# The Problem of Stretching in Persian 

# Calligraphy and a New Type 3 PostScript 

## Nastaliq Font

by

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## AUTHOR'S DECLARATION

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.


#### Abstract

This research is about a typeface for implementing Persian calligraphy called Nastaliq. The main purpose for developing this font was to handle stretching of letters in order to achieve line justification through a dynamic font. Therefore, a PostScript Type 3 font was developed. However, as the research progressed, it came clear that Nastaliq's stretching cannot be implemented in a dynamic font. Therefore, the research's purpose changes to implementing a font containing all the needed glyphs of all needed stretchings of all stretchable letters to allow achieving line justification. For this propose a mathematical formulation to model handwritten Nastaliq was necessary. The result was a PostScript font containing more than 1200 glyphs. To make it possible to use this font in the future, a regular expression grammar was developed to identify and name each glyph as a positioned letter in a particular context. This thesis describes all the steps taken to build the font.


## Acknowledgements

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## Dedication

Hereby, I dedicate my thesis to my lovely wife and to my beloved parents who have supported me in every situation in my life.

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Table 1 - List of All Programs Developed to Make the PostScript Font Containing All Stretched Glyphs.

## Chapter 1

## Introduction

### 1.1 Persian Writing

The spread of computers around the world has resulted in the need for word-processing software in many languages. One of these languages is Persian, which is widely spoken in Iran, Afghanistan, Tajikistan, Uzbekistan, and Bahrain, and has a status of official language in the first three countries under different names [1]. The Persian alphabet has 32 letters and it is written from right to left. Some of the vowels are shown as letters, such as $,(0, u), 1(a)$, and $v(i, y)$ and some others are not written. Each Persian letter can have up to four different shapes according to the position it has in its word. These shapes are mainly known as: "Stand Alone", "Connecting Previous", "Connecting Following" and "Connecting Both". Some letters have fewer than four shapes. Figure 1shows all the four different shapes for letter $ب(\mathrm{Be})$ in different words. The fact that every word can have up to four different shapes makes Persian typography a big challenge.


Figure 1- Different shapes of letter $ب$ (Be) in different words shown within in the red rectangles. A) stand alone, B) connect previous, C) connect both, and D) connect following

Similar to other languages, Persian also has several typefaces. The regular Book-Typing typeface which is the type face already used in Figure 1, is written on a horizontal line. Other typefaces may differ.

### 1.2 Persian Calligraphy

The best known Persian typeface, which is also a kind of calligraphy, is Nastaliq. Nastaliq has some properties which differentiate it from other typefaces. Each letter has both a vertical and a horizontal shift from its neighbours; in most other languages, each letter is positioned either horizontally or vertically with respect to the previous one. Even in regular Persian book typefaces, we just see a horizontal shift. Figure 2 illustrates an instance of a Chinese Text. To write Chinese, traditionally letters are arranged in vertical columns, read from top to bottom down a column, and right to left across columns. As can be seen in this figure, each letter only has a vertical movement from its neighbors in the same column. Figure 3 shows a sample of a Hebrew text. Hebrew language is written on horizontal baselines, the same as English, but is from right to left. As can be seen in this figure, each letter has only a horizontal movement from its neighbors on the same baseline. Figure 4 illustrates an example of two words, written in Book-Typing typeface on the left and their Nastaliq versions. As shown in the figure the Book-type typeface is written on a horizontal baseline; However for the Nastaliq versions the letters have both vertical and horizontal movement. The rectangles in this figure show the first letters in each word and the circles show the last letter in the words. As can be easily seen each rectangle is positioned both vertically and horizontally different for the circle in the same letter, which means there exists both horizontal and vertical movements.

| 是 | － | 我 | 的 | 我 | 我 | 意 | 敬 | 你 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 我 | 檄 | 出 | 标 | 求 | 倾 | 龍 | 願 | 在 |
| 心 | 是 | 惡 | 見 | 你 | 需 | 教 | 你 | 天 |
| 所 | 你 | 兩 | 不 | 免 | 用 | 成 | 的 | 上 |
| 頃 | 的 | 的 | 要 | 我 | 的 | 就 | 圆 | 願 |
| 的 | 直 | 是 | 由 | 的 | 根 | 在 | 膆 | 你 |
| 啊 | 到 | 那 | 我 | 债 | 食 | 地 | 降 | 的 |
|  | 世 | 圆 | 入 | 照 | 求 | 如 | 臨 | 名 |
|  | 世 | 權 | 迷 | 我 | 你 | 在 | 願 | 兒 |
|  | 代 | 勢 | 頋 | 免 | 今 | 天 | 你 | 被 |
|  | 代 | 荣 | 你 | 人 | 日 |  | 的 | 人 |
|  | 這 | 耀 | 営 | 賽 | 給 | 坊 | 旨 | 尊 |

Figure 2－A sample of Chinese Text－Each Letter Has a Vertical Movement From Its Previous One on The Baseline．

$$
\begin{aligned}
& \text { הָתוֹרָה: }
\end{aligned}
$$

Figure 3－A sample of Hebrew Text－Each Letter Has a Vertical Movement From Its Previous One on The Baseline


Figure 4- A Sample of Two Words in Persian Written in Book-Typing Typeface on the Left Side and Nastaliq on the Right Side.

Another issue, that makes Nastaliq different from many typefaces but still similar to others, is stretching. The same as Arabic Naskh and Hebrew, some letters can be stretched in Persian Nastaliq. Figure 5 shows stretching in different languages such as Hebrew, Arabic Naskh and Nastaliq. This stretching can be used for several reasons; the most important one is to replace the extra space at the end of lines to achieve line adjustment, which is a common typography technique. In English this justification can be achieved by increasing the distance between the words in order to align the along the margins. This method does not work for Nastaliq, since in Nastaliq words should have a constant distance from each other.

As stretching could be implemented respecting several calligraphic rules, the main stage of this research was to prepare a set of rules for stretching in Persian Nastaliq. In addition, while

Nastaliq has up to four shapes for each letter, the shape of these shapes can change as a function of the letters before and after them. Therefore, a simple Persian Nastaliq typeface might contain hundreds of glyphs in order to be able to handle all possible letter combinations.
A)
ב
B)

$\square$

Figure 5- Different Examples of Stretching in A) Hebrew, B) Arabic Naskh, and C)

## Nastaliq

### 1.3 Problems with Automating Persian Writing and Persian Calligraphy

This work was originally focused on enhancing ditroff/ffortid [2] to be able to handle Persian Nastaliq calligraphy by using a technique for implementing stretching developed by Abdelouahad Bayar and Khalid Sami [3] and by using a dynamic font [4] to implement the stretching. The reason ditroff/ffortid was chosen as the main editor for typing Nastaliq is that, its
modular architecture described in 2.1.1. For example if we need to make changes to the program handling right to left typing, no other program should be touched. This makes it easier for the developers to add new feature to the editor.

For this purpose, we would need a font containing all the shapes of all glyphs with every possible stretching amount achieved dynamically during printing. For this purpose the user manual for a program called NasTroff was prepared to serve as a requirements specification. However as the research advanced, it became apparent that Nastaliq is much more complicated than other typefaces, and that a dynamic font could not be used to achieve the needed stretching. This complexity can be categorized into existence of both vertical and horizontal movement of the letters, existence of tens of different glyphs for each letter and at last, existence of exceptional cases for most of the glyphs. Therefore, the focus shifted to developing only a font with all stretching amounts of all shapes of all glyphs rather than also making the changes to ditroff /ffortid to handle this font. As a result the user's manual for NasTroff was left incomplete. However, the developed font was designed to meet the requirements of being used in NasTroff. Since the font is written in PostScript, it can be used even without any word-processor.

### 1.4 Notational Conventions

In order to make the content of this thesis easier to understand, here, I provide the list of notational conventions. Different fonts have been used to show different concepts.

- Times New Roman is used for the body of the article.
- Bold Arial is used for chapters and sections names.
- Cambria Italic is used for mathematical formulae and variables
- Palatino Linotype is used for showing rules in Persian Nastaliq.
- Comic Sans is used for the formal grammar of the regular languages.
- Calibri is used for Algorithms.
- Each Persian Letter is first shown in Persian, followed by its English name in parenthesis.
- For two points $K_{1}, K_{2}$, the notation $K_{1}<K_{2}$ means $K_{1}$ is located at the left side of $K_{2}, K_{1}>K_{2}$ means $K_{1}$ is located at the right side of $K_{2}$, and $K_{1}=K_{2}$ means $K_{1}$ is located in the same horizontal location as $K_{2}$. Also $K_{1}>=K_{2}$ means that $K_{1}$ is either located on the right side of $K_{2}$ or in the same horizontal location as $K_{2}$, and $K_{1}<=K_{2}$ means that $K_{1}$ either located on the left side of $K_{2}$ or in the same horizontal location as $K_{2}$.

The following chapters include Chapter 2 - Related and previous work on this subject, Chapter 3 - Rules and Instructions for Persian Calligraphy and Nastaliq,Chapter 4 - Research leading to this thesis, Chapter 5- Modeling the Stretching, Chapter 6 - The Nastaliq Font, and Chapter 7 - Future Work.

## Chapter 2

## Related and Previous Work

### 2.1 Formatting Bi-Directional Text

There are several programs that handle both right-to-left and left-to-right typesetting. We restrict our attention to programs that handle Arabic, Persian and related languages. Almost any program that can typeset Arabic can also typeset Persian. The following subsections describe programs for general bidirectional typesetting or for typesetting Persian.

### 2.1.1 Ditroff/ffortid

The bidirectional version of ditroff, ditroff/ffortid, was built in a modular manner by adding a post-processor, ffortid, to an unchanged ditroff[2]. ffortid is responsible for printing right-to-left text from right to left, while ditroff treats all text as if it were written from left to right. Because ditroff was not modified at all, all ditroff pre-processors and macro packages work for ditroff/ffortid. Moreover, since ffortid output looks like ditroff output, all ditroff postprocessors work for ditroff/ffortid. Johny Srouji and Daniel Berry describe an Arabic extension to ditroff/ffortid that can handle much of Persian as well [5]. The algorithm they use for reversing left to right text to achieve right to left text in ditroff/ffortid, is as given below.

To understand this algorithm better, Daniel Berry and Johny Srouji, first explain how ffortid works. The main job of ffortid is to reorganize the characters so that the text is in visual order, in which the text of each language is written in its own direction and the flow of the single-directional chunks in each line is consistent with the current document direction. The
current document direction is left to right or right to left as the user decides. To be more specific, current document direction is usually the direction of the main language used in the document. For example, typically, an English book is a left-to-right document even if it contains a lot of text in right-to-left text. ffortid works by totally reformatting each line as a function of the current document direction and the direction of each character. At the top level of abstraction, it reads the characters of each line, delimited by the end-of-line marker, and permutes characters so that the uni-directional chunks of a line flow in the current document direction while the characters in each chunk flow in the chunk's own internal direction.

```
for each line in the file do
        if the Current-Document-Direction is left to right then
            reverse each contiguous sequence of right-to-left characters in the line
        else /* (the current document direction is right to left)*/
            reverse the whole line;
            reverse each contiguous sequence of left-to-right characters in the line
    fi
    od
```


### 2.1.2 TEX/XET

The most ambitious $\mathrm{T}_{\mathrm{E}} \mathrm{X}$-based project aimed at formatting Arabic is the $\mathrm{T}_{\mathrm{E}} \mathrm{X} / \mathrm{X}_{\mathrm{E}} \mathrm{T}$ program developed by Pierre MacKay by modifying $\mathrm{T}_{\mathrm{E}} \mathrm{X}$ itself to be bidirectional [6]. The program does the same reversal of text in designated right-to-left fonts that ffortid does, but inside the modified $\mathrm{T}_{\mathrm{E}} \mathrm{X}$ using the internal data structures of the program rather than the dvi output. It assumes separate letters but does nothing about stretching.

### 2.1.3 Microsoft Word

The 2002 Microsoft Office system provided a right-to-left functionality and features for entering, editing and displaying right-to-left or combined right-to-left and left-to-right text [7]. Microsoft Word 2007 provides support for many more languages and has solved many of the bugs. On Windows XP, MirEmad [8] allows Microsoft Word to typeset Nastaliq, making Microsoft Word one of the few applications so far handling typesetting Nastaliq and bidirectional formatting at the same time. However, the amount of stretch is constant and there is just one stretched version of each letter.

### 2.1.4 Persian Only Programs

There are programs that can typeset Persian with several fonts. However, these applications do not also typeset left-to-right languages. Therefore, they are not bi-directional. NameNegar [9], and Maryam [10] are examples of this kind of program.

### 2.2 Keshideh

In Persian, stretching is commonly known as keshideh. Keshideh has different types as explained next.

### 2.2.1 Different Versions of Stretching

### 2.2.1.1 Stretching the Letters

This case occurs when we stretch the stand-alone and connecting-previous version of the
 $s(\mathrm{Kaf})$, and $⿶(\mathrm{Gaf})$.

### 2.2.1.2 Stretching the Connections

This case occurs when we have connected letters in a word. In this case, respecting the rules introduced in Chapter 3, we can stretch at most one connection in a word.

### 2.3 Pre-complied Variations and Dynamic Stretching

There are two ways considered in this research to implement stretching. First, in the dynamic fonts, each glyph is described by a parameterized procedure that draws the glyph's outline in a variation determined by the values of the parameters passed to the procedure. Notice that there can be an infinite number of versions for each glyph since each parameter can be any value. This technique is called dynamic stretching. The other solution is to construct a finite set of stretched versions for each letter. In this case, we can design all the stretched amounts of the letter and select the correct glyph.

### 2.4 Dynamic Font for stretching letters

Daniel Berry has implemented stretching of letters in Arabic, Persian, and Hebrew[2]. Although the Persian typeface he uses is different from Nastaliq, many parts of this work are
useful for stretching Nastaliq. The outline of any character is a series of elements, each of which is a line, a cubic Bézier curve, a circle, or an arc. Daniel Berry stretches the Bézier curves which do not affect the nature of the letter. Figure 6 illustrates the stretching of a cubic Bézier curve.


## Figure 6- Stretching a Four Point Curve Using the Method of Daniel Berry[2]

To stretch a 4 point Bézier curve by $A$ units, simply add $A$ to the $x$ values of the two righthand points and to all points to the right of the left-hand one of these.

It is a problem to stretch through the shared end point of two Bézier curves whose tangents are the same at the shared point. As shown in Figure 7, adding $A$ to the $x$ values to the three rightmost points of the right-hand curve in order to stretch between the first and second point introduces a corner into what was a smooth meeting at the shared point.



Figure 7- Cornered Shared End Point Caused by Stretching

As shown in Figure 8, the corner can be avoided by preserving the slope in the left-hand tangent of the right-hand curve by increasing both the $x$ and $y$ values by amounts consistent with the slope of the tangent. However, now the right end of the combined curve does not have the same $y$ value as before. A proper solution requires redesign of the two adjacent Bézier curves into one or three adjacent curves so that the place of stretch is in the middle of one cubic Bézier segment.


Figure 8-Fixing the Corner

One special case of stretching through a shared end point works, specifically when the tangents through the shared end points are completely horizontal, as suggested by Figure 9.


## Figure 9-Special Case, Horizontal Tangents

### 2.5 Problem with Daniel Berry's Solution

Although this kind of stretching works and provides a stretched letters, it still has some problems. As can be seen in Figure 10, Daniel Berry provides some various attempts to stretch the Stand-Alone Alif Maksura in Arabic which is the same as the letter Ye in Persian.


Figure 10 Various Attempts to Stretch the Stand-Alone Alif Maksura

Daniel Berry has manually made some different version of letter Alif Maksura in Arabic. As can be seen in Figure 10, the thickness of the horizontal part of the letter " "(Alif Maksura) shown in the right side of the figure, differs in all variations. These different looking versions of stretching are symptoms of a single fundamental problem described in the next section, namely
that Daniel Berry's implementation of stretching does not model the way stretching is done in hand-written Persian or Arabic writing.

### 2.6 Abdelouahad Bayar and Khalid Sami's Observations

Abdelouahad Bayar and Khalid Sami [3] describe the problem that Daniel Berry's method faces. Daniel Berry's system offers the possibility of stretching characters horizontally to a degree. However in Persian and Arabic calligraphy a vertical stretching might also be required.

In each of Persian and Arabic calligraphy, some parts of letters are written, directly with the nib head and other parts are drawn in a way that the contour is set first, with the right up corner of the nib head (the "Qalam's tooth") and afterwards, it is darkened with a brush[11, 12, 13]. The following three figures show an instance of letter Reh. The whole nib head is used for the black part, and a corner of the nib head is used for the other part.


Figure 11 - An Example of Different Uses of the Nib Head in One Letter


Figure 12- a) Sample Nib Head, b) The Stand-Alone $\lrcorner$ (Reh) Using the Nib Head, c) The Written Part of the Letter $\lrcorner($ Reh $)$, d) The Drawn Part of the Letter $\lrcorner$ (Reh)

The problem with Daniel Berry's method is illustrated in Figure 13 andFigure 14. The thickness of the stretched letter changes and does not respect the constant size of the nib head. In some places, it is more and in some places the thickness is less than the nib head. Thus Daniel Berry's method does not model hand-written Arabic writing.


Figure 13- Unstretched Stand-Alone ق (Qaf)


Figure 14-Long Stretching of the Stand-Alone ق (Qaf) in Daniel Berry's Method

In the Arabic Naskh style, the nib head behaves as a rectangle of length $l$ and width $l / 6[3]$. This rectangle moves with a constant inclination angle of about 70 degrees from the baseline. A nib's lead with $l=12 \mathrm{~mm}$ is shown in Figure 15:


## Figure 15-Nib Head in Naskh Style

Figure 16 shows how a correct stretching should look. As can be seen, the thickness of the stretched letter is constant and equal to the length of the nib head. Notice that this thickness is not calculated vertically, but along lines that have a 70 degree angle from the baseline.


Figure 16- A Sample of Correct Stretching in Arabic Naskh Respecting the Constant Thickness of the Nib Head

## Chapter 3

## Rules and Instructions for Persian Calligraphy and Nastaliq

### 3.1 Persian Nastaliq and Different rules for typesetting

In order to make a model for handwritten Nastaliq, it is necessary to provide a set of rules that should be respected. In this chapter, many rules have been gathered which would be useful in order to typeset Persian. It is important to know that there are also many other rules existing for Nastaliq, but the ones provided in this chapter are the only ones important for the purpose of this thesis. Another important point to mention is that the master calligraphers of Nastaliq have provided exceptional cases for each rule they have mentioned in order to make the text look better, and this responsibility for the judgment of the choosing what is more beautiful is given to the author. In order to make these rules applicable I had to make a set of strict rules and therefore, I simplified them to make them implementable in a software application.

### 3.2 Rules

These rules are derived from several instructions and tutorials [12, 13, 14, 15, 16].
Rule 1:

For regular lines, up to 3 stretchings in a line of text is always acceptable. More than 3 stretchings is possible at the writer's discretion to help keep balance in the line. Rule 2:

It is not desirable to have two adjacent stretched letters.

Rule 3:

The space between two adjacent letters that are not connected should be one dot.

Figure 17 shows the Stand-Alone letter $ب$ (Be), which contains a typical dot which is identified by a rectangle .

## SA_Be



Figure 17-Stand-Alone Letter ب(Be). The Typical Dot is Shown in the Rectangle.

Rule 4:

There are three reasons to stretch letters of which only the third is important for typesetting software, since the each of the first two needs a human to choose what is more beautiful and the second one needs a human to distinguish the letters by the meanings of the whole words containing them:

I - look more beautiful,
II- avoid confusing between similar words, and
III- justify lines and fill spaces

## Rule 5:

Different amounts of stretching are allowed by different master calligraphers. We adopt the convention that a letter can be stretched by an amount of from one to five dots.

Rule 6:

A letter stretched by an amount of two to five dots should not appear at the beginning or end of a sentence.

Rule 7:
No two stretchings in two neighboring lines should be located vertically close to each other. If there are no other choices, it is permitted to disobey this rule.

## Rule 8:

No stretches may appear at the beginning and end of a line. However, if there is no other choice and we really need to stretch, we can stretch in these situations. Rule 9:

Stretching is acceptable in the following eight conditions and not in other contexts:
a) $\operatorname{after}\rangle_{(\mathrm{He}) \text { or }}^{(\mathrm{He}) \text { as in } 1 人}$
b) after (Sad) or (Ta) as in



d) after (Fe or Ghe) or (Fe or Ghe) as in

e) after $(\mathrm{Mim})$ as in
f) after $\boldsymbol{\infty}$ (Ha) as in
g) after $\boldsymbol{\mathcal { ( H a } ) \text { as in }}$
h) $\quad$ after $\boldsymbol{( H a})$ as in , and




Rule 10:

The letters $س$ and cannot be stretched in the following conditions:
a) before $\sum_{(\mathrm{Jim}, \mathrm{Che}, \mathrm{He} \text { or Khe), }} \sum_{(\mathrm{Jim}, \mathrm{Che}, \mathrm{He} \text { or Khe), or }}^{\text {(Jim, Che, He or }}$ Khe-Re),
b) before another stretched version of the same letter,
c) before $\boldsymbol{\zeta}_{(\mathrm{Mim}),} \boldsymbol{\sigma}_{(\mathrm{Mim}-\mathrm{Alef}) \text {, or }}$,
d) before ${ }_{(\mathrm{Ha})}$,
e) before (Ye),

Rule 11:

In every line, it is suggested by Nastaliq Masters to have 1 stretched letter or two half-stretched ones, instead of having many stretched letters. A half stretched letter is a letter which is not stretched by the maximum permitted amount. Sometimes in order to make achieve line alignment, we may decide to put more than one stretched letters in it. Rule 12:

It is important to check each stretchable letter's location in its line before stretching it. When there is more than one stretchable letter in a line, it is required that there be some distance between every two stretched letters. To be more specific, divide each line into 5 equal-width sections from right to left. If one letter ends up in two sections, it must be considered as a member of the section that contains a larger portion
of the letter. If the letter is equally in two sections, it can be considered to be in either section.

If there is to be one stretched letter in a line, the best place for it to be is Section 3. If no letter in Section 3 is stretchable, the next candidate is Section 4, and if no letter in Section 4 is stretchable, the next candidates are Sections 2, 5 and 1, in that order.

If there are to be 2 stretched letters in a line, the preferred places are Sections 3 and 5. If neither section has a stretchable letter, or they have stretchable letters but some rules might be violated by stretching these letters, the next candidates are the couples of Sections 2 and 5, Sections 2 and 4, and Sections 4 and 1 in that order.

### 3.3 How to stretch a letter in Persian Nastaliq

As discussed in Section 2.2.1, there are two kinds of stretching in Persian. We can either stretch the connection between two letters or a letter itself respecting the rules introduced in Section Chapter 3. Note that the amount of stretching is a multiple of the width of a dot. Figure 18 shows a stand-alone version of letter $ب$ (Be), with a size of 5 dots. A dot is shown at the bottom of the letter in this figure. Note that for the font used in this research, the initial size of the unstretched version of many letters such as $ب(\mathrm{Be})$, $\quad(\mathrm{Pe}), \because(\mathrm{Te})$, and $ث(\mathrm{Se})$ is a little more than 5 dots. In order to calculate the exact size of a dot used in this font, I divided the total length of such a letter by 5 and calculated the exact amount of the dots used in this font which is 135 points in PostScript. Therefore, a letter's length might not look exactly equal to the size we
expect. Figure 19 also shows a stand-alone version of letter Be that is stretched by the amount of four dots.

## SA_Be



Figure 18- Unstretched Stand-Alone Letter (Be) with Size 5 Dots. The Typical Dot Can Be Seen in the Bottom of the Letter.

## SA_Be9



Figure 19- Stretched letter $ب$ (Be) of Figure 18 by the Amount of 4 Dots.

Use of the nib head in Persian Nastaliq, is not the same as in Arabic Naskh. There are two different styles of writing with the nib. First is called the weak style, which is seen mostly in the beginnings and the middles of some letters and in the ends of most of the letters like $ب(\mathrm{Be})$, $\jmath(\operatorname{Re}), \omega(\mathrm{Sin})$, and $\tau(\mathrm{He})$, in which not all of the nib head is placed on the paper. Only part of the nib is used to draw the weak part of the letter. The rectangle in Figure 20, show the usage of nib head in weak mode. As shown in Figure 20, the nib head is using the weak style everywhere inside the larger rectangle at top of the figure. In this area the nib head is larger than the drawn letter's width which means that not the whole head is used to draw some part of the letter. To achieve using just some part of the nib head, one places enough of the corner of the nib head on the paper and moves it on the paper. The smaller rectangles stands for the nib head.


Figure 20- Use of the Weak Style for Letter $\underset{\text { 〒 (Jim) - Shown within the Rectangle. }}{\text { - }}$

The second style is called the power style in which the entire nib- head is placed on the paper. There are also two modes for the angle of the nib head for this style. In solid mode which
 head's angle does not change, and stays almost the same in order to write the letters. However in practice it may rotate by 20 degrees. Figure 21 shows a version of letter $\because(\mathrm{Te})$ stretched by the amount of 3 dots. The section within the larger rectangle is drawn using the solid mode. The other mode is called rotate mode in which the nib head rotates as in letters $\boldsymbol{\text { ج (Jim), 飞 (Che), ح }}$ (He), $\dot{\boldsymbol{c}}$ (Khe), ق (Ghaf), and $(\mathrm{Ne})$. This rotation occurs to draw the curved shape of the letter. Figure 22 shows letter He. As shown in the figure, the nib head rotates up to 20 degrees.


Figure 21- Nib Head Using the Power Style and Solid Mode in Letter ت (Te)


Figure 22- Nib Head Using Power Style and Rotate Mode in Letter $₹$ (He).

Stretching a letter is done using the power style and in the solid mode and as for the Arabic Naskh style, the angle of the nib head is about 70 degrees from the horizontal axis. It is
also necessary to mention that in Nastaliq, the part of the letters that are written using rotate mode never get stretched, however this might happen in Arabic Naskh

## Chapter 4

## Research Leading to this Thesis

### 4.1 Original Goal:

Our main purpose was to modify ffortid in order that it could support Nastaliq writing, obeying calligraphic rules, with dynamic stretching, respecting both vertical and horizontal movements of letters.

### 4.2 Method to Achieve Goal

For this purpose I decided first to write a user's manual for a new version of ffortid called NAS that would handle the Nastaliq writing. This user's manual would serve as a requirements specification.

### 4.3 The Process of Preparing the User's Manual for NAS

It was first a little bit unusual for me to start with just writing the user's manual, but after many discussions with my supervisor, who played the customes and insisted on preparing the user's manual first, I decided to start with working on three issues:

- The rules for Persian Nastaliq
- The available Nastaliq fonts
- Reading and understanding the code for ffortid

First, in order to prepare the rules for Nastaliq, I had to find reliable sources. For this purpose, I took advantage of a trip I had made to Iran. I met Mr. Majid Hosseinzadeh, the Head of the Association of Calligraphers in Iran, and explained to him what my research is about and
asked him for the best available resources to find and learn rules for Nastaliq. He suggested that I buy a book written many years ago which is known as the bible of Nastaliq for calligraphers [11]. My problem was that this book was no longer sold in public stores for some political reason. At last, I found the book in Tehran's University Library and copied all the pages of that book. I also found and purchased all the other available books for learning Nastaliq in Iran. The next stage was to find out if any other software applications handled the features of Nastaliq that I was studying. The next stage was to find and purchase all the available Nastaliq typesetting software applications. By installing and using all of them I found out that none of them could handle the features I wanted to implement. The next step was to read the references I found and gather all the rules which would affect my software. I had also to find out which available font would best be useful for my purpose. As mentioned in Section 6.2, the IranNastaliq font was the one chosen to proceed with. As explained in the next section, I had to stop and rescope the whole idea of my research before starting to read the code for ffortid.

I learned the importance of writing some requirements document before starting to code. In this case, the user's manual served as my requirements document. If I had started coding before preparing the manual, I would have wasted a lot of time writing and changing code before I ended up learning that it was impossible to reach my goal. Therefore, spending about 6 months of working on gathering information for and preparing the user manual, saved me about one year of reading the entire ditroff/ffortid code, getting familiar with any already existing processors handling the features that I am interested in, and writing new classes and methods.

### 4.4 Rescoping the Research

In the middle of writing the manual, I realized that the goal could not be achieved in a time reasonable for a Masters thesis. I learned that stretching could not be achieved entirely by use of a dynamic font. The reason was that human's interaction was necessary for many glyphs in order to recognize which part of the glyph and which Bézier curves had to be stretched. Therefore, hundreds of particular instances of stretched letters would have to be pre-compiled into the font. As a result, I, with my supervisor's approval, rescoped my thesis to implementing only the font.

The development of a new ffortid processor was therefore abandoned and the project changed to that of developing a PostScript font with hundreds of glyphs, including up to five different amounts of stretching of several glyphs, a PostScript font that eventually could be used with a modified ffortid to be written in the future to work with this font.

The requirements of the new font are derived from the requirements of the modified ffortid. Note that it implements many but not all rules of Persian calligraphy and Nastaliq as mentioned in Section3.1.

## Chapter 5

## Modeling the stretching

It is necessary to describe the work of Abdelouahad Bayar and Khalid Sami[3] in detail, since it is the basis for the research of this thesis.

Abdelouahad Bayar and Khalid Sami first developed a model of the way the Persian characters are written by hand. They determined that the outlines have to follow the path determined by the four edges of the nib head. This means that the thickness of the letter along any axis parallel to the angle of the nib head must be equal to the height of the nib head. Therefore, a mathematical modeling is needed for the software to achieve the correct simulation of Nastaliq handwriting.

For example, In Figure 23 there are two curves $B_{1}$ and $B_{2}$, which is a translation of $B_{1}$ by vector $\vec{u}$.


Figure 23- Surface Rased with Edge $\boldsymbol{l}_{\mathbf{1}}$ Shown in Grey

In Figure 23, $B_{1}$ is a Bézier curve with 4 control points $M_{10}, M_{11}, M_{12}$, and $M_{13}$. Consider $B_{2}$, the Bézier curve with the four control points $M_{20}, M_{21}, M_{22}$, and $M_{23}$ such that:
$M_{2 i}=t \underset{u}{\rightarrow}\left(M_{1 i}\right), I \in\{0,1,2,3\}$ where $t \vec{u}$ is the translation of vector $\vec{u}$ such that $\vec{u}=$ $\overrightarrow{(l . \cos (\alpha), l . \sin (\alpha))}$

This thesis provides a review on Abdelouahad Bayar and Khalid Sami's work on modeling and supporting keshideh in Arabic calligraphy:

### 5.1 Keshideh, the Mathematical Model:

I use the notation $\left[M_{0}, M_{1}, M_{2}, M_{3}\right.$ ] to denote the Bézier curve with control points $M_{0}, M_{1}$, $M_{2}, M_{3}$.

Figure 24 shows the set $\beta_{1}$, the set of Bézier curves $\left[M_{0}, M_{1}, M_{2}, M_{3}\right]$ with an invariant concavity verifying:

$$
\overrightarrow{M_{2} M_{3}}=\lambda \vec{l}, 0 \leq \operatorname{Ang}\left(\overrightarrow{M_{2} M_{3}}, \vec{\imath}\right) \leq \frac{\pi}{2} \text {, Where } \vec{l} \text { is the axis } x \text { director vector and the }
$$ $\operatorname{Ang}(\vec{u}, \vec{v})$ function gives the angle between vectors $\vec{u}, \vec{v}$ in terms of parameters respecting the positive orientation of the coordinate system.



Figure 24-Curves of Type $1, \boldsymbol{\beta}_{1}$

Figure 25 shows the set $\beta_{2}$, the set of Bézier curves $\left[M_{0}, M_{1}, M_{2}, M_{3}\right]$ with an invariant concavity verifying:

$$
\overrightarrow{M_{0} M_{1}}=\lambda \vec{i}, 0 \leq \operatorname{Ang}\left(\overrightarrow{-\overrightarrow{-i}, M_{2} M_{3}}\right) \leq \frac{\pi}{2}
$$



Figure 25-Curves of Type $\mathbf{2}, \boldsymbol{\beta}_{\mathbf{2}}$

Keshideh is a juxtaposition of two Bézier curves, $B_{1}$ and from the set $\beta_{1}$ and $B_{2}$ from the set $\beta_{2}$. If $L_{0}, L_{1}, L_{2}, L_{3}$ are control points of $B_{1}$ and $R_{0}, R_{1}, R_{2}, R_{3}$ are the control points of $B_{2}$ then $L_{3}$ and $R_{0}$ are equal. So, the definition of $E_{b e}$ that stretches the curves in $\beta_{1}$ and $E_{a f}$ that stretches the curves in $\beta_{2}$ are:
$E_{b e}: \beta_{1} \times\left[0, h_{m}\right] \times\left[0, v_{m}\right] \rightarrow \beta_{1}$
$(B, h, v) \rightarrow E_{b e}(B, h, v)$
Note that $h$ represents the horizontal stretching amount and let $v$ represents the vertical stretching amount.

The transformation $E_{b e}$ stretches curves in $\beta_{1}$. The details of its definition are given below:

Let $B_{1}=\left[M_{10}, M_{11}, M_{12}, M_{13}\right]$ and $B_{2}=\left[M_{20}, M_{21}, M_{22}, M_{23}\right]$ be two curves in $\beta_{1}$. Let $(h, v) \in\left[0, h_{m}\right] \times\left[0, v_{m}\right]$.

### 5.1.1 $E_{b e}$ Transformation:

$B_{2}=E_{b e}\left(B_{1}, h, v\right)$ if and only if the control points of $B_{2}$ are:

$$
\begin{aligned}
& M_{20}=M_{10}-(h, 0) \\
& M_{23}=M_{13}-(0, v) \\
& M_{21}=\left(1-c_{1}\right) M_{20}+c_{1} J \\
& M_{22}=\left(1-c_{2}\right) J+c_{2} M_{23},
\end{aligned}
$$

with $c_{1}, c_{2}$ satisfying :

$$
\begin{aligned}
& M_{11}=\left(1-c_{1}\right) M_{10}+c_{1} J \\
& M_{12}=\left(1-c_{2}\right) I+c_{2} M_{13},
\end{aligned}
$$

where : $\{I\}=\left(M_{10} M_{11}\right) \cap\left(M_{12} M_{13}\right)$
and $\{J\}=\Delta_{1} \cap \Delta_{2}$ with:
$\Delta_{1}$ being the parallel to $\left(M_{10} M_{11}\right)$ passing through the point $M_{20}$ and $\Delta_{2}$ being the parallel to ( $M_{12} M_{13}$ ) passing through the point $M_{23}$.

An example of a stretching using the function $E_{b e}$ is presented in Figure 26.


Figure 26- Stretching a Curve Belonging to the Set $\beta_{1}$ with $E_{b e}\left(B_{1}\right)=B_{2}$

### 5.1.2 $E_{a f}$ Transformation:

Let $E_{a f}$ be the stretching function defined as follows:
$E_{a f}: \beta_{2} \times\left[0, h_{m}\right] \times\left[0, v_{m}\right] \rightarrow \beta_{2}$
$(B, h, v) \rightarrow E_{a f}(B, h, v)$
The transformation $E_{a f}$ stretches curves in $\beta_{2}$. The details of its definition are given below:

Let $B_{1}=\left[M_{10}, M_{11}, M_{12}, M_{13}\right]$ and $B_{2}=\left[M_{20}, M_{21}, M_{22}, M_{23}\right]$ be two curves in $\beta_{2}$. Let $(h, v) \epsilon\left[0, h_{m}\right] \times\left[0, v_{m}\right]$.
$B_{2}=E_{a f}\left(B_{1}, h, v\right)$ if and only if the control points of $B_{2}$ are:
$M_{20}=M_{10}-(h, v)$
$M_{23}=M_{13}$
$M_{21}=\left(1-c_{1}\right) M_{20}+c_{1} J$
$M_{22}=\left(1-c_{2}\right) J+c_{2} M_{23}$,
with $c_{1}, c_{2}$ satisfying:
$M_{11}=\left(1-c_{1}\right) M_{10}+c_{1} J$
$M_{12}=\left(1-c_{2}\right) I+c_{2} M_{13}$,
where: $\{I\}=\left(M_{10} M_{11}\right) \cap\left(M_{12} M_{13}\right)$
and $\{J\}=\Delta_{1} \cap \Delta_{2}$ with:
$\Delta_{1}$ being the parallel to $\left(M_{10} M_{11}\right)$ passing through the point $M_{20}$ and $\Delta_{2}$ being the parallel to $\left(M_{12} M_{13}\right)$ passing through the point $M_{23}$.

An example of $E_{a f}$ stretching is given in Figure 27:


Figure 27 -Stretching a Curve Belonging to the Set $\beta_{2}$ with $E_{a f}\left(B_{1}\right)=B_{2}$

Abdelouahad Bayar and Khalid Sami divide the total stretch value of a letters into two parts and pass each half to one of the two functions $E_{a f}$, and $E_{b e}$.

### 5.2 Stretching a Letter in Persian Nastaliq

Abdelouahad Bayar and Khalid Sami talk about how to stretch a Bézier curve with its control points and an amount of stretch so that we respect the constant size of the nib head. For a quick review, Assume that $B_{1}$ is a Bézier curve with four control points $M_{10,} M_{11}, M_{12}, M_{13}$.

They provide a formula which gives us $B_{2}$ which is a curve respecting the constant size of the nib head.

Given four initial control point $M_{10}, M_{11}, M_{12}$ and $\mathrm{M}_{13}$, we can get the stretched curve's control points.
$M_{20}=M_{10}-(h, 0)$
$M_{23}=M_{13}-(0, v)$
$M_{21}=\left(1-c_{1}\right) M_{20}+c_{1} J$
$M_{22}=\left(1-c_{2}\right) J+c_{2} M_{23}$,
with $c_{1}, c_{2}$ satisfying:
$M_{11}=\left(1-c_{1}\right) M_{10}+c_{1} J$
$M_{12}=\left(1-c_{2}\right) I+c_{2} M_{13}$,
where: $\{I\}=\left(M_{10} M_{11}\right) \cap\left(M_{12} M_{13}\right)$
and $\{J\}=\Delta_{1} \cap \Delta_{2}$ with:
$\Delta_{1}$ being the parallel to ( $M_{10} M_{11}$ ) passing through the point $M_{20}$ and $\Delta_{2}$ being the parallel to
( $M_{12} M_{13}$ ) passing through the point $M_{23}$.
They conjectured that the new curve, with its four Bézier control points, respects the spacing issue of respecting the constant size of the nib head between the curves in letters or in other words, if we stretch the curves in the original glyph using the transformations they have provided, the resulting glyph containing the new stretched curves would have the same thickness as the original one. The proof of this conjecture was not supplied by Abdelouahad Bayar and Khalid Sami. So, I provide this proof as a part of the research for this thesis. Mona Mojdeh has also derived an independent proof that is not presented here. To prove this claim, assume there are two curves spaced $l$ from each other along the line having a 70 degree angle from the baseline. It is necessary show that after applying the formula, the resulting curve's control points are spaced the same distance.

Step 1) Simplifying the formula
Assume that current curves are made with control points [ $M_{10}, M_{11}, M_{12}, M_{13}$ ] and $\left[M_{20}, M_{21}, M_{22}, M_{23}\right]$. Denote the variables for the shifted version of the curve with a $t$ notation at superscripted at the top of its metric variable. So $M_{20}=M_{10}^{t}, M_{21}=M_{11}^{t}, M_{22}=$ $M_{12}^{t}, M_{23}=M_{13}^{t}$

By considering just the $x$ value of each $M_{i j}$ we get:

$$
\begin{aligned}
& X_{11}=\left(1-C_{1}\right) X_{10}+C_{1} I_{x} \\
& X_{12}=\left(1-C_{2}\right) I_{x}+C_{2} X_{13} \\
& C_{1}=\frac{X_{11}-X_{10}}{I_{x}-X_{10}}
\end{aligned}
$$

$$
\begin{aligned}
& C_{2}=\frac{X_{12}-I_{x}}{X_{13}-I_{x}} \\
& M_{11}=\left(1-\frac{X_{11}-X_{10}}{I_{x}-X_{10}}\right)\left(X_{10}-h, Y_{10}\right)+\left(\frac{X_{11}-X_{10}}{I_{x}-X_{10}}\right)\left(I_{x}, I_{y}\right) \\
& M_{12}=\left(1-\frac{X_{12}-I_{x}}{X_{13}-I_{x}}\right)\left(I_{x}, I_{y}\right)+\frac{X_{12}-I_{x}}{X_{13}-I_{x}} \cdot\left(X_{13}, Y_{13}\right)
\end{aligned}
$$

Step 2) Calculating $I$ for the shifted curve
Denote again that the variables for the shifted version of the curve with a t notation at superscripted at the top of its metric variable. So, For $I_{x}^{t}$ : Translate every $x$ to $x+I \cdot \cos (\alpha)$ and $y$ to $y+1 \cdot \sin (\alpha)$. So we get:

$$
I_{X^{\prime}}^{t}=\frac{\left(X_{10}+l \cdot \cos (\alpha)\right) \frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}\left(X_{12}+l . \cos (\alpha)\right) \frac{Y_{13}-Y_{12}}{X_{13}-X_{12}} Y_{10}+Y_{12}}{\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}-\frac{Y_{13}-Y_{12}}{X_{13}-X_{12}}}=l \cdot \cos (\alpha)+I_{X}
$$

With the same result, we can get: $I_{y}^{t}=I_{y} \cdot \sin (\alpha)+I_{x}$
Step 3) Calculating the metrics for the stretched curve, Denote that single apostrophe stands for the stretched version.

$$
\begin{aligned}
& M_{10^{\prime}}=M_{10}-(h, 0) \\
& M_{13^{\prime}}=M_{13}-(0, v) \\
& M_{11^{\prime}}=\left(1-C_{1}\right) M_{10^{\prime}}+C_{1} J \\
& M_{12^{\prime}}=\left(1-C_{2}\right) J+C_{2} M_{13^{\prime}}
\end{aligned}
$$

Presenting the control points of the shifted curve by vector $\vec{U}=(I \cdot \cos (\alpha), I \cdot \sin (\alpha))$ with $X^{t}$ and $Y t$, we know that:

$$
\begin{array}{ll}
X_{11}=X_{11}+1 \cdot \cos (\alpha) & X_{12}=X_{12}+1 \cdot \cos (\alpha) \\
Y_{11}=Y_{11}+1 \cdot \sin (\alpha) & Y_{11}=Y_{11}+1 \cdot \sin (\alpha)
\end{array}
$$

$$
\begin{array}{ll}
X_{12}=X_{12}+1 \cdot \cos (\alpha) & X_{13}=X_{13}+1 \cdot \cos (\alpha) \\
Y_{12}=Y_{12}+1 \cdot \sin (\alpha) & Y_{13}=Y_{13}+1 \cdot \sin (\alpha) \\
M_{10}=\left(X_{10}, Y_{10}\right), M_{11}=\left(X_{11}, Y_{11}\right), M_{12}=\left(X_{12}, Y_{12}\right), M_{13}=\left(X_{13}, Y_{13}\right) \\
M_{20}=\left(X_{10}+1 \cdot \cos (\alpha), Y_{10}+1 \cdot \sin (\alpha)\right), M_{21}=\left(X_{11}+1 \cdot \cos (\alpha), Y_{11}+1 \cdot \sin (\alpha)\right) \\
M_{22}=\left(X_{12}+1 \cdot \cos (\alpha), Y_{12}+1 \cdot \sin (\alpha)\right), M_{23}=\left(X_{13}+1 \cdot \cos (\alpha), Y_{13}+1 \cdot \sin (\alpha)\right)
\end{array}
$$

Step 4) Calculating Metrics for the stretched curves. Note again that $X_{I}$ means the $x$ value of $I$ and $Y_{I}$ means the $y$ value of $I$.

Now the stretched version of curve 1 is (remind that in the following formulae, a single apostrophe stands for a new stretched point):

$$
\begin{aligned}
& M_{10}^{\prime}=\left(X_{10}-h, Y_{10}\right) \\
& M_{13}^{\prime}=\left(X_{13}, Y_{10}-V\right)
\end{aligned}
$$

The line from $\mathrm{M}_{10}$ to $\mathrm{M}_{11}$ is: $\frac{y-Y_{10}}{x-X_{10}}=\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}$
The line from $\mathrm{M}_{12}$ to $\mathrm{M}_{13}$ is: $\frac{y-Y_{12}}{x-X_{12}}=\frac{Y_{13}-Y_{12}}{X_{13}-X_{12}}$

$$
\begin{aligned}
& \{I\}=\xrightarrow[M_{10} M_{11}]{\longrightarrow} \cap \xrightarrow[M_{12} M_{13}]{ } \\
\Rightarrow & \frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}\left(x-X_{10}\right)+Y_{10}=\frac{Y_{13}-Y_{12}}{X_{13}-X_{12}}\left(x-X_{12}\right)+y 12 \\
\Rightarrow & \quad X_{I}=\frac{\left(X_{10} 0 \frac{Y_{11}-Y_{10}}{X_{11}-X_{10}-X_{12} Y_{13}-Y_{12}}-Y_{10}+Y_{12}\right)}{\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}-\frac{Y_{13}-Y_{12}}{X_{13}-X_{12}}} \\
\Rightarrow & Y_{I}=\left(\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}\right)\left(\frac{\left(X_{10} \frac{Y_{11}-Y_{10}}{X_{11}-X_{10}-X_{12} Y_{13}-Y_{12}} Y_{12}+Y_{10}\right)}{\frac{Y_{11}-Y_{10}-X Y_{13}-Y_{12}}{X_{11}-X_{10}} \frac{Y_{13}-X_{12}}{X_{12}}}-X_{10}\right)+Y_{10}
\end{aligned}
$$

$\{J\}=\Delta_{1} \cap \Delta_{1}$ with:
$\Delta_{1}$ :being the parallel to $\left(M_{10} M_{11}\right)$ passing through the point $M_{20}$.
$\Delta_{2}$ : being the parallel to $\left(M_{12} M_{13}\right)$ passing through the point $M_{23}$.

$$
\begin{aligned}
& \Rightarrow \frac{y-Y_{10}}{x-X_{10}+h}=\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}} \\
& \& \frac{y-Y_{13}+v}{x-X_{13}}=\frac{Y_{13}-Y_{12}}{X_{13}-X_{12}} \\
& \Rightarrow \quad X_{J}=\frac{\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}\left(X_{10}+h\right)-Y_{10}-\frac{Y_{13}-Y_{12}}{X_{13}-X_{12}} X_{13}+Y_{13}-v}{\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}-\frac{Y_{13}-Y_{12}}{X_{13}-X_{12}}} \\
& \& Y_{J}=\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}\left(\frac{\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}\left(X_{10}+h\right)-Y_{10}-\frac{Y_{13}-Y_{12}}{X_{13}-X_{12}} X_{13}+Y_{13}-v}{\frac{Y_{11} 1-Y_{10}}{X_{11}-X_{10}}-\frac{Y_{13}-Y_{12}}{X_{13}-X_{12}}}-X_{10}+h\right)+Y_{10}
\end{aligned}
$$

Step 5) Calculating Metrics for the stretched version of shifted curve. Note again that $X_{I}$ means the $x$ value of $I$ and $Y_{I}$ means the $y$ value of $I$.

Now we shift the result curve with the space equal to $l$ and angle $\alpha$, and replace $X_{1 i}$ by $X^{t}{ }_{1 i}$ to get the metrics of the shifted curve with $\vec{U}$.

$$
\begin{aligned}
& X_{11}^{*}=X_{11}+l \cdot \cos (\alpha) \quad X_{12}^{*}=X_{12}+l \cdot \cos (\alpha) \quad Y_{11}^{*}=Y_{11}+l \cdot \sin (\alpha) \quad Y_{11}^{*}=Y_{11}+l \cdot \sin (\alpha) \\
& X_{12}^{*}=X_{12}+l \cdot \cos (\alpha) \quad X_{13}^{*}=X_{13}+l \cdot \cos (\alpha) \quad Y_{12}^{*}=Y_{12}+l \cdot \sin (\alpha) \quad Y_{13}^{*}=Y_{13}+l . \sin (\alpha)
\end{aligned}
$$

The resulting $X_{J}^{t}$ will be:

$$
\begin{aligned}
& X_{J}^{t}= \\
& \frac{\frac{Y_{11}+l \cdot \sin (\alpha)-Y_{10}-l \cdot \sin (\alpha)}{X_{11}+l \cdot \cos (\alpha)-X_{10}-l \cdot \cos (\alpha)}\left(X_{10}+l \cdot \cos (\alpha)+h\right)-\left(Y_{10}+l \cdot \sin (\alpha)\right)-\frac{Y_{13}+l \cdot \sin (\alpha)-Y_{12}-l \cdot \sin (\alpha)}{X_{13}+l \cdot \cos (\alpha)-X_{12}-l \cdot \cos (\alpha)} \cdot\left(X_{13}+l \cdot \cos (\alpha)\right)+Y_{13}+l \cdot \sin (\alpha)-v}{\frac{Y_{11}+l \cdot \sin (\alpha)-Y_{10}-l \cdot \sin (\alpha)}{X_{11}+l \cdot \cos (\alpha)-X_{10}-l \cdot \cos (\alpha)}-\frac{\left.Y_{13}+l \cdot \sin (\alpha)-1\right)_{12}-l \cdot \sin (\alpha)}{X_{13}+l \cdot \cos (\alpha)-X_{12}-l \cdot \cos (\alpha)}} \\
& \quad=X_{J}+l \cdot \cos (\alpha)
\end{aligned}
$$

And the resulting $Y^{t}$ will be:

$$
\begin{aligned}
& \quad \text { Remember: } Y_{J}=\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}\left(\frac{\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}\left(X_{10}+h\right)-Y_{10}-\frac{Y_{13}-Y_{12}}{X_{13}-X_{12}} X_{13}+Y_{13}-v}{\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}-Y_{130}-Y_{12}} X_{10}+h\right)+Y_{10} \\
& \quad=\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}\left(X_{J}-X_{10}+h\right)+Y_{10} \\
& \Rightarrow \quad Y_{J}^{t}=\frac{Y_{11}+l \cdot \sin (\alpha)-Y_{10}-l \cdot \sin (\alpha)}{X_{11}++l \cdot \cos (\alpha)-X_{10}-+l \cdot \cos (\alpha)}\left(X_{J}+l \cdot \cos (\alpha)-X_{10}-+l \cdot \cos (\alpha)+h\right)+Y_{10}+l \\
& \quad \cdot \sin (\alpha)=Y_{J}+l \cdot \sin (\alpha)
\end{aligned}
$$

The new $X_{I}^{t}$ will be:
$X_{I}=$

$$
\begin{aligned}
& \frac{\left(\left(X_{10}+l \cdot \cos (\alpha) \frac{Y_{11}+l . \sin (\alpha)-Y_{10}-l \cdot \sin (\alpha)}{X_{11}+l \cdot \cos (\alpha)-X_{10}-l . \cos (\alpha)}-\left(X_{12}+l . \cos (\alpha)\right) \frac{Y_{13}+l . \sin (\alpha)-Y_{12}-l \cdot \sin (\alpha)}{X_{13}+l \cdot \cos (\alpha)-X_{12}-l \cdot \cos (\alpha)}-Y_{10}+l \cdot \sin (\alpha)+Y_{12}\right)-l \cdot \sin (\alpha)\right.}{\frac{Y_{13}-Y_{12}}{X_{11}+l \cdot \cos (\alpha)-X_{10}-l \cdot \cos (\alpha)} \frac{\overline{X_{13}+l \cdot \cos (\alpha)-X_{12}-l \cdot \cos (\alpha)}}{}}=X_{I}+l \cdot \cos (\alpha)
\end{aligned}
$$

The new $Y_{I}^{\prime}$ will be:

$$
\begin{aligned}
& \text { Remember } Y_{I}=\left(\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}\right)\left(\frac{\left(X_{10} \frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}-X_{12} \frac{Y_{13}-Y_{12}}{X_{13}-X_{12}}-Y_{10}+Y_{12}\right)}{\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}-\frac{Y_{13}-Y_{12}}{X_{13}-X_{12}}}-X_{10}\right)+Y_{10}= \\
&\left(\frac{Y_{11}-Y_{10}}{X_{11}-X_{10}}\right)\left(X_{I}-X_{10}\right)+Y_{10} \\
& \Rightarrow Y_{I}^{t}=\left(\frac{Y_{11}+l \cdot \sin (\alpha)-Y_{10}-l \cdot \sin (\alpha)}{X_{11}+l \cdot \cos (\alpha)-X_{10}-l \cdot \cos (\alpha)}\right)\left(X_{I}+l \cdot \cos (\alpha)-X_{10}-l \cdot \cos (\alpha)\right)+Y_{10}+l \cdot \sin (\alpha) \\
& \Rightarrow Y_{I=}^{\xi} \\
& Y_{I}+l \cdot \sin (\alpha)
\end{aligned}
$$

Now Remember:
$M^{t}{ }_{10}=\left(X_{10}+1 \cdot \cos (\alpha), Y_{10}+l \cdot \sin (\alpha)\right), M^{t_{11}}=\left(X_{11}+1 \cdot \cos (\alpha), Y_{11}+1 \cdot \sin (\alpha)\right)$,
$M^{t}{ }_{12}=\left(X_{12}+l \cdot \cos (\alpha), Y_{12}+l \cdot \sin (\alpha)\right), M_{13}=\left(X_{13}+l \cdot \cos (\alpha), Y_{13}+l \cdot \sin (\alpha)\right)$
and : $M_{10^{\prime}}=M_{10}-(h, 0) \quad M_{13^{\prime}}=M_{13}-(0, v)$
$M_{11^{\prime}}=\left(1-C_{1}\right) M_{10^{\prime}}+C_{1} J \quad M_{12^{\prime}}=\left(1-C_{2}\right) J+C_{2} M_{13^{\prime}}$

Step 6) Proving that the stretch shifted version is the same as shifted stretched version
Now we need to check if the stretched curves have the same distance from each other. In this case we need to check if the control points of the stretched curves also respect the $\vec{U}$ transformation or not.

$$
\begin{aligned}
& M^{t}{ }_{10^{\prime}}=M^{t}{ }_{10}-(h, 0) \\
& M^{t}{ }_{13^{\prime}}=M^{t}{ }_{13}-(0, v) \\
& M^{t} 11^{\prime}=\left(1-C_{1}{ }_{1}\right) M^{t}{ }_{10}{ }^{+}+C^{t_{1}}{ }^{\prime} J \\
& M^{t}{ }_{12^{\prime}}=\left(1-C^{t}\right) J+C^{t_{2}} . M^{t}{ }_{13^{\prime}} \\
& \Rightarrow M^{t}{ }_{10^{\prime}}=\left(X_{10}+l \cdot \cos (\alpha), Y_{10}+l \cdot \sin (\alpha)\right),-(h, 0) \\
& M^{t}{ }_{13^{\prime}}=\left(X_{13}+l \cdot \cos (\alpha), Y_{13}+l \cdot \sin (\alpha)\right)-(0, v) \\
& M^{t}{ }_{11^{\prime}}=\left(1-C_{1}{ }_{1}\right) M^{t_{10}}+C^{t}{ }_{1} \cdot J^{\prime} \\
& M^{t}{ }_{12^{\prime}}=\left(1-C^{t} 2\right) J^{\prime}+C^{t_{2}} M^{t_{13}} \\
& \Rightarrow \quad X_{10^{\prime}}=X_{10}+l \cdot \cos (\alpha)-h=X_{10}+l \cdot \cos (\alpha)-h \\
& \Rightarrow \quad Y_{10^{\prime}}=Y_{10^{0}}+l \cdot \sin (\alpha)=Y_{10}+l \cdot \sin (\alpha) \\
& \Rightarrow \quad X_{13^{\prime}}=X_{13}+l \cdot \cos (\alpha)=X_{13}+l \cdot \cos (\alpha) \\
& \Rightarrow \quad Y_{13^{\prime}}=Y_{13}+l \cdot \cos (\alpha)-v=Y_{13}+l \cdot \sin (\alpha)-v \\
& X^{t} 1_{1}=\left(1-C^{t_{1}}\right) X^{t_{10}}{ }^{\prime}+C^{t_{1}} X_{J} \\
& \text { and remember: } \\
& C_{1}=\frac{X_{11}-X_{10}}{I_{x}-X_{10}} \\
& C_{2}=\frac{X_{12}-I_{x}}{X_{13}-I_{x}} \\
& \text { So } C^{t_{1}}=\frac{X_{11}+l \cdot \cos (\alpha)-X_{10}-l \cdot \cos (\alpha)}{I_{x}+l \cdot \cos (\alpha)-X_{10}-l \cdot \cos (\alpha)}=C_{1} \text {, }
\end{aligned}
$$

and, $C_{2}{ }_{2}=\frac{X_{12}+l \cdot \cos (\alpha)-I_{x}-l \cdot \cos (\alpha)}{X_{13+l \cdot \cos (\alpha)}-I_{x}++l \cdot \cos (\alpha)}=C_{2}$
So, $\quad X^{t}{ }_{11^{\prime}}=\left(1-C_{1}^{t}\right) X^{t}{ }_{10^{\prime}}+C^{t} X_{J}=\left(1-C_{1}\right)\left(X_{10}+l \cdot \cos (\alpha)-h\right)+C_{1}\left(X_{J}+l \cdot \cos (\alpha)\right)=(1-$
$\left.C_{1}\right)\left(X_{10}-h\right)+C_{1}\left(X_{J}\right)+l \cdot \cos (\alpha)=X_{11^{\prime}}+l \cdot \cos (\alpha)$
With the same approach $Y_{11^{\prime}}=Y_{11}+l \cdot \sin (\alpha)$.
At last, $M^{t}{ }_{12^{\prime}}=\left(1-C^{t} 2\right) J^{\prime}+C^{t} M^{t}{ }_{13^{\prime}}$
So $X_{12^{\prime}}=\left(1-C t_{2}\right) X_{J}+C_{2} X_{13^{\prime}}=\left(1-C_{2}\right)\left(X_{J}+l \cdot \cos (\alpha)\right)+C_{2}\left(X_{13}+l \cdot \cos (\alpha)\right)=\left(1-C_{2}\right.$ $) X_{J}+C_{2} X_{13}+l \cdot \cos (\alpha)=X_{12^{\prime}}+l \cdot \cos (\alpha)$

With the same approach: $Y_{22^{\prime}}=Y_{22}+l \cdot \cos (\alpha)$
The proof shows that, for every Bézier curve, if you stretch it and then shift it with $\vec{U}=(l \cdot \cos (\alpha), l \cdot \sin (\alpha))$, the result is the same as shifting with $U$ first and then stretching it, using the stretching technique of Abdelouahad Bayar and Khalid Sami. This result proves that with this method, even after the stretching, the stretched curves of a glyph maintain the initial distance between parallel curves according to the size of the nib head, its angle $\alpha$ with the horizontal axis, and its length $l$. It is important to mention that even when adding or decreasing constants to the $x$ or $y$ value of the points in the initial stage of stretching, Abdelouahad Bayar and Khalid Sami`s method continues to be valid. A similar proof shows that the $E_{a b}$ and $E_{a f}$ transformations end in the desired results.

The stretching for Persian Nastaliq uses almost the same method. However, for the IranNastaliq font used in this research, it was concluded that the vertical shift would not be necessary. So we can set $v$ to be zero and work just with $h$.

## Chapter 6

## The Nastaliq Font:

### 6.1 Why Develop a PostScript Type 3 Font?

Recall that the main purpose of this research was to make it possible to typeset Nastaliq using ditroff/ffortid and a dynamic font. The main point is that ditroff/ffortid uses fonts in PostScript Types 3 and 1. But only Type 3 fonts are able to handle dynamic stretching. This difference occurs because the Type 3 PostScript code of a glyph can contain parameters, while for type one there can only be constants. Dynamic stretching means that only a single parameterized outline per stretchable character is needed; the actual outline of any stretching of a letter can be determined at printing time by the PostScript interpreter. A Type 1 font has a fixed outline for each character. A separate outline is needed in the font for each stretching amount of each stretchable form of each letter.

Therefore, I developed a PostScript Type 3 font; however I did not make it dynamic, but containing all the pre-compiled variations of all stretched glyphs. To build this font, I had to find an existing Nastaliq font and convert it to Type 3.

### 6.2 The Font Used in this Research

The Nastaliq font used in this research is called IranNastaliq. It is a TrueType font and is free to download and use in Microsoft Word under Windows. However, when using this font in other applications or operating systems, several problems exist. For instance, when using Microsoft Office with Mac OS X, only the stand-alone version of each letter shows up and letters cannot connect. Several fonts were reviewed in order to chose the best
to be converted to a PostScript font. These fonts include the IranNastaliq, WM_Nastaliq by Maryam [8], and NamehNegar[9]. The reason that IranNastaliq was chosen is that this font uses absolute values for coordinates when converted to PostScript, unlike the other two fonts. It is easier to understand absolute coordinates and see how to modify them. Also, as mentioned in the next section, the conversion failed for the NamehNegar font.

### 6.3 Conversion Tools

I needed to convert the TrueType IranNastaliq font to a PostScript font. To convert this font, I used a program called ttf2pt1that converted its input TrueType (.ttf) font to a PostScript Type 1 font, together with a .afm file, which contains standard font metrics. The ttf 2 pt 1 program was applied to all three fonts mentioned above. The result was that only IranNastaliq and WM_Nastaliq were successfully converted. The resulting font metrics file gives only horizontal movements, which is for each character the distance to the next. For normal Latin fonts, only the horizontal movement is needed since all lines of text are horizontal. In a font for slanted baseline writing, the vertical movement of the next character might also be needed. One big problem was that no font conversion program would generate the vertical movement for the metrics file. Even FontForge which is one of the most well known applications for font conversion in Mac OS X, does not compute vertical movements.

### 6.4 Converting t1a to Type 3

It was necessary then to convert each Type1 font to a Type 3 font. This conversion required changing the commands and getting rid of the hinting commands, which provide
information that is useless for Type 3 fonts. In order to be able to see what I was doing, I converted the Type 3 font into a PostScript program that displayed all its defined glyphs.

### 6.5 Glyph Identification

I was able to see that the initial font contained about 1366 glyphs which had nothing to do with Nastaliq. I had to remove all the unnecessary non-Persian letter glyphs. I mentioned in the first chapter that, Persian has 32 letters and at most 4 positions per letter in normal typography, Therefore, there should not be more than 128 glyphs. After removing unnecessary glyphs, there remained 878 glyphs from the original 2244 glyphs. Thus, in Nastaliq, every letter has about 27 different glyphs. Some of these shapes were so similar to each other that it was hard to determine if all but one of a set of similar glyphs could be thrown out or each similar but not identified glyph served different purpose. Figure 28, 29, and 30 show the letter ق in three very similar shapes. The first shape is when ق (Ghaf) is connected following to (Mim), while (Mim) itself is not connected to anything else. The second shape is when the ق (Ghaf) is connected to following (Mim) and the following after (Mim) is connected to another letter, and the third shape is used when (ghaf) is connected
 in the three combinations described in the previous three figures.

Next I had to rename the 878 glyphs. The glyphs in the original fonts had random names which make it hard to use them normally to write PostScript programs that would print Persian text. A sample of naming in the original file has been shown in Figure 31. As can be seen the name provides no information about the glyph. Therefore, I had to identify
each glyph and its possible context. The name of any glyph is an encoding of its letter, its position and all of the possible contexts. I defined a regular expression for naming the glyphs by this encoding. It was easy to envision either a person or a software application choosing any glyph by determining its name, given its letters, connectivity, and current context.

## CF Ghaf_Jim|Che|He|Khe



Figure 28- Connect Following Letter (Ghaf) Followed by One of the Letters (Jim), ๔ (Che), 兀 (He), خ (Khe)

## CF_Ghaf_Mim>None



Figure 29- Connect Following Letter ق (Ghaf) Followed by Letter (Mim) Which is Not Followed by Anything

## CF Ghaf Mim>All



Figure 30 Connect Following Letter ق (Ghaf) Followed by Letter (Mim) Which is Followed by Another Letter


Figure 31 - Different Combinations of Letter Ghaf

## afii57449



Figure 32- Random Naming for Glyphs in the Original Font- Letter $\mathcal{G}$ (Ye)

The possible connectivities are SA for stand alone, CB for connecting both, CP for connecting previous, and CF for connecting following.

### 6.6 Naming the Glyphs

To help understand the naming process, Figures 33, 34, and 35 provide three examples of letters with their complete names.

Figure 33 shows the connecting-previous position of letter $\succcurlyeq(\mathrm{Pe})$ stretched by 1 dot to the total amount of 6 dots that is written after one of the letters $\underset{\text { を }}{ }(\mathrm{Jim}), 匹(\mathrm{Che}), \tau(\mathrm{He}), \tau$ (Khe), س (Sin), ش(Shin), ص(Sad), ض(Zad), b(Ta), ط(Za), ع (Ein), غ (Ghein), „(Mim), or o(Ha). First comes CP for "connecting-previous". Then comes the previous context
"Jim|Che $\mid$ He $\mid$ Khe $\mid$ Sin $\mid$ Shin $|S a d| Z a d|T a| Z a|E i n| G h e i n|M i m| H a ", ~ t h e n ~ c o m e s ~ t h e ~ l e t t e r ~ " P e ", ~$ Finally comes the stretch amount " 6 ". If the glyph had a following context, it would come after the " 6 ".

## CP_JimlChelHelKhelSinlShinlSadlZadlTalZalEinlGheinlMimlHa_Pe6



Figure 33- A Sample Naming for Stretched connecting-previous پ(Pe)
Figure 34 shows the shape of the connecting-following $ب$ (Be) with no stretch that is followed by one of the letters $ب(\mathrm{Be}), \succcurlyeq(\mathrm{Pe}), \quad \because(\mathrm{Te})$, or $\doteq(\mathrm{Se})$ which is itself followed by one


Figure 35 Shows the shape of the connecting-both Te with no stretching amount that is placed after any letter and followed by $\mathrm{a}_{\mathrm{p}}$ (Mim) that is not connected following to anything. Its name is CB_All_Te_MimQNone. The "All" here means that the Te follows any letter, and the "MimQ none" means that the Mim which comes after the Te is not connected following to any letter.

## CF_Be_OBe $|\mathrm{Pe}| \mathrm{Te}|\mathrm{SeCQOBe}| \mathrm{Pe}|\mathrm{Te}| \mathrm{SeC}$



Figure 34- Non-Stretched Letter $ب$ (Be), Which is Followed by One of the Letters
 $\because(\mathrm{Te})$, or ث (Se).

CB_All_Te_MimQNone $N$


Figure 35- Non-Stretched Letter ت (Te), Which is Followed by a Letter (Mim) Which is Not Connected Following to Any Other Letters.

The naming process contains the following Grammar:
| is for "or"
O and C are for grouping -O is '(' and C is ' $)^{\prime}$
" $\{x\}$ " means " $x \mid \varepsilon$ "

* is for zero or more occurrences of what * is applied to
$\varepsilon$ is empty string.

Nonterminals are in Comic Sans Ms.

Terminals are Calibri.

There are no spaces in the strings generated by this grammar; spacing in these rules is for the readability of the grammar.

```
GlyphName \(\rightarrow\) Connectivity_\{ContextPrevious_\}Glyph\{_ ContextFollowing\}
Connectivity \(\rightarrow\) SA | CP | CF | CB
Glyph \(\rightarrow\) Letter \{Length\} o- Letter \{Length\}c*
Length \(\rightarrow\) NumeralAtLeastOne | K
NumeralAtLeastOne \(\rightarrow 1|2| 3|4| \ldots \mid 9\)
Letter \(\rightarrow\) UpperCaseLetter LowerCaseLetter OLowerCaseLetterC*
Digit \(\rightarrow 0|1| 2|3| 4|\ldots| 9\)
UpperCaseLetter \(\rightarrow \mathrm{A}|\mathrm{B}| \mathrm{C}|\mathrm{D}| \ldots|\mathrm{Y}| \mathrm{Z}\)
LowerCaseLetter \(\rightarrow \mathrm{a}|\mathrm{b}| \mathrm{c}|\mathrm{d}| \ldots|\mathrm{y}| \mathrm{z}\)
```

```
ContextPrevious \(\rightarrow\) Context
ContextFollowing \(\rightarrow\) Context
Context \(\rightarrow\) Any \(\mid\) Rest \(\mid\) ListOfChoices \(\mid\) Null
ListOfChoices \(\rightarrow\) Choice ORSign OChoiceC*
Choice \(\rightarrow\) LettterOrNone OQ LetterOrNonec*
LetterOrNone \(\rightarrow\) Letter | None
ORSign \(\rightarrow{ }^{\prime}{ }^{\prime}\)
```

This Grammar was developed to generate strings that encode glyphs in Nastaliq .
Since "(", ")", "\{", "\}", "[", and "]" are meaningful characters in PostScript, they cannot be used in the naming of glyphs. Therefore, I have used "O" instead of "(" and "C" instead of ")" for grouping. The Letter Q means that the current letter is followed by another letter or group of letters. Since the or sign " $\mid$ " is not a meaningful character in PostScript, it can be used to denote "or" in a glyph name.

### 6.7 Calculating Font Metrics

The next step was to calculate each glyph's bounding box and movements. This was important because without these information the font could not be used as expected. To calculate a glyph's bounding box it is necessary to determine the minimum and maximum
values of both the $x$ and $y$ coordinates. The horizontal movement was taken from the .afm file that was generated by ttf 2 pt 1 . As discussed in Section 6.3, the vertical information was not given by any conversion tools. Therefore, all work on the vertical movement of each glyph is postponed until the time that, a conversion tool handling both vertical and horizontal information is developed.

To help me determine bounding boxes, I modified the font definition so that it would draw a small circle at the start and end points of each curve and line. It drew the first circle in each glyph larger than the others. I could trace the path to draw the glyph.

### 6.8 Identification of Stretchable Curves

Next, I had to identify the stretchable glyphs. There are two conditions here. 1) The glyph contains 2 curves that can be stretched and 2) the glyph contains four curves that can be stretched. Figure 36 and 31 show examples of the two conditions. I had to first identify the condition of each glyph. I had to examine the code for each glyph, identify its stretchable curves, and find their start and end points. For some of the glyphs, each condition should be considered. For these glyphs either condition would have worked, and I had to choose the one that would look better. It is important to mention that this stretching is not dynamic. There is a separate glyph in the font for each stretch amount. I should notice that the font initially contained at most one stretched sample of stretchable glyphs, but these glyphs could not be accessed with an editor and just existed in the font with no use. At this stage, I learned that the glyph for some specific amounts of stretching do not exist in the original font. So, I
had to make some glyphs manually. The process involved copying each part of a glyph from a similar glyph and binding these parts together to get the required glyph.

### 6.9 Implementing the Font:

To implement this font, I developed several Java programs, which helped me to do various tasks in order to save my time and simplify the work for me. One program translated the original names of each glyph in the font to its new name which encodes the purpose of the glyph. For this purpose, I had to implement a parser, that parses the font file, and finds each instance of a glyph name while leaving everything else untouched.

Abdelouahad Bayar and Khalid Sami's formula had to be implemented to generate the stretched glyphs from the existing ones. This program is given the control points of each curve as input and outputs the control points of the stretched glyph. For this purpose, two conditions have to be taken care of. First, the start point of the curve which is going to be stretched must be is on the right side of the end point and second, the start point must be on at the left side of the end point. I refer to curves of the first kind as Type 1 and the second kind as Type 2. Here is the algorithm my program uses to stretch a glyph. As mentioned previously, each letter can be stretched minimum by the amount of from one to five dots.

Let call the file containing the glyphs PostScript code, fontFile.Ps,
Read from fontFile.Ps and get condition and number of control points from the user:
If there are four points, this means two points are located at the left side and two are located at the right side as shown in Figure 36. $X_{1}, X_{2}$ are left and $X_{3}$ and $X_{4}$ are right.

If a current point is at the left side of $X_{1}$ and $X_{2}$, leave the curve or line starting at current point untouched.

If the current point is either $X_{1}$ or $X_{2}$, then we need to check where the curve ends.


Figure 36- A 4-Point-Stretchable Curve Glyph

1. Let $C p$ be the current control point
2. If $C p<X_{1}$ or $C p<X_{2}$ then do nothing /*don't move the curve or line starting $C p^{*} /$
3. Else If $C p=X_{1}$ or $C p=X_{2}$ then
4. Call the end point of the curve starting at $C p, E p$
5. If $E p<=X_{1}$ or $<=X_{2}$ then do nothing /*don't move the curve or line starting Cp*/
6. If $E p=X_{3}$ or $E p=X_{4}$ then apply stretch 1
7. Else If $C p>X_{3}$ or $C p>X_{4}$ then add the stretch amount to the $x$ value of start and end point of the curve. /* move the curve's or line's starting at $C p$ by stretch amount horizontally*/
8. Else If $C p=X_{3}$ or $C p=X_{4}$ then
9. Let $E p$ be the end point of the curve starting at $C p$
10. If $E p>=X_{3}$ or $>=X_{4}$ then add the stretch amount to the $x$ value of start and end point of the curve or line $/ *$ move the curve or line starting $C p$ by stretch amount*/ horizontally.
11. If $E p=X_{3}$ or $E p=X_{4}$ then stretch 2
12. Else if $\left\{C p<X_{3}\right.$ or $\left.X_{4}\right\}$ add the stretch amount to the $x$ value of the start and end point of the curves /*move the curve or line starting $C p$ by stretch amount*/

Method Stretch 1: Add the stretch amount just to the end point of the curve. /*the start point does not get shifted but the end point gets shifted by the stretch amount horizontally*/

Method Stretch 2: Add the stretch amount just to the start point of the curve. /*the start point gets shifted by the stretch amount horizontally but the end point does not move.*/

There are several cases to consider. For example, what happens if a glyph contains more than one closed paths? How does the program know which path is the main path. What happens to the other closed paths after stretching?

To write this code, I had to come up with a profile for different characters. It is amazing that almost every character has its own properties so I need to consider all of them in order to let the user handle them. This issue is explained completely after the 6 point glyphs are introduced.


Figure 37- A 6-Point-Stretchable Curve Glyph

Stretching a 6-point glyph is the same as stretching a 4-point glyph. There are more cases that need to be considered.

1. Call the current point $C p$
2. If $C p<X_{1}$ or $C p<X_{2}$ then don't move the curve or line starting $C p$
3. Else If $C p=X_{1}$ or $C p=X_{2}$ then
4. Call the end point of the curve starting at $C p, E p$
5. If $E p<=X_{1}$ or $<=X_{2}$ then don't move the curve or line starting $C p$
6. If $E p=X_{5}$ or $E p=X_{6}$ then stretch 3
7. Else if $C p>X_{3}$ or $C p>X_{4}$ then move the curve or line starting $C p$ by the stretch amount horizontally
8. Else If $C p=X_{3}$ or $C p=X_{4}$ then
9. Call the end point of the curve starting at $C p, E p$
10. If $E p>=X_{3}$ or $>=X_{4}$ then move the curve or line starting $C p$ by the stretch amount horizontally.
11. If $E p=X_{5}$ or $E p=X_{6}$ then stretch 4
12. Else if $\left(\left(C p<X_{3}\right)\right.$ or $\left.\left(C p<X_{4}\right)\right)$ and $\left(\left(C p!=X_{5}\right)\right.$ or $\left.\left(C p!=X_{6}\right)\right)$ then move the curve or line starting $C p$ by the stretch amount horizontally.
13. Else if $C p=X_{5}$ or $C p=X_{6}$ then
14. Call the end point of the curve starting at $C p, E p$
15. If $E p=X_{3}$ or $E p=X_{4}$ then stretch 5
16. Else Stretch 6

Now let us see how these stretch methods are different.
From Sections 5.1 and 5.2 we remember Bayar-Sami's transformation:

Given 4 initial control point $M_{10}, M_{11}, M_{12}$ and $M_{13}$ we can get the new stretched curve Control Points:

For stretch 1: Apply these changes to Bayar-Sami Formula:
$M_{20}=M_{10}$
$M_{23}=M_{13}+($ StretchAmount, 0$)$
$M_{21}=\left(1-C_{1}\right) M_{20}+C_{1 J}$
$M_{22}=\left(1-C_{2}\right) J+C_{2} M_{23}$
For stretch 2: Apply these changes to Bayar-Sami Formula:
$M_{20}=M_{10}+($ StretchAmount, 0$)$
$M_{23}=M_{13}$
$M_{21}=\left(1-C_{1}\right) M_{20}+C_{1} J$
$M_{22}=\left(1-C_{2}\right) J+C_{2} M_{23}$
For stretch 3: Apply these changes to Bayar-Sami Formula:
$M_{20}=M_{10}$
$M_{23}=M_{13}+($ StretchAmount $/ 2,0)$
$M_{21}=\left(1-C_{1}\right) M_{20}+C_{1} J$
$M_{22}=\left(1-C_{2}\right) J+C_{2} M_{23}$
For stretch 4: Apply these changes to Bayar-Sami Formula:
$M_{20}=M_{10}+($ StretchAmount, 0$)$
$M_{23}=M_{13}+($ StretchAmount $/ 2,0)$
$M_{21}=\left(1-C_{1}\right) M_{20}+C_{1} J+($ StretchAmount $/ 2,0)$
$M_{22}=\left(1-C_{2}\right) J+C_{2} M_{23}+($ StretchAmount $/ 2,0)$
For stretch 5: Apply these changes to Bayar-Sami Formula:
$M_{20}=M_{10}+($ StretchAmount $/ 2,0)$
$M_{23}=M_{13}+($ StretchAmount, 0$)$
$M_{21}=\left(1-C_{1}\right) M_{20}+C_{1} J+($ StretchAmount $/ 2,0)$
$M_{22}=\left(1-C_{2}\right) J+C_{2} M_{23}+($ StretchAmount $/ 2,0)$
For stretch 6: Apply these changes to Bayar-Sami Formula:
$M_{20}=M_{10}+($ StretchAmount $/ 2,0)$
$M_{23}=M_{13}$
$M_{21}=\left(1-C_{1}\right) M_{20}+C_{1} J$
$M_{22}=\left(1-C_{2}\right) J+C_{2} M_{23}$

### 6.9.1 Exceptional Cases:

i) $\quad ب(\mathrm{Be}), \succcurlyeq(\mathrm{Pe}), \because(\mathrm{Te})$, and $(\mathrm{Se})$ and not stand-alone version of $\dot{(\mathrm{Ne}), ~}(\mathrm{He})$, and (Ee):

When stretching such a letter, the dot or Hamza that is shown in letter and makes it to look different from leter $\varsigma$, should be shifted by half of the total stretch amount to the right. This shift should always place the dot or Hamza at the horizontal middle of the stretched part of the glyph.


Figure 38- Stretching and Component Shifts, Letter $ب$ (Be)
ii) Letters (Kaf), گ (Gaf):


Figure 39- Stretching and Component Shifts, Letter Kaf

As shown in Figure 39, there are two closed paths in this glyph which are connected to each other at point $R$. The closed path located at the left side of $R$ is the main part that needs to be stretched and the right path is just the identifier shape for this letter which identifies if this letter is $\checkmark$ (Kaf) or $\Xi_{(G a f) \text {. In this case, the identifier just needs to be }}$ shifted by the amount of the stretched letter. For the letter $\smile$ (Gaf), as shown in Figure 40, the identifier shape contains two parts which means both of them should be shifted.


Figure 40-Stretching and Component Shifts, Letter gaf

The stand-alone version of letter has a Hamza that needs to get shifted by the stretch amount. It is important to notice that for the stand-alone Letter $ك$ (Kaf), there exists no rule explaining how much the Hamza should be moved and the decision is left for the writer to where to place it horizontally above the letter.


Figure 41- Stand-Alone Letter $ك$ (Kaf), Not stretched


Figure 42- Stretched Letter $ك$ (Kaf)
 (Ghein), the dots need to get shifted by the stretch amount.



Figure 43-Connecting-Both Letter (Fe) of the Size 5 Points


## Figure 44-The Connecting-Both Letter ف (Fe), Stretched Version of Figure 43 by the Amount of 4 Points.

I used the BlueJ [12] programming environment to implement this software. Table 1 presents a list of the programs and their descriptions.

| Program Name | Description |
| :--- | :--- |
| Parse | This file gets a font and a list of useful glyph names and parses the fonts <br> Ppdate <br> Ulyphs names derived from Parse drives the code for the mentioned glyphs. |
| ShowCurves | This program goes over the glyphs in the font and identifies the start <br> point of each curve or line in each glyph which a small circle. The very <br> first circle, which is the start position of drawing the glyph and the |
| second one are respectively shown with a larger and smaller circle. |  |


| BoxCalculator | This program goes over each glyph in the font and calculates its font <br> bounding box coordinates. These coordinates are needed when <br> assigning memory to each glyph. Also this program finds the FontBBox <br> which is the minimal bounding box which can contain all the glyphs. |
| :--- | :--- |
| Stretch | This program gets the control points of a Bézier curve and the amount <br> of stretch size of the curve as input and returns the control points of the <br> stretched curve. |
| StretchGlyph4 | This program gets the PostScript code of a glyph together with the <br> coordinates of the four points of the stretchable Bézier curve, identifies <br> two stretchable curves in the glyphs, and stretches the glyphs using the <br> Class Stretch and returns the code for the stretched glyph. |
| StretchGlyph6 | This program gets the PostScript code of a glyph together with the <br> coordinates of the six control points of the stretchable Bézier curve, <br> identifies four stretchable curves in the glyphs and stretches the glyphs |
| using Class Stretch and returns the code for the stretched glyph. |  |

Table 1 - List of All Programs Developed to Make the PostScript Font

## Containing All Stretched Glyphs.

In the end, the font had 1247 glyphs which look very good to any Persian reader. I added 369 glyphs to the 878 glyphs that were already in the font.

### 6.10 The Problem of Misplaced Curves

One other problem that came up in this research was stretching any glyph that did not have proper curves. I define a proper curve, as one whose start and end points are positioned in its glyph's path so that it is easy to decide if the glyph is a 6-point or a 4-point stretchable glyph. For instance, the glyph $\because(\mathrm{Pe})$, shown in Figure 45, can be easily determined as a 6point stretchable glyph. Curves $A, B, C$, and $D$ are the four proper curves in this figure. Also, the glyph $\boldsymbol{\text { , }}$, shown in Figure 46, can be easily determined as a 4-point stretchable glyph, with curves $A$ and $B$ in the figure being the two stretchable curves.

On the other hand, the glyph $\checkmark$ in Figure 47 is one for which it is not easy to decide what type of stretchable glyph it is. As a result, due to the unsuitable position of the two control points of the curves showed within the larger circles in the outline, neither of the two decisions would look perfect after stretching the letters. Figure 48 and 49 show the two decisions made. The problem with Figure 48 is that the curves $A$ and $C$ are located higher than where they should be. In Figure 49, curves $B$ and $D$ are too close to each other.

As can be seen, the start and end points of a curve in a glyph are proper when their vertical vector is small in comparison to their horizontal vector. There is no fixed ratio to define this comparison. Therefore, human interaction is needed to define whether the curve is a proper one or not by choosing which one looks more beautiful.

## SA_Pe



Figure 45- A 6-Point-Stretchable Letter $\because(\mathrm{Pe})$, an Easily Decidable 6-PointStretchable Glyph


Figure 46- A 4-Point-Stretchable Letter (Fe), an Easily Decidable 4-PointStretchable Glyph


Figure 47-Letter (Kaf) a Glyph with Misplaced Start Points of Curves Shown in Larger Circles


Figure 48-Stretched Stand-Alone Version of Letter ك (Kaf) introduced in Figure 41 with Curves A and C Stretched


Figure 49-Stretched Stand-Alone Version of Letter ك (Kaf) introduced in Figure 41 with Curves B and D Stretched

## Chapter 7

## Conclusions and Future Work

### 7.1 Summary

The research described in this thesis has focused on the problem of stretching for Persian calligraphic typography. The first intention was to implement a dynamic font which would work with ditroff/ffortid modified as necessary to allow formatting of calligraphic Persian language. However, after attempting to specify the software for the task, it became clear that due to the complexity of Nastaliq, it would be impossible to implement its stretching in a dynamic font. So the goal of the research was changed to provide a PostScript font which would contain all the allowed variations of stretching of each stretchable glyph so the font would be usable on its own. Several TrueType Nastaliq fonts were converted to PostScript to find out which one would include all the features I needed. I ended up working with a free font called IranNastaliq. Due to the complexity of the calligraphy, the font contains about 900 glyphs which are used in writing. Then, each glyph was named by its letter, position, stretch amount, and previous and following contexts. This naming process was necessary to make the identification process of each glyph easier. It was necessary to prove Abdelouahad Bayar and Khalid Sami's conjecture[3]. Also, their formulations for Arabic Naskh Stretching had to be changed so that they would work for Persian Nastaliq. The last stage was to implement this formulation to achieve all the desired stretched sizes of each glyph. There are still some problems left for future work.

### 7.1.1 List of Contributions

The contributions achieved in this thesis can be summarized to four categories:
i) Development of a new regular language for naming Glyphs in a Nastaliq Font

As explained in Section 6.6, a regular expression was developed to name each glyph in the font to make it easier for the user to identify it and use it in the right context.
ii) Proof of the validity of Abdelouahad Bayar and Khalid Sami's stretching model Another achievement of this research was proving the validity of Abdelouahad Bayar and Khalid Sami's conjecture about modeling the handwritten Arabic Naskh. This proof was necessary to make sure a similar process would be valid for modeling Nastaliq.
iii) A stretching model for Nastaliq

A stretching formulation based on Abdelouahad Bayar and Khalid Sami's model was developed in this research to model handwritten Nastaliq.
iv) A Nastaliq Type3 font containing all the stretched version of stretchable glyphs.

A PostScript Type3 font was developed containing 1247 glyphs including all the possible stretched versions of all possible stretchable letter.

### 7.2 Lessons Learned

There were lessons that I learned in this research. Most important, before starting any software development, first, prepare a software specification or a user's manual for it. The main reason as mentioned earlier is that a good software requirements specification, in my case a user's manual, is always a good way to discover flaws in one's software earlier. I also
learned that I had to trust my supervisor, because if it wasn't because of his insisting on writing the manual, I would have wasted a lot of time on coding without getting any result. I also learned that research takes more time than what we expect. This is because we do not know all the aspects and therefore, at any stage you might find important issues neglected that would need to be considered.

### 7.3 Future Work and Existing Problems

A remaining problem is that no available program for working with TrueType fonts can handle fonts with both vertical and horizontal movements. As mentioned earlier many convertors can be used to work with TrueType fonts such as ttf2pt1, FontForge, CrossFont, TransType, etc. None of them could handle horizontal and vertical movement at the same time. For future work it is recommended to work on a program that can handle both vertical and horizontal movements in TrueType fonts.

Another important issue is handling the problem misplaced start and end points of curves in the glyph as mentioned in Section 0. The solution for this problem is changing the stretchable Bézier curves so that the start and end position of each curve is in a suitable place for stretching. This change requires cutting a Bézier curve into two new curves given a break point or combining two adjacent Bézier curves. The combination is usually helpful to reduce the number of curves in the layout of the glyph in order to achieve a better look. Figure 50, illustrate two proper curves $B_{2}$ and $D_{2}$.As shown in this figure, by breaking the curves $B$ and $D$ in Figure 47, we can achieve two curves, $B_{2}$ and $D_{2}$, which their start points and end points have much less vertical distance.


Figure 50-Breaking Curves $B$ and $D$ in Figure 47 to Achieve Proper Curves in letter $ك$ (Kaf)

### 7.4 Breaking a Bézier curve into Two Bézier Curves Given its four Points and a Point Placed on the Curve:

It would be enough to show that if any Bézier curve is broken at any point into two curves, these two curves would be Bézier curves. It is clear that if you break any polynomial curve, the two curves are still polynomials with the same degree. We know that every Bézier curve is a polynomial of degree 3 . Since every polynomial curve of degree at most 3 can be expressed as a Bézier curve [10], by breaking this polynomial into two polynomial curves of the same degree, we can get two Bézier curves. This process also needs direct human intervention to choose the right break point on the glyph's layout to achieve a more beautiful glyph after stretching.

### 7.4.1 Combining Two Bézier Curves in Order to Get a New Curve:

As a matter of fact, it is not possible to combine two Bézier curves of the same degree and get a single Bézier curve covering the same points with the same degree in general. But there are special cases in which this can be done. For instance, if the starting curve is the result of the splitting operation descried in Section 7.4, it would be possible to combine the curves.

More generally, we can divide every curve into many break points. Assume we have $n$ break points, $K_{l,} K_{2}, K_{3}, \ldots, K_{n}$. The trick is to calculate a polynomial $P(n)$ of degree 3, which would minimize the sum of the square distances between $P(n)$ and every $K_{i}$ for $i \epsilon\{1,2, \ldots, n\}$. This method is called the least squares method and is applied extensively in optimization models. The resulting curve might not be exactly what we want, but it might still be useful improving the appearance of stretched glyphs.

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## Appendix A

## NAS Troff User's Manual

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## [NAS TROFF USER’S

## MANUAL

This Document contains the User Manual for NAS , a modification to ffortid to let Troff support the Persian calligraphy called Nastaliq and also support its stretching mode.

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## 1 Introduction:

### 1.1 Product Overview

NAS is a Troff postprocessor for handling the Persian calligraphy called Nastaliq. NAS lets Troff documents contain Nastaliq as well as automatically handling the stretching problem while both right and left justification. In Chapter 2, the required rules for Persian Nastaliq as well as the rules handling the stretching problem are described. Chapter 3 describes some basic use cases about stretching. Chapter 4 provides the algorithms and procedure NAS uses to stretch letter. This software is meant to run on an Apple's Mac OS X . We assume the user uses the free editor program called Textedit which is pre-installed on all Mac OS $\times$ systems and will build the result using Troff, the plan 9 version- modified by the author of this article.

NAS was decided to handle these as follows:

1. A Persian font would provide the different forms of each letter as independent characters and each character that is to be connected on any side would be designed to be flush to the bounding box on that side at precisely the same place relative to the baseline.
2. A pre-processor, called ptrn, would do letter form and ligature identification on letter-only input to yield output with each glyph to be printed, be it a form of a letter or a form of a ligature. The letter-only input would be according to a standard encoding for the language being processed, and the output would be according to the font's encoding for the glyphs. Thus, ditroff would format input consisting of the glyphs to be printed. If the input to the pre-processor has diacritical marks, then they will be translated into their glyph codes surrounded by instructions to place them in the proper vertical position with respect to the character with which it is associated.
3. The ffortid postprocessor would be modified to stretch connections to last letters of words and/or lines in order to achieve one kind of Keshidah.

Figure 1 describes the flow of how this software works. The user inputs Unicode text in the editor. The next state is ptrn's turn which would do letter form and ligature identification on letter-only input to yield output with each glyph to be printed. Also shape changes which may occur are handled in this section. The result which is Unicode together with some ligatures will be delivered to ditroff/ffortid. Now with the changes implied to the ffortid class, the software calculates where to places each letter and also how much each letter should be stretched. The last step before the output is generating the post script code first and then applying the change in shapes which will be done to the post script code of each letter. Figure 2 also provides a big picture of the flow of data in the software.


Figure 1- The flow of data from keyboard to output result


Figure 2- High level view of NAS and troff Processors

### 1.2 A sample Run

Writing Nastaliq can be easily achieved by using NAS. First, you need to run the Textedit on your Mac OS X. NAS not only handles documents only in Nastaliq but it also handles making documents containing both left to right and right to left languages. In order to write Nastaliq, we first need to change the language of the keyboard. This can be easily done be clicking on the language bar at the top left side of the screen and choose Persian - ISIRI 2901 from the menu. If this language does not exist in the menu, simply click on the system preferences on the dock at the bottom of the page and choose international-Input Menu - Persian-ISIRI 2901.

Below you can see a sample Troff file handling Persian Nastaliq.
Here is a sample set of Troff command you need to start typing Persian. /F PN marks the beginning of using Nastaliq font. \fP Marks its end. .OA means "other abstract header", .lp means "left adjusted paragraph".
. OA
.lp
|f(PN مقاله اين يك مثال اسـت برای امتحان نوشتن زبان فارسـى به كمك \f \Hditroff/ffortid\fPN در اين برنامه ما از يك نگارنده به نام \ff

استفاده ميكنيم (Apple Mac OS X)\*PN كه در سـيستم های Textedit)\fPN) كه \fP ، دارد وجود

It is severely hard to show the time sequence in the editor used to type this manual for a text containing both right to left and left to right languages. So it is just enough to remind that Farsi, unlike English is right-to-left.


Figure 3 Sample Persian-Nastaliq editing with Troff

## 2 Chapter 2 <br> Conventions

### 2.1 User assumption

2.1.1 •The User of NAS is assumed to have a good knowledge of Troff. If not please refer to the user manual of Troff Plan 9[5].
2.1.2 •Nastaliq has many different rules used by different Masters in Iran. Here we obey mainly the rules from the sources we mention in the bibliography[1,2,3,4].

### 2.2 National Conventions

- Times is used for regular text, Headings
- Times Italic is used for Terms and emphasis.
- Comic Sans is used for program names and code.
- Tahoma is used for Sample Input.
- Century is used for sample output.


### 2.3 Terms

NAS - The name of the program

You - The person who uses NAS, addressed by "you".

Dot - A square shape found in some Persian alphabet such as several letters in the alphabet such as "ق، ق، ف، ب،...". The size of a dot is defined to be $\sqrt{ } 2$ * (side of a dot).

Keshidah- A procedure applicable to some letters in Persian alphabet which will result in stretching of the letter. A stretched letter's length should be between 7 to 11 dots , giving the author the right to choose what is the most proper length.

Persian : (Also known as Farsi) The language spoken in Iran, Tajikistan and Afkanistan.



Most of the Persian letters have up to 4 different shapes:

- Stand alone
- Connected before
- Connected after
- Connected both before and after

Nastaliq - an Old Persian calligraphy style.

Spacing - The required space we need between not connected letters and also words in Persian Nastaliq. It is mainly equal to one dot.

Base line- In Nastaliq the baseline of each character is determined according to the position of the character in the word containing it and whether it is going to be connected or not. As a necessity, it is the case that we start every maximal group of connected
letters from a height higher than the original baseline and we come down while writing on a slanted line so that the last letter in this group is either tangent to the baseline or under it due to the nature of the letter.

Shape Changes: In Nastaliq, Letters "س" and "ش" might change shapes to a shape different from the four ones already introduced. When applying Keshidah, most letters will become just a little bit more stretched, but some letters, such as letter " $\omega$ ", lose all their teeth. This case happens only to letters "س" and "ش". The Iran Nastaliq font used in this software does not support the stretch glyph; however, by a modification to the PostScript code of existing letters, it is possible to produce the desired shaped.

Ptrn- A pre-processor, which would do letter form and ligature identification on letteronly input to yield output with each glyph to be printed. Also shape changes which may occure are handles in this sections.
ditroff - troff's processor which handles the left to right languages
ffortid - troff's post-processor which handles the right to left languages including stretching.
psdit - converts the ditroff or ffortid output to PostScript.

Unicode : A standard encoding allowing computer based systems to consistently represent and manipulate almost any existing character in many language or formula. There are several versions of Unicode as follows:

UTF-8 : uses 1 byte for all ASCII characters, which have the same code values as in the standard ASCII encoding, and up to 4 bytes for other characters

UCS-2 : uses 2 bytes for all characters, but does not include every character in the Unicode standard

UTF-16 : extends UCS-2, using 4 bytes to encode characters missing from UCS-2

Textedit - The default text editor on Mac OS X Systems which uses Unicode with UTF-8 and UTF-16. ${ }^{[5]}$

MAC OS X: The current (2009) version of operating system implemented on Apple Macintosh machines.

### 2.4 Rules in Persian Nastaliq (This section just contains the actual rules in Persian Nastaliq- Chapter 4 is about what I will and can implement!)

Rules needed to be taken care of:

Rule 1: Page 23 Rules of Calligraphy by Amirkhani -Adaabol khatte Amikhani ${ }^{[2]}$

For regular lines, up to 3 stretches can be still acceptable (leaving the decision for the author) and can help to keep the balance of the text. Stretched letter should not be selected from the beginning of each line. It is also not desirable to have stretched letters beside each other.

Rule 2 : Page 9 Learning Nastaliq, selected parts of secrets of Nastaliq ${ }^{[4]}$

For not connected letters, which come after each other in a line, we should consider a space of a dot between them.

If the ending part of the previous letter or the starting part of the next letter is thin, they can be shifted/ closer to each other until they look connected.

Rule 3: Page 92- Learning Persian Calligraphy ${ }^{[1]}$
There are three reasons to stretch letters which the third one is important for us in NAS since the first two needs human interaction to choose what is more beautiful:

I - To look more beautiful

II- To avoid confusing between similar words

III- To justify lines and fill empty spaces

Rule 4: Learning Persian Calligraphy

A full stretch is between 9 to 11 points and a short one is from 4 to 5 points.

Rule 5 : Learning Persian Calligraphy

A Full stretch does not appear in the beginning or the end of a sentence.

Rule 6: Learning Persian Calligraphy

No two stretches in two neighbor lines should be located vertically close to each other.

Rule 7: Learning Persian Calligraphy
Neither full nor short stretches may appear at the beginning and end of the line.

Rule 8: Learning Persian Calligraphy

Stretching is acceptable in the following 7 conditions:


After $\rightarrow$ as in,


(

After $\sim$ as in,


Rule 9: Learning Persian Calligraphy

We also are not allowed to stretch letters"س" and" شس"in following conditions:
Note: The slanted line drawn on the stretched letter means that it is a mistake to stretch it.

Before another stretched same letters, as in,


Before
 , as in


Before $\mathcal{G}^{\mathcal{G}}$, as in

Rule 10: Page 14, A look into cheminstry in Nastaliq[3]

In every line it is better to have 1 stretched letter or 2 half stretched ones. Sometimes in order to make the line look better we may decide to put more than one stretched letters in
every line. It is better to not decide the stretched letters in lines so that they do not locate beneath each other. This helps the text to not look more crowded in some parts of a page than other parts.

For a letter, even it is stretchable, it is important to check its location in the line before stretching it, due to calligraphy rules in Persian. It is important that in every line we consider at least one stretched letter, and with respect to the kind of the letters, we can put more stretches in each line; however, it is required that we consider some space between every two stretched letter. To be more specific, let's divide each line into 5 sections as shown below. We can divide every line to five sections just considering the line, not caring about the words in it. Also if one letter is in two sections (when we divide the text into 5 sections, it is possible that a dividing line divides a letter into two sections. In the following figure, letter "ظ" faces such a condition, but since it is mostly contained in section 4 than section 3, we assume it is in section 4, not 3.), it can be considered as a member of the section which contains a larger vertical portion of the letter. If the letter is equally in two sections, it can be considered to be in any of those two.


When we just want to have one stretched letter in a line, the best place to choose its location is section 3. If the letters in this section are not stretchable, our next candidate is
section 4, and again if section 4 does not contain such a letter, the next candidates are section 2,5 and 1 in that order.

If we want to choose 2 stretched letters in one line, the preferred situations are $(3,5)$. If it is not possible to choose stretched letters from those sections the next candidates are $(2,5),(2,4)$ and $(4,1)$ in that order.

Rule 11:When fully stretched, for some letters such as "ب" the beginning of the letter should be replaced one dot higher than usual.

## 3 Chapter 3 - Basic Use Cases:

## 3.1 - An overview of stretching

In this section the instruction for stretching letters is provided. This software provides two different Ways to stretch letters: Automatic and Manual.

### 3.1.1 - Automatic:

To obtain an automatic stretching, the user has to choose the both side-justified paragraph by the ".ad .p" command. In this way, the software automatically chooses the suitable letters to stretch using the provided rules in chapter 2 and stretches them automatically by the algorithm and amount explained again in chapter 2.

### 3.1.2 - Manual :

In this way the user has to exit the automatic mode by the ".na command" and enter the Manual mode. Then by the .strm command before any word, the user can ask for the maximum stretch of the last possible letter in the word. The user has to use the .nstrm command to cancel the manual stretching mode.

Note: Entering the Manual stretching mode would also turn off the automatic stretching mode automatically.

## 4 Design Decision

### 4.1 Overview

In this chapter a set algorithms will be provided which are already implemented in the NAS. These algorithms are included in this manual to help the user understand the rules of Nastaliq together with the functionality of NAS better.

### 4.2 Algorithms

### 4.2.1 Automatic Calculating

1- Calculating the length of existing line: The length of the existing line would be the addition of the width of all Maximal connected group of letters plus the space between the not connected letters or Maximal connected group of letters. The default size of this space is known to be $\sqrt{ } 2 *$ side of a dot.
2- Calculating the available amount for stretching
The amount to calculate the free space we have in a line would be equal to :
Default Length of a line - length of the current line
3- When the available amount is calculated we need to find out which letters are stretchable. According to the font we use in NAS, most of the letters are
 the stand-alone, the connect-before or connect-both position. Letters such as
 after positions. These letters are stretchable if they obey rule 2.4.8.

4- Since we prefer to have not many stretched letters, and according to rules 10 in 2.3 , we would rather to have stretched letters in specific places in a line, we also need an algorithm to find the suitable letters and stretch them.

1- If the existing line is the last line in a paragraph, i.e. it contains a period and there is nothing after the period, automatic stretching would not apply to this line; if not move to next step.
2- Divide the length of line into 5 section, call them $\mathrm{S} 1, \mathrm{~S} 2, \mathrm{~S} 3, \mathrm{~S} 4, \mathrm{~S} 5$ from left to right, which means that the most left section is S1 and the most right one is S 5 .

3- If the majority of a letter is in one section, but some of it is also in another section. We would consider the section containing the majority as the main section containing the letter. If it is the case that a letter is equally in two sections, we would consider it to be in the first one.

4- Check Section S3, if there exists a stretchable letter which would not contradict rule 8 in 2.3 , stretch it to the maximum amount up to 11 points so that the line is aligned. If there still exists spare space in the line. If there are no stretchable letters in section S3, check sections S4, S2, S5 and 1 in the given order. If the line still has spare amount, we would undo any changes done in section S3. (NAS first checks if it is possible to make the changes, if so it makes the changes, if not it will move forward to the next step without any changing in order to save time. There is a golden rule provided here to make our job easier to find the suitable letter for stretching in a section containing more than one stretchable letter. This golden rule is just a pseudo code for implementing rule 6 in chapter 2.4. The golden rule says: if there are more than one stretchable letters in a section and we need to choose one to stretch, we would stretch the first stretchable one. The only case this does not apply is if on the above line in the same paragraph, there exists a stretched letter exactly located at the top of
our choice. If this is the case we move forward the next choice in the section. If the next choice also is problematic or there is no other choice in the following sections, we will move forward with the choice we made first.

5- Now we know that for the line we are in, more than one stretch is needed. So by rule 10 in 2.3 , check sections (S3, S5), find the stretchable letters respecting 4.2.1.3 and 2.8 - rule 8 in these two sections (each should contain at least 1 stretchable letter), if there exists such letters in these 2 sections, choose them using the golden rule and stretch them by the equal amount up to 11 points so that the line is filled. If these two sections do not both contain the suitable letters, check section (S2, S5), (S2, S4), (S4, S1) respectively. If the stretching in none of the group of sections given above would not fill the line, it will mean than we need more than 2 stretches.
6- Check sections S1, S3, S5. If each contains a stretchable letter, by the golden rule, choose a stretchable letter in each section and stretch them by the equal amount up to 11 point as much as the line is filled. If the line cannot be filled and there is still spare space, there is nothing more we can do. So we move to the next line.

### 4.2.2 Manual calculating

In manual calculating, the user will have the choice to ask for a maximum stretching mode. The applied rules and uncommon cases and have been introduced in chapter 3 already. Notice that manual stretching can still be applied to stretchable letter defined in 4.2.1.3.

## 5 References:

1- Learning Persian Calligraphy - Fazaeli, Habibbollah -Publisher: Yasavoli

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3- "A look into cheminstry in Nastaliq" - Falsafi, Amir- ISBN : 964-306-092-6 ; Publisher: Yasavoli, $15^{\text {th }}$ Edition

4- "Learning Nastaliq, selected parts of secrets of Nastaliq" - Mahmoodi, Seyyed Mahdi- ISBN : 964-90656-6-0; Publisher: Moallef - $6^{\text {th }}$ Edition

5- Unicode and Multilingual Editors and Word Processors for Mac OS X, http://alanwood.net/unicode/utilities_editors_macosx.html

