Service System Design with Immobile Servers, Stochastic Demand and Economies of Scale

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

The service system design problem seeks to locate facilities, determine their capacity, and assign customers to them in order to improve the service quality and the customers' experience while minimizing the capacity acquisition cost, the customer access cost, and the average waiting cost. While the centralization of facilities will lead to economies of scale, decentralizing them will lead to faster response times. Traditionally, the capacity acquisition costs were assumed linear with a fixed setup cost. In this work, we explicitly account for economies of scale by modeling the cost as a concave function of capacity.

In this thesis, we model and provide solution methodologies for the service system design problem with immobile servers, stochastic demand and economies of scale. We start by reformulating the problem, and then provide solution approaches based on piece-wise linearization, Second Order Cone Programming (SOCP), and Lagrangian Relaxation. Extensive numerical testing on a standard data set is provided and the results analyzed.

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Dedication

This is dedicated to my beloved Qianyao Tan.

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Chapter 1

Introduction

In the service system design problem, the costs minimized include linear capacity costs, transportation costs, and customer waiting costs. Capacity costs include the costs to open service facilities and to assign enough service capacity to them. The customer waiting costs are often measured by the average time per customer, the number of customers that can be served within a time limit, or the probability to serve all customers within a desired time period. For large service facilities, economies of scale can be achieved, which pushes for the establishment of fewer and larger facilities. This, on the other hand, will force access cost and waiting times to increase. Thus, it is important for the decision makers to accurately monitor all the costs, and to balance the trade off between the capacity costs and customer waiting costs to minimize the total cost.

In this thesis, we consider a service system design problem with immobile servers, stochastic demand, and economies of scale. We will tackle the problem with different approaches. First, we will provide a new formulation based on a Benders-type idea and propose a piecewise linear approximation based on Special Order Set of Type 2 (SOS2) constraints.

Next, we focus on the special case with a square root capacity function, and propose a Second Order Cone Programming (SOCP) approach. New auxiliary variables and a facility utilization factor variable are introduced to reformulate the problem. A SOCP based Lagrangian Relaxation is proposed where the subproblems are SOCP problems. The Lagrangian bounds along with feasible solutions are generated and compared with the other method.

A small example with 3 facilities and 6 customers is used to illustrate the different models and solution approaches introduced. In the beginning, a complete enumeration is used to get the optimal solution. As new approaches are being introduced, this small example is used again to give a better illustration of the algorithms.

This thesis is organized as follows: Chapter 2 presents the literature review for the facility location and the service system design problems. Chapter 3 describes the problem formulations. Chapter 4 presents different solution methodologies, including the SOS2 and SOCP approaches. Chapter 5 provides the numerical testing on a standard data set. Chapter 6 concludes the thesis.

Chapter 2

Literature Review

The service system design problem with immobile servers, stochastic demand and congestion has been activity studied in recent decades. Amiri [3, 4] was among the first to consider the objective. He assumed an infinite buffer capacity, Poisson arrival and exponential service time. Their assumptions became standard assumption in most subsequent work. Elhedhli [11] considered a service system design problem modelled as a network of M/M/1 queues. He transformed the nonlinear model to a linear mixed integer problem with a large set of constraints and solved it using a cutting plane method.

2.1 The Facility Location Problem

Facility location problems seek to locate facilities to serve customer demand with the objective of minimizing facility opening and operating costs as well as transportation costs. Hamacher and Nickel [15] provided a classification scheme for facility location problems.

Klose and Drexl [18] studied the distribution system design problem, and provide a summary of continuous facility location and network design problems. Revelle et al. [19] reviewed the p-median plant location problem as well as the p-center and the covering problems.

2.1.1 The Uncapacitated Facility Location Problem

Let's consider a problem with J potential facilities and I customers. The demand for customer i is λ_i, $i \in I$. The fixed cost to open a facility j is f_j, $j \in J$, and the unit cost of serving from facility j to customer i is c_{ij}. Let x_{ij} take value of 1 if customer i is served by facility j and 0 otherwise. Let y_i take value of 1 if facility j is open. The UFLP model is:

$$\text{[UFLP]:} \quad \min \sum_{j=1}^{J} f_j y_j + \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} \lambda_i x_{ij}$$

$$\text{s.t.} \sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1, ..., I \tag{1}$$

$$\sum_{i=1}^{I} x_{ij} \leq y_j \qquad j = 1, ..., J \tag{2}$$

$$x_{ij}, y_j \in \{0, 1\} \qquad i = 1, ..., I; j = 1, ..., J \tag{3}$$

The objective function minimizes the total costs, which include the facility opening costs and the variable transportation costs. Constraints (1) ensure that every customer's demand is being met whereas constraints (2) guarantee that customers are assigned to the open facilities.

In the following sections, we will review some of the literature that modelled the two cost components we model in this work: concave capacity costs due to economies of scale and convex cost due to waiting time.

2.2 Capacity Costs

Capacity costs are often assumed to have a fixed set up cost and linear acquisition cost. In the real world, however, the capacity cost may be nonlinear due to economies of scale. The facilities will thus have a cost advantage as the size increases. The cost per unit of output usually decreases as volume increases because fixed cost per unit diminishes. Therefore, a concave capacity cost function is a more reasonable assumption than a linear one. Based on this, Florian and Klein [13] provided a mathematical model to solve a multi-period single commodity production planning problem with concave production and storage cost and capacity. They considered both the backlog and the no backlog cases. A dynamic programming method was employed with the assumption of constant capacities over periods.

Zangwill [22] worked on the minimum concave cost solution for acyclic single source multiple destination networks, acyclic single source single destination networks, as well as acyclic multiple source single destination networks. The author presented theories to describe the extreme point solutions, but did not provide any numerical testing. Cohen and Moon [6] considered a model to deal with the integrated plant loading problem with the consideration of economies of scale. The authors presented a mathematical formulation with concave production costs and fixed operating costs. They adopted the Benders decomposition method to solve the piecewise linear concave cost function. Dasci and Verter

[8] dealt with the concave capacity cost function using a progressive piecewise linear underestimation technique. Hajiaghayi et al. [14] modelled an uncapacitated facility location problem with a concave facility cost function, which was a function of the number of clients assigned to it. They used a greedy algorithm to achieve an approximation ratio of 1.861. Following their research, Romeijn et al. [20] considered an uncapacitated facility location problem with a concave facility cost function, which was a function in proportion to the amount of demand assigned to the facility. They tackled the problem with a greedy algorithm along with the idea of cost-scaling and reached an approximation factor of 1.52. Dupont [9] derived heuristic algorithms and a branch and bound method to solve the facility location problem with the limitation of a pre-determined service distance.

2.3 Customer Waiting Time

Customer waiting time or cost is an important component of service system design problems. Amiri [3, 4] presented a combined model of the problem, established an integer programming formulation of the problem, and proposed two heuristic solution methods based on Lagrangian Relaxation. Eskigun et al. [12] included lead time into the supply chain network design problem and proposed a Lagrangian heuristic to solve it. Aboolian et al. [1] accounted for the elasticity of customer demand, taking transportation and congestion delay costs into consideration. Vidyarthi et al. [21] presented models for make-to-order and assemble-to-order supply chains under Poisson demand.

Chapter 3

Problem Formulation

In this chapter, we present the formulation for the service system design problem with immobile servers, stochastic demand and economies of scale. We then provide a reformulation of the problem and provide a small numerical example.

3.1 Mathematical Model

First of all, let us define the following indices and parameters:

i : index for customers, $i = 1, 2, ..., I$;

j : index for potential facility locations, $j = 1, 2, ..., J$;

c_{ij} : unit cost of serving customers i from facility j (\$/unit);

t : response time cost per unit time per customer (\$/period/customer);

λ_i : mean demand rate for customer i (units/period), $i = 1, 2, ..., I$;

and the following decision variables:

$$x_{ij} = \begin{cases} 1 & \text{if customer } i \text{ is assigned to facility } j,\ j = 1, ..., J \\ 0 & \text{otherwise} \end{cases}$$

μ_j : the mean service rate at facility j (units/period).

The problem can be formulated as:

$$\text{[FLM]:} \quad \min \sum_{j=1}^{J} f_j(\mu_j) + \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij}\lambda_i x_{ij} + t\sum_{j=1}^{J} \frac{\sum_{i=1}^{I}\lambda_i x_{ij}}{\mu_j - \sum_{i=1}^{I}\lambda_i x_{ij}}$$

$$\text{s.t.} \sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1, ..., I \tag{1}$$

$$\sum_{i=1}^{I} \lambda_i x_{ij} - \mu_j \leq 0 \qquad i = 1, ..., I \tag{2}$$

$$x_{ij} \in \{0, 1\}; \mu_j \geqslant 0 \qquad i = 1, ..., I; j = 1, ..., J \tag{3}$$

The first term in the objective function captures the capacity cost μ_j at facility $j, j = 1, ..., J$. The function $f_j(\mu_j)$ is assumed to be concave, increasing to capture the economies of scale. For example, $f_j(\mu_j)$ could take the form $a\mu^b$ where $a > 0$ and $0 < b < 1$. The second term accounts for the variable access cost for customers at facilities. The third term is the expected total response time at facility j assuming an (M/M/1) queuing system. Constraints (1) guarantee each customer is assigned to exactly one of the facilities. Constraints (2) ensure the assigned capacity of a facility will not exceed its service rate, and only open facilities will be assigned to customers. Constraints (3) restrict the assignment variable x_{ij} to be binary, and the facility's service rate to be greater or equal to 0.

3.2 Problem Reformulation

The above problem is nonlinear with concave and convex terms. In this section, we provide a transformation to help solve it. Let us define a new auxiliary variable, $R_j = \frac{\mu_j - \sum_{i=1}^{I} \lambda_i x_{ij}}{\sum_{i=1}^{I} \lambda_i x_{ij}}, \quad j = 1, ..., J$. By rearranging the terms, we get $\mu_j = (R_j+1) \sum_{i=1}^{I} \lambda_i x_{ij}, \quad j = 1, ..., J$.

We can rewrite the formulation [FLM] as:

$$
\begin{aligned}
\text{[FLM2]:} \quad & \min \sum_{j=1}^{J} f_j(\sum_{i=1}^{I} \lambda_i x_{ij}(R_j + 1)) + \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} \lambda_i x_{ij} + \sum_{j=1}^{J} \frac{t}{R_j} \\
& \text{s.t.} \sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1, ..., I \qquad (4) \\
& \mu_j = (R_j+1) \sum_{i=1}^{I} \lambda_i x_{ij} \qquad j = 1, ..., J \qquad (5) \\
& x_{ij} \in \{0, 1\}; R_j \geqslant 0; \mu_j \geqslant 0 \qquad i = 1, ..., I; j = 1, ..., J \qquad (6)
\end{aligned}
$$

Note that constraints $\sum_{i=1}^{I} \lambda_i x_{ij} - \mu_j \leq 0$ are redundant in the presence of constraints (5).

Given the difficulty in solving [FLM2] directly, we first try to explore its structure. By fixing variables x to $\bar{x}$, the problem reduces to:

$$
\min \sum_{j=1}^{J} f_j(\sum_{i=1}^{I} \lambda_i \bar{x}_{ij}(R_j + 1)) + \sum_{j=1}^{J} \frac{t}{R_j}
$$

$$\text{s.t. } \mu_j = (R_j + 1)\sum_{i=1}^{I} \lambda_i \bar{x}_{ij} \qquad j = 1, ..., J$$

$$\mu_j, R_j \geqslant 0; \qquad j = 1, ..., J$$

which decomposes to j smaller problems:

$$\min\ f_j(\sum_{i=1}^{I} \lambda_i \bar{x}_{ij}(R_j + 1)) + \frac{t}{R_j}$$

$$\text{s.t. } \mu_j = (R_j + 1)\sum_{i=1}^{I} \lambda_i \bar{x}_{ij}$$

$$\mu_j, R_j \geqslant 0$$

Note that the constraint linking μ_j to R_j can be safely eliminated. So the problem reduces to:

$$\min\ f_j(\sum_{i=1}^{I} \lambda_i \bar{x}_{ij}(R_j + 1)) + \frac{t}{R_j}$$

$$\text{s.t. } R_j \geqslant 0$$

We will next explore the solution of this problem. The analysis will focus on the square root function for f_j. However, the model is able to deal with any form of functions in the form $a\mu^b$ where $a > 0; 0 < b < 1$. For brevity, let us write the previous model as $\min\limits_{R \geqslant 0} \sqrt{C(R+1)} + \dfrac{t}{R}$. The concave term, $f(C(R+1))$, represents a concave function with scaling parameter C. The convex term, $\frac{t}{R}$, represents a convex function with a scaling parameter of t.

Figure 3.1 displays the objective function $\sqrt{C(R+1)} + \dfrac{t}{R}$ for $C = 1$ and $t = 1$. As seen, the graph is first convex then concave. The global minimum is reached at $R = 1.8351$ and the objective is 2.2287. As the function is unimodular, the global minimum is found by taking the derivative and setting it to 0.

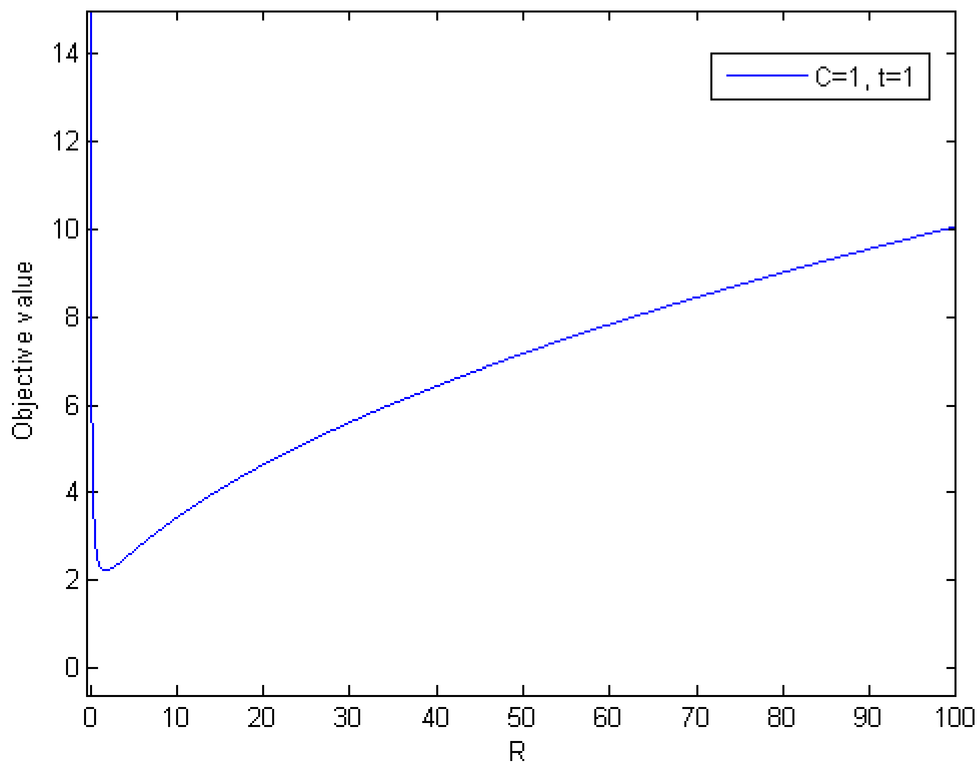

Figure 3.1: The objective function $\sqrt{C(R+1)} + \dfrac{t}{R}$ for $C = 1, t = 1$

Next, we analyze the function as the scaling parameters, C and t, change.

Case 1: Changing the concave scaling factor

As we can see from Figure 3.2 and Table 3.1, as C increases, the optimum decreases and the optimal objective increases. This relationship explains the effect of diseconomies of scales whereby, as the cost of variable input increases while holding other factors constant, the optimal production level decreases and the total production cost increases. As R increases further, the difference between the total costs under different scenarios will be magnified.

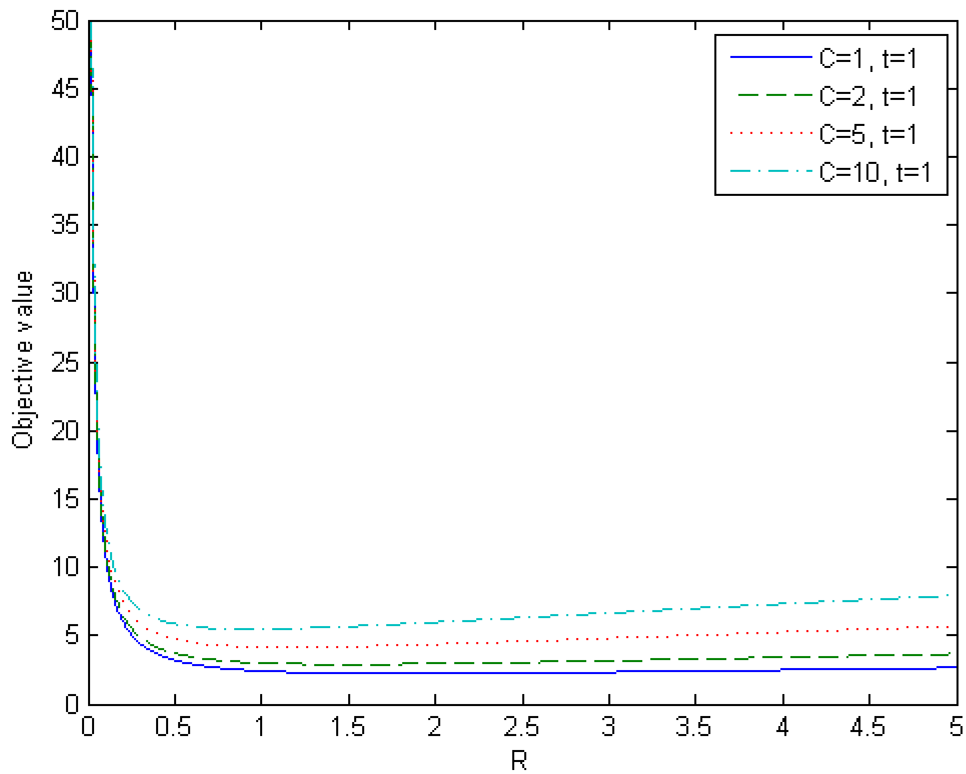

Figure 3.2: The effect of the scaling parameter, C, on the objective function

Case 2: Changing the convex scaling factor

C	t	Global Minimum	
		R value at minimum	Objective value
1	1	1.8351	2.2287
2	1	1.4945	2.9027
5	1	1.1445	4.1483
10	1	0.9384	5.4684

Table 3.1: The optimal solution and objective for different (C, t) combinations

Figure 3.3 and Table 3.2 show the effect of varying t values while fixing the rest. As t increases, both the optimum R and the optimal objective function increase. The intuition behind this is that as the fixed cost increases while holding others constant, the optimal production level increases and the total production cost increases. However, as R increases even more, the increased fixed cost effect will diminish. As R approaches infinity, the total cost tends to reach a common value.

C	t	Global Minimum	
		R value at minimum	Objective value
1	1	1.8351	2.2287
1	2	2.7907	2.6636
1	5	4.9360	3.4494
1	10	7.6751	4.2483

Table 3.2: The optimal solution and objective for different (C, t) combinations

As the function $\sqrt{C(R+1)} + \dfrac{t}{R}$ is first convex and then concave with the minimum achieved at the convex part, it is safe to take the derivative and set it to zero to find the global minimum. For that, we get:

$$\frac{\partial(\sqrt{\sum_{i=1}^{I} \lambda_i \bar{x}_{ij}(R+1)} + \frac{t}{R})}{\partial R} = 0$$

which implies

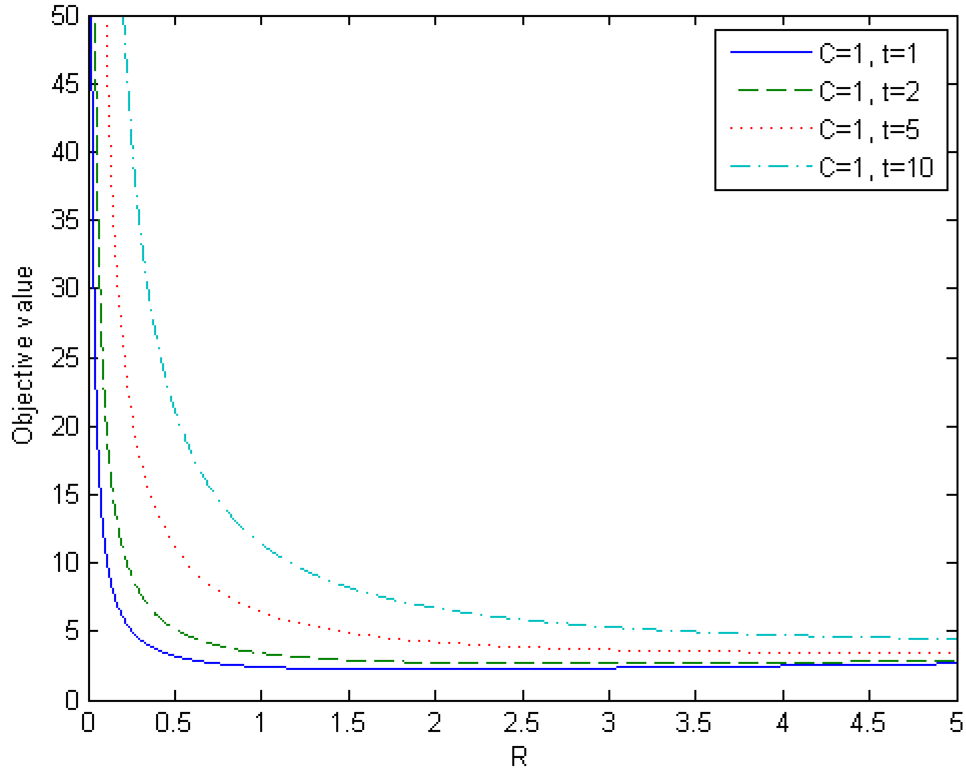

Figure 3.3: The effect of the scaling parameter, t, on the objective function

$$\frac{\sqrt{\sum_{i=1}^{I} \lambda_i \bar{x}_{ij}}}{2\sqrt{(R_j+1)}} = \frac{t}{R_j^2}$$

$$\sqrt{\sum_{i=1}^{I} \lambda_i \bar{x}_{ij}} = \frac{2t\sqrt{(R_j+1)}}{R_j^2}$$

$$\sum_{i=1}^{I} \lambda_i \bar{x}_{ij} = \frac{4t^2(R_j+1)}{R_j^4}$$

$$\sum_{i=1}^{I} \frac{\lambda_i}{4t^2}\bar{x}_{ij} = \frac{1}{R_j^3}+\frac{1}{R_j^4} \tag{*}$$

By substituting (*) into the original problem, we can rewrite [FLM2] as:

$$\min \sum_{j=1}^{J} \frac{2t(1+R_j)}{R_j^2} + \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij}\lambda_i x_{ij} + \sum_{j=1}^{J} \frac{t}{R_j}$$

Rearranging the terms leads to:

$$\text{[FLM3]:} \quad \min \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij}\lambda_i x_{ij} + \sum_{j=1}^{J} \frac{2t}{R_j^2} + \sum_{j=1}^{J} \frac{3t}{R_j}$$

$$\text{s.t.} \sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1, ..., I \tag{7}$$

$$\sum_{i=1}^{I} \frac{\lambda_i}{4t^2} x_{ij} = \frac{1}{R_j^3}+\frac{1}{R_j^4} \qquad j = 1, ..., J \tag{8}$$

$$x_{ij} \in \{0,1\}; R_j \geqslant 0 \qquad i = 1, ..., I; j = 1, ..., J \tag{9}$$

Defining a new variable, $P_j = \frac{1}{R_j}$; then [FLM3] can be written as:

$$\text{[FLM4]:} \quad \min \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij}\lambda_i x_{ij} + \sum_{j=1}^{J} 2tP_j^2 + \sum_{j=1}^{J} 3tP_j$$

$$\text{s.t.} \sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1, ..., I \tag{10}$$

$$\sum_{i=1}^{I} \frac{\lambda_i}{4t^2} x_{ij} = P_j^3 + P_j^4 \qquad j = 1, ..., J \tag{11}$$

$$x_{ij} \in \{0,1\}; P_j \geqslant 0 \qquad i = 1, ..., I; j = 1, ..., J \tag{12}$$

We have successfully transformed our mathematical model to [FLM4], which is one of the contributions of this thesis. In the following chapter, we will show how to solve [FLM4] using piecewise linearization with Special Order Set Type 2 (SOS2) constraints.

3.3 A Combinatorial Benders Approach

Based on the idea of fixing x_{ij}, a Combinatorial Benders approach is derived. The idea is to start with a certain realization $\bar{x}_{ij}$, find the corresponding R_j and μ_j values by solving:

$$
\begin{aligned}
\text{[SP]:} \quad & \min \sum_{j=1}^{J} f_j\Big(\sum_{i=1}^{I} \lambda_i \bar{x}_{ij}(R_j+1)\Big) + \sum_{j=1}^{J} \frac{t}{R_j} \\
& \text{s.t. } \mu_j = (R_j+1)\sum_{i=1}^{I} \lambda_i \bar{x}_{ij} \qquad j = 1, ..., J \\
& \mu_j, R_j \geqslant 0; \qquad j = 1, ..., J
\end{aligned}
$$

A feasible solution is then obtained.

To generate a different x_{ij} realization, the following cut is added: $\sum_{\bar{x}^h=0} x_{ij}^h + \sum_{\bar{x}^h=1}(1 - x_{ij}^h) \geqslant 1 \qquad h = 1, ..., H$

The Combinatorial Benders Master problem is:

$$
\begin{aligned}
\text{[MP]:} \quad & \min \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij}\lambda_i x_{ij} \\
& \text{s.t. } \sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1, ..., I && (13) \\
& \sum_{\bar{x}^h=0} x_{ij}^h + \sum_{\bar{x}^h=1}(1 - x_{ij}^h) \geqslant 1 \qquad h = 1, ..., H && (14) \\
& x_{ij} \in \{0,1\}; \theta_j \geqslant 0 \qquad i = 1, ..., I; j = 1, ..., J && (15)
\end{aligned}
$$

3.3.1 An Illustrative Example

We build a small example assuming that there are 3 facilities serving 6 Customers. The response time cost per unit time per customer, t, is set to 25. The unit cost of serving customers i from facility j and the mean demand rate for the product from customer i are shown in Table 3.3.

		c_{ij} Facility 1	2	3	λ
Customer	1	5	15	25	10
	2	30	20	10	20
	3	5	35	15	30
	4	10	10	8	25
	5	20	5	25	15
	6	8	12	20	5

Table 3.3: The unit costs and mean demand rates for the illustrative example

The results are shown in Table 3.4.

By looking at Table 3.5, we observe that the Combinatorial Benders method will find the lowest cost solution within two iterations. It has to go through 729 iterations to prove optimality. This shows the inefficiency of the method as it will typically go through a complete enumeration. To fix this, a better cut set should be devised.

		Facilities		
		1	2	3
Customers	1	10		
	2			20
	3	30		
	4			25
	5		15	
	6	5		
Rj		4.1034	5.8027	4.1034
μ_j		229.6546	102.0411	229.6546
Objective value (SPj)		21.2468	14.4098	21.2468
Objective value (Total)			771.9035	
Number of iterations			729	
Total runtime			74.2773	

Table 3.4: Solutions of the illustrative example

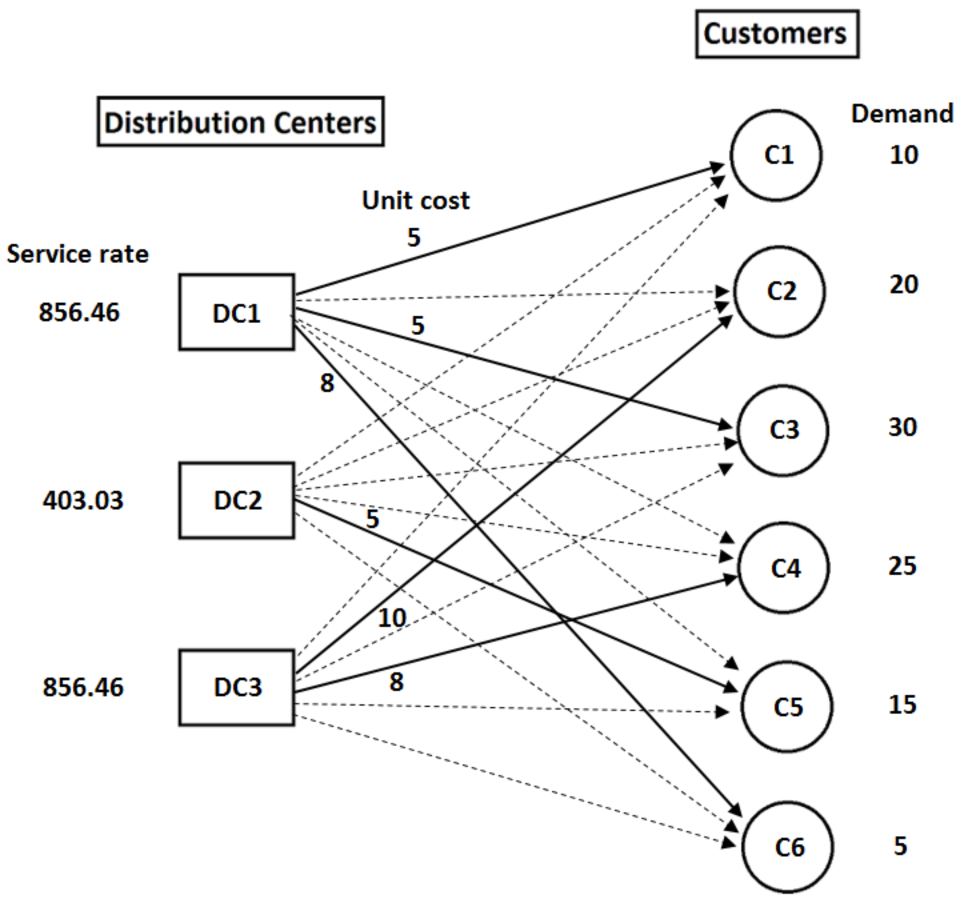

Figure 3.4: Solutions of the illustrative example

	Objective			Cost on Facility		
Iteration	Subproblem	Master problem	Total	Facility 1	Facility 2	Facility 3
1	28.81	1575.00	1603.81	-	-	-
2	56.90	715.00	771.90	33.84%	11.58%	54.57%
3	57.56	735.00	792.56	27.81%	19.04%	53.15%
4	55.24	765.00	820.24	62.77%	10.90%	26.33%
5	57.56	765.00	822.56	31.76%	41.99%	26.25%
6	56.84	775.00	831.84	26.49%	10.75%	62.76%
7	56.12	785.00	841.12	56.38%	17.95%	25.67%
8	57.56	785.00	842.56	26.16%	48.22%	25.63%
9	57.92	815.00	872.92	23.99%	27.75%	48.26%
10	55.89	825.00	880.89	53.84%	10.15%	36.01%
11	57.99	825.00	882.99	24.96%	39.11%	35.93%
12	58.04	835.00	893.04	18.86%	33.97%	47.17%
13	56.74	865.00	921.74	50.29%	26.28%	23.43%
14	57.43	865.00	922.43	22.70%	53.89%	23.41%
15	57.70	875.00	932.70	18.05%	25.97%	55.97%
16	57.16	885.00	942.16	44.88%	32.20%	22.92%
17	57.16	885.00	942.16	17.87%	59.21%	22.92%
18	57.92	915.00	972.92	26.85%	50.82%	22.33%
19	56.67	915.00	971.67	21.55%	9.20%	69.24%
20	57.32	925.00	982.32	43.04%	24.66%	32.30%
350	57.70	1565.00	1622.70	56.21%	14.93%	28.87%
351	53.03	1565.00	1618.03	65.89%	10.04%	24.07%
352	43.91	1565.00	1608.91	19.17%	0.00%	80.83%
353	57.70	1565.00	1622.70	40.50%	30.63%	28.87%
354	52.59	1565.00	1617.59	3.08%	16.52%	80.40%
355	53.96	1575.00	1628.96	3.84%	25.53%	70.63%
356	56.74	1575.00	1631.74	37.75%	14.85%	47.41%
357	55.81	1575.00	1630.81	47.34%	9.96%	42.69%
358	57.16	1585.00	1642.16	37.51%	33.97%	28.52%
359	58.04	1585.00	1643.04	53.03%	18.47%	28.51%
360	56.84	1585.00	1641.84	47.02%	29.26%	23.72%
720	57.43	2535.00	2592.43	23.76%	43.57%	32.67%
721	56.67	2550.00	2606.67	35.27%	50.75%	13.98%
722	56.35	2550.00	2606.35	45.03%	40.99%	13.98%
723	57.70	2565.00	2622.70	34.78%	40.74%	24.49%
724	57.32	2565.00	2622.32	25.06%	50.44%	24.49%
725	57.16	2575.00	2632.16	23.40%	40.59%	36.01%
726	56.74	2585.00	2641.74	23.32%	52.37%	24.31%
727	57.92	2585.00	2642.92	32.97%	42.73%	24.30%
728	57.16	2625.00	2682.16	22.96%	49.32%	27.72%
729	58.04	2625.00	2683.04	32.47%	39.82%	27.71%

Table 3.5: Detailed results for the Combinatorial Benders methodology on the illustrative example

Chapter 4

Solution Approaches

In this chapter, we propose two solution methodologies to solve [FLM4]. The first is a linearization based on SOS2 constraints andthe second is based on Second Order Cone Programming (SOCP) approach.

4.1 A Linearization Based on SOS2 Constraints

Let us recall the model [FLM4]:

$$\min \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} \lambda_i x_{ij} + \sum_{j=1}^{J} 2tP_j^2 + \sum_{j=1}^{J} 3tP_j$$

$$\text{s.t.} \sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1, ..., I \tag{10}$$

$$\sum_{i=1}^{I} \frac{\lambda_i}{4t^2} x_{ij} = P_j^3 + P_j^4 \qquad j = 1, ..., J \tag{11}$$

$$x_{ij} \in \{0,1\}; P_j \geqslant 0 \quad i = 1,...,I; j = 1,...,J \tag{12}$$

In order to estimate $P^2, P^3, or P^4$, we generate a set of cuts for the function $f(P)$ where the generated piecewise linear approximation $\hat{f}$ satisfies $0 \leqslant f(P) - \hat{f}(P) \leqslant \epsilon$ on every point(Elhedhli [10]).

To illustrate this, let us assume that the approximation $\hat{f}$ has $n+1$ breakpoints located at $P_0, P_1, ..., P_n$, and each line segment is tangent to the origin function f at the $n+1$ points $p_0, p_1, ..., p_n$ where $P_{k-1} \leqslant p_k \leqslant P_k$. We can recursively determine the P_k and p_k values, given the fact that $\hat{f}$ is linear between $P_{k-1} \leqslant p_k \leqslant P_k$. The slopes of $\hat{f}$ can be also determined, which are $f'(p_k)$. Thus, given the values of $\hat{f}(P_{k-1})$ and P_{k-1} as well as using the fact that $f(P_k) = P^2, P^3, or P^4$, we can find p_k by using:

$$\hat{f}(P_{k-1}) = f(p_k) + f'(p_k)(P_{k-1} - p_k)$$

Then using the fact that p_k and $\hat{f}(P_k) = f(P_k) + \epsilon = f(P_k) + \epsilon$, we can find P_k by using:

$$\hat{f}(P_k) = f(p_k) + f'(p_k)(P_k - p_k)$$

Then, P_k can be used as the start point, which is P_{k-1} of the next piecewise linear segment. For P^2, the procedure is:

Step 1: We initialize $P_{k-1} = 0$.

Step 2: The line equation to solve for p_k is:

$$\hat{f}(P_{k-1}) = p_k^2 + 2p_k(P_{k-1} - p_k)$$

Since the maximum error is limited to ϵ, we have:

$$\hat{f}(P_{k-1}) = P_{k-1}^2 - \epsilon$$

Solve the following to find p_k:

$$p_k^2 - 2P_{k-1}p_k + P_{k-1}^2 - \epsilon = 0$$

Step 3: The line equation to solve for P_k is:

$$\hat{f}(P_k) = p_k^2 + 2p_k(P_k - p_k)$$

Since the maximum error is limited to ϵ, we have:

$$\hat{f}(P_k) = P_k^2 - \epsilon$$

To find P_k, solve:

$$P_k^2 - 2p_kP_k + p_k^2 - \epsilon = 0$$

We repeat steps 2 and 3 until $P_k \geqslant \sqrt[2]{\frac{\sum_{i=1}^{I}\lambda_i}{8t^2}}$, which is the highest value possible for P.

The number of linear segments will depend on the error term ϵ. As we can see from Table 4.1 and Table 4.2, the number of SOS2 constraints increases as ϵ decreases.

	Iter 1	Iter 2
P_{k-1}	0.0000	0.2000
p_k	0.1000	0.3000
P_k	0.2000	0.4000

Table 4.1: Approximation parameters for the P^2 case with $\epsilon = 0.01$, $t = 25$ and total demand 105 units.

	Iter 1	Iter 2	Iter 3	Iter 4	Iter 5
P_{k-1}	0.0000	0.0632	0.1265	0.1897	0.2530
p_k	0.0316	0.0949	0.1581	0.2214	0.2846
P_k	0.0632	0.1265	0.1897	0.2530	0.3162

Table 4.2: Approximation parameters for the P^2 case with $\epsilon = 0.001$, $t = 25$ and total demand 105 units.

Similarly for $P^3 + P^4$, the linearization parameters are determined as follows:

Step 1: We initialize $P_{k-1} = 0$.

Step 2: The line equation to solve for p_k is:

$$\hat{f}(P_{k-1}) = p_k^4 + p_k^3 + (4p_k^3 + 3p_k^2)(P_{k-1} - p_k)$$

Since the maximum error is limited to ϵ, we have:

$$\hat{f}(P_{k-1}) = P_{k-1}^4 + P_{k-1}^3 - \epsilon$$

To find p_k, solve:

$$3p_k^4 + (2 - 4P_{k-1})p_k^3 - 3P_{k-1}p_k^2 + P_{k-1}^4 + P_{k-1}^3 - \epsilon = 0$$

Step 3: The line equation to solve for P_k is:

$$\hat{f}(P_k) = p_k^4 + p_k^3 + (4p_k^3 + 3p_k^2)(P_k - p_k)$$

Since the maximum error is limited to ϵ, we have:

$$\hat{f}(P_k) = P_k^4 + P_k^3 - \epsilon$$

To find P_k, solve:

$$P_k^4 + P_k^3 + (-4p_k^3 - 3p_k^2)P_k + 3p_k^4 + 2p_k^3 - \epsilon = 0$$

We repeat steps 2 and 3 until $P_k \geqslant \sqrt[3]{\frac{\sum_{i=1}^{I} \lambda_i}{8t^2}}$.

In a similar way, the number of SOS2 constraints being generated will depend on the error term ϵ. As seen in Table 4.3 and Table 4.4, the number of SOS2 constraints increases as ϵ decreases.

	Iter 1	Iter 2	Iter 3	Iter 4
P_{k-1}	0.0000	0.2695	0.4203	0.5393
p_k	0.1592	0.3487	0.4816	0.5913
P_k	0.2695	0.4203	0.5393	0.6410

Table 4.3: Approximation parameters for the $P^3 + P^4$ case with $\epsilon = 0.01$, $t = 25$ and total demand 105 units.

	Iter 1	Iter 2	Iter 3	Iter 4	Iter 5	Iter 6	Iter 7	Iter 8	Iter 9	Iter 10
P_{k-1}	0.0000	0.1309	0.2075	0.2690	0.3222	0.3699	0.4136	0.4541	0.4922	0.5281
p_k	0.0765	0.1710	0.2391	0.2961	0.3464	0.3920	0.4341	0.4733	0.5103	0.5454
P_k	0.1309	0.2075	0.2690	0.3222	0.3699	0.4136	0.4541	0.4922	0.5281	0.5623

Table 4.4: Approximation parameters for the $P^3 + P^4$ case with $\epsilon = 0.001$, $t = 25$ and total demand 105 units.

From Table 4.5, we can see that the number of SOS2 constraints increase, as ϵ decrease. However, as ϵ decreases, the concave and convex terms are more accurately estimated. Thus, we need to find a trade-off between accuracy and efficiency. We have completed different tests in the numerical results section to illustrate this effect.

	SOS2 Constraints	
ϵ	P^2	$P^4 + P^3$
0.01	2	4
0.001	5	10
0.0001	15	31
0.00001	46	97
0.000001	145	304
0.0000001	1450	962
0.00000001	4583	3039
0.000000001	14492	9610

Table 4.5: Number of SOS2 cuts with different ϵ for the P^2 and, $P^4 + P^3$ cases.

The piecewise approximations are displayed in Figure 4.1 and Figure 4.2.

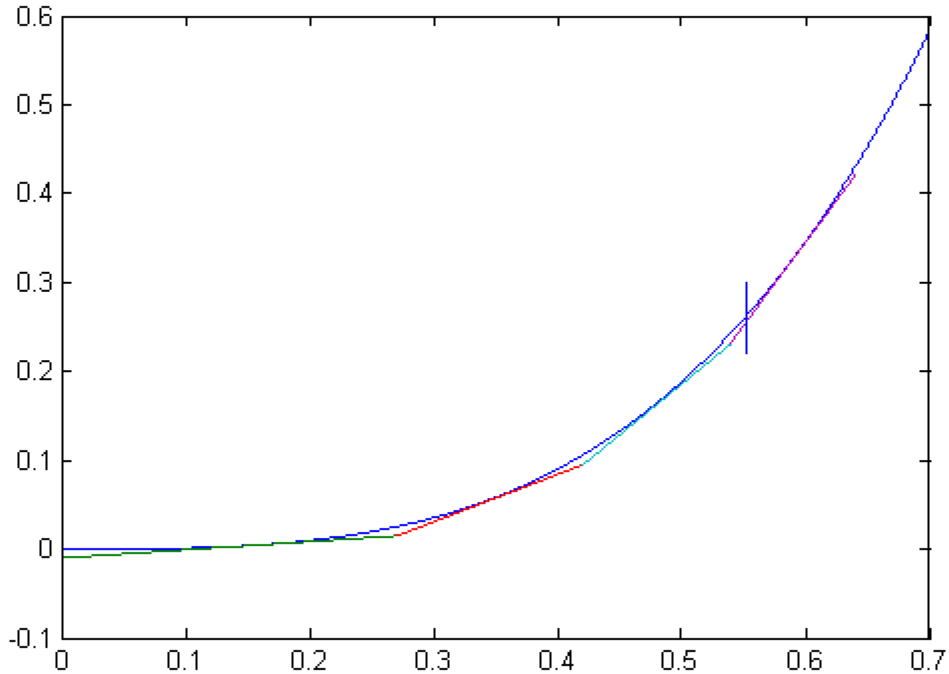

Figure 4.1: Piecewise approximations for $P^4 + P^3$ case with $\epsilon = 0.001$

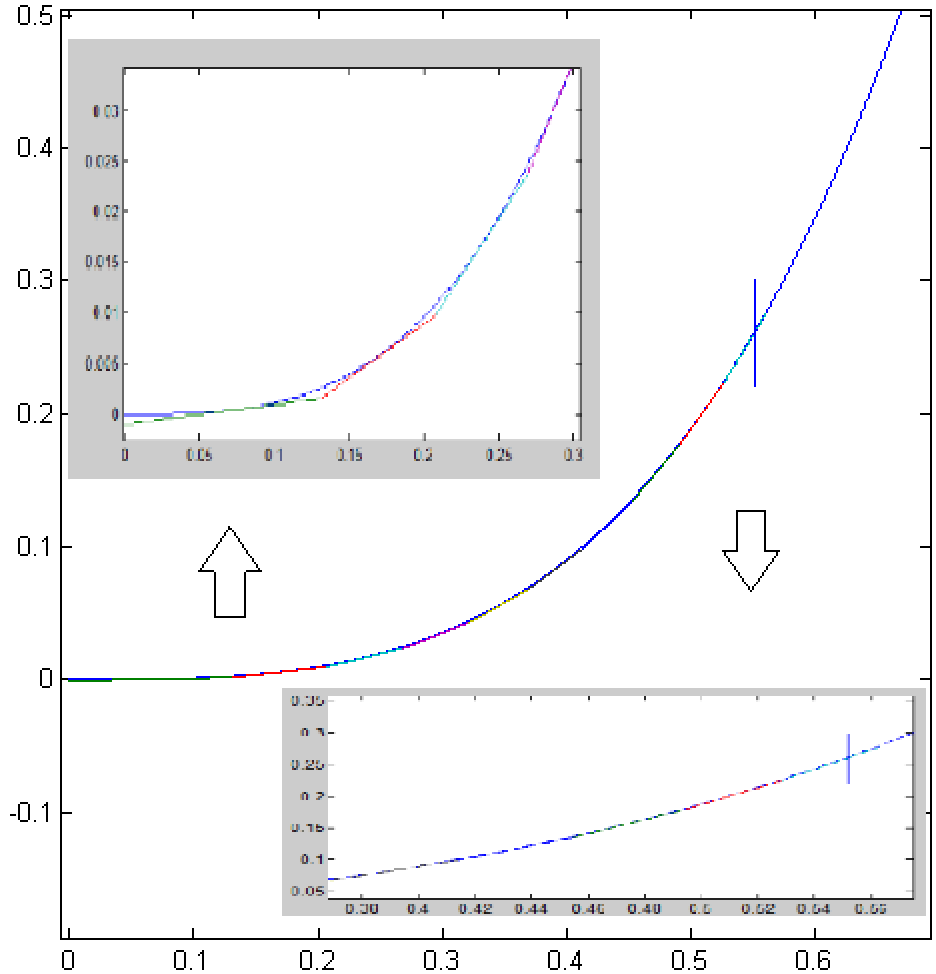

Figure 4.2: Piecewise approximations for $P^4 + P^3$ case with $\epsilon = 0.001$

4.1.1 The Linearized Formulation

The linearization of [FLM4] using the previous procedure is:

$$\text{[FLM5]:} \quad \min \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij}\lambda_i x_{ij} + \sum_{j=1}^{J} 2t\theta_j + \sum_{j=1}^{J} 3tP_j$$

$$\text{s.t.} \sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1, ..., I \tag{16}$$

$$P_j = \sum_{k=1}^{K} \beta_k^{(j)} \hat{p}_k \qquad j = 1, ..., J \tag{17}$$

$$\theta_j = \sum_{k=1}^{K} \beta_k^{(j)} \hat{y}_k \qquad j = 1, ..., J \tag{18}$$

$$\sum_{k=1}^{K} \beta_k = 1 \tag{19}$$

$$P_j = \sum_{l=1}^{L} \delta_l^{(j)} \hat{p}_l \qquad j = 1, ..., J \tag{20}$$

$$\sum_{i=1}^{I} \frac{\lambda_i}{4t^2} x_{ij} = \sum_{k=1}^{L} \delta_l^{(j)} \hat{y}_l \qquad j = 1, ..., J \tag{21}$$

$$\sum_{l=1}^{L} \delta_l = 1 \tag{22}$$

$$x_{ij} \in \{0, 1\}; \theta_j, P_j \geqslant 0, \qquad i = 1, ..., I; j = 1, ..., J \tag{23}$$

$$0 \leqslant \beta_k, \delta_l \leqslant 1 \qquad k = 1, ..., K; l = 1, ..., L \tag{24}$$

Constraints (16), (17), and (18) are the SOS2 constraints used to estimate P^2 term in the objective function of [FLM4]. Constraints (20), (21), and (22) are the SOS2 constraints used to estimate $P^4 + P^3$ terms in the constraints in [FLM4]. The variables β_k and δ_l are the ordered sets of binary SOS2 variables, of which at most two consecutive ones can be non-zero.

The results of the SOS2 approximation tested on the illustrative example are displayed in Table 4.7 and Table 4.8 for $\epsilon = 0.001$ and $\epsilon = 0.0001$ respectively. For comparison, we display the optimal solution in Table 4.6.

It is clear that the approximation is very efficient in finding the optimal solution in very competitive times.

		Facilities		
		1	2	3
Customers	1	10		
	2			20
	3	30		
	4			25
	5		15	
	6	5		
Rj		4.1034	5.8027	4.1034
μ_j		229.6546	102.0411	229.6546
Objective value (SPj)		21.2468	14.4098	21.2468
Objective value (Total)			771.9035	
Number of iterations			729	
Total runtime			74.2773	

Table 4.6: Complete enumeration results

$\epsilon = 0.001$		Facilities		
		1	2	3
Customers	1	10		
	2			20
	3	30		
	4			25
	5		15	
	6	5		
P_j		0.2438	0.1723	0.2438
$R_j (=1/P_j)$		4.1017	5.8023	4.1017
Objective value (Total)			771.8641	
Total runtime			0.0362	
Total cost			771.9246	
Gap			0.0078%	

Table 4.7: SOS2 results with $\epsilon = 0.001$

$\epsilon = 0.0001$		Facilities		
		1	2	3
Customers	1	10		
	2			20
	3	30		
	4			25
	5		15	
	6	5		
P_j		0.2441	0.1724	0.2441
$R_j (=1/P_j)$		4.0968	5.8011	4.0968
Objective value (Total)			771.9896	
Total runtime			0.0409	
Total cost			771.9863	
Gap			0.0005%	

Table 4.8: SOS2 results with $\epsilon = 0.0001$

The SOS2 linearization approach is able to provide very accurate results for smaller error (ϵ). However, as ϵ decreases, the number of SOS2 constraints increases dramatically, and the total solution time may be very long. Thus, there is a trade off between accuracy and speed. We will demonstrate this trade off in the numerical testing chapter.

4.2 A SOCP Approach

In this section, we tackle [FLM4] using a Second Order Cone Programming (SOCP) approach. Commercial software such as Cplex can solve reasonable size SOCP problems efficiently.

A Second Order Cone Program is an optimization problem of the form:

$$\min f^T x$$

$$\text{s.t. } \|A_i x + b_i\|_2 \leq c_i^T x + d_i \qquad i = 1, ..., m$$

4.2.1 The SOCP Approach Reformulation

We tackled the Second Order Cone Programming Approach from our [FLM]:

$$\text{[FLM]:} \quad \min \sum_{j=1}^{J} f_j(\mu_j) + \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} \lambda_i x_{ij} + t \sum_{j=1}^{J} \frac{\sum_{i=1}^{I} \lambda_i x_{ij}}{\mu_j - \sum_{i=1}^{I} \lambda_i x_{ij}}$$

$$\text{s.t. } \sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1, ..., I \tag{1}$$

$$\sum_{i=1}^{I} \lambda_i x_{ij} - \mu_j \leq 0 \qquad i = 1, ..., I \tag{2}$$

$$x_{ij} \in \{0,1\}; \mu_j \geqslant 0 \qquad i = 1,...,I; j = 1,...,J \tag{3}$$

We define two new variables, $P_j = \sum_{j=1}^{J} \frac{\sum_{i=1}^{I} \lambda_i x_{ij}}{\mu_j - \sum_{i=1}^{I} \lambda_i x_{ij}}$, $v_j = \sqrt{u_j}$, and substitute it into the objective function and constraints of [FLM], we get:

$$\text{[FLM6]:} \quad \min \sum_{j=1}^{J} v_j + \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} \lambda_i x_{ij} + t \sum_{j=1}^{J} P_j$$

$$\text{s.t.} \sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1,...,I \tag{25}$$

$$\sum_{i=1}^{I} \lambda_i x_{ij}^2 \leq v_j^2 \qquad j = 1,...,J \tag{26}$$

$$\sum_{i=1}^{I} \lambda_i x_{ij} = \frac{P_j}{1+P_j} v_j^2 \qquad j = 1,...,J \tag{27}$$

$$x_{ij} \in \{0,1\}; P_j \geqslant 0; v_j \geqslant 0 \qquad i = 1,...,I; j = 1,...,J \tag{28}$$

Since x_{ij} is binary, we can rewrite $x_{ij} = x_{ij}^2$. In order to simplify our model, we introduce a new auxiliary variable, $\alpha_j = \frac{P_j}{1+P_j}$, which is indeed the facility utilization factor. Our model becomes:

$$\text{[FLM7]:} \quad \min \sum_{j=1}^{J} v_j + \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} \lambda_i x_{ij} + \sum_{j=1}^{J} t \frac{\alpha_j}{1-\alpha_j}$$

$$\text{s.t.} \sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1,...,I \tag{29}$$

$$\sum_{i=1}^{I} \lambda_i x_{ij}^2 \leq v_j^2 \qquad j = 1,...,J \tag{30}$$

$$\sum_{i=1}^{I} \lambda_i x_{ij}^2 = \alpha_j v_j^2 \qquad j = 1,...,J \tag{31}$$

$$x_{ij} \in \{0,1\}; 0 \leqslant \alpha_j < 1; v_j \geqslant 0 \qquad i = 1,...,I; j = 1,...,J \qquad (32)$$

Constraints (30) are redundant given the existence of constraints (31).

Two different approaches are derived to solve [FLM7]. First, we solve it with a fixed facility utilization factor, α. Next, a Lagrangian Relaxation approach is used to find α_j for each facility.

4.2.2 The SOCP Approach with a Fixed Facility Utilization Factor, α

In this section, we solve [FLM7] by using a fixed facility utilization factor, α. We try different values of $\bar{\alpha} = 0.1, 0.2, 0.3, ..., 0.9$. Then, $P_j = \dfrac{\bar{\alpha}}{1-\bar{\alpha}} = \dfrac{0.1}{1-0.1} = 0.11, 0.25, 0.43, ..., 9$ correspondingly. Next, we solve the following model with IBM Cplex function cplexqcp.

$$\text{[FLM8]:} \quad \min \sum_{j=1}^{J} v_j + \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij}\lambda_i x_{ij}$$

$$\text{s.t.} \sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1,...,I \qquad (33)$$

$$\sum_{i=1}^{I} \lambda_i x_{ij}^2 \leqslant \alpha v_j^2 \qquad j = 1,...,J \qquad (34)$$

$$x_{ij} \in \{0,1\}; 0 \leqslant \alpha < 1; v_j \geqslant 0 \qquad i = 1,...,I; j = 1,...,J \qquad (35)$$

Once a list of results is generated, we add tP_j back to the objective value and select the one with the smallest total cost.

This approach, is tested on the illustrative example. The results are displayed in Table 4.9 where v_j are the capacity costs. We can see that $\alpha = 0.9$ corresponding to a total cost of 736.00 is the minimum. The corresponding x_{ij} match the optimal solution.

	Alpha									
Variable	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x11	1	1	1	1	1	1	1	1	1	1
x21	0	0	0	0	0	0	0	0	0	0
x31	1	1	1	1	1	1	1	1	1	1
x41	0	0	0	0	0	0	0	0	0	0
x51	0	0	0	0	0	0	0	0	0	0
x61	1	1	1	1	1	1	1	1	1	1
x12	0	0	0	0	0	0	0	0	0	0
x22	0	0	0	0	0	0	0	0	0	0
x32	0	0	0	0	0	0	0	0	0	0
x42	0	0	0	0	0	0	0	0	0	0
x52	1	1	1	1	1	1	1	1	1	1
x62	0	0	0	0	0	0	0	0	0	0
x13	0	0	0	0	0	0	0	0	0	0
x23	1	1	1	1	1	1	1	1	1	1
x33	0	0	0	0	0	0	0	0	0	0
x43	1	1	1	1	1	1	1	1	1	1
x53	0	0	0	0	0	0	0	0	0	0
x63	0	0	0	0	0	0	0	0	0	0
v_1	21.2132	15	12.24745	10.6066	9.486833	8.660254	8.017837	7.5	7.071068	6.708205
v_2	12.24745	8.660254	7.071068	6.123725	5.477226	5	4.629101	4.330127	4.082483	3.872985
v_3	21.2132	15	12.24745	10.6066	9.486833	8.660254	8.017837	7.5	7.071068	6.708204
Obj Value w/o t*1/R	769.6739	753.6603	746.566	742.3369	739.4509	737.3205	735.6648	734.3301	733.2246	732.2894
Obj Value with t*1/R	994.6739	853.6603	804.8993	779.8369	764.4509	753.9872	746.3791	740.5801	736.0024	-

Table 4.9: SCOP results with α and $epsilon = 0.1$

However, the three α for all facilities are constant in this case. In reality, facilities are often operating in different utilization levels. In the next section, we use a heuristic approach to find the different utilization factors, α_j, for each facility.

4.2.3 A SOCP Based Lagrangian Relaxation

In this section, we devise a Lagrangian approach in order to explore different utilization levels α_j. Let us start from [FLM6]:

$$\text{[FLM6]:} \quad \min \sum_{j=1}^{J} v_j + \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij}\lambda_i x_{ij} + t \sum_{j=1}^{J} P_j$$

$$\text{s.t.} \sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1, ..., I \tag{25}$$

$$\sum_{i=1}^{I} \lambda_i x_{ij}^2 = \frac{P_j}{1+P_j} v_j^2 \qquad j = 1, ..., J \tag{27}$$

$$x_{ij} \in \{0,1\}; P_j \geqslant 0; v_j \geqslant 0 \qquad i = 1, ..., I; j = 1, ..., J \tag{28}$$

We relax constraints (25) using Lagrangian multipliers, β_i. It leads to the following subproblems:

$$\text{[SP]:} \quad \min \sum_{j=1}^{J} v_j + \sum_{i=1}^{I}\sum_{j=1}^{J}(c_{ij}\lambda_i - \beta_i)x_{ij} + t\sum_{j=1}^{J} P_j$$

$$\text{s.t.} \sum_{i=1}^{I} \lambda_i x_{ij}^2 = \frac{P_j}{1+P_j} v_j^2 \qquad j = 1, ..., J \tag{36}$$

$$x_{ij} \in \{0,1\}; P_j \geqslant 0; v_j \geqslant 0 \qquad i = 1, ..., I; j = 1, ..., J \tag{37}$$

which can be further decomposed to j subproblems, [SPj]:

$$\text{[SPj]:} \quad \min v_j + \sum_{i=1}^{I}(c_{ij}\lambda_i - \beta_i)x_{ij} + tP_j$$

$$\text{s.t.} \sum_{i=1}^{I} \lambda_i x_{ij}^2 = \frac{P_j}{1+P_j} v_j^2 \tag{36}$$

$$x_{ij} \in \{0,1\}; P_j \geqslant 0; v_j \geqslant 0 \qquad i = 1, ..., I \tag{37}$$

The subproblems can be solved by going through all possible values of P_j and solving the resulting SOCP. We do this approximately for $\alpha_j = 0.1, 0.2, 0.3, ..., 0.9$ which correspond to $P_j = \dfrac{\alpha_j}{1-\alpha_j} = \dfrac{0.1}{1-0.1} = 0.11, 0.25, 0.43, ..., 9$ respectively. Next, we solve the remaining SOCP:

$$\text{[SPj]:} \quad \min v_j + \sum_{i=1}^{I}(c_{ij}\lambda_i - \beta_i)x_{ij} + tP_j$$

$$\text{s.t.} \sum_{i=1}^{I} \lambda_i x_{ij}^2 \leqslant \alpha_j v_j^2 \tag{36}$$

$$x_{ij} \in \{0,1\}; v_j \geqslant 0 \qquad i = 1, ..., I \tag{37}$$

Next, we update the objective values by adding tP_j, and update the best solution found so far up to this iteration. The solutions to the subproblems yield the following lower bound:

$$LB = \sum_{j=1}^{J} z_{SPj} + \sum_{i=1}^{I} \beta_i$$

and the best Lagrangian lower bound is updated for each iteration:

$$LB^* = \max_{\beta_i}\{\sum_{j=1}^{J} z_{SPj} + \sum_{i=1}^{I} \beta_i\}$$

In order to update the Lagrangian multipliers, we take the Lagrangian dual of the subproblems:

$$\max_{\beta_i}\{\sum_{j=1}^{J} z_{SPj} + \sum_{i=1}^{I} \beta_i\}$$

The master problem is:

$$\text{[LMP]:} \quad \max \sum_{j=1}^{J} \theta_j + \sum_{i=1}^{I} \beta_i$$

$$\text{s.t. } \theta_j + \sum_{i=1}^{I} x_{ij}^h \beta_i \leqslant v_j^h + \sum_{i=1}^{I} c_{ij}\lambda_i x_{ij}^h + tr_j^h \quad j = 1, ..., J; h = 1, ..., H$$

$$\theta_j, \beta_i \text{ free} \qquad i = 1, ..., I; j = 1, ..., J$$

The solution to the master problem will generate an upper bound, UB, as well as a new set of multipliers β_i, $i = 1, ..., I$. The new set of β_i are used for the next iteration and the subproblems will generate another set of solutions. The solutions generated by the subproblems will add an additional set of cuts to the master problem.

While the Lagrangian Relaxation method generates the Lagrangian bounds, it does not consider the customer assignment problem. Thus, we need to use a heuristic to generate a feasible solution.

The set covering formulation derived from the master problem is:

$$[\text{HP}]: \quad \min \sum_{j=1}^{J}\sum_{h=1}^{H}(v_j^h + \sum_{i=1}^{I} c_{ij}\lambda_i x_{ij}^h + tr_j^h)\alpha_{jh}$$

$$\text{s.t.} \sum_{j=1}^{J}\sum_{h=1}^{H} x_{ij}^h \alpha_{jh} \geqslant 1 \qquad i = 1, ..., I \tag{40}$$

$$\sum_{h=1}^{H} \alpha_{jh} \geqslant 1 \qquad j = 1, ..., J \tag{41}$$

$$\alpha_{jh} \in \{0, 1\} \qquad j = 1, ..., J; h = 1, ..., H \tag{42}$$

Solving this will lead to a feasible solution.

Numerical Example

Iteration					β_i	LB	UB	α_{jh}		
1	x_{ij}	0	0	0	1000000	8.3333	1000000	0	0	0
		0	0	0	1000000					
		0	0	0	1000000					
		0	0	0	1000000					
		0	0	0	1000000					
		0	0	0	1000000					
2	x_{ij}	1	1	1	1392.778	8.3333	1398.3	0	0	0
		1	1	1	0					
		1	1	1	0					
		1	1	1	0					
		1	1	1	0					
		1	1	1	0					
3	x_{ij}	1	1	1	0	8.3333	1398.3	0	0	0
		0	0	0	1392.778					
		0	0	0	0					
		0	0	0	0					
		0	0	0	0					
		0	0	0	0					
4	x_{ij}	0	0	0	52.77778	8.3333	1398.3	0	0	0
		1	1	1	0					
		0	0	0	1340					
		0	0	0	0					
		0	0	0	0					
		0	0	0	0					
5	x_{ij}	1	0	0	52.77778	8.3333	1398.3	0	0	0
		0	0	0	0					
		1	1	1	0					
		0	0	0	0					
		0	0	0	1340					
		0	0	0	0					
6	x_{ij}	1	0	0	0	8.3333	1398.3	0	1	0
		0	0	0	200					
		0	0	0	200					
		0	0	0	915					
		1	1	1	75					
		0	0	0	0					
7	x_{ij}	0	0	0	52.77778	8.3333	1398.3	0	0	0
		0	0	0	200					
		1	0	0	150					
		1	1	1	200					
		0	0	0	75					
		0	0	0	715					
8	x_{ij}	1	0	0	2.777778	8.3333	748.8889	0	0	0
		0	0	0	202.7778					
		0	0	0	197.2222					
		0	0	0	202.7778					
		0	0	0	77.77778					
		1	1	1	62.77778					
9	x_{ij}	0	0	0	51.38889	673.3333	724.7222	0	0	1
		0	0	1	200					
		1	0	0	151.3889					
		0	0	1	200					
		0	1	0	75					
		1	1	0	41.38889					
10	x_{ij}	1	0	0	50	723.3333	723.3333	1	0	0
		0	0	0	200					
		1	0	0	150					
		0	0	0	200					
		0	0	0	75					
		1	0	0	42.77778					
Feasible solution										
1	0	0								
0	0	1								
1	0	0								
0	0	1								
0	1	0								
1	0	0								

Table 4.10: Lagrangian Relaxation for the illustrative example. $\alpha = 0.1, 0.2, ..., 0.9$

Chapter 5

Numerical Results and Comparison

In this chapter, we evaluate our proposed solution methodologies and present our computational results. The proposed solution procedures are coded in Matlab and solved using IBM ILOG Cplex 12.6. The tests are done on a Dell Optiplex 9020 with Intel Core i7-4770 3.2GHz CPU with 8GB RAM. The data sets used are due to Holmberg [16]. We have tested 55 instances with 10 to 30 facilities and 30 to 150 customers.

5.1 Test Instances

The test instances we use are the benchmark Holmberg [16] instances, which has five categories as seen in Table 5.1. Test instances p1-12 are considered as small, and their $\frac{K}{D}$ (total capacity to demand) ratios are also relatively small. Test instances p13-24 have a medium size, and their $\frac{K}{D}$ ratios range from 2.77 to 3.50. Test instances p25-40 are the largest with 30 facilities and 150 customers. Test instances p41-49 contain extremely low or high $\frac{K}{D}$ ratios. The last instances p50-55 are used to explore the $\frac{K}{D}$ ratios' impact.

	Instances				
	p1-12	p13-24	p25-40	p41-49	p50-55
Number of Facilities (n)	10	20	30	10-30	10-20
Number of customers(m)	50	50	150	70-90	100

Table 5.1: Test Instances

5.2 Comparison Between the Two Approaches

Table 5.2 provides a comparison between the SOS2 linearization and the SOCP based Lagrangian. It is clear that the SOS2 algorithm is able to give decent results with much shorter runtime, while the SOCP algorithm generate almost perfect Lagrangian bounds for smaller instances.

For test instances p1-12 with 10 facilities and 30 customers, both methods are able to generate results with less than 1% gaps. However, the average runtime of the SOS2 method is significantly less. For the test instances p13-24 with 20 facilities and 50 customers, the average gap between the SOS2 method increases to around 3% while the SOCP method is still able to solve to optimality. The runtimes for the SOS2 method are still notably less. For the large instances p25-40 with 30 facilities and 150 customers, the SOS2 method provides smaller gaps with shorter runtimes. We run few instances from p25-40 with the SOCP method and allowed the runtime to reach 20,000 seconds. The average gap generated was around 14%.

				SOS2						SOCP					
Instance feature				$t = 100, \epsilon = 0.008$		$t = 50, \epsilon = 0.03$		$t = 25, \epsilon = 0.1$		$t = 100, \epsilon = 0.25$		$t = 50, \epsilon$=0.25		$t = 25, \epsilon = 0.25$	
#	n	m	K/D	Runtime	Gap	Runtime	Gap	Runtime	Gap	Runtime	Gap	Runtime	Gap	Runtime	Gap
p1	10	50	1.74	0.02	0.53%	0.02	0.30%	0.23	0.15%	129.99	0.00%	109.87	0.00%	118.02	0.00%
p2			1.74	0.01	0.53%	0.02	0.30%	0.05	0.15%	128.36	0.00%	108.93	0.00%	115.54	0.00%
p3			1.74	0.02	0.53%	0.02	0.30%	0.45	0.15%	127.47	0.00%	109.43	0.00%	118.75	0.00%
p4			1.74	0.02	0.53%	0.02	0.30%	0.25	0.15%	127.09	0.00%	110.43	0.00%	115.34	0.00%
p5			1.37	0.02	0.51%	0.02	0.27%	0.08	0.12%	123.59	0.00%	110.89	0.00%	115.60	0.00%
p6			1.37	0.02	0.51%	0.02	0.27%	0.08	0.12%	131.20	0.00%	110.68	0.00%	115.96	0.00%
p7			1.37	0.02	0.51%	0.02	0.27%	0.15	0.12%	133.78	0.00%	110.63	0.00%	115.46	0.00%
p8			1.37	0.02	0.51%	0.02	0.27%	0.14	0.12%	121.22	0.00%	109.64	0.00%	119.32	0.00%
p9			2.06	0.01	0.53%	0.01	0.30%	0.02	0.15%	132.25	0.00%	111.52	0.00%	116.80	0.00%
p10			2.06	0.01	0.53%	0.02	0.30%	0.02	0.15%	126.31	0.00%	109.94	0.00%	117.02	0.00%
p11			2.06	0.01	0.53%	0.01	0.30%	0.02	0.15%	124.85	0.00%	111.33	0.00%	116.54	0.00%
p12			2.06	0.01	0.53%	0.01	0.30%	0.02	0.15%	123.06	0.00%	108.24	0.00%	116.36	0.00%
p13	20	50	2.77	0.65	0.62%	0.37	0.57%	0.47	0.50%	91.43	0.00%	76.87	0.00%	78.92	0.00%
p14			2.77	0.37	0.62%	0.45	0.57%	0.45	0.50%	93.50	0.00%	74.70	0.00%	78.36	0.00%
p15			2.77	0.42	0.62%	0.30	0.57%	0.48	0.50%	103.19	0.00%	75.92	0.00%	77.54	0.00%
p16			2.77	0.53	0.62%	0.44	0.57%	0.42	0.50%	99.25	0.00%	75.73	0.00%	78.42	0.00%
p17			2.80	0.55	0.62%	0.34	0.57%	0.55	0.52%	94.59	0.00%	76.14	0.00%	77.47	0.00%
p18			2.80	0.43	0.62%	0.37	0.57%	0.55	0.52%	94.13	0.00%	76.15	0.00%	77.84	0.00%
p19			2.80	0.40	0.62%	0.38	0.57%	0.55	0.52%	92.67	0.00%	75.57	0.00%	78.79	0.00%
p20			2.80	0.42	0.62%	0.44	0.57%	0.53	0.52%	97.29	0.00%	75.06	0.00%	81.45	0.00%
p21			3.50	0.43	0.62%	0.39	0.57%	0.50	0.52%	93.02	0.00%	75.20	0.00%	79.93	0.00%
p22			3.50	0.61	0.62%	0.45	0.57%	0.63	0.52%	94.12	0.00%	74.93	0.00%	77.40	0.00%
p23			3.50	0.47	0.62%	0.55	0.57%	0.53	0.52%	94.92	0.00%	75.96	0.00%	77.33	0.00%
p24			3.50	0.50	0.62%	0.50	0.57%	0.53	0.52%	101.94	0.00%	76.25	0.00%	77.76	0.00%
p25	30	150	4.12	1.00	0.71%	1.39	0.46%	2.68	0.89%	2057.13	13.77%	2827.82	23.34%	2987.44	46.70%
p26			4.12	1.07	0.71%	1.38	0.46%	2.85	0.89%	2124.29	13.77%	2910.27	23.34%	2997.72	46.70%
p27			4.12	1.21	0.71%	1.44	0.46%	2.96	0.89%	2040.27	13.77%	2846.69	23.34%	2996.72	46.70%
p28			4.12	1.02	0.71%	1.41	0.46%	2.56	0.89%	2101.49	13.77%	2843.54	23.34%	2993.39	46.70%
p29			3.03	1.19	0.73%	1.52	0.55%	1.52	0.66%	2106.78	13.77%	2840.87	23.34%	2998.12	46.70%
p30			3.03	1.12	0.73%	1.41	0.55%	1.44	0.66%	2107.98	13.77%	2885.84	23.34%	2990.18	46.70%
p31			3.03	1.15	0.73%	1.57	0.55%	1.49	0.66%	2112.01	13.77%	2873.44	23.34%	2984.04	46.70%
p32			3.03	1.14	0.73%	1.52	0.55%	1.46	0.66%	2111.91	13.77%	2848.32	23.34%	2977.91	46.70%
p33			4.04	0.98	0.71%	1.35	0.64%	14.88	0.85%	2110.65	13.77%	2846.83	23.34%	2980.82	46.70%
p34			4.04	0.93	0.71%	1.31	0.64%	16.17	0.85%	2105.46	13.77%	3058.83	23.34%	2990.24	46.70%
p35			4.04	1.03	0.71%	1.30	0.64%	15.23	0.85%	2114.35	13.77%	2908.14	23.34%	2986.74	46.70%
p36			4.04	0.98	0.71%	1.34	0.64%	15.47	0.85%	2104.20	13.77%	2843.41	23.34%	2979.21	46.70%
p37			6.06	3.12	0.71%	7.74	0.48%	11.57	1.05%	2107.68	13.77%	2830.42	23.34%	2981.76	46.70%
p38			6.06	3.22	0.71%	6.99	0.48%	11.84	1.05%	2103.74	13.77%	2835.61	23.34%	2973.91	46.70%
p39			6.06	3.24	0.71%	7.22	0.48%	12.05	1.05%	2016.88	13.77%	2825.85	23.34%	2978.76	46.70%
p40			6.06	3.46	0.71%	7.18	0.48%	12.69	1.05%	2096.77	13.77%	2838.40	23.34%	2997.29	46.70%
p41	10	90	2.12	0.07	0.71%	0.13	0.36%	0.42	0.15%	2291.58	14.51%	2013.09	12.76%	2044.70	-
p42	20	80	4.99	0.04	0.41%	0.04	0.09%	0.53	0.37%	1267.72	0.84%	1387.22	1.56%	1166.52	2.11%
p43	30	70	8.28	0.07	0.29%	0.07	0.05%	0.59	0.79%	423.25	1.38%	453.77	1.86%	485.82	2.69%
p44	10	90	1.76	0.10	0.68%	0.10	0.44%	0.66	0.33%	3520.86	-	2826.53	4.05%	2130.53	20.23%
p45	20	80	4.14	0.04	0.45%	0.05	0.17%	0.56	0.28%	2428.02	1.98%	2179.24	-	2195.38	-
p46	30	70	7.10	0.06	0.31%	0.07	0.07%	0.58	0.60%	553.14	0.01%	2018.91	0.03%	2049.01	0.06%
p47	10	90	1.76	0.03	0.59%	0.03	0.42%	0.44	0.31%	3070.19	34.13%	2130.43	-	2097.35	-
p48	20	80	4.06	0.06	0.44%	0.07	0.21%	0.59	0.37%	2145.54	6.65%	2038.66	12.89%	3230.95	-
p49	30	70	7.08	0.27	0.32%	0.22	0.10%	0.65	0.65%	2208.67	-	3736.84	1.91%	2473.64	2.12%
p50	10	100	1.89	0.08	0.87%	0.07	0.57%	0.67	0.43%	2844.96	-	2011.51	10.18%	2600.98	11.82%
p51	20	100	3.98	0.09	0.57%	0.08	0.26%	0.64	0.44%	2269.25	4.13%	2639.45	1.78%	2070.02	0.65%
p52	10	100	1.60	0.17	0.63%	0.05	0.40%	0.46	0.24%	2811.21	5.41%	2005.40	-	2086.32	6.12%
p53	20	100	3.37	0.07	0.52%	0.06	0.27%	0.56	0.32%	2058.60	-	2228.15	2.75%	2178.28	8.99%
p54	10	100	1.52	0.16	0.68%	0.13	0.59%	0.39	0.49%	2025.91	3.71%	2165.38	25.24%	2067.41	7.56%
p55	20	100	3.21	0.10	0.58%	0.05	0.52%	0.59	0.68%	2008.99	27.88%	2459.85	32.61%	2693.14	-

Table 5.2: Comparison: SOS2 vs SOCP

Chapter 6

Conclusion

This thesis considers a service system design problem with capacity economies of scale and customer waiting costs. The problem minimizes the capacity cost associated with the facilities, the access costs, and the customer waiting time in an M/M/1 queuing network. A formulation composed of both concave and convex terms in the objective function is developed. Due to the non-linearity nature of the model, we tackled the problem with different approaches.

The first approach was a piecewise linearization based on SOS2 constraints. We first reformulated the problem using Benders-type decomposition idea.

The second approach was based on SOCP reformulation that was tackled through Lagrangian Relaxation. A Lagrangian bound was generated and was shown to be pretty sharp. Larger instances were tested and the results were compared with the other SOS2 approach. In general, the SOS2 algorithm was able to give decent results with shorter runtimes, while the SOCP algorithm could generate almost high quality solutions and bounds for smaller instances.

There are a number of future research directions that can be explored. The first is the solution of the new formulation provided. The second is the enhancement of the SOCP-based Lagrangian approach to handle large problems.

APPENDICES

Appendix A

The Numerical Results for the SOS2 Approach

t=	100	$\epsilon=$	0.01											
Instance Feature					Cost structure		Facility		Capacity Utilization					
#	n	m	K/D	capacity%	transportation%	waiting%	open	max%	min%	avg%	Runtime (s)	LB	UB	Error
p1	10	50	1.74	24.26%	64.80%	10.94%	10	99.56%	29.35%	61.25%	0.02	1359.56	1364.57	0.37%
p2			1.74	24.26%	64.80%	10.94%	10	99.56%	29.35%	61.25%	0.02	1359.56	1364.57	0.37%
p3			1.74	24.26%	64.80%	10.94%	10	99.56%	29.35%	61.25%	0.01	1359.56	1364.57	0.37%
p4			1.74	24.26%	64.80%	10.94%	10	99.56%	29.35%	61.25%	0.01	1359.56	1364.57	0.37%
p5			1.37	24.28%	64.85%	10.86%	10	95.50%	32.50%	72.80%	0.02	1366.49	1371.36	0.36%
p6			1.37	24.28%	64.85%	10.86%	10	95.50%	32.50%	72.80%	0.02	1366.49	1371.36	0.36%
p7			1.37	24.28%	64.85%	10.86%	10	95.50%	32.50%	72.80%	0.02	1366.49	1371.36	0.36%
p8			1.37	24.28%	64.85%	10.86%	10	95.50%	32.50%	72.80%	0.02	1366.49	1371.36	0.36%
p9			2.06	24.26%	64.80%	10.94%	10	74.67%	17.33%	48.53%	0.01	1359.56	1364.57	0.37%
p10			2.06	24.26%	64.80%	10.94%	10	74.67%	17.33%	48.53%	0.01	1359.56	1364.57	0.37%
p11			2.06	24.26%	64.80%	10.94%	10	74.67%	17.33%	48.53%	0.01	1359.56	1364.57	0.37%
p12			2.06	24.26%	64.80%	10.94%	10	74.67%	17.33%	48.53%	0.01	1359.56	1364.57	0.37%
p13	20	50	2.77	29.90%	55.30%	14.80%	18	91.28%	0.00%	38.09%	0.03	2060.97	2072.68	0.57%
p14			2.77	29.90%	55.30%	14.80%	18	91.28%	0.00%	38.09%	0.03	2060.97	2072.68	0.57%
p15			2.77	29.90%	55.30%	14.80%	18	91.28%	0.00%	38.09%	0.03	2060.97	2072.68	0.57%
p16			2.77	29.90%	55.30%	14.80%	18	91.28%	0.00%	38.09%	0.03	2060.97	2072.68	0.57%
p17			2.80	29.90%	55.30%	14.80%	18	81.00%	0.00%	35.69%	0.03	2060.97	2072.68	0.57%
p18			2.80	29.90%	55.30%	14.80%	18	81.00%	0.00%	35.69%	0.03	2060.97	2072.68	0.57%
p19			2.80	29.90%	55.30%	14.80%	18	81.00%	0.00%	35.69%	0.03	2060.97	2072.68	0.57%
p20			2.80	29.90%	55.30%	14.80%	18	81.00%	0.00%	35.69%	0.03	2060.97	2072.68	0.57%
p21			3.50	29.90%	55.30%	14.80%	18	64.80%	0.00%	28.55%	0.03	2060.97	2072.68	0.57%
p22			3.50	29.90%	55.30%	14.80%	18	64.80%	0.00%	28.55%	0.03	2060.97	2072.68	0.57%
p23			3.50	29.90%	55.30%	14.80%	18	64.80%	0.00%	28.55%	0.03	2060.97	2072.68	0.57%
p24			3.50	29.90%	55.30%	14.80%	18	64.80%	0.00%	28.55%	0.03	2060.97	2072.68	0.57%
p25	30	150	4.12	39.49%	37.75%	22.76%	25	97.61%	0.00%	25.64%	0.23	1993.59	2004.88	0.57%
p26			4.12	39.49%	37.75%	22.76%	25	97.61%	0.00%	25.64%	0.22	1993.59	2004.88	0.57%
p27			4.12	39.49%	37.75%	22.76%	25	97.61%	0.00%	25.64%	0.21	1993.59	2004.88	0.57%
p28			4.12	39.49%	37.75%	22.76%	25	97.61%	0.00%	25.64%	0.22	1993.59	2004.88	0.57%
p29			3.03	39.87%	37.53%	22.61%	26	99.33%	0.00%	33.00%	0.35	1994.04	2005.23	0.56%
p30			3.03	39.87%	37.53%	22.61%	26	99.33%	0.00%	33.00%	0.29	1994.04	2005.23	0.56%
p31			3.03	39.87%	37.53%	22.61%	26	99.33%	0.00%	33.00%	0.22	1994.04	2005.23	0.56%
p32			3.03	39.87%	37.53%	22.61%	26	99.33%	0.00%	33.00%	0.24	1994.04	2005.23	0.56%
p33			4.04	39.18%	37.93%	22.90%	25	84.25%	0.00%	24.75%	0.23	1992.59	2004.34	0.59%
p34			4.04	39.18%	37.93%	22.90%	25	84.25%	0.00%	24.75%	0.20	1992.59	2004.34	0.59%
p35			4.04	39.18%	37.93%	22.90%	25	84.25%	0.00%	24.75%	0.21	1992.59	2004.34	0.59%
p36			4.04	39.18%	37.93%	22.90%	25	84.25%	0.00%	24.75%	0.19	1992.59	2004.34	0.59%
p37			6.06	39.18%	37.93%	22.90%	25	56.17%	0.00%	16.50%	0.21	1992.59	2004.34	0.59%
p38			6.06	39.18%	37.93%	22.90%	25	56.17%	0.00%	16.50%	0.20	1992.59	2004.34	0.59%
p39			6.06	39.18%	37.93%	22.90%	25	56.17%	0.00%	16.50%	0.20	1992.59	2004.34	0.59%
p40			6.06	39.18%	37.93%	22.90%	25	56.17%	0.00%	16.50%	0.21	1992.59	2004.34	0.59%
p41	10	90	2.12	38.32%	44.24%	17.44%	10	99.47%	12.45%	62.73%	0.25	833.01	837.11	0.49%
p42	20	80	4.99	49.30%	22.65%	28.05%	20	79.82%	1.83%	26.72%	0.05	1005.44	1008.07	0.26%
p43	30	70	8.28	50.56%	13.92%	35.52%	29	55.12%	0.00%	14.00%	0.06	1283.67	1285.92	0.17%
p44	10	90	1.76	29.26%	57.70%	13.04%	10	100.00%	28.76%	68.39%	0.05	1161.23	1166.99	0.50%
p45	20	80	4.14	40.42%	37.37%	22.20%	20	85.64%	2.11%	30.35%	0.04	1255.90	1259.56	0.29%
p46	30	70	7.10	45.42%	23.72%	30.87%	28	76.70%	0.00%	17.62%	0.06	1437.26	1440.04	0.19%
p47	10	90	1.76	23.96%	65.07%	10.97%	10	99.17%	4.22%	68.26%	0.16	1386.57	1392.59	0.43%
p48	20	80	4.06	32.11%	47.90%	19.99%	16	97.09%	0.00%	32.09%	0.06	1442.27	1446.53	0.30%
p49	30	70	7.08	39.23%	31.61%	29.16%	24	95.74%	0.00%	20.33%	0.07	1551.06	1554.18	0.20%
p50	10	100	1.89	35.64%	47.45%	16.91%	10	99.16%	1.27%	67.49%	0.05	898.42	903.92	0.61%
p51	20	100	3.98	45.91%	27.28%	26.82%	18	99.12%	0.00%	34.20%	0.06	1092.86	1097.02	0.38%
p52	10	100	1.60	24.22%	65.13%	10.66%	10	99.17%	32.70%	72.37%	0.09	1456.44	1462.99	0.45%
p53	20	100	3.37	34.94%	46.03%	19.03%	18	98.35%	0.00%	36.53%	0.06	1494.40	1499.75	0.36%
p54	10	100	1.52	23.06%	66.38%	10.56%	10	99.17%	6.94%	73.52%	0.06	1506.17	1514.25	0.54%
p55	20	100	3.21	31.14%	49.89%	18.97%	16	99.17%	0.00%	32.51%	0.05	1564.11	1571.27	0.46%

Table A.1: SOS2 Test results with $t = 100, \epsilon = 0.01$

t=	100	ϵ=	0.008											
Instance Feature					Cost structure		Facility		Capacity Utilization					
#	n	m	K/D	capacity%	transportation%	waiting%	open	max%	min%	avg%	Runtime (s)	LB	UB	Error
p1	10	50	1.74	24.32%	64.81%	10.87%	10	99.56%	29.35%	61.25%	0.02	1354.03	1361.20	0.53%
p2			1.74	24.32%	64.81%	10.87%	10	99.56%	29.35%	61.25%	0.01	1354.03	1361.20	0.53%
p3			1.74	24.32%	64.81%	10.87%	10	99.56%	29.35%	61.25%	0.02	1354.03	1361.20	0.53%
p4			1.74	24.32%	64.81%	10.87%	10	99.56%	29.35%	61.25%	0.02	1354.03	1361.20	0.53%
p5			1.37	24.35%	64.86%	10.79%	10	95.50%	32.50%	72.80%	0.02	1360.96	1367.94	0.51%
p6			1.37	24.35%	64.86%	10.79%	10	95.50%	32.50%	72.80%	0.02	1360.96	1367.94	0.51%
p7			1.37	24.35%	64.86%	10.79%	10	95.50%	32.50%	72.80%	0.02	1360.96	1367.94	0.51%
p8			1.37	24.35%	64.86%	10.79%	10	95.50%	32.50%	72.80%	0.02	1360.96	1367.94	0.51%
p9			2.06	24.32%	64.81%	10.87%	10	74.67%	17.33%	48.53%	0.01	1354.03	1361.20	0.53%
p10			2.06	24.32%	64.81%	10.87%	10	74.67%	17.33%	48.53%	0.01	1354.03	1361.20	0.53%
p11			2.06	24.32%	64.81%	10.87%	10	74.67%	17.33%	48.53%	0.01	1354.03	1361.20	0.53%
p12			2.06	24.32%	64.81%	10.87%	10	74.67%	17.33%	48.53%	0.01	1354.03	1361.20	0.53%
p13	20	50	2.77	30.05%	55.29%	14.66%	20	91.28%	0.00%	38.15%	0.65	2054.07	2066.87	0.62%
p14			2.77	30.05%	55.29%	14.66%	20	91.28%	0.00%	38.15%	0.37	2054.07	2066.87	0.62%
p15			2.77	30.05%	55.29%	14.66%	20	91.28%	0.00%	38.15%	0.42	2054.07	2066.87	0.62%
p16			2.77	30.05%	55.29%	14.66%	20	91.28%	0.00%	38.15%	0.53	2054.07	2066.87	0.62%
p17			2.80	30.05%	55.29%	14.66%	20	81.00%	0.00%	35.69%	0.55	2054.07	2066.87	0.62%
p18			2.80	30.05%	55.29%	14.66%	20	81.00%	0.00%	35.69%	0.43	2054.07	2066.87	0.62%
p19			2.80	30.05%	55.29%	14.66%	20	81.00%	0.00%	35.69%	0.40	2054.07	2066.87	0.62%
p20			2.80	30.05%	55.29%	14.66%	20	81.00%	0.00%	35.69%	0.42	2054.07	2066.87	0.62%
p21			3.50	30.05%	55.29%	14.66%	20	64.80%	0.00%	28.55%	0.43	2054.07	2066.87	0.62%
p22			3.50	30.05%	55.29%	14.66%	20	64.80%	0.00%	28.55%	0.61	2054.07	2066.87	0.62%
p23			3.50	30.05%	55.29%	14.66%	20	64.80%	0.00%	28.55%	0.47	2054.07	2066.87	0.62%
p24			3.50	30.05%	55.29%	14.66%	20	64.80%	0.00%	28.55%	0.50	2054.07	2066.87	0.62%
p25	30	150	4.12	39.92%	37.81%	22.27%	25	97.61%	0.00%	25.80%	1.00	1959.97	1973.81	0.71%
p26			4.12	39.92%	37.81%	22.27%	25	97.61%	0.00%	25.80%	1.07	1959.97	1973.81	0.71%
p27			4.12	39.92%	37.81%	22.27%	25	97.61%	0.00%	25.80%	1.21	1959.97	1973.81	0.71%
p28			4.12	39.92%	37.81%	22.27%	25	97.61%	0.00%	25.80%	1.02	1959.97	1973.81	0.71%
p29			3.03	40.29%	37.60%	22.12%	26	99.33%	0.00%	33.00%	1.19	1960.00	1974.36	0.73%
p30			3.03	40.29%	37.60%	22.12%	26	99.33%	0.00%	33.00%	1.12	1960.00	1974.36	0.73%
p31			3.03	40.29%	37.60%	22.12%	26	99.33%	0.00%	33.00%	1.15	1960.00	1974.36	0.73%
p32			3.03	40.29%	37.60%	22.12%	26	99.33%	0.00%	33.00%	1.14	1960.00	1974.36	0.73%
p33			4.04	39.87%	37.83%	22.30%	25	84.25%	0.00%	24.75%	0.98	1959.55	1973.53	0.71%
p34			4.04	39.87%	37.83%	22.30%	25	84.25%	0.00%	24.75%	0.93	1959.55	1973.53	0.71%
p35			4.04	39.87%	37.83%	22.30%	25	84.25%	0.00%	24.75%	1.03	1959.55	1973.53	0.71%
p36			4.04	39.87%	37.83%	22.30%	25	84.25%	0.00%	24.75%	0.98	1959.55	1973.53	0.71%
p37			6.06	39.86%	37.83%	22.30%	25	56.17%	0.00%	16.50%	3.12	1959.56	1973.50	0.71%
p38			6.06	39.86%	37.83%	22.30%	25	56.17%	0.00%	16.50%	3.22	1959.56	1973.50	0.71%
p39			6.06	39.86%	37.83%	22.30%	25	56.17%	0.00%	16.50%	3.24	1959.56	1973.50	0.71%
p40			6.06	39.86%	37.83%	22.30%	25	56.17%	0.00%	16.50%	3.46	1959.56	1973.50	0.71%
p41	10	90	2.12	38.56%	44.21%	17.23%	10	99.47%	12.45%	62.30%	0.07	825.32	831.17	0.71%
p42	20	80	4.99	50.02%	22.82%	27.16%	20	79.82%	1.83%	26.50%	0.04	970.25	974.25	0.41%
p43	30	70	8.28	52.00%	14.01%	33.99%	29	55.12%	0.00%	14.00%	0.07	1221.56	1225.05	0.29%
p44	10	90	1.76	29.48%	57.56%	12.96%	10	100.00%	33.97%	68.99%	0.10	1157.65	1165.49	0.68%
p45	20	80	4.14	41.04%	37.46%	21.50%	20	85.64%	2.11%	30.99%	0.04	1223.99	1229.45	0.45%
p46	30	70	7.10	46.94%	23.68%	29.38%	29	76.70%	0.00%	17.43%	0.06	1377.59	1381.85	0.31%
p47	10	90	1.76	24.03%	65.08%	10.90%	10	99.17%	4.22%	69.27%	0.03	1385.97	1394.21	0.59%
p48	20	80	4.06	31.96%	48.49%	19.55%	15	97.09%	0.00%	32.20%	0.06	1410.76	1417.00	0.44%
p49	30	70	7.08	39.73%	32.07%	28.20%	23	95.74%	0.00%	19.62%	0.27	1491.46	1496.19	0.32%
p50	10	100	1.89	35.79%	47.43%	16.78%	10	100.00%	1.27%	67.84%	0.08	893.33	901.12	0.87%
p51	20	100	3.98	46.60%	27.37%	26.03%	18	99.12%	0.00%	34.20%	0.09	1061.74	1067.83	0.57%
p52	10	100	1.60	24.20%	65.16%	10.64%	10	99.17%	32.70%	72.43%	0.17	1457.67	1466.81	0.63%
p53	20	100	3.37	35.40%	46.09%	18.51%	18	98.35%	0.00%	37.18%	0.07	1467.55	1475.19	0.52%
p54	10	100	1.52	23.52%	66.03%	10.44%	10	99.17%	20.83%	75.71%	0.16	1515.02	1525.24	0.68%
p55	20	100	3.21	32.20%	49.73%	18.07%	17	99.17%	0.00%	33.40%	0.10	1551.39	1560.41	0.58%

Table A.2: SOS2 Test results with $t = 100, \epsilon = 0.008$

t=	50	$\epsilon=$	0.05											
Instance Feature					Cost structure		Facility		Capacity Utilization					
#	n	m	K/D	capacity%	transportation%	waiting%	open	max%	min%	avg%	Runtime (s)	LB	UB	Error
p1	10	50	1.74	21.46%	69.33%	9.20%	10	99.56%	29.35%	61.25%	0.23	1279.60	1280.04	0.03%
p2			1.74	21.46%	69.33%	9.20%	10	99.56%	29.35%	61.25%	0.06	1279.60	1280.04	0.03%
p3			1.74	21.46%	69.33%	9.20%	10	99.56%	29.35%	61.25%	0.06	1279.60	1280.04	0.03%
p4			1.74	21.46%	69.33%	9.20%	10	99.56%	29.35%	61.25%	0.08	1279.60	1280.04	0.03%
p5			1.37	21.48%	69.38%	9.14%	10	95.50%	32.50%	72.80%	0.17	1286.53	1286.85	0.02%
p6			1.37	21.48%	69.38%	9.14%	10	95.50%	32.50%	72.80%	0.09	1286.53	1286.85	0.02%
p7			1.37	21.48%	69.38%	9.14%	10	95.50%	32.50%	72.80%	0.10	1286.53	1286.85	0.02%
p8			1.37	21.48%	69.38%	9.14%	10	95.50%	32.50%	72.80%	0.09	1286.53	1286.85	0.02%
p9			2.06	21.46%	69.33%	9.20%	10	74.67%	17.33%	48.53%	0.02	1279.60	1280.04	0.03%
p10			2.06	21.46%	69.33%	9.20%	10	74.67%	17.33%	48.53%	0.09	1279.60	1280.04	0.03%
p11			2.06	21.46%	69.33%	9.20%	10	74.67%	17.33%	48.53%	0.09	1279.60	1280.04	0.03%
p12			2.06	21.46%	69.33%	9.20%	10	74.67%	17.33%	48.53%	0.02	1279.60	1280.04	0.03%
p13	20	50	2.77	27.02%	60.28%	12.69%	18	91.28%	0.00%	38.09%	0.08	1902.30	1904.91	0.14%
p14			2.77	27.02%	60.28%	12.69%	18	91.28%	0.00%	38.09%	0.07	1902.30	1904.91	0.14%
p15			2.77	27.02%	60.28%	12.69%	18	91.28%	0.00%	38.09%	0.08	1902.30	1904.91	0.14%
p16			2.77	27.02%	60.28%	12.69%	18	91.28%	0.00%	38.09%	0.23	1902.30	1904.91	0.14%
p17			2.80	27.02%	60.28%	12.69%	18	81.00%	0.00%	35.69%	0.06	1902.30	1904.91	0.14%
p18			2.80	27.02%	60.28%	12.69%	18	81.00%	0.00%	35.69%	0.06	1902.30	1904.91	0.14%
p19			2.80	27.02%	60.28%	12.69%	18	81.00%	0.00%	35.69%	0.06	1902.30	1904.91	0.14%
p20			2.80	27.02%	60.28%	12.69%	18	81.00%	0.00%	35.69%	0.06	1902.30	1904.91	0.14%
p21			3.50	27.02%	60.28%	12.69%	18	64.80%	0.00%	28.55%	0.27	1902.30	1904.91	0.14%
p22			3.50	27.02%	60.28%	12.69%	18	64.80%	0.00%	28.55%	0.06	1902.30	1904.91	0.14%
p23			3.50	27.02%	60.28%	12.69%	18	64.80%	0.00%	28.55%	0.06	1902.30	1904.91	0.14%
p24			3.50	27.02%	60.28%	12.69%	18	64.80%	0.00%	28.55%	0.11	1902.30	1904.91	0.14%
p25	30	150	4.12	36.56%	42.83%	20.61%	25	97.61%	0.00%	25.50%	1.25	1783.88	1787.99	0.23%
p26			4.12	36.56%	42.83%	20.61%	25	97.61%	0.00%	25.50%	1.23	1783.88	1787.99	0.23%
p27			4.12	36.56%	42.83%	20.61%	25	97.61%	0.00%	25.50%	1.38	1783.88	1787.99	0.23%
p28			4.12	36.56%	42.83%	20.61%	25	97.61%	0.00%	25.50%	1.44	1783.88	1787.99	0.23%
p29			3.03	37.14%	42.46%	20.40%	26	99.33%	0.00%	33.00%	1.36	1784.94	1788.42	0.20%
p30			3.03	37.14%	42.46%	20.40%	26	99.33%	0.00%	33.00%	1.43	1784.94	1788.42	0.20%
p31			3.03	37.14%	42.46%	20.40%	26	99.33%	0.00%	33.00%	1.38	1784.94	1788.42	0.20%
p32			3.03	37.14%	42.46%	20.40%	26	99.33%	0.00%	33.00%	1.38	1784.94	1788.42	0.20%
p33			4.04	36.11%	43.11%	20.78%	24	91.75%	0.00%	24.75%	0.37	1783.03	1787.20	0.23%
p34			4.04	36.11%	43.11%	20.78%	24	91.75%	0.00%	24.75%	0.48	1783.03	1787.20	0.23%
p35			4.04	36.11%	43.11%	20.78%	24	91.75%	0.00%	24.75%	0.47	1783.03	1787.20	0.23%
p36			4.04	36.11%	43.11%	20.78%	24	91.75%	0.00%	24.75%	0.43	1783.03	1787.20	0.23%
p37			6.06	36.11%	43.11%	20.78%	24	61.17%	0.00%	16.50%	0.52	1783.03	1787.20	0.23%
p38			6.06	36.11%	43.11%	20.78%	24	61.17%	0.00%	16.50%	0.37	1783.03	1787.20	0.23%
p39			6.06	36.11%	43.11%	20.78%	24	61.17%	0.00%	16.50%	0.48	1783.03	1787.20	0.23%
p40			6.06	36.11%	43.11%	20.78%	24	61.17%	0.00%	16.50%	0.59	1783.03	1787.20	0.23%
p41	10	90	2.12	35.26%	49.36%	15.37%	10	99.47%	12.45%	62.73%	0.53	756.51	756.77	0.03%
p42	20	80	4.99	46.67%	26.63%	26.70%	20	94.74%	1.83%	26.57%	0.45	884.04	885.33	0.15%
p43	30	70	8.28	48.59%	16.66%	34.75%	29	59.06%	0.00%	14.09%	0.22	1116.46	1119.52	0.27%
p44	10	90	1.76	26.29%	62.61%	11.10%	10	100.00%	28.76%	68.39%	0.44	1078.81	1079.48	0.06%
p45	20	80	4.14	37.47%	42.29%	20.24%	20	85.64%	2.11%	30.71%	0.11	1129.25	1130.45	0.11%
p46	30	70	7.10	43.50%	27.42%	29.08%	29	76.70%	0.00%	17.41%	0.18	1266.12	1268.91	0.22%
p47	10	90	1.76	20.87%	69.88%	9.25%	10	99.17%	2.11%	67.34%	0.36	1303.52	1304.76	0.10%
p48	20	80	4.06	28.99%	53.15%	17.86%	16	97.09%	0.00%	32.33%	0.16	1314.98	1316.67	0.13%
p49	30	70	7.08	35.80%	36.64%	27.55%	23	95.74%	0.00%	19.93%	0.31	1379.84	1383.01	0.23%
p50	10	100	1.89	32.70%	52.57%	14.73%	10	99.16%	1.27%	67.59%	0.55	818.41	819.30	0.11%
p51	20	100	3.98	43.46%	31.56%	24.98%	18	99.12%	0.00%	34.29%	0.76	964.91	966.36	0.15%
p52	10	100	1.60	21.49%	69.62%	8.89%	10	99.17%	32.70%	72.37%	0.61	1370.58	1371.33	0.06%
p53	20	100	3.37	31.86%	51.19%	16.95%	18	98.35%	0.00%	36.19%	0.50	1360.64	1362.09	0.11%
p54	10	100	1.52	20.18%	71.02%	8.80%	10	99.17%	2.31%	73.31%	0.31	1418.11	1420.09	0.14%
p55	20	100	3.21	28.13%	55.08%	16.79%	16	99.46%	0.00%	33.55%	0.35	1426.95	1430.06	0.22%

Table A.3: SOS2 Test results with $t = 50, \epsilon = 0.05$

t=	50	ϵ=	0.04											
Instance Feature				Cost structure			Facility	Capacity Utilization						
#	n	m	K/D	capacity%	transportation%	waiting%	open	max%	min%	avg%	Runtime (s)	LB	UB	Error
p1	10	50	1.74	21.59%	69.34%	9.07%	10	99.56%	29.35%	61.25%	0.02	1272.10	1273.45	0.11%
p2			1.74	21.59%	69.34%	9.07%	10	99.56%	29.35%	61.25%	0.02	1272.10	1273.45	0.11%
p3			1.74	21.59%	69.34%	9.07%	10	99.56%	29.35%	61.25%	0.02	1272.10	1273.45	0.11%
p4			1.74	21.59%	69.34%	9.07%	10	99.56%	29.35%	61.25%	0.02	1272.10	1273.45	0.11%
p5			1.37	21.61%	69.39%	9.00%	10	95.50%	32.50%	72.80%	0.02	1279.04	1280.21	0.09%
p6			1.37	21.61%	69.39%	9.00%	10	95.50%	32.50%	72.80%	0.02	1279.04	1280.21	0.09%
p7			1.37	21.61%	69.39%	9.00%	10	95.50%	32.50%	72.80%	0.02	1279.04	1280.21	0.09%
p8			1.37	21.61%	69.39%	9.00%	10	95.50%	32.50%	72.80%	0.02	1279.04	1280.21	0.09%
p9			2.06	21.59%	69.34%	9.07%	10	74.67%	17.33%	48.53%	0.01	1272.10	1273.45	0.11%
p10			2.06	21.59%	69.34%	9.07%	10	74.67%	17.33%	48.53%	0.01	1272.10	1273.45	0.11%
p11			2.06	21.59%	69.34%	9.07%	10	74.67%	17.33%	48.53%	0.01	1272.10	1273.45	0.11%
p12			2.06	21.59%	69.34%	9.07%	10	74.67%	17.33%	48.53%	0.02	1272.10	1273.45	0.11%
p13	20	50	2.77	27.16%	60.33%	12.51%	18	91.28%	0.00%	38.09%	0.03	1886.76	1891.70	0.26%
p14			2.77	27.16%	60.33%	12.51%	18	91.28%	0.00%	38.09%	0.03	1886.76	1891.70	0.26%
p15			2.77	27.16%	60.33%	12.51%	18	91.28%	0.00%	38.09%	0.03	1886.76	1891.70	0.26%
p16			2.77	27.16%	60.33%	12.51%	18	91.28%	0.00%	38.09%	0.03	1886.76	1891.70	0.26%
p17			2.80	27.16%	60.33%	12.51%	18	81.00%	0.00%	35.69%	0.03	1886.76	1891.70	0.26%
p18			2.80	27.16%	60.33%	12.51%	18	81.00%	0.00%	35.69%	0.03	1886.76	1891.70	0.26%
p19			2.80	27.16%	60.33%	12.51%	18	81.00%	0.00%	35.69%	0.03	1886.76	1891.70	0.26%
p20			2.80	27.16%	60.33%	12.51%	18	81.00%	0.00%	35.69%	0.03	1886.76	1891.70	0.26%
p21			3.50	27.16%	60.33%	12.51%	18	64.80%	0.00%	28.55%	0.03	1886.76	1891.70	0.26%
p22			3.50	27.16%	60.33%	12.51%	18	64.80%	0.00%	28.55%	0.03	1886.76	1891.70	0.26%
p23			3.50	27.16%	60.33%	12.51%	18	64.80%	0.00%	28.55%	0.03	1886.76	1891.70	0.26%
p24			3.50	27.16%	60.33%	12.51%	18	64.80%	0.00%	28.55%	0.03	1886.76	1891.70	0.26%
p25	30	150	4.12	36.97%	42.95%	20.08%	25	97.61%	0.00%	25.50%	0.34	1747.89	1753.50	0.32%
p26			4.12	36.97%	42.95%	20.08%	25	97.61%	0.00%	25.50%	0.22	1747.89	1753.50	0.32%
p27			4.12	36.97%	42.95%	20.08%	25	97.61%	0.00%	25.50%	0.44	1747.89	1753.50	0.32%
p28			4.12	36.97%	42.95%	20.08%	25	97.61%	0.00%	25.50%	0.21	1747.89	1753.50	0.32%
p29			3.03	37.58%	42.56%	19.86%	26	99.33%	0.00%	33.00%	0.25	1748.95	1753.69	0.27%
p30			3.03	37.58%	42.56%	19.86%	26	99.33%	0.00%	33.00%	0.23	1748.95	1753.69	0.27%
p31			3.03	37.58%	42.56%	19.86%	26	99.33%	0.00%	33.00%	0.32	1748.95	1753.69	0.27%
p32			3.03	37.58%	42.56%	19.86%	26	99.33%	0.00%	33.00%	0.22	1748.95	1753.69	0.27%
p33			4.04	36.46%	43.28%	20.26%	24	98.50%	0.00%	24.75%	0.19	1747.04	1752.91	0.34%
p34			4.04	36.46%	43.28%	20.26%	24	98.50%	0.00%	24.75%	0.19	1747.04	1752.91	0.34%
p35			4.04	36.46%	43.28%	20.26%	24	98.50%	0.00%	24.75%	0.19	1747.04	1752.91	0.34%
p36			4.04	36.46%	43.28%	20.26%	24	98.50%	0.00%	24.75%	0.20	1747.04	1752.91	0.34%
p37			6.06	36.46%	43.28%	20.26%	24	65.67%	0.00%	16.50%	0.20	1747.04	1752.91	0.34%
p38			6.06	36.46%	43.28%	20.26%	24	65.67%	0.00%	16.50%	0.20	1747.04	1752.91	0.34%
p39			6.06	36.46%	43.28%	20.26%	24	65.67%	0.00%	16.50%	0.20	1747.04	1752.91	0.34%
p40			6.06	36.46%	43.28%	20.26%	24	65.67%	0.00%	16.50%	0.21	1747.04	1752.91	0.34%
p41	10	90	2.12	35.54%	49.38%	15.08%	10	99.47%	12.45%	62.73%	0.16	747.50	748.34	0.11%
p42	20	80	4.99	47.90%	26.59%	25.51%	20	79.82%	1.83%	26.72%	0.04	852.15	852.60	0.05%
p43	30	70	8.28	50.08%	16.77%	33.15%	29	55.12%	0.00%	13.91%	0.06	1062.10	1063.30	0.11%
p44	10	90	1.76	26.53%	62.52%	10.95%	10	100.00%	28.76%	68.76%	0.05	1072.39	1073.99	0.15%
p45	20	80	4.14	38.09%	42.40%	19.51%	20	85.64%	2.11%	30.40%	0.04	1099.66	1100.45	0.07%
p46	30	70	7.10	44.58%	27.60%	27.83%	29	76.70%	0.00%	17.92%	0.06	1213.48	1214.61	0.09%
p47	10	90	1.76	20.94%	69.92%	9.14%	10	99.17%	2.11%	67.30%	0.03	1297.02	1299.93	0.22%
p48	20	80	4.06	29.47%	53.30%	17.23%	16	97.09%	0.00%	32.52%	0.26	1285.68	1287.03	0.10%
p49	30	70	7.08	38.03%	36.14%	25.83%	25	95.74%	0.00%	19.55%	0.08	1327.24	1328.74	0.11%
p50	10	100	1.89	32.88%	52.59%	14.52%	10	99.16%	1.27%	67.49%	0.06	810.94	812.86	0.24%
p51	20	100	3.98	44.19%	31.68%	24.13%	18	99.12%	0.00%	34.29%	0.07	935.87	937.05	0.13%
p52	10	100	1.60	21.56%	69.62%	8.82%	10	99.17%	32.70%	72.37%	0.04	1365.67	1367.86	0.16%
p53	20	100	3.37	32.29%	51.28%	16.42%	18	98.35%	0.00%	36.19%	0.05	1334.17	1335.77	0.12%
p54	10	100	1.52	20.20%	71.04%	8.76%	10	99.17%	2.31%	73.31%	0.12	1414.03	1418.02	0.28%
p55	20	100	3.21	28.88%	54.91%	16.21%	17	99.46%	0.00%	32.29%	0.05	1401.32	1405.27	0.28%

Table A.4: SOS2 Test results with $t = 50, \epsilon = 0.04$

t=	50	ϵ=	0.03											
Instance Feature					Cost structure		Facility		Capacity Utilization					
#	n	m	K/D	capacity%	transportation%	waiting%	open	max%	min%	avg%	Runtime (s)	LB	UB	Error
p1	10	50	1.74	21.63%	69.35%	9.02%	10	99.56%	29.35%	61.25%	0.02	1267.72	1271.46	0.30%
p2			1.74	21.63%	69.35%	9.02%	10	99.56%	29.35%	61.25%	0.02	1267.72	1271.46	0.30%
p3			1.74	21.63%	69.35%	9.02%	10	99.56%	29.35%	61.25%	0.02	1267.72	1271.46	0.30%
p4			1.74	21.63%	69.35%	9.02%	10	99.56%	29.35%	61.25%	0.02	1267.72	1271.46	0.30%
p5			1.37	21.65%	69.39%	8.96%	10	95.50%	32.50%	72.80%	0.02	1274.65	1278.14	0.27%
p6			1.37	21.65%	69.39%	8.96%	10	95.50%	32.50%	72.80%	0.02	1274.65	1278.14	0.27%
p7			1.37	21.65%	69.39%	8.96%	10	95.50%	32.50%	72.80%	0.02	1274.65	1278.14	0.27%
p8			1.37	21.65%	69.39%	8.96%	10	95.50%	32.50%	72.80%	0.02	1274.65	1278.14	0.27%
p9			2.06	21.63%	69.35%	9.02%	10	74.67%	17.33%	48.53%	0.01	1267.72	1271.46	0.30%
p10			2.06	21.63%	69.35%	9.02%	10	74.67%	17.33%	48.53%	0.02	1267.72	1271.46	0.30%
p11			2.06	21.63%	69.35%	9.02%	10	74.67%	17.33%	48.53%	0.01	1267.72	1271.46	0.30%
p12			2.06	21.63%	69.35%	9.02%	10	74.67%	17.33%	48.53%	0.01	1267.72	1271.46	0.30%
p13	20	50	2.77	27.18%	60.38%	12.44%	18	91.28%	0.00%	38.09%	0.37	1877.15	1887.86	0.57%
p14			2.77	27.18%	60.38%	12.44%	18	91.28%	0.00%	38.09%	0.45	1877.15	1887.86	0.57%
p15			2.77	27.18%	60.38%	12.44%	18	91.28%	0.00%	38.09%	0.30	1877.15	1887.86	0.57%
p16			2.77	27.18%	60.38%	12.44%	18	91.28%	0.00%	38.09%	0.44	1877.15	1887.86	0.57%
p17			2.80	27.18%	60.38%	12.44%	18	81.00%	0.00%	35.69%	0.34	1877.15	1887.86	0.57%
p18			2.80	27.18%	60.38%	12.44%	18	81.00%	0.00%	35.69%	0.37	1877.15	1887.86	0.57%
p19			2.80	27.18%	60.38%	12.44%	18	81.00%	0.00%	35.69%	0.38	1877.15	1887.86	0.57%
p20			2.80	27.18%	60.38%	12.44%	18	81.00%	0.00%	35.69%	0.44	1877.15	1887.86	0.57%
p21			3.50	27.18%	60.38%	12.44%	18	64.80%	0.00%	28.55%	0.39	1877.15	1887.86	0.57%
p22			3.50	27.18%	60.38%	12.44%	18	64.80%	0.00%	28.55%	0.45	1877.15	1887.86	0.57%
p23			3.50	27.18%	60.38%	12.44%	18	64.80%	0.00%	28.55%	0.55	1877.15	1887.86	0.57%
p24			3.50	27.18%	60.38%	12.44%	18	64.80%	0.00%	28.55%	0.50	1877.15	1887.86	0.57%
p25	30	150	4.12	36.81%	43.70%	19.48%	25	98.98%	0.00%	25.20%	1.39	1712.94	1720.83	0.46%
p26			4.12	36.81%	43.70%	19.48%	25	98.98%	0.00%	25.20%	1.38	1712.94	1720.83	0.46%
p27			4.12	36.81%	43.70%	19.48%	25	98.98%	0.00%	25.20%	1.44	1712.94	1720.83	0.46%
p28			4.12	36.81%	43.70%	19.48%	25	98.98%	0.00%	25.20%	1.41	1712.94	1720.83	0.46%
p29			3.03	37.91%	42.68%	19.41%	26	99.33%	0.00%	33.00%	1.52	1715.26	1724.77	0.55%
p30			3.03	37.91%	42.68%	19.41%	26	99.33%	0.00%	33.00%	1.41	1715.26	1724.77	0.55%
p31			3.03	37.91%	42.68%	19.41%	26	99.33%	0.00%	33.00%	1.57	1715.26	1724.77	0.55%
p32			3.03	37.91%	42.68%	19.41%	26	99.33%	0.00%	33.00%	1.52	1715.26	1724.77	0.55%
p33			4.04	36.81%	43.40%	19.79%	24	91.75%	0.00%	24.75%	1.35	1713.35	1724.25	0.64%
p34			4.04	36.81%	43.40%	19.79%	24	91.75%	0.00%	24.75%	1.31	1713.35	1724.25	0.64%
p35			4.04	36.81%	43.40%	19.79%	24	91.75%	0.00%	24.75%	1.30	1713.35	1724.25	0.64%
p36			4.04	36.81%	43.40%	19.79%	24	91.75%	0.00%	24.75%	1.34	1713.35	1724.25	0.64%
p37			6.06	36.21%	44.15%	19.64%	24	100.00%	0.00%	16.50%	7.74	1712.41	1720.66	0.48%
p38			6.06	36.21%	44.15%	19.64%	24	100.00%	0.00%	16.50%	6.99	1712.41	1720.66	0.48%
p39			6.06	36.21%	44.15%	19.64%	24	100.00%	0.00%	16.50%	7.22	1712.41	1720.66	0.48%
p40			6.06	36.21%	44.15%	19.64%	24	100.00%	0.00%	16.50%	7.18	1712.41	1720.66	0.48%
p41	10	90	2.12	35.71%	49.38%	14.91%	10	99.47%	12.45%	62.73%	0.13	740.81	743.49	0.36%
p42	20	80	4.99	48.97%	26.67%	24.37%	20	79.82%	1.83%	26.72%	0.04	817.69	818.39	0.09%
p43	30	70	8.28	51.88%	16.85%	31.27%	29	55.12%	0.00%	14.00%	0.07	1000.48	1000.97	0.05%
p44	10	90	1.76	26.43%	62.61%	10.96%	10	100.00%	28.76%	68.39%	0.10	1069.65	1074.33	0.44%
p45	20	80	4.14	38.63%	42.57%	18.79%	20	85.64%	2.11%	30.42%	0.05	1068.70	1070.49	0.17%
p46	30	70	7.10	45.72%	27.83%	26.44%	29	76.70%	0.00%	17.37%	0.07	1154.47	1155.27	0.07%
p47	10	90	1.76	21.49%	69.44%	9.07%	10	99.17%	10.55%	67.83%	0.03	1294.42	1299.81	0.42%
p48	20	80	4.06	30.02%	53.41%	16.57%	16	97.09%	0.00%	32.09%	0.07	1255.14	1257.78	0.21%
p49	30	70	7.08	39.09%	36.39%	24.52%	25	95.74%	0.00%	19.55%	0.22	1268.29	1269.59	0.10%
p50	10	100	1.89	32.90%	52.63%	14.46%	10	99.16%	1.27%	67.49%	0.07	806.58	811.19	0.57%
p51	20	100	3.98	44.92%	31.80%	23.28%	18	99.12%	0.00%	34.29%	0.08	905.75	908.11	0.26%
p52	10	100	1.60	21.51%	69.61%	8.87%	10	99.17%	32.70%	72.37%	0.05	1365.22	1370.75	0.40%
p53	20	100	3.37	32.70%	51.38%	15.92%	18	98.35%	0.00%	36.53%	0.06	1307.95	1311.46	0.27%
p54	10	100	1.52	20.12%	71.05%	8.83%	10	99.17%	2.31%	73.31%	0.13	1414.85	1423.21	0.59%
p55	20	100	3.21	29.13%	55.10%	15.78%	17	99.46%	0.00%	32.36%	0.05	1376.36	1383.58	0.52%

Table A.5: SOS2 Test results with $t = 50, \epsilon = 0.03$

t= 25		ϵ= 0.2												
Instance Feature				Cost structure			Facility	Capacity Utilization						
#	n	m	K/D	capacity%	transportation%	waiting%	open	max%	min%	avg%	Runtime (s)	LB	UB	Error
p1	10	50	1.74	19.24%	73.31%	7.46%	10	99.56%	29.35%	61.25%	0.17	1203.58	1208.11	0.38%
p2			1.74	19.24%	73.31%	7.46%	10	99.56%	29.35%	61.25%	0.06	1203.58	1208.11	0.38%
p3			1.74	19.24%	73.31%	7.46%	10	99.56%	29.35%	61.25%	0.06	1203.58	1208.11	0.38%
p4			1.74	19.24%	73.31%	7.46%	10	99.56%	29.35%	61.25%	0.07	1203.58	1208.11	0.38%
p5			1.37	19.25%	73.35%	7.40%	10	95.50%	32.50%	72.80%	0.08	1210.52	1214.89	0.36%
p6			1.37	19.25%	73.35%	7.40%	10	95.50%	32.50%	72.80%	0.13	1210.52	1214.89	0.36%
p7			1.37	19.25%	73.35%	7.40%	10	95.50%	32.50%	72.80%	0.08	1210.52	1214.89	0.36%
p8			1.37	19.25%	73.35%	7.40%	10	95.50%	32.50%	72.80%	0.08	1210.52	1214.89	0.36%
p9			2.06	19.24%	73.31%	7.46%	10	74.67%	17.33%	48.53%	0.02	1203.58	1208.11	0.38%
p10			2.06	19.24%	73.31%	7.46%	10	74.67%	17.33%	48.53%	0.02	1203.58	1208.11	0.38%
p11			2.06	19.24%	73.31%	7.46%	10	74.67%	17.33%	48.53%	0.02	1203.58	1208.11	0.38%
p12			2.06	19.24%	73.31%	7.46%	10	74.67%	17.33%	48.53%	0.02	1203.58	1208.11	0.38%
p13	20	50	2.77	24.68%	64.86%	10.47%	18	91.28%	0.00%	38.09%	0.13	1750.50	1761.89	0.65%
p14			2.77	24.68%	64.86%	10.47%	18	91.28%	0.00%	38.09%	0.07	1750.50	1761.89	0.65%
p15			2.77	24.68%	64.86%	10.47%	18	91.28%	0.00%	38.09%	0.25	1750.50	1761.89	0.65%
p16			2.77	24.68%	64.86%	10.47%	18	91.28%	0.00%	38.09%	0.17	1750.50	1761.89	0.65%
p17			2.80	24.68%	64.86%	10.47%	18	81.00%	0.00%	35.69%	0.05	1750.50	1761.89	0.65%
p18			2.80	24.68%	64.86%	10.47%	18	81.00%	0.00%	35.69%	0.05	1750.50	1761.89	0.65%
p19			2.80	24.68%	64.86%	10.47%	18	81.00%	0.00%	35.69%	0.06	1750.50	1761.89	0.65%
p20			2.80	24.68%	64.86%	10.47%	18	81.00%	0.00%	35.69%	0.07	1750.50	1761.89	0.65%
p21			3.50	24.68%	64.86%	10.47%	18	64.80%	0.00%	28.55%	0.24	1750.50	1761.89	0.65%
p22			3.50	24.68%	64.86%	10.47%	18	64.80%	0.00%	28.55%	0.05	1750.50	1761.89	0.65%
p23			3.50	24.68%	64.86%	10.47%	18	64.80%	0.00%	28.55%	0.05	1750.50	1761.89	0.65%
p24			3.50	24.68%	64.86%	10.47%	18	64.80%	0.00%	28.55%	0.05	1750.50	1761.89	0.65%
p25	30	150	4.12	34.58%	47.79%	17.63%	25	97.61%	0.00%	25.50%	1.30	1561.46	1586.36	1.59%
p26			4.12	34.58%	47.79%	17.63%	25	97.61%	0.00%	25.50%	1.27	1561.46	1586.36	1.59%
p27			4.12	34.58%	47.79%	17.63%	25	97.61%	0.00%	25.50%	1.36	1561.46	1586.36	1.59%
p28			4.12	34.58%	47.79%	17.63%	25	97.61%	0.00%	25.50%	1.46	1561.46	1586.36	1.59%
p29			3.03	35.13%	47.40%	17.46%	26	99.33%	0.00%	33.00%	1.44	1562.53	1586.68	1.55%
p30			3.03	35.13%	47.40%	17.46%	26	99.33%	0.00%	33.00%	1.44	1562.53	1586.68	1.55%
p31			3.03	35.13%	47.40%	17.46%	26	99.33%	0.00%	33.00%	1.60	1562.53	1586.68	1.55%
p32			3.03	35.13%	47.40%	17.46%	26	99.33%	0.00%	33.00%	1.63	1562.53	1586.68	1.55%
p33			4.04	34.12%	48.11%	17.77%	24	98.50%	0.00%	24.75%	1.18	1560.61	1585.72	1.61%
p34			4.04	34.12%	48.11%	17.77%	24	98.50%	0.00%	24.75%	1.35	1560.61	1585.72	1.61%
p35			4.04	34.12%	48.11%	17.77%	24	98.50%	0.00%	24.75%	1.23	1560.61	1585.72	1.61%
p36			4.04	34.12%	48.11%	17.77%	24	98.50%	0.00%	24.75%	1.23	1560.61	1585.72	1.61%
p37			6.06	34.12%	48.11%	17.77%	24	65.67%	0.00%	16.50%	0.48	1560.61	1585.72	1.61%
p38			6.06	34.12%	48.11%	17.77%	24	65.67%	0.00%	16.50%	0.48	1560.61	1585.72	1.61%
p39			6.06	34.12%	48.11%	17.77%	24	65.67%	0.00%	16.50%	0.38	1560.61	1585.72	1.61%
p40			6.06	34.12%	48.11%	17.77%	24	65.67%	0.00%	16.50%	0.38	1560.61	1585.72	1.61%
p41	10	90	2.12	32.81%	54.24%	12.95%	10	99.47%	12.45%	62.73%	0.44	681.13	686.33	0.76%
p42	20	80	4.99	45.52%	30.80%	23.67%	20	94.74%	1.83%	26.57%	0.19	739.05	758.66	2.65%
p43	30	70	8.28	48.59%	19.78%	31.62%	29	59.06%	0.00%	14.09%	0.24	901.71	936.35	3.84%
p44	10	90	1.76	23.87%	67.02%	9.10%	10	100.00%	28.76%	68.39%	0.45	1002.34	1006.48	0.41%
p45	20	80	4.14	35.37%	47.28%	17.35%	20	85.64%	2.11%	30.71%	0.09	983.31	1001.44	1.84%
p46	30	70	7.10	42.45%	31.75%	25.81%	29	76.70%	0.00%	17.41%	0.12	1050.65	1083.94	3.17%
p47	10	90	1.76	18.69%	73.82%	7.49%	10	99.17%	2.11%	67.30%	0.55	1227.08	1232.21	0.42%
p48	20	80	4.06	26.83%	58.18%	14.99%	16	97.09%	0.00%	32.33%	0.29	1168.92	1187.46	1.59%
p49	30	70	7.08	34.33%	41.66%	24.01%	23	95.74%	0.00%	19.93%	0.19	1164.36	1198.06	2.89%
p50	10	100	1.89	30.29%	57.38%	12.33%	10	99.16%	1.27%	67.59%	0.46	742.39	747.42	0.68%
p51	20	100	3.98	42.00%	36.09%	21.90%	18	99.12%	0.00%	34.29%	0.70	818.74	836.82	2.21%
p52	10	100	1.60	19.27%	73.53%	7.20%	10	99.17%	32.70%	72.37%	0.57	1293.48	1296.82	0.26%
p53	20	100	3.37	29.56%	56.18%	14.26%	18	98.35%	0.00%	36.19%	0.61	1213.39	1229.95	1.36%
p54	10	100	1.52	18.07%	74.82%	7.11%	10	99.17%	2.31%	73.31%	0.25	1340.67	1344.97	0.32%
p55	20	100	3.21	26.01%	59.98%	14.01%	16	99.46%	0.00%	32.32%	0.96	1279.36	1297.38	1.41%

Table A.6: SOS2 Test results with $t = 25, \epsilon = 0.2$

t=	25	$\epsilon=$	0.1											
Instance Feature					Cost structure		Facility	Capacity Utilization						
#	n	m	K/D	capacity%	transportation%	waiting%	open	max%	min%	avg%	Runtime (s)	LB	UB	Error
p1	10	50	1.74	19.29%	73.32%	7.39%	10	99.56%	29.35%	61.25%	0.23	1203.45	1205.27	0.15%
p2			1.74	19.29%	73.32%	7.39%	10	99.56%	29.35%	61.25%	0.05	1203.45	1205.27	0.15%
p3			1.74	19.29%	73.32%	7.39%	10	99.56%	29.35%	61.25%	0.45	1203.45	1205.27	0.15%
p4			1.74	19.29%	73.32%	7.39%	10	99.56%	29.35%	61.25%	0.25	1203.45	1205.27	0.15%
p5			1.37	19.30%	73.36%	7.34%	10	95.50%	32.50%	72.80%	0.08	1210.39	1211.81	0.12%
p6			1.37	19.30%	73.36%	7.34%	10	95.50%	32.50%	72.80%	0.08	1210.39	1211.81	0.12%
p7			1.37	19.30%	73.36%	7.34%	10	95.50%	32.50%	72.80%	0.15	1210.39	1211.81	0.12%
p8			1.37	19.30%	73.36%	7.34%	10	95.50%	32.50%	72.80%	0.14	1210.39	1211.81	0.12%
p9			2.06	19.29%	73.32%	7.39%	10	74.67%	17.33%	48.53%	0.02	1203.45	1205.27	0.15%
p10			2.06	19.29%	73.32%	7.39%	10	74.67%	17.33%	48.53%	0.02	1203.45	1205.27	0.15%
p11			2.06	19.29%	73.32%	7.39%	10	74.67%	17.33%	48.53%	0.02	1203.45	1205.27	0.15%
p12			2.06	19.29%	73.32%	7.39%	10	74.67%	17.33%	48.53%	0.02	1203.45	1205.27	0.15%
p13	20	50	2.77	24.69%	64.95%	10.36%	18	91.28%	0.00%	38.09%	0.47	1748.58	1757.41	0.50%
p14			2.77	24.69%	64.95%	10.36%	18	91.28%	0.00%	38.09%	0.45	1748.58	1757.41	0.50%
p15			2.77	24.69%	64.95%	10.36%	18	91.28%	0.00%	38.09%	0.48	1748.58	1757.41	0.50%
p16			2.77	24.69%	64.95%	10.36%	18	91.28%	0.00%	38.09%	0.42	1748.58	1757.41	0.50%
p17			2.80	24.64%	65.04%	10.32%	18	97.00%	0.00%	35.69%	0.55	1747.68	1756.71	0.52%
p18			2.80	24.64%	65.04%	10.32%	18	97.00%	0.00%	35.69%	0.55	1747.68	1756.71	0.52%
p19			2.80	24.64%	65.04%	10.32%	18	97.00%	0.00%	35.69%	0.55	1747.68	1756.71	0.52%
p20			2.80	24.64%	65.04%	10.32%	18	97.00%	0.00%	35.69%	0.53	1747.68	1756.71	0.52%
p21			3.50	24.64%	65.04%	10.32%	18	77.60%	0.00%	28.55%	0.50	1747.68	1756.71	0.52%
p22			3.50	24.64%	65.04%	10.32%	18	77.60%	0.00%	28.55%	0.63	1747.68	1756.71	0.52%
p23			3.50	24.64%	65.04%	10.32%	18	77.60%	0.00%	28.55%	0.53	1747.68	1756.71	0.52%
p24			3.50	24.64%	65.04%	10.32%	18	77.60%	0.00%	28.55%	0.53	1747.68	1756.71	0.52%
p25	30	150	4.12	32.93%	50.39%	16.69%	22	98.36%	0.00%	24.77%	2.68	1506.49	1519.86	0.89%
p26			4.12	32.93%	50.39%	16.69%	22	98.36%	0.00%	24.77%	2.85	1506.49	1519.86	0.89%
p27			4.12	32.93%	50.39%	16.69%	22	98.36%	0.00%	24.77%	2.96	1506.49	1519.86	0.89%
p28			4.12	32.93%	50.39%	16.69%	22	98.36%	0.00%	24.77%	2.56	1506.49	1519.86	0.89%
p29			3.03	35.78%	47.67%	16.55%	26	99.33%	0.00%	33.00%	1.52	1521.41	1531.52	0.66%
p30			3.03	35.78%	47.67%	16.55%	26	99.33%	0.00%	33.00%	1.44	1521.41	1531.52	0.66%
p31			3.03	35.78%	47.67%	16.55%	26	99.33%	0.00%	33.00%	1.49	1521.41	1531.52	0.66%
p32			3.03	35.78%	47.67%	16.55%	26	99.33%	0.00%	33.00%	1.46	1521.41	1531.52	0.66%
p33			4.04	34.43%	48.77%	16.80%	24	99.75%	0.00%	24.75%	14.88	1514.61	1527.49	0.85%
p34			4.04	34.43%	48.77%	16.80%	24	99.75%	0.00%	24.75%	16.17	1514.61	1527.49	0.85%
p35			4.04	34.43%	48.77%	16.80%	24	99.75%	0.00%	24.75%	15.23	1514.61	1527.49	0.85%
p36			4.04	34.43%	48.77%	16.80%	24	99.75%	0.00%	24.75%	15.47	1514.61	1527.49	0.85%
p37			6.06	31.54%	51.77%	16.70%	21	92.83%	0.00%	16.50%	11.57	1505.79	1521.59	1.05%
p38			6.06	31.54%	51.77%	16.70%	21	92.83%	0.00%	16.50%	11.84	1505.79	1521.59	1.05%
p39			6.06	31.54%	51.77%	16.70%	21	92.83%	0.00%	16.50%	12.05	1505.79	1521.59	1.05%
p40			6.06	31.54%	51.77%	16.70%	21	92.83%	0.00%	16.50%	12.69	1505.79	1521.59	1.05%
p41	10	90	2.12	33.11%	54.25%	12.63%	10	99.47%	12.45%	62.73%	0.42	676.42	677.44	0.15%
p42	20	80	4.99	47.70%	31.02%	21.28%	20	94.74%	1.83%	26.57%	0.53	687.84	690.39	0.37%
p43	30	70	8.28	52.12%	20.13%	27.75%	29	59.06%	0.00%	14.09%	0.59	805.19	811.57	0.79%
p44	10	90	1.76	23.58%	67.41%	9.00%	10	99.17%	24.68%	67.40%	0.66	1002.45	1005.77	0.33%
p45	20	80	4.14	36.71%	47.47%	15.82%	20	85.64%	2.11%	30.71%	0.56	939.03	941.62	0.28%
p46	30	70	7.10	45.12%	32.14%	22.75%	29	76.70%	0.00%	17.41%	0.58	959.32	965.09	0.60%
p47	10	90	1.76	18.65%	73.96%	7.39%	10	99.54%	2.11%	66.74%	0.44	1224.18	1227.98	0.31%
p48	20	80	4.06	27.64%	58.62%	13.74%	16	97.09%	0.00%	32.33%	0.59	1125.49	1129.61	0.37%
p49	30	70	7.08	36.37%	42.37%	21.27%	23	95.74%	0.00%	19.93%	0.65	1073.14	1080.07	0.65%
p50	10	100	1.89	30.30%	57.46%	12.24%	10	99.16%	1.27%	67.62%	0.67	742.32	745.48	0.43%
p51	20	100	3.98	43.48%	36.39%	20.13%	18	99.12%	0.00%	34.29%	0.64	776.12	779.53	0.44%
p52	10	100	1.60	19.16%	73.52%	7.31%	10	99.17%	32.70%	72.37%	0.46	1301.16	1304.28	0.24%
p53	20	100	3.37	30.34%	56.38%	13.28%	18	98.35%	0.00%	36.19%	0.56	1178.52	1182.30	0.32%
p54	10	100	1.52	17.81%	75.08%	7.10%	10	99.46%	2.31%	72.10%	0.39	1341.59	1348.22	0.49%
p55	20	100	3.21	26.50%	60.44%	13.07%	16	99.46%	0.00%	31.98%	0.59	1242.12	1250.60	0.68%

Table A.7: SOS2 Test results with $t = 25, \epsilon = 0.1$

t=	25	ϵ=	0.08											
Instance Feature					Cost structure		Facility		Capacity Utilization					
#	n	m	K/D	capacity%	transportation%	waiting%	open	max%	min%	avg%	Runtime (s)	LB	UB	Error
p1	10	50	1.74	19.16%	73.31%	7.53%	10	99.56%	29.35%	61.25%	0.27	1208.69	1213.22	0.37%
p2			1.74	19.16%	73.31%	7.53%	10	99.56%	29.35%	61.25%	0.19	1208.69	1213.22	0.37%
p3			1.74	19.16%	73.31%	7.53%	10	99.56%	29.35%	61.25%	0.19	1208.69	1213.22	0.37%
p4			1.74	19.16%	73.31%	7.53%	10	99.56%	29.35%	61.25%	0.19	1208.69	1213.22	0.37%
p5			1.37	19.17%	73.35%	7.48%	10	95.50%	32.50%	72.80%	0.05	1215.63	1219.62	0.33%
p6			1.37	19.17%	73.35%	7.48%	10	95.50%	32.50%	72.80%	0.11	1215.63	1219.62	0.33%
p7			1.37	19.17%	73.35%	7.48%	10	95.50%	32.50%	72.80%	0.04	1215.63	1219.62	0.33%
p8			1.37	19.17%	73.35%	7.48%	10	95.50%	32.50%	72.80%	0.05	1215.63	1219.62	0.33%
p9			2.06	19.16%	73.31%	7.53%	10	74.67%	17.33%	48.53%	0.54	1208.69	1213.22	0.37%
p10			2.06	19.16%	73.31%	7.53%	10	74.67%	17.33%	48.53%	0.26	1208.69	1213.22	0.37%
p11			2.06	19.16%	73.31%	7.53%	10	74.67%	17.33%	48.53%	0.21	1208.69	1213.22	0.37%
p12			2.06	19.16%	73.31%	7.53%	10	74.67%	17.33%	48.53%	0.35	1208.69	1213.22	0.37%
p13	20	50	2.77	24.32%	65.41%	10.27%	18	97.28%	0.00%	38.13%	0.85	1746.64	1760.48	0.79%
p14			2.77	24.32%	65.41%	10.27%	18	97.28%	0.00%	38.13%	0.84	1746.64	1760.48	0.79%
p15			2.77	24.32%	65.41%	10.27%	18	97.28%	0.00%	38.13%	0.96	1746.64	1760.48	0.79%
p16			2.77	24.32%	65.41%	10.27%	18	97.28%	0.00%	38.13%	0.86	1746.64	1760.48	0.79%
p17			2.80	24.33%	65.51%	10.17%	18	98.25%	0.00%	35.69%	0.87	1741.83	1754.85	0.75%
p18			2.80	24.33%	65.51%	10.17%	18	98.25%	0.00%	35.69%	0.87	1741.83	1754.85	0.75%
p19			2.80	24.33%	65.51%	10.17%	18	98.25%	0.00%	35.69%	0.84	1741.83	1754.85	0.75%
p20			2.80	24.33%	65.51%	10.17%	18	98.25%	0.00%	35.69%	0.81	1741.83	1754.85	0.75%
p21			3.50	24.33%	65.51%	10.17%	18	78.60%	0.00%	28.55%	0.94	1741.83	1754.85	0.75%
p22			3.50	24.33%	65.51%	10.17%	18	78.60%	0.00%	28.55%	1.04	1741.83	1754.85	0.75%
p23			3.50	24.33%	65.51%	10.17%	18	78.60%	0.00%	28.55%	0.94	1741.83	1754.85	0.75%
p24			3.50	24.33%	65.51%	10.17%	18	78.60%	0.00%	28.55%	0.83	1741.83	1754.85	0.75%
p25	30	150	4.12	33.01%	50.59%	16.40%	22	99.66%	0.00%	24.74%	15.51	1488.83	1503.74	1.00%
p26			4.12	33.01%	50.59%	16.40%	22	99.66%	0.00%	24.74%	14.66	1488.83	1503.74	1.00%
p27			4.12	33.01%	50.59%	16.40%	22	99.66%	0.00%	24.74%	14.26	1488.83	1503.74	1.00%
p28			4.12	33.01%	50.59%	16.40%	22	99.66%	0.00%	24.74%	14.18	1488.83	1503.74	1.00%
p29			3.03	35.52%	47.94%	16.54%	26	100.00%	0.00%	33.00%	173.26	1516.78	1531.94	1.00%
p30			3.03	35.52%	47.94%	16.54%	26	100.00%	0.00%	33.00%	177.91	1516.78	1531.94	1.00%
p31			3.03	35.52%	47.94%	16.54%	26	100.00%	0.00%	33.00%	178.19	1516.78	1531.94	1.00%
p32			3.03	35.52%	47.94%	16.54%	26	100.00%	0.00%	33.00%	170.98	1516.78	1531.94	1.00%
p33			4.04	32.76%	50.65%	16.59%	22	99.75%	0.00%	24.75%	81.94	1491.11	1508.27	1.15%
p34			4.04	32.76%	50.65%	16.59%	22	99.75%	0.00%	24.75%	81.23	1491.11	1508.27	1.15%
p35			4.04	32.76%	50.65%	16.59%	22	99.75%	0.00%	24.75%	81.74	1491.11	1508.27	1.15%
p36			4.04	32.76%	50.65%	16.59%	22	99.75%	0.00%	24.75%	81.39	1491.11	1508.27	1.15%
p37			6.06	30.92%	52.82%	16.26%	20	100.00%	0.00%	16.50%	62.47	1480.22	1496.55	1.10%
p38			6.06	30.92%	52.82%	16.26%	20	100.00%	0.00%	16.50%	61.68	1480.22	1496.55	1.10%
p39			6.06	30.92%	52.82%	16.26%	20	100.00%	0.00%	16.50%	61.83	1480.22	1496.55	1.10%
p40			6.06	30.92%	52.82%	16.26%	20	100.00%	0.00%	16.50%	62.04	1480.22	1496.55	1.10%
p41	10	90	2.12	32.98%	54.23%	12.79%	10	99.47%	12.45%	62.58%	0.45	679.72	682.52	0.41%
p42	20	80	4.99	48.12%	31.05%	20.83%	20	94.74%	1.83%	26.57%	0.56	676.62	678.01	0.20%
p43	30	70	8.28	52.99%	20.20%	26.81%	29	59.06%	0.00%	14.09%	0.68	779.97	783.13	0.41%
p44	10	90	1.76	23.54%	67.42%	9.04%	10	99.17%	24.68%	67.40%	0.69	1002.13	1007.23	0.51%
p45	20	80	4.14	36.92%	47.50%	15.58%	20	85.64%	2.11%	30.71%	0.58	930.76	933.05	0.25%
p46	30	70	7.10	45.75%	32.21%	22.04%	29	76.70%	0.00%	17.41%	0.71	936.31	939.27	0.32%
p47	10	90	1.76	17.98%	74.72%	7.30%	10	99.54%	2.11%	65.83%	0.47	1221.18	1228.61	0.61%
p48	20	80	4.06	25.69%	60.65%	13.66%	14	99.46%	0.00%	31.62%	0.67	1117.22	1123.38	0.55%
p49	30	70	7.08	36.82%	42.54%	20.64%	23	95.74%	0.00%	19.93%	0.48	1050.18	1054.66	0.43%
p50	10	100	1.89	30.07%	57.46%	12.47%	10	99.16%	1.27%	67.59%	0.59	747.58	753.89	0.84%
p51	20	100	3.98	43.66%	36.45%	19.88%	18	99.12%	0.00%	34.29%	0.66	768.56	772.17	0.47%
p52	10	100	1.60	18.95%	73.66%	7.39%	10	100.00%	29.54%	72.16%	0.99	1305.66	1312.21	0.50%
p53	20	100	3.37	29.38%	57.52%	13.10%	18	98.35%	0.00%	35.02%	0.57	1173.73	1179.97	0.53%
p54	10	100	1.52	16.90%	76.09%	7.01%	9	99.46%	0.00%	71.13%	0.59	1335.03	1345.10	0.75%
p55	20	100	3.21	25.93%	61.23%	12.84%	16	99.46%	0.00%	32.68%	0.73	1225.66	1236.02	0.85%

Table A.8: SOS2 Test results with $t = 25, \epsilon = 0.08$

Appendix B

The Numerical Results for the SOCP Approach

t=	100	e=	0.25	runtime=	2000						
Instance feature					Cost structure		Facility status				
#	n	m	K/D	capacity%	transportation%	waiting%	open	Runtime (s)	LB	UB	Error
p1	10	50	1.74	16.37%	60.59%	23.04%	10	129.99	1446.691	1446.691	0.00%
p2			1.74	16.37%	60.59%	23.04%	10	128.36	1446.691	1446.691	0.00%
p3			1.74	16.37%	60.59%	23.04%	10	127.47	1446.691	1446.691	0.00%
p4			1.74	16.37%	60.59%	23.04%	10	127.09	1446.691	1446.691	0.00%
p5			1.37	16.37%	60.59%	23.04%	10	123.59	1446.691	1446.691	0.00%
p6			1.37	16.37%	60.59%	23.04%	10	131.20	1446.691	1446.691	0.00%
p7			1.37	16.37%	60.59%	23.04%	10	133.78	1446.691	1446.691	0.00%
p8			1.37	16.37%	60.59%	23.04%	10	121.22	1446.691	1446.691	0.00%
p9			2.06	16.37%	60.59%	23.04%	10	132.25	1446.691	1446.691	0.00%
p10			2.06	16.37%	60.59%	23.04%	10	126.31	1446.691	1446.691	0.00%
p11			2.06	16.37%	60.59%	23.04%	10	124.85	1446.691	1446.691	0.00%
p12			2.06	16.37%	60.59%	23.04%	10	123.06	1446.691	1446.691	0.00%
p13	20	50	2.77	18.72%	50.85%	30.43%	16	91.43	2190.769	2190.769	0.00%
p14			2.77	18.72%	50.85%	30.43%	16	93.50	2190.769	2190.769	0.00%
p15			2.77	18.72%	50.85%	30.43%	16	103.19	2190.769	2190.769	0.00%
p16			2.77	18.72%	50.85%	30.43%	16	99.25	2190.769	2190.769	0.00%
p17			2.80	18.72%	50.85%	30.43%	16	94.59	2190.769	2190.769	0.00%
p18			2.80	18.72%	50.85%	30.43%	16	94.13	2190.769	2190.769	0.00%
p19			2.80	18.72%	50.85%	30.43%	16	92.67	2190.769	2190.769	0.00%
p20			2.80	18.72%	50.85%	30.43%	16	97.29	2190.769	2190.769	0.00%
p21			3.50	18.72%	50.85%	30.43%	16	93.02	2190.769	2190.769	0.00%
p22			3.50	18.72%	50.85%	30.43%	16	94.12	2190.769	2190.769	0.00%
p23			3.50	18.72%	50.85%	30.43%	16	94.92	2190.769	2190.769	0.00%
p24			3.50	18.72%	50.85%	30.43%	16	101.94	2190.769	2190.769	0.00%
p25	30	150	4.12	16.41%	38.54%	45.05%	13	2057.13	1951.155	2219.773	13.77%
p26			4.12	16.41%	38.54%	45.05%	13	2124.29	1951.155	2219.773	13.77%
p27			4.12	16.41%	38.54%	45.05%	13	2040.27	1951.155	2219.773	13.77%
p28			4.12	16.41%	38.54%	45.05%	13	2101.49	1951.155	2219.773	13.77%
p29			3.03	16.41%	38.54%	45.05%	13	2106.78	1951.155	2219.773	13.77%
p30			3.03	16.41%	38.54%	45.05%	13	2107.98	1951.155	2219.773	13.77%
p31			3.03	16.41%	38.54%	45.05%	13	2112.01	1951.155	2219.773	13.77%
p32			3.03	16.41%	38.54%	45.05%	13	2111.91	1951.155	2219.773	13.77%
p33			4.04	16.41%	38.54%	45.05%	13	2110.65	1951.155	2219.773	13.77%
p34			4.04	16.41%	38.54%	45.05%	13	2105.46	1951.155	2219.773	13.77%
p35			4.04	16.41%	38.54%	45.05%	13	2114.35	1951.155	2219.773	13.77%
p36			4.04	16.41%	38.54%	45.05%	13	2104.20	1951.155	2219.773	13.77%
p37			6.06	16.41%	38.54%	45.05%	13	2107.68	1951.155	2219.773	13.77%
p38			6.06	16.41%	38.54%	45.05%	13	2103.74	1951.155	2219.773	13.77%
p39			6.06	16.41%	38.54%	45.05%	13	2016.88	1951.155	2219.773	13.77%
p40			6.06	16.41%	38.54%	45.05%	13	2096.77	1951.155	2219.773	13.77%
p41	10	90	2.12	20.62%	45.88%	33.50%	9	2291.58	868.8346	994.8947	14.51%
p42	20	80	4.99	18.56%	21.68%	59.76%	10	1267.72	1106.345	1115.593	0.84%
p43	30	70	8.28	15.29%	13.68%	71.03%	11	423.25	1388.605	1407.825	1.38%
p44	10	90	1.76	-	-	-	-	3520.86	1189.287	-	-
p45	20	80	4.14	16.56%	34.72%	48.71%	10	2428.02	1342.017	1368.526	1.98%
p46	30	70	7.10	13.51%	21.67%	64.82%	9	553.14	1542.551	1542.639	0.01%
p47	10	90	1.76	4.94%	74.21%	20.85%	1	3070.19	1191.869	1598.677	34.13%
p48	20	80	4.06	12.84%	43.60%	43.56%	9	2145.54	1435.08	1530.565	6.65%
p49	30	70	7.08	-	-	-	-	2208.67	1609.938	-	-
p50	10	100	1.89	-	-	-	-	2844.96	906.0866	-	-
p51	20	100	3.98	15.98%	27.38%	56.64%	7	2269.25	1130.314	1177.038	4.13%
p52	10	100	1.60	14.59%	63.22%	22.19%	8	2811.21	1425.063	1502.094	5.41%
p53	20	100	3.37	-	-	-	-	2058.60	1567.42	-	-
p54	10	100	1.52	13.57%	64.45%	21.98%	6	2025.91	1315.877	1364.683	3.71%
p55	20	100	3.21	13.91%	47.62%	38.47%	12	2008.99	1355.142	1732.993	27.88%

Table B.1: SOCP Test results with $t = 100, \epsilon = 0.25$, runtime=2000

t=	100	e=	0.2	runtime=	2000						
Instance feature					Cost structure		Facility status				
#	n	m	K/D	capacity%	transportation%	waiting%	open	Runtime (s)	LB	UB	Error
p1	10	50	1.74	18.94%	63.09%	17.97%	10	166.01	1391.28	1391.28	0.00%
p2			1.74	18.94%	63.09%	17.97%	10	165.64	1391.28	1391.28	0.00%
p3			1.74	18.94%	63.09%	17.97%	10	164.68	1391.28	1391.28	0.00%
p4			1.74	18.94%	63.09%	17.97%	10	167.37	1391.28	1391.28	0.00%
p5			1.37	18.94%	63.09%	17.97%	10	166.46	1391.28	1391.28	0.00%
p6			1.37	18.94%	63.09%	17.97%	10	163.87	1391.28	1391.28	0.00%
p7			1.37	18.94%	63.09%	17.97%	10	164.25	1391.28	1391.28	0.00%
p8			1.37	18.94%	63.09%	17.97%	10	164.24	1391.28	1391.28	0.00%
p9			2.06	18.94%	63.09%	17.97%	10	166.48	1391.28	1391.28	0.00%
p10			2.06	18.94%	63.09%	17.97%	10	164.91	1391.28	1391.28	0.00%
p11			2.06	18.94%	63.09%	17.97%	10	163.52	1391.28	1391.28	0.00%
p12			2.06	18.94%	63.09%	17.97%	10	163.68	1391.28	1391.28	0.00%
p13	20	50	2.77	22.12%	53.75%	24.13%	16	89.22	2072.51	2072.51	0.00%
p14			2.77	22.12%	53.75%	24.13%	16	87.61	2072.51	2072.51	0.00%
p15			2.77	22.12%	53.75%	24.13%	16	87.16	2072.51	2072.51	0.00%
p16			2.77	22.12%	53.75%	24.13%	16	87.60	2072.51	2072.51	0.00%
p17			2.80	22.12%	53.75%	24.13%	16	87.41	2072.51	2072.51	0.00%
p18			2.80	22.12%	53.75%	24.13%	16	87.23	2072.51	2072.51	0.00%
p19			2.80	22.12%	53.75%	24.13%	16	89.32	2072.51	2072.51	0.00%
p20			2.80	22.12%	53.75%	24.13%	16	87.44	2072.51	2072.51	0.00%
p21			3.50	22.12%	53.75%	24.13%	16	87.42	2072.51	2072.51	0.00%
p22			3.50	22.12%	53.75%	24.13%	16	87.58	2072.51	2072.51	0.00%
p23			3.50	22.12%	53.75%	24.13%	16	88.81	2072.51	2072.51	0.00%
p24			3.50	22.12%	53.75%	24.13%	16	87.11	2072.51	2072.51	0.00%
p25	30	150	4.12	-	-	-	-	2185.19	1717.25	-	-
p26			4.12	-	-	-	-	2188.21	1717.25	-	-
p27			4.12	-	-	-	-	2169.17	1717.25	-	-
p28			4.12	-	-	-	-	2171.41	1717.25	-	-
p29			3.03	-	-	-	-	2191.66	1717.25	-	-
p30			3.03	-	-	-	-	2184.59	1717.25	-	-
p31			3.03	-	-	-	-	2179.58	1717.25	-	-
p32			3.03	-	-	-	-	2182.09	1717.25	-	-
p33			4.04	-	-	-	-	2185.15	1717.25	-	-
p34			4.04	-	-	-	-	2181.61	1717.25	-	-
p35			4.04	-	-	-	-	2182.13	1717.25	-	-
p36			4.04	-	-	-	-	2181.17	1717.25	-	-
p37			6.06	-	-	-	-	2173.69	1717.25	-	-
p38			6.06	-	-	-	-	2179.78	1717.25	-	-
p39			6.06	-	-	-	-	2170.68	1717.25	-	-
p40			6.06	-	-	-	-	2183.76	1717.25	-	-
p41	10	90	2.12	22.38%	51.47%	26.15%	8	3260.64	762.11	956.07	25.45%
p42	20	80	4.99	-	-	-	-	2013.42	959.36	-	-
p43	30	70	8.28	17.81%	19.13%	63.06%	9	672.32	1160.38	1189.39	2.50%
p44	10	90	1.76	20.22%	61.33%	18.45%	9	2069.66	1100.65	1355.05	23.11%
p45	20	80	4.14	18.31%	41.04%	40.65%	8	2080.79	1194.08	1229.98	3.01%
p46	30	70	7.10	16.81%	26.23%	56.96%	8	881.82	1316.69	1316.77	0.01%
p47	10	90	1.76	-	-	-	-	2019.63	972.82	-	-
p48	20	80	4.06	14.49%	50.61%	34.90%	7	2709.74	1256.59	1432.62	14.01%
p49	30	70	7.08	14.63%	32.91%	52.46%	8	2024.25	1371.65	1429.75	4.24%
p50	10	100	1.89	-	-	-	-	3124.52	821.79	-	-
p51	20	100	3.98	21.69%	32.49%	45.82%	8	5548.73	967.86	1091.30	12.75%
p52	10	100	1.60	16.73%	67.17%	16.10%	7	9178.36	1338.63	1552.37	15.97%
p53	20	100	3.37	18.92%	47.73%	33.35%	11	2009.69	1404.78	1499.36	6.73%
p54	10	100	1.52	16.22%	67.19%	16.58%	8	2115.25	1231.06	1356.84	10.22%
p55	20	100	3.21	5.90%	63.09%	31.01%	1	2020.24	1133.67	1612.23	42.21%

Table B.2: SOCP Test results with $t = 100, \epsilon = 0.2$, runtime=2000

t=	50	e=	0.25	runtime=	2000						
Instance feature					Cost structure		Facility status				
#	n	m	K/D	capacity%	transportation%	waiting%	open	Runtime (s)	LB	UB	Error
p1	10	50	1.74	18.50%	68.48%	13.02%	10	109.87	1279.96	1280.02	0.00%
p2			1.74	18.50%	68.48%	13.02%	10	108.93	1279.96	1280.02	0.00%
p3			1.74	18.50%	68.48%	13.02%	10	109.43	1279.96	1280.02	0.00%
p4			1.74	18.50%	68.48%	13.02%	10	110.43	1279.96	1280.02	0.00%
p5			1.37	18.50%	68.48%	13.02%	10	110.89	1279.96	1280.02	0.00%
p6			1.37	18.50%	68.48%	13.02%	10	110.68	1279.96	1280.02	0.00%
p7			1.37	18.50%	68.48%	13.02%	10	110.63	1279.96	1280.02	0.00%
p8			1.37	18.50%	68.48%	13.02%	10	109.64	1279.96	1280.02	0.00%
p9			2.06	18.50%	68.48%	13.02%	10	111.52	1279.96	1280.02	0.00%
p10			2.06	18.50%	68.48%	13.02%	10	109.94	1279.96	1280.02	0.00%
p11			2.06	18.50%	68.48%	13.02%	10	111.33	1279.96	1280.02	0.00%
p12			2.06	18.50%	68.48%	13.02%	10	108.24	1279.96	1280.02	0.00%
p13	20	50	2.77	22.08%	59.98%	17.95%	16	76.87	1857.44	1857.44	0.00%
p14			2.77	22.08%	59.98%	17.95%	16	74.70	1857.44	1857.44	0.00%
p15			2.77	22.08%	59.98%	17.95%	16	75.92	1857.44	1857.44	0.00%
p16			2.77	22.08%	59.98%	17.95%	16	75.73	1857.44	1857.44	0.00%
p17			2.80	22.08%	59.98%	17.95%	16	76.14	1857.44	1857.44	0.00%
p18			2.80	22.08%	59.98%	17.95%	16	76.15	1857.44	1857.44	0.00%
p19			2.80	22.08%	59.98%	17.95%	16	75.57	1857.44	1857.44	0.00%
p20			2.80	22.08%	59.98%	17.95%	16	75.06	1857.44	1857.44	0.00%
p21			3.50	22.08%	59.98%	17.95%	16	75.20	1857.44	1857.44	0.00%
p22			3.50	22.08%	59.98%	17.95%	16	74.93	1857.44	1857.44	0.00%
p23			3.50	22.08%	59.98%	17.95%	16	75.96	1857.44	1857.44	0.00%
p24			3.50	22.08%	59.98%	17.95%	16	76.25	1857.44	1857.44	0.00%
p25	30	150	4.12	21.90%	49.30%	28.80%	15	2827.82	1407.78	1736.32	23.34%
p26			4.12	21.90%	49.30%	28.80%	15	2910.27	1407.78	1736.32	23.34%
p27			4.12	21.90%	49.30%	28.80%	15	2846.69	1407.78	1736.32	23.34%
p28			4.12	21.90%	49.30%	28.80%	15	2843.54	1407.78	1736.32	23.34%
p29			3.03	21.90%	49.30%	28.80%	15	2840.87	1407.78	1736.32	23.34%
p30			3.03	21.90%	49.30%	28.80%	15	2885.84	1407.78	1736.32	23.34%
p31			3.03	21.90%	49.30%	28.80%	15	2873.44	1407.78	1736.32	23.34%
p32			3.03	21.90%	49.30%	28.80%	15	2848.32	1407.78	1736.32	23.34%
p33			4.04	21.90%	49.30%	28.80%	15	2846.83	1407.78	1736.32	23.34%
p34			4.04	21.90%	49.30%	28.80%	15	3058.83	1407.78	1736.32	23.34%
p35			4.04	21.90%	49.30%	28.80%	15	2908.14	1407.78	1736.32	23.34%
p36			4.04	21.90%	49.30%	28.80%	15	2843.41	1407.78	1736.32	23.34%
p37			6.06	21.90%	49.30%	28.80%	15	2830.42	1407.78	1736.32	23.34%
p38			6.06	21.90%	49.30%	28.80%	15	2835.61	1407.78	1736.32	23.34%
p39			6.06	21.90%	49.30%	28.80%	15	2825.85	1407.78	1736.32	23.34%
p40			6.06	21.90%	49.30%	28.80%	15	2838.40	1407.78	1736.32	23.34%
p41	10	90	2.12	24.70%	54.63%	20.67%	9	2013.09	715.11	806.33	12.76%
p42	20	80	4.99	25.70%	31.84%	42.46%	9	1387.22	773.01	785.06	1.56%
p43	30	70	8.28	22.82%	21.93%	55.24%	10	453.77	888.56	905.07	1.86%
p44	10	90	1.76	20.04%	65.57%	14.38%	8	2826.53	1002.20	1042.76	4.05%
p45	20	80	4.14	-	-	-	-	2179.24	1008.56	-	-
p46	30	70	7.10	19.99%	32.06%	47.96%	9	2018.91	1042.30	1042.64	0.03%
p47	10	90	1.76	-	-	-	-	2130.43	985.93	-	-
p48	20	80	4.06	15.69%	57.13%	27.17%	7	2038.66	1086.64	1226.66	12.89%
p49	30	70	7.08	17.34%	38.54%	44.12%	8	3736.84	1112.18	1133.37	1.91%
p50	10	100	1.89	27.10%	52.63%	20.27%	9	2011.51	746.40	822.39	10.18%
p51	20	100	3.98	24.73%	35.88%	39.39%	9	2639.45	789.89	803.95	1.78%
p52	10	100	1.60	-	-	-	-	2005.40	1265.81	-	-
p53	20	100	3.37	19.98%	53.72%	26.30%	10	2228.15	1233.79	1267.66	2.75%
p54	10	100	1.52	14.61%	73.70%	11.69%	6	2165.38	1138.07	1425.36	25.24%
p55	20	100	3.21	16.51%	59.88%	23.61%	9	2459.85	1064.66	1411.87	32.61%

Table B.3: SOCP Test results with $t = 50, \epsilon = 0.25$, runtime=2000

t=	50	e=	0.2	runtime=	2000						
Instance feature							Facility status				
#	n	m	K/D	capacity%	transportation%	waiting%	open	Runtime (s)	LB	UB	Error
p1	10	50	1.74	20.81%	69.32%	9.87%	10	143.01	1266.28	1266.28	0.00%
p2			1.74	20.81%	69.32%	9.87%	10	144.14	1266.28	1266.28	0.00%
p3			1.74	20.81%	69.32%	9.87%	10	142.46	1266.28	1266.28	0.00%
p4			1.74	20.81%	69.32%	9.87%	10	144.28	1266.28	1266.28	0.00%
p5			1.37	20.81%	69.32%	9.87%	10	140.97	1266.28	1266.28	0.00%
p6			1.37	20.81%	69.32%	9.87%	10	142.58	1266.28	1266.28	0.00%
p7			1.37	20.81%	69.32%	9.87%	10	141.86	1266.28	1266.28	0.00%
p8			1.37	20.81%	69.32%	9.87%	10	143.20	1266.28	1266.28	0.00%
p9			2.06	20.81%	69.32%	9.87%	10	142.50	1266.28	1266.28	0.00%
p10			2.06	20.81%	69.32%	9.87%	10	141.93	1266.28	1266.28	0.00%
p11			2.06	20.81%	69.32%	9.87%	10	142.23	1266.28	1266.28	0.00%
p12			2.06	20.81%	69.32%	9.87%	10	142.55	1266.28	1266.28	0.00%
p13	20	50	2.77	25.16%	61.13%	13.72%	16	108.28	1822.51	1822.51	0.00%
p14			2.77	25.16%	61.13%	13.72%	16	106.18	1822.51	1822.51	0.00%
p15			2.77	25.16%	61.13%	13.72%	16	109.30	1822.51	1822.51	0.00%
p16			2.77	25.16%	61.13%	13.72%	16	106.49	1822.51	1822.51	0.00%
p17			2.80	25.16%	61.13%	13.72%	16	106.90	1822.51	1822.51	0.00%
p18			2.80	25.16%	61.13%	13.72%	16	105.59	1822.51	1822.51	0.00%
p19			2.80	25.16%	61.13%	13.72%	16	105.48	1822.51	1822.51	0.00%
p20			2.80	25.16%	61.13%	13.72%	16	105.20	1822.51	1822.51	0.00%
p21			3.50	25.16%	61.13%	13.72%	16	106.43	1822.51	1822.51	0.00%
p22			3.50	25.16%	61.13%	13.72%	16	107.07	1822.51	1822.51	0.00%
p23			3.50	25.16%	61.13%	13.72%	16	106.63	1822.51	1822.51	0.00%
p24			3.50	25.16%	61.13%	13.72%	16	106.78	1822.51	1822.51	0.00%
p25	30	150	4.12	-	-	-	-	2045.22	1310.12	-	-
p26			4.12	25.96%	49.77%	24.28%	17	2083.08	1310.12	1441.73	10.05%
p27			4.12	25.96%	49.77%	24.28%	17	2119.91	1310.12	1441.73	10.05%
p28			4.12	25.96%	49.77%	24.28%	17	2103.89	1310.12	1441.73	10.05%
p29			3.03	-	-	-	-	2045.87	1310.12	-	-
p30			3.03	-	-	-	-	2053.46	1310.12	-	-
p31			3.03	25.96%	49.77%	24.28%	17	2042.97	1310.12	1441.73	10.05%
p32			3.03	-	-	-	-	2038.23	1310.12	-	-
p33			4.04	-	-	-	-	2031.69	1310.12	-	-
p34			4.04	25.96%	49.77%	24.28%	17	2047.22	1310.12	1441.73	10.05%
p35			4.04	25.96%	49.77%	24.28%	17	2074.29	1310.12	1441.73	10.05%
p36			4.04	-	-	-	-	2040.79	1310.12	-	-
p37			6.06	-	-	-	-	2053.30	1310.12	-	-
p38			6.06	25.96%	49.77%	24.28%	17	2038.56	1310.12	1441.73	10.05%
p39			6.06	-	-	-	-	2050.21	1310.12	-	-
p40			6.06	-	-	-	-	2041.64	1310.12	-	-
p41	10	90	2.12	26.21%	58.03%	15.75%	7	4064.67	664.16	714.15	7.53%
p42	20	80	4.99	-	-	-	-	2102.29	710.45	-	-
p43	30	70	8.28	-	-	-	-	682.07	785.39	-	-
p44	10	90	1.76	21.66%	68.23%	10.11%	8	2290.38	979.42	1236.66	26.26%
p45	20	80	4.14	-	-	-	-	2041.84	944.95	-	-
p46	30	70	7.10	23.51%	36.67%	39.82%	8	2006.82	940.02	941.77	0.19%
p47	10	90	1.76	4.48%	85.07%	10.46%	1	2093.62	965.27	1394.63	44.48%
p48	20	80	4.06	19.08%	59.75%	21.17%	7	2012.72	983.19	1180.67	20.09%
p49	30	70	7.08	19.64%	43.60%	36.76%	7	2526.11	1006.80	1020.26	1.34%
p50	10	100	1.89	26.97%	58.07%	14.96%	7	2097.31	710.77	835.35	17.53%
p51	20	100	3.98	-	-	-	-	2081.27	726.24	-	-
p52	10	100	1.60	-	-	-	-	2014.19	1230.26	-	-
p53	20	100	3.37	22.30%	58.28%	19.42%	10	2327.46	1160.97	1223.09	5.35%
p54	10	100	1.52	19.06%	72.52%	8.43%	9	2063.21	1094.62	1483.11	35.49%
p55	20	100	3.21	4.96%	75.05%	19.98%	1	2356.95	833.11	1355.20	62.67%

Table B.4: SOCP Test results with $t = 50, \epsilon = 0.2$, runtime=2000

t=	25	e=	0.25	runtime=	2000						
Instance feature					Cost structure		Facility status				
#	n	m	K/D	capacity%	transportation%	waiting%	open	Runtime (s)	LB	UB	Error
p1	10	50	1.74	19.79%	73.25%	6.96%	10	118.02	1196.69	1196.69	0.00%
p2			1.74	19.79%	73.25%	6.96%	10	115.54	1196.69	1196.69	0.00%
p3			1.74	19.79%	73.25%	6.96%	10	118.75	1196.69	1196.69	0.00%
p4			1.74	19.79%	73.25%	6.96%	10	115.34	1196.69	1196.69	0.00%
p5			1.37	19.79%	73.25%	6.96%	10	115.60	1196.69	1196.69	0.00%
p6			1.37	19.79%	73.25%	6.96%	10	115.96	1196.69	1196.69	0.00%
p7			1.37	19.79%	73.25%	6.96%	10	115.46	1196.69	1196.69	0.00%
p8			1.37	19.79%	73.25%	6.96%	10	119.32	1196.69	1196.69	0.00%
p9			2.06	19.79%	73.25%	6.96%	10	116.80	1196.69	1196.69	0.00%
p10			2.06	19.79%	73.25%	6.96%	10	117.02	1196.69	1196.69	0.00%
p11			2.06	19.79%	73.25%	6.96%	10	116.54	1196.69	1196.69	0.00%
p12			2.06	19.79%	73.25%	6.96%	10	116.36	1196.69	1196.69	0.00%
p13	20	50	2.77	24.25%	65.89%	9.86%	16	78.92	1690.77	1690.77	0.00%
p14			2.77	24.25%	65.89%	9.86%	16	78.36	1690.77	1690.77	0.00%
p15			2.77	24.25%	65.89%	9.86%	16	77.54	1690.77	1690.77	0.00%
p16			2.77	24.25%	65.89%	9.86%	16	78.42	1690.77	1690.77	0.00%
p17			2.80	24.25%	65.89%	9.86%	16	77.47	1690.77	1690.77	0.00%
p18			2.80	24.25%	65.89%	9.86%	16	77.84	1690.77	1690.77	0.00%
p19			2.80	24.25%	65.89%	9.86%	16	78.79	1690.77	1690.77	0.00%
p20			2.80	24.25%	65.89%	9.86%	16	81.45	1690.77	1690.77	0.00%
p21			3.50	24.25%	65.89%	9.86%	16	79.93	1690.77	1690.77	0.00%
p22			3.50	24.25%	65.89%	9.86%	16	77.40	1690.77	1690.77	0.00%
p23			3.50	24.25%	65.89%	9.86%	16	77.33	1690.77	1690.77	0.00%
p24			3.50	24.25%	65.89%	9.86%	16	77.76	1690.77	1690.77	0.00%
p25	30	150	4.12	26.16%	57.93%	15.90%	20	2987.44	1071.58	1571.98	46.70%
p26			4.12	26.16%	57.93%	15.90%	20	2997.72	1071.58	1571.98	46.70%
p27			4.12	26.16%	57.93%	15.90%	20	2996.72	1071.58	1571.98	46.70%
p28			4.12	26.16%	57.93%	15.90%	20	2993.39	1071.58	1571.98	46.70%
p29			3.03	26.16%	57.93%	15.90%	20	2998.12	1071.58	1571.98	46.70%
p30			3.03	26.16%	57.93%	15.90%	20	2990.18	1071.58	1571.98	46.70%
p31			3.03	26.16%	57.93%	15.90%	20	2984.04	1071.58	1571.98	46.70%
p32			3.03	26.16%	57.93%	15.90%	20	2977.91	1071.58	1571.98	46.70%
p33			4.04	26.16%	57.93%	15.90%	20	2980.82	1071.58	1571.98	46.70%
p34			4.04	26.16%	57.93%	15.90%	20	2990.24	1071.58	1571.98	46.70%
p35			4.04	26.16%	57.93%	15.90%	20	2986.74	1071.58	1571.98	46.70%
p36			4.04	26.16%	57.93%	15.90%	20	2979.21	1071.58	1571.98	46.70%
p37			6.06	26.16%	57.93%	15.90%	20	2981.76	1071.58	1571.98	46.70%
p38			6.06	26.16%	57.93%	15.90%	20	2973.91	1071.58	1571.98	46.70%
p39			6.06	26.16%	57.93%	15.90%	20	2978.76	1071.58	1571.98	46.70%
p40			6.06	26.16%	57.93%	15.90%	20	2997.29	1071.58	1571.98	46.70%
p41	10	90	2.12	-	-	-	-	2044.70	627.54	-	-
p42	20	80	4.99	31.31%	41.77%	26.92%	8	1166.52	606.35	619.14	2.11%
p43	30	70	8.28	31.97%	29.91%	38.12%	11	485.82	638.59	655.76	2.69%
p44	10	90	1.76	20.85%	71.67%	7.48%	9	2130.53	926.75	1114.21	20.23%
p45	20	80	4.14	-	-	-	-	2195.38	839.18	-	-
p46	30	70	7.10	26.29%	42.17%	31.54%	9	2049.01	792.18	792.64	0.06%
p47	10	90	1.76	-	-	-	-	2097.35	971.91	-	-
p48	20	80	4.06	-	-	-	-	3230.95	940.74	-	-
p49	30	70	7.08	22.42%	49.09%	28.49%	8	2473.64	859.31	877.49	2.12%
p50	10	100	1.89	27.27%	61.22%	11.51%	8	2600.98	647.55	724.10	11.82%
p51	20	100	3.98	29.69%	45.26%	25.06%	7	2070.02	627.81	631.91	0.65%
p52	10	100	1.60	16.83%	76.50%	6.67%	7	2086.32	1176.45	1248.49	6.12%
p53	20	100	3.37	21.66%	63.82%	14.52%	9	2178.28	1053.13	1147.78	8.99%
p54	10	100	1.52	15.39%	78.19%	6.42%	5	2067.41	1086.32	1168.50	7.56%
p55	20	100	3.21	-	-	-	-	2693.14	829.07	-	-

Table B.5: SOCP Test results with $t = 25, \epsilon = 0.25$, runtime=2000

t=	25	e=	0.2	runtime=	2000						
Instance feature					Cost structure		Facility status				
#	n	m	K/D	capacity%	transportation%	waiting%	open	Runtime (s)	LB	UB	Error
p1	10	50	1.74	20.94%	73.00%	6.06%	10	174.48	1203.69	1203.69	0.00%
p2			1.74	20.94%	73.00%	6.06%	10	162.87	1203.69	1203.69	0.00%
p3			1.74	20.94%	73.00%	6.06%	10	161.91	1203.69	1203.69	0.00%
p4			1.74	20.94%	73.00%	6.06%	10	163.70	1203.69	1203.69	0.00%
p5			1.37	20.94%	73.00%	6.06%	10	161.70	1203.69	1203.69	0.00%
p6			1.37	20.94%	73.00%	6.06%	10	161.82	1203.69	1203.69	0.00%
p7			1.37	20.94%	73.00%	6.06%	10	169.62	1203.69	1203.69	0.00%
p8			1.37	20.94%	73.00%	6.06%	10	165.47	1203.69	1203.69	0.00%
p9			2.06	20.94%	73.00%	6.06%	10	161.24	1203.69	1203.69	0.00%
p10			2.06	20.94%	73.00%	6.06%	10	159.69	1203.69	1203.69	0.00%
p11			2.06	20.94%	73.00%	6.06%	10	162.28	1203.69	1203.69	0.00%
p12			2.06	20.94%	73.00%	6.06%	10	161.18	1203.69	1203.69	0.00%
p13	20	50	2.77	24.62%	66.14%	9.24%	16	88.77	1690.98	1690.98	0.00%
p14			2.77	24.62%	66.14%	9.24%	16	93.06	1690.98	1690.98	0.00%
p15			2.77	24.62%	66.14%	9.24%	16	88.21	1690.98	1690.98	0.00%
p16			2.77	24.62%	66.14%	9.24%	16	87.89	1690.98	1690.98	0.00%
p17			2.80	24.62%	66.14%	9.24%	16	91.31	1690.98	1690.98	0.00%
p18			2.80	24.62%	66.14%	9.24%	16	88.12	1690.98	1690.98	0.00%
p19			2.80	24.62%	66.14%	9.24%	16	182.05	1690.98	1690.98	0.00%
p20			2.80	24.62%	66.14%	9.24%	16	376.42	1690.98	1690.98	0.00%
p21			3.50	24.62%	66.14%	9.24%	16	88.97	1690.98	1690.98	0.00%
p22			3.50	24.62%	66.14%	9.24%	16	82.66	1690.98	1690.98	0.00%
p23			3.50	24.62%	66.14%	9.24%	16	84.21	1690.98	1690.98	0.00%
p24			3.50	24.62%	66.14%	9.24%	16	86.36	1690.98	1690.98	0.00%
p25	30	150	4.12	-	-	-	-	2906.22	1126.02	-	-
p26			4.12	23.60%	59.24%	17.16%	17	2110.01	1107.15	1456.78	31.58%
p27			4.12	-	-	-	-	2864.30	1126.02	-	-
p28			4.12	-	-	-	-	2869.42	1126.02	-	-
p29			3.03	-	-	-	-	2869.64	1126.02	-	-
p30			3.03	-	-	-	-	2868.17	1126.02	-	-
p31			3.03	-	-	-	-	2873.82	1126.02	-	-
p32			3.03	-	-	-	-	2862.50	1126.02	-	-
p33			4.04	-	-	-	-	2891.67	1126.02	-	-
p34			4.04	-	-	-	-	2880.13	1126.02	-	-
p35			4.04	-	-	-	-	2878.99	1126.02	-	-
p36			4.04	-	-	-	-	2874.26	1126.02	-	-
p37			6.06	-	-	-	-	2879.16	1126.02	-	-
p38			6.06	-	-	-	-	2871.57	1126.02	-	-
p39			6.06	23.60%	59.24%	17.16%	17	2130.96	1107.15	1456.78	31.58%
p40			6.06	23.60%	59.24%	17.16%	17	2198.14	1107.15	1456.78	31.58%
p41	10	90	2.12	21.17%	65.46%	13.37%	6	2691.14	603.96	779.24	29.02%
p42	20	80	4.99	-	-	-	-	2046.28	585.74	-	-
p43	30	70	8.28	37.46%	32.48%	30.06%	10	714.67	597.90	623.82	4.33%
p44	10	90	1.76	16.96%	74.41%	8.63%	7	3023.25	908.73	1206.96	32.82%
p45	20	80	4.14	24.67%	58.40%	16.93%	8	2115.25	816.94	861.59	5.47%
p46	30	70	7.10	29.38%	45.77%	24.86%	8	2067.37	753.27	754.31	0.14%
p47	10	90	1.76	-	-	-	-	2060.50	864.55	-	-
p48	20	80	4.06	17.26%	67.70%	15.04%	7	2129.96	868.53	1038.56	19.58%
p49	30	70	7.08	22.51%	53.91%	23.58%	8	2216.51	815.64	839.41	2.91%
p50	10	100	1.89	-	-	-	-	2066.52	644.10	-	-
p51	20	100	3.98	28.65%	50.71%	20.64%	8	2036.93	591.91	706.62	19.38%
p52	10	100	1.60	15.61%	76.54%	7.85%	8	3279.09	1155.37	1326.29	14.79%
p53	20	100	3.37	20.00%	66.34%	13.66%	9	2346.49	1013.70	1143.46	12.80%
p54	10	100	1.52	15.85%	77.12%	7.03%	6	2056.18	994.82	1096.95	10.27%
p55	20	100	3.21	5.51%	83.38%	11.10%	1	2029.28	688.77	1219.78	77.10%

Table B.6: SOCP Test results with $t = 25, \epsilon = 0.2$, runtime=2000

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