Alternative Electricity Market Systems for Energy and Reserves using Stochastic Optimization

by

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AUTHOR'S DECLARATION FOR ELECTRONIC SUBMISSION OF A THESIS

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

This thesis presents a model that simulates and solves power system dispatch problems utilizing stochastic linear programming. The model features the ability to handle single period, multiple bus, linear DC approximated systems. It determines capacity, energy, and reserve quantities while accounting for N-1 contingency scenarios (single loss of either generator or line) on the network. Market systems applying to this model are also proposed, covering multiple real-time, day-ahead, and hybrid versions of consumer costing, transmission operator payment, and generator remuneration schemes. The model and its market schemes are applied to two test systems to verify its viability: a small 6-bus system and a larger 66-bus system representing the Ontario electricity network.

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Contents

List of Tables

List of Figures

Chapter 1

Introduction

This section outlines the concept of reserves and their role as an ancillary service (A/S) in electricity networks. Introducing this concept will be an overview of electricity networks and the major regulatory agencies in North America that watch over their operation (including defining of ancillary services). Since the Ontario system is simulated as a test system, its governing agencies will also be reviewed. An overview of all recognized A/S will be given followed by an in-depth examination of operating reserves, including different types, roles, and sources in the bulk electric system. Issues pertaining to network security will be discussed at this time.

Concluding this technical glance of the electricity network will be an examination of the associated market concepts, including dispatch methods (merit order, sequential, and simultaneous) and various area pricing schemes.

1.1 Electricity Networks

The electric power system, in its most elementary form, is responsible for delivering energy in the form of electricity from suppliers to consumers (supply and demand) through a network of transmission lines. Buses, which may be interchangeably referred to as nodes, are points in the system where two or more elements connect. These elements consist primarily of generators, loads, and transmission lines. Buses that are connected to generators (which supply electricity to the system) are known as generator buses (supply nodes) and those connected to loads (which consume energy from the system) are similarly called load buses (demand nodes). Buses that are neither connected to generators nor loads may also exist. These generators, loads, and transmission lines have a multitude of characteristics, a selection of which (those used in the model) are discussed in Chapter 3.

1.2 N-1 Contingency

A system with N number of elements, all of which are operational, is said to be operating in the 'N' state. A N-1 contingency occurs when any single element of the system is removed from service (e.g. due to an equipment malfunction). The N-1 contingency criterion states that, in this condition, the electric system should be able to remain secure and operational. This criterion may be extended to encompass the failure of X number of elements, becoming 'N-X contingency criterion.' [2]

The N-1 contingency criterion, encompassing generator and line outages, forms the basis for the stochastic aspect of the presented model.

1.3 Regulatory Agencies

There are two major regulatory agencies that monitor and regulate the bulk electric power system in North America: the North American Electric Reliability Council (NERC), whose domain covers regions in Canada and the U.S.; and the Federal Energy Regulatory Commission, a U.S. agency.

1.3.1 North American Electric Reliability Council

The North American Electric Reliability Council (NERC) is a voluntary not-for-profit organization whose mission is to ensure the reliable, adequate, and secure operation of the bulk electric system within North America. To reach this goal, NERC establishes standards and guidelines and subsequently monitors and enforces their adherence by member organizations. [3]

Ten regional reliability councils make up the members of NERC, whose members in turn represent all sectors and interests in the electric industries, from governments to utilities. Constituents of the Ontario electric system fall under the Northeast Power Coordinating Council (NPCC) [4], discussed in section 1.3.3.

NERC is composed of multiple committees, subcommittees, and working groups who examine, assess, and make policies or recommendations regarding specific areas within the bulk electric system, including reserves.

1.3.2 Federal Energy Regulator Commission

The Federal Energy Regulator Commission (FERC) is the U.S. federal agency mandated to oversee the "energy industries in the economic and environmental interest of the American public" [5] with an outlook toward economic competition within these markets. Concerning the electrical industry, FERC is responsible for the interstate transmission and sale of bulk electricity.

1.3.3 NPCC and IESO Overview and Responsibilities

The Northeast Power Coordinating Council (NPCC) is one of ten regional councils under the auspices of the North American Electric Reliability Council. This council, of which Ontario is a member, also encompasses Quebec and the Maritimes as well as a number of northeastern American States. The NPCC, whose mission is to ensure a reliable interconnected power system, regularly conducts assessments of its members' compliances to its standards and requirements, imposing sanctions if necessary. [4] NPCC's guides and policies follow those of NERC's, with specific requirements and augmentations as required.

In Ontario, market operation falls under the auspices of the Independent Electricity System Operator. The IESO, who is a member of NERC and the NPCC, sets its standards according to their guidelines and policies. In addition, the IESO sets all market related practices. [6]

1.4 Overview of Ancillary Services

Interconnected operations services (IOS) and ancillary services (A/S) have been defined, to a great extent, by the NERC and FERC, respectively. Although their definitions and included services may vary slightly depending on the issuing authority, the terms IOS and ancillary services can be and are used interchangeably, with the latter the most dominant in the industry.

Responsible for the IOS is the IOS Subcommittee (IOSS), who is responsible for the development and maintenance of definitions, policies, practices, and standards of all things regarding IOS. [7] The details of such will be embodied as Policy 10 in NERC's Operating Manual [1], subject to approval.

In FERC's Order 888, a document ordering "sweeping" changes to the electricity industry regarding the unbundling of services, six ancillary services were recognized. In this document, FERC ordered that these particular services be included in an open access transmission tariff. Other services were recognized to exist, but were not identified in this document. [8]

In this section, IOS recognized by NERC will be the focus. NERC defines IOS as the "elementary 'reliability' building blocks from generation (and sometimes load) necessary to maintain bulk electric system reliability". [9] Moreover, they must be capabilities able to be deployed to meet current and future reliability objectives.

The IOS reference document identifies six core IOS, so chosen because they are uniquely measurable and have distinct impacts on system reliability criteria. Each of the services is also affiliated with one of three corresponding reliability objectives, as listed below. [1]

- Resource and Demand Balance
  - Regulation
  - Load Following
  - Contingency Reserve
    - Spinning
    - Supplemental
- Bulk Transmission Reliability
  - Reactive Power Supply from Generation Services
  - Frequency Response
- Emergency Preparedness
  - System Black Start Capability

Under the resource and demand balance objective falls the regulation, load following, and contingency reserve (encompassing spinning and supplemental reserves) services. As the name suggests, these services are responsible for ensuring that there is always enough supply to meet moment by moment demand.

The second reliability objective, bulk transmission, is responsible for ensuring network (transmission system) security. The IOS tasked under that objective are reactive power supply from generation sources and frequency response.

Finally, under emergency preparedness falls system black start capability. This reliability objective addresses the issue of restoring the bulk electric system in the event of a catastrophic failure.

As this thesis focuses on the deployment of reserves, the primary interest is in the first reliability objective, resource and demand balance.

1.4.1 Operating Reserves

As specified in NERC's operating policy 1, operating reserves must be "sufficient to account for such factors as forecasting errors, generation and transmission equipment unavailability, system equipment forced outage rates, maintenance schedules, regulating requirements, and load diversity." [9] These requirements are met through NERC's listed IOS reserve services, detailed in section 1.5.

There are multiple subcategories of operating reserves, each of which can be ordered by their quality (where high quality corresponds to a short time to deployment). These reserves, in descending order of quality, are frequency response, regulation, contingency reserve - spinning, contingency reserve - non-spinning, and load following reserves. Typically higher quality reserves can be used in place of a lower quality reserve, but at a cost. Table 1.1, itemizes the deployment period of each reserve.

Load-serving reserves or backup supplies may also compose the operating reserve; however, they are not identified as IOS as they do not support the reliability of the bulk electric system.

1.5 Resource and Demand Balance

Reiterating the previous section, the resource and demand balance reliability services are responsible for ensuring that supply always matches demand. These services are comprised of a series of reserves under the 'operating reserves' banner.

Table 1.1: Reserve Deployment Periods [1]

Reserve	Deployment Period		
	Seconds	Minutes	Hours
Continuous			
Regulation	X	X	X
Load Following		X	X
Post Contingency			
Frequency Response	<<		
Contingency Reserve - Spinning		X	
Contingency Reserve - Non-Spinning		X	

1.5.1 Regulation and Load Following

During normal (non-contingency) operation of the bulk electric grid, demand will naturally deviate from forecasted loads. Since generation must always match demand, two interconnected operations services are tasked to address these natural deviations: regulation and load following.

There are two different 'types' of normal (non-contingency related) deviations to which these services are addressed, rapid fluctuations and trends in power demand. Figure 1.1 illustrates the differences.

As illustrated in figure 1.1, the regulation service addresses small, unpredictable minute to minute variations in demand. This is natural as consumers often turn on and off load with no particular correlation. In contrast, the load following service addresses imbalances within a longer period of time (the scheduling period - usually an hour) and compensates for large, predictable

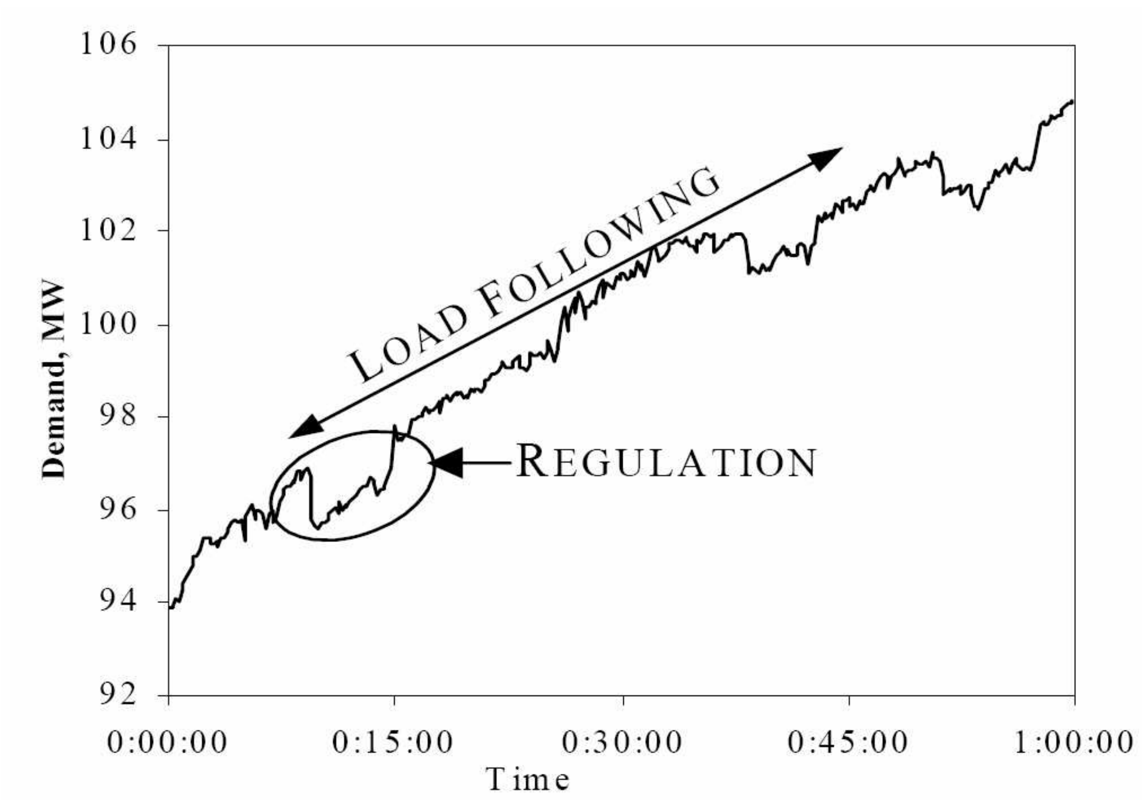

Figure 1.1: Load Following and Regulation [1]

changes in demand. This may occur, for example, if a colder day than predicted occurs and consumers gradually ramp up the heat.

Regulating reserves can be provided for by the same technology as spinning reserves, namely automatic governor controls (AGC), with the difference being that generation reduction (regulation down) is required in addition to generation increases (regulation up). For a further description of the AGC refer to the next section.

The same class of resources used to supply spinning, non-spinning, and regulating reserves, in addition to other long term reserves, may be used to supply the load following reserves. For further information on this, refer to the following sections.

1.5.2 Contingency Reserve

Purpose

Although contingencies are an ‘expected’ occurrence within the bulk electric system, especially given the vast and interconnected systems currently operating within North America, they are not part of normal system operation. They must, however, be prepared for and solutions readied at all times. These contingencies can range from transmission line interruption (e.g. shorting from lines touching brush) to transformer or generator failures.

In the event of a significant contingency, power delivery may be restricted from one area to another (in the event of a line failure), or a power source may be cut out entirely (in the case of a generator failure). Following such possibilities, generation must be replaced and/or redistributed across the grid. This is usually done by the system operator calling upon selected contingency reserves to be activated.

Definition

Contingency reserves are classed into two categories: spinning and non-spinning. Spinning reserves are broadly defined as unloaded capacity, spinning and synchronized to the grid, fully available to the system (i.e. to take on full load) within 10 minutes of being called upon.

Non-spinning reserves encompass generation capacity not connected (or synchronized) to the grid that can be called up within 10 minutes to supply power. This category of reserve may also include any load reduction capacity available to the system operator within 10 minutes. The model presented in Chapter 3 does not differentiate between spinning and non-spinning reserves.

Specific contingency reserve requirements differ between operators belonging to different regional reliability councils. The most common requirement, however, is usually some formulation requiring enough reserve to cover at least the single largest possible contingency. In some cases it may be specified by the council as a percentage of load (typically around 7%) or it may be left up to the system operator to calculate [10] [11]. The model later presented uses the more complex N-1 contingency criterion.

Provision

The provision of spinning reserve capacity by a generator may be attained through changing the set points of its automatic governor control. Figure 1.2 is a simple representation of a generation system and will be used to illustrate the AGC's operation.

The majority of power is generated by turbines driven by steam or water, which in turn drives a synchronous generator thus injecting AC power into the system (represented in the figure as 'load'). The governor is a device forming the feedback loop from the turbine shaft to the steam or water input. In essence, it is one method of controlling the frequency and magnitude of the power generated.

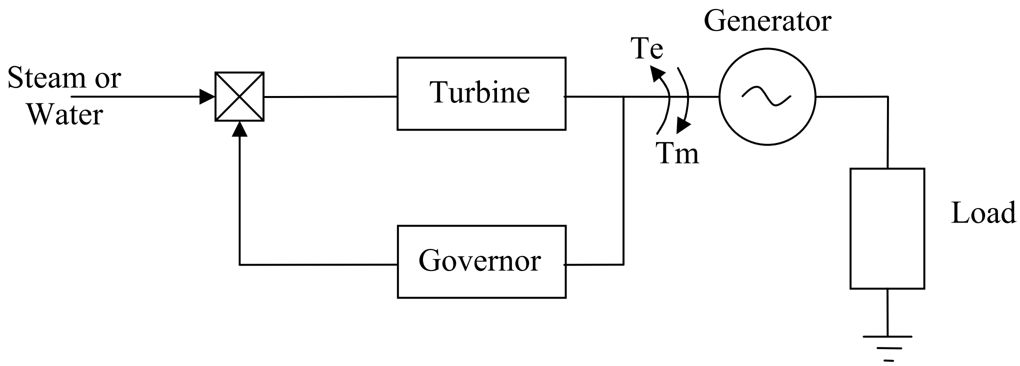

Figure 1.2: Generation Unit with Governor

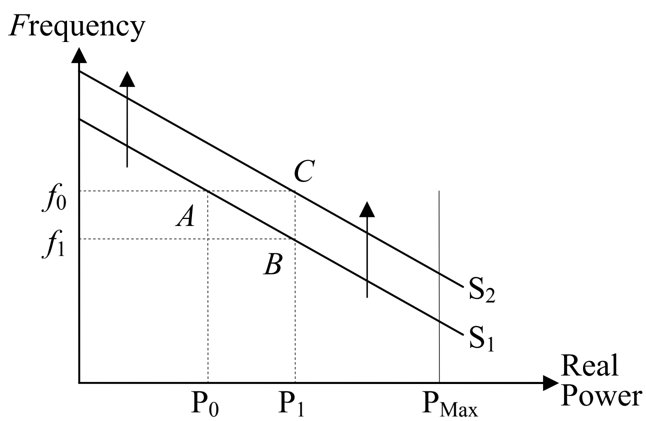

Figure 1.3: Droop Curves and AGC Operation

There are two adjustable ‘characteristics’ associated with the governor: the droop curve and the set point. The droop curve, lines S_1 and S_2 in figure 1.3, is involved with frequency response (the details of this are beyond the scope of this research). However, the AGC is also responsible for automatic adjustments of the governor set, which shifts the droop curve either up or down. Figure 1.3 illustrates this shifting.

This can be used in the provision of reserves. Given an order to increase real power output from P_0 to P_1, the governor set points must be changed from S_1 to S_2. This will maintain a constant frequency of f_0 but result in a change of power, P_1 - P_0. These set points are also representative of generator loading, for example S_1 maybe equivalent to 60% load and S_2 to 95%.

Spinning reserve requirements may also be met though generators connected and spinning but with their turbines un-clutched. Given instructions to supply generation, the turbine can be clutched to the generator, providing synchronized, added capacity [12].

Non-spinning reserves may be provided by equipment not synchronized to the system, for example, thermal units that are 'not started but ready', in addition to generators capable of providing spinning reserves. Furthermore, loads that can be directed to cut their consumption by a specified amount may also be considered under this category. As non-spinning reserves typically take longer to react, they can be regarded as a lower quality source of contingency reserves than spinning.

Costs

The provisioning of all operating reserves (load following, regulation, or contingency) incurs an opportunity cost to the generator. This is the loss of net revenues by not being able to supply the power to the energy markets, as it must be held in reserve. Typically, this would be the market energy price, minus maintenance and fuel costs.

1.6 Dispatch Methods

In competitive markets, the dispatch of contingency reserves is often determined in tandem with that for meeting energy demands. Given a competitive pool market involving these services, there are three principal methods by which dispatch may be determined: merit-order-based dispatch, sequential dispatch, or joint (simultaneous) dispatch [13]. These different systems each have trade-offs between higher co-ordination and greater reliance on private markets [14].

1.6.1 Merit Order

Under merit order dispatch the markets for each product (i.e. energy and reserve) are operated separately and independently of one another. Bidders submit their price/quantity pairs to the appropriate product market, where the operator ranks and selects (dispatches) them based on merit (lowest cost first). Since each product is treated as entirely separate (though both energy and reserves share a generator's capacity), dispatch solutions may be neither optimal nor even feasible [13].

1.6.2 Sequential

In a sequential dispatch system, energy and reserves are recognized as sharing generator capacity. Suppliers submit their price/quantity bid pairs to the market operator who pools then together and then sequentially assigns resources, based on bid price, to required services in order of priority (from high to low). These service requirements, ordered from highest to lowest priority, are energy, regulating reserves, spinning reserves, non-spinning reserves, load following, and backup support. As each product is dispatched, the available capacity in the pool is reduced.

The benefit of this market form is that it is voluntary in nature and is thought to promote market efficiency while avoiding the gaming prevalent in centralized (i.e. simultaneous) dispatch methods [14]. The setbacks of this method include not knowing the best trade-offs for sharing limited resources and capacity. Furthermore, since each market is operated separately, prices for lower quality reserves may exceed those for higher quality reserves, leaving little incentive to bid in the higher quality market. This may result in insufficient allocation of resource to higher priority requirements (such as spinning reserves) while lower quality services (such as load following) may be over-served [15] [16].

1.6.3 Simultaneous (Joint) Dispatch

The simultaneous method assigns resources to system requirements simultaneously, imitating the structure of vertical operations (those seen before deregulation) [14]. Each service is assigned resources within the same consolidated market where higher quality resources can be applied to lower quality services. Essentially, this method is treated as a constrained optimization problem, leading to economic and secure solutions to system needs [16]. This integrated system receives its gains from tighter co-ordination resulting in stronger system reliability.

This dispatch system has superior pricing than the sequential system, with prices accurately reflecting the opportunity cost of scarce resources. Furthermore, it is more likely to assign resources such that all energy and reserve are met than the sequential dispatch method. Unfortunately, its complex and 'black-box' solution and assignment algorithms are vulnerable to gaming by participants within the system [14]. The proposed dispatch model takes the simultaneous dispatch approach.

1.7 Nodal, Uniform, and Zonal Pricing

Three methods are used for assigning pricing in market systems: uniform marginal pricing (UMP), zonal marginal pricing (ZMP), and nodal marginal pricing (NMP). UMP, used in such markets as the UK, Ontario, and Sweden, assigns a single price uniformly across all nodes in the system. In ZMP schemes, used in Norway and Denmark, one price is assigned to nodes within a zone (a set of nodes), but prices may vary from zone to zone. Finally, in NMP, used in Argentina and California, there is a separate price for each node of the system, reflecting congestion and line losses [17] [18].

Chapter 2

Literature Review

In addition to the fundamentals on energy networks and market structure outlined in the introduction, a review of literature more closely pertaining to the topic of this thesis is given. The areas of research reviewed in this section are pricing of energy and reserve under various market structures, dispatching of reserves in deterministic settings, and the optimal allocation of reserves in a stochastic environment. While the former sports a significant quantity of research, less is available on the latter.

2.1 Pricing of Energy and Reserves

Alaywan et al. [14] present a detailed comprehensive AC (non-linear) model for simultaneous auction electricity markets. Through the use of its Lagrangian, they define marginal prices of energy and various ancillary services, including regulation, and spinning reserves. Although applied to a non-linear program, the concept used by Alaywan et al. is similar to the application of duals to find prices in the linear program of Chapter 3.

In Arroyo and Galiana [19], the nodal marginal pricing of energy and multiple reserve types in a detailed simultaneous market are also explored. Two major claims are made by the authors: local reserves should not be pre-specified but remain as decision variables within the constraints; and that there are no differential prices between different types of reserves. This approach, of not pre-specifying reserves, is similar to Chapter 3's method of using the N-1 contingency criterion to determine reserve quantities. Although alike in this manner, Arroyo and Galiana do not include reserve energy costs within the objective function, which is done in the proposed model and other literature, as introduced below.

2.2 Deterministic Dispatching of Reserves

The concept of using a market structure that dispatches energy reserves in consideration of its probability of use is established in Singh [20]. In his paper, Singh describes a single period, one bus system whereby there is a separate auction for each reserve type. Each reserve auction is settled through a cost minimization function, which considers each participant's reserve capacity bids and reserve energy bids (the cost of turning the reserve into energy). Paired with each energy bid is also a probability factor - the likelihood of the reserve being called to use. The function is, of course, subject to the usual maximum output and minimum required quantity constraints.

Singh analyzes three different methods with which the ISO may choose to run the reserve auction. These methods involve assigning the probability of reserve energy utilization to 0, 0 to 1, or 1. Setting probability to 0 forces bids to be ranked based solely on capacity bids, which has the added effect of mitigating gaming. Determining and assigning probabilities between 0 and 1 can be used to pursue a true cost minimization solution. Finally, setting probability to 1 gives an estimation of the maximum payments.

Rashidinejad et al. [21] also propose a single period, one bus system that concerns itself with the probability of reserve energy utilization. However, unlike with Singh's, dispatch of reserves is jointly determined with energy dispatch (simultaneous dispatch). Two frameworks of optimization are presented, one based solely on bids and the other on costs. The authors conclude that, compared to merit order and sequential dispatch, joint energy and reserve dispatch is the most economic and secure. This thesis uses this apporach and expands on it by introducing stochastic, multiple bus systems.

2.3 Stochastic Modelling of Contingencies

Bouffard et al. [22], in a recent conference paper, cite two drawbacks of using deterministic models: (1) required constraints may be unachievable and (2) social welfare may not be optimized. In their paper they use stochastic programs in presenting a multi-period, networked model, considering generation energy, reserve, reserve energy costs, demand benefit, and value of expected load not served (ELNS) in their objective function. The latter, ELNS, represents value lost from load shedding. Since the proposed model is multi-period, unit commitment issues are integrated, including ramp up/ down rates from pre-contingency to post-contingency states. In their second accompanying paper [23], where the model was applied to two test systems, it was shown that there are "potential economic benefits of a stochastic market-clearing formulation through the optimization of the expected costs of reserve deployment and involuntary load shedding" [23]. They also identify possible issues and solutions to solving such extensive optimization problems.

Chapter 3

Model Description

The purpose of this model is to mathematically describe a single period, multiple bus, simultaneous dispatch electrical system using linear programming (LP) techniques. This model expands on the technique, using the probability of reserve usage in allocating dispatch quantities, first used by Singh [20] and then by Rashidenejad et al. [21]. In the prior models the probability of using reserves from each generator was known; however, in this model such data is not predetermined and may only be calculated from the output of the model. Instead, this model uses stochastic programming in determining reserve usage while adhering to the N-1 contingency criterion. Compared to the model to be presented, Bouffard et al. [22] present a more realistic, multi-period model that is faithful to real constraints and costs. On the otherhand, the model of this thesis is more similar to market LP's and pays more attention to markets, pricing, and risks, as described in Chapter 5.

3.1 Nomenclature

All nomenclature used in describing the model and the accompanying market schemes follow below. These include the sets and indices used in distinguishing different nodes/buses and scenarios, the parameters and variables used in the linear program, and the auxiliary variables that are later used to describe the market system. Caution should be taken when interpreting symbols as they are formulated using an economics (as opposed to electrical) point of view. Since the length of the planning period is one hour, unit costs, energy prices, and marginal values are measured in \$/MWh.

3.1.1 Sets and Indices

s: scenario ($s \epsilon S$)

$\hat{s}$: base scenario (most probable, no contingencies)

S: set of scenarios

i, j: node ($i, j \epsilon N$)

N: set of nodes in network

3.1.2 Parameters

π_s: % probability of scenario s, $\sum_{s \epsilon S} \pi_s = 1$

A_i^e: offer price of energy at node i (\$/MWh)

A_i^r: offer price of reserve at node i (\$/MWh)

A_i^d: marginal value of demand at node i (\$/MWh)

M_i^c: generator i capacity (MW)

M_i^d: load i forecast demand (MW)

B_{ij}: line susceptance between ij (Ω^{-1})

f_{ij}: line capacity between ij (MW)

U_{ijs}: binary defining line existence and availability ($U_{ijs} = 1$) and/or non-existence or unavailability ($U_{ijs} = 0$) for scenario s

χ_{is}: availability of generator i, scenario s; $0 \leq \chi_{is} \leq 1$; outage fraction $= 1 - \chi_{is}$

λ_i: fraction of demand, M_i^d, that cannot be shed

3.1.3 Variables of Optimization Model

q_i^c: capacity dispatch from node i (MW)

q_{is}^e: energy dispatch from node i for scenario s (MW)

q_{is}^r: reserve dispatch from node i for scenario s (MW)

q_{is}^d: energy demand at node i for scenario s (MW)

θ_{is}: voltage angle at node i for scenario s (rad)

α_x: dual variable for primal constraints (3.1)-(3.6) respectively, $x = 1, 2, 3, 4, 5, 6$

- α_{1i}: marginal value of capacity at i (\$/MWh)
- α_{2is}: marginal value of energy at i for scenario s (\$/MWh)
- α_{3ijs}: marginal value of line capacity (i, j) for scenario s (\$/MWh)
- α_{4is}: marginal value of capacity allocated in stage 1 at i for scenario s (\$/MWh)
- α_{5is}: marginal value of forecast demand at i for scenario s (\$/MWh)
- α_{6is}: marginal value of minimum demand at i for scenario s (\$/MWh)

3.1.4 Auxiliary Variable Definitions

$\dot{\alpha}_{xis} = \frac{1}{\pi_s}\alpha_{xis}$: probability removed dual used for pricing, $x = 3, 4, 5, 6$ (\$/MWh)

$p_{is}^e = \dot{\alpha}_{2is} = \frac{1}{\pi_s}\alpha_{2is}$: energy price at i for scenario s (\$/MWh)

$\bar{p}_i^e = \sum_{s \epsilon S} \pi_s p_{is}^e = \sum_{s \epsilon S} \alpha_{2is}$: expected energy price at i (\$/MWh)

$\overline{p_i^e q_i^e}$ $\equiv \sum_{s \epsilon S}(\pi_s p_{is}^e q_{is}^e)$: expected payment to generator i (\$)

$\hat{p}_i^e$: energy price at i (\$/MWh)

$\hat{p}_i^r$: reserve price at i (\$/MWh)

$\bar{q}_i^x$ $= \sum_{s \epsilon S}(\pi_s q_{is}^x)$: expected value of q_{is}^x, $x = e, r, c, d$ (MW)

Π_{is}^X: generator i profit in scheme X for scenario s (\$)

$\bar{\Pi}_i^X$: generator i expected value of profit in scheme X (\$)

3.2 Model

The model in its entirety, to aid in examination, is presented below. A description of each equation in the model follows in the next section.

$$\min_{\substack{q_i^c, q_{is}^e, q_{is}^r, \\ q_{is}^d, \theta_{is}}} : \sum_{s \epsilon S} \sum_{i \epsilon N} \pi_s (A_i^e q_{is}^e + A_i^r q_{is}^r - A_i^d q_{is}^d) \tag{3.0}$$

Such that

(First Stage)

$$q_i^c \leq M_i^c \qquad \forall i \epsilon N \qquad (\alpha_{1i}) \tag{3.1}$$

(Second Stage)

$$q_{is}^e - q_{is}^d - \sum_{\substack{j \epsilon N \\ \text{s.t.} \\ U_{ijs}=1}} B_{ij}(\theta_{is} - \theta_{js}) = 0 \qquad \forall i \epsilon N, \forall s \epsilon S \qquad (\alpha_{2is}) \tag{3.2}$$

$$B_{ij}(\theta_{is} - \theta_{js}) \leq f_{ij} \qquad \forall s \epsilon S, \forall i \epsilon N \text{ \& for } j \text{ s.t. } U_{ijs} = 1 \qquad (\alpha_{3ijs}) \tag{3.3}$$

$$q_{is}^e + q_{is}^r - \chi_{is} q_i^c = 0 \qquad \forall i \epsilon N, \forall s \epsilon S \qquad (\alpha_{4is}) \tag{3.4}$$

$$q_{is}^d \geq \lambda_i M_i^d \qquad \forall i \epsilon N, \forall s \epsilon S \qquad (\alpha_{5is}) \tag{3.5}$$

$$q_{is}^d \leq M_i^d \qquad \forall i \epsilon N, \forall s \epsilon S \qquad (\alpha_{6is}) \tag{3.6}$$

(Variable Sign Restrictions)

$$q_i^c \geq 0 \qquad \forall i \epsilon N \tag{3.7}$$

$$q_{is}^e \geq 0 \qquad \forall i \epsilon N, \forall s \epsilon S \tag{3.8}$$

$$q_{is}^r \geq 0 \qquad \forall i \epsilon N, \forall s \epsilon S \tag{3.9}$$

3.3 Model Description

The market system model is composed of two parts, the objective function, (3.0), and constraints, (3.1) to (3.9), described in subsections 3.3.1 and 3.3.2, respectively.

3.3.1 Objective Function

Under the premise of this market system the ISO will take, from each generator at node i, offers of energy and reserve at price A_i^e and A_i^r respectively, and a maximum quantity, M_i^c, that the sum of energy and reserve cannot exceed. Following established procedures, the ISO estimates the marginal value of demand, A_i^d at each demand node i, and the probability of each scenario π_s. This value, also known as the 'value of lost load,' has been the subject of numerous studies and is usually higher than cost and market value. The ISO then chooses, for each possible scenario s and all nodes i, the quantity of energy, q_{is}^e, and reserves, q_{is}^r, to be dispatched, along with the quantity of demand, q_{is}^d, to be met. The quantities, q_{is}^e, q_{is}^r, and q_{is}^d, are chosen in such a way as to maximize the expected value of social welfare (i.e., minimize expected costs offset by consumer benefit). The variables q_i^c and θ_{is} are discussed in the next section.

$$\min_{\substack{q_i^c, q_{is}^e, q_{is}^r, \\ q_{is}^d, \theta_{is}}} : \sum_{s \epsilon S} \sum_{i \epsilon N} \pi_s (A_i^e q_{is}^e + A_i^r q_{is}^r - A_i^d q_{is}^d) \tag{3.0}$$

While maximizing social welfare, the objective function takes into consideration all single contingencies in addition to the base case, where nothing goes wrong. In this model a single contingency means the complete failure of one element, either a generator or a line.

3.3.2 Constraints

This section is divided into first and second stage constraints. The first stage constraints, (3.1), correspond to decisions in the day-ahead market. The second stage constraints, (3.2)-(3.6), model real-time operation of the network for the base case and all contingency scenarios. For each constraint, a dual variable is denoted by (α) with subscripts 1-6. The first stage variables, q_i^c, are set before the state of the world is known. The second stage variables, q_{is}^e, q_{is}^r, q_{is}^d and the voltage angles at node i, θ_{is}, are resolved for each scenario, s, subject to the earlier chosen q_i^c.

First Stage

$$q_i^c \leq M_i^c \qquad \forall i \epsilon N \qquad (\alpha_{1i}) \tag{3.1}$$

The total capacity (energy plus reserve), q_i^c, that a generator will be required to supply in a day's time is restricted by the maximum capacity of the generator, M_i^c, submitted as part of the generator's offer package. The quantity, q_i^c, being dispatched in advance, is independent of any scenario that may play out in real-time. In the proposed market system, this will alert the generator to the total quantity of energy or reserve (or combination thereof) that must be provided in real-time. Note that at this point in time (day-ahead), q_i^c is not differentiated between energy and reserve.

Second Stage

$$q^e_{is} - q^d_{is} - \sum_{\substack{j\epsilon N \\ \text{s.t.} \\ U_{ijs}=1}} B_{ij}(\theta_{is} - \theta_{js}) = 0 \qquad \forall i\epsilon N, \forall s\epsilon S \qquad (\alpha_{2is}) \qquad (3.2)$$

$$B_{ij}(\theta_{is} - \theta_{js}) \leq f_{ij} \qquad \forall s\epsilon S, \forall i\epsilon N \qquad (\alpha_{3ijs}) \qquad (3.3)$$

$$\& \text{ for } j \text{ s.t. } U_{ijs} = 1$$

$$q^e_{is} + q^r_{is} - \chi_{is} q^c_i = 0 \qquad \forall i\epsilon N, \forall s\epsilon S \qquad (\alpha_{4is}) \qquad (3.4)$$

$$q^d_{is} \geq \lambda_i M^d_i \qquad \forall i\epsilon N, \forall s\epsilon S \qquad (\alpha_{5is}) \qquad (3.5)$$

$$q^d_{is} \leq M^d_i \qquad \forall i\epsilon N, \forall s\epsilon S \qquad (\alpha_{6is}) \qquad (3.6)$$

In the second stage, all possible scenarios (encompassing the base and all single contingency scenarios) that could occur in real-time are solved (in consideration of the objective function). Thus, for any given scenario that does play out in real-time, feasible quantities will have been determined and will be ready for dispatch to generators (or from loads, in the case of shedding).

The network is represented by a lossless, second order DC approximation, (3.2), used by Fuller [17], Hogan [24], and many others. This equation enforces, for every scenario, network load flows and supply and demand matching. It takes B_{ij}, the susceptance of line (i, j), as parameters. The voltage angles, θ_{is}, vary accordingly depending on load flow in or out of node i. Note that the summation only includes nodes j such that the line (i, j) exists and is not removed from service for the scenario in question, i.e. $U_{ijs} = 1$.

Network line flow limits are enforced by (3.3), with each in-service line (i, j) having a maximum carrying capacity of $U(i, j)$. An outage on any given line (i, j), for any scenario, s, is simulated by assigning its binary $U_{ijs} = 0$, therefore removing the corresponding inequality (in (3.2) and (3.3)) from the model.

By definition, the sum of a generator's energy and reserve product must equal the total capacity dispatched by the ISO to the generator in the day-ahead decision, as in (3.4). If a contingency (either complete or, in the case of larger stations, partial) occurs on a generator at bus i, in any given scenario s, its capacity will be reduced to the fraction χ_{is} of maximum dispatch capacity, q_i^c, where $0 \leq \chi_{is} \leq 1$. This equality is modeled in (3.4). A single contingency, as dictated by the N-1 contingency criterion, is represented by either forcing a line outage, $U_{ijs} = 0$, or generator outage, $\chi_{is} < 1$.

In order to accommodate scenarios where it is impossible to meet all forecasted demand, M_i^d, a variable demand is introduced. This demand has a maximum (ideal) value of M_i^d, the forecast, and a minimum amount, $\lambda_i M_i^d$, that must be met in all scenarios, where $0 \leq \lambda_i \leq 1$. These limits on demand, q_i^d, are enforced by (3.5) and (3.6) and set by the system operator. In essence, this constraint defines a minimum load that cannot be shed and a remaining load, defined as 'sheddable,' that can be dropped in emergencies. To prevent load shedding in non-emergency situations (the norm), a significantly high value to consumers, A_i^d, is introduced into the objective function. As signified by the bus index i, this value can vary by node, which may be the case when comparing rural loads to that of a city's downtown core. Bouffard, et al [22] substantially explore this area in detail, including security, expected load not served, and its value.

The sign restrictions of variables are as follows:

$$q_i^c \geq 0 \qquad \forall i \epsilon N \tag{3.7}$$

$$q_{is}^e \geq 0 \qquad \forall i \epsilon N, \forall s \epsilon S \tag{3.8}$$

$$q_{is}^r \geq 0 \qquad \forall i \epsilon N, \forall s \epsilon S \tag{3.9}$$

The voltage angles, θ_{is}, are unrestricted in sign.

Chapter 4

Dual Model

The dual form of the model is shown below to facilitate discussions in Chapter 5 on market schemes. Since the susceptance line parameter is independent of line flow, the equality $B_{ij} = B_{ji}$ is used in deriving and simplifying (4.5).

$$\max_{\substack{\alpha_{1i},\alpha_{2ij},\alpha_{3ijs}\\ \alpha_{4is},\alpha_{5is},\alpha_{6is}}} : \sum_{i\epsilon N} \alpha_{1i} M_i^c + \sum_{s\epsilon S} \sum_{i\epsilon N} \Big(\sum_{\substack{j\epsilon N\\ \text{s.t.}\\ U_{ijs}=1}} \alpha_{3ijs} f_{ij} + \alpha_{5is} \lambda_i M_i^d + \alpha_{6is} M_i^d \Big) \tag{4.0}$$

such that

$$\alpha_{1i} - \sum_{s\epsilon S} \chi_{is} \alpha_{4is} \leq 0 \qquad \forall i\epsilon N \qquad (q_i^c) \tag{4.1}$$

$$\alpha_{2is} + \alpha_{4is} \leq \pi_s A_i^e \qquad \forall i\epsilon N, \forall s\epsilon S \qquad (q_{is}^e) \tag{4.2}$$

$$\alpha_{4is} \leq \pi_s A_i^r \qquad \forall i\epsilon N, \forall s\epsilon S \qquad (q_{is}^r) \tag{4.3}$$

$$-\alpha_{2is} + \alpha_{5is} + \alpha_{6is} \leq -\pi_s A_i^d \qquad \forall i\epsilon N, \forall s\epsilon S \qquad (q_{is}^d) \tag{4.4}$$

$$\sum_{\substack{j\epsilon N\\ \text{s.t.}\\ U_{ijs}=1}} B_{ij}(-\alpha_{2is} + \alpha_{2js} + \alpha_{3ijs} - \alpha_{3jis}) = 0 \qquad \forall i\epsilon N, \forall s\epsilon S \qquad (\theta_{is}) \tag{4.5}$$

$$\alpha_{1i} \leq 0 \tag{4.6}$$

$$\alpha_{3ijs} \leq 0 \tag{4.7}$$

$$\alpha_{5is} \geq 0 \tag{4.8}$$

$$\alpha_{6is} \leq 0 \tag{4.9}$$

Both α_{2is} and α_{4is} are unrestricted in sign.

4.1 Complementary Slackness

To assist in proofs developed in the next chapter, complementary slackness is applied to (3.1), (3.5)-(3.6), and (4.1)-(4.4), leading to the following expressions:

$$q_i^c \alpha_{1i} = M_i^c \alpha_{1i} \tag{4.10}$$

$$q_{is}^d \alpha_{5is} = \lambda_i M_i^d \alpha_{5is} \tag{4.11}$$

$$q_{is}^d \alpha_{6is} = M_i^d \alpha_{6is} \tag{4.12}$$

$$\alpha_{1i} q_i^c = \sum_{s \epsilon S} \chi_{is} \alpha_{4is} q_i^c \tag{4.13}$$

$$\alpha_{2is} q_{is}^e = \pi_s A_i^e q_{is}^e - \alpha_{4is} q_{is}^e \tag{4.14}$$

$$\alpha_{4is} q_{is}^r = \pi_s A_i^r q_{is}^r \tag{4.15}$$

$$(-\alpha_{2is} + \alpha_{5is} + \alpha_{6is}) q_{is}^d = -\pi_s A_i^d q_{is}^d \tag{4.16}$$

Chapter 5

Market Schemes

This chapter proposes multiple schemes that can be used in the design of a market when applying the model described in Chapter 3. The approaches introduced in this chapter comprise consumer costing, transmission operator payment, and generator compensation schemes. Besides defining the energy and reserve pricing and quantities of each scheme, economic characteristics such as expected profit and variance of profit will be detailed, with a benefit analysis of each.

All schemes are derived from the dual model and/or complementary slackness conditions of the model from Chapter 4, starting with use of the strong duality property [25], which states that, given an optimal solution, the objective functions of the primal and dual can be equated. By equating (3.0) and (4.0), and shifting the supply-related terms to the left-side and demand-related terms to the right, the basis for a possible market design forms.

$$\sum_{s\epsilon S}\sum_{i\epsilon N}\pi_s(A_i^e q_{is}^e + A_i^r q_{is}^r) - \sum_{i\epsilon N}\Big(\alpha_{1i}M_i^c + \sum_{s\epsilon S}\sum_{\substack{j\epsilon N\\ \forall j \text{ s.t.}\\ U_{ijs}=1}}(\alpha_{3ijs}f_{ij})\Big)$$
$$= \sum_{s\epsilon S}\sum_{i\epsilon N}(\pi_s A_i^d q_{is}^d + \alpha_{5is}\lambda_i M_i^d + \alpha_{6is}M_i^d) \quad (5.1)$$

The terms, $\sum_{s\epsilon S}\sum_{i\epsilon N}\pi_s(A_i^e q_{is}^e + A_i^r q_{is}^r) - \sum_{i\epsilon N}\alpha_{1i}M_i^c$, on the left side, form the basis for generator compensation. The remaining terms on the left, $\sum_{s\epsilon S}\sum_{\substack{j\epsilon N\\ \forall j \text{ s.t.}\\ U_{ijs}=1}}(\alpha_{3ijs}f_{ij})$, are associated with transmission owner payments. All terms on the left are non-negative due to (3.8), (3.9), (4.6), and (4.7). The right-side terms, $\sum_{s\epsilon S}\sum_{i\epsilon N}(\pi_s A_i^d q_{is}^d + \alpha_{5is}\lambda_i M_i^d + \alpha_{6is}M_i^d)$, form the basis for consumer costs. The first, second, and third terms in the summation on the right are non-negative, non-negative, and non-positive, respectively.

Examining these terms suggests real-time or expected value consumer cost, transmission operator payment, and generator compensation schemes. In this thesis, proposed schemes will center on real-time, day-ahead, and hybrid generator remuneration schemes; however, real-time and day-ahead schemes for consumer costing and transmission operator payment will also be examined.

All schemes presented are based, at least partially, on the expected value of related real-time variables. This research assumes that any expected value, over all scenarios, equals the long-run average that would be observed over time when the same day-ahead marked conditions are repeated. However, short run disparities between income from consumers and payments to generators and the transmission operator implies that, in the real world, the market operator would be required to keep a buffer or reserve to ensure continued payouts.

All pricing schemes contain or relate to the probability-removed dual, $\dot{\alpha_{2is}} = \alpha_{2is}/\pi_s$ of (3.2), henceforth referred to as p_{is}^e.

5.1 Consumer Costs

This section introduces two consumer payment schemes for goods and services (transmission, energy, and reserves) rendered to the consumer. The two payment schemes are real-time or expected value derivations of the right (consumer-cost side) of (5.1). Although energy may be seen as the only 'good' from a consumer point of view, both transmission network use and reserves are necessary services that must be paid for. Since they are all equated to the left (resource-side) of (5.1), these schemes will cover the expected value of all costs encountered by the network.

5.1.1 Real-time

Energy cost to load i: $p^e_{is} \times q^d_{is}$

The real-time consumer payment at node i, when the state of the world is as in scenario s, is the real-time price times the real-time quantity consumed, where the price is the dual of (3.2).

Theorem 5.1.1. *For every demand node i and scenario, s,*

$$p^e_{is}q^d_{is} = (\pi_s A^d_i q^d_{is} + \alpha_{5is}\lambda_i M^d_i + \alpha_{6is}M^d_i)/\pi_s$$

Proof.

$$\begin{aligned}
&(\pi_s A^d_i q^d_{is} + \alpha_{5is}\lambda M^d_i + \alpha_{6is}M^d_i)/\pi_s \\
&\quad = (\pi_s A^d_i q^d_{is} + \alpha_{5is}q^d_{is} + \alpha_{6is}q^d_{is})/\pi_s && \text{from (4.11),(4.12)} \\
&\quad = (\alpha_{2is}q^d_{is})/\pi_s && \text{from (4.16)} \\
&\quad = p^e_{is}q^d_{is}
\end{aligned}$$

□

By Theorem 5.1.1, the expected value of total consumer payments equals the right (consumer costs) side of (5.1); thus this scheme provides a feasible cost recovery method. As previously noted, although payments from the consumer may not match generator and transmission owner compensation in the short run, they are expected to be equal in the long run, i.e. have equal expected values.

The cost per unit of energy is (from the proof of Theorem 5.1.1)

$$p^e_{is} = \dot{\alpha}_{2is} = A^d_i + \dot{\alpha}_{5is} + \dot{\alpha}_{6is} \tag{5.2}$$

However, since $\dot{\alpha}_{5is} \neq 0$ and $\dot{\alpha}_{6is} \neq 0$ are mutually exclusive, as constraints (3.5) and (3.6) cannot both be active, and $\dot{\alpha}_{5is} \geq 0$ and $\dot{\alpha}_{6is} \leq 0$, p^e_{is} will be non-positive in cases where A^d_i is not sufficiently high to offset $\dot{\alpha}_{6is}$. Hence, although p^e_{is} is normally positive, $A^d_i < |\dot{\alpha}_{6is}|$ will result in a negative price for that scenario. If in the rare case there is a negative price, it would be the result of problems distributing power and its persistence would suggest making improvements to the network.

5.1.2 Day-ahead

Energy cost to load i: $\sum\limits_{s \epsilon S}(\alpha_{2is} \times q^d_{is}) = \sum\limits_{s \epsilon S} \pi_s p^e_{is} q^d_{is}$

This pricing schedule is based on expected values: costs are assessed to the individual demand nodes based on the expected state of the network for the hour under consideration. Loads pay the expected costs of their node for their consumption, regardless of the specific consumption and price that happen in real-time. Unlike the real-time schedule, there is no variance in payments from the consumer for any given hour.

5.2 Transmission Owner Compensation

Two methods of compensating transmission network owners, as suggested by the transmission terms of (5.1), are covered in this section: real-time and day-ahead.

5.2.1 Real-time

Payment to Transmission Operator: $\sum\limits_{\substack{i,j \epsilon N \\ \text{s.t.} \\ U_{ijs}=1}} (-\dot{\alpha}_{3ijs} f_{ij})$

This is a real-time scheme; payments are made to the transmission operator based upon the actual state of the system for the hour in question. According to complementary slackness conditions on (3.3), payments to the transmission operator occur with line congestion. This payment should act as a signal to expand network capacity on the affected lines.

5.2.2 Day-ahead

Payment to Transmission Operator: $\sum\limits_{s \epsilon S} \sum\limits_{\substack{i,j \epsilon N \\ \text{s.t.} \\ U_{ijs}=1}} (-\alpha_{3ijs} f_{ij})$

This scheme is similar to that of real-time, except that it pays the transmission network operators the expected value of the real-time payments (over all scenarios).

5.3 Generator Compensation

The various generator compensation mechanisms, the focus of this chapter, are based on three different approaches. These approaches differ in the timing that their prices are rooted upon: pure real-time, pure day-ahead, and hybrid (a combination of the previous two). All of the schemes, except for day-ahead scheme B (explained later), have the same expected value of revenue and

Table 5.1: Generator Payments for Schemes with Explicit Energy Prices

<table>
<tr><th rowspan="2">Scheme</th><th colspan="2">Energy</th><th colspan="2">Reserve</th></tr>
<tr><th>Price</th><th>Quantity</th><th>Price</th><th>Quantity</th></tr>
<tr><td>Real-Time</td><td>p^e_{is}</td><td>q^e_{is}</td><td>–</td><td>–</td></tr>
<tr><td>Day-Ahead B</td><td>$\bar{p}^e_i$</td><td>$\bar{q}^e_i$</td><td>–</td><td>–</td></tr>
<tr><td>Day-Ahead C</td><td>$\bar{p}^e_i$</td><td>$\bar{q}^e_i$</td><td>$\dfrac{\overline{p^e_i q^e_i} - \bar{p}^e_i \bar{q}^e_i}{\bar{q}^r_i}$</td><td>$\bar{q}^r_i > 0$</td></tr>
<tr><td>Day-Ahead D</td><td>$p^e_{i\hat{s}}$</td><td>$q^e_{i\hat{s}}$</td><td>$\dfrac{\overline{p^e_i q^e_i} - p^e_{i\hat{s}} q^e_{i\hat{s}}}{q^r_{i\hat{s}}}$</td><td>$q^r_{i\hat{s}} > 0$</td></tr>
<tr><td rowspan="2">Hybrid C-HY</td><td>$\bar{p}^e_i$</td><td>$\bar{q}^e_i$</td><td>$\dfrac{\overline{p^e_i q^e_i} - p^e_{i\hat{s}} q^e_{i\hat{s}}}{q^r_{i\hat{s}}}$</td><td>$\bar{q}^r_i > 0$</td></tr>
<tr><td colspan="4">constrained on/off adjustment: $A^e_i(q^e_{is} - \bar{q}^e_i) + A^r_i(q^r_{is} - \bar{q}^r_i)$</td></tr>
<tr><td rowspan="2">Hybrid D-HY</td><td>$p^e_{i\hat{s}}$</td><td>$q^e_{i\hat{s}}$</td><td>$\dfrac{\overline{p^e_i q^e_i} - A^e_i(\bar{q}^e_i - q^e_{i\hat{s}}) - A^r_i(\bar{q}^r_i - q^r_{i\hat{s}}) - p^e_{i\hat{s}} q^e_{i\hat{s}}}{q^r_{i\hat{s}}}$</td><td>$q^r_{i\hat{s}} > 0$</td></tr>
<tr><td colspan="4">constrained on/off adjustment: $A^e_i(q^e_{is} - q^e_{i\hat{s}}) + A^r_i(q^r_{is} - q^r_{i\hat{s}})$</td></tr>
</table>

profit. Summaries of each scheme and their characteristics are provided in Tables 5.1, 5.2, and 5.3.

Table 5.1 details the quantities and prices charged for energy and reserves delivered by generators for those schemes that carry explicit energy and reserve prices. Table 5.2 details the total payments made for energy and reserves delivered by generators for those schemes that do not carry explicit energy and reserve prices, as well as subsets of those schemes in table 5.1 that, under certain conditions, bear no explicit prices.

Table 5.3 summarizes all the major economic characteristics of the generator compensation schemes. Economically, there are two important criteria for establishing a successful market:

Table 5.2: Generator Payments for Schemes with Integrated Payment

Scheme	Payment	Notes
Day-Ahead A	$\overline{p_i^e q_i^e}$	
Day-Ahead C	$\overline{p_i^e q_i^e}$	$\bar{q}_i^r = 0$
Day-Ahead D	$\overline{p_i^e q_i^e}$	$q_{i\hat{s}}^r = 0$
Hybrid E	$A_i^e q_{is}^e + A_i^r q_{is}^r + (-\alpha_{1i} q_i^c)$	
Hybrid C-HY	$\overline{p_i^e q_i^e}$	$\bar{q}_i^r = 0$
	constrained on/off adjustment: $A_i^e(q_{is}^e - \bar{q}_i^e) + A_i^r(q_{is}^r)$	
Hybrid D-HY	$\overline{p_i^e q_i^e} - A_i^e(\bar{q}_i^e - q_{i\hat{s}}^e) - A_i^r(\bar{q}_i^r - q_{i\hat{s}}^r)$	$q_{i\hat{s}}^r = 0$
	constrained on/off adjustment: $A_i^e(q_{is}^e - q_{i\hat{s}}^e) + A_i^r(q_{is}^r)$	

Table 5.3: Summary of Generator Payment Scheme Characteristics

Scheme	Variance of Profit for i (Risk to Generator i)	Obeys Pricing Rule	Accounts for Real-time Variance	Explicit Energy Price	Explicit Reserve Price
Real-Time	$\sum_{s \epsilon S} \pi_s (p_{is}^e q_{is}^e - A_i^e q_{is}^e - A_i^r q_{is}^r + \alpha_{1i} q_i^c)^2$	Yes	Yes	Yes	No
Day-Ahead A	$\sum_{s \epsilon S} \pi_s (\overline{p_i^e q_i^e} - A_i^e q_{is}^e - A_i^r q_{is}^r + \alpha_{1i} q_i^c)^2$	N/A	No	No	No
Day-Ahead B	*Not Considered*	No	No	Yes	No
Day-Ahead C	$\sum_{s \epsilon S} \pi_s (\overline{p_i^e q_i^e} - A_i^e q_{is}^e - A_i^r q_{is}^r + \alpha_{1i} q_i^c)^2$	No	No	Yes[1]	Yes
Day-Ahead D	$\sum_{s \epsilon S} \pi_s (\overline{p_i^e q_i^e} - A_i^e q_{is}^e - A_i^r q_{is}^r + \alpha_{1i} q_i^c)^2$	Yes[2]	No	Yes[2]	Yes
Hybrid E	0	N/A	Yes	No	No
Hybrid C-HY	0	No	Yes	Yes[1]	Yes
Hybrid D-HY	0	Yes[2]	Yes	Yes[2]	Yes

[1] When $\bar{q}_i^r > 0$

[2] When $q_{i\hat{s}}^r > 0$

sufficient revenue to cover all costs, and pricing at or above marginal cost for any generator that supplies energy. The latter is referred to as the 'pricing rule' in the table, with the former being met for all schemes except day-ahead B. Variance of profit, also indicated in this table, is a measure of risk.

Each scheme is discussed below in detail.

5.3.1 Real-time Generator Pricing

In this real-time pricing scheme, compensation to generators is based solely on the scenario that occurs in the hour under consideration, i.e. the actual state of the system. Although providing a payment for energy, p^e_{is}, there is no explicit payment for reserves. This is not uncharacteristic of markets, as reserves are not routinely seen as a 'product' by consumers.

Payment to generator i for energy: $p^e_{is} q^e_{is}$

This scheme satisfies both of the criteria important for establishing a market: sufficient revenue and the pricing rule.

Theorem 5.3.1. *Under real-time pricing, for each generator i, the expected revenue will equal or exceed expected costs, i.e.,* $\sum_{s \epsilon S} \pi_s p^e_{is} q^e_{is} \geq \sum_{s \epsilon S} \pi_s (A^e_i q^e_{is} + A^r_i q^r_{is})$.

Proof.

$$\begin{aligned}
&\sum_{s\epsilon S}\pi_s p^e_{is} q^e_{is} \\
&\quad = \sum_{s\epsilon S} \alpha_{2is} q^e_{is} \\
&\quad = \sum_{s\epsilon S} \pi_s A^e_i q^e_{is} - \sum_{s\epsilon S} \alpha_{4is} q^e_{is} && \text{from (4.14)} \\
&\quad = \sum_{s\epsilon S} (\pi_s A^e_i q^e_{is} + \alpha_{4is} q^r_{is}) - \sum_{s\epsilon S} \alpha_{4is} \chi_{is} q^c_i && \text{from (3.4)} \\
&\quad = \sum_{s\epsilon S} (\pi_s A^e_i q^e_{is} + \pi_s A^r_i q^r_{is}) - \alpha_{1i} q^c_i && \text{from (4.15), (4.13)} \\
&\quad \geq \sum_{s\epsilon S} (\pi_s A^e_i q^e_{is} + \pi_s A^r_i q^r_{is})
\end{aligned}$$

□

This theorem states that a generator providing energy capacity (in the form of either energy or reserves) can expect to, at a minimum, break even (if not make a profit).

The next theorem states that the price of energy supplied by a generator is equal to or greater than the marginal cost of energy for that generator, i.e. the price for energy supplied is greater than or equal to the difference between the cost of energy and the cost of reserve. This marginal cost is identified in stage two, when the state of the world is known. Capacity dispatch. q^c_i, is fixed, and so the cost of generator i is $A^e_i q^e_{is} + A^r_i q^r_{is} = A^e_i q^e_{is} + A^r_i(\chi_{is} q^c_i - q^e_{is}) = (A^e_i - A^r_i) q^e_{is} + A^r_i \chi_{is} q^c_i$. Thus, the marginal cost of generation in stage two is $A^e_i - A^r_i$.

Theorem 5.3.2. *Under real-time pricing, for each generator, i, and scenario, s, $p^e_{is} \geq A^e_i - A^r_i$ when $q^e_{is} > 0$.*

Proof.

$$\begin{aligned} \alpha_{2is} + \alpha_{4is} &= \pi_s A_i^e && \text{from (4.2)} \\ \therefore \alpha_{2is} - \pi_s A_i^e &= -\alpha_{4is} \geq -\pi_s A^r && \text{from (4.3)} \\ \therefore \alpha_{2is} &\geq \pi_s (A_i^e - A_i^r) \\ \therefore p_{is}^e &\geq A_i^e - A_i^r \end{aligned}$$

□

Theorem 5.3.2 allows prices to be low enough, in some scenarios, that a generator cannot recover all of its costs from producing energy and reserves. However, Theorem 5.3.1 ensures that all costs are covered in the expected value (long-run average) sense.

The profit per scenario,

$$\Pi_s^{RT} = p_{is}^e q_{is}^e - A_i^e q_{is}^e - A_i^r q_{is}^r,$$

may be positive or negative. The expected profit is

$$\bar{\Pi}^{RT} = -\alpha_{1i} q_i^c$$

(see proof of Theorem 5.3.1). Finally, the variance in profit is

$$\sum_{s \epsilon S} \pi_s (p_{is}^e q_{is}^e - A_i^e q_{is}^e - A_i^r q_{is}^r + \alpha_{1i} q_i^c)^2.$$

5.3.2 Day-ahead Pricing

Compensation schemes based on day-ahead pricing consider only the expected state of the system for the time period in question, regardless of what the actual state of the system ends up being. Obviously, all schemes in this category have the drawback of not paying for any real-time variations in dispatch. Four schemes, A, B, C, and D falling under the day-ahead category are presented.

Scheme A

Payment to generator i: $\overline{p_i^e q_i^e} \equiv \sum_{s \epsilon S}(\pi_s p_{is}^e q_{is}^e)$

In this scheme, compensation is the expected value of the real-time payment. There is no explicit price for energy, nor, like the previous real-time method, is there an explicit payment for reserves.

The profit (or loss) per scenario, Π_s^A, is

$$\overline{p_i^e q_i^e} - A_i^e q_{is}^e - A_i^r q_{is}^r.$$

The expected profit, $\bar{\Pi}^A$, is identical to the real-time scheme, $-\alpha_{1i} q_i^c$. Variance in profit is

$$\sum_{s \epsilon S} \pi_s(\overline{p_i^e q_i^e} - A_i^e q_{is}^e - A_i^r q_{is}^r + \alpha_{1i} q_i^c)^2.$$

Scheme B

Payment to generator i: $\bar{p}_i^e \bar{q}_i^e$

This scheme is an extension of the one used in Bouffard, et al [23], where price is based on $\bar{p}_i^e$, the expected value of the dual of the real-time price. Extending this concept further, total compensation is the product of expected price and expected quantity delivered.

This compensation scheme has a major drawback: $\bar{p}_i^e \bar{q}_i^e \neq \overline{p_i^e q_i^e}$. It could be that for a generator providing reserves in many scenarios, p_{is}^e and q_{is}^e will be positively correlated; therefore $\overline{p_i^e q_i^e} > \bar{p}_i^e \bar{q}_i^e$, with the difference being the value of reserves. Because of this drawback, this scheme will not be further explored. Rather, based on this thinking, the next scheme presented includes a payment for reserves.

Scheme C

If $\bar{q}_i^r > 0$,

Payment to generator i for energy: $\bar{p}_i^e \bar{q}_i^e$

Payment to generator i for reserve: $\hat{p}_i^r \bar{q}_i^r$

where $\hat{p}_i^r = \frac{\overline{p_i^e q_i^e} - \bar{p}_i^e \bar{q}_i^e}{\bar{q}_i^r}$

Else if $\bar{q}_i^r = 0$,

Payment to generator i: $\overline{p_i^e q_i^e}$

Scheme C has two alternatives, contingent on whether the average quantity of reserve dispatched, $\bar{q}_i^r$, is zero or greater than zero. If $\bar{q}_i^r > 0$, this scheme presents two distinct prices: one for energy and one for reserve. The price of energy, $\bar{p}_i^e$, is non-negative but, as in scheme B, may violate the pricing rule of Theorem 5.3.2 that is normally expected of any market. Reserve price, $\bar{p}_i^r$, is unrestricted in sign; positive if p_{is}^e and q_{is}^e are positively correlated, and negative if p_{is}^e and q_{is}^e are negatively correlated. The latter could arise with the failure of a very large generator significantly reducing q_{is}^e and thus causing a price spike in p_{is}^e. Negative reserve prices can be interpreted as a built-in correction for the overpayment of energy supplied that may occur.

When $\bar{q}_i^r = 0$, an integrated payment is made to the generator for energy supplied, identical to that of scheme A, thus meeting the sufficient revenue requirement and pricing rules. Regardless, profit, expected profit, and the variance of profit are identical to that of day-ahead scheme A.

The next scheme, D, modifies this scheme to reduce the likelihood of violating Theorem 5.3.2's pricing rule, by taking the energy price and quantity from the most probable (base) scenario, $\hat{s}$.

Scheme D

If $q^r_{i\hat{s}} > 0$,

Payment to generator i for energy: $p^e_{i\hat{s}} q^e_{i\hat{s}}$

Payment to generator i for reserve: $\hat{p}^r_i q^r_{i\hat{s}}$

where $\hat{p}^r_i = \frac{\overline{p^e_{is} q^e_{is}} - p^e_{i\hat{s}} q^e_{i\hat{s}}}{q^r_{i\hat{s}}}$

Else if $q^r_{i\hat{s}} = 0$,

Payment to generator i: $\overline{p^e_i q^e_i}$

Like scheme C, there are two possible subsets used for pricing: $q^r_{i\hat{s}} > 0$ and $q^r_{i\hat{s}} = 0$. In the first subset, which makes payments based on the most probable scenario, $\hat{s}$, both sufficient revenues and pricing rules (Theorems 5.3.1 and 5.3.2, respectively) are met. However, this scheme does not entirely eliminate infractions of the pricing rule, with such instances possibly occurring within the $q^r_{i\hat{s}} = 0$ subset when the payment becomes $\overline{p^e_i q^e_i}$ and p^e_i is undefined.

Finally, since this scheme uses the most likely scenario rather than expected values for pricing and quantity, it may benefit from being more understandable to producers who operate and plan based on 'likely' scenarios - it is similar to some existing markets.

5.3.3 Hybrid Generator Pricing

Hybrid compensation schemes are composed of both real-time and day-ahead pricing components. In this section three are presented: E, which is derived from the left side of (5.1); and C-HY and D-HY, which are variations of the day-ahead schemes C and D with added real-time adjustments, commonly referred to as 'constrained on/off' payments. All hybrid schemes have the advantage of zero variance and therefore no risk.

Scheme E

Advance day-ahead payment to generator i: $-\alpha_{1i} q_i^c$

Payment to generator i for energy: $A_i^e \times q_{is}^e$

Payment to generator i for reserve: $A_i^r \times q_{is}^r$

This scheme is derived from the generation terms on the left of (5.1), modified by (4.10). It includes a payment in advance and real-time compensation for actual energy and reserve delivered by the generators.

The advance payment may be viewed as a guaranteed profit to generators regardless of what the real-time state of the world ends up being. The value α_{1i} can be non-zero only if the generator is called upon to provide full capacity ($q_i^c = M_i^c$). Thus, this advance payment can be considered an incentive for 'at capacity' generators to expand. Generators are always compensated for the cost of energy and reserve delivered; thus they will never lose money for delivery of such goods.

The drawback of this method is that no explicit market price for either energy or reserve can be made, since the ISO contracts with each generator separately.

The expected profit for each generator is the same as for the real-time scheme, $-\alpha_{1i} q_i^c$, with zero variance.

5.3.4 Scheme C-HY

If $\bar{q}_i^r > 0$,

Payment to generator i for energy: $\bar{p}_i^e \bar{q}_i^e$

Payment to generator i for reserve: $\hat{p}_i^r \bar{q}_i^r$

where $\hat{p}_i^r = \frac{\overline{p_i^e q_i^e} - \bar{p}_i^e \bar{q}_i^e}{\bar{q}_i^r}$

Adjustment to generator i for constraining on/off: $A_i^e(q_{is}^e - \bar{q}_i^e) + A_i^r(q_{is}^r - \bar{q}_i^r)$

Else if $\bar{q}_i^r = 0$,

Payment to generator i: $\overline{p_i^e q_i^e}$

Adjustment to generator i for constraining on/off: $A_i^e(q_{is}^e - \bar{q}_i^e) + A_i^r(q_{is}^r)$

This scheme is identical to that of scheme C except for the constrained on/off adjustment. The adjustment is made in real-time and compensates generators for producing more energy or reserve than expected while penalizing for producing less (energy or reserve) than expected, by the amount A_i^e or A_i^r, respectively, per unit.

The profit per scenario is

$$\Pi_s^{C-HY} = \Pi^C + A_i^e(q_{is}^e - \bar{q}_i^e) + A_i^r(q_{is}^r - \bar{q}_i^r).$$

As the expected values of $A_i^e(q_{is}^e - \bar{q}_i^e)$ and $A_i^r(q_{is}^r - \bar{q}_i^r)$ are zero, the expected profit remains the same as for C. The major benefit of this scheme over that of the original day-ahead scheme C is the generators' variance in profit being reduced to zero.

5.3.5 Scheme D-HY

If $q_{i\hat{s}}^r > 0$,

Payment to generator i for energy: $p_{i\hat{s}}^e q_{i\hat{s}}^e$

Payment to generator i for reserve: $\hat{p}_i^r q_{i\hat{s}}^r$

where $\hat{p}_i^r = \frac{\overline{p_i^e q_i^e} - A_i^e(\bar{q}_i^e - q_{i\hat{s}}^e) - A_i^r(\bar{q}_i^r - q_{i\hat{s}}^r) - p_{i\hat{s}}^e q_{i\hat{s}}^e}{q_{i\hat{s}}^r}$

Adjustment to generator i for constraining on/off: $A_i^e(q_{is}^e - q_{i\hat{s}}^e) + A_i^r(q_{is}^r - q_{i\hat{s}}^r)$

Else if $q_{i\hat{s}}^r = 0$,

Payment to generator i: $\overline{p_i^e q_i^e} - A_i^e(\bar{q}_i^e - q_{i\hat{s}}^e) - A_i^r(\bar{q}_i^r - q_{i\hat{s}}^r)$

Adjustment to generator i for constraining on/off: $A_i^e(q_{is}^e - q_{i\hat{s}}^e) + A_i^r(q_{is}^r)$

Similar to C-HY, D-HY complements day-ahead scheme D with a constrained on/off adjustment, and thus inherits most of its characteristics. The additional $-A_i^e(\bar{q}_i^e - q_{i\hat{s}}^e) - A_i^r(\bar{q}_i^r - q_{i\hat{s}}^r)$ is used to preserve the expected profit seen in all schemes (except B).

The profit per scenario, with no variance, is

$$\Pi_s^{D-HY} = \Pi^D + A_i^e(q_{is}^e - \bar{q}_i^e) + A_i^r(q_{is}^r - \bar{q}_i^r).$$

Chapter 6

Examples

Two test systems are used with the model to verify and examine its applicability and that of the accompanying market system. The first test system is a small network containing six buses; the second is a much larger system containing 66 buses, representing Ontario's electrical system. The examples will illustrate the model's resource and demand allocation and the payment and compensation schedules for loads, transmission operator, and generators. To solve the two systems, the model and each system's network and market data were first coded into GAMS programs, included under appendix A, and solved on a UNIX workstation using the CPLEX solver.

6.1 6-bus Test System

6.1.1 Description

The 6-bus test system used in this example has been modified from the test system used in [17]. These changes were necessary to ensure that there was sufficient data (particularly pricing) to apply the model and to ensure feasibility, especially during contingencies. This model is for one

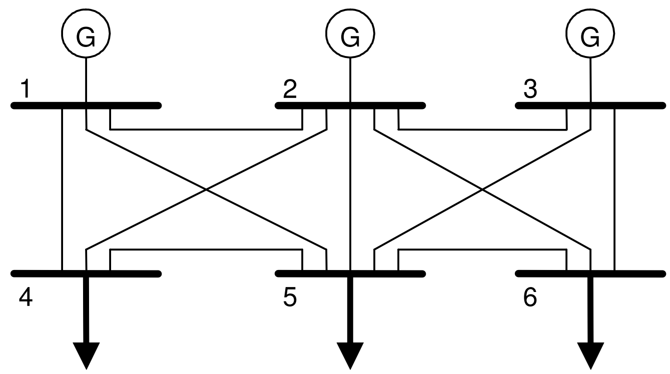

Figure 6.1: 6-Bus Test System

Table 6.1: 6-Bus Test System - Generator Data

Generator	M_i^c (MW)	A_i^e (\$/MWh)	A_i^r (\$/MWh)
1	113	8	4
2	167	2	0.5
3	82	21	16

hour and assumes constant flows over the hour (for example, a load with a 1 MW demand will consume 1 MWh of energy over the hour). The network configuration of the test system is shown in figure 6.1.

This system consists of three generators (nodes 1, 2, and 3), three loads (nodes 4, 5, and 6), and eleven transmission lines, all of whose characteristics are in tables 6.1, 6.2, and 6.3, respectively.

The 6-bus system features three generators of largely varying capacities (between 82 MW and 167 MW) and marginal (incremental) costs (\$2 to 21/MWh for energy and \$0.5 to 16/MWh for reserve). Demand also varies significantly between loads, with forecasted consumption between 30 and 80 MWh. The variable, M_i^d, is the maximum (forecasted) energy consumption level;

Table 6.2: 6-Bus Test System - Load Data

Load	M_i^d (MW)	$\lambda_i M_i^d$ (MW)	A_i^d (\$/MWh)
4	80	40	1500
5	30	15	1500
6	62	31	1500

Table 6.3: 6-Bus Test System - Transmission Line Data

Node i	Node j	$B_{ij}(\Omega^{-1})$	Capacity (MW)
1	2	4.00	65
1	4	4.71	60
1	5	3.11	55
2	3	3.85	35
2	4	8.00	70
2	5	3.00	40
2	6	4.45	45
3	5	3.17	20
3	6	9.62	50
4	5	2.00	50
5	6	3.00	30

Table 6.4: 6-Bus System - Scenarios

Scenario	Element Removed	Probability of Occurrence
1	N/A (Base)	95%
2-4	Generator	0.2% (each scenario)
5-15	Line	0.4% (each scenario)

$\lambda_i M_i^d$ is the minimum consumption level. For this example, the same fraction of non-sheddable load, $\lambda_i = 0.5$, was used for all loads. Transmission lines are, as always, bi-directional between points ij, with susceptance $Y_{ij} = Y_{ji}$. Line capacity was chosen such that, under some scenarios, congestion would occur. The details of each scenario are outlined in table 6.4.

There are a total of 15 scenarios: 1 base scenario (where all elements are working) and 14 single contingency scenarios. Of the contingency scenarios, 3 cover all possible single generator outages and 11 cover all possible transmission line outages. The base scenario has a 95% chance of occurrence, with 0.2% and 0.4% for each generator and line outage, respectively. These probabilities are arbitrarily assigned.

6.1.2 Results

The following set of tables (6.5 to 6.18) detail the consumer cost, transmission payment, and generator compensation schemes of chapter 5 when applied to the 6-bus test system. The first set, tables 6.5 and 6.6, deal with consumer real-time and day-ahead costing schemes.

Examining table 6.5, it is seen that the real-time scheme has a large variance in costs, ranging from \$3.95-4.57/MWh in the base scenario to \$1500/MWh (the marginal value of demand, A_i^d) in scenario 3. Consumption within each load does not vary between scenarios except when its cost of energy is at the marginal value of demand, at which point it is more economical to shed load. For reference, the \$1500/MWh cost correlates to absolute cost of \$60,000 24,000 and 93,000 in scenario 3 compared to \$315, 128 and 283 in scenario 1 for loads 4, 5, and 6, respectively. This method of pricing may not be palatable to consumers, who may shy away from seeing a \$1500/MWh charge on their bill, regardless of energy's long time average value. The day-ahead scheme in table 6.6 alleviates this problem.

The costs in the day-ahead scheme are much more muted compared to those of possible real-time costs, being above the base case but much lower than the maximum charge. Table 6.7, details payments to the transmission operator for congestion. Similar to the consumer scheme, payments in real-time have large variance. For stability in payments, the system operator may prefer to receive the day-ahead amount.

Table 6.5: 6-Bus System - Real-Time Consumer Costs

Load	Scenario	Consumption (MW)	Cost $p^e_{is}q^d_{is}$ (\$)	Cost p^e_{is} (\$/MWh)	Sc	Consumption (MW)	Cost $p^e_{is}q^d_{is}$ (\$)	Cost p^e_{is} (\$/MWh)
4	1	80	316	3.95	9	69.18	103,771	1500.00
5	1	30	129	4.30	9	30	17,744	591.48
6	1	62	283	4.57	9	62	43,248	697.54
4	2	80	56,836	710.45	10	80	259	3.24
5	2	30	21,313	710.45	10	30	251	8.37
6	2	62	44,048	710.45	10	62	618	9.97
4	3	40.00	60,000	1500.00	11	80	16,009	200.12
5	3	16.00	24,000	1500.00	11	30	23,651	788.37
6	3	62	93,000	1500.00	11	49.71	74,556	1500.00
4	4	80	259	3.24	12	80	320	4.00
5	4	30	251	8.37	12	30	120	4.00
6	4	62	767	12.36	12	62	248	4.00
4	5	80	336	4.20	13	80	259	3.24
5	5	30	111	3.70	13	30	251	8.37
6	5	62	239	3.86	13	62	1,686	27.19
4	6	75.88	111,827	1500.00	14	80	312	3.90
5	6	30	7,785	259.49	14	30	131	4.36
6	6	62	6,810	109.84	14	62	284	4.59
4	7	80	323	4.04	15	80	317	3.96
5	7	30	133	4.45	15	30	127	4.24
6	7	62	287	4.63	15	62	288	4.64
4	8	80	259	3.24				
5	8	30	251	8.37				
6	8	62	1,031	16.64				

Table 6.6: 6-Bus System - Day-Ahead Consumer Costs

Load	Cost $\sum_{s \epsilon S} \pi_s p_{is}^e q_{is}^d$ (\$)
4	1,478.48
5	416.87
6	1,062.12

Table 6.7: 6-Bus System - Transmission Operator Payments

Scheme	Scenario	Payment ($)
Real-time	1	68
Real-time	2	-
Real-time	3	-
Real-time	4	990
Real-time	5	370
Real-time	6	127,368
Real-time	7	58
Real-time	8	1,131
Real-time	9	159,128
Real-time	10	693
Real-time	11	73,047
Real-time	12	-
Real-time	13	1,727
Real-time	14	65
Real-time	15	64
Day-ahead	N/A	1,521

Tables 6.8 to 6.16 detail the application of all generator compensation schemes (except day-ahead B) to the 6-bus system. Their profits and variances are shown in tables 6.17 and 6.18.

Besides detailing prices and compensation to generators, table 6.8 lists the dispatch quantities of energy and reserves for all scenarios. Under the base scenario, dispatch of energy and reserve is as what is expected: the cheapest generator is used to capacity in providing energy, with the second and third supplying the necessary remaining energy. The bulk of reserve is provided by generator 2, the second cheapest provider of reserve after 1 (which is already at full capacity). The largest price spikes are seen in scenarios 2 and 3 when generators 1 and 2, respectively, are removed from the system. In the latter scenario, load must be shed in order to maintain system security.

Day-ahead schemes A, C, and D, as shown by tables 6.9, 6.10, and 6.11, respectively, have identical total payments. Unlike A, schemes C and D have explicit prices for energy and reserve. In scheme C's scenario 2, there is a negative reserve price, however, the net payment is still the same. Scheme D's generator compensation prices are all non-negative, avoiding the negative prices seen in C.

Table 6.8: 6-Bus System - Real-Time Generator Compensation

Generator	Sc	Dispatch (MW)		p^e_{is}	Payment	Sc	Dispatch (MW)		p^e_{is}	Payment
		q^e_{is}	q^r_{is}	(\$/MWh)	$p^e_{is}q^e_{is}$ (\$)		q^e_{is}	q^r_{is}	(\$/MWh)	$p^e_{is}q^e_{is}$ (\$)
1	1	3.90	109.10	4.00	15.60	9	54.70	58.30	4.00	218.81
2	1	167.00	0	3.83	639.47	9	101.48	65.52	1.50	152.22
3	1	1.100	3.90	5.00	5.50	9	5.00	0	1052.59	5,262.95
1	2	-	-	710.45	-	10	45.03	67.97	4.00	180.13
2	2	167.00	0	710.45	118,664.62	10	121.97	45.03	1.50	182.95
3	2	5.00	0	710.45	3,552.23	10	5.00	0	1052.59	71.96
1	3	113.00	0	1500.00	169,500.00	11	113.00	0	287.68	32,507.82
2	3	-	-	1500.00	-	11	41.7	125.29	1.50	62.57
3	3	5.00	0	1500.00	7,500.00	11	5.00	0	1721.92	8,609.61
1	4	11.43	101.57	4.00	45.74	12	5.00	108.00	4.00	20.00
2	4	160.57	6.43	1.50	240.85	12	167.00	0	4.00	668.00
3	4	-	-	18.64	-	12	0.00	5.00	4.00	-
1	5	21.09	91.91	4.00	84.38	13	84.36	28.64	4.00	337.45
2	5	149.28	17.72	1.50	223.93	13	87.64	79.36	1.50	131.46
3	5	1.62	3.38	5.00	8.10	13	0.00	5.00	4.60	-
1	6	97.22	15.78	4.00	388.87	14	4.42	108.58	4.00	17.69
2	6	65.67	101.33	1.50	98.50	14	167.00	0	3.84	641.03
3	6	5.00	0	113.28	566.41	14	0.58	4.42	5.00	2.89
1	7	0.25	112.75	4.00	1.00	15	2.66	110.34	4.00	10.64
2	7	167.00	0	3.96	660.70	15	167.00	0	3.86	645.33
3	7	4.75	0.25	5.00	23.75	15	2.34	2.66	5.00	11.69
1	8	34.76	78.2	4.00	136.06					
2	8	132.27	34.8	1.50	198.35					
3	8	5.00	0	14.59	72.95					

Table 6.9: 6-Bus System - Day-Ahead A Generator Compensation

Generator	Payment ($)
1	489.54
2	859.93
3	85.85

Table 6.10: 6-Bus System - Day-Ahead C Generator Compensation

Generator	$\bar{p}^e_i$ ($/MWh)	$\bar{q}^e_i$ (MW)	$\hat{p}^r_i$ ($/MWh)	$\bar{q}^r_i$ (MW)	Payment ($)
1	9.54	5.80	4.06	106.97	489.54
2	8.17	164.78	-257.12	1.89	859.93
3	20.99	1.20	16.00	3.79	85.85

Table 6.11: 6-Bus System - Day-Ahead D Generator Compensation

Generator	$p^e_{i\hat{s}}$ ($/MWh)	$q^e_{i\hat{s}}$ (MW)	$\hat{p}^r_i$ ($/MWh)	$q^r_{i\hat{s}}$ (MW)	Payment ($)
1	4.00	3.90	4.34	109.10	489.54
2	5.15	167.00	0	0	859.93
3	5.00	1.10	20.60	3.90	85.85

Tables 6.12 and 6.13 detail the real-time and advance generator payments, respectively, of hybrid scheme E. Total payment to the generator is the advance plus the real-time compensation for the occurring scenario. As predicted, advance payments are made to generators 1 and 2, who are running at capacity.

Table 6.14 details the results of hybrid scheme C-HY and tables 6.15 and 6.16 detail hybrid scheme D-HY. In order to keep expected values static, the payments before adjustments for D-HY will differ from the day-ahead D payments. After adjustment, the expected values of the total payments are identical to the other schemes.

Tables 6.17 and 6.18 provide a summary of all the generation compensation schemes. Generator profit is presented in table 6.17, with the maximum, minimum, and base scenario profits given for each scheme. The benefit of the hybrid schemes is best seen here, where profit will remain positive regardless of the scenario playing out. As expected, variances decrease in the order of real-time, day-ahead, and hybrid schemes, as detailed in table 6.18.

Table 6.12: 6-Bus System - Hybrid E Generator Compensation (Real-Time)

Generator	Scenario	Payment $A_i^e q_{is^e} + A_i^r q_{is^r}$ (\$)	Sc	Payment $A_i^e q_{is^e} + A_i^r q_{is^r}$ (\$)	Sc	Payment $A_i^e q_{is^e} + A_i^r q_{is^r}$ (\$)
1	1	436.40	6	840.87	11	904.00
2	1	334.00	6	182.00	11	146.07
3	1	85.50	6	105.00	11	105.00
1	2	-	7	453.00	12	472.00
2	2	334.00	7	334.00	12	334.00
3	2	105.00	7	103.75	12	80.00
1	3	904.00	8	591.06	13	789.45
2	3	-	8	281.85	13	214.96
3	3	105.00	8	105.00	13	80.00
1	4	497.74	9	670.81	14	469.69
2	4	324.35	9	235.72	14	334.00
3	4	-	9	105.00	14	82.89
1	5	536.38	10	632.13	15	462.64
2	5	307.43	10	266.45	15	334.00
3	5	88.10	10	105.00	15	91.69

Table 6.13: 6-Bus System - Hybrid E Generator Compensation (Advance)

Generator	Advance $-\alpha_{1i} q_i^c$ (\$)
1	15.22
2	529.43
3	0

Table 6.14: 6-Bus System - Hybrid C-HY Generator Compensation

Gen	Sc	$q^e_{is} - \bar{q}^e_i$ (MW)	$q^r_{is} - \bar{q}^r_i$ (MW)	Real-Time Adjustment ($)	Payment* ($)	Sc	$q^e_{is} - \bar{q}^e_i$ (MW)	$q^r_{is} - \bar{q}^r_i$ (MW)	Real-Time Adj ($)	Payment* ($)
1	1	-1.90	2.13	-6.71	482.83	9	48.90	-48.67	196.50	686.04
2	1	2.22	-1.89	3.50	863.43	9	-63.30	63.63	-94.78	765.15
3	1	-0.10	0.11	-0.35	85.50	9	3.80	-3.79	19.15	105.00
1	2	-5.80	-106.97	-474.32	15.22	10	39.23	-39.00	157.81	647.35
2	2	2.22	-1.89	3.50	863.43	10	-42.81	43.14	-64.05	795.88
3	2	3.80	-3.79	19.15	105.00	10	3.80	-3.79	19.15	105.00
1	3	107.20	-106.97	429.68	919.22	11	107.20	-106.97	429.68	919.22
2	3	-164.78	-1.89	-330.50	529.43	11	-123.07	123.40	-184.43	675.50
3	3	3.80	-3.79	19.15	105.00	11	3.80	-3.79	19.15	105.00
1	4	5.63	-5.40	23.42	512.96	12	-0.80	1.03	-2.32	487.22
2	4	-4.21	4.55	-6.15	853.78	12	2.22	-1.89	3.50	863.43
3	4	-1.20	-3.79	-85.85	0	12	-1.20	1.21	-5.85	80.00
1	5	15.29	-15.06	62.06	551.60	13	78.56	-78.33	315.14	804.67
2	5	-15.49	15.83	-23.07	836.86	13	-77.14	77.47	-115.54	744.39
3	5	0.42	-0.41	2.26	88.10	13	-1.20	1.21	-5.85	80.00
1	6	91.41	-91.19	366.56	856.10	14	-1.38	1.61	-4.63	484.91
2	6	-99.11	99.44	-148.50	711.43	14	2.22	-1.89	3.50	863.43
3	6	3.80	-3.79	19.15	105.00	14	-0.62	0.63	-2.95	82.89
1	7	-5.55	5.78	-21.31	468.23	15	-3.14	3.37	-11.67	477.87
2	7	2.22	-1.89	3.50	863.43	15	2.22	-1.89	3.50	863.43
3	7	3.55	-3.54	17.90	103.75	15	1.14	-1.13	5.85	91.69
1	8	28.96	-28.73	116.74	606.28					
2	8	-32.54	32.88	-48.65	811.29					
3	8	3.80	-3.79	19.15	105.00					

* Day-ahead plus real-time

Table 6.15: 6-Bus System - Hybrid D-HY Generator Compensation (Before Adjustments)

Generator	$p^e_{i\hat{s}}$ (\$/MWh)	$q^e_{i\hat{s}}$ (MW)	$\hat{p}^r_i$ (\$/MWh)	$q^r_{i\hat{s}}$ (MW)	Payment (\$)
1	4.00	3.90	4.28	109.10	482.83
2	5.17	167.00	0	0	863.43
3	5.00	1.10	20.51	3.90	85.50

Table 6.16: 6-Bus System - Hybrid D-HY Generator Compensation (Total)

Gen	Sc	$q^e_{is} - q^e_{i\hat{s}}$ (MW)	$q^r_{is} - q^r_{i\hat{s}}$ (MW)	Real-Time Adjustment ($)	Payment* ($)	Sc	$q^e_{is} - q^e_{i\hat{s}}$ (MW)	$q^r_{is} - q^r_{i\hat{s}}$ (MW)	Real-Time Adj ($)	Payment* ($)
1	1	0	0	0	489.54	9	50.80	-50.80	203.21	692.75
2	1	0	0	0	859.93	9	-65.52	65.52	-98.28	761.65
3	1	0	0	0	85.85	9	3.90	-3.90	19.50	105.35
1	2	-3.90	-109.10	-467.60	21.93	10	41.13	-41.13	164.52	654.06
2	2	0	0	0	859.93	10	-45.03	45.03	-67.55	792.38
3	2	3.90	-3.90	19.50	105.35	10	3.90	-3.90	19.50	105.35
1	3	109.10	-109.10	436.40	925.93	11	109.10	-109.10	436.40	925.93
2	3	-167.00	0	-334.00	525.93	11	-125.29	125.29	-187.93	672.00
3	3	3.90	-3.90	19.50	105.35	11	3.90	-3.90	19.50	105.35
1	4	7.53	-7.53	30.13	519.67	12	1.10	-1.10	4.40	493.93
2	4	-6.43	6.43	-9.65	850.28	12	0	0	0	859.93
3	4	-1.10	-3.90	-85.50	0.35	12	-1.10	1.10	-5.50	80.35
1	5	17.19	-17.19	68.77	558.31	13	80.46	-80.46	321.85	811.39
2	5	-17.72	17.72	-26.57	833.36	13	-79.36	79.36	-119.04	740.89
3	5	0.52	-0.52	2.61	88.45	13	-1.10	1.10	-5.50	80.35
1	6	93.32	-93.32	373.27	862.81	14	0.52	-0.52	2.08	491.62
2	6	-101.33	101.33	-152.00	707.93	14	0	0	0	859.93
3	6	3.90	-3.90	19.50	105.35	14	-0.52	0.52	-2.60	83.24
1	7	-3.65	3.65	-14.60	474.94	15	-1.24	1.24	-4.96	484.58
2	7	0	0	0	859.93	15	0	0	0	859.93
3	7	3.65	-3.65	18.25	104.10	15	1.24	-1.24	6.20	92.05
1	8	30.86	-30.86	123.45	612.99					
2	8	-34.76	34.76	-52.15	807.78					
3	8	3.90	-3.90	19.50	105.35					

* Day-ahead plus real-time

Table 6.17: 6-Bus System - Generator Profit, For Selected Scenarios

Scheme	Generator/ Profit (Π) (\$)		
	1	2	3
Real-Time			
Minimum Profit	-452	-84	-80
Maximum Profit	168,596	118,311	8,505
Base Scenario ($\hat{s}$) Profit	-452	305	-80
Day-Ahead A, C, D			
Minimum Profit	-414	526	-19
Maximum Profit	490	860	86
Base Scenario ($\hat{s}$) Profit	22	526	0
Hybrid E, C-HY, D-HY			
Min = Max = Base Scenario ($\hat{s}$) Profit	15	529	0

Table 6.18: 6-Bus System - Variance of Generator Payment Schemes

Generator	Expected Profit (\$)	Variance of Profit Π		
		Real-Time	Day-Ahead A, C, D	Hybrid E, C-HY, D-HY
1	15	61,046,866	2,863	0
2	529	27,805,004	572	0
3	0	535,964	25	0

6.2 Ontario Test System

6.2.1 Description

The Ontario test system is adapted from data used by the IESO to simulate Ontario's electricity network on a small scale; it is a scaled down, 66 bus version of the full system [26] for one historical hour. It is composed of 53 loads, 171 transmission lines, and 12 generators. The marginal value of demand for each load is the same for all, $1500/MWh. Transmission line limits are set realistically, but not necessarily correct. A description of each generator's details is given in table 6.19.

Table 6.19: Ontario System - Generator Data

Load (Label)	Type	M_i^c (MW)	A_i^e (\$/MWh)	A_i^r (\$/MWh)
1902	Fossil (Gas/Oil)	2140	61.1	16
2901	Nuclear	3524	3.75	0.5
2962	Nuclear	2064	3.74	0.5
4000	TS	8257	4	-
4905	Fossil (Coal)	1140	28	4
6308	Hydro	1290	1	0
6328	Fossil (Coal)	3920	28	4
6902	Nuclear	3076	3.75	0.5
6906	Nuclear	3140	3.75	0.5
7920	Fossil (Coal)	1975	28	4
8110	TS	1479	4	-
9103	TS	773	1	-

The generator data includes the type, capacity, and prices for each unit. Type and capacity were both gathered through sources in the public domain [27], [28]. Marginal costs were realistically assigned based on marginal cost studies [29]; although not exact (as these costs are not in the public domain), they are realistic for the generator type. Generator types include nuclear, hydro, and fossil (oil, natural gas, and coal). Generators designated TS are regarded as aggregate nodes - nodes that don't represent a single generator but a small area of the larger network. These TS generators have been assigned costs arbitrarily and are deemed to be always active, but incapable of providing reserve.

There are 181 scenarios: the base scenario, 9 single generator contingencies (all except the TS) and 171 single line outage contingencies. The base scenario has a 95% probability of occurrence, with the remaining scenarios splitting the other 5%: 0.015% per single generator outage scenario and 0.0285% per single line outage scenario.

6.2.2 Results

A selection of results from applying the market scheme to the Ontario system is presented in tables 6.20 to 6.23.

Real-time consumer costs, presented in table 6.20, vary significantly, even being negative in some scenarios. However, base prices remain reasonable as well as expected costs (used in the day-ahead system).

Again, similar to consumer costs, payments appear much more reasonable when the transmission expected (day-ahead) payments of table 6.21 are used.

Examining the generator pricing schemes of table 6.22, it is clear that hybrid scheme D-HY provides the best approach. Both energy and reserve prices are very reasonable compared to the largely varying amount seen in real-time, yet it also avoids the negative reserve prices contained in C-HY. The benefits of the hybrid schemes, in general, is further proved by their low variance, as seen in table 6.23.

Table 6.20: Ontario System - Consumer Costs, For Selected Loads

Load	Real-Time Cost				Expected
(Label)	Min ($)	Max ($)	Base Scenario ($)	Base Sc ($/MWh)	Cost ($)
1	-49,961	719,890	11,645	23.29	12,988
100	5,758	750,000	11,645	23.29	13,390
101	2,657	348,632	5,413	23.29	6,392
103	1,981	300,000	4,658	23.29	5,289
344	3,374	450,000	6,987	23.29	8,311
359	-9,615	450,000	6,987	23.29	8,147
1001	-79,241	600,000	9,316	23.29	10,433
1104	2,221	300,000	4,658	23.29	5,523
1106	3,161	600,000	9,316	23.29	11,010
1301	2,229	300,000	4,658	23.29	5,512
2002	-6,552	1,050,000	16,303	23.29	18,781
2007	1,950	900,000	13,974	23.29	16,084
2100	2,275	1,050,000	16,303	23.29	18,900
2106	1,950	900,000	2,241	3.74	2,640
3107	4,901	900,000	13,974	23.29	16,263
3108	5,012	900,000	13,974	23.29	16,602

Table 6.21: Ontario System - Transmission Payments

Real-Time Payment			Expected
Min ($)	Max ($)	Base ($)	Payment ($)
0	74,842,794	95,064	134,233

Table 6.22: Ontario System - Generator Pricing, For Selected Generators

Scheme	Price ($ /MWh) at Generator (Label)											
	1902	2901	2962	4000	4905	6308	6328	6902	6906	7920	8110	9103
Real-Time												
Miniumum p_{is}^{e}	0	0	0	-49.54	8.17	1.00	0	0	0	0	-80131.92	-50.28
Maximum p_{is}^{e}	2924.35	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00
Base $p_{i\hat{s}}^{e}$	23.29	3.25	3.74	23.29	23.29	23.29	23.29	23.29	23.29	23.29	23.29	23.29
Day-Ahead C and Hybrid C-HY												
Base $\bar{p}_{i}^{e}$	28.53	3.97	3.97	26.75	27.32	27.02	26.93	26.25	26.25	27.19	4.00	27.23
Base $\hat{p}_{i}^{r}$	0	-345.87	-262.55	-	4.00	-484.19	0	-343.63	-750.76	11.49	-	-
Day-Ahead D and Hybrid D-HY												
Base $p_{i\hat{s}}^{e}$	0	3.75	3.75	26.75	23.29	26.80	0	26.03	26.03	27.17	4.00	27.23
Base $\hat{p}_{i}^{r}$	0	0	0	-	4.15	0	0	0	0	0	-	-

Table 6.23: Ontario System - Variance of Generator Payment Schemes

Generator (Label)	Expected Profit ($\$ \times 10^3$)	Variance of Profit ($\$^2 \times 10^3$)		
		Real-Time	Day-Ahead A, C, D	Hybrid E, C-HY, D-HY
1902	0	0	0	0
2901	0	831,751.5	78.4	0
2962	0	6,563.3	31.2	0
4000	171.9	273,493,197.8	0	0
4905	0	5,207,387.4	3,186.4	0
6308	33.3	7,711,312.0	0.7	0
6328	0	0	0	0
6902	68.5	36,120,111.8	60.2	0
6906	70.0	37,640,229.0	40.4	0
7920	0	3,613,655.0	400.5	0
8110	0	1,635,228,333.0	0	0
9103	12.4	1,216,887.9	0	0

Chapter 7

Conclusion

This thesis presents a model that can be used for electrical energy and reserve markets, utilizing stochastic linear programming. Its primary feature is its ability to take into consideration single generator or line contingencies when determining optimal dispatch. Multiple pricing structures are presented, each with its own sets of advantages and disadvantages. However, given the benefits of generator hybrid schemes C-HY and D-HY (explicit prices for energy and reserves and zero variance) and reasonable results from simulating the test cases, either scheme is an excellent candidate for further development and study.

7.1 Suggestions for Further Research

1. Integrating a more complex AC, lossy network and multi-period market to the model (and efficient algorithms to solve the model in a decent amount of time)
2. Analyzing market power issues introduced in this model
3. Comparative analysis of market dispatch with and without contingency consideration in the objective function (i.e. model only feasibility of contingencies, not their costs).

Appendix A

GAMS Code for 6-bus Model Simulation

```
$eolcom #
$inlinecom { }

*---------------Sets----------------
Sets
         i   buses       / 1, 2, 3, 4, 5, 6 /
         is(i)  supply buses / 1, 2, 3/
         id(i)  demand buses / 4, 5, 6/;

Set
         s n-1 scenarios /1*15/;
* 1 Base Scenario
* 3 Generator Outages
* 11 Line Outages

Alias (i,j);

*---------------Parameters----------------
*-----------Buses------------

Parameter
         GenMax(is)    generator maximum energy and reserve capacity
                / 1     113
                  2     167
                  3     82       /;

Parameter
         Ae(is)        marginal cost of energy
                / 1     8
                  2     2
                  3     21       /;

Parameter
         Arc(is)       marginal cost of reserve
                / 1     4
                  2     0.5
                  3     16      /;
```

```
Parameter
        QdMax(id)     maximum demand
              / 4     80
                5     30
                6     62      /;

Parameter QdVar     ;
        QdVar = 0.5;
* Fraction of demand that cannot be shed

Parameter Ad;
        Ad = 1500;
* Marginal value of demand

*-----------Lines------------

Table B(i,j)    susceptance of the line
                 1       2       3       4       5       6
        1               4.00            4.71    3.11
        2                       3.85    8.00    3.00    4.45
        3                                       3.17    9.62
        4                                       2.00
        5                                               3.00
        6                                                       ;

Table F(i,j)    line capacity
                 1       2       3       4       5       6
        1               50              55      50
        2                       24      65      40      45
        3                                       20      45
        4                                       50
        5                                               30
        6                                                        ;

*-----------Scenarios------------
Parameter Chi(s,is)      generator scenarios
*Chi is actually 1-Chi
```

```
                / 2.1 1
                  3.2 1
                  4.3 1       /  ;


Parameter U(s,i,j)       line scenarios
*U is actually 1-U
                / 5.1.2   1
                  6.1.4   1
                  7.1.5   1
                  8.2.3   1
                  9.2.4   1
                  10.2.5  1
                  11.2.6  1
                  12.3.5  1
                  13.3.6  1
                  14.4.5  1
                  15.5.6  1   /  ;

Parameter Pi(S)          scenario probabilities
                / 1  0.95
                  2  0.002
                  3  0.002
                  4  0.002
                  5  0.004
                  6  0.004
                  7  0.004
                  8  0.004
                  9  0.004
                  10 0.004
                  11 0.004
                  12 0.004
                  13 0.004
                  14 0.004
                  15 0.004   /  ;

Variables
         snb             social welfare ($)
```

```
        t(s,i)          theta at bus i (voltage angle in radians);

Positive variables
        qp(is)          quantity of energy capacity reserved (pre-dispatched)
        qe(s,is)        quantity of energy dispatched at is  (MW)
        qrc(s,is)       quantity of reserve capacity dispatched at is (MW)
        qd(s,id)        quantity of energy demanded at id (MW);

Equations
        welfare                 define objective function
        poweralloc(is)          power allocation
*       ==simultaneous==

        powerflows(s,is)        supply node balance - realtime
        powerflowd(s,id)        demand node balance - realtime
        powerflowe(s,i)         neither a demand or node bus balance - realtime
        limit(s,i,j)            line power transfer limits d1 - realtime
        genlimit(s,is)          generator maximum output - realtime
        dminlimit(s,id)         demand lower limit
        dmaxlimit(s,id)         demand upper limit

        swingdef(s,i)           swing bus definition;


*Model
welfare..      snb =e= sum(s, sum(is, pi(s)*Ae(is)*qe(s,is)))
                       + sum(s, sum(is, pi(s)*Arc(is)*qrc(s,is)))
                       - sum(s, sum(id, pi(s)*Ad*(qd(s,id)-(QdVar*QdMax(id)))));
*-------First Stage-------
poweralloc(is)..        qp(is) =l= GenMax(is) ;

*-------Second Stage-------
powerflows(s,is)..      qe(s,is) =e= sum(j$((F(is,j)>0 or F(j,is)>0) and not
                        (U(s,is,j)=1 or U(s,j,is)=1)),(B(is,j)+B(j,is))*(t(s,is)-t(s,j)))*100;
powerflowd(s,id)..      -qd(s,id) =e= sum(j$((F(id,j)>0 or F(j,id)>0) and not
                        (U(s,id,j)=1 or U(s,j,id)=1)),(B(id,j)+B(j,id))*(t(s,id)-t(s,j)))*100;
powerflowe(s,i)$((not is(i)) and (not id(i))).. 0 =e= sum(j$((F(i,j)>0 or F(j,i)>0) and not
                                                (U(s,i,j)=1 or U(s,j,i)=1)),
```

```
                                                   (B(i,j)+B(j,i))*(t(s,i)-t(s,j)))*100;
*ignored in U=1 cases... because if line's down there is no connection (so theta's don't matter)

limit(s,i,j)$((F(i,j)>0 or F(j,i)>0) and not (U(s,i,j)=1 or U(s,j,i)=1))..
                        ((B(i,j)+B(j,i))*(t(s,i)-t(s,j)))*100
                        =l= (1-U(s,i,j)-U(s,j,i))*(F(i,j)+F(j,i));

genlimit(s,is)..         qe(s,is) + qrc(s,is) =e= (1-Chi(s,is))*qp(is);
dminlimit(s,id)..        qd(s,id) =g= QdVar*QdMax(id);
dmaxlimit(s,id)..        qd(s,id) =l= QdMax(id);

swingdef(s,'1')..        t(s, '1') =e= 0;

Model network /all/;
*cplex on watmims (preferred)
*minos on watems
option lp=cplex;
*for cplex
network.OptFile = 1;
network.reslim = 21600;
option iterlim = 100000;
Solve network using lp minimizing snb;
option decimals=8;

Parameter alpha1(is);
alpha1(is) = poweralloc.m(is);

Parameter alpha2a(s,is);
alpha2a(s,is) = powerflows.m(s,is);
Parameter alpha2ac(s,is);
alpha2ac(s,is) = powerflows.m(s,is)/Pi(s);

Parameter alpha2b(s,id);
alpha2b(s,id) = powerflowd.m(s,id);
Parameter alpha2bc(s,id);
alpha2bc(s,id) = powerflowd.m(s,id)/Pi(s);

Parameter alpha2c(s,i);
```

```
alpha2c(s,i)$((not is(i)) and (not id(i))) = powerflowe.m(s,i);
Parameter alpha2cc(s,i);
alpha2cc(s,i)$((not is(i)) and (not id(i))) = powerflowe.m(s,i)/Pi(s);

Parameter alpha3(s,i,j);
alpha3(s,i,j) = limit.m(s,i,j);
Parameter alpha3c(s,i,j);
alpha3c(s,i,j) = limit.m(s,i,j)/Pi(s);

Parameter alpha4(s,is);
alpha4(s,is) = genlimit.m(s,is);
Parameter alpha4c(s,is);
alpha4c(s,is) = genlimit.m(s,is)/Pi(s);

Parameter alpha5(s,id);
alpha5(s,id) = dminlimit.m(s,id);
Parameter alpha5c(s,id);
alpha5c(s,id) = dminlimit.m(s,id)/Pi(s);

Parameter alpha6(s,id);
alpha6(s,id) = dmaxlimit.m(s,id);
Parameter alpha6c(s,id);
alpha6c(s,id) = dmaxlimit.m(s,id)/Pi(s);

Parameter zed(s,i,j);
zed(s,i,j) = (1-U(s,i,j)-U(s,j,i));

Parameter chix(s,is);
chix(s,is) = (1-chi(s,is));

Parameter Qdmin(id);
Qdmin(id) = QdVar*QdMax(id);

**********************Construction of output files**********************
file onedim_file /_predispatch.csv/;
put onedim_file;
onedim_file.nd=10;
onedim_file.nz=1.0e-10;
```

```
*put 'Obj Value and q(ip)'//;
*put 'Objective Value: ' welfare.l/;
put 'Gen, M_i, qp(is), alpha1'/;
loop(is, put is.tl, ',' GenMax(is), ',' qp.l(is), ',' alpha1(is)/);

file twodim_is_file /_supply_nodes.csv/;
put twodim_is_file;
twodim_is_file.nd=10;
twodim_is_file.nz=1.0e-10;
put '2D varaibles - Is'//;
put 'Scenario, Bus, qe(is), qrc(is), Ae, Arc,  theta(is), M_i, Chi(is), ScenPr, Alpha2a, Alpha2ac'/;
loop(s, loop(is, put s.tl, ',' is.tl, ',' qe.l(s,is),  ',' qrc.l(s,is), ',' Ae(is), ',' Arc(is) ','
  t.l(s,is), ',' GenMax(is), ',' Chix(s,is), ',' Pi(s), ',' alpha2a(s,is), ',' alpha2ac(s,is)/;));

file twodim_id_file /_demand_nodes.csv/;
put twodim_id_file;
twodim_id_file.nd=10;
twodim_id_file.nz=1.0e-10;
put '2D varaibles - Id'//;
put 'Scenario, Bus, QdVar, Ad, Qdmax, Qdmin, qd, theta(id), ScenPr, Alpha2, Alpha2c, Alpha5, Alpha5c,
  Alpha6, Alpha6c'/;
loop(s, loop(id, put s.tl, ',' id.tl, ',' QdVar, ',' Ad, ',' QdMax(id), ',' QdMin(id), ','
  qd.l(s,id), ',' t.l(s,id), ',' Pi(s), ',' alpha2b(s,id), ',' alpha2bc(s,id), ','
  alpha5(s,id), ',' alpha5c(s,id), ',' alpha6(s,id), ',' alpha6c(s,id)/;));

file twodim_i_file /_trans_nodes.csv/;
put twodim_i_file;
twodim_i_file.nd=10;
twodim_i_file.nz=1.0e-10;
put '2D varaibles - I'//;
put 'Scenario, Bus, theta(i), ScenPr, Alpha2, Alpha2c'/;
loop(s, loop(i$((not is(i)) and (not id(i))), put s.tl, ',' i.tl, ',' t.l(s,i), ',' Pi(s), ','
  alpha2c(s,i), ',' alpha2cc(s,i)/;));

file threedim_file /_lines.csv/;
put threedim_file;
threedim_file.nd=10;
threedim_file.nz=1.0e-10;
```

```
put ’line/node varaibles’//;
put ’s, i, j, line usage, line capacity, theta(i), theta(j), U(sij), Pi(s), alpha3, alpha3c’/;

loop(s, loop(i, loop(j$((F(i,j)>0 or F(j,i)>0) and not (U(s,i,j)=1 or U(s,j,i)=1) and
  limit.l(s,i,j) > 0), put s.tl, ’,’ i.tl, ’,’ j.tl, ’,’ limit.l(s,i,j), ’,’ limit.up(s,i,j), ’,’
  t.l(s,i), ’,’ t.l(s,j), ’,’ zed(s,i,j), ’,’ Pi(s), ’,’ alpha3(s,i,j), ’,’ alpha3c(s,i,j)/;)));
loop(s, loop(i, loop(j$((F(i,j)>0) and not (U(s,i,j)=1 or U(s,j,i)=1)
  and (limit.l(s,i,j)=0 or limit.l(s,j,i)=0 )), put s.tl, ’,’ i.tl, ’,’ j.tl, ’,’ limit.l(s,i,j), ’,’
  limit.up(s,i,j), ’,’ t.l(s,i), ’,’ t.l(s,j), ’,’ zed(s,i,j), ’,’ Pi(s), ’,’ alpha3(s,i,j), ’,’
  alpha3c(s,i,j)/;)));
loop(s, loop(i, loop(j$((F(i,j)>0 or F(j,i)>0) and (U(s,i,j)=1)), put s.tl, ’,’ i.tl, ’,’ j.tl, ’,’
  ’out’, ’,’ ’out’, ’,’ t.l(s,i), ’,’ t.l(s,j), ’,’ zed(s,i,j), ’,’ Pi(s)/;)));


*******Alpha Values*******

file alpha4_file /_alpha4.csv/;
put alpha4_file;
alpha4_file.nd=10;
alpha4_file.nz=1.0e-10;
put ’Alpha4’//;
put ’s, Gen, alpha4, alpha4c’/;
loop(s, loop(is, put s.tl, ’,’ is.tl, ’,’ alpha4(s,is), ’,’ alpha4c(s,is)/;));
```

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