Task Optimization and Workforce Scheduling

by

Mahsa Shateri

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Mahsa Shateri

Abstract

This thesis focuses on task sequencing and manpower scheduling to develop robust schedules for an aircraft manufacturer. The production of an aircraft goes through a series of multiple workstations, each consisting of a large number of interactive tasks and a limited number of working zones. The duration of each task varies from operator to operator, because most operations are performed manually. These factors limit the ability of managers to balance, optimize, and change the statement of work in each workstation. In addition, engineers spend considerable amount of time to manually develop schedules that may be incompatible with the changes in the production rate.

To address the above problems, the current state of work centers are first analyzed. Then, several deterministic mathematical programming models are developed to minimize the total production labour cost for a target cycle time. The mathematical models seek to find optimal schedules by eliminating and/or considering the effect of overtime on the production cost. The resulting schedules decrease the required number of operators by 16% and reduce production cycle time of work centers by 53% to 67%. Using these models, the time needed to develop a schedule is reduced from 36 days to less than a day.

To handle the stochasticity of the task durations, a two-stage stochastic programming model with heuristic algorithm is developed to minimize the total production labour cost and to find the number of operators that are able to work under every scenario. The solution of the two-stage stochastic programming model finds the same number of operators as that

of the deterministic models, but reduces the time to adjust production schedules by 88%.

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that I dedicate this thesis.

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Chapter 1

Introduction

Manufacturers of large size products, e.g., aerospace and railway vehicle industries, face many production challenges as the products typically move through a series of workstations, involve many tasks, and use a significant number of operators. Just-in-time movement of parts between workstations is essential to reduce backlog in the production line. In addition, the complexity and size of the product requires considerable space and many task interactions. Moreover, due to the limited available space, only few parts can be stored at each workstation at all times. Therefore, an optimal schedule of a production line, considering all the above constraints will definitely help managers and engineers better monitor the production variables, increase utilization, and evaluate different what-if scenarios. The what-if scenarios may include verifying the number of operators per shift, adjusting the number of tasks assigned to an operator, finding an efficient sequence of jobs, and evaluating the importance of each activity in production line.

Direct labour accounts for 10% to 13% of the commercial aircraft production cost which forces aircraft manufacturers to increase worker utilization. Moreover, each production work center contains between 50 to 100 interactive work tasks that are mostly non-repetitive and manual. This and the absence of an automation system to control the production speed contribute to the variability of cycle times, which ranges from 4 to 20 days.

Currently, schedules are established by continually adjusting tasks in a spread sheet until all task requirements are met. It takes on average 1.5 months (approximately 36 business days) for Bombardier to create a manual schedule for a workstation that consists of 150 tasks. This time excludes the required time to gather input information. In addition, production rates vary between workstations due to changes in manpower, material shortage, and exchange of tasks between workstations. To adjust the workstation schedule for every variation in production, engineers need approximately one month. This work proposes and solves a set of mathematical models to find work schedules that take the above issues into account.

1.1 Introduction to Bombardier

This work is done on a scheduling problem faced by Bombardier Aerospace. Bombardier is a global transportation company that designs and manufactures two leading businesses, aerospace and rail transportation. Bombardier has 66,900 employees and its network is spread in 29 countries around the globe.

Bombardier Aerospace, located in Toronto Canada, is ranked as the world's third largest civil aircraft manufacturer due to its outstanding performance aircraft and services in several markets such as business aircraft, commercial aircraft, amphibious aircraft, jet travel solutions, specialized aircraft solutions, and aircraft services and training. The assembly facility on site assembles business and commercial aircrafts. The three main business jets built by Bombardier are Learjet aircraft, Challenger aircraft, and Global business jet and Q400 is an example of its commercial aircrafts.

1.2 Problem Description

In the problem under study, the aim is to find the best number of operators and the optimal sequence of schedules to perform the required production tasks for A4D0300 workstation. This workstation is located in the fuselage subassembly where the Q400 commercial aircraft is produced. The fuselage subassembly consists of two workstations: A4D0300 and A4D0400. A large number of tasks are assigned to each workstation. Most tasks follow precedence relationship. Each workstation has few number of work areas and every work area has limited space for operators. Thus, a limited number of operators can work at a workstation at any given time. Bombardier's goal is to complete the tasks assigned to a workstation in a four day cycle time.

Currently, operators work in three equal shifts. Due to union policies and regulations, most operators must be scheduled to work in morning shifts. Operators receive different salaries depending on their work experience with Bombardier and their working shift.

Bombardier has used Maynard Operation Sequence Technique (MOST) to calculate the processing time for each operation. MOST is a time measurement technique that concentrates on the movements involved in a particular task. Major work elements in MOST analysis are action distance, body motion, gain control, and placement [20]. Based on the results of MOST analysis and the nature of tasks, a set of operations is scheduled for each operator. Based on the result of interviews with operators, the actual operation times are different from those of MOST. Actual processing time is between the MOST normal time and 1.6 times that time. Therefore, a 60% tolerance is given to operators to complete the assigned tasks.

In addition, some workstations involve critical tasks in which no other operations are allowed during the execution of these tasks. Operators work based on their experience and usually deviate from the sequence of tasks assigned to them. For this reason, it is difficult for managers to follow which tasks are being performed at a certain time. Moreover, managers don't have enough information to order the required material at the right time. These problems make scheduling very challenging at Bombardier.

1.3 Literature Review

According to Graves [13], an optimized schedule uses production resources over a certain time cycle that best satisfies a set of constraints. There are three stages to classify a scheduling problem: "requirement generation", "processing complexity", and "scheduling criteria".

Large scale components at each workstation make it difficult to store complete products. In addition, high cost of production and customized manufacturing operations require costumer's detailed order before the production stage [13]. The components move through a series of operations with a significant number of operators at each workstation. Just-in-time transportation of parts is essential, because it will reduce backlog in the production line. In addition, the complexity and size of the product require considerable space and many interactions. Due to the limited available space, only few parts can be stored at each subassembly at any given time. Graves [13] named this type of processing complexity, a "Single stage, flow shop" problem. The third stage of specifying a scheduling problem is "Scheduling Criteria". According to Graves [13], maximizing performance and minimizing cost of production are two criteria for a scheduling problem.

According to Ernst et al. [10], the first step in classifying a scheduling problem is to identify its process model. The process model varies based on the nature of the problem. In addition, it is important to develop a schedule to determine the number of operators needed at different shifts in a cycle time. This scheduling process is called "shift based demand" [10]. Moreover, it is essential to consider manufacturer's policies and limitation with number of operators per each shift. This type of scheduling process is called "shift scheduling". Finally, most operations follow a specific sequence. This scheduling process is called "task assignment" [10].

Furthermore, task sequencing and manpower scheduling are two types of scheduling approaches. Task sequencing considers precedence relationship between tasks and working zones to assign sequence of operations to each workstation [7]. Manpower scheduling focuses on assigning operators to sequence of tasks based on their skill level [10]. The defined problem requires schedules that integrate both approaches. In addition, artificial intelligence, constraint programming, and mathematical programming are different methods of solving scheduling problems with similar objectives. Artificial intelligence has been used to assist schedulers in initial scheduling construction and in dealing with disruptions in making a crew scheduling system for Indian airlines [1]. Constraint programming has been used to solve for an optimal result for problems that are highly constrained [10]. Azarmi and Abdulhameed [3] used this approach to minimize the total cycle time and amount of travelling time that workers spend to change locations for performing tasks for a workforce with several tasks having a particular order, different durations, and locations.

Mathematical programming is a commonly used approach to model and solve scheduling problems [10]. Various methods such as stochastic, probabilistic or chance constraints, and fuzzy programming methods have been developed to address problems in scheduling under uncertainty [24].

Two-stage stochastic programming is one of the most common approach used to solve problems with uncertainties. According to Ierapetritou and Li [15], in the first stage of the two-stage stochastic programming, variables are decided before considering any uncertain parameters. In second stage, decisions will be certain for every uncertain parameter. Bassett et al. [6] used stochastic method to find schedules for every instance of uncertain processing times. Presman et al. [23] used stochastic programming on serial machines that are subjected to breakdown or repair. Denton and Gupta [9] applied two-stage stochastic programming on scheduling appointments for operating rooms. Li and Ierapetritoui [19] addressed uncertain task durations by using stochastic programming method.

Heuristic approaches can be used to adjust schedules based on uncertain parameters. This method was used in literature [8, 14]. Probabilistic or chance constraints have been addressed in several scheduling problems. The probabilistic obtained results are feasible for scenarios that follow a specific distribution. Therefore, decisions are not feasible for those scenarios that are not included in the best fit distribution [17]. The example of chance constraints is scheduling problem for uncertain processing time with risk of violation for certain distribution [22]. Fuzzy programming method was used in scheduling problems for flow shops with mixed integer programming model (MIP) and uncertain processing times [5].

Janak et al. [17] used robust optimization approach to solve MIP scheduling problems. Based on this approach, robust solutions for uncertain coefficient or right hand side parameters in an inequality constraint that are described by a known probability distribution can be obtained. This approach has been proved for uncertain variable with uniform, normal, difference of normal, general discrete, and binomial distributions. However, this approach applies to only continuous uncertain parameters. Janak et al. [16] also summarized robust optimization for short-term scheduling in three main categories. Robust optimization for uncertainty in processing times, uncertainty in product demands, and uncertainty in prices of products and/or raw materials are the three main categories.

Branch-and-bound method using several bounding strategies was often used in the literature. A branch-and-bound algorithm for flow-shop scheduling problems [2], a general bounding scheme for the permutation flow-shop problems [18], and an algorithm for the dynamic lot-size problem with time-varying production capacity constraints [4] are examples of scheduling problems that were solved using the branch-and-bound algorithms.

1.4 Contributions

In this thesis, several optimization models are used to develop and optimize schedules for the A4D0300 workstation at Bombardier. Most scheduling techniques employ only one approach to optimize schedules for high volume manufacturers. However, this study integrates task sequencing and manpower scheduling approaches to develop schedules for low volume manufacturers, such as aircraft industries, with the goal of minimizing the manufacturing cost. This cost is associated with engineering time to make schedules, labour cost, inventory, and late delivery of the final product.

Mathematical models are developed in two phases. In phase I, the objective is to minimize the production labour cost by eliminating overtime while achieving a target cycle time. It is important to create mathematical models that are applicable to every workstation in the production. Hence, several interviews have been conducted with operators of various workstations as well as managers of different production plants to assure that mathematical models consider all possible constraints involved in the production of an aircraft.

In phase II, the objective is to minimize the production labour cost by allowing overtime. Since the duration of each task can be potentially affected by several factors, two-stage stochastic programming is implemented to find the number of operators needed for completing every task under certain scenarios.

The results of the proposed mathematical models from phase I demonstrate that the required number of operators can be reduced from 32 to 27 workers and the cycle time can be reduced by 53% to 67%. The solution of the proposed models from phase II show that the same number of operators is required to complete all assigned tasks under possible scenarios. However, engineer's time to develop a feasible schedule reduces by 88%.

1.5 Structure of this Thesis

Following this introductory chapter, data collection and current state analysis of the system is shown in chapter 2. Mathematical programing models are developed in chapter 3 to minimize the production labour cost. In chapter 4, a two-stage stochastic programming model is proposed to handle uncertainty in processing times. Chapter 5 concludes this thesis.

Chapter 2

Analysis of the Current System

Currently, Bombardier uses Flexsim® to develop schedules for its workstations. This software requires information such as the number and skill of operators needed for each operation, the number of operators that could work at each working zone at a certain time (capacity of work zone), the priority and sequence of operations in the workstation, and the standard time for each operation (task duration) [12].

2.1 Description of the Process Under Study

The workstation used in this study is located in the fuselage production subassembly. Fuselage subassembly consists of two main workstations, A4D0300 and A4D0400. A4D0300 requires structural skilled operators for its tasks and A4D0400 requires operators with structural and electrical skills. Most A4D0300 operations are predecessors of A4D400 tasks. The structural and electrical categorized operations are further divided into the sub-skill tasks. An operator with structural skill can perform every sub-skill task that is under structural skilled criteria. Moreover, an electrical operator can perform every sub-skill operation that is characterized as electrical. Operators prefer to work on the tasks which require the sub-skill of their expertise. Due to the limited time of this project and because both workstations were under continues production change, Bombardier limited data information for this project to only A4D0300 workstation.

Operators work in three equal shifts and the number of operators working in each shift is different. The number of operators for different shifts is based on the union policies and the financial planning for Bombardier. The union policies assure the comfort of the operators by making them work in their preferred shifts. Most operators prefer to work in the morning shift. In addition, the operators working in the morning shift receive the least amount of salary in comparison to the afternoon and night shift operators. The operators working in the night shift receive the highest wage. Moreover, operator's salaries also depend on employee's skill type in the company.

Currently, 52 operators work in the fuselage subassembly. The fuselage subassembly consists of two sets of aircraft components. Each set of aircraft components is done in three main work areas: flight compartment, forward joint, and rear joint. Each set stays in the subassembly for eight days. Components leave the subassembly in order of their arrival. In the A4D0300 workstation, operators start with one set of aircraft components. Three main components are pushed into one another to form the body of the aircraft. More operations are performed inside a joint element in the A4D0400 workstation. Therefore, there is no physical movement between components of an aircraft in this subassembly. Operators perform more than 260 operations in fuselage subassembly. Figure A.1 in Appendix A demonstrates the physical layout of this workstation. In this figure, flight compartment, forward joint, and rear joint work areas are shown in green, yellow, and blue respectively.

Operations are divided between operators based on the skill required for each task. Currently, 40 operators have structural skill and 12 operators work on electrical operations. Twenty three workers work in the morning shift performing structural and electrical jobs, 19 operators are scheduled in the afternoon shift to conduct structural jobs, and 10 workers work in the night shift working on structural tasks. In total, 32 operators are scheduled to work in the A4D0300 workstation and 20 operators are assigned to work on the A4D0400 workstation. In the A4D0300 workstation, 16, 11, and 5 operators are assigned to work in the morning, afternoon, and night shift, respectively. Eight operators work on the flight compartment, twelve operators work on the forward joint, and twelve operators work on the rear joint. The operators in the flight compartment work centre work in two equal shifts.

When components arrive to the A4D0300 workstation, few predecessor tasks such as "load" must be completed before successor tasks can start. Most operations follow prece-

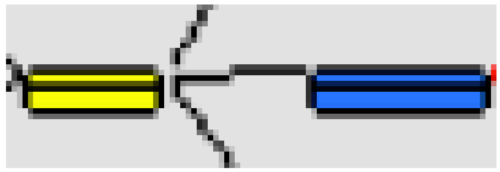


Figure 2.1: Precedence relationship between tasks

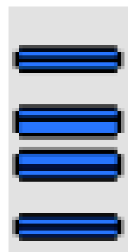


Figure 2.2: Floater tasks

dence relationship in the fuselage subassembly. The relationship between operations is shown in Figure A.2 in Appendix A. In this figure, rectangular nodes represent operations and arcs represent the relationship between tasks. Figure 2.1 shows a sample of the relationship between tasks. In this figure, the yellow coloured node is the predecessor task for the blue coloured operation. In addition, some tasks have to be performed simultaneously and some operations don't have a relation with other tasks (floaters). Floater tasks can be performed at any time in the assembly. Figure 2.2 is an example of floater tasks.

In total, 165 operations are assigned to the A4D0300 workstation. Operations are divided into 3 independent work centers; flight compartment, forward joint, and rear joint. These work centers don't share a common task and independent set of operations are assigned to each work center. Thus, three models were developed to analyze the A4D0300 workstation. 35 tasks are assigned to flight compartment, 58 tasks are assigned to forward joint, and 72 tasks are required for rear joint. Figures A.3, A.4, and A.5 in Appendix A show the precedence relationship between tasks in flight compartment, forward joint, and rear joint work centers.

Bombardier requires every operation in the fuselage subassembly to be completed in 8 days. Currently, operators must complete the tasks in the A4D0300 workstation in first four days and work on the operations of the A4D0400 workstation in the second four days. Tables B.2, B.5, and B.8 in Appendix B illustrate the number of tasks per zone for the flight compartment, forward joint, and rear joint work centers. Tables B.3, B.6, and B.9 in Appendix B display the simultaneous tasks assigned to each work center, and tables B.4, B.7, and B.10 in Appendix B show the precedence relationship between tasks for each work center.

The tasks in the A4D0300 have been categorized in 30 main packages and operations in the A4D0400 have been categorized in 39 main packages. Load, align, pre-trim, pre-drill, drill, rivet, and seal are main operations for flight compartment, forward joint, and rear joint in the A4D0300 workstation. Table B.1 in Appendix B shows the main operations of the A4D0300 workstation.

The A4D0300 workstation has a limited number of work areas (zones). A maximum of six operators can work in the flight compartment at any time; three internally and three externally. Figure A.6 in Appendix A demonstrates the division of the zones in the flight compartment work center. Only two operators could work on the zone coloured in green, two operators could work on the zone coloured in yellow and two operators can work on the zone coloured in dark blue at any given time. Internal and external operators work simultaneously at every work center. To save space, a bench is used as preparation task for some operations in this workstation. About 10% to 20% of particular tasks can be completed on the bench. This could save some areas for other operations to be performed on the actual work center. In Figure A.2 in Appendix A, the operation coloured in orange is the bench work. Figure A.4 in Appendix A shows the division of zones for the A4D0300 workstation. Each node represents a zone and each major zone can be divided into its left, right, center, exterior, and interior sub-zones. Predecessor boxes are the major zones and successor boxes are the sub-zones. Figures A.8, A.9, and A.10 in Appendix A show the division of zones for the flight compartment, forward joint, and rear joint work centers, respectively.

2.2 Analysis and Results of the Current System

Several models were developed to analyze the current state of each work center. Flexsim is used to model and visualize any process for complex workstation assemblies [12]. It requires input information from work centers in general and tasks in particular. The input information from work centers are the number and skill level of employees for each shift, the precedence relationship between tasks, the precedence relationship of work areas, and the maximum capacity of operators for each work area. The input information from operations are the process time for each task, the location and work area of each operation in workstation, the required number of operators for each task, and the percentage of overtime allowed by company. Flexsim provides solutions such as the operation's start and completion time, the percent of task that is completed at each time division in the schedule, and the expected number of operators from each skill type for every shift[11].

Data used for Flexsim and the mathematical models is collected through the interviews and questionnaires from operators and lead managers of different workstations, and management groups of two different plant locations. Currently, the operators follow task sequences based on their personal experience with the workstation.

Questions asked at interviews and questionnaires are categorized as follows:

- Operator's skill type required for each task.
- Required number of operators needed for each task.
- Work task sequence network for each work center.
- Technical difficulties involved with each task.
- Work area of each operation on the aircraft.
- Zoning sequence network for each work center.

- Ergonomic access and location constraints.

- Possibility of tasks breaking down into smaller sub-tasks.

- Identifying simultaneous tasks and critical tasks for each work center.

Table 2.1 is an example of data information gathered from operators.

Table 2.1: A4D0300 workstation: Sample of data information for flight compartment

Book	Book Description	Processing Time	Zone	Priority
Task 1	Locate and Drill Straps	4 (hr)	FCI_Left	1
Task 2	Locate and Drill Straps	4 (hr)	FCI_Right	1
Task 3	Join and CNSK	2.5 (hr)	FCI_Left	2
Task 4	Install and CNSK Stringers	5.5 (hr)	FCI_Left	3

Some operations are performed over several work areas. In Flexsim, these operations are divided into sub-operations with corresponding time, work zone, and operator's skill level assigned to each sub-operation. There is a precedence relationship between sub-operations of a particular task.

Figure A.11 in Appendix A shows the result of interviews for a particular task, "DHS410". In this example, the orange marked areas represent the location where DHS410 is being processed. This "sealing" task is categorized as "591" or the structural operation. It takes 8 hours to proceed and it requires 1 person to complete this task.

Moreover, it is important to design tasks by considering the ergonomic access in work centers. Figure A.12 in Appendix A shows the number of operators that are able to work internally at any work zone at each time. One person can work on right, one person can work on left, and another person can fit in the center to perform the tasks assigned to the floor and ceiling.

The output results from Flexsim are the cycle time, the Gantt chart illustrating the work sequence and the utilization of operators, the critical path, and the bottleneck task for each workstation.

Flexsim assigns tasks to operators heuristically. Depending on the input value for the number of operators, the cycle time will vary for each workstation. Each model in Flexsim is compiled 30 times with allowing the operators to work in three equal shifts and having five percent overtime. The best number of operators and cycle time for the A4D0300 workstation is 32 operators and 8 days respectively, with 2 shifts for the flight compartment, 11 days for the forward joint, and 12 days for the rear joint.

Figure A.13 in Appendix A is an example of the schedule, shown as a Gantt chart. The descriptions on theb left side of the table show the name and order in which the tasks are being processed. The columns represent days and the rows represent sequences of tasks. Thus, it takes 8 days and 2 shifts to complete each task in the flight compartment work center. The blue, yellow, and orange boxes illustrate the regular, critical, and bottleneck tasks. For instance, the "A4D0300_DHS420_430_FJ_R" is the first operation in the schedule and must start in day 3. The black-orange coloured triangles show the shortage of resources (work zones and man power) for a particular day. The gray boxes represent full utilization capacity for a certain day. The daily utilization of operators is shown in purple. Based on this Gantt chart, the operators scheduled in the flight compartment work center are more utilized in the first 6 days and their utilization drops significantly in the remaining 2 days and 2 shifts. Figure A.14 in Appendix A is another example of the result provided by Flexsim. This table represents information regarding to the operator, working shift, start, end, hours worked, and progress percentage of each task for the flight compartment. This information could further be transformed into a graphical work-stream as in Figure A.15 in Appendix A.

Chapter 3

Mathematical Models

In this chapter, several mathematical programming models are developed to build schedules for the tasks required in A4D0300 workstation in a four day cycle time. The decisions include finding the optimal number of operators and the optimal sequencing of tasks. The objective is to minimize the labour cost. The main issues to model are relationships between tasks such as simultaneous or prerequisite tasks, skill specifications, and workspace capacity.

The developed models build discrete schedules that use 30 minute time intervals. The decision to use intervals of this length was made with the agreement of the industrial partner Bombardier. It is also based on the fact that the durations of all tasks are multiples of 30 minutes.

All the models developed in this chapter are solved using Gurobi. Gurobi is a solver and is called from MATLAB to optimize developed programs. Gurobi has been used to solve large scale MIP for various applications. It uses Branch and Bound technique to solve MIP [21].

3.1 Formulation 1: Finding feasible schedules

It is essential to examine the possibility of completing every task in A4D0300 workstation in a four day cycle time (168 time intervals). Formulation 1 is developed to find a feasible schedule. The sets and indices are:

- $l \in L = \{1, 2\}$ is the index for operator's skill type.
- $j \in J^z = \{1, 2, 3, ..., 165\}$ is the index for tasks that need processing in zone z.
- $z \in Z = \{1, 2, 3, ..., 31\}$ is the index for zones.
- $t \in T = \{1, 2, 3, ..., 168\}$ is the index for time intervals.
- $I_1 = (i, j) : i, j \in J^z, z \in Z$ where tasks i and j must be performed simultaneously.
- $I_2 = (i, j) : i, j \in J^z, z \in Z$ where tasks i and j cannot be performed simultaneously.
- $I_3 = (i, j) : i, j \in J^z, z \in Z$ where task i is a predecessor of task j.

The decision variables are:

1. CT_j: Completion time of task j; $j \in J^z$

2.
$$X_{jt} = \begin{cases} 1, & \text{If task } j \text{ is performed in time interval } t;\ t \in T,\ j \in J^z; \\ 0, & \text{Otherwise.} \end{cases}$$

The parameters are:

- C_{lj}: Number of required operators with skill type l to perform task j; $l \in L$, $j \in J^z$.
- C^z: Available space for total number of operators in zone z; $z \in Z$.
- q_j: Duration of task j; $j \in J^z$.

The mathematical model is:

$$\min \quad \sum_{t \in T} \sum_{l \in L} \sum_{j \in J^z} C_{lj} X_{jt} \tag{3.1}$$

$$\text{s.t.} \quad \sum_{l \in L} \sum_{j \in J^z} C_{lj} X_{jt} \leq C^z \qquad \forall t \in T \tag{3.2}$$

$$\sum_{t \in T} X_{jt} = q_j \qquad \forall j \in J^z \tag{3.3}$$

$$CT_j - tX_{jt} \geq 0 \qquad \forall j \in J^z,\ t \in T \tag{3.4}$$

$$X_{it} - X_{jt} = 0 \qquad \forall (i,j) \in I_1 \tag{3.5}$$

$$M(1 - X_{jt}) + tX_{jt} - CT_i \geq 0 \qquad \forall (i,j) \in I_3 \tag{3.6}$$

$$X_{it} + X_{jt} \leq 1 \qquad \forall (i,j) \in I_2 \tag{3.7}$$

$$CT_j \geq 0 \qquad \forall j \in J^z \tag{3.8}$$

$$X_{jt} = 0, 1 \qquad \forall j \in J^z,\ t \in T \tag{3.9}$$

In this model, all employees are treated equally in respect to their salaries. The objective function 3.1 calculates the total number of operators working in all time intervals. Note that this is a constant $\sum_{l\in L}\sum_{j\in J^z} C_{lj}q_j$ since constraint 3.3 sets the total number of intervals that takes for a task to be processed to the task's duration. Constraint 3.2 restricts the number of operators working at a work area to be less than or equal to its available capacity at any time interval. Constraint 3.4 finds the completion time of each task. Constraint 3.5 assures simultaneous tasks i and j to proceed at the same time intervals. Constraint 3.6 restricts predecessor task i to complete before successor task j can start. Constraint 3.7 stops every task j while the critical task i is being processed. Constraints 3.8 and 3.9 limit decision variables CT_j to be non-negative and X_{jt} to be binary.

3.1.1 Results of Formulation 1

Formulation 1 was implemented to find feasible schedules for three work centers at A4D0300 workstation. The flight compartment has 35 tasks, the forward joint has 58 operations, and the rear joint has 72 tasks. Table 3.1 gives the summary of results for formulation 3.1, consisting of the cycle time intervals for completing every task at each work center, the total number of operators required to work at each work center, and the solution time in seconds. The objective value is the summation of number of operators working at each time interval in the cycle time.

Table 3.1: Formulation 1: Summary of results

	Cycle Time (# of Time Intervals)	Objective Value	Computation Time (s)
Flight Compartment	96	386	2
Forward Joint	126	572	4
Rear Joint	134	758	7

The resulting schedules for flight compartment, forward joint, and rear joint work centers are shown in Figures A.16, A.17 , and A.18 in Appendix A. In these figures, the orange coloured rows represent time intervals and the blue coloured columns illustrate existing tasks at each work center. The light purple boxes show times in which a particular task is being processed and the red coloured column demonstrates the completion time of each task. The following Figure 3.1 is a small snapshot of an actual schedule.

The above formulation finds a feasible schedule for a certain cycle time. However, the solution may use different number of skilled operator for each 30 minute time interval. The

	Time 1	Time 2	Time 3	Time 4	Time 5	Time 6	Time 7	Time 8	Time 9	Time 10	Time 11	Time 12	Time 13	Time 14	Time 15	Time 16	Time 17	Time 18	Completi
Task 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	18
Task 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22
Task 3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	18
Task 4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22
Task 5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	23
Task 6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	70
Task 7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	81
Task 8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	109
Task 9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	122
Task 10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	130
Task 11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	23
Task 12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	70
Task 13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	81
Task 14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	109
Task 15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	122
Task 16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	130
Task 17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	110
Task 18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	23
Task 19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	81
Task 20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	88
Task 21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	85
Task 22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	70
Task 23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	81
Task 24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	109
Task 25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	122
Task 26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	130
Task 27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	72
Task 28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	68
Task 29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	62
Task 30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	23

Figure 3.1: Example of a final schedule

variation in number of operators for two consecutive time intervals may be large. However, the number of employees per each shift is fixed. Having a different number of operators for each time interval will lead to a significant un-utilized time for certain operators. Therefore, it is essential to minimize the number of skilled operators per working shift. Note that Bombardier pays higher salary to the night shift operators than the evening shift ones. The day shift operators are paid the lowest.

3.2 Minimizing the Required Number of Operators per Shift

To minimize the required number of operators from each skill type for every working shift in a workstation, we made the following modifications.

The decision variables are:

1. e_{sl}: Required number of skilled operator l working in shift s; $s \in S$, $l \in L$
2. CT_j: Completion time of task j; $j \in J^z$
3. $$X_{jdst} = \begin{cases} 1, & \text{If task } j \text{ is performed in day } d\text{, shift } s \text{ at time interval } t\text{; } s \in S,\ d \in D; \\ 0, & \text{Otherwise.} \end{cases}$$

This model uses the same indices and parameters as 3.1 and the following indices:

- $s \in S = \{1, 2, 3\}$ is the index for working shifts of a day.
- $d \in D = \{1, 2, 3, 4\}$ is the index for the days required for every task to complete in each work center.

The mathematical model is:

$$\min \quad \sum_{s\in S}\sum_{l\in L} e_{sl} \tag{3.10}$$

$$\text{s.t.} \quad \sum_{l\in L}\sum_{j\in J^z} C_{lj}X_{jdst} \leq C^z \qquad \forall d \in D,\ s \in S,\ t \in T \tag{3.11}$$

$$\sum_{d\in D}\sum_{s\in S}\sum_{t\in T} X_{jdst} = q_j \qquad \forall j \in J^z \tag{3.12}$$

$$CT_j - (S(T(d-1)) + T(s-1) + t)X_{jdst} \geq 0 \qquad \forall j \in J^z,\ d \in D,\ s \in S,\ t \in T \tag{3.13}$$

$$X_{idst} - X_{jdst} = 0 \qquad \forall (i,j) \in I_1 \tag{3.14}$$

$$M(1 - X_{jdst}) + ((S(T(d-1)) + T(s-1) + t)$$

$$X_{jdst}) - CT_i \geq 0 \qquad \forall (i,j) \in I_3 \tag{3.15}$$

$$e_{sl} - e_{s+1l} \geq 0 \qquad \forall s \in S,\ l \in L \tag{3.16}$$

$$X_{it} + X_{jt} \leq 1 \qquad \forall (i,j) \in I_2 \tag{3.17}$$

$$\sum_{j \in J^z} C_{lj} X_{jdst} - e_{sl} \leq 0 \qquad \forall l \in L,\, d \in D,\, s \in S,\, t \in T \tag{3.18}$$

$$e_{sl} \geq 0, \text{ Integer} \qquad \forall s \in S,\, l \in L \tag{3.19}$$

$$CT_j \geq 0 \qquad \forall j \in J^z \tag{3.20}$$

$$X_{jdst} = 0, 1 \qquad \forall j \in J^z,\, d \in D,\, s \in S,\, t \in T \tag{3.21}$$

In the above model, the parameters S and T refer to the total number of shifts per day and the total number of time intervals in a target cycle time, respectively. The objective function 3.10 finds the minimum total number of skilled operators for every shift in a work center. Constraint 3.11 limits the number of operators working at each area to be less than or equal to total available zone capacity at any time interval. Constraint 3.18 links the required number of skilled operators for every shift to decision variable X_{jdst}. Constraint 3.12 restricts the processing time of each operation to its duration. Constraint 3.13 finds the completion time of each operation. Constraint 3.14 limits simultaneous tasks i and j to process at same time intervals. Constraint 3.15, restricts predecessor task i to complete before successor task j can start. Constraint 3.16 assures that the total number of operators assigned to the earlier shifts to be greater than or equal to the total number of operators assigned to the later shifts. Constraint 3.17 stops task j while the critical task i is being processed. Constraints 3.19, 3.20 and 3.21 restrict decision variables e_{sl} to be integer and non-negative, CT_j to be non-negative, and X_{jdst} to be binary.

3.2.1 Results of Formulation 2

The formulation 3.2, was solved to find the minimum number of operators for three work centers at A4D0300 workstation. The cycle time was limited to four days with three equal eight hour shifts. Table 3.2 shows the summary of results for formulation 3.2 consisting of the total required number of operators for every work center, and the computation time of the program in seconds. The optimal result concludes that 27 operators are required to complete every task in a four day cycle time. Table 3.3, gives the total required number of operators for each working shift for every work center at the A4D0300 workstation.

Table 3.2: Formulation 2: Summary of results

	Objective Value	Computation Time (s)
Flight Compartment	6	48
Forward Joint	10	242
Rear Joint	11	149

Table 3.3: Formulation 2: Required number of skilled operator for each shift

	Shift 1	Shift 2	Shift 3
Flight compartment work center	2	2	2
Forward joint work center	4	3	3
Rear joint work center	4	4	3

Due to the fact that operators at the evening and night shifts are paid more than operators at the day shift, the above mathematical model can further be improved by assigning different penalties to operators of each shift. The value of penalties assigned to each shift reflects the difference in pay and depend on the rules and regulations of the company. Section 3.3 explains this approach in detail.

3.3 Using Penalties to Minimize the Required Number of Operators per Shift

The operator's salaries vary depending on the policies of the employer Bombardier. Thus, more accurate results can be obtained by using penalties assigned to the employees of various shifts. In this section, the penalties are used to encourage more work done during the day shifts.

The following formulation uses the same decision variables and parameters as formulation 3.2 with the additional parameters:

- p_{sl}: Penalty associated with the number of operators with skilled level l in shift s.

The mathematical model is:

$$\min \quad \sum_{s\in S}\sum_{l\in L}\sum_{s\in S} p_{sl}e_{sl} \tag{3.22}$$

$$\text{s.t.} \quad \sum_{l\in L}\sum_{j\in J^z} C_{lj}X_{jdst} \leq C^z \qquad \forall d \in D,\ s \in S,\ t \in T$$

$$\sum_{j\in J^z} C_{lj}X_{jdst} - e_{sl} \leq 0 \qquad \forall l \in L,\ d \in D,\ s \in S,\ t \in T$$

$$\sum_{d\in D}\sum_{s\in S}\sum_{t\in T} X_{jdst} = q_j \qquad \forall j \in J^z$$

$$CT_j - (S(T(d-1)) + T(s-1) + t)X_{jdst} \geq 0 \quad \forall j \in J^z,\ d \in D,\ s \in S,\ t \in T$$

$$X_{idst} - X_{jdst} = 0 \qquad \forall (i,j) \in I_1$$

$$M(1 - X_{jdst}) + ((S(T(d-1)) + T(s-1) + t)$$

$$X_{jdst}) - CT_i \geq 0 \qquad \forall (i,j) \in I_3$$

$$X_{idst} + X_{jdst} \leq 1 \qquad \forall (i,j) \in I_2$$

$$e_{sl} \geq 0, \text{ Integer} \qquad \forall s \in S,\ l \in L$$

$$CT_j \geq 0 \qquad \forall j \in J^z$$

$$X_{jdst} = 0, 1 \qquad \forall j \in J^z,\ d \in D,\ s \in S,\ t \in T$$

The objective function 3.22 finds the minimum production labour cost in a workstation. The number of operators scheduled to every shift depends on the value of penalty assigned to operators of the particular shift. Other constraints of this model are similar to constraints 3.11, 3.18, 3.12, 3.13, 3.14, 3.15, and 3.17 and description of these constraints can be found in Section 3.2.

This formulation is solved for three work centers at A4D0300 workstation. The arbitrary penalties assigned to the operators working in morning shift, afternoon shift, and night shift are 20, 22, and 30. The total production labour cost for flight compartment work center is \$120, forward joint is \$198, and rear joint is \$360. Table 3.4 displays the results consisting of the total required number of operators for each working shift for every work center. 27 operators are required to complete every task in a four day cycle time for A4D0300 workstation. These results meet the expectation of Bombardier for the required number of operators working at each shift. However, the results will vary if assigned penalties are different.

Table 3.4: Formulation 3: Summary of results and required number of skilled operator for each shift

	Objective Value	Computation Time (s)	Shift 1	Shift 2	Shift 3
Flight Compartment	6	52	2	2	2
Forward Joint	9	502	4	3	2
Rear Joint	12	382	4	4	4

The resulting schedules for flight compartment, forward joint, and rear joint work centers are shown in Figures A.19, A.20, and A.21 in Appendix A. In these figures, the blue coloured rows represent the time intervals for shifts that are separated by the green rows. Each green block illustrates the start of a new shift. The columns demonstrate existing tasks at each work center and the red coloured row show the completion time of each task. The light purple boxes display times in which a particular task is being processed. The following Figure 3.2 is a small snapshot of an actual schedule.

The mathematical models developed in this chapter do not consider overtime. However, Bombardier as a manufacturer faces large amount of backlogs in its tasks processing time. Thus, it is important to allow some overtime for every working shift. The following MIP is developed to minimize the total regular and overtime production labour costs and to determine the total required number of operators for every shift for each work center.

	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Task 7	Task 8	Task 9	Task 10	Task 11	Task 12	Task 13	Task 14	Task 15	Task 16	Task 17	Task 18	Task 19	Task 20	Task 21	Task 22	Task 23	Task 24
Day 1/Shift 1																								
Time 1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Time 2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Time 3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Time 4	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Time 5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Time 6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Time 7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Time 8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Time 9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
Time 10	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
Time 11	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time 12	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
Time 13	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
Time 14	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time 15	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
Time 16	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
Day 1/Shift 2																								
Time 1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
Time 2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
Time 3	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time 4	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time 5	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time 6	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time 7	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time 8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
Time 9	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time 10	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time 11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
Time 12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
Time 13	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time 14	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time 15	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time 16	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Completion Time	8	17	51	77	128	169	191	18	51	70	128	164	185	66	9	18	51	70	133	175	192	56	81	140

Figure 3.2: Example of final schedule

3.4 Minimizing the Required Number of Operators per Shift by Allowing Overtime

In this section, a MIP model is developed to minimize the total number of regular and overtime employees. The decision variables and parameters are the same as formulation 3.3 with changes in the definition of the decision variable X_{jsdt} and parameter p_{sl}. These changes are due to the consideration of both regular and overtime working hours.

1.

$$XO_{jst} = \begin{cases} 1, & \text{If task } j \text{ is performed in shift } s \text{ at overtime interval } t;\ s \in S,\ j \in J^z; \\ 0, & \text{Otherwise.} \end{cases}$$

2.

$$XR_{jst} = \begin{cases} 1, & \text{If task } j \text{ is performed in shift } s \text{ at regular time interval } t;\ j \in J^z; \\ 0, & \text{Otherwise.} \end{cases}$$

3. PO_{stl}: is the overtime cost of operators with skill level l in shift s at overtime interval t.

4. PR_{sl}: is the regular time cost for skilled operator l in shift s.

The mathematical model is:

$$\min \quad \sum_{s \in S} \sum_{l \in L} (PR_{sl} e_{sl}) + \sum_{s \in S} \sum_{t \in T} \sum_{l \in L} PO_{stl} XO_{jst} \tag{3.23}$$

$$\text{s.t.} \quad \sum_{l \in L} \sum_{j \in J^z} C_{lj} (XO_{js-1t} + XR_{jst}) \leq C^z \qquad \forall s \in S,\ t \in T \tag{3.24}$$

$$\sum_{j \in J^z} C_{lj} XO_{jst} - e_{sl} \leq 0 \qquad \forall l \in L,\ s \in S,\ t \in T \tag{3.25}$$

$$\sum_{j \in J^z} C_{lj} XR_{jst} - e_{sl} \leq 0 \qquad \forall l \in L,\ s \in S,\ t \in T \tag{3.26}$$

$$\sum_{s \in S} \sum_{t \in T} (XO_{jst} + XR_{jst}) = q_j \qquad \forall j \in J^z \tag{3.27}$$

$$CT_j - (T(s-1)+t)XR_{jst} \geq 0 \qquad \forall j \in J^z,\, s \in S,\, t \in T \tag{3.28}$$

$$CT_j - (T(s)+t)XO_{jst} \geq 0 \qquad \forall j \in J^z,\, s \in S,\, t \in T \tag{3.29}$$

$$XR_{ist} - XR_{jst} = 0 \qquad \forall (i,j) \in I_1 \tag{3.30}$$

$$XO_{ist} - XO_{jst} = 0 \qquad \forall (i,j) \in I_1 \tag{3.31}$$

$$M(1 - XR_{jst}) + (T(s-1)+t)XR_{ist} - CT_i \geq 0 \qquad \forall (i,j) \in I_3 \tag{3.32}$$

$$M(1 - XO_{jst}) + (T(s)+t)XO_{ist} - CT_i \geq 0 \qquad \forall (i,j) \in I_3 \tag{3.33}$$

$$XR_{ist} + XR_{jst} \leq 1 \qquad \forall (i,j) \in I_2 \tag{3.34}$$

$$XO_{ist} + XO_{jst} \leq 1 \qquad \forall (i,j) \in I_2 \tag{3.35}$$

$$XO_{jst} - XO_{jst+1} \geq 0 \qquad \forall j \in J^z,\, s \in S,\, t \in T \tag{3.36}$$

$$e_{sl} \geq 0, \text{ Integer} \qquad \forall s \in S,\, l \in L \tag{3.37}$$

$$CT_j \geq 0 \qquad \forall j \in J^z \tag{3.38}$$

$$XO_{jst}, XR_{jst} \in 0,1 \qquad \forall j \in J^z,\, s \in S,\, t \in T \tag{3.39}$$

The objective function 3.23 minimizes the total production labour cost by allowing two half-hour overtime intervals. Constraint 3.24 limits the number of operators working at each area to be less than or equal to the total available zone capacity at any time interval. Constraints 3.25 and 3.26 link the required number of skilled operators for every shift to the decision variable X_{jst}. Constraint 3.27 restricts the processing time of each operation to its duration. Constraints 3.28 and 3.29 find the completion time of each operation.

Constraints 3.30 and 3.31 restrict simultaneous tasks i and j to proceed at same time intervals. Constraints 3.32 and 3.33 limit predecessor task i to complete before successor task j can start. Constraints 3.34 and 3.35 stop every task j while the critical task i is being processed. Constraint 3.36 assures that the overtime operators complete their tasks in earlier overtime intervals to maximize their utilization. Constraints 3.37, 3.38, and 3.39 restrict decision variables e_{sl} to be integer and non-negative, CT_j to be non-negative, XO_{jst}, and XR_{jst} to be binary.

3.4.1 Results of Formulation 4

One overtime hour was allowed for each shift for every work center. The previous formulation was solved with different overtime costs for operators working in morning, afternoon, and night shifts. Bombardier allows different rates of operators working overtime based on its variation in production. In the following sections, the results of two different cases are explained in detail. In Case 1, we study the effect of arbitrarily assigning three different penalties to the operators working overtime at each shift. In Case 2, we analyse the results by assigning similar overtime penalties to each shift.

Due to the size and complexity of the MIP model, the computation time of each model was limited to eight hours. Thus, the objective value shows the best feasible solution found during the computation time of the program [21].

Case One:

The regular time penalties assigned to operators working in morning, afternoon, and night shifts are 20, 22, and 30. The overtime penalties assigned to each overtime interval for morning, afternoon, and night shifts are 1.0, 1.5, and 2.0. Table 3.5 displays the total required number of operators for each working shift for every work center and the percentage gap from optimality of the result. 27 operators are required to complete every task in a four day cycle time for the A4D0300 workstation and the total production labour cost for flight compartment work center is $120, forward joint is $198, and rear joint is $360.

Table 3.5: Formulation 4: Summary of results and required number of skilled operator for each shift

	Objective Value	% Gap from Optimality	Shift 1	Shift 2	Shift 3
Flight Compartment	6	2	2	4	0
Forward Joint	9	8	5	2	2
Rear Joint	12	11	6	4	2

The resulting schedules for flight compartment, forward joint, and rear joint work centers are shown in Figures A.22, A.23, and A.24 in Appendix A. In these figures, the blue coloured rows represent the time intervals for shifts that are separated by green coloured rows. Each green block illustrates the start of a new shift and each orange coloured row is the overtime interval allowed for the model. The columns display the existing tasks at each work center and the red coloured row shows the completion time of each task. The light purple boxes display the times in which a particular task is being processed. The following figure 3.3 is a small snapshot of an actual schedule.

	Time 1	Time 2	Time 3	Time 4	Time 5	Time 6	Time 7	Time 8	Time 9	Time 10	Time 11	Time 12	Time 13	Time 14	Time 15	Time 16	Time 17	Time 18	Completi
Task 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	18
Task 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22
Task 3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	18
Task 4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22
Task 5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	23
Task 6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	70
Task 7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	81
Task 8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	109
Task 9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	122
Task 10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	130
Task 11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	23
Task 12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	70
Task 13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	81
Task 14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	109
Task 15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	122
Task 16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	130
Task 17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	110
Task 18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	23
Task 19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	81
Task 20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	88
Task 21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	85
Task 22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	70
Task 23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	81
Task 24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	109
Task 25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	122
Task 26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	130
Task 27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	72
Task 28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	68
Task 29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	62
Task 30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	23

Figure 3.3: Example of a final schedule

Case Two:

The regular time penalties assigned to operators working in morning, afternoon, and night shifts are 20, 22, and 30. The overtime penalties assigned to each overtime interval for morning, afternoon, and night shifts are 0.3, 0.3, and 0.3. Table 3.6 displays the total required number of operators for each working shift for every work center and the percentage gap from optimality of the result. 26 operators are required to complete every task in a four day cycle time for the A4D0300 workstation and the total production labour cost for flight compartment work center is \$120, forward joint is \$198, and rear joint is \$330.

Table 3.6: Formulation 4: Summary of results and required number of skilled operator for each shift

	Objective Value	% Gap from Optimality	Shift 1	Shift 2	Shift 3
Flight Compartment	6	2	2	4	0
Forward Joint	9	4	4	3	2
Rear Joint	11	8	6	3	2

The resulting schedules for flight compartment, forward joint, and rear joint work centers are shown in Figures A.25, A.26, and A.27 in Appendix A. In these figures, the blue coloured rows represent the time intervals for shifts that are separated by the green coloured rows. Each green block shows the start of a new shift and each orange coloured row is the overtime interval allowed for the model. The columns demonstrate the existing tasks at each work center and the red coloured row show the completion time of each task. The light purple boxes display the times in which a particular task is being processed. The percent utilization of the operators are shown in Table 3.7.

Table 3.7: Percent utilization of operators for each work center

	Formulation 3.2	Formulation 3.3	Formulation 3.4 Case 1	Formulation 3.4 Case 2
Flight Compartment Morning Shift	96.1	96.1	81.1	81.1
Flight Compartment Afternoon Shift	98.44	98.44	86.46	86.46
Flight Compartment Night Shift	99.22	99.22	0	0
Forward Joint Morning Shift	90.23	90.23	77.6	89.8
Forward Joint Afternoon Shift	94.3	94.27	83.55	76.3
Forward Joint Night Shift	65.63	99.22	84.67	82.7
Rear Joint Morning Shift	91.8	91.8	88.38	86.71
Rear Joint Afternoon Shift	94.14	94.14	91.64	98.84
Rear Joint Night Shift	83.27	96.88	91.3	92.63
	Formulation 3.1			
Flight Compartment	57.14			
Forward Joint	60.43			
Rear Joint	68			

Chapter 4

Uncertain Task Processing Times

The mathematical models developed in Chapter 3 are based on the assumption that the duration of each task is known. However, the actual processing times for each task are uncertain. For each task, a historical data under 30 scenarios is given. In this chapter we propose a two-stage stochastic programming approach for the problem. This is motivated by the data that Bombardier provided. The data represents the duration of each task in production of 30 products. It consists of 30 scenarios with different processing times per task. We use the same set of parameters, indices, and decision variables as Section 3.4. We define an additional index $\delta \in \Delta = \{1, , 30\}$: for scenarios.

The formulation is:

$$\min \quad \sum_{s\in S}\sum_{l\in L}(PR_{sl}e_{sl}) + E(\sum_{s\in S}\sum_{t\in T}\sum_{l\in L}PO^{\delta}_{stl}XO^{\delta}_{jst}) \tag{4.1}$$

$$\text{s.t.} \quad \sum_{l\in L}\sum_{j\in J^z}C_{lj}(XO^{\delta}_{j(s-1)t} + XR^{\delta}_{jst}) \leq C^z \qquad \forall s\in S,\, t\in T,\, \delta\in\Delta \tag{4.2}$$

$$\sum_{j\in J^z}C_{lj}XO^{\delta}_{jst} \leq e_{sl} \qquad \forall l\in L,\, s\in S,\, t\in T,\, \delta\in\Delta \tag{4.3}$$

$$\sum_{j\in J^z}C_{lj}XR^{\delta}_{jst} \leq e_{sl} \qquad \forall l\in L,\, s\in S,\, t\in T,\, \delta\in\Delta \tag{4.4}$$

$$\sum_{s\in S}\sum_{t\in T}(XO^{\delta}_{jst} + XR^{\delta}_{jst}) = q^{\delta}_j \qquad \forall j\in J^z,\, \delta\in\Delta \tag{4.5}$$

$$CT^{\delta}_j - (T(s-1)+t)XR^{\delta}_{jst} \geq 0 \qquad \forall j\in J^z,\, s\in S,\, t\in T,\, \delta\in\Delta \tag{4.6}$$

$$CT^{\delta}_j - (T(s)+t)XO^{\delta}_{jst} \geq 0 \qquad \forall j\in J^z,\, s\in S,\, t\in T,\, \delta\in\Delta \tag{4.7}$$

$$XR^{\delta}_{ist} - XR^{\delta}_{jst} = 0 \qquad \forall (i,j)\in I_1,\, s\in S,\, t\in T,\, \delta\in\Delta \tag{4.8}$$

$$XO^{\delta}_{ist} - XO^{\delta}_{jst} = 0 \qquad \forall (i,j) \in I_1,\ s \in S,\ t \in T,\ \delta \in \Delta \tag{4.9}$$

$$M(1 - XR^{\delta}_{jst}) + (T(s-1)+t)XR^{\delta}_{ist} - CT^{\delta}_{i} \geq 0 \quad \forall (i,j) \in I_3,\ s \in S,\ t \in T,\ \delta \in \Delta \tag{4.10}$$

$$M(1 - XO^{\delta}_{jst}) + (T(s)+t)XO^{\delta}_{ist} - CT^{\delta}_{i} \geq 0 \qquad \forall (i,j) \in I_3,\ s \in S,\ t \in T,\ \delta \in \Delta \tag{4.11}$$

$$XR^{\delta}_{ist} + XR^{\delta}_{jst} \leq 1 \qquad \forall (i,j) \in I_2,\ s \in S,\ t \in T,\ \delta \in \Delta \tag{4.12}$$

$$XO^{\delta}_{ist} + XO^{\delta}_{jst} \leq 1 \qquad \forall (i,j) \in I_2,\ s \in S,\ t \in T,\ \delta \in \Delta \tag{4.13}$$

$$XO^{\delta}_{jst} - XO^{\delta}_{js(t+1)} \geq 0 \qquad \forall j \in J^{z},\ s \in S,\ t \in T,\ \delta \in \Delta \tag{4.14}$$

$$e_{sl} \geq 0 \qquad \forall s \in S,\ l \in L \tag{4.15}$$

$$CT^{\delta}_{j} \geq 0 \qquad \forall j \in J^{z},\ \delta \in \Delta \tag{4.16}$$

$$XO^{\delta}_{jst}, XR^{\delta}_{jst} \in 0,1 \qquad \forall j \in J^{z},\ s \in S,\ t \in T,\ \delta \in \Delta \tag{4.17}$$

In the above model, the expected value for each scenario in the objective function is

1/30, because the probability distribution of scenarios is assumed to be uniform.

The objective function 4.1 minimizes the total production labour cost by considering the regular cost for skilled operator as well as the expected overtime cost over all scenarios. Constraint 4.2 limits the number of operators working at each area including both regular operators assigned to the similar shift as well as the overtime operators from previous shift, to be less than or equal to the total available zone capacity at any time interval for each scenario. Constraints 4.3 and 4.4 link the required number of skilled operators for every shift to the decision variable X^{δ}_{jst} for every scenario. Constraint 4.5 restricts the processing time of each operation to its duration for every scenario. Constraints 4.6 and 4.7 find the completion time of each operation for every scenario. Constraints 4.8 and 4.9 make sure that tasks i and j are processes at the same time in each scenario. Constraints 4.10 and 4.11 make sure that predecessor task i is completed before successor task j. Constraints 4.12 and 4.13 ensure that task j is stopped while critical task i is being processed. Constraint 4.14 ensure that an operator can not be idle in the first time interval and busy in the second. Constraints 4.15, 4.16, and 4.17 restrict decision variables e_{sl} to be integer and non-negative, CT^{δ}_{j} to be non-negative, XO^{δ}_{jst}, and XR^{δ}_{jst} to be binary for every scenario. Solving the two-stage stochastic programming directly is very time-consuming. Hence, we propose the following construction heuristic that adjusts the solution of the most-likely scenario to find feasible solutions for all scenarios.

1. Separate tasks by skill level required.

2. For each skill level, add the task durations for each scenario.

3. Select scenarios with the largest duration for each skill level.

4. For every skill level l, use e_{sl} of the most-likely scenario.

5. While $\sum_{l\in L} e_{s_1 l} \leq C^z$:

   - Start from $l = 1$ to $l = L$, solve formulation 3.4 for the scenario with the largest duration for each skill type.

     – If the solution is infeasible or unbounded, add 1 employee to skill level $e_{s_1 1}$.

     – Continue adding one employee to skill level $e_{s_1 l}$, until $\sum_{l\in L} e_{s_1 l} = C^z$ or the model is feasible.

   - If the solution is feasible, use the results for $e_{s_1 l}$ and the values of $e_{s_2 l}$ and $e_{s_3 l}$ from the most-likely scenario and solve the stochastic model to find schedules for every scenario.

6. If $\sum_{l\in L} e_{s_1 l} = C^z$, and the solution for scenarios with the largest duration is infeasible:

   - While $\sum_{l\in L} e_{s_2 l} \leq C^z$:

     – Repeat step 5 to find $e_{s_2 l}$.

     – If the solution is feasible, use the results for $e_{s_1 l}$ and $e_{s_2 l}$ and the value of $e_{s_3 l}$ from the most-likely scenario and solve the stochastic model to find schedules for every scenario.

7. If $\sum_{l \in L} e_{s_2 l} = C^z$, and the solution for scenarios with the largest duration is still infeasible:

   - Repeat step 5 to find $e_{s_3 l}$.

8. Use the results for $e_{s_1 l}$, $e_{s_2 l}$, and $e_{s_3 l}$ to solve the stochastic model and find schedules for every scenario.

$e_{s_1 l}$ is required to have the largest number of employees because of the cost assigned to each shift in the objective function of the stochastic formulation.

4.1 Results of the Two-stage Stochastic Programming

We apply the above heuristic to three work centers at the A4D0300 workstation. One overtime hour was allowed to each shift for every work center. The flight compartment has 35 tasks, the forward joint has 58 operations, and the rear joint has 72 tasks.

Figures A.28, A.29, and A.30 provide scenarios for flight compartment, forward joint, and rear joint work centers. In these figures, the green coloured column represents the scenarios and the blue coloured row represents the tasks assigned to each work center. The yellow coloured column shows the total task durations for every scenario and the red coloured row displays the scenario with the largest duration.

The regular time costs assigned to operators working in morning, afternoon, and night shifts are 20, 22, and 30. The overtime costs assigned to each overtime interval for morning, afternoon, and night shifts are 0.1563, 0.234, and 0.469. Table 4.1 displays the total required number of operators for each shift for every work center and the total production labour cost. Table B.11 shows the overtime production labour cost for every scenario at each work center.

The following figure 4.1 is a small snapshot of an actual schedule.

	Task 14	Task 15	Task 16	Task 17	Task 18	Task 19	Task 20	Task 21	Task 22	Task 23	Task 24	Task 25	Task 26	Task 27	Task 28	Task 29	Task 30	Task 31	Task 32	Task 33	Task 34	Task 35	SUM
Scenario 1	9	8	5	11	3	19	24	8	4	4	2	4	11	19	8	11	19	8	11	19	8	14	369
Scenario 2	9	5	5	9	3	19	24	5	4	4	2	4	9	19	5	9	19	5	9	19	5	14	336
Scenario 3	9	8	3	11	1	19	24	8	4	4	2	4	11	19	8	11	19	8	11	19	8	11	354
Scenario 4	9	6	5	11	3	16	24	6	4	4	2	4	11	16	6	11	16	6	11	16	6	14	337
Scenario 5	9	6	5	8	3	19	24	6	4	4	2	4	8	19	6	8	19	6	8	19	6	12	335
Scenario 6	9	7	5	11	3	19	21	7	4	4	2	4	11	19	7	11	19	7	11	19	7	13	352
Scenario 7	9	8	4	11	3	19	24	8	2	2	1	2	11	19	8	11	19	8	11	19	8	14	358
Scenario 8	9	8	5	10	3	17	22	8	4	4	2	4	10	17	8	10	17	8	10	17	8	11	342
Scenario 9	9	8	5	11	3	18	23	8	4	4	2	4	11	18	8	11	18	8	11	18	8	13	359
Scenario 10	8	7	4	9	2	17	20	7	3	3	1	3	9	17	7	9	17	7	9	17	7	12	318
Scenario 11	9	8	1	11	1	18	21	8	2	2	1	2	11	18	8	11	18	8	11	18	8	14	335
Scenario 12	9	9	5	12	2	19	24	9	3	3	2	3	12	19	9	12	19	9	12	19	9	15	377
Scenario 13	9	10	6	11	3	21	25	10	4	4	2	4	11	21	10	11	21	10	11	21	10	14	401
Scenario 14	9	8	5	11	4	19	24	8	6	6	4	6	11	19	8	11	19	8	11	19	8	17	383
Scenario 15	9	11	5	10	3	19	24	11	4	4	2	4	10	19	11	10	19	11	10	19	11	16	386
Scneario 16	9	12	8	11	3	20	20	12	4	4	2	4	12	20	12	11	20	12	11	20	12	14	401
Scneario 17	9	8	7	11	6	19	24	8	5	5	1	5	11	19	8	11	19	8	11	19	8	15	387
Scenario 18	9	10	5	11	3	22	25	10	4	4	2	4	11	22	10	11	22	10	11	22	10	17	407
Scenario 19	9	11	6	13	5	20	26	11	7	7	4	7	13	20	11	13	20	11	13	20	11	16	436
Scenario 20	11	10	7	13	5	21	26	10	6	6	4	6	13	21	10	13	21	10	13	21	10	16	437
Scenario 21	10	9	6	12	4	20	25	9	5	5	3	5	12	20	9	12	20	9	12	20	9	15	403
Scenario 22	8	7	4	10	2	18	23	7	3	3	1	3	10	18	7	10	18	7	10	18	7	13	335
Scenario 23	7	6	3	9	1	17	22	6	2	2	1	2	9	17	6	9	17	6	9	17	6	12	302
Scenario 24	9	7	5	14	2	19	24	7	4	4	2	4	14	19	7	14	19	7	14	19	7	14	378
Scenario 25	9	9	5	11	3	20	24	9	4	4	2	4	11	20	9	11	20	9	11	20	9	16	384
Scenario 26	9	8	5	11	3	19	24	8	5	5	3	5	11	19	8	11	19	8	11	19	8	13	372
Scenario 27	9	8	8	11	4	19	24	8	3	3	2	3	11	19	8	11	19	8	11	19	8	15	379
Scenario 28	9	8	5	11	3	22	23	8	4	4	2	4	11	22	8	11	22	8	11	22	8	14	384
Scenario 29	9	7	5	11	3	19	24	7	3	3	2	3	11	19	7	11	19	7	11	19	7	14	359
Scenario 30	9	8	5	11	1	19	24	8	4	4	1	4	11	19	8	11	19	8	11	19	8	14	362

Figure 4.1: Sample of the historical data provided by Bombardier

According to the results of section 3.4, 27 operators are required to work at the A4D0300 workstation in order to complete every task with most-likely duration. However, 32 operators are needed when considering the task uncertainty. Thus, it is important for the company to share the 5 extra employees with other workstations in order to maximize operator's utilization.

Table 4.1: The required number of operators per Shift and total labour cost per compartment

	Morning	Afternoon	Night	Total Cost
Penalty Per Regular Working Shift	20	30	60	
Overtime Penalty Per Time Interval	0.1563	0.234	0.469	
Flight Compartment	3	4	0	180.0156
Forward Joint	6	3	2	330.1682
Rear Joint	6	4	4	480.3541

4.2 Comparison

The results of the five proposed models as applied to the A4D0300 workstation are compared. Three criteria are used: the required number of operators, the computation time, and the total production labour cost. Table 4.2 provides a summary.

Table 4.2: Summary of the results

	Total Number of Operators	Computation Time (s)	Total Production Cost ($)
Formulation 3.1	1712(over all time intervals)	13	N/A
Formulation 3.2	27	439	N/A
Formulation 3.3	27	936	678
Formulation 3.4	26	86400	648
Formulation 4	32	109188	990.5379

The two-stage stochastic programming model, Formulation 4, requires the largest number of operators as well as a significant computation time, because it evaluates all possible scenarios. These solutions show that as more complexities are taken into the calculation in the objective function, the solutions are more efficient but the computation time of the model increases significantly.

As expected, the two-stage stochastic programming model, Formulation 4, requires the largest number of operators as well as a significant computation time, because it evaluates all possible scenarios.

Chapter 5

Conclusion

In this thesis, a realistic scheduling assignment problem for an airline manufacturer was investigated. The models developed incorporated most, if not all, practical constraints that affect the schedules.

Flexsim was first used to analyze the current state of three work centers: the flight compartment, the forward joint, and the rear joint. It was found that 32 operators are required to work in three work centers with 5% overtime allowance. In addition, a cycle time of 8 days and 2 shifts for flight compartment, 11 days for forward joint, and 12 days for rear joint are required to complete every task.

To improve the current schedule, several deterministic MIP models were developed to evaluate the possibility of completing every task in 4 days. The objective was to build schedules that minimize the total number of skilled operators per working shift by eliminating and/or considering the effect of overtime on the production cost. The results reveal that, 27 operators were needed to complete every task in a target cycle time. Therefore, the schedules obtained from mathematical models reduced the number of operators by 5 and decreased production cycle time by 53% for flight compartment, 64% for forward joint, and 67% for rear joint work centers with approximately no overtime hours. In addition to the significant savings in production cost, it takes 1.5 months (approximately 36 days) for engineers to make manual schedules. However, each mathematical model could provide an optimal solution in less than a day.

As most operations are performed manually, the task durations are typically uncertain. Currently, it takes approximately one month (24 days) for engineers to adjust the production variations for each schedule. To improve this, a two-stage stochastic programming approach was proposed to find the required number of operators that are capable of completing every task in each work center under different scenarios. Based on the results, 32 operators are needed to complete every task in all work centers. Most processing times are between the minimum and most-likely durations, thus, it is important to share the extra five operators with other workstations in the production to increase their utilization.

Appendix A

Figures

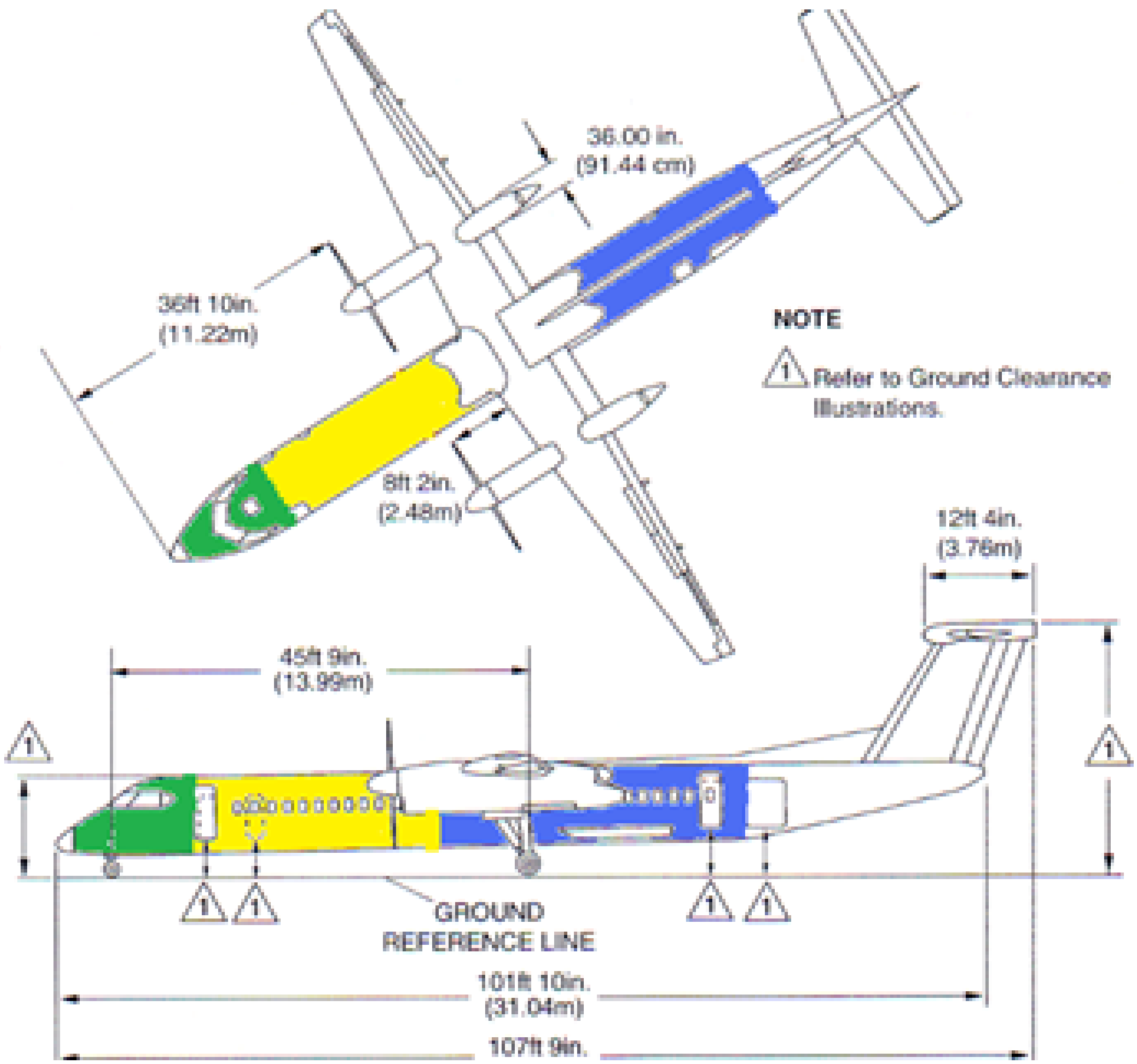


Figure A.1: A4D0300 workstation: Physical layout and division of work areas

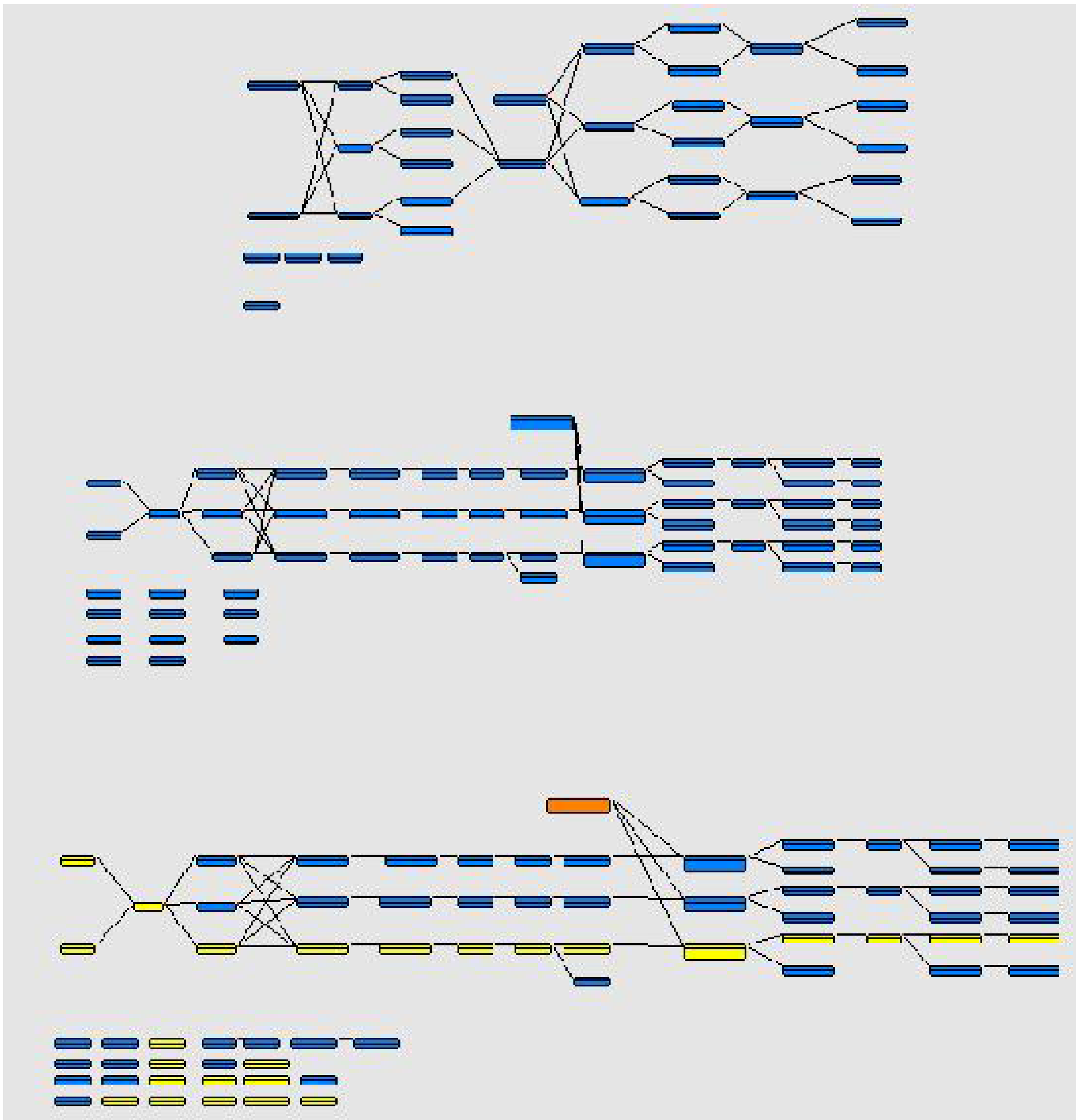


Figure A.2: A4D0300 workstation: Precedence relationship of operations

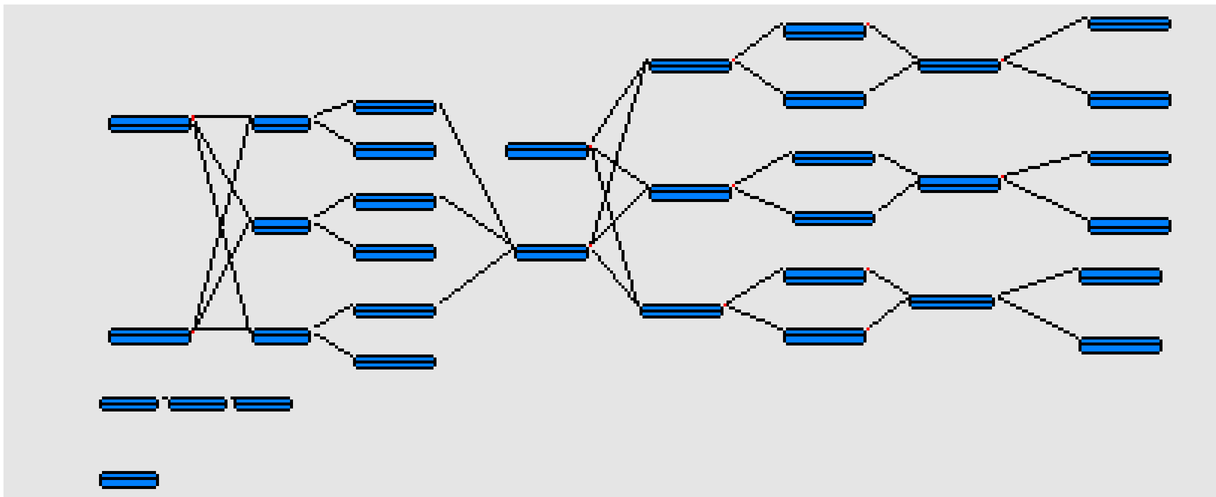


Figure A.3: Precedence relationship of operations in flight compartment work center

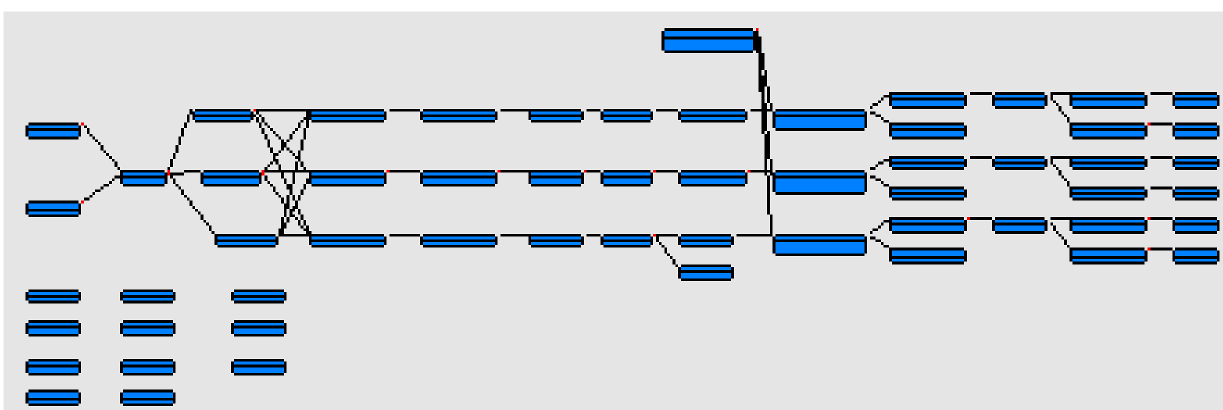


Figure A.4: Precedence relationship of operations in forward joint work center

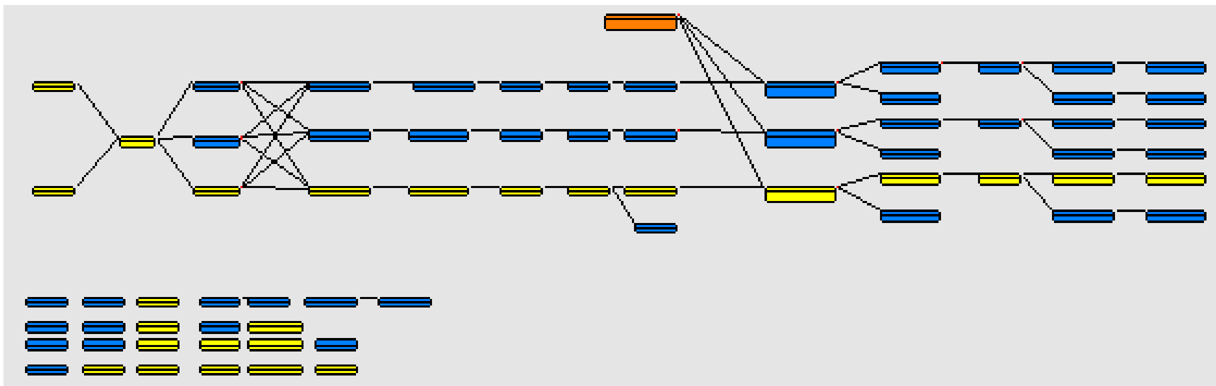


Figure A.5: Precedence relationship of operations in rear joint work center

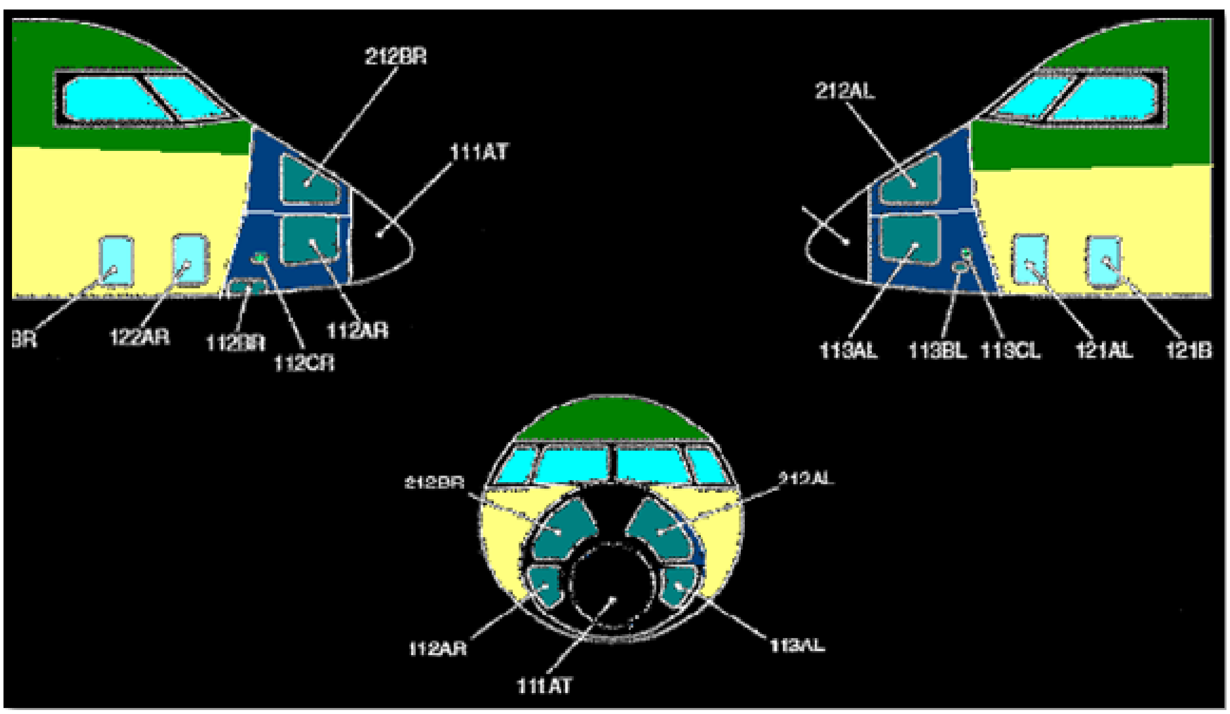


Figure A.6: Flight compartment subassembly: Physical layout and division of work areas

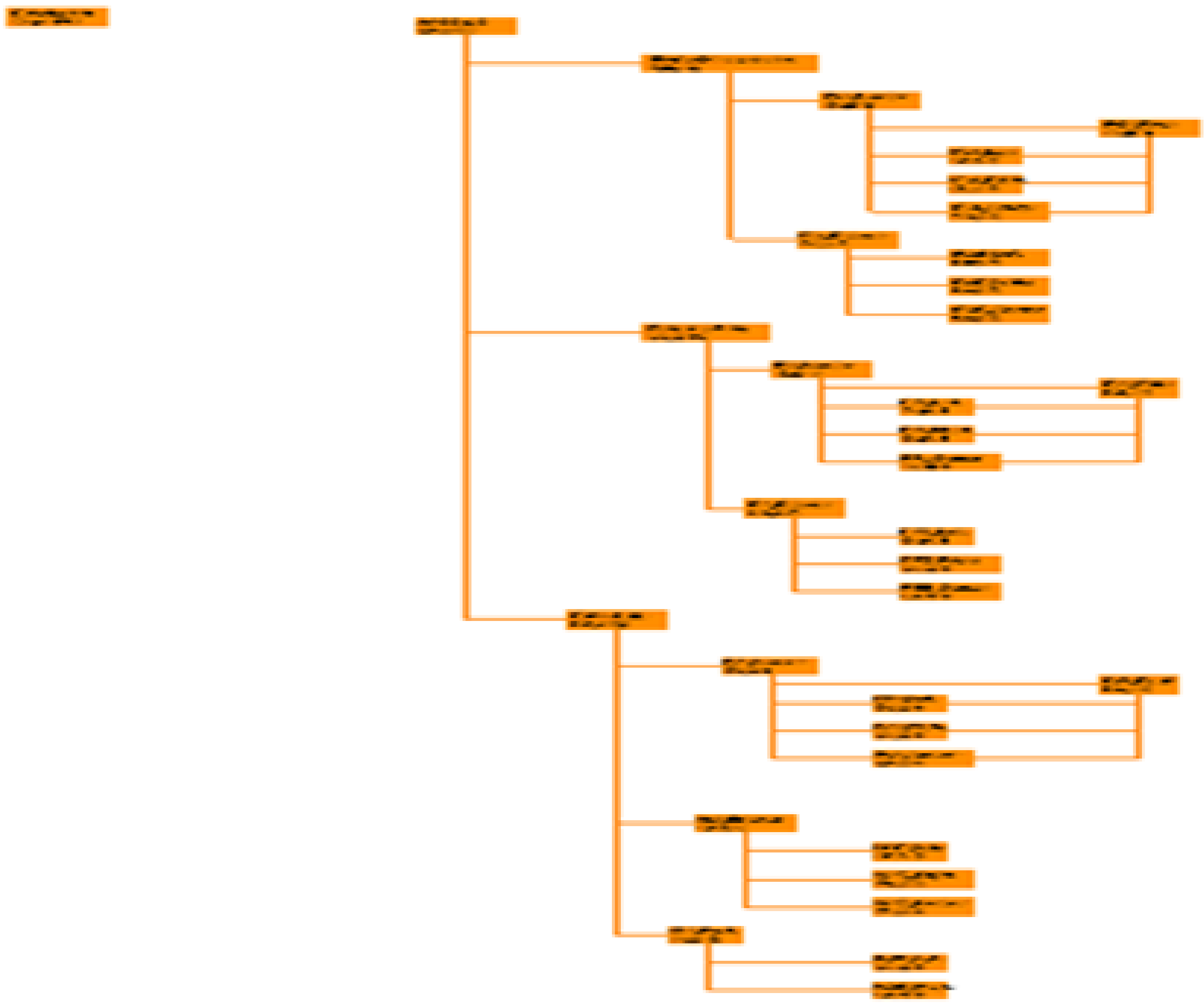


Figure A.7: A4D0300 workstation: Precedence relationship of work areas

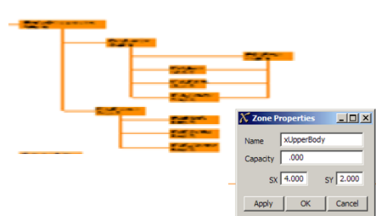


Figure A.8: Flight compartment work center: Precedence relationship of work areas

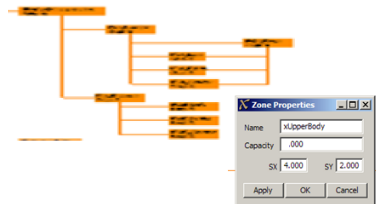


Figure A.9: Forward joint work center: Precedence relationship of work areas

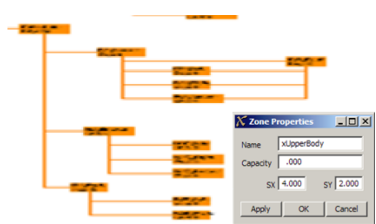


Figure A.10: Rear joint work center: Precedence relationship of work areas

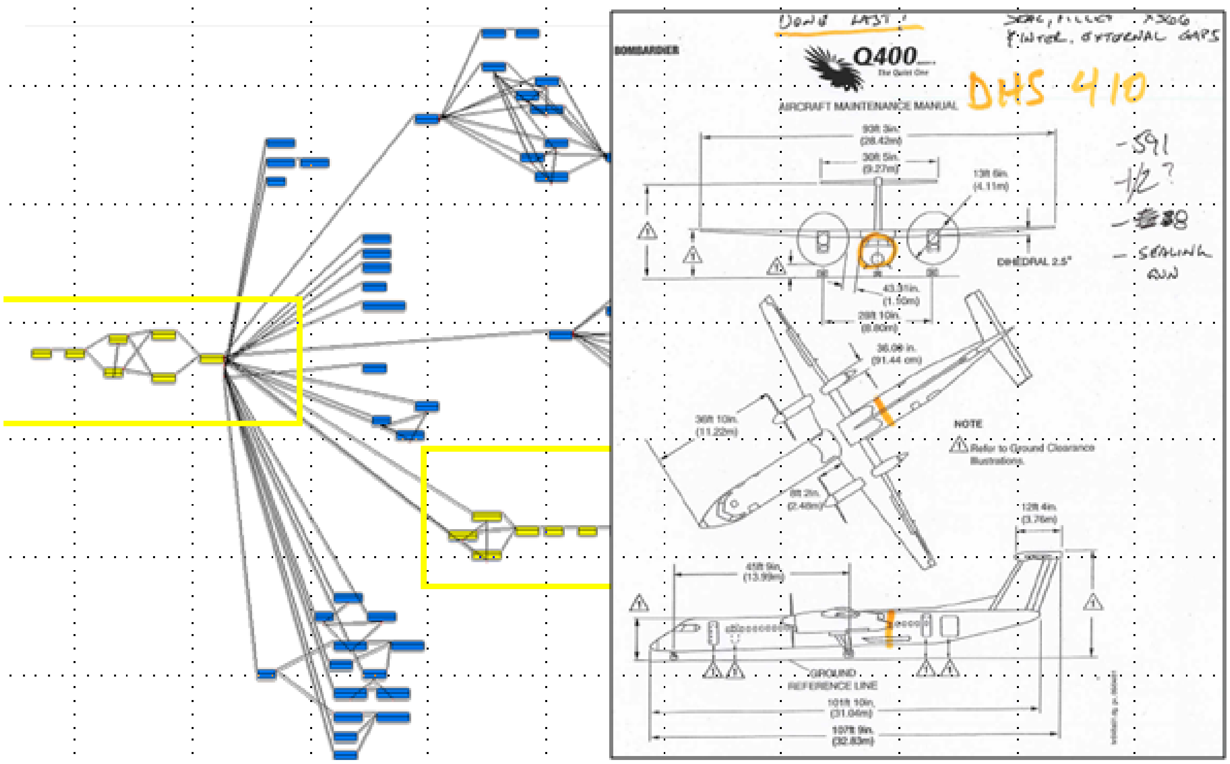


Figure A.11: Data gathering: Sample result of interviews

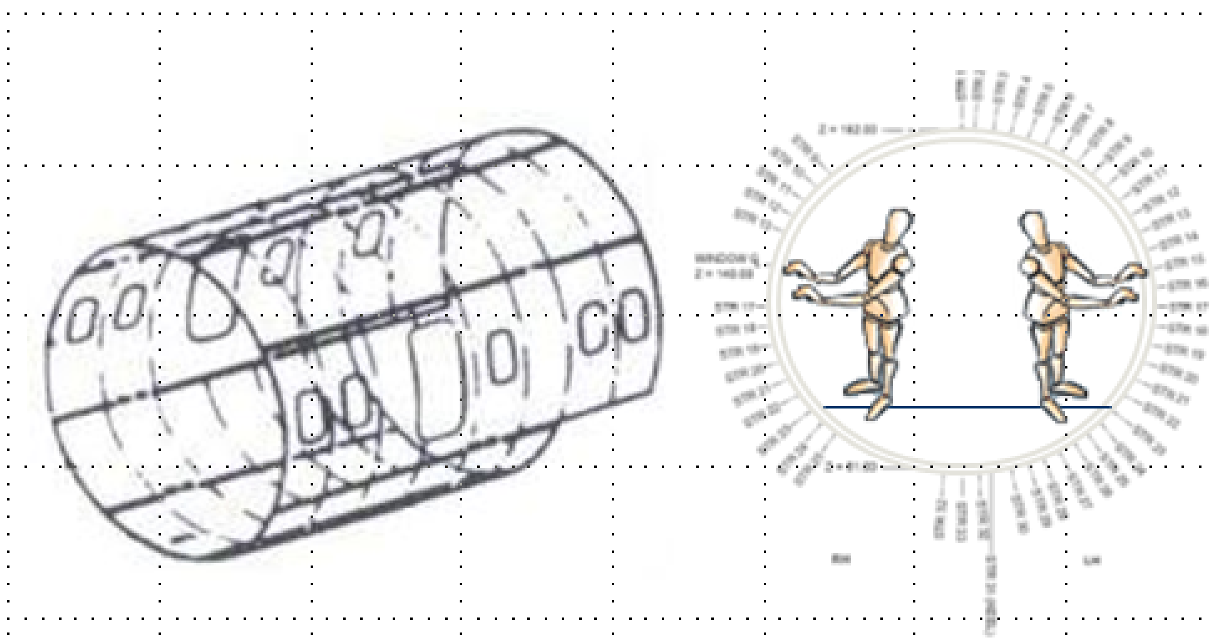


Figure A.12: Data gathering: Sample result of Ergonomic access

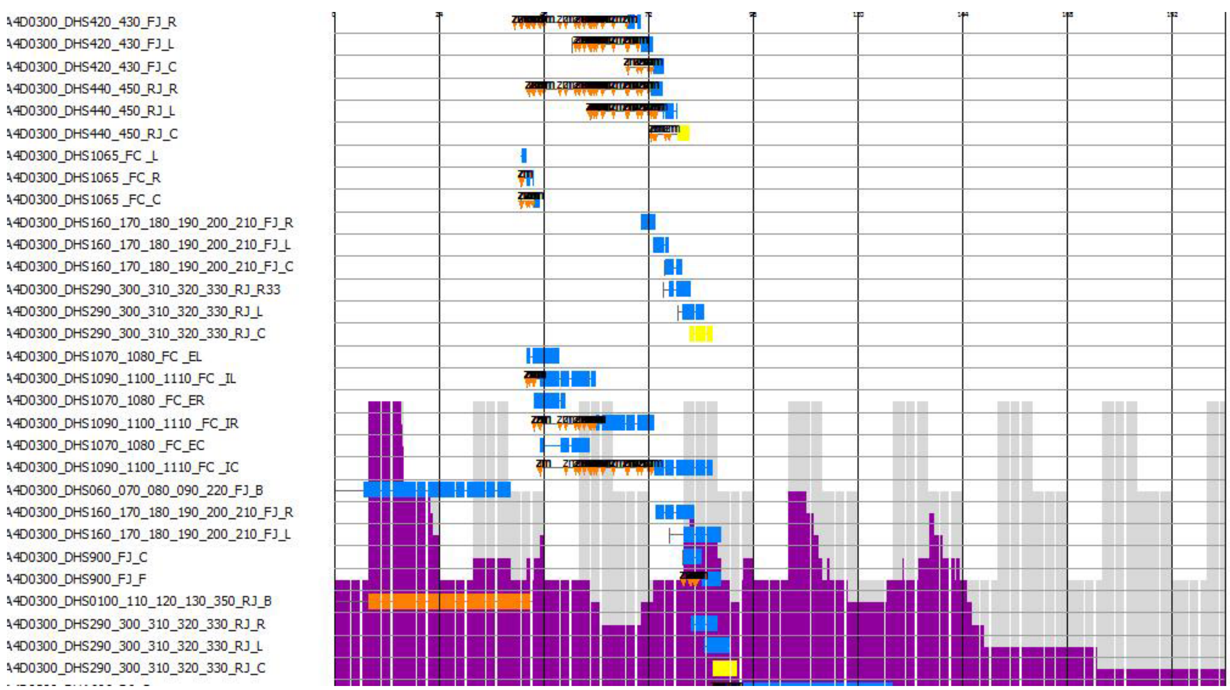


Figure A.13: A4D0300 workstation: Sample Gantt chart

Worker	WO Name	Start	End	Hrs Worked	Progress Hrs	Progress Percent
A4D0300_1_DayShift	A4D0300_DHS140_150_FJ	8	10.75	2.75	5.5	68.75
A4D0300_1_DayShift	A4D0300_DHS140_150_FJ	11	12.25	1.25	8	100
A4D0300_1_DayShift	A4D0300_DHS060_070_080_090_FJ_L	12.25	12.92	0.67	0.67	100
A4D0300_1_DayShift	A4D0300_DHC220_FJ_I	12.92	13	0.08	0.08	1
A4D0300_1_DayShift	A4D0300_DHC220_FJ_I	13.5	16	2.5	2.58	32.25
A4D0300_1_DayShift	A4D0300_DHS600_RJ_IC	81.44	82.75	1.31	1.31	43.667
A4D0300_1_DayShift	A4D0300_DHS600_RJ_IC	83	84.69	1.69	3	100
A4D0300_1_DayShift	A4D0300_DHS640_RJ_IR	84.69	85	0.31	0.31	15.5
A4D0300_1_DayShift	A4D0300_DHS640_RJ_IR	85.5	87.19	1.69	2	100
A4D0300_1_DayShift	A4D0300_DHA206_RJ_IF	87.19	88	0.81	0.81	81
A4D0300_1_DayShift	A4D0300_DHA206_RJ_IF	104	104.19	0.19	1	100
A4D0300_1_DayShift	A4D0300_DHS360_370_380_390_400_RJ_IR	104.19	106.75	2.56	2.56	28.992
A4D0300_1_DayShift	A4D0300_DHS360_370_380_390_400_RJ_IR	107	109	2	4.56	51.642
A4D0300_1_DayShift	A4D0300_DHS360_370_380_390_400_RJ_IR	109.5	112	2.5	7.06	79.955
A4D0300_1_DayShift	A4D0300_DHS230_240_250_260_270_280_FJ_ER	130.36	130.75	0.39	0.39	7.317
A4D0300_1_DayShift	A4D0300_DHS230_240_250_260_270_280_FJ_ER	131	133	2	2.39	44.841
A4D0300_1_DayShift	A4D0300_DHS230_240_250_260_270_280_FJ_ER	133.5	136	2.5	4.89	91.745
A4D0300_2_AfternoonShift	A4D0300_DHS610_RJ_IL	16	17.42	1.42	2	100
A4D0300_2_AfternoonShift	A4D0300_DHS290_300_310_320_330_RJ_R	17.42	18.75	1.33	1.33	33.25
A4D0300_2_AfternoonShift	A4D0300_DHS290_300_310_320_330_RJ_R	19	21	2	3.33	83.25
A4D0300_2_AfternoonShift	A4D0300_DHS290_300_310_320_330_RJ_R	21.5	22.17	0.67	4	100
A4D0300_2_AfternoonShift	A4D0300_DHS290_300_310_320_330_RJ_L	22.17	24	1.83	1.83	45.75
A4D0300_2_AfternoonShift	A4D0300_DHS1070_1080_FC _EL	44.2	45	0.8	0.8	11.429
A4D0300_2_AfternoonShift	A4D0300_DHS1070_1080_FC _EL	45.5	48	2.5	3.3	47.143

Figure A.14: A4D0300 workstation: Sample work-stream

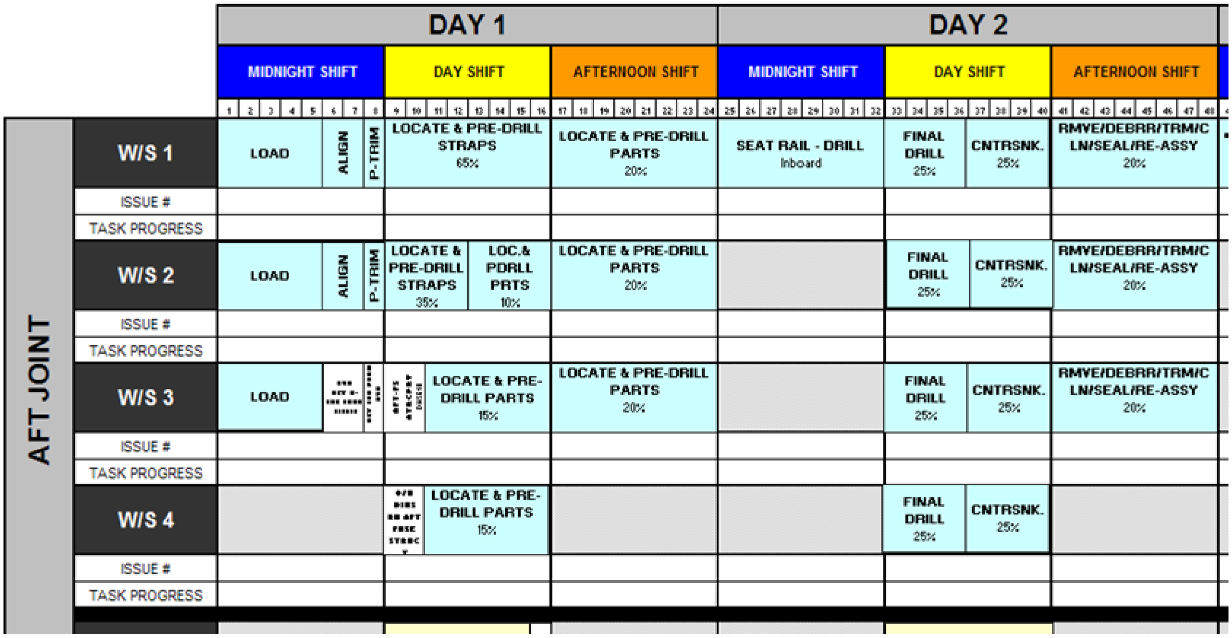


Figure A.15: Sample workstream

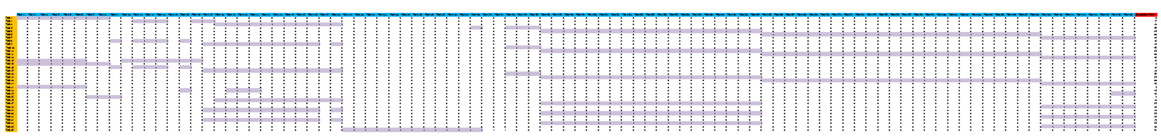


Figure A.16: Schedule for flight compartment work center

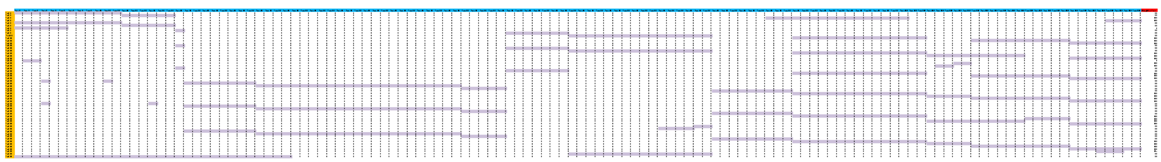


Figure A.17: Schedule for forward joint work center

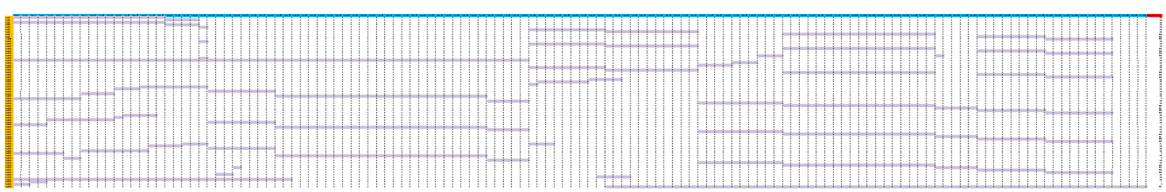


Figure A.18: Schedule for rear joint work center

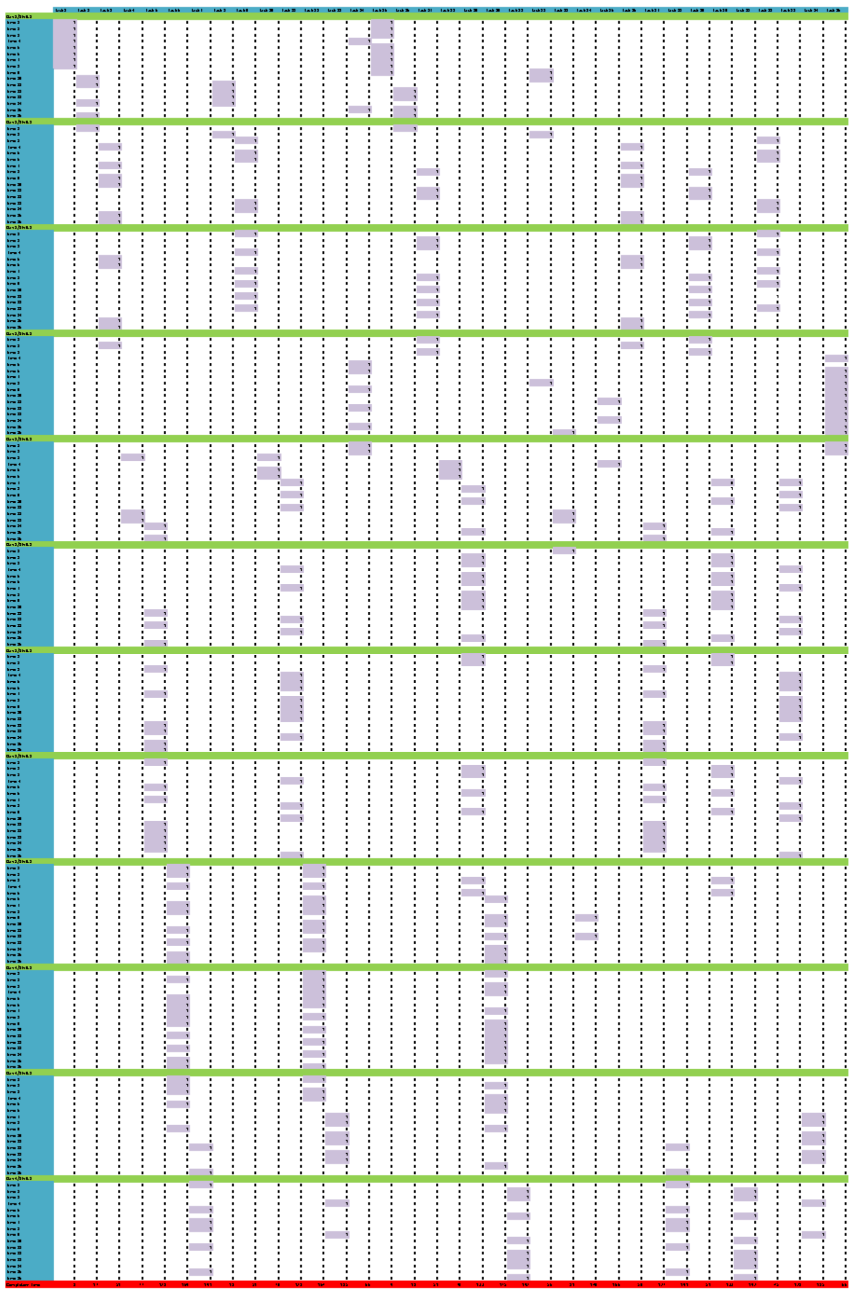


Figure A.19: Shift schedule for flight compartment work center

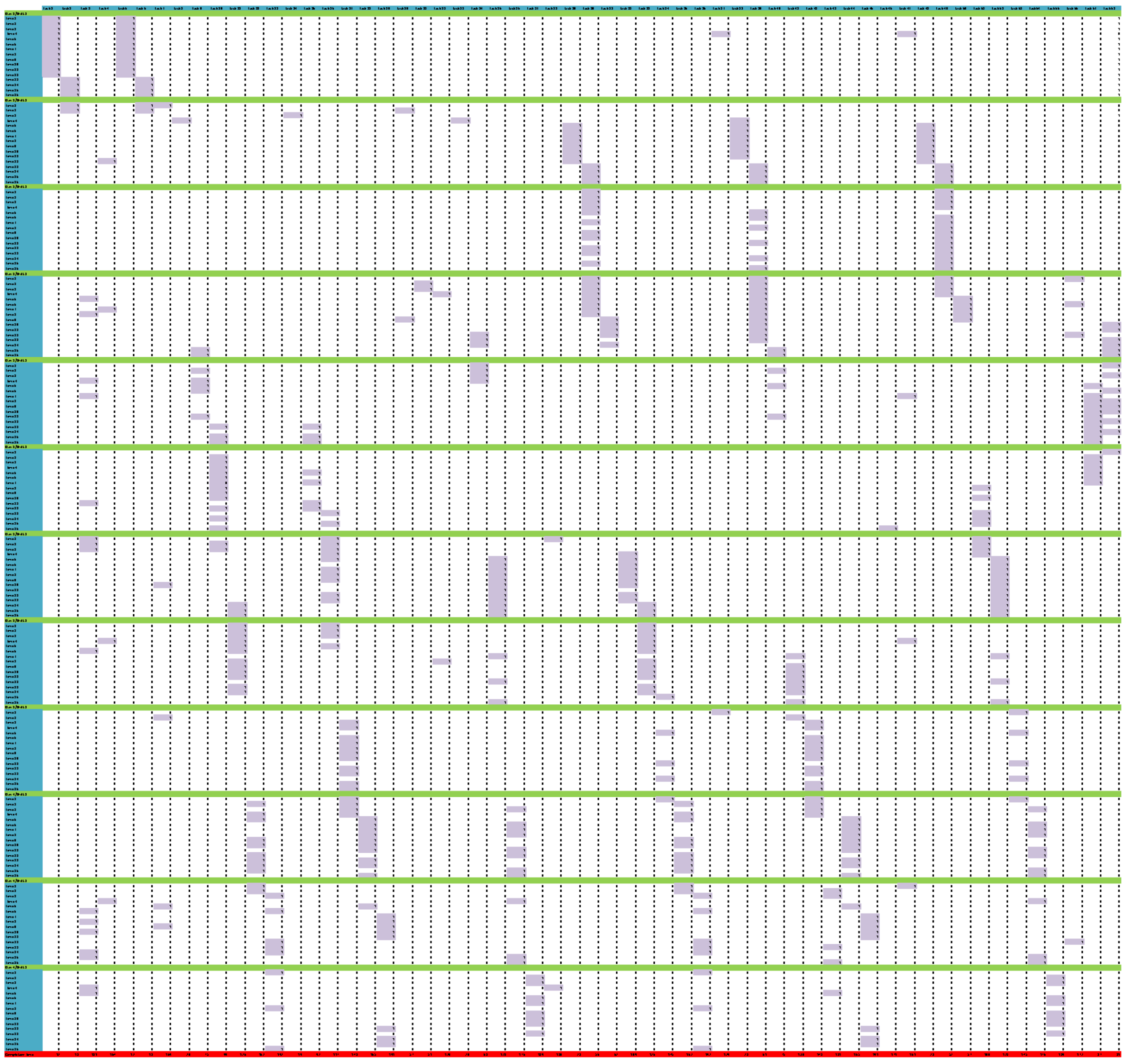


Figure A.20: Shift schedule for forward joint work center

Figure A.21: Shift schedule for rear joint work center

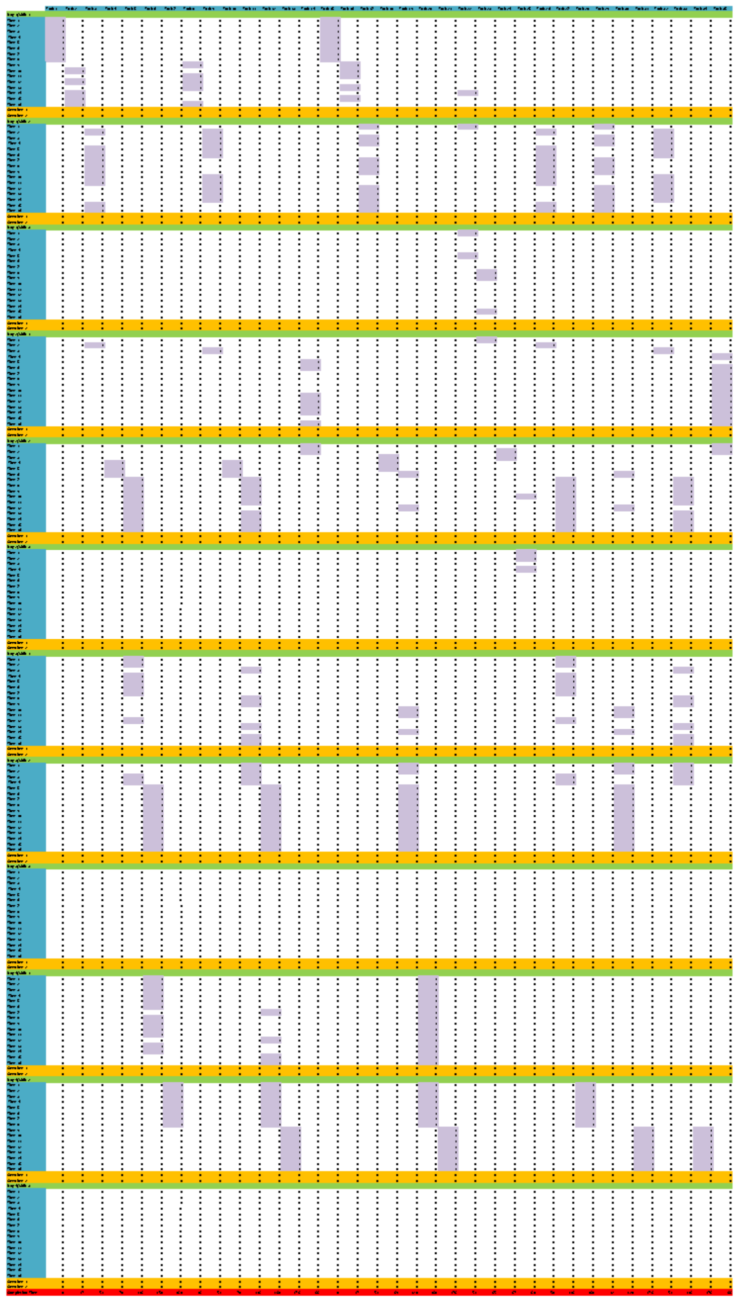


Figure A.22: Overtime Schedule for Flight Compartment Work Center

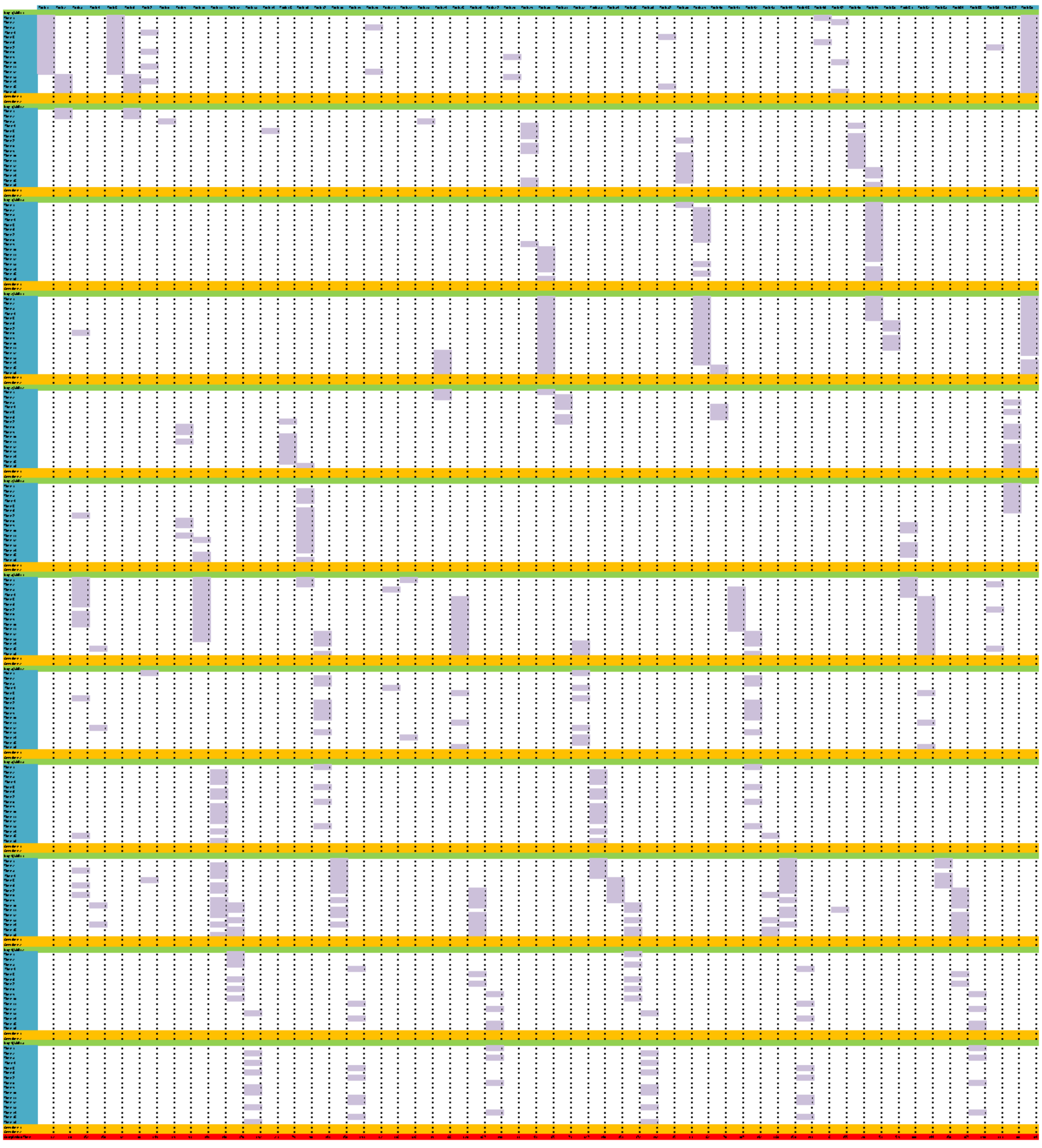


Figure A.23: Overtime schedule for forward joint work center

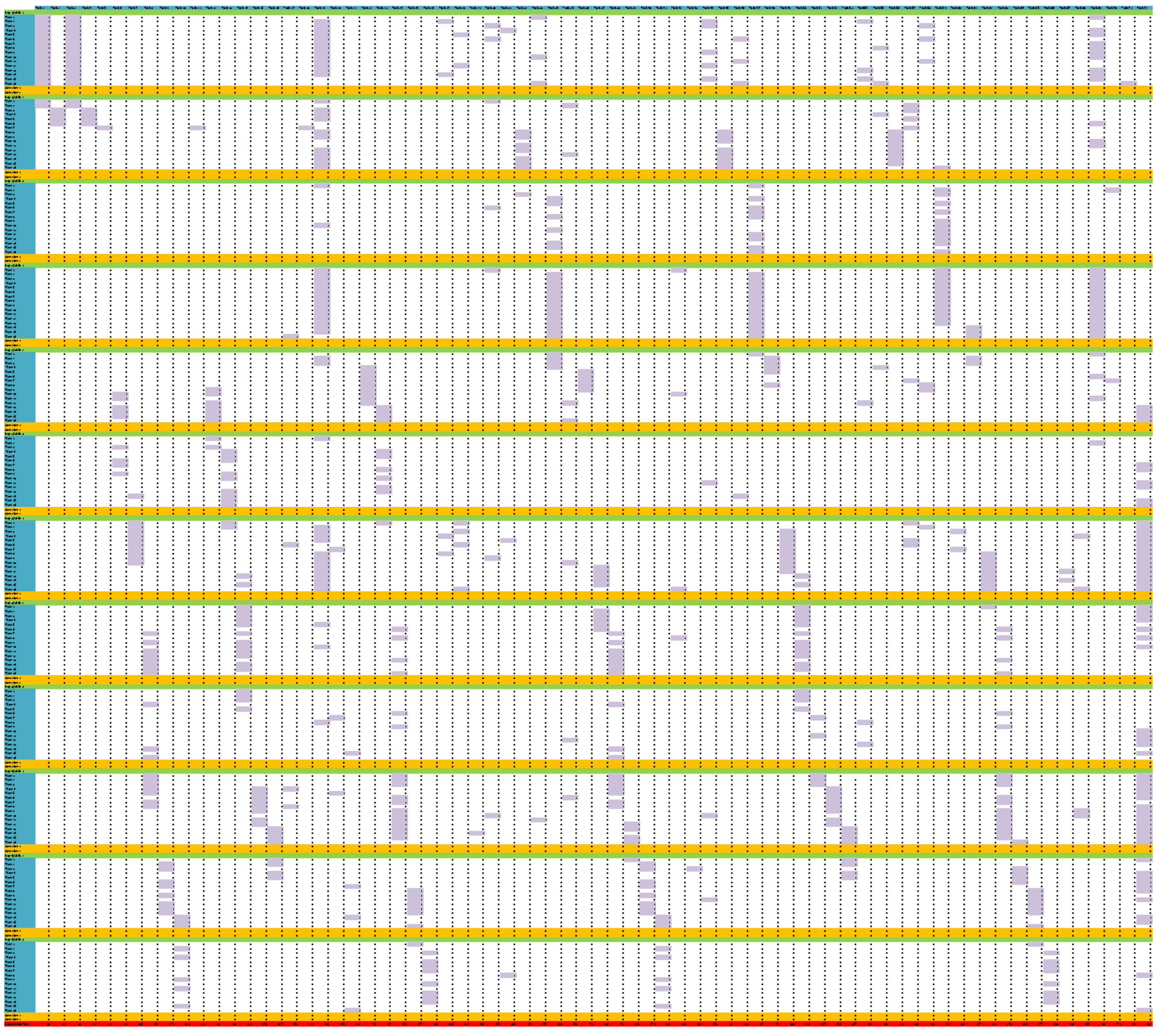


Figure A.24: Overtime schedule for rear joint work center

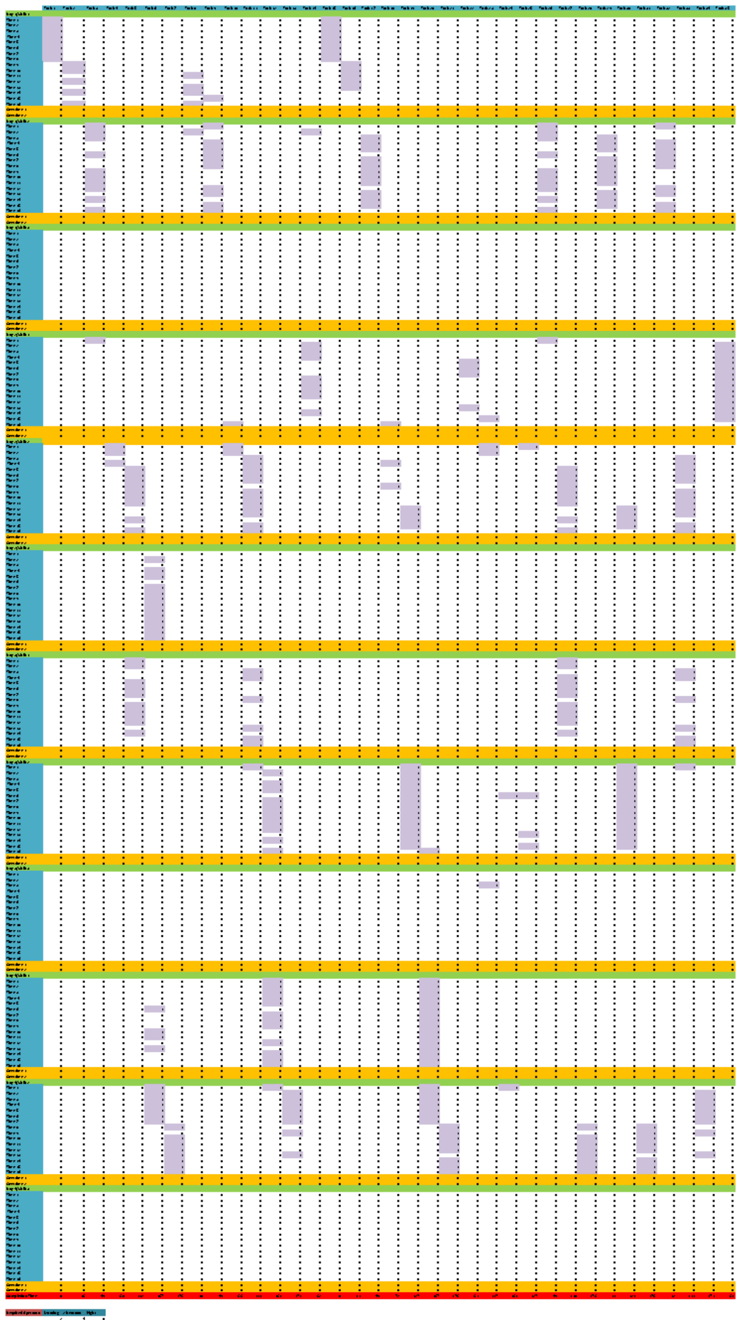


Figure A.25: Overtime schedule for flight compartment work center

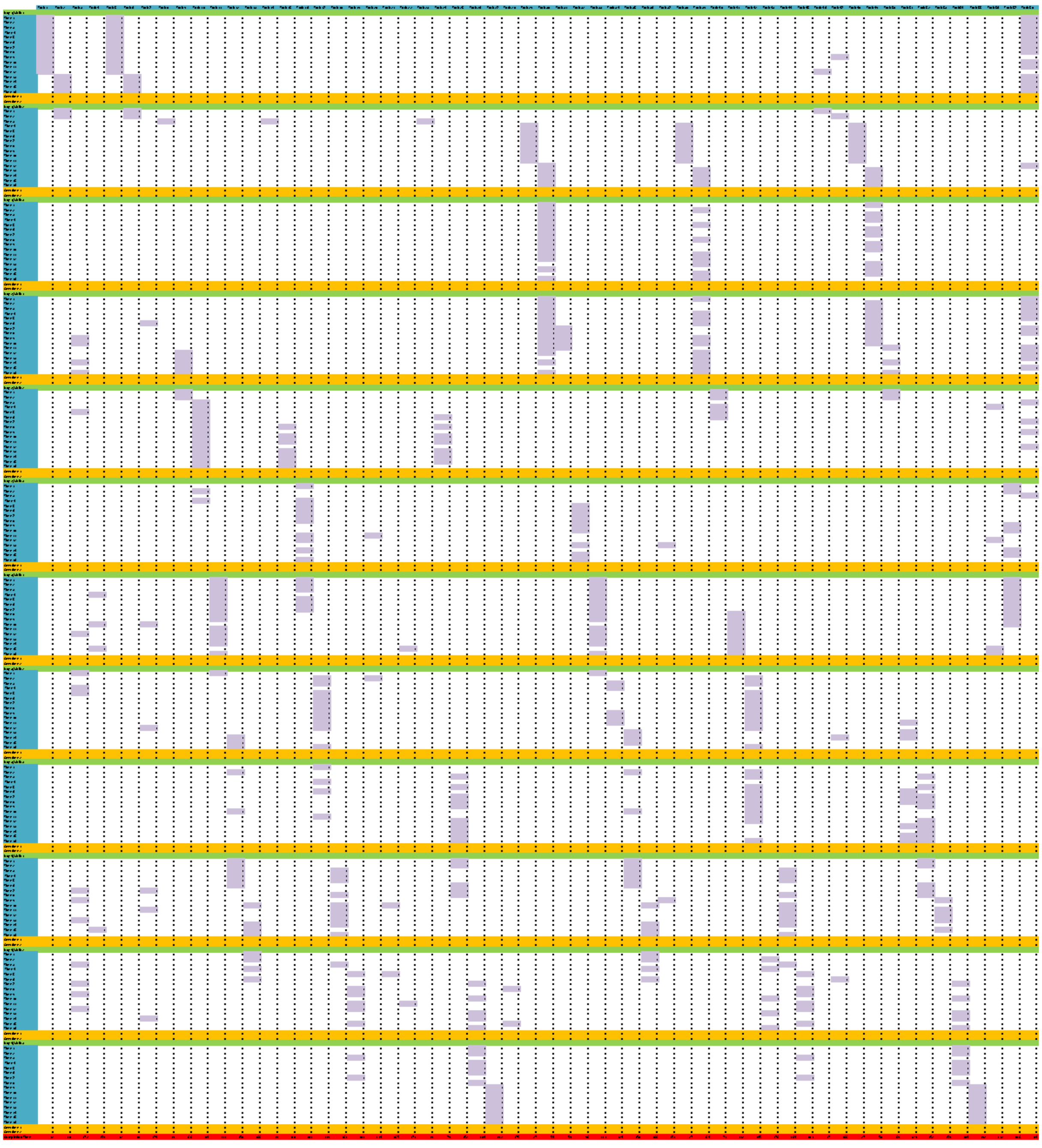


Figure A.26: Overtime schedule for forward joint work center

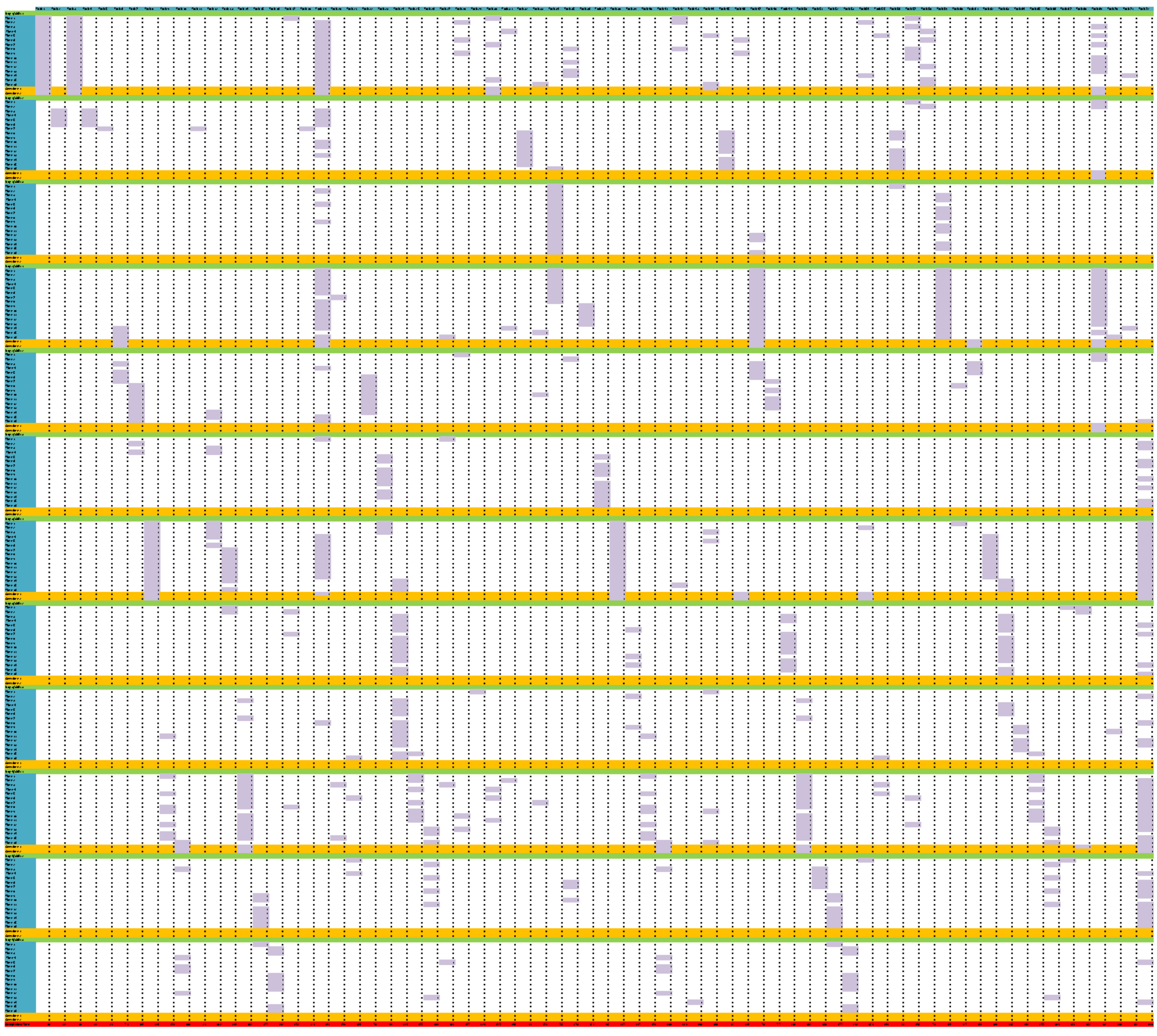
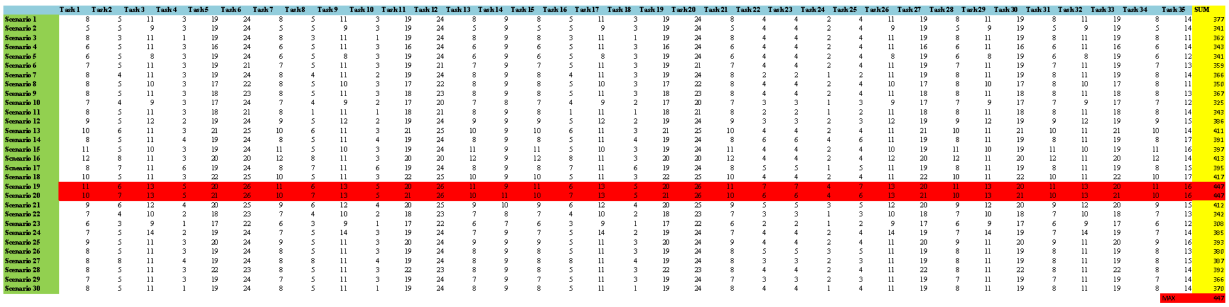


Figure A.27: Overtime schedule for rear joint work center



	Task 1	Task2	Task 3	Task 4	Task5	Task 6	Task 7	Task8	Task 9	Task 10	Task11	Task 12	Task 13	Task 14	Task 15	Task 16	Task17	Task 18
Scenario 1	8	5	11	3	19	24	8	5	11	3	19	24	8	9	8	5	11	3
Scenario 2	5	5	9	3	19	24	5	5	9	3	19	24	5	9	5	5	9	3
Scenario 3	8	3	11	1	19	24	8	3	11	1	19	24	8	9	8	3	11	1
Scenario 4	6	5	11	3	16	24	6	5	11	3	16	24	6	9	6	5	11	3
Scenario 5	6	5	8	3	19	24	6	5	8	3	19	24	6	9	6	5	8	3
Scenario 6	7	5	11	3	19	21	7	5	11	3	19	21	7	9	7	5	11	3
Scenario 7	8	4	11	3	19	24	8	4	11	2	19	24	8	9	8	4	11	3
Scenario 8	8	5	10	3	17	22	8	5	10	3	17	22	8	9	8	5	10	3
Scenario 9	8	5	11	3	18	23	8	5	11	3	18	23	8	9	8	5	11	3
Scenario 10	7	4	9	3	17	24	7	4	9	2	17	20	7	8	7	4	9	2
Scenario 11	8	5	11	3	18	21	8	1	11	1	18	21	8	9	8	1	11	1
Scenario 12	9	5	12	2	19	24	9	5	12	2	19	24	9	9	9	5	12	2
Scenario 13	10	6	11	3	21	25	10	6	11	3	21	25	10	9	10	6	11	3
Scenario 14	8	5	11	4	19	24	8	5	11	4	19	24	8	9	8	5	11	4
Scenario 15	11	5	10	3	19	24	11	5	10	3	19	24	11	9	11	5	10	3
Scneario 16	12	8	11	3	20	20	12	8	11	3	20	20	12	9	12	8	11	3
Scneario 17	8	7	11	6	19	24	8	7	11	6	19	24	8	9	8	7	11	6
Scenario 18	10	5	11	3	22	25	10	5	11	3	22	25	10	9	10	5	11	3
Scenario 19	11	6	13	5	20	26	11	6	13	5	20	26	11	9	11	6	13	5
Scenario 20	10	7	13	5	21	26	10	7	13	5	21	26	10	11	10	7	13	5
Scenario 21	9	6	12	4	20	25	9	6	12	4	20	25	9	10	9	6	12	4
Scenario 22	7	4	10	2	18	23	7	4	10	2	18	23	7	8	7	4	10	2
Scenario 23	6	3	9	1	17	22	6	3	9	1	17	22	6	7	6	3	9	1
Scenario 24	7	5	14	2	19	24	7	5	14	3	19	24	7	9	7	5	14	2
Scenario 25	9	5	11	3	20	24	9	5	11	3	20	24	9	9	9	5	11	3
Scenario 26	8	5	11	3	19	24	8	5	11	3	19	24	8	9	8	5	11	3
Scenario 27	8	8	11	4	19	24	8	8	11	4	19	24	8	9	8	8	11	4
Scenario 28	8	5	11	3	22	23	8	5	11	3	22	23	8	9	8	5	11	3
Scenario 29	7	5	11	3	19	24	7	5	11	3	19	24	7	9	7	5	11	3
Scenario 30	8	5	11	1	19	24	8	5	11	1	19	24	8	9	8	5	11	1

	Task 19	Task20	Task 21	Task 22	Task23	Task 24	Task 25	Task26	Task 27	Task 28	Task29	Task 30	Task 31	Task32	Task 33	Task 34	Task 35	SUM
Scenario 1	19	24	8	4	4	2	4	11	19	8	11	19	8	11	19	8	14	377
Scenario 2	19	24	5	4	4	2	4	9	19	5	9	19	5	9	19	5	14	341
Scenario 3	19	24	8	4	4	2	4	11	19	8	11	19	8	11	19	8	11	362
Scenario 4	16	24	6	4	4	2	4	11	16	6	11	16	6	11	16	6	14	343
Scenario 5	19	24	6	4	4	2	4	8	19	6	8	19	6	8	19	6	12	341
Scenario 6	19	21	7	4	4	2	4	11	19	7	11	19	7	11	19	7	13	359
Scenario 7	19	24	8	2	2	1	2	11	19	8	11	19	8	11	19	8	14	366
Scenario 8	17	22	8	4	4	2	4	10	17	8	10	17	8	10	17	8	11	350
Scenario 9	18	23	8	4	4	2	4	11	18	8	11	18	8	11	18	8	13	367
Scenario 10	17	20	7	3	3	1	3	9	17	7	9	17	7	9	17	7	12	325
Scenario 11	18	21	8	2	2	1	2	11	18	8	11	18	8	11	18	8	14	343
Scenario 12	19	24	9	3	3	2	3	12	19	9	12	19	9	12	19	9	15	386
Scenario 13	21	25	10	4	4	2	4	11	21	10	11	21	10	11	21	10	14	411
Scenario 14	19	24	8	6	6	4	6	11	19	8	11	19	8	11	19	8	17	391
Scenario 15	19	24	11	4	4	2	4	10	19	11	10	19	11	10	19	11	16	397
Scneario 16	20	20	12	4	4	2	4	12	20	12	11	20	12	11	20	12	14	413
Scneario 17	19	24	8	5	5	1	5	11	19	8	11	19	8	11	19	8	15	395
Scenario 18	22	25	10	4	4	2	4	11	22	10	11	22	10	11	22	10	17	417
Scenario 19	20	26	11	7	7	4	7	13	20	11	13	20	11	13	20	11	16	447
Scenario 20	21	26	10	6	6	4	6	13	21	10	13	21	10	13	21	10	16	447
Scenario 21	20	25	9	5	5	3	5	12	20	9	12	20	9	12	20	9	15	412
Scenario 22	18	23	7	3	3	1	3	10	18	7	10	18	7	10	18	7	13	342
Scenario 23	17	22	6	2	2	1	2	9	17	6	9	17	6	9	17	6	12	308
Scenario 24	19	24	7	4	4	2	4	14	19	7	14	19	7	14	19	7	14	385
Scenario 25	20	24	9	4	4	2	4	11	20	9	11	20	9	11	20	9	16	393
Scenario 26	19	24	8	5	5	3	5	11	19	8	11	19	8	11	19	8	13	380
Scenario 27	19	24	8	3	3	2	3	11	19	8	11	19	8	11	19	8	15	387
Scenario 28	22	23	8	4	4	2	4	11	22	8	11	22	8	11	22	8	14	392
Scenario 29	19	24	7	3	3	2	3	11	19	7	11	19	7	11	19	7	14	366
Scenario 30	19	24	8	4	4	1	4	11	19	8	11	19	8	11	19	8	14	370
																	MAX	447

Figure A.28: Scenarios for flight compartment work center

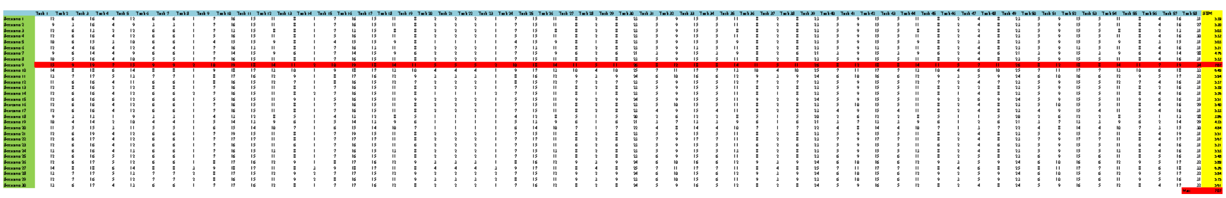


Figure A.29: Scenarios for forward joint work center

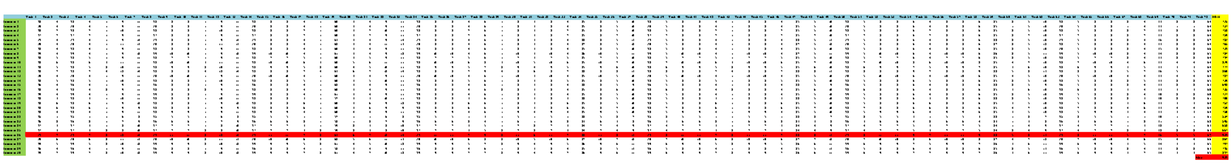


Figure A.30: Scenarios for rear joint work center

Appendix B

Tables

Table B.1: A4D0300 Workstation: List of critical tasks

Load	Align	Pre-trim	Clean	Locate	Straps	Deburr	Final Drill
Rivet	Seat Rail	Seal	Cut Frame	Join	Install	Countersink	

Table B.2: Number of tasks per zone in flight compartment work center

Zone	1	2	3	4	5	6	7	8	9
Number of Tasks	7	6	1	7	3	4	3	3	1

Table B.3: Simultanous tasks in flight compartment work center

Task i	3	17	9	19	11	5	7	21	13
Task j	26	29	32	30	33	27	28	31	34

Table B.4: Precedence relationship between tasks in flight compartment work center

Successor Task	2	8	16	2	8	16	23	24	3	26	17	29	9	32	35
Predecessor Task	1	1	1	15	15	15	22	23	2	2	8	8	16	16	3
Successor Task	35	4	18	10	4	18	10	5	27	19	30	11	33	6	7
Predecessor Task	9	14	14	14	35	35	35	4	4	18	18	10	10	5	6
Successor Task	20	21	31	12	13	34	35	28							
Predecessor Task	19	20	20	11	12	12	17	6							

Table B.5: Number of tasks per zone in forward joint work center

Zone	1	2	3	4	5	6	7	8	9	10
Number of Tasks	4	2	7	6	8	9	9	10	2	1

Table B.6: Simultanous tasks in forward joint work center

Task i	2	33	35	36	17	18	19	52	54	55
Task j	6	11	12	13	42	44	45	25	26	27

Table B.7: Precedence relationship between tasks in forward joint work center

Successor Task	2	6	6	2	9	14	24	29	30	31	9	10	32	33	34
Predecessor Task	1	1	5	5	6	6	6	8	29	30	31	9	10	32	33
Successor Task	35	36	32	41	51	38	39	40	15	16	41	42	43	44	45
Predecessor Task	34	35	58	58	58	14	38	39	40	15	16	41	42	43	44
Successor Task	48	49	50	24	57	51	52	53	54	55					
Predecessor Task	23	48	49	50	24	57	51	52	53	54					

Table B.8: Number of tasks per zone in rear joint work center

Zone	1	2	3	4	5	6	7	8	9	10	11	12
Number of Tasks	2	2	6	6	10	15	12	13	1	1	1	3

Table B.9: Simultanous tasks in rear joint work center

Task i	2	24	25	26	8	9	10	14	15	16
Task j	4	63	65	66	38	40	41	50	52	53

Table B.10: Precedence relationship between tasks in rear joint work center

Successor Task	4	2	4	5	11	18	32	34	36	6	7	37	38	39	40
Predecessor Task	1	1	3	4	4	4	5	32	34	36	6	7	37	38	39
Successor Task	41	37	49	62	45	47	48	12	13	49	50	51	52	53	56
Predecessor Task	40	69	69	69	11	45	47	48	12	13	49	50	51	52	18
Successor Task	59	61	23	72	62	63	64	65	66	22	21	60			
Predecessor Task	56	59	22	22	23	62	63	64	65	61	19	58			

Table B.11: Total overtime cost of labour per scenario

Penalties	Scenario1	Scenario2	Scenario3	Scenario4	Scenario5
Flight Compartment	0	0	0	0	0
Forward Joint	0	0	0	0	0
Rear Joint	0	0	0	0	0
Flight Compartment	0	0	0	0	0
Forward Joint	0	0	0	5.0453	0
Rear Joint	0	0	0	0	0
Penalties	Scenario11	Scenario12	Scenario13	Scenario14	Scenario15
Flight Compartment	0	0	0	0	0
Forward Joint	0	0	0	0	0
Rear Joint	0	0	0	0	0
Penalties	Scenario16	Scenario17	Scenario18	Scenario19	Scenario20
Flight Compartment	0	0	0	0.469	0
Forward Joint	0	0	0	0	0
Rear Joint	0	0	0	0	0
Penalties	Scenario21	Scenario22	Scenario23	Scenario24	Scenario25
Flight Compartment	0	0	0	0	0
Forward Joint	0	0	0	0	0
Rear Joint	0	0	0	0	0
Penalties	Scenario26	Scenario27	Scenario28	Scenario29	Scenario30
Flight Compartment	0	0	0	0	0
Forward Joint	0	0	0	0	0
Rear Joint	8.279	2.345	0	0	0

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