Cooperative Water Resources Allocation among Competing Users

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

A comprehensive model named the Cooperative Water Allocation Model (CWAM) is developed for modeling equitable and efficient water allocation among competing users at the basin scale, based on a multiperiod node-link river basin network. The model integrates water rights allocation, efficient water allocation and equitable income distribution subject to hydrologic constraints comprising both water quantity and quality considerations. CWAM allocates water resources in two steps: initial water rights are firstly allocated to water uses based on legal rights systems or agreements, and then water is reallocated to achieve efficient use of water through water transfers. The associated net benefits of stakeholders participating in a coalition are allocated by using cooperative game theoretical approaches.

The first phase of the CWAM methodology includes three methods for deriving initial water rights allocation among competing water uses, namely the priority-based multiperiod maximal network flow (PMMNF) programming, modified riparian water rights allocation (MRWRA) and lexicographic minimax water shortage ratios (LMWSR) methods. PMMNF is a very flexible approach and is applicable under prior, riparian and public water rights systems with priorities determined by different criteria. MRWRA is essentially a special form of PMMNF adapted for allocation under the riparian regime. LMWSR is designed for application under a public water rights system, which adopts the lexicographic minimax fairness concept. The second step comprises three sub-models: the irrigation water planning model (IWPM) is a model for deriving benefit functions of irrigation water; the hydrologic-economic river basin model (HERBM) is the core component of the coalition analysis, which searches for the values of various coalitions of stakeholders and corresponding optimal water allocation schemes, based on initial water rights, monthly net benefit functions of demand sites and the ownership of water uses; the sub-model cooperative reallocation game (CRG) of the net benefit of the grand coalition adopts cooperative game solution concepts, including the nucleolus, weak nucleolus, proportional nucleolus, normalized nucleolus and Shapley value, to perform equitable reallocation of the net benefits of stakeholders participating in the grand coalition. The economically efficient use of water under the grand coalition is achieved through water transfers based on initial water rights.

Sequential and iterative solution algorithms utilizing the primal simplex method are developed to solve the linear PMMNF and LMWSR problems, respectively, which only include linear water quantity constraints. Algorithms for nonlinear PMMNF and LMWSR problems adopt a two-stage approach, which allow nonlinear reservoir area- and elevation-storage relations, and may include nonlinear water quality constraints. In the first stage, the corresponding linear problems, excluding nonlinear constraints, are solved by a sequential or iterative algorithm. The global optimal solution obtained by the linear programming is then combined together with estimated initial values of pollutant concentrations to be used as the starting point for the sequential or iterative nonlinear programs of the nonlinear PMMNF or LMWSR problem. As HERBM adopts constant price-elasticity water demand functions to derive the net benefit functions of municipal and industrial demand sites and hydropower stations, and quadratic gross benefit functions to find the net benefit functions of agriculture water uses, stream flow demands and reservoir storages, it is a large scale nonlinear optimization problem even when the water quality constraints are not included. An efficient algorithm is built for coalition analysis, utilizing a combination of the multistart global optimization technique and gradient-based nonlinear programming method to solve a HERBM for each possible coalition.

Throughout the study, both the feasibility and the effectiveness of incorporating equity concepts into conventional economic optimal water resources management modeling are addressed. The applications of CWAM to the Amu Darya River Basin in Central Asia and the South Saskatchewan River Basin in western Canada demonstrate the applicability of the model. It is argued that CWAM can be utilized as a tool for promoting the understanding and cooperation of water users to achieve maximum welfare in a river basin and minimize the damage caused by water shortages, through water rights allocation, and water and net benefit transfers among water users under the regulated water market or administrative allocation mechanism.

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Glossary

$\boldsymbol{\alpha}$	real vector
$\alpha(j,t)$	parameter of constant elasticity demand function
$\boldsymbol{\beta}$	real vector
$\beta(j,t)$	price elasticity of constant elasticity demand function
$\varepsilon(j,t)$	elasticity of water demand function
η	turbine efficiency (%)
$\boldsymbol{\theta}$	map ordering the coordinates of vectors in a nonincreasing order
$\boldsymbol{\theta}(\mathbf{f}(\mathbf{x}))$	nonincreasing ordered outcome vector
$\boldsymbol{\theta}\left(\mathbf{f}(\mathbf{x}^*)\right)$	lexicographically smallest nonincreasing ordered outcome vector
λ_l	lower limit of total effective precipitation and irrigation
λ_u	upper limit of total effective precipitation and irrigation
μ	number of uses, $\mu = \lvert U \rvert$
ξ	the largest index of pollutant types
π	permutation
σ	numerical coefficient, $\sigma = 0.00273$ 10^6kWh/10^6m^3·m
τ	the largest index of time periods
υ	characteristic function relating each coalition S to a real number $\upsilon(S)$
$\upsilon(S)$	aggregate value gained by the members of coalition S
$\upsilon(\{1\}), \upsilon(\{2\}), \cdots, \upsilon(\{n\})$	values of individual stakeholders acting in isolation
$\omega(j,t)$	weight for water shortage ratio of water demand node j during period t
Γ	noncooperative water allocation game $\Gamma = \left\langle \bar{Q}_1, \cdots, \bar{Q}_n; NB_1, \cdots, NB_n \right\rangle$
$\Delta H(j,t)$	effective water head of hydropower generation (m)
Δt	period length
$\boldsymbol{\Theta}$	map ordering the coordinates of vectors in a nondecreasing order
$\boldsymbol{\Theta}(\mathbf{y})$	nondecreasing ordered outcome vector $\boldsymbol{\Theta}(\mathbf{y}) = (\Theta_1(\mathbf{y}), \Theta_2(\mathbf{y}), \cdots, \Theta_m(\mathbf{y}))$
Ω	feasible set
$\boldsymbol{\Omega}_s$	set of feasible solutions at the sth iteration
$\mathbf{a}$	vector of the attribute types of the demand sites
$\mathbf{a}_{\mathrm{j}}$	vector of attributes of each demand site, $a_j = (a_{j1}, a_{j2}, \cdots, a_{jn})$
a_i (i=1,···, 9)	estimated coefficients of the crop yield-water and salinity function
$a_s(j,t)$	production activity level of sector s within demand node j
$aft_{j,cp}$	average stage yields by crop cp and demand site j
A	water surface area of the reservoir k
A_0 , A_1 , A_2 , A_3	coefficients of reservoir area-storage curve
A_j	total available crop area of demand site j
$AF_{j,cp}$	crop areas (ha)

$AF^{l}{}_{j,cp}$	lower limit of crop field area
$AF^{u}{}_{j,cp}$	upper limit of crop field area
AGR	set of agricultural nodes
AQU	set of aquifer nodes
$b_0, b_1, b_2, b_{3p}, b_{4p}, b_{5p}$	coefficients of the quadratic benefit functions of water uses
$B_{ijt}(\cdot)$	benefit function for demand node j
$\widetilde{B}_j$	total profit of irrigation water use at demand site j (10^6\$)
$c_{0\,j,cp}$, $c_{1\,j,cp}$	coefficients of fertilizer application function
$c^{p}{}_{j,cp,t}$	concentration of pollutant p in the irrigation water to crop field (j, cp)
$cc_{j,cp}$	cultivation cost (\$/mt)
$\mathbf{C}$	vector of network flow variables $C(k, t)$
$C_{ijt}(\cdot)$	cost function for demand node j
$\widetilde{C}^{p}{}_{j}$	pollutant concentration of the total available irrigation water
$C^{p}{}_{j,t}$	mixed concentration of pollutant p in irrigation water to node j
$C_p(k,t)$	concentration of pollutant p at storage node k at the end of period t
$C_p(k_1,k,t)$	concentration of pollutant p in the link flow from node k_1 to k during period t
$C_{pa}(k,t)$	pollutant concentration of the inflow adjustment $Q_a(k, t)$
$C_{p\max}(k,t)$	maximum concentration of pollutant p for storage node k
$C_{p\max}(k_1,k,t)$	maximum concentration of pollutant p in link (k_1, k)
$C_{pN}(k,t)$	mixed concentration of pollutant p in the total inflow to nonstorage node k
$C_{pN\max}(k,t)$	maximum mixed concentration of pollutant p in the inflows to non-storage node k
$C_{pout}(k,t)$	concentration of pollutant p in outflow from an outlet
$C_{pout\max}(k,t)$	maximum concentration of pollutant p in outflow from an outlet
$CDN^{p}{}_{j,t}$	concentration of monthly return drainage from irrigation node j
$CDP^{p}{}_{j,t}$	concentration of deep percolation from the whole irrigation node
$C(N, \upsilon)$	core of the cooperative game
CP	set of crop types
$CPN^{p}{}_{j,cp,t}$	pollutant concentration of percolation
$DN_{j,t}$	return flow (drainage) from crop field (10^6m^3)
$DP_{j,t}$	deep percolation from an irrigation demand site to the underlined aquifer (10^6m^3)
$e_L(k_1,k,t)$	total water loss coefficient of link (k_1, k)

$e_N(k, t)$	water loss coefficient at a demand node (except reservoir) or treatment plant
$e_{pL}(k_1,k,t)$	pollutant loss coefficient for link (k_1, k)
$e_{pN}(k,t)$	removal ratio of pollutants at node k
$e_{pSL}((k_1,k_2),k,t)$	pollutant loss coefficient for each seepage flow $Q((k_1,k_2),k,t)$
e_r	permitted relative error
$e_{sL}((k_1,k_2),k,t)$	water loss coefficient for each seepage flow $Q((k_1,k_2),k,t)$
$e(S,\boldsymbol{x})$	excess of coalition S with respect to payoff vector x
$e^*(S,\boldsymbol{x})$	optimal value of $e(S,\boldsymbol{x})$ found in previous solution loops
$e_w(S,\boldsymbol{x})$	excess adopted by weak nucleolus concept
$e_p(S,\boldsymbol{x})$	excess adopted by proportional nucleolus concept
$e_n(S,\boldsymbol{x})$	excess adopted by normalized nucleolus concept
E	set of coalitions for which the corresponding upper bounds of excesses are fixed to their optimal values
$E_R(k,t)$	evaporation rate at a reservoir node k
$EDN_{j,cp}$	drainage efficiency
$EIP_{j,cp}$	effective precipitation ratio
$EIR_{j,cp}$	irrigation efficiency
$EP_{j,cp,t}$	effective precipitation (mm)
$EPC_{j,cp}$	pollutant consumption ratio of a crop field
$ETm_{j,cp}$	maximum seasonal potential evapotranspiration of a crop field during whole growing season (mm)
$ETm_{j,cp,t}$	maximum potential evapotranspiration requirements of specific growth stage (mm)
$f_{ijt}(\cdot)$	net benefit function, $f_{ijt}(\cdot)=B_{ijt}(\cdot)-C_{ijt}(\cdot)$
$f_j(\mathbf{x})$	the jth objective function
$f_{jt}(\mathbf{x})$	performance function of demand node j during period t, $f_{jt}(\mathbf{x})=\omega(j,t)\cdot R(j,t)$
f_r^*	optimal value found for $f_r(\mathbf{x})$ in previous sequential solution
$fc_{j,cp}$	fixed cost of crop production (\$/ha)
$ft_{j,cp,t}$	stage yield deficit of crop field
$\mathbf{f}(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X_s})$	vector of multiple objectives of the general water allocation problem
$\mathbf{f}(\mathbf{x})$	vector of multiple objectives, $\mathbf{f}(\mathbf{x})=\left(f_1(\mathbf{x}),f_2(\mathbf{x}),\cdots f_m(\mathbf{x})\right)$
$\mathbf{f}^{(m)}(\mathbf{x})$	vector of multiple objectives of a PMMNF problem, whose elements are ordered from the highest priority to lowest
$\mathbf{f}^{(\mu\tau)}(\mathbf{x})$	vector of the multiple objectives of a LMWSR problem, whose elements are sorted in a nonincreasing order
F	allocation criterion or allocation method

FD	set of possible crop fields
FR	set containing index pairs of (j, t) for which the corresponding upper bounds of $\omega(j,t)R(j,t)$ are fixed to their optimal values
FR_s	set of FR at the sth iteration
$FT_{j,cp}$	variable fertilizer application (kg/ha)
$\mathbf{g}(\mathbf{Q},\mathbf{S},\mathbf{C}) \geq \mathbf{0}$	non-equality constraints for network type variables Q, S and C
$\mathbf{g_s}(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X_s}) \geq \mathbf{0}$	non-equality constraints for both network type decision variables Q, S and C and non-network type decision variables $\mathbf{X_s}$
$G(V, L)$	directed network of a river basin
$\mathbf{h}(\mathbf{Q},\mathbf{S},\mathbf{C}) = \mathbf{0}$	equality constraints for network type variables
$\mathbf{h_s}(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X_s}) = \mathbf{0}$	equality constraints for both network type decision variables Q, S and C and non-network type decision variables $\mathbf{X_s}$
H	elevation of the water surface above a given reference level
H_0, H_1, H_2, H_3	coefficients of reservoir elevation -storage curve
$H_{tw}(j,t)$	elevation of tail water (m)
HPP	set of hydropower plant nodes
HPP_{res}	set of storage hydropower plants directly linked to reservoirs
HPP_{riv}	set of run-of-river hydropower plants with constant water heads
$i \in N$	stakeholder i
I	set of subscripts of multiple objectives, $I = \{1, 2, \cdots, m\}$
IN	set of inflow nodes
$INDEM$	set of instream demand nodes
$IP_{j,cp,t}$	infiltrated precipitation (mm)
j	water demand node
(j, cp)	crop field
J	set of irrigation nodes
JUN	set of junction nodes
(k_1, k_2)	link from node k_1 to k_2
$K_p(k,t)$	first order decay rate coefficient of pollutant p
L	set of links of the network
L_{in}	set of inflow links
L_{out}	set of outflow links
L_{ret}	set of return flow links
L_{seep}	set of seepage links
L_{with}	set of withdrawal links
$memb(S, i)$	set of members of coalition S
$mft_{j,cp}$	minimum stage yields by crop cp and demand site j

$mp_{j,cp}$	on-farm irrigation management practice factor of the total moisture required for optimal yields
M	real variable representing the maximal weighted water shortage ratio
M_{jt}^{*}	minimax value of $\alpha(j,t)R(j,t)$ found in previous iterative solution
M_s^{*}	minimum value of M at the sth iteration
MI	set of municipal and industrial nodes
N	set of stakeholders participating in the grand coalition, $N = \{1,2,\cdots, n\}$
$\lvert N \rvert$	cardinality of the grand coalition N
$\mathcal{N}$	set of all possible coalitions, $\mathcal{N} = \{S_1, S_2, \cdots, S_{2^n}\}$
NB_{ijt}	net benefit of stakeholder i's demand node j during period t
NB_{it}	net benefit of stakeholder i during period t
$NB_i(\mathbf{x})$	net benefit for the player i
$NB(S)$	net benefit of coalition S
$\mathbf{NB}(\mathbf{x})$	vector of net benefits of stakeholders
NR	set containing index pairs of (j, t) for which the corresponding upper bounds of $\omega(j,t)R(j,t)$ are not fixed
NR_s	set of NR at the sth iteration
NST	nonstorage node set
(N, v)	n-person cooperative game
$ord(r')$	relative position of priority r' in the priority rank set $PR=\{r_1, r_2, \cdots, r_m\}$ listed in an order from the highest to lowest
$owner(sh, j)$	set of water use ownerships
$OFFDEM$	set of offstream demand nodes
OUT	set of outlet nodes
$\mathbf{O}(\boldsymbol{x})$	vector of excesses arranged in nonincreasing order
p	index of pollutant types
P_0	choke price of inverse water demand function
$pcp_{j,cp}$	crop price (\$/mt)
pen_j	penalty item (10^6\$)
$P(j,t)$	price of willingness to pay for additional water at full use (\$/m^3)
$P_D(j,t)$	power demand from HPP j
$PN_{j,cp,t}$	percolation of a crop field (10^6m^3)
$POW(j,t)$	power generated (10^6kWh)
PR	set of priority ranks which are ordered from the highest to lowest
$P(S, S-\{i\})$	probability of stakeholder i joining the coalition S
q^{*}	Nash equilibrium of noncooperative water allocation game
$q_i \in \overline{Q}_i$	strategy of player i

$q_{-i} \in \bar{Q}_{-i}$	strategies of other players
$q_{j,cp,t}$	irrigation water allocated to crop field (*j, cp*)
$q_s(j,t)$	water use rate of sector *s* within demand node *j*
$\bar{Q}$	set of strategies of all players, $\bar{Q} = \prod_{i=1}^{n} \bar{Q}_i$
Q	vector of network flow variables $Q(k_1,k,t)$
$(\mathbf{Q}^*, \mathbf{S}^*)$	global solution of linear problem PMMNF_QL
$(\mathbf{Q}^*, \mathbf{S}^*, \mathbf{C}^*)$	starting point for the solution of the nonlinear PMMNF problem
Q_0	choke quantity of inverse water demand function
$Q_c(k,t)$	water consumed at node *k*
$Q_a(k,t)$	inflow adjustment to node *k* during period *t* (excluding river reach seepages)
$Q_D(j,t)$	water demand of nonstorage node
$Q_D(k,j,t)$	withdrawal (or diversion) demand
$Q^r_{D,z}(k,j,t)$	withdrawal demand of the sublink *z* of link (*k, j*) with a priority of *r*
$Q_g(k,t)$	gain of inflow adjustment at node *k* during period *t*
$\bar{Q}_i$	strategy set of player *i*
$Q_{IN}(k,t)$	flow to inflow (source) node *k* from the outside of river network
$\tilde{Q}_j$	total water volume limit
$Q_{j,t}$	irrigation water allocated to demand node *j*
$Q(j,t)$	total inflow to demand node *j* during period *t*
$Q(k_1,k,t)$	flow from node k_1 to *k* during period *t*
$Q_l(k_1,k,t)$	conveyance losses of the flow from node k_1 to *k*
$Q_{\max}(k_1,k,t)$	maximum flow from k_1 to *k*
$Q_{\min}(k_1,k,t)$	minimum flow from k_1 to *k*
$Q_{minireq}(k,j,t)$	minimum stream flow
$Q_{out\min}(k,t)$	minimum outflow from an outlet
$Q_{out\max}(k,t)$	maximum outflow from an outlet
$Q_{OUT}(k,t)$	total outflow from outlet node *k* to the outside of river network
$\{Q_R(k_1,j,t),\ C_{pR}(k_1,j,t),\ Q_R(j,k_2,t),\ C_{pR}(j,k_2,t)\}$	set of water rights for $j \in U \backslash RES$
$Q^r_z(k,j,t)$	inflow of the sublink *z* of link (*k, j*) with a priority of *r*
r	priority rank
r′	priority rank that the current programming aims for
R(*j, t*)	water shortage ratio of demand site *j*
RES	set of reservoir nodes
s	salinity of the irrigation water (dS/m)
$s_N(k, k_2, t)$	coefficient of node seepage from node *k* to node k_2

S	coalition of stakeholders, $S \subseteq N$
$\mathbf{S}$	vector of network flow variables $S(k, t)$
$\overline{S}$	average storage of reservoir j during period t
$\lvert S \rvert$	cardinality of coalition S
$S(k,t)$	storage volume of reservoir (or aquifer) at the end of period t
$\overline{S}(k,t)$	average storage during period t
$S_D(j,t)$	water demand of reservoir j
$S_{D,z}^r(j,t)$	storage demand of the subzone z of reservoir j with a priority of r
$s_L((k_1,k_2),k,t)$	coefficient of link seepage from the river reach (k_1, k_2) to aquifer k
$S_{\max}(k,t)$	maximum water volume of a storage node k
$S_{\min}(k,t)$	minimum water volume for a storage node k
$\{ S_R(j,t),\ C_{pR}(j,t) \}$	set of water rights for $j \in RES$
$S_{target}(j,t)$	target water storage of reservoir j
$S_z^r(j,t)$	storage of the subzone z of reservoir j with a priority of r
SFR	set of stream flow requirement nodes
SH	set of all stakeholders in a river basin
$SM_{j,cp}$	soil moisture in the seasonal full root zone at the beginning of the crop growing season (mm)
SRC	set of nonstorage nodes that are simplified to provide water supplies to some demand sites and receive corresponding return flows from them
ST	set of possible crop growing stages
STO	storage node set
t	index of time periods
T	set of time periods
TP	set of water treatment plants
u	irrigation uniformity
$u(\mathbf{f}(\mathbf{x}))$	social utility function of multiple objectives
U	set of water demand sites
U_i	water demand nodes owned by stakeholder i
U_S	set of water demand nodes of coalition S
$v(X)$	nucleolus of cooperative game (N, υ)
V	set of nodes, $V=\{v_1, v_2, \cdots, v_v\}$
$vc_{j,cp}$	cost of fertilizer ($/kg)
w	ratio of total available water to maximum seasonal potential evapotranspiration of the crop field
w_i	weight of weighted sum aggregation
$wc(k,j,t)$	water supply cost ($/m^3)

$WA_{j,cp}$	total water available to a crop field during whole growing season (mm)
$WE_{j,cp,t}$	Total of effective precipitation and effective water application to a crop field during period t (mm)
$\mathbf{x}$	vector representing all the control variables
$\mathbf{x}^*$	solution for general optimization problem
$\boldsymbol{x}$	payoff or reward vector, $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$
$x(S)$	aggregate reward payoff to members of coalition S
$x(N)$	aggregate reward payoff to members of the grand coalition N
X	set of n-vectors (payoff vectors), $X = \left\{\boldsymbol{x} : \sum_{i=1}^{n} x_i = \upsilon(N)\right\}$
$\mathbf{X}_s$	vector of non-network type decision variables
$\boldsymbol{y}$	real vector, $\mathbf{y} = (y_1, y_2, \cdots y_m)$
Ya	actual crop yield (metric tons (mt)/ha)
Ym	maximum potential crop yield (mt/ha)
$Z_{pc}(k,t)$	removal of pollutant p at node k
$Z_{pd}(k,t)$	pollutant discharge from the production activities at node k
$z_{pd0}(k,t)$, $z_{pd1}(k,t)$, $z_{pd2}(k,t)$	coefficients of quadratic functions of pollutant loads
$Z_{pg}(k,t)$	total amount of pollutant p added to node k during period t
$Z_{pl}(k_1,k,t)$	conveyance losses of pollutant p in the water flow from node k_1 to k

Chapter 1
Introduction

1.1 Background

Water is fundamental to the survival and prosperity of human beings and societies. Mankind's quest for a better use of water resources is as old as civilization itself. Due to concerns with the increasing scarcity of water resources, degradation of the water environment and climatic change, more and more public attention and academic research are being devoted to water resources management and policies, especially water allocation.

There are several key problems concerning water allocation and management:

(1) Precipitation is geographically and temporally unevenly distributed over different areas of the world. Some places have exceptionally abundant precipitation, exceeding 1500-3000 mm annually, whereas desert regions receive less than 100 mm (Al Radif, 1999). Fluctuations in temperature and other meteorological conditions greatly affect the variation in the magnitude and timing of hydrologic events such as the distribution of stream flow. For example, increasing temperature accompanying climate change may result in more snowmelt and increasing stream flows during early spring, but lead to decreasing flows in other months due to higher evaporation (Westmacott and Burn, 1997). Water demands are also time-dependent variables. For instance, water demands for irrigating crops occur during the growing season and vary according to growth phases of plants (Doorenbos and Kassam, 1979). The high water consumption period often does not coincide with times of abundant rainfall or stream flow.

(2) Water demand is driven by the rapid increase of world population and other stresses. World population reached 5.38 billion in 1996 and will probably increase to around 7.9 billion by the year 2020, 9.9 billion in 2050 and 10.4 billion by 2100 (UN, 1998). It is projected that by the middle of this century at worst 7 billion people in 60 coun-

tries will face water scarcity, and at best 2 billion in 48 countries (UNWWAP, 2003). This quick rate of growth brings severe consequences that result from high stresses on fresh water resources and their unprecedented impacts on socio-economic development. Stresses include unparalleled demands for agricultural irrigation, domestic water supply and sanitation, industrial usages, energy production, and environmental requirements; as well as changes in the patterns of consumption as a result of industrialization, rural/urban shifts, and migration; and unaccounted-for water losses (UNCSD, 1994).

(3) Water scarcity is now a common occurrence in many countries. It has been estimated that currently more than 2 billion people are affected by water shortages in over forty countries among which 1.1 billion do not have sufficient drinking water. The situation is particularly serious in many cities located in developing countries (UNWWAP, 2003). The major reasons are high water demand from population growth, degraded water quality and pollution of surface and groundwater sources, and the loss of potential sources of fresh water supply due to old and unsustainable water management practices.

(4) Conflicts often arise when different water users (including the environment) compete for limited water supply in both intra-country and international river basins. For example, the competition for water has led to disputes between Arabs and Israelis, Indians and Bangladeshes, Americans and Mexicans, and among all 10 Nile basin coriparians (Wolf, 1999). Water is argued to be a public good, which should be equitably utilized. However, fair water allocation may not mean the efficient use of water resources.

(5) Water allocation is central to the management of water resources. The need to establish appropriate water allocation methodologies and associated management institutions and policies has been recognized by researchers, water planners and governments. Many studies have been carried out in this domain, but there are still many obstacles to reaching equitable, efficient and sustainable water allocations (Dinar *et al.,* 1997; McKinney *et al.*, 1999; Syme *et al.*, 1999; UNESCAP, 2000).

1.2 Objectives of the Research

The overall objective of the research is to develop a methodology for policy makers and water managers that will allow for allocating water in an equitable, efficient and environmentally sustainable manner. We aim to devise and implement an integrated water allocation modeling approach that promotes equitable cooperation of relevant stakeholders to achieve optimal economic and environmental utilization of water, subject to hydrologic and other constraints. The model is developed as a generic tool that can be applied to any river basin for short-term water resources management planning with multiple time periods to account for the time variations of water availabilities and demands.

1.3 Outline of the Thesis

The thesis is organized into seven chapters. Chapter 1 gives a brief introduction to the background behind the research. Chapter 2 reviews the objectives and principles of water allocations, water rights systems for allocating water rights in intra-country situations and generalized principles for inter-country basins, and the approaches and models for water allocation that have been used in the literature. Next, Chapter 3 develops the framework of the cooperative water allocation model (CWAM). CWAM allocates water resources in two steps: water rights are initially allocated to water uses based on a node-link river basin network and legal rights systems or agreements, and then water and net benefits are reallocated to achieve efficient use of water through water and net benefit transfers. Three methods, priority-based multiperiod maximal network flow programming (PMMNF), modified riparian water rights allocation (MRWRA) and lexicographic minimax water shortage ratios (LMWSR), are proposed for determining the initial water rights allocation. The associated net benefit reallocation is carried out by application of cooperative game theoretical approaches. Chapter 4 presents the sequential programming algorithms for both the linear and nonlinear PMMNF problems, and the iterative programming algorithms for the linear and nonlinear LMWSR problems. A two-stage approach is designed for solving nonlinear problems, which adopts a strategy of solving with good starting points and thereby increases the possibility to find approximate global optimal solutions. The algorithms developed are applied to the Amu Darya River.

In Chapter 5, an integrated hydrologic-economic river basin model (HERBM) for optimal water resources allocation is designed adopting monthly net benefit functions of various water demand sites. A separate irrigation water planning model (IWPM) at farm level is developed to estimate the monthly net benefit functions of irrigation water uses. The hydrologic-economic river basin model is extended for analyzing the values of various coalitions of stakeholders. Based on the results of coalition analysis, cooperative game theoretical approaches are used to derive the fair reallocation of the net benefit gained by stakeholders participating in the grand coalition. The algorithm for coalition analysis utilizing a multistart global optimization technique is presented following the model design. Chapter 6 applies the developed CWAM to a case study of the South Saskatchewan River Basin in western Canada. The computation results of the initial water rights allocation and the subsequent water and net benefits reallocation are interpreted and analyzed in detail. Chapter 7 summarizes the results and original contributions of this research, and lists some recommended directions for future research.

Chapter 2
Perspectives on Water Allocation

The first part of this chapter describes the objectives and principles of water allocation, and the relationship between the hydrologic cycle and water uses, since water allocation is essentially an activity for allocating available water to demanding users subject to the constraints of hydrological balances. Then water rights systems for allocating water resources in intra-country basins and generalized principles of transboundary water allocation in inter-country basins are reviewed. Subsequently, four major institutional mechanisms for water allocation, water transfers and externalities are discussed. The second part of the chapter reviews approaches and models for water allocation that have been used in practice or proposed. Water rights based methods, simulation and optimization models, and application of cooperative game theory in water resources management are discussed.

2.1 Principles and Mechanisms of Water Allocation

2.1.1 Objectives and Principles of Water Allocation

What is water allocation? "The simplest definition of water allocation is the sharing of water among users. A useful working definition would be that water allocation is the combination of actions which enable water users and water uses to take or to receive water for beneficial purposes according to a recognized system of rights and priorities"(UNESCAP, 2000). Because of water's time-varying characteristics and its extreme importance to humans and society, as well as the complex relationships among climate, hydrology, the environment, society, economics and sustainable development, water allocation is a complex task.

Water allocation does not mean merely the right of certain users to abstract water from sources but also involves other aspects. Table 2.1 lists a number of activities involved in a comprehensive and modern water allocation scheme.

Table 2.1 Elements of water allocation

Element	Description
Legal basis	Water rights and the legal and regulatory framework for water use
Institutional base	Government and non-government responsibilities and agencies which promote and oversee the beneficial use of water
Technical base	The monitoring, assessment and modeling of water and its behavior, water quality and the environment
Financial and economic aspects	The determination of costs and recognition of benefits that accompany the rights to use water, facilitating the trading of water
Public good	The means for ensuring social, environmental and other objectives for water
Participation	Mechanisms for coordination among organizations and for enabling community participation in support of their interests
Structural and development base	Structural works which supply water and are operated, and the enterprises which use water

*Adapted from UNESCAP (2000), p.4.

The overall objective of water allocation is to maximize the benefits of water to society. However, this general objective implies other more specific objectives that can be classified as social, economic and environmental in nature as shown in Table 2.2. As can be seen in this table, for each classification there is a corresponding principle: equity, efficiency and sustainability, respectively.

Table 2.2 Objectives and principles of water allocation

Objective	Principle	Outcome
Social objective	Equity	Provide for essential social needs: • Clean drinking water • Water for sanitation • Food security
Economic objective	Efficiency	Maximize economic value of production: • Agricultural and industrial development • Power generation • Regional development • Local economies
Environmental objective	Sustainability	Maintain environmental quality: • Maintain water quality • Support instream habitat and life • Aesthetic and natural values

*Adapted from UNESCAP (2000), p.33.

Equity means the fair sharing of water resources within river basins, at the local, national, and international levels. Equity needs to be applied among current water users, among existing and future users, and between consumers of water and the environment. Since equity is the state, quality, or ideal of being just, impartial, and fair, and different people may have different perceptions for the same allocation (Young, 1994), it is important to have pre-agreed rules or processes for the allocation of water, especially under the situations where water is scarce. Such agreements and methodologies should reflect the wishes of those affected sufficiently to be seen to be equitably and accountably applied.

Efficiency is the economic use of water resources, with particular attention paid to demand management, the financially sustainable use of water resources, and the fair compensation for water transfers at all geographical levels. Efficiency is not so easy to achieve, because the allocation of water to users relates to the physical delivery or transport of water to the demanding points of use. Many factors are involved in water transfers, one of which is the conflict with equitable water rights. For example, a group of farmers should have permits to use certain amounts of water for agricultural irrigation. However, agriculture is often a low profit use; some water for irrigation will be transferred to some industrial uses if policy makers decide to achieve an efficiency-based allocation of water. In this case, farmers should receive fair compensation for their losses.

Sustainability advocates the environmentally sound use of land and water resources. This implies that today's utilization of water resources should not expand to such an extent that water resources may not be usable for all of the time or some of the time in the future (Savenije and Van der Zaag, 2000).

2.1.2 Hydrological Cycle, Water Demand and Water Allocation

Water allocation is essentially an exercise in allocating available water to demanding users. Two obvious major sources of supply are surface water and groundwater. The hydrologic cycle illustrated in Figure 2.1 provides a conceptual description about the process of transformation and transportation of water on the earth. Although the complete hydrological cycle is global in nature, a rational and suitable water resources modeling and management unit is river basin (McKinney *et al*., 1999). In order to make wise operational decisions regarding solutions to sharing water in a watershed, a fundamental scientific understanding of hydrologic constraints and conditions is required.

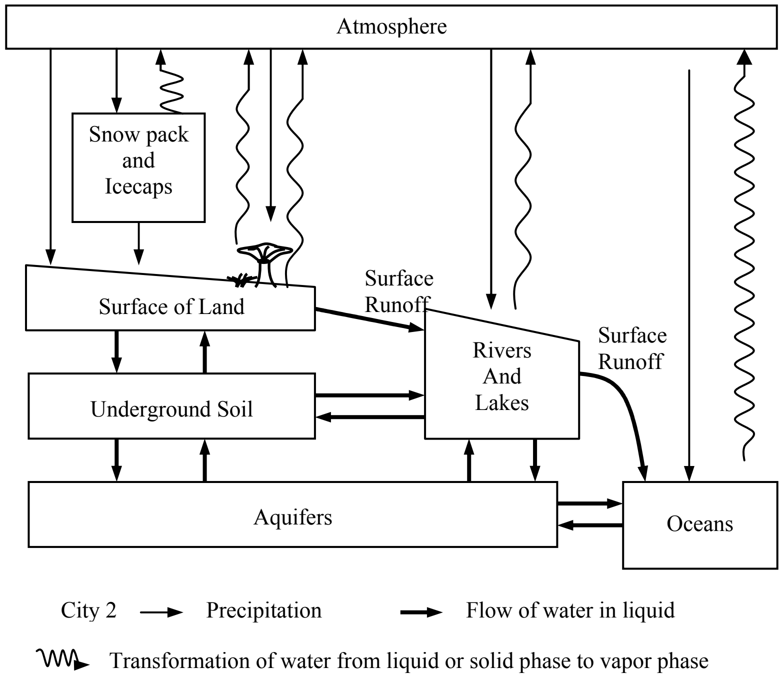

Figure 2.1 The hydrological cycle (Hipel and McLeod, 1994, p.21)

The need for a water allocation activity arises from demands for water. The demand of a water use is determined by social, economic and environmental needs (number of households, hectares of irrigated areas and crop types, minimum stream flows, etc.) and the water use rate of each activity. Where resources are restricted compared to demands, as for irrigation in some regions, conflicts can arise among competing users. Table 2.3 describes a hierarchical classification of water uses. In the table, water uses are grouped into five demand groups: municipal and industrial use, agriculture, hydropower, navigation and other demands including flood storage, recreation, ecological uses and even bulk water export.

A schematic diagram illustrating water uses and hydrological balances in a typical subwatershed is shown in Figure 2.2. An operational water allocation plan should be based on the hydrologic constraints and linkages between demanding uses and water sources. Figure 2.3 and Figure 2.4 are schematic diagrams showing the typical demand for water supply of a city and demand of agriculture, respectively, and their inflows and outflows. Figure 2.5 depicts common water supply systems showing how water is conveyed to the distribution systems of a city. Figure 2.6 is a schematic diagram of a typical reservoir with storages for multiple purposes and uses.

Table 2.3 Hierarchical classification of water uses

Water uses			Objectives	Effects
Municipal and industrial use	Municipal uses (O)	Domestic	Use for cooking, washing, watering lawns, and air conditioning	Remove water from system, adds pollution to river and underground aquifer
		Public	Use in public facilities and for fire fighting	
		Commercial	Use in shopping centers, hotels, and laundries	
		Small industrial uses not having a separate water supply system	Use for industrial production	
		Conveyance losses in the distribution system	—	
	Industrial uses (O)		Use for large water-using industries such as steel, paper, chemicals, textiles and petroleum refining	
	Waste dilution (I)		Serve as the source for self-purification of the stream	
Demand for Agriculture (O)	Irrigation		Use for raising crops	Remove water from system, adds sediment, nutrients and agricultural chemicals to river and underground aquifer
	Factory farm uses		Use for livestock	
	Conveyance losses and waste		—	
Demand for hydropower (I)	Hydropower generation		Produce hydropower	Helps regulate river flow
Demand for navigation (I)	River regulation		Water release from upstream reservoirs to raise water depth	Keeps water in river
	Lock-and-dam		Increase water depth for navigation through ship locks and dams	
	Artificial canalization		Use for artificially constructed channels with a number of ship locks	
Other demands	Flood storage (I)		Control floods	Provide downstream flood protection
	Recreation (I)		Provide a place for swimming, fishing and other recreation activities	Keeps water in river
	Water export (O)		Large diversion and export for commercial purposes	Remove water from system
	Ecological uses (I & O)		Conservation of scare aqua lives, use for forestry, filling wetlands, etc.	Keeps water in river, or remove water from system

*Adapted from Gupta (2001). I: In-stream use (nonconsumptive); O: Off-stream use (consumptive)

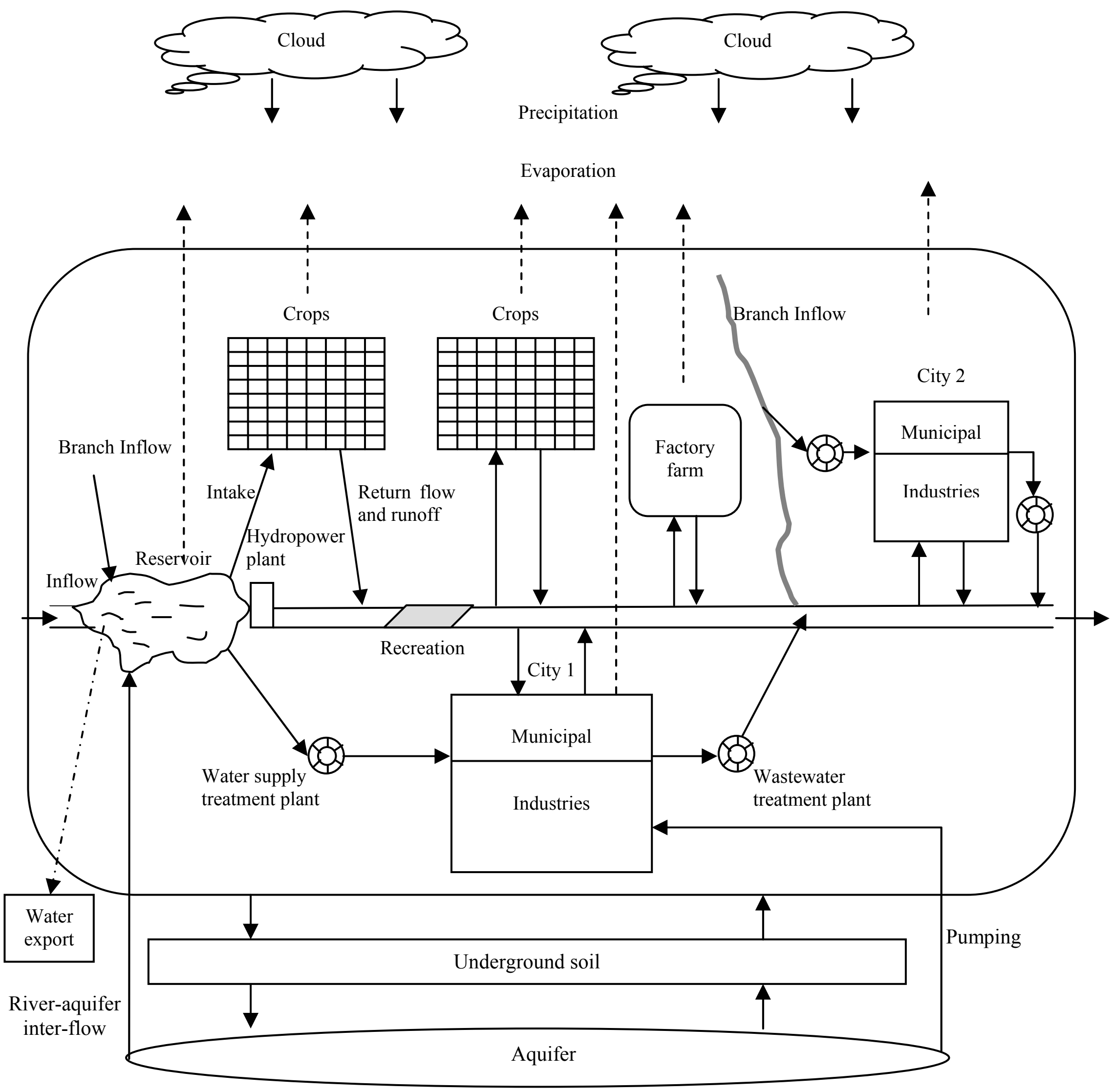

Figure 2.2 Water uses and hydrological balance

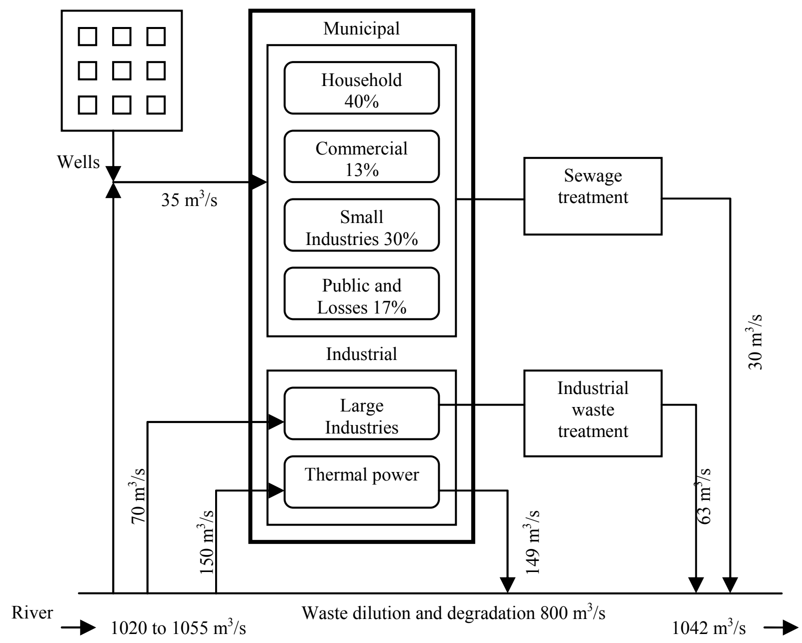

Figure 2.3 Demands for water supply of a city (adapted from Gupta, 2001, p.4)

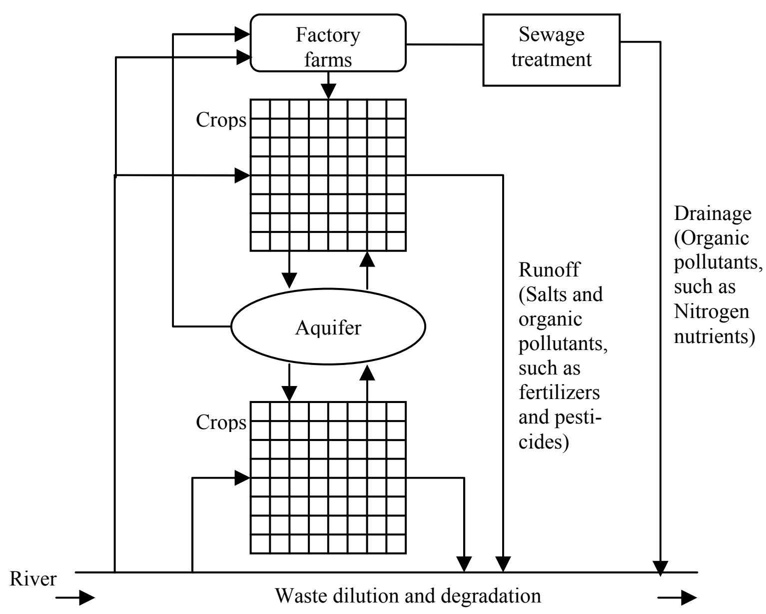

Figure 2.4 Demands of agriculture

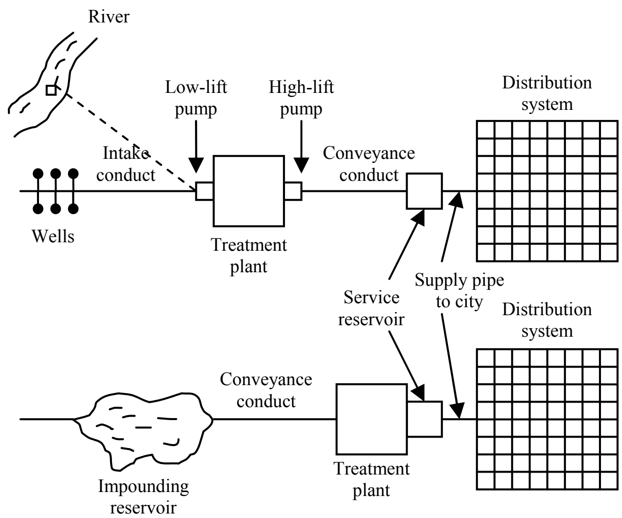

Figure 2.5 Water supply system (Gupta, 2001, p.12)

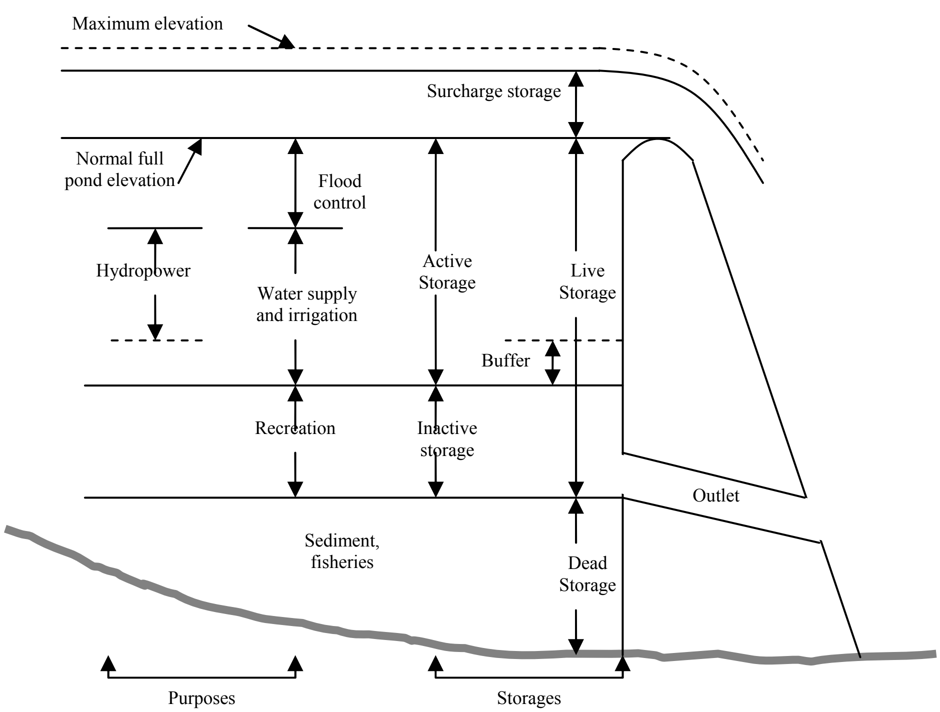

Figure 2.6 Reservoir storages (Gupta, 2001, p.489)

2.1.3 Water Rights Systems and Generalized Principles of Transboundary Water Allocation

The question of "whose water is it" is a fundamental issue of water allocation (Green and Hamilton, 2000). In many countries, the State is the owner of all or nearly all water and allocates water permits or user rights (water rights) (Jain and Singh, 2003). Water rights are often granted for a limited period of time, while some may be granted in perpetuity. Countries have developed their own specific water rights systems to solve the issues of planning, developing, allocating, distributing and protecting their water resources. As shown in Table 2.4, various systems of water rights can be grouped into three basic doctrines: riparian rights, prior (appropriative) rights and public allocation.

Table 2.4 The most important systems of water rights (Savenije and Van der Zaag, 2000, p. 25)

Water rights system	Description
Riparian rights	Links ownership or reasonable use of water to ownership of the adjacent or overlying lands, and are derived from Common Law as developed in England. Therefore, these principles are mainly found in countries that were under the influence of the British Empire.
Prior (Appropriative) rights	Are based on an appropriation doctrine, under which a water right is acquired by actual use over time. The system is developed in the western part of the USA, a typical (semi-) arid 'frontier zone'.
Public allocation	Involves administrated distribution of water, and seems to occur mainly in so called 'civil law' countries, that derive their legal systems from the Napoleonic Code, such as France, Italy, Spain, Portugal, The Netherlands and their former spheres of influence.

The common law riparian rights system treats water as a common property, and was developed in humid regions where water is abundant and water allocation did not cause major problems for individual water users. The riparian rights system has evolved into two basic doctrines: reasonable use and correlative rights. The *reasonable use doctrine* means that a riparian landowner can divert and use any quantity of water for use on riparian lands, as long as these diversions and uses do not interfere with reasonable use of other riparian landowners. There is no sharing of a shortage in available water except as a court determines whether continuing a use is unreasonable (Dellapenna and Stephen, 2002). The *correlative rights doc-*

trine requires that riparian landowners must share the total flow of water in a stream, and may withdraw only their "share" of water for reasonable use. For example, the proportion of use allocated to each riparian is based on the amount of waterfront property owned along a stream and creates equal rights for riparians (Cech, 2002).

The prior (appropriative) rights regime treats water as private property. Water is appropriated according to "first in time, first in right". In cases of water scarcity, there is no sharing of the shortage in water. Junior users are allocated water after the senior users have been satisfied.

The public allocation regime treats water as a public property, and the state is the owner of waters. In this system, water rights are administratively allocated to users through water permits from governments. As the water demands increase and begin to compete for available water supplies during times of water scarcity, the need for active public management of water has been recognized. Today, in the United States, there is no state relying only on "pure" riparian rights, since it will cause the "tragedies of the commons" without some forms of regulatory management. The introduction of water management through a regulatory permit system is increasingly common among states. This modified system is named as "regulated riparianism" and the rights are called "regulated riparian rights". The regulated riparianism treats water as a public property, and hence is a kind of public allocation water rights regime (Dellapenna, 1994, 2000; Dellapenna and Stephen, 2002).

For international river basins between countries, generally there is no formal inter-country water rights system but international water agreements defining ownership of the water resources. To mitigate problems of water allocation, the international legal community has established generalized, global legal and economic principles for inter-country river basins, which include: (1) absolute sovereignty, (2) absolute riverine integrity, (3) limited territorial sovereignty, and (4) economic criteria, as listed in Table 2.5. Of the four principles, absolute sovereignty and absolute riverine integrity are the extreme doctrines; limited territorial sovereignty is more moderate; and allocating water based on its economic value is a more recent addition to water conflict resolution. While water markets have received considerable attention and have been applied in a number of intrastate settings, water markets have not yet developed at an international scale due to concerns over equity issues of water rights (Wolf, 1999).

Table 2.5 Generalized principles of transboundary water allocation

Principles	Description
Absolute sovereignty	Based on hydrography and implies unilateral control over waters within a nation's territory. It is often the initial claim by upstream riparians during treaty negotiations.
Absolute riverine integrity	Suggests that every riparian has a right to the waters that flow through its territory. It emphasizes the importance of historical usage, or chronology. This doctrine is often the initial bargaining position for downstream riparians.
Limited territorial sovereignty	Reflects the right to reasonable and equitable use of international waters while inflicting no significant harm on any other riparians.
Economic Criteria	Under this principle, the market is used to allocate water among competing users in an economically efficient manner.

* Based on Buck *et al.* (1993), Wolf (1999), and Giordano and Wolf (2001).

2.1.4 Institutional Mechanisms for Water Allocation

The intra-country water rights systems or generalized principles of transboundary water allocation in inter-country basins provide the basis for various institutional mechanisms for water resources allocation. Dinar *et al.* (1997) discuss the concepts, advantages, and disadvantages of four basic institutional mechanisms: administrative allocation, user-based allocation, marginal cost pricing and water markets allocation. In practice, most countries have some combinations of institutional mechanisms.

Administrative allocation mechanism is broadly employed in most of the countries where the governments allocate and distribute water permits as water use rights to different uses. The allocation rules of administrative allocation mechanism can be based on historical facts (such as *prior rights*), on equitable shares in available water volumes (such as *regulated riparian rights*), on individual requirements, or even based on political pressure. The disadvantage is that an administrative allocation mechanism often leads to inefficient use of water and failure to create incentives for water users to conserve water, improve water use efficiency and allow tradable water transfer to achieve best benefits in a whole river basin. In practice, administrative allocation mechanisms typically consist of various inefficient water pricing schemes. Flat rates or fixed charges are common, simple to manage and easy to be understood by users.

User-based allocation mechanism is employed in community wells, farmer-managed irrigation systems, and systems managed by water and sanitation associations (Tang, 1992; Pitana, 1993; Ostrom *et al.*, 1994). "User-based allocation requires collective action institutions with authority to make decisions on water rights" (Dinar *et al.*, 1997). A major advantage of user-based allocation is the potential flexibility to adapt water delivery patterns to meet local needs. However, the effect depends on the content of local norms and the strength of local institutions. While empirical studies of common pool resource management have shown that such institutions can develop spontaneously or through an external catalyst, the institutions are not always in place or strong enough to allocate water efficiently (Meinzen-Dick *et al.*, 1997).

Marginal cost pricing (*MCP*) *mechanism* sets a price for water to equal the marginal cost of supplying the last unit of that water. The advantage of MCP is that it is theoretically efficient. "Not only are the marginal costs and benefits equal, but also at the efficient price the difference between the total value of water supplied and the total cost is at a maximum" (Dinar *et al.*, 1997). One of the principal limitations of MCP is it is difficult to collect sufficient information to correctly estimate and subsequently monitor benefits and costs (Saunders *et al.*, 1977). MCP also tends to be unfair. If water prices increase to a sufficient level, low-income groups may be negatively affected. Given the above disadvantages and difficulties in implementation, there are few good examples of MCP applications to water allocation in reality.

Water markets mechanism allocates water by means of tradable water–use rights and promotes efficient water uses through allowing users to sell and buy freely their water rights. It requires intervention of government to create necessary conditions before markets become operational, including (a) defining the original allocation of water rights, (b) creating the institutional and legal frameworks for trade, and (c) investing in basic necessary infrastructure to allow water transfers (Holden and Thobani, 1996). Water markets mechanism is a relatively new concept in many countries, but water markets do exist in Australia (Pigram *et al.*, 1992), Spain (Reidinger, 1994), California (Howe and Goodman, 1995), Chile (Hearne and Easter, 1995), and India (Saleth, 1996). Water markets have attractive potential benefits such as distributing secure water rights to users, providing incentives to efficiently use water and gaining additional income through the sale of saved water. However, there are some challenges in the design of a well-functioning water market. The difficulties include: measuring water, well defined water rights taking into account the variable flows and hydrological constraints, sale-for-cash by poor farmers, externality and third party effects. Furthermore, water can be argued to be a public property and markets cannot work for raw water. Dellapenna (2000) maintains that water markets are rare in reality and are not true free markets.

2.1.5 Water Transfer and Externalities

As facts turn out, the water allocation merely based on a certain water rights approach usually does not make efficient use of water by achieving maximum overall benefits for the whole river basin. In fact, water markets and banks are often promoted to achieve more efficient water use by allowing temporary transfer of water rights or permits among agricultural, urban and environmental uses (Holden and Thobani, 1996; Mahan *et al*., 2002).

What one user does with his/her water may affect the water supply of other users through the hydrological linkages of aquifers and return flows. For example, excess water from irrigating one field may supply water to another field via surface return flows; the runoff from factory farms or agricultural farms can bring a large amount of nutrients and pesticides into the water body. So there are unavoidable physical and economic externalities and third-party impacts associated with a certain water allocation plan. In other words, the allocation of water to one user may affect others' availability of water and production.

2.1.6 Integrated Water Allocation Planning Procedure

The integrated water resources management is a multiple dimension process centered around the need for water, the policy to meet the needs and the management to implement the policy, which requires integration of various components including physical, biological, chemical, ecological, environmental, health, social, and economic (Singh, 1995). Water allocation plans may be made at three levels from national to local. At the *level of water rights*, a water allocation plan deals with the interacting obligations of water users and the regulatory authorities. It may indicate the cumulative rights that are intended to be issued, and it may include the criteria for management at other levels. At the *operational level*, a water allocation plan is concerned with shorter-term, usually annual, management of reservoir storage, river flows, and diversions. At the *local level*, the distribution rules and priorities are set out (UNESCAP, 2000).

A general comprehensive water allocation procedure at the operational level is proposed in Figure 2.7. This procedure starts with setting objectives under certain regulations and institutions governing water rights policy and water allocation mechanisms. Then physical and social investigations, together with hydrological modeling, water quality modeling, economic analysis, and social analysis should be carried out to have a comprehensive water resources assessment. The water resources assessment phase generates the possible options for water allocation. Then a water allocation plan can be obtained by evaluating the possible options utilizing certain criteria considering the factors of water availability, need, cost and benefit. After a plan is made, and its proposals are agreed upon by the representatives of water users and others, it needs to be implemented. To evaluate the performance of the plan, monitoring and reporting are required. Each feedback in this process can provide more highlights in the next iteration. The water allocation plan made at the operational level determines the water flow or volumes for distribution at the local level.

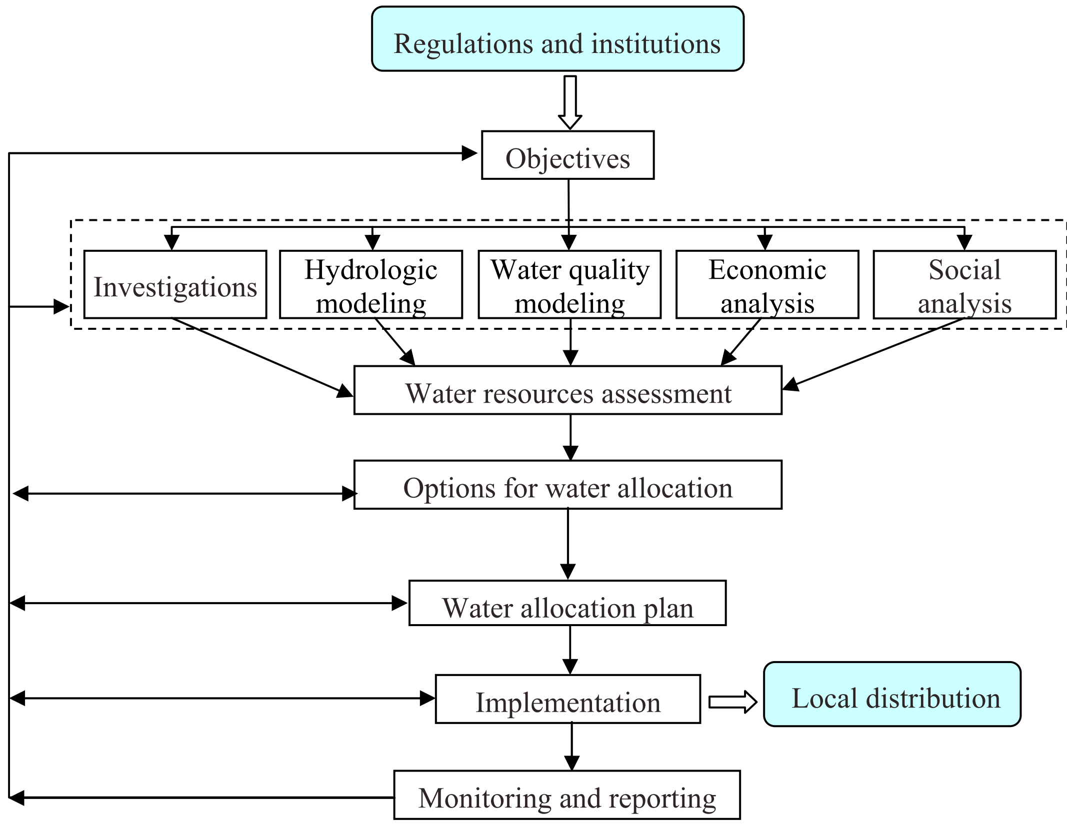

Figure 2.7 Water allocation planning procedure at the operational level

2.2 Approaches and Models for Water Allocation

From the point of view of sovereignty of river basins, models and algorithms can be grouped into two basic categories of intra-country and inter-country water rights allocation. Models and algorithms can also be classified as simulation and optimization models according to modeling techniques. There are also some applications of cooperative game theory in water resources management, but mostly in the area of cost allocation for joint projects.

2.2.1 Intra-Country Water Rights Allocation Models and Algorithms

The reasonable use riparian doctrine is applicable when there is no stress of water resources, while the correlative rights doctrine can be used for equitably allocating water so that every user receives a proportionate share of all available water. Some eastern states in the US are abandoning the traditional riparian system and are adopting a permit system that distributes water resources under the regulated riparian rights doctrine (Cech, 2002). However, few quantitative algorithms or models can be found in the literature.

Simulation and optimization models have been formulated to handle prior water rights allocations (Wurbs, 1993). The conventional simulation models, in the sense that no formal mathematical programming algorithms are used, such as the Water Rights Analysis Package (WRAP) (Wurbs, 2001) or MIKE BASIN (DHI, 2001), first calculate naturalized flows covering all time steps of a specified hydrologic period of analysis for all nodes, then subsequently distribute water to demands according to priority order in turn for each time step. At each priority step within the water rights computation loops, water is allocated to nodes with the same priority from upstream to downstream. If a source is connected to many demands with the same priority, water is allocated simultaneously by proportion to those demands. New water availabilities of all nodes should be updated after each allocation. Since no optimization technique is used, simulation models cannot achieve optimal outcomes over multiple periods.

Linear programming and network flow models have been used extensively to model prior water rights allocation. Many models are formulated as weighted sum multi-objective optimization problems in terms of linear program, where the weights reflect the priorities or importance of objectives (Diba, *et al.*, 1995). Certain types of models are also formulated as linear minimum-cost network flow problems and solved by network flow algorithms, in which the negative cost coefficients represent the priorities or importance of the link flows. The most commonly used method is the minimum-cost capacitated pure network-flow model, which can be solved using efficient linear programming algorithms such as the out-of-kilter algorithm (Wurbs, 1993). The pure flow network is a circular network having no storage at nodes and no gain or loss in the links, which can be converted from a river basin network by adding some pseudo accounting nodes and links for carry-over storages, reservoir evaporation and channel losses. Models utilizing this type of algorithm include ACRES (Sigvaldason, 1976), MODSIM (Labadie *et al.*, 1986), WASP (Kuczera and Diment, 1988), CRAM (Brendecke *et al.*, 1989), DWRSIM (Chung *et al.*, 1989), and KOM (Andrews *et al.*, 1992). The general form of the linear programming problem is as follows (Wurbs, 1993):

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} q_{ij}$$

$$\textit{subject to}$$

$$\sum q_{ij} - \sum q_{ji} = 0 \quad \textit{for each node}$$

$$l_{ij} \le q_{ij} \le u_{ij} \quad \textit{for each link}$$

where, n is the number of nodes; i and j are the indices of nodes; and flows (q_{ij}) represent stream flows, diversions, and carry-over storages associated with link (i, j). Each link has three parameters: unit cost coefficient c_{ij} for q_{ij}, lower bound l_{ij} on q_{ij}, and upper bound u_{ij} on q_{ij}.

Although pure network flow algorithms are computationally efficient, they have a major limitation. Non-network-type constraints and variables that may be included in a linear program or generalized network problem, have to be excluded from the network structure since they cannot be transformed into a pure network representation (Hsu and Cheng, 2002). For the linear problems associated with a large-scale regional water-supply system that contains non-network-type constraints and non-network-type variables, a network simplex method along with matrix partition is reported to be more efficient (Sun *et al.*, 1995; Hsu and Cheng, 2002). With some additional computational cost, minimum-cost generalized network models (Hsu and Cheng, 2002) and mixed integer linear programming models (Tu *et al.*, 2003) have also been developed to explicitly represent the priorities, gains, losses and other system policies.

A common shortcoming of the existing prior water rights allocation methods is that most of them are weighted sum linear programming or minimum-cost network flow models, and their weight factors and unit cost coefficients are intuitively set through an intuitive or trial-and-error approach. Hence, it is difficult to ensure that water is allocated in the priority order, because return flows, instream flows and reservoir storages with junior priorities can be reused by downstream uses (Israel and Lund, 1999). Israel and Lund (1999) presented a general linear program algorithm for determining values for unit cost coefficients that reflect water use priorities for generalized network flow programming models of water resource system, but reservoir evaporation and channel loss are neglected in the algorithm. Furthermore, models in terms of weighted sum program are not equitable water allocation methods from a theoretic point of view (Ogryczak *et al*., 2003).

Models for public allocation are either simulation or optimization models that treat water as a public property. In the past decades, many mathematical simulation and optimization models for water quantity, quality and/or economic management have been developed and applied to problems at both the subsystem level and the river basin level, such as reservoir operation, groundwater use, conjunctive use of surface water and groundwater, and irrigation and drainage management (McKinney *et al.*, 1999). The details can be found in Section 2.2.3 *Simulation and Optimization Models*.

2.2.2 Inter-Country Water Rights Allocation Models and Algorithms

The international community has drawn from the generalized doctrine of limited territorial sovereignty to devise international laws concerning the equitable allocation of water resources between countries. For example, the International Law Association adopted the *Helsinki Rules* in 1966 (International Law Association, 1967); the United Nations adopted the *Convention on the Law of the Non-Navigational Uses of International Watercourses* in 1997 (UN, 1997). The *Helsinki Rules* list factors to be considered in the determination of equitable and reasonable allocation of water, which include hydrology, geography and climate of the river basin, population, past and existing uses of water, economic and social needs of each states, availability of alternative resources and their comparative cost, efficiency of water use and the practicality of compensating other riparians. The UN's *Convention* defines the obligation not to cause appreciable harm and the reasonable and equitable use as co-equal criteria for the allocation of water between riparian countries. However, the seemingly fair and simple principles or guidelines of reasonable and equitable use are difficult to be applied directly in practice. An obvious reason for that impracticality is the different interpretations of these principles by different riparian countries. Measurable criteria and models need to be designed and used to achieve fair apportionment of water in light of water shortages (Postel, 1992; Seyam *et al.*, 2000; Van der Zaag *et al.*, 2002).

Seyam *et al.* (2000) derived four algorithms for allocating the waters of a shared river between riparian countries using population as a distribution factor. Van der Zaag *et al*. (2002)

proposed six algorithms for transboundary water allocation, which are similar to those of Seyam *et al*. (2000), but the proportion factors expand to population, and each country's area. However, all of these algorithms or conceptual models are based on average flows and disregard the environmental requirements of instream flows. In real time, the river flows fluctuate. Hence, to be operational in the real world, the variability in water availability in both space and time should be taken into account.

2.2.3 Simulation and Optimization Models

Keep in mind that, the classification of simulation and optimization models is according to modeling techniques, and the purpose of this section is to review them from the perspective of modeling techniques rather than from water rights systems or water allocation mechanisms. Actually, they have been applied in modeling water allocation under various water rights regimes, water agreements and institutional mechanisms. In the following review, we will focus on the connections between water quantity/quality models and water allocation problems.

2.2.3.1 Simulation Models

Simulation models simulate water resources behavior in accordance with a predefined set of rules governing allocations and infrastructure operations. They are used to model water quantity, quality, economic and social responses for a set of alternative allocation scenarios.

River basin flow simulation models have been successfully applied to manage water resources systems. Some of them are designed for particular specific basins, not designed to be general tools applicable in other systems, such as the Colorado River Simulation System (CRSS) (Wurbs, 1995). Other simulation models are generalized tools such as AQUATOOL (Andreu *et al.*, 1996), allowing for user-defined nodes, links, operation rules, and targets. A rich range of time series models have been employed for simulating hydrological and water quality sequences (see, for instance, Hipel and McLeod (1994)).

River basin water quality simulation models range from dimensionless to one-dimension, two-dimensions and three-dimensions, and from static models to dynamic models. Many water quality variables including temperature, dissolved oxygen (DO), biochemical oxygen demand (BOD), nutrients, coliforms and specific chemicals can be simulated with comprehensive models such as the Enhanced Stream Water Quality Model (QUAL2E) (EPA, 1996), Water Quality Analysis Simulation Program Modeling System (WASP) (EPA, 1993) and Water Quality for River-Reservoir Systems (WQRRS) (USACE, 1985).

Comprehensive river basin simulation systems emerge with the rapid development of information technology. Traditional simulations are enhanced with interactive and advanced graphic user interfaces, allowing on-screen configuration of the simulations, and display of results. WaterWare, developed by a consortium of European Union-sponsored research institutes (Jamieson and Fedra, 1996a,b), integrates Geographic Information System (GIS) functions and incorporates embedded expert systems with a number of simulation and optimization models and related tools. For addressing water allocation, conjunctive use, reservoir operation or water quality issues, MIKE BASIN couples the power of ArcView GIS with comprehensive hydrologic modeling to provide basin-scale solutions (DHI, 2001).

2.2.3.2 Optimization Models

Optimization models optimize and select allocations and infrastructure operations based on objectives and constraints. These models must have a simulation component to calculate hydrologic flows and constituent mass balance. However, the simulation models coupled in optimization water allocation generally have to be rudimentary in order to possess reasonable numerical calculation ability and time constraints. Whereas the assessment of system performance can be best addressed with simulation models, optimization models are more useful if improvement of the system performance is the main goal (McKinney *et al.*, 1999).

Agricultural use is an important factor in water allocation, since irrigation water demand generally consumes most of the water available in a region. Amir and Fisher (1999) introduce an optimizing linear model for analyzing agricultural production under various water quantities, qualities, timing, pricing and pricing policies. The model serves as the "agricultural sub-model" (AGSM) incorporated into their Water Allocation System (WAS). AGSM is formulated at the level of a district. Its objective function is the net agricultural income of the district, which is maximized by selecting the optimal water consuming activities. In this procedure, the decision variables are the land areas of different activities (Fisher, 1997).

Because of the significant capabilities of reservoirs to handle irrigation, hydropower generation and flow adjustment for ecological use and controlling flood and drought, many models governing the reservoir operations and water allocation have been developed (Vedula and Mujumdar, 1992; Vedula and Kumar, 1996; Chatterjee *et al*., 1998). Umanahesh and Sreenivasulu (1997) designed a stochastic dynamic programming model for the operation of irrigation reservoirs under a multicrop environment. The model considers stochastic reservoir inflows with variable irrigation demands and assumes the soil moisture and precipitation to be deterministic. The demands vary from period to period and are determined from a soil moisture balance equation. An optimal allocation process is incorporated into the model to determine the allocations to individual crops during an intraseasonal period whenever competition for available water exists among the crops. Thus, the model integrates the reservoir release decisions with the irrigation allocation decisions with respect to each crop in each period.

Due to the complexity of water allocation at the regional or basin level, three-level or three-layer optimization models based on an economic efficiency criterion are proposed by Reca *et al*. (2001a,b) and Shangguang *et al*. (2002). Level one models optimize irrigation timing for a single crop; level two models optimize water and land resources allocation for the cropping pattern on an irrigation area scale, which estimate the optimal benefit function for agricultural uses at different areas; level three models optimize water allocation among all types of demanding uses on a hydrologic system (region or basin) scale. The overall optimization model is defined in a deterministic way, assuming that the climatic variables and inflows to the system are known. To take into account the stochastic character of these variables, the optimization processes are repeated for such scenarios (Reca *et al*., 2001a).

The optimization models mentioned above mainly focus on the economic optimization while have simple water balance constraints. The Colorado River basin water resources study is one of the earliest research projects aiming to develop an optimization model for investigating performance of alternative market institutions for water allocation by incorporating a basin scale flow network (Booker and Young, 1994). In this study, both offstream (irrigation, municipal, and thermal energy) and instream (hydropower and water quality) uses are represented by empirical marginal benefit functions. The combination of increased hydrologic and demand-side details, permits this model to capture more completely who might gain and who might lose from institutional changes, thereby allowing interstate and intersector water markets in the basin. Cai *et al*. (2003) develop a holistic optimization model integrating hydrologic, agronomic, and economic relationships in the context of a river basin. The model reflects the interrelationships between these components and can be applied to explore both economic and environmental consequences of various policy choices. AQUARIUS is a state-of-the-art computer model devoted to the temporal and spatial allocation of water flows among competing traditional and non-traditional water uses in a river basin (Diaz *et al*., 1997). The model is driven by an economic efficiency operational criterion requiring the re-allocation of stream flows until the net marginal return in all water uses is equal. All these hydrology-inferred optimization models search for economic optimal water allocation at the river basin level, but do not take equity into consideration.

2.2.3.3 Integrated Simulation and Optimization Models

Simulation and optimization models can be complementary tools to solve water allocation problems with competition over scare water resources. Although detailed simulation models cannot be coupled within the optimization process, they can be used to assess the feasibility of the water allocation policies determined by optimization models, with regard to infrastructure operations and the water resources system responses under extreme conditions (Fedra *et al.*, 1993; Faisal *et al.*, 1994).

In an extensive study of the impact of a severe, sustained drought on water resources in the Colorado River basin in the United States, several models were adapted and applied to account for interstate water rights in accordance with the 1992 Colorado River Compact and related legislation. The Colorado River Network Model (CRM) is capable of simulation and optimization; it was adapted to simulate priority-based water allocations (Harding *et al.*, 1995). In this study, the Colorado River Institution Model (CRIM) was developed to simulate and optimize water allocations under a variety of market and nonmarket arrangements (Booker, 1995)

2.2.4 Application of Cooperative Game Theory in Water Management

Water resources and environmental management problems often engage multiple stakeholders with conflicting interests (Hipel and Fang, 2005; Hipel *et al.*, 1997 and 2003; Fang *et al.,* 1988 and 2002; Kilgour *et al.*, 1988). The multi-decision maker situation characterized by the lack of formal property rights and the existence of externalities can be adequately handled by means of game-theoretic notions and models.

Non-cooperative game theory, a main branch of game theory, addresses each decision maker's decision in a conflict situation, asking what choices would be "rational" for a decision maker, and what combinations of choices by all decision makers would be "stable". While there are applications of non-cooperative game theory focusing on international transboundary pollution problems (Hipel *et al*., 1997; Kilgour *et al.*, 1992), there are few applications in the area of water allocation. Part of the Colorado River drought study includes an interactive, gaming simulation of the drought, where riparian states and the federal government are represented by players with information on the status of water resources resulting from their own real-time decisions regarding intrastate water allocation (Henderson and Lord, 1995). Using the AZCOL river model that is based on the CRM simulation model, three games were played with rules based on existing compact agreements, a hypothetical interstate basin commission, and water markets. Based on water market rules, a win-win situation was identified where lower basin states could buy long-term water rights from the upper basin with legally enforceable provision that upper basin short-term deficits would be covered with water purchased back from the lower basin.

Water can be argued to be a public good. As the literature on the tragedy of the commons suggests, under situations lacking formal property rights, stakeholders will choose solutions to maximize their individual gain without concern for others so that the common resources may not be used efficiently to achieve an overall best benefit (Hardin, 1968; Dawes, 1973). This is so called common-pool resource (CPR) management problems. It is noticed that stakeholders can achieve a win-win outcome by cooperation to plan and implement a joint venture (Crane and Draper, 1996; Ostrom, 1996). If stakeholders decide to cooperatively share the common resources, the remaining question is how to allocate the costs and benefits of the cooperation among the stakeholders so that there is concrete incentive for each stakeholder or subgroup of stakeholders to participate in the joint venture and cooperation, and all the stakeholders view the allocation solution as fair. Another branch of game theory, cooperative game theory, can contribute to the fair allocation of CPR.

Cooperative game theory in the form of the characteristic function provides rigorous mathematical models, fulfilling the requirements of individual rationality, group rationality and joint efficiency (Owen, 1995). The theory has been successfully applied to the following types of problems in water resources management: (1) cost allocation of water resources development projects, including joint waste water treatment and disposal facility (Giglio and Wrightington, 1972; Dinar and Howitt, 1997), and water supply development projects (Young *et al.*, 1982; Driessen and Tijs, 1985; Dinar *et al.*, 1992; Lejano and Davos, 1995; Lippai and Heaney, 2000); (2) equitable allocation of waste loads to a common receiving medium (Kilgour *et al.*, 1988, Okada and Mikami, 1992); (3) allocation of water rights (Tisdell and Harrison, 1992).

Cost allocation of water resources projects is fairly well explored by cooperative game theory. The previous studies show how the joint cost of a development project can be allocated to each stakeholder equitably. For example, Young *et al.* (1982) compare three traditional methods (proportional to population, proportional to demand, and separable costs-remaining benefits method) and four methods from cooperative game theory (Shapley value, least nucleolus, weak nucleolus and proportional nucleolus) by application to an actual municipal cost allocation problem in Sweden. Other concepts to solve a cost allocation game are also proposed, such as minimum costs-remaining savings (MCRS) (Heaney and Dickinson, 1982), nonsepearable cost gap (NSCG) (Driessen and Tijs, 1985) and normalized nucleolus (Lejano and Davos, 1995). Dufournaud and Harrington (1990; 1991) develop a cooperative game theory approach for estimating "fair" and equitable division of benefits and costs resulting from a three-riparian, two-time-period, joint-basin development project.

Compared with the applications in cost allocation for water resources development projects, there are only a limited number of applications using cooperative game theory in water allocation. In a cost allocation game, each stakeholder can decrease his cost by joining a coalition or the grand coalition to gain a certain benefit. However, in water allocation problems, both the cost and benefit of each stakeholder change, the net benefit of each stakeholder should increase if he or she joins a coalition or the grand coalition. Dinar *et al.* (1986) investigate the allocation of cost and benefits from regional cooperation, with respect to reuse of municipal effluent for irrigation in the Ramla region of Israel. Different allocations based on marginal cost pricing and schemes from cooperative game theory like the core, Shapley value, generalized Shapley value and nucleolus are applied. Although no method has been found to be preferred, the marginal cost pricing was found to be unacceptable by the participants. This study extends the cost allocation to cost and benefit (net benefit) allocation for a joint water resources development project. Yaron and Rater (1990) present an analysis of the economic potential of regional cooperation in water uses in irrigation under conditions characterized by a general trend of increasing salinity. The related income distribution schemes with and without side payments are solved with the aid of cooperative game theory. Tisdell and Harrison (1992) use a number of different cooperative games to model the efficient and socially equitably reallocation of water among six representative farms in Queensland, Australia. Rogers (1969) uses linear programming to compute the optimum benefits of six strategies of India and East Pakistan (now Bangladesh) (acting singly or in cooperation) in the international Ganges-Brahmaputra river basin, and then analyzes the strategies by a nonzero-sum game for the two countries. Incorporating Nepal into his analysis, Rogers (1993a, b) outlines the applicability of cooperative game theory and Pareto frontier analyses to water resources allocation problems. Okada and Sakakibara (1997) also apply a hierarchical cooperative game model to analyze the cost/benefit allocation in a basin-wide reservoir redevelopment as part of water resources reallocation. Kucukmehmetoglu and Guldmann (2004) use cooperative game theory concepts (core and Shapely value) to identify the distribution of the total joint benefit of cooperation which is calculated by a linear programming model that allocates the waters of the Euphrates and Tigris rivers to agricultural and urban uses in the three riparian countries – Turkey, Syria and Iraq. One major deficiency of these cooperative game theory based models is they do not consider hydrological constraints or only simple ones. Another deficiency is that they focus on fair distribution of joint benefits but do not investigate how to fairly allocate water rights which are the status quo of a cooperative water allocation game.

Chapter 3
Cooperative Water Allocation Model

3.1 Introduction

Both nationally and internationally, the issue of water resources allocation has become the focus of many conflicts. The competition for water is evident not only in terms of quantity but also quality. Irrigation, urban, industrial, recreational and environmental uses are competing for their "fair" share of water. Many negotiations begin with the parties basing their initial positions in terms of rights – the sense that a riparian is entitled to a certain allocation based on intra-country water rights regimes or international river basin agreements (Gioradano and Wolf, 2001). The basic underlying theme in water allocation relates to what are "fair" or "equitable" water rights.

For water rights allocation inside a country, various water rights systems can be grouped into three basic doctrines: riparian rights, prior rights and public allocation (Savenije and Van der Zaag, 2000). In the past decades, many mathematical simulation and optimization models for water quantity, quality and economic management have been developed. Unfortunately, most models and applications do not incorporate fairness concepts in their quantitative calculations, except for prior water allocation models which interpret fairness in that senior users owning higher priorities have more privileges to withdraw water than junior users owning lower priorities. Conventional simulation (Wurbs, 2001), minimum-cost pure (Fredericks *et al.*, 1998) and generalized (Hsu and Cheng, 2002) network flow, and mixed integer linear programming models (Tu *et al.*, 2003) have been developed for prior water allocation. However, simulation models cannot provide either spatially or temporally optimal allocations due to structural limitations. The minimum-cost network flow models and linear programming formulations also have a common shortcoming in that they lack systematic and formal methods to set proper unit cost coefficients to ensure that water is allocated in the priority order when return flows, instream uses, or reservoir storage rights are included in the programming, because return and instream flows and reservoir storages can be reused by junior downstream uses (Israel and Lund, 1999).

For international river basins between countries, there is no formal inter-country water rights system but there are international water agreements defining ownership of the water resources. The agreements may be reached by following principles including absolute sovereignty, absolute riverine integrity, limited territorial sovereignty, and economic criteria (Wolf, 1999; Giordano and Wolf, 2001). Although international water laws assert that the water should be equitably allocated, they provide no well-defined, transferable and measurable criteria for water rights allocation, and few models concerning fair water rights for transboundary basins exist in the literature.

Many recent studies are concerned about increasing the efficiency and effectiveness of water resources management, and center around economic and market mechanisms to promote efficiency from an economic perspective (McKinney *et al.*, 1999; Mahan *et al*., 2002). As Fisher *et al*. (2002) argue, water markets are not truly free and competitive markets, which are usually regulated by the government and lack large numbers of independent small sellers and buyers. Furthermore, for a free market to lead to an efficient allocation, social costs must coincide with private costs, and social benefits must be in line with private ones. However water uses have "externalities", affecting water quantity and quality for others. Such externalities do not typically enter the calculations of individual costs and benefits, but increase the social costs. Many countries reveal by their policies that they regard water for certain uses (often agriculture) being a public value that exceeds its private one. While water markets cannot be expected to lead to socially optimal allocations automatically, it is possible to build economic optimization models to guide water policy and allocations to reach optimal social benefits.

In order to achieve sustainable development and a secure society, institutions and methodologies for water allocation should be reformed for regions with water resources shortages. Water allocation should consider three principles: equity, efficiency and sustainability (Wang *et al*., 2003a,b). By equity, it is meant that water resources within river basins should be fairly shared by all of the stakeholders. Efficiency means the economic use of water resources with respect to minimizing costs and maximizing benefits. Under sustainability, water is utilized economically both now and in the future such that the environment is not harmed.

Due to the different production abilities of water users in the real world, water allocations merely based on a water rights approach usually do not make efficient use of water for the whole river basin. Meanwhile, an economic efficient water allocation plan generally is not an equitable one for all water users or stakeholders, and an economic water allocation plan cannot be implemented if the involved participants or stakeholders do not regard it as being fair. To achieve equitable and efficient water allocation requires all stakeholders' cooperation in sharing water resources. However, there are few studies that jointly consider both aspects of efficiency and equity in water allocation.

The purpose of this research is to design a comprehensive methodology for equitable and efficient cooperative water resources allocation, which integrates water rights allocation, efficient water allocation and fair income distribution within the context of realistic hydrologic constraints at the river basin level. In this chapter, the basic idea and framework of the cooperative water allocation model (CWAM) are presented. The model allocates water resources in two steps: initial water rights are firstly allocated to water uses based on an abstracted node-link river basin network and legal rights systems or agreements, and then water is reallocated to achieve efficient use of water through water transfers. The associated net benefit reallocation is carried out by application of cooperative game theoretical approaches. The next section describes the configuration of CWAM, the river basin network model, water balances and constraints, and the water allocation problems. Section 3.3 presents the three methods proposed for the initial water rights allocation, including priority-based multiperiod maximal network flow (PMMNF) programming, modified riparian water rights allocation (MRWRA) and lexicographic minimax water shortage ratios (LMWSR). Section 3.4 defines the cooperative water allocation game, and presents the cooperative game theoretical approaches for equitable reallocation of the net benefits obtained under the grand coalition. The final part illustrates an application of CWAM through a simple example.

3.2 Cooperative Water Allocation Model

3.2.1 Configuration of the Model

The Cooperative Water Allocation Model (CWAM) is designed as a comprehensive model for modeling equitable and efficient water resources allocation at the basin scale based on a node-link river basin network, whose configuration is plotted in Figure 3.1. The model consists of two big blocks: the first one is the initial water rights allocation, and the second one is the reallocation of water and net benefits, which correspond to the two steps of the cooperative water allocation procedure. The first block includes the priority-based multiperiod maximal network flow (PMMNF) programming, modified riparian water rights allocation (MRWRA) and lexicographic minimax water shortage ratios (LMWSR) methods for deriving equitable initial water rights allocation among competing uses. PMMNF is a very flexible approach and is applicable under prior, riparian and public water rights systems. MRWRA is essentially a special form of PMMNF adapted for allocation under the riparian regime. LMWSR is designed for a public water rights system, which adopts the concept of lexicographic minimax fairness. The second block comprises three sub-models: the irrigation water planning model (IWPM) is a model for deriving benefit functions of irrigation water; the hydrologic-economic river basin model (HERBM) is the core component of the coalition analysis, which is a tool for finding optimal water allocation schemes and net benefits of various coalitions of stakeholders. The input includes hydrologic and water demand data, initial water rights, water demand curves and benefit functions, and sets of stakeholders, coalitions and ownership; the sub-model cooperative reallocation game (CRG) of the net benefit of the grand coalition adopts cooperative game theoretical approaches to perform equitable allocation of the net benefits of the grand coalition. The economically efficient use of water under the grand coalition is achieved through water transfers (water reallocation) based on initial water rights.

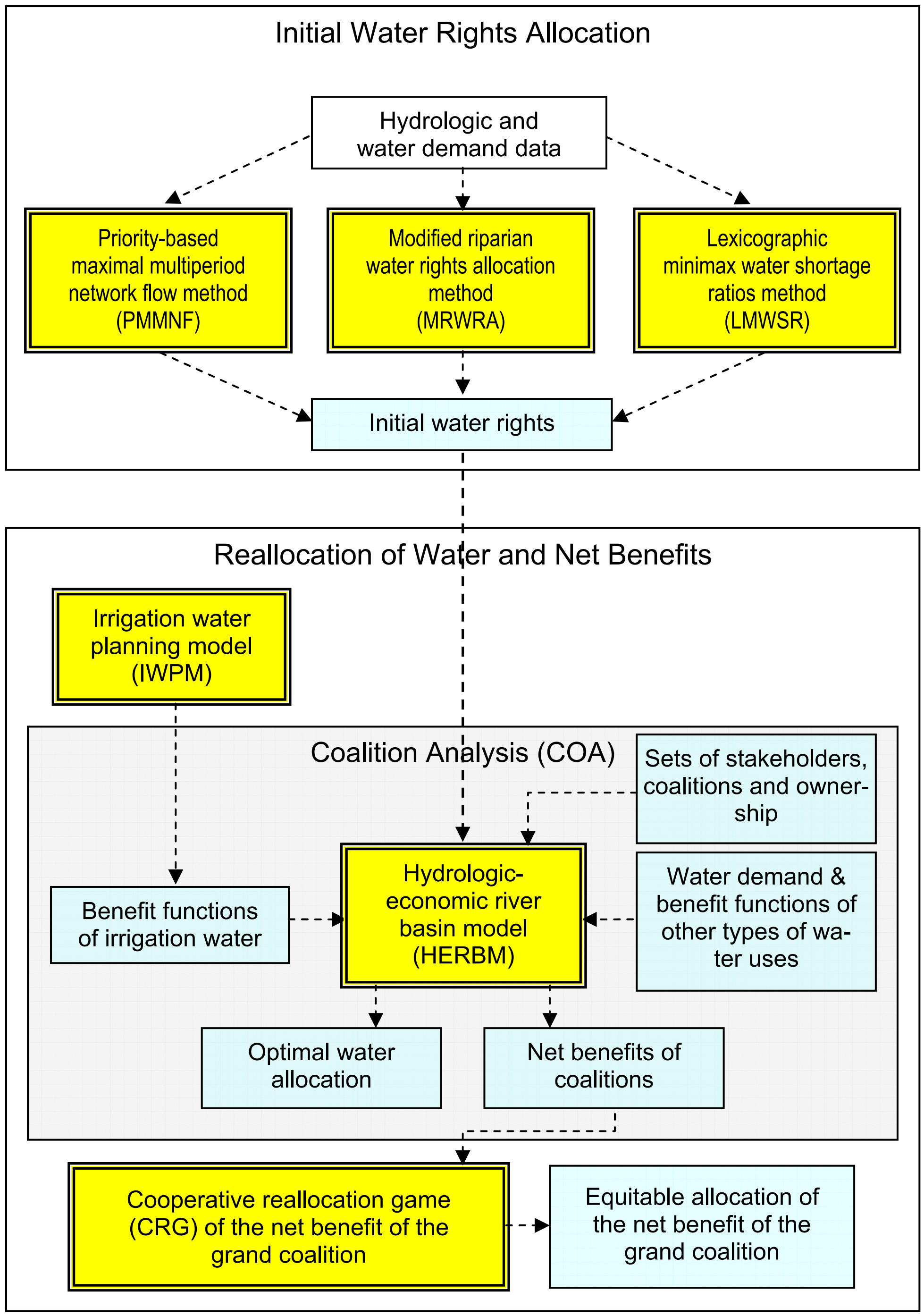

Figure 3.1 Components and data flows of the Cooperative Water Allocation Model (CAWM)

3.2.2 River Basin Network Model

3.2.2.1 Directed Node-link Network

A node-link river basin network is an abstracted graphical model for describing a river basin or watershed. Water users and uses are aggregated into stakeholders (water user associations) and water demand nodes, respectively, according to geography and ownership. A node is symbolized as a dot, circle, triangle, or rectangle, representing a physical component of interest such as inflow, natural or man-made junction, intake structure, water or wastewater treatment plant, aquifer, reservoir, natural lake, dam, weir, or water demand site. A link represents a natural or man-made water conduit such as river channel, canal or pipeline between two different nodes, but can also stand for any flow of water such as the seepage between a demand site and an aquifer. The links include river reaches, diversions, transmission and return flow links. A sample flow network is shown in Figure 3.2.

Let $G(V, L)$ be the directed network of a river basin, where $V = \{v_1, v_2, \cdots, v_v\}$ is the set of nodes, (k_1, k_2) denotes the arc or link from node k_1 to k_2, and $L = \{(k_1, k_2)\colon\ k_1, k_2 \in V$ and $k_1 \neq k_2\}$ is the set of links of the network. Water users are grouped into stakeholders whose set is defined by $N = \{1, 2, \cdots, n\}$. A number of water use sites which take water and discharge return flows, including offstream and instream economic uses, minimum environmental flow requirements as well as reservoirs, are abstracted as a set of water demand nodes in the node-link river basin network model, where $U = \{j \in V\colon j$ is a water demand node$\}$. The demand set of stakeholder i, $i \in N$, can be defined as $U_i = \{j \in V\colon j$ is a water demand node of stakeholder $i\}$. Thus, $U_i \subseteq U \subseteq V$. Note that a stakeholder may have several water demand nodes, and a water demand node may be associated with a number of water uses and users, such as agricultural farms and farmers. Reservoirs are considered as a kind of demand nodes since they have water storage demands and rights to maintain water head for directly linked storage hydropower plants, on-site recreational activities, controlling floods, and storing water for future needs.

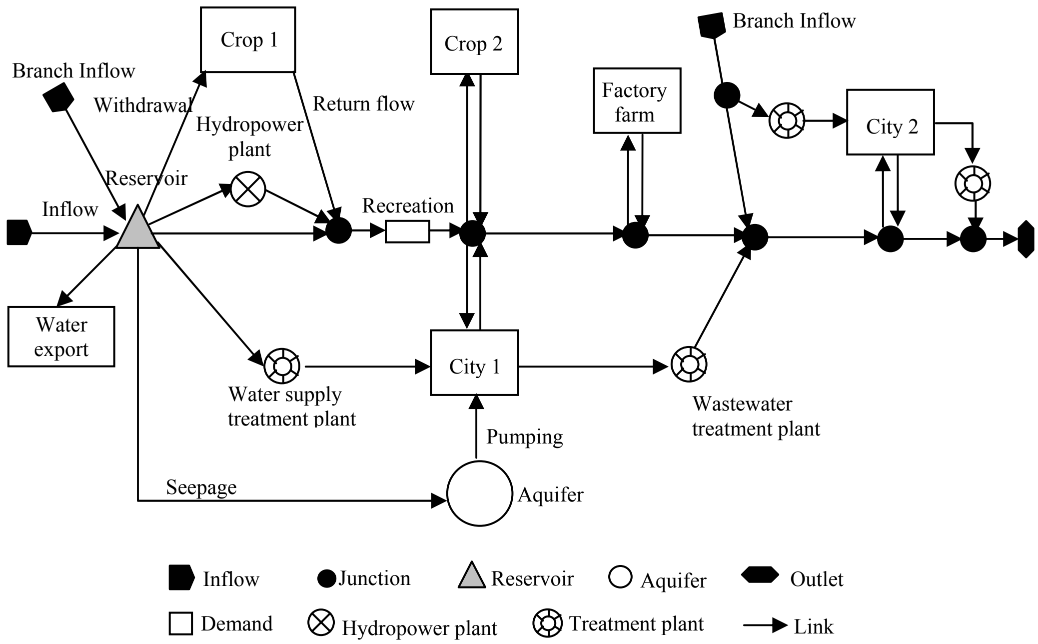

Figure 3.2 An example of a river basin network

3.2.2.2 Node and Link Types

Since different types of nodes and links have different hydrological and economic properties, subsets of the general set of nodes V are defined for the mathematical formulation of the model.

Nonstorage node set: $NST = IN \cup OUT \cup JUN \cup TP$

where, IN = {$k \in V$: k is an inflow node}, OUT = {$k \in V$: k is an outlet node}, JUN = {$k \in V$: k is a junction node}, TP = {$k \in V$: k is a treatment plant}. A junction node can be a simple, confluence, diversion or offtake (withdrawal) node. Treatment plants include water and wastewater treatment facilities.

Storage node set: $STO = AQU \cup RES$

where, AQU = {$k \in V$: k is an aquifer}, RES = {$k \in V$: k is a reservoir}.

Demand node set: $U = OFFDEM \cup INDEM$

where, $OFFDEM = AGR \cup MI$ = {$k \in V$: k is an offstream demand node},

$INDEM = RES \cup HPP \cup SFR$ = {$k \in V$: k is an instream demand node},
AGR = {$k \in V$: k is an agricultural node}, MI = {$k \in V$: k is a municipal and Industrial node}, HPP = {$k \in V$: k is a hydropower plant}, HPP_{res} = {$k \in HPP$: k is a storage hydropower plant directly linked to a reservoir}, HPP_{riv} = {$k \in HPP$: k is a run-of-river hydropower plant with constant water head}, SFR = {$k \in V$: k is a node with a stream flow requirement for in-stream uses}. The HPP_{res} set includes both fixed-head and variable-head hydropower plants. A fixed-head storage hydropower plant is connected to one reservoir by a canal or open-channel structure; a variable-head storage hydropower plant is built as an integral part of the reservoir or is connected to the reservoir by tunnel and penstock (Diaz, *et al*. 1997). The *SFR* set includes all kinds of instream environmental uses nodes such as fishery, wetland, recreation, and ecology.

Based on the ten subsets defined above, *IN*, *OUT, JUN, TP, AQU, RES, SFR*, *AGR, MI,* and *HPP*, form a partition of the general network node set *V*. Note, a node can belong to only one subset. To maintain the partition structure, a junction node automatically becomes an *SFR* node if there is an imposed flow requirement for instream use at that node.

Several subsets of the general link set *L* are defined below:

Inflow link: $L_{in} = \{(k_1, k): (k_1, k) \in L\}$

Outflow link: $L_{out} = \{(k, k_2): (k, k_2) \in L\}$

Withdrawal link: $L_{with} = \{(k_1, j): (k_1, j) \in L\}$

Return flow link: $L_{ret} = \{(j, k_2): (j, k_2) \in L\}$

Seepage link: $L_{\mathrm{seep}} = \{(k, k_2): (k, k_2) \in L, k \in V \text{ and } k_2 \in AQU, \text{ or } k \in AQU \text{ and } k_2 \in V\}$

3.2.2.3 Schematizing a River Basin

Schematizing a river basin is the first job in the model development. It is important to define a flexible schematization reflecting the overall natural conditions and based on the objectives of the modeling, the availability of information and the required output spatial resolution. Different types of spatial schematizations may be introduced (DHI, 2001), such as: (1) Lumping of smaller rivers into a single branch upstream of an intake point; (2) Combining small irrigation areas into a single node with one intake point; (3) Lumping town supply and industrial water supply into one entity.

A demand site is best defined as a set of uses that share a physical distribution system that are within a defined region, or that share an important withdrawal supply point. An ideal schematization should represent the activities at the level of detail that is desired, while at the same time lump homogeneous demands and resources for which no further differentiation is required. The level of the aggregation, in practice, is generally constrained by the level of detail water uses data available and may also depend on the level of detail desired for analysis. A properly abstracted river basin network has the following network properties:

(1) A river basin network is a *simple directed graph*, in which there are no multiple links with a same direction between any pair of nodes (*multiarcs*), and there is no link starting and ending at a same node (*loop*).

(2) Let the directed graph be $G(V,L)$, and $P = \{v_1, l_1, v_2, l_2, \cdots, v_{k-1}, l_{k-1}, v_k\}$ be a sequence of alternant nodes and links, where $v_1, v_2, \cdots, v_k$ are nodes and $l_1, l_2, \cdots, l_{k-1}$ are links. Then P is called a *directed route* from node v_1 to v_k, if $l_i = (v_i, v_{i+1})$ for all $i=1, 2, \cdots, k-1$.

(3) $v_1 \in IN$ is called a *source node* of the river basin, $v_k \in OUT$ or v_k being a *SFR* node with zero outflow is called a *sink node*. For each source node v_1, there is at least one directed route $P = \{v_1, l_1, v_2, l_2, \cdots, v_{k-1}, l_{k-1}, v_k\}$ leading to a sink node, where $v_i \in IN \cup JUN \cup OUT \cup AQU \cup RES \cup SFR$ for all $i=1, 2, \cdots, k-1$.

These network properties mean that every upstream inflow of a river basin can always flow through natural hydrological channels to reach an outlet node or a totally consumptive instream use node of the basin. To assure this, the schematization should be carried out step by step: abstract the natural hydrological network first before adding demand nodes and relevant inflow and outflow links.

3.2.3 Water Balances and Constraints

There are three kinds of constraints in the water allocation model: physical constraints, policy constraints and system control constraints. Some key assumptions are made in the following formulation of the hydrological models.

(1) The overall modeling concept is to find stationary (quasi-steady state) solutions for each period. The time series of input and output are presumed to be constant and flux-averaged values during any time period, except that the water quantity and quality within reservoirs and aquifers are changing within a period.

(2) The typical period for modeling is one month, while it can also be specified as a week or year.

(3) The hydrological responses of aquifers and reservoirs are modeled by a linear reservoir model.

(4) Time lag is not considered for the transport of flows including return flows from demand sites.

(5) Both water flows and pollution loads enter a river basin network only at nodes.

(6) The transport of substances affecting water quality is modeled as by advection only. Dispersion is not taken account of. The degradation process of substances is assumed to be first-order decay.

(7) Water and substances in an aquifer or reservoir are completely mixed.

The above assumptions allow the size and computation time of the optimization problems of water allocation to be controllable and reasonable. However, the approximation made for a stationary solution becomes weaker when the states of a process change notably during the period of the simulation. For example, the simulation results for reservoir and variable-head storage hydropower production may become poor approximations if the water level changes significantly during a period, because power generation depends on the water level in the reservoir. Due to the stationarity assumption, the hydrological models based on a river basin network scheme are best utilized to simulate water quantity and quality in a slowly changing system. For fully dynamic modeling, detailed water quantity and quality simulation models may be coupled with the optimization models of water allocation through an iterative approach. But it might be intractable because of the very heavy load for computation and lack of input information for full dynamic modeling.

3.2.3.1 Physical Constraints

The physical constraints are mass balances and capacity limits. Figure 3.3 shows the water balance for a general node. Note that the link losses of evaporation and leakage, node gains from local catchment's drainage, and losses of evaporation and consumption at demand nodes (dot arrows) are not explicitly plotted in the river basin network model, but they are included in the mathematical model.

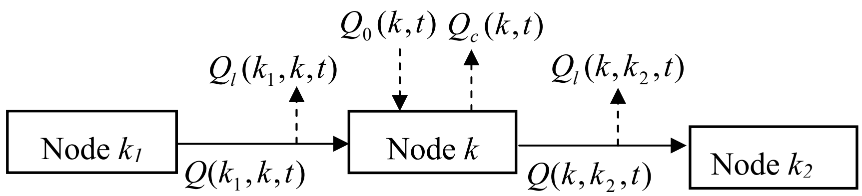

Figure 3.3 Water balance of a general node *k*

The water and pollutant balance equations for a general node k during each period t can be written as:

$$S(k,t)-S(k,t-1)=\sum_{(k_1,k)\in L} Q(k_1,k,t)-\sum_{(k_1,k)\in L} Q_l(k_1,k,t)+Q_g(k,t)-Q_c(k,t) - \sum_{(k,k_2)\in L} Q(k,k_2,t),\ \forall k\in V \tag{3.1}$$

$$C_p(k,t)S(k,t)-C_p(k,t-1)S(k,t-1)=\sum_{(k_1,k)\in L} C_p(k_1,k,t)Q(k_1,k,t)-\sum_{(k_1,k)\in L} Z_{pl}(k_1,k,t) +Z_{pg}(k,t)-Z_{pc}(k,t)-\sum_{(k,k_2)\in L} C_p(k,k_2,t)Q(k,k_2,t),\ \forall k\in V \tag{3.2}$$

where, t – index of time periods (period length is Δt), $t\in T=\{1, 2, \cdots, \tau\}$ (τ is the largest index of periods); $S(k,t)$ – storage volume for storage node (reservoir or aquifer) $k\in STO$ at the end of period t; $Q(k_1,k,t)$ – flow from node k_1 to k during period t; $Q_l(k_1,k,t)$ – conveyance losses because of evaporation, leakage and seepage of the flow from node k_1 to k;

$Q_g(k,t)$ – gain of inflow adjustment at node k during period t for discharges from small tributaries, local catchment drainages, river reach seepages or flows from other sources; $Q_c(k,t)$ – water consumed at node k because of economic activities and evaporation; p – index of pollutant types, $p \in P = \{1, 2, \cdots, \xi\}$($\xi$ is the largest index of pollutants); $C_p(k,t)$ – concentration of pollutant p at storage node k at the end of period t; $C_p(k_1,k,t)$ – concentration of pollutant p in the water flow from node k_1 to k during period t; $Z_{pl}(k_1,k,t)$ – conveyance losses of pollutant p in the water flow from node k_1 to k; $Z_{pg}(k,t)$ – total amount of pollutant p added to node k during period t because of inflow adjustment $Q_g(k,t)$ and of water use activities; $Z_{pc}(k,t)$ – removal of pollutant p at node k. Note, $S(k,t) = 0,\ \forall k \in V \setminus STO$. For source node $k \in IN$, the total inflow received from the outside of river network is represented as $Q_{IN}(k,t)$, while $Q(k_1,k,t)$ represents flows from other nodes to k. For outlet node $k \in OUT$, the total outflow from node k to the outside of river network is represented as $Q_{OUT}(k,t)$, and there is no $Q(k,k_2,t)$ anymore. Throughout the thesis, without pointing out specifically, the units for flow and storage are million cubic meters (mcm), and the units for concentration and discharge are g/l and thousand tonnes (kton), respectively.

The model itself does not include sub-models simulating precipitation infiltration, runoff, groundwater flow and evaporation processes. Drainage and various coefficients of seepage and link loss are input parameters to be estimated from corresponding statistics or external hydrologic modeling (Singh, 1988; Singh and Woolhiser, 2002). The effective rain precipitations to crop fields are not included in the river basin network model, but are accounted for in calculating the irrigation water demands and the benefits of crop production.

Besides the general mass balance equations for each node, there are mass balance constraints for some natural physical response processes. These include:

(1) Link losses

The evaporation, leakage and seepage losses can be specified as percentages of link flows (SEI, 2001). Given the total water loss coefficient $e_L(k_1,k,t)$ for link (k_1, k), then

$$Q_l(k_1,k,t) = e_L(k_1,k,t)Q(k_1,k,t),\ \forall k \in V \tag{3.3}$$

The pollutant loss in a link is approximated as a percentage of the inflow pollutant amount,

$$Z_{pl}(k_1,k,t) = e_{p_L}(k_1,k,t)C_p(k_1,k,t)Q(k_1,k,t),\ \forall k \in V \tag{3.4}$$

where, $e_{p_L}(k_1,k,t)$ is the pollutant loss coefficient for link (k_1, k).

In order to account for the seepages of link flow to aquifers, a link seepage coefficient $s_L((k_1,k_2),k,t)$ should be specified for every pair of the source river reach (k_1, k_2) and destination aquifer k. At the same time, the corresponding water loss coefficient $e_{sL}((k_1,k_2),k,t)$ and pollutant loss coefficient $e_{pSL}((k_1,k_2),k,t)$ should be specified for each seepage flow $Q((k_1,k_2),k,t)$. Note that a link may have seepages to several aquifers. The $e_{pL}(k_1,k,t)$ and $e_{pSL}((k_1,k_2),k,t)$ can be directly input as data or estimated by a first order one-dimensional steady state water quality model (see Appendix A).

(2) Node inflow adjustments

The inflow adjustments to a surface node come from the local catchment drainage and small tributaries ($Q_a(k,t)$), while the inflow adjustments to an aquifer also include seepages from surface river reaches, such that

$$Q_g(k,t) = Q_a(k,t),\ \forall k \in V \setminus AQU \tag{3.5}$$

$$Q_g(k,t) = Q_a(k,t) + \sum_{(k_1,k_2)\in L} \left[1 - e_{SL}((k_1,k_2),k,t)\right] \cdot s_L((k_1,k_2),k,t)Q(k_1,k_2,t),\ \forall k \in AQU \tag{3.6}$$

Given the pollutant concentration $C_{pa}(k,t)$ of the inflow adjustment $Q_a(k,t)$, then

$$Z_{pg}(k,t) = C_{pa}(k,t)Q_a(k,t),\ \forall k \in IN \cup OUT \cup JUN \tag{3.7}$$

$$\begin{aligned} Z_{pg}(k,t) = C_{pa}(k,t)Q_a(k,t) + \sum_{(k_1,k_2)\in L} \left[1 - e_{pSL}((k_1,k_2),k,t)\right] \cdot \\ s_L((k_1,k_2),k,t)C_{pL}(k_1,k_2,t)Q(k_1,k_2,t),\ \forall k \in AQU \end{aligned} \tag{3.8}$$

$$Z_{pg}(k,t) = C_{pa}(k,t)Q_a(k,t) + Z_{pd}(k,t), \quad \forall k \in AGR \cup MI \cup TP \cup RES \cup HPP \cup SFR \tag{3.9}$$

where $Z_{pd}(k,t)$ is the pollutant discharge from the production activities at a node. It is assumed to be zero for $\forall k \in IN \cup OUT \cup JUN \cup AQU$.

For $\forall k \in AGR \cup MI \cup TP \cup RES \cup HPP \cup SFR$, the pollutant loads produced by the demand site are assumed to be quadratic functions of inflows.

$$\begin{aligned} Z_{pd}(k,t) &= z_{pd0}(k,t) + z_{pd1}(k,t) \sum_{(k_1,k)\in L} (1 - e_L(k_1,k,t))Q(k_1,k,t) \\ &+ z_{pd2}(k,t) \left[\sum_{(k_1,k)\in L} (1 - e_L(k_1,k,t))Q(k_1,k,t) \right]^2 \\ &\forall k \in AGR \cup MI \cup TP \cup RES \cup HPP \cup SFR \end{aligned} \tag{3.10}$$

where, $z_{pd0}(k,t)$, $z_{pd1}(k,t)$, $z_{pd2}(k,t)$ are coefficients. Normally $z_{pd2}(k,t)$ is negative, and the discharged amount increases when inflow grows, but the increased rate becomes smaller when the total inflow gradually approaches the critical point of the quadratic curve.

(3) Node losses, consumption and pollutant discharges

It is assumed that there is no water consumed at inflow, outlet, junction or aquifer nodes,

$$Q_c(k,t) = 0, \ \forall k \in IN \cup OUT \cup JUN \cup AQU \tag{3.11}$$

All of the water losses at a demand node (except a reservoir) or treatment plant are lumped into a consumption coefficient $e_N(k, t)$ and the total loss is

$$\begin{aligned} Q_c(k,t) = e_N(k,t) \left(Q_a(k,t) + \sum_{(k_1,k)\in L} (1 - e_L(k_1,k,t))Q(k_1,k,t) \right) \\ \forall k \in AGR \cup MI \cup TP \cup HPP \cup SFR \end{aligned} \tag{3.12}$$

The physical characteristics of a reservoir are defined by two basic functions: the area-storage curve, which represents the surface water areas enclosed by the reservoir for different storages; and the elevation-storage curve, which defines the variation of water surface elevation with reservoir storage. The curves are approximated by the following polynomial functions:

$$A(S(k,t)) = A_0 + A_1 S(k,t) + A_2 S^2(k,t) + A_3 S^3(k,t)$$
$$H(S(k,t)) = H_0 A_0 + H_1 S(k,t) + H_2 S^2(k,t) + H_3 S^3(k,t)$$

where A is water surface area of the reservoir k, H is the elevation of the water surface above a given reference level, and A_0 , A_1 , A_2 , A_3 , H_0 , H_1, H_2, and H_3 are parameters computed from measured area-storage and elevation-storage data. In most cases, the nonlinear power functions can be reasonably approximated as linear relationships (Diaz *et al.*, 1997; Cai *et al.*, 2001c):

$$A(S(k,t)) = A_0 + A_1 S(k,t)$$
$$H(S(k,t)) = H_0 + H_1 S(k,t) .$$

Given the evaporation rate $E_R(k,t)$, the loss at a reservoir node k is

$$Q_c(k,t) = E_R(k,t) \cdot A\left(\bar{S}(k,t)\right), \quad \forall k \in RES \tag{3.13}$$

where $\bar{S}(k,t)$ is the average storage during period t, $\bar{S}(k,t) = \frac{1}{2}\left(S(k,t-1) + S(k,t)\right)$.

The removal of pollutants at nodes is calculated by the following equations. It is assumed that there is no consumption of pollutant at any node $k \in IN \cup OUT \cup JUN \cup HPP$, and, thus

$$Z_{pc}(k,t) = 0, \quad \forall k \in IN \cup OUT \cup JUN \cup HPP \tag{3.14}$$

Pollutants are assumed to be removed at a ratio of $e_{pN}(k,t)$ at an off-stream node, and, hence

$$Z_{pc}(k,t) = e_{pN}(k,t)\left(Z_{pg}(k,t) + \sum_{(k_1,k)\in L} (1 - e_{pL}(k_1,k,t)) C_p(k_1,k,t) Q(k_1,k,t) \right) \tag{3.15}$$
$$\forall k \in AGR \cup MI \cup TP \cup SFR$$

For $k \in AQU \cup RES$, pollutants are assumed to be fully mixed and of first order decay, and the consumption of pollutants in a storage node during each period is approximated to be proportional to the mass in the storage node at the end of period t. Therefore,

$$Z_{pc}(k,t) = e_{pN}(k,t)S(k,t)C_p(k,t), \quad \forall k \in RES \cup AQU \tag{3.16}$$

where, $e_{pN}(k,t) = 1 - \exp(-K_p(k,t) \cdot \Delta t)$, $K_p(k,t)$ is the first order decay rate coefficient. This approximation is valid when the value of $S(k,t)C_p(k,t)$ does not change much during period *t*. For full dynamic modeling, simulation models incorporating numerical solutions of differential equations with small time steps have to be utilized. The limitation of quasi-steady state of the river system is consistent for the purpose of water resources planning.

(4) Outflows

Given the node seepage coefficient $s_N(k, k_2, t)$, the seepages from an inflow or nonstorage node k to node k_2 is

$$Q(k,k_2,t) = s_N(k,k_2,t)\left(Q_{IN}(k,t) + Q_a(k,t) + \sum_{(k_1,k)\in L} (1 - e(k_1,k,t))Q(k_1,k,t) \right) \quad \forall k \in IN, \quad (k,k_2) \in L_{seep} \tag{3.17}$$

$$Q(k,k_2,t) = s_N(k,k_2,t)\left(Q_a(k,t) + \sum_{(k_1,k)\in L} (1 - e(k_1,k,t))Q(k_1,k,t) \right) \quad \forall k \in V \setminus (STO \cup IN), \quad (k,k_2) \in L_{seep} \tag{3.18}$$

A linear reservoir model is applied to reservoirs and aquifers, and the seepages from a reservoir k or discharges from an aquifer k is represented as

$$Q(k,k_2,t) = s_N(k,k_2,t)\overline{S}(k,t), \ \forall k \in RES \cup AQU, \quad (k,k_2) \in L_{seep} \tag{3.19}$$

The outflows from a given node to multiple destinations are assumed to have the same concentrations. The concentrations in outflows from a storage node are approximated to be equal to the concentration in that node at the end of period *t*. Therefore,

$$C_p(k,k_2,t) = C_p(k,k_2',t),\ \forall k \in V,\ \forall (k,k_2),\ (k,k_2') \in L \tag{3.20}$$

$$C_p(k,k_2,t) = C_p(k,t),\ \forall k \in RES \cup AQU,\ \forall (k,k_2) \in L \tag{3.21}$$

In addition to water and pollutant balance equations, there are some capacity limits for storage nodes and links, such as:

maximum water volume of a storage node k:

$$S(k,t) \le S_{\max}(k,t),\ \forall k \in RES \cup AQU \tag{3.22}$$

maximum flow from k_1 to k:

$$Q(k_1,k,t) \le Q_{\max}(k_1,k,t),\ \forall (k_1,k) \in L \tag{3.23}$$

and maximum outflow from at an outlet:

$$Q_{out}(k,t) \le Q_{out\max}(k,t),\quad \forall k \in OUT \tag{3.24}$$

3.2.3.2 Policy Constraints

Policy constraints are hydrological, economic and social restrictions governing water allocations. Typical policy constraints set the demand limits for storage nodes or links, together with capacity limits forming the lower and upper bounds for storages and flows, such as:

minimum flow from k_1 to k:

$$Q(k_1,k,t) \ge Q_{\min}(k_1,k,t),\ \forall (k_1,k) \in L \tag{3.25}$$

maximum total inflow for demand node j:

$$\sum_{(k_1,j)\in L\setminus L_{Seep}} (1-e_L(k_1,j,t))Q(k_1,j,t) \le \max\left(Q_D(j,t)-Q_a(j,t),\ \ 0\right) \quad \forall j \in AGR \cup MI \cup HPP \tag{3.26}$$

minimum volume for a storage node k:

$$S(k,t) \ge S_{\min}(k,t),\ \forall k \in RES \cup AQU \tag{3.27}$$

minimum outflow from an outlet:

$$Q_{out}(k,t) \ge Q_{out\min}(k,t),\quad \forall k \in OUT \tag{3.28}$$

maximum concentration of pollutant p in link (k_1,k):

$$C_p(k_1,k,t) \le C_{p\max}(k_1,k,t), \quad \forall (k_1,k) \in L \tag{3.29}$$

maximum mixed concentration of pollutant *p* in the inflows to nonstorage node *k*:

$$0 \le C_{pN}(k,t) \le C_{pN\max}(k,t), \ \forall k \in V \setminus (RES \cup AQU) \tag{3.30}$$

maximum concentration of pollutant *p* for storage node *k*:

$$C_p(k,t) \le C_{p\max}(k,t), \quad \forall k \in RES \cup AQU \tag{3.31}$$

and maximum concentration of pollutant *p* in outflow from at an outlet:

$$C_{pout}(k,t) \le C_{pout\max}(k,t), \quad \forall k \in OUT \tag{3.32}$$

where, $Q_D(j,t)$ is the total water demand of demand node *j*, and $C_{pN}(k,t)$ is the mixed concentration of pollutant *p* in the inflows to nonstorage node *k*, satisfying

$$\begin{aligned} C_{pN}(k,t)\left[Q_a(k,t) + \sum_{(k_1,k)\in L}(1-e(k_1,k,t))Q(k_1,k,t)\right] &= C_{pa}(k,t)Q_a(k,t) \\ + \sum_{(k_1,k)\in L}(1-e_{pL}(k_1,k,t))C_p(k_1,k,t)Q(k_1,k,t), \ &\forall k \in V \setminus (RES \cup AQU) \end{aligned} \tag{3.33}$$

Note, water quality constraints should be used carefully and only when necessary, since strict imposition of constraints may result in improper or infeasible solutions.

3.2.3.3 System Control Constraints

Sometimes using only the physical and policy constraints will not completely constrain the solutions as represented in the real world situations, because of the simplified abstraction of the river network, hydrological or social-economic processes. The system control constraints are used to compensate for this. For example, if one lets $SRC \in V \setminus STO$ be the set of nonstorage nodes that are simplified to provide water supplies to some demand sites and receive corresponding return flows from them, then the total inflow to any node $k \in SRC$, excluding the return flows, should exceed the total diversions from *k* to them, because in reality those return flows are not available for diversion at that node. Let *j* be a water demand node, then,

$$\sum_{\substack{(k,j)\in L\\ \text{and}\\ (j,k)\in L}} Q(k,j,t) \le Q_a(k,t) + Q_{IN}(k,t) + \sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))Q(k_1,k,t) - \sum_{\substack{(j,k)\in L\\ \text{and}\\ (k,j)\in L}} (1-e_L(j,k,t))Q(j,k,t), \quad \forall k \in (IN \cap SRC) \tag{3.34}$$

$$\sum_{\substack{(k,j)\in L\\ \text{and}\\ (j,k)\in L}} Q(k,j,t) \le Q_a(k,t) + \sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))Q(k_1,k,t) - \sum_{\substack{(j,k)\in L\\ \text{and}\\ (k,j)\in L}} (1-e_L(j,k,t))Q(j,k,t), \quad \forall k \in SRC \setminus IN \tag{3.35}$$

3.2.4 Water Allocation Problems

3.2.4.1 Generalized Multiperiod Network Flow Programming

Water allocation constitutes a supply-demand water resources system planning process. Supply, demand, water rights, and allocation methods are the major topics in a water allocation. The river basin network model and associated constraints presented above describe water supply and hydrological relationships among the supplies, demands and river system. In the following, definitions for water demand and water rights are provided, and water allocation is summarized as a generalized network flow programming problem for which various specific water allocation problems may be formulated.

Definition 3.1 For $j \in U \setminus RES$, the *water demand* is the target amount of total inflow ($Q_D(j,t)$) into the demand node from its withdrawal ($Q(k_1,k,t)$) and inflow adjustment ($Q_g(k,t)$) to satisfy its need; for $j \in RES$, the *water demand* is the target storage ($S_D(j,t)$).

Demands may be set according to historical diversions, or projected demands estimated by empirical functions or more complex models (such as the IWPM model developed in Chapter 5 which generates estimation of irrigation water demands). Some example estimations are given below:

$$
\begin{aligned}
&Q_D(j,t) = \sum_s a_s(j,t) q_s(j,t), \quad \forall j \in AGR \cup MI \\
&Q_D(j,t) = \frac{P_D(j,t)}{\sigma\eta\Delta H(j,t)}, \quad \forall j \in HPP \\
&Q_D(j,t) = Q_{minireq}(j,t), \quad \forall j \in SFR \\
&S_D(j,t) = S_{target}(j,t), \quad \forall j \in RES
\end{aligned}
\tag{3.36}
$$

where, $a_s(j,t)$ and $q_s(j,t)$ are the production activity level and water use rate of sector s within an *AGR* or *MI* node *j*, respectively; $P_D(j,t)$ is the power demand from *HPP j*, σ is power generation coefficient, η is the generation efficiency, and $\Delta H(j,t)$ is the water head of *HPP j*; $Q_{minireq}(k,j,t)$ is the minimum stream flow required to maintain normal activity or ecology quality; $S_{target}(j,t)$ is the target water storage of reservoir *j* to meet current and future needs. Note, for $j \in HPP_{riv}$ or $j \in HPP_{res}$ with open tunnels, $\Delta H(j,t)$ is nearly fixed over all of the time periods. For other $j \in HPP_{res}$ with a variable water head, the $\Delta H(j,t)$ is dependent on the elevation difference between the storage surface of linked reservoir *k* and the tail-water level, $\Delta H(j,t) = H(S(k,t)) - H_{tw}$, $k \in RES, j \in HPP_{res}$. In the model, η and H_{tw} are assumed to be constant or insignificant by leaving out dependencies of machine efficiency on head differences and dependencies of tail water levels on discharges. To simplify the estimation of $Q_D(j,t)$ for the HPP_{res} nodes, a target $\Delta H(j,t)$ is firstly approximately estimated based on historical operational data and subsequently this $\Delta H(j,t)$ value is used to estimate $Q_D(j,t)$.

Definition 3.2 For $j \in U$, the *withdrawal* (or *diversion*) *demand* is the water requirement $Q_D(k,j,t)$ for the diversion or routing flow through the link (*k*, *j*) to satisfy the need of a demand site *j*.

Generally, the actual water demand is not equal the sum of all its withdrawal demands, because there are diversion losses and additional gains from rainfall or return flows. In reality, $Q_D(k,j,t)$ is normally set according to historical or empirical data.

Traditional definitions for water rights only consider water quantity. However, water has characteristics of both quantity and quality. A better definition of water rights allocated to each stakeholder under certain hydrologic conditions should be in terms of the water quantity and quality for all inflows, storages and return flows from his or her water use nodes within each specific period t.

Definition 3.3 For $j \in U \setminus RES$, the *water rights* are a set of volume and pollutant concentration limits for all inflows and outflows, $\{Q_R(k_1, j, t)$, $C_{pR}(k_1, j, t)$, $Q_R(j, k_2, t)$, $C_{pR}(j, k_2, t)\}$, where $(k_1, j) \in L$, and $(j, k_2) \in L$, $t \in T$. For $j \in RES$, the *water rights* are a set of reservoir storage and pollutant concentration limits, $\{S_R(j,t)$, $C_{pR}(j,t)\}$.

Note that the subscript R means the corresponding variables are allocated as water rights.

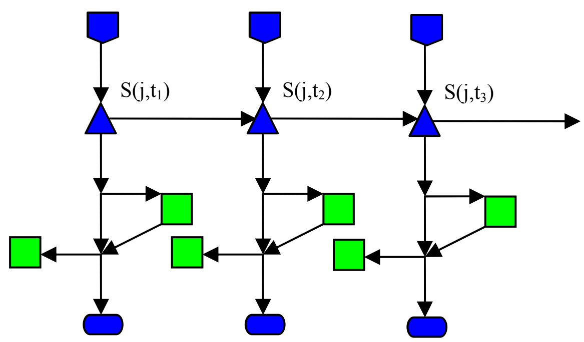

Figure 3.4 Multiperiod network configuration

In regional water resources planning, the river basin network can be represented in a multiperiod network configuration being connected by the reservoir carry-over storage links, as shown in Figure 3.4. Thus, we have the following definition.

Definition 3.4 Water allocation at the basin level is a *generalized multiperiod network flow* (*GMNF*) programming problem, which can be mathematically expressed as a multiple objective optimization problem:

$$
\begin{aligned}
&\max/\min \mathbf{f}(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X_s}) \\
&\textit{Subject to} \\
&\quad \mathbf{h}(\mathbf{Q},\mathbf{S},\mathbf{C}) = \mathbf{0} \\
&\quad \mathbf{h_s}(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X_s}) = \mathbf{0} \\
&\quad \mathbf{g}(\mathbf{Q},\mathbf{S},\mathbf{C}) \geq \mathbf{0} \\
&\quad \mathbf{g_s}(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X_s}) \geq \mathbf{0} \\
&\quad \mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X_s} \geq \mathbf{0}
\end{aligned} \tag{3.37}
$$

where, $\mathbf{f}(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X_s})$ is a vector of multiple objectives, $\mathbf{f} = (f_1, f_2, \cdots, f_m)$; **Q**, **S** and **C** are the vectors of network flow variables $Q(k_1, k, t)$, $S(k, t)$, $C(k_1, k, t)$ and $C(k, t)$; X_s is the vector of non-network type decision variables (side variables), which may be water prices, water transport costs, pollution control costs, crop types, irrigation areas, product prices, *etc*.; $\mathbf{h}(\mathbf{Q},\mathbf{S},\mathbf{C}) = \mathbf{0}$ and $\mathbf{g}(\mathbf{Q},\mathbf{S},\mathbf{C}) \geq \mathbf{0}$ represent the equality and non-equality constraints for network type variables **Q, S** and **C**, respectively; $\mathbf{h_s}(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X_s}) = \mathbf{0}$ and $\mathbf{g_s}(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X_s}) \geq \mathbf{0}$ represent the equality and non-equality constraints for both network type decision variables **Q**, **S** and **C** and non-network type decision variables $\mathbf{X_s}$.

The general water allocation is a highly nonlinear program, because it includes many constraints for the nonlinear water quantity and quality relationships. If the pollutant associated variables and constraints are ignored, it is converted into a program with water quantity constraints only. Further linear simplification for the reservoir area- and elevation–storage curves will convert it into a linear program.

Let a vector **x** represent all of the control variables, Ω denote the feasible set defined by the constraints in Problem GMNF, and $\mathbf{a} = (\mathbf{a_j}: j \in U)$ be the vector of the attribute types of the demand sites, where the type of each demand site is defined as a vector of attributes, $\mathbf{a_j} = (a_{j1}, a_{j2}, \cdots, a_{jn})$. Then the water allocation problem can be viewed in a more generic form as:

$$
\mathrm{F}(\mathbf{x} \in \Omega, \mathbf{a}) \tag{3.38}
$$

where F is the allocation criterion or allocation method. Various forms for F can be found in literature. Simulation and optimization models have been developed for water supply-demand planning and management (Wurbs, 1993). In typical formulations a larger value of the outcome means a better effect (higher service quality or client satisfaction). Otherwise, the outcomes can be replaced with their complements such as, shortage ratios. Therefore, without loss of generality, we can assume that each individual outcome is to be maximized, so F is rewritten as the following generic multiple objective optimization problem:

$$\max\left[\mathbf{f}(\mathbf{x}) : \mathbf{x} \in \Omega\right] \tag{3.39}$$

where $\mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots f_m(\mathbf{x})\right)$, $f_j(\mathbf{x})$ is the jth objective function, $j \in I = \{1, 2, \cdots, m\}$.

Some common types of objectives include: satisfy existing or projected water demands, minimize the difference in water deficits among all demand sites, maximize the flow to downstream river nodes, maximize economic production, minimize the concentration of salts in the system, and minimize water diverted from other basins (McKinney and Karimov, 1997). When satisfying water demands, there are multiple objectives associated with a number of water uses, each competing for maximum water withdrawal.

While the weighted sum method, also known as the parametric approach, is the most common method used for solving multiobjective problems, there exist numerous methods, such as the lexicographic approach, goal programming, genetic or evolutionary algorithms, and tabu search algorithms (Marler and Arora, 2004). It is well known that the set of solutions (Pareto frontier) of a multiple objective optimization problem have Pareto optimality.

Definition 3.5 A solution $\mathbf{x}^* \in \Omega$ is said to be *Pareto optimal* (noninferior) for problem $\max\left[\mathbf{f}(\mathbf{x}) : \mathbf{x} \in \Omega\right]$ if and only if there exists no $\mathbf{x} \in \Omega$ such that $f_j(\mathbf{x}) \geq f_j(\mathbf{x}^*)$ for $j = 1, 2, \cdots, m$, with strict inequality holding for at least one j.

3.2.4.2 Noncooperative and Cooperative Water Allocation

Let NB_{ijt} be the net benefit of stakeholder i's demand node j during period t. Then

$$NB_{ijt} = f_{ijt}(Q(k_1, j, t), C_p(k_1, j, t), S(j, t), C_p(j, t), Q(j, k_2, t), C_p(j, k_2, t)),\ (k_1, j) \in L, (j, k_2) \in L \tag{3.40}$$

The net benefit function of demand node j, $f_{ijt}(\cdot)$, is determined by $f_{ijt}(\cdot) = B_{ijt}(\cdot) - C_{ijt}(\cdot)$. The $B_{ijt}(\cdot)$ and $C_{ijt}(\cdot)$ are the benefit function and cost function for demand node j, respectively. The $f_{ijt}(\cdot)$ can be estimated from statistics or obtained through optimization models with control variables in water use such as use type, area, user's technology and skill level, price, and other economic and policy factors (Booker and Young, 1994; Diaz *et al.*, 1997).

The total net benefit NB_i of stakeholder i is the sum of the net benefits of all owned uses during all time periods such that

$$NB_i = \sum_{t \in T} NB_{it} = \sum_{t \in T} \sum_{j \in U} NB_{ijt} \tag{3.41}$$

where, $NB_{it} = \sum_{j \in U} NB_{ijt}$ is the net benefit of stakeholder i during period t.

If net benefits of stakeholders are taken as the multiple objectives, then the water allocation problem pursuing maximum net benefits may be expressed as:

$$\max\left[\mathbf{NB}(\mathbf{x}) : \mathbf{x} \in \Omega\right] \tag{3.42}$$

where $\mathbf{NB}(\mathbf{x}) = \left(NB_1(\mathbf{x}), NB_2(\mathbf{x}), \cdots NB_n(\mathbf{x})\right)$, and stakeholder $i \in N = \{1, 2, \cdots, n\}$.

Definition 3.6 A water allocation $\mathbf{x}^* \in \Omega$ is *economic efficient* (Pareto optimal) for problem $\max\left[\mathbf{NB}(\mathbf{x}) : \mathbf{x} \in \Omega\right]$, if and only if there exists no $\mathbf{x} \in \Omega$ such that $NB_i \geq NB_i^*$ for $i \in N$, with strict inequality holding for at least one stakeholder i.

Definition 3.7 *Noncooperative water allocation* is an n-person noncooperative game $\Gamma = \left\langle \bar{Q}_1, \cdots, \bar{Q}_n; NB_1, \cdots, NB_n \right\rangle$, where $\bar{Q}_i$ is the strategy set of player i subject to constraints $\mathbf{x} \in \Omega$, $NB_i(\mathbf{x})$ is the vector of net benefit functions of players (stakeholders), and $i \in N = \{1, 2, \cdots, n\}$ represents stakeholder i.

Note, the hydrologic flow scheme ($\mathbf{x}$) and net benefit for the player i ($NB_i(\mathbf{x})$) depends on the strategies of all other players as well as on his or her own strategy. Let $\bar{Q} = \prod_{i=1}^{n} \bar{Q}_i$ be the set of strategies of all players, $q_i \in \bar{Q}_i$ be the strategy of player *i*, and $q_{-i} \in \bar{Q}_{-i}$ be the strategies of other player. Then, the equilibrium of the noncooperative water allocation game can be defined as:

Definition 3.8 A vector $q^* \in \bar{Q}$ is a *Nash equilibrium* if for every stakeholder $i \in N$ and $q_i \in \bar{Q}_i$, $NB_i(q^*) \geq NB_i(q_i, q_{-i}^*)$, *i.e.* player *i* has no incentive to deviate from q^* when other players play q_{-i}^*.

A Nash equilibrium q^* for noncooperative water allocation game may not necessarily be Pareto optimal, and a Pareto optimal solution $\mathbf{x}^*$ may not be a Nash equilibrium. However, even Pareto optimal solutions normally have total net benefits (i.e. social welfare) less than the maximum that may be obtained by the cooperation of all players. This structural inefficiency of the noncooperative equilibrium is interpreted as an incentive to promote stakeholders to cooperate in order to gain maximum social welfare.

The above analysis shows why cooperative water allocation is more attractive to produce more social welfare, but a key issue, the fairness, still needs to be dealt with carefully in order to have cooperation. The question arises as to how to assure that the demands and rights of all stakeholders are fairly treated while pursuing more social welfare? In the next two sections, a two-step procedure is designed for equitable and efficient cooperative water allocation.

3.3 Initial Allocation of Water Rights

At the stage of initial water allocation, water rights is the main focus and the objective is to satisfy water demands of all uses subject to priorities, existing water rights systems, water management agreements and policies, and economic factors. The control variables of the problem are water flows, storage and pollutant concentrations, while other factors are set as fixed and are treated as the attributes of demands.

Based on a river basin network, three methods are formulated: priority-based maximal multiperiod network flow (PMMNF) programming, modified riparian water rights allocation

(MRWRA), and lexicographic minimax water shortage ratios (LMWSR). Each of the three methods may be formulated as linear or nonlinear problems according to whether nonlinear hydrologic relations and water quality constraints are included or not. Wang *et al*. (2004) and Fang *et al*. (2005) present brief descriptions the linear programming version of the LMWSR and PMMNF methods, respectively, in conference proceedings. The discussions in their papers focus on intra-country water rights allocation. However, the proposed methods are also applicable to transboundary water rights allocation depending on inter-country rights systems and agreements among countries.

3.3.1 Priority-based Maximal Multiperiod Network Flow Programming Method

The priority-based maximal multiperiod network flow programming (PMMNF) method is devised for water allocation under a prior water rights allocation regime. Prior allocation is based on the *Doctrine of Prior Appropriation*, which requires that water is firstly delivered to senior water rights holders. Junior uses are allocated after senior uses have been satisfied as fully as possible subject to hydrologic constraints. A prior water rights system is developed and used in drought regions, such as most western US states and western Canada. Priority is normally assigned to uses according to "first in time, first in right", and according to the functional importance of water uses in some cases (Savenije and Van der Zaag, 2000).

3.3.1.1 Reservoir Operation Rule Curves, Hedging Rule and Water Rights

Reservoir operation rules are usually established at the planning stage of the proposed reservoir, and provide guidelines for reservoir releases to meet various demands. Figure 3.5 shows the three rule curves, firm, target and flood control storage curves, associated with a typical multipurpose reservoir, which divide the live storage of a reservoir into four zones from the bottom to top: conservation, reduced operating, normal operating and flood control zones. When reservoir storage falls in the conservation zone during a drought period, water is released only to downstream minimum instream flow demands; when storage lies within the reduced operating zone, the downstream offstream demands cannot be fully met and are subject to reduction; when storage falls in the normal operating zone, all downstream offstream demands are fully satisfied; the flood control zone serves as storage buffer to diminish the impacts of high floods. These rules dealing with the trade-offs between meeting ongoing demands and maintaining adequate reservoir storage when inflow is insufficient are called hedging rules (Hsu and Cheng, 2002).

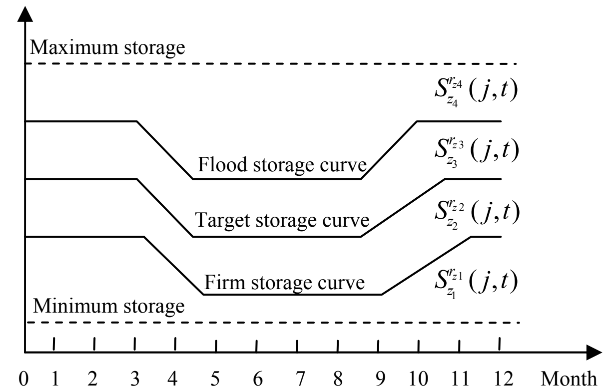

Figure 3.5 Reservoir operation rule curves (Hsu and Cheng, 2002)

In general, a reservoir can be divided into *m* subzones and each has a storage demand $S_{D,z}^{r}(j,t)$ with priority *r*. The priority ranks are positive integers, and the smaller the value the higher is the priority. Let $S_z^{r_z}(j,t)$ be the storage right of the subzone *z* of reservoir *j*, then $0 \le S_z^r(j,t) \le S_{D,z}^r(j,t),\ z = z_1,\ z_2,\ \cdots,\ z_m$, and $S(j,t) = \sum_{z=z_1}^{z_m} S_z^r(j,t)$.

Let $Q_{D,z}^r(k,j',t)$ $(z = z_1,\ z_2,\ \cdots,\ z_n)$ be withdrawal demands of an offstream demand or instream flow requirement node j' through link (k,j'), with corresponding priorities $r_{z1}(k,j',t)$, $r_{z2}(k,j',t)$, $\cdots$ and $r_{zn}(k,j',t)$, respectively. The $Q_z^r(k,j',t)$ denotes the allocated diversion right, then, $0 \le Q_z^r(k,j',t) \le Q_{D,z}^r(k,j',t)$, and $Q(k,j',t) = \sum_{z=z_1}^{z_n} Q_z^r(k,j',t)$.

3.3.1.2 The Priority-based Multiperiod Maximal Network Flow (PMMNF) Method

Considering the difficulty in assigning proper unit cost coefficients for generalized network flow programming, we propose a maximal network flow programming method for multiperiod prior water allocation.

The basic ideas of the method are:

(1) Every inflow link to a demand node may have one or more withdrawal demands with various priorities. Thus, in a prior water allocation method, each inflow link to a demand node is viewed as to consist of one or several dummy sublinks and each sublink has a withdrawal demand and corresponding priority (refer to Figure 3.6).

(2) If more than one sublinks with the same supply priority are connected to an identical source node and no water demand limits exist, flows are allocated simultaneously in proportion to their withdrawal demands in every time period.

(3) Each inflow link to a stream flow requirement node ($j \in SFR$), is separated into a bypass sublink in addition to sublinks for stream flow requirements with various priorities. Note that priority ranks are assigned to all sublinks except for bypass sublinks (refer to Figure 3.6c).

(4) The storage of every reservoir is divided into several subzones according to reservoir operating rules. Each sub-zone has a storage and corresponding priority (refer to Figure 3.6d).

(5) Water is allocated to meet inflow and storage demands according to priorities.

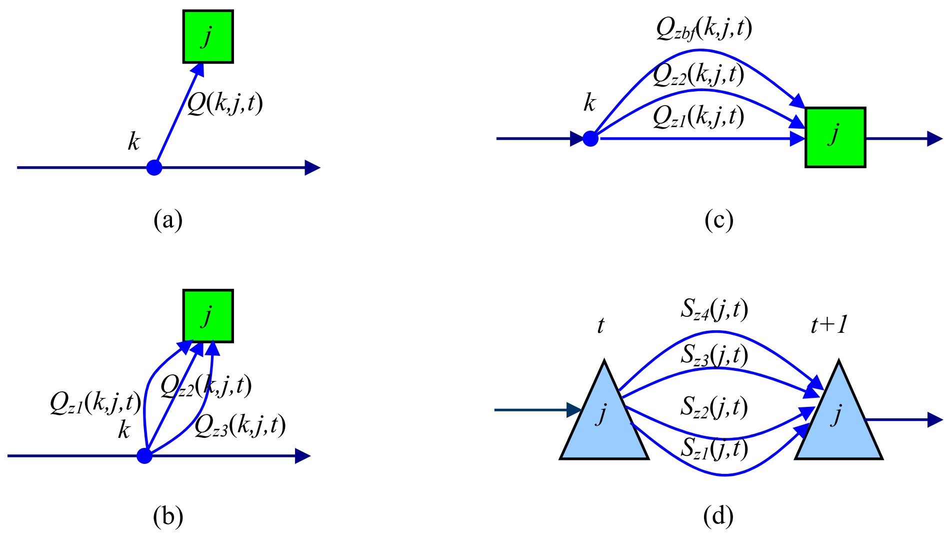

Figure 3.6 Sublinks and subzones

(a) Single water-right flow sublink, (b) Multiple water-right flow sublinks, (c) Water-right and by-pass flow sublinks to a stream flow demand node, (d) Reservoir carry-over storage sublinks.

PMMNF is formulated as the following problem with multiple ordered objectives:

$$
\begin{aligned}
&\max\left[\mathbf{f}^{(m)}(\mathbf{x})\right] \\
&\textit{subject to} \\
&\quad \mathbf{h}(\mathbf{Q},\mathbf{S},\mathbf{C}) = \mathbf{0} \\
&\quad \mathbf{g}(\mathbf{Q},\mathbf{S},\mathbf{C}) \geq \mathbf{0} \\
&\quad Q(k,j,t) = \sum_{z=z_1}^{z_n} Q_z^r(k,j,t),\ \forall j \in U \\
&\quad S(j,t) = \sum_{z=z_1}^{z_m} S_z^r(j,t),\ \forall j \in RES \\
&\quad Q_z^r(k,j_1,t)/Q_z^r(k,j_2,t) = Q_{D,z}^r(k,j_1,t)/Q_{D,z}^r(k,j_2,t), \\
&\qquad\qquad \forall j_1, j_2 \in U,\ \ j_1 \neq j_2,\ Q_D(j_1,t) = \varnothing,\ Q_D(j_2,t) = \varnothing \\
&\quad \sum_{t\in T}\sum_{j\in U}\left[S_z^r(j,t) + \left(1 - e_L(k,j,t)\right)Q_z^r(k,j,t)\right] = f_r(\mathbf{x}),\ \ \forall j \in U,\ \forall r \in PR \\
&\quad 0 \leq Q_z^r(k,j,t) \leq Q_{D,z}^r(k,j,t),\ \ \forall (k,j) \in L,\ Q_{D,z}^r(k,j,t) > 0 \\
&\quad 0 \leq S_z^r(j,t) \leq S_{D,z}^r(k,j,t)\ ,\ \ \forall j \in RES \\
&\quad \mathbf{Q},\mathbf{S},\mathbf{C} \geq \mathbf{0}
\end{aligned}
\tag{3.43}
$$

where, $\mathbf{f}^{(m)}(\mathbf{x}) = \left(f_{r_1}(\mathbf{x}), f_{r_2}(\mathbf{x}), \cdots, f_{r_m}(\mathbf{x}) \right)$, r is the priority assigned to a reservoir subzone or inflow sublink, and *PR* is the set of priority ranks whose elements are ordered from the highest to lowest, $r \in PR = \{r_1, r_2, \cdots, r_m\}$. Note that the priority ranks are positive integers, and the smaller the value the higher is the priority. The $S_z^r(j,t)$ and $S_{D,z}^r(j,t)$ are the storage variable and storage demand of the subzone *z* of reservoir *j* with a priority of *r*, respectively; the $Q_z^r(k,j,t)$ and $Q_{D,z}^r(k,j,t)$ are the inflow variable and withdrawal demand of the sublink *z* of link *(k, j)* with a priority of *r*, respectively. The first two constraints are the general water quantity and quality constraints for every node of a river basin network. The third constraint indicates each link flow to a demand is the sum of allocated flows to its sublinks with various priorities. The fourth constraint means that the reservoir storage volume is the sum of all allocated storages for subzones with various priorities. The fifth constraint makes sure that water is allocated proportionally in every time period to withdrawal demands with the same priority and directly connected to the same supply node, if no inflow demand is set for both demand nodes. The sixth constraint defines the objective function $f_r(\mathbf{x})$ as the total value of the storages and effective inflow volumes for all demands with the priority rank *r*. The next two constraints set the lower and upper bounds for all sublink flows and subzone storages which have various priorities. The last constraint defines positive network flow, storage and pollutant concentration vectors. Note that, for each nonstorage demand site, either the inflow demand ($Q_D(j,t)$) or sublink withdrawal demands ($Q_{D,z}^r(k,j,t)$) or both should be input to the model.

If a vector **x** is used to represent all of the control variables and Ω is utilized to denote the feasible set defined by the constraints in the PMMNF problem, the problem can be expressed in a more compact form as:

$$\max\left[\mathbf{f}^{(m)}(\mathbf{x}) : \mathbf{x} \in \Omega\right] \tag{3.44}$$

3.3.2 Modified Riparian Water Rights Allocation Method

Riparian rights systems are mainly found in humid regions such as the eastern US and Canada, where water is abundant and water allocation does not cause major problems for water users (Savenije and Van der Zaag, 2000). Under a traditional riparian system, the water at a node is distributed to water demands immediately connected to it proportionally at every period as fully as possible in an order from upstream to downstream. Local priorities may also be set for all demands connected to the same node (DHI, 2001). A traditional riparian water allocation problem can be converted into a special prior water allocation problem, if higher priorities are strictly assigned to upstream nodes and lower priorities to downstream nodes.

The traditional riparian water rights allocation system works well when water is abundant, but poorly in terms of fairness during water shortage times, because the downstream uses may receive little water while upstream demands are satisfied as fully as possible. In order to assure reasonable water uses and no extreme harm to downstream uses, the minimum demands of all uses in a river basin should be met as far as possible. So the traditional riparian allocation method described above is modified to a fairer method called modified riparian water rights allocation (MRWRA).

The MRWRA method can be viewed as a special form of PMMNF and thus has the same mathematical formulation and algorithm. The difference lies in the criteria to assign the priority ranks. In MRWRA the higher priority ranks are assigned to the group of minimum water demands $Q_{D,\min}(k,j,t)$ and $S_{D,\min}(j,t)$, and lower priority ranks are assigned to the group of surplus water demands $Q_{D,sur}(k,j,t)$ and $S_{D,sur}(j,t)$ in a river basin. The surplus water demands are the differences between corresponding minimum and maximum water demands, $Q_{D,sur}(k,j,t) = Q_{D,\max}(k,j,t) - Q_{D,\min}(k,j,t)$, and $S_{D,sur}(j,t) = S_{D,\max}(j,t) - S_{D,\min}(j,t)$. In each group of priority ranks, an upstream use is assigned a higher priority than a downstream use.

3.3.3 Lexicographic Minimax Water Shortage Ratios Method

The public allocation regime treats water as a public property, and the state is the owner of waters. In this system, water rights are administratively allocated to users through water permits from governments. As water demands increase and begin to compete for available water supplies during times of water scarcity, the priorities to get water are usually assigned by governmental authorities according to the importance of uses. For such cases, the PMMNF method can be applied.

Another possible approach for allocating equitable initial water rights is to have water allocated among all demands in the sense that no shortage ratio can be decreased further without either violating a constraint or increasing an already equal or worse-off shortage ratio value that is associated with another demand. This equitable water sharing can be formulated as a lexicographic minimax multiperiod resource allocation problem (Wang *et al.*, 2003b).

The lexicographic minimax solution concept is always Pareto-optimal to a multiple objective problem and simultaneously satisfies equity principles (Luss, 1999). As the standard minimax solution concept considers only the largest outcomes and minimizes them, it is criticized from the Pareto optimality viewpoint as well as from the perspective of inequity minimization. Besides minimization of the largest outcomes, the lexicographic minimax concept sequentially minimizes the second largest outcomes (provided that the largest one remains as small as possible), the third largest (provided that the two largest remains as small as possible), and so on. The lexicographic minimax water shortage ratios (LMWSR) approach is formulated based on lexicographic order and lexicographic minimax concepts defined as follows.

Definition 3.9 Given real vectors $\mathbf{\alpha}=(\alpha_1, \alpha_2,\cdots,\alpha_m)$ and $\mathbf{\beta}=(\beta_1, \beta_2,\cdots,\beta_m)$, vector $\boldsymbol{\alpha}$ is called *lexicographically smaller* than vector $\boldsymbol{\beta}$ (denotes as $\boldsymbol{\alpha} <_{\mathrm{L}} \boldsymbol{\beta}$), if there is an integer k, $1 \le k \le m$, such that $\alpha_l = \beta_l$ for $1 \le l < k$, and $\alpha_k < \beta_k$.

The above definition introduces the strong lexicographic inequality relation $<_{\mathrm{L}}$. The weak lexicographic inequality is then defined with the relation $\boldsymbol{\alpha} \le_{\mathrm{L}} \boldsymbol{\beta}$, if either $\boldsymbol{\alpha} <_{\mathrm{L}} \boldsymbol{\beta}$ or $\boldsymbol{\alpha} = \boldsymbol{\beta}$. Let $\mathbf{\theta}:R^m \rightarrow R^m$ be the map which orders the coordinates of vectors in a nonincreasing order, i.e., $\mathbf{\theta}(\mathbf{y}) = (\theta_1(\mathbf{y}), \theta_2(\mathbf{y}), \cdots, \theta_m(\mathbf{y}))$, if $\theta_1(\mathbf{y}) \ge \theta_2(\mathbf{y}) \ge \cdots \ge \theta_m(\mathbf{y})$, $\mathbf{y} = (y_1, y_2, \cdots, y_m)$, and there exists a permutation π of set I such that $\theta_i(y) = y_{\pi(i)}$ for all $i \in I = \{1, 2, \cdots, m\}$.

Definition 3.10 A feasible allocation $\mathbf{x}\in\Omega$ is the *lexicographic minimax* solution if the nonincreasing ordered outcome vector $\boldsymbol{\theta}(\mathbf{f}(\mathbf{x}))$ is lexicographically smaller than or equal to any other feasible allocation, i.e., $\boldsymbol{\theta}\left(f_1(\mathbf{x}^*), f_2(\mathbf{x}^*),,\cdots,f_m(\mathbf{x}^*)\right) \leq_L \boldsymbol{\theta}\left(f_1(\mathbf{x}), f_2(\mathbf{x}),,\cdots,f_m(\mathbf{x})\right)$, for all $\mathbf{x}\in\Omega$.

That means, the lexicographic minimax solution is an optimal solution of the following lexicographic problem

$$\operatorname{lex}\min\left[\boldsymbol{\theta}\left(\mathbf{f}(\mathbf{x})\right):\mathbf{x}\in\Omega\right] \tag{3.45}$$

Now, let $R(j,t)$ be the water shortage ratio of demand site j,

$$R(j,t)=\begin{cases} \dfrac{S_D(j,t)-S(j,t)}{S_D(j,t)}, \quad \forall j\in RES \\ \dfrac{Q_D(j,t)-\left(\sum\limits_{(k,j)\in L}(1-e_L(k,j,t))Q(k,j,t)+Q_g(j,t)\right)}{Q_D(j,t)}, \\ \qquad\qquad \forall j\in U,\ \forall j\notin RES \end{cases} \tag{3.46}$$

then the equitable allocation of water storage, diversion and routing water rights may be obtained through the following the minimax water shortage ratios programming,

$$\operatorname{lex}\min\left[\mathbf{f}^{(\mu\tau)}(\mathbf{x}):\mathbf{x}\in\Omega\right] \tag{3.47}$$

where, μ is the number of uses, $\mu=|U|$; $\mathbf{f}^{(u\tau)}(\mathbf{x})$ is the vector of $\mu\tau$ elements $f_{jt}(\mathbf{x})$,where these elements are sorted in a nonincreasing order; $f_{jt}(\mathbf{x})=\omega(j,t)\cdot R(j,t)$ is the performance function of demand node j during period t, $\forall j\in U, \forall t\in T$; the $\omega(j,t)$ is the weight for corresponding water shortage ratio.

The solution algorithms for PMMNF and LMWSR methods and equity concepts lying within them are discussed in Sections 4.2, 4.3, and 4.5, respectively. It is argued that the initial water rights allocated by the proposed PMMNF method are priorly equitable, and those allocated by LMWSR are perfectly equitable. Note that the proposed methods may also be applied to international basins.

3.4 Reallocation of Water and Net Benefits

Stakeholders are able to gain additional benefits if they cooperate to reallocate water properly based on their water rights. Theoretically, two major groups of ways are open to cooperation: (1) The cooperation involves both water quotas exchange and side payments, i.e. direct income transfers. (2) The cooperation is restricted to exchange of water quotas only, and the distribution of income is determined solely by water transfers. Because water exchange generally causes the water giver to lose money, and the receiver to gain money, the second type of cooperative water allocation without side payments cannot provide incentive for a water user to participate in the cooperation. Thus, the first type of cooperative water allocation mechanism is adopted in the model.

3.4.1 Water and Net benefits Reallocation Game with Side Payments

Recall that $N = \{1,2,\cdots,n\}$ is the set of water stakeholders or players competing for water in the concerned river basin or sub-watershed, and $i \in N$ a typical stakeholder. A group of stakeholders $S \subseteq N$ entering a cooperative agreement and working together is called a coalition. N itself is called the grand coalition, the coalition consisting of all stakeholders. A coalition structure is a partition $\{S_1, S_2, \cdots, S_m\}$ of the n stakeholders, in which $\bigcup_1^m S_i = N$ and for all $i \neq j$, $S_i \cap S_j = \emptyset$. For a game with n players, 2^n coalitions are possible, or 2^n-1 if the null coalition is excluded. The expression $\upsilon(S)$ is used to represent the aggregate value gained by the members of coalition S, while the values of individual stakeholders acting in isolation are represented as $\upsilon(\{1\})$, $\upsilon(\{2\})$, $\cdots$, $\upsilon(\{n\})$.

Definition 3.11 Reallocation of water and net benefits through cooperation of all stakeholders is an n-person *cooperative game* (N, υ), where N is the set of players, and υ is the characteristic function relating each coalition $S \subseteq N$ to a real number $\upsilon(S)$, representing the total value (net benefit) which S is able to generate through internal cooperation, with the convention that $\upsilon(\emptyset) = 0$.

Definition 3.12 The *value* $\upsilon(S)$ generated by the coalition *S* is the maximum total net benefit that coalition *S* can gain based on coalition members' water rights over the entire planning period, subject to not decreasing the water flows and not increasing the pollutant concentrations in the flows to other stakeholders not taking part in coalition *S*.

$$
\begin{aligned}
&\upsilon(S) = \max NB(S) = \max\left(\sum_{i\in S}\sum_{j\in U_i}\sum_{t\in T} NB_{ijt}\right) \\
&\textit{subject to:} \\
&\quad \mathbf{x}\in\Omega \\
&\quad \sum_{(k,j)\in L} Q(k,j,t) \le \sum_{(k,j)\in L} Q_R(k,j,t),\ \forall j\in AGR\cup MI\cup HPP \textit{ and } j\in U_S \\
&\quad Q(k,j,t) \ge Q_R(k,j,t),\ \forall j\in U\setminus RES \textit{ and } j\notin U_S \\
&\quad S(j,t) \ge S_R(j,t), \forall j\in RES \textit{ and } j\notin U_S \\
&\quad C_p(k,j,t) \le C_{pR}(k,j,t),\ \forall j\in U\setminus RES \textit{ and } j\notin U_S \\
&\quad C_p(j,t) \le C_{pR}(j,t),\ \forall j\in RES \textit{ and } j\notin U_S
\end{aligned}
\tag{3.48}
$$

where, $NB(S)$ is the net benefit of coalition *S*, and $U_S = \bigcup_{i\in S} U_i$ is the set of water demand nodes of coalition *S*. The first inequality constraint ensures that the total diversion to all off-stream demand sites and hydropower plants involved in a coalition (or water trade) does not exceed the total of their initial rights of diversion. To protect the water rights of water uses not participating in the coalition *S*, the next two inequalities set lower bounds of the water allocations and the last two inequalities set the upper bounds of pollution concentrations to corresponding initial water rights, respectively. Under more cooperative environment or regulated policies, the pollution constraints in above definitions may be ignored.

A "solution" to a game is a vector of the payoffs received by each stakeholder. This pay-off or reward vector after a trade can be written as $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$. The payoff vector is called an imputation to the cooperative game. The central stability concept in cooperative game theory is the core, $C(N, \upsilon)$, defined as follows (Young *et al.*, 1982; Tisdell and Harrison, 1992).

Definition 3.13 For a cooperative game (N, υ), a payoff vector $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$ is in the *core* $C(N, \upsilon)$, if

$$\sum_{i \in N} x_i = \upsilon(N) \tag{3.49}$$

$$\sum_{i \in S} x_i \geq \upsilon(S), \textit{ for all } S \subset N \tag{3.50}$$

The first equality condition of this definition ensures that the payoff vector is feasible (the so-called joint efficiency condition) for the grand coalition *N*. The second inequality condition introduces a stability requirement which states that no coalition *S* by acting on its own can achieve an aggregate value higher than the share that it receives under the payoff vector. For coalitions with multiple players, it is called group rationality condition; for singleton coalitions which consist of only one player, it is called individual rationality requirement.

Therefore, once an allocation from the core has been selected, no coalition on its own can improve the payoff of all its members. However, the core of a cooperative game may be empty. If it exists, there is no guarantee that it has a unique feasible solution.

3.4.2 Solution Concepts

Many fair allocation concepts have been proposed based on cooperative game theory to find a unique payoff vector, which can be classified into two categories, core-based and noncore-based concepts (Owen, 1995). Perhaps the best known are the nucleolus (core-based) and the Shapley value (noncore-based).

3.4.2.1 Nucleolus and Related Solutions

The nucleolus is the reward vector for which excesses for all coalitions are as small as possible, an excess being the amount by which the worth of a coalition exceeds the aggregate payoff to its members in isolation. If we let $x(S) = \sum_{i \in S} x_i$, and $x(N) = \sum_{i \in N} x_i$, then the excess of *S* with respect to $\boldsymbol{x}$ is $e(S, \boldsymbol{x}) = \upsilon(S) - x(S) = \upsilon(S) - \sum_{i \in S} x_i$. The excess $e(S, \boldsymbol{x})$ is a measure of the inequity (dissatisfaction) of an imputation $\boldsymbol{x}$ for a coalition *S*, which can be interpreted as a tax or punishment to force the subcoalition stakeholders to participate the grand coalition when $e(S, \boldsymbol{x}) > 0$, and as subsidy to encourage participation in the grand coalition when

$e(S, \boldsymbol{x}) < 0$. The nucleolus minimizes the maximum excess of any coalition S lexicographically (Schmeidler, 1969), which is defined as follows.

For a given n-persona game (N, υ) and payoff vector $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$, define $\mathbf{O}(\boldsymbol{x})$ as the 2^n-vector of excesses arranged in nonincreasing order, i.e., $\mathbf{O}(\boldsymbol{x}) = (O_1(\boldsymbol{x}), O_2(\boldsymbol{x}), \cdots, O_{2^n}(\boldsymbol{x}))$, if $O_k(\boldsymbol{x}) \geq O_{k+1}(\boldsymbol{x})$, and $O_k(\boldsymbol{x}) = e(S_{\pi(k)}, \boldsymbol{x})$ for all $k \in \{1, 2, \cdots, 2^n\}$, where $S_1, S_2, \cdots, S_{2^n}$ are the subsets of N, and π is a permutation of $\{k\}$.

Definition 3.14 Let (N, υ) be the n-persona game, and let $X = \left\{\boldsymbol{x} : \sum_{i=1}^{n} x_i = \upsilon(N)\right\}$ be the set of n-vectors (payoff vectors). Then, the *nucleolus* of (N, υ) over the set X is the set $v(X)$ defined by $v(X) = \left\{\boldsymbol{x} \mid \boldsymbol{x} \in X, \text{ and } \mathbf{O}(\boldsymbol{x}) \leq_{\mathrm{L}} \mathbf{O}(\boldsymbol{y}) \text{ for all } \boldsymbol{y} \in X\right\}$.

The main properties of the nucleolus are stated without proof in the following theorem. See the book of Owen (1995) for a discussion.

Theorem 3.1 The nucleolus $v(X)$ of a cooperative game exists and consists of a unique point. The nucleolus satisfies the joint efficiency and individual rationality conditions. If the core is not empty, the nucleolus is in the core.

Thus, the nucleolus is the optimal solution of the following lexicographic problem

$$\operatorname{lex}\min\left[\mathbf{O}(\boldsymbol{x}) : x(N) = \upsilon(N)\right] \tag{3.51}$$

The solution approach for the lexicographic minimax problem is to iteratively solve a series of minimax programs as the following formulation,

$$\begin{aligned}
&\min\ e \\
&\textit{subject to}: \\
&\qquad x(N) = \upsilon(N) \\
&\qquad e(S, \boldsymbol{x}) \leq e, \quad \forall\ S \in \mathcal{N} \setminus E \\
&\qquad e(S, \boldsymbol{x}) \leq e^*(S, \boldsymbol{x}), \quad \forall S \in E
\end{aligned} \tag{3.52}$$

where, $\mathcal{N}=\{S_1,S_2,\cdots,S_{2^n}\}$ are the set of all possible coalitions, $\mathcal{N}\setminus E$ is the set of coalitions for which the corresponding upper bounds of $e(S,\boldsymbol{x})$ are not fixed, and, on the contrary, E is the set of coalitions for which the corresponding upper bounds of $e(S,\boldsymbol{x})$ are fixed to their optimal values $e^*(S,\boldsymbol{x})$ found in previous solution loops. The algorithm starts with an empty set E, and once a minimax problem is solved, constraints $e(S^*,\boldsymbol{x})=e^*(S,\boldsymbol{x})$ are identified and the corresponding coalitions are removed from the set $\mathcal{N}\setminus E$. At subsequent iterations, the upper bounds of these $e(S,\boldsymbol{x})$ are set to their optimal values. Each iteration further narrows the solution space until a unique point is ultimately reached.

Variations of the nucleolus are obtained by changing the definition of the excess function as shown in Table 3.1, while the optimization algorithm remains the same. The weak nucleolus concept (Young *et al.*, 1982) replaces the excess with the average excess where $|S|$ is the cardinality of coalition S; proportional nucleolus (Young *et al.*, 1982) replaces the excess with the ratio of excess to net benefit of coalition S; normalized nucleolus (Lejano and Davos, 1995) replaces the excess with the ratio of excess to imputation of coalition S. The nucleolus and related variation approaches can reduce or expand the core to obtain a unique solution in both cases of empty core and large core (Dinar *et al.*, 1986).

Table 3.1 Various nucleolus solution concepts and coalition excesses

Solution concept	Coalition excess
Nucleolus	$e(S,x)=\upsilon(S)-x(S)$
Weak Nucleolus	$e_w(S,\boldsymbol{x})=(\upsilon(S)-x(S))/\lvert S\rvert$
Proportional Nucleolus	$e_p(S,\boldsymbol{x})=(\upsilon(S)-x(S))/\upsilon(S)$
Normalized Nucleolus	$e_n(S,\boldsymbol{x})=(\upsilon(S)-x(S))/x(S)$

3.4.2.2 Shapley Value

With the Shapley value solution concept, each stakeholder's reward or value to the game, should equal a weighted average of the contributions the stakeholder makes to each coalition of which he or she is a member. The weighting depends on the number of total stakeholders and the number of stakeholders in each coalition.

Theorem 3.2 There exists a unique payoff vector $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$ satisfying the Shapley axioms, of which the payoff to ith stakeholder is given by the Shapley value (Shapley, 1971),

$$x_i = \sum_{\substack{S \subseteq N \\ i \in S}} \frac{(|S|-1)!(|N|-|S|)!}{|N|!} \left[\upsilon(S) - \upsilon(S - \{i\}) \right], \ (i \in N) \tag{3.53}$$

where, $|S|$ and $|N|$ are the cardinalities of coalition S and the grand coalition N, respectively. The proof of the above theorem is given in the book by Owen (1995). The Shapley value shown above assumes equal probability for the formulation of any coalition of the same size. This assumption has been relaxed and generalized by Loehman *et al.* (1979). The generalized Shapley allocation to stakeholder i is given by

$$x_i = \sum_{\substack{S \subseteq N \\ i \in S}} P(S, S - \{i\}) \cdot \left[\upsilon(S) - \upsilon(S - \{i\}) \right] \tag{3.54}$$

where $P(S, S - \{i\}) = P(S | S - \{i\}) \cdot P(S - \{i\})$ is the probability of stakeholder i joining the coalition S. Note that the Shapley value is not always in the core. It may fail to be in the core even if the core is not empty.

3.4.2.3 Value of participation in the grand coalition and side payment

Once the payoff vector is obtained, the values for players to participate in the grand coalition and associated side payments can be estimated.

Definition 3.15 For stakeholder i, the *value* (or *gain*) *of participation* in the grand coalition equals $-e(\{i\}, \boldsymbol{x}) = x(i) - \upsilon(\{i\})$. The *side payment* to other stakeholders is the difference of the value gained by participating in the grand coalition and allocated payoff, which is equal to $NB(i) - x(i)$. Note, a negative side payment means receiving payment from others.

3.5 An Illustrative Example

Suppose there are three stakeholders: Irrigation Water Association (IWA), City 1 and City 2, along a river as shown in the node-link river basin network in Figure 3.7. The IWA has two crop areas located upstream. The return flow coefficients of both crop areas are 20%. The return flow coefficients from both cities are 90%. The minimum demands from the Crop 1, Crop 2, City 1, and City 2 are 40, 50, 20 and 25 mcm/yr, respectively, while the maximum demands are 100, 120, 40 and 50 mcm/yr, respectively. The statistical functions of net benefits and salinity in return flows given in Table 3.2 are determined according to experience published in the literature (Booker and Young, 1994). Now we develop the water allocation plan for a hypothetical 5-year drought period with a series of annual upstream flows, $Q(1,2,t)$, having values 280, 260, 240, 240 and 260 mcm and corresponding average salinity concentrations $C_p(1,2,t)$ of 400, 410, 420, 430, and 410 mg/l, respectively. The problem is modeled as a 5-year plan with an annual period. Inflow adjustments and evaporation losses are not considered in the following analysis.

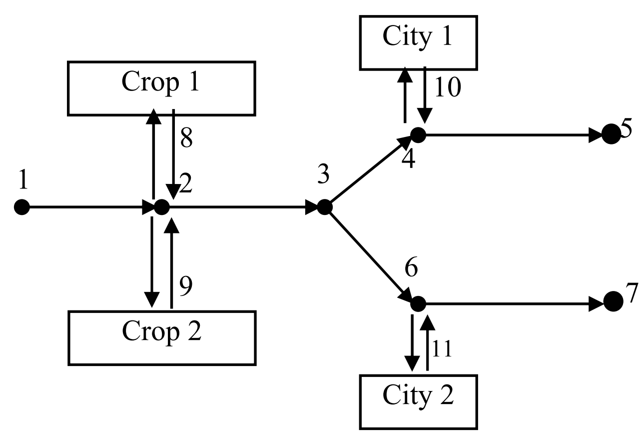

Figure 3.7 Flow network and water uses

Table 3.2 Net benefit and return flow salinity functions

Stake-holder	Water use	Net benefit function NB_{jt} (10^3\$)	Return flow salinity $Z(j, k, t)$ (10^6 kg)
1. IWA	Crop 1	$-1000 + 60Q(2,8,t) - 0.2Q(2,8,t)^2$	$0.3Q(2,8,t) - 0.0008Q(2,8,t)^2$
	Crop 2	$-1100 + 60Q(2,9,t) - 0.2Q(2,9,t)^2$	$0.3Q(2,9,t) - 0.0008Q(2,9,t)^2$
2. City 1	City 1	$700Q(4,10,t) - 0.3Q(4,10,t)^2 - 0.25Q(4,10,t) \times \max\left(C_p(4,10,t) - 400,\ 0\right)$	$2.5Q(4,10,t) - 0.0008Q(4,10,t)^2$
3. City 2	City 2	$680Q(6,11,t) - 0.3Q(6,11,t)^2 - 0.25Q(6,11,t) \times \max\left(C_p(6,11,t) - 400,\ 0\right)$	$2.5Q(6,11,t) - 0.0008Q(6,11,t)^2$

*Units of flow and salinity are *mcm* and *mg/l*, respectively.

3.5.1 Initial Water Rights and Optimal Economic Allocation

Three initial allocation methods are employed in the analysis. The maximal network flow programming method is utilized for prior allocation by assigning both of the two cities priority 1 and both crops priority 2. The weighting factors are set equal for all of the four water uses in the lexicographic minimax demand shortage ratios method. The results show that, except for the first year, there are water shortages for the four water uses. Since the prior water rights system gives higher priorities to the downstream cities, but the riparian system allocates water according to natural flow directions, cities get more water under the prior allocation than modified riparian allocation, while crops get more water in the modified riparian allocation. Under the lexicographic minimax demand shortage ratios allocation method, the four uses share the water shortage with an equal ratio of 10% in years 3 and 4, and 2% in years 2 and 5. Under the grand coalition of economic optimal allocation, crops reduce their water diversions, and cities get more water than the initial allocated water rights, as shown in Figure 3.8. As an example, Table 3.3 shows the water flows allocated using a modified dual-priority riparian water rights allocation method.

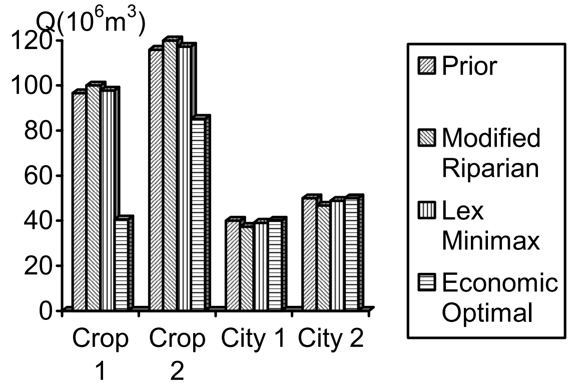

Figure 3.8 Initial allocation by different methods for year 2

Table 3.3 Upstream inflows and initial water rights allocations based on initial water rights allocated by the modified riparian water rights allocation (MRWRA) method

Period	Year 1	Year 2	Year 3	Year 4	Year 5
$Q(1,2,t)$ (mcm)	280.00	260.00	240.00	240.00	260.00
$C_p(1,2,t)$ (mg/l)	400.00	410.00	420.00	430.00	410.00
$Q(2,8,t)$ (mcm)	100.00	100.00	100.00	100.00	100.00
$C_p(2,8,t)$ (mg/l)	400.00	410.00	420.00	430.00	410.00
$Q(2,9,t)$ (mcm)	120.00	120.00	120.00	120.00	120.00
$C_p(2,9,t)$ (mg/l)	400.00	410.00	420.00	430.00	410.00
$Q(4,10,t)$ (mcm)	40.00	37.33	28.44	28.44	37.33
$C_p(4,10,t)$ (mg/l)	677.69	748.57	857.50	860.63	748.57
$Q(6,11,t)$ (mcm)	50.00	46.67	35.56	35.56	46.67
$C_p(6,11,t)$ (mg/l)	677.69	748.57	857.50	860.63	748.57
$Q(4,5,t)$ (mcm)	42.22	33.60	25.60	25.60	33.60
$C_p(4,5,t)$ (mg/l)	2437.98	2744.59	2752.49	2752.49	2744.59
$Q(6,7,t)$ (mcm)	52.78	42.00	32.00	32.00	42.00
$C_p(6,7,t)$ (mg/l)	2430.40	2736.30	2746.17	2746.17	2736.30
$NB_{1t}(10^6\$)$	6.22	6.22	6.22	6.22	6.22
$NB_{2t}(10^6\$)$	24.74	22.46	16.42	16.39	22.46
$NB_{3t}(10^6\$)$	29.78	27.01	19.73	19.70	27.01
Total Net Benefit ($10^6\$$)	60.74	55.70	42.37	42.32	55.70

3.5.2 Coalition, Water Transfers and Benefits

Based on the initially allocated water rights, salinity concentrations and net benefits of individual stakeholders, the water flows are allocated to gain maximum net benefit $v(S)$ for individual stakeholders and every possible coalition S. Table 3.4 lists values of allocated water flows and corresponding salinity concentrations and net benefits in every period (year here) under the grand coalition based on initial water rights allocated by MRWRA, and Table 3.5 summarizes the overall net benefits during the 5-year period for each independent stake-holders and coalitions under the situations of initial water rights allocation, optimization within stakeholders and various coalitions.

Table 3.4 Optimal water allocations and values under the grand coalition

Period	Year 1	Year 2	Year 3	Year 4	Year 5
$Q(1,2,t)$ (mcm)	280.00	260.00	240.00	240.00	260.00
$C_p(1,2,t)$ (mg/l)	400.00	410.00	420.00	430.00	410.00
$Q(2,8,t)$ (mcm)	40.94	40.58	40.34	40.34	40.48
$C_p(2,8,t)$ (mg/l)	400.00	410.00	420.00	430.00	410.00
$Q(2,9,t)$ (mcm)	94.91	85.22	74.21	75.23	85.29
$C_p(2,9,t)$ (mg/l)	400.00	410.00	420.00	430.00	410.00
$Q(4,10,t)$ (mcm)	40.00	40.00	40.00	40.00	40.00
$C_p(4,10,t)$ (mg/l)	524.55	537.36	548.30	558.12	537.28
$Q(6,11,t)$ (mcm)	50.00	50.00	50.00	50.00	50.00
$C_p(6,11,t)$ (mg/l)	524.55	537.36	548.30	558.12	537.28
$Q(4, 5,t)$ (mcm)	53.56	50.47	48.38	47.56	50.46
$C_p(4,5,t)$ (mg/l)	2015.19	2110.08	2180.77	2211.21	2110.48
$Q(6,7,t)$ (mcm)	108.77	99.90	90.98	90.98	99.93
$C_p(6,7,t)$ (mg/l)	1438.39	1526.58	1629.05	1634.01	1526.17
$NB_{1t}(10^6\$)$	3.91	3.67	3.35	3.38	3.66
$NB_{2t}(10^6\$)$	26.27	26.15	26.04	25.94	26.15
$NB_{3t}(10^6\$)$	31.69	31.53	31.40	31.27	31.53
$\upsilon(1,2,3)_t(10^6\$)$	61.88	61.35	60.78	60.59	61.34
Total Net Benefit (10^6\$)	61.88	61.35	60.78	60.59	61.34

Table 3.5 Overall net benefits during the 5-year planning period based on initial water rights allocated by the modified riparian water rights allocation (MRWRA) method

Net Benefit (10^6\$)	Initial Allocation	Individual optimal	1,2 Coalition	1,3 Coalition	2,3 Coalition	Grand Coalition
NB_1	31.10	31.26	27.42	25.94	31.10	17.97
NB_2	102.47	102.47	125.06	108.73	117.55	130.54
NB_3	123.24	123.24	129.66	152.22	108.68	157.43
$\upsilon(1,2)$			152.48			
$\upsilon(1,3)$				178.17		
$\upsilon(2,3)$					226.22	
$\upsilon(1,2,3)$						305.94
Total	256.82	256.98	282.14	286.89	257.32	305.94

Figure 3.9 displays the flow allocation for year 2 under coalitions based on initial water rights allocated by the MRWRA method. Since irrigation has lower marginal net benefits than the cities, water is transferred from IWA to City 1 and City 2 when they form coalitions. For example, at the last points of abstracting water according to their water rights during the drought year 2, Crop 1 and Crop 2 have very low marginal net benefits given as 0.02 and 0.01 $\$/m^3$, respectively, while City 1 and City 2 have higher marginal net benefits calculated as 0.59 and 0.56 $\$/m^3$, respectively. Therefore, water is transferred from crops to cities when they form coalitions. However, not all water reduced by crops is taken by the cities. There is pollutant trading among the crops and cities. In the grand coalition situation, Crop 1, Crop 2, City 1 and City 2 have marginal net benefits 0.04, 0.03, 0.64 and 0.62 $\$/m^3$, respectively. The reason why the cities' marginal net benefits increase as the amount of water received increases is the water quality becomes better when Crop 1 and Crop 2 use less water as shown in Figure 3.10. For the same reason, although only an additional 2.67 and 3.33 mcm of water are received by City 1 and City 2 to obtain the maximum grand coalition net benefit, the amount of water received by Crop 1 and Crop 2 is reduced by 59.42 and 34.78 mcm, respectively. This implies that although the cities obtain a small additional volume of water to meet their maximum demands, their water quality is improved and they gain more benefit when the upstream crops reduce a relative larger volume of water, as depicted in Figure 3.11.

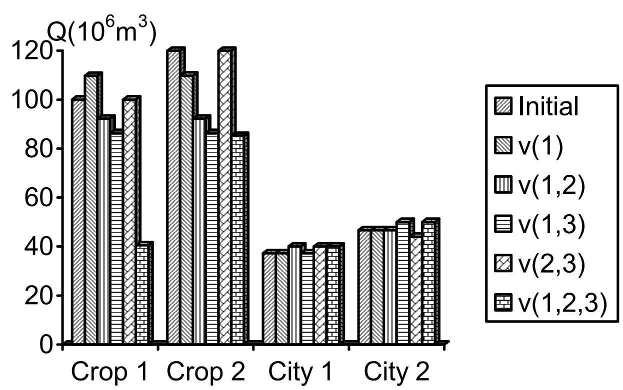

Figure 3.9 Flow allocation for year 2 under coalitions based on initial water rights allocated by the modified riparian water rights allocation (MRWRA) method

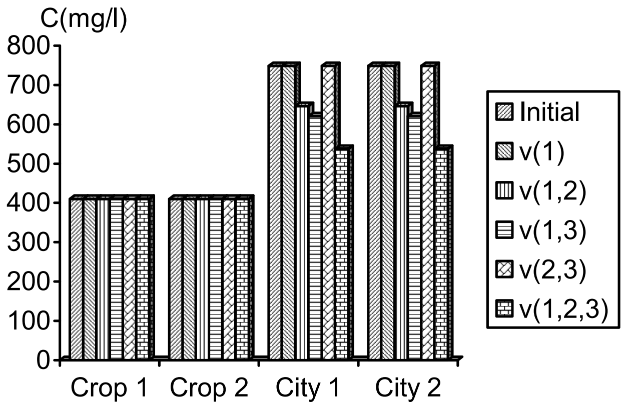

Figure 3.10 Pollutant limit allocation for year 2 under coalitions based on initial water rights allocated by the modified riparian water rights allocation (MRWRA) method

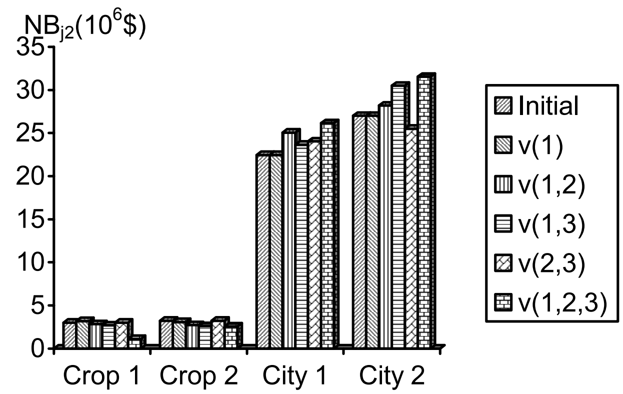

Figure 3.11 Net benefits in year 2 before reallocation based on initial water rights allocated by the modified riparian water rights allocation (MRWRA) method

3.5.3 Reallocation of the Grand Coalition Net Benefit

Figure 3.12 shows the set of all possible nonnegative allocations of the total net benefit ($305.94 million) of the grand coalition among competing stakeholders, but only the shaded area, the core, is the subset of allocations satisfying individual and group rationality.

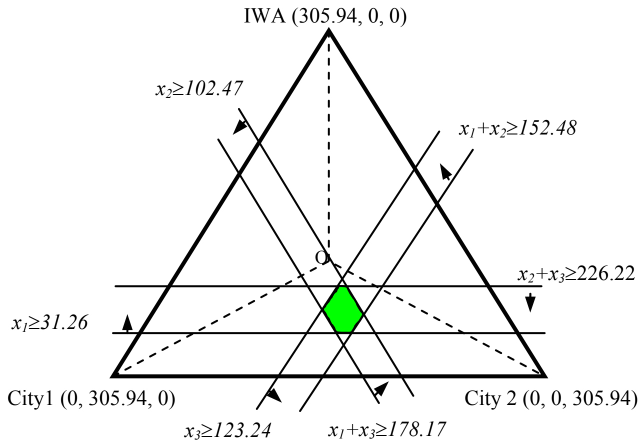

Figure 3.12 The core of a cooperative water allocation game based on initial water rights allocated by the modified riparian water rights allocation (MRWRA) method

Solving with the nucleolus and variation solution concepts and Shapley value, we get the overall 5-year and individual annual schedules of equitable and efficient allocation of net benefits in this cooperative water allocation project. The results based on the initial water rights allocated by MRWRA method are shown in Table 3.6. These Pareto optimal schedules provide the alternatives needed for further negotiation or reaching a final decision. Figure 3.13 shows the consequence of net benefit reallocation by the Shapley Value method based on initial water rights allocated by MRWRA: by participating in the grand coalition for the 5-year period, IWA decreases 482.48 mcm of total water diversion, but receives a side payment of $36.51 million from Cities 1 and 2, and gains an additional $23.22 million; City 1 gets 28.44 mcm more of total water diversion, but has to pay a side payment of $16.42 million to IWA, and gains an additional $11.64 million; City 2 gets 35.56 mcm more of total water diversion, but has to pay a side payment of $20.09 million to IWA, and gains an additional $14.10 million.

Table 3.6 Equitable allocations of the net benefits of the grand coalition based on the initial water rights allocated by the modified riparian water rights allocation (MRWRA) method (10^6\$)

Period	Stake-holder	Nucleolus	Weak Nucleolus	Proportion Nucleolus	Normalized Nucleolus	Shapley Value
Overall 5 Years	IWA	53.67	61.03	57.88	62.30	54.48
	City 1	113.41	109.77	110.36	110.91	114.12
	City 2	138.86	135.14	137.69	132.73	137.34
Year 1	IWA	10.86	12.35	11.71	12.60	11.02
	City 1	22.94	22.20	22.32	22.43	23.08
	City 2	28.09	27.33	27.85	26.85	27.78
Year 2	IWA	10.76	12.24	11.61	12.49	10.92
	City 1	22.74	22.01	22.13	22.24	22.88
	City 2	27.84	27.10	27.61	26.61	27.54
Year 3	IWA	10.66	12.13	11.50	12.38	10.82
	City 1	22.53	21.81	21.93	22.03	22.67
	City 2	27.59	26.85	27.35	26.37	27.29
Year 4	IWA	10.63	12.09	11.46	12.34	10.79
	City 1	22.46	21.74	21.86	21.96	22.60
	City 2	27.50	26.76	27.27	26.29	27.20
Year 5	IWA	10.76	12.24	11.61	12.49	10.92
	City 1	22.74	22.01	22.13	22.24	22.88
	City 2	27.84	27.10	27.61	26.61	27.54

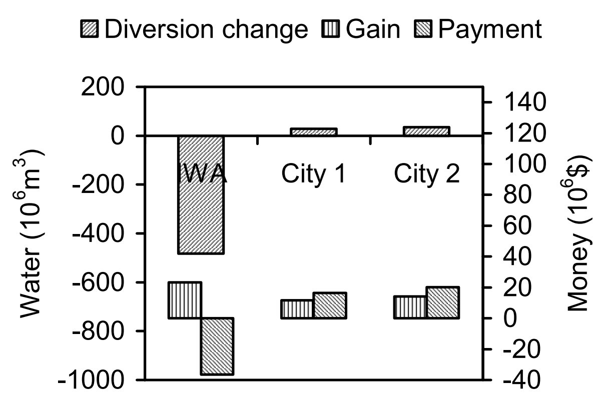

Figure 3.13 Changes of total water diversions, gains of participation in the grand coalition and side payments of stakeholders

3.6 Summary

The framework of the comprehensive Cooperative Water Allocation Model (CWAM) for equitable and efficient water allocation at the basin level is presented in this chapter. The model adopts a two-step allocation approach: initial water rights are firstly allocated to competing uses based on legal water rights systems or agreements, and then water and net benefits are fairly reallocated by hydrologic-economic modeling and cooperative game theoretical approaches. CWAM envisions a river basin and water uses as a node-link river basin network subject to hydrological, policy and system control constraints, and searches water distributions over multiple time periods (typical period is one month). The model integrates knowledge and sub-models from hydrology, economics and cooperative game theory.

The three initial water rights allocation methods are designed for application under different water rights regimes or agreements. PMMNF is a very flexible approach and is applicable under prior, riparian and public water rights systems. MRWRA is essentially a special form of PMMNF adapted for allocation under the riparian regime. LMWSR is designed for use under a public water rights system, which adopts the concept of lexicographic minimax fairness. Solution concepts such as the nucleolus, weak nucleolus, proportional nucleolus, normalized nucleolus and Shapley value are adopted to solve the cooperative water reallocation game. The primary illustrative application shows its potential to provide decision support to reach fair and efficient water allocation among competing uses with multiple stakeholders in an operational way.

CWAM may be applied to an entire river basin or a sub-watershed, although it is at the basin level that the goal to make equitable, efficient and sustainable water allocation plan may be best realized. The output of the model includes initial water rights, shadow prices of water at various demand sites, water allocation schemes, coalition values, payoffs to stakeholders, gains of participation in the grand coalition, and side payments. The model is designed to serve as a tool for simulating water trade, and promoting the equitable cooperation of stakeholders to achieve economic efficient use of water, under either the water market or administrative allocation mechanism. The optimal water allocations to individual uses at the local level (inside a stakeholder) may be implemented through a water market mechanism, or performed by the stakeholder.

The modeling framework is designed mainly for short-term water allocation planning and enforcement. Since the model design is based on a multiperiod river basin network structure, the spatial and temporal variability of water are accounted for in the water allocations. However, because the model itself is designed as a deterministic one, the effects of stochasticity or uncertainty of hydrologic events, water demands and other input parameters have to be evaluated through sensitivity analyses. A typical time step is one month, although other time steps can be employed. The use of smaller time intervals is restricted by computational capacity, data availability and the assumed quasi-static nature of the river basin network model and the hydrologic processes.

Chapter 4
Water Rights Allocation by Priority-based and Lexicographic Minimax Programming

4.1 Introduction

As stated in Chapter 3, the priority-based maximal multiperiod network flow (PMMNF) method is a very flexible method that can be applied under prior, riparian and public water rights regimes, and generates water rights allocation schemes while strictly preserving priority order. The modified riparian water rights allocation (MRWRA) method is actually a special case of PMMNF. The lexicographic minimax water shortage ratios (LMWSR) method is a method adopting the lexicographic minimax fairness concept, and can be applied for equitable water rights allocation under the public water rights regimes. Thus, the proposed three methods are essentially to solve two types of mathematical programming problems, PMMNF and LMWSR.

The purpose of Chapter 4 is to build the algorithms for PMMNF and LMWSR. A sequential programming approach is used to solve both the linear and nonlinear PMMNF problems, while the linear and nonlinear LMWSR problems are solved by iterative programming. For nonlinear problems, considering nonlinear water quality constraints is solved by a two-stage approach, which adopts a strategy of solving with good starting points and increasing the possibility to find approximate global optimal solutions of the nonlinear problems. The developed methods are applied to the Amu Darya river basin located in the central eastern Asia. Finally, the equity principles and concepts for generic resource allocation problems with multiple objectives are reviewed from a theoretical perspective and quantitatively, and the fairness concepts underlying PMMNF and LMWSR are discussed subsequently.

4.2 Algorithms for Priority-based Multiperiod Maximal Network Flow (PMMNF) Programming

4.2.1 Sequential Linear Algorithm for PMMNF

4.2.1.1 Problem PMMNF_QL

Consider the generic PMMNF problem $\max\left[\mathbf{f}^{(m)}(\mathbf{x}):\mathbf{x}\in\Omega\right]$. If only the water quantity constraints are included, and only linear surface area- and elevation-storage relationships are applied to reservoirs, the problem will be a linear program called PMMNF_QL.

4.2.1.2 Solution method

The prior water allocation can be accomplished by sequentially solving the following maximal network flow program for each priority rank r', from the highest to lowest priority:

$$\begin{aligned}
&\max f_{r'}(\mathbf{x}) \\
&\textit{subject to} \\
&\qquad \mathbf{x}\in\Omega \\
&\qquad f_r(\mathbf{x})\geq f_r^*, \forall r<r'
\end{aligned} \tag{4.1}$$

where r' is the priority rank that the current programming aims for, and f_r^* is the optimal value found for $f_r(\mathbf{x})$ in previous sequential solution.

The model PMMNF_QL is firstly formulated by defining the number of sets describing the configuration and input data for the concerned river basin network, priority of demands, equations of water quantity constraints, and objective functions. Then, the model defined in a network form is converted to a sequential series of standard linear programs, and solved by the *primal simplex method*. The algorithm is coded in GAMS, a general algebraic modeling system, for mathematical programming problems (Brooke *et al*., 1998), and solver MINOS is utilized for solving the linear problem. MINOS is a powerful solver for linear and nonlinear programming, and can efficiently deal with sparse constraints and large scale problems

(Murtagh *et al*., 2002). Without repeating again in the following contexts, all the algorithms are coded in GAMS and solved by MINOS.

For example, the river basin network in the Amu Darya case study, the node set of the network is defined as

```
*Define the river basin network
Set k Set of general nodes /
   I1*I9,
   J1*J14,
   Aq1*Aq9,
   R1*R6,
   Ag1*Ag12,
   H1,
   S1 /;
```

There are 9 inflow nodes, namely I1, I2, ···, I9; 14 junction nodes namely J1, J2, ···, J14; 9 aquifers namely Aq1, Aq2, ···, Aq9; 6 reservoirs R1, R2, ···, R6; 12 agricultural demand sites Ag1, Ag2, ···, Ag12; one hydropower plant H1 and one stream flow requirement node S1 defined in the general node set.

Accordingly, the set of links is defined as follows, by using the symbol "." connecting the starting and ending nodes for each link.

```
Set link(k1, k) Directed links k1.k between nodes in general node set k
/ I1.J1, I1.Ag1, I2.R1, I2.Aq1, I3.J3, I4.Aq3, I4.Ag3, I4.J4, I5.Ag4, I5.R2, I6.Ag6,
I6.Aq5, I6.J8, I7.Aq6, I7.R3, I8.Aq6, I8.R3, I8.R4, I9.R4, I9.R5, I9.Aq8,
J1.Ag1, J1.J2, J2.J3, J3.J4, J4.J5, J5.J6, J5.Ag4, J6.J7, J7.Ag5, J7.J8, J8.Ag6, J8.J9,
J9.Ag7, J9.J10, J10.Ag8, J10.J11, J11.R6, J11.Ag9, J12.Ag10, J12.J13, J13.Ag11,
J13.J14, J14.Ag12, J14.S1,
Aq1.Ag2, Aq2.Ag1, Aq3.Ag3, Aq4.Ag4, Aq5.Ag6, Aq6.Ag7, Aq7.Ag9, Aq8.Ag8,
Aq9.Ag10, Aq9.Ag11, Aq9.Ag12,
Ag1.J1, Ag2.R1, Ag2.I2, Ag3.J4, Ag4.I5, Ag6.J8, Ag6.I6, Ag7.I8, Ag7.R6, Ag8.R4,
Ag8.R5, Ag8.R6, Ag9.J11, Ag10.J12, Ag11.J13, Ag12.J14,
R1.Ag2, R1.Aq1, R1.H1, R1.J2, R2.Ag4, R2.J6, R3.Aq6, R3.J9, R3.Ag7, R4.Ag8,
R4.J9, R5.Ag8, R5.J10, R6.J12,
H1.J2 /;
```

4.2.1.3 Steps of the Algorithm for PMMNF_QL

1. Define the feasible set Ω;
2. Initialize $r' = r_1$;
3. Solve problem PMMNF_QL, $\max\left[f_{r'}(\mathbf{x}) : \mathbf{x} \in \Omega, f_r(\mathbf{x}) \geq f_r^*, \forall r < r'\right]$, for priority r', and let f_r^* denote the optimal value of $f_r(\mathbf{x})$;
4. If $ord(r') \neq ord(r_m)$, set $r' = r_{ord(r')+1}$, and go to step 3; else stop.

Note that $ord(r')$ is the relative position of priority r' in the priority rank set $PR = \{r_1, r_2, \cdots, r_m\}$ listed in the order from the highest to lowest. The contents of step 1 include: defining the river basin network in terms of the general node set, node subsets, directed link set, node seepage link subset, and link seepage set; defining the set of time steps; inputting hydrological data including the inflows, loss coefficients, and parameters for the linear *A-S* and *H-S* relationships of reservoirs, node and link capacities, and policy constrained bounds; defining the priority set $\{r_1, r_2, \cdots, r_m\}$; defining the sets and priorities for single-right links, sublinks of multiple-right links, and reservoir subzones; inputting water demands for all single-right links, sublinks and reservoir subzones; defining control variables for link and sublink flows, and reservoir subzone storages; setting lower and upper bounds; and defining linear water quantity constraints; defining objective functions for each priority r.

The PMMNF_QL algorithm has several notable characteristics: (1) The river basin network is defined for multiple periods, and thus in times of water shortage water may be stored in reservoirs and aquifers for future uses with senior priority, regardless of the current demands of those with junior priority. (2) As the limiting case of a weighted sum multiobjective optimization problem, the algorithm allocates water to meet demands strictly according to priority ranks. Junior demands receive water only after senior demands are met as fully as possible subject to hydrological constraints. However, this does not mean that a junior demand always has a lower supply/demand satisfaction ratio than senior ones. Some demands may have higher satisfaction ratios than some demands with the same priority or even with senior priority, because they are instream uses which receive return flows from those senior demand sites or they have additional local water supply that is unavailable to those senior demands. (3) The algorithm is designed to be flexible. The demand constraints could be set by total-demand control, link flow control or both of them.

4.2.2 Two-Stage Sequential Nonlinear Algorithm for PMMNF

4.2.2.1 Nonlinear PMMNF Problems

Two types of nonlinear PMMNF problems are classified according to their characteristics. One is called PMMNP_QNL, in which only water quantity constraints are included, and nonlinear surface area- and elevation-storage relationships are applied to reservoirs. Another one is PMMNF_QC, which considers water quality constraints also, in addition to the constraints in PMMNF_QNL.

4.2.2.2 Solution Method

Despite advancements in computing technology in modeling and scientific calculation software, it is still a difficult task to solve large scale nonlinear optimization problems, especially in early stages of the modeling process when good starting points are unknown. Cai *et al*. (2001c) presented a method based on *generalized benders decomposition* (GBD) to search an approximate global solution for large-scale nonlinear water allocation problems with bilinear constraints (i.e. all nonlinear terms are products of two variables). However, for the GBD master problem to be tractable, the original model must have a special structure such that it must be possible to solve each subproblem and master problem globally. More generic global solution methods such as genetic algorithm have also been applied to large water resources management models, but the problem needs to have a special structure and the solution converges slower when the number of complicating variables increases. Furthermore, it may require a long computational time and cannot guarantee to converge to the global solution due to the stochastic natures of genetic algorithms (Cai *et al*., 2001b).

A simple but effective domain decomposition approach called the “two-stage” approach is utilized to solve the large-scale nonlinear water rights allocation problems with existing commercial NLP solvers. The approach adopts a strategy of solving nonlinear programs from good starting points, and is similar to Cai *et al*.’s (2001a) “piece-by-piece” approach. The difference is that the “two-stage” approach consists of only two stages and the problem in the second stage is not formed by simply adding another piece to the problem in the first stage. To simplify the explanation, consider the nonlinear PMMNF problem:

$$
\begin{aligned}
&\max f_{r'}(\mathbf{x}) \\
&\textit{subject to} \\
&\quad \mathbf{h}(\mathbf{Q},\mathbf{S},\mathbf{C}) = \mathbf{0} \\
&\quad \mathbf{g}(\mathbf{Q},\mathbf{S},\mathbf{C}) \geq \mathbf{0} \\
&\quad f_r(\mathbf{x}) \geq f_r^*, \forall r < r' \\
&\quad \mathbf{Q},\mathbf{S},\mathbf{C} \geq \mathbf{0}
\end{aligned}
\tag{4.2}
$$

The first stage of the "two-stage" approach firstly searches a corresponding simplified linear program PMMNF_QL,

$$
\begin{aligned}
&\max f_{r'}(\mathbf{x}) \\
&\textit{subject to} \\
&\quad \mathbf{h}'(\mathbf{Q},\mathbf{S}) = 0 \\
&\quad \mathbf{g}'(\mathbf{Q},\mathbf{S}) \geq 0 \\
&\quad f_r(\mathbf{x}) \geq f_r^*, \forall r < r' \\
&\quad \mathbf{Q},\mathbf{S} \geq 0
\end{aligned}
\tag{4.3}
$$

which considers only linear water quantity constraints and constant demands. The demands of hydropower plants with variable water heads are estimated according to some approximate water heads. Nonlinear items of area-storage relationships are ignored in the first stage. Let the solution be ($\mathbf{Q}^*$, $\mathbf{S}^*$). Set the initial value of **C** to the estimated initial values for $\mathbf{C}^*$, then ($\mathbf{Q}^*$, $\mathbf{S}^*$, $\mathbf{C}^*$) is used as the starting point for the first nonlinear program of the sequential series of PMMNF problems. Since ($\mathbf{Q}^*$, $\mathbf{S}^*$) is the global solution of PMMNF_QL, the ($\mathbf{Q}^*$, $\mathbf{S}^*$, $\mathbf{C}^*$) should be a good starting point near the final solution of the nonlinear PMMNF problem.

In the second stage, each original nonlinear PMMNF program is solved by the *projected Lagrangian method*. The solution process of each nonlinear problem involves a sequence of major iterations, each of which requires the solution of a projected linearly constrained subproblem by the *augmented Lagrangian Method* (Murtagh *et al*., 2002). During the second stage, if a nonlinear program encounters an infeasible solution, a relaxed problem with relaxed upper bounds on pollutant concentrations is formulated and solved, which generates a new better starting point for the original nonlinear program.

4.2.2.3 Steps of the nonlinear algorithm for PMMNF

1. Define the feasible set **Ω**;
2. Formulate and solve the corresponding linear problem PMMNF_QL, and denote the global optimal solution as ($\mathbf{Q}^*$, $\mathbf{S}^*$);
3. Initialize $r' = r_1$, and let ($\mathbf{Q}^*$, $\mathbf{S}^*$, $\mathbf{C}^*$) be the starting point;
4. Solve the nonlinear problem PMMNF_QNL or PMMNF_QC, and let f_r^* denote the optimal value of $f_r(\mathbf{x})$;
5. If $ord(r') \neq ord(r_m)$, set $r' = r_{ord(r')+1}$, and go to step 4; else stop.

Note that $ord(r')$ is the relative position of priority r' in the priority set $\{r_1, r_2, \cdots, r_m\}$. Although it cannot be guaranteed that the two-stage approach will find the global solution, the strategy to search for good starting points both in the first and second stages will enable the algorithm to find an approximate global optimal solution. This is evidenced in the Amu Darya case study.

4.3 Algorithms for Lexicographic Minimax Water Shortage Ratios (LMWSR) Method

4.3.1 Iterative Linear Algorithm for LMWSR

4.3.1.1 LMWSR_QL Problem

The generic LMWSR problem $\text{lexmin}\left[\mathbf{f}^{(\mu\tau)}(\mathbf{x}) : \mathbf{x} \in \Omega\right]$ is a water-quantity-only water rights allocation problem if only the water quantity constraints are included. Furthermore, if only linear area- and elevation-storage relationships are applied to reservoirs, the problem is a linear program called LMWSR_QL.

4.3.1.2 Solution Method

The solution of the lexicographic minimax programming is a refinement of the standard minimax concept. The idea for finding the lexicographic minimax solution is to sequentially identify all of the minimax solutions and to sort their achievement vectors in weakly decreasing order to identify the lexicographically minimal one. This approach is naive but it may lead to quite efficient procedures for many allocation problems.

The generic solution approach for the lexicographic minimax program is to repeatedly solve a series of minimax programs:

$$\begin{aligned}
&\min M \\
&\textit{subject to}: \\
&\quad \mathbf{x} \in \Omega \\
&\quad \omega(j,t)R(j,t) \le M, \quad \forall (j,t) \in NR \\
&\quad \omega(j,t)R(j,t) \le M_{jt}^{*}, \quad \forall (j,t) \in FR
\end{aligned} \tag{4.4}$$

where, M is a real variable; NR is the set containing index pairs of (j,t) for which the corresponding upper bounds of $\omega(j,t)R(j,t)$ are not fixed; and, on the contrary, FR is the set containing index pairs of (j,t) for which the corresponding upper bounds of $\omega(j,t)R(j,t)$ are fixed to their optimal values M_{jt}^{*} found in previous solution loops. The algorithm starts with an empty set FR, and once a minimax problem is solved, constraints $\omega(j^{*},t^{*})R(j^{*},t^{*}) = M^{*}$ are identified and the corresponding index pairs of (j, t) are removed from the set NR. At subsequent iterations, the upper bounds of these $\omega(j,t)R(j,t)$ are set to their optimal values. Iterations stop when the optimal values for all decision variables are identified.

4.3.1.3 Steps of the Algorithm for LMWSR_QL

The steps of the algorithm for linear LMWSR reads as follows:

1. Define the feasible set $\mathbf{\Omega}$;
2. Initialize $s = 0$, $\mathbf{\Omega}_0 = \mathbf{\Omega}$, and $NR_0 = (j, t)$. NR_0 is the set containing index pairs of j and t for which the upper bounds of corresponding $R(j,t)$ are not fixed;
3. For the current loop s, solve the problem P_s: $\min\limits_{x,M}\{M : \mathbf{x} \in \Omega_s,\ \textit{and}\ f_{jt}(\mathbf{x}) \le M\ \textit{for}\ \forall (j,t) \in NR_s,\ \forall t \in T\}$, where $f_{jt}(\mathbf{x}) = \omega(j,t)R(j,t)$. Let M_s^{*} denote the optimal value of P_s;
4. Identify $FR_s = \{(j, t) \in NR_s$: $\mathrm{abs}(f_{jt}(\mathbf{x}) - M_s^{*}) \le e_r \cdot \mathrm{abs}(M_s^{*})$ for all $(j, t) \in NR_s\}$ and put $NR_{s+1} = NR_s - FR_s$. Define $\mathbf{\Omega}_{s+1}=\{\mathbf{x} \in \mathbf{\Omega}_s$: $f_{jt}(\mathbf{x}) \le M_s^{*}$ for $(j, t) \in FR_s\}$;
5. If $NR_{s+1} = \emptyset$, then go to step 6. Otherwise increase s by 1 and return to step 3;
6. Stop. The final set $\mathbf{\Omega}_{s+1}$ is the last set of all the lexicographic minimax solutions.

where, e_r is a parameter representing permitted relative error of weighted water shortage $f_{jt}(\mathbf{x})$ to the maximum M_s^* at each iteration loop. Normally a number less than *0.001* is small enough. Note that the algorithm is well defined for linear problems, because Ω_s is a convex polyhedron at each iteration and a unique optimal value M_s^* can be found easily. Therefore, each index set FR_s is not empty. Moreover, while the algorithm is implemented by using the *primal simplex method*, sets FR_s can be easily identified, and the modifications of FR_s may be implemented by fixing the upper bounds of $f_{jt}(x)$, whose $(j, t) \in FR_s$.

4.3.2 Two-Stage Iterative Nonlinear Algorithm for LMWSR

4.3.2.1 Nonlinear LMWSR problems

Two types of nonlinear LMWSR problems are classified according to their characteristics. One is called LMWSR_QNL, in which only water quantity constraints are included, and nonlinear surface area- and elevation-storage relationships are used for reservoirs. Another one is LMWSR_QC, which considers water quality constraints also, in addition to the constraints in LMWSR_QNL.

4.3.2.2 Solution Methods

A "two-stage" approach similar to that for nonlinear PMMNF is utilized to solve the nonlinear LMWSR problems,

$$
\begin{aligned}
&\min M \\
&\textit{subject to} \\
&\qquad \mathbf{h}(\mathbf{Q},\mathbf{S},\mathbf{C}) = \mathbf{0} \\
&\qquad \mathbf{g}(\mathbf{Q},\mathbf{S},\mathbf{C}) \geq \mathbf{0} \\
&\qquad \omega(j,t)R(j,t) \leq M, \forall (j,t) \in NR \\
&\qquad \omega(j,t)R(j,t) \leq M^*, \forall (j,t) \in FR \\
&\qquad \mathbf{Q},\mathbf{S},\mathbf{C} \geq \mathbf{0}
\end{aligned}
$$

The first stage of the "two-stage" approach firstly searches a corresponding simplified linear program LMWSR_QL,

$$
\begin{aligned}
&\min M \\
&\textit{subject to} \\
&\quad \mathbf{h}'(\mathbf{Q},\mathbf{S}) = \mathbf{0} \\
&\quad \mathbf{g}'(\mathbf{Q},\mathbf{S}) \geq \mathbf{0} \\
&\quad \omega(j,t)R(j,t) \leq M, \forall (j,t) \in NR \\
&\quad \omega(j,t)R(j,t) \leq M^*, \forall (j,t) \in FR \\
&\quad \mathbf{Q},\mathbf{S} \geq 0
\end{aligned}
$$

which considers only linear water quantity constraints and constant demands. The demands of hydropower plants with variable water heads are estimated according some approximate water heads. Nonlinear items of area-storage relationships are ignored in the first stage.

4.3.2.3 Steps of the Nonlinear Algorithm for LMWSR

The steps of the algorithm for nonlinear LMWSR are as follows:

1. Define the feasible set $\mathbf{\Omega}$;
2. Formulate and solve the corresponding linear problem LMWSR_QL, and denote the global optimal solution as ($\mathbf{Q}^*$, $\mathbf{S}^*$);
3. Initialize $s = 0$, $\mathbf{\Omega}_0 = \mathbf{\Omega}$, and $NR_0 = (j, t)$. NR_0 is the set containing index pairs of j and t for which the upper bounds of corresponding $R(j,t)$ are not fixed. Let ($\mathbf{Q}^*$, $\mathbf{S}^*$, $\mathbf{C}^*$) be the starting point;
4. For the current loop s, solve the problem P_s: $\min_{x,M}\{M : \mathbf{x} \in \Omega_s,\ \textit{and}\ f_{jt}(\mathbf{x}) \leq M\ \textit{for}\ \forall (j,t) \in NR_s,\ \forall t \in T\}$, where $f_{jt}(\mathbf{x}) = \omega(j,t)R(j,t)$. Let M_s^* denote the optimal value of P_s;
5. Identify $FR_s = \{(j, t) \in NR_s$: $\text{abs}(f_{jt}(\mathbf{x}) - M_s^*) \leq e_r \cdot \text{abs}(M_s^*)$ for all $(j, t) \in NR_s\}$ and put $NR_{s+1} = NR_s - FR_s$. Define $\mathbf{\Omega}_{s+1} = \{\mathbf{x} \in \mathbf{\Omega}_s: f_{jt}(\mathbf{x}) \leq M_s^*$ for $(j, t) \in FR_s\}$;
6. If $NR_{s+1} = \emptyset$, then go to step 7. Otherwise increase s by 1 and return to step 4;
7. Stop. The final set $\mathbf{\Omega}_{s+1}$ is the last set of all the lexicographic minimax solutions.

Remarks: Each of the iterative algorithms for LMWSR problems produces a series of optimal values $\{M_1^*, M_2^*, \cdots, M_s^*, \cdots\}$ listed in decreasing order.

Proof. It is obvious that $M_{s+1}^* \neq M_s^*$ for any s. Assume $M_{s+1}^* > M_s^*$, at least one index pair (j,t) can be found by the iterative linear or two-stage algorithm such that $f(j,t) = M_{s+1}^* > M_s^*$, where $(j,\ t) \in NR_{s+1}$. However, M_s^* is the optimal solution of P_s,

$f(j,t) \le M_s^*$, where $(j,\ t) \in NR_s$. Because $NR_{s+1} \subset NR_s$, we get $f(j,t) \le M_s^*$, where $(j,\ t) \in NR_{s+1}$. This contradicts the assumption.

4.4 Case Study: Initial Allocation of Water Rights in the Amu Darya River Basin

4.4.1 Background

In order to achieve sustainable development and a secure society, institutions and methodologies for water allocation should be reformed for regions with water resources shortages, like the Aral Sea basin in Central Asia. The objective of this case study is to apply the PMMNF and LMWSR methods for water rights allocation to the Amu Darya river basin in the Aral Sea basin. The challenges for achieving a fair, efficient and sustainable water allocation are discussed.

The Aral Sea basin lies within northern Afghanistan and the five independent states of the former Soviet Union including the area of two southern oblasts (regions) of Kazakhstan, three oblasts of the Kyrgyz Republic, the whole territory of Tajikistan and Uzbekistan, and four velayats (regions) of Turkmenistan, as shown in Figure 4.1 (IFAS and UNEP, 2000).

The Aral Sea basin covers an area of about 1.9 million km^2 (UNFAO, 2005), having a sharply continental climate characterized by high evapotranspiration and severely arid conditions. Annual precipitation is less than 100 mm in the southwest deserts and about 200 mm approaching the foothills of the southeastern mountains (Raskin *et al*., 1992). The region provides favorable thermal conditions for the growth of cotton and other heat-loving crops: the noontime temperature during growing seasons (May-September) ranges from 20 to 45°C and the average daily temperature in July is 35°C (Raskin *et al*., 1992).

The Amu Darya (2,574 km, draining 1,327,000 km^2) and Syr Darya (2,337 km, draining 484,000 km^2) rivers, with an average total annual flow of 116.5 km^3, are the two major rivers of the region and supply the Aral Sea with bulk water. The population in the basin has grown from 13 million in 1960 to more than 40 million at present. Annual water diversions have increased from 60 to 105 km^3 and irrigated lands rose from 4.5 million ha to just over 8 million ha (McKinney, 2003b).

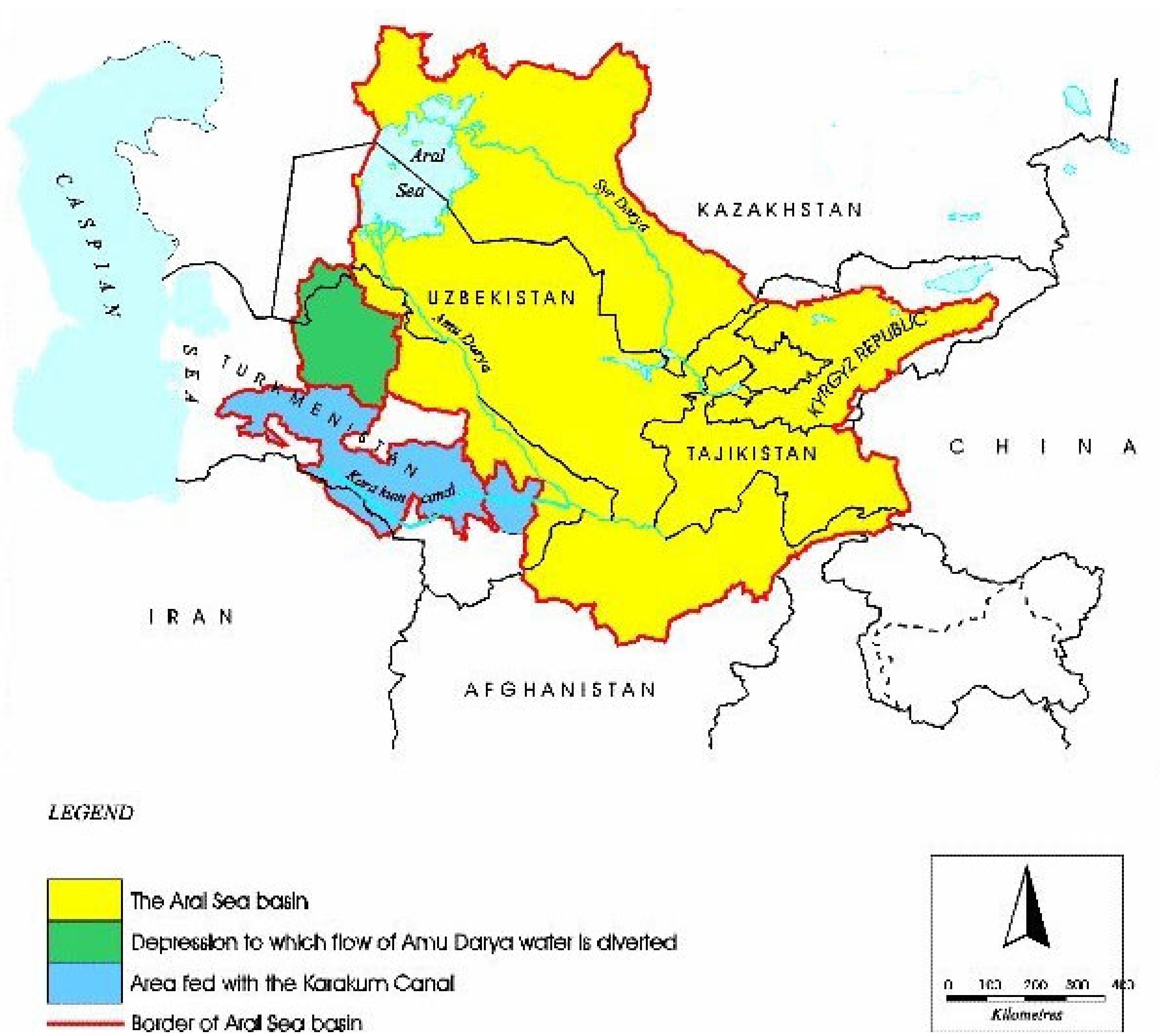

Figure 4.1 Central Asian states and Aral Sea basin

(Source: http://www.fao.org/ag/agl/aglw/aquastat/regions/fussr/fussrfig16.htm)

The Aral Sea was once the fourth largest lake in the world by area. However, it is now facing a serious ecological crisis. The causes for the disaster date back to the introduction of advanced irrigation techniques in the region in the mid-1920's and the collectivization in the

1930's by the then newly established Soviet government. Small-scale traditional irrigation systems were replaced by large state-run enterprises. The change led to destabilization of irrigation and lowered water use efficiency. Irrigation efficiencies were further decreased by expansion and mechanization of irrigation in the 1950's. During this period, the potential shrinkage of and damage to the Aral Sea were recognized. However, the large irrigation water diversions continued up to the 1990's (Micklin, 1992). As a result, the average annual flow to the Aral Sea dropped from over 50 km^3 before 1960 to 10 km^3 since 1975 (Gleick, 1993). Today, the Aral Sea is nearing half of its surface area and less than one-third of its volume existing in 1960.

The significant reduction of inflows to and the massive shrinkage of the Aral Sea created devastating ecological impacts, including:

- The salinity has been significantly increased.
- The fishing industry in the Aral Sea has greatly shrunk.
- There is a huge area (about 30,000 km^2) of salt on the former lake bed which is toxic to human beings and deleterious to crops. The salt can be whipped up by winds and carried over wide areas.
- Requirements for fresh water irrigation are increased to alleviate the affects from soil salinization and salt storm deposits.
- The moderating effect of the Aral Sea on local climate is reduced, resulting in hotter summers, colder winters, and a decreased growing season.
- The ecology of the river deltas has been seriously degraded as the river flow diminishes and surrounding water table falls along with the sea shrinkage.

4.4.2 Water Uses and Allocation in the Aral Sea Basin

According to an estimate for 1987 by Raskin *et al*. (1992), the total annual water demand in the Aral Sea region is 97.32 km^3, with 53.55 km^3 in the Amu Darya basin and 43.77 km^3 in the Syr Darya basin. The agricultural sector is the primary user of water resources, accounting for 82% of the total demand. The major crop is cotton, accounting for 51% of the agricultural water use in the Amu Darya basin and 34% in the Syr Darya basin. Fodder crops account for the second largest requirement, at 29% and 19% of the agricultural water demand in the Amu Darya and Syr Darya basins, respectively. Water-intensive rice production accounts for 19% and 12%, respectively, of the agricultural water demand. Industrial water uses are far less than agricultural requirements, accounting for approximately 6% in the Amu Darya basin and 12% in the Syr Darya basin. The municipal water demands are about 6% of the total demand. Water demands for livestock are quite small, while fisheries account for some 3% of the total demand (Raskin *et al*., 1992). During the 1940's to 1970's, several agreements were reached between the Soviet Union and Afghanistan regarding the waters of the Amu Darya river, allocating an annual supply of 9 km^3 to Afghanistan. However, no more than 2 km^3 per year has been diverted to Afghanistan so far (McKinney, 2003b).

According to the International Fund for the Aral Sea (IFAS) and United Nations Environment Program (UNEP) (IFAS and UNEP, 2000), the average long-term surface water supply in the Aral Sea basin is about 116 km^3, as shown in Table 4.1. This table also gives the source of the surface waters from the Aral Sea basin nations: Kazakhstan, Kyrgyz Republic, Tajikistan, Turkmenistan, Uzbekistan, Afghanistan, and Iran. The annual surface water supply in the Amu Darya river basin is about 80 km^3, about 8% of which is formed in Afghanistan. Only about 10% of the Amu Darya's surface water supply reaches the Aral Sea. In the Syr Darya river basin, the annual surface water supply is about 40 km^3. The Syr Darya river no longer reaches the Aral Sea. All of its discharge is now being used for irrigation and other purposes.

Table 4.1 Average annual long-term resources of surface water in the Aral Sea basin (km^3) (IFAS and UNEP, 2000)

State	Basin of the river		Total in the Aral Sea basin
	Syr Darya	Amu Darya	
Kazakhstan	2.426	0	2.426
Kyrgyz Republic	27.605	1.604	29.209
Tajikistan	1.005	59.898	60.903
Turkmenistan	0	1.549	1.549
Uzbekistan	6.167	4.736	10.903
Afghanistan and Iran	0	11.593	11.593
Total	37.203	79.28	116.483

Table 4.2 shows available ground water resources in the Aral Sea basin. As can be seen, except for Uzbekistan, the quantities available to use are less than one km^3 per year. A deficit of water resources is observed in the whole region except Kyrgyzstan and Tajikistan, caused by an arid climate and water losses especially in irrigation systems (IFAS and UNEP, 2000).

Table 4.2 Annual ground water resources in the Aral Sea basin (km^3) (IFAS and UNEP, 2000)

State	Evaluation of regional resources	Exploitational resources approved to use	Available to use
Kazakhstan	1.846	1.224	0.173
Kyrgyzstan	0.862	0.67	0.291
Tajikistan	6.65	2.2	0.975
Turkmenistan	3.36	1.22	0.42
Uzbekistan	18.455	7.796	6.959
Aral Sea basin	31.173	13.11	8.071
Syr Darya basin	16.42	7.428	4.76
Amu Darya basin	14.753	5.682	3.311

Intensive development of irrigation and drainage in the Aral Sea basin has had two major impacts on water quantity and quality in the rivers: a major freshwater uptake for irrigation, and generation of polluted return water of elevated salinity. The reduction of water quantity and increase of salinity of inflows caused the increase of salinity in Aral Sea. Today, an estimated 200,000 tones of salt and sand are being carried away by wind and discharged within a radius of 300 km every day (Shalpykova, 2002).

> "Fresh water of less than 1 g/l is present only in the upper catchments of the rivers and tributary streams and in the upper parts of the middle courses of major rivers. Further downstream all rivers and associated lakes receive return waters. From 1960 to 1995 water salinity in the lower Amu Darya increased from 0.5 to 1.2 g/l. Today water salinity near Termez city, which is situated between the upper and middle courses of the Amu-Darya, is about 0.3-0.5 g/l. Further downstream it rises to 1.2 g/l. During the last decades water salinities in rivers ranged from 0.5 to 2.0 g/l, in reservoirs from 0.5 to 2.5 g/l, and in lakes formed from residual waters and in natural lakes receiving such waters then were from 3 to 20 g/l." (Petr *et al.*, 2003)

The quality of water available to agriculture is as important as the quantity, and water with salinity over 1 mg/l is considered unsuitable for irrigating usual crops (Petr *et al*., 2003). Depending on the crop, crop production decreases with increasing salt concentration in water and soil. The same applies to freshwater fish, though less is known about harmful salinity levels.

Under the Soviet system, water allocation and conflict resolution in the basin were an intra-national issue and water use strategies were developed to maximize the perceived benefits to the entire region, in which the cost of environmental damage was assumed to be minimal (Micklin, 1991). Dramatic changes took place after the breakup of the Soviet Union in 1991. Each independent republic of the region has been struggling to realize its full potential since then. The political conditions in this region make water resources management very complex and international treaties have been negotiated to resolve conflicts over water resources.

The countries in the Aral Sea basin have recognized the need to develop fair and rational agreements for sharing and using their water and energy resources. Some progress has been made in achieving regional cooperation and attracting international financial and technical support for solving Aral Sea water problems, including:

- Creation of the Interstate Coordinating Water Commission (ICWC) in 1992,
- Establishment of the Interstate Fund for the Aral Sea in 1993 (IFAS and UNEP, 2000),
- Signing of the Syr Darya Water and Energy Resources Framework Agreement in 1998 (Antipova *et al*., 2002), and
- Initiation of various Aral Sea basin studies and model developments for decision support.

Studies of the Aral Sea problem have found that the Aral Sea cannot be restored to its former grandeur without totally disrupting the economies of the basin states (McKinney, 2003b). The main task facing the region is to preserve the Aral Sea's current size and ameliorate adverse impacts, which would require an inflow of 35 km^3/yr (Micklin, 1992). Another reality is that the present method of water allocation among the Aral Sea basin countries is still based on Soviet era rules, disregarding the emerging priorities of the independent republics. The present method is not suitable for interstate water allocation and thus inevitably creates resources management conflicts. Intrastate water allocation is also in a similar situation, leading to inefficient and wasteful delivery and use of water (McKinney, 2003a).

To achieve sustainable water use, the water allocation for competing uses in the Aral Sea basin should be carried out in an integrated and cooperative manner. Water resources policies in the Aral Sea basin need to be restructured. Equitable intrastate water rights systems, interstate agreements, and corresponding institutions must be developed. Proper decisions should be made on the introduction of water conservation techniques, water pricing, how to account for water demands from various types of uses, and how water should be apportioned among competing users.

Regional water resources management tools may assist the republics in the Aral Sea basin in making better decisions. Mathematical simulation (Raskin *et al*., 1992) and optimization (Antipova *et al*., 2002), (McKinney and Karimov, 1997) models of the Aral Sea water resource systems have been reported. While many factors are addressed in the current planning effort for this region, some aspects, such as stakeholder participation, water rights and fairness of water allocation, are not given enough consideration and not explicitly included in models.

In the following, the PMMNF and LMWSR methods are applied to the transboundary Amu Darya river basin according to the absolute sovereignty and limited territorial sovereignty principles, respectively. The applications are only meant to be illustrative in nature with a focus on testing and comparing the performance of the developed algorithms. Comprehensive case studies can be carried out if more extensive documentation and data about the Amu Darya river basin were available.

4.4.3 River Basin Network Scheme and Input Data

Figure 4.2 portrays the network scheme of the Amu Darya river basin water resource system. This representation is based on the work of Raskin *et al*. (1992) and McKinney and Karimov (1997). In the network, there are seven types of nodes: 9 inflow, 14 junction, 9 aquifer, 6 reservoir, 1 hydropower plant, 12 off stream demand and 1 stream flow requirement nodes. The number of links is 87. Each off stream demand node aggregates water demands of the water users in a common geographic area with shared water sources and includes water for irrigation, livestock, municipal and industrial uses. The off stream demand nodes are considered as agricultural nodes, since irrigation is the dominant water use in this basin. The furthest node downstream represents the Aral Sea, which is modeled as a stream flow requirement node. Treating the Aral Sea as a demand node rather than an outlet of the network makes it possible to analyze the Aral Sea's water demands and their effects on water allocation.

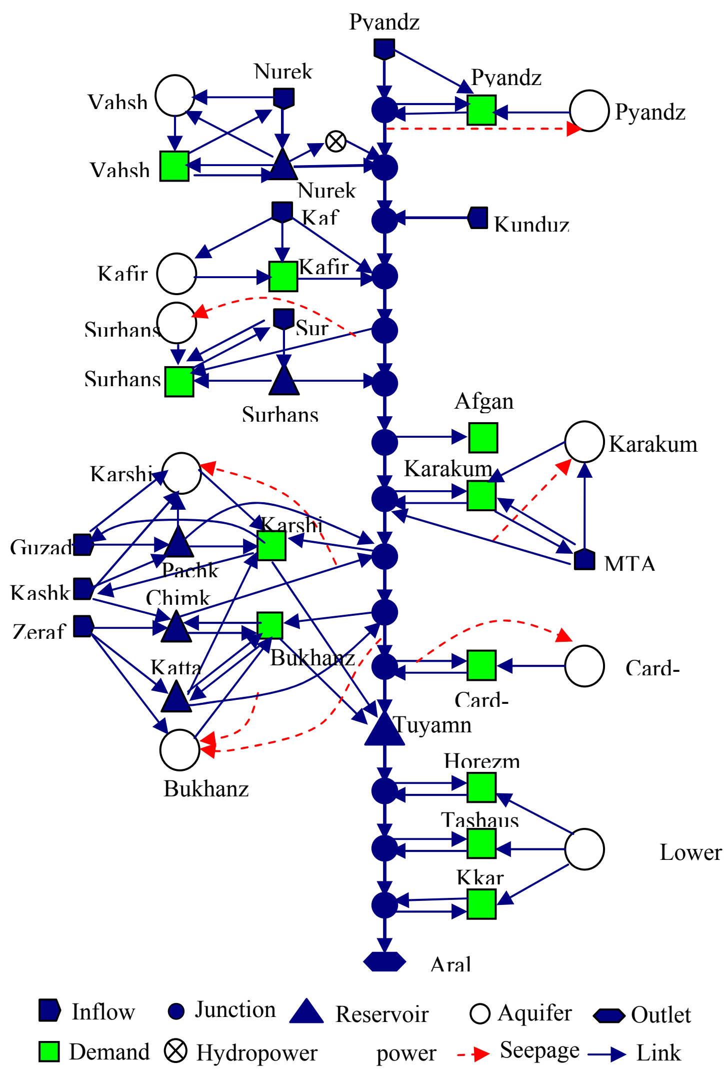

Figure 4.2 Amu Darya river basin network

Every type of node has its own hydrological characteristics. Reservoir evaporation, node losses, and inflow adjustments, seepages and return flows, are considered as hydrological constraints in addition to the general water balance. Each reservoir is divided into four zones with different storage priorities according to the operation rules. Other assumptions made for the nodes include linear surface area-storage relationships for reservoirs, and the linear reservoir response sub-model for simulating seepages from the reservoirs and aquifers. Links between nodes represent river reaches, diversion pipes, seepages and return flows. Capacity constraints, transmission losses as well as the node and link seepages to aquifers are also considered by utilizing various loss coefficients. The input data for surface water supply, ground water supply, major reservoirs, and demands at various sites in the Amu Darya river basin mostly come from previous studies of the Aral Sea basin (McKinney and Karimov, 1997; Raskin *et al*., 1992).

The estimated tributary inflows on a monthly basis for a typical dry year are given in Table 4.3. The total annual basin inflow is 39.2 km^3 in this scenario. Due to limited data, the initial available ground water volume of each aquifer is taken to be one twelfth of the total annual available ground water resources. The monthly inflow adjustments into aquifers are estimated by the proportions of the total annual available ground water resources according to the distribution of monthly inflows of the corresponding nearest tributaries. The initial storage volume of each reservoir is assumed to be corresponding to active operational storage. The monthly inflow adjustments for each reservoir or agricultural demand node are estimated by the proportions of the product of annual available precipitation and area with respect to the distribution of monthly inflows of the corresponding nearest tributary. The various loss coefficients are estimated according to common experience. For example, the water loss (consumption) coefficients at all agricultural demand nodes are assumed to be 70% in all of the time periods; coefficients of node seepages are set to 1% where there are seepages from the nodes to aquifers; water loss coefficients of all links in the river basin are taken to be 5%, and all link seepage coefficients are set as 1%. The monthly total inflow water demands at various demand sites in the Amu Darya river basin are given in Table 4.4. The Aral Sea's demand of water from the Amu Darya river is assumed to be 23 km^3 per year. Its monthly distribution is estimated as being proportional to the monthly total available water supply of the Amu Darya basin. In Table 4.4, a zero entry means that there is no inflow demand for the corresponding demand site and month.

Table 4.3 Dry year monthly surface water supply (km^3) (Raskin *et al*., 1992)

Tributary	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
Nurek	0.205	0.186	0.268	0.512	0.958	1.615	1.993	1.873	0.958	0.501	0.358	0.286	9.713
Pyandz	0.506	0.457	0.954	1.373	1.82	2.709	3.188	2.598	1.652	0.993	0.784	0.587	17.621
Kunduz	0.058	0.06	0.07	0.128	0.165	0.301	0.291	0.242	0.137	0.091	0.066	0.059	1.668
Kaf	0.08	0.083	0.269	0.478	0.677	0.701	0.531	0.332	0.167	0.162	0.127	0.102	3.709
Surdarya	0.043	0.044	0.153	0.33	0.475	0.482	0.291	0.183	0.08	0.076	0.065	0.048	2.27
Guzadarya	0.002	0.001	0.009	0.036	0.018	0.016	0.009	0.003	0.003	0.005	0.004	0.004	0.11
Kashkadarya	0.016	0.018	0.078	0.13	0.115	0.122	0.093	0.05	0.027	0.019	0.017	0.015	0.7
Zeraf	0.052	0.042	0.079	0.105	0.189	0.436	0.575	0.513	0.248	0.127	0.093	0.072	2.531
Murgab, Tegen and Atrek (MTA)	0.048	0.037	0.097	0.232	0.118	0.084	0.034	0.034	0.035	0.06	0.061	0.07	0.91
Total	1.01	0.928	1.977	3.324	4.535	6.466	7.005	5.828	3.307	2.034	1.575	1.243	39.232

Table 4.4 Monthly total inflow water demands (km^3) (Raskin *et al*., 1992)

Demand Site	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
Pyandz	0	0.01	0.04	0.04	0.04	0.04	0.06	0.08	0.04	0.02	0	0	0.37
Vahsh	0	0.1	0.53	0.4	0.39	0.46	0.71	0.96	0.53	0.24	0	0.08	4.4
Kafir	0	0.08	0.36	0.29	0.26	0.33	0.51	0.66	0.36	0.18	0	0.04	3.07
Afgan	0	0	0	0.01	0	0.01	0.01	0.01	0	0	0	0	0.04
Surhans	0	0.12	0.63	0.47	0.46	0.55	0.85	1.12	0.63	0.29	0	0.09	5.21
Karakum	0.67	0.62	0.66	0.84	1.76	1.89	1.98	1.84	1.44	1.19	0.89	0.75	14.53
Karshi	0.58	0.43	0.42	0.74	1.02	1.06	1.11	1.09	0.87	0.73	0.68	0.65	9.38
Bukharaz	1.19	1.09	0.9	0.7	1.5	2.49	3.09	2.64	1.22	0.74	0.11	0.26	15.93
Cardzou	0.03	0.13	0.29	0.24	0.52	0.86	1.11	0.87	0.21	0.07	0.01	0	4.34
Horezm	0.02	0.11	0.32	0.1	0.39	0.48	0.58	0.56	0.08	0	0	0.01	2.65
Tashaus	0.18	0.2	0.37	0.32	0.42	0.53	1	0.87	0.06	0	0	0.26	4.21
Kkar	0.56	0.31	1.4	0.34	2.05	2.52	3.51	2.74	0.26	0.01	0.23	0.71	14.64
Total	3.23	3.2	5.92	4.49	8.81	11.22	14.52	13.44	5.7	3.47	1.92	2.85	78.77

To demonstrate the capability of the water rights allocation methods comprising quantity and quality considerations, salinity control constraints are included in the Amu Darya case study. Due to limited data available, some assumptions are made in constructing the salinity constraints. Initial concentrations are assumed to range from upstream at 0.5 g/l to downstream at 1.5 g/l in reservoirs, and from upstream at 1.0 g/l to downstream at 10.0 g/l for aquifers. All inflows to the basin have a salinity of 0.5 g/l. The salt concentration of precipitation is 0.1 g/l during all time periods, and the salinity of all precipitation infiltration to aquifers is equal to its corresponding initial value. The salt addition to demand sites is assumed to all come from irrigation water and precipitation. The salt loss ratios of links are set to be 0.05, equal to their corresponding water loss ratios, while the salt loss ratio of any link seepage to an aquifer is assumed to be 0.01. Salt loss ratios at demand sites are 0.3, except that the nodes representing Afghanistan and the Aral Sea are set to 1, assuming salt is totally consumed by the nodes.

4.4.4 Results of PMMNF

Utilizing the data described above, PMMNF is applied to find the water rights allocations for demand nodes in this basin. Both linear and nonlinear programs are considered, depending on whether salinity constraints are included or not. Although the public water allocation system terminated after the break-up of the former Soviet Union and there currently exist no formal and clear water rights systems in this basin, the situation among the independent states competing for water resources can be analyzed by using the prior water allocation method. As pointed out before, the priority based maximal multiple-period network flow water rights allocation method is very flexible. It can be applied not only under a strict priority water rights regime but also under riparian water rights and even public allocation, if the priorities are properly assigned according to the existing water rights system or agreements. In this study, the priorities are set as the follows:

- The minimum water volume required by each reservoir to meet the water head for hydropower production, ecology and the future needs is assigned the highest priority.
- Off stream demands are assigned priorities in the order of upstream to downstream. All inflow links to a same demand node are set to be equal.
- Two cases for the Aral Sea's inflow demand are considered. In Case P1, it is assigned the lowest priority while in Case P2 it is given the second highest priority.

According to the solution report from MINOS, the linear PMMNF_QL programming consists of 18 iterations, and there are 1352 equations and 1556 control variables in every iteration. It requires about 2.5 seconds to run the program on a 3GHz Intel Pentium 4 CPU, and about 5 seconds on a 1.7 GHz Intel Pentium 4 CPU.

As mentioned above, the program searches optimal flow control schedules and gets the water rights allocations for demand sites (including water-right inflows and reservoir storages) subject to hydrological constraints and priority assignments. Ratios of water supply to demand are used to represent the satisfaction of demand for each demand site under the water right allocation. Each ratio is determined by dividing the total value of inflow adjustment and water-right inflows or storages by the node water demand for cases 1 and 2, and each month of the year. Table 4.5 lists results for the 12 agricultural demand sites, the hydropower plant and Aral Sea. Keep in mind that, if a demands node has a ratio of one in a month, it means its demand is fully met in that period. The lower the ratio, the less is the satisfaction of the demand. As can be seen in Table 4.5, for Case P1, the demands upstream from the middle node Karakum are nearly fully satisfied, and the downstream demands are less satisfied except for Karshi, Bukharaz and Horezm. This is because the upstream demands are assigned higher priorities such that they have the advantage of being able to take the upstream source water. For Case P2, Pyandz, the farthest upstream demand, and the Nurek hydropower plant are satisfied, and the Aral Sea's satisfaction ratios amount to 100%. However most of the others have very low supply/demand ratios. The relatively high ratios of Afgan are due to their low inflow demands and the contributions from inflow adjustments. The inflows to the Aral Sea for both cases are plotted in Figure 4.3. Since the Aral Sea is given the lowest priority in the basin in Case P1, the Aral Sea can only get less than 2.3% of its monthly water demands. In Case P2, the Aral Sea gets much higher monthly ratios. However, the upstream demands get much lower monthly water supply to demand ratios.

The accuracy of input data also greatly affects the results of initial water rights allocations. The effects of uncertainty of parameters on system performance can be estimated by sensitivity analysis, which is very frequently utilized approach in water resources systems modeling (Jain and Singh, 2002). A sensitivity analysis of various water loss coefficients shows that water loss (consumption) coefficients at agricultural demand sites play an important role in water allocation, because they are the major factors to determine how much return flow is available for downstream uses. Figure 4.4 shows the inflows to the Aral Sea from the Amu Darya river with various water loss (consumption) coefficients used as inputs. An 8.33% increase of loss coefficients will result in a 60.87% decrease of the flow to the Aral Sea in December, and a 16.67% increase causes a 69.57% decrease in the same month.

Table 4.5 Water supply/demand ratios for Cases P1 and P2

Demand Site	Case	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Pyandz	P1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	P2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Vahsh	P1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	P2	1.000	1.000	0.780	0.546	0.812	1.000	0.807	0.116	0.088	0.648	1.000	1.000
Nurek HPP	P1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	P2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Kafir	P1	1.000	0.977	0.718	1.000	1.000	1.000	0.981	0.903	1.000	1.000	1.000	1.000
	P2	1.000	0.070	0.050	0.105	0.153	0.129	0.068	0.034	1.000	0.854	1.000	1.000
Surhans	P1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	P2	1.000	0.098	0.065	0.167	0.228	0.201	0.089	0.044	0.220	1.000	1.000	0.729
Afgan	P1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	P2	1.000	1.000	1.000	0.804	1.000	0.600	0.375	0.375	1.000	1.000	1.000	1.000
Karakum	P1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	P2	0.081	0.068	0.153	0.253	0.076	0.052	0.021	0.022	0.029	0.102	0.334	0.131
Karshi	P1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	P2	0.027	0.018	0.146	0.279	0.124	0.108	0.061	0.022	0.694	0.582	0.096	0.100
Bukharaz	P1	0.926	0.086	0.937	1.000	0.846	0.667	0.528	0.456	0.582	0.654	1.000	1.000
	P2	0.021	0.026	0.124	0.232	0.111	0.074	0.047	0.030	0.546	0.087	0.399	0.161
Cardzou	P1	0.167	0.037	0.033	0.051	0.042	0.059	0.060	0.393	1.000	1.000	1.000	1.000
	P2	0.167	0.037	0.033	0.051	0.042	0.059	0.060	0.393	1.000	1.000	1.000	1.000
Horezm	P1	1.000	1.000	1.000	1.000	1.000	1.000	0.536	0.199	0.554	1.000	1.000	1.000
	P2	0.231	0.043	0.027	0.099	0.051	0.091	0.098	0.091	0.252	1.000	1.000	0.444
Tashaus	P1	0.073	0.173	0.251	0.127	0.295	0.331	0.155	0.108	0.524	1.000	1.000	0.158
	P2	0.037	0.029	0.029	0.042	0.058	0.100	0.072	0.074	0.362	1.000	1.000	0.037
Kkar	P1	0.025	0.057	0.029	0.091	0.035	0.054	0.045	0.047	0.207	0.940	0.100	0.036
	P2	0.019	0.028	0.012	0.061	0.019	0.035	0.033	0.038	1.000	1.000	0.381	0.034
Aral Sea	P1	0.019	0.020	0.015	0.008	0.011	0.017	0.021	0.023	0.020	0.013	0.014	0.018
	P2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

(All agricultural demand node loss coefficients are set to be 70%)

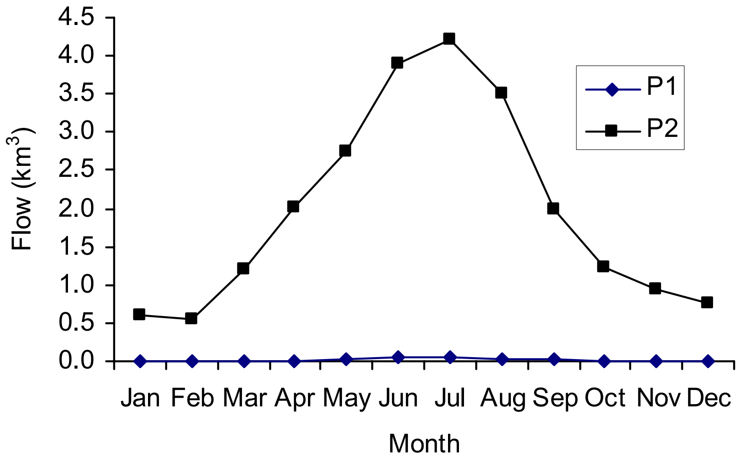

Figure 4.3 Inflows to the Aral Sea from the Amu Darya river allocated by linear priority-based multiperiod maximal network flow programming (PMMNF_QL)

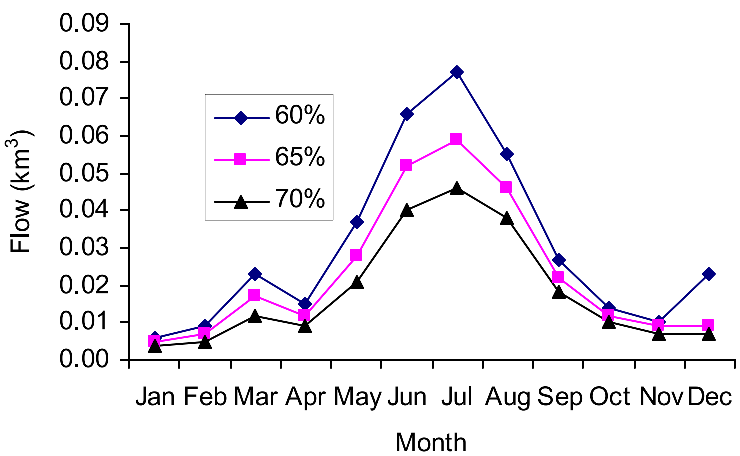

Figure 4.4 Inflows to the Aral Sea from the Amu Darya river with various water loss (consumption) coefficients input at agricultural demand sites (Case P1)

Table 4.6 provides the statistics for computational times for the nonlinear PMMNF_QC algorithm with different starting points. The starting point has a significant effect on the solving speed. By setting the initial values of water quantity related control variables to be the solution of the corresponding linear PMMNF_QL, and assigning zero as the initial value of all water quality control variables, the PMMNF_QC will be able to find an optimal solution in 16.416 seconds. According to the solution report from MINOS, the PMMNF_QC programming consists of 18 iterations, and there are 3908 equations and 3284 control variables in every iteration. It consumes about 24 seconds in total to run the program on a 3GHz Intel Pentium 4 CPU as shown in Table 4.7.

Table 4.6 Solving nonlinear priority-based multiperiod maximal network flow (PMMNF) program (PMMNF_QC) from different starting points (Case P1)

Starting point[a]	1st priority iteration time (s)[b]	Total solving time (s)[b]
Q(link,t)=0 g/l, C(link,t)=0 g/l, C_N(k,t)=0 g/l	22.527	61.832
Q given by solution of the 1st loop of PMMNF_QL, C(link,t)=0 g/l, C_N(k,t)=0 g/l	5.020	51.729
Q given by PMMNF_QL, C(link,t)=0 g/l, C_N(k,t)=0 g/l	3.859	16.416
Q given by PMMNF_QL, C(link,t)=1 g/l, C_N(k,t)=1 g/l	7.867	33.281
Q given by PMMNF_QL, C(link,t)=2 g/l, C_N(k,t)=2 g/l	11.488	41.838
Q given by PMMNF_QL, C(link,t)=4 g/l, C_N(k,t)=4 g/l	14.332	44.403
Q given by PMMNF_QL, C(link,t)=6 g/l, C_N(k,t)=6 g/l	25.172	50.636

a. Q(link,t) and C(link,t) are the water flows and pollutant concentration in a link during period t, respectively. C_N(k,t) is the and mixed pollutant concentration in an aquifer, reservoir, or total inflow to a demand site.
b. The time here only consists of the resource-usage from model statistics in GAMS output files, using a 3GHz Pentium 4 PC.

Table 4.7 Statistics of algorithms for linear (PMMNF_QL) and nonlinear (PMMNF_QC) priority-based multiperiod maximal network flow (PMMNF) programming

(Case P1, initial concentrations: $C(link, t)$ = 0 g/l, $C_N(k, t)$ = 0 g/l)

Model	Number of equations[a]	Number of variables[a]	Number of non-zero elements[a]	Number of non-linear elements[a]	Number of priority iterations	Total running time(s)[b]
PMMNF_QL	1352	1556	4912	0	18	2.474
PMMNF_QC	3908	3284	15154	6714	18	24.105

a. From model statistics in GAMS output files.
b. Sum of the times of mode generation, execution and resource-usage at each loop from GAMS output files, using a 3GHz Pentium 4 PC. The PMMNF_QC modeling time includes the corresponding PMMNF_QL running time also.

The inclusion of salinity constraints not only makes the water rights allocation model a large scale nonlinear program hard to be solved, but also requires the cautious setting of concentration limits. For example, if the inflow mixed concentration limits for agricultural demand sites are set loosely (e.g. 100 g/l), the solution of PMMNF _QC will be the same as the water-quantity-only PMMNF_QL. As the limits decrease, the water flows and pollutant concentrations of the water rights allocation scheme will be changed accordingly. However, it is shown in this study that the quantity and salinity of the inflows to the agricultural demand sites and the Aral Sea (Figure 4.5 to Figure 4.7) change slightly when inflow mixed concentration limits for agricultural demand sites are set to be 100 g/l and 6 g/l, respectively. If the inflow mixed concentration limits for all agricultural demand sites are set to 2 g/l, the upstream agricultural demands will not be affected, but the agricultural demand sites below Bukharaz will have smaller satisfaction ratios. As shown in Figure 4.8, if the inflow mixed concentration limits for all agricultural demand sites are set to be 6 g/l, the concentrations of salt in the inflows to the Aral Sea with a second highest priority will be much lower than the case having the lowest priority. This is because more water is allocated to the Aral Sea, while the water quantity allocated to upstream demand sites is reduced.

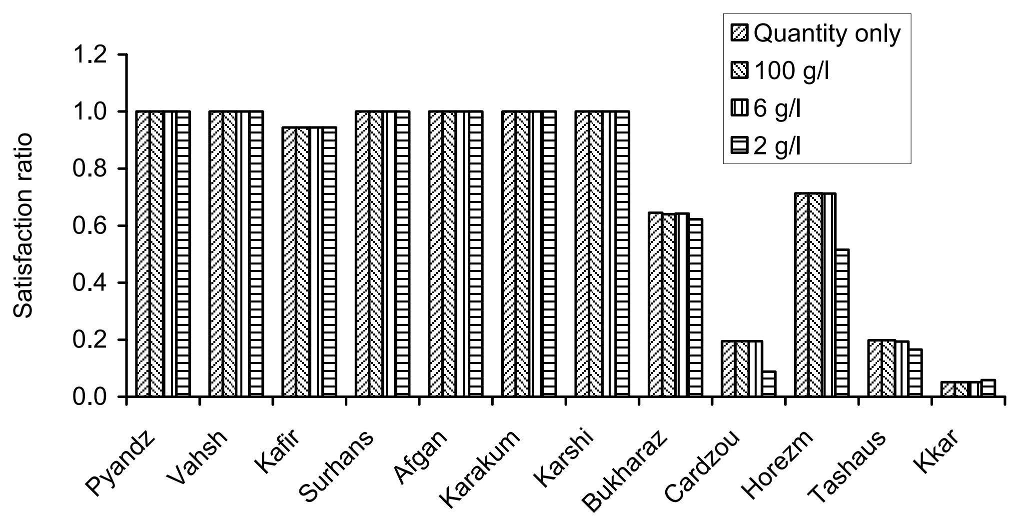

Figure 4.5 Over-all satisfaction ratios of demand nodes under different inflow mixed concentration limits for agricultural demand sites (Case P1)

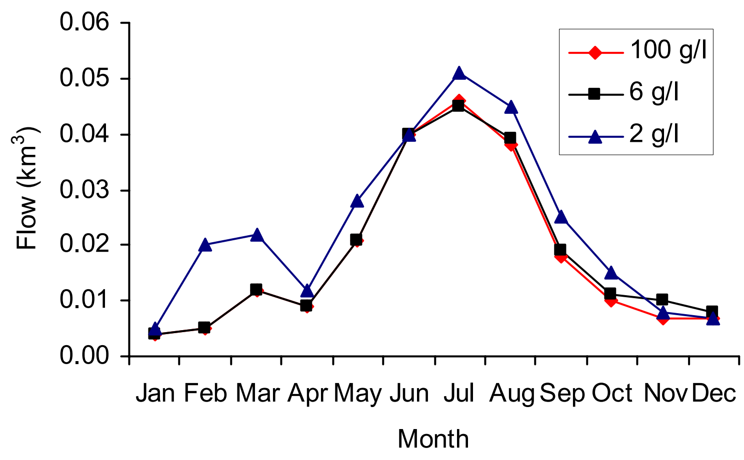

Figure 4.6 Inflows to the Aral Sea (Case P1) under inflow mixed concentration limits for agricultural demand sites

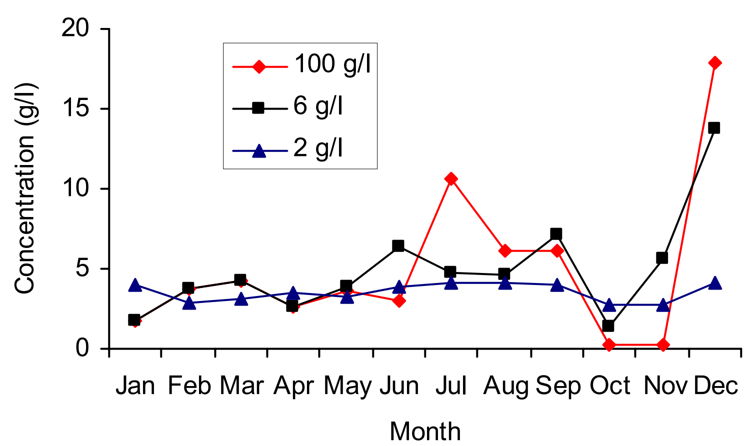

Figure 4.7 Salinity of the inflows to the Aral Sea (Case P1) under inflow mixed concentration limits for agricultural demand sites

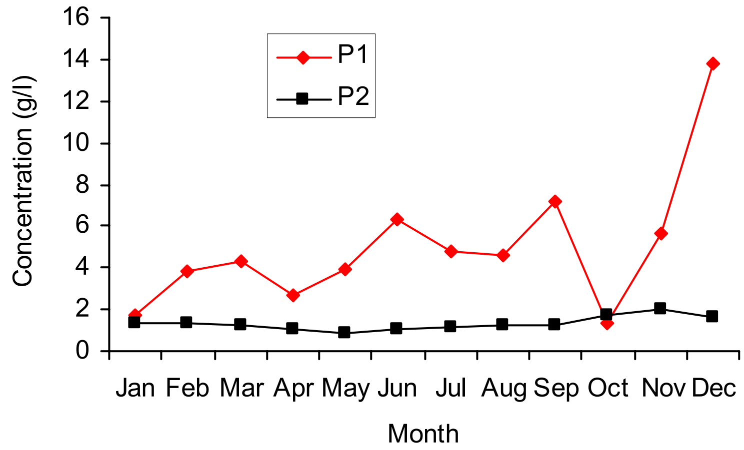

Figure 4.8 Salinity of inflows to the Aral Sea with different priority

(inflow mixed concentration limits for agricultural demand sites are 6 g/l)

4.4.5 Results of LMWSR

Two cases for the assignments of the weighting factors for demands are considered. In Case W1, all agricultural and hydropower demands are assigned a weight value of 10, and the weights of the reservoirs and the Aral Sea are set to be 1. For Case W2, all demand nodes are assigned an equal value of 1. All agricultural demand node loss coefficients are set to be 70% in both cases. According to the solution report from MINOS, the linear LMWSR_QL programming has 1465 control variables in each iteration, and 1716 equations at the first lexicographic iteration loop and 1477 at the last loop. The number of iterations and computational times depend on the input data to the modeling. For this case study, a scenario having a 10:1 weight assignment has 47 iterations requiring about 5 computer seconds in total to run on a 3GHz Intel Pentium 4 CPU; while the problem with a 1:1 weight assignment has 23 iterations taking about 3 seconds.

Table 4.8 lists the ratios of water supply to demand for the 12 agricultural demand sites, the hydropower plant and Aral Sea for cases W1 and W2. Keep in mind that, if a demand node has a ratio of one in a month, it means its demand is fully met in that period. The lower the ratio, the less is the satisfaction of the demand. As can be seen in Table 4.8, for Case W1, all agricultural and hydropower demands have relatively higher satisfaction ratios than the Aral Sea. This is because the demand of the Aral Sea is assigned a lower weight. As we can see for Case W2, the Aral Sea is satisfied as most agricultural and hydropower demands at a ratio of 42.5%. Generally speaking, the variations among all objectives in the lexicographic optimization are caused by the weights and constraints of the feasible set. The differences among water demands in Case W2 (equal weights) come from the hydrological constraints of the link flows and node storages. The values in Table 4.8 also reveal that the upstream and downstream demand sites are fairly dealt with and there is no preference given to upstream nodes. Should the weight factors affecting allocation results be properly assigned based on analysis of the attributes of all the demand sites, the lexicographic approach would be able to provide equitable water allocations.

The inflows to the Aral Sea for both cases are plotted in Figure 4.9. Since the Aral Sea is given the lower weight in Case W1, the Aral Sea can only receive less than 13% of its total annual water demands. In Case W2, the Aral Sea has an equal weight with all other demands, so it obtains much more water in every month than in Case W1.

The sensitivity of the effects of the assignment of weights on the allocation is investigated, where all agricultural and hydropower demands are assigned equal weight values and the reservoir and Aral Sea are assigned another equal weight value. As the weight of agricultural and hydro-power demands are greater than 3 times that of one of the reservoirs and the Aral Sea, the allocation results will tend to be identical. The allocation is rather insensitive to the assignment of weights for this situation. The inflows to the Aral Sea from the Amu Darya River as depicted in Figure 4.10, illustrate this phenomenon.

The accuracy of input data also greatly affects the results of initial water rights allocations. A sensitivity analysis of various water loss coefficients shows that water loss (consumption) coefficients at agricultural demand sites play an important role in water allocation, because they are the major factors to determine how much return flow is available for downstream uses. Figure 4.11 shows the inflows to the Aral Sea from the Amu Darya River with various water loss (consumption) coefficients used as inputs. An 8.33% increase of loss coefficients from 0.60 will result in a 16.79% decrease in the flow to the Aral Sea in July, and a

16.67% increase of loss coefficients from 0.60 will result in a 31.93% decrease. Both decrease quickly with the increase of loss coefficients.

Table 4.8 Water supply/demand ratios for Cases W1 and W2

Demand	Case	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Pyandz	W1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	W2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Vahsh	W1	1.000	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	1.000	0.639
	W2	1.000	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	1.000	0.693
Nurek HPP	W1	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639
	W2	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425
Kafir	W1	1.000	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	1.000	0.820
	W2	1.000	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	1.000	0.820
Surhans	W1	1.000	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	1.000	0.639
	W2	1.000	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	1.000	0.425
Afgan	W1	1.000	1.000	1.000	0.804	1.000	0.639	0.639	0.639	1.000	1.000	1.000	1.000
	W2	1.000	1.000	1.000	0.804	1.000	0.600	0.425	0.425	1.000	1.000	1.000	1.000
Karakum	W1	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639
	W2	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425
Karshi	W1	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639
	W2	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425
Bukharaz	W1	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639
	W2	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425
Cardzou	W1	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	1.000	1.000
	W2	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	1.000	1.000
Horezm	W1	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	1.000	1.000	0.639
	W2	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	1.000	1.000	0.444
Tashaus	W1	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	1.000	1.000	0.639
	W2	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	1.000	1.000	0.425
Kkar	W1	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.639	0.804	0.639	0.639
	W2	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.730	0.425	0.425
Aral Sea	W1	0.181	0.114	0.220	0.036	0.142	0.128	0.166	0.159	0.039	0.016	0.054	0.183
	W2	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425	0.425

(All agricultural demand node loss coefficients are set to be 70%)

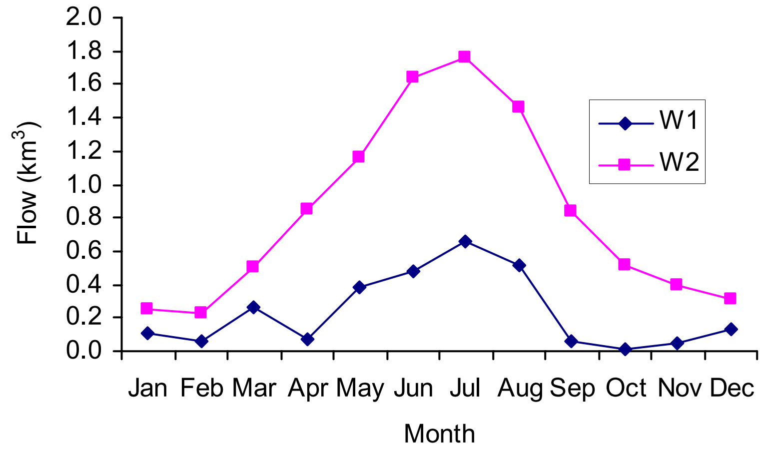

Figure 4.9 Inflows to the Aral Sea from the Amu Darya river

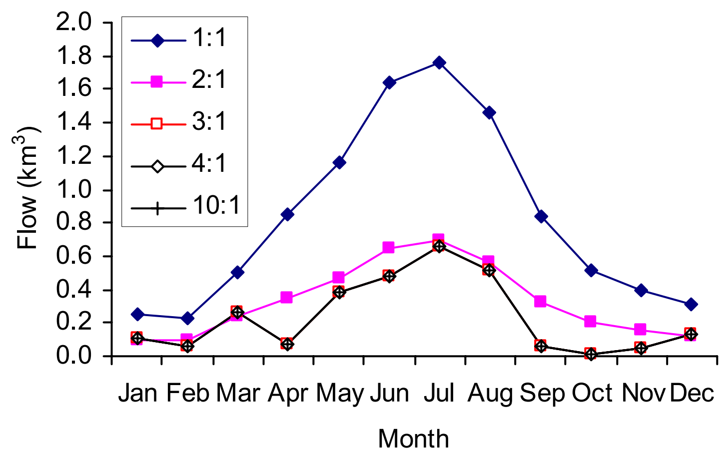

Figure 4.10 The flows to the Aral Sea with assignments of various ratios of weights between the agricultural and hydropower demand sites to the reservoirs and the Aral Sea

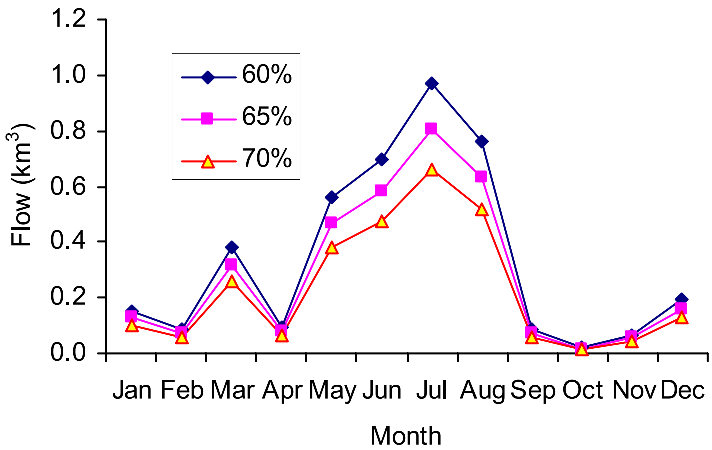

Figure 4.11 Inflows to the Aral Sea from the Amu Darya River with various water loss (consumption) coefficients input at agricultural demand sites

Table 4.9 Solving nonlinear lexicographic minimax water shortage ratios (LMWSR_QC) program from different starting points (Case W1)

Starting point[a]	1st lexicographic iteration time (s)[b]	Total solving time (s)[b]
Q(link,t)=0, C(link,t)=0, $C_N(k,t)$=0	Locally infeasible (infeasibilities: 673, sum: 61.554)	–
Q given by solution of the 1st loop of LMWSR_QL, C(link,t)=0, $C_N(k,t)$=0	16.141	60.096 (47 loops)
Q given by LMWSR_QL, C(link,t)=0, $C_N(k,t)$=0	13.750	58.610 (47 loops)
Q given by LMWSR_QL, C(link,t)=1, $C_N(k,t)$=1 g/l	19.141	56.339 (47 loops)
Q given by LMWSR_QL, C(link,t)=2, $C_N(k,t)$=2 g/l	18.219	62.501 (47 loops)
Q given by LMWSR_QL, C(link,t)=4, $C_N(k,t)$=4 g/l	17.109	33.369 (49 loops)
Q given by LMWSR_QL, C(link,t)=6, $C_N(k,t)$=6 g/l	20.414	60.452 (47 loops)

a. Q(link,t) and C(link,t) are the water flows and pollutant concentration in a link during period t, respectively. $C_N(k,t)$ is the and mixed pollutant concentration in an aquifer, reservoir, or total inflow to a demand site.
b. The time here only consists of the resource-usage from model statistics in GAMS output files, using a 3GHz Pentium 4 PC.

Table 4.9 contains the statistics of computational times for the LMWSR_QC algorithm with different starting points. By setting the initial values of water quantity related control variables to be the solution of the corresponding linear LMWSR_QL, and assigning the initial value of 4 g/l to all water quality control variables, the nonlinear LMWSR_QC will be able to find a optimal solution in 33.369 seconds. According to the solution report from MINOS, the LMWSR_QC programming consists of 49 lexicographic iterations, and there are 4272 equations at the first lexicographic iteration loop and 3193 control variables at every lexicographic iteration loop. It costs totally about 45 seconds in total to run the program on a 3GHz Intel Pentium 4 CPU as shown in Table 4.10.

Table 4.10 Statistics of models: linear (LMWSR_QL) and nonlinear (LMWSR_QC) lexicographic minimax water shortage ratios programming

(Case W1, initial concentrations: $C(link, t)$ = 4 g/l, $C_N(k, t)$ = 4 g/l)

Model	1st lexicographic iteration				Number of lexicographic iterations	Total running time(s)[b]
	Number of equations[a]	Number of variables[a]	Number of non-zero elements[a]	Number of non-linear elements[a]		
LMWSR_QL	1716	1465	4999	0	47	4.734
LMWSR_QC	4272	3193	15241	6714	49	45.650

a. From model statistics for the first lexicographic iteration loop in GAMS output files.
b. Sum of the times of mode generation, execution and resource-usage at each loop from GAMS output files, using a 3GHz Pentium 4 PC. The LMWSR_QC modeling time includes LMWSR_QL running time also.

The inclusion of salinity constraints not only makes the water rights allocation model a large scale nonlinear program hard to be solved, but also requires the cautious setting of concentration limits and penalty parameters. For example, as shown in Figure 4.12, if the inflow mixed concentration limits for agricultural demand sites are set loosely (e.g. 100 g/l), the solution of LMWSR_QC will be the same as the water-quantity-only linear LMWSR. As the limits decrease, the water flows and pollutant concentrations of the water rights allocation scheme will be changed accordingly. However, it is shown, for instance, in Figure 4.13 that the quantity and salinity of the inflows to the agricultural demand sites and the Aral Sea change slightly when inflow mixed concentration limits for agricultural demand sites are set to be 100 g/l and 6 g/l, respectively. If the inflow mixed concentration limits for all agricultural demand sites are set to 2 g/l, all of the agricultural demand sites, except for the most upstream Pyandz, will have reduced satisfaction ratios and the reduction is evenly distributed among them. Figure 4.14 shows that if the inflow mixed concentration limits for all agricultural demand sites are set to be 6 g/l, the concentrations of salt in the inflows to the Aral Sea with a high weight will much lower than the case of the low weight. This is because more water is allocated to the Aral Sea, while the water quantity allocated to upstream demand sites is reduced.

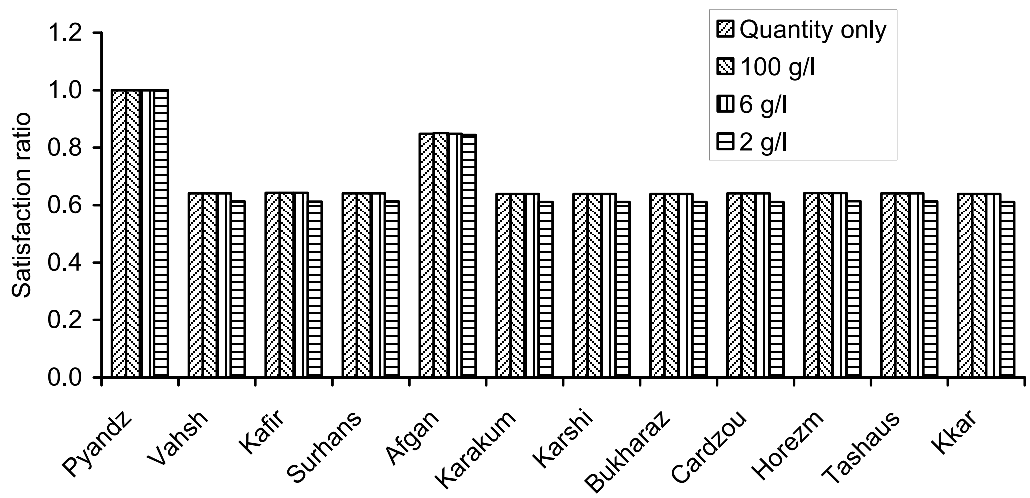

Figure 4.12 Over-all satisfaction ratios of demand nodes under different inflow mixed concentration limits for agricultural demand sites (Case W1)

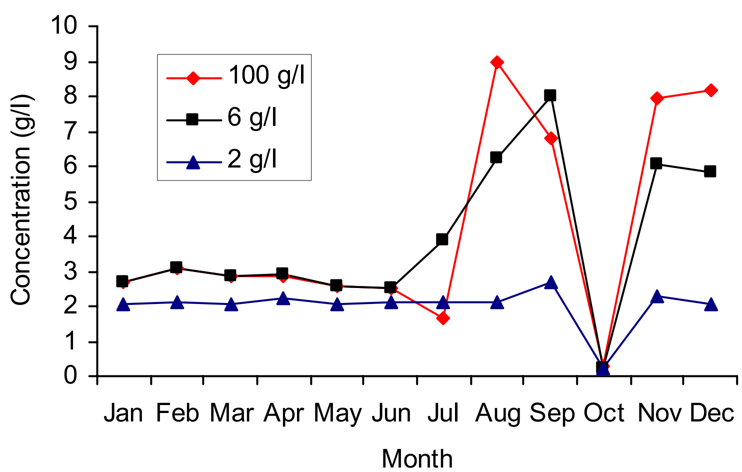

Figure 4.13 Salinity of the inflows to the Aral Sea (Case W1) under different inflow mixed concentration limits for agricultural demand sites

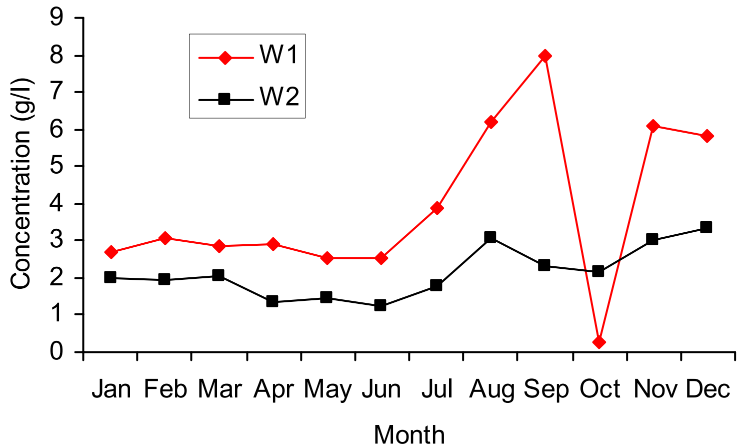

Figure 4.14 Salinity of inflows to the Aral Sea with different priority

(inflow mixed concentration limit for agricultural demand sites is 6 g/l)

4.5 Equity Principles and Fairness Concepts Embedded in PMMNF and LMWSR

4.5.1 Principles for Multiple Objective Fair Resource Allocation Problems

Before investigating the fairness concepts underlying PMMNF and LMWSR, the equity principle and concepts of the generic resource allocation problem should be reviewed from a theoretical perspective and quantitatively presented. Consider a generic resource allocation problem defined as a multiple objective optimization problem:

$$\max\left[\mathbf{f}(\mathbf{x}) : \mathbf{x} \in \Omega\right] \tag{4.5}$$

where $\mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots f_m(\mathbf{x})\right)$, $f_j(\mathbf{x})$ is the jth objective function, $j \in I = \{1, 2, \cdots, m\}$. The function vector $\mathbf{f}(\mathbf{x})$ maps the decision space $X = R^n$ into the outcome space $Y = R^m$, $\mathbf{y} = (y_1, y_2, \cdots y_m)$. The $\mathbf{x}$ denotes the vector of decision variables, and $\mathbf{\Omega}$ is the feasible set defined by the constraints of the optimization problem.

The generic resource allocation problem only specifies that it is interested in maximization of all objective functions, but says nothing about solution concepts on how to find an efficient and equitable allocation scheme **x*** by employment of suitable methods. Typical solution concepts are defined based on the aggregation functions of multiple objectives to be maximized. Since equity is essentially an abstract social-political concept implying fairness and justice (Young, 1994), we may let $u(\mathbf{f}(\mathbf{x}))$ be the social utility function. Typical solution concepts for the generic multiple objective resource allocation problem is restated as

$$\max u\left(\mathbf{f}(\mathbf{x}) : \mathbf{x} \in \Omega\right) \tag{4.6}$$

In order to guarantee fairness of the solution concept, the social utility function aggregated from the individual objective functions must follow three fundamental fairness principles proposed by Ogryczak *et al*. (2003).

(1) Monotonicity

To assure the consistency of the aggregated problem with the maximization of all individual objective functions in the original generic resource allocation problem, the utility function must be strictly increasing with respect to every coordinate. Accordingly, for all $i \in I$, whenever $y_i' < y_i$

$$u(y_1, \cdots, y_{i-1}, y_i', y_{i+1}, \cdots, y_m) < u(y_1, \cdots, y_{i-1}, y_i, y_{i+1}, \cdots, y_m) \tag{4.7}$$

Since a feasible solution $\mathbf{x} \in \Omega$ is called Pareto-optimal (or efficient) if no solutions dominate it (i.e. no $\mathbf{y}' = \mathbf{f}(\mathbf{x}') > \mathbf{y} = \mathbf{f}(\mathbf{x})$ exits), the monotonicity property of the aggregation function means that the maximization of it will produce Pareto-optimal solutions.

(2) Impartiality

The utility function is impartial (symmetric) if for any permutation π of *I*,

$$u(y_{\pi(1)}, \cdots, y_{\pi(i)}, \cdots, y_{\pi(m)}) = u(y_1, \cdots, y_i, \cdots, y_m) \tag{4.8}$$

where $y_{\pi(i)}$ is a permutation of y_i, and $y_{\pi(i)} = y_i$. However the coordinates are different.

(3) Equitability

A utility function is equitable if it satisfies the principle of rational transfers for any $0 < \varepsilon < y_{i'} - y_{i''}$,

$$u(y_1, \cdots, y_{i'} - \varepsilon, \cdots, y_{i''} + \varepsilon, \cdots, y_m) > u(y_1, \cdots, y_{i'}, \cdots, y_{i''}, \cdots, y_m) \tag{4.9}$$

The functions satisfying this strictly inequality relationship are also called strictly Scgur-concave functions (Ogryczak *et al*., 2003).

Sometime, a resource is allocated according to the priority ranks of the individual demands, such as the realization of water rights allocation under a prior allocation regime. The priority principle and fairness concept are formally formulated as follows.

(4) Priority

A utility function is based on a priority rule if it satisfies the following principle of transfers

$$u(y_1, \cdots, y_{i'} + \varepsilon_1, \cdots, y_{i''} - \varepsilon_2, \cdots, y_m) > u(y_1, \cdots, y_{i'}, \cdots, y_{i''}, \cdots, y_m) \tag{4.10}$$

for any $\varepsilon_1 > 0,\ \varepsilon_2 > 0$, and $y_{i'}$ has a higher priority rank over $y_{i''}$.

For the case of a utility function meeting the monotonicity, impartiality and equitability principles, it is called a *perfectly equitable* utility function for the generic resource allocation problem. The solution of the maximization of a *perfectly equitable* utility function will produce a *perfectly equitable* resource allocation scheme. If a utility function satisfies the monotonicity and priority principles, it is called a *priorly equitable* utility function for the generic resource allocation problem. The solution of the maximization of a *priorly equitable* utility function will produce a *priorly equitable* resource allocation scheme.

Various solution concepts are defined by utility functions with different forms of aggregation of the individual objective functions. The simplest utility function commonly used is defined as the weighted sum of outcomes

$$u(\mathbf{y}) = \sum_{i=1}^{m} w_i y_i \tag{4.11}$$

or the worst outcome

$$u(\mathbf{y}) = \min_{i \in I} y_i \tag{4.12}$$

The weighted sum aggregation is a strictly increasing function, and therefore, the maximization of this function will always generate a Pareto-optimal solution. However, the weighted sum aggregation violates the requirements of impartiality and equality, as it assigns the weights to individual outcomes. The minimum aggregation form makes the generic resource allocation problem a maximin optimization problem. Because the minimum aggregation utility function is only non-decreasing, the maximization of the worst outcome may not generate a Pareto-optimal solution. Although it obeys the impartiality principle, the minimum aggregation function does not satisfy the equitability principle. Hence, the maximization of these two forms of aggregation functions for social utility will not generate a *perfectly equitable* solution for the resource allocation problem under study.

Assuming that the weights in the weighted sum aggregation satisfy $w_1 > w_2 > \cdots > w_m$, when the differences among weights tend to infinity, the maximization of the weighted sum aggregation becomes a priority-based multiple objective resource allocation optimization problem,

$$\max\left[f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots f_m(\mathbf{x}) \right] \tag{4.13}$$

where, $f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots f_m(\mathbf{x})$ are ordered from the highest priority to the lowest one. As a limiting case of the weighted sum aggregation, the priority-based multiple objective resource allocation optimization problem is not a *perfectly equitable* allocation method. Actually, it satisfies the monotonicity and priority principles, and thus is a *priorly equitable* allocation method.

Yager (1988) introduced the so-called ordered weighted averaging (OWA) aggregation. In the OWA aggregation, the weights are assigned to the ordered values. This can be mathematically formalized as follows. First, introduce the ordering map Θ: $R^m \rightarrow R^m$, such that $\Theta(\mathbf{y}) = (\Theta_1(\mathbf{y}), \Theta_2(\mathbf{y}), \cdots, \Theta_m(\mathbf{y}))$, where $\Theta_1(\mathbf{y}) \le \Theta_2(\mathbf{y}) \le \cdots \le \Theta_m(\mathbf{y})$, and there exists a permutation π of set I such that $\Theta_i(y) = y_{\pi(i)}$ $for\ i = 1, 2, \cdots, m$. Further, the weighted sum aggregation is applied to the ordered achievements vector $\Theta(\mathbf{y})$. Thus, the utility obtained by OWA aggregation is

$$u(\mathbf{y}) = \sum_{i=1}^{m} w_i \Theta_i(\mathbf{y}) \tag{4.14}$$

When applying OWA aggregation to the generic resource allocation problem, we get

$$\max\left[\sum_{i=1}^{m} w_i \Theta_i(\mathbf{f}(\mathbf{x})) : \mathbf{x} \in \Omega\right] \tag{4.15}$$

This OWA aggregation function has been proven to satisfy the principles of monotonicity, impartiality and equitability (Ogryczak *et al*., 2003). Therefore, the solution of this weighted sum of ordered outcome problem is a perfectly equitable solution for the generic resource allocation problem. Furthermore, when the differences among weights tend to infinity, the OWA aggregation approximates the lexicographic ranking of the ordered outcome vectors (Yager, 1997). This means, as the limiting case of the OWA problem, the lexicographic maximization problem

$$\text{lex}\max\left[\mathbf{f}(\mathbf{x}) : \mathbf{x} \in \Omega\right] \tag{4.16}$$

is a specific formulation of the generic resource allocation problem, whose solution is a *perfectly equitable* allocation scheme.

4.5.2 Equity Concepts Underlying in the PMMNF and LMWSR

PMMNF and LMWSR consider fairness in different ways. PMMNF can be viewed as the limiting case of an optimization problem with a weighted sum of individual objective functions or outcomes. As discussed above, the weighted sum aggregation violates the principles of impartiality and equitability and, hence, PMMNF is not a *perfectly equitable* water rights allocation method. However, the PMMNF method is designed based on priority (i.e. the sum of outcomes with a higher priority rank is maximized before those with a lower priority rank.). Because it satisfies the monotonicity and priority principles, it is a *priorly equitable* aggregation function, and the PMMNF will generate a *priorly equitable* water rights allocation scheme.

LMWSR constitutes a lexicographic minimax formulation of water shortage ratios. It is equivalent to the lexicographic maximin formulation of water satisfaction ratios. Therefore, LMWSR is a water rights allocation method that can generate a *perfectly equitable* allocation scheme.

4.6 Summary

A sequential solution approach is used to solve PMMNF problems, in which the difficulty in assigning proper coefficients (or weights) for network flow programming to reflect priority ranks is avoided. The LMWSR problems are solved by an iterative procedure. The linear PMMNF and LMWSR problems consider only linear water quantity constraints, which are solved by the primal simplex method. The nonlinear PMMNF and LMWSR problems allow for nonlinear reservoir area-storage curves and hydropower plants with variable water heads, and may include nonlinear water quality constraints, which can be efficiently solved by the proposed two-stage approach. In the first stage, the corresponding linear problem excluding nonlinear constraints is solved by a sequential or iterative algorithm and the global optimal solution can be reached. The global optimal solution is then used as part of the starting point for the nonlinear program of the nonlinear PMMNF or LMWSR problem. Reasonable initial values for pollutant concentrations should be estimated.

The case study of the Amu Darya river basin shows, although one cannot guarantee the accuracy of input data in this case study, that the PMMNF method is a useful tool for water rights allocation under various rights systems. Under the situation of water shortage in a river basin, the water rights system and associated priority assignments are key factors to consider in achieving fair water allocation for a secure society. The method developed can be utilized to test different water rights systems and priority assignments. Once a sound water rights system and the associated priority assignment are constructed, water can be allocated to users. Based on the initial water rights allocation, the economic optimal water reallocation can be carried out in a cooperative and sustainable manner (Wang *et al*., 2003a). The applicability of LMWSR is demonstrated by effectively solving the large linear programming problem for the Amu Darya river basin by the GAMS coded algorithm for the lexicographic minimax water shortage ratios approach. It is shown that the upstream and downstream demand sites are fairly handled, and there is no preference given to upstream nodes. The Aral Sea ecological crisis demonstrates that the establishment of equitable intrastate water right systems and interstate agreements to facilitate regional cooperation on water resources management among the countries in a river basin is the foundation for achieving fair water allocation and is, indeed, a difficult challenge.

By using the social utility function to aggregate individual objectives or outcomes, equity principles for generic resource allocation problems are reviewed from a theoretic perspective and quantitatively presented in the last part of this chapter. Various solution concepts are described for implementing these equity principles. It is shown that the weighted sum aggregation and minimum aggregation functions for social utility will not generate perfectly equitable solution for the resources allocation problem. When the differences among weights tend to infinity, the maximization of the weighted sum aggregation becomes a priority-based multiple objective resource allocation optimization problem. The priority-based multiple objective resource allocation optimization problem has an aggregated social utility function satisfying the monotonicity and priority principles and, thus, is a priorly equitable allocation method. The ordered weighted averaging aggregation function satisfies the principles of monotonicity, impartiality and equitability. Therefore, the weighted sum of ordered outcome problem is a specific formulation for the generic resource allocation problem, which generates perfectly equitable solutions. When the differences among weights of an ordered weighted averaging aggregation tend to infinity, the weighted sum of ordered outcome problem is transformed into an extreme case, a lexicographic maximization problem.

Based upon these equity principles and fair solution concepts for the generic resource allocation problem, we conclude that PMMNF is a water rights allocation method that generates a priorly equitable water rights allocation scheme, while LMWSR is a method that can generate a perfectly equitable allocation scheme. Thus, of the three methods for water rights allocation that are formulated based on various water rights systems, PMMNF and MRWRA are *priorly equitable*, while LMWSR is *perfectly equitable*.

Chapter 5
Hydrologic-economic Modeling and Reallocation of Water and Benefits

5.1 Introduction

The second step of the cooperative water allocation model (CWAM) is the reallocation of water and benefits. In this step, water is reallocated according to the optimal flow scheme obtained under the grand coalition, such as cooperation of all stakeholders in a concerned region or river basin. Then the net benefit of the grand coalition is reallocated to the stakeholders by cooperative game theoretical allocation methods. The net benefits of various stakeholder coalitions subject to water quantity and quality constraints are obtained by hydrologic-economic modeling. The most fundamental problems of hydrologic-economic modeling for water resources management are how to estimate the net benefits of water uses at demand sites and how to integrate the hydrologic and economic components.

A commonly used methodology is that the net benefit functions of water demand sites are first estimated and then are included in the hydrologic-economic model at the basin scale. Net benefit functions are often derived from empirical water demand functions, which may be obtained by econometric approaches (Diaz *et al*., 1997; Rosegrant *et al*., 2000; Ringler, 2001). The most widely used models have a constant-elasticity power function, yielding a water demand function that is convex to the origin. The water demand and net benefit functions can also be estimated by external simulation or optimization models which consider more details of the production processes and characteristics within demand sites. For example, optimization models maximizing revenue at the farm level have been applied to representative farms to derive water demand functions of the irrigation farms (Booker and Young, 1994; Mahan, 1997, Reca *et al*., 2001a) by solving for irrigation water input under assumed water availability (yielding estimates of marginal water value). Water demand functions of each representative farm are then extrapolated to model efficient allocations across all the corresponding farm land in a given agricultural region.

An alternative to the common methodology is that the models for production processes within demand sites are directly included and combined with the hydrologic component to form more complex river basin models. For example, in the studies of water allocation in the Mapio River Basin (Rosegrant *et al*., 2000), empirical agronomic crop production functions expressing the input-output relation between water and crop production are estimated by an external crop-water simulation model, and then are directly included in the river basin model. Ringler (2001) adopts the Food and Agriculture Organization of the United Nations (FAO)'s crop yield-evapotranspiration linear relationship model in the hydrologic-economic optimization model for the Meikong River Basin. Cai *et al*. (2003) proposed a holistic modeling framework for economic optimal water allocation integrating water and salinity balances at the basin, farm and crop field levels, and agronomic sub-models for crop production into one consistent optimization model. The direct inclusion of sub-models of the production processes inside demand sites may make the information transfer between the river basin model and production sub-models easy and consistent, but solving the merged large model may be hard or time consuming.

Most of the previous water allocation models only consider water quantity, subject to seasonal or yearly water quota water constraints, and do not include water quality constraints (Mahan, 1997; Reca *et al*., 2001a,b). It is assumed there is no stage constraint and water can always be optimally distributed among the stages within a season or year, and the optimal crop yield functions are utilized in the basin scale water allocation model. This is not true in the real world. Water supplies may be limited and shortages may occur in some stages within a growth season or year. There are a few research reports addressing this problem in the literature. Rosegrant *et al*.(2000), Ringler (2001) and Cai *et al*. (2003), for example, explored economic optimal water allocation models with monthly time periods, taking into account both the quantity and salinity aspects.

Considering the complexity of the integrated water allocation problem and the subsequent large number of scenarios of coalition analysis of net benefits at the reallocation stage, CWAM adopts the common approach to formulate the integrated hydrologic-economic river basin model (HERBM) based on derived net benefit functions of demand sites, which is presented in the following sections in this chapter. Monthly net benefit functions of water uses at municipal and industrial, hydropower generation, reservoir, and stream flow requirement demand sites, are estimated by econometric approaches. Monthly net benefit functions of irrigation water uses at agricultural demand sites are estimated by an offline external irrigation water planning model (IWPM) at the farm level. Based on a series of runs of the agronomic model with various water and salinity inputs, monthly benefit functions can be regressed and monthly water demand functions can be derived accordingly. The design of the agronomic model as an external offline model rather than a component merged within the hydrologic-economic model at the basin scale has two major advantages: (1) reducing the size of optimization problems; and (2) the monthly demand and benefit functions can be elicited, making it possible to compare them and analyze water trading in a more explicit way. The algorithms for the hydrologic-economic river basin model, coalition analysis, and reallocation of net benefits are coded in GAMS.

5.2 Integrated Hydrologic-Economic River Basin Model

The integrated hydrologic-economic river basin model is formulated as :

$$\max\left(\sum_{j}\sum_{t} NB_{jt} : \mathbf{x} \in \Omega\right) \tag{5.1}$$

where, NB_{jt} is the net benefit of demand j during period t; and $\mathbf{x} \in \Omega$ represents the hydrologic and economic constraints of the program. The objective of the model is to maximize the annual net benefit of water uses in the basin. The estimation of net benefit functions for each demand site should be carefully carried out taking account of the characteristics of water uses.

In the following, the net benefit of each demand site is derived as the profit of the total inflows to the demand node minus various supply costs. All inflows to a demand site are assumed to be fully mixed and are available to all competing uses within the demand site. Keeping in mind that water demand and benefit functions of demand nodes are defined over total inflows to demand nodes, they must be derived in harmony with the boundaries of demand sites. In short, the integrated hydrologic-economic river basin model (HERBM) treats each demand node as a single block. The subsystems and processes inside demand nodes are not simulated in the basin-scale model, whose performances are only represented by water consumption coefficients, pollution removal ratios, and net benefit functions.

5.2.1 Net Benefit Functions of Municipal and Industrial Demand Sites

In CWAM, all diversions received by a municipal and industrial demand node are assumed to be treated. Depending on the aggregation level of a river basin schematization, the treatment process may represented by a water treatment plant node, or may be implicitly accounted for in terms of link loss. Municipal and industrial consumers value water that is delivered and treated. However, the value of this commodity is not directly comparable with hydropower generation and other instream water uses. To value untreated raw water for municipal and industrial uses, the costs of conveying and treating water must be deducted from the value of delivered and treated water.

Empirical studies indicate that the quantity of water demanded by the municipal and industrial (*MI*) sector is sensitive to price but not as sensitive as irrigation demand. Compared with agricultural uses, the *MI* sector requires limited quantities of water but is willing to pay relatively higher prices. *MI* demands tend to be relatively inelastic, whose elasticity value is normally larger than -1 (Diaz *et al*., 1997).

Assume that the water demand functions of the *MI* demand curves during period *t* follow the constant price-elasticity form:

$$Q(j,t) = \alpha(j,t) P(j,t)^{\beta(j,t)} \tag{5.2}$$

where $Q(j,t) = Q_a(j,t) + \sum_{(k,j)\in L} Q(k,j,t)\big(1 - e(k,j,t)\big)$ is the total inflow to demand node *j* during period *t* (10^6m^3); $Q(k,j,t)$ is the water quantity of inflow from link (*k, j*) during period *t* (10^6m^3); $e(k,j,t)$ is the water loss coefficient for link (*k, j*) in period *t*; $P(j,t)$ is the price of willingness to pay for additional water at full use (\$/m^3); $\alpha(j,t)$ is a parameter for the constant elasticity demand function $\left(\alpha(j,t)>0\right)$; and $\beta(j,t)$ is the price elasticity of demand $\left(\beta(j,t)<0\right)$. According to the general definition of elasticity, $\varepsilon=(\partial Q/Q)/(\partial P/P)$, it can be concluded that $\varepsilon=\beta(j,t)$ remains constant during period *t*, but is seasonally variable for different periods.

The inverse demand function for effective inflow arriving at demand site *j* in period *t* is expressed as:

$$P(j,t)=\left(Q(j,t)/\alpha(j,t)\right)^{1/\beta(j,t)} \tag{5.3}$$

As the available effective inflow decreases, the price of willingness to pay for additional water increases. Generally, when the price goes up to some amount, the demand site may sort water from alternative sources. This price is the so called "choke price" (Mahan *et al*., 2002). By introducing the concepts of choke price P_0 and corresponding choke quantity Q_0, the inverse water demand function consists of a horizontal line segment and the curve of constant price-elasticity joined at the choke point (Q_0, P_0). A sample inverse water demand function with constant price-elasticity and choke price is shown in Figure 5.1, and can be expressed using the general mathematical function as follows:

$$P(j,t)=\begin{cases}P_0(j,t), & \left(0\le Q(j,t)\le Q_0(j,t)\right)\\ \left(Q(j,t)/\alpha(j,t)\right)^{1/\beta(j,t)}, & \left(Q(j,t)>Q_0(j,t)\right)\end{cases} \tag{5.4}$$

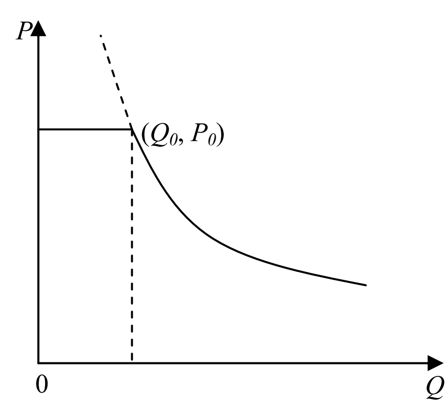

Figure 5.1 Inverse water demand function with constant price-elasticity and choke price

Note, if individual sub-sectors inside a demand site are considered, such as domestic, industrial, commercial and institutional water uses, the above overall inverse demand function for effective inflow to the demand site may be aggregated from demand functions of the individual sub-sectors by the horizontally aggregation approach (Diaz *et al*. 1997).

The gross benefit of total inflow to an *MI* demand site j (B_{jt} , 10^6\$) is derived from its inverse demand function:

$$B_{jt} = \begin{cases} P_0(j,t)Q(j,t), & \left(0 \le Q(j,t) \le Q_0(j,t)\right) \\ P_0(j,t)Q_0(j,t) + \dfrac{\left(1/\alpha(j,t)\right)^{1/\beta(j,t)}}{1+1/\beta(j,t)}\left[Q(j,t)^{(1+1/\beta(j,t))} - Q_0(j,t)^{(1+1/\beta(j,t))}\right], & \left(Q(j,t) > Q_0(j,t), \beta(j,t) \ne -1\right) \\ P_0(j,t)Q_0(j,t) + \alpha(j,t)\left[\ln Q(j,t) - \ln Q_0(j,t)\right], & \left(Q(j,t) > Q_0(j,t), \beta(j,t) = -1\right) \end{cases} \tag{5.5}$$

Thus, the net benefit function of an *MI* demand site is

$$NB_{jt} = B_{jt} - \sum_{(k,j)\in L} Q(k,j,t)wc(k,j,t), \ \forall j \in MI \tag{5.6}$$

where, $Q(k,j,t)$ is the water quantity diverted from k to j during period t (10^6m^3); $wc(k,j,t)$ is the water supply cost (\$/m^3) of diversion.

Since the prices of willingness to pay on the segment of inverse demand curve with constant elasticity is lower than the corresponding choke price, the maximization of above non-smooth net benefit function can be converted into maximization of a smooth objective function by splitting the water quantity into two parts and introducing a mass conservation constraint, as shown in the following program:

$$\begin{aligned} &\max NB_{jt} \\ &\textit{subject to}: \\ &\mathbf{x} \in \Omega \\ &B_{jt} = P_0(j,t)Q_1(j,t) + \frac{\left(1/\alpha(j,t)\right)^{1/\beta(j,t)}}{1+1/\beta(j,t)}\left[\left(Q_0(j,t)+Q_2(j,t)\right)^{(1+1/\beta(j,t))} - Q_0(j,t)^{(1+1/\beta(j,t))}\right], \ \left(\beta(j,t) \ne -1\right) \\ &B_{jt} = P_0(j,t)Q_1(j,t) + \alpha(j,t)\left[\ln\left(Q_0(j,t)+Q_2(j,t)\right) - \ln Q_0(j,t)\right], \ \left(\beta(j,t) = -1\right) \\ &Q(j,t) = Q_1(j,t) + Q_2(j,t) \\ &0 \le Q_1(j,t) \le Q_0(j,t) \\ &Q_2(j,t) \ge 0 \end{aligned} \tag{5.7}$$

where, $x \in \Omega$ represents other constraints of the problem.

5.2.2 Net Benefit Functions for Hydropower Plants

The production of hydroelectric energy during a period at a hydropower plant (*HPP*) is dependent on the installed plant capacity, the flow through the turbines, the average productive storage head, the number of hours in the period, and production efficiency. Assuming the density of water to be 1000 kg/m^3 for all time periods, hydropower generation can be estimated using a standard approach based on effective hydraulic head, turbine discharge volume and efficiency, as the following form (Loucks *et al*., 1981):

$$POW(j,t) = \sigma \eta Q(j,t) \Delta H(j,t) \tag{5.8}$$

where, $POW(j,t)$ is the power generated (10^{6}kWh); $\sigma = 0.00273$, is an numerical coefficient to conserve units (10^{6}kWh/10^{6}m^{3}m); η is turbine efficiency (%); $Q(j,t)$ is the rate of discharge (10^{6}m^3); and $\Delta H(j,t)$ is the effective water head of hydropower generation (m). For run-of-river hydropower stations, $\Delta H(j,t)$ is a constant parameter. For a hydropower plant j directly attached to an unique reservoir k, $\Delta H(j,t) = \frac{1}{2}\left(H(k,t-1) + H(k,t)\right) - H_{tw}(j,t)$, where $H_{tw}(j,t)$ is the elevation of tail water (m).

Electricity selling prices are so heavily regulated that they cannot be used as the basis for deriving the values of water input for hydropower plants (Diaz, *et al*., 1997). The value of hydroelectric energy is therefore commonly estimated using the alternative cost technique: assume that electricity not produced at the hydropower plants is produced at the next more expensive alternative. The demand functions for hydropower are represented as constant elasticity downward sloping demand curves, similar to municipal and industrial demands,

$$P(j,t) = \begin{cases} P_0(j,t), & \left(0 \le POW(j,t) \le POW_0(j,t)\right) \\ \left(POW(j,t)/\alpha(j,t)\right)^{1/\beta(j,t)}, & \left(POW(j,t) > POW_0(j,t)\right) \end{cases} \tag{5.9}$$

where, P_0 and POW_0 are the choke price (\$/kWh) and corresponding choke quantity (10^{6}kWh) for hydropower, respectively.

The gross benefit of total inflow to a hydropower plant j (B_{jt}, 10^6\$) is derived from its inverse demand function:

$$B_{jt} = \begin{cases} P_0(j,t)POW(j,t), \quad \left(0 \le POW(j,t) \le POW_0(j,t)\right) \\ P_0(j,t)POW_0(j,t) + \dfrac{\left(1/\alpha(j,t)\right)^{1/\beta(j,t)}}{1+1/\beta(j,t)}\left[POW(j,t)^{(1+1/\beta(j,t))} - POW_0(j,t)^{(1+1/\beta(j,t))}\right], \\ \qquad \left(POW(j,t) > POW_0(j,t), \beta(j,t) \ne -1\right) \\ P_0(j,t)POW_0(j,t) + \alpha(j,t)\left[\ln POW(j,t) - \ln POW_0(j,t)\right], \quad \left(POW(j,t) > POW_0(j,t), \beta(j,t) = -1\right) \end{cases} \tag{5.10}$$

Then, the net benefit function NB_{jt} (10^6\$) of a hydropower plant can be represented in the following general form

$$NB_{jt} = B_{jt} - POW(j,t)pc(j,t) - \sum_{(k,j)\in L} Q(k,j,t)wc(k,j,t), \forall j \in HPP \tag{5.11}$$

where $pc(j,t)$ is the power production cost (\$/kWh) for hydropower station j during period t.

5.2.3 Net Benefit Functions of Agricultural Demand Sites

Agricultural water uses include irrigation and livestock watering. If the effect of water quality is not considered, the value of water for agricultural uses can be expressed by the inverse demand function in the constant elasticity form like *MI* demands (Mahan *et al*., 2002). Quadratic functions are also often used, which may also include water quality items. For example, Booker and Young (1994) use quadratic functions to estimate irrigation profits at varying water diversion and salt discharge levels. It should be pointed out that irrigation profit functions in the literature are normally derived over the annual or seasonal period, and they are extended to smaller time periods (growing stages or months) to be used by the hydro-economic river basin model. In this model, the water quality factor is considered, and the gross benefit functions of effective inflow (B_{jt}, 10^6\$) to agricultural demand sites (*AGR*) are assumed to be in the following quadratic form:

$$\begin{aligned} B_{jt} &= b_0(j,t) + b_1(j,t)Q(j,t) + b_2(j,t)Q(j,t)^2 \\ &+ \sum_p \left[b_{3p}(j,t)C_{pN}(j,t) + b_{4p}(j,t)C_{pN}(j,t)^2 + b_{5p}(j,t)Q(j,t)C_{pN}(j,t)\right], \ \forall j \in AGR \end{aligned} \tag{5.12}$$

where, $C_{pN}(j,t) = \frac{1}{Q(j,t)}\left[Q_a(j,t)C_{pa}(j,t) + \sum_{(k,j)\in L} Q(k,j,t)C_p(k,j,t)\left(1 - e_{pL}(k,j,t)\right)\right]$ is the mixed concentration of pollutant *p* of the total inflow to the demand node j, $C_p(k,j,t)$ is the concentration of pollutant *p* in link flow (g/l), and $e_{pL}(k,j,t)$ is the pollutant *p* loss coefficient for link (*k, j*) in period *t*. $C_{pa}(j,\ t)$ is the pollutant *p* concentration in node adjustment. The coefficients b_0 to b_{5p} may be obtained by econometric methods or by regression analysis from the output of external simulation or optimization models. The net benefit NB_{jt} (10^6\$) of an agricultural demand site is:

$$NB_{jt} = B_{jt} - \sum_{(k,j)\in L} Q(k,j,t)wc(k,j,t), \forall j \in AGR \tag{5.13}$$

where, $wc(k,j,t)$ is the water supply cost (\$/m^3).

5.2.4 Net Benefit Functions of Stream Flow Requirement Demand Sites

Because instream recreational opportunities, aquatic ecology and environment quality are not generally goods sold in a market, estimating the benefits from water use for stream flow demands requires unique economic valuation approaches such as the travel cost method and the contingent valuation method (Brown *et al*., 1991). The analytical form of the demand curve for stream flow requirement (SFR) demand sites adopted in this model is adapted from Diaz *et al*. (1997). The marginal value of stream flow, expressed in dollars per million cubic meter (\$/10^6m^3), is assumed to a linear function of flow and pollutant concentration. The general form of the gross benefit function is represented as a quadratic function, which is same as that of agricultural demands. The net benefit function is also the same as that of agricultural demand.

5.2.5 Net Benefit Functions of Reservoirs

The reservoir recreation benefit may be expressed as a hyperbolic tangent function as well as a quadratic function of the water stored in the reservoir (Diaz *et al*., 1997). In order to include the water quality items in the function also, quadratic form functions are adopted in this model. Assuming that the aggregate value of reservoir recreation activity, aquatic ecology and environmental quality is related to water level in the reservoir, the gross benefit function of reservoir (*RES*) storage is expressed in the quadratic form similar to those of agricultural and stream flow demand sites, except that the items of flow volumes are changed to reservoir storages:

$$\begin{aligned} B_{jt} &= b_0(j,t) + b_1(j,t)\bar{S}(j,t) + b_2(j,t)\bar{S}(j,t)^2 \\ &+ \sum_p \left[b_{3p}(j,t)C_p(j,t) + b_{4p}(j,t)C_p(j,t)^2 + b_{5p}(j,t)\bar{S}(j,t)C_p(j,t) \right], \forall j \in RES \end{aligned} \tag{5.14}$$

where, $\bar{S} = \left(S(j,t-1) + S(j,t)\right)/2$ is the average storage of reservoir *j* during period *t*; $C_p(j,t)$ is the mixed concentration of pollutant *p* in the reservoir. The coefficients b_0 to b_{5p} may be obtained by econometric methods. The net benefit function of reservoir storages is same as that of agricultural demands.

Assuming pollutant concentrations remain constant with small changes of water withdrawal, inverse demand functions (marginal values) for withdrawal of raw water can be obtained by derivation of the net benefit functions, $P(j,t) = \partial NB(j,t) / \partial \bar{S}(j,t)$ for reservoirs, and $P(j,t) = \partial NB(j,t) / \partial Q(k,j,t)$ for other types of demand sites, respectively.

5.3 Irrigation Water Planning Model

The irrigation water planning model (IWPM) at farm level is designed with a one-year horizon and 12-month periods, which maximizes the total profit of irrigated crop productions within an irrigation district by adopting quadratic empirical crop yield-water and salinity functions. When field survey data are limited, the economic benefit functions for irrigation water use may be derived using this model. The model is similar to other agricultural production models commonly used in irrigation water planning and management, but provides monthly benefit functions. The estimation of monthly benefit functions adopts a method similar to that utilized in the State Agricultural Production model developed by Draper *et al.* (2003).

5.3.1 Balances among Crop Fields

Every irrigation node is considered to consist of a number of crop fields during each time period (month), and each field is characterized with a representative crop type. Let J be the set of irrigation nodes, CP be the set of crop types, and T be the set of time periods. Then the sets of possible crop fields and crop growing stages can be defined as $FD = \{(j,cp): j \in J, cp \in CP, AF^u{}_{j,cp} > 0\}$ and $ST = \{(j,cp,t): j \in J, cp \in CP, t \in T, AF^u{}_{j,cp} > 0, \text{and } ETm_{j,cp,t} > 0\}$, respectively. $AF^u{}_{j,cp}$ is the upper limit of crop field area, and $ETm_{j,cp,t}$ is the maximum potential evapotranspiration of crop field during each time period. Note that, irrigation nodes may plant different types of crops, and most crop growing seasons are shorter than a year. Without being specified, all subscripts of variables and parameters presented in the following equations are subject to the predefined patterns of crop fields and growing stages, and, hence, $(j,cp) \in FD$, and $(j,cp,t) \in ST$.

Within an irrigation demand site, water diverted from rivers, reservoirs and aquifers is mixed, and then allocated to each crop field (*j, cp*). Figure 5.2 illustrates water balances of a small farm with four crop fields. The total inflow is allocated to crop fields within the irrigation farm *j*, subject to the following balance equations:

$$Q_{j,t} = \sum_{cp} q_{j,cp,t} \tag{5.15}$$

$$C^p_{j,t} = c^p_{j,cp,t} \tag{5.16}$$

where, $Q_{j,t}$ is irrigation water allocated to demand node *j* ($10^6 m^3$); $C^p_{j,t}$ is the mixed concentration of pollutant *p* in irrigation water allocated to demand node *j* (g/l); $q_{j,cp,t}$ is the irrigation water allocated to crop field (*j*, *cp*) ($10^6 m^3$); $c^p{}_{j,cp,t}$ is concentration of pollutant *p* in the irrigation water allocated to crop field (*j, cp*) (g/l).

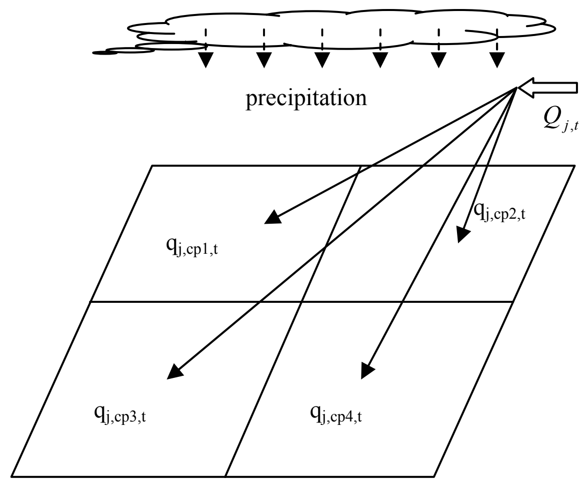

Figure 5.2 Water balances within a simple farm

5.3.2 Crop Production Functions: Yield - Water and Salinity Relationships

Crop-water relationships are very complicated and not all management issues have been addressed in one comprehensive model. Crop production functions can be identified as having four categories: evapotranspiration and transpiration models, simulation models, estimated models, and hybrid models (McKinney *et al*., 1999). Evapotranspiration models utilize linear yield-evapotranspiration relationships, which assume crop production is a high producing variety, well-adapted to the growing environment, and growing in fields where optimum agronomic and irrigation practices except for water are provided. Although evapotranspiration and transpiration models capture important aspects of crop-water relationships, they have limited ability to capture the impacts of non-water inputs, and are of limited use for policy analysis. The simulation models simulate the crop production process in detail, while hybrid models combine aspects of the other three types. Among these model types, estimated production functions are more flexible than other types of models, and polynomial or quadratic functions are most widely used. Theoretically, each type of crop in an irrigation region has its own specific crop-water function, since it is derived subject to a series of hydrological, physical and policy constraints. The functions can be estimated through regression methods based on the results of a simulation model on a number of inputs.

The quadratic polynomial form of crop production functions proposed by Dinar and Letey (1996) is adopted here, but the water and salinity inputs are redefined to be quantity and mixed concentrations of the seasonally total water available to a crop field. The quadratic function is expressed as follows:

$$Ya/Ym = f(w,s,u) = a_0 + a_1 w + a_2 s + a_3 u + a_4 w\cdot s + a_5 w\cdot u + a_6 s\cdot u + a_7 w^2 + a_8 s^2 + a_9 u^2 \quad (5.17)$$

where, Ya is the actual crop yield (metric tons (mt)/ha); Ym is the maximum potential crop yield (mt/ha); $w = WA/ETm$, is ratio of the total available water WA (mm) to maximum seasonal potential evapotranspiration of the crop ETm(mm); s is salinity of the irrigation water (dS/m), 1dS/m=0.64 ~0.8 g/l; u is irrigation uniformity; and a_i (i=1,···, 9) are estimated coefficients.

Note that the subscripts (*j, cp*) of variables and parameters in the above crop production equation are omitted for simplicity. The parameter of uniformity (u) is used as a surrogate for both irrigation technology and irrigation management. The uniformity values are about 50, 70, 80 and 90 for flood irrigation, furrow irrigation, sprinklers and drip irrigation, respectively (Rosegrant *et al*., 2000). The total water available for a crop field includes effective precipitation and effective irrigation water application as well as pre-season soil moisture. Ya/Ym is maximized when w is greater than unity. One of the reasons for this is at any time when the total available soil moisture is less than a fraction of the soil water depth between the soil water content at field capacity and the soil water content at wilting point, actual evapotranspiration *ETa* will not be able to reach *ETm* (Doorenbos and Kassam, 1979). Non-optimum crop growing environment, agronomic and irrigation practices may also cause this.

Assuming the crop areas and irrigation efficiency maintain constant throughout the growing season, then (Mahan, 1997):

$$WA_{j,cp} = SM_{j,cp} + \sum_t WE_{j,cp,t} = SM_{j,cp} + \sum_t \left[EP_{j,cp,t} + \left(q_{j,cp,t} / AF_{j,cp} \right) \cdot EIR_{j,cp} \times 10^5 \right] \quad (5.18)$$

where, $WA_{j,cp}$ is the total water available to a crop field during whole growing season (mm); $SM_{j,cp}$ is soil moisture in the seasonal full root zone at the beginning of the crop growing season (mm); $WE_{j,cp,t}$ is effective precipitation and effective water application to a crop field during period t (mm); $EIR_{j,cp}$ is irrigation efficiency; $AF_{j,cp}$ is crop areas (ha); and $EP_{j,cp,t}$ is effective precipitation (mm). The water application efficiency of irrigation $EIR_{j,cp}$ is defined as the ratio of average depth of water stored in the zone root over average applied depth (Cai *et al*., 2003). The effective precipitation $EP_{j,cp,t}$ is the precipitation infiltrated into the root zone and available for crop use. *EP* depends on total precipitation, soil moisture content, reference crop evapotranspiration, and soil characteristics (moisture at field capacity, hydraulic conductivity, etc.), and can be estimated by the evapotranspiration/precipitation ratio method (USDA, 1967). For simplicity, if it is assumed the infiltrated precipitation can be effectively used by crops at a ratio of $EIP_{j,cp}$, then the effective precipitation is approximated as:

$$EP_{j,cp,t} = IP_{j,cp,t} \cdot EIP_{j,cp} \tag{5.19}$$

Based on the concepts of irrigation efficiency and effective precipitation, return flows, deep percolation and irrigation water demand can be approximately estimated by the following simplified balances. Percolation ($PN_{j,cp,t}$, 10^6m^3) of a crop field is the amount of water leaving root zones to downward soil layers and is inaccessible to plant roots. The percolation includes excessive irrigation water and excessive water from infiltrated precipitation ($IP_{j,cp,t}$, mm).

$$PN_{j,cp,t} = q_{j,cp,t}(1 - EIR_{j,cp}) + (IP_{j,cp,t} - EP_{j,cp,t}) \times AF_{j,cp} / 10^5 \tag{5.20}$$

Part of the percolation forms the return flow to the river system, which may be in the forms of surface drainage or sub-surface drainage. Let $EDN_{j,cp}$ be the drainage efficiency, the ratio of drainage over percolation from the root zone of crop field. Then the total return flow ($DN_{j,t}$, 10^6m^3) can be approximated as:

$$DN_{j,t} = \sum_{cp} \left(PN_{j,cp,t} \cdot EDN_{j,cp} \right) \tag{5.21}$$

Thus, the total deep percolation from an irrigation demand site to the underlined aquifer ($DP_{j,t}$, 10^6m^3) is the difference between percolation and return drainage:

$$DP_{j,t} = \sum_{cp} PN_{j,cp,t} - DN_{j,t} \tag{5.22}$$

The corresponding node consumption, return flow and deep percolation coefficients of the irrigation demand nodes which are used in the river basin scale water allocation model can be derived as the following equations, respectively:

$$e_N(j,t) = \sum_{cp} q_{j,cp,t} EIR_{j,cp} / Q(j,t) \tag{5.23}$$

$$e_{RE}(j,t) = \sum_{cp} q_{j,cp,t}(1 - EIR_{j,cp}) EDN_{j,cp} / Q(j,t) \tag{5.24}$$

$$e_{DP}(j,t) = 1 - e_N(j,t) - e_{re}(j,t) \tag{5.25}$$

Assuming all the actual evapotranspiration comes from the effective precipitation and irrigation, then the irrigation water demand ($QD_{j,t}$, 10^6m^3) of an agricultural demand site during period t can be approximated as:

$$QD_{j,t} = \sum_{cp} \max\left(mp_{j,cp} \cdot ETm_{j,cp,t} - EP_{j,cp,t}, 0\right) \cdot AF_{j,cp} \Big/ \left(10^5 EIR_{j,cp}\right) \tag{5.26}$$

where, $mp_{j,cp}$ is the on-farm irrigation management practice factor of the total moisture required for optimal yields. Typical irrigation practice factors are normally at about 80% evapotranspiration (ET) level, and are less than 90% ET level, since it may be uneconomical to divert at a higher level (AIPA, 2002).

Assuming the pollutant consumption ratio of a crop field is $EPC_{j,cp}$, and ignoring the pollutants in precipitation, then the mass balance of pollutant p at the crop field is approximated as:

$$PN_{j,cp,t} \cdot CPN^p{}_{j,cp,t} = q_{j,cp,t} \cdot C^p{}_{j,cp,t}(1 - EPC^p{}_{j,cp}) \tag{5.27}$$

where, $CPN^p{}_{j,cp,t}$ represents the pollutant concentration of percolation (g/l). Assuming the concentrations of monthly return drainage and deep percolation from the crop field are to be equal to $CPN^p{}_{j,cp,t}$, then the concentrations of monthly return drainage ($CDN^p{}_{j,t}$) and deep percolation ($CDP^p{}_{j,t}$) from the whole irrigation node can be estimated as follows:

$$CDN^p{}_{j,t} = \sum_{cp} \left(PN_{j,cp,t} \cdot CPN^p{}_{j,cp,t} \cdot EDN_{j,cp} \right) \Big/ DN_{j.t} \tag{5.28}$$

$$CDP^p{}_{j,t} = \sum_{cp} \left[PN_{j,cp,t} \cdot CPN^p{}_{j,cp,t} \cdot (1 - EDN_{j,cp}) \right] \Big/ DP_{j,t} \tag{5.29}$$

5.3.3 Irrigation Water Planning Model

The irrigation water planning model searches for the optimal irrigation water allocation among crop fields to maximize the profit from irrigation at a given agricultural demand site. The total profit ($\widetilde{B}_j$, 10^6\$) of irrigation water use at demand site j is calculated based on the empirical seasonal crop-water yields functions, which is expressed as follows:

$$\tilde{B}_j = 10^{-6} \left[\sum_{cp} \left(pcp_{j,cp} - cc_{j,cp} \right) Ya_{j,cp} \cdot AF_{j,cp} - \sum_{cp} fc_{j,cp} \cdot AF_{j,cp} - \sum_{cp} vc_{j,cp} \cdot FT_{j,cp} \cdot AF_{j,cp} \right] \tag{5.30}$$

where, $pcp_{j,cp}$ is the crop price (\$/mt); $cc_{j,cp}$ is the cultivation cost (\$/mt); $fc_{j,cp}$ is the fixed cost of crop production (\$/ha), which consists of expenditures on machinery, labor, irrigation, and other production costs; $vc_{j,cp}$ is the cost of fertilizer (\$/kg); $FT_{j,cp}$ is the variable fertilizer application (kg/ha), which causes the variable cost on fertilization. The fertilizer application is assumed to be a linear function of water available to crop, $FT_{j,cp} = c_{0\,j,cp} + c_{1\,j,cp} \cdot WA_{j,cp}$, where $c_{0\,j,cp}$ and $c_{1\,j,cp}$ are coefficients.

Although the seasonal crop yield function drives the optimal seasonal water allocation among crops, it cannot distribute the water within the crop growth season according to the maximum potential evapotranspiration requirements of specific growth stages ($ETm_{j,cp,t}$). In order to achieve consistency between the seasonal yield function and the water balance of all time periods, a penalty item is introduced into the objective function to minimize the difference between the minimum and average crop stage yields of each crop field, which may occur if the seasonal irrigation water application is not sufficient. The objective function to maximize the penalized total profit of irrigated crop production of demand *j* is specified as:

$$\max \widetilde{B}_j - pen_j \tag{5.31}$$

The penalty item (pen_j, 10^6\$) is defined as:

$$pen_j = 10^{-6}\left[\sum_{cp} pcp_{j,cp} Ym_{j,cp} \cdot AF_{j,cp}\left(aft_{j,cp} - mft_{j,cp}\right)\right] \tag{5.32}$$

where $aft_{j,cp}$ and $mft_{j,cp}$ are the average and minimum stage yields by crop *cp* and demand site *j*, respectively. The stage yield deficits are calculated by seasonal crop yield-water function *f* as shown in (5.17), but the item of total available soil water is changed to the effective precipitation and irrigation,

$$ft_{j,cp,t} = f\left(\frac{WE_{j,cp,t}}{ETm_{j,cp,t}}, s_{j,cp,t}, u_{j,cp}\right) \tag{5.33}$$

where $s_{j,cp,t}$ is the salinity of irrigation water to the crop field (*j, cp*) during period *t*, and $u_{j,cp}$ is the irrigation uniformity parameter of the crop field. This approximation is acceptable due to the absence of stage crop yield-water functions and the assumption that all the actual crop evapotranspiration comes from the effective precipitation and irrigation.

The objective function is subject to the following sets of constraints in addition to the water and salinity balance equations and crop yield functions:

(1) Total land limit (A_j, ha)

$$\sum_{cp:(j,cp,t)\in ST} AF_{j,cp} \le A_j \qquad (\forall t \in T) \tag{5.34}$$

(2) Minimum and maximum area limits for each crop

$$AF^{l}{}_{j,cp} \le AF_{j,cp} \le AF^{u}{}_{j,cp} \quad (5.35)$$

(3) Total water volume limit ($\widetilde{Q}_j$, $10^6 m^3$)

$$\sum_t Q_{j,t} \le \widetilde{Q}_j \quad (5.36)$$

(4) Polluant concentration restriction

$$C^{p}{}_{j,t} = \widetilde{C}^{p}{}_{j} \quad (5.37)$$

(5) Limits on stage (monthly) effective precipitation and water application available for crop use

$$\lambda_l ETm_{j,cp,t} \le WE_{j,cp,t} \le \lambda_u ETm_{j,cp,t} \quad (5.38)$$

where, A_j is the total available crop area of demand site j; $AF^{l}{}_{j,cp}$ and $AF^{u}{}_{j,cp}$ are the lower and upper limits of crop areas, respectively; $\widetilde{Q}_j$ and $\widetilde{C}^{p}{}_{j}$ are the total available irrigation water and corresponding pollutant concentration; λ_l and λ_u are the lower and upper limits of effective precipitation and irrigation. While the first, third and fourth constraints are obligatory, the second and fifth constraints are optional. However, the planner can introduce them to take into account other inexplicitly defined limitations such as technique or market constraints.

Since the IWPM model searches for an optimal schedule of irrigation not only among crop fields but also for all growing stages, the maximization of benefit of crop production will make the water allocated to the same irrigation water demand site during different time periods have equal marginal benefits, as long as water allocations $Q_{j,t}$ are not at their lower and upper bounds.

With a series of inputs of annual available amounts and pollutant concentrations of irrigation water, the optimal total profit of crop productions, monthly irrigation quantities allocated to the demand site and corresponding pollutant concentration for each period are obtained by solving the model. The benefits of irrigation water are calculated by deducting the total profits with the base profit of crop production that would be achieved only with precipitation and no irrigation input. The benefits of irrigation water are then regressed to estimate the coefficients of the quadratic benefit functions of irrigation water that are utilized in the hydrologic-economic river basin model. Although the regression by normal multiple regression using the least squares method can produce stage (monthly) benefit functions that fit the total profit very well, they may not correctly represent the real relationships between monthly profits and irrigation. To solve this problem, constraints on the equality of marginal benefits of irrigation ($Q_{j,t}$) are introduced into the least squares optimization. The number of constraints on equal marginal benefits is less than the number of parameters to be calibrated, but is large enough. The negative sum of squares of the differences of marginal benefits between any two months under every water availability scenario is also added as a penalty to the objective of the regression program.

5.4 Coalition Analysis and Reallocation of Water and Net Benefits

Normally hydrologic-economic models are used in maximizing the total net benefit of a group of water uses in a river basin and searching for optimal water allocation schedules. The second stage of the cooperative model developed in the thesis not only aims for economic optimal water allocation but also investigates how this can be achieved by the stakeholders of the water uses in an equitable way through economic incentive or water trade in the water market. This process of reallocation of water and net benefits is analyzed by the application of cooperative game theoretical methodologies, in which coalition analysis plays a key role. In cooperative water allocation games, the values (net benefits in the model) of various coalitions are estimated by the hydrologic-economic optimal river basin model (HERBM).

Recall that the payoff $v(S)$ of a coalition S is defined as the maximum total net benefit, $NB(S)$, that coalition S can gain based on coalition members' water rights over the entire planning period, subject to not decreasing the water flows and not increasing the pollutant concentrations in the flows to other stakeholders not taking part in coalition S. Under more cooperative environment or regulated policies, the upper bound limits of pollution concentrations in the above definition may be ignored.

The algorithm for coalition analysis is quite computationally intensive. Ideally, all potential coalitions of stakeholders should be considered for the cooperative game analysis. However, the large number of potential coalitions among stakeholders in a river basin would make the gaming analysis unrealistic if each stakeholder is considered as a totally independent individual. Recall that there are 2^n-1 possible nonempty coalitions for a game involving *n* stakeholders. In cases where the number of stakeholders is large, individual stakeholders have to be classified into stakeholder-groups according to the types of water uses. In the algorithm, the coalition value is assumed to be equal to the one obtained with initial water rights and no water reallocation performed, if all the water uses involved in a coalition have been allocated initial water rights satisfaction ratios greater than 99.9%. Furthermore, if all the withdrawals and corresponding net benefits of a stakeholder obtained under the river basin optimal allocation situation are the same with (or have very small change from) those obtained with its initial water rights, then the stakeholder may be excluded from the coalition analysis. The reduction of the number of stakeholders would drastically decrease the computational effort and time.

The following inequality expresses a property of a coalition in which the value of a coalition should not be less than that can be obtained by the initially allocated water rights of its members.

$$NB(S) \geq \sum_{i\in S} \sum_{j\in U_i} \sum_{t\in T} NB_{Rijt} \tag{5.39}$$

where, NB_{Rijt} is the net benefit of water use obtained based on the initially allocated water right. This relationship is added as a constraint to help the algorithm to find proper solutions.

As shown in the flowchart in Figure 5.3, the algorithm for coalition analysis consists of the following steps:

1. Define the primary set of stakeholders *SH*, and the set of water use ownerships *owner*(*sh, j*);
2. Select the set of stakeholders participating in the grand coalition, $N=\{1,2,\cdots, n\}$;
3. Generate coalition indices. The number of coalitions is 2^n-1;
4. Convert the coalition indices into binary numbers. Every digit of a binary number represents, if the corresponding orderly indexed stakeholder is participating in the

coalition (*1-Yes*), or not (*0-No*). For example, if the grand coalition consists of eight stakeholders, there are 255 coalitions in total, and the tenth coalition, *S10*, is converted into a binary number *00001010*, which specifies that *S10* consist of the second and the fourth stakeholders;

5. Define the set of membership, *memb*(*S, i*), for all possible coalitions according to the binary represented coalition indices;
6. Set the option whether the net benefits of coalitions will be calculated by strict or relaxed limits on water quality rights protection;
7. Read the input of hydrologic data, initial water rights, water demand and benefit functions;
8. Sequentially solve the river basin hydrologic-economic model (HERBM) for each coalition utilizing the multistart global optimization technique. Output the optimal water allocation scheme and optimal net benefits of water uses under each coalition scenario;
9. All the net benefits of coalitions are saved in an external file for further reallocation of the grand coalition net benefit by cooperative game solution concepts.

The algorithm is coded in GAMS, and the highly nonlinear HERBM is solved by the OQNLP and MINOS solvers. OQNLP utilizes a multistart global optimization technique to make heuristic scatter search for good starting points, and then MINOS is called to find local optima with the projected Lagrangian method for each trial point. Based on all the local optima found for a coalition, the maximal one is selected as the approximate global optimal net benefit of the coalition. The combination of search and gradient-based solvers in a multistart procedure by OQNLP solver takes advantages of the ability of a search method to locate an approximation to a good local solution (often the global optimum), and the strength of a gradient-based NLP solver to find accurate local optimal solutions rapidly (GAMS, 2005).

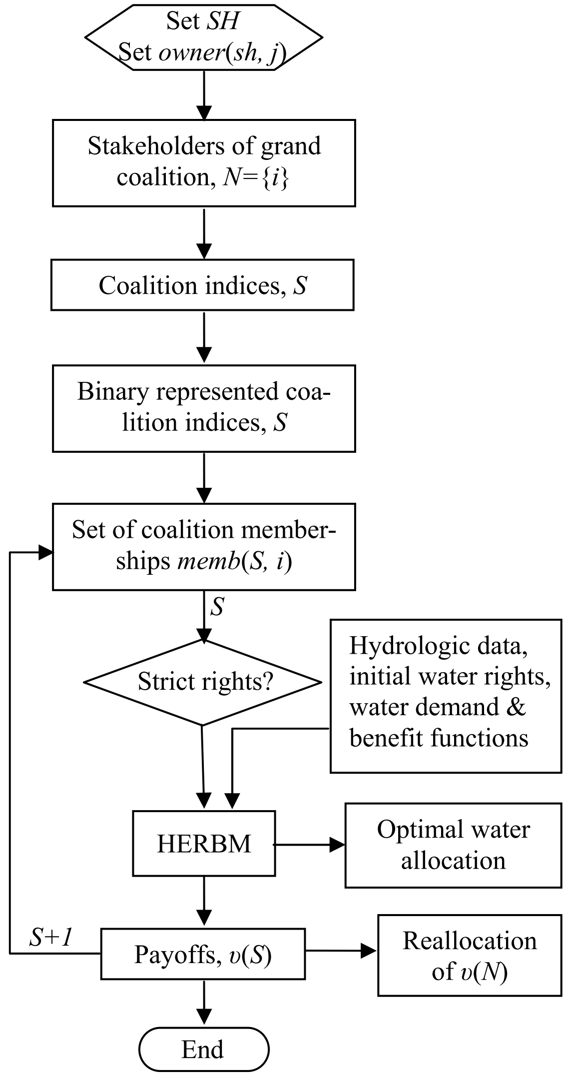

Figure 5.3 Flowchart of the algorithm for coalition analysis

5.5 Summary

The chapter firstly describes an integrated hydrologic-economic river basin model (HERBM) for economic optimal water resources allocation among competing uses within a region. HERBM searches the optimal economic net benefits of demand sites with monthly net benefit functions of various demand sites. The constant price-elasticity water demand functions are utilized to derive the gross benefit functions of water uses of municipal and industrial demand sites, and hydropower stations. Quadratic benefit functions accounting for both the water quantity and quality aspects are adopted for agriculture water uses, stream flow demands and reservoir storages. An irrigation water planning model (IWPM) has been designed as an offline external tool to derive the monthly gross profit functions of effective irrigation water uses. Different from other models, the hydrologic-economic river basin model developed in this thesis adopts net benefit functions for all uses including irrigation with monthly periods rather than seasonal or annual periods. This allows one to integrate the two models at the different scales through a “compartment modeling” approach, which greatly reduces the size of the basin scale water allocation problem and corresponding computational effort, and makes model formulation more flexible so that the monthly benefit and demand functions can be estimated by econometric methods or through more detailed sub-models for some demand sites. The estimation of parameters of benefit functions of water uses should be carefully carried out. Since the assumption of the IWPM is that the annual water can be optimally allocated to all crop fields and for all time periods, subject to land area and irrigation water limits, the application of monthly benefit functions of irrigation water by the model should follow these assumption and limits. As reservoirs can carry over water storages to meet water demand at different time periods, the derived optimal monthly benefit functions are normally applicable to river basins with storage reservoirs.

The hydrologic-economic river basin model (HERBM) is modified and extended to estimate the net benefits of various coalitions. Theoretically, various coalitions of stakeholders should be considered at the cooperative game modeling stage, and then cooperative game methods are used to derive the fair reallocation of the benefit of the grand coalition gained for the whole basin. However, the large number of potential coalitions among stakeholders would make the gaming analysis unrealistic if each stakeholder is considered as a totally independent individual, and thus individual stakeholders need to be aggregated into stakeholder-groups according to the types of water uses. Both the IWPM and HERBM are coded in GAMS. IWPM is solved by the MINOS solver with the projected Lagrangian method, while HERBM is solved by the combination of OQNLP and MINOS solvers utilizing a multistart global optimization technique. The models and algorithms developed in this chapter will be examined and applied to the case study in Chapter 6.

Chapter 6
Cooperative Water Resources Allocation in the South Saskatchewan River Basin

6.1 Introduction

This chapter presents an application of the Cooperative Water Allocation Model (CWAM) developed in previous chapters to the South Saskatchewan River Basin in southern Alberta, Canada, investigating how to fairly and efficiently allocate water resources to competing demand sites at the river basin scale. The South Saskatchewan River Basin (SSRB) comprises four sub-basins: the Red Deer, Bow and Oldman River sub-basins and the portion of the South Saskatchewan River sub-basin located within Alberta, as shown in Figure 6.1. This area accounts for only seven percent of Alberta's total annual flow but supports about half the province's population (Dyson *et al*., 2004). Water uses including agricultural, municipal, industrial, hydropower generation, fisheries, recreation, and effluent dilution are competing for limited water resources. Furthermore, the downstream province Saskatchewan also relies on the flow of the South Saskatchewan River to support its agriculture. Population growth and economic expansion are increasingly stressing water resources in the South Saskatchewan River Basin. Improving efficiency in water use, such as by the adoption of market mechanisms, may reduce this stress. The Government of Alberta has carried out several research projects on management of water resources in this basin, and completed the SSRB Water Management Plan. Phase One of the Water Management Plan was approved in June 2002 and the plan authorizes water allocation transfers within the SSRB, subject to Alberta Environment approval and conditions (Alberta Environment, 2002a). Phase Two addresses water management issues, and Alberta Environment's Water Resources Management Model has been utilized to simulate and evaluate the water allocations in the SSRB by matching the water supplies during the historical period 1928-1995 with demands on a weekly basis under different scenarios (Alberta Environment, 2003b). However, the SSRB study lumps the large number of withdrawal licenses into only two categories: senior and junior. Furthermore, the model used is a simulation model, which is not able to search optimal allocation schemes over multiple time periods and at the basin scale.

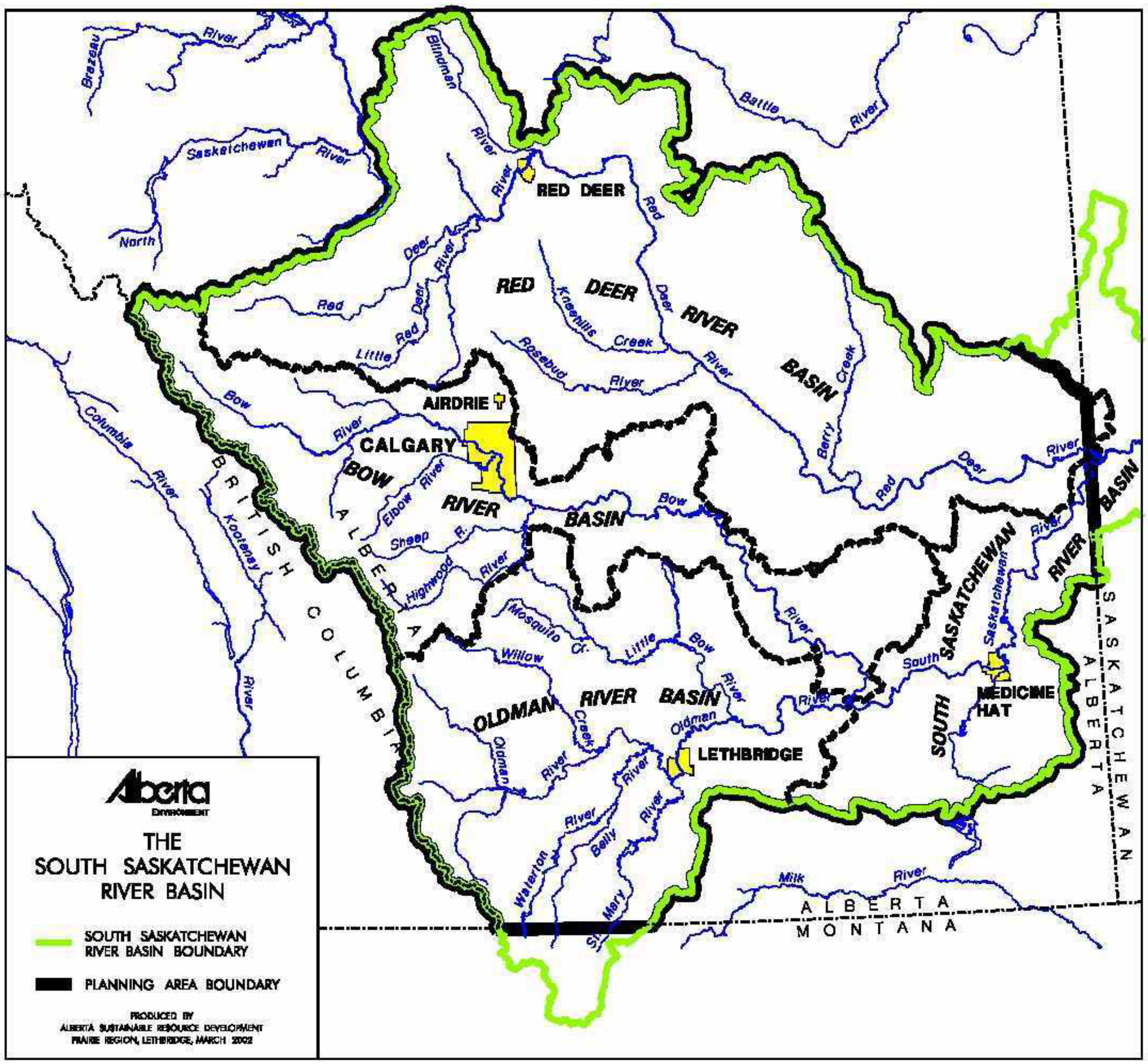

Figure 6.1 The South Saskatchewan River Basin within the Province of Alberta (Alberta Environment, 2002a)

As water transfers are proposed to mitigate shortages in times of low water supplies and high demands, the fairness in the water transfers and trades is a major concern. The fairness issue arises in two aspects: one is the equity in water rights allocation, and the other is the equity in water trade, i.e., in the water and associated net benefits reallocation (Wang *et al.*, 2003b). Section 6.2 gives a brief introduction to the river basin, including water availability, uses and allocation policy. Section 6.3 describes the river basin network, data sources and estimation of input data. Section 6.4 firstly specifies six case scenarios of water allocation under wet, normal, and assumed drought hydrologic conditions, and then discusses the major results of initial water rights allocation by the Priority-based Maximal Multiperiod Network Flow (PMMNF) programming method and Lexicographic Minimax Water Shortage Ratios (LMWSR) method. Based on the allocated initial water rights, values of various coalitions are modeled by the coalition analysis component of the CWAM, and equitable reallocation of the net benefits of all stakeholders participating in the grand coalition are carried out using cooperative game theoretic approaches.

6.2 The South Saskatchewan River Basin (SSRB)

6.2.1 Geography, Climate and Land Uses

The South Saskatchewan River Basin in southern Alberta drains about 120,000 square kilometers. Elevations range from 3,600 meters at the region's highest peak to less than 700 meters in the southeast prairie (Dyson *et al*., 2004). Headwaters of the basin originate from snow packs in the Rocky Mountains of southwestern Alberta and eventually flow into Hudson Bay. The Red Deer River sub-basin is located in the north, whose main surface water source is the Red Deer River. The Bow River sub-basin is bounded by the Red Deer River sub-basin to the north, the Oldman River sub-basin to the south, the Rocky Mountains to the west, and narrows to the South Saskatchewan River sub-basin in the east. The main upstream tributaries of the sub-basin are the Bow and Elbow Rivers. The Oldman River sub-basin receives flows from four main upstream tributaries: the Oldman River, The Waterton River, the Belly River and the St. Mary River. The South Saskatchewan River sub-basin's main water source is the South Saskatchewan River, which aggregates discharges from the Bow River, Oldman River and smaller tributaries.

The climate on Southern Alberta's prairie landscape is primarily semi-arid, characterized by abundant sunshine, and strong, dry winds. The average annual precipitation is 300 to 450 millimeters, less than half falling during the growing season lasting 160 to 185 days long

(Dyson *et al*., 2004). Snowmelt from the Rocky Mountains and flanking foothills supplies bulk river flows across southern Alberta in the spring, with levels declining in the summer and remaining low during the winter.

The area of the SSRB is predominantly a grassland ecosystem, except for the narrow swath of aspen parkland in the north and the Rocky Mountains and foothill landscapes to the west. The SSRB land use is primarily large and medium scale agriculture, producing commercial crops such as wheat and canola. Livestock production is also a main agricultural activity with large areas left for pasture. There are more than 1.5 million people living in the SSRB in 2004, about 81 percent living in urban centers along its major rivers and two principal highways. Canada's fifth-largest city, Calgary, has about one million residents in its greater metropolitan area. The other cities are Lethbridge (about 73,000 people), Red Deer (73,000 people), and Medicine Hat (51,000 people). All these urban centers have experienced considerable growth during the past decade, especially Calgary's satellite communities. The SSRB occupies less than 20 percent of Alberta's total area. However, it contributes about half of the province's economic revenue (Dyson *et al*., 2004). Petroleum, agriculture and manufacturing are major drivers in this economy, along with the growth of knowledge-based and value-added processing industries, particularly in the larger cities.

6.2.2 Water Availability and Uses

Historical natural flows in the SSRB have been computed based on daily recorded stream flows and project adjustments. The average annual amount from 1912 to 2001 is about nine billion cubic meters, representing seven percent of Alberta's total river flow (Dyson *et al*., 2004). During the more recent period from 1975 to 1995 the SSRB had a mean annual flow of about 8.4 billion cubic meters, with 17.9%, 43.6%, 38% and about 0.5% originating in the Red Deer, Bow, Oldman and South Saskatchewan sub-basins, respectively (Alberta Environment, 2002b). As shown in Figure 6.2, the natural flows have significant variability from one year to another. As water demands keep increasing the available water supply in some years will be unable to meet the high demands. For example, 2001 is one of the driest years, in which the total available natural flow is about 5 billion cubic meters and is less than the total allocation volume.

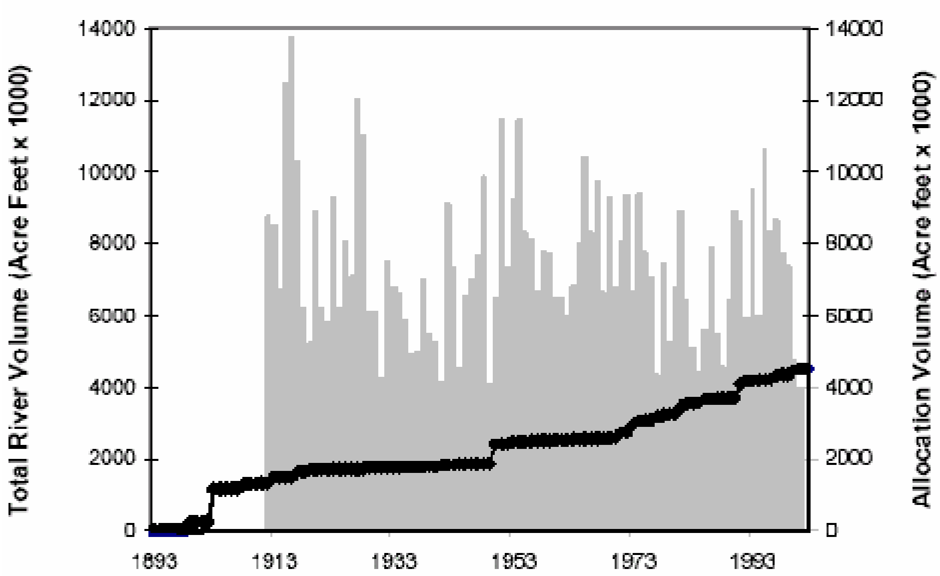

Figure 6.2 Historical annual natural flows (1912~2001) and allocation volumes (1893~2001) of the South Saskatchewan River Basin (Alberta Environment, 2003a)

Historic records also indicate that the 20th century was wetter than usual and that more prolonged droughts might thus be expected in the future, particularly if climate change warnings prove accurate (Dyson *et al.*, 2004). For example, the 1990s was the warmest decade on record in the southern Canadian prairies, with average temperatures at least one degree Celsius above the historic norm. In the critical headwaters of the SSRB, warmer temperatures mean glaciers are receding, snow packs are melting faster and more winter-spring precipitation is falling as rain instead of snow. On the prairies, higher temperatures mean more evaporation from rivers, reservoirs and fields.

Water uses in the SSRB are subject to two water-sharing agreements, the *1909 Boundary Waters Treaty* between Great Britain and the United States of America and the *1969 Master Agreement on Apportionment* for the South Saskatchewan River between Alberta and Saskatchewan. The first international agreement entitles the USA to have 25% of the flow in the St. Mary River for flows less than 666 cubic feet per second and 50% of the flow in excess of 666 cubic feet per second during the April 1 to October 31 irrigation season. The *1969 Master Agreement on Apportionment* requires that at least 50% of the annual natural flow of the SSRB in South Alberta shall be passed to Saskatchewan, and the instantaneous flow to Saskatchewan should be not less than half of the instantaneous natural flow of the SSRB in South Alberta or 42.5 m^3/s, whichever is less. The only exception to this general rule is, when annual natural flow falls below 5.2 billion cubic meters, Alberta can take more than 50% of the natural flow up to a maximum 2.6 billion cubic meters provided that the instantaneous outflow to Saskatchewan does not fall below 42.5 m^3/s at any time throughout the year (Alberta Environment, 2002b).

A large number of reservoirs exist in the SSRB, which are important components of both the headworks and the irrigation district infrastructure. For example, the Onstream Waterton, St. Mary and Oldman Reservoirs operated by Alberta Environment are large enough to store water during the high flow periods usually from early May to mid-July and to carry storage over to supply controlled releases to meet instream flow needs, and for consumptive uses and interprovincial apportionment commitments during low flow periods. The 8 offstream storage reservoirs operated by Alberta Environment and the 38 reservoirs owned and operated by the irrigation districts are all offstream reservoirs, which are used to accommodate seasonal variations in supply and demand (AIPA, 2002).

Irrigated agricultural water use is the dominant type of consumption sector. As shown in Figure 6.3, the storage reservoirs and canals form a complex water supply system for the 13 large irrigation districts. These irrigation districts cover more than 500,000 hectares of land relying almost exclusively on surface water: otherwise they would be exposed to considerable periods of drought. The Western headworks system diverts water at Calgary to Chestermere Lake and supply water for the Western Irrigation District. The Carseland-Bow River headworks system diverts water from the Bow River at Carseland to the Lake McGregor, Travers and Little Bow Reservoirs, and then to the Bow River Irrigation District. The Eastern headworks system takes water at the Bassano Dam on the Bow River to the Crawling Valley Reservoir and Lake Newell, and then supplies water to the Eastern Irrigation District. The Lethbridge Northern headworks system diverts water on the Peigan Reserve at Oldman River to Keho Reservoir, and is the supply source for the Lethbridge Northern Irrigation District. In the south of the Oldman River sub-basin, the Waterton St. Mary, Mountain View, Leavitt, Aetna and United Headwork systems divert and interconnect water from the Waterton, Belly and St. Mary Rivers. These canal systems provide water supply for Mountain View, Leavitt, Aetna and United Irrigation Districts, and for the downstream Magrath, Raymond, St. Marry River and Taber Irrigation Districts. The Canvan Lake headworks system diverts water from the Gros Ventre Creek to Cavan Reservoir and provides water to the Ross Creek Irrigation District (Alberta Environment, 2004). Irrigation infrastructure has similarly provided water for about 50 rural communities, thousands of farms and small rural industries (Dyson *et al*., 2004).

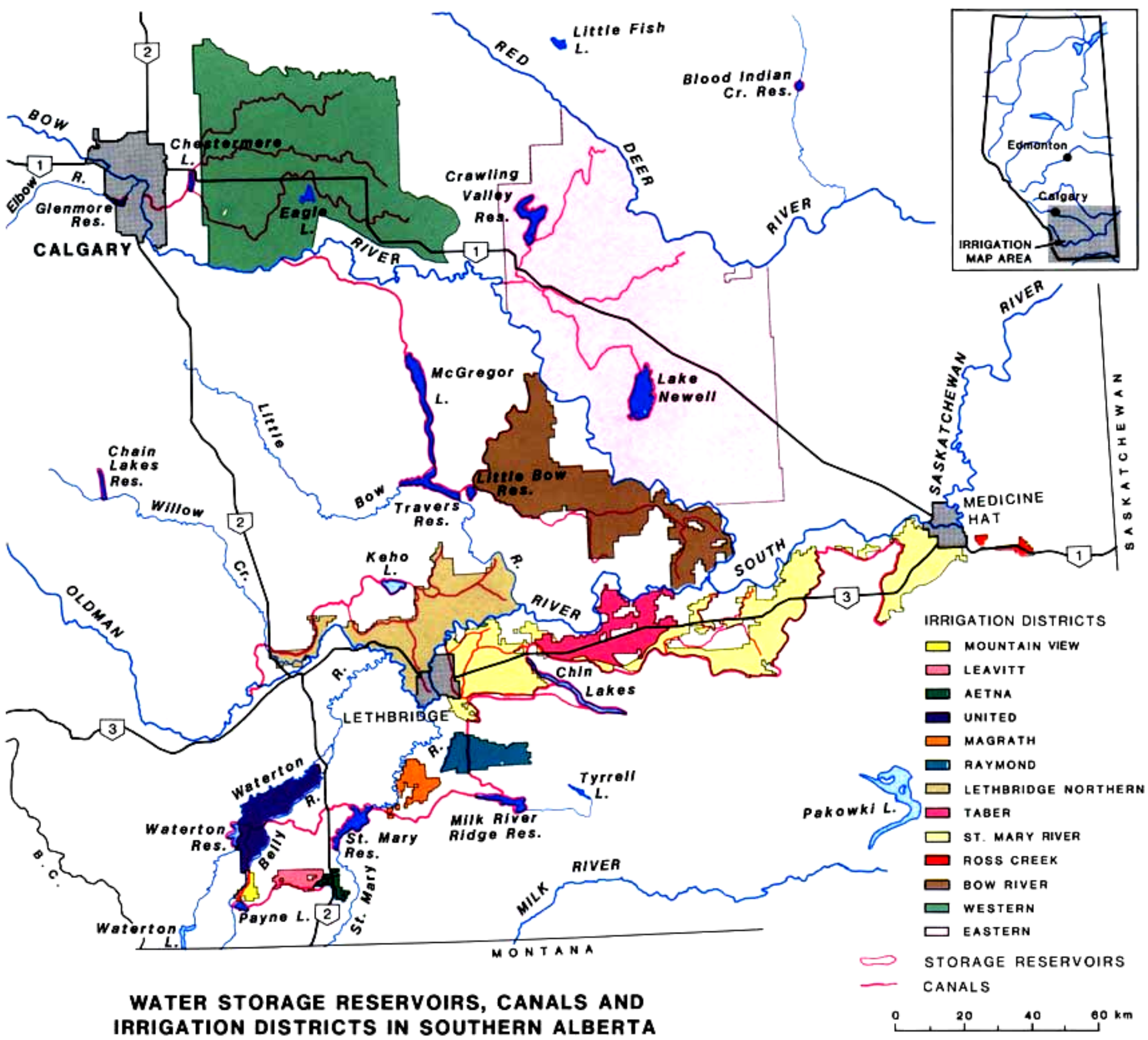

Figure 6.3 Water storage reservoirs, canals and irrigation districts within the South Saskatchewan River Basin (Mitchell and Prepas, 1990)

Larger municipalities have been major water users for municipal and industrial consumption. The region's four largest cities account for 90 percent of municipal withdrawals of surface water, while 100 rural communities depend on groundwater. The petroleum industry, Alberta's primary economic activity, extracts considerable amounts of water for processing oil and natural gas. The surface and ground water used for enhanced recovery amounts to 47.5 million cubic meters in 2001. Other growing industries, such as food processing and manufacturing plants, and several large petrochemical complexes, also withdraw water from the area's major rivers.

Non-consumptive water demands in the SSRB include hydropower generation, recreation, aquatic ecosystem, and effluent dilution. Some 10 hydroelectric dams have been built on the upper Bow River system since the 1890s, providing important electricity supply, especially during the peak demand periods. Visitors, including Albertans, participate in water-based recreation activities such as boating, swimming and fishing in the basin. The Bow River immediately downstream of Calgary and the Oldman River and its tributaries support world-class trout fisheries (Dyson *et al*., 2004).

6.2.3 Policy for Water Allocation in SSRB

Once international and interprovincial agreements are met, the remaining water in the SSRB is available for allocation. Under the *Water Act*, the Government of Alberta owns the rights to all waters within its borders and licenses are required for all appropriations or diversions other than those for traditional agricultural uses (6,250 cubic meters/year) and domestic uses (1,250 cubic meters/year). The licenses are assigned the maximum amounts of withdrawals and priorities on a first-in-time, first-in-right basis. In times of shortage, licenses owning higher priorities have privilege to divert water over those with lower priorities, regardless of the purposes of the water uses. There are approximately 20,000 licenses registered in the SSRB. In the SSRB, irrigation accounts for 75 percent of the allocation volume, followed by municipalities (13 percent), industry (3.7 percent) and other agriculture (1.7 percent) (Alberta Environment, 2003a). By August 1999, the volume of water licensed to the 13 irrigation districts was 3,434 million cubic meters. Including the allocation of about 375 million cubic meters to the more than 110,000 hectares of private irrigation, the total allocation for irrigation in the SSRB would be in the order of 3.8 billion cubic meters (AIPA, 2002). Compared to irrigation water use, the licensed diversion for stockwatering and feedlots is smaller, with a total of 77.9 million cubic meters in the whole basin as of 1996 (Hydroconsult and CRE, 2002).

6.3 The Modeling Approach and Input Data

The time horizon of the modeling is one year with 12 monthly time periods. The major data sources are the Water Survey of Canada’s HYDAT database (Environment Canada, 2002), the reports of previous studies (Mitchell and Prepas, 1990; Mahan, 1997; AIPA, 2002; Golder Associates Ltd., 2003; Alberta Environment, 2004), and the online information system of Alberta's River Basins. The schematization of the SSRB river basin network, and estimation of input data including monthly water demands and withdrawal permits, monthly water demand curves and benefit functions, and other parameters, are summarized in this section. Additional information on model input data is presented in Appendix B.

6.3.1 River Basin Network

The SSRB network is depicted in Figure 6.4. The network has 55 nodes in total, including 10 inflow (IN1~IN10), 1 outlet (O1), 17 reservoirs (R1~R17), 9 irrigation (A1~A9), 4 domestic (D1~D4), 4 general (G1~G4), 4 industrial (I1~I4), 2 hydropower plants (H1, H2), and 4 instream flow requirement (S1~S4) demand nodes. Note that the general demand refers to municipal excluding domestic demand. The directed links between the inflow, junction, reservoir, hydropower plant and stream flow demand nodes represent river reaches or channels. The directed links to offstream irrigation, domestic, general and industrial demand nodes are diversion canals, while the reversely directed links from them to nodes on streams represent the return flow routes. Since irrigation, major urban and industrial users in SSRB are extremely reliant upon surface water sources and groundwater is mostly abstracted for rural domestic uses, groundwater sources are not considered in this case study. The river basin network is schematized to represent significant supply sources and water demand sites. Inflows from other small tributaries and water uses of towns, rural areas and villages are accounted for in the flow adjustments for associated nodes. All the demand nodes in the SSRB network are listed in Table 6.1.

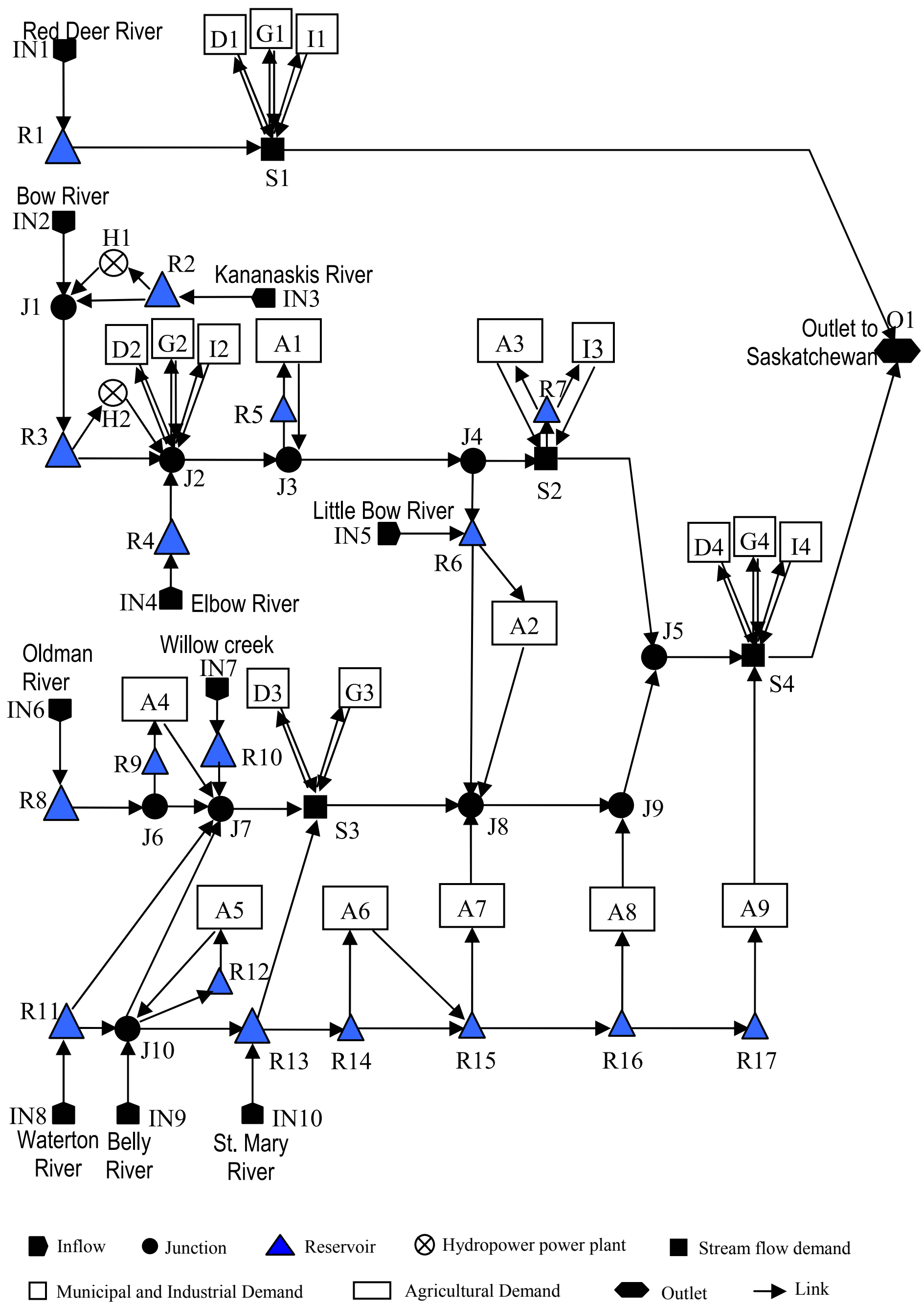

Figure 6.4 Network of the South Saskatchewan River Basin in Southern Alberta

Table 6.1 Demand nodes in the South Saskatchewan River Basin network

Sub-basin	Node	Demand node name	Type	Usage
Red Deer	D1	City of Red Deer - Domestic	MI	Domestic
	G1	City of Red Deer - General	MI	General
	I1	City of Red Deer - Industrial	MI	Industrial
	R1	Gleniffer Lake	RES	Storage
	S1	Red Deer River at Red Deer	SFR	Aquatic & Recreation
Bow River	A1	Western Irrigation Region	AGR	Irrigation
	A2	Bow River Irrigation Region	AGR	Irrigation
	A3	Eastern Irrigation Region	AGR	Irrigation
	D2	City of Calgary - Domestic	MI	Domestic
	G2	City of Calgary - General	MI	General
	I2	City of Calgary - Industrial	MI	Industrial
	I3	Eastern Industrial Region-Industrial	MI	industrial
	H1	Hydropower Plants on Kananaskis River	HPP	Hydropower
	H2	Hydropower Plants on Bow River	HPP	Hydropower
	R2	Barrier, Interlakes and Pocaterra Aggregate Reservoir	RES	Storage
	R3	Bearspaw, Horseshoe, Ghost, Upper and Lower Kananaskis Aggregate Reservoir	RES	Storage
	R4	Glenmore Reservoir	RES	Storage
	R5	Chestermere and Eagle Lakes Aggregate Reservoir	RES	Storage
	R6	Bow, McGregor Lake and Travers Aggregate Reservoir	RES	Storage
	R7	Crawling Valley, Lake Newell and Snake Lake Aggregate Reservoir	RES	Storage
	S2	Bow River at Bassano Dam	SFR	Aquatic & Recreation
Oldman River	A4	Lethbridge Northern Irrigation Region	AGR	Irrigation
	A5	Mountain View, Leavitt, Aetna, United Irrigation Region	AGR	Irrigation
	A6	Raymond and Magrath Irrigation Region	AGR	Irrigation
	A7	St. Mary River Irrigation Region-West	AGR	Irrigation
	A8	Taber Irrigation Region	AGR	Irrigation
	A9	St. Mary River Irrigation Region-East	AGR	Irrigation
	D3	City of Lethbridge - Domestic	MI	Domestic
	G3	City of Lethbridge - General	MI	General
	R8	Oldman Reservoir	RES	Storage
	R9	Keho Lake	RES	Storage
	R10	Chain Lakes and Pine Coulee Aggregate Reservoir	RES	Storage
	R11	Waterton Reservoir	RES	Storage
	R12	Cochrane Lake and Payne Lake Aggregate Reservoir	RES	Storage
	R13	St. Mary Reservoir	RES	Storage
	R14	Jensen and Milk River Ridge Aggregate Reservoir	RES	Storage
	R15	Chin Lakes	RES	Storage
	R16	Fincastle, Horsefly and Taber Lake Aggregate Reservoir	RES	Storage
	R17	Forty Mile and Sauder Aggregate Reservoir	RES	Storage
	S3	Oldman River at St. Mary River Confluence	SFR	Aquatic & Recreation
South Saskatche-wan River	D4	City of Medicine Hat - Domestic	MI	Domestic
	G4	City of Medicine Hat - General	MI	General
	I4	City of Medicine Hat - Industrial	MI	Industrial
	S4	South Saskatchewan River at Medicine Hat	SFR	Aquatic & Recreation

6.3.2 Water Supplies

Monthly river flow data are complied from the Water Survey of Canada's HYDAT database (Environment Canada, 2002) for the period from 1912 to 2001. Standard deviations of monthly flows of most tributaries vary from 25% to 100% of corresponding averages. Monthly water supplies during the crop growing season from May to September are much higher than those in winter. The long term averaged annual inflow of the ten major tributaries is about 4.4 billion cubic meters, much lower than the average annual natural inflow of about 9 billion cubic meters. Hence, it is very important to account in the node adjustments for small tributaries and drainages that are not explicitly expressed in the river basin network. In this study, node adjustments are obtained by multiplying together the respective monthly precipitations, sub-watershed areas corresponding to river reaches (Golder Associates, 2003), and approximate run-off rates. The total annual inflow of the ten major tributaries and the natural flow of the SSRB in 1995 are about 5.7 and 13.2 billion cubic meters, respectively. Therefore, 1995 is a wet year.

Reservoir data are compiled from AIPA (2002) and the online reports "*Water Supply Outlook for Alberta*" (Alberta Environment, 2000). Ten out of the 17 aggregate reservoirs have storage capacities larger than 100 mcm. In the drought year 2000, most reservoirs had significantly varied live storages among months with levels less than 80%, except that some offstream irrigation reservoirs, including Crawling Valley, Lake Newell and Snake Lake (R7) and Keho Lake (R9), maintained relatively stable storages at levels of more than 80% of storage capacities. The large onstream reservoirs Oldman (R8), Waterton (R11) and St. Mary (R13) have low storages, whose mean monthly storage percentages are at 25%, 50%, and 28 %, respectively.

6.3.3 Water Demands

6.3.3.1 Agricultural Water Demands

Since irrigation is the predominant agricultural water use in the SSRB, only irrigation water demands are considered in this study. All water demands of irrigation regions are assumed to occur during the growing season. Water demands for irrigation water are determined by the difference between crop potential evapotranspiration and effective precipitation. Irrigation water demands usually increase when precipitation reduces. The IWPM model is applied to estimate effective precipitation and irrigation water demands. Irrigated agriculture in the SSRB includes 13 irrigation districts and a number of smaller privately-owned irrigation systems. Many gravity and sprinkler irrigation systems are used in the SSRB. The center pivot sprinkler system, the most common irrigation system in the SSRB, is assumed to be the representative irrigation system. In the modeling, irrigation districts are aggregated into 9 irrigation regions according to surface water sources and agroclimate zones, and individual crops are aggregated into 6 representative crop categories based on similarities in evapotranspiration rates, crop yields, crop values and nutrient requirements. The 6 crop categories are special cereals (SC), traditional cereals (TC), feed grains (FG), oilseeds (OS), vegetables (VG) and alfalfa (AL). For each category, a representative crop is selected as a proxy for the other crop types. They are soft wheat, hard spring wheat, barley, canola, potatoes and alfalfa, respectively. The crop production data utilized in the above estimation are provided by Mahan (1997).

The monthly irrigation water demands in future years under the long-term mean and other precipitation conditions are estimated with assumed increases of cropping areas and the same cropping patterns as of 1995. Table 6.2 shows the projected monthly irrigation water demands in 2021 under the assumed half long-term mean monthly precipitations. The monthly gross consumptions of irrigation water in 1995 are obtained by proportionally splitting the seasonal gross consumptions of diverted and distributed water (Mahan, 1997), according to the monthly distribution patterns estimated by IWPM, assuming that the irrigation efficiency is 65% and irrigation levels are at 80% of the monthly potential evapotranspiration rates required for maximum yields under the crop patterns and cropping areas as of 1995.

Permits of annual withdrawals of raw water as of 1999 are compiled from licenses of irrigation districts (AIPA, 2002), which determine the maximal amount of raw water that could be diverted from supply sources in one year. The priority of a water permit in the SSRB is expressed as the license application date plus its application order in that date. Monthly licensed maximal amount of withdrawals with different priority ranks are split from the licensed annual withdrawal according to the monthly distribution patterns of maximum crop evapotranspiration of an irrigation region, and are set as the upper bounds of inflows. Withdrawal amounts licensed after 1999 are estimated as the increases of the maximum crop evapotranspiration due to cropping area expansions and are split into monthly limits in a similar way.

Table 6.2 Monthly irrigation water demands under assumed half long-term mean monthly precipitation conditions (mcm)*

Node	Irrigation regions	May	Jun	Jul	Aug	Sep	Growth season
A1	Western	15.332	38.462	47.352	32.119	17.380	150.645
A2	Bow River	27.221	147.970	209.984	118.812	38.340	542.327
A3	Eastern	77.054	228.020	291.392	178.108	86.933	861.507
A4	Lethbridge Northern	33.971	101.871	127.641	79.743	38.417	381.643
A5	Mountain View, Aetna, United, Leavitt	6.353	12.338	18.648	11.852	6.384	55.575
A6	Raymond and Magrath	2.443	22.287	45.180	9.623	1.729	81.262
A7	St. Mary River -West	21.768	89.185	155.901	69.427	25.805	362.086
A8	Taber	12.851	59.718	80.034	51.842	20.424	224.869
A9	St. Mary -East	34.537	150.904	216.540	131.288	42.559	575.828

*Estimated by the IWPM model at the 80% level of maximum crop potential evapotranspiration rates, assuming 20% increases of crop areas over 1995 and with the same crop patterns. Return flow and consumption ratios of the irrigation regions are assumed to be 25% and 75% respectively (65% for effective irrigation, 10% for deep percolation). 1 mcm means 1 million cubic meters.

6.3.3.2 Domestic and General Water Demands

The total gross consumption of delivered and treated municipal water during the growing season (May to September) of 1995 (Mahan, 1997), combined with monthly distribution percentages of municipal water uses, are used to estimate the monthly water demands of domestic and general water uses in 1995, as shown in Figure 6.5 and Figure 6.6, respectively. Note that the general water uses refer to municipal water uses except for domestic use. The monthly distributions of the domestic and general annual demands are estimated roughly according to the relative percentages plotted by Hydroconsult and CRE (2002). The domestic and general water demands also have relatively higher amounts during the crop growing season. The domestic and general water demands for other years are estimated as the products of projected population and corresponding per capita water demands, and follow the same monthly distribution patterns as in 1995.

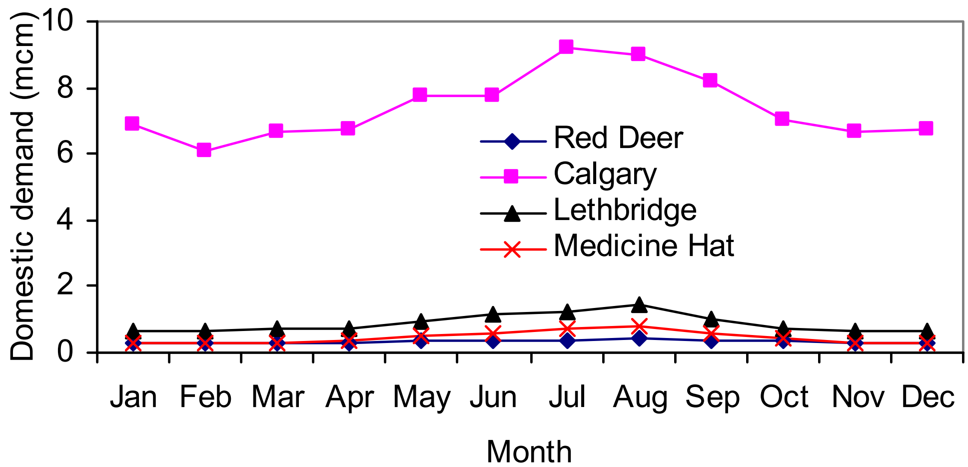

Figure 6.5 Monthly distribution of domestic water demands in 1995

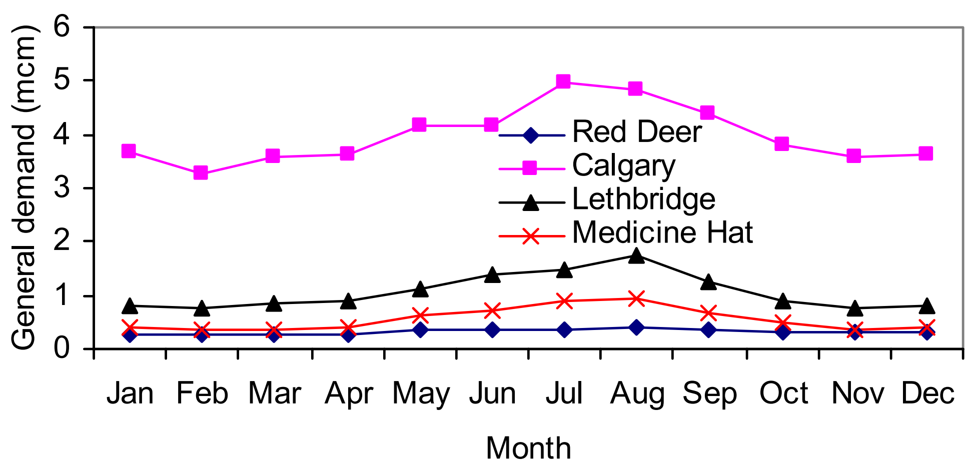

Figure 6.6 Monthly distribution of general water demands in 1995

Since it is not possible or practical to model all of the large numbers of licenses individually in the SSRB, licenses with different priority numbers need to be lumped into groups that preserve the priorities of water demands as far as possible. Due to the shortage of first-hand data of the water licenses, withdrawals of cities licensed in past time periods are estimated by assuming that each city has a constant per capita withdrawal over the time periods and the actual municipal withdrawals are 80% of licensed withdrawals (Hydroconsult and CRE, 2002). The 1995 domestic and general water demands are firstly adjusted by considering the diversion loss ratio of 12% to obtain the 1995 gross withdrawals, and then divided by 80% to get the licensed withdrawals as of 1995. The licensed annual withdrawals are proportionally split into different time periods according to corresponding populations of municipalities. Projected increases of municipal water demands after 1996 are assumed to be fully licensed in addition to the existing permits.

6.3.3.3 Industrial Water Demands

Monthly water demands of industrial uses in 1995 as shown in Figure 6.7 are obtained by splitting the seasonal consumption provided by Mahan (1997) according to the monthly percentages plotted by Hydroconsult and CRE (2002). Similar to irrigation and municipal demands, industrial demands are higher during the crop growing season from May to September. The monthly industrial water demands in future years are assumed to follow the same distribution patterns as of 1995, and are split from the annual demands projected by Hydroconsult and CRE (2002).

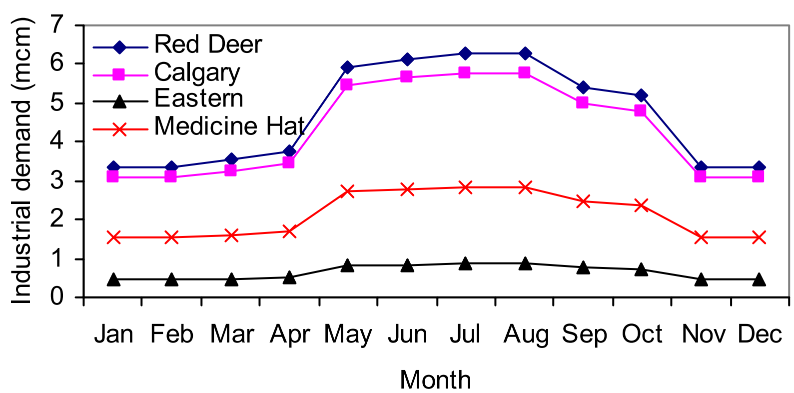

Figure 6.7 Monthly distribution of industrial water demands in 1995

Industrial water withdrawals licensed in past time periods are estimated in the same way as for municipal withdrawals. According to statistics of water licenses in Hydroconsult and CRE (2002), many water use factors of actual withdrawals over licensed ones are between

65% and 80%. A conservative water use factor of 65% and constant per capita withdrawals over time are assumed. The 1995 industrial water demands are firstly adjusted by considering the diversion loss ratio 12% to obtain the 1995 gross withdrawals, and then divided by 65% to get the total licensed withdrawals as of 1995, which are finally proportionally split into different time periods according to the corresponding populations of the industrial regions. Projected increases of industrial water demands after 1996 are assumed to be fully licensed in addition to the existing permits.

6.3.3.4 Hydropower Generation, Stream Flow and Live Storage Demands

The total hydropower generation capacity is 325 megawatts, about 5% of the power generated in Alberta (Mitchell and Prepas, 1990). Water demand of hydropower generation plants over the Bow River is reported to be 100 mcm during the period of May to September (Mahan, 1997). It is assumed that the monthly water demands of the aggregated hydropower generation node are 20 mcm per month over the years. Water demands of the small hydro plants installed at other dams in the basin are not considered in this study.

A stream flow requirement demand site requires a certain level of instream flow to be maintained for recreation activities, riparian ecology and fish habitats. Based on previous studies (Mahan, 1997; Alberta Environment and ASRD, 2003), the instream flow requirements during the crop growing season are set as follows: Red Deer River at Red Deer city (S1) at 13.2 m^3/s (35 mcm/month), Bow River at Bassano (S2) at 57.1 m^3/s (150 mcm/month), Oldman River at Lethbridge (S3) at 25.2 m^3/s (67 mcm/month), and South Saskatchewan River at Medicine Hat (S4) at 28.5 m^3/s (74 mcm/month). For other months, the instream flow requirements are assumed to be half of those during the crop growing season. The target live storage volumes of all reservoirs are assumed to be 80% of the corresponding live storage capacities throughout the year. According to the *Master Agreement on Apportionment,* the Province of Saskatchewan's requirement for receiving at least 50% of the annual natural flow of the SSRB is formulated as a constraint. Constraints are also set at the minimum instantaneous outflow of the outlet of SSRB, which is set to be the lesser of 50% natural flows or 42.5 m^3/s (110 mcm/month).

6.3.4 Water Loss Coefficients

According to the typical irrigation water use coefficients obtained by a previous study (AIPA, 2002), 7% of water withdrawals are lost in the diversion canals and the distribution systems to farms. Thus, most of the link loss ratios are set to be about 3%, and the losses of reservoirs are estimated by multiplying the reservoir surface areas with the corresponding monthly evaporation. The water consumption ratios of water diverted onto farm lands are approximated to be 75%, and the return flow ratios are 25%. Among the 75% of water consumed at irrigation nodes, 65% is stored as soil moisture and utilized through evapotranspiration, and 10% is lost through deep percolation.

For municipal water uses, the loss ratios at the untreated water treatment stage and distribution stage are 4% and 8%, respectively (Mahan, 1997). Thus, loss ratios of all links to municipal and industrial uses are set to be 12%. The node consumption ratios are 15% and 25% for domestic and general water uses, respectively. The water treatment and distribution processes for industrial water uses are assumed to be the same as municipal water uses, and loss ratios of all inflow links are 12%. The consumption ratios of industrial nodes are set as follows: City of Red Deer at 3.5%, Calgary at 5.1%, Eastern Irrigation Region at 4.2%, and Medicine Hat at 3.5% (Mahan, 1997). A 0.5% loss is assigned to each return flow link from the municipal and industrial nodes.

6.3.5 Salt Concentrations and Loads

Most of the water used for irrigation in the SSRB originates as snowmelt in the Rocky Mountains and has good quality, with total dissolved salt of less than 0.355 g/l (Alberta Environment, 1982). It is assumed that the salt concentrations of all inflows and node adjustments are 0.355 g/l. The initial salt concentrations in reservoirs are also set to be 0.355 g/l. The salt losses due to deep percolation and evapotranspiration at irrigation regions are assumed to be at the rate of 10%. It is reported that salinity added by water consumers during normal municipal and industrial (M&I) use is typically in the order of 0.25 to 0.30 g/l for total dissolved solids in communities which have strong controls on industrial discharges, and no brackish groundwater or seawater infiltration to sewers (BEE, 1999). In this study, salt addition to the water diverted by municipal and industrial nodes is approximated at the rate of 0.25 g/l.

Since salinity and mineral concentrations are generally not reduced during conventional primary, secondary, and tertiary wastewater treatments (BEE, 1999), the salt loss ratios of links are simply assumed to be equal to the water loss ratios of the links.

6.3.6 Benefit Functions of Irrigation Water Uses

The benefit functions of agricultural demand sites are estimated by 50 runs of the Irrigation Water Planning Model (IWPM) with different annual water availability and regressed by the method described in Chapter 5. The input data for the model includes crop types, cropping areas, maximum crop evapotranspiration, spring soil moisture, coefficients of nonlinear crop yield-effective irrigation functions, product prices, production costs, water supply costs, and water charges, which are tabulated in Appendix B. Coefficients of the crop yield-effective irrigation functions are estimated from the corresponding coefficients of crop yield-raw water functions estimated by UMA (1982), and Viney *et al*. (1996), by accounting for the irrigation efficiencies. Water is normally licensed without charge in the SSRB, but irrigation districts do charge for water delivery cost on a per-hectare basis rather than on a quantity-usage basis (Mahan, 1997). The charges for irrigation water range from $21.00 to $36.45 per hectare (the units for all monetary items in this study are in 1995 dollars). These per-hectare water charges are added to the fixed farm production costs in the modeling. In contrast, the pumping costs of farm production are variable with water volumes. Thus, they are treated as water supply costs in the hydrologic-economic river basin model, and are not included in the evaluation of irrigation water value.

Note that irrigation benefit is the remaining value of the crop production profit after deduction of the base benefit that could be gained with precipitation only. Thus, the demand and benefit functions for irrigation water are variable with precipitation, crop pattern and areas. Each irrigation region has different irrigation benefit functions under different agroclimate and crop production conditions. A sample monthly benefit function of irrigation water at the Western Irrigation Region (A1) is shown in Figure 6.8, assuming half of the long-term precipitation and 20% expansion of all crop areas over 1995. The correlation of the predicted seasonal irrigation benefits and outputs from IWAP is presented in Figure 6.9, which shows that the multiple regression fits well with the square of the correlation coefficient set at 0.9966.

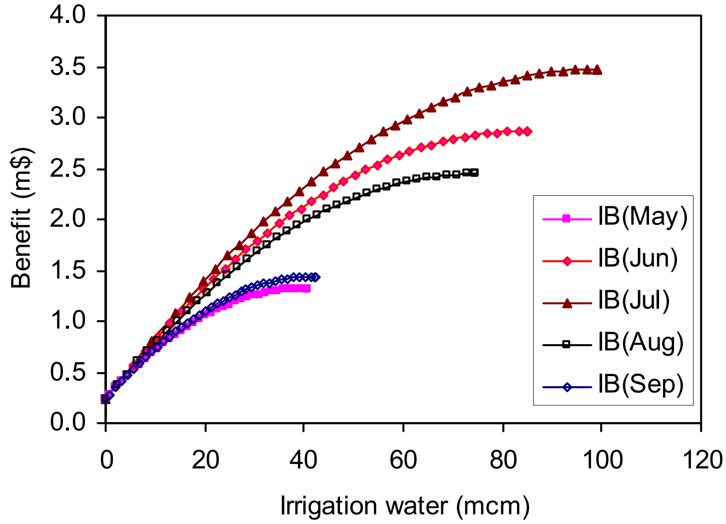

Figure 6.8 Regressed monthly benefit functions of irrigation water use of the Western Irrigation Region (A1) (Assuming half of the long-term precipitation, same cropping pattern, and 20% expansion of all crop areas over 1995)

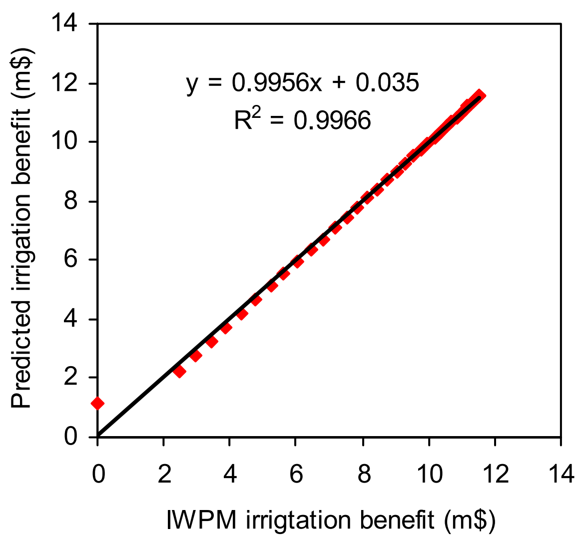

Figure 6.9 Correlation of predicted seasonal irrigation benefits (IB) of Western Irrigation Region (A1) and outputs from the irrigation water planning model (IWPM)

6.3.7 Municipal Water Demand Functions

Studies found that the municipal water demand elasticity differs by season, with elasticity being greater in summer than in other seasons. This is due to a higher degree of outdoor use, largely for lawn watering and garden sprinkling, occurring in the warmer months (Diaz *et al*., 1997). For this study, monthly water demand functions of domestic and general (commercial and industrial) uses are assumed to be in the constant elasticity form with an elasticity of -0.5 during the time period of May through September, and -0.4 for other months. All the monthly constant-elasticity water demand functions for all domestic and general demand sites during the year 1995 are estimated from observed reference quantities, reference prices and the assumed constant elasticity values. A reference price is defined as an observed price (marginal value) of water on a demand curve which corresponds to an observed quantity (i.e. reference quantity). The monthly reference quantities as of 1995 are roughly estimated as the monthly gross consumption, while reference water prices are taken from Mahan (1997) which are estimated as the sum of volumetric water utility and sanitary sewer charges. Choke prices are arbitrarily set at \$5.00/m^3 for domestic water uses and \$3.50/m^3 for general water uses.

The same methodology is applied to estimate the coefficients of the municipal water demand functions for other years. The coefficients of the monthly domestic and general water demand functions for the year 2021 are estimated in the case study, assuming the reference prices, choke prices and per capita water demands remain the same as in 1995, and with medium population growth rates for each municipality (Hydroconsult and CRE, 2002). Figure 6.10 and Figure 6.11 show the domestic and general water demand functions for Calgary, the largest municipality in the SSRB.

More detailed data about reference quantities, reference prices and constant elasticity values are listed in Appendix B. Different cities have different unit costs for water treatment, distribution, and wastewater treatment. These types of costs are not accounted for in the municipal water demand functions, but are inputs as water supply costs to the hydrologic-economic river basin model.

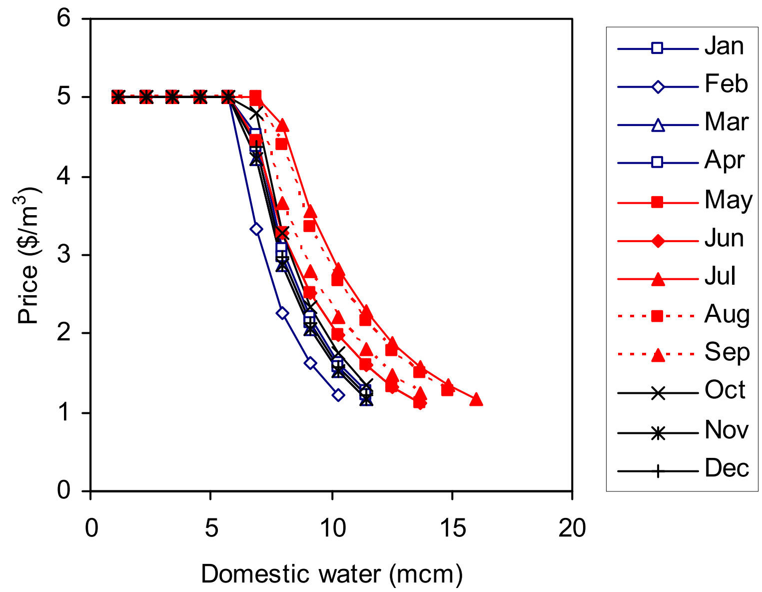

Figure 6.10 Monthly domestic water demand curves of Calgary as of 2021

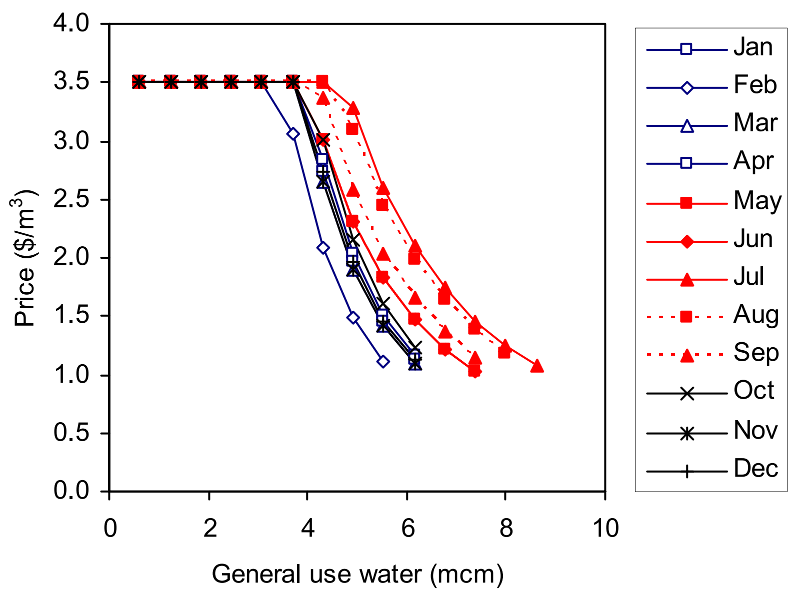

Figure 6.11 Monthly general water demand curves of Calgary as of 2021

6.3.8 Industrial Water Demand Functions

Industrial water demand functions are estimated by the same methodology applied to municipal water demands. The monthly reference quantities as of 1995 are roughly estimated as the monthly gross consumption, while reference water prices are taken from Mahan (1997). Choke prices are set at $2.50/m^3 for all industrial water uses. The coefficients of the monthly industrial water demand functions in the year 2021 are estimated, assuming the reference prices and choke prices remain the same as in 1995, while industrial water demands will grow at medium rates (Hydroconsult and CRE, 2002). Figure 6.12 shows the monthly industrial water demand functions for Calgary. The unit costs for water supply (water treatment, distribution, and wastewater treatment) to industrial uses are assumed to be the same as those estimated for the nearest major urban center.

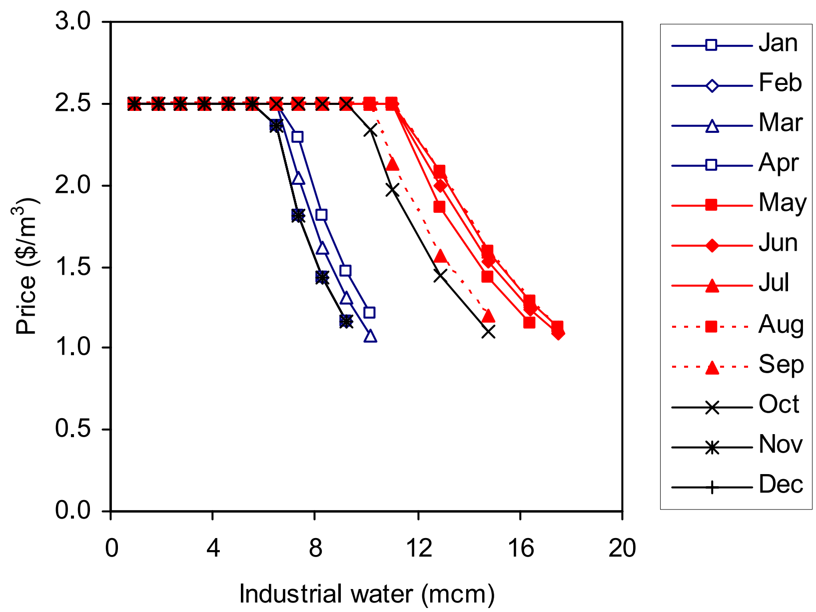

Figure 6.12 Monthly industrial water demand curves of Calgary as of 2021

6.3.9 Hydropower Demand Functions

The hydropower generation efficiencies of the two aggregate hydropower stations are assumed to be 80%, and their effective cumulative water heads are both at 200 meters throughout the year. It is assumed the hydropower generation has little effect on the power price on the regional power market. Thus, the hydropower demand curve remains at a constant price throughout the year. The unit hydropower value is estimated by the alternative cost approach, compared to thermal generation, at \$0.05/kWh. The fixed and variable costs for hydropower generation are estimated to be \$25/MWh and \$0.85/MWh (Mahan, 1997), respectively. Based on the above assumptions, the value of water for hydropower generation at both aggregate stations are estimated to be about 0.011 \$/m^3. Since the power generated by the each aggregated hydropower station with fully satisfied water demand is estimated to be 9.5 million kWh per month, the choke quantity may be set to any value greater than 9.5 million kWh. The choke price for hydropower generation is set to be 0.05 \$/m^3, and the elasticity parameter can be set freely. In this case study, hydropower demand functions are assumed to the same for different years. The identical monthly constant price hydropower demand function for both aggregate hydropower stations (H1 and H2) is shown in Figure 6.13.

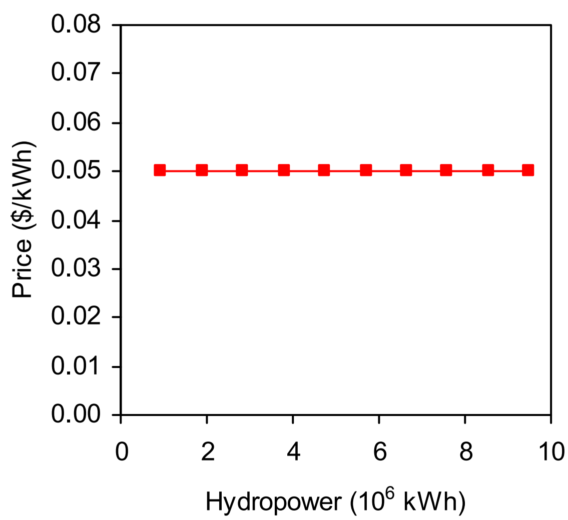

Figure 6.13 Monthly hydropower demand curve of aggregate hydropower stations

6.4 Modeling Scenarios and Result Analysis

As most of the data in the model are compiled from secondary sources and some data are estimated according to common knowledge, and the model has not been fully calibrated, the results do not necessarily fully reflect the real situation in the SSRB. Furthermore, the basin economy is not fully represented as some users, such as the values of instream flow and reservoir storage demands are not included in the hydrologic-economic modeling and coalition analysis. However, the instream flow and reservoir storage demands are considered in the initial water rights allocation, and their initial water rights are preserved in the coalition analysis by introducing hydrologic constraints on them. The case study focuses more on the types of analyses that can be carried out by the model and less on specific numbers.

6.4.1 Case Scenarios

Six case scenarios are designed according to the combination of water demands, hydrologic conditions, and the methods for initial water rights allocation, as shown in Table 6.3.

Table 6.3 Case scenarios

Case	Water demands	Inflows	Method*
A (1995 Wet & PMMNF)	1995 demand	1995	PMMNF
B (2021 normal & PMMNF)	2021 forecast	Long term mean	PMMNF
C (2021 drought & PMMNF)	2021 forecast	Drought	PMMNF
D (1995 Wet & LMWSR)	1995 demand	1995	LMWSR
E (2021 normal & LMWSR)	2021 forecast	Long term mean	LMWSR
F (2021 drought & LMWSR)	2021 forecast	Drought	LMWSR

* Method for initial water rights allocation

In cases A, B and C, initial water rights are allocated by the PMMNF method reflecting Alberta's existing prior water rights system. In cases D, E and F, initial water rights are allocated by the LMWSR method, which sequentially minimizes the maximum weighted water shortage ratios. Case A (1995 Wet & PMMNF) and Case D (1995 Wet & LMWSR) represent the actual situations of water demands, tributary inflows and node adjustments in 1995. Case B (2021 normal & PMMNF) and Case E (2021 normal & LMWSR) consider the forecasted water demands in 2021, and the long term mean (1912-2001) tributary inflows and node adjustments. Case C (2021 drought & PMMNF) and Case F (2021drought & LMWSR) explore water allocations under the forecasted water demands in 2021, and the hydrologic conditions of an assumed drought year. Since the Case A represents the real situations as of 1995, it is also used for calibrating model parameters such as water loss coefficients, and node adjustments.

Some assumptions are made in the scenarios:

(1) The tributary inflows and node adjustments in the drought year Cases C and F are assumed to be 50% of the long term mean values.

(2) For all cases, the initial storages of Oldman, St. Mary, and Waterton reservoirs are set to be 25%, 25% and 50% of their full live storages, respectively. Other reservoirs are set to 80% of their respective full storages. The minimum monthly live storages of all reservoirs are assumed to be 5% of storage capacities.

(3) Constraints for annual and monthly outflows to Saskatchewan at the border are set according to the *Master Agreement on Apportionment* and corresponding natural flows of the SSRB.

(4) For all cases, the salt concentrations of tributary inflows and river node adjustments are assumed to be 0.355 g/l.

(5) The water demands of hydropower generation in 2021 are assumed to remain the same as those in 1995.

(6) The cropping areas of all irrigation districts are assumed to increase 20% over 1995, while the cropping patterns remains the same, and the irrigation is maintained at the 80% evapotranspiration level. The projected irrigation water demands in 2021 for normal and drought year scenarios are listed in Appendix B.

(7) The domestic and general demands in 2021 are approximated as the products of population and per capita water demands in 2021. Adopting the medium population growth rates forecasted by Hydroconsult and CRE (2002), the populations of Red Deer, Calgary, Lethbridge and Medicine Hat in 2021 will increase about 47%, 67%, 32%, and 26% over 1996, respectively. Assuming the per capita water demands of each city remain constant over the years, then the annual domestic water demands of

Red Deer, Calgary, Lethbridge and Medicine Hat in 2021 are projected to be 6.025, 147.792, 14.148 and 7.043 mcm, respectively. The annual general water demands of Red Deer, Calgary, Lethbridge and Medicine Hat in 2021 are projected to be 5.692, 79.590, 16.771 and 8.346 mcm, respectively. The monthly distribution of the forecasted domestic and general demands follows the same patterns as in 1995.

(8) According to Hydroconsult and CRE (2002), the medium growth rates of industrial water uses of Red Deer, Calgary, the Eastern Industrial area and Medicine Hat from 1996 to 2021 are 150%, 200%, 100% and 100%, *i.e.*, their annual industrial demands will be 139.673, 154.143, 15.383, and 50.990 mcm, respectively. The monthly distribution of the forecasted industrial demands follows the same patterns as in 1995.

(9) In the PMMNF method applied to Cases A, B and C, 10 priority ranks are assigned to all the demands in the SSRB: all domestic water demands have the highest priority rank; licensed withdrawals of irrigation, municipal, and industrial demands with license application dates during the corresponding five time intervals, "Before 1982", "1982 to 1986", "1987 to 1991", "1992 to 1996" and "1997 to 2021" are assigned priority ranks 2, 3, 4, 7 and 8, respectively; hydropower generation water demands and stream flow requirements are set to priority ranks 5 and 6, respectively; each reservoir is divided into two zones, the target and surplus storage zones, whose demands are assigned the priority ranks 9 and 10, respectively.

(10) In the LMWSR method applied to Cases D, E and F, weights of water uses are set based on the "equivalent weighted shortages" rule, i.e., water shortage should be shared subject to equivalent weighted water shortage ratios. The higher the social utility or the lower water-shortage endurance the use has, the larger is the weight. Weights for demands are set as follows: domestic at 20, other offstream and hydropower generation water demands at 10, stream flow requirement at 3, reservoir target storage at 1. This means that, without other constraints, if in a month a reservoir storage is short of 90% of its target storage (i.e. satisfaction ratio at 10%), then the domestic, other offstream and hydropower generation water demands, and stream flow demands directly linking to and receiving outflows from it should share the shortage at ratios of 4.5%, 9% and 30%, respectively.

The above assumptions are based on the results of previous studies carried out in the SSRB, which reflect the typical estimation of inputs. It should be pointed out that the input parameters, especially the water availability, demands, priority and weights, affect the results of initial water rights allocation, and thus influence the reallocation of net benefits.

6.4.2 Initial Water Rights Allocation

6.4.2.1 Results of PMMNF (Cases A, B and C)

Solving the model

The first stage of the algorithm solves 10 linear PMMNF_QL programs which only consider water quantity constraints in the sequential order of priority ranks. The solution report from MINOS shows that each PMMNF_QL program has 2615 control variables and 1620 equations, and the whole process takes about one second to run on a 3 GHz Intel Pentium 4 CPU. The second stage of the algorithm is similar to the first stage, but the considered PMMNF_QC programs have nonlinear salinity balance constraints added. Hence, the sizes of programs increase and each has 4523 control variables and 4212 equations.

The selection of different starting points for optimization causes variations of computing times but all are short, when proper option parameters for the MINOS solver and algorithm are utilized. For example, for Case A, by setting the initial values of water quantity related control variables to be the solution of the linear PMMNF_QL programming at first stage, and assigning zero as the initial value of all salt concentrations, the algorithm for nonlinear PMMNF_QC programming can find an optimal solution in 3.5 seconds, including the running times of both stages. Because the global solution of the linear PMMNF_QL programming at the first stage provides a good starting point for the second stage of the nonlinear PMMNF_QC programming, the final solutions are identical to or approximately the global solutions for most times as long as the initial values for pollutant concentrations in rivers and reservoirs are set in a reasonable range.

Water rights allocation

Annual water supply/demand satisfaction ratios for all demand sites under the Cases A, B and C are shown in Figure 6.14. In the wet (Case A) and normal (Case B) hydrologic years, all offstream and hydropower generation water demands are satisfied. In Case C, the Bow River (A2), Lethbridge Northern (A4), St. Mary River-West (A7), Taber (A8) and St. Mary River-East (A9) irrigation regions, the general (G2) and industrial (I2) demands of Calgary, the hydropower plants on Kananaskis River (H1), and the Oldman River at St. Mary River confluence (S3) will have water shortages, the satisfaction ratios ranging from 0.966 (96.6%) to 0.475 (47.5%). The stream flow demand sites usually receive more inflows than requirements since unused natural flows bypass them to the downstream, except that the annual flow at the Oldman River at St. Mary River confluence (S3) is 73.7% of its demand under the drought Case C. As natural flows decrease from a wet year to a normal or dry year, the satisfaction ratios of stream flow requirement sites and reservoirs normally decrease. In the wet year, fifteen of the seventeen reservoirs have annual average storages more than their target storage volumes, while only ten reservoirs exceed this percentage under the normal year scenario. Under the drought year scenario, only the Gleniffer Lake (R1) and the Crawling Valley, Lake Newell and Snake Lake Aggregate Reservoir (R7) are satisfied.

As the PMMNF method allocates water rights by sequentially maximizing the total effective inflow for all demands owning the same priority rank from the highest priority to the lowest one, the allocation results are determined by three major factors: the water availability, demand amounts, and priority ranks of demands. The demands owning higher priority have privileges to receive water, and an upstream demand has more advantage to take water than a downstream demand having the same priority rank. For example, as shown in Figure 6.15, the irrigation regions consisting of the St. Mary River Irrigation Region-West (A7), Taber Irrigation Region (A8), and St. Mary River Irrigation Region-East (A9) cannot be satisfied as fully as the Raymond and Magrath Irrigation Region (A6) (refer to Figure 6.14), although they divert water from the same water headworks system. The reason is that A6's monthly demands can be fully satisfied by utilizing its licensed withdrawal at the priority rank 2, while the other three districts have to resort to their withdrawal licenses with lower priority ranks of 4 and 8.

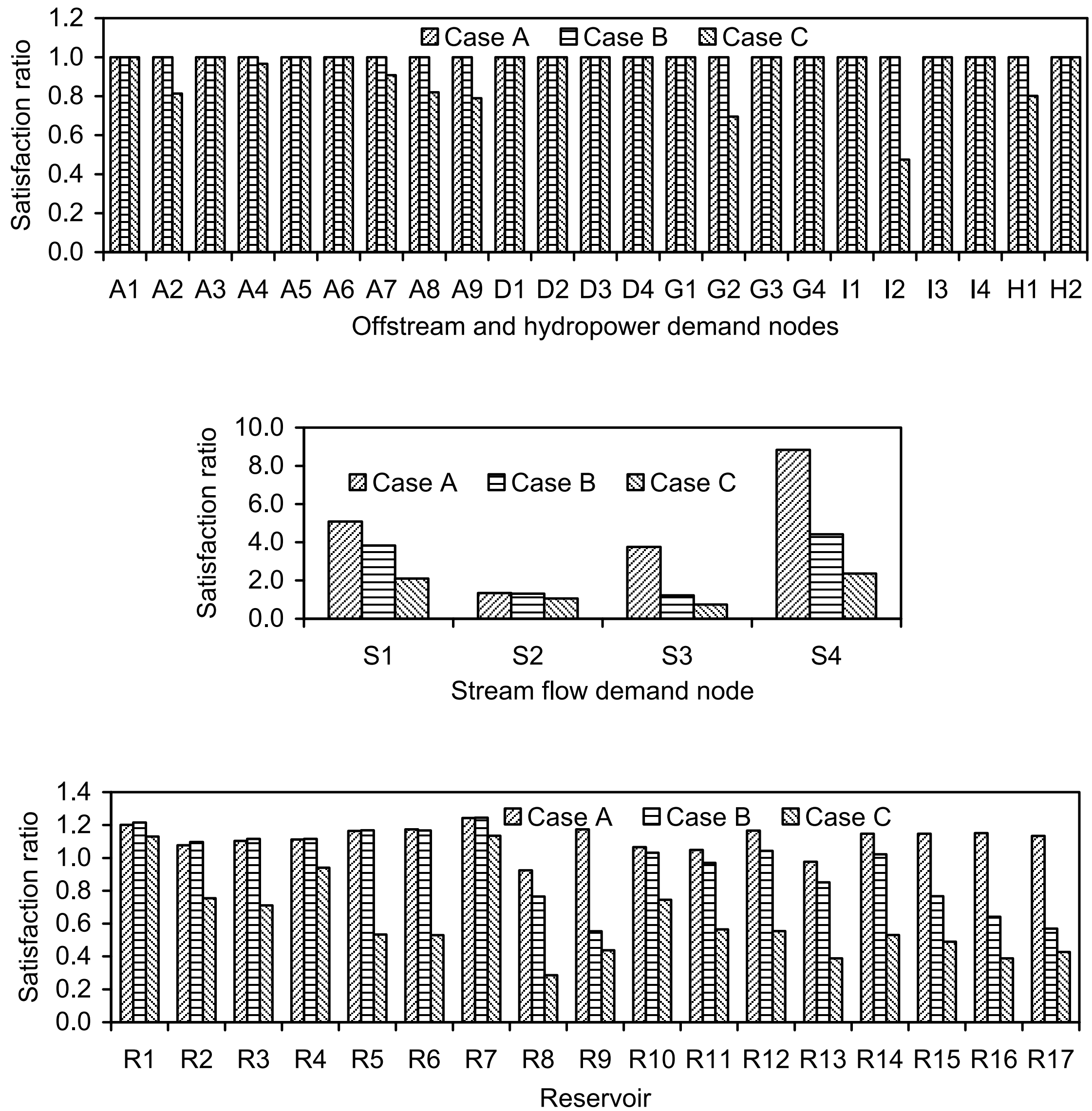

Figure 6.14 Annual water supply/demand satisfaction ratios allocated by the priority-based multiperiod maximal network flow (PMMNF) programming

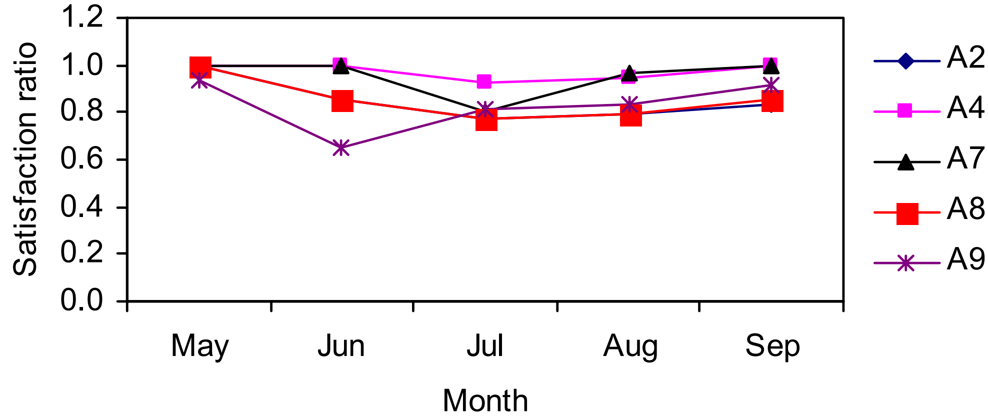

Figure 6.15 Monthly water supply/demand satisfaction ratios of irrigation regions with water shortages (Case C)

Another example showing the effects of priority setting is the water allocation among the domestic, general and industrial demands of the large municipality of Calgary under the drought Case C. As shown in Figure 6.16, all monthly domestic demands are satisfied even if there is severe drought, because under Alberta's existing prior water rights system they are always assigned to the highest priority no matter when the application for withdrawals are submitted. The uniformly distributed satisfaction ratios of the general (G2) and industrial (I2) water demands at 0.695 and 0.475, respectively, are because the potential increased demands in future development are licensed with lower priority ranks. Calgary would have to yield the rights to utilize these withdrawal licenses with low priority in the drought scenario.

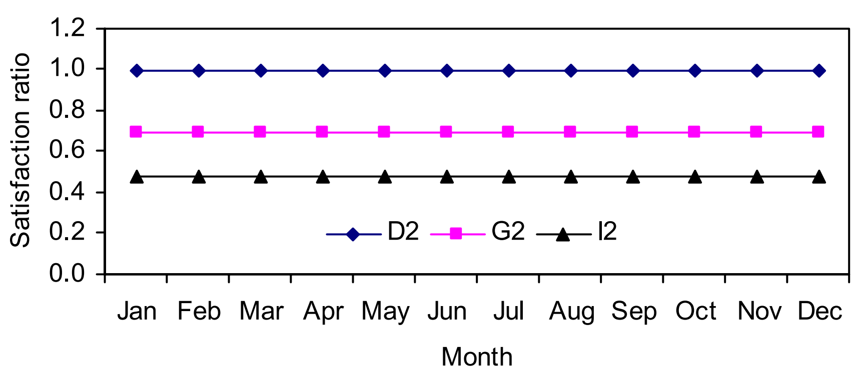

Figure 6.16 Monthly water supply/demand satisfaction ratios of domestic, general and industrial demands at Calgary (Case C)

The monthly satisfaction ratios of stream flow demands under the drought scenario Case C at the Red Deer River at Red Deer (S1), Bow River at Bassano Dam (S2), Oldman River at St. Mary River confluence (S3) and the South Saskatchewan River at Medicine Hat (S4) are plotted in Figure 6.17. Due to the large water demands of the extensive irrigation districts lying in the Bow and Oldman River sub-basins, the satisfaction ratios of S2 and S3 are rather lower than those of S1 and S4. The shortage of stream flow is even more serious in the Oldman River sub-basin, which almost extends throughout the growing season.

Salinity is considered in this study in order to explore the effects of water allocations on the salt concentrations in the river system and on the benefits of crop production. Since the salinity of the headwaters of the SSRB is rather low, the salinity of most river reaches and all reservoirs vary from 0.355 to 1 g/l. Most of the return flows from demand sites and a few river reaches have salinity of more than 1 g/l, but all less than 2g/l. Figure 6.18 displays the mixed salt concentrations of the monthly inflows to irrigation districts, where most vary between 0.3 and 0.5g/l. The higher salt concentrations of inflows to the Eastern Irrigation Region (A3) are due to two reasons: its return flow link is connected to the same point of water diversion, and the upstream inflows in corresponding months are relatively lower.

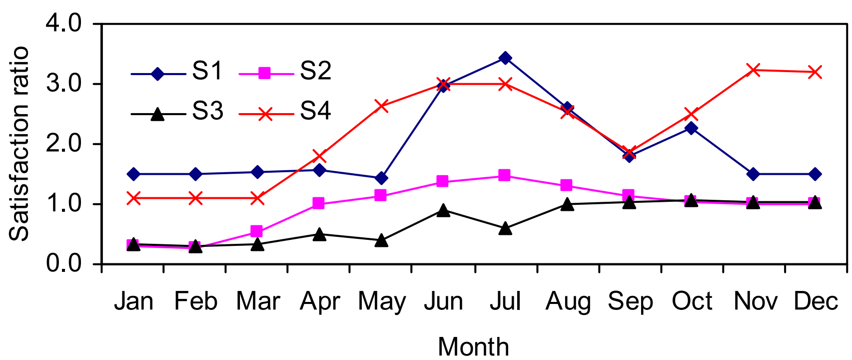

Figure 6.17 Monthly water supply/demand satisfaction ratios of stream flow requirements (Case C)

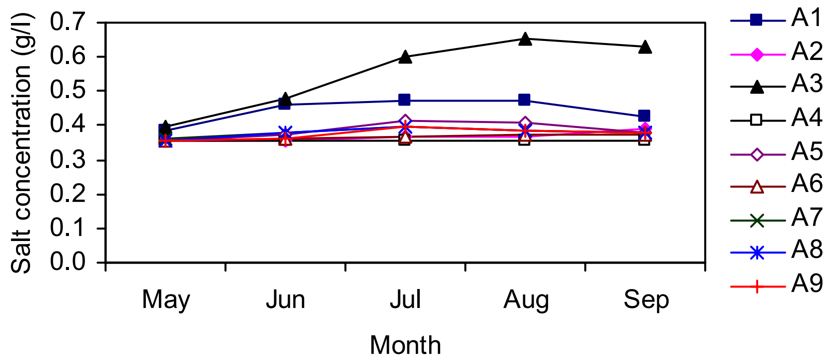

Figure 6.18 Mixed salt concentrations of the inflows to irrigation regions (Case C)

Reservoir operation

Onstream reservoirs serve as important tools for water resources management of water supplies to the multiple purpose uses in the river basin. The monthly releases of the five major onstream reservoirs in the SSRB under the drought scenario Case C are plotted in Figure 6.19, in which each legend symbolizes a river reach or diversion canal. For example, "R1.S1" means the reach from R1 (Gleniffer Lake) to S1 (Red Deer). While all the monthly releases of reservoirs help to meet the demands of the domestic, municipal, irrigation, hydropower generation, stream flow requirements for ecological uses and pollution dilution, releases are relative higher during the crop growing season from May to September. The Gleniffer Lake (R1), and the Bearspaw, Horseshoe, Ghost, Upper and Lower Kananaskis Reservoirs above Calgary (R3) maintain releases throughout the year to S1 (Red Deer) and H2 (Hydropower plants on Bow River), respectively, which play key roles to meet the downstream monthly flow requirements of the Red Deer, Bow, and South Saskatchewan River sub-basins, and the agreement of monthly outflows to Saskatchewan. Complementary to releases, the large onstream reservoirs, such as the Oldman (R8), Waterton (R11) and St. Mary (R13) Reservoirs store water and carry over for releases during July, August and September to meet the high irrigation demands during these months. Water stored in many onstream reservoirs follows the declining trend until the end of the year as shown in Figure 6.20.

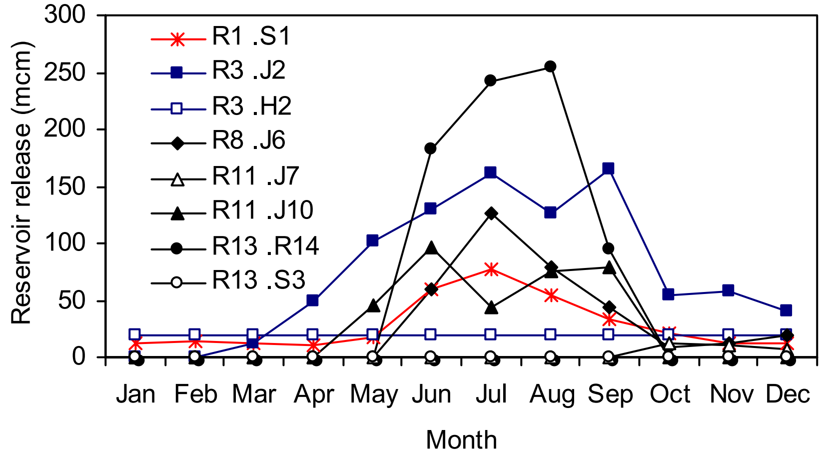

Figure 6.19 Monthly releases of onstream reservoirs (Case C)

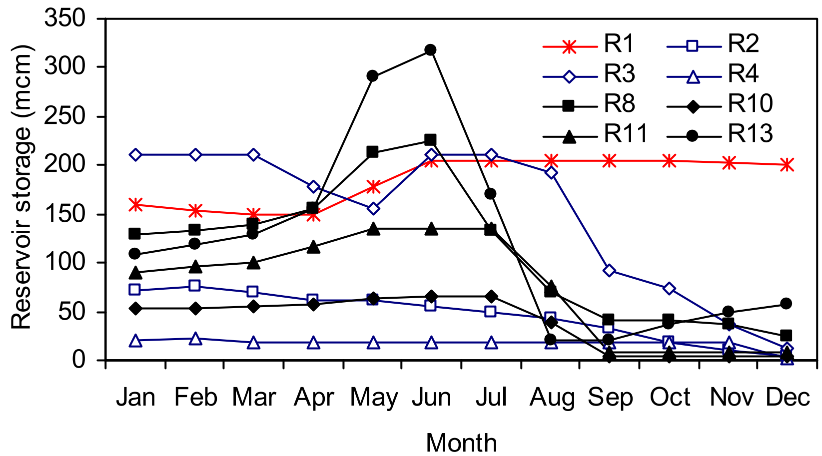

Figure 6.20 Monthly storages of onstream reservoirs (Case C)

The major purpose of the operations of offstream reservoirs is to secure water supplies for irrigation. Figure 6.21 displays the monthly irrigation diversions to the nine aggregate irrigation regions, all happening during the growing season. Accordingly, the storages of offstream reservoirs decrease from May to September, and maintain low levels after then except for R7 (Crawling Valley, Lake Newell and Snake Lake Aggregate Reservoir) as shown in Figure 6.22.

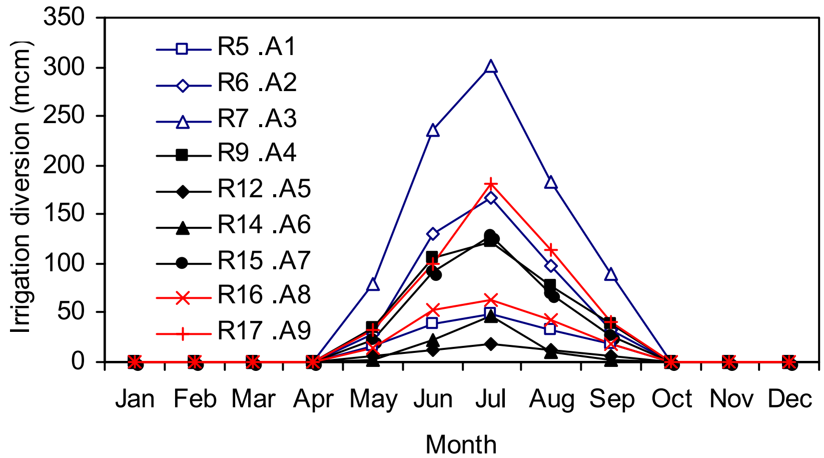

Figure 6.21 Monthly irrigation diversions from offstream reservoirs (Case C)

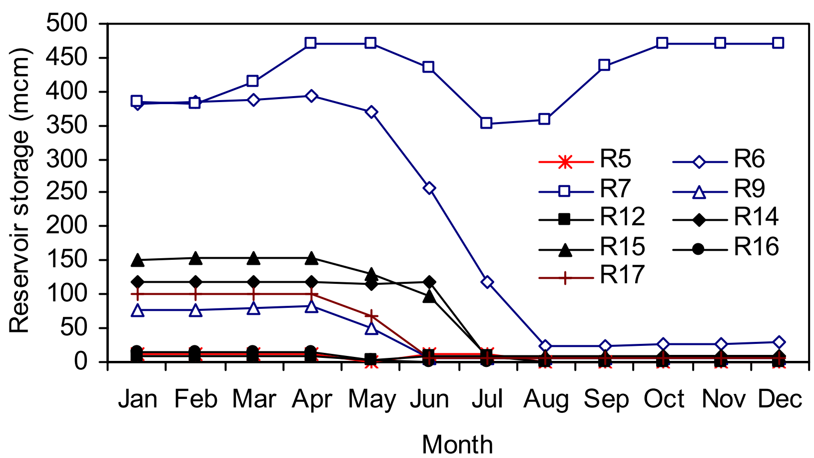

Figure 6.22 Monthly storages of offstream irrigation reservoirs (Case C)

Outflows to the Province of Saskatchewan

Monthly outflows from the SSRB to Saskatchewan at the provincial border are plotted in Figure 6.23. The outflows are higher during the growing season than other months, due to higher precipitations and upstream tributaries inflows during that period. In total, there are about 9.473, 5.335, and 2.292 billion cubic meters of water discharges under the Cases A (1995 wet), B (2021 normal) and C (2021 drought), respectively, which are 71.6%, 59.2% and 59.8% of the corresponding annual natural flows of the SSRB. The discharges of the

SSRB satisfy the *Master Agreement on Apportionment* between the Provinces of Alberta and Saskatchewan.

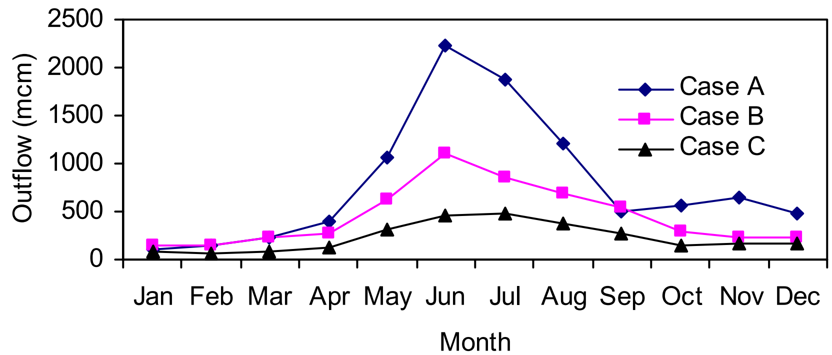

Figure 6.23 Monthly outflows to the Province of Saskatchewan.

Sensitivity analysis

Sensitivity analyses are carried out for three important parameters: initial reservoir storages, node loss (consumption) coefficients, and priority ranks of stream flow demands. The results for the drought scenario (Case C) are shown in Figure 6.24 to Figure 6.26. A 10% decrease of the initial storages of all reservoirs reduces the annual demand satisfaction ratios of St. Mary River Irrigation Region-East (A9), hydropower plants on the Kananaskis River (H1), Red Deer River at Red Deer (S1), Bow River at Bassano Dam (S2), Oldman River at St. Mary River confluence (S3), and South Saskatchewan River at Medicine Hat (S4) by 11.3%, 3.1%, 2.5%, 4.7%, 5.6%, and 8.7%, respectively. The annual satisfaction ratios of other demands remain the same. A 10% increase of the node loss coefficients of demand sites leads to decreases of 5.6%, 6.4% and 12.9% of the annual satisfaction ratios of S2, S3 and S4, respectively. If priority of the four stream flow requirement sites is lowered from the rank 6 to rank 8, and the water demands with license application dates during the time intervals, “1992 to 1996” and “1997 to 2021” are assigned priority ranks 6 and 7, the Bow River Irrigation Region (A2), Calgary-General (G2), and Calgary - Industrial (I2) will have their annual satisfaction ratios increased by 22.9%, 43.9% and 110.5%, respectively. However, the S2 and S4 nodes will have their annual satisfaction ratios decreased by 9.2% and 7.0%, respectively. For the wet (Case A) and normal (Case B) scenarios, the above changes of input parameters would not cause any changes to the satisfaction ratios of all offstream and hydropower generation water demands, except for small changes of the four stream flow demand sites. In summary, allocations to most demands are not sensitive to parameters described above. However, when water supplies are not sufficient, the parameter changes may cause considerable variations for A2, A9, G2, and I2.

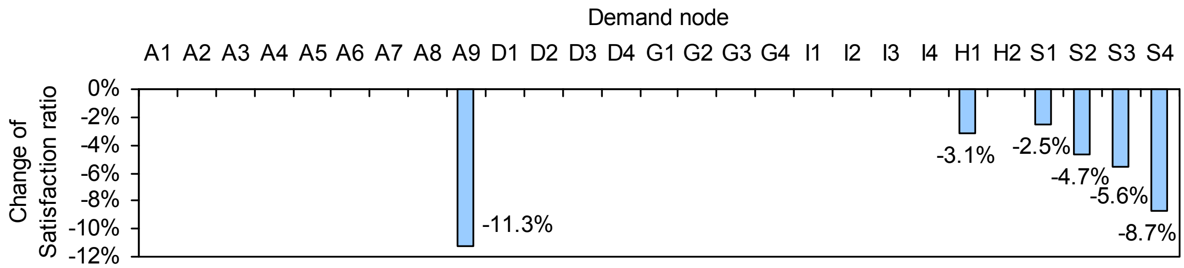

Figure 6.24 Changes of annual satisfaction ratios with a 10% decrease of initial reservoir storages (Case C)

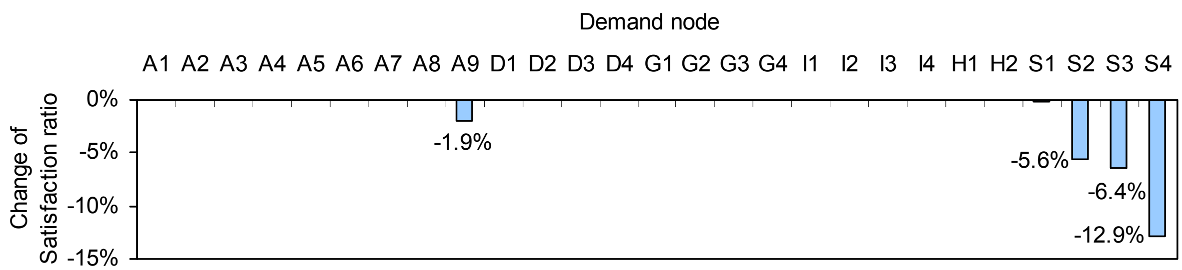

Figure 6.25 Changes of annual satisfaction ratios with a 10% increase of node loss coefficients (Case C)

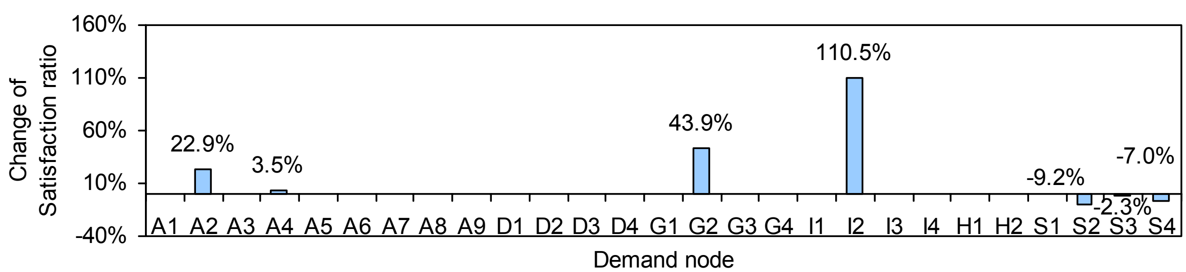

Figure 6.26 Changes of annual satisfaction ratios when priority ranks of stream flow requirement sites are set to be lower than all other non-storage demands (Case C)

6.4.2.2 Results of LMWSR (Cases D, E and F)

Solving the model

Similar to PMMNF, the first stage of the algorithm for the LMWSR_QC method is to iteratively solve a series of LMWSR_QL programs, which consider linear water quantity constraints only. The number of iterations and computational times depend on the input data. Take Case D as an example, in which the LMWSR_QL program solved at every iteration has 1825 control variables in each iteration. When default parameters of the solver and algorithm are utilized, the programming iterates 180 loops, with 2098 equations at the first iteration loop and 1634 equations at the last one, costing 7 seconds to run on a 3 GHz Intel Pentium 4 CPU. At the second stage of the solution algorithm, each of the iteratively solved nonlinear LMWSR_QC programs has 3733 control variables. For Case D, when the initial values of all salt concentrations are set to be zero, the nonlinear LMWSR_QC programming iterates 150 times, with 4690 equations at the first iteration loop and 4226 equations at the last one. It takes about 42 seconds in total to find an optimal solution on a 3 GHz Intel Pentium 4 CPU.

Water rights allocation

Annual water supply/demand satisfaction ratios for all demand sites under the Cases D, E and F are shown in Figure 6.27. The annual satisfaction ratios of all offstream and the hydropower generation water demands are equal to or less than one but all are larger than 0.93 in the wet (Case D) and normal (Case E) hydrologic years, and varying from 0.802 to 1 in the assumed drought scenario (Case F). Compared to the PMMNF method, the allocations by the LMWSR method leads to more evenly distributed satisfaction ratios. The stream flow demand sites normally have annual satisfaction ratios larger than one, except that the Bow River at Bassano Dam (S2) and Oldman River at St. Mary River confluence (S3) under the drought Case F receive only 76% and 65% of their required annual flows, respectively. In the wet year, seven of the seventeen reservoirs can have annual satisfaction ratios equal to or more than one, while five reservoirs may exceed this percentage under the normal year scenario. Under the drought year scenario, only the Gleniffer Lake (R1) and the Crawling Valley, Lake Newell and Snake Lake Aggregate Reservoir (R7) can store target volumes.

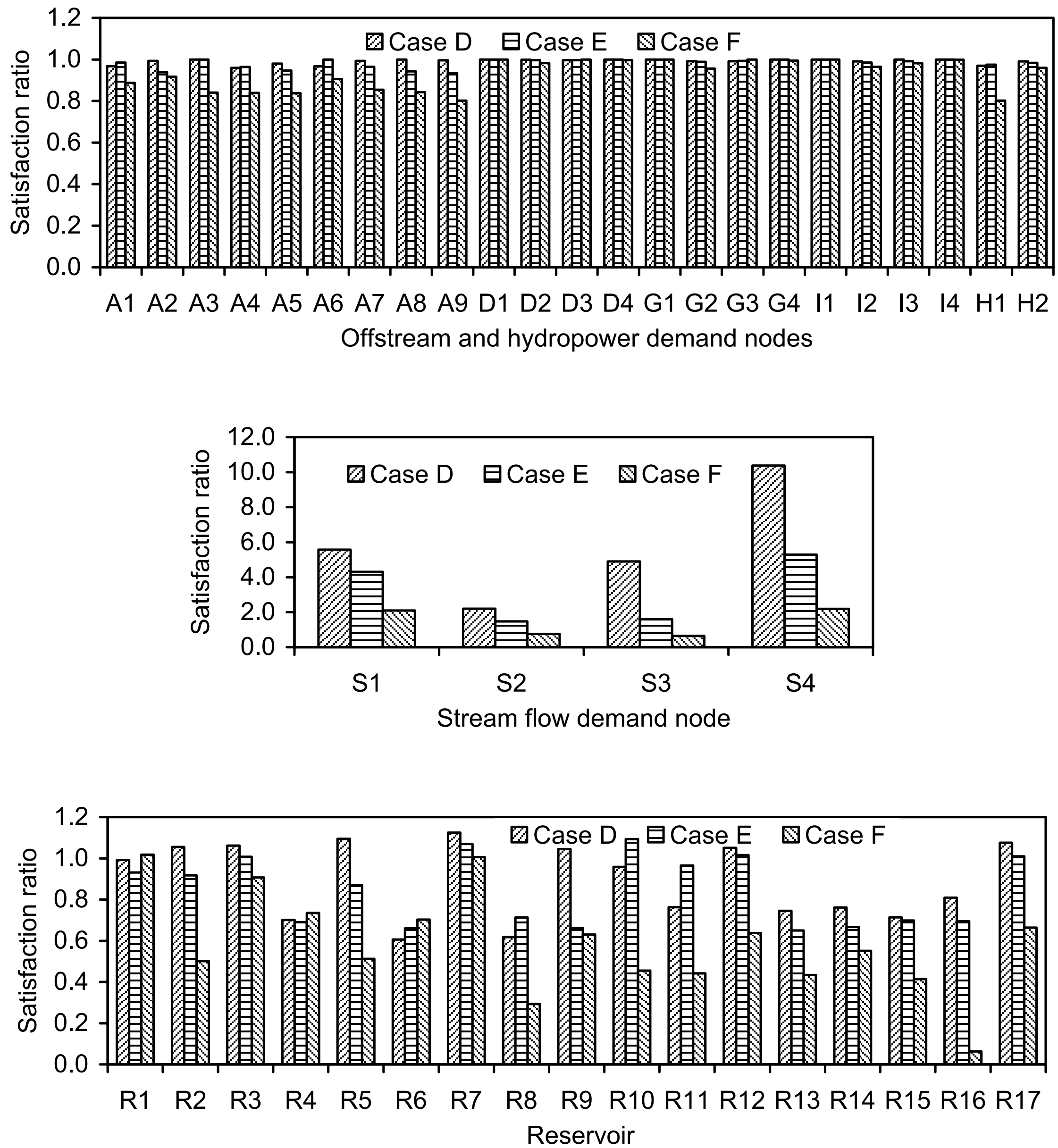

Figure 6.27 Annual water supply/demand satisfaction ratios allocated by the lexicographic minimax water shortage ratios (LMWSR) method

As the LMWSR method for water rights allocation searches for the vector of lexicographic minimax fair distribution of weighted shortage ratios by iteratively finding the minimax water shortage ratios and then fixing their upper bounds, the allocation results are determined by three major factors: the water availability, demand amounts, and weights of demands. The demands owning higher weights have the privilege to receive water, and all demands of the same weight are equitably treated no matter if they are located upstream or downstream. As shown in Figure 6.28, all irrigation regions share the water shortages with monthly satisfaction ratios varying from 0.802 to 1, but most are at 0.802 and 0.906. The monthly demands of Raymond and Magrath Irrigation Region (A6) from May to August cannot be fully satisfied as the case utilizing the PMMNF method.

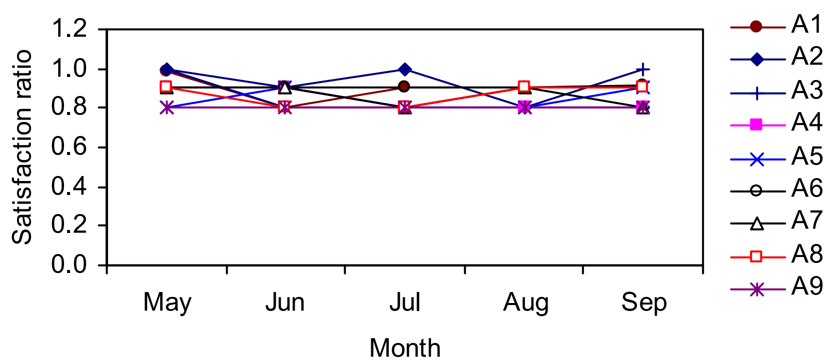

Figure 6.28 Monthly water supply/demand satisfaction ratios of irrigation regions with water shortages (Case F)

Figure 6.29 shows that the domestic use (D2) of Calgary usually has higher monthly satisfaction ratios than general (G2) and industrial (I2) uses under the drought year scenario, since its demands are assigned larger weights.

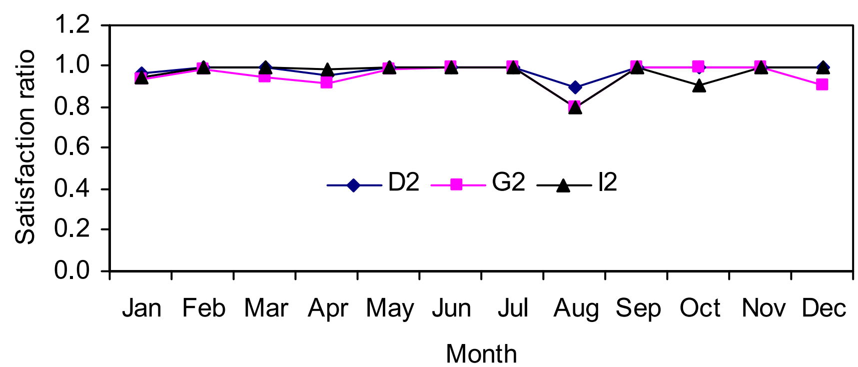

Figure 6.29 Monthly water supply/demand satisfaction ratios of domestic, general and industrial demands at Calgary (Case F)

The monthly satisfaction ratios of the four stream flow sites under the drought scenario Case F are plotted in Figure 6.30. Similar to the results of PMMNF, the satisfaction ratios of S2 and S3 are rather lower than those of S1 and S4, due to the large water demands of the extensive irrigation districts located in the Bow and Oldman River sub-basins.

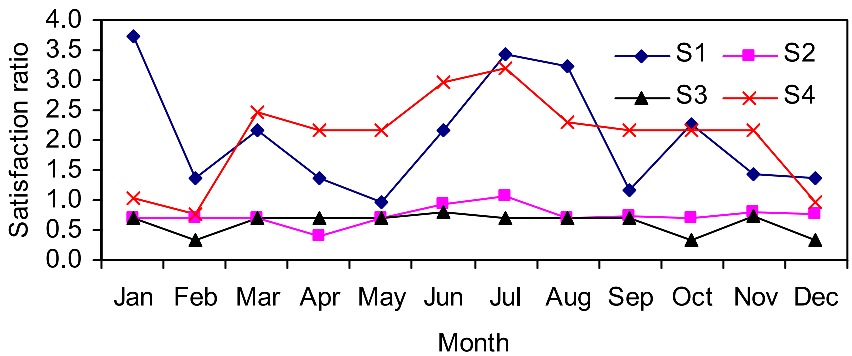

Figure 6.30 Monthly water supply/demand satisfaction ratios of stream flow requirements (Case F)

Similar to the results for PMMNF, the salt concentrations of river reaches and all reservoirs are rather low. Figure 6.31 displays the mixed salt concentrations of the monthly inflows to irrigation districts, where most vary between 0.3 and 0.5g/l. The salt concentrations of inflows to the Eastern Irrigation Region (A3) during August and September are relative higher at about 0.7 g/l.

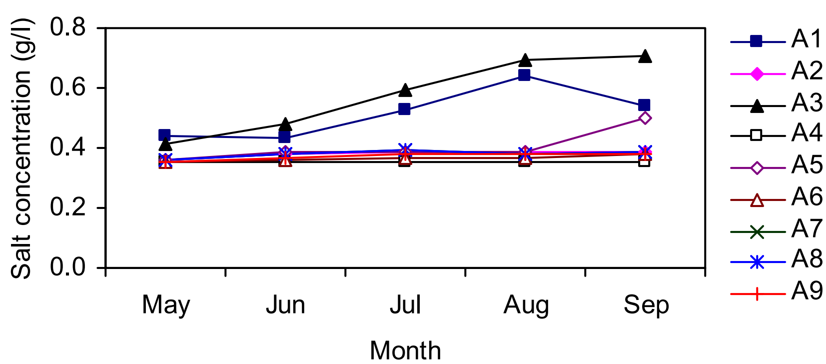

Figure 6.31 Salt concentrations in the mixed inflows to irrigation regions (Case F)

Reservoir operation

Monthly releases and storages of the five major onstream reservoirs in the SSRB under the drought scenario Case F are plotted in Figure 6.32 and Figure 6.33. The distribution patterns of monthly releases and storages are quite similar to those obtained by PMMNF, except that the release from the St. Mary reservoir (R13) to Jenson Reservoir (R14) during July is much higher at 522 mcm, and thus causes a resulting fast decrease of storage. Figure 6.34 displays the monthly irrigation diversions to the nine aggregate irrigation regions, whose distribution is similar to the results obtained by PMMNF. Accordingly, the storages of offstream reservoirs decrease from May to September, and maintain low levels after then except for R7 (Crawling Valley, Lake Newell and Snake Lake Aggregate Reservoir) as shown in Figure 6.35.

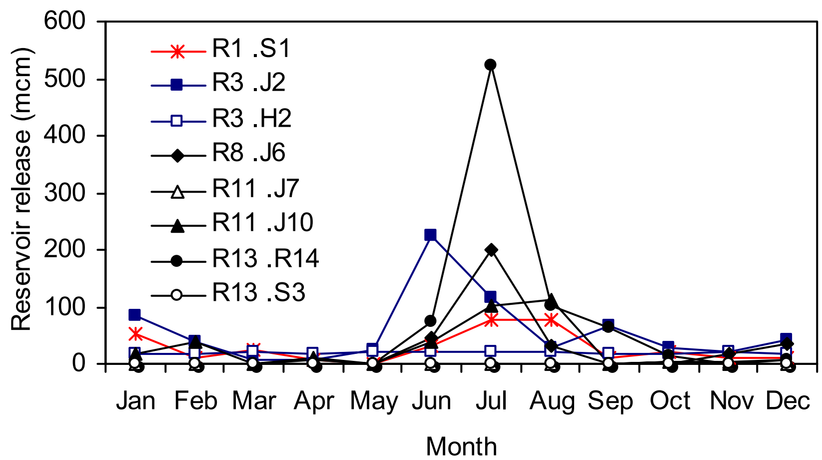

Figure 6.32 Monthly releases of onstream reservoirs (Case F)

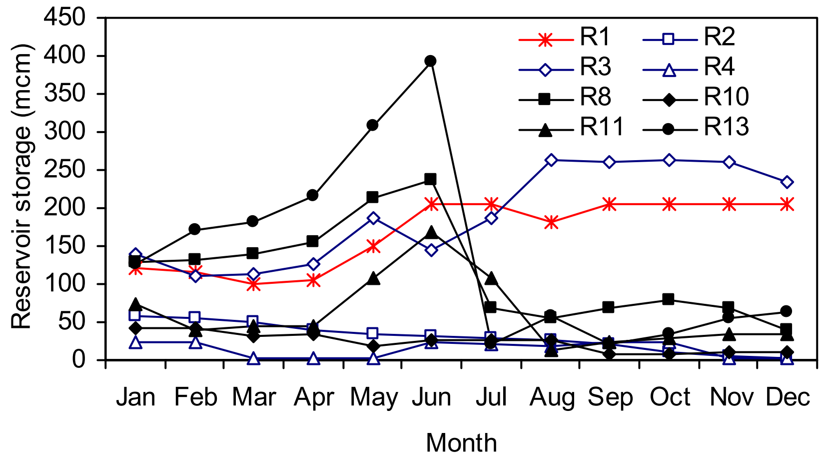

Figure 6.33 Monthly storages of onstream reservoirs (Case F)

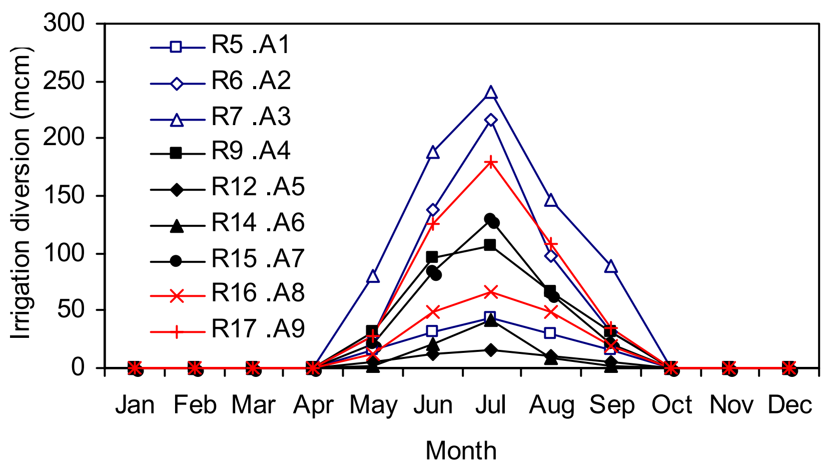

Figure 6.34 Monthly releases of offstream irrigation reservoirs (Case F)

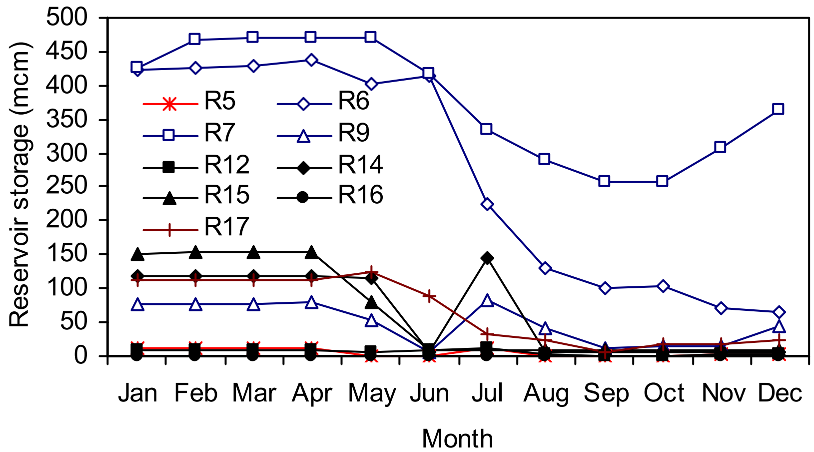

Figure 6.35 Monthly storages of offstream irrigation reservoirs (Case F)

Outflows to the Province of Saskatchewan

Monthly outflows from the SSRB to Saskatchewan are plotted in Figure 6.36, which are higher during the growing season than other months. In total, there are about 10.573, 6.009, and 2.585 billion cubic meters of water discharges under the Cases D (1995 wet), E (2021 normal) and F (2021 drought), respectively, which are 80.0%, 66.7% and 57.4% of the corresponding annual natural flows of the SSRB.

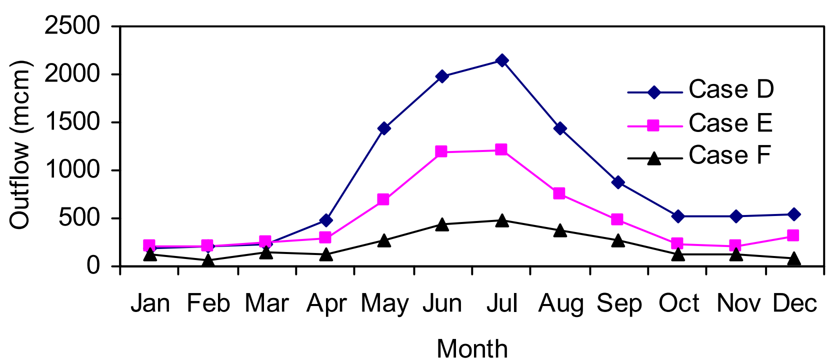

Figure 6.36 Monthly outflows to Saskatchewan.

Sensitivity analysis

Results of sensitivity analyses for initial reservoir storages, node loss (consumption) coefficients, and weights of stream flow demands under the drought scenario (Case F) are shown in Figure 6.37 to Figure 6.39. A 10% decrease of the initial storages of all reservoirs causes decreases of annual satisfaction ratios of the Bow River irrigation region (A2), St. Mary River Irrigation Region-West (A7) and South Saskatchewan River at Medicine Hat (S4) by 10.5%, 7.0% and 6.9%, respectively. But the Eastern Irrigation Region (A3) and Oldman River at St. Mary River Confluence (S3) will have their annual satisfaction ratios increased by 13.9% and 5.3%, respectively. Other demands have small changes of satisfaction ratios. A 10% increase of the node loss coefficients of demand sites leads to decreases of 7.1%, 7.4% and 6.3% of annual satisfaction ratios at the Western (A1), Bow (A2), and Raymond and Magrath (A6) irrigation regions, but increases of 7.1% for A3 and 7.0% for S4. Figure 6.39 shows the changes of annual satisfaction ratios if the weights setting for the domestic, other offstream and hydropower generation, stream flow, and reservoir storage demands are changed from 20:10:3:1 to 20:10:4:1, 20:10:5:1 and 20:10:10:1, respectively. As the weights of stream flow demands increase, more water passes stream flow demand sites, causing reductions of inflows to most other demands except for A3. If the weights for the stream flow demands are assigned to 10, the same weights as the general, industrial and hydropower generation demands, S2 (Bow River at Bassano Dam), S3, S4 and A3 will have satisfaction ratios increased by 16.4%, 34.3%, 19.0% and 19.0%, while those of A1, A2 and A3 are decreased by 9.7%, 9.5%, and 11.5%, respectively. In summary, allocations to most demands are not sensitive to the parameters described above. However, when water supplies are not sufficient, the parameter changes may cause considerable variations for A1, A2, A3, A6, and the stream flow demand sites (S2, S3 and S4).

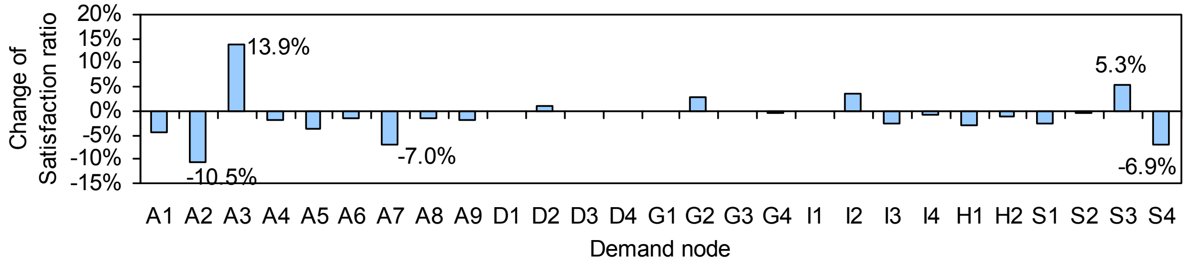

Figure 6.37 Changes of annual satisfaction ratios with a 10% decrease of initial reservoir storages (Case F)

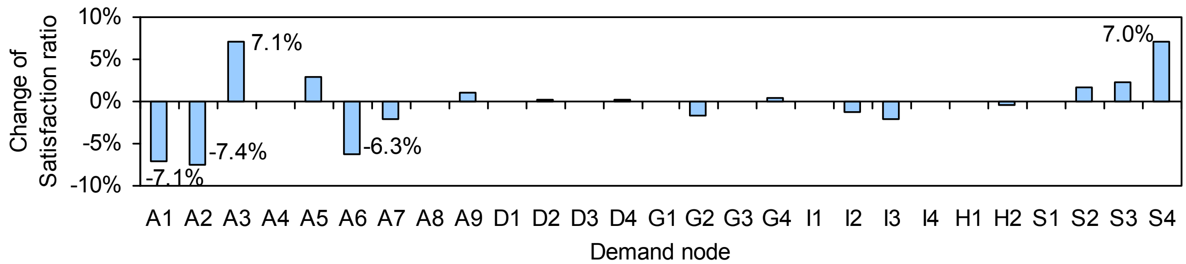

Figure 6.38 Changes of annual satisfaction ratios with a 10% increase of node loss coefficients (Case F)

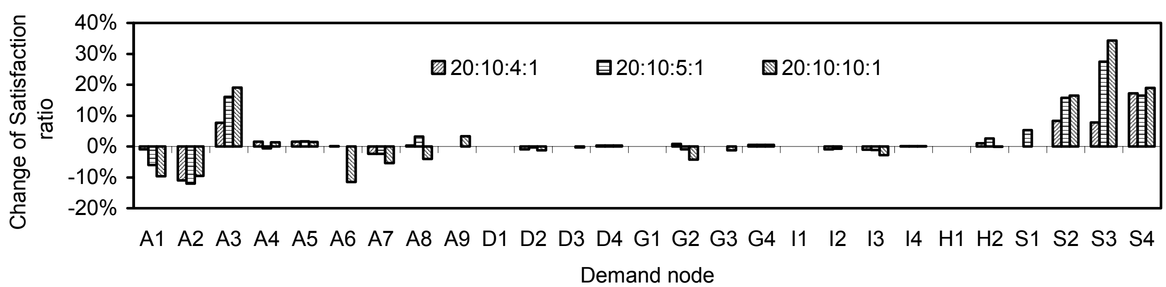

Figure 6.39 Changes of annual satisfaction ratios under different weights (Case F)

6.4.3 Reallocation of Water and Net Benefits

Water uses in the SSRB are grouped under the ownership of nine stakeholders: City of Red Deer (RD)- domestic, general and industrial; Bow River hydropower stations and associated reservoirs (BH); City of Calgary (CA) - domestic, general and industrial; Eastern Industrial Region (EIN); Irrigation regions and associated offstream irrigation reservoirs in the Bow River Sub-basin (BIR); City of Lethbridge (LB) – domestic and general; Irrigation regions and associated offstream reservoirs in the Oldman River Sub-basin (OIR); City of Medicine Hat (MH) - domestic, general and industrial; and Alberta Environment (AE) – stream flow demand sites and onstream reservoirs. Due to data limitation, the economic values of stream flows and reservoir storages are not explicitly included in the objective functions of the hydrologic-economic river basin modeling and coalition analysis. Instead their water rights are preserved through hydrologic constraints.

6.4.3.1 Net Benefits of Initial Water Rights

Annual inflows to non-storage demand sites and corresponding net benefits based on initial rights allocated under all the six case scenarios are listed in Table 6.4 and Table 6.5, respectively. Irrigation, the dominant consumptive water use in the SSRB, receives large amounts of water, but produces low net benefits. Hydropower generation is the second largest water use, and has a low water value also. Although hydropower stations are non-consumptive users, they do compete for river flows with upstream uses and reservoir carry-over storages for future uses. The differences of water values among different water uses are also been evident by the monthly marginal net benefits of raw water based on initial withdrawal rights to non-storage demand sites which are almost evenly distributed among months as displayed in Table 6.6 (Case C). Recall that the initial water rights are allocated by the PMMNF method in this case. The marginal values of raw water withdrew from the junction node J2 to the general (G2) and industrial (I2) demands of Calgary are more than \$1.8 per cubic meter during all months, and are significantly higher than other uses, even the domestic use of Calgary. The domestic use of Calgary has monthly marginal water values at \$0.833 per cubic meter. Domestic, general and industrial water uses at other municipalities have marginal water values between \$0.357 and \$0.746 per cubic meter. The marginal values of irrigation withdrawals are between \$0.015 and \$0.051 per cubic meter, while those of hydropower stations are at a constant of 0.011 per cubic meter throughout the year.

Table 6.7 lists the marginal values of raw water withdrawals based on the initial water rights allocated by the LWMSR method (Case F). These marginal values are similar to those allocated by the PMMNF method, except that those of the domestic, general and industrial uses of Calgary are more evenly distributed between $0.745 and $1.312 per cubic meter. As shown in Figure 6.40, no matter what the wet, normal or drought year cases are, the total annual inflows of non-storage demand sites in the SSRB based on the initial rights allocations by the LMWSR method are a little smaller than that allocated by the PMMNF method, and more water is left in the river system and passed onto the downstream Province of Saskatchewan. However, total net benefits based on the results of LMWSR are nearly as large as that by PMMNF, and even greater under the hydrologic drought scenario. This means that the LMWSR method can produce water allocations that are not only equitable but also as economic efficient as PMMNF.

Table 6.4 Annual inflows to non-storage demand sites based on initial rights (mcm)

Water use type	Demand site	Case A	Case B	Case C	Case D	Case E	Case F
Irrigation	A1	52.529	112.107	150.645	50.865	110.478	133.784
	A2	245.951	398.043	441.443	244.21	373.784	497.327
	A3	361.36	640.171	861.507	361.36	640.171	723.347
	A4	135.301	308.088	368.678	129.832	296.983	320.221
	A5	9.38	43.725	55.575	9.193	41.41	46.52
	A6	22.34	63.972	81.262	21.602	63.972	73.663
	A7	123.81	280.704	328.625	122.915	270.801	309.184
	A8	89.761	179.436	184.445	89.761	169.142	189.399
	A9	276.03	474.417	455.132	274.847	443.172	461.772
Domestic	D1	4.1	6.027	6.027	4.1	6.027	6.027
	D2	88.685	147.792	147.792	88.553	147.363	145.258
	D3	10.726	14.149	14.149	10.697	14.1	14.149
	D4	5.59	7.042	7.042	5.59	7.042	7.023
General	G1	3.874	5.692	5.692	3.874	5.692	5.692
	G2	47.76	79.589	55.275	47.375	78.741	76.12
	G3	12.714	16.772	16.772	12.61	16.652	16.772
	G4	6.625	8.348	8.348	6.625	8.348	8.303
Industrial	I1	55.871	139.674	139.674	55.871	139.674	139.674
	I2	51.383	154.142	73.188	50.922	151.754	148.799
	I3	7.694	15.381	15.381	7.694	15.26	15.098
	I4	25.494	50.987	50.987	25.494	50.987	50.941
Hydropower	H1	240	240	192.462	232.863	234.046	192.462
	H2	240	240	240	237.842	236.093	230.317
Total		2116.978	3626.258	3900.101	2094.695	3521.692	3811.852

Table 6.5 Annual net benefits based on initial water rights of demand sites (million $)

Water use type	Demand site	Case A	Case B	Case C	Case D	Case E	Case F
Irrigation	A1	1.976	3.328	5.567	1.937	3.303	5.269
	A2	18.609	25.153	42.341	18.544	24.511	44.461
	A3	16.016	24.25	40.515	16.016	24.25	37.24
	A4	6.821	13.27	19.092	6.68	13.054	17.959
	A5	0.264	1.2	1.764	0.26	1.163	1.604
	A6	1.008	3.688	5.511	0.986	3.688	5.267
	A7	6.372	17.837	29.024	6.353	17.591	28.479
	A8	8.478	14.395	21.673	8.478	14.11	21.89
	A9	26.088	36.813	50.436	26.04	35.919	50.926
Domestic	D1	12.592	18.52	18.52	12.592	18.52	18.52
	D2	315.101	525.112	525.112	314.975	524.687	522.448
	D3	35.978	47.454	47.454	35.955	47.415	47.454
	D4	17.013	21.431	21.431	17.013	21.431	21.42
General	G1	8.724	12.825	12.825	8.724	12.825	12.825
	G2	124.799	207.975	172.774	124.444	207.179	204.368
	G3	30.014	39.592	39.592	29.947	39.516	39.592
	G4	14.652	18.454	18.454	14.652	18.454	18.431
Industrial	I1	71.002	177.506	177.506	71.002	177.506	177.506
	I2	99.293	297.877	160.032	98.871	295.659	292.182
	I3	15.383	30.761	30.761	15.383	30.644	30.486
	I4	36.172	72.327	72.327	36.172	72.327	72.308
Hydropower	H1	2.69	2.69	2.157	2.61	2.623	2.157
	H2	2.69	2.69	2.69	2.666	2.646	2.581
Total		871.735	1615.148	1517.558	870.3	1609.021	1675.373

Table 6.6 Monthly marginal net benefits of raw water based on initial withdrawal rights to non-storage demand sites (Case C) ($\$/m^3$)

Diversion (or release)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
J2.D2	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833
J2.G2	2.263	2.264	2.264	2.263	1.841	1.841	1.841	1.841	1.84	2.264	2.264	2.263
J2.I2	1.924	1.924	1.924	1.924	1.924	1.924	1.924	1.924	1.924	1.924	1.924	1.924
R2.H1	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011
R3.H2	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011
R5.A1					0.015	0.015	0.015	0.015	0.015			
R6.A2					0.024	0.036	0.045	0.038	0.030			
R7.A3					0.018	0.019	0.02	0.019	0.018			
R7.I3	0.746	0.746	0.746	0.746	0.745	0.745	0.745	0.745	0.746	0.744	0.746	0.746
R9.A4					0.017	0.017	0.023	0.021	0.021			
R12.A5					0.015	0.015	0.015	0.015	0.015			
R14.A6					0.022	0.03	0.031	0.02	0.018			
R15.A7					0.026	0.025	0.041	0.027	0.023			
R16.A8					0.027	0.038	0.046	0.043	0.034			
R17.A9					0.029	0.051	0.044	0.040	0.030			
S1.D1	0.418	0.422	0.418	0.418	0.421	0.418	0.419	0.419	0.421	0.419	0.418	0.418
S1.G1	0.386	0.383	0.386	0.382	0.386	0.383	0.383	0.385	0.384	0.381	0.387	0.384
S1.I1	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384
S3.D3	0.668	0.668	0.669	0.669	0.67	0.67	0.67	0.669	0.67	0.669	0.668	0.668
S3.G3	0.494	0.493	0.492	0.495	0.493	0.493	0.493	0.493	0.492	0.495	0.494	0.492
S4.D4	0.457	0.452	0.452	0.454	0.454	0.457	0.455	0.456	0.454	0.454	0.454	0.457
S4.G4	0.357	0.357	0.357	0.358	0.358	0.358	0.358	0.358	0.359	0.358	0.357	0.357
S4.I4	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358

Table 6.7 Monthly marginal net benefits of raw water based on initial withdrawal rights to non-storage demand sites (Case F) ($/m^3)

Diversion (or release)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
J2.D2	0.921	0.848	0.848	0.974	0.845	0.833	0.833	1.090	0.833	0.833	0.833	0.833
J2.G2	0.917	0.773	0.886	0.990	0.767	0.745	0.745	1.311	0.745	0.745	0.745	1.030
J2.I2	0.856	0.745	0.745	0.767	0.745	0.745	0.745	1.312	0.745	0.967	0.745	0.745
R2.H1	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011
R3.H2	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011
R5.A1					0.015	0.021	0.018	0.017	0.017			
R6.A2					0.024	0.033	0.029	0.038	0.031			
R7.A3					0.018	0.026	0.028	0.025	0.018			
R7.I3	0.746	0.746	0.746	0.746	0.745	0.850	0.745	0.745	1.073	0.744	0.897	0.891
R9.A4					0.019	0.020	0.027	0.026	0.026			
R12.A5					0.019	0.017	0.021	0.019	0.017			
R14.A6					0.024	0.033	0.036	0.025	0.020			
R15.A7					0.028	0.030	0.041	0.030	0.030			
R16.A8					0.030	0.041	0.044	0.035	0.032			
R17.A9					0.033	0.042	0.045	0.043	0.035			
S1.D1	0.418	0.422	0.418	0.418	0.421	0.418	0.419	0.419	0.421	0.419	0.418	0.418
S1.G1	0.386	0.383	0.386	0.382	0.386	0.383	0.383	0.385	0.384	0.381	0.387	0.384
S1.I1	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384
S3.D3	0.668	0.668	0.669	0.669	0.670	0.670	0.670	0.669	0.670	0.669	0.668	0.668
S3.G3	0.494	0.493	0.492	0.495	0.493	0.493	0.493	0.493	0.492	0.495	0.494	0.492
S4.D4	0.550	0.452	0.452	0.454	0.454	0.457	0.455	0.456	0.454	0.454	0.454	0.457
S4.G4	0.533	0.357	0.357	0.358	0.358	0.358	0.358	0.358	0.359	0.358	0.357	0.357
S4.I4	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358

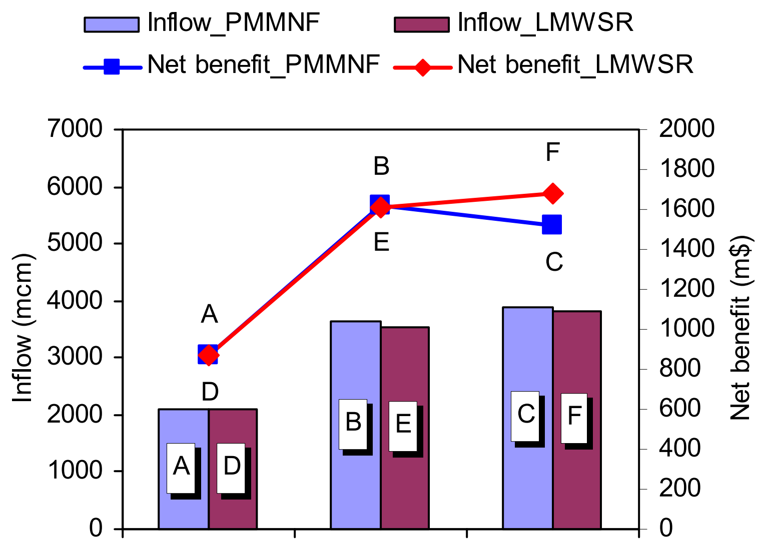

Figure 6.40 Total annual inflows and corresponding total net benefits of non-storage demand sites in the South Saskatchewan River Basin based on the initial rights allocations under the six case scenarios

6.4.3.2 River Basin Optimal Water Allocation

Figure 6.41 and Figure 6.42 show the annual inflows to non-storage demand sites under Case C and corresponding net benefits gained by water demand sites. Under the whole river basin optimal allocation scenario under Case C, the irrigation regions A1, A2, A5, A6, A7, and hydropower stations H1 yield inflow amounts of 119.398, 5.158, 21.218, 2.006, 40.658 and 13.797 mcm, respectively (202.235 mcm in total), while A8, A9, G2, and I2 receive extra 9.103, 45.311, 24.314, 80.954 mcm of water (159.682 mcm in total) in addition to their inflow rights. As noted, however, not all inflow yields by stakeholders can be received by others. The differences are lost during the water transportation in the river system and diversion systems. The reductions of raw water withdrawals and increments of withdrawals at different locations are computed by the cooperative water allocation model (CWAM). The water allocation schemes and marginal benefits of raw water withdrawals can be used as references for water resources planning or water trade.

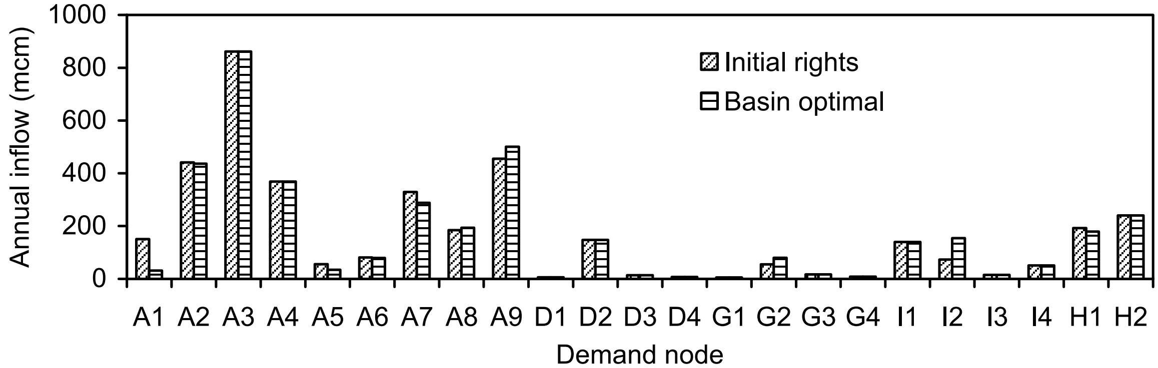

Figure 6.41 Annual inflows to non-storage demand sites (Case C)

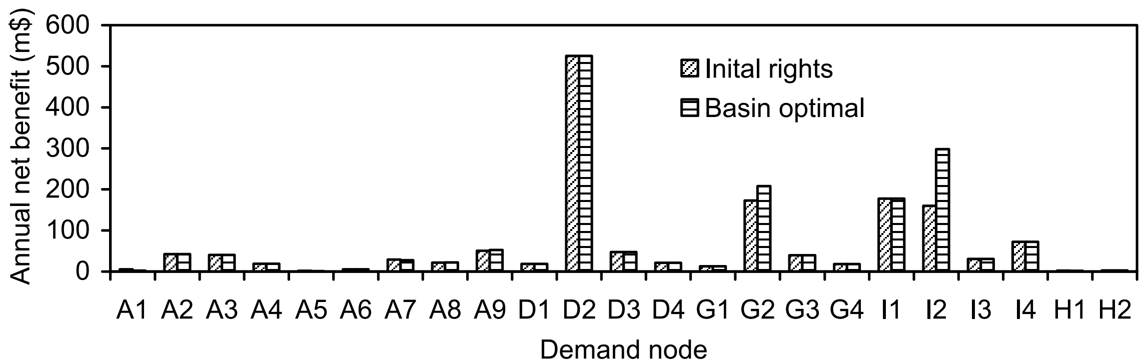

Figure 6.42 Annual net benefits of inflows allocated to demand sites (Case C)

Annual net benefits of stakeholders under the initial and river basin optimal scenarios in Case C are summarized in Table 6.8, which shows the water trade (including both intra- and inter-stakeholder trade) will lead to an increase in the total net benefit of the SSRB in the amount of 170.5 million dollars. Marginal net benefits of water uses under the river basin optimal allocation scenario in Case C are tabulated in Table 6.9, which are evenly distributed among months due to the economic optimization covering all the time periods. For example, the monthly marginal water values of the domestic, general and industrial uses at Calgary city are \$0.833, \$0.745 and \$0.745 per cubic meter, respectively. The differences among different types of water uses are due to differences in their water demand or benefit functions and the upper bounds set on monthly water demands.

Table 6.8 Annual net benefits of water use by stakeholders under Case C (million $)

Stakeholder	Demand site	Water use net benefit		Stakeholder net benefit	
		Initial rights	Basin optimal	Initial rights	Basin optimal
RD (Red Deer City)	D1	18.520	18.520	208.851	208.851
	G1	12.825	12.825		
	I1	177.506	177.506		
BH (Hydrostations on Bow River)	H1	2.157	2.002	4.847	4.692
	H2	2.690	2.690		
CA (Calgary City)	D2	525.112	525.112	857.918	1030.963
	G2	172.774	207.975		
	I2	160.032	297.877		
EIN (Eastern Industrial Region)	I3	30.761	30.761	30.761	30.761
BIR (Irrigation Regions in Bow River Subbasin)	A1	5.567	2.427	88.423	85.328
	A2	42.341	42.386		
	A3	40.515	40.515		
LB (Lethbridge City)	D3	47.454	47.454	87.046	87.046
	G3	39.592	39.592		
OIR (Irrigation Regions in Oldman River Subbasin)	A4	19.092	19.092	127.499	128.205
	A5	1.764	1.313		
	A6	5.511	5.463		
	A7	29.024	27.781		
	A8	21.673	22.080		
	A9	50.436	52.476		
MH (Medicine Hat City)	D4	21.431	21.431	112.212	112.212
	G4	18.454	18.454		
	I4	72.327	72.327		
Total		1517.557	1688.059	1517.557	1688.059

Table 6.9 Marginal net benefits of withdrawals to non-storage demand sites under the river basin optimal allocation scenario of Case C ($/m^3)

Diversion (or release)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
J2.D2	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833
J2.G2	0.745	0.745	0.745	0.745	0.745	0.745	0.745	0.745	0.745	0.745	0.745	0.745
J2.I2	0.745	0.745	0.745	0.745	0.745	0.745	0.745	0.745	0.745	0.745	0.745	0.745
R2.H1	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011
R3.H2	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011
R5.A1					0.035	0.036	0.036	0.036	0.038			
R6.A2					0.038	0.038	0.038	0.038	0.038			
R7.A3					0.018	0.019	0.020	0.019	0.018			
R7.I3	0.746	0.746	0.746	0.746	0.745	0.745	0.745	0.745	0.746	0.744	0.746	0.746
R9.A4					0.017	0.017	0.023	0.021	0.021			
R12.A5					0.015	0.027	0.027	0.026	0.020			
R14.A6					0.028	0.030	0.031	0.026	0.020			
R15.A7					0.038	0.038	0.038	0.036	0.027			
R16.A8					0.039	0.039	0.039	0.037	0.028			
R17.A9					0.029	0.040	0.040	0.038	0.029			
S1.D1	0.418	0.422	0.418	0.418	0.421	0.418	0.419	0.419	0.421	0.419	0.418	0.418
S1.G1	0.386	0.383	0.386	0.382	0.386	0.383	0.383	0.385	0.384	0.381	0.387	0.384
S1.I1	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384
S3.D3	0.668	0.668	0.669	0.669	0.670	0.670	0.670	0.669	0.669	0.669	0.668	0.668
S3.G3	0.494	0.493	0.492	0.495	0.493	0.493	0.493	0.493	0.492	0.495	0.494	0.492
S4.D4	0.457	0.452	0.452	0.454	0.454	0.457	0.455	0.456	0.454	0.454	0.454	0.457
S4.G4	0.357	0.357	0.357	0.358	0.358	0.358	0.358	0.358	0.359	0.358	0.357	0.357
S4.I4	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358	0.358

The annual inflows to non-storage demand sites under Case F and corresponding net benefits gained by water demand sites are shown in Figure 6.43 and Figure 6.44. Under the river basin optimal allocation scenario in Case F, large amounts of water are exchanged among irrigation regions, rather than to a more valuable user, the City of Calgary. Although the irrigation regions A5 and A7, as well as hydropower stations H1 and H2 yield inflow in the total amount of 272.126 mcm, while A1, A2, A3, A4, A6, A8, A9, D2, G2 and I2 receive extra 263.925 mcm in total in addition to their inflow rights, the gain of water trade is only $15.53 million. This is caused by the more evenly water satisfaction ratios of initial water rights obtained by the LMWSR method.

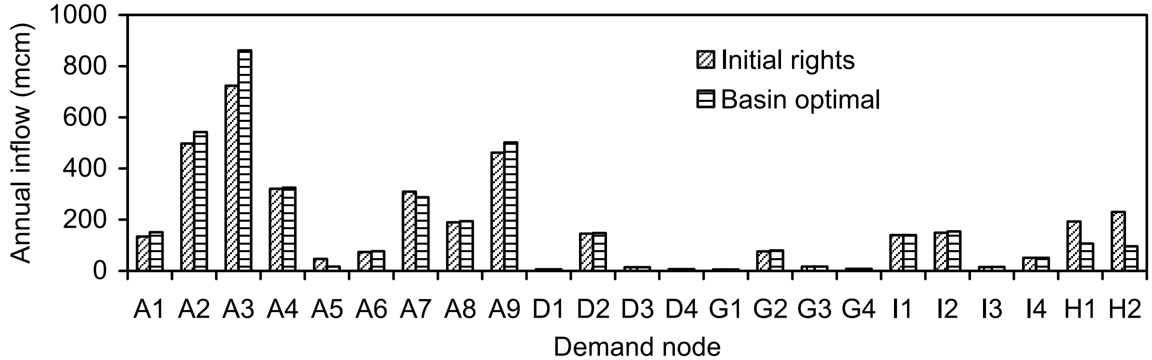

Figure 6.43 Annual inflow to non-storage demand sites (Case F)

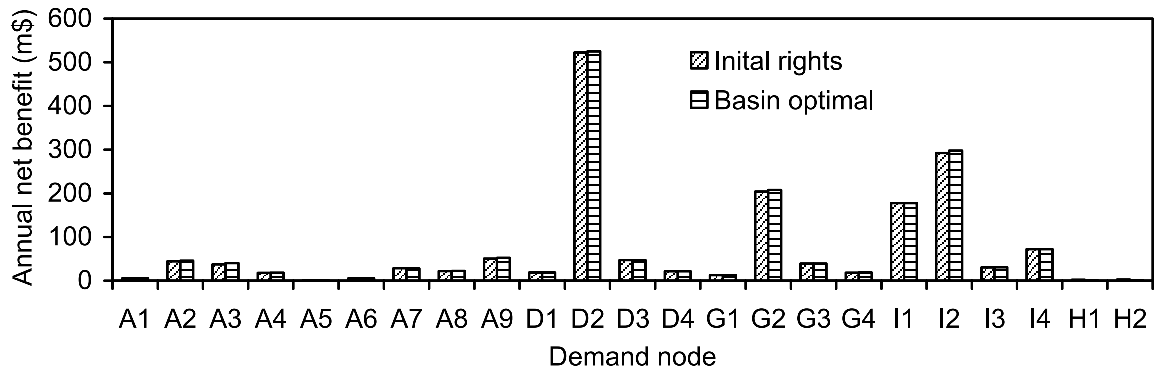

Figure 6.44 Annual net benefits of inflows allocated to demand sites (Case F)

6.4.3.3 Reallocation of Net Benefits among Stakeholders

The eight stakeholders considered in the hydrologic-economic river basin modeling and coalition analysis lead to 255 nonempty coalitions. It would require extensive computation effort to solve so many large NLP problems. However, the analysis of the initial rights and basin optimal scenarios under both Cases C and F has shown that only four stakeholders (BH, CA, BIR and OIR) have significant changes of inflows and net benefits, while others are either nil or very small. Thus, the coalition analysis just needs to consider these four players. The reduction of stakeholders drastically decreases the coalition number to only 15. For every coalition, the multistart global optimization algorithm for coalition analysis utilizes the OQNLP solver to generate hundreds of scatter trial points, select good starting points, and call the gradient-based nonlinear solver MINOS for further optimization and final solution determination. According to solver reports, each run of the hydrologic-economic river basin model, there are 4801 control variables, 4947 constraints, and 5927 non-linear elements. With the default solver option parameters and the generation of 500 trial points for each coalition, the total computing time for all the 15 coalitions may take over 20 minutes on a 3 GHz Intel Pentium 4 CPU.

Stakeholders may have different capacities of water withdrawal and different gains of water uses when participating in different coalitions. For example, as shown by the net benefits of coalitions under Case C listed in Table 6.10, if BH and CA pursue the intra-optimal allocation, they may only get nil and \$28.110 million more than those obtained by their initial rights, respectively. But they can gain an increase of \$38.642 million if they work cooperatively. The different capability of gains while involving different coalitions makes it necessary to analyze all the possible coalitions of stakeholders in order to promote the grand coalition and equitably allocate the net benefits (side payments). The cores of the cooperative net benefit reallocation games under both Cases C and F are nonempty, which means there are infinite possible allocations satisfying the equity rationalities as long as they are located in the cores. The allocations by various nucleolus and Shapley values are just instances for reference.

Value of participation in the grand coalition for each stakeholder is represented as the additional gain over the independent optimal (intra-stakeholder optimal) net benefit that can be produced based on his or her initial water rights. The additional gains under Case C after reallocation of water and net benefits with different cooperative game solution concepts are summarized in Table 6.11 and Figure 6.45. Under Case C, Calgary is normally allocated most of the additional gain over the intra-optimal scenario benefit, with ranges from 67 to 138 million dollars, since it is the major contributor to the grand coalition. The Bow River Irrigation Regions (BIR) makes additional gains from about 3 to 60 million dollars, by receiving a side payment from Calgary for the water trade among them. The values of participation in the grand coalition under Case F are plotted in Figure 6.46. The total additional gain over intra-optimal allocation is only \$13.815 million. Most of the gain is allocated to Calgary due to its significant contribution to the grand coalition. BIR gains from \$0.553 to \$2.421 million, since it withdraws more water under the grand coalition scenario than its initial water rights. Bow River hydropower stations (BH) make additional gains from about 2 to 3 million dollars, by receiving side payments from Calgary and the Bow River Irrigation Regions (BIR) for water rights transferred to them.

Table 6.10 Net benefits of coalitions under Case C (million $)

Coalition	Coalition members	Initial rights	Coalition value	Increase
1	BH	4.847	4.847	0
2	CA	857.918	886.028	28.110
3	BH, CA	862.765	901.407	38.642
4	BIR	88.423	88.996	0.573
5	BH, BIR	93.270	93.844	0.574
6	CA, BIR	946.341	1080.674	134.333
7	BH, CA, BIR	951.188	1118.456	167.268
8	OIR	127.499	127.810	0.311
9	BH, OIR	132.346	132.657	0.311
10	CA, OIR	985.417	1023.823	38.406
11	BH, CA, OIR	990.264	1031.058	40.794
12	BIR,OIR	215.922	216.915	0.993
13	BH, BIR,OIR	220.769	221.762	0.993
14	CA, BIR,OIR	1073.840	1208.173	134.333
15	BH, CA, BIR,OIR	1078.687	1249.188	170.501

Table 6.11 Values of participation in the grand coalition for stakeholders under Case C reallocated with different cooperative game solution concepts (million $)

Stakeholder	Intra-optimal	Additional gain over Intra-optimal				
		Nucleolus	Weak Nucleolus	Proportional Nucleolus	Normalized Nucleolus	Shapley Value
BH	4.847	16.467	0.731	0.011	0.011	12.863
CA	886.028	74.284	107.762	138.269	130.433	67.213
BIR	88.996	49.294	32.283	2.927	10.763	59.723
OIR	127.810	1.462	0.731	0.300	0.300	1.708
Total	1107.681	141.507	141.507	141.507	141.507	141.507

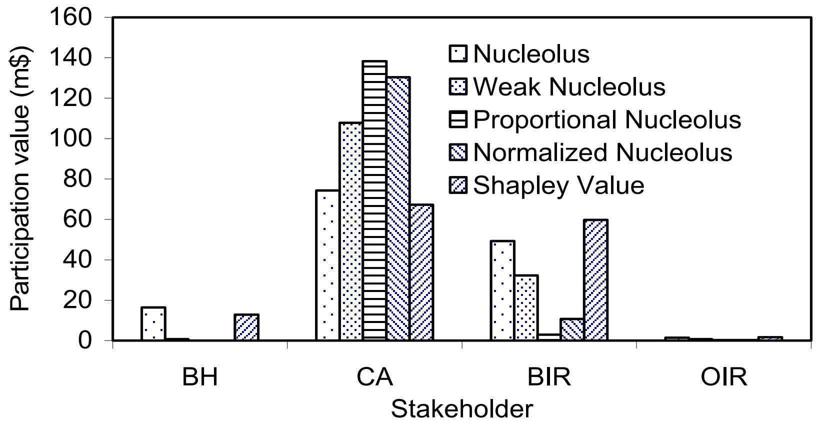

Figure 6.45 Values of participation in the grand coalition for stakeholders under Case C reallocated with different cooperative game solution concepts

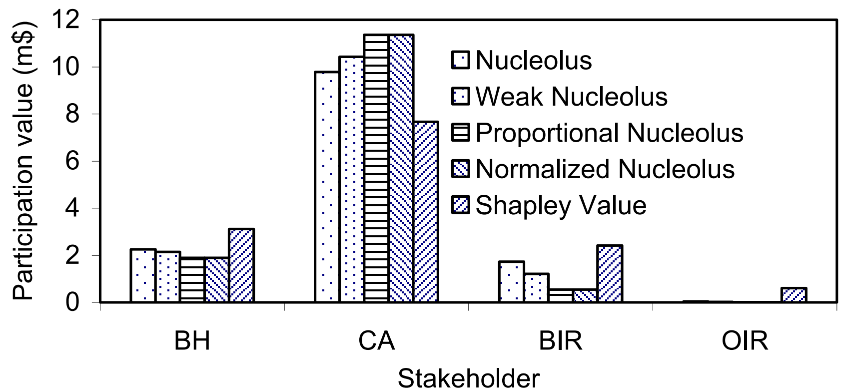

Figure 6.46 Value of participation in the grand coalition for stakeholders under Case F reallocated with different cooperative game solution concepts

6.4.3.4 Sensitivity Analysis

Sensitivity analyses are carried out for the changes of economic parameters under the hydrologic drought year Cases C and F, including crop prices, elasticities and choke prices of domestic, general and industrial demands. As shown in Table 6.12, a 20% decrease or increase of crop prices causes less than a 4.0% reduction or increase of the total net benefit in the SSRB. A 20% decrease of demand elasticities of municipal and industrial uses causes less than a 3.7% reduction of the total net benefit in the SSRB, while a 20% increase leads to less than a 6.5% increase of total net benefit. A 50% decrease or increase of choke prices of municipal and industrial uses causes less than a 35.0% reduction or increase of the total net benefit. The total net benefits of water uses and gains of water trade are not very sensitive to the changes of these economic parameters, as well as to the hydrologic parameters changes which has been indicated by the sensitivity analyses carried out for initial water rights.

Table 6.12 Sensitivity analysis of the total net benefits of water uses and gains of water trade (absolute value unit: million $)

Parameter	Case C			Case F		
	Initial rights	Grand coalition	Trade gain	Initial rights	Grand coalition	Trade gain
Base parameters	1517.558	1688.059	170.501	1675.373	1690.904	15.531
20% decrease of crop prices	-4.0%	-3.8%	-2.3%	-3.5%	-3.6%	-16.1%
20% increase of crop prices	4.0%	3.3%	-2.7%	3.5%	3.6%	11.8%
20% decrease of demand elasticities of municipal and industrial uses	-3.2%	-3.7%	-7.9%	-3.5%	-3.6%	-7.2%
20% increase of demand elasticities of municipal and industrial uses	5.9%	5.4%	0.3%	6.5%	6.4%	3.6%
50% decrease of choke prices of municipal and industrial uses	-35.0%	-33.9%	-23.6%	-34.1%	-33.8%	-1.9%
50% increase of choke prices of municipal and industrial uses	35.0%	33.7%	22.1%	34.1%	33.8%	-2.9%

6.5 Summary

The application to the South Saskatchewan River Basin (RRSB) in Southern Alberta verifies the applicability of the cooperative water allocation model (CWAM) developed in this thesis for fair and efficient allocation of water resources at the basin scale.

The results of PMMNF show that in the wet and normal hydrologic years, all non-storage demands can be satisfied. With the forecasted water demand in 2021 and the assumed drought conditions (50% of the long-term normal natural flows), the Bow River (A2), Lethbridge Northern (A4), St. Mary River-West (A7), Taber (A8) and St. Mary River-East (A9) irrigation regions, the general (G2) and industrial (I2) demands of Calgary, hydropower plants on Kananaskis River (H1), and Oldman River at St. Mary River confluence (S3) will have water shortages, with the annual satisfaction ratios varying from 0.966 to 0.475. The general and industrial water demands of Calgary, the largest municipality in the SSRB, are supplied with rather low ratios at 0.695 and 0.475, respectively. This is because the newly issued withdrawal licenses for their water demands expanding in the near future would have low priority ranks according to the existing prior water allocation system.

The results of LMWSR show that in the wet and normal hydrologic years, all irrigation regions have annual satisfaction ratios larger than one, and other non-storage demands have ratios equal to or less than one but all larger than 0.93. Under the scenario of forecasted demands in 2021 and the assumed drought conditions, the annual satisfaction ratios of all off-stream and the hydropower generation water demands vary from 0.802 to 1.

Sensitivity analyses show that the changes of initial reservoir storages, node loss coefficients, priority ranks and weights, will not cause significant changes in water allocations under a wet or normal hydrologic year. However, they do cause considerable variations for some irrigation regions and the stream flow requirement sites under dry years. Since the PMMNF method is based on the priority concept adopted by the prior rights regime, its application usually produces large differences in the satisfaction ratios among uses in times of water shortages. Compared to the PMMNF method, the allocations by LMWSR lead to more evenly distributed satisfaction ratios, because LMWSR is based on the lexicographic minmax equity concept. LMWSR sounds fairer in having water shortages shared among all the monthly demands of various types of uses while taking account of the weights reflecting the relative importance to water uses and their endurance to water shortages.

Eight stakeholders are considered in the economic analysis. The value of stream flows and reservoir storages are not explicitly included due to data limitation, and their water rights are preserved through hydrologic constraints. Total net benefits based on the initial rights allocated by LMWSR are nearly as large as that by PMMNF, and even greater under the hydrologic drought scenario. This shows that the LMWSR method can produce water allocations that are not only equitable but also as economically efficient as PMMNF.

Water transfer (or trade) under the river basin optimal allocation scenario in the assumed 2021 drought year will lead to an increase of total net benefit of the SSRB in the amount of $170.5 million over that obtained based on initial water rights allocated by PMMNF, but only

$15.5 million over that allocated by LMWSR. This is because of the more even water satisfaction ratios of water uses under the LMWSR water allocation.

The analysis of the initial rights and basin optimal scenarios under both Cases C and F shows that only four stakeholders (BH, CA, BIR and OIR) have significant changes of inflows and net benefits, while others are either nil or very small. Thus, only these four stakeholders are chosen for the coalition analysis. The reduction in the number of stakeholders drastically decreases the computational load for the coalition analysis.

The cores of the cooperative net benefit reallocation games under both Cases C and F are nonempty, which means there are infinite possible allocations satisfying the equity rationalities as long as they are located in the cores. The reallocation of the net benefit of the grand coalition with different cooperative game solution concepts shows that the additional gains of Calgary, the major contributor to the grand coalition, are usually higher than other stakeholders in the SSRB.

Sensitivity analyses show that, under the hydrologic drought year Cases C and F, the total net benefits of water uses and gains of water transfer are not very sensitive to the changes in economic parameters including crop prices, elasticities and choke prices of domestic, general and industrial demands, as well as to hydrologic parameters.

Chapter 7
Conclusions and Future Research

7.1 Conclusions

The overall contribution of this dissertation is the development of the Cooperative Water Allocation Model (CWAM) for equitable and efficient water allocation among competing users at the river basin scale. In the following, the modeling methodology, components, case studies, and main contributions are summarized.

Cooperative Water Allocation Methodology

- CWAM adopts a two-step methodology: initial water rights are firstly allocated to competing uses based on legal water rights systems or agreements, and then water is reallocated to achieve economically efficient use through water transfers among uses. The associated net benefits of stakeholders are reallocated by cooperative game theoretical approaches. The two-step cooperative water allocation methodology allocates water rights and utilizes them as the basis to promote fair cooperation of stakeholders to pursue maximum social welfare in a river basin.
- The model carries out water rights assignment, efficient water allocation and fair income distribution subject to both water quantity and quality constraints, and produces output information including initial water rights, water transfers, shadow prices of water at various demand sites, and corresponding water allocation schemes.
- The model is designed based on an abstract river basin network and the modeling framework is generic, flexible and expandable. Thus, it is applicable to any river basin or sub-watershed.
- The model may serve as a decision support tool to guide water trade under either a regulated water market or an administrative allocation mechanism, which has been demonstrated through the results of the preliminary applications to the Amu Darya

River Basin in Central Asia and the South Saskatchewan River Basin in western Canada.

Initial Water Rights Allocation

- Three methods are developed for initial water rights allocation under different water rights regimes or agreements. The priority-based multiperiod maximal network flow (PMMNF) programming allocates water strictly according to the priority ranks of water uses, and is a very flexible approach that is applicable under prior, riparian and public water rights systems. The modified riparian water rights allocation (MRWRA) is essentially a special form of PMMNF adapted for allocation under the riparian regime. The lexicographic minimax water shortage ratios (LMWSR) method adopts the lexicographic minimax fairness concept, and is designed for fair allocation under the public water rights system. The developed methods may also be applied to international basins as long as water demands and associated priorities or weights are set to reflect international water allocation principles and agreements.
- Sequential algorithms are developed to solve both the linear and nonlinear PMMNF problems. The difficulty unsolved by previous studies in assigning proper weights or unit cost coefficients for network flow programming is avoided by the sequential approach for each priority rank. The LMWSR problems are solved by an iterative procedure.
- The linear PMMNF and LMWSR problems, containing only linear water quantity constraints, can be efficiently solved by the primal simplex method. The nonlinear PMMNF and LMWSR problems, which allow nonlinear reservoir area and elevation-storage curves and water quality constraints, can be efficiently solved by conventional gradient-based nonlinear programming techniques through the proposed two-stage approach. The first stage solves the corresponding linear problems excluding nonlinear items and constraints by a sequential or iterative algorithm. The obtained global optimal solution is then combined with initial values for pollutant concentrations to form a good starting point for the nonlinear programming of the nonlinear PMMNF or LMWSR problem.

- The case study of the Amu Darya River basin shows that the water rights system and associated priority assignments are key factors when employing the PMMNF method to search for fair water rights allocations, while the weights setting determines the results of LMWSR. Thus it is important to have water rights systems or to reach agreements with harmonized and fair distribution rules before the implementation of water allocation, since they are the legal bases.
- Equity principles for generic resource allocation problems are reviewed, by using the social utility function aggregating individual objectives or outcomes. The priority-based multiple objective resource allocation optimization is a priorly equitable allocation method, because it has an aggregated social utility function satisfying the monotonicity and priority principles. The ordered weighted averaging aggregation function has been proved to satisfy the principles of monotonicity, impartiality and equitability. Thus, the weighted sum of ordered outcome optimization is a perfectly equitable resource allocation method. This means that the lexicographic maximization is a specific formulation of the generic resource allocation problem producing perfectly equitable allocation schemes.
- Based upon these equity principles and fair solution concepts for the generic resource allocation problem, it is concluded that PMMNF and MRWRA are priorly equitable, while LMWSR is perfectly equitable.

Irrigation Water Planning Model

- Quadratic empirical crop yield-water and salinity relationship functions have been utilized to develop the irrigation water planning model (IWPM), which optimizes the monthly distribution of the annual water available to an agricultural demand site and searches for the optimal annual profit of irrigation. The outputs of a series of simulations under different water availability are used in a regression model to obtain the monthly gross profit functions of irrigation water. In order to derive the quadratic functions correctly representing the real relationships between monthly profits and irrigation, the normal least squares method for multiple regression is modified by introduction of constraints reflecting equality of marginal benefits between months.

- The default assumption of IWPM is that the total available water can be optimally allocated to all crop fields and for all time periods, subject to land area and irrigation water limits. Therefore, the application of monthly benefit functions of irrigation water derived by the model should follow these assumption and limits. The derived optimal monthly benefit functions are normally applicable to river basins with storage reservoirs.

Hydrologic-economic River Basin Model

- The integrated hydrologic-economic river basin model (HERBM) is developed for finding the optimal total net benefit of demand sites. The constant price-elasticity water demand functions are utilized to derive the gross benefit functions of water uses of municipal and industrial demand sites, and hydropower stations. Quadratic benefit functions consisting of both water quantity and quality items are adopted for agriculture water uses, stream flow demands and reservoir storages.
- HERBM is based on monthly net benefit functions of various demand sites which are estimated by econometric methods or through more detailed sub-models. This "compartment modeling" approach integrating sub-models at the field or node scale into HERBM at the basin scale reduces the size of the hydrologic-economic water allocation optimization problem, and makes the model more flexible and less computationally intensive.

Coalition Analysis, Water and Net Benefits Reallocation Game

- HERBM is modified and extended to estimate the net benefits of various coalitions of stakeholders. The highly nonlinear HERBM is solved by the combination of MINOS and OQNLP solvers utilizing a multistart global optimization technique.
- Solution concepts from cooperative game theory, including the nucleolus, weak nucleolus, proportional nucleolus, normalized nucleolus and Shapley value, are adopted to solve the water and net benefits reallocation game. The solutions provide reallocation alternatives for the grand coalition.

South Saskatchewan River Basin Case Study

- The results of PMMNF show that in the wet and normal hydrologic years, all non-storage demands can be satisfied. With the forecasted water demands in 2021 and the assumed drought conditions (50% of the long-term normal natural flows), some non-storage demands cannot be fully satisfied, whose annual satisfaction ratios range from 0.966 (96.6%) to 0.475 (47.5%). The general and industrial water demands of Calgary, the largest municipality in the SSRB, are supplied with rather low ratios at 0.695 (69.5%) and 0.475 (47.5%), respectively. This is because the newly issued withdrawal licenses for their water demands expanding in the near future would have low priority ranks according to the existing prior water rights system.
- The results of LMWSR show that in the wet and normal hydrologic years, all off-stream and the hydropower generation water demands have annual satisfaction ratios equal to or less than one but all larger than 0.93 (93%). Under the scenario of forecasted demands in 2021 and the assumed drought conditions, the annual satisfaction ratios of all offstream and the hydropower generation water demands vary from 0.802 (80.2%) to 1 (100%).
- Sensitivity analyses show that the changes of initial reservoir storages and node loss coefficients will not cause significant changes for water allocations under a wet or normal hydrologic year. However, they do cause considerable variations for some irrigation regions and the stream flow requirement sites under dry years.
- Since the PMMNF method is based on the priority concept adopted by the prior rights regime, its application usually produces large difference of the satisfaction ratios among uses in times of water shortages. Compared to the PMMNF method, the allocations by LMWSR lead to more evenly distributed satisfaction ratios, because LMWSR is based on the lexicographic minimax equity concept. LMWSR sounds fairer in having water shortages shared among all the monthly demands of various types of uses while taking into account the weights reflecting the relative importance to water uses and their endurance to water shortages.
- Eight stakeholders are considered in the economic analysis. The values of stream flows and reservoir storages are not explicitly included due to data limitations, and

their water rights are preserved through hydrologic constraints. Total net benefits based on the results of LMWSR are nearly as large as that by PMMNF, and even greater under the hydrologic drought scenario. This shows that the LMWSR method can produce water allocations that are not only equitable but also as economically efficient as PMMNF.

- Water trade under the river basin optimal allocation scenario in the assumed 2021 drought year will lead to an increase of the total net benefit of the SSRB in the amount of $170.5 million over that obtained based on initial water rights allocated by PMMNF, but only $15.5 million over that allocated by LMWSR. This is because of the more evenly distributed water satisfaction ratios of water uses under the LMWSR water allocation.
- The analysis of the initial rights and basin optimal scenarios under both Cases C and F shows that only four stakeholders (BH, CA, BIR and OIR) have significant changes of inflows and net benefits, while others are either nil or very small. Thus, only these four stakeholders are chosen for the coalition analysis. The reduction of the number of stakeholders drastically decreases the computational load for the coalition analysis.
- The cores of the cooperative net benefit reallocation games under both Cases C and F are nonempty, which means there are infinite possible allocations satisfying the equity rationalities as long as they are located in the cores. The reallocation of the net benefit of the grand coalition with different cooperative game solution concepts shows that the additional gains of Calgary, the major contributor to the grand coalition, are usually higher than other stakeholders in the SSRB.
- Sensitivity analyses show that, under the hydrologic drought year Cases C and F, the total net benefits of water uses and gains of water transfer are not very sensitive to the changes of economic parameters including crop prices, elasticities and choke prices of domestic, general and industrial demands, as well as to hydrologic parameters.

7.2 Future Research

There are a number of assumptions and limitations contained in this research:

- Typical time steps of the hydrologic and economic sub-models are one month, which may also be one year. Smaller time steps are not realistic, because: (1) The river basin network is a quasi-static model, in which the flows and storages are periodically averaged, the storage changes in river reaches are ignored, and no time lag is considered for return flows, and (2) The use of smaller time intervals for hydrologic processes in an optimization model is restricted by computational capacity and data availability.
- The model heavily depends on some empirical relationships in hydrologic, agronomic, and economic components. The parameters should be calibrated for the study area, and the assumptions with these relationships should be checked carefully according to specific conditions in the study area.
- This kind of modeling framework requires spatially and temporally variable input data from hydrology, economics and other disciplines. Some input data of the model may have to be estimated according to similar studies and common knowledge due to the limitation of data availability.
- If there are many stakeholders to be considered in the grand coalition, they need to be aggregated into stakeholder-groups according to the types of water uses, because the large number of potential coalitions among stakeholders would make the gaming analysis unrealistic if every stakeholder is considered as a totally independent individual.
- Similar to other water allocation models at the basin scale, CWAM is a deterministic one which does not explicitly include uncertainty perspectives in the structure. The uncertainties in the hydrologic, economic, social and political areas have to be taken into consideration through scenario analysis.
- In the SSRB case study, the values of stream flows and reservoir storages are not included because their benefit functions are not available. Therefore, their water rights are preserved through hydrologic constraints.

- The model contains numerous input data with both spatial and temporal distributions for the water resources system at the basin scale, and employs large-scale nonlinear optimization algorithms. In particular, the cooperative game coalition analysis is very computationally intensive. It is necessary to develop a powerful integrated decision support system to facilitate the data collection, model formulation and post-solution processing.

The applications of CWAM to the Amu Darya and South Saskatchewan River Basins demonstrate its capability to provide reliable results for short-term water resources management planning with monthly time periods. To improve the applicability of the methodology, some potential directions for future research are as follows:

- Check all assumptions involved in the different components of the model according to conditions existing in the real world; verify the interrelationships between the hydrologic, agronomic, environmental, and institutional components integrated into the model.
- Investigate methodologies on how to specify the weights in LMWSR affecting fairness of allocations among water uses.
- Explore other possible definitions of the characteristic functions of cooperative water reallocation games by investigating concepts such as maximin and Nash equilibrium, and developing corresponding algorithms.
- Develop other solution algorithms and verify model output. For example, develop algorithms for PMMNF and LMWSR by utilizing the multistart or other global optimization techniques, and compare the effectiveness and efficiency of the other algorithms to the two-stage algorithms developed in this research.
- Define and run more scenarios for the SSRB case study, such as: update the crop-yield relationship to include salinity in the benefit functions for irrigation water, estimate net benefit functions for stream flow and reservoir storage demands, and update the coalition analysis.
- Develop a GIS-based decision support system for CWAM. GIS technology can be utilized to facilitate the management of numerous spatial and attribute data, and to

enable the formulation of models and analysis of spatial equity among water users in a river basin.

- The basic allocation methodology can be refined for application to other resource allocation problems such as supplying electricity to different kinds of customers over time and space.

Appendix A Hydrological Constraints of the Generalized Multiperiod Network Flow (GMNF) Program

By eliminating intermediate variables, the hydrological constraints $\boldsymbol{g}(\boldsymbol{Q},\boldsymbol{S},\boldsymbol{C})$ and $\mathbf{h}(\boldsymbol{Q},\boldsymbol{S},\boldsymbol{C})$ of the generalized multiperiod network flow (GMNF) program for water allocation can be reduced as follows.

Water quantity constraints:

$$\sum_{(k,k_2)\in L} Q(k,k_2,t) - \sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))Q(k_1,k,t) = Q_{IN}(k,t) + Q_a(k,t), \quad \forall k \in IN$$

$$Q_{OUT}(k,t) - \sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))Q(k_1,k,t) = Q_a(k,t), \quad \forall k \in OUT$$

$$\sum_{(k,k_2)\in L} Q(k,k_2,t) - \sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))Q(k_1,k,t) = Q_a(k,t), \quad \forall k \in JUN$$

$$\sum_{(k,k_2)\in L} Q(k,k_2,t) - \left(1 - e_N(k,t)\right) \sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))Q(k_1,k,t) = \left(1 - e_N(k,t)\right) Q_a(k,t),$$
$$\forall k \in AGR \cup MI \cup HPP \cup SFR \cup TP$$

$$S(k,t) + \sum_{(k,k_2)\in L} Q(k,k_2,t) - S(k,t-1) - \sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))Q(k_1,k,t)$$
$$+ E_R(k,t)\cdot A\left(\frac{1}{2}\left(S(k,t-1)+S(k.t)\right)\right) = Q_a(k,t), \ \forall k \in RES$$

$$S(k,t) + \sum_{(k,k_2)\in L} Q(k,k_2,t) - S(k,t-1) - \sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))(Q(k_1,k,t)$$
$$- \sum_{(k_1,k_2)\in L} \left[1 - e_{SL}((k_1,k_2),k,t)\right]\cdot s_L((k_1,k_2),k,t))Q(k_1,k_2,t) = Q_a(k,t), \ \forall k \in AQU$$

$$Q(k,k_2,t) = s_N(k,k_2,t)\left(Q_{IN}(k,t) + Q_a(k,t) + \sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))Q(k_1,k,t) \right)$$
$$\forall k \in IN, \quad (k,k_2) \in L_{seep}$$

$$Q(k,k_2,t) = s_N(k,k_2,t)\left(Q_a(k,t) + \sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))Q(k_1,k,t) \right)$$
$$\forall k \in V \setminus (STO \cup IN), \quad (k,k_2) \in L_{seep}$$

$$Q(k,k_2,t) = s_N(k,k_2,t)\left[\frac{1}{2}\left(S(k,t-1)+S(k,t)\right)\right], \forall k \in RES \cup AQU$$

$$\sum_{\substack{(j,k)\in L\\ and\\ (k,j)\in L}} Q(k,j,t) \le Q_a(k,t) + Q_{IN}(k,t) + \sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))Q(k_1,k,t)$$

$$- \sum_{\substack{(j,k)\in L\\ and\\ (k,j)\in L}} (1-e_L(j,k,t))Q(j,k,t), \quad \forall k \in IN \cap SRC$$

$$\sum_{\substack{(j,k)\in L\\ and\\ (k,j)\in L}} Q(k,j,t) \le Q_a(k,t) + \sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))Q(k_1,k,t)$$

$$- \sum_{\substack{(j,k)\in L\\ and\\ (k,j)\in L}} (1-e_L(j,k,t))Q(j,k,t), \quad \forall k \in SRC \setminus IN$$

$$\sum_{(k_1,j)\in L\setminus L_{seep}} (1-e_L(k_1,j,t))Q(k_1,j,t) \le \max(Q_D(j,t) - Q_a(j,t), \quad 0), \quad \forall j \in AGR \cup MI \cup HPP$$

$$Q_{\min}(k_1,k,t) \le Q(k_1,k,t) \le Q_{\max}(k_1,k,t), \quad \forall (k_1,k) \in L$$

$$S_{\min}(k,t) \le S(k,t) \le S_{\max}(k,t), \quad \forall k \in RES \cup AQU$$

$$Q_{pout\min} \le Q_{pout}(k,t) \le Q_{pout\max}(k,t), \; \forall k \in OUT$$

Water quality constraints:

$$\sum_{(k,k_2)\in L} C_p(k,k_2,t)Q(k,k_2,t) - \sum_{(k_1,k)\in L} (1-e_{pL}(k_1,k,t))C_p(k_1,k,t)Q(k_1,k,t)$$

$$= C_{p_{IN}}(k,t)Q_{IN}(k,t) + C_{pa}(k,t)Q_a(k,t), \forall\, k \in IN$$

$$C_{p_{OUT}}(k,t)Q_{OUT}(k,t) - \sum_{(k_1,k)\in L} (1-e_{p_L}(k_1,k,t))C_p(k_1,k,t)Q(k_1,k,t) = C_{pa}(k,t)Q_a(k,t), \forall\, k \in OUT$$

$$\sum_{(k,k_2)\in L} C_p(k,k_2,t)Q(k,k_2,t) - \sum_{(k_1,k)\in L} (1-e_{pL}(k_1,k,t))C_p(k_1,k,t)Q(k_1,k,t) = C_{pa}(k,t)Q_a(k,t),$$

$$\forall\, k \in JUN \cup HPP$$

$$\sum_{(k,k_2)\in L} C_p(k,k_2,t)Q(k,k_2,t) - \sum_{(k_1,k)\in L} (1-e_{pL}(k_1,k,t))C_p(k_1,k,t)Q(k_1,k,t)$$

$$- \left\{ \begin{array}{l} z_{pd0}(k,t) + z_{pd1}(k,t) \displaystyle\sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))Q(k_1,k,t) \\ + z_{pd2}(k,t) \left[\displaystyle\sum_{(k_1,k)\in L} (1-e_L(k_1,k,t))Q(k_1,k,t) \right]^2 \end{array} \right\} + Z_{pc}(k,t)$$

$$= C_{pa}(k,t)Q_a(k,t), \; \forall\, k \in AGR \cup MI \cup TP \cup SFR$$

$$C_p(k,t)S(k,t)+\sum_{(k,k_2)\in L} C_p(k,k_2,t)Q(k,k_2,t)-C_p(k,t-1)S(k,t-1)$$
$$-\sum_{(k_1,k)\in L}(1-e_{pL}(k_1,k,t))C_p(k_1,k,t)Q(k_1,k,t)+Z_{pc}(k,t)=C_{pa}(k,t)Q_a(k,t),\ \forall\ k\in RES$$
$$C_p(k,t)S(k,t)+\sum_{(k,k_2)\in L} C_p(k,k_2,t)Q(k,k_2,t)-C_p(k,t-1)S(k,t-1)$$
$$-\sum_{(k_1,k)\in L}(1-e_{pL}(k_1,k,t))C_p(k_1,k,t)Q(k_1,k,t)$$
$$-\sum_{(k_1,k_2)\in L}\left[1-e_{pSL}((k_1,k_2),k,t)\right]\cdot s_L((k_1,k_2),k,t)C_p(k_1,k_2,t)Q(k_1,k_2,t)$$
$$+Z_{pc}(k,t)$$
$$=C_{pa}(k,t)Q_a(k,t),\ \forall\ k\in AQU$$
$$C_{pN}(k,t)\left[Q_a(k,t)+\sum_{(k_1,k)\in L}(1-e(k_1,k,t))Q(k_1,k,t)\right]=C_{pa}(k,t)Q_a(k,t)$$
$$+\sum_{(k_1,k)\in L}(1-e_{pL}(k_1,k,t))C_p(k_1,k,t)Q(k_1,k,t),\ \forall k\in V\setminus(RES\cup AQU)$$
$$Z_{pc}(k,t)=0,\quad \forall k\in IN\cup OUT\cup JUN\cup HPP$$
$$Z_{pc}(k,t)=e_{pN}(k,t)\left\{\begin{array}{l} C_{pa}(k,t)Q_a(k,t)+\sum_{(k_1,k)\in L}(1-e_{pL}(k_1,k,t))C_p(k_1,k,t)Q(k_1,k,t)\\ +z_{pd0}(k,t)+z_{pd1}(k,t)\sum_{(k_1,k)\in L}(1-e_L(k_1,k,t))Q(k_1,k,t)\\ +z_{pd2}(k,t)\left[\sum_{(k_1,k)\in L}(1-e_L(k_1,k,t))Q(k_1,k,t)\right]^2 \end{array}\right\},$$
$$\forall k\in AGR\cup MI\cup TP\cup SFR$$
$$Z_{pc}(k,t)=e_{pN}(k,t)S(k,t)C_p(k,t),\ \ \forall k\in RES\cup AQU$$
$$C_p(k,k_2,t)=C_p(k,k_2',t),\forall k\in V,\forall(k,k_2),(k,k_2')\in L$$
$$C_p(k,k_2,t)=C_p(k,t),\ \forall k\in RES\cup AQU,\ \forall(k,k_2)\in L$$
$$0\le C_p(k_1,k,t)\le C_{p\max}(k_1,k,t),\ \forall(k_1,k)\in L$$
$$0\le C_p(k,t)\le C_{p\max}(k,t),\ \forall k\in RES\cup AQU$$
$$0\le C_{pN}(k,t)\le C_{pN\max}(k,t),\ \forall k\in V\setminus(RES\cup AQU)$$
$$0\le C_{pout}(k,t)\le C_{pout\max}(k,t),\ \forall k\in OUT$$

Estimating pollutant loss coefficients for links and link seepages:

Consider the one-dimensional partial differential equation describing the concentration C_p in a river reach

$$\frac{\partial C_p}{\partial t} = E\frac{\partial^2 C_p}{\partial x^2} - \frac{\partial (uC_p)}{\partial x} - K_p C_p$$

where t represents time (rather than time step in the river basin model), x is the distance from the upstream node, E is the dispersion coefficient, u is the flow velocity, and K_p is the temperature dependent first order decay coefficient of pollutant p. $K_p(T) = K_p(20^\circ C)\cdot\theta^{(T'-20)}$, where T' is the water temperature and θ is the temperature coefficient ranging from 1.02 ~1.06 (Loucks *et al.*, 1981). The first order decay assumption is proper for pollutants such as BOD, COD, TP, TN, E. Coli count and toxic compounds (DHI, 2001).

Let U be the mean flow velocity and $C_p(0)$ be the concentration of pollutant p at the upstream point (x = 0). The steady state solution ($\partial C_p / \partial t = 0$) for the point ($x$ >0) at the downstream is

$$C_p(x) = \frac{C_p(0)}{m}\exp\left[\frac{U}{2E}(1-m)x\right]$$

where $m = \sqrt{1+\frac{4K_pE}{U^2}}$. Ignoring E for simplicity, the steady state solution is

$$C_p(x) = C_p(0)\exp\left(-K_p x/U\right) = C_p(0)\exp\left(-K_p t_d\right)$$

where t_d is the traveling time of flow from the upstream node to x, which is also called hydraulic detention time.

Assuming the point and non-point pollution loads are input to nodes of the river basin network and applying the above one-dimensional steady state water quality model, the pollutant loss coefficient $e_{pL}(k_1,k,t)$ for link (k_1, k) is approximately estimated as

$$e_{pL}(k_1,k,t) = e_L(k_1,k,t) + \left[1 - e_L(k_1,k,t)\right]\left(1 - \exp\left(-K_p(k_1,k,t)t_d(k_1,k,t)\right)\right)$$

where $K_p(k_1,k,t)$ is the temperature dependent first order decay rate coefficient of pollutant p within link (k_1,k) during period t and $t_d(k_1,k,t)$ is the target hydraulic detention time of link (k_1,k) with the allocated flow. For river reaches or open ditches, $t_d = Ldw/Q(k_1,k,t)$, where length L and width w are assumed to be constant, and depth d can be estimated from empirical relation $Q = \alpha d^{\beta}$, where α and β are equation parameters. For pipeline, $t_d = LA/Q(k_1,k,t)$, where A is the section area of pipeline. Since the target flow and hydraulic detention time are assumed to be constant in the river basin network model, the corresponding $e_{pL}(k_1,k,t)$ is a constant coefficient. Note that the river basin network model is designed for water resources planning and management, and any attempt to know the actual process should be accomplished by more detailed dynamic simulation models.

Similarly, we have

$$e_{pSL}((k_1,k_2),k,t) = e_{SL}((k_1,k_2),k,t) + \left[1 - e_{SL}((k_1,k_2),k,t)\right]\left(1 - \exp\left(-K_p((k_1,k_2),k,t)t_d((k_1,k_2),k,t)\right)\right)$$

Appendix B Input Data of the South Saskatchewan River Basin (SSRB) Case Study

Water Supplies

Table B.1 Long-term mean surface water supplies, 1912 to 2001 (mcm*)

Node	Node Name	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
IN1	Red Deer River below Burnt Timber Creek	13.821	12.362	13.365	18.481	80.084	156.038	136.063	95.351	58.320	37.498	20.606	15.990
IN2	Bow River at Banff	24.106	19.741	20.597	27.216	137.402	326.592	286.589	179.185	101.606	62.942	37.325	28.123
IN3	Kananaskis River above Pocaterra Creek	27.588	22.934	20.463	11.508	12.455	20.995	21.749	18.829	14.178	13.606	20.969	27.052
IN4	Elbow River at Bragg Creek	7.633	6.822	8.624	12.208	38.837	62.986	41.783	24.909	20.425	17.061	11.923	9.267
IN5	Little Bow River at Carmangay	0.937	1.887	6.267	6.895	5.544	5.728	4.794	3.187	3.007	2.678	1.700	1.162
IN6	Oldman River near Waldron's Corner	6.509	5.661	8.223	22.447	104.458	128.563	54.907	23.597	16.356	14.356	10.627	7.848
IN7	Willow Creek above Chain Lakes	0.295	0.302	0.729	2.618	8.973	8.294	3.428	1.886	1.348	1.141	0.669	0.429
IN8	Waterton River at 12 km upstream of Waterton Reservoir	10.687	8.685	10.312	28.771	132.581	195.696	89.191	30.266	21.332	22.981	19.492	12.910
IN9	Belly River up-stream of the con-fluence of Mami Creek	5.035	4.572	5.625	15.889	58.657	78.797	43.658	18.910	12.493	13.044	9.953	6.455
IN10	St. Mary River up-stream of Woolford Provincial Park	11.758	10.378	14.303	34.733	127.760	189.475	99.636	45.533	36.029	34.284	25.039	15.401

*1 mcm =1 million cubic meters.

Table B.2 Capacities and full-storage surface areas of aggregated reservoirs

Node	Aggregated reservoirs	Capacity (mcm)	Full-storage surface area (ha)
R1	Gleniffer Lake	204	1705*
R2	Barrier, Interlakes and Pocaterra	76	1867*
R3	Bearspaw, Horseshoe, Ghost and Kananaskis	263	2195*
R4	Glenmore	24	909*
R5	Chestermere and Eagle Lakes	13	505
R6	Little Bow, McGregor Lake and Travers	447	7895
R7	Crawling Valley, Lake Newell and Snake Lake	469	9115
R8	Oldman Reservoir	490	2425
R9	Keho Lake	96	2350
R10	Chain Lakes and Pine Coulee	59	1444*
R11	Waterton	111	1095
R12	Cochrane Lake and Payne Lake	12	330
R13	St. Mary	369	3765
R14	Jensen and Milk River Ridge	146	1615
R15	Chin Lakes	190	1590
R16	Fincastle, Horsefly and Taber Lake	19	1155
R17	Forty Mile and Sauder	124	1990

* Estimated

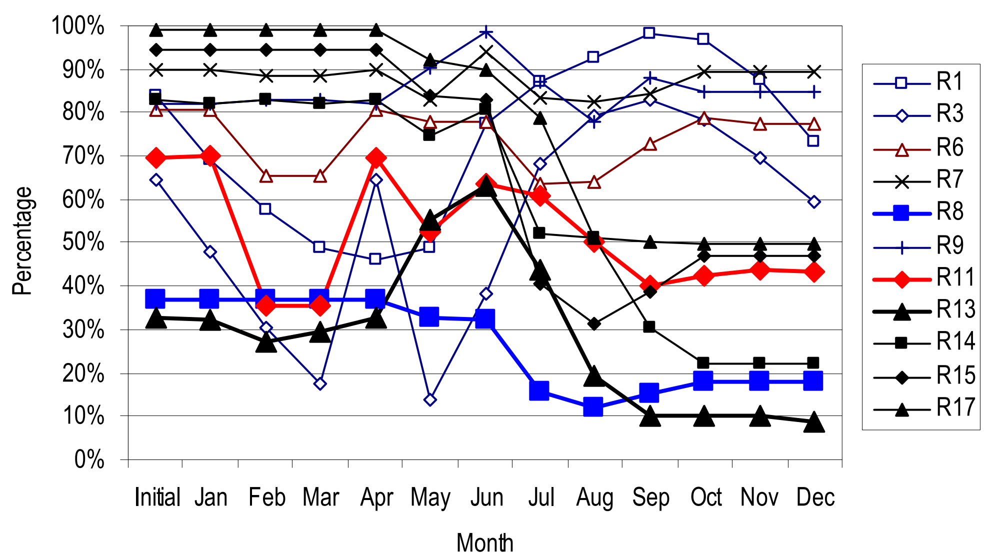

Figure B.1 Percentages of the monthly live storages over capacities in 2000

Water Demands

Table B.3 Licensed withdrawals and gross consumptions of irrigation regions (mcm)

Node	Irrigation Region	Licenses as of 1999		Gross consumption during the growing season in 1995
		Priority	Annual withdrawal	
A1	Western	1903090401	197.853	52.53
A2	Bow River	1908102702	185.025	245.95
		1913032501	185.025	
		1953062501	98.680	
		1992020510	86.345	
A3	Eastern	1903090402	939.927	361.36
A4	Lethbridge Northern	1917111601	185.025	135.30
		1974110401	82.645	
		1982041501	61.675	
		1991082301	61.675	
A5	Mountain View, Aetna, United, Leavitt	1919032401	62.909	9.38
		1923071003	9.251	
		1939061701	9.560	
		1945063001	6.784	
		1991121702	10.176	
		1993051701	20.970	
A6	Raymond and Magrath	1899020703	15.098	22.34
		1899020704	11.324	
		1950053108	25.682	
		1950053114	52.744	
		1991082204	4.934	
		1991082302	32.071	
A7	St. Mary River -West	1899020701	84.603	123.81
		1950053107	166.932	
		1991082309	111.682	
A8	Taber	1899020702	41.939	89.76
		1950053117	143.086	
		1991082602	9.868	
A9	St. Mary -East	1899020701	122.838	276.03
		1950053107	242.377	
		1951030201	3.701	
		1991082309	162.155	

* Gross consumption refers to the diverted and treated water at the consumer's meter.

Table B.4 Effective precipitation in 1995* (mcm)

Node	Irrigation regions	May	Jun	Jul	Aug	Sep	Total
A1	Western	9.598	6.736	20.703	4.099	3.185	44.321
A2	Bow River	33.758	29.492	90.651	17.905	7.748	179.554
A3	Eastern	54.993	40.997	126.015	23.769	15.941	261.715
A4	Lethbridge Northern	30.553	40.433	23.533	11.106	6.717	112.342
A5	Mountain View, Aetna, United, Leavitt	6.126	7.050	6.206	2.052	0.723	22.157
A6	Raymond and Magrath	5.945	16.585	14.598	2.141	0.199	39.468
A7	St. Mary River -West	30.639	61.457	54.095	13.914	3.242	163.347
A8	Taber	15.873	26.684	15.531	7.669	4.034	69.791
A9	St. Mary -East	26.599	44.063	17.143	22.894	12.950	123.649

*Estimated by the IWPM model.

Table B.5 Monthly distribution of the gross consumption of irrigation water in 1995 (mcm)

Node	Irrigation regions	May	Jun	Jul	Aug	Sep	Total
A1	Western	3.30	17.78	8.95	14.74	7.75	52.53
A2	Bow River	7.42	90.43	53.67	71.91	22.52	245.95
A3	Eastern	19.99	127.66	67.86	98.92	46.94	361.36
A4	Lethbridge Northern	0.32	26.20	53.00	37.52	18.26	135.30
A5	Mountain View, Aetna, United, Leavitt	0.00	1.32	2.49	3.37	2.20	9.38
A6	Raymond and Magrath	0.00	2.29	13.20	5.59	1.26	22.34
A7	St. Mary River -West	0.00	15.36	44.25	43.93	20.28	123.81
A8	Taber	0.13	14.99	36.74	27.15	10.74	89.76

Table B.6 Monthly irrigation water demands under long-term mean precipitation conditions* (mcm)

Node	Irrigation regions	May	Jun	Jul	Aug	Sep	Total
A1	Western	9.275	27.030	37.625	25.613	12.564	112.107
A2	Bow River	15.717	97.914	167.392	90.395	26.625	398.043
A3	Eastern	46.335	158.436	232.185	140.385	62.830	640.171
A4	Lethbridge Northern	22.274	78.909	110.328	67.121	29.456	308.088
A5	Mountain View, Aetna, United, Leavitt	4.385	7.529	18.012	9.288	4.511	43.725
A6	Raymond and Magrath	1.151	10.974	43.684	6.948	1.215	63.972
A7	St. Mary River -West	13.629	47.265	150.358	52.043	17.409	280.704
A8	Taber	8.097	44.563	68.608	43.126	15.042	179.436
A9	St. Mary -East	20.996	113.239	192.111	115.258	32.813	474.417

*Estimated by the IWPM model at the 80% level of maximum crop potential evapotranspiration rates. Return flow and consumption ratios of the irrigation regions are assumed to be 25% and 75% respectively (65% for effective irrigation, and 10% for deep percolation).

Table B.7 Monthly irrigation water demands under assumed half long-term mean precipitation conditions* (mcm)

Node	Irrigation regions	May	Jun	Jul	Aug	Sep	Total
A1	Western	15.332	38.462	47.352	32.119	17.380	150.645
A2	Bow River	27.221	147.970	209.984	118.812	38.340	542.327
A3	Eastern	77.054	228.020	291.392	178.108	86.933	861.507
A4	Lethbridge Northern	33.971	101.871	127.641	79.743	38.417	381.643
A5	Mountain View, Aetna, United, Leavitt	6.353	12.338	18.648	11.852	6.384	55.575
A6	Raymond and Magrath	2.443	22.287	45.180	9.623	1.729	81.262
A7	St. Mary River -West	21.768	89.185	155.901	69.427	25.805	362.086
A8	Taber	12.851	59.718	80.034	51.842	20.424	224.869
A9	St. Mary -East	34.537	150.904	216.540	131.288	42.559	575.828

*Estimated by the IWPM model at the 80% level of maximum crop potential evapotranspiration rates. Return flow and consumption ratios of the irrigation regions are assumed to be 25% and 75% respectively (65% for effective irrigation, and 10% for deep percolation).

Table B.8 Gross consumptions of municipal water uses during the growing season in 1995 (mcm) (Mahan, 1997)

Node	City	Domestic	General
D1	Red Deer	2.17	2.05
D2	Calgary	47.11	25.37
D3	Lethbridge	6.31	7.48
D4	Medicine Hat	3.35	3.97

Table B.9 Historical population of cities and per capita water demands in 1995

Node	City	Historical population				Per capita water demands in 1995 (m^3/person/year)	
		1981	1986	1991	1996	Domestic	General
D1	Red Deer	46,393	54,425	58,145	60,075	69	65
D2	Calgary	592,808	636,843	710,795	768,082	117	63
D3	Lethbridge	54,558	58,841	60,974	63,053	171	203
D4	Medicine Hat	40,700	41,823	43,625	46,783	121	144

Table B.10 Estimated withdrawals of cities licensed in past time periods (mcm)

Node	City	Usage	Before 1982	1982 -1986	1987-1991	1992-1996
D1	Red Deer	Domestic	4.460	0.772	0.358	0.186
D2	Calgary	Domestic	97.278	7.226	12.135	9.401
D3	Lethbridge	Domestic	13.079	1.027	0.511	0.498
D4	Medicine Hat	Domestic	6.903	0.190	0.306	0.536
G1	Red Deer	General	4.214	0.730	0.338	0.175
G2	Calgary	General	52.387	3.891	6.535	5.063
G3	Lethbridge	General	15.504	1.217	0.606	0.591
G4	Medicine Hat	General	8.180	0.226	0.362	0.635

Table B.11 Gross consumptions of industrial uses in 1995 (mcm) (Mahan, 1997)

Node	Industrial Region	1995 gross consumption during growing season
I1	Red Deer	30.00
I2	Calgary	27.59
I3	Eastern	4.13
I4	Medicine Hat	13.69

Table B.12 Industrial withdrawals of cities licensed in different time periods* (mcm)

Node	Industrial Region	Before 1982	1982 -1986	1987-1991	1992-1996
I1	Red Deer	74.823	12.954	6.000	3.113
I2	Calgary	69.365	5.153	8.653	6.703
I3	Eastern	10.401	0.523	0.659	2.087
I4	Medicine Hat	38.741	1.069	1.715	3.006

*Assume the ratios of actual withdrawal /licensed withdrawal equal 65% and constant per capita withdrawals for each industrial region.

Crop Production of Irrigation Regions

Table B.13 Aggregated irrigation regions (Mahan, 1997)

Demand Node	Aggregated irrigation region	Original irrigation districts	Agroclimate zone
A1	Western Irrigation Region (WIR)	Western Irrigation District	ZC
A2	Bow River Irrigation Region IBRIR)	Bow River Irrigation District	ZA2
A3	Eastern Irrigation Region (EIR)	Eastern Irrigation District	ZA2
A4	Lethbridge Northern Irrigation Region (LNIR)	Lethbridge Northern Irrigation District	ZB
A5	Mountain View, Aetna, United, Leavitt Irrigation Region (MAULR)	Mountain View, Aetna, United, Leavitt Irrigation Districts	ZC
A6	Raymond and Magrath Irrigation Region (RMIR)	Raymond and Magrath Irrigation Dis-tricts	ZA2
A7	St. Mary River Irrigation Region-West (SMRIRW)	St. Mary River Irrigation District-West	ZA2
A8	Taber Irrigation Region (TIR)	Taber Irrigation District	ZA1
A9	St. Mary River Irrigation Region-East (SMRIRE)	St. Mary River Irrigation District-East	ZA1

Table B.14 Crop categories and representative crops (Mahan, 1997)

Crop classification	Dominant crop types	Representative crops
Special Cereals (SC)	Soft, medium, and winter wheat	Soft wheat
Traditional Cereals (TC)	Hard spring and durum wheat	Hard spring wheat
Feed Grains (FG)	Barley, oats, rye, and grain corn	Barley
Oilseeds (OS)	Canola, flaxseed, mustard seed, and sunflower	Canola
Vegetables (VG)	Potatoes, sugar beets, and other vege-tables	Potatoes
Alfalfa (AL)	Alfalfa, hay, grass, and pasture	Alfalfa

Table B.15 Cropping areas of representative crop types (ha)* (Mahan, 1997)

Node	Irrigation regions	Special Cereals (SC)	Traditional Cereals (TC)	Feed Grains (FG)	Oilseeds (OS)	Vegetables (VG)	Alfalfa (AL)	Total
A1	Western (WIR)	503	129	2329	1597	647	12039	17244
A2	Bow River (BRIR)	24283	3032	8710	8623	10456	20401	75505
A3	Eastern (EIR)	11158	2579	15981	11756	3342	60144	104960
A4	Lethbridge Northern (LNIR)	4483	1451	8371	2329	2025	25214	43873
A5	Mountain View, Aetna, United and Leavitt (MAULIR)	286	0	539	226	50	4621	5722
A6	Raymond and Magrath (RMIR)	3691	197	1559	6730	71	1213	13461
A7	St. Mary River-West (SMRIRW)	10031	2636	11806	4465	6578	14364	49880
A8	Taber (TIR)	6220	328	5297	750	7193	9166	28954
A9	St. Mary River-East (SMRIRE)	22010	3528	5877	8602	11578	20829	72424

* The cropping areas are the annually average values from 1992 to 1995.

Table B.16 Potential evapotranspiration rates at maximum yields of crops (mm) (Mahan, 1997)

Irrigated crop type	Representative crops	May	Jun	Jul	Aug	Sep	Growth season total
Special Cereals (SC)	Soft wheat*	37.5	187.5	262.5	150	0	637.5
Traditional Cereals (TC)	Hard spring wheat	30	150	210	120	0	510
Feed Grains (FG)	Barley	30	180	200	0	0	410
Oilseeds (OS)	Canola	30	150	230	0	0	410
Vegetables (VG)	Potatoes	30	120	160	180	90	580
Alfalfa (AL)	Alfalfa	120	210	230	200	120	880

* Soft wheat is adjusted upward by 25% of hard wheat based on UMA (1982).

Table B.17 Estimated maximum potential yield (Y_m) (mt/ha) (Mahan, 1997)

Irrigated crop type	Agroclimate zones				
	ZA1	ZA2	ZB	ZC	ZD
Special Cereals (SC)	8.34	8.34	8.34	7.50	6.75
Traditional Cereals (TC)	6.67	6.67	6.67	6.00	5.40
Feed Grains (FG)	6.42	6.42	6.42	6.30	5.10
Oilseeds (OS)	3.50	3.55	3.40	3.40	3.20
Vegetables (VG)	56.09	56.09	56.09	54.00	44.80
Alfalfa (AL)	11.12	11.12	11.12	10.10	9.00

Table B.18 Estimated coefficients (Y_a/Y_p) of nonlinear crop yield- effective irrigation function* (Mahan, 1997)

Irrigated crop type	A_0	a_1	a_2	R^2
Special Cereals (SC)	-0.191	1.809	-0.688	0.552
Traditional Cereals (TC)	-0.191	1.809	-0.688	0.552
Feed Grains (FG)	-0.199	1.844	-0.795	0.370
Oilseeds (OS)	0.031	1.246	-0.444	0.382
Vegetables (VG)	-0.518	2.741	-1.252	0.698
Alfalfa (AL)	-0.097	1.413	-0.386	0.474

*These coefficients of crop yield-effective irrigation functions are estimated from the corresponding coefficients of crop yield-raw water functions, by accounting the irrigation efficiencies.

Table B.19 Linear nitrogen fertilizer-water functions of representative crop types (Mahan, 1997)

Irrigated crop type	Nitrogen fertilizer - water functions*
Special Cereals (SC)	$FT_{j,SC} = -32.61 + 0.3561 WA_{j,SC}$
Traditional Cereals (TC)	$FT_{j,TC} = -50.41 + 0.3561 WA_{j,TC}$
Feed Grains (FG)	$FT_{j,FG} = -41.51 + 0.3561 WA_{j,FG}$
Oilseeds (OS)	$FT_{OS} = -32.61 + 0.3561 WA_{j,OS}$
Vegetables (VG)	$FT_{j,VG} = -10.00 + 0.2000 WA_{j,VG}$
Alfalfa (AL)	N/A

* $FT_{j,cp}$ and $WA_{j,cp}$ are the nitrogen fertilizer application rate (kg/ha and total available water (mm/season) during the growing season fc each representative crop, respectively.

Table B.20 Low, expected, and high crop prices (1995 dollars/mt) (Mahan, 1997)

Irrigated crop type	Low	Expected	High
Special Cereals (SC)	124.02	155.02	186.02
Traditional Cereals (TC)	198.19	247.74	297.29
Feed Grains (FG)	94.11	117.63	141.16
Oilseeds (OS)	246.38	307.98	369.57
Vegetables (VG)	50.00	50.00	60.00
Alfalfa (AL)	64.92	64.92	77.90

Table B.21 Fixed costs of farm production (1995 dollars/hectare/growth season)

(Mahan, 1997)

Cost	Special Cereals (SC)	Traditional Cereals (TC)	Feed Grains (FG)	Oilseeds (OS)	Vegetables (VG)	Alfalfa (AL)
Seed	29.44	28.90	19.65	22.94	474.28	10.70
Equipment fuel	33.35	35.81	40.69	26.85	33.25	30.80
Chemicals	43.57	39.32	28.74	56.02	390.06	2.50
Pivot operation	8.4	8.4	8.4	8.4	N/A	8.4
Machinery operation	19.8	19.8	19.8	19.8	170.5	19.8
Operating interest	63.74	57.21	37.75	54.16	161.79	7.47
Hail/crop insurance	15.44	14.33	5.96	6.53	N/A	N/A
Repairs: Machinery & buildings	75.79	79.77	69.68	59.54	78.62	67.69
Total	289.53	283.54	230.67	254.24	1308.50	147.26

Table B.22 Variable costs of farm production* (Mahan, 1997)

Cost	Special Cereals (SC)	Traditional Cereals (TC)	Feed Grains (FG)	Oilseeds (OS)	Vegetables (VG)	Alfalfa (AL)
Pumping cost ($\$/m^3$)	0.0177	0.0177	0.0177	0.0177	0.0177	0.0177
Nitrogen fertilizer ($/kg)	0.7055	0.7055	0.7055	0.7055	0.7055	0.7055
Cultivation($/mt)	3.29	3.29	4.12	3.53	3.29**	3.29**

* All costs are estimated in 1995 Canadian dollars.

Table B.23 Regional water charges* (Mahan, 1997)

Demand Node	Aggregated irrigation region	Water charges (1995 dollars/hectare)
A1	Western Irrigation Region (WIR)	36.45
A2	Bow River Irrigation Region IBRIR)	29.65
A3	Eastern Irrigation Region (EIR)	21.00
A4	Lethbridge Northern Irrigation Region (LNIR)	34.59
A5	Mountain View, Aetna, United, Leavitt Irrigation Region (MAULR)	29.65
A6	Raymond and Magrath Irrigation Region (RMIR)	29.65
A7	St. Mary River Irrigation Region-West (SMRIRW)	32.74
A8	Taber Irrigation Region (TIR)	29.65
A9	St. Mary River Irrigation Region-East (SMRIRE)	29.65

* Water is charged on a per-acre basis rather than on a quantity-usage basis.

Water Demand Curves and Benefit Functions

Table B.24 Coefficients of benefit functions of irrigation water under the 2021 drought scenarios (Cases C and F)

Node	Demand name	Coefficient	May	Jun	Jul	Aug	Sep
A1	Western Irrigation Region	b0	0.230153	0.230153	0.230153	0.230153	0.230153
		b1	0.053995	0.062003	0.065451	0.059508	0.057043
		b2	-0.00067	-0.00036	-0.00033	-0.0004	-0.00067
A2	Bow River Irrigation Region	b0	2.829335	2.829335	2.829335	2.829335	2.829335
		b1	0.066023	0.106155	0.121086	0.101128	0.079436
		b2	-0.00043	-0.0002	-0.00018	-0.00023	-0.00048
A3	Eastern Irrigation Region	b0	1.776998	1.776998	1.776998	1.776998	1.776998
		b1	0.059268	0.073148	0.078829	0.067823	0.062871
		b2	-0.00014	-7.7E-05	-6.9E-05	-8.4E-05	-0.00015
A4	Lethbridge Northern Irrigation Region	b0	1.628245	1.628245	1.628245	1.628245	1.628245
		b1	0.04837	0.052747	0.053225	0.05486	0.052424
		b2	-0.00028	-6.4E-05	-4.6E-05	-0.00016	-0.00029
A5	Mountain View, Aetna, United, Leavitt Irrigation Region	b0	0.036262	0.036262	0.036262	0.036262	0.036262
		b1	0.055279	0.057256	0.066163	0.057929	0.055583
		b2	-0.00172	-0.00097	-0.00088	-0.00103	-0.00174
A6	Raymond and Magrath Irrigation Region	b0	0.217594	0.217594	0.217594	0.217594	0.217594
		b1	0.061151	0.089639	0.10818	0.074374	0.060097
		b2	-0.00447	-0.00096	-0.00066	-0.00135	-0.00707
A7	St. Mary River Irrigation Region-West	b0	1.94627	1.94627	1.94627	1.94627	1.94627
		b1	0.072447	0.096147	0.124216	0.09364	0.076967
		b2	-0.00064	-0.00029	-0.00026	-0.00035	-0.00068
A8	Taber Irrigation Region	b0	1.848034	1.848034	1.848034	1.848034	1.848034
		b1	0.074669	0.111216	0.123449	0.112035	0.093436
		b2	-0.0011	-0.00053	-0.00047	-0.00061	-0.00114
A9	St. Mary River Irrigation Region-East	b0	3.681897	3.681897	3.681897	3.681897	3.681897
		b1	0.077329	0.113412	0.128868	0.114731	0.088377
		b2	-0.00045	-0.00022	-0.00018	-0.00025	-0.0005

Table B.25 Reference quantities*, prices, and elasticities of domestic water uses in 2021**

Demand Node	Parameters	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
City of Red Deer	Reference quantity	0.445	0.414	0.445	0.451	0.528	0.577	0.583	0.607	0.552	0.493	0.457	0.475
	Reference price	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	Coefficient α	0.443	0.413	0.443	0.449	0.526	0.574	0.580	0.604	0.550	0.491	0.455	0.473
	Elasticity β	-0.4	-0.4	-0.4	-0.4	-0.5	-0.5	-0.5	-0.5	-0.5	-0.4	-0.4	-0.4
City of Calgary	Reference quantity	11.424	10.109	11.129	11.277	12.887	12.887	15.385	14.942	13.626	11.720	11.129	11.277
	Reference price	1.26	1.26	1.26	1.26	1.26	1.26	1.26	1.26	1.26	1.26	1.26	1.26
	Coefficient α	12.531	11.088	12.207	12.369	14.466	14.466	17.270	16.772	15.296	12.855	12.207	12.369
	Elasticity β	-0.4	-0.4	-0.4	-0.4	-0.5	-0.5	-0.5	-0.5	-0.5	-0.4	-0.4	-0.4
City of Lethbridge	Reference quantity	0.893	0.825	0.934	0.989	1.263	1.538	1.648	1.951	1.374	0.989	0.866	0.879
	Reference price	1.19	1.19	1.19	1.19	1.19	1.19	1.19	1.19	1.19	1.19	1.19	1.19
	Coefficient α	0.957	0.884	1.001	1.060	1.378	1.678	1.798	2.128	1.499	1.060	0.928	0.942
	Elasticity β	-0.4	-0.4	-0.4	-0.4	-0.5	-0.5	-0.5	-0.5	-0.5	-0.4	-0.4	-0.4
City of Medicine Hat	Reference quantity	0.409	0.368	0.396	0.444	0.673	0.763	0.958	0.985	0.694	0.541	0.402	0.409
	Reference price	0.83	0.83	0.83	0.83	0.83	0.83	0.83	0.83	0.83	0.83	0.83	0.83
	Coefficient α	0.380	0.341	0.367	0.412	0.613	0.696	0.873	0.898	0.632	0.502	0.373	0.380
	Elasticity β	-0.4	-0.4	-0.4	-0.4	-0.5	-0.5	-0.5	-0.5	-0.5	-0.4	-0.4	-0.4

* Reference quantities (mcm) represent the gross volume of treated water that is consumed by the domestic water use consumers.

** Reference prices (1995$/$m^3$) represent the sum of volumetric water utility charge and volumetric sanitary sewer charge.

Table B.26 Reference quantities*, prices, and elasticities of general water uses in 2021**

Demand Node	Parameters	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
City of Red Deer	Reference quantity	0.420	0.392	0.420	0.426	0.499	0.545	0.550	0.573	0.522	0.466	0.431	0.448
	Reference price	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95
	Coefficient α	0.412	0.384	0.412	0.417	0.487	0.531	0.536	0.559	0.509	0.456	0.423	0.439
	Elasticity β	-0.4	-0.4	-0.4	-0.4	-0.5	-0.5	-0.5	-0.5	-0.5	-0.4	-0.4	-0.4
City of Calgary	Reference quantity	6.152	5.444	5.993	6.073	6.940	6.940	8.285	8.047	7.338	6.311	5.993	6.073
	Reference price	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16
	Coefficient α	6.529	5.777	6.360	6.444	7.475	7.475	8.924	8.666	7.903	6.698	6.360	6.444
	Elasticity β	-0.4	-0.4	-0.4	-0.4	-0.5	-0.5	-0.5	-0.5	-0.5	-0.4	-0.4	-0.4
City of Lethbridge	Reference quantity	1.058	0.978	1.107	1.172	1.498	1.823	1.954	2.313	1.629	1.172	1.026	1.042
	Reference price	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	Coefficient α	1.054	0.974	1.102	1.168	1.490	1.814	1.944	2.301	1.620	1.168	1.022	1.037
	Elasticity β	-0.4	-0.4	-0.4	-0.4	-0.5	-0.5	-0.5	-0.5	-0.5	-0.4	-0.4	-0.4
City of Medicine Hat	Reference quantity	0.485	0.436	0.469	0.527	0.798	0.905	1.135	1.168	0.822	0.641	0.477	0.485
	Reference price	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72
	Coefficient α	0.425	0.382	0.411	0.462	0.677	0.768	0.963	0.991	0.698	0.562	0.418	0.425
	Elasticity β	-0.4	-0.4	-0.4	-0.4	-0.5	-0.5	-0.5	-0.5	-0.5	-0.4	-0.4	-0.4

* Reference quantities (mcm) represent the gross volume of treated water that is consumed by the general water use consumers.

** Reference prices (1995$/m^3) represent the sum of volumetric water utility charge and volumetric sanitary sewer charge.

Table B.27 Unit cost of water treatment, distribution, and wastewater treatment (1995$/m^3)

(Mahan, 1997)

Demand Node	Water treatment	Water distribution	Wastewater treatment
City of Red Deer	0.1404	0.1092	0.2028
City of Calgary, Eastern Industrial Region	0.0673	0.0659	0.1426
City of Lethbridge	0.0898	0.1197	0.1684
City of Medicine Hat	0.0662	0.0691	0.1403

Table B.28 Reference quantities*, prices, and elasticities of monthly demands of industrial water uses in 2021**

Demand Node	Parameters	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
City of Red Deer	Reference quantity	8.345	8.345	8.866	9.388	14.812	15.334	15.647	15.647	13.561	13.039	8.345	8.345
	Reference price	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95
	Coefficient α	7.846	7.846	8.336	8.827	13.927	14.417	14.711	14.711	12.750	12.259	7.846	7.846
	Elasticity β	-1.202	-1.202	-1.202	-1.202	-1.202	-1.202	-1.202	-1.202	-1.202	-1.202	-1.202	-1.202
City of Calgary	Reference quantity	9.209	9.209	9.785	10.361	16.347	16.922	17.268	17.268	14.965	14.390	9.209	9.209
	Reference price	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16
	Coefficient α	9.919	9.919	10.539	11.159	17.606	18.226	18.598	18.598	16.118	15.498	9.919	9.919
	Elasticity β	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5
Eastern Industrial Region	Reference quantity	0.919	0.919	0.976	1.034	1.631	1.689	1.723	1.723	1.493	1.436	0.919	0.919
	Reference price	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16
	Coefficient α	0.969	0.969	1.029	1.090	1.719	1.780	1.816	1.816	1.574	1.513	0.969	0.969
	Elasticity β	-0.354	-0.354	-0.354	-0.354	-0.354	-0.354	-0.354	-0.354	-0.354	-0.354	-0.354	-0.354
City of Medicine Hat	Reference quantity	3.046	3.046	3.237	3.427	5.407	5.598	5.712	5.712	4.950	4.760	3.046	3.046
	Reference price	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72
	Coefficient α	2.335	2.335	2.481	2.627	4.145	4.291	4.379	4.379	3.795	3.649	2.335	2.335
	Elasticity β	-0.809	-0.809	-0.809	-0.809	-0.809	-0.809	-0.809	-0.809	-0.809	-0.809	-0.809	-0.809

* Reference quantities (mcm) represent the gross volume of treated water that is consumed by the industrial water users in one month.

** Reference prices (1995$/$m^3$) represent the sum of volumetric water utility charge and volumetric sanitary sewer charge.

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