Optimization Models and Techniques for Implementation and Pricing of Electricity Markets

by

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Abstract

Vertically integrated electric power systems extensively use optimization models and solution techniques to guide their optimal operation and planning. The advent of electric power systems re-structuring has created needs for new optimization tools and the revision of the inherited ones from the vertical integration era into the market environment.

This thesis presents further developments on the use of optimization models and techniques for implementation and pricing of primary electricity markets. New models, solution approaches, and price setting alternatives are proposed. Three different modeling groups are studied. The first modeling group considers simplified continuous and discrete models for power pool auctions driven by central-cost minimization. The direct solution of the dual problems, and the use of a Branch-and-Bound algorithm to solve the primal, allows to identify the effects of disequilibrium and different price setting alternatives over the existence of multiple solutions. It is shown that particular pricing rules worsen the conflict of interest that arise when multiple solutions exists under disequilibrium. A price-setting alternative based on dual variables is shown to diminish such conflict. The second modeling group considers the unit commitment problem. An interior-point/cutting-plane method is proposed for the solution of the dual problem. The new method has better convergence characteristics and does not suffer from the parameter tuning drawback as previous methods. The robustness characteristics of the interior-point/cutting-plane method, combined with a non-uniform price setting alternative, show that the conflict of interest is diminished when multiple near optimal solutions exist. The non-uniform price setting alternative is compared to a classic average pricing rule. The last modeling group concerns to a new type of linear network-constrained clearing system models for daily markets for power and spinning reserve. A new model and solution approach is proposed. The model considers bids for supply and demand and bilateral contracts and a direct current model for the transmission network. The use of an interior point method that can take advantage of the special structure of the Newton's system is proposed.

The use of optimization models in a market environment is still facing several challenges; this thesis presents developments that help understand the use of complex optimization models for electricity markets; new models, solution approaches and price setting alternatives are proposed.

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This thesis is dedicated for a new generation of Mexicans eager to find the way out the Labyrinth, eager to break minds' ties and be free of paternalism and archaic structures.

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Glossary of Terms

B&B	Branch-and-Bound.
CNR	Cost not recovered.
CP	Cutting plane.
CS	Clearing system.
DC	Direct current.
DP	Dynamic programming.
EWPP	England and Wales Power Pool.
IPM	Interior point method.
IP/CP	Interior-point/cutting-plane.
IPM-PP	Interior point method for the potential problem.
ISO	Independent system operator.
LR	Lagrangian relaxation.
MO	Market operator.
PB	Penalty bundle.
PDIIPM	Primal dual infeasible interior point method.
PJM	Pennsylvania-New Jersey-Maryland.
PP	Potential problem.
PX	Power exchange.
SG	Sub-gradient.
SMP	System Marginal Price.
UC	Unit commitment.

Chapter 1

Introduction

A re-structuring process of the power industry, that started in 1978, gave birth to the first competitive market for electricity generation in Chile in 1982 [1]. The order to privatize the United Kingdom electricity industry in 1988 concluded with the creation, in 1992, of the England and Wales Power Pool. In 1992, the approval of the Electricity Policy Act (EPAct) ordered *Open Access* to transmission networks as the basis for the introduction of competition in the electricity industry in the U.S.A. Ever since, a variety of electricity markets have been created, among them, the *Pennsylvania-New Jersey-Maryland* (PJM) Pool that started operations in 1997; the Californian market that started operations in 1998 and the New York Power Pool in the same year. Alberta and Ontario, in Canada, have also re-structured their systems to a market-oriented basis. Several other countries have also implemented electricity markets, or are in the process to do so, among them, Argentina, Australia, Brazil, Mexico, Germany, Norway and Spain.

The world-wide re-structuring process of the electric energy industry that begun more than twenty years ago, has deeply accelerated creating a political and technical turmoil. The operation of large investor- or state-owned utilities is being transformed from an rate-of-return basis to a competitive basis by the creation of electricity markets.

The reasons for such re-structuring process are varied. In well developed countries, the introduction of competition is believed to be a bridge to achieve more efficiency in the industry, to equate price differences among regions and, eventually, to reduce the energy prices. In developing countries, re-structuring is sometimes linked to a privatization process of the state-owned utilities. The need to acquire funds to build the systems expansions required to cope with the rapid load growth, is one of the reasons for the re-structuring of their systems. It is also believed that the introduction of competition and private participation can make the state-owned utilities more efficient.

1.1 Structure and Implementation of Electricity Markets

The electricity industry has peculiarities that make the design of an electricity market a challenging task. Electric energy cannot be stored in large quantities and, therefore, the generation supply has to match the demand at every time instant; transmission and system interactions have to be observed to guarantee the secure and reliable operation of the system. Such complexities need to be taken into account in what can be identified as the three main design components of an electricity market: (i) the design of the primary electricity markets; (ii) the design of transmission management procedures; and (iii) the design of procedures for the provision of ancillary services. The first design component refers to the creation of markets to competitively buy and sell real power in different time and structural frames. The second component refers to the definition of rules and procedures to provide access to the transmission system, including the congestion and pricing protocols. The last design component deals with the procedures to be implemented for the provision of the ancillary services that are necessary to support the reliable and secure operation of the system such as voltage support, frequency regulation and operating reserves. Even though transmission and ancillary services are not always provided in a market oriented basis, the creation of primary markets is a constant in all the re-structured systems.

There is a great diversity of market designs; practically, not two equal market designs exist. A classification given by R. Wilson [2, 1999] identifies three groups of market models: (i) centralized models; (ii) decentralized models; and (iii) hybrid models. Centralized models are characterized by the creation of an *independent system operator* (ISO) that executes a central cost-minimization scheduling of generation which constitutes the primary market. Alternatively, the same ISO is involved in the operation of the transmission system and procurement of ancillary services. On the other hand, decentralized models neither rely on central optimization of generation nor on a single ISO. In these designs, there is a *Market Operator* (MO) in charge of running primary markets by using simple models such as standard auctions; transmission security and congestion management are executed by an ISO in coordination with the MO. Centralized and decentralized models represent opposite poles of existing market designs; new hybrid designs are being recently created. These designs do not implement primary markets by standard auctions or centralized cost-minimization in the same modeling level as centralized designs.

This thesis deals with the use of optimization models and techniques for implementation and pricing of primary electricity markets. The implementation of primary electricity markets has inherited several optimization tools from the vast experience of the vertical integration era. The transition of the cost-minimization models to a market environment has not been an easy process. There is a need for new models and optimization techniques, as well as the revision and improvement of existing ones. In the following section, representative market models are described. At the same time, the challenges that the implementation of optimization-based primary electricity markets is facing are described.

1.2 Optimization Models in Primary Electricity Markets

1.2.1 Centralized Models and the Use of Unit Commitment

Unit commitment (UC) problems have long been used in vertically-integrated utilities to determine the short term (24 to 168 hours) economic operation of power system with considerable amounts of thermal generation. A classic UC is a large non-linear mixed-integer programming problem whose solution gives the commitment of generators and their respective power outputs so that a forecasted load demand and system reserve are satisfied; and, at the same time, satisfying the operative limits of the generation units [3].

A UC model has been used by the *England and Wales Power Pool* (EWPP) as the mechanism to implement a daily market for real generation; i.e., a power Pool auction. The Pool receives bids from generators that contain a cost function and a set of operational limitations. The cost functions contain no-load, start-up and variable cost coefficients. The operative limits include minimum and maximum power outputs, ramp constraints and minimum shut down constraints, among others [4]. The Pool uses a Lagrangian relaxation algorithm that determines the minimum cost solution to the UC problem. Once the UC problem is solved, the Pool computes several price components to determine the price for suppliers and consumers; among these components is the *System Marginal Price* (SMP), which defines the price for real power and is computed in such a way that scheduled generators recover the costs submitted in their bids [4].

The cost functions submitted by generators do not necessarily need to reflect their true values; it is expected that the competitive forces would drive generators to submit a cost as low as their actual cost in order to be scheduled at the solution to the UC problem; and, therefore, receive revenues from the power they produce. However, in the experience of the EWPP, large increases in the SMP and other price components have been observed since 1992, and became more pronounced afterwards. Empirical evidence shows that the large increases in prices were mainly due to duopoly market power by two major generator companies acting in the Pool [5, 1997] [6, 1999]; and, also, to the strategic selection of the start-up, and variable cost parameters included in the bids [7, 1999].

There have been other concerns related to the use of unit commitment models for primary electricity markets. Using a simplified unit-commitment model, Jacobs [8, 1997] shows that cost-minimization and uniform pricing based on averages, as used in the EWPP, fails to produce lower prices for consumers. The author shows that different feasible solutions, that do not minimize cost, can result in lower prices for consumers. The same author and related research by Hao et al. [9, 1999] propose the use of price-minimization unit-commitment power pool auctions. In this model, the objective is to find a schedule that minimizes a uniform price and, at the same time, guarantees cost-recovery for all suppliers. However, as identified by the authors, the decomposability of the optimization model is lost, which makes it harder to solve; numerical results show that price minimization leads to lower energy prices but requires the use of more expensive generators.

In a publication by Johnson, Oren and Svoboda [10, 1997], equity or fairness concerns related to the use of unit commitment for competitive markets are raised. Using a unit commitment and hydro thermal coordination program, they show that slight variations in the tuning parameters of the Lagrangian relaxation-based scheduling program lead to different near-optimal solutions. Despite that such multiple near-optimal solutions represent equally acceptable cost-minimizing solutions, they can represent very different profits for individual suppliers, which generates a conflict of interest since particular parameter setting could be favoring a particular generator. Additional work by Sheblé et al. [11, 1999] [12, 1999] further explores on the existence of multiple solutions in unit commitment models.

Other more recently developed electricity markets, such PJM and the NY ISO, also based on forms of UC models to execute their primary markets. At the same time, the transmission network is considered along with the primary market, constituting a highest centralized model. The equity concerns are not considered substantial to change their market models and they rely on settlement systems to manage no-load and start-up cost as a counter-measure to avoid the possible strategic behavior [13,14]. The EWPP is undergoing a new re-structuring process that has split generation companies, and considers the utilization of simpler models to execute the primary market [15].

1.2.2 Decentralized Models and the Use of Standard Auctions

The U.K. experience and the concerns that arise due to the complexity of the UC strongly influenced the design of the Californian market. In the California market, the *Power Exchange* (PX) implements a daily market where both suppliers and consumers submit simple price-quantity bids in order to compete to sell and buy real power [16]. The PX performs an ordering of the bids to construct a supply/demand curve whose intersection defines the schedules and a sinlge market price. Other systems that implement similar daily markets are Spain [17] and Alberta in Canada [18].

The PX model can be classified as an standard uniform double-sided auction market. Uniform, since a single price is set for the product; and double-sided, since both consumers and suppliers participate in the auction. An auction is a market mechanism to allocate goods and determine their price based on the bids submitted by participants [19]. The use of auctions as a general principle for price determination in a deregulated power industry has been first considered by G. B. Sheblé in [20, 1994] and Post et al. [21, 1995].

The use of simple models to implement electricity markets intends to provide a transparent and pure market-oriented trading floor, where the determination of schedules and prices is not made inside a "black box" optimization algorithm [2].

Generation companies acting in this type of markets need to rely on their bidding strategies to recover all their cost components; at the same time, such strategies have to be designed so that the most probable outcome of the auction is in accordance to the operational restrictions of their generation units. The design of bidding strategies that involve the self-commitment of generation units, acting in this type of markets, is an increasing research area of interest [22–24]. There is, therefore, a trade-off among reliability and market transparency. Unit commitment models take into consideration the operative limits of the suppliers and perform a central coordination of the resources.

Even though the use of standard auctions in electricity markets can be supported by the known theoretical and experimental properties of standard auctions as applied to other markets, the properties of such auctions in the context of electricity markets are not well known. Reviews on the theory of auctions can be found in [11, 19]. The basic result on the theory of uniform double-sided auctions specifies that participants best behavior is to reveal their true valuation of the object, i.e., reveal their true cost of generation, in the context of an electricity market.

Recent work by Elmaghraby and Oren [25, 1999] considers a PX standard-auction as compared to a simplified unit-commitment model; the research concludes that, under specific behavioral assumptions, the PX auction is not able to achieve economic dispatch in equilibrium. That is, the behavior of bidders in a PX auction is such that cost minimization is never achieved, which, in a unit-commitment like auction, is a possibility if participants submit their true costs. In general, if an efficient competitive-incentive auction for electricity markets exists and can be implemented, is still an open question [26, 1998] [27, 2000].

As in the U.K., price increases due to market power have also been diagnosed in the California market [28, 1999].

1.2.3 Hybrid Models and Linear Optimization

In both the California PX market and the U.K. Pool. the daily market for power is executed without considering the transmission network. The operators of both markets coordinates with a separated operator who determines the feasibility of the resulting schedules and determines the corrective actions based on the established design principles for transmission management. In California, such an operator is called the ISO [16], and in the case of the U.K., is the National Grid Company [4].

Hybrid designs have recently been created; these designs do not allow the inclusion of unit commitment in the primary market, but allow certain time-dependent operative limits to be expressed in the bids. In this new type of models, a simplified representation of the transmission network is considered. The time-dependent optimization models that arise from these new models result in large, but easily solvable, linear optimization models. The basis for these models is the spot pricing theory of electricity developed by Schweppe et at. [29]. A benefit maximization dispatch problem is formulated and its solution provides the schedules for suppliers and consumers. At the same time, the optimal dual variables related to the power demand constraints in the optimization model determine the locational prices that are also used for tra nsmission pricing.

Examples of these new structures are the New Zealand Market [30, 1998], the proposed structure for the Mexican market (under evaluation) [31, 1999]. The final report of the Ontario Market Design Committee [32, 1998] recommends the introduction of locational marginal pricing for a second stage on the development of the Ontario market. The rapid appearance of these alternative designs has created an increasing need for the identification of mathematical models and solution approaches for what is called transmission constrained market clearing [33, 1999] [34, 1999].

1.3 Research Motivation

While the strategic behavior of participants in an auction deserves investigation, and perhaps more practical experience, the use of complex optimization models (such as unit commitment) to conduct electricity auctions is not well understood yet and deserves further investigation [35, 1999]. The search for better solution algorithms that can be valuable in both vertical-utility operation or market-environment is of great importance. At the same time, the search for better alternative pricing schemes needs investigation. New models and solution approaches are required for recently created or proposed electricity markets [33, 1999] [34, 1999].

In particular: (i) the study of simplified continuous and discrete models and direct solutions approaches can provide information for the understanding and design of price setting alternatives in power pool auctions driven by more complex optimization models; (ii) the search for new solution approaches to unit commitment problems is still needed; methods that do not rely on parameter tuning and their use to design price setting alternatives are desirable; and (iii) the modeling and proposal of solution approaches to market clearing systems that arise from newly created market structures is also required.

The structure of the thesis along with the principal contributions is described in the next section; additional relevant bibliographical reviews are given along each chapter.

1.4 Structure of the Thesis and Main Contributions

- In Chapter 2, a generic cost-minimization power Pool auction model is described. Using Lagrangian duality, the conditions for the existence of an equilibrium are presented. As illustrative examples, simplified continuous models that represent a standard auction and a economic dispatch problem are presented. For both models, direct solution approaches to solve the primal and dual problems are presented. The relation between standard auctions and pricing with dual variables is illustrated.

- In Chapter 3, two simplified discrete models for power pool auctions are studied. The models consider bids with startup cost and linear and quadratic variable cost. For both models, a closed form solution to the dual problem is derived and enumerative Branch-and-Bound methods are developed to find the solution to the primal problem. The non-existence of equilibrium and its effect on different pricing alternatives is presented. It is shown that average pricing and price minimization worsens the conflict of interest that arises when multiple solutions exist. A derivation shows that, under disequilibrium, dual variables used as prices do not generate enough revenues to recover all the submitted cost of participants. The cost not recovered is bounded above by the duality gap. Based on this observation, a non-uniform price setting alternative using dual variables is proposed. The alternative is simple, avoids the price spikes that can easily happen with average pricing, and shows that it reduces the conflict of interest when multiple solutions exist.

- In Chapter 4, a unit commitment model and its solution by Lagrangian relaxation is presented. A new *interior-point/cutting-plane* (IP/CP) method to solve the dual problem is proposed. This method, which has been used in other engineering applications, has better convergence characteristics and does not suffers from the parameter tuning drawback as previous approaches. The interior-point/cutting-plane method requires the solution of a potential problem, for which an infeasible primal-dual interior-point method is proposed. Some implementation details are described in the chapter.

- In Chapter 5, a numerical evaluation of the performance of the IP/CP method to solve the unit commitment problem are presented. Its convergence characteristics and robustness to parameter changes are proven by numerical simulation. In the same chapter, we study the application of the IP/CP to execute unit commitment power pool auctions. Numerical results show that the robustness characteristics of the IP/CP method, combined with the non-uniform price setting alternative, diminishes the conflict of interest that can arise from the existence of near-optimal solutions. The non-uniform price setting alternative is compared to an average pricing rule derived from the an average price setting as used in the EWPP.

- In Chapter 6, a hybrid model for an energy and spinning reserve network-constrained market clearing system is described. The model considers demand side bidding and bilateral contracts and a direct current representation of the transmission network. An interior-point method is proposed for its solution; this method can take advantage of the special structure of the Newton's system. The model represents an experimental development related to recently created and proposed electricity markets.

- The last chapter of the thesis provides closing comments, conclusions and recommendations for future work.

Chapter 2

Central Cost Minimization and Competitive Market Equilibrium

Whenever a market exists or has to be implemented, a basic question arises on the existence of competitive market equilibrium in such a market. Electric power cannot be stored; balance among power supply and demand has to take place at every time instant in the system. For these reasons, power markets need to be created and implemented in computerized auctions with specific modeling assumptions. In Section 2.1 of this chapter, the equilibrium theory of production economies is applied to study the existence and uniqueness of equilibrium in power markets that are executed by central optimization models or optimization-based power pool auctions. The conditions for the existence of an equilibrium are derived from the dual problem to a generic power pool auction model. The derivations in this chapter are used in subsequent chapters.

The dual approach is used to present two simplified continuous optimization models for power pool auctions. The first model, presented in Subsection 2.2.1, deals with the simple-bid type of auctions that are used in markets such as California and Spain; the relation between dual variables and standard auctions pricing is described. In Subsection 2.2.2, a second model considers the classic economic dispatch as an optimization-based auction; a direct solution approach is derived based on duality.

2.1 The Existence and Uniqueness of Competitive Market Equilibrium

In a competitive market, none of the participants (suppliers or consumers) is able to affect the market price of the commodities being traded. Suppliers and consumers take prices as given and act to maximize their profit and utility, respectively. The intersection of the competitive supply and demand curves gives a market equilibrium. A market equilibrium point is a price vector for the commodities, and schedules of production and consumption that result in a "state of rest"; that is, no participant has incentives to depart from that point. This definition is due to Walras [36].

If an electricity market for power generation is implemented by a power pool auction, the pool or market operator can be viewed as the Walrasian auctioneer that determines such an equilibrium point. Here, it is considered an auction for power generation based on cost-minimization where the demand is inelastic; that is, demand does not respond to the prices. For the description of the equilibrium concepts, perfect competition is assumed and the cost functions suppliers submit to the pool are assumed to represent their true cost functions. The following general cost-minimization power pool auction model is considered:

$$f^* = \min \quad \textstyle\sum_i c_i(\boldsymbol{p}_i), \tag{2.1a}$$

$$\text{s.t} \quad p_d^t - \textstyle\sum_i p_i^t = 0, \qquad \forall t \tag{2.1b}$$

$$\boldsymbol{p}_i \in \mathcal{P}_i, \qquad \forall i \tag{2.1c}$$

where, for simplicity, $\sum_i$ denotes the sum over the n suppliers considered in the market; i.e., $\sum_{i=1}^n$. The column vector $\boldsymbol{p}_i = [p_i^1, p_i^2, \cdots, p_i^m]^T$ denotes the power outputs (schedule) for supplier i at each of the m periods in the market; $c_i(\boldsymbol{p}_i) = \sum_t c_i(p_i^t)$ denotes the total variable cost of supplier i; $\sum_t$ denotes $\sum_{t=1}^m$; p_d^t is the inelastic demand for period t, $\boldsymbol{p}_d = [p_d^1, p_d^2, \cdots, p_d^m]^T$; $\mathcal{P}_i$ denotes the set of operative limits that are included in the bids, e.g., allowable power outputs, ramp limits and time constraints; and $\forall t$ is short hand notation for $t = 1, \cdots, m$ and the same applies for i. The vector $\boldsymbol{p}$ can contain as many variables as required for modeling; for instance, commitment variables $\boldsymbol{u}$.

Problem (2.1) defines a generic central cost-minimization problem that can be used to represent different modeling levels for a power pool auctions. In general, the solution to this problem gives the power outputs that satisfy the demand at minimum cost while satisfiying the operative limits of the suppliers.

The conditions under which the solution to (2.1) represent a competitive market equilibrium point are derived next.

In a competitive market, each supplier acts to maximize its profits which can be modeled by the following maximization problem:

$$\psi_i(\rho) = \max_{p_i \in \mathcal{P}_i} [\rho^T p_i - c_i(p_i)] \tag{2.2}$$

where $\rho = [\rho^1, \rho^2, \cdots, \rho^m]^T$ is a vector that defines the price for power at each period of the market. Equation (2.2) specifies that, given a price vector ρ, a supplier produces power in such a way that profits (revenues minus cost) are maximized; at the same time, the power output has to be feasible to the operative limits set $\mathcal{P}$. For a given price vector ρ, in order for a power output vector, p_i, to be optimal in (2.2), the following necessary conditions need be satisfied:

$$\begin{aligned} \rho - \nabla_{p_i} c_i(p_i) &= 0, \\ p_i &\in \mathcal{P}_i. \end{aligned} \tag{2.3}$$

The solution to problem (2.2) must satisfy (2.3); it gives the i-th bidder supply function denoted by

$$p_i(\rho) = \begin{pmatrix} p_i^1(\rho) \\ p_i^2(\rho) \\ \vdots \\ p_i^m(\rho) \end{pmatrix} \tag{2.4}$$

The profit function (2.2) can then be written as

$$\psi_i(\rho) = \rho^T p_i(\rho) - c_i(p_i(\rho)) \tag{2.5}$$

The total supply function is defined as the sum of all the bidders supply functions, i.e.,

$$\phi(\rho) = \textstyle\sum_i p_i(\rho) = \begin{pmatrix} \phi^1(\rho) \\ \phi^2(\rho) \\ \vdots \\ \phi^m(\rho) \end{pmatrix} \tag{2.6}$$

A price vector ρ^*, for which the supply function intersects the demand $\phi(\rho^*) = p_d$, is a market equilibrium price (see Figure 2.1). Some conditions need be satisfied to guarantee the existence of such an equilibrium point [36]. These conditions can be summarized as follows: (i) zero output is a feasible allocation for each supplier $\mathbf{0} \in \mathcal{P}_i$; (ii) the sets $\mathcal{P}_i$ are

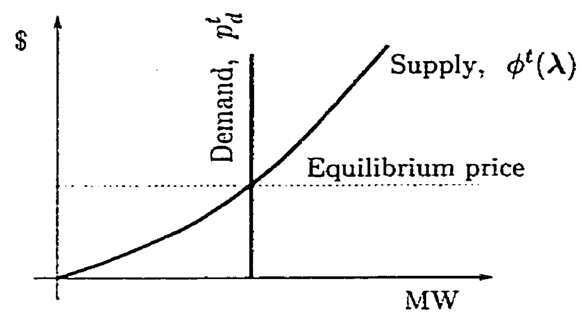


Figure 2.1: Market equlibrium with inelastic demand, at time t

bounded and a feasible allocation exists; i.e., there are $p_i \in \mathcal{P}_i$ so that $\sum_i p_i = p_d$; and (iii) the supply functions $p_i(\lambda)$ are continuous.

The first condition ensures that a supplier can always go off the market so that, eventually, is not forced to operate at negative profits which, in general, can be achieved. The power operative limits or production sets $\mathcal{P}_i$ are always bounded and a feasible solution is assumed to exist. However, the continuity of the supply function cannot be guaranteed if the cost functions $c_i(p_i)$ are not differentiable or the set $\mathcal{P}_i$ is non-convex [37]. Specially, the later condition cannot be guaranteed for most of the unit-commitment like electricity auctions since the set $\mathcal{P}_i$ is discrete.

When an equilibrium point exists, it can be determined by solving the dual to the cost-minimization power-pool auction problem in (2.1). The (Lagrangian) dual problem (see [38]) is given by

$$\psi^* = \max \quad \psi(\lambda) \tag{2.7}$$

where λ is the vector of dual variables to the demand constraint (2.1b). The dual function, denoted by ψ, is

$$\psi(\lambda) = \min_{p_i \in \mathcal{P}_i} \textstyle\sum_i c_i(p_i) + \lambda^T(p_d - p), \tag{2.8}$$

where $p = [\sum_i p_i^1, \cdots, \sum_i p_i^m]^T$. Regardless of the structure of $\mathcal{P}_i$ and the cost functions $c_i(p_i)$, this dual function has been proven to be piece-wise concave and non-differentiable [38], and can be rewritten as

$$\psi(\lambda) = \lambda^T p_d - \textstyle\sum_i \psi_i(\lambda) \tag{2.9}$$

where $\psi_i(\lambda)$ are the profit functions as defined in (2.2). Using (2.5) into (2.9), the dual function can be written as

$$\psi(\lambda) = \lambda^T p_d - \textstyle\sum_i \lambda^T p_i(\lambda) + \sum_i c_i(p_i(\lambda)). \tag{2.10}$$

Since the dual function is concave, but not differentiable, the necessary and sufficient condition (see [39]) for a vector λ^* to be an optimal solution to the dual problem (2.7) is

$$0 \in \partial\psi(\lambda^*), \tag{2.11}$$

where $\partial\psi(\lambda)$ denotes the sub-differential (the set of sub-gradients) of the dual function at λ. Equation (2.11) specifies that the dual function achieves its maximum at the point where its sub-differential contains a zero sub-gradient. This is analogous to the point where the gradient of a concave differentiable function is zero. Some definitions and properties of non-differentiable functions are presented in Appendix A. From (2.10), and using the property (A.4) in the Appendix A, (2.11) is equivalent to

$$p_d \in \partial \textstyle\sum_i \lambda^{*T} p_i(\lambda^*) - \partial \textstyle\sum_i c_i(p_i(\lambda^*)) \tag{2.12}$$

If the supply functions are continuous at λ^*, then (2.12) can be written as

$$p_d \in \textstyle\sum_i p_i(\lambda^*) + \textstyle\sum_i \lambda^{*T} \partial p_i(\lambda^*) - \textstyle\sum_i \nabla^T_{p_i} c_i(p_i) \partial p_i(\lambda^*), \tag{2.13}$$

Using (2.6) and (2.3), (2.13) results in

$$p_d = \phi(\lambda^*). \tag{2.14}$$

That is, if the supply functions are continuous at the optimal solution of the dual problem, λ^*, the demand intersects the supply. Hence, the optimal dual variable is a market equilibrium price $\rho = \lambda^*$; furthermore, the supply $p_i(\lambda^*)$ gives an optimal solution to the primal problem,

$$p_i^* = p_i(\lambda^*). \tag{2.15}$$

From (2.10), the optimal value of the dual function is

$$\psi^* = \lambda^{*T} p_d - \textstyle\sum_i \lambda^{*T} p_i(\lambda^*) + c_i(p_i(\lambda^*)) = \textstyle\sum_i c_i(p_i(\lambda^*))$$

which is equal to the primal objective function

$$f^* = \textstyle\sum_i c_i(p_i^*) = \textstyle\sum_i c_i(p_i(\lambda^*)) = \psi^*. \tag{2.16}$$

Whenever there is a market equilibrium point, there is no duality gap $f^* - \psi^* = 0$, and the optimal dual vector is a market equilibrium (clearing) price. Alternatively, it needs be mentioned that optimal dual variables in the absence of duality gap are Lagrange multipliers [38].

The Lagrange multipliers vector may not be unique; this happens if the concave dual function is "flat" on top (which occurs when the demand intersects the supply function at a "flat" section [36]). Under equilibrium, there is no other feasible schedule (optimal or not) that can be preferred by all the suppliers, in terms of profits (2.2). That is, a Walrasian equilibrium is Pareto optimal; this is known as the "first theorem of microeconomics" [36].

If the continuity on the supply functions is not satisfied, the optimal dual variables are not a market equilibrium. There is no price that can equate supply and demand; at the optimal dual solution, the demand does not intersect the supply,

$$p_d \neq \phi(\lambda^*). \tag{2.17}$$

The conditions for the existence and uniqueness of competitive market equilibrium are a classic development in microeconomic theory due to Debreu [36]- [40]. In this thesis, they are developed in the context of the cost-minimization power pool auctions and help illustrate price setting alternatives and their implications. When an equilibrium does not exist, the optimal dual variables are not market clearing prices. Given an adopted pricing rule, and the final schedule implemented, conflict of interest may arise among suppliers.

In a vertically integrated industry, cost minimization is the basis of operation. Numerical algorithms that solve (2.1) to obtain optimal (or near optimal) solutions do not arise any conflict of interest. Any optimal or near optimal solution does not bring conflict among generators that are owned by the same utility and, even more, prices are determined in a rate-of-return basis by government regulation and are not a results of the optimization algorithm.

The closer the models in (2.1) to the physical characteristics of the power plants are (i.e., unit commitment models), the more difficult is the optimization problem to solve. However, the outcome of the auction gives schedules that are feasible to the specified operative limits; this is considered to be a more reliable market. The simpler the models are (i.e., standard auctions), the easiest is the optimization problem to solve; the auction becomes a more transparent market where the determination of schedules and prices is more visible to everybody in the market. Most of the conflicts that arise when complex optimization models are used are avoided.

2.2 Simplified Power Auctions as Continuous Optimization Models

2.2.1 The Simple-Bids Linear Model

Due to the problems and previous experience with the use of complex optimization models, markets such as California and Spain have opted for the use of simpler or standard auction models. In standard auctions, simple price-quantity bids are used in the formulation (2.1). For each period in the market, suppliers and consumers submit to the market operator one or several simple bids that contain the prices and quantities of power they are willing to sell or buy. The market operator constructs a supply/demand curve (see Figure 2.2) to determine the market clearing price and schedules from its intersection. Suppliers are ordered in increasing bid price, and consumers in decreasing prices order.

For the inelastic demand case, the market price ρ is the bid price of the last bidder that supplies the power to cover demand, which is know as the standard first-price auction [19]. A second-price auction sets the price as the bid price of the first supplier in the curve that was not necessary to cover the demand. This type of simple-bids auction can be formulated

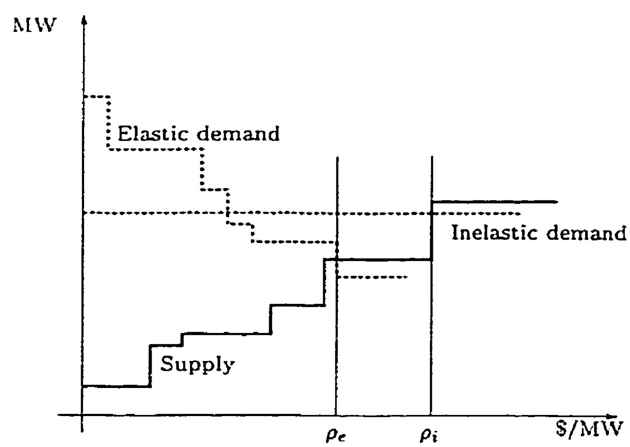


Figure 2.2: Supply/demand curve, elastic and inelastic demand

using the model (2.1), for which we considered the case of inelastic demand.

The cost function is defined as $c_i(p_i) = \beta_i p_i$ and the set of operative limits by $\mathcal{P}_i = \{p_i \mid 0 \leq p_i \leq \overline{p}_i\}$, where β_i is the bid price in \$/MW and $\overline{p}_i$ is the offered amount. For the market or pool operator, these bid prices represent costs to be minimized. The optimization model that describes the auction is given by

$$
\begin{aligned}
f^* = \min \quad & \textstyle\sum_i \beta_i p_i \\
\text{s.t} \quad & p_d - \textstyle\sum_i p_i = 0 \\
& 0 \le p_i \le \overline{p}_i
\end{aligned}
\tag{2.18}
$$

The dual function (2.9) is now given by

$$\psi(\lambda) = \lambda p_d - \textstyle\sum_i \max_{0 \le p_i \le \overline{p}_i} (\lambda - \beta_i) p_i. \tag{2.19}$$

This function is piece-wise concave, as schematically shown in Figure 2.3.

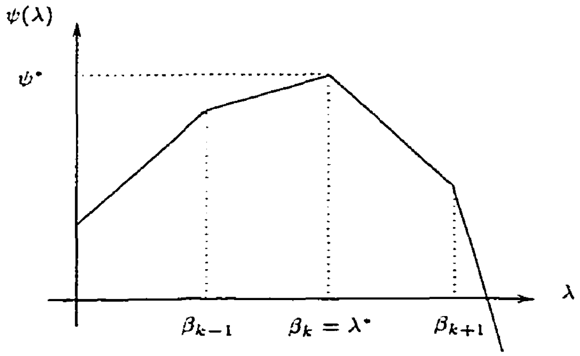


Figure 2.3: Piece-wise concave dual function

As derived in Section 2.1, an optimal dual variable can be found by searching the point at which the dual function has a zero sub-gradient. Let us assume that: (i) the bids are re-ordered in a non-decreasing cost order: $\beta_1 \le \beta_2 \cdots \le \beta_n$; (ii) k is the smallest index so that $\sum_{i=1}^{k} \overline{p}_i \ge p_d$; (iii) $\mathcal{O} = \{i \mid \beta_i = \lambda^*, i = 1, \cdots, n\}$; (iv) $r = \min\{\mathcal{O}\} - 1$; and (v) $p_c = \sum_{i=1}^{r} \overline{p}_i$. Then, the possible solutions for (2.18) can be summarized as follows:

1. *Unique primal and dual optimal solutions.* If $|\mathcal{O}| = 1$ and $p_c < \overline{p}_k$, the unique primal and unique dual optimal solutions are $\boldsymbol{p}^* = [\overline{p}_1, \cdots, \overline{p}_r, p_c, 0_{k+1}, \cdots, 0_n]^T$ and $\lambda^* = \beta_k$, respectively.

2. *Unique primal and multiple dual solutions.* If $p_c = \sum_{i \in \mathcal{O}} \overline{p}_i$, the optimal primal solution is unique but degenerated; that is, there are multiple dual solutions. The primal solution is given by $\boldsymbol{p}^* = [\overline{p}_1, \cdots, \overline{p}_{r+|\mathcal{O}|}, 0_{r+|\mathcal{O}|+1}, \cdots, 0_n]^t$, and $\lambda^* = [\beta_k, \beta_{r+|\mathcal{O}|+1}]$.

3. *Multiple primal and unique dual solution.* If $|\mathcal{O}| > 1$ and $p_c < \sum_{i \in \mathcal{O}} \overline{p}_i$, there are multiple primal solutions given by $\boldsymbol{p}^* = [\overline{p}_1, \cdots, \overline{p}_r, p_{r+1}, \cdots, p_{r+|\mathcal{O}|}, 0_{r+|\mathcal{O}|+1}, \cdots, 0_n]^T$;

that is, the power p_c can be allocated among bidders in the set $\mathcal{O}$, in multiple ways. The optimal dual solution is $\lambda^* = \beta_k$.

All these situations can be verified by evaluating the primal and dual objective function values, which results in $f^* = \psi^*$. All the solutions represent a market equilibrium point for the auction. The Lagrange multipliers are market clearing prices. Pricing with β_k is equivalent to a first-price auction, and pricing with β_{k+1} is equivalent to a second-price auction.

When there are multiple primal solutions –case 3 above– all these represent market equilibrium schedules; independently on the distribution of p_c among bidders in the set $\mathcal{O}$, their profits result the same (zero) in all the cases. Under this situation, bids in the set $\mathcal{O}$ are selected based on a priority order, until the load p_c is supplied. For instance, in the Spanish electricity market [17], the following priority criteria is used to select these bids: (i) the bid that arrives first on time to the market operator; (ii) the bid whose offered quantity is larger; and (iii) the bid whose name has alphabetical precedence. These, or any other criteria, can be represented by a tie-breaking priority order assigned to each bid, say $o(i)$. Bids in the top of the list are considered first to be part of the schedule; conversely, bids on the bottom of the list are not likely to be part of the final schedule.

Numerical Example

The data for a five-bidders case is shown in Table 2.1; the optimal primal and dual solutions for different demand levels are summarized in Table 2.2. The supply function and demand levels considered are shown in Figure 2.4. Table 2.2 also summarizes the cost, revenues and profits $\pi_i = \rho p_i - c_i(p_i)$ for each of the solutions. The price is set up by $\rho = \beta_k$ in all the cases, which corresponds to the first-price auction. All the values are in appropriate units \$, \$/MW and MW. In the first demand case (Table 2.2), there is a unique primal and

Table 2.1: Five simple-bidders example

i	1	2	3	4	5
$\overline{p}_i$ (MW)	50	50	100	100	100
β_i (\$/MW)	5	10	20	30	30

dual solution. For $p_d = 100$ MW, there are multiple Lagrange multipliers; the dual function (see Figure 2.5) is flat on top; the supply function is intersected by the demand in a flat

Table 2.2: Optimal solutions, cost, revenues and profits

p_d	i	p_i^*	c_i	$\rho \times p_i^*$	π_i	$f^* = \psi^*$	λ^*
70	1	50	250	500	250	450	10
	2	20	200	200	0		
100	1	50	250	500	250	750	[10,20]
	2	50	500	500	0		
250	1	50	250	1500	1250	4250	30
	2	50	500	1500	1000		
	3	100	2000	3000	1000		
	4	50	1500	1500	0		
	1	50	250	1500	1250	4250	30
	2	50	500	1500	1000		
	3	100	2000	3000	1000		
	5	50	1500	1500	0		

segment, as seen in Figure 2.4. In this case, the multiple equilibrium prices vary from 10 to 20 $/MW; it has to be seen that suppliers 1 and 2 are at their maximum output values and, therefore, the largest equilibrium price is given by the bid price of supplier 3 (the next less expensive bid to be used).

When $p_d = 250$ MW, there are two bidders in the set $\mathcal{O} = \{4, 5\}$; either 4 or 5 is selected by the priority order to supply 50 MW. Whichever final schedule is implemented, both solutions represent the same profit ($\pi_i = \rho p_i^* - c_i(p_i^*) = 0$) for bidders 4 and 5.

2.2.2 Simplified Continuous Quadratic Model

The model considered now represents the classic loss-less economic dispatch problem of power systems [3]. This classic economic dispatch problem has been solved by a number of methods including the gradient method, Newton's and the λ-iteration techniques [41]. Using the dual approach of Section 2.1, it is shown that its solution does not require a numerical-iterative algorithm; instead a simple interpolation over the supply function finds the optimal primal and the possible multiple dual solutions. The cost function and operative limits set are, respectively

$$c_i(p_i) = \beta_i p_i + \gamma_i p_i^2, \tag{2.20}$$

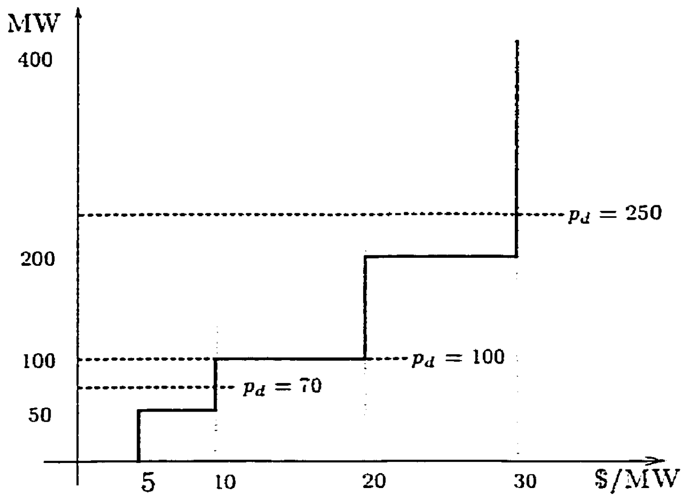


Figure 2.4: Supply curve and demand levels

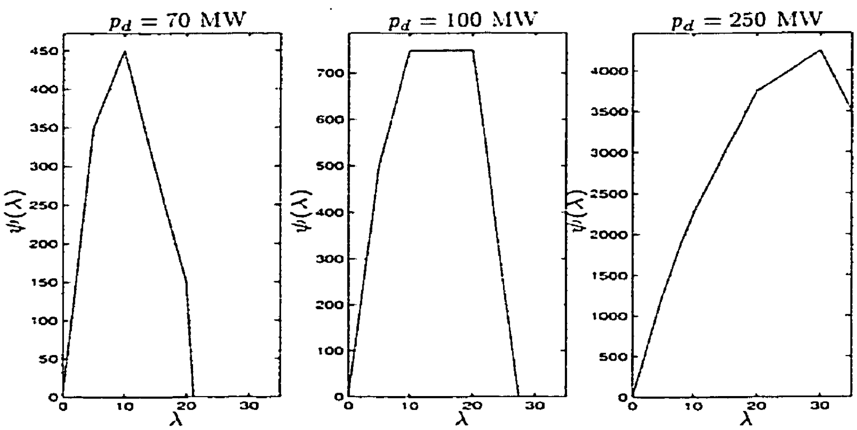


Figure 2.5: Dual functions

$$\mathcal{P}_i = \{p_i \mid \underline{p}_i \leq p_i \leq \overline{p}_i\}. \tag{2.21}$$

The cost function is assumed strictly convex, i.e., $\gamma_i > 0$. It should be noticed that a constant cost is not included in (2.20). The solution to the profit maximization subproblems (2.2) gives the following supply function for each supplier:

$$p_i(\lambda) = \begin{cases} \underline{p}_i, & 0 \leq \lambda < \beta_i + 2\gamma_i \underline{p}_i \\ (\lambda - \beta_i)/(2\gamma_i), & \beta_i + 2\gamma_i \underline{p}_i \leq \lambda < \beta_i + 2\gamma_i \overline{p}_i \\ \overline{p}_i, & \lambda \geq \beta_i + 2\gamma_i \overline{p}_i \end{cases} \tag{2.22}$$

and the profit function

$$\psi_i(\lambda) = \begin{cases} \lambda \underline{p}_i - c_i(\underline{p}_i), & 0 \le \lambda < \beta_i + 2\gamma_i \underline{p}_i \\ -(\lambda/2\gamma_i)(\beta_i - \lambda/2) + \beta_i^2/(4\gamma_i), & \beta_i + 2\gamma_i \underline{p}_i \le \lambda < \beta_i + 2\gamma_i \overline{p}_i \\ \overline{p}_i \lambda - c_i(\overline{p}_i), & \lambda \ge \beta_i + 2\gamma_i \overline{p}_i \end{cases} \tag{2.23}$$

These functions are shown schematically in Figure 2.6. As can be seen in Figure 2.6, the profit functions have a negative segment that ends at the minimum average cost $\lambda = c_i(\underline{p}_i)/\underline{p}_i$. If the optimal dual variable does not exceed this value, the supplier is forced to operate at negative profits since $0 \notin \mathcal{P}_i$.

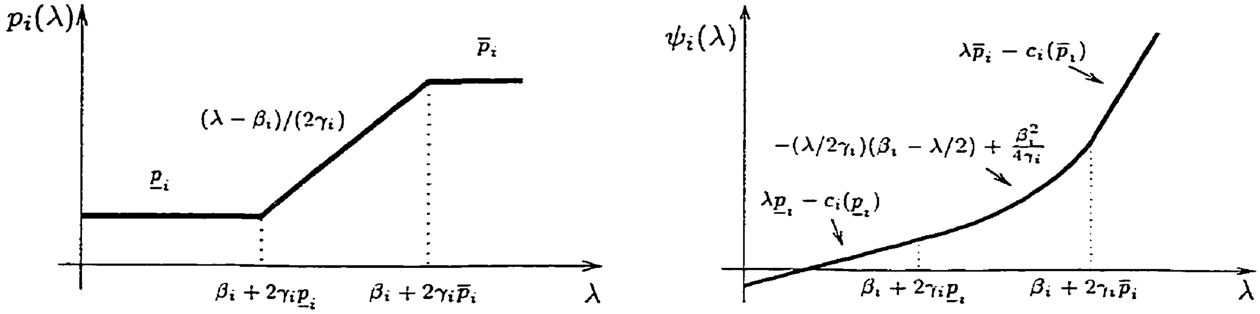
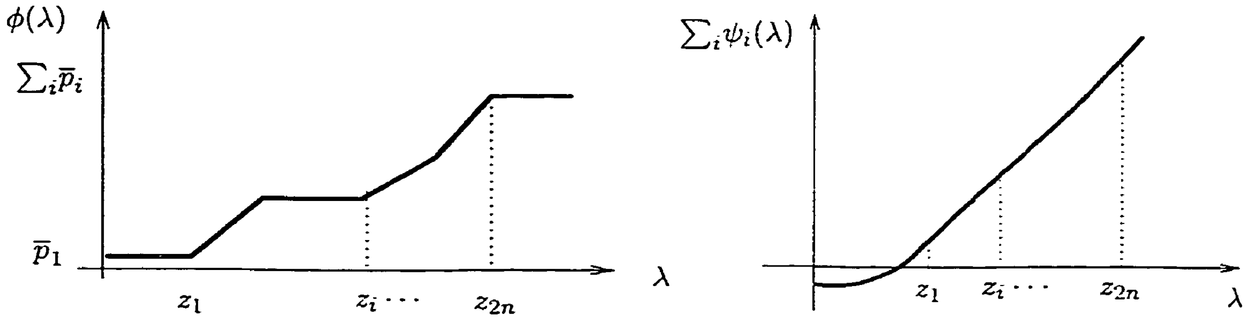


Figure 2.6: Supply and profit functions



(a)

Figure 2.7: Total supply and profits

Each bidder supply function, Figure 2.6, is non-decreasing and has two non-differentiable points; hence, the total supply function is also non-decreasing with, at most, $2n$ non-differentiable points, as sketched in Figure 2.7. The non-differentiable points of the supply function are given by

$$z = \{\beta_1 + 2\gamma_1 \underline{p}_1, \beta_1 + 2\gamma_1 \overline{p}_1, \cdots, \beta_n + 2\gamma_n \underline{p}_n, \beta_n + 2\gamma_n \overline{p}_n\} \tag{2.24}$$

Introducing, for convenience, $z_0 = 0$ and reordering these points in a non-decreasing order, $z_0 \leq z_1 \leq z_2 \leq \cdots \leq z_{2n}$, then

$$\phi(z_0) \geq \phi(z_1) \geq \phi(z_2) \geq \cdots \geq \phi(z_{2n}) \tag{2.25}$$

An optimal solution to the dual problem is given by

$$\lambda^* = z_k + [p_d - \phi(z_k)] \frac{z_k - z_{k-1}}{\phi(z_k) - \phi(z_{k-1})} \tag{2.26}$$

where k be the smallest index so that $\phi(z_k) \geq p_d$. If $\phi(z_{k+1}) = \phi(z_k)$, then there are multiple optimal dual solutions given by

$$\lambda^* = [z_k,\ z_{k+1}] \tag{2.27}$$

which happens if the demand intersect the supply in a flat segment; see Figure 2.7. The optimal primal solution is given by

$$p_i^* = p_i(\lambda^*), \tag{2.28}$$

which is evaluated from (2.22). The optimal dual and primal objective function values are the same; therefore, there is no duality gap. The problem is strictly convex and there is always equilibrium with a unique schedule with possible multiple prices, as indicated in (2.27). A direct interpolation over the supply function (2.26) and the evaluation of (2.22) gives the solution to the classic economic dispatch problem.

Numerical Example

A data set for an auction with four bidders with quadratic cost functions is presented in Table 2.3. The minimum output constraint is not considered (set to zero) and the results of the cost minimization power pool auction are as given in Table 2.4.

For this model, the supply function is continuous everywhere and, therefore, there is always a unique market equilibrium schedule (see Figure 2.8). For $p_d = 40$ MW, there are multiple equilibrium prices as can be noticed in the correspondent dual function in Figure 2.9; the demand intersects the supply at one of its flat portions (Figure 2.8). The costs, revenues and profits for each case are summarized in Table 2.4. In all the cases, the price is set up using the minimum Lagrange multiplier $\rho = \min\{\lambda^*\}$.

Table 2.3: Four quadratic-cost bidders example

i	1	2	3	4
$\overline{p}_i$ (MW)	20.0	20.0	50.0	40.0
β_i (\$/MW)	10.0	15.0	40.0	45.0
γ_i (\$/MW^2)	0.1	0.2	0.5	0.4

Table 2.4: Optimal solutions, costs, revenues and profits

p_d	i	p_i^*	c_i	$\rho \times p_i^*$	π_i	$f^* = \psi^*$	λ^*
40	1	20.0000	240.0000	460.0000	220.0000	620.0000	[23.0, 40.0]
	2	20.0000	380.0000	460.0000	80.0000		
100	1	20.0000	240.0000	1388.8889	1148.8889	3979.7222	69.4444
	2	20.0000	380.0000	1388.8889	1008.8889		
	3	29.4444	1611.2654	2044.7531	433.4877		
	4	30.5556	1748.4568	2121.9136	373.4568		
125	1	20.0000	240.0000	1700.0000	1460.0000	5872.5000	85.0000
	2	20.0000	380.0000	1700.0000	1320.0000		
	3	45.0000	2812.5000	3825.0000	1012.5000		
	4	40.0000	2440.0000	3400.0000	960.0000		

2.3 Lagrange Multipliers and Marginal Prices

Lagrange multipliers, provided they exist, represent the equilibrium prices. For the two continuous models presented in this section, these prices always exist. The term *marginal* or *shadow* price has classically been used to define the Lagrange multipliers related to the power demand constraint (2.1b) that appears in most power dispatch problems [3, 29]. However, since Lagrange multipliers may not exist or can have multiple values, the use of the term marginal price is avoided. In general, marginal prices are computed to measure a rate of change of the objective function with respect to changes in the constraints; they are used mainly to perform sensitivity analysis.

Analytical formulations for marginal prices, and their relation to dual variables, have been studied for linear programs using directional derivatives [42, 43] and for linear mixed-integer problems using price functions [44, 45].

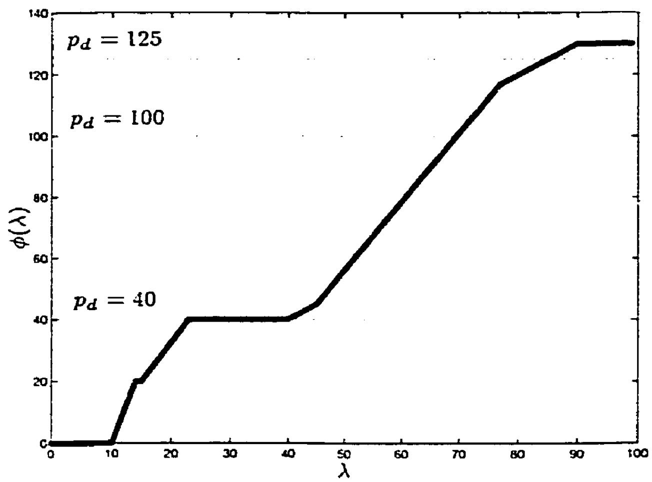


Figure 2.8: Total supply function

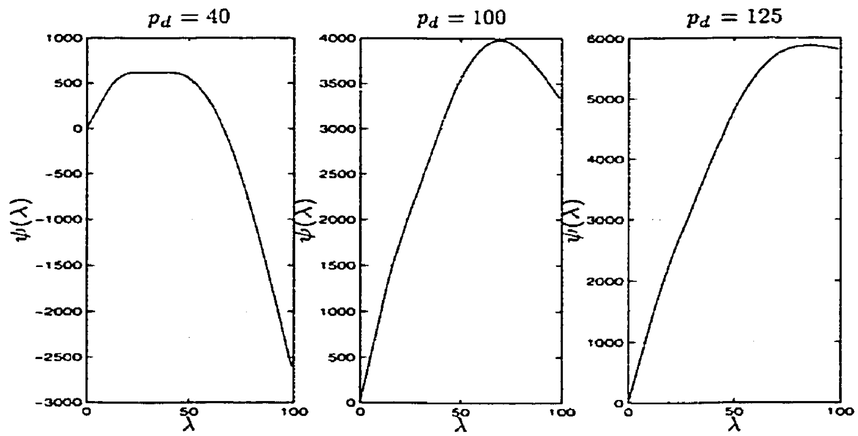


Figure 2.9: Dual functions

2.4 Summary

This chapter presents the conditions for the existence of competitive market equilibrium for power pool auctions driven by a central optimization model. The dual problem to a general cost minimization power pool auction is used as a general framework to illustrate the relation between central optimization and equilibrium. The following remarks are made:

- An equilibrium exists if the supply functions are continuous, which can be guaranteed only if the cost functions are differentiable and the operative limit sets are convex.
- When an equilibrium exists, the optimal solution to the dual problem provides Lagrange multipliers that are market clearing prices and also provides an equilibrium schedule.
- Multiple equilibrium prices happen if the demand intersects the supply at a flat region; under equilibrium, multiple optimal schedules are equally preferred by all suppliers and do not represent any conflict of interest.
- If an equilibrium does not exist, the optimal dual variables do not represent market clearing prices and multiple cost-minimizing solutions may exist. Under this situation, as presented in subsequent chapters, the selection of the pricing rule and the final schedule needs be done carefully in order to avoid unreasonable prices and conflict of interest among suppliers.

As an illustration of the derivations presented in the first section, two simplified optimization models for power auctions are presented and solved. For both models, equilibrium always exists and its multiplicity in price and schedule can directly be identified.

- The first model corresponds to the simple-bid type of auctions used in markets such as California and Spain; the relation between dual variables and a first-price auction is presented.
- The second model deals with the classic economic dispatch problem used as an auction; for this quadratic model, a direct solution approach that does not require the use of an iterative algorithm is developed [41].

Chapter 3

Simplified Discrete Models for Power Pool Auctions

In this chapter, simplified discrete models for power pool auctions are presented. The models consider that the cost functions, beside the linear and quadratic components, include a startup cost term. The inclusion of startup cost requires the introduction of binary variables that make the models very suitable to understand electricity auctions run by more complex optimization models, such as unit commitment.

Elmaghraby and Oren [25, 1999] use cost functions with linear and startup cost terms to represent a simplified unit commitment model and study the strategic behavior and efficiency of standard auctions. Jacobs [8, 1997] uses a linear model with startup cost to identify different solutions to the cost minimization auction. Dekrajangpetch and Sheblé [12, 1999] use simplified models to study Lagrangian relaxation in the context of an auction. Recently, Radinskaia and Galiana [46, 2000] have also investigated the analytical solution to simplified unit commitment models.

Using the derivations in Chapter 2, we use the simplified models to investigate price setting alternatives under disequilibrium and their effects on multiple solutions. Section 3.1 presents a model with linear and startup cost, and in Section 3.2, a model with quadratic and startup cost. For both models, an analytical solution to the dual problem is derived and the primal problems are solved using a *Branch-and-Bound* (B&B) algorithm.

In Section 3.3, price setting alternatives such as maximum average cost and price minimization are analyzed. It is shown that the conflict of interest arising from the existence of multiple solutions are worsened by such pricing rules. Average pricing can cause unreasonable price increases without any strategic behavior assumption. The observations are used to propose a non-uniform price setting alternative based on dual variables. Numerical examples are presented in Sections 3.3 and 3.4.

3.1 Linear Model With Startup Cost

In this model, a constant startup cost α_i ($) is added to a linear cost function. The startup cost requires the introduction of binary variables in the model; these variables, denoted by $u_i \in \{0,\ 1\}$, are one when the bid is selected and zero otherwise. The introduction of binary variables transforms the cost-minimization power pool auction (2.1) into a unit-commitment like problem. The cost function that includes a constant startup cost is defined by

$$c_i(p_i, u_i) = \alpha_i u_i + \beta_i p_i, \tag{3.1}$$

The operative limits set is

$$\mathcal{P}_i = \{(p_i, u_i) \mid 0 \leq p_i \leq u_i \overline{p}_i,\ u_i \in \{0, 1\}\}. \tag{3.2}$$

In 3.2, if the power output is greater than zero, then $u_i = 1$ and the startup cost takes effect in the objective function; if $u_i = 0$, all the components in the cost function are zero. With cost functions and operative limits defined by (3.1) and (3.2), the dual problem to (2.1) can be solved in a closed form. The solution to the dual problem also provides lower bounds that makes B&B an attractive alternative to find multiple solutions to the primal problem.

3.1.1 Closed Form Solution to the Dual Problem

Following the notation in Section 2.1, the solution to the profit maximization subproblems (2.2) is given by

$$p_i(\lambda) = \begin{cases} 0, & 0 \leq \lambda < \overline{\lambda}_i \\ 0 \text{ or } \overline{p}_i, & \lambda = \overline{\lambda}_i \\ \overline{p}_i, & \lambda > \overline{\lambda}_i \end{cases} \tag{3.3}$$

where $\overline{\lambda}_i = \alpha_i/\overline{p}_i + \beta_i$, i.e., the average cost at maximum output. Using (3.3) in (2.5), the i-th bidder profit function is

$$\psi_i(\lambda) = \begin{cases} 0, & 0 \leq \lambda \leq \overline{\lambda}_i \\ \overline{p}_i\lambda - (\alpha_i + \beta_i\overline{p}_i), & \lambda > \overline{\lambda}_i \end{cases} \tag{3.4}$$

The supply and profit functions are schematically shown in Figure 3.1; as it can be noted in this figure and in equation (3.3), the supply functions for each supplier have a discontinuity at $\overline{\lambda}_k$; this makes the total supply function, Figure 3.2, also a discontinuous function.

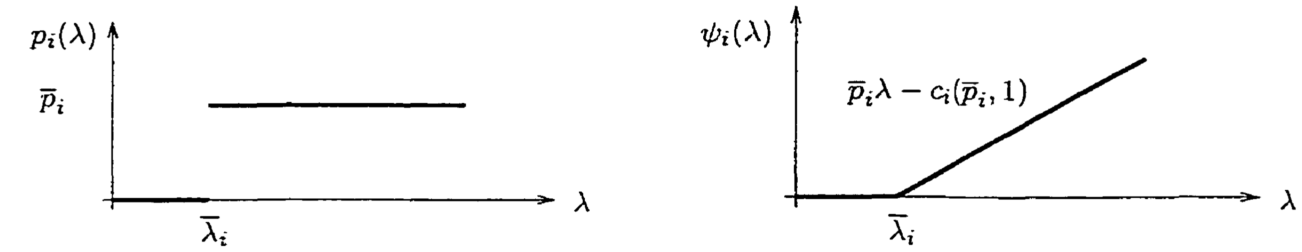


Figure 3.1: i-th bidder supply and profit functions

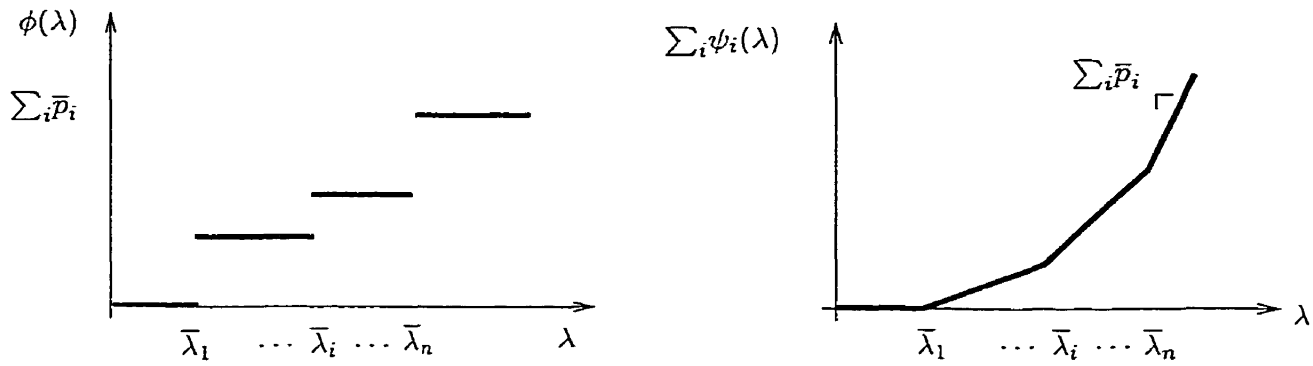


Figure 3.2: Total supply and total profit functions

In order to find an optimal dual variable, it suffices to find the point where a subgradient of the dual function equals the demand. Let us re-order the bids in the following non-decreasing fashion:

$$\overline{\lambda}_1 \leq \overline{\lambda}_2 \cdots \leq \overline{\lambda}_n \tag{3.5}$$

Following the curve forms in Figure 3.2 and since $\psi(\lambda) = \lambda p_i - \sum_i \psi_i(\lambda)$, an optimal solution to the dual problem is given by

$$\lambda^* = \overline{\lambda}_k \tag{3.6}$$

where k is the smallest index so that

$$\textstyle\sum_{i=1}^{k} \overline{p}_i \geq p_d \tag{3.7}$$

If the equality holds in (3.7), then the dual function is "flat" on the top and there are multiple dual variables in the interval

$$\lambda^* = [\lambda_k, \lambda_{k+1}] \tag{3.8}$$

If the last situation happens, the demand intersects the supply and the Lagrange multipliers (3.8) are the equilibrium prices. Moreover, an optimal primal solution is given by $p^* = [\overline{p}_1, \cdots, \overline{p}_k, 0, \cdots, 0]^T$. All other possible multiple primal solutions could be found by enumeration of the 2^n commitment combinations $u_i \in \{0, 1\}$; however, complete enumeration is prohibitive even for very small n [47]. A B&B enumerative approach is developed in next section for this purpose.

3.1.2 A Branch-and-Bound Algorithm to Solve the Primal Problem

Branch-and-Bound algorithms can be described as intelligent enumerative approaches [47]. A B&B is a much more inexpensive alternative to complete enumeration and is able to obtain multiple near-optimal or optimal solutions. The practical success of a B&B method depends on two factors [47,48]: first, the computational effort it takes to obtain tight bounds to partitions in the original problem; and second, the existence of information that can be used to design the partitioning (branching) rules that speed up the algorithm. For the power auction model (2.1), with the linear and start-up cost function (3.1), both upper and (tight) lower bounds are readily available from the solution of the dual problem and a good branching order can be easily obtained.

A binary tree for a problem with size $n = 3$ is shown in Figure 3.3. Each node k of the tree is a partition of the original problem (2.1). A partition represents the problem in a smaller feasible set, where a particular set of binary variables is fixed to zero, one , or left free, as indicated by 0, 1 and X in Figure 3.3.

If $\overline{f}_k$ and $\underline{f}_k$ are an upper and lower bounds to each node subproblem, then an upper and lower bound to the optimal objective value of the original problem is given by $\overline{f} = \min\{\overline{f}_k\}$ and $\underline{f} = \min\{\underline{f}_k\}$, respectively [47]. The B&B algorithm, based on the branching order, constructs new nodes of the tree and either updates the bounds, cuts the branch or determines that an optimal solution have been found. At a particular node, $k+1$, no optimal solution can be found if: (i) no feasible solution is contained on it, or (ii) its lower bound is larger than the best actual upper bound $\underline{f}_{k+1} > \overline{f}$. In this situation, the tree branch is not further explored (it is cut). If none of these conditions are met, the bounds are updated: $\overline{f} = \min\{\overline{f}, \overline{f}_k\}$ and $\underline{f} = \min\{\underline{f}, \underline{f}_k\}$.

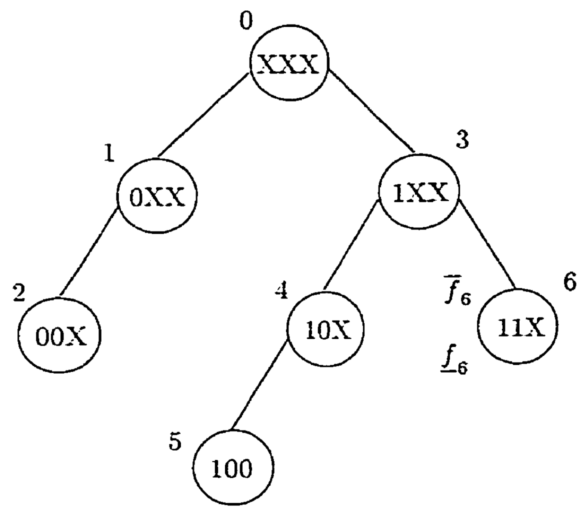


Figure 3.3: Binary tree for the Branch-and-Bound algorithm

If only a good near-optimal solution is required, the B&B algorithm can be stopped at any node that satisfies $\overline{f}_k - \underline{f}_k \leq \epsilon$, where ϵ measures the desired quality of the solution (complementarity gap). If the algorithm is stopped until there are no more nodes to visit, all the nodes whose $\overline{f}_k = \overline{f}$ are multiple near-optimal solutions. Only if $\overline{f} = \overline{f}_k = \underline{f}_k$, the solutions can be assured to be optimal.

The upper and the lower bound for each tree node are computed as follows.

Determination of a lower bound

The optimal dual objective function value at each node subproblem provides a lower bound for the subproblem $\psi_k^* \leq f_k^*$ [38]. Since the dual problem can be solved in a closed form, the computation of this lower bound is very inexpensive. At node k, let $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ contain the indices of bids, with u_i free, $u_i = 1$ and $u_i = 0$, respectively; then, the dual function (2.9) of a subproblem at node k can be written as

$$\psi_k(\lambda) = \lambda p_d - \sum_{i \in \mathcal{B}} \{-\alpha_i + \max_{0 \leq p_i \leq \overline{p}_i} (\lambda - \beta_i) p_i\} - \sum_{i \in \mathcal{A}} \max_{\substack{0 \leq p_i \leq u_i \overline{p}_i \\ u_i \in \{0,1\}}} \{(\lambda - \beta_i) p_i - \alpha_i u_i\}. \tag{3.9}$$

At node k, the optimal solution to the dual problem is denoted as

$$\psi_k^* = \max \quad \psi_k(\lambda) \tag{3.10}$$

This optimal value can be found by evaluating (3.9) at λ^*, where λ^* is obtained from (3.6) where $\overline{\lambda}_i = \alpha_i/\overline{p}_i + \beta_i$ is replaced by $\overline{\lambda}_i = \hat{\alpha}_i/\overline{p}_i + \beta_i$, with

$$\hat{\alpha}_i = \begin{cases} \alpha_i, & \forall i \in \mathcal{A} \\ 0, & \forall i \in \mathcal{B} \\ \infty, & \forall i \in \mathcal{C} \end{cases} \tag{3.11}$$

For bids in $\mathcal{C}$, $\hat{\alpha}_i = \infty$ is introduced as an indication of the absence of bid i at node k ($u_i^l = 0$). If the optimal dual variable is equal to ∞, the dual problem is unbounded; therefore, the primal problem is infeasible.

Determination of an upper bound

At every node of the tree, an upper bound to the primal problem is given by the objective function value of any feasible solution. A tight upper bound can be computed if a primal feasible solution is constructed by considering the bids already committed, $u_i = 1$, and the bids in $\mathcal{A}$ that satisfy $p_i(\lambda^*) \geq 0$, where λ^* is the optimal dual solution to a dual subproblem at node k. The details of the algorithm are summarized below.

Algorithm 3.1 Determination of an upper bound

1. *Initialization.* Set $p_i = \overline{p}_i$ for all $i \in \mathcal{B}$ and compute $p_r = p_d - \sum_{i \in \mathcal{B}} \overline{p}_i$.

2. *Infeasibility check.* If $p_r > \sum_{i \in \mathcal{A}} \overline{p}_i$, stop, the problem at node k is infeasible; otherwise, continue.

3. *Select bids in $\mathcal{A}$ to commit.* With only bids in $\mathcal{A}$ and the remaining demand p_r, solve the dual problem (3.9) using (3.6). From (3.3), if $p_i(\lambda^*) \geq 0$, set $u_i = 1$.

4. *Linear dispatch.* Considering the demand p_d and bids with $u_i = 1$, solve simple-bids dispatch problem by the procedure in Subsection 2.2.1.

5. *Set the upper bound.* Set $\overline{f}_k = \sum_{\{i|u_i=1\}} \alpha_i + \beta_i p_i$ using any p_i among the possible multiple linear solutions.

Determination of the branching order

The branching order decides the new node to be created when expanding the binary tree. It is desirable to create nodes that eliminate as fast as possible bigger portions of the tree.

For instance, see Figure 3.3, if it is found that the lower bound of the subproblem at node 1 is larger than the upper bound at node 0 ($\underline{f}_1 > \overline{f}_0$), then the whole left portion of the tree is eliminated when the first node is created. Considering the non-decreasing average cost order $\overline{\lambda}_1 \leq \overline{\lambda}_2 \leq, \cdots, \leq \overline{\lambda}_n$, since bid 1 most probably is part of the optimal solution, setting $u_1 = 0$ in the first node of the tree is likely to result in the elimination of its left portion. For the next node, node 3, if we set $u_2 = 0$, it is again likely that the left branch out of node 3 is eliminated. The process is likewise continued for bids $3, 4 \cdots n$.

Therefore, the branching order is taken as $1, 2, \cdots, n$ where $\overline{\lambda}_1 \leq \overline{\lambda}_2 \leq, \cdots, \leq \overline{\lambda}_n$. Any branching rule can be used; however, a good selection considerably speed ups the B&B algorithm.

3.1.3 Numerical Example

Table 3.1 contains the data for five bidders with linear and constant startup cost. The cost minimization auction is solved for the following demand levels: 52, 130, 190 and 210 MW. The primal and dual solution(s) for each of the demand levels are presented in Table 3.2.

Table 3.1: Five bidders with linear plus startup cost function

i	1	2	3	4	5
α_i ($)	30	40	70	35	35
β_i ($/MW)	10	15	25	25	25
$\overline{p}_i$ (MW)	50	50	90	20	20

In the same table, the values $\overline{\lambda_i}$ are presented for convenience. All the quantities are in appropriate units MW, $/MW and $. Complete enumeration of the $2^5 = 32$ commitment possibilities can confirm that the solutions presented are optimal in all the demand cases. The total supply function is shown in Figure 3.4; as can be seen, for the demand levels of 52, 110 and 130 MW, there is no market equilibrium and the optimal dual variable is unique. For $p_d = 110$ MW, there are two multiple solutions. For $p_d = 130$, multiple solutions exist; only two of these are shown. The first solution contains bids 4 and 5, and the second solution contains bid 3. As can be noticed in the first solution, 30 MW can be supplied in an infinite number of ways suppliers 4 and 5; that is, $p_4^* = x$ and $p_5^* = 30 - x$ for all $10 \leq x \leq 20$, lead to same objective function value $ 2140.

For the demand levels of 190 and 210 MW, there exists a market equilibrium point as can be identified in Figure 3.4. For $p_d = 190$ MW, the primal solution is unique but there

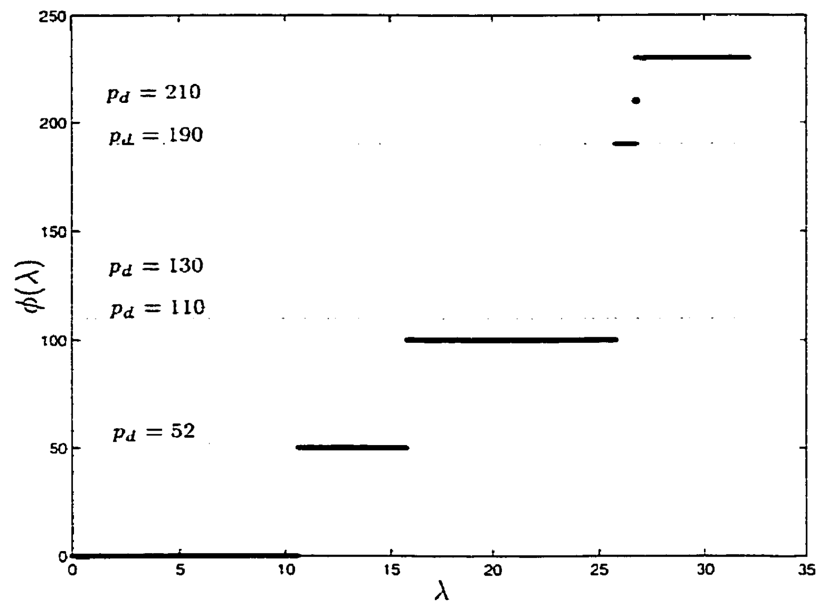


Figure 3.4: Total supply function

Table 3.2: Solution to the cost minimization auction

p_d	p_1^*	p_2^*	p_3^*	p_4^*	p_5^*	λ^*	f^*	ψ^*	dg
52	50	2	0	0	0	15.80	600.00	561.6000	38.4000
110	50	50	0	10	0	25.78	1605.00	1577.7778	27.2222
	50	50	0	0	10				
130	50	50	0	20	10	25.78	2140.00	2093.3333	46.6667
	50	50	30	0	0				
190	50	50	90	0	0	[25.78,26.75]	3640.00	3640.0000	0.0000
210	50	50	90	20	0	26.75	4175.00	4175.0000	0.0000
	50	50	90	0	20				
$\overline{\lambda}_i$	10.6	15.8	25.78	26.75	26.75				

are multiple equilibrium prices. For $p_d = 210$ MW, there is a unique equilibrium price and two optimal primal solutions.

Table 3.3 summarizes the costs, revenues and profits for each of the solutions when the price is set up by the smallest optimal dual variable,

$$\rho = \min\{\lambda^*\} \tag{3.12}$$

In Table 3.3, when an equilibrium exists, pricing with the optimal dual variable leads to

Table 3.3: Costs, revenues and profits. Pricing with dual variable $\rho = \min\{\lambda^*\}$

p_d	i	p_i^*	c_i^*	ρ	$\rho \times p_i^*$	π_i	CNR
52	1	50	530.0	15.80	790.000	260.000	
	2	2	70.0	15.80	31.600	-38.400	-38.400
110	1	50	530.0	25.78	1288.889	758.889	
	2	50	790.0	25.78	1288.889	498.889	
	4	10	285.0	25.78	257.778	-27.222	-27.222
	1	50	530.0	25.78	1288.889	758.889	
	2	50	790.0	25.78	1288.889	498.889	
	5	10	285.0	25.78	257.778	-27.222	-27.222
130	1	50	530.0	25.78	1288.889	758.889	
	2	50	790.0	25.78	1288.889	498.889	
	4	20	535.0	25.78	515.556	-19.444	
	5	10	285.0	25.78	257.778	-27.222	-46.667
	1	50	530.0	25.78	1288.889	758.889	
	2	50	790.0	25.78	1288.889	498.889	
	3	30	820.0	25.78	773.333	-46.667	-46.667
190	1	50	530.000	25.78	1288.889	758.889	
	2	50	790.000	25.78	1288.889	498.889	
	3	90	2320.000	25.78	2320.000	0.000	0.000
210	1	50	530.000	26.75	1337.500	807.500	
	2	50	790.000	26.75	1337.500	547.500	
	3	90	2320.000	26.75	2407.500	87.500	
	4	20	535.000	26.75	535.000	0.000	0.000
	1	50	530.000	26.75	1337.500	807.500	
	2	50	790.000	26.75	1337.500	547.500	
	3	90	2320.000	26.75	2407.500	87.500	
	5	20	535.000	26.75	535.000	0.000	0.000

positive profits for all bidders; the pricing rule in (3.12) is analogous to a first-price auction in the sense that when equilibrium exists, the "last" bidder breaks even at zero profit. For the cases in disequilibrium, some of the bidders do not recover their cost; in all these cases the total cost not recovered is equal to the magnitude of the duality gap. For $p_d = 130$ MW, all the other possible optimal solutions to supply 30 MW among bids 4 and 5, lead to the same cost not recovered of \$ 46.667 among these bids.

For $p_d = 210$ MW, there is equilibrium with two optimal schedules; both solutions result in the same profit. Even though bidders 4 and 5 are not scheduled in one of the solutions, it makes no difference for them which schedule is finally implemented.

In all the disequilibrium cases, there are incentives to move away from the optimal schedules. For instance, for $p_d = 52$ MW, at a price of 15.80 \$/MWh, bidder two would like to increase its output to 50 MW in order to maximize its profits. For $p_d = 110$ MW, since 25.78 < 26.75, bidders 4 and 5, in each solution, would like to reduce their output and operate at zero profit.

For $p_d = 130$ MW, in the first optimal solution, at the price 25.78 \$/MW, bidders 4 and 5 would like to reduce their output to zero in order to avoid negative profit (since 25.78 < 26.75). In the second solution, bidder 3 would like to increase to maximum output in order to maximize profit. The cases in disequilibrium are used in subsequent sections to describe the problems that arise in selecting the final schedule and the pricing rule.

3.2 Quadratic Model With Startup Cost

Additionally to the linear term in the cost function of the previous model (3.1), a quadratic term, $\gamma_i > 0$, is added to the cost function submitted with the bids, i.e.,

$$c_i(p_i, u_i) = \alpha_i u_i + \beta_i p_i + \gamma_i p_i^2 \tag{3.13}$$

The set of operative limits remains unchanged

$$\mathcal{P}_i = \{(p_i, u_i) \mid 0 \le p_i \le u_i \overline{p}_i,\ u_i \in \{0, 1\}\}. \tag{3.14}$$

The dual problem can still be solved in a closed form way, and the primal problem is solved by the B&B algorithm of Subsection 3.1.2.

3.2.1 Closed Form Solution to the Dual Problem

As may be expected, the supply and profit functions for this model have the combined characteristics of the models with pure quadratic cost functions (Subsection 2.2.2) and the linear model with startup-cost (Section 3.1). For this case, in order for $p_i(\lambda)$ to be optimal to the profit maximization subproblems (2.2), it has to satisfy

$$p_i(\lambda) = \max\,[0, \min\,\{\overline{p}_i, \frac{\lambda - \beta_i}{2\gamma_i}\}]. \tag{3.15}$$

And he profit function is

$$\psi_i(\lambda) = \max \{0,\ (\lambda - \beta_i)p_i(\lambda) - \gamma_i p_i^2(\lambda) - \alpha_i\}. \tag{3.16}$$

The substitution of (3.15) into (3.16) gives two possible solutions; if $\sqrt{\alpha_i/\gamma_i} < \overline{p}_i$, the supply and profit functions are given by

$$p_i(\lambda) = \begin{cases} 0, & \lambda < \lambda_{ai} \\ 0 \text{ or } \sqrt{\alpha_i/\gamma_i}, & \lambda = \lambda_{ai} \\ \frac{\lambda-\beta_i}{2\gamma_i}, & \lambda_{ai} < \lambda < \lambda_{bi} \\ \overline{p}_i & \lambda \geq \lambda_{bi} \end{cases} \tag{3.17}$$

$$\psi_i(\lambda) = \begin{cases} 0, & \lambda < \lambda_{ai} \\ \lambda p_i(\lambda) - c_i(p_i(\lambda), 1), & \lambda_{ai} \leq \lambda < \lambda_{bi} \\ \overline{p}_i\lambda - c_i(\overline{p}_i, 1), & \lambda \geq \lambda_{bi} \end{cases} \tag{3.18}$$

and, if $\sqrt{\alpha_i/\gamma_i} \geq \overline{p}_i$, by

$$p_i(\lambda) = \begin{cases} 0, & \lambda < \lambda_{ci} \\ 0 \text{ or } \overline{p}_i, & \lambda = \lambda_{ci} \\ \overline{p}_i, & \lambda > \lambda_{ci} \end{cases} \tag{3.19}$$

$$\psi_i(\lambda) = \begin{cases} 0, & \lambda < \lambda_{ci} \\ \lambda\overline{p}_i - c_i(\overline{p}_i, 1), & \lambda \geq \lambda_{ci} \end{cases} \tag{3.20}$$

where in (3.17) and (3.19),

$$\begin{aligned} \lambda_{ai} &= \beta_i + 2\sqrt{\alpha_i\gamma_i} \\ \lambda_{bi} &= \lambda_{ci} = c_i(\overline{p}_i, 1)/\overline{p}_i = \alpha_i/\overline{p}_i + \beta_i + \gamma_i\overline{p}_i \end{aligned} \tag{3.21}$$

Variable λ_{ci} is introduced to distinguish among the two possible forms of the supply function in Figures 3.5 and 3.6. The combination of a large startup cost and small non-linear term makes $\sqrt{\alpha_i/\gamma_i}$ bigger, which causes the supply function to behave in the same way when only a linear term is included in the cost function. The total supply and profit functions

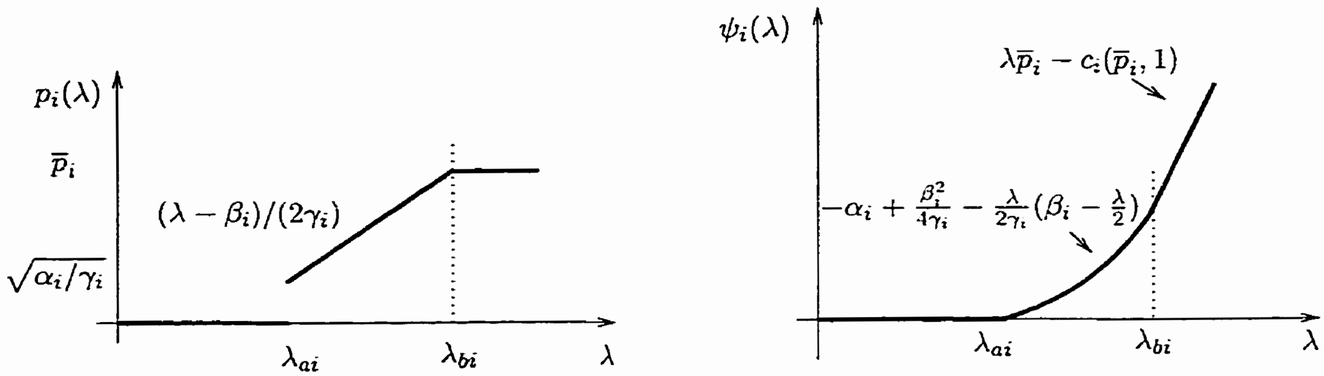


Figure 3.5: i-th bidder supply and profit functions for $\sqrt{\alpha_i/\gamma_i} < \overline{p}_i$

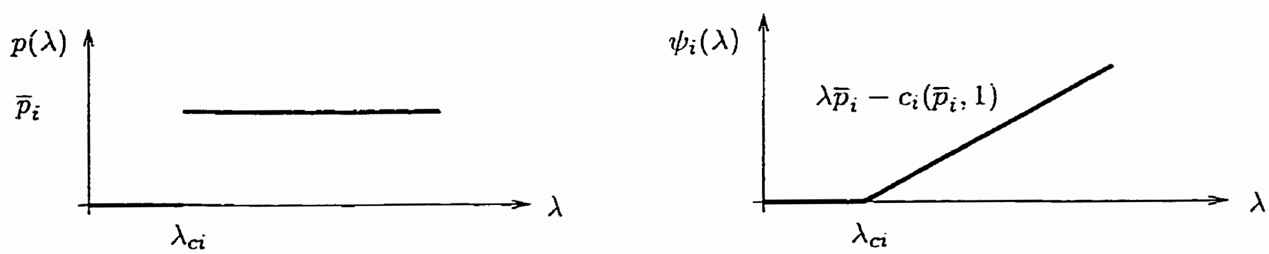


Figure 3.6: i-th bidder supply and profit functions for $\sqrt{\alpha_i/\gamma_i} \geq \overline{p}_i$

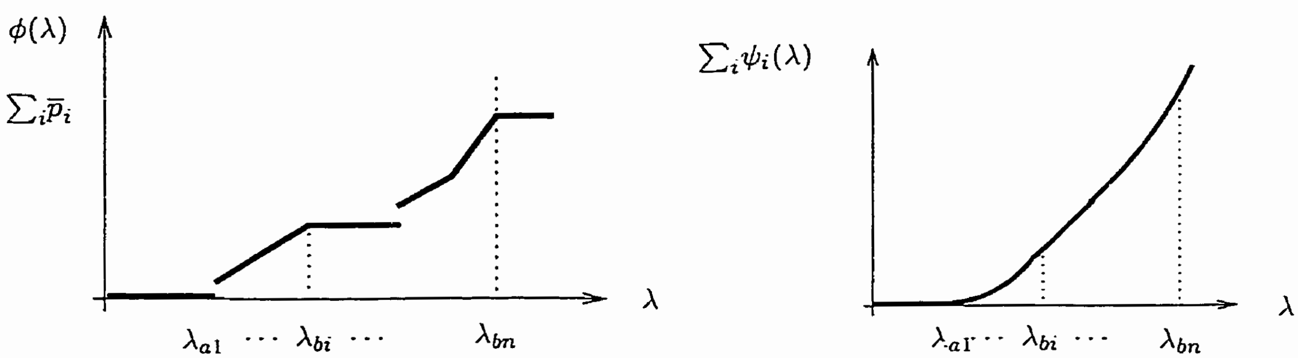


Figure 3.7: Total supply and total profits

have the forms depicted in Figure 3.7. If the points $\lambda_{ai}, \lambda_{bi}$ and λ_{ci} are gathered into the vector z, whose components are then reordered in a non-decreasing way $z_0 \leq z_1 \leq z_2 \cdots$;

where $z_0 = 0$. An optimal solution to the dual problem is given by

$$\lambda^* = \begin{cases} z_k, & \text{if } z_k = \lambda_{ak} \text{ or } z_k = \lambda_{ck} \\ z_k + [p_d - \overline{\phi}(z_k)]\dfrac{z_k - z_{k-1}}{\overline{\phi}(z_k) - \overline{\phi}(z_k)}, & \text{if } z_k = \lambda_{bk} \end{cases} \tag{3.22}$$

where $\overline{\phi}(z_k) = \sum_{i=1}^{k} \overline{p}_i(z_k)$, and $\overline{p}_i(z_k)$ is used to indicate the maximum supply when z_k is a discontinuity point in the supply; i.e., λ_{ai} and λ_{ci}. In the same equation, k is the smallest index so that $\overline{\phi}(z_k) \geq p_d$; if the equality holds with $z_k = \lambda_{bk}$, then there are multiple optimal dual variables given by

$$\lambda^* = [z_k, z_{k+1}]$$

If $\overline{\phi}(\lambda^*) = p_d$, λ^* is an equilibrium price and the optimal primal solution is given by $p = [\overline{p}_1(\lambda^*), \cdots, \overline{p}_k(\lambda^*), 0, \cdots, 0]^T$. Other primal solutions can be found using the B&B algorithm.

3.2.2 Branch-and-Bound for the Primal Problem

The B&B algorithm described in Subsection 3.1.2 is also used to solve the primal problem to the quadratic model in this section. The computation of lower and upper bounds is summarized below.

Determination of a lower bound

Considering the same description as in Subsection 3.1.2 for the sets $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$, the dual function at node k is given by

$$\begin{aligned} \psi_k(\lambda) = \lambda p_d - \sum_{i \in \mathcal{A}} & \max_{\substack{0 \leq p_i \leq u_i \overline{p}_i \\ u_i \in \{0,1\}}} (\lambda - \beta_i)p_i - \gamma_i p_i^2 - u_i \alpha_i \\ & - \sum_{i \in \mathcal{B}} -\alpha_i + \max_{0 \leq p_i \leq \overline{p}_i} (\lambda - \beta_i)p_i - \gamma_i p_i^2 \end{aligned} \tag{3.23}$$

The optimal solution to problem (3.23) is obtained from (3.22); but considering $\hat{\alpha}_i$ instead of α_i for the computation of $\lambda_{ai}, \lambda_{bi}, \lambda_{ci}$ in (3.21),

$$\hat{\alpha}_i = \begin{cases} \alpha_i, & i \in \mathcal{A} \\ 0, & i \in \mathcal{B} \\ \infty, & i \in \mathcal{C} \end{cases} \tag{3.24}$$

The optimal dual function at node k is evaluated from (3.23), with λ^* from (3.22). And the lower bound is set up $\underline{f}_k = \psi_k^*$.

Determination of an upper bound

The determination of the upper bound follows a similar procedure to the linear case, as described in the following algorithm.

Algorithm 3.2 Determination of an upper bound

1. *Initialization.* Set $p_i = \overline{p}_i$ for all $i \in \mathcal{B}$; compute $p_r = p_d - \sum_{i \in \mathcal{B}} \overline{p}_i$.
2. *Infeasibility check.* If $p_r > \sum_{i \in \mathcal{A}} \overline{p}_i$, stop; the problem at node k is infeasible; otherwise, continue.
3. *Select bids in $\mathcal{A}$ to commit.* With bids in $\mathcal{A}$ and demand p_r, solve the dual problem (3.23) using (3.22). Depending on $\sqrt{\alpha_i/\gamma_i}$, use (3.17) or (3.19); if $p_i(\lambda^*) \geq 0$ set $u_i = 1$.
4. *Perform a quadratic economic dispatch.* Considering the demand p_d and bids with $u_i = 1$, solve a quadratic dispatch with the analytical procedure in Subsection 2.2.2.
5. *Compute the upper bound.* Set $\overline{f}_k = \sum_{\{i|u_i=1\}} \alpha_i + \beta_i p_i + \gamma_i p_i^2$.

Determination of the branching order

For each of the forms of the supply function in Figures 3.5 and 3.6, a value $\hat{z}_i$ is assigned to each bid. For bids with $\sqrt{\alpha_i/\gamma_i}$ the value is set at the mid point between λ_{ai} and λ_{bi}; that is, $z_i = \lambda_{ai} + (\lambda_{bi} - \lambda_{ai})/2$. For bids with $\sqrt{\alpha_i/\gamma_i} \geq \overline{p}_i$ the value is set to $\hat{z}_i = \lambda_{bi}$. The indexing that results from the ordering $\hat{z_1} \leq \hat{z_2} \leq \cdots \leq \hat{z_n}$ defines the branching order. Setting $u_i = 0$ in this order, it is more likely to eliminate large portions of the tree.

3.2.3 Numerical Example

A data set for a five bidders is presented in Table 3.4. The cost minimization model is solved for $p_d = 30, 80, 85, 95$ and 140 MW. The total supply function is presented in Figure 3.8. The costs, revenues and profits for each case are summarized in Table 3.6. For $p_d = 30, 85$ and 95 MW, there is no market equilibrium point. In the first case, there is a cost not

Table 3.4: Five bidders with quadratic and constant startup cost

i	1	2	3	4	5
α_i (\$/MW)	20.0	80.0	80.0	90.0	100.0
β_i (\$/MW)	10.0	12.0	12.0	30.0	35.0
γ_i (\$/MW^2)	0.4	0.6	0.6	0.8	1.0
$\overline{p}_i$ (MW)	20.0	30.0	30.0	50.0	60.0

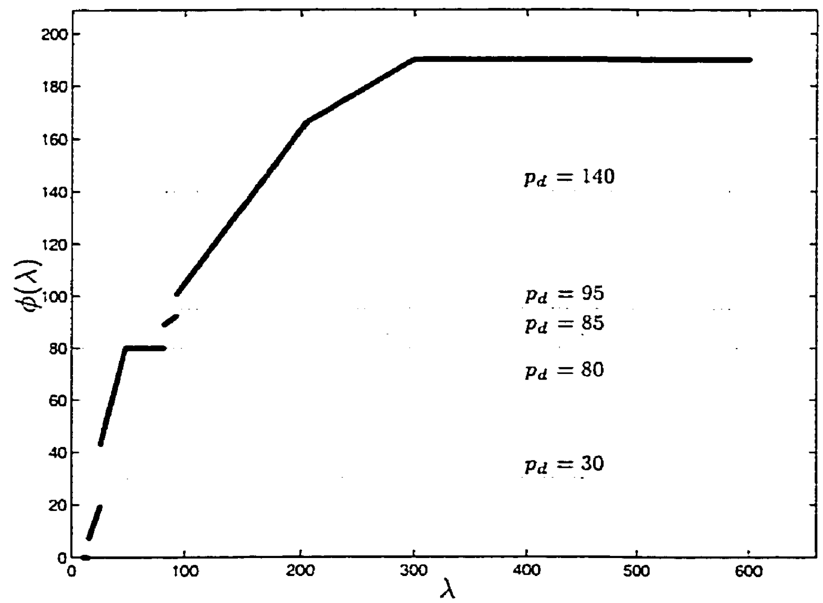


Figure 3.8: Total supply function

recovered that is smaller than the duality gap (0.180 < 0.4488); in the second case, the cost not recovered is equal to the duality gap; and, in the last case, the cost not recovered is zero. For $p_d = 30$, there are two optimal primal solutions. For $p_d = 80$ and 140 MW, there is a market equilibrium point; in the first case, there are multiple equilibrium prices, and in the second, it is unique. The inclusion of the quadratic term in the cost function makes the total supply function smoother, which results in more chances for equilibrium to exist at different demand levels.

Table 3.5: Solution to cost minimization auction

p_d	p_1^*	p_2^*	p_3^*	p_4^*	p_5^*	λ^*	f^*	ψ^*	dg
30	19	11	0	0	0	25.86	639.0000	638.5512	0.4488
	19	0	11	0	0				
80	20	30	30	0	0	[48.00, 81.83]	2340.0000	2340.0000	0.0000
85	20	30	30	5	0	81.83	2772.5000	2749.1641	23.3359
95	20	30	30	15	0	92.25	3622.5000	3612.4871	10.0129
140	20	30	30	35	25	160.00	9102.5000	9102.5000	0.0000

Table 3.6: Costs, revenues and profits. Pricing with optimal dual variable

p_d	i	p_i^*	c_i^*	ρ	$\rho \times p_i^*$	π_i	CNR
30	1	19.000	354.400	25.856	491.272	136.872	
	2	11.000	284.600	25.856	284.420	-0.180	-0.180
	1	19.000	354.400	25.856	491.272	136.872	
	3	11.000	284.600	25.856	284.420	-0.180	-0.180
80	1	20.000	380.000	48.000	960.000	580.000	
	2	30.000	980.000	48.000	1440.000	460.000	
	3	30.000	980.000	48.000	1440.000	460.000	0.000
85	1	20.000	380.000	81.832	1636.656	1256.656	
	2	30.000	980.000	81.832	2454.984	1474.984	
	3	30.000	980.000	81.832	2454.984	1474.984	
	4	5.000	432.500	81.832	409.164	-23.336	-23.336
95	1	20.000	380.000	92.249	1844.981	1464.981	
	2	30.000	980.000	92.249	2767.471	1787.471	
	3	30.000	980.000	92.249	2767.471	1787.471	
	4	15.000	1282.500	92.249	1383.735	101.235	0.000
140	1	20.000	380.000	160.000	3200.000	2820.000	
	2	30.000	980.000	160.000	4800.000	3820.000	
	3	30.000	980.000	160.000	4800.000	3820.000	
	4	35.000	3882.500	160.000	5600.000	1717.500	
	5	25.000	2880.000	160.000	4000.000	1120.000	0.000

3.3 Price Setting Alternatives

When an equilibrium exists, the Lagrange multipliers are market equilibrium prices and recover the costs of all the scheduled bids. In this situation, multiple optimal primal solutions are Pareto optimal, therefore, none of these solutions is preferred by the bidders. Therefore, under equilibrium, the Lagrange multipliers are market clearing prices, and multiple optimal solutions do not represent any conflict of interest.

When an equilibrium does not exist, the optimal dual variables may not recover part of the cost of bidders in the margin of the optimal primal solutions. Since multiple primal solutions may exist, the selection of one or other as the final schedule may bring conflict of interest if the pricing rule results in different profits for a participant when one or other solution is selected as the final schedule.

After the experience in the England and Wales Power Pool with the use of average pricing, there has not been enough research that tries to identify price setting alternatives for unit commitment like power pool auctions. In [8, 1996], it is recognized that the cost-minimization auction under average pricing does not always leads to lower prices. Based on this observation, the use of a price-minimization auction is proposed in [9, 1997]. In [49, 1999], the minimum uniform price increment that is necessary to recover the cost of all the suppliers, is added to the dual variables.

In this section these alternatives are reviewed and the effects they have on the prices and on the selection of the final schedule are discussed. Based on the observations made, a non-uniform price setting alternative based on dual variables is proposed.

3.3.1 Maximum Average Cost

In this alternative, the price is set up as the largest average cost among the bids scheduled in the solution of the cost minimization auction; that is,

$$\rho_{ave} = \max\{c_i^*/p_i^* \mid \forall\ u_i = 1\} \tag{3.25}$$

where, for short, $c_i^* = c_i(p_i^*, 1)$. This price ensures that all the bids recover their cost. This average pricing alternative, extended to account for all the periods in a unit commitment auction, has been used in the England and Wales Power Pool to set the price for power; this is the so-called *System Marginal Price* (SMP) [4, 1998].

Let us consider the results in Table 3.3 for the auction with linear and startup cost. The maximum average cost for each of the demand levels, and each of the multiple solutions, is presented in Table 3.7. In the same table, the price given by the smallest optimal dual variable is presented. The results in this table can illustrate the three mayor factors that

Table 3.7: Dual variable and maximum average price; test cases from Table 3.3

p_d (MW)	52	110		130		190	210	
$\rho = \min\{\lambda^*\}$ (\$/MW)	15.80	25.78	25.78	25.78	25.78	25.78	26.75	26.75
$\rho = \rho_{ave}$ (\$/MW)	35.00	28.50	28.50	28.50	27.33	25.78	26.75	26.75

have raised conflict on the use of unit commitment power pool auctions. The first factor, as recognized by J. Jacobs in [8], consists on the fact that multiple cost minimizing solutions lead to different maximum average prices. For instance, consider $p_d = 130$ MW, the average price of the second solution is lower than the average price of the first (27.33 < 28.5 \$/MW). The author notes that the price is not an equilibrium and considers that the Pool operator is "failing" to obtain prices that are the best for consumers. Based on this observation, related work [9] proposes the substitution of the cost-minimization model by a price-minimization model.

The second factor, based on the actual experience of the England and Wales Power Pool, deals with suppliers' strategic behavior. The final report that investigates the price spikes in the SMP [7], states that some suppliers were able to strategically choose their cost coefficients submitted on the bids, so that these bids became price setter at very high average prices. In the EWPP, the cost curve contains three linear segments plus a startup and no-load cost [7]. A simple linear segment and startup cost, as in our model, can describe the situation.

Consider Table 3.7; for the cases in equilibrium, the minimum Lagrange multiplier is equal to the maximum average price. However, for the cases in disequilibrium, the average price takes values above equilibrium prices that correspond to higher demand levels. For instance, consider $p_d = 130$ MW, the average price of any of the multiple solutions (28.50 and 27.33 \$/MW) is higher than the equilibrium price (26.75 \$/MW) when the demand is 210 MW. Even more, for $p_d = 52$ MW, the demand is just above a discontinuity of the supply (Figure 3.4) and the average price results in 35 \$/MW; that is, 30% larger than the equilibrium price when the demand is 4.03 times higher; i.e., 26.75 \$/MW for 210 MW. These price spikes do not necessarily need be a direct consequence of strategic behavior. In a competitive situation, where suppliers are assumed to submit their true cost functions, it suffices that the demand be located just above a discontinuity of the supply function to set the average price at a high value. If a supplier "learns" to locate one of the bids just below the demand, the price spikes can consistently occur; to answer if this situation can happen or be easily created, it is required to study the strategic behavior of all the participants incorporating demand predictions.

Table 3.8 presents the cost, revenues and profits for each of the demand levels and multiple solutions for the same test cases in Table 3.3. The last factor, is that multiple optimal or near-optimal solutions to the cost minimization problem represent different profits for bidders, which creates conflict of interest for the selection of the final schedule. This observation has been made first in [10]. For instance, consider $p_d = 130$ MW, in the first solution, bidder 3 makes \$ 35; however, in the second solution, bid 3 is not scheduled and makes \$ 0. Bid 3 looses if the pool operator chooses the second solution as the final schedule. The difference in profits among different solutions is worsened because average prices are different.

In [49], after the cost minimization model is solved, a minimum price increment that ensures cost recovery it is added to the dual variable. As can be noted from Table 3.3, this leads to the average prices.

For the cases in equilibrium, if multiple solutions exist (i.e., $p_d = 210$) they do not raise conflict of interest since the multiple solutions result in the same profits for the bidders involved.

3.3.2 Price Minimization Auction

The price minimization version for the auction with linear and startup cost in Section 3.1 can be formulated as follows:

$$\rho^*_{ave} = \min \quad \rho \tag{3.26a}$$

$$\text{s.t.} \quad \rho \geq (\alpha_i/p_i + \beta_i)u_i, \tag{3.26b}$$

$$p_d - \textstyle\sum_i p_i = 0, \tag{3.26c}$$

$$0 \leq p_i \leq \overline{p_i}u_i, \; u_i \in \{0,1\} \tag{3.26d}$$

The problem consists on finding a schedule that satisfies the demand at minimum average price. The problem can be transformed into the following equivalent form that does not

Table 3.8: Costs, revenues and profits. Pricing with maximum average cost $\rho = \rho_{ave}$

p_d	i	p_i^*	c_i^*	ρ	$\rho \times p_i^*$	π_i	$\sum_i \pi_i$	$\rho \times p_d$
52	1	50	530.0	35.00	1750.000	1220.000		
	2	2	70.0	35.00	70.000	0.000	1220.00	1820.00
110	1	50	530.0	28.50	1425.000	895.000		
	2	50	790.0	28.50	1425.000	635.000		
	4	10	285.0	28.50	285.000	0.000	1530.00	3135.00
	1	50	530.0	28.50	1425.000	895.000		
	2	50	790.0	28.50	1425.000	635.000		
	5	10	285.0	28.50	285.000	0.000	1530.00	3135.00
130	1	50	530.0	28.50	1425.000	895.000		
	2	50	790.0	28.50	1425.000	635.000		
	4	20	535.0	28.50	570.000	35.000		
	5	10	285.0	28.50	285.000	0.000	1565.00	3705.00
	1	50	530.0	27.33	1366.667	836.667		
	2	50	790.0	27.33	1366.667	576.667		
	3	30	820.0	27.33	820.000	0.000	1413.33	3553.33
190	1	50	530.000	25.78	1288.889	758.889		
	2	50	790.000	25.78	1288.889	498.889		
	3	90	2320.000	25.78	2320.000	0.000	1257.78	4897.78
210	1	50	530.000	26.75	1337.500	807.500		
	2	50	790.000	26.75	1337.500	547.500		
	3	90	2320.000	26.75	2407.500	87.500		
	4	20	535.000	26.75	535.000	0.000	1442.50	5617.50
	1	50	530.000	26.75	1337.500	807.500		
	2	50	790.000	26.75	1337.500	547.500		
	3	90	2320.000	26.75	2407.500	87.500		
	5	20	535.000	26.75	535.000	0.000	1442.50	5617.50

require the introduction of binary variables:

$$\rho_{ave}^* = \min \quad \rho \tag{3.27a}$$

$$\text{s.t.} \quad p_d - \textstyle\sum_i p_i = 0, \tag{3.27b}$$

$$\alpha_i p_i + \beta_i p_i^2 - \rho p_i^2 \leq 0, \tag{3.27c}$$

$$0 \leq p_i \leq \overline{p_i}. \tag{3.27d}$$

Besides the complexity that the solution to (3.27) represents, price minimization auctions have another drawback that has not been pointed previously in the literature. The non-convexities in constraint (3.27c) can cause the existence of multiple solutions, which further complicate the selection of the final schedule. Since the price for each of the solutions is the same, but the schedules are strongly different, the conflict of interest for the selection of the final schedule are emphasized. Table 3.9 summarizes the solution to the price minimization auction for the same data system using the demand levels in Table 3.3. Except for the first demand level, the price minimization model is able to obtain a price at the same level of the optimal dual variable (see Table 3.3). For all the cases, the schedules change considerably as compared to the cost minimization results; more expensive suppliers are loaded, resulting in a large total cost of the solution which considerably reduces the profits for suppliers. For the disequilibrium cases, there are several multiple optimal solutions with strongly different schedules. For instance, for $p_d = 130$ MW, all the solutions have the same price; however, the selection of the first or second leaves bidder 1 or 2 without any profits. The same situation happens for $p_d = 110$ MW. In fact, for this two demand levels there are more alternate solutions than the one presented on the table. For $p_d = 130$, $p^* = [10, 30, 90, 0, 0]^T$ is also a multiple solution.

The results in Table 3.9 have been obtained by repetitively changing the initial condition of an interior-point for non-convex non-linear programming [50]. In this case, simple inspection can also be used to obtain the several multiple solutions.

3.3.3 Dual Variables, Duality Gap and Cost Recovery

As observed in the test cases, the cost not recovered by dual variables under disequilibrium results smaller than the duality gap; the bids that do not recover the cost are in the margin of the solution, and are the bids that appear (or do not appear) in multiple solutions and set the high average prices. In this section, it is proven that the total *cost not recovered* (CNR) by the optimal dual variables is bounded above by the magnitude of the duality gap.

Let $\mathcal{B}_-$ be the set of bidders at any multiple optimal primal solution whose profits are negative when λ^* is used to set the price; and let $\mathcal{B}_+$ be the set of bidders that have positive profits. The optimal objective function and the demand can be written as

$$f^* = f_{\mathcal{B}_-} + f_{\mathcal{B}_+}$$

$$p_d = p_{\mathcal{B}_-} + p_{\mathcal{B}_+}$$

Table 3.9: Costs, revenues and profits. Pricing with price minimization auction $\rho = p^*_{ave}$

p_d	i	p^*_i	c^*_i	ρ	$\rho \times p^*_i$	π_i	$\sum_i \pi_i$	$\rho \times p_d$
52	1	5.1253	81.2528	15.85	81.2500	0.000		
	2	46.8747	743.1208	15.85	743.1208	0.000	0.000	824.20
110	1	20	230.00	25.78	515.56	285.56		
	3	90	2320.00	25.78	2320.0	0.000	285.56	2835.80
	2	20	340.00	25.78	515.56	175.56		
	3	90	2320.00	25.78	2320.00	0.00	175.56	2835.80
	1	10	130.00	25.78	257.78	127.78		
	2	10	190.00	25.78	257.78	67.78		
	3	90	2320.00	25.78	2320.00	0.00	195.56	2835.80
130	1	40	430.00	25.78	1031.11	601.11		
	3	90	2320.00	25.78	2320.00	0.00	601.11	3351.40
	2	40	640.00	25.78	1031.11	391.11		
	3	90	2320.00	25.78	2320.00	0.00	391.11	3351.40
	1	20	230.00	25.78	515.56	285.56		
	2	20	340.00	25.78	515.56	175.56		
	3	90	2320.00	25.78	2320.00	0.00	461.12	3351.40
190	1	50	530.000	25.78	1288.889	758.889		
	2	50	790.000	25.78	1288.889	498.889		
	3	90	2320.000	25.78	2320.000	0.000	1257.78	4897.78
210	1	50	530.000	26.75	1337.500	807.500		
	2	50	790.000	26.75	1337.500	547.500		
	3	90	2320.000	26.75	2407.500	87.500		
	4	20	535.000	26.75	535.000	0.000	1442.50	5617.50
	1	50	530.000	26.75	1337.500	807.500		
	2	50	790.000	26.75	1337.500	547.500		
	3	90	2320.000	26.75	2407.500	87.500		
	5	20	535.000	26.75	535.000	0.000	1442.50	5617.50

where $f_{\mathcal{B}_-} = \sum_{i\in\mathcal{B}_-} c_i(p^*_i)$ and $p_{\mathcal{B}_-} = \sum_{i\in\mathcal{B}_-} p^*_i$; and p^*_i represents the optimal output of bidder i; the same notation applies for $p_{\mathcal{B}_+}$. With these definitions, the optimal dual function value (2.9) can be written as

$$\psi^* = \lambda^{*T} p_d - \sum_i \psi_i(\lambda^*) = \lambda^{*T}(p_{\mathcal{B}_-} + p_{\mathcal{B}_+}) - \sum_{i\in\mathcal{B}_+} \psi_i(\lambda^*) - \sum_{i\in\mathcal{B}_-} \psi_i(\lambda^*)$$

The duality gap is given by

$$\begin{aligned} dg =& f^* - \psi^* \\ =& \left[\textstyle\sum_{i\in\mathcal{B}_+} \psi_i(\lambda^*) - (\lambda^{*T} p_{\mathcal{B}_+} - f_{\mathcal{B}_+})\right] + (f_{\mathcal{B}_-} - \lambda^{*T} p_{\mathcal{B}_-}) + \left[\textstyle\sum_{i\in\mathcal{B}_-} \psi_i(\lambda^*)\right] \end{aligned} \tag{3.28}$$

For any bid, we have

$$\psi_i(\lambda^*) \geq \lambda^{*T} p_i^* - c_i(p_i^*) \geq 0, \tag{3.29}$$

where $\psi_i(\lambda^*)$ is the maximum profit (bounded below by zero), as given by the solution to the profit maximization subproblems (2.2). Hence, the two terms in the squared brackets of (3.28) are positive and, therefore, the cost not recovered, $CNR = (f_{\mathcal{B}_-} - \lambda^{*T} p_{\mathcal{B}_-})$, is bounded above by the duality gap

$$CNR \leq dg \tag{3.30}$$

Since the magnitude of the duality gap tends to zero as the number of separable components in the dual function increases [38], it can be expected that the duality gap reduces as the number of participants in the auction increases. It has to be noticed that the derivation holds similar for a pair of feasible primal and dual solutions. Let $\hat{f}$ be the objective function value at any primal feasible solution and $\psi(\lambda)$ the dual objective function value at any λ. Following the same derivation, the complementarity gap ($cg = \hat{f} - \psi(\lambda)$) gives the bound for the cost not recovered; that is, $CNR \leq cg$. A particular proof of the latter relationship is presented by Madrigal and Quintana in [51].

3.3.4 Non-Uniform Pricing Based on Dual Variables

The two previous uniform pricing alternatives present clear drawbacks for pricing under disequilibrium; multiple solutions represent quite different profits. Price spikes can occur even in the absence of strategic behavior, and the price minimization auction considerable reduces prices for suppliers since costly bids tend to be used. The alternative described here, consist on paying their cost to these marginal bidders with negative profits $\mathcal{B}_-$, and paying the value of dual variable to the rest of the bidders. The cost not recovered is compensated by adding an equal price increment and decrement to suppliers and load (consumers), as follows:

$$\rho_c = \lambda^* + \Delta\lambda_c, \tag{3.31}$$

$$\rho_{si} = \lambda^* - \Delta\lambda_{si} \qquad \forall i \mid \pi_i > 0, \tag{3.32}$$

$$\rho_{si} = \lambda^* + \Delta\lambda_{si} \qquad \forall i \mid \pi_i < 0, \tag{3.33}$$

where the price increment and decrement are given by

$$\Delta\lambda_c = \kappa \times CNR \times \frac{1}{p_d}, \tag{3.34}$$

$$\Delta\lambda_{si} = (1-\kappa) \times CNR \times \frac{\pi_i}{\sum_{i|\pi_i>0} \pi_i} \qquad \forall i \mid \pi_i > 0, \tag{3.35}$$

$$\Delta\lambda_{si} = -\frac{\pi_i}{p_i} \qquad \forall i \mid \pi_i < 0. \tag{3.36}$$

where κ is a factor that determines the distribution of the cost not recovered among suppliers and load. Ideally, this constant is 0.5 so that suppliers and load generate equally the cost not recovered. In very trivial cases, such as an auction with only one supplier in disequilibrium, the total positive profits $\sum_{i|\pi_i>0} \pi_i$ may be smaller than the cost not recovered. Therefore, constant k is computed as follows:

$$\kappa = \min\left\{0.5, \frac{\sum_{i|\pi_i>0} \pi_i}{CNR}\right\} \tag{3.37}$$

Even for cases with a small number of bidders (i.e., five as in the examples) the total positive profit is much larger then the magnitude of the duality gap, which is enough to guarantee that $\kappa = 0.5$ from (3.37). The price increment, for suppliers that do not recover their cost, (3.36), is computed so that they exactly receive their cost. The price decrement for each supplier whose original profit is positive, (3.35), is amortized according to its profit. This guarantees that the decrement does not causes the profit of any supplier to go below zero (also a very improbable situation). Other amortizations, for instance, based on offered quantities, total costs or a priority selection could be used.

Due to the size of the duality gap, it may also be reasonable to generate the CNR by any other administrative means; for instance, using settlement systems [2].

Table 3.10 presents the results of applying the non-uniform pricing rule for each of the solutions and demand levels considered for the data set in Table 3.1. The following observations can be made: (i) The reduction in revenues for suppliers, and the increase in payments by the demand, adds up to the cost not recovered in each case; (ii) for the cases in equilibrium, no adjustment is made since the cost not recovered is zero; (iii) for the cases in disequilibrium, the profits of suppliers are the same for all the alternate solutions.

Table 3.10: Costs, revenues and profits. Non-uniform pricing based on dual variables

p_d	i	p_i^*	c_i^*	ρ_{si}	$\rho_{si} \times p_i^*$	π_i	$\sum_i \pi_i$	ρ_c	$\rho_c \times p_d$
52	1	50	530.0	15.416	770.800	240.800			
	2	2	70.0	35.000	70.000	0.000	240.800	16.170	840.8000
110	1	50	530.0	25.614	1280.677	750.677			
	2	50	790.0	25.670	1283.490	493.490			
	4	10	285.0	28.500	285.000	0.000	1244.1667	25.901	2849.1667
	1	50	530.0	25.614	1280.677	750.677			
	2	50	790.0	25.670	1283.490	493.490			
	5	10	285.0	28.500	285.000	0.000	1244.1667	25.901	2849.1667
130	1	50	530.0	25.496	1274.811	744.811			
	2	50	790.0	25.593	1279.634	489.634			
	4	20	535.0	26.750	535.000	0.000			
	5	10	285.0	28.500	285.000	0.000	1234.4444	25.957	3374.4444
	1	50	530.0	25.496	1274.811	744.811			
	2	50	790.0	25.593	1279.634	489.634			
	3	30	820.0	27.333	820.000	0.000	1234.4444	25.957	3374.4444
190	1	50	530.000	25.778	1288.889	758.889			
	2	50	790.000	25.778	1288.889	498.889			
	3	90	2320.000	25.778	2320.000	0.000	1257.78	25.778	4897.78
210	1	50	530.000	26.750	1337.500	807.500			
	2	50	790.000	26.750	1337.500	547.500			
	3	90	2320.000	26.750	2407.500	87.500			
	4	20	535.000	26.750	535.000	0.000	1442.50	26.750	5617.50
	1	50	530.000	26.750	1337.500	807.500			
	2	50	790.000	26.750	1337.500	547.500			
	3	90	2320.000	26.750	2407.500	87.500			
	5	20	535.000	26.750	535.000	0.000	1442.50	26.750	5617.50

The last observation brings an advantage of non-uniform pricing to deal with the conflict of interest that arises with the existence of multiple solutions. Since the marginal bidders are paid their cost, they make zero profit. The rest of the bids, that are likely to appear in all alternate solutions, receive the same price and, therefore, the conflict of interest that may arise from the selection of the final schedule is minimized. See for instance, in Table

3.10, the cases for $p_d = 110$ and $p_d = 130$ MW, in both solutions suppliers receive the same profit; the selection of any of these solutions does not bring a conflict of interest.

The non-uniform pricing alternative is also applied for the test case with quadratic and startup cost functions in Table 3.4, for the same demand levels as in Table 3.5. The result are summarized in Table 3.11. Since the supply functions tend to be smoother, the duality gap and cost not recovered tend to be smaller. The increments and decrements to load and suppliers prices are smaller.

Table 3.11: Costs, revenues and profits. Non-uniform pricing based on dual variables

p_d	i	p_i^*	c_i^*	ρ_s	$\rho_s \times p_i^*$	π_i	ρ_c	$\rho_c \times p_d$
30	1	19.000	354.400	25.852	491.182	136.782		
	2	11.000	284.600	25.873	284.600	0.000	25.851	775.7820
	1	19.000	354.400	25.852	491.182	136.782		
	3	11.000	284.600	25.873	284.600	0.000	25.851	775.7820
80	1	20.000	380.000	48.000	960.000	580.000		
	2	30.000	980.000	48.000	1440.000	460.000		
	3	30.000	980.000	48.000	1440.000	460.000	48.000	3840.0000
85	1	20.000	380.000	81.659	1633.171	1253.171		
	2	30.000	980.000	81.696	2450.893	1470.893		
	3	30.000	980.000	81.696	2450.893	1470.893		
	4	5.000	432.500	86.500	432.500	0.000	81.683	6967.457
95	1	20.000	380.000	92.249	1844.981	1464.981		
	2	30.000	980.000	92.249	2767.471	1787.471		
	3	30.000	980.000	92.249	2767.471	1787.471		
	4	15.000	1282.500	92.249	1383.735	101.235	92.249	8763.658
140	1	20.000	380.000	160.000	3200.000	2820.000		
	2	30.000	980.000	160.000	4800.000	3820.000		
	3	30.000	980.000	160.000	4800.000	3820.000		
	4	35.000	3882.500	160.000	5600.000	1717.500		
	5	25.000	2880.000	160.000	4000.000	1120.000	160.000	22400.000

3.3.5 Comparison of Alternatives

For the data in Table 3.1, the mean values of the total profit and load payment for each of the pricing alternatives are shown in Figure 3.9. The mean values are computed considering all the multiple solutions for each demand level in disequilibrium (p_d = 52, 110 and 130 MW). All the values are normalized to the profit and payment that result from average price setting. In each graph, the pricing used is denoted by ρ_{ave}, the maximum average cost alternative; ρ^*_{ave}, the price minimization auction; and ρ_i, the non-uniform pricing alternative.

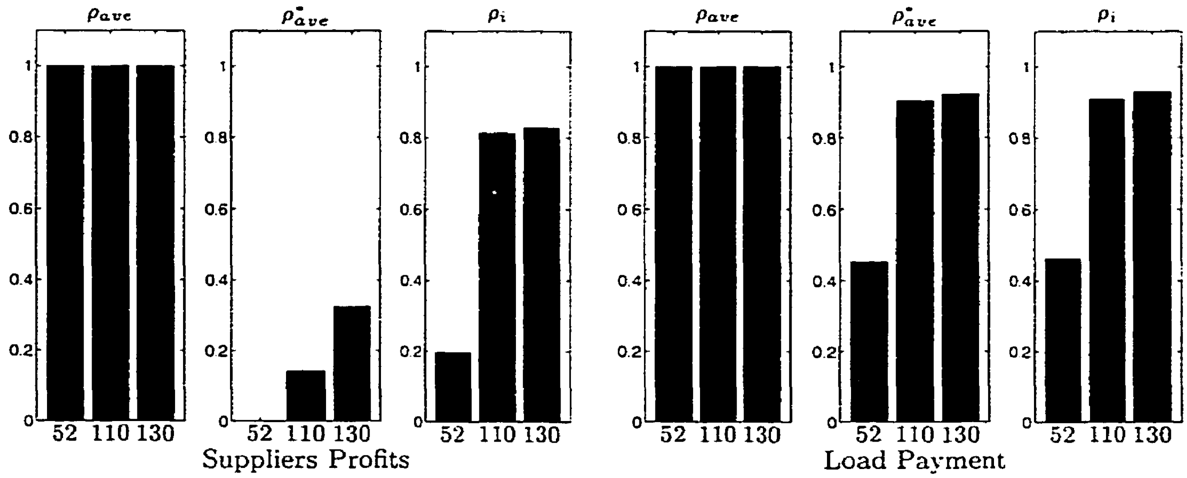


Figure 3.9: Normalized mean value of profits and payments under disequilibrium

Pricing with maximum average cost leads to higher prices which results in larger profits for suppliers and larger payments for consumers. The use of price minimization leads to low profit for suppliers since more costly bids are used. The non-uniform pricing alternative leads to profit and payment that behave more with the demand; it is a result that favors neither suppliers nor consumers.

The standard deviation of total profit and payment for each disequilibrium case are presented in Figure 3.10. Pricing with maximum average causes different solutions to have very different profit and payment. The price minimization model leads to different solutions that mean different profits, specially for suppliers. The non-uniform pricing alternative leads in all the cases to the same profit and payment.

As noted in the numerical examples, when an equilibrium does not exist, multiple solutions bring conflict of interest that are enlarged or diminished by the particular pricing

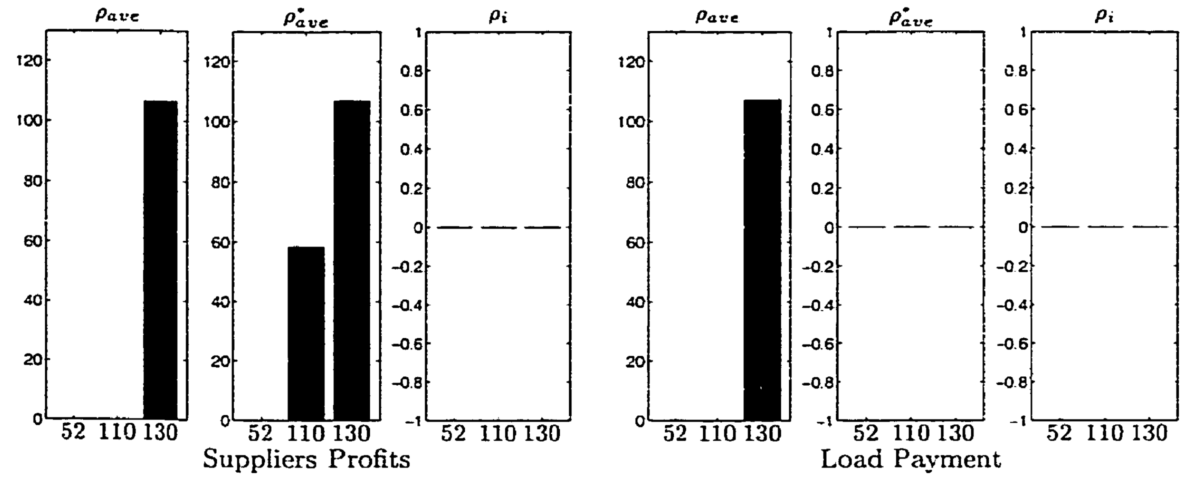


Figure 3.10: Standard deviation of total profits and payments under disequilibrium

rule utilized. Pricing with maximum average cost and the price minimization auction enhance these conflicts; multiple solutions have either different prices or different schedules, that result in considerable profit differences. Non-uniform pricing based on dual variables, diminishes the conflict of interest. Therefore, even though all the multiple solutions to the cost minimization model cannot in general be found, it is known in advance, that if the price is set using (3.31)-(3.33), the conflicts are diminished.

3.4 Larger, Randomly Generated Test Cases

Four data sets with larger number of bidders are constructed randomly in order to simulate more realistic situations. It is considered that bids come from coal, gas and nuclear stations. This is simulated by considering the startup-cost/variable-cost/capacity as: high/low/large, low/high/small and medium/medium/medium, respectively. For each type of generation, clusters of identical units are formed to induce multiple solutions.

Table 3.12 summarizes the characteristics and solution of four different test cases. as follows: (i) the number of bidders n, number of clusters with different bids n_c, the maximum power output and the demand; (ii) the best primal objective function, the optimal dual variables, the complementarity gap, cost not recovered and number of multiple primal solutions n_s; (iii) the load price, the mean value and variance of suppliers price, the total load payments and the total profits; (iv) the total number of unit commitment combinations, the number of nodes visited by the B&B algorithm n_v and the time required to find the solutions. All the values are in appropriate units, MW, \$, and \$/MW. The B&B algorithm is implemented in MATLAB, running on a 200Mhz personal computer.

Table 3.12: Summarized results for larger auctions with linear and startup cost functions

	A	B	C	D
n	75	105	105	75
n_c	75	105	10	15
$\sum_i \overline{p}_i$	8108.00	11408.00	11297.00	8306.00
p_d	6292.00	4563.20	8473.00	4020.00
$\overline{f}$	470735.00	265282.20	638411.00	237012.00
λ^*	[99.33,101.21]	76.34	[94.14,103.22]	72.29
cg	0.00	63.34	0.00	46.65
CNR	0.00	63.34	0.00	46.65
n_s	1	1	1	4
ρ_c	99.3333	76.3478	94.1392	72.2786
$\overline{\rho}_s$	99.3333	76.3290	94.1392	72.2956
$\sigma(\rho_s)$	0.0000	0.2585	0.0000	0.4875
$\rho_c \times p_d$	625005.3333	348390.5068	797641.7848	290628.3061
$\sum_i \pi$	154270.3333	83108.3068	159230.7848	53616.3061
2^n	3.7779e+22	4.0565e+31	4.0565e+31	3.7779e+22
n_v	151	111	211	193
Time (sec)	9.6632	10.6080	16.6058	12.0530

For cases A and C, a competitive market equilibrium point exists. For cases B and D, an equilibrium does not exist; for B, there is only one cost-minimizing solution; however, for C there are four multiple solutions. For the last case, since there are only 15 clusters of different units it can be expected that multiple solutions arise. It is clearly seen, for the last case in Figure 3.11, that demand does not intersect the supply function. The cost not recovered is very small for larger cases; the non-uniform price that suppliers receive is very similar for all of them, as shown by the standard deviation $\sigma(\rho_s)$. The number of nodes the B&B algorithm visits is very small in all the cases as compared to all possible combinations. This is due to the good branching order and tight bounds available for the simplified models. The time required to find the solution is only in the order of seconds; a complete enumeration would require in the order of 10^{15} years (based on an estimation that uses the times in Table 3.12).

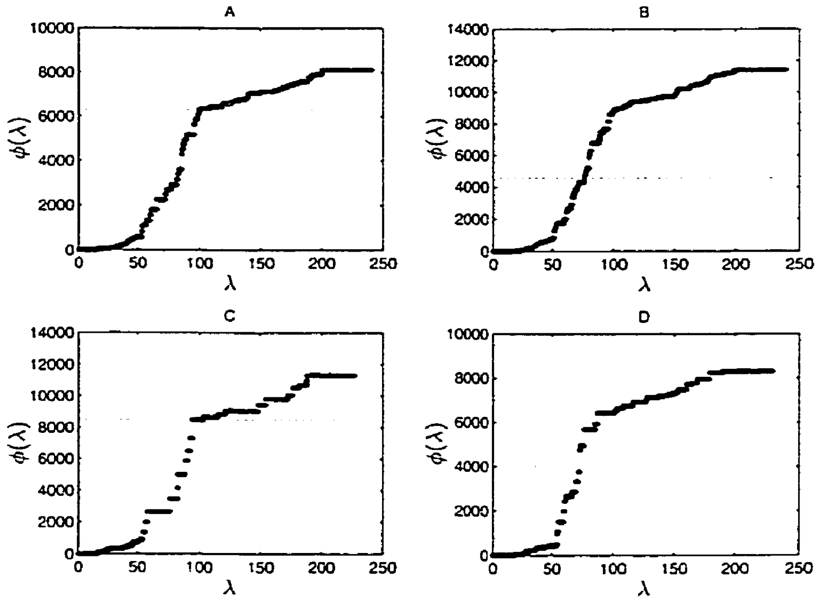


Figure 3.11: Total supply functions randomly generated test cases

Table 3.13 shows the bids that are involved in the multiple solutions; they are bids 39, 40, 41, 42 which have the same characteristics. There is only one bid that does not recover the cost; this is bid number 28 and appears in all the solutions. Since the bids that appear in all the multiple solutions are marginal, they make zero profit in all the solutions when the price is setup using non-uniform pricing. However, if the price would have been set up by the maximum average cost, it is setup by bid 28 to 75.4 \$/MW which causes the profit and load payments to increase. The other effect is that bidders 39, 40, 41 and 42 would receive different profits in each solution.

Table 3.13: Bids in the margin. Case D

i	α_i	β_i	$\overline{p}_i$	p_i^*	$c_i(p_i^*)/p_i^*$	λ^*	$\lambda^* p_i^*$	π_i
(1,2,3,4) 28	36.0	73.0	207	15.0	75.400	72.290	1084.347	-46.653
⋮								
(1) 39	71.0	72.0	245	245.0	72.290	72.290	17711.000	0.000
(2) 40	71.0	72.0	245	245.0	72.290	72.290	17711.000	0.000
(3) 41	71.0	72.0	245	245.0	72.290	72.290	17711.000	0.000
(4) 42	71.0	72.0	245	245.0	72.290	72.290	17711.000	0.000

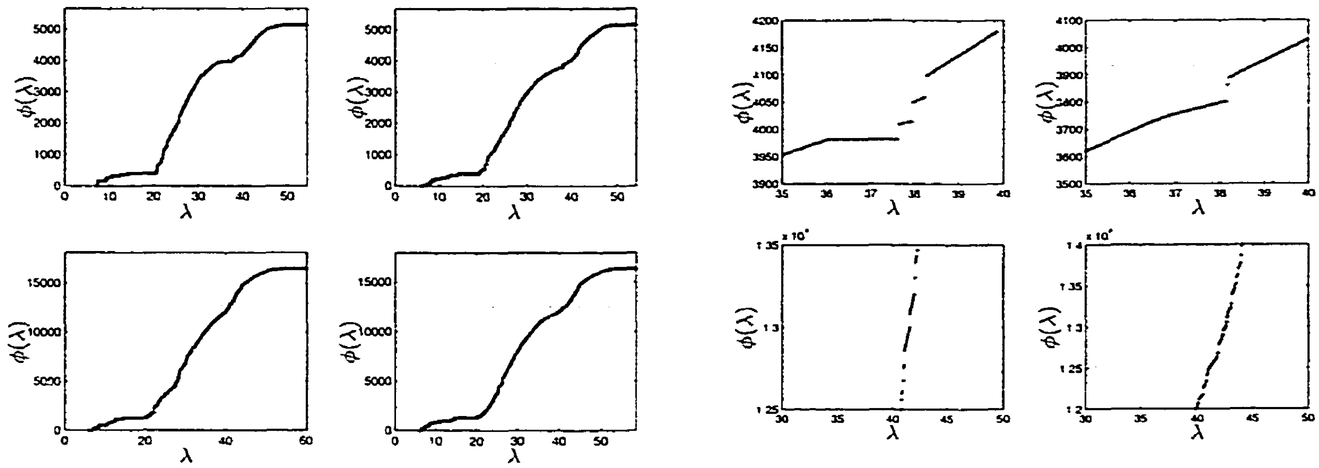
Table 3.14 presents four larger cases for auction with bids that contain quadratic and startup cost functions. For cases B and C, there is a market equilibrium point. For cases A and B, there is no market equilibrium point. in the latter two cases, there is a cost not recovered, equal to the duality gap in one case, and smaller in the other. The inclusion of the quadratic cost term smoothes out the supply functions, which tends to reduce the duality gap; see Figure 3.12. In the same figure, the zoomed plots show that in cases A and C the demand does not intersects the supply.

Table 3.14: Summarized results for larger auctions with quadratic and startup cost functions

	A	B	C	D
n	48	48	150	150
n_c	8	12	25	30
$\sum_i \overline{p}_i$	5142	5148.00	16441.00	16304.00
p_d	3992.60	3933.00	12728.00	12517.00
$\overline{f}$	99591.1900	101411.4000	377920.8133	357144.3735
λ^*	37.6160	38.7624	40.9839	41.1910
cg	0.5390	0.0000	0.2863	0.0000
CNR	0.5390	0.0000	0.0073	0.0000
n_s	2	1	2	1
ρ_c	37.6161	38.7624	40.9840	41.1910
$\overline{\rho}_s$	37.6159	38.7624	40.9840	41.1910
$\sigma(\rho_s)$	0.0089	0.0000	0.0000	0.0000
$\rho_c \times p_d$	150186.0520	150357.5661	521644.2022	515587.9861
$\sum_i \pi$	50594.8589	48946.1695	143723.3889	158443.6125
2^n	2.8147e+14	2.8147e+14	1.4272e+45	1.4272e+45
n_v	335	75	479	299
Time (sec)	163.7400	41.9256	779.5643	577.8281

The bids that are in the margin for each of the alternate solutions in cases A and C are shown in Tables 3.15 and 3.16, respectively.

For both cases, pricing with maximum average cost would increase the price to 37.667 and 40.984 \$/MW, respectively in each case. As can be noted in the same tables, the bids that appear in both alternate solutions correspond to the bids that do not recover their cost. The selection of either of the solutions does not bring conflict of interest.



(a) Original (b) Zoom

Figure 3.12: Total supply functions randomly generated test cases

Table 3.15: Bids on the margin. Case A

i	α_i	β_i	γ_i	$\overline{p}_i$	p_i^*	$c_i(p_i^*)/p_i^*$	ρ	ρp_i^*	π_i
(1) 39	11.0	36.0	0.059	81.000	10.600	37.667	37.6160	398.730	-0.539
(2) 40	11.0	36.0	0.059	81.000	10.600	37.667	37.6160	398.730	-0.539

Table 3.16: Bids on the margin. Case C

i	α_i	β_i	γ_i	$\overline{p}_i$	p_i^*	$c_i(p_i^*)/p_i^*$	ρ	ρp_i^*	π_i
(1) 109	83.0	36.0	0.075	57.0	33.620	40.984	40.9839	1377.874	-0.0073
(2) 110	83.0	36.0	0.075	57.0	33.620	40.984	40.9839	1377.874	-0.0073

3.5 Summary

In this chapter, two simplified discrete models for power pool auctions are studied. The models consider linear and quadratic cost functions with constant startup cost. The introduction of binary variables, and the methods developed to solve the primal and dual problems, make the models very suitable to understand the effects of disequilibrium and price setting alternatives in centralized power pool auctions. As far as we are aware, these issues have not been thoroughly investigated in the existing literature, specially in the framework of electricity power pool auctions. The following individual remarks are made.

- For both models, their dual problem has a closed form solution. A Branch-and-Bound enumerative approach that can find multiple near-optimal or optimal solutions is designed for the solution of the primal problem.
- When an equilibrium exists, there can be multiple Lagrange multipliers that represent market equilibrium prices. In equilibrium, multiple primal solutions do not represent conflict of interest.
- When an equilibrium does not exist, it has been shown that the optimal dual variables do not recover an amount of cost that is bounded above by the duality gap.
- Also, when an equilibrium does not exist, multiple solutions bring conflict of interest that are enlarged or diminished by the particular pricing rule adopted. Pricing with maximum average cost and the price minimization auction, enhance these conflicts; multiple solutions have either different prices or different schedules, that result in considerable profit differences.
- Pricing with maximum average cost can create unreasonable high prices even in the absence of strategic behavior. Price minimization uses more expensive resources that considerably reduce the profit for suppliers.
- A non-uniform price setting alternative based on dual variables is proposed. The alternative produces prices that resemble more the behavior of the demand and diminishes the conflict of interest when multiple solutions arise. Since the cost not recovered is bounded above by the duality gap, the deviations in price are negligible; load payment and suppliers profit do not increase as compared to the other alternatives.

Chapter 4

An Interior-Point/Cutting-Plane Method for Unit Commitment

A *unit commitment* (UC) problem consists on determining the power generators that need be committed and their production levels to supply the forecasted short-term (24 or, at most, 168 hours) demand and spinning reserve requirements, at a minimum cost. Units operation is subject to several constraints. UC models are very large non-linear mixed-integer (therefore, non-convex) programming problems. The formulation of a unit commitment problem is as varied as the number of approaches that have been used to solve it. Enumerative approaches, dynamic programming, genetic programming, neural networks, and simulated annealing are among the techniques that have been used to solve this problem; comprehensive reviews on UC literature can be found in [52, 1994] [53, 1998].

The most accepted and successful approach to solve UC problem is *Lagrangian relaxation* (LR); the LR technique was first introduced in [54, 1977] and became very well established with subsequent developments [55, 1983] [56, 1988]. The key idea in LR-based approaches is to solve the dual problem instead of the primal problem. The dual problem has a separable structure, i.e., in a per thermal-unit basis, which permits its easy evaluation and, at the same time, provides a primal (not necessarily feasible) solution. The dual function is concave but not differentiable. Therefore, non-differentiable optimization techniques are required to solve the dual problem. Pioneering work on LR-based UC solution approaches has used *sub-gradient* (SG) methods as the dual maximization engine [54, 56]. Despite their bad convergence characteristics, they still are being used due to its easy implementation and low per-iteration computer effort. Several *Cutting-plane* (CP) variants to solve non-differentiable optimization have been employed to solve the dual to unit commitment or other power scheduling problems. For instance, in [57, 1996] a *penalty-bundle* (PB) method is used to solve the UC problem. In [58, 1997], a reduced complexity bundle method is introduced to solve the dual of a power scheduling problem. In [59, 1999], a CP with dynamically adjusted constraints is used to solve a hydro-thermal coordination problem. All these cutting-plane variants still have the disadvantage that parameters need be carefully tuned; these parameters define a *stabilization scheme* that prevents unboundedness in the maximization of the dual function and help improve convergence [60, 1994].

In this chapter, we formulate a UC model and propose the use of an *interior-point/cutting-plane* (IP/CP) method to solve the dual problem. IP/CP methods have been used to successfully solve non-differentiable problems in other engineering applications, such as lot sizing [61, 1997] and multi-commodity flow problems [62, 1994]. An IP/CP method has two advantages over previous approaches: First, it has better convergence and robustness characteristics; second, it does not suffer from the parameter tuning drawback of previous approaches. These two characteristics make the IP/CP an attractive alternative when the UC model is used to execute power pool auctions.

Section 4.1 formulates a UC model. Section 4.2 presents the LR algorithm. In Section 4.3, two existing techniques to solve the dual problem, and the IP/CP method are presented. In Section 4.4, a primal-dual interior-point method is proposed to solve the potential problem that arises in the IP/CP method. Sections 4.5 and 4.6 deal with the solution of the profit maximization subproblems and the primal feasibility search phase. The last section briefly describes some implementation issues. Numerical results on the IP/CP testing and the use of the UC as a power pool auction mechanism are presented in Chapter 5.

4.1 The Unit Commitment Problem

A unit commitment problem consists on determining the power generators that need be committed and their production levels to supply the forecasted short-term demand, at minimum cost. In a classical unit commitment model, not only the demand needs be satisfied, (2.1b), but also a power reserve constraint has to be observed; i.e., to the model in (2.1) the reserve constraint is added, that is,

$$
\begin{aligned}
f^* = \min \quad & \textstyle\sum_i c_i(p_i), && && (4.1a)\\
\text{s.t} \quad & p_d^t - \textstyle\sum_i p_i^t = 0, && \forall t && (4.1b)\\
& r_d^t - \textstyle\sum_i r_i^t \le 0, && \forall t && (4.1c)\\
& p_i \in \mathcal{P}_i, && \forall i && (4.1d)
\end{aligned}
$$

where r_d^t is the power reserve requirement for period t, and r_i^t is the reserve contribution of supplier i at period t. Both the cost function $c_i(p_i)$ and the set of operative limits $\mathcal{P}_i$ depend on inter-temporal effects that make problem (4.1) non-separable in time. The UC belongs to the class of NP-hard problems, as proven in [63]. The cost function in (4.1a) contains fuel or variable costs and startup-costs,

$$c_i^t = c_{vi}^t + c_{si}^t \tag{4.2}$$

Each of the cost components is given by

$$c_{vi}^t = u_i^t \alpha_{0i} + \beta_i p_i^t + \gamma_i {p_i^t}^2 \tag{4.3}$$

$$c_{si}^t = u_i^t (1 - u_i^{t-1})[\alpha_{1i} + \alpha_{2i}(1 - e^{-x_{di}^{t-1}/\tau_i})] \tag{4.4}$$

where α_{0i} (\$) is a *no-load* cost, β_i and γ_i are the linear and quadratic cost components in \$/MW and \$/MW^2, respectively. The startup cost function contains a constant α_{1i} and a variable term α_{2i} (\$); the variable term depends on the number of hours the unit has been de-committed (off) up to sub-period $t-1$ (x_{di}^{t-1}), and on the cooling constant τ_i. The commitment variables are again represented by u_i^t, which take the value of one if unit i is committed at time t, and zero otherwise.

The reserve contribution of each thermal plant in (4.1c) is given by $r_i^t = u_i^t(\overline{p}_i - p_i^t)$, where $\overline{p}_i$ is the maximum allowable power output.

The set of operative limits $\mathcal{P}_i$ in (4.1c) can contain a variety of restrictions [52, 53]; among them: (i) minimum and maximum power output; (ii) minimum up and down times; (iii) ramp constraints; and (iv) crew and must-run constraints. The first three groups of constraints are written as follows:

$$u_i^t \underline{p}_i \le p_i^t \le u_i^t \overline{p}_i, \qquad \forall i,\ \forall t \tag{4.5}$$

$$u_i^t \in \{0, 1\} \qquad \forall i,\ \forall t \tag{4.6}$$

where $\underline{p}_i$ and $\overline{p}_i$ are the minimum and maximum allowable power outputs, respectively. The minimum *up*- and minimum *down*-time constraints are written as

$$u_i^{t-1}(1-u_i^t)(x_{ci}^{t-1}-\overline{t}_i) \geq 0 \qquad \forall i,\ \forall t \tag{4.7}$$

$$u_i^t(1-u_i^{t-1})(x_{di}^{t-1}-\underline{t}_i) \geq 0 \qquad \forall i,\ \forall t \tag{4.8}$$

where x_{ci}^{t-1} and x_{di}^{t-1} are the number of periods unit i has been committed and decommitted, respectively. The minimum number of periods a unit can be committed or decommitted is represented by $\overline{t}_i$ and $\underline{t}_i$, respectively.

Analytic expressions for x_{di}^t and x_{ci}^t are, for instance, $x_{ci}^t = x_{ci}^{t-1}u_i^t + u_i^t$ and $x_{di}^t = x_{di}^{t-1}(1-u_i^t)+(1-u_i^t)$. Initial time conditions are always specified and denoted by x_{ci}^0 and x_{di}^0; one of them has to be zero. Ramp constraints can be written as

$$u_i^{t-1}p_i^{t-1} - u_i^t p_i^t \leq \underline{\Delta}p_i \qquad \forall i,\ \forall t \tag{4.9}$$

$$u_i^t p_i^t - u_i^{t-1}p_i^{t-1} \leq \overline{\Delta}p_i \qquad \forall i,\ \forall t \tag{4.10}$$

where $\underline{\Delta}p_i$ and $\overline{\Delta}p_i$ are the maximum ramping-down and ramping-up constraints.

4.2 Dual-Primal Solution by Lagrangian Relaxation

The Lagrangian relaxation technique, as applied to solve UC problems, is a two phase optimization approach. In the first phase, the (Lagrangian) dual problem to the UC model (4.1) is solved until a stopping criterion is satisfied; in the second phase, a primal feasible solution is constructed from the dual solution.

The dual objective function is given as in (2.9) but additionally including the reserve constraints (4.1), as follows:

$$\psi(\lambda) = \textstyle\sum_t \left(\lambda_p^t p_d^t + \lambda_r^t r_d^t\right) - \sum_i \psi_i(\lambda) \tag{4.11}$$

where

$$\psi_i(\lambda) = \max_{p_i \in \mathcal{P}_i} \textstyle\sum_t (\lambda_p^t p_i^t + \lambda_r^t r_i^t - c_{vi}^t - c_{si}^t) \tag{4.12}$$

and λ is the vector of dual variables related to the power and reserve constraints; that is $\lambda = [\lambda_p, \lambda_r]^T = [\lambda_p^1, \cdots, \lambda_p^m, \lambda_r^1, \cdots, \lambda_r^m]^T$. The dual UC problem is given by

$$\psi^* = \max_{\lambda \geq 0} \quad \psi(\lambda) \tag{4.13}$$

The dual variables of the power-balance equality constraints are not necessarily constrained to be positive; however, since the cost always increases as real power generation increases, they always take positive values. The evident separable nature of the dual function $\psi(\boldsymbol{\lambda})$ is what makes the LR method a successful technique to solve the UC problem in a two-stage approach: it is summarized in the following algorithm.

Algorithm 4.1 The Lagrangian Relaxation Algorithm

---

1. **Initialization.** Obtain an initial dual vector $\boldsymbol{\lambda}^0$, and set $k = 0$.
2. **Phase 1.** Solution to the dual problem.
   (a) **Evaluation of the dual function.** From (4.11), evaluate the dual objective function $\psi(\boldsymbol{\lambda}^k)$ by solving the n individual profit-maximization subproblems, (4.12). Let $\boldsymbol{p}(\boldsymbol{\lambda}^k)$ be the optimal primal variables (u_i^t and p_i^t) to the dual problem.
   (b) **Convergence test.** If a convergence criterion is satisfied, go to Phase 2; otherwise, continue.
   (c) **Improved dual solution.** Using a non-differentiable optimization technique, find an improved dual solution vector, $\boldsymbol{\lambda}^{k+1}$. Set $k = k + 1$ and go to Step (a).
3. **Phase 2.** Feasibility search. Map $\boldsymbol{p}(\boldsymbol{\lambda}^k)$ to a primal feasible solution $\boldsymbol{p}(\boldsymbol{\lambda}^k) \rightarrow \hat{\boldsymbol{p}}$:
   (a) Find a reserve feasible solution
   (b) Dispatch generation to exactly match the power demand

There are several alternatives for the selection of initialization and stopping criteria. The selection of an alternative is linked to the method used to solve the dual problem. The technique used to solve the profit maximization subproblems (4.12) depends on the type and number of restrictions that are included in the set $\mathcal{P}_i$. The procedures (usually heuristic) to find a primal feasible solution also vary depending on the modeling detail in $\mathcal{P}_i$. The solution to the dual problem is the most time-consuming part in the LR algorithm; there is a number of non-differentiable optimization techniques that have been used to solve the dual problem.

4.3 Solution to the Dual Problem

The dual function in (4.11) is concave and non-differentiable; optimization techniques for non-differentiable optimization to solve the maximization problem (4.13) depend on the computation of sub-gradients to the dual function. As proven in [38], a sub-gradient to the concave function (4.11) $\xi \in \mathbb{R}^{2m}$ is given by

$$\xi = \left[p_d^1 - \textstyle\sum_i p_i^1, \ldots, p_d^m - \sum_i p_i^m, r_d^1 - \sum_i r_i^1, \ldots, r_d^m - \sum_i r_i^m\right]^T \tag{4.14}$$

This sub-gradient represents a *miss-match* vector of the power demand and reserve constraints, (4.1b) and (4.1c), respectively .

4.3.1 Sub-Gradient Method

A *sub-gradient* (SG) method is an extension of gradient methods for smooth optimization; it uses a sub-gradient as the search direction vector. In the widely known Polyak's sub-gradient method [64], a sub-gradient is used to obtain a normalized search direction; and the combination of two parameters, κ_1 and κ_2, define the *step length*. The updated dual vector is

$$\lambda^{k+1} = \lambda^k + \max[0, \frac{1}{\kappa_1 + k \times \kappa_2} \frac{\xi^k}{\|\xi^k\|}], \tag{4.15}$$

where ξ^k stands for the sub-gradient at λ^k, as given by (4.14). Despite its poor and final oscillatory convergence characteristics and the fact that the tuning of parameters κ_1 and κ_2 is a non trivial task [65], sub-gradient methods are still extensively used to solve different power-system scheduling problems due, mainly, to its easy implementation and low per-iteration computational effort [52, 56, 58, 63, 66–68].

4.3.2 Penalty-Bundle Method

The *penalty-bundle* (PB) method is one of several variants of cutting-plane methods [60]; both methods are briefly described next. A *bundle* is a collection of: (i) dual vectors $\{\lambda^1, \ldots, \lambda^k\}$; (ii) their corresponding dual objective function values $\{\psi(\lambda^1), \ldots, \psi(\lambda^k)\}$; and (iii) the sub-gradients $\{\xi^1, \ldots, \xi^k\}$. A *cutting-plane* approximation, $\hat{\psi}_k$, of the dual function ψ that is associated to this bundle is

$$\hat{\psi}_k(\lambda) = \min_{j=1,\ldots,k} \{\psi(\lambda^j) + (\lambda - \lambda^j)^T \xi^j\} \tag{4.16}$$

The approximation in (4.16) overestimates the dual function, that is, $\hat{\psi}_k(\boldsymbol{\lambda}) \geq \psi(\boldsymbol{\lambda})$. The maximization of the original non-differentiable dual function (4.11) is replaced by the maximization of the cutting-plane approximation (4.16). The maximization of (4.16) is equivalent to solving the linear program

$$\begin{aligned} \max \quad & z \\ \text{s.t.} \quad & z \leq \psi(\boldsymbol{\lambda}^j) + (\boldsymbol{\lambda} - \boldsymbol{\lambda}^j)^T \boldsymbol{\xi}^j, \quad j = 1, \cdots, k \\ & \boldsymbol{\lambda} \geq \mathbf{0} \end{aligned} \tag{4.17}$$

In the *pure* cutting-plane method, the solution to (4.17), $\boldsymbol{\lambda}^{k+1}$, is used as an improved dual solution to maximize the original dual function. This new vector is then used to update the cutting-plane approximation by adding a *new cut*, $z \leq \psi(\boldsymbol{\lambda}^{k+1}) + (\boldsymbol{\lambda} - \boldsymbol{\lambda}^{k+1})^T \boldsymbol{\xi}^{k+1}$, in (4.17). The updated approximation is again maximized and a new improved dual vector is obtained; this process is repeated until a stopping criterion is satisfied. As iterations proceed, the size of the linear problem (4.17) increases.

This *pure* cutting-plane procedure has severe drawbacks; for instance, at the first iteration, (4.17) is unbounded; and, in general, at iteration k, the solution to (4.17), may be far away in the unbounded optimization region. To avoid this problem, *stabilization schemes* that prevent the search to go far from the actual approximation can be used [60]. The most simple stabilization scheme, Kelley's cutting plane method [60], is to add the constraint $\boldsymbol{\lambda} \leq \overline{\boldsymbol{\lambda}}$ in (4.17) so that the dual search space is bounded; $\overline{\boldsymbol{\lambda}}$ is an upper bound of the dual variables, which in most engineering applications is at hand. One of the most used stabilization techniques is the penalty function method. In this technique, a penalty function is added to the objective function z in (4.17); then an improved point is obtained from the solution to the quadratic problem

$$\begin{aligned} \max \quad & z - \tfrac{1}{2}\kappa_3 \|\boldsymbol{\lambda} - \boldsymbol{\lambda}^k\|^2 \\ \text{s.t.} \quad & z \leq \psi(\boldsymbol{\lambda}^j) + (\boldsymbol{\lambda} - \boldsymbol{\lambda}^j)^T \boldsymbol{\xi}^j, \quad j = 1, \cdots, k \\ & \boldsymbol{\lambda} \geq \mathbf{0} \end{aligned} \tag{4.18}$$

The introduction of a penalty term causes the solution to (4.18) to be in a region *close* to the previous solution point $\boldsymbol{\lambda}^k$; and hence, it has the effect of bounding the search space. The penalty parameter κ_3 controls the *search region*, which can be viewed as a *sphere* with center $\boldsymbol{\lambda}^k$, and a *radius* controlled by κ_3. Although the theoretical convergence properties of the PB and SG method are similar [69], careful tuning of the penalty parameter can lead to considerable improvements in LR-based schemes to solve unit commitment [57, 1996] and hydro-thermal scheduling problems [59, 1999]. This method is usually addressed as the penalty-bundle method.

Heuristic rules can be designed to set the parameter κ_1, κ_2 and κ_3 for some specific problems [60,69,70]. The choice of these parameters strongly affects the convergence characteristics of both methods. In most power-systems applications, these parameters are set up using trial-and-error runs or ad-hoc techniques.

4.3.3 An Interior-Point/Cutting-Plane Method

Interior-Point/Cutting-Plane Method (IP/CP) methods for non-differentiable optimization have been first introduced in [71, 1992]. In the recent years, IP/CP methods have been very successful to solve engineering applications such as multi-commodity flow problems [62], lot sizing problems [72, 1994], stochastic programming [73, 1997] and also to compute market equilibria of international trade permits [74, 1997]. The theoretical convergence properties of an IP/CP method is sightly superior to other cutting plane methods [74, 1997]; however, in practice, quoting from the survey [69, 1999], "the method is not always the fastest, but is constantly good and is by far the most stable". IP/CP methods have never been explored before to solve power-system scheduling problems.

IP/CP methods strongly differ to other cutting plane methods; the latter ones maximize a cutting plane approximation over a stabilization region to obtain an improved dual solution vector. IP/CP methods take the analytic center of a *localization set* as the new improved dual solution. The localization set is a convex closed region denoted by Ω^k, and defined as

$$\begin{aligned} \Omega^k = \{ \ (z, \boldsymbol{\lambda}) \mid \ & z \le \psi(\boldsymbol{\lambda}^j) + (\boldsymbol{\lambda} - \boldsymbol{\lambda}^j)^T \xi^j, \quad j = 1, \cdots, k \\ & z \ge \underline{z}, \\ & \mathbf{0} \le \boldsymbol{\lambda} \le \overline{\boldsymbol{\lambda}} \ \} \end{aligned} \tag{4.19}$$

The boundaries of the localization set are given by: (i) The cutting-plane approximation of the UC dual function (4.16); (ii) a lower bound $\underline{z}$ to the dual objective function value; (iii) the known dual variables lower bounds $\boldsymbol{\lambda} \ge \mathbf{0}$; and (iv) a *box constraint* $\boldsymbol{\lambda} \le \overline{\boldsymbol{\lambda}}$. The constraints (iii) and (iv) limit the localization set from the *left* and *right*, respectively; and the constraints (i) and (ii) limit the localization set from *above* and *below*, respectively. As pointed out in [72], the selection of the box constraint $\overline{\boldsymbol{\lambda}}$ has a limited influence on the convergence characteristics of the IP/CP method; and, in practice, any *large enough* number based on knowledge of the problem can be chosen. For the UC problem, the same convergence characteristics have been observed; the selection of a very wide range of values $\overline{\boldsymbol{\lambda}}$

does not affect the convergence characteristics of the IP/CP methods. For the UC problem, the box constraint can be set up based on units' cost coefficients. A lower bound of the dual objective function is readily available from the evaluation of the dual function at previous iterations, i.e., $\overline{z} = \max\{\psi(\lambda^1), \ldots, \psi(\lambda^k)\}$.

The localization set in (4.19) can be rewritten as

$$\Omega^k = \{y \mid s = b - \mathbf{A}^T y,\ s > 0\} \tag{4.20}$$

where

$$\mathbf{A}^T = \begin{pmatrix} 1 & -\xi^{1^T} \\ \vdots & \vdots \\ 1 & -\xi^{k^T} \\ -1 & 0 \\ 0 & \mathbf{I} \\ 0 & -\mathbf{I} \end{pmatrix}, \quad b = \begin{pmatrix} \psi(\lambda^1) - \lambda^{1^T}\xi^1 \\ \vdots \\ \psi(\lambda^k) - \lambda^{k^T}\xi^k \\ \underline{z} \\ \overline{\lambda} \\ \mathbf{0} \end{pmatrix}$$

and $y = [z, \lambda^T]$. In the same equation, $\mathbf{A}^T \in \mathbb{R}^{\overline{m} \times \overline{n}}$; $s,\ b \in \mathbb{R}^{\overline{m}}$, and $\mathbf{I} \in \mathbb{R}^{2\overline{m} \times 2\overline{m}}$ is an identity matrix; the dimension are given by $\overline{m} = 4m + k + 1$ and $\overline{n} = 2m + 1$. The localization set Ω^k has an iteration-increasing size. The *analytic center* of Ω^k, denoted by y^{k+1}, is defined as the unique point that solves the potential problem

$$\begin{aligned} \max \quad & \textstyle\sum_{i=1}^{\overline{m}} ln(s_i) \\ \text{s.t.} \quad & s = b - \mathbf{A}^T y \\ & s > 0 \end{aligned} \tag{4.21}$$

The term $\sum_{j=1}^{\overline{m}} ln(s_j)$ defines a potential function whose maximum is achieved at a point centered in the localization set; for example, a *non-centered* point close enough to a hyperplane j has $s_j \to 0$, and the associated potential component $ln(s_l) \to -\infty$. From y^{k+1}, λ^{k+1} is taken as an updated dual solution vector, which is used to evaluate the dual function. The localization (4.19) is updated to Ω^{k+1} by adding a new cut, $z \leq \psi(\lambda^{k+1}) + (\lambda - \lambda^{k+1})^T \xi^{k+1}$, and by replacing the dual-function lower bound approximation by $\underline{z} = \max\{\psi(\lambda^{k+1}), \underline{z}\}$. The analytic center of the updated localization set is obtained and the process in repeated again until a stopping criterion is satisfied.

The difference between the IP/CP approach and previous cutting-plane methods is graphically illustrated in Figure 4.1 for $\lambda \in \mathbb{R}^1$. The figure includes: (a) a pure CP method with bounds on dual variables; (b) a PB cutting plane method; and (c) the IP/CP method.

In all the cases, it is assumed that two cuts have already been obtained (solid straight lines). In cases (a) and (b), the improved point is taken from the maximization of the cutting plane approximation, which is limited by bounds (dotted vertical lines) in dual variables and by a penalty function (dotted circle), respectively. In IP/CP methods, the analytic center of the localization set is taken as the improved point. The lower bound for the dual objective function and the box constraint are represented by the dotted horizontal and vertical lines, respectively.

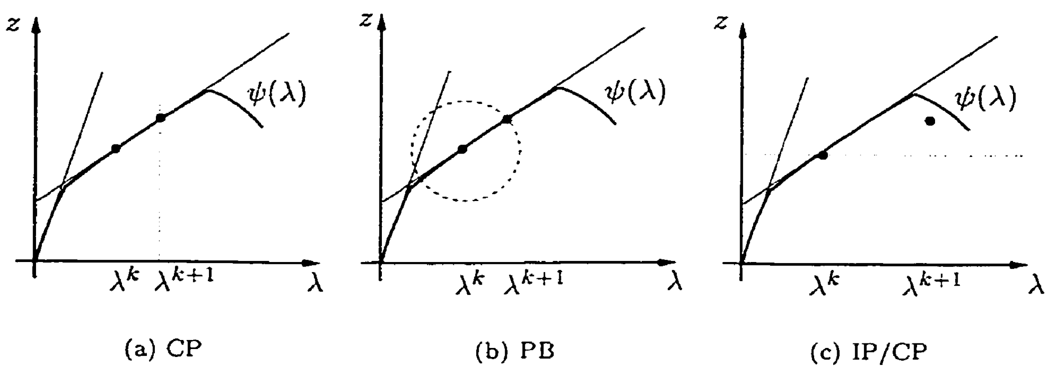


Figure 4.1: Cutting plane methods

Figure 4.2 schematically depicts the third, fourth and fifth iterations of an IP/CP method; the dot inside each updated localization set (shaded region) represents the analytic center; the horizontal dotted line represents the lower bound $\underline{z}$; and the bold curved line represents the dual function. This figure depicts the classic behavior of the IP/CP method; cuts generated from the analytic center are deeper and the localization set rapidly shrinks towards a single point that corresponds to the optimal value.

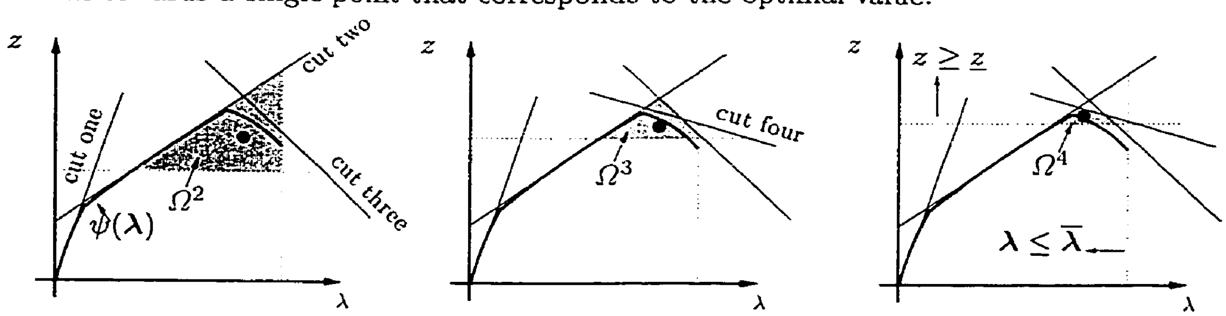


Figure 4.2: Illustrative iterates of an IP/CP method

All cutting plane methods require the solution of a linear or non-linear optimization problem whose size increases with the number of iterations. For the PB method, a quadratic problem (4.18) has to be solved. This problem is very suitable for interior-point methods for quadratic programming. For the IP/CP method, the *potential problem* (PP) (4.21) has to be solved. In [61, 1997], a damped Newton method is used to solve this problem. Recently in [75, 1999], an infeasible primal-dual Newton method is used to find the analytic center. The solution to problem (4.21) has received less attention than linear programming problems; as stated in [69, 1999], "no method can be credited superiority". In the next section, an infeasible log-barrier primal-dual interior-point-method to solve the potential problem is developed.

4.4 A Primal-Dual Interior-Point Method to Solve the Potential Problem

An infeasible primal-dual *interior-point-method* (IPM) to solve the concave non-linear problem (4.21) can be derived as an extension of primal-dual IPM's for linear programming [76]. Following Fiacco and McCormick's logarithmic barrier function approach [77], the transformed problem to (4.21) is given as

$$\begin{aligned} \max \quad & \textstyle\sum_{i=1}^{\overline{m}} ln(s_i) + \mu^v \sum_{i=1}^{\overline{m}} ln(s_i) \\ \text{s.t.} \quad & \boldsymbol{s} = \boldsymbol{b} - \mathbf{A}^T \boldsymbol{y} \end{aligned} \tag{4.22}$$

where $\mu^v > 0$ is a barrier parameter that is monotonically driven to zero: $\mu^0 > \mu^1 > \cdots > \mu^\infty = 0$. The sequence of solutions $\{\boldsymbol{y}(\mu^v), \boldsymbol{s}(\mu^v)\}$, that solve problem (4.22) for each μ^v, defines a barrier trajectory that converges to the unique maximizer of the original problem (4.21). The necessary and sufficient conditions for optimality of (4.22) can be derived from the barrier Lagrangian function

$$\mathcal{L}_{\mu^v} = (1+\mu^v)\textstyle\sum_{i=1}^{\overline{m}} \ln(s_i) - \boldsymbol{x}^T(\boldsymbol{s} - \boldsymbol{b} + \mathbf{A}^T\boldsymbol{y}) \tag{4.23}$$

where $\boldsymbol{x} \in \mathbb{R}^{\overline{m}} > 0$ is the vector of Lagrange multipliers to the constraints $\boldsymbol{s} = \boldsymbol{b} - \mathbf{A}^T\boldsymbol{y}$. The maximum of (4.22) is given by a stationary point of (4.23); which has to satisfy the following first-order necessary conditions:

$$\boldsymbol{r}_s = -\mathbf{SX}\boldsymbol{e} + (1+\mu^v)\boldsymbol{e} = \mathbf{0} \tag{4.24}$$

$$\boldsymbol{r}_x = -\mathbf{A}^T\boldsymbol{y} - \boldsymbol{s} + \boldsymbol{b} = \mathbf{0} \tag{4.25}$$

$$\boldsymbol{r}_y = -\mathbf{A}\boldsymbol{x} = \mathbf{0} \tag{4.26}$$

where $\mathbf{S} = \text{Diag}(\boldsymbol{s})$, $\mathbf{X} = \text{Diag}(\boldsymbol{x})$ and $\boldsymbol{e} \in \mathbb{R}^{\overline{m}}$ is a vector of ones. From an initial condition $\boldsymbol{s}^0 > \mathbf{0}$, $\boldsymbol{x}^0$ and $\boldsymbol{y}^0$ that does not necessarily satisfies the equality constraints, the primal-dual interior-point method generates iterates of the form

$$\begin{pmatrix} \boldsymbol{s} \\ \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix}^{v+1} = \begin{pmatrix} \boldsymbol{s} \\ \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix}^{v} + \alpha^v \begin{pmatrix} \boldsymbol{\Delta s} \\ \boldsymbol{\Delta x} \\ \boldsymbol{\Delta y} \end{pmatrix} \tag{4.27}$$

where the search directions $\boldsymbol{\Delta s}$, $\boldsymbol{\Delta x}$ and $\boldsymbol{\Delta y}$ are computed by solving one Newton's iteration that advances towards the solution of the optimality conditions (4.24)-(4.26). The common step length α^v is computed so that variables $\boldsymbol{s}$ and $\boldsymbol{x}$ remain strictly positive. A first-order Taylor-series expansion of (4.24)-(4.26) gives the augmented Newton's system, from which the $\boldsymbol{\Delta x}$ and $\boldsymbol{\Delta y}$ directions are computed

$$\begin{pmatrix} -\mathbf{X}^{-1}\mathbf{S} & \mathbf{A}^T \\ \mathbf{A} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \boldsymbol{\Delta x} \\ \boldsymbol{\Delta y} \end{pmatrix} = \begin{pmatrix} \boldsymbol{r}_x - \mathbf{X}^{-1}\boldsymbol{r}_s \\ \boldsymbol{r}_y \end{pmatrix} \tag{4.28}$$

The $\boldsymbol{s}$ direction is given by

$$\boldsymbol{\Delta s} = \mathbf{X}^{-1}(\boldsymbol{r}_s - \mathbf{S}\boldsymbol{\Delta x}) \tag{4.29}$$

The step length is computed as $\alpha^v = \min\{\alpha_x, \alpha_s\}$, where

$$\alpha_x = \min\{1, -\varrho \frac{x_i^v}{\Delta x_i^v} \mid \forall \Delta x_i^v < 0\}$$
$$\alpha_s = \min\{1, -\varrho \frac{s_i^v}{\Delta s_i^v} \mid \forall \Delta s_i^v < 0\}$$

The use of separate step lengths in IPM's for non-linear programming, does not always improve convergence as happens with linear programming [78]. In our experience with the potential problem (4.21), a common step length has performed well. The safety factor is set to the typical value of $\varrho = 0.99995$ [76].

4.4.1 Initialization, Barrier Parameter Reduction and Stopping Criteria

The initialization of variables for the IPM to solve the potential problem (IPM-PP) is done as follows. The initial value for the vector $\boldsymbol{y}^0$ is set up from the upper level k-iterations of the LR algorithm by $\boldsymbol{y}^0 = [\psi(\boldsymbol{\lambda}^{k-1}), \boldsymbol{\lambda}^{k-1}]$. This initialization allows a simple *hot-restart* of the IPM-PP that is equal to the analytic center of the previous localization set.

More elaborated hot-restart strategies are described in [69]. Once y^0 is set, variables s are initialized by

$$s^0 = |b - \mathbf{A}^T y^0|$$

The dual variables are all initialized to one, i.e.,

$$x^0 = e$$

From (4.24), the following barrier parameter reduction strategy can be derived

$$\mu^{k+1} = \sigma(\textstyle\sum_{i=1}^{\overline{m}} s_i^k x_i^k - 1) \qquad\qquad 0 < \sigma < 1$$

where a typical value for the parameter σ is given by 0.2 [78]. The convergence criteria of the IPM-PP are set by the following normalized residuals:

$$\frac{||r_s||}{1+\overline{m}} \leq \epsilon_1, \quad \frac{||r_x||}{1+||b||} \leq \epsilon_2, \quad \frac{||r_y||}{1+||x||} \leq \epsilon_3 \tag{4.30}$$

These convergence criteria are set to $\epsilon_1 = \epsilon_2 = 10^{-8}$ and $\epsilon_3 = 10^{-6}$. The parameter setting here presented has been found very efficient in all our tests performed.

4.5 Solution to the Profit Maximization Subproblems

The evaluation of the dual function for a given dual vector λ^k is required at each iteration of the LR algorithm. In order to evaluate (4.11), it is required to solve the profit maximization subproblems (4.12). These subproblems are non-linear and mixed-integer, and contain inter-temporal constraints. The supply function, that results from their solution, is no longer available in closed form. The profit-maximization sub-problems can be efficiently solved using *dynamic programming* (DP). A DP approach can easily handle non-linear objective functions and minimum time constraints. The inclusion of ramp constraints needs more considerations; for the purpose of this research, the inclusion of such constraints does not affect the development of the new approach or provide valuable information; the alternatives to include them have been studied in several other works [63, 67, 79, 80].

A DP approach to solve the subproblems (4.12) consists on a forward enumeration of all states that are feasible to the minimum time constraints, (4.7)-(4.8), and to the binary commitment conditions (4.6). In order to illustrate the DP approach, consider the graph shown in Figure 4.3. The graph corresponds to a unit that has been already committed for 1 hour; that is $x_{ci}^0 = 1, x_{di}^0 = 0$ and whose minimum time constraints are $\overline{t}_i = 3$ and $\underline{t}_i = 2$.

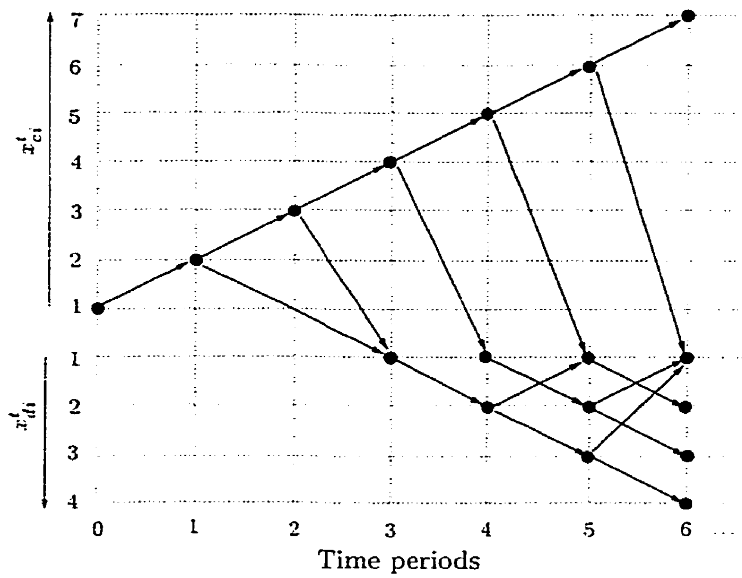


Figure 4.3: Dynamic programming graph

The graph is constructed by visiting only the feasible states to these constraints until $t = 6$. In the DP graph, upper nodes(states) correspond to *on* states $u_i^t = 1$, and lower nodes to *off* states $u_i^t = 0$. Since the DP graph implicitly handles minimum time constraint in its construction, it is not necessary to use the non-linear expressions (4.7) and (4.8). To each *on*-state, a profit π_i^t is associated. This profit is the maximum possible value that satisfies the minimum and maximum output constraints (4.5),

$$\begin{aligned} \pi_i^t \;&=\; \max \quad \lambda_p^{t\,k} p_i^t + \lambda_r^{t\,k}(\overline{p}_i - p_i^t) - (\alpha_{0i}^t + \beta_i p_i^t + \gamma_i {p_i^t}^2) \\ &\quad\;\; \text{s.t.} \quad \underline{p}_i \le p_i^t \le \overline{p}_i^t \end{aligned} \tag{4.31}$$

The solution to problem (4.31) is given by

$$p_i^t = \max\{\underline{p}_i, \min\{\overline{p}_i, p^*\}\} \tag{4.32}$$

where

$$p^* = \frac{\lambda_p^{t\,k} - \lambda_r^{t\,k} - \beta_i}{\gamma_i} \tag{4.33}$$

To each arc in the DP graph, a transition cost is associated; this cost is given by the start-up cost c_{si}^t. The total profit created by arriving to state x_i at stage t is computed using the recurrent formula

$$F(x_i^t) = \max_{x_j^{t-1} \in \mathcal{X}^{t-1}} [F(x_j^{t-1}) + \pi_i^t - c_{si}^t] \tag{4.34}$$

where $\mathcal{X}^{t-1}$ represents all the feasible states at stage $t-1$ in the DP graph. The optimal to the profit maximization subproblems, given by the vector $[u_i^1, p_i^1, \cdots, u_i^m, p_i^m]$, and represented by $p(\lambda^k)$, is obtained by back-tracking from the node in the last stage of the DP graph whose $F(x_i^m)$ is maximum. If there are multiple nodes in the last stage with maximum profit, the trajectory that results in more power output is selected. This helps to reach primal feasibility faster since more power is being committed at the dual optimization phase.

4.6 Primal Feasibility Search and Stopping Criteria

After the dual maximization phase is terminated, by any stopping criterion, a primal solution $p(\lambda^k)$ is available. However, this solution in most of the cases does not satisfy the power demand and reserve constraints (4.1c) and (4.1c). The primal feasibility search phase in the LR algorithm takes $p(\lambda^k)$ and maps it to a primal feasible solution using cost-based heuristics. The solution obtained after the primal feasibility search, $\bar{p}$, usually results in very small complementarity gaps.

A simple procedure to obtain a primal feasible solution, derived from [56] and generalized in [63], consists on first generating a reserve feasible solution, and afterwards on dispatching the committed units to satisfy the real power demand. The reserve feasible solution is constructed by successively increasing the dual variables to the reserve constraint at the time periods when the reserve is not met. The subproblems (4.12) are then solved for each increased vector of dual values until the reserve constraint is satisfied. The procedure is outlined in the following algorithm.

Algorithm 4.2 Reserve feasibility search

1. Set $j = 0$ and $\hat{\lambda}_r^j = \lambda_r^k$ (from the last iteration of the dual maximization phase).
2. With $\hat{\lambda}^j = [\lambda_p^k, \hat{\lambda}_r^j]$, solve the profit maximization sub-problems (4.12).
3. If $p(\hat{\lambda}^j)$ is feasible to constraints (4.12), stop; otherwise, continue.
4. Set $\hat{\lambda}_r^{j+1} = \hat{\lambda}_r^j + \epsilon_r^j \times \min\{\mathbf{0}, [r_d^1 - \sum_i r_i^1, \ldots, r_d^m - \sum_i r_i^m]^T\}$
5. Set $j = j + 1$ and go to step 2.

In Step 4 of the algorithm, the power reserve dual variables are increased for the periods where the reserve constraint is not met. The increase is made based on a sub-gradient update; as long as it is positive, the parameter ϵ_r^j guarantees that a finite number of iterations j are needed in order to achieve reserve feasibility [56, 63]. However, a large value can commit more units than necessary; on the other hand, a small increase in the value can require more iterations to achieve feasibility. The reserve feasibility search does not, in general, represent a time consuming task as compared to the dual maximization phase.

Once the reserve constraint is met, a simplified quadratic economic dispatch is solved for every period t, considering only the units whose $u_i^t = 1$. Only under very uncommon practical situations, as described in [63], a reserve feasible solution cannot result in a feasible dispatch; that is, the inequalities $\sum_i u_i^t \underline{p}_i^t \leq p_d^t \leq \sum_i u_i^t \overline{p}_i^t$ cannot be satisfied. The economic dispatch problem to be solved at each time t is

$$\begin{aligned} \min \quad & \textstyle\sum_{i|u_i^t=1} \beta_i p_i^t + \gamma_i {p_i^t}^2 \\ \text{s.t} \quad & p_d^t - \textstyle\sum_{i|u_i^t=1} p_i^t = 0 \\ & \underline{p}_i \leq p_i^t \leq \overline{p}_i, \qquad \forall i \mid u_i = 1 \end{aligned} \tag{4.35}$$

This problem can be solved by the direct approach presented in Subsection 2.2.2. The commitment obtained after the reserve feasibility search and the dispatch obtained from the solution to (4.35), represents the feasible schedule that is denoted by $\hat{p}$.

The quality of the solution $\hat{p}$ can be measured by the *complementarity gap* (cg) or *relative complementarity gap* (rcg):

$$cg = \hat{f} - \psi(\lambda^k) \tag{4.36}$$

$$rcg = \frac{\hat{f} - \psi(\lambda^k)}{\hat{f}} \times 100 \tag{4.37}$$

where $\hat{f}$ denotes the primal objective function value at any primal feasible solution $\hat{p}$. Experience on solving UC problems [53, 56, 63] has shown that the rcg can be reduced to values about 1-2%, especially as the number of units in the UC increases. The smaller the cg is, the closest it is to the dg. Since the dg is not know in advance, it may happen that a duality gap exists for the particular instance of the UC model and, therefore, the rcg cannot be reduced to zero; in this situation, a small rcg does not necessarily mean $\hat{p}$ is a sub-optimal solution. Only when rcg is equal to zero, it can be assured that the solution at hand is optimal.

The computation of a primal feasible solution at each iteration of the LR algorithm, in order to use (4.37) as an stopping criterion, can turn expensive especially when the method to solve the dual problem has poor convergence characteristics. Stopping criteria for the SG algorithm are not easy to implement due to its final oscillatory characteristics. In most of the cases, the SG algorithm is stopped when a given maximum iteration count is reached [12, 57, 63, 66, 67].

Cutting plane methods provide better stopping criterion that only require the use of dual quantities. At each iteration k of the LR algorithm, the values z^k from the solution of (4.18) or (4.21), in the PB or IP/CP methods, is an upper bound to the optimal dual value $z^k \geq \psi^*$. Also, since the evaluation of the dual function (4.11) at λ^k is a lower bound to optimal dual objective function $\psi^* \geq \psi(\lambda^k)$, the following *dual gap* stopping criterion can be implemented for these methods:

$$\Delta\psi = \frac{z^k - \psi(\lambda^k)}{z^k} \times 100 \leq \epsilon_d \tag{4.38}$$

Due to the robustness of IP/CP methods, the dual gap criterion can be set to values as low as $\epsilon_d = 10^{-6}$.

4.7 Some Practical Implementation Issues

The base programming language for the LR implementation is C, using the GNU's gcc compiler (v2.7) [81], all under the Linux operative system. The implementation of a DP subroutine uses dynamic memory allocation; the efficient sparse implementation of the IPM to solve the potential problem strongly improved initial implementations. Some of the key implementation issues of these two components in the LR algorithm are described in the next subsections.

4.7.1 Dynamic Programming Data Structure

A dynamic memory allocation data structure has been designed to generate the DP graph in Figure 4.3. The structure is made of a static array whose components are pointers to linked lists. The linked lists consist of registers that contain the information of each node, at a given stage (see Figure 4.4). The structure is defined as follows :

```
typedef struct STATE{
  double PROFIT ;
  double POWER ;
  int TIME ;
  int INDEX ;
  int FROM ;
  struct STATE *NEXT ;
 }STATES ;
typedef STATES *DPGRAPH[m];
```

The DP graph is contained in the variable DPGRAPH which is a static array with length m (the number of time periods). Each component of the graph, DPGRAPH[t], contains a pointer to a linked list, STATES, that contains all the states at stage t. Each STATE in the linked list contains the following information:

PROFIT	The maximum profit of arriving to this node
POWER	The optimal power at this node
TIME	The time the unit has been off (-) or on (+)
INDEX	A consecutive number assigned to each node as they are created
FROM	The INDEX of the preceding state at stage $t-1$
*NEXT	A pointer to the next state in the same stage t

Starting from stage $t = 0$, and given the initial conditions for the time constraints (4.7) and (4.8), new nodes are created as long as the time constraints are satisfied. Four possibilities exist: (i) the unit continues on $u_i^t = 1, u_i^{t+1} = 1$; (ii) the unit continues off $u_i^t = 0, u_i^{t+1} = 0$; (iii) the unit is decommitted $u_i^t = 1, u_i^{t+1} = 0$; or (iv) the unit is committed $u_i^t = 0, u_i^{t+1} = 1$. For case (iv), when the unit is committed, the startup cost (4.4) is subtracted to the profit (4.31), as indicated in equation (4.34). The process is continued for stages $t = 1, \cdots, m$; at the final stage, the node with maximum profit is searched. An optimal solution is obtained by backtracking using the variables FROM in the data structure. If there are multiple nodes in the last stage with the same maximum profits, the solution with larger total power output is selected. For each stage p_i^t, in (4.32), is given by the variable POWER; and the commitment variable is $u_i^t = 0$ if at stage t the variable TIME≤ 0, and 1 otherwise.

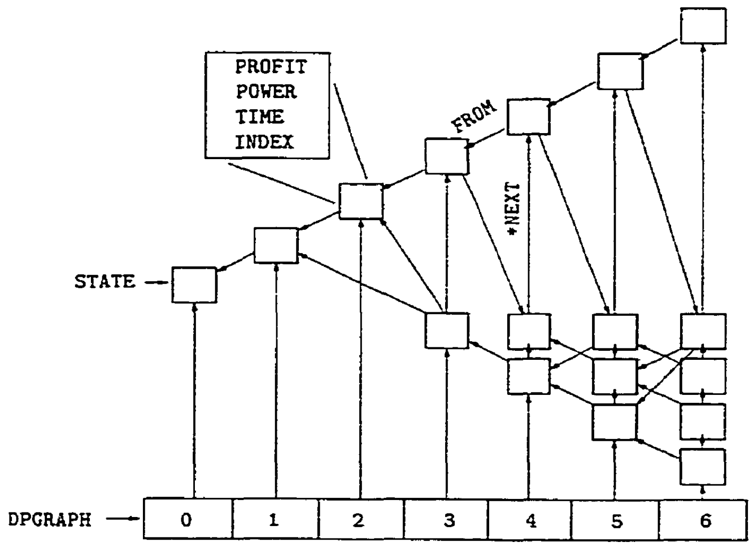


Figure 4.4: DP data structure

4.7.2 Implementation Issues for the PB and the IP/CP Method

A first experimental implementation of the IP/CP method to solve the dual UC problem used a semidefinite programming solver to solve the potential problem (4.21) [82]. The implementation of the primal-dual interior-point method, in Section 4.4, considerably reduced the computation times to solve the potential problem.

The major computational effort for the IPM-PP to solve the potential problem, in Section 4.4, lies on the solution of the augmented symmetric indefinite Newton's system (4.28). The symmetric indefinite Newton's system (4.28) is sparse, which represents an advantage; but its solution requires partial pivoting to preserve numerical stability. The Newton's system is solved using the public domain solver SuperLU [83]. The software contains C-callable subroutines to perform factorization of general non-symmetric non-definite sparse matrices. It implements the factorization $\mathbf{P}_r\mathbf{W}\mathbf{P}_c^T = \mathbf{L}\mathbf{U}$, where $\mathbf{P}_r$ is determined from partial pivoting to guarantee stable pivots; and $\mathbf{P}_c$ is a permutation matrix to preserve sparsity. The sparsity preserving permutation order can be set-up by the user in order to exploit the specific structure of matrix $\mathbf{W}$. In our case, since $\mathbf{W}$ is symmetric, a simple minimum degree ordering [84] works well and avoids the costly internal symbolic computation $\mathbf{W}\mathbf{W}^T$ that is used to obtain an ordering algorithm for general matrices.

Additionally, the products between sparse-matrices and full vector required to compute the residual vectors (4.24)-(4.26) and the $\boldsymbol{\Delta s}$ search direction (4.29) are performed using the C-callable libraries of the SparseBlas package. This package contains several high-performance libraries for basic linear algebra operations with sparse matrices; the package is public domain software [85].

The quadratic problem (4.18) for the PB method has been solved using LOQO. LOQO is an implementation of a primal-dual predictor-corrector interior-point-method for non-convex non-linear programming. The method is based on successive quadratic programming approximations [86].

4.7.3 Summary

This chapter presents the unit commitment problem and its solution by Lagrangian relaxation. The LR algorithm is a two phase solution procedure for the UC model. The first phase consists on the solution to the dual problem; the second phase, consists on a primal feasibility search. The following remarks are made.

- Previous techniques to solve the dual problem have extensively used the sub-gradient method [54,56]; other work has used variations of cutting plane methods; among them, the penalty bundle method [57, 1996], the reduced-complexity bundle method [58, 1997], and a dynamically adjusted cutting plane method [59, 1999]. All these cutting plane methods are based on the maximization of an stabilized cutting-plane approximation of the dual function. Both sub-gradient and cutting plane methods are strongly dependent on parameter setting; which makes the methods problem dependent.

- Interior-point/cutting-plane methods for non-differentiable optimization have been first introduced in [71, 1992]. The IP/CP methods has been applied to solve other engineering application such as multi-commodity flow problems [62, 1994], lot sizing problems [72, 1994], stochastic programming problems [73, 1997]. IP/CP methods do not maximize an stabilized cutting plane approximation; instead, they find the analytic center of a localization set that contains the dual optimum.

- In this chapter, the use of an IP/CP method to solve the dual UC problem is introduced. A primal-dual interior point to solve the potential problem has been developed. The method derives from the primal-dual approaches for non-linear programming. Previous methods to solve the potential problem consider damped Newton methods [61, 1997], and recently primal-dual Newton methods [75, 1999].

- A reserve feasibility search algorithm derived from [63] has been presented. Some implementation details of the LR algorithm have been described. Emphasis is given to the solution of the profit maximization subproblems, and to the solution of the Newton's system from the IPM to solve the potential problem.

Chapter 5 shows that the good convergence characteristics and robustness of the IP/CP are also present on for the solution of the UC problem. This allows us to consider the IP/CP method as a viable free-of-tuning alternative to obtain stable prices and design better pricing alternatives for unit-commitment power pool auctions.

Chapter 5

Performance Evaluation and the Unit Commitment Power Pool Auction

This first part of this chapter presents a numerical evaluation for the performance of the IP/CP to solve the UC problem. In Section 5.2, a performance comparison of the methods to solve the dual problem is presented. In Subsection 5.2.1, the robustness of the IP/CP method against changes in the initialization of dual variables and changes in the box constraint setting is tested. In Subsection 5.2.2, the convergence characteristics of the primal-dual interior-point method to solve the potential problem are illustrated.

In the second part of the chapter, Section 5.3, the use of the UC model as a real-power pool auction is analyzed. In Subsection 5.3.1, initial considerations are given. In Subsection 5.3.2, the non-uniform price setting alternative of Subsection 3.3.4 is extended to the UC power pool auction. In Subsection 5.3.3, a numerical evaluation of the deviation of profits due to parameter changes is performed. Additional numerical results in Subsection 5.3.4 compare the non-uniform price setting alternative with average pricing.

5.1 Evaluation of the IP/CP to Solve the UC Problem

Four unit commitment data sets are used along this chapter. The first system, extracted from [87], contains 26 units. The second system is obtained from [63], it contains 32 generation units. Two other larger data sets containing 67 and 104 generations units are generated from the 67 units system. The characteristics of the systems are summarized in Table 5.1. The detailed parameters for UC − 26 and UC − 32 are presented in Appendix B.

Table 5.1: Characteristics of the test systems

System	No. Units	Minimum Demand (MW)	Maximum Demand (MW)	Total Capacity (MW)
UC − 26	26	1824	2850	3105
UC − 32	32	1596	2850	3561
UC − 67	67	3192	5700	7343
UC − 104	104	4788	8550	11726

A first group of results deals with the comparison of the non-differentiable techniques to solve the dual problem.

5.2 Comparative Performance of Dual Maximization Techniques

A base value for the initialization of the dual variables $\lambda_0 = [\lambda_p^0, \lambda_r^0]$ is obtained by solving a quadratic economic dispatch problem (Subsection 2.2.2) for each time period, considering all the units with the minimum power output constraint relaxed to zero, $\underline{p}_i = 0$. From the solution to the quadratic economic dispatch problem, the optimal dual variable in (2.26) is used to set the initial condition for the power demand constraint dual variables, i.e., $\lambda_p^{t\,0} = \lambda^*$. The dual variables related to the reserve constraint are all set to zero, $\lambda_p^{t\,0} = 0$

The box constraint for the IP/CP method is initialized at the base value $\overline{\lambda} = \overline{\kappa}$, where the latter is based on following cost measure:

$$\overline{\kappa} = \max\{(\alpha_{0i} + \alpha_{i1} + \alpha_{2i}) + (\beta_i \overline{p}_i + \gamma_i \overline{p}_i{}^2) \mid \forall i\} e \tag{5.1}$$

where $e \in \mathbb{R}^{2m}$ is a ones vector. Constant $\overline{\kappa}$ is a measure of the largest possible total startup cost, plus total no-load and fuel cost among the units on the system. Such a high value represents a "safe" setting of the box constraint. Since dual variables for real and reactive power "measure" the rate of change of the objective with respect to changes in demand and reserve, the value in (5.1) is never likely to be reached. This value is left fixed in all the runs to be presented in this section.

For the PB and SG algorithms, a set of preliminary trial runs is executed for each system in order to determine the parameter settings that best perform on maximizing the dual function within a range of 250 iterations. These values are presented in Table 5.2.

Table 5.2: Parameter setting

	SG		PB
System	κ_1	κ_2	κ_3
UC − 26	0.20	0.60	1.00
UC − 32	0.10	0.06	0.10
UC − 67	0.15	0.06	1.5
UC − 104	0.15	0.04	0.2

In a first set of tests, the dual stopping criterion, (4.38), for the PB and IP/CP algorithms is set to $\Delta\psi \leq \epsilon_d = 10^{-3}$. Additionally, a maximum number of 250 iterations for all the systems, including the SG method, is used to stop the dual maximization phase. The results of these tests are summarized in the first row-block of results in Table 5.3; as can be seen, for all the cases, the IP/CP method takes around 96 iterations to reduce the dual gap to the stopping criterion (the solution times are in seconds and the objective values in \$ 10^2). Both the SG and PB methods are not able to achieve the same objective function values before the iteration limit is reached. However, the execution times of the IP/CP method are almost double than those of th SG method.

Even though the PB method achieves better objective values than the SG in the first two cases, the times required for the solution are much higher. As mentioned in Section 4.7, the quadratic problem (4.18) is being solved by LOQO, which required the interaction between two separate programs; this is already considered in Table 5.3 by subtracting the time that has been required to read and write the text files.

In order to compare under the same objective function achievement, a second test is performed for the PB and IP/CP methods using the dual values obtained at the last iteration of the SG method $\psi_{SG}(\lambda^k)$; that is, the dual maximization phase is stopped at any iteration i when $\psi_{PB}(\lambda^i) \geq \psi_{SG}(\lambda^k)$ and $\psi_{IP/CP}(\lambda^i) \geq \psi_{SG}(\lambda^k)$, respectively. The results of these runs are presented in the second part of Table 5.3. It can be observed in these results that the IP/CP method is able to obtain the same dual objective values of the SG algorithm in less computation time; this is due to the low number of iterations it takes. The PB method achieves better results only for the UC − 32 case.

Figure 5.1 shows the evolution of the dual function for the UC − 104 case and the three

Table 5.3: Comparative performance, solution to the dual problem

	SG			PB			IP/CP		
System	Iter.	Time	$\psi_{SG}(\lambda^k)$	Iter.	Time	$\psi_{PB}(\lambda^k)$	Iter.	Time	$\psi_{IP/CP}(\lambda^k)$
UC − 26	250	41	7252.24	250	296	7269.26	97	71	7273.00
UC − 32	250	43	9150.43	250	342	9172.84	96	54	9184.36
UC − 67	250	40	18159.67	250	419	18021.40	96	63	18251.59
UC − 104	250	61	27123.12	250	555	27026.11	96	72	27132.94
UC − 26				135	126	7253.86	50	29	7256.36
UC − 32				93	23	9151.69	59	27	9155.00
UC − 67				250	419	18021.40	46	23	18161.75
UC − 104				250	555	27026.11	83	56	27124.91

solution methods. As can be seen, the IP/CP method rapidly reaches better dual function values as compared to the PB and SG methods. Better dual values can be achieved by the IP/CP method, at the expense of computational time; however, such values cannot be achieved by the PB and SG methods, as observed in Table 5.3. Table 5.4 shows the

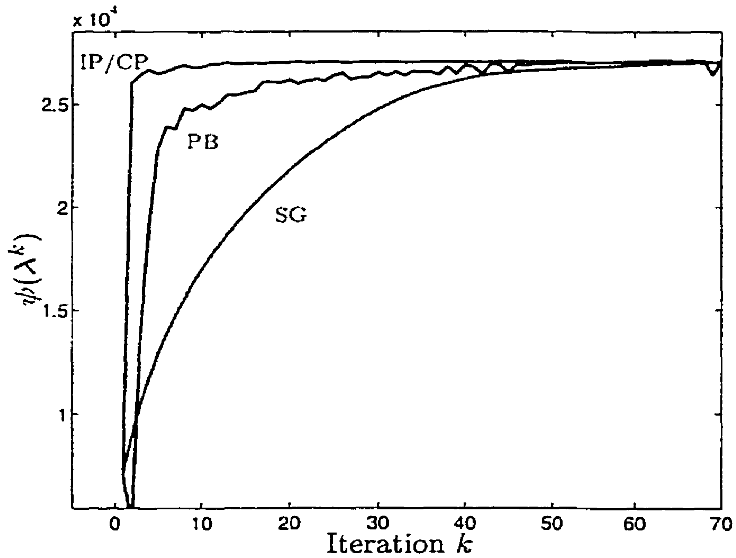


Figure 5.1: Classic convergence pattern

results achieved by the IP/CP method when the stopping criteron is further reduced to $\epsilon_d = 10^{-6}$. In the same table, the primal and dual objective function values after the feasibility search are also presented. The relative complementarity gap is presented in the last column. The dual objective function values have been further improved as compared to Table 5.3. In all the UC cases, Phase 2 of the LR algorithm takes computational times

Table 5.4: Best solution for each UC case

System	Iter.	Time	$\psi(\lambda^k)$	$\hat{f}$	$\psi(\hat{\lambda}^j)$	rcg
UC – 26	230	430.00	7274.1415	7448.45911	7271.6186	2.43192
UC – 32	206	293.00	9186.9381	9278.07658	9183.1654	1.03353
UC – 67	205	318.00	18255.7956	18409.84884	18254.8772	0.84893
UC – 104	199	320.00	27135.4740	27502.59396	27134.6996	1.35580

of less than two seconds. The reserve requirement in all the cases is set to 7% of the system demand. Parameter ϵ_r in the reserve feasibility phase is set to $0.005, 0.4, 0.45$ and 0.55 in each UC case, respectively.

5.2.1 Effects of Initialization and Box Constraint Setting

More than the speed characteristics, its robustness properties have made the IP/CP an attractive method for several other applications as mentioned in Section 4.3.3. In this section, we perform several tests to confirm the limited influence that the initialization and parameter setting have on the method. In Table 5.5, summarized results of six different runs of the IP/CP method are presented. In the first three cases, the box constraint is set up to 1, 4 and 6 times the base value $\overline{\kappa}$. In the other three runs, the initial dual vector is set to 1/10, 1/4 and 2 times the base value λ_0. For all the cases, the stopping criterion

Table 5.5: Effect of box constraint and initialization on the IP/CP method

	UC – 26		UC – 32		UC – 67		UC – 104	
Parameter	Iter	$\Delta\psi$	Iter	$\Delta\psi$	Iter	$\Delta\psi$	Iter	$\Delta\psi$
$1 \times \overline{\kappa}$	94	0.00091	94	0.00085	96	0.00089	98	0.00076
$4 \times \overline{\kappa}$	95	0.00079	100	0.00092	98	0.00100	96	0.00100
$6 \times \overline{\kappa}$	95	0.00079	100	0.00092	98	0.00100	96	0.00100
$0.10 \times \lambda_0$	97	0.00097	96	0.00088	96	0.00080	94	0.00095
$0.25 \times \lambda_0$	97	0.00082	96	0.00098	96	0.00084	94	0.00094
$2.00 \times \lambda_0$	94	0.00091	94	0.00085	96	0.00089	98	0.00076

selected is $\Delta\psi \leq \epsilon_d = 10^{-3}$. As can be seen, the IP/CP achieves convergence within the same number of iterations in all of the cases.

5.2.2 IPM Performance in Solving the Potential Problem

For each test sytem, Figure 5.2 shows the number of iterations v that the primal-dual IPM (Section 4.4) requires to solve the potential problem (PP) at each iteration, k, of the LR algorithm. It is important to note that the number of iterations required to solve the

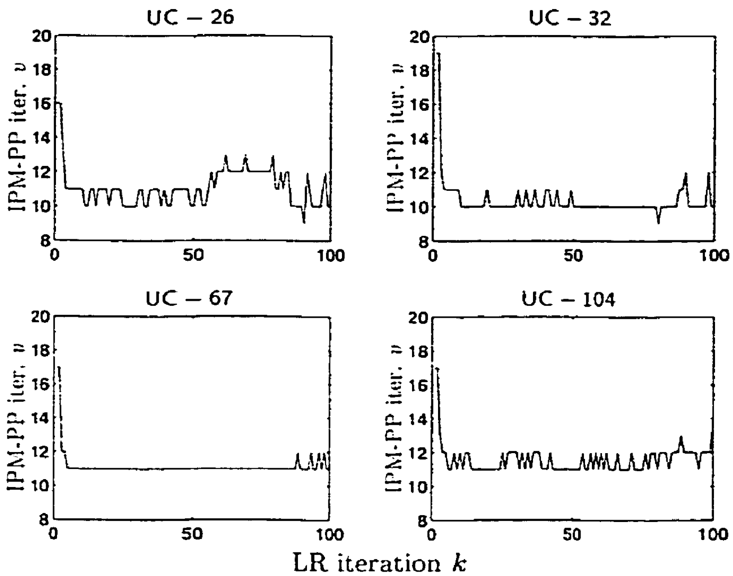


Figure 5.2: Iterations required to solve the potential problem

potential problem is slightly larger in the first few k-iterations of the LR algorithm; this is due to the larger size of the localization set -refer to Figure 4.2. After these few k-iterations, the v-iterations required by the IPM to find the analytic center remain fairly constant. The low number of iterations required by the IPM method allows the achievement of better dual function values in reasonable computational times. These results also confirm the robustness of the parameter setting for the IPM which has not been changed for any of the test systems.

5.3 Unit Commitment as a Power Pool Auction

Chapters 3 describes the problems that arise with the use of discrete models to conduct power pool auctions. The non-existence of an equilibrium combined with particular pricing rules can bring conflict of interest for the selection of the final schedule. A non-uniform price setting alternative, based on dual variables, has been shown to diminish the conflict of interest when multiple optimal solutions exist under disequilibrium. For those simplified models, the optimal primal and dual solutions can be identified.

Even though LR is a successful technique to solve UC problems, it cannot guarantee that an optimal primal and dual solution are obtained. Moreover, the identification of multiple solutions for such type of non-linear mixed-integer problems is an impossible task in practical terms. Experience indicates that only good near-optimal solutions to the UC problems are obtained in most of the cases. In this section, we investigate the applicability of the developed IP/CP method and the use of the non-uniform price setting alternative to implement and price real power pool auctions based on our UC model. The analysis performed in this section is only applied to real power as classically done in most of the previous studies that investigate the use of UC models to conduct electricity auctions [8–10, 12].

The IP/CP method is able to obtain more stable prices that, combined with the non-uniform pricing rule, minimize the conflict of interest that arises when multiple near-optimal solutions are obtained due to parameter variation, as identified in [10] or when multiple optimal solutions exists [12]. The non-uniform price setting is generalized for the UC model and compared to a maximum average cost alternative. A small example and the UC data sets are used mainly to present numerical results.

5.3.1 Introductory Considerations

The LR algorithm can be interpreted as a decentralized price-driven auction [11,56,63,88]. An auctioneer proposes a set of prices λ^k and the suppliers react to these prices by proposing the supply $p(\lambda^k)$ that maximizes their profit for the given set of prices; that is, the solution to sub-problems (4.12). Based on the mismatch between demand and supply, an auctioneer increments or decrements the prices until the supply eventually meets the demand, as done by a sub-gradient optimization (4.15). If suppliers respond with the true values $p(\lambda^k)$ that solve (4.12), and if an equilibrium does not exist, the sub-gradient updating procedure permanently oscillates since there is no price vector that matches supply and demand. The IP/CP method can also be interpreted as an auctioneer; in such a case, the IP/CP proposes prices and suppliers respond with both $p(\lambda^k)$ and $\psi_i(\lambda^k)$. The IP/CP method does not oscillate if an equilibrium does not exist; however, in a decentralized operation the profits, $\psi_i(\lambda^k)$, for each proposed price vector need be known by the auctioneer (the IP/CP).

The same situation happens if the UC model is solved in a centralized fashion; a vector of equilibrium prices may not exist. If it exists, the solution to optimality of the dual problem gives an equilibrium price (out of the possible multiple ones, Section 2.1). Since the dual UC problem cannot be solved analytically to obtain these prices, it is desirable that the numerical optimization technique be robust enough to approach the optimal dual solution. To illustrate this, let us consider the simplified linear discrete model of Section 3.1, and the example in Table 3.1, with results in Table 3.2. When the demand is $p_d = 130$ MW, there is no equilibrium price, and the optimal dual variable is 25.78 \$/MW. If the SG algorithm is used to numerically solve the dual problem, it permanently oscillates around the solution; the IP/CP method can realiably obtain the dual optimum, even when different parameter initializations are selected.

Figure 5.3 shows the final value of the dual function and dual variable obtained for several combinations of parameter κ_2, the initial dual vector λ^0 and the box constraint $\overline{\lambda}$ (shown in Table 5.6). As can be seen, independently on the parameter initialization, the IP/CP method always arrives to the optimal dual solution (in 10 iterations in all these cases). The SG method (even with 1000 iterations in this small case) does not obtain the optimal dual variable, and arrives to different values for different parameter settings.

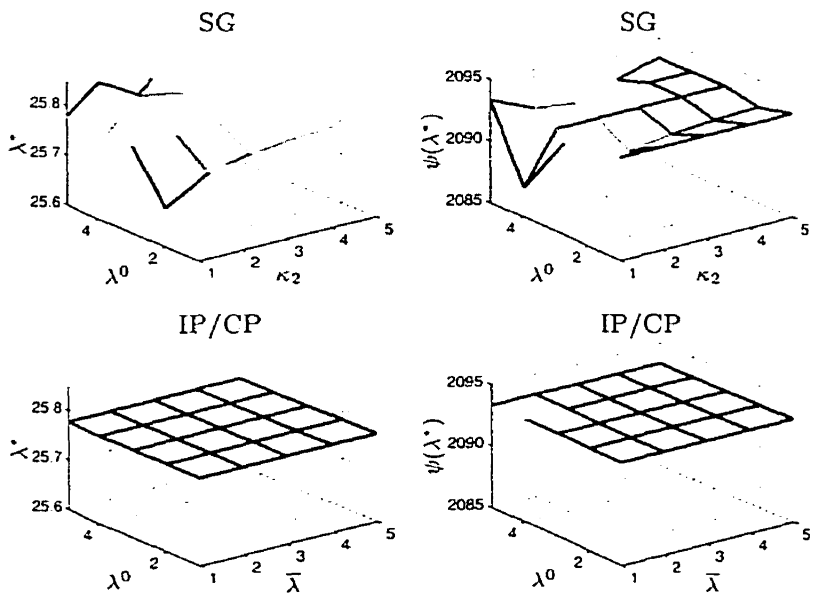


Figure 5.3: SG vs IP/CP method

When an equilibrium exists, i.e., $p_d = 190$ MW, in the same Table 3.2, both the IP/CP and the SG methods (after parameter tuning) arrive to one of the multiple equilibrium prices. The SG arrives to 26.7448 \$/MW, and the IP/CP to 26.3533 \$/MW. The IP/CP method arrives to a point "close" to the center (26.265 \$/MW) of the flat region in the dual function. The numerical solution of the dual problem does not guarantee that a specific dual variable, among the optimal, is found.

Table 5.6: Different initialization parameters

	1	2	3	4	5
$\overline{\lambda}$	52	78	104	130	156
κ_2	0.0020	0.0040	0.0060	0.0080	0.0010
λ^0	1	27	53	79	105

For the UC problem, once the dual problem is solved, a Phase 2 is performed to find a primal feasible solution from $p(\lambda^k)$, as obtained in the last iteration of the dual maximization phase. The following simplified heuristics are used to generate a primal feasible solution to the real power demand constraint.

Algorithm 5.1 Final heuristic

1. Initialize $t = 0$.

2. Set $t = t + 1$, and compute $\underline{p} = \sum u_i^t \underline{p}_i^t$ and $\overline{p} = \sum u_i^t \overline{p}_i^t$. If $\underline{p} \leq p_d \leq \overline{p}$, go to Step 5. If $\underline{p} > p_d$, go to Step 3. If $p_d > \overline{p}$, go to Step 4.

3. De-commit units. According to the minimum time constraints (4.7)-(4.8), determine the set of units that can be de-committed at period t. For each of these units $\forall j \in \mathcal{F}$, evaluate its total cost (4.2) at minimum output; i.e. $\underline{c}_j^t$. Order the units $j \in \mathcal{F}$ in the non-decreasing order $\underline{c}_1^t \geq \underline{c}_2^t \geq \cdots \geq \underline{c}_{|\mathcal{F}|}^t$. De-commit units in the order $\underline{c}_1^t \geq \underline{c}_2^t \geq \cdots \geq \underline{c}_r^t$ until $p_d \geq \sum_i u_i \underline{p}_i$. If $t < m$, go back to Step 2; otherwise, go to Step 5.

4. Commit units. Determine the set of units that can be committed at period t. For each of these units $\forall j \in \mathcal{F}$, evaluate its total cost (4.2) at minimum output; i.e. $\underline{c}_j^t$. Order the units $j \in \mathcal{F}$ in the non-increasing cost order $\underline{c}_1^t \leq \underline{c}_2^t \leq \cdots \leq \underline{c}_{|\mathcal{F}|}^t$. Commit units in the order $\underline{c}_1^t \leq \underline{c}_2^t \leq \cdots \leq \underline{c}_r^t$ until $\sum_i u_i \overline{p}_i \geq p_d$. If $t < m$, go back to Step 2; otherwise, go to Step 5.

5. Economic dispatch. For each t, with all the units whose $u_i^t = 1$, solve the quadratic economic dispatch problem (4.35), using the procedure in Subsection 2.2.2.

More elaborate heuristics that include unit de-commitment have been developed in [63] and [89]. These heuristics are used to further search for units that can be de-committed and result in total cost reduction. For evaluation purposes of the IP/CP and the price setting alternative, only the simplified heuristics are considered in this section. Additionally, the initial time conditions of all the units are considered so that they can always be shut down at the first period; that is, no unit can force its entry to the auction.

5.3.2 Price Setting Alternatives

The maximum average cost rule

The most documented pricing rule for power pool auctions executed by unit commitment models is the average cost pricing rule that has been used in the England and Wales Power Pool [4]. The price for real power at each period is defined as the *System Marginal Price* (SMP). The SMP is computed as the maximum average cost among the scheduled units. It is computed so that all scheduled units recover their variable, no-load and start-up costs along all the periods in the auction [4]:

$$\begin{aligned} \rho_{av}^t &= \max \quad \rho_i^t & & \qquad (5.2)\\ \rho_i^t &= (c_{vi}^t - \alpha_{0i})/\hat{p}_i^t + \frac{\sum_t c_{si}^t + \alpha_{0i}}{\sum_{t \in A} \hat{p}_i^t} & & t \in A \\ \rho_i^t &= (c_{vi}^t - \alpha_{0i})/\hat{p}_i^t & & \text{other } t \end{aligned}$$

where ρ_{ave}^t is used to denote the SMP, and A denotes the "Table A" periods, which correspond to the high demand periods in the auction. This pricing rule separates the variable cost from the no-load and startup-costs. Startup cost is distributed (amortized) on the basis of the power output that each unit produces in the "Table A" periods. In [90], several alternatives for the distribution of the no-load and start-up costs are analyzed. For comparison purposes, we only consider the alternative where all periods in "Table A".

Non-uniform pricing based on dual variables

In Subsection 3.3.4, a non-uniform price setting alternative is presented for the simplified discrete models. The formulation is extended here for the unit commitment auction. The total profits for each supplier can be written as

$$\pi_i = \sum_t \pi_i^t \tag{5.3}$$

$$\pi_i^t = \lambda_p^{t\,k} \hat{p}_i^t - c_{vi}^t - c_{si}^t \tag{5.4}$$

where $\lambda_p^{t\,k}$ is the dual variable obtained from the dual maximization phase. Let us define the cost not recovered by the dual variables as $CNR = \sum_{i|\pi_i<0} \pi_i$. Under this non-uniform pricing rule, bidders with negative profits are paid their cost. The amount necessary to generate these revenues is obtained by applying a non-uniform price for suppliers and consumers,

$$\rho_i^t = \lambda_p^{t\,k} - \Delta\lambda_i^t \tag{5.5}$$

$$\rho_c^t = \lambda_p^{t\,k} + \Delta\lambda_c \tag{5.6}$$

The price increments and decrements are given by

$$\Delta\lambda_i^t = \kappa \times CNR \times \frac{\pi_i}{\sum_{i|\pi_i>0} \pi_i} \times \frac{\max\{\pi_i^t, 0\}}{\sum_{t|\pi_i^t>0} \pi_i^t} \times \frac{1}{\hat{p}_i^t}, \quad i \mid \pi_i > 0 \tag{5.7}$$

$$\Delta\lambda_i^t = \pi_i \times \frac{1}{\sum_t \hat{p}_i^t}, \quad i \mid \pi_i < 0 \tag{5.8}$$

$$\Delta\lambda_c = \kappa \times CNR \times \frac{1}{\sum_t p_d^t}, \tag{5.9}$$

where κ has the same meaning as in (3.37); since CNG is very small compared to the total profits, it results in 0.5 for all tests performed. The cost not recovered is again distributed in the basis of profits. In (5.7), the amount passed to each supplier is distributed in the periods with positive profits.

5.3.3 Deviation of Profits Among Sub-Optimal Solutions Due to Parameter Changes

The unit commitment data set with 26-units, UC − 26, is used throughout this section. Five different initializations are considered; the initial dual vector is varied from 0.4, 0.6, 1.0, 1.4 and 1.8 times the base value λ^0. The stopping criterion for the SG algorithm is 250 iterations, and the stopping criterion for the IP/CP method is $\epsilon_d \leq 10^{-6}$.

All the runs for the different settings result in very acceptable near-optimal primal solutions; this is justified by the small complementarity gaps of the solution after Phase 2, as summarized in Table 5.7. In all the cases, the relative complementarity gap is below 1%. It has to be noticed that the cost not recovered is smaller than the complementarity gap.

Table 5.7: Complementarity gaps and cost not recovered

Parameter	SG			IP/CP		
Setting	rcg	cg	CNR	rcg	cg	CNR
$0.4 \times \lambda^0$	0.8920	65.0101	44.8894	0.2602	19.0794	9.2626
$0.6 \times \lambda^0$	0.3030	22.2134	13.9534	0.2603	19.0866	10.9695
$1.0 \times \lambda^0$	0.2961	21.7136	13.3118	0.3349	24.5563	15.8987
$1.4 \times \lambda^0$	0.2007	14.7204	8.0929	0.2603	19.0866	10.9700
$1.8 \times \lambda^0$	0.4651	34.0653	5.0230	0.2328	17.0729	9.0597

The dual variables obtained by each method in each of the runs are graphically presented in Figure 5.4. As noted in the graph, the IP/CP method arrives practically to the same dual vector; the SG method arrives to considerable different values.

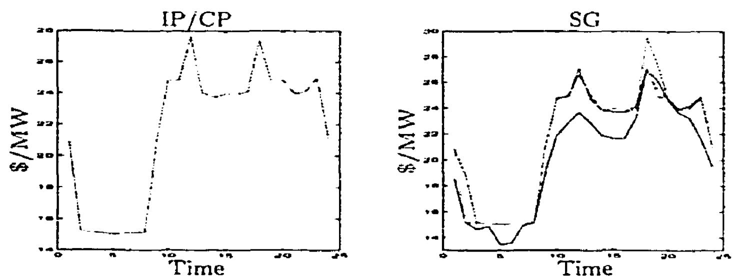


Figure 5.4: Dual variables with five different initializations

In Figure 5.5, the left-hand side graphs show the mean value $\overline{\pi}_i$ of the total profit for each scheduled unit (10-26). The mean value is computed from the five different runs with different initial dual vector. The right-hand side plots of the same figure show the standard deviation of the profits. In this case, the price is setup using average pricing (5.2). For both cases, when the dual problem is solved using the IP/CP and the SG methods, considerable standard deviation values of the profits are observed. The deviations are more pronounced in the case where the SG algorithm is used.

The same quantities are shown in Figure 5.6, but in this case the price is set by the non-uniform pricing rule (5.5)-(5.6). When the IP/CP method is used, the standard deviation in profits is negligible as compared to the SG method. This is due to two factors: (i) the dual variables for every run are quite similar, as seen in Figure 5.4; and (ii) the solutions found by Phase 2 differ only on some marginal units whose profits result negative; such negative profits are set to zero by the non-uniform pricing rule, which tends to equalize the profits in all the solutions (see Table 5.8). The situation in these two figures can be

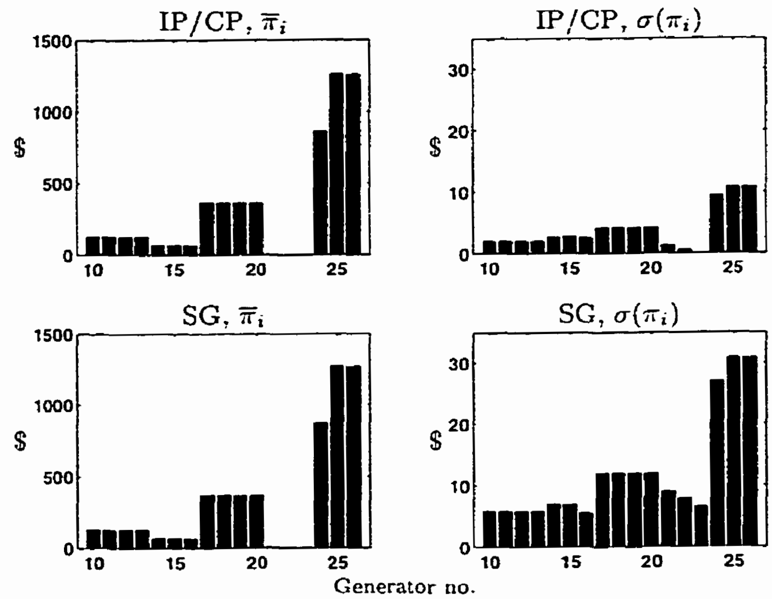


Figure 5.5: Mean and standard deviation of profits, pricing with average cost

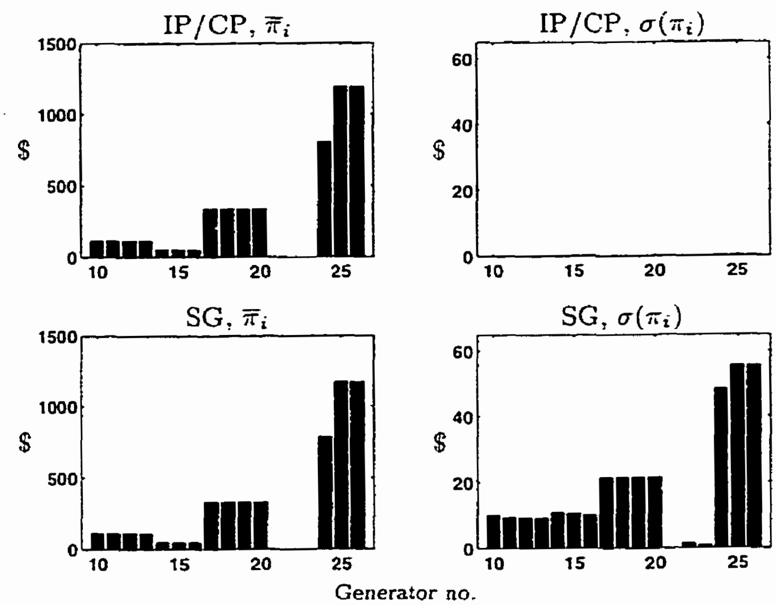


Figure 5.6: Mean and standard deviation of profits, non-uniform pricing

compared to the results in Tables 3.8 and 3.10. Average pricing worsens the deviation of profits for each solution; non-uniform pricing tends to equate the profits of each multiple, optimal or near-optimal, solution.

In order to better appreciate the impact of different solutions in the profits of each

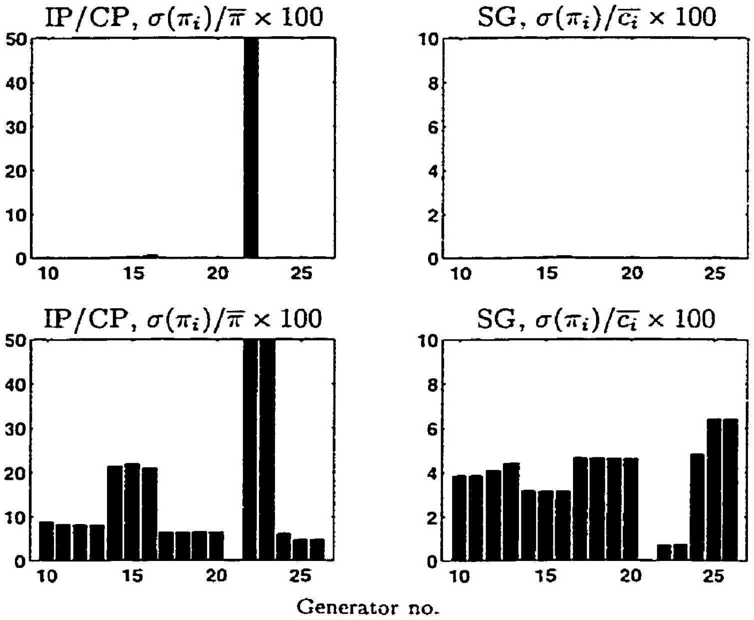


Figure 5.7: Standar deviation of profits in percent of mean profit and cost

supplier. Figure 5.7 shows the standard deviation of profits as a percent of the mean profits. In the same figure, the right hand side plots show the standard deviation of profits as a percent of the mean cost. As can be seen in these last plots, the deviation of profits as compared to the cost are considerable (in the order of 6%) with the SG method: while with the IP/CP method, the deviations cannot be distinguished.

In the left-hand side plots, a large deviation in profits for supplier 22 can be seen in the IP/CP plot, and two large deviations for suppliers 22 and 23 in the SG plot. Excluding these large values for the moment, it can be seen that the deviation of profits for the rest of the suppliers is in the order of 20 and 10 % with the SG method; while with the IP/CP, these deviations are not present.

The large deviations in the left-hand side plots correspond to the marginal suppliers (21, 22, 23); however, these values do not reflect the fact that in most of the alternate solution these bidders operate at zero profits. For these marginal bidders, their cost c_i, revenue r_i and profit π_i, for each of the five different runs, are presented in Table 5.8. The values are the ones obtained by the IP/C method. As can be seen, except for unit 22 in runs 2, 4 and 5, all the units operate at zero profit. Since unit 22 goes from 0.15 to 0 profit in different runs, the standard deviation results in a value close to the mean. Judging for the magnitude of costs and revenues, these large values should not represent a concern for the marginal suppliers; refer to the right-hand side plots in Figure 5.7.

Table 5.8: Cost, revenue and profit of suppliers on the margin

y	Gen.	1	2	3	4	5	$\overline{y}$	$\sigma(y)$	$\sigma(\pi)/\overline{y} \times 100$
	21	432.79	496.24	450.59	496.24	465.32	468.24	25.08	0
c_i	22	245.12	192.32	217.02	192.32	206.86	210.73	19.57	0.05
	23	158.68	141.69	169.63	141.69	156.06	153.55	10.70	0
	21	432.79	496.24	450.59	496.24	465.32	468.24	25.08	0
r_i	22	245.12	192.47	217.02	192.47	207.11	210.84	19.50	0.05
	23	158.68	141.69	169.63	141.69	156.06	153.55	10.70	0
	21	0	0	0	0	0	0	0	-
π_i	22	0	0.15	0	0.15	0.26	0.11	0.10	88.48
	23	0	0	0	0	0	0	0	-

The results presented in this section show that the IP/CP method is a robust mean to compute dual variables as compared to the classic SG algorithm. This, combined with the non-uniform pricing rule, tends to considerably reduce the deviation of profits among all the near-optimal solutions. Average pricing worsens the profit deviation among the alternate solutions.

5.3.4 Additional Numerical Examples

Table 5.9 presents the summarized results on the application of the non-uniform pricing rule for each of the UC cases. The total load payment LP and total suppliers profits SP for each of following price setting alternatives (denoted by the subscripts) are presented: (i) λ, pricing with dual variables; (ii) ρ_i, non-uniform pricing based on dual variables; and (iii) ρ_{ave}, pricing with maximum average cost. In the same table, the following information is presented: (i) $\Delta LP_{\rho_i/\lambda}$, the percent increment in load payment necessary to cover half of the cost not recovered; (ii) $-\Delta SP_{\rho_i/\lambda}$, the percent decrement in suppliers profit necessary to cover the other half of CNR; and (iii) $\Delta LP_{\rho_{ave}/\rho_i}$, $\Delta SP_{\rho_{ave}/\rho_i}$, the increment in load payments and suppliers profits, respectively, if the average pricing is used; these increments are computed taking non-uniform pricing as the base values.

It can be seen that when the non-uniform pricing rule is used, the total increments (decrements) in load-payment (suppliers-profit), necessary to compensate for the small amount of cost not recovered, is negligible as compared to the overall increases in load payment and suppliers profits if the price is setup with the average rule (5.2). In the latter case, the uniform price leads to increases in load payment around 4%, which can represent supplier profit increases above 30 %.

Figures 5.8 and 5.9 show the dual variables, average prices, non-uniform suppliers and load prices, for the UC − 26 and UC − 104 cases. As can be seen, the average pricing rule tends to smooth the prices; in the second case, two marginal units can be clearly identified. These units set the average price, which causes a global increase in load payment and supplier profits; as seen in last column in Table 5.9.

Table 5.9: Summarized results for large unit commitment models

	UC − 26	UC − 32	UC − 67	UC − 104
$\hat{f}$	7355.95143	9137.15826	18109.15853	26802.93762
$\psi(\lambda^k)$	7331.39505	9081.76754	18044.64874	26785.32911
rcg	0.33494	0.60991	0.35750	0.06573
cg	24.55638	55.39072	64.50978	17.60851
CNR	15.89879	43.32062	63.27989	5.04465
LP_{λ}	12513.8768	10360.7465	20700.7083	30215.9431
LP_{ρ_i}	12521.8262	10382.4068	20732.3482	30218.4654
$\Delta LP_{\rho_i/\lambda}$	0.0636	0.2091	0.1528	0.0083
SP_{λ}	5173.8242	1266.9089	2654.8297	3418.0501
SP_{ρ_i}	5165.8747	1245.2486	2623.1897	3415.5277
$-\Delta SP_{\rho_i/\lambda}$	0.1536	1.7097	1.1918	0.0738
$LP_{\rho_{av}}$	13030.0875	10831.3917	21357.4119	32121.4577
$\Delta LP_{\rho_{ave}/\rho_i}$	4.1913	4.5426	3.1724	6.3063
$SP_{\rho_{av}}$	5674.1361	1694.2335	3248.2534	5318.5201
$\Delta SP_{\rho_{ave}/\rho_i}$	9.6701	33.7297	22.3526	55.6010
Time	203.00	163.00	171.00	224.00
Iters	176	159	156	150

The cases so far studied in this section deal only with sub-optimal solutions obtained by the LR algorithm; multiple optimal solutions can also exist but cannot be easily identified. An interesting small example that shows the possibility of multiple optimal solutions in unit commitment like power pool auctions is presented in [12]. Minimum time constraints are not considered and the variable startup cost is considered zero. The data of the problem is presented in Table 5.10; there are four units and four periods in the auction, with demands

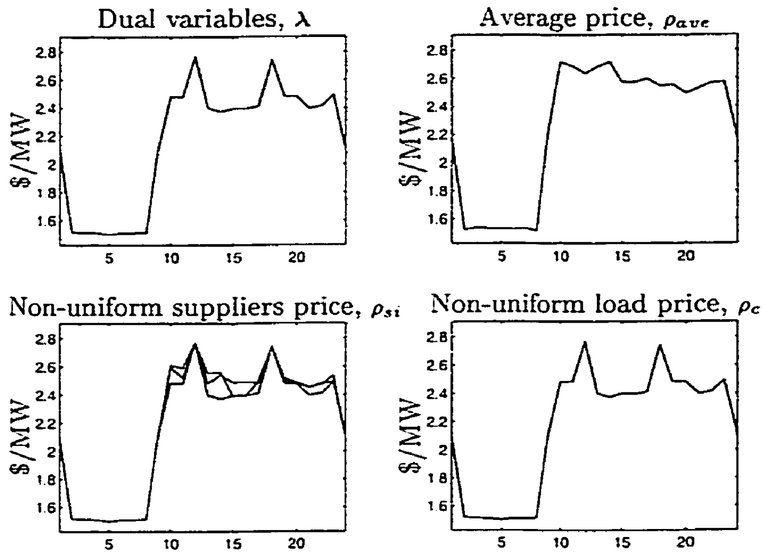


Figure 5.8: Prices for UC – 26

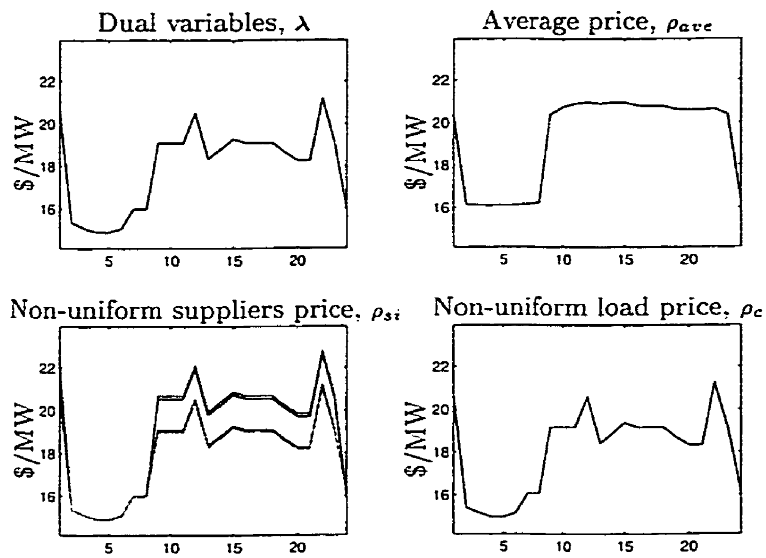


Figure 5.9: Prices for UC – 104

of $p_d = [170, 520, 1100, 1000]^T$.

Table 5.11 presents the primal solutions found by Phases 1 and 2 of the LR algorithm; the last column presents a multiple optimal solution that cannot be found by LR. Both solutions are optimal, with primal objective function $f^* = 30801.20$ \$, and dual objective function $\psi^* = 30225.92$ \$. The duality gap is $dg = 575.25$ \$ and the cost not recovered results in \$ 573.52. The solution obtained in Phase 1 satisfies the condition $\underline{p} \leq p_d \leq \overline{p}$, in Step 2 of Algorithm 5.1 which drives the algorithm to the economic dispatch phase in Step 5. The solution to the economic dispatch results in an optimal solution at the end of Phase 2. However, there is another optimal solution as denoted in the last part of Table 5.11. This solution cannot be obtained by Phase 2 algorithm.

Table 5.10: Small UC problem with multiple optimal solutions

i	α_{0i}	β_i	γ_i	α_{02}	$\underline{p}_i$	$\overline{p}_i$
1	500	10	0.002	3300.7	100	600
2	300	8	0.0025	0	100	400
3	100	6	0.005	0	50	200
4	542	9.88	0.002	3324.7	100	600

Table 5.11: Multiple optimal solutions

	Phase 1 Solution				Phase 2 Solution				Multiple Solution			
t	1	2	3	4	1	2	3	4	1	2	3	4
1	0	0	169.93	0	0	0	170	0	0	0	170	0
2	0	346.41	200.00	0	0	320	200	0	0	320	200	0
3	0	400.00	200.00	600.00	0	400	200	500	500	400	200	0
4	0	400.00	200.00	520.57	0	400	200	400	400	400	200	0

Table 5.11 summarizes the costs, revenues and profits for each unit in each of the alternate solutions. The second and third row show the revenues and profits if the optimal dual variables are used to set the price. The fourth and fifth row show the same quantities but applying non-uniform pricing. Despite the fact the multiple solution cannot be obtained by the LR algorithm, it has to be noticed that either supplier 1 or 4 become marginal in one of the solutions and end up with the zero profit; this makes no difference for suppliers which solution is selected by the algorithm. Average pricing, in this case, does not worsen the situation since it also results in the same profits for all suppliers.

5.4 Summary

This chapter presents a numerical evaluation on the performance of the IP/CP method to solve the unit commitment and its application as a real power pool auction. The following

Table 5.12: Pricing the multiple solutions

	Phase 2 Solution				Multiple Solution			
	1	2	3	4	1	2	3	4
$\sum_t c_i^t$	0.00	10916.00	5764.50	14120.70	14120.70	10916.00	5764.50	0.00
$\sum_t \lambda_p^t p_i^t$	0.00	14908.98	9152.66	13547.17	13547.17	14908.98	9152.66	0.00
π_i	0.00	3992.98	3388.16	-573.527	-573.52	3992.98	3388.16	0.00
$\sum_t c_i^t$	0.00	10916.00	5764.50	14120.70	14120.70	10916.00	5764.50	0.00
$\sum_t \rho_{si}^t p_i^t$	0.00	14753.82	9021.21	14120.70	14120.70	14753.82	9021.21	0.00
π_i	0.00	3837.82	3256.62	0.00	0.00	3837.82	3256.62	0.00

remarks are made:

- Even though the IP/CP method has a larger per-iteration computational effort, its convergence characteristics are such that it can achieve the same objective function values of the SG and PB methods in less total computation time.

- The principal advantages of the IP/CP stem from its good convergence and robustness characteristics. It does not suffer from parameter tuning and can achieve a very tight optimality condition in the solution of the dual problem. The initialization of the dual vector and the selection of the box constraint do not alter its convergence characteristics.

- The primal-dual IPM proposed for the solution of the potential problem has shown its good convergence characteristics. It requires a fairly constant number of iterations to solve the potential problem in each of the LR iterations. This allows the achievement of tight optimality bounds in the dual maximization phase within reasonable times (seconds to few minutes).

- The robustness of the IP/CP method allows the computation of dual variables that are very stable to parameter changes. Even though the SG method can obtain comparable solutions in terms of complementarity gaps, the dual vectors it generates considerably changes with parameter variations.

- The non-uniform pricing alternative based on dual variables obtained from the IP/CP method reduces the conflict of interest when multiple near-optimal solutions exists. The average pricing alternative further emphasizes the profit variations. For large UC problems, the non-uniform price setting alternative tends to avoid the overall increases in load payment and suppliers profits.

- Although the LR algorithm can find good near-optimal solutions in most of the practical cases, it cannot identify multiple optimal solutions. Multiple optimal solutions, given a pricing rule, can represent conflict of interest. In such situations, the conflicts can be diminished with the use of the non-uniform pricing alternative.

Chapter 6

A DC Network-Constrained Clearing System

This chapter presents a network-constrained clearing system that corresponds to hybrid market structures. In this type of structures, central dispatch is performed to implement the primary market. At the same time, a direct current (DC) representation of transmission network is included. Even though unit commitment decisions are not specified by suppliers, certain type of temporal operative limits are expressed in the bids. An actual market that performs this type of network or security-constrained market clearing is New Zealand [30, 1998]. Recently, the proposal for the Mexican market [31, 1999] and a second stage in the market for Ontario [32, 1998] consider this type of market model.

The inclusion of a DC representation of the transmission network has the intention to consider real power flow transmission limits and generate locational prices for power that give price signals for the correct expansion of generation and transmission. Locational or nodal prices are used in some market designs as the basis for transmission pricing [29]. The inclusion and pricing of other security aspects, such as voltage and frequency support is usually left outside the main power pool auction [2].

Classic power dispatch problems that contain temporal constraints, i.e., *dynamic economic dispatch*, have been solved by a number of methods, among them, the Simplex method [91], Lagrangian relaxation [92] and gradient projection methods [93]. Irisarri et al. [94, 1998] present the first application of an interior point method for quadratic programming to solve a dynamic economic dispatch problem. Interior point methods have proven to be suitable for the solution of several classic power systems problems; among them, constrained-economic dispatch, optimal power flows and hydro-thermal coordination [95].

Developments that include modeling considerations to reflect specific effects of deregulation in dispatch problems are varied. Dekrajangpetch and G.B. Sheblé [96, 2000] present an affine-scaling interior point method to implement a one-hour transmission-constrained auction with bids for supply and demand of real power. Fahd et al. [97, 1992] present a model for the implementation of brokerage systems using linear programming. Ferrero and Shahidehpour [98, 1996] [99, 1997] present formulations of dynamic economic dispatch problems to evaluate import and export transactions.

Models that deal with daily electricity markets in hybrid designs have simultaneously appeared recently. Madrigal and Quintana [100, 1998] [101, 1999] present a model for a daily market for power and spinning reserve. The model includes supply and demand bids and a *direct current* (DC) model for the transmission system; the problem is solved using a primal-dual interior point method. Alvey et al. [30, 1998] present a similar development for the New Zealand market, including outage constraints and multiple bid segments.

In this chapter, a network-constrained clearing-system for a daily market for power and spinning reserve is presented. The model is related to hybrid structures where the market operator also considers a DC transmission network model and allows the specification of operative limits such as ramp and energy constrains. Additionally to our previous models, it includes bilateral contracts and the model is solved by an interior point method taking advantage of the special structure of the Newton's system. The model is described in Section 6.1, and its solution in Section 6.2; numerical results on the experimental implementation are presented in Section 6.3.

6.1 The Clearing System Model

The model considers a daily market for power and spinning reserve, where: (i) n_s suppliers submit offers for power and spinning-reserve; (ii) n_c consumers bid to purchase power; (iii) n_b bilateral contracts submit schedules with incremental and decremental price information; and (iv) a DC transmission network model is included. The *Clearing System* (CS) can be formulated as the problem

$$\begin{array}{lll} \text{(CS)} & \max & f_S + f_C + f_B \\ & \text{s.t} & \boldsymbol{h}_S \leq \mathbf{0},\ \boldsymbol{h}_C \leq \mathbf{0} \\ & & \boldsymbol{h}_B \leq \mathbf{0},\ \boldsymbol{g} = \mathbf{0} \end{array} \tag{6.1}$$

The objective function in (6.1) defines a benefit maximization over the m periods of the market session, i.e., in a daily market, $m = 24$. The maximization is subject to: (i) Suppliers constraints specifications, h_S; (ii) consumers constraints specifications, h_C; (iii) a bilateral contracts model, h_B; and (iii) system and transmission network constraints, g. The solution to CS gives the schedule decisions for suppliers, consumers and, if necessary, the revised schedules for bilateral contracts. At the same time, in this type of market structues, the dual variables that are provided by the model are used for locational pricing and pricing of transmission services. Each of the components is described next.

6.1.1 Suppliers Model

Suppliers submit bids for power and spinning reserve supply along with the amounts offered and the operative limits. Suppliers' bid information is summarized by: (i) The offer price (\$/MWh) for power β_i^t and spinning reserve γ_i^t, for all $t = 1, \cdots, m$; (ii) combined (power p_i^t and spinning-reserve r_i^t) maximum output $\overline{p}_i^t$; (iii) up, $\overline{\Delta}_{pi}$, and down, $\underline{\Delta}_{pi}$, ramp rates; (iv) maximum energy supply in the trading day $\overline{e}_{pi}$; and (v) the node in the system, i.e., z, where the power is injected.

In the objective function (6.1), the suppliers component is given by

$$f_S = -\sum_i \sum_t (\beta_i^t p_i^t + \gamma_i^t r_i^t) \tag{6.2}$$

which corresponds to total cost minimization, as given by the prices for power and spinning reserve. Maximum output, ramp rates and maximum energy supply are described, respectively, by the following equations:

$$p_i^t + r_i^t \le \overline{p}_i^t \qquad \forall i,\ \forall t \tag{6.3}$$

$$p_i^{t-1} - p_i^t \le \underline{\Delta}_{pi} \qquad \forall i,\ \forall t \tag{6.4}$$

$$p_i^t - p_i^{t-1} \le \overline{\Delta}_{pi} \qquad \forall i,\ \forall t \tag{6.5}$$

$$\sum_t p_i^t \le \overline{e}_{pi} \qquad \forall i \tag{6.6}$$

Constraints (6.3)-(6.6) define the set h_S in (6.1). This set can be transformed into equality constraints by adding appropriate slack variables.

The contribution of suppliers to nodal power injection at node z of the transmission network is denoted as P_z^t, and is given by

$$P_z^t = \sum_{i \in z} p_i^t \qquad \forall z \tag{6.7}$$

where the $i \in z$ defines all suppliers i whose connection node to the network is z.

6.1.2 Consumers Model

Consumers submit bids for demand of real power containing: (i) The amount of power required $\overline{d}_j^t$ from which η_j% is dispatchable-load (curtailable-load) and $(1-\eta_j)$% is non-dispatchable (non-curtailable); (ii) the price γ_j^t the consumer is willing to pay for the dispatchable-load; and (iii) the network node at which the load is to be withdrawn. Consumers part, f_C, in the CS objective function is given by

$$f_C = \sum_j \sum_t \gamma_j^t d_j^t \tag{6.8}$$

The component h_C, in (6.1), is given by

$$0 \leq d_j^t \leq \eta_j \overline{d}_j^t \qquad \forall j,\ \forall t \tag{6.9}$$

At any node z, the total power withdrawn, by all consumers connected to it, is given by

$$D_z = \sum_{j \in z}((1-\eta_j)\overline{d}_j^t + d_j^t) \qquad \forall z \tag{6.10}$$

The signs in the objective function components (6.2) and (6.8) define a benefit maximization problem; suppliers with lower prices are used and loads with higher bid prices are served.

6.1.3 Bilateral Contracts Model

Bilateral contracts for power are represented by three components: (i) a *schedule* of hourly transaction power amounts that have been agreed between a bilateral-seller, v, and bilateral-buyer, $\overline{v}$: $b_v^t = b_{\overline{v}}^t$; (ii) A schedule-percentage *incremental*; that is, a percentage $\overline{\xi}_v$ of the schedule the seller is able to sell to the market at bid price ϑ_v; (iii) a schedule-percentage *decremental*; that is, a percentage $\underline{\xi}_v$ of the schedule the seller will not produce but buy from the market at bid price ν_v^t; and (iv) the seller and buyer connection nodes to the network: z and $\overline{z}$, respectively. The last part of the objective function, in the CS, is

$$f_B = \sum_v \sum_t (\nu_v^t \nabla b_v^t - \vartheta_v^t \Delta b_v^t) \tag{6.11}$$

At any node z, the nodal power injection from bilateral contracts is given by

$$T_z = \sum_{v \in z}(b_v^t + \Delta b_v^t - \nabla b_v^t) - \sum_{\overline{v} \in z} b_{\overline{v}}^t, \qquad \forall z \tag{6.12}$$

The incremental and decremental quantities are limited by the following constraints:

$$0 \leq \Delta b_v^t \leq \overline{\xi}_v b_v^t \qquad \forall v,\ \forall t \tag{6.13}$$

$$0 \leq \nabla b_v^t \leq \underline{\xi}_v b_v^t \qquad \forall v,\ \forall t \tag{6.14}$$

Constraints (6.13) and (6.14) define the set h_B in (6.1). With this model, bilateral-buyers always get their schedule satisfied; the sellers satisfy the schedule totally with their own power or with components ∇b_v^t from the market. This provides the hedging mechanism for contracts dealt as *contracts by differences*. In this type of contracts, if the selling price of the contract (only known among seller and buyer) is above the market price, the buyer pays the difference; if the market price is below the contract price, the seller reimburses the difference.

6.1.4 Network and System Constraints

The system nodal real power balance is expressed in terms of a *direct current* (DC) model of the electrical network [3]. Taking into account the individual effect of suppliers, consumers and bilateral contracts, the power balance equation for each node z is given by

$$P_z^t + T_z^t = D_z^t + \mathbf{B}_z \delta^t \qquad \forall z,\ \forall t \tag{6.15}$$

In (6.15), the contributions of suppliers, bilateral transactions and consumers are given by (6.7), (6.12) and (6.10), respectively; $\mathbf{B}_z$ is the z-th row of the susceptance matrix $\mathbf{B}$ and δ^t is the vector of nodal voltage angles. Nodal voltage angles are free variables that can be handled by adding the artificial bounds $-2\pi \leq \delta_i \leq 2\pi$ for all nodes ($\pi = 3.141516$), except a reference $\delta_1 = 0$.

Real power-flow transmission-line limits are expressed by the set of constraints

$$\mathbf{X}\delta^t \leq \overline{q} \qquad \forall t \tag{6.16}$$

where $\mathbf{X}$ is the reactance matrix of the transmission lines [3]. The maximum allowable real power flow on the transmission lines is denoted by $\overline{q}$. The amount of spinning reserve to be acquired from suppliers has to satisfy a system requirement defined by $\overline{r}^t$; that is,

$$\textstyle\sum_i r_i^t = \overline{r}^t \qquad \forall t \tag{6.17}$$

The set of network and system constraints g, in (6.1), is given by (6.15), (6.16) and (6.17).

6.2 Solution by an Interior Point Method

As defined in the previous section, the CS (6.1) is a large and sparse linear programming problem. The large dimension stems from the time-dependent constraints and the inclusion of a network model on it. The CS problem is solved using the *primal-dual infeasible-interior-point* (PDIIPM) algorithm by Kojima et al. [102]; the derivation is briefly described along with the special structure of the CS problem.

Including the necessary slack variables, the problem (6.1) can be transformed into the standard (primal) linear programming problem,

$$\min \quad \{c^T x \mid \mathbf{A}x = b, x \geq 0\} \tag{6.18}$$

where x and $\mathbf{c} \in \mathbb{R}^{\overline{n}}$; b and $y \in \mathbb{R}^{\overline{m}}$. The constraint matrix, $\mathbf{A} \in \mathbb{R}^{\overline{n}\times\overline{m}}$, has the following structure

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_1 & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \dots & \mathbf{A}_m & \mathbf{0} \\ \mathbf{B}_1 & \dots & \mathbf{B}_m & \mathbf{B}_0 \end{pmatrix} \tag{6.19}$$

where blocks $\mathbf{A}_t$'s contain constraints that only relate variables at time t in the CS, that is, all constraints except ramp (6.5)-(6.4) and the energy (6.6) constraints, which are included in the last row of $\mathbf{A}$ in sub-matrices $\mathbf{B}_t$'s. The special structure of $\mathbf{A}$ can be preserved in the Newton's system. The dual problem to (6.18) is given by

$$\max \quad \{b^T y \mid \mathbf{A}^T y + z = c, \ z \geq \mathbf{0}\} \tag{6.20}$$

And its associated barrier-Lagrangian by

$$\mathcal{L}_{\mu^k} = b^T y + \mu^k \textstyle\sum_i \ln(z_i) - x^T(\mathbf{A}^T y + z - c) \tag{6.21}$$

where $\mu^0 > \mu^1 > \cdots > \mu^\infty = 0$ is the barrier parameter and k the iteration index. The sequence of stationary points $\{x(\mu^k), y(\mu^k), z(\mu^k)\}$ to (6.21) define the central trajectory that converges to a solution of the original problem (6.20). Starting from an initial point, $x^k > \mathbf{0}, \ z^k > \mathbf{0}$, the PDIIPM generates a series of points of the form:

$$\begin{aligned} x^{k+1} &= x^k + \alpha_x^k \boldsymbol{\Delta} x \\ y^{k+1} &= y^k + \alpha_y^k \boldsymbol{\Delta} y \\ z^{k+1} &= z^k + \alpha_z^k \boldsymbol{\Delta} z \end{aligned} \tag{6.22}$$

where the search directions $\boldsymbol{\Delta}x$, $\boldsymbol{\Delta}y$ and $\boldsymbol{\Delta}z$ are computed using a one-step Newton's iteration that moves the current point towards the solution of the first-order necessary optimality conditions for (6.21), given by

$$\begin{aligned} r_d &= c - \mathbf{A}^T y - z = 0 \\ r_p &= b - \mathbf{A}x \qquad = 0 \\ r_c &= \mu^k e - \mathbf{X}\mathbf{Z}e \quad = 0 \end{aligned} \tag{6.23}$$

where $\mathbf{X} = \text{Diag}(x)$, $\mathbf{Z} = \text{Diag}(z)$ and e is a vector of ones. A first-order Taylor linearization of the optimality conditions (6.23) leads to the Newton's system whose solution provides the x- and y-search directions,

$$\begin{pmatrix} -\mathbf{D}^{-2} & \mathbf{A}^T \\ \mathbf{A} & 0 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} f_x \\ f_y \end{pmatrix} \tag{6.24}$$

where $\mathbf{D}^{-2} = \mathbf{Z}^{-1}\mathbf{X}$ is also diagonal; $f_x = \mathbf{X}^{-1}r_c - r_d$ and $f_y = -r_p$. From (6.23), the z-search direction is computed by

$$\Delta z = -r_d - \mathbf{A}^T \Delta y \tag{6.25}$$

The step lengths α_x^k, α_y^k and α_z^k in (6.22) are computed so that the new point remains strictly interior. That is,

$$\alpha_x^k = \min\{1, \varrho \frac{-x_i^k}{\Delta_{xi}^k} \mid \forall \Delta_{xi}^k < 0\} \tag{6.26}$$

$$\alpha_y^k = \alpha_z = \min\{1, \varrho \frac{-z_i^k}{\Delta_{zi}^k} \mid \forall \Delta_{zi}^k < 0\} \tag{6.27}$$

The barrier parameter is reduced using

$$\mu^{k+1} = \sigma(c^T x^k - b^T y^k)/\overline{n}, \quad 0 < \sigma < 1 \tag{6.28}$$

Typical values for the safety factor and barrier parameter are $\varrho = 0.99995$ and $\sigma = 0.2$, respectively [76]. The algorithm is stopped when the following criteria are satisfied:

$$\frac{||r_p||}{1+b} \leq \epsilon, \quad \frac{||r_d||}{1+||c||} \leq \epsilon, \quad \frac{|c^T x - b^T y|}{1+|c^T x|} \leq \epsilon \tag{6.29}$$

6.2.1 Solution to the Newton's System

If variables x and y are partitioned as follows:

$$\Delta x = [\Delta x^1, \cdots, \Delta x^m, \Delta x^0]^T \tag{6.30}$$

$$\Delta y = [\Delta y^1, \cdots, \Delta y^m, \Delta y^0]^T \tag{6.31}$$

The structure of the Newton's system (6.24) can be rewritten as

$$\begin{pmatrix} \mathbf{W}^1 & \dots & 0 & \hat{\mathbf{B}}_1^T \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & \mathbf{W}^m & \hat{\mathbf{B}}_m^T \\ \hat{\mathbf{B}}_1 & \dots & \hat{\mathbf{B}}_m & \hat{\mathbf{B}}_0 \end{pmatrix} \begin{pmatrix} \Delta^1 \\ \vdots \\ \Delta^m \\ \Delta^0 \end{pmatrix} = \begin{pmatrix} f^1 \\ \vdots \\ f^m \\ f^0 \end{pmatrix} \tag{6.32}$$

where $\Delta^t = [\Delta x^t, \Delta y^t]^T$, $f^t = [f_x^t, f_y^t]^T$, and

$$\mathbf{W}^t = \begin{pmatrix} -\mathbf{D}_t^{-2} & \mathbf{A}_t{}^T \\ \mathbf{A}_t & 0 \end{pmatrix}, \quad \hat{\mathbf{B}}_t = \begin{pmatrix} 0 & 0 \\ \mathbf{B}_t & 0 \end{pmatrix}, \quad \hat{\mathbf{B}}_0 = \begin{pmatrix} -\mathbf{D}_0^{-2} & \mathbf{B}_0^T \\ \mathbf{B}_0 & 0 \end{pmatrix} \tag{6.33}$$

The solution to the system (6.32) can be accomplished by factoring only the matrices $\mathbf{W}^t$'s and one with the dimension of $\hat{\mathbf{B}}_0$. The recognition of the special structure of the Newton's system has also been used in [103] for a hydro-thermal coordination problem and in [94] for a dynamic economic dispatch problem. Performing a block elimination in (6.32), the following system of equations for the solution of Δ^0 is obtained:

$$\bar{\mathbf{B}}_0 \Delta^0 = \bar{f}^0 \tag{6.34}$$

where

$$\bar{\mathbf{B}}_0 = \hat{\mathbf{B}}_0 - \textstyle\sum_t \hat{\mathbf{B}}_t \bar{\mathbf{B}}_t \tag{6.35}$$

$$\bar{f}^0 = f^0 - \textstyle\sum_t \hat{\mathbf{B}}_t \bar{f}^t \tag{6.36}$$

Matrices $\bar{\mathbf{B}}_t$ and vector $\bar{f}^t$ are the solution to

$$\mathbf{W}^t \bar{\mathbf{B}}_t = \hat{\mathbf{B}}_t^T \qquad \forall t \tag{6.37}$$

$$\mathbf{W}^t \bar{f}^t = f^t \qquad \forall t \tag{6.38}$$

The solution for each Δ^t is given by

$$\Delta^t = \bar{f}^t - \bar{\mathbf{B}}_t \Delta^0 \qquad \forall t \tag{6.39}$$

The solution of all the variables only requires the factorization of the m matrices $\mathbf{W}^t$'s and the matrix $\bar{\mathbf{B}}_0$ in (6.34).

6.3 Numerical Examples

The model and solution approach to the CS have been experimentally implemented using MATLAB. The sparsity features of MATLAB are used throughout the implementation, and in the special solution of the Newton's system. Tests runs are performed on a 200Mz PC, running on LINUX operative system.

6.3.1 Illustrative Small System

The test system is shown in Figure 6.1, and the related data in Table 6.1. It is assumed that consumers submit only dispatchable loads for one trading period ($m = 1$), and that the *incremental* and *decremental* percent for the bilateral contracts are $\overline{\xi} = \underline{\xi} = 0.5$, with their respective prices shown in Table 6.1. All units are in MW and \$/MWh. The system reserve requirement is $\overline{r} = 5$ MW. Transmission lines reactances are $x_{21} = 0.04$, $x_{23} = 0.02$, and $x_{31} = 0.04$, all in p.u. of 100 MW. In Figure 6.1, the left node is 1, the right is 2 and bottom is 3. The dual variables to the real power balance constraint (6.15) are used to define the nodal prices and are shown at each node in Figure 6.1. The price in all the nodes results in 20 \$/MWh. Since there is no congestion, this result can also be found by a simple ordering of the bids on the supply and demand side. The bilateral schedule, denoted by the thick arrows, remains unchanged. The price for spinning reserve is 4 \$MW/h and is being provided by supplier 2. The resulting nodal angles are $\delta = [0, 0.0241, 0.0160]'$ (rad).

Table 6.1: Small test system data

Suppliers	β	γ	$\overline{p}$
1	10.0	2.0	20.0
2	20.0	4.0	35.0
Consumers	γ	η	$\overline{d}$
1	40	1.0	30.0
2	30	1.0	10.0
Bilaterals	ϑ	ν	T
3 − 2	80.0	15.0	10.0

In a second case, the offer price of supplier 2 is reduced from 20 to 12 \$/MWh; the new results are shown in Figure 6.2. Since the new price for power goes down to 12 \$/MWh, and the price for the decremental schedule submitted by the bilateral is 15, the bilateral-seller gets 5 MW from the market and, therefore, its schedule is adjusted to 5 MW. The price for spinning reserve stills is 4 \$/MWh but is now provided by supplier 1, which has reached its maximum combined power and reserve output.

Let us now assume that the transmission limit on line 2 − 3 is 5 MW. The results for this case are shown in Figure 6.3. Due to congestion, nodal prices are different in the system; the bilateral contract reacts to the congestion prices and no longer can take 5 MW from the market. The spinning-reserve price is again 4 \$/MWh and is produced by supplier 2. In

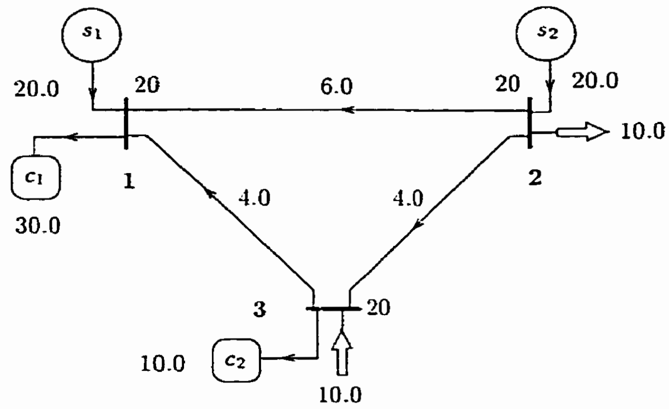


Figure 6.1: Results first case

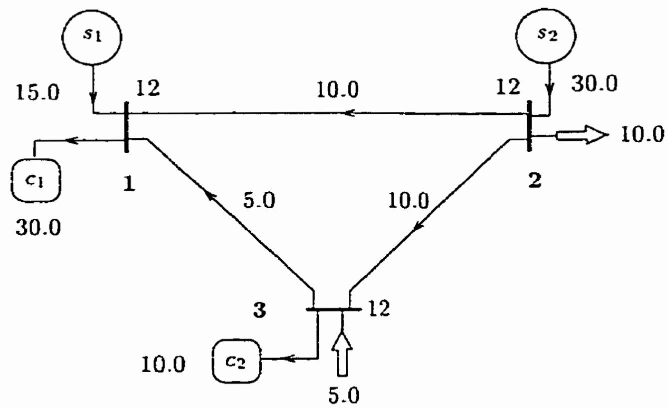


Figure 6.2: Results with lower prices

all these cases, the PDIIPM takes 14 iterations to find the optimal solution. The stopping criterion is $\epsilon = 10^{-10}$; the initial condition for all variables is set to 1.0.

6.3.2 14-Consumer, 7-Supplier, 2-Bilateral Case

For this simulation, the classic IEEE 14-node system is used to represent the transmission network [104]. It is assumed that there is a consumer that submits bids for power at every node. Prices submitted by consumers for the 24 hours are decreasing as the node index increases, and vary in time. There are 7 suppliers submitting offers for power and spinning reserve located at nodes 1, 2, 4, 6, 9, 11 and 13; the offered prices are time varying. In the same system, there are 2 bilateral contracts, with seller nodes 3, 2, and respective consumer nodes 10 and 6. Two different simulations are performed with this system. In the first case,

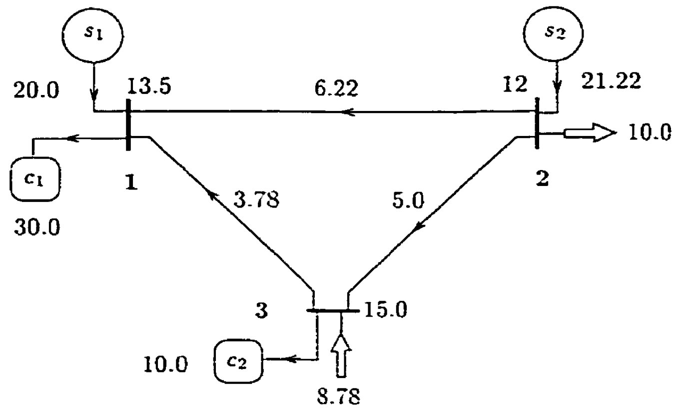


Figure 6.3: Results with transmission constraints

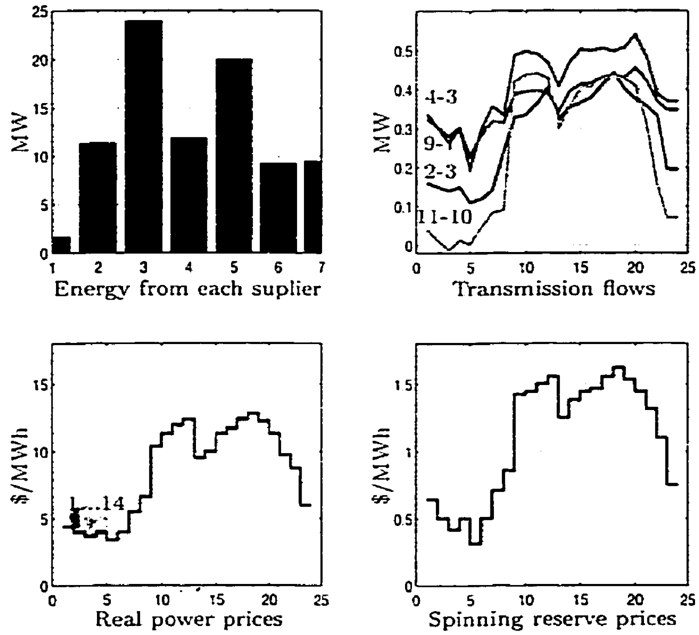


Figure 6.4: Results without congestion

all maximum transmission limits are assumed 60 MW. Some of the results obtained by the CS are summarized in Figure 6.4; all values are in per unit of 100 MW.

In the first graph of Figure 6.4, the total energy purchased from every supplier is presented; the energy limit on supplier 5, 2000 MW, has been reached. The real power flows of some of the transmission lines (connecting nodes 2-3, 4-3, 9-7 and 11-10) in the system are shown in the second graph of the same figure. Since there is no congestion on the system the nodal prices for power are equal in all the nodes (1 to 14) on the system, as can be seen in the third graph. Both the prices for real power and spinning reserve (last graph) vary during the day according to the demand behavior.

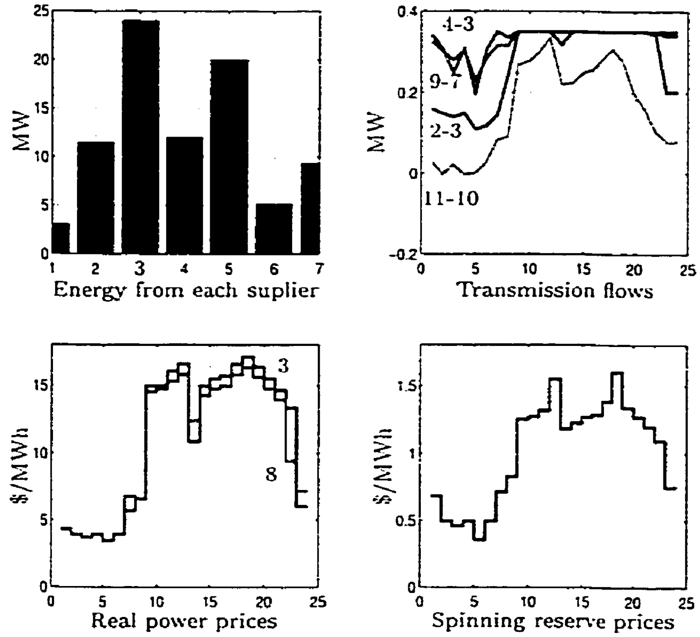


Figure 6.5: Results with congestion, part A

In a second simulation case, maximum transmission limits are reduced to 35 MW and the new results are as shown in Figure 6.5. Congestion (in lines connecting nodes 2-3, 4-3, 9-7 and 11-10) is clearly seen in the second graph of Figure 6.5, specially during high load periods. The supply from 6 and 7 considerably changes, as seen in the first graph of the same figure. Nodal prices increase and take different values in each node of the system due to the congestion; these prices for nodes 3, 8 are presented in the third graph. In the last graph it can be seen that the spinning reserve prices slightly change during the congestion periods. For this simulation case (as seen in the first graph of Figure 6.6) not all the requested demand is served due to the congestion; parts of the load are not served. The supply/demand curve for hour 18 is shown in the third graph of the same figure; as can be seen, enough supply exists for that loading condition at a price of 13 $/MWh; however, the congestion does not allow the supply to be transmitted, which leads to prices above 18 $/MWh. In the second graph, the schedules for both bilateral contracts are shown; one of which is not re-scheduled and the other is incremented. The increments happen when the price is above 15 $/MWh which correspond to its incremental bid price $\vartheta_1 = 15$. The extra power produced by that bilateral-seller is transmitted to feed some load. In the last graph of Figure 6.6, for consumers at nodes 3 and 8, at time $t = 18$, each bar represents: the requested load, the served load, the submitted bid price and the nodal price, respectively. As can be seen, at these nodes with higher prices, the load is not fully served. The same

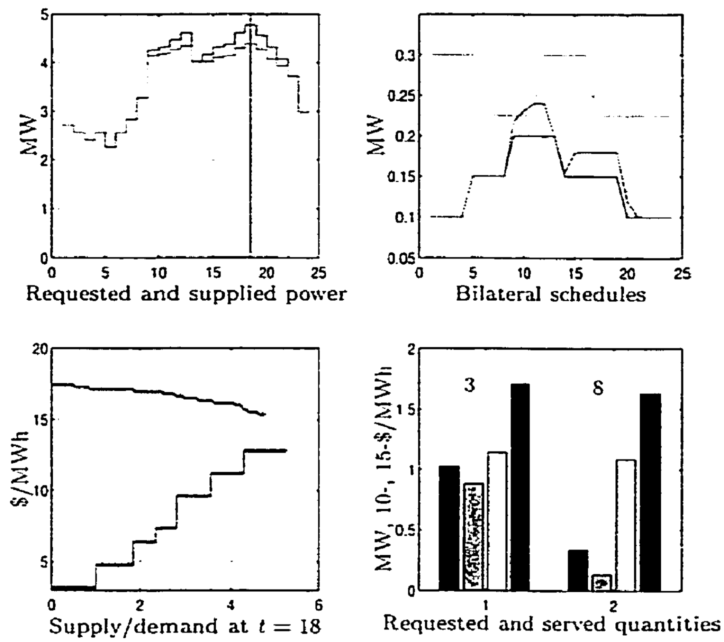


Figure 6.6: Results with congestion, part B

initialization and stopping criteria of the last example haven been used. In the last case, the time required by the PDIIPM is 24.5 seconds and 17 iterations if the Newton's system is solved block-wise. If the Newton's system is solved without taking into account its block structure, the time required is 34.38 seconds and 25 iterations. The difference in iterations is explained by the larger substitution round-off error when solving the full system. Since the sparsity is exploited in both cases, the solution times are comparable. Even for this small case, the dimension of $\mathbf{W}$ is 3878×3878 and the dimension of the blocks $\mathbf{W}^t$ is 147×147; see Figure 6.7.

6.4 Summary

A model and solution approach to a network-constrained clearing-system for a daily market for power and spinning reserve has been proposed in this chapter. The model is related to new hybrid types of market designs such as the proposed structure for the market in Mexico and the proposed extensions for the Ontario market. Bidders specify operative limits in

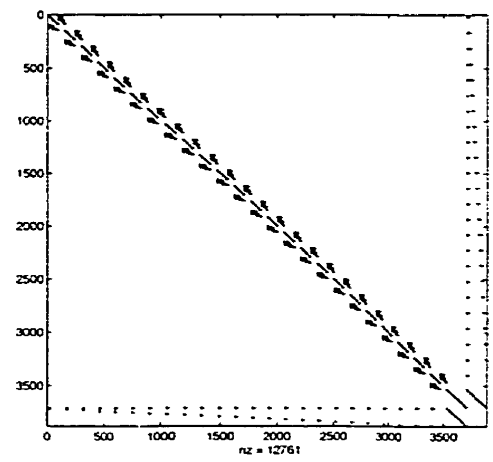


Figure 6.7: Newton's system matrix **W**

their bids but the unit commitment decisions are not made by the market operator. A DC transmission model is included in the market clearing to consider real power transmission flow limits. The following remarks are made:

- Few models and solution approaches have recently appeared for this specific type of problems. Related work includes an interior point for dynamic economic dispatch [94, 1998]; a network-constrained one-hour auction with bids for supply and demand, solved by an affine-scaling interior point [96, 2000]. And [30, 1998], a daily market for power and spinning reserve related to the New Zealand market.

- Our model considers: (i) Offers for power and spinning reserve with ramp and energy constraints; (ii) bids for the demand of power; (iii) bilateral contracts with incremental and decremental prices to reflect contracts by differences; (iv) a DC transmission model for the transmission system.

- The use of PDIIPM for the clearing system problem is presented; the special structure of the restrictions matrix is exported to the Newton's system which allows a more efficient solution.

- Numerical results on small problems show the validity of the model and the proposed solution approach. Its implementation on a large-scale basis and other extensions are recommended in the conclusions chapter.

Chapter 7

Conclusions

7.1 Summary and Contributions

Optimization tools have long been used to successfully help on the operation and planning of classic vertically integrated power systems; the use of optimization tools in electricity markets is still undergoing a development phase. The experiences from the first electricity markets in the world and the fast development of new structures. has created the need to review, and propose new models and techniques for the efficient implementation and pricing of electricity markets.

This thesis elaborates on the use and development of optimization models and techniques for implementation, and pricing of electricity markets. Observations and mathematical derivations that provide more insight into the use of optimization models for electricity markets are presented; new models, solution approaches and pricing alternatives are discussed. The conclusions and contributions of this research are as follows.

In Chapter 2, a generic cost-minimization power pool auction model is described. Using Lagrangian duality. the conditions for the existence of an equilibrium are presented. As illustrative examples, simplified continuous models that represent a standard auction and a quadratic economic dispatch problem are presented. For both models, closed form solutions are presented. The main contribution in this chapter are

- The conditions for the existence of an equilibrium in production economies are very well known. However, as far as we are aware, their derivation and, more importantly, its interpretation in the context of a power pool auction driven by a central cost-minimization model, have not been presented elsewhere.

- The dual formulation of the linear and quadratic models allows the solution of both primal and dual problems in a closed form. The simplified quadratic economic dispatch problem has been around for more than forty years, and is a component in many power system optimization applications. Its solution has always been carried out by a number of iterative methods; in this research work, a direct solution approach is presented [41, 2000].

In Chapter 3, two simplified discrete models for power pool auctions are studied; the models are introduced with the intention to provide more insight on the consequences of disequilibrium and different pricing rules for unit-commitment like power pool auctions. The contributions made in this chapter are as follows:

- Direct solution approaches for the dual problems, and the application of an enumerative Branch-and-Bound algorithm to find multiple solutions to the primal problem.
- The non-existence of an equilibrium and its effect on different pricing alternatives are presented through numerical examples. It is shown that average pricing and price minimization worsens the conflict of interest that arises when multiple solutions exist.
- A mathematical derivation shows that, under disequilibrium, dual variables used as prices do not recover a cost amount that is bounded above by the duality gap. Based on this observation, a non-uniform price setting alternative using dual variables is proposed.
- The non-uniform price setting alternative is simple, avoids the price spikes that can easily happen with average pricing, and shows that it reduces the conflict of interest when multiple solutions exist.

In Chapter 4, a unit commitment model and its solution by Lagrangian relaxation is presented. Lagrangian relaxation is the most accepted numerical-optimization based approach to solve UC problem. The major computational effort on solving UC problems is the dual maximization phase; in this respect the contributions made are:

- The application of an *interior-point/cutting-plane* (IP/CP) method to solve the dual unit commitment problem. Even though IP/CP methods have recently been used to solve other engineering applications, they have not been explored before in power scheduling applications [82, 2000]. The IP/CP has two major advantages over previous approaches: (i) it has better convergence characteristics; and (ii) it is a robust algorithm that is not affected by parameter tuning, as other approaches.

- The interior-point/cutting-plane method requires the solution of a potential problem, for which an infeasible primal-dual interior-point method is developed. Implementation details of the IP/CP method are also described.

In Chapter 5, a numerical evaluation of the performance of the IP/CP method to solve the unit commitment problem is presented. In the same chapter, we study the application of the IP/CP to execute unit commitment real power pool auctions. With respect to the characteristics of the IP/CP method the following remarks are made:

- The IP/CP method has better convergence characteristics than the previous approaches such as sub-gradient and penalty-bundle methods. Even though its per-iteration computational effort is higher, it requires far less iterations to achieve good dual objective function values.

- The IP/CP method can achieve tight optimality bounds in the solution of the dual problem. The tests performed show that the convergence characteristics of the IP/CP are not affected by the initialization of the dual vector or the selection of the box constraint; the method is problem independet.

- The primal dual interior point method to solve the potential problem has also stable convergence characteristics. The number of iterations it requires to solve the potential problem remains fairly constant after few iterations of the LR algorithm.

With respect to the use of the UC and IP/CP as a real power pool auction, the following remarks and contributions are made:

- The use of average pricing worsens the conflict of interest that can arise from the existence of multiple solutions. If the sub-gradient method is used to solve the dual problem and obtain prices, the deviation in profit is further emphasized.

- The numerical results show that the robustness characteristics of the IP/CP method, combined with the proposed non-uniform price setting alternative for the UC model, diminishes the conflict of interest that can arise from the existence of near-optimal solutions.

- The non-uniform price setting alternative, as compared to average pricing, avoids the overall increases in consumer payments and suppliers profit .

In Chapter 6, a model for a daily market clearing for power and spinning reserve is presented. The model is related to recently developed hybrid market structures where unit commitment decisions are left to the suppliers but some operative limits are allowed to be specified. Few developments have treated on the solution of this new type of models [96, 2000] [30, 1998] [101, 1999]. The conclusions an contributions in this chapter are

- A model for the daily market clearing system that includes: (i) bids for supply of power and spinning reserve with temporal limits; (ii) bids for power demand; (iii) bilateral contracts; and (iv) a direct current model for the transmission network.
- The use of a primal dual interior point method for its solution is proposed. The special structure of the constraint matrix is exploited in the solution of the Newton's system.
- Although the implemented presented is at an experimental level, the results on two small systems show the validity of the models and solution approach.

7.2 Research Recommendations

7.2.1 On Simplified Discrete Models for Power Pool Auctions

- The inclusion of elastic demand (demand side bidding) in discrete models for power pool auction is recommended. Although demand side bidding has been included in unit commitment models [105, 1999], its implications on the existence of equilibrium and price setting alternatives should be investigated.
- Semidefinite programming is an evolving research field on the mathematical programming arena. Semidefinite programming can be applied to solve combinatorial problems [106, 1996]. Initial research has been conducted on the solution of discrete power dispatch problem [80, 1999]. The evolution of semidefinite programming optimization may provide reliable solution approaches to discrete power dispatch problems.
- The non-uniform price setting alternative that has been proposed can be compared with (non-linear) two-part tariffs used in regulated industries [107]. Two part tariffs, are said to be Pareto-improving if they do not tend to favor a specific customer, which, by analogy, we observe in our simulations. Related pricing alternatives, such

as Ramsey pricing which requires the addition of revenue constraints in the cost minimization models, are recommended to be studied.

- Any non-uniform price setting alternative may always be controversial [2]. Whenever equilibrium prices and schedules are not found, the alternatives have to be designed so that they give good global results [108]. The investigation of alternative ways to handle the cost not recovered should be investigated. Even though settlement systems are not considered market-oriented approaches [2], they represent an alternative.

- A recent study for the California PX [109, 1999] presents an evaluation on the implementation feasibility of a multi-round simple-bids auction (iterative bidding); one of the motives for such a study is that muti-round bidding would give small generators more flexibility to design strategies to recover their startup and no-load costs through the simple-bids auction. Simplified discrete models with linear and constant startup cost (as presented in Chapter 3) could be an alternative for the same purpose. In [110, 2000], a combinatorial model for one of the ancillary services market in California is shown to avoid price spikes that recently appeared in such markets; however, complete enumeration is the obstacle to implement such an auction. The use of dual variables and alternative solution algorithms (i.e., Branch-and-Bound) in ancillary services markets should be investigated.

7.2.2 Unit Commitment Models and the IP/CP Method

- Further improvements can be made to the IP/CP method to speed up its performance [69, 1999] : (i) the efficient removal of cuts; as the LR iterations proceed, redundant cuts can be removed from the localization set which reduces the size of the potential problem; (ii) generation of multiple and *deep* cuts; at each iteration, more than one cut or *deeper* cuts are added to the localization set, consequently, it can faster shrink to the optimal solution. It is recommended to study if these improvements can further speed up the implementation of the IP/CP to solve the UC problem or other scheduling applications.

- Solving transmission-constrained unit commitment problems by LR requires the solution to m optimal power flow problems at each iteration [53]. The low number of iterations required by the IP/CP method to achieve good dual function values can considerably reduce the total computation effort to solve this type of problems. Any other scheduling application solved by LR can benefit from the IP/CP method. Following our derivation, applications to hydro-thermal coordination [111, 2000] and inter-utilities power-exchange coordination problems [112, 2000] have recently appeared.

- In order to study reserve pricing alternatives, the inclusion of a cost component that is directly related to the power reserve is recommended. Phase 2 algorithms that do not rely on dual variables modification need be investigated for this purpose. At the same time, pre-processing algorithms that can identify multiple solutions are also of interest.

7.2.3 Security-Constrained Clearing Systems and Interior Point Methods

- Mehrotra's predictor-corrector algorithm is one of the most successful interior point methods implemented in different software codes [76]. The large-scale implementation of the predictor-corrector algorithm exploiting the special structure of the Newton's should be pursued in order to generate large scale tools for the analysis of market behavior.

- Infeasibility detection in network-constrained clearing systems can be of importance under high loading conditions. Although some work has bee done in this respect for a network-constrained auction using an affine-scaling method [96, 2000], we recommend the investigation of *homogeneous self-dual* interior point formulations to detect infeasibility in daily clearing systems. Homogeneous self-dual methods have comparable performance to primal dual methods [76].

- Develop quantitative studies on the effects that numeric format precision and different stopping criteria have over the final prices and schedules determined by the clearing system. The extent of such effects has not been thoroughly analyzed or quantified. Related work should include the effects of restoring to an optimal basis after the interior-point termination [76] and detection of multiple dual solutions. Some work has been done in this direction [96, 2000].

- The clearing system models can be extended to include other different types of reserves that can be defined as separated ancillary services and security limits. This involves the inclusion of non-linear network models in this type of market clearing models.

7.2.4 Optimization Models for Power Pool Auctions and Market Power

- The use of dual variables for pricing in unit commitment models is recently being considered by other works. In [88, 1999], the authors outline the possibility of including revenue adequacy constraints in a decentralized pool dispatch model. A yet unpublished manuscript [113, 2000] investigates on the use of cutting plane approaches to solve unit commitment problems and generate dual variables for pricing purposes. Other forms of coordination and price setting alternatives can still be investigated.

- Although the study of strategic behavior does not used to lie directly on the electrical engineering or power engineering field, the theoretical study of auctions to detect gaming that can lead to market power in electricity markets should be pursued. The theoretical study of combinatorial auctions is still in its initial development phase [11, 114]; and few developments have been made in that direction with respect to power pool auctions [27, 2000] [25, 1999]. Other experimental or game-theoretical approaches can also be used [36]. Data mining techniques could also help to describe behavioral patterns of participants in an electricity auction [115].

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Appendix A

Some Properties of Non-Differentiable Functions

The sub-differential of a function $f(x) : \mathbb{R}^m \to \mathbb{R}$, is is defined as the set

$$\partial f(x) := \{\xi \mid f^o(x, v) \geq \xi^T v \quad \forall v \in \mathbb{R}^m\} \tag{A.1}$$

Where $f^o(x, v)$ is the generalized directional derivative of f at x in the direction of v

$$f^o(x, v) = \limsup_{y \to x, r \downarrow 0} \frac{f(y + rv) - f(y)}{r} \tag{A.2}$$

Each element of the sub-differential, $\xi \in \mathbb{R}^m$, is called a sub-gradient. If f is concave then the sub-differential is simply given by

$$\partial f(x) = \{\xi \mid f(v) \leq f(x) + \xi^T (v - x) \quad \forall v \geq 0\} \tag{A.3}$$

Let $f_1(x), \ldots, f_n(x)$ be a set of functions $f_i(x) : \mathbb{R}^m \to \mathbb{R}$ and k_i any scalars, then

$$\partial \textstyle\sum_i k_i f_i(x) \subset \sum_i k_i \partial f_i(x) \tag{A.4}$$

This is, all the sub-gradients of $\partial \sum_i k_i f_i(x)$ are contained in $\sum_i k_i \partial f_i(x)$.

Appendix B

Unit Commitment Data Sets

Table B.1: 26 Units sytem, load data

t	1	2	3	4	5	6	7	8
p_d^t(MW)	2223.00	2052.00	1938.00	1881.00	1824.00	1825.00	1881.00	1995.00
t	9	10	11	12	13	14	15	16
p_d^t(MW)	2280.00	2508.00	2565.00	2593.00	2565.00	2508.00	2479.00	2479.00
t	17	18	19	20	21	22	23	24
p_d^t(MW)	2593.00	2850.00	2821.00	2764.00	2679.00	2622.00	2479.00	2308.00

Table B.2: 32 Units dytem, load data

t	1	2	3	4	5	6	7	8
p_d^t(MW)	1824.00	1710.00	1653.00	1596.00	1596.00	1653.00	1824.00	2166.00
t	9	10	11	12	13	14	15	16
p_d^t(MW)	2479.50	2707.50	2821.50	2850.00	2821.50	2850.00	2850.00	2764.50
t	17	18	19	20	21	22	23	24
p_d^t(MW)	2736.00	2736.00	2650.50	2622.00	2622.00	2650.50	2479.50	2052.00

Table B.3: 26 Units system, generators data

i	$\underline{p}_i$ (MW)	$\overline{p}_i$ (MW)	$\underline{t}_i$ (h)	$\underline{t}_i$ (h)	x_0 (h)	α_{0i} ($)	β_i ($/MW)	γ_i ($/MW^2)	α_{1i} ($)	α_{2i} ($)	τ_i (h)
1	2.4	12.0	0	0	-1	24.3891	25.5472	0.0253	0.0	0.0	1.0
2	2.4	12.0	0	0	-1	24.4110	25.6753	0.0265	0.0	0.0	1.0
3	2.4	12.0	0	0	-1	24.6382	25.8027	0.0280	0.0	0.0	1.0
4	2.4	12.0	0	0	-1	24.7605	25.9318	0.0284	0.0	0.0	1.0
5	2.4	12.0	0	0	-1	24.8882	26.0611	0.0286	0.0	0.0	1.0
6	4.0	20.0	0	0	-1	117.7551	37.5510	0.0120	20.0	20.0	2.0
7	4.0	20.0	0	0	-1	118.1083	37.6637	0.0126	20.0	20.0	2.0
8	4.0	20.0	0	0	-1	118.4576	37.7770	0.0136	20.0	20.0	2.0
9	4.0	20.0	0	0	-1	118.8206	37.8896	0.0143	20.0	20.0	2.0
10	15.2	76.0	3	2	3	81.1364	13.3272	0.0088	50.0	50.0	3.0
11	15.2	76.0	3	2	3	81.2980	13.3538	0.0089	50.0	50.0	3.0
12	15.2	76.0	3	2	3	81.4641	13.3805	0.0091	50.0	50.0	3.0
13	15.2	76.0	3	2	3	81.6259	13.4073	0.0093	50.0	50.0	3.0
14	25.0	100.0	4	2	-3	217.8952	18.0000	0.0062	70.0	70.0	4.0
15	25.0	100.0	4	2	-3	218.3350	18.1000	0.0061	70.0	70.0	4.0
16	25.0	100.0	4	2	-3	218.7752	18.2000	0.0060	70.0	70.0	4.0
17	54.25	155.0	5	3	5	142.7348	10.6940	0.0046	150.0	150.0	6.0
18	54.25	155.0	5	3	5	143.0288	10.7154	0.0047	150.0	150.0	6.0
19	54.25	155.0	5	3	5	143.3179	10.7367	0.0048	150.0	150.0	6.0
20	54.25	155.0	5	3	-4	143.5972	10.7583	0.0049	150.0	150.0	6.0
21	68.95	197.0	5	4	-4	259.1310	23.0000	0.0026	200.0	200.0	8.0
22	68.95	197.0	5	4	-4	259.6490	23.1000	0.0026	200.0	200.0	8.0
23	68.95	197.0	5	4	-4	260.1760	23.2000	0.0026	200.0	200.0	8.0
24	140.0	350.0	8	5	10	177.0575	10.8616	0.0015	300.0	300.0	8.0
25	100.0	400.0	8	5	10	310.0021	7.4921	0.0019	500.0	500.0	10.0
26	100.0	400.0	8	5	10	311.9102	7.5031	0.0020	500.0	500.0	10.0

Table B.4: 32 Units system, generators data

i	$\underline{p}_i$ (MW)	$\overline{p}_i$ (MW)	$\underline{t}_i$ (h)	$\underline{t}_i$ (h)	x_0 (h)	α_{0i} ($)	β_i ($/MW)	γ_i ($/MW^2)	α_{1i} ($)	α_{2i} ($)	τ_i (h)
1	4.00	20.00	1	2	1	63.999	20.000	0.000000001	40	0	1
2	4.00	20.00	1	2	2	63.999	20.000	0.000000001	40	0	1
3	10.0	76.00	6	12	6	133.919	16.193	0.01508	45	0	1
4	10.0	76.00	6	12	-2	133.919	16.193	0.01508	45	0	1
5	4.00	20.00	1	2	1	63.999	20.000	0.000000001	40	0	1
6	4.00	20.00	1	2	2	63.999	20.000	0.000000001	40	0	1
7	10.0	76.00	6	12	10	133.919	16.193	0.01508	45	0	1
8	10.0	76.00	6	12	-2	133.919	16.193	0.01508	45	0	1
9	15.0	100.0	10	20	10	199.124	12.468	0.01532	45	0	1
10	15.0	100.0	10	20	10	199.124	12.468	0.01532	110	0	1
11	15.0	100.0	10	20	10	199.124	12.468	0.01532	110	0	1
12	20.0	197.0	12	24	12	209.546	13.928	0.002085	100	0	1
13	20.0	197.0	12	24	12	209.546	13.928	0.002085	100	0	1
14	20.0	197.0	12	24	12	209.546	13.928	0.002085	100	0	1
15	3.00	12.00	2	4	4	21.145	16.193	0.09553	30	0	1
16	3.00	12.00	2	4	-9	21.145	16.193	0.09553	30	0	1
17	3.00	12.00	2	4	4	21.145	16.193	0.09553	30	0	1
18	3.00	12.00	2	4	3	21.145	16.193	0.09553	30	0	1
19	3.00	12.00	2	4	4	21.145	16.193	0.09553	30	0	1
20	20.0	155.0	12	24	-20	275.606	12.360	0.08898	100	0	1
21	20.0	155.0	12	24	12	275.606	12.360	0.08898	100	0	1
22	40.0	400.0	48	60	48	577.537	14.253	0.0007365	440	0	1
23	40.0	400.0	48	60	48	577.537	14.253	0.0007365	440	0	1
24	10.0	76.00	6	12	-2	133.919	16.193	0.01508	45	0	1
25	10.0	76.00	6	12	-2	133.919	16.193	0.01508	45	0	1
26	10.0	76.00	6	12	-2	133.919	16.193	0.01508	45	0	1
27	10.0	76.00	6	12	-2	133.919	16.193	0.01508	45	0	1
28	10.0	76.00	6	12	-2	133.919	16.193	0.01508	45	0	1
29	10.0	76.00	6	12	-2	133.919	16.193	0.01508	45	0	1
30	20.0	155.0	12	12	12	275.606	12.360	0.008898	100	0	1
31	20.0	155.0	12	12	12	275.606	12.360	0.008898	100	0	1
32	35.0	350.0	24	24	24	517.669	11.892	0.005220	250	0	1