Optimal Execution Strategies

A Computational Finance Approach

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

In today's competitive business environment, strategies relating to market forecasting, decision making and risk management have received a lot of attention. The empirical result-s reveal that the market movement is not neutral to large orders. This makes the investors suffer from prohibitive execution costs. Hence, effective optimal execution strategies that assist investors on controlling market reaction are desperately demanded. However, most existing methods for analysing these strategies suffer from a serious weakness in that they fail to consider the impact of large orders on market price. In this thesis, the analysis of optimal execution strategies is conducted from the perspective of agent-based computational finance.

This thesis introduces an artificial stock market composed of agents assigned with information sharing and trading strategies, and analyses the market impact and reaction when agents are assigned with optimal trading strategies, including minimum risk volume-weighted average price (VWAP) and implementation shortfall (IS) strategies. In addition, refinement has also been made to the IS strategy by replacing the linear temporary impact function with a quadratic one.

Key Words: Artificial Stock Market, Market Impact, Optimal Execution Strategy, VWAP, Implementation Shortfall, Quadratic Temporary Impact Function

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Chapter 1

Introduction

1.1 Motivation

The study of algorithmic trading always starts with studying the optimal execution strategies. Instead of seeking speculative opportunities, optimal execution strategies focus on executing orders with minimized cost. From the perspective of individual investors, their orders would not significantly affect the market price, due to the relatively small volume of either buying or selling positions. But orders from institutional investors are generally considered as the driving force of market price, and generally lead to a gap (or in other words, execution cost) between the market price and the real captured price.

The size of execution cost directly determines the profitability of an institutional investor. One example is the famous Value-Line Funds (as in [1]). From 1965 to 1986, the average annually book return of Value-Line Funds was over 20%, but the real captured return turned out to be only 2.5% above the market growth. This tremendous gap arose directly from the execution cost.

Before the widespread of algorithmic trading strategies, institutional investors who wanted to liquidize their orders had to hire broker-dealers to find a counter-party to prevent the market impact from executing their orders. When a qualified counter-party was not available, some special strategies were conducted by broker-dealers to gradually liquidize their large positions in the market, so that the market impact would be diminished at a low level. Therefore, these strategies built the foundation of optimal execution strategies. Nowadays, the advance in algorithmic trading have enabled institutional traders to manage their execution costs. This has sparked the academic interest in optimal execution strategies.

Recently, extensive studies have been conducted on acquiring an optimal execution strategy for institutional investors. As for measuring the risk and further quantitative analysis, most studies start with modelling the market price movement [2, 3, 4]. Often, they share a common assumption that the market price movement follows Geometric Brownian Motion, ignoring the fact that an order may impact the market. This assumption is widely accepted in most of the pricing models or trading strategies. However, the Geometric Brownian Motion assumption is too restrictive when the model admits the market impact from large orders. In this case, the market price would follow a different type of distribution. To solve this, Almgren and Chriss postulated that the price impact is also a part of the driving force of the random walk process[3].

1.2 Research Methodology

In this thesis, we will explore the use of a new methodology in addressing the aforementioned problems. They require a new model to incorporate the impact caused by a large order placement into the price formation process from both the supply and demand perspectives. This leads us to the area of agent-based computational finance.

The past few decades have witnessed a rapid development of agent-based computational finance. At the same time, criticism has also been levied against this field, since it has so far been unable to mimic the real financial behaviours in the market. Despite this, the agent-based computational finance method offers a way to simulate all possible scenarios. By assigning different parameters, an agent-based computational system models a typical pattern of pricing process replicating the price formation mechanism in the real world.

1.3 Research Objectives

In the following paragraphs, we will highlight our contributions to the study of the Implementation Shortfall (IS) strategy by using the agent-based simulation method. In addition, we will analyse its market-wide impact on the artificial stock market system and focus on the following objectives:

- Research Objective 1: *Construct the Minimum Risk Volume Weighted Average Price (VWAP) model and the Implementation Shortfall models based on linear and non-linear temporary impact assumptions*

We will present the optimal volume weighted average price (minimum risk VWAP) model proposed by Hizuru Konishi[2] and implementation shortfall (IS) model introduced by Almgren and Chirss[3]. Moreover, we will also make an improvement to the IS model by replacing the linear temporary impact assumption with a quadratic one. Calculations and comparisons will also be made based on these three models.

- Research Objective 2: *Construct an Artificial Stock Market*

  In order to carry out our analyses of optimal execution strategies, we will construct an artificial stock market. Order formation, market clearing and information propagation mechanisms will be designed for that market.

- Research Objective 3: *Analyse market effects of linear and non-linear temporary impact IS model*

  We will study the market quality indicators and agents' performances under various market circumstances, such as markets with differing levels of information availability or proportions of investors.

The remainder of this thesis is organized as follows. In chapter 2, we selectively review the literature and background knowledge on agent-based computational models and optimal execution strategies. In chapter 3, we will introduce the mathematical model of minimum risk VWAP and implementation shortfall algorithms. We will also present an improved implementation shortfall model. In chapter 4, we will discuss our agent-based model, and present the information sharing and market price formation mechanisms. Empirical analysis will be performed on the impact of different algorithms with different parameter settings in chapter 5. We will also fulfil the third research objective we proposed above. Finally, we will discuss the empirical findings, limitations, and implications for further study in chapter 6. Detailed descriptions about the genetic algorithm for numerical optimizations are given in the appendix.

Chapter 2

Theoretical Background and Literature Review

2.1 Algorithmic Trading and Optimal Execution Strategies

2.1.1 Overview

Algorithmic trading, also known as automated trading, was introduced a few decades ago. Although it is relatively new, its promising profitability and automatic nature have already made it popular in the capital market. According to a NYSE report in 2000, 22% of trading activities were conducted via trading algorithms, comparing to 11.6% in 1995. This number went up to 50.6% in 2004. Being free from the interference of emotional behaviour and inconsistent decisions made by human beings, algorithmic trading is able to accurately execute the pre-determined trading strategies without any hesitation and exploit the arbitrage opportunities with a reaction time of mini-seconds. Traders may be tempted to deviate from their trading strategies once those strategies appear to be malfunctioning with non-preferable returns. Such deviations may cause cancellation of the otherwise successful trading strategies or result in a large risk exposure to the portfolios. Trading algorithms take market data such as price and volume as their inputs and give orders to buy or sell as outputs. On the other hand, trading strategies are formulated mathematically and programmed by utilizing algorithmic trading methods, thus algorithmic trading is immune to any bias or deviation caused by human nature.

Algorithms can be categorized into two branches based on the particular objectives: optimal execution algorithms and speculative algorithms. The aim of the optimal execution algorithms is to minimize the market impact and opportunity cost of order execution, while speculative algorithms are designed to hedge market risk and seek potential profits. Although speculative strategies are of great importance in both academic and industrial society, we will mainly focus on optimal execution strategies. Developers of optimal execution strategies often have to compromise between opportunity cost and market impact risk. Traditionally, by bridging the development environment with trading infrastructure, the algorithm will be applicable for being tested with real-time data from the financial market. The trading system extracts market data, such as price, volume and volatility, into the database. The algorithm processes the input data in order to generate an optimal trading strategy. The system will record the performance, risk exposure and trading cost, and, in return, refine the trading algorithm. The optimal algorithm is reached after the iterative refinement has taken place by training it in a cyclic way.

Optimal execution strategies take into account a variety of current market conditions as well as characteristics of the order to be processed: order type, size, and frequency [5]. Bertsimas and Lo(1998) developed the strategies to take advantage of instantaneous price change when executing an order [6]. Kissell and Malamut(2006) investigated the adaptation process of the speed of order processing according to traders' current beliefs about the impending direction of market prices [7]. Capital markets algorithmic trading costs are an important factor that affects an optimal execution strategy. Generally, algorithmic trading costs mainly consider two factors: proportional transaction costs and liquidity cost. Pham et al [8] studied the permanent market impact and fixed discrete transaction costs based on an assumption that the price process follows Brownian motion. Junca [9] analysed the case where no transaction cost is involved and how this formulation relates with a singular control problem and provided a viscosity solution characterization. Almgren and Chriss (2001) studied the execution of portfolio transactions with the aim of minimizing a combination of opportunity cost and transaction cost, and introduced the implementation shortfall strategy, which sparked a great deal of interest from both researchers and investors [3]. Apart from optimizing the execution cost, algorithms that optimize liquidity supply, hedge positions and even monitor position changes in the marketplace, have also been developed [5]. In the following sections, we will introduce two common forms of optimal execution algorithms: the minimal cost algorithm and the volume participation algorithm.

Seminal works like [10, 11] were the first to suggest that informed investors should trade in a strategy that precludes other participants from recognizing their orders, in case of being front-run by others. Chakravarty(2001) investigated the impact of stealth trading upon the market price and found that medium-sized orders may always result in a disproportionally large market impact [12]. By slicing a large or medium-sized order into smaller ones and executing them subsequently, an investor is able to control his trading aggressiveness and market impact.

2.1.2 Minimal Cost Algorithms

Aggressive execution strategies with large orders can lead to disastrous effects on both market price and volume, although the traders may benefit from locking their profit in advance if the market goes in tandem with their expectations. On the other hand, traders using relatively passive executing strategies and aiming to minimize the market impact may suffer a large opportunity risk.To balance between an aggressive and passive execution, Almgren and Chriss [3] structured the problem as:

$$\min_{\alpha} \quad Cost(\alpha) + \lambda Risk(\alpha) \tag{2.1}$$

In this optimization problem, they minimize the linear combination of trading cost and opportunity cost. α is the trading rate, which can be used as a measurement of market aggressiveness, while λ represents the coefficient of risk aversion of investors. It was suggested by [13] that market aggressiveness varies with respect to the usage of market or limit orders. Contrary to a limit order, a market order is more likely to be executed in a relatively short time, raising the market aggressiveness considerably.

The cost and risk functions in (2.1) are defined as:

$$Cost(\alpha) = E_0[S(\alpha) - S_b], \tag{2.2}$$

$$Risk(\alpha) = \sigma(\varepsilon(\alpha)), \tag{2.3}$$

$$S(\alpha) = S + h(X, \alpha) + g(X) + \varepsilon(\alpha), \tag{2.4}$$

where E_0 denotes the ex-ante expectation at the beginning of the trading period, S_b the benchmark execution price, $S(\alpha)$ the realized execution price and $\varepsilon(\alpha)$ a random deviation of the trading outcome with $E[\varepsilon(\alpha)] = 0$ and $Var[\varepsilon(\alpha)] = \sigma^2(\alpha)$. S is the market price when the order is placed and X is the order size. $h(X, \alpha)$ is the temporary market impact function due to the liquidity demand of trading, while $g(X)$ is the permanent price impact function due to the change in market confidence or information leakage during the order execution.

The minimal cost approach proposed by Almgren and Chriss is known as the Implementation Shortfall (IS) strategy. The idea of IS was first introduced by Perold[14]. It is a standard ex-post measure of transaction costs used in performance valuations, defined as the difference $XS_0 - \sigma n_k \tilde{S}_k$ between the initial book price and the captured price. Almgren and Chriss modelled the temporary and permanent impact function of the market and, from that, they derived a price sequence. The objective of their strategy can be presented as:

$$\min_{x:V(x)\leqslant V_\star} E(x), \tag{2.5}$$

where $E(x)$ is the expectation of IS while $V(x)$ is the variance. We will introduce their work in the coming chapter, along with a slight improvement to their impact functions.

2.1.3 Volume Participation Algorithms

Another execution approach, which is relatively passive compared to the minimal cost algorithms, is to issue a certain portion of orders according to the expected volume on the trading day.

A widely accepted implementation of volume participation algorithms is the volume-weighted average price (VWAP) algorithm. VWAP is one of the most popular measures of trading cost. It is based on a simple straightforward idea of the weighted moving average of the trading horizon. In the real world, a broker may guarantee an execution of an order with the VWAP algorithm and have a computer program to place orders for exploiting the trader's commission and create P&L. This is called a guaranteed VWAP execution. VWAP balances execution with volume, and it is calculated according to the following formulas:

$$P_{VWAP} = \frac{\sum_j P_j \cdot Q_J}{\sum_j Q_j}, \tag{2.6}$$

where P_{VWAP} is a volume weighted average price, P_j the price of each individual trade and Q_j the quantity of each individual trade.

Konishi [2] introduced a static optimal execution strategy for a VWAP trade. Specifically, it is an order slicing strategy based on traders' forecasts of the distribution of volatility and market trading volume during a trading day. The objective of this strategy can be formulated as:

$$\min_{x(t)} E[(VWAP - vwap)^2], \tag{2.7}$$

where $VWAP$ stands for the market VWAP while $vwap$ is the VWAP of a trader's execution strategy. $x(t)$ denotes the trader's remaining position at time t. We know that, since $vwap$ is a function of trader's trading volume during the whole trading period, it also can be seen as a function of the trader's remaining position $x(t)$. (2.7) expresses the main idea of a VWAP execution strategy: the trader will try his best to get his $vwap$ as close to the market VWAP as possible, thereby executing the minimum risk VWAP trading strategy. In that paper, the authors modelled the price process as a Brownian motion without a drift term. McCulloch and Kazakov(2007) proved that a semi-martingale price process $P_t = A_t + M_t + P_0$ in which A_t is the drift, will also be applicable since the price drift does not contribute to VWAP risk [15]. Moreover, McCulloch and Kazakov generalized Konishi's minimum VWAP risk trading problem [2] to an optimal VWAP trading problem, by using a mean-variance framework with a user-defined coefficient of risk aversion λ. This can be expressed as a mean-variance optimization:

$$\max_{x(t)}[E(\nu(x(t))) - \lambda Var(\nu(x(t)))], \tag{2.8}$$

where $\nu(x(t))$ denotes the difference between traded VWAP and market VWAP as a function of the trading strategy $x(t)$ and λ is the coefficient of risk aversion. We will introduce these strategies in the coming chapters.

2.2 General Framework of ACE and Its Applications

2.2.1 Study of Market Micro-structure

A market is a complex environment where exchanges between two parties take place along with the activities of bargaining and negotiating. It allows prices and quantities to be adjustable based on demands and supplies. It also makes the existence of perfectly competitive economies possible. In 1874, Léon Walras [16] published *Elements of Pure Economics* with an explanation of the General Equilibrium theory. In a General Equilibrium framework, the allocation of products and service is determined by price signals, and at the same time, market supplies and demands will automatically be equalized. As pointed out in [17, 18], there are three assumptions required to apply the General Equilibrium theory to a modern economy: (1) all agents face the same prices; (2) all agents are price takers, namely they take prices as given and do not consider that their purchasing decisions will move prices; (3) markets exist for all goods, and agents can freely participate in these markets. If we add another weak assumption that the desire of consumers can never be fully satisfied, we will get the first fundamental theorem of welfare economics, which states that any Walrasian (or competitive) equilibrium leads to a Pareto efficient allocation of resources [19].

The weakest point of General Equilibrium theory rests on these three aforementioned assumptions. The existence of the Walrasian Auctioneer may prevent any form of strategic behaviour. It assumes that information only exists in price, and communication between participants is not allowed. Since the only exogenous factors utilized by consumers to make their decisions are price and dividend, the decision-making process reduces to a relatively simple optimization problem. By assuming the existence of a Walrasian Auctioneer, a system can reach its equilibrium.

Although General Equilibrium theory already clarified the assumptions required for an optimal allocation of resources, it is unable to explain why manufacturing, pricing and trading activities interact with each other only through procurement processes. As stated in [20], the procurement process consists of various factors, since consumers need to decide the purchase quantity while manufacturers need to determine the price and quantity that are most likely to be welcome in the market. Moreover, allies may be formed among consumers or among manufacturers in order to maximize their own profits.

A theoretical framework serves as a simplification of our real world, and the General Equilibrium theory serves to simplify a procurement process. Especially in an economic system with an attainable long-term equilibrium, such a simplification is necessary and applicable since the procurement process will not affect the system in the long run. However, if no attainable long-term equilibrium can be reached, the General Equilibrium theory may turn out to be insufficient in describing the behaviour of participants. In fact, the procurement process is a deterministic force that manipulates the whole system.

It is only relatively recent that researchers have started exploring alternative models without the Walrasian Auctioneer assumption in order to study market behaviour and market micro-structure. Studies start with the pricing models based on participants' activities and their reactions to their competitors. Some further studies also reveal difficulties arising from the complexity of the dynamics in the market. This includes information asymmetry, strategic interaction, and so on. In order to model these factors and construct a more general model, an agent-based computational method (ACM) is introduced by researchers. The agent-based computational method requires a complex adaptive system which consists of a great number of interacting agents.

2.2.2 The Evolution of ACM

In the past few decades, fuelled by the expansion of computational capacity, the agent-based computational method has been applied to research fields such as economics, management science, game theory and finance. The first attempt of the agent based computational method was made in [21], in which Axelrod extensively used a set of computational simulations to study the strategic behaviour of the Iterated Prisoner's Dilemma. By introducing this new computational idea, that paper is viewed as providing the foundation of a new branch in game theory.

In the 1980s, a new field called Artificial Life (AL) was introduced by treating life as a computational model. This name was coined in 1986 by Christopher Langton, an American computer scientist. Meanwhile, Santa-Fe Institute also pioneered research in integrating computational approaches with the social sciences. One of these studies implemented the agent-based computational method in finance, based on bounded rationality and inductive reasoning theories. In this framework, an artificial stock market is constructed as a multi-agent system using a large-scale computational process as well as artificial intelligence theories. It is shown to be an efficient method for testing theoretical models when an analytical solution is not an option. Just as Cristiano Castelfranchi said in [22]:" Agent-based Social Simulation will be crucial for the solution of one of the hardest problems of economic theory: the spontaneous organization of dynamic social orders that cannot be planned but emerge out of intentional planning agents."

Lovric et al [23] studied the evolution of a decentralized market under controlled experimental conditions. By setting an initial population of agents, such as traders and financial institutions, the system is constructed with determined initial position and information for each agent. Without any interference from its modeller, the system evolves over time, driven by the effects of its endogenous factors, the interactions between agents for instance.

Agents in stock markets are usually modelled mathematically by specifying heterogeneity, bounded rationality and interaction through a certain trading mechanisms. Sometimes, possible learning and evolving mechanism may also be incorporated into the model, as well as trading and portfolio strategies.

The Gode & Sunder model was created in 1993 [24]. It assumes traders have zero-intelligence without learning and no logical decision-making mechanisms. As a result, they randomly trade in a double auction house. Although the Gode & Sunder model is very simple, it did lay the foundation for modelling an artificial stock market and allow further incorporation of learning and evolving mechanisms into the model. Following such a conceptual framework, in 1994, Routledge [25] developed a new model to study the uncertainty and information in a financial market. Later, the Arifovic model [26] was introduced to model the foreign exchange market based on the General Equilibrium theory known as the multiple-point general equilibrium theory. In 1996, the artificial neural network was introduced into economics and financial modelling [27] as a price forecasting algorithm.

The Santa-Fe framework [28] was introduced in 1997. It was designed to analyse the stock pricing process and price volatility. Agents in this model are assumed to be bounded rationale, making decisions based on their anticipation of future prices and dividends. The resulting forecasts will determine their demand functions, featuring different parameters for their learning and adapting processes. After that, Microscopic Simulation was introduced from physical science into the financial markets, serving as a tool for studying complex systems by simulating many interacting microscopic elements [29]. It is believed that microscopic simulation models could be used to extend existing analytical models in finance, by either replacing more realistic assumptions or building new ones, such as models that incorporate various technical and fundamental strategies observed in experiments and real markets and dynamic models with heterogeneous investors that can learn and change their strategies [23].

In recent years, along with the fast development of computer technology, even more models have been introduced in different supporting platforms. Table 2.1 contains a list of some existing platforms that can be used for stock market analysis, such as the very first agent-based computational platform called Swarm.

2.2.3 General Framework of Agent-based Computational Method

2.2.3.1 Economic System

Under the framework of ACM, an economic system is a complex adaptive system [30]. A complex adaptive system is one special case of a complex system, and a complex system that can be defined as (1) a system consisting of multiple interactive units, and (2) a system that exhibits emergent properties. That is to say, a system that can acquire properties from the interactions of its units and not from the units themselves.

As argued in [31, 32], there is still no universal definition for the complex adaptive system. Here are three representative definitions that are widely accepted:

Table 2.1: Agent-based computational platforms

Language	Java	C++	C	Logo Dialects	Visual Languages
Platform	ADK AnyLogic Ascape Cougaar DOMAR-J jEcho ECJ FAMOJA Jade JAS JASA JCA-Sim jES JESS Mason Moduleco Omonia Swarm VSEit	DeX LSD MOOSE Jade's Sim++ SimBioSys	Echo MAML	NetLogo MacStarLogo StarLogo OpenStarLogo StarLogoT StarLogo TNG	MAML PS-I RepastS SeSAm SimPlusPlus SME Zeus StarLogo TNG AgentSheets

- **Definition 1**: A complex adaptive system is a complex system that consists of multiple units with reactive ability, which means that its units may change their characteristics according to exogenous factors.
- **Definition 2**: A complex adaptive system is a complex system that consists of multiple goal-directed units, which means that its units may react to exogenous factors in order to achieve their goals.
- **Definition 3**: A complex adaptive system is a complex system that consists of multiple planner units, which means its units may interact or even control exogenous factors in order to achieve their goals.

2.2.3.2 Agent

In an artificial stock market, agents are abstractions of traders, brokers, financial institutions and retail investors. Agents are sealed software systems with algorithms to process data. Some of their data is private while other data is public or partially public. With the public-available data, agents communicate and interact based on the information at hand.

An artificial stock market is initialized by setting the initial dataset for agents and specifying decision-making algorithms as well as data publicity. The data contains various aspects of an agent's attributions, such as type (institutions, individuals, etc.) and structural attributions (such as social network and utility function). Behaviours of agents are categorized into social behaviour and strategic behaviour. Social behaviour is determined by forming relations with other agents while strategic behaviour is the outcome of private decision-making algorithms, such as pricing, learning and portfolio strategies.

2.2.3.3 Learning

Learning is a process in which individuals or agents collect and process information after they make their decisions. In the artificial stock market simulations, agents start with a pre-specified set of information and parameters, which shape their behavioural pattern. The greedy agents without equipping themselves with a learning mechanism would suffer from continuous loss when the parameters are ill-posed, while the agents with a learning mechanism would learn to adapt their strategies according to the current market status. Currently, the subject of learning algorithms has become a hot topic, and the most frequently used algorithms are the artificial neural network, learning classifier systems and the genetic algorithm.

2.2.3.4 Objective and Application

The artificial stock market provides a new approach to mimic market phenomena under various market scenarios, and also brings insights into understanding investors' decision-making mechanism.

Furthermore, apart from the out-sample empirical tests, the artificial stock market introduces another method for investors to explore and test new trading and executing strategies. In the traditional empirical analysis, the price sequences are taken as the realized market price without any impact from large orders. When the order size is large enough to impact the market, it may turn out to be useless or sometimes even misleading since the market impact from large orders is no longer negligible. Hence, only an approach based on market simulation could facilitate the investors to incorporate both the price sequence and the market reactions.

2.3 Genetic Algorithm and Numerical Optimization

2.3.1 A Brief Introduction to Genetic Algorithm

In 1950s, a few computer scientists started doing their research on 'Artificial Evolutionary System' independently, expecting that evolution could be used as a new approach for optimizing engineering problems. They built the early prototype of genetic algorithm. In early 1960s, Rechenberg and Schwefel implemented the idea of mutating into a wind tunnel experiment. After that, their further researches gave birth to a branch of evolutionary system: Evolutionary Strategy (ES), which was an early form of genetic algorithm.

Another breakthrough was made by Holland in mid 1960s. He proposed a method which uses the bit-string encoding technology. This new method enabled the implementation of both mutation and crossover operations. Moreover, Holland stated that it is crossover, not mutation, that should play a main role when information is passed among generations.

An analysis using genetic algorithm starts with a population that may contain a potential optimal solution to the target functionThe first step is map explicit information into digit sequences. A common method is bit string encoding. After the first generation created, selection is performed in order to select individuals to reproduce the new generation. Gene sequence can reproduce with crossover and mutation operations. The individual with best fitness among one generation are most likely to be one of the potential solutions to the target function. Figure 2.1 shows the basic procedure of genetic algorithm.

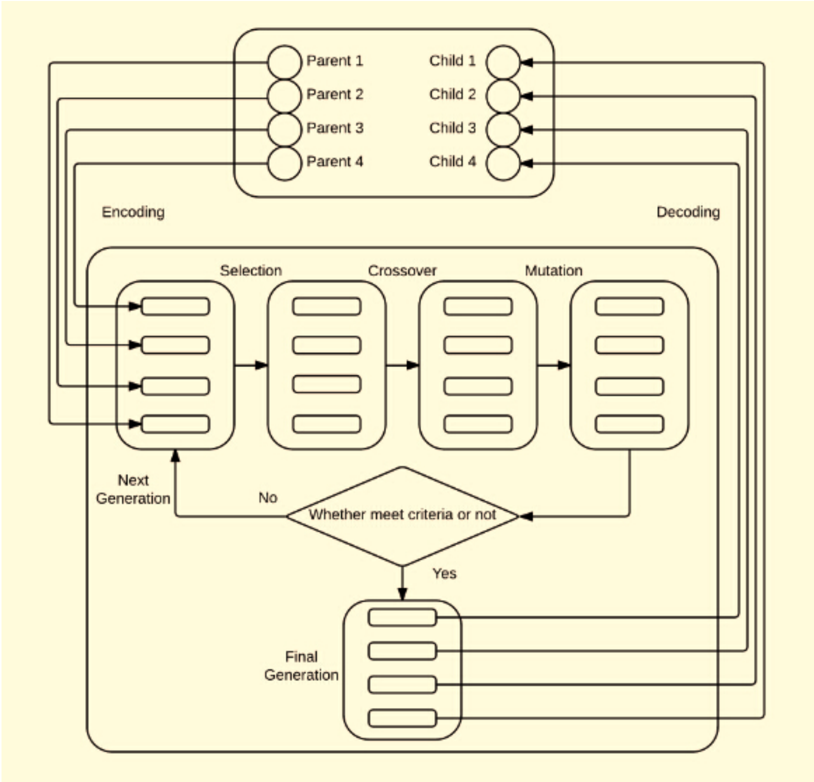

Figure 2.1: Procedure of GA

Algorithm begins with N random parents carrying their own information from a random initiation. After calculating their fitness functions, we have the fitness of the first generation. The second step is to generate their offspring and test the optimization criteria. The parents recombine their gene (crossover) in order to produce new offspring, with a infinitesimal yet positive possibility to mutate. The performance of a new generation is evaluated by calculating fitness functions. The second step is repeated until a new generation fits optimization criteria.

Three Basic Operations:

Genetic algorithm consists three basic operations: selection, crossover and mutation.

Selection:

Selection is used to select individuals for recombination or crossover, and to determine the number of children for the given parents. The selection process gradually improves the overall performance of fitness for each new generation. The first step of selection is calculating fitness according to:

- proportional fitness assignment;
- rank-based fitness assignment.

After calculating fitness for each individuals, we may select from the parent generation using:

- roulette wheel selection;
- stochastic universal sampling;
- local selection;
- truncation selection;
- tournament selection;

Crossover / Recombination:

Genetic recombination is reproducing new-generation individuals using information passed from the parent generation. Algorithms are classified according to various encoding methods:

- real valued recombination;
  - discrete recombination;
  - intermediate recombination;
  - linear recombination;
  - extended linear recombination;
- binary valued crossover;
  - single-point crossover;
  - multiple-point crossover;
  - uniform crossover;

– shuffle crossover;

– crossover with reduced surrogate;

Mutation:

New-born individuals have a small chance to mutate. This will have a disruption on its gene sequence. Algorithms vary according to different encoding methods:

- real valued mutation;

- binary valued mutation.

Despite of the prosperity of choices in each step, a practical combination proposed in [33] uses roulette wheel selection, single-point crossover and mutation.

Figure 2.2 shows the information carried by each individual from the first generation with their fitness in bracket, where larger fitness means closer approximation to the optima.

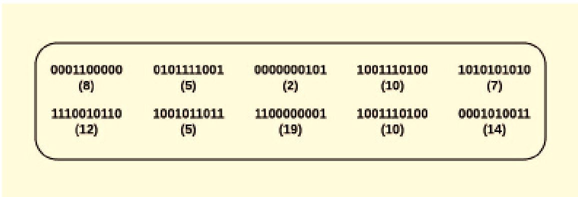

Figure 2.2: The first generation and gene

By using the roulette wheel selection, each individual is assigned a probability proportional to their fitness, like a space on a roulette wheel as shown in Figure 2.3 and Table 2.2. Since there are 10 individuals in one generation, the system will sample 10 uniform random draw in order to select parents of the next generation.

As a practical example, the ten random numbers are: 0.070221, 0.545929, 0.783567, 0.446931, 0.507893, 0.291198, 0.71634, 0.272901, 0.371435, 0.854641. The next step matches these random numbers with aggregate probability of each individual to see which one is picked. In this simulation, individuals labelled as 1,8,9,6,7,5,8,4,6,10 are picked, and individuals No.8 and No.6 are picked twice. Individuals who fit the environment better may have a higher chance to survive and be picked as parents. Individuals No.2 and No.3 are extinct because of either their low fitness and a bad luck. Their positions are taken by No.8 and No.6, which means better individual on fitness has a higher chance to survive and

Individual	Chromosome	Fitness	Probability	Aggregate Probability
1	0001100000	8	0.086957	0.086957
2	0101111001	5	0.054348	0.141304
3	0000000101	2	0.021739	0.163043
4	1001110100	10	0.108696	0.271739
5	1010101010	7	0.076087	0.347826
6	1110010110	12	0.130435	0.478261
7	1001011011	5	0.054348	0.532609
8	1100000001	19	0.206522	0.739130
9	1001110100	10	0.108696	0.847826
10	0001010011	14	0.152174	1.000000

Table 2.2: Chromosome and probability

reproduce more offsprings. The crossover follows after the reproduction, which is called hybridization in biology and is regarded as a key step of evolution. In the single-point crossover, parents are randomly chosen from the population and exchange parts of their chromosome. Children are born with inherited gene from the recombination of the parents, as shown in Figure 2.4. In the single-point crossover, the cross-point is randomly chosen and parents exchange parts of their gene sequences on the right hand side of the cross-point.

However, crossover should not be the only mechanism in evolution, since children from a relatively good ancestor will quickly dominate the whole system with high-quality gene, and a prematurely converged solution can not stand for the global optimized solution. One solution to this problem is to add mutation into the system. Mutation is a crucial step in keeping the bio-diversity at an acceptable level while genes from better ancestors still have higher chance to survive. By turning one random digit from 0 to 1 or from 1 to 0, we have the mutation mechanism implemented into our genetic system, as shown in Figure 2.5. In fact, the probability that a mutation takes place is quite low.

In general, evolution from one generation to another includes selection, crossover and mutation. If we put everything mentioned above together, the whole evolution procedure can be shown by Figure 2.6. Starting from selecting parents in the first generation based on the fitness, individuals No.8 and No.6 are picked out so that they take the place of those with smaller fitness, which are No.2 and No.3 in this case. Next, 4 pairs of parents are chosen following certain probability distribution for the operation of the single-point crossover, in order to produce 4 pairs of children. After that, children mutate, which means a random digit of their chromosome have a small probability to invert. Generation

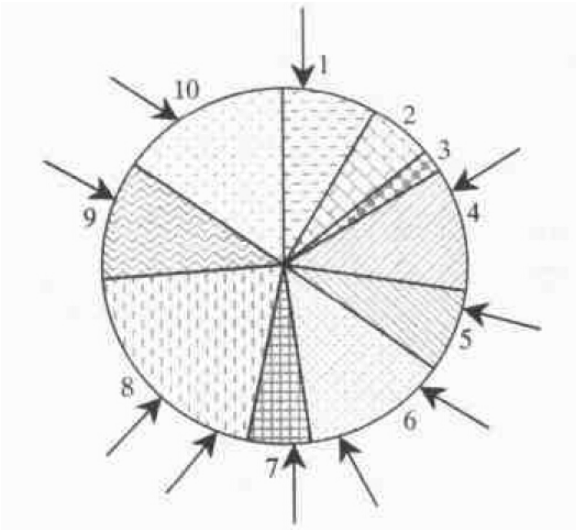

Figure 2.3: Roulette Wheel Selection

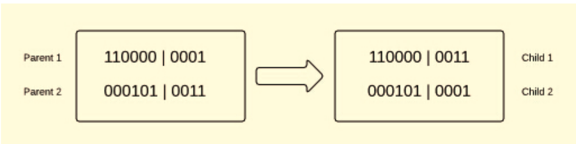

Figure 2.4: Single-point Crossover

by generation, the system searches for increasingly closer optimal solution, and the optimal individual among the last generation is the optimal solution. The number of generations needed to get the optimal solution depends on the convergence property of our objective problem.

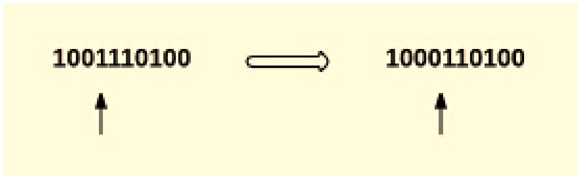

Figure 2.5: Mutation

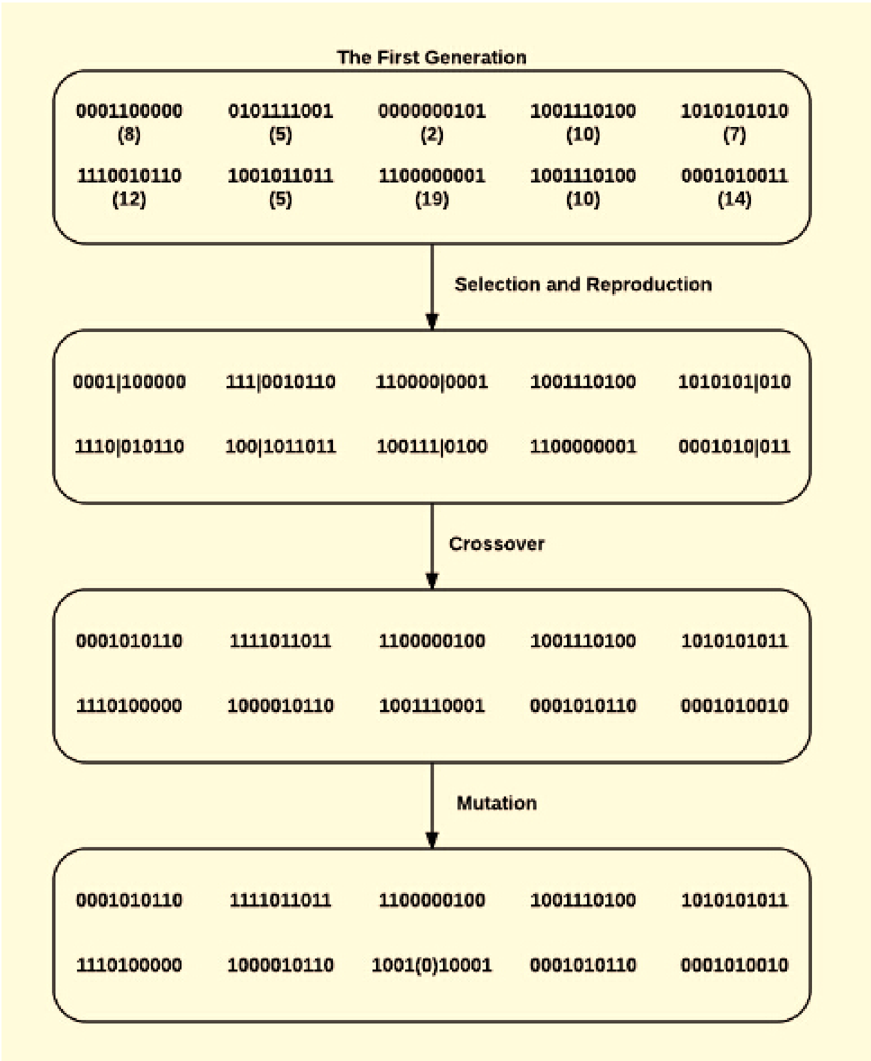

Figure 2.6: Evolution procedure by genetic algorithm

Chapter 3

Optimal Execution Strategies

3.1 Volume Participation Algorithms: Minimum Risk VWAP

Volume-weighted average price has been used as a benchmark for measuring the trading cost and the execution quality. When liquidizing a large order, traders and brokers are most likely to set their objective as beating the market VWAP. An important contribution to this field was proposed by Konishi [2] who developed a solution to the minimum risk VWAP trading strategy. By following Konishi [2], we introduce the minimum risk VWAP trading strategy for single-stock cases.

The main purpose for a trader to implement VWAP strategies when executing a block of shares X is to achieve captured prices as close to the market VWAP as possible during the trading period. Firstly, we construct a probability space (ω, F, Q) with a filtration F_t which satisfies the usual conditions. Let $V(t)$ be accumulated market trading volume excluding the trader's at time t. Also, $v(t)$ denotes the trader's accumulative trading volume from time 0 to time t. Here, $V(t)$ is an F_t-adapted process and $v(t)$ is a controllable variable. At the start, $v(0) = V(0)$=0 and the trading strategy v(t) is already given. The stock price $S(t)$ follows a stochastic process:

$$dS(t) = \sigma(t, V(t))dB(t),$$

where $\sigma(t, V(t))$ is a positive F_t-adapted process in L^2 space and $B(t)$ is a standard Brownian motion. Secondly we assume that opposite-directional operations are not allowed in this market, which means that no buying action is allowed when a trader is liquidizing a sell order. By assuming this, $v(t)$ becomes a non-decreasing function. Therefore, the trader's own VWAP at time t can be calculated as

$$vwap = \frac{\int_0^t S(s)dv(s)}{v(t)}$$

Also, the market VWAP is written as

$$VWAP = \frac{\int_0^t S(s)d(V(s) + v(s))}{V(t) + v(t)} = \frac{\int_0^t S(s)dV(s) + v(t) \cdot vwap}{V(t) + v(t)}$$

The main objective of this is to minimize the expected squared error of the trader's VWAP to market VWAP, which is due to the unpredictable price movement and error in volume forecast. We represent our objective as

$$\min_{v(t)} E[VWAP - vwap]^2$$

Next, we rewrite the expected squared error of the VWAP execution in an explicit form as

$$WE[D] + Cov\left(\left(\frac{V(T)}{V(T) + v(T)}\right)^2, D\right), \tag{3.1}$$

where

$$\begin{aligned}
D &= \int_0^T ((X(t) - x(t))dP(t))^2, \\
X(t) &= \frac{V(T) - V(t)}{V(T)}, \\
x(t) &= \frac{v(T) - v(t)}{v(T)}, \\
W &= E\left[\left(\frac{V(T)}{V(T) + v(T)}\right)^2\right],
\end{aligned}$$

$x(t)$ here denotes the ratio of the remaining position of a trader at time t against his initial position, namely an execution trajectory or execution strategy. $X(t)$ represents the market remaining trading volume against its total trading volume through the entire process. Furthermore, since $V(T)$ denotes the whole market trading volume, we assume that, compared to $V(T)$, $v(T)$ is much smaller, and $V(T)/(V(T)+v(T))$ has a value which is close to 1. The covariance term of (3.1) is negligible. Taking into account that W is calculated according to the trader's expectation, we can rewrite the objective problem as

$$\min_{x(t)} \int_0^T E[\sigma(t,V)^2(X(t)-x(t))^2]dt, \tag{3.2}$$

and this can be simplified as

$$\begin{aligned}
&\min \int_0^T E[\sigma(t,V)^2(X(t)-x(t))^2]dt \\
&= \int_0^T E[\sigma(t,V)^2 X(t)^2]dt \\
&\quad + \min \int_0^T (-2E[\sigma(t,V)^2 X(t)]x(t) + E[\sigma(t,V)^2]x(t)^2)dt \\
&= \int_0^T \left(E[\sigma(t,V)^2 X(t)^2] - \frac{E[\sigma(t,V)^2 X(t)]^2}{E[\sigma(t,V)^2]} \right) dt \\
&\quad + \min \int_0^T \left(\frac{E[\sigma(t,V)^2 X(t)]}{E[\sigma(t,V)^2]} - x(t) \right)^2 E[\sigma(t,V)^2]dt
\end{aligned} \tag{3.3}$$

Next, we slice the whole execution time into $v(T)$ units. Here, we need to introduce a new index t_k ($k = 0, 1, \cdots, v(T)$). We have t_0=0 and $t_{v(T)}$=T. The execution trajectory can be rewritten as

$$x(t) = 1 - \frac{k}{v(T)} \qquad \text{if} \quad t_k < t \leqslant t_{k+1} (k = 0, 1, \cdots, v(T) - 1)$$

In this way, the determination of the optimal execution trajectory $x^*(t)$ is equivalent to the determination of an optimal execution time $t^*(k)$ for each units of our order. So $x^*(t)$ becomes a step function, which is continuous but not differentiable for all points within its domain. According to (3.3), we notice that the optimization problem has turned out to be an optimization that finds the step function $x(t)$ that best fits a continuous and differentiable curve $\frac{E[\sigma(t,V)^2X(t)]}{E[\sigma(t,V)^2]}$. From (3.3), we have

$$\begin{aligned}
&\min\int_0^T(-2E[\sigma(t,V)^2X(t)]x(t)+E[\sigma(t,V)^2]x(t)^2)dt \\
&=\min[\sum_{k=1}^{v(T)}\int_{t_{k-1}}^{t_k}(-2E[\sigma(t,V)^2X(t)]x(t)+E[\sigma(t,V)^2]x(t)^2)dt]
\end{aligned} \tag{3.4}$$

The first half of (3.4) can be rewritten as

$$\begin{aligned}
& \sum_{k=1}^{v(T)}\int_{t_{k-1}}^{t_k}(-2E[\sigma(t,V)^2X(t)]x(t))dt \\
= & \sum_{k=1}^{v(T)}\int_{t_{k-1}}^{t_k}\left(-2E[\sigma(t,V)^2X(t)](1-\frac{k-1}{v(T)})\right)dt \\
= & \sum_{k=1}^{v(T)}(v(T)-k+1)\int_{t_{k-1}}^{t_k}\left(-\frac{2E[\sigma(t,V)^2X(t)]}{v(T)}\right)dt \\
= & \sum_{k=1}^{v(T)-1}(v(T)-k)\int_{t_{k-1}}^{t_k}\left(-\frac{2E[\sigma(t,V)^2X(t)]}{v(T)}\right)dt \\
& +\int_0^{t_{v(T)}}\left(-\frac{2E[\sigma(t,V)^2X(t)]}{v(T)}\right)dt \\
= & \cdots \\
= & \sum_{k=1}^{v(T)-i}(v(T)-k-i+1)\int_{t_{k-1}}^{t_k}\left(-\frac{2E[\sigma(t,V)^2X(t)]}{v(T)}\right)dt \\
& +\sum_{k=v(T)-i+1}^{v(T)}\int_0^{t_k}\left(-\frac{2E[\sigma(t,V)^2X(t)]}{v(T)}\right)dt \\
= & \cdots \\
= & \sum_{k=1}^{v(T)}\int_0^{t_k}\left(-\frac{2E[\sigma(t,V)^2X(t)]}{v(T)}\right)dt,
\end{aligned} \tag{3.5}$$

and the second half of (3.4) as

$$
\begin{aligned}
& \sum_{k=1}^{v(T)} \int_{t_{k-1}}^{t_k} (E[\sigma(t,V)^2]x(t)^2)dt \\
= \quad & \sum_{k=1}^{v(T)} \int_{0}^{t_k} (E[\sigma(t,V)^2]x(t_{k-1})^2)dt - \sum_{k=1}^{v(T)} \int_{t_0}^{t_{k-1}} (E[\sigma(t,V)^2]x(t_{k-1})^2)dt \\
= \quad & \sum_{k=1}^{v(T)} \int_{0}^{t_k} (E[\sigma(t,V)^2]x(t_{k-1})^2)dt - \sum_{k=0}^{v(T)-1} \int_{0}^{t_k} (E[\sigma(t,V)^2]x(t_k)^2)dt \\
= \quad & \sum_{k=1}^{v(T)} \int_{0}^{t_k} (E[\sigma(t,V)^2]x(t_{k-1})^2)dt \\
& -[\sum_{k=1}^{v(T)} \int_{0}^{t_k} (E[\sigma(t,V)^2]x(t_k)^2)dt + \int_{0}^{t_0} (E[\sigma(t,V)^2]x(t_0)^2)dt] \\
& - \int_{0}^{t_{v(T)}} (E[\sigma(t,V)^2]x(t_{V(t)})^2)dt] \\
= \quad & \sum_{k=1}^{v(T)} \int_{0}^{t_k} (E[\sigma(t,V)^2][x(t_{k-1})^2 - x(t_k)^2])dt \\
= \quad & \sum_{k=1}^{v(T)} \int_{0}^{t_k} \left(E[\sigma(t,V)^2] \left[\left(\frac{v(T)-k+1}{v(T)} \right)^2 - \frac{v(T)-k}{v(T)} \right] \right) dt \\
= \quad & \sum_{k=1}^{v(T)} \int_{0}^{t_k} \left(E[\sigma(t,V)^2] \frac{2(v(T)-k)+1}{v(T)^2} \right) dt. \qquad (3.6)
\end{aligned}
$$

Therefore, (3.4) can be written as

$$
\min \sum_{k=1}^{v(T)} \int_{0}^{t_k} \left(E[\sigma(t,V)^2] \left(\frac{2(v(T)-k)+1}{v(T)^2} - \frac{2E[\sigma(t,V)^2 X(t)]}{v(T)E[\sigma(t,V)^2]} \right) \right) dt.
$$

Here, we replace the integrand with $f_k(t)$. It is easy to observe that this function is monotonically increasing with respect to t. Moreover, we have

$$f_k(t_{k-1}^*) = f_{k-1}(t_{k-1}^*) - \frac{2}{v(T)^2} < f_{k-1}(t_{k-1}^*) \leqslant 0,$$
$$f_k(T) = \frac{2(v(T)-k)+1}{v(T)} > 0,$$

from which we can infer that this function will turn from negative to positive only once in $[t_{k-1}, T]$, and the turning point we are looking for is at t_k^*.

Hence, t_k^* satisfies

$$\left(\frac{v(T)-k+1}{v(T)}\right)^2 - \frac{2E[X(t_k^*)]}{v(T)} - \left(\frac{v(T)-k}{v(T)}\right)^2 \leqslant 0,$$
$$\lim_{t \downarrow t_k^*} -\frac{2E[X(t)]}{v(T)} + \left(\frac{v(T)-k+1}{v(T)}\right)^2 - \left(\frac{v(T)-k}{v(T)}\right)^2 \geqslant 0,$$

and

$$E[X(t_k^*)] \geqslant \frac{2(v(T)-k)+1}{2v(T)} \geqslant \lim_{t \downarrow t_k^*} E[X(t)]$$

By assuming that $E[X(t)]$ is continuous at t_k^* we have

$$E[X(t_k^*)] = \frac{2(v(T)-k)+1}{2v(T)},$$

so that,

$$x(t_k^*) = 1 - \frac{k}{v(T)} = \frac{2v(T)E[X(t_k^*)]-1}{2v(T)}$$

Note that, $x^*(t)$ takes a constant value over the range of

$$E[X(t)] \leqslant E[X(t_k^*)] < E[X(t)] + \frac{1}{v(T)}$$

Therefore, $x^*(t)$ lies in the range of

$$E[X(t)] - \frac{1}{2v(T)} \leqslant x^*(t) < E[X(t)] + \frac{1}{2v(T)}$$

from which we can write $x^*(t)$ as

$$x^*(t) = \left\lfloor \frac{[(2v(T)E[X(t)] + 1)/2]}{v(T)} \right\rfloor, \tag{3.7}$$

where $\lfloor\rfloor$ represents the integer part.

For the case when $E[X(t)]$ is not continuous at t_k^*, the same argument also applies. In most real cases, $E[X(t)]$ can be measured by taking the average $X(t)$ of the empirical data in the past 3 days or 1 week, depending on the preference of the trader.

3.2 A Minimal Cost Algorithm: Implementation Shortfall

While trying to minimize the expected squared error of VWAP of the trading strategy to the market VWAP, Konishi [2] made several assumptions, including a stochastic pricing process and a negligible market impact, and these may cause an unwanted inaccuracy in modelling the real market. Alternatively, we can introduce the implementation shortfall (IS) model proposed by Almgren and Chriss [3] based on a price process generated by temporary and permanent impact functions.

Following Almgren and Chriss [3] and the assumption on the single-asset market, we define the initial position to be M. We divide the given time period T into N intervals, each with length $\tau = T/N$. Also, we define $t_k = k\tau$ for each subsection, so that a trading strategy, which is represented by the remaining position of the trader, is $m_k, (k = 1, ..., N)$. We have $m_0 = M$, $m_N = 0$, and for any $k \in \{0, \cdots, N\}$

$$M - m_k = v\left(\frac{kT}{N}\right).$$

Moreover, we specify a trade list $n_1, \cdots, n_N$, which represents the portion that a trader has to liquidate at each trading time. More specifically, for any $k \in \{0, \cdots, N\}$, we have

$$m_k = M - \sum_{j=0}^{k} n_j.$$

Next, we assume the initial market price to be S_0, and that the price process evolves according to one exogenous factor: volatility, as well as two endogenous factors: the temporary market impact and the permanent market impact. We assume the execution period is measured by days, so that we do not need to include any drift terms into this price process. In this way, we can write the price process as

$$\begin{aligned} S_k &= S_{k-1} + \sigma\sqrt{\tau}\xi_k - \tau g\left(\frac{n_k}{\tau}\right) && (3.8)\\ &= S_0 + \sum_{j=1}^{k}\left(\sigma\tau^{\frac{1}{2}}\xi_j - \tau g\left(\frac{n_j}{\tau}\right)\right), && (3.9)\end{aligned}$$

where t=$(1, \cdots, N)$, σ represents the yearly standard deviation of the asset, and ξ_t's are iid normal random variables. $g(v)$ denotes the permanent impact function of the average trading rate $v = n_t/\tau$ within the period t_{k-1} to t_k.

Furthermore, we model the temporary market impact, which is also a function of average trading rate. We are not going to include the temporary impact function directly into the market price process since the impact is temporary and we assume that the market will recover to its equilibrium state by the next trading period. In this way, the temporary impact effect will only be expressed as a difference between the market price and the captured price. Here, we use $\widetilde{S_k}$ to denote the captured price and $h\left(\frac{n_t}{\tau}\right)$ as the temporary impact function, whose value depends on the trading speed $\frac{n_t}{\tau}$:

$$\widetilde{S_k} = S_{k-1} - h\left(\frac{n_t}{\tau}\right).$$

Therefore, the total capture of trading, which is calculated by summing the product of the actual trading price $\tilde{S}_k$ and the number of shares n_k we trade during each time interval, can be written as

$$
\begin{aligned}
\sum_{k=1}^{N} n_k \widetilde{S_k} &= \sum_{k=1}^{N} n_k \left(S_k - h\left(\frac{n_k}{\tau}\right)\right) \\
&= MS_0 + \sum_{k=1}^{N} \left(\sigma\tau^{\frac{1}{2}}\xi_k - \tau g\left(\frac{n_k}{\tau}\right)\right) m_k - \sum_{k=1}^{N} n_k h\left(\frac{n_k}{\tau}\right) \\
&= MS_0 + \sum_{k=1}^{N} \sigma\tau^{\frac{1}{2}}\xi_k m_k - \sum_{k=1}^{N} \tau g\left(\frac{n_k}{\tau}\right) m_k - \sum_{k=1}^{N} n_k h\left(\frac{n_k}{\tau}\right). \qquad (3.10)
\end{aligned}
$$

The left hand side is the capture of the strategy upon completion of this execution. From (3.10), we notice that, the first term MS_0 on the right hand side stands for the initial market value of the order. The second term $\sum_{k=1}^{N} \sigma\tau^{\frac{1}{2}}\xi_k m_k$ represents the total effect of market volatility, and the third term $\sum_{k=1}^{N} \tau g(\frac{n_k}{\tau})m_k$ is the total effect of the permanent impact, which is always represented as a loss since the price tends to move against traders, noticeably or not. The fourth term $\sum_{k=1}^{N} n_k h(\frac{n_k}{\tau})$ measures the total temporary market impact of the execution, and it is also the difference of the total captured revenue against the market price, or in other words, the error revenue from execution against the theoretical earning.

In this way, the total trading cost, which is calculated as the difference between the initial book value and the total capture, can be written as

$$
MS_0 - \sum_{k=1}^{N} n_k \widetilde{S_k} = \sum_{k=1}^{N} n_k h\left(\frac{n_k}{\tau}\right) - \sum_{k=1}^{N} \left(\sigma\tau^{\frac{1}{2}}\xi_k - \tau g\left(\frac{n_k}{\tau}\right)\right) m_k. \qquad (3.11)
$$

This measure of total trading cost was introduced by Perold (1988), and is called *implementation shortfall (IS)*. At the beginning of the execution period, we calculate the expectation and variance of IS, and these can be written as

$$
\begin{aligned}
E(m) &= \sum_{k=1}^{N} \tau m_k g\left(\frac{n_k}{\tau}\right) + \sum_{k=1}^{N} n_k h\left(\frac{n_k}{\tau}\right) \qquad (3.12) \\
V(m) &= \sigma^2 \sum_{k=1}^{N} \tau m_k^2 \qquad (3.13)
\end{aligned}
$$

In order to study this model and its characteristics further, we need to assume an explicit form of each impact function. Firstly, we start with the permanent impact function. In the real markets, executing a large order may result in a dramatic movement in the opposite direction of the order. In the long run, although the market may recover to an equivalent state, these operations still will undermine the confidence of other investors and cause the market's new equivalent state to deviate from its previous state. However, compared to the market fluctuation and temporary impact effect, the permanent impact effect is not as important. For this reason, we can write it as a linear function

$$g\left(\frac{n_k}{\tau}\right) = \gamma \frac{n_k}{\tau}, \tag{3.14}$$

where γ is a pre-determined constant that can be measured empirically or determined according to a trader's expectation. By plugging (3.14) into (3.10), we have the permanent impact term rewritten as

$$\begin{aligned}
\sum_{t=1}^{N} \tau g(\frac{n_t}{\tau}) m_t \quad &= \gamma \sum_{k=1}^{N} m_k n_k \\
&= \gamma \sum_{k=1}^{N} m_k (m_{k-1} - m_k) \\
&= \frac{1}{2} \gamma \sum_{k=1}^{N} (m_{k-1}^2 - m_k^2 - (m_k - m_{k-1})^2) \\
&= \frac{1}{2} \gamma \left(M^2 - \sum_{t=1}^{N} n_t^2 \right).
\end{aligned}$$

Also, the price process can be rewritten as

$$S_k = S_0 + \sigma \sum_{j=1}^{k} \tau^{\frac{1}{2}} \xi_j - \gamma (M - m_k).$$

Similarly, we can make a further assumption that the temporary impact function fits a linear function as well. So we assume a temporary impact function of the form

$$h\left(\frac{n_t}{\tau}\right) = \epsilon sgn(n_t) + \frac{\eta}{\tau} n_t, \tag{3.15}$$

in which, ϵ and η are pre-determined constants, and $sgn(\cdot)$ is the sign function defined as

$$sgn(x) = \begin{cases} 1 & \text{if x} > 0 \\ 0 & \text{if x} = 0 \\ -1 & \text{if x} < 0 \end{cases} \tag{3.16}$$

A reasonable estimation for ϵ is the fixed costs of selling, such as half the bid-ask spread plus transaction fees.

In this way, we rewrite the expectation of impact cost as

$$E(m) = \frac{1}{2}\gamma M^2 + \epsilon \sum_{k=1}^{N} |n_k| + \frac{\tilde{\eta}}{\tau} \sum_{k=1}^{N} n_k^2, \tag{3.17}$$

where $\tilde{\eta} = \eta - \frac{1}{2}\gamma\tau$.

Here, we construct our optimal strategy by minimizing the expectation of our trading cost with a given maximum level of variance V_*:

$$\min_{m:V(m) \leqslant V_*} E(m). \tag{3.18}$$

We solve the constraint optimization problem in (3.18) by introducing a Lagrangian multiplier λ and convert it to an unconstrained problem

$$\min_m (E(m) + \lambda V(m)). \tag{3.19}$$

λ can be interpreted as the trader's coefficient of risk aversion. We set

$$U(m) = E(m) + \lambda V(m).$$

According to (3.17), the expected transaction cost is strictly convex. Moreover, for $\lambda > 0$, (3.19) is also strictly convex. Therefore, by setting its partial derivatives to zero, we can obtain the unique global minimum of $U(m)$

$$\frac{\partial U}{\partial m_k} = 2\tau\left(\lambda\sigma^2 m_k - \tilde{\eta}\frac{m_{k-1} - 2m_k + m_{k+1}}{\tau^2}\right) = 0.$$

So, we have

$$\frac{1}{\tau^2}(m_{k-1} - 2m_k + m_{k+1}) = \frac{\lambda\sigma^2}{\tilde{\eta}}m_k = \tilde{\kappa}^2 m_k. \tag{3.20}$$

We notice that the solution to this linear differential equation according to the optimal trading trajectory m_k can be written as a combination of the exponentials $\exp(\pm\kappa t_k)$, where κ satisfies

$$\frac{2}{\tau^2}(cosh(\kappa\tau) - 1) = \tilde{\kappa}^2, \tag{3.21}$$

$\tilde{\eta}$ and $\tilde{\kappa}$ denote an infinitesimal asymptotic correction of η and κ: as $\tau \to 0$, we have $\tilde{\eta} \to \eta$ and $\tilde{\kappa} \to \kappa$. The optimal trading trajectory can be written as

$$m_k = \frac{sinh(\kappa(T - t_k))}{sinh(\kappa T)}M \qquad k = 0, \cdots, N,$$

and the associated trading portion for each period is

$$n_k = \frac{2sinh(\frac{1}{2}\kappa\tau)}{sinh(\kappa T)}cosh(\kappa(T - t_{k-\frac{1}{2}}))M, \qquad j = 1, \cdots, N,$$

where $t_{k-\frac{1}{2}} = (k - \frac{1}{2})\tau$.

Therefore, the expectation and variance of trading cost of the optimal execution strategy for a given initial order size M can be expressed as

$$E(M) = \frac{1}{2}\gamma M^2 + \epsilon M + \tilde{\eta}M^2\frac{tanh(\frac{1}{2}\kappa\tau)(\tau sinh(2\kappa T) + 2T sinh(\kappa\tau))}{2\tau^2 sinh^2(\kappa T)}$$
$$V(M) = \frac{1}{2}\sigma^2 M^2\frac{\tau sinh(\kappa T)cosh(\kappa(T - \tau)) - T sinh(\kappa\tau)}{sinh^2(\kappa T)sinh(\kappa\tau)}.$$

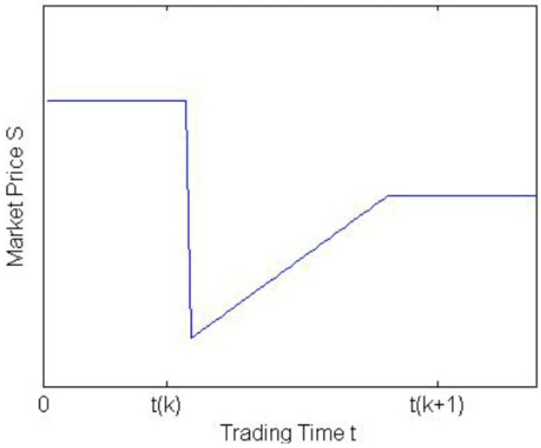

Figure 3.1: Linear Temporary Impact Effect

3.3 An Alternative IS Strategy with Quadratic Temporary Impact

In (3.15), we follow the work of Almgren and Chriss [3] and made a strong assumption that the temporary impact effect is a linear function proportional to trading rate $\frac{n_t}{\tau}$, with an analytical solution to the optimization problem. Figure 3.1 shows an example of the modelled linear temporary impact effect. We notice that there exists a sharp turning point where the market rapidly switches from a recovering state to an equilibrium state. If the depth of the temporary impact effect exceeds a certain level or the recovering period is relatively short, the linear function loses its efficiency in modelling the temporary impact effect due to the sharp turning point, since the real market is a collection of numerous heterogeneous individuals and abrupt reversal is not realistic. In order to smoothly approximate the market impact, we introduce an alternative strategy by replacing the linear temporary impact model with a quadratic one:

$$h\left(\frac{n_k}{\tau}\right) = \left(\epsilon + \eta\frac{n_k^2}{\tau}\right) sgn(n_k).$$

So the total temporary impact term of (3.12) can be written as

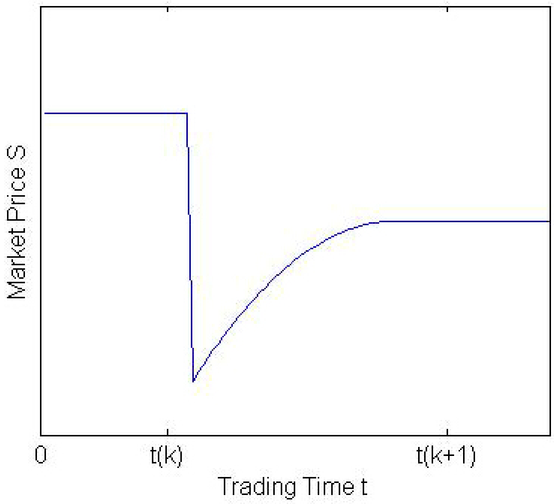

Figure 3.2: Quadratic Temporary Impact Effect

$$\begin{aligned}\sum_{k=1}^{N} n_k h(\frac{n_k}{\tau}) &= \sum_{k=1}^{N} n_k sgn(n_k) \left(\epsilon + \eta \frac{n_k^2}{\tau} \right) \\ &= \sum_{k=1}^{N} \left[\epsilon |n_k| + \eta \frac{|n_k|^3}{\tau} \right],\end{aligned}$$

and the expectation of implementation shortfall can be written as

$$E(m) = \frac{1}{2}\gamma M^2 - \frac{1}{2}\gamma \sum_{k=1}^{N} n_k^2 + \frac{\eta}{\tau} \sum_{k=1}^{N} |n_k|^3.$$

Here, we follow Algmren and Chriss [3] by assuming that no reversal operation is allowed, which means that all n_k have identical signs. We further assume that $n_k > 0$ for $k \in (1, \cdots, N)$, so that we have

$$E(m) = \frac{1}{2}\gamma M^2 - \frac{1}{2}\gamma \sum_{k=1}^{N} n_k^2 + \frac{\eta}{\tau} \sum_{k=1}^{N} n_k^3,$$

and the target function turns out to be

$$\begin{aligned} U(m) &= E(m) + \lambda V(x) \\ &= \frac{1}{2}\gamma M^2 - \frac{1}{2}\gamma \sum_{k=1}^{N} n_k^2 + \frac{\eta}{\tau} \sum_{k=1}^{N} n_k^3 + \lambda\sigma^2 \sum_{k=1}^{N} \tau m_k^2. \end{aligned} \tag{3.22}$$

By replacing the temporary impact function with a quadratic one, the final target function (3.22) becomes a non-convex cubic function.

For simplicity, we apply the genetic algorithm to solve this analytically-intractable optimization problem. Note that, in order to implement numerical methods, we need to add some constraints into the original optimization problem. We rewrite the final constraint optimization problem as

$$\begin{array}{lll} \min & U(m) & \\ s.t & m_{k-1} \leqslant m_k & k \in (1, \cdots, N) \end{array}$$

3.3.1 Genetic Algorithm for Numerical Optimization

In order to explain the procedure of solving this optimization problem numerically using the genetic algorithm, we need to set up an initial value for parameters γ, η, λ and σ. Following the study of Almgren and Chirss [3], we set parameters $\gamma = 2.5 \times 10^{-7}$, $\eta = 2.5 \times 10^{-6}$, $\lambda = 1 \times 10^{-6}$ and $\sigma = 1.9$.

- Genetic Operation: Selection

  As we have mentioned before, selection is the first step of the reproduction process. It is done randomly with a probability depending on the corresponding fitness of the individuals so that the best ones are more likely to be chosen for reproduction over the poor ones [34].

  In this case, we choose the roulette wheel selection method as our selection algorithm. Firstly, we set the population of each generation to be 200, 5% of which comes directly from the top 5% elites of the previous generation, 76% from crossover of their parents and 19% from mutation. The number of parents needed to reproduce the next generation is $152 \times 2 + 38 = 342$, each of them contains 10 numbers that represent the 10 steps trading strategy. We next assign each individual of the first generation with a probability corresponding to their fitness score, and then randomly draw 342 times from this population of 200 to select a parents set.

- Genetic Operation: Crossover

  Next, we will perform a crossover operation in order to reproduce the next generation. We randomly pick two individuals from the parents set, namely, r_1 and r_2. In Table 3.1, we present an example of a pair of parents selected from one simulation. We generate a binary string, where 0 means that we select the genes at the corresponding position from parent r_1 and 1 represents a selection from parent r_2. In doing this, we have a new string of numbers representing an individual in the next generation, and a new trading trajectory at the same time. According to Table 3.1, by crossover, the new individual is lucky enough to have a lower score, which means a better fitness than both of its parents. In most cases, the new individual will not be that lucky to have such an improvement over its parents, and will be abandoned as the iteration goes on.

	Genome of Individuals					Score
r_1	0.1982	0.1962	0.1156	0.1159	0.1027	9.5218e+10
	0.0998	0.0582	0.0446	0.0365	0.0338	
r_2	0.1598	0.1565	0.1226	0.1172	0.1168	1.0373e+11
	0.1157	0.0859	0.0496	0.0452	0.0312	
Kid	0.1982	0.1565	0.1226	0.1172	0.1027	9.4824e+10
	0.1157	0.0582	0.0496	0.0452	0.0338	

Table 3.1: Crossover Operation

- Genetic Operation: Mutation

  The third step is mutation, where 19% of the population is randomly chosen using a roulette wheel selection method and mutate into new individuals. A new 10-digit string composed of 1, 0 and -1 will be randomly generated according to a pre-set mutate possibility, which in this case is set to be 10%. It means that each digit of this string has a 5% probability to mutate from 0 to 1 and a 5% probability from 0 to -1. With a reasonable step size, chromosomes will mutate toward the direction determined by the direction matrix. In the example shown in Table 3.2, the step size is set to be 1. We can see that the score of the mutated new individual is slightly higher than its parent. This means that mutation causes a negative effect to the individual, and this individual will most likely be washed out as the iteration goes on.

	Genome of Individuals					Score
r_1	0.3002	0.2126	0.1639	0.1079	0.0481	1.2331e+11
	0.0443	0.0425	0.0424	0.0235	0.0146	
Kid	0.3002	0.2126	0.1639	0.1080	0.0481	1.2290e+11
	0.0442	0.0425	0.0425	0.0234	0.0146	

Table 3.2: Mutate Operation

- Optimization

  By genetic operations, a generation will pass its out-standing genes to the next generation or generate new genome through crossover and mutation. A new genome which improves the fitness will be more likely to survive while others will have a higher possibility of being abandoned. In Table 3.3, we present the best individual that fits the environment in selected generations, and the corresponding fitness score. Among the 200 randomly-generated individuals in the first generation, the individual with a score of 9.7581×10^8 shows the best fitness. Individuals with the top 10 scores will be able to reproduce 10 off-springs, each of them carrying exactly the same information as its parent. Crossover and mutation will occur among the rest of the 190 individuals from the first generation when they are reproducing their kids. After no more than 21 generations, individuals with not-so-flat trading curves are produced and proven to have a better survivability. One example of such an individual is the one we presented at the 22_{th} generation. In this way, individuals keep evolving until the 956_{th} generation. After 200 more generations we find that no significantly better individual has been produced. So we come to the conclusion that the individual with a fitness score of 8.2253×10^7 is one of the best fits for this environment, and this means that 8.2253×10^7 is a numerical solution to this optimization problem which we are looking for.

Generation	Genome of Individuals					Score
1	0.4459	0.2348	0.1956	0.1011	0.0188	9.7581e+08
	0.0038	0.0000	0.0000	0.0000	0.0000	
8	0.9989	0.0004	0.0003	0.0002	0.0001	2.4500e+08
	0.0000	0.0000	0.0000	0.0000	0.0000	
22	0.7219	0.2768	0.0005	0.0005	0.0002	1.2014e+08
	0.0001	0.0001	0.0000	0.0000	0.0000	
50	0.7188	0.2775	0.0023	0.0006	0.0006	1.1941e+08
	0.0002	0.0001	0.0000	0.0000	0.0000	
250	0.6620	0.2825	0.0254	0.0080	0.0070	1.0928e+08
	0.0052	0.0038	0.0038	0.0021	0.0000	
500	0.6198	0.2827	0.0429	0.0128	0.0123	1.0110e+08
	0.0094	0.0080	0.0073	0.0046	0.0000	
816	0.5474	0.2738	0.0731	0.0269	0.0208	8.6858e+07
	0.0170	0.0156	0.0145	0.0110	0.0000	
1000	0.5207	0.3251	0.0742	0.0231	0.0144	8.2302e+07
	0.0129	0.0113	0.0101	0.0082	0.0000	
1016	0.5222	0.3246	0.0738	0.0228	0.0142	8.2253e+07
	0.0135	0.0110	0.0099	0.0081	0.0000	

Table 3.3: Best individuals among their generations

Chapter 4

Framework of the Artificial Stock Market

As we have mentioned before, most existing methods for analysing optimal execution strategies fail to consider the potential impact of large orders on market price data that they use. In this chapter, we introduce an artificial stock market system, which is able to include the market impact and reaction into its price formation process.

The most fundamental and important part of an artificial stock market is the price formation mechanism. In this thesis, this mechanism is built based on a simple logic that matches the market demands and supplies and finds an equilibrium price in a limit-order book.

As for agents, a traditional way to model agents' behaviours is to divide the population into several groups, and assign each group with a specific trading strategy, such as fundamental analysis, technical analysis or noise trading. Raberto and Cincotti [35] found that, in such a framework, there always exists one group that greatly out-performs others, and this leads to convergence of the market wealth distribution and the market is determined to reduce its volatility since the out-standing group may take over the whole market. To solve this,agent aggregation based on an agent clustering mechanism can be applied according to the model in Cont and Bouchaud [36]. Moreover, agents' uncertainty about the market price can also be modelled by introducing a random variable into the decision making process.

In this thesis, we follow Peter [37] and assume that the market evolves in discrete steps. We assign N traders into the market with the i-th trader denoted by its subscript i. t represents the current time and the corresponding market price is denoted as p(t). $C_i(t)$

and $A_i(t)$ stand for the amount of cash and assets held by the i-th trader at time t. These traders will place buy orders with probability P_i or sell orders with possibility $1 - P_i$ at each trading time.

4.1 Order Formation Mechanism

Firstly, we assume that the i-th trader places a sell order with size a_i^s at time t+1, which is a random portion of his stock position at time t:

$$a_i^s = \lfloor r_i A_i(t) \rfloor,$$

where r_i is a random number uniformly drawn from the interval [0,1] and $\lfloor\rfloor$ denotes the integer part.

Secondly, we assign a limit price s_i to each sell order:

$$s_i = p(t) LN_i(\mu_s, \sigma_i),$$

where $LN_i(\mu_s, \sigma_i)$ denotes a random draw from a lognormal distribution with average μ_s and standard deviation σ_i. Here, following Raberto [4], we set $\mu_s = 0.99$. This means, without taking other market information into consideration, that the mean value of all sell orders is more likely to be slightly lower than the market price p(t). It is a realistic assumption since rational investors always tend to place sell orders at slightly lower prices in order to liquidize their orders more quickly. Also, with $\mu_s = 0.99$, we manually set up a spread between bid and ask prices. Also, trading in a market with only limit-order book, agents may try to execute their orders at prices no lower than their expectation.

Thirdly, the investors' uncertainty about the market price may increase in a nervous market, and this will result in a higher volatility of stock price:

$$\sigma_i = k\sigma(f_i),$$

where k denotes a constant and f_i stands for a factor calculated by the i-th trader using empirical volatility data. Note that this factor varies from different agents since each investor analyses the market condition in his own way. The individual volatility

$$\sigma_t^2(f_i) = \sum_{j=1}^{J} \alpha_{ij} u_{t-j}^2,$$

where α_{ij} denotes the attached weight for the i^{th} agent and u_t denotes the return, with expression $u_t = \log \frac{p(t)}{p(t-1)}$. Here we take the uncertainty as a form of exponentially weighted moving average, which means that the weight α_j can be written as

$$\alpha_{i(j+1)} = e^{-f_i}\alpha_{ij},$$

We notice that, with an f_i close to 0, the agent assigns the volatility in the past J days almost the same weight. This means that, when analysing the market information, the agent will put more weight on the market trend instead of on the recent market event. With an f_i close to 1, the agent pays more attention on the new market information in the past few days and only take the trend of volatility for reference.

Conversely, a buy order is generated in a symmetrical way with

$$c_i = r_i C_i(t),$$

which denotes the amount of cash an agent may spend in building up his position at time t. r_i is a random draw from a uniform distribution in the interval [0,1]. Also, the associated limit price of this buy order can be written as

$$b_i = \frac{p(t)}{LN_i(\mu_b, \sigma_i)},$$

where $LN_i(\mu_s, \sigma_i)$ denotes a random draw from a log-normal distribution with average μ_s and standard deviation σ_i. The minimum number of shares that the agent is able to buy is calculated as

$$a_i^b = [\frac{c_i}{b_i}].$$

r_i and $N_i(\mu, \sigma)$ will be generated independently for each agent at each time step. By analysing empirical volatility of market price under the constraint of limited resources of each agent, we hope to be able to capture the heterogeneous random behaviour on the part of the investors.

4.2 Market Clearing Mechanism

The price formation mechanism is modelled by finding the equilibrium price of the supply and demand curves.

To compute the supplies and demands curves, we have to assume that the agents have placed U buy orders and V sell orders at time t+1. We denote the pair (a_u^b, b_u), $u = 1, 2, \cdots, U$ to be the quantity of shares to buy and the associated limit price. Likewise, we also set the pair (a_v^s, s_v), $v = 1, 2, \cdots, V$ to be the quantity of shares to sell and the associated limit price. Then, we can define the demand function at time t+1 to be

$$D_{t+1}(p) = \sum_{u=1|b_u \geqslant p}^{U} a_u^b,$$

where $D_{t+1}(p)$ denotes the total amount of shares that the market will demand when the equilibrium price is p. It shows a decreasing trend with respect to price p: with a higher price, fewer buyers can be satisfied, and fewer buy order can be executed. When p is greater than the maximum value of limit buy price, no buy order will be executed.

Similarly, the supply function can be written as the number of shares that will be sold when the equilibrium price is p:

$$S_{t+1} = \sum_{v=1|s_v \leqslant p}^{V} a_v^s,$$

In our case the equilibrium price p^* sometimes can be referred to as the market clearing price, since it represents an optimal point at which the market will execute most orders. It is determined by finding the intersection of the demand and the supply curves.

Figure 4.1 shows the supply and demand curve based on a sample simulation. We notice that, both the demand and the supply curves are step functions. Hence, in most cases, the equation $D(p^*) = S(p^*)$ does not hold. Under this circumstance, we need to introduce a market maker, whose capital size is infinitely large so as to absorb any small amount of orders due to the demand and supply difference at the clearing price. It is a widely accepted but strong assumption since the market will not be completely closed under this assumption. After each period, orders with limit prices not compatible with the clearing price will be discarded, others will be executed, and the cash and stock position of their agents will thus be updated.

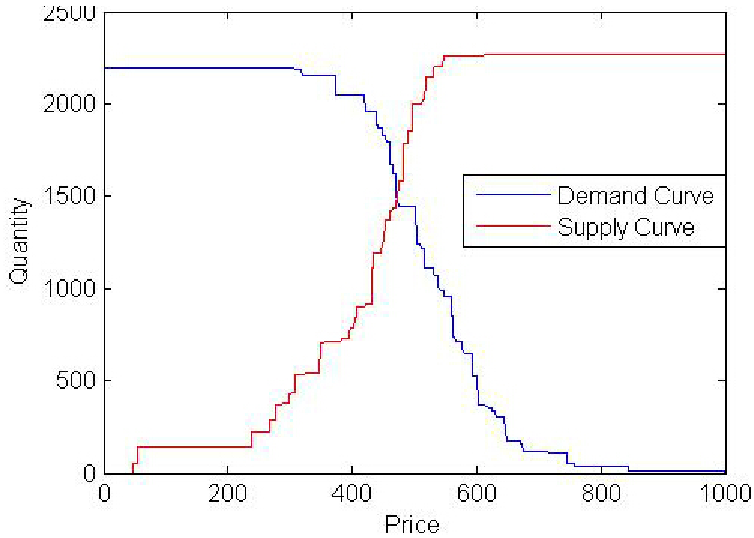

Figure 4.1: Demand and Supply Curve

4.3 Information Propagation Mechanism

Usually, agents are modelled with absolute independence, producing a log return that follows a normal distribution. The associated long-term price behaviour will also follow a random walk [38].

In order to simulate pricing behaviours in the real market, such as market return with fat tails following a Pareto-Levi distribution [39, 40], we introduce the random graph approach for constructing an agents' social network [41]. Following the research of Raberto and Bouchaud [4, 41], we stipulate that the cluster size distribution follows an inverse power law. Instead of having a constant probability P_i for each agent to place a buy order, we introduce an opinion propagation mechanism that updates the decision-making algorithm dependent on the clustering effect. In the following, we provide an explanation of this mechanism.

Firstly, at the beginning of each trading period, clusters will form between two random agents with probability P_a. One agent can be engaged into different clusters simultaneously. Different clusters containing the same agent can be treated as a large single cluster as a whole. This mechanism is designed to simulate the market public information. Interpretations of a same public information may vary among agents, and agents in the same cluster share similar opinions on that information.

Secondly, after forming market-wise clusters, private information such as rumours or insider information, is introduced into the market with probability P_c, and it will be acquired by random clusters. Once informed, agents that belong to chosen clusters will be over-confident with the information they acquire. They will make bold and not-so-rational decisions: either buying stocks using 90% of their cash positions or selling 90% of their stock positions. After positions of agents have been updated across the market, clusters will be disbanded and all agents will become independent again.

We will present our results of empirical studies based on this market structure in the next chapter.

Chapter 5

Results of Empirical Analysis

5.1 Choice of Parameters

5.1.1 Parameters of Artificial Stock Market

In order to examine the simulation results and statistical properties of the artificial stock market, we will start by initiating the system parameters. The market price of the traded stock p_0 is set to be \$100.00 per share for simplicity. The number of agents involved in the simulation N is set to be 100 as a compromise to computational capacity of hardware due to the exponentially increasing computational complexity.

At the beginning, each agent holds a cash position of \$10,000.00 and a stock position of 100 shares. The parameters that determine the fluctuations in limit prices (k_i, f_i, μ_i) do not have a direct influence on the time-series properties of our system. By tuning the system, we finally reach an optimal value for k_i equals to 3.5. The time window of each trader to conduct analysis T_i and the interval within the individual factor f_i for each agent are specified as $T_i = 30$ and $f_i \in [0.5, 0.9]$. In each trading interval, the probability for an independent agent to place a buy order P_i is set to be 50%, so the probability for placing place a sell order $1 - P_i$ is also 50%. Following the work of Raberto [4], we set the spread of the average limit price for buy and sell orders to be 0.02, the probability of forming a link between two agents is $P_a = 0.1$, and the probability of information leaking to a certain cluster is set to be 0.05.

5.1.2 Parameters of Optimal Execution Strategies

A numerical example of the model provides deeper insight into optimal execution strategies. Following Almgren [3], we firstly assume that the single stock in the market has an initial price $S_0 = \$100.00$ and the initial order size is $M = 5,000$ shares.

Secondly, we assume that the annual volatility of the stock is $\sigma_a = 0.3$, and the annual return $r_a = 0.1$. Also, the stock has a bid-ask spread with a value of 1/8 and a median daily trading volume of 5,000,000 shares. Given that a year consists of 250 trading days, the absolute parameter σ and α can be calculated as

$$\begin{aligned} \sigma &= 100 \times \tfrac{0.3}{\sqrt{250}} = 1.90 \\ \alpha &= 100 \times \tfrac{0.1}{250} = 0.04 \end{aligned}$$

Thirdly, we assume that investors want their orders to be executed within less than 10 time steps. This means T=10, $\tau = 1$ and N=10. The time window for the investor to estimate $E[X(t)]$ is also set to be $T_e = 10$ trading periods.

Fourthly, we stipulate the fixed part of the temporary impact in (3.15) to be half of the bid-ask spread, $\epsilon = 1/16$, and assume that every investor incurs a price impact equals to the bid-ask spread for every 1% of the daily trading volume. For instance, trading at a rate of 5% of the daily volume incurs a cost of 5/8 on each trade. In this way, η defined in (3.15) is calculated as

$$\eta = \frac{1}{8} \times \frac{1}{0.01 \times 5 \times 10^6} = 2.5 \times 10^{-6}$$

Fifthly, as for the permanent impact, we assume that the impact becomes significant if an investor places an order with a minimum size of 10% of the daily trading volume. A significant impact means that the price goes against the trade direction by one bid-ask spread, and the effect is assumed to be linear regardless of the order's size. Thus we calculate γ which is first seen in (3.14) as

$$\gamma = \frac{1}{8} \times \frac{1}{0.1 \times 5 \times 10^6} = 2.5 \times 10^{-7}$$

At last, we choose the coefficient of the investors' risk aversion in (3.19) to be $\lambda = \lambda_u = 1 \times 10^{-6}$ and κ is calculated as

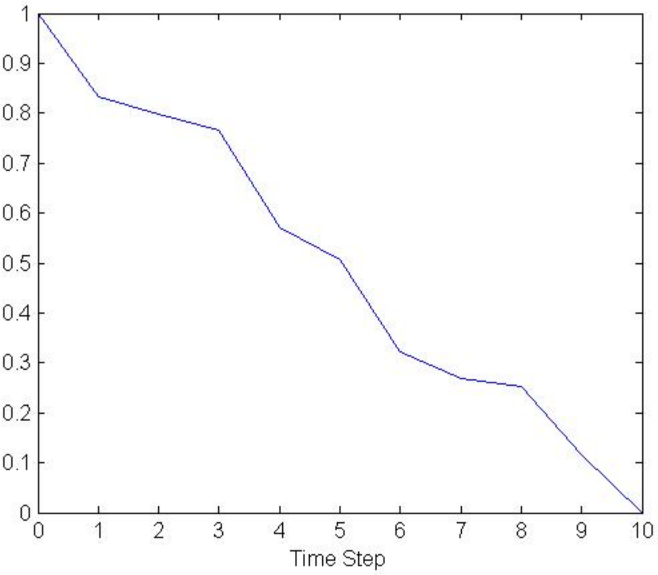

Figure 5.1: Trading Trajectory of Minimum Risk VWAP Algorithm

$$\kappa \approx \tilde{\kappa} + \Omega(\tau^2) \approx \sqrt{\frac{\lambda\sigma 2}{\eta}} + \Omega(\tau) \approx \sqrt{\frac{1\times 10^{-6}\sigma 2}{2.5\times 10^{-6}}} \approx 1.27$$

Figures 5.1, 5.3 and 5.4 are examples of simulated trading trajectories of the minimum risk VWAP strategy and IS strategies with the linear and quadratic temporary impact functions.

We notice from Figure 5.1 that the execution trajectory of the minimum risk VWAP exhibits a slight fluctuation while attaining a monotonically decreasing trend, for the reason that the VWAP algorithm aims to minimize the difference of executed VWAP and market VWAP. Moreover, the curve of the execution trajectory does not have a constant convexity since the trajectory is dependent on the market trading volume in the past few days. Note that, here we use a sample of simulated trading volume data from the artificial stock market in order to implement the minimum risk VWAP algorithm. Figure 5.2 presents the comparison between the market VWAP and the investor's predicted VWAP during the same period of time. A similar trend can be clearly observed from the figure. Also, by showing small deviations in the first several steps and relatively larger but still acceptable differences in the last three steps, Figure 5.2 indicates that the investor's prediction is reliable during the execution period, with more accuracy at the first few steps.

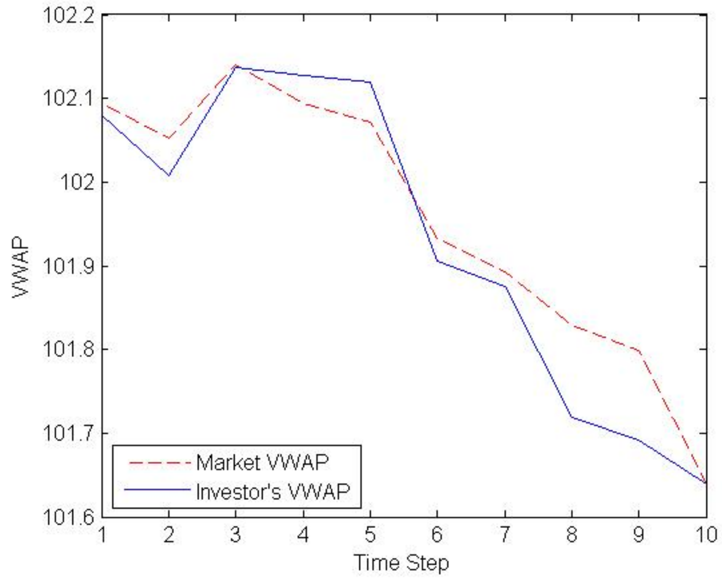

Figure 5.2: Market VWAP and Investor's VWAP

Figures 5.3 and 5.4 depict the simulated results of the IS model using the linear and quadratic temporary impact functions. According to the formulas we introduced earlier, both of these algorithms are sensitive to the agents' coefficients of risk aversion, or in other word, their risk preferences. We will include the outcomes of these two strategies based on different risk preferences, but for now, by setting both of the coefficients to be $\lambda = 1 \times 10^{-6}$, we are able to have a clearer comparison of them.

From Figures 5.3 and 5.4, we see that both models lead to convex trading trajectories with higher trading speed at the beginning and a lower speed in the end. It is consistent with our assumption that the curve will show convexity, reinforcing the notion that the greater portion of the order should be executed within the first few steps of an execution process, because the longer apart, the higher the opportunity costs. Moreover, the trading trajectory with the linear impact implementation shortfall model shows a much deeper curve, which means that those investors are less sensitive to impact risk. They tend to liquidize their orders as fast as possible when the market price moves against them. On the other hand, the curve shown in Figure 5.4 suggests that investors using the quadratic impact implementation shortfall model care more about the market impact, for the reason that the market impact grows quadratically against the trading speed. In order to control the execution cost, investors choose to liquidize their orders in a gentler way.

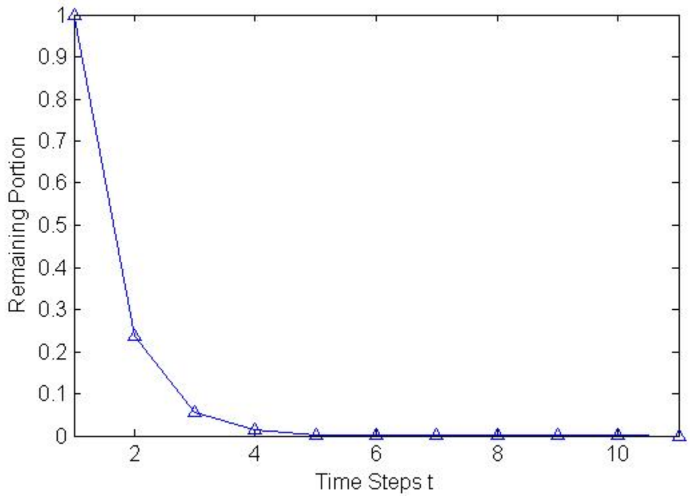

Figure 5.3: Trading Trajectory of Linear Impact Implementation Shortfall Algorithm

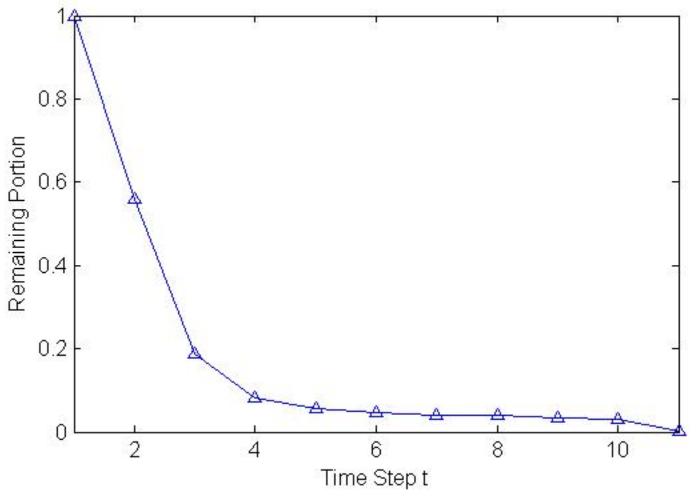

Figure 5.4: Trading Trajectory of Quadratic Impact Implementation Shortfall Algorithm

5.2 Investor's Preference

In order to further study the trading trajectories of the implementation shortfall strategies, agents will be assigned with different risk preferences. We are not able to include this diversity into the minimum risk VWAP case since the model does not contain any factor concerning risk adjustment. However, the risk preference is already incorporated in the model since λ measures risk aversion, in both the linear-impact and quadratic-impact implementation shortfall cases. Judging from the basic assumptions of these two models, the curve of the optimal trading trajectory should bend more with a larger coefficient of risk aversion, for the reason that a large coefficient of risk aversion leads to a lower risk tolerance and a faster trading speed.

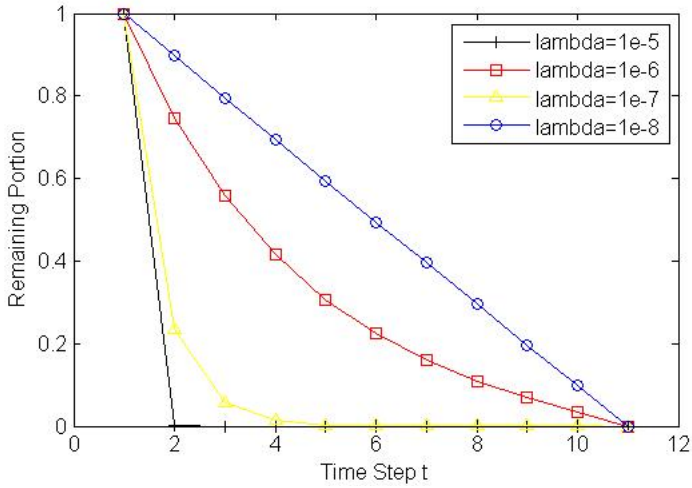

Figure 5.5: Trading trajectories of Linear-impact Implementation Shortfall model

In Figures 5.5 and 5.6, we present the corresponding trading trajectories of the linear-impact and quadratic-impact implementation shortfall model with the coefficient of risk aversion λ being 1×10^{-5}, 1×10^{-6} ,1×10^{-7} and 1×10^{-8} respectively.

From Figures 5.5 and 5.6, we see that the trajectories of the linear-impact IS strategy are more sensitive to the changes of the coefficients of risk aversion. With λ equalling to 1×10^{-8}, the model ignores the risk preference of the investors and poses an approximately linear trajectory with a minimum market impact risk. When λ is equal to 1×10^{-5}, the corresponding trading strategy is to execute the whole order at the first trading date, ignoring the ensuing market impact. As for the case of the quadratic-impact implementa-

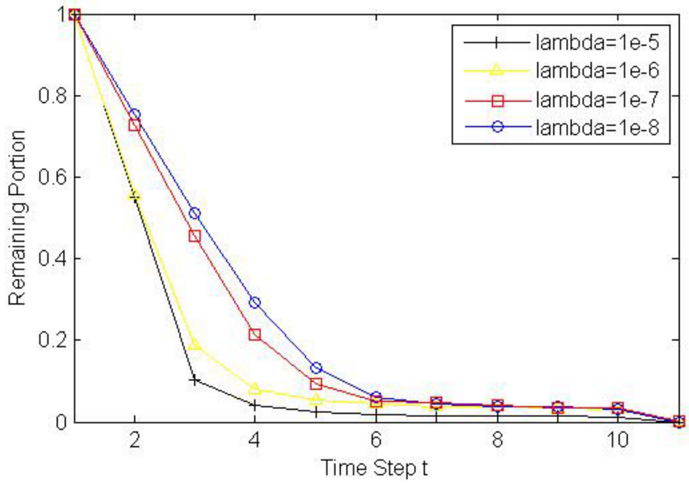

Figure 5.6: Trading trajectories of Quadratic-impact Implementation Shortfall model

tion shortfall model, the model places more weight on the market impact than the market fluctuation and this results in conservative trading strategies.

In the following section, we will implement these strategies based on the artificial stock market we constructed, and present the simulation results as well as the statistical properties of our market when a certain number of investors are assigned with optimal execution strategies.

5.3 Descriptive Statistics

Firstly, we start with a simulation without optimal execution strategies. Figure 5.7 shows the price process of the artificial stock market with 5,000 time steps. The maximum price is \$122.59 and the minimum is \$45.27. Figures 5.8 and 5.9 present the daily return and absolute daily return of the simulated market respectively.

A pattern of mean reversion is observed in these time series, suggesting that the time series are likely to be stationary. Also, there appear to be signs of uneven fluctuations over the sample period, providing the evidence of clustered volatility in various sub-sample periods. Moreover, there is also a strong evidence of persistence of the squared and absolute return as indicated in the plots, measuring volatility of return from different perspectives.

Figure 5.11 is the histogram of the market daily return and Figure 5.12 is the quantile-

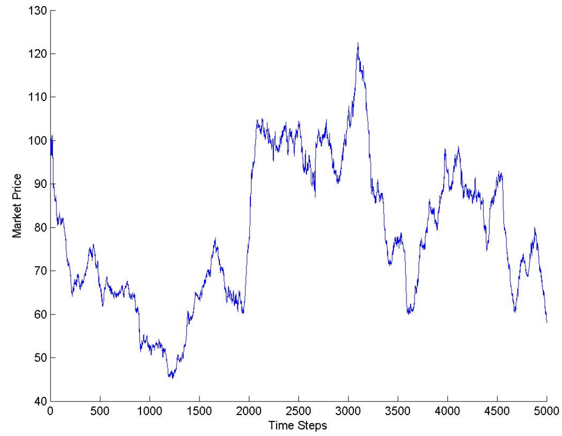

Figure 5.7: Market Price Process

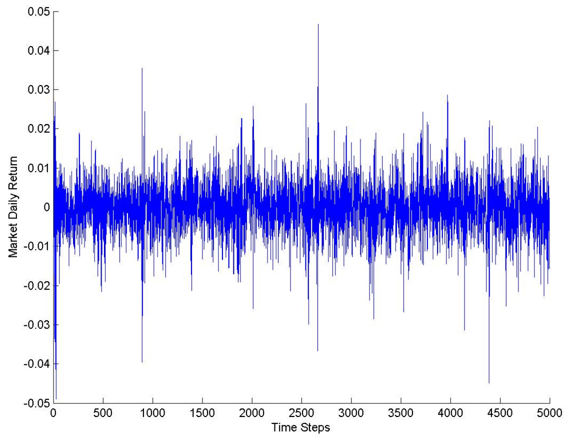

Figure 5.8: Market Daily Return

quantile plot of the market daily return against normal distribution. Both the histogram and QQ plot indicate a significant departure from normality on both tails.

In Table 5.1 we present descriptive statistics for the daily return of our simulated market, the Jarque-Bera test for normality, and Ljung-Box-Pierce Q(K) test for the joint insignificance of autocorrelation coefficients with $K = 5$. The distribution of the daily return is clearly non-normal with negative skewness and pronounced excess kurtosis, and

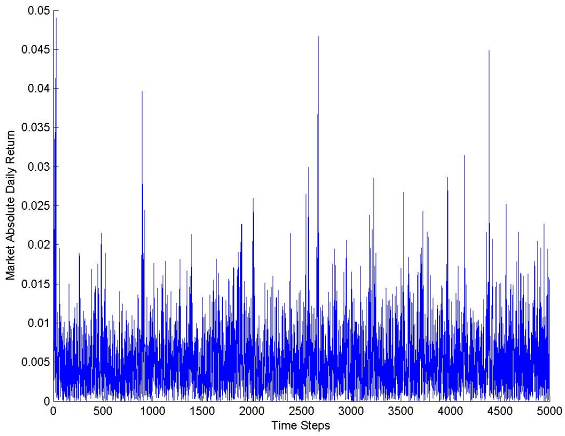

Figure 5.9: Market Absolute Daily Return

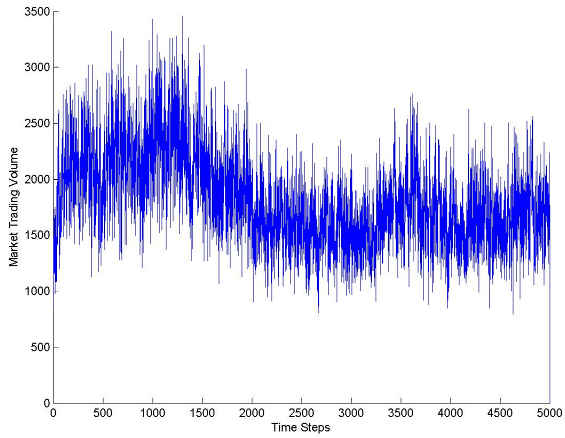

Figure 5.10: Market Trading Volume

this is consistent with the observations from the US market data.

The Q(K) test does not reject H_0 that the sample autocorrelation coefficients at the first-lag are jointly 0, but rejects H_0 for the last 4 lags. The results from the joint tests confirm the results reported above on the evidence of non-normality at the 95% confidence level.

Secondly, after a quick glance at the market statistics, we move on to comparing the

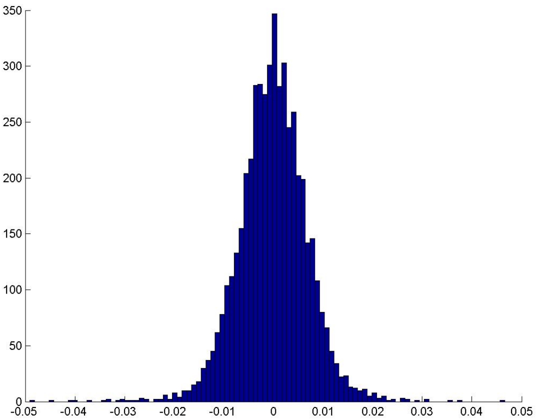

Figure 5.11: Histogram of Market Daily Return

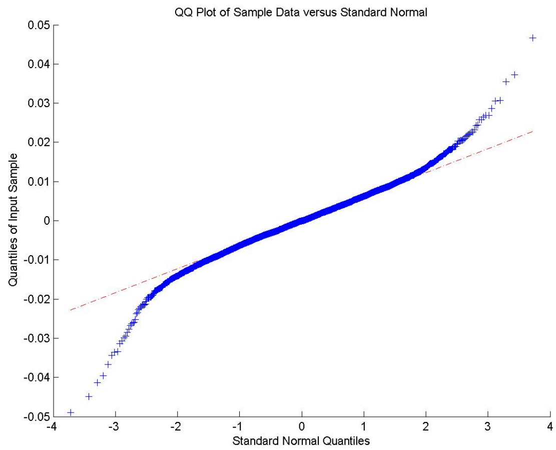

Figure 5.12: QQ-Plot of Market Daily Return

impact of the optimal execution strategies on the artificial market as well as the profitability of investors. The percentage of institutional traders that implement the optimal execution strategies is set to be 10% and will remain constant. Table 5.2 presents the descriptive statistics of the simulated market daily return data. We see that the changes of statistics of the artificial market remain at a low level. The market with the minimum risk VWAP strategy out-performs the other two in terms of volatility of the market return. This indicates a smaller market impact driven by the minimum risk VWAP strategy.

Table 5.1: Descriptive Statistics and Tests

Descriptive Statistics of Market Daily Return			
Mean	Standard Deviation	Skewness	Kurtosis
-8.1410×10^{-5}	0.0098	-0.1927	6.0870

Jarque-Bera Test		
Test Statistic	p-value	Decision
2.0163×10^{3}	0.001	Reject H_0 Strongly

Ljung-Box-Pierce Q(K) test					
Lag	1	2	3	4	5
Test Statistic	3.1007	14.0892	29.9518	36.0237	50.9611
p-value	0.0723	0.0009	0.0000	0.0000	0.0000
Reject H_0?	Not at 6% or less	Yes	Yes	Yes	Yes

Moreover, the performances of investors who use the minimum risk VWAP strategy are the same as that of the market, since the main idea of the minimum risk VWAP strategy is following the market trading pace in order to hide their orders and minimize the market impact. Whereas, those two implementation shortfall strategies are seen to create small excess returns over the market. Given this, the question is how the market and return will vary under different market circumstances when the institutional investors are using those two implementation shortfall strategies and if there are any advantages or disadvantages between these two strategies.

5.4 Impact on Market Quality

Liquidity, volatility, clarity, efficiency, reliability and fairness are six generally accepted measures of market quality. Considering that efficiency, clarity, reliability and fairness are qualitative measures and hard to compare, we follow [42] and focus on the study of the change of market liquidity and volatility to reveal the impacts of IS strategies on our artificial stock market.

5.4.1 Impact on Liquidity

According to [43], liquidity is the cost that a trader has to pay in order to complete his order immediately. In a more liquidized market, a trader will need to pay less for a fast

Table 5.2: Comparison Among Different Strategies

Market Daily Return Without Optimal Execution Strategies			
Mean	Standard Deviation	Skewness	Kurtosis
-8.141×10^{-5}	0.0098	-0.1927	6.0870
Profitability	Market Return	Institution Return	Noise Return
	-33.43%	-33.26%	-33.49%
Market Daily Return With Minimum Risk VWAP Strategy			
Mean	Standard Deviation	Skewness	Kurtosis
-5.2317×10^{-5}	0.0050	-0.0664	5.6874
Profitability	Market Return	Institution Return	Noise Return
	-23.01%	-22.96%	-23.05%
Market Daily Return With Linear-impact Implementation Shortfall Strategy			
Mean	Standard Deviation	Skewness	Kurtosis
-2.0734×10^{-4}	0.0078	-0.1704	5.8604
Profitability	Market Return	Institution Return	Noise Return
	-62.16%	-58.27%	-64.49%
Market Daily Return With Quadratic-impact Implementation Shortfall Strategy			
Mean	Standard Deviation	Skewness	Kurtosis
-1.1499×10^{-4}	0.0074	-0.2182	6.4555
Profitability	Market Return	Institution Return	Noise Return
	-42.77%	-36.63%	-44.95%

execution, and vice versa. In that way, liquidity has been regarded as the most important indicator of market quality. Optimal market depth (OMD) is used in [44] to measure the market liquidity.

$$OMD = \frac{Q_{call} + Q_{bid}}{2},$$

where Q_{call} is the size of sell orders at the clear price and Q_{bid} is the size of buy orders at the clear price. We also follow the study of [45] and introduce the Amihud illiquidity measure (ILQ) into our analysis.

$$ILQ_t = \frac{1}{t}\sum_{i=1}^{t} \frac{|R_i|}{Vol_t},$$

where R_i is the daily return and Vol_t is the daily trading volume of a stock. The Amihud illiquidity measure represents the average daily absolute stock return over trading volume, and it serves as a rough measure of the stock price impact.

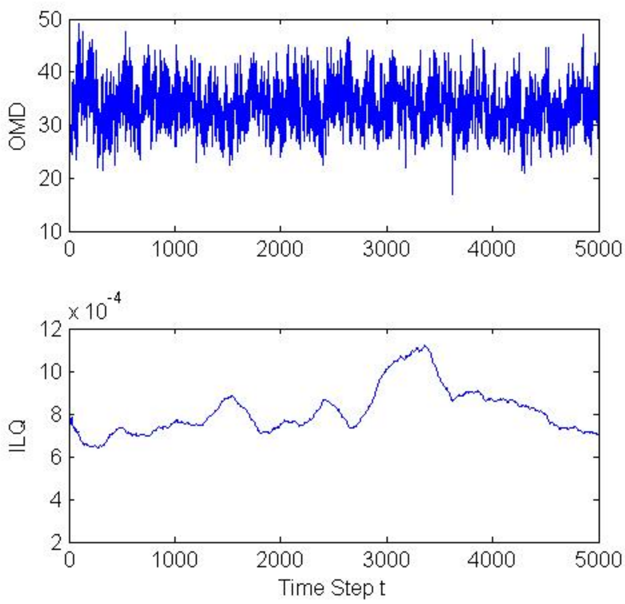

Figure 5.13: Liquidity Measure Without IS Strategy

Here, we present comparison results of the market impact of the two IS strategies on the quality of the artificial stock market. Figures 5.13, 5.14 and 5.15 present the optimal market depth and the Amihud measure in different markets. From these graphs, we notice that after carrying out the IS strategies, the curves of the Amihud illiquidity measure show a trend that shifts downwards, indicating a lower average daily return to trading volume ratio and higher liquidity. As for the optimal market depth indicator, no significant change can be observed for the reason of its high volatility. So in Table 5.3, we present the statistical features of both the optimal market depth and Amihud illiquidity measure for comparison.

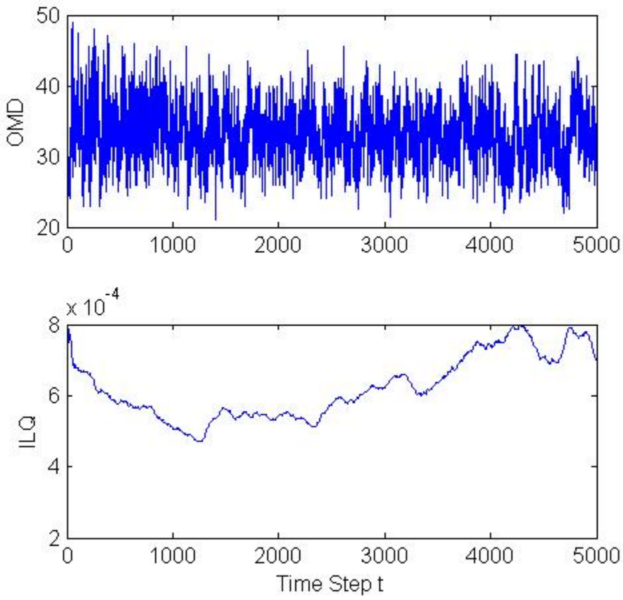

Figure 5.14: Liquidity Measure With Linear IS Strategy

Table 5.3: Market Liquidity Measures

	Optimal Market Depth			Amihud illiquidity Measure		
	Mean	StdDev	SEM	Mean	StdDev	SEM
Without Slicing	32.0520	4.0998	0.0580	7.4618e-04	5.9196e-05	1.8719e-06
Linear IS	33.2556	4.0532	0.0573	5.4262e-04	8.8816e-05	1.2560e-06
Quadratic IS	34.0563	4.0746	0.0576	3.9559e-04	8.7700e-05	1.2403e-06

Table 5.3 presents the statistical features of the corresponding indicators. It indicates that, with 10% of agents using IS strategies, the market with the quadratic-impact IS

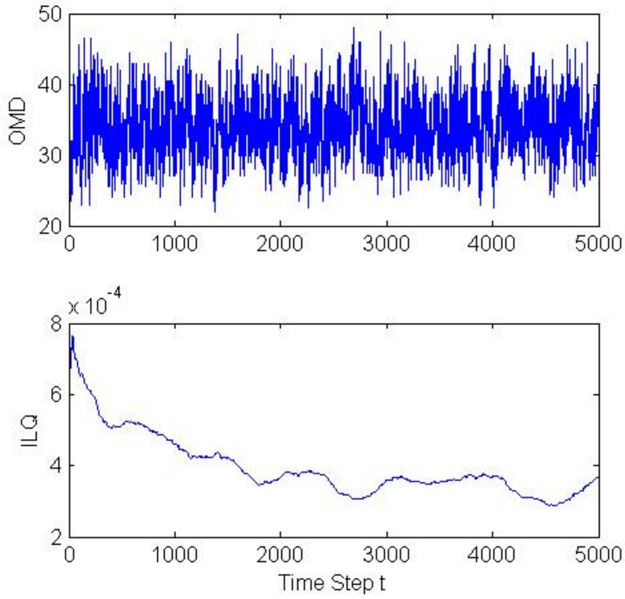

Figure 5.15: Liquidity Measure With Quadratic IS Strategy

strategy shows the highest optimal market depth and the lowest illiquidity measure. And the market with no optimal execution strategy has the lowest optimal market depth and the highest illiquidity measure. This result is consistent with our assumption since both the high optimal market depth and low illiquidity measure indicate a highly liquidated market.

We think that the main reason IS strategies improve market liquidity is that these strategies slice large orders into fragments, so that not only is the market impact diluted, but those orders also provide extra liquidity to the market when agents would otherwise not conduct any trading activities. Moreover, since the direction of the order is already determined, agents using order slicing strategies will be continuously executing a small fragment of the order in the following several trading periods, even when the market is moving against them. This feature ensures some extra liquidity when the market is booming or collapsing. And at the same time it ensures the agents to receive a preferable price when their gambles turn out to be wrong in the short term. Just as Foucault, Kadan and Kandel(2008)[46] explain, an algorithmic trading strategy profits from providing liquidity to the market. Since the order size is significantly small and trading frequency is high, traders will have a better chance in successfully executing an order and will be more likely to capture rather than pay the market spread [47].

Furthermore, we also notice that the standard deviation of the Amihud illiquidity measure of the market without the IS strategies is significantly smaller than that of the market with the two IS strategies. We think that this is due to the settings of our artificial stock market since even the institutional investors will not perform any special trading strategies within the first 30 trading periods.

5.4.2 Impact on Volatility

Apart from liquidity, volatility is also an important indicator of market quality, and it measures the risk of holding an asset. It is accepted that a lower volatility indicates better market stability. We introduce the daily volatility of return (DVR) as a measure of the market volatility:

$$DVR = StdDev(\ln P_t - \ln P_{t-1}) \tag{5.1}$$

Since the profitability of an optimal execution strategy comes from its low market impact of large orders, we can expect a lower market volatility from the market with the IS strategies. In Table 5.4 we present the statistics of market price and market daily log return as a measure of market volatility.

	Price StdDev	Daily Log Return		
		Mean	StdDev	SEM
Without Slicing	22.9961	-8.1410e-05	0.0098	1.3811e-04
Linear IS	17.7575	-2.0734e-04	0.0078	1.1880e-04
Quadratic IS	15.2661	-1.1499e-04	0.0074	1.0469e-04

Table 5.4: Market Volatility Measures

According to Table 5.4, both markets with the two IS strategies feature lower volatility of market price and daily log return, compared to the market without the optimal execution strategy. In terms of market volatility, the market with the quadratic-impact implementation shortfall again outperforms the one with the linear-impact implementation shortfall strategy.

In summary, the existence of the implementation shortfall strategy reduces volatility of both market price and daily log return, providing evidence that the market becomes more stable due to the IS strategies. Moreover, the IS strategy with a quadratic temporary impact function performs better in stabilizing the market than with a linear temporary impact function.

5.5 Impact on Performance and Trading Cost

The artificial stock market has a higher liquidity and volatility, with 10% of its agents using the implementation shortfall strategies. However, the original objective of developing these optimal execution strategies is not to improve the market quality, but to help agents to reduce the trading cost incurred in executing large orders, and to increase their profitability.

In the artificial stock market of a single stock asset with agents sharing the same pricing and timing strategies, the average buy/sell price (ABP/ASP) can be regarded as a measurement of the average trading cost. By comparing the average ratio of the buy/sell price between institutional investors who are using the IS strategies and the rest of the market (noise investors), we are able to see the impact of the IS strategies on the trading cost.

Table 5.5: Average Trade Prices and Profitabilities

	Noise Return	Institution Return	Difference
Linear IS	-0.6449	-0.5827	0.0622
Quadratic IS	-0.4495	-0.3663	0.0832

Table 5.6: Average Trading Cost Comparison

Average Buy Price	Institutional Investors			Noise Investors			$\frac{ABP_{Institution}}{ABP_{Market}}$
	Mean	StdDev	SEM	Mean	StdDev	SEM	
Linear IS	54.4100	2.0272	0.6411	53.4854	2.1045	0.2218	1.0173
Quadratic IS	41.8484	0.4674	0.1478	41.5377	0.4173	0.0440	1.0075
Average Sell Price	**Institutional Investors**			**Noise Investors**			$\frac{ASP_{Institution}}{ASP_{Market}}$
	Mean	StdDev	SEM	Mean	StdDev	SEM	
Linear IS	54.3821	1.9989	0.6321	51.2352	2.0123	0.2121	1.0614
Quadratic IS	41.8096	0.4471	0.1414	39.7226	0.4589	0.0484	1.0525

Table 5.5 presents the portfolio return of the institutional investors and the noise investors. The results shown in Table 5.6 indicate that institutional investors who use the linear and quadratic temporary impact IS strategies have higher average ratios. Both the

linear and quadratic temporary impact IS strategies endow investors with excess returns of around 7% above noise investors. Most of the excess profits come from a relatively higher average selling price. In both cases, the average selling price of the institutional investors exceeds those of the noise investors by around 5% to 6% while the average buying price is much closer. The higher average sell price suggests that the implementation shortfall strategy enables investors to capture a better sell price, even in a bear market where the market value shrinks by more than 40%.

However, there is no significant difference between the performances of the linear temporary impact IS strategy and that of quadratic temporary impact IS strategy.

5.6 Competitive Analysis with Variant Market Parameters

5.6.1 Investors' Composition

Under the condition that 10% of the agents in the artificial stock market are institutional investors, the agents trading with the quadratic temporary impact implementation shortfall strategy contribute more to the improvement of market quality. In this subsection, we will further study the market reaction when the market composition changes.

The deterministic factor that affects the profitability of a trading algorithm is the composition of the market. Special and effective algorithms usually come from specific designs of market composition, and the impact of an algorithm upon different markets may also vary. The market generally consists of two basic factors, the population of participants and the choice of algorithm.

Table 5.7: Market Quality Indicators

	Linear IS				Quadratic IS			
	OMD	ILQ	StdPrice	StdRet	OMD	ILQ	StdPrice	StdReturn
5%	33.6795	5.3050e-04	16.3953	0.0083	34.7375	3.3920e-04	8.6718	0.0079
10%	33.2556	5.4262e-04	17.7575	0.0078	34.0563	3.9559e-04	15.2661	0.0074
15%	34.4665	6.9537e-04	14.5339	0.0074	33.4780	4.1415e-04	12.9438	0.0090
30%	33.7595	8.4370e-04	18.1877	0.0099	34.8605	4.9368e-04	9.9963	0.0097

As mentioned earlier, the artificial stock market consists of the noise investors and a certain number of institutional investors who are capable of using the optimal execution

Table 5.8: Portfolio Return

Linear Temporary Impact Implementation Shortfall							
	Institutional Investors			Noise Traders			R_{excess}
	AveBuy	AveSell	Return	AveBuy	AveSell	Return	
5%	68.1343	68.1715	-0.2186	67.9113	66.4779	-0.2697	0.0511
10%	54.4100	54.3821	-0.5827	53.4854	51.2352	-0.6449	0.0622
15%	72.9946	73.2216	-0.1252	69.9244	66.8318	-0.1917	0.0665
30%	110.3743	111.5039	0.3288	110.3744	105.5913	0.2332	0.0956
Quadratic Temporary Impact Implementation Shortfall							
Percentage	Institutional Investors			Noise Traders			R_{Excess}
	AveBuy	AveSell	Return	AveBuy	AveSell	Return	
5%	105.2643	105.2981	0.2020	103.5146	101.8151	0.0892	0.1128
10%	41.8484	41.8096	-0.3663	41.5377	39.7226	-0.4495	0.0832
15%	148.7310	149.1622	0.3935	144.2607	137.8926	0.3239	0.0696
30%	117.4134	118.7752	0.2417	116.1921	111.3274	0.1335	0.1082

strategies. When the number of institutional investors changes, the profitability of agents moves in two opposite directions: with a higher percentage of homogeneous competitors, the institutional investors have more advantages in betting on the market direction; on the other hand, since the potential profit is limited in such a closed market, a higher number of skilled investors are likely to dilute the limited excess return. We are curious about which force will dominate when the number of institutional investors changes, and we will conduct a series of simulations to examine this.

Table 5.7 presents the market quality indicators of both markets where agents are using the linear or quadratic temporary impact IS strategies. Table 5.8 compares the profitability indicators of two kinds of agents.

Firstly, the optimal market depth and the Amihud illiquidity measure in Table 5.7 indicate slightly different results in terms of market liquidity. The Amihud illiquidity measure becomes larger as the percentage of the institutional investors grows, while the optimal market depth indicator barely changes. Considering the Amihud illiquidity measure is much more sensitive in measuring market liquidity, we believe that a growing number of institutional investors will slightly decrease the market liquidity. Moreover, the volatility of market price and daily return data remains at a low level, without displaying an obvious pattern. It is rigorous to conclude that, the market will always benefit from the IS strategies in terms of market volatility, with the population of institutional investors no more than 30%.

Secondly, Table 5.8 shows that, no matter what the population of the institutional investors is, and which direction the market price goes, those institutional investors will always earn higher excess returns then the noise traders. Moreover, a relatively higher excess return can be observed in both cases when the proportion of institutional investors reaches 30%. This indicates that, when the number of homogeneous investor is large enough, the benefit of information sharing will over-whelm the profit-diluting effect.

5.6.2 Private Information

Apart from the composition of investors, the effect of private information on market quality and institutional investors' profitability are also of great interests. Market information can be divided into public information and private information. Public information is available to every participant in the market, although sometimes investors will not react rationally. Private information is the resource available only to a small number of investors. According to the DHS model in [48], investors who possess private information will always act over-confidently and sometimes irrationally. The DHS model indicates that, investors may possess private information through various channels and they are most likely to be over-confident about their information. If the information turns out to be true and the market reacts rationally, they are able to make a satisfying profit and act even more confidently in the future.

In the structure of our artificial stock market, we set private information as a variable. At the beginning of every simulation period, small portions of investors would form sub-groups with small probability P_c and share their private information, some of which might be material but some merely rumours. They will act overconfidently according to their information, betting most of their fortune on it. We are interested in how the market quality and institutional investors' profitability reacts in response to the change of the amount of private information available in the market. In the previous sections, we performed our simulations with $P_c = 0.5\%$. In this section, we will conduct a further test with P_c varying from 0.05% to 1%.

Similarly, Table 5.9 lists the market statistics for both cases. No clear trend can be observed when the private information parameter is changed. The market quality and volatility indicators are both stable, thus we are able to conclude that the market liquidity and volatility are not sensitive to the private information parameter. As for the investors' performance indicators, there is no clear trend to be seen either. The only thing that needs to be mentioned is that, in both cases, the excess return of the institutional investors drops rapidly when the private information parameter is set at $P_c = 1\%$. This is an interesting

	Linear IS				Quadratic IS			
	OMD	ILQ	StdPrice	StdRet	OMD	ILQ	StdPrice	StdReturn
0.05%	34.0140	6.5651e-04	11.4723	0.0093	34.5120	3.5324e-04	16.5700	0.0085
0.1%	33.9945	6.0564e-04	14.2208	0.0093	33.4780	4.1415e-04	12.9438	0.0090
0.5%	33.2556	5.4262e-04	17.7575	0.0078	34.0563	3.9559e-04	15.2661	0.0074
1%	34.6165	6.2698e-04	15.9663	0.0072	33.3435	4.3024e-04	17.0546	0.0084

Table 5.9: Market Quality Indicators

Linear Temporary Impact Implementation Shortfall							
	Institutional Investors			Noise Traders			R_{excess}
	AveBuy	AveSell	Return	AveBuy	AveSell	Return	
0.05%	102.9471	102.9886	1.0213	102.5951	97.8924	0.9246	0.0967
0.1%	78.9216	78.9653	0.7857	78.0414	74.4056	0.6808	0.1049
0.5%	54.4100	54.3821	-0.5827	53.4854	51.2352	-0.6449	0.0622
1%	98.1567	98.1946	1.0764	97.8375	94.0425	1.0554	0.0210
Quadratic Temporary Impact Implementation Shortfall							
Percentage	Institutional Investors			Noise Traders			R_{excess}
	AveBuy	AveSell	Return	AveBuy	AveSell	Return	
0.05%	96.7649	100.0015	1.1241	99.9778	92.7263	1.0855	0.0586
0.1%	128.7462	133.2054	1.1075	133.1451	123.0873	1.0814	0.0762
0.5%	41.8484	41.8096	-0.3663	41.5377	39.7226	-0.4495	0.0832
1%	117.2791	117.2331	1.1507	111.9456	106.5438	1.1033	0.0273

Table 5.10: Portfolio Return

phenomenon as we expected that the excess return would remain at the same level as P_c varies. Two possible explanations can be made: 1) with greater availability of private information, the participants are more likely to place large orders, which will increase the market liquidity to a high level, where institutional investors can no longer profit from providing extra liquidity; 2) with a greater availability of private information, the participants are more likely to share common opinions and will weaken the advantage of the implementation shortfall strategies.

The first explanation, while intuitively appealing, is incorrect, and can be proven to be false by examining the illiquidity measure. The insignificant increase in market illiquidity measure indicates that, even with a higher number of orders, the effective liquidity does not enjoy any notable improvement. Here, we are able to conclude that, the decrease in performance could be partially ascribed to the fact that the institutional investors have no priority in possessing private information in this setting. As a result, the advantage of the implementation shortfall strategies is weakened, when the private information parameter increases.

Chapter 6

Conclusions and Further Researches

6.1 Conclusions

It is of benefit to examining the financial market in a broader perspective and to the test of optimal trading strategies under the framework of computational finance. Instead of simply analysing the real market data, we constructed an artificial stock market in this thesis and provided an application for the agent-based computational finance theories.

Based on the structural setting of the artificial markets, we tested the minimum risk VWAP and the IS strategies with linear temporary impact. Moreover, we replaced the linear temporary impact function of IS strategy with a quadratic one, and the new model turned out to be able to capture the complexity of the financial market better despite the fact that it has an intractable analytical solution. In this thesis, we introduced the genetic algorithm to solve the corresponding optimization problem numerically.

The statistical analysis of our artificial stock market reinforced some well-established patterns of financial asset returns which are reported for the US stock market, such as fat-tail and volatility clustering effects. Agents with the minimum risk VWAP strategy are shown to succeed by following the market and in slicing/hiding their orders. When 10% of the agents choose to use the minimum risk VWAP strategy, the market shows a significantly lower volatility than before. This indicates that agents with the minimum risk VWAP strategy are able to provide extra liquidity while executing their orders. However, these agents have no advantages over the rest of the market in terms of profitability. No notable excess return can be observed from the 20-year-long market simulation.

Moreover, IS strategies have also made their contributions to the improvement of market volatility, although they are not as good as the minimum risk VWAP. The slightly higher optimal market depth indicator and significantly lower Amihud illiquidity measure both imply higher market liquidity. The significantly lower standard deviation of market return indicates an improvement on market volatility. Furthermore, the market with the quadratic implementation shortfall strategy shows a better market liquidity than the market with the linear strategy.

Our study on two IS strategies indicates that, both of these two strategies are able to produce more than 7% excess return over the 20-year simulation period under most circumstances. With the number of homogeneous investors increasing from 5 to 30, no clear trend is observed in terms of the market quality. However, a relatively higher excess return is captured by both strategies as the number of competitors reaches 30. This indicates that when the number of homogeneous investors is large enough, a common opinion will be more likely to be formed and this leads to an advantage in their trading activities.

As the private information availability increases from 0.05% to 1%, the quadratic-impact IS strategy shows an advantage according to the market illiquidity measure. This can be seen as evidence of improvement in market liquidity. As for the performances, a significantly low excess return is observed in both cases when the private information parameter reaches 1%. This could be partially ascribed to the fact that the institutional investors have no priority in possessing private information in this market. As a result, the advantage of the optimal execution strategies is weakened as the private information parameter increases.

This novel approach in the optimal execution and test on trading algorithms can be appealing for institutional traders whose order size might be large relative to the market trading volume.

6.2 Future Research

The main purpose of this thesis is to study the optimal execution strategies, and to illustrate how to apply a new approach in empirical testing of trading strategies. In this thesis, we gave a brief overview of this new research approach on optimal execution strategies. The accuracy and efficiency of this approach depend greatly on the structure of the artificial stock market and how well the market is able to mimic the real stock market.

The strongest assumption of our artificial stock market is the single-asset market. Agents in our stock market will not face any stock selection, timing or risk management mechanisms. Although it is still sufficient for the purpose of testing our strategies, it might turn out to be ineffective in testing more complicated strategies, since our optimal execution strategies are mostly based on measuring and minimizing the market impact. Therefore further research is needed to introduce more security into the market, and design better schemes for stock selection, timing and explicit risk management strategies for agents.

The second possible improvement in the artificial stock market structure lies in the agents' learning and evolving mechanism. In fact, during the research, we tried to implement the genetic algorithm into the agents' learning mechanism, but it did not work well. By adding new learning mechanisms and social network structures, such as the genetic algorithm and the neural network algorithm, the simulated artificial stock market will probably perform better in mimicking the real financial market.

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