Essays in Risk Management for Crude Oil Markets

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

This thesis consists of three essays on risk management in crude oil markets. In the first essay, the valuation of an oil sands project is studied using real options approach. Oil sands production consumes substantial amount of natural gas during extracting and upgrading. Natural gas prices are known to be stochastic and highly volatile which introduces a risk factor that needs to be taken into account. The essay studies the impact of this risk factor on the value of an oil sands project and its optimal operation. The essay takes into account the co-movement between crude oil and natural gas markets and, accordingly, proposes two models: one incorporates a long-run link between the two markets while the other has no such link. The valuation problem is solved using the Least Square Monte Carlo (LSMC) method proposed by Longstaff and Schwartz (2001) for valuing American options. The valuation results show that incorporating a long-run relationship between the two markets is a very crucial decision in the value of the project and in its optimal operation. The essay shows that ignoring this long-run relationship makes the optimal policy sensitive to the dynamics of natural gas prices. On the other hand, incorporating this long-run relationship makes the dynamics of natural gas price process have a very low impact on valuation and the optimal operating policy.

In the second essay, the relationship between the slope of the futures term structure, or the forward curve, and volatility in the crude oil market is investigated using a measure of the slope based on principal component analysis (PCA). The essay begins by reviewing the main theories of the relation between spot and futures prices and considering the implication of each theory on the relation between the slope of the forward curve and volatility. The diagonal VECH model of Bollerslev et al. (1988) was used to analyze the relationship between of the forward curve slope and the variances of the spot and futures prices and the covariance between them. The results show that there is a significant

quadratic relationship and that exploiting this relation improves the hedging performance using futures contracts.

The third essay attempts to model the spot price process of crude oil using the notion of convenience yield in a regime switching framework. Unlike the existing studies, which assume the convenience yield to have either a constant value or to have a stochastic behavior with mean reversion to one equilibrium level, the model of this essay extends the Brennan and Schwartz (1985) model to allows for regime switching in the convenience yield along with the other parameters. In the essay, a closed form solution for the futures price is derived. The parameters are estimated using an extension to the Kalman filter proposed by Kim (1994). The regime switching one-factor model of this study does a reasonable job and the transitional probabilities play an important role in shaping the futures term structure implied by the model.

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I would like to thank the department of Economics for providing the support and facilities I have needed to produce and complete my thesis.

Dedication

To my parents... See you soon.

To my wife... Here you go.

To my little twins... Here I am.

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Chapter 1

Introductory Chapter

Interest in energy-related investments and risk management has been growing in recent years. Among the important energy commodities is crude oil, which is characterized by highly uncertain and volatile prices. Crude oil is an important component of economic and business activities in any economy. Thus, understanding its price movement is crucial for successful economic and business decisions. Moreover, crude oil, and energy commodities in general, have become one of the most active alternative assets[1] during recent years. Most energy products, such as crude oil and natural gas, have very liquid futures contracts that are traded in exchanges. Moreover, several investment vehicles tied to their prices are available in the market, ranging from small mutual funds and exchange-traded funds (ETF), to large over-the-counter contracts (e.g SWAP contracts). In addition to financial investments, recent increases in energy prices have induced a large inflow of capital into energy projects. For example, according to the Canadian Energy Research Institute (CERI), the oil sands industry in Alberta attracted in excess of $18 billion of investment

[1] Alternative assets are alternative to the traditional investments such as publicly-traded stocks, bonds and mutual funds, see Anson (2002)

in the year 2007 and it is forecast to reach beyond $300 billion over the period from 2008 to 2030 [2]

This thesis attempts to contribute to the existing understanding of risk management in crude oil markets through three not unrelated essays. An important focus of the thesis is the pricing of crude oil futures contracts. Futures contracts are fundamental tools for pricing and risk management in energy markets, as they are in most commodity and financial markets. A futures contract is an agreement between two parties to buy or sell an asset at a certain future time for a set price agreed on today. Futures contracts are traded on exchanges, with certain standardized features and for different delivery dates ranging from few months to more than 5 years. Understanding the dynamics of the relation between spot and futures prices is very important as it helps in better dealing with the uncertainty in the market, in devising the appropriate models for the price process and in better valuation of related contingent claims.

In the first essay (chapter 2), the valuation of an oil sands project is studied. Unlike conventional crude oil extraction, oil sands production consumes substantial amounts of natural gas during extracting and upgrading. Natural gas prices are known to be stochastic and highly volatile. This introduces a significant stochastic component in the extraction cost. The essay studies the impact of this stochastic component in valuing oil sands projects. The valuation is done using real options methods. The motivation to use real options methods is the fact that, using these methods, operational flexibilities can be taken into consideration when valuing the project. Introducing stochastic extraction cost makes the valuation more complicated due to the fact that not only does the movement of the output and input prices need to be considered, but also the type of the co-movement

[2] See McColl and Slagorsky "Canadian Oil Sands Supply Costs and Development Projects (2008-2030)" Canadian Energy Research Institute, November 2008

of the two prices must be taken into account. Given this fact, the essay begins with an investigation of the empirical literature about the nature of the co-movement between crude oil and natural gas prices. In particular, I consider whether there is a long-term effect that results from an economic relationship between crude oil and natural gas or whether the co-movement arises only from a short-term effect associated with the correlation of the energy prices. For the valuation section of this essay, the stochastic dynamics of the oil and gas prices are modeled using the two-factor model of Schwartz and Smith (2000) in a multi-commodity framework. In general, two-factor models have proved to capture the historical dynamics and the term structure of commodities futures fairly well. More importantly, the Schwartz and Smith (2000) framework allows us to distinguish between the long- the short-run movements of the commodity prices and thus enables us to model the long- and the short-run co-movements in the two markets in an explicit way. The valuation problem is solved by the Least Square Monte Carlo (LSMC) method proposed by Longstaff and Schwartz (2001) for valuing American options. The LSMC proved to be an efficient tool for valuing high order problems, where the number of stochastic factors are large as it is the case in this essay where there are four factors: two for crude oil prices and two for natural gas prices.

In the second essay (chapter 3), the relationship between the slope of the futures term structure and volatility in the crude oil market is investigated. The futures term structure, or simply the forward curve[3], is a plot of the futures prices against their corresponding maturities at a specific point in time. Generally, the forward curve can take many shapes. However, there are two main shapes that market participants usually pay attention to: a positive slope forward curve which is known as contango and a negative slope forward curve which is known as backwardation. Many studies have documented the significance

[3] Both terms, the futures term structure and the forward curve will be used interchangeably.

of the slope of the forward curve for predicting the volatility of the market. However, these studies use the basis, which is the spread between the futures and spot prices or between two futures prices, as a measure of the slope of forward curve. In this measure, the choice of maturities of the futures price is arbitrary. In this essay, I use another measure of the slope based on principal component analysis (PCA) used in Borovkova (2006). The advantage of using PCA is that all futures prices are used in calculating the slope of the forward curve. In this essay, I begin by reviewing the main theories on the relation between spot and futures prices and extract the implication of each theory for the relation between the slope of the forward curve and volatility. Both the literature in commodities prices modeling and the literature in exhaustible resources pricing contain theories which have some implications for the equilibrium state of this relationship. Five main theories are presented and their implications are compared. Futures contracts are commonly used as a hedging tool by producers, consumers and risk averse investors. To illustrate the usefulness of the prediction power of the forward curve slope, the essay studies whether exploiting this prediction power will improve hedging performance using futures contracts.

Modeling the stochastic nature of commodities prices is a crucial step for valuing financial and real contingent claims related to commodities prices. The notion of convenience yield, defined as the benefits accruing to the owner of the physical commodity due to the flexibility in handling shocks in the market, plays a central role in commodities prices modeling as it derives the relationship between futures and spot prices in the commodities markets. The third essay (chapter 4) attempts to model the spot price process of crude oil by the notion of convenience yield in a different way. The existing convenience yield models assume the convenience yield to have either a constant value, such as Brennan and Schwartz (1985), or to have a stochastic behavior with mean reversion to one equilibrium level, such as Gibson and Schwartz (1990), Schwartz (1997) and Casassus et al. (2005).

The model of this essay extends the Brennan and Schwartz (1985) model to allows for regime switching in the convenience yield. The motivations behind this choice of modeling are the following. Theoretically, the convenience yield is seen as a function of the level of the commodity inventory in the economy which is in turn a function of the supply and demand conditions. Moreover, macroeconomic conditions which run through different cycles of booms and busts are likely to have impacts on the commodities markets especially for crucial commodities such as crude oil. Given that, it is unlikely that there is only one equilibrium state the commodity market should revert to. From the empirical side, estimating the Gibson and Schwartz (1990) model using crude oil futures in different periods of time produces very different values of the equilibrium level of convenience yield.

The regime switching approach to modeling provides a natural way to relax this restrictive assumption about the level of the convenience yield. Regime-switching models are time-series models in which parameters are allowed to take on different values in each of some fixed number of regimes or states. A stochastic process assumed to have generated the regime shifts is included as part of the model, which allows for model-based forecasts that incorporate the possibility of future regime shifts. The primary use of these models in econometrics has been to describe changes in the dynamic behavior of macroeconomic and financial time series (Hamilton (1994) and Dai et al. (2007)).

The model of this essay is different from those of Chen and Forsyth (2010) and Chen (2010) who take regime switching approach to model energy prices in three main ways. First, the regime switching model proposed in their studies is based on the one-factor model applied in Schwartz (1997) where the commodity price reverts to different levels with different volatilities. In this essay, the convenience yield switches to different levels with different volatilities. Second, unlike their studies, the model of this essay allows for pricing the risk of switching between the regimes. Third, they calibrate the parameters of the model by solving the partial deferential equation (PDE) characterizing the futures price numerically and calibrate the solution to the observed futures prices using least square methods. The model of this essay is estimated using an extension to the Kalman filter procedure proposed by Kim (1994). The choice of the Kalman filter procedure for estimating the model is motivated by the Monte Carlo study of Duffee and Stanton (2004) in estimating the term structure of interest rates where Kalman filtering procedure is found to be a tractable and reasonably accurate estimation technique. To judge the performance of the model of this study, it is compared with Gibson and Schwartz (1990) two- factor model.

Overall, the dissertation contributes to our understanding about risk management in crude oil markets in a number of ways.

- The thesis contributes to the literature about the co-movement of crude oil and natural gas prices by investigating the type of the co-movement using the futures prices of the two markets and proposing a way of modeling the two types of the co-movement that can be easily estimated by the term structure of futures prices in the two markets.

- The thesis contributes to the literature of real options valuation of exhaustible resources by studying the value of an oil sands project. In particular, the thesis studies how a stochastic extraction cost can affect the value of an exhaustible resource.

- It also contributes to the understanding of the relation between volatility and the slope of the forward curve in two ways: by extracting the implications of various theoretical work on this relation; and by analyzing the relation empirically using a more appropriate measure of the forward curve slope extracted by PCA.

- The thesis also contributes to the literature about commodity prices modeling by proposing a regime switching model that is more appropriate for convenience yield modeling especially for long-run valuation purposes. Moreover, the thesis shows how a closed form solution for the futures price formula can be obtained.

The main results of the dissertation are as follows:

- The analysis of the first essay shows that higher natural gas price volatility reduces the value of the project. It also shows that not only the dynamics of oil and natural gas prices are important, but also the nature of the co-movement of the two prices is an important factor to take into consideration in valuation and optimal operation. While the economic links between the two markets, i.e. being substitutes as sources of energy, suggests the existence of a long-run relationship between the two prices, the empirical evidence is weak especially if one incorporates the recent divergence in the two price series. The valuation results show that incorporating a long-run relationship between the two markets is a very crucial decision in valuing the project and in its optimal operation. It is shown that ignoring this long-run relationship makes the optimal policy more sensitive to the dynamics of natural gas prices.

- In the second essay, it is found that the forward curve slope has no significant linear impact on the variances of the futures and spot prices and the covariance between them. However, the slope of the forward curve does have a significant quadratic impact not only on the variance of spot and futures price returns as Carlson et al. (2007) and Kogan et al. (2009) found, but also in the covariance between the two prices. Moreover, it is shown that incorporating the slope of the forward curve quadratically produces a significant improvement in the hedging performance using futures contracts.

- Compared to the performance of the Gibson and Schwartz (1990) two-factor model, the regime switching one-factor model of the third essay does a reasonable job. In particular, the model outperforms the Gibson and Schwartz (1990) model for fitting the prices of far maturities contracts. Moreover, the transitional probabilities have been found to play an important role in producing various shapes of the futures term structure that are commonly seen in the market.

Chapter 2

The Impact of Stochastic Extraction Cost on the Value of an Exhaustible Resource: the Case of the Alberta Oil Sands

2.1 Introduction

Traditionally, valuing a natural resource project, or any project in general, is based on the simple net present value method. Using this method, expected future cash flows from operating a project are discounted to the current time using a constant risk adjusted discount rate and added up to give the value of the project. This procedure has been criticized for ignoring possible flexibilities in starting or operating the project. Examples of such ignored flexibilities are: the flexibility in starting the investment (option to delay) and the flexibility to switch between different mode of operations (option to switch). In addition, the use of a constant risk adjusted discount rate is known to be inappropriate for valuing projects [1].

On the other hand, in the real options valuation approach, managerial flexibilities are taken into consideration when valuing a project. In general, the real options approach is based on the analogy between financial options and investment projects, and thus it uses the valuation tools developed for financial options. For more details on this method and its features, see Dixit and Pindyck (1994) and Schwartz and Trigeorgis (2004).

In their seminal paper, Brennan and Schwartz (1985) set the ground for using contingent claims analysis for valuing an exhaustible natural resources when the decision-maker has flexibility to choose from multiple modes of operation. The uncertainty in their model has only one source, the output price. They assumed fixed extraction cost and that the price follows Geometric Brownian Motion (GBM)[2].

Many papers account for more realistic assumptions about the sources of the uncertainty faced by an exhaustible resource. Cortazar et al. (2008) and Tsekrekos et al. (2010) extended Brennan and Schwartz (1985) valuation problem under different output price model dynamics. Cortazar et al. (2001) studied the valuation of natural resource exploration investments when there is joint price and geological-technical uncertainty. Armstrong et al. (2004) accounts for the uncertainty in the reserve.

However, one aspect that seems to be ignored in this literature, valuing exhaustible

[1] For valuating a copper mine using the real options approach, Brennan and Schwartz (1985) showed that the risk of the mine is function of the spot price of copper which is stochastic. Thus, the instantaneous rate of return required by investors should be stochastic, showing the inappropriateness of assuming a constant discount rate in the present value analysis.

[2] Brownian motion is a continuous-time stochastic process that has independent increments of normal distribution with mean of zero and variance of the time difference, i.e. if $z(t)$ is a Brownian motion then $dz(t) \sim N(0, dt)$. For more details see Klebaner (2005)

resources using contingent clams analysis, is the possibility that production cost or part of it, along with other state variables, is stochastic and volatile as well. An exception is Slade (2001) who used yearly panel data about 21 copper mines in Canada from the 1980 to 1993 period and found that average costs, which include the costs of mining, milling, smelting, refining, shipping, and marketing, to be highly variable. Using these data, Slade (2001) then applied the real options theory to Canadian mining investments and studied the impact of copper price, average cost and resource reserve uncertainties under different assumptions about the stationarity of the stochastic processes.

The lack of studies that account for stochastic cost is possibly because of the difficulty of obtaining enough data on cost variables as is the case in Slade (2001). This makes the variability in cost hard to appreciate. A perfect example where the uncertainty of extraction cost appears to be salient is the oil sands industry. The oil sands industry consumes substantial amounts of natural gas during production and upgrading activities. According to the Canadian Energy Research Institute (CERI), natural gas, its price being highly volatile, contributes more than 25 percent of the total per barrel supply cost[3]. "In 2007, the oil sands industry accounted for approximately 1.0 billion cubic feet per day (bcf/d) of natural gas demand, slightly more than 40 percent of Alberta total natural gas demand of 2.7 bcf/d"[4].

Two features characterize the source of uncertainty about extraction cost in the oil sands industry. First, data about natural gas prices is readily available on a daily basis. Second, crude oil and natural gas markets are linked together and thus, for better valuation and risk management decisions, modeling the nature of their co-movement should be considered.

[3]The supply cost is the constant dollar price needed to recover all capital expenditures, operating costs, royalties, taxes, and earn a specified return on investment

[4]see McColl and Slagorsky, "Canadian Oil Sands Supply Costs and Development Projects (2008-2030)" Canadian Energy Research Institute, 2008

Accordingly, this chapter examines the nature of the co-movement of crude oil and natural gas markets and then studies the impact of the stochastic extraction cost on the valuation of an oil sands project. In particular, two extensions of the Schwartz and Smith (2000) model to specify the stochastic dynamics of the two prices are suggested and the Brennan and Schwartz (1985) valuation problem is solved.

As shown in Brennan and Schwartz (1985), an analytical solution to such a problem is unavailable, so they solve the problem using a finite difference numerical methods. Recent developments in valuing American options using simulation based methods enable researchers to explore more realistic extensions to the Brennan and Schwartz (1985) model that proved to be impractical to solve using the prevailing numerical methods such as finite difference or lattice methods. The Least Square Monte Carlo (LSMC) method developed by Longstaff and Schwartz (2001) has proved to be an efficient tool for valuing complex real options problems. Gamba (2003) provides a comprehensive overview on how LSMC could be used to value various types of real options. Accordingly, LSMC is used for the purpose of this chapter. Cortazar et al. (2008) and Tsekrekos et al. (2010) also used this method for solving real options valuation problems.

This chapter is organized as follows: section 2.2 gives a background on the oil sands industry. Section 2.3 reviews the empirical literature on the co-movement of natural gas and crude oil markets with some recent results. Sections 2.4 and 2.5 specify the modeling procedures of the state variables and the oil sands project to be used in estimation and simulation. Data description and results are given in sections 2.6 and 2.7 respectively. The last section is for concluding remarks.

2.2 Oil Sands Background

The oil sands are unevenly spread over 140,000 km^2 (54,000 square miles) in Northern Alberta, Canada. The area contains an estimated 1.7 trillion barrels (initial volume-in-place) of an extremely heavy crude oil referred to as bitumen[5]. This reserve is believed to be a valuable energy source given its size, the current and expected high prices of crude oil and the state of the global supply and demand of the oil market. According to Canadian Association of Petroleum Producers (CAPP), capital expenditure in oil sands projects has risen from $4.2 billion in 2000 to $11.2 billion in 2009. [6]

Approximately 20 percent of of Alberta's oil sands can be found close enough under the surface (generally less than 75 meters) to permit mining production. On the other hand, around 80 percent of this reserve is found too deep below the surface for feasible mining operations. Bitumen in such deep deposits (typically 400 meters below the surface) needs to be recovered from the *in situ* (Latin: in place) position, similar to conventional oil, but by using a variety of special production techniques.

In *in situ* extraction techniques, a high temperature steam is injected inside the bitumen deposit through horizontal or vertical wells to reduce its viscosity and make it easier to be pumped up to the surface. The steam generators used within the process use natural gas as a fuel source. According to CERI, a rule-of-thumb commonly used in the industry is that 1 Mcf (thousand cubic feet) of natural gas is required to produce a barrel of bitumen. It is estimated that natural gas usage amounts to about 45 percent of total per-barrel operating cost. Table 2.1 shows the per-barrel of bitumen operating cost for a typical *in*

[5]Crude bitumen, or bitumen, is a term that reflects the heavy and highly viscous oil in the oil sands areas. The term "oil sands" includes the crude bitumen, minerals, and rocks that are found together with the bitumen (www.ERCB.com)

[6] See 2011 Statistical Handbook in (http://www.capp.ca).

Table 2.1. Operation Cost for Bitumen In situ Production (Canadian Dollars)

Operating Cost (Excluding Energy)	
Fixed Operation Cost	\$ 47.6 Million per year
Variable Operating Cost	\$ 6.6 per barrel
Natural Gas Cost	\$ 7.5 per barrel
Total (for capacity of 30,000 barrel per day)	\$ 18 per barrel
Total (WTI equivalent)	\$ 35 per barrel

Source: Canadian Energy Research Institute (CERI), 2008

situ project.

A typical *in situ* oil sands plant consists of multiple well pads containing a group of wells where bitumen is extracted and a central processing facility (CPF) where the extracted bitumen is processed to meet certain specifications. Steam from the CPF is transported by pipelines to the well pads and distributed to the various wells. Produced water and bitumen from the wells are then taken back for processing in the CPF. The majority of the bitumen is upgraded to produce synthetic crude oil (SCO). Given this heavy dependency on natural gas in bitumen production, uncertainty in natural gas prices is an important risk factor that needs to be accounted for. Natural gas prices are characterized by high volatility and high correlation with other energy prices especially with the oil prices (see Pindyck (2004), Geman (2005) and Brown and Yucel (2007)). Figure 2.1 shows the price of natural gas at Henry Hub, a major trading point located in the south of the US on the Gulf of Mexico, along with the price of WTI crude oil from 1997 until 2010. A casual inspection of the graph indicates that the price of natural gas tends to move with the price of oil, but not always. The next section studies this co-movement in detail.

In this chapter, I study the impact of this risk factor on the value and the optimal operation of an oil sands project. While the application is for oil sands industry, the analysis and insights are applicable to a variety of large natural resource projects that requires a significant amount of a volatile input with a volatile price.

2.3 Co-movement of Crude Oil and Natural Gas Prices

In general, there are two sources of co-movement among commodities as explained in Casassus et al. (2010). The first one is a short-term effect associated with the correlation of commodity prices while the second source arises from a long-term effect that results from an economic relationship such as a production relationship where one commodity is produced from another one and substitution relationship where two commodities are substitutes in consumption. Figure 2.1 shows the time series of the price of crude oil and natural gas. It appears from the graph that the two commodity prices tend to move together. The correlation coefficient is 0.26 between their (log) differences and 0.75 between their levels.

Villar and Joutz (2006) identify several economic factors that link natural gas and crude oil prices, from both supply and demand sides. One of the main links is the competition between natural gas and petroleum products which occurs principally in the industrial and electric generation sectors. Industry and electric power generators switch back and forth between natural gas and residual fuel oil, using whichever energy source is least expensive.

Some empirical studies confirm this fact, finding a long-run relationship between the two commodity price series. Villar and Joutz (2006) studied the co-movement of the two prices over the period from 1989 through 2005 and found oil and natural gas prices to be co-integrated with a trend. Brown and Yucel (2007) examined the relationship between weekly prices over the period from January 7, 1994 through July 14, 2006. Their analysis revealed

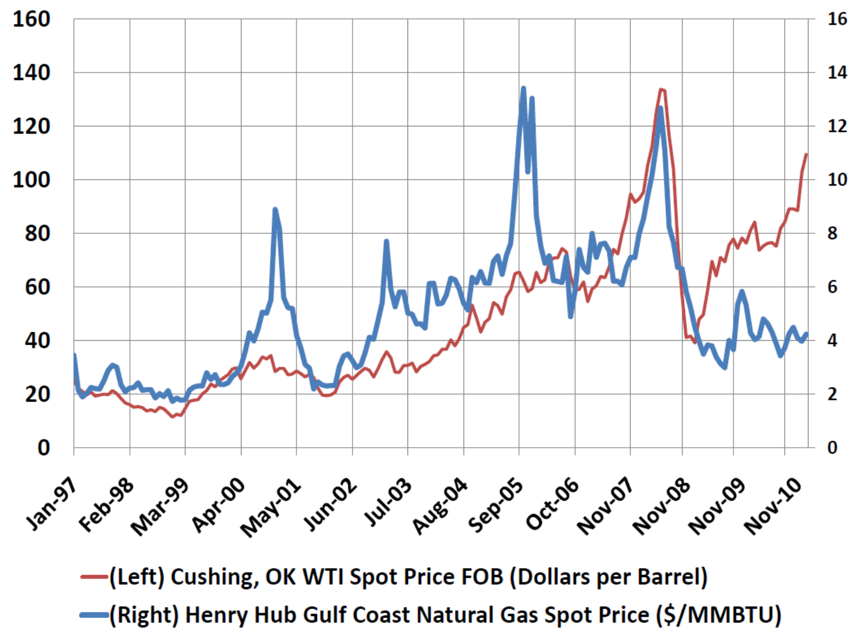


Figure 2.1. WTI Crude Oil and HH Natural Gas Prices

that weekly oil and natural gas prices have a strong relationship, but the relationship is conditioned by weather, seasonality, natural gas storage and shut in production in the Gulf of Mexico. Hartley et al. (2008) examined the relation between natural gas and crude oil prices by studying the relation between natural gas and residual fuel oil, the main product of crude oil that is viewed as a substitute for natural gas. They used monthly data from February 1990 through October 2006. They demonstrated the existence of a long-run co-integrating relationship between natural gas and residual fuel oil. Moreover, they found that changes in electricity generating technology can explain the apparent drift in this long-run relationship seen after 2000.

Given the fact that oil prices are determined internationally, a relationship such as that found in these studies led to the use of rules of thumb in the energy industry that relate natural gas prices to those for crude oil. For example, the Canadian Energy Research

Institute (CERI) in its 2009 report about Canadian oil sands supply costs and development projects[7] assumed that there is a 10:1 ratio between the price of oil in \$/barrel and the price of natural gas in mm Btu[8]. Other rule of thumbs have also been used as shown in Brown and Yucel (2007).

However, other empirical studies find a weak or no long-run relationship between the two prices. Serletis and Rangel-Ruiz (2004) explored the strength of shared trends and shared cycles between natural gas and crude oil markets. Using daily data from January 1991 to April 2001, their results show that there has been a decoupling of the prices of these two sources of energy and they explained that this was a result of oil and gas deregulation. Bachmeier and Griffin (2006) found that the degree of the co-integration between the two prices was very weak during the period from 1990 to 2004. Mohammadi (2009) analyzed annual and monthly data of the period from 1970 to 2007 and found a lack of co-integration relationship in the annual data and a weak one in the monthly data. Moreover, he examined the possibility of co-integration with asymmetric adjustments using threshold autoregressive (TAR) models. The results again fail to reject the null hypothesis of no co-integration.

Figure 2.2 shows the correlation coefficient between the daily returns of the two commodity prices in each month. It is clear from the graph that the correlation between the two price movements has gone through up and down cycles. In the late 90's, the correlation was relatively low, around 0.1. From 2003 to 2008, one can identify a cycle of a high co-movement, the correlation coefficient was around 0.4 on average. This cycle has

[7] See McColl, Mei, Millington and Slagorsky "Canadian Oil Sands Supply Costs and Development Projects (2009-2043)" Canadian Energy Research Institute, November 2009

[8] mm Btu stands for 10,000 million British thermal units. Natural gas can also be measured in gigajoule(GJ) and thousand cubic feet (Mcf). NYMEX Henry Hub natural gas prices are quoted in mm Btu. The relation between these three measures are: 1 mm BTU = 1.027 Mcf =1.05 GJs

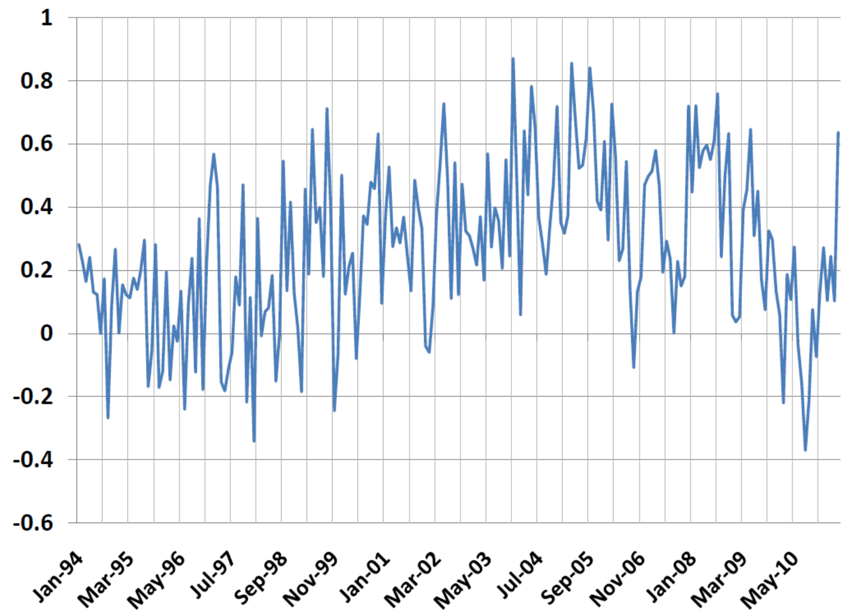


Figure 2.2. Monthly Correlation of Daily Returns of WTI Crude Oil and HH Natural Gas.

been attributed to two sources[9]: (1) to the large demand for energy products from emerging economies, such as China and India, which have experienced a very rapid economic growth during the period, and (2) to the financial market demand for commodities index investments which are designed to get exposure to commodities prices for diversification purposes and/or better risk-return opportunities. A cycle of low correlation is clearly seen recently, which is mainly attributed to strong growth in shale gas production [10]. This is also clear from the divergence in the two prices seen since the end of 2008 as shown in Figure 2.1.

[9] There is a large amount of research work on 2004-2008 increase in energy prices: whether it is caused by fundamentals (supply and demand factors) or by a bubble resulted from the large inflow of index investments. Refer to Irwin and Sanders (2011) for an excellent survey of the subject.

[10] For more details see *The 2011 Annual Energy Outlook* prepared by the U.S. Energy Information Administration available at http://www.eia.gov

Table 2.2. The Long-run Slope of Crude Oil and Natural Gas

		Number of Weeks	
WTI Crude oil	HH N.Gas	**F12-F01**	**F24-F01**
Positive	Negative	53	42
Negative	Positive	211	160
	total	814	761

crude oil and natural gas futures prices from January 1995 to August 2010 was used

Regarding the long-term relationship, I found empirical support for the result that there is no long-term relationship between oil and natural gas prices. Table 2.3 shows the Johansen's maximum-likelihood tests of co-integration[11]. The results fail to reject the null hypothesis of no co-integration in futures prices for different maturities except the results of the trace statistic in the first month futures prices. However, the results for the long-term futures suggest no long-run relationship. Moreover, Table 2.2, shows that more than 25% of time, the long-run slope of the forward curves of both oil and gas futures, measured by the difference between the one year or the two years futures price and the first month futures price, have different signs which indicates that the two markets lack a strong long-run relationship. Casassus et al. (2010) shows that commodities with economic links exhibit an upward-sloping curve in their correlation term structure, i.e. the correlation coefficient between futures price returns as a function of maturities. Figure 2.3 shows the correlation term structure between natural gas and crude oil prices and it is clear that the upward-sloping is absent indicating a lack of long-run relationship.

In summary, the economic links between two markets suggest the existence of a long-

[11]Using the Augmented Dickey-Fuller test, the null hypothesis of non-stationarity could not be rejected in the levels but can be in the first difference for all futures prices of both commodities. This result is standard in the literature and it is not shown here.

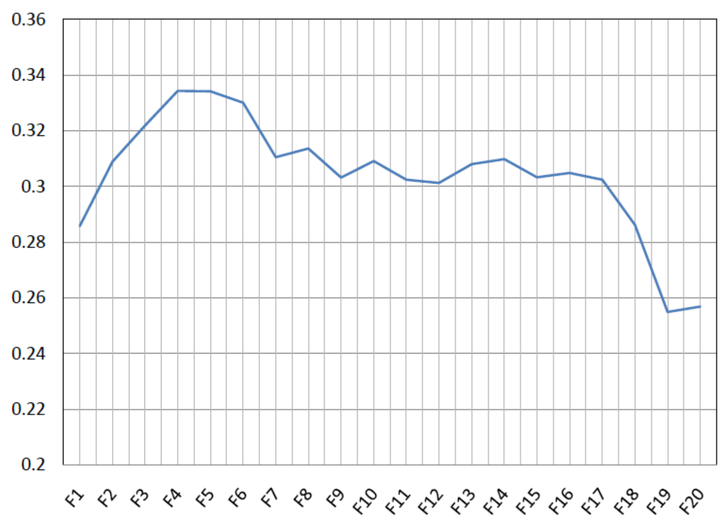


Figure 2.3. The Correlation Term Structure of HH N. Gas and WTI Crude Oil Futures

relationship between the two prices but the empirical evidence is weak especially if one incorporates the recent divergence in the two price series. Accordingly, in modeling the dynamics of the two prices, two models are proposed, one incorporates a long-run link between the two markets while the other has no such link.

2.4 Modeling the Dynamics of Natural Gas and Crude Oil Prices

The models presented in this section can be seen as an extension to the Schwartz and Smith (2000) model. Denote $S_{1,t}$ and $S_{2,t}$ to be the time t spot price of one unit of crude oil and natural gas respectively. Assume that the spot price of both commodities is decomposed

Table 2.3. Johansen's Maximum-Likelihood Tests of Co-Integration

	Null ($r \equiv$ No. of Cointegrations)	Alternative	Statistic	Prob.**
		F01		
Trace	$r = 0$	$r > 1$	28.272	0.025
	$r \leq 1$	$r > 1$	10.466	0.108
Max-Eigen	$r = 0$	$r = 1$	17.807	0.084
	$r = 1$	$r = 2$	10.466	0.108
		F04		
Trace	$r = 0$	$r > 1$	24.159	0.081
	$r \leq 1$	$r > 1$	8.203	0.236
Max-Eigen	$r = 0$	$r = 1$	15.955	0.147
	$r = 1$	$r = 2$	8.203	0.236
		F07		
Trace	$r = 0$	$r > 1$	20.4042	0.2062
	$r \leq 1$	$r > 1$	5.0447	0.5900
Max-Eigen	$r = 0$	$r = 1$	15.359	0.175
	$r = 1$	$r = 2$	5.045	0.590
		F10		
Trace	$r = 0$	$r > 1$	19.130	0.273
	$r \leq 1$	$r > 1$	3.226	0.849
Max-Eigen	$r = 0$	$r = 1$	15.904	0.149
	$r = 1$	$r = 2$	3.226	0.849
		F15		
Trace	$r = 0$	$r > 1$	17.866	0.353
	$r \leq 1$	$r > 1$	4.119	0.725
Max-Eigen	$r = 0$	$r = 1$	13.747	0.272
	$r = 1$	$r = 2$	4.119	0.725

The sample is from 3/20/1995 to 8/02/2010 (803 observations). A linear deterministic trend is included in the VAR system with maximum lag interval of 10 and the optimal lag is chosen by AIC.

into three components as following[12]:

$$Log(S_{i,t}) = X_{i,t} + x_{i,t} + g_i(t), \quad i = 1, 2, \tag{2.1}$$

where:

$X_{i,t}$ is a non-stationary stochastic process corresponding to the long-run movement in the price of commodity i,

$x_{i,t}$ is a mean-reverting stochastic process. It accounts for the short-term variations in the price of commodity i around its long-run component, and

$g_i(t)$ is a deterministic function corresponding to the seasonal movement in the price of commodity i. It will be specified later.

In specifying the stochastic behavior of the long-run and the short-run components, two specifications are considered. I will denote them as *Model I* and *Model II* respectively.

Model I

In this model, the behavior of the long-run and the short-run stochastic components, $X_{i,t}$ and $x_{i,t}$ respectively, is given by the the following stochastic differential equations under the physical measure:

[12] Given this choice of modeling, the oil price behavior becomes exogenous to the oil sand industry. This is not unreasonable because the impact of oil extraction from oil sands on the price of oil is negligible. Oil prices have been increasing recently even with the rise of the supply from oil sands industry, which reflects the fact that oil sands supply is not yet to affect on oil prices.

$$Log(S_{i,t}) = X_{i,t} + x_{i,t} + g_i(t), \quad \text{i=1,2,}$$

$$\begin{cases} dX_{1,t} = & \mu_1 dt + \sigma_1 dW_{1,t} \\ dX_{2,t} = & \mu_2 dt + \sigma_2 dW_{2,t} \\ dx_{1,t} = & -\kappa_1 x_{1,t} dt + \gamma_1 dZ_{1,t} \\ dx_{2,t} = & -\kappa_2 x_{2,t} dt + \gamma_2 dZ_{2,t}. \end{cases} \tag{2.2}$$

where μ_i denotes the rate of growth of the long-run component of commodity i, σ_i denotes the volatility of the long-run component of the price of commodity i, κ_i denotes the speed of mean reversion in the short-run component of the price of commodity i, γ_i denotes the volatility of the short-run component of the price of commodity i, and $dW_{i,t}$ and $dZ_{i,t}$ are four possibly correlated increments of Brownian motions.

The system can be written in the following matrix form:

$$d\mathbf{Y}_t = (\mathbf{M} + \mathbf{\Psi}\mathbf{Y}_t)dt + \mathbf{\Sigma} d\mathbf{B}_t, \tag{2.3}$$

where:

$$\mathbf{Y}_t = \begin{bmatrix} X_1 \\ X_2 \\ x_1 \\ x_2 \end{bmatrix}, \mathbf{M} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ 0 \\ 0 \end{bmatrix}, \mathbf{\Psi} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\kappa_1 & 0 \\ 0 & 0 & 0 & -\kappa_2 \end{bmatrix}, \mathbf{\Sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & \gamma_1 & 0 \\ 0 & 0 & 0 & \gamma_2 \end{bmatrix}$$

and

$$\mathbf{B}_t = \begin{bmatrix} W_{1,t} \\ W_{2,t} \\ Z_{1,t} \\ Z_{2,t} \end{bmatrix}.$$

In this model, the co-movement between the two commodity prices is only captured through the correlation structure of the Brownian motions increments.

Model II

In this model, motivated by the rule of thumb used in the natural gas market, we let the long-run component of the natural gas price depend on its deviation from the long-run component of the crude oil price as follows:

$$Log(S_{i,t}) = X_{i,t} + x_{i,t} + g_i(t), \quad i = 1, 2,$$

$$\begin{cases} dX_{1,t} = & \mu_1 dt + \sigma_1 dW_{1,t} \\ dX_{2,t} = & \alpha(X_{1,t} - X_{2,t} - \chi)dt + \sigma_2 dW_{2,t} \\ dx_{1,t} = & -\kappa_1 x_{1,t} dt + \gamma_1 dZ_{1,t} \\ dx_{2,t} = & -\kappa_2 x_{2,t} dt + \gamma_2 dZ_{2,t} \end{cases} \tag{2.4}$$

In this specification, the long-run component of natural gas reverts to a level of $e^{-\chi}$ from the long-run component of crude oil price. The parameter χ dictates the equilibrium ratio between the two long-run prices. That is, in equilibrium, $S_{2,t} = e^{-\chi} \cdot S_{1,t}$. Temporary deviation from this long-run ratio (because of demand and supply imbalances caused by macro-economic factors and inventory shocks, etc.) will be corrected over the long-run.

Note that the long-run component of oil price, $X_{1,t}$, is assumed not to depend on the price of natural gas. This reflects the empirical result that crude oil prices are determined internationally while natural gas prices are determined regionally (see Villar and Joutz (2006) and Mohammadi (2009)).

The matrix form for this model is the same as equation (2.3) except that the vector $\mathbf{M}$ and the matrix $\mathbf{\Psi}$ are defined as follows:

$$\mathbf{M} = \begin{bmatrix} \mu_1 \\ -\alpha \cdot \chi \\ 0 \\ 0 \end{bmatrix} \text{ and } \mathbf{\Psi} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \alpha & -\alpha & 0 & 0 \\ 0 & 0 & -\kappa_1 & 0 \\ 0 & 0 & 0 & -\kappa_2 \end{bmatrix}$$

2.4.1 Seasonality

The third component, $g_i(t)$ corresponds to the seasonal movement in the price of commodity i. Following Harvey (1989), $g_i(t)$ is modeled by trigonometric functions of the form:

$$g_i(t) = A_i sin(2\pi ft) + B_i cos(2\pi ft) \tag{2.5}$$

where A_i and B_i are constants correspond to the size of the seasonality effect and f is the frequency of the seasonality per year[13]

[13]Trigonometric functions for seasonality are well known in natural gas derivatives pricing, examples are: Xu (2004), Casassus et al. (2010) and Chen and Forsyth (2010).

2.4.2 Futures Pricing

Denote the futures price at time t for one unit of commodity i delivered in τ period by $F_{i,t}(\tau, \mathbf{Y_t})$, where $\mathbf{Y_t}$ is the vector of the risk factors that affect the price of commodity i as specified above. For derivative pricing, one should specify the stochastic processes in the risk neutral measure denoted as Q measure[14]. To achieve that, constant market prices of risk are assumed and the change of measure is thus of the following form:

$$d\mathbf{B_t}^Q = d\mathbf{B_t} + \mathbf{\Lambda}\Sigma^{-1}dt \tag{2.6}$$

where $\mathbf{\Lambda}$ is 4 by 1 vector of constant market prices of risk. That is, $\mathbf{\Lambda} = [\lambda_{X_1} \;\; \lambda_{X_2} \;\; \lambda_{x_1} \;\; \lambda_{x_2}]^\top$, where λ_j is the market price of risk associated with the process j.

Therefore, the dynamics of the state vector under the risk-neutral measure would be:

$$d\mathbf{Y_t} = (\mathbf{M^Q} + \mathbf{\Psi}\mathbf{Y_t})dt + \mathbf{\Sigma} d\mathbf{B_t^Q} \tag{2.7}$$

where

$$\mathbf{M^Q} = \mathbf{M} - \mathbf{\Lambda} = \begin{bmatrix} \mu_1 - \lambda_{X_1} \\ \mu_2 - \lambda_{X_2} \\ -\lambda_{x_1} \\ -\lambda_{x_2} \end{bmatrix} = \begin{bmatrix} \mu_1^Q \\ \mu_2^Q \\ -\lambda_{x_1} \\ -\lambda_{x_2} \end{bmatrix}$$

[14] The risk neutral measure, as opposed to the physical measure, is the measure implied by the market prices of the derivative contracts. This measure adjusts for the risk as market participants adjust for risk when they set the derivative prices. Details on deriving the risk neutral process for the purpose of derivative pricing can be found in Björk (2003).

for *Model I*, and

$$\mathbf{M^Q} = \mathbf{M} - \mathbf{\Lambda} = \begin{bmatrix} \mu_1 - \lambda_{X_1} \\ \alpha \cdot \chi - \lambda_{X_2} \\ -\lambda_{x_1} \\ -\lambda_{x_2} \end{bmatrix} = \begin{bmatrix} \mu_1^Q \\ -\alpha \cdot \chi^Q \\ -\lambda_{x_1} \\ -\lambda_{x_2} \end{bmatrix}$$

for *Model II*

From Björk and Landen (2000), the futures price of commodity i at time t for delivery in τ periods, $F_{i,t}(\tau)$, should satisfy the following partial differential equation (PDE):

$$-\frac{\partial F_{i,t}(\tau)}{\partial \tau} + \frac{\partial F_{i,t}(\tau)^\top}{\partial \mathbf{Y}_t}(\mathbf{M}^Q + \mathbf{\Psi})\mathbf{Y}_t + \frac{1}{2}Tr\left(\frac{\partial^2 F_{i,t}(\tau)}{\partial \mathbf{Y}_t^2}\mathbf{\Omega}\right) = \mathbf{0} \tag{2.8}$$

where $Tr(\cdot)$ is the matrix trace and $\mathbf{\Omega}$ is the covariance matrix of $\mathbf{\Sigma} d\mathbf{B}$ which is then given by:

$$\mathbf{\Omega} = dt \cdot \begin{bmatrix} \sigma_1^2 & \rho_{w_1w_2}\sigma_1\sigma_2 & \rho_{w_1z_1}\sigma_1\gamma_1 & \rho_{w_1z_2}\sigma_1\gamma_2 \\ \rho_{w_1w_2}\sigma_1\sigma_2 & \sigma_2^2 & \rho_{w_2z_1}\sigma_2\gamma_1 & \rho_{w_2z_2}\sigma_2\gamma_2 \\ \rho_{w_1z_1}\sigma_1\gamma_1 & \rho_{w_2z_1}\sigma_2\gamma_1 & \gamma_1^2 & \rho_{z_1z_2}\gamma_1\gamma_2 \\ \rho_{w_1z_2}\sigma_1\gamma_2 & \rho_{w_2z_2}\sigma_2\gamma_2 & \rho_{z_1z_2}\gamma_1\gamma_2 & \gamma_2^2 \end{bmatrix},$$

where $\rho_{i,j}$ denote to the instantaneous correlation between Brownian motions i and j.

The spot price of the commodity i at time t can be seen as the futures price at time t for immediate delivery (i.e. $\tau = 0$). Thus, the above PDE has the following boundary condition:

$$\text{Log}\big(F_{i,T}(0, \mathbf{Y_T})\big) = X_{i,T} + x_{i,T} + g(T). \tag{2.9}$$

Since the two models are in the affine framework, the solution of the above PDE has the following form as shown by Dai and Singleton (2000) and Tian (2003):

$$\mathrm{Log}\big(\mathbf{F}_{i,t}(\tau, \mathbf{Y}_t)\big) = \alpha_i(t,\tau) + \beta_i(t,\tau)\mathbf{Y}_t, \tag{2.10}$$

where $\alpha(t,\tau)$ and $\beta(t,\tau)$ solve the following system of ordinary differential equations:

$$\begin{aligned} -\frac{\partial \alpha_i(t,\tau)}{\partial \tau} + \beta_i(t,\tau)\mathbf{M}^Q + \frac{1}{2}\beta(t,\tau)\mathbf{\Omega}\beta(t,\tau)^\top &= 0 && (2.11)\\ \frac{\partial \beta(t,\tau)}{\partial \tau} - \beta(t,\tau)\mathbf{\Psi} &= 0, && (2.12) \end{aligned}$$

with boundary conditions:

$$\begin{aligned} \alpha_i(T,0) &= g_i(T) && (2.13)\\ \beta_i(T,0) &= [1\ 0\ 1\ 0] \ \text{ if } \ i=1 && (2.14)\\ &= [0\ 1\ 0\ 1] \ \text{ if } \ i=2. && (2.15) \end{aligned}$$

Integrating (2.12) and then plug it into (2.11), one gets:

$$\begin{aligned} \beta_i(t,\tau) &= \beta_i(T,0)e^{(\mathbf{\Psi}\tau)} && (2.16)\\ \alpha_i(t,\tau) &= g_i(T) + \int_0^\tau \left(\beta_i(t,u)\mathbf{M}^Q + \frac{1}{2}\beta(t,u)\mathbf{\Omega}\beta(t,u)^\top\right) du. && (2.17) \end{aligned}$$

Thus, the futures prices for *Model I* is given by:

$$\begin{aligned} \mathrm{Log}(F_{i,t}(\tau)) &= g_i(T) + X_i + x_i e^{\kappa_i \tau} + \mu_i^Q \tau + \frac{1}{2}\sigma_i^2 \tau \\ &\quad + \left(\frac{\lambda_{x_i}}{\kappa_i} - \frac{\sigma_i \gamma_i \rho_{w_i z_i}}{\kappa_i}\right)\left(e^{-\kappa_i \tau} - 1\right) - \frac{\gamma_i^2}{4\kappa_i}(e^{-2\kappa_i \tau} - 1), && (2.18) \end{aligned}$$

where $i = 1, 2$.

For *Model II*, while the futures price for crude oil ($i = 1$) is the same as that of *Model I*, the natural gas futures ($i = 2$) is given by:

$$
\begin{aligned}
\mathrm{Log}(F_{2,t}(\tau)) \;\; = \;\; & g_2(T) + X_1 + \chi + \frac{1}{2}\sigma_1^2\tau + \mu_1^Q\tau + (X_2 - X_1 - \chi)e^{-\alpha\tau} + x_2 e^{-\kappa_2\tau} \\
& + \left(\frac{\lambda_{x_2}}{\kappa_2} - \frac{\rho_{w_1 z_2}\sigma_1\gamma_2}{\kappa_2}\right)\left(e^{-\kappa_2\tau} - 1\right) - \frac{\gamma_2^2}{4k_2}(e^{-2k_2\tau} - 1) \\
& + \left(\frac{\mu_1^Q}{\alpha} + \frac{\sigma_1^2}{2\alpha} + \frac{\rho_{w_1 w_2}\sigma_1\sigma_2}{\alpha}\right)(e^{-\alpha\tau} - 1) \\
& + \left(-\frac{\sigma_2^2}{4\alpha} - \frac{\rho_{w_1 w_2}\sigma_1\sigma_2}{2\alpha}\right)(e^{-2\alpha\tau} - 1) \\
& + \left(\frac{\rho_{w_1 z_2}\sigma_1\gamma_2}{\alpha + \kappa_2} - \frac{\rho_{w_2 z_2}\sigma_2\gamma_2}{\alpha + k_2}\right)(e^{-(\alpha+\kappa_2)\tau} - 1).
\end{aligned}
\qquad (2.19)
$$

2.4.3 Estimation Procedure

The two models can be estimated using quasi maximum likelihood through the Kalman filter. The state space form is the appropriate procedure to deal with situations in which the state variables are not observable, but are known to be generated by a Markov process, as is the case in this chapter. Once a model has been cast in state space form, the Kalman filter may be applied to estimate the parameters of the model and the time series of the unobservable state variables. The Kalman filter is a recursive procedure for computing the optimal estimator of the state vector at time t, based on the information available at time t, and it enables the estimate of the state vector to be continuously updated as new information becomes available. When the disturbances and the initial state vector are normally distributed, the Kalman filter enables the likelihood function to be calculated, which allows for the estimation of any unknown parameters of the model and provides the basis for statistical testing and model specification. For a detailed discussion of state space models and the Kalman filter see Chapter 3 in Harvey (1989).

To cast the models in the state space form, one needs to specify the *transition equation* that governs the dynamic of the state variables and the *measurement equation* that relates the observable variables to the state variables.

The transition equation can be deduced from Equations (3) to get:

$$\mathbf{Y}_{t+\Delta t} = (\mathbf{\Psi}\Delta t + \mathbf{I})\, Y_t + \mathbf{M}\Delta t + \mathbf{e}_{t+\Delta t}, \quad \mathbf{e}_{t+\Delta t} \sim N(0, \mathbf{\Omega}\Delta t). \tag{2.20}$$

At each time, a vector of (log) future prices of both commodities for different maturities is observed. Assuming that these prices are observed with measurement error (these errors may be caused by bid-ask spreads, the non-simultaneity of the observations, etc. see Schwartz (1997)), the measurement equation will then be :

$$\begin{bmatrix} \mathrm{Log}\big(F_{1,t}(\tau_1)\big) \\ \mathrm{Log}\big(F_{1,t}(\tau_2)\big) \\ \vdots \\ \mathrm{Log}\big(F_{2,t}(\tau_1)\big) \\ \mathrm{Log}\big(F_{2,t}(\tau_2)\big) \\ \vdots \end{bmatrix} = \begin{bmatrix} \alpha_{1,t}(\tau_1) \\ \alpha_{1,t}(\tau_2) \\ \vdots \\ \alpha_{2,t}(\tau_1) \\ \alpha_{2,t}(\tau_2) \\ \vdots \end{bmatrix} + \begin{bmatrix} \beta_1(\tau_1) \\ \beta_1(\tau_2) \\ \vdots \\ \beta_2(\tau_1) \\ \beta_2(\tau_2) \\ \vdots \end{bmatrix} \mathbf{Y}_t + \omega_t, \quad \omega_t \sim N(0, \upsilon^2 \mathbf{I}) \tag{2.21}$$

$$= \mathbf{A} + \mathbf{B}\mathbf{Y}_t + \omega_t. \tag{2.22}$$

where ω_t represents the measurement errors in the futures prices.

2.5 Oil Sands Valuation Model

In this section, Brennan and Schwartz (1985) modeling procedure is extended to account for a stochastic extraction cost. Consider a competitive firm that operates an oil sands project to extract bitumen from known inventory of Q units. The project is assumed to be currently operating which means that initial cost to build the facility is sunk.

When the project is operating, the profit flow rate generated by selling the produced amount from t to $t + dt$ is given by:

$$\Pi_t = q_t\big(S_t - c_1 - v_t\big) - c_2 - \tau ax, \tag{2.23}$$

where q_t is the optimal rate of production in barrels per unit of time which is assumed to be known to the management, S_t is the price of one barrel of bitumen, c_1 is a deterministic variable cost, v_t is a stochastic variable cost, c_2 is the fixed cost and τax is the total taxes consisting of income tax plus royalties.

Since most bitumen is upgraded to crude oil, the dynamic of the bitumen price will imitate the dynamic of the crude oil price. Thus, the dynamic of S_t is given by the dynamic of $S_{1,t}$ specified in section 2.4.

In the case of the oil sands industry, v_t corresponds mainly to the cost of natural gas purchases in order to produce one unit of bitumen. Thus, v_t is governed by the dynamics of $S_{2,t}$ specified in section 2.4.

Depending on profitability, the decision-maker has the option to switch between different modes of operation. When the price drops low enough, the decision-maker can incur a fixed cost, K_{oc}, and suspend the operation until the price level goes back up to profitable levels. During suspension, the decision-maker should also incur a flow of maintenance cost,

M. If the price drops dramatically to very low levels, the decision-maker has the option to abandon the project permanently. On the other hand, if the project is currently closed and the price recovers to a profitable level, the decision-maker has the option to reopen the field again by paying another fixed cost of K_{co}. These options have value and option pricing theory can be used to find their values. Since the options are of the American type, their values are the the sum of all expected future cash flows from pursuing the optimal exercise policy discounted at the risk-free rate. The above project can be seen as an American option where the underlying assets are the price of crude oil and natural gas modeled in the last section. Thus, the value of the project is the sum of all expected future cash flows discounted at the risk free rate, provided that the optimal policy of switching between operation modes is pursued.

For a small time step of Δt, the value of the project is then be governed by the following two Bellman equations for currently open and closed projects respectively[15]:

$$\mathbf{V}_{open}\big(Y_t, Q, t\big) =$$

$$max \begin{cases} \Pi_t \Delta t + e^{-(r+\tau_o)\Delta t} E_t\big[\mathbf{V}_{open}(Y_{t+\Delta t}, Q - q\Delta t, t + \Delta t)\big] & \text{open} \\ -M\Delta t - K_{oc} + e^{-(r+\tau_c)\Delta t} E_t\big[\mathbf{V}_{closed}(Y_{t+\Delta t}, Q, t + \Delta t)\big] & \text{close} \\ 0 & \text{abandon} \end{cases} \tag{2.24}$$

$$\mathbf{V}_{closed}\big(Y_t, Q, t\big) =$$

$$max \begin{cases} \Pi_t \Delta t - K_{co} + e^{-(r+\tau_o)\Delta t} E_t\big[\mathbf{V}_{open}(Y_{t+\Delta t}, Q - q\Delta t, t + \Delta t)\big] & \text{re-open} \\ -M\Delta t + e^{-(r+\tau_c)\Delta t} E_t\big[\mathbf{V}_{closed}(Y_{t+\Delta t}, Q, t + \Delta t)\big] & \text{close} \\ 0 & \text{abandon,} \end{cases} \tag{2.25}$$

[15] In continuous time, and given that the prices are modeled without jumps, an open project can not be abandoned directly without being temporary closed. Thus, in continuous time setting, the third line in equation 2.24 should be dropped.

where τ_i , $i = o$ or c, is the property tax rate proportional to project value when it is open and when it is closed respectively. r is the risk free rate. As mentioned above, the expectations are taken under the risk-neutral measure.

Analytical solutions to equations (2.24) and (2.25) are unavailable, thus numerical methods should be used. Several numerical methods have been proposed for such problem. Among them are: finite difference and lattice methods. The Least Square Monte Carlo (LSMC) method developed by Longstaff and Schwartz (2001) has proved to be an efficient and simple tool for such problems (see Cortazar et al. (2008) and Tsekrekos et al. (2010)).

The LSMC procedure starts by simulating a large number of paths of Y_t from the current time to time T when the project is over. Then, backward recursion is carried out starting from time T up to the current time using the two Bellman equations stated above. The essence of the LSMC method is in the way it calculates the expectation of the project values in each simulated path at each time step. It achieves this task by path-wise regression of the project value at each node, on a linear combination of basis functions of the state variables at the same node across all paths. That is, the following regression is estimated at each time step for open and closed projects:

$$\mathbf{V}_{i,t+\Delta t}(\omega) = \sum_{j=1}^{N} a_j \Psi_j\big(\mathbf{Y}_t(\omega)\big) + error(\omega), \tag{2.26}$$

where $i =$ open or closed, ω is a simulated path, $\Psi_j(\cdot)$ is a set of N basis functions and a_j are their corresponding coefficients. Note that $\mathbf{V}_{i,t+\Delta t}(\omega)$ is known at time step t since we are moving backward.

The expectation of the project value at each ω is then approximated using the estimated parameters of a_j as follows:

$$E_t[\mathbf{V}_{i,t+\Delta}](\omega) = \sum_{j=1}^{N} \hat{a}_j \Psi_j(\mathbf{Y}_t(\omega)). \tag{2.27}$$

Although the choice of the basis functions is arbitrary, Tsekrekos et al. (2010) shows that the procedure is robust to different choices and that simple power functions are enough for reasonable results.

2.6 Data Description for Estimation and Simulation

Given the fact that most of extracted bitumen gets upgraded to crude oil, the bitumen price should move closely with the crude oil price. Thus, in estimating the parameters of the bitumen price process, we relied on the reported prices of the crude oil.

To estimate the parameters of the two models, we use weekly data of West Texas Intermediate (WTI) crude oil futures and Henry Hub (HH) natural gas futures. Both contracts are traded on the New York Mercantile Exchange (NYMEX). The WTI crude oil futures contract is for delivery at Cushing, Oklahoma and its price is considered a worldwide benchmark for crude oil prices. The HH natural gas futures contract is for delivery at Henry Hub, a natural gas pipeline located in Erath, Louisiana. Natural gas prices at Henry Hub are considered benchmarks for the entire North American natural gas market. The data consists of weekly futures prices for the period from the beginning of 1995 to the end of August 2010. The above data-set was obtained from Datastream.

To construct a continuous series of futures prices, following the literature, futures prices are sorted each week according to the contract horizon with "first month" contract being the contract with the earliest delivery date with futures price denoted as $F01$, the "second month" contract being the contract with the next earliest delivery date with futures price

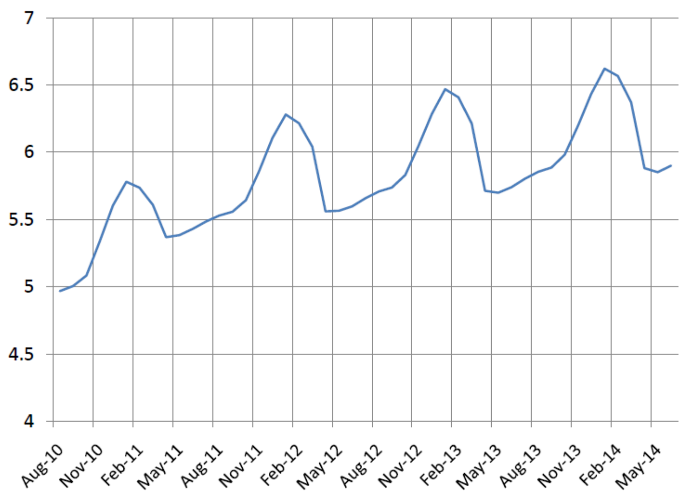


Figure 2.4. HH Natural Gas Future Curve at June 7th, 2010

denoted as $F02$, and so on. Since futures contracts have fixed delivery dates[16], the time to maturity changes as the time progresses. However, it remains within narrow range for each contract. For estimating the two models, the price of five futures contracts are used for both crude oil and natural gas processes which correspond to 1 month, 4 months, 7 months, 12 months and 15 months futures contracts.

Crude oil prices do not show seasonality, which is consistent with the literature on oil futures, such as Schwartz (1997). However, natural gas prices are well-documented to have strong seasonality as can be seen clearly from the forward curve in figure 2.4

Seasonality in natural gas prices results primarily from demand fluctuations driven by weather related factors. Cold winter results in above average consumption since natural gas is the main residential and commercial heating fuel. Thus, demand for natural gas

[16]For WTI, trading in the current delivery month ceases on the third business day prior to the twenty-fifth calendar day of the month preceding the delivery month. For natural gas, the trading of any delivery month ceases three business days prior to the first day of the delivery month. More details can be seen in http://www.cmegroup.com.

is typically high in winter and since storage facilities are limited, winter-maturing futures tend to be higher than those maturing in summer as is clear from figure 2.4. Since the seasonality is yearly, f in equation (2.5) is set equal to 1. For more details on the seasonal behavior of gas prices, see Xu (2004).

To accomplish the objective of this study, a hypothetical *in situ* oil sands project with a capacity of 5 million barrel per year is considered [17]. The decision-maker, for simplicity, is assumed to have four opportunities per year to switch between operating modes[18]. In the CERI 2009 report about supply cost in oil sands[19], variable cost is assumed to be \$6.8 per barrel, which is our estimate for c_1. In the same report, for a capacity of 30,000 barrels per day, the report estimated the annual average of the capital cost (excluding the initial cost of building the facility) to be 36.5 million dollars and the fixed operation costs to be 61.2 million dollars. Dividing the sum by the capacity assumed in the report and multiplying the result by \$5 millions barrel per year, the capacity assumed in this study, we get \$41 millions per year of fixed cost, our estimate for c_2. Maintaining the project while closed is assumed to be 10% of the fixed operating cost which is going to be around \$4 million per year. For simplicity, switching costs are assumed to be zero, i.e. the operator can switch to closed mode without incurring a cost. This implies that the value of the project is the same whether it is open or suspended as it is clear from the two bellman equations. The impact of switching costs on the value of a natural resource has been studied in Mason

[17] This choice coincides with some of the existing projects, see CERI 2008 report. Higher project capacities also exist, but considering them will be at the cost of the speed of simulation without much impact on the nature of the results.

[18] This assumption is used to make the size of the numerical calculation manageable. Increasing the switching opportunities frequency will increase the size of the working matrices exponentially. It certainly makes the values to be more precise but will not change the pattern of the results. This assumption has shown up in literature as well, for example: Cortazar et al. (2008) assumed only 3 opportunities to switch.

[19] See Table 3.1 in: McColl, Mei, Millington and Slagorsky "Canadian Oil Sands Supply Costs and Development Projects (2009-2043)" Canadian Energy Research Institute, November 2009

(2001)[20].

For tax parameters, CERI 2009 report assumption of constant income tax at nineteen percent (federal) and ten percent (provincial) is assumed. The royalties system of the oil sand industry relates the applicable royalties to the price of WTI crude oil and whether the project has reached its payout. Project payout would be said to have occurred when accumulated revenues first exceeded accumulated capital and operating expenditures. The rule is to apply a base royalties of 1% if WTI $\leq$ \$55, 9% if WTI $\geq$ \$120 with linear interpolation when WTI is in between until the project payout; thereafter, the royalties will be the greater of the base royalty or a net revenue royalty of 25% if WTI$\leq$ \$55 and 40% if WTI $\geq$ \$120 with linear interpolation when WTI is in between. To avoid adding additional state variables, we assume that the project is past the payout. Property tax is applied to the oil sands project at a rate of 1%. More details can be found in http://www.energy.alberta.ca. Finally, I follow CERI report in applying 2.5% rate of inflation. This rate of inflation is applied to the deterministic variable and fixed costs (c_1 and C_2) and the maintenance cost (M). Table 2.4 summaries these parameter that are used in simulation.

2.7 Results

Table 2.5 shows main descriptive statistics for the (log) returns of different futures prices of different delivery months for both commodities. It is clear from the table that the natural gas returns exhibit higher volatility than crude oil returns. The natural gas market is more sensitive to fluctuating factors such as inventory and weather related factors. Volatility of

[20]As Mason (2001) shows, greater switching costs cause firms to be less inclined to change status. However, non-zero switching costs will not change the way the project value and the optimal switching prices react to the dynamic of natural gas prices, the main focus of the paper.

Table 2.4. Hypothetical Oil Sands Project Characteristics

Parameter	Value
Maximum Project Life (T)	50 years
Deterministic variable Cost (c_2)	\$6.8 per barrel
Deterministic Fixed Cost (c_1)	\$41 millions per year
Maintenance Cost (M)	\$6 Millions per year
Production Rate (q)	5 Million Barrel per year
Income Tax and Royalties (τax)	Income Tax : 29%
	Royalties: maximum of (a) 1% if WTI $\leq$ \$55, 9% if WTI $\geq$ \$120 and linear interpolation in between of gross revenue and (b)25% if WTI $\leq$ \$55, 40% if WTI $\geq$ \$120 and linear interpolation in between of net revenue
Switching Costs (K_{oc} and k_{co})	Assumed 0 for simplicity
Property Taxes (τ_o and τ_c)	1%
Inflation Rate	2.5%

both commodities declines with maturity; an observation known in futures literature as Samuelson's effect. The table also shows that the distributions of all returns are skewed to the left and exhibit high kurtosis.

2.7.1 Estimation Results

Table 2.6 shows the results of the Kalman filter quasi maximum likelihood estimation. Most of the parameters in the two models are significant. In particular, the parameters of the long-run relationship in *Model II*, α and χ, are highly significant. By having one more parameter in *Model II*, the likelihood has increased by 185 units. In terms of the fitting error, Table 2.7 shows the mean error (ME) and the root squared mean error (RSME) of the five contracts used in estimation. For crude oil, the errors are almost the same for both

Table 2.5. Descriptive Statistics of Crude Oil and Natural Gas Log Returns

(Weekly data from January of 1995 to August of 2010 have been used)

	Crude Oil					Natural Gas				
	F01	F04	F07	F12	F15	F01	F04	F07	F12	F15
Mean	0.009	0.010	0.010	0.010	0.010	0.007	0.007	0.007	0.006	0.007
Std. Dev.	0.211	0.172	0.151	0.133	0.126	0.330	0.229	0.183	0.145	0.139
Skewness	-0.482	-0.510	-0.494	-0.417	-0.360	0.183	-0.104	-0.296	-0.207	-0.507
Kurtosis	5.149	5.025	5.128	5.374	5.501	4.844	3.543	4.296	4.671	4.983
Observations	813	813	813	813	813	813	813	813	813	813

models because the crude oil process has the same dynamics in both models. For natural gas, *Model II* does slightly better than *Model I.*

In *Model I* both markets have same long-run component volatility, σ_1 and σ_2, but volatility of the short-run component in gas market, γ_2, has almost double magnitude than that of oil market, γ_1. This is to due the fact that natural gas market is known to be very sensitive to weather-related and inventories factors. Moreover, higher κ_2 indicates that a shock to gas market will return faster than the case in oil market. The correlation between the long-run components of the two markets, $\rho_{w_1 w_2}$, is 0.48 indicating a significant co-movement in the two markets.

In *Model II*, both α and χ are significant. Given the values of χ^Q and χ, the equilibrium ratio of the gas price to the crude oil one turns to be 1 to $e^{\chi^Q} = e^{2.359} = 10.581$ in the risk neutral measure and 1 to $e^{\chi} = e^{1.853} = 6.400$ in the true measure. That is, market participants in the natural gas market adjust for risk by setting the equilibrium ratio to be higher than what is seen historically. The speed at which the gas price reverts to this equilibrium ratio from oil is very slow, $\alpha = 0.2257$. This might explain the difficulty of rejecting the null hypothesis of no co-integration between the two markets as shown in section 2.3.

Table 2.6. The Kalman Filter Quasi Maximum Likelihood Estimation

	Model I		***Model II***	
Parameter	**Value**	**SE**	**Value**	**SE**
μ_1	0.0882	0.0547	0.0978	0.0550
μ_2	0.0564	0.0501		
μ_1^Q	-0.0501	0.0091	-0.0479	0.0094
μ_2^Q	-0.0660	0.0026		
λ_{x_1}	-0.0229	0.0443	-0.0154	0.0475
λ_{x_2}	-0.2215	0.1057	-0.1300	0.1104
κ_1	1.3075	0.0859	1.1802	0.0708
κ_2	2.0084	0.0235	2.4984	0.0326
σ_1	0.1796	0.0058	0.1758	0.0060
σ_2	0.1797	0.0064	0.2302	0.0076
γ_1	0.2359	0.0130	0.2466	0.0133
γ_2	0.5001	0.0151	0.4915	0.0151
$\rho_{w_1w_2}$	0.4797	0.0687	0.2991	0.0776
$\rho_{w_1z_1}$	0.3507	0.0839	0.2969	0.0879
$\rho_{w_1z_2}$	0.1784	0.0740	0.2369	0.0800
$\rho_{w_2z_1}$	0.2785	0.0787	0.2711	0.0747
$\rho_{w_2z_2}$	0.3086	0.0588	0.3068	0.0582
$\rho_{z_1z_2}$	0.2556	0.0777	0.2073	0.0789
υ	0.030	0.0005	0.0292	0.0005
A_2	0.0647	0.0006	0.0650	0.0006
B_2	0.0219	0.0005	0.0223	0.0005
α			0.2257	0.0038
χ			1.853	0.2850
χ^Q			2.359	0.0364
No. of Obs.	812		812	
LL	14643		14828	

data from January of 1995 to August of 2010 have been used. The price of 1 month, 4 months, 7 months, 12 months and 15 months futures contracts have been used from both markets in estimation. LL is the logarithm of the likelihood evaluated at the estimated values of the parameters.

Table 2.7. Fitting Error of *Model I* and *Model II*

	Mean Error(ME)				Root Mean Squared Error (RMSE)			
	Crude Oil		Natural Gas		Crude Oil		Natural Gas	
	Model I	Model II	Model I	Model II	Model I	Model II	Model I	Model II
F1	-0.0003	-0.0004	-0.0010	-0.0009	0.0183	0.0184	0.0285	0.0238
F4	0.0018	0.0018	0.0027	0.0018	0.0124	0.0123	0.0446	0.0432
F7	-0.0002	-0.0002	-0.0012	-0.0010	0.0099	0.0098	0.0393	0.0380
F12	-0.0013	-0.0013	-0.0013	-0.0006	0.0085	0.0087	0.0320	0.0316
F15	0.0003	0.0002	0.0007	0.0006	0.0124	0.0125	0.0294	0.0293
All	0.0003	0.0003	0.0000	-0.0001	0.0615	0.0616	0.1739	0.1659

The volatility of the long-run component of the natural gas price, σ_2, has risen from 0.18 in *Model I* to 0.23 in *Model II*. This is mainly because of the fact that the free movement of the gas price in *Model I* is restricted in *Model II* and this restriction increases the volatility. Moreover, in *Model II*, the correlation between the long-run components of the two commodities, $\rho_{w_1 w_2}$, dropped to almost half than its value in *Model I*. This is due to the fact that the link in the expected values in the *Model II* has captured some of the co-movement.

A clearer picture about the two models can be seen in their implied forward curves as shown in Figure 2.5. The forward curve is the graph of the futures prices as a function of their maturities. In the figure, the initial values of the long-run components, $X_{1,0}$ and $X_{2,0}$ is set to be log(60) and log(6) respectively and the short-run components of the two prices are set to zero. This means that two the prices are now in equilibrium ratio under the risk neutral measure. Note that crude oil curve in the graph is scaled down by factor of 10 to ease the comparison.

The slope of the natural gas forward curve in *Model I* is lower than that in *Model II*. This is because the slope of the forward curve in *Model II* converges to that of the crude oil

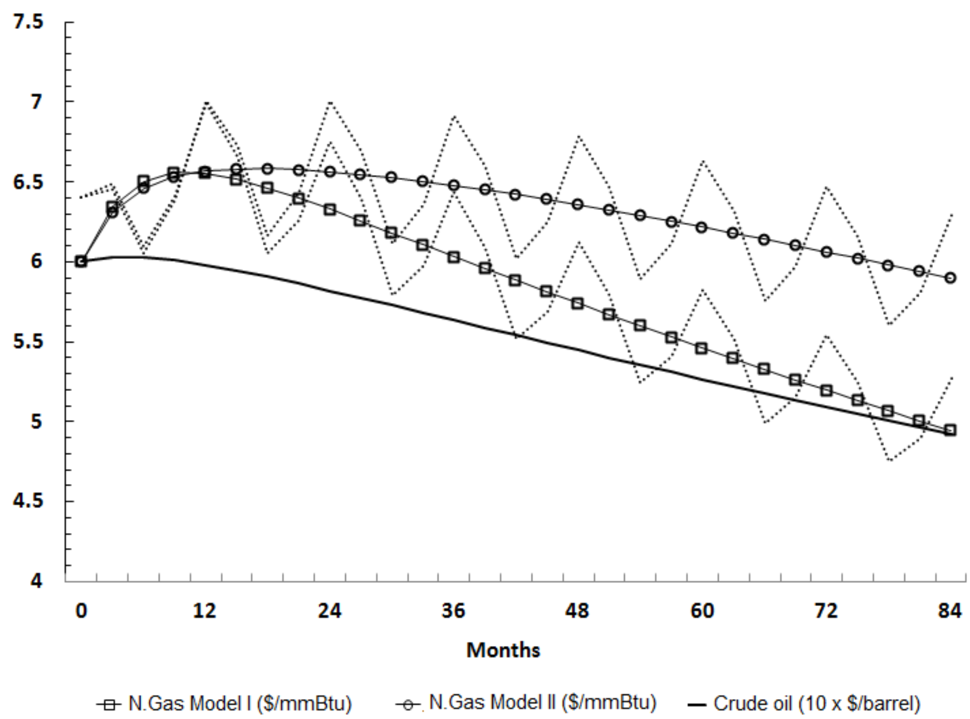


Figure 2.5. Implied Forward Curves

forward curve which is lower than the slope of natural gas in *Model I* given the estimated parameters. To see that, differentiate equations (2.18) and (2.19) with respect to τ and set τ to ∞. In *Model I*, the slope of natural gas curve in the long-run is then given by:

$$\mu_2^Q + \frac{1}{2}\sigma_2^2, \tag{2.28}$$

while in *Model II* it is:

$$\mu_1^Q + \frac{1}{2}\sigma_1^2. \tag{2.29}$$

That is, *Model II*, forward curve of natural price converges to that of the crude oil as one moves further along the forward curve.

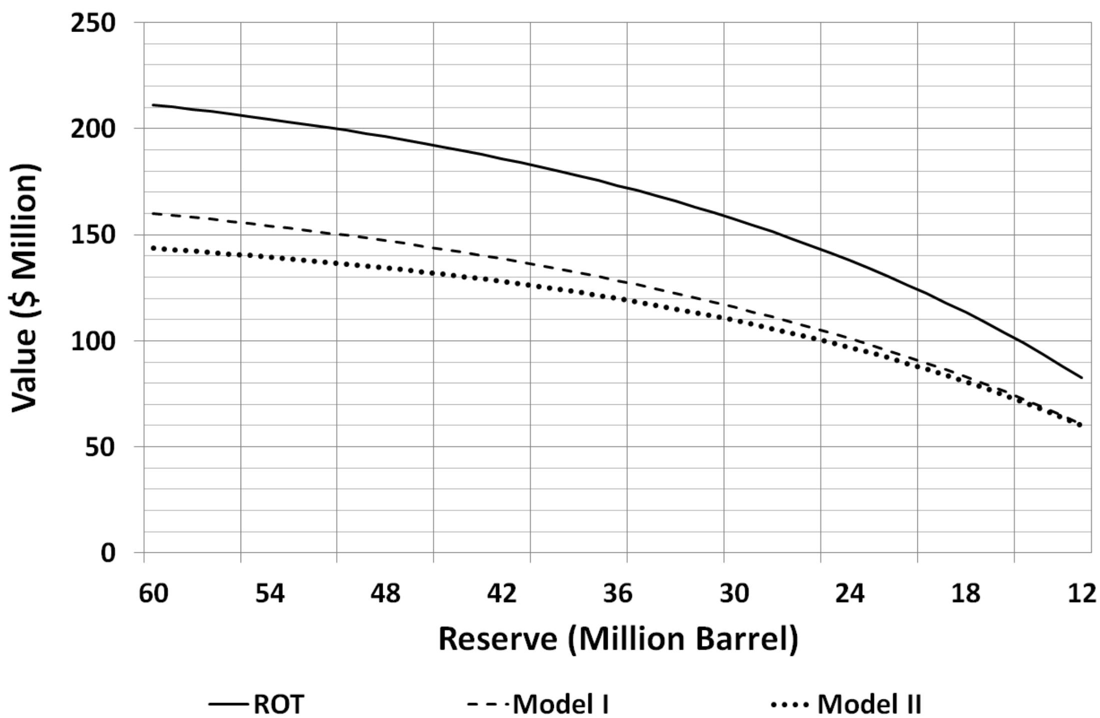


Figure 2.6. Oil Sand Project Value Under the Three Models

Moreover, in *Model II* a futures price of natural gas is a function of the deviation between natural gas and crude oil long-run components, $X_{2,t} - X_{1,t}$, as one can see from equation (2.19). If this deviation is higher (lower) than χ, the gas price is expected to move upward (downward) in the risk neutral measure. This has a significant impact on valuation, as shown in the next section, given the slow rate of convergence of the long-run component of the gas price to the equilibrium ratio from the long-run component of the oil price.

Table 2.8. Oil Sands Project Value (Values in Millions)

$S_0 = \$30$ (\$60 WTI), $v_0 = \$6$ and $Q = 60$ barrels.
$\hat{\sigma_2}$ and $\hat{\rho}_{w_1 w_2}$ are the estimated values in table 2.6.

	Model I				*Model II*			
	(1)	(2)	(3)	(2)-(3)	(4)	(5)	(6)	(5)-(6)
	$\hat{\sigma_2}$	$\sigma_2 = 0.3$	$\sigma_2 = 0.1$		$\hat{\sigma_2}$	$\sigma_2 = 0.3$	$\sigma_2 = 0.1$	
$\hat{\rho}_{w_1 w_2}$	160.16	144.52	168.26	23.74	143.65	139.37	150.46	11.09
0.2	165.74	155.52	170.96	15.44	145.41	141.71	151.22	9.52
0	169.58	162.41	172.96	10.55	148.82	146.41	152.68	6.27
-0.2	173.15	168.49	174.82	6.33	152.27	150.80	154.08	3.28

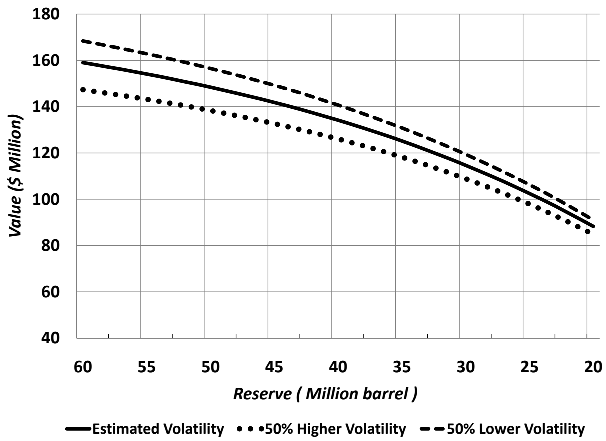
2.7.2 Valuation Results

Figure 2.6 shows the value of the hypothetical oil sands project as a function of the remaining reserve. The initial values of the long-run components are set to be \$30 for bitumen (around \$60 WTI) and \$6 for natural gas, which gives the equilibrium ratio under the risk neutral measure. The short-run components are set to zero. Moreover, I plot the value of the project under the pure 10:1 rule of thumb (ROT) commonly used by industry along with the value of the project under the two models[21]. It is clear from the graph that the rule of thumb significantly overestimates the value of the project. The reason is that the natural gas forward curve is much lower under the 10:1 rule of thumb than it is under both models as is clear from Figure 2.5.

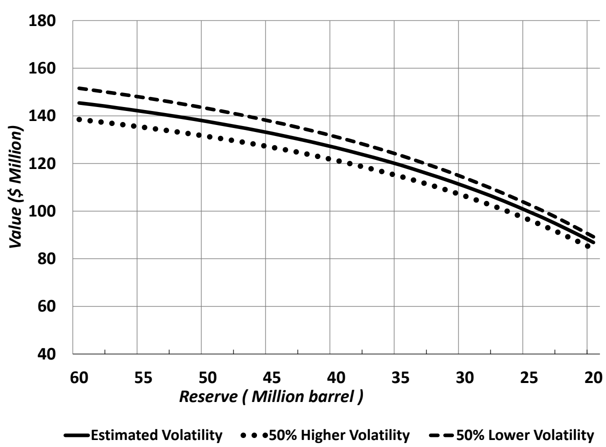
Comparing the value of the oil sands project under the two models, *Model II* gives slightly lower value than *Model I*. This is because the estimated value of σ_2 is higher in *Model II*(0.230) than its value in *Model I* (0.179) and this higher volatility lifts up the

[21]CERI has applied this rule of thumb in their report of 2009, see McColl, Mei, Millington and Slagorsky "Canadian Oil Sands Supply Costs and Development Projects (2009-2043)" Canadian Energy Research Institute, November 2009.

Figure 2.7. The Impact of N. Gas Long Term Component Volatility.
$X_{1,0} = Log(60)$, $X_{2,0} = Log(6)$ **and** $x_{1,0} = x_{2,0} = 0$



(a) *Model I*



(b) *Model II*

natural gas forward curve in *Model II* , as can be seen from equation (2.19). This shift in the forward curve reduces the expected cash flows and, in turn, the project value.

Figures 2.7 shows the impact of the long-run component volatility of the natural gas price on the value of the project in both models which is also shown in Table 2.8 at $Q = 60$. The graph and the table show that volatility has a higher impact under *Model I* than it has under *Model II*. The reason is that in *Model II* the natural gas price reverts after a shock to a mean value based on the price of crude oil. This restriction reduces the sensitivity of the gas price to volatility. As shown in equation (2.29), what matters for the forward curve of natural gas in the long-run is the volatility of the long-run component of crude oil, σ_1, not the volatility of natural gas long-run component, σ_2. Moreover, the same table shows that the value will drop by relatively more when volatility increases if the correlation between oil and gas is high. This is due the fact that higher correlation makes the gas price (and the cost) and oil price (and the revenue) to move more together which reduces the expected cash flows and then the current value.

Figure 2.8 shows the impact of different starting values for natural gas price which are different from the equilibrium values determined in relation to the crude oil price. It shows the value of the project under both models for different values of the long-run component of natural gas. The oil price is set at \$30 (\$60 WTI). At a gas price of \$6, which is around the equilibrium ratio under *Model II*, the two models give same values. When gas price is higher than \$6, the value under *Model II* is higher because gas price needs to adjust downward to the equilibrium ratio which makes future costs lower and project value higher. On the other hand, when gas prices are below \$6, *Model II* gives lower value than *Model I* because, in this case, gas price needs to adjust upward to its equilibrium ratio from oil price which makes future costs higher and project value lower.

Also of interest are critical prices at which it is optimal for the owner to switch from

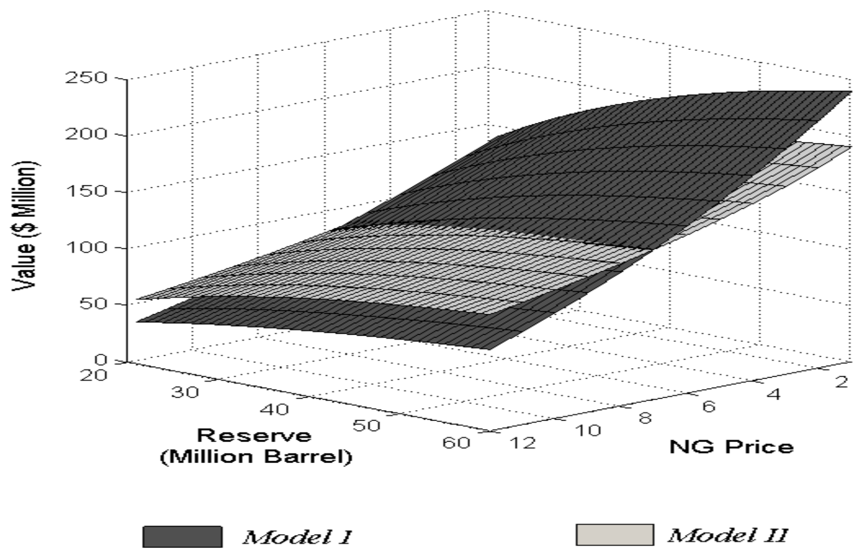


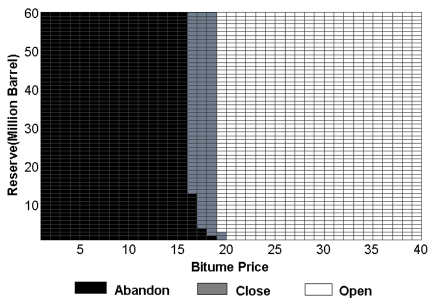
Figure 2.8. Value of the Project as a Function of N. Gas Long Term component. $X_{1,0} = Log(60)$ **and** $x_{1,0} = x_{2,0} = Log(0)$

being open to closed (i.e shutting down production), from closed to open (i.e resuming the production after a temporary shutdown) or abandoning the production. Figures from 2.9 to 2.12 shows these critical prices as a function for different values of the remaining reserve. These prices are determined using the Bellman equations (equations 2.24 and 2.25) as follows: the Bellman equations are solved for different values of the current price of oil. The prices that equate the value of the project in two adjacent modes (i.e from open to closed or from closed to abandon) will give the prices at which it is optimal to switch between these two modes.

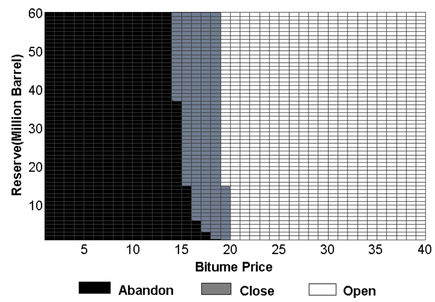
Figures 2.9 and 2.10 show that the critical switching prices are almost the same for different scenarios of the natural gas long-run component volatility, σ_2, and the correlation between the long-run components of the two commodities, $\rho_{w_1 w_2}$. The figures show that there is almost no impact of the dynamics of the natural gas long-run component on the

Figure 2.9. Impact of N. Gas Process on the Switching Prices Under *Model I*: Backwardation Case

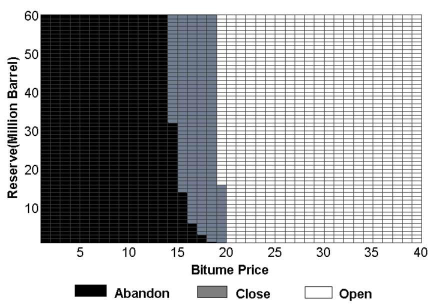
$\hat{\sigma}_2$ and $\hat{\rho}_{w_1 w_2}$ are the estimated values in Table 2.6



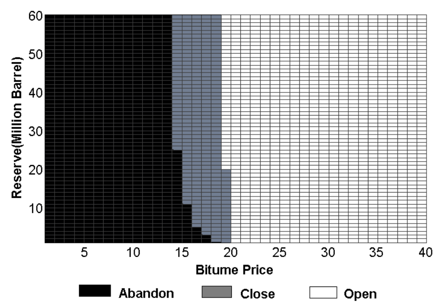
(a) $\sigma_2 = 1.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 1.5\hat{\rho}_{w_1 w_2}$



(b) $\sigma_2 = 1.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 0.5\hat{\rho}_{w_1 w_2}$



(c) $\sigma_2 = 0.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 1.5\hat{\rho}_{w_1 w_2}$



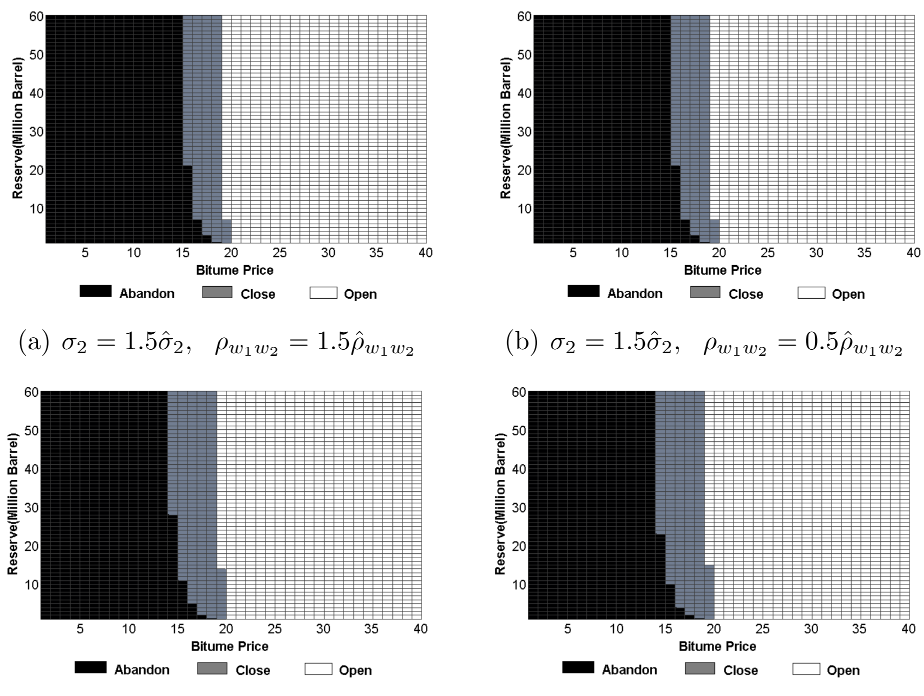
(d) $\sigma_2 = 0.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 0.5\hat{\rho}_{w_1 w_2}$

switching prices. The reason behind this absence is that, given the estimated parameters, crude oil prices are expected to fall in the risk neutral measure and since gas price is a small component of the total average cost, C_t, the impact of gas prices on the switching prices is dominating by the falling oil prices.

To gain more insight into the impact of stochastic cost on the switching prices and the optimal policy, I let the market for oil to be in contango, i.e the forward curve to be upward sloping. The contango situation has been observed in oil markets in 336 out of 820 weeks throughout the sample. Within the modeling of this chapter, contango in forward

Figure 2.10. Impact of N. Gas Process on the Switching Prices Under *Model II*: Backwardation Case

$\hat{\sigma}_2$ and $\hat{\rho}_{w_1 w_2}$ are the estimated values in Table 2.6



(a) $\sigma_2 = 1.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 1.5\hat{\rho}_{w_1 w_2}$

(b) $\sigma_2 = 1.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 0.5\hat{\rho}_{w_1 w_2}$

(c) $\sigma_2 = 0.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 1.5\hat{\rho}_{w_1 w_2}$

(d) $\sigma_2 = 0.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 0.5\hat{\rho}_{w_1 w_2}$

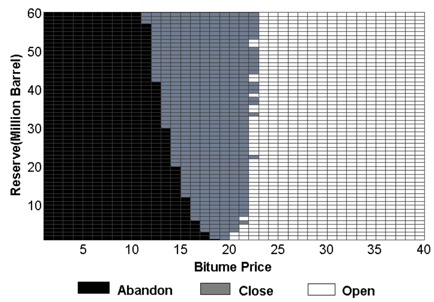
curve can be achieved by having higher μ_1 or higher σ_1. Since the estimated value for σ_1 is already high, I increase the value of μ_1. To make the analysis more valid, I apply the same increase in the rate of return of the gas long-run component in *model I*.

Figures 2.11 and 2.12 show the impact of stochastic cost on the optimal policy as a function of the oil price for different scenarios of the natural gas long-run component volatility, σ_2, and the correlation between the long-run components of the two commodities, $\rho_{w_1 w_2}$.

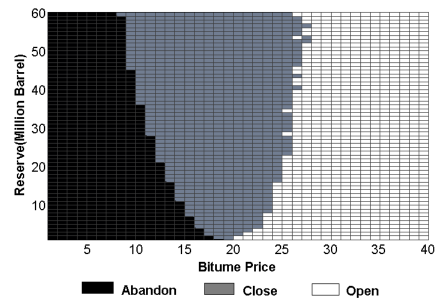
For *Model I*, as shown in figure 2.11, a higher σ_2 increases the slope of the forward

Figure 2.11. Impact of N. Gas Process on the Switching Prices Under *Model I*: Contango Case

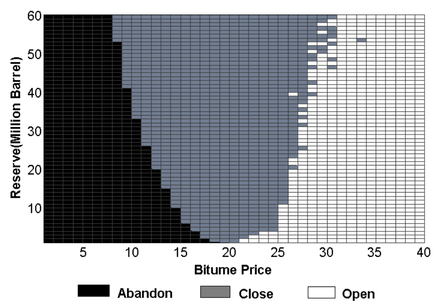
$\hat{\sigma}_2$ and $\hat{\rho}_{w_1 w_2}$ are the estimated values in Table 2.6



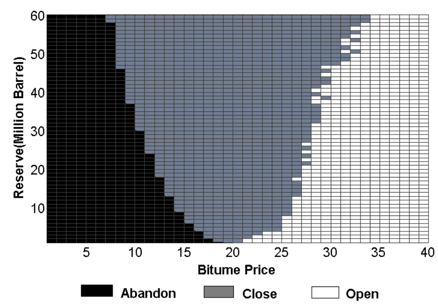
(a) $\sigma_2 = 1.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 1.5\hat{\rho}_{w_1 w_2}$



(b) $\sigma_2 = 1.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 0.5\hat{\rho}_{w_1 w_2}$



(c) $\sigma_2 = 0.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 1.5\hat{\rho}_{w_1 w_2}$



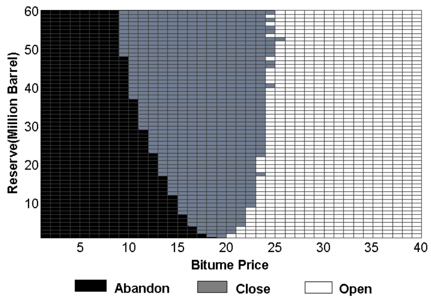
(d) $\sigma_2 = 0.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 0.5\hat{\rho}_{w_1 w_2}$

curve of natural gas and then reduces the expected future profit flows. The reverse is true too, a lower σ_2 decreases the slope of the forward curve and increases the expected future profit flows. Therefore, it is optimal to switch to open mode at lower prices when σ_2 is high, figures 2.11(a) and figures 2.11(b), than the case when it is low, 2.11(c) and figures 2.11(d).

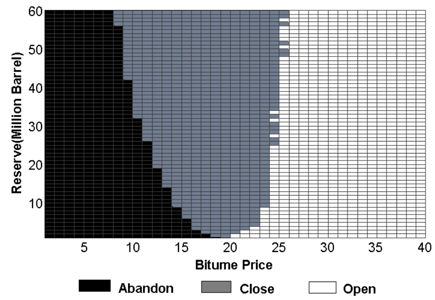
Turning to the impact of ρ_{w1w2}, the correlation between the long-run components of the two commodities, a higher ρ_{w1w2} reduces the future profit flows and the project value. The reverse is also true, a lower ρ_{w1w2} makes the two prices to move less together and

Figure 2.12. Impact of N. Gas Process on the Switching Prices Under *Model II*: Contango Case

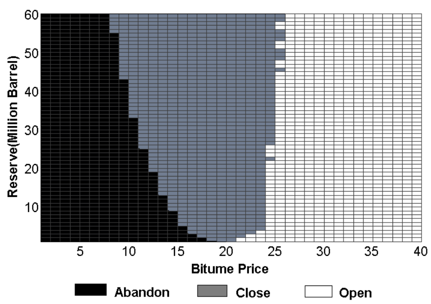
$\hat{\sigma}_2$ and $\hat{\rho}_{w_1 w_2}$ are the estimated values in Table 2.6



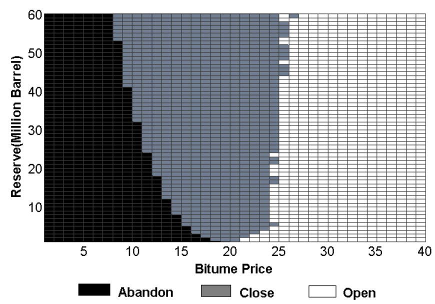
(a) $\sigma_2 = 1.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 1.5\hat{\rho}_{w_1 w_2}$



(b) $\sigma_2 = 1.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 0.5\hat{\rho}_{w_1 w_2}$



(c) $\sigma_2 = 0.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 1.5\hat{\rho}_{w_1 w_2}$



(d) $\sigma_2 = 0.5\hat{\sigma}_2, \quad \rho_{w_1 w_2} = 0.5\hat{\rho}_{w_1 w_2}$

this increases the future cash flows and then the project value. Therefore, it is optimal to switch to open mode at lower prices when ρ_{w1w2} is high than the case when it is low.

However, the case is different under *Model II* as shown in figure 2.12. The impact of the dynamics of natural gas is almost gone. The optimal switching prices is almost same under the four scenarios. This is due the fact that, under this model, natural gas price is tied to follow the crude oil price and this link makes the value of the project and the optimal policy less sensitive to the dynamic of natural gas prices.

2.8 Concluding Remarks

In this chapter, I study the impact of having a stochastic and a volatile component in the extraction cost on project valuation in the oil sand industry where a substantial amount of natural gas is used to produce oil. I show that a higher natural gas price volatility reduces the value of the project. The chapter also shows that not only the dynamics of oil and natural gas prices are important, but the co-movement of the two prices are also an important factor to take in consideration in valuation and optimal operation. While the economic links between the two markets, i.e being substitutes as sources of energy, suggests the existence of a long-run relationship between the two prices, the chapter shows that the empirical evidence is weak especially if one incorporates the recent divergence in the two price series. The valuation results show that incorporating a long-run relationship between the two markets is a very crucial decision in valuing an oil sands project and in determining its optimal operation. The chapter shows that ignoring this long-run relationship makes the optimal policy sensitive to the dynamic of natural gas prices. On the other hand, incorporating this long-run relationship makes the dynamic of natural gas price process have a vary low impact on valuation and the optimal operating policy.

Chapter 3

The Relationship Between Volatility and the Forward Curve in Crude Oil Markets

3.1 Introduction

Forward and futures contracts are fundamental tools for pricing and risk management in energy markets, as they are in most commodities and financial markets. A forward contract is an agreement between two parties to buy or sell an asset at a certain future time for a set price agreed on today. A futures contract has the same general features as a forward contract but is an exchange-traded contract, with certain standardized features. Understanding the dynamics of the relation between spot and futures (or forward) prices is crucial for a sound risk management analysis. For example, futures prices are usually used as an indication of which direction the spot prices will take in the future, and the

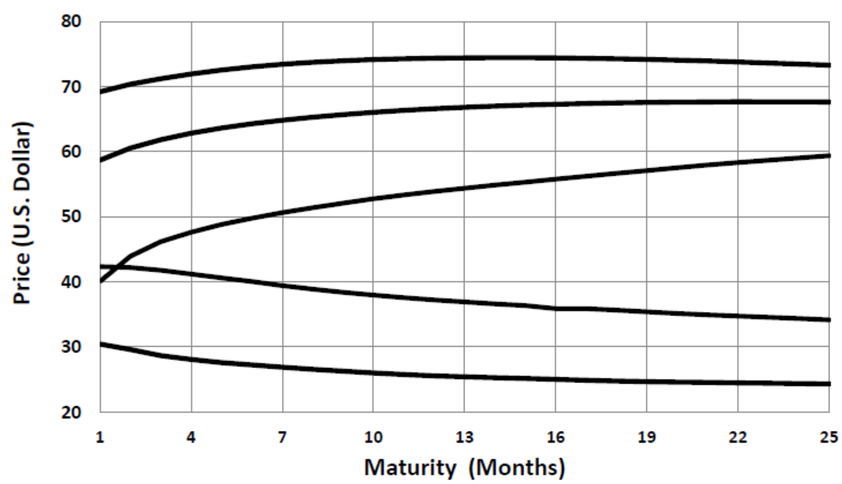


Figure 3.1. WTI Crude Oil Forward Curve at Different Points in Time

difference between spot and futures prices may give an indication about the volatility of future spot prices.

Among the aspects of the relationship between spot prices and futures (and forwards) prices is the term structure of the futures prices or simply the forward curve[1]. The forward curve is a term given to the plot of futures prices against their corresponding maturities at a specific time. Generally, the forward curve can takes many shapes. However, there are two main shapes that market participants usually pay attention to: a positively sloped forward curve which is known as contango and a negatively sloped forward curve which is known as backwardation. Figure 3.1 shows the forward curves of the West Texas Intermediate (WTI) crude oil futures prices at different points in time.

Many researchers have studied the relationship between the forward curve and the market volatility. Fama and French (1988) test the implication of the theory of storage on industrial metals and found that, as the theory predicts, volatility is high when the

[1]Both terms, the futures term structure and the forward curve will be used interchangeably.

slope of the curve is negative. Ng and Pirrong (1994) also found similar results for the daily prices of the industrial metals during the period from September 1986 to September 1992. Litzenberger and Rabinowitz (1995) found that, in the crude oil market, volatility is positively associated with the level of backwardation. On the other hand, Carlson et al. (2007) and Kogan et al. (2009) found that the relation between the slope of the forward curve and volatility is non-monotonic and has a V-shape. That is, the higher the volatility, the steeper the slope of the curve is regardless of whether it is positive or negative.

This chapter contributes to this strand of literature by investigating empirically the relationship between the slope of the forward curve and expected variances of the spot and futures prices and the covariance between them in the crude oil market using another measure of the slope extracted using principal component analysis (PCA). When the relation between the forward curve slope and volatility is studied in the literature, the commonly used measure of the slope is the spread between the spot and futures prices. However, there is no consensus in the literature as to what maturity date to use in calculating the spread series. For example, Ng and Pirrong (1994) and Carlson et al. (2007) used the spread of the third month futures price and the spot; Lien and Yang (2008) used the second month futures contract; and, Kogan et al. (2009) used spread between the sixth month and the third month futures prices. Borovkova (2006) shows that PCA can be used to find a very reasonable measure of the slope of the forward curve. PCA has been used in commodity future pricing to uncover the factors behind the movement of the forward curve (see for example Cortazar and Schwartz (1994)). A forward curve of practically any shape can be constructed by combining three simple shapes: the so-called level, slope and curvature. Mathematically, these three basic shapes correspond to the first three principal components of the array of the futures prices time series for all maturities.

Moreover, this chapter studies whether the relationship between the forward curve and volatility can be used to improve the hedging performance using futures contracts. Few researchers take account for the slope of forward curve in calculating the optimal hedge ratio. In metal prices, Ng and Pirrong (1994) estimated a bivariate GARCH when the slope of the forward curve, measured by the interest rate adjusted spread between spot and futures prices, is included as a regressor in their variance and covariance equations. They showed that the optimal hedging ratio is a function of the slope of the forward curve. However, they have not conducted any systematic performance analysis on their results. Recently, Lien and Yang (2008) estimated a similar model for a wide range of commodities, including crude oil, but allowed for the asymmetric effect of the slope on variances and covariances. They found that the in-sample and out-of-sample results both reveal that incorporating the asymmetry effect of the slope asymmetrically into the estimated volatility leads to better hedging in terms of risk reduction.

This chapter is organized as follows: section 3.2 is devoted to a review of the theoretical work on the forward curve and the implications of various theories of the relation between the slope of the forward curve and the market volatility. Section 3.3 shows how the slope measure is constructed using PCA. The following section is for specifying the model used to estimate the relation between volatilities and the forward curve slope. Data description, estimation methodology and the estimation results are shown is Sections 3.5 to 3.7. Application to minimum variance hedge ratio is shown in section 3.8. The last section is devoted to concluding remarks.

3.2 Theoretical Background

Many theories have been proposed to unveil the factors that govern the equilibrium relationship between spot and futures (or forward) prices and to explain the shape of forward curve. The following section is a review of the most popular theories: theory of storage, insurance perspective and hedging pressure theory, real option theory and exhaustible resources theories. Each theory is explained and the implication of the theory on the shape of forward curve is highlighted. Note that the terms forward and futures are used interchangeably here since the theories apply to both of them, although they are not the same in terms of pricing if the interest rate is assumed to be stochastic.

The theory of storage has two versions: the first one relies on the notion of convenience yield and was developed by Working (1949), Telser (1958) and Brennan (1958). This version adjusts the known free-arbitrage relationship between the spot price and forward price to account for a "convenience" yield received by the commodity inventory holders. Commodity inventory holders (whether they are producers, consumers or speculators) receive an implicit stream of benefits when they hold inventories of the commodity. This stream of benefits comes from the fact that they can respond flexibly and efficiently to supply and demand shocks. The theory posits that the marginal value of this yield declines as the total level of accumulated inventory increases. That is, the benefit of inventory holding is high when the total level of inventory in the economy is low. Consequently, the arbitrage relation between spot and forward prices should be adjusted to account for this yield. If $F_t(\tau)$ is the forward price at time t for delivery in τ periods, S_t is the spot price of the commodity at time t, w_t is the cost rate of storing the commodity as a percentage of the spot price, r_t is the interest rate for one period and I_t is the accumulated level of inventory at time t, then the arbitrage relation between spot and forward is given by:

$$F_t(\tau) = S_t e^{(r_t + w_t - c_t(I_t))\tau}, \tag{3.1}$$

where c_t is the marginal convenience yield for holding one unit of the commodity in storage and $c_t(I_t)$ is a decreasing function of I_t. Thus, when the accumulated inventory level is high, convenience yield becomes less than the interest rate plus the storage costs. In such cases, the forward price goes higher than the spot price and contango, an upward sloping curve, is observed. On the other hand, when the accumulated inventory level is low, convenience yield is greater than the interest rate plus the storage costs causing the forwards price to be lower than the spot price and backwardation, a downward sloping curve, is observed.

The second version of the theory of storage is due to Williams and Wright (1991) and Deaton and Laroque (1996) among others. Routledge et al. (2002) extends the theory to study the implied forward curve equilibrium. The theory relies on the fact that aggregate inventory levels can not be reduced to be less than zero which limits the trading ability of inventory holders. If Q_t is the amount that risk neutral inventory holders, according to their expectation of the future prices, hold in storage to be available at time $t + \tau$, then equilibrium price and inventory holding must satisfy:

$$E_t(S_{t+\tau}) = S_t e^{(r_t+w_t)\tau} \quad \text{if } Q_t > 0 \tag{3.2a}$$

$$E_t(S_{t+\tau}) \leq S_t e^{(r_t+w_t)\tau} \quad \text{if } Q_t = 0. \tag{3.2b}$$

Equations (3.2a) and (3.2b) must hold in the equilibrium because of the optimal trading of risk neutral inventory holders. That is, if the expected price is greater than the current price adjusted for interest rate and storage costs, inventory holders would add to their inventories, driving the current price up until equality is reached. On the other hand, if the expected price is less than current price adjusted for interest rate and storage costs, inventory holders would sell from their inventories, driving spot price down until equality is reached or inventory is stocked out.

Under risk neutrality, the forward price is equal to the expected future spot price.

Therefore

$$F_t(\tau) = S_t e^{(r_t + w_t)\tau} \quad \text{if } Q_t > 0 \tag{3.3a}$$

$$F_t(\tau) \leq S_t e^{(r_t + w_t)\tau} \quad \text{if } Q_t = 0. \tag{3.3b}$$

Thus, from the above equations, contango is observed in the market if aggregate inventory is positive and backwardation may be observed if aggregate inventory is reduced to zero.

In both versions of the theory, the slope of forward curve is a function of the inventory level. A higher level of inventory is associated with the curve being in upward sloping (contango) and a lower level of inventory is associated with the curve being in downward sloping (backwardation). Since the volatility of the commodity price is supposed to be a non-increasing function of the level of inventory, the theory also predicts a negative relationship between volatility and the slope of the curve (see Ng and Pirrong (1994)).

The theory of insurance perspective is due to Keynes (see Fama and French (1987), Bodie and Rosansky (1980), Kolb (1992) and Lautier (2005)). In this theory, producers of a commodity would hedge against the uncertainty in the spot prices in the future by entering into a forward contract to sell the commodity at a specified date and price. Speculators, on the other end of the contract, take the price risk and ask for a premium by setting the forward price less than the expected future spot price to compensate for the risk they bear. In other words, the theory predicts that

$$F_t(\tau) = E_t(S_{t+\tau}) - m_t(\tau) \tag{3.4}$$

where $m_t(\tau)$ is the risk premium due to accepting the risk of price fluctuation from t up to $t+\tau$. Rearranging terms in equation (3.4), one gets:

$$F_t(\tau) - S_t = E_t(S_{t+\tau} - S_t) - m_t(\tau). \tag{3.5}$$

If the market is efficient, i.e. the information about expected spot prices is (or at least mostly) reflected in the current spot price, the risk premium would dominate the difference between forward price and current spot price. Since the risk premium is higher for bearing the price risk for a longer time, i.e. $m_t(\tau)$ is an increasing function of τ, the market finds itself in backwardation.

Hedging pressure theory can be seen as a continuation of the insurance perspective theory (see Change (1985), Bessembinder (1992), Roon et al. (2000), Dinceler et al. (2003) and Gorton et al. (2007)). The assumption under the insurance perspective theory is that producers of the commodity dominate the hedging positions in forward or futures markets. In hedging pressure theory, the reverse might happen. In some markets, consumers of the commodity dominate the hedging positions of forward contracts. In this case, to induce risk-averse speculators, hedgers would offer a higher price than what is expected in the future. Thus, the risk premium, $m_t(\tau)$, would be negative and, under market efficiency, contango is observed in the market. In summary, the theory predicts that the net position of hedgers (the hedging pressure) determines the sign of the risk premium $m_t(\tau)$ and, in turn, the slope of the forward curve. Moreover, since higher volatility implies higher risk premium, the theory also predicts that a steeper slope is associated with higher volatility whether in backwardation or contango.

The third theory is **the real option theory** proposed by Litzenberger and Rabinowitz (1995). The authors distinguish between *strong* backwardation when futures prices are lower than the spot ones, i.e. $F_t(\tau) < S_t$, and *weak* backwardation when *discounted* futures prices are lower than the spot ones, i.e. $F_t(\tau)e^{-r\tau} < S_t$. They observed that between February 1984 and April 1992 around 90 percent of the time the oil forward curve was in weak backwardation and around 77 percent of time it was in strong backwardation. Accordingly, they proposed a theory of backwardation to resolve this empirical observation. They assume that oil reserves are owned by a continuum of price taking oil producers with heterogeneous extraction costs that rise at the rate of interest. Each oil reserve is characterized as a call option where the owner has the option to produce now and pay the cost of extraction (and get the payoff of price minus the extraction cost) or wait to produce in the next period (keep the option alive). Denote x^i as the marginal extraction cost at time $t+1$ for producer i and $C(k)$ as a call option where k is the strike price. Moreover, assume that extraction cost rises at the rate of interest (i.e $x_t^i = x^i e^{-r}$) then, the optimal rule for each producer is:

$$\begin{array}{lll} \text{Produce at time } t & \text{if} & S_t - x^i e^{-r} > C_t(x^i) \\ \text{Produce at time } t+1 & \text{if} & S_t - x^i e^{-r} < C_t(x^i) \\ \text{Indifferent} & \text{if} & S_t - x^i e^{-r} = C_t(x^i). \end{array}$$

The authors showed that there is one extraction cost, x^m, such that producers for whom their extraction costs are above this value will defer production to the next period and producers for whom their extraction costs are below this value will produce in the current period. Thus, part of the oil reserves will be produced at time t and part will be delayed for future production. In the equilibrium $S_t - x^m e^{-r} = C_t(x^m)$ should hold. Combining this equation with the put and call parity, $C_t(x^m) = P_t(x^m) + e^{-r}(F_{t,t+1} - x^m)$, one gets

$$\text{Degree of Weak Backwardation} = S_t - e^{-r}F_{t,t+1} = P_t(x^m). \tag{3.6}$$

That is, the degree of weak backwardation is equal to the value of a put option with an exercise price equal to the extraction cost of the marginal producer. Equation (3.6) implies that in order to have non-zero production, the market should be in weak backwardation, i.e. $S_t - e^{-r}F_{t,t+1}$ is strictly positive. If it is zero then the option value will be zero, which means that the extraction cost of the marginal producer, x^m, is so high that no production will take place. Thus, the theory predicts that the market should be all the time in weak backwardation and that weak backwardation is necessary for production. Strong backwardation is observed when volatility is high. The intuition for the theory is that when the market is volatile, the value of delaying production increases, causing current prices to increase relative to futures prices. However, this theory has been brought to question when recent data showed that being in contango is actually quite frequent. Specifically, from the beginning of 1996 to the end 2008, the market was in contango 40 percent of the time.

The fourth theory is due to **Hotelling (1931)**. He postulated a theory of the price movement of an exhaustible resource. The theory shows that if competitive risk-neutral producers with zero marginal extraction costs can make costless supply adjustments, then the spot price should rise at the risk-free interest rate. Since under risk neutrality, the expected future spot price is equivalent to the forward price, the theory predicts that forward prices should be higher than the spot ones by the risk free rate. In other words, the slope of the forward curve is always positive and equal to the risk-free rate. The predictions of the earlier literature based on Hotelling's theory are clearly inconsistent with the data since, in reality, forward curves can be either in backwardation or in contango. Many researchers extend the basic model and introduce some features of resource markets that would cause the price predicted by theory to have better reconciliation with the observed data (see Gaudet (2007) and Slade and Thille (2009) for reviews). However, the main result of this literature is that exhaustibility of a commodity leads to a rising prices at least in the long-run.

In the fifth theory, Carlson et al. (2007) proposed an equilibrium model for exhaustible resources, such as crude oil, where supply adjustment is not costless. In their model, increasing the supply above historical average is costly but not decreasing it. Thus, a supply adjustment to a positive demand shock is limited and backwardation may be observed. When there is an adverse demand shock, since decreasing supply is not costly, supply adjusts accordingly and exhaustibility causes the market to be in contango, as in Hotelling-based models. Therefore, whether the market is in backwardation or contango depends on whether the demand shock is positive or negative and whether production is below its historical average or not. Moreover, the theory predicts that a high level of both backwardation and contango is associated with a high level of demand shocks and, in turn, high level of volatility in the price change.

Table 3.1 summarizes the implications of each of the above theories on shape of the forward curve.

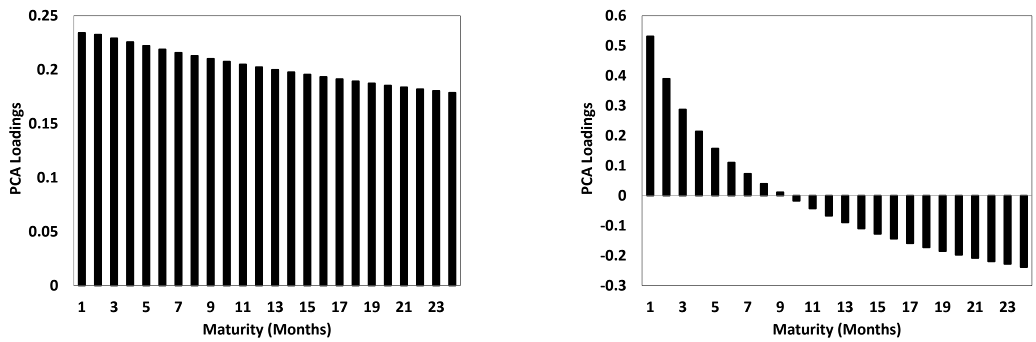
3.3 PCA Slope Measure

Principal component analysis (PCA) is a statistical technique that deals with a large number of (correlated) variables and reduces them to a smaller number of uncorrelated linear combinations, called principal components, that account for the most variability in the original variables. More details about PCA can be found in Jolliffe (2002).

Applying PCA to crude oil futures returns, Figure (3.2) shows the loadings of the first and the second principal components of the crude oil futures returns which together account for about 99% of the variation in all futures returns. It is clear from the figure that

Table 3.1. Theories on forward prices and their implications

Theory	Implication for forward curve
Theory of storage (both versions)	A negative (positive) slope is associated with lower (higher) level of inventory which, in turn, is associated with higher (lower) volatility.
Hedging Pressure Theory	The slope is positive (negative) if hedgers are net long (net short) and is steeper, regardless of its sign, when volatility is high.
Real Option Theory	The slope is always negative and it is steeper if volatility is high.
Hotelling Theory	The slope is always positive and constant at risk free rate.
Production with Adjustment Cost Theory	The slope is positive if there is negative demand shock (low demand). On the other hand, it is negative if there is a positive demand shock (high demand) and supply adjustment cost. Moreover, a high volatility is associated with a steeper slope, regardless of its sign.



(a) First PCA (b) Second PCA

Figure 3.2. PCA Loadings of the Futures Price Returns

the second factor is responsible for the slope of the forward curve. This is because it gives opposite loadings for the two ends of the curve. That is, a shock to this factor will move the two ends of the forward curve in opposite directions. Borovkova (2006) constructs from this factor a leading indicator of a transition in the forward curve from backwardation to contango, and visa versa. In this study, time series of this factor is used as a measure of the slope of the forward curve and incorporate it when modeling the volatilities of crude oil spot and futures returns.

This study focuses on the second principal component when the ends of the forward curve move in opposite directions. Following Borovkova (2006), to construct a time series that corresponds to the level and the direction of the forward curve slope, PCA is applied to the centered forward curve defined below; then the first component of this centered curve is used. That is, if the forward curve at time t is denoted by

$$\big(F_t(1), F_t(2), ...\big),$$

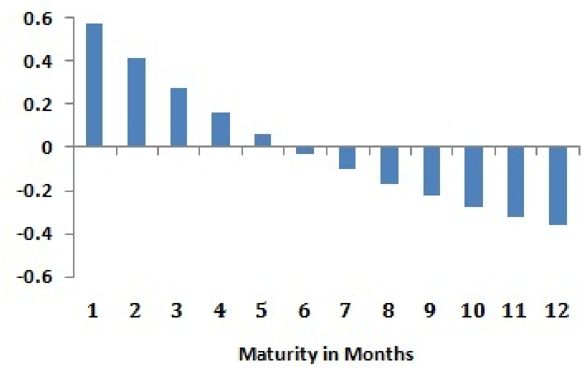


Figure 3.3. Loadings of the 1^{st} PC of the Centered Futures Prices
Prices of the first 12 nearby futures contracts are used
in calculating the principal component loadings.

then the centered forward curve is constructed by:

$$\big(F_t(1) - \bar{F}_t, F_t(2) - \bar{F}_t, ...\big),$$

where $\bar{F}_t$ is the average of all futures prices observed at time t. Since the whole forward curve is centered, the first component of this centered curve would correspond to the slope of the original curve as it is clear from Figure 3.3.

Now, let $L_1(\tau)$ denote the loading of this first principal component that corresponds to the centered futures price of τ maturity. Then, the time series of the slope could be constructed as follows:

$$Slope_t = \sum_{\tau=1}^{N} L_1(\tau) \cdot (F_t(\tau) - \bar{F}_t(\tau)), \tag{3.7}$$

where N is the number of futures contracts used to construct the factor. To remove the effect of the oil price level, $Slope_t$ is divided by the oil spot price at time t. That is:

$$I_t = Slope_t / S_t \tag{3.8}$$

Thus, I_t measures the relative slope of the forward curve at time t. Positive (negative) values of this variable corresponds to the forward curve having a positive (negative) slope. The size of the variable indicates the steepness of the forward curve. In this analysis, at each time t, the slope factor, $Slope_t$, is extracted using the futures term structure of the past year. Thus, I_t represents the one-year rolling relative slope. Figure (3.4) shows the time series of I_t extracted using the first 12 nearby contracts prices.

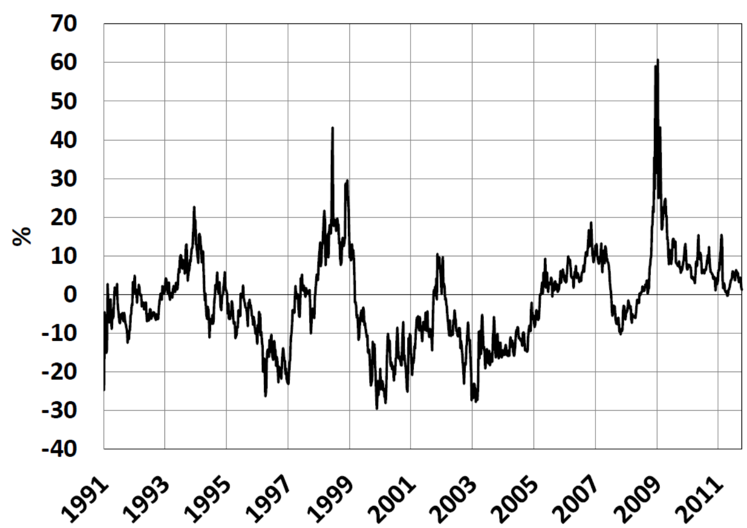


Figure 3.4. Time Series of the Relative Slope Factor I_t

3.4 Volatilities Model Specification

To study whether the forward curve slope is related to the second moments of the spot and futures prices of crude oil, a bivariate version of the diagonal VECH model of Bollerslev et al. (1988) with the slope factor entering as additional regressors is proposed. Formally, the conditional mean of the bivariate system is specified as:

$$r_{s_t} = \mu_s + \sum_{i=1}^{p} a_{s,i} r_{s,t-i} + \sum_{i=1}^{q} b_{s,i} r_{f,t-i}(\tau) + c_s E_{t-1} + u_{s,t} \tag{3.9a}$$

$$r_{f,t}(\tau) = \mu_f + \sum_{i=1}^{p} a_{f,i} r_{s,t-i} + \sum_{i=1}^{q} b_{f,i} r_{f,t-i}(\tau) + c_f E_{t-1} + u_{f,t}, \tag{3.9b}$$

where $r_{s,t}$ and $r_{f,t}(\tau)$ are the percentage return of the spot price and futures price of delivery in τ periods respectively. E_{t-1} is the error correction term to account for the co-integration between the spot and futures prices. The inclusion of the lag returns is to account for the possible serial correlation.

The error terms, $u_{s,t}$ and $u_{f,t}$, are assumed to have a bivariate student's t-distribution with v degree of freedom. That is,

$$\begin{bmatrix} u_{s,t} \\ u_{f,t} \end{bmatrix} \sim \text{Student-}t \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{s,t} & \sigma_{sf,t} \\ \sigma_{sf,t} & \sigma^2_{f,t} \end{bmatrix}, v \right). \tag{3.10}$$

The choice of student's t-distribution is to account for the fat tail feature commonly seen in commodities returns. $\sigma^2_{s,t}$, $\sigma^2_{f,t}$ and $\sigma_{sf,t}$ are the conditional variance of the spot return, the conational variance of the futures return and the conditional covariance between the spot and the futures returns. They have the following specification:

$$\sigma_{s,t}^2 = \omega_s + \alpha_s u_{s,t-1}^2 + \beta_s \sigma_{s,t-1}^2 + \gamma_s I_{t-1} + \kappa_s I_{t-1}^2 \tag{3.11a}$$

$$\sigma_{f,t}^2 = \omega_f + \alpha_f u_{f,t-1}^2 + \beta_f \sigma_{f,t-1}^2 + \gamma_f I_{t-1} + \kappa_f I_{t-1}^2 \tag{3.11b}$$

$$\sigma_{sf,t} = \omega_{sf} + \alpha_{sf} u_{s,t-1} u_{f,t-1} + \beta_{sf} \sigma_{fs,t-1} + \gamma_{sf} I_{t-1} + \kappa_{sf} I_{t-1}^2. \tag{3.11c}$$

The above specification allows the slope factor, I_t to affect the variances and the covariance linearly and quadratically.

Assuming stationarity of the slope series, the stationarity conditions for the conditional variance equations are the same as the stationarity conditions for each equation since the spot variance does not enter the equation of the futures variance, and visa versa. These conditions are $\alpha_j + \beta_j < 1$ where $j = s, f, sf$.

Without any further restrictions, this specification does not ensure the conditional covariance matrix to be positive definite at each t. However, positive definite was not an issue in the empirical analysis since the estimated unrestricted conditional covariance matrix at each time was positive definite.

3.5 Data Description

Weekly data of West Texas Intermediate (WTI) crude oil are used for the purpose of this study. WTI crude oil futures contracts for more than four years maturities are traded in New York Mercantile Exchange (NYMEX). WTI futures contracts are very liquid and are among the most traded commodity futures traded worldwide. The data consists of weekly futures prices for the period from the January 1995 to August 2011. The above data-set was obtained from Datastream and the Energy Information Administration (US Department of Energy).

To construct continuous series of futures prices, following the literature, futures prices are sorted each week according to the contract horizon with "first month" contract being the contract with the earliest delivery date, the "second month" contract being the contract with the next earliest delivery date, etc. The return series for a given contract is created by using a roll-over strategy. For instance, for "first month" series a position is taken in the nearest-to-maturity contract until before expiry where the position changes to the following contract, which then becomes the nearest-to-maturity contract. For this study, 3rd, 6th, 9th and 12th months futures prices are used. The return series are then calculated as the percentage change of these continuous series. That is, the return of X_t is $100 \times \frac{X_t - X_{t-1}}{X_{t-1}}$.

To construct the time series of the slope measure using PCA, the first year contracts (total of 12 futures prices at each week) are used since they are much more liquid than the longer ones. I construct a one year rolling PCA at each week. That is, I_t is the slope component of the past 52 weeks including the current week.

To compare the PCA slope measure of this study with the ones used in the literature, the following regression is estimated:

$$\sigma_{a,t} = \beta_0 + \beta_1 \sigma_{a,t-1} + \beta_2 z_{t-1} + \beta_3 z_{t-1}^2, \tag{3.12}$$

where $\sigma_{a,t}$ is the actual volatility of the spot price return in week t. It is the square root of the daily returns variance in week t. z_{t-1} is the measure of the forward curve slope at the end of the last week. In addition to the PCA slope factor, I_t, the spread between the second month futures price and the spot, the spread between the third month futures price and the spot, and the spread between the sixth month futures price and the third month

Table 3.2. Prediction Power of Different Slope Measures

The OLS estimation results for :

$$\sigma_{a,t} = \beta_0 + \beta_1 \sigma_{a,t-1} + \beta_2 z_{t-1} + \beta_3 z_{t-1}^2,$$

$\sigma_{a,t}$ is the actual volatility of the spot price return in week t

	z_t							
	I_t		$100 \times \frac{F_t(2m)-S_t}{S_t}$		$100 \times \frac{F_t(3m)-S_t}{S_t}$		$100 \times \frac{F_t(6m)-F_t(3m)}{F_t(3m)}$	
	Coeff.	Std.E.	Coeff.	Std.E.	Coeff.	Std.E.	Coeff.	Std.E.
β_0	1.1733	0.0786	1.2007	0.0776	1.1975	0.0775	1.1520	0.0788
β_1	0.4003	0.0311	0.4360	0.0309	0.4304	0.0310	0.4001	0.0313
β_2	0.0073	0.0030	0.0075	0.0135	0.0070	0.0090	0.0477	0.0108
β_3	0.0010	0.0001	0.0062	0.0012	0.0033	0.0006	0.0154	0.0021
R^2	0.2875		0.2634		0.2680		0.2843	

futures price have been used. Table 3.2 shows that I_t has the highest R^2 among the other measures indicating that it has more prediction power than the other measures.

Table 3.3 reports descriptive statistics for the spot, 3rd, 6th, 9th and 12th months contracts: unconditionally and conditional on the slope factor being positive or negative.

Some observations can be drawn from the table. First, the average returns is higher in backwardation than their values in contango. This is natural result of the nature of the market in both regimes. In contango, where futures price is an increasing function of maturity, prices are expected to decline as maturity decreases. In backwardation, where futures price is a decreasing function of maturity, prices are expected to increase as maturity increases. Second, volatilities of the farther contracts prices are lower than those of the closer ones. This observation confirms the regularity of futures markets known in futures

Table 3.3. Descriptive Statistics

The return series are calculated as the percentage change of the continuous price series. That is, the return of X_t is $r_x = 100 \times \frac{X_t - X_{t-1}}{X_{t-1}}$.

	r_s	r_{f3}	r_{f6}	r_{f9}	r_{f12}
			Unconditional		
Mean (%)	0.3485	0.3512	0.3379	0.3300	0.3132
Std. Dev. (%)	5.4418	4.3846	3.7980	3.4305	3.1836
Skewness	-0.2410	-0.3322	-0.3440	-0.3019	-0.2688
Kurtosis	6.1725	5.2687	5.5427	5.8417	6.0703
Observations	836				
	Conditional on Backwardation ($I_t < 0$)				57%
Mean (%)	0.7112	0.9621	0.8365	0.7686	0.7075
Std. Dev. (%)	4.8631	3.9192	3.2565	2.8396	2.5882
Skewness	-0.5204	-0.5140	-0.4565	-0.2190	-0.0273
Kurtosis	4.2314	4.4186	4.6761	4.6948	4.5607
Observations	479				
	Conditional on Contango ($I_t > 0$)				43%
Mean (%)	-0.1381	-0.4685	-0.3311	-0.2585	-0.2159
Std. Dev. (%)	6.1064	4.8262	4.3371	4.0213	3.7800
Skewness	0.0403	-0.0393	-0.0791	-0.1148	-0.1407
Kurtosis	6.8990	5.7510	5.5265	5.5008	5.5953
Observations	357				

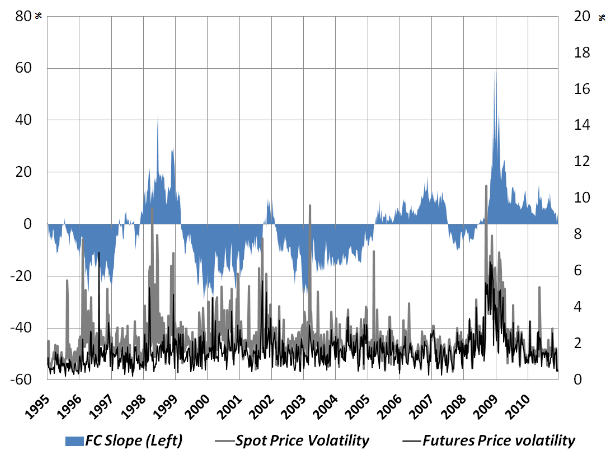


Figure 3.5. Time Series of the Slope Factor, I_t, and the Actual Spot Returns Volatility, σ_a

(For futures series, the sixth month contract is used)

literature as Samuelson's effect which states that the volatilities of futures returns decline with maturity. This reflects the fact that closer contracts prices are more sensitive to current information than the farther ones. Third, one can see clearly that volatilities are higher when the market is in contango than their values when it is in backwardation. This result is at odds with both the theory of storage and real option theory. Fifth, the kurtosis of the returns is high which indicates that crude oil returns tend to have large size of movements. More interestingly, the kurtosis is higher in contango than its value in backwardation. That is, large movements of returns are seen more in contango regimes than in backwardation ones. This observation might explain why the volatility is also higher in contango regimes.

Figure 3.5 shows the time series of the actual volatility, $\sigma_{a,t}$, in each week along with the slope factor at the Friday of the previous week. From the figure, one can observe that high values of the slope factor is associated with high volatility regardless of its sign. In other words, regardless of being in contango or in backwardation, when the forward curve

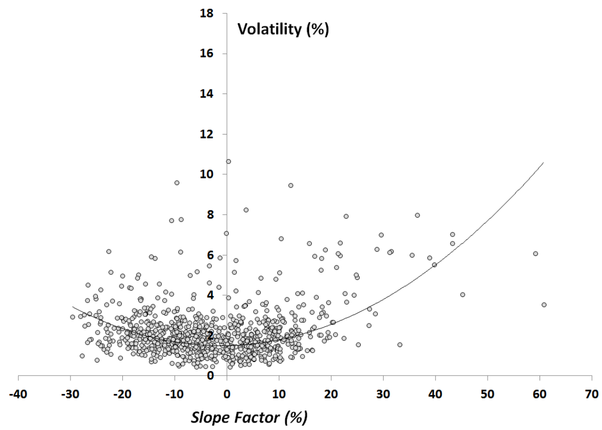


Figure 3.6. Scatter Plot of the Slope Factor, I_t, and the Actual Spot Returns Volatility, σ_a

of the oil futures is observed to be steep at the end of the week, the volatility of spot and futures prices tend to be higher in the coming week.

A clearer picture emerges from figure 3.6 which depicts a scatter plot of the slope factor at the end of each week (in the horizontal axis) and the actual volatility of the next week (in the vertical axis). The observed U shape confirms the above observation seen in the last figure.

Before estimating the bivariate volatility model explained in the last section, unit root and co-integration test have been conducted. Table 3.4 shows the unit root tests results. It shows that the null hypothesis of having a unit root in the price level series can not be rejected. However, it is strongly rejected in the return series. The same results also have been obtained, although not reported, for the futures prices. This results is in keeping with the literature. For the relative slope series, I_t, the same table shows that the null hypothesis of having a unit root is rejected at 5% level. Therefore, no transformation is applied to I_t. Also, the results indicate that the squared slope factor, I_t^2, is strongly stationary.

Table 3.4. Unit Root Test Results

Null Hypothesis: The variable has a unit root.
Constant and time trend are included in the regression.
For ADF, lag Length is based on SIC when maximum lag is 20.
For P-P, Bandwidth is chosen based on Newey-West method using Bartlett kernel)

	S_t	r_S	I_t	I_t^2
			The Augmented Dickey-Fuller (ADF) Test	
Test Statistic	-2.5080	-31.9690	-3.2940	-6.0624
Prob*	0.3242	0.0000	0.0155	0.0000
			The Phillips-Perron (PP) Test	
Test Statistic	-3.1120	-32.0462	-3.5224	-13.3754
Prob*	0.1041	0.0000	0.0077	0.0000

*MacKinnon (1996) one-sided p-values.

Table 3.5 shows the results of Johansen's maximum-likelihood tests of co-integration. It shows that the spot price and the futures prices are co-integrated. This result is expected given the fact that both prices, cash and futures, lie in the same market and thus are subject to the same risk factors. This result justifies the inclusion of an error correction term in the means specification in equations (3.9).

3.6 Estimation Methodology

The model in equations (3.9) and (3.11) are estimated using maximum likelihood (ML) method. Preliminary analysis of the autocorrelations, using the AIC and the significance of the lag returns coefficients, reveals that lag returns of greater than one are not needed. Thus, p and q in equation (3.9) are set to one. The sample likelihood is constructed from the bivariate distribution of the error terms, $u_{s,t}$ and $u_{f,t}$, which has a bivariate student's t-distribution. Let:

Table 3.5. Johansen's Maximum-Likelihood Tests of Co-Integration

Statistic	Null	Alternative	Value	5% C.V.	Prob.
	($r \equiv$ No. of Cointegrations)				
			S and F3		
Trace	$r = 0$	$r > 1$	34.6594	25.8721	0.0031
	$r \leq 1$	$r > 1$	11.6759	12.5180	0.0688
Max. EV	$r = 0$	$r = 1$	22.9835	19.3870	0.0143
	$r = 1$	$r = 2$	11.6759	12.5180	0.0688
			S and F6		
Trace	$r = 0$	$r > 1$	31.1898	25.8721	0.0099
	$r \leq 1$	$r > 1$	10.0666	12.5180	0.1242
Max. EV	$r = 0$	$r = 1$	21.1233	19.3870	0.0277
	$r = 1$	$r = 2$	10.0666	12.5180	0.1242
			S and F9		
Trace	$r = 0$	$r > 1$	28.9907	25.8721	0.0198
	$r \leq 1$	$r > 1$	9.5100	12.5180	0.1512
Max. EV	$r = 0$	$r = 1$	19.4807	19.3870	0.0485
	$r = 1$	$r = 2$	9.5100	12.5180	0.1512
			S and F12		
Trace	$r = 0$	$r > 1$	34.4560	25.8721	0.0033
	$r \leq 1$	$r > 1$	9.5959	12.5180	0.1467
Max. EV	$r = 0$	$r = 1$	24.8602	19.3870	0.0072
	$r = 1$	$r = 2$	9.5959	12.5180	0.1467

The sample is from Jan. 1995 to Aug. 2010 (836 observations). A linear deterministic trend is included in the VAR system with maximum lag interval of 10 and the optimal lag is chosen by AIC.

$\mathbf{u}_t = \begin{bmatrix} u_{s,t} \\ u_{f,t} \end{bmatrix}$, and $\mathbf{\Sigma_t} = \begin{bmatrix} \sigma_{s,t}^2 & \sigma_{sf,t} \\ \sigma_{sf,t} & \sigma_{f,t}^2 \end{bmatrix}$, then:

$$l(\phi; \mathbf{u_t}) = \log \left(\frac{\Gamma\left(\frac{v+M}{2}\right) v^{M/2}}{(v\pi)^{M/2} \Gamma\left(\frac{v}{2}\right) (v-2)^{M/2} |\mathbf{\Sigma_t}|^{\frac{1}{2}} \left[1 + \frac{\mathbf{u}_t^\top \mathbf{\Sigma}_t^{-1} \mathbf{u}_t}{v-2}\right]^{\frac{(v+M)}{2}}} \right) \tag{3.13}$$

where $l(\phi; \mathbf{u}_t)$ is the conditional likelihood of observing $\mathbf{u_t}$, M is the size of vector $\mathbf{u_t}$, which is 2 in our case, and ϕ is the parameter vector to be estimated. $\mathbf{u_t}$ is observed through the mean equations in (3.9). v is the degree of freedom which is also to be estimated, i.e $v \in \phi$. $\Gamma(\cdot)$ is the gamma function.

Having the likelihood of each observation, the parameters set ϕ can then be estimated by maximizing the likelihood of the sample, that is:

$$\hat{\phi} = \arg\max_\phi \sum_{t=1}^{T} l(\phi; \mathbf{u}_t).$$

3.7 Estimation Results

MLE results are shown in Table 3.6. The main interest of this study is in the variance equations parameters. The table shows that the slope factor has no significant *linear* impact on the variance of the spot returns and the variance of the futures returns for all contracts used in the estimation. γ_s and γ_f and are all insignificant in all used futures contracts. However, the slope factor *does* have significant *quadratic* impacts on the variances of the spot and futures returns as shown by significant κ_s and κ_f of all used futures contracts. The result indicates that volatility of the spot and futures markets are high when the slope of the forward curve is steep regardless of whether it is in contango or in backwardation.

Table 3.6. Maximum Likelihood Estimation Results

	r_s and $r_{f(3)}$		r_s and $r_{f(6)}$		r_s and $r_{f(9)}$		r_s and $r_{f(12)}$	
	Coeff.	t-value	Coeff.	t-value	Coeff.	t-value	Coeff.	t-value
				Mean Equations				
μ_s	0.6608	4.5207	0.3838	2.7297	0.2391	1.7495	0.1769	1.3015
a_s	-0.0987	-1.1872	-0.0533	-0.7393	-0.0373	-0.5890	-0.0083	-0.1461
b_s	0.0892	0.9111	0.0280	0.2908	-0.0242	-0.2662	-0.0805	-0.9316
c_s	-0.0105	-0.0951	-0.0212	-0.3693	-0.0245	-0.5847	-0.0214	-0.6191
μ_f	0.6085	4.6256	0.3817	3.5323	0.2717	2.9239	0.2141	2.5378
a_f	0.0616	0.9330	0.0428	0.8869	0.0238	0.6499	0.0292	1.0040
b_s	-0.0716	-0.8719	-0.0645	-0.9374	-0.0730	-1.2457	-0.1000	-1.9603
c_f	-0.2779	-2.8719	-0.1499	-3.5624	-0.1187	-4.3082	-0.1009	-4.8015
				Variance Equations				
ω_s	6.3656	4.4832	2.5522	4.0272	1.3177	3.5224	1.2485	3.3869
α_s	0.1893	5.4770	0.1414	5.9139	0.1260	6.4113	0.1296	6.3966
β_s	0.4863	7.0668	0.7124	19.3964	0.7883	31.2088	0.7907	31.5423
γ_s	0.0894	0.9452	0.0378	0.7787	0.0220	0.6926	0.0109	0.3605
κ_s	0.0261	3.3258	0.0145	3.4572	0.0095	3.4736	0.0089	3.3616
ω_f	4.9692	3.8313	1.0471	3.5578	0.3760	2.9917	0.2713	3.0320
α_f	0.1521	4.7170	0.0909	4.9248	0.0820	5.1874	0.0814	5.1671
β_f	0.5387	6.5304	0.8119	24.9506	0.8682	38.0600	0.8777	42.0999
γ_f	0.0814	1.1886	0.0277	1.4139	0.0147	1.4663	0.0106	1.4251
κ_f	0.0139	2.7173	0.0037	2.6032	0.0018	2.5687	0.0012	2.2968
ω_{sf}	5.4922	4.2121	1.5456	3.8067	0.6217	3.2461	0.5006	3.2171
α_{sf}	0.1674	5.1831	0.1093	5.5537	0.0962	5.9937	0.0960	6.0095
β_{sf}	0.5173	7.0280	0.7719	23.0401	0.8406	37.4398	0.8485	41.5888
γ_{sf}	0.0897	1.1439	0.0351	1.2047	0.0203	1.2641	0.0151	1.1595
κ_{sf}	0.0174	2.9002	0.0065	2.9167	0.0035	2.9015	0.0026	2.7091
v	6.0572	9.3367	5.9741	8.2928	6.9854	6.7879	6.9113	6.6114
Log L	-3827.9170		-3932.5850		-3956.6180		-3987.3360	

LogL in the last raw refer to the logarithm of the likelihood evaluated at the estimated coefficients. To assure that the estimated coefficients are not for local minimum, several initial values of the coefficients have been tried. Moreover, simulation analysis has been done using the estimated coefficients. The unconditional volatilities and the unconditional kurtosis of the simulated data are similar unconditional volatilities and unconditional kurtosis seen in the data.

This result supports the hedging pressure theory and the adjustment cost theory and it also in line with the empirical work of Carlson et al. (2007) and Kogan et al. (2009). The results are at odds with the theory of storage and the real option theory as shown section 3.2.

More interestingly, the results show that not only variances are function of the steepness of the forward curve slope, but the covariance between the spot and futures returns is also impacted by of the steepness of the slope quadratically as evidenced by the significant κ_{sf}. That is, the higher the slope of the forward curve the closer the spot and futures markets are expected to be regardless whether the market is in contango or backwardation. This result indicates that when high uncertainty is expected, both spot and futures prices respond similarly to a shock in the market[2].

To further investigate the above results, the Wald's χ^2 test is used to test the following two null hypothesis: the first one is that the linear impact is zero, i.e $\gamma_s = \gamma_{sf} = \gamma_f = 0$ and the second one is that the quadratic impact is zero, i.e. $\kappa_s = \kappa_f = \kappa_{sf} = 0$. Table 3.7 shows the results of the Wald's χ^2 test. The test results fail to reject the first one but strongly reject the second one.

Thus, the above results shows that the slope of forward curve has a significant power to predict the volatility in the market. Within the framework of the model of this study, the contribution of the slope factor to the innovations in variances and the covariance can be calculated as shown by Ng and Pirrong (1994), who estimated a similar model for metals prices. From the equations in (3.11), there are two kinds of shocks that contribute

[2]The table also shows that the persistence of the variances (which is reflected in the value of β_i where $i = s, f$ and sf) is increasing with the maturity of the contract. On the other hand, the impact of the innovation on the variance (which is reflected in the value of α_i where $i = s, f$ and sf) is decreasing with the maturity. This result has to do with Samuelson's effect where futures variances decrease with the contract maturity. In other words, futures prices for longer maturity response less to the shocks in the current market.

Table 3.7. Wald's Test Results

Wald test			
Null Hypothesis	Value	df	Probability
	r_s and $r_{f(3)}$		
$\gamma_s = \gamma_f = \gamma_{sf} = 0$	4.3054	3	0.2324
$\kappa_s = \kappa_f = \kappa_{sf} = 0$	24.844	3	0
	r_s and $r_{f(6)}$		
$\gamma_s = \gamma_f = \gamma_{sf} = 0$	3.6713	3	0.2964
$\kappa_s = \kappa_f = \kappa_{sf} = 0$	16.730	3	0.0009
	r_s and $r_{f(9)}$		
$\gamma_s = \gamma_f = \gamma_{sf} = 0$	4.0260	3	0.2570
$\kappa_s = \kappa_f = \kappa_{sf} = 0$	14.525	3	0.0041
	r_s and $r_{f(12)}$		
$\gamma_s = \gamma_f = \gamma_{sf} = 0$	4.8032	3	0.1934
$\kappa_s = \kappa_f = \kappa_{sf} = 0$	12.6720	3	0.0087

Table 3.8. Contribution of the Forward Curve Slope on the Second Moments

The reported numbers are the averages of $y_t^{(j)} = \frac{\kappa_j I_{t-1}^2}{\alpha_j u_{j,t-1}^2 \kappa_j I_{t-1}^2}$ when the model is re-estimated with $\gamma_s = \gamma_f = \gamma_{sf} = 0$.

	r_s and r_{f3}	r_s and r_{f6}	r_s and r_{f9}	r_s and r_{f12}
$j = s$	0.4917	0.4411	0.4004	0.3920
$j = f$	0.427	0.3023	0.2334	0.1942
$j = sf$	0.4470	0.3560	0.2939	0.2622

to variances and the covariance at each time: the slope of the forward curve, I_t, and the squared residual returns, $u_{j,t}$ where $j = s, f, sf$. The lag of the variances and covariance terms reflect the persistence of the effects of these two shocks. From the equations in (3.11), the contribution of the slope factor to the total shock in the time t second moments can be calculated as:

$$y_t^{(j)} = \frac{\gamma_j I_{t-1} + \kappa_j I_{t-1}^2}{\alpha_j u_{j,t-1}^2 + \gamma_j I_{t-1} + \kappa_j I_{t-1}^2}, \tag{3.14}$$

where $j = s, f$ or sf.

Since the linear impact of I_t is not significant, the model is re-estimated with only the significant quadratic impact, i.e. $\gamma_s = \gamma_f = \gamma_{sf} = 0$, then $y_t^{(j)}$ is calculated at each time and the average values are reported in Table 3.8. The numbers show that the slope of the forward curve explains a large fraction of the innovations in the second moments, especially for the near months contracts.

3.8 Application: Minimum Variance Hedge Ratio

In commodities (and financial) futures literature, calculating the optimal hedge ratio, which is the value of futures sales divided by the value of the cash positions[3], has been the focus of much research (See Lien and Tse (2002) and Chena et al. (2003) for reviews of this literature). If the hedger's objective is to reduce the variance of the hedging portfolio return, the optimal hedge ratio is then the ratio of the covariance between spot and futures returns to the variance of futures return.

Denote P_t to be the hedging portfolio at time t which is formed by having x_s units of long positions in the spot market and x_f units of short positions in the futures market of the same commodity. The return on the hedge portfolio, denoted by $r_{P,t}$, is then:

$$r_{P,t+1} = \frac{P_{t+1} - P_t}{P_t} = \frac{x_s \cdot S_t \cdot r_{s,t+1} - x_f \cdot F_t(\tau) \cdot r_{f_{t+1}(\tau)}}{x_s \cdot S_t} = r_{s,t+1} - h_t r_{f,t+1}(\tau), \tag{3.15}$$

where $h_t = \frac{x_f \cdot F_t(\tau)}{x_s \cdot S_t}$. Note that the value of P_t comes only from the spot positions since investing in futures does not require an initial outlay of capital. To minimize the variance of $r_{P,t+1}$, the optimal hedge ratio, h^*, is given by :

$$h_t^* = \frac{Cov\left(r_{f_{t+1}}, r_{f_{t+1}(\tau)}\right)}{Var\left(r_{f_{t+1}(\tau)}\right)}. \tag{3.16}$$

Consequently, the minimum variance hedge ratio, h^*, depends on how the second moments (variances and covariance) of spot and futures returns are modeled. Early research, such as Myers and Thompson (1989), assumed constant variances and covariance and, as

[3] It can also be defined as the number of futures contracts needed to hedge against the spot price of one unit of the commodity. This definition is more convenient when working with price changes rather than price returns. However, the definition in the text is more convenient when working with price returns as it is the case in this study. For more details, see Chena et al. (2003).

a result, the optimal hedge ratio would be constant and could be easily estimated by a least-square regression. However, the assumption of constant second moments is not realistic given the documented evidence that most asset prices, including commodities, have time-varying volatilities. This implies that the optimal hedge ratio should be time varying as well. Many researchers have recently employed various specifications to model the second moments. For example Baillie and Myers (1991), Kroner and Sultan (1993) and Ng and Pirrong (1994) among others used bivariate GARCH models. Lien and Wilson (2001) suggested a stochastic volatility model. Alizadeh et al. (2008) used the Markov Regime Switching approach.

This section studies how much improvement in hedging performance can be obtained when the significant impact of the forward curve shape is incorporated in modeling the second moments as seen in the last section. Again, given the insignificance of the linear impact of the slope factor, the model in equations (3.11) are re-estimated with the linear impacts omitted, that is $\gamma_s = \gamma_f = \gamma_{sf} = 0$. The minimum hedge ratio is then calculated as :

$$h_t^* = \frac{\sigma_{sf,t+1}}{\sigma^2_{f,t+1}}. \tag{3.17}$$

Data from January 1995 to December 2010 are used for the estimation and data from January 2011 to October 2011 are used to evaluate the hedge performance. The 3rd month futures contract is used in the hedging portfolio. The hedging performance is measured by the variance of $r_{p,t}$. For comparison, the hedging performance of two other models are computed. The first one is the pure diagonal VECH which obtain by sitting $\kappa_s = \kappa_f = \kappa_{sf} = 0$. The second one is using the OLS hedge ratio obtained by regressing the spot returns on the furfures returns. The hedge ratios of these two models are denoted by $h_t^{(Bgarch)}$ and $h_t^{(OLS)}$ respectively. The results are shown in figure 3.7 which shows the time series of the hedging portfolios and Table 3.9 which shows the variance of the hedging

Table 3.9. Hedge Portfolio Performance

Hedging Portfolio	Variance
$r_{s,t} - h_t^* r_{f(3),t}$	0.9388
$r_{s,t} - h_t^{(Bgarch)} r_{f(3),t}$	1.0620
$r_{s,t} - h^{(OLS)} r_{f(3),t}$	1.0791
$r_{s,t}$	22.8390

portfolios.

The figure shows that the hedging portfolio designed by using h_t^* has less variability than the other two portfolios. This is also clear from the variances of the four portfolios calculated in Table 3.9. This result implies that incorporating the information of the shape of the forward curve in calculating the hedge ratio does help in improving the hedging performance in term of risk reduction.

3.9 Concluding Remarks

In this chapter, the main theories on the relation between spot and futures markets are reviewed and the implication of each reviewed theory on the relationship between the shape of the forward curve and the volatility of the market is extracted. The focus of this study was in crude oil markets. Moreover, the relationship between the slope of the forward curve and the expected variances of the spot and futures prices and the covariance between them was investigated using a bivariate GARCH model augmented with a measure of the forward curve slope extracted by principal component analysis (PCA) as suggested by Borovkova (2006).

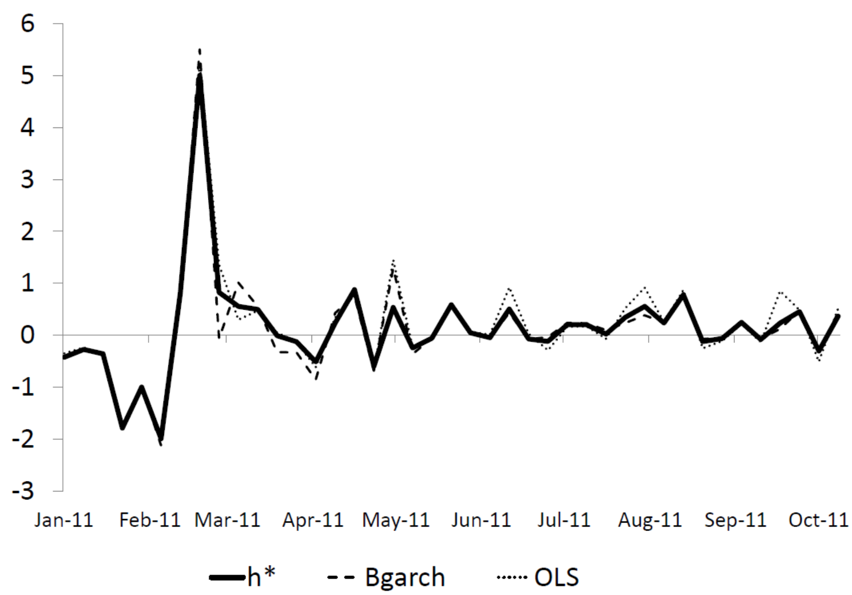


Figure 3.7. Time Series of The Hedging Portfolio Return

The slope of the forward curve was found to have a significant quadratic impact on the variances and the covariance of the spot and futures returns in the crude oil prices. Moreover, the study shows that exploiting this relation in designing the minimum variance hedge ratio leads to better results in term of risk reduction. A potential direction for futures research is to study the hedging performance, when the prediction power of the slope factor is taken into consideration, more thoroughly using different hedging goals other than variance reduction.

Chapter 4

Contango and Backwardation in the Crude Oil Market: A Regime Switching Approach

4.1 Introduction

Modeling the stochastic nature of commodities prices is a crucial step for valuing financial and real contingent claims related to commodities prices. The notion of convenience yield, defined as the benefits accruing to the owner of the physical commodity due to the flexibility in handling shocks in the market, plays a central role in commodities price modeling as it derives the relationship between futures and spot prices in the commodities markets. Early models of commodities prices, such as Brennan and Schwartz (1985), include a constant convenience yield with a one-factor geometric Brownian motion (GBM) to model the movement of the spot price. Many recent models extend this model by adding more factors. Gibson and Schwartz (1990) modeled the convenience yield as a stochastic mean reverting process and found the model able to generate various kinds of futures term structures that are commonly seen in the market. Schwartz (1997) studied the implication of a three-factor model where the third factor is the interest rate. Casassus et al. (2005) studies an extension to these models and found the importance of convenience yield being a function of the spot price and the interest rate levels. Liu and Tang (2011) introduce a stochastic volatility in the convenience yield process.

Most of these models assume a mean reverting process to model the convenience yield. That is, the convenience yield process is specified to revert to a certain level, or an equilibrium level, at a certain speed. This specification is somehow restrictive. Theoretically, convenience yield is derived as a function of the inventory level and the supply and demand conditions. Accordingly, it is to restrictive to assume that there is only one state that the market should revert to all the time. Empirically, the estimated value of this equilibrium level is very unstable as shown below. This assumption may not have much impact on the short-term pricing. However, given the large estimate of the speed of mean reversion in the convenience yield process found in papers such as Gibson and Schwartz (1990) and Schwartz and Smith (2000), this assumption may have a significant impact in the long-run.

Given that the shifts in the convenience yield level are both temporary and recurrent, the regime switching approach to modeling changes in the dynamic behavior provides a potential way, as opposed to models with structural breaks, to relax this restrictive assumption about the level of the convenience yield. Regime-switching models are time-series models in which parameters are allowed to take on different values in each of some fixed number of regimes or states. A stochastic process assumed to have generated the regime shifts is included as part of the model, which allows for model-based forecasts that incorporate the possibility of future regime shifts. The primary use of these models in econometrics has been to describe changes in the dynamic behavior of macroeconomic and financial time series (Hamilton (1994) and Dai et al. (2007)).

In general, temporary and recurrent shifts in the behavior of a time series can also be modeled using the threshold models where the regime shifts are triggered by the level of observed variables in relation to an unobserved threshold. However, for the purpose of modeling commodities prices, the ability to get a closed form solution for the futures price formula makes the regime switching framework more attractive.

Markov switching models were introduced by Hamilton (1989) to capture nonlinearities in GNP growth rates arising from discrete jumps in the conditional mean. Regime switching models are well developed for bond pricing and the term structure of interest rates (Bansal and Zhou (2002) and Dai et al. (2007)) and electricity futures prices (Blochlinger (2008)). However, they are less explored in studying the futures term structure of other commodities. Much of the attention in this literature is given to capturing the time series properties of the observed commodities prices. For example, Fong and See (2003) modeled the conditional volatility of crude oil futures returns as a regime switching process. The model features transition probabilities that are functions of the basis, the spread between the spot and futures prices. Alizadeh et al. (2008) purposed a regime switching conditional volatility model and studied its implication on the optimal hedge ratio and the hedging performance. Chiarella et al. (2009) modeled the evolution of the gas forward curve using regime switching.

Chen and Forsyth (2010) proposed a one-factor regime-switching model for the risk adjusted natural gas spot price and studied the implications of the model on the valuation and optimal operation of natural gas storage facilities. They solved the partial differential equation governing the futures prices numerically and used a least squares approach to calibrate the model parameters. Chen (2010) proposed a regime switching model for crude oil prices in order to capture the historically observed periods of lower but more stable prices followed by periods of high and volatile prices. The study modeled the crude oil spot price as a mean reverting process that reverts to different levels and exhibits different volatilities within each regime.

In this study, regime switching framework is exploited to study the movement in crude oil futures term structures. In particular, the Brennan and Schwartz (1985) one-factor model has been extended to accommodate shifts in the convenience yield level and, in turn, in the futures term structure in a discrete time setting. Unlike Chen and Forsyth (2010) and Chen (2010), the model of this study allows for pricing the regime switching risk as well as the market price risk. Moreover, a closed form solution for the futures prices is derived and an extension to the Kalman filter suggested by Kim (1994) is used to estimate the model parameters.

The chapter is organized as follows: a background of convenience yield modeling is given in section 4.2. In the following section, the regime switching model is specified. Section 4.4 is devoted to futures price formula derivation. In 4.5 section, an estimation method based on the Kalman filter is propped in detail. Data description and the model estimation results for the crude oil market are given in the next two sections. The last section is devoted to concluding remarks.

4.2 Convenience Yield in Commodities Price Modeling

The convenience yield is defined as the stream of benefits received by holding an extra unit of the commodity in storage rather than buying the unit in the futures market. This stream of benefits comes from the fact that holding commodity in storage enables the holder to respond flexibly and efficiently to supply and demand shocks. This concept has been introduced to reduced form modeling of commodities prices. Expressing the net convenience yield, net from storage cost, as a fraction to the commodity price, i.e. the net convenience yield $= \delta \cdot S_t$, where S_t is the commodity spot price, Brennan and Schwartz (1985) introduced a constant δ to a process of Geometric Brownian motion to model price stochastic movements, that is :

$$\frac{dS_t}{S_t} = (\mu - \delta)dt + \sigma_s dz_{s,t}, \tag{4.1}$$

where μ is the total rate of return of holding one unit of the commodity[1] and σ_s is the volatility of the commodity price. $dz_{s,t}$ is a Brownian motion increment to account for the stochastic movement in the commodity price. This simple model implies only one shape of the futures term structure depending on δ, the net convenience yield[2]. In reality, the futures term structure is seen in different shapes. Gibson and Schwartz (1990) shows that the assumption of constant convenience yield is very restrictive. Accordingly, driven by the numerical properties of the convenience yield implied by the futures prices, they introduced a mean reverting stochastic process for the convenience yield movement as follows:

$$\frac{dS_t}{S_t} = (\mu - \delta_t)dt + \sigma_s dz_{s,t} \tag{4.2a}$$

$$d\delta_t = \kappa(\theta - \delta_t)dt + \sigma_\delta dz_{\delta,t}, \tag{4.2b}$$

[1] The return on holding one unit of the commodity, μ, comes from two sources: the rate of change in the commodity price (the capital gain) and the convenience yield. Thus, $\mu - \delta$ corresponds to the rate of change in the commodity price (the capital gain)

[2] Brennan and Schwartz (1985) shows that the futures price for delivery in τ periods is given by $F_t(\tau) = S_t e^{(r-\delta)\tau}$. Thus, the term structure has positive (negative) slope if $r > \delta (r < \delta)$

where δ_t reverts to a long-run (or an equilibrium) value of θ at a speed of κ with volatility of σ_δ. In this setting, the near end of the futures term structure can take any shape depending on how far δ_t is from its long-run level, θ, and on the speed of the reversion, κ. However, the far end of the futures term structure converges to one shape depending on the value of θ compared to the risk free rate. Many of recent models can be seen as an extension to Gibson and Schwartz (1990) model. For example, Casassus et al. (2005) allows the convenience yield to depend on the spot and the interest rate. Liu and Tang (2011) introduces a stochastic volatility in the convenience yield process to account for the heteroscedasticity observed in the implied convenience yield.

The assumption that δ_t reverts to a certain level in the long-run is somehow restrictive. From the theoretical side, the convenience yield is seen as a function of the level of the commodity inventory in the economy[3] which is in turn a function of the supply and demand conditions. Moreover, macroeconomic conditions which run through different cycles of booms and busts are likely to have an impact on commodities markets especially for crucial commodities such as crude oil. Given that, it is unlikely that there is only one equilibrium state the commodity market should revert to. From the empirical side, estimating the Gibson and Schwartz (1990) model, the model in equation 4.2, using crude oil futures in different periods of time produces very different values of θ. For example, estimating the model using weekly WTI crude oil futures price from 01/01/1992 to 01/01/2000 and from 01/01/2000 to 6/10/2011 yields the value of θ equal to 0.0392 and 0.1149 respectively[4]. These two values imply very different shapes of the futures term structure as shown in figure 4.1.

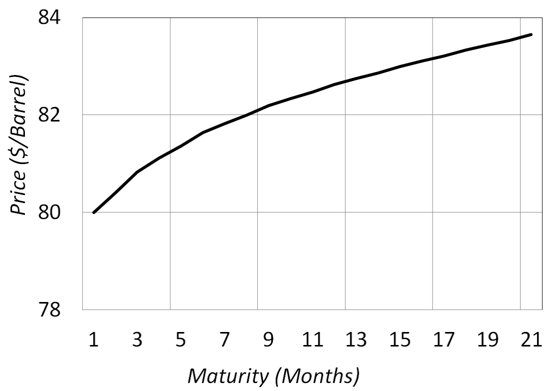
Markov switching models provide a good venue to take to relax this assumption. In the

[3] Refer to Chapter 2 for detailed explanation about the theory of storage and the notion of convenience yield.

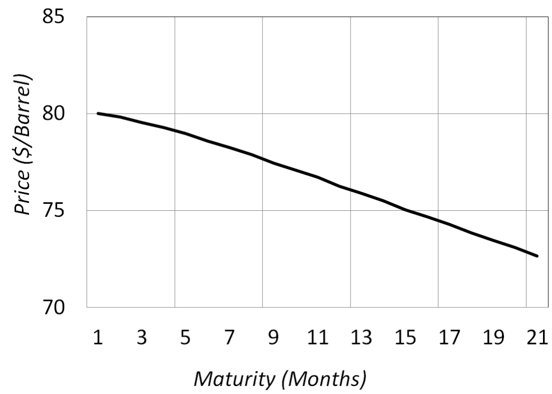
[4] The full estimation results can be seen in Table 4.3.

Figure 4.1. The Implied Forward Curve of Gibson and Schwartz (1990) Model

The two curves show the crude oil futures term structure implied from the estimated parameters in Table 4.2. Initial value of the spot price is \$80 and the initial value of the convenience yield is equal to $\hat{\theta}$.



(a) Parameters are estimated using WTI futures from Jan. 1992 to Jan. 2000 ($\hat{\theta} = 0.0392$)



(b) Parameters are estimated using WTI futures from Jan. 2000 to Aug. 2011 ($\hat{\theta} = 0.1149$)

next section, a regime switching model based on Brennan and Schwartz (1985) one-factor model is specified.

4.3 Regime Switching Model Specification

Let P_t be the spot price of the commodity at time t and let x_t be the logarithm of the spot price, i.e. $x_t = \log(P_t)$. Assuming that there are a number of regimes the commodity market could run through, the dynamics of x_t in each regime under the objective measure is given by:

$$\Delta x_t = x_{t+1} - x_t = (\mu_{s_t} - \delta_{s_t})\Delta t + \sigma_{s_t}\sqrt{\Delta t}\ \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N(0,1). \tag{4.3}$$

μ_{s_t} is the total expected return from holding one unit of the commodity. δ_{s_t} is the net convenience yield, i.e the accrued benefits of holding one unit of the commodity in storage minus the storage cost. σ_{s_t} is the volatility of the commodity price change. The values of all three parameters are functions of which regime the market is in, which is indicated by the subscript s_t where s_t is the process that determines which regime the market is in at time t. This specification can be seen as an extension to the continuous time Brennan and Schwartz (1985) model but with regime dependent parameters. Thus, we call this model the Brennan and Schwartz regime switching (B&S-RS) model.

As in Hamilton (1994), s_t is modeled as an S-state discrete time Markov chain process which is assumed to be independent of x_t. The evolution of s_t is governed by the transitional probability matrix which specifies the probability of switching from one regime to another. In this study we are interested in the case where $S = 2$, i.e. there are only two regimes in the market. For a two-state Markove chain, the transitional matrix under the objective measure $\mathbb{P}$ is then given by:

$$\mathbf{\Pi}^{\mathbb{P}}_{t,t+1} = \begin{bmatrix} \pi^{\mathbb{P}}_{(1,1)} & 1 - \pi^{\mathbb{P}}_{(1,1)} \\ 1 - \pi^{\mathbb{P}}_{(2,2)} & \pi^{\mathbb{P}}_{(2,2)} \end{bmatrix}, \tag{4.4}$$

where $\pi^{\mathbb{P}}_{(i,i)}$ is the $\mathbb{P}$-measure probability to stay in regime i at $t+1$ given the market is in regime i at t where $i = 1, 2$.

The B&S-RS model of this chapter is going to be compared with the Gibson and Schwartz (1990) two-factor model (G&S). The discrete time version of G&S model can be written as follows:

$$\Delta x_t = (\mu - \delta_t)\Delta t + \sigma_s \epsilon_{1,t} \tag{4.5}$$

$$\Delta \delta_t = \kappa(\theta - \delta_t)\Delta t + \sigma_\delta \epsilon_{2,t}, \tag{4.6}$$

and

$$\begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{x\delta} \\ \rho_{x\delta} & 1 \end{bmatrix} \Delta t\right). \tag{4.7}$$

x_t is again the log of the spot price which has a total expected return of μ with volatility of σ_x. δ_t is the net convenience yield and is modeled as a mean reverting process. It reverts to a long-run level of θ at a speed of κ with volatility of σ_δ. $\rho_{x\delta}$ is the instantaneous correlation between the shocks of the two processes.

For later development, the above two models, B&S-RS and G&S, are written in the following form:

$$X_{t+1} = \alpha^{(s_t)} + \beta^{(s_t)} X_t + \Sigma^{(s_t)} \epsilon_{t+1}, \tag{4.8}$$

where for the B&S-RS model: $X_{t+1} = x_{t+1}$, $\alpha^{(s_t)} = (\mu_{s_t} - \delta_{s_t})\Delta t$, $\beta^{(s_t)} = 1$, $\Sigma^{(s_t)} = \sigma_{s_t}\sqrt{\Delta t}$ and $\epsilon_{t+1} \sim N(0, 1)$; and for the G&S model:

$$X_t = \begin{bmatrix} x_t \\ \delta_t \end{bmatrix}, \alpha^{(s_t)} = \begin{bmatrix} \mu \\ \kappa \cdot \theta \end{bmatrix} \Delta t, \beta^{(s_t)} = I_{2\times 2} + \begin{bmatrix} 0 & -1 \\ 0 & -\kappa \end{bmatrix} \Delta t,$$

$$\Sigma^{(s_t)} = \begin{bmatrix} \sigma_x\sqrt{1-\rho_{x\delta}^2} & \rho_{x\delta}\sigma_x \\ 0 & \sigma_\delta \end{bmatrix} \sqrt{\Delta t}, \text{ and } \epsilon_t \sim N\left(\begin{bmatrix} 0 & 0 \end{bmatrix}^\top, I_{2\times 2}\right).$$

If the market does not allow arbitrage opportunities, then according to the fundamental theory of asset pricing (see (Björk, 2003, Theorem 3.8)), there exists a positive stochastic discount factor, denoted by $\mathcal{M}_{t,t+1}$, underlying the time-t valuation of the payoff of any contingent claim paid at date $t + 1$. That is, if $\mathcal{G}_{t+1}$ is the payoff of the contingent claim at $t + 1$, then:

$$\text{Price}_t(\mathcal{G}_{t+1}) = E(\mathcal{M}_{t,t+1}\mathcal{G}_{t+1}).$$

Following Dai et al. (2007), to allow for pricing the risk of the regime shift, $\mathcal{M}_{t,t+1}$ is parametrized as follows:

$$\mathcal{M}_{t,t+1} \equiv \mathcal{M}(X_t, s_t; X_{t+1}, s_{t+1}) = \exp\left(-r_{t,t+1} - \gamma_{t,t+1} - \frac{1}{2}\Lambda_t\Lambda_t^\top - \Lambda_t\epsilon_{t+1}\right), \qquad (4.9)$$

where $\gamma_{t,t+1} \equiv \gamma(X_t, s_t; s_{t+1})$ is the market price of risk associated with regime shift from regime s_t at time t to regime s_{t+1} at time $t+1$ and it can be a function of the state variable X_t. $\Lambda_t \equiv \Lambda(X_t, s_t)$ is the market price of risk associated with the stochastic movement of X_t and it is also regime and state dependent. $r_{t,t+1}$ is the risk free rate at time t for one period which is assumed to be deterministic, i.e. $r_{t,t+1} = r \cdot \Delta t$[5].

The existence of a stochastic discount factor under the absence of arbitrage implies an equivalent martingale measure, $\mathbb{Q}$ measure, under which the price of any contingent claim would be the expectation of the discounted payoff. That is, there exists a measure $\mathbb{Q}$ such that:

$$\text{Price}(\mathcal{G}_{t+1}) = E_t^{\mathbb{Q}}(e^{-r\Delta t}\mathcal{G}_{t+1}), \qquad (4.10)$$

where $E_t^{\mathbb{Q}}[\cdot]$ denotes the conditional expectation under the $\mathbb{Q}$ measure.

Given equation (4.10) and the specification of the stochastic discount factor in equation (4.9), the equivalent $\mathbb{Q}$ measure is then defined by (see the derivation in Appendix A.1):

$$\frac{\mathbb{Q}(dX_{t+1}, s_{t+1} = k|X_t, s_t = j)}{\mathbb{P}(dX_{t+1}, s_{t+1} = k|X_t, s_t = j)} = e^{-\gamma_{t,t+1}^{(j,k)} - \frac{1}{2}\Lambda_t^{(j)}\Lambda_t^{(j)\top} - \Lambda_t^{(j)}\epsilon_{t+1}}, \qquad (4.11)$$

where $\gamma_{t,t+1}^{(j,k)} \equiv \gamma(X_t, s_t = j, s_{t+1} = k)$ and $\Lambda_t^{(j)} \equiv \Lambda(X_t, s_t = j)$.

[5] Many researchers, such as Miltersen (2003), Carmona and Ludkovski (2004) and Tang (2009) , have pointed out that the stochastic risk free rate does not play a substantial role in commodity futures modeling due to its low volatility. Thus, there is a very limited gain by introducing a stochastic risk free rate compared to the resulted complexity of modeling and estimating.

Assuming $\gamma_{t,t+1}^{(j,k)}$ to be constant, i.e. $\gamma_{t,t+1}^{(j,k)} = \gamma^{(j,k)}$, then the regime switching probabilities under $\mathbb{Q}$ are given by:

$$\pi_{j,k}^{\mathbb{Q}} = E_t^{\mathbb{Q}}[1_{s_{t+1}=k} | s_t = j] = \pi_{j,k}^{\mathbb{P}} \cdot e^{-\gamma^{(j,k)}}, \tag{4.12}$$

where $1_{s_{t+1}=k}$ is an indicator function that equals to 1 if the subscript is true and zero otherwise.

Moreover, assuming a constant market price of risk within each regime, i.e. $\Lambda_t^{(s_t)} = \left(\Sigma^{(s_t)}\right)^{-1} \Lambda^{(s_t)}$, where $\Lambda^{(s_t)}$ is vector of constant within each regime, then it is shown in Appendix A.2 that the behavior of X_t in the $\mathbb{Q}$ measure is given by:

$$X_{t+1} = \hat{\alpha}^{(s_t)} + \beta^{(s_t)} X_t + \Sigma^{(s_t)} \epsilon_{t+1}, \tag{4.13}$$

where: $\hat{\alpha}^{(s_t)} = \alpha^{(s_t)} - \Lambda^{(s_t)}$.

For B&S-RS model, $\Lambda^{(s_t)} = \lambda^{(i)} \Delta t, i = 1, 2$, while for G&S model, since it is regime-independent, $\Lambda^{(s_t)} = [\lambda_x \Delta t \ \lambda_\delta \Delta t]^\top$.

If the commodity is a traded asset, then absence of arbitrage implies that the total expected return of holding a unit of the commodity should be equal to the risk-free rate. This is due the fact that one can design a portfolio of the commodity and the derivatives and choose the weights to eliminate the risk. Approximating the return on the commodity by the difference in the logarithm[6], i.e. Δx_t, this dynamic hedging implies:

$$E^Q \left[\Delta x_t | J_t, s_t = j\right] \approx (r - \delta^{(j)}) \Delta t, \tag{4.14}$$

[6] The approximating is reasonable for small time step. In this chapter, weekly data is used, i.e. $\Delta t = 0.0192$. Thus the error is negligible.

for B&S-RS one factor model, and

$$E^{Q}\left[\Delta x_t | J_t\right] \approx (r - \delta_t)\Delta t, \tag{4.15}$$

for the G&S two factor model.

Thus, for the B&S-RS one factor model:

$$\hat{\alpha}^{(s_t)} = (r - \delta^{(s_t)})\Delta t$$

and, for the G&S two factor model:

$$\hat{\alpha}^{(s_t)} = \left[\begin{array}{c} r \\ \kappa \cdot \theta - \lambda_\delta \end{array}\right] \Delta t$$

.

4.4 Futures Pricing

Denote $F_{t,n} \equiv F_n(X_t, s_t)$ to be the futures price at time t of a unit of the commodity delivered in n periods. A futures contract entered at time t has a payoff at time $t+1$ of $F_{t+1,n-1} - F_{t,n}$. Since, there is no payment at the inception at time t, this payoff must have a price of zero, that is:

$$0 = e^{-r\Delta t} E_t^{\mathbb{Q}}\left[F_{t+1,n-1} - F_{t,n}\right], \tag{4.16}$$

which implies:

$$F_{t,n} = E_t^{\mathbb{Q}}\left[F_{t+1,n-1}\right]. \tag{4.17}$$

Appendix A.3 shows that the futures price is an exponential affine function of the state variable within each regime. Specifically:

$$F_n\left(X_t, s_t = j\right) = e^{A_n^{(j)} + B_n X_t}, \tag{4.18}$$

where for B&S-RS model:

$$\begin{aligned} A_n^{(j)} &= \operatorname{Log}\left(\sum_{k=1}^{S} \pi_{jk} \cdot e^{A_{n-1}^{(k)}}\right) + B_{n-1}\hat{\alpha}^{(j)} + \frac{1}{2} B_{n-1} \Sigma^{(j)} \Sigma^{(j)\top} B_{n-1}^{\top} && (4.19) \\ B_n &= B_{n-1}\beta, && (4.20) \end{aligned}$$

with $A_0^{(i)} = 0$ for $i = 1, 2$ and $B_0 = 1$. For the G&S model, $S = 1$ which implies:

$$A_n = A_{n-1} + B_{n-1}\hat{\alpha} + \frac{1}{2} B_{n-1} \Sigma \Sigma^{\top} B_{n-1}^{\top} \tag{4.21}$$

$$B_n = B_{n-1}\beta, \tag{4.22}$$

with $A_0 = 0$ and $B_0 = [1 \quad 0]$.

4.5 Estimation Methodology

As the focus of this study is on the crude oil markets, the B&S-RS one factor model can be estimated using the time series of the crude oil spot price or, if not available, the first contract of futures prices as a proxy. However, the parameters estimated this way would not correspond to the $\mathbb{Q}$ measure which is the relevant measure for pricing contingent claims. Moreover, using such methods, the convenience yield, δ_i where $i = 1, 2$, cannot be identified.

In the bond pricing literature, where regime switching models have been studied extensively, several estimation methods have been proposed. Bansal and Zhou (2002) used efficient method of moments (EMM). Dai et al. (2007) relied on maximum likelihood (ML) which involves inverting equation (4.18) to extract the states vector, X_t, which has Gaussian conditional likelihood. However, this requires one to chose a number of futures contracts or to design a number of portfolios of futures contracts that is the same as the number of factors to be extracted. The contracts choice or the portfolio weights are chosen arbitrarily. Duffee and Stanton (2004) compared the performance of the three methods in estimating affine term structure models: ML, EMM and methods based on Kalman filter. They found that ML is a good method for simple term structure models and the performance of EMM (a commonly used method for estimating complicated models) is poor even in the simple term structure models. According to Duffee and Stanton (2004), Kalman filtering procedure is found to be a tractable and reasonably accurate estimation technique that they recommend in settings where ML is impractical.

Thus, in this chapter I use an extension to the Kalman filter for estimating the parameters of the B&S-RS model proposed in Kim (1994). Blochlinger (2008) used this procedure to estimate electricity price models in regime switching framework.

The Kalman filter is a recursive procedure for computing the optimal estimator of the state vector at time t, based on the information available up to time t, and it enables the estimate of the state vector to be continuously updated as new information becomes available. When the disturbances and the initial state vector are normally distributed, the Kalman filter enables the likelihood function to be calculated, which allows for the estimation of any unknown parameters of the model and provides the basis for statistical testing and model specification. For a detailed discussion of state space models and the Kalman filter see Chapter 3 in Harvey (1989).

The first step in using Kalman filtrating procedure is to cast the model in the state space form. To do this, one needs to specify the *transition equation* that governs the dynamic of the state variables and the *measurement equation* that relates the observable variables to the state variables.

The transition equation is represented by equation (4.8), which is:

$$X_{t+1} = \alpha^{(s_t)} + \beta^{(s_t)} X_t + \Sigma^{(s_t)} \epsilon_{t+1}$$

where for B&S-RS model,:

$$\alpha^{(s_t)} = \begin{cases} (r + \lambda_1 - \delta_1)\Delta t & if \quad s_t = 1 \\ (r + \lambda_2 - \delta_2)\Delta t & if \quad s_t = 2 \end{cases},$$

$$\Sigma^{(s_t)} = \begin{cases} \sigma_1 \sqrt{\Delta t} & if \quad s_t = 1 \\ \sigma_2 \sqrt{\Delta t} & if \quad s_t = 2 \end{cases}$$

and $\beta^{(s_t)} = 1$ in both regimes, and for the G&S model, the matrices $\alpha^{(s_t)}$, $\beta^{(s_t)}$ and $\Sigma^{(s_t)}$ are same as defined in section 4.3. At each time, a vector of (log) future prices of the commodity for different maturities is observed. Assuming that these prices are observed with measurement error (these errors may be caused by bid-ask spreads, the non-simultaneity of the observations, etc. see Schwartz (1997)), the measurement equation will then be :

$$\begin{bmatrix} f_t(n_1) \\ f_t(n_2) \\ \vdots \end{bmatrix} = \begin{bmatrix} A_{n_1}^{(s_t)} \\ A_{n_2}^{(s_t)} \\ \vdots \end{bmatrix} + \begin{bmatrix} B_{n_1} \\ B_{n_2} \\ \vdots \end{bmatrix} X_t + \begin{bmatrix} e_{1,t} \\ e_{2,t} \\ \vdots \end{bmatrix} \tag{4.23}$$

$$Y_t = A^{(s_t)} + BX_t + e_t \qquad e_t \sim N(0, Q_{s_t}), \tag{4.24}$$

where e_t represent the measurement error in the futures prices. It is assumed that the measurement errors are not correlated and have regime independent volatilities. That is, $Q_{s_t} = Q$ where the off diagonal elements of Q and are zeros and the diagonal elements, denoted by v_i^2, are to be estimated.

The Kim (1994) filter extends the Kalman filter to accommodate state space models with regime switching. To ease the explanation of the algorithm, let $J_t \equiv (Y_t, Y_{t-1}, Y_{t-2}, \ldots, Y_1)$ and define the following:

$$\begin{aligned} X_{t|t-1}^{(i,j)} &= E\left[X_t | J_{t-1}, s_t = j, s_{t-1} = i\right] \\ P_{t|t-1}^{(i,j)} &= E\left[(X_t - X_{t|t-1})(X_t - X_{t|t-1})^\top | J_{t-1}, s_t = j, s_{t-1} = i\right] \\ X_{t|t}^{(j)} &= E\left[X_t | J_t, s_t = j\right] \\ P_{t|t}^{(j)} &= E\left[(X_t - X_{t|t})(X_t - X_{t|t})^\top | J_{t-1}, s_t = j\right] \end{aligned}$$

That is, $X_{t|t-1}^{(i,j)}$ is the prediction of X_t based on information up to time $t-1$ conditional on s_t being in the regime j and on s_{t-1} being on regime i and $P_{t|t-1}^{(i,j)}$ is the associated mean square error. On the other hand, $X_{t|t}^{(j)}$ is the prediction about X_t based on information up to time t, given that s_t is in regime j and $P_{t|t}^{i}$ is the associated mean square error.

Given these definitions, the algorithm goal is to start with $X_{t-1|t-1}^{i}$ and $P_{t-1|t-1}^{i}$ from the previous step to produce $X^i_{t|t}$ and $P^i_{t|t}$ of the current step using the above model and the current observation of time t. Specifically, it goes by:

$$X^{(i,j)}_{t|t-1} = \alpha_i X^{(i)}_{t-1|t-1} \tag{4.25}$$

$$P^{(i,j)}_{t|t-1} = \alpha_i P^{(i)}_{t-1|t-1}\alpha_i^\top + \Sigma_i\Sigma_i^\top \tag{4.26}$$

$$\eta^{(i,j)}_{t|t-1} = Y_t - (A_j + B_j X^{(i,j)}_{t|t-1}) \tag{4.27}$$

$$H^{(i,j)}_t = B_j P^{(i,j)}_{t|t-1} B_j^\top + Q^j \tag{4.28}$$

$$K^{(i,j)}_t = P^{(i,j)}_{t|t-1} B_j^\top \left[H^{(i,j)}_t\right]^{-1} \tag{4.29}$$

$$X^{(i,j)}_{t|t} = X^{(i,j)}_{t|t-1} + K^{(i,j)}_t \eta^{(i,j)}_{t|t-1} \tag{4.30}$$

$$P^{(i,j)}_{t|t} = (I - K^{(i,j)}_t) P^{(i,j)}_{t|t-1}. \tag{4.31}$$

These step constitutes the Kalman filtering procedure and given the normality assumption of the pricing errors, the likelihood of observing Y_t conditional on J_{t-1} and on $s_t = j$ and $s_{t-1} = i$ can be evaluated as follows:

$$f(Y_t|s_{t-1} = i, s_t = j, J_t) = (2\pi)^{-N/2}|H^{(i,j)}_t|^{-1/2}\exp(-\frac{1}{2}\eta^{(i,j)\top}_{t|t-1} H^{(i,j)}_t \eta^{(i,j)}_{t|t-1}), \tag{4.32}$$

where N is the size of X_t. For Gibson and Schwartz (1990) model, where the model is regime independent (i.e. $S = 1$), the above likelihood reduced to $f(Y_t|J_{t-1})$.

However, if the number of the regimes is $S > 1$ (in our case $S = 2$), then the results of the above filtration procedure is S^2 predictions, $X^{(i,j)}_{t|t}$, and S^2 associated forecast errors, $P^{(i,j)}_{t|t}$. Thus, each iteration would require S-fold of cases to consider. Kim (1994) suggested the following approximation, in each iteration, to collapse the S^2 forecasts and their associated forecast errors to only S cases:

$$X_{t|t}^{(j)} = \frac{\sum_{i=1}^{S} Pr\left[s_{t-1} = i, s_t = j|J_t\right] \cdot X_{t|t}^{(i,j)}}{Pr\left[s_t = j|J_t\right]} \tag{4.33}$$

$$P_{t|t}^{(j)} = \frac{\sum_{i=1}^{S} Pr\left[s_{t-1} = i, s_t = j|J_t\right] \cdot \left(P_{t|t}^{(i,j)} + (X_{t|t}^{(j)} - X_{t|t}^{(i,j)})(X_{t|t}^{(j)} - X_{t|t}^{(i,j)})^{\top}\right)}{Pr\left[s_t = j|J_t\right]}. \tag{4.34}$$

Kim (1994) gives the details behind this approximation of this procedure. The outputs $X_{t|t}^{(j)}$ and $P_{t|t}^{(j)}$ are then used as inputs to the Kalman filtration procedure in the next step.

To achieve this recursive procedure, one needs to calculate the probabilities terms appearing in equations (4.33) and (4.34). Kim (1994) suggested to use the Hamilton (1994) procedure to obtain these probabilities recursively. This procedure is explained in detail in Appendix A.4.

As shown in Appendix A.4, as a by product of the Hamilton (1994) filtration procedure, the conditional likelihood of each iteration, $f(Y_t; \psi|J_{t-1})$ is obtained, where ψ is the set of parameters to be estimated.

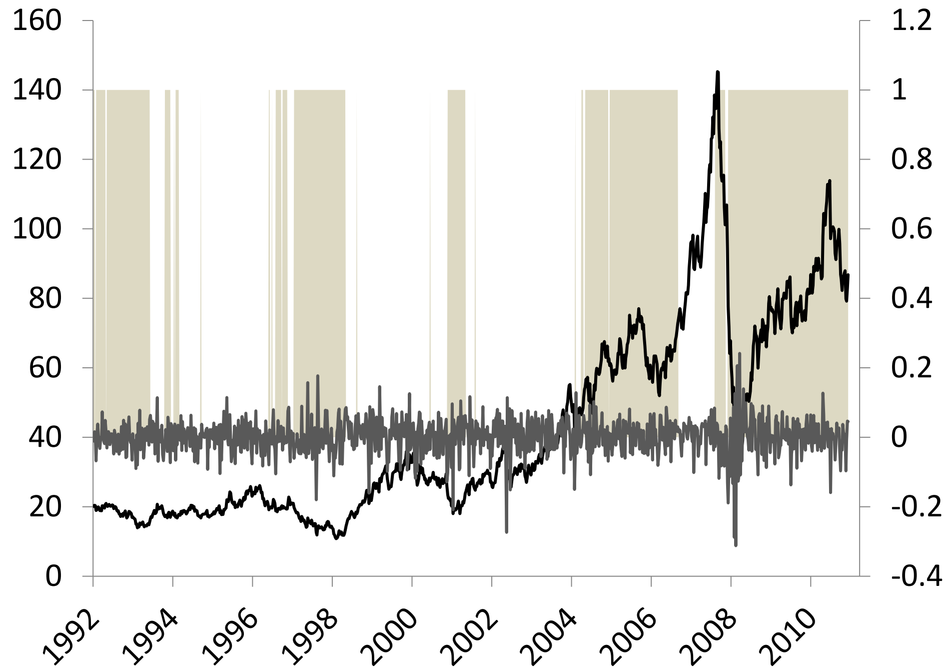
Having the likelihood of each observation, the parameters of the two models can then be estimated by maximizing the likelihood of the sample, that is:

$$\hat{\psi} = \arg\max_{\psi} \sum_{t=1}^{T} log(f(Y_t; \psi|J_{t-1})).$$

4.6 Data Description

To estimate the parameters of the two models, weekly data of West Texas Intermediate (WTI) crude oil futures are used. WTI crude oil futures contracts for more than four years

Figure 4.2. Crude Oil Price and Return Series



Right scale is for WTI oil spot price, left scale is for its log return and shaded areas are for the periods of contango markets.

maturities are traded in New York Mercantile Exchange (NYMEX). WTI futures contracts are very liquid and are among the most traded commodity futures worldwide. Data from January 1992 to the August of 2011 has been obtained from Datastream and the Energy Information Administration (US Department of Energy). For the risk free rate, the average of the 3 months U.S. treasury bill is used.

To construct continuous series of futures prices, following the literature, futures prices are sorted each week according to the contract horizon with "first month" contract being the contract with the earliest delivery date, the "second month" contract being the contract with the next earliest delivery date, etc. Each contract will switch to the next one just before it expires[7]

The performance of our specification of the B&S-RS one-factor model is compared with the G&S two-factor model. The estimation and performance analysis are done for the whole sample period and for two sub-periods, namely: from January 1992 to January 2000 and from January 2000 to August 2011.

Table 4.1 reports descriptive statistics for the weekly returns of the spot, 6th, 12th, 17th and 20th months contracts: unconditionally and conditional on the slope of the futures term structure being positive or negative. The table shows that crude oil market visits backwardation regime and contango regime half of the time in the whole sample period and in the two sub-samples. Moreover, periods of backwardation generate higher returns. The market has higher volatility when being in contango than the case when it is being in backwardation. It is also clear that volatility declines with maturity; an observation known in futures prices literature as Samuelson's effect.

[7]For WTI, trading in the current delivery month ceases on the third business day prior to the twenty-fifth calendar day of the month preceding the delivery month. More details can be seen in http://www.cmegroup.com.

Table 4.1. Descriptive Statistics

Descriptive statistics for the crude oil (log) returns for the whole sample and for the two sub-samples. Contango and backwardation is defined by the sign of the difference between $F6$ and $F1$. Positive sign indicates contango market and negative sign indicates backwardation market.

	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
	From January 1992 to the August of 2011					
	Unconditional		Contango (49.4%)		Backwardation (50.6%)	
F1	0.00199	0.05307	0.00044	0.05999	0.00318	0.04866
F6	0.00188	0.03866	-0.00007	0.04413	0.00378	0.03421
F12	0.00184	0.03256	-0.00022	0.03794	0.00393	0.02752
F17	0.00182	0.02991	-0.00021	0.03513	0.00394	0.02471
F20	0.00181	0.02868	-0.00018	0.03387	0.00392	0.02336
	From January 1992 To January 2000					
	Unconditional		Contango (41.3%)		Backwardation (58.7%)	
F1	0.00220	0.04686	-0.00031	0.05333	0.00363	0.04456
F6	0.00131	0.02930	-0.00145	0.03190	0.00343	0.02850
F12	0.00076	0.02244	-0.00211	0.02298	0.00302	0.02268
F17	0.00044	0.01978	-0.00240	0.01989	0.00272	0.02014
F20	0.00030	0.01855	-0.00239	0.01863	0.00251	0.01885
	From January 2000 to August 2011					
	Unconditional		Contango (53%)		Backwardation (47%)	
F1	0.00184	0.05555	0.00068	0.06212	0.00281	0.05070
F6	0.00211	0.04205	0.00039	0.04751	0.00390	0.03682
F12	0.00230	0.03605	0.00041	0.04172	0.00438	0.02970
F17	0.00242	0.03332	0.00051	0.03885	0.00458	0.02675
F20	0.00246	0.03204	0.00055	0.03755	0.00465	0.02534

4.7 Estimation Results

Table 4.2 shows the results of the Kim filter estimation and Figure 4.3 shows the implied term structure of the futures prices for the whole sample and the two sub-samples. For comparison purposes, the futures term structures implied by the Gibson and Schwartz (1990) two-factor model for the three periods are included.

Regarding the estimated parameters of B&S-RS model, all the parameters are significant except those for the market price of risk[8].

Regime one is characterized by negative convenience (δ_1) and higher volatility (σ_1). On the other hand, regime two is characterized by positive convenience (δ_2) and lower volatility (σ_2). It is clear from the figure that the first regime corresponds to a positive slope of the futures term structure while the second regime corresponds to a negative slope of the futures term structure. The volatility estimates of both regimes are high reflecting the higher variability of crude oil markets as it is the case in energy markets and commodity markets in general (see for example Deaton and Laroque (1996)). However, this result is at odds with the theory of storage prediction which asserts that higher convenience yield is associated with low level of inventory and high volatility

The classification of observation into the two regimes coincides with the observed slope of the futures term structure as shown in figure 4.4. There are two methods in literature

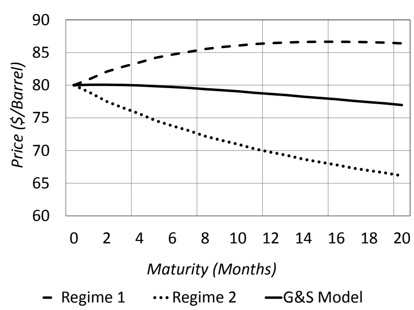
[8]Standard test statistics (such as Lagrange Multiplier, Likelihood Ratio and Wald test statistics) cannot be used to test whether there is one regime or two regimes in the data. This is because the parameters related to the second regime are not identified under the null of no regime-switching. In this case, standard asymptotic distribution theory cannot be applied and the usual test statistics are not chi-squared distributed asymptotically. Hansen (1996) proposed a standardized likelihood ratio statistic to conduct the test in such situations. The idea is to concentrate the identified parameters out of the likelihood function and take the supremum of the concentrated likelihood function over the possible unidentified parameters after standardizing it. The optimization can be done by using a grid search across plausible values for the nuisance parameters which is computationally extremely expensive unless the model is very simple.

Table 4.2. Kim Filter Estimation Results of the B&S-RS Model

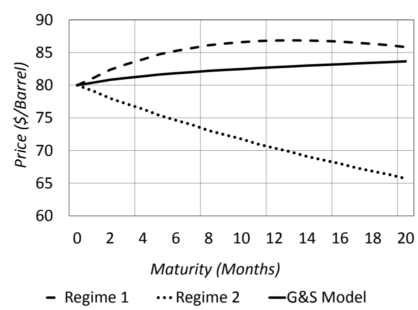
From	01/01/1992		01/01/1992		01/01/2000	
To	6/10/2011		01/01/2000		6/10/2011	
	($r = 0.0315$)		($r = .0455$)		($r = 0.0205$)	
	F1, F3, F7 & F12		F1, F3, F7 & F12		F1, F3, F7 & F12	
	value	SE	value	SE	value	SE
δ_1	-0.0916	0.0259	-0.1321	0.0411	-0.1155	0.0297
σ_1	0.2783	0.0082	0.2629	0.0248	0.3137	0.0106
λ_1	-0.1290	0.0988	-0.3115	0.2580	-0.1210	0.1251
δ_2	0.2425	0.0179	0.1777	0.0332	0.1983	0.0145
σ_2	0.2223	0.0072	0.1659	0.0122	0.2514	0.0084
λ_2	0.3723	0.0870	0.2023	0.0959	0.3406	0.0998
$\pi_{11}^{\mathbb{Q}}$	0.9850	0.0008	0.9846	0.0014	0.9812	0.0010
$\pi_{22}^{\mathbb{Q}}$	0.9920	0.0005	0.9922	0.0010	0.9965	0.0004
$\pi_{11}^{\mathbb{P}}$	0.9905	0.0061	0.9815	0.0290	0.9937	0.0036
$\pi_{22}^{\mathbb{P}}$	0.9889	0.0082	0.9921	0.0125	0.9879	0.0090
v_1	0.0512	0.0002	0.0558	0.0004	0.0501	0.0002
v_2	0.0317	0.0004	0.0342	0.0006	0.0279	0.0002
v_3	0.0001	0.0000	0.0002	0.0000	0.0001	0.0000
v_4	0.0220	0.0002	0.0268	0.0003	0.0219	0.0002
LL	6914		2605		4084	
	F1, F4, F9 & F13		F1, F4, F9 & F13		F1, F4, F9 & F13	
	value	SE	value	SE	value	SE
δ_1	-0.0842	0.0269	-0.1194	0.0428	-0.1089	0.0326
σ_1	0.2685	0.0079	0.2556	0.0252	0.3047	0.0103
λ_1	-0.1205	0.0946	-0.2933	0.2552	-0.1122	0.1212
δ_2	0.2310	0.0192	0.1707	0.0329	0.1900	0.0157
σ_2	0.2093	0.0067	0.1574	0.0129	0.2393	0.0077
λ_2	0.3598	0.0808	0.1913	0.0951	0.3291	0.0928
$\pi_{11}^{\mathbb{Q}}$	0.9861	0.0009	0.9863	0.0014	0.9823	0.0011
$\pi_{22}^{\mathbb{Q}}$	0.9927	0.0006	0.9926	0.0009	0.9969	0.0005
$\pi_{11}^{\mathbb{P}}$	0.9906	0.0065	0.9814	0.0325	0.9937	0.0038
$\pi_{22}^{\mathbb{P}}$	0.9889	0.0084	0.9921	0.0130	0.9878	0.0085
v_1	0.0541	0.0001	0.0468	0.0004	0.0419	0.0001
v_2	0.0267	0.0002	0.0268	0.0002	0.0219	0.0002
v_3	0.0001	0.0000	0.0002	0.0000	0.0001	0.0000
v_4	0.0220	0.0002	0.0268	0.0003	0.0319	0.0002
LL	6924		2607		4089	

Figure 4.3. The Implied Term Structure of Futures

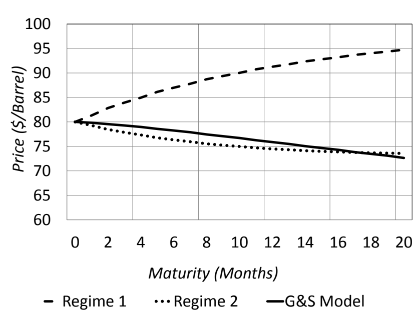
For both models, the sport price is set equal to 80\$. For G&S model, the initial value of convenience yield is set to the delirium level, that is $\delta_0 = \theta$



(a) The Whole Sample



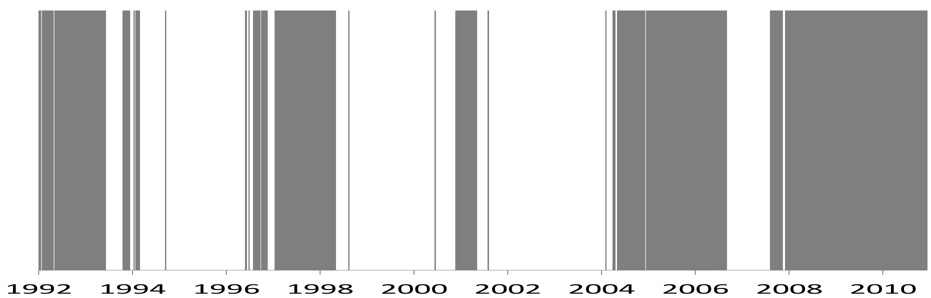
(b) The First Sub-sample



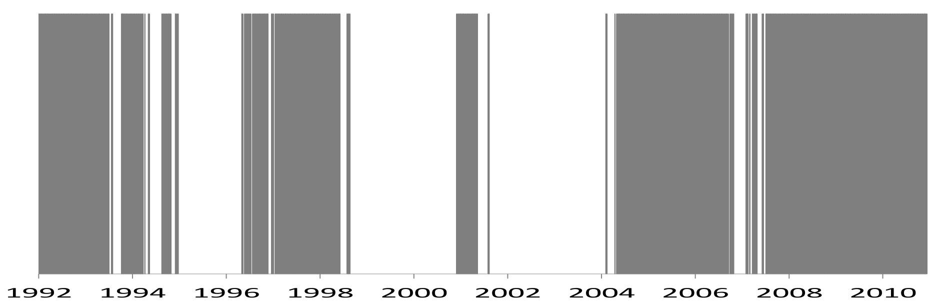
(c) The Second Sub-Sample

Figure 4.4. Observation Classification into Regimes

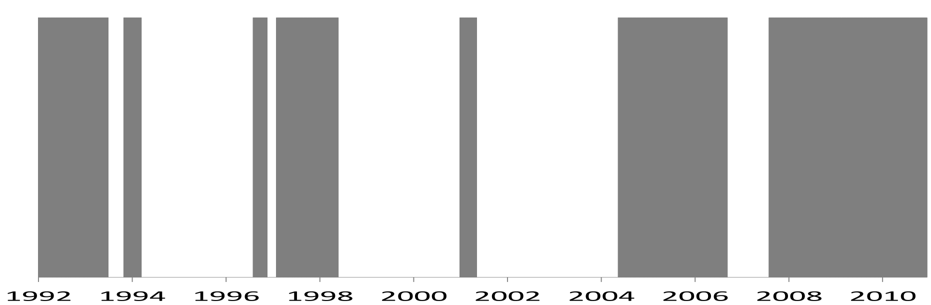
Second Sub-period



(a) The sign of $F6 - F1$: shaded area is for positive sign



(b) Implied Classification Using Minimum Error Method: Shaded area for regime 1



(c) Implied Classification Using Smoothed Probability Method: Shaded area for regime 1 to classify the observation into its regime. The first one is to use the smoothed regime probabilities, i.e. $Pr[s_t = j|J_T]$. If $Pr[s_t = j|J_T] > .5$, then t is classified into regime j. This method was suggested by Hamilton (1994). The other method is to compare the fitted prices of each regime with the actual prices at each time and classify the observation to the regime that has the least error. That is, if the fitted futures price at time t for regime i is closer to the actual price, then, s_t is set to equal i. This method has been used by Bansal and Zhou (2002).

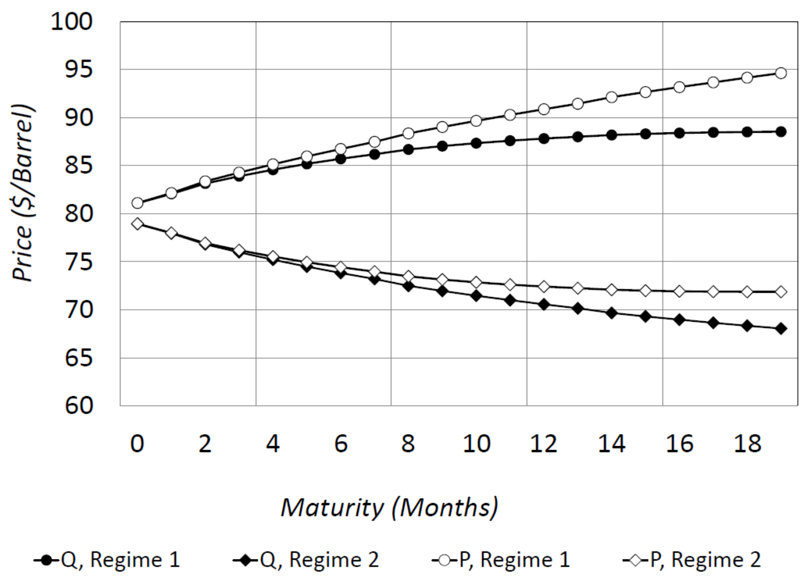
From figure 4.4, comparing 4.4(a) with 4.4(b) and 4.4(c), it is clear that the first regime corresponds to the market having positive slope while the second one corresponds to the market having a negative slop. In other words, the first regime shows the market when being in contango and the second regime shows the market when being in backwardation.

Transitional probabilities, $\pi_{11}^{\mathbb{P}}$ and $\pi_{22}^{\mathbb{P}}$, in both regimes reflect the persistence of each regime. Note that the reported values are the probabilities to switch between the two regimes in one week. The values are very high for both regimes because the a period of one week is too short to allow for switching. To make a clearer picture, the corresponding annual transitional probabilities are calculated as follows :

$$\begin{bmatrix} 0.9905 & 1 - 0.9905 \\ 1 - 0.9889 & 0.9889 \end{bmatrix}^{(52)} = \begin{bmatrix} 0.6951 & 0.3049 \\ 0.3563 & 0.6437 \end{bmatrix}.$$

That is, the probability to stay in the same regime after one year is 0.6951 for regime 1 and 0.6437 for regime 2. The result shows that both regimes are highly persistence. Moreover, the market stays slightly longer in the first regime where it has lower volatility and positive convenience yield. The recent persistence in regime one at the end of the financial crisis and afterward, as shown in Figure 4.4, contributes to the fact that $\pi_{1,1}^{\mathbb{P}}$ is slightly higher than $\pi_{22}^{\mathbb{P}}$ although the market visits regime two more than regime one in the estimation period, as shown in Table 4.1.

Figure 4.5. The Impact of the Market Price of Regime Switching Risk



The transitional probabilities in the pricing measure, measure $\mathbb{Q}$, is given by $\pi_{11}^{\mathbb{Q}}$ and $\pi_{22}^{\mathbb{Q}}$. These estimates reflects how the market reacts to the risk of the regime shift. Again, the annual transitional probabilities are calculated as follows:

$$\begin{bmatrix} 0.9850 & 1-0.9850 \\ 1-0.9920 & 0.9920 \end{bmatrix}^{(52)} = \begin{bmatrix} 0.5423 & 0.4577 \\ 0.2441 & 0.7559 \end{bmatrix}.$$

These number shows that the risk of switching from regime one to regime two in one year is higher in $\mathbb{Q}$ than in the actual measure $\mathbb{P}$ $(0.4577 > 0.3049)$. On the other hand, the risk of switching from regime two to regime one in one year is lower in $\mathbb{Q}$ than in the actual measure $\mathbb{P}$ $(0.2441 < 0.3563)$. If the market is currently in regime one, where futures prices are higher than the spot price, risk-averse market participants set the switching risk (the probability that futures prices will drop lower than the spot price) to be higher than actual risk. On the other hand, if the market is currently in regime two, where futures prices are lower than the spot price, they set the switching risk (the probability that futures prices will jump up above the spot price) to be lower than the actual risk. In both cases, they achieve that by setting futures prices lower than what is expected to be if they are risk neutral in term of regime shift. Figure 4.5 explains this fact by depicting the implied futures term structure in each regime using the transitional probabilities of both measures. it shows that risk averse market participants, to account for the risk of regime shift, set the futures prices lower than what spot price is expected to be at expiry. Moreover, the reduction increases as the maturity of the futures contract increases.

Table 4.2 also shows that the first regime has a negative market price of risk, $\lambda_1 = -0.129$, while the second regime has a high and a positive market price of risk, $\lambda_2 = 0.37$. That is, in the first regime, market participants trade futures contracts at higher prices than expected at maturity as a reward of bearing the price risk. On the other hand, in the second regime, they trade futures contracts at lower prices than expected at maturity. This is also confirmed by the observation from Table 4.1 where the returns are negative in contango (regime one according to the estimates of our model) and positive in backwardation (regime two according to the estimates of our model). However, the standard errors of the market price of risk (MPR) parameters, λ_1 and λ_2, are relatively high compared with the other parameters. The reason for that is, as explained in Schwartz and Smith (2000), the MPR parameters cannot be directly identified from futures prices.

Table 4.2 shows that the the same pattern of results appears also in the two sub-samples reflecting the stability of the parameters.

Table 4.3. Kalman Filter Estimation Results of the G&S Two-factor Model

	From 01/01/1992 To 6/10/2011		From 01/01/1992 To 01/01/2000		From 01/01/2000 To 6/10/2011	
	F1, F3, F7 & F12		F1, F3, F7 & F12		F1, F3, F7 & F12	
	value	SE	value	SE	value	SE
θ	0.0827	0.0023	0.0392	0.0032	0.1149	0.0030
κ	1.3994	0.0219	1.6254	0.0348	1.3637	0.0240
σ_x	0.3718	0.0052	0.3186	0.0088	0.3873	0.0072
σ_δ	0.3892	0.0080	0.4327	0.0150	0.4063	0.0084
$\rho_{x\delta}$	0.8721	0.0085	0.9516	0.0080	0.7567	0.0108
λ_x	0.1360	0.0764	0.0789	0.0902	0.1002	0.1073
λ_δ	0.0166	0.0839	0.1642	0.1502	-0.1029	0.1359
υ	0.0107	0.0001	0.0108	0.0001	0.0096	0.0001
LL	9802.6		3727.2		5645.4	

4.7.1 Model Comparison

The Kalman filter estimation results for the G&S model two-factor model are shown in Table 4.3. The G&S model implies that there is only one equilibrium long-run slope of the futures term structure that is dictated by the parameter θ. That is, the possibility that the futures term structure shift its long-run slope in the future is ignored. If the speed to return to this long-run slope, which is dictated by the parameter κ is fast (which is the case for estimated κ here and is also shown by Gibson and Schwartz (1990)), this would have significant impact on long-term forecasting, investment and risk management decisions where the futures term structure is used as a risk-adjusted forecast for the future spot prices. Figure 4.3 shows how the equilibrium slope of the G&S model is different across the two subperiods which reflect the instability of θ, the parameter that dictates the equilibrium slope of the futures term structure.

In term of comparing the performance of B&S-RS model with the G&S model in fitting the futures term structure, figure 4.6 shows that although the G&S model outperforms B&S-RS model for short-term maturities, B&S-RS model outperforms the G&S model for long-term maturities. Figures 4.7 and 4.8 show the same pattern for the two sub-periods. This result is an implication of the assumption of the G&S model that the convenience yield reverts to only one equilibrium level which implies that the futures term structure should revert to one slope all the time.

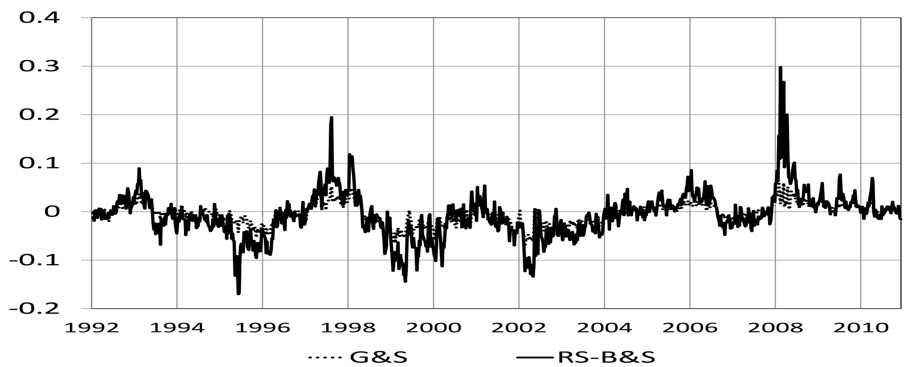
4.7.2 The Impact of the Transitional Probabilities

Within the framework of our regime switching model, the transitional probabilities $\pi_{11}^{\mathbb{Q}}$ and $\pi_{22}^{\mathbb{Q}}$ have an important role in shaping the term structure of the futures prices. Figure 9 shows how the term structure changes with different values of the transitional probabilities and for each regime.

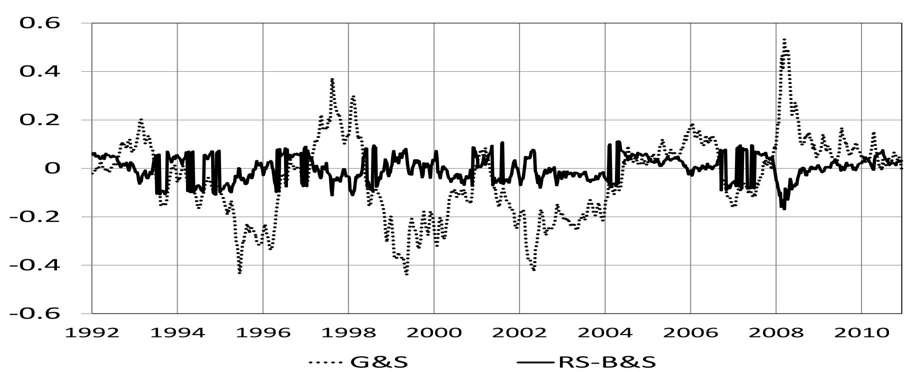
For easy presentation, denote $p = \pi_{11}^{\mathbb{Q}}$ and $q = \pi_{22}^{\mathbb{Q}}$. Consider the market to be currently in regime one where the slope of the forward curve is positive. Figure 4.9(a) shows that as p decrease relative to q, the far end of the curve bends down. This is because the probability to be in regime 2, which is characterized by having negative slope, is now higher. Figure 4.9(b) shows that as q decreases, the far end of the curve bends up because the probability to be in regime 1, which is characterized by having positive slope, is now higher. However, the effect of changing q is much smaller than the effect of changing p because the market is now in regime one. The same pattern is also seen when the market is currently in regime 2 where the forward curve has a negative slope. figure 4.9(d) shows that as q decrease

Figure 4.6. Model Performance Compared to Gibson and Schwartz (1990) Model

Tho Whole Period



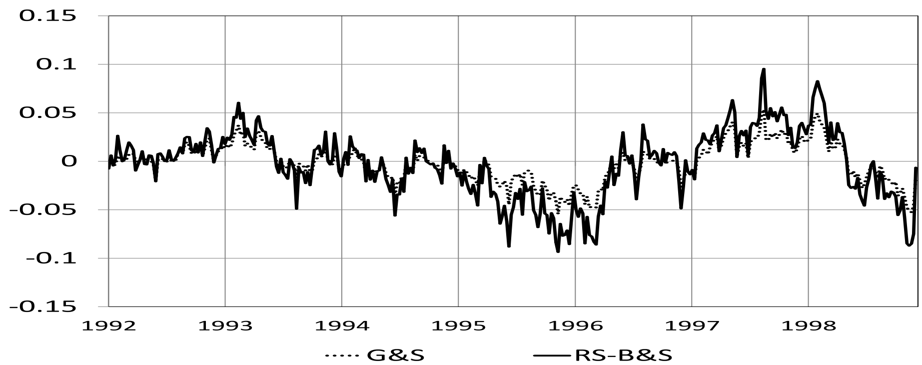
(a) Pricing Error for log(F2)



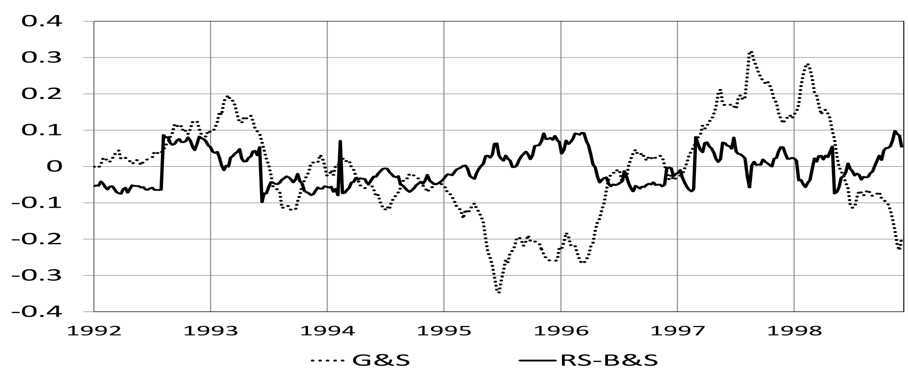
(b) Pricing Error for log(F15)

Figure 4.7. Model Performance Compared to Gibson and Schwartz (1990) Model

First Sub-period



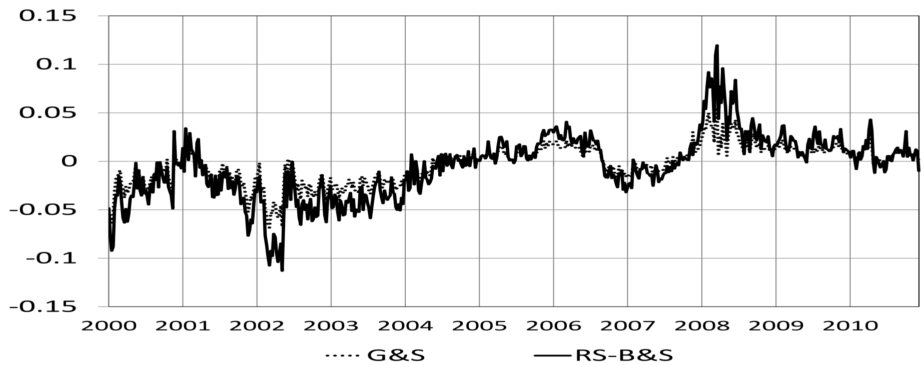
(a) Pricing Error for log(F2)



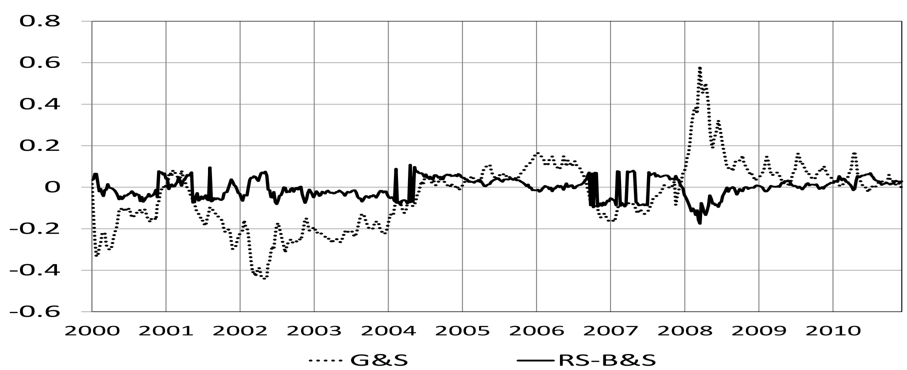
(b) Pricing Error for log(F15)

Figure 4.8. Model Performance Compared to Gibson and Schwartz (1990) Model

Second Sub-period



(a) Pricing Error for log(F2)



(b) Pricing Error for log(F15)

relative to p, the far end of the curve bends up because the probability to be in regime 1, which has positive slope, is now higher. Figure 4.9(c) shows that as p increases, the far end of the curve bends up because the probability to be in regime 1. However, the effect of changing p is much smaller than the effect of changing q because the market is currently in regime two.

The estimates of the transitional probabilities are assumed to be exogenous and constant and thus the estimated transitional probabilities reflect the persistence of the regimes during the estimation period, which is not necessary true for future periods. One way to improve the regime switching model prediction is to have time varying transitional probabilities. An appealing feature of B&S-RS model of this study is that the regimes that the market runs through correspond to the slope of futures term structure. A large amount of research has been conducted to explain the factors behind the changes in the shape of the futures term structure, especially what factors make the market being in contango or backwardation. Examples of such factors are: inventory level and volatility of the market (see Williams and Wright (1991) and Deaton and Laroque (1996)); hedging pressure, the net hedging position of short and long sides of the market, (see Roon et al. (2000), Dinceler et al. (2003) and Gorton et al. (2007)). These factors can be exploited to improve the estimation about the probability of switching from a regime to another.

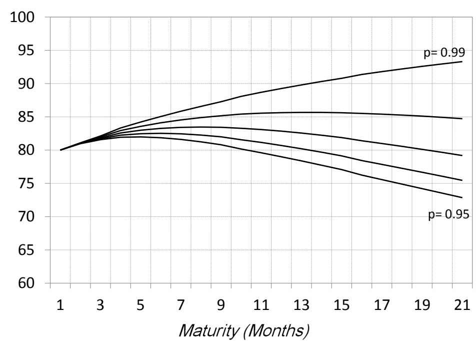
4.8 Concluding Remarks

In this chapter, we exploit regime switching models to study the movement in the crude oil futures term structures. In particular, we extend Brennan and Schwartz (1985) one-factor model to account for shifts in in the futures term structure between contango and backwardation and the reverse in discrete time setting. We allow for the regime shift risk as well as the market price risk. Moreover, we derive a closed from solution for the futures prices and used an extension to the Kalman filter suggested by Kim (1994) to estimate the model parameters in discrete time. Compared to the performance of the Gibson and Schwartz (1990) two-factor, the regime switching one-factor model of this study did a reasonable job. In particular, the model outperforms Gibson and Schwartz (1990) model for fitting the prices of longer maturities contracts. Moreover, it is found that the transitional probabilities played an important role in shaping the futures term structure implied by the model.

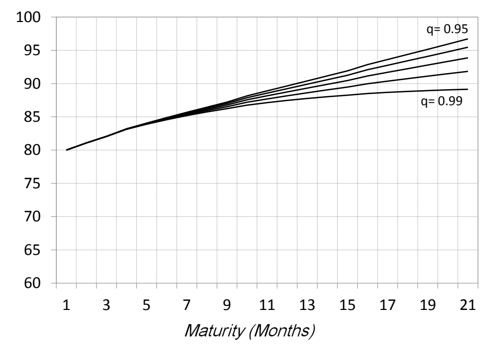
As a future extension to the model, one might add more state factors other than the spot price movement. In this case, one needs to be careful in choosing the added factors as an important assumption in deriving the futures formula is that the state variables are independent from the Markov chain process governing the regime switching. Another direction is to test the improvement of the model when the transitional probabilities are endogenous and using the variables that the literature finds have explanation power in determining the shape of the futures term structure such as inventory level and the net hedging position.

Figure 4.9. The Impact of the Transitional Probabilities

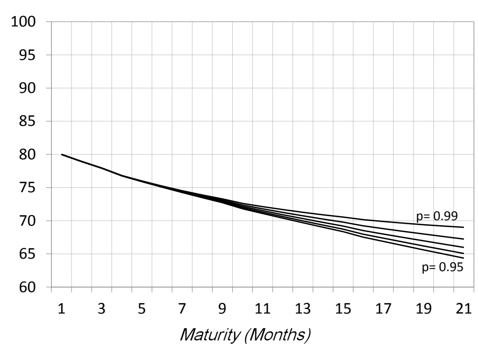
For easing the display of the graphs, $\pi_{11}^{\mathbb{Q}} = p$ and $\pi_{22}^{\mathbb{Q}} = q$



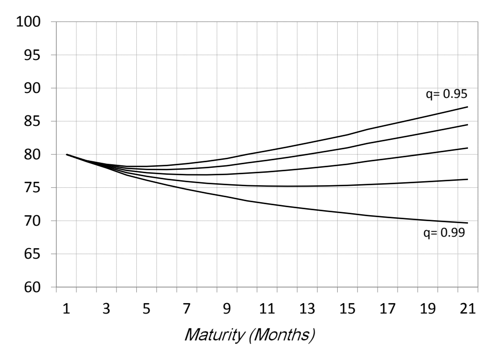
(a) Regime One: p changes and q is at estimated value



(b) Regime One: q changes and p is at estimated value



(c) Regime Two: p changes and q is at estimated value



(d) Regime Two: q changes and p is at estimated value

Chapter 5

Conclusion

This thesis attempts to contribute to the existing understanding of risk management in crude oil markets through three essays. In the first essay (chapter 2), the valuation of an oil sands project is studied using real options approach. Oil sands production consumes substantial amount of natural gas during extracting and upgrading. Natural gas prices are known to be stochastic and highly volatile which introduces a significant stochastic component in the extraction cost. The essay studies the impact of this stochastic component on the value of an oil sands project and its optimal operation. Before studying the valuation problem, the essay investigated the co-movement of crude oil and natural gas prices. It is found that although there are economic links between the two commodities, the empirical evidence in detecting a long-run relationship is weak especially if one incorporates the recent divergence in the two price series. Accordingly, in modeling the dynamics of the two prices, two models are proposed, one incorporates a long-run link between the two markets while the other has no such link. The valuation problem is solved using the Least Square Monte Carlo (LSMC) method proposed by Longstaff and Schwartz (2001) for valuing American options. The valuation results show that incorporating a long-run relationship between the two markets is a very crucial decision in the value of the project and in its optimal operation. The chapter shows that ignoring this long-run relationship makes the optimal policy sensitive to the dynamics of natural gas prices. On the other hand, incorporating this long-run relationship makes the dynamics of natural gas price process have a very low impact on valuation and the optimal operating policy. Natural gas is the primary fuel source for the oil sands industry. An interesting extension is to study the impact of having a new technology that replaces the natural gas as the source of fuel in oil sands extraction and upgrading. Moreover, natural gas prices have fallen recently due to the emergence of a large supply from gas shale deposits. An interesting extension is to study the potential impact of this development on the volatility of natural gas prices, on the co-movement of natural gas and crude oil prices, on the search for other alternative fuel sources and in turn on the value of an oil sands project.

In the second essay (chapter 3), the relationship between the slope of the futures term structure and volatility in the crude oil market is investigated using a measure of the slope based on principal component analysis (PCA) proposed by Borovkova (2006). The advantage of using PCA is that all futures prices are used in calculating the slope of the forward curve. I show in the essay that this slope measure has more power to predict the market volatility than the measures commonly used in literature. The essay begins by reviewing the main theories of the relation between spot and futures prices and considering the implication of each theory on the relation between the slope of the forward curve and volatility. Both the literature in commodities price modeling and the literature in exhaustible resources pricing contain theories which have some implications for the equilibrium state of this relationship. Five main theories are presented and their implications are compared. After that, the diagonal VECH model of Bollerslev et al. (1988) was used to analyze the relationship between of the forward curve slope and the variances of the spot and futures prices and the covariance between them. The results show that there is a significant quadratic relationship. That is, the variances are expected to be higher whenever the slope is steeper regardless of its sign. Moreover, the essay shows that exploiting this relation improves the hedging performance using futures contracts.

The third essay (chapter 4) attempts to model the spot price process of crude oil using the notion of convenience yield in a regime switching framework. The notion of convenience yield plays a central role in commodities price modeling as it derives the relationship between futures and spot prices in the commodities markets. Unlike the existing studies, which assume the convenience yield to have either a constant value or to have a stochastic behavior with mean reversion to one equilibrium level, the model of this essay extends the Brennan and Schwartz (1985) model to allows for regime switching in the convenience yield along with the other parameters. In the essay, a closed form solution for the futures price is derived. The parameters are estimated using an extension to the Kalman filter proposed by Kim (1994). The regime switching one-factor model of this study did a reasonable job. In particular, the model outperforms the Gibson and Schwartz (1990) two factor model for fitting the prices of longer maturities contracts. Moreover, I found that the transitional probabilities played an important role in shaping the futures term structure implied by the model. As a future extension to the model, more state factors other than the spot price movement might be added. In this case, one needs to be careful in choosing the added factors as an important assumption in deriving the futures formula is that the state variables are independent from the Markov chain process governing the regime switching. Moreover, the model can be improved by relaxing the assumption of constant traditional probabilities. Better results might be obtained if the transitional probabilities are time varying. However, this will be at the cost of making the model more complex in terms of finding a closed form solution for the futures price and in terms of estimation

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APPENDICES

Appendix A

Appendix to Chapter 4

A.1 The Definition of the $\mathbb{Q}$ Measure

$$
\begin{aligned}
\text{Price}_t(\mathcal{G}_{t+1}|X_t, s_t = j) &= E^{\mathbb{P}}(\mathcal{M}_{t,t+1}\mathcal{G}_{t+1}|X_t, s_t = j) \\
&= e^{-r_t}\sum_{k=1}^{S}\int \frac{\mathcal{M}_{t,t+1}^{j,k}\mathcal{G}_{t+1}}{e^{-r_t}}\mathbb{P}(dX_{t+1}, s_{t+1} = k|X_t, s_t = j) \\
&= e^{-r_t}\sum_{k=1}^{S}\int e^{-\gamma_{t,t+1}^{j,k}-\frac{1}{2}\Lambda_t^{j\top}\Lambda_t^{j}-\Lambda_t^{j\top}\epsilon_{t+1}}\mathcal{G}_{t+1}\mathbb{P}(dX_{t+1}, s_{t+1} = k|X_t, s_t = j) \\
&= e^{-r_t}\sum_{k=1}^{S}\int \mathcal{G}_{t+1}\mathbb{Q}(dX_{t+1}, s_{t+1} = k|X_t, s_t = j) \\
&= e^{-r_t}E^{\mathbb{Q}}[\mathcal{G}_{t+1}]
\end{aligned}
$$

A.2 X_t in the $\mathbb{Q}$ Measure

Denote $E^{\mathbb{P}}_{t,j}[\cdot] \equiv E^{\mathbb{P}}[\cdot|X_t, s_t = j]$. Before deriving the dynamic of X_t in measure $\mathbb{Q}$, observe that the no arbitrage price of a zero coupon bond that pays \$1 at $t+1$ is:

$$e^{-r_t} = E^{\mathbb{P}}_{t,j}[1 \cdot \mathcal{M}_{t,t+1}] = E^{\mathbb{P}}_{t,j}\left[e^{-r_t - \gamma_{t,t+1} - \frac{1}{2}\Lambda_t^\top \Lambda_t - \Lambda_t^\top \epsilon_{t+1}}\right],$$

which implies:

$$1 = E^{\mathbb{P}}_{t,j}\left[e^{-\gamma_{t,t+1}}\right] = \sum_{k=1}^{S} \pi_{j,k} e^{-\gamma^{(j,k)}}$$

The conditional moment generation function (MGF) of X_{t+1} given $s_t = j$ is

$$\begin{aligned}
E^{\mathbb{Q}}\left[e^{u^\top X_{t+1}}|X_t, s_t = j\right] &= e^{r_t} E^{\mathbb{P}}\left[\mathcal{M}_{t,t+1} e^{u^\top X_{t+1}}|X_t, s_t = j\right] \\
&= E^{\mathbb{P}}_{t,j}\left[e^{-\gamma_{t,t+1} - \frac{1}{2}\Lambda_t^\top \Lambda_t - \Lambda_t^\top \epsilon_{t+1}} \cdot e^{u^\top X_{t+1}}\right] \\
&= E^{\mathbb{P}}_{t,j}\left[e^{-\gamma_{t,t+1} - \frac{1}{2}\Lambda_t^\top \Lambda_t - \Lambda_t^\top \epsilon_{t+1}} \cdot e^{u^\top(\alpha^{(s_t)} + \beta^{(s_t)} X_t) + u^\top \Sigma^{(s_t)} \epsilon_{t+1}}\right] \\
&= E^{\mathbb{P}}_{t,j}\left[e^{-\gamma_{t,t+1}}\right] \cdot E^{\mathbb{P}}_{t,j}\left[e^{-\frac{1}{2}\Lambda_t^\top \Lambda_t - \Lambda_t^\top \epsilon_{t+1}} \cdot e^{u^\top(\alpha^{(s_t)} + \beta^{(s_t)} X_t) + u^\top \Sigma^{(s_t)} \epsilon_{t+1}}\right] \\
&= E^{\mathbb{P}}_{t,j}\left[e^{-\frac{1}{2}\Lambda_t^\top \Lambda_t + u^\top(\alpha^{(s_t)} + \beta^{(s_t)} X_t) + (u^\top \Sigma^{(s_t)} - \Lambda_t^\top)\epsilon_{t+1}}\right] \\
&= e^{-\frac{1}{2}\Lambda_t^{(j)\top} \Lambda_t^{(j)} + u^\top(\alpha^{(j)} + \beta^{(j)} X_t) + \frac{1}{2}\left(u^\top \Sigma^{(j)} - \Lambda_t^{(j)\top}\right)\left(u^\top \Sigma^{(j)} - \Lambda_t^{(j)\top}\right)^\top} \\
&= e^{u^\top(\alpha^{(j)} + \beta^{(j)} X_t - \Sigma^{(j)} \Lambda_t^{(j)}) + \frac{1}{2} u^\top \Sigma^{(j)} \Sigma^{(j)\top} u} \\
&= e^{u^\top(\alpha^{(j)} - \Lambda^{(j)} + \beta^{(j)} X_t + \frac{1}{2} u^\top \Sigma^j \Sigma^{j\top} u} \\
&= e^{u^\top(\hat{\alpha}^j + \beta^j X_t) + \frac{1}{2} u^\top \Sigma^j \Sigma^{j\top} u},
\end{aligned}$$

which implies that the behavior of X_{t+1} is as follows:

$$X_{t+1} = \hat{\alpha}^{(s_t)} + \beta^{(s_t)} X_t + \Sigma^{(s_t)} \epsilon_{t+1}$$

A.3 Futures Price Formula Derivation

$$
\begin{aligned}
F_{t,n} &= E^{\mathbb{Q}}\left[F_{t+1,n-1}|X_t, s_t = j\right] \\
F_n\left(X_t, s_t\right) &= E^{\mathbb{Q}}\left[F_{n-1}\left(X_{t+1}, s_{t+1}\right)|X_t, s_t = j\right] \\
e^{A_n^{(j)}+B_n^{(j)}X_t} &= E^{\mathbb{Q}}\left[e^{A_{n-1}^{(s_{t+1})}+B_{n-1}^{(s_{t+1})}X_{t+1}}|X_t, s_t = j\right] \\
&= \sum_{k=1}^{S}\pi_{jk}E^{\mathbb{Q}}\left[e^{A_{n-1}^{(k)}+B_{n-1}^{(k)}X_{t+1}}\right] \\
&= \sum_{k=1}^{S}\pi_{jk}E^{\mathbb{Q}}\left[e^{A_{n-1}^{(k)}+B_{n-1}^{(k)}\left(\hat{\alpha}^{(j)}+\hat{\beta}^{(j)}X_t+\Sigma^{(j)}\epsilon_{t+1}\right)}\right] \\
&= \sum_{k=1}^{S}\pi_{jk}e^{A_{n-1}^{(k)}+B_{n-1}^{(k)}\left(\hat{\alpha}^{(j)}+\hat{\beta}^{(j)}X_t\right)}E^{\mathbb{Q}}\left[e^{B_{n-1}^{(k)}\Sigma^{(j)}\epsilon_{t+1}}\right] \\
&= \sum_{k=1}^{S}\pi_{jk}\cdot e^{A_{n-1}^{(k)}+B_{n-1}^{(k)}\left(\hat{\alpha}^{(j)}+\hat{\beta}^{(j)}X_t\right)+\frac{1}{2}B_{n-1}^{(k)}\Sigma^{(j)}\Sigma^{(j)\top}B_{n-1}^{(k)\top}} \\
A_n^{(j)}+B_n^{(j)}X_t &= \log\left(\sum_{k=1}^{S}\pi_{jk}\cdot e^{A_{n-1}^{(k)}+B_{n-1}^{(k)}\left(\hat{\alpha}^{(j)}+\hat{\beta}^{(j)}X_t\right)+\frac{1}{2}B_{n-1}^{(k)}\Sigma^{(j)}\Sigma^{(j)\top}B_{n-1}^{(k)\top}}\right)
\end{aligned}
$$

since $\beta^{(1)} = \beta^{(2)}$ for both B&S-RS and G&S models and setting $B(n)^{(1)} = B(n)^{(2)} = B(n)$, one gets:

$$
A_n^{(j)} + B_nX_t = B_{n-1}\hat{\beta}X_t + \text{Log}\left(\sum_{k=1}^{S}\pi_{jk}\cdot e^{A_{n-1}^{(k)}+B_{n-1}\hat{\alpha}^{(j)}+\frac{1}{2}B_{n-1}\Sigma^{(j)}\Sigma^{(j)\top}B_{n-1}^{\top}}\right)
$$

Wich implies:

$$A_n^{(j)} = \text{Log}\left(\sum_{k=1}^{S} \pi_{jk} \cdot e^{A_{n-1}^{(k)}}\right) + B_{n-1}\hat{\alpha}^{(j)} + \frac{1}{2}B_{n-1}\Sigma^{(j)}\Sigma^{(j)\top}B_{n-1}^{\top}$$
$$B_n = B_{n-1}\beta.$$

A.4 Hamilton (1994) Filtration Procedure

Step 1 Calculate $Pr[s_{t-1} = i, s_t = j|J_{t-1}]$as following:

$$\begin{aligned} Pr[s_{t-1} = i, s_t = j|J_{t-1}] =& Pr[s_t = j|s_{t-1}] \cdot \sum_{h=1}^{S} Pr[s_{t-2} = h, s_{t-1} = i|J_{t-1}] \\ =& \pi_{t-1,t}^{(i,j)} \cdot \sum_{h=1}^{S} Pr[s_{t-2} = h, s_{t-1} = i|J_{t-1}], \end{aligned}$$

where $\pi_{t-1,t}^{(i,j)}$ is the transitional probability and can be taken rom the transitional matrix.

Step 2 Calculate the joint density function of y_t and (s_{t-1}, s_t) as following:

$$f(y_t, s_{t-1} = i, s_t = j|J_t) = f(y_t|s_{t-1} = i, s_t = j, J_t) \cdot Pr[s_{t-1} = i, s_t = j|J_{t-1}]$$

where

$$f(y_t|s_{t-1} = i, s_t = j, J_t) = (2\pi)^{-N/2}|H_t^{(i,j)}|^{-1/2}\text{exp}(-\frac{1}{2}\eta_{t|t-1}^{(i,j)\top}H_t^{(i,j)}\eta_{t|t-1}^{(i,j)}).$$

Step 3 Calculate :

$$Pr[s_{t-1} = i, s_t = j | J_t] = \frac{f(y_t, s_{t-1} = i, s_t = j | J_{t-1})}{f(y_t | J_{t-1})} \tag{A.1}$$

where

$$f(y_t | J_{t-1}) = \sum_{j=1}^{S} \sum_{i=1}^{S} f(y_t, s_{t-1} = i, s_t = j | J_{t-1}).$$