The relation between math anxiety and basic numerical and spatial processing

by

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A thesis
presented to the University of Waterloo
in fulfilment of the
thesis requirement for the degree of
Doctor of Philosophy
in
Psychology

Waterloo, Ontario, Canada, 2011

Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Math anxiety refers to the negative reaction that many people experience when placed in situations that require mathematical problem solving (Richardson & Suinn, 1972). This reaction can range from seemingly minor frustration to overwhelming emotional and physiological disruption (Ashcraft & Moore, 2009). In fact, it has been argued that math anxiety can be considered as a genuine phobia given that it is a state anxiety reaction, shows elevated cognitive and physiological arousal, and is a stimulus-learned fear (Faust, 1992). Math anxiety has been associated with many negative consequences, the most pertinent of which is poor achievement in math. This negative consequence is of central importance in today's society as people's mathematical abilities have been shown to strongly influence their employability, productivity, and earnings (Bishop, 1989; Bossiere, Knight, Sabot, 1985; Riviera-Batiz, 1992)

A large literature exists demonstrating a negative relation between math anxiety and performance on complex math. That said, there is currently no published research (outside of that presented in this thesis) which investigates whether math anxiety is also related to the basic processes that serve as the foundations for that complex math. In this thesis I examine the relation between math anxiety and three of these basic processes that support complex mathematical problem solving. Specifically, in a series of experiments, I demonstrate that, in addition to their difficulties with complex math, high math anxious adults perform more poorly than their low math anxious peers on measures of counting (Experiments 1 and 2), numerical comparison (Experiment 3 and 4), and spatial processing (Experiment 5 and 6). My findings are then discussed with respect to their implications for our understanding of math anxiety and for potential remediation programs.

Acknowledgments

Seven years ago, a scared and shy teenager, I walked into my TA’s office in the hopes of finding volunteer work in a Cognition lab. This TA agreed and took it upon himself to teach me the ins and outs of research. His enthusiasm for science is infections to say the least, and, as a result, he ignited within me an insatiable appetite for knowledge and an obsession with research. He taught me that passion and effort are more important than innate ability and that almost any goal is attainable so long as you pick yourself up after every time that you fall. Thank you for showing me what it is that I love to do.

While Evan Risko can be credited for introducing me to academia, there are several other people who have helped to get me where I am today. This would not have been possible if not for the infinite support of my husband. Thank you Mark for sharing in my excitement and frustration these last eight years. Of course, thank you for tolerating the long hours and geographical uncertainty that comes with academia. The fact that you continuously support me and have always made sacrifices to put my career first makes my accomplishments as much yours as they are mine.

Thank you Mom and Dad. You have always encouraged education and you have made every sacrifice necessary for your kids to achieve our goals (educational and otherwise). My Dad once told me that if you are comfortable in what you are doing then you are not challenging yourself enough. This advice often influences the choices I make.

Thank you Jonathan Fugelsang. You allowed me intellectual freedom and creativity while providing gentle guidance during my graduate career. You have been, for me, the **perfect** supervisor. Our relationship just “makes sense” because “that’s how we roll”. I look forward to our many collaborations in the future (because you can’t get rid of me this easily).

Thank you Daniel Ansari. You have acted as an unofficial supervisor to me over the last few years and for that I am very grateful. I have enjoyed the many projects that we have done together as well as the lab meetings that you so graciously let me attend. I, of course, appreciate our many google-chat conversations. Thank you.

Thank you to my Ph.D. committee members, Derek Besner, Mike Dixon, Mark Ashcraft, and my “unofficial committee member” Colin McLeod. You have always provided thoughtful and constructive advice and have served as excellent role models to us students.

Rita Cherkewski, day in and day out you work diligently behind the scenes to make sure that everything runs smoothly. Thank you.

Finally, to the many people with whom I have worked over the course of this degree: Jonathan Fugelsang, Evan Risko, Daniel Ansari, Stephanie Waechter, Nathaniel Barr, Sherif Soliman, Frank Preston, Veronica D’Aprile. Thank you for making this fun.

Dedication

For my family.

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Chapter 1: Introduction

Researchers have known for many years that, in industrialized and in developing nations, people's mathematical abilities strongly influence their employability, productivity, and earnings. In fact, mathematical competence has an even stronger influence on earning potential than level of literacy, years of schooling, and intelligence (Bishop, 1989; Bossiere, Knight, Sabot, 1985; Riviera-Batiz, 1992). It is thus no surprise that, in industrialized societies, quantitative competencies are at an economic premium (Paglin & Rufolo, 1992). For instance, Paglin and Rufolo, (1992) report a direct relation between the quantitative demands of various careers and their associated wages such that the more math intensive the occupation, the higher the entry-level and subsequent wages. The development and preservation of numerical and mathematical competencies is thus of crucial importance for individuals within these societies and for the society as a whole (Geary, 2000).

Impairments in mathematical skills can result from a number of factors, one of which is math anxiety. Math anxiety manifests as negative emotions about mathematics (Richardson & Suinn, 1972) and reactions can range from seemingly minor frustration to overwhelming emotional and physiological disruption (Ashcraft & Moore, 2009). In fact, it has even been argued that math anxiety can be classified as a genuine phobia given that it is a state anxiety reaction, shows elevated cognitive and physiological arousal, and is a stimulus-learned fear (Faust, 1992).

This thesis examines the relation between math anxiety and basic numerical and spatial processing. I present data from several converging lines of research that reveal more basic performance differences between high and low math anxious individuals than have been previously investigated. Specifically, in a series of experiments, I demonstrate that high and low math anxious adults perform more poorly than their low math anxious peers on measures of counting, numerical comparison, and mental rotation ability. These findings are then discussed in terms of their implications for our understanding of math anxiety and for potential remediation programs.

History of Math Anxiety Research

The formal study of math anxiety was arguably born in the 1950's. In an anecdotal report, Gough (1954), a school teacher, noted that many of her students, predominantly the females, were exhibiting emotional difficulties with math and were failing to learn the material. Gough (1954) termed this emotional reaction "mathemaphobia" and credited it as the cause of many of her student's failures in mathematics. Shortly after, Dreger and Aiken (1957) published a report on "numerical anxiety" among college students. In this seminal paper, Dreger and Aiken (1957) undertook the first attempt at introducing standardized assessments into the study of math anxiety. With the addition of three simple math anxiety related questions (e.g., Many times when I see a math problem I just "freeze up") to the Taylor Manifest Anxiety Scale (Taylor, 1953), the first formalized assessment of math anxiety was created.

While Dreger and Aiken (1957) provided the first attempt at creating a standardized assessment of math anxiety, it was Richardson and Suinn's (1972) Mathematics Anxiety Rating Scale (MARS) that served as the catalyst spurring a plethora of research into the topic. The MARS is a 98-item Likert-type scale with response options ranging from 1 (not at all) to 5 (very much). On this scale, participants rank how anxious they feel in a variety of educational (e.g., registering for a math course) and everyday life (e.g., totaling a bill) situations. Subsequent work on math anxiety has relied heavily on the MARS and its descendants. For example, the MARS (or the MARS-A, a version adapted for use with adolescents) was used in over half of the studies in Hembree's (1990) meta-analysis of math anxiety in college aged students and in Ma's (1999) meta-analysis of math anxiety in pre-college aged students. While the MARS has many positive features (e.g., the wide scope of the scale's items and its high test-retest reliability; Brush, 1978), its one major shortcoming is its length. As a result, two new and more efficient scales have emerged. In 1989, Alexander and Martay published a shorter version of the MARS (the Abbreviated MARS, sometimes called sMARS for "shortened MARS"), which is a 25-item Likert-type scale that correlates very strongly with the original MARS (r = .97). Fourteen years later, Hopko, Mahadevan, Bare, and Hunt (2003) published the Abbreviated Math Anxiety Scale (AMAS). This 9-item questionnaire is also highly correlated with the original MARS (r = .85) and has a high test-retest reliability (r = .85; Hopko et al., 2003). Furthermore, this scale is reputed as being "an excellent, and far more efficient, substitute for the full-length math anxiety instruments" and "appears to be the test of choice for future work on math anxiety" (Ashcraft & Ridley, 2007; p. 316). As such, the AMAS is the scale used in each of the experiments that comprise this thesis.

Etiology of Math Anxiety

While much research has focused on developing reliable tools to assess math anxiety, there is considerably less work on its causes. With respect to the development of math anxiety, we do know that it peaks at about Grade 9 or 10 and then essentially plateaus thereafter (Hembree, 1990). That said, almost no research has examined children younger than the fourth grade (Hembree, 1990; Ma, 1999): Consequently nothing is known about the onset of math anxiety. However, studies assessing the implications of various teaching styles may provide insight into the development of math anxiety. For example, Turner et al. (2002) examined factors that lead children to avoid math and concluded that one cause of avoidance was having a teacher who conveys a high demand for correctness in math but provides little cognitive or motivational support during lessons. Turner et al. speculated that students with such teachers may feel "vulnerable to public displays of incompetence" (p. 101). Consistent with this claim, Ashcraft (2002) reports that his participants often cite public embarrassment in math as a cause of their math anxiety. Further research by Beilock, Gunderson, Ramirez, and Levine (2010) suggests that highly math anxious teachers can actually cause their students to endorse negative stereotypes about mathematics. Beilock et al., (2010) assessed math anxiety in first- and second-grade female teachers as well as the math achievement and gender stereotype endorsement of the students in these teachers' classrooms. There was no relation between a teacher's math anxiety and her students' math achievement at the beginning of the school year. However, by the end of the year the more math anxious a teacher was the lower the female students' math achievement, and the higher the likelihood that they would endorse the stereotype that "boys are good at math, and girls are good at reading". In contrast, the male students' math achievement and stereotype endorsement was unrelated to their teachers' level of math anxiety. Beilock et al., (2010) did not assess the students' levels of math anxiety, thus, we cannot know whether having a math anxious teacher leads to increased math anxiety. Nonetheless, it is plausible that such characteristics in teachers (i.e., a high demand for correctness with little cognitive or emotional support, a high degree of math anxiety themselves) are risk factors for the development of math anxiety in their students.

Sex Effects in Math Anxiety

Conventional wisdom suggests that there is a strong sex effect in both math and math anxiety such that women are worse at math and are more likely to be math anxious than their male counterparts (e.g., Ashcraft & Ridley, 2007). Interestingly, the data on these issues remain rather unclear. For example, while boys tend to outperform girls on math achievement tests in industrialized countries (e.g., Harnisch, Steinkamp, Tsai, & Walberg, 1986), girls tend to outperform boys on basic arithmetic tests (e.g., Hyde, Fannema, & Lamon, 1990). With respect to math anxiety, females consistently score higher on indices of math anxiety than their male counterparts (Hembree, 1990). However, within the high math anxious population, there does not seem to be a sex difference with respect to achievement. In other words, while females tend to be more anxious than males, high math anxious females are not worse at math than high math anxious males (Hembree, 1990). The observation that females are more likely to be math anxious than males is typically attributed to social factors such as stereotypes about women's mathematical abilities (Beilock, Rydell, & McConnell, 2007) and the possibility that women are more likely to report anxiety (Ashcraft, 2002). Evidence from the stereotype threat literature illustrates just how intimately tied together sex stereotypes and math performance can be. For example, simply telling women that men are superior at math can cause women to perform at a level below that of which they are capable (e.g., Beilock et al., 2007; Spencer, Steel, & Quinn, 1999). This effect is thought to arise because, on some level, women believe the stereotype. Understanding the sex effect in math anxiety is important in that it is certainly associated with the underrepresentation of women in math-heavy fields (e.g., physics and engineering; Chipman, Krantz, & Silver, 1992).

Consequences of Math anxiety

Perhaps the most thoroughly researched area of math anxiety is that of its consequences. Articulate summaries of these consequences can be found in the meta-analyses conducted by Hembree (1990) and Ma (1999). Probably the most commonly discussed consequence of math anxiety is its association with poor math achievement. As student's math anxiety scores increase, their grades in high school and college math courses decrease (e.g., Betz, 1978). Contrary to what was once assumed, this deficit in mathematics achievement is not a result of low intelligence (Dreger & Aiken, 1957). Interestingly, math anxiety is arguably unrelated to overall intelligence. Hembree's (1990) meta-analysis demonstrated that, across different IQ tests, there is a small correlation between math anxiety and IQ (.17; when IQ is treated as a composite score), but the correlation disappears when the quantitative section is removed from the IQ test (.06; i.e., looking solely at verbal intelligence). Thus, the performance deficit associated with math anxiety is argued to be specific to mathematical achievement (as noted by Ashcraft et al., 2007) and is not a general intelligence deficit as was once theorized.

While math anxiety is unrelated to intelligence, it is strongly related to other types of anxieties and, in particular, test anxiety (.52; Hembree, 1990). However, despite the overlap between math anxiety and other types of anxiety, the evidence suggesting that math anxiety is a separate phenomenon is compelling. For instance, correlations between various measures of math anxiety are as high as .85, whereas correlations between math anxiety and other forms of anxiety (e.g., general anxiety and test anxiety) range from .30 to .52 (Ashcraft, 2002). Thus, at least two thirds of the variance in math anxiety is unexplained by test anxiety (the most strongly associated form of anxiety), while two thirds of the variance among different math anxiety assessments is shared (Ashcraft et al., 2007). Furthermore, Faust's (1992) observation of the physiological reactions of high math anxious individuals can also be viewed as persuasive evidence that math anxiety is a separate construct from that of test anxiety. Specifically, Faust (1992) demonstrated that high math anxious individuals experienced increasing physiological arousal (e.g., increases in heart rate) when they performed math tasks of increasing difficulty, but not when they performed a verbal task of increasing difficulty. Interestingly, the low math anxious individuals did not experience increases in physiological arousal in either condition. Given both the large correlations between various measures of math anxiety, the small to moderate correlations between math anxiety and other anxieties, and the compelling physiological evidence from Faust (1992), it is not surprising that researchers now agree that math anxiety is a distinct psychological construct. In fact, Ashcraft (1995, p. 21) states: “it is beyond doubt that math anxiety exists as a bona fide syndrome”.

The correlations between math anxiety and variables such as motivation and self-confidence in math are strongly negative, ranging between -.47 and -.82 (Ashcraft, 2002). As Ashcraft et al. (2007) note, while these relations are not particularly surprising, they are worrisome in terms of personal and educational attainment. Generally speaking, high math anxious individuals express more negative attitudes about math and report less self-confidence in math (Ashcraft et al., 2007). As a consequence, individuals with math anxiety tend to avoid college majors and career paths that depend heavily on math or quantitative skills. Many researchers have suggested that this avoidance behavior acts as a major contributing factor to the achievement deficit associated with math anxiety (e.g., Ashcraft, 2002). Specifically, as a result of their avoiding math-related material, high math anxious individuals are exposed to fewer math courses in school, and spend less time on their math homework (e.g., Fennema, 1989). It is also likely that high math anxious individuals try to avoid mathematical processing in everyday life as well (e.g., doing their own taxes, playing number-related games), consequently taking fewer opportunities to hone their math skills. This avoidance could, of course, have a snowball effect causing the high math anxious individuals to not only cease to attain new mathematical skills beyond their mandatory courses but also to lose proficiency in the skills that they have already learned in their courses.

Research indicates that there are online cognitive consequences of math anxiety as well. Ashcraft and colleagues were the first to note that math anxiety can actually cause people to perform more poorly on math tasks than their abilities should warrant. Ashcraft and colleagues identified speed and accuracy effects of math anxiety on multicolumn addition problems such that the high math anxious participants performed slower and less accurately when standard computerized laboratory tasks were used (Ashcraft & Faust, 1994; Faust et al., 1996). However, when the same stimuli were tested in a paper-and-pencil format designed to minimize the anxiety felt, the high and low math anxious individuals performed equally accurately (Ashcraft & Kirk, 1998). Ashcraft and colleagues thus concluded that, at least for simple math, high and low math anxious individuals are equally competent but, that when under pressure, their anxiety causes the high math anxious individuals to perform below their abilities (Ashcraft & Kirk, 2001). Ashcraft and Kirk (2001) proposed that, when anxious, the high math anxious individuals experience negative thoughts and ruminations and that attending to these thoughts and ruminations requires working memory resources. As a consequence, because anxious individuals devote attention to their intrusive thoughts and worries, they suffer from reduced working memory capacity. They further suggested that math questions that rely more on direct retrieval than on working memory (e.g., basic addition facts; 3+4) would not be affected by math anxiety whereas working memory demanding math questions (e.g., addition with carrying; 18+47) would show large effects of math anxiety.

To test their hypothesis, Ashcraft and Kirk (2001) presented high and low math anxious individuals with addition problems. The questions were divided into "basic fact" questions in which there were two single-digit operands (e.g., 4+3), "medium" questions in which there was a double and a single-digit operand (e.g., 15+2), and "large" questions in which there were two double-digit operands (e.g., 23+11). Furthermore, for half of the questions participants were required to perform a carry operation (making them more working memory demanding). The Participants performed these calculations under high and low verbal working memory loads. They were presented with either two letters (low working memory load) or six letters (high working memory load) before each addition problem, and after participants responded to the problem, they were asked to recall the letters in order. In the more complex (and more working demanding) problems in which the addition problem involved carrying, errors increased significantly more for the high math anxious individuals than the low math anxious individuals. Moreover this was especially true in the high working memory load condition (i.e., six letters). On carry problems, the high math anxious individuals made approximately 40% errors in the high working memory load condition, whereas their low math anxious peers made approximately 20% errors. Both the high and low math anxious groups made approximately 10% errors in the low working memory load condition. In other words, both math anxiety groups performed equally well on the simple (and non-working memory demanding questions). Both math anxiety groups also experienced a performance drop on the complex (and working memory demanding) questions relative to their performance on the simple problems. Importantly, the performance drop experienced by the high math anxious group was larger than that of the low math anxious group. Ashcraft and Kirk (2001) interpreted these findings as evidence that the high math anxious individuals have a decreased working memory capacity relative to their non math anxious peers and thus experience a larger decrement on the working memory demanding (i.e., complex) math questions.

In the aforementioned experiment, Ashcraft and Kirk (2001) also included two measures of working memory capacity: (1) a listening-span task and (2) a computation-span task (Salthouse

& Babcock, 1990). The inclusion of these measures allowed Ashcraft and Kirk (2001) to further test the nature of the reduced working memory capacity in the high math anxious group. Their results indicated that there were no differences between the groups when working memory capacity was measured using the verbal task (i.e., listening span) but there was a between group difference when the arithmetic-based task (i.e., computation span) was used. In order to fully understand Ashcraft and Kirk's interpretation of their data, it is important to first note the distinction between trait and state working memory capacity. While trait working memory capacity refers to one's baseline level of capacity to temporarily remember and manipulate information for a brief amount of time, state working memory capacity refers to one's capacity at any given instance. Trait working memory capacity is thought to remain relatively stable while state working memory capacity can be influenced by exogenous factors (e.g., anxiety). Ashcraft and Kirk (2001) thus concluded that while both groups had the same trait level capacities (because there were no between group differences on the letter span task), the high math anxious individuals suffered from decreased state working memory capacity when mathematical stimuli were used (given the between group differences on the computation span task). In other words, high and low math anxious individuals have the same baseline level of working memory capacity but, as a consequence of their intrusive thoughts and ruminations, the high math anxious individuals have lower state level working memory capacity available to them when they are engaged in mathematical processing. Ashcraft (2002) speculates that, in the case of math anxiety, the negative thoughts and ruminations probably involve preoccupation with one's dislike or fear of math, one's low self-confidence in math, and the like. Ashcraft and Kirk (2001) suggest that math anxiety causes lower math performance because paying attention to these intrusive thoughts acts like a secondary task (analogous to remembering letters), diverting attention away from the math task.

In conclusion, Ashcraft and colleagues found that high and low math anxious individuals perform equally well on measures of simple arithmetic (i.e., single digit addition) indicating that, at least on the most basic of mathematical processing tasks, high and low math anxious individuals appear to be equally competent. Ashcraft and colleagues also find that while the low math anxious participants outperform the high math anxious participants on multi-column addition problems when they are administered via a computer, this is no longer the case if the questions are administered via paper and pencil. Ashcraft and colleagues interpreted these results to mean that high and low math anxious individuals are also equally competent when it comes to multi-column addition problems. However, when under pressure (i.e., during the computerized testing session) the high math anxious participants experience negative thoughts and ruminations, which occupy their working memory. As a result they perform at a level lower than that of which they are technically capable.

Consistent with this theory is Hembree's (1990) observation that effective treatments for math anxiety result in significant improvements in students' math achievement scores. In fact, cognitive-behavior treatments (e.g., systematic desensitization and conditioned inhibition paired with relaxation), have been shown to bring high math anxious students to nearly the performance level shown by students with low math anxiety. Importantly, because the treatments did not involve teaching or practicing math, the improvement could not possibly be due to an increase in math skill. Rather, it appears that the initial assessment of the students' competencies are under representative of their true abilities.

Hembree's findings along with those of Ashcraft and colleagues have been taken to suggest that (1) the negative relation between math anxiety and achievement in mathematics appears to be limited to complex (i.e., working memory demanding) math tasks, and (2) at least some of the high math anxious individual's low achievement is a direct result of the anxiety that they feel. Furthermore, it has been suggested that an anxiety-induced reduction in working memory capacity may be at the root of both of the aforementioned conclusions. Specifically, complex mathematical operations (e.g., addition with carrying) are more working memory demanding than simple operations are (Ashcraft, Donley, Halas, & Vakali, 1992; Hitch, 1978). As such, because math anxiety ties up working memory capacity, the effects of math anxiety are only observed on complex (i.e., highly working memory demanding) math tasks. One important implication of these findings is the theory that math anxiety cannot be caused by competence differences in the building blocks of mathematics, but rather that math anxiety must have some other cause (which has yet to be identified; see Ashcraft & Kirk, 2001 for a discussion on why a low-level competency account of math anxiety is not sufficient).

Hembree's (1990) meta-analysis indicated that alleviating the anxiety felt by high math anxious individuals results in a significant improvement in their math achievement scores. He notes that "Mathematics anxiety reductions by way of [cognitive-behavioral] methods appeared to be related to better performance approaching the level of students with low mathematics anxiety" (Hembree, 1990; p. 43). It is important to note here that Hembree never claims that the anxiety reductions results in equivalent performance of high and low math anxious individuals on math tests but rather it serves to lessen the gap. Thus, it is possible that a competency deficit is present in the high math anxious group that exists above and beyond that caused by the anxiety.

Potential Parallels with Developmental Dyscalculia

As reviewed above, there is a large body of work examining the consequences of math anxiety (e.g., Hembree, 1990), and works by Ashcraft and colleagues (e.g., Ashcraft & Kirk,

2001) have examined how it can cause decreased performance in math tests. However, as mentioned previously, there is surprisingly little work on antecedents to math anxiety. One potentially fruitful avenue for research is to extrapolate from work on other conditions characterized by mathematical deficits: One such condition is Developmental Dyscalculia which is a specific learning disability that affects the acquisition of mathematical skills in children with normal intelligence and age-appropriate school education (e.g., Kucian et al., 2011). Developmental Dyscalculia has been demonstrated to be associated with a wide range of numerical and mathematical processing deficits including problems with complex mathematics as well as on indices of basic number processing (for a review see Ansari & Karmiloff-Smith, 2002). For example, Landerl, Bevan, and Butterworth (2004), demonstrated that children with developmental dyscalculia performed worse than their normal achieving peers on measures of numerical comparison and counting ability. Furthermore, Rotzer et al. (2009) demonstrated that children with dyscalculia also have lower spatial working memory capacity. Importantly, each of the aforementioned skills (i.e., numerical comparison, counting ability, and spatial skills) have been argued to be building blocks of complex mathematics as proficiency in each has been linked to achievement in mathematics (e.g., Geary, 1993; Gelman & Gallistel, 1978; Holloway & Ansari, 2009; Linn & Peterson, 1986). Given the assumption that the skills indexed by these aforementioned tasks provide the foundation upon which higher-level mathematics is built, researchers have suggested that the high-level deficits observed in individuals with Developmental Dyscalculia are intimately tied to their difficulties in these low-level processes (e.g., Landerl et al., 2004; Rotzer et al., 2009). This theory is important in that it suggests the possibility that the math anxiety related deficits that are seen in higher-level math (e.g., addition with carrying; Ashcraft & Kirk, 2001) may also be associated with deficits in lower-level numerical processing skills. Against this background, I set out to investigate whether high math anxious individuals, like those with Developmental Dyscalculia, would exhibit deficits in numerical and spatial processing in addition to their deficits in complex math.

In order to investigate whether math anxiety is associated with deficits in numerical and spatial processing, I compared high and low math anxious individuals' performance on three tasks. To assess numerical processing, I had participants perform enumeration and numerical comparison tasks. To assess spatial processing, I had participants perform a mental rotation task. If in math anxiety, like in Developmental Dyscalculia, the deficits in complex math are associated with lower level deficits, then I expect to see performance decrements in the high math anxious individuals relative to their low math anxious peers even on these simple tasks. Each of the three sets of tasks (enumeration, numerical comparison, and mental rotation) is presented in its own chapter (i.e., chapters 2 through 4 respectively). I then discuss the implications of my findings for our understanding of math anxiety (Chapter 5).

Selection of Participants

For each of the experiments in my thesis (except Experiment 5), participants were invited to participate based on their scores on a measure of math anxiety. Participants were first administered a measure of math anxiety along with several other psychometric tests during a separate mass testing session occurring at the beginning of each semester. Math anxiety was measured using the Abbreviated Math Anxiety Scale (AMAS; Hopko et al., 2003), which is included in the Appendix A. Potential scores on the AMAS range from 9-45 with a higher score being indicative of a higher level of math anxiety. From the pool of participants who were administered the test battery, I selected participants with scores under 20 to constitute the low math anxiety group and participants with scores over 30 to constitute the high math anxiety group. These groups constituted roughly the top and bottom quartiles of the overall distribution (which, of course, was slightly different each term). Eligible participants were then invited into the lab to complete the study but were unaware that they were invited in on the basis of these scores.

Figure 1 represents an example distribution of math anxiety scores from a sample of 2,012 undergraduate students, all of whom were taking at least one psychology class at the time of testing. Specifically, these data were collected between September 2010 and February 2011. Furthermore, this is the distribution from which I recruited the participants who were eligible to participant in Experiment 5. Here the mean AMAS score was 22.6 with a standard deviation of 8.1. Figures 2 and 3 represent the same distribution broken down by gender. For the males (n = 734) the mean AMAS score was 20.2 with a standard deviation of 7.7 (Figure 2). For the females (n = 1, 278) the mean AMAS score was 23.9 with a standard deviation of 8.0 (Figure 3). While each term yielded a slightly different distribution (with different means and standard deviations), the general pattern of the distribution remained consistent.

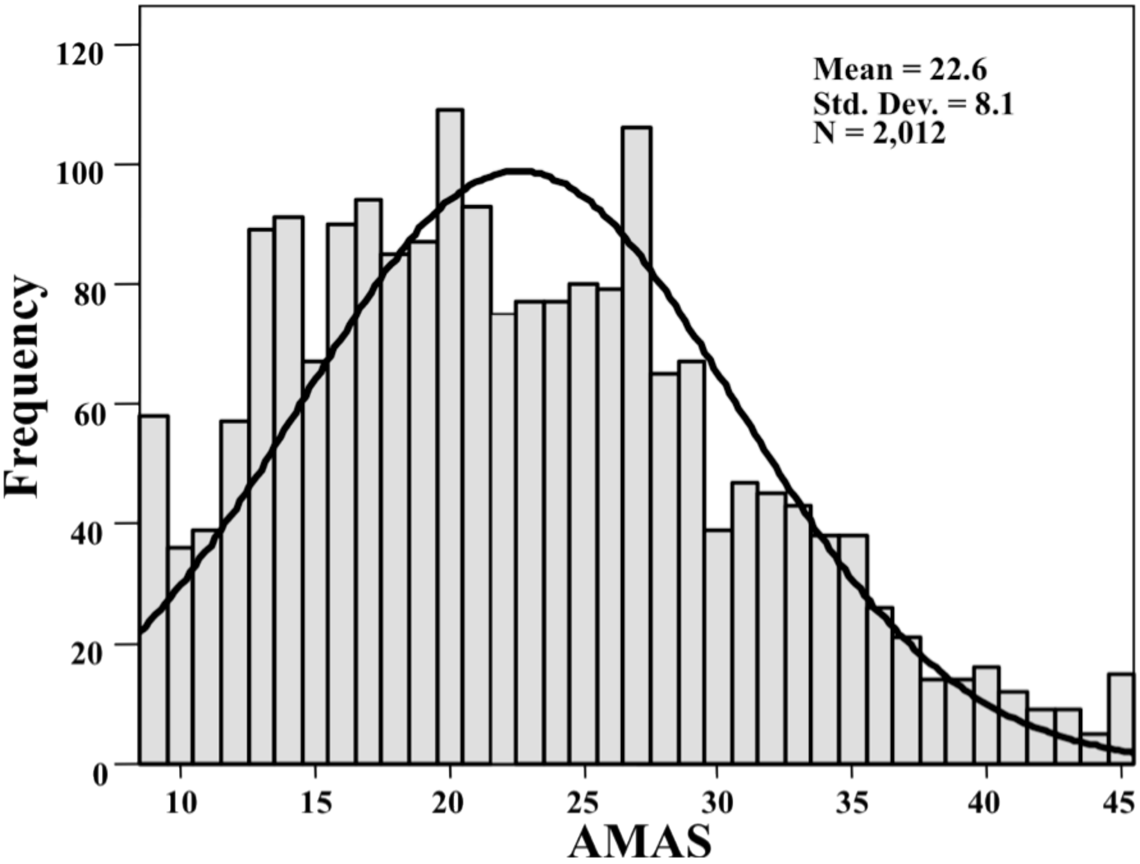

Figure 1. Sample distribution of AMAS scores collapsed across sex.

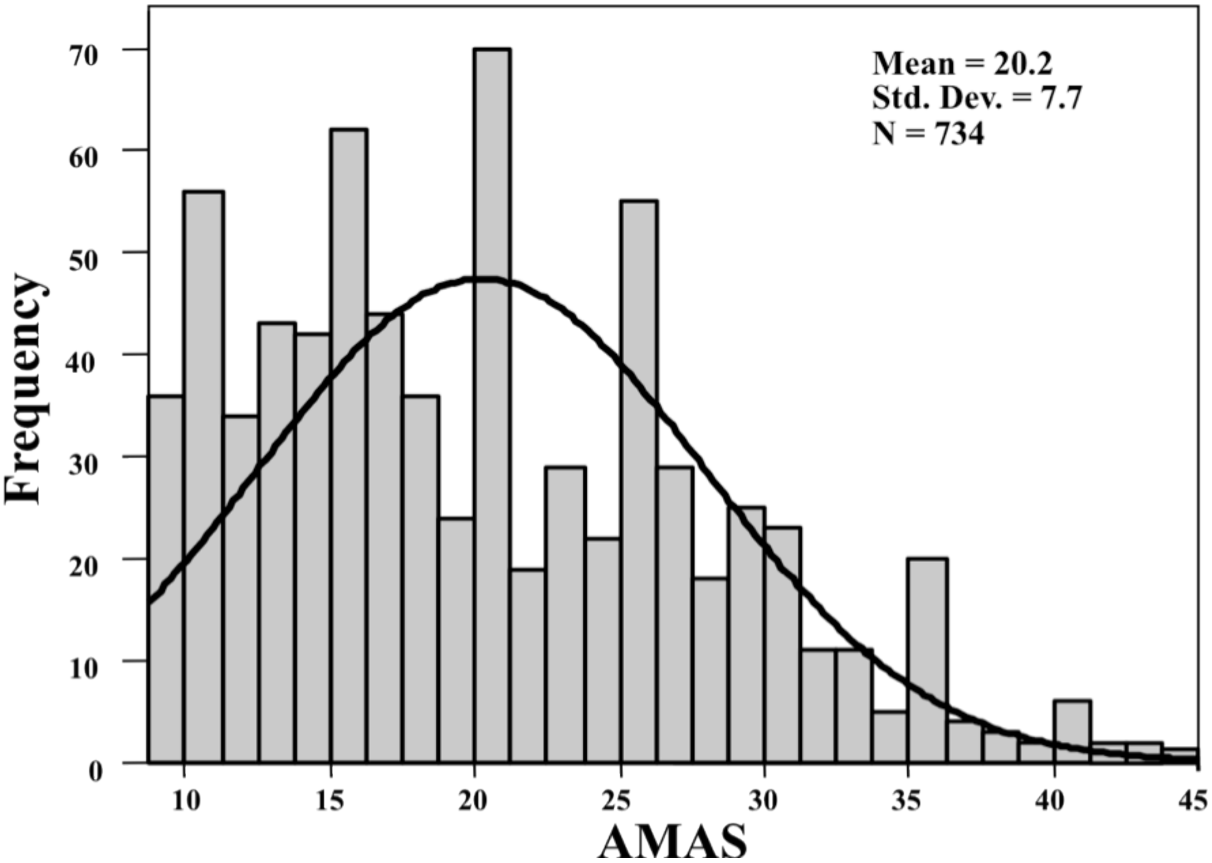

Figure 2. Sample distribution of AMAS scores for male participants.

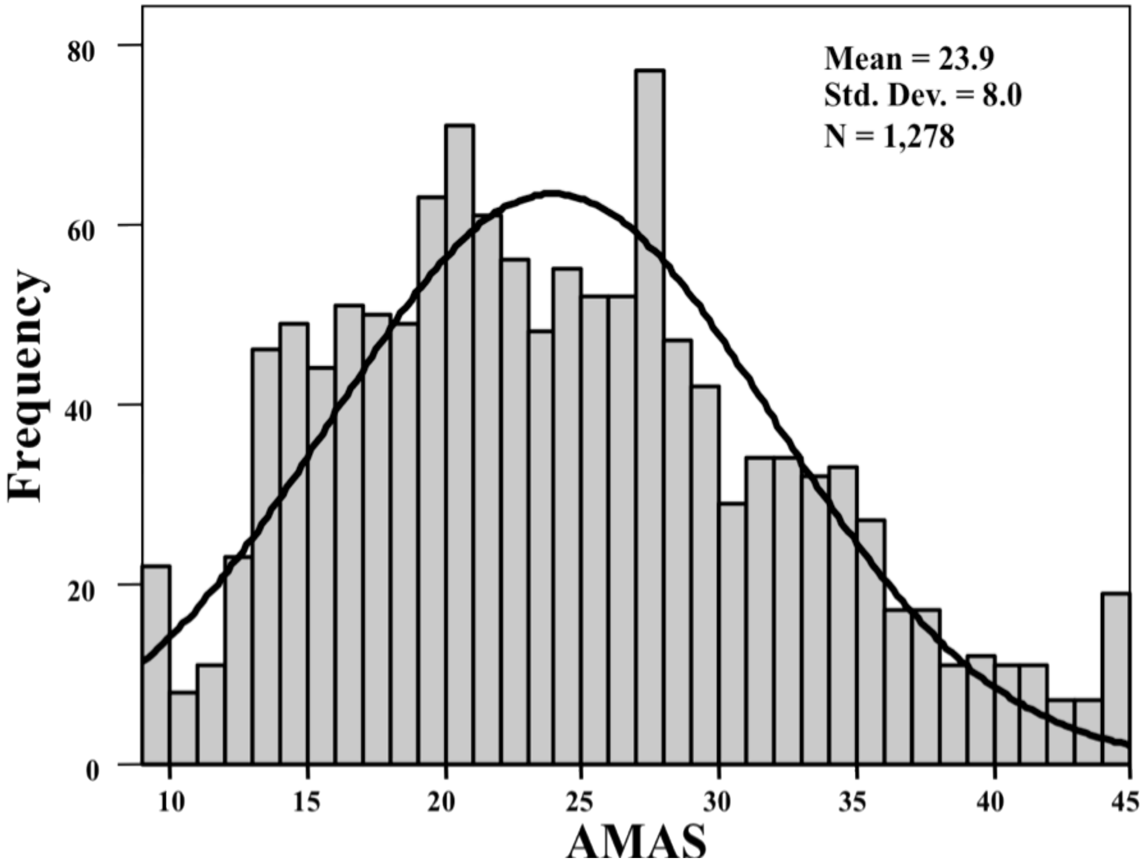

Figure 3. Sample distribution of AMAS scores for female participants.

Chapter 2: Enumeration

The ability to enumerate (count) objects has been identified as a low-level ability that is argued to be a building-block of complex math (e.g., Geary, 1993; Gelman & Gallistel, 1978); meaning that if one does not master counting then they are also expected to experience difficulty with more complex mathematics (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004). One potential explanation for this link between early counting ability and performance on complex math is that counting is considered to be a backup strategy in the acquisition of arithmetic (Siegler 1986; see also Lemaire & Siegler, 1995). When a child is initially presented with 6 objects and then another 4 objects and asked to identify the total number, the child will first count each of the ten objects and conclude that 6 + 4 = 10. However, after many successful counts, re-presentation of the problem leads to a strong activation of the answer, leading children to rely increasingly on the retrieval of the solution instead of counting. If the child repeatedly makes counting errors, then either incorrect solutions may become associated with a specific problem or the correct solutions may become only weakly associated resulting in a longer time needed to generate the correct answer (Siegler, 1986; see also Temple & Sherwood, 2002).

To test whether those high in math anxiety differ from those low in math anxiety with respect to basic counting ability, a visual enumeration task was used. In this task, participants are presented with a display containing multiple objects (typically between 1 and 9) and are instructed to identify the number of objects presented. When enumerating visually presented objects, two distinct patterns of performance emerge. For 1 to 4 items, performance is fast and accurate with only a small increase in response times (RTs) and typically no decrease in accuracy as a function of the increase in the number of stimuli presented. This is commonly called ‘subitizing’ (Kaufman, Lord, Reese, & Volkmann 1942). Conversely, for 5+ items, RTs increase and accuracy decreases as the number of stimuli presented increases (e.g., Trick & Pylyshyn, 1993). This is referred to as counting. Many account for the subitizing versus counting distinction by postulating the existence of two different cognitive mechanisms, one dedicated to small sets of objects and one deployed during the enumeration of large sets of objects (Feigenson, Dehaene, & Spelke, 2004; Revkin, Piazza, Izard, Cohen, & Dehaene, 2008). Furthermore, it is thought by some that the ability to subitize represents a visual process in that we can visually track up to four items in parallel (e.g., Trick & Pylyshyn, 1993; 1994).

The visual enumeration task is commonly used to index the presence/absence of a numerical processing deficit. Even in basic tasks such as dot counting, evidence suggests that children with Developmental Dyscalculia have particular deficits in producing the correct numerosity of objects. Landerl et al. (2004) observed that the increase of response latencies in counting 4 to10 items was steeper for children with developmental Dyscalculia compared to typically developing children. Moreover, even within the subitizing range of 1 to 3 items the RT slopes of children with Developmental Dyscalculia were steeper than the slopes for control children (see also Moellera, Neuburgera, Kaufmann, Landerl, & Nuerka, 2009). Enumeration tasks also have the added benefit that the two numerical processing skills (subitizing and counting) are thought to differentially tap working memory. Specifically, counting is thought to put greater demands on working memory than subitizing is. This allows us to test Ashcraft and colleagues' theory of the online effects of math anxiety. Recall that Ashcraft and colleagues posit that, when engaged in mathematical problem solving, high math anxious individuals experience negative thoughts and ruminations that reduce their state-level working memory capacity. Specifically, they theorize that the more working memory demanding a math task is, the more susceptible performance on that task is to the effects of math anxiety. Thus, a deficit in the counting range but not in the subitizing range would be consistent with Ashcraft and colleagues' theory. More generally, a relation between math anxiety and negative performance on either subitizing or counting would provide evidence that math anxiety is associated with deficits in, at least one of, the building blocks of complex mathematics. Furthermore, this observation would provide evidence that math anxiety is related to performance deficits in skills much more simplistic than what has been observed to date.

Experiment 1

Experiment 1 served to investigate whether math anxiety was associated with a numerical processing deficit in either the subitizing or counting range of a visual enumeration task. Thus, both high and low math anxious participants were presented with displays of squares that ranged in number from 1 to 9 and were simply instructed to say aloud the number of items on the screen.

Methods

Participants. Thirty-two undergraduate students (16 high math anxious and 16 low math anxious) from the University of Waterloo were each granted experimental credit towards a course for participation.

Apparatus and Procedure. The data were collected on a Pentium 4 PC computer running E-Prime 1.1 (Schneider, Eschman, & Zuccolotto, 2001). Stimuli were displayed on a 17" monitor. Vocal responses were collected using a Plantronics LS1 microphone headset. Participants were instructed that when a display appeared, their task was to say aloud the number of squares on the screen. Each trial began with a fixation point that remained on the screen for 500 ms. A display containing from one through nine square boxes was then centrally presented at fixation until a vocal response was detected. Area, density and the individual sizes of stimuli were varied between displays to ensure that none of these variables were correlated with numerosity.

Responses were coded online as correct, incorrect, or mistrial (i.e., microphone mis-triggered). A set of nine practice trials served to familiarize the participant with the task and allowed the experimenter to adjust the microphone sensitivity to minimize spoiled trials. There were four blocks, each block containing the same 78 trials making for a total of 312 observations per subject. Each subject received a different random order of displays within each block. Items were presented six times in each block with the exception of the numbers 3 and 4 which were presented one less and one more time respectively. This programming error was corrected in Experiment 2 and proved to be inconsequential.

Results

Response times (RTs) and errors were analyzed across participants with number of items in the display treated as a within-participants variable and math anxiety group treated as a between-participants variable. Trials on which there was a mistrial (0.7%) were removed prior to analysis.

Response Times and Errors

Trials on which there was an incorrect response (3.8%) were removed prior to RT analysis. The remaining RTs were submitted to a recursive data trimming procedure (Van Selst & Jolicoeur, 1994) using a 2.5 standard deviation cut-off in each cell resulting in an additional 4.1% of the RT data being removed.

Figure 4 depicts the relation between mean RTs (ms) and number of items for the low and high math anxiety groups. Table 1 depicts the relation between enumeration accuracy (proportion error) and number of items presented for the low and high math anxiety groups.

A 9 (Number: 1 to 9) x 2 (Math Anxiety Group: high vs. low) ANOVA yielded a main effect of Number, $F(8,240) = 257$, $MSE = 53752.8$, $p < .01$, and no main effect of Math Anxiety

Group, *F*(1,30) = 2.9, *MSE* = 469401.9, *p* > .05. Critically, there was a Number x Math Anxiety Group interaction, *F*(8,240) = 3.4, *MSE* = 53752.8, *p* < .01. A parallel ANOVA conducted on the error data yielded a main effect of Number, *F*(8,240) = 21.8, *MSE* = 23.1, *p* < .01, and no main effect of Math Anxiety Group, *F*(1,30) = 2.0, *MSE* = 85.8, *p* > .05. Critically, there was a Number x Math Anxiety Group interaction, *F*(8,240) = 2.1, *MSE* = 23.1, *p* < .05. In order to further unpack the Number by Math Anxiety group interaction, I next conducted separate ANOVAs on the subitizing and counting ranges.

Subitizing Range. A 4 (Number: 1 to 4) x 2 (Math Anxiety Group: high vs. low) ANOVA conducted on data within the subitizing range yielded a main effect of Number, *F*(3,90) = 117.9, *MSE* = 1398, *p* < .01, but no effect of Math Anxiety group (*F*<1). Critically, there was no Number x Math Anxiety Group interaction (F<1). A parallel ANOVA conducted on the error data yielded no main effect of Number, *F*(3,90) = 1.6, *MSE* = .01, *p* > .05, and no main effect of Math Anxiety Group (*F*<1). There was no Number x Math Anxiety Group interaction (F<1).

Counting Range. A parallel ANOVA was conducted on data within the counting range (Numbers: 5 to 9). A 5 (Number: 5 to 9) x 2 (Math Anxiety Group: high vs. low) ANOVA yielded a main effect of Number, *F*(4,120) = 169, *MSE* = 37899, *p* < .01, and a marginal effect of Math Anxiety Group, *F*(1,30) = 3.1, *MSE* = 219527, *p* =.09. Critically, there was a Number x Math Anxiety Group interaction, *F*(4,120) = 4.0, *MSE* = 37899, *p* < .01, in which the high math anxiety group responded more slowly as a function of increasing number than the low math anxiety group. A parallel ANOVA conducted on the errors yielded a main effect of Number, *F*(4,120) = 9.5, *MSE* = .01, *p* < .01, such that as number increased, so did the number of errors. There was no main effect of Math Anxiety Group, *F*(1,30) = 2.1, *MSE* = .01, *p* >.05, however,

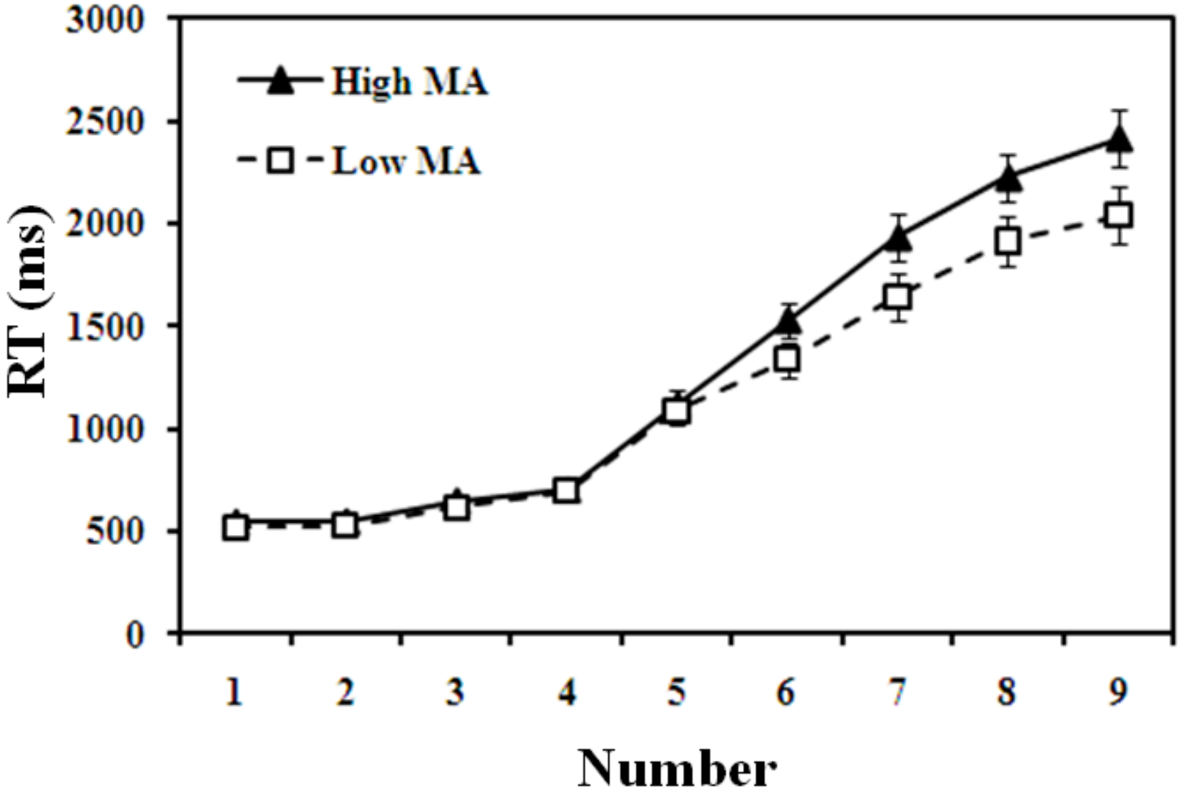

Figure 4. Relation between mean response times (ms) and number of items presented for the high math anxiety and low math anxiety groups (Experiment 1). Error bars represent the standard error of the mean.

	Number								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Low MA	0.0	0.3	0.6	0.4	1.9	5.0	3.2	7.0	9.4
High MA	0.0	0.3	0.4	0.6	2.9	5.8	10.6	10.6	10.5

Table 1. Relation between percentage error and number of items presented for the low math anxiety and high math anxiety groups (Experiment 1).

there was a marginal Number x Math Anxiety Group interaction, *F*(4,120) = 2.2, *MSE* = .01, *p* =.08, in which the high math anxiety group made more errors as a function of increasing number than the low math anxiety group.

Discussion

The primary purpose of Experiment 1 was to determine whether individuals with math anxiety have, in addition to their difficulties with complex mathematics, a difficulty with enumeration, a building block of complex mathematics. Here I clearly show that in the context of a visual enumeration task, high math anxious individuals perform significantly worse in the counting range (5 to 9 items) than their low math anxious peers. This was true in the RT data and also to some extent the error data. Furthermore, this numerical deficit does not extend to the subitizing range (enumeration of 1 to 4 items). The data presented here suggest that individuals with math anxiety do, in fact, have a basic processing deficit in, at least, one of the building blocks of complex mathematics.

Given that counting has been shown to rely more on working memory resources than subitizing does (Tuholski et al., 2001); these data are thus consistent with Ashcraft and colleagues' theory that math anxiety leads to impaired task performance through compromising working memory resources. According to this account, the performance deficits observed in high math anxious individuals are caused by anxiety-induced ruminations that limit the working memory capacity available to perform mathematical tasks. Consequently, only working demanding tasks should show an effect of math anxiety.

Experiment 2

While the data from Experiment 1 are compelling, I decided to conduct an additional experiment to replicate and extend the findings. Specifically, Experiment 2 served to (1) allow me to replicate the previous pattern of data, (2) test the relation between math anxiety, enumeration, and working memory capacity, and (3) rule out the possibility that my effects were driven by a slight coding error in Experiment 1. Thus, Experiment 2 was identical to Experiment 1 with two simple changes. First, I included two backwards span tasks as measures of working memory capacity to allow me to further examine the degree to which the math anxiety x counting interaction is related to working memory capacity. Second, I corrected a slight programming error that had resulted in the unequal presentation of each display. While I saw no reason for this to impact the outcome of the experiment, I nevertheless thought it prudent to replicate Experiment 1's results with each display presented an equal number of times.

Methods

Participants. Twenty-eight undergraduate students (14 high math anxious and 14 low math anxious) from the University of Waterloo participated and were either granted experimental credit towards a course or were paid $6.00. Neither of these participants participated in Experiment 1.

Apparatus and Procedure. The apparatus and procedure were the same as Experiment 1 except for three simple changes. In Experiment 2 there were seven blocks each with 54 stimuli (each number was presented 6 times) making for a total of 378 stimuli. Also, exact responses were coded rather than just whether the response was correct or incorrect. In addition, after performing the enumeration task, two measures of working memory capacity were administered (a backwards digit span task and a backwards letter span task; Weschler, 1997; see Appendix B) in a counterbalanced order. In these tasks participants heard a series of letters or digits presented at a rate of approximately 1 item per second. Participants then had to report the items back to the experimenter in the reverse order of what they were originally presented. The test continued with the addition of one item every second trial until the point at which participants made errors on two trials in a row. Each participant's score was the highest number of digits on which he or she made no errors.

Results

Trials on which there was a mistrial (2.1%) were removed prior to analysis. The data from one participant was discarded and replaced by another participant due to high error rates (63% incorrect).

Response Times and Errors

Figure 5 depicts the relation between mean RTs (ms) and number of items presented for the high math anxiety and low math anxiety groups. Table 2 depicts the relation between enumeration accuracy (proportion error) and number of items presented for the low and high math anxiety groups.

Trials on which there was an incorrect response (5.6%) were removed prior to RT analysis. The remaining RTs were submitted to a recursive data trimming procedure (Van Selst & Jolicoeur, 1994) using a 2.5 standard deviation cut-off in each cell resulting in an additional 4.8% of the RT data being removed.

A (Number: 1 to 9) x 2 (Math Anxiety Group: high vs. low) ANOVA yielded a main effect of Number, $F(8,208) = 182$, $MSE = 4891$, $p < .01$, and a marginal effect of Math Anxiety Group, $F(1,26) = 3.9$, $MSE = 403685$, $p = .06$. Critically, there was a Number x Math Anxiety Group interaction, $F(8,208) = 3.4$, $MSE = 4891$, $p < .01$. A parallel ANOVA conducted on the error data yielded a main effect of Number, $F(8,208) = 14.9$, $MSE = 77$, $p < .01$, no effect of Math Anxiety Group ($F<1$) and no Number x Math Anxiety Group interaction ($F<1$). As Experiment 1, I next conducted separate ANOVAs on the subitizing and counting ranges.

Subitizing Range[1]. A 4 (Number: 1 to 4) x 2 (Math Anxiety Group: high vs. low) ANOVA conducted on data within the subitizing range yielded a main effect of Number, $F(3,78) = 58.0$, $MSE = 4169$, $p < .01$, but no effect of Math Anxiety Group ($F<1$). Critically, there was no Number x Math Anxiety Group interaction (F<1). A parallel ANOVA conducted on the error data yielded a main effect of Number, $F(3,78) = 8.4$, $MSE = .01$, $p < .01$, but no main effect of Math Anxiety Group ($F < 1$). There was no Number x Math Anxiety Group interaction (F < 1).

Counting Range. A 5 (Number: 5 to 9) x 2 (Math Anxiety Group: high vs. low) ANOVA conducted on data within the counting range yielded a main effect of Number, $F(4,104) = 113$, $MSE = 31038$, $p < .01$, and no effect of Math Anxiety Group, $F(1,26) = 2.9$, $MSE = 628879$, $p > .05$. Critically, there was a Number x Math Anxiety Group interaction, $F(4,104) = 2.8$, $MSE = 31038$, $p < .05$, in which the high math anxiety group responded more slowly as a function of increasing number than the low math anxiety group. A parallel ANOVA conducted on the error data yielded a main effect of Number, $F(4,104) = 10.7$, $MSE = .01$, $p < .01$, such that as the number of items increased, so did the number of errors. There was no main effect of Math Anxiety Group and no Number x Math Anxiety Group interaction, (*F*s < 1).

Working Memory and Enumeration

There was no significant difference between the mean backwards digit spans of individuals in the low math anxiety group (6.8) and those in the high math anxiety group (6.3), $t(26) = 1.1$, $p > .05$, but there was a significant difference between the mean backwards letters spans of the individuals in the low math anxiety group (6.1 items) and those in the high math anxiety group (5.0 items), $t(26) = 2.4$, $p < .05$. To determine the relation between working

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[1] **If the subitizing range is defined as 1 to 3 items, the results are qualitatively similar for both experiments.**

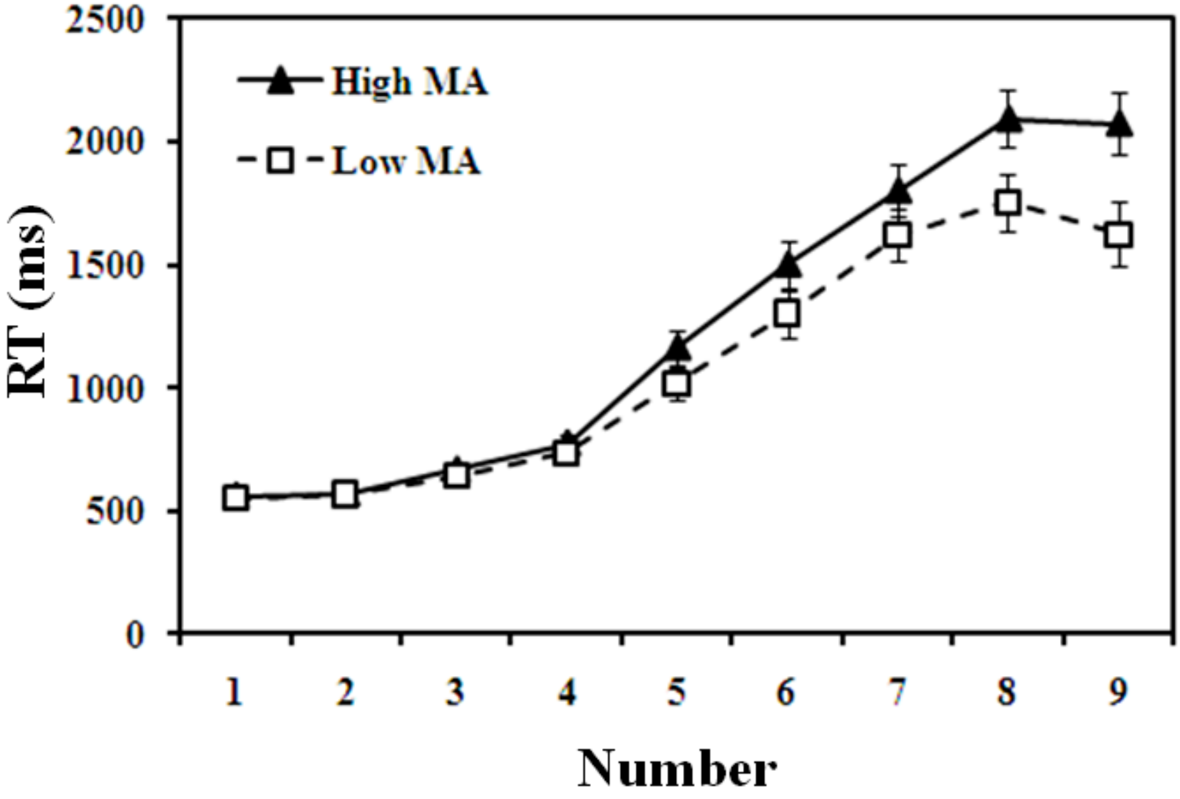
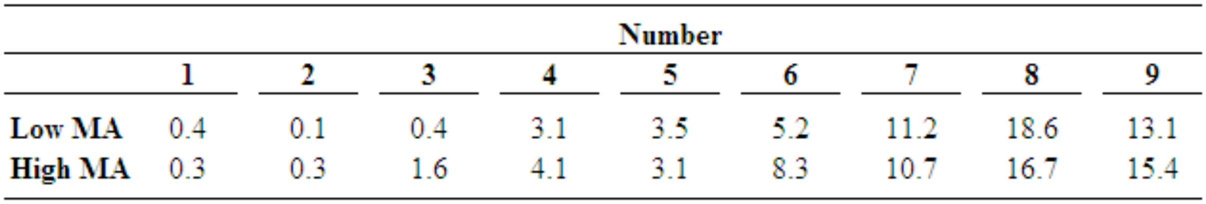


Figure 5. Relation between mean response times (ms) and number of items presented for the high math anxiety and low math anxiety groups (Experiment 2). Error bars represent the standard error of the mean.



	Number								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Low MA	0.4	0.1	0.4	3.1	3.5	5.2	11.2	18.6	13.1
High MA	0.3	0.3	1.6	4.1	3.1	8.3	10.7	16.7	15.4

Table 2. Relation between percentage error and number of items presented for the low math anxiety and high math anxiety groups (Experiment 2).

memory and performance in the visual enumeration task I created a composite working memory capacity measure (WMC) comprising the average of each individual's backwards digit span and backwards letter span scores. I then conducted an analysis parallel to the initial analyses with WMC as a covariate.

Subitizing Range. A 4 (Number: 1 to 4) x 2 (Math Anxiety Group: high vs. low) ANCOVA with WMC as a covariate conducted on response time data within the subitizing range yielded a main effect of Number, $F(3,75) = 3.5$, $MSE = 4228$, $p < .05$, but no effect of Math Anxiety group ($F<1$) and no Number x Math Anxiety Group interaction (F<1).

Counting Range. A 5 (Number: 5 to 9) x 2 (Math Anxiety Group: high vs. low) ANCOVA with WMC as a covariate conducted on the counting range yielded a main effect of Number, $F(4,100) = 4.9$, $MSE = 31373$, $p < .01$, and no effect of Math Anxiety Group, $F(1,25) < 1$. Critically, the aforementioned Number x Math Anxiety Group interaction is no longer significant, $F(4,100) = 1.3$, $MSE = 31373$, $p > .05$, suggesting that working memory capacity differences between the two groups may mediate the different performance for participants in the high and low math anxiety groups.

Summary

The primary purpose of Experiment 2 was to replicate the pattern of data observed in Experiment 1 while simultaneously investigating the relation between math anxiety, counting, and working memory capacity. Critically, the observed pattern in Experiment 1 replicated completely in an independent sample of individuals. As in Experiment 1, the high math anxious individuals demonstrated poorer performance relative to their low math anxious peers in the counting range (5 to 9) of the visual enumeration task but not in the subitizing range (1 to 4). Again, this pattern was present in the RT data and was not contradicted by the error data. These data suggest that, in addition to their difficulties with higher-level mathematical tasks, individuals with math anxiety also exhibit deficits in enumeration. That said, this deficit is specific to the counting range as there was no difference between the high and low math anxious individuals in the subitizing range.

Experiment 2 also served to investigate whether working memory mediates the interaction between math anxiety and number in the counting range. Two critical patterns emerged from this analysis. First, high math anxious individuals had a smaller working memory capacity than low math anxious individuals as indexed by a backwards letter span task. Second, the interaction between math anxiety and number in the counting range was eliminated when the effect of working memory capacity is controlled. Thus, working memory mediates the impact of math anxiety on counting. Furthermore, the results from Experiment 2 also indicate that the math anxiety by counting interaction observed in Experiment 1 was, in no way, driven by the unequal presentation of numerosities.

Discussion

The finding that high math anxious individuals perform worse than their low math anxious counterparts in the counting range but not in the subitizing range is important in two ways. First, the reported findings indicate that the problems experienced by high math anxious individuals exist at a very low level (i.e., numerical processing), substantially below the level previously hypothesized or tested (Ashcraft et al., 2007). The present data therefore demonstrate that math anxiety is not limited to complex mathematical processing and problem solving but also extends to counting objects, a building block of mathematical processing (e.g., Geary, 1993; Gelman & Gallistel, 1978).

Secondly, the dissociation between subitizing and counting is consistent with Ashcraft and colleagues' theory that math anxiety leads to impaired task performance through compromising working memory resources. According to this account, the performance deficits observed in high math anxious individuals are caused by anxiety-induced ruminations that limit the working memory capacity available to perform mathematical tasks. As such, Ashcraft and colleague's account predicts that math anxiety would compromise the working memory dependent counting process, but not the working memory independent process of subitizing (Tuholski et al., 2001). The pattern of data reported above is consistent with this hypothesis.

In addition, the results of Experiment 2 are also consistent with the idea that the processing limitations in math anxiety are intimately linked to working memory capacity limitations. When differences in working memory capacity were controlled for, the difference in performance in the counting range was eliminated. It is important to note that Ashcraft and colleagues posit a *transitory* effect of math anxiety on working memory capacity such that while performing a math related task, a portion of the working memory capacity of high math anxious individuals is tied up handling negative thoughts and ruminations. Thus, when high and low math anxious individuals are not performing math related tasks their working memory capacities should be equivalent. However, Ashcraft and Kirk (2001) do show a small non-math related difference in working memory capacity on a listening span task (Salthouse & Babcock, 1990) between low math anxious (3.7 items) and high math anxious (2.6 items) individuals.

Given that my working memory measure was administered directly following a math task, it is unclear whether the group differences observed here reflect a stable individual difference between low and high math anxious individuals or a transitory effect of anxiety induced by the performance of the math task. Nonetheless, what is clear is that the differences between low and high math anxious individuals in the counting range are mediated by working memory, be it via a stable individual difference or a transitory effect of anxiety induced ruminations.

Conclusion

I have demonstrated, using a visual enumeration paradigm, that individuals scoring high in math anxiety differ from their low math anxious peers on enumeration of items in the counting range but not the subitizing range. Furthermore, I have demonstrated that these differences appear to stem from differences in working memory capacity. These data are taken as evidence that the effect of math anxiety extends beyond the level of mathematical processing and into that of numerical processing thus demonstrating that the problems experienced by math anxious individuals exist at a level lower than was previously thought.

Chapter 3: Numerical Comparison

In chapter 2, I presented low and high math anxious participants with displays containing from 1 to 9 squares and asked them to enumerate the squares. I found that both groups enumerated equally quickly and accurately when there were anywhere from 1 to 4 squares presented. However, as the number of squares increased from 5 to 9, the effect of math anxiety also increased whereby the high math anxious individuals were increasingly slower to enumerate than their low math anxious peers. These data constitute compelling evidence that math anxiety is associated not only with difficulties in complex math, but also with difficulties in, at least one of the building blocks of mathematics - enumeration. In light of the evidence for a numerical processing deficit, it is important to investigate whether math anxiety is associated with deficits in other processes that are also considered to be building blocks of mathematical cognition. Thus, in this chapter I present the results of investigations into the relation between math anxiety and numerical comparison (another task thought to tap numerical processing ability). Specifically, I provide evidence that high math anxious individuals perform more poorly than their low math anxious peers on indices of numerical comparison ability (Experiments 3 and 4).

In the numerical cognition literature it is frequently posited that numerical magnitudes are represented mentally on an internal number line and that this mental number line underlies ‘number sense’, that is, the fundamental ability to efficiently process numerical magnitude information (Dehaene, 1997). Each number is thought to hold a specific place on the number line and share representational features with the numbers close to it. Furthermore, the degree of overlap between numerical magnitude representations is thought to vary between individuals. For example, someone with a precise representation of number will have relatively less overlap

between close numbers compared with an individual who exhibits less precise numerical representations (Dehaene, Dehaene-Lambertz, & Cohen, 1998; Holloway & Ansari, 2009).
The processing of numerical magnitude is most commonly assessed using different variants of the numerical comparison task. For example, participants may be asked to compare a single digit to a fixed standard, or to compare two digits presented simultaneously. While there are different variants of the task, all yield a similar pattern of data. This pattern is known as the numerical distance effect (NDE), referring to an inverse relation between numerical distance and response times. In other words, participants are faster and more accurate at indicating which of two numbers is larger (or smaller) when the numerical distance separating the two numbers is relatively large (e.g., 2 vs. 9), compared to when it is comparatively small (e.g., 8 vs. 6; Dehaene, Dupoux, & Mehler, 1990; Moyer & Landauer, 1967). The most dominant theoretical model of the NDE posits that this effect reflects the relative overlap of numerical magnitude representation on a mental number line, where numbers that are positioned closely to one another share more representational overlap and are thus harder to discriminate during numerical comparison than those that are far apart[2].

Number comparison is a core numerical skill. In fact, McCloskey (1992) takes the ability to select the larger of two numbers to be *the* criterion of understanding numbers. Evidence that one's ability to compare two numbers serves as a building block for complex mathematics comes from developmental data as well as patient studies. For example, Holloway and Ansari (2009)

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[2] **An alternative account is that the NDE indexes the *comparison process* involved in numerical comparison, rather than *numerical representation* per se (e.g., Van Opstal, Gevers, De Moor, & Verguts, 2008). While I acknowledge that there is presently a debate with respect to what the NDE indexes, it is not important for the present investigation whether the NDE is an index of numerical representation or numerical comparison processes. Rather, what is important is that we accept that the NDE is indexing numerical magnitude at a very basic level. Nonetheless, the present thesis will adhere to the widely accepted view that the NDE indexes overlap of numerical representations in order to facilitate the communication of the findings.**

demonstrated that there exists a relation between one's ability to compare Arabic digits and their fluency in mathematics, such that poorer comparison (evidenced by a larger NDE) is associated with poorer fluency in mathematics (see also Mundy & Gilmore, 2009). Even more compelling are longitudinal data collected by De Smedt, Verschaffel, and Ghesquiere (2009) showing that accuracy and speed of number comparison predict future math achievement. In addition to the developmental data, evidence for the link between number comparison and math ability also comes from observations from special populations. For example, children with Developmental Dyscalculia have been shown to perform more poorly on tasks of basic numerical magnitude processing relative to typically developing children (Landerl, Bevan, & Butterworth, 2004; Mussolin, Mejias, & Noel, 2010; Paterson, Girelli, Butterworth, & Karmiloff-Smith, 2006; Simon, Bearden, Mc-Ginn, & Zackai, 2005).

The Present Investigation

In light of the above data suggesting that the NDE can serve as an indicator of individual differences in the representation of numerical magnitude, in the following experiments I examined both high and low math anxious individual's performance on two symbolic variants of the numerical comparison task. If high math anxious individuals do, in fact, have less precise representations of numerical magnitude than their low math anxious peers, then high math anxious individuals should exhibit a larger NDE than those low in math anxiety.

Experiment 3

Methods

Participants. Forty-eight undergraduate students (24 high math anxious and 24 low math anxious) from the University of Waterloo participated and were either granted experimental credit or were paid $5.00. As in the previous experiments, math anxiety was measured using the

AMAS (Hopko et al., 2003). In addition, as with the previous experiments, I selected participants with scores under 20 to constitute our low math anxious group and participants with scores over 30 to constitute my high math anxious group.

Stimuli, apparatus and procedure. The data were collected on a Pentium 4 PC computer running E-Prime 1.1 (Schneider, Eschman, & Zuccolotto, 2001). Stimuli were displayed on a 19" monitor. Numerical distance was measured using a comparison-to-a-standard variant of the numerical comparison task. Each trial began with a fixation point that remained on the screen for 500 ms. A display containing a single Arabic digit in 35-point Arial font was presented at fixation. Numbers ranged from 1 to 4 and from 6 to 9. Participants were told to identify whether the presented number was lower than '5' or higher than '5' by pressing the "A" key to denote lower and the "L" key to denote higher. The number remained on the screen until the participants made a button press. There were a total of 160 trials. The numerical distance between the stimuli and the number 5 ranged from 1 to 4, with 40 comparison trials total per distance. Stimulus displays were presented randomly.

Results

RTs and errors were analyzed across participants with numerical distance as a within-subject variable and math anxiety group as a between-subject variable. Trials on which there was an incorrect response were removed prior to RT analysis (2.7%). The remaining RTs were submitted to a 2.5 standard deviation recursive outlier removal procedure, resulting in the removal of an additional 4.0% of the data. See Figure 6 for the RT data, and Table 3 for the error data.

A 4 (Distance: 1 to 4) x 2 (Math Anxiety Group: low vs. high) ANOVA conducted on the RT data yielded a main effect of Distance, *F*(3,138) = 28.9, *MSE* = 1087.3, $p < .01$, and a main

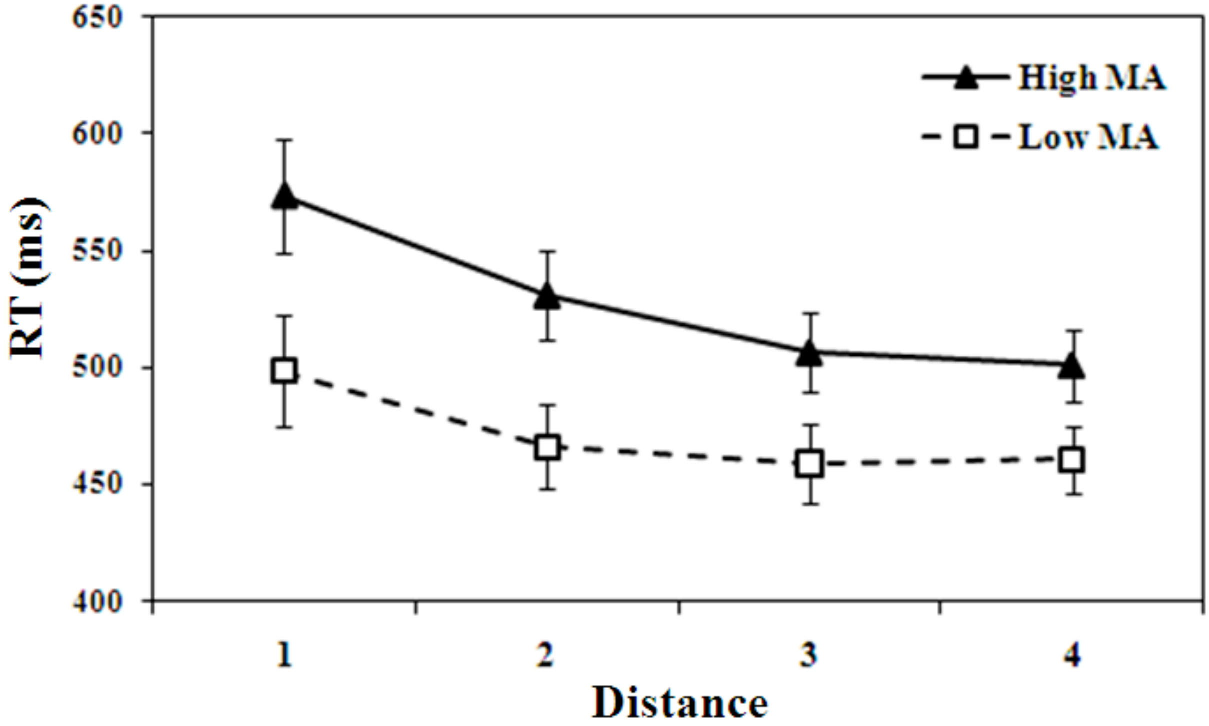

Figure 6. Math Anxiety Group by Numerical Distance in the comparison to a standard variant (Experiment 3). The error bars depict the standard error of the mean.

	Distance			
	1	**2**	**3**	**4**
Low MA	7.3	3.7	2.5	2.1
High MA	9.3	5.6	6.0	6.1

Table 3. Relation between percentage error and Distance for the low math anxiety and high math anxiety groups in the comparison to a standard variant (Experiment 3).

effect of Math Anxiety Group, *F*(1,46) = 5.0, *MSE* = 312233.4, $p < .05$, whereby the high math anxious group were slower overall to respond than the low math anxious group. Importantly, there was a Distance x Math Anxiety Group interaction, *F*(3,138) = 2.8, *MSE* = 1087.3, $p <.05$, whereby the effect of distance was larger for the high math anxiety group than the low math anxiety group. A t-test on the slopes of the distance effects for high and low math anxious participants supported this interpretation by revealing a steeper NDE for the high math anxious group than the low math anxious group, *t*(46) = 2.0, $p = .05$. A parallel analysis on the error data yielded a main effect of Distance, *F*(3,138) = 15.0, *MSE* = 126.8, $p < .01$, no main effect of Math Anxiety Group, $F < 1$, and no interaction, *F*(3,138) = 1.1, *MSE* = 126.8, $p > .05$

Discussion

The primary purpose of the present experiment was to determine whether individuals high in math anxiety represent numbers differently than their low math anxious peers. Here it appears that, in the context of a comparison to a standard task, numerical distance interacts with math anxiety whereby the effect of numerical distance is larger for high math anxious than low math anxious individuals. These data suggest that the high math anxious individuals have less precise representations of number than their low math anxious counterparts.

While the comparison to a standard variant of the numerical comparison task is commonly used, there are nonetheless several variations of this task. For example participants may also be asked to compare two simultaneously presented digits or to state whether two simultaneously presented digits are the same or different. While these variants all produce the characteristic NDE, Maloney, Risko, Preston, Ansari, and Fugelsang (2010) have recently found that some variants of the numerical comparison task are more reliable than others. Specifically, Maloney et al. (2010) reported that, when using Arabic digits, the NDE that arises in the comparison to a standard variant, the one used in Experiment 3, is not statistically reliable (i.e., the size of an individual's NDE in the first half of the experiment does not predict the size of their NDE in the second half of the experiment). The simultaneous presentation variant (two digits presented side by side) however, was statistically reliable. Furthermore, Maloney et al. (2010) identified that the NDE that arises in the comparison to a standard variant does not strongly correlate with the NDE found using a simultaneous comparison variant, suggesting that these variants might be differentially indexing numerical magnitude. Against this background, I wanted to replicate the Math Anxiety Group by NDE interaction using the more reliable simultaneous presentation variant of the numerical comparison task.

Experiment 4

Methods

Participants. Forty-four undergraduate students (22 in each math anxiety group) from the University of Waterloo participated and were either granted experimental credit or were paid $6.00.

Stimuli, apparatus and procedure. The stimuli, apparatus, and procedure (i.e., trial composition) were identical to that used in Experiment 3, with the following exception. Following the 500ms fixation point, a display containing two Arabic digits in 35-point Arial font was presented. Participants were told to identify which of the two numbers was numerically larger by pressing the "A" key to denote that the number on the left was larger and the "L" key to that the number on the right was larger.

Results

RTs and errors were analyzed across participants with Distance as a within-subject factor and Math Anxiety Group as a between-subject factor. Trials on which there was an incorrect response were removed prior to RT data analysis (3.2%). The remaining RTs were submitted to a 2.5 standard deviation recursive outlier removal procedure (Van Selst & Jolicoeur, 1994), resulting in the removal of an additional 3.1% of the data.

A 4 (Distance: 1 to 4) x 2 (Math Anxiety Group: low vs. high) ANOVA conducted on the RT data yielded a main effect of Distance, $F(3,126) = 53.3$, $MSE = 574.1$, $p < .01$, and no main effect of Math Anxiety Group, $F < 1$. Importantly, replicating the key finding in Experiment 3, there was a Distance x Math Anxiety Group interaction, $F(3,126) = 2.7$, $MSE = 574.1$, $p <.05$, whereby the NDE was larger for high math anxious than low math anxious participants. A t-test on the slopes of the distance effects for the high and low math anxiety groups revealed a marginally steeper NDE for the high math anxious group than the low math anxious group, $t(42) = 1.8$, $p = .07$. A parallel analysis on the error data yielded a main effect of Distance, $(3,126) = 32.3$, $MSE = 111.3$, $p < .01$, no main effect of Math Anxiety Group, and no interaction, F's < 1. See Figure 7 for the RT data and Table 4 for the error data.

Experiments 3 and 4 Combined Analyses

It is possible that the comparison to a standard variant of the number comparison task used in Experiment 3 may require more working memory resources than the simultaneous presentation variant used in Experiment 4, given that in the comparison to a standard variant one must keep the standard (5) online in working memory while performing the task. Ashcraft and colleague's account of math anxiety would predict that, if the math anxiety by NDE interaction is driven by an online reduction in working memory capacity, then the more working memory demanding variant (i.e., the comparison to a standard variant) should show a larger interaction with math anxiety than the simultaneous presentation variant. Thus, I conducted a combined analysis of Experiments 3 and 4, including Task Variant as a between participants factor. If a

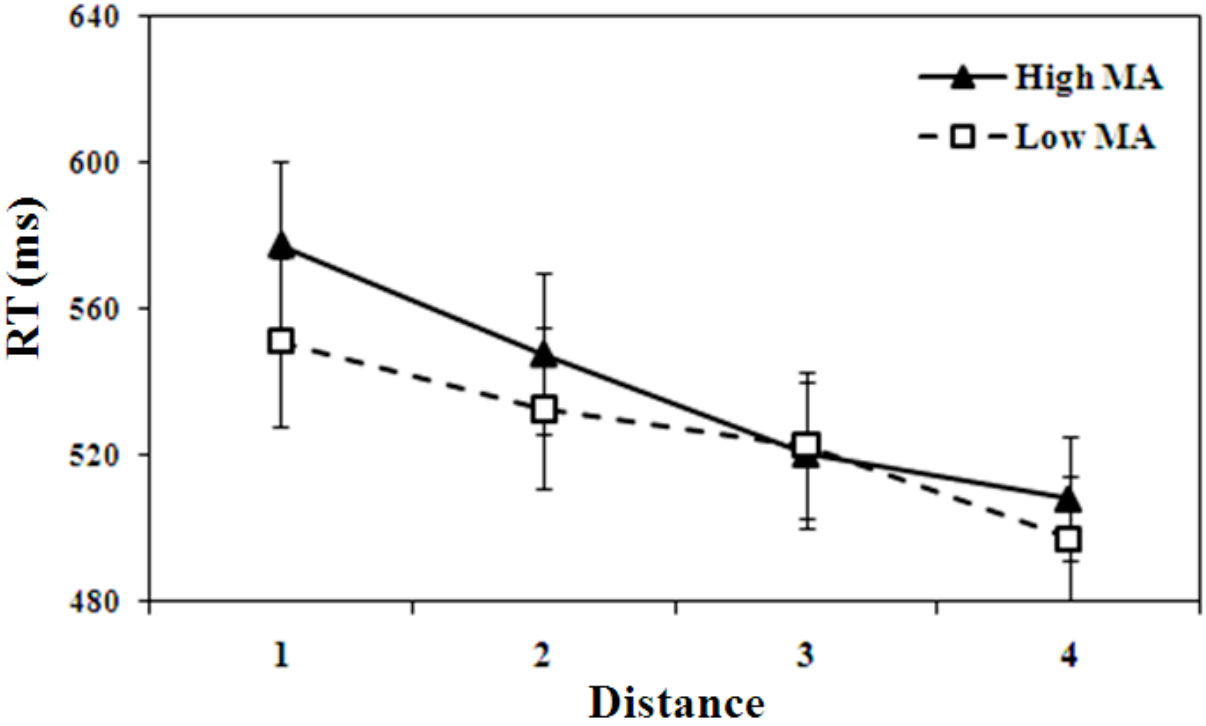

Figure 7. Math Anxiety Group by Numerical Distance in the simultaneous presentation variant (Experiment 4). The error bars depict the standard error of the mean.

	Distance			
	1	**2**	**3**	**4**
Low MA	7.3	5.2	2.3	1.6
High MA	7.6	5.8	1.8	1.1

Table 4. Relation between percentage error and Distance for the low math anxiety and high math anxiety groups in the simultaneous presentation variant (Experiment 4).

decrease in working memory capacity is what is driving the Math Anxiety Group by Distance interaction, then I should see a larger effect of Math Anxiety Group on Numerical Distance in the comparison to a standard variant than in the two-digit comparison variant. This 2 (Math Anxiety Group: high vs. low) x 2 (Task Variant: comparison to a standard vs. two-digit comparison) x 4 (Distance: 1 to 4) ANOVA yielded a main effect of Distance, $F(3,264) = 71.2$, $MSE = 842.4$, $p < .01$, a marginal effect of Math Anxiety Group, $F(1,88) = 3.3$, $MSE = 33396.2$, $p = .07$, no Math Anxiety Group x Task Variant interaction, $F(1,88) = 1.4$, $MSE = 33396.2$, $p > .05$, and a Distance x Math Anxiety Group interaction, $F(3,264) = 4.7$, $MSE = 842.4$, $p <.01$. A follow up t-test on the slopes revealed that the slope of the NDE was larger for high math anxious than low math anxious participants, $t(90) = 2.6$, $p = .01$. Importantly, there was no Distance x Math Anxiety Group x Task Variant interaction, $F < 1$. Therefore, the results of the combined analysis indicate that the Math Anxiety Group x Distance interaction does not vary as a function of Task Variant, and as such, does not vary as a function of the working memory demands of the task.

Summary

The results from Experiment 4 clearly replicated the math anxiety by NDE interaction from Experiment 3. Therefore, in a second experiment, using a different variant of the symbolic comparison task, with a new set of high and low math anxious participants, we demonstrate that size of the NDE is larger for high math anxious participants than for low math anxious participants. Furthermore, the size of the math anxiety by NDE interaction did not vary as a function of task type even though the comparison to a standard task arguably requires more working memory resources.

Discussion

In the present investigation, I examined the numerical comparison abilities of high math anxious individuals relative to their low math anxious peers. In two independent experiments, I found that the high math anxious individuals had a larger NDE than their low math anxious peers in a symbolic numerical comparison task. Furthermore, this effect did not seem to vary as a function of the differential working memory demands of the task[3]. As the NDE is thought to provide an index of the precision of one's mental number line, these data suggest that high math anxious individuals have less precise representations of number than do low math anxious individuals. This finding is important in at least two ways: First, as was the case with Experiments 1 and 2, the reported findings indicate that the deficits exhibited by high math anxious individuals are not limited to complex mathematical processing and problem solving. Rather, I present another example of a building block of complex mathematics on which high math anxious individuals perform more poorly than their low math anxious peers.

Secondly, evidence suggests that the math anxiety effects in the numerical comparison task are not likely to be driven by a high math anxious individuals' decreased working memory capacities as the size of the Math Anxiety Group by Distance interaction did not vary as a function of the working memory demands of the comparison tasks (see also footnote #3). Thus, the observation that high math anxious individuals have a larger NDE than low math anxious individuals calls for an amendment to Ashcraft and colleagues' theory, which suggests that all

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[3] **Other studies in my lab have examined the effects of an increased verbal working memory load on numerical comparison. Specifically, in the comparison to a standard task, increases in verbal working memory load (i.e., the need to remember 6 letters) leads to an increase in the size of the NDE. Conversely, the same working memory load, applied to the simultaneous presentation task, results in an elimination of the NDE. Thus, these data suggest that while the interaction between math anxiety and the NDE in the comparison to a standard task may be the result of an online reduction in working memory capacity in the high math anxious group, the same cannot be said for the simultaneous presentation task.**

math anxiety related deficits are due to an online reduction in working memory capacity. Rather, the theory should now state that while it appears that most of the math anxiety related deficits can be accounted for by reduced online working memory capacity, at least one deficit (i.e., the larger NDE) seems to exist outside of the context of these working memory reductions.

Conclusion

Experiments 3 and 4 indicated that high math anxious individuals have larger NDEs than their low math anxious peers. Given that the NDE is thought to provide an index of the precision of an individual's mental number line (e.g., Holloway & Ansari, 2009), these data are taken to suggest that high math anxious individuals have less precise representations of number than their low math anxious peers.

Chapter 4: Spatial Processing

In Chapters 2 and 3, I presented data indicating that high math anxious participants performed worse than their low math anxious peers on two distinct types of tasks that are thought to tap aspects of numerical processing that are considered building block of complex math. Specifically, in Chapter 2, I demonstrated that high math anxious individuals perform worse on measures of enumeration when there are 5 or more objects presented. In Chapter 3, I demonstrated that high math anxious individuals have larger NDEs than their low math anxious peers. These data are taken as evidence that math anxiety is associated not only with difficulties in complex math but also with difficulties in basic numerical processing. In light of the evidence for a numerical processing deficit, I felt it important to investigate whether math anxiety is associated with deficits in other non-numerical processes that have also been linked to achievement in math. Thus, in this chapter I present the results of investigations into the relation between math anxiety and spatial processing. Specifically, I provide evidence that high math anxious individuals both report lower aptitude for spatial processing and perform more poorly on a measure of mental rotation ability than their low math anxious peers (Experiments 5 and 6).

Spatial processing refers to skill in representing and transforming symbolic, non-linguistic information (Gardner, 1983). The link between spatial processing and math abilities has been widely studied (e.g., Akerman & Dykman, 1995; Rourke, 1993; Rourke & Finlayson, 1978; Share, Moffitt, & Silva, 1988; Linn & Petersen, 1986; Casey, Perez, & Nuttall, 1992). However, although there is a plethora of research investigating this link, the story is, unfortunately, anything but simple. There certainly seems to be some link between poor spatial skills and low math achievement. For example, Rotzer and colleagues (2009) demonstrated that children with Developmental Dyscalculia presented with both lower performance on a spatial working memory task (the Corsi Block Tapping task) and decreased neural activation in the brain regions associated with spatial working memory (i.e., right intraparietal sulcus, right insula and right inferior frontal lobe) relative to normal achieving children. Rotzer et al. (2009) argued that poor spatial working memory processes may inhibit the formation of spatial-number representations (i.e., the mental number line) in addition to the storage and retrieval of arithmetic facts which form the basis for complex math. In a similar vein, Assel, Landry, Swank, Smith, and Steelman, (2003) argued that children, when first learning to count, often use arrays of objects to represent the cardinal value of the sets to be counted. These spatial representations of the counting task help children to regulate their counting (e.g., keep track of the number of items they have already counted and the items yet to be counted). Thus, a spatial deficit would lead to a counting deficit which would lead to problems further down the line (i.e., in more complex mathematics).

Some researchers have suggested that the relation between spatial skills and achievement in math only exists early in development (e.g., Hartje, 1987) and for underachieving populations (as evidenced by the research on Developmental Dyscalculia). For example, Hartje (1987) argued that spatial skills are important for learning to count and for the solution of arithmetic problems but that this importance declines as facts, procedures, and rules, become subsumed by propositional knowledge and become directly retrievable from memory. Thus, while spatial skills are necessary for the development of number knowledge, they may not be related to mathematical achievement when tested in adults. Nonetheless, given the clear link between spatial skills and populations who tend to underachieve in math (e.g., Developmental Dyscalculics), in the present investigation, I set out to determine if high math anxious individuals also demonstrate a deficit in spatial processing. Specifically, in Experiment 5, I assessed the relation between participant's levels of math anxiety and a self-report of their aptitude for processing spatial information. In Experiment 6, I tested whether the ability to mentally rotate objects (a spatial skill) was related to math anxiety. I also investigated the relations between spatial ability, math achievement, and math anxiety in adults.

Experiment 5

To determine whether there is indeed a relation between self-reported spatial processing ability and math anxiety, I first conducted an online study composed of questionnaires assessing spatial processing ability and math anxiety. Spatial processing was assessed via the Spatial scale of the Object Spatial Imagery Questionnaire (OSIQ; Blajenkova et al., 2006; Appendix C), a 30-item questionnaire consisting of two 15-item scales (object imagery and spatial imagery). The Spatial scale is a measure of individuals' aptitude and preference for processing and imaging schematic images and the spatial relations between objects, whereas the object scale is a measure of individuals' aptitude and preference for imaging colorful, picture-like images. The spatial and object subscales of the OSIQ are typically administered together. Thus, even though I had no theoretical grounds on which to include the object subscale, I nonetheless decided to include it for exploratory purposes. Experiment 5 therefore tested the relation between self-reported spatial processing skills and math anxiety as well as self-reported object imagery ability and math anxiety. A negative correlation between math anxiety and the spatial processing subscale of the OSIQ, indicating that higher math anxiety is associated with poorer spatial processing scores, would be evidence for an association between math anxiety and poor spatial processing.

Methods

Participants. One hundred and eighteen (80 female, 38 male) undergraduate students from the University of Waterloo participated and were granted experimental credit in return. Unlike the previous Experiments in this thesis, in this Experiment, participants were not selected based on their AMAS scores. Instead, everyone in the participant pool was eligible to participate and math anxiety score (i.e., AMAS score) was treated as a continuous variable rather than as a dichotomous variable.

Procedure. Participants completed both the OSIQ (Blajenkova et al., 2006) and the AMAS (Hopko et al., 2003) online. They also identified their gender. Thirteen participants skipped one or two items on one or more questionnaires, resulting in 21 missing item scores in the full data set (less than 0.04% of total data). Missing scores were replaced by scale means. Individuals' total scores were computed for the AMAS; individuals' mean scores were computed for the OSIQ Spatial scale and the OSIQ Object scale. This represents the standard way that scores on these measures are calculated in the literature.

Results

In this sample, scores on the AMAS ranged from 9 to 45 (which represents both the lowest and highest possible scores) and the mean score was 19.8. Thus, the distribution of AMAS scores in this sample is representative of the overall distribution obtained in mass testing (see Figure 1).

To test the prediction that math anxiety is related to scores on the OSIQ Spatial subscale, a Pearson correlation was computed between OSIQ-Spatial scores and AMAS scores. As predicted, participants' AMAS scores were significantly negatively correlated with their OSIQ-Spatial scores, $r(116) = -.50$, $p < .01$, (see Figure 8). These data suggest that as math anxiety increases the ability to spatially represent data decreases. Interestingly, AMAS scores were also

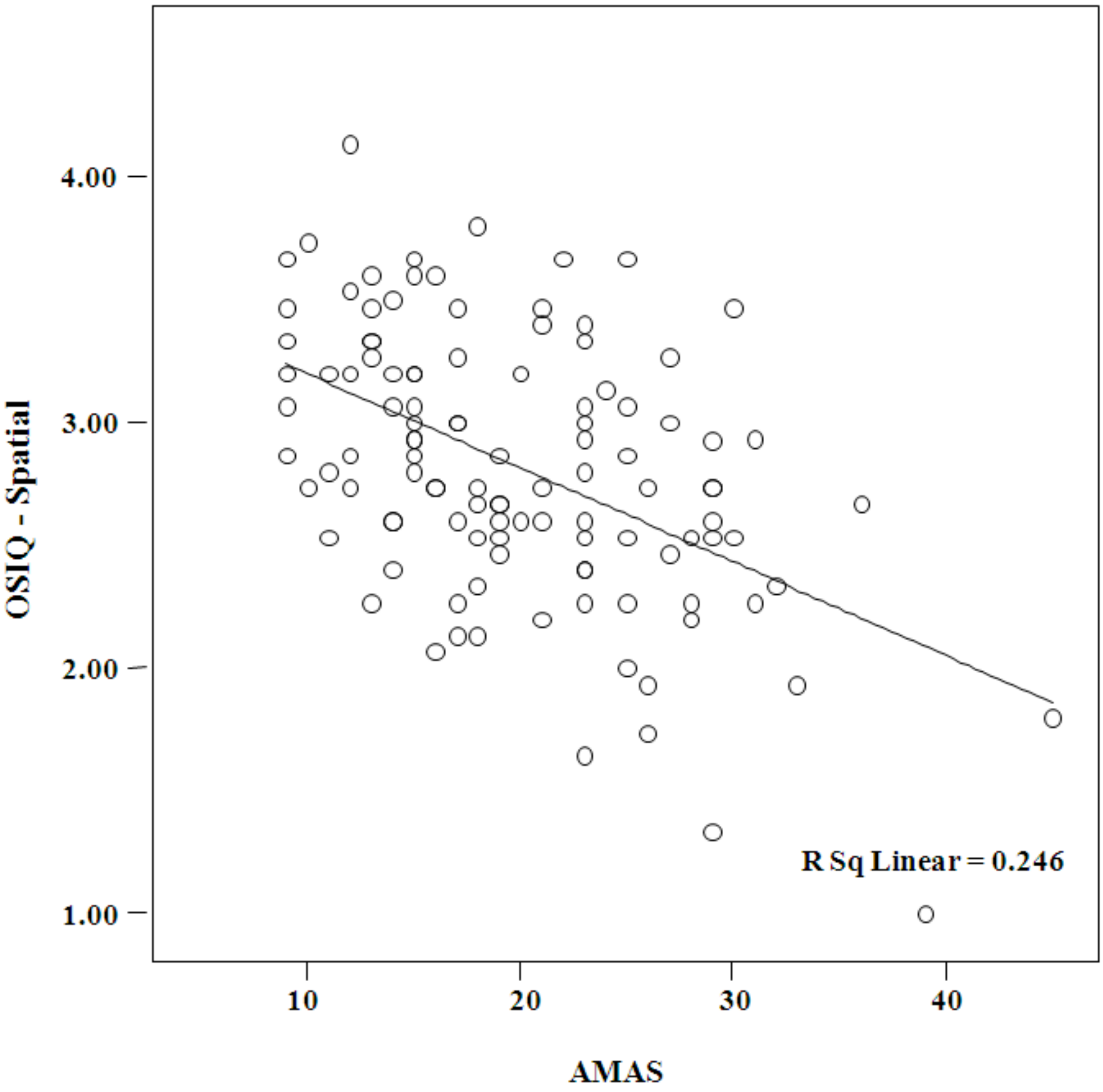

Figure 8. Relation between AMAS scores and OSIQ-Spatial scores (Experiment 5).

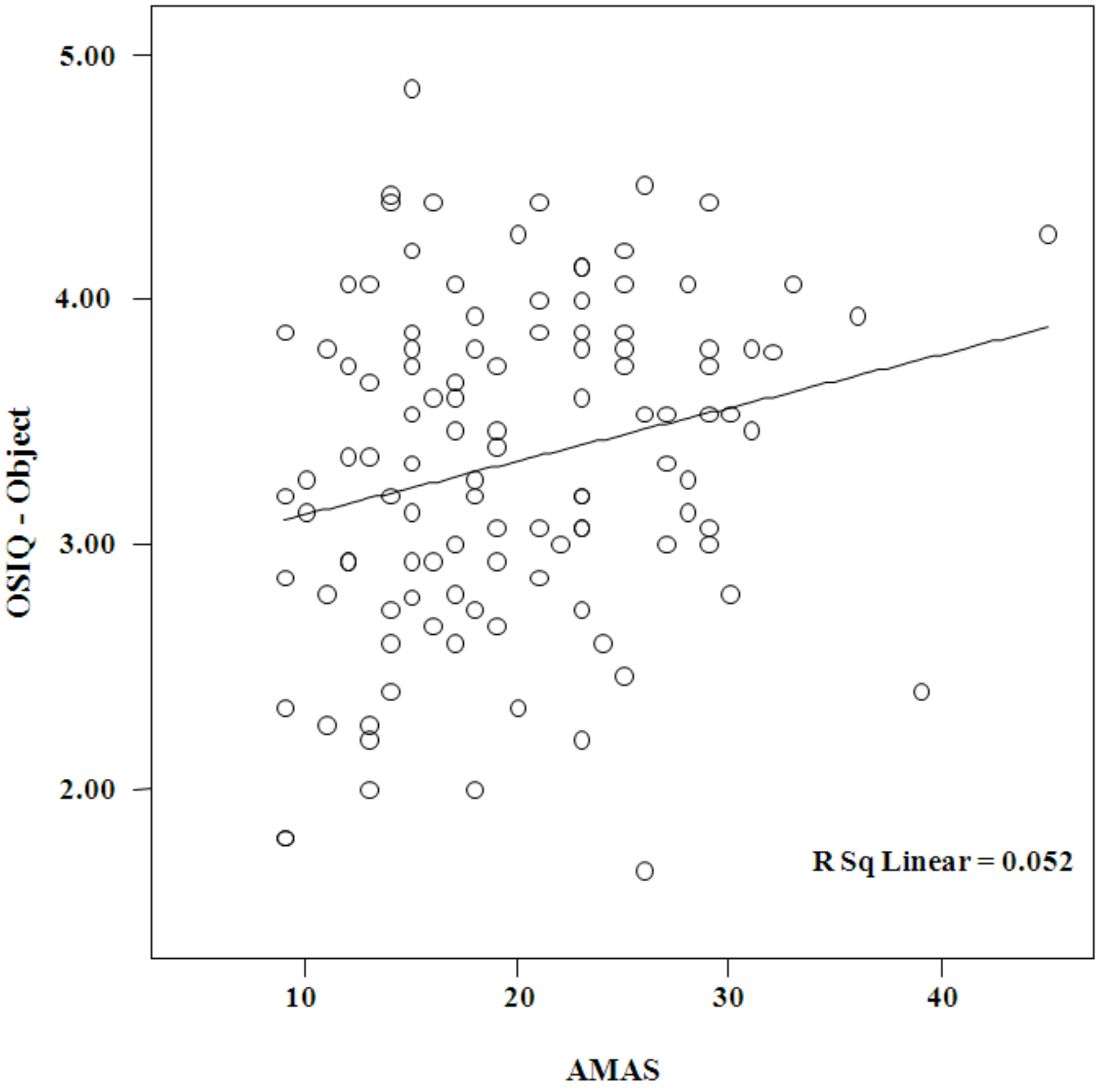

Figure 9. Relation between AMAS scores and OSIQ-Object scores (Experiment 5).

significantly positively correlated with OSIQ-Object scores, *r(116)* = .278, *p* < .01, suggesting that as math anxiety scores increase, so does the ability to vividly represent objects (see Figure 9).

A strong sex difference has been previously reported in the literature in both math anxiety (such that females report higher levels of math anxiety than males; Hembree, 1990) and spatial processing (such that males outperform females on measures of spatial processing; e.g., Linn & Petersen, 1986). Indeed, in my sample, females scored higher (M= 21.14) on the AMAS than males (M= 16.78), *t*(116) = 3.20, *p* < .01; and males scored higher (M= 3.16) on the OSIQ Spatial Scale than females (M= 2.69), *t* (116) = 4.78, *p* < .01, indicating that in my sample, females indeed reported higher levels of math anxiety and lower spatial processing abilities than males. As such, I re-conducted my analysis controlling for sex to rule out the possibility that the relation between math anxiety and self-reported spatial processing ability is attributable to a sex difference. I thus computed a partial correlation between OSIQ-Spatial scores and AMAS scores controlling for sex. Again, participants' AMAS scores were negatively correlated with their OSIQ-Spatial scores, *r(116)* = - .449, *p* < .01. Thus, the relation between math anxiety and spatial processing cannot be explained by recourse to both factors being related to sex. Interestingly, after partialling out spatial processing ability from math anxiety scores, there was no longer a sex difference in math anxiety, *t*(116) = 1.00, *p* > .05.

Discussion

The results of Experiment 5 demonstrate a clear relation between self-reported levels of math anxiety and self-reported levels of spatial processing ability. Specifically, high math anxious individuals reported lower aptitude for spatial processing than their low math anxious peers. These data indicate that high math anxious individuals at least *perceive* themselves as being worse at spatial processing than do low math anxious individuals. Furthermore, there was a significant correlation between self-reported levels of math anxiety and self-reported levels of object imagery ability. These data indicate that high math anxious individuals also perceive themselves as being more skilled at conjuring up vivid images in their mind's eye than do their low math anxious peers.

The sex difference in math anxiety is typically attributed to social stereotypes about women's abilities in math (Beilock et al., 2007) or to women's increased likelihood of reporting anxiety (Ashcraft, 2002). However, the present data suggest that the sex difference is not due simply to social stereotypes or to women's willingness to report anxiety, but are instead the result of real or perceived deficits in visual-spatial ability. Understanding the root of the sex difference in math anxiety is important as the sex difference in math anxiety certainly contributes to the underrepresentation of women in math heavy fields (Chipman et al., 1992).

In Experiment 6, I set out to determine whether this perceived difference in spatial processing ability translates into an actual performance difference on a classic spatial processing task, specifically, a mental rotation task. Furthermore, I also set out to replicate the correlations between AMAS and the two OSIQ sub-scales, and to examine whether differences in mental rotation ability can account for the sex difference in math anxiety.

Experiment 6

The purpose of Experiment 6 was threefold. First, I wanted to replicate the finding from Experiment 5, showing that math anxiety and self-reported spatial and object processing ability are related. Second, I wanted to investigate whether high math anxious individuals' perception that they are poor at spatial processing tasks translates to poorer performance on a measure of spatial processing ability. Third, I wanted to investigate whether differences in mental rotation ability could explain the sex difference in math anxiety. If high math anxious individuals demonstrate poorer spatial processing scores than their low math anxious peers, this would provide the first objective evidence that math anxiety is associated with deficits that extend beyond mathematics.

In the present investigation, high and low math anxious participants completed the AMAS (Hopko et al., 2003), the OSIQ (Blajenkova et al., 2006), and a test of spatial processing ability, the Mental Rotations Test (the MRT-A; Vandenberg & Kruse, 1978). Participants also completed a test of mathematical competence. If the high math anxious individuals do, in fact, have a spatial processing deficit, then this should manifest as lower scores on the MRT-A. The mathematics task was included to replicate the well-established finding that high math anxious individuals perform more poorly on mathematical tasks than their low math anxious peers and to investigate the relation between spatial ability and math performance in adults. Finally, in light of the effects of sex on mental rotation and math anxiety observed in the literature (e.g., Hembree 1990; Linn & Petersen, 1992) and in Experiment 5, sex was included as a factor in the reported analyses.

Methods

Participants. Seventy undergraduate students participated in this study in exchange for course credit. Eight participants were excluded from the analysis because their mass testing AMAS score indicated that they were in one category (e.g., high math anxious) while their in-study AMAS scores indicated that they belonged to the other category (e.g., low math anxious). Final analysis included a total of 62 (22 male, 40 female) undergraduate students, with 31 participants in each math anxiety group (15 male, 16 female in the low math anxious group; 7 male and 24 female in the high math anxious group).

Procedure. As with Experiments 1 through 4, I selected participants with scores under 20 to constitute my low math anxious group and participants with scores over 30 to constitute my high math anxious group. Participants were run in groups ranging from one to six participants.

Mental Rotations Test (MRT-A; Vandenberg & Kruse, 1978; see Appendix D). The MRT-A is a 24-item test assessing mental rotation ability. For each item, participants were presented with a 3-dimensional target object. Participants were instructed to mark the two rotated versions of the target object out of four options. Participants had three minutes to complete the first twelve trials, a four-minute break, and then three minutes to complete the last twelve trials.

Math problems (see Appendix E*)*. I developed a 25-item test to assess participants' math abilities. Math problems included two- and three-digit addition and subtraction problems (e.g., "178 – 19 = "; "898 + 449 = "), multiplication and division problems (e.g., "186 × 274 = "; "756 ÷ 6 = "), problems involving fractions and exponents (e.g., "3/4 – 1/3 = "; "$3^2 + 5^2$ = "), and simple algebra problems ("[x + 1] [x – 2] = 10; x = ?"). Participants were given twenty minutes to complete the problems. They were allowed to use pen and paper, but no calculator, as aids.

Participants also completed the AMAS (Hopko et al., 2003) and the OSIQ (Blajenkova et al., 2006) in session.

Results

Figure 10 presents OSIQ-Spatial and Object scale scores, MRT-A scores, and Math Test scores by Sex and Math Anxiety Group. Table 5 presents correlations for each of the measures. Univariate ANOVAs were conducted to compare the effects of Math Anxiety Group and Sex on the OSIQ scales and the MRT-A.

Math Anxiety and Spatial and Object Processing

For the OSIQ spatial scale, there was a main effect of Math Anxiety Group, such that low

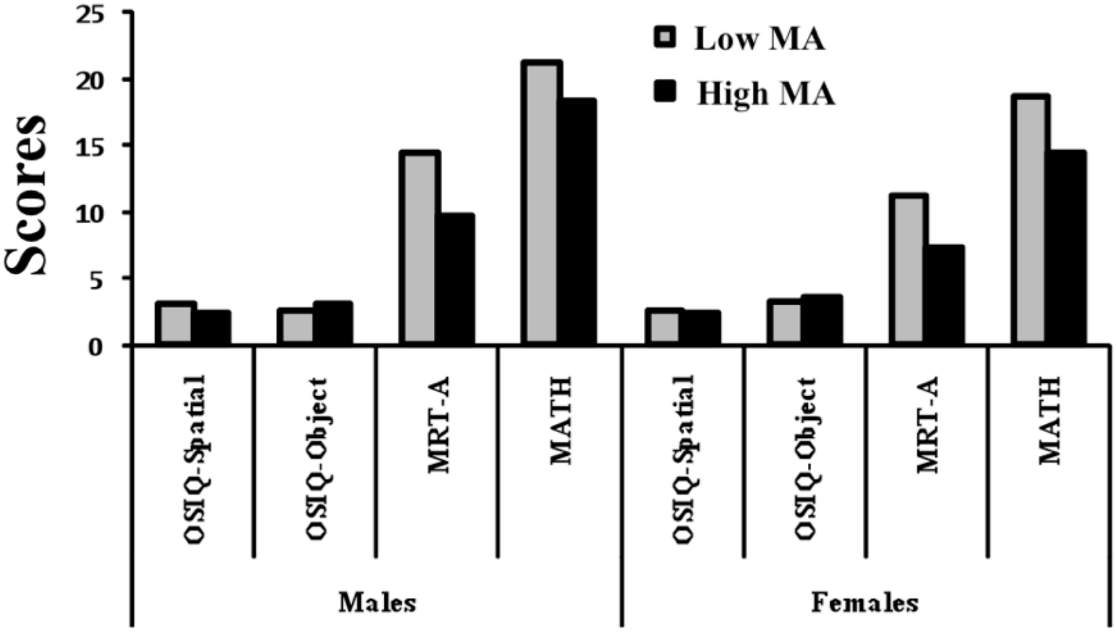

Figure 10. OSIQ-Spatial and Object scale scores, MRT-A scores, and Math Test scores by Sex and Math Anxiety Group (Experiment 6).

	OSIQ-Object	AMAS	MATH	MRT-A
OSIQ-Spatial	-0.30*	-0.53*	0.40*	0.51*
OSIQ-Object		0.36*	-0.42*	-0.40*
AMAS			-0.53*	-0.51*
MATH				0.39*

*Correlation is significant at the $p < .05$ level

Table 5. Correlations between OSIQ-Object, OSIQ-Spatial, AMAS, Math, and MRT-A scores (Experiment 6).

math anxious participants (M= 2.94) scored higher than high math anxious participants (M= 2.40), *F*(1, 60) = 12.4, *MSE* = 3.8, *p* < .01. There was also a marginally significant main effect of sex, such that males (M= 2.82) scored higher than females (M= 2.52), *F*(1, 60) = 3.6, *MSE* = 1.1, *p* = .06. There was no Math anxiety Group by Sex interaction, *F*(1, 60) = 2.3, *MSE* = .693, *p* > .05, indicating that the effect of math anxiety on self-reported spatial imagery ability did not differ significantly by sex.

For the OSIQ object scale, there was a main effect of Math Anxiety Group, such that low math anxious participants (M= 3.0) scored lower than high math anxious participants (M= 3.4); *F*(1, 60) = 5.8, *MSE* = 0.29, *p* < .01. There was also a significant main effect of Sex, such that males (M= 2.9) scored lower than females (M= 3.6), *F*(1, 60) = 11.9, *MSE* = 0.29, *p* < .05. There was no Math Anxiety Group by Sex interaction, *F*(1, 60) < 1, indicating that the effect of math anxiety on self-reported object imagery ability did not differ significantly by sex.

For the mental rotation task, there was a main effect of Math Anxiety group, such that low math anxious participants (M= 12.8) outperformed high math anxious participants (M= 7.9); *F*(1, 60) = 10.57, *MSE* = 238.4, *p* < .01. There was also a main effect of Sex, such that males (M= 12.9) outperformed females (M= 8.9), *F*(1, 60) = 4.2, *MSE* = 94.8, *p* < .05. There was no Math Anxiety Group by Sex interaction, *F*(1, 60) < 1, indicating that the effect of math anxiety on mental rotation performance did not differ significantly by sex.

In Experiment 5 the sex effect in math anxiety was driven by a sex effect in spatial processing ability. To determine whether this was again the case, I conducted a univariate ANCOVA in which Sex predicted AMAS score and OSIQ Spatial was a covariate. In this analysis, the effect of Sex on Math Anxiety was no longer significant, *F*(1, 60) = 2.2, *MSE* =

88.8, $p > .05$, indicating that, consistent with the results from Experiment 5, the sex effect in math anxiety is driven by a sex effect in self-reported spatial processing ability.[4]

Math Anxiety and Mathematics Performance

An important finding in the math anxiety literature is the observation that high math anxious individuals perform worse on measures of mathematical ability than their low math anxious peers. A one-way ANOVA in which Math Anxiety Group predicted Math Task Scores indicated that, in the present data set, this was again the case, $F(1, 60) = 13.3$, $MSE = 311.6$, $p < .01$, as low math anxious participants (M= 19.9) outperformed high math anxious participants (M= 15.4).

Spatial Processing and Mathematics Performance

In order to test whether spatial processing ability was related to math performance I conducted a Pearson correlation between MRT-A scores and Math Task Scores. This analysis yielded a significant correlation, $r(60) = .34$, $p < .01$, such that higher MRT-A scores were associated with higher Math Task Scores. Interestingly, this relation was no longer significant when controlling for AMAS score, $r(59) = .15$, $p > .05$. These data thus suggest that the relation between spatial processing ability and math performance, in this sample, was driven by math anxiety. Furthermore, the relation between math anxiety and math ability still persists even after controlling for MRT-A scores, $r(59) = -.42$, $p < .01$.

Summary

The results from Experiment 6 are clear. As in Experiment 5, the high math anxious individuals reported lower aptitude for spatial processing than their low math anxious peers.

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[4] **This pattern of results is also true when MRT-A scores are entered as the covariate rather than OSIQ Spatial scores.**

Perhaps more importantly, high math anxious individuals also performed significantly worse on the MRT-A than their low math anxious peers. Thus, not only do high math anxious individuals *feel* like they are less adept at spatial processing tasks, they actually perform worse on a measure of spatial processing. Critically, this difference between high math anxious and low math anxious individuals was observed in a task that, for all intents and purposes, is not a math task. Furthermore, the correlations observed between AMAS, MRT-A, and the OSIQ Spatial scale remained significant even when controlling for sex differences between the groups. These data indicate that, above and beyond any sex differences in spatial processing and math anxiety, high math anxious individuals both report lower aptitude for spatial processing and perform worse on a measure of spatial processing.

The results from the present experiment also served to replicate and extend the finding from Experiment 5 that math anxiety is positively related to object imagery ability. Thus, again, the high math anxious participants reported better aptitude for conjuring up vivid mental images than their low math anxious peers. The results from the present experiment also served to replicate the finding from Experiment 5 that the sex effect in math anxiety (i.e., the finding that females are more likely to be math anxious than males) is driven by differences in spatial processing ability. Furthermore, these results also indicate that, at least in this sample, a relation exists between mental rotation ability and math performance but, importantly, that this relation is mediated by math anxiety. Finally, these results indicate that math anxiety is related to math performance above and beyond mental rotation abilities. The theoretical implications of these results are discussed below.

Discussion

In the present investigation I set out to determine if individuals with math anxiety demonstrate a deficit in spatial processing. The results are clear. High math anxious individuals are worse at spatial processing than their low math anxious peers. The data from the present studies also indicate that the spatial processing deficit observed in math anxiety exists above and beyond any sex differences in either spatial processing or math anxiety. In fact, the results suggest that the much-studied sex difference in math anxiety may actually be driven by a sex difference in spatial processing (given that the sex difference in math anxiety is eliminated when controlling for self-reported spatial processing ability). This finding runs counter to the way in which sex differences in math anxiety are typically discussed. Specifically, psychologists have attributed the sex difference in math anxiety to social factors such as stereotypes about women's mathematical abilities (Beilock, Rydell, & McConnell, 2007) and the possibility that women are more likely to disclose anxiety (Ashcraft, 2002). While these mechanisms could conceivably also contribute, the present data strongly imply that the sex difference in math anxiety stem, at least in part, from sex differences in spatial processing ability.

Interestingly, in the present sample the relation between spatial abilities and math performance was mediated by math anxiety. This finding may shed light onto the issue of inconsistent findings regarding the relation between spatial abilities and math performance in normal achieving adults. Perhaps studies that find correlations between spatial skills and math performance have a disproportionately high number of high math anxious individuals in their sample. As this is the first study to examine the relations between spatial processing, math performance, and math anxiety, I cannot possibly say with any certainty whether math anxiety is responsible for other findings in the literature. I can merely suggest that, in light of the present data, this is a possibility.

The final issue to address here is the replication of the positive correlation between math anxiety and object imagery ability. The results from both Experiments 5 and 6 indicate that high math anxious individuals perceive themselves as being particularly skilled at conjuring up vivid mental images. While there is no direct evidence from the literature to suggest why object imagery is related to math anxiety, research examining imagery skills in math problem solving may serve to shed some light. Specifically, Van Garderen and Montague (2003) demonstrated that, when trying to solve written math problems, children who use pictorial imagery (imagery that encodes the persons, places, or things described in the problem) perform significantly worse on those problems than children who use schematic (i.e., spatial) imagery. I did not ask participants which type of imagery they are most likely to use when solving written math problems. However, if the students who are particularly good at conjuring up detailed images of people, places, and things (i.e., the good object imagers) are also more likely to use this type of imagery when solving math problems, and this type of imagery is associated with poor performance in math, then this may explain why math anxiety is related to object imagery.

Conclusion

The present data indicate that high math anxious individuals both report lower aptitude for spatial processing and perform worse on a measure of spatial processing ability than their low math anxious peers. Furthermore, this relation between spatial processing and math anxiety is not driven by sex differences in either spatial processing or math anxiety. These data are consistent with the data reported in Chapters 2 and 3 in that they too indicate that high math anxious individuals have difficulty in, at least some of, the basic building blocks of complex math.

Critically, these data indicate that not only is math anxiety related to performance on measures of complex math and numerical processing, but math anxiety is also related to performance on a non-numerical task. The implications of these data, as well as the data from Chapters 2 and 3, are discussed next in Chapter 5.

Chapter 5: General Discussion

The primary purpose of this thesis was to investigate whether high math anxious individuals differed from their low math anxious peers on indices of numerical and spatial processing abilities. Given that math anxiety is associated with poor performance in math, research aimed at understanding this condition is important as people's mathematical abilities have an enormous impact on their employability, productivity, and earnings (Bishop, 1989; Bossiere et al., 1985; Riviera-Batiz, 1992). In this thesis, numerical processing ability was assessed via performance on an enumeration task (Chapter 2) and two numerical comparison tasks (Chapter 3). Spatial processing was assessed via a measure of self-reported ability and via performance on a mental rotation task (Chapter 4). High math anxious individuals were found to perform worse than their low math anxious peers on each of the aforementioned measures. The results from each of these investigations thus indicate that high math anxious individuals do, in fact, have difficulty in basic numerical and spatial processing which are arguably the building blocks of complex mathematics.

The present results have important implications for the current theoretical understanding of math anxiety. To date, math anxious individuals have been found to have difficulties with complex math (e.g., addition with carrying; Ashcraft & Kirk, 2001). However, the data presented in this thesis indicate that, in addition to their difficulties with complex math, math anxious individuals also have deficits in both basic numerical processing and spatial processing.

The present data speak to Ashcraft and colleague's theory of the online effects of math anxiety. According to this account, the performance deficits observed in math anxiety are caused by anxiety-induced ruminations that limit the working memory capacity available to high math anxious individuals to perform math tasks. The dissociation reported in the two enumeration experiments (Experiments 1 and 2 in Chapter 2) between subitizing and counting is consistent with this claim, as counting is thought to require working memory resources whereas subitizing is not. Additional support for Ashcraft and colleagues claim was provided by my observation that when differences in working memory capacity were controlled for, the math anxiety difference in performance in the counting range was eliminated. That said, the data from the two numerical comparison experiments (Experiments 3 and 4 in Chapter 3) suggest that, while the relation between math anxiety and complex mathematics may be mediated by differences in working memory capacity, not all of the relations between math anxiety and processing deficits can be explained by recourse to working memory differences. As such, these data call for a reconceptualization of the current theory of math anxiety to include (1) that a relation does exist between math anxiety and basic low level numerical and spatial processing, and (2) why this relation exists.

Inconsistencies with Previous Research

Perhaps the most obvious question that this research raises surrounds the issue of why I find a math anxiety related deficit on numerical and spatial processing (which are arguably the building blocks of even simple mathematics) while Ashcraft and colleagues did not find differences on tasks such as single digit addition. Here, I provide a few potential explanations. One potential reason for why I find effects of math anxiety on tasks that are arguably more basic than those on which Ashcraft and colleagues have not found effects is that, in my Experiments the participants involved represent the extremes of the math anxiety distribution. Specifically, whereas Ashcraft and colleagues administer their assessment of math anxiety and then categorize participants as either low, moderate, or high anxiety; I first assess math anxiety in a large subset of the undergraduate population (often around 1500 students; see Figure 1) and then invite only the participants who fall within the extreme ranges to participate in the experiments. This sampling technique affords me a very high degree of power to detect between group differences.

Another possible reason as to why I find math anxiety differences on such simple tasks concerns the fact that the majority of my experiments, specifically Experiments 1 to 4, are administered via computer which allows me to collect accuracies as well as precise response time measurements. Although Ashcraft and colleagues found no differences between the high and low math anxious groups on single digit addition independent of whether it was tested via computer or paper and pencil, they did find effects of math anxiety on two digit addition when it was tested via computer but not when it was tested via paper and pencil. There are, at least, two reasons why effects were found via one test modality but not the other. First, it is possible that the high and low math anxious participants did actually perform differently on two digit addition but that this difference was in the response times rather than the accuracies. Some of the largest effects of math anxiety appear on standardized tests (Hembree, 1990) which tend to be timed. Thus, effects of math anxiety on response times can be as important as effects on accuracies. After all, tests are rarely graded on the basis of your accuracy on what you have time to complete but rather are graded on your accuracy across all of the questions.

A third potential reason as to why I find math anxiety related differences on tasks that are arguably simpler than two column addition also involves the medium via which the questions are administered. Recall that Ashcraft and colleagues posit that math anxiety consumes working memory resources and that it is this reduction in working memory capacity that causes the high math anxious individuals to perform worse than their low math anxious peers. Ashcraft and colleagues suggest that, given the lack of a math anxiety effect on the paper and pencil version of their test, the paper and pencil version must be less anxiety inducing than their computerized version of the same test (Ashcraft & Kirk, 1998). However, it is also possible that both tests are equally anxiety invoking (after all the same material is being tested) but that the paper and pencil version of the test is actually *less* working memory demanding. Consistent with this notion, research indicates that performing mathematics on paper allows students to offload some of the working memory demands onto the paper (i.e., rather than carrying a "1" in their working memory, students can write the "1" down reminding them of the need to carry it) making the task less working memory demanding than when they cannot use paper (Cary & Carelson, 2001). Thus, if the accuracy differences between high and low math anxious participants are due to the high math anxious individual's decreased working memory capacities, then the fact that they can offload some of the working memory demands may explain the lack of between group differences on paper and pencil versions of the math task.

Antecedents to Math Anxiety

As was discussed in the introduction, the etiology of math anxiety is not yet known. That said, these data provide insight into one potential cause. Similar to claims made about Developmental Dyscalculia, one could suggest that the deficits in numerical and spatial processing observed here are the primary *cause* of math anxiety. We know that children who are poor at numerical and spatial processing are likely to be poor at math (Holloway & Ansari, 2008; Rotzer et al., 2009). As such these students may encounter additional negative experiences with math than their more numerically and spatially competent peers. The degree of emotional reaction to these negative experiences with math may also be related to their teachers teaching style (Ashcraft, 2002; Beilock et al., 2010; Turner, 2002). Furthermore, we know that math anxiety is related to general anxiety (Hembree, 1990). Thus, perhaps it is the children who present with a deficit in the basic building blocks of mathematics and who also have a predisposition to anxiety who develop math anxiety. Then, as a consequence of their math anxiety, these children both experience a reduction in online working memory capacity when engaging in mathematical processing (which causes them to perform even more poorly on complex math; Ashcraft & Kirk, 2001) and avoid honing their math skills (Hembree, 1990). Thus, by the time these children become adults, their math deficits are a combination of both their low-level deficits and their online anxiety reactions. The heterogeneous nature of the math anxiety-related deficit would explain why, even when anxiety is successfully reduced using cognitive-behavioral therapy, math performance is never quite equated between the high and low math anxiety groups (i.e., that some of the deficit is attributable to differences in the fundamental building blocks). For a schematic representation of this theory see Figure 11.

While the above interpretation is certainly consistent with the data, a second, equally consistent interpretation exists. Specifically, math anxiety may cause poor numerical and spatial processing abilities. As noted above, high math anxious individuals tend to avoid math, presumably because of their anxiety toward it (Hembree, 1990). For example, individuals high in math anxiety are exposed to fewer math courses in school, and spend less time on their math homework (e.g., Fennema, 1989). Thus, as a consequence of their anxiety, high math anxious individuals may have never taken the time and made the effort to hone their numerical and spatial processing skills. This theory, of course, requires that basic numerical and spatial processing skills are malleable and that avoidance of math can influence one's ability to compare two numbers, to enumerate up to nine objects, and to mentally rotate objects. Nonetheless, independent of the cause and effect relation between math anxiety and numerical and spatial processing deficits, the finding of these deficits can have strong implications for not only our

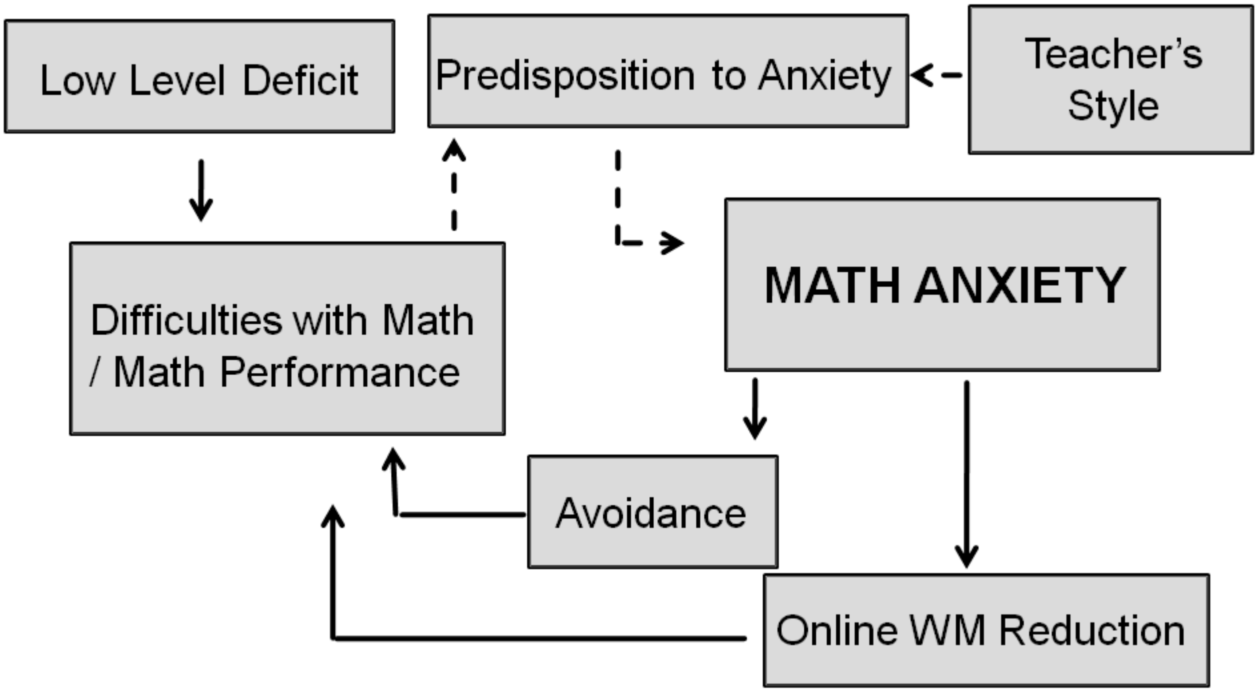

Figure 11. Schematic depicting one potential account of the development of and the effects of math anxiety.

understanding of the nature of math anxiety but also for how to go about remedying and preventing the condition.

Implications for the Prevention and Remediation of Math Anxiety

My finding that math anxiety is associated with poor numerical and spatial processing could have significant implications for the prevention and remediation of math anxiety. For example, if these processing deficits precede math anxiety, then early identification of children with poor numerical and spatial processing skills may help to prevent the development of math anxiety in the future if these children are given effective basic skills training. On the other hand, if math anxiety causes the numerical and spatial processing deficits and these in turn cause problems in complex math then it is important that remediation programs work to both (1) eliminate the math anxiety and (2) improve numerical and spatial processing abilities. Hembree (1990) demonstrated that cognitive behavioral therapy paired with relaxation therapy can serve to significantly reduce math anxiety and performance deficits on math tests (but cannot completely eliminate the between groups performance differences). As mentioned above, it is possible that the performance deficits that exist after an anxiety reduction are due to deficits in basic numerical and spatial processing. If this is the case, then in addition to alleviating the math anxiety, if a remediation program is to be completely effective, it will also need to improve basic skills. In this vein, recent evidence indicates that intensive training of numerical comparison skills can result in significant increases in some areas of math for children with Developmental Dyscalculia (e.g., subtraction; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). Specifically, Wilson et al. (2006) had 7 to 9 year old children play “The Number Race”, a game designed to provide adaptive training on numerical comparison with the intent of strengthening core number skills. The children trained for half an hour a day, four days a week over a period of five-weeks. The results were promising, suggesting that the training software was successful in increasing numerical skills in children with Developmental Dyscalculia.

In addition to promising results with numerical comparison training, spatial training such as playing action video games (Feng, Spence, & Pratt, 2007), sketching 3-D objects (Sorby, 2009) and even practicing spatial tests (Lohman & Nichols, 1990) has also been demonstrated to robustly improve spatial skills. Importantly, research shows that not only can spatial skills be improved with training but also that intensive training of spatial skills can actually translate to increased performance in subject-areas in which spatial skills are found to be helpful for performance. For example, Sorby (2009) found that spatial training was associated with higher grades in future courses including calculus, and physics (see also Blasko & Holliday-Darr, 2010). Given these promising results, I feel that the training of basic numerical skills (i.e., numerical comparison) and of spatial skills may represent an exciting and effective intervention for the prevention and remediation of math anxiety. It thus appears that the most promising remediation for math anxiety likely involves a combination of Cognitive Behavioral Therapy and basic numerical and spatial skills training.

Conclusion

The data constituting this thesis present a case arguing for a math anxiety related deficit in numerical and spatial processing by demonstrating that high math anxious individuals perform worse than their low math anxious peers on measures of enumeration, numerical comparison, and spatial processing. Furthermore, these data serve to reopen the door to the possibility that math anxiety may result from a low-level deficit (a possibility that had previously been abandoned given the null effects found between math anxiety and basic addition and multiplication problems; Ashcraft & Faust, 1994; Faust et al., 1996). Fortunately, given the encouraging research to date on the training of some of these building block skills (e.g., numerical comparison and spatial processing ability), the uncovering of this putatively early deficit in math anxiety may serve as a promising step in terms of its prevention and remediation.

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Appendix A: Abbreviated Math Anxiety Scale (AMAS)

Please rate each item in terms of how anxious you would feel during the event specified. Use the following scale and record your answer in the space to the left of the item:

Scale:

1 = Low Anxiety

2 = Some Anxiety

3 = Moderate Anxiety

4 = Quite a bit of Anxiety

5 = High Anxiety

____ 1. Having to use the tables in the back of a math book.

____ 2. Thinking about an upcoming math test one day before.

____ 3. Watching a teacher work an algebraic equation on the blackboard.

____ 4. Taking an examination in a math course.

____ 5. Being given a homework assignment of many difficult problems which is due the next class meeting.

____ 6. Listening to a lecture in math class.

____ 7. Listening to another student explain a math formula.

____ 8. Being given a “pop” quiz in a math class.

____ 9. Starting a new chapter in a math book.

Appendix B: Working Memory Tasks used in Experiment 2

Backward Letter Span

Description: The experimenter presents letters starting with sequence length 2 and increasing to a maximum of 9 orally. The participant must repeat the letters immediately, in reverse order. The task is discontinued when the participant makes two consecutive failures. The highest number of correctly recalled letters defines the backward letter span score (i.e. between 2 and 9).

Instructions (to be read to the participant):

"I will read out loud a sequence of letters. Your task is to tell me the letters in reverse order. For example, if I say 'J – H – Q', the correct answer would be to say 'Q – H – J'. What would your answer be if I say 'B – H'? (*Let the participant say 'H – B').* That is correct. Do you have any questions about this task? (*Let the participant enough time to give you a clear No… Otherwise, clarify any question before beginning.)* Then, let's begin."

Scoring:

1) Read the sequence slowly and clearly.
2) On the answer sheet, write down the participant's answer (each letter s/he mentions).
3) Compare the participant's answer to the answer key (printed on the answer sheet).
4) Indicate whether or not the sequence was correctly repeated in reverse order (correct: check under 'Pass'; incorrect: check under 'Fail').
5) The task is discontinued when the participant makes two consecutive failures.
6) The participant's score is defined by the highest number of correctly recalled letters (i.e. between 2 and 9).

Backward Letter Span

Description: The experimenter presents letters starting with sequence length 2 and increasing to a maximum of 9 orally. The participant must repeat the letters immediately, in reserve order. The task is discontinued when the participants makes two consecutive failures. The highest number of correctly recalled letters defines the backward letter span score (i.e. between 2 and 9).

2) J - X

2) M - R

3) L - M - J

3) H - Q - B

4) L - X - M - B

4) R - F - X - M

5) Q - J - B - X - F

5) F - M - B - L - Q

6) B - X - J - R - M - Q

6) R - M - X - B - Q - F

7) L - B - J - F - X - M - R

7) M - F - L - X - H - R - Q

8) B - J - H - X - F - L - M - R

8) R - H - B - X - M - J - L - F

9) B - J - Q - F - R - H - M - L - X

9) J - L - H - M - B - Q - F - X - R

Participant #: ____________________________________ Date: __________________

RA: _______________________

(Stop testing after two "fail" in a row)*

	Participant's answers	Pass	Fail
2) [X-J]			
2) [R-M]			
3) [J-M-L]			
3) [B-Q-H]			
4) [B-M-X-L]			
4) [M-X-F-R]			
5) [F-X-B-J-Q]			
5) [Q-L-B-M-F]			
6) [Q-M-R-J-X-B]			
6) [F-Q-B-X-M-R]			
7) [R-M-X-F-J-B-L]			
7) [Q-R-H-X-L-F-M]			

8) [R-M-L-F-X-H-J-B]			
8) [F-L-J-M-X-B-H-R]			
9) [X-L-M-H-R-F-Q-J-B]			
9) [R-X-F-Q-B-M-H-L-J]			

Score (range: 2 to 9): ________

Backward Digit Span

Description: The experimenter presents digits starting with sequence length 2 and increasing to a maximum of 9 orally. The participant must repeat the digits immediately, in reverse order. The task is discontinued when the participant makes two consecutive failures. The highest number of correctly recalled digits defines the backward digit span score (i.e. between 2 and 9).

Instructions (to be read to the participant):

"I will read out loud a sequence of digits. Your task is to tell me the digits in reverse order. For example, if I say '4 – 3 – 7', the correct answer would be to say '7 – 3 – 4'. What would your answer be if I say '1 – 3'? (*Let the participant say '3 – 1').* That is correct. Do you have any questions about this task? (*Let the participant enough time to give you a clear No… Otherwise, clarify any question before beginning.)* Then, let's begin."

Scoring:

1) Read the sequence slowly and clearly.
2) On the answer sheet, write down the participant's answer (each digit s/he mentions).
3) Compare the participant's answer to the answer key (printed on the answer sheet).
4) Indicate whether or not the sequence was correctly repeated in reverse order (correct: check under 'Pass'; incorrect: check under 'Fail').
5) The task is discontinued when the participant makes two consecutive failures.
6) The participant's score is defined by the highest number of correctly recalled digits (i.e. between 2 and 9).

Backward Digit Span

Description: The experimenter presents digits starting with sequence length 2 and increasing to a maximum of 9 orally. The participant must repeat the digits immediately, in reserve order. The task is discontinued when the participants makes two consecutive failures. The highest number of correctly recalled digits defines the backward digit span score (i.e. between 2 and 9).

2) 4 – 9

2) 6 – 8

3) 5 – 6 – 4

3) 3 – 7 – 1

4) 5 – 9 – 6 – 1

4) 8 – 2 – 9 – 6

5) 7 – 4 – 1 – 9 – 2

5) 2 – 6 – 1 – 5 – 7

6) 1 – 9 – 4 – 8 – 6 – 7

6) 8 – 6 – 9 – 1 – 7 – 2

7) 5 – 1 – 4 – 2 – 9 – 6 – 8

7) 6 – 2 – 5 – 9 – 3 – 8 – 7

8) 1 – 4 – 3 – 9 – 2 – 5 – 6 – 8

8) 8 – 3 – 1 – 9 – 6 – 4 – 5 – 2

9) 1 – 4 – 7 – 2 – 8 – 3 – 6 – 5 – 9

9) 4 – 5 – 3 – 6 – 1 – 7 – 2 – 9 – 8

Participant #: ____________________________________ Date: __________________

RA: _______________________

(Stop testing after two "fail" in a row)*

	Participant's answers	Pass	Fail
2) [9-4]			
2) [8-6]			
3) [4-6-5]			
3) [1-7-3]			
4) [1-6-9-5]			
4) [6-9-2-8]			
5) [2-9-1-4-7]			
5) [7-5-1-6-2]			
6) [7-6-8-4-9-1]			
6) [2-7-1-9-6-8]			
7) [8-6-9-2-4-1-5]			
7) [7-8-3-9-5-2-6]			
8) [8-6-5-2-9-3-4-1]			
8) [2-5-4-6-9-1-3-8]			
9) [9-5-6-3-8-2-7-4-1]			
9) [8-9-2-7-1-6-3-5-4]			

Score (range: 2 to 9): ________

Appendix C: Object-Spatial Imagery Questionnaire (OSIQ; used in Experiments 5 and 6)

The participants were asked to read all of the questionnaire items and rate each of them on a 5-point scale with 1 labelled 'totally disagree' and 5 labelled 'totally agree'.

1. I was very good in 3-D geometry as a student. S

2. If I were asked to choose between engineering professions and visual arts, I would prefer engineering. S

3. Architecture interests me more than painting. S

4. My images are very colourful and bright. O

5. I prefer schematic diagrams and sketches when reading a textbook instead of colourful and pictorial illustrations. S

6. My images are more like schematic representations of things and events rather than detailed pictures. S

7. When reading fiction, I usually form a clear and detailed mental picture of a scene or room that has been described. O

8. I have a photographic memory. O

9. I can easily imagine and mentally rotate 3-dimensional geometric figures. S

10. When entering a familiar store to get a specific item, I can easily picture the exact location of the target item, the shelf it stands on, how it is arranged and the surrounding articles. O

11. I normally do not experience many spontaneous vivid images; I use my mental imagery mostly when attempting to solve some problems like the ones in mathematics. S

12. My images are very vivid and photographic. O

13. I can easily sketch a blueprint for a building that I am familiar with. S

14. I am a good Tetris player. S

15. If I were asked to choose between studying architecture and visual arts, I would choose visual arts. O

16. My mental images of different objects very much resemble the size, shape and colour of actual objects that I have seen. O

17. When I imagine the face of a friend, I have a perfectly clear and bright image. O

18. I have excellent abilities in technical graphics. S

19. I can easily remember a great deal of visual details that someone else might never notice. For example, I would just automatically take some things in, like what colour is a shirt someone wears or what colour are his/her shoes. O

20. In high school, I had less difficulty with geometry than with art. S

21. I enjoy pictures with bright colours and unusual shapes like the ones in modern art. O

22. Sometimes my images are so vivid and persistent that it is difficult to ignore them. O

23. When thinking about an abstract concept (e.g. ‘a building’) I imagine an abstract schematic building in my mind or its blueprint rather than a specific concrete building. S

24. My images are more schematic than colourful and pictorial. S

25. I can close my eyes and easily picture a scene that I have experienced. O

26. I remember everything visually. I can recount what people wore to a dinner and I can talk about the way they sat and the way they looked probably in more detail than I could discuss what they said. O

27. I find it difficult to imagine how a 3-dimensional geometric figure would exactly look like when rotated. S (reverse)

28. My visual images are in my head all the time. They are just right there. O

29. My graphic abilities would make a career in architecture relatively easy for me. S

30. When I hear a radio announcer or a DJ I’ve never actually seen, I usually find myself picturing what he or she might look like. O

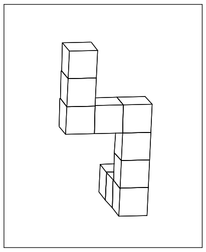
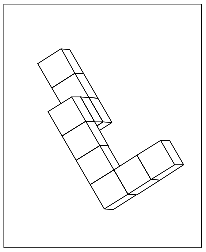
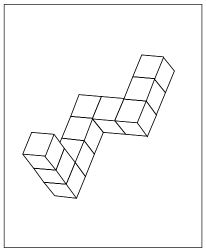
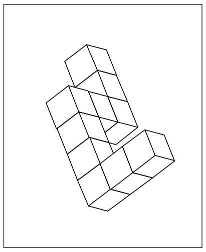
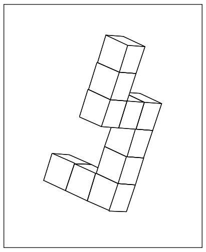
*** The “O” and “S” were not included in the version administered to participant. They imply indicate for the researchers whether the item belongs to the “Object” or the “Spatial” scale respectively.**

Appendix D: Mental Rotations test (MRT-A)

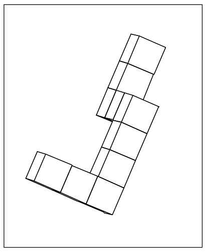
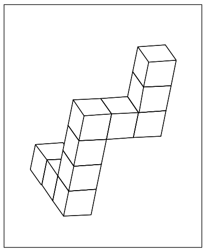
This test is composed of the figures provided by Shepard and Metzler (1978), and is, essentially, an Autocad- redrawn version of the Vandenberg & Kuse MRT test.

©Michael Peters, PhD, July 1995

Please look at these five figures

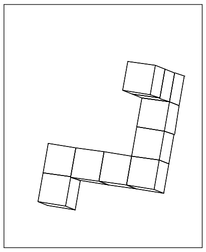
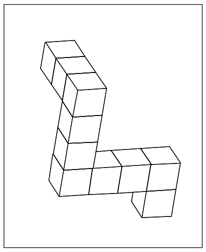
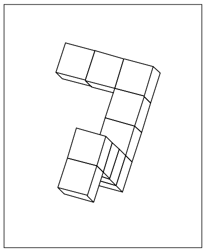
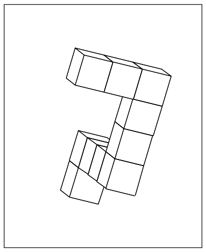
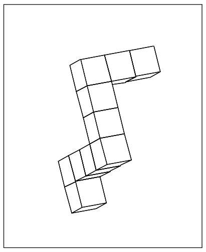
Note that these are all pictures of the same object which is shown from different angles. Try to imagine moving the object (or yourself with respect to the object), as you look from one drawing to the next.

Here are two drawings of a new figure that is different from the one shown in the first 5 drawings. Satisfy yourself that these two drawings show an object that is different and cannot be "rotated" to be identical with the object shown in the first five drawings.

Now look at
this object:
1.

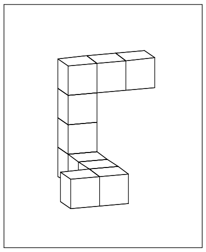
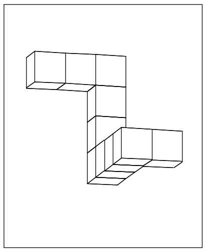
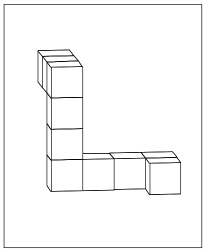
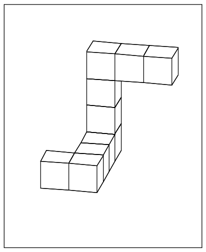
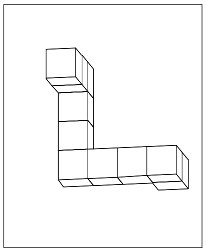
Two of these four drawings show the same object.
Can you find those two? Put a big X across them.

If you marked the first and third drawings, you made the correct choice.

Here are three more problems. Again, the target object is shown twice in each set of four alternatives from which you choose the correct ones.

2.a

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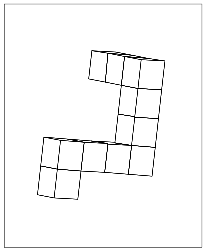
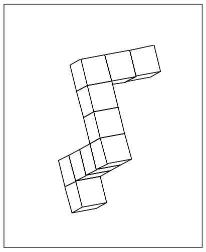
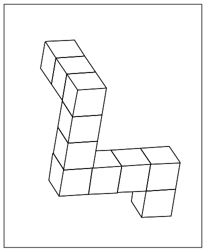
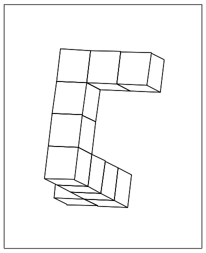
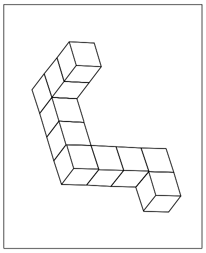

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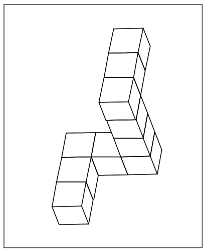
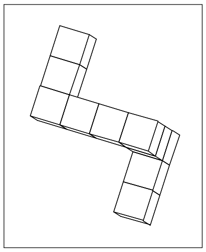
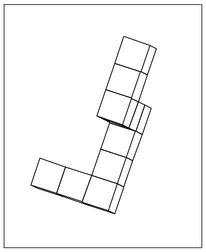
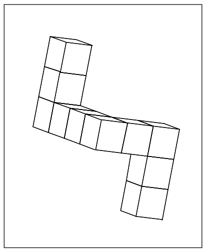
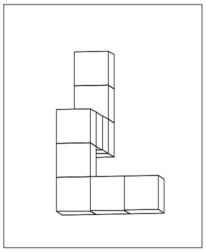






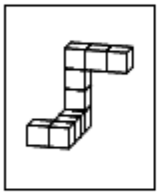
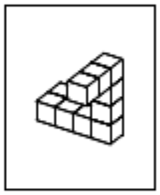
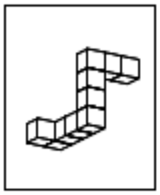
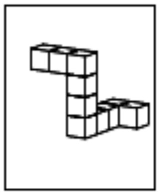
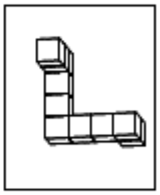
Correct Choice:

2: second and third
3: first and fourth
4: first and third

When you do the test, please remember that for each problem set there are two and only two figures that match the target figure.

You will only be given a point if you mark off both correct matching figures, marking off only one of these will result in no marks.

1.a

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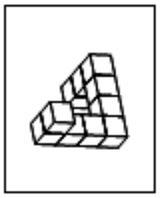
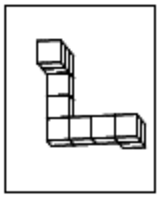
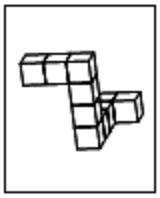
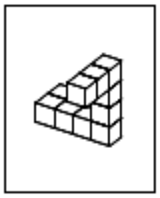
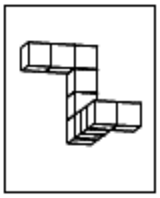

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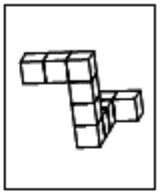
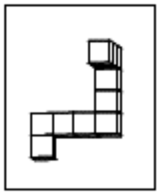
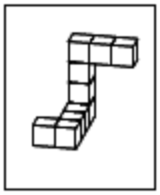
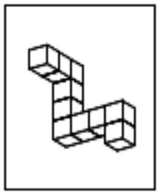
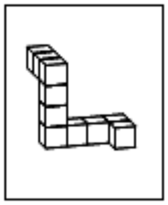

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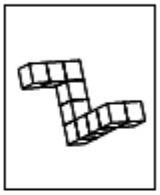
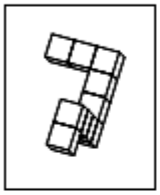
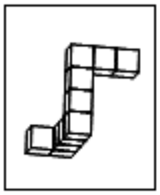
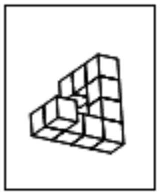
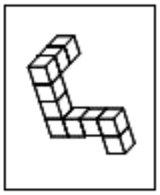

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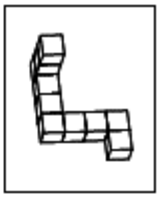
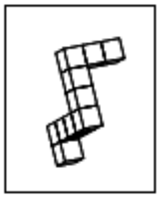
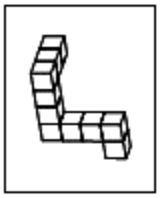
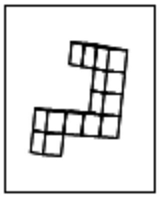
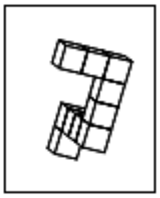

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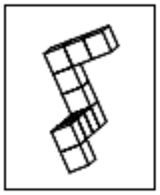
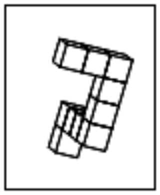
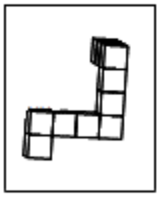
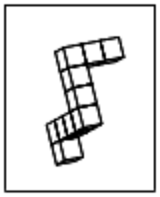
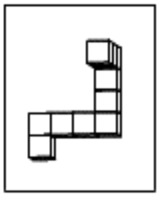

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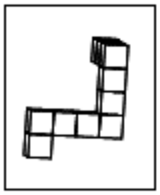
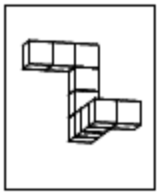
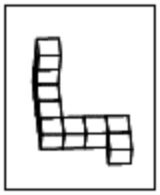
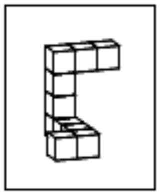
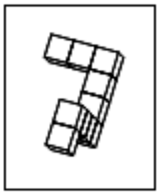

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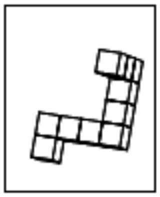
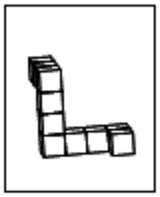
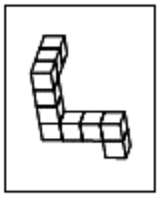
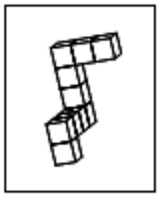
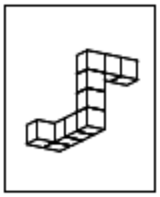

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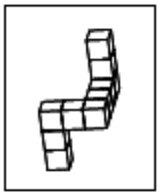
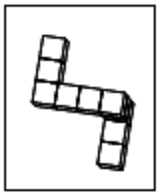
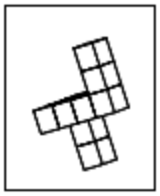
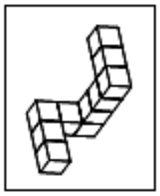
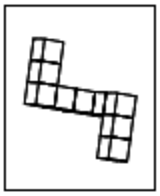

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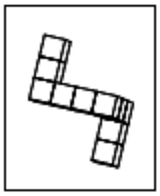
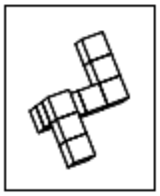
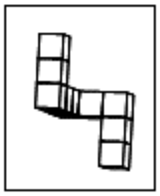
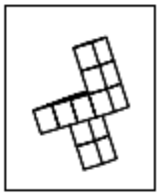
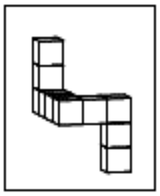

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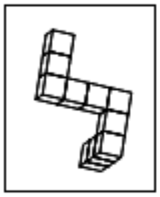
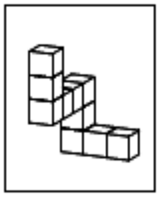
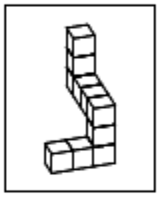
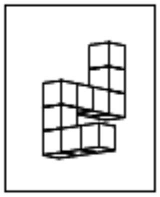
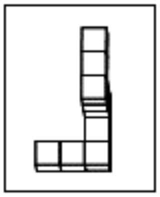

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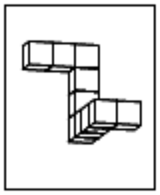
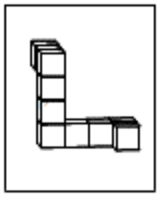
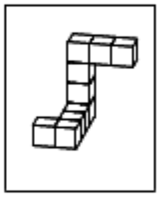
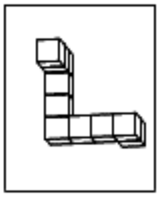

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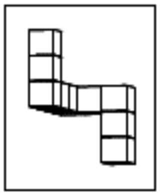
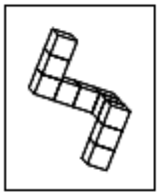
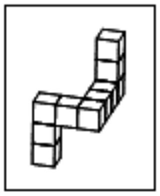
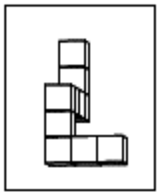
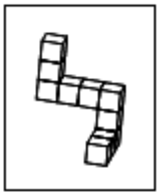

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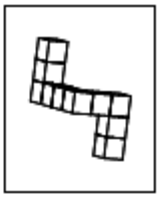
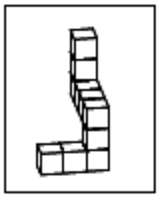
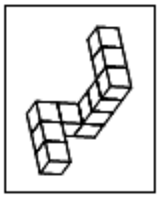
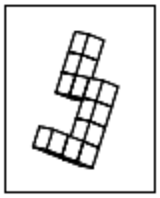
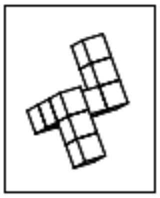

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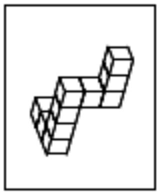
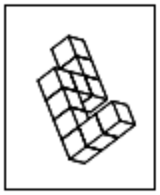
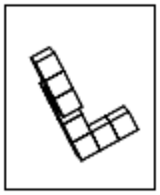
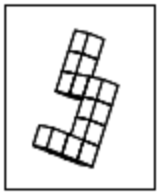
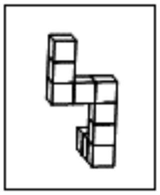

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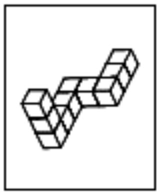
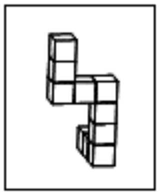
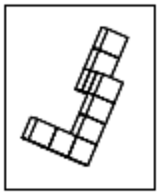
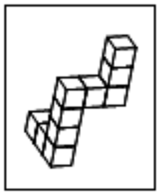
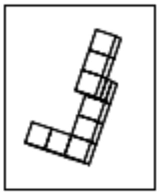

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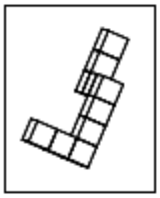
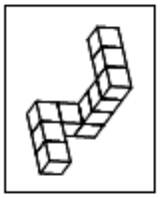
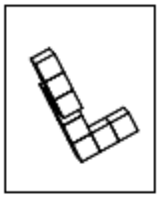
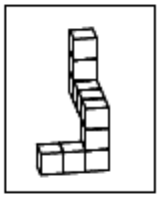
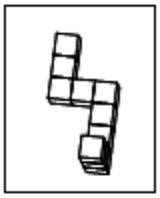

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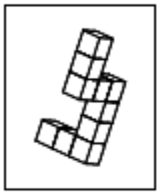
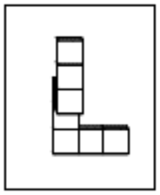
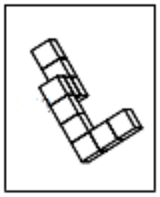
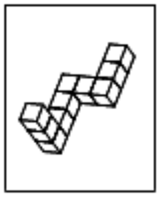
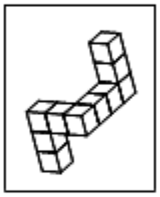

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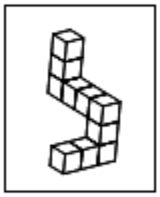
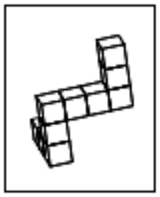
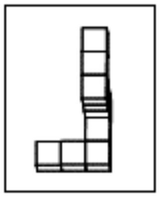
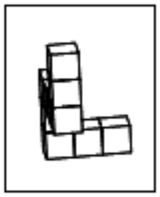
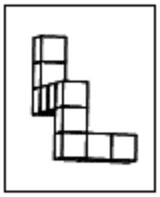

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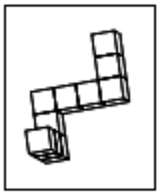
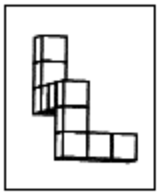
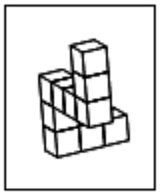
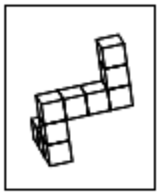
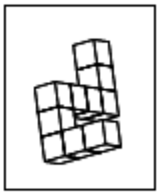

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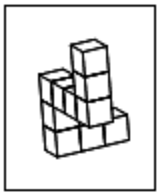
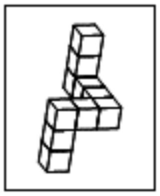
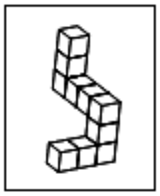
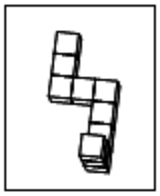
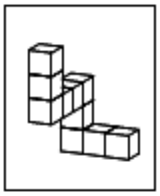

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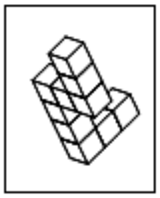
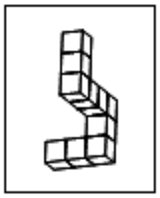
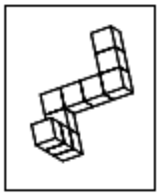
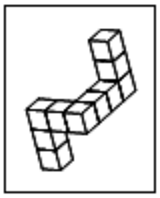

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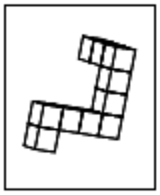
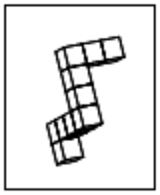
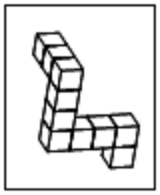
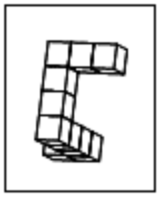
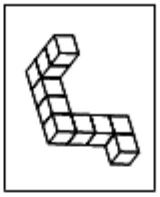

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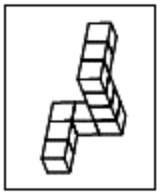
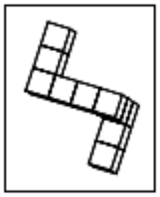
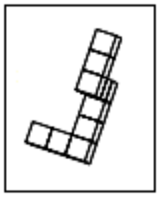
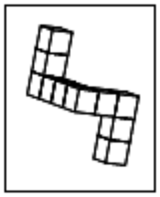
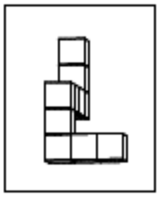

Appendix E: Math Problems (used in Experiment 6)

1. $\begin{array}{r} 122 \\ \underline{-\ 16} \end{array}$

2. $\begin{array}{r} 898 \\ \underline{+\ 449} \end{array}$

3. $\begin{array}{r} 711 \\ \underline{-\ 488} \end{array}$

4. 7 X 8 =

5. $\begin{array}{r} 26 \\ \underline{\text{X}\ 5} \end{array}$

6. 26 ÷ 3 =

7. 756 ÷ 6 =

8. $\begin{array}{r} 55 \\ 38 \\ 91 \\ \underline{+\ 42} \end{array}$

9. 0.3 + 0.7 =

10.

$\frac{11}{12} - \frac{3}{12} =$

11.

$\frac{3}{4} - \frac{1}{3} =$

12. 12 – 0.739 =

13. 306
$\underline{\text{X } 209}$

14. $10^4 =$

15. $3^2 + 5^2 =$

16. 5.5 X .04 =

17. -33 + (-17) =

18.

$\frac{3}{7} \div \frac{4}{5} =$

19. $2y + 14 = 6$

$y =$ ___

20. $3y – 15 = 21 + y$

$y =$ ___

21.

$\frac{13}{17} \div 3 =$

22. $(x + 1)(x – 1) = 10$

$x =$ ____

23. $5x + y = 3$

$x + 4y = 6$

$x =$ ___

$y =$ ___

24. $y^9 \div y^3 =$

25. $60 - 10 \div 5 =$