A Multistage Stochastic Mixed-Integer Model for Perishable Capacity Expansion Problem

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

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Abstract

We study a multi-stage capacity expansion problem under demand uncertainty. We consider the problem where there are multiple resources to be expanded at each stage. Moreover, the resources have limited life time after acquisition. Our goal is to determine the time and size of each resource to be expanded so that the expected expansion cost of capacities is minimized. Therefore, we formulate the problem as a multi-stage stochastic mixed-integer program. Capacity shortage and excess are allowed subject to a joint chance constraint.

We apply the multi-stage stochastic mixed-integer model to formulate vaccine vial opening decisions in the health clinics. This formulation enables us to find the optimal combination of vial sizes to be opened. Additionally, a trade off between vaccine wastage and shortage can be addressed using the chance constraint.

We provide a branch and price algorithm based on a nodal decomposition to solve the model. In addition, a heuristic algorithm is proposed to solve the subproblems where the life time of the resources is limited to one period. We implement the branch and price algorithm assuming continuous capacity expansion decisions. Computational results are presented for the vaccine vial opening problem with three vial sizes; 1-, 5-, and 10-doses. The primary results indicate the strength of the proposed algorithm in solving problems with large dimensions. Moreover we report results that indicate the usage of 10-dose vials and the portion of 10-dose vials in the total vaccine usage increases with the arrival rate. Although the total usage of 1- and 5- dose vials increase with the arrival rate, their portion in the total vaccine usage decreases. This implies that vaccination wastage or shortage can be managed by keeping moderate amount of smaller size vials while supplying most of the demand using larger vial sizes to benefit from the economies of scale.

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Chapter 1

Introduction

1.1 Motivation and Background

We provide a novel framework for capacity-expansion problem with multiple perishable capacities and stochastic demand. We apply this framework to find the optimal vaccine vials opening policy under the availability of multi-dose vials with varying sizes.

In any industry, there is a set of vital resources. The two main aspects of these resources are their type and capacity. Firms need to enhance the level or type of their resources at many times in order to stay competitive in the market.

Increasing the capacity of the resources, upgrading them, or replacing them require enormous amount of investment in most cases. Additionally, as reported by Dixit [9], these investments are irreversible meaning that once invested, they cannot be retracted. This point becomes more important as we consider the level of uncertainty that originates from different sources such as future operational costs or demand. Therefore, the decision of investing in any resource capacity expansion needs to be taken after a careful assessment. Furthermore, decision makers usually face multiple investment opportunities. For example, an electronic company may want to invest in designing and developing a brand new production system or invest in improving and advancing its current one. Similarly, a company may need to decide between acquiring an expensive equipment with high production or service rate (to benefit from economies of scale) and a cheaper equipment with moderate production or service rate (for flexibility) or a combination of them.

Aside from the type and size of expansion, it is sometimes necessary to determine when the capacity should be expanded. For example, imagine a company is expecting the demand for a product to increase. Therefore, in addition to the size of capacity acquisition, one should know when to implement the expansion. That is why the capacity expansion problem is typically studied in multiple time periods.

1.1.1 Objectives

Many studies have addressed the capacity expansion problem with different settings such as single-period or multi-period decisions, decisions with perfect information about future or decisions under uncertainty. However, in all of them, it is assumed that the capacities infinitely endure. In this thesis, we consider a problem where capacities have a limited lifetime. A decision maker deals with an investment option that will provide the company benefits in the short run, but it will most likely be outdated by a new product in the near future. Therefore, a trade-off needs to be established between benefits and costs of investment and the future adjustments.

The level of demand satisfaction is also an important factor which we further discuss in the literature review section. In most cases, it is assumed that all the demand should be satisfied [3]; while in others, backlogging is allowed [18]. In this thesis, we allow both capacity shortage and excess. We deal with this concern with a chance constraint model.

We then use the proposed capacity expansion framework to examine the vaccine vial opening problem in health clinics. Vaccination is the most effective method to prevent infectious diseases especially for infants and against periodic epidemics. Therefore, many optimization methods have been employed by researchers to address the challenges in determining the optimal vaccination strategies.

Vaccines are perishable products; therefore, it is important to plan how the available vaccine vials should be used to maximize efficiency and effectiveness. Using the capacity expansion model enables us to address these concerns.

The remainder of this thesis is organized as follows. Chapter 2 provides a literature review on capacity expansion and vaccine administration problems. Chapter 3 describes the capacity expansion problem and the model formulation. Moreover, we provide an application of the model in vaccine administration. In chapter 4, Dantzig-Wolf decomposition of the model and solution method is described. Chapter 5 includes the computational results, and chapter 6 provides the conclusion as well as future research ideas.

Chapter 2

Literature Review

2.1 Capacity Expansion Problem

In many applications, capacity expansion in order to meet growing demand level is an essential part of the strategic level decision making. This problem requires decision makers to determine which facilities to be expanded and what is the optimal size and time of expansion. since 1950s, several studies have used quantitative methods. Manne [19] formulated a capacity expansion problem for heavy industries in India. Demand was known and increasing in the next 30 years, and capacity adjustments had to be done in order to meet the demand. His goal was to determine the optimal time and size of expansion such that the present worth of all costs was minimized.

Eppen et al.[11] studied a capacity planning problem at General Motors. The problem consisted of multiple locations, each having multiple expansion opportunities. Moreover, demand and investment costs were stochastic. Eppen et al. [11] developed a two-stage stochastic integer program. Their goal was to determine the best choice of expansion and its size at each of the locations so that the profit is maximized.

Swaminathan [28] studied the capacity expansion problem in the semiconductors industry where there were demand uncertainty. Swaminathan [28] developed a two-stage stochastic model with recourse to formulate the problem. The main goal was to determine the expansion size before demand realization and to allocate the new capacities to different products. The objective function was to minimize the shortage cost.

Examples of the capacity expansion models in service industries, such as restaurants and hotels can be found in Herman and Ganz [13].

2.1.1 Deterministic Models

We provide a literature review on the deterministic methods in capacity planing problem. Manne [18] proposed a deterministic optimization model to address the capacity planning problem for superhighways and/or pipelines. In this problem, demand was given for the next 20 years, and was linearly growing. The author also assumed that the capacities should satisfy the demand or exceed it. Also, there was a significant cost advantage for large expansions. Therefore, the goal was to find a trade-off between investment costs and economies of scale savings.

Luss [17] introduced a deterministic capacity expansion model for cable sizing problem in communication networks. He considered two types of expansion opportunities: either installing a new cable or converting an existing one. There were three cost elements, installation, conversion and holding which were nondecreasing and concave. The goal was to find the time, type, and size of cables to be invested to minimize the total cost.

2.1.2 Stochastic Models

We next give a review of the articles that propose stochastic optimization models.

Sen et al. [25] studied the capacity planning problem for networks that have private line customers under demand uncertainty. The authors developed a two-stage linear stochastic model with recourse. In a two-stage model, the capacity expansion decision is made in the first time stage before the demand is known. In the second stage, demand is realized and therefore recourse actions such as outsourcing can be taken. Sen et al. [25] targeted to meet demand under all scenarios. However, with the presence of higher potential errors in the demand estimation in the long run, satisfying this condition would require enormous investment costs. Therefore the authors studied the problem over a short time horizon which was long enough to cover the lead time. Sen et al[25] solved the model with a sampling based stochastic decomposition algorithm.

See Liu et al. [16], and Eppen et al. [11] for more literature review on two-stage stochastic models. We next provide a literature review of some studies that are closely related to our work.

Ahmed et al. [2] studied the multi-period capacity expansion problem where demand and costs were stochastic. They considered multiple types of capacity to be acquired and determined the size and time of each resource to be expanded over a finite horizon. The fixed-charge expansion costs were also taken into account which infers the presence of the economies of scales. Ahmed et al. [2] represented the uncertainties with a scenario tree and formulated a multi-stage stochastic integer program. They [2] proposed a problem reformulation which resulted in a tighter LP relaxation and developed a heuristic to obtain a good quality feasible integer solution. Then, they used the integer feasible solution in a branch and bound algorithm to accelerate the convergence of the algorithm.

The most related article to our research is that of Singh et al. [27]. They suggested a multi-stage stochastic mixed-integer programming model for capacity expansion problem under uncertainty. They wanted to minimize the fixed-charge cost of expansions and operating costs. They also assumed that capacities should meet or excess the operating requirement which is one of the main differences from our work. Singh et al. [27] provided a variable disaggregation approach to enable the Dantzig-Wolf decomposition, which is then solved by column generation method.

2.2 Vaccination Strategies

Operation Research applications in health care is a growing literature due to the recent peak in interest in the health care domain. Several recent Operation Research applications are proposed to address challenges in clinical decision making such as cancer screening [12], chronic disease treatment [6], infectious disease prevention [20], and health care delivery [22]. Determining an effective vaccine administration policy can be categorized into health care delivery literature but it is also related to medical decision making literature as the performance of the model is likely to affect health outcomes as well.

We now provide a literature review on the vaccination polices. Determining the best vaccination strategies has always been a great concern for health authorities and administrators in health clinics. Therefore, mathematical modeling and analysis have been widely applied by researchers to address questions such as what is the most effective policy to order vaccine vials in health clinics? [23]; which strategies should be followed when implementing immunization programs? [8]; which vaccine vial sizes should be produced or ordered? [15] Early studies have used deterministic methods to primarily evaluate current vaccination polices [26]. Later, simulation and stochastic methods have been incorporated by majority of researchers [10].

2.2.1 Deterministic Models

We first provide a literature review on deterministic models. Most deterministic models are in the form of compartmental models that represent the progression of infectious diseases and measure the effectiveness of vaccination policies. The decisions analyzed usually involve how much vaccine inventory should be hold and which patient groups (based on location, risk factors, etc.) should be prioritized to maximize the health outcomes and minimize the costs.

For instance, Waaler et al. [30] proposed a mathematical model to evaluate the prevalence of the tuberculosis and the effectiveness of the control polices including vaccination programs. They considered the problem over a twenty-year horizon. In particular, they investigated the effectiveness of the current vaccination strategies in reducing the infection prevalence over the years.

Note that there are a few agent-based simulation models and some stochastic process and stochastic dynamic programming modes proposed of the same problem. However, we consider unlimited available inventory and homogeneous patients, therefore, this literature is not very related to the current work. See Brogger [5] and Revelle et al. [24] for more literature review of deterministic modeling in vaccination strategies.

2.2.2 Stochastic and Simulation Models

Rajgopal et al.[23] studied the effect of ordering policies on the vaccination program performance. They assumed that the demand is unknown and the forecasting and estimation methods are not available in the clinics. Therefore, the average arrival rate in the past sessions were used to make the ordering decisions. Another factor is the vaccine vial size. They also investigated the need for a buffer stock. They developed a spreadsheet simulation model to examine these factors. They measured the performance by the amount of satisfied demand and did not take wastage into account. Moreover, they never considered ordering a combination of different vial sizes.

Lee et al.[15] developed a computational model to examine how single dose and multi dose vials affect the total cost of health clinics and wastage. There were three cost elements: cost of vaccine usage, cost of diposal of vaccine vials, and storage cost of vaccine vials. They considered five childhood vaccines, and assumed that they have limited life time after being opened. They also did a sensitivity analysis to determine how wastage and cost changes by vial costs and open-vial life time. Additionally, they evaluated the effect of two arrival processes: Poisson and Uniform distribution.

Dhamodharan and Proano [8] developed a Monte Carlo simulation model to determine the optimal vial size and optimal reorder point in a vaccination center. They assumed that the demand is stochastic and the lead times are known and constant. Their goal was to find the vial size that minimizes the purchase and wastage costs. For a given vial size, they generated demand instances and developed a mixed integer model to find the number of vials to be opened and the resulting wastage. The vial size with the minimum cost was selected. Their goal was to meet hundred percent of the demand, and their model had the limitation of selecting only one vial size to order.

Tanner et al.[29] proposed stochastic programming approach along with chance constraints to determine the optimal vaccination policy for an epidemic disease. They described three situations in which finding the optimal vaccination strategy is necessary. In the first case, their goal was to find a minimum cost vaccination policy for which the probability that the reproduction number after vaccination is less than one, is at least equal to the reliability parameter. The second case described the situation in which the vaccination budget is limited. Therefore, they searched for a policy that minimizes the probability of reproduction number being greater than or equal to one.

In the third case, in addition to the vaccination policy, the reliability was also a variable, and the goal was to minimize the costs of vaccination strategy which takes the reliability into account as well.

Chapter 3

Problem Definition

We study the capacity expansion problem where capacities are outdated after a certain number of periods. We consider the planning problem over a finite horizon, and we also assume that at each time period, there are a set of new capacities to be acquired. Moreover, due to the uncertain nature of demand estimations in the long run, we consider demand to be probabilistic. We use a scenario tree to represent the uncertainty. An example of a scenario tree with two periods is shown in Figure 3.1. It is a rooted tree with k possible outcomes of the random element, e.g., demand in the second stage. Our goal is to answer the following questions: When a new capacity is needed to be available? What is the best type of capacity to acquire? For each type of capacity, how much should be acquired? In addition to the capacity expansion decisions, we want to determine how the available capacity should be distributed in the successor nodes of the scenario tree. Since the newly expanded capacities will be expired in the future periods, managing capacity excess is one

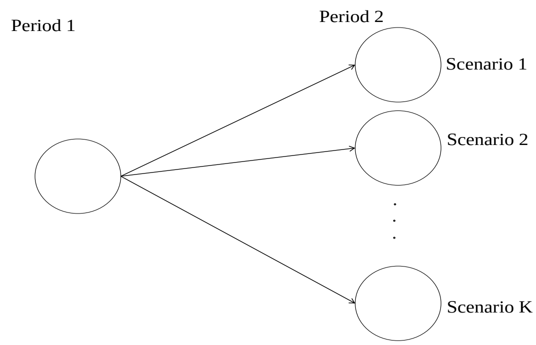

Figure 3.1: A two stage scenario tree

of our concerns. We want to avoid the extra capacities as much as possible rather than eliminating them entirely. As a result, when there are several expansion opportunities, there is a trade off between unit acquisition cost of the capacities and excess capacity. Instead of setting a constraint to enforce the model to generate zero wastage, we employ a chance constraint to keep unused capacities less than a certain threshold with some probability.

Another issue that we address is the demand coverage. Do we want to meet demand under each scenario? We deal with this concern as we did with the wastage. Some realizations of the demand have very small probability; therefore, we can allow the optimal solution to have shortage in those scenarios if we need to avoid costly choices. So, we set a chance constraint to assure that unsatisfied demand does not exceed a certain level with a reliability parameter.

The chance constraint keeps track of scenarios in which either capacity shortage or excess violates the requirement, and guarantees that the probability of all violated constraints is less than one minus a reliability parameter.

3.1 Chance Constraint

A chance or probabilistic constraint is used when it is not necessary to satisfy a certain constraint for all the realizations of a random element. According to Birge and Louveaux [4], when a random element is discrete, the general format of a chance constraint will be:

$$P(\ g(x, y(w), \xi(w)) \leq 0\ ) \geq\ \alpha, \tag{3.1}$$

where x is the vector of first stage decision variables, w is a random event, $\xi(w)$ is the random variable, and $y(w)$ is a second stage variable. For instance, x can capture the decision of how much to produce in a manufacturing system under demand uncertainty. Then w, $y(w)$, and $\xi(w)$ would respectively represent the demand, second stage variables (e.g. how much to sell) that is determined after knowing the demand, and the vector of demand realizations.

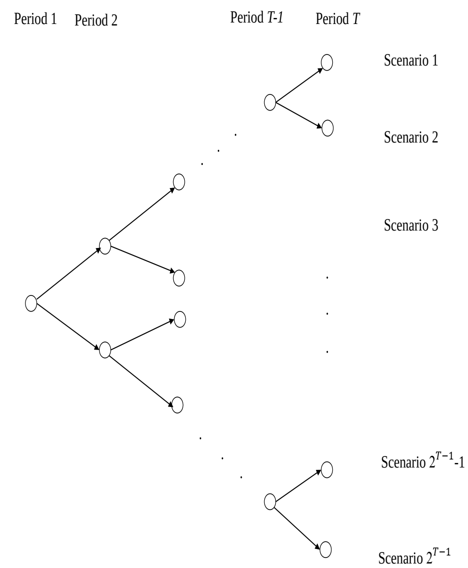

Figure 3.2: A scenario tree with T stages and $K = 2$ scenarios at each stage.

3.1.1 Notation

We use a scenario tree with T time stages to represent the uncertainty of the demand. Figure 3.2 shows a scenario tree with T periods. Each path of the scenario tree, starting from node 0 and ending with a node in the last stage, represents a scenario. The root node is in the first stage and is refered to as node 0. In node 0, no demand has been realized. In the second stage, there are K possible outcomes of demand. Each outcome has a probability of $p_1(k)$, and the summation of probabilities over all the outcomes is equal to one. For each node in the second stage, there are K possible realizations of demand in the third stage, and this continues until stage T, in which there are K^{T-1} scenarios.

Table 3.1: Sets and parameters

Parameters	Description
I	The set of available resources
d_m	Demand at node $m \in N$
p_m	The probability of node $m \in N$
c_i	Operating cost of capacity $i \in I$
v_i	Operating capacity that becomes available by expanding one unit of capacity $i \in I$
β_s, β_w	The maximum amount of shortage and wastage allowed
α	Reliability rate
ν	Life time of acquired capacities in terms of number of periods

The probability of each scenario is calculated as below:

$$\begin{aligned} P(Scenario\ 1) = &\quad p_1(1) \times p_2(1) \times \cdots \times p_T(1), \\ P(Scenario\ 2) = &\quad p_1(1) \times p_2(1) \times \cdots \times p_T(2), \\ &. \\ &. \\ &. \\ P(Scenario\ K^{T-1}) = &\quad p_1(K) \times p_2(K) \times \cdots \times p_T(K) \end{aligned} \tag{3.2}$$

Note that $\sum_{i=1}^{K} P(Scenario\ i) = 1$.

A brief description of the parameters in the model is given in Table 3.1. Besides the parameters, we need to define a few sets on the scenario tree nodes. Let N be the set of all nodes in the scenario tree and N_t be the nodes in time stage t. Also, $\mathscr{P}_{mn}$ is the set of all the scenario tree nodes in the path from node m to node n. In order to take the life time

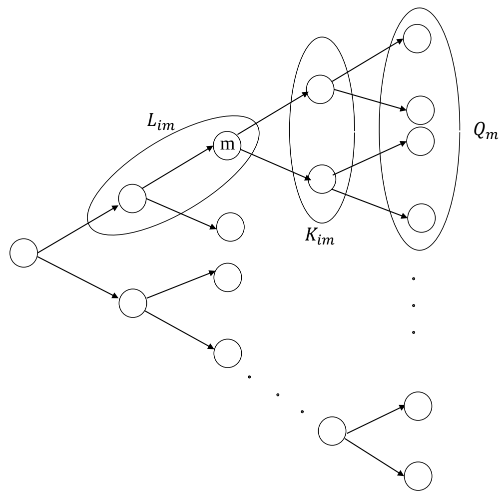

Figure 3.3: Notation illustration when $T = 5$ and $\nu = 1$.

restrictions into account, we define K_{im} as the leaf nodes of the subtree where capacity i expanded at node m can be used, and L_{im} as nodes where capacity of type i used at node m can come from. Moreover, Q_m represents the leaf nodes of the scenario tree which have node m in their path. The sets L_{im}, K_{im}, and Q_m are illustrated in Figure 5.1.

There are two types of decisions to be made. First, we want to determine when and how much of a certain kind of capacity should become available. Therefore, we define variable x_{im} as the number of capacity type i that is expanded at node m. Note that this decision results in the availability of the total amount of $v_i x_{im}$ operating capacity from type i at node m. Once some new capacities are acquired, they can be used either in the current node or in the successor nodes as long as they have not expired. So, we define the variable

y_{imn} as the amount of capacity type i that become available at node m and is used at node n.

3.1.2 Formulation

We formulate the problem as a multi-stage stochastic mixed integer model.

$$\min \quad \sum_{m \in N} \sum_{i \in I} p_m c_i x_{im} \tag{3.3a}$$

$$\text{subject to} \tag{3.3b}$$

$$P \begin{pmatrix} \sum_{i \in I} v_i x_{im} - \sum_{i \in I} \sum_{k \in \mathscr{P}_{mn}} y_{imk} \geq \beta_w \quad m \in N, n \in K_{im} \\ \text{or} \\ d_m - \sum_{i \in I} \sum_{r \in L_{im}} y_{irm} \geq \beta_s \quad m \in N \end{pmatrix} \leq 1 - \alpha \tag{3.3c}$$

$$v_i x_{im} - \sum_{k \in \mathscr{P}_{mn}} y_{imk} \geq 0 \qquad i \in I, m \in N, n \in K_{im} \tag{3.3d}$$

$$x_{im} \in \mathbb{Z}^+, y_{imk} \geq 0 \qquad i \in I,\ m \in N, k \in \mathscr{P}_{mn}, n \in K_{im} \tag{3.3e}$$

The objective function minimizes the expected cost of newly expanded capacities over all the nodes. Constraint ($3.3c$) is the joint chance constraint that manages the amount of unused capacities and unsatisfied demand. As a result of this constraint, with reliability rate of α, wastage is not greater than β_w and unsatisfied demand is less than or equal to

β_s. Constraints ($3.3d$) make sure that the utilized amount of a capacity does not exceed the available amount. Constraints 3.3e enforce the integrality conditions on variables x_{im} and y_{imn}.

Using the probability constraint causes nonlinearity in the model. We follow Birge and Louveaux [4] to reformulate the model to obtain its linear equivalent. To do so, we introduce a binary variable (ψ_q) for each scenario in the scenario tree. This binary variable is 1 if the chance constraint is violated in at least one node in that scenario. It is zero if both constraints are satisfied in all the nodes of that scenario. The mixed integer reformulation of the chance constraint is:

$$\sum_{i \in I} v_i x_{im} - \sum_{i \in I} \sum_{k \in \mathscr{P}_{mn}} y_{imk} \leq \beta_w + U_w \psi_q \qquad m \in N, n \in K_{im}, q \in Q_m \tag{3.4a}$$

$$d_m - \sum_{i \in I} \sum_{k \in L_{im}} y_{ikm} \leq \beta_s + d_m \psi_q \qquad m \in N, q \in Q_m \tag{3.4b}$$

$$\sum_{q \in N_T} p_q \psi_q \leq 1 - \alpha \tag{3.4c}$$

U_m indicates the upper bound of the left hand side in ($3.4a$). So if the total wasted capacity is less than β_w, the binary variable will be zero, and if the constraint is violated at any node of that scenario, the binary variable will be one. Note that d_m is the upper bound of the shortage in ($3.4b$). Constraints ($3.4b$) can be explained in the same way as the wastage constraints ($3.4a$). Constraint ($3.4c$) assures that the probability of all the violated scenarios does not exceed $1-\alpha$. Using this reformulation, the mixed integer equivalent of the stochastic model is:

$$[\text{EF}] \quad \min \quad \sum_{m \in N} \sum_{i \in I} p_m c_i x_{im} \tag{3.5a}$$

$$\text{subject to} \quad \sum_{i \in I} v_i x_{im} - \sum_{i \in I} \sum_{k \in \mathscr{P}_{mn}} y_{imk} \leq \beta_w + U_w \psi_q \qquad m \in N, n \in K_{im}, q \in Q_m \tag{3.5b}$$

$$v_i x_{im} - \sum_{k \in \mathscr{P}_{mn}} y_{imk} \geq 0 \qquad i \in I, m \in N, n \in K_{im} \tag{3.5c}$$

$$d_m - \sum_{i \in I} \sum_{k \in L_{im}} y_{ikm} \leq \beta_s + d_m \psi_q \qquad m \in N, q \in Q_m \tag{3.5d}$$

$$\sum_{q \in N_T} p_q \psi_q \leq 1 - \alpha \tag{3.5e}$$

$$x_{im} \in \mathbb{Z}^+, y_{imk} \geq 0, \psi_q \in \{0, 1\} \qquad m \in N, k \in \mathscr{P}_{mn}, n \in K_{im}, q \in N_T \tag{3.5f}$$

3.2 An Application to Vaccine Administration

We employ the described capacity expansion model to formulate the optimal vaccine vial opening problem in health clinics. Vaccine administration can be a challenge at the immunization centers, especially in rural areas, for several reasons. Vaccines are normally perishable, and once a multi dose vial is opened, it will be wasted if it is not fully used by a certain deadline. Therefore, vaccine wastage in the immunization sessions is a great concern. This concern becomes more important in areas where regularly ordering new batches is not possible. One of the main aspects contributing to the high amount of wastage is the vaccine vial sizes. Regardless of the expiration date of vaccines, when a vial with multiple doses is opened, the remaining doses will be wasted at the end of the session.

In this section, we consider an immunization clinic, and assume demand is being realized gradually. So, we divide each vaccination session into several periods. At each period, demand for future periods is unknown. We also assume the arrival rate is λ.

Furthermore, we assume that there is a pool of different vaccine vials, and there is unlimited number of available vials from each size. We want to minimize the total expected cost of purchasing, and determine how many vials from each size should be opened. Variables x_{im} will serve this purpose. Variables y_{imn} indicate the usage decisions. Note that I is the set of vial sizes and c_i is the purchase cost of vial size i.

The model (3.5a) can be used to formulate the vaccine vial opening problem.

Chapter 4

Solution Method

Even for a relatively small number of stages and demand realizations, the model (3.5a) can become a large-scale mixed integer program, and by increasing these factors, the problem size exponentially grows. Therefore, a commercial optimization tool can only solve instances that are considered to be very small compared to the real world problems. We present a branch and price method based on a nodal decomposition approach to address this difficulty.

4.1 Reformulation

We first provide a reformulation of the model to enable the decomposition of the model. We define a new binary variable for each node (z_m)and use it to keep track of the wastage constraint (3.5b). The reformulated model is:

$$
\begin{aligned}
\text{[REF]} \quad \min \quad & \sum_{m \in N} \sum_{i \in I} p_m c_i x_{im} && && \text{(4.1a)} \\
\text{subject to} \quad & \sum_{i \in I} v_i x_{im} - \sum_{i \in I} \sum_{k \in \mathscr{P}_{mn}} y_{imk} \leq \beta_w + U_w z_m && m \in N, n \in K_{im}, && \text{(4.1b)} \\
& v_i x_{im} - \sum_{k \in \mathscr{P}_{mn}} y_{imk} \geq 0 && i \in I, m \in N, n \in K_{im}, && \text{(4.1c)} \\
& d_m - \sum_{i \in I} \sum_{k \in L_{im}} y_{ikm} \leq \beta_s + d_m \psi_q && m \in N, q \in Q_m, && \text{(4.1d)} \\
& \psi_q \geq z_m && m \in N, q \in Q_m && \text{(4.1e)} \\
& \sum_{q \in N_T} p_q \psi_q \leq 1 - \alpha && && \text{(4.1f)} \\
& x_{im} \in \mathbb{Z}^+, y_{imk} \geq 0, \psi_q, z_m \in \{0,1\} && m \in N, k \in \mathscr{P}_{mn}, n \in K_{im}, q \in N_T && \text{(4.1g)}
\end{aligned}
$$

This formulation allows us to do a Lagrangian relaxation of constraint (4.1d) which leads to a nodal decomposition of the model.

4.2 Dantzig-Wolf Decomposition

Constraints (4.1b) and (4.1c) include the opening decisions at each node, and utilizing decisions for the successor nodes. Therefore they can independently be solved for each node. However, constraint (4.1d) are the complicating constraints which link the decision variables from the predecessors of a node. Thus, we decompose the reformulated model into two sets: the set of constraints that can be solved for each node (4.1b and 4.1c) and the linking constraint (4.1d).

We define the set $\mathcal{X}_m$ as follows:

$$\begin{aligned}\mathcal{X}_m = \{(x_{im}, y_{imn}, z_m)_{i \in I, n \in K_{im}} \mid & \sum_{i \in I} v_i x_{im} - \sum_{i \in I} \sum_{k \in \mathscr{P}_{mn}} y_{imk} \leq \beta_w + U_w z_m, \\ & v_i x_{im} - \sum_{k \in \mathscr{P}_{mn}} y_{imk} \geq 0, \\ & x_{im} \in \mathbb{Z}^+, \; y_{imn} \geq 0, z_m \in \{0, 1\}\},\end{aligned} \tag{4.2}$$

which represents feasible solutions of the constraints 4.1b and 4.1c at each node. Now, let $\mathcal{F}_m$ be the index set of feasible solutions at node m and $(\bar{x}_{im}, \bar{y}_{imn}, \bar{z}_m)^j$ be an element of $\mathcal{X}_m$. Then, $\mathcal{X}_m$ can be rewritten as $\{(\bar{x}_{im}, \bar{y}_{imn}, \bar{z}_m)^j \mid j \in \mathcal{F}_m\}$.

Each member of $\mathcal{X}_m$ can be represented as:

$$
\begin{aligned}
(x_{im}, y_{imn}, z_m) &= \sum_{j \in \mathcal{F}_m} (\bar{x}_{im}, \bar{y}_{imn}, \bar{z}_m)^j w_m^j, \\
\sum_{j \in \mathcal{F}_m} w_m^j &= 1, \quad w_m^j \in \{0, 1\}.
\end{aligned} \tag{4.3}
$$

By relaxing constraints 4.1b and 4.1c, and replacing variables x_{im}, y_{imn}, and z_m with the equation 4.3, we obtain an equivalent formulation as follows:

$$
\begin{aligned}
\text{[MP]} \quad \min \quad & \sum_{i \in I} \sum_{m \in N} \sum_{j \in \mathcal{F}_m} p_m c_i \bar{x}_{im}^j w_m^j && && \text{(4.4a)} \\
\text{subject to} \quad & d_m - \sum_{i \in I} \sum_{k \in L_{im}} \sum_{j \in \mathcal{F}_k} \bar{y}_{ikm}^j w_k^j \leq \beta_s + d_m \psi_q && m \in N, q \in Q_m \ (\pi_{mq}) && \text{(4.4b)} \\
& \psi_q \geq \sum_{j \in \mathcal{F}_m} \bar{z}_m^j w_m^j && m \in N, q \in Q_m, \ (\gamma_{mq}) && \text{(4.4c)} \\
& \sum_{j \in \mathcal{F}_m} w_m^j = 1 && m \in N, \ (\mu_m) && \text{(4.4d)} \\
& \sum_{q \in N_T} p_q \psi_q \leq 1 - \alpha, && && \text{(4.4e)} \\
& w_m^j, \psi_q \in \{0, 1\} && m \in N, j \in \mathcal{F}_m, q \in N_T && \text{(4.4f)}
\end{aligned}
$$

The objective function minimizes the expected opening cost. Constraints (4.4b) enforce the shortage chance constraint for the selected feasible solutions. Constraints (4.4c) make sure that the binary variable for each scenario is at least equal to the binary variable of each node in that scenario. Therefore, if wastage constraint has been violated at node m, the binary variable for all the scenarios containing node m needs to be one. Constraints (4.3) are the convexity constraints which make sure that exactly one feasible solution is selected for each node. Note that π_{mq}, γ_{mq}, and μ_m are the dual variables of the constraints 4.4b, 4.4c, and 4.4d, respectively.

Even for the small sizes of the problem, the master problem has a huge number of variables which makes it impracticable to generate all the variables at the beginning and solve the master problem. Therefore, we start solving the master problem with an initial limited number of variables. Then we attempt to find new variables with negative reduced cost, and add them to the restricted master problem.

In order to find a non-basic variable in $\mathcal{X}_m$, we solve the following subproblem for each node.

$$[\text{SP(m)}] \quad \min \quad \sum_{i\in I} p_m c_i x_{im} - \sum_{i\in I}\sum_{n\in K_{im}}\sum_{k\in \mathscr{P}_{mn}}\sum_{\ell\in Q_k} \bar{\pi}_{k\ell} y_{imk} + \sum_{q\in Q_m} \bar{\gamma}_{mq} z_m - \bar{\mu}_m \tag{4.5a}$$

$$\text{subject to} \quad \sum_{i\in I}\left(v_i x_{im} - \sum_{k\in \mathscr{P}_{mn}} y_{imk}\right) \leq \beta_w + M z_m \qquad n \in K_{im}, \tag{4.5b}$$

$$v_i x_{im} - \sum_{k\in \mathscr{P}_{mn}} y_{imk} \geq 0 \qquad i \in I, n \in K_{im}, \tag{4.5c}$$

$$x_{im} \in \mathbb{Z}^+,\ y_{imn} \geq 0, z_m \in \{0,1\}, \qquad i \in I, n \in K_{im}. \tag{4.5d}$$

The subproblem (4.5a) searches for the minimum reduced cost for node m. Therefore if the objective function is negative, a new variable will be added to the restricted master problem.

4.3 Solving the Master Problem

As described previously, the master problem has typically a large number of variables. Therefore, we apply the column generation method to solve the master problem. We first consider a restricted master problem (RMP) in which only a subset of the variables ($\mathcal{F}'_m \subset \mathcal{F}_m$) are present. Then we solve the linear programing relaxation of the restricted master problem, and obtain the optimal dual solutions. The optimal solution of the restricted master problem can be considered as a feasible solution of the master problem in which the value of the variables not included in the master problem is zero. Therefore, we need to evaluate the reduced cost of the nonbasic variables, and pick a variable with negative reduced cost to be added to the restricted master problem.

To do so, we use the optimal dual solutions of the restricted master problem, and solve the subproblem for each node. The objective function of each subproblem (4.5a) is in fact the reduced cost of the variables corresponding to that node. Moreover, the feasible region of a subproblem is the set $\mathcal{F}_m$. In other words, subproblems seek for a variable in $\mathcal{F}_m$ that has the most negative reduced cost. Therefore, if the objective function of the subproblem is negative, the variable will be added to the restricted master problem. The steps of solving the restricted master problem, obtaining the dual values, and solving the subproblems are repeated until no variable exist with negative reduced cost, i.e. when the objective function of all the subproblems are non negative. At this point, the master problem is solved optimally, and the solution of the restricted master problem is the optimal solution of the LP relaxation of the master problem.

4.4 Solving the Subproblems

The subproblems (4.5a) are all mixed integer program which can be solved by any optimization tool. However, by increasing the problem size, the size of each subproblem as well as the number of subproblems to be solved grow. This results in a high computational time in the column generation method and thus in the branch and price algorithm.

We provide a heuristic algorithm to solve the subproblems where the opened vial life time is one period, i.e. once a multi-dose vial is opened, it can be used at either current node or the successor nodes at the next stage.

To describe the heuristic, let $g_{mn} = \sum_{l \in Q_n} \bar{\pi}_{nl}$ be the coefficient of y_{imn}, $\forall i \in I, n \in \mathscr{P}_{mk}$, $k \in K_m$ in the objective function of the subproblem (4.5a) of the node m. Also, let $g'_k = \max\{c_{mn}, \forall n \in \mathscr{P}_{mk},\}$, $\forall k \in K_m$. Note that at each subproblem, there are $|I|$ opening decision variables. Let $b_i = \sum_{i \in I} p_m c_i$ be the coefficient of the x_i, $\forall i \in I$ in the objective function of the subproblem. Figure 5.2 demonstrates a given node (m) and its successor nodes (1 to K). Any vial opened at node m can be used at node m and 1, or node m and 2, and so on. Moreover, there is one binary variable (z) associated with the wastage chance constraint of the node m. Let $d = \sum_{q \in Q_m} \bar{\gamma}_{mq}$ be the coefficient of the z in the objective function.

There are two main cases to investigate: 1: g'_k are all positive, 2: There is at least one

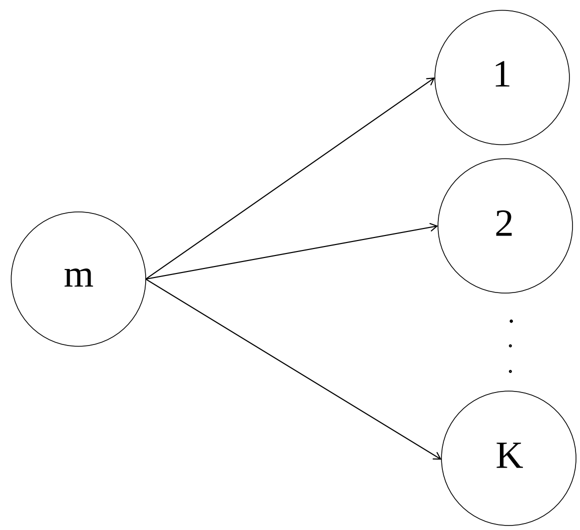

Figure 4.1: The current node (m) and its successors.

non positive $g'_k, \forall k \in K_m$.

It is easy to verify that if g'_k are all positive, the optimal solution should include opening the maximum possible amount, and using the maximum amount in the node at which g'_k happens.

If there is at least one non positive $g'_k, \forall k \in K_m$, it is necessary to find a trade off between not utilizing the opened vials in some scenarios of the current node and setting the variable z to one.

In both cases, there is a condition to be checked. All the opening and using decisions are taken as long as $b_i - v_i g'_k \leq 0$, i.e. the immediate effect of opening some vials and using them on the objective function is negative. Otherwise, one of the following three options should be selected: not opening vial i, opening vial i and fully using them or opening but not using them at scenarios violating $b_i - v_i g'_k \leq 0$. Note that the latter results in the variable z to be one. The key steps of the algorithm are summarized as follows:

STEP 1: Calculate $b_i, \forall i \in I$, $g'_k, \forall k \in K_m$ and d.

STEP 2 : If all g'_k are non-positive, let

$$\begin{aligned}
&x_i = x_{lb},\\
&y_{imm} = y_{lb},\ y_{im1} = \cdots = y_{imK} = 0,\\
&z = 0,
\end{aligned}$$

and stop.

STEP 3: For all $i \in I$, if $b_i - v_i g'_k \leq 0$,

- If all g'_k are positive, one of the followings is the optimal solution:

$$\begin{aligned}
&x_i = x_{ub}, & & x_i = x_{ub},\\
&y_{imm} = y_{ub}, y_{im1} = \cdots = y_{imK} = 0, \quad or & & y_{imm} = 0, y_{im1} = \cdots = y_{imK} = y_{ub},\\
&z = 0 & & z = 0
\end{aligned}$$

- If at least one of g'_k is non-positive, for instance g'_1, one of the followings is the optimal solution:

$$x_i = x_{ub}, \qquad\qquad x_i = x_{ub},$$

$$y_{imm} = y_{im1} = 0,\ y_{im2} = \cdots = y_{imK} = y_{ub}, \quad or \quad y_{imm} = 0,\ y_{im1} = \cdots = y_{imK} = y_{ub},$$

$$z = 1 \qquad\qquad z = 0$$

4.5 Branching

After the LP relaxation of the master problem is solved to optimality, we check the integrability of the binary variables associated with the chance constraint. We apply two branching scheme to make sure that all the integrability requirements are satisfied.

First we consider all the binary variables associated with the chance constraints in the master problem. Note that there is a binary variable corresponding to each scenario in the master problem. Let ψ_0 be the binary variable for the first scenario and not integer. We create two branches and apply the bound changes on the master problem of the child nodes. At the node where the variable is fixed to zero, i.e. its upper bound is changed to zero, we apply the same upper bound in all the subproblems of the first scenario. The reason is because ψ_0 being equal to zero implies that the chance constraint for all the nodes in that scenario is satisfied; therefore, their binary variable should also be fixed to zero.

Furthermore, we consider the binary variables in the extended format (3.5a) to be integer. Therefore, we calculate their value every time the master problem is optimally solved. Let z_{m_0} be the one with fractional value. We create two child nodes and apply bound changes in the subproblem of the node m_0. In addition, we evaluate the existing columns in the master problem and fix them to zero if they do not comply with the branching decisions. Similar branching strategy can be taken into account to address the integrality requirements of the variables x_{im}. However, the implementation of this branching rule is not provided in this thesis.

Chapter 5

Computational Results

In this chapter, we present computational experiments using the solution method described in Chapter 4. We consider the vaccine vial opening problem where there are three vial sizes. Vial sizes are similar to those used in the analysis of Rajgopal et al. [23]. Table 5.1 represents the vial sizes and the corresponding costs. We implement the branch and price method where the capacity expansion decisions are continuous, i.e. x_{im} variables are assumed to be continuous. In the first experiment, we evaluate the performance of the branch and price algorithm, and then compare it with that of solving the extensive formulation (EP)(3.5a) with SOPlex-2.0.1. In the second experiment, we investigate the

Table 5.1: Vial sizes

Vial size	Cost ($)
1	1
5	4
10	7

Table 5.2: Problem dimensions

Problem	T	K	N. of Scenarios	α	Binary	Continuous	Constraints
$P\_3\_2\_0$	3	2	4	0.95	4	48	49
$P\_3\_2\_1$				0.85			
$P\_3\_2\_2$				0.75			
$P\_3\_3\_0$	3	3	9	0.95	9	132	127
$P\_3\_3\_1$				0.85			
$P\_3\_3\_2$				0.75			
$P\_4\_2\_0$	4	2	8	0.95	8	120	129
$P\_4\_2\_1$				0.85			
$P\_4\_2\_2$				0.75			
$P\_4\_3\_0$	4	3	27	0.95	27	342	415
$P\_4\_3\_1$				0.85			
$P\_4\_3\_2$				0.75			
$P\_5\_2\_0$	5	2	16	0.95	16	264	305
$P\_5\_2\_1$				0.85			
$P\_5\_2\_2$				0.75			
$P\_5\_3\_0$	5	3	81	0.95	81	1071	1441
$P\_5\_3\_1$				0.85			
$P\_5\_3\_2$				0.75			
$P\_6\_2\_0$	6	2	32	0.95	32	552	689
$P\_6\_2\_1$				0.85			
$P\_6\_2\_2$				0.75			
$P\_6\_3\_0$	6	3	243	0.95	243	3258	4843
$P\_6\_3\_1$				0.85			
$P\_6\_3\_2$				0.75			
$P\_7\_2\_0$	7	2	64	0.95	64	1125	1521
$P\_7\_2\_1$				0.85			
$P\_7\_2\_2$				0.75			
$P\_8\_2\_0$	8	2	128	0.95	128	2280	3313
$P\_8\_2\_1$				0.85			
$P\_8\_2\_2$				0.75			

impact of mean arrival rate on the combination of vial sizes that are opened. We assume that the set of the possible demand values is $\left\{\lambda - \left\lceil \frac{K}{2} \right\rceil, \ldots, \lambda + \left\lfloor \frac{K}{2} \right\rfloor\right\}$ in which λ is the mean arrival rate per period and K is the number of demand realizations. We then calculate their probability as follows:

$$P(\lambda - \left\lceil \frac{K}{2} \right\rceil) = \sum_{i=0}^{\left\lceil \frac{K}{2} \right\rceil} p(\lambda - i),$$
$$P(\lambda + \left\lfloor \frac{K}{2} \right\rfloor) = \sum_{i=\left\lfloor \frac{K}{2} \right\rfloor} p(\lambda + i),$$

where $(\lambda \pm i)$ has Poisson distribution. The probability of the other possible values of the demand is calculated according to the Poisson distribution.

All experiments are done on a system with 11 GB of RAM and 3.60GHz processor. The branch and price algorithm is implemented in C++ using SCIP-3.1.1 [1] as the framework and SoPlex-2.0.1 [31] as the optimization software.

5.1 Performance of the Branch and Price Algorithm

In this experiment, we consider the vaccine vial opening problem with 10 different problem sizes in which the number of stages and demand realizations varies. For each problem size, we generate 3 problem instances by changing the reliability rate (α). Table 5.2 summarizes the problem sizes, the values of α, number of binary and continuous variables and constraints in the extensive formulation (EF) (3.5a). Note that in this table, T indicates

Table 5.3: Branch and price performance

Problem	Obj. Value ($)	EF CPU Time (sec)	B&P CPU Time (sec)	Time Reduction (%)
$P\_3\_2\_0$	6.22	0.957	0.72	24.76
$P\_3\_2\_1$	5.29	1.804	0.73	59.53
$P\_3\_2\_2$	4.98	0.913	0.99	-8.43
$P\_3\_3\_0$	6.22	1.001	0.73	27.07
$P\_3\_3\_1$	5.81	1.947	0.12	93.83
$P\_3\_3\_2$	5.12	1.001	0.11	89.01
$P\_4\_2\_0$	1.09	2.028	0.77	62.03
$P\_4\_2\_1$	1.03	1.032	0.9	12.79
$P\_4\_2\_2$	9.02	1.068	0.9	15.73
$P\_4\_3\_0$	1.59	2.436	5.47	-124.54
$P\_4\_3\_1$	13.6	1.464	7.63	-421.17
$P\_4\_3\_2$	11.4	0.516	4.48	-768.21
$P\_5\_2\_0$	21.8	2.184	1.08	50.54
$P\_5\_2\_1$	19.9	1.248	3.86	-209.29
$P\_5\_2\_2$	17.4	1.284	4.01	-212.30
$P\_5\_3\_0$	42.8	19.428	8.01	58.77
$P\_5\_3\_1$	35.8	71.1	9.72	86.32
$P\_5\_3\_2$	28.6	21.564	9.11	57.75
$P\_6\_2\_0$	42.6	1.512	2.92	-93.12
$P\_6\_2\_1$	38.3	5.268	3.87	26.53
$P\_6\_2\_2$	33.6	5.496	3.91	28.85
$P\_6\_3\_0$	966.67	(5.95%)	1076.2	-
$P\_6\_3\_1$	872.46	(7.27%)	1141.05	-
$P\_6\_3\_2$	761.38	(8.05%)	1094.66	-
$P\_7\_2\_0$	84.6	9.444	5.05	46.52
$P\_7\_2\_1$	74.7	35.052	7.32	79.11
$P\_7\_2\_2$	64.4	46.932	7.91	83.14
$P\_8\_2\_0$	167.38	650.43	540.8	16.85
$P\_8\_2\_1$	105.38	2394.03	821.56	65.68
$P\_8\_2\_2$	94.4	2875.95	874.52	69.59

the number of stages and K indicates the number of demand realizations emanating from each node.

We compare the performance of the proposed branch and price algorithm with the results of solving the extensive format (3.5a), with the variables x_{im} as continuous, using SCIP-3.1.1. Table 5.3 presents the objective function value, the CPU time for both methods, and the reduction in solving time. As can be seen, the proposed branch and price algorithm provides a lower solving time in majority of the instances. A CPU time limit of 3600 seconds was imposed while running all instances. For instances with 243 scenarios, the extensive format is not solved to optimality in the time limit. Thus the optimality gap is reported for them. The largest instance that we have solved has 243 scenarios; larger instances would require more than 3600 seconds of CPU time. We report the time reduction in percentage for instances up to 128 scenarios. the positive values show the reduction, and the positive values represent an increase in the solution time.

5.2 Effect of the Mean Arrival Rate on the Optimal number of opened Vial

In the second experiment, we solve the extensive format (3.5a) for the vaccine vial opening problem using SoPlex-2.0.1. We consider 6 instances of the vaccine vial opening problem with 4 stages and 2 realizations of the demand emanating from each scenario tree node, i.e.

Table 5.4: Problem parameters

Reliability rate (α)	90% (\$)
Maximum allowable wastage and shortage (β_s, β_w)	1 dose
Opened vials' life time (ν)	1 period

8 scenarios in total. In these instances, we change the mean arrival rate per period from 5 to 30, and examine the number of opened vials from each vial size. Table 5.4 summarizes the problem parameters used in this experiment.

Our goal is to investigate how the number of vials from each vial size changes by different arrival rates. Table 5.5 shows the optimal number of each vial size that is opened. As can be observed, when λ is less than 10, no 10-dose vial is opened in order to avoid wastage. Yet, the overall usage of 10 dose vials is increasing with the mean arrival rate. However, 5-dose vials do not show a monotonic behavior; as when λ is divisible by 5, number of opened 5-dose vials is always 1. While for the other values of λ, more than one vial is opened. This can be explained by the desire to balance between the opening cost and the wastage.

We next define the ratio R_i which measures the portion of the total vaccine usage that is provided by each vial size. R_i is calculated as follows:

$$R_i = \frac{v_i x_i}{\sum_{i \in I} v_i x_i}, \quad \forall i \in I.$$

Table 5.5: The optimal number of vials

λ per period	1-dose	5-dose	10-dose	Total (dose)
5	2.33	1.806	0	11.36
10	1.54	1	2	26.54
15	3.39	1.53	3	41.04
20	1.53	1	5	56.53
25	1.53	2	6	71.53
30	1.52	1	8	86.52

Table 5.6: The values of R_i, i $= 1, 5, 10$

λ per period	R_1	R_5	R_{10}
5	20.51	79.48	0
10	5.80	18.83	75.35
15	8.26	18.64	73.09
20	2.70	8.84	88.44
25	2.13	13.98	83.88
30	1.75	5.77	92.46

R_1, R_2, and R_3 refers to the ratio of 1, 5, and 10-dose vials, respectively. The values of R_i are summarized in Table 5.6 .

Figure 5.1 depicts how the ratio (R_i) changes with the mean arrival rate. As can be seen, the ratio of 10-dose vials increase with the arrival rate in a non monotonic way. It can be observed that at arrival rates 15 and 25, R_{10} has dropped, and instead 5-dose usage has increased. This implies that with higher value of λ, higher portion of the demand is satisfied by 10-dose vials. However, when the mean arrival rate is a multiple of 5, the contribution of the 5-dose vials in satisfying the demand increases. This behavior can be interpreted as a prevention against wastage.

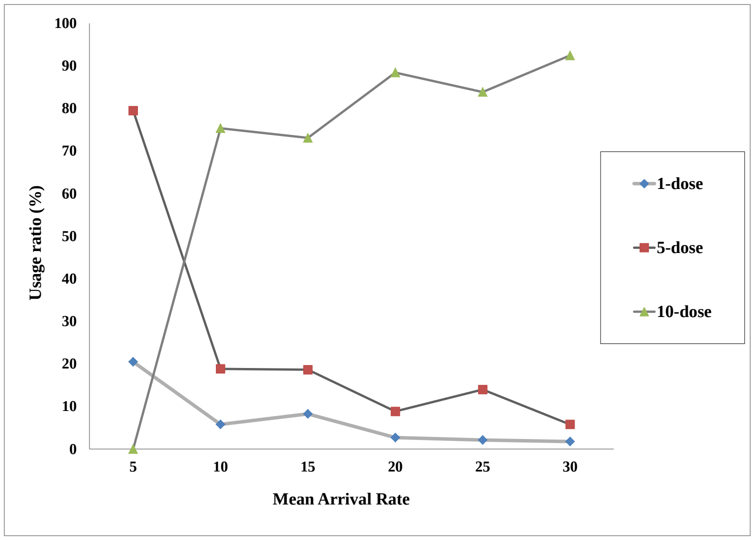

Figure 5.1: The ratio (R_i), $i = 1, 2$, and 3

5.3 Effect of the Reliability Rate (α) on the Costs

In the third experiment, we investigate the effect of α on the total cost. Therefore, we solve the vaccine vial opening problem with three vial sizes, 1-, 5-, and 10-doses. We choose a problem size of 8 scenarios, i.e. 4 stages and 2 demand outcomes emanating from each node. Moreover, we use a maximum allowable wastage and shortage of 1 unit.

We solve the described problem where α changes from 0.7 to 1 for the mean arrival rate of 5 and 10 per period. Note that $\alpha = 1$ makes the chance constraint equivalent to two ordinary constraints that require all the scenarios to comply with the maximum allowable amount of shortage and wastage. Table 5.7 summarizes the value of the objective function

Table 5.7: Expected cost's changes with α

	Mean arrival rate per period = 5	Mean arrival rate per period = 10
α	Obj. Value	Obj Value
0.7	9016.69	16929.56
0.75	9020.00	17448.61
0.8	9658.27	20241.15
0.85	10300.00	20241.15
0.9	10891.65	21035.60
0.95	10900.00	21865.38
1	10900.65	21865.38

(costs) for different value of α. As can be seen, the objective function increases with α for both mean arrival rates.

We also represent the results in Figure 5.2 which shows increasing the objective function with α. As α increases, the chance constraint enforces the model to satisfy ordinary wastage and shortage constraints for more scenarios. Therefore, in order to satisfy the shortage constraint, higher capacity expansion costs are imposed.

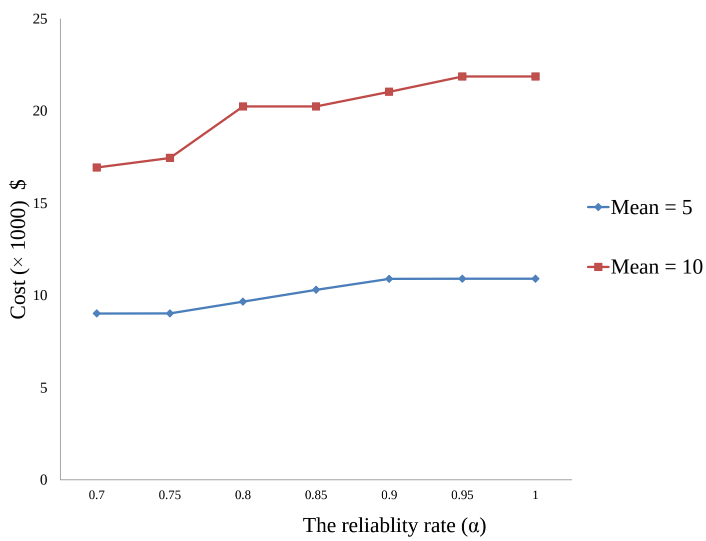

Figure 5.2: Expected cost's changes with α

Chapter 6

Conclusions and Future Work

We considered a multi-stage capacity expansion problem where demand is unknown and capacities are perishable. We developed a multi-stage stochastic programming model to formulate the problem. A scenario tree was used to represent the uncertainty of the demand. The objective function was to minimize the expansion cost of capacities. Our main purpose was to determine when a capacity should be expanded and the size of expansion. In our model, the capacity requirements were determined by demand under all scenarios. We used a probabilistic constraint to address the trade-off between capacity shortage and capacity excess.

To the best of our knowledge, perishable capacities have not been applied in the stochastic capacity expansion problem in the literature.

We employed binary variables to reformulate the chance constraint, and obtain a deterministic mixed-integer equivalent of the model. As an application, we proposed to formulate the vaccine vial opening problem. This formulation enabled us to address important concerns regarding the vaccine administration. We considered different vial sizes as the resources that can be used during a vaccination session. We assumed that the demand is realized during multiple time periods. Our goal was to minimize the purchase cost of vials, and hedge against the vaccine wastage and shortage using a chance constraint.

We developed a reformulation of the mixed-integer model to enable a nodal decomposition. Then, we decomposed the model into a Dantgiz-Wolf master problem and a subproblem for each node. We proposed a heuristic to solve the subproblems, and implemented a branch and price algorithm where capacity expansion decisions were continuous. The results supported the performance of the algorithm compared to solving the extensive format using SoPlex-2.0.1. Implementing the branch and price algorithm with integer capacity expansion decisions can be a potential future work.

We also reported that the total usage of 5 and 10 dose vials increases with the mean arrival rate. However, unlike the 10 dose vials, the portion of 1 and 5 dose vials in the total vaccine usage decreases with the arrival rate.

In addition, we examined the behavior of the objective function with the changes in the reliability rate and mean arrival rate. We reported the results suggesting that the expected purchase cost increase with the reliability rate.

In this study, we assumed that there is unlimited amount of each resource to be expanded.

This assumption can be removed in future studies. Moreover, a fixed charge cost can be considered by which economies of scales can be taken into account. This assumption can make the model more complex, however, the proposed decomposition approach would still be valid.

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