Graph-Based Model For Distribution Systems: Application To Planning Problem

by

Haytham Labrini

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Master of Applied Science
in
Electrical and Computer Engineering

Waterloo, Ontario, Canada, 2015

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

Distribution system engineers analyze distribution systems and operate them to minimize the costs of delivery of power while satisfying customers and imposed constraints such as voltage limits, congestion, system losses, substation/transformers operational loading limits, budget and such. It is hence a relatively complex and reasonably challenging task.

Researchers approached the problems that arise in the distribution system such as loss reduction and congestion mitigation using different methods. The most accurate one to find the optimal solution for any problem is the extensive search (ES). This search evaluates each and every possibility and chooses the best option or options depending on the objective of the study. The only drawback of this method is the very large search space that makes it inefficient, especially for operational and on-line applications.

In order to decide on the feasibility of the solution, an evaluation function is chosen to discriminate between the different solutions. The power flow (PF) and optimal power flow (OPF) are the most widely used in literature; they describe the distribution system using the exact formulas, making them very accurate but time-expensive. PF and OPF are perfect for longterm planning as there is no time constraints, and to some extent to operational planning. It is, however, very difficult to apply them to on-line/abnormal application such as restoration and reconfiguration as time is a critical component. A second problem is that most of these approaches do not take full advantage of the structure of the distribution system which changes with time.

This thesis proposes dynamic graph-based method as an approach and applies it to a distribution planning problem. The main objective is the cost of the upgrades of the different DG units and line reinforcement to demonstrate the effectiveness of the method. The results show that taking account of the future changes in the system improves the benefits from the various installations. This approach has the potential to be extended to other problems distribution network may face.

Acknowledgments

I would like to express my gratitude Dr. Elshatshat and Prof. Salama for their supervision, belief and engagement through the learning process of this master thesis. FurthermoreI would like to thank Dr. Gad for his help during the writing process.I would like to thank all my family, friends and loved ones, who have supported me throughout the entire process.

Dedication

This is dedicated to the one I love.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Preamble

Today, reliable energy supply is one of the biggest concerns in many countries around the world. Up to now, electrical energy has mainly been generated by centralized large scale power plants, either nuclear or fossil fuel-based plants. However, due to the depleting of oil reserves, increase in oil price, uncertainties related to political issues in oil producing countries, combined with environmental concerns related to CO2 emissions, several countries are encouraging the use and investment in Distributed Generation (DG). DG is defined under IEEE Std 1547.3-2007 as an electric generation facility connected to an electric power system (EPS) through a point of common coupling (PCC); and is a subset of Distributed Resources (DR) . DR are resources of electric power that are not directly connected to bulk power transmission systems and include both generators and energy storage facilities. DG is also known in the literature as any generation unit with a maximum capacity of 5 MW to 10 MW [1] , and they are usually connected to the distribution network.

Their installation in distribution networks may have either positive or negative impact on distribution networks operation depending on the size, the site and the type of the DG units. DG units are usually categorized into 2 types: Combined Heat and Power (CHP) and Renewable Energy Sources (RES). CHP, or what is known as cogeneration, includes power plants where electricity is the primary product and heat is a byproduct or the other way around. Reciprocating engines, Micro-turbines and Fuel cells are some examples that can be used as CHP plants. On the other hand, RES refers to distributed generation that makes use of natural energy resources such as the irradiation of the sun and the wind energy to generate power. The main DG technologies are wind turbines, photovoltaic arrays, solar thermal power, and geothermal power. Out of the above, wind turbines and photovoltaic panels are the most commonly used technology in industry. The main difference between them and CHP is the intermittency of their output which can be forecasted but is not dispatchable unless accompanied with storage devices or employed more complex designs and controls.

The main objective of distribution systems operators is to improve the performance of the system at a minimal cost by, for example, minimizing the losses and improving the voltage profile. These operations are traditionally carried out at a centralized level. However, with the introduction of communication networks and power electronics, the grid is becoming smart. This shift makes the grid more robust and responsive to changes while delivering high quality of power. New distributed algorithms need to be developed in order to manage the grid operations. DG units and storage devices play a major role in the development of smart grid as they provide local generation and control. They may have many advantages including loss reduction, voltage profile improvement, congestion relief and operation cost reduction.

The planning and allocation of the DG units is another important step in the improvement of the distribution network performance. Poor allocation of DG units in the system can have negative effects on the network such as line-congestion, voltage problems and may also result in an increase the line losses. This may happen in the case of demand growth; allocating a DG unit upstream the congested line may provide local power but will not mitigate the congestion in that line. The voltage may also rise if the DG is too large compared to the local power consumption.

1.2 Motivation

The operations and planning activities have been addressed by many researchers and have been formulated as a min/max optimization problem. This problem seeks to optimize a given cost, planning, or reliability objective while meeting some engineering and technical constraints such as bus voltages and equipment limits. Different techniques have been proposed depending on the complexity of the problem. There are mainly two classes that are used: Analytical approaches - Linear Programming(LP) , Non-Linear Programming (NLP) and Mixed Integer Non-Linear Programming (MINLP)- and meta-heuristic approaches (Genetic algorithm, Simulated annealing and Particle swarm optimization). Many of these techniques are very efficient in the planning stage since there is no time limit, they are, however, very time-consuming specifically for online applications and do not cope with the dynamicity of the system topology which is the main feature of smart grids. Therefore,it seems there is a need for an alternative method that address the above concerns while suitable for smart grid paradigm.

This thesis proposes a method that is based on Graph Theory to manage and control the operation and planning of a particular distribution network. Simple, yet, efficient to manage and control the grid operations, it takes into account the dynamicity of the network topology and the above mentioned constraints. As a case study, this thesis focuses on developing a framework to analyze a long term planning problem and can be easily extended to operational and online applications. DG integration and line reinforcement are some options that might be considered to meet the load growth in the distribution system planning horizon. The objective is to minimize the investment cost while meeting all technical constraints such as power balance, voltage limits and line capacity limits. This method outputs a set of solutions (size, site and time of integration) for DG units and lines depending on the needs of the operator.

1.3 Objective

This thesis introduces an efficient technique that is based on graph theory to manage and control the planning of a particular distribution network. To show its practicality, it is used to allocate DG units in a long term plan so as to improve the overall quality of the distribution network performance. The following is a list of objective centered on the use of graph-theory to solve this problem.

- Presenting graph theory and the different methods that can be applied to study, analyze and optimize the distribution networks. The different components of the distribution network such as lines and DG units are described in the graph context.
- Introducing an algorithm for DG allocation for Loss minimization. It takes the graph as input and places the DG accordingly. This DG unit is supplying with active power to support the grid. The rule is then used further for DG planning.
- Constructing a dynamic graph to represent the distribution network throughout the different states (years) to be used for optimization purposes. The Successive Shortest Path (SSP) method is used to find the best combination of DG units and lines to meet the demand and all different constraints that need to be respected.
- Simulating these methods to evaluate their efficiency on the IEEE 69 bus test system. Different scenarios are discussed while several options are provided to the operator depending on the needs.

1.4 Thesis outline

This thesis consists of 7 chapters. Following this introductory chapter, Chapter 2 presents a literature review on DG allocation and distribution networks planning. Chapter 3 provides an introduction to graph theory concepts used in this thesis. Chapter 4 constructs a method for sitting and sizing DG sources to minimize losses and relieve congestion. Chapter 5 builds up on the Chapter 4 an algorithm to mitigate congestion and extend it to multi-year study. In chapter 6, this method is evaluated through the simulation of the IEEE 69 bus test system. Conclusion and possible future work will be presented in chapter 7.

Chapter 2

Literature review on DS planning

2.1 Preamble

DG units are small DG sources connected to the distribution network to provide power directly to the consumer. They hence have many advantages compared to traditional sources. They reduce the transmission losses and improve the voltage profile, provide local loads with power, and reduce possible congestion in the system. It is, hence, very important to allocate them properly and operate them as to take full benefits of them [2]. Several methods have been developed to solve the following distribution network problems such as network reconfiguration, optimal operations, optimal planning and expansion, power restoration plans, etc.

Distribution networks are different from transmission networks; they exhibit particular characteristics and challenges that dictate the objective, constraints and ultimately the optimization method. Many methods have been developed to solve these problems; the following section discusses these methods and presents the relevant literature.

2.2 Distribution network planning with DG units

DG planning as a part of distribution system planning has been formulated as an optimization problem in which the DG size and site are determined to achieve a set of objectives and subject to a set of constraints, as installing DG units, became the most common way to improve the quality of distribution networks in the literature. The main objectives for

DG planning are voltage profile improvement and loss and cost minimization [3]. Different formulations and methods have been presented.

Several studies incorporate both distribution system planning and DG planning problems. The main goal is to improve the behavior of a system by introducing components that allow it to comply with present and future requirements. The objective is usually to minimize the cost associated with the installation of new components to the system without violating all the technical and engineering constraints such as the voltage and line capacity limits. The solution of this problem usually involves identifying the site, size and year of allocation of each component (lines, storage and DG units as examples). The following is a survey of some of the literature that addresses this problem.

In [4], a long-term optimization approach for distribution network is presented; it allows substation, feeder, and DG upgrades while also considering line limits, technology limitations, varying energy prices, and environmental limits. Paper [5] presents a reliability model to determine the optimal DG locations and sizes. The paper concludes that DG may become a cost effective solution to long-term planning problems, with the right allocation and capital deferral credit. Ref [6] describes a combination of the steepest descent and the simulated annealing approaches for radial distribution network planning. Capital recovery, energy loss and undelivered energy costs are all taken into account in this formulation.

The work in [7] proposes a multi-objective model for DG allocation under load uncertainty. This approach minimizes the economic cost, technical risk and economic risks. The output is a set of pareto-optimal solutions that planners need to choose from. In [8], GA is used to minimize the costs of network expansion, power losses, and served and unserved energy. The solution is the best compromise solution that satisfies the planners requirements. In [9], El-Khattam et al. presents a heuristic approach to DG investment planning from the perspective of the Local Distribution Company (LDC). The benefit-to-cost ratio is used to privilege some DG units over others. While investment and operating costs, energy import costs, unserved power costs, and losses are all included in the objective function, the model does not incorporate options other than DG units nor does provide planning over time. proposes a similar approach that includes substation and line upgrades using binary variables.

While the previous models have been proposed, with particular emphasis on DG placement and sizing, Wong et al. [10] proposed a distribution system planning model that is suitable for examining the impact of regulatory policies on DG unit investments. By examining these investments, it is possible to determine the effects of the policies on long-run energy dispatch and purchases; thus predict the role the policies play on distribution system economics and environmental emissions. A method is presented in [11] for coordinating the approval process of DG proposals submitted by multiple, competing, and private investors to achieve maximum investor participation while complying with the technical operational limits of the local distribution company. The proposed model utilizes a feedback mechanism between the LDC and Private investors to maximize their participation and the penetration of DG-units into the distribution system.

Investment and operation costs have been addressed in [12]. In [13], the deferment of grid reinforcement is studied upon using DG units. Reliability enhancement is considered as the main objective in [14] whereas [15] discusses the tradeoff between the conflicting interests of the utility and DG owners. Ref [16] uses a market based approach to maximize the social welfare and DG owner benefits. Losses are taken as an objective function in [17, 18]. In [19], the authors tackle losses and investment cost concurrently by providing the operator with optimal pareto solutions to choose from.

In Ref [20], the author proposes an analytical expression to calculate the optimal DG size and an effective methodology based on the exact loss formula to identify the corresponding optimum location for DG placement for minimizing the total power losses in primary distribution systems, whereas [21] presents and evaluates another analytical method to determine the optimal placement and sizing of DG suitable for radial systems. Power flow is used to calculate the losses without having to calculate the flow after each DG siting and sizing. Ref [22] presents an analytical approach to determine the optimal DG location in both radial and networked systems that will minimize power losses using a non-iterative algorithm; acceptable results are obtained compared to the exhaustive search method even though some constraints have been neglected. Ref [23] presents another analytical methods to size and site DG units for loss minimization. Given that the DG source can generate active and reactive power, the power factor of DG units is set closer to the power factor of combined load in the respective system.

The second set of methods is based on meta-heuristic methods such as the GA and the PSO method. In [24], a multi-objective optimization approach based on GA for optimal allocation of different types of DG units in the distribution network has been presented. It maximizes the savings in the system-upgrades investment deferral, cost of annual energy losses, and cost of interruption; all objective functions are converted into cost. The proposed method generates probabilistic and Monte Carlo Simulation models for the combined generation-load included so as to account for all possible operating conditions. [25] proposes an approach to maximize the system loading margin as well as the profit of the distribution companies over the planning period, while taking into account the system constraints. The objective functions are fuzzied into a single multi-objective function, and subsequently solved using GA. The fuzzy controller is used to dynamically adjust the crossover and mutation rates to maintain the proper population diversity (PD) during GA operation, which effectively overcomes the premature convergence problem of the Simple Genetic Algorithm (SGA).

2.3 Application of Graph Theory (GT) in Distribution Systems

Distribution networks can also be approached from their topology using graph theory. A graph is a mathematical structure used to model pairwise relations between objects from certain collections. It is typically formed of vertices or nodes connected by edges or arcs. In the distribution network context, the nodes represent the buses while the arcs represent the branches. Graph theory, hence, has been employed in other fields to analyze the flow of commodities, relying on the structure and characteristics of the graph. GT methods solve different types of problems; the main ones are the shortest path problem, the max flow/mincut problem and the minimum cost flow.

Graph theory has slowly been introduced to the power system field. However, it has been mainly used to support evolutionary optimization methods by providing a means to validate certain architecture constraints, mainly to validate the radiality of a distribution network as in [26]. The Scaling Push-Relabel algorithm [27] has been used to solve the problem of congestion mitigation. Ref [28] used the k-shortest path as a mechanism to restore power in a grid after a blackout, whereas [29] used graph theory to design a communication network to self-organize photovoltaic generators in a distribution network using a distrusted control.

Graph theory has many methods that can be applied to power systems to solve many of the issues operators face. It is even more urgent to implement them as the grid is evolving toward the smart grid, which features communication networks and a high penetration level of DG units. This shift will require fast response times and efficient algorithms t control all these devices in the system while focusing on the topology as it has a large impact on the operations and performances of the distribution networks. All new networks need also to be designed from that perspective.

Chapter 3

Graph Theory

3.1 Graph Theory Definition

Graph theory is defined as the study of graphs. A graph is a mathematical structure used to model pairwise relations between objects from a certain collection. In this context, the graph is formed of vertices or nodes connected by edges or arcs. It may have different configurations: directed and undirected. Directed graphs are formed of edges allowing the flow only in one direction, while the undirected graphs do not have any restriction on the flow direction. Figure 3.1 shows an example of undirected graph.

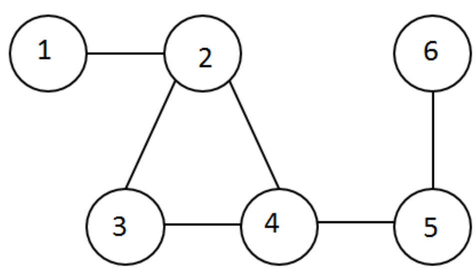


Figure 3.1: Undirected Graph

This graph is formed of 6 nodes and 7 arcs. It is undirected which means that the information can leave Node 1 to Node 5, or Node 5 to Node 1. In the case of a directed

graph, an arrow shows the direction of the edge. As an example, the flow from Node 1 to Node 5 can be allowed while Node 5 to Node 1 may not be allowed. This will have significant behavioral constraints on the flows.
In this thesis, the studied graphs are directed graphs (V;E) where V is an n-set of nodes and E is an m-set of directed arcs. A distribution network can be seen as a directed graph from the substation/sources to the loads. The following are the definition of different terms used in graph theory.

- **Length:** The shortest path and minimum mean cycle problems use a length function $l : E \rightarrow R$. The length $l(v, w)$ is the distance from node v to node w. We denote the length of a cycle (path) Gama by $l(\gamma) = \sum l(e)$.
- **Costs:** The minimum cost flow problem uses a cost function $c : E- > R$. The cost $c(v, w)$ is the unit shipping cost for arc (v, w). We denote the cost of a cycle γ by $c(\gamma) = \sum c(e)$.
- **Capacities:** The maximum flow, minimum cost flow, and generalized maximum flow problem uses a capacity function$u : E \rightarrow R^+$. The capacity $u(v, w)$ limits the amount of flow we are permitted to send into arc (v, w).
- **Symmetry:** For the maximum flow and minimum cost flow problems, we assume the input network is symmetric, i.e the arc (v, w) in E then the arc (w, v) also in E. This is without loss of generality, since the opposite arc can be added with zero capacity. And without loss of generality, the costs are assumed antisymmetric, i.e $c(v, w) = -c(w, v)$ for every arc (v, w) in E.
- **Pseudoflow:** A pseudo flow $f : E \rightarrow R$ is a function that satisfies the capacity constraints and the anti-symmetry cost constraints.
- **Mass Balance Constraints:** At each node, the sum of the coming flow and sent flow is equal to the supply/demand of the node. It is the same as in power balance at each node of the network.
- **Path:** a path is a sequence of vertices $\{v_1, v_2, ..., v_k\}$ from a vertex v_1 to another vertex v_k. We say that a cycle exist in a graph when there exist a path between a node and itself.

Figure 3.1 shows a simple undirected graph with cycles.

Different problems have been defined in graph theory. The following is a list of the problems that graph theory aims at solving.

- **Shortest Path Problem:** the goal is to find a simple path between two different nodes, so as to minimize the total length. An instance of the shortest path problem is a network $G = (V, E, s, t, l)$, where s in V is called a source, t in V is called a sink and l is a length function. Number of polynomial-time algorithms for the problem exists such as Bellman-Ford and Dijkstra, each one is designed for a specific type of networks.

- **Maximum Flow Problem:** The goal is to send as much flow as possible between two nodes, subject to arc capacity limits. An instance of the maximum power flow problem is a network $G = (V, E, s, t, u)$ where s in V is the source, t in V is called the sink, and u a capacity function. A flow is a pseudo flow that satisfies the flow conservation constraints. The objective is hence to find the flow of maximum value; that will use all the capacities allowed in the different branches until we can no more transmit and get congestion in several lines.

- **Minimum Cut Problem:** This problem is intimately related to the Maximum Flow Problem. The input is the same, the goal however differs: it aims at finding a partition of nodes that separates the source from the sink, so that the total capacity of arcs going from the source to the sink is minimum. The relation between this problem and the former one is that if the arcs are found, the maximum flow will be the sum of their capacities.

- **Minimum Cost Flow Problem:** The goal is to send flow from supply nodes to demand nodes as cheaply as possible, subject to capacity constraints. An instance of the minimum cost flow problem is a network $G = (V, E, b, u, c)$, where b is a supply function, u a capacity function and c a cost function. v has a supply if $b(v) > 0$ and demand if $b(v) < 0$. The total supply is supposed to equal the total demand, the sum of the supply is equal to the sum of demands. A flow is a pseudoflow if it satisfies the mass balance constraints.

3.2 Matrix Representation of Graphs

Graphs can be directed or undirected graphs. As defined before, a graph G(V,E) is composed of pairs of nodes connected by an edge. If the edge has no defined direction, then the graph is undirected, otherwise it is directed. Let e=(u,v) an edge in E; if G is undirected then the edge e'=(v,u) also exist in E, otherwise it does not. Figure 3.2 shows a directed graph and undirected graph. An edge in a graph has many characteristics such as upper and lower limit. These limits can be used to transform a graph from an undirected to a directed graph by stating to each direction a limit. However, if the edges carry no information, then directed graph can be constructed by creating edges in both directions instead of the undirected edge.

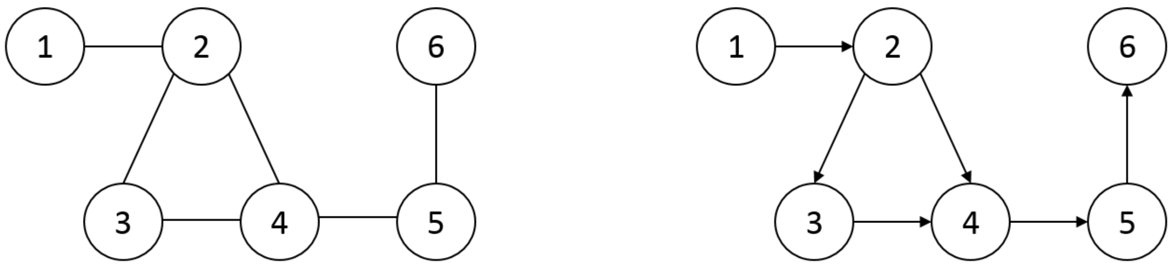


Figure 3.2: a)Undirected graph and b) directed graph

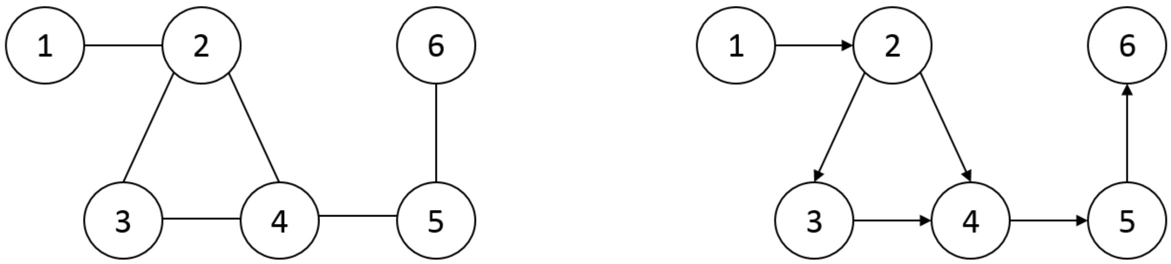
3.2.1 Adjacency matrix

The adjacency matrix of the graph $G = (V, E)$ is an $n \times n$ matrix $D = (d_{ij})$, where n is the number of vertices in G, $V = \{v_1, \cdots, v_n\}$ and d_{ij} = number of edges between v_i and v_j . when $d_{ij} = 0$, (v_i, v_j) is not an edge in G. The matrix D of an undirected graph is symmetric, i.e. $D^T = D$.

In the case of a directed graph, the same definition remains. The only difference is that the matrix D is no more symmetric but depends on the edges direction. Figure 3.3 shows two graphs, one directed and the other one undirected and their matrix representations.

3.2.2 All-vertex incidence matrix

The all-vertex incidence matrix of an undirected graph $G(V, E)$ is an $m \times n$ matrix $\underline{A} = (a_{ij})$, where n is the number of vertices in G, m is the number of edges in G and a_{ij} is equal to 1 if v_i is an end vertex of e_j and 0 otherwise.
The all-vertex incidence matrix of an directed graph $G(V, E)$ is an $m \times n$ matrix $\underline{A} = (a_{ij})$, where n is the number of vertices in G, m is the number of edges in G and a_{ij} is equal to 1 if v_i is an initial vertex of e_j, a_{ij} is equal to -1 if v_i is a terminal vertex of e_j and 0 otherwise.



$$\mathbf{Adj_{Undirected}} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \qquad \mathbf{Adj_{Directed}} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure 3.3: a)Undirected graph and adjacency representation and b) directed graph and adjacency representation

The following matrix shows the all-vertex incidence matrix of the directed graph used in 3.2.

$$\mathbf{E} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

3.2.3 Cut Matrix

A cut is basically the set of edges connected to a certain vertex. Hence if all the cuts of an undirected graph $G = (V, E)$ are $I_1, \cdots, I_t$, then the cut matrix of G is a $n \times m$ matrix $Q = (q_{ij})$, where m is the number of edges in G and $q_{ij} = 1$ if $e_j \in I_i$ and 0 otherwise.

In a directed graph, the orientation counts and hence, there will be a difference between an edge going out of the vertice and coming to vertice. The incoming and outgoing edges are interpreted in a cut as $+1$ and -1 repectively.

The following matrix shows the all-vertex incidence matrix of the directed graph used in 3.2.

$$\mathbf{V} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

3.2.4 Circuit Matrix

As definined before, a circuit is a closed walk with no repetitions of vertices or edges allowed, other than the repetition of the starting and ending vertex. It follows that if $G = (V, E)$ is an undirected graph with l circuits $C_1, \cdots, C_l$, the circuit matrix is an $l \times m$ matrix $\underset{\sim}{B} = (b_{ij})$ where $b_{ij} = 1$ if e_j is in the circuit C_i, and 0 otherwise.

In the case of a directed graph $G(V, E)$ that contains circuits $C_1, \cdots, C_l$, circuits are givent an arbitrary direction. Once the orientation for each circuit is chosen, the circuit matrix $\underset{\sim}{B} = (b_{ij})$ is defined as follows. $b_{ij} = 1$ if the edge e_j is in the circuit C_i and same direction, $b_{ij} = -1$ if the edge e_j is in the circuit C_i and opposite direction, and 0 otherwise.

In the directed graph shown in 3.2, there exists only one circuit composed of the edges $/(2, 3), (3, 4), (4, 2)/$. The following matrix shows the all-vertex incidence matrix of this graph.

$$\mathbf{C} = \begin{pmatrix} 0 & 1 & -1 & 1 & 0 & 0 \end{pmatrix}$$

3.3 Electrical Circuits and Solving Methods

Since the problem is related to electrical circuit, studying stationary linear networks is important. Stationary Linear network is a directed graph G that satisfies the following conditions:

1. G is connected, i.e all vertices are connected.
2. every arc of G belongs to some circuit.
3. every edge e_j in G is associated with a number i_j called the $through - quantity$ or $flow$. If there are m edges in G, then the through-vector is :

$$\mathbf{i} = \begin{pmatrix} i_1 \\ \vdots \\ i_m \end{pmatrix}$$

4. Every vertex v_i in G is associated with a p_i called potential. The potential difference of the arc $e_j = (v_{i1}, v_{i2})$ is

$$u_j = p_{i2} - p_{i1}$$

   If there are n vertices and m edges in G, then potential vector and accross vector are respectively

$$\mathbf{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_n \end{pmatrix} \text{ and } \mathbf{u} = \begin{pmatrix} u_1 \\ \vdots \\ u_m \end{pmatrix}$$

5. Every edge e_j represents one of the following:
   (a) component which represents an impedance r_j such as $u_j = i_j r_j$.
   (b) a through-source, for which the through quantity i_j is fixed.
   (c) an accross-source, for which the accross-quantity u_j is fixed.
6. The sum of the through-quantities of an oriented cut of G is zero when the cut is interpreted as an edge set and the sign of a through-quantity is changed if the directions of a cut and an edge are different: (Kirchhoff s Through-Quantity Law)
7. The sum of the across-quantities of an oriented circuit of G is zero when the sign of an across-quantity is changed if the directions of a circuit and an edge are different.(Kirchhoff s Across-Quantity Law)

Now, since the different characteristics of a linear stationary network are defined, let us calculate the through-vector and accross-vector.

$$\mathbf{Ei} = \mathbf{0} \ , \ \mathbf{Vu} = \mathbf{0}$$

In the other side, we have

$$S(i) = \begin{cases} r_j i_j - u_j = 0 & \text{if the edge j is a normal edge} \\ u_j = s_j & \text{if the accross-value is constant} \\ i_j = s_j & \text{if the through-value is constant} \end{cases}$$

or in a matrix form

$$\begin{pmatrix} \mathbf{D_I} & \mathbf{D_U} \end{pmatrix} \begin{pmatrix} \mathbf{I} \\ \mathbf{U} \end{pmatrix} = \mathbf{S}$$

And if the 3 equations are combined into one matrix after eliminating all the non relevant rows:

$$\begin{pmatrix} \mathbf{D_I} & \mathbf{D_U} \\ \mathbf{V} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} \end{pmatrix} \begin{pmatrix} \mathbf{I} \\ \mathbf{U} \end{pmatrix} = \begin{pmatrix} \mathbf{S} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

The solution of this system is as follow:

$$\begin{pmatrix} \mathbf{I} \\ \mathbf{U} \end{pmatrix} = \begin{pmatrix} \mathbf{D_I} & \mathbf{D_U} \\ \mathbf{V} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{S} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

3.4 Graph Theory Measures and Algorithms

In this section, some of graph theory measures and algorithms are presented and described.

3.4.1 Measures and indices

Several measures and indices can be used to analyze the network efficiency. Many of them were initially developed by Kansky and can be used for:

- Expressing the relationship between values and the network structures they represent.
- Comparing different transportation networks at a specific point in time.
- Comparing the evolution of a transport network at different points in time.

Outside the description of the network size by the number of nodes and edges, and its total length and traffic, several measures are used to define the structural attributes of a graph; the diameter, the number of cycles and the order of a node.

- **Diameter (d)**. The length of the shortest path between the most distanced nodes of a graph is the diameter. d measures the extent of a graph and the topological length between two nodes.

- **Number of Cycles (u)**. The maximum number of independent cycles in a graph. This number (u) is estimated through the number of nodes (v), links (e) and of sub-graphs (p) that are induced by a set of vertices and arcs of the original graph. Trees and simple networks have a value of 0 since they have no cycles. The more complex a network is, the higher the value of u, so it can be used as an indicator of the level of development and complexity of a transport system.

$$u = e - v + p$$

- **Average shortest path length (s)**. A measure of efficiency that is the average number of stops needed to reach two distant nodes in the graph. The lower the result, the more efficient the network in providing ease of circulation. In comparison, the diameter is the maximum length of all possible shortest paths.

$$l_g = \frac{1}{n(n-1)\sum_{i,j} d(v_i, v_j)}$$

Numerous measures exist for highlighting the situation of a node in a network. Some of them are made at the "local level" based on links with adjacent nodes, while others on the "global level" consider the node's situation in the whole network.

- **Order (degree) of a Node (o)**. The number of its attached links and is a simple, but effective measure of nodal importance. The higher its value, the more a node is important in a graph as many links converge to it. Hub nodes have a high order, while terminal points have an order that can be as low as 1. A perfect hub would have its order equal to the summation of all the orders of the other nodes in the graph and a perfect spoke would have an order of 1.The percentage of nodes directly connected in the entire graph is thus a measure of reachability. An isolate is a node without connections (degree equals to 0). The difference between in-degree and out-degree in a directed graph (digraph) may underline interesting functions of some nodes as attractors or senders. The order may be calculated at different depths: adjacent nodes (depth 1), adjacent nodes of adjacent nodes (depth 2), etc. If x_{ij} is the weight of the edge (i, j), then the weighted degree is simply:

$$k_i = C_D(i) = \sum_{j}^{N} x_{ij}$$

- **Koenig number (or associated number, eccentricity)**. A measure of farness based on the number of links needed to reach the most distant node in the graph.

$$e(x) = max_{y \in X} d(x, y)$$

- **Shimbel Index (or Shimbel distance, nodal accessibility, nodality)**. A measure of accessibility representing the sum of the length of all shortest paths connecting all other nodes in the graph. d_{ij} represents the shortest path cost between node i and node j.

$$A_i = \sum_{j=1}^{N} d_{ij}$$

- **Hub Dependence (hd)**. A measure of node vulnerability that is the share of the highest traffic link in total traffic (weighted degree). Weak nodes depending on few links will have a high hub dependence, especially if they locate in the neighborhood of a large node, while hubs will have a more even traffic distribution among their connections. It indicates to what extent removing the largest traffic link would affect the node's overall activity. The measure can be extended to more links (2, 3 ... 10 largest flow links). Average nearest neighbors degree (knn). A measure of neighborhood indicating the type of environment in which the node situates. A node with low order (degree) may be surrounded by a variety of other nodes, small or large, which has a direct influence on its own centrality and growth potential. A network is assortative or disassortative depending on the similarity of the order (degree) among neighboring nodes, which can be tested by means of Pearson correlation (assortativity coefficient). Neighbor connectivity is the correlation between the order (degree) of nodes and the average order (degree) of their neighbors.

3.4.2 Shortest Path

Shortest path is one of the most famous problems in graph theory. The goal is to find the shortest path between two different vertices in a graph G in terms of weights w. It is the path such as $\sum_{i=1}^{n} w(e_i)l(e_i)$ is minimum, where $l(e_i)$ is equal to 1 if e_i is in the path and 0 otherwise. The most famous algorithm is the Dijkstra's algorithm. This algorithm can be used in any directed graph (undirected can be transformed to a directed graph by defining a direction to all arcs and then adding arcs in the opposite direction with same weight).

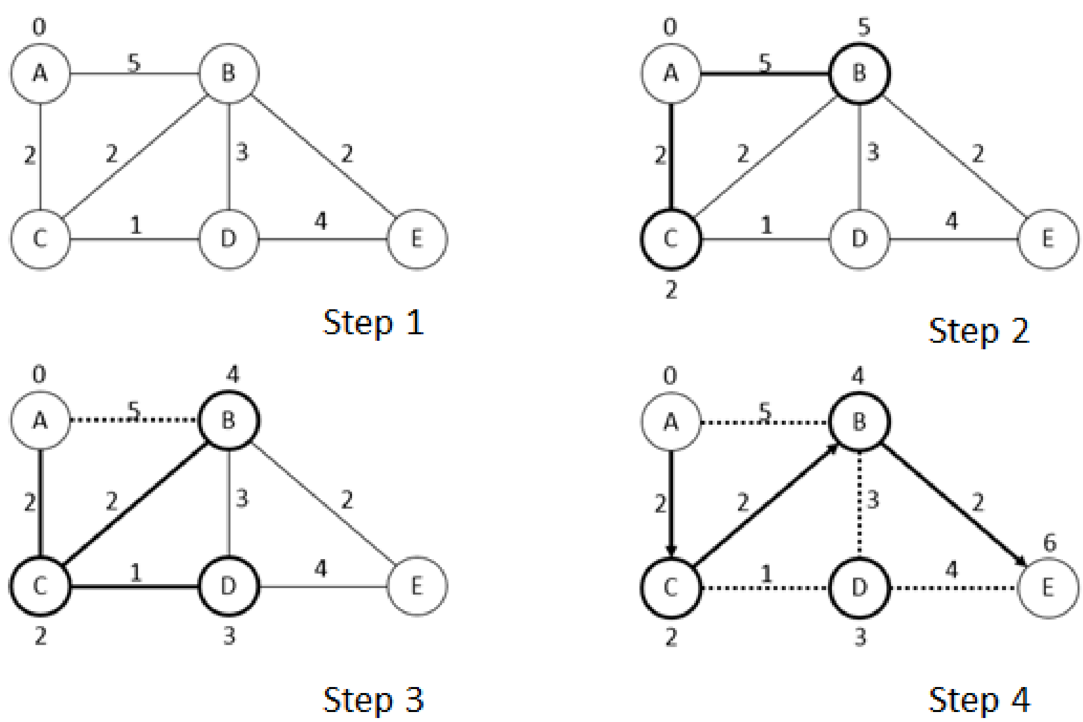


Figure 3.4: Djikstra Algorithm

Dijkstra's algorithm is basically a breath-first search with labeling to keep track of all visited vertices. It can also be extended to look for lightest path from one vertex to all vertices by not stopping until all vertices at reach are visited and labeled. Different other algorithms exist to find the lightest path such as the Floyd's Algorithm and A^* algorithm.

The Dijkstra algorithm is used in figure 3.4. The starting point is the node$A$ and ending node is node E. When there are 2 different paths to the same node, the shortest ones are kept while the other ones are dashed. The final length is 6 going from $A - C - B - E$.

3.4.3 Successive Shortest Paths (SSP) for Cost Minimization

Given a graph $G(V; E)$, we would like to minimize the cost of a particular flow from a source s to a sink t. Let us assume that every edge is having a cost/weight and a capacity as defined earlier. The objective is to send x units from the source s to the sink t in an efficient manner. The successive shortest path uses the shortest path to determine which path to use until exhaustion of one of the edge, i.e hitting the capacity of the edge. There are several ways to implement it, the following are 2 different ways:

Method 1

1. s is the starting vertex and t is the destination vertex. $C(e_i)$ is the capacity of the edge e_i which is positive while f is the number of sent units.
2. run the shortest path algorithm, if such path exists go to 3., otherwise stop.
3. send one unit at the time and update

   $$C(e_i) \rightarrow C(e_i) - 1 \text{ and } f \rightarrow f + 1$$

   and repeat until one of the capacities C becomes 0 or f becomes x.
4. repeat 2.

Method 2

1. s is the starting vertex and t is the destination vertex. $C(e_i)$ is the capacity of the edge e_i which is positive while f is the number of sent units. initialize $Cost = 0$ and $f = 0$.

2. run the shortest path algorithm, if such path exists go to 3., otherwise stop.

3. find

$$C^* = \min\{\min_{e_i \text{in path}}\{C(e_i)\}, x - f\}$$

   and update

$$C(e_i) \to C(e_i) - C^* \ , \ f \to f + C^* \text{ and } Cost \to Cost + \sum_{i=1}^{n} w(e_i) l(e_i) C^*$$

   and repeat until one of the capacities C becomes 0 or f becomes x.

4. if $f = x$, stop, otherwise repeat 2.

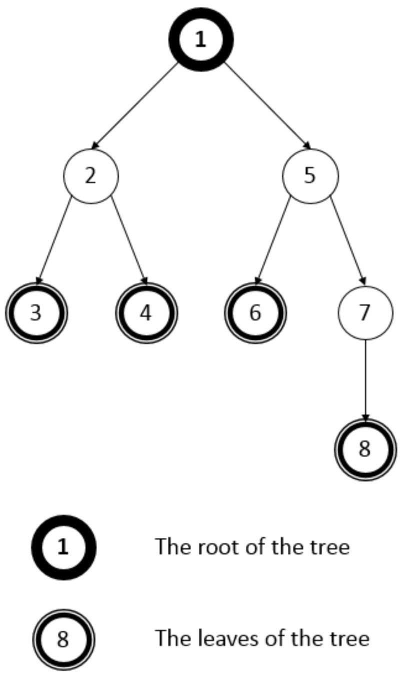
The difference between both approaches is the way the flow is updated after each shortest path. The complexity will depend on the different characteristics of the problem that is treated: the number of units needed to be sent x, the number of paths from s to t and the mean number of edges between s and t.

3.5 Trees

A tree is a connected graph that has no circuits (also called an acyclic graph). It is a very important concept in graph theory since it is the simplest type of graphs to analyse. In a directed tree G, there exist exactly one directed path from any two different vertices, which means that there are no circuits in a tree. The circuit matrix for a tree does not exist. Since there is exactly one path between a vertex and all other vertexes, two types of vertexes can be found.

- **leaf** is a vertex of degree 1, i.e connected to exactly one edge.
- **root** is a vertex that is connected to all other vertexes in the tree through outgoing edges.

Figure 3.5 shows a tree with highlighted root and leaves, and corresponding adjacency matrix. By analyzing the adjacency matrix of the tree, we notice that there are rows that are composed only of zeros. These rows describe nodes that have no outgoing links. These are the leaves. This particularity of the tree allows the representation of the tree in a different form. By writing the tree in the following form, the storage and different operations on the tree can be simplified.



$$\mathbf{Adj_{Tree}} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure 3.5: A tree with Adjacency matrix

The following algorithm describes the steps to represent the tree in this new form, and applies it to the example in 3.5:

- write all edges in the form : (start vertex, end vertex)

start vertex	end vertex
1	2
2	3
2	4
1	5
5	6
5	7
7	8

The start node is called the parent of the end vertex.

- find the root, and state its start vertex as being vertex 0.

start vertex	end vertex
0	1

- By combining both tables, and sorting the end vertexes in the ascending order, we can write the following vector $[0, 1, 2, 2, 1, 5, 5, 7]$ that represents the parents of its index in the vector. This is an easier way to store the trees, and it also simplify different operations such as shortest problem..

In the following part, the different measures and indexes are used to describe the tree example.

As to use the different measures and indexes, weights are assigned to the edges and vertexes of the tree. Figure 3.6 shows the different weights. The path from node 1 to node 3 is highlighted.

Vertex index	Vertex Parent	edge weight	Vertex weight
1	0	0	0
2	1	2	1
3	2	1	2
4	2	2	1
5	1	3	1
6	5	1	2
7	5	1	1
8	7	1	2

First, the Shimbel matrix for the undirected tree equivalent is created. Each element s_{ij} of the matrix represents the distance between the ith node and the jth node.

$$\mathbf{S} = \begin{pmatrix} 0 & 2 & 3 & 4 & 3 & 4 & 4 & 5 \\ 2 & 0 & 1 & 2 & 5 & 6 & 6 & 7 \\ 3 & 1 & 0 & 3 & 6 & 7 & 7 & 8 \\ 4 & 2 & 3 & 0 & 7 & 8 & 8 & \mathbf{9} \\ 3 & 5 & 6 & 7 & 0 & 1 & 1 & 2 \\ 4 & 6 & 7 & 8 & 1 & 0 & 2 & 3 \\ 4 & 6 & 7 & 8 & 1 & 2 & 0 & 1 \\ 5 & 7 & 8 & 9 & 2 & 3 & 1 & 0 \end{pmatrix}$$

These are the different measure and indexes related to this tree

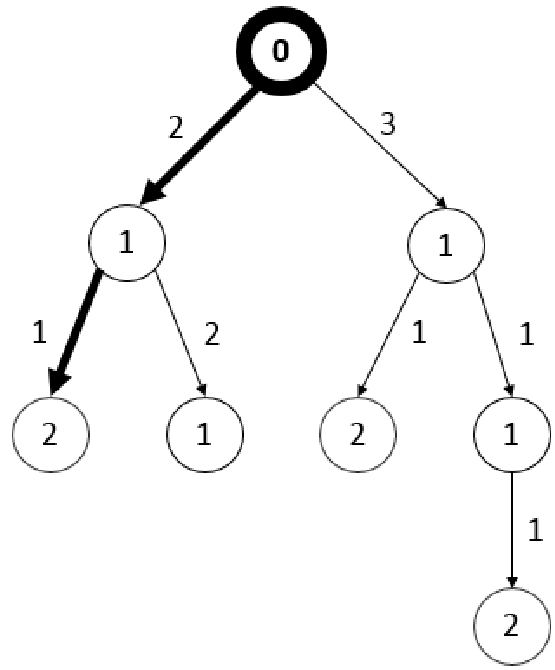


Figure 3.6: A weighted tree with highlighted path

- The diameter in this case is the maximum distance between two nodes in the tree which is 9 in this case.
- the number of cycles is zero by definition of the tree.
- The degree and eccentricity of the nodes are:

Node	Degree	Eccentricity
1	2	5
2	3	7
3	1	8
4	1	9
5	3	7
6	1	8
7	2	8
8	1	9

- The closeness of the node to the other nodes is:

Node	closeness
1	25
2	29
3	35
4	41
5	25
6	31
7	29
8	35

This measure shows how centralized is the node in the graph when only the length is taken into consideration. In the case the cost and the consumption itself is taken into consideration, the closeness is calculated differently.

1. The first step is to find the different trees codes if the root travels through all the graph. The only elements that altered from a tree to the other are the elements part of the path from the root to the new root.

Node	Tree code
1	[0,1,2,2,1,5,5,7]
2	[2,0,2,2,1,5,5,7]
3	[2,3,0,2,1,5,5,7]
4	[2,4,2,0,1,5,5,7]
5	[5,1,2,2,0,5,5,7]
6	[5,1,2,2,6,0,5,7]
7	[5,1,2,2,7,5,0,7]
8	[5,1,2,2,7,5,8,0]

2. Find the flow in the different trees taking into account the weight of each node. This operation can be done while rebuilding the adjacency matrix from the tree code.

   (a) Find the leaves : the indexes that are not in the code $\rightarrow \{3, 4, 6, 8\}$

   (b) add their weight to their parent weight and prune them. Repeat until no node is left in the tree.

   The following matrix presents the flows in the different edges depending on the considered root.

Roots	1-2	2-3	2-4	1-5	5-6	5-7	7-8
1	4	2	1	6	2	3	2
2	6	2	1	6	2	3	2
3	6	8	1	6	2	3	2
4	6	2	9	6	2	3	2
5	4	2	1	4	2	3	2
6	4	2	1	4	8	3	2
7	4	2	1	4	2	7	2
8	4	2	1	4	2	7	8

The cost of the flow in the linear case and quadratic case are respectively:

$$CL_F = \sum_{e \in E} C_e \times F_e$$

$$CQ_F = \sum_{e \in E} C_e \times F_e^2$$

we can hence calculate both the linear and quadratic cost for the tree in the different cases :

Roots	CL	CQ
1	37	163
2	41	203
3	47	263
4	57	363
5	31	103
6	37	163
7	35	143
8	41	203

The table shows that the optimal transport operations happen when the root is at node 5 as it does reduce the quadratic and the linear cost.

- the shortest path in the case of trees is very simple when the code method is used. As there is only one path from any node to the other, merging the paths from node X to the root and node Y to the root gives the solution to the problem.

3.6 Dynamic Graphs

The different graphs that have been discussed are static; the value of the flow do not change as the node weight and the flow cost does not change. However, and in most real life applications, the flow value on an arc may change over time. In the case of distribution systems, this is due to the variability of the different nodes consumption and the cost of the power; the flow value on each arc should adjust to these changes. If we consider a tree composed of 4 nodes coded $code = \{0, 1, 2, 3\}$ where Node 1, 2 and 3 are named A, B and C respectively. The problem is proposed for 4 different times or states $T = \{1, 2, 3, 4\}$.

Figure 3.7 shows the problem in its static formulation. A source lumps all the different sources at different states and supplies the flow to the same tree for 4 different states $T = \{1, 2, 3, 4\}$.

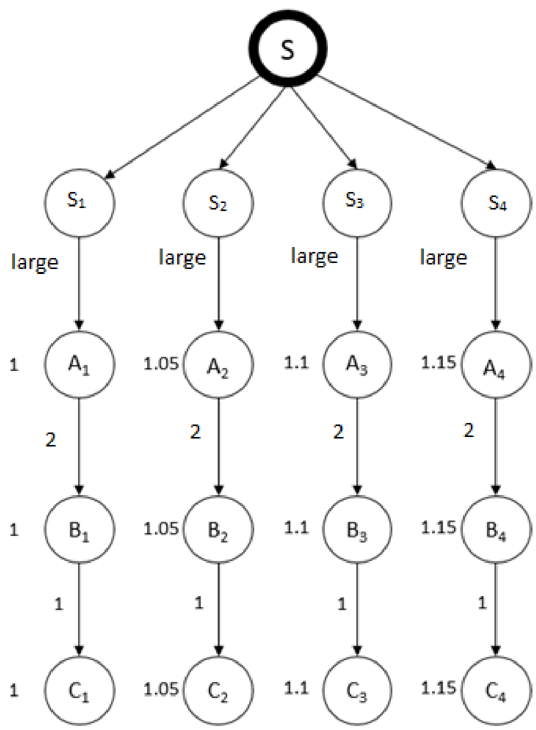


Figure 3.7: 4 states graph

Replacing the sources at different T with one large source that provides to all 4 states allows to solve one problem instead of 4 problems. The methods to solve this problem become very similar to the ones used in the previous sections.

3.7 Conclusion

This chapter presents an introduction to graph theory and the main concepts that are used in this thesis. Almost all distribution networks are operated as radial networks and hence can be represented as tree in this study. The next chapter is presenting different concepts that are used to create the algorithm for DG planning.

Chapter 4

DG siting and sizing problem

4.1 Preamble

DG siting and sizing is a very important stage in the improvement of the distribution network performance. DG units usually provide support and several advantages to the distribution network, however, if misplaced, can deteriorate the network by increasing the losses, congesting lines or increasing the voltage over the limits. Other than the technical issues, DG should also provide a return on investment as to be viable.

4.2 Mathematical Formulation

DG placement and sizing is formulated as an optimization problem with several objectives such as loss minimization while subject to different constraints.

4.2.1 Objective Functions

When studying an optimization problem, the objective function is the most important function. It evaluates the different options so that the algorithm outputs the best one. In our case, there will be two main objectives that need to be discussed.

- **Loss**
  Several factors, in distribution systems, impact the level of the power losses: losses

in the transmission and distribution lines, transformers, capacitors, insulators, etc. The resistance of the lines in the distribution system causes the real power and the reactive impedance (inductances) causes the reactive power losses. High real power losses affect the efficiency of transferring energy. However, the reactive is not less effective than real power because it affects the voltage level and hence the voltage profile. Practically the main generation plants are usually located far from the demand which also plays a role in increasing the power losses. Several methods are used to calculate the losses in a DS including:

- **Power Flow-based method**: the PF uses the power formula in a distribution system to calculate the voltages and currents in each and every line. The power losses can then be derived perfectly. This is the most accurate way to to deduce the losses, it is however computationally expensive. Whenever a new DG is included in the system, the power flow is ran again to find the currents in the lines and hence deduce the losses.
- **Exact losses formula**: The exact formula is derived directly from the above real power formulas which calculates all voltages and angles in the system.

  $$P_L = \sum_{i=1}^{N}\sum_{j=1}^{N}[\alpha_{ij}(P_iP_j + Q_iQ_j) + \beta_{ij}(Q_iP_j - P_iQ_j)]$$

  where $\alpha_{ij} = \frac{r_{ij}}{V_iV_j}cos(\delta_i - \delta_j)$,$\beta_{ij} = \frac{r_{ij}}{V_iV_j}sin(\delta_i - \delta_j)$ and $Z_{ij} = r_{ij} + ix_{ij}$ are the ijth element of $[Z_{bus}]$ matrix with $[Z_{bus}] = [Y_{bus}]^{-1}$ where P_i and Q_i are the active and reactive power consumption of bus i and δ_i is the angle of bus i. It has been assumed that α_{ij} and β_{ij} are constant when a DG unit is added to the system.
- **Approximated losses formula**: Let assume the load bus consumption is P_{load} and Q_{load}, while its voltage is V_{load} and R is the resistance of the line as in figure 4.1. The current can be written as:

  $$I = \frac{P_{load} + jQ_{load}}{V_{load}}$$

  The losses in the lines hence are:

  $$P_L = R \times I^2 = R \times |\frac{P_{load}+jQ_{load}}{V_{load}}|^2$$

The approximate formula will be used to calculate the losses in the distribution system and allocate the DG units for loss reduction. The voltage is assumed to be

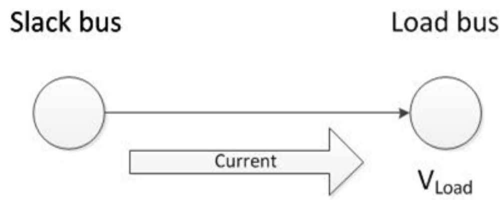


Figure 4.1: one bus diagram

1pu. The Calculating the current in the lines of the radial system will be described in the following sections.

4.2.2 Constraints

The following are the different constraints that a distribution system should respect at any time.

- **Power Balance:** it represents the equality between the power supplied by the substation and DG units, and the power consumed by the different customers. The losses are also taken into account in the study. For the whole system,

$$\sum_{i=1}^{n} P_{Gi} = \sum_{i=1}^{n} P_{Di} + P_L$$

$$\sum_{i=1}^{n} Q_{Gi} = \sum_{i=1}^{n} Q_{Di} + Q_L$$

  where P_{Gi} and Q_{Gi} are the active and reactive power generated by the bus i, P_{Di} and Q_{Di} are the active and reactive power consumed power by the bus i and P_L and Q_L are the active and reactive power loss in the system.

- **Feeder Capacity Limits:** the power flow in any feeder F should not exceed the thermal capacity of this feeder.

$$|S_F| < C_{F,max}$$

  where $C_{i,max}$ is the thermal limit of the feeder i, and S_i the power flow in that same feeder, where,

$$S_F{}^2 = P_F{}^2 + Q_F{}^2$$

- **Voltage limits:** The voltage at any bus of the system should remain between limits.

$$V_{min} < |V_i| < V_{max}$$

where V_i is the voltage at bus i. In IEEE standard Std 1547.3-2007, the values of V_{min} and V_{max} depend on the operator, and are taken as :

$$V_{min} = 0.95pu \text{ and } V_{max} = 1.05pu$$

- **Substation capacity limits:** Although the substation have also a capacity to respect, it will not be taken into account in this thesis.

- **DG penetration limit:** The total DG penetration level should generally not exceed a certain fraction of the distribution network consumption Pen

$$\sum_{i=1}^{N_{DG}} P_{DG,i} < Pen \times P_t$$

where Pen is the penetration of DG allowed in the system and P_t is the total consumption of the distribution system. N_{DG} is the number of DG units allowed in the system. This limitation usually associated with renewable DG units; they may deteriorate the system voltage if the penetration is high.The range of P_{en} is not limited.

- **DG output:** The DG output should be a multiple of the $P_{DG,u}$:

$$P_{DG} = k \times P_{DG_u}$$

where P_{DG} is the output of the DG unit, $P_{DG,u}$ is the set DG unit output, and k an integer. $P_{DG,u}$ depends on the system and is of 1kw in this case.

- **DG power factor:** The DG power factor is unity. The DG units are usually constrained to provide only active power in distribution networks.

4.3 Graph Theory Based Method for Loss Reduction

In radial distribution systems, the current is usually flowing from the slack bus to the end of the network. The introduction of the DG units disturbs the flow and its direction may change with time depending on the consumption of the loads and generation of the DG units. The following sections describe the proposed method; a space reduction method that keeps only the best DG potential sites for loss reduction.

4.3.1 Objective

Distribution networks are composed of several buses; all these buses are potential sites for DG allocation. DG allocation is an important step as the benefits of the DG unit depends on it. It is hence of major importance to identify the most promising/beneficial buses. This section proposes a method that reduces the number of potential buses to a set of promising ones. The objective function in this chapter is loss reduction. The relationship between the losses and the current, as described before, is :

$$P_L = R \times |\tfrac{P_{load}+jQ_{load}}{V_{load}}|^2$$

It is about reducing the value of the current in the branches. Reducing the current in lines directly reduce the voltage drop in the affected lines too. Congestion can also be mitigated by reducing the current in the saturated lines. To solve this problem, the distribution network is first distributed into a graph.

The verices represent buses while edges represent lines. The nodes or vertices can be characterized by their supply or demand of active power, depending on the node, and voltage. The lines, however, can be characterized by two values: cost and capacity. The cost in this case is the resistance, while the limit is the capacity of the line.

The graph has now all the elements needed to describe the distribution system and provides very good indications of the best location for DG, as the results will show. In a radial system, the power is provided by the slack bus. The flow is in one direction: from the slack bus toward the nodes. Each node can be seen as a slack bus for all the downstream nodes connected to it. By lumping all the down the stream nodes into that particular node, we will have the value of power transmitted by the network to this particular node.

For the number of potential nodes to be reduced, they need to be ranked. The ranking is based on the impact of a DG of capacity P_{DG} as will be explained in the following sections. Divide and Conquer algorithm decides on which nodes are to be compared and ranked first as to speed up the process of ranking.

4.3.2 Divide and Conquer Algorithm

The Divide and Conquer algorithm solves a problem by :

1. breaking the problem into smaller subproblems that are easier to solve,
2. solving these subproblems one after the other,
3. combine the answers in a proper way to solve the original problem.

The computational time can hence be reduced using the "divide and conquer" algorithm; it is used to match the nodes that can be locally compared and hence eliminate all non relevant comparisons. The algorithm divides the system into small groups of nodes that are close from each others; it hence allows to compare the impact of DG units installed at these nodes since their voltages are very close.

4.3.3 Flow in The System

In order to calculate the losses, the first task is to find the current and voltage in the lines. Let us consider the 2 bus system in Figure 4.2.

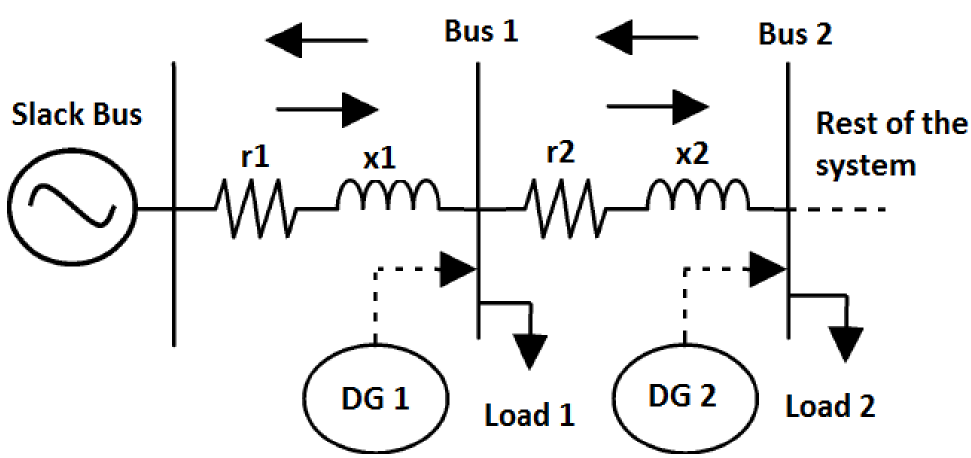


Figure 4.2: simple 2 bus system

The total active and reactive power consumption are P and Q respectively, while the voltage at bus 1 is V. The current in the line is:

$$|I| = |\frac{(P+jQ)}{V}| = \frac{\sqrt{P^2+Q^2}}{V}$$

To simply explain the algorithm, we assume that the power factor is one, and that the voltage is equal to 1pu. It implies that:

$$I = P$$

Let us assume that if Node N_i is the parent of Node N_j, then the line connecting both nodes is Line L_j. And hence, The flow in the lines can be calculated using the following steps.

For all nodes N_i in labels

1. find the shortest path,
2. add the power P_i to all the flows in the lines of the path.

To find the shortest path between i and the root, we use the following algorithm:

1. store i,
2. calculate $PARENT(i)$, and store it,
3. calculate $PARENT(PARENT(i))$, and store,
4. repeat then stop when 0 is reached.

Let us use this method on the graph represented in figure 4.3.

the different labels are:

$$\{1, 2, 3, 4, 5, 6, 7\}$$

and the corresponding parents are:

$$\{0, 1, 2, 2, 1, 5, 5\}$$

Running the described algorithm provides the following flow.

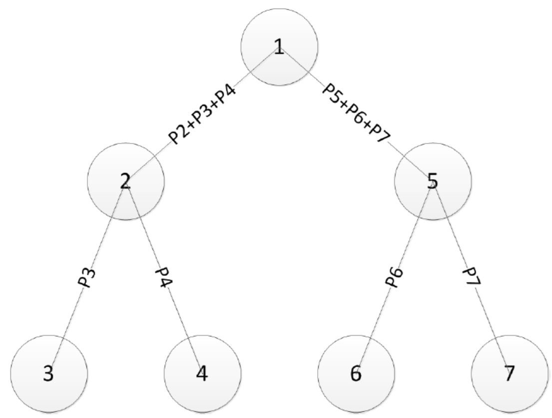


Figure 4.3: 7 bus graph

1. path $1, 2 \rightarrow \{1, 2\} \longrightarrow PL_2 = P_2$
2. path $1, 3 \rightarrow \{1, 2, 3\} \longrightarrow PL_2 = P_2 + P_3, PL_3 = P_3$
3. path $1, 4 \rightarrow \{1, 2, 4\} \longrightarrow PL_2 = P_2 + P_3 + P_4, PL_4 = P_4$
4. path $1, 5 \rightarrow \{1, 5\} \longrightarrow PL_5 = P_5$
5. path $1, 6 \rightarrow \{1, 5, 6\} \longrightarrow PL_5 = P_5 + P_6, PL_6 = P_6$
6. path $1, 7 \rightarrow \{1, 5, 7\} \longrightarrow PL_5 = P_5 + P_6 + P_7, PL_7 = P_7$

And hence, the flow is calculated in a very simple way. The next section discusses the general impact of a DG unit on the losses of a line.

4.3.4 Impact of DG allocation on losses

The following example present a simple system as shown in Figure 4.2, the load consumption is 900 kW and power factor (PF) is 0.9 lagging. The active losses are of 3.26kW. A DG is introduced at the load bus; its capacity is then increased from 0kW to 2000kW. Figure 4.4 shows the result of and losses. The power flow has been used for this first example.

The optimal DG capacity which minimizes the losses is 900kW. However, increasing the capacity over 900kW will just increase the losses until the point where the losses with

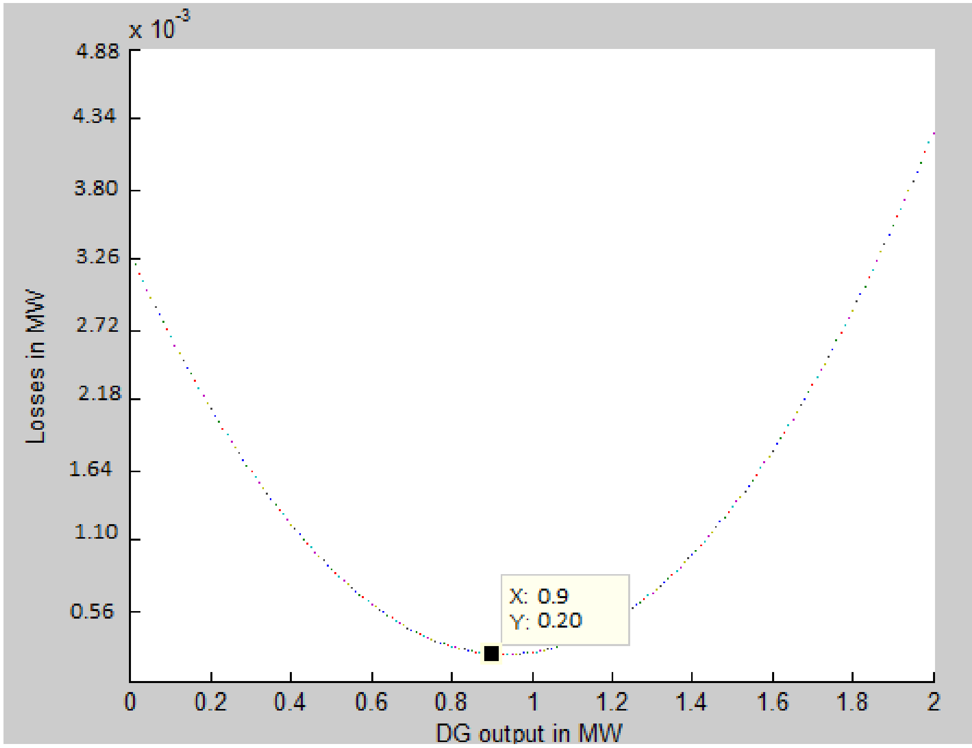


Figure 4.4: Losses in a 2 bus system with DG

DG are even higher than the losses without DG. A DG can also have negative impact on distribution system losses. Figure 4.5 shows the improvement of the voltage profile after adding a DG. The improvement is clear since the voltage is increasing with DG capacity, even after exceeding the slack bus voltage. The voltage needs however to remain between $^{+}/_{-}5\%$; a very large DG may push it then to over the limit and create problems in the system.

From this simple example, a DG can have advantages in reducing losses and improving the voltage profile, however, it can also worsen the situation if the DG is larger than a certain value. This value can be seen as a threshold which will limit the DG capacity to not worsen the situation in a system. Adding a DG to bus 1 means that the power consumed by the bus lower by PDG. The following equation describes the new formula which is used when studying the system with DG.

$$P_L = \frac{R}{V_{Load}^2}((P_{Load} - P_{DG})^2 + Q_{Load}^2)$$

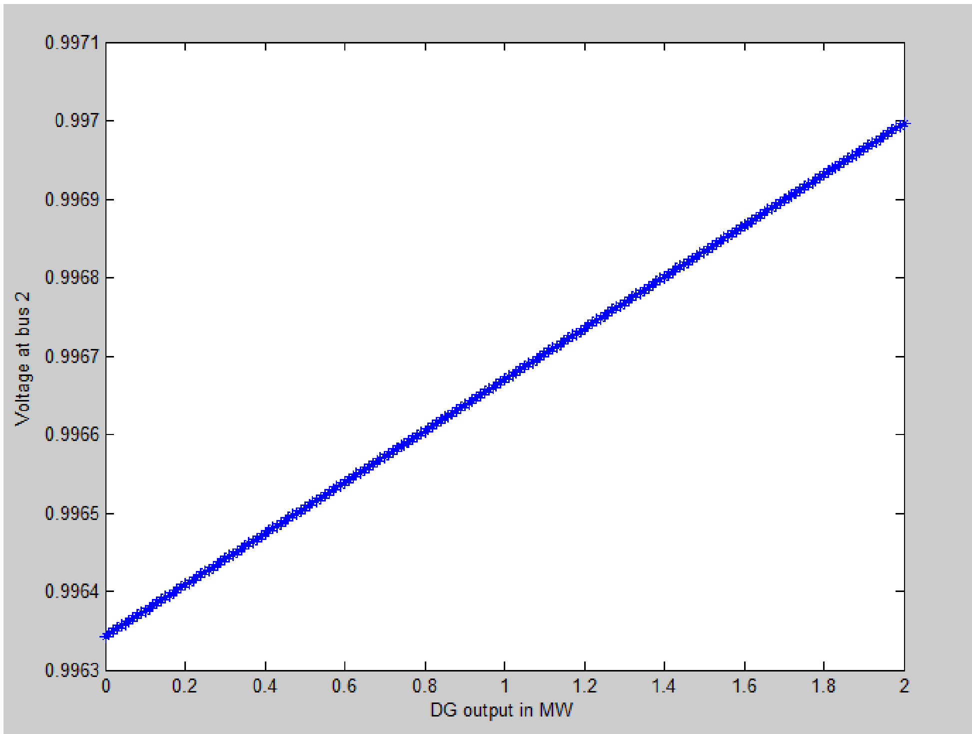


Figure 4.5: Voltage in a 2 bus system with DG (Bus 2 = Load bus)

This formula will hit the minimum when P_{DG} is equal to P_{Load}. The threshold is twice P_{Load} in this case. And hence, using a DG of capacity over this threshold will only increase the losses and is not beneficial to the system. These results are the same as the ones found using the power flow.

$$P_{DG,threshold} = 2P_{Load}$$

The next section discusses the different methods to compare simple bus configurations before applying them to larger systems. The DG used in the next section are supplying active power only.

4.3.5 Generic system study case

This section studies two generic combinations of buses. Figure 4.6 and 4.7 shows both combinations. Bus 1 and 2 consume 400kW and 200kW, respectively. The PF is of 0.9.

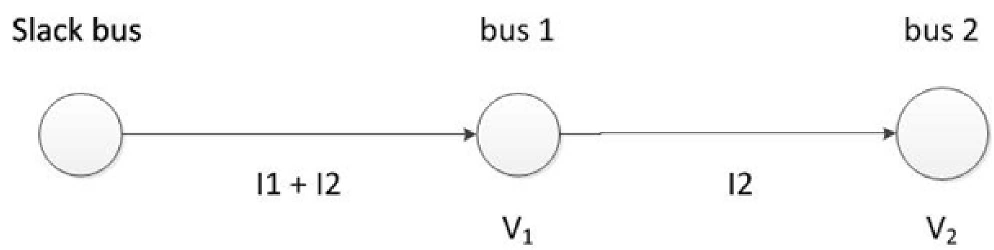


Figure 4.6: Generic system 1

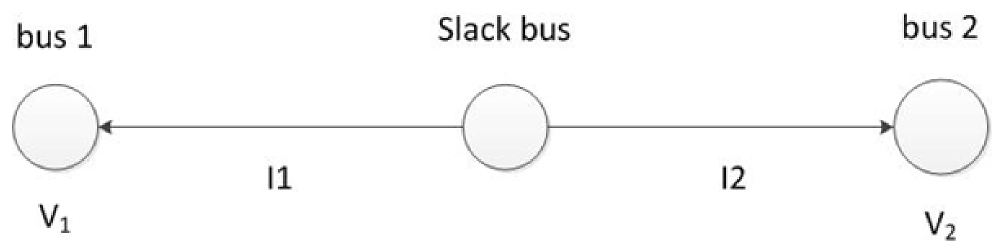


Figure 4.7: Generic system 2

Generic system 1

This section studies the first combination of buses, and the DG thresholds that will decide on which capacity should be put in the bus to get better results.

The following equation describes the losses in the system when a DG is introduced at bus 1. In all following formulas, the losses in line 2 are not considered when calculating the flow in line 1.

$$P_{L,DG\ at\ Bus1} = \frac{R_1}{V_1^2}((P_1 + P_2 - P_{DG})^2 + (Q_1 + Q_2)^2) + \frac{R_2}{V_2^2}((P_2)^2 + (Q_2)^2)$$

In order to simplify those equations, voltages are supposed to be the same when in reality they are close to each others making the approximation possible without loss of accuracy. Taking $L_1 = \frac{R1}{V_1^2}$ and $L_2 = \frac{R2}{V_2^2}$, the final formula is shown in the following equation.

$$P_{L,DG\ at\ Bus1} = L_1((P_1 + P_2 - P_{DG})^2 + (Q_1 + Q_2)^2) + L_2((P_2)^2 + (Q_2)^2)$$

By adding the assumption that the voltage V_1 and V_2 change very slowly due to the DG, the following equation can be written. The losses when the DG is in bus 1 then in bus 2 are compared.

$$P_{L,DG\ at\ Bus1} = L_1((P_1 + P_2 - P_{DG})^2 + (Q_1 + Q_2)^2) + L_2((P_2)^2 + (Q_2)^2)$$

$$P_{L,DG\ at\ Bus2} = L_1((P_1 + P_2 - P_{DG})^2 + (Q_1 + Q_2)^2) + L_2((P_2 - P_{DG})^2 + (Q_2)^2)$$

$$P_{L,Diff} = L_2(P_2)^2 - L_2(P_2 - P_{DG})^2$$

$$P_{L,Diff} = P_{L,DG\ at\ Bus1} - P_{L,DG\ at\ Bus2} = L_2 P_{DG}(2P_2 - P_{DG})$$

And since P_{DG} and L_2 are positive, $P_{L,Diff}$ is positive (Bus 2 better than Bus 1) if:

$$P_{DG} < 2P_2$$

$P_{L,Diff}$ does not depend on reactive power of the load, and hence, in all this work, equations will not contain any reference to reactive power. $P_{L,Diff}$ can be studied the same way as earlier; the threshold is twice the load active consumption at Bus2. If the DG capacity is higher than that threshold, the DG will have less benefits and hence better sit it in the other bus. Figure 4.8 shows the results of the system using the power flow. The decision is based on the threshold.

The result shows that the right decision has been taken for every DG capacity. For DG above 400 Kw, the DG benefit at bus 2 is lower than if the DG is at bus 1. This threshold can be used whenever 2 buses are in this particular combination.

Generic system 2

This section studies the second combination of buses, and the DG thresholds that will decide on which capacity should be put in the bus to get better results. The formulas corresponding to adding the DG at bus 1 and bus 2 are described.

$$P_{L,DG\ at\ Bus1} = L_1((P_1 - P_{DG})^2 + (Q_1)^2) + L_2((P_2)^2 + (Q_2)^2)$$

$$P_{L,DG\ at\ Bus2} = L_1((P_1)^2 + (Q_1)^2) + L_2((P_2 - P_{DG})^2 + (Q_2)^2)$$

$$P_{L,Diff} = P_{DG}(L_2(2P_2 - P_{DG}) - L_1(2P_1 - P_{DG}))$$

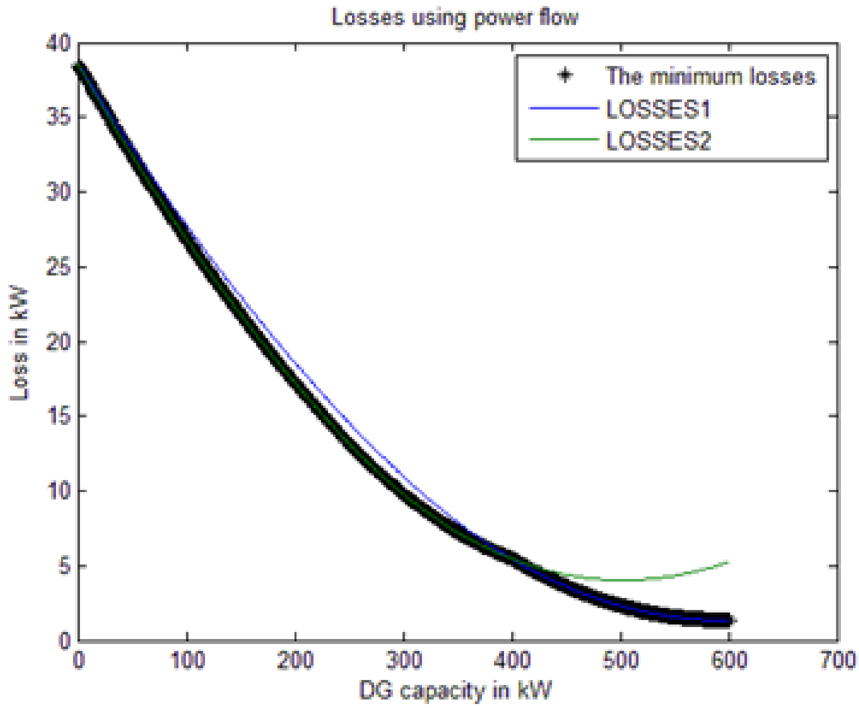


Figure 4.8: Losses in Generic system 1 (LOSSES1 and LOSSES2 are losses when DG at bus 1 and 2 respectively)

Again, $P_{L,Diff}$ does not depend on reactive power as earlier. In this case, the threshold is, however, more complicated to find. $P_{L,Diff}$ is equal to 0 when the DG capacity is zero or the threshold. After some calculation we find:

- **Case 1:** $L_1 = L_2$

$$P_{DG} placedatmax(P_1, P_2)$$

- **Case 2:** $L_1 \neq L_2$

$$P_{DG} < 2\frac{(L_2P_2 - L_1P_1)}{L_2 - L_1}$$

The above equations show that that in the case where $L1 = L2$ the DG unit is always placed at the bus with higher load. However, when $L1 \neq L2$, the DG unit is placed at

Bus1 if lower than the threshold, otherwise placed at bus 2. At 150kW, the DG benefit at bus 2 is lower than if the DG is at bus 1. This threshold method can be used whenever 2 buses are in this combination.

4.3.6 General rules

The general formula of losses in a graph is:

$$P_L = \sum_{i=2}^{n} R_i PLine_i^2$$

If a DG unit is introduced in this graph at Node T, the flow is affected as follows: the flow is altered only in the lines involved in the path between the root and the node T. And hence, the Losses formula can be written in the following way:

$$P_L = \sum_{i \in Pth_T} R_i PLine_i^2 + \sum_{i \notin Pth_T}^{n} R_i PLine_i^2$$

where Pth_T is the lines in the path between the root and the node T.

By adding the DG, the formula becomes:

$$P_{L,T} = \sum_{i \in Pth_T} R_i (PLine_i - P_{DG})^2 + \sum_{i \notin Pth_T}^{n} R_i PLine_i^2$$

if we need to compare this potential node T to another potential node S, the formula regarding S is:

$$P_{L,S} = \sum_{i \in Pth_S} R_i (PLine_i - P_{DG})^2 + \sum_{i \notin Pth_S}^{n} R_i PLine_i^2$$

where Pth_S is the lines in the path between the root and the node S.

In order to calculate the difference between $P_{L,S}$ and $P_{L,T}$, we define the following node sets:

- $Pth_{S,T} = Pth_T \cap Pth_S$

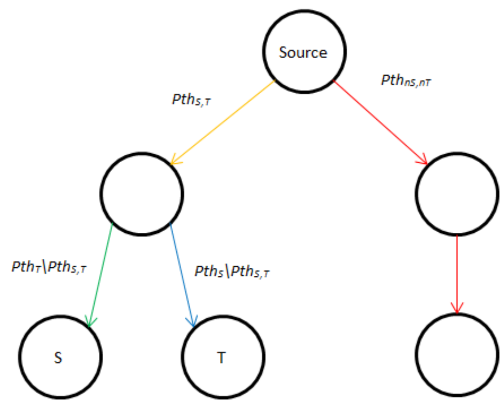


Figure 4.9: Different sets in the system

- $Pth_{nS,nT} = {Pth_T}^C \cap {Pth_S}^C$

Using these different sets, $P_{L,T}$ and $P_{L,S}$ is rewritten as:

$$P_{L,T} = \sum_{i \in Pth_{S,T}} R_i(PLine_i - P_{DG})^2 + \sum_{i \in (Pth_S \backslash Pth_{S,T})} R_i(PLine_i - P_{DG})^2 + \sum_{i \in Pth_{nS,nT}} R_i(PLine_i)^2 + \sum_{i \in (Pth_S^C \backslash Pth_{nS,nT})} R_i PLine_i^2$$

$$P_{L,S} = \sum_{i \in Pth_{S,T}} R_i(PLine_i - P_{DG})^2 + \sum_{i \in (Pth_T \backslash Pth_{S,T})} R_i(PLine_i - P_{DG})^2 + \sum_{i \in Pth_{nS,nT}} R_i(PLine_i)^2 + \sum_{i \in (Pth_T^C \backslash Pth_{nS,nT})} R_i PLine_i^2$$

And hence, the different:

$$P_{L,T} - P_{L,S} = \sum_{i \in (Pth_S \backslash Pth_{S,T})} R_i(PLine_i - P_{DG})^2 - \sum_{i \in (Pth_T \backslash Pth_{S,T})} R_i(PLine_i - P_{DG})^2 + \sum_{i \in (Pth_S^C \backslash Pth_{nS,nT})} R_i PLine_i^2 - \sum_{i \in (Pth_T^C \backslash Pth_{nS,nT})} R_i PLine_i^2$$

And using the fact that:

- $(Pth_T^C \setminus Pth_{nS,nT}) = (Pth_S \setminus Pth_{S,T})$
- $(Pth_T \setminus Pth_{S,T}) = Pth_S^C \setminus Pth_{nS,nT})$

The formula is rewritten as:

$$P_{L,T} - P_{L,S} =$$

$$\sum_{i \in (Pth_S \setminus Pth_{S,T})} (R_i(PLine_i - P_{DG})^2 - R_i PLine_i^2) - \sum_{i \in (Pth_T \setminus Pth_{S,T})} (R_i(PLine_i - P_{DG})^2 - R_i PLine_i^2)$$

it can further formulate as:

$$\sum_{i \in (Pth_S \setminus Pth_{S,T})} R_i(-P_{DG}(2PLine_i - P_{DG})) - \sum_{i \in (Pth_T \setminus Pth_{S,T})} R_i(P_{DG}(2PLine_i - P_{DG}))$$

And hence, the Bus T is better than Bus S when:

- Case $(\sum_{i \in (Pth_T \setminus Pth_{S,T})} L_i - \sum_{i \in (Pth_S \setminus Pth_{S,T})} L_i) \neq 0$

$$P_{DG} < 2 \frac{(\sum_{i \in (Pth_T \setminus Pth_{S,T})} R_i PLine_i - \sum_{i \in (Pth_S \setminus Pth_{S,T})} R_i PLine_i)}{\sum_{i \in (Pth_T \setminus Pth_{S,T})} R_i - \sum_{i \in (Pth_S \setminus Pth_{S,T})} R_i)}$$

- Case $(\sum_{i \in (Pth_T \setminus Pth_{S,T})} R_i - \sum_{i \in (Pth_S \setminus Pth_{S,T})} R_i) = 0$

$$\sum_{i \in (Pth_T \setminus Pth_{S,T})} R_i PLine_i - \sum_{i \in (Pth_S \setminus Pth_{S,T})} R_i PLine_i > 0$$

4.3.7 Conclusion

This chapter described the rules that are used for comparing different nodes in the system. These rules are hence used to reduce the number of potential nodes for DG allocation a distribution network.

Chapter 5

Long Term DG Planning

Distribution systems change throughout time due to the increase of loads from one year to another and hence require planning. This increase can alter the state of the system by worsening the voltage profile, increasing the losses and creating congestion. This chapter will discuss these different problems. The long term planning is a DG and line sizing and siting problem over many years; it can hence be interpreted as a dynamic graph. The formulation of the problem is is then similar to the previous chapter, but taking the time component into account as can be seen in the next section.

5.1 Mathematical Formulation

This section present the general formulation of the problem.

5.1.1 Objective functions

The main objective of this optimization is to minimize the cost of installation of news lines and DG for congestion mitigation.

5.1.2 Constraints

The following are the different constraints that a distribution system should respect at any time. Long term DG planning introduce a time component in the active power and reactive power. These constraints are the same as in the former section with the addition of the time component y representing the years.

- **Power Balance:** it represents the equality between the power supplied by the substation and DG units, and the power consumed by the different customers. The losses are also taken into account in the study. For every node of the system,

$$\sum_{i=1}^{n} P_{Gi}(y) = \sum_{i=1}^{n} P_{Di}(y) + P_L(y)$$

$$\sum_{i=1}^{n} Q_{Gi}(y) = \sum_{i=1}^{n} Q_{Di}(y) + Q_L(y)$$

  where $P_{Gi}(t)$ and $Q_{Gi}(t)$ are the active and reactive power generated by the bus i, $P_{Dj}(y)$ and $Q_{Dj}(y)$ are the active and reactive power consumed power by the bus j and $P_L(y)$ and $Q_L(y)$ are the active and reactive power loss in the system.

- **Feeder Capacity Limits:** the power flow in any feeder should not exceed the thermal capacity of this feeder.

$$|S_i(y)| < C_{i,max}$$

  where $C_{i,max}$ is the thermal limit of the feeder i, and S_i the power flow in that same feeder.

$$S_i(y)^2 = P_i(y)^2 + Q_i(y)^2$$

- **Voltage limits:** The voltage at any bus of the system should remain between limits at any moment.

$$V_{min} < |V_i| < V_{max}$$

  where V_i is the voltage at bus i. The values of V_{min} and V_{max} depends on the standard used and are taken here as: :

$$V_{min} = 0.95pu \text{ and } V_{max} = 1.05pu$$

- **Substation capacity limits:** Although the substation have also a capacity to respect, it will be not be taken into account in this thesis.

- **DG penetration limit:** The total DG penetration level should not exceed a percentage of the distribution system power consumption.

$$\sum_{i=1}^{N_{DG}} P_{DG,i}(y) < Pen \times P_t(y)$$

where Pen is the penetration of DG allowed in the system and $P_t(y)$ is the total consumption of the distribution system. N_{DG} is the number of DG units allowed in the system.There is no limit on Pen.

- **DG power factor:** The power factor is set to 1. The DG units are usually constrained to provide only active power in distribution networks.

5.2 Description of Proposed Method

The proposed model presented in the previous chapter is applied to the 3-bus radial distribution system represented by (Si,Ai,Bi,Ci) in Figure 5.2 .The total system peak demand is 3 units in year-0 and assumed to grow by 0.15 annually. Each feeder segment is 1 unit long. It is assumed that a non limited budget is allocated to mitigate congestion throughout the years in this system.

The system peak for year 0 is met. However, congestion does not allow the substation supply the missing power to the DS starting year 1. Figure 5.2 provides a view of the flow and the congestion that the systems will face through the following years. In order to mitigate this problem, line reinforcement can be installed to follow the demand increase rate. Table 5.1 shows the line improvement at each year.

Table 5.1: Line reinforcement with no DG

Lines	Capacity
Line $A_2 - B_2$	0.1
Line $B_2 - C_2$	0.05
Line $A_3 - B_3$	0.2
Line $B_3 - C_3$	0.1
Line $A_4 - B_4$	0.3
Line $B_4 - C_4$	0.15

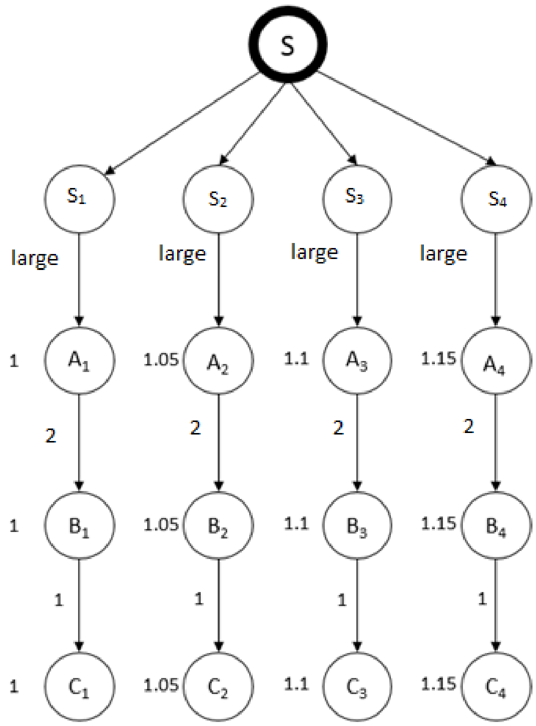


Figure 5.1: 4 year DS planning

Line improvement can be very expensive due to all the costs it implies (permits, installation cost, time). DG units are then used to mitigate the problems locally. Table 5.2 shows the DG installation per year while Figure 5.3 shows the flow in the lines during the different years.

Table 5.2: DG installation with no line reinforcement

Node	Capacity
C_2	0.1
C_3	0.2
C_4	0.3

In this section, a simple DG planning problem is solved incorporating the method

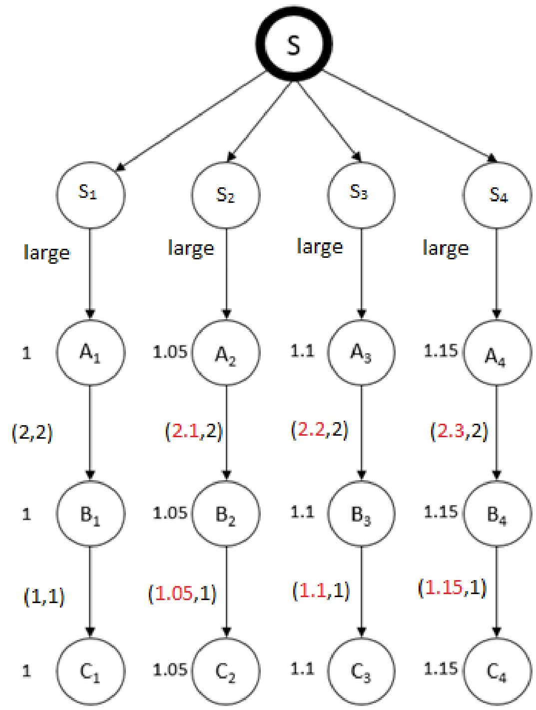


Figure 5.2: 4 year DS planning, (flow,line capacity)

described in Chapter 4. The DG units are installed at the leaves allowing the system to meet its demands and mitigate the congestion. The objective is however to mitigate the congestion at the minimum cost possible, and hence each line needs to be assigned a cost. Table 5.3 presents the price of each improvement that occurs in the system where $C_{LineImpr}$ is the cost of improving a line per MW per unit, C_{DGinst} is the cost of installing a DG of $1MW$, $C_{DGmaint}$ is the cost of maintaining and running the DG at $1MWh$, t is the number of hours at a certain power output, i is the interest rate and n is the year in which the installation/improvement has been made. These costs represent the net present value of the different resources. In this example, the DG unit is a gas turbine with fix output for simplicity. The cost of the power imported from the substation and fuel for the DG are assumed equal.

By assigning each line its appropriate cost and then running the SSP, the minimum cost flow can be found and the improvement plan deduced. This model optimizes each

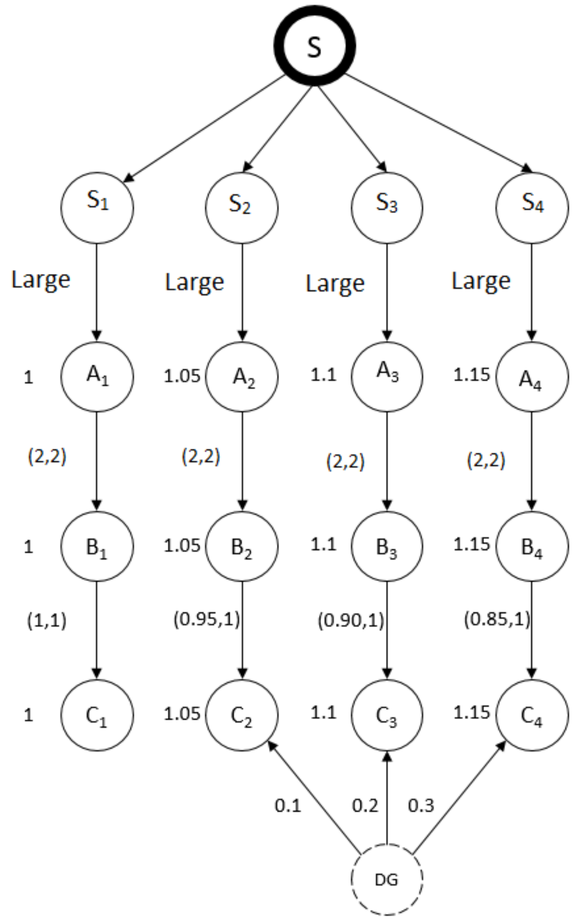


Figure 5.3: 4 year DS planning with DG

Table 5.3: Cost formulas of the DG installation and line reinforcement

type of improvement	Cost per flow
Line improvement	$\frac{1}{(1+i)^n} C_{LineImpr} \$/MW/unit$
DG installation and maintenance	$\frac{1}{(1+i)^n} C_{DGinst} \$/MW + C_{DGmaint} \$/MW \times 8760$
Regular lines	0$

state. In a general case, the DG node is connected to all nodes of the system; however, following the rules of the previous chapter, only certain nodes are connected to the DG units.

In this thesis, the cost regarding the different options are in Table 5.2. The cost of the power from the substation and the fuel for the DG units are supposed to be same in what follows.

Table 5.4: Capital cost of available resources[30]

	Capital
Feeder	$200,000\$/MW$
Gas turbine	$780,000\$/MW$

In order to find the best solution for the planning problem, the algorithm is divided into two different phases. Phase 0 initialize all variables, phase I in figure 5.4 finds the best DG allocation for each year and phase II combines theses solutions to find the best overall solution for the problem while taking into account the planner concerns and results in 3 different approaches:

- **Forward DG allocation:** In this case, the t+1 period DG allocation is built directly on t period DG allocation. This means that whatever allocation found in t period is used to allocate the DG units at period t+1.
- **Backward DG allocation:** In this case, the t period DG allocation is built directly on t+1 period DG allocation. This means that whatever allocation found in t+1 period is used to allocate the DG units at period t by eliminating the least effcient DG units for period t. It can also be bundled by installing the DG units the earliest possible as to reduce the number of DG improvements over the horizon.

5.3 Simulation of a 14-bus system

The following system is for simulation purposes and to show how the algorithm runs. It is composed of 14 buses connected by 13 lines. The load increase rate is of 10% the original load consumption per year. The cost of the DG unit is set at twice the cost of the line as to show the logic behind the choices between DG units and lines installation. Figure

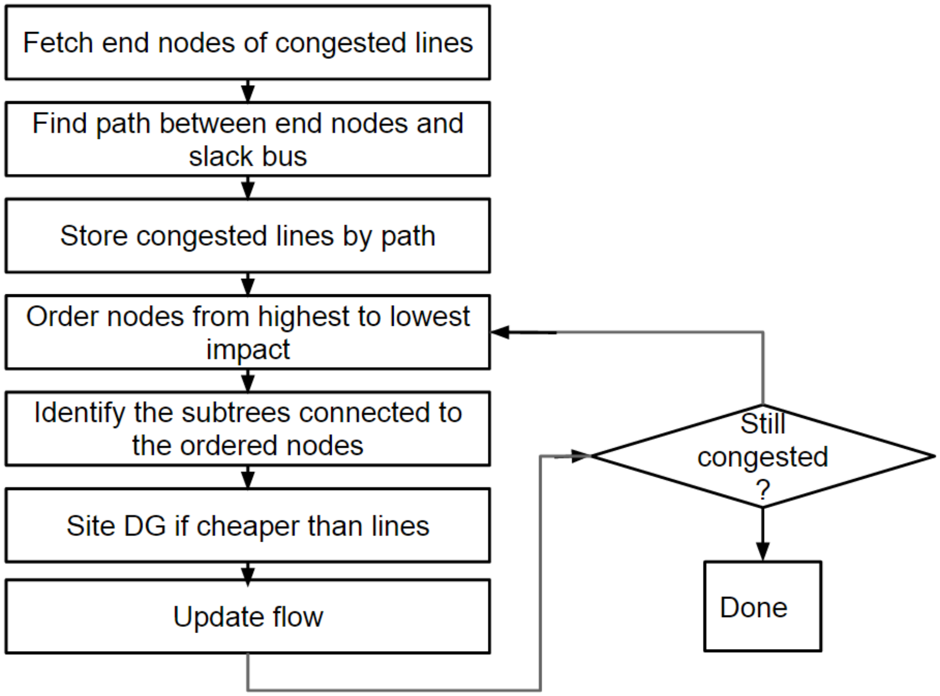


Figure 5.4: Phase 1

5.5 shows the system with the flow in the different lines. This is a snapshot after running phase 0. There is no congestion during year 0. The congestion will happen starting year 5 onward.

5.3.1 DG allocation in year 5

The system faces congestion on year 5, and this section illustrates the different steps the algorithm goes through to allocate the different DG units and lines. In order to place all the DG units, it took 4 steps for the algorithm.

Table 5.5 presents the results for each individual year from year 1 to year 10.

Remarks

- We can notice that the system experiences congestion starting year 5,

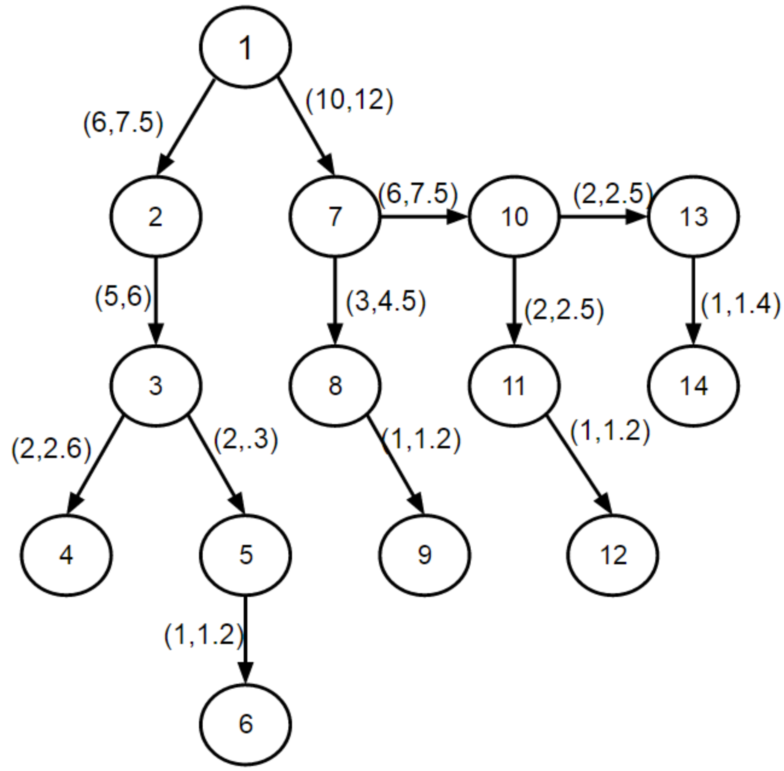


Figure 5.5: Study case, 14 bus system

Table 5.5: DG Allocation : Results all years

DG lines	Y5	Y6	Y7	Y8	Y9	Y10
4	0.05	0.15	0.2	0.45	0.6	0.65
6	0.05	0.1	0.15	0.2	0.25	0.3
9	0.05	0.1	0.6	0.7	0.9	1
12	0.05	0.15	0.2	0.45	0.6	0.85
14				0.2	0.35	0.5
2--3	0.2	0.2	0.1	0.1		
1--7	0.35	0.6	0.65	0.9	1.55	1.2

- 2-3 is used during the period 5-8 then becomes obsolete,
- Line 1-7 is using during all the period 5-10, but its capacity goes down by year 10,

5.3.2 Continuous Forward(figure 5.7)

DG allocation is ran for year 1 and the solution is applied to the system, then it is ran again for year 2 and so on until all congestion for the whole horizon is mitigated. Table 5.5 shows the results for the continuous forward method on this system. The issue of this method is that some lines may be obsolete after some time as in this example. A line has been installed at year 5 to mitigate congestion for these 2 years. However, a DG installation at year 7 mitigate the other lines congestion solves at the same time the congestion on line 2-3 which makes that line installation obsolete. figure 5.7.

Table 5.6: Planning over horizon, Continuous forward

DG lines	Y5	Y6	Y7	Y8	Y9	Y10
4	0.05	0.01	0.15	0.15	0.15	0.05
6	0.05	0.05	0.05	0.05	0.05	0.05
9	0.05	0.05	0.5	0.1	0.2	0.1
12	0.05	0.1	0.05	0.25	0.15	0.25
14				0.2	0.15	0.15
2--3	0.2	0.2				
1--7	0.35	0.25	0.05	0.25	0.65	

5.3.3 Continuous Backward(figure 5.8)

DG allocation is ran for year 10 and the solution is applied to the system, then it is ran again for year 9 by eliminating all the DG/lines that are no more necessary for that year, and so on until all congestion for the whole horizon is mitigated. Table 5.6 shows the results for the continuous backward method on this system. The results show that the line 2-3 has not been used as the DG has been installed earlier to avoid congestion making it more efficient. As it can be noticed, the Continuous backward and Continuous forward provide the same last year result but have different plans and different costs. This method can be extended further by bundling the DG units at the higher points in a path tor educe the number f installed DG units.

Table 5.7: Planning over horizon, Continuous backward

DG lines	Y5	Y6	Y7	Y8	Y9	Y10
4	0.05	0.1	0.15	0.15	0.2	
6	0.05	0.05	0.05	0.05	0.05	0.05
9	0.05	0.05	0.5	0.1	0.3	
12	0.05	0.1	0.05	0.25	0.4	
14				0.2	0.15	0.15
2--3						
1--7	0.35	0.25	0.05	0.35	0.3	

As it can be noticed, the Continuous backward and Continuous forward provide the same last year result but have different plans and different costs. The cost of the CF plan is 1.67M$ while the one of CB plan is 1.6M$ for a difference of 4.35%.

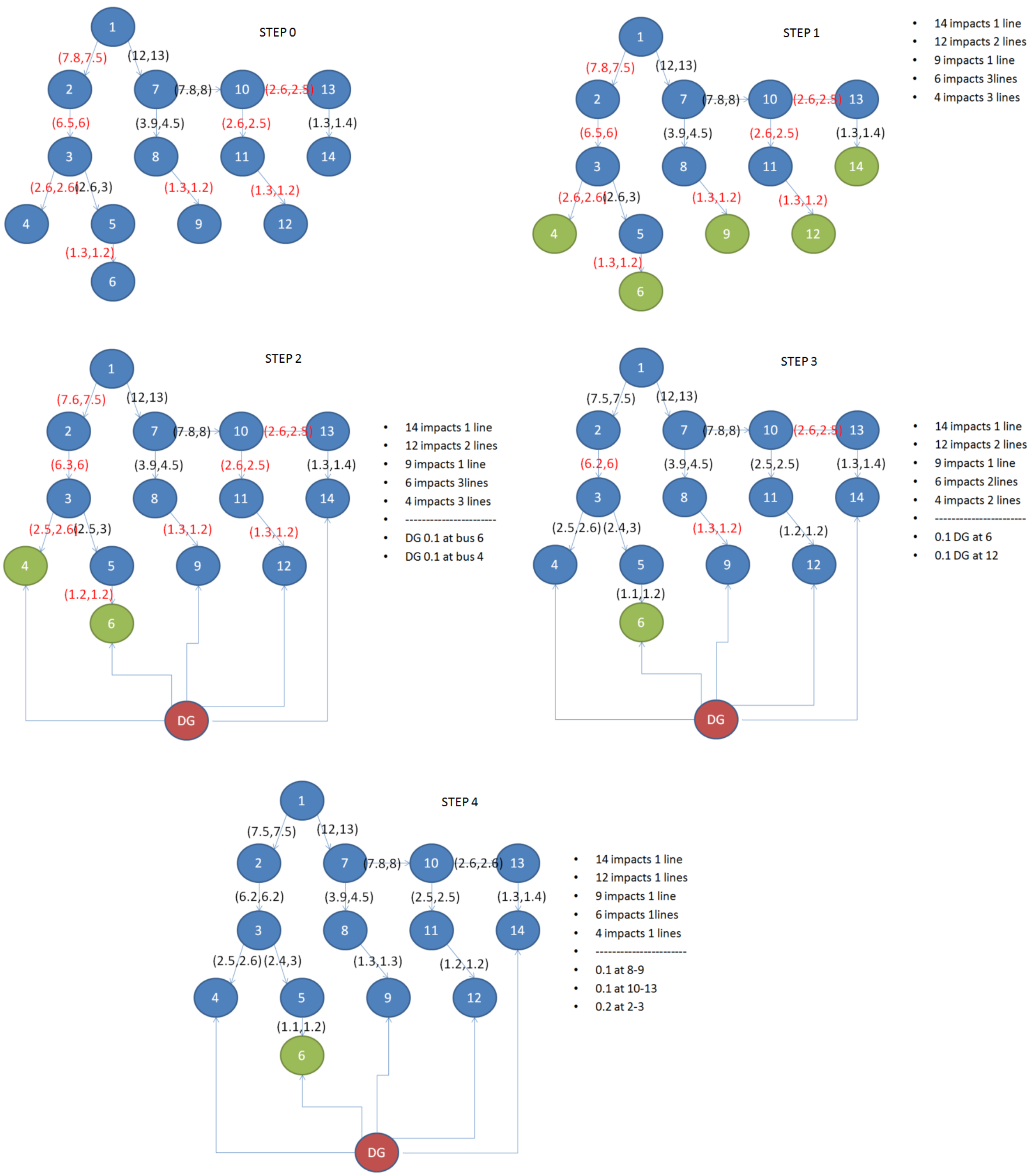


Figure 5.6: DG allocation : year 5 all steps

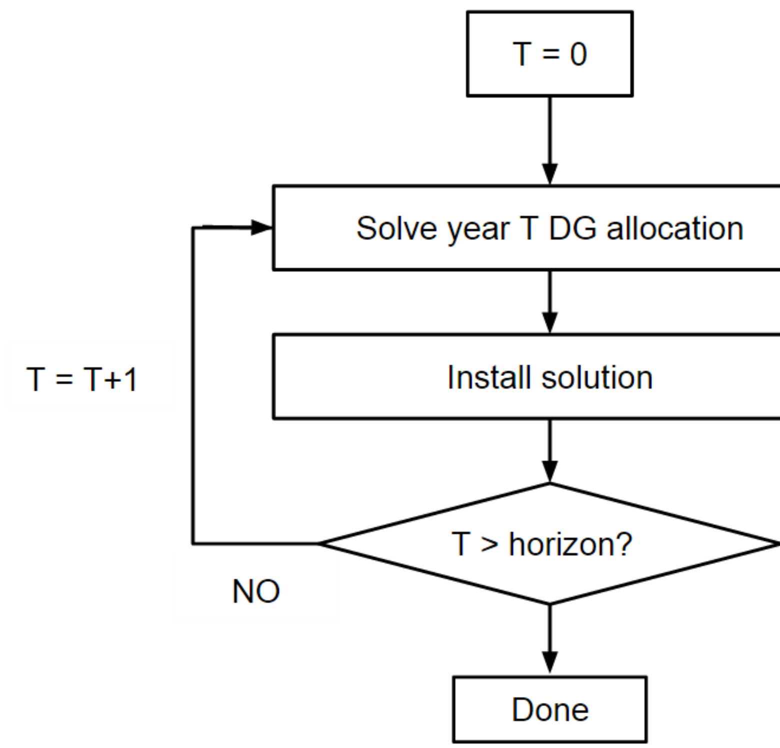


Figure 5.7: Continuous Forward Algorithm

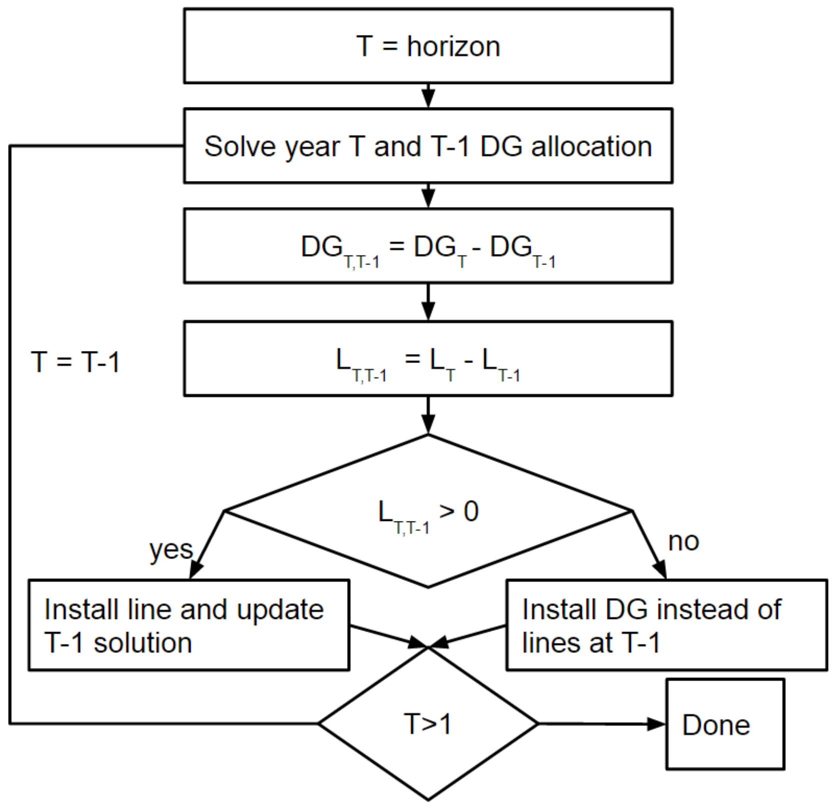


Figure 5.8: Continuous Backward Algorithm

Chapter 6

Simulations

This chapter discusses the application and comparison of the different methods that have been presented in the previous chapter.

6.1 Description of the 69 Bus Distribution Test System

The proposed model presented in the previous chapters is applied to the 69-bus IEEE distribution system shown in figure 6.1 . The system comprises 69 buses, split among several branches with a grid-connected substation at bus-1. The total system peak demand is 3.8MW and 2.7Mvar in year-0 and assumed to grow 5% annually. Each feeder segment is 1 km long for simplification. Appendix A lists all details of associated with the components of the system.The considered test system was simulated and the proposed method executed in the Matlab environment in order to determine the optimal set of recommendations for a 10 year investment plan state by state. The outcome from this model provides the optimal size, location and period of commissioning of distribution system component upgrades along with DG units.

6.2 69-bus IEEE Bus Distribution Test System

This section discusses the DG allocation in the 69 bus system.

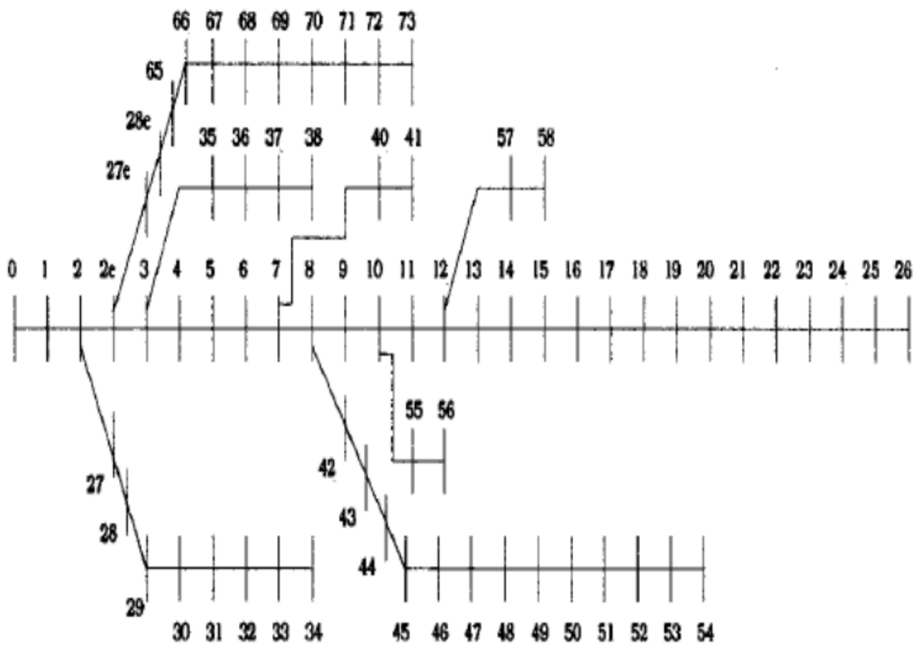


Figure 6.1: IEEE 69 bus system

Table 6.1 presents a comparison between the proposed method and the extensive search when placing a DG unit of a particular size - in this case a percentage of the total load. they both led to the same results which shows the efficiency of the proposed method using different DG sizes.

Table 6.1: results of allocation of different DG sizes

DG capacity	Extensive search result	Graph method result
5%	64	64
10%	64	64
15%	64	64
20%	61	61
25%	61	61
30%	61	61

6.3 69-bus IEEE Bus Distribution Test System

6.3.1 Individual yearly plan

Table 6.3 shows the optimal plan for each year. The system faces undervoltage and congestion during the 10 years period. DG units and lines are installed following the proposed method. Improvement of the voltage is noticed as the DG units are installed in majority in the end of the feeders and hence highly impact the voltage profile in all the system. The same happens with losses. More DG units are installed, more impact on voltage and losses is noticed. The flow method is hence a very simple and fast method to upgrade the system, mitigating the congestion while keeping the losses as low as possible. The cost of the upgrades reduces with time since each DG impact more lines congestion.

The optimization results in table 6.2 represents the phase I of the proposed method. For year 1, DG units are installed at the end of the feeder (buses 64,65) in order to mitigate the congestion in the path 1-65. However, the congestion intensifies every year saturating new lines and hence new nodes in the path are used(61,62,63). In year 9, the path 1-27 is also congested and hence DG units are installed at the buses (24,25,26,27). In addition to the DG units, several lines had to be upgraded. The cost of this upgrade in the last year is $1.89M$.

6.3.2 Forward planning

Table 6.3 shows the results related to this case. Different DG units needed to be installed and lines to be reinforced. one problem with this method is that some reinforced lines become obsolete and not needed in later years, which makes that investment not worth it. In years 1-4, the line 53 is needed as there is few congested lines. As the flow increases by year 5, one DG can mitigate all the lines in that path, and hence the upgrade at line 53 is no more needed and becomes obsolete. It is hence better to place a DG in early stages to provide the power even though it is expensive, as it will target many congested lines in the future.

6.3.3 Backward planning

Table 6.4 shows the results of applying this method. The number of DG units and line reinforcement is much lower than the forward method case. This is due to the fact that the lines are only reinforced if they are mandatory for the end of the planning horizon.

Table 6.2: individual yearly upgrade plan (DG and line capacity in MW)

	1	2	3	4	5	6	7	8	9	10
26	0	0	0	0	0	0	0	0	0	0.055224
61	0	0	0	0	0.914198	1.163892	1.358512	1.570328	1.807641	2.090544
62	0	0	0	0.690759	0	0	0	0	0	0
64	0.249207	0.390886	0.537841	0	0	0	0	0	0	0
DG cost	0.194382	0.304891	0.419516	0.538792	0.713075	0.907836	1.059639	1.224856	1.40996	1.673699
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0.040246	0.185998	0.326716	0.455712	0.499767
3	0	0	0	0	0	0.040065	0.18579	0.326478	0.455439	0.499452
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0.005015	0.063205	0.067283
53	0.102523	0.112192	0.122889	0.134889	0.084946	0.019199	0.021827	0.025314	0.030526	0.040697
54	0.091322	0.099782	0.109126	0.119589	0.067865	0	0	0	0	0
55	0.044246	0.048353	0.052878	0.057927	0	0	0	0	0	0
Lines cost	0.047618	0.052065	0.056979	0.062481	0.030562	0.019902	0.078723	0.136705	0.200976	0.22144
Total cost	0.242	0.356956	0.476494	0.601273	0.743637	0.927738	1.138362	1.361561	1.610936	1.895139

Hence, the early stages are mostly using DG units. There is however a need for line 53 to help the DG target the lines that are not mitigated as it is cheaper.

6.3.4 Comparison between both upgrades

In this section the distribution system plans from the both proposed approaches are compared. It is noted that the second proposed approach (bundled) results in a distribution system expansion plan of lower cost due to the installed lines become obsolete and hence are no more needed. The backward and bundled method both depend on the horizon that is being planned for as further in future it is, better and cheaper the plan is.

Table 6.3: Forward method plan

	1	2	3	4	5	6	7	8	9	10
26	0	0	0	0	0	0	0	0	0	0.055224
61	0	0	0	0	0.223439	0.473133	0.667753	0.879569	1.116882	1.399785
62	0	0	0	0.152918	0.152918	0.152918	0.152918	0.152918	0.152918	0.152918
64	0.249207	0.390886	0.537841	0.537841	0.537841	0.537841	0.537841	0.537841	0.537841	0.537841
Cost DG	0.194382	0.304891	0.419516	0.538792	0.713075	0.907836	1.059639	1.224856	1.40996	1.673699
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0.040246	0.185998	0.326716	0.455712	0.499767
3	0	0	0	0	0	0.040065	0.18579	0.326478	0.455439	0.499452
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0.005015	0.063205	0.067283
53	0.102523	0.112192	0.122889	0.134889	0.134889	0.134889	0.134889	0.134889	0.134889	0.134889
54	0.091322	0.099782	0.109126	0.119589	0.067865	0.119589	0.119589	0.119589	0.119589	0.119589
55	0.044246	0.048353	0.052878	0.057927	0.057927	0.057927	0.057927	0.057927	0.057927	0.057927
Cost lines	0.047618	0.052065	0.056979	0.062481	0.052136	0.078543	0.136839	0.194123	0.257352	0.275781
Total cost	0.242	0.356956	0.476494	0.601273	0.765211	0.986379	1.196478	1.418979	1.667312	1.94948

Table 6.4: Backward method plan

	1	2	3	4	5	6	7	8	9	10
26	0	0	0	0	0	0	0	0	0	0.055224
61	0.384775	0.539021	0.699845	0.868275	0.982064	1.163892	1.358512	1.570328	1.807641	2.090544
62	0	0	0		0	0	0	0	0	0
64	0	0	0	0	0	0	0	0	0	0
Cost DG	0.300125	0.420436	0.545879	0.677254	0.76601	0.907836	1.059639	1.224856	1.40996	1.673699
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0.040246	0.185998	0.326716	0.455712	0.499767
3	0	0	0	0	0	0.040065	0.18579	0.326478	0.455439	0.499452
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0.005015	0.063205	0.067283
53	0.102523	0.112192	0.122889	0.134889	0.084946	0.019199	0.021827	0.025314	0.030526	0.040697
54	0	0				0	0	0	0	0
55	0	0			0	0	0	0	0	0
Cost lines	0.020505	0.022438	0.024578	0.026978	0.016989	0.019902	0.078723	0.136705	0.200976	0.22144
Total cost	0.320629	0.442875	0.570457	0.704232	0.782999	0.927738	1.138362	1.361561	1.610936	1.895139

Chapter 7

Conclusions and Future Work

7.1 Summary

Deregulation of the power industry, policy changes and advancements in DG technologies affected the design and the planning of distribution system. The main goal is to mitigate congestion at minimum cost in the distribution system for the whole planning horizon while taking into account the losses and dynamicity of the network. The proposed method is based on graph theory, and more precisely on the Successive Shortest Path (SSP). The algorithm has for aim to incorporate DG units and feeders upgrades as to meet the demand for the planning period. This approach is very efficient and can be adapted to the different needs of the planner. The method has been tested on a radial 14-bus system to show its simplicity. It has also been tested on the 69-bus IEEE system and shows satisfying results.

7.2 Future Work

Further research can be conducted based on the work presented in this thesis. Some ideas are presented below:

- In this thesis, parameters, such as market prices and future capital costs, etc. are assumed to be deterministic; however, in future research their uncertainty may be considered. Consequently, stochastic optimization, or robust programming techniques, can be applied to the proposed framework to mitigate the effects of this uncertainty.

- Gas turbine DG is considered in the proposed framework; however, different DG technologies such as renewable-energy based DG and power electronics based may be considered in future research.
- It may be useful to examine the role that DG units have providing ancillary services such as reactive power and control capabilities, and consider them in determining optimal placement. Reactive power provide support to the voltage profile whereas the control capabilities will allow more flexibility and responsiveness to the system in case of abnormal events.

APPENDICES

Appendix A

IEEE 69 bus data

The data of the IEEE69 bus test system is presented in the following tables[31].

Table A.1: Load consumption in IEEE69 bus test system

Bus No	Load $(MW, Mvar)$	Bus No	Load $(MW, Mvar)$	Bus No	Load $(MW, Mvar)$
1	(0,0)	2	(0,0)	3	(0,0)
4	(0,0)	5	(0,0)	6	(0.0026,0.0022)
7	(0.0404,0.03)	8	(0.075,0.054)	9	(0.03,0.022)
10	(0.028,0.019)	11	(0.145,0.104)	12	(0.145,0.104)
13	(0.008,0.0055)	14	(0.008,0.0055)	15	(0,0)
16	(0.0455,0.03)	17	(0.06,0.035)	18	(0.06,0.035)
19	(0,0)	20	(0.001,0.0006)	21	(0.114,0.081)
22	(0.0053,0.0035)	23	(0,0)	24	(0.028,0.02)
25	(0,0)	26	(0.014,0.01)	27	(0.014,0.01)
28	(0.026,0.0186)	29	(0.026,0.0186)	30	(0,0)
31	(0,0)	32	(0,0)	33	(0.014,0.01)
34	(0.0195,0.014)	35	(0.006,0.004)	36	(0.026,0.01855)
37	(0.026,0.01855)	38	(0,0)	39	(0.024,0.017)
40	(0.024,0.017)	41	(0.0012,0.001)	42	(0,0)
43	(0.006,0.0043)	44	(0,0)	45	(0.03922,0.0263)
46	(0.03922,0.0263)	47	(0,0)	48	(0.079,0.0564)
49	(0.3847,0.2745)	50	(0.3847,0.2745)	51	(0.0405,0.0283)
52	(0.0036,0.0027)	53	(0.00435,0.0035)	54	(0.0264,0.019)
55	(0.024,0.0172)	56	(0,0)	57	(0,0)
58	(0,0)	59	(0.1,0.072)	60	(0,0)
61	(1.244,0.888)	62	(0.032,0.023)	63	(0,0)
64	(0.227,0.162)	65	(0.059,0.042)	66	(0.018,0.013)
67	(0.018,0.013)	68	(0.028,0.02)	69	(0.028,0.02)

Table A.2: Lines in IEEE69 bus test system - 1

Bus 1	Bus 2	Resistance(Ohm)	Reactance(Ohm)	Line Capacity (MVA)
1	2	3.1e-005	7.5e-005	3.798
2	3	3.1e-005	7.5e-005	3.798
3	4	9.4e-005	0.000225	3.798
4	5	0.001566	0.001834	3.798
5	6	0.022836	0.01163	3.798
6	7	0.023778	0.01211	3.798
7	8	0.005753	0.002932	3.798
8	9	0.003076	0.001566	3.798
9	10	0.051099	0.01689	1.266
10	11	0.01168	0.004311	1.266
11	12	0.044386	0.014668	1.266
12	13	0.064264	0.021213	1.266
13	14	0.065138	0.021525	1.266
14	15	0.066011	0.021812	1.266
15	16	0.012266	0.004056	1.266
16	17	0.02336	0.007724	1.266
17	18	0.000293	0.0001	1.266
18	19	0.02044	0.006757	1.266
19	20	0.01314	0.004343	1.266
20	21	0.021313	0.007044	1.266
21	22	0.000873	0.000287	1.266
22	23	0.009927	0.003282	1.266
23	24	0.021607	0.007144	1.266
24	25	0.04672	0.017127	1.266
25	26	0.019273	0.00637	1.266
26	27	0.010806	0.003569	1.266
3	28	0.000275	0.000674	1.266
28	29	0.003993	0.009764	1.266
29	30	0.02482	0.008205	1.266
30	31	0.00438	0.001448	1.266
31	32	0.0219	0.007238	1.266
32	33	0.052347	0.01757	1.266

Table A.3: Lines in IEEE69 bus test system - 2

Bus 1	Bus 2	Resistance(Ohm)	Reactance(Ohm)	Line Capacity (MVA)
33	34	0.10657	0.035227	1.266
34	35	0.091967	0.030404	1.266
3	36	0.000275	0.000674	1.266
36	37	0.003993	0.009764	1.266
37	38	0.00657	0.007674	1.266
38	39	0.001897	0.002215	1.266
39	40	0.000112	0.000131	1.266
40	41	0.04544	0.05309	1.266
41	42	0.019342	0.022605	1.266
42	43	0.002558	0.002982	1.266
43	44	0.000574	0.000724	1.266
44	45	0.006795	0.008566	1.266
45	46	5.6e-005	7.5e-005	1.266
4	47	0.000212	0.000524	1.899
47	48	0.00531	0.012996	1.899
48	49	0.018081	0.044243	1.899
49	50	0.005129	0.012547	1.899
8	51	0.00579	0.002951	1.899
51	52	0.020708	0.006951	1.899
9	53	0.010856	0.005528	1.899
53	54	0.012666	0.006451	1.899
54	55	0.017732	0.009028	1.899
55	56	0.017551	0.008941	1.899
56	57	0.099204	0.033299	1.899
57	58	0.048897	0.016409	1.899
58	59	0.01898	0.006277	1.899
59	60	0.02409	0.007312	1.899
60	61	0.031664	0.016128	1.899
61	62	0.006077	0.003095	1.899
62	63	0.009047	0.004605	1.899
63	64	0.04433	0.02258	1.899

Table A.4: Lines in IEEE69 bus test system - 3

Bus 1	Bus 2	Resistance(Ohm)	Reactance(Ohm)	Line Capacity (MVA)
64	65	0.064951	0.033081	1.899
11	66	0.012553	0.003812	1.899
66	67	0.000293	8.7e-005	1.899
12	68	0.046133	0.015249	1.899
68	69	0.000293	0.0001	1.899

References

[1] M.F. Shaaban, Y.M. Atwa, and E.F. El-Saadany. DG allocation for benefit maximization in distribution networks. Power Systems, IEEE Transactions on, 28(2):639-649, May 2013.

[2] Yaosuo Xue, Liuchen Chang, and J. Meng. Dispatchable distributed generation network - a new concept to advance DG technologies. In Power Engineering Society General Meeting, 2007. IEEE, pages 1-5, June 2007.

[3] P. Siano, L.F. Ochoa, G.P. Harrison, and A. Piccolo. Assessing the strategic benefits of distributed generation ownership for DNOs. Generation, Transmission Distribution, IET, 3(3):225-236, March 2009.

[4] S. Wong, K. Bhattacharya, and J.D. Fuller. Electric power distribution system design and planning in a deregulated environment. Generation, Transmission Distribution, IET, 3(12):1061-1078, December 2009.

[5] A.A. Chowdhury, S.K. Agarwal, and D.O. Koval. Reliability modeling of distributed generation in conventional distribution systems planning and analysis. Industry Applications, IEEE Transactions on, 39(5):1493-1498, Sept 2003.

[6] J.M. Nahman and D.M. Peric. Optimal planning of radial distribution networks by simulated annealing technique. Power Systems, IEEE Transactions on, 23(2):790-795,May 2008.

[7] M. Haghifam, H. Falaghi, and O.P. Malik. Risk-based distributed generation placement. Generation, Transmission Distribution, IET, 2(2):252-260, March 2008.

[8] G. Celli, E. Ghiani, S. Mocci, and F. Pilo. A multiobjective evolutionary algorithm for the sizing and siting of distributed generation. Power Systems, IEEE Transactions on, 20(2):750-757, May 2005.

[9] W. El-Khattam, K. Bhattacharya, Y. Hegazy, and M.M.A. Salama. Optimal investment planning for distributed generation in a competitive electricity market. Power Systems, IEEE Transactions on, 19(3):1674-1684, Aug 2004.

[10] S.Wong, K. Bhattacharya, and J.D. Fuller. Long-term effects of feed-in tariffs and carbon taxes on distribution systems. Power Systems, IEEE Transactions on, 25(3):1241 1253, Aug 2010.

[11] S. Wong, K. Bhattacharya, and J.D. Fuller. Coordination of investor-owned DG capacity growth in distribution systems. Power Systems, IEEE Transactions on, 25(3):1375-1383, Aug 2010.

[12] D.T.-C.Wang, L. Ochoa, and G. Harrison. DG impact on investment deferral: Network planning and security of supply. In Power and Energy Society General meeting, 2010 IEEE, pages 1-1, July 2010.

[13] S. Abapour, K. Zare, and B. Mohammadi-ivatloo. Maximizing penetration level of distributed generations in active distribution networks. In Smart Grid Conference (SGC), 2013, pages 113-118, Dec 2013.

[14] M. Fotuhi-Firuzabad and A. Rajabi-Ghahnavie. An analytical method to consider DG impacts on distribution system reliability. In Transmission and Distribution Conference and Exhibition: Asia and Pacific, 2005 IEEE/PES, pages 1-6, 2005.

[15] Y.M. Atwa and E.F. El-Saadany. Optimal allocation of ESS in distribution systems with a high penetration of wind energy. Power Systems, IEEE Transactions on, 25(4):1815-1822, Nov 2010.

[16] N. Kumar, P. Dutta, and Le Xie. Optimal DG placement for congestion mitigation and social welfare maximization. In North American Power Symposium (NAPS), 2011, pages 1-5, Aug 2011.

[17] Y.P. Patel and A.G. Patel. Placement of DG in distribution system for loss reduction. In Power India Conference, 2012 IEEE Fifth, pages 1-6, Dec 2012.

[18] M.F. Shaaban and E.F. El-Saadany. Optimal allocation of renewable DG for reliability improvement and losses reduction. In Power and Energy Society General Meeting, 2012 IEEE, pages 1-8, July 2012.

[19] N. Jain, S.N. Singh, and S.C. Srivastava. Planning and impact evaluation of distributed generators in Indian context using multi-objective particle swarm optimization. In Power and Energy Society General Meeting, 2011 IEEE, pages 1-8, July 2011.

[20] S. Kansal, B.B.R. Sai, B. Tyagi, and V. Kumar. Optimal placement of wind-based generation in distribution networks. In Renewable Power Generation (RPG 2011), IET Conference on, pages 1-6, Sept 2011.

[21] M. Fotuhi-Firuzabad and A. Rajabi-Ghahnavie. An analytical method to consider DG impacts on distribution system reliability. In Transmission and Distribution Conference and Exhibition: Asia and Pacific, 2005 IEEE/PES, pages 1-6, 2005.

[22] Duong Quoc Hung, N. Mithulananthan, and R.C. Bansal. Analytical expressions for DG allocation in primary distribution networks. Energy Conversion, IEEE Transactions on, 25(3):814-820, Sept 2010.

[23] R.K. Hosseini and R. Kazemzadeh. Optimal DG allocation by extending an analytical method to minimize losses in radial distribution systems. In Electrical Engineering (ICEE), 2011 19th Iranian Conference on, pages 1-1, May 2011.

[24] V. Calderaro, A. Piccolo, and P. Siano. Maximizing DG penetration in distribution networks by means of GA based reconfiguration. In Future Power Systems, 2005 International Conference on, pages 6 pp.-6, Nov 2005.

[25] M.F. Akorede, H. Hizam, I. Aris, and M.Z.A. Ab Kadir. Effective method for optimal allocation of distributed generation units in meshed electric power systems. Generation, Transmission Distribution, IET, 5(2):276-287, February 2011.

[26] A. Vahidnia, G. Ledwich, A. Ghosh, and E. Palmer. An improved genetic algorithm and graph theory based method for optimal sectionalizer switch placement in distribution networks with DG. In Universities Power Engineering Conference (AUPEC), 2011 21st Australasian, pages 1-7, Sept 2011.

[27] P.H. Nguyen, W.L. Kling, and P.F. Ribeiro. Agent-based power routing in active distribution networks. In Innovative Smart Grid Technologies (ISGT Europe), 2011 2nd IEEE PES International Conference and Exhibition on, pages 1-6, Dec 2011.

[28] J. Quiros-Tortos and V. Terzija. A graph theory based new approach for power system restoration. In PowerTech (POWERTECH), 2013 IEEE Grenoble, pages 1-6, June 2013.

[29] Huanhai Xin, Zhihua Qu, J. Seuss, and A. Maknouninejad. A self-organizing strategy for power flow control of photovoltaic generators in a distribution network. Power Systems, IEEE Transactions on, 26(3):1462-1473, Aug 2011.

[30] S. M. Wong, "Some Aspects of Distribution System Planning in the Context of Investment in Distributed Generation," , PhD Thesis, University of Waterloo, 2004.

[31] Ritu Parasher, " Low Flow Analysis Of Radial Distribution Network Using Linear Data Structure," , Master's Thesis, Rajasthan Technical University, 2013