Optimal Trading Strategies for an Asset with Disordered Return

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

We explore various trading strategies from a mathematical and practical perspective. Using a geometric Brownian motion with a disorder to model asset price bubbles, we apply this model to multiple periods and explore the trading strategies on real market data (S&P500, NASDAQ and SSE Composite). We find that the mathematical model is sucessful in predicting large market events such as the 2008 crisis, however fails to generate gains over times of healthy market growth. Three practical models are also presented and show that beneficial trading can be performed using some simple deterministic assumptions. The mathematical and practical methods are compared.

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Thanks to Bin Li for being a great and supportive supervisor and being a pleasure to work with. Thanks to family friends and especially Laura for giving up her weekends with me to get this puppy done. Yahoo!

Dedication

All Done!

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Chapter 1

Introduction

1.1 Motivation

When entering into a long position of a stock, the main motivation for the investor is to maximize their gains buy collecting dividends and reselling this asset at a higher price than they purchased. Ideally this increased value should be able to cover any transaction costs incurred by executing the trade, as well as being the best use of one's capital. It is always possible to come out of a position with a nominal benefit, such as simply investing your money in a fixed instrument, or even keeping your money in a low-interest savings account. The real challenge is to determine the optimal way to deploy your capital in the market in order to maximize your return subject to an acceptable risk. While the concept is simple to understand, the vastness and complexity of the financial world makes this problem non-trivial.

What makes this problem, and all problems in financial statistics, even more challenging is the fact that financial markets are not purely statistical systems. In a truly statistical system we may apply all of the mathematical machinery to a given problem and be very confident that the assumptions we make will never be violated in our lifetimes. In finance, however, there are a finite amount of market participants, some of which carry a large amount of capital which can cause a market to change its behaviour dramatically thereby violating statistical assumptions. Not only this, other global factors play a very large role in moving the markets. If we wish to truly predict tomorrow's market prices we must also consider the global economy, politics, weather, or even hype driven by social media. A problem of this size is of course not feasible to handle in its entirety, so there has been much theoretical and practical research done into trying to understand and predict these markets.

Some strategies try to predict when a stock has hit its rock bottom and purchase it, then wait for the stock to rise once again and resell at a higher price. This could take advantage of certain periodic trends in seasonality (yearly), fiscal (quarterly) or even political (4 year presidential terms) sectors [31, 17] . These types of methods are known as market timing strategies and when the investor does not plan on repurchasing the asset, this is known as a Buy and Hold type strategy. The problem with these types of methods are that it is difficult to determine the difference between a temporary market correction or an actual transition to a bear/bull market. Another strategy is to predict if we are entering a bull market from a bear market and take a position only after we are confident that the market will continue to go up. While we lose out on buying at the lowest lows and selling at the highest highs, we are able to be more confident in our decision. It makes it much less likely that we purchase a stock only to see it fall lower when we were expecting a rise in price. A related method is to determine and predict the presence of an asset bubble. As an example we can look at the dot-com crash in 2000 (See Figure 1.1)

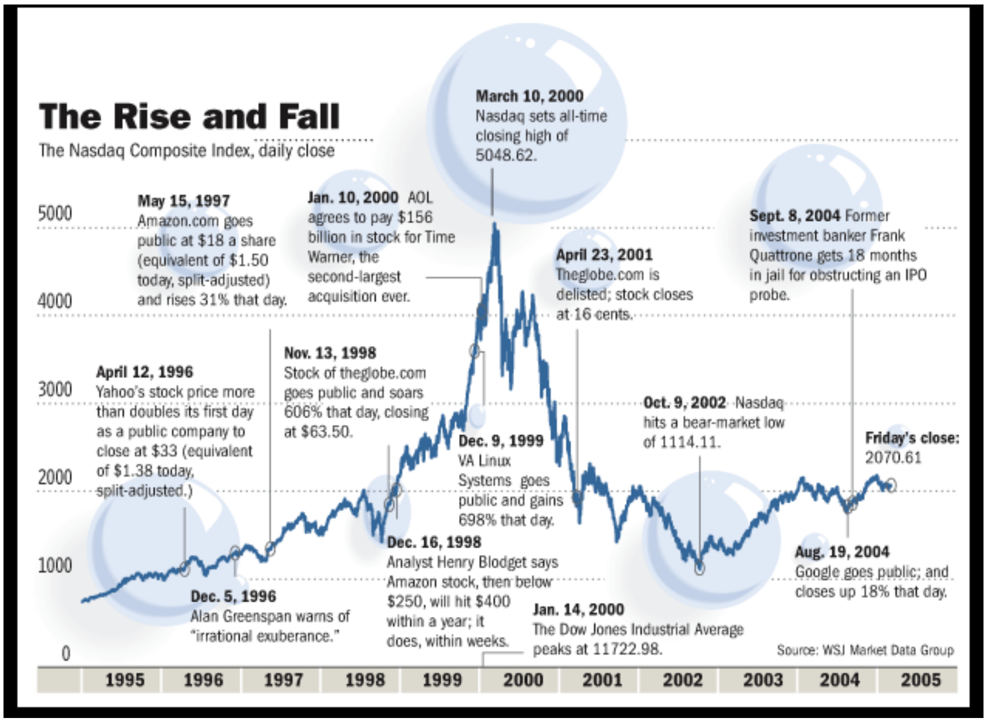

Figure 1.1: Diagram showing the steps leading up to the dot-com market collapse in the NASDAQ. As you can see, a large speculative growth was followed by a steep decline in market price. (Tom Fleerackers: From https://flatworldbusiness.wordpress.com/flat-education/previously/web-1-0-vs-web-2-0-vs-web-3-0-a-bird-eye-on-the-definition/dotcom-bubble/)

A speculative bubble of this nature occurs where market participants believe that the stock price will increase simply because it is increasing. The effect when many people buy the stock is to raise its price even higher which of course benefits those who own it, and also entices more investors to buy. This process continues until some percentage of investors sell off their position which drops the price, triggering the rest of the market rapidly trying to liquidate their position before they lose all of the gains they have made. This occurs until the value of the underlying companies and assets start to align with the market price and finally the bubble ends. The overall effect is a very large increase in price followed by a rapid decrease. The goal in this strategy is to determine whether a drop in the asset's price will trigger a collapse, or if it is just a temporary fluctuation.

The overall goal of all the above strategies is to take advantage of market fluctuations and adjustments in order to maximize financial gain. In the next section we will highlight some the practical and mathematical methods used to determine the optimal times to buy and sell and also take a look at the three markets we will study which are the S&P500, NASDAQ and SSE Composite index.

1.2 Background

In order to study financial markets much work has been done in applying mathematical models in order to quantify and predict the behaviour of certain stocks and markets. Sometimes these methods are simply done to predict a distribution of possible asset paths in order to calculate fair market pricing, while in other cases it may be done to generate a model for finding the probability that a market will transition from a bull to a bear market. In the latter case, these mathematical models rely on parameterization methods to quantify some of their inputs based on available historical data, as well as methods and definitions to even determine what classifies as a bull and bear market.

In general we can categorize trading strategies into two general groups which are buy-low-sell-high (BLSH) and momentum strategies. These two methods can be subdivided into those that look at a finite or infinite trading horizon and those that allow for multiple trades over time or a single decision. In a buy-low-sell-high strategy, the goal is to predict the minimum of the stock price in some range, and sell it when a maximum is hit at some other time. There are many examples of this in the literature considering a variety of different models, time horizons and strategies. For example in [43, 28] the authors consider a mean reversion type model to characterize the price of the asset and determine a set of stopping time for both buying and selling the asset in order to maximize return. Other papers modify the optimization condition to try and minimize the difference between selling price and the global maximum [6, 5, 27], or for the terminal wealth [8].

In momentum strategies, the goal is not to try to predict when we hit a global minimum or maximum, rather we aim to determine if we are currently in a market upswing or in a bull market. There has been much research into the reality of momentum in asset price, and it is accepted that when a market is truly in an upswing, it will continue to do well for a certain amount of time [19, 20, 35]. The goal here is to first determine if we are in a bull market given past asset data, and then determine the optimal time to sell under some condition. As in the BLSH strategies there is a plethora of different optimization conditions and models. For example Dai et. al. [7] looks at optimally determining if we are currently in a bear or bull market using a regime switching type model while works by Shiryaev et. al. [34, 33] lays down a model for minimizing the relative difference between the current asset value and global maximum when closing a momentum trade, and goes on to apply such a model using real market data [36].

Another very interesting model was developed by Ekstrom and Lindberg [11] where they model the asset behaviour as a Brownian motion with a disorder at some unknown time θ wherein after that time the return of the asset switches from positive to negative. This model was meant to study asset bubbles which carry this behaviour and are somewhat non-standard events. Later in this thesis we will look at their ([11]) derivation and extend it to handle multiple trading events and apply this method to historical market data for backtesting. In order to parameterize the model (and similarly in the cases of other methods) we rely on characterizing historical data based on trend analysis. The problem of trend analysis itself has been tackled in various ways in the literature such as Markov chain methods [21, 26], and what are known as dating algorithms [25]. Aside from these models there are a wide array of different perspectives and strategies available in the literature (See. [16, 42, 18, 44]).

While mathematical models provide us with the tools to analyze the markets, they are also limited based on their own inherent assumptions. For example, how to treat the return, volatility and which market model to use, under what time horizon and in what subsection of the global market does a given model actually apply to. This is why there are also various practical models in the literature that seek to predict markets based on different factors and assumptions. In a paper by Jagadeesh et. al. [19] the authors look at a strategy where we simply purchase stocks that are performing well and sell those performing poorly. In performing this simple model they are able to generate gains over a 3-12 month window. In a model by Lleo et. al. [24] they focus on macroeconomic factors in order to predict market crashes/bubbles, and Egan [10] explores the autocorrelation in market prices.

It is clear that both the mathematical and practical methods are important for different reasons all working toward the same overall goal. In this thesis I plan to extend the work of [11] and apply it to the S&P500, NASDAQ and SSE Composite Index. The challenges will be determining how to calculate the parameters on a limited dataset. I will also compare three more practical trading strategies aimed to maximize returns through tracking the market and applying a deterministic set of trading rules based on recent asset performance. I will then compare the results and highlight the benefits of both the mathematical and practical approaches.

In the next section I will outline some characteristics and details about the datasets we will be using for our historical testing. Chapter 2 will outline and extend the model proposed by Ekstrom and Lindberg [11]. In this section there will also be a discussion on the difficulty of parameterization of such models and results will be given. In Chapter 3 we will describe and explore the results of the three practical models used to generate returns that beat the market on average. Chapter 4 will contrast and compare the practical methods as well as the mathematical results. Chapter 5 will give a recap and conclude.

1.3 Historical Market Data

In order to apply our trading strategies we must first choose some datasets to test on. For this we decided to choose the S&P500, NASDAQ and SSE Composite indices. The reason for our choice of these three has to do with their correlations, market environments and behaviours. When applying our strategies it is important to understand the dataset in order to properly interpret the results. In this section we give a brief history of each index as well as some key metrics and known behaviour and details. We will also compare between the indices and explore the correlations and market composition difference between them. With this in mind we will be able to explain better the results we attain. For all calculations done for the remainder of this thesis, market data was extracted from historical records from Yahoo! Finance [13, 12, 14].

1.3.1 S&P500 Index

The Standard and Poor's 500 (S&P 500) is a large American stock index comprising the top 500 companies with the largest market capitalization. These stocks are a mix between the NYSE and NASDAQ markets, and therefore it is not surprising that there is a large correlation between the movements of the S&P500 and NASDAQ in our results. The market capital in total is around 20 trillion making it a large-cap index and it is comprised mostly of Information Technology (19.8%), Financials (16.8%) and Health Care (15.6%) sectors [2, 23]. Historically the average annualized returns have been around 10% with significant changes in times of market bubbles and crashes. This index is a rather diverse cross section of the U.S. market.

1.3.2 NASDAQ

The National Association of Securities Dealers Automated Quotations (NASDAQ) is the world's second largest stock exchange second only to the New York Stock Exchange (NYSE). Stared in 1972 the NASDAQ currently has a market capitalization of 8.5 trillion with annualized returns around 10% [41]. In addition to the market crash of 2008, the NASDAQ also played host to the dot-com crash in 2000 where a large speculative bubble caused a loss of market capital of around 5 trillion (more than half of the current total capitalization) by 2002 [15]. Since this market has suffered from two large collapses in recent history, it makes it a useful dataset to capture this type of behaviour.

1.3.3 SSE Composite

The SSE Composite is an index composed of the top stocks listed on the Shanghai Stock Exchange (SSE) comprised largely of government run institutions [1]. As such it is heavily tied to the economy of China as a whole, and in this way is significantly different than the S&P500 and the NASDAQ which includes non-government corporations. There are a few other interesting characteristics of this index as well. Firstly, the opening times of the SSE and American exchanges are opposite to one another, also the SSE has also had a few significant crashes. The first crash occurred in 2007 where the index fell 9% due to rumours of a rise in interest rates as well as a lockdown on speculative trading[4]. The most recent actually started in June of this year (2015) and is ongoing. In just three weeks the index fell by 30% forcing the government to take action such as restricting trading. [30, 9, 40]

Chapter 2

Mathematical Methods and Approach

In the standard Black-Scholes model it is assumed that the expected return as well as the volatility of a stock are constant over time [37, 38]. Even though this simplified model has been used widely in both academia and industry, it is unable to capture many features that are present in true markets. The issue of relevance for us is the lack of a time dependent return $\mu(t)$. By allowing the return to take on a time dependence we have more flexibility to model a situation where the markets may have some sort of trend reversal. This model is known as a Brownian motion with a disorder. Under this model we can write the differential of the asset price as,

$$dS_t = \mu(t)S_t dt + \sigma S_t dW_t \tag{2.1}$$

where σ is the volatility of the process and W_t is a standard Brownian motion. We also define the expected return to be,

$$\mu(t) = \mu_+ - (\mu_+ - \mu_-)\mathbf{1}_{\{t<\theta\}} \tag{2.2}$$

where $\mu_- < r < \mu_+$ are the bear, risk-free and bull market returns, respectively. The time θ is called the disorder time which is a random time in which a market trend change occurs. In general the time θ is not directly observable and it is independent of the current stock price S_t, so the challenge is to try and determine if we have crossed this time given only the filtration generated by the observable stock path $\mathcal{F}_t^S$. With this model, the goal is to try and maximize the discounted return of the stock over some time. The maximization condition on this interval can be written as,

$$V = \sup_{\tau \in \mathcal{T}} \mathbf{E} e^{-r\tau} S_\tau \tag{2.3}$$

where τ is some stopping time in the set of all possible stopping times $\mathcal{T}$. In order to fully define the maximization, the probability of the disorder time must be established. We let $\mathcal{P}(\theta = 0) = \pi$ which means that there is a non-zero probability π that the disorder has occurred at time zero, and

$$\mathcal{P}(\theta > t | \theta > 0) = e^{-\lambda t} \tag{2.4}$$

which defines the probability that the disorder has not occurred yet given that it did not already occur before we entered the interval. This characterization will become important when we evaluate multiple optimization intervals, since we will have more information as to the state of the process before entering the next interval.

The challenge here is that the disorder time is not directly observable, however by defining the posterior probability process $\Pi_t = \mathcal{P}(\theta \leq t | \mathcal{F}_t^S)$ and using results from filtering theory [29] we can show that Π_t satisfies the differential

$$d\Pi_t = \lambda(1 - \Pi_t)dt - \omega \Pi_t (1 - \Pi_t) d\bar{W}_t, \; \Pi_0 = \pi \tag{2.5}$$

where $\omega = (\mu_+ - \mu_-)/\sigma$ and

$$d\bar{W}_t = \frac{1}{\sigma} \left(\frac{dS_t}{S_t} - (\Pi_t \mu_- + (1 - \Pi_t)\mu_+) dt \right). \tag{2.6}$$

We can combine the two above expressions to represent Π_t directly in terms of the observable price process S_t. Plugging in Eq. 2.6 into Eq. 2.5 and combining dt and dS_t terms we end up at

$$d\Pi_t = f(\Pi_t, t)dt - \frac{\omega \Pi_t (1 - \Pi_t)}{\sigma S_t} dS_t \tag{2.7}$$

where

$$f(\Pi_t, t) = \frac{\omega}{\sigma} \Pi_t^2 (1 - \Pi_t) \mu_- - \frac{\omega}{\sigma} \Pi_t (1 - \Pi_t)^2 \mu_+ + \lambda(1 - \Pi_t) \tag{2.8}$$

When we look at some numerical results we will convert Equation 2.7 into a discretized form in order to convert the historical stock prices into the probability process. The authors in [11] go on to apply a generalized Girsanov's theorem to change the optimization problem from a two dimensional to one dimensional problem. Under the new measure, the optimization conditions are the solutions to a free boundary problem. The ordinary differential equation admits analytical solutions, and the free boundary become a simple constant which is the solution to the equation

$$2\int_0^\infty e^{-at}t^{(b+\gamma-3)/2}(1+Bt)^{(\gamma-b+1)/2}dt \tag{2.9}$$

$$= (\gamma-b+1)(1+B)\int_0^\infty e^{-at}t^{(b+\gamma-1)/2}(1+Bt)^{(\gamma-b-1)/2}dt \tag{2.10}$$

where

$$a = \frac{2\lambda}{\omega^2} \tag{2.11}$$

$$b = \frac{2}{\omega}\left(\frac{\lambda}{\omega} - \sigma\right) \tag{2.12}$$

$$c = \frac{2(\lambda + r - \mu_+)}{\omega^2} > 0 \tag{2.13}$$

$$\gamma = \sqrt{(b-1)^2 + 4c} \tag{2.14}$$

and the lower bound is

$$B > \frac{\mu_+ - r}{r - \mu_-} \tag{2.15}$$

They go on to show that given the boundary B, the optimal stopping time is defined as

$$\tau* = \inf\left\{t \geq 0 : \Pi_t \geq \frac{B}{B+1}\right\} \tag{2.16}$$

Using this stopping time the value function can be solved to be

$$V = \begin{cases} \frac{(1+B)\psi(\phi)}{(1+\phi)\psi(B)} S_0, & \text{if } \phi < B \\ S_0, & \text{otherwise} \end{cases} \tag{2.17}$$

where $\phi = \pi/(1-\pi)$ and S_0 are the initial states of the probability and price process and

$$\psi(\phi) = \int_0^\infty e^{-at} t^{(b+\gamma-3)/2} (1+\phi t)^{(\gamma-b+1)/2} dt \tag{2.18}$$

For a more detail derivation of the above results please see [11]. The above treatment takes care of the offloading of an asset, however we can also find the optimal time to go short an asset. In a similar treatment we find the solution for A, the optimal level to short by solving the equation,

$$2\int_0^\infty e^{-at} t^{(b+\gamma-3)/2} (1+At)^{(\gamma-b+1)/2} dt \tag{2.19}$$

$$= (\gamma - b + 1)(1+A)\int_0^\infty e^{-at} t^{(b+\gamma-1)/2} (1+At)^{(\gamma-b-1)/2} dt \tag{2.20}$$

where

$$a = \frac{2\lambda}{\omega^2} \tag{2.21}$$

$$b = \frac{2}{\omega}\left(\frac{\lambda}{\omega} + \sigma\right) \tag{2.22}$$

$$c = \frac{2(\lambda + r - \mu_+)}{\omega^2} \tag{2.23}$$

$$\gamma = \sqrt{(b-1)^2 + 4c} \tag{2.24}$$

The optimal stopping time becomes,

$$\tau_A * = \inf\left\{ t \geq 0 : \Pi_t \geq \frac{A}{A+1} \right\} \tag{2.25}$$

With these optimal stopping conditions we are able to determine the optimal sell and buying time using historical datasets.

2.1 Method Application

Now that we are able to optimally sell and go short based on the above method, we are in a position to apply the strategy to a real asset. We start with the assumption that we currently hold a single share of the asset, and when we sell of this asset, we immediately go short one share. We then determine the best time to get off the short position (purchase and give back asset) and at the same time we do this we also use the proceeds to purchase as many shares of the assets as possible. We then repeat this process.

We begin by selecting a calibration dataset which is needed to adjust our free parameter; which in this case is the decay time λ from Eq. 2.4. In our case we will be calibrating on the S&P500, NASDAQ and SSE Composite from the year 2000-2005. On this set we adjust the free parameter λ such that the mean relative return (MRR) defined by,

$$MRR = \frac{1}{n} \sum_{i=1}^{n} \frac{w_i}{S_i} \tag{2.26}$$

where n is the number of trade days in the time period, w_i/S_i is the current wealth/stock price at time i; is maximized. We then use this adjusted parameter and apply the calibrated method to the dataset from year 2005-2015.

To perform these calculations we must first solve for the two boundaries (A, B) shown above. This is simply done by applying a numerical integration scheme to both sides of Equations (2.10,2.20) and perform a Newton iteration until we reach equality. After this is done we must convert the stock price process S_t into the probability process Π_t. Since we are dealing with a discrete dataset we must first discretize Equation 2.7. This discretization is shown as,

$$\begin{aligned} \Pi_{t+1} - \Pi_t &= f(\Pi_t, t)\Delta t - \frac{\omega \Pi_t (1 - \Pi_t)}{\sigma S_t}(S_{t+1} - S_t) \\ \Pi_{t+1} &= \Pi_t + f(\Pi_t, t)\Delta t - \frac{\omega \Pi_t (1 - \Pi_t)}{\sigma}(\frac{S_{t+1}}{S_t} - 1) \end{aligned} \tag{2.27}$$

In the above equation we use the previous stock state S_t, probability state Π_t and the current stock state S_{t+1} to determine the next value in the probability path. This is then applied to the total stock path with the initial condition that $\Pi_0 = 0$. An example of this is shown in Figure 2.1.

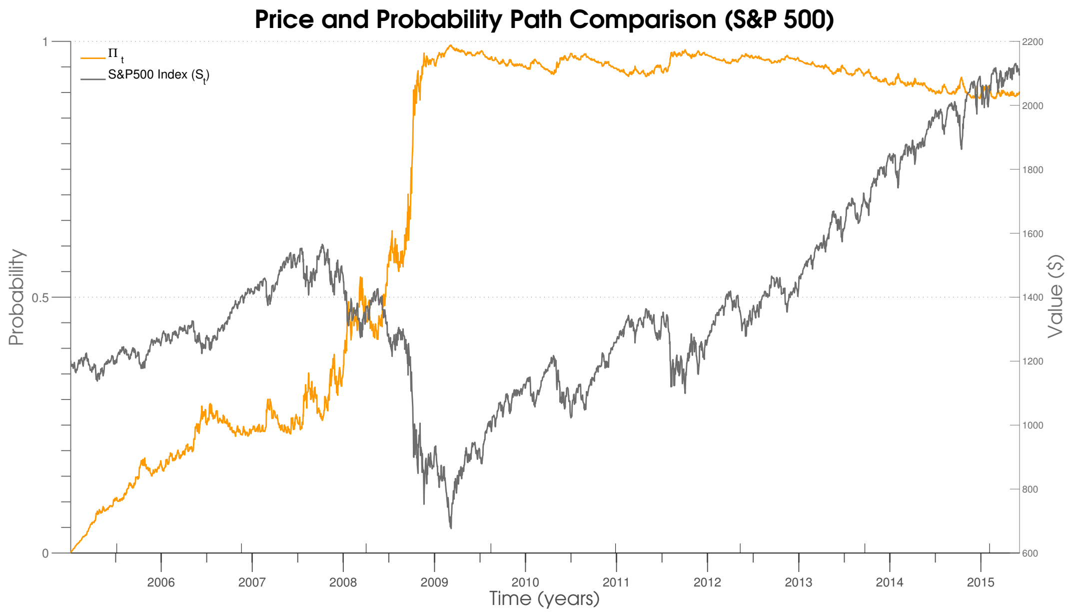

Figure 2.1: The stock price path and corresponding probability path for the S&P500 from 2005-2015. The probability path Π_t rapidly increases to one during the 2008 financial crisis, which in turn makes it very likely that a disorder has occurred.

Now that we are able to calculate the boundaries (A, B) and the probability path Π_t we can apply our strategy. By finding the time τ_1, when the probability path crosses the sell boundary ($\Pi_t > B/(B+1)$) we issue a sell order and go short an asset. We then reset the probability path at τ_1 to zero and start again ($\Pi_{\tau_1} = 0$), this time looking for the optimal time to buy and close our short position when $\Pi_t > A/(A+1)$ which occurs at some time $\gamma_1 > \tau_1$. We continue this until the end of the dataset in 2015. In the end we are left with a stock purchase process and wealth process which we can use to infer the benefits and downfalls of the strategy.

2.2 Market Results

In this section we use the above treatment on the S&P500, NASDAQ and SSE Composite index to see how our strategy performs. We choose for the S&P500 and NASDAQ a rate or return of $\mu = 10\%$ and a volatility of $\sigma = 20\%$ which is in line with historical market data when calculating the average year over year return and volatility. The SSE Composite is roughly double in both respects giving $\mu = 20\%$ and $\sigma = 40\%$. In our treatment we assume that when the market goes into a downturn that the rate of return simply flips sign, so in the case of the SSE Composite the bearish sections of the market we assume have a return of $\mu_- = -20\%$.

Taking historical market data from 2000-2005 we chose a range of possible λ values which acts as the only free parameter in our model. The challenge here is to find the value of λ that optimizes the MRR metric in the calibration dataset. Since the numerical calculations were relatively computationaly cheap, we simply selected parameters in the range of $\lambda \in [1, 10]$ and selected the values that maximized the MRR. These values are shown in Table 2.2 for the three datasets. At this point we should make note that one of the major downfalls/challenges of this method is the selection of the parameter λ. In the model we are trying to capture the probability that a momentum trade persists through time until the bubble bursts. Since these are unlikely events in history, a proper calibration is difficult. To highlight this fact we take the NASDAQ as our example. We will see later that the results of this method seem to fail when applied to the NASDAQ even though the S&P500 (which is highly correlated) performs rather well. This is due to a difficulty in determining the best λ from the available market data.

	S&P 500	**NASDAQ**	**SSE Composite**
λ	2.7	7.3	4.0

Table 2.1: Calibrated λ parameter for the S&P500, NASDAQ and SSE Composite indices. The results show that the NASDAQ has a significantly higher calibrated λ value which may explain its poor performance with this method.

As a matter of fact it is clear in Table 2.2 that the optimal λ value for the NASDAQ is much larger than that of the S&P500 and the SSE Composite which both did well in the final results.

The calibrated parameters were used to calculate the optimal boundaries for selling and purchasing an asset (B, A) and in turn used to generate a trading strategy. The results for the three datasets are summarized in Table 2.2.

	S&P 500	NASDAQ	SSE Composite
$\frac{A}{A+1}$	0.6241	0.6089	0.5656
$\frac{B}{B+1}$	0.4178	0.4065	0.4642
$\frac{A}{A+1} - \frac{B}{B+1}$	0.2063	0.2024	0.1014
MRR	1.3939	DEFAULT	1.4542

Table 2.2: Results for the S&P500, NASDAQ and SSE Composite indices. The results show that only the NASDAQ met with financial ruin, while the other indices had a rather large MRR return.

As you can see, both the S&P500 and SSE Composite were able to produce significant gains on average over a buy and hold method (where one purchases the asset and holds onto it). As discussed above, when the method was applied to the NASDAQ, we met with financial ruin when our wealth eventually dwindled to zero. It is interesting to compare the difference between the buy and sell boundaries $\frac{A}{A+1} - \frac{B}{B+1}$. When comparing the NASDAQ and S&P500 the difference is almost identical except that the absolute values in the S&P500 are shifted upward by about 0.01. When comparing this to the SSE Composite, the Buy-Sell band is twice as tight. The significance of this is that a tighter band leads to a large amount of trades in that asset which is exemplified in the following figures. Note that in our results we do not include the cost of transactions, however we can infer the importance by the number of trades performed.

Figures 2.2,2.3,2.4 show the price S_t, stock held and wealth paths when applying the strategy to the S&P500, NASDAQ and SSE Composite. The figures led us to investigate in further detail the benefits and downsides of the trading strategy.

Starting with the S&P500 shown in Figure 2.2 we can see that the major benefit is the result of the sale of the asset just before the 2008 crisis and the prompt repurchasing in 2009 leading to a large gain over this period.

Figure 2.2: Asset price, stock held and wealth processes using our strategy of the S&P500. A large initial gain is due to the optimal sale of the asset before the 2008 financial crisis, followed by a prompt repurchasing of the asset in 2009.

The benefit of this strategy prevails until a sudden drop in asset price mid-2011 which forced the method to sell off the asset and go short one share. When this occurred, the continued rise in the asset was not sudden enough to trigger the closing of the short position until the very end of 2015 leading to heavy losses in the later part of the dataset. Overall, the method generated excellent returns for the majority of the time, especially in large market swings.

We can now contrast these results with that of the NASDAQ shown in Figure 2.3.

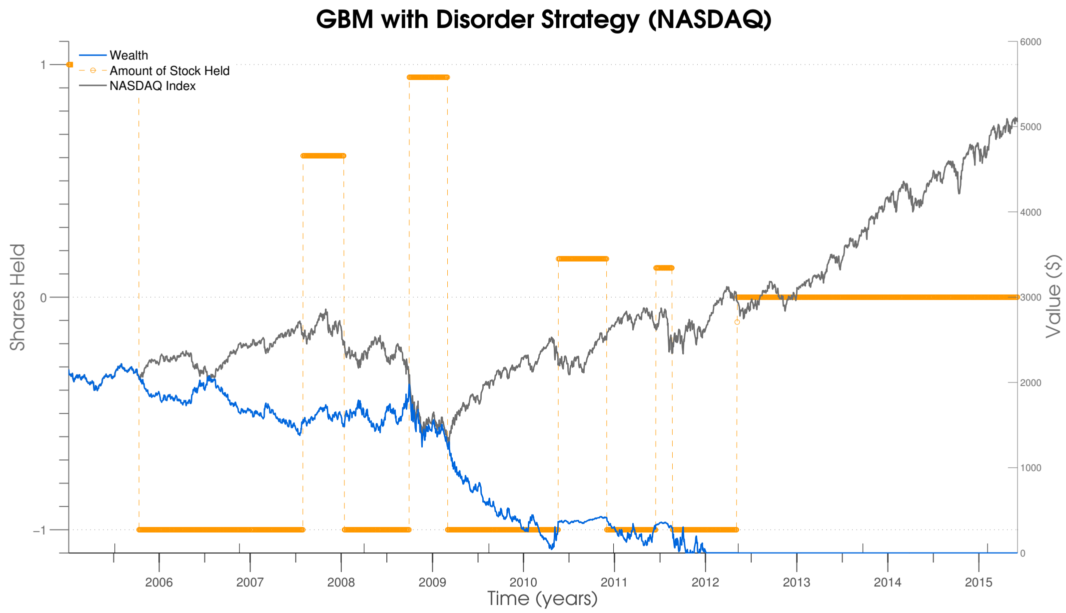

Figure 2.3: Asset price, stock held and wealth processes using our strategy of the NASDAQ. Due to poor calibration the application of the trading strategy eventually leads to a series of poor trades and financial ruin in 2012.

In this case we see the effect of poor parameterization on the execution of a trading strategy. We noted earlier that the NASDAQ has a decay parameter of $\lambda = 7$, which was more than twice the size as the very similar S&P500. This difference leads to terribly poor performance due to an early decision to exit the market at any small change in market conditions. Essentially this means that the strategy was overly sensitive to a temporary market correction early on which lead to a reduction in the ability to purchase assets and eventually lead to entering a short position in mid-2009 when the recession was undergoing a recovery. This massive drop in wealth eventually lead to a default, which we define as the point where personal wealth becomes zero. At this point we stop the trading strategy as we do not allow the borrowing of assets without claimable wealth.

For the SSE Composite index, the situation is rather different. Due to the smaller difference between the optimal buy and sell levels that were calculated, there was a higher propensity to trade. Shown in Figure 2.4 we can clearly see that the number of trades is much larger than that of the last two datasets.

Figure 2.4: Asset price, stock held and wealth processes using our strategy of the SSE Composite. The optimal trading boundaries are much closer for the SSE Composite which leads to a larger number of trades. The sale of the asset and short positioning during the large market drop generates a soaring wealth.

Once again we see that the early sale and shorting of the asset around 2008 leads to a large generation of wealth, however in the current case, there exists a larger number of profitable trades at later dates. For example from 2011 and onward there is a pattern of bear markets and temporary corrections which flatten out the asset price. The method was able to take advantage of these smaller changes and generate vast wealth until around 2014 where the beginning of China's speculative bubble begins.

2.3 Discussion

It is clear from the presented data that the mathematical model presented by [11] can indeed be used to generate significant gains during times of unbounded market growth. In the S&P500 and SSE Composite cases, the ability to exit out of the 2008 crisis early, and repurchase at optimal times lead to vast wealth growth over that period. This scenario is exactly what the model was designed to capture and it did so well. The major issue however is when the market returns to a non-volatile state of healthy growth. In all cases the method was not able to continue generating wealth after the market corrections finished. This result is perhaps not at all surprising due to the model and its assumptions.

In the model we specify that the bull expected rate of return (μ_+) and the bear rate of return (μ_-) were symmetric. This assumption holds true in times where speculation drives the market and leads to large gains and fast drops. However, in times of healthy market growth, these assumptions do not hold. It may be the case that in these times the model would need to be changed to allow for a non-symmetric disorder to occur, or necessitate the use of a completely different market model.

Another difficulty as highlighted in the dreadful performance of the strategy applied to the NASDAQ is that poor parameterization can cause the method to become useless. This issue is inherent with any models that need calibration to market data. Since we chose calibration datasets that contained market crashes, the parameterization was pushed to capture these events, which partially lead to the poorer performance during strong market growth.

Chapter 3

Application to Real Markets

In this section we focus on developing a variety of trading strategies which will be applied and calibrated to real market data from the S&P500, NASDAQ and SSE Composite indices. In each of these models we use market data from 2000-2005 as our calibration data set and optimize the various parameters for each model. We then use these parameters to apply our strategy from 2005-present. For each model we assume an all or nothing type trading strategy, where we can only hold either a position in stock or a risk-free asset that gives some return ($r = 2\%$). At the start of the method we assume that we currently hold a single share in the market, and we must make a trading decision in some set period of time to either continue to hold the stock or sell the stock and invest all returns at the risk-free rate. If at some later point in time we decide to repurchase some stock, we must extract all wealth from the risk-free account and purchase W_t/S_t shares where W_t defines our wealth process and S_t is the stock process. Note that at the initial time ($t = 0$) we hold one share and therefore $W_0 = S_0$.

3.1 Relative Return Strategy

When an investor is presented with the opportunity to buy or sell an asset they must take many things into account. For example, what is their current position, how risky is the asset and how has this asset been performing in the past in response to certain situations. In order to characterize this type of logic, the Relative Return strategy compares the historical returns between two differently size time windows Δt_h and Δt_p. We define Δt_h as the size of the historical time window which acts as a baseline of comparison to $\Delta t_p < \Delta t_h$ which characterized the asset performance in the past. The idea is to look at how the stock has done overall and compare this to how it did in the last period. If we calculate the mean return in these two windows as μ_h and μ_p, we can say that if $\mu_p > \mu_h$, the recent performance of the asset is better than the historical expectation which means the stock is trending upwards and it is time to buy the next time we have that option. In the case that $\mu_p < \mu_h$ the asset has recently been performing poorly and if we currently have a position in the asset it may be time to liquidate into a risk-free investment. In the current set up we can also set some limit on what we define to be a significant change. Define $r^*_\mu = (\mu_p - \mu_h)/\mu_h$ as the relative return boundary which can be adjusted and optimized for a given asset or index. Using this set up we can write a simple algorithm in pseudo code to solve this problem outlined in Algorithm 1.

In the relative mean algorithm we first define the historical date range Δt_h and the date range of the partition Δt_p. These are specified as the amount of days to look back upon. For example, if we made the presumption that the return of an asset over the last one year is correlated in some way to the return over the previous month, which would tell us something about the current trend of the asset, we would assign $\Delta t_h = 365$ and $\Delta t_p = 30$ in calendar days. We then calculate the return over these two horizons as μ_h and μ_p. The relative value between these two returns r^t_μ is calculated at each time step and compared to our defined significance boundaries r^+_μ and r^-_μ. If $r^t_\mu \in [-\infty, r^-_\mu]$ then we expect that the current performance of the asset is poor compared to its history, and we should make a trading decision accordingly. If $r^t_\mu \in [r^+_\mu, \infty]$ the asset has recently been performing well and this may indicate an upswing in the asset. In the case $r^t_\mu \in (r^-_\mu, r^+_\mu)$ then the performance is in line with past performance and we cannot extract much new information.

In the algorithm we define S_t, a_t and w_t as the stock price, asset state and wealth state at time t. The asset state is the total number of shares that we currently hold at time t. Whenever a trading decision is made we either liquidate all of our assets at the market value and place it into a risk-free holding, or convert all of our holdings into purchasing the maximal amount of assets we can buy at that time.

In the end we are left with the wealth and stock position over time which can then be analyzed and optimized based on some given measure (terminal wealth, mean time wealth etc.).

We apply our method to the S&P500, NASDAQ and SSE Composite indexes using historical adjusted closing data from 2000 to the present day. To begin we must make some assumptions in our model. First we assume that at the start of any given trading day we have the ability to make a trading decision. This condition is equivalent to setting $\Delta t_p = 1$ in days. Next we select three historical date ranges to compare our last daily

Algorithm 1 Relative Mean Algorithm

```
procedure runRMA
    Set $\Delta t_p$, $\Delta t_h$
    Set $r_\mu^+$, $r_\mu^-$
    for $t \le t_{end}$ do
        Calculate $\mu_p^t$, $\mu_h^t$
        $r_\mu^t = (\mu_h^t - \mu_p^t)/\mu_h^t$
        if $r_\mu^t$ > $r_\mu^+$ then
            $j = t$
            if $a_t$==0 then
                $a_t = w_t/S_t$
                for $j \le t + \Delta t_p$ do
                    $a_j = a_{j-1}$
                    $w_j = a_j * S_j$
            else
                for $j \le t + \Delta t_p$ do
                    $a_j = a_{j-1}$
                    $w_j = a_j * S_j$
        else if $r_\mu^t$ < $r_\mu^*$ then
            $j = t$
            if $a_t$==0 then
                for $j \le t + \Delta t_p$ do
                    $w_j = w_{j-1} * (1 + r)$
                    $a_j = 0$
            else
                for $j \le t + \Delta t_p$ do
                    $w_j = w_{j-1} * (1 + r)$
                    $a_j = 0$
        else
            $j = t$
            if $a_t$==0 then
                for $j \le t + \Delta t_p$ do
                    $w_j = w_{j-1} * (1 + r)$
                    $a_j = 0$
            else
                for $j \le t + \Delta t_p$ do
                    $a_j = a_{j-1}$
                    $w_j = a_j * S_j$
        $t = t + \Delta t_p$
```

return (μ_p) to the current historical return (μ_h) which is rolled over each day. By rolled over it is meant that the mean return is recalculated as we move forward in time such that we are always calculating μ_h on the last Δt_h days. For this study we choose three different ranges which are $\Delta t_h = \{250, 90, 5\}$ representing roughly a year, four months and one week of trading. The last parameter in this method is the relative return cutoff value r^*_μ. Since this is a somewhat free parameter we calibrate our model using the closing data from the years 2000-2005 optimizing the value defined by,

$$MRR = \frac{1}{n}\sum_{i=1}^{n}\frac{w_i}{S_i} \tag{3.1}$$

where n is the number of trade days in the time period, w_i/S_i is the current wealth/stock price at time i. This definition is the average ratio of the wealth compared to the asset price over time. This is a proxy for comparison between our strategy and a buy and hold type method. The parameter is then applied to the data from 2006-2015. The reason for choosing to optimize on average wealth is because as an investor, it is important to be able to fulfill obligations at any given time. This is a kind of liquidity condition that is place on the investor. If we chose to optimize on the maximum wealth at the end of the year 2015 then it would be possible that our wealth could have been very low for the majority of time and simply jumped at the end which of course limits our value over time.

Ideally, in order to calculate our optimal relative return (r^*_μ) which maximizes the average wealth on the calibration set we would use something like a Newton iteration method to find the root of the derivative of the wealth as a function of r_μ. However when working with actual historical data the outcome is not always a smooth function, therefore such methods are not appropriate for this problem. In light of this we simply use a set of equally spaced points of $r_\mu \in [0, 10]$ and choose the maximizing parameter. Once this is found we can apply our parameter set on the testing set and look at the results. Below is a table showing the sets of calibrated parameters for SP500, NASDAQ and the SSE Composite indices at the three Δt_h values (Table 3.1).

	S&P 500			**NASDAQ**			**SSE Composite**		
Δt_h **(days)**	250	90	5	250	90	5	250	90	5
r^*_μ	0.05	8.4	8.75	6.56	9.88	1.35	2.67	7.27	3.07

Table 3.1: Calibration parameters for the three indices and three historical baseline (Δt_h). The above results use $\Delta t_p = 1$ and the calibration is done over the range of $r_\mu \in [0, 10]$. The results vary greatly between the indices tested and don't show a specific trend.

Using the calibrated parameters, we now apply the same strategy starting from 2005. This is essentially an investor living in 2005 and using all of the historical data available in order to make a decision for the next day. An extension to this would be to recalibrate the model every Δt_c days in order to keep up with current trends and have the most recent calibrations, however in the current example we only calibrate once and proceed until 2015. In Figures 3.1, 3.3 and 3.5 we see the three outcomes of the strategy using the calibrated parameters in Table 3.1.

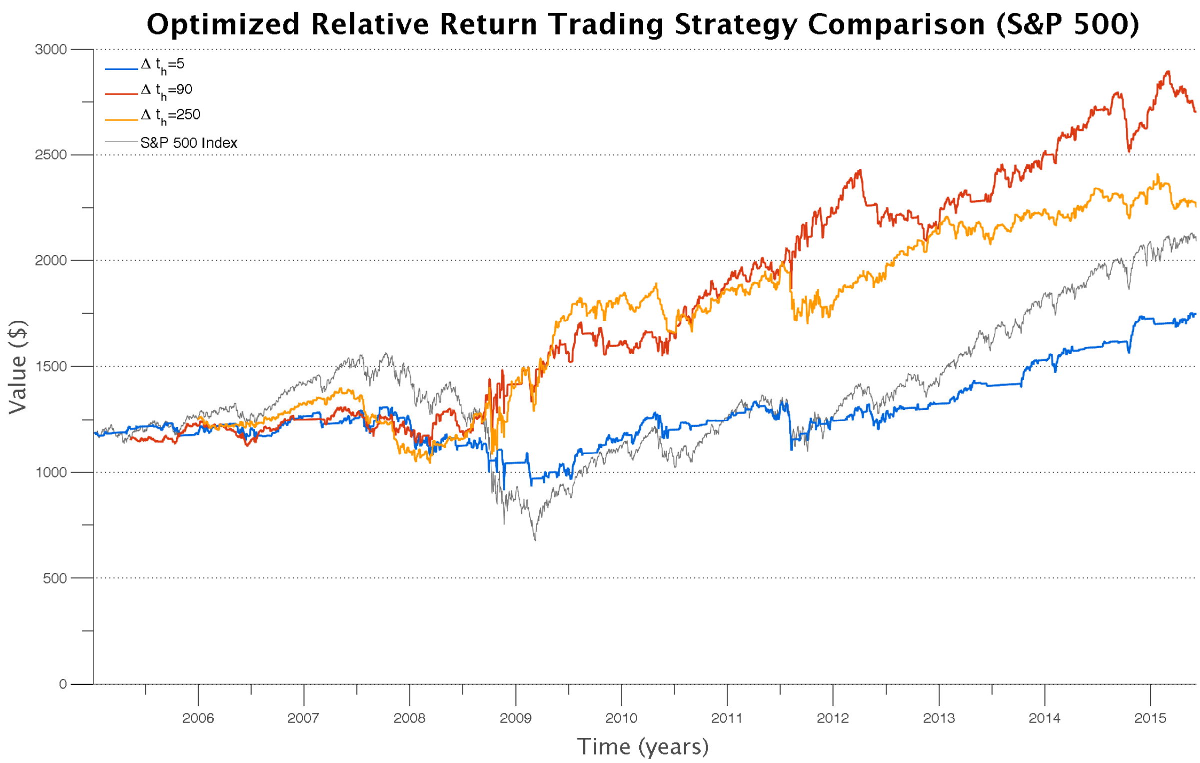

Figure 3.1: Application of the Relative Return strategy on the S&P500 index starting from the year 2005. The grey shows the index value over time which is equivalent to a buy and hold type of strategy. In the case of $\Delta t_h = 90, 250$ we see a larger gain over time signifying that we are able to beat the market in these situations. This also indicates that short term fluctuations (for example 5 days or less) are not indicative of future outcomes. The details of these results are discussed later in this work.

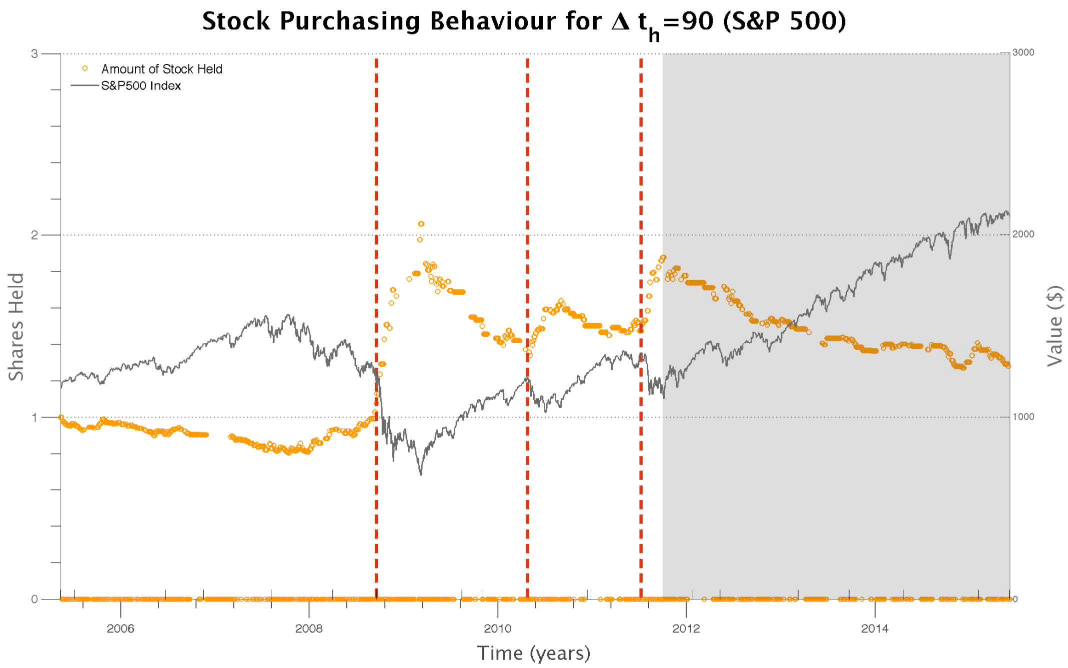

Figure 3.2: Stock holdings for the Relative Return strategy on the S&P500 index. The plot shows a comparison between the index price and the amount shares that have been purchased at a given time. The red bars indicate three times where the index began a downtrend, and the corresponding response in the trading strategy. In times of downturn we are able to repurchase shares at a discount and resell at a later time when the stock recovers. In the grey region the index is doing well and our methods performance is impacted negatively.

When applying the strategy to the S&P500 index we see that we are able to beat the market when using a reference time of $\Delta t_h = 90, 250$. In Table 3.2 we show the MRR defined in Equation 3.1 as well as the number of transactions that occurred for each Δt_h.

	S&P 500		
Δt_h **(days)**	250	90	5
# of Transaction	1266	682	224
MRR (Mean Return Ratio)	1.1394	**1.2637**	0.9249
	NASDAQ		
Δt_h **(days)**	250	90	5
# of Transaction	798	578	917
MRR (Mean Return Ratio)	1.2066	**1.2176**	0.9080
	SSE Composite		
Δt_h **(days)**	250	90	5
# of Transaction	1025	511	441
MRR (Mean Return Ratio)	0.7247	0.8055	**1.3453**

Table 3.2: This table shows the results of the Relative Return strategy applied to the S&P500, NASDAQ and SSE Composite indices for $\Delta t_h = \{250, 90, 5\}$. In each case we tabulate the number of transactions that occurred in the 10 years of trading as well as the MRR. The bolded MRR's highlight the parameter set that resulted in the best performance.

The value in this strategy is shown when there are times of large market drops, for example around the 2008 crisis. In this situation when looking at the history for the last 3 months to a year it is clear that the market is in a downtrend and it is time to exit our position. However it is also able to capture rebounds in this downtrend and recover some of the value lost in previous periods. Taking the results for $\Delta t_h = 90$ we can look at the stock position over time of the applied strategy (Fig. 3.2). In the graph we see that the major value we attain comes from the cheap purchasing of stock in situations of an economic downturn. Not only during the 2008 crisis do we see this, we also see it during the partial market dips in 2010 and 2012 (Red bars).

While this cheap repurchasing during times of financial downturn allows us to turn a profit when the stock recovers, in times of stability and growth the method is not able to gain an advantage leading to a dwindling of the stock position as shown in the greyed section past 2012. In light of this, a method that ties the probability of being in a large downturn with this strategy would enable us to apply our method only in the appropriate time frames. Another issue with this method is that there is a large amount of transactions occurring which would drive our profits down. If we were to apply our method taking into account a transaction cost, the calibration would have given us a much larger cutoff value r_μ^* in order to reduce the number of transactions we perform.

Given that the S&P500 is a rather large index with many institutional investors our method may not be best suited for this type of environment.

We can also apply the method to the NASDAQ to see how the strategy performs. In Figures 3.3 and 3.4 we see a very similar strategy being applied as the S&P500.

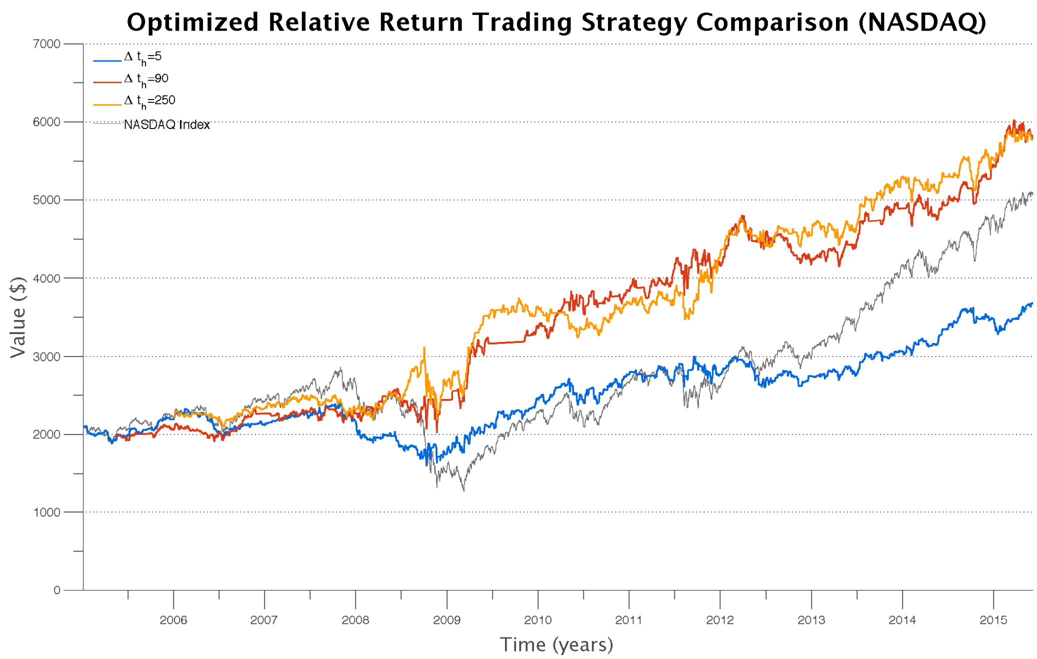

Figure 3.3: Application of the Relative Return strategy on the NASDAQ index starting from the year 2005. The grey shows the index value over time which is equivalent to a buy and hold type of strategy (ignoring dividends). In the case of $\Delta t_h = 90, 250$ we see a larger gain over time signifying that we are able to beat the market in these situations. This also indicates that short term fluctuations (for example 5 days or less) are not indicative of future outcomes. The details of these results are discussed later in this work.

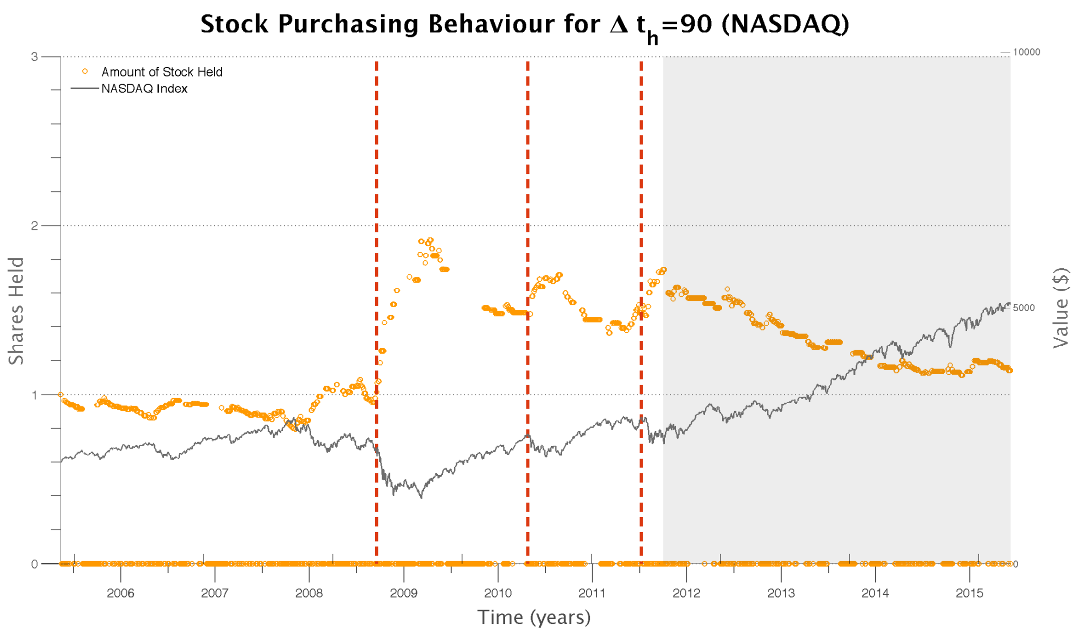

Figure 3.4: Stock holdings for the Relative Return strategy on the NASDAQ index. The plot shows a comparison between the index price and the amount of shares that have been purchased at a given time. The red bars indicate three times where the index began a downtrend, and the corresponding response in the trading strategy. In times of downturn we are able to repurchase shares at a discount and resell at a later time when the stock recovers. In the grey region the index is doing well and our method's performance is impacted negatively.

This is very much expected due to how highly correlated these two indices are to one another (about 0.93 according to Ref [39]). In order to see how the method can be applied to a different type of market we investigate next the SSE Composite index.

Recently the SSE Composite Index has been in a large market downturn and is generally a more volatile environment. Below we look at the method as applied to the SSE Composite and contrast and compare the outcome. Figure 3.5 and the corresponding Table 3.2 shows that our method applied to the SSE Composite Index only shows an increase in performance when the historical trending is applied to $\Delta t_h = 5$. Shown in the accompanying table, in this case the strategy produced an average value 135% of the index with somewhat low trading volume. What this seems to indicate is a view on how the collective market participants act in certain market conditions. As said earlier, the S&P500 index has a larger number of institutional investors, while the SSE Composite is heavily burdened by private investors whom are more weary of and sensitive to market conditions. In the case of an institutional investor, a certain position is usually not a speculative investment in the hopes of making a quick profit, instead the positions held are meant to fulfill a larger more diversified portfolio. This means that a drop in the market in the short term doesn't result in a quick response, either because the current position as a whole is still a benefit, or because of certain liquidation issues if the company is holding a vast position in a certain investment. In the case of a private investor, the responsiveness is much quicker since liquidation of one's position is easy, and the position is not always a part of a larger portfolio.

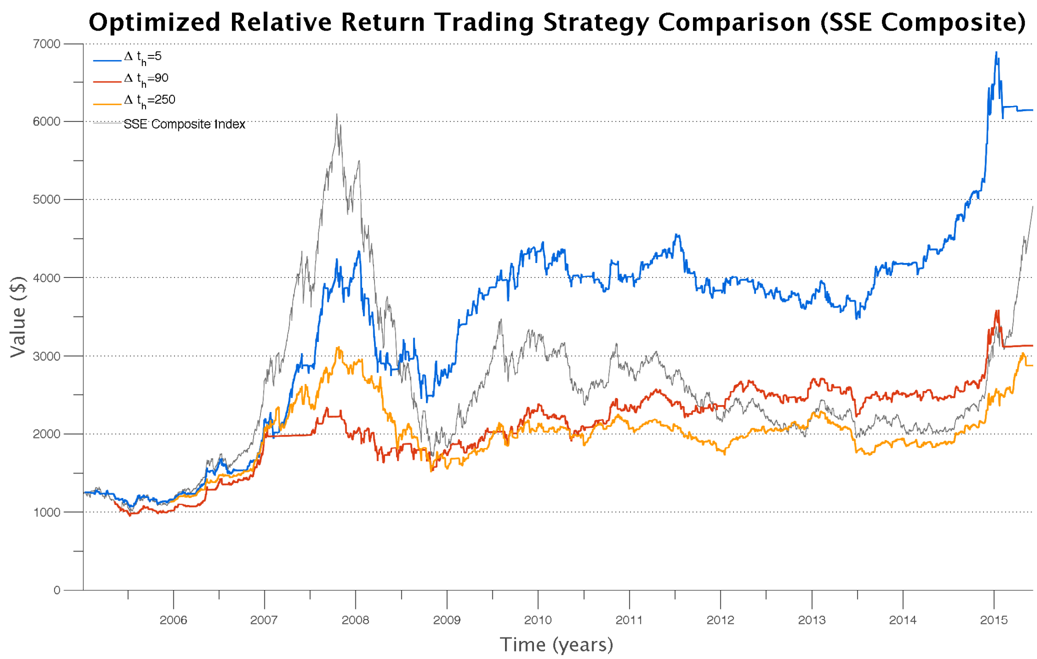

Figure 3.5: Application of the Relative Return strategy on the SSE Composite index starting from the year 2005. The grey shows the index value over time which is equivalent to a buy and hold type of strategy. In the case of $\Delta t_h = 5$ we see a larger gain over time signifying that we are able to beat the market in these situations. The short term outlook on the market indicates that the market dynamics in the SSE are very different from the S&P 500 possibly due to the composition of institutional and private investors in each market.

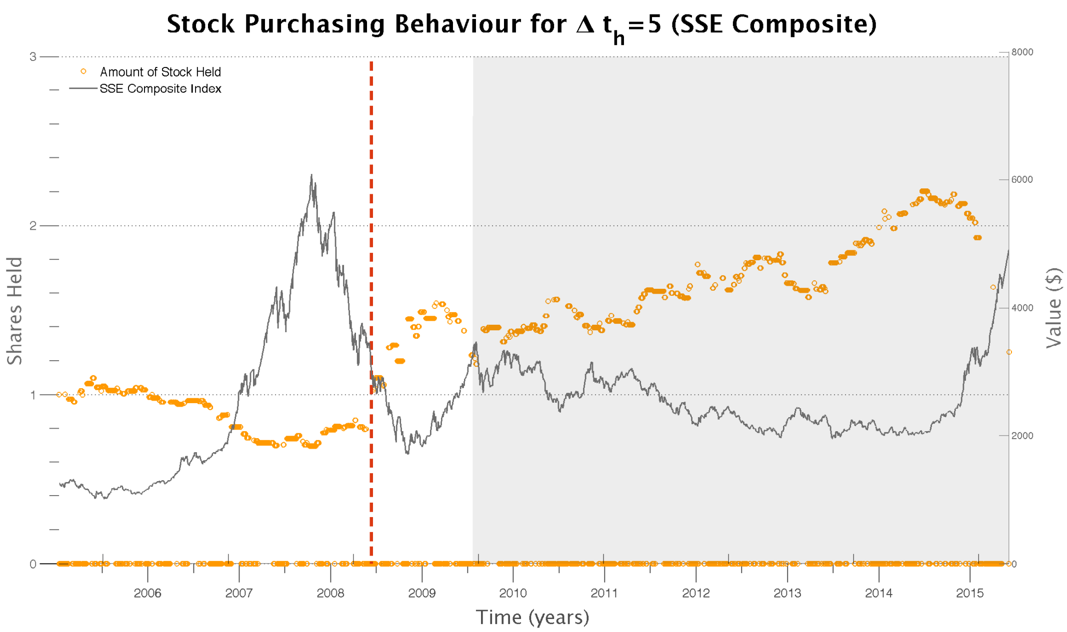

Figure 3.6: Stock holdings for the Relative Return strategy on the SSE Composite index. The plot shows a comparison between the index price and the amount of shares that have been purchased at a given time. The red bars indicate the time where the index began a downtrend, and the corresponding response in the trading strategy. In the grey region the index is doing poor and our methods performance is impacted for the better.

3.2 Drawdown Strategy

When trying to determine when to buy or sell an asset another aspect to look at is how large the asset fluctuations have been in recent history. While looking just at the return of an asset gives some indication of future performance, if in that same period there were large fluctuations then it is possible that the large return was perhaps an artifact of this high volatility in this period and perhaps our decisions may be effected by this. In order to capture this behaviour we define the drawdown factor as,

$$d = \frac{\mu_{\Delta t_h}}{S^+_{\Delta t_h} - S^-_{\Delta t_h}} \tag{3.2}$$

where Δt_h is the look back time, $\mu_{\Delta t_h}$ is the return over the look back time and $S^{+(-)}_{\Delta t_h}$ are the max and min stock prices in the time period Δt_h. This factor weighs the return in an asset against its lowest and highest points in the period. Therefore, in a given period Δt_h we may have $\mu_{\Delta t_h}$ be rather large, however if $S^{+}_{\Delta t_h} - S^{-}_{\Delta t_h}$ is also large, then the impact is less and d takes on a smaller value.

We employ a similar strategy to the Relative Mean algorithm (Algorithm 1) with some modifications. We first note that on every trading day we are able to make a decision based on the drawdown factor calculated in the previous period Δt_h. We also must specify a boundary on what we define as a significant factor to drive our decisions (δ_d). This is a free parameter which will be calibrated using historical data before being applied to the strategy similar to the Relative Mean method. Once again we calibrate this parameter on the data from 2000-2005 for the S&P500, NASDAQ and SSE Composite indices and apply the calibrations starting from 2005 to 2015. We use the same MRR factor to quantify the success of the method. For details on the method see Algorithm 2 below.

Starting with the calibrations of δ_d on the calibration dataset we have compiled the optimal drawdown factors in Table 3.3. The calibration was done until the number of trades went to zero and $\Delta t_h = \{250, 90, 30\}$.

	S&P 500			**NASDAQ**			**SSE Composite**		
Δt_h **(days)**	250	90	30	250	90	30	250	90	30
δ_d (10^{-6})	0.2	4.4	21.4	0	2.1	1.7	0.1	1.3	8.4

Table 3.3: Calibration of the drawdown cutoff parameters for the three indices and three historical baseline (Δt_h). This table seems to show a trend that an increase in the number of historical days Δt_h seems to cause the calibrated parameters to go to zero.

We now apply the calibrated parameters to our test datasets starting from the year 2005. Table 3.4 provides a summary of the major results of this trading strategy.

Algorithm 2 Drawdown Algorithm

procedure RUNDA
    `Set` Δt_h
    `Set` δ_d
    for $t \leq t_{end}$ **do**
        `Calculate` $\mu^t_{\Delta t_h}$
        `Calculate` $S^+_{\Delta t_h}$`,` $S^-_{\Delta t_h}$
        $d = \mu^t_{\Delta t_h}/(S^+_{\Delta t_h} - S^-_{\Delta t_h})$
        if $d > \delta_d$ **then**
            if a_t`==0` **then**
                $a_t = w_t/S_t$
                $a_{t+1} = a_t$
                $w_{t+1} = a_{t+1} * S_{t+1}$
            else
                $a_{t+1} = a_t$
                $w_{t+1} = a_{t+1} * S_{t+1}$
        else if $d < -\delta_d$ **then**
            if a_t`==0` **then**
                $w_{t+1} = w_t * (1 + r)$
                $a_{t+1} = 0$
            else
                $w_{t+1} = w_t * (1 + r)$
                $a_{t+1} = 0$
        else
            if a_t`==0` **then**
                $w_{t+1} = w_{j-1} * (1 + r)$
                $a_{t+1} = 0$
            else
                $a_{t+1} = a_t$
                $w_{t+1} = a_{t+1} * S_{t+1}$
        $t = t + 1$

	S&P 500		
Δt_h **(days)**	250	90	30
# of Transaction	11	17	21
MRR (Mean Return Ratio)	1.0827	**1.1106**	1.1026
	NASDAQ		
Δt_h **(days)**	250	90	30
# of Transaction	19	15	109
MRR (Mean Return Ratio)	0.9887	**1.0193**	0.8844
	SSE Composite		
Δt_h **(days)**	250	90	30
# of Transaction	19	25	41
MRR (Mean Return Ratio)	0.9270	**1.5908**	1.3058

Table 3.4: Number of transactions and MRR for using the Drawdown strategy with the S&P500, NASDAQ and SSE Composite indices. The bolded numbers indicate the best results for a given index. A rather large benefit is found for $\Delta t_h = 90$ in the SSE Composite result.

The results show that in all three test datasets we are able to generate a MRR value greater than one, indicating that this strategy produces more wealth than simply holding the asset. In all cases the optimal historical timeframe to calculate the drawdown parameters was $\Delta t_h = 90$. In the case of the S&P500 and NASDAQ we see very similar results. The S&P500 found an MRR of 1.1106 as compared to the NASDAQ with an MRR of 1.0193. In addition, the number of trades executed in both cases were also very close at 17 and 15 for the S&P500 and NASDAQ, respectively. The fact that the strategy produces such similar results indicates that the drawdown method taps into similar behaviours of both indices.

A major difference between the Drawdown strategy and the Relative Return is in the stock purchasing behaviour. In the Relative Return we see a large amount of transactions occurring in order to generate a good return, however in the Drawdown strategy we only execute a trade once or twice a year. In Figures 3.7 ,3.8 and 3.9, 3.10 we see the wealth and stock purchasing behaviour for the S&P500 and NASDAQ.

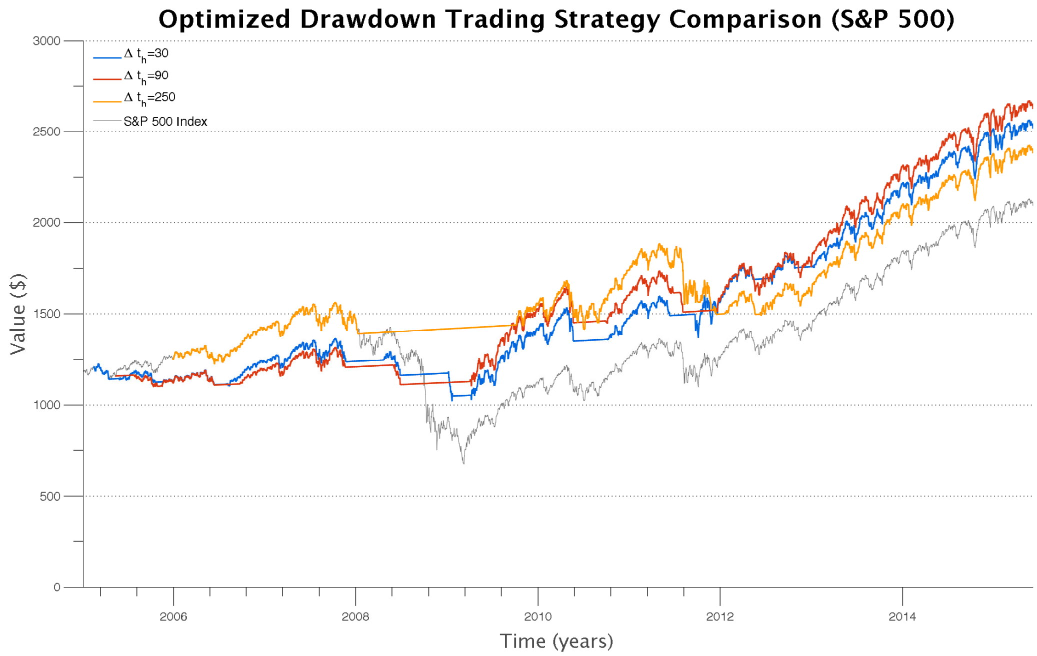

Figure 3.7: Returns generated by applying the drawdown trading strategy on the S&P500 index. In all cases the method is able to on average produce a greater return than a simple buy and hold method.

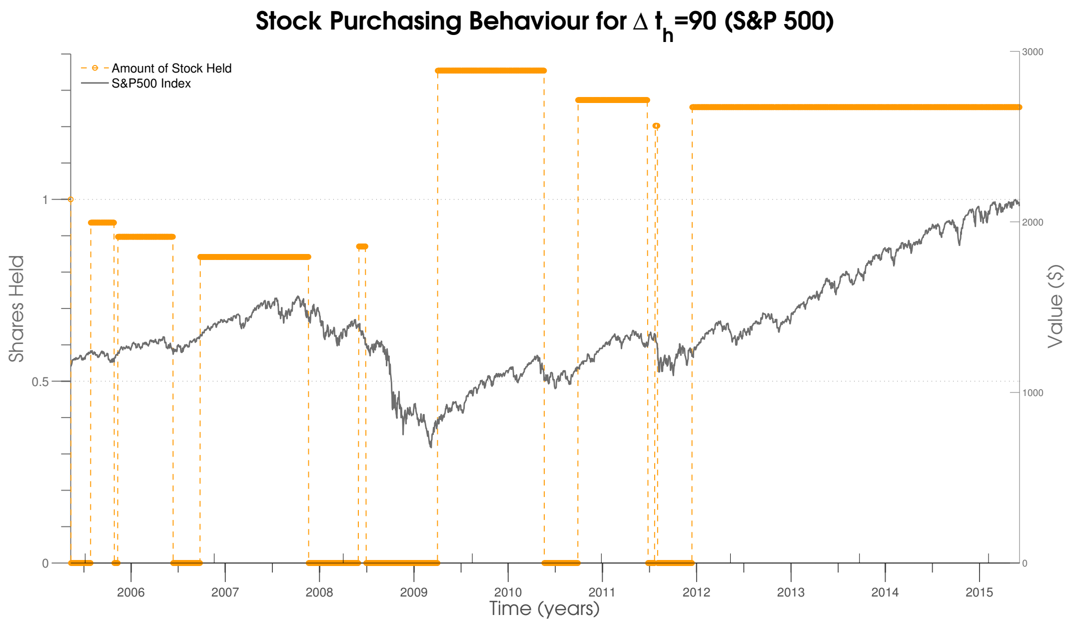

Figure 3.8: Choosing the best outcome of $\Delta t_h = 90$ we see some key features of the drawdown strategy. As opposed to the Relative Return method, the drawdown has a very small number of trades only at key times. It is clear that in times where the index is performing poorly we liquidate our position in exchange for a risk free asset.

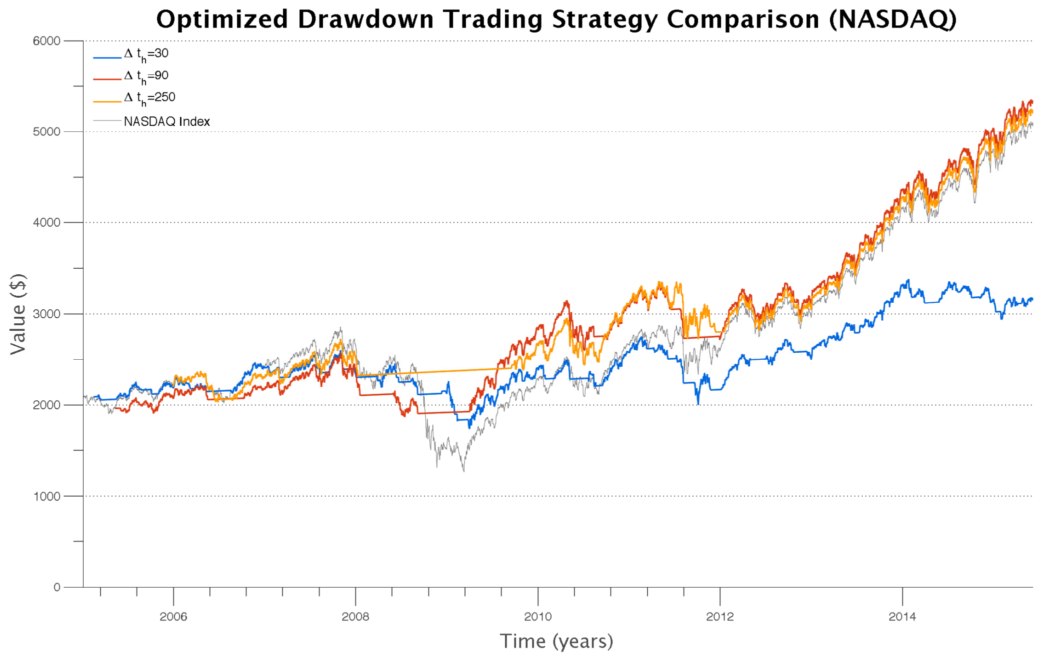

Figure 3.9: Returns generated by applying the drawdown trading strategy on the NASDAQ index. Only in the case of $\Delta t_h = 90$ do we see a benefit from this strategy.

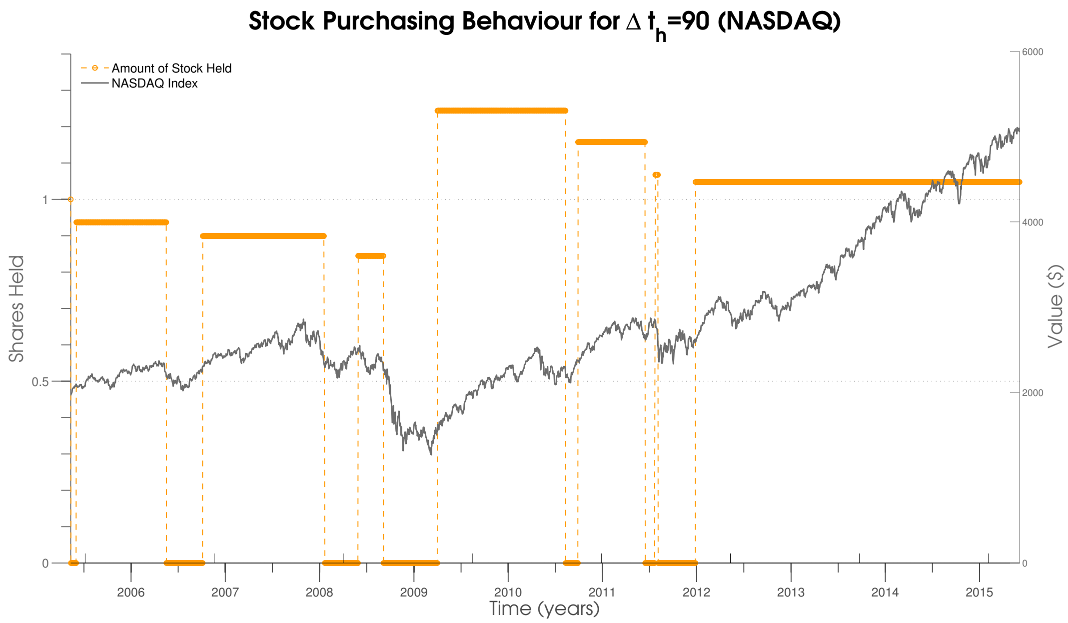

Figure 3.10: Choosing the best outcome of $\Delta t_h = 90$ we see some key features of the drawdown strategy. As opposed to the Relative Return method, the drawdown has a very small number of trades only at key times. It is clear that in times where the index is performing poorly we liquidate our position in exchange for a risk free asset. This is very similar to the results for the S&P500 due to the large correlation between these two indices.

It is clear when looking at the stock purchasing process that we are able to detect large drops in the asset value quite early on. This lets us sell off the assets before the value plummets and repurchase the asset at a large discount. For example observe the stock process around the 2008 financial collapse and see how we sell off our position very early into the drop. This also occurs for minor drops as well, shown around mid-2011. The only downside of this method is in times of stable market growth. In this scenario, we more often jump the gun and sell off our position expecting a large drop, only to have the asset price rebound and leave us at a loss.

A similar behaviour is also observed in the Chinese market shown in Figures 3.11 and 3.12.

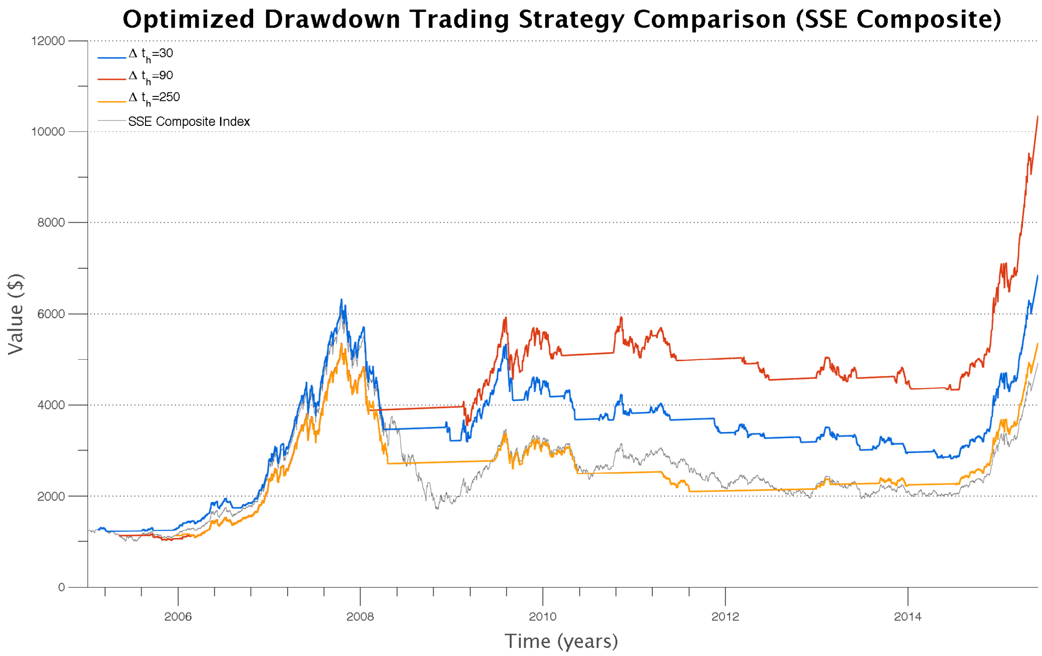

Figure 3.11: Returns generated by applying the drawdown trading strategy on the SSE index. The outcome here is largely due to the massive bubble occurring in 2008. Since the method capitalizes on exiting before an extremely large deficit has occured, we were able to repurchase the stock at a discount and scale our wealth accordingly.

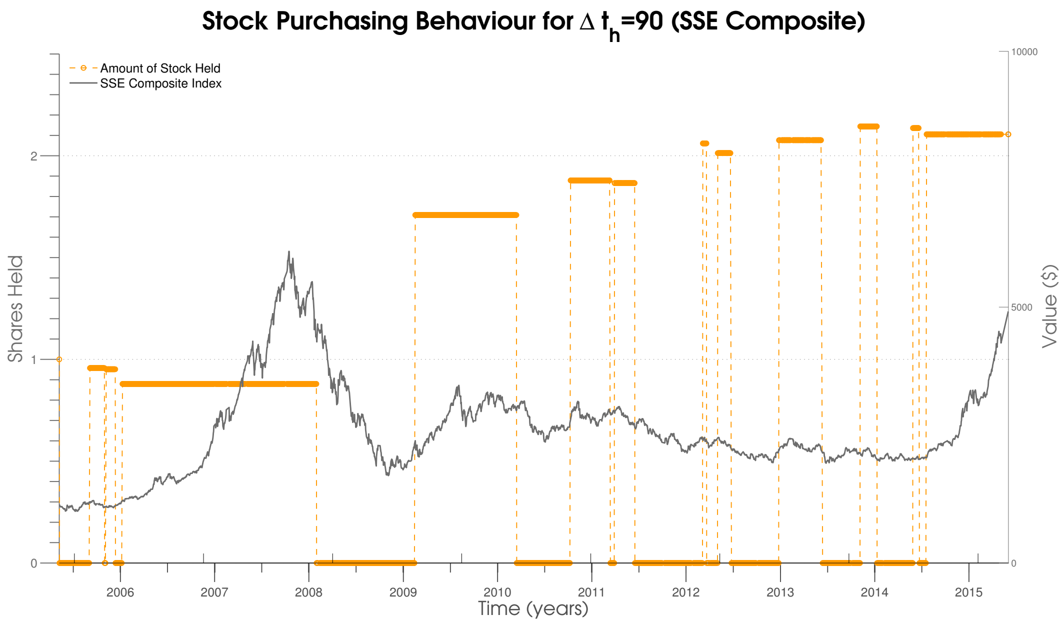

Figure 3.12: Choosing the best outcome of $\Delta t_h = 90$ we see some key features of the drawdown strategy. As opposed to the Relative Return method, the drawdown has a very small number of trades only at key times. It is clear that in times where the index is performing poorly we liquidate our position in exchange for a risk free asset.

3.3 Max/Min Strategy

The Max/Min strategy is by far the most simplistic method we present in this section. The idea is that when looking at the past history of a stock asset for some lookback time Δ_l, if the maximum in that time region has occurred more recently than the minimum, we expect that we are in a market upswing. If we see the minimum occurs more recently than the maximum then the market is in a downturn and we should sell the asset. For a more detailed description please see Algorithm 3.

Since this model is so simple, we use historical data to help us determine what the best lookback time size should be using the calibration data from 2000-2005. This calibration is shown in Table 3.5.

Algorithm 3 Max/Min Algorithm

procedure RUNMM
    `Set` Δt_p
    `Set` δ_d
    for $t \leq t_{predict}$ **do**
        `Calculate` t_p^+, t_p^-
        if $t_p^+ > t_p^-$ **then**
            $j = t$
            if a_t`==0` **then**
                $a_t = w_t / S_t$
                for $j \leq t + \Delta t_p$ **do**
                    $a_j = a_{j-1}$
                    $w_j = a_j * S_j$
            else
                for $j \leq t + \Delta t_p$ **do**
                    $a_j = a_{j-1}$
                    $w_j = a_j * S_j$
        else
            $j = t$
            if a_t`==0` **then**
                for $j \leq t + \Delta t_p$ **do**
                    $w_j = w_{j-1} * (1 + r)$
                    $a_j = 0$
            else
                for $j \leq t + \Delta t_p$ **do**
                    $w_j = w_{j-1} * (1 + r)$
                    $a_j = 0$
        $t = t + \Delta t_p$

	S&P 500	**NASDAQ**	**SSE Composite**
Δt_l **(days)**	87	16	20

Table 3.5: Calibration of the Max/Min strategy for the S&P500, NASDAQ and SSE Composite. The values of Δt_l represent the optimal lookback times which generated the greatest returns on the calibration datasets (2000-2005).

Using these values we now look at the performance of the trading strategy in comparison to the standard buy and hold method. A comparison between this strategy and the stock price as well as the stock purchasing process is shown for the S&P500, NASDAQ and SSE Composite in Figures (3.13,3.14), (3.15, 3.16) and (3.17,3.18), respectively. A summary of the MRR and number of executed trades for this method is shown in Table 3.6.

	S&P 500	**NASDAQ**	**SSE Composite**
Number of trades	27	145	103
MRR (Mean Return Ratio)	1.0086	0.8464	1.0985

Table 3.6: Results of applying the optimized Max/Min trading strategy on the three indices. The overall performance was rather poor, only producing marginal gains in the case of the S&P500 and SSE Composite index. The number of trades in all datasets were varied indicating that this trading strategy may be too simplistic to generate reasonable performance.

Looking at the S&P500 results we see a marginal increase in wealth compared to the buy and hold strategy. While the method was able to capture the market drop in 2008, the overall performance was rather poor.

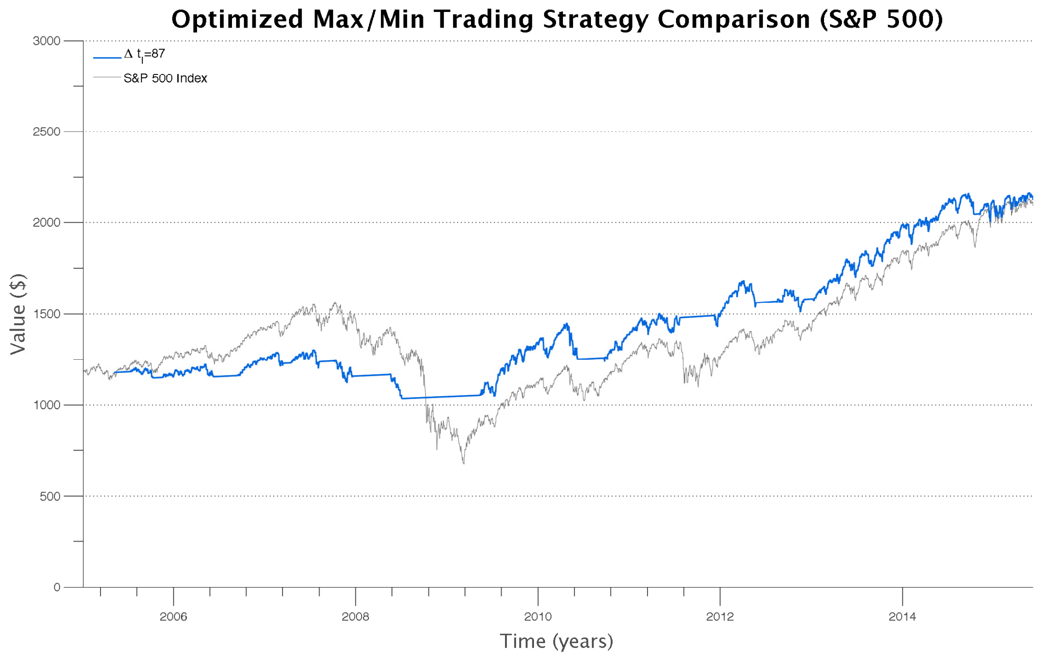

Figure 3.13: A comparison between the S&P500 market index (grey) and the optimized Max/Min strategy for a loopback time of $\Delta t_l = 87$. We can see that the overall performance is weak, however early exits out of the market in 2008 and the end of 2011 lead to some significant gains.

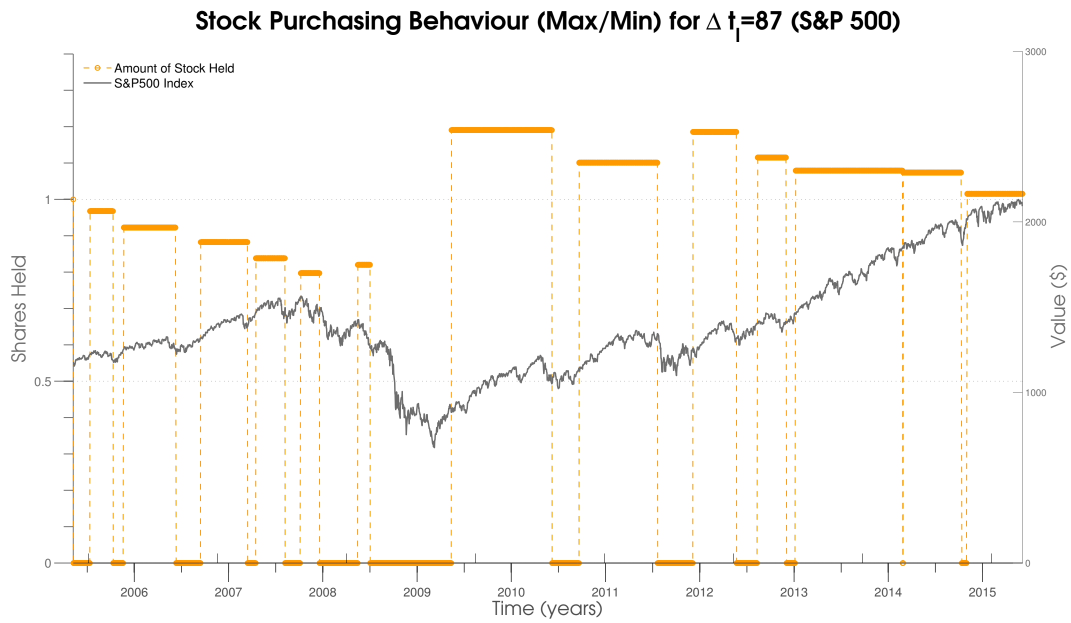

Figure 3.14: The stock purchasing process for the S&P500 with a lookback time of $\Delta t_l = 87$. This figure clarifies the benefits arising from the 2008 slump where the method exited the position before the majority of the downturn. After this however the method does not bring much of a benefit.

Looking at the stock process for the S&P500 in Figure 3.14 we see that the only real gains that occur were from the cheap repurchasing of the stock after the 2008 crisis.

Moving on to the NASDAQ results we see that the Max/Min method actually produced a worse outcome than simply holding the index. In contrast to the outcome in the S&P500, the strategy was not able to capitalize on an early exit from the market during the 2008 financial crisis. Moreover, there was a very large amount of trading going on (145) during the entire period.

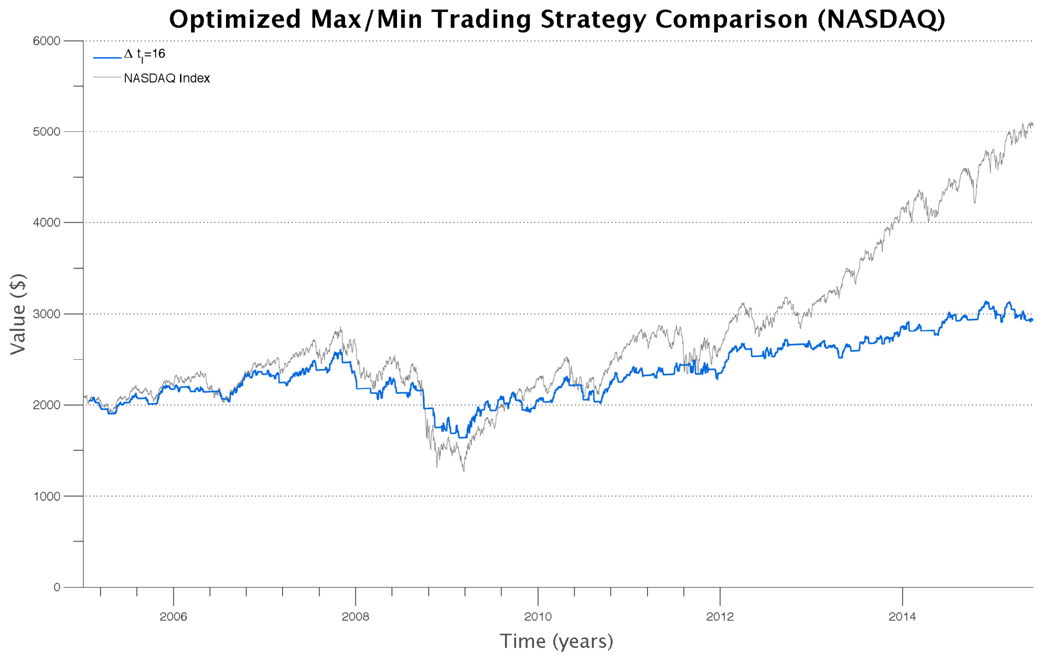

Figure 3.15: A comparison between the NASDAQ market index (grey) and the optimized Max/Min strategy for a lookback time of $\Delta t_l = 16$. We can see that the overall performance is weak and the number of trades performed was much higher than that of the S&P500. This increased trading during 2008 generated a loss of benefit and eventually large underperformance of the method.

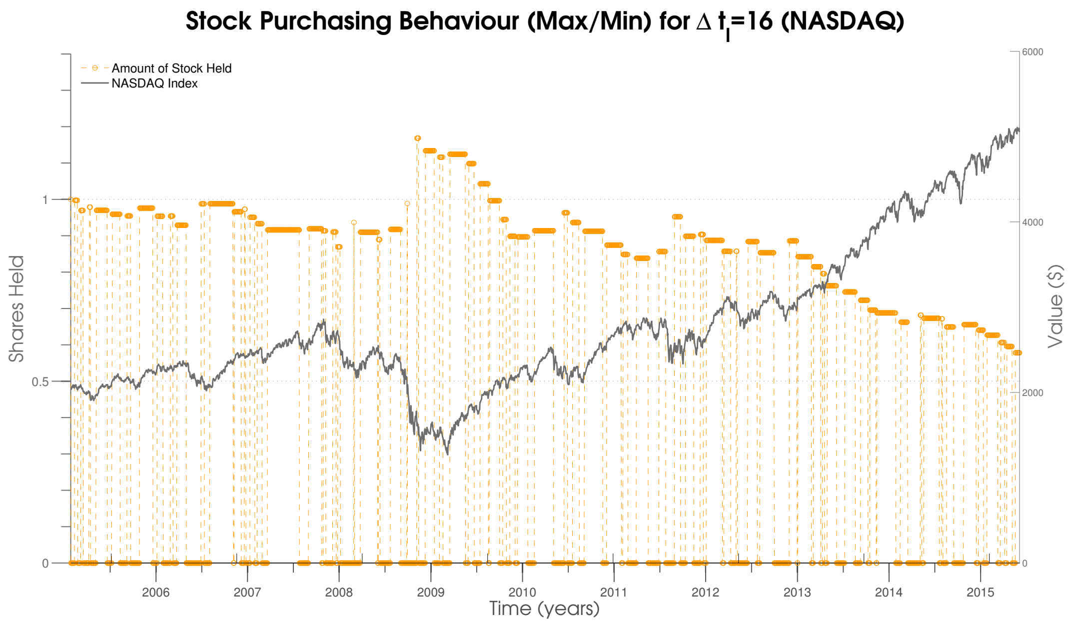

Figure 3.16: The stock purchasing process for the NASDAQ with a lookback time of $\Delta t_l = 16$. This figure makes it much more clear the amount of trades occurring. While we see a small increase in stock position during the 2008 crisis, the large amount of trading quickly diminished these gains.

What is most interesting about this result is that it is so different from the outcome of the S&P500. In general we would expect that since the general market dynamics between the two are so similar, that the trading strategy should have been able to tap into a similar trend. This was true in the previous two methods (Relative Return and Drawdown). This may indicate that the Max/Min model is just too simplistic to extract any real value.

Finally we look at the results from the SSE Composite index. Once again we see a similar result where we have a large amount of trading (103) and a failure to capture the large market drop in 2008 (Figures 3.17 and 3.18). The only difference here is the gains acquired in the continuous fall from 2011-2015.

Figure 3.17: A comparison between the SSE Composite market index (grey) and the optimized Max/Min strategy for a lookback time of $\Delta t_l = 20$. While the method was not able to capture the large market drop in 2008 as well as previous strategies, it was able to gain traction in the constant decline from 2011-2015.

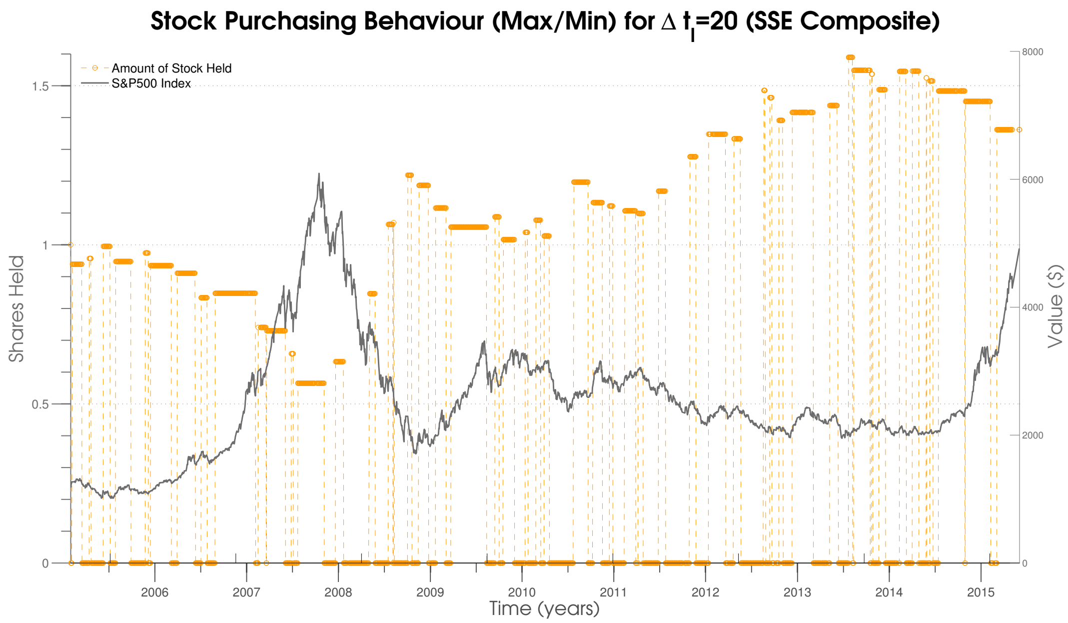

Figure 3.18: The stock purchasing process for the SSE Composite with a lookback time of $\Delta t_l = 20$. This figure makes it much more clear the amount of trades occurring. The consistent rise in stock from 2011-2015 indicates that this method may be best suited for times of constant economic decline.

It is interesting to note that the gain during 2011-2015 is the first time we have seen one of the methods actually produce a consistent improvement during less volatile market conditions. While this method may not be suited to describe asset bubbles and financial crises, it may find use in more stable market environments.

3.4 Strategy Comparisons

In this chapter we explored the application of three different trading strategies which varied in complexity. The Relative Return strategy which applied a trading rule based on the day over day return relative to some historical benchmark performed well in all cases and was capable of constant gains compared to the buy and hold strategy. The only major downside of this strategy was the amount of trades performed. All datasets had the number of trades to be around 500 and above which means that over a 10 year interval we were trading weekly. While this may not seem to be very frequent, every trade incurs a cost which drives our wealth down.

The drawdown method added an extra layer of complexity since it also characterized a measure of volatility. Applying this method to the S&P500, NASDAQ and SSE Composite lead to a better return than the buy and hold strategy with a massive reduction in the number of trades performed over the same time frame. Since the drawdown was able to take advantage of market drops early on and repurchase the assets at a discount it was able to generate significant gains during major market events. In contrast to the Relative Return method however, it was not sensitive to small market shifts later on. This can be attributed to the extra information we attain by looking at the drawdown over the time period.

The last method (Max/Min) showed very poor performance. The model's oversimplification lead to erratic trading behaviour and inconsistent results. As opposed to the other two methods, it was not able to characterize the trading behaviour in a sensible way. This is obvious when comparing the performance between the S&P500 and the NASDAQ. In the Max/Min method, the trading strategies between these two datasets were very different, even though the stocks are very correlated and are likely driven by similar factors.

In comparing the three models we have found that in order to generate the best advantage over the buy and hold strategy it may be necessary to incorporate a measure of the relative volatility of the asset. Since this was implemented within the Drawdown strategy, this method gave the best returns while minimizing the number of transactions needed. In the future it would be interesting to study the effect of transaction costs on the three methods in order to quantify the effect this reduction in trades has.

Chapter 4

Summary and Conclusion

In this thesis we investigated the application of a mathematically driven trading strategy, as well as three simplistic deterministic trading rules. The mathematical method extended the results of Ref. [11] to multiple trading periods and was applied to real market data of the S&P500, NASDAQ and SSE Composite indices. The original intent of the mathematical method was to allow the modelling of asset bubbles by modelling the asset price as a geometric Brownian motion with a disorder. The assumption was that in times of speculative growth, followed by collapse, the expected return exhibited symmetric behaviour. By applying this model we found that the strategy was very successful in telling us when to exit large market corrections such as the 2008 crisis and other major speculative bubbles, however it was unable to generate gains in situations where the market was exhibiting healthy growth. This was shown to be true in all of the tested markets. While this is somewhat disappointing it is not surprising since the model was initially designed with the intent to model asset bubbles only, and by this measure it performed well.

Another issue that was found with the mathematical model was the specification of the momentum decay parameter λ was very poor in the case of the NASDAQ. Upon calibration of this dataset the λ value was found to be roughly twice the size of the very similar S&P500. This misspecification lead to a trading strategy which eventually caused the investor's wealth to hit zero around 2012 (the investor defaulted). It was discussed that the selection of correct calibration data is difficult since the model tries to capture very rare market events.

To contrast with the mathematical model, we also look at three simple deterministic trading strategies called the Relative Return, Drawdown and Max/Min strategies. These models provided simple sets of rule for executing a trade under certain conditions. The strategies here did not use advanced mathematical techniques, rather they were based on common sense and understanding of the markets. It was found that these practical methods were able to produce similar trading behaviours and returns when compared to the mathematical method. However when looking at the final MRR results, the S&P500 and SSE Composite generated much better wealth in the mathematical model than any of the practical methods. For example under the GBM with disorder method the MRR for the S&P500 was 1.3939 while for the best of the practical methods (Relative Return) only produced an MRR of 1.2637 (and with fewer trades). In the case of the NASDAQ however, the mathematical model lead to a default scenario, where the wealth generated by applying the strategy lead to zero value. This goes back to the issue of mis-parameterization.

Overall this research has shown that we were able to extend the works of Ref. [11] to apply the method to multiple periods and real market data and we were able to generate returns that beat the market. We also compared these results to the practical method which were also able to consistently produce $MRR > 1$. From this we can see that there is room for both mathematical as well as practical approaches when developing trading strategies. In the future it would be interesting to explore the effect of modifying the mathematical model to allow for a non-symmetric disorder parameter to be allowed. This would bring a time dependence to the μ_+ and μ_- and would perhaps lead to better performance during health market growth. It would also be useful to quantify the effect of transaction costs on the practical methods in order to calculate the benefits of using the less transaction heavy Drawdown method in comparison to the Relative Return strategy. Finally, another issue that was not addressed was the effect of dividends paid on a held stock. It was mentioned early that we ignored dividends for this work, however it would be very interesting to see how this addition would play into the optimization and performance of these methods.

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