3.13	13.61	16.16	7.48
15	-0.65	26.26	64.90	65.63	30.55	11.53	15.87	-8.49	0.62	9.15	13.83	-2.63
16	15.03	34.67	65.63	109.81	111.32	49.63	36.31	25.83	-1.11	22.87	12.54	3.04
17	35.08	40.65	30.55	111.32	416.08	151.29	82.33	79.97	14.20	36.56	-3.75	-1.36
18	11.25	17.53	11.53	49.63	151.29	106.65	68.13	71.94	14.94	18.34	1.93	8.80
19	20.12	14.06	15.87	36.31	82.33	68.13	174.45	155.08	46.91	44.43	22.61	27.29
20	38.16	28.72	-8.49	25.83	79.97	71.94	155.08	245.14	58.00	37.32	4.31	20.59
21	15.99	3.13	0.62	-1.11	14.20	14.94	46.91	58.00	38.64	14.37	6.16	8.74
22	21.54	13.61	9.15	22.87	36.56	18.34	44.43	37.32	14.37	51.07	42.90	20.11
23	22.85	16.16	13.83	12.54	-3.75	1.93	22.61	4.31	6.16	42.90	62.87	31.75
24	27.52	7.48	-2.63	3.04	-1.36	8.80	27.29	20.59	8.74	20.11	31.75	42.87