Coordinating the Optimal Discount Schedules of Supplier and Carrier

by

Yi Ke

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Transportation is important in making supply chain decisions. With the careful consideration of transportation expenses, the performance of each supply chain member, as well as the entire supply chain, could be improved significantly. The purpose of this research is: 1) to explore and identify the various situations that relate to replenishment and transportation activities; and 2) to reveal the strength of the connection between purchase quantity and transportation discounts, and integrate the two discounts to enhance supply-chain coordination. The problem is analyzed and categorized into four representative cases, depending on transportation. To aid the supplier or the carrier to determine the discount that should be offered, in light of the buyer's reaction to that discount, decision models are proposed under three different circumstances.

First, assuming a single product, we investigate the quantity discounts from the *supplier's* perspective, via a noncooperative game-theoretical approach and also a joint decision model. Taking into account the price elasticity of demand, this analysis aids a sole supplier in establishing an all-unit quantity discount policy in light of the buyer's best reaction. The Stackelberg equilibrium and the Pareto-optimal solution set are derived for the noncooperative and joint-decision cases, respectively. Our research indicates that channel efficiency can be improved significantly if the quantity discount decision is made jointly rather than noncooperatively. Moreover, we extend our model in several directions: (a) the product is transported by a private fleet; (b) the buyer may choose to offer her customers a different percentage discount than that she obtained from the supplier; and

(c) the case of multiple (heterogeneous) buyers. Numerical examples are employed, here and throughout the thesis, to illustrate the practical applications of the models presented and the sensitivity to model parameters.

Secondly, we consider a situation with a family of SKUs for which the supplier will offer a quantity discount, according to the aggregate purchases of the product group. Management of those items is based on the modified periodic policy. From the *supplier's* point of view, what are the optimal parameters (breakpoint and discount percentage)? For deterministic demand, we discuss the cases in which demand is both constant and price-sensitive. First as a noncooperative Stackelberg game, and then when the two parties make the discount and replenishment decisions jointly, we illustrate the impact of price-sensitivity and joint decision making on the supplier's discount policy.

The third approach studies the case in which transportation of the goods by a common carrier (a public, for-hire trucking company) is integrated in the quantity discount decisions. In reality, it is quite difficult for the carrier to determine the proper transportation discount, especially in the case of LTL (less-than-truckload) trucking. This is not only because of the "phantom freight" phenomenon, caused by possible over-declaration of the weight by the shipper, but also due to the fact that the discount relates to both transportation and inventory issues. In this research, we study the problem of coordinating the transportation and quantity discount decisions from the perspectives of the parties who *offer* the discounts, rather than the ones that take them. By comparison of the noncooperative and cooperative models, we show that cooperation provides better overall results, not only to each party, but also to the entire supply chain. To divide the extra payoffs gained from that cooperation, we further conduct a coalition analysis, based upon the concept of "Shapley Value." A detailed algorithm and numerical examples are provided to illustrate the solution procedure.

Finally, the thesis concludes with comprehensive remarks. We summarize the contributions of this thesis, show the overall results obtained here, and present the directions that our research may take in the future.

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This thesis contains not only research topics; it is a significant milestone in my life and represents more than two decades of studies and works. Throughout these years, I have learned that there are so many people dedicating their lives to science and aspiring to share their brilliant ideas with the world. They are the initial reason of starting my research career; I wish to acknowledge them, along with numerous other remarkable individuals who keep on inspiring me in various ways.

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Dedication

This is dedicated to Enlong Ke and Xinqing Wang, my beloved parents.

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Chapter 1

Introduction

A Supply Chain is a complicated system that physically manufactures and transports products to customers. Interacting with other social systems, a typical supply chain involves multiple stages, diverse participants, and many other factors. As the essential link between stages, transportation plays a vital role in any supply chain system (Chopra and Meindl, 2010). This research targets the truck mode, as an important case, to focus on determination of the optimal discount schedule for transportation.

1.1 Problem statement

To transport products to another supply chain member, a company may either ship by its own vehicle (the case of "private fleet") or employ a public, for-hire transportation service provider (called a "common carrier"). When a private fleet is used, whoever is in charge of the shipment needs to incorporate the transportation expenses in his/her own operational decision making, while, in the other case, it is the common carrier's responsibility to determine how to charge for the transportation services. Details about the different decisions and cost structures related to these two situations are discussed in Section 1.3.

More specifically, there are two types of common-carrier trucking services, Less-than-truckload (LTL) and truckload (TL) freight. These are offered by companies that are quite distinct. A TL carrier moves goods directly from origin to destination. An LTL carrier, however, requires a network of terminals which permit collection of products from more than one customer. Loads are combined at an origin terminal, delivered on a single vehicle to a destination terminal, and deconsolidated there for local delivery to the ultimate consignees (buyers or receivers of the goods)(Figure1.1).

1.2 Why does transportation matter?

In the recent few decades, significant attention has been paid to the coordination among members in a supply chain. Quantity discounts, having been proved to be a useful coordinating mechanism, have been broadly analyzed and explored, from the perspective of both operations management and marketing (Choi et al. 2005). Not only the two parties involved in a purchasing activity, but also the entire distribution channel can benefit

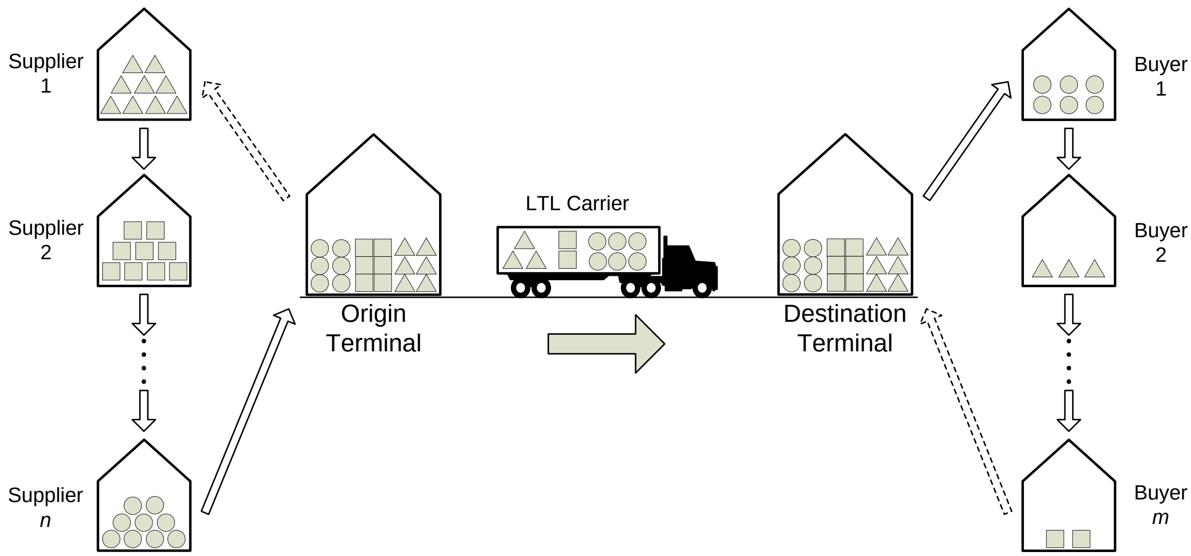


Figure 1.1: Illustration of an LTL transportation operation

from employing quantity discounts. However, although it is a major portion of the total relevant costs, the transportation expense has frequently been omitted or assumed fixed. Consideration of transport has often been neglected, both when the supplier makes pricing decisions, and when the buyer decides replenishment quantities. These omissions may result in unexpected losses related to the reordering decisions.

To illustrate the impact of transportation cost on lot-sizing decisions, consider Example 1.1. This simple distribution channel contains three parties: a supplier, a buyer, and a carrier, as depicted in Figure 1.2. The supplier, denoted by S, manufactures a certain product and sells it according to an established quantity-discount schedule: the product is sold at a unit price of \$ 400 for any order less than or equal to 39 units, and \$ 360 per

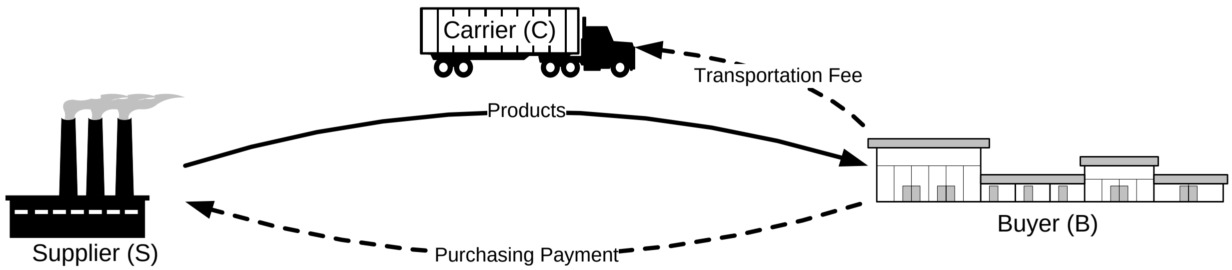


Figure 1.2: A simple distribution channel

unit for any order of 40 or more units, respectively. The buyer, denoted by B, purchases 120 units of this product annually from S. The carrier, denoted by C, is a common carrier that provides trucking service to ship the product from S to B, based on a predetermined freight tariff. (Here, we assume that B pays for the transportation.) Suppose that each unit of the product weights 500 pounds. The transportation cost per hundredweight (100 pounds), i.e. cwt, is \$ 10 for shipping a weight of less than 30 000 pounds, but \$ 7/cwt for any larger weight. Note that this is an all-unit discount.

Suppose B's ordering cost is \$ 300 per order, and inventory holding cost per unit is 20% of the purchase price. Then, according to the traditional economic order quantity (EOQ) model, B's EOQ based on the undiscounted and discounted unit purchase prices can be calculated as 30 units and approximately 32 units per order, respectively. Naturally, the calculated value of 32 units is not eligible for the purchase-quantity discount.

Now consider the situation that B is aware only of S's quantity discount but not C's transportation discount schedule. This is because S, and not B, discussed the transportation arrangements with C. To determine the optimal replenishment quantity, the relevant

Table 1.1: Cost information for Example 1.1 *

Order quantity	30 units	40 units	50 units	60 units
Ordering cost	$ 1 200	$ 900	$ 720	$ 600
Inventory cost	$ 1 200	$ 1 440	$ 1 800	$ 2 160
Purchase cost	$ 48 000	$ 43 200	$ 43 200	$ 43 200
Transportation cost	$ 6 000	$ 6 000	$ 5 040	$ 4 200
Total relevant cost	$ 56 400	$ 51 540	$ 50 760	$ 50 160

* All costs are annual figures.

costs that need to be taken into account include those of ordering, inventory holding, and purchasing. By comparing the total relevant cost (TRC) calculated by the undiscounted EOQ (30 units) and the breakpoint (40 units), B would determine a reorder quantity of 40 units, with the relevant costs of $ 45 540. At this quantity, the transportation cost is $ 6 000, which brings B's total costs to $ 51 540. However, we may also notice that, when B increases the replenishment quantity to 50 units, TRC decreases to $ 50 760, which is $ 780 lower than the value obtained in the previous situation. Actually, if B had the knowledge of both discounts, and had included the consideration of transportation cost in the first place, an optimal order quantity of 60 units would have been found. At this replenishment level, the total relevant costs are minimized at $ 50 160. This is $ 1 308 lower than when Q = 40 units, and $ 600 less than even TRC for a quantity of 50 units. The details of the cost information for this example can be found in Table 1.1.

This improvement is due to a specific feature of the transportation discount, the so-called “bumping clause” phenomenon. As stated previously, the transportation rates offered by C are \$ 10/cwt for lots which weigh less than 30 000 pounds, and \$ 7/cwt for any greater weight. Then the transportation costs per shipment and per year are as depicted in Figure 1.3. Note that when the shipping weight is between 210 cwt and 300 cwt, it is advantageous to over-declare the weight to be 300 cwt, which would result in a lower transportation charge per year.

The savings caused by this over-declaration is demonstrated by the difference between EF and EG, which, reflecting the annual transportation charge, is the downward-sloped line MN. The amounts of 210 cwt and 300 cwt are defined as WBT (smallest weight where over- declaration is advantageous) and MWT (stated minimum weight to obtain discount), respectively. The phenomenon of over-declaration is referred to as evoking the “bumping clause,” or as the shipment of “phantom freight.” Because of the bumping clause, it becomes much more complicated for the carrier to determine the LTL discount schedule that should be offered, than it is for a supplier to set the traditional purchase-quantity discount. The details regarding this problem are explained later in this chapter.

As demonstrated by this simple example, there is a strong relationship between inventory and transportation decisions. With the consideration of this relationship, buyers would choose better lot-sizing alternatives than if only inventory-related issues were considered.

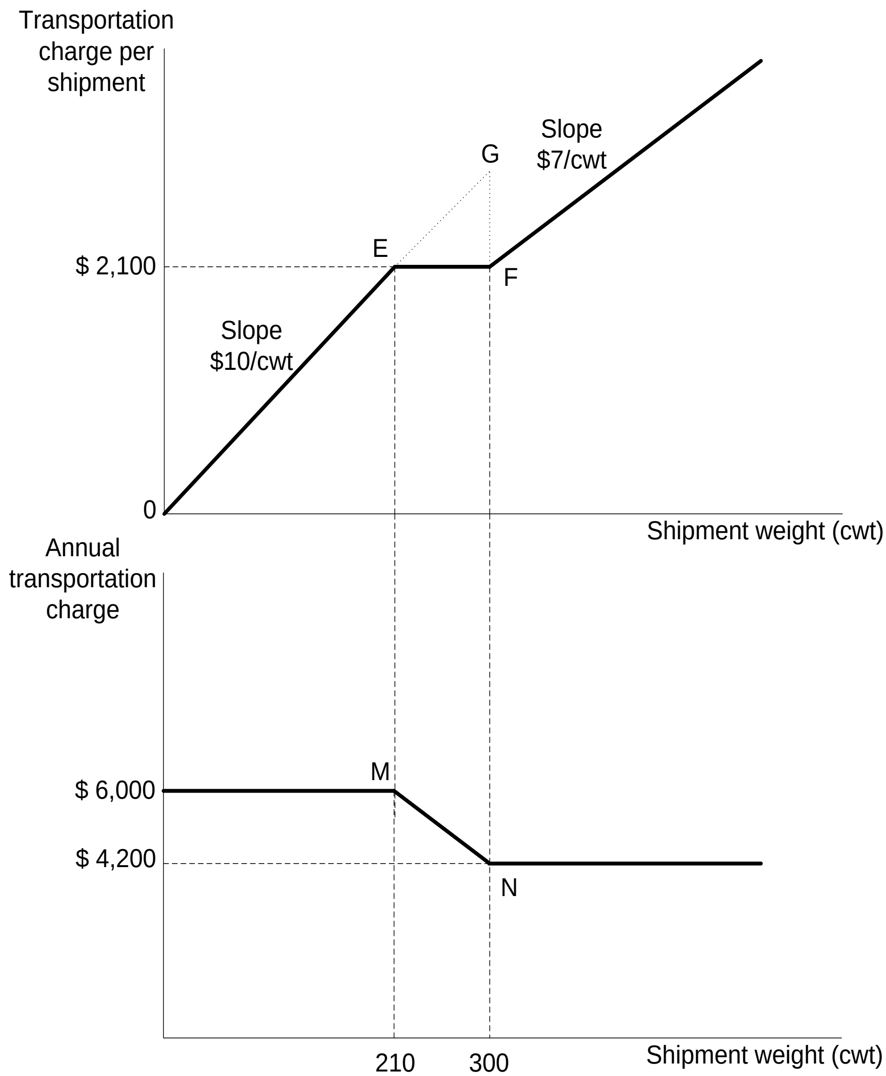


Figure 1.3: Transportation charges for the simple numerical example

1.3 Motivation and objectives

As mentioned in the previous sections, integration of the purchase-quantity and transportation discounts is vital to buyers. In fact, the impacts are not restricted only to buyers. Thinking from the supplier's point of view, the increase of order sizes would be undoubtedly welcome. With the same order frequency, larger order sizes would result in higher total annual profits. On the other hand, with no change in annual demand, smaller order frequency would result in lower annual order processing and/or fixed manufacturing setup charges. Another benefit to the supplier is that the buyer's revised order pattern may reduce the supplier's opportunity cost of holding inventory (Lee and Rosenblatt, 1986). Monahan (1984) also mentioned that larger order sizes would cause shifts in the magnitude and timing of the invoice payments by buyers. These types of shifts might give suppliers the possibility of using the buyers' money, not only more, but also earlier.

Similarly, LTL carriers could potentially benefit, too, not just because of the reduction in annual fixed operating expenses caused by fewer shipments each year. The shipment-consolidation-related costs could also be decreased. Shipment consolidation is a general technique employed by LTL carriers to combine several smaller shipments into one or more truckloads. It is obvious that a pattern of larger shipments would make it more straightforward to consolidate a full truckload.

Moreover, the achievable improvement of channel efficiency and effectiveness is another motivation. Recall that a supply chain network consists of many parties that provide products and services to fulfill customers' requests. The essential purpose of a successful supply chain is to maximize the overall generated value, which is the difference between the revenue created from end-customers and the costs of the entire supply chain (Chopra and Meindl, 2010). Evidently, the optimization of this value and its division among all involved parties really needs the coordination and cooperation across the supply chain. An exploration of the carrier's role and the effect of its transportation rate would be challenging, yet helpful in this manner.

Another emerging motivation of this research is the "Green" issue. Critical environmental problems confront countries all over the world. Shipment of larger loads can reduce the negative impacts that transportation has on the environment.

In light of the above motivations, the main objectives of this research are:

1) To explore the various situations that relate to replenishment and transportation activities, and thus, to identify the characteristics of each situation; and

2) To reveal the strength of the connection between quantity and transportation discounts, and integrate the setting of these discounts, to enhance the benefits to supply chain members.

1.4 Four cases of transportation problems

Two key players need to be considered in any transportation problem. The shipper (referred to as "supplier" in this thesis) is the party who sends the products, while the consignee (referred to as "buyer") is the party who receives the goods. Recall that products may be transported between successive nodes of a supply chain, i.e. from the supplier to the buyer, by a private fleet or a common carrier. Under the condition of a private fleet, depending upon negotiations between the supplier and buyer, either party may provide the vehicle and be responsible for the transportation decisions and transportation cost.

However, when a common carrier is employed to move the product, the transportation-pricing decisions are made by the carrier. These include decisions on consolidation as well as price and discount-schedule determination. Depending upon circumstances, either the supplier or buyer may select the outside carrier, and then pay that firm for providing the transportation services.

Consequently, the actual situations in reality may involve two players, supplier and buyer, or three players, the supplier, buyer and carrier. This thesis classifies those situations into four cases according to players' participations and decisions. Table 1.2 shows the details for the four cases of any combined transportation-ordering problem.

In Case I, the supplier provides the private fleet. So, with the consideration of transportation expense, the supplier decides his production plan as well as the selling price and relevant quantity discount. The buyer then takes that selling price schedule and determines the order quantity. In Case II, the buyer provides the private fleet. Hence, different from the first case, the buyer needs to consider the transportation expense when making replenishment decisions. Case III reflects the situation that the supplier pays for common-carrier transportation services. In this case, the carrier makes all transportation-pricing decisions

Table 1.2: Classification of decisions for the four cases

Case #	C^1 or P^2	$Trans^3$ paid by	Buyer	Supplier	Carrier
I	P	Supplier	a) Order quantity considering EOQ and quantity discount	a) Selling price and relevant quantity discount; b) Transportation costs; c) Production plan	N/A
II	P	Buyer	a) Order quantity considering EOQ, quantity discount, and transportation costs	a) Selling price and relevant quantity discount; b) Production plan	N/A
III	C	Supplier	a) Order quantity considering EOQ and quantity discount	a) Selling price and relevant quantity discount; b) Production plan and shipment plan	a) Transportation discount policy; b) Transportation costs; c) Consolidation, vehicle routing and other issues
IV	C	Buyer	a) Order quantity considering EOQ, quantity discount, and transportation payments	a) Selling price and relevant quantity discount; b) Production plan	a) Transportation discount policy; b) Transportation costs; c) Consolidation, vehicle routing and other issues

1. 'C' denotes common carrier;
2. 'P' stands for private fleet; and
3. 'Trans' stands for transportation.

and offers a transportation discount schedule. Then, the supplier takes the transportation discount schedule, determines his own product plan, and offers the selling-price schedule. The buyer's decisions are the same as those in the first case. In Case IV, the buyer pays for the service of a public, for-hire trucking company. Thus, the buyer determines the replenishment policy based on the transportation and selling discount schedules offered by the common carrier and the supplier, respectively. The motivating example of Table 1.1 is one instance of this case.

During any decision-making process, cost is always a vital factor that impacts the final outcomes. Corresponding to our four cases, the relevant cost structures that need to be considered by each player are listed in Table 1.3. As shown, the main costs considered in our research consist of those related to production, ordering, inventory holding, and transportation. Sahin et al. (2009) discussed the details of various transportation-related costs. For our purposes, we classify those costs into two portions, fixed and variable costs. Further details will be given when these costs appear in our models.

On the basis of the four cases in Table 1.2, our research intends to develop decision models. In each case, conditions will be found, such that the entire distribution channel and every party involved would benefit.

Table 1.3: Cost analysis for the four cases

Case #	Parties	Production cost	Ordering cost	Inventory cost	Transportation fixed cost	Transportation variable cost
I	Buyer	-	✓	✓	-	-
	Supplier	✓	✓	✓	✓	✓
	Carrier	-	-	-	-	-
II	Buyer	-	✓	✓	✓	✓
	Supplier	✓	✓	✓	-	-
	Carrier	-	-	-	-	-
III	Buyer	-	✓	✓	-	-
	Supplier	✓	✓	✓	-	✓*
	Carrier	-	-	-	✓	✓
IV	Buyer	-	✓	✓	-	✓*
	Supplier	✓	✓	✓	-	-
	Carrier	-	-	-	✓	✓

* Transportation fees paid to the carrier.

1.5 Organization of the thesis

Figure 1.4 outlines the organization of this thesis. Chapter 1 has begun with a brief introduction of this research, including problem statement, motivation and objectives. Then the focal research was classified into four cases, which captures the essential relationships among the players involved, and yields a better understanding of the relevant decisions and cost structure for each party. The literature review of Chapter 2 exhibits historical and

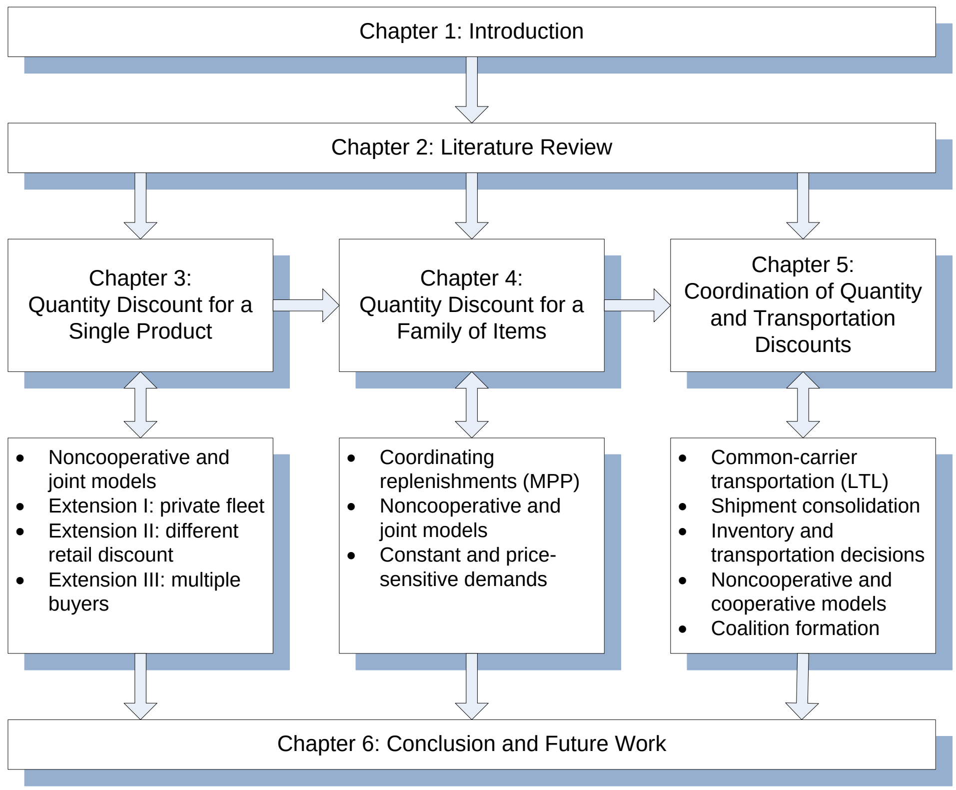


Figure 1.4: Organization of the thesis

contemporary approaches related to this topic, as well as their advantages and disadvantages. Also, it reflects on how those previous approaches inspire out research. Chapters 3 though 5 report our research contributions. The topic is investigated from three different aspects: the quantity discount for a single product, for a family of items, and coordinating the quantity and transportation discounts. Numerical examples are employed to illustrate the proposed models throughout the thesis. Finally, Chapter 6 emphasizes the findings of this research and its hoped-for significance. That chapter summaries the work we have done and drafts a blueprint for future research.

Chapter 2

Literature Review

This chapter presents an overview of the literature in analyzing both purchasing and transportation discounts. The first section discusses achieving supply chain coordination by using a quantity discount, specifically, the optimal quantity discount problem mainly from the supplier's standpoint. Then, the second portion of the review includes the efforts to optimize replenishment decisions with consideration of transportation expenses. Finally and more specifically, a typical game-theoretical framework, a Stackelberg game model, is introduced along with a review of applications in discount pricing decisions.

2.1 Supply chain coordination with quantity discounts

Supply chain coordination deals with the relationships among the supply chain members, and concerns how to achieve the benefits of overall goals with joint efforts. Many different coordination mechanisms have been studied in the literature. In this review, we omit parts of the relevant literature, such as supply chain coordination with contracts (Cachon and Zipkin, 1999; Cachon and Lariviere, 2001, 2005; Cachon and Kök, 2010), quick response (Iyer and Bergen, 1997; Krishnan et al., 2010), accurate response to early sales (Fisher and Raman, 1996), continuous replenishment planning (Vergin and Barr, 1999; Raghunathan and Yeh, 2001; Yao and Dresner, 2008), vendor managed Inventory (Gümüş et al., 2008; Darwish and Odah, 2010; Bookbinder et al., 2010), and information sharing within a supply chain (Aviv and Federgruen, 1998; Gavirneni et al., 1999; Lee et al., 2000; Lee and Whang, 2000). Instead, we focus on the quantity discounts, as it is the main concern of our research.

The quantity discount problem is prevalent in the inventory management realm. Traditional analyses of this problem considered primarily the viewpoint of the buyer: calculating the optimal order quantity, which minimizes the buyer's total relevant costs. However, by switching to the perspective of the *supplier*, the situation becomes significantly different.

Monahan (1984) presented a quantity discount pricing model to maximize a vendor's incremental net profit and cash flow by adjusting the pricing structure to encourage buyers to increase the order size. His model was described, however, in comments by Banerjee (1986) and Joglekar (1988), as a single-item, single-customer, and single-vendor model based on several "unreasonable implicit assumptions". Therefore, Lee and Rosenblatt (1986) generalized Monahan's model and developed an algorithm to solve the supplier's joint ordering and price discount problem.

Lal and Staelin (1984) addressed this problem by constructing a unified pricing policy which motivates the buyer to increase the order quantity, so that the costs for both the buyer and supplier are decreased. Characterizing the range of order sizes and prices which reduce costs for buyer and supplier, Dada and Srikanth (1987) studied and developed a model based upon Lal and Staelin's model. The model was extended to allow a price-dependent inventory holding cost, for which the corresponding calculation is more complicated than for an independent and constant holding cost.

Chakravarty and Martin (1988) developed a discount pricing model, in which a sole supplier has an infinite production rate, and a single buyer utilizes a periodic review system. Assuming a constant and uniform demand rate, a negotiation mechanism was introduced to allocate a sharing of the savings. A few years later, Chakravarty and Martin (1991) relaxed the assumption of constant demand to a demand that varies with respect to the retail price, which is addressed as an additional decision variable independent of the discounted wholesale price.

Weng (1995) attempted to combine the research of marketing and operations literature. He proposed a coordination mechanism using an order-quantity discount and a periodic franchise fee. Chen et al. (2001) considered a two-echelon system, in which a supplier distributes a single product to multiple retailers who in turn sell to consumers. They showed that Weng's scheme is not guaranteed to coordinate the channel when the retailers are not identical. Through comparison of a centralized and decentralized systems, they illustrated that the optimum level of channelwide profit can be also achieved when the system is decentralized, but only via periodically charged fixed fees and a nontraditional discount pricing scheme. That nontraditional discount is the sum of three discount components based on retailer's: 1) annual sales volume, 2) order quantity, and 3) order frequency.

Li and Liu (2006) developed a discount model for a supplier-buyer system with multiple periods and probabilistic customer demand. They showed the benefit of making joint decisions, and designed a method to divide this extra benefit between the buyer and supplier. That method can be employed to obtain the optimal quantity discount policy. Lau et al. (2008) studied a situation in which a manufacturer sells products to a very large number of retailers. This manufacturer does not need to coordinate the replenishment cycle with the retailers, and the handling charge paid by the retailers can be reduced when orders are sufficiently large.

This problem has also been discussed from the perspective of game theory. Kohli and Park (1989) studied the quantity discount problem under the framework of a cooperative two-player bargaining game. Their research aimed to maximize the joint cost saving (efficiency gain) of the monopolistic supplier and the single buyer (or a homogeneous group of buyers), and then, divide the cost-saving to the two players corresponding to each player's "bargaining power" (the degree to which one party has the capability to dominate the situation over the other). More recently, a specific game model, a Stackelberg model, has been broadly applied in determining the optimal quantity discount schedule. The details about the basic facts and implementation of this game-theoretic model are addressed in Section 2.3.

2.2 Joint decisions on inventory and transportation

Most of the literature reviewed in the last section did not take transportation costs into account when determining the quantity discount schedule. Normally, the inventory and transportation decisions are made by different segments of an organization. However, as illustrated in the first chapter, the interaction between these two decisions is crucial to minimize an organization's total relevant costs, and thus, to improve the overall performance. The impacts of this interaction are gradually emerging in the practical decision process. Many interested parties and scholars have come to realize the importance of integrating transportation issues into inventory decisions, especially those on replenishment (Russell and Cooper, 1992; Carter et al., 1995a; Carter and Ferrin, 1996). This section covers research on those joint decisions of inventory and transportation.

Meyer et al. (1959) were the first to consider the relationship between the two decisions and integrate the inventory approach in determining the values of different transportation modes. Constable and Whybark (1978) presented a mathematical model including transportation attributes and inventory policy parameters, where backorders are allowed.

Dissimilar to other literature, Burns et al. (1985) emphasized the distribution procedure, and developed an analytic method for minimizing all transportation and inventory costs involved in shipping products from a supplier to many customers. Two distribution strategies, direct shipping and peddling, were investigated and compared. In addition to the exact joint determination, a heuristic procedure was presented and evaluated. Buffa and Munn (1989) proposed a model whose freight-rate involved shipping distance and weight. Their algorithm obtained the optimal reordering time, such that the total logistic costs, consisting of inventory and transportation costs, were minimized. Bookbinder et al. (1989) employed a spreadsheet simulation model and a linear programming model to obtain the best decision policy in inventory, warehousing, and transportation for a Canadian fine-paper distributor.

Russell and Krajewski (1991) presented an analytical algorithm for finding the optimal order quantity that reflects both transportation economics and quantity discounts. (This algorithm was adjusted by Carter et al. (1995b) in adapting to anomalous cases existing in the freight rate schedule.) One year later, Russell and Krajewski (1992) proposed a mixed integer linear programming model to obtain a coordinated replenishment policy including multiple items from a common supplier, with the consideration of both quantity and transportation discounts.

Tersine and Barman (1991) structured the quantity and freight discounts into replenishment decisions, concerning dual discount situations with an all-unit or incremental quantity discount, and all-weight or incremental freight discounts. Arcelus and Rowcroft (1992) compared the impacts of three cases of freight rate structures and quantity discounts on a profit-maximizing firm, with or without disposals. Considering the supplier's standpoint, Martin (1993) presented a model to obtain the quantity discounting policy based on the assumption of a single-buyer situation, a step-functional shipping cost with multiple break points, and an annual demand linearly related to the unit price. With the consideration of possible frequency or time consolidations, Speranza and Ukovich (1994) aimed to derive a shipping strategy for multiple products on a single link by analyzing the tradeoff between transportation and inventory costs, which are both influenced by the frequency of shipment. Based on the classical EOQ model, Tersine et al. (1995) analyzed a firm's lot-sizing problem by integrating quantity and transportation discounts into a restructured discount schedule. Decomposition rules were provided, so that the composite model can be disaggregated to other specific cases.

Çetinkaya and Lee (2000) and Chaouch (2001) proposed an analytical model for joint inventory and transportation decisions in a vendor-managed inventory system. Swenseth and Godfrey (2002) noticed the opportunity of over-declaring the shipment weight, so that a TL shipment or another LTL weight-break can be reached. They incorporated two freight rate functions, the inverse (a constant charge per shipment) and the adjusted inverse (TL freight rate plus an LTL-adjusting term), into the determination of a firm's inventory replenishment decision. In the work of Zhao et al. (2004), the multiple uses of vehicles were considered along with the transportation cost in optimizing the long-term average costs for a supplier-retailer logistic system. Without ignoring the over-declaration options in shipping products, Abad and Aggarwal (2005) developed a model to determine the reseller's selling price in addition to the regular lot-sizing decisions. Elhedhli and Benli (2005) included a carload discount schedule in an optimal lot-sizing procedure and analyzed the resulting total cost function.

Wang et al. (2006) proposed a mixed-integer programming mathematical model to deal with a multi-item, single-vendor, single-buyer problem such that the cost of the entire system is minimized. Cardós and Garcia-Sabater (2006) accounted for the operational and client service constraints in designing an inventory replenishment policy for a consumer products retail chain. Rieksts and Ventura (2008) made the efforts of studying TL and LTL simultaneously to obtain further reductions in the overall average costs. Darwish (2008) examined the integrated effects of purchasing and transportation issues on inventory decisions under the condition of a stochastic demand rate. Four different combinations of quantity and transportation discounts for a continuous review system are presented and discussed in that paper. Ouyang et al. (2008) analyzed an integrated inventory system with a price-sensitive demand rate when both trade credit and freight rate are linked to the order quantity.

Toptal (2009) presented a replenishment decision model with the consideration of a stepwise freight cost and an all-unit quantity discount. Then, the model was applied to a single-period problem under several scenarios. Hwang (2009) analyzed a dynamic lot-sizing model considering production and inbound transportation to the VMI warehouse. He showed that the most important parameter influencing the system is the minimum replenishment quantity, and that the minimum replenishment quantity policy is successful only when large-size demands are guaranteed based on collaborations. Chang (2011) showed that the existing algorithm for determining the optimal lot sizes, incorporating quantity and freight discounts, may lead to a suboptimal solution. Key steps of the algorithms were then modified to achieve a global optimal solution.

Quantity and transportation discounts have thus been studied and discussed for decades, as noted by our survey in this section. However, not much literature integrates decisions by the *carrier* into their investigation. The coordination of the discount schedules for supplier and carrier is thus the main purpose of our research. In order to perform the analysis, a game theoretical framework is introduced and employed. Some basics of this specific game-theory model, as well as its implementations in the quantity discount problems, are explained in the next section.

2.3 Implementation of the Stackelberg Model

2.3.1 Basics of the Stackelberg game

Developed by von Stackelberg (1934), such a game is a dynamic model of duopoly in which a dominant player (leader) moves first and a subordinate (follower) moves subsequently (Gibbons, 1992), having been informed of the dominant player's move. The Stackelberg equilibrium analyzes and specifies the behaviors of the players when one of them has the ability to enforce his/her strategy on the other. Acting independently and noncooperatively, both players try to maximize their own payoffs (Chiang et al., 1994).

"Perfect information" is one of the most important properties of a Stackelberg game. Under this condition, the follower has an absolute knowledge about the leader's action, and the leader is also aware of the follower's reaction.

As applied in economics, the equilibrium of a Stackelberg game corresponds to solutions of the maximization problem:

$$\max_{a_L \in A_L} u_L(a_L, R_F(a_L)), \tag{2.1}$$

subject to:

$$R_F(a_L) \in \arg\max_{a_F \in A_F} u_F(a_L, a_F). \tag{2.2}$$

where A_i is the set of actions for player i; $a_i \in A_i$ is the specific action of player i; u_i is the payoff function that represents player i's preferences; $R_F(a_L)$ is the follower's reaction to the leader's action; player L refers to the leader and player F refers to the follower (Gibbons, 1992; Osborne and Rubinstein, 1994). This type of model can be solved by a certain methodology called "backward induction," which starts from the follower getting the move at the second stage of the game. Given the action a_L previously chosen by the leader, the follower is now facing the problem shown in Equation (2.2).

Suppose that for each $a_i \in A_i$, there is only one optimal solution, $R_F(a_L)$, to this follower's problem. That is the follower's reaction, and also the best response to the leader's action. With perfect information, the leader identifies this follower's game and the optimal solution to it. So, knowing the follower's reaction, the leader's problem at the first stage becomes the optimization problem depicted by Equation (2.1).

Assume that this problem for the leader also has a unique solution, a_L^*. Then, the backward-induction outcome of this game is $(a_L^*, R_F(a_L^*))$ (Gibbons, 1992).

2.3.2 Application of the Stackelberg model to the discount-pricing problem

In the past few decades, the Stackelberg model has been applied to various areas in supply chain management, such as inventory and production issues, outsourcing in dynamic environments, as well as pricing strategies. This section reviews the utilization of this model in generating the supplier's quantity discount decisions. Because of the strong connection to our research, it is helpful to carry out a thorough retrospection of these applications.

Table 2.1 summarizes a few of the Stackelberg model implementations in quantity discount pricing decisions. Note that the summary is carried out on the basis of five characteristic criteria. First of all, for a leader-follower approach, such as the Stackelberg model, the power structure and the dominant player of the focal situation become very important. Two examples of that power structure are: 1) the "manufacturer-Stackelberg" ("[mS]") game, in which a manufacturer, or supplier, is the dominant player that acts as the leader in the Stackelberg game; and 2) the "retailer-Stackelberg" ("[rS]") game, in which the dominant player is the retailer instead. Next, the model size is always a vital criterion in formulating an optimization problem, as it may significantly affect the difficulty of the solution procedure.

Thirdly, various types of discounts involve different cost information, which would greatly diversify the model formulations. Basically, two types of discount schedules exist in Business-to-Business transactions: lot-size-based and volume-based quantity discounts. And the discount based on the lot size can be further divided into two schemes: all-unit and incremental discounts. Moreover, the characteristics of demand information may have great impacts on the outcome of a pricing decision, not to mention the resulting complications. Lastly, the cost structures for both supplier and buyer need to be considered in modeling the discount problem. As some of the literature has been reviewed in the first section of this chapter, brief discussions of others are conducted as follows.

Chiang et al. (1994) investigated the problem in both noncooperative and cooperative models. Stackelberg equilibrium and Pareto Optimality criteria were respectively employed in their analysis to find a set of optimal strategies. Parlar and Wang (1994) studied the discount decisions for a supplier with a group of homogeneous customers. Starting from the Stackelberg equilibrium using a general quantity discount schedule, the paper indicated that both the supplier and buyers can benefit from the discount, as long as the supplier offers such a discount schedule that induces the buyers to order more than the EOQ.

Wang (2002) extended the work of Parlar and Wang (1994) to the case of *multiple* groups of homogeneous buyers. Under the framework of the Stackelberg game, the supplier's discount schedule can be formulated and calculated from a simple non-linear programming model with the consideration of minimizing the buyer's total costs. The supplier

Table 2.1: Summary of application of Stackleberg model in supply chain discount pricing decisions

Papers[1]	Model sizes[2]		Discount types	Number of break points	Demand types	Cost Structures	
	# of suppliers	# of buyers				Suppliers	Buyers
Rosenblatt and Lee (1985)	1	1	Linear	N/A	Constant, continuous	Ordering, Inv. holding, Acquisition	Purchasing, Ordering, Inv. Holding
Chiang et al. (1994)	1	1	All-unit	1	Deterministic and constant	Ordering, Inv. Holding	Purchasing, Ordering, Inv. Holding
Parlar and Wang (1994)	1	N	All-unit	N	Linear, price-sensitive	Set-up, Ordering, Inv. Holding, Acquisition	Purchasing, Ordering, Inv. Holding
Wang (2002)	1	N	All-unit	N	Deterministic and constant	Set-up, Ordering, Inv. Holding, Acquisition	Purchasing, Ordering, Inv. Holding
Viswanathan and Wang (2003)	1	1	Quantity, Volume, Combined	1	Price-sensitive	Set-up, Ordering, Inv. Holding, Acquisition	Purchasing, Ordering, Inv. Holding
Wang and Wang (2005)	1	N	All-unit, Incremental	N	Price-sensitive	Ordering, Acquisition, Inv. related	Purchasing, Ordering, Inv. Holding
Yang and Zhou (2006)	1	N	All-unit, Incremental	1	Linear, price-sensitive	Operation costs without specifying	Operation costs without specifying
Lau et al. (2007)	1	1	Volume	1	Linear and Stochastic, price-sensitive	Manufacturing (uncertain)	Purchasing
Qin et al. (2007)	1	1	Volume	1	Linear, price-sensitive	Set-up, Ordering, Inv. Holding	Purchasing, Ordering, Inv. Holding, Franchise fees
Zhou (2007)	1	1	Regular, Fixed percentage, Incremental volume, and Fixed marginal-profit-rate	1	Stochastic, price-sensitive	Acquisition (manufacturing)	Purchasing
Chen (2010)	1	1	one-time wholesale	0	Stochastic	Acquisition	Purchase, Shortage
Hsieh et al. (2010)	1	N	Short-term [6]	1	Linear vs Price-sensitive	Ordering, Inventory	Purchase, Inv holding, Ordering

1. All papers are [mS] games with the supplier dominant, except that Lau et al. (2007) is an [rS] game with the retailer dominant.
2. The number of products is eliminated because all models only consider a single product.
3. Supplier's order cost refers to supplier's cost of processing buyer's orders.
4. Supplier's opportunity cost of capital.
5. "Inv." is short for "Inventory".
6. This short-term discount is offered to encourage retailers to place their orders other than at the beginning of the replenishment cycle.

then provides this unified discount schedule to multiple homogeneous buyers, each with a different demand and cost structure. Several critical features of the model were studied and explained.

Under the condition of deterministic but price-sensitive demand, Viswanathan and Wang (2003) investigated a single-supplier, single-buyer distribution channel. Both the quantity and volume discounts, as well as a simultaneous combination of the two discounts, were studied in that paper. Wang and Wang (2005) integrated the research of Wang (2002) and Viswanathan and Wang (2003). Analyzed in this 2005 paper is the situation of one supplier selling a single product to multiple groups of homogeneous buyers with price-sensitive demand.

Yang and Zhou (2006) focused on a two-echelon system with one monopoly supplier and several groups of homogeneous buyers. Aiming to improve both the channel profit and each player's profit, two unified discount pricing schemes, the regular quantity discount and the incremental volume discount, were developed based on a manufacturer-Stackelberg game. Qin et al. (2007) considered a supply chain consisting of a single buyer (or a group of homogeneous buyers) with price-sensitive demand, purchasing one product from a sole supplier. Assuming a linear inverse demand function, this paper compared the system profit obtained from the volume discount policy when the supplier and buyer(s) work independently, to that policy in the case that they work jointly. Considering a stochastic price-sensitive demand situation, Zhou (2007) proposed four quantity-discount pricing policies. Those policies were compared with respect to the two parties' profits and the channel efficiency.

Chen (2010) analyzed a newsvendor problem, in which a returns policy with a wholesale-price-discount scheme is proposed to achieve supply chain coordination. He showed how the manufacturer sets the discounted wholesale price in a returns-discount contract, and how this contract can improve the supply chain efficiency. Hsieh et al. (2010) examined a short-term discount offered at the beginning of the replenishment cycle to multiple retailers in a two-stage distribution system. Results of the authors revealed that the distributor's profit improvement due to coordination is decreasing in the number of retailers and the inventory holding cost rate, but increasing in the price elasticity.

As a unique publication, Lau et al. (2007) analyzed the discount pricing problem by applying the [rS] game rather than the usual [mS] game. They studied a dominant-retailer scenario, under which the retailer (buyer) has relatively higher market power over the manufacturer (supplier). Also, the paper proposed a so-called "reverse quantity discount" scheme, which is offered by the retailer to the supplier to improve the former's profit and coordinate the channel.

2.4 Summary

This chapter has reviewed the existing literature related to the quantity and transportation discount problems. Quantity discounts have been examined for a long time. Our review first focused on the supplier's decision about determining the quantity discount, such that the supplier would gain extra profit, and the buyer would not be worse off. From the buyer's perspective, it is beneficial to make the inventory and transportation-purchase decisions jointly. Those research efforts have solved some problems on a wide scale. Nevertheless, in answering these questions, we end up with more questions: Would the carrier's pricing decision affect the problem? In other words, what would be the impact on the situation of including that firm's procedure to determine the transportation discount? Can decisions on quantity and transportation discounts be coordinated to improve the overall supply chain performance? If the answer is "yes", how can this coordination be done? These are questions that motivate our research.

Another main content of this chapter has been to introduce the particular leader-follower structure of the Stackelberg game model, and its applications to quantity discount problems. The relevant literature exhibits its strength in solving the quantity discount problem. More specifically, the thorough cost analysis and the game-theoretic model proposed by Wang (2002) shows us the opportunities, i.e. that the Stackelberg game framework can be used to serve our purpose. Subsequently, the ideas of this game will be employed to develop our decision models that follow in Chapters 3 to 5.

Chapter 3

Supplier's Optimal Quantity Discount Policy for a Single Item [1]

3.1 Introduction

As discussed in Chapter 2, significant attention has been paid in the past few decades toward the coordination among the members of a supply chain. As a useful coordinating mechanism, quantity discounts have been broadly analyzed and explored, from the perspective of both operations management and marketing (Choi et al. 2005). Not only the two parties involved in a purchasing activity, but also the entire distribution channel, can benefit from employing quantity discounts. In the realm of inventory management,

[1] This chapter is an extended version of Ke and Bookbinder (2012a)

traditional analyses of the quantity discount problem primarily consider the viewpoint of the buyer: calculating the optimal order quantity, which minimizes the buyer's total relevant costs. However, upon switching to the perspective of the supplier, the situation may become significantly different.

Taking into account the price elasticity of demand, the models proposed in this chapter aid a sole supplier in establishing a discrete all-unit quantity discount policy without ignoring the buyer's payoff. The quantity discount problem is investigated, first noncooperatively under the framework of a Stackelberg game, and then jointly based on the multiobjective decision making process.

Li and Wang (2007) reviewed the coordination mechanisms of supply chain systems. According to their categorization, our research applies to deterministic demand in both decentralized and centralized systems. Our work differs from previous approaches in the following ways. First of all, we integrate the *price elasticity of demand* into the determination of a supplier's optimal quantity discount policy. To be specific, the demand is formulated as a function of price elasticity rather than of price. This way of defining demand focuses more on the relationship between the demand and the quantity discount. Moreover, by slightly shifting the product's price within a small range, the elasticity of demand is rather easy to calculate. Secondly, considering the maximization of each party's payoff as the two objectives, our joint decision model is developed based on multiobjective decision making: Pareto-optimal solution sets can thus be obtained and analyzed. Thirdly, we obtain cost savings in transportation. Then, we extend our model to the case in which the buyer offers her customers a retail discount, one whose parameters may differ from the supplier's wholesale discount policy. This is expressed by a multiplicative factor, enabling the buyer to choose a favorable retail discount that maximizes her profit. Moreover, we present an approach for the case of heterogeneous buyers that differs from existing research. Rather than offering distinct discount percentages to each group of buyers, we illustrate that significant improvements can be achieved when some of the groups are combined to one discount level.

The rest of this chapter is organized as follows. A noncooperative game-theoretic model based on the Stackelberg equilibrium is developed and analyzed first. Correspondingly, the determination of a quantity discount policy from the viewpoint of the supplier is presented. Next, a joint decision model is proposed to maximize the combined gains for both the supplier and buyer. For each of the extensions discussed above, we present numerical examples and sensitivity analyses to show the impacts of various parameters. Finally, we conclude the chapter with summary remarks.

3.2 Assumptions and notation

We begin with the following assumptions (the first and third of which are later relaxed):

1. There is a sole supplier, one buyer (or a group of homogeneous buyers), and a single break-point.

2. Just one product is involved. Accordingly, a unique (positive) value of price elasticity of demand is considered; demand is higher when price drops.

3. The buyer is assumed to sell the product at the same discount percentage from her regular price that she received from the supplier.

4. Both buyer and supplier employ the economic order quantity (EOQ) model.

5. An all-unit quantity discount situation is assumed.

6. Both players can estimate the relevant information, such as the product's price elasticity of demand, annual demand rate, the inventory holding cost of each party, as well as their ordering cost, product acquisition cost, and selling price.

7. The buyer is assumed to buy wisely, i.e., she would accept the chance to gain more profit when and wherever possible, for example by taking the discounted price.

Note that our research starts at the point that the market is in equilibrium, i.e. the supplier offers a unit selling price to the buyer, and the buyer orders the products at her EOQ level. This assumption has been extensively used in many relevant papers, for example, Wang (2002), Rubin and Benton (2003), and Qin et al. (2007), to name a few. Our models employ the following notation.

D	Buyer's annual demand.
P	Unit acquisition cost without quantity discount for buyer.
R	Unit selling price for buyer.
v	Unit acquisition cost for supplier. The supplier, as a wholesaler or distributor,

pays this price for the product; when supplier is a manufacturer, this is the production cost per unit. Either way, no discount is available at this stage.

A_i	Order processing cost: $i = B$ or S represents the buyer or supplier, respectively.
H_i	Unit inventory holding cost per year (\$/unit/year).
η	The absolute value of the product's price elasticity of demand.
Q	Buyer's optimal order quantity (EOQ) before discount. $Q = \sqrt{2DA_B/H_B}$.
q	Buyer's actual order quantity, at the discounted price.
q'	Buyer's *best* order quantity, when a discount is possible.
ρ	Price discount ($0 < \rho < 1$).
φ	Quantity break point. When ordering a quantity less than φ, buyer pays the original price P without discount; otherwise, buyer pays the discounted price, $(1-\rho)P$.
TC_j	Buyer's total annual costs: $j = 0$ or 1 represents the value without or with a quantity discount.
π_{ij}	Profit gained from the product by player i for the case j.
Π_i	Payoff gained by the buyer, taking advantage of the quantity discount.

Since, usually, demand increases (decreases) as price decreases (increases), we have $\eta = -\%\Delta D/\%\Delta R$. Also, according to the third assumption, buyer sells the product at the same discount percentage from her regular price that she received from the supplier, hence we have $\Delta R/R = \Delta P/P$. Therefore, $\eta = -\%\Delta D/\%\Delta P$, where $\%\Delta D = \frac{D_{after} - D_{before}}{D_{before}} \times 100\%$

and $\%\Delta P = \frac{P_{after} - P_{before}}{P_{before}} \times 100\%$. Correspondingly, when the supplier offers a discount ρ, the discounted selling price is $(1-\rho)P$; at that price, and considering elasticity, buyer's annual demand becomes $(1+\eta\rho)D$.

Occasionally, we will employ a superscript, s, applying to certain notation, such as ρ, q, and Π_i. $s = (N), (J)$, (t), and (r) will denote values obtained from the noncooperative model, joint decision model, models with transportation considerations, and models that consider a different retail discount, respectively.

3.3 Payoff analysis

In our models, the *payoffs* represent the profits gained or costs saved by each party due to the quantity discount.

3.3.1 Buyer's payoff

From the preceding assumptions and notation, the buyer's total cost before discount is

$$TC_0 = DP + \frac{A_B D}{Q} + \frac{QH_B}{2}. \tag{3.1}$$

where $C = \sqrt{2DA_BH_B}$ is the minimal total cost when the buyer employs an EOQ policy.

Correspondingly, the buyer's profit can be calculated by subtracting total cost from the revenue of selling the product, i.e.,

$$\pi_{B0} = DR - TC_0 = DR - DP - C. \tag{3.2}$$

With discount, the new price is $(1-\rho)P$, and annual demand increases to $(1+\eta\rho)D$. Thus, buyer's total cost is

$$TC_1 = (1+\eta\rho)D(1-\rho)P + \frac{A_B(1+\eta\rho)D}{q} + \frac{q(1-\rho)H_B}{2}. \tag{3.3}$$

As assumed, when purchasing at a discounted price, the buyer establishes her selling price as $(1-\rho)R$. So the buyer's profit with discount is $\pi_{B1} = (1+\eta\rho)D(1-\rho)P - TC_1$. After simplification,

$$\pi_{B1} = (1+\eta\rho)D(1-\rho)(R-P) - \frac{A_B(1+\eta\rho)D}{q} - \frac{q(1-\rho)H_B}{2}. \tag{3.4}$$

Therefore, the buyer's payoff after discount is $\Pi_B(q,\rho) = \pi_{B1} - \pi_{B0}$, or

$$\Pi_B(q,\rho) = D(R-P)\rho(\eta - \eta\rho - 1) + \frac{C}{2}\left[2 - \frac{Q}{q}(1+\eta\rho) - \frac{q}{Q}(1-\rho)\right]. \tag{3.5}$$

3.3.2 Supplier's payoff

Total costs for the supplier include his own acquisition cost, plus the costs of handling the buyer's orders, of holding inventory, and the long-term costs related to inventory. When products are sold to the buyer at the original price without any discount, the profit is

$$\pi_{S0} = DP - Dv - \frac{A_S D}{Q} - \frac{QH_S}{2}. \tag{3.6}$$

With discount, the supplier's profit becomes

$$\pi_{S1} = (1+\eta\rho)D(1-\rho)P - (1+\eta\rho)Dv - \frac{A_S(1+\eta\rho)D}{q} - \frac{qH_S}{2}. \tag{3.7}$$

Then, supplier's payoff after the quantity discount is $\Pi_S(q,\rho) = \pi_{S1} - \pi_{S0}$, or

$$\Pi_S(q,\rho) = -DP\rho(1-\eta+\eta\rho) - Dv\eta\rho + A_S D\left(\frac{1}{Q} - \frac{1+\eta\rho}{q}\right) + \frac{H_S}{2}(Q-q). \tag{3.8}$$

This formulation of the supplier's inventory-related cost was first introduced by (Dolan, 1978), then employed and extended by others, such as Lal and Staelin (1984), Dada and Srikanth (1987), and Wang (2002). Wang (2002) notes that, although the supplier's optimal replenishment policies are not considered, the preceding formulation reasonably approximates the gain to a supplier who uses stationary inventory replenishment polices.

3.4 Noncooperative model

In reality, after the supplier determines his quantity discount policy, the buyer reacts to that policy and chooses her order quantities and schedules. That is why the Stackelberg equilibrium is used to analyze this noncooperative game-theoretic model.

The Stackelberg game is a dynamic model of duopoly in which a dominant player (leader) moves first and a subordinate (follower) moves sequentially (Gibbons, 1992), having been informed of the dominant player's move. Such a framework contains the concept of a "hierarchical equilibrium solution," which analyzes and specifies the behaviors of the players when one of them has the ability to enforce his/her strategy on the other. In our model, the Stackelberg equilibrium allows the supplier, considered as the leader, to construct the quantity discount policy. The supplier thus maximizes his own payoff, taking account that the buyer, considered as the follower, is attempting to maximize her payoff.

3.4.1 Model development

Starting from buyer's perspective, equating $\partial \Pi_B / \partial q$ to zero, we have

$$q' = \sqrt{\frac{2DA_B(1+\eta\rho)}{H_B(1-\rho)}} = Q\sqrt{\frac{1+\eta\rho}{1-\rho}}, \tag{3.9}$$

where Q is the EOQ at the non-discounted price. Note that $q' > Q$.

Similarly, from supplier's point of view, differentiating with respect to the discount rate, ρ, and substituting q' for q, we have

$$\frac{\partial \Pi_S(q',\rho)}{\partial \rho} = -2DP\eta\rho - DP(1-\eta) - Dv\eta$$
$$-\sqrt{\frac{1}{(1+\eta\rho)(1-\rho)}}\left[\frac{H_S Q}{4}\left(\frac{1+\eta}{1-\rho}\right) + \frac{DA_S}{2Q}(\eta - 1 - 2\eta\rho)\right] = 0. \tag{3.10}$$

Then, from Eq.(3.10), the price discount ρ can be written as a function of elasticity, $\rho = \Phi(\eta)$, with other parameters (D, P, v, H_S, and A_S) assumed known and constant.

Once ρ is determined, the quantity break point φ can be decided by maximizing the supplier's profit at this particular discount. The supplier's payoff is

$$\Pi_S(\varphi) = -DP\rho(1-\eta+\eta\rho) - Dv\eta\rho + D\left(\frac{1}{Q} - \frac{\eta\rho}{\varphi}\right)A_S + \frac{1}{2}(Q-\varphi)H_S. \tag{3.11}$$

Solving $\dfrac{d\Pi_S(\varphi)}{d\varphi} = \dfrac{DA_S(1+\eta\rho)}{\varphi^2} - \dfrac{1}{2}H_S = 0$, we have

$$\varphi = \sqrt{\frac{2DA_S(1+\eta\rho)}{H_S}}. \tag{3.12}$$

Eq.(3.12) can be expressed in terms of Q as

$$\varphi = Q\sqrt{\frac{A_S H_B}{A_B H_S}(1+\eta\rho)}. \tag{3.13}$$

Eq.(3.13) indicates that the supplier reaches his maximum payoff when buyer orders the quantity $\varphi = Q\sqrt{\dfrac{A_S H_B}{A_B H_S}(1+\eta\rho)}$ each time. However, only when the buyer's payoff exceeds that in the original EOQ model, would she change her order quantity from Q to φ and take the discount.

We discuss separately the cases $\varphi > q'$ and $\varphi \leq q'$. When $\varphi > q'$,

$$\rho < 1 - \frac{A_B H_S}{A_S H_B}. \tag{3.14}$$

Replacing the q in Eq.(3.5) by φ, we have

$$\Pi_B(\varphi, \rho) = D(R-P)\rho(\eta - \eta\rho - 1) + \frac{C}{2}\left[2 - \frac{Q}{\varphi}(1+\eta\rho) - \frac{\varphi}{Q}(1-\rho)\right]. \tag{3.15}$$

If $\Pi_B(\varphi, \rho) \geq 0$, buyer orders φ units at the discount price; otherwise, buyer keeps original EOQ without taking discount (See Figure 3.1(a) and Example 3.1 below).

When $\varphi \leq q'$, and thus $\rho \geq 1 - \dfrac{A_B H_S}{A_S H_B}$, the buyer orders q' units at the discount price. Since the optimal discount rate, ρ, is obtained by maximizing $\Pi_S(q', \rho)$, , the supplier would also have a positive payoff, $\Pi_S^{(N)}(q', \rho)$, under this scenario (Figure 3.1(b)).

Models for the fixed-demand problem (Chiang et al., 1994; Wang, 2002) take no account of elasticity of demand. Our model, by contrast, computes payoffs based on *profits* of each party, rather than considering only the total costs for buyer. When demand varies as a result of changes in price, the buyer's revenue, as well as her total cost, are both impacted; the traditional fixed-demand model no longer applies. Note also, for price-sensitive demand, the supplier may still gain profit even when buyer orders her EOQ at a discounted price. This results from extra demand due to price elasticity. Detailed numerical illustrations follow.

3.4.2 Procedure for noncooperative model

The supplier can thus determine his optimal quantity discount policy as follows:

Step 1 Calculate buyer's EOQ before discount, and the corresponding profit;

Step 2 Compute the optimal discount rate, ρ, by Eq.(3.10);

Step 3 Solve for the optimal price break point, φ, by Eq.(3.13) with ρ as in Step 2;

Step 4 Compare the values of φ and q' to obtain the final policies and respective payoffs, $\Pi_B^{(N)}$ and $\Pi_S^{(N)}$, for each party. Then make sure both payoffs ≥ 0. If the supplier's payoff < 0, he will not offer a quantity discount; while if the buyer's payoff < 0, she would decline the discount and keep her original order pattern.

3.4.3 Example 3.1

To demonstrate the procedure of determining a given supplier's optimal all-unit quantity discount policy, the following parameters are assumed: $D = 1000$ units, $\eta = 2$, $P = \$35$, $R = \$50$, $A_B = \$500$, $H_B = \$10$, $v = \$10$, $A_S = \$400$, and $H_S = \$3$.

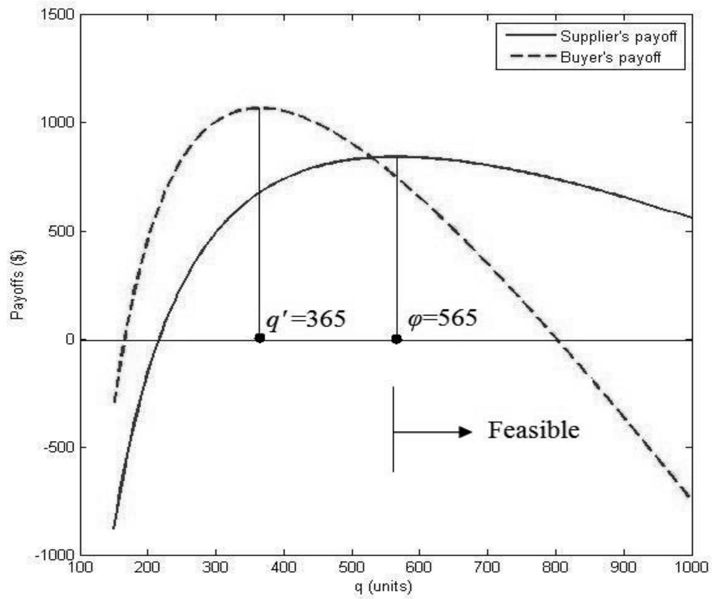
The EOQ can be calculated as $Q = 316$ units. From Eq.(3.10), the optimal percentage discount $\rho = 9.91\%$. Thus, we obtain the corresponding values of q' as 365 units and

$\varphi = 565$ units. This is the optimal break point for that supplier, i.e., if buyer's order quantity is greater than 565 units, the unit price should be set at $\$35 \times (1 - 9.91\%) = \$31.53$; otherwise, the price is \$35 per unit. Under this quantity discount policy, supplier's payoff is $\Pi_S^{(N)} = \Pi_S(\varphi = 565) = \$843$. For the buyer, since she has to order 565 units each time to be able to obtain the discount, she achieves a payoff of $\Pi_B^{(N)} = \Pi_B(\varphi = 565) = \$748$. Figure 3.1(a) illustrates the payoffs for supplier and buyer at various order quantities.

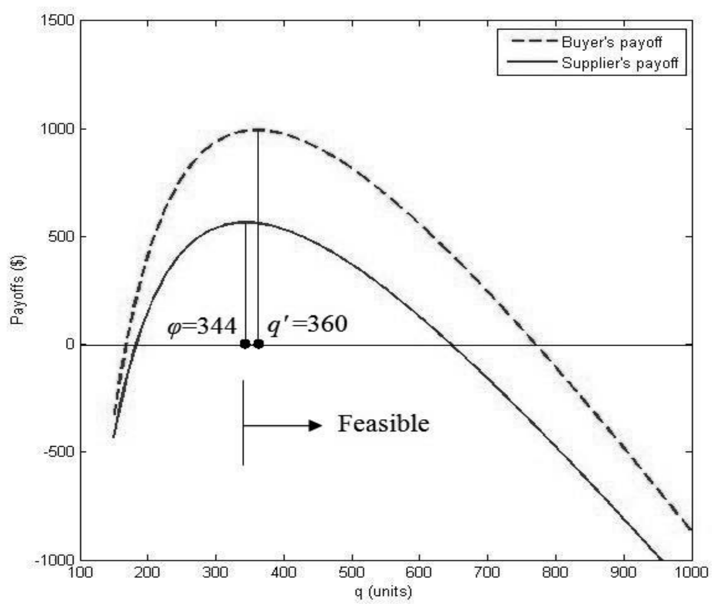
We now modify the value of H_S to \$8, keeping other parameters unchanged. With the same EOQ, the new values for q', ρ and φ become 360 units, 9% and 344 units, respectively. Thus, if $\varphi = 344 (< q')$ is employed as the quantity break point by the supplier, the buyer would order $q = 360$ units each time. The supplier's payoff is then \$562 and the buyer's payoff \$992. Figure 3.1(b) shows the payoffs for each party at various order quantities in this case.

3.5 Joint decision model

Let us now assume that the buyer and supplier agree to make the quantity discount decisions together. Instead of separately maximizing the two individual payoffs, the joint benefit of the two parties is maximized. Specifically, the supplier agrees to offer the determined discount percentage, and the buyer agrees to an order quantity equal to the discount break point. Then, the two parties share the joint benefit (no extra charges).



(a) $\varphi > q'$



(b) $\varphi \leq q'$

Figure 3.1: Payoffs for buyer and supplier as a function of order quantities (noncooperative)

3.5.1 Model development

We thus maximize the payoffs of each party as the two objectives when making the joint decision. In what follows, $\lambda \in [0, 1]$ is a weight used to integrate the two objectives. The Pareto-optimal solution set for this problem is obtained by varying the value of λ (Cohon, 1978). The joint payoff Π_J gained from the discount decision is therefore

$$\Pi_J(q, \rho) = \lambda \Pi_B(q, \rho) + (1 - \lambda)\Pi_S(q, \rho). \tag{3.16}$$

To maximize Π_J, we set $\dfrac{\partial \Pi_J(q, \rho)}{\partial q} = 0$ and get

$$q = \sqrt{\frac{2D(1 + \eta\rho)[\lambda A_B + (1 - \lambda)A_S]}{\lambda H_B(1 - \rho) + (1 - \lambda)H_S}}. \tag{3.17}$$

Note that

$$q(\lambda = 1) = \sqrt{\frac{2DA_B(1 + \eta\rho)}{H_B(1 - \rho)}}; \tag{3.18}$$

and

$$q(\lambda = 0) = \sqrt{\frac{2DA_S(1 + \eta\rho)}{H_S}}. \tag{3.19}$$

Comparison of expressions (3.18) with (3.9), and (3.19) with (3.12), shows that $q(\lambda = 1) = q'$ and $q(\lambda = 0) = \varphi$, respectively the buyer's optimal order quantity under the discount and the quantity discount point obtained in the noncooperative model.

Now, let us take the first derivative of $\Pi_J(q,\rho)$ with respect to ρ. We have

$$\frac{\partial \Pi_J(q,\rho)}{\partial \rho} = \lambda \frac{\partial \Pi_B(q,\rho)}{\partial \rho} + (1-\lambda)\frac{\partial \Pi_S(q,\rho)}{\partial \rho},$$

or

$$\begin{aligned}\frac{\partial \Pi_J(q,\rho)}{\partial \rho} &= \lambda DR(\eta-1) + (1-2\lambda)DP(\eta-1) - 2\lambda DR\eta\rho \\ &\quad - 2(1-2\lambda)DP\eta\rho - (1-\lambda)Dv\eta - \frac{D\eta}{q}[\lambda A_B + (1-\lambda)A_S] \\ &\quad + \frac{1}{2}\lambda H_B q. \end{aligned} \tag{3.20}$$

It is easy to see that the second derivative

$$\frac{\partial^2 \Pi_J(q,\rho)}{\partial \rho^2} = -2\lambda DR\eta - 2(1-2\lambda)DP\eta < 0. \tag{3.21}$$

So $\Pi_J(q,\rho)$ is maximized at the value that satisfies $\dfrac{\partial \Pi_J(q',\rho)}{\partial \rho} = 0$.

Replacing q by the expression (3.17), we find

$$\begin{aligned}\frac{\partial \Pi_J(q,\rho)}{\partial \rho} &= -2D\eta[\lambda R + (1-2\lambda)P]\rho + D(\eta-1)[\lambda R + (1-2\lambda)P] \\ &\quad - (1-\lambda)Dv\eta - \eta\sqrt{\frac{D[\lambda A_B + (1-\lambda)A_S][\lambda H_B(1-\rho) + (1-\lambda)H_S]}{2(1+\eta\rho)}} \\ &\quad + \frac{\lambda H_B}{2}\sqrt{\frac{2D(1+\eta\rho)[\lambda A_B + (1-\lambda)A_S]}{\lambda H_B(1-\rho) + (1-\lambda)H_S}} = 0. \end{aligned} \tag{3.22}$$

Solving Eq.(3.22) gives us the optimal discount percentage that both players agree to. The buyer's corresponding order quantity can then be calculated by Eq.(3.17).

Besides the fact that we include price elasticity of demand, there are three other differences between our model and that of Chiang et al. (1994). Firstly, they assume neither the supplier nor buyer would be worse off after the cooperation. This reasonable assumption is based on the concept of "individual rationality". However, our research found that the benefit of cooperation would be very limited if each player considers his/her gain separately. Instead, we assume a cooperative environment in which the two players can share the benefit from making the decision together, and both achieve much higher payoffs (see Example 3.2). The second difference is that Chiang et al. (1994) restrict the buyer's cost by a budget limit; we do not. Furthermore, those authors use an integer variable, n, to represent the multiple of buyer's order quantity such that nQ is the supplier's order or production quantity. In our case, the supplier's concern is only about the proportion of his payoff that results from the change of buyer's order quantity due to the quantity discount.

3.5.2 Example 3.2

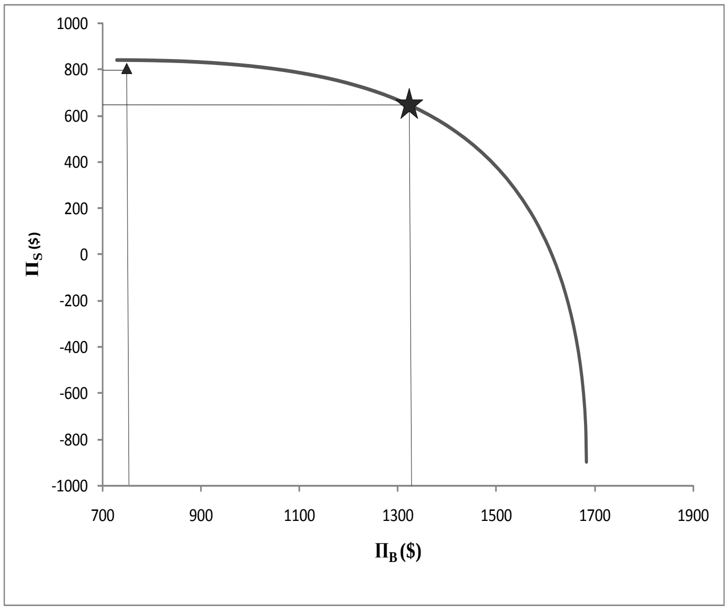
Suppose that the buyer and supplier, with the same parameters as in Example 3.1, now make the quantity discount decisions jointly. By varying the value of λ between 0 and 1, the Pareto frontier can be achieved. Figures 3.2(a) and 3.2(b) show the Pareto efficient frontiers for the cases $\varphi > q'$ and $\varphi \leq q'$, respectively. As λ increases, the buyer's payoff,

Π_B, increases, and the supplier's payoff, Π_S, decreases. This is quite intuitive because the value of λ indicates the importance of the buyer's payoff in making the joint decision. Results obtained naturally favour the buyer for larger λ.

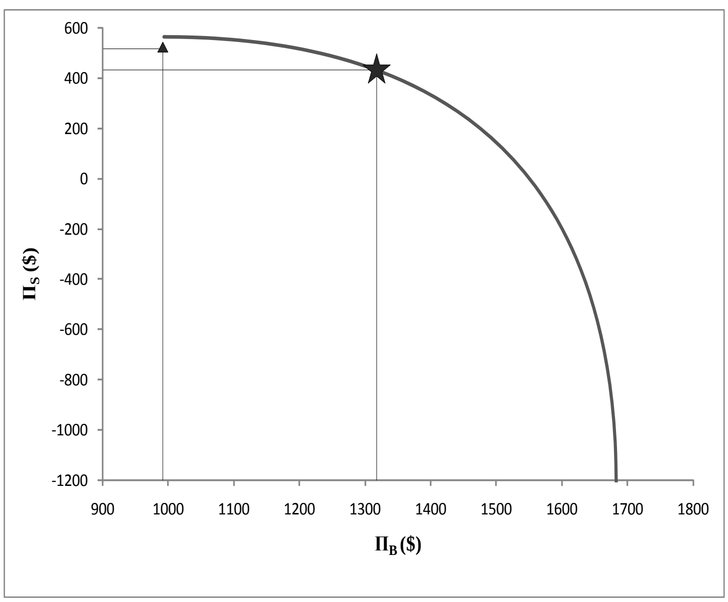
Figure 3.2 also illustrates the relationship between the Pareto optimal solution sets obtained from the joint decision model and the best noncooperative solutions. The triangular-shaped points indicate solutions of the noncooperative model, and the star-shaped points are the best solutions of the joint model with $\lambda = 0.5$. Figures 3.2(a) and 3.2(b) both show that the noncooperative solutions are very close to the Pareto frontiers. This observation tells us that, although both player's payoffs can be improved by moving northeasterly to the Pareto curves, the improvements would be fairly small (no more than 2% in this example).

Nevertheless, the results can be significantly boosted if we first maximize the total channel payoffs under the joint decision-making environment, then divide the extra gain between buyer and supplier. Numerically, when $\varphi > q'$, $\Pi_J(\lambda = 0.5) = \$1972$, while, from the noncooperative model, we have $\Pi_B^{(N)} + \Pi_S^{(N)} = 748 + 843 = \$1591$. The total improvement due to making the decision jointly is \$ 381. Assuming the buyer and supplier would divide this improvement equally between them, we get $\Pi_B^{(J)} = 939$ and $\Pi_S^{(J)} = 1033$. Compared to the noncooperative model, the payoffs of the buyer and supplier are enhanced by 25% and 23%, respectively. Similarly, when $\varphi \leq q'$, the payoffs of the buyer and supplier after the division are \$1090 and \$660, improved by around 10% and 17%, respectively.

Additionally, compared to Example 3.1, we find that the quantity discount percentage



(a) $\varphi > q'$



(b) $\varphi \leq q'$

Figure 3.2: Pareto frontier for the joint decision model

Table 3.1: Comparison of the noncooperative and joint decision models *

		$\varphi > q'$	$\varphi \leq q'$
Noncooperative model	$\rho^{(N)}$	9.91%	9%
	$q^{(N)}$	565	360
	$\Pi_B^{(N)}$	\$748	\$992
	$\Pi_S^{(N)}$	\$843	\$562
Joint decision model	$\rho^{(J)}$	14.1%	13.5%
	$q^{(J)}$	446	371
	$\Pi_J(\lambda = 0.5)$	\$1972	\$1750
	$\Pi_B^{(J)}$	\$939	\$1090
	$\Pi_S^{(J)}$	\$1033	\$660

* Here and in the following tables, the demand after discount is $D(1 + \eta\rho)$, where $D = 1000$ units is the base demand.

when $\varphi > q'$ is 14.1%, which is deepened by 42%, and the order quantity is 446 units, a decrease of 21%. For the case $\varphi \leq q'$, comparison shows that the discount percentage is raised from 9% to 13.5%, while the order quantity is only slightly increased. These differences show that, by making decisions together, both players may greatly benefit, enjoying a deeper discount without much increase in the order quantity (See Table 3.1).

3.6 Transportation considerations

In some cases, the buyer or supplier may offer their own vehicle to transport the products. Then, in addition to the impact on purchasing, inventory and ordering costs, quantity discounts may also yield transportation cost savings for the responsible party. Our model can deal with that situation by simply modifying one input parameter.

3.6.1 Model extension 1

We assume that the buyer provides a private fleet, thus transporting products in her own truck. With C_{bf} and C_{bv} as fixed and variable transportation costs, buyer's total cost is

$$TC_0^{(t)} = DP + (A_B + C_{bv})D/Q + QH_B/2 + C_{bf}. \tag{3.23}$$

Accordingly, the buyer's payoff after the discount can be reformulated as

$$\Pi_B^{(t)}(q,\rho) = D(R-P)\rho(\eta - \eta\rho - 1) + \frac{C^{(t)}}{2}\left[2 - \frac{Q}{q}(1+\eta\rho) - \frac{q}{Q}(1-\rho)\right], \tag{3.24}$$

where $C^{(t)} = \sqrt{2DH_B(A_B + C_{bv})}$.

Comparison of Eqs. (3.24) and (3.5) shows that this model can account for transportation costs by adding the variable portion to the ordering cost, i.e, replacing the buyer's ordering cost A_B by $A_B^{(t)} = A_B + C_{bv}$. Note for $A_B^{(t)}$, we still need to determine whether $\varphi > q'$ or $\varphi \leq q'$; the same conditions as for A_B must also be satisfied by $A_B^{(t)}$.

Similar modification can also be made if, instead, it is the supplier's vehicle that will be employed. In this case, we substitute $A_S^{(t)} = A_S + C_{sv}$ for the supplier's ordering cost A_S, where C_{sv} represents the supplier's variable transportation cost per trip.

3.6.2 Example 3.3

In this section, four sets of parameters are used to illustrate the practicability of our extended models. Table 3.2 lists the related parameters as well as results from the corresponding calculation. We consider the private fleet provided by the buyer or supplier, and both cases of $\varphi > q'$ and $\varphi \leq q'$ for each situation.

Take the first column for example. This column shows the situation with buyer's private fleet, under the case of $\varphi > q'$. For the noncooperative and joint models, the discount percentages and order quantities are 9.76% and 565 units, as well as 13.86% and 555 units, respectively. Compared to the noncooperative model, the joint decision model gives a deeper discount and a lower order quantity, but in the meantime, results in higher respective payoffs for both buyer and supplier ($ 1074 and $ 756 jointly versus $ 986 and $ 669 noncooperatively). From the table, we can see that these advantages of the joint decision model also hold for other situations.

3.7 Different retail discount

So far, we assumed that the buyer sells the item at the same discount percentage that she obtained when purchasing that product at a discounted price. In practice, the buyer may choose a selling price that maximize her profit (or payoff), rather than simply offer the same discount after she receives it. In this section, we assume that the retail discount percentage offered by the buyer to end customers is a certain multiple x times the discount

Table 3.2: Results for transportation by private fleet (four sets of parameters)

		Buyer's private fleet		Supplier's private fleet	
		$\varphi > q'$	$\varphi \leq q'$	$\varphi > q'$	$\varphi \leq q'$
Transportation cost		$C_{bv} = \$500$ $C_{bf} = \$1000$	$C_{bv} = \$1000$ $C_{bf} = \$1000$	$C_{sv} = \$200$ $C_{sf} = \$1000$	$C_{sv} = \$200$ $C_{sf} = \$1000$
H_S		\$3	\$3	\$3	\$12
Noncooperative model	$\rho^{(Nt)}$	9.76%	9.6%	9.77%	8.13%
	$q^{(Nt)}$	565	629	690	356
	$\Pi_B^{(Nt)}$	\$986	\$955	\$338	\$915
	$\Pi_S^{(Nt)}$	\$669	\$645	\$1095	\$456
Joint decision model	$\rho^{(Jt)}$	13.86%	13.67%	14%	12.91%
	$q^{(Jt)}$	555	645	493	366
	$\Pi_J^{(t)}(\lambda = 0.5)$	\$1830	\$1759	\$2059	\$1592
	$\Pi_B^{(Jt)}$	\$1074	\$1035	\$651	\$1026
	$\Pi_S^{(Jt)}$	\$756	\$724	\$1408	\$566

that she received from the supplier. Taking into account this multiple, the supplier may change his quantity discount policy accordingly.

3.7.1 Model extension 2

Suppose the buyer offers a discount of $x\rho$ to the end customer, where ρ is the discount she receives from the supplier, and $x \geq 0$. Since $-\eta = (\Delta D/D)/(\Delta R/R)$, we have

$$\Delta D = -\eta \cdot \frac{\Delta R}{R} \cdot D = -\eta(-x\rho)D = x\eta\rho D. \tag{3.25}$$

Let the new demand rate be $D(1 + x\eta\rho) = D(1 + \bar{\eta}\rho)$, where $\bar{\eta} = x\eta$. Thus, we can rewrite the payoff functions for the buyer and supplier. Specifically, buyer's profit after the discount is

$$\begin{aligned}\pi_{B1}^{(r)}(x, q, \rho) =& D(1 + \bar{\eta}\rho)R(1 - x\rho) - D(1 + \bar{\eta}\rho)P(1 - \rho) \\ &- \frac{A_B D(1 + \bar{\eta}\rho)}{q} - \frac{1}{2}qH_B(1 - \rho).\end{aligned} \tag{3.26}$$

Here the superscript (r) refers to the case in which the retailer (buyer) may offer a different discount. With the updated payoff functions, varying the value of x gives the corresponding discount policies for the supplier. And then, the buyer's retail discount can be determined by choosing that value x which maximize her payoff.

In particular, the payoff in the joint model can be expressed as

$$\max_{x,q,\rho} \Pi_J^{(r)}(x,q,\rho) = \lambda\Pi_B^{(r)}(x,q,\rho) + (1-\lambda)\Pi_S^{(r)}(x,q,\rho)], \tag{3.27}$$

s.t.

$$\Pi_B^{(r)}(x,q,\rho) \geq 0;$$

$$\Pi_S^{(r)}(x,q,\rho) \geq 0;$$

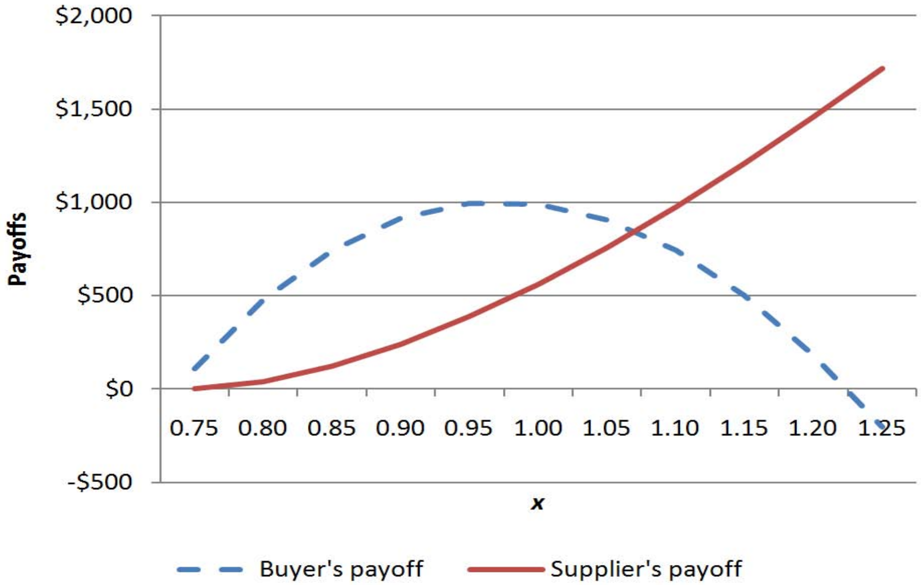
$$0 \leq \rho \leq 1;$$

$$x \geq 0.$$

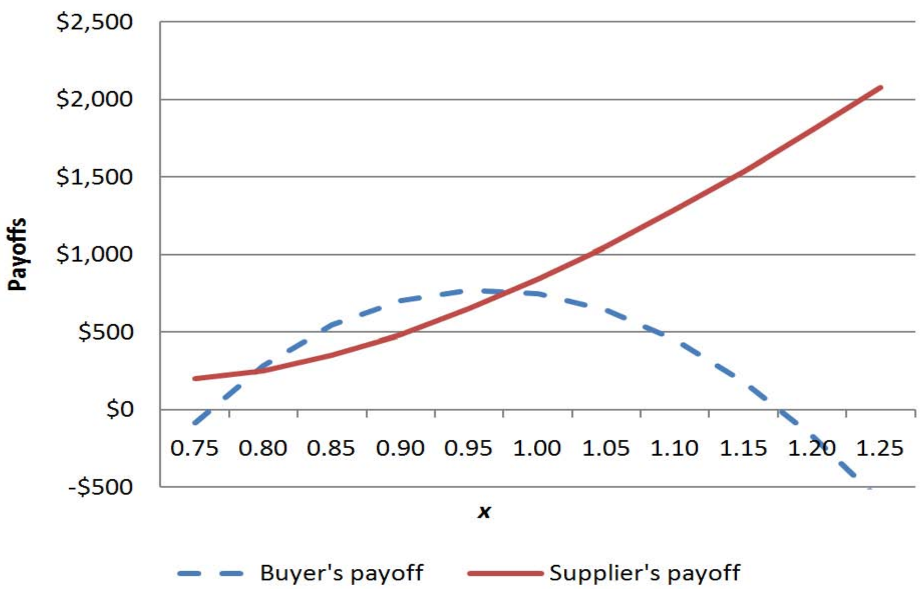
Solving this maximization problem gives the supplier's optimal discount policy, as well as the buyer's retail discount schedule.

3.7.2 Example 3.4

To illustrate the impact of buyer's independent retail discount policy on the supplier's quantity discount determination, we allow x to vary from 0.75 to 1.20 for the noncooperative cases, and show the payoff curves for the buyer and supplier in Fig. 3.3. Consider Fig. 3.3(a), $\varphi > q'$. Note that when $x < 0.75$, the supplier and buyer both lose because the profit gained in additional demand cannot cover the losses from the discounts. When $x > 1.20$, the buyer still loses due to the large retail discount she offers. Therefore, it is only when $0.75 \leq x \leq 1.15$ that both parties would like to consider the possibility of offering discounts. The range that is mutually favourable to discounts changes to $0.75 \leq x \leq 1.20$ when $\varphi \leq q'$ in Fig. 3.3(b). The optimal results are listed in Table 3.3.



(a) $\varphi > q'$



(b) $\varphi \leq q'$

Figure 3.3: Payoffs for buyer and supplier as a function of x (noncooperative)

Table 3.3: Optimal results when the retail discount $= x\rho$

		$\varphi > q'$	$\varphi \leq q'$
Noncooperative model	$\rho^{(Nr)}$	8.78%	8.27%
	$x^{(N)}$	0.965	0.972
	$x^{(N)}\rho^{(Nr)}$	8.48 %	8.04 %
	$q^{(Nr)}$	558	356
	$\Pi_B^{(Nr)}$	\$750	\$1005
	$\Pi_S^{(Nr)}$	\$707	\$461
Joint decision model	$\rho^{(Jr)}$	15.41%	14.34%
	$x^{(J)}$	0.905	0.932
	$x^{(J)}\rho^{(Jr)}$	13.94%	13.36%
	$q^{(J)}$	450	371
	$\Pi_J(\lambda = 0.5)$	\$2004	\$1768
	$\Pi_B^{(J)}$	\$1023	\$1156
	$\Pi_S^{(J)}$	\$981	\$612

3.8 Case of heterogeneous buyers

In the previous sections, we have shown that both the supplier and buyer can benefit from the quantity discount. However, the circumstances become much more complicated when more than one buyer is involved. In this section, we extend our consideration to a situation in which a sole supplier sells a single product to m heterogeneous groups of buyers ($k = 1, 2, \cdots, m$). Each group of buyers has its own demand and cost structure.

3.8.1 Model extension 3

Assume that there are w discount levels ($l = 1, 2, \cdots, w; w \leq m$). We develop a heuristic solution procedure that is based on our noncooperative model and a few ideas presented by Wang (2002) (Fig. 3.4).

More specifically, as stated by Wang, "a larger buyer is given a higher (at least not lower) break point." This is reasonable and understandable. But in practice, even after ranking buyers in ascending order by their EOQ before any discount, it is still very challenging to make sure that a larger buyer receives a higher discount level. Therefore, we employed Proposition 5 of Wang (2002): the payoff of a certain buyer (or group of buyers) at the corresponding discount level is the same as at the next lower break point under the supplier's optimal discount schedule. This condition guarantees that a consistent discount policy (i.e. having the same multiple breakpoints and discount percentages) is available to every retailer.

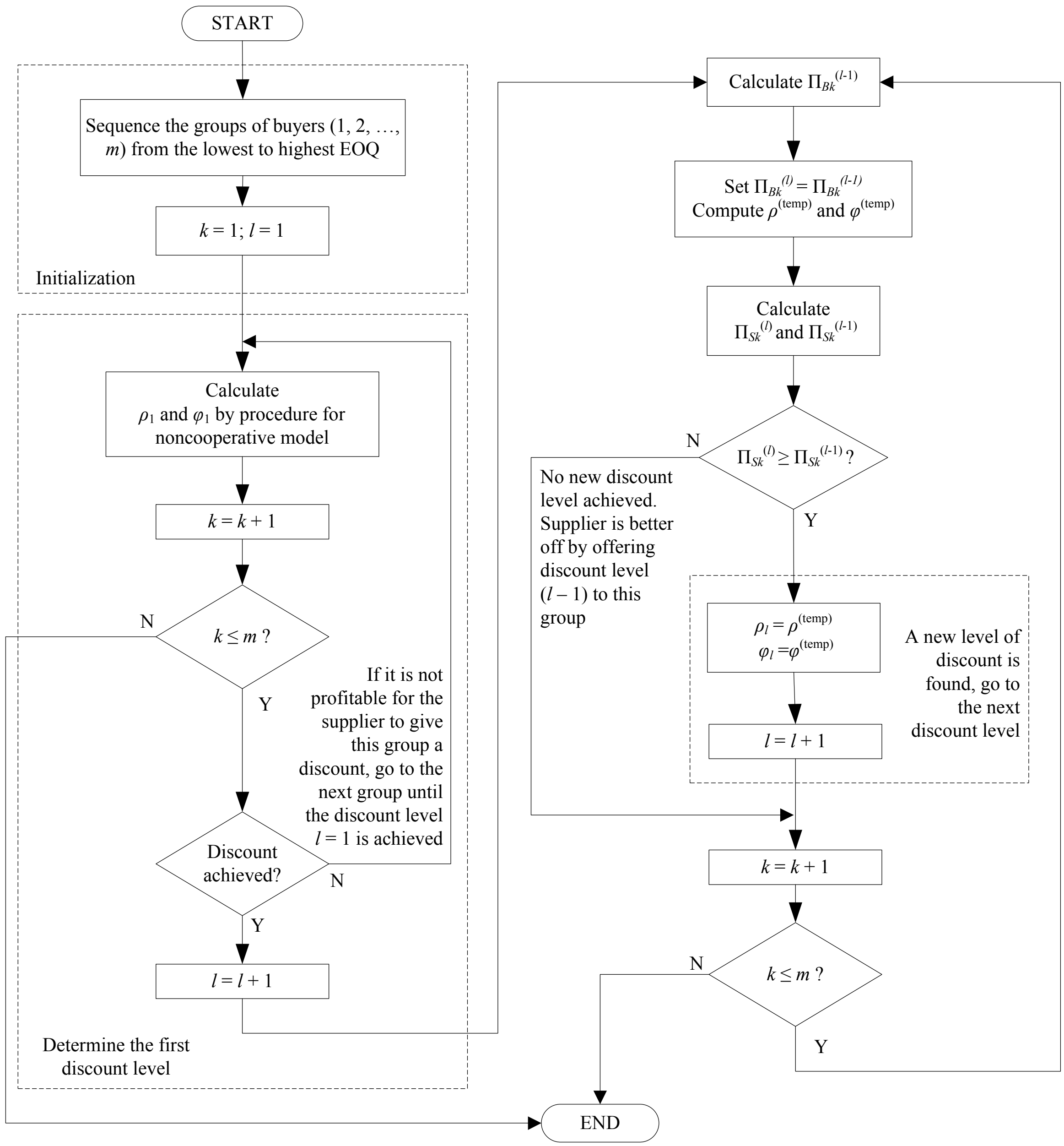


Figure 3.4: Heuristic solution procedure for the case of heterogeneous buyers

However, while buyers' payoffs are assured, the supplier also has to gain more by offering a higher discount. So another condition is introduced in our procedure: comparing the supplier's payoff at a newly-derived discount level to that at the next lower level. If this new level gives higher payoff, the supplier confirms it and continues to further calculate a successive level; otherwise, the supplier would like this group of buyers to use the most recent discount level. Thus, situations can appear in which more than one group of buyers share the same discount level.

Despite all the adaptation from Wang (2002), our approach still differs from it in many ways. First of all, Wang's model treats constant demand; ours considers price-sensitivity. Secondly, Wang assumed that each group of buyers fits into its own discount level. But we allow more than one group of buyers to possibly share a given discount level. So the number of discount levels does not have to be the same as the number of groups of buyers. Thirdly, in determination of the first discount level: Wang's solution method permits the payoff to the first group of buyers to equal 0, while in our method, the payoffs to that group are maximized. Finally, according to our calculation, the supplier may still benefit when buyers order at their EOQ. Results are shown and compared in the following numerical illustration.

Table 3.4: Demand and cost parameters (Based upon Lal and Staelin (1984)and Wang (2002))

$H_S = 3; \eta = 2; P_0 = 35$

Group k	n_k	D_k	H_{Bk}	A_{Bk}	A_{Sk}	Q_k
1	632	362	11.60	145	81	96
2	363	1 658	11.00	283	147	292
3	85	4 191	10.50	407	305	570
4	30	9 228	10.00	526	445	985
5	8	14 966	9.50	634	573	1 413
6	4	18 565	9.25	1 176	1 063	2 173
7	6	25 346	9.00	1 305	1 275	2 711

3.8.2 Example 3.5

To demonstrate the application and advantages of our model, the instance employed by Wang (2002) (originally from Lal and Staelin (1984)) is used here. A large US manufacturer (supplier) sells one product to 1128 buyers with a wholesale price of $35. All buyers were divided by Lal and Staelin (1984) into seven homogeneous groups based on their demands and estimated cost structures. Information regarding these groups is listed in Table 3.4; n_k gives the number of buyers in each group. Additionally, we assume that the price elasticity of demand for this product is 2. Both our procedure and that of Wang (2002) are applied to this example, with respective results in Tables 3.5 and 3.6.

Our optimal discount schedule contains four discount levels (break points) instead of seven. The discount percentage varies from 9.96% to 13.95% for us, while Wang's is

Table 3.5: Optimal quantity discount schedule obtained by our method

Group k	$\rho(\%)$	φ	q'	q	$\Pi_{Sk}{}^*$	$\Pi_{Bk}{}^*$	$\Pi_{Bk}^{(l-1)}{}^*$	$n_k\Pi_{Sk}$	$n_k\Pi_{Bk}$
1	9.96	153	110	153	280	328	0	177 174	207 296
2	10.82	445	341	445	1 271	1 858	1 858	461 373	674 273
3	12.53	1 032	682	1 032	3 271	5 082	5 082	278 031	432 010
4			1 178	1 178	6 721	12 549	11 329	201 617	376 457
5	13.95	2 704	1 723	2 704	11 002	20 468	20 468	88 019	163 748
6			2 649	2 704	12 507	27 022	25 233	50 026	108 086
7			3 305	3 305	17 030	37 047	34 589	102 178	222 280
				Column sum	52 081	104 353	98 559	1 358 419	2 184 149

$*$ Π_{Sk} = payoff that supplier gains from each member of buyer group k; Π_{Bk} = corresponding payoff of this buyer; $\Pi_{Bk}^{(l-1)}$ = payoff gained by each member of group k at discount level $(l-1)$.

Table 3.6: Optimal quantity discount schedule obtained by the method of Wang (2002)

Group k	$\rho(\%)$	φ	q'	q	Π_{Sk}	Π_{Bk}	$\Pi_{Bk}^{(l-1)}$	$n_k\Pi_{Sk}$	$n_k\Pi_{Bk}$
1	1.65	142	98	142	106	0	0	66 992	0
2	2.38	413	303	413	561	370	370	203 643	134 310
3	3.64	956	601	956	2 130	1 358	1 358	181 050	115 430
4	4.77	1 732	1 057	1 732	5 585	4 512	4 512	167 550	135 360
5	5.71	2 524	1 536	2 524	10 014	9 281	9 281	80 112	74 248
6	6.88	3 896	2 401	3 896	13 739	13 529	13 529	54 956	54 116
7	7.52	4 978	3 024	4 978	19 610	20 334	20 334	117 660	122 004
				Column sum	51 745	49 384	49 384	871 963	635 468

relatively lower with a range of 1.65% to 7.52%. Although the supplier gains lower payoffs from our two largest groups, his total payoff is now \$ 1 358 419, an increase of 56% compared to results of Wang's method. From the buyers' side, each group shows a higher payoff, and total payoffs to all buyers increase to \$ 2 184 149, more than 3.4 times that in Wang (2002).

In addition, let us further compare our results to the joint model. Assume that each buyer group negotiates separately with the supplier about the discount policy in the joint case. The discount schedule obtained is listed in Table 3.7. The total payoffs, to the buyer groups plus supplier from the joint model, are \$ 3 802 165. This shows a slight increase, \$ 259 597 or about 7.33%, compared to the noncooperative case (sum of last two columns of Table 3.5).

These results may be understood as follows. Relative to Table 3.6 (Wang's model), both parties are much better off in Table 3.5 (our model). Table 3.7's outcomes (joint model) are in turn just a little better for the buyer and supplier than in Table 3.5. That change is somewhat small (7.33%) because the buyer benefits to such a great extent from our model over Wang's. The net effect is that, for these parameters, the outcome of the proposed noncooperative procedure (Table 3.5) is close to that of the joint model (Table 3.7).

Table 3.7: Optimal quantity discount schedule obtained by the joint model ($\lambda = 0.5$)

Group k	$\rho(\%)$	φ	$\Pi_{Jk}{}^*$	$n_k\Pi_{Jk}$
1	14.25	128	717	453 143
2	14.52	385	3 435	1 246 924
3	14.62	803	8 900	756 477
4	14.70	1 418	19 902	597 050
5	14.74	2 053	32 491	259 930
6	14.68	3 142	39 966	159 866
7	14.79	4 248	54 796	328 776
	Column sum		160 207	3 802 165

$*$ Π_{Jk} denotes the joint payoff of the supplier and each member of buyer group k.

3.9 Numerical analyses

With three examples, we have compared results obtained from the noncooperative and joint decision models. Here we discuss further impacts of a few main parameters on two key outcomes: the discount percentage and channel efficiency, defined as the total payoffs of the two players gained from the quantity discount, i.e., $\Pi_B + \Pi_S$. Unless clarified otherwise, parameter values are as in previous examples.

Firstly, the relationship between η and ρ from Eq. (3.10) is examined. Fig. 3.5 shows the function $\rho = \Phi(\eta)$ when η is varied from 1 to 5. It turns out that when $\eta < 1.5$, the value $\rho = \Phi(\eta)$ calculated directly from Eq. (3.10) is negative. Since we must have $\rho \geq 0$, ρ is assigned the value of 0: no quantity discount. ρ is increasing in η when $\eta \geq 1.5$. The larger value of η indicates that demand grows more quickly than the price has dropped, i.e., product demand increases by a greater percentage. The result is higher profit for the supplier, with the buyer no worse off.

Fig. 3.6 shows $\rho = \Phi(\eta)$ for $v = 0$, \$10, and \$20. As expected, the lower the price the supplier pays for the product, the greater the discount he provides the buyer. And when buyer's annual demand D goes up, the channel efficiency improves (Fig. 3.7). Joint decision making yields a clear benefit, one that increases in D. The joint decision model gives a deeper discount (Examples 3.2 and 3.3), but the two models have similar discount patterns.

Fig. 3.8 depicts the respective impacts of A_B/A_S and H_B/H_S on channel efficiency for the noncooperative model. Each curve first increases, then decreases. The reason is that the buyer's order quantity may equal φ or q' in different portions of the curve, depending upon parameter values. Finally, Table 3.8 summarizes the effects of varying the preceding parameters on channel efficiency and on ρ for both the noncooperative and the joint models.

3.10 Summary

Real world complications motivated us to ask, Would switching the perspective, from the player who takes the discount to the one who sets it, improve the effectiveness and efficiency of a supply chain? Generally, the answer is *yes*.

From the supplier's perspective, and including the price elasticity of demand, a non-

Figure 3.5: Sample relation $\rho = \Phi(\eta)$ between ρ and η

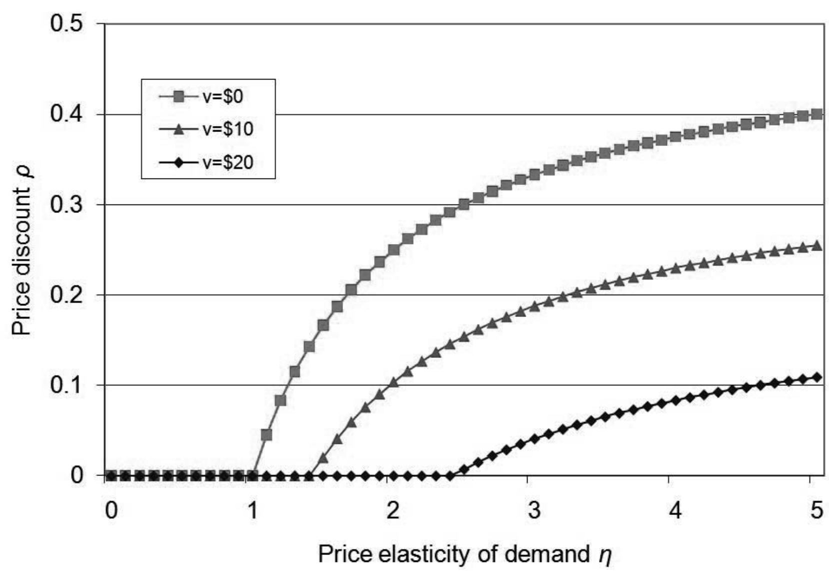


Figure 3.6: Curves for $\rho = \Phi(\eta)$ for several values of v

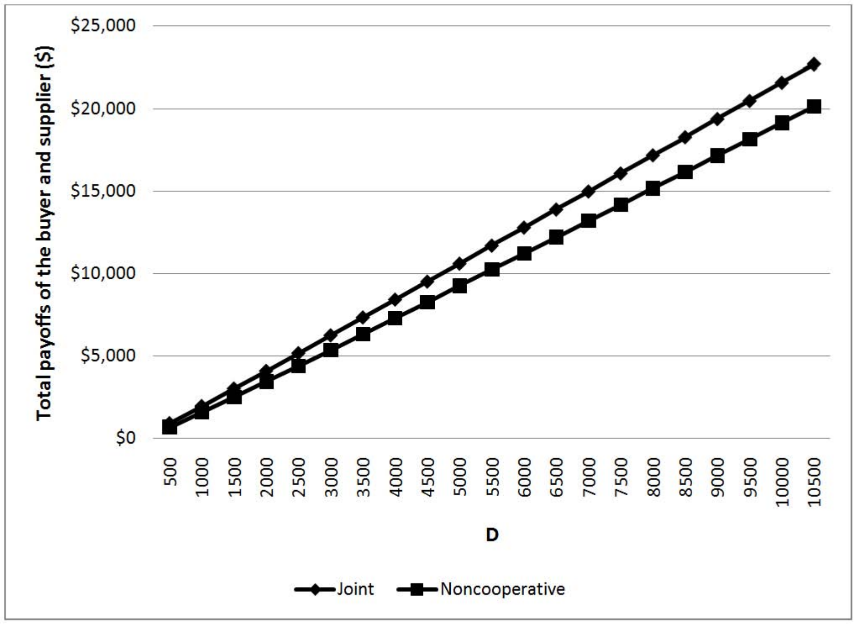


Figure 3.7: Impact of D on the channel efficiency

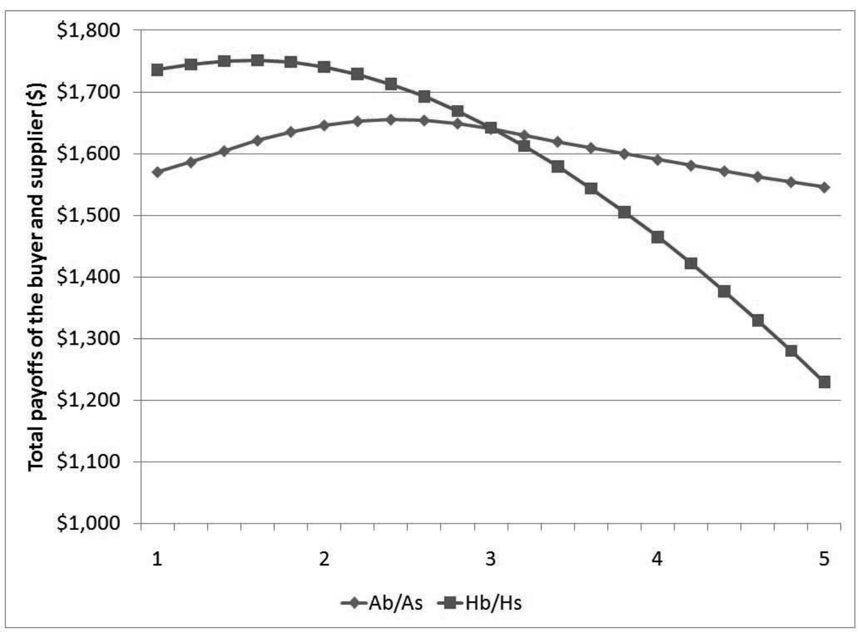


Figure 3.8: Effects of A_B/A_S and H_B/H_S on channel efficiency for noncooperative model

Table 3.8: Impacts of varying parameters on two key results

	Channel efficiency ($\Pi_B + \Pi_S$)		Discount percentage (ρ)	
	Noncooperative	Joint	Noncooperative	Joint
$A_B/A_S \uparrow$	First $\uparrow$, then $\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$H_B/H_S \uparrow$	First $\uparrow$, then $\downarrow$	$\uparrow$	$\uparrow$	Almost the same
$D \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$v \uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\eta \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$

cooperative game-theoretic approach and a joint decision model were developed. These should aid a sole supplier in determining an all-unit quantity discount for a single-buyer distribution channel. Pareto optimality can be achieved by jointly considering the benefits of both the supplier and buyer. The importance of transportation led us to extend our models to the case of product shipment by the private fleet of either the supplier or buyer. We also considered the fact that the buyer may wish to select best retail price to maximize her own payoff. Additionally, a heuristic solution procedure (Fig. 3.4) showed the possibility of significant improvements (Tables 3.5 and 3.6) by condensing the heterogeneous buyer-groups to a smaller number of discount levels. Numerical instances showed application of the models.

Recall that we introduced four cases for the provision of transportation. One of the model extensions in this chapter showed the solution structure of Cases I and II, the two private-fleet cases. In the next chapter, we will discuss the situation in which the buyer orders a *family* of items from the supplier.

Chapter 4

Supplier's Optimal Quantity Discount Policy for a Family of Items [1]

4.1 Introduction

In Chapter 3, we have investigated the quantity discount problem involving one item, a single supplier and one or multiple buyers. However, in a real supply chain system, there always exist situations in which a buyer needs to procure several items from the same supplier. Any purchase order with a certain supplier, even for a single SKU, may incur a fixed

[1] This chapter is an extended version of Ke and Bookbinder (2012b)

cost; but each additional SKU on that same order adds only a much smaller fixed cost. The grouping of a family of items may be based upon different factors, such as their functions (for example, periodic maintenance of a machine may require a systematic replacement of a few items), their characteristics (various items may have similar inventory review policies or may be stored at the same location), or upon vendor-imposed requirements (e.g. a publisher may be required by the printer to order at least four products together). When this is the case, the coordinated replenishment of these items may benefit the buyer in terms of savings on purchase price and the costs of transportation and ordering. That coordination may also make relevant scheduling decisions easier.The modified periodic policy (MPP) is a method to determine the replenishment policy when dealing with the multi-item situation. In the MPP (e.g. Silver et al. (1998)), one or more SKUs are considered "base items," replenished every T periods; all other SKUs in the group are replenished less often, at an order quantity that will last some multiple of T periods.

With the consideration of quantity discounts, coordinating replenishment decisions for multiple items becomes more complicated. The question of a quantity discount has usually been analyzed from the perspective of the buyer: For a given discount scheme, should that buyer purchase a large enough amount, to be able to take advantage of a reduced cost per unit? Research for this purpose has been done by authors such as Chakravarty (1984), Russell and Krajewski (1992), Li and Huang (1995), Silver et al. (1998), Xu et al. (2000), Cha and Moon (2005), Moon and Cha (2006), Moon et al. (2008), Cha and Park (2009), and Shi and Zhang (2010), to name a few. A comprehensive review of related works can be found in Khouja and Goyal (2008).

However, by switching the viewpoint from the buyer to *supplier*, the situations may be significantly different. Actually, the quantity discount has been employed as a coordination mechanism to improve the performance not only of each single party, but also of the overall supply chain (in this chapter, buyer plus supplier). As mentioned in Chapter 3, the case of a single item has been broadly investigated in the literature. Studies regarding this matter have been thoroughly reviewed in Chapter 2.

Chen and Chen (2005a,b) employed a saving-sharing mechanism, through a quantity discount scheme adopted from Lal and Staelin (1984), to achieve the Pareto improvement. That discount approach was originally developed to deal with a problem in which only one product is distributed from the supplier to the buyer(s), but the pricing structure of this product does not alter its ultimate demand.

In this chapter, we aim to aid the supplier in *setting* the quantity discount policy. That discount scheme will be determined according to the aggregate purchases of the product group. Both constant and price-sensitive demands will be examined and compared, to illustrate the impact that the price-sensitivity may have on the supplier's discount policy.

The remainder of Chapter 4 is organized as follows. Based on the assumptions and notation, we develop a noncooperative model based on the Stackelberg game (Model I), separately considering the payoffs to the supplier and buyer. That model is analyzed from the supplier's perspective in assisting him to set the group discount for a family of items with constant and deterministic demand. Next, we present a joint model (Model II), which maximizes the total payoffs of the two parties. The model then is extended to analyze the case of price-sensitive demand, both noncooperatively (Model III) and jointly (Model IV). Numerical examples are employed through the chapter to show the application of our approaches.

4.2 Assumptions and notation

Our research considers a situation in which a buyer purchases, from a single supplier, products belonging to a family of n items. The buyer and supplier respectively employ the MPP and EOQ as the inventory policy. We start at the point that the two parties have already formed a buyer-seller relationship in the market, i.e, the supplier has set a selling price for each item, and sells these products to the buyer; and the buyer orders the group of products at a pattern that is determined by MPP.

The demand D_i for each item i is assumed to be constant per unit time for Models I and II, and price-sensitive for Models III and IV. This supplier offers a group discount to the buyer: a certain discount percentage is offered on each unit ordered when the total dollar value of a single replenishment exceeds a predetermined breakpoint. When the buyer purchases the products at the discounted prices, she is assumed to sell these items at the same discount percentage from her regular prices.

Additionally, we assume that the supplier is aware of all relevant information of the buyer, such as her annual demand rate for each item, her inventory holding cost, and ordering cost. The following notation will be employed.

n	Number of items in the family (item index $i = 1, 2, ..., n$).
D_i	Annual demand rate of item i in the absence of a discount.
P_i	The purchase price of product i, paid by the buyer to supplier.
R_i	Buyer's selling price of item i.
η_i	Price elasticity of demand of item i.
A_B	Buyer's major ordering cost for the family, when at least one item is ordered.
a_{Bi}	Buyer's minor ordering cost, when item i is included in a group replenishment.
r	Buyer's inventory holding cost (% of purchase price).
A_S	Supplier's major setup cost for the family.
a_{Si}	Supplier's minor setup cost for item i, when that item is included in a production batch.
H_i	Supplier's holding cost for inventory of item i.
v_i	Supplier's acquisition price of product i.
ρ	Fractional discount applicable to aggregate orders from the group ($0 < \rho < 1$).
m_i	The integer number of base intervals that the replenishment quantity of item i will last.
T_0	The base item's EOQ before discount, expressed as a time supply.
T_B	Buyer's best order policy after discount (as a time supply).
T_S	The order quantity to maximize supplier's payoff (as a time supply).

T_J The order policy to maximize the joint payoff of the two parties (as a time supply).

T_b The breakpoint of the group discount, expressed as a time supply.

V_b The breakpoint in terms of the total value of replenishment.

Π_B^s Buyer's payoff gained from the discount. $s = [\mathbb{C}]$ and $[\mathbb{S}]$ denote, respectively, the cases of constant and price-sensitive demands.

Π_S^s Supplier's payoff gained from the discount.

Π_J^s Joint payoffs of the supplier and buyer.

4.3 Constant demand case

In our analyses, the *payoff* to the particular party refers to the profits gained or the costs saved by the group discount.

4.3.1 Buyer's payoff

Before the group discount is offered by the supplier, there is a time interval T_0 between replenishments of the family. A set of multipliers m_i has been determined and is employed, again before the discount. So for each item i, the replenishment quantity is $D_i m_i T_0$, hence the average inventory level is $D_i m_i T_0/2$.

The buyer's major ordering cost A_B is incurred every T_0 units of time, whereas the cost a_{Bi} is incurred only once in every $m_i T_0$ periods. Therefore, the buyer's total annual relevant cost before the discount can be expressed as

$$TRC_0^{[\mathbb{C}]} = \sum_{i=1}^{n} D_i P_i + \left(A_B + \sum_{i=1}^{n} \frac{a_{Bi}}{m_i} \right) \frac{1}{T_0} + \frac{1}{2} r T_0 \sum_{i=1}^{n} m_i D_i P_i, \tag{4.1}$$

where $D_i P_i$ is the purchase expense for each item i.

Setting $\partial TRC_0^{[\mathbb{C}]} / \partial T_0 = 0$ gives the expression of T_0, that is

$$T_0 = \sqrt{\frac{2(A_B + \sum_{i=1}^{n} a_{Bi}/m_i)}{r \sum_{i=1}^{n} m_i D_i P_i}}.$$

Substitution of T_0 back into Eq. (4.1) gives the lowest costs for a give set of m_i's:

$$TRC_0^{[\mathbb{C}]} = \sum_{i=1}^{n} D_i P_i + \frac{\left(A_B + \sum_{i=1}^{n} \frac{a_{Bi}}{m_i} \right)}{\sqrt{\frac{2(A_B + \sum_{i=1}^{n} a_{Bi}/m_i)}{r \sum_{i=1}^{n} m_i D_i P_i}}} + \sqrt{\frac{2(A_B + \sum_{i=1}^{n} a_{Bi}/m_i)}{r \sum_{i=1}^{n} m_i D_i P_i}} \times \frac{r \sum_{i=1}^{n} m_i D_i P_i}{2},$$

which can be simplified to

$$TRC_0^{[\mathbb{C}]} = \sum_{i=1}^{n} D_i P_i + \sqrt{2\left(A_B + \sum_{i=1}^{n} \frac{a_{Bi}}{m_i}\right) r \sum_{i=1}^{n} m_i D_i P_i}.$$

Thus, the integers m_i must be equal to the values that minimize

$$\left(A_B + \sum_{i=1}^{n} \frac{a_{Bi}}{m_i}\right) r \sum_{i=1}^{n} m_i D_i P_i.$$

The derivation of m_i for each item i can be found in Silver et al. (1998), Section 11.2.2.

According to those authors, the items are first numbered such that $a_i/(D_i P_i)$ is smallest for item 1. We set $m_1 = 1$. The other values for m_i's are

$$m_i = \sqrt{\frac{a_{Bi}}{D_i P_i} \times \frac{D_1 P_1}{A_B + a_{B1}}}.$$

(Throughout, the calculated values of m_i, and later m_i', are to be rounded to the nearest integer, as required by the MPP.)

With the discount fraction of ρ, buyer's total annual relevant cost is

$$TRC_1^{[\mathbb{C}]} = (1-\rho)\sum_{i=1}^{n} D_i P_i + \left(A_B + \sum_{i=1}^{n} \frac{a_{Bi}}{m_i}\right)\frac{1}{T_B} + \frac{1}{2} r T_B (1-\rho) \sum_{i=1}^{n} m_i D_i P_i.$$

Therefore, buyer's payoff due to the discount is $\Pi_B^{[\mathbb{C}]} = TRC_0^{[\mathbb{C}]} - TRC_1^{[\mathbb{C}]}$. After simplification, we have

$$\Pi_B^{[\mathbb{C}]} = \rho \sum_{i=1}^{n} D_i P_i + \left(A_B + \sum_{i=1}^{n} \frac{a_{Bi}}{m_i}\right)\left(\frac{1}{T_0} - \frac{1}{T_B}\right) + \frac{1}{2} r \sum_{i=1}^{n} m_i D_i P_i \Big[T_0 - T_B(1-\rho)\Big]. \quad (4.2)$$

4.3.2 Supplier's payoff

As to the supplier's cost structure, we consider his costs incurred with only this buyer, including his own production acquisition cost, the cost of handling the buyer's orders, and the corresponding inventory holding cost.

Similar to the buyer's ordering costs, the supplier also has a major cost A_S for handling orders of this family of products and a minor cost a_{Si} for each item i. So before the discount, the supplier's profit is

$$\pi_{S0}^{[\mathbb{C}]} = \sum_{i=1}^{n} D_i P_i - \sum_{i=1}^{n} D_i v_i - \left(A_S + \sum_{i=1}^{n} \frac{a_{Si}}{m_i}\right)\frac{1}{T_0} + \frac{1}{2} T_0 \sum_{i=1}^{n} m_i D_i H_i.$$

Then, the profit after the discount is

$$\pi_{S1}^{[\mathbb{C}]} = (1-\rho) \sum_{i=1}^{n} D_i P_i - \sum_{i=1}^{n} D_i v_i - \left(A_S + \sum_{i=1}^{n} \frac{a_{Si}}{m_i}\right)\frac{1}{T_S} + \frac{1}{2} T_S \sum_{i=1}^{n} m_i D_i H_i.$$

So supplier's payoff after applying the group discount is $\Pi_S^{[\mathbb{C}]} = \pi_{S1}^{[\mathbb{C}]} - \pi_{S0}^{[\mathbb{C}]}$, which can be rewritten as

$$\Pi_S^{[\mathbb{C}]} = -\rho \sum_{i=1}^{n} D_i P_i + \left(A_S + \sum_{i=1}^{n} \frac{a_{Si}}{m_i}\right)\left(\frac{1}{T_0} - \frac{1}{T_S}\right) + \frac{1}{2}\sum_{i=1}^{n} m_i D_i H_i \Big[T_0 - T_S\Big]. \tag{4.3}$$

4.3.3 Model I - Noncooperative model with constant demand

The Stackelberg game is a dynamic game model in which the player with dominant power (leader) moves first, and the other player (follower) observes the leader's movements and reacts to these movements based upon the follower's best interests. This is a close representation of what happens in a real discount problem. Therefore, our noncooperative model employs a Stackelberg equilibrium, which allows the supplier (leader) to construct a quantity discount policy by maximizing his payoff, in light of the buyer's (follower) best reaction of attempting to maximize her own payoff.

To maximize buyer's payoff, set $\partial \Pi_B^{[\mathbb{C}]} / \partial T_B = 0$. (Note that this objective function, Eq. (4.2), is concave in T_B.) Then, the buyer's best policy is to order a time supply of

$$T_B = \sqrt{\frac{2\left(A_B + \sum_{i=1}^{n} \frac{a_{Bi}}{m_i}\right)}{r(1-\rho)\sum_{i=1}^{n} m_i D_i P_i}} = \frac{T_0}{\sqrt{1-\rho}}. \tag{4.4}$$

LEMMA 4.1. *Suppose the demand rate is deterministic and constant. The supplier is then worse off if buyer orders her EOQ at the discounted price. That is,*

$\Pi_S^{[\mathbb{C}]} = \pi_{S1}^{[\mathbb{C}]}(T_B) - \pi_{S0}^{[\mathbb{C}]}(T_0) < 0.$

Proof. If the buyer orders $T_B = T_0/\sqrt{1-\rho}$, the supplier's payoff is, from Eq. (4.3),

$$\Pi_S^{[\mathbb{C}]} = -\rho \sum_{i=1}^{n} D_i P_i + \left(A_S + \sum_{i=1}^{n} \frac{a_{Si}}{m_i}\right)\left(\frac{1}{T_0} - \frac{\sqrt{1-\rho}}{T_0}\right) + \frac{1}{2}\sum_{i=1}^{n} m_i D_i H_i \left[T_0 - \frac{T_0}{\sqrt{1-\rho}}\right].$$

After some algebra,

$$\frac{\partial \Pi_S^{[\mathbb{C}]}}{\partial \rho} = -\frac{1}{1-\rho} \times \left[\sum_{i=1}^{n} D_i P_i (1-\rho) - \frac{1}{2T_0}\left(A_S + \sum_{i=1}^{n} \frac{a_{Si}}{m_i}\right) - \frac{T_0}{4}\sum_{i=1}^{n} m_i D_i H_i\right].$$

Let $K = \sum_{i=1}^{n} D_i P_i (1-\rho) - \frac{1}{2T_0}\left(A_S + \sum_{i=1}^{n} \frac{a_{Si}}{m_i}\right) - \frac{T_0}{4}\sum_{i=1}^{n} m_i D_i H_i$.

If $K \le 0$, we must have $\Pi_S^{[\mathbb{C}]} < 0$. If $K > 0$, $\partial \Pi_S^{[\mathbb{C}]}/\partial \rho < 0$. And since $\Pi_S^{[\mathbb{C}]}(\rho = 0) = 0$, we again find $\Pi_S^{[\mathbb{C}]} < 0$ for any positive value of ρ.

Therefore, the supplier loses if buyer orders her EOQ at the discounted price. □

From this lemma, we find that the supplier has to set the group discount in such a way that the buyer orders more than her EOQ, so that he can gain profit from offering the discount. Now, to maximize the supplier's payoff, set $\partial \Pi_S^{[\mathbb{C}]}/\partial T_S = 0$. We then have

$$T_S = \sqrt{\frac{2\left(A_S + \sum_{i=1}^{n} \frac{a_{Si}}{m_i}\right)}{\sum_{i=1}^{n} m_i D_i H_i}}. \tag{4.5}$$

This indicates that supplier would like to encourage the buyer to order at intervals of $T_b = T_S$ periods, by offering a group discount applied to her total purchasing value.

Note that the group discount is offered on *each* replenishment. Besides, the smallest replenishment size corresponds to the total replenishment value of all items having $m_i = 1$. Therefore, the position of the breakpoint, in terms of the total value of replenishment, can then be calculated as:

$$V_b = T_b \sum_{i \in I} D_i P_i. \tag{4.6}$$

where I denotes the set of all items having $m_i = 1$.

We also notice that the payoff for supplier is a monotone decreasing function of ρ. To offer this discount, the supplier has to ensure that he gains a positive payoff, i.e., $\Pi_S^{[\mathbb{C}]}(T_b) \geq 0$. After simplification, we have

$$\rho \leq \rho^U = \left(\frac{1}{T_0} - \frac{1}{T_b}\right) \frac{2\left(A_S + \sum_{i=1}^{n} \frac{a_{Si}}{m_i}\right) - T_0 T_b \sum_{i=1}^{n} m_i D_i H_i}{2 \sum_{i=1}^{n} D_i P_i}. \tag{4.7}$$

Similarly, the buyer's payoff is a monotone increasing function of ρ. So she would only take the discount if she can save on her total relevant costs, i.e., $\Pi_B^{[\mathbb{C}]}(T_b) \geq 0$. After manipulating,

$$\rho \geq \rho^{L} = \left(\frac{1}{T_0} - \frac{1}{T_b}\right) \frac{rT_0T_b \sum_{i=1}^{n} m_i D_i P_i - 2\left(A_B + \sum_{i=1}^{n} \frac{a_{Bi}}{m_i}\right)}{2\sum_{i=1}^{n} D_i P_i + rT_b \sum_{i=1}^{n} m_i D_i P_i}. \tag{4.8}$$

Thus, we have found an upper bound, ρ^U, and also a lower bound, ρ^L, on the percentage of discount the supplier should offer. (Note that $\rho^U \geq \rho^L$ is both necessary and sufficient for a feasible discount percentage to exist.) Both the supplier and the buyer will benefit from the group discount policy for any discount percentage between ρ^U and ρ^L. Realize that when $\rho = \rho^U(\rho^L)$, the payoff to the supplier (buyer) is zero. By examining the degree to which his payoff would be shared with the buyer, the supplier may determine his discount percentage within this range. Numerical examples will follow in Sec. 4.3.5.

4.3.4 Model II - Joint model with constant demand

Now we assume a centralized system in which the supplier and buyer agree to make the decisions on quantity discount and replenishment together. The joint payoffs are maximized and shared between the two parties with no extra charges.

The joint payoff of the buyer and supplier is defined as

$$
\begin{aligned}
\Pi_J^{[\mathbb{C}]} = \Pi_B^{[\mathbb{C}]} + \Pi_S^{[\mathbb{C}]} \quad = \quad & \left(A_B + \sum_{i=1}^{n} \frac{a_{Bi}}{m_i}\right)\left(\frac{1}{T_0} - \frac{1}{T_J}\right) + \frac{1}{2} r \sum_{i=1}^{n} m_i D_i P_i \left[T_0 - T_J(1-\rho)\right] \\
& + \left(A_S + \sum_{i=1}^{n} \frac{a_{Si}}{m_i}\right)\left(\frac{1}{T_0} - \frac{1}{T_J}\right) + \frac{1}{2} \sum_{i=1}^{n} m_i D_i H_i \left[T_0 - T_J\right].
\end{aligned}
$$

After manipulation,

$$
\begin{aligned}
\Pi_J^{[\mathbb{C}]} \quad = \quad & \left(A_B + A_S + \sum_{i=1}^{n} \frac{a_{Bi} + a_{Si}}{m_i}\right)\left(\frac{1}{T_0} - \frac{1}{T_J}\right) + \frac{1}{2}\left(r \sum_{i=1}^{n} m_i D_i P_i + \sum_{i=1}^{n} m_i D_i H_i\right) T_0 \\
& - \frac{1}{2}\left[r(1-\rho) \sum_{i=1}^{n} m_i D_i P_i + \sum_{i=1}^{n} m_i D_i H_i\right] T_J. \qquad (4.9)
\end{aligned}
$$

To maximize the joint payoff, set $\partial \Pi_J^{[\mathbb{C}]} / \partial T_J = 0$, so we have

$$
T_J = \sqrt{\frac{2\left(A_B + A_S + \sum_{i=1}^{n} \frac{a_{Bi} + a_{Si}}{m_i}\right)}{r(1-\rho) \sum_{i=1}^{n} m_i D_i P_i + \sum_{i=1}^{n} m_i D_i H_i}}. \qquad (4.10)
$$

Since $\partial^2 \Pi_J^{[\mathbb{C}]} / \partial T_J^2 = -2\left(A_B + A_S + \sum_{i=1}^{n} \frac{a_{Bi} + a_{Si}}{m_i}\right) \Big/ T_J^3 < 0$, the solution of T_J in Eq. (4.10) is optimal.

Set the breakpoint $T_b = T_J$. Then, replacing T_J in Eq. (4.9) by T_b, the value of ρ may be obtained by maximizing $\Pi_J^{[\mathbb{C}]}$. Thus, the joint model can be written as the following nonlinear optimization problem:

$$\begin{aligned}\max_{T_b,\rho} \Pi_J^{[\mathbb{C}]}(T_b,\rho) &= \left(A_B + A_S + \sum_{i=1}^{n} \frac{a_{Bi}+a_{Si}}{m_i}\right)\left(\frac{1}{T_0} - \frac{1}{T_b}\right) \\ &+\frac{1}{2}T_0\left(r\sum_{i=1}^{n} m_i D_i P_i + \sum_{i=1}^{n} m_i D_i H_i\right) \\ &-\frac{1}{2}T_b\left[r(1-\rho)\sum_{i=1}^{n} m_i D_i P_i + \sum_{i=1}^{n} m_i D_i H_i\right]. \qquad (4.11)\end{aligned}$$

$s.t.$

$$\Pi_B^{[\mathbb{C}]}(T_b,\rho) \geq 0;$$

$$\Pi_S^{[\mathbb{C}]}(T_b,\rho) \geq 0;$$

$$0 < \rho < 1.$$

Solving this maximization problem gives the supplier's best choice of T_b and ρ for the group discount policy. The breakpoint V_b, i.e., the total replenishment value, can then be computed from Eq. (4.6).

LEMMA 4.2. *Compared to the noncooperative approach, the optimal policy is to order more frequently in the joint model. That is, as a time supply,* $T_J < T_S$.

Proof. The order quantity that maximizes the supplier's payoff is higher than that which maximizes the buyer's. Thus $T_S > T_B$, so we have:

$$T_J = \sqrt{\frac{2\left(A_B + A_S + \sum_{i=1}^{n} \frac{a_{Bi} + a_{Si}}{m_i}\right)}{r(1-\rho)\sum_{i=1}^{n} m_i D_i P_i + \sum_{i=1}^{n} m_i D_i H_i}}$$

$$< \sqrt{\frac{2\left(A_S + \sum_{i=1}^{n} \frac{a_{Si}}{m_i}\right)\left[1 + \sum_{i=1}^{n} m_i D_i H_i \Big/ r(1-\rho)\sum_{i=1}^{n} m_i D_i P_i\right]}{r(1-\rho)\sum_{i=1}^{n} m_i D_i P_i + \sum_{i=1}^{n} m_i D_i H_i}}$$

$$< \sqrt{\frac{2\left(A_S + \sum_{i=1}^{n} \frac{a_{Si}}{m_i}\right)}{\sum_{i=1}^{n} m_i D_i H_i}} = T_S.$$

□

LEMMA 4.3. *In the case of constant demand, when the joint payoffs are maximized, only the buyer's payoff is positive. The supplier needs to share the payoff gained by the buyer.*

Proof. From Eq. (4.11), we notice that $\Pi_J^{[\mathbb{C}]}$ is monotonically increasing with respect to ρ. Hence $\Pi_J^{[\mathbb{C}]}$ is maximized when ρ is at its highest value.

The upper limit of ρ can be obtained by setting $\Pi_S^{[\mathbb{C}]}(T_b, \rho) = 0$. Therefore, at the

Table 4.1: Numerical example 4.1 - Parameters

$A_B = 200; r = 20\%; A_S = 1800$			
	Item 1	Item 2	Item 3
D_i	1200	120	70
P_i	50	20	10
a_{Bi}	120	120	120
a_{Si}	80	80	80
H_i	7	2	1

optimal solution, the only party that gains positive payoff (cost saving) is the buyer, while the supplier has zero payoff and needs to share the payoff gained by the buyer. □

Lemmas 4.2 and 4.3 will be further illustrated by the following numerical example.

4.3.5 Numerical example 4.1 - Constant demand

In this section, we consider a case where the buyer purchases three items. Table 4.1 lists the values of relevant parameters. Item 1 has the smallest value of $a_i/(D_i v_i)$, so according to MPP, we set $m_1 = 1$. Then the values of m_2 and m_3 can be calculated as 3 and 6, respectively. Using these m_i values, we get $T_0 = 0.23$ year = 84 days.

From the supplier's perspective, solving Eq. (4.5) gives $T_S = 0.63$ years $\approx$ 232 days. By setting $T_b = T_S$, we calculate the value of the replenishment break point as $V_b =$ \$ 38 066. So the buyer can take advantage of the group discount only when the total value of a

Table 4.2: Numerical example 4.1 - Results

% of payoff shared by the buyer	0	25%	50%	75%	100%
ρ	2.71%	3.34%	3.98%	4.65%	5.34%
$\Pi_B^{[\mathbb{C}]}$	0	422	858	1310	1778
$\Pi_S^{[\mathbb{C}]}$	1659	1266	858	437	0

replenishment exceeds V_b.

From Eqs. (4.7) and (4.8), the upper and lower bounds of the discount are $\rho^U = 5.34\%$ and $\rho^L = 2.71\%$. Figure 4.1 illustrates the payoffs for buyer and supplier at different discount percentages between the bounds. Now, by examining the percentage of payoffs he would allocate to the buyer, the supplier can determine his discount policy. Table 4.2 shows the discount policy and corresponding payoffs according to the degree to which the supplier is willing to share the payoff. The first row of the table is the percentage of the payoff which goes to the buyer. For example, assuming that the supplier shares half the total profit with the buyer, by our calculation, we have $\rho = 3.98\%$. Under this discount policy, the payoffs of supplier and buyer are $\Pi_S^{[\mathbb{C}]} = \Pi_B^{[\mathbb{C}]} = \$\ 858$.

Next, we apply the joint model with the same set of parameters. The optimal discount policy obtained from the optimization model (4.11) is $\rho = 4.74\%$ and $T_b = 0.44$ years ≈ 160 days. The breakpoint for the replenishment value V_b can be thus calculated as \$ 26 750. At that breakpoint, the joint payoff achieves \$ 2 400. This improves the total payoff by \$ 684, compared to solution of the first model.

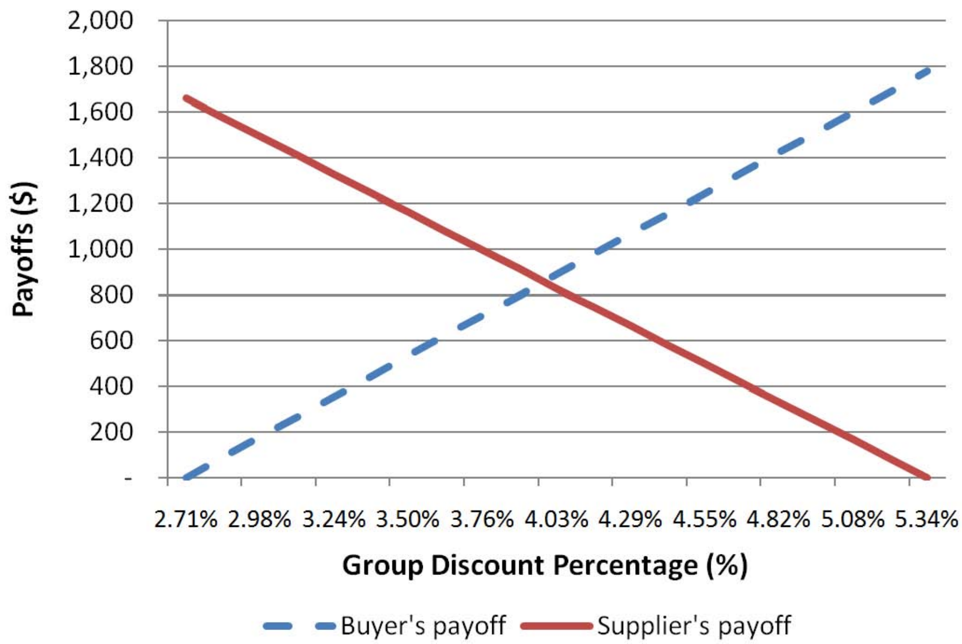


Figure 4.1: Payoffs for buyer and supplier as a function of discount percentage (Numerical example 4.1)

Examining the two models, the joint model provides the higher discount percentage (4.74% vs 3.98%) and lower breakpoint ($ 26 750 vs $ 38 066), but a higher joint payoff. If the supplier and buyer were to divide this payoff improvement equally between them, both would be much better off (each would receive profits of $ 1 200 from the joint decisions).

4.4 Price-sensitive demand case

So far, we discussed the case in which the demands for all items are constant and deterministic. However, in practice, the demand rate increases by ΔD as the price decreases ($\Delta P < 0$). So we extend our model to consider the impact on demand due to a change in price.

Instead of a regular demand function, we introduce the price elasticity of demand, denoted by η_i, for each item i. The buyer is assumed to sell the products at the same discount percentage from her regular price, when she purchases the group of items at the discounted prices. Therefore, we have

$$\eta_i = -\frac{\Delta D_i/D_i}{\Delta R_i/R_i} = -\frac{\Delta D_i/D_i}{\Delta P_i/P_i}.$$

4.4.1 Buyer's payoff

Before any discount, the buyer's total annual cost is

$$TRC_0^{[\mathbb{S}]} = \sum_{i=1}^{n} D_i P_i + \left(A_B + \sum_{i=1}^{n} \frac{a_{Bi}}{m_i} \right) \frac{1}{T_0} + \frac{1}{2} r T_0 \sum_{i=1}^{n} m_i D_i P_i,$$

where

$$m_i = \sqrt{\frac{a_{Bi}}{D_i P_i} \times \frac{D_1 P_1}{A_B + a_{B1}}},$$

which is exactly the same as in the constant-demand case. However, taking into account the price sensitivity of demand, not only the buyer's annual cost, but also her revenue, changes due to the discount she receives. Therefore, we now consider the annual *profit* of the buyer, rather than only cost. The buyer's profit before discount is

$$\pi_{B0}^{[\mathbb{S}]} = \sum_{i=1}^{n} D_i R_i - TRC_0^{[\mathbb{S}]}.$$

Buyer's total relevant cost after discount is

$$TRC_1^{[\mathbb{S}]} = (1-\rho) \sum_{i=1}^{n} D_i (1+\eta_i \rho) P_i + \left(A_B + \sum_{i=1}^{n} \frac{a_{Bi}}{m_i'} \right) \frac{1}{T_B} + \frac{1}{2} r T_B (1-\rho) \sum_{i=1}^{n} m_i' D_i (1+\eta_i \rho) P_i,$$

where

$$m_i' = \sqrt{\frac{a_{Bi}}{D_i(1+\eta_i\rho)P_i(1-\rho)} \times \frac{D_1(1+\eta_1\rho)P_1(1-\rho)}{A_B + a_{B1}}} = m_i \sqrt{\frac{(1+\eta_1\rho)}{(1+\eta_i\rho)}}. \tag{4.12}$$

Note that when the η_i are the same for each item, we have $m_i' = m_i$.

So the profit of buyer who benefits from a discount fraction ρ is

$$\pi_{B1}^{[\mathbb{S}]} = \sum_{i=1}^{n} D_i(1+\eta_i\rho)R_i(1-\rho) - TRC_1^{[\mathbb{S}]}.$$

Correspondingly, the buyer's payoff from the discount is $\Pi_B^{[\mathbb{S}]} = \pi_{B1}^{[\mathbb{S}]} - \pi_{B0}^{[\mathbb{S}]}$. After simplification, we have

$$\begin{aligned}\Pi_B^{[\mathbb{S}]} \quad = \quad & \rho\sum_{i=1}^{n} D_i(R_i-P_i)(\eta_i-\eta_i\rho-1) + A_B\left(\frac{1}{T_0}-\frac{1}{T_B}\right) + \left(\frac{1}{T_0}\sum_{i=1}^{n}\frac{a_{Bi}}{m_i} - \frac{1}{T_B}\sum_{i=1}^{n}\frac{a_{Bi}}{m_i'}\right) \\ & + \frac{1}{2}r\left[T_0\sum_{i=1}^{n} D_iP_im_i - T_B(1-\rho)\sum_{i=1}^{n} D_i(1+\eta_i\rho)P_im_i'\right]. \qquad (4.13)\end{aligned}$$

4.4.2 Supplier's payoff

Supplier's profit before the discount is identical to the case with constant demand:

$$\pi_{S0}^{[\mathbb{S}]} = \sum_{i=1}^{n} D_iP_i - \sum_{i=1}^{n} D_iv_i - \left(A_S + \sum_{i=1}^{n}\frac{a_{Si}}{m_i}\right)\frac{1}{T_0} + \frac{1}{2}T_0\sum_{i=1}^{n} m_iD_iH_i.$$

With the consideration of price elasticity of demand, his profit after the discount is

$$\pi_{S1}^{[\mathbb{S}]} = (1-\rho)\sum_{i=1}^{n} D_i(1+\eta_i\rho)P_i - \sum_{i=1}^{n} D_i(1+\eta_i\rho)v_i - \left(A_S + \sum_{i=1}^{n}\frac{a_{Si}}{m_i'}\right)\frac{1}{T_S} + \frac{1}{2}T_S\sum_{i=1}^{n} m_i'D_i(1+\eta_i\rho)H_i.$$

So supplier's payoff after applying the group discount is $\Pi_S^{[\mathbb{S}]} = \pi_{S1}^{[\mathbb{S}]} - \pi_{S0}^{[\mathbb{S}]}$, which can be rewritten as

$$
\begin{aligned}
\Pi_S^{[\mathbb{S}]} &= \rho\sum_{i=1}^{n} D_i P_i(\eta_i - \eta_i\rho - 1) - \rho\sum_{i=1}^{n} D_i\eta_i v_i + A_S\left(\frac{1}{T_0} - \frac{1}{T_S}\right) \\
&+ \left(\frac{1}{T_0}\sum_{i=1}^{n}\frac{a_{Si}}{m_i} - \frac{1}{T_S}\sum_{i=1}^{n}\frac{a_{Si}}{m_i'}\right) \\
&+ \frac{1}{2}\left[T_0\sum_{i=1}^{n} D_i H_i m_i - T_S\sum_{i=1}^{n} D_i(1+\eta_i\rho)H_i m_i'\right]. \qquad (4.14)
\end{aligned}
$$

4.4.3 Model III - Noncooperative model with price-sensitive demand

Given a discount ρ, the buyer's best response is to select an order pattern that maximizes her payoff. It is easy to show that $\Pi_B^{[\mathbb{S}]}$ in Eq. 4.13 is concave with respect to T_B. So we set $\partial\Pi_B^{[\mathbb{S}]}/\partial T_B = 0$ to maximize this function, and obtain

$$
T_B = \sqrt{\frac{2\left(A_B + \sum_{i=1}^{n}\frac{a_{Bi}}{m_i'}\right)}{r(1-\rho)\sum_{i=1}^{n} m_i' D_i(1+\eta_i\rho)P_i}}. \qquad (4.15)
$$

When the η_i are the same for each item, and thus when $m_i' = m_i$,

$$
T_B = \frac{T_0}{\sqrt{(1-\rho)(1+\eta_i\rho)}}
$$

The supplier will take advantage of the buyer's best response, to determine the discount percentage that should be offered. Substituting T_B for T_S in Eq. (4.14), the supplier's *optimal* discount percentage ρ can be obtained by solving the maximization problem:

$$\begin{aligned}
\max_{\rho} \Pi_S^{[\mathbb{S}]}(\rho, T_B, m_i') \quad &= \quad \rho \sum_{i=1}^{n} D_i P_i (\eta_i - \eta_i \rho - 1) - \rho \sum_{i=1}^{n} D_i \eta_i v_i + A_S \left(\frac{1}{T_0} - \frac{1}{T_B} \right) \\
&\quad + \left(\frac{1}{T_0} \sum_{i=1}^{n} \frac{a_{Si}}{m_i} - \frac{1}{T_B} \sum_{i=1}^{n} \frac{a_{Si}}{m_i'} \right) \\
&\quad + \frac{1}{2} \left[T_0 \sum_{i=1}^{n} D_i H_i m_i - T_B \sum_{i=1}^{n} D_i (1 + \eta_i \rho) H_i m_i' \right], \qquad (4.16)
\end{aligned}$$

$s.t.$

$$\Pi_B^{[\mathbb{S}]}(\rho, T_B, m_i') \geq 0;$$

$$\Pi_S^{[\mathbb{S}]}(\rho, T_B, m_i') \geq 0;$$

$$0 < \rho < 1.$$

where T_B and m_i' can be found by Eqs. (4.15) and (4.12), respectively.

Once ρ is determined, the quantity breakpoint T_b can be determined by maximizing the supplier's profit at this particular discount.

Solving $\partial \Pi_S^{[\mathbb{S}]} / \partial T_S = 0$, we have

$$T_b = T_S = \sqrt{\frac{2\left(A_S + \sum_{i=1}^{n} \frac{a_{Si}}{m'_i}\right)}{\sum_{i=1}^{n} m'_i D_i (1 + \eta_i \rho) H_i}} \tag{4.17}$$

Accordingly, the breakpoint in terms of the total value of replenishment can be calculated by Eq. (4.6).

A solution procedure is introduced to solve this model (Fig. 4.2). We start from the situation that no discount is applied. The buyer determines her replenishment pattern, i.e. m_i and T_0, for the group of n items according to MPP. By employing m_i as the initial value of m'_i, T_B is calculated by Eq. (4.15). In light of buyer's best order interval, the supplier computes the best discount percentage ρ for this group and the corresponding breakpoint T_b. Then, the value of m'_i is updated with ρ. If this newly obtained value of m'_i is identical or very close to the previous one, we have found the optimal discount policy; otherwise, we use this new value and re-do the calculations of T_B, ρ, and T_b, until the integer m'_i does not change any more. Note that we keep on checking the payoffs for supplier and buyer in each iteration, to make sure both parties can benefit from the group discount.

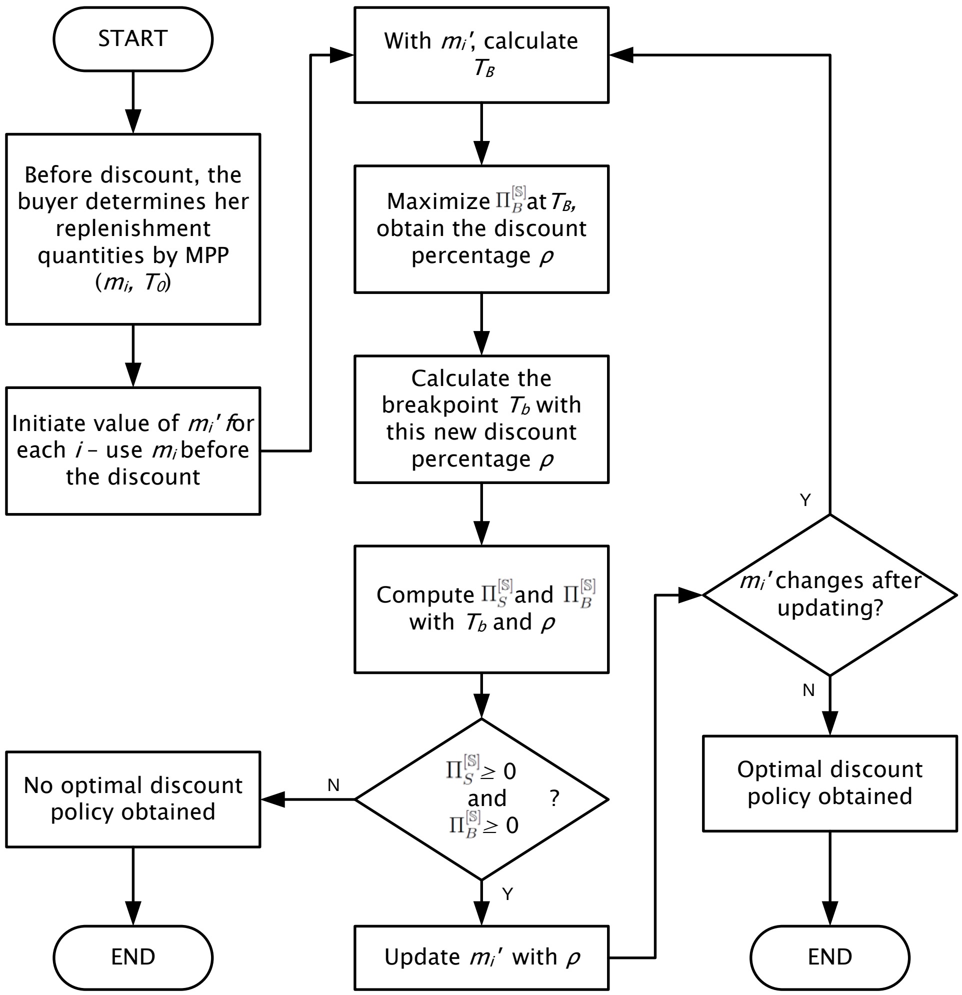


Figure 4.2: Solution procedure for the noncooperative model with price-sensitive demand (Model III)

4.4.4 Model IV - Joint model with price-sensitive demand

Now we define the *joint* payoff of the buyer and supplier, as the summation of the supplier and buyer's payoffs. After simplification, we have

$$\begin{aligned}
\Pi_J^{[\mathbb{S}]}(T_J,\rho) &= \Pi_B^{[\mathbb{S}]}(T_J,\rho)+\Pi_S^{[\mathbb{S}]}(T_J,\rho) \\
&= \rho\sum_{i=1}^{n} D_iR_i(\eta_i-\eta_i\rho-1)-\rho\sum_{i=1}^{n} D_i\eta_i v_i \\
&\quad +\frac{1}{T_0}\left(A_B+A_S+\sum_{i=1}^{n}\frac{a_{Bi}+a_{Si}}{m_i}\right)-\frac{1}{T_J}\left(A_B+A_S+\sum_{i=1}^{n}\frac{a_{Bi}+a_{Si}}{m_i'}\right) \\
&\quad +\frac{1}{2}T_0\left(r\sum_{i=1}^{n} D_iP_im_i+\sum_{i=1}^{n} D_iH_im_i\right) \\
&\quad -\frac{1}{2}T_J\left[r\sum_{i=1}^{n} D_i(1+\eta_i\rho)P_i(1-\rho)m_i'+\sum_{i=1}^{n} D_i(1+\eta_i\rho)H_im_i'\right]. \qquad (4.18)
\end{aligned}$$

Let $\partial\Pi_J^{[\mathbb{S}]}(T_J,\rho)/\partial T_J=0$, to find

$$T_J=\sqrt{\frac{2\left(A_B+A_S+\sum_{i=1}^{n}\frac{a_{Bi}+a_{Si}}{m_i'}\right)}{r\sum_{i=1}^{n} D_i(1+\eta_i\rho)P_i(1-\rho)m_i'+\sum_{i=1}^{n} D_i(1+\eta_i\rho)H_im_i'}}. \qquad (4.19)$$

Set $T_b=T_J$, then calculate the discount fraction ρ by solving $\partial\Pi_J^{[\mathbb{S}]}(T_J,\rho)/\partial\rho=0$. So the joint model can be written as the following maximization problem:

$$
\begin{aligned}
\max_{T_b,\rho} \Pi_J^{[\mathbb{S}]}(T_b,\rho) \quad = \quad & \rho\sum_{i=1}^{n} D_i R_i(\eta_i-\eta_i\rho-1)-\rho\sum_{i=1}^{n} D_i\eta_i v_i \\
& +\frac{1}{T_0}\left(A_B+A_S+\sum_{i=1}^{n}\frac{a_{Bi}+a_{Si}}{m_i}\right)-\frac{1}{T_b}\left(A_B+A_S+\sum_{i=1}^{n}\frac{a_{Bi}+a_{Si}}{m_i'}\right) \\
& +\frac{1}{2}T_0\left(r\sum_{i=1}^{n} D_i P_i m_i+\sum_{i=1}^{n} D_i H_i m_i\right) \\
& -\frac{1}{2}T_b\left[r\sum_{i=1}^{n} D_i(1+\eta_i\rho)P_i(1-\rho)m_i'+\sum_{i=1}^{n} D_i(1+\eta_i\rho)H_i m_i'\right]. \qquad (4.20)
\end{aligned}
$$

$s.t.$

$$\Pi_B^{[\mathbb{S}]}(T_b,\rho)\geq 0;$$

$$\Pi_S^{[\mathbb{S}]}(T_b,\rho)\geq 0;$$

$$0<\rho<1.$$

To find the solution for this joint model, we employ a procedure (Fig. 4.3) similar to that of the noncooperative case. First, for each item i, we initialize the m_i' with the value of m_i obtained before the discount, and thus solve the maximization problem given by Eq. (4.20) for T_b and ρ. Compute the joint payoff $\Pi_J^{[\mathbb{S}]}$ and update the values of m_i' accordingly. Then the maximization problem is re-solved with those updated values, until no additional change occurs.

Notice that it may happen that the joint payoff decreases after updating the m_i' value. This is because the updating process is carried out by the *buyer*, who could gain positive

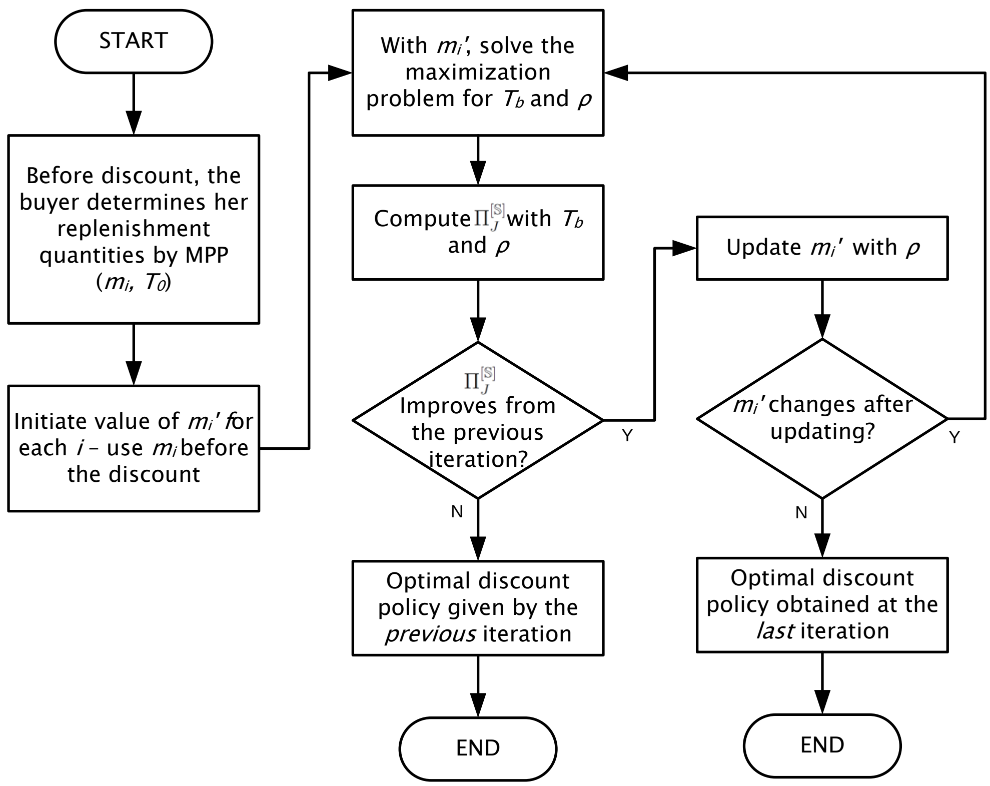


Figure 4.3: Solution procedure for joint model with price-sensitive demand (Model IV)

payoff from that process. However, the updating may have negative impact on the supplier's payoff, and accordingly, on the joint payoff. If this situation does occur, we assume that the buyer would not continue the updating: In Model IV, the joint payoff receives the highest consideration.

4.4.5 Numerical example 4.2 - Price-sensitive demand

To demonstrate the application of the two models with price-sensitive demand, we again employ a numerical example considering a group of three items, but now each has a (distinct) price elasticity of demand. Respective parameters are listed in Table 4.3. Before the discount, item 1 is set as the base item ($m_1 = 1$), due as before to its lowest value of $a_i/(D_i v_i)$; and correspondingly, we have $m_2 = 3$ and $m_3 = 6$. Then, as in Example 4.1, the value of T_0 is computed as 0.23 years $\approx$ 84 days.

Table 4.4 shows the procedure in solving the noncooperative case. The first iteration is based on the original m_i values before discount. We obtain the discount policy with a discount percentage $\rho = 8.35\%$ and the breakpoint in terms of total replenishment value, $V_b =$ \$ 23 129. After updating m_i', we enter the second iteration, which gives a discount policy of $\rho = 8.36\%$ and $V_b =$ \$ 23 272 (or $T_b = 131$ days in time value). Since no further change occurs in m_i' after updating, we stop here. The optimal discount policy that the supplier should offer is found in the last iteration (iteration 2). This policy gains the supplier and buyer extra profits of \$ 1297 and \$ 766, respectively.

Table 4.3: Numerical example 4.2 - Parameters

$A_B = 200; r = 20\%; A_S = 600$

	Item 1	Item 2	Item 3
D_i	1200	120	70
η_i	2	0.5	3
P_i	50	20	10
R_i	65	26	13
v_i	15	6	3
a_{Bi}	120	120	120
a_{Si}	80	80	80
H_i	7	2	1

Fig. 4.4 illustrates the payoffs for buyer and supplier as a function of the replenishment interval. We see that the buyer's payoff is maximized at $T_B = 83$ days, but she has to order every 131 days to obtain the group discount offered by the supplier. Even though her payoff is lower than that indicated by the level T_B, the buyer can still gain positive payoff, which is the motivation for her to increase order quantities and take the discount.

We now consider the case in which the supplier and buyer make the discount and replenishment decisions jointly. Table 4.5 shows the procedure whereby the optimal joint decision is obtained. Similar to the noncooperative case, we start our calculations from the m_i' value before the discount, and continue until the updating brings no more change or negligible change to this value for each item i. However, through comparison of results

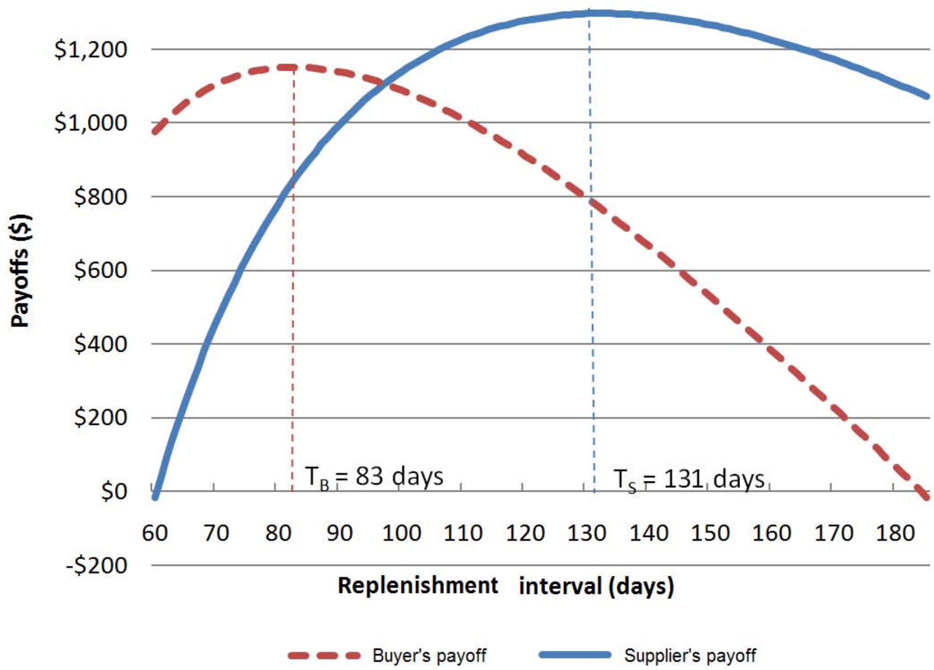


Figure 4.4: Payoffs for buyer and supplier as a function of replenishment interval (Numerical example 4.2)

Table 4.4: Solution procedure for Model III

Iterations	m'_i (items 1,2,3)	ρ	T_B	T_S	T_b	V_b	$\Pi_B^{[\$]}$	$\Pi_S^{[\$]}$	Updated m'_i
1	1,3,6	8.35%	0.224 yrs 82 days	0.361 yrs 132 days	0.361 yrs 132 days	23 129	756	1 306	1,3,5
2	1,3,5	8.36%	0.226 yrs 83 days	0.358 yrs 131 days	0.358 yrs 131 days	23 272	766	1 297	1,3,5

attained from the two iterations, we realize that the first iteration provides a higher joint payoff. Therefore, the buyer would not update his order pattern (the value of m'_i for each item i). Instead, she would agree to keep the original m_i values to make sure that system maximization is achieved. The optimal joint decisions are: 1) the supplier offers a discount of 11.76% on this group of items; and 2) the buyer places her order every 104 days, with a total replenishment value for each order of $ 18 167.

Accordingly, the joint payoff gained from these decisions is $ 2426, an increase of $ 363 compared to the total payoffs for the supplier and buyer in the noncooperative case. If both parties agree to divide this extra payoff equally, the buyer and supplier would receive profits of $ 948 and $ 1478 from the joint decisions, respectively. We also notice that the optimal replenishment policy obtained by making decisions jointly is to place orders every 104 days, which is less than the 131 days by the noncooperative model.

We performed an extensive data analysis based upon more than 5,000 parameters sets. The following relations were found between the solutions of the noncooperative and joint

Table 4.5: Solution procedure for Model IV

Iterations	m_i' (items 1,2,3)	ρ	T_b	V_b	$\Pi_J^{[\mathbb{S}]}$	Updated m_i'	Notes
1	1,3,6	11.76%	0.284 yrs 104 days	18 167	2 426	1,3,5	Optimal
2	1,3,5	11.77%	0.287 yrs 105 days	18 774	2 424	1,3,5	no need to update m_i

models in the case of price-sensitive demand:

1. It is optimal to order more frequently in the joint model than in the noncooperative model. As a time supply, one has $T_J < T_S$.

2. The quantity discount percentage obtained from the joint model is greater than the one from the noncooperative model.

3. The total payoffs for the supplier plus the buyer are significantly improved when they agree to make decisions jointly, compared to when each makes an individual decision.

Finally, we compare the numerical results of Examples 4.1 and 4.2. Note that, except for a few extra parameters related to the price-sensitivity, we assume a larger value of A_S in the constant-demand case ($A_S = 1800$ in Table 4.1 but $A_S = 600$ in Table 4.3). This is for illustrative purposes: The larger value of A_S results in a deeper discount percentage. However, even then (see Table 4.6), the optimal discount percentage for the constant-demand case is still much lower than in the case of price-sensitive demand (3.98% compared

Table 4.6: Comparisons of the four models

Model	I	II	III	IV
ρ	3.98%	4.74%	8.36%	11.76%
T_b (days)	232	160	131	104
V_b	38 066	26 750	23 272	18 167
Π_S	858	1 200	1 297	1 478
Π_B	858	1 200	766	948
Π_J	1 716	2 400	2 063	2 426

to 8.36% for the noncooperative models, and 4.74% compared to 11.76% for the joint models).

We also notice from Fig. 4.4 that, when the demand rate is price-sensitive, the supplier's payoff is positive even if the buyer orders her EOQ at the discounted price. This is quite the opposite to the cases with constant demand rate (Lemma 4.1). These comparisons illustrate the huge impacts of price elasticity on the supplier's optimal discount policy. The reason for those impacts is very straightforward. When demand is constant, the payoffs are gained from savings in the ordering costs. However, when demand changes with the price fluctuation, those payoffs result from increases in sales, in addition to the ordering-cost savings. Therefore, we can say that the price elasticity of demand is an important motivator for a discount.

4.5 Summary

The research in this chapter concerns a situation regarding a buyer purchasing multiple items from a single supplier. We defined what is meant by the *optimal* discount scheme, and showed how to determine it. With the group discount, a buyer's best decision could be to order a larger amount, one that will satisfy demand during a longer interval, a greater multiple of the base period T. However, that interval between replenishments (i.e. the quantity breakpoint for the group discount) is chosen by maximizing the supplier's payoff function. The supplier offers that discount, to encourage the buyer to take it, by providing a positive payoff to her.

We first assumed that the demand rate of each item is deterministic and constant. As functions of the particular replenishment intervals involved, we determined an upper bound and also a lower bound on the percentage of discount the supplier could offer. We demonstrated that, for any discount percentage between the two bounds, both the supplier and buyer will benefit from the group discount policy. Additionally, another model maximizing the joint payoff of the supplier and buyer was presented. In a simple numerical example, the joint model showed a better overall result than did the case of separate payoffs to buyer and supplier. That is, each party is better off in the case of the joint model.

Moreover, we developed a model considering the fact that demand can change due to fluctuations in price. Two solution procedures were developed for the noncooperative and joint models, accompanying the corresponding maximization problems. Through comparison of the numerical results from the two examples, we showed the impacts of price elasticity on the supplier's optimal discount policy, and illustrated that price elasticity of demand is one of the most important motivations for both the supplier and buyer, to offer and accept a discount.

We have conducted extensive graphical analyses over the full feasible range for parameters in the functions $\Pi_S^{[\$]}$ [Eq. (4.16) for Model III] and $\Pi_J^{[\$]}$ [Eq. (4.20) for Model IV]. For example, $\Pi_J^{[\$]}$ [Eq. (4.20)] was found to be unimodal in T for fixed ρ, and unimodal in ρ for fixed T. This unimoldality explains why, in the various numerical tests, our algorithms (Figs. 4.2 and 4.3) converge rapidly.

Note that our models can also be implemented when more than one buyer purchases from this single supplier. In such a case, the supplier would offer a group discount separately to each buyer, rather than offering a common discount to all buyers. More specifically, a second buyer may purchase items which are only partially the same as those of the first buyer. The supplier would then negotiate with the second buyer using the same approach, but likely obtain a slightly different policy, because of the distinct products with correspondingly changed demands and cost structures.

Our next step, in Chapter 5, will be to coordinate decisions on transportation with the determination of a quantity discount policy. In addition to a private fleet, goods movement can also be done by common carrier (a public, for-hire trucking company). In that case, however, the situation becomes more complicated due to the LTL transportation pricing scheme with "bumping clause." This is the over-declaration of the shipment weight, when advantageous, to obtain the all-unit transportation discount. In Chapter 5, we will conduct comprehensive analyses, both noncooperatively and cooperatively, regarding the problem of coordinating the quantity and transportation discount schedules for the supplier and carrier.

Chapter 5

Coordinating the Discount Decisions of Carrier and Supplier

5.1 Introduction

In Chapters 3 and 4, we have discussed the quantity discount problem from the supplier's point of view. However, a supply chain is a complicated system that not only physically manufactures products but also delivers them to customers. Transportation between successive nodes thus plays a vital role in any supply chain. In spite of its significant impacts, the transportation expense is often omitted or assumed fixed, either when the supplier makes pricing decisions, or when the buyer decides replenishment quantities. This neglect of transportation can easily overwhelm any savings related to good inventory management.

In the past few decades, the interaction between transportation and inventory management has gradually grown in influence in the practical decision process. Many scholars have come to realize the importance of integrating transportation issues into those involving inventory, especially replenishment decisions. Taking into consideration both the quantity and transportation discounts, researchers tried to determine the optimal replenishment policy for a buyer. Publications in this area include Russell and Krajewski (1991, 1992), Carter et al. (1995a), Tersine et al. (1995), Darwish (2008), and Toptal (2009), to name a few. Çetinkaya (2005) has conducted a detailed review regarding this topic.

Nevertheless, by switching the perspective from the party who takes the discount to the parties who determine the discounts, the situations may become significantly different. Various studies have illustrated the quantity discount as a coordination mechanism to improve the performance of a distribution channel involving buyer(s) and supplier. Chapters 3 and 4 proposed several analytic models to assist the supplier in determining the quantity discount policy in the cases of a single buyer, multiple buyers or a family of items. Through examining the payoffs derived, those works showed that not only the supplier, but also the buyer, can benefit from the discounts. Details and a comprehensive review of related studies can be found in Chapter 2.

Now let us take a look at a situation involving discounts in both the purchase quantity and in transportation. Consider a simple supply chain, in which a for-hire Less-than-Truckload carrier provides transportation services. The freight rate schedule per hundredweight (100 pounds), i.e. cwt, is \$ 48/cwt for shipping a weight of 120 cwt or more, but

$ 50/cwt for any smaller weight. A supplier manufactures and sells a product, each unit weighting 1 cwt, to a buyer according to a quantity discount schedule. As the one who is responsible for the transportation cost, the supplier would like to take the advantage of this all-unit transportation discount. He therefore offers a purchase quantity discount as follows: the unit selling price is $ 200, but the buyer can receive 5% off this regular price when she orders 120 units or more each time.

Suppose the order processing costs for the buyer and supplier are $ 300 and $ 800 per order, respectively. The inventory holding costs are 20% of the dollar value of the inventory for the buyer and supplier, and $ 50 per cwt per year for the carrier. The cost incurred by the carrier to transport an order from the supplier to the buyer is $ 400 per trip. Then, according to the EOQ model, the best order quantity of the buyer is 42 units before any discount. At this moment, the buyer's annual relevant cost is $ 25 697. Now if the buyer would like to take that 5% discount, i.e. increasing the order quantity to 120 units, her annual cost can be further reduced by $ 197. It is obviously profitable for the buyer to make this change in her orders. So far, it seems that the two discounts work perfectly together, as the buyer is better off by taking them.

Next, let us examine the revenues and cost structures for the supplier and carrier. When the buyer orders at her EOQ level, the profits for the supplier and carrier are $ 15 313 and $ 3808, respectively. However, these two profit numbers respectively decrease by $ 272 and $ 1448, due to the buyer's decision of taking the discount schedules that each party determined and offered by themselves. In this case, the overall performance of the supply chain (buyer plus supplier plus carrier) is *worse*, after buyer obtains the discount.

This example reveals the importance of determining the optimal discount schedules of supplier and carrier. From the supplier's perspective, the possibility of taking advantage of a transportation discount is definitely helpful in terms of cost saving and increased profit. But how should he readjust the quantity discount according to that transportation discount? On the other hand, how should the carrier set a transportation discount that would benefit both carrier and supplier? Most importantly, would the integration of quantity and transport discounts further improve the overall performance of the supply chain? In this research, we aim to coordinate the purchase-quantity and transportation discount decisions. Both noncooperative and cooperative models are analyzed from the viewpoints of the supplier and carrier.

The rest of this chapter is organized as follows. On the basis of the assumptions and notation, the payoffs of the three parties are examined. Next, we develop a noncooperative game-theoretic model under the framework of the Stackelberg game, separately considering the payoffs to the buyer, supplier, and carrier. Then, a cooperative model assuming transferable utilities is proposed to maximize the joint payoffs of the three parties. We further conduct a coalition analysis based upon the concept of Shapley Value to fairly divide the extra payoffs gained from the cooperation. Numerical examples are employed to show the applications of approaches introduced in this chapter, followed by concluding remarks.

5.2 Assumptions and notation

Consider a situation involving three parties: a single buyer purchases one product from a supplier, and a common carrier is responsible for transporting the goods to the buyer. Both buyer and supplier employ the economic order quantity (EOQ) model, while the carrier uses the economic shipping weight (ESW) approach.

The ESW approach is an alternative formulation of the economic shipment quantity (ESQ) model (Brennan, 1981; Burns et al., 1985; Abdelwahab and Sargious, 1990; Higginson, 1995), the most-employed model in determining the target shipment quantity. The basic idea of ESQ is to accumulate the orders received until a predetermined target size is reached. Then the carrier dispatches all those orders as one consolidated load. Minimizing the sum of transportation and inventory holding costs, the simplest case of ESQ can be expressed as

$$ESQ = \sqrt{\frac{2\hat{a}F}{r}},$$

where $\hat{a}$ is the order arrival rate, F is the sum of all fixed costs associated with a vehicle dispatch, and r is the inventory holding cost per unit item per time period. Transferring the unit expression from number of orders (as in ESQ) to weight, we have

$$ESW = \sqrt{\frac{2\hat{a}FE(W)}{r_w}},$$

where $E(W)$ is the expected weight of a customer order and r_w is the inventory holding cost per unit weight per time period. Let $\lambda = \hat{a}E(W)$ represent the build-up rate, i.e. the total weight accumulated in the time period corresponding to $\hat{a}$. ESW can then be expressed as

$$ESW = \sqrt{\frac{2F\lambda}{r_w}}.$$

In this chapter, we further assume that this supplier's shipments have no impact on λ. After adding in those shipments, the transportation capacity of the carrier is still not exceeded.

We start our study at the point that the market is in an equilibrium, such that both the supplier and carrier respectively offer standard rates for the product and for transportation, and the buyer orders at her EOQ level. The supplier offers an all-unit quantity discount based on buyer's replenishment quantity, and the carrier offers an all-unit transportation discount on the corresponding shipment weight each time. The demand rate of the product is assumed constant, and does not change according to any variations in price.

Additionally, all players are assumed to be able to estimate the relevant information, such as the annual demand rate, the inventory holding cost of each party, as well as the corresponding ordering cost, product acquisition cost, transportation related costs, and selling price. Also, all parties are assumed to act wisely, i.e., they would accept the chance to gain more profit when and wherever possible. The following notation is employed.

Index:

i $\quad$ $i = B$, S or C indicates the buyer, supplier or carrier, respectively.

j The stage that the problem is in. $j = 0$, 1 or 2 represents the stages in which no discount, one discount or two discounts have been offered, respectively.

k $k = N$ or V respectively denotes the rate before and after the transportation discount (i.e. for non-volume and volume shipments, respectively).

Product's notation:

D Buyer's annual demand.

w Unit weight of the product.

Buyer's notation:

x Unit selling price offered by the buyer to end-customers.

A_B Buyer's order processing cost.

r_B Buyer's interest rate for holding inventory per time period.

Q_j Buyer's order quantity at Stage j.

Supplier's notation:

p_j The unit selling price offered by the supplier to buyer at Stage j.

v Unit acquisition cost for supplier. The supplier, as a wholesaler or distributor, pays this price for the product; when supplier is a manufacturer, this is the production cost per unit.

A_S Supplier's order processing cost.

r_S Supplier's interest rate for holding inventory per time period.

φ	Quantity break point. When ordering a quantity less than φ, buyer pays the original price p without discount; otherwise, buyer pays the discounted price, $(1-\rho)p$, where ρ is the percentage discount offered ($0 < \rho < 1$).

Carrier's notation:

λ	The mean build-up rate for the carrier's loads.
r_w	The variable cost of carrying inventory per unit weight per time period.
F	The sum of all fixed costs associated with a vehicle load.
MWT	Stated minimum weight to obtain discount, i.e. breakpoint for the transportation discount.
f_k	The freight rate offered by the carrier: when the weight of the shipment $\leq$ MWT, the freight rate is f_N; otherwise, the freight rate is f_V.

Other notation:

TC_j	Buyer's total annual costs for Stage j.
π_{ij}	Profit gained from the product by player i for Stage j.
Π_i	Payoff gained by the buyer taking the advantage of the quantity discount.

Occasionally, we will employ a superscript, s, added to certain notation such as p, q, and Π_i. $s = (N)$, (J), $(PJ1)$, $(PJ2)$ and $(PJ3)$ will denote values obtained from the noncooperative model; the joint or cooperative model; and the models that consider three distinct coalition situations (buyer-supplier, supplier-carrier, and buyer-carrier), respectively. (Those coalitions are treated in Sec. 5.6)

5.3 Payoff analysis

In our models, the payoffs represent the profits gained or costs saved by each party due to the discounts.

5.3.1 Buyer's payoff

From the preceding assumptions and notation, buyer's total cost per year at Stage j is

$$TC_{Bj} = Dp_j + \frac{D}{Q_j}A_B + \frac{1}{2}Q_j r_B p_j. \tag{5.1}$$

When no discount is offered, $Q_0 = Q_B = \sqrt{2DA_B/(r_B p_0)}$, i.e. this is the optimal order quantity that minimizes buyer's total costs.

5.3.2 Supplier's payoff

Herein, we consider the supplier's profits gained only from this particular buyer. More specifically, we account for the supplier's own acquisition cost, his costs to process buyer's orders and to hold inventory, and the transportation cost paid to the carrier. Therefore, we can write the supplier's profit at Stage j as

$$\pi_{Sj} = Dp_j - Dwf_k - Dv - \frac{D}{Q_j}A_S - \frac{1}{2}Q_j r_S v. \tag{5.2}$$

where $k = V$ when $j = 2$, otherwise $k = N$.

This formulation shows the supplier's inventory-related cost (Dolan, 1978). Despite not taking the supplier's optimal replenishment policies into consideration, it is a reasonable approximation of the supplier's gain when stationary inventory replenishment polices are employed (Wang, 2002).

5.3.3 Carrier's payoff

Before the transportation discount, carrier's profit gained from that certain supplier is

$$\pi_{C0} = \pi_{C1} = Dwf_N - \frac{D}{Q_B}F - \frac{1}{2}Q_B w r_w. \tag{5.3}$$

Note that when D and r_w are values *per year*, this is the carrier's annual profit.

Now consider shipment consolidation, and with a mean build-up rate of λ. the carrier's best dispatch weight is

$$ESW = \sqrt{\frac{2F\lambda}{r_w}}. \tag{5.4}$$

So, the time between dispatches can be written as

$$T = \frac{ESW}{\lambda} = \sqrt{\frac{2F}{r_w \lambda}}. \tag{5.5}$$

After the discount, i.e. after the carrier has consolidated supplier's shipments into the regular loads, carrier's profit from shipments made by this supplier is

$$\pi_{C2} = Dwf_V - \frac{1}{2}Q_2 wr_w. \tag{5.6}$$

Notice from this equation that, once consolidation is possible, the fixed transportation costs related to each dispatch is saved. That is exactly the motivation for the carrier to offer a transportation discount.

5.4 Noncooperative model

In this model, we analyze the situation in which the three parties all act on their own, i.e. without any cooperation with the other two parties. Again, a Stackelberg game is used to analyze this noncooperative approach. The parties who set the prices (the supplier at Stage 1 and carrier at Stage 2), considered as the "leaders", will construct the discount policy. As before, these parties thus maximize their respective payoffs, in light of the best reactions by the parties that take the discount (the buyer at Stage 1 and supplier at Stage 2), considered as the "followers" in the game.

5.4.1 Model development

At Stage 1, before receiving the transportation discount, the supplier and buyer have already formed a stable relationship based on the supplier's pricing policy. With only two parties involved, this policy can be determined as follows.

From the buyer's perspective, given the supplier's quantity discount, her total annual cost is

$$TC_{B1} = Dp_1 + \frac{D}{Q}A_B + \frac{1}{2}Qr_Bp_1. \tag{5.7}$$

To minimize this cost, the buyer's optimal order quantity $Q = \sqrt{2DA_B/(r_Bp_1)}$. As proven by Wang (2002), the supplier loses if buyer order her EOQ at the discounted price. Therefore, the supplier has to set the discount in such a way that the buyer orders *more* than her EOQ; that way, he can gain profit by offering this discount.

The supplier's profit is

$$\pi_{S1} = Dp_1 - Dwf_N - Dv - \frac{D}{Q_j}A_S - \frac{1}{2}Q_1r_Sv, \tag{5.8}$$

which is maximized when

$$Q_1 = Q_S = \sqrt{\frac{2A_SD}{r_Sv}}. \tag{5.9}$$

The selling price p_1 is set to make sure that both supplier and buyer are not worse off. Also note that supplier's profit is a monotonically decreasing function with respect to the price of the product. So the lower bound of p_1 can be obtained by setting $\Pi_S^{1-0} = \pi_{S1} - \pi_{S0} \geq 0$, i.e.

$$p_1 \geq p_1^{[L]} = p_0 + \left(\frac{1}{Q_S} - \frac{1}{Q_B}\right)A_S + \frac{1}{2D}r_Sv(Q_S - Q_B). \tag{5.10}$$

Also, letting $\Pi_B^{1-0} = TC_{B0} - TC_{B1} \geq 0$, we have the upper bound of p_1,

$$p_1 \leq p_1^{[U]} = \frac{Dp_0 + \left(\dfrac{1}{Q_B} - \dfrac{1}{Q_S}\right) DA_B + \dfrac{1}{2} Q_B r_B p_0}{D + \dfrac{1}{2} Q_S r_B}. \tag{5.11}$$

The value of p_1 can be chosen between $p_1^{[U]}$ and $p_1^{[L]}$, depending upon how the supplier would like to share the total payoffs gained from this discount with the buyer. Note that a condition for offering such a quantity discount is $p_1^{[U]} \geq p_1^{[L]}$.

Let us now move to Stage 2. The carrier dispatches every $T = \sqrt{2F/(r_w \lambda)}$ time units. To fit the supplier's order shipment into carrier's dispatching schedule, the carrier would like to encourage the supplier to send an order every nT units of time, where T is that time between carrier's dispatches, and n is an integer.

PROPOSITION 5.1. *If the supplier's current shipments can already be fitted into the carrier's regular total loads, it would not be profitable for the carrier to offer a transportation discount.*

Proof. One of the motivations for the carrier to offer such a discount is to save the fixed transportation costs related to each load. When the supplier's current shipments would already fit in the carrier's regular total loads, those fixed costs have previously been accounted for. Therefore, it would be counterproductive for the carrier to encourage the supplier change his shipping pattern. The transportation discount would only cause decreases in revenue, rather than any savings in the carrier's costs. □

Therefore, the problem of determining MWT (i.e. buyer's replenishment pattern, or supplier's shipment pattern) is to find a best value for n. Ordering every nT units of time, the amount of each buyer's replenishment is $Q_2 = nTD$.

From Eq (5.6), we can see that π_{C2} is a monotonically decreasing function with respect to n. Therefore, the best value of n can be found by solving

$$n - 1 < \frac{Q_1}{TD} < n \tag{5.12}$$

where Q_1 is the optimal order quantity for Stage 1, in which only the quantity discount is offered.

Thus, if the supplier dispatches a shipment every nT units of time, the amount of each replenishment is $Q_2 = nTD$. Accordingly, MWT can be set as $nTDw$.

Since the carrier's payoff gained from this supplier has to be non-negative, i.e. $\Pi_C^{2-1} = \pi_{C2} - \pi_{C1} \geq 0$, we can calculate the lower bound of f_V as

$$f_V \geq f_V^{[L]} = f_N + \frac{1}{2}\left(nT - \frac{Q_s}{D}\right) r_w - \frac{F}{Q_S w}. \tag{5.13}$$

Note that the carrier must make sure that the supplier takes the discount, and ships exactly the weight of MWT every time, i.e. sends out shipments every nT units of time. Otherwise, the carrier could not consolidate this supplier's shipment with the regular loads because of the mismatched schedule.

f_V also has to be set to guarantee that both other parties can benefit, i.e. the payoffs of both the supplier and buyer should be non-negative.

From the supplier's perspective, the profit after receiving the transportation discount is

$$\pi_{S2} = Dp_2 - Dwf_V - \frac{A_S}{nT} - \frac{1}{2}nTDr_S v. \tag{5.14}$$

Letting $\Pi_S^{2-1} = \pi_{S2} - \pi_{S1} = 0$, the upper bound of f_V is

$$f_V \leq f_V^{[U]} = f_N + \frac{1}{w}\left[\left(p_2^{[U]} - p_1\right) - \left(\frac{1}{nTD} - \frac{1}{Q_S}\right)A_S - \frac{1}{2}r_S v\left(nT - \frac{Q_S}{D}\right)\right]. \tag{5.15}$$

Here $p_2^{[U]}$ can be obtained by setting $\Pi_B^{2-1} = TC_{B1} - TC_{B2} = 0$, i.e.

$$p_2^{[U]} = \frac{Dp_1 + \left(\frac{D}{Q_S} - \frac{1}{nT}\right)A_B + \frac{1}{2}Q_S r_B p_1}{D + \frac{1}{2}nTDr_B}. \tag{5.16}$$

Similarly, from $\Pi_S^{2-1} = \pi_{S2} - \pi_{S1} \geq 0$, we can get the lower bound of p_2 as

$$p_2 \geq p_2^{[L]} = p_1 + w(f_V - f_N) + \left(\frac{1}{nTD} - \frac{1}{Q_S}\right)A_S + \frac{1}{2}r_S v(nT - \frac{Q_S}{D}). \tag{5.17}$$

However, it is also possible that the carrier may set WBT to be less than Q_1, i.e.

$$\frac{f_V}{f_N}nTD \leq Q_1. \tag{5.18}$$

In this case, after analyzing the freight rates of the carrier, the supplier may choose to over-declare the weight of his current shipment.

PROPOSITION 5.2. *The carrier is worse off if the supplier over-declares the shipment weight, and sends out shipments according to his own schedule.*

Proof. If the supplier over-declares the weight of his current shipments, the carrier's revenue decreases. However, the costs incurred by the carrier, due to this supplier, are unchanged: the supplier keeps his original shipment schedule. So the carrier's profit decreases, i.e. the carrier's payoff from the transportation discount is negative. □

Therefore, the carrier's freight tariff and discount rate should be determined such that the supplier would not be able to profitably over-declare the weight of his current shipments. Rather, the supplier should be encouraged to take the schedule that the carrier "assigns". In other words, the carrier has to make sure that supplier's cost at MWT is not higher than his cost at Q_1 (with the over-declaration), i.e.

$$Dwf_V + \frac{A_S}{nT} + \frac{1}{2}nTDr_Sv \leq \frac{D}{Q_1}nTDwf_V + \frac{DA_S}{Q_1} + \frac{1}{2}Q_1r_Sv. \tag{5.19}$$

5.4.2 Additional consideration

As mentioned in the simple example in Chapter 1 (Example 1.1), a specific characteristic of common-carrier freight rates is the "bumping clause," or the shipping of "phantom freight."

When the shipment weight is less than MWT, but greater than a certain weight $WBT \equiv MWTf_V/f_N$, it is advantageous to over-declare the weight to be MWT, which would result in a lower transportation cost. WBT is the smallest weight where over declaration is advantageous. Figure 1.3 in Chapter 1 illustrates this phenomenon.

Because of the bumping clause, the carrier also requires that, with the given transportation discount, the supplier's new EOQ at the discounted freight rate should be between WBT and MWT. That new EOQ is

$$Q'_s = \sqrt{\frac{2D(A_S + f_V MWT)}{r_S v}} = \sqrt{\frac{2D(A_S + f_N WBT)}{r_S v}}. \tag{5.20}$$

In this case, the supplier would obviously send out a shipment every Q'_S/D units of time. Based on the supplier's choices, the carrier then has to set the transportation discount such that $nTD = Q'_S$, i.e.

$$f'_V = \frac{1}{2w} nTr_S v - \frac{A_S}{nTDw}. \tag{5.21}$$

To summarize, if $f_V^{[L]} \leq f'_V \leq f_V^{[U]}$, then f'_V is the freight rate the carrier should set; otherwise, choose the freight rate f_V, as described above.

5.4.3 Example 5.1

A numerical example is employed to illustrate the procedure of determining the quantity and transportation discounts. The relevant parameters are shown in Table 5.1.

Table 5.1: Parameters for Examples 5.1-5.3

Product	D	120	units/year
	w	1	cwt/unit
Buyer	A_B	\$300	/order
	r_B	0.2	/\$/year
Supplier	A_S	\$800	/order
	r_S	0.2	/\$/year
	v	\$100	/unit
	p_0	\$200	/unit
Carrier	λ	10 000	cwt/year
	F	\$400	/trip
	r_w	\$50	/cwt/year
	f_N	\$50	/cwt

Before any discount has been offered, the buyer orders $Q_0 = Q_B = 42$ units each time. Entering Stage 1, the supplier encourages buyer to increase her order quantity to $Q_1 = Q_S = 98$ units by offering her a quantity discount. Analyzing the payoff functions for the buyer and supplier, the upper and lower bounds of the discounted prices are found to be \$ 195.14 (i.e. a discount of 2.427%) and \$ 193.94 (3.031%), respectively. Both the supplier and buyer can benefit from any quantity discount between 2.427 and 3.031%.The supplier then determines a quantity discount policy within this range, through examination of the degree that he would like to share the payoff with the buyer. Table 5.2 lists the discount policy at this stage and the corresponding payoffs according to the extent of sharing. For example, assuming 50% of the total payoff is shared, the price is determined as \$ 194.57 per unit (2.717%), and we have $\Pi_S^{1-0} = \$\ 75$ and $\Pi_B^{1-0} = \$\ 75$.

Table 5.2: Example 5.1 - Supplier's discount policy for Stage 1

% of payoff allocated to buyer	0%	25%	50%	75%	100%
ρ	2.427%	2.569%	2.717%	2.871%	3.031%
Π_B^{1-0}	0	\$37	\$75	\$115	\$157
Π_S^{1-0}	\$145	\$111	\$75	\$38	0

At Stage 2, we first consider the carrier's perspective. With 300 working days per year, the carrier's ESW can be obtained as 400 cwt, which gives a transportation policy of dispatching every 0.04 years, i.e. 12 days. From Eq (5.12), we can calculate $n = 21$, which determines the carrier's $MWT = nTDw = 100$ cwt, i.e. the weight of supplier's shipments have to be greater than 10 000 lbs to receive the transportation discount. To make sure both the carrier and supplier can gain payoffs from this discount, the upper and lower bounds of f_V are obtained from Eqs (5.15) and (5.13) as \$ 49.65 and 46.50 per cwt, respectively. Table 5.3 lists the transportation discount policy at this stage and the corresponding payoffs according to the degree to which the carrier is willing to share the payoff with the supplier. For instance, assuming that the carrier would like to keep 50% of the payoffs, we get $f_V =$ \$ 48.08 per cwt. By examining the supplier's over-declaration condition described by Eq (5.19), this rate is feasible.

Now taking this transportation discount, the supplier would like to revise his quantity discount policy, aiming to encourage the buyer to further increase her order quantity to $Q_2 = nTD = 100$ units. Similar to Stage 1, we can calculate $p_2^{[U]} = 194.22$ per unit

Table 5.3: Example 5.1 - Carrier's discount policy for Stage 2

% of payoff allocated to supplier	0%	25%	50%	75%	100%
f_V	\$49.65	\$48.86	\$48.08	\$47.29	\$46.50
Π_S^{2-1}	0	\$94	\$189	\$284	\$378
Π_C^{2-1}	\$378	\$284	\$189	\$94	0

(2.888%) and $p_2^{[L]} = 192.65$ per unit (3.675%). As before, the supplier then determines the quantity discount to offer, through the percentage of total payoffs shared with the buyer.

Assuming the supplier decides to evenly divide those payoffs with the buyer, the price should be 193.47 per unit (3.266%). This price guarantees additional payoffs of \$ 98 for both the supplier and buyer. From this example, we find that, with a discounted freight rate of \$ 48.08 per cwt and a quantity discount of 3.266%, the total payoffs for the buyer, supplier and carrier can be calculated as \$ 173, \$ 173, and \$ 189, respectively.

5.5 Cooperative model

This section studies a situation where the buyer, supplier, and carrier agree to make their inventory and discount decisions jointly. We propose a cooperative game model with "transferable utility." In this type of game, each player has the option to give any amount of gains to another player. The net monetary flow decreases the giving player's utility payoff, and increases the receiving player's utility payoff (Myerson, 1991). Therefore, the objective becomes to maximize the total joint profits of the three parties, rather than each individual payoff. Then, all parties share the benefits with no extra cost.

5.5.1 Model development

We define the joint profit as

$$\begin{aligned}\pi_J &= \pi_{B2} + \pi_{S2} + \pi_{C2} \\ &= Dx - \left(\frac{A_B}{nT} + \frac{1}{2}nTDr_Bp_2\right) - \left(\frac{A_S}{nT} + \frac{1}{2}nTDr_Sv\right) - \frac{1}{2}nTDr_w. \qquad (5.22)\end{aligned}$$

Assuming buyer keeps the same selling price, we can rewrite the above equation as an optimization problem that minimizes the joint cost. At the same time, the payoffs for all parties should be positive, i.e.

$$\min_{n,p,f_V} TC_J = \frac{A_B}{nT} + \frac{1}{2}nTDr_Bp_2 + \frac{A_S}{nT} + \frac{1}{2}nTDr_Sv + \frac{1}{2}nTDr_w; \qquad (5.23)$$

$$\texttt{s.t.}$$

$$\Pi_B^{2-0} = \pi_{B2} - \pi_{B0} \geq 0;$$

$$\Pi_S^{2-0} = \pi_{S2} - \pi_{S0} \geq 0;$$

$$\Pi_C^{2-0} = \pi_{C2} - \pi_{C0} \geq 0;$$

$$n \ \texttt{integer}; p, f_V \geq 0.$$

Note that we skip $j = 1$ (the middle stage) in this case. That is because when all parties make their decisions together, they don't need to consider the case of a single discount.

LEMMA 5.1. *By making decisions jointly, neither the purchasing price nor the transportation price is higher than the result in the noncooperative model.*

Proof. From Eqs (5.23), we notice that TC_J is monotonically increasing in p. So when p is at its lowest value, TC_J is minimized. That lower bound of p can be obtained by setting $\Pi_S^{2-0} = 0$, i.e.

$$p^{(J)} = p^{[L]} = p_0 + w(f_V - f_N) + \left(\frac{1}{nTD} - \frac{1}{Q_B}\right)A_S + \frac{1}{2}r_S v\left(nT - \frac{Q_B}{D}\right). \tag{5.24}$$

Also, from $\Pi_S^{2-0} \geq 0$, we find that p is minimized when f_V is at its lowest value. This lower bound $f_V^{[L]}$ can be found by setting $\Pi_C^{2-0} = 0$, i.e.

$$f_V^{(J)} = f_V^{[L]} = f_N + \frac{1}{2}\left(nT - \frac{Q_B}{D}\right)r_w - \frac{F}{Q_B w}. \tag{5.25}$$

Since both $p^{(J)}$ and $f_V^{(J)}$ are at their lowest levels, we have $p^J \leq p^{[N]}$ and $f_V^J \leq f_V^{[N]}$, respectively. $\square$

LEMMA 5.2. *By making decisions jointly, the value of the integer n is lower than in the noncooperative case.*

Proof. From the objective function of the joint model, letting $\partial TC_J/\partial n = 0$, we have

$$n^{(J)} = \frac{1}{T}\sqrt{\frac{2(A_B + A_S)}{D(r_B p + r_S v + r_w w)}}. \tag{5.26}$$

Note that the order quantity that maximizes the supplier's payoff is higher than that which maximizes the buyer's. Therefore, comparing to the noncooperative model:

$$n^{(J)} < \frac{1}{T}\sqrt{\frac{2(A_B + A_S)}{D(r_B p + r_S v)}} < \frac{1}{T}\sqrt{\frac{2A_S\left(1 + \dfrac{r_B p}{r_S v}\right)}{D(r_B p + r_S v)}} = \frac{1}{T}\sqrt{\frac{2A_S}{D r_S v}} = \frac{Q_S}{T} \leq \left\lceil \frac{Q_S}{T} \right\rceil = n^{(N)}.$$

□

Furthermore, because $n^{(J)} < n^{(N)}$, the time between supplier's shipments is shorter, and accordingly, the weight of each shipment is lower. This fact would make it easier for the carrier to consolidate these shipments to the regular loads with fewer worries about exceeding transportation capacity.

PROPOSITION 5.3. *When the joint payoffs are maximized, only the buyer's payoff is positive. Both other parties need to share the payoff gained by the buyer.*

Proof. From the previous proofs, we have the solutions for the joint decision model as:

$$n^{(J)} = \frac{1}{T}\sqrt{\frac{2(A_B + A_S)}{D(r_B p + r_S v + r_w w)}};$$

$$p^{(J)} = p_0 + w(f_V - f_N) + \left(\frac{1}{nTD} - \frac{1}{Q_B}\right)A_S + \frac{1}{2}r_S v\left(nT - \frac{Q_B}{D}\right);$$

$$f_V^{(J)} = f_N + \frac{1}{2}\left(nT - \frac{Q_B}{D}\right)r_w - \frac{F}{Q_B w}.$$

Note that $p^{(J)}$ and $f_V^{(J)}$ are obtained by letting $\Pi_S^{2-0} = 0$ and $\Pi_C^{2-0} = 0$, respectively. Therefore, at the optimal solution, the only party that gains payoff (cost saving) is the buyer, while the other two parties each have zero profit. Therefore, at this point, all three parties need to share the payoff gained originally by the buyer. □

PROPOSITION 5.4. *By making decisions jointly, the total payoffs for the buyer, supplier, and carrier together are higher than the aggregate individual payoffs from the noncooperative model.*

Proof. For both the noncooperative and cooperative models, the aggregate payoffs for the buyer, supplier, and carrier together are the profit gained from the discount policies, compared to the situation when no discount is offered, i.e. Stage 0. Therefore, whichever case has a higher total profit gives a higher payoff.

We also notice that the cooperative model is solved to maximize this total profit [Eq. (5.22)]. So no matter how payoffs are allocated among the players in the noncooperative model, the corresponding aggregate profit is less than the optimal solution obtained from Eq. (5.23), i.e. the maximum profit of Eq. (5.22). □

5.5.2 Example 5.2

Suppose that all three parties, with the same parameters listed in Table 5.1, are now making decisions jointly. Solving Eqs (5.23) gives $n^{(J)} = 10$, $p^{(J)} = \$\ 189.94$ per unit, and $f_V^{(J)} = \$\ 42.03$ per cwt. Relative to the noncooperative model, the values of p and f_V are higher, and the value of n is lower.

These results boost the joint payoff to the level of \$ 1237, about 2.3 times that of the total payoffs obtained from the noncooperative model ($\Pi_J^{(N)} = \Pi_B^{(N)} + \Pi_S^{(N)} + \Pi_C^{(N)} = 173 + 173 + 189 = \$\ 535$).

5.6 Coalition analysis

From Examples 5.1 and 5.2, we realize that total payoffs for the three parties are significantly improved from cooperation. However, can the three parties fairly divide the extra payoffs among them? To effectively solve this allocation problem, a coalition analysis based on the idea of *Shapley Value* is conducted in this section. We introduce a profit-sharing mechanism in which each player shares an additional amount of payoff equaling to his/her Shapley value.

Coalition formation is the most emphasized topic in cooperative game theory. When more than two players are involved in a game, a subset of all the players may choose to act cooperatively and share the payoffs gained from this coalition among them. Note that our cooperative model introduced in the last section is a special case of coalition, that involving all three players. That can thus be called the "grand coalition."

The Shapley Value is the value to a player of the opportunity to play a cooperative game. This method was proposed by Shapley (1953) to calculate a unique expected payoff allocation for the players in each coalition form. Through the Shapley value to each player of the game, one can obtain a measure of the utility that this player can reasonably be expected to receive in the game (Myerson, 1991). More specifically, given a utility-sharing game, let the players join the coalition one at a time. Each player's contribution is the incremental value that this player adds to the payoff at joining. The Shapley Value of a player is his average utility contribution over all possible orderings of the players (Young, 1994). A principle of sharing based on contribution gives each player an incentive to cooperate.

Let $\mathcal{N}$ be the player set, i.e. $\mathcal{N} = \{B, S, C\}$, where B, S, and C respectively denote the buyer, supplier and the carrier. The characteristic function $u(\mathcal{J})$ specifies the maximum value that can be realized by coalition $\mathcal{J}$ ($\mathcal{J} \subseteq \mathcal{N}$). This function represents the amount of transferable utility that the members of $\mathcal{J}$ could earn without any help from the players outside of $\mathcal{J}$. We always have $u(\emptyset) = 0$. The Shapley Value is defined as a unique value, $\phi(.)$, for all characteristic functions $u(.)$. This value can be computed using

$$\phi_i(u) = \sum_{\mathcal{J} \subseteq \mathcal{N}-i} \frac{|\mathcal{J}|!(|\mathcal{N}| - |\mathcal{J}| - 1)!}{|\mathcal{N}|!} [u(\mathcal{J} \cup i) - u(\mathcal{J})], \tag{5.27}$$

where $i = B$, S or C, respectively represents the three parties.

Note that when $|\mathcal{J}| = 1$, the values of $u(\{B\})$, $u(\{S\})$, and $u(\{C\})$ can be obtained in the noncooperative model with *no* sharing among the parties, i.e. $u(\{B\}) = \Pi_B^{(N)}$, $u(\{S\}) = \Pi_S^{(N)}$, and $u(\{C\}) = \Pi_C^{(N)}$. Also, the value of $u(\mathcal{N})$ can be computed by the joint payoffs from the cooperative approach, i.e. $u(\mathcal{N}) = \Pi_J$. The next three subsections study the coalitions between two of the players and calculate the value of $u(\mathcal{J})$ when $|\mathcal{J}| = 2$.

5.6.1 Case I: $\mathcal{J}_1 = \{Buyer, Supplier\}$

The buyer and supplier act together, while the carrier is on its own. Two stages exist, in addition to the starting stage $j = 0$: in Stage 1, the buyer and supplier make their joint decisions about the quantity discount; and then, the carrier's transportation discount decision comes into action in Stage 2.

The best solution from the buyer-supplier perspective is obtained by solving the optimization problem:

$$\begin{aligned}\max_{Q,p} \pi_{BS1} &= \pi_{B1} + \pi_{S1} \\ &= Dx - \left(\frac{D}{Q}A_B + \frac{1}{2}Qr_Bp\right) - \left(\frac{D}{Q}A_S + \frac{1}{2}Qr_Sv + Dwf_N\right).\end{aligned}$$

This is equivalent to

$$\min_{Q,p} TC_{BS1} = \frac{D}{Q}A_B + \frac{1}{2}Qr_Bp + \frac{D}{Q}A_S + \frac{1}{2}Qr_Sv + Dwf_N; \tag{5.28}$$

$\texttt{s.t.}$

$$\Pi_B^{1-0} = \pi_{B1} - \pi_{B0} \geq 0;$$

$$\Pi_S^{1-0} = \pi_{S1} - \pi_{S0} \geq 0;$$

$$Q, p \geq 0.$$

Solving this problem gives us:

$$Q_1^{(PJ1)} = \sqrt{\frac{2D(A_B + A_S)}{r_Bp_1^{(PJ1)} + r_Sv}}; \tag{5.29}$$

$$p_1^{(PJ1)} = p_0 + \left(\frac{1}{Q_1^{(PJ1)}} - \frac{1}{Q_B}\right)A_S + \frac{1}{2D}r_Sv(Q_1^{(PJ1)} - Q_B). \tag{5.30}$$

Now from the carrier's standpoint, let

$$n - 1 < \frac{1}{T}\sqrt{\frac{2(A_B + A_S)}{D(r_Bp_1^{(PJ1)} + r_Sv)}} < n, \tag{5.31}$$

or

$$n^{(PJ1)} = \left\lceil \frac{1}{T}\sqrt{\frac{2(A_B + A_S)}{D(r_Bp_1^{(PJ1)} + r_Sv)}} \right\rceil \tag{5.32}$$

LEMMA 5.3. *The value of the integer $n^{(PJ1)}$ in the partial joint model (Case I) is larger than in the joint model, but smaller than in the noncooperative model.*

Proof. Compare the three integer values n from Eqs. (5.12), (5.26), and (5.32). Here there are three different discounts, respectively for the noncooperative, the partially-joint, and the joint models. Although the value of p differs in each case, the resulting change in the buyer's inventory holding cost is insignificant and can be neglected, compared to the supplier's holding cost. It is then easy to see that $n^{(J)} < n^{(PJ1)} < n^{(N)}$. □

Similarly to the noncooperative model, with this value of $n^{(PJ1)}$, we can calculate the lower bound of f_V as

$$f_V \geq f_V^{(PJ1)[L]} = f_N + \frac{1}{2}\left(n^{(PJ1)}T - \frac{Q_1^{(PJ1)}}{D}\right)r_w - \frac{F}{Q_1^{(PJ1)}w}, \tag{5.33}$$

and the upper bound of f_V as

$$\begin{aligned} f_V \leq f_V^{(PJ1)[U]} &= f_N + \frac{1}{w}\left[\left(p_2^{(JP1)[U]} - p_1^{(PJ1)}\right) - \left(\frac{1}{n^{(PJ1)}TD} - \frac{1}{Q_1^{(PJ1)}}\right)A_S \right. \\ & \left. -\frac{1}{2}r_S v\left(n^{(PJ1)}T - \frac{Q_1^{(PJ1)}}{D}\right)\right], \end{aligned} \tag{5.34}$$

where

$$p_{2}^{(PJ1)} \leq p_{2}^{(PJ1)[U]} = \frac{Dp_{1}^{(PJ1)} + \left(\frac{D}{Q_{1}^{(PJ1)}} - \frac{1}{n^{(PJ1)}T}\right)A_{B} + \frac{1}{2}Q_{1}^{(PJ1)}r_{B}p_{1}^{(PJ1)}}{D + \frac{1}{2}n^{(PJ1)}TDr_{B}}. \tag{5.35}$$

Also, we can obtain the lower bound of $p_{2}^{(PJ1)}$ as

$$\begin{aligned} p_{2}^{(PJ1)} \geq p_{2}^{(PJ1)[L]} &= p_{1}^{(PJ1)} + w(f_{V} - f_{N}) + \left(\frac{1}{n^{(PJ1)}TD} - \frac{1}{Q_{1}^{(PJ1)}}\right)A_{S} \\ &\quad + \frac{1}{2}r_{S}v\left(n^{(PJ1)}T - \frac{Q_{1}^{(PJ1)}}{D}\right). \end{aligned} \tag{5.36}$$

LEMMA 5.4. *In the case of $\mathcal{J}_1 = \{Buyer, Supplier\}$, the transportation discount does not change the coalition payoffs for the buyer and supplier.*

Proof. Acting independently to the coalition parties (supplier and buyer), the carrier would like to maximize its own payoff in this case rather than sharing with others. Therefore, we can determine $f_{V}^{(PJ1)} = f_{V}^{(PJ1)[U]}$. The value of $f_{V}^{(PJ1)[U]}$ is given by Eqs. (5.34) and (5.35).

In light of $f_{V}^{(PJ1)}$, the supplier modifies the discounted price to make sure his own payoff is positive, i.e. setting $p^{(PJ1)} = p_{2}^{(PJ1)[L]}$. Note from Eqs. (5.34)-(5.36), it is easy to show that when $f_{V}^{(PJ1)} = f_{V}^{(PJ1)[U]}$, $p_{2}^{(PJ1)[L]} = p_{2}^{(PJ1)[U]}$. Therefore, at this value of $f_{V}^{(PJ1)[U]}$, payoffs for both the buyer and supplier stay the same as in Stage 1. $\square$

With the optimal values obtained for $n^{(PJ1)}$, $p^{(PJ1)}$, and $f_{V}^{(PJ1)}$, we can compute the coalition payoff for the buyer and supplier (the optimal value of $\Pi_B^{1-0} + \Pi_S^{1-0}$ in Eq. (5.28)), which is determined as $u(\mathcal{J}_1)$, where $\mathcal{J}_1 = \{Buyer, Supplier\}$.

5.6.2 Case II: $\mathcal{J}_2 = \{Supplier, Carrier\}$

The second case of the partial joint model describes a situation that the supplier and carrier make decisions together first, and then the supplier offers the determined quantity discount to the buyer. The buyer then decides whether to take the advantage of the discount or not.

For the supplier and carrier, they would like to maximize their joint profit, i.e.

$$\begin{aligned} \max_{n,p,f_V} \pi_{SC2} &= Dp - \left(\frac{1}{nT}A_S + \frac{1}{2}nTDr_S v\right) - \frac{1}{2}nTDwr_w \qquad (5.37) \\ \texttt{s.t.} & \\ & \Pi_S^{2-0} = \pi_{S2} - \pi_{S0} \geq 0; \\ & \Pi_C^{2-0} = \pi_{C2} - \pi_{C0} \geq 0; \\ & n \ \texttt{integer;} \ \ p, f_V \geq 0. \end{aligned}$$

Solving this optimization problem, we have

$$n^{(PJ2)} = \frac{1}{T}\sqrt{\frac{2A_S}{D(r_S v + wr_w)}}, \qquad (5.38)$$

$$f_V^{(PJ2)} = f_N + \frac{1}{2}\left(n^{(PJ2)}T - \frac{Q_B}{D}\right)r_w - \frac{F}{Q_B w}, \tag{5.39}$$

and the lower bound of p is

$$\begin{aligned} p^{(PJ2)[L]} &= p_0 + \frac{1}{2}\left(n^{(PJ2)}T - \frac{Q_B}{D}\right)wr_w - \frac{F}{Q_B} + \left(\frac{1}{n^{(PJ2)}TD} - \frac{1}{Q_B}\right)A_S \\ &\quad + \frac{1}{2}r_S v\left(n^{(PJ2)}T - \frac{Q_B}{D}\right). \end{aligned} \tag{5.40}$$

LEMMA 5.5. *The value of the integer $n^{(PJ2)}$ in the coalition model (Case II) is lower than in the noncooperative model.*

Proof. The proof is omitted due to its similarity to the proof of Lemma 5.3. □

To encourage the buyer to place orders every $n^{(PJ2)}T$ units of time, supplier must offer a price that guarantees $\Pi_B^{2-0} \geq 0$. This gives the upper bound of selling price:

$$p^{(PJ2)[U]} = \frac{Dp_0 + (\frac{D}{Q_B} - \frac{1}{n^{(PJ2)}T})A_B + \frac{1}{2}r_B Q_B p_0}{D + \frac{1}{2}r_B n^{(PJ2)}TD} \tag{5.41}$$

In this case, due to the coalition between the supplier and carrier, they would like to maximize their joint payoffs and not share any gain with the buyer. Therefore the supplier sets the selling price at its upper bound, i.e. $p^{(PJ2)} = p^{(PJ2)[U]}$, as determined by Eq. (5.41). That joint payoff for the supplier and carrier [the optimal value of $\Pi_S^{2-0} + \Pi_C^{2-0}$ from Eq.(5.37)] is the value of $u(\mathcal{J}_2)$, where $\mathcal{J}_2 = \{Supplier, Carrier\}$.

5.6.3 Case III: $\mathcal{J}_3 = \{Buyer, Carrier\}$

The last case considers the situation in which the buyer and carrier cooperate and make decisions together. In this case, at the first stage, the supplier sets and offers the quantity discount. At Stage 2, the buyer and carrier negotiate with each other to determine the order pattern and transportation discount, and agree to share the extra payoff gained from this coalition. The supplier then takes the transportation discount and readjusts his quantity discount.

This is a special case of coalition, compared to the previous two cases. The reason is that the buyer and carrier are connected through the supplier, specifically, the selling price that supplier offers. Not cooperating with any party in this case, the supplier would act exactly the same as in the noncooperative situation, i.e. offer a quantity discount that will maximize his own profit. The buyer could choose to take the discount if she can gain non-negative payoffs from it, or discard it otherwise. This quantity discount can be determined by the procedure proposed in the noncooperative model. Therefore, from the supplier's point of view, at Stage 1, the discount breakpoint is $Q_S = \sqrt{2A_S D / r_S v}$, and the discounted price is

$$p_1^{(PJ3)} = \frac{Dp_0 + \left(\dfrac{1}{Q_B} - \dfrac{1}{Q_S}\right) DA_B + \dfrac{1}{2} Q_B r_B p_0}{D + \dfrac{1}{2} nTDr_B}. \tag{5.42}$$

Now moving to Stage 2, the joint decision of the buyer and carrier can be expressed as the following maximization problem:

$$
\begin{aligned}
\max_{n,f_V} \Pi_{BC3} \quad &= \quad Dx - \left(Dp + \frac{1}{nT}A_B + \frac{1}{2}nTDr_Bp\right) + Dwf_V - \frac{1}{2}nTDwr_w \qquad (5.43)\\
\texttt{s.t.} \quad & \\
& \Pi_B^{2-1} = \pi_{B2} - \pi_{B1} \geq 0;\\
& \Pi_S^{2-1} = \pi_{S2} - \pi_{S1} \geq 0;\\
& \Pi_C^{2-1} = \pi_{C2} - \pi_{C1} \geq 0;\\
& n \geq \frac{1}{T}\sqrt{2A_S/Dr_Sv};\\
& n \ \texttt{integer;} \ f_V \geq 0.
\end{aligned}
$$

Note that, compared to the models (5.28) and (5.37), we introduce an additional constraint regarding the feasible range of n, generated from the breakpoint $Q_S = \sqrt{2A_SD/r_Sv}$. The reason is that the buyer could only take advantage of the quantity discount when her order quantity is greater than this breakpoint.

LEMMA 5.6. *The value of the integer $n^{(PJ3)}$ in the partial joint model (Case III) is the same as in the noncooperative model.*

Proof. Solving Eq. (5.43), we have

$$n = \frac{1}{T}\sqrt{\frac{2A_B}{D(r_Bp + wr_w)}}. \tag{5.44}$$

However, because

$$n = \frac{1}{T}\sqrt{\frac{2A_B}{D(r_B p + w r_w)}} < \frac{1}{T}\sqrt{\frac{2A_B}{D r_B p}} < \frac{1}{T}\sqrt{\frac{2A_S}{D r_S v}}, \tag{5.45}$$

this value of n is not feasible. Thus we must have

$$n^{(PJ3)} = \left\lceil \frac{1}{T}\sqrt{\frac{2A_S}{D r_S v}} \right\rceil. \tag{5.46}$$

$\square$

With this transportation discount breakpoint, the supplier can now readjust his selling price as

$$p^{(PJ3)} = \frac{D p_0 + \left(\frac{1}{Q_B} - \frac{1}{nTD}\right) D A_B + \frac{1}{2} Q_B r_B p_0}{D + \frac{1}{2} n T D r_B}. \tag{5.47}$$

Note that Π_{BC3} is a monotonically increasing function of f_V. Therefore, the best value of f_V is its upper bound, found from $\Pi_S^{2-1} = \pi_{S2} - \pi_{S1} = 0$, i.e.

$$\begin{aligned} f_V^{(PJ3)} \leq f_V^{(PJ3)[U]} &= f_N + \frac{1}{w}\left[\left(p^{(JP3)} - p_1\right) - \left(\frac{1}{n^{(PJ3)}TD} - \frac{1}{Q_S}\right) A_S \right. \\ &\quad \left. - \frac{1}{2} r_S v \left(n^{(PJ3)} T - \frac{Q_S}{D}\right)\right]. \end{aligned} \tag{5.48}$$

Thus, the joint payoff for the supplier and carrier can be calculated using the value of

$n^{(PJ3)}$, the selling price for the product $p^{(PJ3)}$, and the transportation rate $f_V^{(PJ3)}$ respectively obtained from Eqs. (5.46), (5.47), and (5.48). We notice that the solutions for this case are exactly the same as in the noncooperative model. The reason for this fact is actually very intuitive: the supplier is a "bridge" connecting the other two parties, there is no way for the buyer and carrier to cooperate without this bridge.

This payoff [the optimal value of $\Pi_B^{2-1}+\Pi_C^{2-1}$ from Eq.(5.43)] is the value of $u(\mathcal{J}_3)$, where $\mathcal{J}_3 = \{Buyer, Carrier\}$ in this case.

5.6.4 Solution procedure using Shapley Value

Now having obtained $u(.)$, the Shapley Values of this problem can be calculated by Eq. (5.27). The procedure to compute the Shapley Values now follows.

Step 1 Calculate $u(.)$ based on the noncooperative model, cooperative model, and the three cases of coalition models. To be specific, $u(\{B\}) = \Pi_B^{(N)}$, $u(\{S\}) = \Pi_S^{(N)}$, $u(\{C\}) = \Pi_C^{(N)}$, $u(\{BS\}) = \Pi_{BS}$ from coalition case I, $u(\{SC\}) = \Pi_{SC}$ from coalition case II, $u(\{BC\}) = \Pi_{BC}$ from coalition case III, and $u(\mathcal{N}) = u(\{BSC\}) = \Pi_J$ from the cooperative model, respectively.

Step 2 Fill these results in the corresponding cells of Table 5.4. Note that the left column lists 6 permutations of $\mathcal{N} = \{B, S, C\}$, with names of the players across the top of the table. Each cell shows that player's contribution to the coalition it joins. This

Table 5.4: Calculation of the Shapley Values

	Payoffs		
Permutation	**Buyer**	**Supplier**	**Carrier**
BSC	$\Pi_B^{(N)}$	$\Pi_{BS} - \Pi_B^{(N)}$	$\Pi_J - \Pi_{BS}$
BCS	$\Pi_B^{(N)}$	$\Pi_J - \Pi_{BC}$	$\Pi_{BC} - \Pi_B^{(N)}$
SBC	$\Pi_{BS} - \Pi_S^{(N)}$	$\Pi_S^{(N)}$	$\Pi_J - \Pi_{BS}$
SCB	$\Pi_J - \Pi_{SC}$	$\Pi_S^{(N)}$	$\Pi_{SC} - \Pi_S^{(N)}$
CBS	$\Pi_{BC} - \Pi_C^{(N)}$	$\Pi_J - \Pi_{BC}$	$\Pi_C^{(N)}$
CSB	$\Pi_J - \Pi_{SC}$	$\Pi_{SC} - \Pi_C^{(N)}$	$\Pi_C^{(N)}$

contribution is the value added by the particular player (indicated by the column) when he or she joins the preceding coalition in a permutation (specified by the row).

For example, in the permutation BSC, the buyer joins the set $\emptyset$, so the buyer is credited with $u(\{B\}) - u(\emptyset) = \Pi_B^{(N)}$. Then the supplier enters and is credited with $u(\{BS\}) - u(\{B\}) = \Pi_{BS} - \Pi_B^{(N)}$. Finally the carrier joins, and the grand coalition is now formed. The carrier's contribution to the resulting payoff is $u(\{BSC\}) - u(\{BS\}) = \Pi_J - \Pi_{BS}$.

Step 3 For each player's column, compute the average over the six permutations. The figure obtained is the Shapley Value for each player, which provides a fair share of the joint payoffs gained from the cooperation.

5.6.5 Example 5.3

Let us continue employing the previous parameters (Table 5.1) to study the effects of coalitions. In Case I, the buyer and supplier first act together to make the quantity discount decisions. By solving Eq. (5.28), we have $Q_1^{(PJ1)} = 68$ units and $p_1^{(PJ1)} = \$\ 195.08$ per unit, with a quantity discount percentage of 2.462%. Then, the carrier analyzes the situation at this stage, and tries to determine its transportation discount. From Eq. (5.32), the integer $n^{(PJ1)}$ can be calculated as 15. Additionally, the carrier would like to maximize its own payoff and at the same time make sure that the payoffs of the buyer and supplier are not negative. So the value of f_V can be determined as $f_V^{(PJ1)} = f_V^{(PJ1)[U]} = \$\ 49.94$ per cwt. In light of this discounted freight rate, the supplier and buyer readjust their joint decision to ordering each time the quantity $Q^{(PJ1)} = \lceil n^{(PJ1)} T \rceil = 72$ units at the price of $p^{(PJ1)} = \$\ 194.66$ per unit(2.672%). Under this policy, the joint payoff for the buyer and supplier is \$ 437, and the carrier's payoff is \$ 593. So we have $u(\mathcal{J}_1) = u(\{BS\}) = \$\ 437$. Checking the condition (Eq. (5.19)) confirms the feasibility of this rate.

For Case II, the supplier and carrier first make their discount decisions together. Solving Eqs. (5.37) gives $n^{(JP2)} = 11$ and $f_V^{(PJ2)} = \$\ 42.64$ per cwt. The order quantity is thus 52 units. This coalition gives the supplier and carrier a joint payoff of \$ 1177, i.e. $u(\mathcal{J}_2) = u(\{SC\}) = \$\ 1177$. Taking the preceding freight rate into consideration, the supplier can respectively calculate from Eqs. (5.40) and (5.41) the lower and upper bounds of the discounted selling price as $p^{(PJ2)[L]} = \$\ 189.89$ per unit (5.054%) and $p^{(PJ2)[U]} = \$\ 199.70$

per unit (0.151%). Note that, in this case, the objective of the coalition pair (supplier and carrier) is to maximize their joint payoffs under the condition that the buyer does not lose. Therefore the selling price is determined at the upper bound, i.e. $p^{(PJ2)} = \$\ 199.70$. The buyer's payoff is zero at this price.

Case III considers the condition that the buyer cooperates with carrier, while the supplier acts independently. The best coalition policy can be obtained from Eqs. (5.46) through (5.48) as $n^{(PJ3)} = 21$, $p^{(PJ3)} = \$\ 194.80$, and $f_V^{(PJ3)} = \$\ 49.65/cwt$, respectively. This policy gives a joint payoff of \$378 and the supplier's payoff of \$145. Therefore, $u(\mathcal{J}_3) = u(\{BC\}) = \$\ 378$.

Table 5.5 summarizes the results of all numerical examples in Section 5.6. Note that the results computed for the noncooperative model are based on the situation that all players act independently and maximize their own payoffs, i.e. no sharing among the players. Specifically, $u(\{B\}) = \Pi_B^{(N)} = 0$, $u(\{S\}) = \Pi_S^{(N)} = \$\ 145$, and $u(\{C\}) = \Pi_C^{(N)} = \$\ 378$ (Tables 5.2 and 5.3). Also, from the cooperative model, $u(\mathcal{N}) = u(\{BSC\}) = \$\ 1237$.

We note that the total payoffs for each of the three coalition cases are greater than the result from the noncooperative model. These facts indicate that as the degree of joint decision making grows, the total payoffs for the buyer, supplier, and carrier increase, i.e. the channel efficiency can be significantly improved by coordinating with other players. The payoffs to each party demonstrate how they can all benefit by cooperating with the others. Those benefits increase as the degree of cooperation increases. That is,

Table 5.5: Numerical example - Results concerning coalitions

	Noncooperative Model	**Coalition**			**Cooperative Model (Grand Coalition)**
		Buyer-Supplier (Case I)	**Supplier-Carrier (Case II)**	**Buyer-Carrier (Case III)**	
p	\$194.80	\$194.66	\$199.70	\$194.80	\$189.94
ρ	2.599%	2.672%	0.151%	2.599%	5.029%
f_V	\$49.65	\$49.94	\$42.64	\$49.65	\$42.03
n	21	15	11	21	10
Q	100	72	52	100	49
Π_B	0	-	0	-	-
Π_S	\$145	-	-	\$145	-
Π_C	\$378	\$593	-	-	-
Π_{BS}	-	\$437	-	-	-
Π_{SC}	-	-	\$1177	-	-
Π_{BC}	-	-	-	\$378	-
Π_J	-	-	-	-	\$1237

LEMMA 5.7. *The total payoffs obtained in each of the three cases of coalitions are less than the value from the grand-coalition model.*

Proof. As in the proof of Proposition 5.4, the optimal solution of the cooperative model, i.e. the grand coalition, yields the maximal total profit, i.e. the maximization of total payoffs. So the aggregate payoffs for any other cases, including the noncooperative model and coalitions involving two of the players, are *feasible* solutions to the problem of maximizing the joint profit Π_J. But the aggregate payoffs for those other cases cannot exceed those of the cooperative model. □

Employing the numerical results from Table 5.5 in Table 5.4 yields the values in Table 5.6. The Shapley Value for each player is calculated by averaging the six permutations for each column. According to the table, the three players can divide the joint payoff of $1237 (Table 5.5) as $69, $540, and $628, respectively, to the buyer, supplier, and carrier.

There are three pair-wise coalitions. According to Table 5.6, the joint payoffs to the players in each case is less than the sum of what those could earn by being in the grand coalition. Hence, the grand coalition is stable in a long run.

5.7 Summary

When it comes to the coordination of transportation and inventory decisions, most research has been conducted from the buyer's perspective. Given both quantity and transportation

Table 5.6: Numerical example - Shapley Value calculation

	Payoffs		
Coalitions	**Buyer**	**Supplier**	**Carrier**
BSC	0	\$437	\$800
BCS	0	\$859	\$378
SBC	\$292	\$145	\$800
SCB	\$60	\$145	\$1032
CBS	\$0	\$859	\$378
CSB	\$60	\$799	\$378
Average	\$69	\$540	\$628

discounts, the buyer determines the replenishment policy to minimize her total costs or maximize her own profit. Considering the four cases of transportation problems, this is the case in which buyer chooses the carrier and pays the relevant transportation cost (Case IV). Different from previous work, the approaches proposed in this chapter analyze Case III, in which the *supplier* pays for the common-carrier transportation service.

The purpose of our research has been to coordinate the quantity and transportation discounts offered by the supplier and carrier. We first studied the payoffs that each actor obtained from the coordination. Distinct from the payoffs to the buyer and supplier, calculation of the carrier's payoff required taking into account the transportation consolidation decision by employing the ESW model. We also revealed the motivation for the carrier to offer a transportation discount, namely to encourage that the supplier ship his orders according to (consistent with) the carrier's regular loads. This would save the carrier the fixed transportation expenses related to an additional dispatch.

Subsequently, we developed two models for considering the discount coordination problem under the situation that each party makes decision noncooperatively or cooperatively. In the noncooperative model, we determined the upper and lower bounds on the selling price and the freight rate offered by the supplier and carrier, respectively. Through a simple numerical example, and as we proved in general (Proposition 5.4), the cooperative model showed a better overall result of the joint payoffs than the sum of separate payoffs from the noncooperative model.

Furthermore, we conducted a coalition analysis based on the concept of the Shapley Value. The three possible cases of coalition were discussed and integrated. A procedure to apply the Shapley Value to fairly divide the extra gains from the cooperation was proposed and illustrated by numerical examples.

Chapter 6

Conclusions and Future Research

This chapter summarizes the contributions of this thesis. We conclude with the concentration and orientation that our ongoing research will continue in the future.

6.1 Conclusions

Although discounts have been used and analyzed for decades, there were still so many things about discounts that needed to be explored. The complications of reality urged us to consider this problem in a different way. Switching the perspective from the participant who takes the discount, to the one who *sets* the discount, has led to a better resolution for improving the effectiveness and efficiency of that supply chain. However, our objective then took this even one step further, involving both the quantity and transportation discounts.

We were motivated to ask, "What would be the results of coordinating these two discounts for the supplier and the carrier," as well as "How to achieve this coordination and make it profitable?"

Our research started from a simple case, in which the buyer's total costs may be significantly reduced by the joint consideration of quantity and transportation discounts. Then the transportation problems were classified into four distinctive cases. Through the analyses of decisions and costs, the problem structure was enhanced, and thus, we gained a better understanding of the motivation and objectives. We have conducted our research in three phases.

First of all, assuming a single product, we investigated the quantity discounts from the *supplier's* perspective, and including the price elasticity of demand. A noncooperative game-theoretic approach and a joint decision model were developed to aid a sole supplier in establishing an all-unit quantity discount policy in light of the buyer's best reaction. The Stackelberg equilibrium and Pareto optimal solution set were derived for the noncooperative and joint-decision cases, respectively. Our research indicated that channel efficiency can be improved significantly if the quantity discount decision is made jointly rather than noncooperatively. Moreover, these two basic models were extended in three directions.

- The importance of transportation led us to extend our models to the case of product shipment by the private fleet of either the supplier or buyer.

- We also considered the fact that the buyer may wish to set the best retail price to maximize her own payoff.

- A heuristic solution procedure showed the possibility of significant improvements by condensing the heterogeneous-buyer groups to a smaller number of discount levels.

Numerical case studies were employed in Chapter 3 and throughout the thesis to illustrate the practical applications of the models presented, and the sensitivity to model parameters.

Next, in Chapter 4, we considered a situation with a *family* of SKUs for which the supplier will offer a quantity discount, according to the aggregate purchases of the product group. Management of those items was based on the modified periodic policy. We defined what is meant by the optimal discount scheme, and showed how to determine it.

With the group discount, a buyer's best decision could be to order a larger amount, one that will satisfy demand during a longer interval, i.e. for a greater multiple of the base period T. However, that interval between replenishments (equivalent to the quantity breakpoint for the group discount) should be chosen by maximizing the supplier's payoff function. The supplier offers that discount, to encourage the buyer to take it, by providing a positive payoff to her. We demonstrated that, for any discount percentage between the upper and lower bounds that we calculated, both the supplier and buyer will benefit from the group discount policy. Additionally, another model maximizing the joint payoff of the supplier and buyer was presented. In a simple numerical example, the joint model showed a better overall result than did the case of separate payoffs to buyer and supplier. That is, each party was better off in the case of the joint model.

In that chapter, we developed another model, considering the fact that demand can change due to variations in price. Two solution procedures were developed for the non-cooperative and joint models, accompanying the corresponding maximization problems. Through comparison of numerical results from the two examples, we showed the impacts of price elasticity on the supplier's optimal discount policy. We thus illustrated that price elasticity of demand is one of the most important motivations, both for the supplier to offer a discount and for the buyer to accept it.

In Chapter 5, our third approach studied the case in which determination of the freight discount offered by the common carrier (a public, for-hire trucking company) is integrated into the quantity discount decisions. The transportation discount, especially an LTL discount schedule, is extremely hard to set in practice. This is not only because of the "phantom freight" phenomenon caused by over-declaration, but also due to the fact that such a discount relates to both transportation and inventory issues. In this research, we studied the problem of coordinating the transportation and inventory decisions both noncooperatively and cooperatively. More specifically, the transportation and quantity discount decisions were again analyzed from the perspectives of the party who offers the discounts, rather than the one that takes them. We showed that the cooperative approach provides better overall results, compared to the noncooperative model. To divide the extra payoffs gained from this cooperation, we further conducted a coalition analysis based upon the concept of Shapley Value.

6.2 Future research

Through our three main chapters of research results, the benefits of coordination were obvious and attractive. However, these are only tips of the iceberg. There are many directions that can be explored in the future.

First of all, the assumptions on demand rate can be relaxed. In all inventory-related problems, demand is perhaps the most important issue. A firm's inventory policy can significantly vary due to demand situations that are being faced. We have discussed the possibility of extending the carrier's decision model presented in Chapter 5 to incorporate in that model the fact that demand varies with respect to price. We will again compare the results for this extended case with those obtained in Chapter 5, and draw conclusions on price-sensitivity analogous to those in Chapter 4.

Stochastic demand is another possible direction. Most existing discount literature does not consider the uncertainty of demand. Randomness is important in the newsvendor model. Although that deals with a single-period case. the newsvendor model works well in planning perishable goods or high-fashion items, and is the foundation of the inventory control considering uncertain demand. So our future research will extend to allow stochastic demand rate. We will seek the impact of demand uncertainty on the discount decisions, on replenishment policies, and on the overall supply chain performance.

Secondly, the size of a model is always critical for its development. In our research, model sizes refer to the numbers of buyers, products, as well as discount break points.

We have examined the quantity discount problem with extension to multiple buyers or multiple products (a family of items). What if the carrier provides services to multiple shippers (or suppliers, in this thesis)? Would this affect the carrier's shipment-consolidation decisions, and thus affect the LTL discount schedule? If yes, what about the vendor and the buyer? What should be their policies corresponding to carrier's decisions? All these very interesting questions lie in this path.

Moreover, up to now, we have analyzed only a simplified payoff function from the carrier's perspective. Actual transportation situations are far more complicated. Many issues need to be taken into account when we are dealing with those problems. For example, it is necessary to consider the restriction of transport capacities, both weight and volume. The two main concerns about the maximum number of items that can be carried per shipment, "weigh out" and "cube out," need to be integrated into the transportation-pricing procedures.

It is also possible for us to study the different relationships among the supplier, carrier and buyer. Our models assume that the party who offers discounts has the dominant power over the other party. Therefore, the manufacturer-Stackelberg game is employed. However, in specific situations, the party who receives discounts is dominant. In this case, the retailer-Stackelberg game is the appropriate framework. Moreover, there also exist situations that no player is powerful enough to dominate. A Vertical-Nash game, different from the Stackelberg game, is then applicable. In the Vertical-Nash game, both parties make decisions and act simultaneously. Relevant literature includes Jeuland and Shugan

(1983), Choi (1991), as well as Chu and Messinger (1997), among others. A comparison among the three types of game models would provide us with interesting insights.

In addition, the availability of information is a critical issue worth considering. In this thesis, one of the most important assumptions is “perfect information”, which is essential to a Stackelberg game model. However, in reality, although the supplier can get the exact information of a buyer’s annual demand and EOQ directly from this buyer’s order pattern, estimating a buyer’s ordering and inventory holding costs is extremely challenging. The situation becomes even more complicated when the demand is stochastic. Therefore, it is necessary to analyze the case without full information. Corbett and de Groote (2000) derived an optimal quantity discount policy under asymmetric information and compared it to the situation where the supplier has full information. Other research that examined the problem of supply chain coordination with asymmetric information includes Sucky (2006), Burnetas et al. (2007), and Esmaeili and Zeephongsekul (2010), to name a few. It will be inspiring to read through these articles.

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