MODELS OF EFFICIENT CONSUMER PRICING SCHEMES IN ELECTRICITY MARKETS

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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ABSTRACT

Suppliers in competitive electricity markets regularly respond to prices that change hour by hour or even more frequently, but most consumers respond to price changes on a very different time scale, i.e. they observe and respond to changes in price as reflected on their monthly bills. This thesis examines mixed complementarity programming models of equilibrium that can bridge the speed of response gap between suppliers and consumers, yet adhere to the principle of marginal cost pricing of electricity. It develops a computable equilibrium model to estimate the time-of-use (TOU) prices that can be used in retail electricity markets. An optimization model for the supply side of the electricity market, combined with a price-responsive geometric distributed lagged demand function, computes the TOU prices that satisfy the equilibrium conditions. Monthly load duration curves are approximated and discretized in the context of the supplier's optimization model. The models are formulated and solved by the mixed complementarity problem approach. It is intended that the models will be useful (a) in the regular exercise of setting consumer prices (i.e., TOU prices that reflect the marginal cost of electricity) by a regulatory body (e.g., Ontario Energy Board) for jurisdictions (e.g., Ontario) where consumers' prices are regulated, but suppliers offer into a competitive market, (b) for forecasting in markets without price regulation, but where consumers pay a weighted average of wholesale price, (c) in evaluation of the policies regarding time-of-use pricing compared to the single pricing, and (d) in assessment of the welfare changes due to the implementation of TOU prices.

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1. Introduction

System operations in many electric power industries worldwide have been experiencing dramatic changes due to the restructuring of the industry. In developing countries, the poor performance of the vertically integrated monopolies of power system (low labor productivity, poor service quality, high system losses, inadequate investment incentives and lower prices that could not cover costs and support investments) was the main reason for these restructuring and liberalization efforts (Joskow, 2003b). In developed countries, the performance was generally better, but high operating costs, construction costs overruns of new facilities, political pressures that drove costly programs and high retail costs to cover these costs stimulated the restructuring efforts (Joskow, 1998). These vertically integrated monopolies (generation, transmission, distribution and retail supply) have been deregulated[1] to establish a competitive market structure. Now, generally, the industry is comprised of two main markets: retail and wholesale. The retail market is often regulated and the consumers pay a flat per unit price for electricity, whereas the wholesale market is open to competition where retail suppliers, generator companies, distributors and others (i.e. arbitragers, transmission owners, large industrial customers, etc.) act as players. The price in the wholesale market is very volatile and time-varying as opposed to the price

[1] The deregulation programs have included privatization, separation of vertical segments that are potentially competitive, creation of competitive wholesale and retail markets and application of performance based regulatory mechanisms. See Joskow (2003b) for details.

in the retail market, where it has been usually fixed by regulatory bodies. Joskow (2003a) asserted the following analogy for this problem:

> "Charging the same price for kWh regardless of when it is consumed is like a supermarket charging for a cart of groceries on the average cost per pound of groceries … rather than based on the individual items in the cart."

The underlying objective of restructuring and introducing competition into these markets is to motivate efficiency improvements and price reductions. The literature on the subject is now vast, because the overall restructuring process appears to be unusually difficult[2].

The wholesale and retail markets are still incomplete and inefficient (Joskow, 2003b). The incompleteness is inevitable because of the unique properties of electricity: non-storability[3], instant supply-demand balance requirement and uncontrollable flow over lines[4]. The inefficiency stems from several reasons such as oligopolistic behavior (exercise of market power), barriers to entry (high capital investment requirement), capital intensive production, various supply and demand conditions (technical constraints on supply side and short-term inelasticity of demand) and finally, prices that do not reflect the marginal cost of electricity (Joskow, 2003b; Wilson, 2002).

Especially in liberalized electricity market design, eliminating all inefficiencies in the market is almost impossible. It is therefore inevitable to trade one inefficiency for another in the practice of electricity market design (Daxhelet and Smeers, 2001).

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[2] For a comprehensive discussion of the origins of these difficulties, see Wilson (2002).
[3] Methods to store electricity are not very efficient.
[4] There have been some improvements in controlling the electricity flows over lines with 'phase shifting'.

Therefore, this study only deals with the inefficiency that exists especially in the retail electricity markets because of fixed pricing structure that has also been a practice of regulated monopolies in the last decades.

The primary goal of this thesis is to develop a policy analysis tool to examine different pricing schemes in electricity markets. The thesis examines mixed complementarity programming models of equilibrium that can bridge the speed of response gap between suppliers and consumers, yet adhere to the principle of marginal cost pricing of electricity.

It is intended that the proposed models would be useful for jurisdictions (e.g., Ontario) where consumers' prices are regulated, but suppliers offer into a competitive market. These models may also be used for forecasting in markets where there is no consumer price regulation, but consumers pay weighted average wholesale price. Regulatory bodies (e.g.., Ontario Energy Board) can analyze the different settings of consumer prices (i.e., TOU prices that reflect the marginal cost of electricity), and evaluate the policies regarding TOU pricing compared to the single pricing. Furthermore, the welfare changes due to implementation of these policies can be assessed.

The thesis is organized as follows: In sections 2 and 3, a background on general and partial equilibrium theory followed by an overview of pricing schemes and advanced metering technologies required for these schemes are given. In Section 4, the computable equilibrium model and its underlying assumptions are introduced. In Section 5, the mathematical model with illustrative numerical examples followed by extensions of the model (fixed pricing model, representative weekday model and welfare analysis) is presented. The thesis is concluded with Section 6, in which the results are summarized and directions for future research are suggested.

2. General Equilibrium Modeling

2.1 Theory of General Equilibrium

The notion of "equilibrium" in economics literature was first introduced by Adam Smith in his well-known book, "The Wealth of Nations" in 1776. He introduced the concept of *invisible hand* as the force that brings the markets to an equilibrium, where supply meets the demand and efficient allocation of resources is achieved. However, he did not provide any "careful statements" or specific arguments about the efficiency proposition in a competitive market (Arrow and Hahn, 1971, p.2). Therefore, many economists attributed the full recognition and major contribution of the general equilibrium theory to Leon Walras (Arrow and Hahn, 1971, p.3; Mas-Collel *et al.*,1995; Shoven and Whalley, 1992). After Walras, many economists and researchers contributed to the literature: Pareto's optimal allocation analysis, Edgeworth's contract curve (Edgeworth Box), Arrow-Debreu's existence of equilibrium (1954), Scarf's computation of general equilibrium (1973) were the major cornerstones in general equilibrium theory. For a comprehensive history and review of general equilibrium theory, see Arrow and Kahn (1971), Kirman (1998), Varian (1978), Shoven and Whalley (1992), Scarf (1998) and Mas-Collel *et al.* (1995).

The theory of general equilibrium determines the prices and quantities in a perfectly competitive system. It is referred to as *"Walrasian equilibrium"* (Mas-Colell *et* *al.,* 1995)[5]. Arrow-Debreu (1954) proved the existence of equilibrium and Scarf (1973) used a fixed point theorem to compute the equilibrium. Thereafter, many researchers found algorithms to compute the equilibrium[6]. Dafermos and Nagurney (1984) used variational inequalities approach to formulate a general equilibrium model involving spatial networks. Mathiesen (1985) paraphrased the computation problem as a nonlinear complementarity problem and developed a computational algorithm using a sequence of linear complementarity problems. Dirkse and Ferris (1996) used a variant of Newton's method with a robust path-search algorithm that involves a piecewise linearization of path from current iteration to next point. They also authored a mixed complementarity solver accessible via General Algebraic Modeling System (GAMS), PATH (also accessible from other modeling languages, such as AMPL).

A lot of effort has gone into developing conditions that guarantee the uniqueness of the equilibrium. However, many of these conditions are found to be too restrictive for applied models (Kehoe, 1998).

2.2 *Computable General Equilibrium Modeling*

In applied economic research, the numerical methods (i.e., quantitative simulations) provide the decision makers and policy analyzers with the information that reveals the inherent complexities of interactions in economical models. They also allow monitoring the impact of structural policy changes (Bohringer and Rutherford,

[5] The definition and proof of the equilibrium conditions are beyond the scope of this thesis. The reader can refer to Manne (1985), Shoven and Whalley (1992) and Scarf (1998) for a comprehensive analysis.
[6] For the recent developments in computing equilibrium, see Scarf (1998)

2004). Computable equilibrium models have become prevalent in economic policy analysis, because they remove the need for working on small dimensional analytic models and incorporate much more details and complexities than analytical models. For example, the simultaneous impacts of several taxes can be observed as tax-policy models (Shoven and Whalley, 1984). Such models permit the evaluation of proposed changes on a tax policy.

Bohringer and Rutherford (2004), focus on the potential usefulness of computable equilibrium models for energy policy analysis. They provide an example of computable equilibrium model that bridge the gap between “bottom-up energy system models” and “top-down general equilibrium models”. Bottom-up energy system models are typically optimization problems that describe the energy system in detail. Top-down general equilibrium models, on the other hand, are general equilibrium models that capture the interactions, inefficiencies and income flows between energy markets and the remainder of the economy. This is useful because energy policies not only affect the energy markets, but also the other markets through indirect spillovers (i.e. double dividend from energy taxation, changes in international prices triggered by energy policy constraints and technological change induced by energy policies). If the indirect spillover effects of energy policies on non-energy markets are omitted, the model becomes a partial equilibrium model, which may yield very different results than a general equilibrium model (Bohringer and Rutherford, 2004).

Policy analysis and research in economic modeling for markets and games required the development of modeling languages and computer programs that allows the formulation of economic equilibrium models. The theoretical and practical developments in algorithms for computable equilibrium models yield alternative solvers and programs. GAMS (General Algebraic Modeling System), which was originally developed to assist economists at the World Bank in the quantitative analysis of economic policy questions (Rutherford, 1995), has several MCP (mixed complementarity problem) solvers that can handle large-scale equilibrium models. The most common MCP solvers in GAMS are PATH, NLPEC and MILES (a Mixed Inequality and non-Linear Equation Solver). These solvers are examined briefly in section 4.3.

2.3 Partial Equilibrium Modeling

A partial equilibrium model usually deals with a sector of an economy, and assumes that all prices other than the price of the commodity being studied are assumed to remain fixed (Varian, 1978). On the contrary, in general equilibrium models, all prices are variables and all markets clears. Thus, partial equilibrium models do not allow any interactions with other markets.

Cournot, Marshall and later neoclassical economists extensively used partial equilibrium analysis for a single market. The demand and supply of a single commodity is assumed to be a function of the price of that commodity. The equilibrium price is set such that demand and supply are equal. Therefore, partial equilibrium analysis is a special case of general equilibrium analysis (one commodity and one market) (Arrow and Hahn, 1971). However, it can be extended to a multi-commodity case[7].

Samuelson (1952) was the first economist to formulate a partial equilibrium model as a mathematical programming problem. He applied a general procedure in solving a problem of spatial equilibrium, using Enke's (1951) formulation. Samuelson's formulation showed that the problem of maximizing "net social payoff" (consumers' plus producers' surpluses in different regional markets minus the transportation costs) subject to regional commodity balance equations generates a set of optimality conditions that define the equilibrium in each regional market. Takayama and Judge (1971) used a linear price dependent demand and supply function to define a "quasi welfare function". They extended Samuelson's spatial equilibrium model to determine the prices, production, allocation and consumption for all regional commodities within the model. Moreover, they proposed a quadratic programming algorithm to obtain the competitive equilibrium solution. An example is given to clarify their approach (Thompson and Thore, 1992).

Consider a spatial network economy where the supply and demand markets are spatially separated and the competition is perfect. In equilibrium a commodity produced in plants ($k=1,\ldots,K$) are transported to the demand regions ($l=1,\ldots,L$) if the supply price plus the unit transportation cost is equal to the demand price. Let the

[7] See Mas-Collel *et al*. (1995, p.314) and Arrow and Hahn (1971) for the proof.

demand price in region l is a linear function of quantity demanded in region l, $\mathbf{q} = q_1,...,q_L$.

Demand price in region $l = \alpha_l - \beta_l q_l$ where α_l and β_l are positive constants.

Similarly, assume that the supply price (marginal cost) of plant k is a linear function of the supply quantity, $\mathbf{s} = s_1,...,s_K$.

Supply price in plant $k = \gamma_k + \delta_k s_k$ where δ_k is a positive constant.

Finally, let matrix $\mathbf{z} = \begin{bmatrix} z_{11} & \cdots & z_{1L} \\ \vdots & \ddots & \vdots \\ z_{K1} & \cdots & z_{KL} \end{bmatrix}$ denotes the quantity of commodity to be shipped from plant k to demand region l and $T(\mathbf{z})$ is the function of total transportation costs (e.g., $T(\mathbf{z}) = \sum_{k=1}^{K}\sum_{l=1}^{L} c_{kl} z_{kl}$, where c_{kl} is the unit cost of transporting from plant k to demand region l). $W(\mathbf{q}, \mathbf{s})$ is the "quasi welfare function" (Takayama and Judge, 1971, p.117), which is a measure of consumers' plus producers' surpluses. The following mathematical programming problem computes the equilibrium prices and quantities (Thompson and Thore, 1992).

$$\begin{array}{llr} \max\limits_{q_k,\, s_l,\, z_{kl}} & W(\mathbf{q}, \mathbf{s}) - T(\mathbf{z}) & [Dual] \\ \text{subject to} & \sum_{l=1}^{L} z_{kl} \le s_k \quad \forall \ k = 1,...,K & [u_k] \\ & \sum_{k=1}^{K} z_{kl} \ge q_l \quad \forall \ l = 1,...,L & [v_l] \\ & z_{kl},\ q_k,\ s_l \ge 0 \ \forall \ k = 1,...,K \ \ and \ \ l = 1,...,L & \end{array}$$

The dual variable u_k is the negative of the "imputed equilibrium price" of the commodity at the dock of plant k and the dual variable v_l is interpreted as the equilibrium price of the commodity in demand region l. Karush-Kuhn-Tucker (KKT) conditions for this optimization problem can be used to find the equilibrium quantities and prices.

The quasi welfare function, $W(\mathbf{q}, \mathbf{s})$, can be defined as follows:

$$W(\mathbf{q}, \mathbf{s}) = \textit{(areas under demand curve)} - \textit{(areas under supply curve)}$$
$$= \sum_{l=1}^{L} \left(\int (\alpha_l - \beta_l q_l) dq_l \right) - \sum_{k=1}^{K} \left(\int (\gamma_k + \delta_k s_k) ds_k \right)$$

Instead of maximizing the objective function above, a minimization of the "economic potential function", as explained below, can be used (Thompson and Thore, 1992, p.43).

$$\min_{q_k,\, s_l,\, z_{kl}} \; \sum_{k=1}^{K} \left(\int (\gamma_k + \delta_k s_k) ds_k \right) + T(\mathbf{z}) - \sum_{l=1}^{L} \left(\int (\alpha_l - \beta_l q_l) dq_l \right)$$

$$\min_{q_k,\, s_l,\, z_{kl}} \; \sum_{k=1}^{K} \left(\gamma_k s_k + 0.5 \delta_k s_k^2 \right) + \sum_{k=1}^{K} \sum_{l=1}^{L} c_{kl} z_{kl} - \sum_{l=1}^{L} \left(\alpha_l q_l - 0.5 \beta_l q_l^2 \right)$$

The first term of the objective function is the total costs of all plants and the second term is the total transportation costs. The third term has no direct economic interpretation as claimed by Thompson and Thore (1992). But the negative of the whole objective function is the quasi welfare function minus the transportation costs. The KKT conditions for the problem using this economic potential function are as follows.

$$
\begin{array}{lll}
u_k^* \le 0 & u_k^*\left(\sum_{l=1}^{L} z_{kl}^* - s_k^*\right) = 0 & \forall\ k = 1,..,K \\
u_k^* + v_l^* \le c_{kl} & z_{kl}^*\left(c_{kl} - u_k^* - v_l^*\right) = 0 & \forall\ k = 1,..,K\ and\ l = 1,..,L \\
\sum_{k=1}^{K} z_{kl}^* - q_l^* \ge 0 & v_l^*\left(\sum_{k=1}^{K} z_{kl}^* - q_l^*\right) = 0 & \forall\ l = 1,..,L \\
v_l^* - \alpha_l + \beta_l q_l^* \ge 0 & q_l^*\left(v_l^* - \alpha_l + \beta_l q_l^*\right) = 0 & \forall\ l = 1,..,L
\end{array}
$$

These KKT conditions along with the primal constraints characterize the optimum point, which is the equilibrium for this model. The fourth condition states that each retail market should clear and the fifth condition ensures that the demand price in each market cannot exceed the market price, and when demand is positive, the demand price and the market price are equal.

The above optimization problem can be extended to a multi-commodity case with independent linear supply and demand functions. If the demand functions are not symmetric, or in other words, integrability conditions[8] are not satisfied, a quasi welfare function cannot be constructed. For example, cross price effects (i.e., interdependent demand or supply functions) can cause the integrability conditions to fail. Empirically estimated systems are unlikely to satisfy these conditions. However, complementarity problems overcome this shortcoming. Rutherford (1995) has demonstrated that any neoclassical demand system can be used with recently developed solvers for complementarity problems. Section 4.3 explains the complementarity problems in detail.

To sum up, especially for energy markets, which have spillover effects on the rest of the economy, partial equilibrium models do not seem adequate for policy analysis.

[8] See Takayama and Judge (1971, p.116) for these conditions.

However, partial equilibrium models allow concentrating on a particular sub-section of the economy, with all other variables in other markets being treated constant. This concentration makes it possible to model a particular market (commodity) with many details and much care when compared to general equilibrium models.

3. Overview of Pricing Schemes

3.1 Demand Response

"Demand response is about increasing responsiveness of electricity demand to changes in wholesale electricity prices" (Harrington, 2004, p.30). In the deregulation of many electricity markets only wholesale markets have been open to competition. Often, the state or provincial public utility commissions have insulated the retail market from competitive pricing (an average consumer pays a fixed price for electricity), which indeed made consumers indifferent to electricity prices or uninterested in power usage during price spikes in wholesale markets. Thus, the link between the wholesale and retail markets was disconnected. In that respect, demand response becomes a matter of retail marketers` concern.

Increased demand response is expected to accomplish a more stable and efficient market. If even a small fraction of retail electric customers participate in bulk-power markets, along with power suppliers, large spikes in wholesale price of electric power, such as those that hit markets in California and New York in summer 2000, can be flattened (Hirst, 2001, p.41). It is expected that the price-sensitive consumers would change their consumption patterns such as moving consumption from peak hours to off peak hours (because of the expensive electricity in peak hours) or consuming more in off-peak hours and less in peak hours.

Besides these benefits, demand response is able to provide system reliability, cost reduction, market efficiency, risk management, market power mitigation and environmental benefits. Moreover, national benefits of demand response alone could be $15 billion for the U.S.A., as calculated by McKinsey and Company (Barrett and Violette, 2002, p.18).

3.2 *Efficient Pricing of Electricity*

From the viewpoint of standard economic theory, efficient pricing occurs when marginal cost of supply is equal to marginal value of demand, which also ensures to maximize consumers' plus producers' surpluses (Dewees, 2001, p.9). This has been accomplished by several wholesale electricity markets where an auction mechanism establishes the equilibrium of supply and demand on an hourly basis. In this mechanism, the marginal costs of generation plants are acquired by a bidding process of generators for every hour. The generators are assumed to bid at their short-run marginal cost (if the competitive forces are effective). The offers from consumers are collected and processed by the Independent System Operator (ISO) to find an equilibrium price, which is also called the wholesale spot price of electricity. This wholesale price, which is based on marginal cost, should achieve efficient electricity production and consumption in the absence of other costs (e.g., transaction costs) (Dewees, 2001, p.9).

Nevertheless, because of the unique properties of electricity (non-storability, demand variation, marginal cost differences among different type of generators), the short-run marginal costs of generators vary, sometimes substantially (Hirst, 2001, p.39). Therefore the prices in a wholesale market are very volatile on an hourly basis. The following figure illustrates the volatility of the wholesale spot prices in the Ontario market for June 2004.

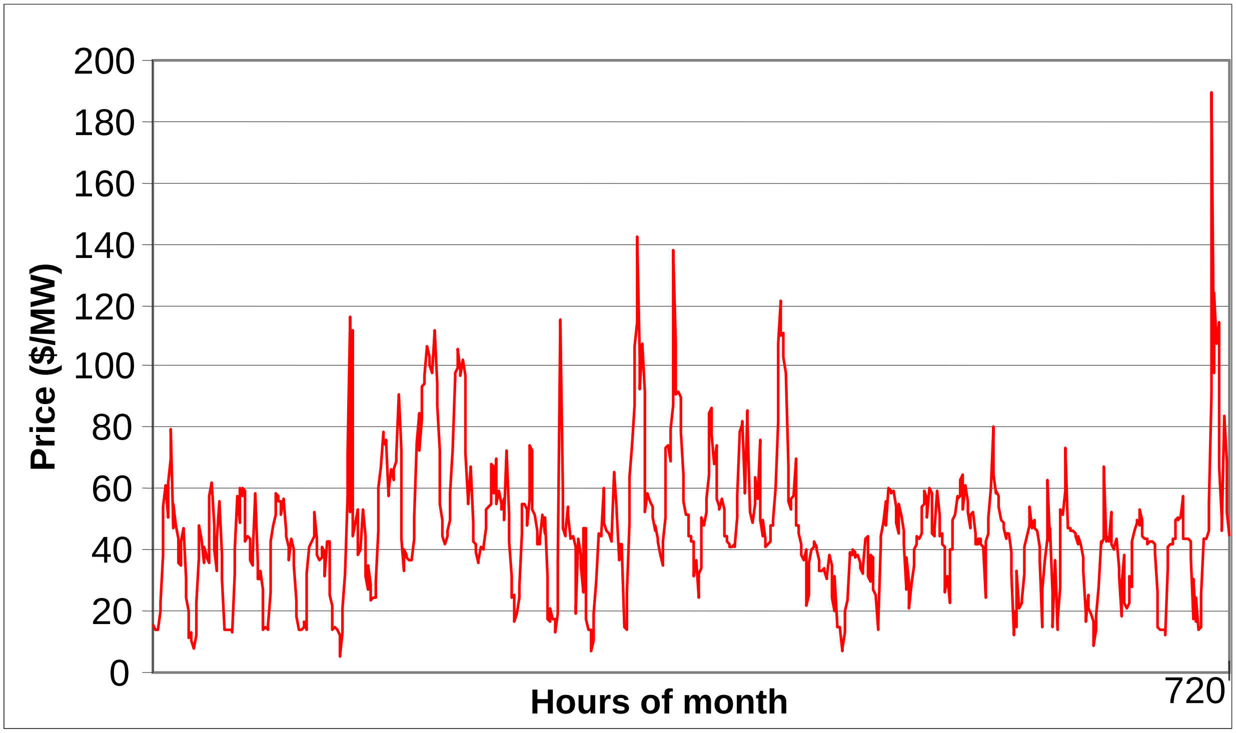

Figure 3-1: Wholesale spot prices in Ontario market for June 2004. (Source: IESO)

As depicted in the figure above, the price spikes are inevitable due to changes in demand and the marginal cost of the generators that supply the last unit of energy demanded.

3.3 Pricing Schemes

There are several pricing schemes that have been used all over the world. They can be classified in two groups:

- fixed (i.e., flat rate regardless of time and system load);
- time differentiated, i.e., dynamically changing over time (e.g., by hour) (time of use pricing, critical peak pricing, real time pricing).

This chapter emphasizes the major pricing schemes that have been experienced, or considered for use: fixed pricing, time-of-use (TOU) pricing, critical peak pricing and real-time pricing. Also, metering requirements required for each scheme are evaluated. Thereafter, a comparison of these schemes is presented. Lastly, a literature review on TOU pricing experiments and econometric studies is summarized.

3.3.1 Fixed Pricing

The most common retail pricing practice all over the world before deregulation and even after deregulation is the fixed pricing per kWh of energy consumed. Regulated rates for small commercial and residential consumers in the United States and Canada are usually fixed for a year (Dewees, 2001, p.10). Under this scheme, the sale price of electricity does not vary with time (e.g., 4.3¢/kWh for the Ontario market before May 2004). Some other applications of fixed pricing are also available and used in some jurisdictions such as the current two-tier pricing for the Ontario market:

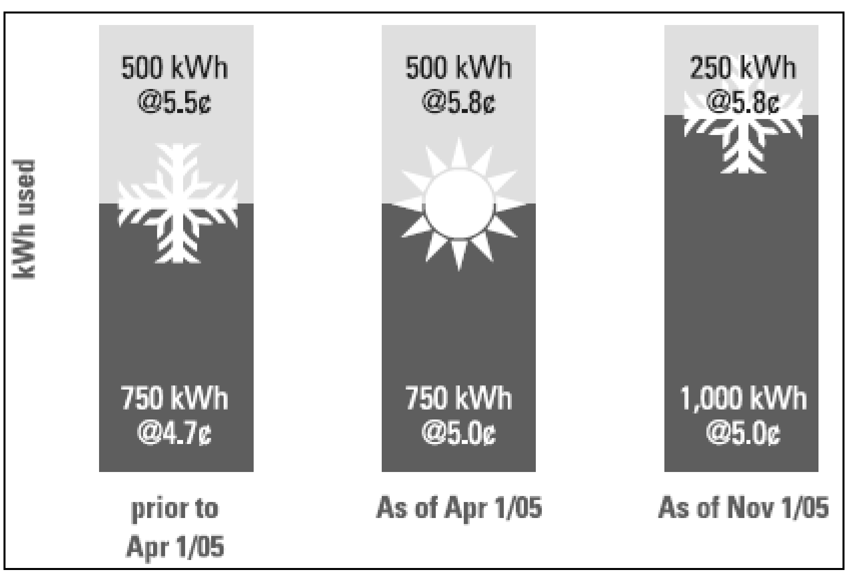

Figure 3-2: Residential Rates and Thresholds by Season for Ontario (OEB, 2005)

As of April 1, 2005, residential customers pay 5.0¢/kWh for the first 750kWh and 5.8¢/kWh for each additional kWh over the 750kWh threshold (for summer months May-September). In winter months (November-April), the threshold is increased to 1000kWh (OEB, 2005).

The main criticism about fixed pricing is that it is usage-based rather than time-based. Customers are billed on their cumulative consumption over a period. A traditional meter[9] that records the usage is read at intervals of one to three months. In the fixed pricing scheme, the price that consumers pay for electricity is time-invariant. Therefore, consumers are protected from the price changes in the wholesale market that occur in real-time and hence their monthly bills are usually stable over the course of the effective fixed rate. Also, the retailers are able to meet their revenue requirements due to this stability.

The problem today is that consumers are indifferent to electricity prices or have no interest in cutting power use during the price spikes, primarily because state or provincial regulatory bodies insulate them from price volatility. Therefore, the consumers become insensitive to price changes in electricity markets. Most of the inefficiency and incompleteness of the wholesale market stems from this insensitive demand.

Another problem with fixed pricing is its unfairness. The electricity costs more to produce at peak hours. Plants that produce the necessary electricity to meet peak

[9] A traditional meter measures the aggregate consumption of electricity in the billing period. It usually keeps the consumption level in one register that can be read manually.

demand are more expensive to run than the nuclear or hydro plants that meet the off-peak (or base) load. A fixed pricing scheme blends these costs of producing electricity in different plants into a fixed price, which causes the off-peak users to subsidize the consumption of peak users. (OEB, 2004)

3.3.2 Time-of-Use (TOU) Pricing

In TOU pricing, both prices and time periods are known a priori and are fixed for some duration (e.g., a season). An example of a TOU rate with three prices and four-time periods is Pacific Gas & Electric's summer commercial TOU prices (Borenstein et al. 2002, p.5):

- off-peak (weekdays 21.30-8.30 and all weekends, holidays) 5.62¢/kWh
- shoulder (weekdays 8.30-12.00, 18.00-21.30) 10.29¢/kWh
- peak (weekdays 12.00-18.00) 23.26¢/kWh

The prices for each block (or the time blocks) are reset only two or three times a year to reflect seasonal variation of prices. This property of TOU pricing does not consider the peak of the system and therefore the variation in real-time is not captured accurately. TOU pricing uses the same price for same periods regardless of system load, condition and wholesale price which more accurately reflect the real marginal cost of electricity. Therefore, customers do not have any more incentive to reduce their loads in peak hours than in average hours, even though load reductions in these hours have substantially higher value to the system.

Another problem with TOU pricing arises when this pricing scheme is implemented on a voluntary basis. Then, only those customers who can lower their bills by going to TOU rates would select it. However, this may lead to a revenue loss for the utility that would have to recover its costs within the form of higher average rates from all customers. (Faruqui and George, 2004)

A major requirement for a TOU pricing scheme is the TOU metering devices. These devices usually have two to six registers (two registers may be for off-peak and peak hours) that can record usage in different time of hours in a day by switching from one to another. These meters can be read manually. As an alternative, a communication device can handle the meter reading and send the consumption data to utilities, but it is not necessarily needed. Typically, a residential TOU meter is as much as three to four times more costly than a traditional residential meter. (Waters, 2004, p.56)

3.3.3 Critical Peak Pricing (CPP)

A critical peak pricing (CPP) scheme is a new form of pricing that has been developed to overcome the limitations of time-of-use and real-time pricing schemes. It is a traditional TOU pricing scheme which is in effect all year except for 50-100 critical peak hours, the timing of which is unknown and where a much higher price (e.g., 10 times higher) is in effect for the peak and shoulder periods. Customers are informed well before the critical peak hours (hours ahead or a day ahead) and that way they can respond to price changes in critical periods.

Small-scale pilot programs conducted by two utilities, Georgia Power Utilities (GPU) and American Electric Power, give very convincing results in favor of CPP schemes. These two utilities use a two-way[10] communication and control device called "TransText" that informs consumers about an approaching critical period. Moreover, it can be programmed so that the consumer's thermostat is automatically adjusted when prices exceed a certain level. The American Electric Power pilot program estimated the demand reductions of 2-3 kW per consumer during on-peak periods and of 3.5-6.6 kW per consumer during critical peak periods, which stands for almost 60 percent of the average consumer's peak load during the winter period. GPU also found similar results showing elasticities of substitution[11] that ranged from -0.31 to -0.4, which are significantly higher than the elasticities associated with traditional TOU rates (Faruqui and George, 2002, p.49). Besides these advantages of CPP over TOU, CPP is based on system conditions (rather than normal user peak) and it can reflect the wholesale price when the system is in a critical period.

However, CPP has some weaknesses that should be mentioned. Like TOU pricing, even for critical periods, the prices do not change in line with the wholesale price. First of all, prices are limited and their levels are preset. Secondly, the number of critical peak periods to be invoked by utilities is limited in a year (50-100 hours a year).

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[10] For utilities, a one way communication (consumer to utility) is sufficient to collect consumption data during critical peak hours. Utilities may choose to inform the consumers by another mode of communication (internet, phone, TV broadcasts, etc.)

[11] Elasticity of substitution is defined as the reciprocal of the degree to which the substitutability of two factors, that is the marginal rate of substitution, varies as the ratio of the two inputs varies and output is held constant.

Finally, the utilities protect consumers only from very high prices by CPP (Borenstein et al. 2002, p.15).

For implementation of CPP, a TOU meter with an additional critical peak register is required. A one-way communication device is enough for utilities to send the critical peak hour information to the metering device to initiate and to end the record of critical peak consumption. The cost of CPP meters is as much as TOU meters, however, they need communication devices (i.e. wireless GSM or CDMA) that have both installation and operation costs.

3.3.4 Real-Time Pricing (RTP)

In real-time pricing scheme, prices vary on an hourly basis. Generally, prices are fixed and known only on a day-ahead or hour-ahead basis. These pricing schemes can be used to effectively influence customer usage in peak hours. It reflects the wholesale prices (the marginal cost of electricity), weather conditions, generator failures, scarcity of generation and other contingencies in a wholesale electricity market. Utilities can charge different retail prices for different hours of the day and for different days.

It has been successfully implemented by Gulf Power in Florida for medium/large industrial and commercial customers (1639 customers as of June 2002). Gulf Power has found that a relatively small fraction of customers are extremely price-responsive, with price elasticities in the range of –0.1 to –0.25, whereas, a third of customers are modestly responsive and almost half of the customers appear to be not responsive at all (Borenstein et al. 2002, Appendix A).

From the viewpoint of generators, RTP reduces the total payments to generators in wholesale markets, because of the reduction in peak demand when prices are very high. Also, RTP can reduce the ability of generators to exercise market power. When generators tend to increase the wholesale price by withholding capacity, retail prices also increase and thus, reduce the demand for power. Then, profitability of price increases is reduced by demand response (i.e., the price increase can be offset by the decrease in demand and this can reduce the profitability) and exercise of market power is discouraged. Finally, RTP can reduce the need for excess capacity by either shifting consumption from peak hours to off-peak hours or by reducing consumption at peak hours. (Borenstein et al. 2002, p.10-11)

Although RTP is a major conceptual advancement over TOU and CPP schemes, it usually has elusive benefits. The uncertainty and volatility of prices transfer the price risk to customers and consequently, this has failed to attract many customers. On the other hand, the incremental metering and billing costs associated with the implementation of RTP can discourage customers and utilities. A study for Pacific Gas and Electric Company estimated these costs around one billion dollars (Faruqui and George, 2004).

As Joskow and Tirole (2004) asserted, final consumers may not react to real-time prices for two main reasons. Firstly, the cost of monitoring and evaluation of hourly prices and constantly optimizing the use of equipment are enormous for small consumers. Secondly, adjusting consumption freely may not be possible for consumers due to physical attributes and configuration of the distribution network, in particular;

most directed interruptions (due to a shortage in supply) that can be controlled by the distribution network operator usually occurs at the level of zones. This means that individual consumers cannot have their preferred priority for being served by the system operator.

A mandatory requirement to implement an RTP scheme is the interval metering technologies, which can measure the consumption of users on an interval basis. Interval meters can record a separate consumption measurement for each hour in a billing period. As in CPP, it needs a communication device, but the communication should be two-way: to send consumption data from consumers to utilities and to send pricing information to consumers from utilities[12]. Thus, utilities can accomplish their metering and billing process and customers can gather the pricing information. The cost of a residential interval meter is typically six times the cost of a traditional residential meter and a commercial interval meter is about twice the cost of the residential interval meter (Waters, 2004, p.56).

3.3.5 Comparison of Pricing Schemes

Time differentiated pricing schemes are found to be more efficient than the fixed pricing schemes, because of two reasons: (a) they reflect the marginal cost of electricity (partially in TOU pricing or fully in RTP) more accurately than fixed pricing schemes and therefore improve the efficiency of resource allocation and (b) they motivate

[12] Sending price information is not necessarily required and may be accomplished by broadcasting over media, automated telephone systems, internet, etc. However, for automated response systems that can be installed by individual customers, it is more efficient to have two-way communication between utilities and customers.

customers to reduce their electricity consumption in peak loads and shift to off-peak periods and therefore reduce the need for new capacity requirement and substantially reduce the reliance on peak capacity that mainly uses fossil fuels (Boiteux, 1949; Houthakker, 1951; Faruqi and Malko, 1983; Aigner, 1984). However, they have not been extensively used in any competitive electricity market.

Among all pricing schemes mentioned, critical peak pricing (CPP) seems the best for especially small/medium commercial and residential customers. It is easy for customers to respond compared to RTP and moreover, it is less costly to implement when the costs of RTP are considered. Thus, as an alternative to fixed pricing, CPP can accomplish much of what RTP offers. (Borenstein et al. 2001, p.29).

Compared with TOU pricing, CPP gives more incentives to reduce peak demand, but modeling it would require a stochastic component in the model. Moreover, some utilities and regulatory bodies (e.g., Ontario Energy Board) have no plans for CPP, but they are considering TOU pricing (OEB, 2005). Therefore, in this thesis a TOU pricing scheme is modeled instead of a CPP scheme. A stochastic model that can consider the critical peak hours is left for future research.

Figure 3-3 illustrates the TOU, CPP and fixed pricing schemes. Note that, for the CPP scheme, there is a prior notice that informs customers about the approaching critical peak period.

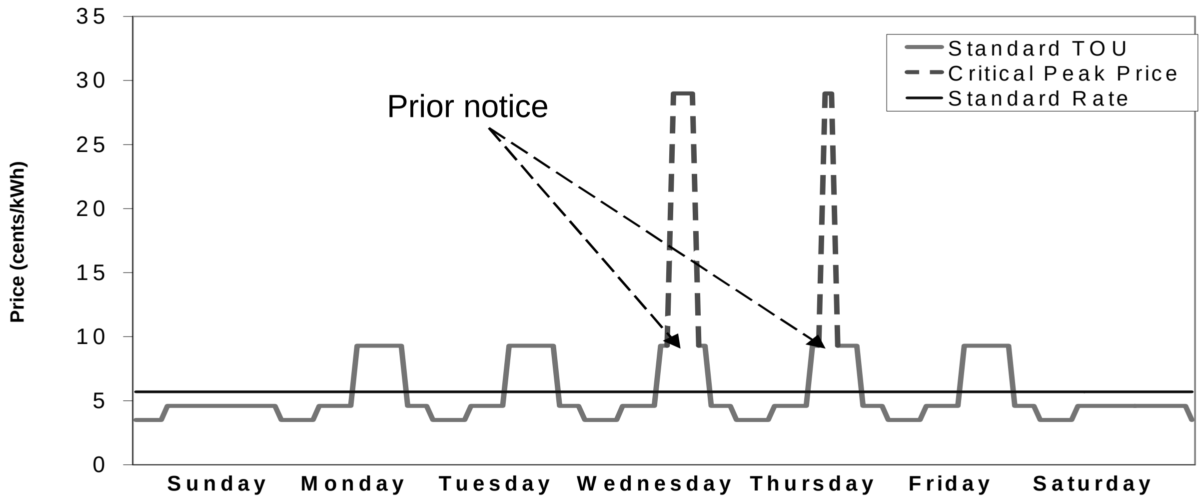


Figure 3-3: TOU, CPP and Fixed pricing schemes

TOU and CPP pricing schemes attempt to give consumers more incentives to either reduce their demand when the system is at its peak, or shift the demand to times when it is off-peak. Much research has found evidence in favor of this assumption. See Faruqui and Malko (1983), Aigner (1984) and King et al. (2003) for a comprehensive survey.

However, the results are widely varying from one jurisdiction to another. As Faruqui and George (2002) conclude, each utility should conduct its own research to estimate the net benefits of time-differentiated prices based on incremental metering costs, usage pattern, supply behavior and other key drivers (i.e., weather conditions, electric appliance usage such as heating, air conditioning, etc.). Also, the estimates of the price elasticities should be conducted under well designed and controlled experiments to implement a wide-scale time differentiated pricing scheme.

3.4 Review of Literature on TOU Pricing

The literature on TOU pricing is vast and it is based on the previous studies on "Peak Load Pricing"[13] or "Marginal Cost Pricing". Peak loads and their pricing have been a concern because of the capacity requirements for these loads. The seminal paper concerning peak load pricing is Boiteux (1949) and theoretical contributions have been made by Houthakker (1951), Steiner (1957), Williamson (1966) and Turvey (1968).

Steiner (1957) considered a firm with single production technology (with constant operating cost and capital cost) and demand with two classes: off-peak and peak (two off-peak and one peak period, without any cross price elasticity). He maximized the social net benefit function and found that all capacity costs are charged to peak demand users. This is the classical peak load pricing result. However, it has been criticized, since off-peak demand users also need and use the capacity. A justification has been made by Wenders (1976), by allocating a part of the capacity cost to off-peak demand users. However, Turvey (1968) argued the relevance of the capacity costs in the short term (e.g., a year).

This theoretical body of literature was not able to give practical answers to the problem and a need for large-scale experimental studies about peak-load pricing and TOU pricing emerged. There have been many experiments conducted with TOU pricing over the past three decades. These experiments yield insights about the impact of TOU pricing on customers and utilities as well as on welfare of the society.

[13] See Crew et al. (1995) for a survey of the theory.

Many of these experiments were done in the late seventies and early eighties, including projects sponsored by the U.S. Federal Energy Administration (now part of the U.S. Department of Energy). A survey of this research can be found in a special *Annals* issue of the "Journal of Econometrics" (Aigner, 1984). A more recent survey is published by King *et al.* (2003). These experiments all collected data that allows econometricians to estimate an electricity demand function with many explanatory variables (single demand function e.g., linear, double log or other; or demand system models e.g., translog, generalized Leontief or other functions) as well as the own and cross price elasticities, elasticities of substitution and lag elasticities. A survey of twelve TOU experiments by Faruqui and Malko (1983) drew the following conclusions for TOU pricing.

a) TOU rates reduce the electricity consumption in peak-periods, whereas electricity consumption in off-peak periods either stays constant or increases by small amounts.

b) Load shifting is rarely observed and TOU rates generally cause an overall reduction in daily consumption.

c) Peak users typically respond more than off-peak users.

d) Peak and off-peak own price elasticities range from 0 to -0.4. These elasticities vary among experiments due to variation in total usage, climate, rate level, etc. The difference between elasticity estimates derived from the single equation and demand system models are negligible.

They estimated that the elasticity of substitution between peak and off-peak periods for an average customer living in a typical climate was 0.14. For customers living in a hot climate who had all major electric appliances in their home, the elasticity rose to 0.25, and for those living in cool climates without any major electric appliances in the home, the elasticity of substitution fell to 0.09. But, these elasticity estimates may not be valid now, because they were developed during the early eighties when electricity prices and schemes were quite different.

However, Mountain and Lawson (1995) conducted a comprehensive experiment for the Ontario market[14]. They empirically estimated the variation in responsiveness of the Canadian consumers to TOU electricity prices by time of day and by month of year. They estimated the two-period (off-peak, peak) and three-period (off-peak, peak, super-peak) own and cross price elasticities by using 16 different rate structures. The range of their estimates for two-period and three-period price structures is summarized in the following tables.

Table 3-1: Range of own and cross price elasticities for three period price structure

		Price		
		Off-peak	Peak	Super-peak
Quantity	Off-peak	*-0.033 to -0.136*	*0.014 to 0.141*	*-0.05 to 0.023*
	Peak	*0.010 to 0.043*	*-0.018 to -0.059*	*0.006 to 0.018*
	Super-peak	*-0.002 to 0.016*	*0.006 to 0.024*	*-0.017 to -0.022*

Table 3-2: Range of own and cross price elasticities for two period price structure

		Price	
		Off-peak	Peak
Quantity	Off-peak	*-0.003 to -0.100*	*0.003 to 0.101*
	Peak	*0.009 to 0.088*	*-0.009 to -0.088*

[14] So far, it is the only experiment conducted for the Ontario market. Previous Canadian studies had to rely on the elasticity estimates from U.S. studies (Mountain and Lawson, 1995)

Although these estimates are lower than any other estimates found in different studies, the study did show significant demand reductions and load shifting. Therefore it can be used for experimental models especially when dealing with the Ontario market.

4. The Elements of a New Model of Pricing in Electricity Markets

The model proposed in chapter 5 of this thesis is a computable equilibrium model based on a network structure that can represent the interactions between decision-makers in terms of quantity of energy flows and electricity prices. It is a multi-period computable equilibrium model for retail electricity market that seeks an efficient pricing scheme. The potential uses of the model are (a) to compute the prices that will be the regulated (fixed or TOU) prices based on a marginal cost principle; (b) to forecast prices (fixed or TOU) that can happen at equilibrium in an unregulated market. The model consists of two parts: the demand side and the supply side. This chapter explains the basic concepts used in the model.

The supply side is basically a cost minimization problem of generators. The demand side is represented by a demand equation that uses the prices and lagged demand as independent variables. Such models are usually called process models (Wu and Fuller, 1995). If the demand functions are elastic and integrable, the objective function of the supply model could be converted from cost minimization to welfare maximization (sum of producers' plus consumers' surpluses), because integrability allows the first order conditions of the mathematical program to satisfy the equilibrium conditions. However, integrability is not a common situation when demand functions depend on other commodities' prices (Bohringer and Rutherford, 2004), (e.g., when off-peak demand depends, in part, on the peak price). In such cases, the demand function cannot be converted into a utility function. Therefore, this problem in process models cannot be handled by a single optimization framework (Bohringer and Rutherford, 2004).

There are several algorithms to solve process models, such as the PIES algorithm (Ahn and Hogan, 1982), the decoupling algorithm (Wu and Fuller, 1995), and algorithms for complementarity problems (Mathiesen, 1985; Dirkse and Ferris, 1996; Ferris et al. 2001; Manne, 1985) or variational inequalities (Nagurney, 1993). The former two algorithms are based on a sequence of integrable optimization problems, whereas the latter two are more general and recognized approaches.

The model proposes a network structure to simulate the electricity market. The supply nodes represent the generators with different technologies of production (i.e., nuclear, hydro, coal, gas and oil, indexed by i). On the other hand, the demand nodes represent the demand for different time blocks, such as off-peak, mid-peak, on-peak demand indexed by j . Figure 4-1 displays the basic electricity network model. The index $h = 1,...,H_j$ stands for the different hours in time block j .

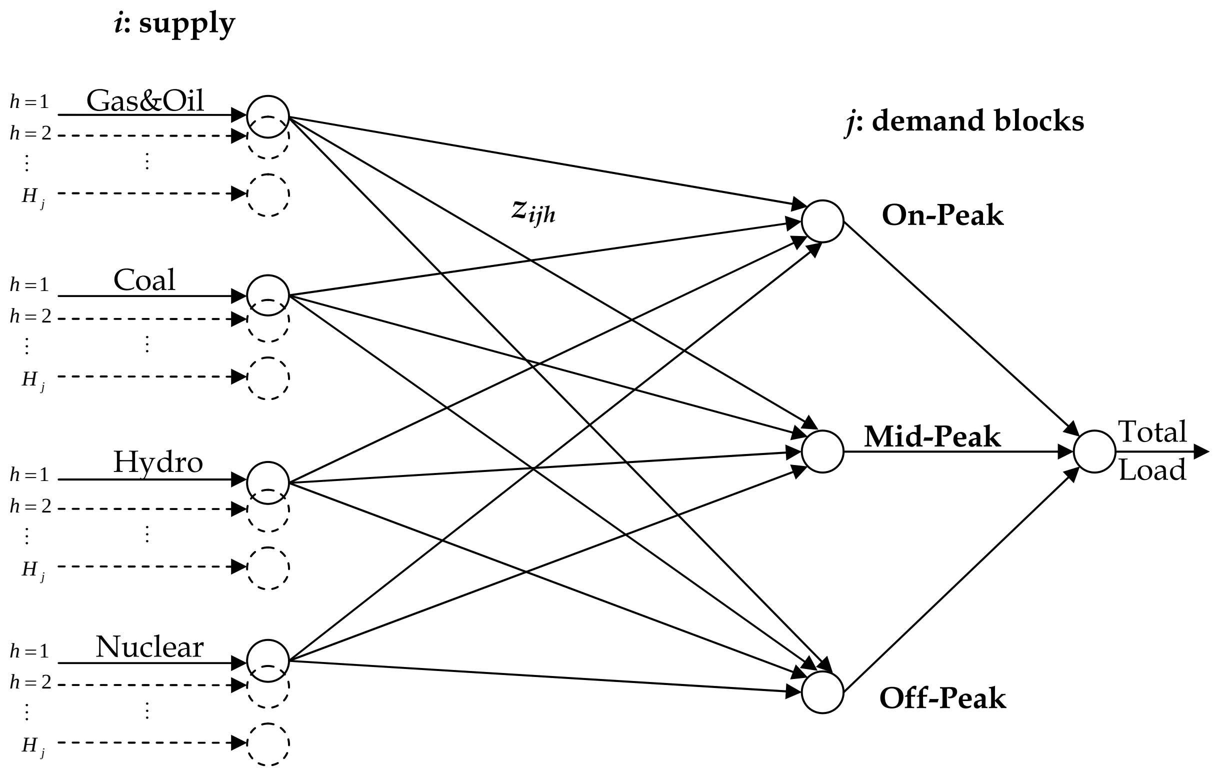
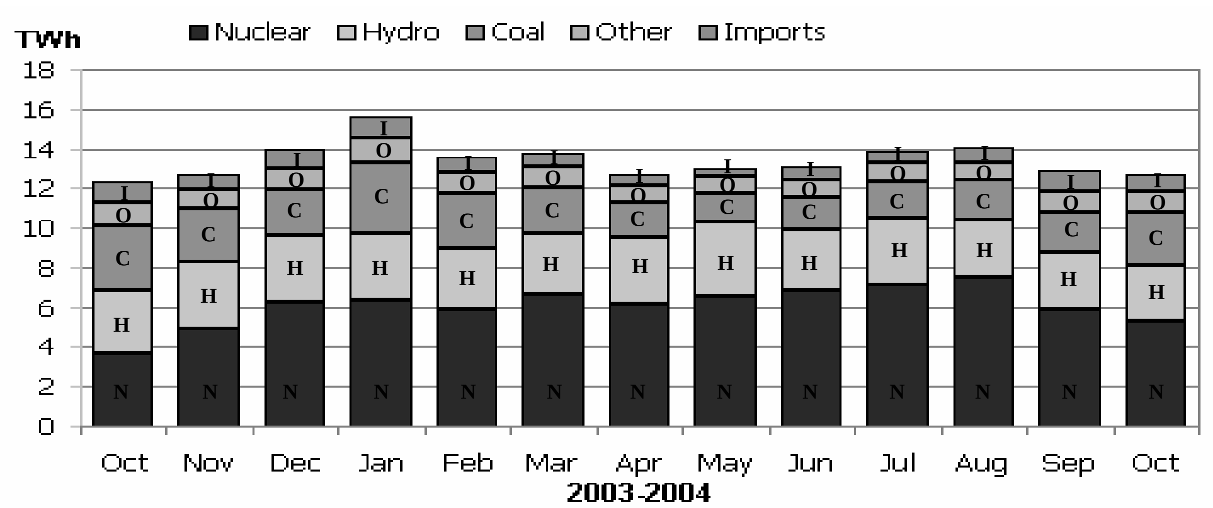


Figure 4-1: Basic electricity network

The objective in representing the different production technologies is to consider the various variable cost structures. The model does not consider the ramp-up times/limits, for simplicity, but they could be included, in principle, for future research. Different production technologies can serve different demand blocks to be more economical. For example, nuclear plants can serve any demand block, because their per unit costs are lower, whereas ramp-up time for a nuclear plant is long. Therefore, it is more economical for nuclear plants to produce power all the time. On the other hand, gas and oil plants can serve on-peak demand block, which requires a quick response. Because their ramp-up time is very short, they can respond to rapidly varying loads. Hydro plants can technically supply power for all demand blocks and coal plants can supply energy to mid-peak and on-peak loads. Due to low marginal costs for hydro and nuclear plants, medium marginal cost for coal plants and high marginal cost of gas and oil plants, the optimal solution (to the cost minimization problem) has nuclear and hydro plants supplying to all demand blocks, coal plants supplying to mid-peak and on-peak demand blocks and gas and oil plants supplying to on-peak demand block. This is consistent with the ramp-up times/limits; therefore, the absence of the ramp-up times/limits is not of great importance.



N	Supply By Fuel Type for September 2004			
Nuclear (N)	Hydro (H)	Coal (C)	Other (O) (gas, oil, etc.)	Imports (I)
42%	22%	21%	8%	7%

Figure 4-2: Energy Supply in Ontario by Fuel Type in 2003-2004 (source: IESO)

The figure above presents the energy supply of Ontario by power production technologies (i.e., by fuel types) in the 2003-2004 period. Nuclear power generators are the main source of energy in the Ontario market followed by hydro and coal generators.

Other production technologies –gas, oil, solar, wind, etc.– constituted eight percent of the total energy supply in September 2004.

As Turvey (1968) argued, the capacity costs are not relevant in the short term (i.e. a year), because the need for extra capacity is not significant in the short term. The fluctuation of demand in the short term generally does not exceed the fluctuation of available generation capacity due to maintenance. Therefore, only variable operating costs of the generators are included in the model because they can accurately reflect the marginal cost of electricity for the short term. This convention treats all fixed and capacity costs as sunk and thus irrelevant to the present analysis.

A study by Johnston (1960) developed the short-run cost functions of the electric generators in Great Britain. Seventeen different firms were examined to validate a cubic polynomial cost function. However, the results of the study did not support a nonlinear cubic or a quadratic form, but rather favored a typical linear cost function.

The model employed in this thesis, therefore, uses a linear short-term cost function. In other words, in the short-run (normal operating range for the generators, i.e. a year) the marginal cost for different production technologies is assumed to be constant for output between zero and installed capacity. Operating cost (or marginal cost) of each production technology is estimated as follows (Wong, 2005):[15]

Table 4-1: Estimates of marginal cost of power production technologies

Production Technology	Hydro	Nuclear	Coal	Gas and Oil
$/MWh	*1*	*3.75*	*28*	*61*

[15] These marginal cost data are from Wong (2005) and are estimates from different sources such as www.opg.com, www.brucepower.com.

Since the cost information is kept confidential by firms and regulatory bodies, only crude estimates can be used. Actual costs may be different. Nevertheless, these estimates can be used for modeling and test purposes.

Another major assumption is about the network structure. Transmission constraints such as line and voltage limits are not included in the model for simplicity. The transmission network is ignored in analyses. This means that there is a single price at any given time, as is now the case in Ontario. Geographically differentiated prices (i.e., nodal, zonal pricing) would require a representation of the transmission network in the model.

Time-of-use pricing practices usually differentiate between the weekday and weekend, because normally the consumption is lower in weekends (e.g., peak hours are less likely) and higher in weekdays. However, to simplify the model and the analysis, the distinction between consumption levels in weekdays and weekends are ignored.

Lastly, the suppliers (generators) are assumed to behave in a competitive manner (no exercise of market power). Oligopolistic models and market power issues are beyond the scope of this thesis, but they can be incorporated into the model, in future research.

4.1 Geometric Distributed Lag Demand

Process models usually use demand functions that are only functions of the current period's prices. However, reaction of demand to changes in price is a process in time. Especially in energy markets, the adjustment to varying prices can occur after some periods rather than instantaneously (Wu and Fuller, 1995, p.648). As an example, assume a residential electricity consumer with a monthly billing period and varying monthly electricity prices. The response of this consumer to price changes can be plotted as in Figure 4-3.

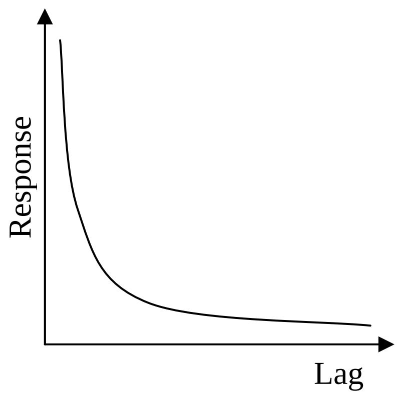


Figure 4-3: Response and lag relation in GDL

The figure depicts that the response is spread over time and it declines by time. This is called the "time-lagged" effect. The reaction of the consumer is not at a point of time but rather distributed over time. The main reasons for this response can be categorized in two groups. Firstly, usage patterns (i.e. habits) and imperfect information about the market preclude the instantaneous adjustment to prices. If the consumer is unaware of the monthly prices, the adjustment of consumption due to price changes may occur in the next periods after the consumer understands the information on the bill. Secondly, the need for some services is not interruptible and the demand may be linked to durable equipment (Wu and Fuller, 1995). For example, a price increase in electricity may not affect the usage of an old heating furnace. The need for heating and capital cost of new and efficient furnaces may prevent the immediate reaction of demand to price changes.

A geometric distributed lag demand can represent this response process in time. As Dhrymes (1981, p.2) explains in further details, a basic distributed lag demand function can be:

$$y^{(t)} = v^{(t)} + \sum_{i=0}^{\infty} w_i x^{(t-i)} + u^{(t)} \tag{1}$$

where $y^{(t)}$ is the dependent variable (i.e., demand) in period t (i.e., month, year), $v^{(t)}$ is a constant, $x^{(t)}$ is the exogenous variable (e.g., prices) and $u^{(t)}$ is the random residual term which is independent of $x^{(t)}$ and has a distribution with mean zero and constant variance.

Another expression is the exponential form, which is also called the constant elasticity model.

$$y^{(t)} = m^{(t)} \prod_{i=0}^{\infty} \left[x^{(t-i)}\right]^{\alpha_i} \tag{2}$$

where $m^{(t)}$ is a constant for period t and α_i is the elasticity of exogenous variables (i.e., lag elasticity). As with (1), the model requires an infinite number of parameters. However, in practice it is not required to use all the history terms, because the lagged independent variable $x^{(t)}$ has a decreasing influence on the dependent variable $y^{(t)}$ as the lag increases, and as the lag goes to infinity the influence is close to zero. Therefore, these lag terms that do not affect the independent variable can be truncated at some point (n) (Wu and Fuller, 1995):

$$y^{(t)} = m\prod_{i=0}^{n}\left[x^{(t-i)}\right]^{\alpha_i} \tag{3}$$

The number of parameters, α_i, is usually reduced by assuming a form of dependence on the lag, i. For example, a one commodity lagged demand model is as follows.

$$d^{(t)} = a^{(t)}\left[p^{(t)}\right]^{b}\left[d^{(t-1)}\right]^{e} \tag{4}$$

where $d^{(t)}$ is the demand of electricity in period t (t=1,2,…T), $a^{(t)}$ is a constant representing non-price effects (e.g. the appliance stocks, weather conditions, socio-demographic factors), $p^{(t)}$ is the price of electricity at period t, $d^{(t-1)}$ is the lagged demand, b is the constant price elasticity and e is the lag elasticity. This is also called a constant elasticity model, which is widely used in econometric studies and also in the model of this thesis.

By taking natural logarithm of both sides of the equation (4), we can get

$$\ln(d^{(t)}) = \ln(a^{(t)}) + b\ln(p^{(t)}) + e\ln(d^{(t-1)}) \tag{5}$$

With successive substitution and letting $\overline{d}^{(t)} = \ln(d^{(t)})$, $\overline{a}^{(t)} = \ln(a^{(t)}) + e^{t}\ln(d^{(0)})$ and $\overline{p}^{(t)} = \ln(p^{(t)})$ equation (5) becomes (where $0 < e < 1$)

$$\begin{bmatrix} \overline{d}^{(1)} \\ \vdots \\ \overline{d}^{(t)} \\ \vdots \\ \overline{d}^{(T)} \end{bmatrix} = \begin{bmatrix} \overline{a}^{(1)} \\ \vdots \\ \overline{a}^{(t)} \\ \vdots \\ \overline{a}^{(T)} \end{bmatrix} + \begin{bmatrix} b & \cdots & & 0 \\ \vdots & & & \\ e^{t-1}b & b & & \\ \vdots & & & \\ e^{T-1}b & e^{T-1}b & \cdots & b \end{bmatrix} \begin{bmatrix} \overline{p}^{(1)} \\ \vdots \\ \overline{p}^{(t)} \\ \vdots \\ \overline{p}^{(T)} \end{bmatrix} \tag{6}$$

Equation (6) can be extended to a multi-commodity case (i.e., several demand blocks), with both own and cross-price elasticities. This extension is presented in chapter 5.

4.2 *Monthly Load Duration Curve*

The monthly load duration curve, which is obtained by arranging the hourly loads in descending order (as in the annual load duration curve), is a representation of the variation in monthly electricity demand by time. It differs from the annual load duration curve only by the time period. The expected shape of the monthly duration curve is similar to the annual load duration curve and the area under this curve represents the total energy requirement in a month. The shape of the curve is often the same but it moves up and down with varying demand in each month. The following figure displays the annual (2004) and monthly (May 2002, May 2003 and May 2004) load duration curves with Ontario market data.

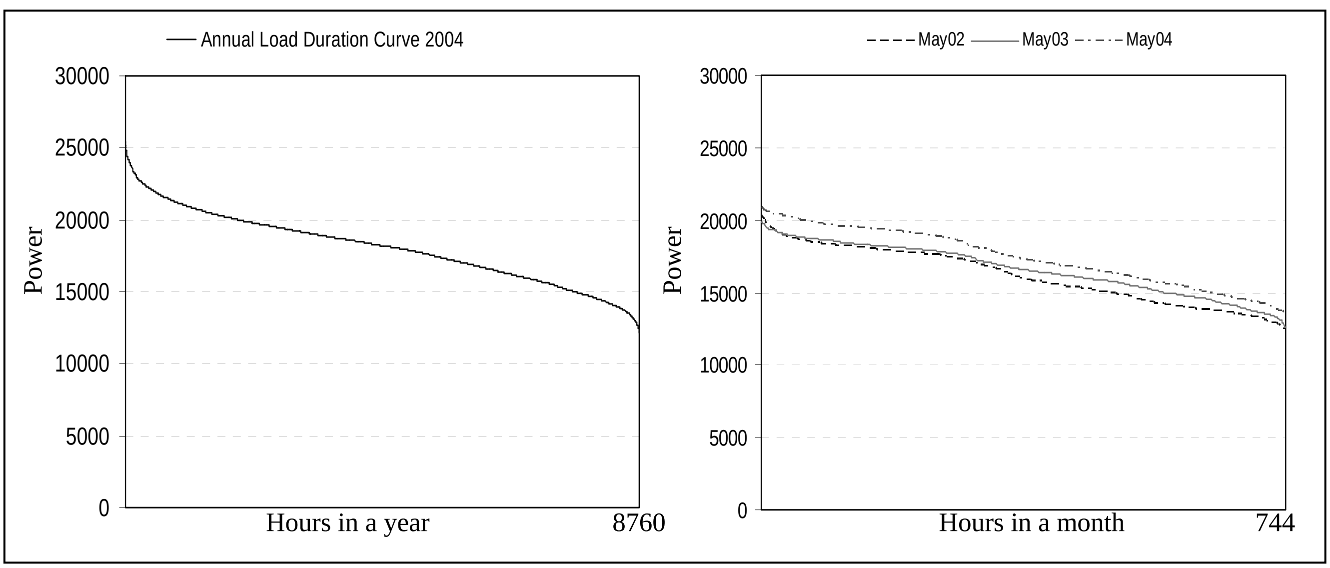


Figure 4-4: Yearly and monthly annual load duration curves (source: IESO)

The load duration curve can be used to model different load blocks (i.e., off-peak, mid-peak, on-peak). The monthly load duration curve can be discretized and approximated by horizontal or vertical strips. Horizontal strips refer to the various types of load (such as seasonal peak, daily peak, cycling and base) in a month, whereas vertical strips refer to the load in various time intervals (such as on-peak, mid-peak and off-peak). A utility planning model as described by Sherali et al. (1982) for each of the discretization methods (horizontal and vertical) can be modeled. This can capture the supply side of the equilibrium model. Figure 4-5 illustrates a three-step vertical approximation of the monthly load duration curve, the type of approximation used in the demand side of the model of this thesis.

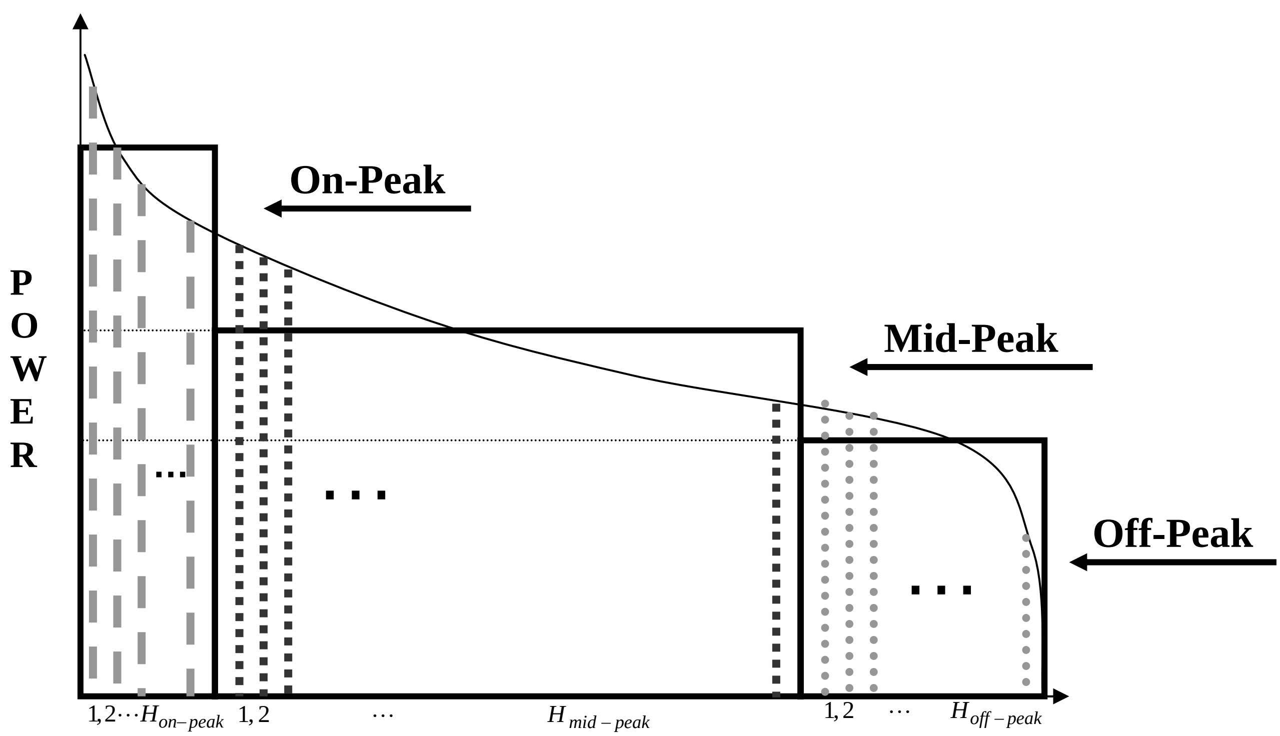


Figure 4-5: 3-step vertical approximation of the monthly load duration curve

A further approximation can be made by modeling the hourly loads as seen in Figure 4-5. The supply side of the model uses the hourly loads and the demand side uses the three-step vertical approximation that is based on monthly loads. The sum of the hourly vertical strips for demand block j (e.g., sum of hourly on-peak demands) equals the demand side vertical strip area (e.g., monthly on-peak demand).

4.3 Solution Procedure

As mentioned before, there are several algorithms to solve the process models, such as the PIES algorithm (Ahn and Hogan, 1982), the decoupling algorithm (Wu and Fuller, 1995), and algorithms for complementarity problems (Mathiesen, 1985; Dirkse and Ferris, 1996; Ferris et al. 2001; Manne, 1985) or variational inequalities (Nagurney, 1993).

The model proposed in this thesis is represented and solved by a mixed complementarity problem (MCP) approach. This approach is becoming more widely used in a variety of application areas, such as restructured electricity markets, engineering mechanics, optimal control, asset pricing, etc. (Ferris et al. 2001). Also, it can capture the details in real world applications; e.g., in deregulated electricity markets, generation options, variable demand, and the transmission grid can be modeled. Lastly, a rich body of theory about complementarity problems allows analyses of model properties (i.e., solution existence and uniqueness) (Hobbs and Helman, 2004, p.70).

A complementarity condition between a non-negative variable x_i and a non-positive function $G_i(x)$ where $x=\{x_i\}$ (a vector of variables) can be written as (Hobbs and Helman, 2004, p.71-72):

$$x_i \geq 0; \quad G_i(x) \leq 0; \quad x_i G_i(x) = 0$$

Also, this can be written as:

$$0 \leq x_i \perp G_i(x) \leq 0$$

In general, a complementarity problem is defined as follows:

CP: find x such that $0 \leq x \perp G(x) \leq 0$

where x and G are vector valued.

This complementarity problem is "square" if the number of individual conditions (equations) equals the number of variables x. A more general form is the mixed complementarity problems (MCP) where y is introduced as a second vector of variables, and $H(x,y)$ as a vector-valued function with the same dimensions as y:

MCP: find x, y such that $0 \leq x \perp G(x,y) \leq 0$ and $H(x,y)=0$

The term "mixed" reflects that the formulation includes equality constraints as well as inequality constraints. The term "complementarity" refers to the complementary slackness between variables and the constraints (Bohringer and Rutherford, 2004).

GAMS (General Algebraic Modeling System) is a modeling language that has access to several solvers that can solve MCP (mixed complementarity problem) models. The most common MCP solvers in GAMS are PATH and MILES (a Mixed Inequality and non-Linear Equation Solver).

Both solvers use Newton type algorithms, but they differ in the adjustment process when the initial solution is far away from the equilibrium. PATH uses a path search algorithm whereas MILES uses a backtracking line search based on Mathiesen's (1985) algorithm (Rutherford, 1993). A benchmark study by Rutherford (1995) showed that PATH solver was generally more efficient than MILES for large dimensional MCP models.

NLPEC is another GAMS solver that reformulates the complementarity constraints of MCP and MPEC (Mathematical Programs with Equilibrium Constraints) models and solves by existing NLP (Non-Linear Programming) solvers, e.g. MINOS 5.0. Actually it is designed to solve MPEC models but a MCP model can be considered as a MPEC model with a constant objective (GAMS Corp., 2004).

5. The Mathematical Model

5.1 *Supply Side*

The supply side of the model is a cost minimization problem for all hours' generation given the demands that must be met. It can be formulated as follows.

Parameters

set of generation facilities: $(i = 1,...,m)$
set of demand blocks: $(j = 1,...,n)$
set of periods (months): $(t = 1,...,T)$
set of hours in demand block j $(h = 1,...,H_j^{(t)})$
$c_i^{(t)}$ = operating cost per unit of energy for facility i in period t (\$/kWh)
$r =$ discount factor (r^t: discount factor to the power of t)
$d_{jh}^{(t)} =$ energy demand for demand block j in hour h in period t (kWh)
$K_i^{(t)}$ = capacity of facility i in period t (kW)

Decision variables

$z_{ijh}^{(t)}$ = the energy flowing from facility i to demand block j for hour h in period t (kWh)

$$\begin{array}{lll}
\min \sum_{t=1}^{T} \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{h=1}^{H_j^{(t)}} r^t c_i^{(t)} z_{ijh}^{(t)} \right) & & \\
\textit{subject to} & & [\textit{dual}] \\
\sum_{i=1}^{n} z_{ijh}^{(t)} \geq d_{jh}^{(t)} & \forall\, h, j \textit{ and } t & [p_{jh}^{(t)}] \\
-z_{ijh}^{(t)} \geq -K_i^{(t)} & \forall\, h, i, j \textit{ and } t & [u_{ijh}^{(t)}] \\
z_{ijh}^{(t)} \geq 0 & \forall\, h, i, j \textit{ and } t &
\end{array} \qquad (7)$$

Note that, in (7), expressions such as "$\forall\, h, i, j \textit{ and } t$" are short, more readable versions of the more accurate expressions such as "$\forall\, h = 1,..., H_j^{(t)}$, $i = 1,..., m$, $j = 1,..., n$, $t = 1,..., T$". The objective function of the model minimizes the total operating costs of all facilities (nuclear, hydro, coal, gas and oil) that inject energy to an arc connecting i and j . It is discounted with r to reflect the time value of money. However, there is no harm to assume that the discount rate is zero. The first set of constraints ensures that electricity supply is sufficient to meet demand; at an optimal solution, these constraints are binding equalities. The second set of constraints are the capacity constraints for each generation facility; they are written in the " $\geq$ " form to ensure that the dual variables are non-negative, in order to ease the interpretation of these duals. The dual variables $p_{jh}^{(t)}$ have the interpretation of the marginal cost of increasing the energy demand of the j^{th} vertical strip for hour h by a unit. Hence, they give the marginal cost of hourly energy demanded in the various demand blocks. The dual variables $u_{ijh}^{(t)}$ can be interpreted as the marginal cost of reducing the capacity of facility i by a unit for demand block j and hour h. It is the "scarcity rent", as economists call it. The Karush-Kuhn-Tucker (KKT) conditions for the supply side of the model are as follows:

$$\boxed{\begin{array}{ll} z_{ijh}^{(t)} \geq 0 \perp r^t c_i^{(t)} - p_{jh}^{(t)} + u_{ijh}^{(t)} \geq 0 & \forall\ h, i, \ j \, and \ t \\ p_{jh}^{(t)} \geq 0 \perp \sum_{i=1}^{n} z_{ijh}^{(t)} - d_{jh}^{(t)} \geq 0 & \forall\ h, \ j \, and \ t \\ u_{ijh}^{(t)} \geq 0 \perp K_i^{(t)} - z_{ijh}^{(t)} \geq 0 & \forall\ h, i, \ j \, and \ t \end{array}} \tag{8}$$

5.2 *Full MCP with Demand Side*

Equation (6) can be extended to a multi-commodity case where each commodity is the electricity demand in different times of day (i.e., demand blocks on-peak, mid-peak, off-peak)

$$\ln(D^{(t)}) = A^{(t)} - B\ln(P^{(t)}) + E\ln(D^{(t-1)}), \quad or$$

$$let\ \overline{D}^{(t)} = \ln(D^{(t)}) \quad and \quad \overline{P}^{(t)} = \ln(P^{(t)}), \tag{9}$$

$$\begin{bmatrix} \overline{D}^{(1)} \\ \vdots \\ \overline{D}^{(t)} \\ \vdots \\ \overline{D}^{(T)} \end{bmatrix} = \begin{bmatrix} A^{(1)} \\ \vdots \\ A^{(t)} \\ \vdots \\ A^{(T)} \end{bmatrix} - \begin{bmatrix} B & \cdots & & 0 \\ \vdots & & & \\ E^{t-1}B & B & & \\ \vdots & & & \\ E^{T-1}B & E^{T-2}B & \cdots & B \end{bmatrix} \begin{bmatrix} \overline{P}^{(1)} \\ \vdots \\ \overline{P}^{(t)} \\ \vdots \\ \overline{P}^{(T)} \end{bmatrix}$$

$A^{(t)}$ = vector of the factors representing non-price effects at period t,
$D^{(t)}$ = vector of all demands for electricity in period t (i.e. on-peak, mid-peak, off-peak demand)
$P^{(t)}$ = vector of all electricity prices in period t (i.e. on-peak, mid-peak, off-peak prices)
$B =$ a square matrix of the constant price elasticities (i.e. own-price and cross-price)
$E =$ a square diagonal matrix of the constant lag elasticities

The demand side along with the supply side's first-order optimality conditions as in (8) can be solved as a mixed complementarity problem. The MCP problem is formulated as follows:

MCP: Find $z_{ijh}^{(t)}$, $p_{jh}^{(t)}$, $p_j^{(t)}$, $u_{ijh}^{(t)}$, $d_{jh}^{(t)}$, $d_j^{(t)}$ that satisfy

$$\begin{array}{ll}
z_{ijh}^{(t)} \geq 0 \perp r^t c_i^{(t)} - p_{jh}^{(t)} + u_{ijh}^{(t)} \geq 0 & \forall\ h, i, j\ and\ t \\
p_{jh}^{(t)} \geq 0 \perp \sum_{i=1}^{n} z_{ijh}^{(t)} - d_{jh}^{(t)} \geq 0 & \forall\ h, j\ and\ t \\
u_{ijh}^{(t)} \geq 0 \perp K_i^{(t)} - z_{ijh}^{(t)} \geq 0 & \forall\ h, i, j\ and\ t \\
\ln(d_j^{(t)}) = a_j^{(t)} + \sum_{k=1}^{n} b_{jk} \ln(p_k^{(t)}) + e_{jj} \ln(d_j^{(t-1)}) & \forall\ j\ and\ t \\
d_{jh}^{(t)} = \delta_{jh}^{(t)} d_j^{(t)} & \forall\ h, j\ and\ t \\
p_j^{(t)} = \sum_{h=1}^{H_j^{(t)}} \delta_{jh}^{(t)} p_{jh}^{(t)} & \forall\ j\ and\ t
\end{array} \tag{10}$$

where $a_j^{(t)}$, $d_j^{(t)}$ and $p_j^{(t)}$ are the j^{th} elements of vector $A^{(t)}$, $D^{(t)}$ and $P^{(t)}$, respectively. Similarly, b_{jk} and e_{jj} are the elements of matrices B and E, respectively. The first three conditions are the supply side conditions and the fourth equation is the geometric distributed lagged demand equation. The fourth conditions are replaced by their exponential form in some computational experiments:

$$d_j^{(t)} = \exp(a_j^{(t)}) \prod_{k=1}^{n} \left[(p_k^{(t)})^{b_{jk}} \right] (d_j^{(t-1)})^{e_{jj}} \qquad \forall\ j \ and\ t\ .$$

The last two equations are the connection between the hourly and monthly time scales. The parameter $\delta_{jh}^{(t)}$ is the weight of hourly demands within month t, for demand block j. These weights can be estimated from data for load duration curves in the same months of past years. They have the following property (i.e., sum of the weights in each demand blocks equals to 1):

$$\sum_{h=1}^{H_j^{(t)}} \delta_{jh}^{(t)} = \sum_{h=1}^{H_j^{(t)}} \frac{d_{jh}^{(t)}}{d_j^{(t)}} = \frac{\sum_{h=1}^{H_j^{(t)}} d_{jh}^{(t)}}{d_j^{(t)}} = 1$$

The fifth equation ensures that the demand variation within a block follows the shape of the load duration curve. The fifth and sixth equations together ensure that the revenue requirement of suppliers for demand block j is met by revenue collected from consumers:

$$\sum_{h=1}^{H_j^{(t)}} p_{jh}^{(t)} d_{jh}^{(t)} = \sum_{h=1}^{H_j^{(t)}} p_{jh}^{(t)} \delta_{jh}^{(t)} d_j^{(t)} = p_j^{(t)} d_j^{(t)}$$

Note that the fifth and the sixth sets of equations do not impose the historical shape of the entire month's load duration curve. The historical parameters $\delta_{jh}^{(t)}$ impose the historical shape of the load duration curve within the hours of demand block *j*, but if prices differ from historical ones, then the monthly load duration curve of the solution can have a shape that is different from the historical shape.

5.3 *Illustrative Example*

An illustrative example is given in this section to clarify the structure and the solution methodology of the process model. This example consists of four periods (T1, T2, T3, T4), four types of generation facilities (nuclear, hydro, coal, gas and oil), and three demand blocks (on-peak, mid-peak and off-peak electricity). The data for GDL demand equations for this illustration are in Table 5-1.

Table 5-1: Own and Cross Price Elasticities (Mountain and Lawson, 1995)

		Price	
b_{jk}	**off-peak**	**mid-peak**	**on-peak**
Quantity **off-peak**	-0.037	0.014	0.023
Quantity **mid-peak**	0.01	-0.027	0.018
Quantity **on-peak**	0.008	0.009	-0.017

These elasticities are taken from Mountain and Lawson (1995). Unfortunately, in their experiment, they did not use any lagged demand term. Therefore, an estimation of the lag elasticity is retrieved from an econometric study by Shin (1985). The lag elasticities for all GDL demand functions are set to 0.75 ($e_j = 0.75$). Although this is acceptable for illustrative purposes, the use of the model for policy purposes would require careful econometric estimation of all elasticities. Also, note that if there are seasonal TOU hours (e.g., winter, summer), the lag elasticities will be affected accordingly.

Table 5-2: Demand Data from April 2004 (T0) to August 2004 (T4). (IESO, 2005)

$d_j^{(t)}$ (MWh)	**off-peak**	**mid-peak**	**on-peak**	**Total**
T0	*4,238,361*	*4,322,465*	*4,084,028*	***12,644,854***
T1	4,261,461	4,406,394	4,279,331	**12,947,186**
T2	4,249,603	4,451,510	4,385,893	**13,087,006**
T3	4,477,074	4,714,803	4,628,355	**13,820,233**
T4	4,525,873	4,743,658	4,688,461	**13,957,993**
Total	**17,514,012**	**18,316,365**	**17,982,041**	**53,812,418**

The hourly demand data for the Ontario market for each month from April 2004 to August 2004 are sorted in descending order. Then, the demand data for each day are grouped into 9 hours of off-peak, 8 hours of mid-peak and 7 hours of on-peak demand, in a day as in the proposed Ontario TOU pricing scheme. However, no weekend and weekday distinction is made for illustrative purposes. Table 5-2 shows this aggregated data. The demands for T0 are needed as the lagged demands for the first period of the model, T1. The model is solved for periods T1 to T4, and the solution is compared with the historical data in Table 5-2. Differences can be attributed to the effects of TOU pricing, or to model errors.

The parameters $\delta_{jh}^{(t)}$ are calculated using the above data and the hourly demands for each group. Each hourly demand in a period is divided by its total demand for the demand block in that period (e.g., each hourly off-peak demand in a month is divided by the total off-peak demand in that month). This gives a crude estimate of the weights of each hour in total monthly demand blocks (off-peak, mid-peak, on-peak).

To estimate the $a_j^{(t)}$ parameters in the lagged demand model, the historical fixed price (e.g., 5.0cents/kWh, which is $50/MWh) and historical demand data (e.g., demand blocks for 2004) are used. The following formula is used to estimate the $a_j^{(t)}$ parameters.

$$a_j^{(t)} = \ln \frac{d_j^{(t)}}{\prod_{k=1}^{n} \left[\left(p_k^{(t)}\right)^{b_{jk}} \right] \left(d_j^{(t-1)}\right)^{e_{jj}}}$$

Table 5-3: Estimates of parameters $a_j^{(t)}$

$a_j^{(t)}$	**off-peak**	**mid-peak**	**on-peak**
T1	3.8204	3.8352	3.8524
T2	3.8135	3.8309	3.8419
T3	3.8677	3.8807	3.8773
T4	3.8395	3.8437	3.8498

Table 5-4: Generator cost data for each month ($/MWh)

	Hydro	**Nuclear**	**Coal**	**Gas and Oil**
T1	1.04	3.79	28.2	61
T2	1.05	3.8	28.4	61.2
T3	1.06	3.81	28.6	61.4
T4	1.07	3.82	28.8	61.6

Operating costs of generators are changing each month to reflect some variation in the economy (e.g., increase in gas and oil prices etc.)[16]. However, the discount rate is assumed to be zero to simplify the illustration, but there is no difficulty to use a positive discount rate. If a positive discount rate is used, $p_j^{(t)}$ in the demand function should be scaled by r^t (i.e., $p_j^{(t)}/r^t$), because econometricians usually use nominal values of prices rather than discounted values, when estimating the demand function.

[16] Generators may avoid these fluctuations in economy by hedging or long-term supply contracts.

Table 5-5: Capacity of facility *i* (available resources) (MW)

Hydro	Nuclear	Coal	Gas and Oil
6,984	9,901	6,882	4,527

The capacity of facilities for different generation technologies is presented in Table 5-5. Available resources for the beginning of year 2004 are assumed to be fixed throughout the year. However, there is no harm to use changing capacities for each period *t*.

The model is coded in GAMS and solved by the MCP solver, PATH (MILES was not as efficient as PATH in computation time). The GAMS code is in the Appendix A. Initial solutions (other than zero) are provided for the PATH solver to avoid execution errors (e.g., a flat start, where all variables in the model are set to zero, causes execution errors because of the logarithmic or exponential terms in the model). The Network-Enabled Optimization System (NEOS) server for optimization (Czyzyk *et al.*, 1998) provides an online PATH solver for mixed complementarity problems. GAMS code can be submitted online and the results can be obtained in the browser (or sent by e-mail) after the computations are done. Both the logarithmic form and the exponential form of the GDL demand functions were submitted to the NEOS server to examine any possible differences in computation time spent on GAMS/PATH.

Though the PATH solver was robust enough to find the equilibrium solution for a 4-month model, it was unable to find a solution for a 12-month model due to time limitations (28800 seconds=8 hours) on the NEOS server. To overcome this difficulty, an iterative solution procedure is introduced. One month is solved by itself and fixed for the next month (e.g., this month's demand at equilibrium is fixed to the next month's lagged demand). In other words, instead of solving the model at once for all 4 months, the iterative solution procedure solves each month separately, which is expected to reduce the computation time.

Solutions of all models (exponential GDL, logarithmic GDL and the iterative model with logarithmic GDL) were identical. The only difference was the computation times. The following tables display the results.

Table 5-6: $z_{ij}^{(t)}$ [17] Energy (MWh) flowing from facility i to demand block j for each month

$z_{ij}^{(t)}$	(MWh)	**off-peak**	**mid-peak**	**on-peak**
T1	**Hydro**	1,948,536	1,732,032	1,515,528
T1	**Nuclear**	2,482,923	2,443,492	2,148,517
T1	**Coal**	10,325	201,492	566,285
T2	**Hydro**	1,885,680	1,676,160	1,466,640
T2	**Nuclear**	2,512,731	2,376,240	2,079,210
T2	**Coal**	87,877	346,691	780,548
T3	**Hydro**	1,948,536	1,732,032	1,515,528
T3	**Nuclear**	2,653,770	2,455,448	2,148,517
T3	**Coal**	147,099	460,293	898,628
T3	**Gas and Oil**			42
T4	**Hydro**	1,948,536	1,732,032	1,515,528
T4	**Nuclear**	2,680,126	2,455,448	2,148,517
T4	**Coal**	165,762	485,864	960,978

Table 5-7: $d_j^{(t)}$ Energy (MWh) demand for demand block j

$d_j^{(t)}$ (MWh)	**off-peak**	**mid-peak**	**on-peak**	**Total**
T1	4,441,784	4,377,017	4,230,330	**13,049,130**
T2	4,486,288	4,399,091	4,326,398	**13,211,777**
T3	4,749,405	4,647,773	4,562,714	**13,959,893**
T4	4,794,423	4,673,344	4,625,023	**14,092,790**
Total	**18,471,900**	**18,097,225**	**17,744,465**	**54,313,590**

[17] Note that $z_{ij}^{(t)} = \sum_{h=1}^{H_j} z_{ijh}^{(t)}$

Table 5-8: $p_j^{(t)}$ Marginal cost (TOU prices, \$/MWh) for demand block j

$p_j^{(t)}$ (\$/MWh)	**off-peak**	**mid-peak**	**on-peak**
T1	8.49	22.71	28.20
T2	15.21	28.40	28.40
T3	17.45	28.60	28.77
T4	20.10	28.80	28.80

Table 5-9: Computation times (seconds)

CPU Times (seconds)			
Logarithmic GDL	***7,886.68***	**Exponential GDL**	***3,330.73***

Iterative model	**T1**	**T2**	**T3**	**T4**	**Total**
(with logarithmic GDL)	69.22	62.14	73.65	74.89	**279.9**

The above results showed that TOU prices for all models are as expected, (i.e., higher prices for on-peak, lower prices for off-peak). For some months, the TOU prices are exactly equal to the operating cost of the generator that serves the last unit of energy (i.e., the marginal cost of production). The demand for off-peak block is higher than the actual demand values in Table 5-2, whereas, both mid-peak and on-peak demands are lower than the actual demand values. This is expected since the off-peak price is less than mid-peak and on-peak prices, which in turn increases the demand during the off-peak period. Table 5-10 summarizes the changes in demand for each demand block j.

Table 5-10: Percentage change in $d_j^{(t)}$ Energy (MWh) demand for demand block j (compared to actual demand values in Table 5-2)

$d_j^{(t)}$ (MWh)	**off-peak**	**mid-peak**	**on-peak**	**Total**
T1	4.23%	-0.67%	-1.15%	**0.79%**
T2	5.57%	-1.18%	-1.36%	**0.95%**
T3	6.08%	-1.42%	-1.42%	**1.01%**
T4	5.93%	-1.48%	-1.35%	**0.97%**
Total	**5.47%**	**-1.20%**	**-1.32%**	**0.93%**

Table 5-10 depicts that there is about 5.5% increase in off-peak demand when compared to actual off-peak demand in 2004. The mid-peak and on-peak demands are decreased about 1.2% when compared to actual mid-peak and on-peak demands. Overall demand over the 4 months has increased by almost 1% with the implementation of TOU prices. This increase is expected since the TOU prices are lower than the actual fixed price of 50$/MWh (5cents/kWh).

The generation capacity in the market is assumed to be fixed for the entire model scope. In reality, regular maintenance and unexpected generator failures may lower this fixed capacity and capacity shortages may force more expensive resources (e.g., gas and oil) to be used.

The computation time of the exponential form is less than that of the logarithmic form. But different instances have shown that neither the logarithmic form nor the exponential form has any advantage over each other (sometimes the exponential form and sometimes the logarithmic form is faster). The iterative model has the best performance in terms of computation time (more than 12 times faster than the model with exponential GDL demand).

The iterative solution procedure is employed for a 12-month (yearly) TOU pricing model. The hourly demand data for the Ontario market from November 2003 to December 2004 are used to compute the parameters $\delta_{jh}^{(t)}$ and $a_j^{(t)}$. The same data for elasticities, which are used in the 4-month model, are used for the 12-month model. The results are presented in Appendix B. Similar conclusions, as for the 4-month model, can be drawn from these results.

5.4 *Extensions*

5.4.1 Fixed Pricing Model

Instead of TOU pricing, some consumers may prefer a fixed price or regulator bodies in electricity markets may choose to implement a fixed pricing scheme. In this case, consumers' prices need not vary by month, nor by time of day. It is possible to model a single price for the whole day, but which varies by month, or season. It is also possible to define time-of-use prices that are the same in every month. We illustrate by showing how to model a single price that is the same at all times of day and in all months.

It is easy to incorporate the revenue requirements of the suppliers or retailers to the model, by modifying the fourth and sixth sets of equations in (10) to reflect the revenue requirements of suppliers and retailers.

$$\boxed{\begin{array}{ll}
z_{ijh}^{(t)} \geq 0 \perp r^t c_i^{(t)} - p_{jh}^{(t)} + u_{ijh}^{(t)} \geq 0 & \forall\ h,\ i,\ j \text{ and } t \\
p_{jh}^{(t)} \geq 0 \perp \sum_{i=1}^{n} z_{ijh}^{(t)} - d_{jh}^{(t)} \geq 0 & \forall\ h,\ j \text{ and } t \\
u_{ijh}^{(t)} \geq 0 \perp K_i^{(t)} - z_{ijh}^{(t)} \geq 0 & \forall\ h,\ i,\ j \text{ and } t \\
\ln(d_j^{(t)}) = a_j^{(t)} + \sum_{k=1}^{n} b_{jk} \ln(P_f) + e_{jj} \ln(d_j^{(t-1)}) & \forall\ j \text{ and } t \\
d_{jh}^{(t)} = \delta_{jh}^{(t)} d_j^{(t)} & \forall\ h,\ j \text{ and } t \\
\sum_{t=1}^{T}\sum_{j=1}^{n} P_f d_j^{(t)} = \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{h=1}^{H_j^{(t)}} p_{jh}^{(t)} d_{jh}^{(t)} &
\end{array}} \qquad (11)$$

The fourth equation is the geometric distributed lagged demand with fixed price P_f. The sixth equation ensures that the revenue requirements of suppliers over all periods are met. Fixed price multiplied by the total demand (i.e., sum of demand blocks in all periods) is equal to the revenues collected from hourly prices and demands over all hours, demand blocks and periods. Note that the fixed price P_f is actually a weighted average of hourly prices:

$$P_f = \frac{\sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{h=1}^{H_j^{(t)}} p_{jh}^{(t)} d_{jh}^{(t)}}{\sum_{t=1}^{T}\sum_{j=1}^{n} d_j^{(t)}}$$

The fixed pricing model cannot be solved by the aforementioned iterative procedure because the price is fixed over all periods. It should be solved as one model over all months to find the fixed price for all months. No special procedures are required for the 4-month model, but for the 12-month model, we make the model much smaller by an approximation called the "representative weekday model". An illustrative example is given in sub-section 5.4.2.

The fixed pricing model (equation (11)) is coded in GAMS and solved by the PATH solver with the same data and parameters that are used in the TOU pricing model in section 5.3. Solution of the fixed pricing model for a 4-month period is presented in the following tables.

Table 5-11: $z_{ij}^{(t)}$ Energy (MWh) flowing from facility i to demand block j for each month

$z_{ij}^{(t)}$	(MWh)	off-peak	mid-peak	on-peak
T1	**Hydro**	1,948,536	1,732,032	1,515,528
T1	**Nuclear**	2,313,107	2,448,590	2,148,517
T1	**Coal**		222,321	615,451
T2	**Hydro**	1,885,680	1,676,160	1,466,640
T2	**Nuclear**	2,354,318	2,376,240	2,079,210
T2	**Coal**	9,766	392,728	840,067
T3	**Hydro**	1,948,536	1,732,032	1,515,528
T3	**Nuclear**	2,492,604	2,455,448	2,148,517
T3	**Coal**	35,935	518,153	963,640
T3	**Gas and Oil**			762
T4	**Hydro**	1,948,536	1,732,032	1,515,528
T4	**Nuclear**	2,541,366	2,455,448	2,148,517
T4	**Coal**	36,149	545,117	1,024,242
T4	**Gas and Oil**			99

Table 5-12: $d_j^{(t)}$ Energy (MWh) demand for demand block j

$d_j^{(t)}$ (MWh)	off-peak	mid-peak	on-peak	Total
T1	4,261,643	4,402,943	4,279,496	**12,944,082**
T2	4,249,764	4,445,128	4,385,917	**13,080,809**
T3	4,477,075	4,705,633	4,628,447	**13,811,155**
T4	4,526,051	4,732,597	4,688,386	**13,947,034**
Total	**17,514,533**	**18,286,302**	**17,982,246**	**53,783,080**

Table 5-13: P_f Fixed Price ($/MWh) and computation time (seconds)

P_f Fixed Price ($/MWh)	CPU Time (seconds)
21.79	***18,467.7***

The fixed price is a weighted average of hourly prices and the demand for all periods are very close to actual values for the same period (Table 5-2). Note that the computation time is more than 5 hours, which is too long for this small-size problem. The computation time limit (8 hours) on the NEOS server does not allow the computation of some other instances and variations of the fixed pricing model, such as changing generator capacities by month instead of fixed generator capacities, and long-term models (12 months or so).

This extension of the model can be used to charge consumers who are not on a TOU pricing scheme while meeting the revenue requirements of the suppliers. Moreover, a comparative welfare analysis for TOU and fixed pricing can be done. The welfare effects of TOU pricing on consumers and suppliers can be examined. This is done in section 5.5.

5.4.2 Representative Weekday Model

Similar analysis can be done for a fairly small, but nonetheless representative model. A representative weekday of the month or an average of all hourly demands of weekdays within a month can be used in such analysis. Instead of using all hourly demands in a month (720 hours for a 30-day month), one weekday (24 hours) can represent all weekdays in a month. Electricity demands in weekends are usually lower than that of the weekdays and generally, all demands in weekends are assumed to be off-peak. Therefore, a real application of this procedure would require representative weekend days, too, but for simplicity of the illustration, we ignore the weekend differences and use one representative weekday for the whole month.

Average demands for each 24 hours in each weekday of a week can be used to model a representative weekday of the month. A key assumption in this modeling approach is that the weekdays in a specific month are identical (i.e., one weekday represents all the days in a month). The averages of each 24 hours for each weekday in all months in the year 2004 for the Ontario market are presented in Table B-7 in Appendix B. This table also shows the time intervals (e.g., off-peak, mid-peak and on-peak) for summer and winter months. In earlier experiments, the proposed definitions of the time intervals for the Ontario market caused some anomalies with mid-peak and on-peak prices. Particularly, mid-peak prices were higher than the on-peak prices, because mid-peak demands were usually higher than the on-peak demands according to proposed time intervals. Therefore, another procedure is used to define this time intervals for TOU pricing.

The average hourly demand data for a representative weekday for the Ontario market from January 2004 to December 2004 are sorted in descending order. Then, for summer (winter) months[18] the demand data for each weekday in each month are grouped into 9 hours (9 hours) of off-peak, 8 hours (7 hours) of mid-peak and 7 hours (8 hours) of on-peak demand, in a weekday[19]. This grouping gives an idea about the intervals for TOU pricing for weekdays. The averages of hourly demands in a weekday are grouped in way that for summer months (as well as for winter months) specific TOU pricing intervals are found. Table B-7 in Appendix B shows the approximate TOU intervals calculated for summer and winter months in 2004 for the Ontario market. The TOU intervals for weekdays are as follows:

- off-peak hours: 11pm-7am for all months (summer and winter)

[18] Summer months are from April to September (6months), and winter months are from October to March (6months).

[19] For summer months, the highest 7 hours (8 hours for winter) are marked as on-peak. The lowest 9 hours (9hours for winter) are marked as off-peak. The rest of the hours are marked as mid-peak, which is 8 hours (7 hours for winter) for summer months.

- mid-peak hours: 8am-9am, 1pm-4pm and 10pm for winter and 8am-10am, 6pm-10pm for summer
- on-peak hours: 10am-12am and 5pm-9pm for winter and 11am-5pm for summer

Note that these TOU intervals are approximated only from 2004 data for the Ontario market. A more accurate analysis can be performed with previous years' data, and TOU intervals for a weekday can be estimated more carefully.

The parameters $\delta_{jh}^{(t)}$ are calculated by using $\delta_{jh}^{(t)} = d_{jh}^{(t)} / d_{j}^{(t)}$ ($d_{j}^{(t)}$ parameters are the total off-peak, mid-peak and off-peak demand). The $a_{j}^{(t)}$ parameters were estimated by using the same procedure in section 5.3. Both the TOU pricing model (from MCP (10)) and the fixed pricing model (from MCP (11)) are solved in GAMS/PATH solver with the same data given in section 5.3. GAMS codes are in Appendix A and results are summarized in Appendix B. The TOU pricing model and the fixed pricing model were solved as mentioned in previous sub-section 5.4.1. Prices, computation times and demands for both models are given in the following tables.

Table 5-14: $p_j^{(t)}$ TOU prices for demand block j and P_f fixed price ($/MWh)

$p_j^{(t)}$ ($/MWh)	off-peak	mid-peak	on-peak
T1	27.40	27.40	31.66
T2	27.60	27.60	27.60
T3	17.63	27.80	27.80
T4	12.44	28.00	28.00
T5	13.28	28.20	28.20
T6	18.06	28.40	28.40
T7	21.56	28.60	28.60
T8	23.57	28.80	28.80
T9	12.94	29.00	29.00
T10	13.21	29.20	29.20
T11	21.35	29.40	29.40
T12	29.60	29.60	29.60

P_f ($/MWh)
24.356

Table 5-15: Computation times (seconds) for TOU and Fixed Pricing Models

CPU Times (seconds)	
TOU pricing	**4.94**
Fixed Pricing	**13.7**

Table 5-16: $d_j^{(t)}$ Energy (MWh) demand for demand block j for each month for the TOU and Fixed Pricing models for the Representative Weekday

	TOU Pricing				**Fixed Pricing**			
$d_j^{(t)}$ (MWh)	**off-peak**	**mid-peak**	**on-peak**	**Total**	**off-peak**	**mid-peak**	**on-peak**	**Total**
T1	175,493	154,977	183,566	**514,036**	174,911	154,557	184,017	**513,485**
T2	165,250	144,523	170,424	**480,197**	164,839	144,211	170,738	**479,788**
T3	155,759	138,035	162,461	**456,255**	152,870	138,422	163,279	**454,571**
T4	152,674	149,264	132,264	**434,202**	146,094	150,775	133,628	**430,496**
T5	152,421	149,779	135,468	**437,668**	143,416	152,032	137,339	**432,788**
T6	157,418	155,702	143,365	**456,485**	147,891	158,147	145,373	**451,410**
T7	158,033	157,330	146,840	**462,203**	149,234	159,604	148,716	**457,554**
T8	159,139	160,321	149,137	**468,597**	151,320	162,354	150,805	**464,479**
T9	150,949	152,055	140,030	**443,034**	141,078	154,715	142,117	**437,910**
T10	150,208	128,430	151,473	**430,111**	138,649	131,124	154,138	**423,911**
T11	159,622	133,832	160,165	**453,620**	148,550	136,341	162,690	**447,581**
T12	172,875	144,183	173,941	**491,000**	163,801	146,178	175,994	**485,972**
Total	**1,909,841**	**1,768,432**	**1,849,133**	**5,527,407**	**1,822,651**	**1,788,461**	**1,868,835**	**5,479,947**

Similar conclusions as for the 4-month model can be drawn from the tables above. The tables in Appendix B (B-10, B-11) compare the TOU pricing demand, fixed pricing demand and the actual demand for the representative weekday model. Fixed pricing demand is almost the same as the actual demand for all demand blocks and all months. TOU pricing off-peak demand is about 5% more than the fixed pricing off-peak demand. Both mid-peak and on-peak demands of TOU pricing model are around 1% less than that of the fixed pricing model. It is safe to say that these results are consistent with the 4-month model.

Figure B-1 in Appendix B compares the TOU prices and hourly prices for a representative weekday of T5 (May 2004). TOU prices move consistently with the hourly prices, since TOU prices are weighted averages of the hourly prices.

The representative weekday model is very useful and fast in estimating the TOU prices and the fixed price. Instead of using hourly averages for the weekday, a representative week model within a month can be selected and equilibrium prices and quantities can be computed. This whole week representation for a specific month instead of a weekday representation can be utilized to find the TOU prices for weekends. This would allow the weekday and weekend distinction, which is more suitable and more accurate for TOU pricing.

5.4.3 Welfare Analysis: TOU vs. Fixed Pricing

In this sub-section a welfare analysis for TOU pricing versus fixed pricing is presented. The economic impact of TOU pricing scheme is measured. The following table compares the fixed and TOU pricing models' equilibrium solutions for the 4-month model.

Table 5-17: Percent change in Prices and Demand after the implementation of TOU Pricing Scheme for the 4-month model ([TOU-Fixed]/Fixed)

	% Change in Prices				**% Change in Demand**			
	off-peak	**mid-peak**	**on-peak**		**off-peak**	**mid-peak**	**on-peak**	**Total**
T1	-61.06%	4.23%	29.42%	**T1**	4.23%	-0.59%	-1.15%	**0.81%**
T2	-30.21%	30.34%	30.34%	**T2**	5.57%	-1.04%	-1.36%	**1.00%**
T3	-19.94%	31.25%	32.04%	**T3**	6.08%	-1.23%	-1.42%	**1.08%**
T4	-7.78%	32.17%	32.17%	**T4**	5.93%	-1.25%	-1.35%	**1.05%**
				Total	**5.47%**	**-1.03%**	**-1.32%**	**0.99%**

Equilibrium TOU prices for the off-peak demand blocks for the 4-month model are significantly lower than the equilibrium fixed price. On the other hand, equilibrium TOU prices for mid-peak and on-peak demand blocks are higher than the fixed price.. The off-peak demands for TOU scheme are about 5% higher than that of the fixed pricing scheme. The change in demand for mid-peak and on-peak hours is around -1%. Some amount of the on-peak and mid-peak demand is shifted to off-peak hours. More accurately, this can be attributed partially to a shift in consumption from mid- peak and on-peak hours to off-peak hours and partially to a reduction in consumption at mid-peak and on-peak hours. This was expected, because many studies have reported findings that show either a shift in consumption from peak hours to off-peak hours or a demand reduction in peak hours. Total demand for each period has increased almost 1%. This can be attributed to lower prices for the off-peak hours which cause an increase in total demand. The results reported in this thesis are, therefore, consistent with real experiments in TOU pricing.

For welfare analysis, since the demand function is not symmetric (non-integrable), a consumer utility function cannot be derived from the GDL demand function. Therefore, an approximation method, which is introduced by Arnold Harberger (1971), is used to estimate the change in consumers' total value. He used a Taylor series approximation for the change in total value for a single consumer as follows.

$$\frac{\Delta U}{\lambda + \frac{1}{2}\Delta\lambda} \cong \sum_{j=1}^{n}\left(p_j + \frac{1}{2}\Delta p_j\right)\Delta X_j$$

The left side is the change in total value in monetary units. If the changes in prices (Δp_j), quantities demanded (ΔX_j) and marginal utility of income $\Delta\lambda$ is small enough to ignore the third order terms in Taylor expansion, this expression is fairly accurate (Fuller, 1996).

Change in consumers' surplus is, then given by the expression below, change in consumers' total value minus change in consumers' payments (Fuller, 1996).

$$\text{Change in Consumers' Surplus} \cong \sum_{i=1}^{n}\left(p_j + \frac{1}{2}\Delta p_j\right)\Delta X_j - \left[(p_j + \Delta p_j)(X_j + \Delta X_j) - p_j X_j\right]$$

Change in producers' surplus can be calculated by change in profits, or equivalently, change in suppliers' revenues minus change in suppliers' costs. Table 5-18 summarizes the welfare analysis after the implementation of TOU prices.

Table 5-18: Welfare Analysis for the 4-Month model (changes are "TOU-Fixed Price") (T1:May 04…T4:August 04)

	Change in Consumers' Total Value ($)	Change in Consumer' Payments ($)	Change in Consumers' Surplus ($)	Change in Suppliers' Revenues ($)	Change in Suppliers' Costs ($)	Change in Suppliers' Surplus ($)	Change in Total Surplus ($)
T1	921,188	-25,656,845	26,578,033	-25,654,900	-1,058,390	-24,596,510	**1,981,523**
T2	1,726,405	30,996,037	-29,269,632	30,997,200	-177,490	31,174,690	**1,905,058**
T3	2,222,900	46,108,469	-43,885,569	46,110,900	234,950	45,875,950	**1,990,381**
T4	2,518,739	60,230,948	-57,712,209	60,231,400	728,230	59,503,170	**1,790,961**
Total	**7,389,231**	**111,678,609**	**-104,289,378**	**111,684,600**	**-272,700**	**111,957,300**	**7,667,922**

Change in total surplus can be calculated by adding changes in consumers' surplus and producers' surplus. It can be concluded that TOU pricing scheme after a fixed pricing scheme increases the consumers' total value, but decreases consumers' surplus because of higher consumer payments to suppliers. On the other hand, the suppliers' are better off with the TOU pricing since their revenues increase considerably. The net welfare to the society is increased by TOU prices. Gains on suppliers' surplus compensated the loss in consumers' surplus.

Similar analysis can be performed for the representative weekday model. Comparison of TOU pricing model results with the fixed pricing model results are presented in Appendix B. Table 5-19 displays the welfare analysis for the representative weekday model. Similar to 4-month model, representative weekday model also shows an increase in the net welfare. Note that these increases are based on an average weekday of each month; there are some welfare losses in winter months (T1:January, T2:February, T12:December) and most of the welfare gains are from summer and fall months.

Table 5-19: Welfare Analysis for the Representative Weekday Model (changes are "TOU-Fixed Price") (T1: January04…T12: December 04)

	Change in Consumers' Total Value	**Change in Consumer' Payments**	**Change in Consumers' Surplus**	**Change in Suppliers' Revenues**	**Change in Suppliers' Costs**	**Change in Suppliers' Surplus**	**Change in Total Surplus**
T1	13,310	2,359,964	-2,346,654	2,359,710	13,188	2,346,522	**-133**
T2	10,637	1,567,733	-1,557,096	1,567,500	11,302	1,556,198	**-898**
T3	29,212	27,968	1,244	27,730	14,957	12,773	**14,018**
T4	45,811	-702,677	748,488	-702,931	1,195	-704,126	**44,362**
T5	61,073	-472,410	533,482	-472,590	-16,150	-456,440	**77,042**
T6	84,575	341,130	-256,556	340,900	17,230	323,670	**67,114**
T7	92,094	961,624	-869,530	961,330	24,093	937,237	**67,707**
T8	88,989	1,350,122	-1,261,133	1,349,880	30,341	1,319,539	**58,407**
T9	57,453	-241,385	298,838	-241,600	-23,179	-218,421	**80,417**
T10	73,596	-167,515	241,111	-167,720	-29,455	-138,265	**102,846**
T11	117,734	1,150,479	-1,032,745	1,150,220	32,695	1,117,525	**84,780**
T12	135,622	2,697,243	-2,561,621	2,697,000	148,803	2,548,197	**-13,424**
Total	**810,106**	**8,872,278**	**-8,062,172**	**8,869,429**	**225,019**	**8,644,409**	**582,237**

Note that this welfare analysis is for an average weekday; therefore the values are much smaller than that of the welfare analysis for the 4-month model, which represents a whole month, including weekends. Also note that in this analysis T5:May and T8:August corresponds to the previous welfare analysis for the 4-month model

These welfare analyses can be used by regulatory bodies in determining whether to pursue TOU prices or fixed prices. The welfare gains from TOU prices can be compared with the investment in metering technology and communication infrastructure.

6. Conclusion

Pricing is the most fundamental aspect of electricity markets whose design must balance objectives that often conflict with each other. Economic theory dictates that efficient pricing is achieved when electricity is priced at the marginal cost of supplying the last increment of electricity demand. However, in retail electricity markets, it requires strenuous efforts to implement a pricing scheme that reflects this marginal cost of electricity.

In this thesis, different pricing schemes are examined which can be used for many electricity retail markets. A computable equilibrium model is developed to estimate the time-of-use (TOU) prices. This model is significantly different from any other pricing model for electricity markets, because of the consideration of the time-differentiated pricing concept in an optimization and equilibrium framework. Furthermore, it overcomes very important shortcomings in electricity market models: existing models either ignored the demand response to changing prices, or, at the other extreme, they assumed that the full demand response occurred within one hour. The model considers the demand side effect in pricing. It may be a useful tool to forecast the TOU prices and analyze the welfare changes before the implementation.

However, the model has some weaknesses. First of all, it requires carefully estimated demand functions with significant own and cross price elasticities and as well as lag elasticities. However, the number of econometric studies on TOU prices for Ontario market is very limited (only one study by Mountain and Lawson (1995) in the last decade). Therefore, estimating the model parameters is another task to accomplish in order to reach the model objectives.

Another problem with the model is that it does not consider the transmission grid, and therefore many limitations of transmission lines are not examined. Line and voltage limits affect the flow of energy from generators to demand nodes and congestion is an important problem in transmission lines. Models that take into account such problems and reflect the costs of these issues may estimate prices more precisely and accurately.

Beyond these weaknesses, the model has a bright future and there are further research venues to explore. As declared by Ontario Energy Board (2004) on December 7th, the Ontario government plans to implement a regulated TOU scheme for medium/small commercial and residential customers in Ontario.[20] The proposed model can be helpful in estimating the TOU prices and assessing the outcomes of a pricing reform. The representative weekday model is very promising in computation times and provides very close solutions to models where all hours in a year are represented.

There are many ways in which this foundational basic framework can be extended. Through the introduction of multiple-firm structure, strategic interactions between competing firms can be analyzed, in order to explore the potential for large firms to "game" the market, in the context of a model that more realistically represents

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[20] OEB plan includes the customers who have less than 250,000kWh yearly consumption, such as institutions, schools, universities, hospitals. Also, new residential customers are mandated to have TOU meters. Board has planned to install 800,000 TOU meters by the end of 2007. It is estimated that the capital and operating costs of implementing a TOU scheme is about 1.07 billion, which is an incremental cost $3-$4 on an average customer's monthly bill. (discounted estimate in a 15-year horizon)

consumers' responses to changing prices. Also, the model can be made more realistic by the introduction of linearized DC network in the model. Moreover, stochastic components such as generator failures, weather conditions and other factors can be examined. A CPP pricing scheme can be implemented with a stochastic component of price spikes.

Appendix A

GAMS CODES

TOU Pricing -4 Month Model- (model with logarithmic or exponential forms of the GDL Demand Function)

```
****LOGARITHMIC FORM
***********************NEOS Job#     : 628921 Password : HpYiGeSo **************************

****EXPONENTIAL FORM
***********************NEOS Job#     : 632302   Password : xMDFafNn  **************************

SETS
       H   set of hours     /h1*h279/
       I   equipment type   / Hydro, Nuclear, Coal, Gas/
       J   load  / offpeak, midpeak, onpeak/
       T   periods /T1*T4/
ALIAS (J,K);

  PARAMETERS
     KAP(I) capacity of facility i    (MW-ontario available resources)
         /    hydro       6984
              nuclear     9901
              coal        6882
              gas         4527/;

  TABLE DELTA(T,J,H)  weight of each hour group   (total =1)
* ---------------add Delta(t,j,h)s here --------------------------- ;

TABLE D0(T,J) demand at time 0  (MWh)
          offpeak           midpeak          onpeak
T1        4238360.62        4322465.3        4084027.76;


  TABLE A(T,J)  factors representing non-price effects
*rounded     estimated by fixed price 5c/kwh
          offpeak       midpeak       onpeak
T1        3.8204        3.8352        3.8524
T2        3.8135        3.8309        3.8419
T3        3.8677        3.8807        3.8773
T4        3.8395        3.8437        3.8498

TABLE C(T,I)  Operating cost per unit of energy for facility i  ($\MWh)
          Hydro       Nuclear     Coal        Gas
T1        1.04        3.79        28.2        61
T2        1.05        3.8         28.4        61.2
T3        1.06        3.81        28.6        61.4
T4        1.07        3.82        28.8        61.6   ;


  TABLE B(J,K) price elasticities own-cross
              offpeak       midpeak       onpeak
offpeak        -0.037        0.014         0.023
midpeak         0.01        -0.027         0.018
onpeak          0.008        0.009        -0.017   ;

  TABLE E(J,J) lag elasticities
                      offpeak   midpeak    onpeak
        offpeak       0.75
        midpeak                  0.75
         onpeak                             0.75 ;

  POSITIVE VARIABLES
       Zh(T,I,J,H) quantity of energy flowing from facility i for each hour h=1...Hj
       Ph(T,J,H)   marginal cost\price of electricity for hourly demand J.H (hourly TOU price)
       U(T,I,J,H)  scarcity rent of facilities  ;
```

```
  FREE VARIABLES
       D(T,J)  demand corresponding to vertical strip j
       Dh(T,J,H)  hourly demand
       P(T,J)     marginal cost\price of electricity for demand J (TOU price);

*initial quesses*
 D.l(T, J)=10000;
 P.l(T, J)=20;
 Dh.l(T, J, H)=5000;
 Ph.l(T, J, H)=20;

EQUATIONS
COMP(T,I,J,H)   dual complementarity condition
DEMBAL(T,J,H)  demand balance
CAPBAL(T,I,J,H)    capacity balance
DEMAND(T,J)    GDL demand equation
DEMANDh(T,J,H) hourly demand
PRICE(T,J)     monthly TOU price;



COMP(T,I,J,H)..  C(T,I)-Ph(T,J,H)+U(T,I,J,H)=G=0;
DEMBAL(T,J,H).. SUM(I, Zh(T,I,J,H))-Dh(T,J,H)=G=0;
CAPBAL(T,I,J,H).. KAP(I)-Zh(T,I,J,H)=G=0;
***DEMAND(T,J).. -LOG(D(T,J))+A(T,J)+SUM(K, B(J,K)*LOG(P(T,K)))+E(J,J)*LOG(D0(T,J)+D(T-1,J))=E=0
;
DEMAND(T,J).. -D(T,J)+exp(A(T,J))*PROD(K, P(T,K)**B(J,K))*((D0(T,J)+D(T-1,J))**E(J,J))=E=0  ;

DEMANDh(T,J,H).. Dh(T,J,H)-DELTA(T,J,H)*D(T,J)=E=0;
PRICE(T,J)..     P(T,J)-SUM(H, DELTA(T,J,H)*Ph(T,J,H))=E=0;

MODEL HTOU /COMP.Zh, DEMBAL.Ph, CAPBAL.U, DEMAND.D, DEMANDh.Dh, PRICE.P/ ;
OPTION MCP=PATH;
****Option to reduce solver output
OPTION LIMROW=0;
OPTION LIMCOL=0;
OPTION SOLPRINT=OFF;
*****MAXIMUM ITERLIM****
OPTION ITERLIM=1E+9;
*****RESOURCE LIMIT IN SECONDS****ALSO THE LIMIT ON NEOS SERVER
OPTION RESLIM=28800;
SOLVE HTOU USING MCP;
  PARAMETER
  Z(T,I,J)   quantity of energy flowing from facility i for theta(j) hours
  CPUTIME    CPU-TIME
  REVENUE(T)    Revenue of suppliers
  COST(T)       Cost of suppliers
  TotalCost     total cost
  TotalRevenue  total revenue;

Z(T,I,J)=SUM(H, Zh.l(T,I,J,H));
  CPUTIME=HTOU.resusd;
  REVENUE(T)=SUM(J, P.l(T,J)*D.l(T,J));
  COST(T)=SUM((I,J,H), C(T,I)*Zh.l(T,I,J,H));
DISPLAY  Ph.l, COST, REVENUE, Z, D.l, "TOU PRICEs", P.l, "CPU TIME", CPUTIME;

DISPLAY "RESULTS HERE", COST, REVENUE, Z, D.l, P.l, CPUTIME;

TotalCost=SUM(T,COST(T));
TotalRevenue=SUM(T, REVENUE(T));
DISPLAY TotalCost, TotalRevenue;
```

TOU Pricing -4 Month Model- (Iterative model with logarithmic or exponential forms of the GDL Demand Function)

```
********************** NEOS Job#     : 628920   Password : XzTvautV *************************

SETS
       H   set of hours     /h1*h279/
       I   equipment type   / Hydro, Nuclear, Coal, Gas/
       J   load  / offpeak, midpeak, onpeak/
       T   periods /T1*T4/
       TT(T) dynamic set;
ALIAS (J,K);

  PARAMETERS
     KAP(I) capacity of facility i    (MW-ontario available resources)
         /    hydro       6984
              nuclear     9901
              coal        6882
              gas         4527/;

  TABLE DELTA(T,J,H)  weight of each hour group   (total =1)
* ---------------add Delta(t,j,h)s here -------------------------- ;

TABLE D0(T,J) demand at time 0  (MWh)
          offpeak           midpeak          onpeak
T1        4238360.62        4322465.3        4084027.76
T2        1                 1                1
T3        1                 1                1
T4        1                 1                1       ;
******TO AVOID SINGULARITY, otherwise PATH will give following error:
**** Exec Error at line 168: log: FUNC SINGULAR: x = 0


  TABLE A(T,J)  factors representing non-price effects
*rounded     estimated by fixed price 5c/kwh
          offpeak       midpeak       onpeak
T1        3.8204        3.8352        3.8524
T2        3.8135        3.8309        3.8419
T3        3.8677        3.8807        3.8773
T4        3.8395        3.8437        3.8498


TABLE C(T,I)  Operating cost per unit of energy for facility i  ($\MWh)
          Hydro       Nuclear     Coal        Gas
T1        1.04        3.79        28.2        61
T2        1.05        3.8         28.4        61.2
T3        1.06        3.81        28.6        61.4
T4        1.07        3.82        28.8        61.6   ;


  TABLE B(J,K) price elasticities own-cross
              offpeak       midpeak       onpeak
offpeak        -0.037        0.014         0.023
midpeak         0.01        -0.027         0.018
onpeak          0.008        0.009        -0.017   ;

  TABLE E(J,J) lag elasticities
                      offpeak   midpeak    onpeak
        offpeak       0.75
        midpeak                  0.75
         onpeak                            0.75 ;

 POSITIVE VARIABLES
       Zh(T,I,J,H) quantity of energy flowing from facility i for each hour h=1...Hj
       Ph(T,J,H)   marginal cost\price of electricity for hourly demand J.H (hourly TOU price)
       U(T,I,J,H)  scarcity rent of facilities  ;
  FREE VARIABLES
       D(T,J)  demand corresponding to vertical strip j
       Dh(T,J,H)  hourly demand
       P(T,J)     marginal cost\price of electricity for demand J (TOU price);

*initial quesses*
```

```
 D.l(T, J)=10000;
 P.l(T, J)=20;
 Dh.l(T, J, H)=5000;
 Ph.l(T, J, H)=20;

EQUATIONS
COMP(T,I,J,H)   dual complementarity condition
DEMBAL(T,J,H)  demand balance
CAPBAL(T,I,J,H)    capacity balance
DEMAND(T,J)    GDL demand equation
DEMANDh(T,J,H) hourly demand
PRICE(T,J)     monthly TOU price;

COMP(TT,I,J,H)..  C(TT,I)-Ph(TT,J,H)+U(TT,I,J,H)=G=0;
DEMBAL(TT,J,H).. SUM(I, Zh(TT,I,J,H))-Dh(TT,J,H)=G=0;
CAPBAL(TT,I,J,H).. KAP(I)-Zh(TT,I,J,H)=G=0;
DEMAND(TT,J).. -LOG(D(TT,J))+A(TT,J)+SUM(K, B(J,K)*LOG(P(TT,K)))+E(J,J)*LOG(D0(TT,J))=E=0  ;
**DEMAND(TT,J).. -D(TT,J)+exp(A(TT,J))*PROD(K, P(TT,K)**B(J,K))*(D0(TT,J)**E(J,J))=E=0  ;

DEMANDh(TT,J,H).. Dh(TT,J,H)-DELTA(TT,J,H)*D(TT,J)=E=0;
PRICE(TT,J)..     P(TT,J)-SUM(H, DELTA(TT,J,H)*Ph(TT,J,H))=E=0;



MODEL HTOU /COMP.Zh, DEMBAL.Ph, CAPBAL.U, DEMAND.D, DEMANDh.Dh, PRICE.P/ ;
OPTION MCP=PATH;
****Option to reduce solver output
OPTION LIMROW=0;
OPTION LIMCOL=0;
OPTION SOLPRINT=OFF;
*****MAXIMUM ITERLIM****
OPTION ITERLIM=1E+9;
*****RESOURCE LIMIT IN SECONDS****ALSO THE LIMIT ON NEOS SERVER
OPTION RESLIM=28800;

  PARAMETER
  Z(T,I,J)   quantity of energy flowing from facility i for theta(j) hours
  CPUTIME    CPU-TIME
  REVENUE(T)    Revenue of suppliers
  COST(T)       Cost of suppliers
  TotalCost     total cost
  TotalRevenue  total revenue;

LOOP (T,
TT(T)=YES;
SOLVE HTOU USING MCP;
D0(T+1,J)=D.l(T,J);
  Z(T,I,J)=SUM(H, Zh.l(T,I,J,H));
  CPUTIME=HTOU.resusd;
  REVENUE(T)=SUM(J, P.l(T,J)*D.l(T,J));
  COST(T)=SUM((I,J,H), C(T,I)*Zh.l(T,I,J,H));
DISPLAY Z, Zh.l, D.l, "LAGGED DEMAND", D0, Dh.l, U.l, "HOURLY PRICES", Ph.l, "TOU PRICE", P.l,
"CPU TIME", CPUTIME;
DISPLAY "LAG DEMAND PARAMETER", D0;
DISPLAY Dh.l, "RESULTS HERE", COST, REVENUE, Z, D.l, P.l, CPUTIME;
TT(T)=NO;
);
TotalCost=SUM(T,COST(T));
TotalRevenue=SUM(T, REVENUE(T));
DISPLAY TotalCost, TotalRevenue;
```

FIXED Pricing -4 Month Model- (model with logarithmic or exponential forms of the GDL Demand Function)

```
********************* NEOS Job#     : 632282   Password : DYWerbfO ********************

SETS
       H   set of hours     /h1*h279/
       I   equipment type   / Hydro, Nuclear, Coal, Gas/
       J   load  / offpeak, midpeak, onpeak/
       T   periods /T1*T4/
ALIAS (J,K);

  PARAMETERS
     KAP(I) capacity of facility i    (MW-ontario available resources)
         /    hydro       6984
              nuclear     9901
              coal        6882
              gas         4527/;


  TABLE DELTA(T,J,H)  weight of each hour group   (total =1)
* ---------------add Delta(t,j,h)s here --------------------------- ;

TABLE D0(T,J) demand at time 0  (MWh)
          offpeak           midpeak          onpeak
T1        4238360.62        4322465.3        4084027.76;


  TABLE A(T,J)  factors representing non-price effects
*rounded     estimated by fixed price 5c/kwh
          offpeak       midpeak       onpeak
T1        3.8204        3.8352        3.8524
T2        3.8135        3.8309        3.8419
T3        3.8677        3.8807        3.8773
T4        3.8395        3.8437        3.8498

  TABLE B(J,K) price elasticities own-cross
              offpeak       midpeak       onpeak
offpeak        -0.037        0.014         0.023
midpeak         0.01        -0.027         0.018
onpeak          0.008        0.009        -0.017   ;

  TABLE E(J,J) lag elasticities
                      offpeak   midpeak    onpeak
        offpeak       0.75
        midpeak                  0.75
         onpeak                             0.75 ;

POSITIVE VARIABLES
  Zh(T,I,J,H) quantity of energy flowing from facility i for each hour h=1...Hj
  Ph(T,J,H)   marginal cost\price of electricity for hourly demand J.H (hourly TOU price)
  U(T,I,J,H)  scarcity rent of facilities  ;

FREE VARIABLES
       D(T,J)  demand corresponding to vertical strip j
       Dh(T,J,H)  hourly demand
       Pf     marginal cost\price of electricity (FIXED price);

*initial quesses*
 D.l(T, J)=10000;
 Pf.l=15;
 Dh.l(T, J, H)=5000;
 Ph.l(T, J, H)=20;

EQUATIONS
COMP(T,I,J,H)   dual complementarity condition
DEMBAL(T,J,H)  demand balance
CAPBAL(T,I,J,H)    capacity balance
DEMAND(T,J)    GDL demand equation  (fixed price)
DEMANDh(T,J,H) hourly demand
REVBAL         revenue balance;
```

```
COMP(T,I,J,H)..  C(T,I)-Ph(T,J,H)+U(T,I,J,H)=G=0;
DEMBAL(T,J,H).. SUM(I, Zh(T,I,J,H))-Dh(T,J,H)=G=0;
CAPBAL(T,I,J,H).. KAP(I)-Zh(T,I,J,H)=G=0;
DEMAND(T,J).. -LOG(D(T,J))+A(T,J)+SUM(K, B(J,K)*LOG(Pf))+E(J,J)*LOG(D0(T,J)+D(T-1,J))=E=0 ;
*DEMAND(T,J).. -D(T,J)+exp(A(T,J))*PROD(K, Pf**B(J,K))*((D0(T,J)+D(T-1,J))**E(J,J))=E=0  ;

DEMANDh(T,J,H).. Dh(T,J,H)-DELTA(T,J,H)*D(T,J)=E=0;
REVBAL..     Pf*SUM((T,J,H), Dh(T,J,H))-SUM((T,J,H), Dh(T,J,H)*Ph(T,J,H))=E=0;

MODEL HTOU /COMP.Zh, DEMBAL.Ph, CAPBAL.U, DEMAND.D, DEMANDh.Dh, REVBAL.Pf/ ;
OPTION MCP=PATH;
****Option to reduce solver output
OPTION LIMROW=0;
OPTION LIMCOL=0;
OPTION SOLPRINT=OFF;
*****MAXIMUM ITERLIM****
OPTION ITERLIM=1E+9;
*****RESOURCE LIMIT IN SECONDS****ALSO THE LIMIT ON NEOS SERVER
OPTION RESLIM=28800;
SOLVE HTOU USING MCP;
  PARAMETER
  Z(T,I,J)   quantity of energy flowing from facility i for theta(j) hours
  CPUTIME    CPU-TIME
  REVENUE(T)    Revenue of suppliers
  COST(T)       Cost of suppliers
  TotalCost     total cost
  TotalRevenue  total revenue;

Z(T,I,J)=SUM(H, Zh.l(T,I,J,H));
  CPUTIME=HTOU.resusd;
  REVENUE(T)=SUM(J, Pf.l*D.l(T,J));
  COST(T)=SUM((I,J,H), C(T,I)*Zh.l(T,I,J,H));

DISPLAY  Ph.l, COST, REVENUE, Z, D.l, "FIXED PRICE", Pf.l, "CPU TIME", CPUTIME;

DISPLAY "RESULTS HERE", COST, REVENUE, Z, D.l, Pf.l, CPUTIME;

TotalCost=SUM(T,COST(T));
TotalRevenue=SUM(T, REVENUE(T));
DISPLAY TotalCost, TotalRevenue;
```

TOU Pricing -12-Month Model- (Iterative model with logarithmic or exponential forms of the GDL Demand Function)

```
**********************NEOS Job#     : 634494   Password : iBmGJnWq *************************
SETS
       H   set of hours     /h1*h279/
       I   equipment type   / Hydro, Nuclear, Coal, Gas/
       J   load  / offpeak, midpeak, onpeak/
       T   periods /T1*T12/
       TT(T) dynamic set;
ALIAS (J,K);

  PARAMETERS
     KAP(I) capacity of facility i    (MW-ontario available resources)
         /    hydro       6984
              nuclear     9901
              coal        6882
              gas         4527/;

  TABLE DELTA(T,J,H)  weight of each hour group   (total =1)
* ---------------add Delta(t,j,h)s here -------------------------- ;

TABLE D0(T,J) demand at time 0  (MWh)
          offpeak           midpeak          onpeak
T1        4561590.36        4778176.32       4607383.82
T2        1                 1                1
T3        1                 1                1
T4        1                 1                1
T5        1                 1                1
T6        1                 1                1
T7        1                 1                1
T8        1                 1                1
T9        1                 1                1
T10       1                 1                1
T11       1                 1                1
T12       1                 1                1 ;
******TO AVOID SINGULARITY, otherwise PATH will give following error:
**** Exec Error at line 168: log: FUNC SINGULAR: x = 0

  TABLE A(T,J)  factors representing non-price effects
*rounded     estimated by fixed price 5c/kwh
         offpeak        midpeak       onpeak
T1        3.9558        3.9488        3.9353
T2        3.7553        3.7255        3.7032
T3        3.8308        3.8585        3.8448
T4        3.7539        3.7471        3.7410
T5        3.8204        3.8352        3.8524
T6        3.8135        3.8309        3.8419
T7        3.8677        3.8807        3.8773
T8        3.8395        3.8437        3.8498
T9        3.7299        3.7652        3.7565
T10       3.8089        3.8078        3.8083
T11       3.8654        3.8543        3.8232
T12       3.9509        3.9383        3.9397 ;

TABLE C(T,I)  Operating cost per unit of energy for facility i  ($\MWh)
          Hydro       Nuclear     Coal        Gas
T1        1           3.75        27.4        60.2
T2        1.01        3.76        27.6        60.4
T3        1.02        3.77        27.8        60.6
T4        1.03        3.78        28          60.8
T5        1.04        3.79        28.2        61
T6        1.05        3.8         28.4        61.2
T7        1.06        3.81        28.6        61.4
T8        1.07        3.82        28.8        61.6
T9        1.08        3.83        29          61.8
T10       1.09        3.84        29.2        62
T11       1.1         3.85        29.4        62.2
T12       1.11        3.86        29.6        62.4     ;
  TABLE B(J,K) price elasticities own-cross
              offpeak       midpeak       onpeak
```

```
offpeak        -0.037        0.014         0.023
midpeak         0.01        -0.027         0.018
onpeak          0.008        0.009        -0.017   ;

TABLE E(J,J) lag elasticities
                      offpeak   midpeak    onpeak
        offpeak       0.75
        midpeak                  0.75
         onpeak                             0.75 ;

 POSITIVE VARIABLES
       Zh(T,I,J,H) quantity of energy flowing from facility i for each hour h=1...Hj
       Ph(T,J,H)   marginal cost\price of electricity for hourly demand J.H (hourly TOU price)
       U(T,I,J,H)  scarcity rent of facilities  ;
  FREE VARIABLES
       D(T,J)  demand corresponding to vertical strip j
       Dh(T,J,H)  hourly demand
       P(T,J)     marginal cost\price of electricity for demand J (TOU price);

*initial quesses*
 D.l(T, J)=10000;
 P.l(T, J)=20;
 Dh.l(T, J, H)=5000;
 Ph.l(T, J, H)=20;

EQUATIONS
COMP(T,I,J,H)   dual complementarity condition
DEMBAL(T,J,H)  demand balance
CAPBAL(T,I,J,H)    capacity balance
DEMAND(T,J)    GDL demand equation
DEMANDh(T,J,H) hourly demand
PRICE(T,J)     monthly TOU price;

COMP(TT,I,J,H)..  C(TT,I)-Ph(TT,J,H)+U(TT,I,J,H)=G=0;
DEMBAL(TT,J,H).. SUM(I, Zh(TT,I,J,H))-Dh(TT,J,H)=G=0;
CAPBAL(TT,I,J,H).. KAP(I)-Zh(TT,I,J,H)=G=0;
DEMAND(TT,J).. -LOG(D(TT,J))+A(TT,J)+SUM(K, B(J,K)*LOG(P(TT,K)))+E(J,J)*LOG(D0(TT,J))=E=0  ;
**DEMAND(TT,J).. -D(TT,J)+exp(A(TT,J))*PROD(K, P(TT,K)**B(J,K))*(D0(TT,J)**E(J,J))=E=0  ;

DEMANDh(TT,J,H).. Dh(TT,J,H)-DELTA(TT,J,H)*D(TT,J)=E=0;
PRICE(TT,J)..     P(TT,J)-SUM(H, DELTA(TT,J,H)*Ph(TT,J,H))=E=0;



MODEL HTOU /COMP.Zh, DEMBAL.Ph, CAPBAL.U, DEMAND.D, DEMANDh.Dh, PRICE.P/ ;
OPTION MCP=PATH;
****Option to reduce solver output
OPTION LIMROW=0;
OPTION LIMCOL=0;
OPTION SOLPRINT=OFF;
*****MAXIMUM ITERLIM****
OPTION ITERLIM=1E+9;
*****RESOURCE LIMIT IN SECONDS****ALSO THE LIMIT ON NEOS SERVER
OPTION RESLIM=28800;

  PARAMETER
  Z(T,I,J)   quantity of energy flowing from facility i for theta(j) hours
  CPUTIME    CPU-TIME
  REVENUE(T)    Revenue of suppliers
  COST(T)       Cost of suppliers
  TotalCost     total cost
  TotalRevenue  total revenue;

LOOP (T,
TT(T)=YES;
SOLVE HTOU USING MCP;
D0(T+1,J)=D.l(T,J);
  Z(T,I,J)=SUM(H, Zh.l(T,I,J,H));
  CPUTIME=HTOU.resusd;
  REVENUE(T)=SUM(J, P.l(T,J)*D.l(T,J));
  COST(T)=SUM((I,J,H), C(T,I)*Zh.l(T,I,J,H));
```

```
DISPLAY Z, Zh.l, D.l, "LAGGED DEMAND", D0, Dh.l, U.l, "HOURLY PRICES", Ph.l, "TOU PRICE", P.l,
"CPU TIME", CPUTIME;
DISPLAY "LAG DEMAND PARAMETER", D0;
DISPLAY Dh.l, "RESULTS HERE", COST, REVENUE, Z, D.l, P.l, CPUTIME;
TT(T)=NO;
);
TotalCost=SUM(T,COST(T));
TotalRevenue=SUM(T, REVENUE(T));
DISPLAY TotalCost, TotalRevenue;
```

TOU Pricing -Representative Weekday Model, 12 Months - (model with exponential form of the GDL Demand Function)

```
****EXPONENTIAL FORM
**********************NEOS Job#     : 634056   Password : hSkArGpe *************************

SETS
       H   set of hours     /h1*h24/
       I   equipment type   / Hydro, Nuclear, Coal, Gas/
       J   load  / offpeak, midpeak, onpeak/
       T   periods /T1*T12/
ALIAS (J,K);

  PARAMETERS
     KAP(I) capacity of facility i    (MW-ontario available resources)
         /    hydro       6984
              nuclear     9901
              coal        6882
              gas         4527/;

  TABLE DELTA(T,J,H)  weight of each hour group   (total =1)
* ---------------add Delta(t,j,h)s here -------------------------- ;

TABLE D0(T,J) demand at time 0  (MWh)
       offpeak            midpeak            onpeak
T1     151818.4939        137443.0722        165906.4309;

  TABLE A(T,J)  factors representing non-price effects
*rounded     estimated by fixed price 5c/kwh
     offpeak midpeak onpeak
T1   3.1242  3.0719  3.1084
T2   2.9587  2.9146  2.9558
T3   2.9278  2.9256  2.9673
T4   2.939   3.0418  2.8004
T5   2.9545  2.986   2.9781
T6   2.9991  3.0192  3.0144
T7   2.9851  2.9988  2.9945
T8   2.9922  3.009   2.9914
T9   2.9117  2.948   2.9216
T10  2.9469  2.8187  3.0473
T11  3.0289  2.9818  3.0404
T12  3.0749  3.0222  3.0785

TABLE C(T,I)  Operating cost per unit of energy for facility i  ($\MWh)
          Hydro       Nuclear     Coal        Gas
T1        1           3.75        27.4        60.2
T2        1.01        3.76        27.6        60.4
T3        1.02        3.77        27.8        60.6
T4        1.03        3.78        28          60.8
T5        1.04        3.79        28.2        61
T6        1.05        3.8         28.4        61.2
T7        1.06        3.81        28.6        61.4
T8        1.07        3.82        28.8        61.6
T9        1.08        3.83        29          61.8
T10       1.09        3.84        29.2        62
T11       1.1         3.85        29.4        62.2
T12       1.11        3.86        29.6        62.4  ;

  TABLE B(J,K) price elasticities own-cross
              offpeak       midpeak       onpeak
offpeak        -0.037        0.014         0.023
midpeak         0.01        -0.027         0.018
onpeak          0.008        0.009        -0.017   ;

  TABLE E(J,J) lag elasticities
                      offpeak   midpeak    onpeak
        offpeak       0.75
        midpeak                  0.75
         onpeak                              0.75 ;

  POSITIVE VARIABLES
       Zh(T,I,J,H) quantity of energy flowing from facility i for each hour h=1...Hj
```

```
       Ph(T,J,H)   marginal cost\price of electricity for hourly demand J.H (hourly TOU price)
       U(T,I,J,H)  scarcity rent of facilities  ;
  FREE VARIABLES
       D(T,J)  demand corresponding to vertical strip j
       Dh(T,J,H)  hourly demand
       P(T,J)     marginal cost\price of electricity for demand J (TOU price);

*initial quesses*
 D.l(T, J)=10000;
 P.l(T, J)=20;
 Dh.l(T, J, H)=5000;
 Ph.l(T, J, H)=20;

EQUATIONS
COMP(T,I,J,H)   dual complementarity condition
DEMBAL(T,J,H)  demand balance
CAPBAL(T,I,J,H)    capacity balance
DEMAND(T,J)    GDL demand equation
DEMANDh(T,J,H) hourly demand
PRICE(T,J)     monthly TOU price;

COMP(T,I,J,H)..  C(T,I)-Ph(T,J,H)+U(T,I,J,H)=G=0;
DEMBAL(T,J,H).. SUM(I, Zh(T,I,J,H))-Dh(T,J,H)=G=0;
CAPBAL(T,I,J,H).. KAP(I)-Zh(T,I,J,H)=G=0;
*DEMAND(T,J).. -LOG(D(T,J))+A(T,J)+SUM(K, B(J,K)*LOG(P(T,K)))+E(J,J)*LOG(D0(T,J)+D(T-1,J))=E=0  ;
DEMAND(T,J).. -D(T,J)+exp(A(T,J))*PROD(K, P(T,K)**B(J,K))*((D0(T,J)+D(T-1,J))**E(J,J))=E=0  ;

DEMANDh(T,J,H).. Dh(T,J,H)-DELTA(T,J,H)*D(T,J)=E=0;
PRICE(T,J)..     P(T,J)-SUM(H, DELTA(T,J,H)*Ph(T,J,H))=E=0;

MODEL HTOU /COMP.Zh, DEMBAL.Ph, CAPBAL.U, DEMAND.D, DEMANDh.Dh, PRICE.P/ ;
OPTION MCP=PATH;
****Option to reduce solver output
OPTION LIMROW=0;
OPTION LIMCOL=0;
OPTION SOLPRINT=OFF;
*****MAXIMUM ITERLIM****
OPTION ITERLIM=1E+9;
*****RESOURCE LIMIT IN SECONDS****ALSO THE LIMIT ON NEOS SERVER
OPTION RESLIM=28800;

SOLVE HTOU USING MCP;
  PARAMETER
  Z(T,I,J)   quantity of energy flowing from facility i for theta(j) hours
  CPUTIME    CPU-TIME
  REVENUE(T)    Revenue of suppliers
  COST(T)       Cost of suppliers
  TotalCost     total cost
  TotalRevenue  total revenue;

Z(T,I,J)=SUM(H, Zh.l(T,I,J,H));
  CPUTIME=HTOU.resusd;
  REVENUE(T)=SUM(J, P.l(T,J)*D.l(T,J));
  COST(T)=SUM((I,J,H), C(T,I)*Zh.l(T,I,J,H));
DISPLAY  Ph.l, COST, REVENUE, Z, D.l, "TOU PRICEs", P.l, "CPU TIME", CPUTIME;

DISPLAY "RESULTS HERE", COST, REVENUE, Z, D.l, P.l, CPUTIME;

TotalCost=SUM(T,COST(T));
TotalRevenue=SUM(T, REVENUE(T));
DISPLAY TotalCost, TotalRevenue;
```

FIXED Pricing -Representative Weekday Model, 12 Months - (model with logarithmic form of the GDL Demand Function)

```
****LOGARITHMIC FORM
**********************NEOS Job#     : 634071   Password : XubLpxEG  *************************

SETS
       H   set of hours     /h1*h24/
       I   equipment type   / Hydro, Nuclear, Coal, Gas/
       J   load  / offpeak, midpeak, onpeak/
       T   periods /T1*T12/
ALIAS (J,K);

  PARAMETERS
     KAP(I) capacity of facility i    (MW-ontario available resources)
        /    hydro       6984
             nuclear     9901
             coal        6882
             gas         4527/;

  TABLE DELTA(T,J,H)  weight of each hour group   (total =1)
* ---------------add Delta(t,j,h)s here -------------------------- ;

TABLE D0(T,J) demand at time 0  (MWh)
       offpeak            midpeak            onpeak
T1     151818.4939        137443.0722        165906.4309;

  TABLE A(T,J)  factors representing non-price effects
*rounded     estimated by fixed price 5c/kwh
     offpeak midpeak onpeak
T1   3.1242  3.0719  3.1084
T2   2.9587  2.9146  2.9558
T3   2.9278  2.9256  2.9673
T4   2.939   3.0418  2.8004
T5   2.9545  2.986   2.9781
T6   2.9991  3.0192  3.0144
T7   2.9851  2.9988  2.9945
T8   2.9922  3.009   2.9914
T9   2.9117  2.948   2.9216
T10  2.9469  2.8187  3.0473
T11  3.0289  2.9818  3.0404
T12  3.0749  3.0222  3.0785

TABLE C(T,I)  Operating cost per unit of energy for facility i  ($\MWh)
          Hydro       Nuclear     Coal        Gas
T1        1           3.75        27.4        60.2
T2        1.01        3.76        27.6        60.4
T3        1.02        3.77        27.8        60.6
T4        1.03        3.78        28          60.8
T5        1.04        3.79        28.2        61
T6        1.05        3.8         28.4        61.2
T7        1.06        3.81        28.6        61.4
T8        1.07        3.82        28.8        61.6
T9        1.08        3.83        29          61.8
T10       1.09        3.84        29.2        62
T11       1.1         3.85        29.4        62.2
T12       1.11        3.86        29.6        62.4  ;

  TABLE B(J,K) price elasticities own-cross
              offpeak       midpeak       onpeak
offpeak        -0.037        0.014         0.023
midpeak         0.01        -0.027         0.018
onpeak          0.008        0.009        -0.017   ;

  TABLE E(J,J) lag elasticities
                     offpeak   midpeak    onpeak
        offpeak       0.75
        midpeak                 0.75
         onpeak                            0.75 ;

  POSITIVE VARIABLES
```

```
       Zh(T,I,J,H) quantity of energy flowing from facility i for each hour h=1...Hj
       Ph(T,J,H)   marginal cost\price of electricity for hourly demand J.H (hourly TOU price)
       U(T,I,J,H)  scarcity rent of facilities  ;
  FREE VARIABLES
       D(T,J)  demand corresponding to vertical strip j
       Dh(T,J,H)  hourly demand
       Pf     marginal cost\price of electricity (FIXED price);

*initial quesses*
D.l(T, J)=100000;
 Pf.l=27;
 Dh.l(T, J, H)=500;
 Ph.l(T, J, H)=27;

EQUATIONS
COMP(T,I,J,H)   dual complementarity condition
DEMBAL(T,J,H)  demand balance
CAPBAL(T,I,J,H)    capacity balance
DEMAND(T,J)    GDL demand equation  (fixed price)
DEMANDh(T,J,H) hourly demand
REVBAL         revenue balance;

COMP(T,I,J,H)..  C(T,I)-Ph(T,J,H)+U(T,I,J,H)=G=0;
DEMBAL(T,J,H).. SUM(I, Zh(T,I,J,H))-Dh(T,J,H)=G=0;
CAPBAL(T,I,J,H).. KAP(I)-Zh(T,I,J,H)=G=0;
DEMAND(T,J).. -LOG(D(T,J))+A(T,J)+SUM(K, B(J,K)*LOG(Pf))+E(J,J)*LOG(D0(T,J)+D(T-1,J))=E=0  ;
***DEMAND(T,J).. -D(T,J)+exp(A(T,J))*PROD(K, Pf**B(J,K))*((D0(T,J)+D(T-1,J))**E(J,J))=E=0  ;
DEMANDh(T,J,H).. Dh(T,J,H)-DELTA(T,J,H)*D(T,J)=E=0;
REVBAL..  SUM((T,J), D(T,J)*(Pf-SUM(H, DELTA(T,J,H)*Ph(T,J,H))))=E=0;

MODEL HTOU /COMP.Zh, DEMBAL.Ph, CAPBAL.U, DEMAND.D, DEMANDh.Dh, REVBAL.Pf/ ;
OPTION MCP=PATH;
****Option to reduce solver output
OPTION LIMROW=0;
OPTION LIMCOL=0;
OPTION SOLPRINT=OFF;
*****MAXIMUM ITERLIM****
OPTION ITERLIM=1E+9;
*****RESOURCE LIMIT IN SECONDS****ALSO THE LIMIT ON NEOS SERVER
OPTION RESLIM=28800;

SOLVE HTOU USING MCP;
  PARAMETER
  Z(T,I,J)   quantity of energy flowing from facility i for theta(j) hours
  CPUTIME    CPU-TIME
  REVENUE(T)    Revenue of suppliers
  COST(T)       Cost of suppliers
  TotalCost     total cost
  TotalRevenue  total revenue;

Z(T,I,J)=SUM(H, Zh.l(T,I,J,H));
  CPUTIME=HTOU.resusd;
  REVENUE(T)=SUM(J, Pf.l*D.l(T,J));
  COST(T)=SUM((I,J,H), C(T,I)*Zh.l(T,I,J,H));
DISPLAY  Ph.l, COST, REVENUE, Z, D.l, "FIXED PRICE", Pf.l, "CPU TIME", CPUTIME;

DISPLAY "RESULTS HERE", COST, REVENUE, Z, D.l, Pf.l, CPUTIME;

TotalCost=SUM(T,COST(T));
TotalRevenue=SUM(T, REVENUE(T));
DISPLAY TotalCost, TotalRevenue;
```

Appendix B

Table B-1: $z_{ij}^{(t)}$ Energy (MWh) flowing from facility *i* to demand block *j* for each month 12-Month (Yearly) TOU Pricing Model (iterative)

$z_{ij}^{(t)}$	TOU Pricing	off-peak	mid-peak	on-peak
T1	**Hydro**	1,948,536	1,732,032	1,515,528
T1	**Nuclear**	2,699,718	2,455,448	2,148,517
T1	**Coal**	570,332	1,154,992	1,363,999
T1	**Gas and Oil**			28,855
T2	**Hydro**	1,822,824	1,620,288	1,417,752
T2	**Nuclear**	2,569,470	2,297,032	2,009,903
T2	**Coal**	291,508	704,347	896,110
T3	**Hydro**	1,948,536	1,732,032	1,515,528
T3	**Nuclear**	2,659,706	2,455,448	2,148,517
T3	**Coal**	119,300	528,897	751,093
T4	**Hydro**	1,885,680	1,676,160	1,466,640
T4	**Nuclear**	2,528,617	2,375,243	2,079,210
T4	**Coal**	43,208	224,841	484,650
T5	**Hydro**	1,948,536	1,732,032	1,515,528
T5	**Nuclear**	2,566,835	2,439,463	2,148,517
T5	**Coal**	39,714	188,628	534,371
T6	**Hydro**	1,885,680	1,676,160	1,466,640
T6	**Nuclear**	2,546,157	2,376,240	2,079,210
T6	**Coal**	121,646	338,725	759,791
T7	**Hydro**	1,948,536	1,732,032	1,515,528
T7	**Nuclear**	2,670,872	2,455,448	2,148,517
T7	**Coal**	170,011	456,772	885,282
T8	**Hydro**	1,948,536	1,732,032	1,515,528
T8	**Nuclear**	2,687,656	2,455,448	2,148,517
T8	**Coal**	181,000	485,171	952,345
T9	**Hydro**	1,885,680	1,676,160	1,466,640
T9	**Nuclear**	2,467,048	2,373,158	2,079,210
T9	**Coal**	56,803	278,016	686,547
T10	**Hydro**	1,948,536	1,732,032	1,515,528
T10	**Nuclear**	2,494,814	2,419,665	2,148,517
T10	**Coal**	33,086	133,010	497,804
T11	**Hydro**	1,885,680	1,676,160	1,466,640
T11	**Nuclear**	2,624,724	2,376,240	2,079,210
T11	**Coal**	185,873	393,957	659,620
T12	**Hydro**	1,948,536	1,732,032	1,515,528
T12	**Nuclear**	2,760,810	2,455,448	2,148,517
T12	**Coal**	543,004	800,966	1,103,231
T12	**Gas and Oil**			3,149

Table B-2: $d_j^{(t)}$ Energy demand (MWh) for demand block *j* for 12-Month (Yearly) TOU Pricing Model (iterative)

$d_j^{(t)}$ (MWh)	off-peak	mid-peak	on-peak	Total
T1	5,218,586	5,342,472	5,056,899	**15,617,957**
T2	4,683,803	4,621,667	4,323,766	**13,629,235**
T3	4,727,542	4,716,377	4,415,138	**13,859,057**
T4	4,457,504	4,276,245	4,030,500	**12,764,248**
T5	4,555,085	4,360,123	4,198,416	**13,113,624**
T6	4,553,483	4,391,125	4,305,641	**13,250,248**
T7	4,789,419	4,644,252	4,549,327	**13,982,998**
T8	4,817,192	4,672,651	4,616,390	**14,106,233**
T9	4,409,531	4,327,334	4,232,397	**12,969,262**
T10	4,476,436	4,284,707	4,161,850	**12,922,993**
T11	4,696,277	4,446,357	4,205,470	**13,348,104**
T12	5,252,350	4,988,446	4,770,425	**15,011,221**
Total	**56,637,206**	**55,071,755**	**52,866,217**	**164,575,178**

Table B-3: Actual Energy demand (MWh) for demand block *j* for the year of 2004 (OEB,2005)

$d_j^{(t)}$ (MWh)	off-peak	mid-peak	on-peak	Total
T1	5,156,194	5,322,153	5,089,690	**15,568,037**
T2	4,625,240	4,615,504	4,348,220	**13,588,965**
T3	4,597,877	4,737,976	4,451,646	**13,787,499**
T4	4,238,361	4,322,465	4,084,028	**12,644,854**
T5	4,261,461	4,406,394	4,279,331	**12,947,186**
T6	4,249,603	4,451,510	4,385,893	**13,087,006**
T7	4,477,074	4,714,803	4,628,355	**13,820,233**
T8	4,525,873	4,743,658	4,688,461	**13,957,993**
T9	4,089,288	4,405,554	4,312,401	**12,807,243**
T10	4,101,095	4,349,336	4,265,278	**12,715,708**
T11	4,349,170	4,512,581	4,293,949	**13,155,699**
T12	4,950,564	5,045,588	4,848,763	**14,844,916**
Total	**53,621,799**	**55,627,523**	**53,676,016**	**162,925,338**

Table B-4: Percent Change in $d_j^{(t)}$ Energy demand (MWh) for demand block *j* for the 12-Month (Yearly) TOU Pricing Model (iterative) (compared to actual demand values in Table B-3)

$d_j^{(t)}$ (MWh)	off-peak	mid-peak	on-peak	Total
T1	1.21%	0.38%	-0.64%	**0.32%**
T2	1.27%	0.13%	-0.56%	**0.30%**
T3	2.82%	-0.46%	-0.82%	**0.52%**
T4	5.17%	-1.07%	-1.31%	**0.94%**
T5	6.89%	-1.05%	-1.89%	**1.29%**
T6	7.15%	-1.36%	-1.83%	**1.25%**
T7	6.98%	-1.50%	-1.71%	**1.18%**
T8	6.44%	-1.50%	-1.54%	**1.06%**
T9	7.83%	-1.78%	-1.86%	**1.27%**
T10	9.15%	-1.49%	-2.42%	**1.63%**
T11	7.98%	-1.47%	-2.06%	**1.46%**
T12	6.10%	-1.13%	-1.62%	**1.12%**
Total	**5.62%**	**-1.00%**	**-1.51%**	**1.01%**

Table B-5: $p_j^{(t)}$ (TOU prices, $/MWh) for demand block j for the 12-Month (Yearly) TOU Pricing Model (iterative)

$p_j^{(t)}$ ($/MWh)	off-peak	mid-peak	on-peak
T1	24.09	27.40	37.61
T2	25.10	27.60	27.60
T3	16.91	27.80	27.80
T4	12.36	26.57	28.00
T5	11.78	21.96	28.20
T6	16.95	28.40	28.40
T7	18.73	28.60	28.60
T8	20.96	28.80	28.80
T9	13.02	26.96	29.00
T10	11.20	21.22	29.20
T11	21.76	29.40	29.40
T12	28.86	29.60	30.44

Table B-6: Computation times (seconds) for 12-Month (Yearly) TOU Pricing Model (iterative)

	CPU Times (seconds)												
TOU pricing (iterative)	***T1***	***T2***	***T3***	***T4***	***T5***	***T6***	***T7***	***T8***	***T9***	***T10***	***T11***	***T12***	**Total**
	47.81	48.44	50.04	84.53	60.19	41.25	49.85	49.09	30.55	79.19	44.48	44.25	**629.67**

Table B-7: Average hourly demands (MWh) for the Representative Weekday and TOU Time Intervals (h1: 1am … h24: 12pm)

MONTH	Jan-04	Feb-04	Mar-04	Apr-04	May-04	Jun-04	Jul-04	Aug-04	Sep-04	Oct-04	Nov-04	Dec-04
h1	19,212	18,062	16,783	15,735	15,300	16,026	16,322	16,465	15,131	14,501	16,289	18,134
h2	18,899	17,735	16,324	15,441	15,114	15,704	15,965	16,100	14,852	14,291	15,827	17,622
h3	18,752	17,573	16,105	15,333	15,022	15,318	15,629	15,834	14,563	14,157	15,610	17,380
h4	18,644	17,478	16,045	15,435	15,110	15,332	15,614	15,861	14,657	14,279	15,492	17,247
h5	18,725	17,754	16,238	15,979	15,529	15,842	15,946	16,196	15,157	15,084	15,689	17,391
h6	19,158	18,385	16,954	17,112	16,457	16,870	16,762	17,215	16,497	16,644	16,356	17,907
h7	20,338	19,521	18,314	18,265	18,212	18,374	17,873	18,212	17,841	18,463	17,736	19,086
h8	21,909	20,692	19,462	18,894	18,939	19,440	19,069	19,515	18,665	18,965	19,098	20,275
h9	22,340	21,068	20,026	19,075	19,247	20,090	19,897	20,336	19,202	18,983	19,638	20,790
h10	22,405	21,059	20,119	19,214	19,483	20,455	20,482	20,866	19,751	19,196	19,656	21,104
h11	22,422	21,015	20,246	19,285	19,723	20,764	20,973	21,258	20,007	19,265	19,754	21,324
h12	22,332	20,936	20,200	19,192	19,626	20,792	21,199	21,447	20,124	19,155	19,778	21,390
h13	22,229	20,657	20,046	19,172	19,770	20,873	21,359	21,679	20,310	19,138	19,724	21,185
h14	22,045	20,549	19,922	18,997	19,623	20,778	21,309	21,639	20,314	18,952	19,599	21,080
h15	21,899	20,372	19,761	18,974	19,494	20,680	21,302	21,570	20,346	18,895	19,567	20,897
h16	21,997	20,388	19,748	19,020	19,586	20,820	21,332	21,655	20,532	19,097	19,659	21,013
h17	22,657	20,821	20,093	18,983	19,518	20,663	21,239	21,559	20,483	19,191	20,296	21,801
h18	23,763	21,518	20,427	18,721	19,202	20,191	20,789	21,048	20,000	19,421	21,232	22,997
h19	23,893	22,070	20,833	18,789	18,964	19,828	20,286	20,506	19,902	19,803	21,163	22,947
h20	23,495	21,905	20,900	19,307	19,194	19,751	20,073	20,692	20,207	19,491	20,665	22,437
h21	23,041	21,417	20,457	19,094	19,197	19,859	20,189	20,573	19,401	18,619	20,153	21,996
h22	22,254	20,667	19,690	17,983	18,146	18,914	19,216	19,233	17,989	17,437	19,425	21,342
h23	21,193	19,581	18,635	16,878	16,788	17,822	18,219	18,265	16,745	16,113	18,251	20,121
h24	19,988	18,747	17,474	15,922	15,888	16,610	16,915	17,176	15,632	15,110	17,291	18,903

Total Off-peak	174,909	164,837	152,870	146,100	143,419	147,898	149,247	151,324	141,074	138,644	148,540	163,791
Total Mid-peak	154,673	144,392	138,654	151,077	152,374	158,528	160,000	162,768	155,117	131,468	136,710	146,582
Total On-peak	184,008	170,741	163,274	133,622	137,340	145,370	148,712	150,807	142,115	154,141	162,697	175,997
Total	513,590	479,970	454,798	430,799	433,132	451,796	457,959	464,899	438,305	424,253	447,948	486,370

LEGEND	Off-peak	Mid-peak	On-peak (winter)	On-peak (summer)

Table B-8: $z_{ij}^{(t)}$ Energy (MWh) flowing from facility i to demand block j for each month TOU and Fixed Pricing for the Representative Weekday Model

$z_{ij}^{(t)}$	TOU PRICING	off-peak	mid-peak	on-peak	FIXED PRICING	off-peak	mid-peak	on-peak
T1	**Hydro**	62,856	48,888	55,872	**Hydro**	62,856	48,888	55,872
T1	**Nuclear**	89,109	69,307	79,208	**Nuclear**	89,109	69,307	79,208
T1	**Coal**	23,528	36,783	48,417	**Coal**	22,946	36,362	48,810
T1	**Gas and Oil**			68	**Gas and Oil**			127
T2	**Hydro**	62,856	48,888	55,872	**Hydro**	62,856	48,888	55,872
T2	**Nuclear**	89,109	69,307	79,208	**Nuclear**	89,109	69,307	79,208
T2	**Coal**	13,285	26,328	35,344	**Coal**	12,874	26,016	35,658
T3	**Hydro**	62,856	48,888	55,872	**Hydro**	62,856	48,888	55,872
T3	**Nuclear**	87,503	69,307	79,208	**Nuclear**	86,178	69,307	79,208
T3	**Coal**	5,400	19,840	27,380	**Coal**	3,836	20,227	28,199
T4	**Hydro**	62,856	55,872	48,888	**Hydro**	62,856	55,872	48,888
T4	**Nuclear**	85,866	79,208	69,307	**Nuclear**	81,632	79,208	69,307
T4	**Coal**	3,952	14,184	14,069	**Coal**	1,606	15,695	15,432
T5	**Hydro**	62,856	55,872	48,888	**Hydro**	62,856	55,872	48,888
T5	**Nuclear**	85,533	79,208	69,307	**Nuclear**	79,234	79,208	69,307
T5	**Coal**	4,032	14,699	17,273	**Coal**	1,326	16,952	19,144
T6	**Hydro**	62,856	55,872	48,888	**Hydro**	62,856	55,872	48,888
T6	**Nuclear**	87,769	79,208	69,307	**Nuclear**	82,611	79,208	69,307
T6	**Coal**	6,793	20,622	25,170	**Coal**	2,424	23,067	27,178
T7	**Hydro**	62,856	55,872	48,888	**Hydro**	62,856	55,872	48,888
T7	**Nuclear**	88,422	79,208	69,307	**Nuclear**	84,031	79,208	69,307
T7	**Coal**	6,755	22,250	28,645	**Coal**	2,348	24,524	30,521
T8	**Hydro**	62,856	55,872	48,888	**Hydro**	62,856	55,872	48,888
T8	**Nuclear**	88,671	79,208	69,307	**Nuclear**	85,138	79,208	69,307
T8	**Coal**	7,612	25,241	30,942	**Coal**	3,326	27,274	32,610
T9	**Hydro**	62,856	55,872	48,888	**Hydro**	62,856	55,872	48,888
T9	**Nuclear**	84,090	79,208	69,307	**Nuclear**	77,265	79,208	69,307
T9	**Coal**	4,003	16,975	21,835	**Coal**	956	19,635	23,922
T10	**Hydro**	62,856	48,888	55,872	**Hydro**	62,856	48,888	55,872
T10	**Nuclear**	82,514	69,307	79,208	**Nuclear**	74,214	69,307	79,208
T10	**Coal**	4,838	10,235	16,393	**Coal**	1,579	12,929	19,058
T11	**Hydro**	62,856	48,888	55,872	**Hydro**	62,856	48,888	55,872
T11	**Nuclear**	88,736	69,307	79,208	**Nuclear**	83,068	69,307	79,208
T11	**Coal**	8,031	15,637	25,085	**Coal**	2,626	18,147	27,610
T12	**Hydro**	62,856	48,888	55,872	**Hydro**	62,856	48,888	55,872
T12	**Nuclear**	89,109	69,307	79,208	**Nuclear**	89,109	69,307	79,208
T12	**Coal**	20,910	25,988	38,861	**Coal**	11,836	27,983	40,914

Table B-9: $d_j^{(t)}$ Energy (MWh) demand for demand block *j* for each month TOU and Fixed Pricing for the Representative Weekday Model

	TOU PRICING				FIXED PRICING			
$d_j^{(t)}$ (MWh)	off-peak	mid-peak	on-peak	Total	off-peak	mid-peak	on-peak	Total
T1	175,493	154,977	183,566	**514,036**	174,911	154,557	184,017	**513,485**
T2	165,250	144,523	170,424	**480,197**	164,839	144,211	170,738	**479,788**
T3	155,759	138,035	162,461	**456,255**	152,870	138,422	163,279	**454,571**
T4	152,674	149,264	132,264	**434,202**	146,094	150,775	133,628	**430,496**
T5	152,421	149,779	135,468	**437,668**	143,416	152,032	137,339	**432,788**
T6	157,418	155,702	143,365	**456,485**	147,891	158,147	145,373	**451,410**
T7	158,033	157,330	146,840	**462,203**	149,234	159,604	148,716	**457,554**
T8	159,139	160,321	149,137	**468,597**	151,320	162,354	150,805	**464,479**
T9	150,949	152,055	140,030	**443,034**	141,078	154,715	142,117	**437,910**
T10	150,208	128,430	151,473	**430,111**	138,649	131,124	154,138	**423,911**
T11	159,622	133,832	160,165	**453,620**	148,550	136,341	162,690	**447,581**
T12	172,875	144,183	173,941	**491,000**	163,801	146,178	175,994	**485,972**
Total	**1,909,841**	**1,768,432**	**1,849,133**	**5,527,407**	**1,822,651**	**1,788,461**	**1,868,835**	**5,479,947**

Table B-10: Actual Demand Values for 2004 (average weekday) and Percent Change in $d_j^{(t)}$ Energy demand (MWh) for demand block *j* for the for the Representative Weekday Model (compared to actual demand values in Table B-9)

	Actual Demand Values for 2004 (Average weekday)				% Change (TOU vs. Actual Demand)			
$d_j^{(t)}$ (MWh)	off-peak	mid-peak	on-peak	Total	off-peak	mid-peak	on-peak	Total
T1	174,909	154,673	184,008	**513,590**	0.33%	0.20%	-0.24%	**0.09%**
T2	164,837	144,392	170,741	**479,970**	0.25%	0.09%	-0.19%	**0.05%**
T3	152,870	138,654	163,274	**454,798**	1.89%	-0.45%	-0.50%	**0.32%**
T4	146,100	151,077	133,622	**430,799**	4.50%	-1.20%	-1.02%	**0.79%**
T5	143,419	152,374	137,340	**433,132**	6.28%	-1.70%	-1.36%	**1.05%**
T6	147,898	158,528	145,370	**451,796**	6.44%	-1.78%	-1.38%	**1.04%**
T7	149,247	160,000	148,712	**457,959**	5.89%	-1.67%	-1.26%	**0.93%**
T8	151,324	162,768	150,807	**464,899**	5.16%	-1.50%	-1.11%	**0.80%**
T9	141,074	155,117	142,115	**438,305**	7.00%	-1.97%	-1.47%	**1.08%**
T10	138,644	131,468	154,141	**424,253**	8.34%	-2.31%	-1.73%	**1.38%**
T11	148,540	136,710	162,697	**447,948**	7.46%	-2.11%	-1.56%	**1.27%**
T12	163,791	146,582	175,997	**486,370**	5.55%	-1.64%	-1.17%	**0.95%**
Total	**1,822,653**	**1,792,344**	**1,868,822**	**5,483,820**	**4.78%**	**-1.33%**	**-1.05%**	**0.79%**

Table B-11: Percent Change in $d_j^{(t)}$ Energy demand (MWh) for demand block *j* for the Representative Weekday Model (comparison of demand values in Table B-9 and Table B-10)

	% Change (Fixed vs. Actual Demand)				% Change (TOU vs. Fixed)			
$d_j^{(t)}$ (MWh)	**off-peak**	**mid-peak**	**on-peak**	**Total**	**off-peak**	**mid-peak**	**on-peak**	**Total**
T1	0.00%	-0.07%	0.00%	**-0.02%**	0.33%	0.27%	-0.25%	**0.11%**
T2	0.00%	-0.13%	0.00%	**-0.04%**	0.25%	0.22%	-0.18%	**0.09%**
T3	0.00%	-0.17%	0.00%	**-0.05%**	1.89%	-0.28%	-0.50%	**0.37%**
T4	0.00%	-0.20%	0.00%	**-0.07%**	4.50%	-1.00%	-1.02%	**0.86%**
T5	0.00%	-0.22%	0.00%	**-0.08%**	6.28%	-1.48%	-1.36%	**1.13%**
T6	0.00%	-0.24%	0.00%	**-0.09%**	6.44%	-1.55%	-1.38%	**1.12%**
T7	-0.01%	-0.25%	0.00%	**-0.09%**	5.90%	-1.42%	-1.26%	**1.02%**
T8	0.00%	-0.25%	0.00%	**-0.09%**	5.17%	-1.25%	-1.11%	**0.89%**
T9	0.00%	-0.26%	0.00%	**-0.09%**	7.00%	-1.72%	-1.47%	**1.17%**
T10	0.00%	-0.26%	0.00%	**-0.08%**	8.34%	-2.05%	-1.73%	**1.46%**
T11	0.01%	-0.27%	0.00%	**-0.08%**	7.45%	-1.84%	-1.55%	**1.35%**
T12	0.01%	-0.28%	0.00%	**-0.08%**	5.54%	-1.36%	-1.17%	**1.03%**
Total	**0.00%**	**-0.22%**	**0.00%**	**-0.07%**	**4.78%**	**-1.12%**	**-1.05%**	**0.87%**

Table B-12: $p_j^{(t)}$ TOU prices for demand block j and P_f fixed price ($/MWh)

$p_j^{(t)}$ ($/MWh)	**off-peak**	**mid-peak**	**on-peak**
T1	27.40	27.40	31.66
T2	27.60	27.60	27.60
T3	17.63	27.80	27.80
T4	12.44	28.00	28.00
T5	13.28	28.20	28.20
T6	18.06	28.40	28.40
T7	21.56	28.60	28.60
T8	23.57	28.80	28.80
T9	12.94	29.00	29.00
T10	13.21	29.20	29.20
T11	21.35	29.40	29.40
T12	29.60	29.60	29.60

P_f ($/MWh)
24.356

Table B-13: Comparison of 12-Month (Yearly) Model with the Representative Weekday Model (comparison of prices in Table B-5 and Table B-12)

	Change in TOU Prices (Representative Weekday minus 12-Month)			% Change in TOU Prices (Representative Weekday minus 12-Month)		
	off-peak	**mid-peak**	**on-peak**	**off-peak**	**mid-peak**	**on-peak**
T1	1.76	-8.54	-28.54	6.86%	-23.76%	-47.41%
T2	0	-6.00	-31.86	0%	-17.84%	-53.58%
T3	-3.69	0	0	-17.31%	0%	0%
T4	-1.62	0	0	-11.51%	0%	0%
T5	5.01	12.80	0	60.62%	83.08%	0%
T6	1.19	1.06	0	7.07%	3.86%	0%
T7	3.06	0	0	16.53%	0%	0%
T8	-0.50	0	-0.33	-2.07%	0%	-1.14%
T9	-5.28	0	0	-28.98%	0%	0%
T10	7.27	13.05	0	122.26%	80.80%	0%
T11	6.13	1.42	0	40.30%	5.06%	0%
T12	0.49	0	-0.84	1.68%	0%	-2.76%

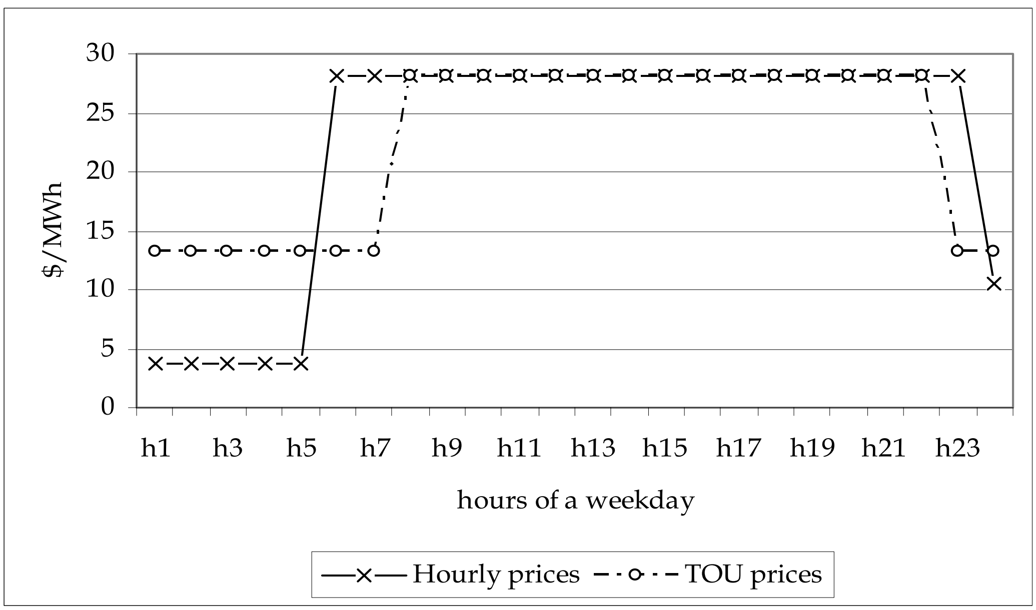

Figure B-1: Comparison of TOU Prices vs. Hourly Prices for T5: May 2004 (Representative Weekday Model)

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